ABSTRACT

DELAY EQUALIZER DESIGN--NUMERICAL ANALYSIS, DIGITAL COMPUTER TECHNIQUES

by Joseph John Lang

The existing delay equalizer design techniques are not completely satisfactory in the sense that they contain many undesirable features such as:

(1) they are very tedious and extremely time consuming;

(2) the design is guided by trial and error process;

(3) the choice of design parameters at each stage of the trial and error process requires skill and experience on the part of the designer.

In recent years, the accelerating demand for higher quality of signal transmission has led to increasingly stringent requirements on delay equalization. As a consequence, the standard curve plotting techniques have become increasingly inadequate and there is a very great need for improved design methods.

In this thesis, the problem of delay equalization is first studied with the specific purpose of locating the drawbacks inherent in the existing delay equalizer design techniques. The results of this study have yielded a new method of delay equalizer design, based on numerical analysis and digital computer techniques. This new design method offers definite improvement over the existing methods:

(1) Given a set of satisfactory initial estimates of the design parameters, the entire design problem is effected in a matter of minutes of computer time;

(2) The thesis method provides a systematic approach to the design problem and eliminates to a large extent the need of skill and experience;

(3) The extensive amount of the trial and error process associated with the existing design techniques is almost eliminated in the thesis design method. The trial and error aspect of the thesis method is reduced to two computer applications;

(4) It appears that the thesis method is capable of producing approximations to a constant delay which are beyond the power of the existing design techniques even though pursued with the utmost skill, experience, and patience over an extended period of time.

The conditions on the delay equalizer network are derived and the problem of delay equalization is defined. The problem of delay equalization is formulated in terms of a system of non-linear algebraic equations. A numerical method for a solution of this system of equations is presented and a general computer program is written. The important consequence of the new design method is demonstrated by means of several solutions to actual equalization problems. The estimate on the number of sections of a delay equalizer is considered and an approach to a solution of this problem is presented. A method for obtaining the initial estimates of the design parameters is given. Formulae for the element values of the delay equalizer, in terms of the design parameters, are presented.

DELAY EQUALIZER DESIGN--NUMERICAL ANALYSIS, DIGITAL COMPUTER TECHNIQUES

By

Joseph John Lang

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Electrical Engineering

1961

ACKNOWLEDGMENTS

The author expresses his indebtedness to his major professor, Dr. Myril B. Reed, for the helpful advice and guidance throughout the preparation of this thesis. Special thanks are due Dr. Herman E. Koenig of the Electrical Engineering Department, Michigan State University, Dr. Gerard P. Weeg and Mrs. Georgia B. Reed of the Computer Laboratory, Michigan State University, and Mr. J. L. Garrison of the Bell Telephone Laboratories, for their many valuable suggestions.

CONTENTS

LIST OF SYMBOLS

Symbol	Description
A_s	Insertion Loss Function
A_{SE}	Insertion loss function of a delay equalizer
A_{SF}	Insertion loss function of a filter
A_{ST}	Total insertion loss function (insertion loss function of a filter and a delay equalizer connected in tandem)
A_{STR}	Total insertion loss requirement (the requirement on the insertion loss function of a filter and a delay equalizer connected in tandem)
B_s	Insertion phase function
B_{SE}	Insertion phase function of a delay equalizer
B_{SF}	Insertion phase function of a filter
B_{ST}	Total insertion phase function (the insertion phase function of a filter and a delay equalizer connected in tandem)
B_{STR}	Total insertion phase requirement (the requirement on the insertion phase function of a filter and a delay equalizer connected in tandem)
d, f_o	Design parameter-pair of one lattice section of a delay equalizer
d_k, f_{ok}	A set of n parameter-pairs for a delay equalizer of n sections $(k=1, 2, 3, \cdots, n)$
e^{P_s}	Insertion factor
f	Frequency (in cycles per second)
f_d	Normalizing frequency
f_i	A set of 2n frequency points from which a system of 2n equations (eq. 3.1.6) is formed. n is the number of lattice sections of the delay equalizer: $(i=1, 2, 3, \cdots, 2n)$
(f_a, f_b)	Delay equalization interval on the frequency axis

Symbol	Description
n	Number of lattice sections of a delay equalizer
P_I	Image transfer function
P_{IE}	Image transfer function of a delay equalizer
P_{IF}	Image transfer function of a filter
P_s	Insertion function
P_{SE}	Insertion function of a delay equalizer
P_{SF}	Insertion function of a filter
P_{ST}	Total insertion function (the insertion function of a filter and a delay equalizer connected in tandem)
R_1	Terminating resistance at the sending (input) terminal-pair of a network
R_2	Terminating resistance at the receiving (output) terminal-pair of a network
T_o	Delay level of the delay equalized filter
T_s	Insertion delay function
T_{SE}	Insertion delay function of a delay equalizer
T_{SER}	Equalizer delay requirement (eq. 2.5.2)
T_{SF}	Insertion delay function of a filter
T_{SFmax}	Maximum value of the insertion delay function of a filter in the equalization interval
T_{SFmin}	Minimum value of the insertion delay function of a filter in the equalization interval
ΔT_{SF}	The difference of the maximum and minimum values of the insertion delay function of a filter in the equalization interval (eq. 5.8)
T_{ST}	Total insertion delay function (the delay function of a filter and a delay equalizer connected in tandem)
x, y	Design parameter-pair of one lattice section of a delay equalizer
x_k, y_k	A set of n parameter-pairs for a delay equalizer of n sections ($k=1, 2, 3, \cdots, n$)
x_k^o, y_k^o	A set of 2n initial estimates for the n design parameter-pairs, x_k, y_k, of a delay equalizer of n sections

Symbol	Description
z	Normalized frequency with respect to the normalizing frequency, f_d
z_i	A set of 2n normalized frequency points from which a system of 2n equations (eq. 3.1.6) is formed. n is the number of lattice sections of the delay equalizer, $(i=1, 2, 3, \cdots, 2n)$
Z_I	Image impedance function
Z_r	Normalized image impedance function with respect to terminating resistance $(r=1, 2, 3, \cdots)$
δ_T	Maximum allowable delay deviation of the total insertion delay function, T_{ST}, from the delay level, T_o
ω	Angular frequency (in radians per second)
t-method	Thesis method of delay equalizer design

I. INTRODUCTION

In the early design of telephone transmission system, only the effects of amplitude distortion in the transmission of signals were taken into consideration. The attenuation (the insertion loss characteristic) within the transmitting band of frequencies was carefully equalized to approximate a constant characteristic to a specified degree of accuracy. However, with the advance of long distance telephone systems and with the demand for improved performance (telephotography, interconnection of radio broadcasting stations over telephone transmission facilities, and finally television), certain disturbing effects were noticed in the transmitted signals. These effects were attributed to insertion phase distortion (deviation of the insertion phase characteristic from linear phase with frequency). The problem of insertion phase equalization appears in literature as early as the 1920's. Thus, a new condition, in addition to constant insertion loss, was found necessary to impose on transmission networks, in the transmitting frequency interval, an insertion phase characteristic which is linear with frequency [8, 27, 31, 36]. This condition of linear insertion phase is given by eq. 2.2.3 and the corresponding condition of constant insertion delay, by eq. 2.5.3.

Filter networks have delay characteristics which are substantially constant over a wide frequency range of the pass-band. Near the cut-off frequency (or frequencies) the delay characteristic increases rapidly. Figures 1.1, 1.2, and 1.3 depict typical delay characteristics of filters with dissipation, for the low-pass, band-pass, and high-pass types, respectively.

The delay of the equalized filter should approximate a constant over a portion or over the entire pass-band of the filter. Consequently, a delay

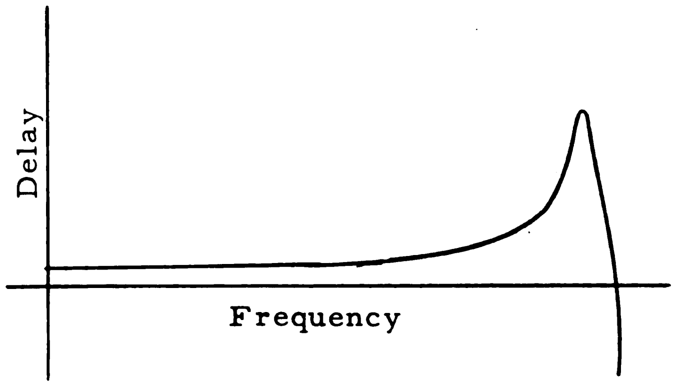


Figure 1.1. Typical Delay Characteristic of a Low-Pass Filter

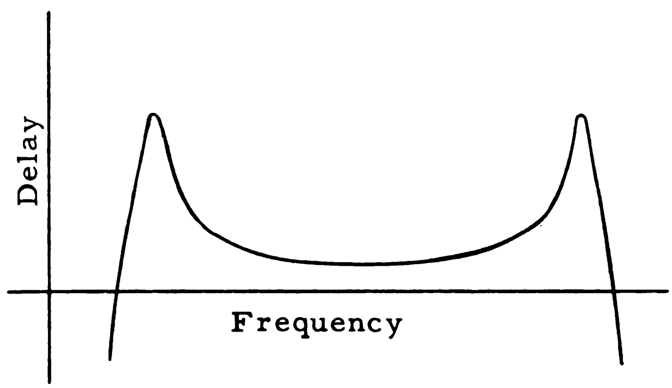


Figure 1.2. Typical Delay Characteristic of a Band-Pass Filter

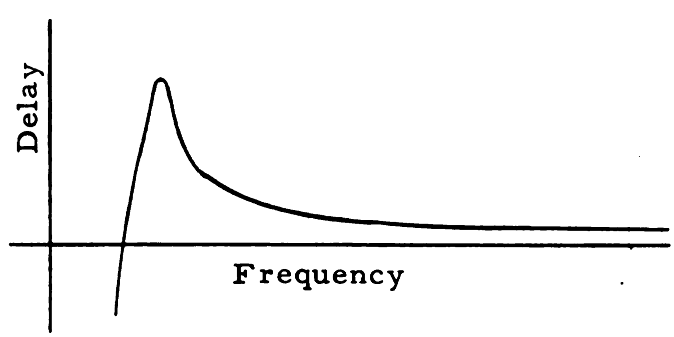


Figure 1.3. Typical Delay Characteristic of a High-Pass Filter

equalizer is a network which is to be connected in cascade with the network to be equalized. This cascade arrangement is to have a delay characteristic approximating a constant to a specified degree of accuracy, throughout a specified interval of equalization. The requirements on the delay equalizer are given in terms of maximum allowable delay deviation of the total delay (the delay of the network to be equalized and the equalizer when connected in tandem) from a constant delay level (see figs. 4.1.1 and 4.3.2). The stringency of the requirements on the delay equalizer depends on the nature of the signals for which the transmission system is intended. Less stringent requirements are imposed on delay equalization of networks in telephone systems, for example, than on those networks in color television systems.

This thesis presents a new method of delay equalizer design which is based on numerical analysis and use of a digital computer. Although both facets (numerical analysis and use of digital computer) have been used extensively in the past, the pattern developed in this thesis combines their use in a new way, leading to a very effective delay equalizer design technique.

II. THE DELAY EQUALIZATION PROBLEM

2.1 Insertion Factor and Insertion Function

The insertion factor, e^{P_s}, of a two terminal-pair network is defined as the ratio of the load current, before insertion of the network between source and load terminations, to the load current after insertion--see figure 2.1.1. The insertion function, P_s, is defined by the equations

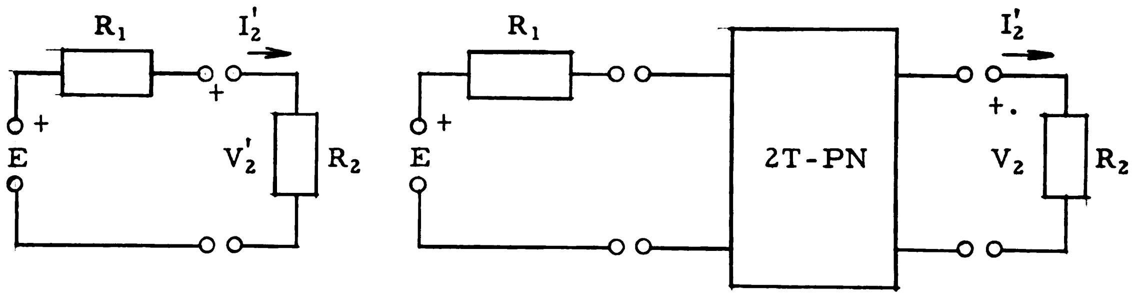


Figure 2.1.1

$$e^{P_s} = \frac{I_2'}{I_2} = \frac{V_2'}{V_2} \qquad (2.1.1)$$

$$P_s = \text{Ln}\left(\frac{I_2'}{I_2}\right) = \text{Ln}\left(\frac{V_2'}{V_2}\right) \qquad (2.1.2)$$

In general the voltage and current variables in the last two equations are complex quantities, hence both the insertion factor and the insertion function are complex functions of frequency ω. The insertion function can be written in the rectangular form as

$$P_s = A_s + jB_s \qquad (2.1.3)$$

where the insertion loss function, A_s, is given in nepers and the insertion phase function, B_s, is expressed in radians.

2.2 Requirements on the Loss and Phase Functions

Figure 2.2.1 (a) shows a two terminal-pair network (a filter) and figure 2.2.1 (b) shows a tandem arrangement of a filter and an equalizer.

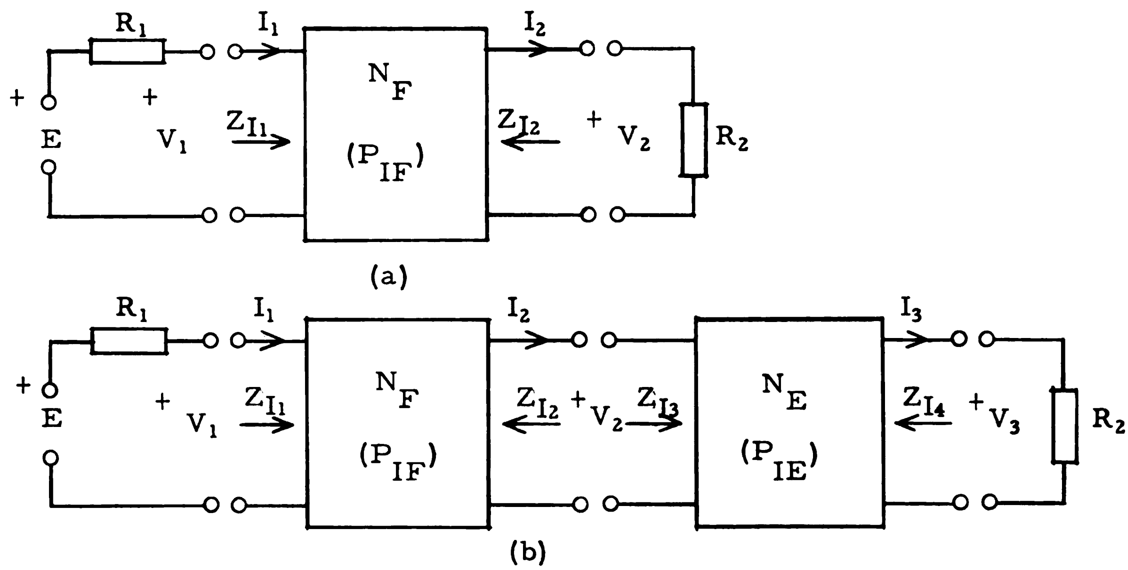


Figure 2.2.1

Let P_{SF} denote the insertion function of the network N_F (the filter) in figure 2.2.1 (a) and let P_{ST} be the insertion function of the tandem arrangement of the two networks (the filter and the equalizer) in figure 2.2.1 (b). Writing the insertion functions in the rectangular form

$$P_{SF} = A_{SF} + jB_{SF}, \quad P_{ST} = A_{ST} + jB_{ST} \tag{2.2.1}$$

The network N_F is considered to be an existing filter, hence the insertion function P_{SF} is a known function of the radian frequency ω. The equalization interval (ω_a, ω_b) on the ω axis may be a portion or the entire passband of the filter. The network N_E (the phase equalizer) is to be designed such that the total insertion loss function, A_{ST}, and the total insertion phase function, B_{ST}, will approximate the total insertion loss requirement, A_{STR}, and the total insertion phase requirement, B_{STR}, respectively, to any specified degree of accuracy:

$$A_{ST} \doteq A_{STR} = A_{SF} + k_1 \tag{2.2.2}$$

such that $|A_{ST} - A_{STR}| < \delta_A$

$$B_{ST} \doteq B_{STR} = k_2\omega + 2\pi k_3 \tag{2.2.3}$$

such that $|B_{ST} - B_{STR}| < \delta_B$

for all ω in (ω_a, ω_b). k_1 and k_2 are positive constants, preferably as small as possible and k_3 is any integer or zero [27, 31, 36, 52].

In order to obtain conditions on the properties of the phase equalizer, N_E, such that eqs. 2.2.2 and 2.2.3 hold over the interval (ω_a, ω_b), a relationship between the functions P_{ST}, P_{SF}, and P_{IE} (the image transfer function of the equalizer) must be found. A particularly convenient relationship would be an expression for P_{IE} as an explicit function of P_{ST} and P_{SF}.

2.3 The Relationship Between the Insertion Functions P_{ST} and P_{SF} and the Image Transfer Function P_{IE}

In the view that the final goal of this thesis is directed towards a new design method of phase equalizers, it is recognized that an involved relationship between the functions might make the design too cumbersome. A very simple relationship can be obtained if two conditions are imposed on the equalizer network. These two conditions and the resulting relationship are stated in the form of a theorem [31, 52]

Theorem: If in figure 2.2.1 (b) the following two conditions hold for all values of ω

$$Z_{I_3} = Z_{I_4} = R_2, \tag{2.3.1}$$

then for all values of ω, the following relationship is true

$$P_{ST} = P_{SF} + P_{IE} \qquad (2.3.2)$$

Proof: The input and output voltage and current variables relationship of the filter network N_F in figure 2.2.1 (a) is given by [33, 42]

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{Z_{I1}}{Z_{I2}}} \cosh P_{IF} & \sqrt{Z_{I1} Z_{I2}} \sinh P_{IF} \\ \frac{1}{\sqrt{Z_{I1} Z_{I2}}} \sinh P_{IF} & \sqrt{\frac{Z_{I2}}{Z_{I1}}} \cosh P_{IF} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (2.3.3)$$

This last equation can be written in symbolic form as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_F & B_F \\ C_F & D_F \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (2.3.4)$$

In addition, from figure 2.2.1 (a)

$$V_2 = R_2 I_2 \qquad (2.3.5)$$

$$V_1 = E - R_1 I_1 \qquad (2.3.6)$$

From eqs. 2.3.4, 2.3.5, and 2.3.6, after eliminating V_1, I_1, and V_2, a relation for I_2 in terms of E is given by

$$I_2 = \frac{E}{C_F R_1 R_2 + A_F R_2 + D_F R_1 + B_F} \qquad (2.3.7)$$

On the other hand, before insertion of the network N_F

$$I_2' = \frac{E}{R_1 + R_2} \qquad (2.3.8)$$

Therefore, the ratio which determines the insertion function, P_{SF}, is

$$e^{P_{SF}} = \frac{I_2'}{I_2} = \frac{C_F R_1 R_2 + A_F R_2 + D_F R_1 + B_F}{R_1 + R_2} \qquad (2.3.9)$$

For convenience, the normalized image impedances with respect to the terminating resistances are defined

$$Z_1 = \frac{Z_{I1}}{R_1} \quad , \quad Z_2 = \frac{Z_{I2}}{R_2} \tag{2.3.10}$$

Substituting the values of A_F, B_F, C_F, and D_F from eq. 2.3.3 into eq. 2.3.9 and eq. 2.3.10 into the result, the insertion factor is obtained [42]

$$e^{P_{SF}} = \frac{\sqrt{R_1 R_2}}{R_1 + R_2} \left[\frac{1 + Z_1 Z_2}{\sqrt{Z_1 Z_2}} \sinh P_{IF} + \frac{Z_1 + Z_2}{\sqrt{Z_1 Z_2}} \cosh P_{IF} \right] \tag{2.3.11}$$

Next, the insertion factor, $e^{P_{ST}}$, for the tandem arrangement of the filter and equalizer in figure 2.2.1 (b) is derived. In addition to the terminal equations for the filter network N_F, given by eq. 2.3.3 and in addition to eqs. 2.3.5 and 2.3.6, there are the terminal equations for the equalizer network N_E, given by [33, 42]

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{Z_{I3}}{Z_{I4}}} \cosh P_{IE} & \sqrt{Z_{I3} Z_{I4}} \sinh P_{IE} \\ \frac{1}{\sqrt{Z_{I3} Z_{I4}}} \sinh P_{IE} & \sqrt{\frac{Z_{I4}}{Z_{I3}}} \cosh P_{IE} \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \tag{2.3.12}$$

This last equation can be written in symbolic form as

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_E & B_E \\ C_E & D_E \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \tag{2.3.13}$$

From eqs. 2.3.3, 2.3.5, 2.3.6, and 2.3.13, after eliminating V_1, I_1, V_2, I_2, and V_3, a relationship for I_3 in terms of E is given by

$$I_3 = \frac{E}{AR_1R_2 + BR_2 + CR_1 + D} \tag{2.3.14}$$

where
$$A = C_F A_E + D_F C_E$$
$$B = A_F A_E + B_F C_E$$
$$C = C_F B_E + D_F D_E$$
$$D = A_F B_E + B_F D_E$$

On the other hand, before insertion of the N_F and N_E networks

$$I_3' = \frac{E}{R_1 + R_2} \tag{2.3.15}$$

Therefore, the ratio which determines the insertion function, P_{ST}, is given by

$$e^{P_{ST}} = \frac{I_3'}{I_3} = \frac{AR_1R_2 + BR_2 + CR_1 + D}{R_1 + R_2} \tag{2.3.16}$$

Again, for convenience the normalized image impedances are defined as follows

$$Z_1 = \frac{Z_{I1}}{R_1}, \quad Z_2 = \frac{Z_{I2}}{Z_{I3}}, \quad Z_3 = \frac{Z_{I4}}{R_2} \tag{2.3.17}$$

Substituting the values of A_F, B_F, C_F, D_F, A_E, B_E, C_E, and D_E into eq. 2.3.16 and eq. 2.3.17 into the result, and insertion factor, $e^{P_{ST}}$, is obtained

$$e^{P_{ST}} = \frac{I_3'}{I_3} =$$
$$= \frac{\sqrt{R_1R_2}}{R_1+R_2}\left[\frac{1 + Z_1Z_2Z_3}{\sqrt{Z_1Z_2Z_3}} \sinh P_{IF} \left(\cosh P_{IE} + \frac{Z_3 + Z_1Z_2}{1 + Z_1Z_2Z_3} \sinh P_{IE}\right)\right.$$
$$\left. + \frac{Z_1 + Z_2Z_3}{\sqrt{Z_1Z_2Z_3}} \cosh P_{IF} \left(\cosh P_{IE} + \frac{Z_2 + Z_1Z_3}{Z_1 + Z_2Z_3} \sinh P_{IE}\right)\right] \tag{2.3.18}$$

The expressions for the insertion factors, $e^{P_{SF}}$ and $e^{P_{ST}}$, are given by eqs. 2.3.11 and 2.3.18, respectively. From the hypothesis of the theorem

$$Z_{I3} = Z_{I4} = R_2 \tag{2.3.19}$$

Substituting eq. 2.3.19 into eq. 2.3.17, the normalized image impedances become

$$Z_1 = \frac{Z_{I1}}{R_1}, \quad Z_2 = \frac{Z_{I2}}{R_2}, \quad Z_3 = 1 \qquad (2.3.20)$$

In this last equation, the normalized image impedances Z_1 and Z_2 are identical with those given by eq. 2.3.10. Substituting eq. 2.3.20 into eq. 2.3.18, the expression for the insertion factor, $e^{P_{ST}}$, becomes

$$e^{P_{ST}} = \frac{\sqrt{R_1 R_2}}{R_1 + R_2}\left[\frac{1 + Z_1 Z_2}{\sqrt{Z_1 Z_2}} \sinh P_{IF} + \frac{Z_1 + Z_2}{\sqrt{Z_1 Z_2}} \cosh P_{IF}\right] e^{P_{IE}} \qquad (2.3.21)$$

Now, from eq. 2.3.11 and from this last equation

$$e^{P_{ST}} = e^{P_{SF}} \cdot e^{P_{IE}} \qquad (2.3.22)$$

hence

$$P_{ST} = P_{SF} + P_{IE} \qquad (2.3.23$$

This proves the theorem.

The theorem states that if the equalizer network is a symmetric, constant resistance network, with the resistance level equal to the receiving (load) resistance, then the insertion function of the tandem arrangement of a filter and an equalizer is given by the sum of the insertion function of the filter and the image transfer function of the equalizer. But under these restrictions on the equalizer, the insertion function of the equalizer, defined with respect to an R_2 load termination, is equal to the image transfer function of the equalizer. Then eq. 2.3.23 may also be written as

$$P_{ST} = P_{SF} + P_{SE} \qquad (2.3.24)$$

These three insertion functions are complex functions of frequency. Equating the real and imaginary parts yields

$$A_{ST} = A_{SF} + A_{SE} \tag{2.3.25}$$

$$B_{ST} = B_{SF} + B_{SE} \tag{2.3.26}$$

From eq. 2.2.2 and 2.3.25

$$A_{SE} = A_{IE} \doteq k_1 \tag{2.3.27}$$

where k_1 is to be made as small as possible. Theoretically, k_1 can be made zero for all values of frequency if an additional restriction is imposed on the equalizer network, namely, restricting the equalizer to a lossless LC structure. Under such restriction

$$A_{SE} = A_{IE} = 0 \qquad \text{for all frequencies} \tag{2.3.28}$$

and then

$$A_{ST} = A_{SF} \qquad \text{for all frequencies} \tag{2.3.29}$$

Thus, the first requirement on the equalizer, given by eq. 2.2.2, is satisfied by restricting the equalizer to a class of symmetric, constant resistance, all pass networks, with the resistance level equal to the receiving resistance [27, 31, 36, 52].

From the second requirement on the equalizer, given by eq. 2.2.3, and from eq. 2.3.26

$$B_{SE} \doteq k_2\omega + 2\pi k_3 - B_{SF} \qquad \omega_a \leq \omega \leq \omega_b \tag{2.3.30}$$

where k_2 is to be as small as possible and k_3 is any integer or zero. The relation 2.3.30 states that in the frequency interval (ω_a, ω_b) the insertion phase of the equalizer must approximate a function which is complementary to the insertion phase function of the filter with respect to a straight line of slope k_2 and intercept $2\pi k_3$ [27, 31, 36, 52].

2.4 The Insertion Delay Function

The insertion delay function, $T_s(\omega)$, used throughout this thesis is defined

$$T_s(\omega) = \frac{d}{d\,\omega} B_s(\omega) \tag{2.4.1}$$

where $B_s(\omega)$ is the insertion phase function of the network in question and is expressed in radians. $T_s(\omega)$ is the insertion delay function, has units of time and is expressed in seconds.

2.5 Requirements on the Insertion Delay Function

The requirement on the insertion phase function, given by eq. 2.3.20, can be written in terms of the delay function as follows

$$T_{SE}(\omega) = \frac{d}{d\,\omega} B_{SE}(\omega) \doteq k_2 - T_{SF}(\omega)\ , \quad \omega_a \leq \omega \leq \omega_b \tag{2.5.1}$$

The relation 2.5.1 states that in the frequency interval (ω_a, ω_b) the insertion delay function of the equalizer must approximate a function which is complementary to the insertion delay function of the filter with respect to a constant k_2. The constant k_2 is commonly referred to as the delay level and is denoted by T_o. The right side of eq. 2.5.1 is called the equalizer delay requirement and is denoted by T_{SER}. The equalizer delay requirement then becomes

$$T_{SER} = T_o - T_{SF} \qquad \omega_a \leq \omega \leq \omega_b \tag{2.5.2}$$

If a given filter is to be delay equalized in the frequency interval (ω_a, ω_b), the insertion delay of the filter, T_{SF}, is a known function of frequency. The insertion delay of the filter can be measured on a test set, if the actual filter is available, or the delay function can be calculated on a digital computer, if only the schematic diagram and element values of the filter network are at hand. Hence, from eq. 2.5.2, given any filter, the equalizer delay requirement, T_{SER}, is known within the delay level T_o.

In practice, the requirements on the total delay, T_{ST}, (the delay of the filter in tandem with the equalizer) are given in terms of the maximum allowable delay deviation, δ_T, from a constant value (from the delay level T_o). Stated mathematically

$$|T_{ST}-T_o| = |T_{SE}+T_{SF}-T_o| = |T_{SE}-T_{SER}| < \delta_T \qquad (2.5.3)$$

for $\omega_a \leq \omega \leq \omega_b$

2.6 The Insertion Delay Function for Matched Cascaded Lattice Sections

From the theorem of section 2.3 and from eq. 2.3.28, it follows that, if the class of networks considered for the delay equalizer is limited to the class of constant resistance, all pass networks, with the resistance level equal to the receiving resistance, R_2, of the filter, then the additive property of the insertion functions of the filter and the equalizer is preserved and the insertion loss of the equalizer is zero.

The fundamental building block of a delay equalizer is a lattice, constant resistance, 360 degree, all pass section, shown in figure 2.6.1 [27, 31, 34, 36, 52].

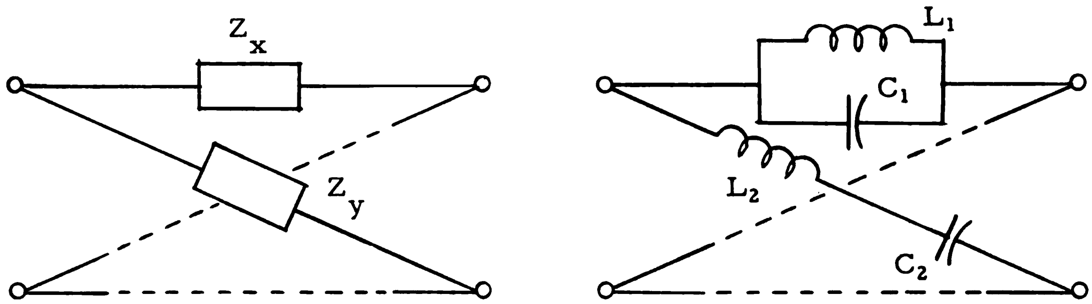


Figure 2.6.1

The series arm and the cross arm impedances of this section are inverse with respect to the terminating resistance, R_2

$$Z_x Z_{\bar{y}} = R_2^2 \qquad (2.6.1)$$

or in terms of the element values

$$\frac{L_1}{C_2} = \frac{L_2}{C_1} = R_2^2 \qquad (2.6.2)$$

When this section is terminated at one port by the resistance, R_2, the input impedance at the other port is R_2, for all values of frequency.

Although the section contains eight elements, only four elements are distinct. Two additional restrictions are imposed on the four element values, L_1, L_2, C_1, and C_2, by eq. 2.6.2, so that one such section contains only two independent parameters. The insertion function with respect to R_2 terminations and the image transfer function are identical and are both pure imaginary. For all values of frequency

$$A_s(\omega) = A_I(\omega) = 0 \qquad (2.6.3)$$

$$B_s(\omega) = B_I(\omega) = 2 \text{ arccot} \sqrt{\frac{L_2}{L_1}} \left[\frac{1}{\omega\sqrt{L_1C_1}} - \omega\sqrt{L_1C_1} \right] \qquad (2.6.4)$$

The insertion delay function of the section is obtained by differentiating this last equation with respect to ω

$$T_s(\omega) = \frac{2\sqrt{\frac{L_2}{L_1}}}{\frac{1}{\sqrt{L_1C_1}}} \left[\frac{1 + \frac{1}{\omega^2 L_1 C_1}}{1 + \frac{L_2}{L_1}\left[\frac{1}{\omega\sqrt{L_1C_1}} - \omega\sqrt{L_1C_1}\right]^2} \right] \qquad (2.6.5)$$

In these last two equations, let

$$d = \sqrt{\frac{L_2}{L_1}} \quad \text{and} \quad f_o = \frac{1}{2\pi\sqrt{L_1C_1}} = \frac{1}{2\pi\sqrt{L_2C_2}} \qquad (2.6.6)$$

The insertion phase and delay functions for one section then become

$$B_s(f) = 2 \text{ arccot } (d) \left(\frac{f_o}{f} - \frac{f}{f_o} \right) \qquad (2.6.7)$$

$$T_s(f) = \frac{d}{\pi f_o} \cdot \frac{1 + (f_o/f)^2}{1 + d^2(f/f_o - f_o/f)^2} \qquad (2.6.8)$$

where

f_o is the value of frequency at which the insertion phase function, $B_s(f)$, takes on the value of π radians; d is a dimesionless constant; $T_s(f)$, the insertion delay function has units of time.

If the impedance level of each section is fixed at the value of the terminating resistance, R_2, then the sections are image impedance matched. Consequently, the insertion function of a cascade arrangement of such sections is equal to the sum of the insertion functions of the individual sections. Then, the insertion delay function of a delay equalizer, composed of n sections connected in cascade, is given by

$$T_{SE}(f) = \sum_{k=1}^{n} \frac{d_k}{\pi f_{ok}} \cdot \frac{1 + \left(\frac{f_{ok}}{f}\right)^2}{1 + d_k^2 \left(\frac{f}{f_{ok}} - \frac{f_{ok}}{f}\right)^2} \tag{2.6.9}$$

2.7 The Design Problem

The design problem is to approximate the equalizer delay requirement, T_{SER}, given by eq. 2.5.2, by a finite sum, given in the last equation.

Stated mathematically

Given the insertion delay function of a filter, $T_{SF}(f)$, and the maximum allowable delay deviation, $\delta_T > 0$, determine the positive constants T_o, d_k, f_{ok}, $(k=1, 2, 3, \cdots, n)$ and the positive integer n, such that

$$|T_{SE}(f) + T_{SF}(f) - T_o| < \delta_T \tag{2.7.1}$$

for all $f_a \leq f \leq f_b$, with $f_a \geq 0$ and $f_b > 0$

III. DELAY EQUALIZER DESIGN BY NUMERICAL METHODS

3.1 The Design Problem in Terms of a Solution to a System of Non-linear Algebraic Equations

If an estimate on the number of sections, n, of the delay equalizer is assumed, i.e., n is taken as known, then the design problem can be formulated in terms of a system of non-linear algebraic equations. A solution to such a system of equations is obtained by numerical techniques in conjunction with the use of a digital computer.

The insertion delay function of a delay equalizer of n sections, given by eq. 2.6.9, in terms of normalized frequency, z, with respect to some arbitrary frequency, f_d ($f_d \neq 0$), and in terms of a new set of parameters, the x and y parameters, is

$$T_{SE}(z) = \frac{1}{\pi f_d} \sum_{k=1}^{n} \frac{z^2 x_k + y_k}{z^2 + (z^2 x_k - y_k)^2} \tag{3.1.1}$$

where

$$z = \frac{f}{f_d}, \quad x = d\frac{f_d}{f_o}, \quad y = d\frac{f_o}{f_d} \tag{3.1.2}$$

The problem is to approximate the equalizer delay requirement, T_{SER}, given by eq. 2.5.2, by $T_{SE}(z)$, in the equalization interval (z_a, z_b), within the maximum allowable delay deviation, δ_T:

$$T_{SE}(z) \doteq T_o - T_{SF}(z) \tag{3.1.3}$$

This last relation contains 2n+1 unknowns: T_o, x_k, and y_k (k=1, 2, 3$\cdots$, n). A system of 2n+1 equations is formed from the relation 3.1.3, one equation for each distinct value of z_i (i=1, 2, 3, $\cdots$, 2n+1), $z_i \neq z_j$ for $i \neq j$, and is given by

$$T_{SE}(z_i, x_k, y_k) - T_o + T_{SF}(z_i) = 0 \tag{3.1.4}$$

This system of 2n+1 equations is reduced to a system of 2n equations, in the unknowns x_k and y_k, by eliminating the unknown T_o. An explicit expression for T_o is obtained from that equation of the set 3.1.4, corresponding to $z_i = 1$ ($f_i = f_d$) and then this expression is substituted into the remaining set of 2n equations, yielding

$$T_{SE}(z_i, x_k, y_k) - T_{SE}(1, x_k, y_k) - T_{SF}(1) + T_{SF}(z_i) = 0 \tag{3.1.5}$$

where

$z_i \neq z_j$, for $i \neq j$ ($i=1, 2, 3, \cdots, 2n$) and $z_i \neq 1$ for all i.

In further detail, this last system of equations becomes

$$\frac{1}{\pi f_d} \sum_{k=1}^{n} \left(\frac{z_i^2 x_k + y_k}{z_i^2 + (z_i^2 x_k - y_k)^2} - \frac{x_k + y_k}{1 + (x_k - y_k)^2} \right) - T_{SF}(1) + T_{SF}(z_i) = 0 \tag{3.1.6}$$

which on multiplying by πf_d and letting

$$\begin{aligned} A_d &= \pi f_d T_{SF}(1), & A_i &= \pi f_d T_{SF}(z_i) \\ B_{ik} &= z_i^2 x_k + y_k, & B_k &= x_k + y_k \\ C_{ik} &= z_i^2 x_k - y_k, & C_k &= x_k - y_k \\ D_{ik} &= 1 + C_{ik}^2, & D_k &= 1 + C_k^2 \end{aligned} \tag{3.1.7}$$

yields

$$\sum_{k=1}^{n} \left(\frac{B_{ik}}{D_{ik}} - \frac{B_k}{D_k} \right) - A_d + A_i = 0 \tag{3.1.8}$$

which is of the general form

$$F_i(X) = 0 \qquad (i=1, 2, 3, \cdots, 2n) \tag{3.1.9}$$

where

$$F_i(X) = F_i(x_1, x_2, x_3, \cdots, x_n, y_1, y_2, y_3, \cdots, y_n)$$

If a solution to this last system of 2n equations is obtained exactly, then the approximation of the equalizer delay requirement, T_{SER}, by the insertion delay function of the equalizer, $T_{SE}(z)$, is exact at 2n+1 points, (at the 2n points, z_i, and at $z = 1$).

3.2 A Numerical Method for a Solution of a System of Non-linear Algebraic Equations

A number of numerical methods for a solution of a system of non-linear algebraic equations appears in literature on numerical analysis [22, 23, 26]. One class of numerical methods, such as the method of steepest descent, the method of Seidel, or the method of relaxation, reduces the problem to that of solving a sequence of single non-linear algebraic equations. The class of functional iteration methods such as the Newton-Raphson method, reduces the problem to that of solving a sequence of systems of linear algebraic equations.

The method of Seidel is here used to obtain a solution of the system of 2n equations, given by eq. 3.1.8. A brief presentation of this method follows.

Consider a system of m non-linear algebraic equations

$$F_i(X) = 0 \quad (i=1, 2, 3, \cdots, m) \tag{3.2.1}$$

where

$$F_i(X) = F_i(x_1, x_2, x_3, \cdots, x_m)$$

Define the function M(X) as

$$M(X) = \sum_{i=1}^{m} F_i^2(X) \tag{3.2.2}$$

If the functions $F_i(X)$ are real functions of real variables $(x_1, x_2, x_3, \cdots, x_m)$, then the function M(X) is either positive or equal to zero, and will be equal to zero only if X is a solution of eq. 3.2.1. Therefore, a solution of eq. 3.2.1 is obtained by finding a value of X at which the function M(X) has an absolute minimum. The method of Seidel is a technique which minimizes the function M(X) by correcting one variable at a time in a fixed order.

If a set of initial estimates

$X_o = (x_1^o, x_2^o, x_3^o, \cdots, x_m^o)$

is sufficiently close to a solution X of eq. 3.2.1, then an improved value, x_1^1, of the first variable, x_1^o, is obtained by the Seidel method, such that $M(X_1) < M(X_o)$,

where $X_1 = (x_1^1, x_2^o, x_3^o, \cdots, x_m^o)$.

An improved value, x_2^1, for the second variable, x_2^o, is obtained such that $M(X_2) < M(X_1)$,

where $X_2 = (x_1^1, x_2^1, x_3^o, \cdots, x_m^o)$.

When all m variables have been corrected once, the set of m cycles is called the Seidel cycle and the process is repeated. Let the letter a denote the number of Seidel cycles, $(a=0, 1, 2, 3, \cdots)$. Then, an improved value, x_j^{am}, for the j^{th} variable, $x_j^{(a-1)m}$, is obtained such that

$$M(X_{am+j}) < M(X_{(a-1)m+j})$$

The improved values of the variables are found in the following manner. To find an improved value, $x_j^{(a+1)m}$, for the j^{th} variable, x_j^{am}, the function M(X) is differentiated with respect to the j^{th} variable, all other variables are considered constant, given by their current estimates. The derivative of M(X) is equated to zero and a solution of this resulting equation for the j^{th} variable, x_j^{am}, yields the improved value, $x_j^{(a+1)m}$

$$\frac{\partial}{\partial x_j^{am}} M(X_{am+j}) = 0 \qquad (3.2.3)$$

This last equation is ordinarily a non-linear equation in x_j^{am} and some method of successive approximation for an equation in one variable must be applied. In the problem of this thesis, a single application of the Newton-Raphson method was selected to obtain an approximate solution for $x_j^{(a+1)m}$

$$x_j^{(a+1)m} = x_j^{am} - \frac{\dfrac{\partial}{\partial x_j^{am}} M(X_{am+j})}{\dfrac{\partial^2}{(\partial x_j^{am})^2} M(X_{am+j})} \qquad (3.2.4)$$

which when written in terms of the $F_i(X)$ functions become

$$x_j^{(a+1)m} = x_j^{am} - \frac{\sum_{i=1}^{m} F_i(X_{am+j}) \dfrac{\partial}{\partial x_j^{am}} F_i(X_{am+j})}{\sum_{i=1}^{m} \left[\left(\dfrac{\partial}{\partial x_j^{am}} F_i(X_{am+j}) \right)^2 + F_i(X_{am+j}) \dfrac{\partial^2}{(\partial x_j^{am})^2} F_i(X_{am+j}) \right]} \qquad (3.2.5)$$

This last equation is written for a general system of m non-linear algebraic equations. The corresponding equation for the special problem at hand is obtained by replacing m by 2n and by taking the functions $F_i(X)$ as defined by eqs. 3.1.9, 3.1.8, 3.1.7, and 3.1.6.

3.3 The Digital Computer Programs

Two computer programs were written in connection with this problem of delay equalization. Both programs are general in the sense that they can be utilized for any delay equalizer design, that is, they are written in terms of the parameter n (the number of sections of the delay equalizer).

Program 1

This program computes a solution to the system of 2n equations, given by eq. 3.1.8, and employs the numerical methods of Seidel and Newton-Raphson, as described in section 3.2. The input data for this program are as follows:

n, the number of sections of the delay equalizer.

(As here exemplified, $n \leq 10$, due to limited storage capacity of the Mistic computer);

x_k^o, y_k^o $(k=1, 2, 3, \cdots, n)$, the initial estimates for a solution of eq. 3.1.8. (These are the initial estimates for the equalizer design parameters--see eq. 3.1.2);

f_d, the normalizing frequency;

$T_{SF}(f_d)$, the value of the insertion delay of the filter at the frequency f_d;

$f_i (i=1, 2, 3, \cdots, 2n)$, The 2n frequencies from which the system of 2n equations is formed, $(f_i \neq f_d$ for all i);

$T_{SF}(f_i)$, the values of the insertion delay of the filter at each frequency f_i;

δ_T, the maximum allowable total delay deviation.

The units of frequency are in cycles per second and the units of delay are in seconds.

The computer prints out all x_j^{am}, y_j^{am}, and all $F_i(X_{am+j})$ after each Seidel cycle. The approximation to a solution of eq. 3.1.8 is considered complete when

$$\frac{1}{\pi f_d} | F_i(X_{am+j})| < \delta_T \qquad \text{for all } i \qquad (3.3.1)$$

and the computer automatically stops when this inequality is satisfied.

Criteria for the choice of the input data are given in chapters V and VI.

A simplified flow diagram of program 1 is presented in figure 3.3.1. Each block in the diagram lists the quantities which are calculated by the various subroutines. The notation used in figure 3.3.1 is consistent with the notation given in eqs. 3.1.6, 3.1.7, 3.1.8, and 3.2.5. The number of times each subroutine is entered during the calculation corresponding to one Seidel cycle is indicated on the diagram in terms of the parameter n.

Program 2

This program computes the sum of the insertion delay of the filter and the equalizer, at specified frequencies f_p. The input data for this program are as follows:

f_p ($p=1, 2, 3, \cdots, r \leq 40$), the frequencies at which the total delay is to be calculated;

$T_{SF}(f_p)$, the insertion delay of the filter at each frequency f_p;

n, the number of sections of the delay equalizer;

x_k, y_k, the delay equalizer design parameters, (see eq. 3.1.2). These parameters are obtained by program 1 as an approximation to a solution of eq. 3.1.8.

All frequency values are given in cycles per second and all delay values in seconds.

The insertion delay of the equalizer is computed from eq. 3.1.1 and added to the insertion delay of the filter.

The format of the output data of program 2 is printed in two columns. The left column with a heading "FREQ." lists the frequencies f_p and the right-hand column headed "TOTAL DELAY" gives the values of the sum of the insertion delay of the filter and the equalizer at the corresponding frequencies.

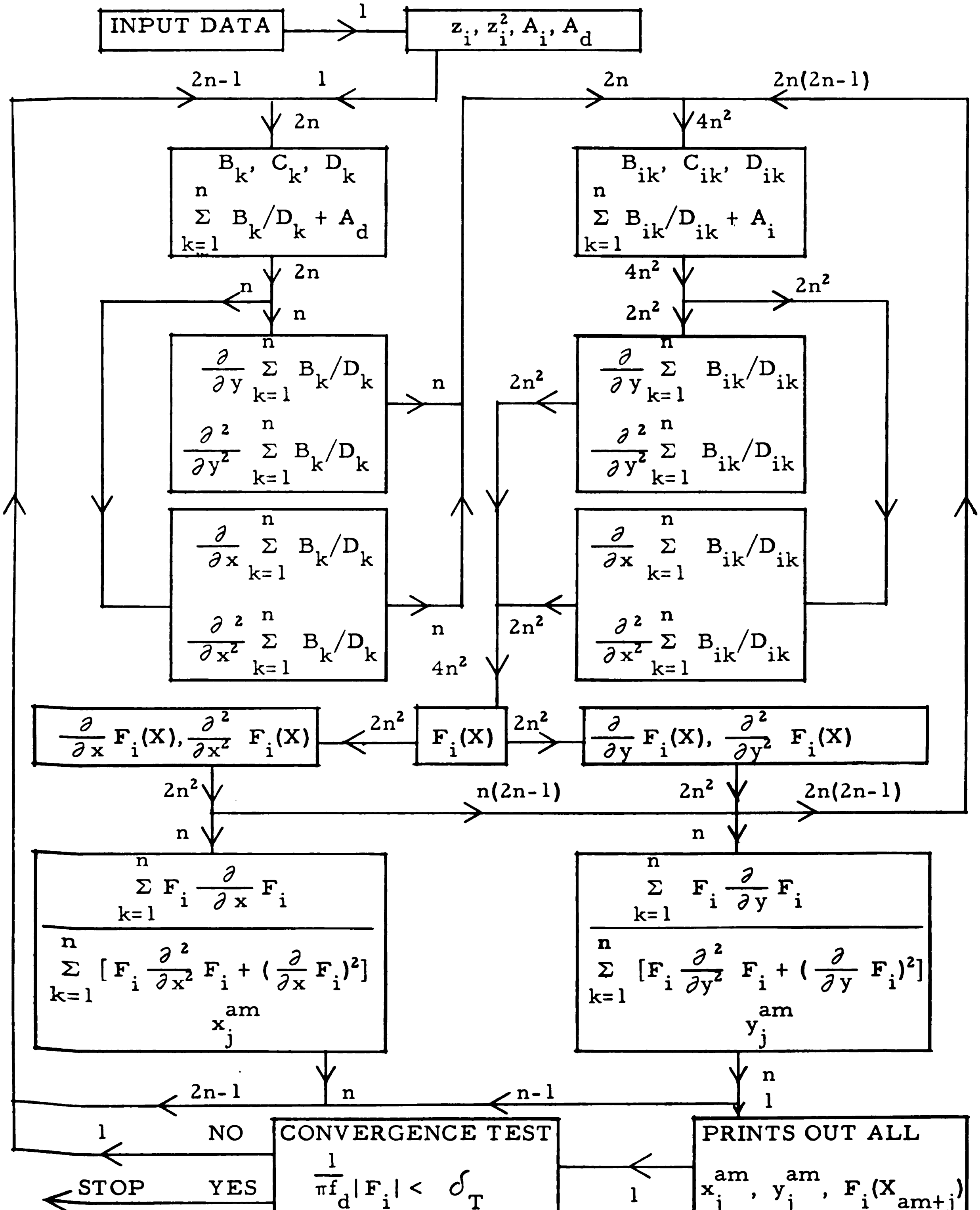


Figure 3.3.1. Flow Diagram of Program 1

IV. SOLUTIONS TO TYPICAL EQUALIZATION PROBLEMS USING THE METHOD OF THIS THESIS

In this chapter, the thesis method of delay equalizer design, hereafter referred to as the t-method for brevity, is applied to a commercial problem of very exceptionally stringent requirements. The quality of delay equalization attained by this new method is demonstrated by means of comparison with a commercial solution to the problem. This commercial solution, obtained by a method described in chapter VI, was considered to be a very good solution. In fact, it was agreed that a possible improvement of the commercial delay characteristic, if any, would not justify the extensive amount of time involved.

4. 1 The Commercial Problem

The delay characteristic of the filter to be equalized as well as the upper and lower limits of the equalized delay are shown in figure 4. 1. 1. The delay equalization interval, in this problem, extends from zero to 7. 2 megacycles. The requirements on the total delay (the sum of the delay of the filter and the delay of the equalizer), in terms of the maximum allowable delay deviation, are given by

0. 0 - 5. 0 megacycles	±	2 millimicroseconds
5. 0 - 5. 5 megacycles	±	4 millimicroseconds
5. 5 - 6. 0 megacycles	±	8 millimicroseconds
6. 0 - 7. 2 megacycles	±	16 millimicroseconds

This set of requirements is drawn at an arbitrary delay level and is shown in figure 4. 1. 1 to indicate the stringency of the requirements.

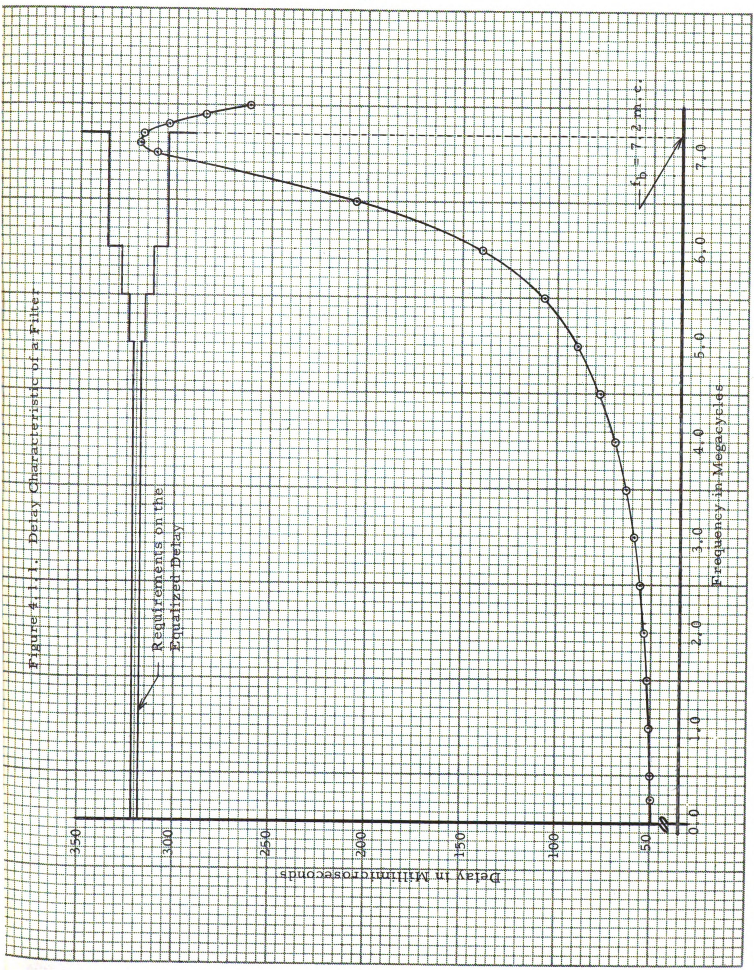


Figure 4.1.1. Delay Characteristic of a Filter

4.2 The Commercial Solution

The commercial solution to this design problem is given in figure 4.2.1, in terms of the total delay characteristic (the sum of the delay characteristics of the filter and the commercially designed equalizer). This commercially designed equalizer consists of six lattice sections of the form shown in figure 2.6.1, connected in cascade. From figure 4.2.1, it is evident that the commercial solution fails to satisfy the requirements on the total delay in the neighborhoods of 5.5 and 5.75 megacycles. Because of the difficulty and time consuming nature of the commercial delay equalizer design technique, this failure to meet the requirements was accepted as the best that could be done.

4.3 Solution by the t-Method

In this section, several solutions to the design problem as obtained by the t-method are presented and the procedure followed in each case is described. Program 1 (section 3.3) was used to calculate the equalizer design parameters, x_k, y_k, given by eq. 3.1.2, and program 2 (section 3.3) was employed to calculate the total delay characteristic.

Solution 1

The first test for the t-method was an investigation of the possibility of improving the commercial solution.

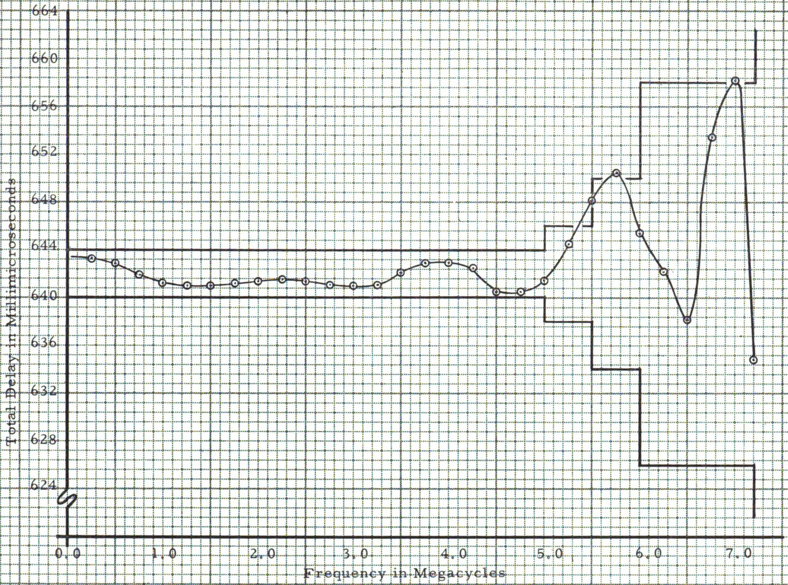
In this case, the input data for program 1 were taken as follows:

The number of sections, n, of the delay equalizer was taken the same as for the commercial design (n = 6).

The initial estimates, x_k^o, y_k^o, for the design parameters of the equalizer were taken as those of the commercial design.

The normalizing frequency, f_d, was chosen as 2 megacycles. The normalizing frequency is taken as any convenient number in the frequency interval where the delay of the filter, T_{SF}, undergoes small variation.

Figure 4.2.1. Total Delay, Commercial Solution
(6 Section Equalizer)



The 2n frequencies, f_i, (12 in this case) were selected as follows: In the interval 1.0 to 5.5 megacycles, nine frequency points were chosen at uniform intervals of 0.5 megacycles. (Note that the 2.0 megacycle point has already been selected by the choice of the normalizing frequency, f_d). In addition, the 0.25 megacycle point was chosen. The remaining two frequency points were selected on the basis of the behavior of the commercial delay characteristic (fig. 4.2.1) in the 5.75 to 7.2 megacycle interval. The points 5.75 and 7.1 megacycles were chosen in expectation of improving the delay characteristic at these points.

In this trial, it was decided to apply the Mistic computer to program 1 for about 30 minutes. In order to prevent the computer from stopping short of this time interval, a very small maximum allowable delay deviation, δ_T, was chosen for the stop order criterion in program 1 (see eq. 3.3.1), namely: $\delta_T = 0.25$ millimicroseconds. The actual computer time was 25 minutes, in which period 22 Seidel cycles were completed. Program 2 was utilized to compute the total delay characteristic, at 0.25 megacycle intervals, on the basis of the design parameters of the 22nd Seidel cycle. The result is shown plotted in figure 4.3.1. The points at which the delay characteristic was calculated are indicated on the plot by circles. The delay values corresponding to the frequency points f_i and the normalizing frequency f_d are identified by double circles.

A comparison of figures 4.2.1 and 4.3.1 clearly reveals that the application of the t-method produced a considerable improvement over the commercial delay characteristic. The new delay characteristic not only satisfies the original stringent requirements, but actually satisfies a set of more stringent requirements, given by

0.00 - 5.75 megacycles	±	1.70 millimicroseconds
5.75 - 6.00 megacycles	±	4.00 millimicroseconds
6.00 - 6.50 megacycles	±	10.00 millimicroseconds
6.50 - 7.20 megacycles	±	12.00 millimicroseconds

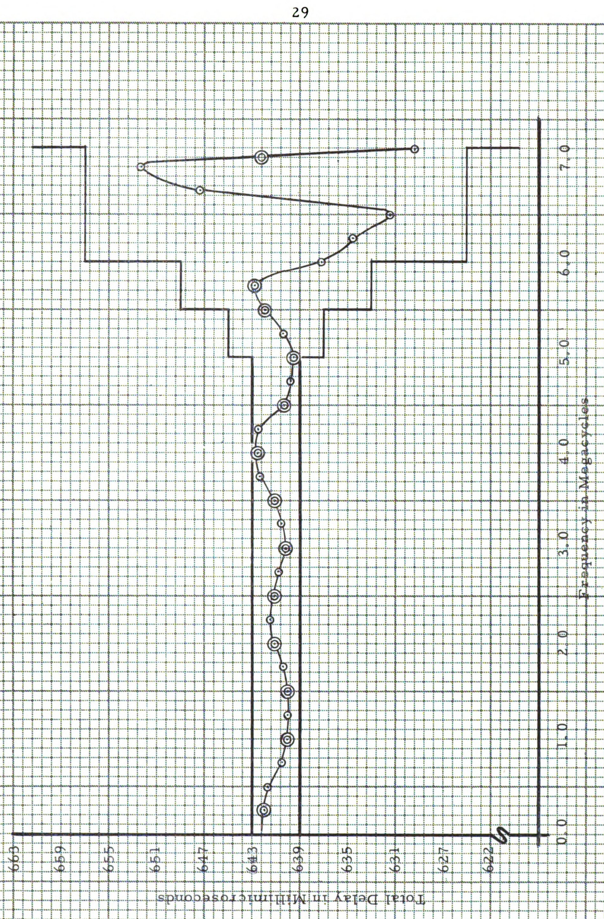


Figure 4.3.1. Total Delay, t-Method, Solution 1
(6 Section Equalizer)

Solution 2

In this solution, an improvement over the delay characteristic obtained by solution 1 (fig. 4.3.1) is sought by selecting a different set of the f_i frequency points. The following basis is used for the choice of the new set of points from the set of points of the previous solution: Those f_i points at which the total delay characteristic has neither a relative maximum or minimum are deleted. Those frequency points at which the total delay characteristic has a relative maximum or minimum or at which a delay improvement is desired are included in the new set of f_i points. Since the number of the f_i frequency points is fixed, for a fixed n, the number of added points is equal to the number of deleted points.

From the total delay characteristic of solution 1, shown in figure 4.3.1, and on the basis of the criterion, given in the last paragraph, a new set of the f_i frequency points was obtained by deleting the points at 2.5 and 3.5 megacycles, from the set of the f_i frequency points of solution 1, and by adding the points at 4.25 and 6.0 megacycles.

The initial estimates, x_k^o, y_k^o, were taken as those obtained by solution 1.

All other input data are as those used in solution 1.

The computer was applied to program 1 for about one hour, in which time 52 Seidel cycles were completed. On the basis of the design parameters of the 52nd Seidel cycle, the total delay characteristic was calculated by program 2 and is shown in figure 4.3.2.

An improvement of this total delay characteristic (fig. 4.3.2), particularly at the 6.0 megacycle point, over that of solution 1 (fig. 4.3.1) should be noted. Solution 2 (fig. 4.3.2) is also well within the original requirements and is actually confined within a set of requirements given by

0.00 - 5.50 megacycles	±	1.5 millimicroseconds
5.50 - 6.00 megacycles	±	3.0 millimicroseconds
6.00 - 6.75 megacycles	±	8.5 millimicroseconds
6.75 - 7.20 megacycles	±	13.5 millimicroseconds

Figure 4.3.2. Total Delay, t-Method, Solution 2
(6 Section Equalizer)

which is a better overall delay characteristic than solution 1 of figure 4.3.1.

Solution 3

The success of solutions 1 and 2 lead into an investigation of the possibility to satisfy the original requirements with a delay equalizer consisting of five sections rather than six.

In this case, the input data for program 1 were taken as follows:

The number of sections, n, was taken as five (n = 5).

The initial estimates, x_k^o, y_k^o, were obtained by the method described in chapter VI. The total delay based on these initial estimates was calculated by program 2 and is shown in figure 4.3.3.

The frequency points f_i were chosen in accordance with criterion 4.5.2, given in section 5 of this chapter, and are indicated in figure 4.3.4.

The computer was applied to program 1 to compute 60 Seidel cycles. The total delay characteristic as calculated by program 2, on the basis of the design parameters of the 60th Seidel cycle, is shown in figure 4.3.4. Inspection of this figure reveals that the delay characteristic does not satisfy the original requirements in the neighborhoods of 6.0 and 6.5 megacycles. At the same time, it is noticed that none of the f_i frequency points were chosen in the interval 5.5 to 7.1 megacycles. To improve this delay characteristic, a new set of the f_i frequency points was chosen according to criterion 4.5.2 and on the basis of the obtained delay characteristic (fig. 4.3.4). The design parameters of the 60th Seidel cycle were taken for the initial estimates of this run. After the application of one Seidel cycle, the total delay was calculated and is shown in figure 4.3.5. From this figure it is seen that the original stringent requirements are satisfied by a delay equalizer of only five sections.

Solutions 1 and 2 (figs. 4.3.1 and 4.3.2) clearly demonstrate that for a delay equalizer of the same number of sections, the original delay requirements are fully met. The quality of delay equalization obtained by

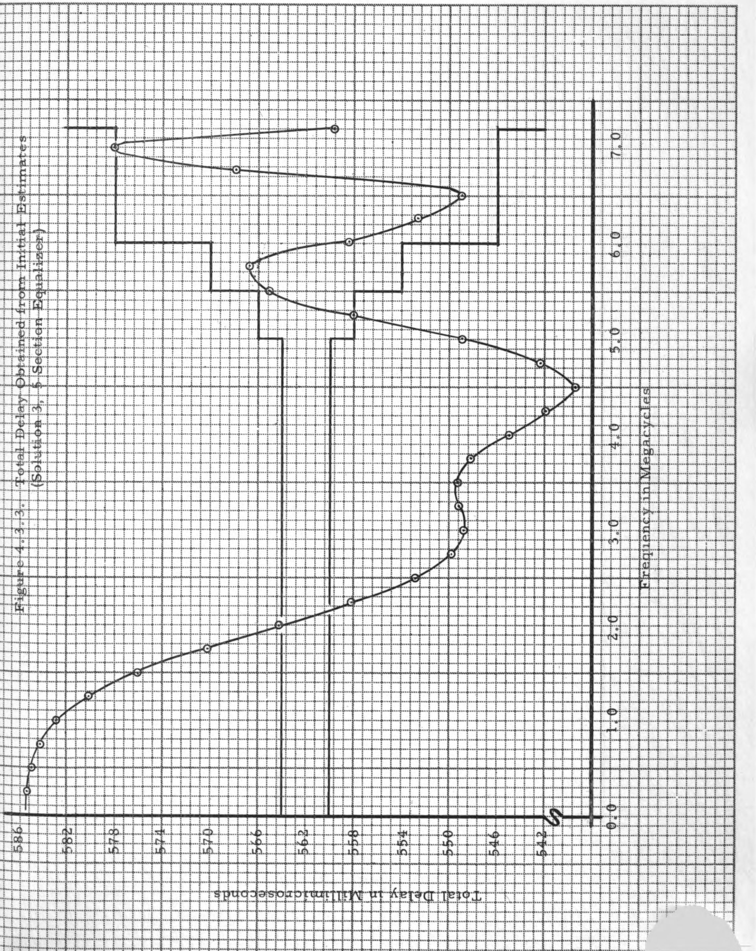


Figure 4.3.3. Total Delay Obtained from Initial Estimates (Solution 3, 5 Section Equalizer)

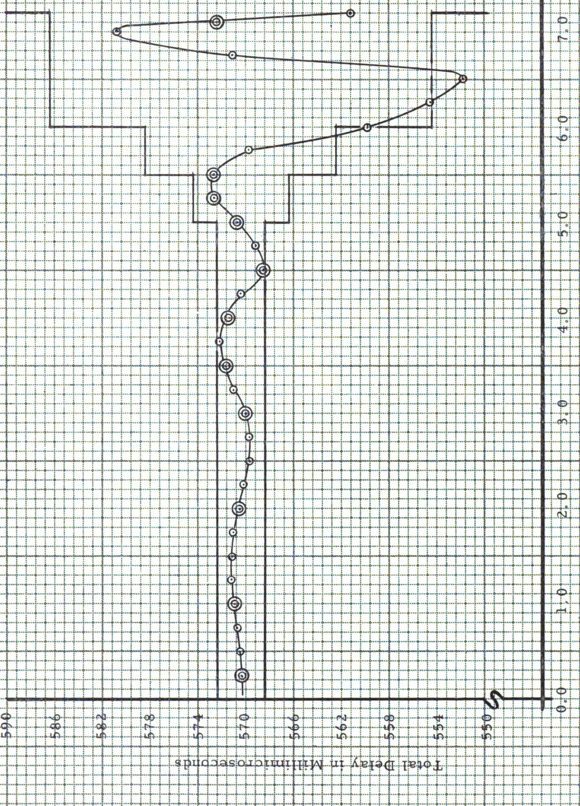


Figure 4.3.4 Total Delay, t-Method, Intermediate Stage
(Solution 3, 5 Section Equalizer)

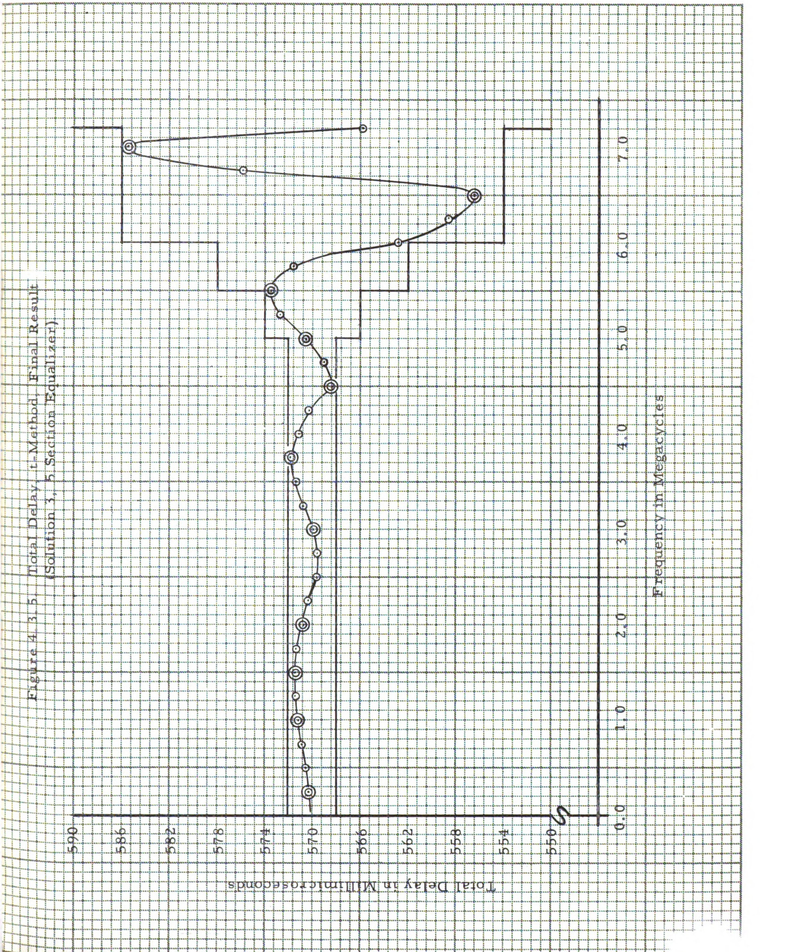


Figure 4.3.5. Total Delay, t-Method, Final Result (Solution 3, 5 Section Equalizer)

the t-method exceeds that of the accepted commercial design (fig. 4.2.1), which actually did not fully meet the original requirements. Solution 3 (fig. 4.3.5) shows that the original requirements can indeed be met not only with an equalizer of six sections, but with an equalizer of one less section than the commercial one, i.e., with five sections.

Another important feature of the t-method should be noted. The total delay characteristic (fig. 4.3.3), based on the initial estimates, x_k^o, y_k^o, varies from 540 to 586 millimicroseconds, in the frequency interval zero to five megacycles. This corresponds to a delay deviation of about ± 23 millimicroseconds. In the same frequency interval, the final result (fig. 4.3.5) is confined within ± 2 millimicroseconds. This example demonstrates that the t-method converged, even though the initial estimates were poorly chosen.

4.4 Another Example of Delay Equalization by the t-Method

The original stringent requirements, given in section 4.1, are for delay equalization of a loss filter in a very special, very high quality transmission system. A set of delay requirements for delay equalization of the same loss filter in a transmission system for color television is the very much relaxed requirement schedule:

0.0 - 5.0 megacycles	±	2 millimicroseconds
5.0 - 5.5 megacycles	±	4 millimicroseconds
5.5 - 6.0 megacycles	±	8 millimicroseconds

with no requirements on the total delay for frequencies greater than six megacycles.

Solution A

In this solution, the input data for program 1 were taken as follows:

The number of sections, n, was taken as four (n = 4). No attempt was made here to arrive at an estimate on the number of sections. Since an equalizer of five sections satisfied the requirements in the interval zero to 7.2 megacycles (solution 3, fig. 4.3.5), an equalizer of fewer sections can be expected to effect the equalization in the interval zero to six megacycles. A few minutes of computer time determines whether the number of equalizer sections can be reduced still further.

The initial estimates, x_k^o, y_k^o, were obtained by the method given in chapter VI. The total delay on the basis of these initial estimates, as calculated by program 2, is shown in figure 4.4.1.

The frequency points f_i were chosen according to criterion 4.5.2 and are indicated in figure 4.4.2.

In 30 minutes of computer time, 68 Seidel cycles were completed. The total delay characteristic on the basis of the design parameters of the 68th Seidel cycle is shown in figure 4.4.2. Inspection of this figure shows that this solution satisfies a set of considerably more stringent requirements than those for color television, namely, a set given by

0.0 - 6.0 megacycles ± 1.1 millimicroseconds

Solution B

The outcome of the previous solution (fig. 4.4.2) suggests that an equalizer of three sections might meet the requirements for color television.

In this solution, the input data for program 1 are as follows:

The number of sections, n, was taken as three (n = 3).

The initial estimates, x_k^o, y_k^o, were obtained by the method described in chapter VI. The total delay characteristic in terms of these initial estimates, as calculated by program 2, is shown in figure 4.4.3.

The frequency points f_i were chosen according to criterion 4.5.2 and are indicated in figure 4.4.4.

In this case, 110 Seidel cycles were completed in about 30 minutes. The total delay characteristic on the basis of the design parameters of the 110th Seidel cycle is shown in figure 4.4.4. It is seen from this figure that the color television requirements are indeed satisfied by a

Figure 4.4.1. Total Delay Obtained from Initial Estimates
(Solution A, 4 Section Equalizer)

Figure 4.4.2. Total Delay, t-Method, Final Result
(Solution A, 4 Section Equalizer)

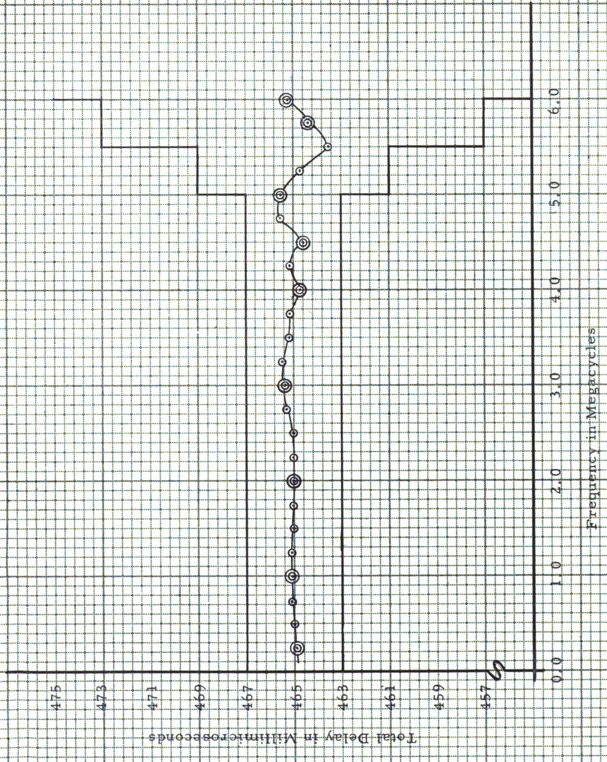


Figure 4.4.3. Total Delay Obtained from Initial Estimates
(Solution B, 3 Section Equalizer)

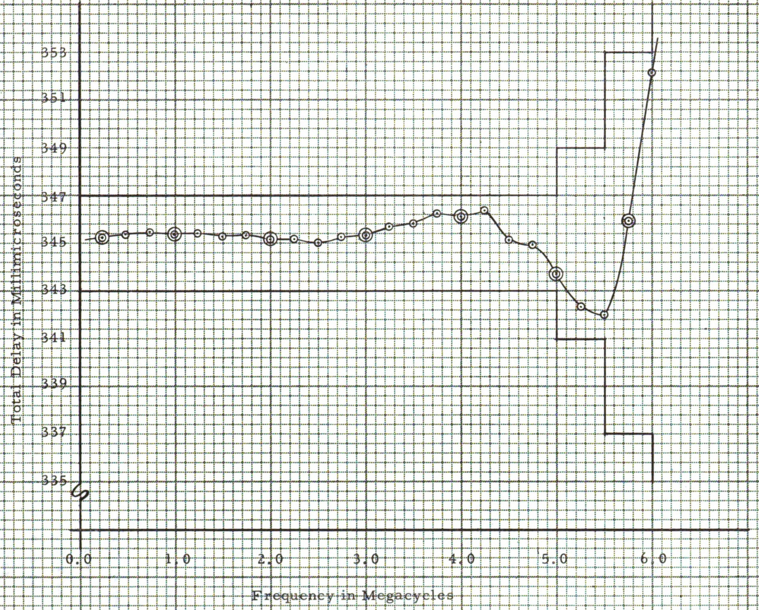


Figure 4.4.4. Total Delay, t-Method, Final Result
(Solution B, 3 Section Equalizer)

delay equalizer of three sections. A selection of a new set of the f_i frequency points (see criterion 4.5.2) and an application of a few additional Seidel cycles might still further improve this delay characteristic. This step was not carried out in this particular example.

The delay equalization of the given filter, satisfying the exceptionally stringent requirements (section 4.1) was effected by a delay equalizer of five sections (fig. 4.3.5), while the color television requirements (section 4.4) were met by an equalizer of three sections (fig. 4.4.4). This is to be contrasted with the commercially designed delay equalizer, consisting of six sections (fig. 4.2.1), which was accepted as a solution to the delay equalization problem for both sets of requirements, in spite of the fact that this commercial design did not fully meet either set of the requirements.

4.5 Criteria for the Choice of Input Data for Program 1

In this section, a discussion on the choice of input data for program 1 is presented and the discussions are summarized in forms of criteria. The estimate on the number of sections, n, of a delay equalizer is treated in chapter V and a method for the selection of the initial estimates, x_k^o, y_k^o, for the design parameters is presented in chapter VI.

Criterion 4.5.1. The normalizing frequency, f_d, is chosen as any convenient number in that frequency interval where the delay of the filter, T_{SF}, undergoes small variation.

Criterion 4.5.2. The 2n frequency points, f_i ($f_i \neq f_d$ for all i), are chosen as follows:

(a) For the first application of the t-method, the f_i frequency points are selected such that the set of the 2n points f_i and the point f_d are uniformly spaced in the equalization interval. (Although this choice is always theoretically possible, the following practical difficulties arise: The t-method requires the functional values of the delay of the filter, T_{SF}, at the frequencies f_i and f_d. If the delay of the filter was measured or calculated beforehand, at a set of frequencies, f_q, the choice of the frequencies f_i and the frequency f_d must be made to coincide with a subset of the f_q frequencies in order to avoid measurement or calculation of the delay of the filter at additional frequencies. Then, in the case where uniform spacing of the f_i and f_d frequencies would lead to a set of points not coincident with a subset of the f_q points, smaller subintervals are selected in that frequency region where the delay requirements are most stringent).

(b) An improvement over the total delay characteristic obtained by the first application of the t-method is often effected by a choice of a new set of the f_i frequencies. This new set of the f_i frequency points is chosen on the basis of the previous set of the f_i points and from the behavior of the total delay characteristic of the first application of the t-method, as follows: Those f_i points in which neighborhoods the total delay characteristic has neither a relative maximum nor minimum are deleted. Those f_i frequency points in which neighborhoods the total delay characteristic has a relative maximum or minimum, or at which point a delay improvement is desired, are included in the new set of the f_i points. Since the number of the f_i frequencies is fixed, for a fixed n, the number of added points is equal to the number of deleted points.

The t-method of delay equalizer design was applied to a number of additional problems over those presented in this chapter, in an attempt to arrive at some convergence criterion in terms of the initial estimates, x_k^o, y_k^o. It was found that in all cases considered, an improvement of the delay characteristic over that of the characteristic based on the initial estimates was effected. However, the success of the t-method depends not only on the improvement of the delay characteristic, but one additional condition must be met, namely, that all of the final design parameters, x_k, y_k, must be positive numbers. This is a necessary condition for the realizability of the equalizer network (see chapter VII). Although positive initial estimates, x_k^o, y_k^o, were used in all cases, it was noted in a few instances that if after the first Seidel cycle one or more of the corrected design parameters took on negative values, invariably these parameters remained negative for all subsequent Seidel cycles. On the basis of this investigation, the following criterion is stated:

Criterion 4.5.3. If after the application of one or a few Seidel cycles, at least one of the design parameters, x_k, y_k, takes on a negative value, a new set of the initial estimates, x_k^o, y_k^o, should be chosen. In such case, program 2 is utilized to calculate the total delay characteristic in terms of the old set of the initial estimates. From the behavior of this delay characteristic and on the basis of the method for the choice of the initial estimates (chapter VI), a new set of initial estimates is obtained.

V. AN ESTIMATE ON THE NUMBER OF SECTIONS

The problem of estimating the number of sections, n, of a delay equalizer is indeed a difficult task. If the estimate is to be of value to the designer, it is necessary to obtain an expression for n in terms of the known quantities, i.e., in terms of

(1) the delay characteristic of the filter, T_{SF};

(2) the equalization interval, (f_a, f_b);

(3) the maximum allowable total delay deviation, δ_T.

In functional notation:

$$n = F(T_{SF}, f_a, f_b, \delta_T) \tag{5.1}$$

From section 2.7, the delay equalization problem is to approximate the delay equalizer requirement, T_{SER}, (eq. 2.5.2) by the delay equalizer function, T_{SE}, (eq. 2.6.9), throughout the equalization interval, (f_a, f_b), to within a specified degree of accuracy:

$$T_{SE}(f) \doteq T_o - T_{SF}(f) \qquad \text{for } f_a \leq f \leq f_b \tag{5.2}$$

Integrating this last equation with respect to frequency, f, over the equalization interval, yields

$$\int_{f_a}^{f_b} T_{SE}(f)df \doteq T_o(f_b - f_a) - \int_{f_a}^{f_b} T_{SF}(f)df \tag{5.3}$$

The integral on the left side of this last expression can be related to the number of sections, n, of the delay equalizer as follows. From the definition of the insertion delay function (eq. 2.4.1) and from eqs. 2.6.7, 2.6.8, and 2.6.9, for an equalizer of n sections

$$\int_0^{\infty} T_{SE}(f)df = \frac{1}{2\pi} \int_0^{\infty} \frac{d}{df} B_{SE}(f)df = \frac{n}{2\pi} (2\pi - 0) = n \tag{5.4}$$

then

$$\int_{f_b}^{f_a} T_{SE}(f)df = n - \int_0^{f_a} T_{SE}(f)df - \int_{f_b}^{\infty} T_{SE}(f)df \tag{5.5}$$

Substituting this last equation into eq. 5.3, yields

$$n \doteq T_o(f_b - f_a) - \int_{f_a}^{f_b} T_{SF} df + \int_0^{f_a} T_{SE} df + \int_{f_b}^{\infty} T_{SE} df \qquad (5.6)$$

Before proceeding further, the three types of equalization intervals are considered separately, i.e.,

(1) equalization of low-pass loss filter: $(0, f_b)$;

(2) equalization of band-pass loss filter: (f_a, f_b);

(3) equalization of high-pass loss filter: (f_a, ∞).

In the case of low-pass equalization, the interval is $(0, f_b)$, hence $f_a = 0$ and eq. 5.6 reduces to

$$n \doteq T_o f_b - \int_0^{f_b} T_{SF}(f) df + \int_{f_b}^{\infty} T_{SE}(f) df \qquad (5.7)$$

The delay functions $T_{SF}(f)$, $T_{SE}(f)$, and the delay level T_o corresponding to the delay equalization solutions as obtained in the previous chapter (figs. 4.4.4 and 4.3.5) are plotted in figures 5.1 and 5.2, for the cases n=3 and n=5, respectively. For any low-pass delay equalization problem, the delay functions, $T_{SF}(f)$ and $T_{SE}(f)$, have typical delay characteristics similar in form to those shown in these two figures. An estimate for each of the two integrals in eq. 5.7 is made on the basis of the shapes of these typical delay characteristics as follows:

First, the following quantities are defined.

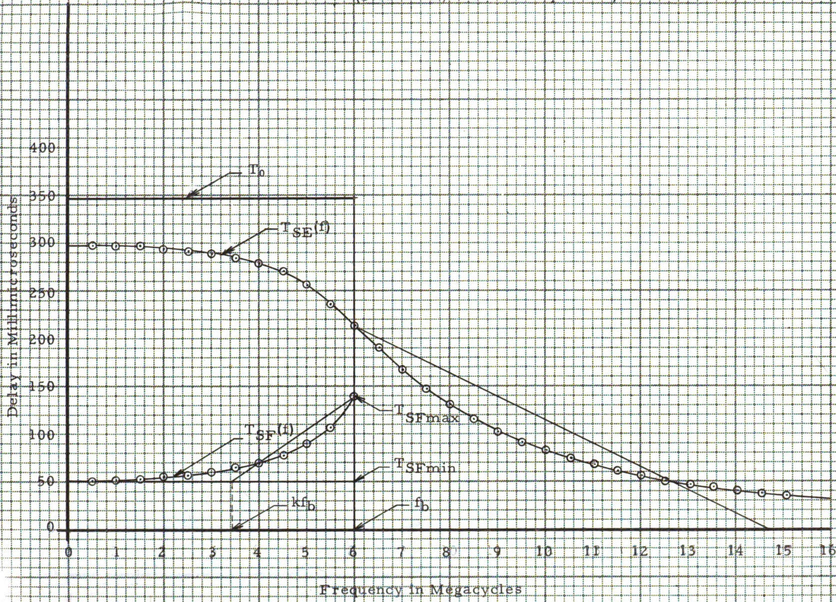
T_{SFmax}: represents the maximum value of the function T_{SF} (the delay of the filter), in the equalization interval;

T_{SFmin}: represents the minimum value of the function, T_{SF}, in the equalization interval;

Also, let $$\Delta T_{SF} = T_{SFmax} - T_{SFmin} \qquad (5.8)$$

The integral corresponding to the second term on the right-hand side of eq. 5.7 is approximated by the sum of the areas of the rectangle and the triangle, shown in figures 5.1 and 5.2. The rectangle has width

Figure 5.1. $T_{SF}(f)$, $T_{SE}(f)$, T_0
(Solution B, 3 Section Equalizer)



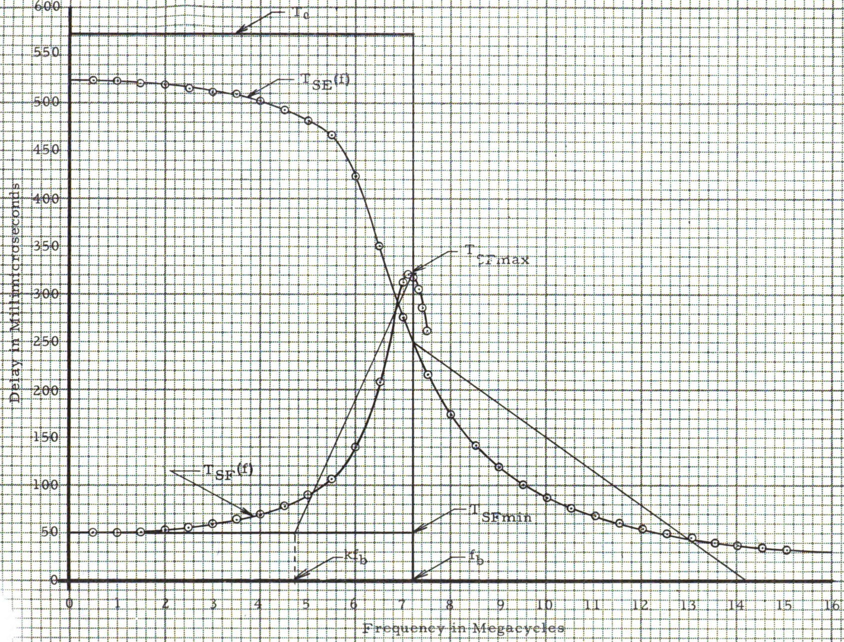


Figure 5.2. $T_{SF}(f)$, $T_{SE}(f)$, T_0
(Solution 3, 5 Section Equalizer)

f_b and height T_{SFmin}, while the triangle has base $(1-k)f_b$ and height ΔT_{SF}. The constant, k $(0 < k < 1)$, is to be determined on the basis of the particular filter delay function, T_{SF}, so that a good approximation of the integral results. Then

$$\int_0^{f_b} T_{SF}(f)df \doteq [T_{SFmin} + \tfrac{1}{2}(1-k)\Delta T_{SF}]f_b \qquad (5.9)$$

The last integral on the right-hand side of eq. 5.7 is approximated by the area of the triangle shown in figures 5.1 and 5.2. It should be remembered, however, that the function, $T_{SE}(f)$ (the equalizer delay), is not known prior to completion of the equalization problem, hence, it is necessary to obtain empirical expressions for the height and the base of this triangle in terms of known quantities, namely, in terms of the quantities, T_{SFmin}, T_{SFmax}, ΔT_{SF}, f_b, and k, associated with the filter delay function, T_{SF}. Such empirical relations should be arrived at from a large number of delay equalization solutions, all of comparable quality of equalization (for example, the solutions could be selected such that in each solution the percent delay deviation is less than x% over more than y% of the delay equalization interval). The estimate on the number of sections, n, so obtained is valid for delay equalizer networks of corresponding delay equalization quality. For an equalizer requiring higher quality of equalization a larger number than that obtained by this estimate would be taken.

To illustrate the above procedure, the integral corresponding to the last term on the right-hand side of eq. 5.7 might be approximated by a triangle as follows: The height of the triangle is taken as $T_{SE}(f_b)$. From eq. 5.2

$$T_{SE}(f_b) \doteq T_o - T_{SF}(f_b) \doteq T_o - T_{SFmax} \qquad (5.10)$$

Inspection of the delay characteristics in figures 5.1 and 5.2 suggests that the base of this approximating triangle is directly proportional to the factor (1-k). Let the constant of proportionality be denoted by k_1. The numerical value of this constant k_1 must be determined from a large number of solutions, as mentioned above. The approximation of the integral then becomes

$$\int_{f_b}^{\infty} T_{SE}(f)df \doteq \frac{1}{2} k_1 (1-k) (T_o - T_{SFmax}) \tag{5.11}$$

Substituting eqs. 5.9 and 5.11 into eq. 5.7, yields

$$n \doteq [T_o - T_{SFmin} - \frac{1}{2}(1-k)\Delta T_{SF}]f_b + \frac{1}{2}k_1(1-k)(T_o - T_{SFmax}) \tag{5.12}$$

This last equation gives an estimate on the number of sections, n, of a delay equalizer. However, the quantity T_o is unknown and an estimate for it in terms of the known quantities must be obtained empirically from a large number of delay equalization solutions.

The estimate on the number of sections, n, as given by eq. 5.12 is computed for the two cases, n=3 and n=5, shown in figures 5.1 and 5.2, respectively. The numerical values of the quantities used in the calculations as well as the results are listed in table 5.1.

Table 5.1: Calculation of Estimates on the Number of Sections, n, from eq. 5.12.

n	T_o	T_{SFmin}	T_{SFmax}	ΔT_{SF}	f_b
3	$.345 \cdot 10^{-6}$	$.5 \cdot 10^{-6}$	$.14 \cdot 10^{-6}$	$.09 \cdot 10^{-6}$	$6.0 \cdot 10^{6}$
5	$.571 \cdot 10^{-6}$	$.5 \cdot 10^{-6}$	$.32 \cdot 10^{-6}$	$.27 \cdot 10^{-6}$	$7.2 \cdot 10^{6}$

n	k	k_1	Calculated Estimates of n:
3	.567	2.10^{7}	2.74
5	.653	2.10^{7}	4.35

The constant of proportionality, k_1, was chosen on the basis of the delay characteristics of figures 5.1 and 5.2. It is seen from this table (table 5.1) that very good estimates on the number of sections are obtained from eq. 5.12, at least for the two cases considered. The number of sections for the delay equalization problem would be taken as the next higher integer with respect to the number obtained from eq. 5.12.

The estimate on the number of sections, n, given by eq. 5.12, applies for delay equalization in the low-pass band. Starting with eq. 5.6, similar procedures would lead to expressions for estimates on the number of sections, n, for the band-pass and high-pass cases.

VI. INITIAL ESTIMATES OF THE DESIGN PARAMETERS

In this chapter, a method for the choice of the initial estimates, x_k^o, y_k^o, for the design parameters is presented. First, some important properties of the insertion phase function, $B_s(f)$, and the insertion delay function, $T_s(f)$, of one lattice section (fig. 2.6.1) are listed.

6.1 Properties of the Insertion Phase Function

From eq. 2.6.7, the insertion phase function, $B_s(f)$, of one lattice section, in terms of the two independent parameters d and f_o, is given by

$$B_s(f) = 2 \text{ arccot } (d)\left(\frac{f_o}{f} - \frac{f}{f_o}\right) \qquad (6.1.1)$$

From this last equation

$$B_s(0) = 0$$
$$B_s(f_o) = \pi$$
$$\lim_{f \to \infty} B_s(f) = 2\pi \qquad (6.1.2)$$
$$B_s(f) > 0 \text{ for } 0 < f < \infty, \; d > 0, \; f_o > 0$$

The insertion phase function, $B_s(f)$, is a monotonically increasing function of frequency, f. It has a value of zero at zero frequency and approaches the value of 2π radians asymptotically as frequency increases without bound.

6.2 Properties of the Insertion Delay Function

From eq. 2.6.8, the insertion delay function, $T_s(f)$, of one lattice section, in terms of the parameters d and f_o, is given by

$$T_s(f) = \frac{d}{\pi f_o} \cdot \frac{1 + (f_o/f)^2}{1 + d^2(f/f_o - f_o/f)^2} \qquad (6.2.1$$

From this last equation

$$T_s(0) = \frac{1}{\pi f_o d}$$

$$T_s(f_o) = \frac{2d}{\pi f_o} \qquad (6.2.2)$$

$$\lim_{f \to \infty} T_s(f) = 0$$

$$T_s(f) > 0 \quad \text{for} \quad 0 < f < \infty,\ d > 0,\ f_o > 0$$

The insertion delay function, $T_s(f)$, has finite and non-zero value at zero frequency (for finite and non-zero values of d and f_o) and approaches zero value asymptotically as frequency approaches infinity.

The insertion delay, $T_s(f)$, is a function of frequency, f, and the two parameters, d and f_o. If the product function, T_p, is considered, given by

$$T_p = T_s(f) \cdot f_o = \frac{d}{\pi} \cdot \frac{1 + (f_o/f)^2}{1 + d^2(f/f_o - f_o/f)^2} \qquad (6.2.3)$$

then T_p can be viewed as a function of f/f_o and d [34]. It is now possible to plot a family of curves of the product function, T_p, versus f/f_o, each curve for a different value of the d parameter. This family of curves is classified into two broad classes, depending on the value of the d parameter, according to whether the product function takes on a maximum value at zero frequency (class A, fig. 6.2.1) or at some finite and non-zero frequency (class B, fig. 6.2.2).

The maximum value of the insertion delay function, $T_s(f)$, of one lattice section, occurs at

$$f = 0, \qquad \text{for } d^2 \leq 1/3, \text{ class A} \qquad (6.2.4)$$

$$f = f_o\sqrt{\sqrt{4 - 1/d^2} - 1}, \qquad \text{for } d^2 > 1/3, \text{ class B} \qquad (6.2.5)$$

Since the radical in this last equation is always less than unity, for $d^2 > 1/3$ and finite, it follows that the insertion delay function takes on a maximum value at some frequency less than f_o.

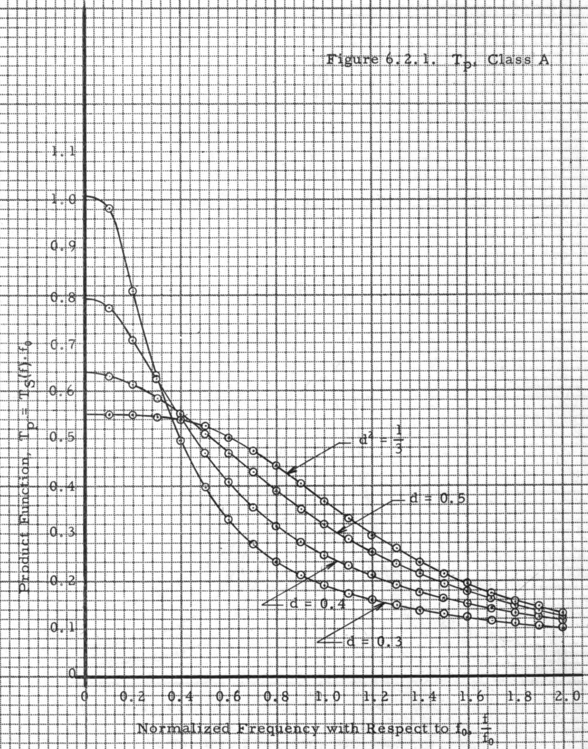


Figure 6.2.1. Tp, Class A
Product Function, Tp = Ts(f)·f0
Normalized Frequency with Respect to f0, f/f0
d² = 1/3
d = 0.5
d = 0.4
d = 0.3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0

Figure 6.2.2. T_p, Class B

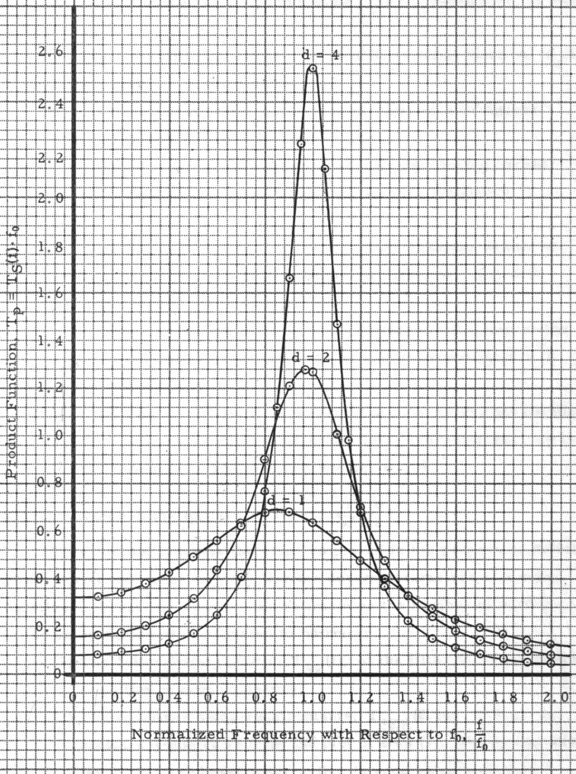


Figure 6.2.3 illustrates the change in the insertion delay function, $T_s(f)$, due to a change in f_o, for one value of the d parameter. The change in $T_s(f)$, due to a change in d, for one value of the f_o parameter, is shown in figure 6.2.4.

A summary of the properties of the insertion delay function, $T_s(f)$, of one lattice section, is given below.

(1) The behavior of the $T_s(f)$ function is illustrated in figures 6.2.1 (class A) and 6.2.2 (class B);

(a) Class A consists of monotonically decreasing curves. They all have maximum value at zero frequency (eq. 6.2.4);

(b) Class B consists of curves having maximum values at finite and non-zero frequencies, given by eq. 6.2.5;

(2) In the case of the class B curves

(a) for a fixed value of f_o, the d parameter determines the sharpness of the peak, as is seen from figures 6.2.2 and 6.2.4;

(b) The parameter f_o is dominant in determining the location of the maximum value of the delay characteristic, as is seen from figure 6.2.3 and eq. 6.2.5.

6.3 The Choice of the Initial Estimates, x_k^o, y_k^o

This section presents the steps employed in the process of selection of the initial estimates, x_k^o, y_k^o, for the design parameters of a delay equalizer. In this procedure, the estimate on the number of sections, n, of the delay equalizer is assumed to be known.

(1) The delay characteristic, $T_{SF}(f)$, of the filter to be equalized is plotted versus frequency;

(2) The equalizer delay requirements, $T_{SER}(f)$ (eq. 2.5.2), is obtained graphically from the plot of step (1). Here, the delay level, T_o, is chosen arbitrarily, (a good value is to take

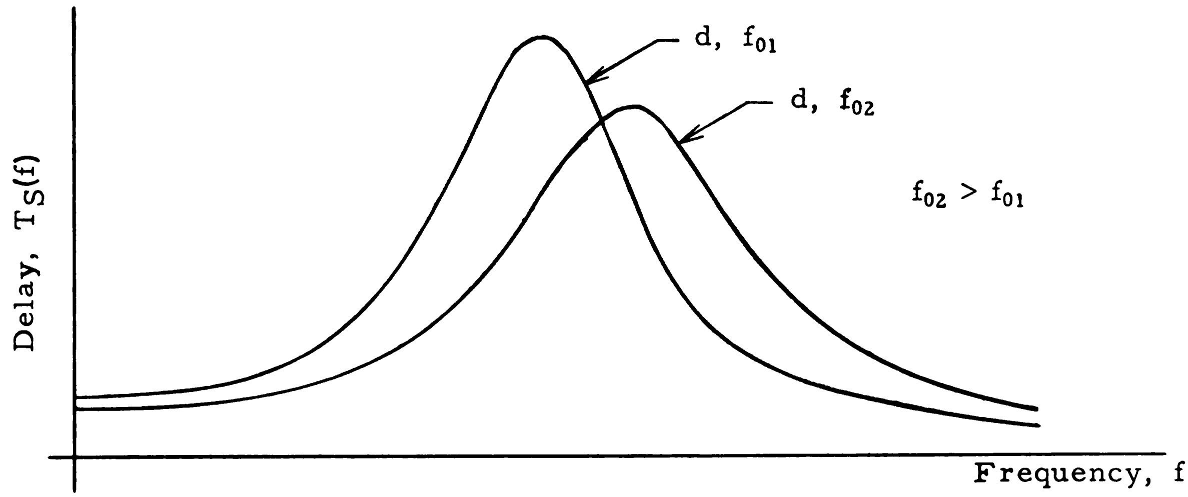


Figure 6.2.3. Change in $T_S(f)$ due to a change in f_0.

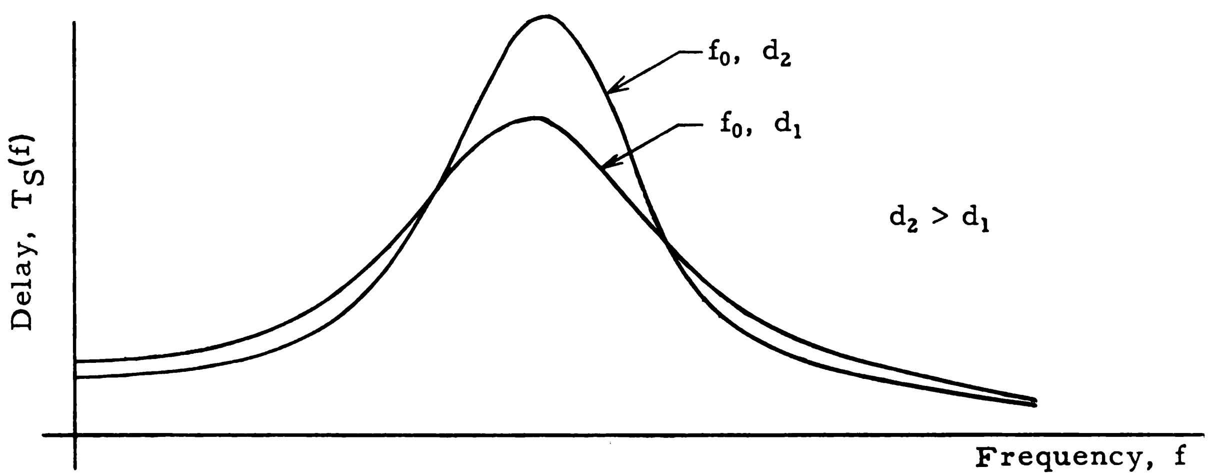


Figure 6.2.4. Change in $T_S(f)$ due to a change in d.

$T_o = 2T_{SFmax}$), since only relative values of the equalizer delay requirement are of interest;

(3) On the basis of the behavior of the insertion delay function, $T_s(f)$, of one section (figs. 6.2.1 and 6.2.2) a set of n parameter pairs, d_k, f_{ok} ($k=1, 2, 3, \cdots, n$), is chosen such that the sum of these n functions, $T_{sk}(f)$, when plotted, approximates the shape of the equalizer delay requirement, $T_{SER}(f)$. This step is a curve fitting problem and the degree of approximation attained depends largely on the skill and experience of the designer. For the initiated, steps (4) and (5) of this procedure might well be omitted;

(4) Program 2 is used to calculate the total delay, $T_{ST}(f)$, (the sum of the delay of the filter and the equalizer, based on the parameters of step 4);

(5) On the basis of the behavior of this total delay characteristic, $T_{ST}(f)$, a new set of the n parameter pairs, d_k, f_{ok}, is obtained as follows: Since the total delay, $T_{ST}(f)$, is to approximate a constant value in the interval of equalization, the d_k parameters of those sections having a delay characteristic which peaks in a neighborhood of a relative maximum of $T_{ST}(f)$ are decreased and the d_k parameters of those sections having a delay characteristic which peaks in a neighborhood of a relative minimum of $T_{ST}(f)$ are increased. The f_{ok} parameters are used to relocate the peaks of the delay characteristics of some sections, if necessary;

(6) Upon the selection of the normalizing frequency, f_d (see criterion 4.5.1), the initial estimates, x_k^o, y_k^o, are calculated in terms of the d_k and f_{ok} parameters form eq. 3.1.2.

It should be pointed out that the steps (1), (2), and (3), of the above procedure, in essence describe the commercial method of delay equalizer design. In the actual design, after each choice of the n parameter pairs,

d_k, f_{ok}, the n functions, $T_{sk}(f)$ $(k=1, 2, 3, \cdots, n)$, are plotted and added graphically, giving the equalizer delay characteristic, $T_{SE}(f)$. This equalizer delay characteristic is compared with the shape of the equalizer delay requirement, $T_{SER}(f)$, and a new set of the parameters, d_k, f_{ok}, is selected on the basis of this comparison. This repetitive trial and error process is continued until such equalizer delay characteristic, $T_{SE}(f)$, is obtained, which approximates the equalizer delay requirement, $T_{SER}(f)$, to the specified degree of accuracy [34].

VII. THE ELEMENT VALUES OF THE LATTICE SECTION

A schematic diagram of one lattice section of a delay equalizer is shown in figure 2.6.1. The element values of this section in terms of the d and f_o design parameters, as obtained from eqs. 2.6.2 and 2.6.6, are given by

$$L_1 = \frac{R_2}{2\pi f_o d} = R_2^2 C_2 \qquad (7.1)$$

$$L_2 = \frac{R_2 d}{2\pi f_o} = R_2^2 C_1 \qquad (7.2)$$

where R_2 is terminating resistance of the filter to be equalized and also the resistance level of the lattice section (eq. 2.6.1).

The element values in terms of the x and y design parameters and in terms of the normalizing frequency, f_d, as obtained from eqs. 3.1.2, 7.1, and 7.2, become

$$L_1 = \frac{R_2}{2\pi f_d y} = R_2^2 C_2 \qquad (7.3)$$

$$L_2 = \frac{x R_2}{2\pi f_d} = R_2^2 C_1 \qquad (7.4)$$

A number of bridged-tee and twin-tee equivalent networks for the lattice section can be derived. Such equivalent networks appear in literature and are not included here [1, 33].

VIII. CONCLUSIONS

The usefulness of the thesis method of delay equalizer design and its advantages over the commercial delay equalizer design technique (section 6.3) are discussed in the following paragraphs.

(1) The commercial method of delay equalizer design, as described in section 6.3, is a trial and error procedure, which is very tedious and extremely time consuming. One of the main advantages of the t-method lies in its time-saving nature. Reference to the five t-method solutions, presented in chapter IV, demonstrates that given a set of satisfactory initial estimates, the entire design problem is effected in a matter of minutes of computer time. More specifically, the average computer time for these five solutions is approximately 35 minutes. Several weeks for one man would be required to effect one such equalization problem by the commercial method.

(2) The commercial design method depends largely on the selection of design parameters, which requires skill and experience on the part of the designer. The t-method provides a systematic approach to the design problem and eliminates to a large extent the need of skill and experience.

(3) The extensive amount of the trial and error process associated with the commercial method is almost eliminated in the t-method. Although no criterion for the choice of the f_i frequency points leading to the best approximation of constant delay is given, criterion 4.5.2 reduces the trial and error aspect of the t-method to two computer applications.

(4) On the basis of the exploration with the t-method, as in chapter IV, it appears that this method is capable of producing approximations to a constant delay which are beyond the power of the commercial method even though pursued with the utmost skill, experience, and patience over an extended period of time.

The five solutions of delay equalization problems, presented in chapter IV, demonstrate the effectiveness of the thesis method of delay equalizer design. It is not meant, however, to imply that the t-method is the final answer to the delay equalization problem, but it definitely indicates that the combined use of numerical analysis and a digital computer has promise in equalizer design.

There is essentially no discussion in the technical literature on the problem of estimating the number of sections of a delay equalizer. One approach to the solution of this problem is presented in chapter V. This approach should be investigated further.

BIBLIOGRAPHY

1) Balabanian, N. Network Synthesis, New York: Prentice-Hall, Inc., 1958.

2) Bennett, B. J. "Synthesis of Electric Filters with Arbitrary Phase Characteristics," Convention Record of the I. R. E., 1953, Part 5, Circuit Theory, pp. 19-26.

3) Blewett, J. P. and Rubel, J. H. "Video Delay Lines," Proc. I. R. E., December, 1947, pp. 1580-1584.

4) Bode, H. W. "A General Theory of Electric Wave Filters," B. S. T. J., April, 1935, p. 211.

5) Bode, H. W. Network Analysis and Feedback Amplifier Design, Princeton, N. J.: D. Van Nostrand Co., Inc., 1945.

6) Bradley, W. E. "Design of a Simple Bandpass Amplifier with Approximate Ideal Frequency Characteristics," I. R. E. Trans., P. G. C. T. 2, December, 1953, pp. 30-38.

7) Brain, A. E. "The Compensation for Phase Errors in Wide-band Video Amplifiers," Proc. I. R. E., July, 1950, pp. 243-251.

8) Brand, S. and Nyquist, H. "Measurement of Phase Distortion," B. S. T. J., July, 1930, pp. 522-543.

9) Bresler, A. D. "On the Approximation Problem in Network Synthesis," Proc. I. R. E., December, 1952, p. 1724.

10) Cannon, W. D. "Delay Distortion Correction," Trans. A. I. E. E., March, 1956, pp. 55-61.

11) Cauer, W. Synthesis of Linear Communication Networks, New York: McGraw-Hill, Inc., 1958.

12) Cherry, E. C. "Application of Electrolytic Tank Techniques to Network Synthesis," Proc. of the Symp. on Modern Network Synthesis, Polytech. Inst. of Brooklyn, Microwave Research Institute, New York, April, 1952, p. 140.

13) Corrington, M. S. and Sonnenfeldt, R. W. "Synthesis of Constant-Time Delay Networks," R.C.A. Review, Vol. 15-2, June, 1954, pp. 163-186.

14) Dagnall, C. H. and Rounds, P. W. "Delay Equalization of Eight-Kilocycle Carrier Program Circuits," B.S.T.J., Vol. 28, April, 1949, pp. 165-220.

15) Darlington, S. "Realization of a Constant Phase Difference," B.S.T.J., January, 1950, pp. 94-104.

16) Darlington, S. "The Potential Analoque Method of Network Synthesis," B.S.T.J., Vol. 30, April, 1951, pp. 315-365.

17) Deutsch, S. "Maximally-Flat Time Delay Ladders," Trans. I.R.E., Circuit Theory, June, 1959, pp. 214-218.

18) Di Toro, M. J. "Phase and Amplitude Distortion in Linear Networks," Proc. I.R.E., January, 1948, pp. 24-36.

19) Floyd, C. F., Corke, R. L., and Lewis, H. "The Design of Linear-Phase Low-pass Filters," Inst. Elect. Engrs., Paper 1295, 11 pp. (Television Convention, 1952).

20) Golay, M. J. E. "The Direct Method of Filter and Delay Line Synthesis," Proc. I.R.E., March, 1954, pp. 585-588.

21) Hebb, M. H. and Hotton, C. W. "On the Design of Networks for a Constant Time Delay," J. Appl. Phys., June, 1949, p. 616.

22) Hildebrand, F. B. Introduction to Numerical Analysis, New York: McGraw-Hill, Inc., 1956.

23) Householder, Alston S. Principles of Numerical Analysis, New York: McGraw-Hill, Inc., 1953.

24) Kallmann, H. E. "Equalized Delay Lines," Proc. I.R.E., September, 1946, pp. 646-657.

25) Kuh, E. S. "Synthesis of Lumped Parameter Precision Delay Line," I.R.E. Nat. Convention Record, 1957, Circuit Theory, Part 2, pp. 160-170.

26) Kunz, K. S. Numerical Analysis, New York: McGraw-Hill, Inc., 1957.

27) Lane, C. E. "Phase Distortion in Telephone Apparatus," B.T.S.J., July 1930, pp. 493-521.

28) Laurent, T. "Basic Theory of Delay and Build-up Times in Electric Filters with Phase Distortion," Arch. Elekt. Uebertragung, March, 1952, pp. 91-98.

29) Linvill, J. G., "The Approximation with Rational Function of Prescribed Magnitude and Phase," Proc. I.R.E., June, 1952, p. 711.

30) MacCall, L. A., "An Analysis of the Distortion of Signals by Linear Systems Having Amplitude and Phase Characteristics of Certain Type," Proc. I.R.E., November, 1931.

31) Mead, S. P. "Phase Distortion and Phase Distortion Correction," B.S.T.J., April, 1928, pp. 195-224.

32) Nuttall, T. C. "Some Aspects of Television Circuit Technique: Phase Correction and Gamma Correction," Bull. Assn. Suisse Electr., August, 1949, pp. 615-622.

33) Reed, M. B. Electric Network Synthesis, New York: Prentice-Hall, Inc., 1955.

34) Rendall, A. R. "Phase Compensation (II), Design of Phase Compensation Networks," Elect. Communication, Vol. 7, 1928-1929, pp. 316-327.

35) Scott, R. E. "Potential Analogs in Network Synthesis," Convention Record of the I.R.E., 1955, Part 2, Circuit Theory, p. 2.

36) Steinberg, J. C. "Effects of Phase Distortion on Telephone Quality," B.S.T.J., July, 1930, pp. 550-566.

37) Storch, L. "An Application of Modern Network Synthesis to the Design of Constant-Time Delay Networks with Low-Q Elements," Convention Record of the I.R.E., 1954, Part 2, pp. 105-117.

38) Storch, L. "Synthesis of Constant-Time Delay Ladder Networks Using Bessel Polynomials," Proc. I.R.E., Vol. 32, November, 1954, pp. 1666-1675.

39) Storer, J. E. Passive Network Synthesis, New York: McGraw-Hill, Inc., 1957.

40) Thomson, W. E. "Delay Networks Having Maximally Flat Frequency Characteristics," Proc. I.R.E., Vol. 96, 1949, pp. 487-490.

41) Thomson, W. E. "Networks with Maximally-Flat Delay," Wireless Engr., Vol. 29, October, 1952, pp. 256-263.

42) Tokad, Y. "Some Improvements of the Image-Parameter Method for the Design of L-C Filters," Ph. D. Thesis, Michigan State University, 1959.

43) Turner, A. H. "Artificial Lines for Video Distribution and Delay," R.C.A. Review, December, 1949, pp. 477-489.

44) Valley, G. E., Jr. and Wallman, H. Vacuum-Tube Amplifiers, New York: McGraw-Hill, Inc., 1948, pp. 84-92.

45) Wallis, C. M. "Design of Low-Frequency Constant Time Delay Lines," A.I.E.E. Trans., Part 1, Communications and Electronics, April, 1952, pp. 135-140.

46) Weinberg, L. "Constant-Resistance All Pass Networks with Maximally Flat Time Delay," Journal of Franklin Institute, January, 1959, pp. 35-53.

47) Wheeler, H. A. "The Interpretation of Amplitude and Phase Distortion in Terms of Paired Echos," Proc. I.R.E., June, 1939, pp. 359-385.

48) Wheeler, H. A. "Wideband Amplifiers for Television," Proc. I.R.E., June, 1939, p. 429.

49) Winkler, S. "The Approximation Problem of Network Synthesis," Trans. I.R.E., P.G.C.T., September, 1954.

50) Wood, I. E., "Note on Maximally Flat Delay Networks," Trans. I.R.E., P.G.C.T., December, 1958, pp. 363-364.

51) Zobel, O.J. "Transmission Characteristics of Electric Wave-Filters," B.S.T.J., October, 1924.

52) Zobel, O.J. "Distortion Correction in Electrical Circuits," B.S.T.J., July, 1928, pp. 438-534.