# A THEORETICAL INVESTIGATION OF ANNULAR MAGNETOHYDRODYNAMIC FLOW WITH A MOVING BOUNDARY

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Dennis C. Kuzma

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## ABSTRACT

### A THEORETICAL INVESTIGATION OF ANNULAR MAGNETOHYDRODYNAMIC FLOW WITH A MOVING BOUNDARY

by Dennis C. Kuzma

The problem considered is the laminar, steady flow of a viscous, incompressible, conducting fluid in the annular space between two infinitely long circular cylinders under the action of a radially impressed magnetic field and an axially impressed electric field when the outer cylinder is given a uniform angular velocity. The conditions of the problem reduce the magnetohydrodynamic equations to three equations in pressure, velocity, and magnetic field. One equation gives the pressure variation in the radial direction and the other two equations are coupled equations for the velocity and the magnetic field. These three equations are functions of one variable, and may be solved in closed form. In the limiting case where the radii become infinite but their difference remains finite, and there is no velocity of the outer cylinder, the solution becomes Hartmann's flow between infinite parallel plates with a transverse magnetic field and a uniform applied electric field.

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By

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## LIST OF SYMBOLS

| | | |
|---|---|---|
| $a$ | = | radius of inner cylinder |
| $b$ | = | radius of outer cylinder |
| $c$ | = | ratio of radii of outer and inner cylinders |
| $\vec{E}$ | = | electric field vector |
| $E_r$, $E_\phi$, $E_z$ | = | components of electric field |
| $E_o$ | = | applied electric field |
| $\vec{H}$ | = | magnetic field vector |
| $h$ | = | dimensionless magnetic field |
| $H_r$, $H_\phi$, $H_z$ | = | components of magnetic field |
| $H_o$ | = | applied magnetic field |
| $\vec{J}$ | = | current density vector |
| $J_r$, $J_\phi$, $J_z$ | = | components of current density |
| $L$ | = | distance from center of channel to plate |
| $M$ | = | Hartmann number |
| $m$ | = | cylindrical Hartmann number |
| $p$ | = | pressure |
| $P$ | = | dimensionless pressure |
| $P_m$ | = | magnetic Prandtl number |
| $r$ | = | radius |
| $R$ | = | dimensionless radius |
| $\vec{s}$ | = | position vector |
| $\vec{S}$ | = | area vector |
| $t$ | = | time |
| $\vec{V}$ | = | velocity vector |
| $V_r$, $V_\phi$, $V_z$ | = | components of velocity |

| | | |
|---|---|---|
| $v$ | = | dimensionless velocity |
| $w$ | = | strength of magnetic source |
| $y$ | = | distance from midchannel |
| $z$ | = | axial coordinate |
| $\mu$ | = | magnetic permeability |
| $\nu$ | = | kinematic viscosity |
| $\rho$ | = | density |
| $\sigma$ | = | electrical conductivity |
| $\phi$ | = | angular coordinate |
| $\omega$ | = | angular velocity |

# INTRODUCTION

When an electrically conducting fluid moves in the presence of a magnetic field, electric currents are induced in the flow. Electric fields applied to the flow also produce electric currents. These electric currents interact with the magnetic field and produce mechanical forces which modify the flow. The friction forces due to the viscosity of the fluid also modify the flow.

Although the equations which describe these phenomena are very complicated nonlinear partial differential equations, some special problems may be solved in closed form. The flow of an electrically conducting fluid in an annular channel with moving boundaries may be solved in closed form in some special cases. Very few problems with moving boundaries have been considered in cylindrical coordinates, other than that of the impulsively accelerated flat plate with a transverse magnetic field as the limiting case of the cylindrical problem.[(1)*] The solution presented here pertains to the case of an annular channel with a moving boundary.

In this study, a viscous, incompressible, conducting fluid is considered in an infinitely long annular channel of inner radius $a$ and outer radius $b$ (Fig. 1). A magnetic field is applied to the channel in the radial direction such that $H_o = w/r$, where $w$ is a constant. A uniform electric field $E_o$ is applied to the channel in the axial direction and the outer cylinder is given a uniform angular velocity $\omega$.

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*Superscript numbers in parentheses refer to the List of References.

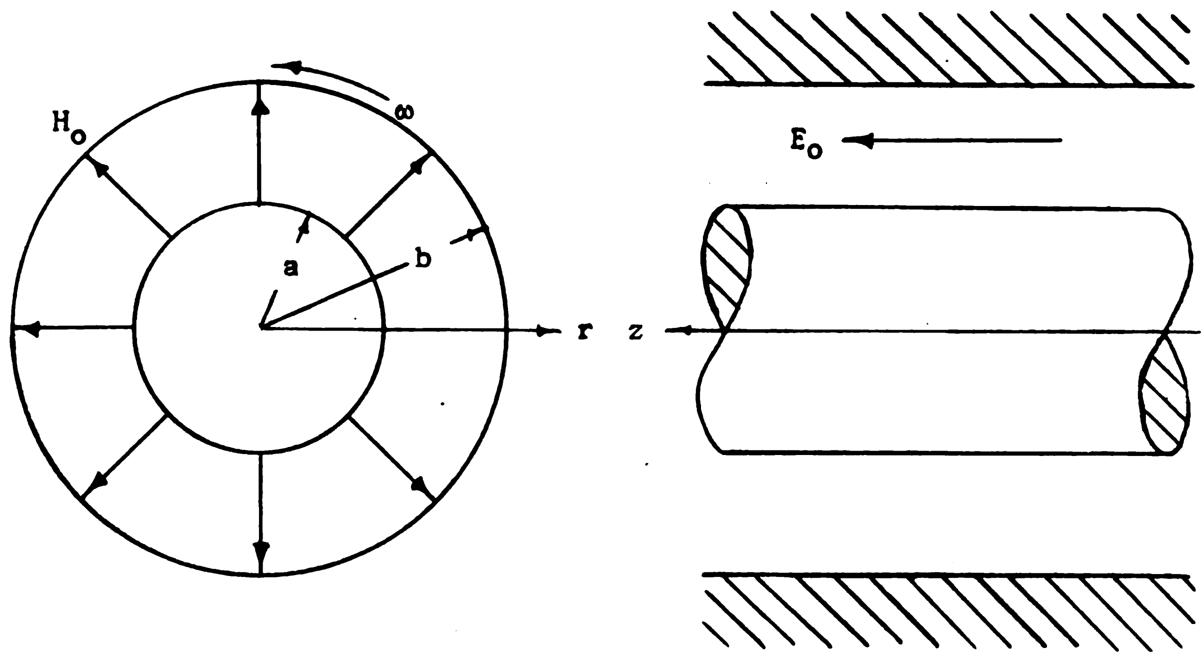


Fig. 1 Annular Channel

The applied magnetic field is a line source magnetic field located on the common axis of the inner and outer cylinders. Although a line source magnetic field does not exist in reality, it is possible to obtain a good approximation to such a field by the use of a core of material with high permeability within the annulus and a cylindrical shell of material with high peameability outside the annulus. The flux lines could close through these permeable paths at long distances from the region of interest (Fig. 2). The source of the flux could be the core itself, if the core is made of a permanantly magnetized material. Since the purpose of this study is the theoretical investigation of the flow and not the experimental problems involved, the details of providing the necessary magnetic field will not be discussed further.

If there were no magnetic field, this problem would be the ordinary flow in the annular space between a rotating outer cylinder and a stationary inner cylinder.[2] This case can be derived as a limiting case of the annular magnetohydrodynamic flow.

If there were no angular velocity, this problem would be a cylindrical analog of Hartmann's flow between infinite parallel plates with a transverse magnetic field and a uniform electric field applied parallel to the plates.[3] It will be shown that Hartmann's flow can be derived as a limiting case of the annular magnetohydrodynamic flow.

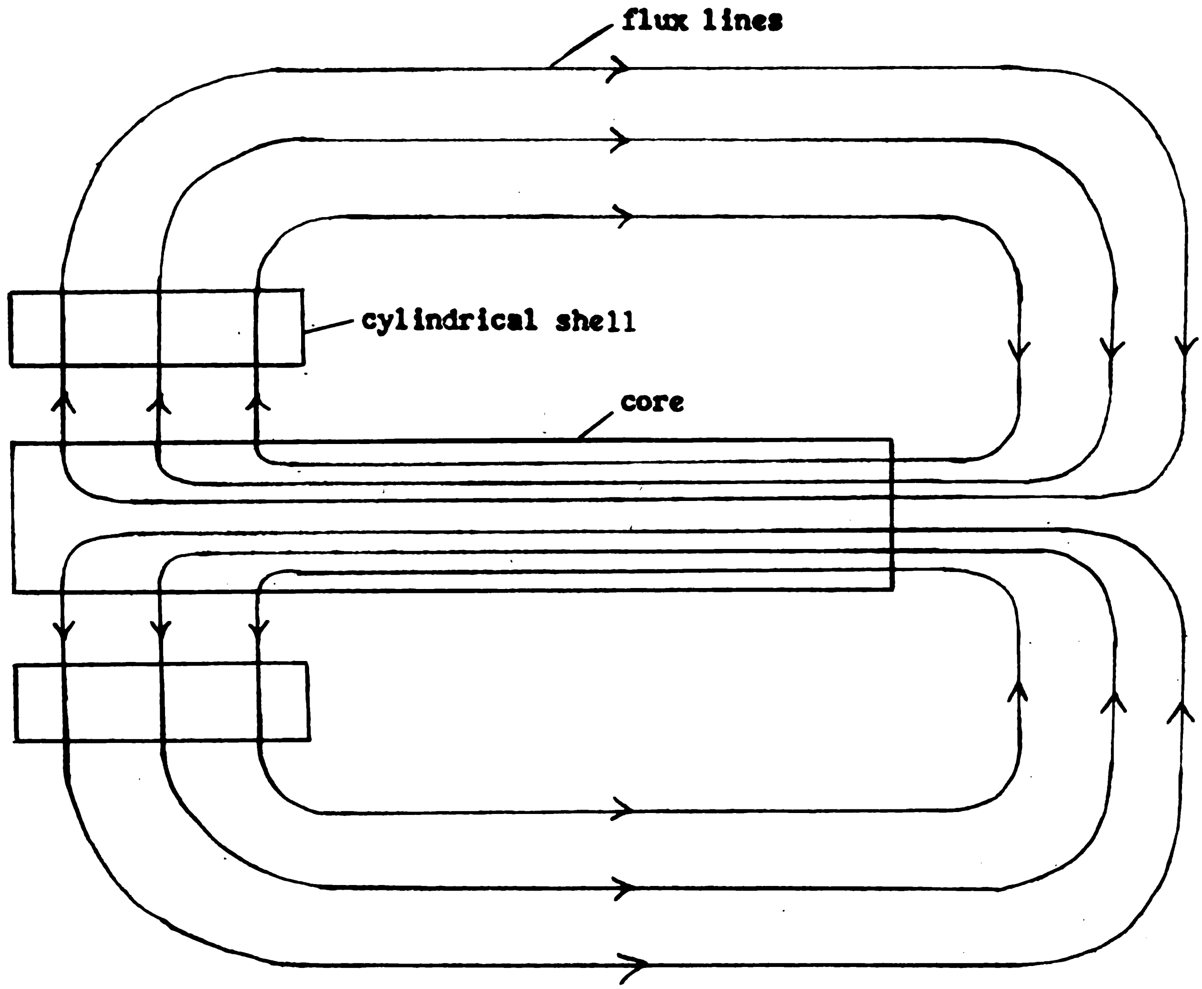


Fig. 2 Method of Obtaining the Magnetic Field

## REVIEW OF PREVIOUS RESEARCH

Because of the great amount of work that has been done in magnetohydrodynamics, no attempt will be made to undertake a complete review of previous research. This review will be confined to works which pertain in some way to the problem presented here.

The results of the first theoretical investigation in the field which is now known as magnetohydrodynamics were published by J. Hartmann[3] in 1937. Hartmann considered the laminar flow of an electrically conducting liquid in a homogeneous magnetic field. This investigation is considered the classic work in magnetohydrodynamics.

A cylindrical analog of Hartmann's flow has been considered by S. Globe.[4] Globe considered the laminar, steady flow of an electrically conducting, incompressible fluid in the annular space between two infinitely long circular cylinders under the action of a radially impressed magnetic field and a constant longitudinal pressure drop.

I. G. Chekmarev[5] also considered the laminar, steady flow of an electrically conducting, incompressible fluid in the annular space between two infinitely long circular cylinders under the action of a radially impressed magnetic field. Chekmarev considered three such problems. The first problem concerned the flow under the action of a constant longitudinal pressure drop and a radial injection of a liquid with constant velocity at the surface of the inner cylinder. The second problem concerned the flow under the action of a uniform axial electric field and a radial injection of a liquid with constant velocity at the surface of the inner cylinder. The third problem concerned the flow with a uniform axial electric field and a constant longitudinal pressure drop.

Few problems in cylindrical coordinates have been considered with moving boundaries. One of the few such problems that has been considered is the work of G. F. Carrier and H. P. Greenspan[1] concerning the flow past an impulsively accelerated flat plate with a transverse magnetic filed as the limiting case of the cylindrical problem.

## EQUATIONS

In order to determine which equations to use, some assumptions must be made about the nature of the fluid. It will be assumed that (1) the fluid is incompressible, (2) the free charge density and the displacement current are negligible, (3) the permeability, conductivity, permittivity, and viscosity are constant scalar quantities, and (4) the Lorentz force is the only body force acting on the fluid.

Under these assumptions, the equations of magnetohydrodynamics in rationalized mks units are[6, 7]:

$$\nabla \times \vec{H} = \vec{J} \tag{1}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{2}$$

$$\nabla \cdot \vec{H} = 0 \tag{3}$$

$$\nabla \cdot \vec{E} = 0 \tag{4}$$

$$\vec{J} = \sigma (\vec{E} + \mu \vec{V} \times \vec{H}) \tag{5}$$

$$\nabla \cdot \vec{V} = 0 \tag{6}$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{V} + \frac{\mu}{\rho} \vec{J} \times \vec{H} \tag{7}$$

Since the physical aspects of the problem suggest the use of cylindrical coordinates, these equations will be used in their component form in cylindrical coordinates. These equations simplify considerably for the following reasons:

(1) Because of symmetry around the axis, $(\partial/\partial\phi) = 0$.

(2) Because of steady flow, $(\partial/\partial t) = 0$.

(3) Because of the infinite dimension in the $z$ direction and the fact that none of the applied fields are functions of $z$, $\vec{H}$ and $\vec{V}$ cannot be functions of $z$.

(4) It is assumed that the applied field $H_o = w/r$ fixes the radial component of the magnetic field at $r = a$ and $r = b$ for all values of $z$.

(5) It is assumed that there is no flow in the $z$ direction. This may be accomplished by making $(\partial p/\partial z) = 0$.

(6) An electric field can only arise from an applied voltage, a time changing magnetic field, or free charges.

The above simplifications lead to the following results when applied to the equations:

(a) Since $\vec{V} = \vec{V}(r)$, equation (6) becomes

$$\frac{d(rV_r)}{dr} = 0 \tag{8}$$

When the conditions that $V_r(a) = V_r(b) = 0$ are applied, it is seen that $V_r = 0$.

(b) Since $\vec{H} = \vec{H}(r)$, it follows from equation (3) that

$$\frac{d(rH_r)}{dr} = 0 \tag{9}$$

Applying the conditions that $H_r(a) = w/a$ and $H_r(b) = w/b$, it is seen that $H_r = w/r$. Thus, the radial component of the magnetic field is not affected by the fluid flowing in the annulus.

(c) Since neither a time changing magnetic field nor free charges exist in the flow, the electric field must be given by the applied field. Thus, $E_r = E_\phi = 0$ and $E_z = E_o$.

(d) Equations (1) and (5) may be combined to eliminate $\vec{J}$. Then

$$\nabla \times \vec{H} = \sigma (\vec{E} + \mu\vec{V} \times \vec{H}) \tag{10}$$

The radial component of this equation, including previous assumptions is

$$V_\phi H_z = 0 \tag{11}$$

But $V_\phi$ is not zero, so $H_z = 0$.

Combining equations (1) and (7) to eliminate $\vec{J}$ and introducing the above simplifications, three scalar equations are obtained from equation (10) and the combination of (1) and (7).

$$\frac{d^2V_\phi}{dr^2} + \frac{1}{r}\frac{dV_\phi}{dr} - \frac{V_\phi}{r^2} = -\frac{\mu w}{\rho \nu r^2}\frac{d(rH_\phi)}{dr} \tag{12}$$

$$\frac{1}{r}\frac{d(rH_\phi)}{dr} = \sigma E_o - \frac{\sigma\mu w}{r} V_\phi \tag{13}$$

$$\frac{dp}{dr} = \frac{\rho V_\phi^2}{r} - \frac{\mu H_\phi}{r}\frac{d(rH_\phi)}{dr} \tag{14}$$

The boundary conditions for the above equations are:

$$V_\phi(a) = 0 \tag{15}$$

$$V_\phi(b) = \omega b \tag{16}$$

$$H_\phi(a) = 0 \tag{17}$$

The boundary conditions for $V_\phi$ are the no slip conditions at the walls. The boundary condition for $H_\phi$ is obtained by requiring that no current flow in the inner cylinder. Equation (1) in integral form becomes

$$\oint \vec{H} \cdot d\vec{s} = \iint \vec{J} \cdot d\vec{S} \tag{18}$$

The path of integration is taken as the circle $r = a$. In the area bounded by $r = a$, $\vec{J}$ is equal to zero. Thus

$$2\pi a\, H_{\phi}(a) = 0 \tag{19}$$

## SOLUTION

The equations and boundary conditions may be obtained in a more convenient form by the introduction of some dimensionless quantities. Let

$$R = r/a \tag{20}$$

$$c = b/a \tag{21}$$

$$v = (V_\phi/E_o a)(\rho\nu/\sigma)^{1/2} \tag{22}$$

$$h = H_\phi(\sigma E_o a)^{-1} \tag{23}$$

$$P = p(E_o{}^2 a^2 \sigma)^{-1} \tag{24}$$

$$m = \mu w(\sigma/\rho\nu)^{1/2} \tag{25}$$

$$P_m = \sigma\mu\nu \tag{26}$$

In the above dimensionless quantities, $m$ is a cylindrical analog of the Hartmann number and $P_m$ is the magnetic Prandtl number.

Introducing the above dimensionless quantities into equations (12), (13), and (14), the equations become:

$$\frac{1}{R}\frac{d(Rh)}{dR} = 1 - \frac{mv}{R} \tag{27}$$

$$\frac{dP}{dR} = \frac{v^2}{R} - P_m \frac{h}{R}\frac{d(Rh)}{dR} \tag{28}$$

$$\frac{d^2v}{dR^2} + \frac{1}{R}\frac{dv}{dR} - \frac{v}{R^2} = -\frac{m}{R^2}\frac{d(Rh)}{dR} \tag{29}$$

The boundary conditions from equations (15), (16), and (17) become:

$$v(1) = 0 \tag{30}$$

$$v(c) = (\omega b/E_o a)(\rho\nu/\sigma)^{1/2} = v_o \tag{31}$$

$$h(1) = 0 \tag{32}$$

When equations (27) and (29) are combined, an equation involving only $v$ is obtained:

$$\frac{d^2v}{dR^2} + \frac{1}{R}\frac{dv}{dR} - (m^2 + 1)\frac{v}{R^2} = -\frac{m}{R} \tag{33}$$

This equation may be solved by the introduction of a new variable. Let

$$R = e^n \tag{34}$$

Then

$$\frac{d}{dR} = e^{-n}\frac{d}{dn} \tag{35}$$

$$\frac{d^2}{dR^2} = e^{-2n}\frac{d^2}{dn^2} - e^{-2n}\frac{d}{dn} \tag{36}$$

When this new variable is substituted into equation (33), the following equation is obtained:

$$\frac{d^2v}{dn^2} - (m^2 + 1)v = -me^n \tag{37}$$

The solution to this equation is well known. It is

$$v = C_1 \sinh \beta n + C_2 \cosh \beta n + e^n/m \tag{38}$$

where

$$\beta = (m^2 + 1)^{1/2} \tag{39}$$

The constants are found by applying the two boundary conditions on v. The constants are:

$$C_1 = \frac{1}{m}\,\frac{mv_o + \cosh(\beta \ln c) - c}{\sinh(\beta \ln c)} \tag{40}$$

$$C_2 = -1/m \tag{41}$$

When these constants are substituted into the equation for the ve-ocity and n is written in terms of R, the following equation is obtained:

$$v = 1/m\,[A \sinh(\beta \ln R) - \cosh(\beta \ln R) + R] \tag{42}$$

where

$$A = \frac{mv_o + \cosh(\beta \ln c) - c}{\sinh(\beta \ln c)} \tag{43}$$

The velocity profiles are plotted for a range of values of the velocity of the outside cylinder with $m = 0, 2, 4,$ and $12$ (Figs. 3-10). The ratio of the inner and outer radii is arbitrarily chosen as 2.

The magnetic field is obtained by inserting equation (42) into equation (27) and integrating. Making use of the fact that $h(1) = 0$, the following is obtained:

$$h(R) = 1/R \int_1^R (x - mv)\, dx \tag{44}$$

Then

$$h(R) = -1/m^2\,(1 + \beta A)\,[\cosh(\beta \ln R) - 1/R]$$

$$+ 1/m^2\,[(\beta + A) \sinh(\beta \ln R)] \tag{45}$$

The pressure distribution is obtained by integrating equation (28). Thus

$$P(R) - P(1) = \int_1^R \frac{v^2}{x}\, dx + P_m \int_1^R \frac{h}{x}\,\frac{d(xh)}{dx}\, dx \tag{46}$$

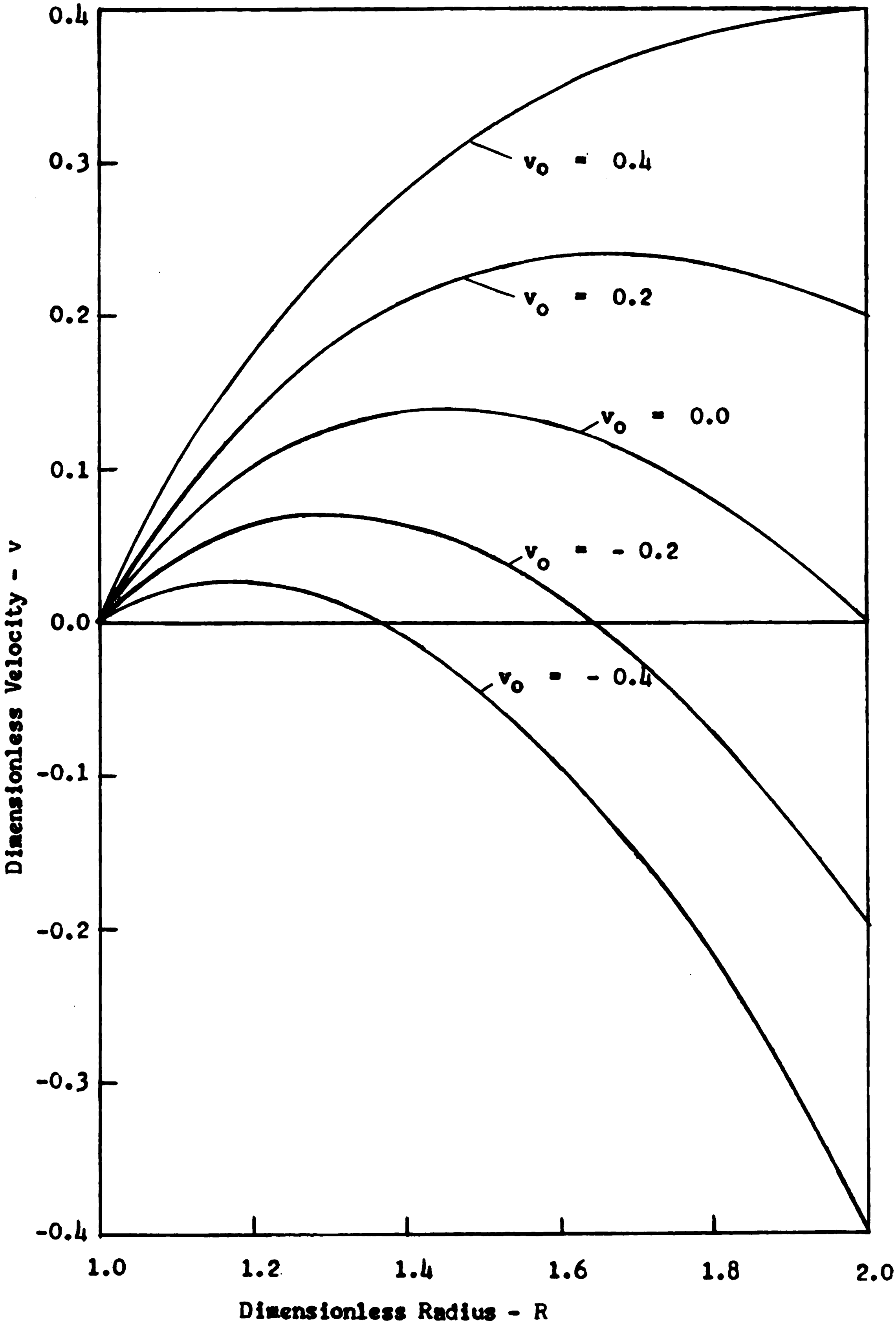


Fig. 3 Velocity Profiles for m = 2

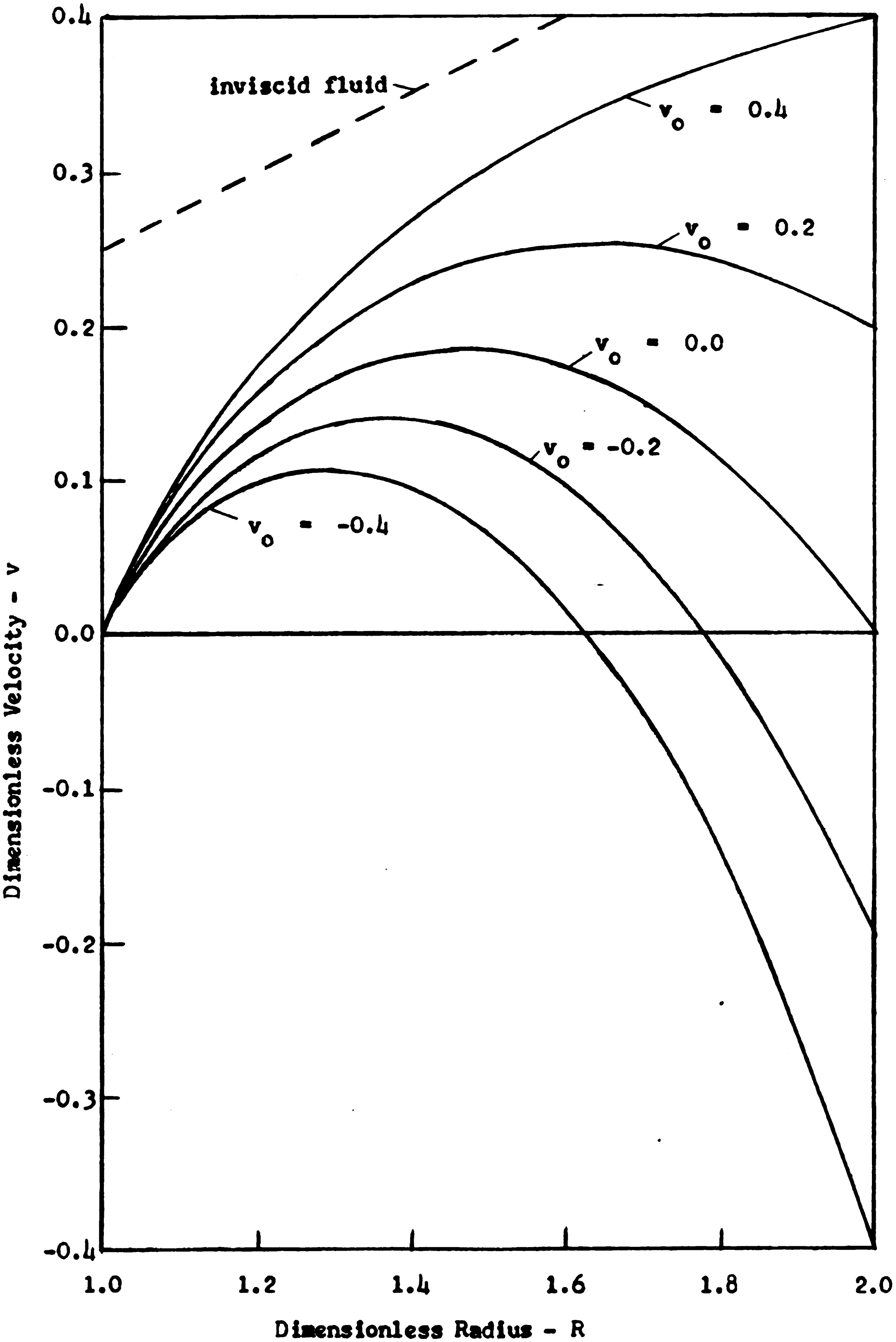


Fig. 4 Velocity Profiles for m = 4

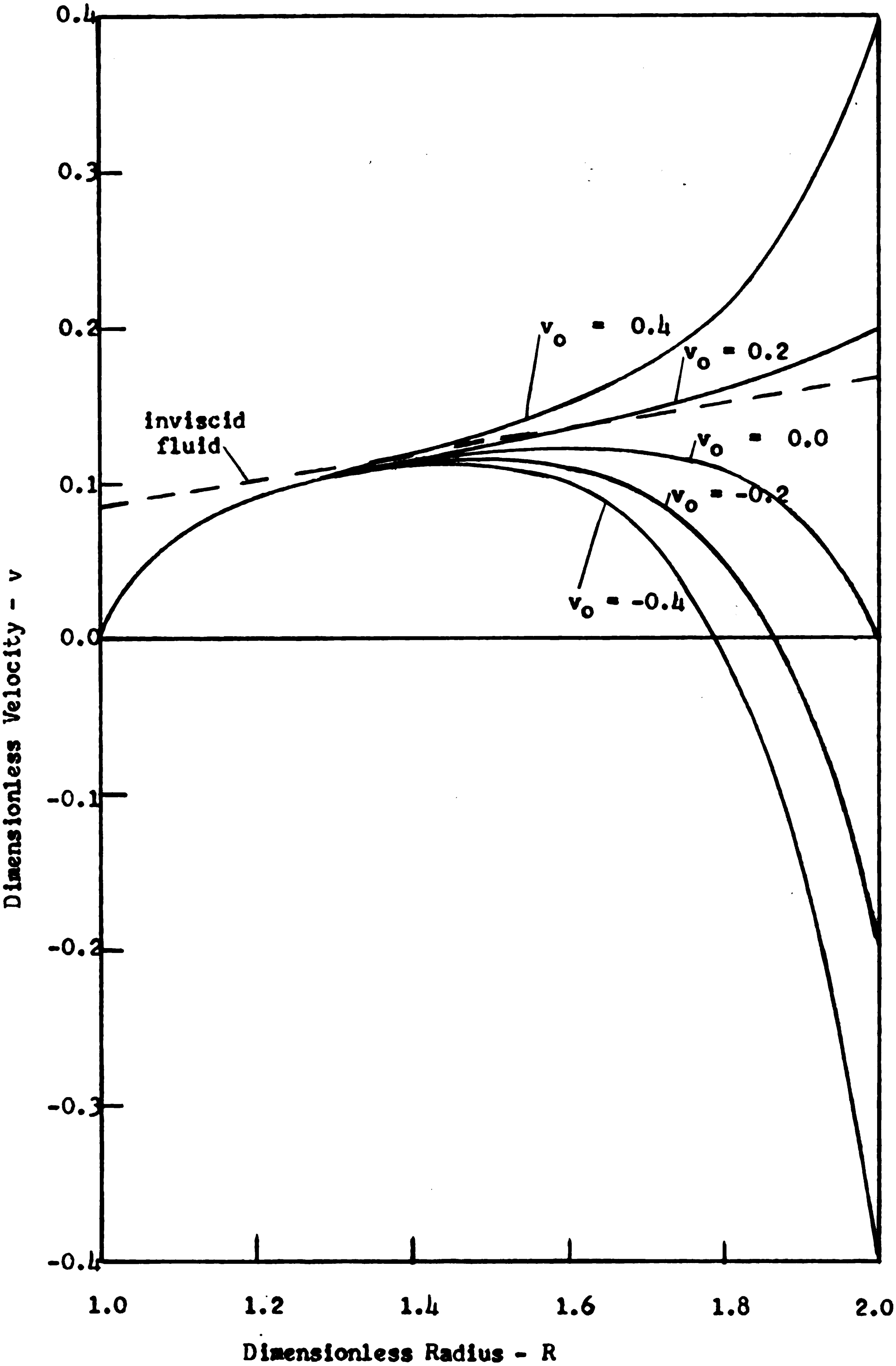


Fig. 5 Velocity Profiles for m = 12

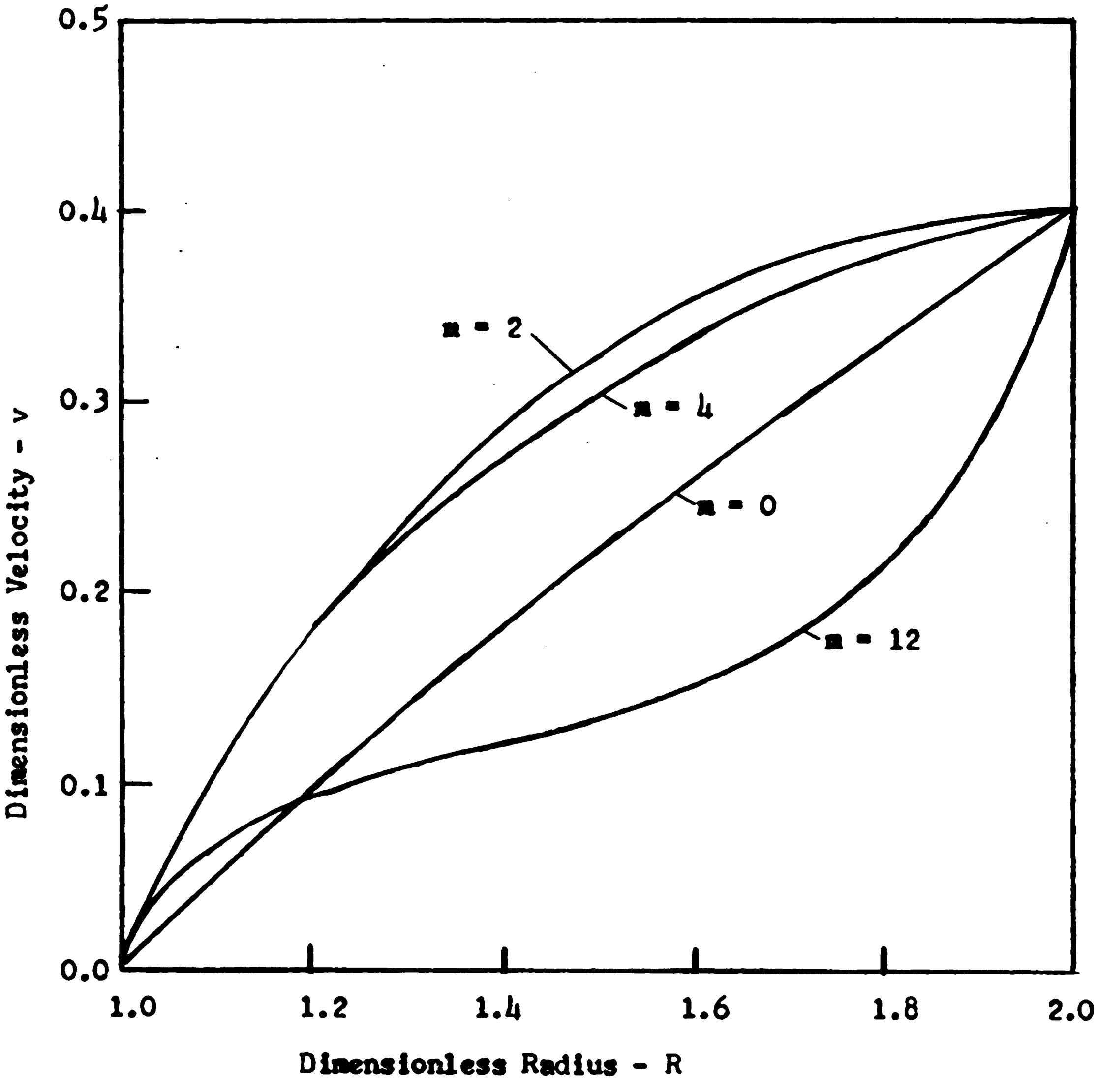


Fig. 6 Velocity Profiles for $v_o$ = 0.4

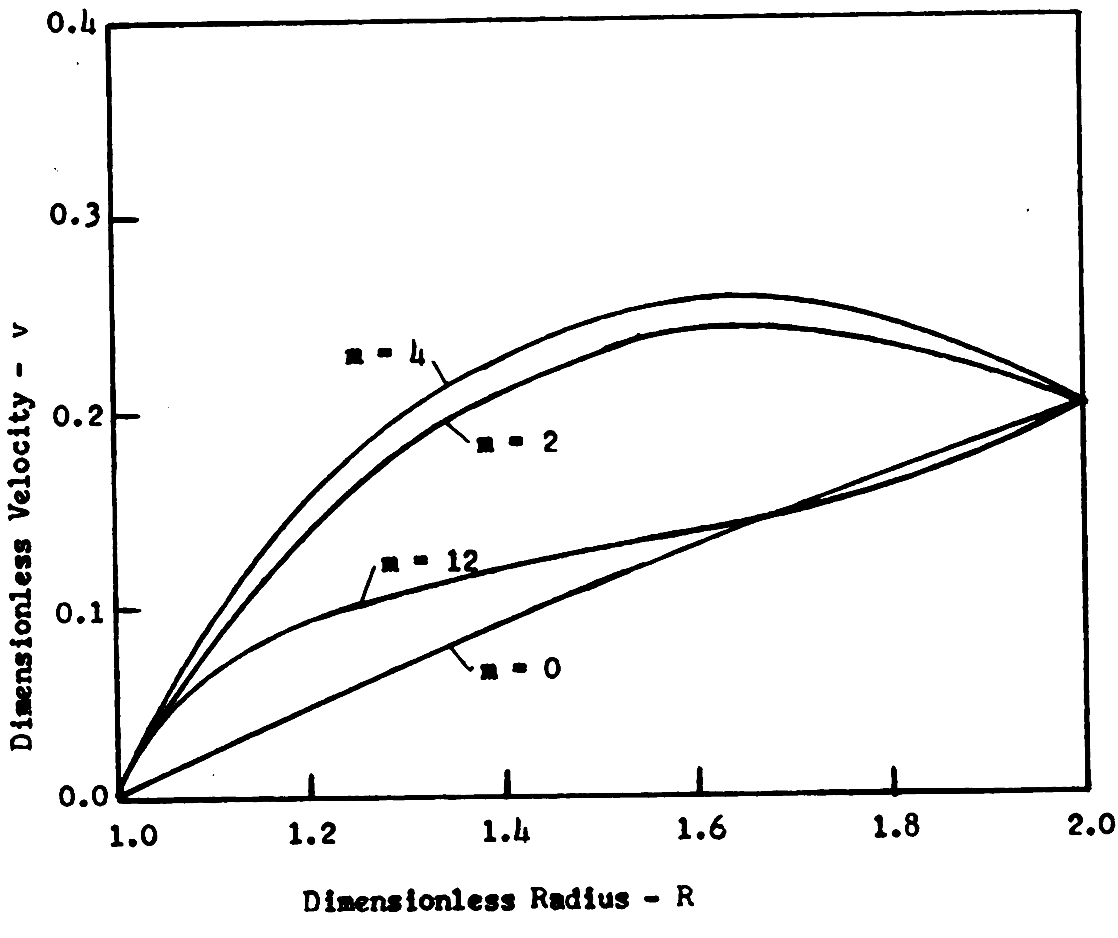


Fig. 7 Velocity Profiles for $v_o = 0.2$

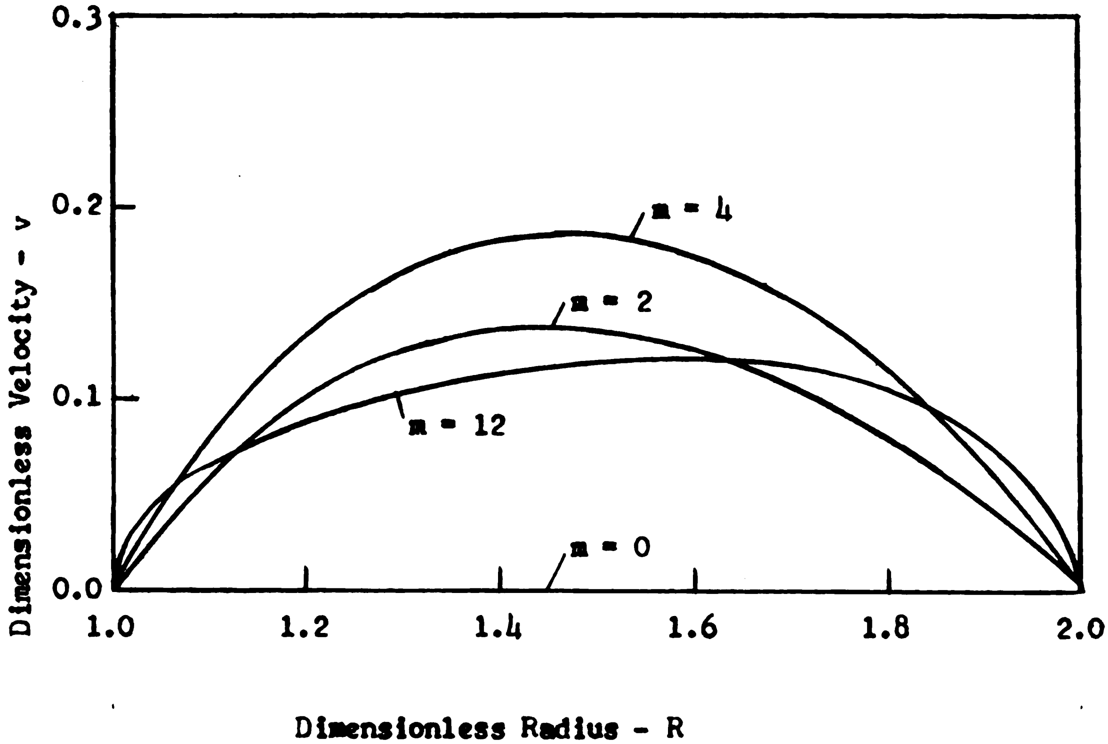


Fig. 8 Velocity Profiles for $v_o = 0$

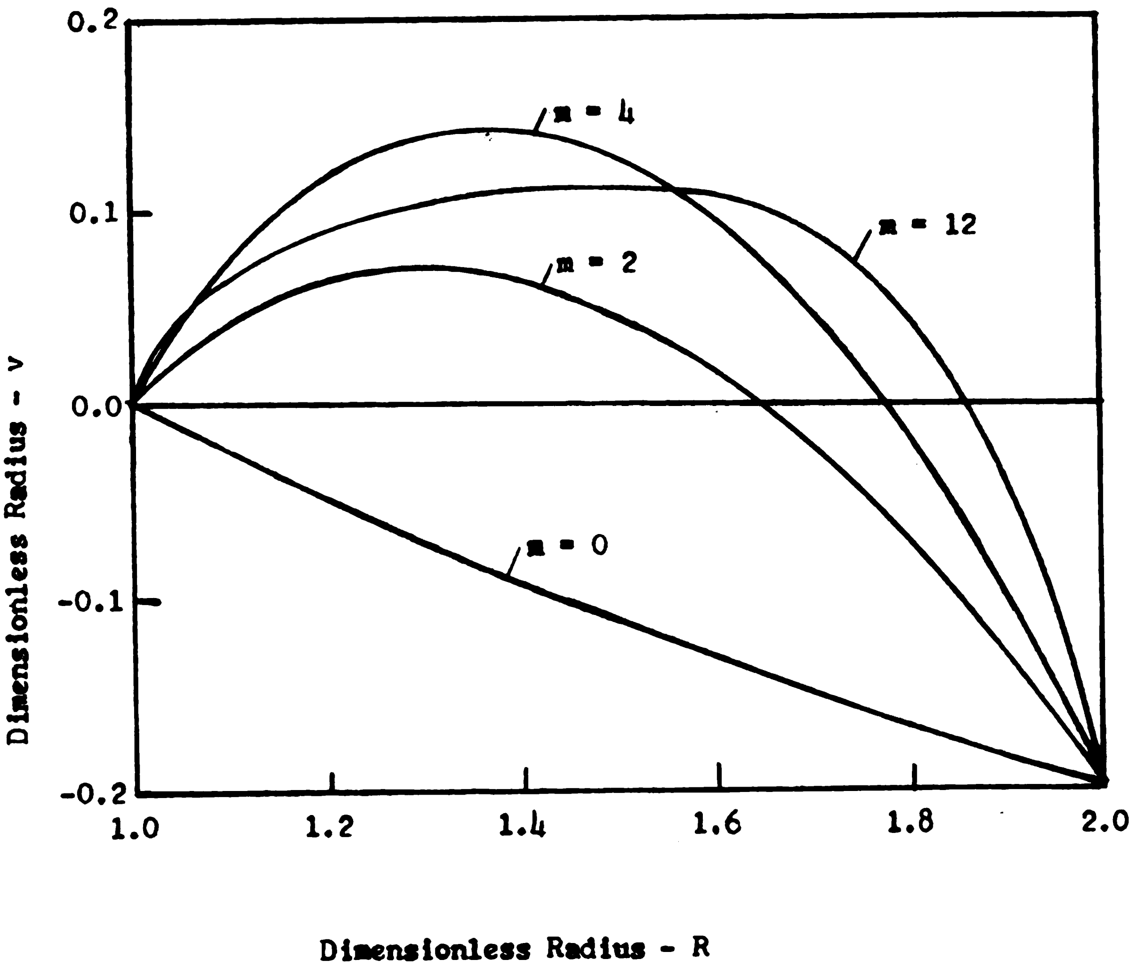


Fig. 9 Velocity Profiles for $v_o = -0.2$

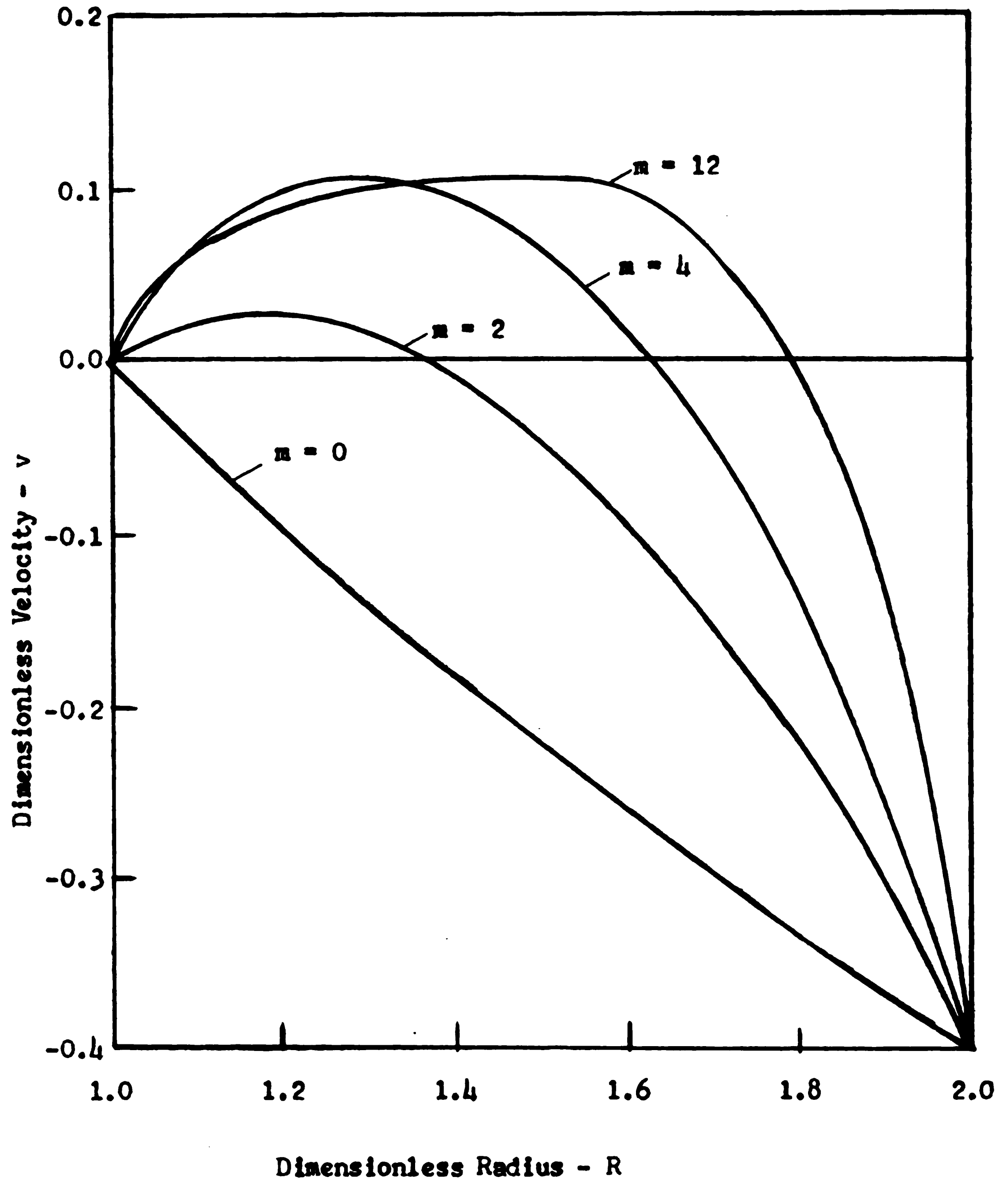


Fig. 10 Velocity Profiles for $v_o = -0.4$

Since the magnetic Prandtl number is invariably small ($10^{-7}$ for mercury), the pressure due to the term containing the magnetic Prandtl number is so small that it cannot be measured. Therefore,

$$P(R) - P(1) = \int_1^R \frac{v^2}{x}\,dx \tag{47}$$

The pressure is found by substituting for the velocity and completing the integration. Thus

$$\begin{aligned} P(R) - P(1) = 1/m^2 \Big[ & \frac{A^2 + 1}{4\beta} \sinh(2\beta \ln R) + \frac{1 - A^2}{2} \ln R \\ & - \frac{A}{\beta} \sinh^2(\beta \ln R) - \frac{2R}{m^2} (\beta + A) \sinh(\beta \ln R) \\ & + \frac{2R}{m^2} (\beta A + 1) \cosh(\beta \ln R) + \frac{R^2}{2} - \frac{1}{2} \\ & - \frac{2}{m^2} (\beta A + 1) \Big] \end{aligned} \tag{48}$$

## LIMITING FORMS OF THE FLOW

In its limiting forms, the flow approaches solutions that have already been determined. Limiting forms of the flow may also be determined for special cases, such as no applied electric field or no viscosity, which have not been previously determined. These special cases will help in understanding the flow processes.

If $m \to 0$, the flow should approach the ordinary flow between rotating cylinders with no magnetic field. Equation (42) may be written in a slightly different form to make the limiting process easier. The equation then becomes

$$v = \frac{v_o \sinh (m^2+1)^{1/2} \ln R}{\sinh (m^2+1)^{1/2} \ln c} - [\cosh (m^2+1)^{1/2} \ln R - R]\, 1/m$$

$$+ \frac{1}{m} \left[\frac{\cosh (m^2+1)^{1/2} \ln c - c}{\sinh (m^2+1)^{1/2} \ln c} \sinh (m^2+1)^{1/2} \ln R\right] \qquad (49)$$

The limit of the above equation as $m \to 0$ is

$$\lim_{m \to 0} v = \frac{v_o \sinh(\ln R)}{\sinh(\ln c)} + \lim_{m \to 0} \frac{1}{m} \Big\{R - \cosh [(m^2+1)^{1/2} \ln R]$$

$$+ \frac{\cosh[(m^2+1)^{1/2} \ln c] - c}{\sinh[(m^2+1)^{1/2} \ln c]} \sinh [(m^2+1)^{1/2} \ln R]\Big\} \quad (50)$$

In order for the limit in equation (50) to be finite, the quantity in brackets must be zero when $m = 0$ so that the limit will be of the form 0/0. If $m = 0$, the quantity in brackets becomes the following:

$$\frac{\cosh(\ln c) - c}{\sinh(\ln c)} \sinh(\ln R) - \cosh(\ln R) + R =$$

$$\frac{c + 1/c - 2c}{c - 1/c} \frac{R - 1/R}{2} - \frac{R + 1/R}{2} + R = 0 \qquad (51)$$

Since the limit is of the form 0/0, L'Hospital's rule may be applied. After applying L'Hospital's rule the limit becomes:

$$\lim_{m \to 0} v = \frac{v_o \sinh(\ln R)}{\sinh(\ln c)} \tag{52}$$

The dimensional quantities may now be inserted. After rearranging, the equation becomes:

$$\lim_{m \to 0} V_\phi = \frac{\omega b^2}{b^2 - a^2} \frac{r^2 - a^2}{r} \tag{53}$$

But this is just the ordinary flow between rotating cylinders and is identical to Eq. 5.15 in Schlichting[2] if the velocity of the inner cylinder is set equal to zero and the necessary changes of symbols are made.

If $a$ and $b$ approach infinity so that their difference is equal to $2L$ and $w/r$ approaches $H_o$ with $v_o = 0$, the velocity distribution should approach that of Hartmann's flow between infinite parallel plates with a transverse magnetic field and a uniform applied electric field parallel to the plates (Fig. 11). Applying the limiting process, the following is obtained:

$$\lim_{a \to \infty} m/a = M/L \tag{54}$$

where $M$ is the regular Hartmann number $\mu H_o L(\sigma/\rho\nu)^{1/2}$.

Let $y$ designate the distance from midchannel so that

$$R = \frac{a + L + y}{a} \tag{55}$$

Then

$$\lim_{a \to \infty} (m^2 + 1)^{1/2} \ln R = M(1 + y/L) \tag{56}$$

and

$$\lim_{a \to \infty} (m^2 + 1)^{1/2} \ln c = 2M \tag{57}$$

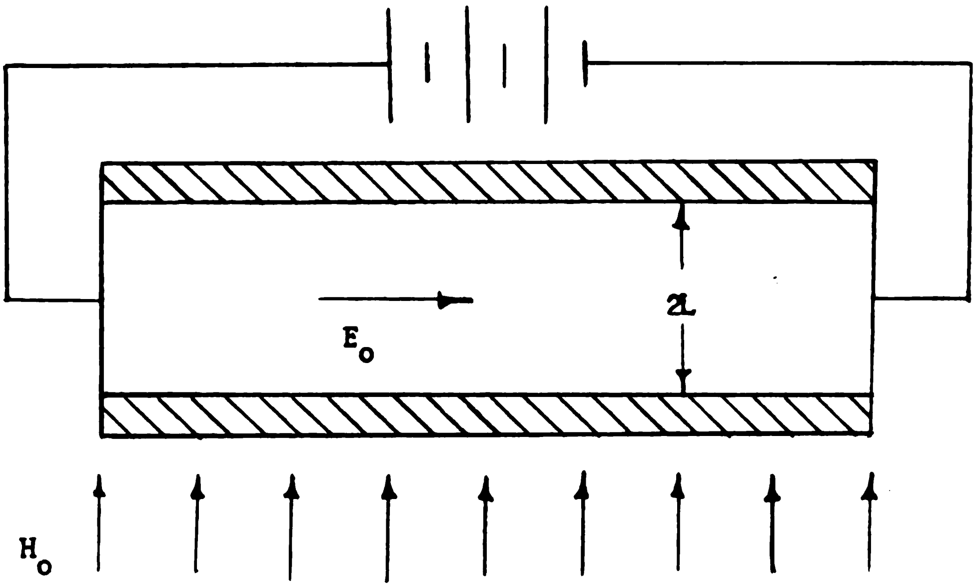


Fig. 11 Rectangular Channel for Hartmann's Flow

and

$$\lim_{a \to \infty} R = 1 \tag{58}$$

Equation (42) is written in terms of the dimensional quantities and the above limits are applied. The equation then becomes the following:

$$\lim_{a \to \infty} V_\phi = \frac{LE_o}{M} (\sigma/\rho\nu)^{1/2} \left[\frac{\cosh 2M - 1}{\sinh 2M} \sinh M(1 + y/L) - \cosh M(1 + y/L) + 1\right] \tag{59}$$

After substituting $\mu H_o L(\sigma/\rho\nu)^{1/2}$ for $M$ and simplifying, the velocity distribution becomes:

$$\lim_{a \to \infty} V_\phi = \frac{E_o}{\mu H_o} \left[1 - \frac{\cosh M(y/L)}{\cosh M}\right] \tag{60}$$

But this is just the velocity distribution for Hartmann's flow and is identical to Eq. 1-29 of Cowling(6) if $P$, the pressure gradient, is set equal to zero in that equation.

For the case of an inviscid fluid, the velocity may be determined from equations (12) and (13) by setting $\nu$ equal to zero. These result in the equation

$$\sigma E_o - \frac{\sigma\mu w}{r} V_\phi = 0 \tag{61}$$

Although $v$ is zero when $\nu = 0$, equation (61) may be written in dimensionless quantities if it is observed that

$$mv = \frac{V_\phi \mu w}{E_o a} \tag{62}$$

The velocity distribution for an inviscid fluid is found by substituting equation (62) into equation (61). The velocity distribution then becomes (Fig. 12):

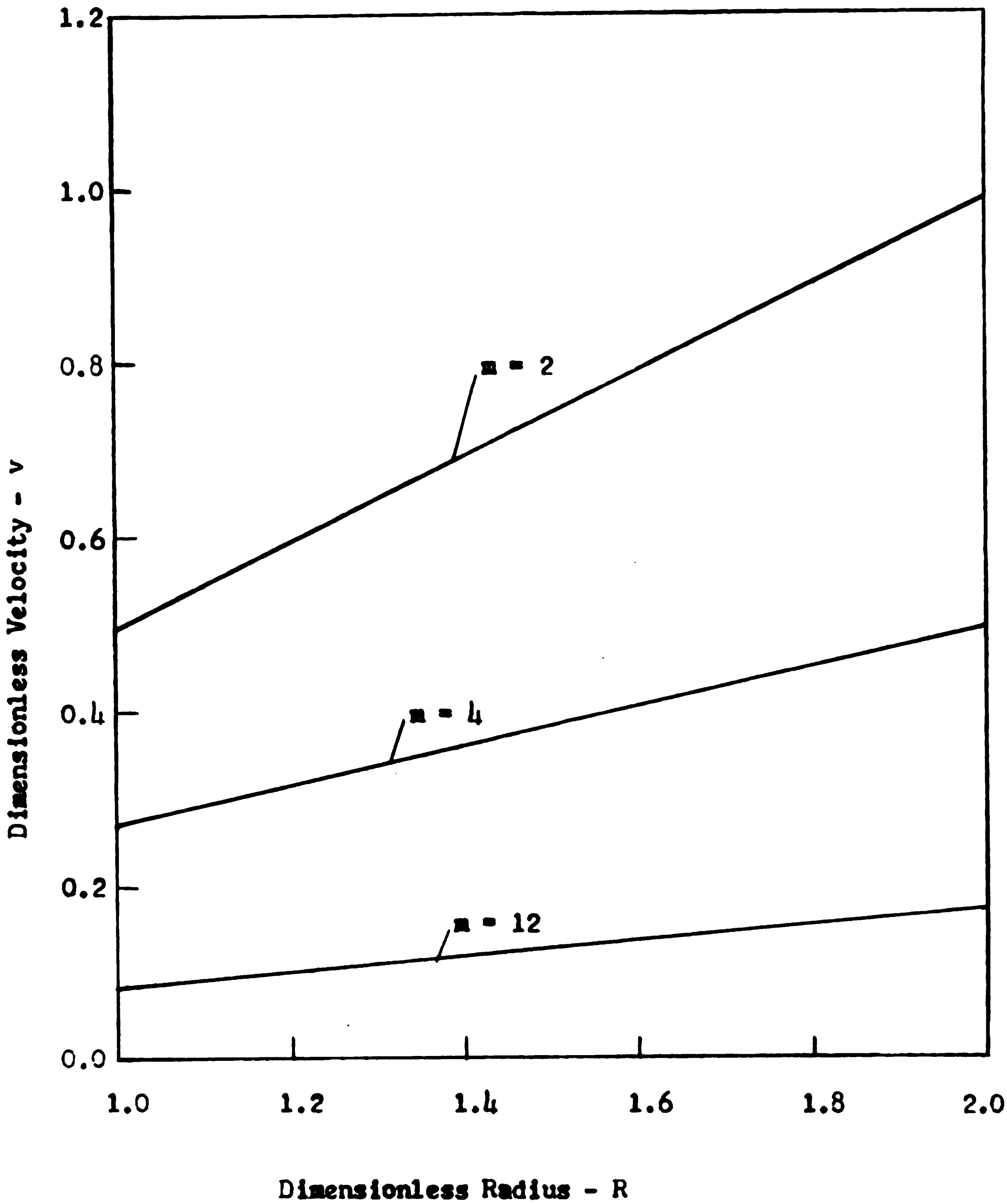


Fig. 12 Velocity Profiles for an Inviscid Fluid

$$v = R/m \tag{63}$$

For the case of no applied electric field, the velocity distribution may be determined from equation (42). In order to do this $v$ and $v_o$ must be written in terms of $V_\phi$ and $\omega$.

$$\frac{V_\phi}{E_o a}(\rho\nu/\sigma)^{1/2} = \frac{\omega b}{E_o a}(\rho\nu/\sigma)^{1/2}\frac{\sinh(\beta \ln R)}{\sinh(\beta \ln c)}$$

$$+ 1/m \left[\frac{\cosh(\beta \ln c) - c}{\sinh(\beta \ln c)} \sinh(\beta \ln R) - \cosh(\beta \ln R) + R\right] \tag{64}$$

When equation (64) is multiplied by $E_o a(\rho\nu/\sigma)^{1/2}$ and $E_o$ is set equal to zero, the velocity distribution with no applied electric field becomes (Fig. 13):

$$V_\phi = \omega b \frac{\sinh(\beta \ln R)}{\sinh(\beta \ln c)} \tag{65}$$

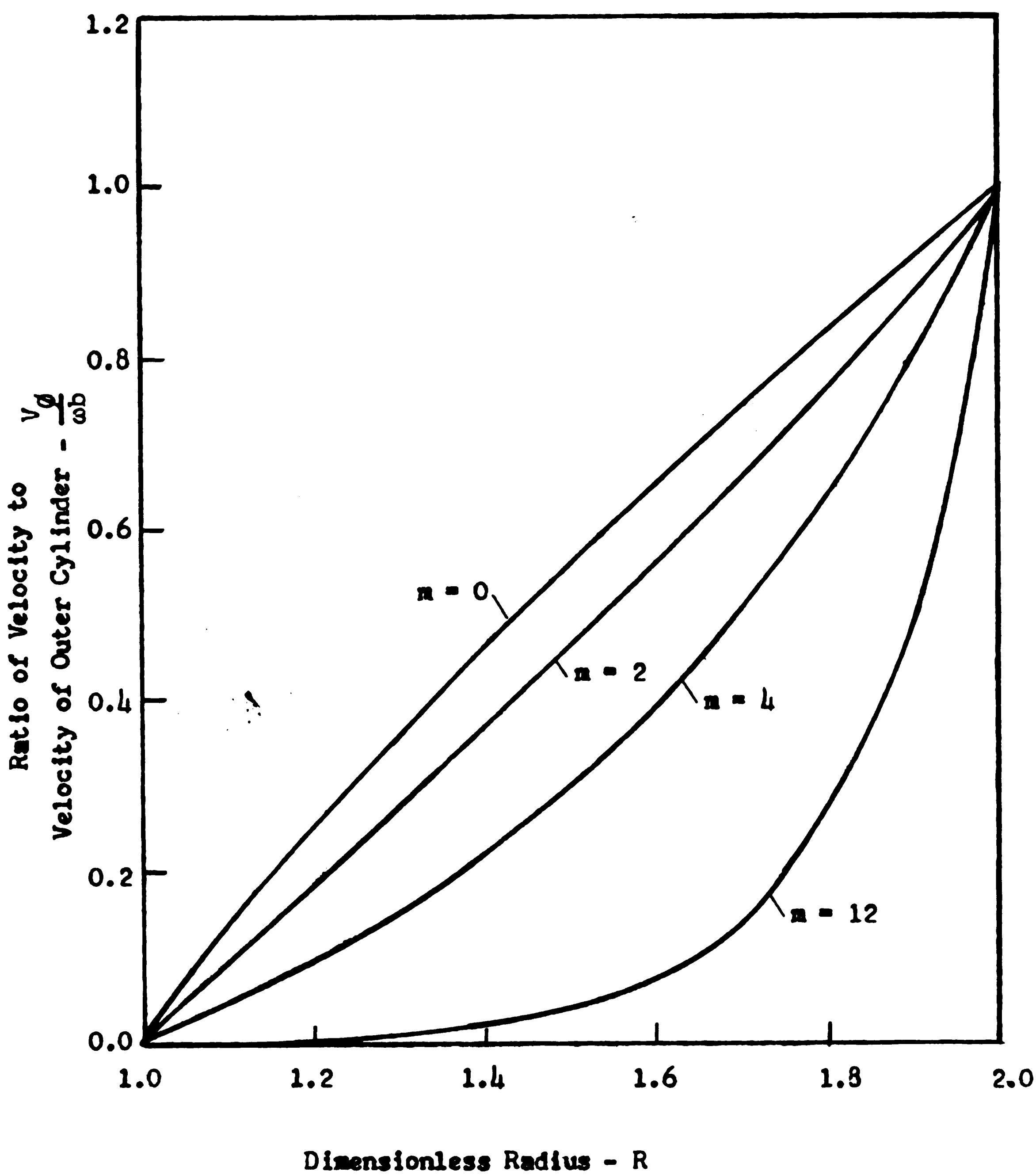


Fig. 13 Velocity Profiles for No Electric Field

## DISCUSSION

From the plots of dimensionless velocity versus the dimensionless radius (Figs. 3-10) it is seen that the velocity profiles approach nearer and nearer to the velocity profiles for inviscid flow as the cylindrical Hartmann number increases. It is also seen that the viscous effects are confined more and more closely to the walls as the cylindrical Hartmann number increases. This substantiates the idea that the viscous effects may be neglected when the Hartmann number is much greater than unity. The cylindrical Hartmann number is of the same order of magnitude as the Hartmann number if $b/a = 2$.

From the plot of the velocity profiles for an inviscid fluid (Fig. 12), it is seen that the magnitude of the velocity increases with decreasing values of the cylindrical Hartmann number. In a viscous fluid the maximum velocity increases and then decreases with increasing cylindrical Hartmann number. This also substantiates the idea that viscous effects are more important at low Hartmann numbers.

From the velocity profiles it is seen that the effects of the magnetic field sometimes oppose the viscous effects of the moving wall. When the wall drags the fluid faster than the current due to the electric field would drive the fluid, then the magnetic field opposes the motion of the fluid. For the case of no applied electric field, (Fig. 13), the magnetic field always opposes the viscous effect of the moving wall.

These results indicate the complexity of the magnetohydrodynamic flow with moving boundaries. They show the interaction of electromagnetic and viscous forces and indicate that moving boundaries further complicate a complex problem.

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