POWER OUTPUT AND EFFICIENCY OF PULSE DURATION MODULATED AMPLIFIERS

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William C. Holm

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William C. Holm

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Major professor

Date August 18, 1961

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ABSTRACT

POWER OUTPUT AND EFFICIENCY OF PULSE DURATION MODULATED AMPLIFIERS

by William C. Holm

The output power capabilities and efficiency of conventional linear untuned amplifiers of the vacuum tube or transistor type are a function of, among other things, the waveform and magnitude of the signal being amplified. A square wave input signal results in highest efficiency and greatest possible output power for a given dissipation rating of the driven device. By modulating the pulse duration of a constant amplitude square wave and using the waveform to drive output tubes or transistors, it is possible to attain high efficiency and high output power regardless of the waveshape of the modulating signal. To recover the modulation signal, a low-pass filter is usually necessary at the output of the driven device. The filter efficiency must be considered in the output efficiency. With certain types of filters, pulse duration modulation (PDM) amplifiers offer no improvement in efficiency over conventional amplifiers. Constant-resistance filters fall in this category; however, distortion is very low, theoretically being independent of the characteristics of the driven device. With the use of a constant-resistance filter, power output capabilities of the PDM amplifier would be higher than for the conventional amplifier because losses in the filter do not contribute to dissipation within the driven tubes or transistors. It is often desirable to obtain high output power for a given dissipation, with efficiency being considered of minor importance.

If high efficiency is desirable, PDM amplifiers with L-C low-pass filters may be used. High efficiency as well as high output power may be obtained by using L-C filters, though usually at some increase in distortion.

This thesis analyzes the performance of PDM amplifiers with constant-resistance low-pass filters. Power output, efficiency, and dissipation equations are derived for various PDM amplifier configurations. These equations are experimentally verified for each type of amplifier. Performance equations are derived for PDM amplifiers with one type of L-C filter, and tests are made which substantiate the contention that high efficiency as well as high output power may be obtained with PDM amplifiers which use L-C filters.

POWER OUTPUT AND EFFICIENCY OF PULSE DURATION MODULATED AMPLIFIERS

By

William C. Holm

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Filter o

o

Maxim

PDM

I.R.

I.R.$_B$

D.R.

D.R.$_I$

α

k

M

DEFINITIONS AND SYMBOLS

Output efficiency: The ratio of a-c signal power delivered to the load over power supply power furnished to the plate or collector circuit. When specified, output efficiency is the ratio of a-c signal power delivered to the load over total input power to the output stages, including base driving power for transistors.

Filter efficiency: The ratio of a-c signal power delivered to the load over total input power to the filter.

Maximum P: Greatest average power.

PDM — Pulse duration modulation.

I. R. — Improvement Ratio. The ratio of maximum power output of a specific class PDM amplifier to the maximum power output of the corresponding conventional amplifier, with neither amplifier exceeding rated tube dissipation at this maximum signal level or any lower signal level.

$I.R._B$ — Improvement Ratio relative to Class B PDM amplifier.

D. R. — Dissipation Ratio. The ratio of tube dissipation of a PDM amplifier to tube dissipation of a conventional amplifier when both amplifiers are operating with the same circuit parameters (same E_{bb}, E_{min}, R, etc.) and delivering the same power to the load.

$D.R._B$ — Dissipation Ratio relative to Class B PDM amplifier.

α — Duty Ratio. The ratio of on-time to the time of one period, the period being that of the repetition frequency. This may be a time function variable due to modulation.

k — Duty ratio with no modulation.

M — A real positive number ($0 \leq M \leq 1$) giving the fraction of maximum permissible amplitude of an a-c signal for a conventional amplifier. With a PDM amplifier M is the modulation factor and represents the fraction of maximum permissible pulse duration swing.

η_{AR}

η_A

η_B

η_t

η_{lR}

η_l

$\eta_{\ell AR}$

$\eta_{\ell A}$

$\eta_{\ell B}$

$\eta_{\ell lR}$

E_s

E_{co}

E_m'

t_m

D_n

E_{bb}

E_{min}

η_{AR} Output efficiency of a Class A series fed amplifier.

η_A Output efficiency of a Class A shunt-fed amplifier.

η_B Output efficiency of a Class B push-pull amplifier.

η_t Output efficiency of driven device (tube or transistor) exclusive of efficiency of filters or coupling networks between device and load. This applies only to PDM amplifiers.

η_{lR} Output efficiency of Class A series-fed amplifier with L-C filter.

η_l Output efficiency of Class A shunt-fed amplifier with L-C filter.

η_{fAR} Filter efficiency for constant-resistance filter with Class A series-fed PDM amplifier.

η_{fA} Filter efficiency for constant-resistance filter with Class A shunt-fed PDM amplifier.

η_{fB} Filter efficiency for constant-resistance filter with Class B PDM amplifier.

η_{flR} Filter efficiency for L-C filter with Class A series-fed PDM amplifier.

E_s Supply voltage for capacitor charging in R-C pulse timing circuit.

E_{co} Cutoff bias for tube.

E_m' Maximum negative going grid voltage excursion of monostable tube.

t_m Time duration of generated pulse.

D_n nth harmonic distortion component.

E_{bb} Plate or collector supply voltage

E_{min} Minimum plate to cathode or collector to emitter voltage. For conventional amplifiers this is at maximum signal level.

$e_{b,min}$	A time function variable, minimum plate to cathode or collector to emitter voltage for PDM amplifiers with L-C filter.
f_m	Modulating frequency
ω_m	2π times the modulating frequency.
f_r	The pulse repetition frequency.
f_c	The filter cutoff frequency.
I_m'	Current intercept point of load line.
I_m	Maximum plate current.
I_{mr}	Maximum rated plate current.
K	The ratio of I_m to E_{min}
β	See Equation (C-12)
P_o	Average signal power per tube delivered to the load for a conventional amplifier.
P_{ot}	Total signal average power delivered to the load for a conventional amplifier.
P_{op}	Average signal power per tube delivered to the load for a PDM amplifier.
P_{pt}	Total signal average power delivered to the load for a PDM amplifier.
P_d	Plate dissipation per tube (or transistor) for a conventional amplifier.
P_{dp}	Plate dissipation per tube for a PDM amplifier.
P_{dr}	Rated plate dissipation per tube.
P_s	Plate power supply power per tube.
P_{st}	Total plate power supply power.

P_{sg}' Screen dissipation per tube for a conventional amplifier.

P_{sg} Screen dissipation per tube for a PDM amplifier.

P_{op}' Average power delivered by a PDM amplifier to the load, including d-c power.

P_t Input power to the filter of a PDM amplifier.

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I. INTRODUCTION

In the design of logic circuits for digital computer applications, the active elements need only exhibit two stable states; on and off. The power dissipation of the active elements is low for both of these states, being high only during the time of switching, and the switching time is purposely made short. Linear untuned voltage and power amplifiers of the vacuum tube or transistor type (audio, control, isolation, and servo amplifiers) must be operated with Class A, Class AB or Class B bias, and the active elements (tubes or transistors) are operated in a region of high dissipation a large percentage of the time for most driving signal wave shapes. Efficiencies for output stages can be derived in terms of class of bias (A, AB, or B), plate or collector supply voltage, minimum instantaneous output voltage, waveform of the signal being amplified, and type of load. These efficiencies are usually expressed for sinusoidal signals. General efficiency expressions are derived in Appendices B, C and D for conventional amplifiers. By setting M=1 and $E_{min} = 0$ it can be seen that the maximum theoretical plate circuit or collector circuit efficiencies for these amplifiers are

$$\text{Maximum } \eta_{AR} = 25\% \qquad (1-1)$$

$$\text{Maximum } \eta_{A} = 50\% \qquad (1-2)$$

$$\text{Maximum } \eta_{B} = \frac{\pi}{4} \times 100\% = 78.5\% \qquad (1-3)$$

These efficiencies can be approached but never achieved in practice. Actual efficiencies in most cases are considerably below these upper efficiency bounds because other factors, such as minimum distortion and maximum power output for a given plate loss, are usually considered more important.

If the driving signal waveform is of a shape that puts the active element into heavy conduction (with low plate or collector drop) during on-time and cuts the device off (little or no plate or collector current) the rest of the time, higher device efficiencies may be achieved than are possible with sinusoidally driven Class A or Class B amplifiers. This type of drive has many of the attributes of Class C operation and the maximum theoretical efficiency is 100 percent. Average conduction current is proportional to the length of time the device is turned on provided the on-current repetitively assumes the same respective value and the off-current is zero. A nonzero off-current will merely add a constant to the output current. Linear amplification may be achieved by using a constant amplitude square wave to turn the output elements on and off, the on-time being determined by the instantaneous value of the signal that is to be linearly amplified. This is pulse duration modulation and for satisfactory results requires a square wave pulse repetition frequency that is high compared to the highest frequency component of the modulating signal. For nonperiodic modulating signals the necessary requirement is that the signal change but a small amount in amplitude in the time of one period of the repetition frequency of the square wave. For most applications this pulse duration modulation method will require an averaging or low-pass filter at the output. The cutoff frequency of the filter would lie between the highest modulating frequency that it is desired to amplify and the repetition frequency of the square wave that turns the output elements on and off.

Many applications for PDM amplifiers suggest themselves. With a pulse repetition frequency of 100 K. C. and a low-pass filter cutoff frequency of 10 K. C., an audio amplifier would result which has a frequency response from d-c to 10 kilocycles. Use of a lower pulse repetition rate, say 1 K. C., would provide a d-c amplifier meeting the requirements of isolation, control, and servo amplifiers in many systems applications. Here d-c amplifiers are often required, and the a-c response need be good only over very low frequencies. In many of these applications linearity of modulation of the pulse generator need not be good, thus allowing simple pulse generator designs and lower cost equipment than would be necessary for linear amplification.

Some applications require no low-pass filter at the output. One such application would be the driving of a resistive heating element in which the heating effect is to be a function of some low power control or error signal that must be amplified.

Tubes and transistors are obvious choices of output elements. Other devices such as silicon controlled rectifiers would find application if conditions unique to their operation are met. For example, if a d-c source is used for silicon controlled rectifiers provision must be made for periodically removing the source from the device so that the trigger element may regain control. Use of an a-c source would result in a nonlinear amplifier, but with some applications this may be satisfactory.

All references to tubes and plate circuits apply also to transistors* and collector circuits even though not specifically mentioned.

In order that PDM amplifiers and conventional amplifiers may be compared on the basis of power output, efficiency and dissipation, the analysis of PDM amplifiers is carried out for sinusoidal modulating signals. In many cases the analysis is valid for d-c signals as well since most of the circuit configurations are d-c amplifiers. In this analysis it is assumed that the frequency of the modulating signal is low compared to the pulse repetition frequency. This restriction is not necessary from a mathematical point of view but it allows many very important relations to be easily established.

From a practical point of view the above restriction is an absolute necessity if the output waveform is to be an accurate reproduction of the input. The waveform of a PDM signal may be represented by a Fourier series.[1] The spectrum components include: a possible d-c

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*With PNP transistors voltage and current polarities are reversed from those expressed in equations.

term (this is not present with some circuit configurations), a term at the frequency of the modulating signal, terms at the pulse repetition frequency and its harmonics, and terms at the sums and differences of these frequencies. Some of the latter terms will lie within the pass band of the low-pass filter and may show up as unwanted components at the load unless their amplitude is small. The coefficients of these terms are small for modulating frequencies that are low relative to the pulse repetition frequency. A finite number of these intermodulation terms lie within the pass band of the filter, and the peak voltage of the waveform represented by their sum is small if the amplitudes of the individual terms in the sum are sufficiently small.

Experimentally it is found that a pulse repetition frequency to modulation frequency ratio of 10:1 or greater is satisfactory and that no zero beat phenomena is apparent for modulation frequencies below this 10 percent figure. Zero beat interference is observed when the frequency of modulation is 12.5 percent of the pulse repetition frequency.

Many advantages result from using a constant-resistance filter for the low-pass output filter.[4] It exhibits a driving point impedance which is resistive and constant at all frequencies. This helps greatly in maintaining waveshape at the input to the filter. This type of filter is essential for output circuits which at times have all driving elements cut off and where there is no provision for a conducting element across the filter input terminals. A disadvantage of using a constant-resistance filter is the fact that it is an R-L-C device and has loss.

As derived in Appendices B, C and D for various PDM amplifier configurations with constant-resistance filters, the efficiency of the output tubes or transistors is

$$\eta_t = 1 - \frac{E_{min}}{E_{bb}} \qquad (1 - 4)$$

This expression is valid no matter what the waveshape of the modulating signal; the modulating signal does not have to be sinusoidal or even periodic. Equation (1 - 4) does not include the efficiency of the output filter.

The following advantages result from this switching method of operation in conjunction with a constant-resistance filter at the output.

1. Most of the circuit configurations are d-c amplifiers. Modulating frequencies may range from d-c to 10 percent of the pulse repetition frequency with good linearity of operation.

2. Greater output power is possible with given components than can be obtained in conventional Class A or Class B amplifiers.

3. Amplifier linearity is independent of the characteristics of the device being driven. The linearity is a function only of the pulse generator and its modulating system, and performance can be made very good. This is extremely important. Output elements with very poor linearity (those unsuited for operation in Class A or Class B linear amplifiers) may be used, or those elements which are normally chosen for Class A and Class B amplifiers may be used over a wider range of values on their volt-ampere characteristic curves than is possible for linear operation in conventional amplifiers. And it is usually possible to use a lower value E_{min}.

4. There is good gain stability as a d-c amplifier and instantaneous recovery from overdrive or clipping. These features are a function of the pulse generator design and its modulating system.

5. There is no degeneration due to screen to cathode voltage variation for cathode follower type output stages and for single-ended push-pull amplifiers.[3]

All of the advantages listed for PDM amplifiers with constant-resistance filters also apply when L-C filters are used except for point 3. The linearity of amplification is not independent of the characteristic curves. The expression

$$\eta_{t1} = 1 - \frac{e_{b,\,min}}{E_{bb}} \qquad (1 - 5)$$

derived in Appendix E takes the place of Eq. (1-4). The term $e_{b,\,min}$ is not a constant as is E_{min}, but varies as a function of the instantaneous value of the modulating signal. This introduces an amplitude distortion. The amplitude distortion may be kept small by keeping the maximum value of $e_{b,\,min}$ small or by attempting to hold $e_{b,\,min}$ at a constant value with clamping diodes or by using feedback to the amplifiers furnishing the driving pulses for the output tubes. Transistors are capable of supporting a very low ratio of maximum $e_{b,\,min}$ to E_{bb}.

Although L-C filters show promise of providing very high plate circuit efficiency, extremely difficult circuit problems arise in their practical application. The driving point impedance must be controlled in the block band as well as the pass band, perhaps to 100 or 200 times the cutoff frequency of the filter. A low driving point impedance at frequencies above cutoff would result in high plate loss and would lower the power handling capability of the amplifier. An L-C filter with a series inductor at its input would provide a desirable high driving point impedance at and above the pulse repetition frequency, but provision must be made for current continuity in the series inductor. This last requisite results in circuit design requirements that seem impossible without recourse to either clamping diodes or to a feedback system which alters the grid or base driving signals so that proper waveshape is maintained at the input to the filter.

The subject of this paper is confined to an investigation of the performance of PDM amplifiers with constant-resistance filters. A research area that shows additional promise is an investigation of the use of L-C filters with PDM amplifiers. For comparison to conventional amplifiers and to PDM amplifiers with constant-resistance filters, the power output and efficiency expressions are derived for a PDM amplifier with one type of L-C filter. A PDM circuit with this type of L-C filter is tested to illustrate the difficulties involved in practical application.

II. GENERATION OF MODULATED PULSES

Two convenient types of duration modulated pulses are those having a no-modulation duty ratio of 50 percent and a no-modulation duty ratio of zero. With the first type, positive going modulating signal would increase the duty ratio, the upper limit on duty ratio being 1.0. Negative going modulating signal would reduce the duty ratio, the lower limit being zero. With the second type two outputs would be needed. The no modulation condition would provide a zero duty ratio (or no output) at both output terminals. Positive going modulating signal would cause the duty ratio to increase at one of the outputs, while the duty ratio of the second output remains zero. Again, the upper limit on duty ratio would be 1.0. Negative going modulating signal would increase the duty ratio of the second output, while that of the first remains zero. The second type of pulse modulation requires that two output elements be driven, whereas the first type may be used to drive either a single output element or, with the addition of an inverting network, two in push-pull.

If the output tubes of a PDM amplifier drive a resistive load such as the input of a constant-resistance low-pass filter, there is an interesting parallel, insofar as power supply requirements are concerned, between the use of these two types of modulated pulses and the operation of Class A and Class B amplifiers. With the first type of pulse duration modulation the d-c power supply current is constant, whether a single stage is being driven or two elements in push-pull. This is identical to the power supply demands of a conventional Class A amplifier, whether it be single stage or push-pull. The second type of pulse duration modulation results in no current with no modulation and an increasing d-c current as modulating signal amplitude increases. This then corresponds to the power supply demands of a conventional push-pull Class B amplifier. Because of this parallel, these two types of pulse duration modulation will be referred to respectively as Class A modulation and Class B modulation.

There are no commercially available pulse generators capable of providing Class B modulation or pulse duration modulation over the range of duty ratios from zero to one. This necessitated, at no small expenditure of time, the design and construction of a pulse generator capable of meeting these requirements. Since this pulse generator is an instrument to provide a particular form of driving signal to various circuits which are to be tested, it is designed for precision of adjustment and versatility, with no consideration for efficiency. The instrument is much more complex (there are 20 tubes) than would be called for in any one application since it must be capable of driving many types of output circuits having either tubes or transistors. Distortion in modulation is held much lower than would usually be required, and pulse amplitude and waveshape are very closely maintained. In order to achieve short rise and fall times and to allow large output voltage excursions, tubes rather than transistors are used. The pulse generator meets the following specifications:

1. The repetition frequency may be anything up to 100 kilocycles.
2. Either Class A push-pull or Class B pulse modulation may be obtained.
3. The pulse duty ratio may be anything between zero and 1.0. The modulation factor may be 100 percent.
4. The voltage excursion of the output pulses is 160 volts (output voltage is either 90 volts or 250 volts).
5. The rise time and fall time at the outputs of the AND and OR stages are in the range of 0.2 microseconds to 0.4 microseconds, depending upon the class of modulation and the load on the output. The 11.5 micromicrofarad input capacitance of the scope probe must be considered part of the load.

6. The modulator is d-c coupled and has an input impedance of approximately 3,000 ohms. A balanced 600 ohm source may be used to drive the modulator. An a-c input voltage of 1.95 volts results in 15 volts a-c signal at the grids of the clamp tubes and provides 100 percent modulation.
7. At 100 percent modulation, the combined distortion of the modulator and Hewlett Packard 200-CD driving oscillator is 0.3 percent second harmonic, 0.1 percent third, and 0.005 percent fourth. These figures are the average for three different amplifier tubes, with and without triggering of the monostable circuits. The distortion of the oscillator at the input of the modulator is 0.25 percent second harmonic, 0.1 percent third, and 0.005 percent fourth.

Figure 2-1 is a block diagram of the pulse generator. Each of the four monostable circuits is adjusted for a 25 percent duty ratio with no modulation. The outputs of OR-1 and NOT-2 constitute a push-pull Class A modulated signal. Likewise so do the outputs of OR-2 and NOT-1. For single ended use, the output of OR-1 or OR-2 alone may be used. A push-pull Class B modulated signal is obtained at the outputs of AND-1 and AND-2.

Figure 2-2 gives the waveforms for Figure 2-1 with various fixed value input signal conditions on the input to the modulator. Figure 2-3 gives the waveforms for Figure 2-1 with sinusoidal modulating signal on the input to the modulator. The binary states are triggered by negative trigger spikes.

The Class B modulated pulses have both leading edge and trailing edge modulation. The centers of the pulses are evenly spaced on the time axis. The Class A modulated pulses at the outputs of either of the OR stages have trailing edge modulation.

Four monostable circuits are used rather than two so that the duty ratio demand on any monostable circuit will be 50 percent or less under

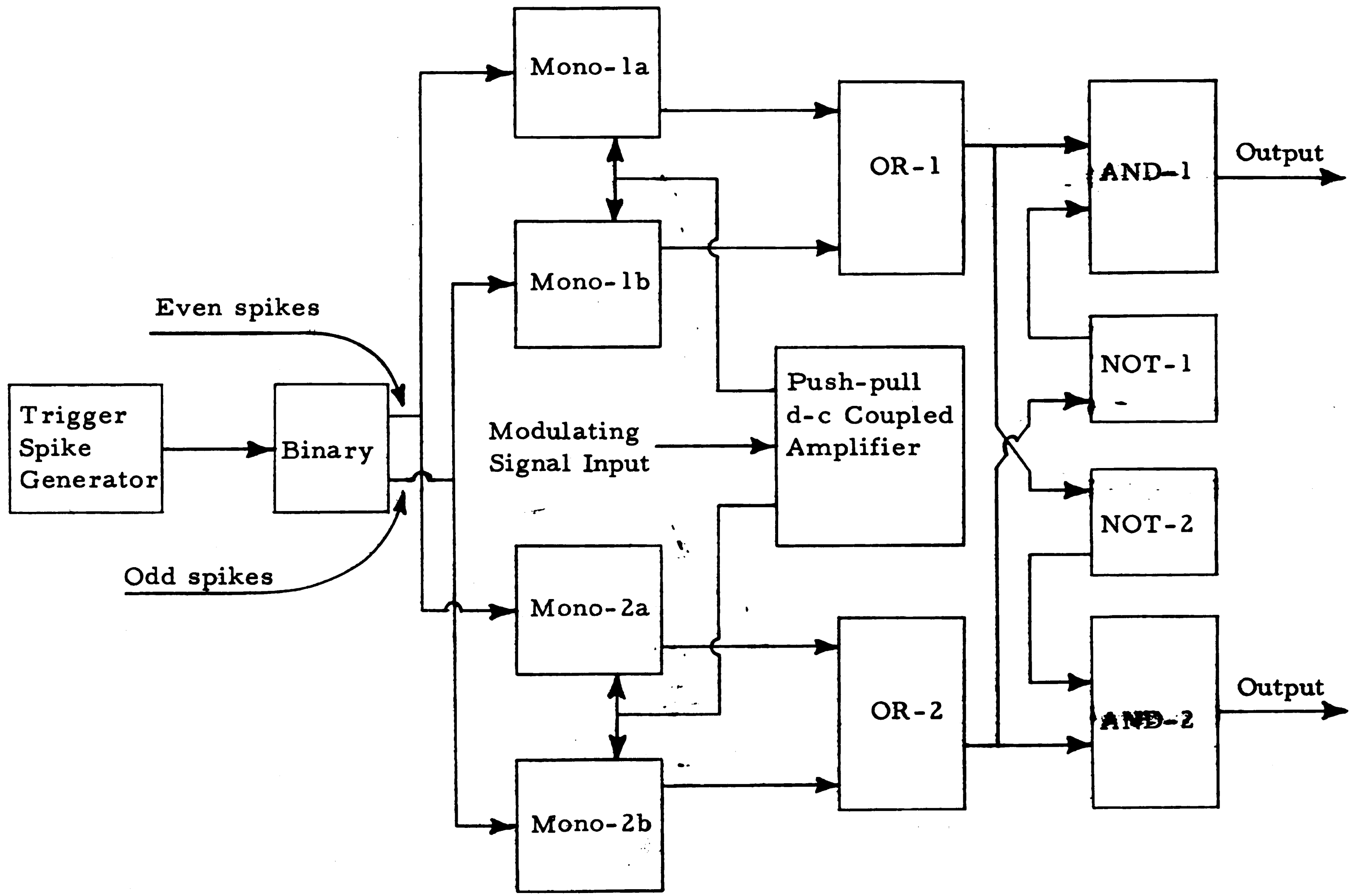


Figure 2-1. Block diagram of pulse generator.

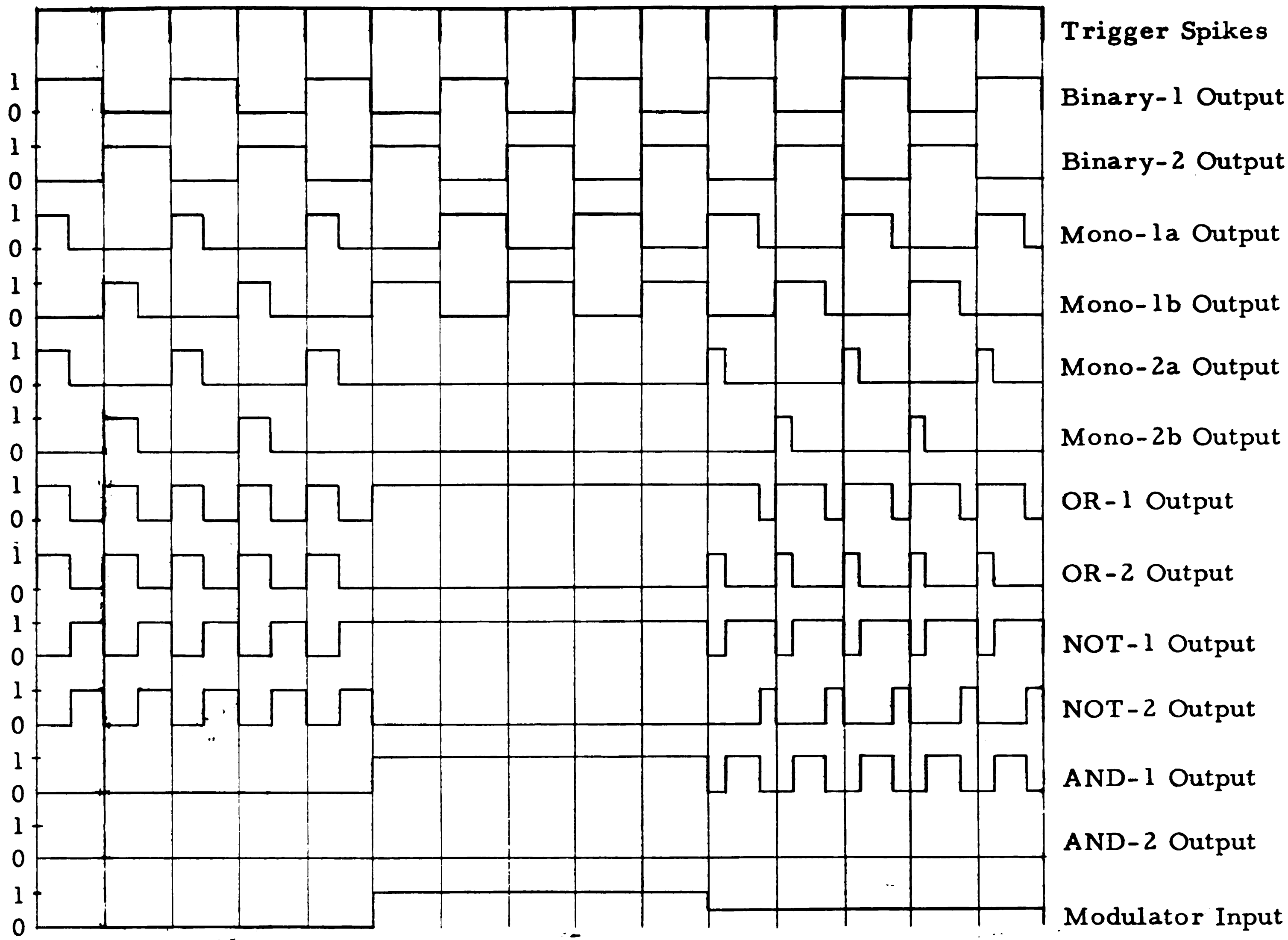


Figure 2-2. Pulse generator waveforms for d-c modulating voltage.

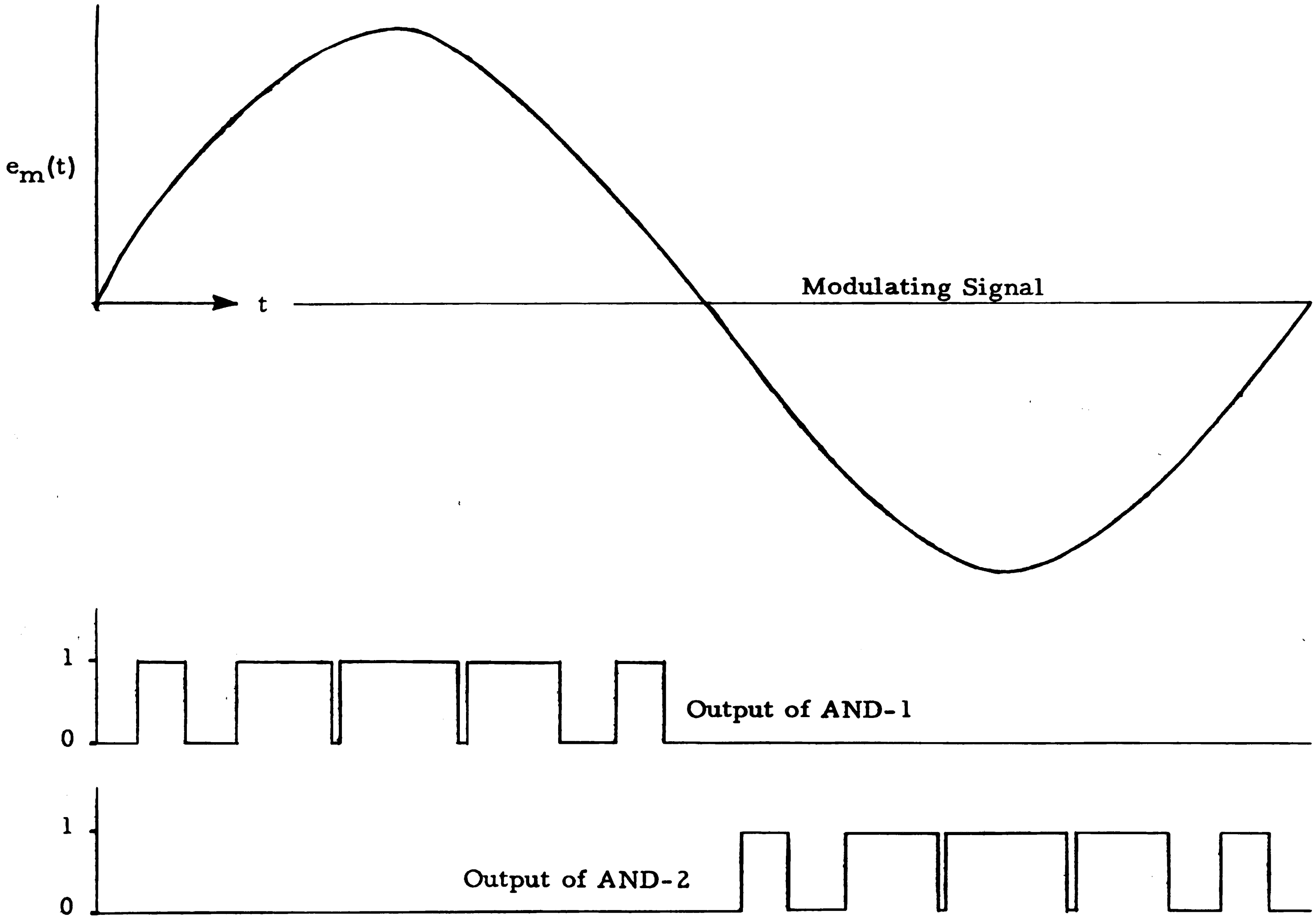


Figure 2-3. Pulse generator output waveforms for sinusoidal modulation.

all modulating conditions. This allows the duty ratio of the AND circuit outputs to be 100 percent at the peaks of the modulating signal, and yet allows 50 percent minimum recovery time for each monostable circuit. Monostable circuits of the type employed in this pulse generator use R-C elements to determine pulse duration, and at the completion of the pulse enough time must be allowed to re-establish the initial conditions on the capacitor before another pulse may be started. The monostable circuits in this pulse generator are capable of a 75 percent duty ratio (recovery time 25 percent of the period of the repetition frequency). A commercially available pulse generator which enjoys extensive use is capable of duty ratios of approximately 70 percent (no provision is made for pulse duration modulation on this unit), and the recommendation of the manufacturer is to not exceed duty ratios of 60 percent.

Figure 2-4 illustrates the method of modulating the pulse duration of the monostable circuits. Clamp tube V3 determines the magnitude of the negative excursion of the V1 plate voltage and hence determines the duration of the generated pulse. The plate voltage of V1 can never drop below the voltage of value given by the instantaneous control grid voltage of V3 plus a voltage of an amount that represents the working bias of V3 when it is conducting.

The linearity of pulse modulation may be made very good by returning R to a high positive voltage and by restricting the voltage change on the capacitor during on-time to a small percentage of this voltage. In Appendix A an equation is derived for the instantaneous time function demodulated signal at the output of the filter as a function of a sinusoidal modulating signal. For the derivation of this equation it is assumed that the period of the modulating signal is long compared to the period of the pulse repetition frequency. This equation may be used in conjunction with the five point method of determining the percent second, third, and fourth harmonic distortion.[2] Choosing $E_s = 400$ volts and $E_{mm} = 60$ volts with 100 percent modulation, the harmonic distortion is 1.746 percent

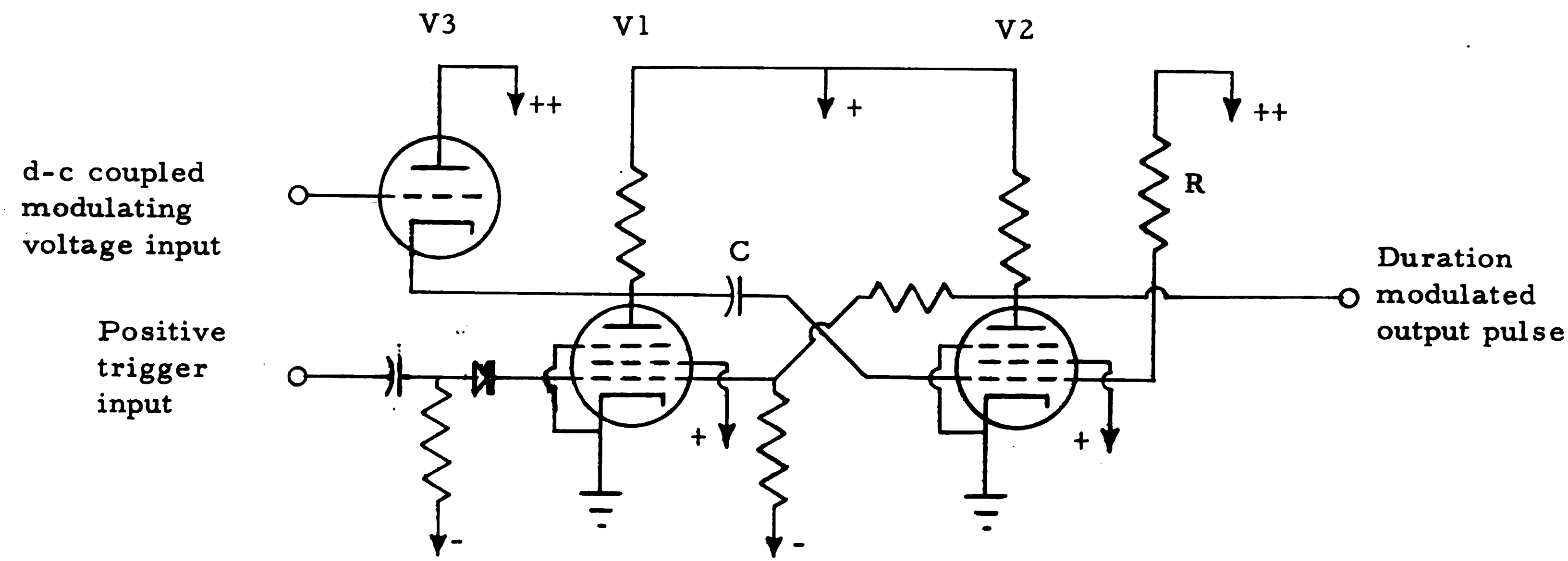


Figure 2-4. Monostable multivibrator.

second, 0.041 percent third, 0.001 percent fourth, and 1.747 percent total harmonic distortion. Modulation factors below 100 percent provide less distortion.

An experimental method of determining the percent harmonic distortion for the first four harmonics makes use of precision d-c voltmeters. The instantaneous value of a modulating cosine voltage is determined at each of the required angles (0, 60, 90, 120, 180 degrees) of the five point method for determining harmonic distortion.[2] The amplitude of this cosine voltage is such as to provide the desired percent modulation. For each calculated instantaneous value a d-c voltage is applied at the input of the modulator and the d-c voltage at the output of the low-pass filter is read. These output voltages are used in the equations of the five point method of determining distortion to determine the percent second, third and fourth harmonic distortion.

Another method of determining the magnitude of the harmonic distortion components is to drive the modulator with a sinusoidal voltage of such amplitude as to provide the desired percent modulation and to determine the magnitude of the harmonics at the output with a wave analyzer.

III. POWER OUTPUT AND EFFICIENCY

The driving point impedance of a constant-resistance low-pass filter is resistive and constant at all frequencies.[4] Apply the periodic voltage waveform of Figure 3-1 to the input terminals of this type of filter.

Figure 3-1. Periodic voltage function applied to filter.

Let

$$\alpha = \frac{T_1}{T} \tag{3-1}$$

The time function current at the input terminals of the filter is

$$i(t) = \frac{e(t)}{R} \tag{3-2}$$

where R is the driving point resistance of the filter. The instantaneous power delivered to the input of the filter is

$$p(t) = e(t)i(t) \tag{3-3}$$

The average power delivered to the filter is

$$\begin{aligned} P_{in} &= \frac{1}{T}\int_0^T p(t)dt = \frac{1}{TR}\int_0^T [e(t)]^2 dt \\ &= \frac{T_1}{T} \times \frac{E^2}{R} = \alpha\frac{E^2}{R} \end{aligned} \tag{3-4}$$

The filter will average the voltage which is applied to the input and provide an output voltage to a load resistor R of

$$E_{ave} = \frac{1}{T}\int_0^T e(t)dt = \frac{T_1}{T} E = \alpha E \tag{3-5}$$

The power delivered to load resistor R is then

$$P_{out} = \frac{E^2_{ave.}}{R} = a^2 \times \frac{E^2}{R} \qquad (3\text{-}6)$$

The filter efficiency factor is

$$\eta_f = \frac{P_{out}}{P_{in}} = a \qquad (3\text{-}7)$$

From Equation (3-7) it is seen that the filter efficiency factor is equal to the duty ratio a. Here a is a constant in the range $0 \leq a \leq 1$. A consequence of Equation (3-7) is that constant-resistance filters cannot be of the L-C type. With Class A and Class B pulse modulation a is a time function variable, and efficiency must be evaluated over one period of the modulation signal. The efficiencies for these classes of modulation with various circuit configurations are evaluated in Appendices B, C, and D. In Appendix F it is proved that the highest efficiency for Class A modulation results when the no-modulation duty ratio is $\frac{1}{2}$.

The constant-resistance filter is designed for whatever load resistor R is to be used. If the load resistor is to be changed to a new value for some reason, such as to increase output power, the filter must be redesigned for the new load value. With ratios of pulse repetition frequency to modulation frequency of 10:1 or greater a filter with an ultimate attenuation slope of 12db per octave is satisfactory. Figure 3-2 shows a constant-resistance low-pass filter of this type.

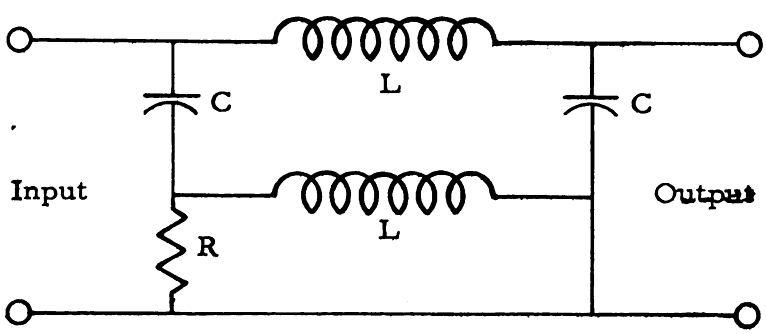


Figure 3-2. Constant-resistance low-pass filter.

$$L = \frac{R\sqrt{2}}{2\pi f_c} \tag{3-8}$$

$$C = \frac{1}{2\pi\sqrt{2}\, f_c R} \tag{3-9}$$

$$\frac{L}{C} = 2R^2 \tag{3-10}$$

The cutoff frequency f_c is the frequency at which the output power is 3db less than the input power. The output is to be terminated in a resistive load R. The driving point impedance of the filter is equal to R at all frequencies.

Any actual filter device that is constructed will of course not have a resistive input impedance of R at all frequencies. However, the pulses of an actual PDM signal will have finite rise and fall times and the relative amplitude of harmonics will be small above a certain frequency. This frequency is several multiples higher than that which would have a period equal to the rise or fall time of the pulse. There is no merit in designing the filter to have a constant input resistance above the frequency of the highest significant harmonic. The choice of cutoff frequency for most of the investigations of this paper is 10 percent of the pulse repetition frequency. If the input resistance of the filter is to be constant for frequencies up through the 10th or 20th harmonics of the pulse repetition frequency, it is required that the input impedance of the filter be controlled to 100 or 200 times the cutoff frequency of the filter.

For a given f_c it can be seen from Equation (3-8) that large R values require large L values in the filter. It would seem that for satisfactory performance, the natural resonant frequency of the inductors should be higher than the frequency up to which the filter input resistance is to be held constant. The coils do not act as inductors of design value L at their natural resonant frequency, and above resonance they present capacitive reactance. The R values generally used for the filters in the investigations of this paper result in L values which are so large that the natural resonance of coils tested for use falls between 5 and 50 times the cutoff frequency of the filter. There is a definite upper limit to the natural resonant frequency that may be achieved for a coil wound to give a specific inductance, regardless of the type of winding used or the means employed to keep interwinding shunt capacitance to a minimum. However, an analysis of the filter of Figure 3-2 shows that it performs very well even with coils that do not maintain the design value inductance over the whole frequency range for which the filter is to be used. The filter consists of two legs connected in parallel at the input terminals. At high frequencies the inductor between input and output presents a high reactance and the filter input impedance is determined essentially by the resistor R within the filter. If the reactance of the inductors is abnormally high, their contribution in parallel with R is little different than it would be if they had the correct reactance. If the impedance of the coils is so high at resonance as to completely remove them from the circuit (the worst possible case), the input impedance of the filter at this frequency f_r will be

$$Z_{in} = R(1 - j\sqrt{2}\,\frac{f_c}{f_r}) \qquad (3\text{-}11)$$

When $f_r = 5\ f_c$

$$Z_{in} = 1.039\ R\angle -15^\circ 45' \qquad (3\text{-}12)$$

When $f_r = 10\ f_c$

$$Z_{in} = 1.01\ R\angle -8^\circ 3' \qquad (3\text{-}13)$$

When $f_r = 50\ f_c$

$$Z_{in} = 1.0004\ R\angle -1^\circ 37' \qquad (3\text{-}14)$$

The voltage transfer function of the filter is affected by the improper L values at high frequencies. However, the result is greater attenuation at these high frequencies than that expressed by the design equations, and this results in an improvement in performance since it is intended that these high frequency components do not pass through the filter. At extremely high frequencies (well above the natural resonant frequency of the coils) the capacitive reactance of the coils becomes low and will affect the filter input impedance and the voltage transfer function. The filter must be designed so that the capacitive reactance of the coils is not low relative to R at frequencies for which voltage components are present at the input of the filter.

In Appendices B, C, and D equations are established for the average plate dissipation of the output tubes. These dissipation expressions are based on the assumption of sinusoidal modulating signal. The peak dissipation is greater than the average, and for those circuit configurations which are used as d-c amplifiers, operating conditions must be chosen so that the peak plate dissipation does not exceed the rated dissipation of the tube.

A problem is encountered when using push-pull Class A modulation with single-ended push-pull circuits.[3] If the switching times of the grid driving pulses are not extremely short compared to half the period of the repetition frequency, there is a common on-time for the tubes. The $2E_{bb}$ supply delivers a large current during this common on-time. This represents a power loss from the supply that does not contribute power to the load, resulting in lower plate circuit efficiency. The tube dissipation is increased and the amplifier has a lower power handling capability.

At high pulse repetition frequencies the switching time of transistors may be very much longer than the switching time of the driving pulses. If the switching time of the transistors is an appreciable fraction of the half period of the repetition frequency, the transistor operating point may be in a region of high dissipation for a greater percent of operating time than is desired for maintaining low dissipation. Unless very fast transistors can be obtained, it is necessary to operate transistor PDM amplifiers at very much lower pulse repetition rates than may be used with tubes.

The complementary pair transistor circuit configuration of Figure 4-3 offers some advantages over push-pull and single-ended push-pull configurations (the diodes are not needed when a constant-resistance filter is used). The base leads may be tied together and driven from a single-ended source; no phase inversion or push-pull drive is needed. No degeneration occurs with either transistor; they both operate as grounded emitter amplifiers. However, when the base leads are tied together, the peak to peak signal driving the transistors must be greater than the peak to peak output signal at the junction of the collectors. This situation can be remedied by driving the bases separately through some d-c coupling element such as a battery or zener diode. An NPN and a PNP transistor may be used together in the complementary pair circuit configuration even if they do not have similar characteristics. They should be able to carry the desired on-current and they should have about the same collector to emitter voltage when on. The base currents may be greatly different for given on-time collector currents.

Tables I, II, and III compare the performance of the various type amplifiers. The equations are derived in Appendices B, C, D, and E. The performance equations for single-ended push-pull are the same as those for push-pull of the same class.

The plate circuit efficiency for each class of PDM amplifier with constant-resistance filter is identical to the same class conventional amplifier. This correspondence between plate circuit efficiencies is further justification for the definitions of Class A and Class B pulse

Table I. Performance of Conventional Amplifiers and PDM Amplifiers with Constant-resistance Filters.

	Class A Series-fed	Class A Shunt-fed and Class A Push-pull	Class B Push-pull
Efficiency	$\eta_{AR} = \frac{M^2}{4}[1 - \frac{E_{min}}{E_{bb}}]$	$\eta_A = \frac{M^2}{2}[1 - \frac{E_{min}}{E_{bb}}]$	$\eta_B = \frac{M\pi}{4}[1 - \frac{E_{min}}{E_{bb}}]$
PDM only	$\eta_{fAR} = \frac{M^2}{4}$	$\eta_{fA} = \frac{M^2}{2}$	$\eta_{fB} = \frac{M\pi}{4}$
	$\eta_t = 1 - \frac{E_{min}}{E_{bb}}$	$\eta_t = 1 - \frac{E_{min}}{E_{bb}}$	$\eta_t = 1 - \frac{E_{min}}{E_{bb}}$
Signal Power Output	$P_o = P_{op} = \frac{M^2(E_{bb} - E_{min})^2}{8R}$	$P_o = P_{op} = \frac{M^2(E_{bb} - E_{min})^2}{2R}$	$P_{oT} = P_{pT} = \frac{M^2(E_{bb} - E_{min})^2}{2R}$
Greatest Possible Power Output ($E_{min} = 0$)	Greatest $P_o = \frac{P_{dr}}{2}$ Greatest P_{op}: no limit See Eq. (B-24)	Greatest $P_o = \frac{P_{dr}}{2}$ Greatest P_{op}: no limit. See Eq. (C-41)	Greatest $P_{oT} = \frac{2\pi P_{dr}}{4 - \pi}$ Greatest P_{pT}: no limit. See Eq. (D-60)
Maximum Plate Dissipation per tube.	Maximum $P_d = \frac{E_{bb}^2 - E_{min}^2}{4R}$ Maximum $P_{dp} = \frac{E_{min}(E_{bb} - E_{min})}{2R}$	Maximum $P_d = \frac{E_{bb}(E_{bb} - E_{min})}{R}$ Maximum $P_{dp} = \frac{E_{min}(E_{bb} - E_{min})}{R}$	See Equations (D-13) and (D-21) Maximum $P_{dp} = \frac{E_{min}(E_{bb} - E_{min})}{\pi R}$
Dissipation Ratio $D.R. = \frac{\text{Maximum } P_{dp}}{\text{Maximum } P_d}$	$D.R. = \frac{2 E_{min}}{E_{bb} + E_{min}}$	$D.R. = \frac{E_{min}}{E_{bb}}$	See Equations (D-68) and (D-69)
Dissipation Ratio Based on Class B PDM Amplifier	$D.R._B = \frac{4 E_{min}}{\pi(E_{bb} + E_{min})}$	$D.R._B = \frac{E_{min}}{\pi E_{bb}}$	
Screen Dissipation per tube.	$P'_{sg} = I'_{sg} E_{sg}$ $P_{sg} = \frac{1}{2} I_{sg} E_{sg}$	$P'_{sg} = I'_{sg} E_{sg}$ $P_{sg} = \frac{1}{2} I_{sg} E_{sg}$	$P'_{sg} \simeq \frac{M I_{sg} E_{sg}}{\pi}$ $P_{sg} = \frac{M I_{sg} E_{sg}}{\pi}$
Power Supply Current	$I_{dc} = \frac{E_{bb} - E_{min}}{2R}$	$I_{dc} = \frac{E_{bb} - E_{min}}{R}$ per tube	$I_{dc} = \frac{2M(E_{bb} - E_{min})}{\pi R}$

Table II. Power Output and Improvement Ratio with Utilization of Full Plate Dissipation Rating.*

	Class A Series-fed	Class A Shunt-fed and Class A Push-pull	Class B Push-pull
Maximum Signal Power Output	Maximum $P_o = \frac{(E_{bb} - E_{min})P_{dr}}{2(E_{bb} + E_{min})}$ Maximum $P_{op} = \frac{1}{4}[\frac{E_{bb}}{E_{min}} - 1]P_{dr}$	Maximum $P_o = \frac{1}{2}[1 - \frac{E_{min}}{E_{bb}}]P_{dr}$ Maximum $P_{op} = \frac{1}{2}[\frac{E_{bb}}{E_{min}} - 1]P_{dr}$	Case 1: Maximum $P_o = \frac{\pi^2}{4}[1 - \frac{E_{min}}{E_{bb}}]^2 P_{dr}$ when $1 - \frac{E_{min}}{E_{bb}} \geq \frac{2}{\pi}$ Case 2: Maximum $P_o = \frac{\pi(E_{bb} - E_{min})P_{dr}}{4E_{bb} - \pi(E_{bb} - E_{min})}$ when $1 - \frac{E_{min}}{E_{bb}} \leq \frac{2}{\pi}$ Maximum $P_{op} = \frac{\pi}{4}[\frac{E_{bb}}{E_{min}} - 1]P_{dr}$
Improvement Ratio $I.R. = \frac{\text{Maximum } P_{op}}{\text{Maximum } P_o}$	$I.R. = \frac{1}{2}[\frac{E_{bb}}{E_{min}} + 1]$	$I.R. = \frac{E_{bb}}{E_{min}}$	Case 1: $I.R. = \frac{E_{bb}^2}{\pi E_{min}(E_{bb} - E_{min})}$ when $1 - \frac{E_{min}}{E_{bb}} \geq \frac{2}{\pi}$ Case 2: $I.R. = \frac{E_{bb}}{E_{min}} - \frac{\pi}{4}[\frac{E_{bb}}{E_{min}} - 1]$ when $1 - \frac{E_{min}}{E_{bb}} \leq \frac{2}{\pi}$
Improvement Ratio Based on Class B PDM Amplifier	$I.R._B = \frac{\pi}{2}[\frac{E_{bb}}{E_{min}} + 1]$	$I.R._B = \frac{\pi E_{bb}}{2E_{min}}$	

*Conventional amplifiers and PDM amplifiers with constant-resistance filters.

Table III. Performance of PDM Amplifiers with L-C Filter.*

	Class A Series-fed	Class A Push-pull
Efficiency	$\eta_{1R} = \eta_{f1R} = \frac{M^2}{2 + M^2}$ Maximum $\eta_{1R} = \frac{1}{3}$	$\eta_1 = \eta_{f1} = 1$
Signal Power Output	$P_{op} = \frac{M^2 E_{bb}^2}{8R}$	$P_{pT} = \frac{M^2 E_{bb}^2}{R}$
Power Supply Current	$I_{dc} = \frac{E_{bb}(2 + M^2)}{8R}$	$I_{dc} = \frac{M^2 E_{bb}}{2R}$ per tube

*This table is based on the assumption that the maximum value of $e_{b, min}$ is negligible compared to E_{bb}. The L-C filter investigated is the low frequency leg of a constant-resistance filter. See Appendix E.

duration modulation. The fact that plate circuit efficiency expressions for PDM amplifiers with constant-resistance filters are no better than for the corresponding classes of conventional amplifiers does not preclude the possibility of obtaining more output power from the PDM amplifiers. The dissipation, except for the contribution due to η_t, is in the filter rather than the output tubes, and more power supply power may be delivered to the circuit without exceeding the dissipation rating of the output tubes than can be handled by a similar class conventional amplifier. Maximum output power for a given plate dissipation is usually considered more important than efficiency. It may be noted from Table III that the maximum theoretical efficiency of a PDM amplifier with L-C filter is 100 percent. In addition it should be noted that the percent efficiency is independent of the percent modulation. Both of these factors are very important when efficiency is a consideration. For many audio amplifier applications the average power output ability of the amplifier should be 50 times the power level necessary to handle the normal program level.[5] Then according to the plate circuit efficiency equations, a Class A conventional amplifier would normally be operating at less than 1 percent efficiency and a Class B conventional amplifier at less than 11 percent. The plate circuit efficiencies of the corresponding classes of PDM amplifiers with constant-resistance filters would be respectively the same, but the plate circuit efficiency of a PDM amplifier with L-C filter is independent of signal level and the efficiency may be close to 100 percent.

In view of the fact that plate circuit efficiency is independent of signal level when an L-C filter is used, an economy in the design of the modulated pulse generator may be effected. In order to obtain high efficiency it is not necessary to swing the duty ratio of the modulated pulses from 0 to 1 at maximum signal level. A simple pulse generator can then be built with allowance for recovery time.

In an actual amplifier circuit it would be difficult to hold $e_{b,min}$ constant at the value E_{min} and there would be a small $i_t^2R_t$ loss within the tubes due to filter input current variation about the established signal current value. This loss does not appear in the equations for PDM amplifier performance with L-C filter because of the forced condition that $e_{b,min} = E_{min}$. Equation (E-23) indicates that there is no power furnished by the power supply when M = 0. However, when $e_{b,min}$ is not held constant at E_{min} during on-time, there will be a power loss in the tubes due to the build up of current in the input inductor of the filter each half cycle of the switching signal. This loss may be made smaller by increasing the size of the input inductor of the filter, but this creates new problems. The cutoff frequency of the filter would be lowered, perhaps putting too low a limit on the highest modulating frequency. In addition it would be necessary to maintain a high natural resonant frequency for the inductor so that a high impedance would be presented to the upper frequency components of the waveform which is applied to the input of the filter, and this requirement would be very difficult to meet.

The expressions for Improvement Ratio and for output power in terms of P_{dr} have no maximum theoretical limit as E_{min} is made smaller. However, the maximum current and voltage ratings of the tube impose a limit on the maximum power obtainable. Expressions relating to these limitations are derived in Appendices B, C, and D. The power output and dissipation equations should be used to determine the operating conditions for PDM amplifiers. There is no reason to refer to practical operating conditions for conventional amplifiers. Parameters chosen for optimum results with a PDM amplifier would very likely result in over dissipation and distortion if employed in a conventional amplifier. In the expressions for Dissipation Ratio and Improvement Ratio the circuit parameters for the two kinds of amplifiers are the same for comparison purposes only. Much greater increases in power handling ability for PDM amplifiers over conventional amplifiers may be achieved than are indicated by the Improvement Ratio expressions.

Since a Class B PDM amplifier has the same linearity as a Class A PDM amplifier, and since this linearity is better than that of a conventional Class A amplifier, a Class B PDM amplifier may replace any class of conventional amplifier with no increase in distortion. To evaluate the improvement in power handling ability when this is done, Improvement Ratios ($I.R._B$) are given in terms of a Class B PDM amplifier. Expressions are also given for Dissipation Ratios ($D.R._B$) in terms of a Class B PDM amplifier. It may be noted from Tables I and II that the D.R. and the I.R. are reciprocals for amplifiers of the same class.

Screen grid dissipation is given by the product of the average screen current and the screen grid voltage. For PDM amplifiers the on-value screen current is I_{sg}. Then for Class A PDM amplifiers the average screen current per tube is $\frac{1}{2}$ I_{sg} and for Class B PDM amplifiers it is $\frac{MI_{sg}}{\pi}$. For conventional amplifiers the average screen current is represented by I_{sg}' and the peak screen current by I_{sg}. The peak screen current of a conventional amplifier may be the same as the on-value screen current of a PDM amplifier, but the average screen currents are not at all likely to be the same. Screen current tends to be very large for low plate voltages (E_{min}), resulting in greater screen dissipation for PDM amplifiers than for the same class conventional amplifiers operating with the same E_{bb}, E_{min}, and load resistance.

IV. EXPERIMENTAL RESULTS AND CONCLUSIONS

All data presented is for a 10 kilocycle repetition frequency, a 1 kilocycle filter cutoff frequency, and a 100 c.p.s. modulation frequency. Multiplication of all of the above frequencies by a factor of 10 results in the same power output, efficiency, and distortion characteristics for the PDM amplifiers; however, the rise and fall time of the 100 kilocycle square-waves limits the modulation to 92 or 93 percent. Circuit operation with M = 1 is instrumental in demonstrating the validity of the derived equations for power output and efficiency.

Table XIII indicates how well experimental results compare with theory for amplifiers using constant-resistance filters. For applications using an L-C filter, no comparison of measured results is made with the theoretical equations of Table III because these equations do not take into consideration the losses associated with the circuit which was tested.

Tables IV through XI give the performance of each type of circuit for which performance equations were derived. For those cases using a constant-resistance filter, these tables also give the theoretical expectation of performance based on the equations appearing in Tables I and II. These equations are derived in the appendices. The upper number of multiple value entries given in the tables for each value of M is the theoretical value. The lower number is the actual value. All theoretical entries for output voltage, power output, and efficiency take into consideration the transfer loss of the filter employed. All filters used, including the L-C, have a voltage transfer function of 0.98 at the frequency of modulation for all signal levels used. The omission of an entry in a harmonic distortion column means that the particular distortion component is so small and erratic as to be unreadable. The magnitudes of frequency components clustered about the repetition frequency and its harmonics are read in decibels below the full power output of the recovered signal. These decibel readings show how well the filters perform in rejecting the unwanted high frequency components. As is seen, the decibel levels of these particular filters are quite satisfactory for most applications, especially when the load is insensitive to signals at these high frequencies. It might be pointed out, though, that these filters all have an ultimate attenuation slope of 12 decibels per octave, and if an application required more filtering than that of the filters used, constant-resistance or L-C filters may be used which have an ultimate attenuation slope of 18 decibels per octave, 24 decibels per octave, or higher.

In Tables X and XI (for transistors) the dissipation column is omitted because the derived equations for dissipation do not take into consideration driving power. A total transistor dissipation may be obtained by subtracting the input power to the filter from the total input power to the transistor circuit.

The 75 microfarad (not electrolytic) capacitor used as a return for the filter in the push-pull circuits does not affect the magnitude or phase of the output voltage. At 100 c.p.s. the magnitude of the signal across this capacitor is 1 percent of the magnitude across the 2,000 ohm load resistor, and the phase angle is such that the capacitor voltage contributes essentially nothing to output voltage observed with respect to ground.

Since the oscilloscope was used to measure the value of E_{min}, several percent error might be expected in the evaluation of theoretical output power and efficiency. For example, in Table V the actual value of E_{min} could easily have been either 45 volts or 50 volts. An E_{min} of 45 volts results in a theoretical efficiency which is 2.5 percent higher than would be the case if E_{min} were 50 volts, and in a theoretical output power which is 5.06 percent higher.

The oscilloscope was used for setting the value of M. The time base was uncalibrated and set so that one period of the repetition frequency covered 10 centimeters. The degree of pulse duration modulation may be observed, the value of M being the fractional part that this is of the 10 centimeters. One realizes then that several percent error may exist in calculated values which are a function of M, especially with expressions that are proportional to M^2.

Actual efficiencies are a function of the average pulse duty ratio or of the pulse duty ratio with no modulation. Variations of these duty ratios from 0.5 would cause actual efficiencies to be somewhat different than the theoretical efficiencies which are based on a 0.5 no-modulation duty ratio. This explains in part the magnitudes of percent deviation from theoretical efficiency in Table XIII. The power output is not affected by a slight variation of no-modulation duty ratio from 50 percent as long as over-modulation does not occur. Then less variation should be expected in percent deviation of actual output power from theoretical than would be expected for percent deviation of efficiency. This is verified by Table XIII.

Precautions were taken in the design of the single-ended push-pull circuits of the vacuum tube type to prevent grid current of the upper tube from contributing power to the load. As with conventional amplifiers, the equations for theoretical efficiency do not take into consideration the contribution of grid driving power. It is desirable to drive both tubes to grid clamp, but excessive grid current is prevented by the use of grid current limiting resistors. These resistors are bypassed with capacitors in order to maintain short switching times. The result is a grid-leak bias which automatically adjusts to the level of grid voltage drive and allows the grid to be driven to clamp voltage with a minimum of grid current. With the transistor circuits the base driving power is considered part of the input power and no effort is made to have it be negligible. As a matter of fact, the on-value base current must be enough to put the transistor into saturation if high output power, high efficiency, and low distortion are to be attained. The single-ended push-pull stages of the vacuum tube type which are tested then have a relatively high E_{min}. This is why the push-pull circuits of Tables VII and IX have a lower output power per tube than do some of the other circuits. These push-pull circuits are capable of more power output merely by increasing the grid drive.

Output power of the upper tube of single-ended push-pull stages is a function of cathode current, and it is immaterial whether this be plate current or grid current. Efficiency would be higher than that given by the theoretical expression because the input power to the grid is not accounted for. In the test of the Class B push-pull circuit (see Tables IX and XIII) it is estimated that the power contribution of the upper grid amounts to about 3.5 percent of the power delivered to the filter input at full modulation. At lower modulation levels the percent contribution would be slightly more. This helps explain the high positive percent deviations of efficiency for the Class B amplifier in Table XIII.

The same effect would tend to make the efficiency deviations of Table XIII for the push-pull Class A amplifier (see Table VII) be positive and high. However, the effect would be much less than in the case of the Class B amplifier because of the larger Class A plate current. An estimate of the contribution of grid current power to the filter for this circuit is 2.25 percent. This percent contribution to the filter input power should be the same at all modulation levels.

The major distortion components for a 200 c.p.s. modulation frequency in Table XII are the same as those for M = 1 in Table IV.

This indicates that the shunt coil reactance at 200 c.p.s. is satisfactory. When a 2,000 ohm load is used, a shunt coil should have at least this much reactance at the lowest modulation frequency for which it is desired to operate the amplifier. Lower values of reactance result in higher distortion, as is indicated in Table XII.

When a 100 percent modulation signal is recovered with a filter connected directly to the output terminals of the pulse generator, there is 1.81 percent total harmonic distortion. The oscillator which provides the modulation signal has 0.2 percent second harmonic distortion and 0.1 percent third, with no other significant harmonics. The calculated total harmonic distortion for pulse generator modulation is 1.747 percent with M = 1 (see p. 14 and p. 16). It is seen then that the distortion produced in modulating the pulse generator is very close to that which is theoretically predicted. A look at the distortion figures in Tables IV through XI shows that the output stages of the tested PDM amplifiers contribute a negligible amount of distortion when they are resistively loaded.

The power output can never exceed the tube dissipation for a conventional Class A amplifier, whether series-fed or shunt-fed. The series-fed Class A PDM amplifier which was tested (Table IV) delivers 7.5 watts of sinusoidal signal to the load with a tube dissipation of 4.2 watts. The rated tube dissipation is 5.5 watts. This strikingly illustrates that more output power may be obtained from a PDM amplifier for a given tube dissipation than may be obtained from a conventional amplifier.

In Table VIII it is seen that the use of an L-C filter with a vacuum tube PDM amplifier results in an efficiency that is independent of signal level, as was theoretically predicted. With the transistor amplifier and L-C filter of Table XI, the efficiency falls off slightly with signal level. This is because of fixed losses associated with driving the transistors to saturation. These losses are not accounted for in the derived efficiency equations for PDM amplifiers with L-C filters. The efficiencies in both of these tables (Table VIII and Table XI) are very high compared to efficiencies of conventional amplifiers, especially at the low distortion levels involved.

The 0.37 milliamperes plate current for no modulation in Table VIII is the plate current due to common on-time of the tubes during switching since it remains the same when the filter and load are removed.

The tube dissipation of a Class A PDM amplifier with constant-resistance filter is constant and does not vary with the signal level. This is not true of a conventional Class A amplifier. Use of an L-C filter with a PDM amplifier gives the lowest tube dissipation.

The transistors in the complementary pair circuit configuration of Figure 4-3 need not have matched or even similar characteristics. It is only necessary that the square-wave at the input furnish the proper base currents to turn each transistor on to saturation. This is a definite advantage over a conventional complementary pair transistor amplifier where a matched pair of transistors must be used in order to obtain low distortion amplification.

An interesting feature of the amplifier of Figure 4-3 is the fact that it puts power into the collector supply voltage source at low signal levels. It may be observed in Table XI that there is a negative collector supply current at zero signal level. This indicates that the base driving power (which is constant for all signal levels) exceeds the circuit losses by 6 milliwatts. It might also be noted that no diodes are needed in conjunction with the L-C filter of the amplifier of Figure 4-3. The transistor junctions themselves provide the necessary diode action.

Though no data is given, it has been observed with a Class A vacuum tube PDM amplifier that there is good linearity between output signal amplitude and plate supply voltage for a given modulation signal level. This affords a means of controlling output signal level other than by adjustment of the modulation level, and efficiency is not sacrificed. A product circuit may be formed for multiplying voltages by using one of the voltages for the plate supply. The other voltage in the product is that used for pulse duration modulation.

In view of the reasons given for the discrepancies in the tables, it may be concluded for PDM amplifiers that:

1. The circuits which were investigated and tested do perform according to the derived theoretical performance equations.
2. Greater output power may be obtained for a given tube or transistor dissipation when amplifying sinusoidal signals than may be obtained with conventional amplifiers. This is true whether constant-resistance or L-C filters are used.
3. The harmonic distortion contribution of the output stage is negligible or extremely low when constant-resistance filters are used or when E_{min} is small.
4. High efficiency, which is independent of signal level, may be obtained with the use of L-C filters. This latter application merits further investigation.

The results of the investigations in this thesis suggest that further work be carried out in the following areas:

1. The use of L-C filters with PDM amplifiers.
2. The design and construction of a simple transistorized pulse generator which is capable of pulse duration modulation. An aid to achieving a simple pulse generator design is the fact that the efficiency of PDM amplifiers with L-C filters is independent of modulation level, therefore the pulse generator need not be capable of 100 percent modulation. Recovery time may then be allowed for the monostable circuits of the pulse generator. An effort should be made to achieve

relatively high efficiency levels for the whole system, including modulator, pulse generator, PDM output stages, and low-pass filter.

Table IV. Performance of Series-fed Class A PDM Amplifier with Constant-resistance Filter.

Circuit Configuration						Filter Configuration					
Figure	Tube Type	P_{dr}	R	E_{bb}	E_{min}	Figure	L	C	f_c	f_m	f_r
B-3	12B4	5.5 w.	2000 ohms	400 v	47v.	3-2	440 mh.	0.055 mfd.	1 K.C.	100 c.p.s.	10 K.C.

M	ma. I_b	watts per tube P_s	P_{dp}	P_{op}	η_{AR}	volts E_o	per cent harmonic distortion D_2	D_3	D_4
1.0	83	33	4.2	7.47	0.211	122.5	1.8	0.67	0.22
				7.5	0.227	122			
0.8	83	33	4.2	4.78	0.135	98	1.5	0.33	0.04
				4.7	0.142	97			
0.5	83	33	4.2	1.87	0.053	61.3	0.85	0.24	
				1.85	0.056	61.			
0	84	33.5	4.2						

M	Decibels below full power output of recovered signal f_r-5f_m	f_r-4f_m	f_r-3f_m	f_r-2f_m	f_r-f_m	f_r	f_r+f_m	f_r+2f_m	f_r+3f_m	f_r+4f_m	f_r+5f_m	$2f_r$
1.0	65.5	57.5	51	48	52.5	45.5	53.5	49	52	59	67.5	62.5
0						36						*

All frequency components clustered about $2f_r$ other than those given are more than 70 decibels below the full power output of the recovered signal.

*All entries of an asterix are more than 72 decibels below the full power output of the recovered signal.

Table V. Performance of Shunt-fed Class A PDM Amplifier with Constant-resistance Filter.

Circuit Configuration						Filter Configuration					
Figure	Tube Type	P_{dr}	R	E_{bb}	E_{min}	Figure	L	C	f_c	fm	f_r
C-4b	12B4	5.5w.	2000 ohms	240v.	48v.	3-2	440mh.	0.055 mfd.	. 1K.C.	100 c.p.s.	10 K.C.

Note: The shunt coil is 25 h. with 154 ohms d-c resistance.
The d-c losses of the shunt coil and filter inductor are accounted for in the calculations of power output, efficiency and dissipation. Use E_{bb} = 224 volts for these calculations. Then $\eta_A = P_{op}/P_s$.

	ma.	watts per tube				volts	per cent harmonic distortion		
M	I_b	P_s	P_{dp}	P_{op}	η_A	E_o	D_2	D_3	D_4
1.0	83	19.9	4.2	7.43	0.385	121.5	2.5	0.54	0.13
				7.2	0.362	120			
0.8	83	19.9	4.2	4.75	0.250	97.5	2.1	0.34	0.06
				4.61	0.231	96			
0.5	84	20.2	4.2	1.86	0.095	60.8	1.35	0.23	0.04
				1.8	0.089	60			
0	84	20.2	4.2						

	Decibels below full power output of recovered signal											
M	f_r-5f_m	f_r-4f_m	f_r-3f_m	f_r-2f_m	f_r-f_m	f_r	f_r+f_m	f_r+2f_m	f_r+3f_m	f_r+4f_m	f_r+5f_m	$2f_r$
1.0	67	58	51.5	48	53.5	45	52	48	51.5	58	67	62
0						36						71

All frequency components clustered about $2f_r$ other than those given are more than 70 decibels below the full power output of the recovered signal.

Table VI. Performance of Series-fed Class A PDM Amplifier with L-C Filter

Circuit Configuration					Filter Configuration					
Figure	Tube Type	P_{dr}	R	E_{bb}	Figure	L	C	f_c	f_m	f_r
E-1	12B4	5.5w.	2000 ohms	400v.	4-1	440 mh.	0.055 mfd.	1 K.C.	100 c.p.s.	10 K.C.

	ma.	watts per tube				volts	per cent harmonic distortion								Average
M	I_b	P_s	P_{dp}	P_{op}	η_{1R}	E_o	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	E_{min}, Volts
1.0	65	26	1.5	7.45	0.286	122	5.5	0.87	0.45	0.44	0.13	0.30	0.05	0.14	23
0.8	59	23.6	1.4	4.7	0.199	97	4.9	0.24	0.12		0.04				23
0.5	53	21.2	1.2	1.86	0.088	61	3.2	0.20	0.07	0.05					23
0	40	16	0.9												

	Decibels below full power output of recovered signal											
M	f_r-5f_m	f_r-4f_m	f_r-3f_m	f_r-2f_m	f_r-f_m	f_r	f_r+f_m	f_r+2f_m	f_r+3f_m	f_r+4f_m	f_r+5f_m	$2f_r$
1.0	65	57.5	51	47.5	52	44.5	52	48	52	59	67	64
0						35.5						72

All frequency components clustered about $2f_r$ other than those given are more than 72 decibels below the full power output of the recovered signal.

Table VII. Performance of Class A Single-ended Push-pull PDM Amplifier with Constant-resistance Filter.

Circuit Configuration						Filter Configuration					
Figure	Tube Type	P_{dr}	R/2	$2E_{bb}$	E_{min}	Figure	L	C	f_c	f_m	f_r
C-4d	12B4	5.5w	2000 ohms	460v.	57v.	3-2	440mh.	0.055 mfd.	1 K.C.	100 c.p.s.	10 K.C.

	ma.	watts per tube				volts	per cent harmonic distortion							
M	I_b	P_s	P_{dp}	P_{op}	η_A	E_o	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9
1.0	40.5	9.3	2.5	3.59 3.48	0.361 0.374	119.5 118	1.9	0.53	0.26	0.03	0.08	0.04	0.05	0.07
0.8	40.5	9.3	2.5	2.29 2.21	0.231 0.227	96 94	1.2	0.37	0.12	0.10	0.10	0.07	0.05	0.05
0.5	40	9.2	2.5	0.897 0.87	0.090 0.095	59.8 59	0.64	0.24	0.02	0.05	0.03	0.03	0.02	0.01
0	40	9.2	2.5											

	Decibels below full power output of recovered signal											
M	f_r-5f_m	f_r-4f_m	f_r-3f_m	f_r-2f_m	f_r-f_m	f_r	f_r+f_m	f_r+2f_m	f_r+3f_m	f_r+4f_m	f_r+5f_m	$2f_r$
1.0	67.5	58.5	51.5	48.5	52.5	45.5	53	49	52.5	59.5	68.5	62.5
0						36.5						73

All frequency components clustered about $2f_r$ other than those given are 68.5 decibels or more below the full power output of the recovered signal.

Table VIII. Performance of Class A Single-ended Push-pull PDM Amplifier with L-C Filter.

Circuit Configuration					Filter Configuration					
Figure	Tube Type	P_{dr}	R/2	$2E_{bb}$	Figure	L	C	f_c	f_m	f_r
*E-2	12B4	5.5 w.	2000 ohms	460 v.	4-1	440mh.	0.055 mfd.	1 K.C.	100 c.p.s.	10 K.C.

*The lower filter input terminal is returned to ground through 75 microfarads. The diodes are type 1N660, selected for high inverse breakdown voltage.

M	ma.	watts per tube				volts	per cent harmonic distortion								Average
	I_b	P_s	P_{dp}	P_{op}	η_1	E_0	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	E_{min}, volts
1.0	20.8	5.02	0.52	3.6	0.72	120	1.6	1.95	0.48	0.33	0.07	0.16	0.11	0.11	25
0.8	13.8	3.17	0.35	2.35	0.74	97	1.5	0.96	0.16	0.04	0.07	0.01	0.06		25
0.5	5.7	1.31	0.11	0.96	0.733	62	1.25	0.34	0.12	0.03	0.12	0.09	0.16	0.05	20
0	**0.37	0.085	0.002												5

**Also 0.37 ma. with filter and load removed.

M	Decibels below full power output of recovered signal											
	f_r-5f_m	f_r-4f_m	f_r-3f_m	f_r-2f_m	f_r-f_m	f_r	f_r+f_m	f_r+2f_m	f_r+3f_m	f_r+4f_m	f_r+5f_m	$2f_r$
1.0	64	56	50	46	50	43.5	50.5	46.5	51	57	65.5	60
0						34						67

All frequency components clustered about $2f_r$ other than those given are 67 decibels or more below the full power output of the recovered signal.

Table IX. Performance of Class B PDM Amplifier with Constant-resistance Filter.

Circuit Configuration						Filter Configuration					
Figure	Tube Type	P_{dr}	R	$2E_{bb}$	E_{min}	Figure	L	C	f_c	f_m	f_r
**D-4b	12B4	5.5 w.	2000 ohms	460 v.	60 v.	3-2	440 mh.	0.055 mfd.	1 K.C.	100 c.p.s.	10 K.C.

** The lower filter input terminal is returned to ground through 75 microfarads rather than to E_{bb}.

	ma.	watts per tube				volts	per cent harmonic distortion							
M	I_b	P_s	P_{dp}	P_{op}	η_B	E_0	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9
1.0	25.8	5.93	1.69	3.48 3.48	0.557 0.585	118 118	0.42	0.85	0.07	0.23	0.02	0.10	0.04	0.11
0.8	20.7	4.76	1.35	2.22 2.23	0.444 0.469	94.3 94.5	0.58	0.68	0.04	0.21	0.02	0.12	0.04	0.12
0.5	12.6	2.9	0.845	0.869 0.87	0.278 0.30	59 59	0.67	0.58	0.07	0.35	0.07	0.26	0.02	0.16
0	0	0	0											

	Decibels below full power output of recovered signal										
M	f_r-5f_m	f_r-4f_m	f_r-3f_m	f_r-2f_m	f_r-f_m	f_r	f_r+f_m	f_r+2f_m	f_r+3f_m	f_r+4f_m	f_r+5f_m
1.0	68	*	51.5	*	52.5	*	53	*	52.5	*	69

All frequency components clustered about $2f_r$ are 69 decibels or more below the full power output of the recovered signal.

*All entries of an asterix are more than 72 decibels below the full power output of the recovered signal.

Table X. Performance of Class A Complementary Pair Transistor PDM Amplifier with Constant-resistance Filter.

Circuit Configuration				Filter Configuration					
Figure	R/2	$2E_{bb}$	E_{min}	Figure	L	C	f_c	f_m	f_r
4-3	750 ohms	20 v.	0.1 v.	3-2	176 mh	0.16 mfd.	1 K.C.	100 c.p.s.	10 K.C.

	ma.	milliwatts			volts	per cent harmonic distortion							
M	I_b	P_{in}	P_{op}	η_A	E_0	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9
1.0	6.0	128.6	62.7	0.476	6.86	1.9	0.26	0.14	0.03		0.05		
			62.5	0.487	6.85								
0.8	6.0	128.6	40.2	0.304	5.49	1.3	0.21	0.04	0.02	0.02			
			40.3	0.313	5.5								
0.5	6.0	128.6	15.7	0.119	3.43	0.68	0.18	0.03	0.05	0.03			
			15.7	0.122	3.43								
0	6.0	128.6											

	Decibels below full power output of recovered signal											
M	f_r-5f_m	f_r-4f_m	f_r-3f_m	f_r-2f_m	f_r-f_m	f_r	f_r+f_m	f_r+2f_m	f_r+3f_m	f_r+4f_m	f_r+5f_m	$2f_r$
1.0	68.5	60	53	50.5	55	47.5	55.5	51	54.5	61.5	70.5	65
0						38.5						*

All entries of an asterix are more than 72 decibels below the full power output of the recovered signal.

Table XI. Performance of Class A Complementary Pair Transistor PDM Amplifier with L-C Filter.

Circuit Configuration			Filter Configuration					
Figure	R/2	$2E_{bb}$	Figure	L	C	f_c	f_m	f_r
4-3	750 ohms	20 v.	4-1	176 mh.	0.16 mfd.	1 K.C.	100 c.p.s.	10 K.C.

M	ma. I_b	milliwatts P_{in}	milliwatts P_{op}	η_1	volts E_0	per cent harmonic distortion D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9
1.0	2.9	66.6	62.5	0.938	6.85	1.9	0.6	0.19	0.06				
0.8	1.7	42.6	40.3	0.948	5.5	1.2	0.36	0.06	0.02	0.02			
0.5	0.45	17.6	15.7	0.892	3.43	0.6	0.18	0.03	0.05				
0	-0.3												

M	Decibels below full power output of recovered signal f_r-5f_m	f_r-4f_m	f_r-3f_m	f_r-2f_m	f_r-f_m	f_r	f_r+f_m	f_r+2f_m	f_r+3f_m	f_r+4f_m	f_r+5f_m	$2f_r$
1.0	68.5	60	53	50.5	55	47.5	55.5	51	54	61.5	70.5	65
0						38.5						*

All entries of an asterix are more than 72 decibels below the full power output of the recovered signal.

Circuit Configuration				Filter Configuration				
Figure	Tube Type	R	E_{bb}	Figure	L	C	f_c	f_r
C-4b	12B4	2000 ohms	240 v.	3-2	440 mh	0.055 mfd.	1 K.C.	10 K.C.

Note: The shunt coil is 25 h. with 154 ohms d-c resistance. The modulation factor is M = 1.0 for all measurements.

c.p.s.	ma.	volts		per cent harmonic distortion									Band of plate voltage values during off-time of tube
f_m	I_b	E_{min}	E_o	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	
50	84	48	118	4.7	0.57	0.21	0.14	0.07	0.17	0.04	0.10	0.02	82 volts
100	84	48	120	2.5	0.51	0.19	0.10	0.05	0.13	0.05	0.06	**	42 volts
200	84	48	120	1.8	0.66	0.12	**						18 volts

** At filter cutoff frequency.

Table XIII. Deviation of Measured Values of Power Output and Efficiency from Theoretical Expectation for Amplifiers with Constant-resistance Filters.

PDM Amplifier Type	Table	M	Per cent deviation from theoretical P_{op}	η
Series-fed Class A	IV	1.0	+0.40	+7.6
		0.8	-1.67	+5.2
		0.5	-1.17	+5.67
Shunt-fed Class A	V	1.0	-3.1	-5.98
		0.8	-2.95	-7.6
		0.5	-3.22	-6.3
Push-pull Class A	VII	1.0	-3.06	+3.6
		0.8	-3.5	-2.08
		0.5	-3.01	+5.55
Push-pull Class B	IX	1.0	0	+5.03
		0.8	+0.45	+5.63
		0.5	+0.115	+7.92
Complementary Pair Transistor Configuration	X	1.0	-0.32	+2.31
		0.8	+0.25	+2.96
		0.5	0	+2.52

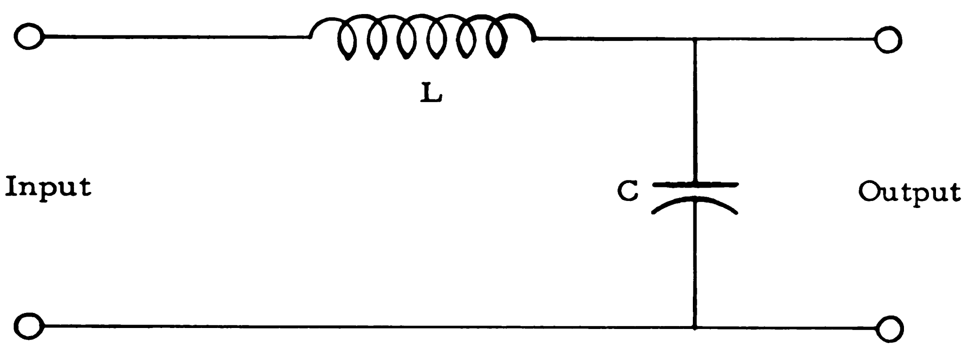


Figure 4-1. L-C low-pass filter.

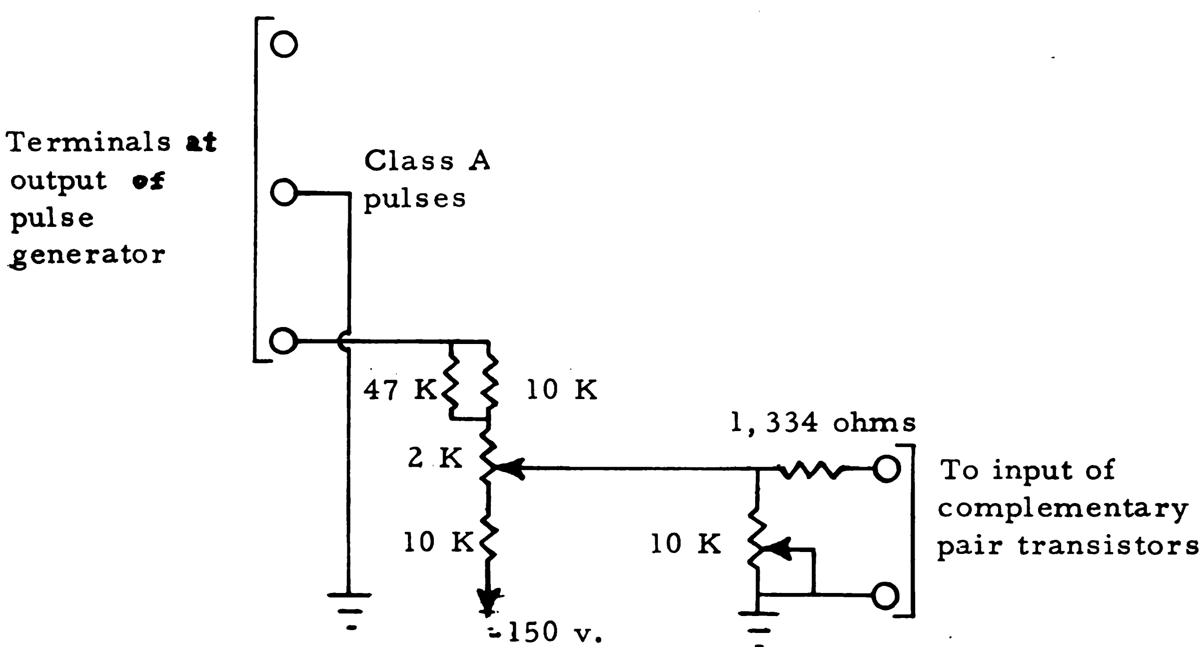


Figure 4-2. Attenuating network between pulse generator and transistor amplifier.

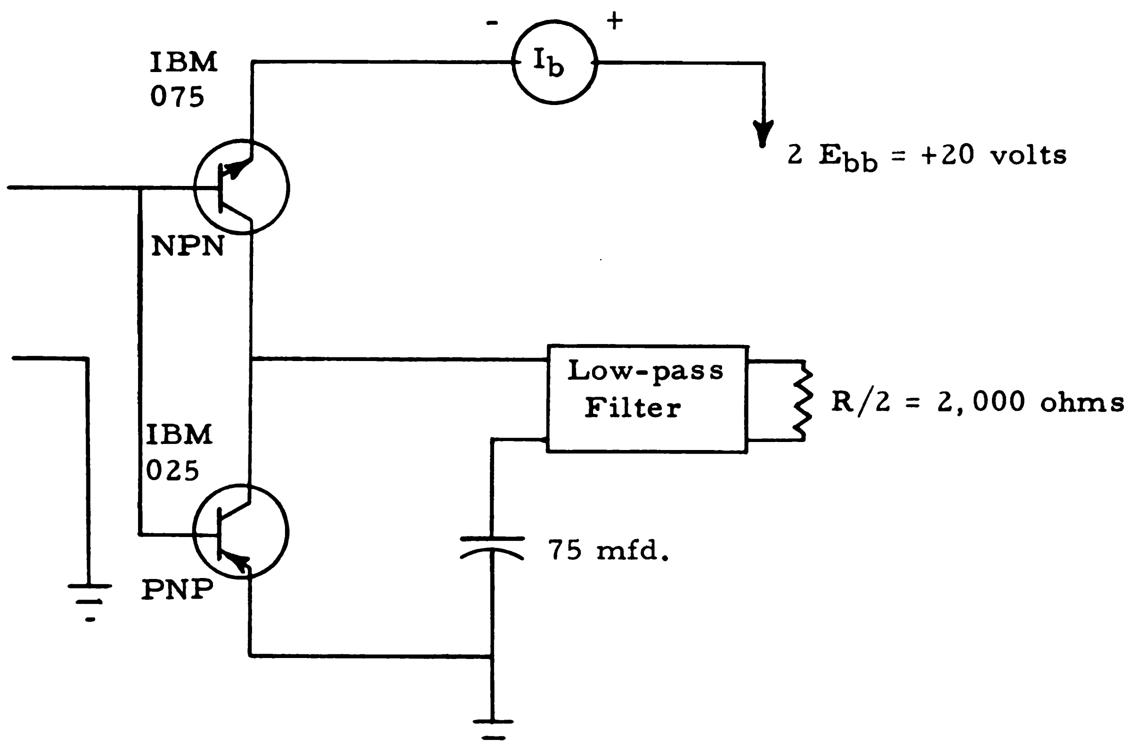


Figure 4-3. Complementary pair transistor configuration.

APPENDIX A

PULSE MODULATION LINEARITY

A-1. Linearity of Modulation of the Monostable Multivibrators (see Figure 2-4).

Circuit parameters are chosen to provide a zero duration pulse when the grid voltage of V3 is at the maximum positive value of an input signal which is of such amplitude as to provide 100 percent modulation. This requires the negative plate voltage excursion of V1 to be equal in magnitude to the cutoff voltage of V2 at this particular instant of the modulation cycle. As the instantaneous value of the modulation voltage at the grid of V3 changes, the clamp voltage for the V1 plate changes. The instantaneous value of the demodulated signal at the output of the low-pass filter is proportional to the pulse duration t_m.

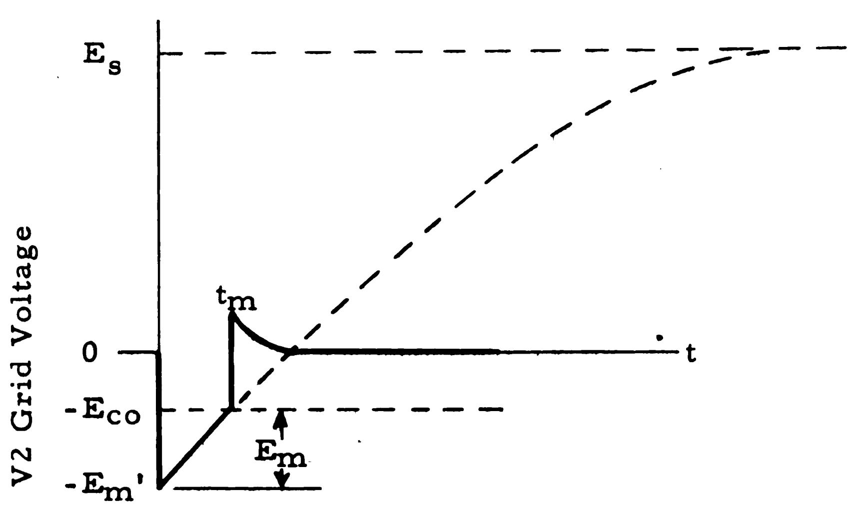


Figure A-1. Grid voltage waveform.

$$E_m = (E_m' - E_{co}) \qquad \text{(A-1)}$$

$$E_m = (E_s + E_{co} + E_m) \; [1 - \epsilon^{-t_m/RC}]$$

Let

$$x_m = t_m/RC \tag{A-2}$$

Then

$$\frac{E_m}{E_s + E_{co} + E_m} = 1 - \epsilon^{-x_m}$$

$$x_m = \ln\left[1 + \frac{E_m}{E_s + E_{co}}\right] \tag{A-3}$$

Let E_{mm} be the maximum value of E_m and let E_m vary sinusoidally in the range $0 \leq E_m \leq E_{mm}$ at a frequency which has a long period compared to the longest pulse duration. Then

$$E_m = \frac{E_{mm}}{2}(1 + \cos \omega_m t) \tag{A-4}$$

$$x_m = \ln\left[1 + \frac{E_{mm}(1 + \cos \theta)}{2(E_s + E_{co})}\right] \tag{A-5}$$

$$t_m = RC \ln\left[1 + \frac{E_{mm}(1 + \cos \theta)}{2(E_s + E_{co})}\right] \tag{A-6}$$

The five point method of finding distortion may be used to find the percent harmonic distortion for the second, third, and fourth harmonics.[2] Let

$$x_1 = x_m\,(0^\circ)$$
$$x_2 = x_m\,(60^\circ)$$
$$x_3 = x_m\,(90^\circ)$$
$$x_4 = x_m\,(120^\circ)$$
$$x_5 = x_m\,(180^\circ)$$

Then since $x_5 = 0$,

$$A_0 = \frac{x_1}{6} + \frac{1}{3}(x_2 + x_4) - x_3$$

$$A_1 = \frac{x_1}{3} + \frac{1}{3}(x_2 - x_4)$$

$$A_2 = \frac{x_1}{4} - \frac{x_3}{2}$$

$$A_3 = \frac{x_1}{6} - \frac{1}{3}(x_2 - x_4)$$

$$A_4 = \frac{x_1}{12} - \frac{1}{3}(x_2 + x_4) + \frac{x_3}{2}$$

$$D_2 = \frac{A_2}{A_1} \times 100\%$$

$$D_3 = \frac{A_3}{A_1} \times 100\%$$

$$D_4 = \frac{A_4}{A_1} \times 100\%$$

$$D = \sqrt{D_2^2 + D_3^2 + D_4^2 + \cdots}$$

In most cases E_{co} may be neglected since it is usually very small compared to E_s.

APPENDIX B

COMPARISON OF SERIES-FED CLASS A AMPLIFIERS

B-1. Series-fed Class A Conventional Amplifier.

It is assumed that the bias is chosen at the value which allows the maximum possible output voltage amplitude for a given R.

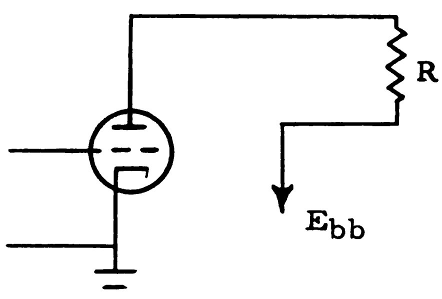


Figure B-1. Series-fed Class A conventional amplifier.

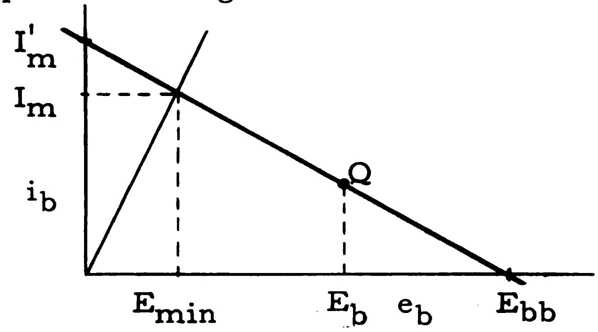


Figure B-2. Load line.

$$I_m = \frac{E_{bb} - E_{min}}{R} \qquad \text{(B-1)}$$

$$E_b = \frac{E_{bb} + E_{min}}{2}$$

Instantaneous signal power delivered to load R is

$$p(t) = \frac{[M\,(\frac{E_{bb} - E_{min}}{2})\,\sin\omega_m t]^2}{R}$$

Let

$$\theta = \omega_m t \qquad \text{(B-2)}$$

Then

$$p(\theta) = \frac{M^2(E_{bb} - E_{min})^2 \sin^2\theta}{4R}$$

Average signal power delivered to R is

$$P_0 = \frac{1}{\pi}\int_0^{\pi} p(\theta)\,d\theta$$

$$= \frac{M^2(E_{bb} - E_{min})^2}{8R} \qquad \text{(B-3)}$$

Average tube current is

$$I_{dc} = \frac{I_m}{2} = \frac{E_{bb} - E_{min}}{2R} \qquad \text{(B-4)}$$

Power supply power is

$$P_s = I_{dc}E_{bb} = \frac{(E_{bb} - E_{min})E_{bb}}{2R} \qquad \text{(B-5)}$$

Plate circuit efficiency is

$$\eta_{AR} = \frac{P_0}{P_s}$$

$$= \frac{M^2}{4}\left[1 - \frac{E_{min}}{E_{bb}}\right] \qquad \text{(B-6)}$$

Maximum P_d occurs when M = 0, and for the best utilization of the tube this maximum P_d should be equal to the rated plate dissipation.

$$P_d = P_s - P_{dc} - P_0 = \frac{E_{bb} - E_{min}}{4R}\left[E_{bb} + E_{min} - \frac{M^2}{2}(E_{bb} - E_{min})\right]$$

$$\text{Maximum } P_d = \frac{(E_{bb} - E_{min})(E_{bb} + E_{min})}{4R} = P_{dr} \qquad \text{(B-7)}$$

$$P_0 = \frac{M^2(E_{bb} - E_{min})}{2(E_{bb} + E_{min})} P_{dr} \qquad \text{(B-8)}$$

$$\text{Maximum } P_0 = \frac{(E_{bb} - E_{min})}{2(E_{bb} + E_{min})} P_{dr} \qquad \text{(B-9)}$$

B-2. Series-fed Class A PDM Amplifier with Constant-resistance Low-pass Filter.

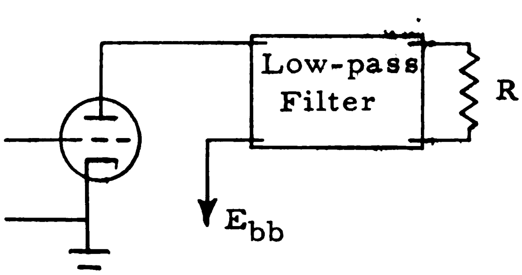


Figure B-3. Series-fed Class A PDM Amplifier.

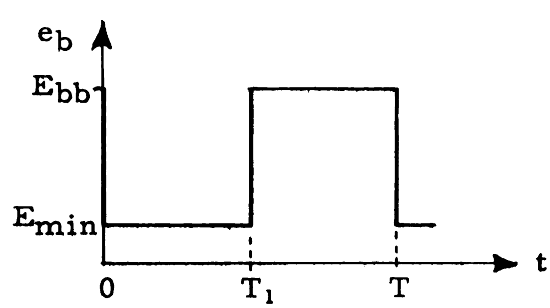


Figure B-4. Plate voltage waveform.

Let

$$a = \frac{T_1}{T} \qquad \text{(B-10)}$$

The voltage waveform at the input of the low-pass filter is identical to the plate voltage waveform of Figure B-4 minus E_{bb}. The time function demodulated voltage at the output of the filter is then

$$e_0(t) = -\ a\ (E_{bb} - E_{min}) \tag{B-11}$$

For Class A sinusoidal pulse duration modulation

$$a = \frac{1}{2}\ (1 + M \sin\theta) \tag{B-12}$$

$$e_0(t) = -\frac{1}{2}(1 + M \sin\theta)(E_{bb} - E_{min}) \tag{B-13}$$

$$i_0(t) = \frac{e_0(t)}{R} \tag{B-14}$$

The average tube current is then

$$I_{dc} = \frac{-1}{2\pi} \int_0^{2\pi} i_0(t)dt$$

$$= \frac{E_{bb} - E_{min}}{4\pi R} \int_0^{2\pi} [1 + M \sin\theta] d\theta$$

$$= \frac{E_{bb} - E_{min}}{2R} \tag{B-15}$$

Power supply power is

$$P_s = I_{dc} E_{bb} = \frac{(E_{bb} - E_{min})E_{bb}}{2R} \tag{B-16}$$

Instantaneous power delivered to load resistor R at the output of the filter is

$$p(t) = \frac{[e_0(t)]^2}{R}$$

$$= \frac{(E_{bb} - E_{min})^2}{4R}\ (1 + 2M \sin\theta + M^2 \sin^2\theta)$$

Average power delivered to R is

$$P'_{op} = \frac{1}{2\pi} \int_0^{2\pi} p(\theta) d\theta$$

$$= \frac{(E_{bb} - E_{min})^2}{8R} (2 + M^2)$$

$$= P_{dc} + P_{op} = I_{dc}^2 R + P_{op}$$

$$= \frac{(E_{bb} - E_{min})^2}{4R} + P_{op}$$

$$\therefore P_{op} = \frac{M^2}{8} \times \frac{(E_{bb} - E_{min})^2}{R} \qquad \text{(B-17)}$$

$$\text{Maximum } P_{op} = \frac{1}{8} \times \frac{(E_{bb} - E_{min})^2}{R} \qquad \text{(B-18)}$$

Plate circuit efficiency is

$$\eta_{AR} = \frac{P_{op}}{P_s} = \frac{M^2}{4} \left[1 - \frac{E_{min}}{E_{bb}} \right] \qquad \text{(B-19)}$$

If E_{min} is set equal to zero, Equation (B-19) expresses the efficiency of the filter and load combination. Therefore

$$\eta_{fAR} = \frac{M^2}{4} \qquad \text{(B-20)}$$

and the tube efficiency is

$$\eta_t = \left[1 - \frac{E_{min}}{E_{bb}} \right] \qquad \text{(B-21)}$$

$$\eta_{AR} = \eta_{fAR} \eta_t \qquad \text{(B-22)}$$

Tube dissipation is independent of M.

The input power to the filter is

$$P_t = \frac{P_{op}}{\eta_{fAR}} = \frac{(E_{bb} - E_{min})^2}{2R}$$

The tube dissipation is

$$P_{dp} = P_s - P_t = \frac{E_{bb} - E_{min}}{2R} [E_{bb} - (E_{bb} - E_{min})]$$

$$= \frac{(E_{bb} - E_{min}) E_{min}}{2R} \qquad \text{(B-23)}$$

r best utilization of the tube, circuit conditions should be chosen so t $P_{dp} = P_{dr}$. Then from Equation (B-17)

$$P_{op} = \frac{M^2(E_{bb} - E_{min})}{4\,E_{min}} \times \frac{(E_{bb} - E_{min})E_{min}}{2R}$$

$$= \frac{M^2}{4}\left[\frac{E_{bb}}{E_{min}} - 1\right] P_{dr} \qquad \text{(B-24)}$$

ssuming that the maximum current rating of the tube is not exceeded, he maximum possible power output is

$$\text{Maximum } P_{op} = \frac{1}{4}\left[\frac{E_{bb}}{E_{min}} - 1\right] P_{dr} \qquad \text{(B-25)}$$

By using the full rated plate dissipation and the same E_{bb} and E_{min} for the two types of amplifiers, the Improvement Ratio for the PDM amplifier over the conventional amplifier for any particular tube is

$$\text{I. R.} = \frac{\text{Maximum } P_{op}}{\text{Maximum } P_0} = \frac{1}{2}\left[\frac{E_{bb}}{E_{min}} + 1\right] \qquad \text{(B-26)}$$

The increase in power handling ability of the PDM amplifier over the conventional amplifier is achieved by using a lower value load resistance. If E_{min}, E_{bb}, R and M are the same in both types of amplifiers, the output power and plate circuit efficiency are the same for both; however, the plate dissipation of the PDM amplifier is less than that of the conventional amplifier by the factor

$$\frac{P_{dp}}{P_d} = \frac{P_{dp}}{(1 - \eta_{AR})P_s} = \frac{E_{min}}{(1 - \eta_{AR})E_{bb}}$$

$$= \frac{2E_{min}}{E_{bb} + E_{min} - \frac{M^2}{2}(E_{bb} - E_{min})} \qquad \text{(B-27)}$$

The tube dissipation of the PDM amplifier is the same for all values of M. The conventional amplifier has its greatest dissipation when M = 0 and it must be designed to handle this. The Dissipation Ratio is

$$D.R. = \frac{\text{Maximum } P_{dp}}{\text{Maximum } P_d} = \frac{2\, E_{min}}{E_{bb} + E_{min}} \qquad \text{(B-28)}$$

From Equation (B-18) it can be seen that the lower the value of R, the greater will be the maximum output power for given values of E_{bb} and E_{min}. The lowest permissible value of R will then be limited either by maximum allowable plate current I_{mr} or by the rated plate dissipation P_{dr}, whichever results in the higher R value. The plate voltage waveform of Figure B-4 and the input resistance R of the low-pass filter result in an on-time tube current of

$$I_m = \frac{E_{bb} - E_{min}}{R} \qquad \text{(B-29)}$$

Since I_m must not be greater than the rated maximum plate current I_{mr},

$$I_m \leq I_{mr}$$

$$\frac{E_{bb} - E_{min}}{R} \leq I_{mr}$$

$$R \geq \frac{E_{bb} - E_{min}}{I_{mr}} \qquad \text{(B-30)}$$

And since P_{dp} must not be greater than P_{dr}, then from Equation (B-23)

$$P_{dp} = \frac{(E_{bb} - E_{min})\, E_{min}}{2R} \leq P_{dr}$$

$$\therefore R \geq \frac{(E_{bb} - E_{min})\, E_{min}}{2\, P_{dr}} \qquad \text{(B-31)}$$

In most cases E_{min} will be partly determined by the value of R. For example it might be convenient to have E_{min} lie on a chosen grid voltage line of the plate characteristic curves such as the zero grid voltage line. Then E_{min} will be determined by the point of intersection of the load line and the chosen grid voltage line. Another line upon which it might be convenient to have E_{min} lie is the diode line.[2] The intersection of this line and the load line would then determine E_{min}. The diode line and grid voltage lines pass through the origin, and each line of practical use is usually straight enough to be expressed algebraically as a line of slope K, where

$$I_m = KE_{min} \tag{B-32}$$

Equations (B-17), (B-29) and (B-32) give

$$P_{op} = \frac{M^2 E_{bb}^2}{8} \times \frac{K^2 R}{(1 + KR)^2} \tag{B-33}$$

Taking the derivative of P_{op} with respect to R and setting it equal to zero shows that the maximum output power occurs when

$$R = \frac{1}{K} \tag{B-34}$$

Power output becomes less as R is made smaller than the value given in Equation (B-34). However, other considerations such as Equation (B-30) and Equation (B-31) might require R to be greater than the value expressed by Equation (B-34). For maximum power output without exceeding the ratings of the tube, R should be chosen as low as possible commensurate with the following inequalities, and E_{bb} should be less than or equal to the maximum allowable plate voltage.

$$R \geq \frac{E_{bb} - E_{min}}{I_{mr}} \tag{B-30}$$

$$R \geq \frac{(E_{bb} - E_{min})E_{min}}{2P_{dr}} \tag{B-31}$$

$$R \geq \frac{1}{K} \tag{B-35}$$

An alternative to the inequality given in Equation (B-35) is the following inequality which results from a combination of Equations (B-29), (B-32) and (B-35).

$$E_{min} \leq \frac{E_{bb}}{2} \qquad \text{(B-36)}$$

It is to be noted that the a-c plate resistance of the tube is no consideration in the choice of R.

If the tube has a screen grid it is necessary to assure that the screen dissipation rating is not exceeded. The on-value screen current may be obtained from the plate characteristic curves once E_{min} and I_m have been determined. Then instantaneous screen grid power is

$$p(t) = \alpha I_{sg} E_{sg}$$

Screen grid dissipation is then

$$P_{sg} = \frac{I_{sg} E_{sg}}{2\pi} \int_0^{2\pi} \alpha \, d\theta = \frac{1}{2} I_{sg} E_{sg} \qquad \text{(B-37)}$$

APPENDIX C

COMPARISON OF SHUNT-FED CLASS A AMPLIFIERS

C-1. Shunt-fed Class A Conventional Amplifier.

It is assumed that the bias is chosen at the value which allows the maximum possible output voltage amplitude for a given R. The circuit configuration of Figure C-1d is known as single-ended Push-pull.[3] When the circuit balance and grid drive are proper for this amplifier, the supply voltage E_{bb} to which the load resistor is returned furnishes no net power but must be able to support currents of both polarities. In cases where d-c amplification is not required, the E_{bb} supply to which the load resistor is returned may be replaced by a large bypass capacitor to ground. This is commonly done. The time constant of the load resistor and capacitor combination would have to be long compared to the period of the lowest frequency to be amplified.

Each tube of the amplifier configurations of Figure C-1 has an effective plate load resistance of R (shunt-fed) and an effective supply voltage of E_{bb}. Therefore Figure C-2 may be used for evaluating the performance of each amplifier in terms of individual tubes. The total output power of any amplifier is then equal to the product of the number of tubes and the power output per tube. At full signal level the maximum current and the minimum plate voltage are the same for all tubes.

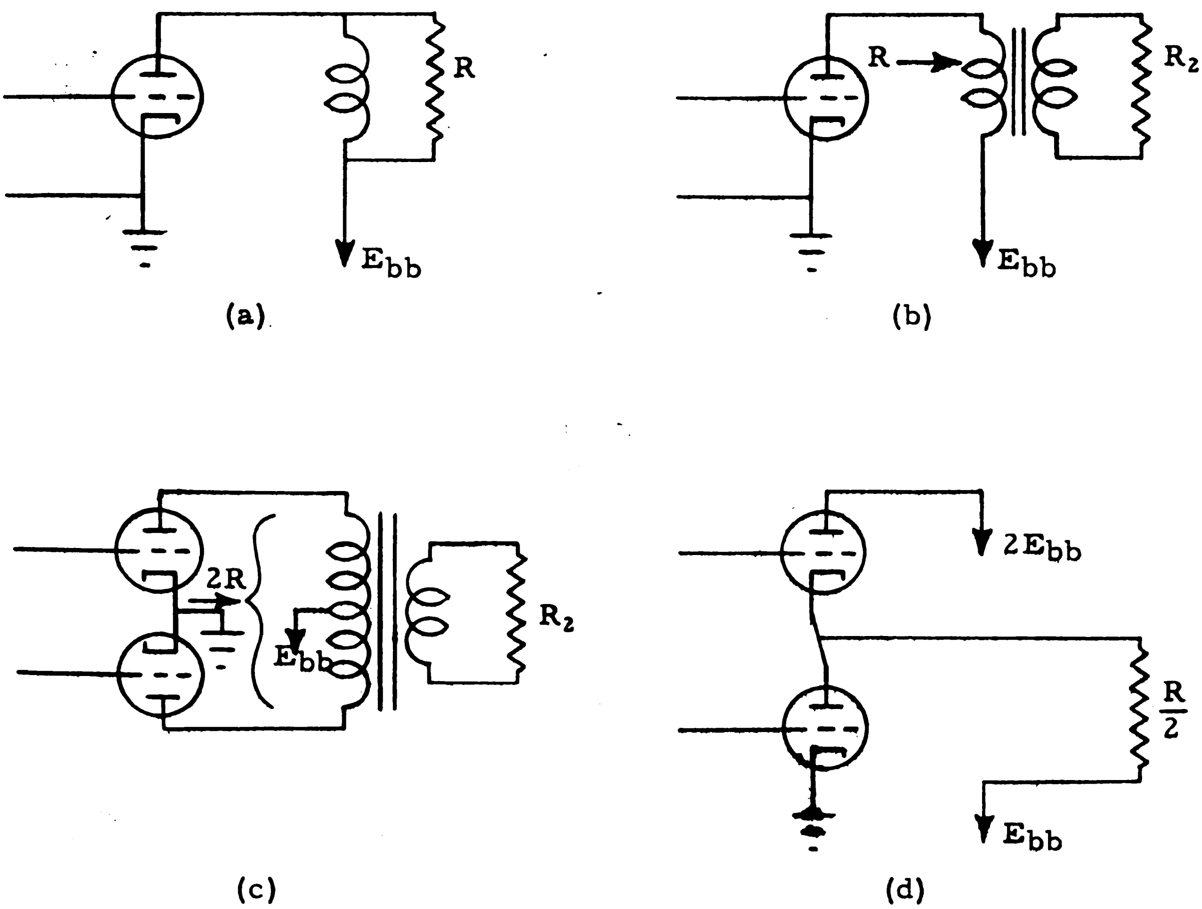


Figure C-1. Shunt-fed Class A conventional amplifier configurations.

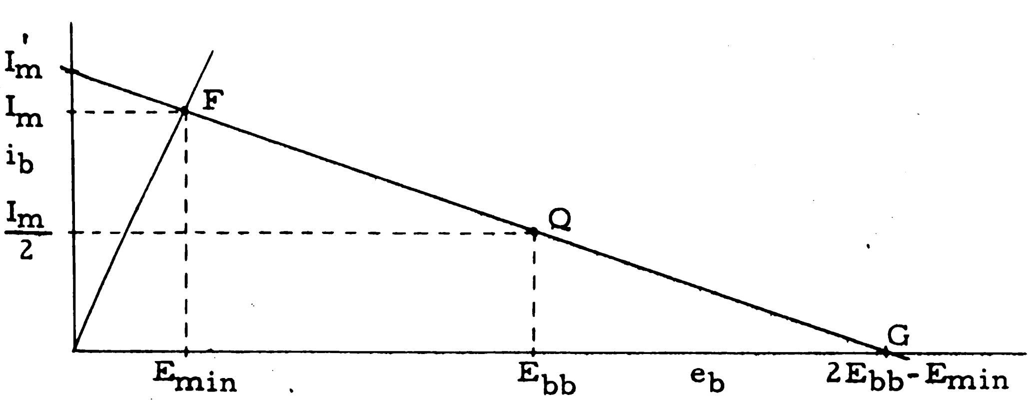


Figure C-2. Load line.

$$R = \frac{2(E_{bb} - E_{min})}{I_m} \tag{C-1}$$

$$I_m = \frac{2(E_{bb} - E_{min})}{R} \tag{C-2}$$

Signal current into R is

$$i(t) = M \frac{I_m}{2} \sin \omega_m t$$

Let

$$\theta = \omega_m t \tag{C-3}$$

Then

$$i(\theta) = M \frac{I_m}{2} \sin \theta$$

Instantaneous output power per tube is

$$p(t) = R \; i^2(t)$$

Average output power per tube is

$$P_0 = \frac{1}{\pi} \int_0^{\pi} p(\theta)\, d\theta = \frac{RM^2 I_m^2}{4\pi} \int_0^{\pi} \sin^2 \theta \, d\theta$$

$$= \frac{M^2 I_m^2 R}{8} = \frac{M^2 (E_{bb} - E_{min})^2}{2R} \tag{C-4}$$

The d-c power supply current per tube is

$$I_{dc} = \frac{I_m}{2} = \frac{E_{bb} - E_{min}}{R} \tag{C-5}$$

The power supply power per tube is

$$P_s = I_{dc} E_{bb} = \frac{E_{bb}(E_{bb} - E_{min})}{R} \tag{C-6}$$

A separate consideration of the circuit of Figure C-1d (or this circuit with the load resistor returned to a capacitor) yields a total power supply power P_{sT} given by the product of $2E_{bb}$ and the d-c current of the upper tube. Then the power supply power per tube is half of P_{sT} and is still given by Equation (C-6).

Plate circuit efficiency is

$$\eta_A = \frac{P_0}{P_s} = \frac{M^2}{2} [1 - \frac{E_{min}}{E_{bb}}] \quad \text{(C-7)}$$

Maximum P_d per tube occurs when M = 0, and for the best utilization of the tubes this maximum P_d should be equal to the rated plate dissipation.

$$\text{Maximum } P_d = P_s = P_{dr} \quad \text{(C-8)}$$

$$P_0 = \eta_A P_s = \frac{M^2}{2} [1 - \frac{E_{min}}{E_{bb}}] P_{dr} \quad \text{(C-9)}$$

$$\text{Maximum } P_0 = \frac{1}{2} [1 - \frac{E_{min}}{E_{bb}}] P_{dr} \quad \text{(C-10)}$$

C-2. Derivation of Average Current Furnished by a Constant-Voltage Source Which is Periodically Switched to a Parallel L-R Combination.

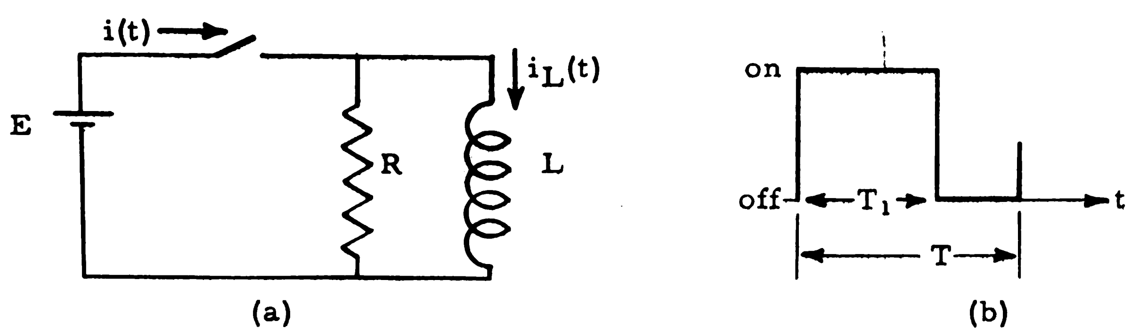


Figure C-3. (a) Switching circuit configuration. (b) Duty cycle for switch.

The voltage across the L-R combination is not zero during the off-time of the switch but is determined by the L-R circuit and the value of coil current at the instant of opening the switch.

$$i(t) = i_L(0) + \frac{E}{R} + \frac{E}{L}\, t \qquad \text{for } 0 < t < T_1$$

$$i(T_1-) = i_L(0) + \frac{E}{R} + \frac{E}{L}\, T_1$$

$$i_L(t) = i_L(T_1)\, \epsilon^{-\frac{R}{L}(t - T_1)} \qquad \text{for } T_1 < t < T$$

$$i_L(T) = i_L(T_1)\, \epsilon^{-\frac{R}{L}(T - T_1)} = i_L(0)$$

$$I_{ave} = \frac{i(0) + i(T_1)}{2} \times \frac{T_1}{T}$$

$$= \frac{i_L(0) + \frac{E}{R} + i_L(0) + \frac{E}{R} + \frac{E}{L}\, T_1}{2} \times \frac{T_1}{T}$$

$$i_L(T_1) = i_L(0) + \frac{E}{L}\, T_1$$

$$i_L(T) = i_L(0) = [i_L(0) + \frac{E}{L}\, T_1]\, \epsilon^{-\frac{R}{L}(T - T_1)}$$

$$i_L(0) = \frac{\frac{E}{L}\, T_1\, \epsilon^{-\frac{R}{L}(T - T_1)}}{1 - \epsilon^{-\frac{R}{L}(T - T_1)}}$$

$$I_{ave} = [i_L(0) + \frac{E}{R} + \frac{E}{2L}\, T_1]\, \frac{T_1}{T}$$

$$= \frac{ET_1}{RT} \left\{ 1 + \frac{RT_1}{2L}\, [1 - \epsilon^{-2\frac{R}{L}(T - T_1)}] \right\} \qquad \text{(C-11)}$$

Let β be defined by

$$T = \beta \times \frac{L}{R} \qquad \text{(C-12)}$$

Let the duty ratio of the switch be

$$a = \frac{T_1}{T} \qquad \text{(C-13)}$$

Then

$$I_{ave} = a\, \frac{E}{R}\, [\, 1 + \frac{a\beta}{2} \times \frac{1 + \epsilon^{-\beta(1-a)}}{1 - \epsilon^{-\beta(1-a)}}\,] \qquad \text{(C-14)}$$

If L is chosen large enough so that the $\frac{L}{R}$ time constant is very large compared to T, then β becomes very small.

Assuming

$$0 \leq a < 1 \tag{C-15}$$

And

$$\frac{L}{R} >> T \tag{C-16}$$

Then

$$I_{ave} = \lim_{\beta \to 0} \left\{ \frac{a E}{R} \left[1 + \frac{a\beta}{2} \times \frac{1 + \epsilon^{-\beta(1-a)}}{1 - \epsilon^{-\beta(1-a)}} \right] \right\}$$

$$= \frac{a E}{R} \left[1 + \frac{a}{2} \times \frac{2}{1-a} \right]$$

$$= \frac{E}{R} \times \frac{a}{1-a} \tag{C-17}$$

The change in coil current during on-time T_1 is given by

$$\Delta i_L(t) = i_L(T_1) - i_L(0) = \frac{E}{L} T_1 = \frac{ET_1T}{LT}$$

$$= a\beta \frac{E}{R} \tag{C-18}$$

It can be seen from Equation (C-18) that the change in coil current is negligible during on-time when Equations (C-15) and (C-16) are satisfied. The current during on-time is then constant and is given by

$$I_m' = \frac{E}{R} \times \frac{1}{1 - a} \tag{C-19}$$

Since the on-current is a constant, the source voltage looks into a resistive load during on-time. Let

$$a = \frac{1}{2} \tag{C-20}$$

Then

$$I_m' = \frac{2E}{R} \tag{C-21}$$

$$I_{ave} = \frac{E}{R} \tag{C-22}$$

Since the on-current I'_m is twice the value that is taken by the resistor alone during on-time, the magnitude of coil current during on-time is equal to the resistor current during on-time. During the time the switch is open the coil current does not change, resulting in a resistor current which is equal in magnitude but opposite in direction to the resistor current during the on-time of the switch. This means that the voltage existing across the L-R combination during off time is -E.
If α is varied sinusoidally about an average value of 1/2 at a rate which is fast enough so that the coil current does not have time to change, the average current will still be given by Equation (C-22) and the magnitude of the voltage excursions across the coil will be the same as with no modulation. When a tube is used for the switch (same circuit as Figure C-1a, but with a square wave grid drive to switch the tube on and off) and when Equations (C-16) and (C-20) are satisfied, the on-time voltage driving the L-R circuit is $-(E_{bb} - E_{min})$ in place of E, the on-time tube current I_m is given by $-I'_m$, and the d-c power supply current is given by $-I_{ave}$. Then

$$I_m = \frac{2(E_{bb} - E_{min})}{R} \qquad (C-23)$$

$$I_{dc} = \frac{(E_{bb} - E_{min})}{R} \qquad (C-24)$$

These equations are identical with Equations (C-2) and (C-5) respectively. If the tube is switched on and off with a Class A PDM signal, Equations (C-23) and (C-24) are still valid. A requirement to be met is that the $\frac{L}{R}$ time constant be long compared to the period of the lowest modulation frequency.

C-3. Shunt-fed Class A PDM Amplifier with Constant-Resistance Low-pass Filter.

The amplifier of Figure C-5b is not a particularly good configuration. A very low repetition rate for the modulated switching voltage

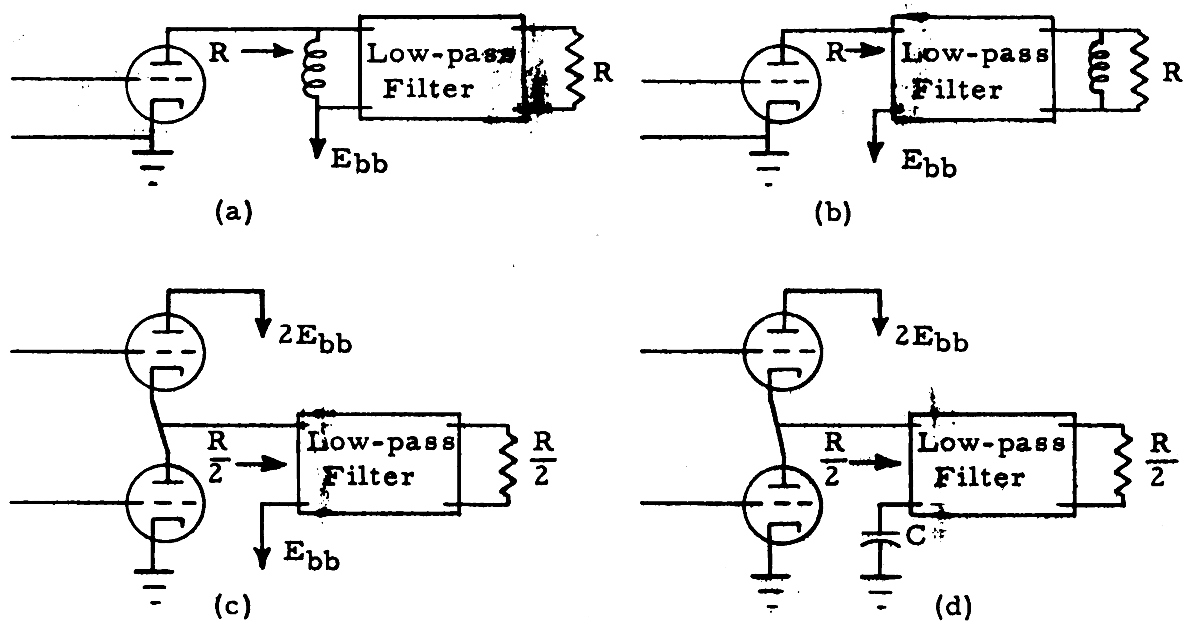


Figure C-4. Shunt-fed Class A PDM amplifier configurations.

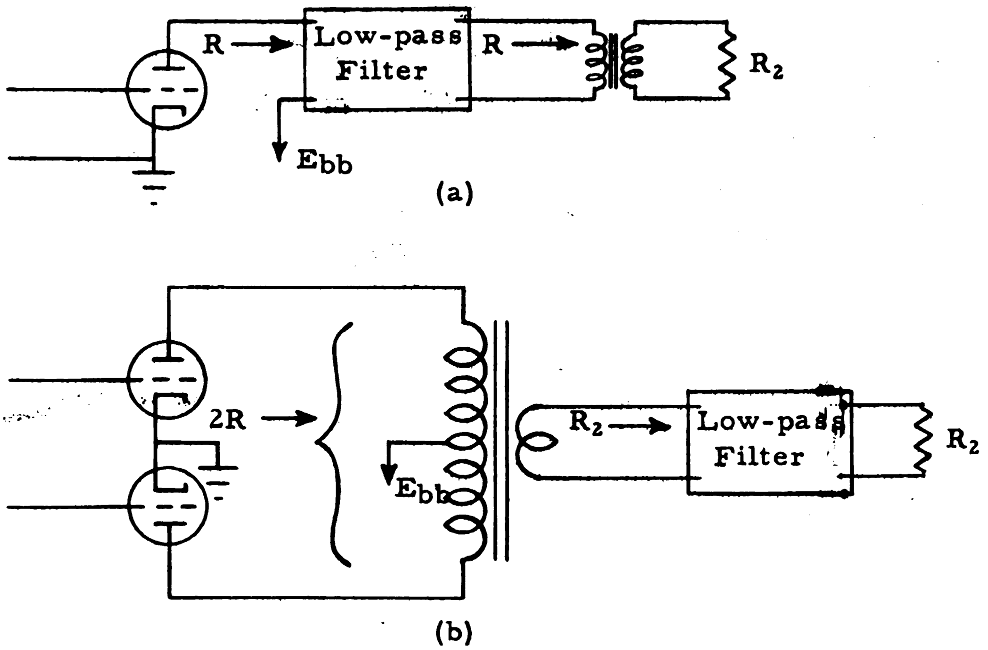


Figure C-5. Transformer coupled push-pull PDM amplifiers.

would be necessary because of output transformer limitations, even with expensive high quality transformers. In addition this is not a d-c amplifier. The range of modulation frequencies is then very limited and the circuit has little utility.

The circuit configuration of Figure C-4b is preferable to that of Figure C-4a. In Figure C-4a the inductor would have to have a large inductance (L/R large relative to the period of the lowest modulating frequency) and an extremely high natural resonant frequency. Shunt capacitance between coil turns presents a low capacitive reactance at frequencies much above resonance, and the inductor must present a high reactance at all frequencies within the spectrum covered by the significant frequency components of the Fourier representation of the PDM signal. The significant frequency components may range to 100 or 200 times the cutoff frequency of the low-pass filter. When placed at the output of the filter, the inductor need present a high reactance only over the range of modulation frequencies, the highest of which is the cutoff frequency of the filter.

The E_{bb} supply to which the filter is returned in Figure C-4c furnishes no net power when the PDM amplifier is modulated with an a-c signal; however, it must be able to handle currents of both polarity. When d-c amplification is not required the circuit of Figure C-4d may be used in place of that of Figure C-4c. The RC product must be long compared to the period of the lowest modulating frequency.

The two single-ended amplifiers of Figure C-4 and the amplifier of Figure C-5a have plate loads which meet the conditions of the derivation of Section C-2. The constant-resistance low-pass filters are the unbalanced type with a common lead between one input terminal and one output terminal. Between the other input terminal and output terminal there is d-c continuity through one or more inductors. The on-current and d-c power supply current are given by Equations (C-23) and (C-24). The on-time plate to cathode voltage for the tube is E_{min}. The off-time plate to cathode voltage is given by the sum of the supply voltage and the voltage E_{bb}-E_{min} which exists across the L-R combination during off-time. The off-time plate voltage is then $2E_{bb}$-E_{min}. It is seen then that during on-time, the operating point of the single-ended PDM amplifiers is given by point F on the load line of Figure C-2. The on-current I_m is the same as the maximum current of a conventional shunt-fed Class A amplifier that has the same plate load resistor R, the same E_{min}, and the same E_{bb}. During off-time the plate to cathode voltage is given by point G and is the same as the plate voltage of the conventional amplifier at the instant the grid is driven to cutoff. The point Q has no meaning for the PDM amplifier since the operating points are either G or F. It can be seen also that the linearity of the demodulated signal at the output of the filter is independent of the characteristic curves of the tube.

Each individual tube of the amplifier configurations of Figure C-4 and Figure C-5 has an effective plate load resistance of R (shunt-fed) and has an effective supply voltage of E_{bb}. For this reason Figure C-2 may be used for evaluating the performance of these amplifiers. Power evaluated on a per tube basis is then multiplied by the number of tubes to obtain total amplifier output power. The plate voltage waveform for Figure C-5a, for the single-ended amplifiers of Figure C-4, and for the lower tubes of the single-ended push-pull amplifiers is given in Figure C-6a. The plate to cathode voltage waveform for the upper tubes of the single-ended push-pull amplifiers is given in Figure C-6b.

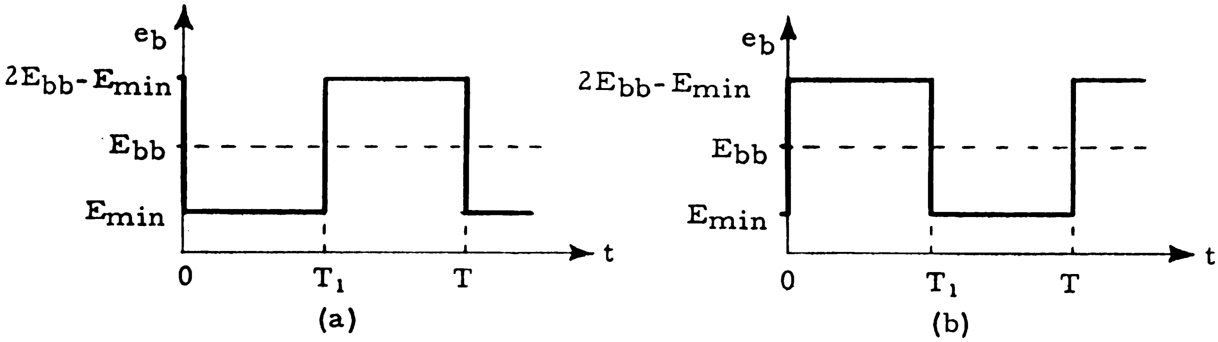


Figure C-6. Plate to cathode voltage waveforms.

$$R = \frac{E_{bb} - E_{min}}{I_m/2} \quad \text{(C-25)}$$

$$I_m = \frac{2(E_{bb} - E_{min})}{R} \quad \text{(C-26)}$$

$$a = T_1/T \quad \text{(C-27)}$$

The voltage waveform at the input of the low-pass filter is identical to the plate voltage waveform of Figure C-6a minus E_{bb}. The time function demodulated voltage at the output of the filter is then

$$e_0(t) = -(2a - 1)(E_{bb} - E_{min}) \quad \text{(C-28)}$$

For Class A sinusoidal pulse duration modulation

$$a = \frac{1}{2}(1 + M\sin\theta) \quad \text{(C-29)}$$

$$e_0(t) = -M(E_{bb} - E_{min})\sin\theta \quad \text{(C-30)}$$

For the single-ended PDM amplifiers the average tube current is given by Equation (C-24).

$$I_{dc} = \frac{E_{bb} - E_{min}}{R} \quad \text{(C-31)}$$

The total power supply power is given by the product of E_{bb} and

Equation (C-31). The power supply power per tube is the same since there is but one tube.

$$P_s = P_{sT} = \frac{E_{bb}(E_{bb} - E_{min})}{R} \qquad \text{(C-32)}$$

The instantaneous power delivered to the load resistor R is

$$p(t) = \frac{[e_0(t)]^2}{R} = \frac{M^2(E_{bb} - E_{min})^2 \sin^2\theta}{R}$$

Average power delivered to load R is

$$P_{op} = \frac{1}{2\pi}\int_0^{2\pi} p(\theta)d\theta$$

$$= \frac{M^2(E_{bb} - E_{min})^2}{2R} \qquad \text{(C-33)}$$

$$\text{Maximum } P_{op} = \frac{(E_{bb} - E_{min})^2}{2R} \qquad \text{(C-34)}$$

Plate circuit efficiency is given by the ratio of P_{op} to P_{sT}.

$$\eta_A = \frac{M^2}{2}\left[1 - \frac{E_{min}}{E_{bb}}\right] \qquad \text{(C-35)}$$

The filter efficiency is found from Equation (C-35) by setting $E_{min} = 0$.

$$\eta_{fA} = \frac{M^2}{2} \qquad \text{(C-36)}$$

The tube efficiency is

$$\eta_t = 1 - \frac{E_{min}}{E_{bb}} \qquad \text{(C-37)}$$

$$\eta_A = \eta_{fA}\,\eta_t \qquad \text{(C-38)}$$

The tube dissipation is independent of M. The input power to the filter is

$$P_t = \frac{P_{op}}{\eta_{fA}} = \frac{(E_{bb} - E_{min})^2}{R}$$

The tube dissipation is

$$P_{dp} = P_s - P_t = \frac{(E_{bb} - E_{min})E_{min}}{R} \qquad \text{(C-39)}$$

For best utilization of the tube, circuit conditions should be chosen so that $P_{dp} = P_{dr}$. Then from Equations (C-33) and (C-39)

$$P_{op} = \frac{M^2}{2}\left[\frac{E_{bb}}{E_{min}} - 1\right]P_{dr} \qquad \text{(C-40)}$$

$$\text{Maximum } P_{op} = \frac{1}{2}\left[\frac{E_{bb}}{E_{min}} - 1\right]P_{dr} \qquad \text{(C-41)}$$

By using the full rated plate dissipation and the same E_{bb} and E_{min} for this amplifier and the conventional shunt-fed single-ended amplifier, the Improvement Ratio for the PDM amplifier over the conventional amplifier for any particular tube is

$$\text{I. R.} = \frac{\text{Maximum } P_{op}}{\text{Maximum } P_o} = \frac{E_{bb}}{E_{min}} \qquad \text{(C-42)}$$

As was the case with the series-fed Class A amplifiers, the increase in power handling ability of the PDM amplifier over the conventional amplifier is achieved by using a lower value load resistance, so that the tube dissipation in both amplifiers is the same. If E_{min}, E_{bb} and M are the same for the two amplifiers and if the load resistor used in the PDM amplifier is the same as that used in the conventional amplifier, then the plate circuit efficiency and power output are the same for both; however, the plate dissipation of the PDM amplifier is less than that of the conventional amplifier by the ratio of P_{dp} of Equation (C-39) to the P_d of the conventional single-ended amplifier.

$$\frac{P_{dp}}{P_d} = \frac{P_{dp}}{(1 - \eta_A)P_s} = \frac{E_{min}}{E_{bb}\left[1 - \frac{M^2}{2}\left(1 - \frac{E_{min}}{E_{bb}}\right)\right]} \qquad \text{(C-43)}$$

The tube dissipation of the PDM amplifier is independent of M, but the tube dissipation of the conventional amplifier is highest when M = 0 and the amplifier must be designed to accommodate this greatest dissipation. The Dissipation Ratio is then

$$D.R. = \frac{\text{Maximum } P_{dp}}{\text{Maximum } P_d} = \frac{E_{min}}{E_{bb}} \tag{C-44}$$

For the push-pull amplifier of Figure C-5b one tube is conducting while the other is off, and either tube during conducting time has an on-current of $\frac{2(E_{bb} - E_{min})}{R}$. The d-c power supply current is then

$$I_{dc} = \frac{2(E_{bb} - E_{min})}{R} \tag{C-45}$$

The total power supply power is given by the product of E_{bb} and the d-c power supply current.

$$P_{sT} = \frac{2E_{bb}(E_{bb} - E_{min})}{R} \tag{C-46}$$

The instantaneous power delivered to the load is

$$p(t) = \frac{2[e_0(t)]^2}{R}$$

Reference to Equation (C-30) yields

$$p(t) = \frac{2M^2(E_{bb} - E_{min})^2 \sin^2\theta}{R}$$

Average power delivered to the load is

$$P_{pT} = \frac{1}{2\pi} \int_0^{2\pi} p(\theta)d\theta$$

$$= \frac{M^2(E_{bb} - E_{min})^2}{R} \tag{C-47}$$

$$\text{Maximum } P_{pT} = \frac{(E_{bb} - E_{min})^2}{R} \tag{C-48}$$

Average output power per tube is

$$P_{op} = \frac{1}{2} P_{pT} = \frac{M^2(E_{bb} - E_{min})^2}{2R} \qquad \text{(C-49)}$$

Plate circuit efficiency is given by the ratio of P_{pT} to P_{sT}

$$\eta_A = \frac{M^2}{2} \left[1 - \frac{E_{min}}{E_{bb}} \right] \qquad \text{(C-50)}$$

This is the same as that given in Equation (C-35). The filter efficiency and the efficiency of the tubes are as given by Equations (C-36) and (C-37) respectively. The dissipation per tube is as given by Equation (C-39) and the output per tube within its dissipation rating is given by Equations (C-40) and (C-41). The Improvement Ratio with maximum utilization of the tubes is given by Equation (C-42). If the PDM amplifier and conventional amplifier are operated with the same R, E_{min}, and E_{bb}, the tube dissipation of the PDM amplifier is less than that of the conventional amplifier by the ratio expressed in Equation (C-43), and the Dissipation Ratio is given by Equation (C-44).

For the single-ended push-pull amplifiers the on-time current for the individual tubes is given by Equation (C-26). The E_{bb} supply furnishes no net power and no d-c current. The average power supply current for the $2E_{bb}$ supply is the same as the average current of the upper tube and is given by

$$I_{dc} = \frac{1}{2\pi} \int_0^{2\pi} I_m(1 - a)d\theta$$

$$= \frac{E_{bb} - E_{min}}{\pi R} \int_0^{2\pi} \left(\frac{1}{2} - \frac{M}{2} \sin\theta \right) d\theta$$

$$= \frac{E_{bb} - E_{min}}{R} \qquad \text{(C-51)}$$

The total power supply power is given by the product of $2E_{bb}$ and the current expressed in Equation (C-51).

$$P_{sT} = \frac{2E_{bb}(E_{bb} - E_{min})}{R} \qquad \text{(C-52)}$$

The instantaneous power delivered to the load is

$$p(t) = \frac{2[e_0(t)]^2}{R}$$

Average power delivered to the load is then the same as is given in Equations (C-47) and (C-48). Average output power per tube is then as given in Equation (C-49). Plate circuit efficiency is as given by Equation (C-50). Filter efficiency is given by Equation (C-36) and the efficiency of the tubes by (C-37). The dissipation per tube is given by Equation (C-39) and the output per tube within its dissipation rating is given by Equations (C-40) and (C-41). Maximum utilization of tubes results in an Improvement Ratio which is the same as that expressed by Equation (C-42). By operating the PDM amplifier with the same R, E_{min}, and supply voltage as is used in a conventional single-ended push-pull amplifier, the tube dissipation of the PDM amplifier is less than that of the conventional amplifier by the ratio expressed in Equation (C-43), and the Dissipation Ratio is the same as that of Equation (C-44).

The following equations then apply to all the PDM amplifier circuits illustrated in Figure C-4 and Figure C-5.

$$I_m = \frac{2(E_{bb} - E_{min})}{R} \qquad \text{(C-53)}$$

$$P_s = \frac{E_{bb}(E_{bb} - E_{min})}{R} \qquad \text{(C-54)}$$

$$P_{op} = \frac{M^2(E_{bb} - E_{min})^2}{2R} = \frac{M^2}{2}\left[\frac{E_{bb}}{E_{min}} - 1\right]P_{dp} \qquad \text{(C-55)}$$

$$\text{Maximum } P_{op} = \frac{(E_{bb} - E_{min})^2}{2R} = \frac{1}{2}\left[\frac{E_{bb}}{E_{min}} - 1\right]P_{dp} \qquad \text{(C-56)}$$

$$\eta_A = \frac{M^2}{2}\left[1 - \frac{E_{min}}{E_{bb}}\right] \quad \text{(C-57)}$$

$$\eta_{fA} = \frac{M^2}{2} \quad \text{(C-58)}$$

$$\eta_t = 1 - \frac{E_{min}}{E_{bb}} \quad \text{(C-59)}$$

$$P_{dp} = \frac{(E_{bb} - E_{min})E_{min}}{R} \quad \text{(C-60)}$$

$$I.R. = \frac{E_{bb}}{E_{min}} \quad \text{(C-61)}$$

$$D.R. = \frac{E_{min}}{E_{bb}} \quad \text{(C-62)}$$

When R is chosen to provide maximum utilization of the plate dissipation rating, the following equations are valid.

$$P_{op} = \frac{M^2}{2}\left[\frac{E_{bb}}{E_{min}} - 1\right]P_{dr} \quad \text{(C-63)}$$

$$\text{Maximum } P_{op} = \frac{1}{2}\left[\frac{E_{bb}}{E_{min}} - 1\right]P_{dr} \quad \text{(C-64)}$$

Equation (C-56) indicates that the lower the value of R, the greater will be the maximum output power for given values of E_{bb} and E_{min}. The lowest permissible value of R is limited by either maximum allowable plate current I_{mr} or by the rated plate dissipation P_{dr}, whichever results in the higher R value. Since

$$I_m \leq I_{mr}$$

Then from Equation (C-53)

$$R \geq \frac{2(E_{bb} - E_{min})}{I_{mr}} \quad \text{(C-65)}$$

Since

$$P_{dp} \leq P_{dr}$$

Then from Equation (C-60)

$$R \geq \frac{(E_{bb} - E_{min})E_{min}}{P_{dr}} \qquad \text{(C-66)}$$

The discussion in Section B-2 of Appendix B relative to Equation (B-32) is equally applicable here. Let

$$I_m = K\,E_{min} \qquad \text{(C-67)}$$

Equations (C-53), (C-55) and (C-67) give

$$P_{op} = \frac{M^2 E_{bb}^2}{2} \times \frac{K^2 R}{(2 + KR)^2} \qquad \text{(C-68)}$$

Taking the derivative of P_{op} with respect to R and setting it equal to zero shows that the maximum output power occurs when

$$R = \frac{2}{K} \qquad \text{(C-69)}$$

Then in order to obtain maximum output power without exceeding the ratings of the tube, R should be chosen as low as possible with due consideration for the following inequalities, and E_{bb} should be chosen in accordance with the recommended plate voltage for the tube. The peak plate voltage $2E_{bb} - E_{min}$ should not exceed the rated peak plate voltage of the tube.

$$R \geq \frac{2(E_{bb} - E_{min})}{I_{mr}} \qquad \text{(C-65)}$$

$$R \geq \frac{(E_{bb} - E_{min})E_{min}}{P_{dr}} \qquad \text{(C-66)}$$

$$R \geq \frac{2}{K} \qquad \text{(C-70)}$$

An alternative to the inequality given in Equation (B-70) is the following inequality which results from a combination of Equations (C-53), (C-67) and (C-70).

$$E_{min} \leq \frac{E_{bb}}{2} \qquad \text{(C-71)}$$

As was pointed out for the series-fed Class A PDM amplifier, the a-c plate resistance of the tube is no consideration in the choice of R.

The considerations for tubes with screen grids are the same as those discussed in Section B-2 of Appendix B for series-fed amplifiers, and Equation (B-37) applies. For single-ended push-pull configurations E_{sg} must be considered to be the screen to cathode voltage of a tube when it is on or conducting.

In conventional single-ended push-pull amplifiers provision must be made for keeping the screen grid of the upper tube at a fixed voltage with respect to the cathode or degeneration will result. This problem does not exist with single-ended push-pull PDM amplifiers since the tube currents and voltages do not change during on-time. This is a definite advantage for PDM amplifiers of this circuit configuration. A disadvantage however is the fact that the screen to cathode voltage of the upper tube is very high during the off-time of the tube. Screen dissipation is no problem during the time the tube is cut off as there is no screen current, but the large interelectrode voltages require the use of tubes with very high screen voltage ratings. In addition, the cutoff voltage for a tube with a given plate to cathode voltage will be higher than would be the case with a lower screen to cathode voltage, and a greater amplitude grid-driving signal will be required in order to assure cutting off the tube.

APPENDIX D

COMPARISON OF CLASS B AMPLIFIERS

D-1. Class B Conventional Amplifier.

Figure D-1 shows the two important circuit configurations for Class B push-pull amplifiers.

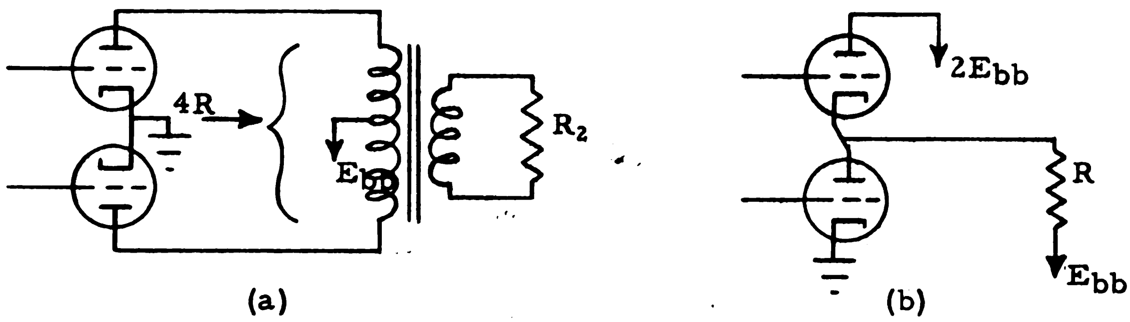


Figure D-1. (a) Common push-pull configuration. (b) Single-ended push-pull.

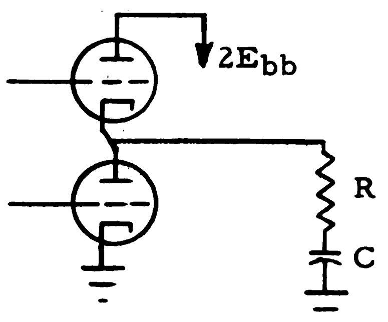


Figure D-2. Single-ended push-pull a-c amplifier.

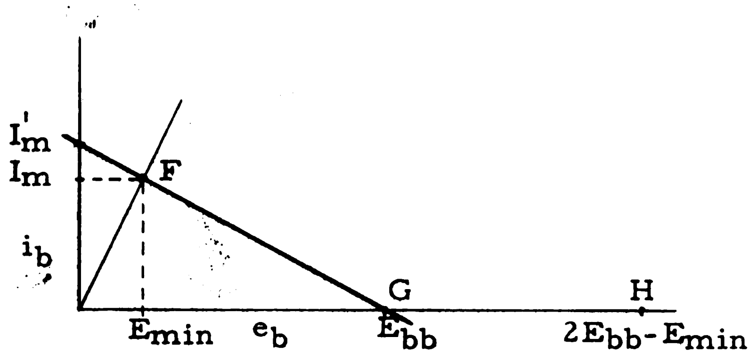


Figure D-3. Load line.

When the circuit of Figure D-1b has an appropriate a-c driving signal, the E_{bb} supply to which the load resistor is returned furnishes no d-c current and no net power; however, it must be able to handle currents of both polarities. When d-c amplification is not necessary, the circuit of Figure D-2 is commonly used in place of that of Figure D-1b. For all the amplifiers shown, each tube individually has an effective plate load resistance of R and an effective supply voltage of E_{bb}.

$$I_m = \frac{E_{bb} - E_{min}}{R} \tag{D-1}$$

The instantaneous power delivered to the load is

$$p(t) = \frac{[M(E_{bb} - E_{min})\sin\omega_m t]^2}{R}$$

Let

$$\theta = \omega_m t \tag{D-2}$$

Then

$$p(\theta) = \frac{M^2(E_{bb} - E_{min})^2 \sin^2\theta}{R}$$

Average power delivered to the load is

$$P_{oT} = \frac{1}{\pi}\int_0^\pi p(\theta)d\theta = \frac{M^2(E_{bb} - E_{min})^2}{2R} \tag{D-3}$$

Average tube current is

$$I_{dc} = \frac{MI_m}{2\pi}\int_0^\pi \sin\theta\, d\theta = \frac{MI_m}{\pi} = \frac{M(E_{bb} - E_{min})}{\pi R} \tag{D-4}$$

The average power supply current for Figure D-1a is twice the value given by Equation (D-4). Total power supply power for Figure D-1a is given by the product of E_{bb} and the average power supply current.

$$P_{sT} = \frac{2M(E_{bb} - E_{min})E_{bb}}{\pi R} \tag{D-5}$$

For the single-ended push-pull configurations the total power supply power is given by the product of the average current for the upper tube and $2E_{bb}$. This power then is also given by Equation (D-5). Plate circuit efficiency is

$$\eta_B = \frac{P_{oT}}{P_{sT}} = \frac{M\pi}{4}\left[1 - \frac{E_{min}}{E_{bb}}\right] \tag{D-6}$$

Total power dissipated in the tubes is

$$P_{dT} = P_{sT} - P_{oT} = \frac{2M(E_{bb}-E_{min})E_{bb}}{\pi R} - \frac{M^2(E_{bb}-E_{min})^2}{2R} \tag{D-7}$$

To find the maximum P_{dt}, set $\frac{dP_{dT}}{dM} = 0$ and obtain

$$M = \frac{2}{\pi\left[1 - \frac{E_{min}}{E_{bb}}\right]} \tag{D-8}$$

Two cases arise, since we must have

$$0 \leq M \leq 1 \tag{D-9}$$

Case 1:

When $\left[1 - \frac{E_{min}}{E_{bb}}\right] \geq \frac{2}{\pi}$, maximum P_{dT} occurs at

$$M = \frac{2}{\pi\left[1 - \frac{E_{min}}{E_{bb}}\right]} \tag{D-10}$$

Then

$$\text{Maximum } P_{dT} = (1 - \eta_B)\, P_{sT} = \frac{1}{2}\, P_{sT} \tag{D-11}$$

$$= \frac{2E_{bb}^2}{\pi^2 R} \tag{D-12}$$

The dissipation P_d per tube is half the total dissipation P_{dT}. Then

$$\text{Maximum } P_d = \frac{E_{bb}^2}{\pi^2 R} \tag{D-13}$$

The dissipation per tube should not exceed the rated dissipation P_{dr} per tube.

$$\text{Maximum } P_d \leq P_{dr} \tag{D-14}$$

$$P_{dr} \geq \frac{E_{bb}^2}{\pi^2 R} \tag{D-15}$$

For best utilization of the tubes the maximum P_d should be equal to P_{dr}.

$$\text{Maximum } P_d = \frac{E_{bb}^2}{\pi^2 R} = P_{dr} \tag{D-16}$$

$$P_0 = \frac{1}{2}\, P_{oT} = \frac{M^2(E_{bb}-E_{min})^2}{4R} \tag{D-17}$$

$$= \frac{M^2 \pi^2}{4} [1 - \frac{E_{min}}{E_{bb}}]^2 \; P_{dr} \tag{D-18}$$

$$\text{Maximum } P_0 = \frac{\pi^2}{4} [1 - \frac{E_{min}}{E_{bb}}]^2 \; P_{dr} \tag{D-19}$$

Case 2:

When $[1 - \frac{E_{min}}{E_{bb}}] \leq \frac{2}{\pi}$, maximum P_{dT} occurs at

$$M = 1 \tag{D-20}$$

Then

$$\text{Maximum } P_{dT} = (1 - \eta_B)P_{sT}$$

$$= \frac{2(E_{bb} - E_{min})}{\pi R} [E_{bb} - \frac{\pi}{4} (E_{bb} - E_{min})] \tag{D-21}$$

The dissipation per tube should not exceed the rated dissipation per tube.

$$P_{dr} \geq \frac{(E_{bb} - E_{min})}{\pi R} [E_{bb} - \frac{\pi}{4} (E_{bb} - E_{min})] \tag{D-22}$$

For best utilization of the tubes the maximum P_d should be equal to P_{dr}.

$$\text{Maximum } P_d = P_{dr} \tag{D-23}$$

$$P_0 = \frac{1}{2} \; P_{0T} = \frac{M^2(E_{bb} - E_{min})^2}{4R} \tag{D-24}$$

$$= \frac{M^2 \pi (E_{bb} - E_{min})}{4[E_{bb} - \frac{\pi}{4} (E_{bb} - E_{min})]} \times \; P_{dr} \tag{D-25}$$

$$\text{Maximum } P_0 = \frac{\pi (E_{bb} - E_{min})}{4[E_{bb} - \frac{\pi}{4} (E_{bb} - E_{min})]} \times \; P_{dr} \tag{D-26}$$

Case 1 and Case 2 both give the same result when

$$[1 - \frac{E_{min}}{E_{bb}}] = \frac{2}{\pi} \tag{D-27}$$

Then

$$P_{dr} = \frac{E_{bb}^2}{\pi^2 R} \tag{D-28}$$

$$P_{oT} = \frac{2M^2 E_{bb}^2}{\pi^2 R} = 2M^2 P_{dr} \tag{D-29}$$

$$P_o = \frac{M^2 E_{bb}^2}{\pi^2 R} = M^2 P_{dr} \tag{D-30}$$

$$P_{sT} = \frac{4M E_{bb}^2}{\pi^2 R} = 4M P_{dr} \tag{D-31}$$

$$P_{dT} = \frac{2(2-M)M E_{bb}^2}{\pi^2 R} = 2M(2-M) P_{dr} \tag{D-32}$$

$$P_d = \frac{(2-M)M E_{bb}^2}{\pi^2 R} = M(2-M) P_{dr} \tag{D-33}$$

$$\text{Maximum } P_{dT} = \frac{2E_{bb}^2}{\pi^2 R} = 2P_{dr} \tag{D-34}$$

$$\text{Maximum } P_d = \frac{E_{bb}^2}{\pi^2 R} = P_{dr} \tag{D-35}$$

$$\text{Maximum } P_o = P_{dr} \tag{D-36}$$

$$\text{Maximum } P_{oT} = 2P_{dr} \tag{D-37}$$

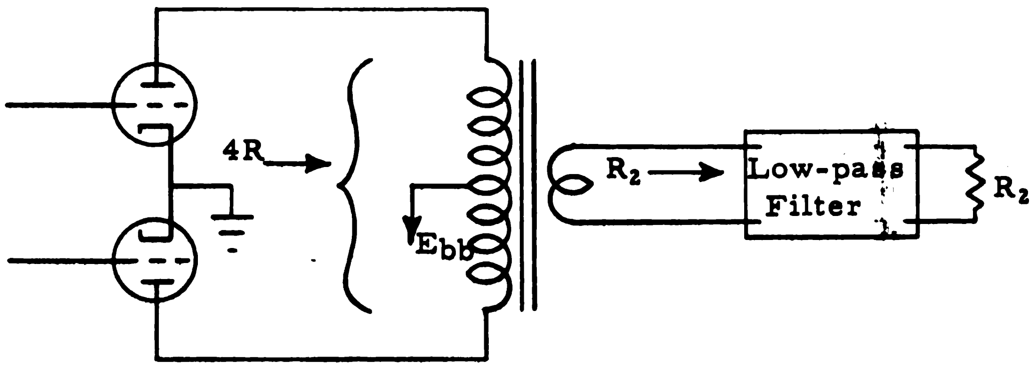
$$\eta_B = \frac{M}{2} \tag{D-38}$$

$$\text{Maximum } \eta_B = \frac{1}{2} \tag{D-39}$$

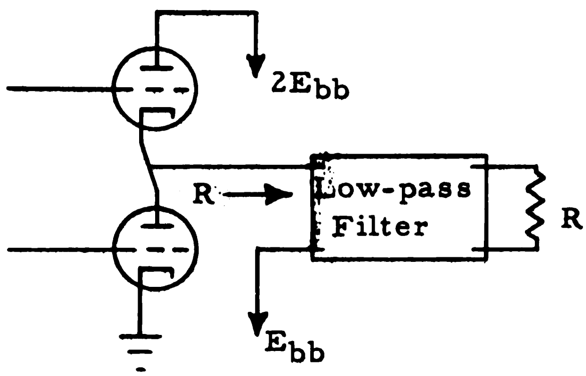
D-2. Class B PDM Amplifier with Constant-resistance Low-pass Filter.

For reasons discussed in Section C-3 in conjunction with Figure C-5b, the PDM amplifier of Figure D-4a is not a particularly good one.

When d-c amplification is not required, the E_{bb} supply to which the filter is returned in Figure D-4b may be replaced by a large capacitor to ground. The discussion of Section C-3 relative to the E_{bb} power supply requirements and the use of a capacitor applies equally well to the circuit of Figure D-4b with Class B modulation.



(a)



(b)

Figure D-4. (a) Transformer coupled Class B PDM amplifier. (b) Single-ended push-pull Class B PDM amplifier.

Each individual tube of the amplifier configurations of Figure D-4 has an effective plate load resistance of R and an effective supply voltage of E_{bb}. Thus Figure D-3 may be used for evaluating the performance of these amplifiers in terms of individual tubes. Output power evaluated on a per tube basis is then doubled to obtain total output power. During on-time the operating point of the PDM amplifier tubes is given by point F on the load line of Figure D-3. The on-current I_m is the same as the maximum current of a conventional Class B amplifier which uses the same load resistor and has the same E_{min} and E_{bb}. During off-time the plate to cathode voltage of the PDM amplifier tubes is given by point G or H, depending upon whether the other tube is conducting or not. As was the case with Class A PDM amplifiers, it can be seen that the linearity of the demodulated signal at the output of the filter is independent of the characteristic curves of the tube. During the half cycle of modulation in which a tube is switched on and off, the on-time plate to cathode voltage is E_{min} and the off-time plate to cathode voltage is E_{bb}. During the other half of the modulation cycle there are plate to cathode voltage excursions between E_{bb} and $2E_{bb}-E_{min}$ but the tube does not conduct.

$$\theta = \omega_m t \tag{D-40}$$

$$a = \frac{1}{2}[\, 1 + M\sin\theta \,] \tag{D-41}$$

The magnitude of voltage excursion at the input to the filter is $E_{bb} - E_{min}$ during both half cycles of modulation, the sign being different during each half cycle. The time function demodulated voltage at the output of the filter is then

$$e_0(\theta) = -(2a - 1)(E_{bb} - E_{min}) \tag{D-42}$$

$$= - M(E_{bb} - E_{min})\sin\theta \tag{D-43}$$

Average tube current is given by

$$I_{dc} = - \frac{1}{2\pi R} \int_0^{\pi} e_0(\theta)d\theta \tag{D-44}$$

$$= \frac{M(E_{bb}-E_{min})}{\pi R} \tag{D-45}$$

The total power supply current for Figure D-4a is twice the value given by Equation (D-45). The product of E_{bb} and the total power supply current yields the total power supply power for Figure D-4a.

$$P_{sT} = \frac{2ME_{bb}(E_{bb}-E_{min})}{\pi R} \qquad \text{(D-46)}$$

The total power supply power for Figure D-4b is given by the product of $2E_{bb}$ and the d-c current of the upper tube since the E_{bb} supply to which the filter is returned furnishes no net power. This total power supply power is then also given by Equation (D-46). The instantaneous power delivered to the load is

$$p(t) = \frac{[e_0(t)]^2}{R} = \frac{M^2(E_{bb}-E_{min})^2 \sin^2\theta}{R}$$

Average power delivered to the load is

$$P_{pT} = \frac{1}{2\pi} \int_0^{2\pi} p(\theta)\, d\theta$$

$$= \frac{M^2(E_{bb}-E_{min})^2}{2R} \qquad \text{(D-47)}$$

$$\text{Maximum } P_{pT} = \frac{(E_{bb}-E_{min})^2}{2R} \qquad \text{(D-48)}$$

Plate circuit efficiency is given by the ratio of P_{pT} to P_{sT}

$$\eta_B = \frac{M\pi}{4} \left[1 - \frac{E_{min}}{E_{bb}}\right] \qquad \text{(D-49)}$$

The filter efficiency is found from Equation (D-49) by setting $E_{min} = 0$.

$$\eta_{fB} = \frac{M\pi}{4} \qquad \text{(D-50)}$$

The efficiency of the tubes is

$$\eta_t = 1 - \frac{E_{min}}{E_{bb}} \qquad \text{(D-51)}$$

$$\eta_B = \eta_{fB}\eta_t \qquad \text{(D-52)}$$

The tube efficiency is independent of M. The input power to the filter is

$$P_t = \frac{P_{pT}}{\eta_{fB}} = \frac{2M(E_{bb}-E_{min})^2}{\pi R} \qquad \text{(D-53)}$$

The tube dissipation is

$$P_{dT} = P_{sT} - P_t = \frac{2M(E_{bb} - E_{min})E_{min}}{\pi R} \qquad \text{(D-54)}$$

$$\text{Maximum } P_{dT} = \frac{2(E_{bb} - E_{min})E_{min}}{\pi R} \qquad \text{(D-55)}$$

This maximum total tube dissipation should not exceed twice the rated dissipation per tube

$$\text{Maximum } P_{dT} \leq 2P_{dr} \qquad \text{(D-56)}$$

$$\text{Maximum } P_{dp} \leq P_{dr} \qquad \text{(D-57)}$$

For best utilization of the tubes, circuit conditions should be chosen so that maximum $P_{dp} = P_{dr}$. Then from Equations (D-47) and (D-55)

$$P_{pT} = \frac{M^2 \pi}{2} \left[\frac{E_{bb}}{E_{min}} - 1\right] P_{dr} \qquad \text{(D-58)}$$

$$P_{op} = \frac{M^2 \pi}{4} \left[\frac{E_{bb}}{E_{min}} - 1\right] P_{dr} \qquad \text{(D-59)}$$

$$\text{Maximum } P_{pT} = \frac{\pi}{2} \left[\frac{E_{bb}}{E_{min}} - 1\right] P_{dr} \qquad \text{(D-60)}$$

$$\text{Maximum } P_{op} = \frac{\pi}{4} \left[\frac{E_{bb}}{E_{min}} - 1\right] P_{dr} \qquad \text{(D-61)}$$

There are two cases to consider in evaluating the Improvement Ratio for the Class B PDM amplifier over the conventional amplifier. In each case the same E_{bb} and E_{min} are used for both amplifiers and the maximum tube dissipation for each amplifier is the same, though the maximum tube dissipation may not occur at the same fraction of maximum permissible signal level in the two amplifiers. As with the Class A amplifiers, the greater output power ability of the Class B PDM amplifier over the conventional Class B amplifier results from using a lower value load resistor.

Case 1:

$$1 - \frac{E_{min}}{E_{bb}} \geq \frac{2}{\pi} \tag{D-62}$$

$$I.R._1 = \frac{\text{Maximum } P_{pT}}{\text{Maximum } P_{oT}} = \frac{E_{bb}^2}{\pi E_{min}(E_{bb} - E_{min})} \tag{D-63}$$

Case 2:

$$1 - \frac{E_{min}}{E_{bb}} \leq \frac{2}{\pi} \tag{D-64}$$

$$I.R._2 = \frac{\text{Maximum } P_{pT}}{\text{Maximum } P_{oT}} = \frac{E_{bb}}{E_{min}}(1 - \frac{\pi}{4}) + \frac{\pi}{4} \tag{D-65}$$

When

$$1 - \frac{E_{min}}{E_{bb}} = \frac{2}{\pi} \tag{D-66}$$

Then

$$I.R. = \frac{E_{bb}}{2E_{min}} = \frac{\pi}{2(\pi - 2)} = 1.376 \tag{D-67}$$

The Improvement Ratio is greater than one under all circumstance since E_{min} can never be chosen greater than E_{bb}. A choice of $E_{min} = E_{bb}$ results in an Improvement Ratio of one, but the output power of either type amplifier is then zero. In a practical amplifier of either type E_{min} will probably never exceed half of E_{bb}. A choice of $E_{min} = 0.1\ E_{bb}$ is practical and results in an I. R. = 3.537. When E_{min}, E_{bb}, and M are the same for both types of amplifiers and when the load resistors are the same, the output power and plate circuit efficiency are the same for both. Two cases must be considered in an evaluation of the dissipation ratio.

Case 1:

$$1 - \frac{E_{min}}{E_{bb}} \geq \frac{2}{\pi} \tag{D-62}$$

$$D.R._1 = \frac{\text{Maximum } P_{dp}}{\text{Maximum } P_d} = \frac{\pi E_{min}(E_{bb} - E_{min})}{E_{bb}^2} \tag{D-68}$$

Case 2:

$$1 - \frac{E_{min}}{E_{bb}} \leq \frac{2}{\pi} \qquad \text{(D-64)}$$

$$D.R._2 = \frac{\text{Maximum } P_{dp}}{\text{Maximum } P_d} = \frac{E_{min}}{E_{bb} - \frac{\pi}{4}(E_{bb} - E_{min})} \qquad \text{(D-69)}$$

When

$$1 - \frac{E_{min}}{E_{bb}} = \frac{2}{\pi} \qquad \text{(D-66)}$$

Then

$$D.R. = 2\frac{E_{min}}{E_{bb}} = 2 - \frac{4}{\pi} = 0.7268 \qquad \text{(D-70)}$$

When $1 - \frac{E_{min}}{E_{bb}} \leq \frac{2}{\pi}$ the maximum tube dissipation occurs at maximum signal level for both types of amplifiers. When $1 - \frac{E_{min}}{E_{bb}} > \frac{2}{\pi}$ the maximum tube dissipation occurs at a signal level which is less than the maximum signal level in the conventional Class B amplifier.

Equation (D-48) indicates that the lower the value of R, the greater will be the maximum output power for given values of E_{bb} and E_{min}. The lowest permissible value of R is limited either by the maximum allowable plate current I_{mr} or by the rated plate dissipation P_{dr}, whichever results in the higher R value. In accordance with Figure D-3 it is seen that the tube current during on-time is

$$I_m = \frac{E_{bb} - E_{min}}{R} \qquad \text{(D-71)}$$

Since

$$I_m \leq I_{mr}$$

Then from Equation (D-71)

$$R \geq \frac{E_{bb} - E_{min}}{I_{mr}} \qquad \text{(D-72)}$$

In addition, the maximum dissipation per tube must not exceed the rated dissipation per tube.

Maximum $P_{dp} \leq P_{dr}$

Then from Equation (D-55)

$$R \geq \frac{(E_{bb} - E_{min})E_{min}}{\pi P_{dr}} \tag{D-73}$$

The discussion in Section B-2 of Appendix B relative to Equation (B-32) is applicable here. Let

$$I_m = KE_{min} \tag{D-74}$$

Equations (D-47), (D-71) and (D-74) give

$$P_{op} = M^2 E_{bb}^2 \times \frac{K^2 R}{(1 + KR)^2} \tag{D-75}$$

Taking the derivative of P_{op} with respect to R and setting it equal to zero shows that the maximum output power occurs when

$$R = \frac{1}{K} \tag{D-76}$$

The load resistor R should be chosen as small as possible in accordance with the following inequalities and E_{bb} should be chosen so that the peak plate to cathode voltage $2E_{bb}-E_{min}$ does not exceed the rated peak plate voltage of the tube.

$$R \geq \frac{E_{bb} - E_{min}}{I_{mr}} \tag{D-72}$$

$$R \geq \frac{(E_{bb} - E_{min})E_{min}}{\pi P_{dr}} \tag{D-73}$$

$$R \geq \frac{1}{K} \tag{D-78}$$

An alternative to the inequality given in Equation (D-78) is the following inequality which results from a combination of Equations (D-71), (D-74) and (D-78)

$$E_{min} \leq \frac{1}{2} E_{bb} \tag{D-79}$$

As pointed out for Class A PDM amplifiers, the a-c plate resistance of the tube is no consideration in the choice of R.

The considerations for tubes with screen grids are the same as those discussed in Section B-2 of Appendix B and in the last two paragraphs of Section C-3 of Appendix C. Equation (B-37) does not apply however. Instantaneous screen grid power for one tube is

$$p(t) = (2a - 1)I_{sg}E_{sg} \text{ for } 0 \leq \theta \leq \pi$$

$$= 0 \quad \text{for} \quad \pi \leq \theta \leq 2\pi$$

The screen grid dissipation per tube is then

$$P_{sg} = \frac{I_{sg}E_{sg}}{2\pi} \int_0^{\pi} (2a - 1)d\theta = \frac{MI_{sg}E_{sg}}{\pi} \qquad \text{(D-80)}$$

APPENDIX E

PDM AMPLIFIERS WITH L-C LOW-PASS FILTER

E-1. General Considerations in the Use of L-C Filters.

The driving point impedance of any L-C low-pass filter which is used with a PDM amplifier should have a pole at infinite frequency so that the quadrature currents delivered by the high frequency voltage components of the Fourier representation of the waveform will be small. This implies a series inductor at the input of the filter. A disadvantage of using any type filter with a series inductor at the input is that provision must be made for continuity of current in this inductor, either through the device driving the filter or through some element (not an R, L or C) connected across the input terminals of the filter. The off-time current at the filter input terminals is not zero. The current at the input terminals of the filter changes during on-time and varies also as a function of the instantaneous modulation signal. None of these disadvantages are encountered when a constant-resistance filter is used. However, a constant-resistance filter is not lossless. Except for small losses associated with the actual elements used in the construction of a filter, an L-C filter has an efficiency of 100 percent. All of the plate circuit loss is then either in tube dissipation or as unused power in the load. The series-fed circuit configuration comes under the category of having unused power in the load since it has a constant d-c load current in addition to the signal for any particular modulation factor M.

Since the on-time current at the filter input is not constant, the minimum plate voltage for the tube or tubes driving the filter is not constant. The characteristic curves of the tube are an important factor in determining the relation between on-time current and on-time voltage. For this reason no general equations can be written for plate circuit efficiency or maximum output power for a given tube dissipation. Expressions for instantaneous efficiency and dissipation can be given in terms of the minimum instantaneous plate voltage $e_{b,min}$. The time function voltage $e_{b,min}$ is unknown until a specific tube and operating conditions are chosen. The instantaneous power delivered by the tube to the filter is

$$p_t(t) = (E_{bb} - e_{b,min})i_s(t)$$

The instantaneous power furnished by the power supply is

$$p_s(t) = E_{bb}i_s(t)$$

Then at any instant during the time of tube conduction the tube efficiency is given by

$$\eta_{t1} = 1 - \frac{e_{b,min}}{E_{bb}} \qquad \text{(E-1)}$$

During the time the tube is not conducting, the power supply furnishes no power and efficiency is of no concern.

E-2. Use of the Low Frequency Leg of a Constant-resistance Filter.

The low frequency leg of a constant-resistance low-pass filter constitutes an L-C low-pass filter. It has d-c continuity between input and output and meets the requirement of having an inductor at the input. In addition it has a voltage transfer function which is identical to that of the constant-resistance filter from which it is taken. This is the type of L-C filter which will be considered, even though the driving point resistance over the pass band may not be as well controlled as with some other types of L-C low-pass filters.

If the maximum value of $e_{b,min}$ is small compared to E_{bb}, the waveform distortion at the output of the low-pass filter will be negligible.

Then a sinusoidally modulated PDM signal will result in an approximately sinusoidal signal current in the resistive load at the output of the filter. If a single-ended circuit configuration is used, it will be necessary to use Class A modulation. Some means must also be provided for current continuity at the input of the filter. A diode connected across the input terminals of the filter will provide for continuity of current in the input inductor and will maintain the waveshape of voltage at the input of the filter during the off-time of the tube. This circuit configuration is shown in Figure E-1.

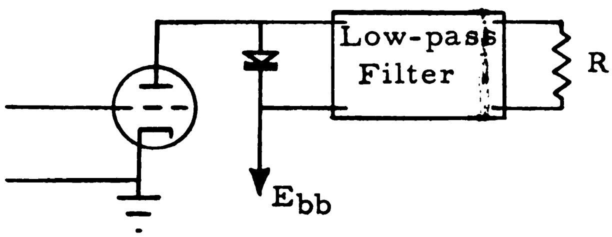


Figure E-1. Series-fed PDM amplifier with L-C filter.

For sinusoidal Class A modulation

$$a = \frac{1}{2}(1 + M\sin\theta) \qquad \text{(E-2)}$$

$$\theta = \omega_m t \qquad \text{(E-3)}$$

To evaluate the performance of the filter and load combination exclusive of the tube, set $e_{b,min} = 0$. The resulting equations will quite accurately specify the performance of the amplifier when the maximum value of $e_{b,min}$ is small compared to E_{bb}. Any exact analysis depends upon the characteristics of the tube and can only be carried out once a specific tube is chosen. Let

$$e_{b,min} = 0 \qquad \text{(E-4)}$$

The instantaneous current in resistor R is

$$i_0(t) = \frac{a\,E_{bb}}{R} = \frac{E_{bb}}{2R}\,(1 + M\sin\theta)$$

The instantaneous power supply current is

$$i_s(t) = a\,[\,-i_0(t)\,] = -\frac{a^2 E_{bb}}{R}$$

The d-c power supply current is then

$$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} i_s(\theta)\,d\theta$$

$$= \frac{E_{bb}}{8R}\,(2 + M^2) \qquad \text{(E-5)}$$

The power supply power is

$$P_s = E_{bb}I_{dc} = \frac{E_{bb}^2}{8R}\,(2 + M^2) \qquad \text{(E-6)}$$

Total power delivered to the load is

$$P'_{op} = \frac{R}{2\pi}\int_0^{2\pi} i_0^2(\theta)\,d\theta$$

$$= \frac{E_{bb}^2}{4R} + \frac{M^2 E_{bb}^2}{8R} = P_{dc} + P_{op} \qquad \text{(E-7)}$$

Signal output power is then

$$P_{op} = \frac{M^2 E_{bb}^2}{8R} \qquad \text{(E-8)}$$

Efficiency of the filter and load combination is

$$\eta_{flR} = \frac{P_{op}}{P_s} = \frac{M^2}{2 + M^2} \qquad \text{(E-9)}$$

$$\text{Maximum } \eta_{flR} = \frac{1}{3} \qquad \text{(E-10)}$$

If $e_{b,min}$ can be held constant at the value E_{min} during the on-time of the tube, the following equations specify performance.

$$i_o(t) = \frac{E_{bb} - E_{min}}{2R} (1 + M\sin\theta)$$

$$i_s(t) = -\frac{a^2(E_{bb} - E_{min})}{R}$$

$$I_{dc} = \frac{E_{bb} - E_{min}}{8R} (2 + M^2) \quad \text{(E-11)}$$

$$P_s = E_{bb} I_{dc} = \frac{E_{bb}(E_{bb} - E_{min})(2 + M^2)}{8R} \quad \text{(E-12)}$$

$$P'_{op} = \frac{(E_{bb} - E_{min})^2}{4R} + \frac{M^2(E_{bb} - E_{min})^2}{8R} = P_{dc} + P_{op} \quad \text{(E-13)}$$

$$P_{op} = \frac{M^2(E_{bb} - E_{min})^2}{8R} \quad \text{(E-14)}$$

$$\eta_{1R} = \eta_{f1R}\eta_t = \frac{P_{op}}{P_s} = \frac{M^2}{(2 + M^2)} [1 + \frac{E_{min}}{E_{bb}}] \quad \text{(E-15)}$$

$$\eta_t = 1 + \frac{E_{min}}{E_{bb}} \quad \text{(E-16)}$$

$$\eta_{f1R} = \frac{M^2}{2 + M^2} \quad \text{(E-17)}$$

If the single-ended amplifier configuration is used with a diode across the input terminals of the filter, it is necessary that the tube and load be series-fed. However, as can be seen by Equation (E-10), the series-fed configuration does not offer as much as might be desired in terms of efficiency. Push-pull and single-ended push-pull configurations eliminate the d-c current in the load and provide higher plate circuit efficiency. There is no merit in using Class B modulation since Class A modulation provides an efficiency which is just as high. When there is no modulation the power supply furnishes no power. The push-pull configuration of Figure C-5b has the same drawbacks when an L-C filter is used as are discussed in Section C-3 of Appendix C. The single-ended push-pull configurations of Figure C-4 would seem to be good choices for use with L-C filters; however, even with one tube presumably on at all times, provision still must be made for current continuity at the input to the filter. It would not be efficient to meet this requirement by allowing or forcing the normally off tube to conduct. During the normal off-time for a tube the plate to cathode voltage is large, and plate current during this time would result in high plate dissipation. This would lower the efficiency. In addition the power handling capability of the amplifier would be greatly reduced by the need to keep the plate dissipation within the tube ratings. The use of diodes as shown in Figure E-2 and Figure E-3 is preferable, but unless the ratio of the maximum value of $e_{b,\,min}$ to E_{bb} is very small, waveform distortion will occur at the input to the filter.

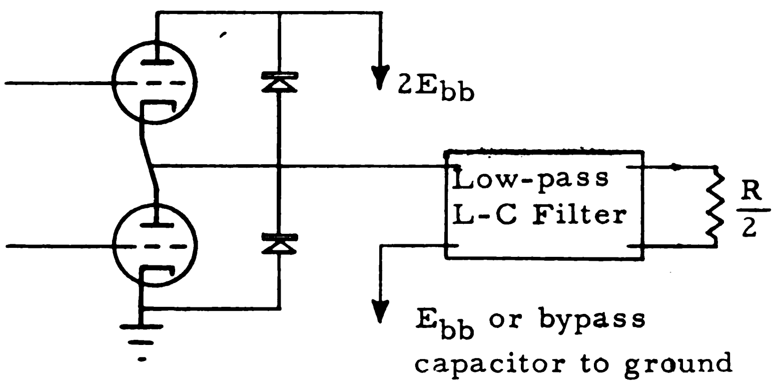


Figure E-2. Single-ended push-pull.

This happens because the diodes permit $e_{b,\,min}$ to be zero during part of the normally on time of the tubes. With transistors the ratio of the maximum value at $e_{b,\,min}$ to E_{bb} is usually very small and waveform distortion is much less severe than with tubes. For the circuits of Figure E-2 and Figure E-3 the instantaneous efficiency of the tubes is given by Equation (E-1). As was the case with the single-ended amplifier, no general equations can be written for plate circuit efficiency or maximum output power for a given tube dissipation.

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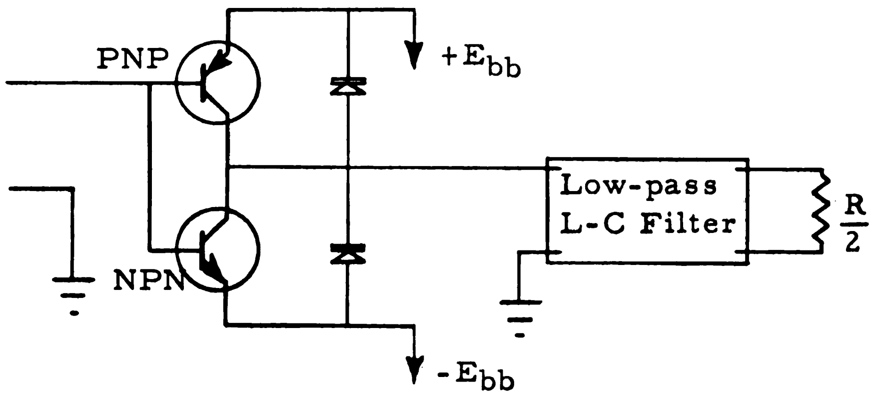

Figure E-3. Complementary pair transistor configuration.

It is necessary to select a tube and determine the functional form of $e_{b,min}$. For sinusoidal Class A modulation Equations (E-2) and (E-3) hold. If $e_{b,min}$ can be held constant at E_{min} during the on-time of the tubes, then the instantaneous voltage across the load resistor is

$$e_o(t) = (2a - 1)(E_{bb} - E_{min}) \tag{E-18}$$

The signal power delivered to the load is

$$P_{pT} = \frac{M^2(E_{bb} - E_{min})^2}{R} \tag{E-19}$$

The output power per tube is

$$P_{op} = \frac{M^2(E_{bb} - E_{min})^2}{2R} \tag{E-20}$$

Since an L-C filter is employed and since there is no d-c current in the load, the efficiency of the filter and load configuration is 100 percent.

$$\eta_{fl} = 1 \tag{E-21}$$

Since $e_{b,min}$ is being held constant at the value E_{min}, Equation (E-1) gives the tube efficiency as

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$$\eta_t = 1 - \frac{E_{min}}{E_{bb}} \tag{E-22}$$

The overall plate circuit efficiency is then

$$\eta_1 = \eta_t \eta_{f1} = 1 - \frac{E_{min}}{E_{bb}} \tag{E-23}$$

The total power supply power is

$$P_{sT} = \frac{P_{pT}}{\eta_1} = \frac{M^2 E_{bb}(E_{bb} - E_{min})}{R} \tag{E-24}$$

The power supply power per tube is

$$P_s = \frac{M^2 E_{bb}(E_{bb} - E_{min})}{2R} \tag{E-25}$$

For the circuit of Figure E-2, the E_{bb} supply to which the filter is returned furnishes no net power for sinusoidal modulation. The $2E_{bb}$ supply then furnishes the power expressed by Equation (E-24). The d-c current furnished by this supply is

$$I_{dc} = \frac{P_{sT}}{2E_{bb}} = \frac{M^2(E_{bb} - E_{min})}{2R} \tag{E-26}$$

The power supplies for the circuit of Figure E-3 share the load equally. Each furnishes half the power given by Equation (E-24). The d-c current furnished by each is then also given by Equation (E-26).

$$P_{dp} = P_s - P_{op} = \frac{M^2(E_{bb} - E_{min})E_{min}}{2R} \tag{E-27}$$

$$\text{Maximum } P_{dp} = \frac{(E_{bb} - E_{min})E_{min}}{2R} \tag{E-28}$$

By referring to Equation (C-8)

$$\text{D. R.} = \frac{\text{Maximum } P_{dp}}{\text{Maximum } P_d} = \frac{E_{min}}{2E_{bb}} \tag{E-29}$$

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If, instead of holding $e_{b,min}$ constant at the value E_{min} during the on-time of the tube, the ratio of the maximum value of $e_{b,min}$ to E_{bb} is made small, the performance of the amplifiers of Figure E-2 and Figure E-3 is expressed by setting $E_{min} = 0$ in Equations (C-19) through (C-26). Then

$$I_{dc} = \frac{M^2 E_{bb}}{2R} \qquad \text{(E-30)}$$

$$P_{sT} = \frac{M^2 E_{bb}^2}{R} \qquad \text{(E-31)}$$

$$P_{pT} = \frac{M^2 E_{bb}^2}{R} \qquad \text{(E-32)}$$

$$P_s = \frac{M^2 E_{bb}^2}{2R} \qquad \text{(E-33)}$$

$$P_{op} = \frac{M^2 E_{bb}^2}{2R} \qquad \text{(E-34)}$$

$$\eta_1 = 1 \qquad \text{(E-35)}$$

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APPENDIX F

DETERMINATION OF NO-MODULATION DUTY RATIO FOR MAXIMUM OUTPUT POWER AND MAXIMUM EFFICIENCY

F-1. Series-fed Class A PDM with Constant-resistance Filter.

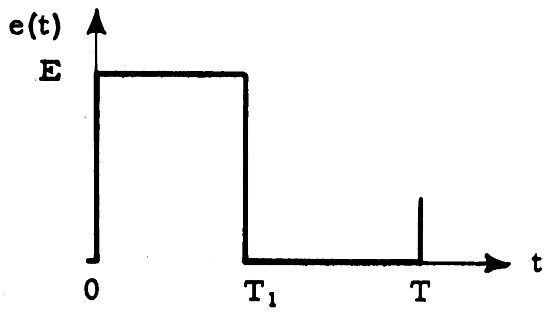

Figure F-1. Waveform at input to filter.

Figure F-2. Filter and load configuration.

$$a = \frac{T_1}{T} \qquad \text{(F-1)}$$

$$0 \leq a \leq 1 \qquad \text{(F-2)}$$

Let the duty ratio with no modulation be

$$k = \frac{T_1}{T} \qquad \text{(F-3)}$$

$$0 \leq k \leq 1 \qquad \text{(F-4)}$$

Let

$$\theta = \omega_m t \qquad \text{(F-5)}$$

Let

$$a = k\,(1 + M\sin\theta) \qquad \text{(F-6)}$$

Then

$$k\,(1 + M) \leq 1 \qquad \text{(F-7)}$$

$$M \leq \frac{1}{k} - 1 \qquad \text{(F-8)}$$

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Power input to the filter is

$$p(\theta) = a\frac{E^2}{R}$$

$$P_t = \frac{1}{2\pi}\int_0^{2\pi} p(\theta)d\theta = k\frac{E^2}{R} \qquad \text{(F-9)}$$

Instantaneous output power to load R is

$$p(\theta) = \frac{(aE)^2}{R}$$

Average output power is

$$P'_{op} = \frac{1}{2\pi}\int_0^{2\pi} p(\theta)d\theta$$

$$= \frac{k^2E^2}{R} + \frac{M^2k^2E^2}{2R} = P_{dc} + P_{op} \qquad \text{(F-10)}$$

The a-c signal output power is then

$$P_{op} = \frac{(MkE)^2}{2R} \qquad \text{(F-11)}$$

The efficiency is

$$\eta_{fAR} = \frac{P_{op}}{P_t} = \frac{kM^2}{2} \qquad \text{(F-12)}$$

Case 1:

$$k \leq \frac{1}{2} \qquad \text{(F-13)}$$

$$0 \leq M \leq 1 \qquad \text{(F-14)}$$

For a fixed value of M the output power and efficiency become greater as k is made larger. The greatest possible output power and efficiency occur when M = 1.

$$k = \frac{1}{2} \qquad \text{(F-15)}$$

$$\text{Maximum } P_{op} = \frac{E^2}{8R} \qquad \text{(F-16)}$$

$$\text{Maximum } \eta_{fAR} = \frac{1}{4} \qquad \text{(F-17)}$$

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Case 2:

$$\frac{1}{2} \leq k \leq 1 \qquad \text{(F-18)}$$

$$M \leq \frac{1}{k} - 1 \qquad \text{(F-8)}$$

$$P_{op} = \frac{(MkE)^2}{2R} \leq \frac{(1 - k)^2E^2}{2R} \qquad \text{(F-19)}$$

$$\eta_{fAR} = \frac{kM^2}{2} \leq \frac{(1 - k)^2}{2k} \qquad \text{(F-20)}$$

The lower the value of k in the range given by Equation (F-18), the greater will be the upper bounds on output power and efficiency. Then the greatest possible output power and efficiency occur when $k = \frac{1}{2}$.

$$k = \frac{1}{2} \qquad \text{(F-21)}$$

$$\text{Maximum } P_{op} = \frac{E^2}{8R} \qquad \text{(F-22)}$$

$$\text{Maximum } \eta_{fAR} = \frac{1}{4} \qquad \text{(F-23)}$$

Therefore the best value of k in the range given by Equation (F-4) is $\frac{1}{2}$. This value of k allows the greatest possible output power and the highest efficiency. Let

$$k = \frac{1}{2} \qquad \text{(F-24)}$$

Then

$$a = \frac{1}{2}\,(1 + M\sin\omega_m t) \qquad \text{(F-25)}$$

$$\eta_{fAR} = \frac{M^2}{4} \qquad \text{(F-26)}$$

F-2. Shunt-fed Class A PDM with Constant-resistance Filter.

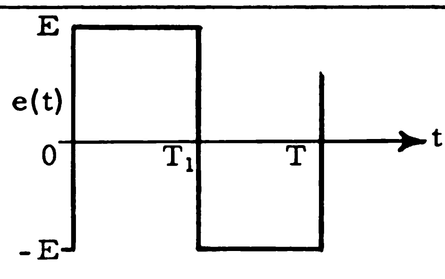

Figure F-3. Waveform at input to filter.

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The filter and load configuration are the same as Figure F-2.

$$a = \frac{T_1}{T} \tag{F-27}$$

$$0 \le a \le 1 \tag{F-28}$$

Let the duty ratio with no modulation be

$$k = \frac{T_1}{T} \tag{F-29}$$

$$0 \le k \le 1 \tag{F-30}$$

Let

$$\theta = \omega_m t \tag{F-31}$$

Let

$$a = k\,(1 + M\sin\theta) \tag{F-32}$$

Then

$$k(1 + M) \le 1 \tag{F-33}$$

$$M \le \frac{1}{k} - 1 \tag{F-34}$$

Power input to the filter is

$$p(\theta) = a\,\frac{E^2}{R} + (1 - a)\,\frac{E^2}{R} = \frac{E^2}{R}$$

$$P_t = \frac{1}{2\pi}\int_0^{2\pi} p(\theta)d\theta = \frac{E^2}{R} \tag{F-35}$$

Instantaneous output power to load R is

$$p(\theta) = \frac{[aE - (1 - a)E]^2}{R} = \frac{(2a - 1)^2E^2}{R}$$

Average output power is

$$P'_{op} = \frac{1}{2\pi}\int_0^{2\pi} p(\theta)d\theta$$

$$= \frac{E^2}{2\pi R}\int_0^{2\pi}[2k + 2kM\sin\theta - 1]^2 d\theta$$

$$= \frac{E^2}{R}[(2k - 1)^2 + 2k^2M^2] = P_{dc} + P_{op} \tag{F-36}$$

The d-c output power is then

$$P_{dc} = \frac{E^2}{R}(2k - 1)^2 \tag{F-37}$$

The a-c signal output power is

$$P_{op} = \frac{2(MkE)^2}{R} \tag{F-38}$$

The efficiency is

$$\eta_{fA} = \frac{P_{op}}{P_t} = 2M^2k^2 \tag{F-39}$$

Case 1:

$$k \leq \frac{1}{2} \tag{F-40}$$

$$0 \leq M \leq 1 \tag{F-41}$$

With M fixed, the greatest output power and efficiency occur when $k = \frac{1}{2}$. The greatest possible output power and efficiency occur when M = 1.

$$k = \frac{1}{2} \tag{F-42}$$

$$\text{Maximum } P_{op} = \frac{E^2}{2R} \tag{F-43}$$

$$\text{Maximum } \eta_{fA} = \frac{1}{2} \tag{F-44}$$

Case 2:

$$\frac{1}{2} \leq k \leq 1 \tag{F-45}$$

$$M \leq \frac{1}{k} - 1 \tag{F-46}$$

$$P_{op} = \frac{2(MkE)^2}{R} \leq \frac{2(1 - k)^2E^2}{R} \tag{F-47}$$

$$\eta_{fA} = 2M^2k^2 \leq 2(1 - k)^2 \tag{F-48}$$

The greatest possible output power and efficiency occur when $k = \frac{1}{2}$.

$$k = \frac{1}{2} \tag{F-49}$$

$$\text{Maximum } P_{op} = \frac{E^2}{2R} \tag{F-50}$$

$$\text{Maximum } \eta_{fA} = \frac{1}{2} \tag{F-51}$$

Therefore, as in Section F-1, the best value of k in the range given by Equation (F-30) is $\frac{1}{2}$. This value of k allows the greatest possible output power and the highest efficiency. In addition, the d-c power loss in the load is zero. Let

$$k = \frac{1}{2} \tag{F-52}$$

Then

$$a = \frac{1}{2} (1 + M \sin \omega_m t) \tag{F-53}$$

$$\eta_{fA} = \frac{M^2}{2} \tag{F-54}$$

$$P_{dc} = 0 \tag{F-55}$$

REFERENCES

1. Black, H. S., Modulation Theory, D. Van Nostrand Company, Inc., New York, 1953.

2. Ryder, J. D., Engineering Electronics, McGraw-Hill Book Company, Inc., New York, 1957.

3. Chai Yeh, "Analysis of a Single-Ended Push-Pull Audio Amplifier," Proc. of I.R.E., June 1953, pp. 743-747.

4. Crowhurst, N. H., "The Basic Design of Constant Resistance Crossovers," Audio Engineering, October, 1953.

5. Reference Data for Radio Engineers, Fourth Edition, International Telephone and Telegraph Corporation, 1956-1957.

6. Landon, Vernon D., "Theoretical Analysis of Various Systems of Multiplex Transmission," RCA Review, Vol. IX, Number 2 and Number 3, June and September, 1948.

7. Feldman, C. B., and Bennett, W. R., "Band Width and Transmission Performance," Bell System Technical Journal, Vol. 28, pp. 490-595, July, 1949.

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