# AN APPROACH TO TEACHING MATHEMATICAL INDUCTION TO ADOLESCENT BOYS

Thesis for the Degree of Ph. D.

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FRANCIS HOWARD HILDEBRAND

1968

THESIS

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Calhoun C. Collier
Major professor

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# ABSTRACT


## AN APPROACH TO TEACHING MATHEMATICAL INDUCTION TO ADOLESCENT BOYS

By

Francis Howard Hildebrand

This study was primarily concerned with developing an approach and materials to promote the achievement of adolescent boys in mathematical induction. A second objective was to determine the relationship between selected response patterns of fifty seventh grade boys and their achievement in mathematical induction.

The design of the study was a one group, pretest--post test type. The two most important facets of this study were the sequence of problems in mathematical induction listed as variable $x(1)$ and the scales of response patterns listed as variable $x(i)$ , $i = 2, 3, 4, 5, 6, 7, 8$ . The statistical design for the study was to answer basic questions of correlation and analysis of variance.

A model for student evaluation was used in an effort to standardize scoring procedures. Partial credit for each induction problem was awarded on the basis of this model.

The materials were designed for use in a relatively unstructured school environment covering 960 minutes of treatment. In such an environment scores of high achieving students

were found to correlate significantly with three scales measuring response patterns. Correlations with an .05 confidence level were found on two scales: One scale x(2), measured the enthusiastic, easygoing, individual who participates in group activities. A second scale x(5), measured the assertive, independent and aggressive individual who was spontaneous. A correlation with an .01 confidence level which measured self confidence was found with a third scale x(7). Such an individual has an adequate self image, and a high regard for his own worth. The materials developed included three basic orientations, algebra, geometry and approximation.

In drawing conclusions many unanswered questions involving the indiscriminate use of physical models as a means of promoting conceptualization and problem solving in mathematical induction occurred. Students who were dependent on models, those who did not move rapidly from the problem statement to representational forms or otherwise translate the problem into their own mode of expression, did not solve the problems.

It was also clear that students need to be trained to collect and organize information as it is generated in the problem solving process. Those problems which were solved by very few students shared the quality of requiring extensive organization of information.

# AN APPROACH TO TEACHING MATHEMATICAL INDUCTION TO ADOLESCENT BOYS

By

Francis Howard Hildebrand

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Finally, I am directly indebted to my wife, Nancy, whose help, encouragement and patience have contributed greatly to my success on this study.

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# CHAPTER I

# INTRODUCTION

## Need for the Study

Organization of the mathematics curriculum is a current problem faced by public schools. Some schools group children homogeneously according to their mathematical achievement. Different curricula provide for very able, average and less able groups. Other schools emphasize flexible intraclass grouping. Materials to handle diverse groups vary from those devised by the University of Illinois Committee on School Mathematics (UICSM) employing the discovery approach[1] to the more formal 'definition--theorem--proof' techniques provided by the School Mathematics Study Group (SMSG).[2]

In the past decade the United States government, through the National Science Foundation has demonstrated its concern with the state of mathematical learning; and in particular has

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[1]University of Illinois Committee on School Mathematics, High School Mathematics, (Revised edition; Urbana: University of Illinois Press, 1962).

[2]School Mathematics Study Group, Mathematics for Junior High School, Volume I, Parts I and II (New Haven: Yale University Press, 1961).

contributed to projects developing text materials, to academic year and summer institutes, and to in-service programs for improvement of mathematics backgrounds for elementary, secondary, and college teachers.

An identifying characteristic of projects such as UICSM and SMSG was that each was subject matter oriented. The primary concern of these professional projects was improving the subject matter of school mathematics.

While mathematics educators and the United States government have shown concern for the organizational aspects of the school, the mathematical background of the teacher, and the development of new materials, they have displayed little concern about the effects of new content or its organization upon the children involved. In short, it appears that school mathematics programs have developed in terms of adult mathematician's goals. The researcher designed this study to prepare materials to teach induction and to examine student responses to the induction material.

Further, motivation for the study grew out of the researcher's observations of students in elementary mathematics who failed as defined by test scores. These students were observed to share certain characteristics or similar ways of responding. One such characteristic was the students' apparent lack of confidence in their efforts in mathematical study. A second characteristic was the students' apparent inclination to withdraw from class participating--they participated less as their test records deteriorated. These characteristics were the result of personal observations; no supporting studies were found.

## Definition of Terms

Specific terms have special meanings in this study as follows:

Autonomy is a display of confidence and is implied by the student's positive response to a question such as: Can you work well, without making mistakes, when you are watched?

Independence involves the student's perceptions of constraints arising in his interpersonal relations and his interaction with the environment. Independence is implied by the student's positive response to a question such as: Do you completely understand what you read?

Mathematical Induction is a study of sequences of particular patterns or cases of symbols which suggest a continuation from which a conjecture arises and may be generalized to a theorem. Mathematical induction as defined here is distinct from other forms of induction. For example, Baconian Induction as a logical method is a process of attaining general statements on the basis of observations, comparisons and experiments through intermediate generalizations and with regard for negative as well as positive instances.[3]

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[3]Websters Third New International Dictionary, unabridged, (Chicago: G and C Merriam Co., 1966), p. 160.

Another example is scientific induction which is a method used in systematic pursuit of intersubjectively accessible knowledge and involving as necessary conditions

1) the recognition and formulation of a problem,

2) the collection of data through observation and if possible experiment,

3) formulation of hypotheses,

4) testing and confirmation of the hypotheses formulated.[4]

Persistence is a display of sustained attention on the part of the student. It is implied by a student's positive response to a question such as: If someone turns on noisy music while you are trying to work, do you continue working?

Play is the simulation of the student's internalized visions. Play is the opportunity for the student to indulge in spontaneous activity with objects, ideas or people. If he deems an activity play, the student is highly responsive.

Response Patterns are constellations of responses where a response is a change in the student associated with or correlated with a stimulus or set of stimuli.

Self Image is a demonstration of the student's conceptualization of himself. This conceptualization is implied by examining the individual subject's response to a question such as: When you write about your personal thoughts, do you enjoy telling about yourself?

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[4]Websters Third New International Dictionary, unabridged, (Chicago: G and C Merriam Co., 1966), p. 2033.

Preference in Problem Structure refers to the perceptual structure a student prefers in problem analysis. An example of such preference is the student's free selection of one of three distinct structures for solving a given problem. The student may choose a geometric, algebraic or verbal approach to problem solution.

## Philosophy and Assumptions of the Study

Funded projects by the United States government such as SMSG and UICSM has shown lively interest in school mathematics curricula during the last decade. A predominating interest of most projects was the improvement of text materials and of teachers' competency in subject matter.

Though this researcher agrees that subject matter is important, his first assumption is that organization of the materials and the response patterns of the individual students are important contributing factors in determining the level of student success in school mathematics.

Important controlling factors which support the learning of mathematics remain essentially unknown, because the methods by which any type of mathematical learning takes place are relatively obscure. Henderson stated:

> The teacher can not depend upon any special type of lesson, such as "supervised study," to guarantee success in teaching and learning . . . . There is no decisive proof that any particular philosophy of teaching . . .

> will guarantee better results than any other method . . . .[5]

In discussing the National Longitudinal Study of Mathematical Abilities, Dessart noted that:

> . . . Only through attempts to find interrelations among the many variables interacting in a learning situation can findings become more than cross sections of a learning situation . . . .[6]

A review of the literature indicates that few of the variables in learning mathematics have been detected, and neither interrelations nor significance of observed phenomena have been sufficiently explored. This suggests that exploration is necessary in terms of identifying fundamental cognitive characteristics and ability expectations in the field of mathematics.

A second assumption of this study is that the child is basically an integrated unit in terms of his emotional structure and his physiology. Further, he tends toward individual autonomy in his cognitive development only if social interaction exists. The tendency toward autonomy in many children is assumed to be constrained by an existing set of response patterns. If the response patterns are adequate for the potential intellectual outcomes of the organism, successful action is applied toward

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[5]Kenneth B. Henderson, "Research on Teaching Secondary School Mathematics," Handbook of Research on Teaching, ed. N. L. Gage (Chicago: Rand McNally and Co., 1965), p. 1025.

[6]Donald J. Dessart, "Mathematics in Secondary Schools," Review of Educational Research, XXXIV (June, 1964), 307.

individual autonomy, particularly in intellectual matters.

A third assumption of this study is that action is necessary for success in mathematics. Mathematics is something children do. Children are not passive about mathematics but rather become involved both intellectually and affectively.

Another basic assumption is that achievement in induction implies both the existence and successful application of a model of intellectual activity. Failure to succeed is no assurance that the use of the model failed since the difficulty may be a matter of motivation.

A final assumption is that mathematics learning is less isolated than many suspect. Such an assumption suggests that learning mathematics is not a simple matter of having a desire to sit down and do something. Desire may be a necessary but far from a sufficient condition for achievement.

A statement about the relationship between the researcher and his subjects seems in order. The experimenter most accurately describes the interaction between himself and the participating children as a form of play. The materials in mathematical induction extend to the youngsters an opportunity to enter into spontaneous activity in abstraction. They are encouraged to explore the material by whatever route their intuition suggests. This approach opens the way for the student to become involved affectively as well as cognitively.

Thus, the induction problems given the subjects, were, within limits, a form of play. The mathematical problem play

involved activity and was limited only by the youngsters' imagination. Some students found such play extremely motivating. They seemed to want the same feeling of power over mathematics that they had over their possessions as children. Those students who fell captive to the problem form of play, lost track of time and no longer classed their efforts as work, since their activity supported developing affective components which generated satisfaction.

Play is highly contagious to most children and consequently when participation does not occur there must be inhibiting factors. Some of the inhibiting factors might be described as a pathology and relate directly to response patterns not adequate for the child's organization of development. Seven response patterns were identified as part of this research. Their specific characteristics are listed in Chapter Two.

From this philosophical orientation comes the underlying purpose of this study. This study was primarily concerned with developing an approach and materials to promote achievement in mathematical induction by adolescent boys. A second objective was to determine the relationship between selected response patterns of fifty seventh-grade boys and their achievement in mathematical induction. For a highly complex intellectual activity, such as success in mathematical induction, appropriate affective components appear to exist in terms of specific response patterns.

## Limitations of the Study

This study was limited in several ways. No taxonomy of problem difficulty was available for sequencing materials in mathematical induction. There was no computer program available employing such tools as step-down F statistics for the item determination of the scales. One investigator can effectively observe only a small number of students in a classroom.

The sample used for the study was not ideal. The selection of the sample was from schools which were willing to cooperate in a research study, rather than selection by a random sampling. The sample was limited to twenty-five male students in each of two classes.

Preparatory learning, critical in any study, is still largely unknown. This area is in need of research.[7] No information was available on the matter of individual preferences for problem structure and what role such a preference plays in strategy formation. The lack of information relating response patterns and preference affinity constitutes a significant limitation of this research.[8]

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[7]Fred T. Tyler, "Issues Related to Readiness to Learn," _The Sixty-third Yearbook of the National Society for the Study of Education_ (Chicago: University of Chicago Press, 1964), p. 236.

[8]The Research and Development Center at Stanford University is currently conducting an investigation in this area.

## Organization of the Study

This research study is organized into five chapters. Chapter One, the Introduction, cites the need for the study, definition of terms, and the philosophy and assumptions on which the study was based. Limitations of the study and organization of the research report are also included.

Chapter Two, Patterns, Problems and Related Research, contains a careful delineation of the hypotheses and a survey of other research related to the study. The structure of the induction materials and descriptive characteristics of the response patterns are also a part of this chapter.

Chapter Three, Purpose and Procedures, reports the design of the study and how the subjects were selected, explains the daily routine and gives student reactions to selected problems.

Chapter Four, Analyses and Results, describes the results of the given sequence of induction material, correlation ratios and related statistics between individual and collective response patterns and achievement in induction. The results of the analyses of data complete this chapter.

The last chapter, Conclusions and Implications, includes the summary and suggestions for further research.

## CHAPTER II

## PATTERNS, PROBLEMS AND RELATED RESEARCH

### Statement of the Problem

This study was primarily concerned with developing an approach and materials to promote the achievement of adolescent boys in mathematical induction. A second objective was to determine the relationship between selected response patterns of fifty seventh-grade boys and their achievement in mathematical induction. The hypotheses to be tested is whether or not significant differences exist for correlation between achievement scales and selected response pattern scales. Specifically the null hypotheses is:

> Between scale $x(1)$ and each scale $x(i)$ , $i = 2, 3, 4, 5, 6, 7, 8$ the correlation is not significant.

### Pertinent Research

The research literature provided data pertinent to the mathematical aspects, identification of responses, and selection of the sample used in this study. Hadamard considered questions of the affective nature of students and their success in the field of mathematics. In a personal reference, Hadamard stated that he knew that powerful emotions

favored different kinds of mental creation.[1] Such kinds of creation are in large measure related through emotion to selective attention.

Hadamard seemed to recognize emotion as potential influence in invention in mathematics but he suggested no measures or materials to determine specific affective responses. Piaget seemed to agree with Hadamard's position about affective influence when he stated that:

> . . . mathematical concepts are not derived from the materials themselves, but from an appreciation of the significance of the operations performed with the materials.[2]

Poincare[3] saw success in invention and problem solving as the result of prior unconscious work that was influenced by affective characteristics. He hypothesized that individuals involved in invention are so emotional in their commitments that fragments remain in their subconscious even when conscious effort is suspended.

In his study "Personality Factors and Success in Mathematics." Kochnower[4] found that his sample leaned toward

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[1]Jacques Hadamard, The Psychology of Invention in the Mathematical Field (Princeton: Princeton University Press, 1945), p. 10.

[2]K. Lovell, The Growth of Basic Mathematical and Scientific Concepts in Children (London: University of London Press, Ltd., 1964), p. 43.

[3]Hadamard, op. cit., p. 19.

[4]W. Kochnower, "Personality Factors and Success in Mathematics," High Points, XLIII (April, 1961), 21.

the Hamlet factor.[5] Kochnower's subjects were highly self-sufficient and individualistic, tended to be loners, displayed obsessive and compulsive behavior and a tendency to react emotionally and excitably. In Kochnower's study the achievers had personalities which rejected imposed cultural demands. Kochnower concluded that a potential profile of a successful mathematics student shows that such a student is sensitive, insecure, introspective, and tends to avoid group activities.

Certain response patterns and spatial ability seem to be inextricably related to success in mathematics. Both Piaget and Lovell found that the child's thinking is largely defined by his perceptions, particularly those formed in the very early stages of school.[6] Lovell also argued that there is a taxonomy for analogies relative to understanding. The problem of defining such a taxonomy includes distinguishing between perceptual space and representational space. Discrimination between a circle and a triangle is an example of a perceptual space. An example of representational space is the mental representation of a circle and a triangle without the aid of a drawing or model.

Intuitive transformations according to Lovell are critical to the formation of representational space from perceptual space.

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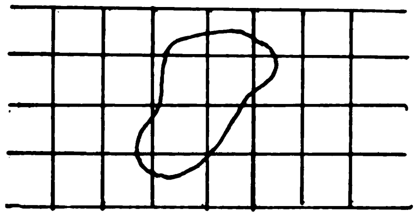
[5]The Hamlet factor is characterized by obstructive, reflective, and individualistic behavior.

[6]Lovell, *op. cit.*, p. 60.

Consider the following problem:



Approximation of Area

Figure 1

The student is asked to find the area of the above bounded region. Lovell described the necessity of a transformation in these words:

> If the pupil has not sufficient mental manoeuvrability to be able to deal with the 'bits and pieces of squares' in his mind . . . there is little one can do to get him to understand.[7]

Related to the need for intuitive transformations and perceptual orientation is the difficulty of converting a given perception into symbolic form for effective mathematical analysis and production. Common physical models and words must eventually be ruled out. The descriptive forms most often successfully substituted are vague internalized images. Such images symbolize the ideas without imposing irrelevant conditions.

Quoting from Les Definitions dans l'Enseignement in Science et Methode, Poincare stated that:

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[7] Ibid., p. 119.

> Almost all . . . wish to know not merely whether all the syllogisms of a demonstration are correct, but why they link together . . . they are not conscious of what they crave, and if they do not get satisfaction they vaguely feel that something is lacking.[8]

Hadamard asserted that every mathematical research compelled him to build a schema necessarily of a vague character so that it was not deceptive.[9]

Engle seemed to support Hadamard's views on vagueness in the individual's production of new ideas. She declared that:

> . . . limiting scientific language to terms that have achieved definition may well cut off a rich source of future knowings.[10]

Though these and other researchers commented on vague internalized models, no support for the use of physical models or their importance in determining student success in the perception of necessary mathematical transformations was found.

In his doctoral dissertation, Schunert indicated that differential assignments were significant in mathematics achievement. He defined achievement from an analysis of the youngsters responses over a given totality of work rather than

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[8]Hadamard, op. cit., pp. 104-5.

[9]Ibid., p. 77.

[10]Mary Engel, "Thesis--Antithesis," American Psychologist, XXI (August, 1966), 786.

an analysis of a very restricted range of problems. His study indicated a need for information on related factors of differential assignments and, by implication, aims specifically at factors for learning.[11]

Another factor that influences success in mathematics is the problem of sex differences. Sex difference in problem solving appears early in development and under different guises. Milton[12] found that males adopt an analytic attitude toward natural events and toward formal problems. McDavid,[13] and Crandall and Robson[14] found that boys appear to be more analytic, more independent and more persistent than girls in problem situations. This difference increases with time, and

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[11]Jim Schunert, "The Association of Mathematical Achievement with Certain Factors Resident in the Teacher, in the Teaching, in the Pupil and in the School," Journal of Experimental Education, XIX (1951), p. 237.

[12]G. A. Milton, "Five Studies of the Relation Between Sex Role Identification and Achievement in Problem Solving," cited by Martin L. and Lois W. Hoffman, eds., Review of Child Development Research (New York: Russell Sage Foundation, 1964), p. 157.

[13]J. W. McDavid, "Imitative Behavior in Preschool Children," cited by Martin L. and Lois W. Hoffman, eds., Review of Child Development Research (New York: Russell Sage Foundation, 1964), p. 157.

[14]V. J. Crandall and A. Robson, "Children's Repetition Choices in an Intellectual Achievement Situation Following Success and Failure," cited by Martin L. and Lois W. Hoffman, eds., Review of Child Development Research (New York: Russell Sage Foundation, 1964), p. 162.

by late adolescence and adulthood, the typical female feels inadequate in problems requiring analytical reasoning. Sweeney's[15] results confirmed this when he found that differences in problem solving can be demonstrated for groups of men and women matched with respect to intelligence. Witkin[16] also found a marked difference between boys and girls in modes of perception, with girls being the more dependent on visual framework. To take into account known factors supported by others' research, the subjects for this study of relationships between response patterns and mathematical achievement were deliberately limited to a sample of boys.

Though certain response patterns such as persistence, independence, autonomy and self image were linked to mathematical success by various researchers, no available evaluative instruments, in toto, were deemed suitable for measuring the desired response patterns. Therefore, modifications of existing instruments were made with the aid of a testing collaborator.[17] Jerome Bruner has examined the question of induction and one point in

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[15]Edward J. Sweeney, "Sex Differences in Problem Solving" (unpublished Ph.D. dissertation, Department of Psychology, Stanford University, 1965), p. 57.

[16]H. A. Witkin, "The Nature and Importance of Individual Differences in Perception," The Journal of Personality, XVIII (December, 1945), 162.

[17]Donna Palonen, Department of Psychology, Michigan State University.

particular is important for this study. The child is postulated as passing through three stages of development. The earliest stage is the enactive level where the child manipulates objects. The second stage is the iconic level where the child deals with mental representation of the physical objects. The third stage is the symbolic level where the child is dealing in mental images.[18]

## Response Pattern Scales

Personality scales which have been published since 1955, and which were commercially available from Western Psychological Service, Beverly Hills, California, were examined. None of the tests seemed to identify the specific response patterns sought, but the Junior Senior High School Personality Questionnaire (HSPQ)[19] was satisfactory in some respects. HSPQ scales were purchased and then modified, both in specific wording and in scoring, to more closely identify the desired response patterns. The resulting scales are identified as $x(i)$ , $i = 2, 3, 4, 5, 6$ .[20] The choice of symbols to identify these specific scales was decided upon since, at this time, no precise words were appropriate

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[18] "Improving Mathematics Education for Elementary Teachers," W. R. Housten, ed., Michigan State University, sponsored by Science and Mathematics Teaching Center, p. 26.

[19] HSPQ Form A, Second Edition (1963) was purchased from The Institute for Personality and Ability Testing, Champaign, Illinois.

[20] Individual scales identifying response patterns can be found in appendix A.

to identify the variable concerned. All interpretations or labels relating to items on the scales used were prepared by the researcher. The author assumes scale reliability and validity for purposes of this study. The following descriptive terms used to help the reader characterize response patterns are those of the author of this research.

The items for scale $x(2)$ were chosen to identify a response pattern which was characterized by a happy, enthusiastic, easygoing, warmhearted extrovert, who enjoys participating in group activities.

Items in scale $x(3)$ were chosen to identify a response pattern characterized by a calm, secure, self-assured, emotionally stable individual, who faces reality.

The twenty items in scale $x(4)$ were chosen to identify a response pattern characterized by a decisive, resourceful, self-sufficient individual, who is relaxed and tolerant rather than aggressive and competitive.

Scale $x(5)$ was characterized by an assertive, independent, aggressive, venturesome individual, who is uninhibited and reacts spontaneously.

The $x(6)$ scale was characterized by an individual who disregards rules, and follows his own urges. He tends to be undependable and has a casual, careless attitude. This person would tend to be impulsive.

Unlike scales $x(2)$ to $x(6)$, scale $x(7)$ was a selection of incomplete sentences designed to identify the individual who tended to be confident, who had a positive self image, and who had a high regard for his own worth. The scale and scoring for response x(7) were modified from The Forer Structured Sentence Completion Test.[21]

Scale x(8), the Draw-A-Person Test without modifications, was used to test whether or not perceptual and motor factors of coordination were related to the individual's achievement in induction.

The scales x(2) through x(6) were administered to the twenty-five subjects in each section, as a group. The subjects responded to scales x(7) and x(8) at the second sitting, on an individual basis. No time limits were set so that the subjects felt no overt pressure to hurry.

Each of the five scales x(2) through x(6) consisted of twenty items, with each item having three possible responses. The students' responses to each item were scored 0, 1 or 2 according to the numerical weight assigned that response.[22]

Scale x(7) contained 15 items for completion scored 0, 1 or 2 . Scale x(8), identified by the Draw-A-Person Test, which was not modified, was scored according to Bodwin and Bruck's procedures.[23]

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[21]Bertram R. Forer, The Forer Structured Sentence Completion Test, (Beverly Hills: Western Psychological Services, 1957).

[22]See Appendix A for the unique integer assigned each scale item by this researcher.

[23]R. F. Bodwin and M. Bruck, "The Adaptation and Validation of the Draw-A-Figure Test as a Measure of Self Concept," (Mimeographed).

Each subject's score on a given scale was the sum of the integers resulting from his selected responses. This range, from 0 to 40 for each of the six scales x(2) - x(7), scattered the individuals responses on a continuum. The higher scores indicated the more consistent patterns.

## Induction Material

A wide variety of mathematical induction problems was developed for assignments. For purposes of this study, mathematical induction was restricted to three primary areas of conceptualization. One conceptualization was basically algebraic and consequently did not fit a simple physical model. Problem 4, given to the students at the fourth meeting, was of this type.

It should be added that the subjects who became involved in the induction process rapidly oriented themselves to perceive what was intended in each new problem. After problems were handed out, the only help provided either as to approach or procedure was in answer to questions or in discussion after all those who wished to, had completed or given up on the problem. Though a casual reader may ponder the intent of such problems as number 4, given below, the participating students were apparently seldom confused about the intent of the problem.

Problem 4

| | |
|---|---|
| $1$ | $= 0 + 1$ |
| $2 + 3 + 4$ | $= 1 + 8$ |
| $5 + 6 + 7 + 8 + 9$ | $= 8 + 27$ |
| $10 + 11 + 12 + 13 + 14 + 15 + 16$ | $= 27 + 64$ |

If this process is continued what will the two numbers be on the right of the equal sign? Can you find a way to write the solution for any given line?

A second conceptualization was basically geometric and was interpretable through physical models, although the orientation did not demand that physical models be used. Some students applied representational space in the form of pencil and paper configurations as opposed to using either physical models or computational forms for orientation to geometric concepts. Problem 7, handed out at the fifth class meeting, was of such a type.

Problem 7

You are to assume that the cube shown on the following page has been immersed in paint and is completely covered. If the cube is then cut through each plane, as indicated by the dashed lines, the question is: In each case how many of the resulting small cubes will be painted on how many faces?

For the general case fill in the table below:

(a) Given n cuts there are _____ cubes with 6 painted faces?

(b) Given n cuts there are _____ cubes with 5 painted faces?

(c) Given n cuts there are _____ cubes with 4 painted faces?

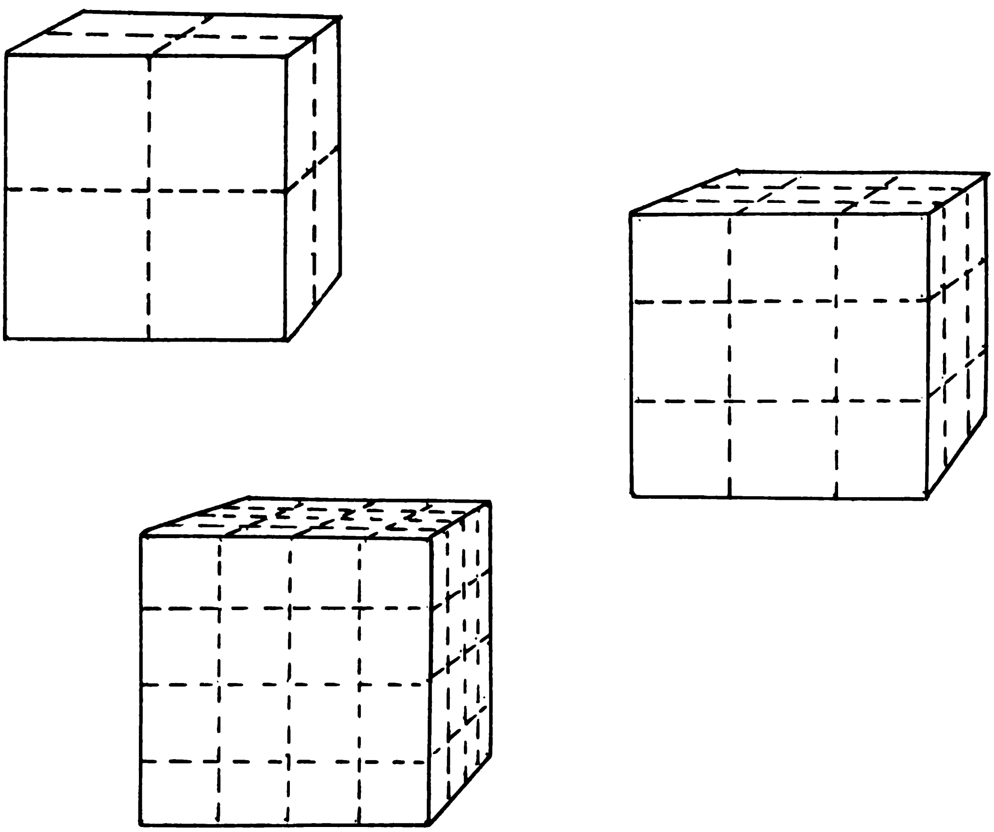
(d) Given n cuts there are _____ cubes with 3 painted faces?

(e) Given n cuts there are _____ cubes with 2 painted faces?

(f) Given n cuts there are _____ cubes with 1 painted face?

(g) Given n cuts there are _____ cubes with 0 painted faces?

<u>Physical Model for Problem 7</u>



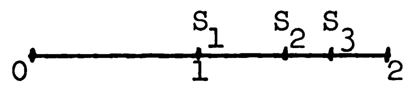
All students who successfully completed Problem 7 worked out specific cases for $n = 1, 2, 3$ and $4$ before they solved the more general case or the $n^{th}$ case.

A third conceptualization appealed to a dimension distinct from either algebra or geometry. This conceptualization involved application of the principle of approximation and extensions of approximation. Experience with induction materials which require methods of approximation are one approach to creating a need for mathematical proof.

A student, confronted with the problem

$$1/2^0 + 1/2^1 + 1/2^2 + 1/2^3 + \ldots + 1/2^n = \square$$

who, through analysis of a number line model




collects successively the partial sums:

$$S_1 = 1/2^0$$

$$S_2 = 1/2^0 + 1/2^1$$

$$S_3 = 1/2^0 + 1/2^1 + 1/2^2$$

and extends the procedure until he believes intuitively that

$$1/2^0 + 1/2^1 + 1/2^2 + 1/2^3 + \ldots + 1/2^n \leqslant 2$$

has established a real need for mathematical proof. An immediate physical model which fits this question is displayed by the following problem.

Problem 10 Part (a)

> A ball is dropped from an original height of 100 feet. Each time it hits the ground it bounces back one half the previous fall. How far does the ball fall?

In order to determine the students' backgrounds for dealing with inductive material involving the three previously discussed conceptualizations, an eight item Pre-test was prepared and administered by this researcher.[24] The subjects had no time limit in which to complete this test. They received one point for each correct answer. Over a period of six weeks, the researcher presented a variety of induction problems[25] for the subjects to study and solve. At the end of the second and fourth weeks, the subjects responded to forty-minute quizzes.[26] Quiz I, containing four items, had a total possible score of four points. Quiz scores were purposely weighted to fit cumulative points on other material. Partial credit was awarded. Quiz II, also contained four items but had a possible score of six points. These quizzes sampled the induction ideas covered in the previous two weeks' work and supplied additional data on the subjects' gain in inductive skills.

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[24]The Pre-test is included in Appendix B.

[25]See Problems in Appendix C.

[26]See Quiz I and Quiz II in Appendix B.

The Post Test was designed to evaluate the gain shown on problems parallel to, if not identical to, items on the Pretest. The gain was 61 points as indicated in Table 1, page 47, of Chapter IV. It should be noted that no item was missed by everyone on the Post Test yet no problem was solved by more than 12 students.

## Summary

In Chapter II, literature pertinent to the study was presented and discussed. Both psychological and mathematical aspects involved in learning induction were included. The experimental studies included were chosen in an effort to clarify some obstacles to learning.

Conceptualizations of the induction material were given with examples from the problems. The complete stock of induction problems given to the students are included in Appendix C. A description of the administration and scoring of tests and quizzes were also presented in this chapter. Though the scales used to identify response patterns and the format for scoring student responses were briefly described in this chapter, the complete set of these scales are included in Appendix A.

# CHAPTER III

## PURPOSE AND PROCEDURE

### Selection of Subjects

The selection of subjects for this study came from two schools. One school was the Laingsburg Community School, the other, the Owosso Junior High School. The schools made modifications to meet the research needs in terms of changing rooms, releasing time for students and disseminating information to parents and other interested individuals in the community.

Originally both schools released their total seventh grade population for this study. The total number of male students in the seventh grade in Laingsburg was 29. Because of anticipated attrition, all the seventh grade boys from the Laingsburg School system were invited to participate. In Owosso all the available seventh graders, 78 boys and girls, were invited to participate. From these, twenty-five males were randomly selected to be the subjects for the study. The subjects were told that the experimental study would not affect their permanent records as no grades would be recorded. Parental consent was obtained for each participant.

From the original fifty-four males selected, 16 in Laingsburg and 24 in Owosso completed the study. Various logistical problems contributed to the elimination of some subjects from the experimental program. For example, some boys missed a psychological examination because of sickness. If they missed such a test and also the makeup test, the students were dropped from the study because of missing data. Certain other students simply failed to come to class.

The Laingsburg students were generally from two types of homes. Some lived on subsistence-type farms. Others came from homes where their fathers were skilled or semi-skilled workers of median income in the Lansing industrial complex.

In general, the Laingsburg boys displayed poor orientation in operating within an evaluation model. A typical Laingsburg boy had not developed a level of sophistication sufficient to generate complex conceptualizations. The average subject in this category found little excitement in problems involving mathematical induction and unhesitatingly remarked, upon inquiry, that everything at school was boring and worthless. To them intellectual matters were of little concern. When further questioned about current aspects of life they found exciting, the subjects were not uniform in their feelings. Even in discussions of mechanical concepts such as might be employed in repairing or designing automobiles, airplanes, missiles and space flights, most of them did not seem particularly enthusiastic. Few of the students could recall facts about current space activity. Individual boys could not recall names of astronauts except John Glenn and none were able to associate details of achievement more specific than "space" with his name.

As a group, the Laingsburg boys had little perception of details in life about them. The overwhelming majority of this group considered reading a chore. School for them was a place of drudgery and work--of listening and doing meaningless things. Observation of the youngsters indicated they had average attention spans of ten minutes for mental activities. During problem sessions, the subjects who chose not to work used the time for social interaction with other youngsters. One could infer that these subjects could not get enough time just to be "listened to". Later, in their own evaluations of this workless period of time the subjects stated that they had been involved in intensely difficult effort.

The majority of students from the Owosso School were primarily from low or medium income families that lived in city dwellings with parents and several siblings. Though they came from dissimilar backgrounds, a majority of the subjects from Owosso responded to intellectual activities in much the same fashion as did the boys from Laingsburg. The question of what the Owosso boys considered play was also interesting. A large, disinterested group did not involve themselves in inductive type play. They did not consider reading a pleasurable activity. Topics in mathematical induction did not interest the majority of this group. Each subject who showed disinterest was also noted to be very weak in steps one and two according to the model used for evaluating achievement.[1] The larger interested group was at Owosso.

## Design of the Study

The basic design of this study is a one group, pre-test--post test type. The two most important facets of this study were the sequence of problems in mathematical induction listed as variable $x(1)$ and the scales of response patterns listed as variable $x(i)$, $i = 2, 3, 4, 5, 6, 7, 8$.

The problems selected for the mathematical portion of the study were based on problems previously used with students of various abilities and at various school levels. These problems were selected with the knowledge that a study of induction is one area of mathematical learning in which a student succeeds according to his own efforts alone.

The mathematical induction problems were framed in terms of anticipated seventh graders' levels of sophistication. The programming was carried out around a total of 960 minutes of contact between the experimenter and the subjects.

The basic day-by-day mechanics for distributing and retrieving materials are indicated in Table 1. The schedule for scaling response patterns related to this research was organized by the school counselor for each school. The test

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[1] See the evaluation model on page 42.

collaborator, Miss Donna Palonen, a Ph.D. candidate in psychology, scaled the response patterns of the students during six weeks of the period set aside for mathematics instruction, on days when no instruction in induction was scheduled. Scales $x(2)$ through $x(6)$ were administered to groups, while scales $x(7)$ and $x(8)$ were administered to individuals. Each scale was hand scored, in one sitting.

The subjects' final response scores on each scale were not submitted to the experimenter until all the subjects' achievement scores in induction had been tallied at the end of the study. All data were then punched on a data deck with a card for each subject and submitted for an analysis of variance routine at the computation center.

## Daily Routine

The basic daily research schedule for each school was a function of the necessary demands imposed by the school's routine. Laingsburg was on a class schedule which readily allowed the experimenter to meet class three mornings a week for fifty minutes, or one-hundred fifty minutes per week, during a period of six weeks and two days. (One meeting was lost due to Easter holidays.)

The Owosso school employed an alternating schedule of A and B days in such a fashion that when Monday was an A day this week it would be a B day the following week. Hence, when the experimental group met on Monday, Wednesday, Friday one week, they met on Tuesday and Thursday the following week. In addition a three day week (a Monday, Wednesday, Friday week) was lost at Easter. The class in Owosso met twenty contact class periods covering nine weeks.

In a letter to the subjects' parents, the school administration invited the subjects to participate in an experiment in mathematics learning. The initial class meeting consisted of orientation and collection of the permission slips which had been signed by the parents. At this same meeting, the experimenter informed the subjects about the anticipated routine which included the sequence of events listed in Figure 2.

The experimenter was the only adult involved with the subjects during the instruction in induction. A typical class period consisted of the following activities: During the first portion of the period students turned in written work from the previous meeting and asked questions about specific problems. Those boys who had no material to hand in or who were without questions were urged to re-examine portions of the problems they had considered previously. While the boys handed in all the previously introduced material, the experimenter distributed a new set of materials. A discussion followed each distribution of new materials and the subjects' questions centered on patterns and structures pertinent to the new problem.

Questions were handled in two ways. In some instances, the questioning involved an individual and in other instances, small groups of subjects. Questions concerning problem solutions

Daily Schedule

Figure 2

| Meeting | Orientation | Pre-test | Problems | 1-2-3 | 4-5-6 | 7-8-9 | 10-11 | 12 | 13-14 | 15-16-17-18 | 19-20 | 21 | 22 | 23-24 | 25-26 | 27-28 | Quiz I | Quiz II | Post Test | Questions and Problem Solving |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | * | | | | | | | | | | | | | | | | | | | |
| 2 | | * | | | | | | | | | | | | | | | | | | |
| 3 | | | | * | | | | | | | | | | | | | | | | |
| 4 | | | | | * | | | | | | | | | | | | | | | |
| 5 | | | | | | * | | | | | | | | | | | | | | |
| 6 | | | | | | | * | | | | | | | | | | | | | |
| 7 | | | | | | | | * | | | | | | | | | * | | | |
| 8 | | | | | | | | | * | | | | | | | | | | | |
| 9 | | | | | | | | | | * | | | | | | | | | | |
| 10 | | | | | | | | | | | * | | | | | | | | | |
| 11 | | | | | | | | | | | | * | | | | | | | | |
| 12 | | | | | | | | | | | | | * | | | | | * | | |
| 13 | | | | | | | | | | | | | | * | | | | | | |
| 14 | | | | | | | | | | | | | | | * | | | | | |
| 15 | | | | | | | | | | | | | | | | * | | | | |
| 16 | | | | | | | | | | | | | | | | | | | | * |
| 17 | | | | | | | | | | | | | | | | | | | | * |
| 18 | | | | | | | | | | | | | | | | | | | | * |
| 19 | | | | | | | | | | | | | | | | | | | | * |
| 20 | | | | | | | | | | | | | | | | | | | * | * |

were not answered directly. Subjects were led to consider either the formation of specific and finite patterns, as in Problem 4 cited on page 22, from which they could organize a structured solution of strategies for testing their intuitive ideas for correctness. As time passed, the work-question sessions involving groups took up most of the class period.

An additional aspect of the study reported here is best identified by analyzing problems. It appears that problem solution is related to cognitive preference in some cases, and training as well as preference in others. Three problems are analyzed in some detail here to exemplify such styles.

Problem 6 may be viewed as basically algebraic--its solution depends on recognition and manipulation of symbolism. Problem 7 may be viewed as basically geometric and depends primarily on spatial understanding. Problem 13 may be viewed as a multi-step transformation problem. A solution requires transformations of words into appropriate symbolic form. If this is properly completed, the solution requires a student to collect sufficient detailed information.

## An Analysis of Problem 6

Consider Problem 6:

$$2 + 4 + 6 + \ldots + 999{,}996 + 999{,}998 + 1{,}000{,}000 = ?$$

This problem was particularly interesting for several reasons. One reason was that the subjects found it difficult to relate it to perceptual space in a meaningful fashion. The students various approaches using representational space were not sufficiently discriminating to evolve a solution. Most of the subjects did not understand why there would be only 250,000 terms left if they added the first term to the last, the second to the next to the last, the third to the third from the last and continued to collapse the terms in this manner until the matchings were complete.[2]

The computational aspects of handling series seemed to dominate the subjects' thinking, so that many of them summed only the six obvious terms, forgetting the meaning of the ellipses for the moment. When the subjects were asked the meaning of the three dots in Problem 6 they answered that these dots represented "missing numerals". When asked to show how they had considered the missing numerals in their computation, most would point to the six given numerals and then, at last, recognize that they had omitted the numerals denoted by the dots. Even then some subjects did not recognize that they had omitted most of the numbers in the series.

Those subjects who failed to solve problems such as Problem 6 were extremely weak in terms of recall which is prerequisite to analyzing problems. In other words, for them, the necessary components of step one of the evaluation model related in Chapter IV were missing. A typical subject in this

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[2] J. J. Gibson and E. J. Gibson, "Perceptual Learning," The Psychological Review, LXII (January, 1955), 40.

category could not compute accurately using two-digit multiplication. Such subjects did not seem to have a particular aversion for multiplication as a process, they simply did not recognize that the principles of multiplication were valuable tools to be used in problem situations.

The students who failed to solve this series problem could not extend the intuitive suggestions imposed by the representational symbols. In a sense, a conceptualization such as an intuitive imposition required a second order abstraction, since the representational objective, in this instance, was a sequence of names for ideas. The three dots were a symbol, representing a sequence of other missing symbols, themselves representative of ideas.

## An Analysis of Problem 7

Problem 7, simply stated was this: A cube is completely covered with paint and cut by orthogonal planes in standard position, parallel to the faces, thus reducing the given cube into eight smaller, regular cubes. Each of the smaller cubes will have paint on some of the surfaces. The problem is to identify how many faces of each of the small cubes are painted. Extend the procedure to answer the same question for two planes regularly spaced through each dimension, then three and finally for *n* planes. (See the model on page 23 for an illustration of this problem.)

Observation of the subjects' cognitive preference in attacking Problem 7 hints at how profound the initial approach may be in mathematical learning. In this study, subjects who played too long with a wooden model apparently penalized themselves. All those who built more than one distinct example of the larger cube by using small wooden cubes, failed to solve the problem, while the successful students neither played with the wooden model more than a few minutes nor reconstructed a larger cube more than once. All successful students drew pictures that showed fewer than five cuts, and then spent considerable time studying their drawings.

Those students who could not construct a three-dimensional perspective of the cube on a piece of paper failed to solve the problem. Each student who solved some portion of the problem, constructed at least one perspective drawing. Successful subjects usually controlled computational demands but depended heavily on intuitive direction to complete their problems.

## An Analysis of An Unsolved Problem

### Problem 13

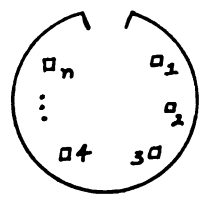
Problem 13, given out at the eighth class meeting went unsolved by both subject groups. The original problem follows:

> Agent 007 has infiltrated the Red Chinese Department of State as a spy for British intelligence. Mao has decided it is time for a purge of his Department of State.
>
> Mao will handle the purge in the following way. He will have all members of the Department of State brought into

a room. There will be n of them and they will be told to sit in a circle. Then Mao's executioner called Chopper will come through and eliminate every other one around the circle until only one remains. Mao wishes to preserve the brightest member and assumes this will determine the brightest.

Suppose you are 007 and you would know what the number n would be just ten minutes before you entered the room. Where should you sit in order to be the lone survivor?

Try some samples if you wish. The following sketch may be helpful.



In this problem the subjects had to determine a scheme whereby they could select the position in a ring or circle of objects that would remain unchanged if someone began at a given initial point and proceeded to eliminate alternate objects continuously around the circle or ring until only one object remained.

Only one subject earned partial credit for this problem and his credit was for an incomplete solution. A detailed inspection of subject responses suggested that a contributing reason for failure rested on the subjects' lack of formal training in data classification and analysis. The subjects gave the additional impression that success on Problem 13 depended on the use of powerful cues from the environment, accommodated by their intuition. If they generated incomplete patterns, either their intuition failed to function or they failed to recognize a critical step in the analysis.

An examination of the partial solution submitted indicated that the subject compiled data to $n < 13$. His data was neither sufficient to impose the subtle nature of the needed sequence on the subject nor sufficient to allow recognition of a possible solution via the binary digit system. Both of these approaches were potentially available to the student in question. After this subject examined the case for 1, 3, 5, 7, 9 and 11 objects, he considered numbers of objects such as 35, 40, and 45 . By extending the classification too rapidly, detail overwhelmed him, and he could not find the pattern.

Five students recognized that only odd numbered positions in the ring needed to be considered. The students who failed to recognize this detail, continued to work from a cognitive preference vested in a geometric space. It is possible to find a geometric pattern but the subjects would have had to take greater care with detail than they applied to the task during their analysis.

One subject, who might be considered talented mathematically, used a geometric approach and was nearly successful but was derailed by a obsession about modifying the original problem. He wished to make a modification based on a unit increase at each step--skip one, remove one; skip two, remove one; skip three, remove one and so on until just one remained.

He might have derived a solution to his modified problem by compiling a finite step-by-step list of examples, but the arithmetic was horrendous and he did not complete his solution.

To be successful on a problem such as Problem 13 the subject must be able to compile detailed lists of data, step-by-step and maintain organization while scanning the data for patterns he believes must exist.

## Summary

Chapter III described the selection of subjects and the design of the study. Two community schools provided subjects for this study--the Laingsburg Community School and the Owosso Junior High School.

The study is basically the one group, Pretest-Post test type. In the course of the research, seven basic response patterns were scaled and compared with the subjects' success in mathematical induction.

In this chapter, daily routine was described in terms of daily objectives and student outcomes, as well as the interaction between experimenter and subjects. Subject responses to three problems were analyzed. In attacking Problem 6 involving series, the primary weakness displayed by subjects was their inability to incorporate representative symbols into intuitive concepts. In handling Problem 7 the subjects' major debilitating factor was their lack of space orientation and their inability to translate their ideas into representational space. In Problem 13 the students lacked training. Their schemes for classifying and analyzing data proved inadequate.

# CHAPTER IV

## ANALYSIS OF DATA

The mathematical induction material was scored according to the correctness at each stage of problem solution. The guide for this evaluation was the evaluation model included below. Partial credit for each problem was awarded where appropriate. The take home problems as well as quizzes and tests were totaled for each subjects' achievement score.

Disregarding the influence of motivation and considering only the student's achievement in induction, this researcher evaluated that achievement according to the following step-by-step model.

(1) Step one was to determine the state of students previous knowledge--his recall of details, rules, facts and terms that were fundamental for solution of the given problem.

(2) Step two was to determine the student's ability to generate transformations from previous knowledge.

(3) Step three was to determine the student's ability to manipulate recalled ideas and generated transformations in order to determine what was meaningful or useful.

(4) Step four was to determine the student's selection of ideas to meet demands of a given problem from his intuitive impressions. This was the beginning of classification.

(5) Step five was to determine the student's ability to organize a given collection of data in terms of existing differences; to recognize the need for additional information or elimination of redundant or extraneous information. Here he begins to develop a need for mathematical proof.

(6) Step six was to determine the student's ability to generalize or organize the results into a coherent structure.

(7) Step seven was to determine the correctness of the solution through testing the result in terms of the original objectives.[1]

Table 1 shows the totality of problems accounting for the achievement scores of the sample. Each problem listed with a score implies that a student received at least one half point credit for that problem. The number in the brackets is the count of the subjects receiving a score for that particular problem. Only those problems for which some subject scored at least a half point or more are listed in the table. Thus under the Pretest in Table 1 only Problems 3 and 5 are listed. They are the only problems on the Pretest solved by any subjects. One subject solved Problem 3 and another solved Problem 5.

The statistical analysis reported herein was designed to answer the basic questions of correlation and analysis of variance related to achievement in mathematical induction and subjects' response patterns. The compiled data which was programmed into the computer can be found in Appendix E.

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[1]The idea for such a model grew out of several sources, including: Benjamin S. Bloom (ed.), Taxonomy of Educational Objectives, Handbook I: Cognitive Domain (New York: David McKay Company, Inc., 1956), p. 18.

Table 2 is a scatter diagram of variables x(1) and x(9). This table shows the composite score of the response patterns and the achievement in induction of the individuals. In addition, the least square linear fit of the scatter is plotted, as well as the mean for the composite and the mean of the achievement scores.

Table 3 lists the simple statistical data for each variable x(i), i = 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 , where the following relations were assumed:

1. The sum of variables $x_i$ is $\sum_{t=1}^{N} x_{it}$ where N is the number of observations in the problem. It is understood that: $\sum_{t=1}^{N} x_{it} = x_{i1} + x_{i2} + \ldots + x_{iN}$ .

2. The mean is denoted by:

$$\bar{x}_i = \frac{\sum_{t=1}^{N} x_{it}}{N} .$$

3. The sum of the squares is denoted by: $\sum_{t=1}^{N} x_{it}^2$ .

4. The sum of the squared deviations from the mean is denoted by:

$$\sum_{t=1}^{N} [x_{it} - \bar{x}_i]^2 .$$

5. The standard deviation is denoted by:

$$STD_i = \sqrt{\frac{\sum_{t=1}^{N} [x_{it} - \bar{x}_i]^2}{N - 1}} .$$

TABLE 1

SCALE SCORES

| Pretest | Quiz I | Quiz II | Post Test | Induction Problems |
|---|---|---|---|---|
| 3. {1} | 1. {18} | 1. {6} | 1. {8} | 1. {10} |
| 5. {1} | 2. {19} | 2. {1} | 2. {8} | 2. {5} |
| | 3. | 3. {3} | 3. | 3. {1} |
| | a) {13} | 4. {1} | a) {7} | 4. {4} |
| | b) {10} | | b) {3} | 5. {3} |
| | c) {6} | | 4. | 6. {1} |
| | 4. {17} | | a) {2} | 7. |
| | | | b) {12} | d) {1} |
| | | | c) {3} | e) {6} |
| | | | 5. {3} | f) {6} |
| | | | 6. {8} | g) {1} |
| | | | 7. {6} | 8. {2} |
| | | | 8. {3} | 9. {3} |
| | | | | 10. |
| | | | | a) {4} |
| | | | | b) {3} |
| | | | | 11. {1} |
| | | | | 12. {1} |
| | | | | 13. {1} |
| | | | | 14. |
| | | | | e) {1} |
| | | | | f) {1} |
| Totals are below this line. | | | | 15. {6} |
| 2 | 83 | 11 | 63 | 61 |

TABLE 2

A SCATTER PLOT

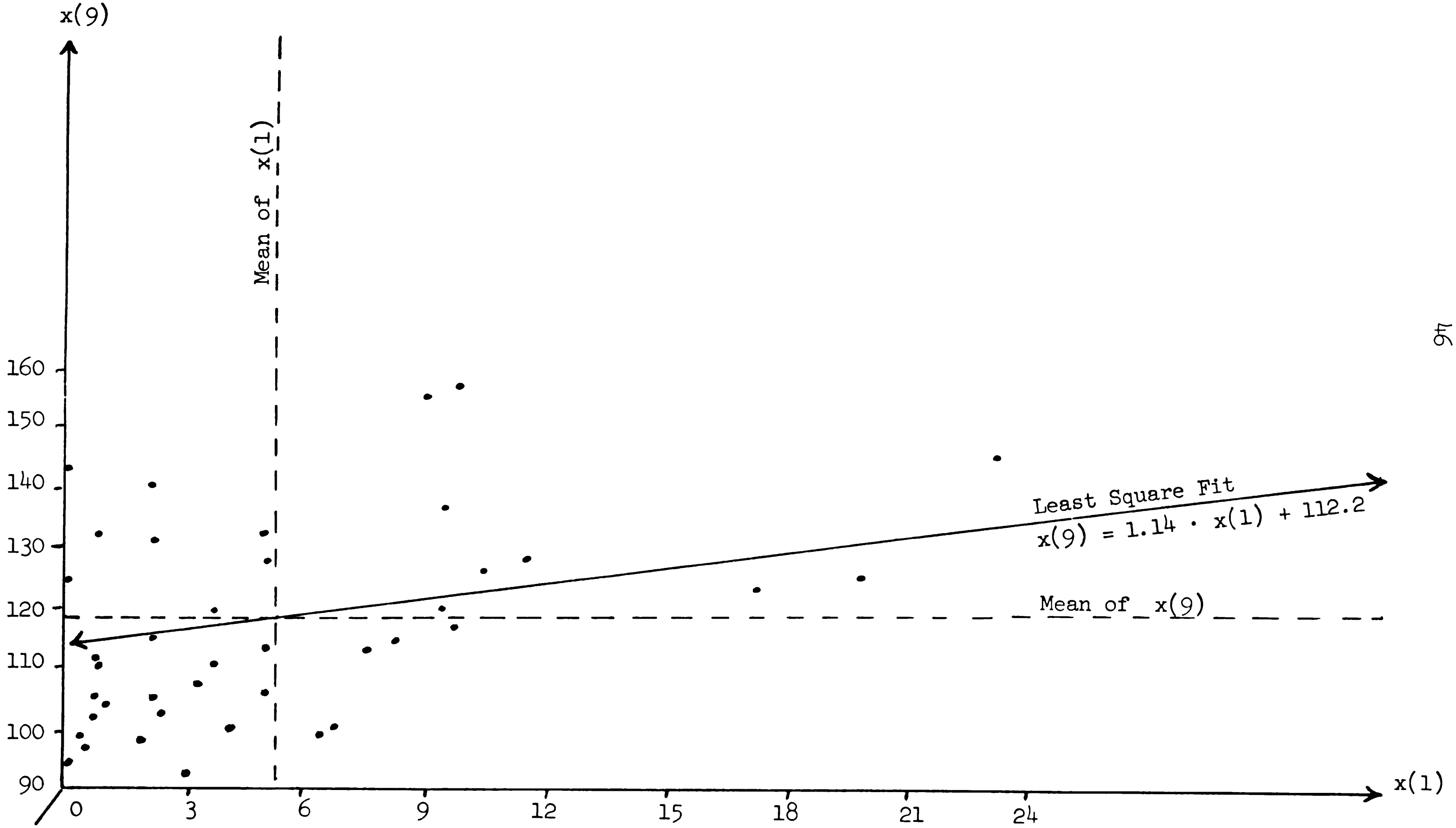

TABLE 3

ANALYSIS OF VARIANCE

| | SUM | MEAN | SUM OF SQUARES | STANDARD DEVIATION | SUM OF SQUARED DEVIATIONS | SKEWNESS | KURTOSIS |
|---|---|---|---|---|---|---|---|
| x(1) | 211 | 5.3 | 2290 | 5.5 | 1182 | 1.5 | 5.1 |
| x(2) | 721 | 18.0 | 14263 | 5.7 | 1267 | .8 | 2.7 |
| x(3) | 834 | 20.9 | 18622 | 5.6 | 1233 | - .6 | 2.9 |
| x(4) | 536 | 13.4 | 7456 | 2.6 | 274 | - .3 | 2.2 |
| x(5) | 781 | 19.6 | 15661 | 3.3 | 412 | .4 | 2.4 |
| x(6) | 341 | 8.6 | 3151 | 2.5 | 244 | .0 | 1.9 |
| x(7) | 532 | 13.3 | 8516 | 6.1 | 1440 | .3 | 2.2 |
| x(8) | 983 | 24.6 | 25575 | 6.0 | 1418 | - .2 | 2.2 |
| x(9) | 4728 | 118.2 | 569540 | 16.6 | 10690 | .6 | 2.7 |
| x(10) | 2034 | 50.9 | 108402 | 11.3 | 4973 | .6 | 2.5 |

6. Skewness is denoted by:

$$\frac{\sum_{t=1}^{N} x_{it}^3 - 3[\sum_{t=1}^{N} x_{it}^2]\bar{x}_i + 3[\sum_{t=1}^{N} x_{it}]\bar{x}_i^2 - N_i x_i^3}{[\sum_{t=1}^{N} x_{it}^2 - N\bar{x}_i^2]^{3/2}}$$

7. Kurtosis is denoted by:

$$\frac{\sum_{t=1}^{N} x_{it}^4 - 4[\sum_{t=1}^{N} x_i^3] + 6[\sum_{t=1}^{N} x_i^2]\bar{x}_i^2 - 4[\sum_{t=1}^{N} x_i]\bar{x}_i^3 + N\bar{x}_i^4}{[\sum_{t=1}^{N} x_{it}^2 - N\bar{x}_i^2]^2} - 3 .$$ [2]

Table 4 lists the simple correlations. These are the Pearson product moment correlations. If $r_{ij}$ denotes the correlation then

$$r_{ij} = \frac{\sum_{t=1}^{N} [x_{it} - \bar{x}_i][x_{jt} - \bar{x}_j]}{\sqrt{\sum_{t=1}^{N} [x_{it} - \bar{x}_i]^2}\sqrt{\sum_{t=1}^{N} [x_{jt} - \bar{x}_j]^2}}$$

where: $x_{it}$ and $x_{jt}$ are the variables and $\bar{x}_i$ , $\bar{x}_j$ the respective means. All elements with value 1.000... are diagonal elements of the matrix. Five of these are of particular interest. Guilford defines a correlation of .304 to be

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[2]The statistical material listed in items 1 through 7 on pages 44 and 48 has been slightly modified and then reproduced from:

William L. Ruble, Donald F. Kiel, and Mary E. Rafter, Calculations of Basic Statistics on the Bastat Routine, STAT Series Description Number 5; (East Lansing, Michigan: Michigan State University Agricultural Experiment Station, 1966), p. 16.

significantly different from zero at the .05 level of confidence given forty degrees of freedom. The .01 level of confidence is defined with a correlation of .393.[3]

The particular five variables of interest are x(2), x(5), x(7), x(9) and x(10). Note that x(1) and x(2) correlate beyond the .05 level of confidence with a correlation of .38. Variables x(5) and x(1) correlate beyond the .05 level of confidence with correlation .31. Variables x(7) and x(1) correlate beyond the .01 level of confidence with correlation .41. Variable x(9), the composite of variables x(2) through x(8), correlates with x(1) beyond the .05 level of confidence with correlation .38, and x(10), the composite of x(2), x(5) and x(7), correlates with x(1) beyond the .01 level of confidence with a correlation of .50.

The Unmatched F Statistics Between Means (UFBM) are listed in Table 5. If the row variable $x_i$ and column variable $x_j$ of the UFBM matrix are denoted by $F\ \bar{x}_i\bar{x}_j$, then $F\ \bar{x}_i\bar{x}_j$ is an F statistic for testing the null hypothesis. $F\ \bar{x}_i\bar{x}_j$ has a numerator of 1 and a denominator of 2N - 2.[4] Column one indicates that every entry exceeds the minimum value necessary for rejecting the null hypothesis with a probability

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[3] J. P. Guilford, Fundamental Statistics in Psychology and Education, (New York: McGraw-Hill Book Company, 1965), p. 581.

[4] Ruble, Ibid., p. 18.

that a Type I error has the following chance of occurring:

| | |
|---|---|
| x(2), | .005 |
| x(3), | .005 |
| x(4), | .005 |
| x(5), | .005 |
| x(6), | .005 |
| x(7), | .005 |
| x(8), | .005 |
| x(9), | .005 |
| x(10), | .005 |

Tables 6 and 7 contain the Least Square Coefficients between all pairs of variables $x_i$, $x_j$ where $i,j = 1, 2, \ldots, 10$. Given the equation form:

$$x_i = a_{ij} + b_{ij}x_j$$

it is understood that:

$x_i$ is the dependent variable,

$a_{ij}$ is the traditional Y-axis intercept,

$b_{ij}$ is the traditional slope coefficient,

$x_j$ is the independent variable.

In addition $a_{ij}$ and $b_{ij}$ are defined by:

$$a_{ij} = \sum_{t=1}^{N} [x_{it} - \bar{x}_i]^2 - b_{ij}\bar{x}_j \; .$$

$$b_{ij} = \frac{\sum_{t=1}^{N} [x_{it} - \bar{x}_j]^2}{\sum_{t=1}^{N} [x_{jt} - \bar{x}_j]^2} \; .$$ [5]

Tables 8 and 9 are composed of the Simple Least Square of Standard Error of Estimate and the Standard Error of Simple

---

[5]Ruble, Ibid., p. 21.

TABLE 4

SIMPLE CORRELATION MATRIX

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x(1) | 1.0 | | | | | | | | | |
| x(2) | 0.38 | 1.0 | | | | | | | | |
| x(3) | 0.21 | 0.14 | 1.0 | | | | | | | |
| x(4) | 0.02 | 0.37 | 0.48 | 1.0 | | | | | | |
| x(5) | 0.31 | 0.48 | 0.42 | 0.05 | 1.0 | | | | | |
| x(6) | -0.01 | 0.09 | -0.28 | 0.5 | -0.14 | 1.0 | | | | |
| x(7) | 0.41 | 0.31 | 0.14 | 0.16 | 0.21 | -0.01 | 1.0 | | | |
| x(8) | -0.09 | -0.06 | -0.00 | 0.06 | 0.07 | -0.05 | 0.06 | 1.0 | | |
| x(9) | 0.38 | 0.65 | 0.55 | 0.60 | 0.63 | 0.04 | 0.61 | 0.38 | 1.0 | |
| x(10) | 0.50 | 0.81 | 0.26 | 0.37 | 0.64 | -0.00 | 0.76 | 0.02 | 0.84 | 1.0 |

TABLE 5

UNMATCHED F STATISTICS BETWEEN MEANS

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x(1) | 0.00 | | | | | | | | | |
| x(2) | 103.75 | 0.00 | | | | | | | | |
| x(3) | 156.94 | 4.98 | 0.00 | | | | | | | |
| x(4) | 70.96 | 21.66 | 57.47 | 0.00 | | | | | | |
| x(5) | 199.10 | 2.10 | 1.66 | 85.37 | 0.00 | | | | | |
| x(6) | 11.64 | 93.18 | 160.44 | 71.63 | 287.77 | 0.00 | | | | |
| x(7) | 38.42 | 12.86 | 33.26 | 0.01 | 32.63 | 21.18 | 0.00 | | | |
| x(8) | 223.81 | 24.93 | 8.17 | 115.18 | 21.74 | 241.83 | 69.39 | 0.00 | | |
| x(9) | 1675.97 | 1309.20 | 1239.92 | 1562.71 | 1368.12 | 1716.11 | 1415.10 | 1129.35 | 0.00 | |
| x(10) | 526.72 | 269.37 | 226.23 | 417.01 | 284.26 | 535.66 | 342.96 | 168.52 | 451.76 | 0.00 |

TABLE 6

COEFFICIENTS FOR LEAST SQUARE FIT $x_i$ on $x_j$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x(1) | 0.00 | | | | | | | | | |
| x(2) | 15.97 | 0.00 | | | | | | | | |
| x(3) | 19.74 | 18.34 | 0.00 | | | | | | | |
| x(4) | 13.36 | 10.26 | 8.64 | 0.00 | | | | | | |
| x(5) | 18.57 | 14.64 | 14.50 | 14.50 | 0.00 | | | | | |
| x(6) | 8.55 | 7.82 | 11.10 | 7.91 | 10.66 | 0.00 | | | | |
| x(7) | 10.94 | 7.33 | 10.24 | 8.25 | 5.54 | 13.59 | 0.00 | | | |
| x(8) | 25.11 | 25.68 | 24.69 | 22.87 | 21.97 | 25.66 | 23.84 | 0.00 | | |
| x(9) | 112.23 | 84.07 | 84.21 | 67.81 | 55.22 | 115.76 | 96.22 | 92.66 | 0.00 | |
| x(10) | 45.48 | 21.97 | 39.79 | 29.97 | 7.29 | 50.98 | 32.16 | 49.84 | -16.67 | 0.00 |

TABLE 7

COEFFICIENTS FOR THE LEAST SQUARE FIT $x_j$ on $x_i$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x(1) | 1.00 | | | | | | | | | |
| x(2) | 0.39 | 1.00 | | | | | | | | |
| x(3) | 0.21 | 0.14 | 1.00 | | | | | | | |
| x(4) | 0.01 | 0.17 | 0.23 | 1.00 | | | | | | |
| x(5) | 0.18 | 0.27 | 0.24 | 0.36 | 1.00 | | | | | |
| x(6) | -0.01 | 0.04 | -0.12 | 0.05 | -0.11 | 1.00 | | | | |
| x(7) | 0.45 | 0.33 | 0.15 | 0.38 | 0.40 | -0.03 | 1.00 | | | |
| x(8) | -0.10 | -0.06 | -0.01 | 0.13 | 0.13 | -0.13 | 0.06 | 1.00 | | |
| x(9) | 1.13 | 1.89 | 1.63 | 3.76 | 3.23 | 0.29 | 1.65 | 1.04 | 1.00 | |
| x(10) | 1.02 | 1.60 | 0.53 | 1.56 | 2.23 | -0.02 | 1.41 | 0.04 | 0.57 | 1.00 |

Least Square Coefficient of Slope respectively.

If $S_{ij}$ is the element in the row corresponding to variable $x_i$ and the column corresponding to $x_j$ of the Standard Error of Estimate matrix, then $S_{ij}$ is the standard error of estimate for the simple least squares equation defined by:

$$S_{ij} = \sqrt{\frac{\left[\sum_{t=1}^{N}(x_{it} - \bar{x}_i)^2 - b_{ij}\sum_{t=1}^{N}(x_{it} - \bar{x}_i)(x_{jt} - \bar{x}_j)\right]}{N - 2}}$$

$S_{ij}^2$ is the error mean square, the error mean square being the error sum of squares divided by the degrees of freedom for error $N - 2$ .

If $S_{b_{ij}}$ is the corresponding element in the Standard Error of Simple Least Square B matrix, then $S_{b_{ij}}$ is the standard error of the slope coefficient for the simple least squares equation. This defines:

$$S_{b_{ij}} = \frac{S_{ij}}{\sqrt{\sum_{t=1}^{N}[x_{it} - \bar{x}_i]^2}} \quad .$$ [6]

Table 10 is an F statistic matrix for testing the null hypothesis of the beta weight $b_{ij}$ as significantly different from zero. The probabilities of a Type 1 error are:

| | |
|---|---|
| x(2), | .02 |
| x(3), | .20 |
| x(4), | .88 |
| x(5), | .06 |
| x(6), | .90 |
| x(7), | .01 |
| x(8), | .58 |
| x(9), | .02 |
| x(10), | .00 |

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[6] Ruble, *Ibid.*, pp. 21-22.

TABLE 8

SIMPLE LEAST SQUARE STANDARD ERROR OF ESTIMATE

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x(1) | 0.00 | | | | | | | | | |
| x(2) | 5.35 | 0.00 | | | | | | | | |
| x(3) | 5.57 | 5.64 | 0.00 | | | | | | | |
| x(4) | 2.68 | 2.49 | 2.35 | 0.00 | | | | | | |
| x(5) | 3.13 | 2.90 | 2.99 | 3.14 | 0.00 | | | | | |
| x(6) | 2.53 | 2.52 | 2.43 | 2.53 | 2.51 | 0.00 | | | | |
| x(7) | 5.63 | 5.85 | 6.10 | 6.07 | 6.02 | 6.16 | 0.00 | | | |
| x(8) | 6.08 | 6.10 | 6.11 | 6.10 | 6.10 | 6.10 | 6.10 | 0.00 | | |
| x(9) | 15.53 | 12.72 | 13.97 | 13.40 | 12.99 | 16.76 | 13.33 | 12.99 | 0.00 | |
| x(10) | 9.92 | 6.73 | 11.03 | 10.65 | 8.77 | 11.44 | 7.49 | 11.44 | 6.25 | 0.00 |

TABLE 9

STANDARD ERROR OF SIMPLE LEAST SQUARE BETA

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x(1) | 0.00 | | | | | | | | | |
| x(2) | 0.16 | 0.00 | | | | | | | | |
| x(3) | 0.16 | 0.16 | 0.00 | | | | | | | |
| x(4) | 0.08 | 0.07 | 0.07 | 0.00 | | | | | | |
| x(5) | 0.09 | 0.08 | 0.09 | 0.19 | 0.00 | | | | | |
| x(6) | 0.07 | 0.07 | 0.07 | 0.15 | 0.12 | 0.00 | | | | |
| x(7) | 0.16 | 0.16 | 0.17 | 0.37 | 0.30 | 0.39 | 0.00 | | | |
| x(8) | 0.18 | 0.17 | 0.17 | 0.37 | 0.30 | 0.39 | 0.16 | 0.00 | | |
| x(9) | 0.45 | 0.36 | 0.40 | 0.81 | 0.64 | 1.07 | 0.35 | 0.41 | 0.00 | |
| x(10) | 0.29 | 0.19 | 0.31 | 0.64 | 0.43 | 0.73 | 0.20 | 0.30 | 0.06 | 0.00 |

TABLE 10

F VALUE FOR SIMPLE LEAST SQUARE BETA

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x(1) | 0.00 | | | | | | | | | |
| x(2) | 6.30 | 0.00 | | | | | | | | |
| x(3) | 1.70 | 0.77 | 0.00 | | | | | | | |
| x(4) | 0.01 | 6.21 | 11.65 | 0.00 | | | | | | |
| x(5) | 3.99 | 11.10 | 7.99 | 3.91 | 0.00 | | | | | |
| x(6) | 0.00 | 0.30 | 3.15 | 0.09 | 0.78 | 0.00 | | | | |
| x(7) | 7.51 | 4.06 | 0.71 | 1.06 | 1.80 | 0.01 | 0.00 | | | |
| x(8) | 0.33 | 0.13 | 0.00 | 0.12 | 0.20 | 0.11 | 0.12 | 0.00 | | |
| x(9) | 6.30 | 28.07 | 16.80 | 21.55 | 25.43 | 0.07 | 22.13 | 6.35 | 0.00 | |
| x(10) | 12.51 | 71.87 | 2.85 | 5.86 | 26.67 | 0.00 | 50.74 | 0.02 | 89.43 | 0.00 |

## Discussion

The material contained here is the conventional analysis of variance. Table 1 implies that the subjects had no formal information about induction since only two points were scored on the Pretest by the entire sample.

It seems reasonable to hypothesize that a contributing factor to the wide difference in the points accumulated on Quiz I and the Post Test was that the Post Test required sustained attention on each problem. That is, the problems became more involved. In none of the three achievement tests did a problem occur that was missed by every student and yet no problem was solved by as many as half the sample. Two points about these tests seem to be apparent. First, each item was within the range of ability of some of the subjects. Second, timed tests apparently penalized some subjects since some of them accumulated more points on short Quiz I than they did on the longer Post Test. The hypothesis is based on the fact that the sample accumulated 83 points from a possible 240 points or 35% of the possible score on Quiz I, while on the Post Test of 11 items they accumulated only 63 points from a possible 440 points or 14%. It is possible that item difficulty was a factor, but there is no taxonomy for item difficulty, thus no further analysis could be made. The difference in scores also suggested that as problem count goes up, some students become overwhelmed and their productivity decreases.

## Summary

Table 2 indicates that 22 subjects scored less than 120 on scale x(9) and less than 8 on scale x(1), while 5 subjects scored greater than 120 on scale x(9) and greater than 10 on scale x(1) . No one who scored less than 120 on scale x(9) also scored 10 or above on scale x(1) .

Table 3 lists the achievement of each subject in mathematical induction. The model used to determine student achievement in induction is also presented in this chapter.

Table 4 lists the correlations among all the variables. Of particular interest is the correlation between x(1) , and x(2) and x(5) which is significantly different from zero at the .05 level of confidence, and the correlation between x(1) and x(7) which is significantly different from zero at the .01 level of confidence.

The F ratio used to evaluate the null hypothesis implied that we can reject the null hypothesis relative to x(2) , x(5) and x(7) as well as x(9) and x(10) with a confidence of .005 . The F statistics which test the null hypothesis of the beta weights show the following probabilities of a Type I error: x(2) , .02 ; x(5) , .06 ; and x(7) , .01 .

# CHAPTER V

# CONCLUSIONS AND IMPLICATIONS

## Conclusions

This study was designed as an exploration in teaching mathematical induction. A second consideration was the hypothesis that there was a constellation of response patterns which had significant correlation with achievement in mathematical induction. The response patterns were identified by specific scales.

The achievement scores for each seventh grader were the result of the specific sequence of induction materials and the evaluation model used in compiling the achievement results. The achievement results of the subjects suggest several conclusions. First, a minimum background in basic factual material, as detailed in step one of the evaluation model, is required if the student is to deal in an effective fashion with materials on induction. This is particularly true if the approach to teaching tends to be unstructured. Students who scored high on scales $x(2)$, $x(5)$ and $x(7)$ were most successful in unstructured learning.

The material as designed in this study was too advanced. The material used here might be viewed as a third level where two simpler levels would be on a finite order. For example, simple manipulative puzzles where a given piece must be moved to a given spot could serve as a first level study. An example of a second level problem is the problem of arranging 4 cubes in such a way that all four sides of the stack show 4 distinct colors. Various patterns may exist on each cube but the problem is still finite.

Achievement in the induction material was compiled in too rigid a manner for the group of subjects in this study. The emphasis was on so-called 'correct' responses to problem conditions. Clearly, the outcome of each problem should be the criterion upon which the achievement mark is made. Achievement is necessarily internalized in the sense that understanding and manipulation precede translation into symbolic forms, and such internalization is difficult to measure. The subjects lacked training in such areas as attention and sustained effort. Yet attention and sustained effort are necessary for internalization and understanding. Piaget might term this a question of accommodation.[1]

In an effort to determine why these subjects generally found induction uninteresting, this researcher uncovered two revealing clues. These students lived in an environment with few activities requiring inductive thinking (in a formal sense), hence they did not believe or feel such a technique had importance

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[1]J. M. Hunt, Intelligence and Experience, (New York: The Ronald Press Company, 1961), p. 258.

or potential. Most of the students also lacked an appropriate sequence of skills and training necessary to handle induction on a sophisticated level.

This research measured the correlation between the selected response patterns of each individual and his achievement in mathematical induction. Successful subjects had to fit their perceptions into a representational reality of logical and predictable symbols for communication and for future manipulation. Some inferences or preferences of style became evident as subjects entertained material of various types. For example, models can suggest direction but as was noted in Problem 7, models can also limit the individual in his solution efforts. Further evidence is desirable if models are to be used as a dominant style for teaching mathematics in the classroom.

This study clearly denotes that a correlation significant at the .01 level of confidence exists between achievement in mathematical induction and the scale $x(7)$ . It is further denoted that a correlation significant at the .05 level of confidence exists between achievement in mathematical induction and scales $x(2)$ and $x(5)$ . This means a significant relationship exists but further study is necessary to determine cause and effect.

## Implications

An additional study is needed to determine the necessary and sufficient conditions for mathematical learning of the type discussed here. There is a need to develop more precise scales for response patterns which are necessary for success in mathematical studies. Refined definitions of the response patterns are in order.

When a study involving response profiles and achievement is considered, two problems of major proportions occur. The first problem is the need to develop adequate scales to carry out the measurement desired. Second, the motivation of the student as a function of his cognitive preference must be considered.

When more appropriate scales are available, further study of response variables necessary for mathematical learning may follow. Such studies should investigate the question of cause and effect. Further, the question of manipulation of materials designed around the individuals' cognitive preference should be included.

Studies designed to establish a taxonomy of problem difficulty in various areas of mathematics learning are needed. Such a taxonomy will be difficult to construct since questions of seriation and classification and their interplay are largely unexplored. In addition, the matter of preference will probably add a new dimension to problem taxonomy.

A study is needed to determine the influence of perception and experience on learning mathematics in terms of a careful discrimination among models, symbols, and written forms of instruction. For example, Piaget discovered that the question of invariance was an important concept in the child's development. Without first teaching the concept of invariance, Piaget considers it pointless to attempt to teach many topics in the school curriculum. In a broader sense there is a need to identify the underlying factors which determine the way that a youngster learns to generalize from specific experiences. It has been shown in the study reported here that some learning in mathematics is associated with the responses that the subject already possesses.

Studies involving adolescents and learning suggest that short-term investigations will probably not be sufficient to yield response profiles that will predict mathematical success with sufficient accuracy. However, if a sufficient number of detailed studies build profiles in a significant number of areas, a pattern for general integration may become apparent. It seems clear that there is a need for research in depth to understand such factors as language orientation and perceptual influence, and their effects on generalization.

If this study were to be repeated modification should include organization of the problems such that each problem may be described by means of an algebraic notation, a geometric notation, or simply written. The material should be organized around three basic levels of sophistication previously described.

# APPENDIX A

# RESPONSE PATTERNS

Response Pattern x(2)

The x(2) variable was determined by scoring the student's responses to the following questionnaire in the fashion indicated. Forty points were the maximum on this questionnaire.

| SCORE | | |
|---|---|---|
| | 1. | At a picnic would you rather spend some time: |
| 0 | a. | exploring the woods alone, |
| 1 | b. | uncertain, |
| 2 | c. | playing around the campfire with the crowd? |
| | 2. | When you write an essay about your personal thoughts and feelings, do you: |
| 2 | a. | enjoy telling about yourself, |
| 1 | b. | uncertain, |
| 0 | c. | prefer to keep some ideas to yourself? |
| | 3. | Have you enjoyed being in drama, such as school plays? |
| 2 | a. | yes, |
| 1 | b. | uncertain, |
| 0 | c. | no. |
| | 4. | Do you feel hurt if people borrow your things without asking you? |
| 0 | a. | yes, |
| 1 | b. | uncertain, |
| 2 | c. | no. |
| | 5. | In dancing or music, do you pick up a few new pieces easily? |
| 2 | a. | yes, |
| 1 | b. | uncertain, |
| 0 | c. | no. |
| | 6. | Do you go out of your way to avoid crowded buses and streets? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |

x(2) continued:

| SCORE | | |
|---|---|---|
| | 7. | In talking with your classmates, do you dislike telling your most private feelings? |
| 0 | a. | yes, |
| 1 | b. | uncertain, |
| 2 | c. | no. |
| | 8. | When you go into a new group, do you: |
| 2 | a. | quickly feel you know everyone, |
| 1 | b. | in between, |
| 0 | c. | take a long time getting acquainted? |
| | 9. | Would you rather live: |
| 0 | a. | in a forest with the song birds, |
| 1 | b. | uncertain, |
| 2 | c. | on a busy street corner? |
| | 10. | When a new teacher comes to your class, does he or she soon notice who you are and remember you? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 11. | Do most people seem to enjoy your company? |
| 2 | a. | yes, |
| 1 | b. | average, |
| 0 | c. | no. |
| | 12. | Do you dislike going into narrow caves or climbing mountains? |
| 0 | a. | yes, |
| 1 | b. | sometimes, |
| 2 | c. | no. |
| | 13. | Are you always ready to show, in front of everyone, how well you can perform some feat? |
| 2 | a. | yes, |
| 1 | b. | maybe, |
| 0 | c. | no. |

x(2) continued:

| SCORE | | |
|---|---|---|
| | 14. | Which would you rather be: |
| 2 | a. | the most popular person in school, |
| 1 | b. | uncertain, |
| 0 | c. | the person with the best grades. |
| | 15. | In a group of people, are you generally one of those who tells jokes and stories? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 16. | When you are ready for a job, would you like one that: |
| 0 | a. | is steady and safe, though hard work, |
| 1 | b. | uncertain, |
| 2 | c. | has lots of change, with lively people? |
| | 17. | In group activities, which do you prefer? |
| 2 | a. | to be a good leader, |
| 1 | b. | in between, |
| 0 | c. | to be a good follower. |
| | 18. | Would you rather: |
| 0 | a. | read a story of wild adventure, |
| 1 | b. | uncertain, |
| 2 | c. | actually have wild adventures? |
| | 19. | Are you best regarded as a person who: |
| 0 | a. | thinks, |
| 1 | b. | in between, |
| 2 | c. | acts? |
| | 20. | Are you very careful not to hurt anyone's feelings or startle anyone? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |

<u>Response Pattern x(3)</u>

The x(3) variable was determined by scoring the student's responses to the following questionnaire in the fashion indicated. Forty points were the maximum on this questionnaire.

| SCORE | | |
|---|---|---|
| | 1. | When you do a foolish thing, do you feel so badly that you wish the earth would swallow you? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 2. | Do you find it easy to keep an exciting secret? |
| 2 | a. | yes, |
| 1 | b. | sometimes, |
| 0 | c. | no. |
| | 3. | Compared to other people, do you make up your mind: |
| 0 | a. | with hesitation, |
| 1 | b. | in between, |
| 2 | c. | with certainty? |
| | 4. | Do you completely understand what you read? |
| 2 | a. | yes, |
| 1 | b. | usually, |
| 0 | c. | no. |
| | 5. | When chalk screeches on the blackboard does it make you cringe? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 6. | Have you always got along really well with your parents and sisters as well as brothers? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |

x(3) continued:

| SCORE | | |
|---|---|---|
| | 7. | Do you often make big plans and get excited only to find they won't work? |
| 0 | a. | yes, |
| 1 | b. | sometimes, |
| 2 | c. | no. |
| | 8. | Are you satisfied that you come up to what others expect of you? |
| 2 | a. | yes, |
| 1 | b. | usually, |
| 0 | c. | no. |
| | 9. | Do you sometimes feel happy and sometimes feel depressed without real reason? |
| 0 | a. | yes, |
| 1 | b. | uncertain, |
| 2 | c. | no. |
| | 10. | If someone asks you to do a new and difficult job, do you: |
| 2 | a. | feel glad and show them how well you do, |
| 1 | b. | in between, |
| 0 | c. | feel you will make a mess of the job? |
| | 11. | Can you always tell what your real feelings are, for instance whether or not you are tired or just bored? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 12. | Do you sometimes feel you are not much good, and that you never do anything worthwhile? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 13. | Do you feel that you are getting along well, and that you do everything that is expected of you? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |

x(3) continued:

| SCORE | | |
|---|---|---|
| | 14. | Do you find yourself humming tunes if someone else started them? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 15. | Which of these changes in school would you rather vote for: |
| 2 | a. | putting slow people in classes together, |
| 1 | b. | uncertain, |
| 0 | c. | doing away with punishment? |
| | 16. | When things are going wonderfully, do you: |
| 0 | a. | actually almost "jump for joy", |
| 1 | b. | uncertain, |
| 2 | c. | feel good inside, but remain calm? |
| | 17. | When you wait in line, do you often: |
| 2 | a. | wait patiently, |
| 1 | b. | undertain, |
| 0 | c. | fidget and think of leaving. |
| | 18. | Do you wish you could learn to be more carefree and light-hearted about school work? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 19. | Do you find it easy to go up and introduce yourself to an important person? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 20. | Do your feelings get so bottled up that you feel you could burst? |
| 0 | a. | often, |
| 1 | b. | sometimes, |
| 2 | c. | seldom. |

Response Pattern $x(4)$

The $x(4)$ variable was determined by scoring the student's responses to the following questionnaire in the fashion indicated. Forty points were the maximum on this questionnaire.

| SCORE | | |
|---|---|---|
| | 1. | Do you think there is a fair chance that you will be a well-known, popular figure when you grow up? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 2. | When you are given higher grades than you usually make, do you feel that the teacher might have made a mistake? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 3. | In first grade, did you always go to school without your mother's having to force you? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 4. | Do you tend to be quiet when out with a group? |
| 2 | a. | yes, |
| 1 | b. | sometimes, |
| 0 | c. | no. |
| | 5. | When a new fad starts, say in dress or manner of speaking, do you: |
| 2 | a. | start early and follow it, |
| 1 | b. | uncertain, |
| 0 | c. | wait and watch before following? |
| | 6. | Would you rather be: |
| 2 | a. | a builder of bridges, |
| 1 | b. | uncertain, |
| 0 | c. | a member of a traveling circus? |

x(4) continued:

SCORE 7. Are you, like a lot of people, slightly afraid of lightning?

0 a. yes,
1 b. perhaps,
2 c. no.

8. Do you think that the average committee of your classmates takes too much time and does poorer work than you alone could do?

2 a. yes,
1 b. perhaps,
0 c. no.

9. Which kind of friends do you like--those who:

2 a. "horseplay",
1 b. uncertain,
0 c. are serious all the time?

10. If you were not a human being, would you rather be:

0 a. an eagle,
1 b. uncertain,
2 c. a seal in a colony by the sea?

11. Do you think that life has been happier and more satisfying for you than for others?

2 a. yes,
1 b. perhaps,
0 c. no.

12. Do you have trouble remembering someone's joke in order to repeat it?

0 a. yes,
1 b. sometimes,
2 c. no.

13. If someone puts on noisy music while you are trying to work, do you continue your work?

2 a. yes,
1 b. perhaps,
0 c. no.

x(4) continued:

| SCORE | | |
|---|---|---|
| | 14. | Would you rather spend time and money on: |
| 0 | a. | a popular dance, |
| 1 | b. | uncertain, |
| 2 | c. | a book on earning more money. |
| | 15. | Do you feel that most of your wants are reasonably well satisfied? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 16. | When you read an adventure story, do you: |
| 0 | a. | get bothered about whether it is going to end happily, |
| 1 | b. | uncertain, |
| 2 | c. | just enjoy the story as it goes along? |
| | 17. | When you do badly in an important game, do you: |
| 2 | a. | say--its just a game, |
| 1 | b. | uncertain, |
| 0 | c. | get angry? |
| | 18. | When you are walking in a quiet street in the dark, do you often get the feeling you are being followed? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 19. | When someone is disagreeing with you, do you: |
| 2 | a. | let him have his way, |
| 1 | b. | uncertain, |
| 0 | c. | tend to interrupt before he finishes? |
| | 20. | Do small troubles, sometimes which are really unimportant, get on your nerves? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |

Response Pattern x(5)

The x(5) variable was determined by scoring the student's responses to the following questionnaire in the fashion indicated. Forty points were the maximum on this questionnaire.

SCORE 1. If a friend's ideas differ from yours, do you keep still to maintain good feelings?

0 a. yes,
1 b. sometimes,
2 c. no.

2. Do you laugh with your friends more in class than other people do?

2 a. yes,
1 b. perhaps,
0 c, no.

3. When you finish school, will you prefer to:

0 a. do something that will make people like you,
1 b. uncertain,
2 c. make a lot of money?

4. Have you told your parents that some teachers are too old-fashioned to understand you and your friends?

2 a. yes,
1 b. perhaps,
0 c. no.

5. Do you prefer having teachers tell you how things should be done?

0 a. yes,
1 b. perhaps,
2 c. no.

6. In a trip with naturalists, would you find it more fun to:

2 a. catch birds and preserve them,
1 b. uncertain,
0 c. make artistic photos and paintings?

x(5) continued:

| SCORE | | |
|---|---|---|
| | 7. | If you accidentally say something odd in public, do you remain uncomfortable long and find it difficult to forget? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 8. | Are you known among your friends for going "for broke" for things you like? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 9. | In school would you rather be: |
| 0 | a. | a librarian, |
| 1 | b. | uncertain, |
| 2 | c. | an athletic director. |
| | 10. | On your birthday, do you prefer: |
| 2 | a. | to be asked in advance about what you would prefer for a gift, |
| 1 | b. | uncertain, |
| 0 | c. | to have the fun of getting a complete surprise? |
| | 11. | Do you sometimes feel, before a big party, that you are not interested in going? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 12. | Can you talk to a group of strangers without being self-conscious? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 13. | Are your feelings easily hurt? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |

x(5) continued:

SCORE 14. Can you work just as well, without making more mistakes, when you are watched?

2 a. yes,
1 b. perhaps,
0 c. no.

15. Do you sometimes feel unwilling to try something, though you know it isn't difficult?

0 a. yes,
1 b. perhaps,
2 c. no.

16. Do you stand up before class without looking nervous?

2 a. yes,
1 b. perhaps,
0 c. no.

17. How would you rate yourself?

0 a. inclined to be moody,
1 b. in between,
2 c. not at all moody.

18. In school, do you feel your teachers:

2 a. approve of you,
1 b. uncertain,
0 c. hardly know you are present?

19. Are you so afraid of consequences that you avoid making decisions one way or the other?

0 a. often,
1 b. sometimes,
2 c. never.

20. Do you have periods of feeling just "run down"?

2 a. seldom,
1 b. sometimes,
0 c. often.

Response Pattern x(6)

The x(6) variable was determined by scoring the student's responses to the following questionnaire in the fashion indicated. Forty points were the maximum on this questionnaire.

| SCORE | | |
|---|---|---|
| | 1. | Which of these says better what you are like? |
| 0 | a. | a dependable leader, |
| 1 | b. | in between, |
| 2 | c. | charming, good looking. |
| | 2. | Do you like to tell people to follow proper rules and regulations? |
| 0 | a. | yes, |
| 1 | b. | sometimes, |
| 2 | c. | no. |
| | 3. | Are you usually patient with people who speak very fast or very slowly? |
| 0 | a. | yes, |
| 1 | b. | sometimes, |
| 2 | c. | no. |
| | 4. | If you found another pupil doing a job you had been told to do, would you: |
| 2 | a. | ask him to turn the job over to you, |
| 1 | b. | uncertain, |
| 0 | c. | let the teacher decide? |
| | 5. | Are you steady and sure in what you do? |
| 2 | a. | seldom, |
| 1 | b. | sometimes, |
| 0 | c. | always. |
| | 6. | With people who take a long time to answer a question, do you: |
| 0 | a. | give them all the time they want, |
| 1 | b. | in between, |
| 2 | c. | try to hurry them, getting angry if they are slow? |

x(6) continued:

| SCORE | | |
|---|---|---|
| | 7. | Do you spend most of your allowance each week for fun? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 8. | Do other people often get in your way? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 9. | If you were working with groups in class, would you rather: |
| 2 | a. | walk around to carry things from one person to another, |
| 1 | b. | uncertain, |
| 0 | c. | specialize in showing people what to do? |
| | 10. | Do you take trouble to be sure you are right before you say anything in class? |
| 0 | a. | always, |
| 1 | b. | generally, |
| 2 | c. | not usually. |
| | 11. | Would you rather be: |
| 2 | a. | a traveling TV actor, |
| 1 | b. | uncertain, |
| 0 | c. | a medical doctor? |
| | 12. | Do people say that you are a person who can always be counted on to do things exactly and methodically? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 13. | Would you like to be extremely good-looking, so that people would notice you? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |

x(6) continued:

| SCORE | | |
|---|---|---|
| | 14. | When something is bothering you, do you think it's better to: |
| 0 | a. | withhold action until you become calm, |
| 1 | b. | uncertain, |
| 2 | c. | explode. |
| | 15. | Do you sometimes say silly things, just to see what people will say? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 16. | Do you ever suggest to the teacher a new subject for the class to discuss? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |
| | 17. | Would you rather spend a break between morning and afternoon classes in: |
| 2 | a. | a card game, |
| 1 | b. | uncertain, |
| 0 | c. | catching up on homework? |
| | 18. | Do you usually: |
| 0 | a. | follow your own ideas, |
| 1 | b. | uncertain, |
| 2 | c. | do the same as other people? |
| | 19. | Do you sometimes go on and do something you very much want to do, even though you feel a bit ashamed of yourself? |
| 2 | a. | yes, |
| 1 | b. | perhaps, |
| 0 | c. | no. |
| | 20. | Do you think that to be polite you must learn to control your feelings? |
| 0 | a. | yes, |
| 1 | b. | perhaps, |
| 2 | c. | no. |

Response Pattern x(7)

The x(7) variable was determined by scoring the student's responses to the following questionnaire in the fashion indicated:

| SCORE PROCEDURE | |
|---|---|
| success 2<br>neutral 1<br>fail 0 | 1. When he was completely on his own, he |
| try 2<br>neutral 1<br>quit 0 | 2. It looked impossible, so he |
| positive 2<br>neutral 1<br>negative 0 | 3. My first reaction to him was |
| stand up 2<br>neutral 1<br>other 0 | 4. When others made fun of him, he |
| act appropriate 2<br>neutral 1<br>scared 0 | 5. When he met his principal, he |
| try 2<br>neutral 1<br>quit 0 | 6. If I think the class is too hard for me, I |
| make it 2<br>think 1<br>avoid 0 | 7. When I have to make a decision, I |
| positive 2<br>neutral 1<br>negative 0 | 8. When they looked at me, I |

x(7) continued:

| SCORE PROCEDURE | | |
|---|---|---|
| cope 2<br>quit 0 | 9. | If I can't get what I want, I |
| handle 2<br>crumble 0 | 10. | When I am criticized, I |
| positive 2<br>neutral 1<br>negative 0 | 11. | People seem to think that I |
| cry 0<br>recover 2 | 12. | After they knocked him down, he |
| give 2<br>neutral 1<br>no 0 | 13. | When they asked my opinion, I |
| take over 2<br>neutral 1<br>avoid 0 | 14. | When they put me in charge, I |
| spoke 2<br>nervous 0 | 15. | When his turn came to speak, he |

Response Pattern x(8)

The x(8) variable was determined by scoring the results of the following directions:

Draw a picture of a person.

What is (his, her) age?

Draw a picture of a person of the opposite sex.

What is (his her) age?

What is the best trait of the first person?

What is the worst trait of the first person?

What is the best trait of the second person?

What is the worst trait of the second person?

What adjective best describes the first person?

What adjective best describes the second person?

What are the wishes of the first person?

What are the wishes of the second person?

The completed responses were scored according to Bodwin and Bruck's procedure:

1. Shading: Light, dim, subtle, and uncertain lines which furtively accent particular parts of the figure. Patterned or stylized shading.

Table of Shading

| (0-20%) | (21-40%) | (41-60%) | (61-80%) | (81-100%) |
|---|---|---|---|---|
| 5 | 4 | 3 | 2 | 1 |
| Markedly absent | | | | Markedly present |

2. Detail in Figure: Unessential features or details added to the figure or background. This analysis follows the table for shading.

x(8) continued:

3. Asymmetry: Imbalanced and lopsided arrangement of the body parts in respect to size, shape, or position on the opposite sides of the center. This analysis follows the table of shading.

4. Mixed age: Disparity in the physiological maturation of various body parts such as breasts emphasized in an otherwise childish body. This analysis follows the table for shading.

5. Immaturity: Drawing is marked by elaborate treatment of the mid-line such as Adam's apple, tie, buttons, buckle, and fly on trousers. There is emphasis on mouth and/or breasts. This analysis follows the table on shading.

# APPENDIX B

## TESTS

## Pretest

1. Compute the following sum:

   $1 + 2 + 3 + \ldots + 498 + 499 + 500 =$

2. Compute the following sum:

   $2 + 2\cdot2 + 2\cdot2\cdot2 + \ldots + 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2 =$

3. What is the name of the following number?

   1,000,000,000,000,000,000

4. Find a set of elements with exactly the same number of elements in it as there are in the set of even counting numbers.

   $\{2, 4, 6, 8, \ldots, 2\cdot n, \ldots\}$

5. If one has 64 feet of fence show which rectangular shape encloses the greatest area.

6. In Alpha Land (the land of pure imagination) the addition and multiplication tables for digits are:

| + | △ | ▽ | □ |
|---|---|---|---|
| △ | △ | ▽ | □ |
| ▽ | ▽ | □ | ▽△ |
| □ | □ | ▽△ | ▽▽ |

| • | △ | ▽ | □ |
|---|---|---|---|
| △ | △ | △ | △ |
| ▽ | △ | ▽ | □ |
| □ | △ | □ | ▽▽ |

Complete the following Alpha addition and multiplication:

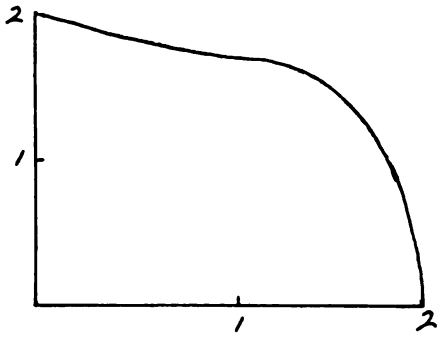
(a)

□ □ ▽<br>
• ▽ □<br>
▽ □ _ □<br>
□ _ ▽ △<br>
▽ _ ▽ _ □

(b)

▽ □ △<br>
□ □ ▽<br>
\+ ▽ △ □<br>
▽ _ □ _

7. What is the approximate area of the bounded region indicated below?




8. Four lines, extended indefinitely far in each of the lines two directions, are placed in a plane in any position you choose. How many regions <u>at most</u> will the plane be cut into?

## Quiz I

1. Find the sum of the series listed below:

   $1 + 2 + 3 + \ldots + 697 + 698 + 699 + 700 =$

2. Find the sum of the series listed below:

   $2 + 4 + 6 + \ldots + 694 + 696 + 698 + 700 =$

3. Part (a)

   What number is represented by $2^5$

   Part (b)

   What number is represented by $2^{10}$

   Part (c)

   What number is represented by $2^{20}$

4. If $n$ is termed a variable and if you are told that $n \in N+$ what do you know about $n$ ?

## Quiz II

1. Find the sum of the following series:

   $2 + 4 + 6 + \ldots + 496 + 498 + 500 =$

2. If a completely painted cube is cut 26 times through each dimension how many resulting cubes will be painted on exactly one side?

3. If a geometrical solid is convex and has 15 faces as well as 10 vertices, how many edges must it have?

4. If there are 370 grains of wheat in a cubic inch, how many grains are in a cubic yard?

Post Test

1. Find the sum:

   $1 + 2 + 3 + \ldots + 1{,}000{,}000 =$

2. Find the sum:

   $2 + 4 + 6 + \ldots + 1{,}000{,}000 =$

3. Given the system: $\triangle,\ \nabla,\ \square,\ \diamond,\ \bowtie,\ \nabla\triangle,\ \nabla\nabla,\ \nabla\square,\ \nabla\diamond,\ \nabla\bowtie, \ldots$

   fill in the blanks

   (a)
$$\begin{array}{r}
\square\ \square\ \bowtie \\
\cdot\ \square\ \diamond \\
\hline
\nabla\ \square\ _\ \square \\
\nabla\triangle\ _\ \diamond\ \triangle\ \ \\
\hline
\nabla\nabla\ _\ \nabla\ \square
\end{array}$$

   (b)
$$\begin{array}{r}
\square\ \square\ \bowtie \\
\diamond\ \square\ \nabla \\
+\ \nabla\ \square\ \bowtie \\
\hline
\nabla\ _\ \square\ _
\end{array}$$

4. If a cube is cut five times each way find:

   (a) How many small cubes result _____ ?

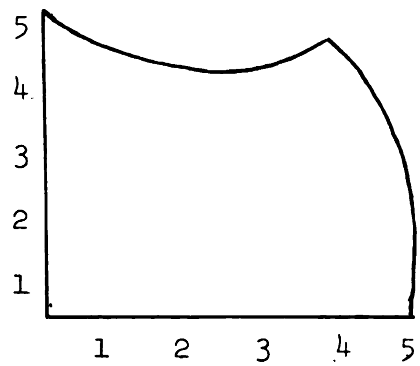
   (b) How many small cubes are painted on three sides _____ ?

   (c) How many small cubes are painted on no sides _____ ?

5. What is the approximate sum of?

   $2^1 + 2^2 + 2^3 + \ldots + 2^{10} =$

6. What is the approximate area of:




7. Fill in the blanks, assuming unique answers are intended: {Just guess what is wanted}.

$\vdash$, T, $\dashv$, $\perp$, T$\vdash$, ___, T$\dashv$, ___, $\dashv\vdash$, ___.

8. If a solid object is convex and has 12 vertices with 7 faces it will have _____ edges?

# APPENDIX C

# PROBLEMS AND DAY-BY-DAY ROUTINE

SECOND MEETING

Preliminary material introduced to and discussed with the student prior to his initial attempts at problem solving included the following:

SETS

$N+ = \{1, 2, 3, \ldots, n, \ldots\}$

VARIABLES

$n$ is a member (a number) of some set and is denoted by $n \in N+$ .

OPERATIONS

If $a \in N+$ and $b \in N+$ then $[a + b] \in N+$ .

If $a \in N+$ and $b \in N+$ then $[a \cdot b] \in N+$ .

SOME SHORTHAND NOTATION

$2 + 2 + 2 = 3 \cdot 2$

$2 + 2 + 2 + 2 = 4 \cdot 2$

$2 + 2 + 2 + \ldots + 2_n = n \cdot 2$

$2 \cdot 2 = 2^2$

$2 \cdot 2 \cdot 2 = 2^3$

$2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2_n = 2^n$

QUESTION

If $a \in N+$ and $n \in N+$ write down what you think $a^n$ means.

COUNTING

Counting means pairing sets 1-1 .

QUESTION

Count the even numbers in $N+$ .

If you have a set of numbers all of which fit the form $2 \cdot n$ and you know $n \in N+$ what do you know about the set of numbers?

THINK

Guess what a related set of numbers looks like in a form similar to the form of the set just discussed.

THIRD MEETING

Add the series:

$1 + 2 + 3 + 4 =$

PROBLEM 1

Try to find the sum of the following series without actually adding each term to the next:

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 =$

PROBLEM 2

If you found a way to add the last series try this one:

$1 + 2 + 3 + \ldots + 98 + 99 + 100 =$

PROBLEM 3

If you were successful on Problem 2 try this one:

$1 + 2 + 3 + \ldots + [n - 2] + [n - 1] + n =$

FOURTH MEETING

PROBLEM 4

| | |
|---|---|
| $1$ | $= 0 + 1$ |
| $2 + 3 + 4$ | $= 1 + 8$ |
| $5 + 6 + 7 + 8 + 9$ | $= 8 + 27$ |
| $10 + 11 + 12 + 13 + 14 + 15 + 16$ | $= 27 + 64$ |

If this process is continued what will the two numbers be on the right of the equal sign? Can you find a way to write the solution for any given line?

PROBLEM 5

$$1 + 2 + 3 + \ldots + 999{,}998 + 999{,}999 + 1{,}000{,}000 =$$

PROBLEM 6

$$2 + 4 + 6 + \ldots + 999{,}996 + 999{,}998 + 1{,}000{,}000 =$$

FIFTH MEETING

PROBLEM 7

You are to assume that the cube shown on the following page has been immersed in paint and is completely covered. If the cube is then cut through each plane as indicated by the dashed lines the question is: (In each case how many of the resulting small cubes will be painted on how many faces?)

(a) Given n cuts there are ___ cubes with 6 painted faces?

(b) Given n cuts there are ___ cubes with 5 painted faces?

(c) Given n cuts there are ___ cubes with 4 painted faces?

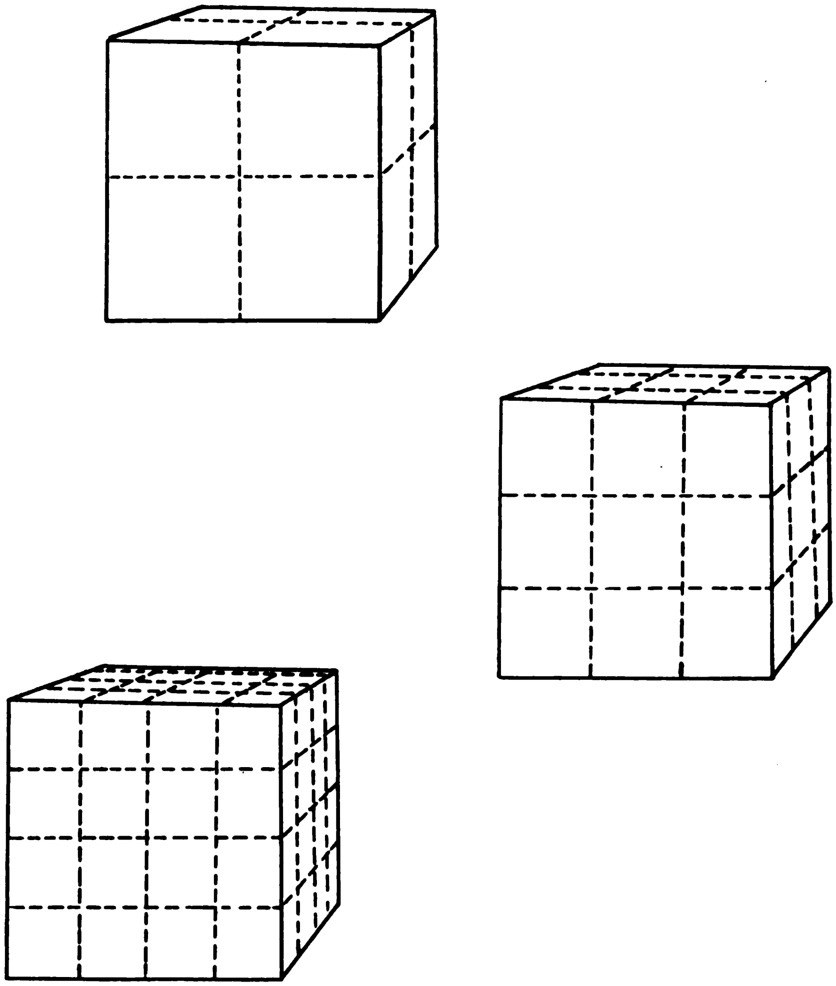
(d) Given n cuts there are ___ cubes with 3 painted faces?

(e) Given n cuts there are ___ cubes with 2 painted faces?

(f) Given n cuts there are ___ cubes with 1 painted face?

(g) Given n cuts there are ___ cubes with 0 painted faces?

(h) Given n cuts there are ___ cubes?



PROBLEM 8

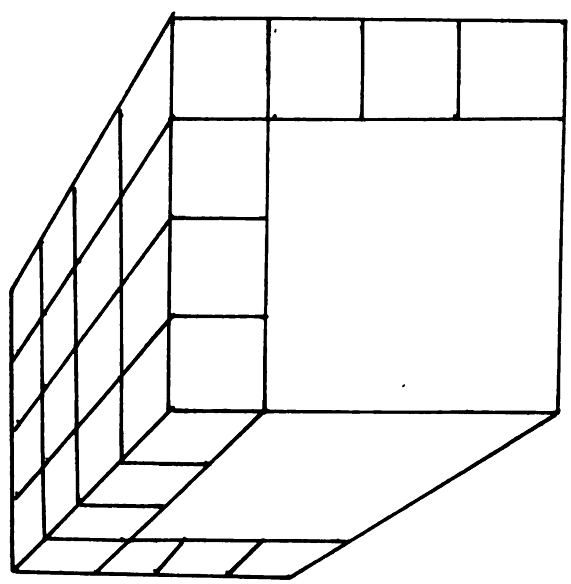
Once upon a time many centuries ago a mathematician in Egypt invented a game played on a board with sixty-four squares. There were eight squares along each edge. The king was so pleased with the game he offered the mathematician any reward he might wish. The mathematician asked to have one grain of wheat placed on the first square, two on the second, four on the third and thus double the quantity on each successive square of the board. The king thought this request trivial and ordered it be carried out. Long before the job was completed the king learned the error he had made and promptly had the mathematician beheaded. It would have taken Egypt thousands of years to have grown all the necessary wheat. If there are 370 grains of wheat in a cubic inch how large would the bin have to have been to hold the wheat required to satisfy the mathematician's request?

If it turns out to be important $2^{65}$ is approximately 37,000,000,000,000,000,000 . Is this number of any use?

PROBLEM 9

A cube may be dissected into 8 subcubes with ease. May a cube be dissected into 9 subcubes? May it be cut into 10, 11, ... ? Consider only those counting numbers less than 54 and identify the number of cubes into which a given cube may be dissected.

Note the example--how many subcubes are there?



SIXTH MEETING

PROBLEM 10

Part (a)

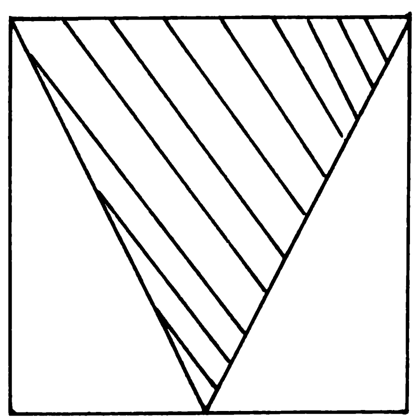
A ball is dropped from an original height of 100 feet. Each time it hits the ground it bounces back 50% of or one half the previous fall. How far does the ball fall?

Part (b)

How far does the ball travel ?

PROBLEM 11

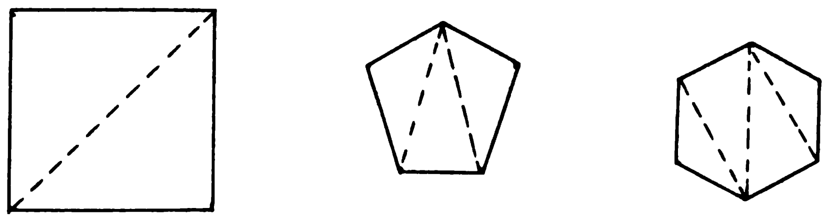
Explain why the shaded area constitutes one half the total area indicated within the bounds of the region below.



## SEVENTH MEETING

### PROBLEM 12

Given an n-gon ; how many 'non-intersecting' lines cut it into how many triangles?



| SIDES | LINES | TRIANGLES |
|---|---|---|
| 4 | 1 | 2 |
| 5 | 2 | 3 |
| 6 | 3 | 4 |
| . | | |
| . | | |
| . | | |
| n | ? | ? |

## EIGHTH MEETING

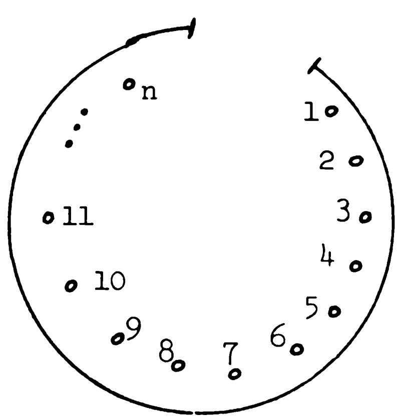
### PROBLEM 13

Agent 007 has infiltrated the Red Chinese department of state as a spy for British intelligence. Mao has decided it is time for a purge of his department of state.

Mao will handle the purge in the following way. He will have all members of the department of state brought into a room. There will be n of them and they will be told to sit in a circle. Then Mao's executioner called chopper will come through and eliminate every other one around the circle until only one remains. Mao wishes to preserve the brightest member and assumes this will determine the brightest.

Suppose you are 007 and you would know what the number n was to be just ten minutes before you entered the room. Where should you sit in order to be the lone survivor?

Try some samples if you wish. The following sketch may be helpful.



PROBLEM 14

Complete the following patterns in at least five different ways and find reasons for your solutions.

(a) 2, 4, 6, 8, ___, ___, ___, ___ .

(b) ⊖, ⦶, ⊘, ⦸, ___, ___, ___, ___ .

(c) $T$, $T$, $\vdash$, $\vdash$, ___, ___, ___, ___ .

The following patterns are so structured that if you read my mind you will find a unique answer for each blank. Read my mind and fill in the blanks.

This suggestion should not surprise you. Much of mathematics is one form of mind reading or another.

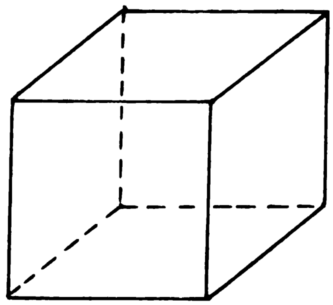
(d) $\triangle$, $\triangledown$, $\square$, $\triangledown\triangle$, $\triangledown\triangledown$, $\triangledown\square$, ___, ___, ___ .

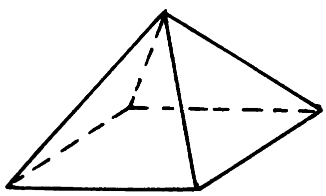
(e) $\perp$, $T$, $\dashv$, $\vdash$, $T\perp$, $TT$, ___, ___, ___ .

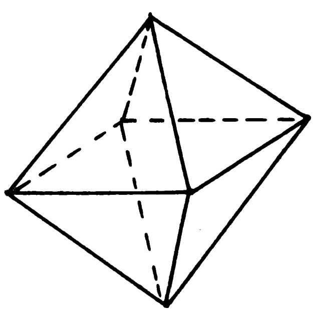
(f) $*$, $\sim$, $\sim *$, $\sim\sim$, $\sim **$, ___, $\sim\sim *$, ___, ___ .

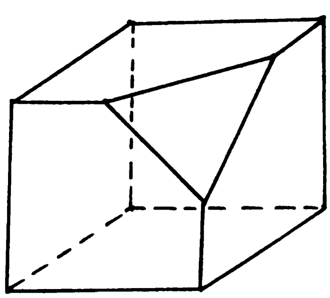
NINTH MEETING

PROBLEM 15









| | FACES | VERTICES | EDGES |
|---|---|---|---|
| CUBE | 6 | 8 | 12 |
| SQUARE PYRAMID | 5 | 5 | 8 |
| OCTAHEDRON | 8 | 6 | 12 |
| TRUNCATED CUBE | 7 | 10 | 15 |

Consider the various convex solids above. Can you find a relationship among the faces, vertices and edges?

PROBLEM 16

$$2 + 4 + 6 + \ldots + [2\cdot n - 2] + 2\cdot n =$$

PROBLEM 17

$$2 + 4 + 6 + \ldots + 2(n - 1) + 2\cdot n =$$

PROBLEM 18

Part (a)

$$2^2 + 4^2 + 6^2 + \ldots + [2(n - 1)]^2 + [2\cdot n]^2 =$$

Part (b)

$$1^2 + 3^2 + 5^2 + \ldots + (2\cdot n - 1)^2 =$$

Part (c)

$$1 + 4 + 7 + \ldots + (3\cdot n - 2) =$$

TENTH MEETING

PROBLEM 19

Part (a)

Show how $\frac{2\cdot n}{n + 1}$ and $2\cdot n - 2\cdot n^2 + 2\cdot n^3 - 2\cdot n^4 \ldots$ are related and show the form of the set builder.

Part (b)

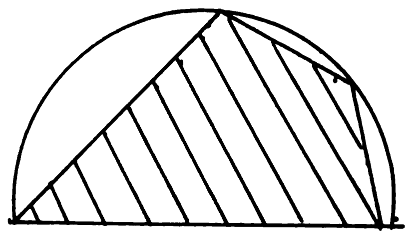
Show what $\frac{8\cdot n}{4\cdot n - 1}$ has for an equivalent form as a series and show the set builder.

Part (c)

Show what $\frac{2\cdot n\ [r]}{n\cdot r \pm 1}$ has for an equivalent form as a series according to n odd or even and show the set builder.

PROBLEM 20

So construct the four sided figure in the half circle below that one side is the diameter and that the area of the figure is largest.



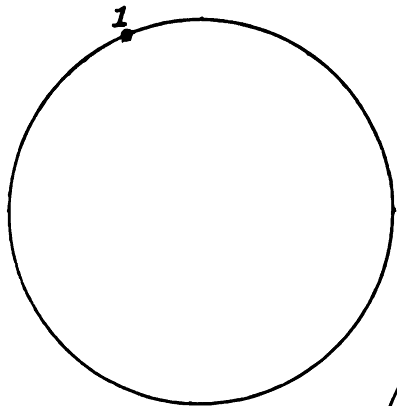
Suppose instead of four sides as above you are asked to fill in the following table:

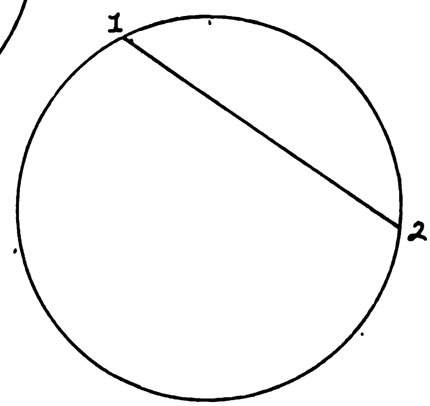
| SIDES | | BEST PROCEDURE |
|---|---|---|
| 2 | | 2 sides equal |
| 3 | (note--this is the above case) | ? |
| 4 | | ? |
| . | | |
| . | | |
| . | | |
| n | | ? |

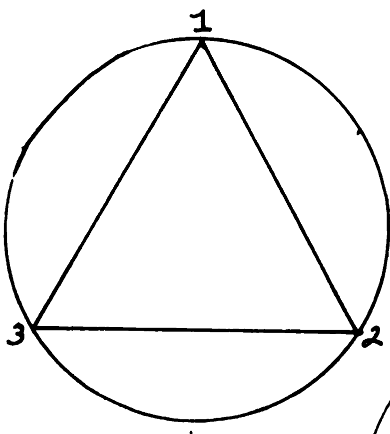
ELEVENTH MEETING

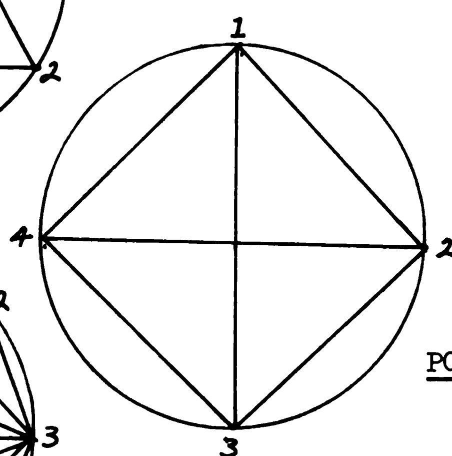
PROBLEM 21

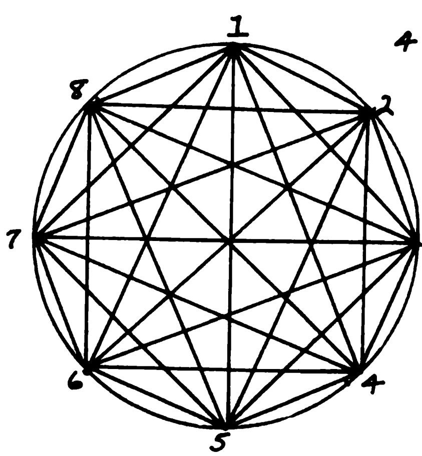
In the following problem count each and every line in each circle exactly once and fill in the accompanying table where indicated:



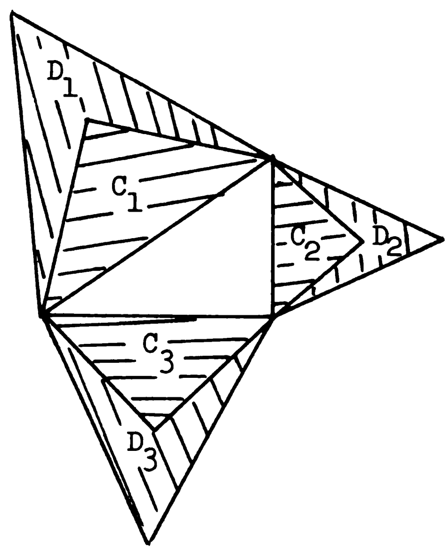

| POINTS | LINES |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 3 | 3 |
| . | |
| . | |
| . | |
| 8 | 28 |
| . | |
| . | |
| . | |
| n | ? |

## THIRTEENTH MEETING

### PROBLEM 23

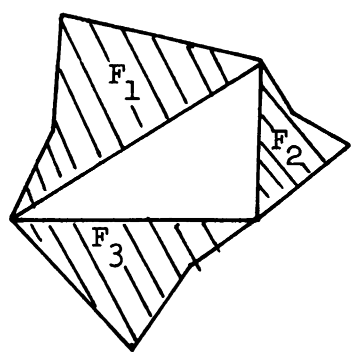
In the figure below determine if:

$C_1 = C_2 + C_3$ and $D_1 = D_2 + D_3$ .




### PROBLEM 24

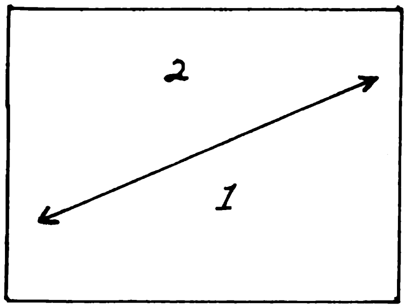
In the figure below determine if $F_1 = F_2 + F_3$ .




Can you make a conjecture you believe at this point? Think about it and draw some pictures.

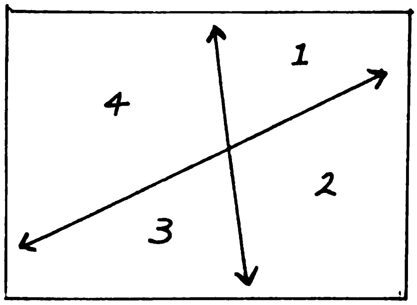
FOURTEENTH MEETING

PROBLEM 25



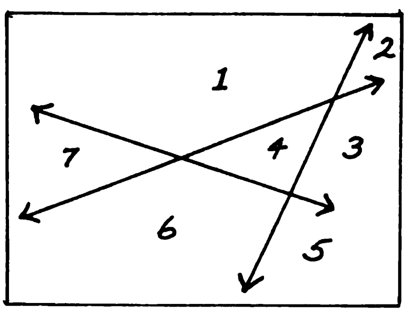

Plane

| LINES | GREATEST NUMBER OF REGIONS |
|---|---|
| 1 | 2 |




Plane

| LINES | GREATEST NUMBER OF REGIONS |
|---|---|
| 2 | 4 |




Plane

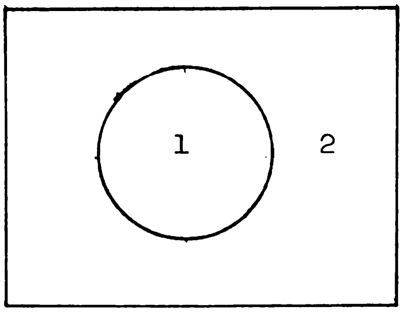
| LINES | GREATEST NUMBER OF REGIONS |
|---|---|
| 3 | 7 |

Suppose you had n lines. How many regions could you form at most?

FIFTEENTH MEETING

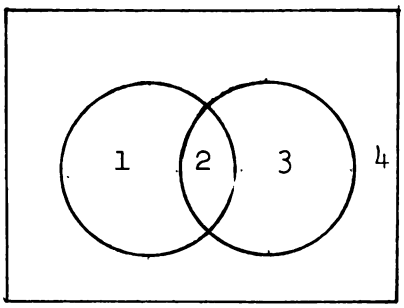
PROBLEM 26

Plane



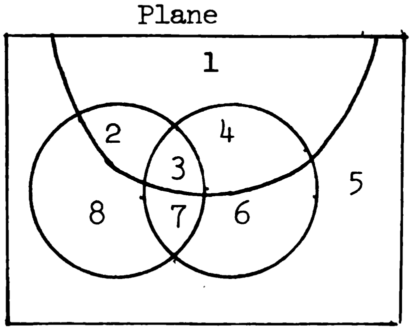

| CIRCLES | GREATEST NUMBER OF REGIONS |
|---|---|
| 1 | 2 |

Plane




| CIRCLES | GREATEST NUMBER OF REGIONS |
|---|---|
| 2 | 4 |

Plane



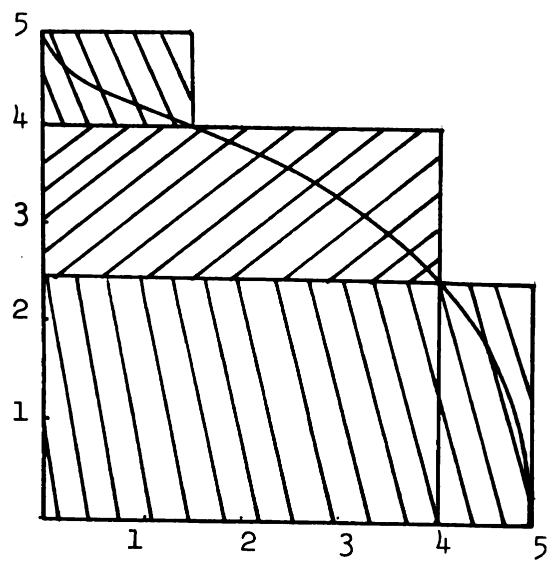

| CIRCLES | GREATEST NUMBER OF REGIONS |
|---|---|
| 3 | 8 |

If n circles were placed in a plane how many regions could be formed at most?

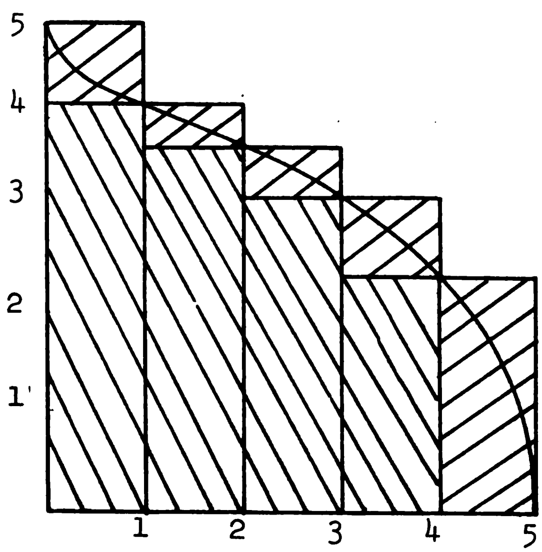
FIFTEENTH MEETING

PROBLEM 27

If you consider the region bounded by the two straight lines and the curve how might you determine its area?

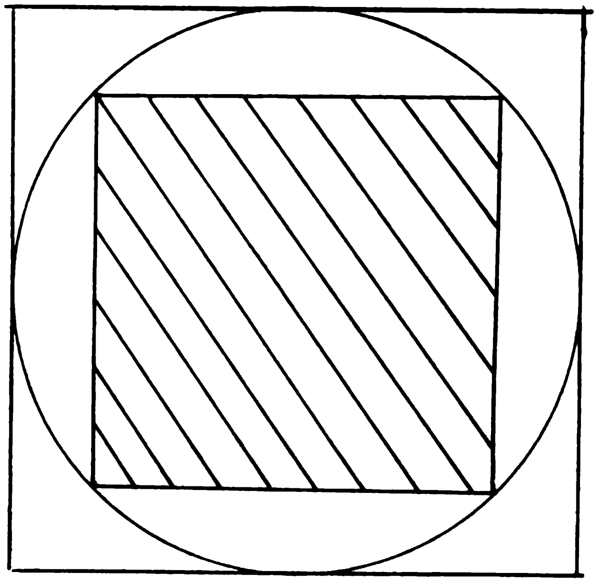



Would this give you a better way of determining the region area? How could you do a better job?

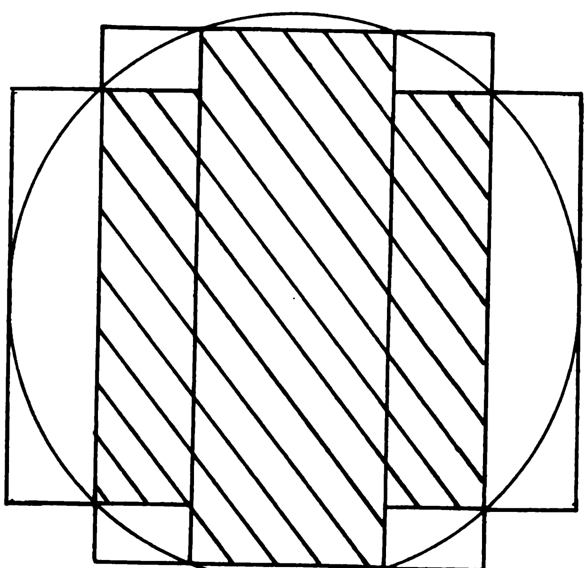


PROBLEM 28

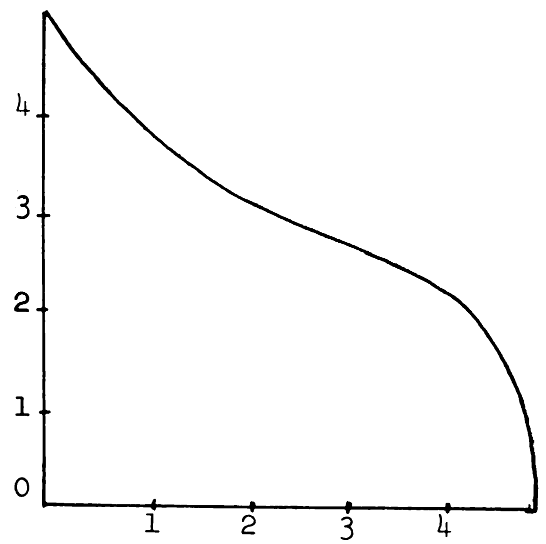
Suppose you wanted to determine the area of the circle below and did not know anything about $\pi$ or did not recall the formula. What could you do?



Would this give a better result? Can you extend this idea?



Make the best estimation of the area of the bounded region below.



# APPENDIX D

## SOLUTION INDEX TO ALL MATHEMATICAL MATERIAL

SOLUTIONS FOR PRETEST:

1. 125,250

2. Acceptable solutions: $\approx$ 2,048 with the exact answer being 2,068 .

3. One quintillion.

4. $\{1, 3, 5, 7, \ldots (2n - 1), \ldots\}$ or $\{1, 2, 3, \ldots n, \ldots\}$
   infinitely many solutions exist.

5. A square of 16' on a side.

6. a) line 1, ▽
      line 2, □
      line 3, ▽, ▽

   b) □, △ .

7. Acceptable solutions: $2 < A < 4$ .

8. 11

SOLUTIONS FOR POST TEST:

1. 500,000,500,000

2. 250,000,500,000

3. a) line 1, ◇
      line 2, △
      line 3, ◇

   b) line 1, □, ⋈.

4. a) 125
   b) 8
   c) $4^3$

5. $\approx 2^{11}$

6. Acceptable solutions: $10 < A < 20$ .

7. $\top\,\top$, $\top\,\perp$, $\dashv\,\top$.

8. 17 .

**SOLUTIONS FOR Quiz I**

1. 245,350
2. 122,850
3. a) 32.
   b) 1024.
   c) 1,048,576.
4. $n$ must be a counting number.

**SOLUTIONS FOR Quiz II**

1. 62,750
2. Acceptable solutions: $6 \cdot 25^2$; $6 \cdot 625$; 3,750 .
3. 23
4. Acceptable answers may vary 1% either way from 17,262,720 or $\pm$ 172,627 .

**SOLUTIONS FOR INDUCTION MATERIAL:**

1. 55
2. 5050
3. $\frac{n[n + 1]}{2}$
4. The $n^{th}$ row has solution: $(n - 1)^3 + n^3$ .
5. 500,000,500,000
6. 150,000,500,000
7. (a) 0 ; (b) 0 ; (c) 0 ; (d) 8 ; (e) $12(n - 1)$ ; (f) $6(n - 1)^2$ ; (g) $(n - 1)^3$ ; (h) $(n + 1)^3$ .
8. 6.65 miles on a side.
9. 1-8-15-20-22-27-29-34-36-38-39-41-43-45-46-48-49-50-51-52-53.

10. a) 200 feet

    b) 300 feet

11. Split the shaded triangle forming two rectangles.

12. n sides, n-3 lines, n-2 triangles.

13. The students were expected to gain solution by forming some classification scheme such as the following finite matrix:

Chair to be selected

| n | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 |
|---|---|---|---|---|---|---|---|---|
| 3 | | * | | | | | | |
| 4 | * | | | | | | | |
| 5 | | * | | | | | | |
| 6 | | | * | | | | | |
| 7 | | | | * | | | | |
| 8 | * | | | | | | | |
| 9 | | * | | | | | | |
| 10 | | | * | | | | | |
| 11 | | | | * | | | | |
| 12 | | | | | * | | | |
| 13 | | | | | | * | | |
| 14 | | | | | | | * | |
| 15 | | | | | | | | * |
| 16 | * | | | | | | | |
| 17 | | * | | | | | | |

Where the pattern now emerges for testing. The pattern to be tested may be examined in the following way:

(1) Only odd numbers occur and the pattern of the solution set fits an exponential sequence.

(2) For 3 people the solution is a one digit sequence or simply 3 .

(3) For n people where $3 < n < 8$ a 4 digit sequence results thus the solutions are 1, 3, 5, 7 , depending on n . This implies if $n = 4$ pick chair 1 ;

if $n = 5$ pick chair $3$ ; if $n = 6$ select $5$ ; if $n = 7$ select $7$ .

(4) For $7 < n < 16$ an $8$ digit sequence results thus the solutions are 1, 3, 5, 7, 9, 11, 13, and 15 . This implies if $n = 8$ select $1$ , and the pattern of selection follows as in item (3).

(5) For $15 < n < 32$ a $16$ digit sequence results thus the solutions are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, and 31 .

(6) Thus the pattern suggested in more formal language is: For any number of people $n$ where $a < n < 2(a + 1)$ implies a sequence of length $(a + 1)$ . Select the chair by counting the odd positive integers where $1$ pairs with $(a + 1)$ .

14. (a) There are an infinity of solutions. For example:

8, 6, 4, 2 . (No rule for formation is given.)

2, 4, 6, 8 . (No rule for formation is given.)

$e^i, e^i, e^i, e^i$ . (No rule for formation is given.)

The other cases follow similarly.

15. Faces + vertices = Edges - 2 .

16. $n(n + 1)$

17. $n(n + 1)$

18. (a) $\frac{2n(n + 1)(2n + 1)}{3}$

(b) $\frac{n(2n + 1)(2n - 1)}{3}$

(c) $\frac{n(3n - 1)}{2}$

19. They are related in that the series is the quotient of the given fraction. Cases b and c follow similarly.

20. The quadralateral must have all sides exclusive of the diameter equal.

21.

| | | |
|---|---|---|
| 1 point | 0 | lines |
| 2 points | 0 + 1 | lines |
| 3 points | 0 + 1 + 2 | lines |
| 4 points | 0 + 1 + 2 + 3 | lines |
| 5 points | 0 + 1 + 2 + 3 + 4 | lines |

| | | |
|---|---|---|
| 1 point | 0 | lines |
| 2 points | 0 + 1 | lines |
| 3 points | 0 + 1 + 2 | lines |
| 4 points | 0 + 1 + 2 + 3 | lines |
| . | | |
| . | | |
| . | | |
| n points | 0 + 1 + 2 + 3 + ... + (n - 1) | lines |

which equal $\frac{n(n-1)}{2}$ .

22. Yes

23. The conditions are valid.

24. The conditions are valid. A conjecture is that a valid given algebraic expression remains valid when multiplied by a constant.

25. $\frac{n^2 + n + 2}{2}$

26. $n^2 - n + 2$

27. The method is by approximation.

# APPENDIX E

## SUBJECTS SCORES SCALE-BY-SCALE

| Ss | x(1) | x(2) | x(3) | x(4) | x(5) | x(6) | x(7) | x(8) |
|---|---|---|---|---|---|---|---|---|
| A | 19.10 | 18 | 27 | 13 | 20 | 4 | 13 | 31 |
| B | 00. | 13 | 27 | 14 | 24 | 8 | 5 | 35 |
| C | 7.5 | 18 | 22 | 14 | 18 | 10 | 14 | 18 |
| D | 10. | 27 | 21 | 14 | 23 | 7 | 15 | 11 |
| E | 1. | 15 | 18 | 12 | 18 | 13 | 18 | 18 |
| F | 4. | 14 | 23 | 16 | 15 | 7 | 11 | 25 |
| G | 10. | 27 | 24 | 19 | 24 | 12 | 20 | 32 |
| H | 3.75 | 18 | 9 | 9 | 16 | 12 | 12 | 26 |
| I | 2. | 11 | 22 | 13 | 20 | 8 | 14 | 18 |
| J | 2.25 | 14 | 22 | 10 | 19 | 6 | 14 | 24 |
| K | 4. | 18 | 27 | 16 | 21 | 10 | 9 | 19 |
| L | 00. | 29 | 20 | 15 | 23 | 8 | 16 | 32 |
| M | 7. | 13 | 16 | 9 | 21 | 10 | 4 | 28 |
| N | 23.5 | 21 | 25 | 12 | 24 | 11 | 25 | 28 |
| O | 17. | 19 | 19 | 14 | 20 | 6 | 22 | 14 |
| P | 9.5 | 29 | 24 | 16 | 17 | 8 | 21 | 22 |
| Q | 11.5 | 29 | 19 | 10 | 22 | 10 | 18 | 21 |
| R | 00.5 | 19 | 17 | 13 | 20 | 10 | 7 | 14 |
| S | 2.6 | 17 | 28 | 14 | 20 | 7 | 24 | 31 |
| T | 8. | 16 | 21 | 15 | 20 | 12 | 6 | 26 |
| U | 2. | 16 | 24 | 16 | 24 | 10 | 15 | 27 |
| V | 00.5 | 11 | 21 | 13 | 16 | 10 | 10 | 18 |
| W | 1. | 14 | 24 | 15 | 12 | 10 | 12 | 28 |
| X | 10.5 | 20 | 27 | 15 | 24 | 10 | 12 | 19 |

| Ss | x(1) | x(2) | x(3) | x(4) | x(5) | x(6) | x(7) | x(8) |
|---|---|---|---|---|---|---|---|---|
| Y | 1.7 | 29 | 22 | 15 | 24 | 6 | 7 | 23 |
| Z | 5. | 16 | 26 | 10 | 19 | 8 | 10 | 25 |
| $A_1$ | 1. | 16 | 11 | 11 | 17 | 11 | 11 | 29 |
| $B_1$ | 1. | 13 | 12 | 7 | 20 | 5 | 18 | 28 |
| $C_1$ | 00.5 | 19 | 23 | 16 | 20 | 5 | 17 | 33 |
| $D_1$ | 1.75 | 12 | 24 | 12 | 14 | 9 | 6 | 22 |
| $E_1$ | 5. | 18 | 33 | 13 | 21 | 6 | 13 | 24 |
| $F_1$ | 00. | 14 | 23 | 14 | 18 | 5 | 4 | 17 |
| $G_1$ | 9.5 | 20 | 19 | 14 | 22 | 5 | 13 | 26 |
| $H_1$ | 3. | 10 | 9 | 11 | 13 | 11 | 19 | 20 |
| $I_1$ | 6.5 | 13 | 12 | 9 | 15 | 6 | 13 | 32 |
| $J_1$ | 1. | 20 | 9 | 15 | 16 | 13 | 5 | 33 |
| $K_1$ | 9. | 30 | 24 | 18 | 22 | 11 | 26 | 25 |
| $L_1$ | 1. | 10 | 24 | 17 | 22 | 6 | 21 | 33 |
| $M_1$ | 5. | 18 | 19 | 12 | 17 | 9 | 5 | 27 |
| $N_1$ | 2.4 | 17 | 17 | 15 | 20 | 6 | 7 | 21 |

# BIBLIOGRAPHY

## BOOKS

Bell, E. T. Men of Mathematics. New York: Simon and Schuster, 1937.

Berlyne, D. E. Soviet Research on Intellectual Processes in Children. Society for Research in Child Development: Monographs, Vol. XXVIII, No. 2. Chicago: University of Chicago Press, 1963.

Bloom, Benjamin S. (ed.). Taxonomy of Educational Objectives. Handbook I: Cognitive Domain. New York: David McKay Company, Inc., 1956.

Bruner, J. S. The Process of Education. Cambridge: Harvard University Press, 1960.

Bruner, J. S., and Rose R. Olver. Development of Equivalence Transformations in Children. Society for Research in Child Development: Monographs, Vol. XXVIII, No. 2. Chicago: University of Chicago Press, 1963.

Bruner, J. S. and others. A Study of Thinking. New York: John Wiley and Sons, Inc., 1965.

Carroll, J. B. Language and Thought. Englewood Cliffs: Prentice Hall, 1964.

Chomsky, Noam. Syntactic Structures. The Hague: Mouton and Company, 1965.

Dienes, Z. P. An Experimental Study of Mathematics Learning. London: Leicester University Press, 1963.

Duncker, Karl. On Problem-Solving. Trans. Lynne S. Lees. Psychological Monographs, Vol. LVIII, No. 5. Washington, D. C.: The American Psychological Association, Inc., 1945.

Flavell, John H. The Developmental Psychology of Jean Piaget. New York: D. Van Nostrand Company, 1963.

Gage, N. L. Handbook of Research on Teaching. Chicago: Rand McNally and Company, 1965.

Gagne, Robert M. The Conditions of Learning. New York: Holt, Rinehart and Winston, 1964.

Hadamard, Jacques. The Psychology of Invention in the Mathematical Field. Princeton: Princeton University Press, 1945.

Hunt, J. McV. Intelligence and Experience. New York: The Ronald Press, 1961.

Johnson, Donald. The Psychology of Thought and Judgment. New York: Harper and Brothers, 1955.

Kaplan, A. The Conduct of Inquiry. San Francisco: Chandler Publishing Co., 1964.

Kennedy, Wallace A. A Multidimensional Study of Mathematically Gifted Adolescents. Society for Research in Child Development: Monographs, Vol. XXV, No. 31. Chicago: University of Chicago Press, 1960.

Lovell, K. The Growth of Basic Mathematical and Scientific Concepts in Children. London: University of London Press, 1964.

Mendelson, Elliott. Introduction to Mathematical Logic. New York: D. Van Nostrand Company, 1964.

Miller, G. A., and others. Plans and The Structure of Behavior. New York: Henry Holt and Company, 1960.

Morrisett, L. N., and J. Vinsonhaler (eds.). Mathematical Learning. Society for Research in Child Development: Monographs, Vol. XXX, No. 1. Chicago: University of Chicago Press, 1965.

Murphy, Gardner. Personality, A Biosocial Approach to Origins and Structure. New York: Harper and Brothers, 1947.

Piaget, Jean. Psychology of Intelligence. Paterson: Littlefield, Adams and Company, 1960.

________. The Language and Thought of The Child. New York: Meridian Books, 1962.

________. The Child's Conception of Geometry. New York: Harper and Row Publishers, 1960.

Piaget, Jean, and Barbel Inhelder. The Growth of Logical Thinking From Childhood to Adolescence. New York: Basic Books, 1958.

Polya, G. Patterns of Plausible Inference. Princeton: Princeton University Press, 1954.

________. Induction and Analogy in Mathematics. Princeton: Princeton University Press, 1954.

Sexton, P. C. Education and Income. New York: The Viking Press, 1961.

School Mathematics Study Group. Mathematics for Junior High School. Vol. I, Parts I and II. New Haven: Yale University Press, 1961.

Spiker, Charles C. Verbal Factors in the Discrimination Learning of Children. Society for Research in Child Development: Monographs, Vol. XXVIII, No. 2. Chicago: University of Chicago Press, 1963.

University of Illinois Committee on School Mathematics. High School Mathematics. Revised edition. Urbana: University of Illinois Press, 1962.

Vygotsky, L. S. Thought and Language. Cambridge: MIT Press, 1962.

Websters Third New International Dictionary. Chicago: G and C Merriam Co., 1966.

Wertheimer, Max. Productive Thinking. New York: Harper and Row, 1959.

## ARTICLES AND PERIODICALS

Aiken, Lewis R. "Personality Correlates of Attitude Toward Mathematics," Journal of Educational Research, LVI (1963), 476-480.

Callicutt, Wade. "Problems of Predicting Success in Algebra," National Association of Secondary School Principals Bulletin, XLV (November, 1961), 107-111.

Cattell, R. B. "Personality Traits Associated with Abilities-II With Verbal and Mathematical Abilities," Journal of Educational Psychology, XXXVI (1945), 475-486.

Crandall, V. J. and A. Robson. "Children's Repetition Choices in an Intellectual Achievement Situation Following Success and Failure," Review of Child Development Research, Martin L. and Lois Hoffman, editors. New York: Russell Sage Foundation, 1964, p. 162.

Cronback, Lee J. "Learning Research and Curriculum Development," Journal of Research in Science Teaching, II, (1964), 204-7.

Dessart, Donald J. "Mathematics in The Secondary School," Review of Educational Research, XXXIV (1964), 298-312.

Duckworth, Eleanor. "Piaget Rediscovered," Journal of Research in Science Teaching, II, (1964), 172-5.

Engel, Mary. "Thesis--Antithesis," American Psychologist, XXI (August, 1966), 786.

Gibson, J. J. and E. F. Gibson. "Perceptual Learning," The Psychological Review, LXII (January, 1955), 40.

Henderson, Kenneth B. "Research on Teaching Secondary School Mathematics," Handbook of Research on Teaching, N. L. Gage, editor. Rand McNally and Company, 1965, pp. 1007-1030.

Hilgard, E. R. "Creativity and Problem-Solving," Creativity and Its Cultivation, Harold H. Anderson, editor. New York: Harper and Row Publishers, 1959, pp. 162-180.

Hills, John R. "Can We Find The Latent Scientist?" School and Society, LXXXV (1957), 30608.

Kochnower, W. "Personality Factors and Success in Mathematics," High Points, XLIII (April, 1961), 65-72.

Lambert, P. "Mathematical Ability and Masculinity," Arithmetic Teacher, VII (January, 1960), 19-21.

Lee, D. M. "A Study of Specific Ability and Attainment in Mathematics," British Journal of Educational Psychology, XXV (November, 1955), 178-89.

McDavid, J. W. "Imitative Behavior in Preschool Children," Review of Child Development Research, Martin L. and Lois W. Hoffman, editors. New York: Russell Sage Foundation, 1964, p. 157.

Milton, G. A. "Five Studies of the Relation Between Sex Role Identification and Achievement in Problem Solving," Review of Child Development Research, Martin L. and Lois W. Hoffman, editors. New York: Russell Sage Foundation, 1964, p. 157.

Mink, Oscar G. "Experience and Cognitive Structure," Journal of Research in Science Teaching, II (1964), 196-203.

Musatti, C. L. "Les Caracteres Perceptifs des Objects et la Theory Mathematique des Groupes," Acta Psychology, XIV (1958), 41-53.

Plank, Emma and Robert. "Emotional Components on Arithmetic Learning as Seen Through Autobiography," The Psychoanalytic Study of the Child, IX (1954), 273-293.

Poffenberger, Thomas and others. "Factors in the Formation of Attitudes Toward Mathematics," Journal of Educational Research, LII (1959), 171-76.

Riffenburgh, R. H. "Personality Traits Associated with Mathematical Ability: A Plea for Research," Journal of Experimental Education, XXIX (December, 1960), 189-92.

Schunert, Jim. "The Association of Mathematical Achievement with Certain Factors Resident in the Teacher, in the Teaching, in the Pupil and in the School," Journal of Experimental Education, XIX (1957), 237.

Sears, P. S. and E. R. Hilgard. "The Teacher's Role in The Motivation of the Learner," Theories of Learning and Instruction, E. R. Hilgard, editor. Chicago: National Society for the Study of Education, 1964, pp. 182-209.

Shulman, L. "Improving Mathematics Education for Elementary Teachers," W. R. Houston, editor. Michigan State University, sponsored by The Science and Mathematics Teaching Center, p. 26.

Smedslund, Jan. "Internal Necessity and Contradiction in Children's Thinking," Journal of Research in Science Teaching, II (1964), 220-221.

Swift, Joan W. "Effects of Early Group Experience: The Nursery School and Day Nursery," Review of Child Development Research, Martin L. and Lois W. Hoffman, editors. New York: Russell Sage Foundation, 1964, pp. 210-228.

Tyler, Fred T. "Issues Related to Readiness To Learn," Theories of Learning and Instruction, E. R. Hilgard, editor. Chicago: National Society for the Study of Education, 1964, pp. 210-239.

Whitkin, H. A. "The Nature and Importance of Individual Differences in Perception," The Journal of Personality, XVIII (December, 1949), 162.

UNPUBLISHED MATERIALS

Hildebrand, F. H. "Some Cognitive Components Involved in Mathematical Induction," Unpublished Paper in Education 982, Michigan State University, 1966. (Mimeographed.)

Knight, Sarah. "Curiosity." Unpublished Paper in Education 982, Michigan State University, 1966. (Mimeographed.)

Rosenfield, I. J. "Mathematical Ability As a Function of Perceptual Field-Dependency and Certain Personality Variables." Unpublished Ph.D. dissertation, University of Oklahoma, 1958.

Sweeney, Edward J. "Sex Differences in Problem Solving." Unpublished Ph.D. dissertation, Department of Psychology, Stanford University, 1965.

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