# AN ECONOMICAL EVALUATION OF THE NUTRITIONAL CONTRIBUTION OF FOOD

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# AN ECONOMICAL EVALUATION OF THE NUTRITIONAL CONTRIBUTION OF FOOD

By

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ABSTRACT

AN ECONOMICAL EVALUATION OF THE
NUTRITIONAL CONTRIBUTION
OF FOOD

by Jose David Langier

Countries or regions of a country whose populations present a deficient nutritional intake also have scarce resources. To improve the nutritional status of these populations, there is a need of an appropriate measure of the nutritional contribution of food to be as economical as possible.

Nutritionists, geographers and economists have developed measures for evaluating the nutritional contributions of foods. These methods fall into three basic categories. The first considers only one nutrient. The second considers all the essential nutrients that food would furnish, but regards all the nutrients as having the same importance. The third method considers the nutrients as being of different importance; however, the weights are arbitrarily assigned. Chapter II reviews these methods and shows that none is appropriate for indicating the food that improves the nutritional status as much as possible.

This thesis suggests four new measures, of which Measure Three (the sum of the nutrients, weighted by the deficiency level) is recommended for general use.

To add the different nutrients, normally measured in different units, their quantities are expressed as percentages of the recommended allowances. The nutritional deficiencies are also expressed as percentages of the allowances.

The measures are examined in their nutritional, geometrical and analytical aspects. Each one implies the existence of a nutritional status function that represents the relation between the intake of nutrients and the nutritional status of the consumer.

Directional derivatives permit the derivation of these measures from the nutritional status functions. The direction is given by the proportions in which the nutrients appear in a food.

A general measure, from a general nutritional status function, is derived as well.

K. Lancaster[1] has developed a version of consumer theory in which the utility comes from the characteristics of the commodities (nutrients, for example). It is shown that the problem studied in this thesis is a natural empirical application of Lancaster's theoretical study, although results of this thesis do not depend upon his theoretical analysis, (The work in this thesis started before the Lancaster's study was published.)

Jose David Langier

A new method of obtaining economical diets, close to the minimum cost diets calculated by linear programming, is developed in this thesis. Also a method is developed for use when the resources available are smaller than the ones required by the least-cost diet. These methods do not require computers and skilled personnel. A desk calculator is sufficient.

The new method is applied to the calculation of economical supplementary diets that remove all nutritional deficiencies, for "poor" and "very poor" families in four different villages in Northeast Brazil. These economical supplementary diets and their costs are compared with the minimum cost supplementary diets computed by linear programming.

Finally, this thesis indicates that a measure of the nutritional contribution of food may be useful in allocating expenditures for the research and development of food, the production of food, food importation and food aid.

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[1]K. J. Lancaster, "A New Approach to Consumer Theory," Journal of Political Economy, Vol. LXXIV, April 1966, No. 2, pp. 132-157.

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## FORWORD

This thesis was made possible by a fellowship from the Economic and Agricultural Development Institute of Michigan State University. I express my gratitude to this Institute for its financial support.

The thesis tries to integrate problems of different areas: nutrition, economics and programming. Therefore, it is advisable to indicate to the specialized reader where he can find the material he is interested in, when first reading this thesis.

The nutritionists will be especially interested in Chapters I, II, Section A of Chapter III, Chapters V and VI. The economist will find Chapters I, III, IV and VI of particular interest. The student of programming methods will want to read particularly Section C of Chapter IV and Chapter V.

Most of all, I express my gratitude to Prof. Victor E. Smith, for his truly patient help in reading and criticizing the early versions of this thesis. To the many others that helped me, I express my sincere gratitude.

Jose David Langier
East Lansing, Michigan

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TABLE OF CONTENTS

LIST OF TABLES

## LIST OF FIGURES

# CHAPTER I

## STATEMENT OF THE PROBLEM

For many low income families and for many inhabitants of underdeveloped countries or regions of a country, the food consumption pattern is such that these people do not achieve one or more of the recommended allowances of the essential nutrients suggested by nutritionists.

The governments of the countries in which such undernutrition exists are trying to improve the nutritional status of their populations by increasing the supplies of existing foods, by introducing new foods into the existing food consumption pattern and by changing food habits. Because the resources of families and countries are scarce, there is a need to be economical. There is a problem of deciding among different alternatives: which production and consumption of food to increase, which new food to introduce.

Nutritionists, geographers and economists have developed measures for evaluating the nutritional contributions of foods. Those methods fall into three basic categories. The first, that I call the conventional method, considered only one nutrient and found the most economical food that would provide that nutrient. The second method considered all the essential nutrients that a food would supply, but regarded all nutrients as having the same importance. The third method considered the nutrients as being of different importance; however, the weights were arbitrarily assigned. In the next chapter, I shall fully examine these methods.

The main problem of this thesis is to find new methods of measuring the nutritional contribution of foods. They will take into account all the essential nutrients and will give them different weights according to their relevance in a particular situation. They can only be applied to find economical foods when there is undernutrition. In Chapter III, I shall discuss these new measures.

There is a mathematical method that gives economical solutions for improving diets, without using a measure of the nutritional contribution of food. The diet obtained by employing this mathematical method, linear programming, eliminates the nutritional deficiencies at minimum cost. Perhaps because it requires mathematics, nutritionists have not yet made much use of this method. V. E. Smith[1] did some interesting applications of linear programming to the problem of human diets. However, this mathematical method is

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[1]Victor E. Smith, Electronic Computation of Human Diets, MSU Business Study, Bureau of Business and Economic Research, Graduate School of Business Administration, Michigan State University, East Lansing, Mich., 1964.

not feasible when the amount of resources available is less than the one required by the linear programming solution. In such case the nutritional objective must be adjusted downward, but linear programming cannot tell us how this should be done.

A measure of the nutritional contribution of food can be used whether or not resources permit the full set of nutritional requirements to be met. Furthermore, a useful measure of the nutritional contribution of food can be easily computed by desk calculator; such a measure requires neither expensive computing equipment nor personnel skilled in linear programming.

## CHAPTER II

## PRESENTATION AND CRITICISM OF PREVIOUS WORKS

### A. Nutritionists

Nutritionists have used different methods in deriving inexpensive diets for low income families. I shall present some of these methods and evaluate their use in improving nutritionally deficient diets.

The conventional method of finding the nutrient-per-dollar ratio is most clearly presented by Tremoliéres, Serville and Jacquot.[1] They group foods into three categories: those foods rich in animal protein, in calories and in vitamin C. To obtain the amount of nutrient-per-dollar of a food, they first find the amount of the edible portion in the food sold at the retail level, then they get the amount of the nutrient furnished in the edible portion and finally divide this quantity by the retail price of the food.

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[1] J. Tremolières, Y. Serville and R. Jacquot, La Pratique de l'Aliméntation, Les Editions Sociales Françaises, Paris, 2nd Ed. 1962, Vol. 3, Chapter 3.

If $E_j$ is the proportion of food j as sold at the retail level that is edible, $N_{ij}$ is the amount of nutrient i obtained from one kilogram of the edible portion of food j and $p_j$ is the price of one kilogram of food j at the retail level, then

$$T_{ij} = \frac{E_j N_{ij}}{p_j}, \quad i = 1.....m \text{ and } j = 1....q,$$

where $T_{ij}$ is the quantity of nutrient i consumed per dollar if food j is purchased. The food with the largest $T_{ij}$ is the most economical source of nutrient i.

Wilson, Fisher and Fuqua[2] compare the percentage of a single nutrient which a given group of foods provides the diet with the percentage which this group of foods contributes to the total cost of the diet. Again the group of foods with the largest ratio is the most economical one providing that nutrient in the diet. They use this measure of efficiency for each of the several nutrients that a group of foods supplies. Wilson, Fisher and Fuqua's measure also applies for a single food instead of a group of foods, indicating the cheapest source of a given nutrient in the diet.

But, if the problem is to improve nutritionally deficient diets and to save resources for uses other than on food, then Tremolières, Serville and Jacquot's method is only appropriate when undernutrition involves only one

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[2]E. D. Wilson, K. H. Fisher and M. E. Fuqua, Principles of Nutrition, John Wiley & Sons, Inc., New York, 1959, Chapter 18.

nutrient. Wilson, Fisher and Fuqua's method is a variant of the conventional method. It also gives a nutrient-per-dollar ratio, with the disadvantage that it can only be applied to foods belonging to the actual diet.[3]

When there is more than one nutritional insufficiency, these two methods are inappropriate because the nutritional contribution of a food includes all the essential nutrients that a food furnishes. To be sure, Tremolières, Serville and Jacquot know that a food provides more than one essential nutrient when consumed, but their method considers only one nutrient as the nutritional contribution of a food. In each case, of course, they attempt to consider that nutrient for which the food is most important.

Because Terroine[4] wants to take into account the fact that a food supplies more than one nutrient, he places the foods in tabular form, in which the rows are the different foods and the columns are the essential nutrients. In the intersection of a row with a column he writes in the $T_{ij}$'s of Tremolières, Serville and Jacquot's method (the i

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[3]V. E. Smith, Electronic Computation of Human Diets, MSU Business Studies, Bureau of Business and Economic Research, Graduate School of Business Administration, Michigan State University, East Lansing, 1964, in pp. 61-67, presents a more elaborate criticism of methods such as that of Wilson, Fisher and Fuqua.

[4]E. F. Terroine, Valeur Aliméntaire et Coût des Denrées," Annales de la Nutrition et de l'Aliméntation, 1962, Vol. 16, pp. 91-172.

represents the nutrient or column and the j represents the food or row). Then he classifies the foods according to the number of times that the food appears as the most or second most economical food furnishing a single nutrient. He ranks the food with the largest number of such appearances in first place, the food with the second largest number in second and so on.

Because Terroine considers only the nutrients that are the most or the second most economically provided by the food he still eliminates other nutritional contributions of a food that may be relevant to improving the nutritional status. To show the distortion in the choice of a food to ameliorate the nutritional status of a family, using Terroine's method, I propose the following example.

A family has three nutritional deficiencies and wants to spend $1.00 for improving its nutritional intake. Suppose that there are two foods, X and Y, such that X is the cheapest source of two deficient nutrients and furnishes only those two nutrients, while Y is the third most economical source of the three deficient nutrients and only provides these three nutrients. Either $.99 expended on X or $1.00 expended on Y will supply the family with a nutritional intake that is equal to the recommended allowances of the two nutrients that X furnishes. However, $1.00 expended on Y will provide a larger amount of the third deficient nutrient that could be provided by the expenditure of the $.01 left from the expenditure on X, on Z the cheapest source of this nutrient. Using Terroine's method, Y will not be considered, although it improves the nutritional status of the family by a larger amount than the combination of X and Z described above.

Years before these works appeared, a forgotten pioneer work (1917), by Sherman and Gillet[5] developed a measure of the "composite value" of the nutrient content of a food. They divided the amounts of the nutrients furnished in the edible portion of one pound of a food by the respective recommended allowances. They weighted these ratios by two different sets of weights and then added them to get the "composite value" for the food. The nutrients they considered were: calories, protein, calcium, phosphorus and iron. (At the time, these were the only nutrients for which nutritionists recommended allowances.) The two sets of weights suggested were: 1) 60, 10, 10, 10 and 10, and 2) 40, 15, 15, 15 and 15, for each of the above nutrients respectively. These weights, they said, were arbitrary, but took into account the fact that calories were often deficient in the diets of poor American families.

This calculation showed remarkable sophistication for the time. However, when undernutrition exists, using

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[5]H. C. Sherman and L. R. Gillet, The Adequacy and Economy of Some City Dietaries, New York Association for Improving the Condition of the Poor, Publication 121, 1917, p. 20. I thank Dr. C. Florencio for bringing this work to my attention.

these weights to evaluate the nutritional contribution of food may distort the choice of the most economical food, because these weights are not adequate measures of the sizes of the deficiencies. It may also distort the choice of the most economical food, because it includes non-deficient nutrients in the nutritional contribution of food.

In 1965, Davis[6] proposed a measure of the multiple contribution of food, using the following method. He took the amount of a nutrient in the edible portion of a food as purchased and divided it by the recommended allowance of this nutrient. He added up these ratios for the vitamins and minerals (vitamins A, D, C, thiamin, riboflavin, niacin, calcium and iron) and divided their sum by the number of vitamins and minerals considered in the study (8). Then he summed this average value with the similar relatives for calories and protein and divided it by three. Finally, he divided this last average by the price of 100 grams of the food as purchased, obtaining an "overall economic-nutritional index."

In 1966, Armstrong[7] calculated such "overall economic-nutritional indices: for Canada using Canadian prices

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[6]J. G. Davis, "The Nutritional Index and Economic Nutritional Index of Foods," Dairy Industries, 1965, Vol. 30, No. 3, pp. 193-197.

[7]J. G. Armstrong, "An Economic-Nutritional Index of Foods," Canadian Nutrition Notes, 1966, Vol. 22, No. 3, pp. 25-39.

and seven instead of eight vitamins and minerals. (He did not consider vitamin D in his study.)

If $N_{cj}$ is the amount of calories in the edible portion of 100 grams of food j as purchased, $N_{pj}$ is the amount of protein in the edible portion of 100 grams of food j as purchased, $N_{ij}$ is the amount of vitamin or mineral i in the edible portion of 100 grams of food j as purchased, $R_c$, $R_p$ and $R_i$ are the recommended allowances of calories, protein and vitamin or mineral i and $p_j$ is the price of 100 grams of food j as purchased, then

$$D_j = \frac{\dfrac{\dfrac{N_{cj}}{R_c} + \dfrac{N_{pj}}{R_p} + \dfrac{1}{m+v} \sum_{i=1}^{m+v} \dfrac{N_{ij}}{R_i}}{3}}{p_j},$$

$j = 1 \ldots q$,

$m$ = number of minerals in the study,

$v$ = number of vitamins in the study,

where $D_j$ is the "overall economic-nutritional index" of food j as purchased, using Davis' method. The food with the largest $D_j$ has the largest nutritional contribution per dollar, by Davis' method.

To sum the essential nutrients of a food implies giving them weights that represent their nutritional importance. By considering the vitamins and minerals as one nutrient, the "protective" nutrient, Davis and Armstrong end by giving weights to the vitamins and minerals that are different from the ones given to the calories and protein, although the "protective" nutrient as a whole has the same weight. For Davis, one percent of the recommended allowance of a vitamin or of a mineral is equivalent to one-eighth percent of the recommended allowance of calories or protein. For Armstrong, one percent of the recommended allowance of a vitamin or of a mineral is equivalent to one-seventh percent of the recommended allowance of calories or protein. It seems that the weight (nutritional importance) of vitamins and minerals should not depend upon the number of them in a given study!

When Davis' method is applied to find the food with the largest nutritional contribution, to improve the nutritional status of families with deficient intake, it considers nutrients of which the consumption may have already reached the recommended allowance and, therefore, distorts the choice of the food to consume. Certain deficiencies of nutrients may be larger than others and because of this, they may be more important than others. This was pointed out by Davis himself in the following passage: "Indeed, some nutritionists consider that protein is by far the most important aspect because it is in protein that the diets of the poorest nations are the most deficient." (p. 194.)

In summary, certain nutritionists' approaches to measuring the nutritional contribution of food fail to consider all the essential nutrients that a food provides. Others that take into account all the essential nutrients do not give them appropriate weights, relevant to the particular situation.

B. Geographers and Economists

During World War I, Cooper and Spillman[8] compared the efficiency of land when used for the production of different foods. First they found the average quantity of a food produced on one acre of land, dividing the total production of this food by the total acreage utilized to obtain this output. Then they calculated the amount of this average that is edible and finally, they obtained the amount of calories and protein in the edible portion.

To increase the availability of calories they suggested increasing the output of the food with the largest calories-per-acre ratio. They proposed the same for protein. Because it considers only one nutrient and disregards the other nutritional contributions of the food, the conventional method as applied by Cooper and Spillman is not suitable when there is need to increase the output of more than one nutrient at the same time.

In 1958 Stamp[9] used the output of calories as the single indicator of the nutritional contribution of food to measure the efficiency of land used for the production of different foods. However, he considered that the output of

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[8] M. C. Cooper and W. J. Spillman, Human Food from an Acre of Staple Farm Products, Farmers' Bulletin 877, U. S. Department of Agriculture, U. S. Government Printing Office, Washington, D. C., 1917.

[9] L. D. Stamp, "The Measurement of Land Resources," Geographical Review, 1958, Vol. 48, pp. 1-15.

foods presents such variety that all the other nutrients would also be provided. Actually he avoided the problem of measuring the nutritional contribution of food, because he did not consider all the other essential nutrients that are furnished by foods.

In 1943, twenty-two years before Davis proposed his method, an economist, Christensen,[10] suggested a method of measuring the total nutritional value of food that takes into account all the essential nutrients contained in a food. His method divides the amount of a nutrient in a food by the recommended allowance of this nutrient, adds up all such relatives for a given food, and divides the sum by the number of all nutrients considered in the study.

If $N_{ij}$ is the amount of nutrient i obtained from a kilogram of the edible portion of food j, $R_i$ is the recommended allowance of nutrient i and m is the number of all nutrients considered, then

$$C_j = \frac{1}{m} \sum_{i=1}^{m} \frac{N_{ij}}{R_i}, \ j = 1..q,$$

where $C_j$ is the total nutritional value of one kilogram of the edible portion of food j, according to Christensen's method. To get the total nutritional value of food j as

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[10] R. P. Christensen, Using Resources to Meet Food Needs, U. S. Department of Agriculture, Bureau of Agricultural Economics, U. S. Government Printing Office, Washington, D. C., 1943.

purchased, it is enough to multiply $C_j$ by $E_j$, where $E_j$ is the proportion of food j as sold at the retail level that is edible.

Christensen applied his measure to find the productivity of the resources employed in the production of different foods in terms of his nutritional unit.[11] In 1948 he used the same method to find the productivity of the same resources, when he revised and expanded his 1943 report.[12]

However, in 1944 Mighell and Christensen,[13] after proposing the same measure,[14] presented some estimates of so-called "marginal food values" that would take into account nutritional needs, consumer preferences and production possibilities.[15] They presented these estimates for only eight foods.[16] How they got their values is not clear, nor

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[11] *Ibid.*, p. 71.

[12] R. P. Christensen, *Efficient Use of Land Resources in the United States*, Technical Bulletin 963, U. S. Department of Agriculture, U. S. Government Printing Office, Washington, D. C., 1948, Tables 20-37.

[13] R. L. Mighell and R. P. Christensen, "Measuring Maximum Contributions to Food Needs by Producing Areas," *Journal of Farm Economics*, 1944, Vol. 26, pp. 181-195.

[14] *Ibid.*, p. 182.

[15] *Ibid.*, pp. 187-189, for a description of their "marginal food values."

[16] *Ibid.*, p. 189, Table 2.

was it clear to the discussants of their paper, C. A. Bonnen and G. A. Pond.[17]

Black[18] and Black and Kiefer,[19] knowing the first work of Christensen, used a combination of the conventional and Christensen's methods to obtain the productivity of land used in the production of different foods. They employed the conventional method to obtain the productivity of land in terms of calories and protein separately. Utilizing Christensen's method they formed two general groups: vitamins (vitamin A, C, thiamin, riboflavin and niacin) and minerals (calcium, phosphorus and iron). Finally they calculated the productivity of land in terms of these two general groups.

Finally, in 1961, Zobler[20] presented a method that is similar to Christensen's, the only difference being that

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[17] Ibid., in C. A. Bonnen's discussion: "I am sure, however, that most of us do not understand the method followed in the computation of these so-called marginal food values." (p. 193). In G. A. Pond's discussion: "I am afraid the average reader would have difficulty accepting these marginal food values without more light on how they were determined. They appear to represent the subjective judgment of the authors." (p. 194).

[18] J. D. Black, Food Enough, Science for War and Peace Series, The Jacques Cattell Press, Lancaster, Penn., 1943, Chapter 12.

[19] J. D. Black and M. E. Kiefer, Future Food and Agriculture Policy, McGraw-Hill Book Co. Inc., New York, London, Toronto, 1948, Chapter 14.

[20] L. Zobler, "A New Measure of Food Production Efficiency," Geographical Review, 1961, Vol. 51, pp. 549-569.

he did not divide the sum of the nutritional relatives by the number of essential nutrients considered in the study.

If $N_{ij}$ is the amount of nutrient i obtained from a kilogram of the edible portion of food j and $R_i$ is the recommended allowance of nutrient i, then

$$Z_j = \sum_{i=1}^{m} \frac{N_{ij}}{R_i}, \; j = 1....q,$$

where $Z_j$ is the nutritional contribution of one kilogram of the edible portion of food j, given by Zobler's method. To allow comparison between foods Zobler suggested a standard food that would have $Z_j$ equal to m and would supply each nutrient with the same amount as the recommended allowance ($N_{is} = R_i$, for all i = 1...m, where s stands for the standard food).

Using his measure Zobler found that one hectare of land in Japan is 7.7 times more efficient than in the United States, when employed for food production.[21] Zobler also computed the nutritional needs of both countries, based on their populations and the recommended allowances of each essential nutrient. But land is so much more abundant in the United States than in Japan that when Zobler took into account the nutritional needs, the food production in the United States satisfied all essential needs except the riboflavin need, while in Japan only the ascorbic acid need was satisfied.[22]

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[21] Ibid., p. 565.

[22] Ibid., p. 568.

Although Zobler considered all essential nutrients of a food in his method, it is possible that some nutrients are more important than others in a particular situation. For example, if the Japanese government wants to import foods to improve the nutritional intake of the Japanese population, using Zobler's method to choose the imported food might result in a very small or even zero improvement in the nutritional status, in relation to the recommended allowances, because his method includes ascorbic acid in the measure of the nutritional contribution of food.

Suppose that there is a food W that provides only ascorbic acid and it has an international price $p_w$ such that using Zobler's method, the ratio $E_w Z_w / p_w$, where $E_w$ is the proportion of food W as purchased in the international market that is edible, is the largest of all possible imported foods. Importing food W is shown to be the most economical by Zobler's method, but it would not decrease undernutrition in Japan, because ascorbic acid is not deficient and food W contains only this nutrient!

Because of the relation between Christensen's and Zobler's methods,[23] $Z_j = mC_j$, my criticism of Zobler's method also applies to Christensen's. In summary, Christensen's and Zobler's methods take into account all the essential

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[23]Zobler knew the works of Christensen (1948) and Mighell and Christensen, because footnotes 1 and 7 refer to these studies. See L. Zobler, *op. cit.*, pp. 549 and 551.

nutrients of a food, but consider each of the nutrients as having the same importance. This may lead to an uneconomical choice, as shown by the hypothetical example described above.

Christensen[24] (1943) and Mighell and Christensen,[25] conscious of this weakness, suggested evaluating foods in nutritional terms, using weights for the nutrients that are based on the nutritional deficiencies, as shown by the following passages respectively:

> This assumption [equal weight for each nutrient] overlooks differences between nutrients with respect to current deficiencies. Additional units of certain products may be worth more than additional units of others, although the value of each as measured by this method [equal weight for each nutrient] are the same, because they contain more of the nutrients especially short in supply. (p. 69)

> The National Research Council recommendations for the individual nutrients can be given equal or variable weights and the values obtained for the nutrients in each product added together to determine the nutrient value. . . . Adjustment in weighting can be made if desired, to allow for the relative scarcity of the nutrients. (pp. 186-187)

Unfortunately they made no application following this line.

It is important to note that during a period when resources had to be economized in the production of food, because of the War efforts, Cooper and Spillman in 1917, Christensen in 1943 and Mighell and Christensen in 1944

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[24] R. P. Christensen, op. cit., 1943.

[25] R. L. Mighell and R. P. Christensen, op. cit.

proposed to measure the nutritional contribution of food, to increase the efficiency of the resources employed in food production. This is quite similar to the present situation in underdeveloped countries. These countries need to utilize their factors of production as efficiently as possible to increase their rate of economic growth.

Since 1947, when Dantzig developed the simplex method that gives the solution of a linear programming problem, it is possible to suggest minimum cost diets satisfying the recommended allowances of all essential nutrients without measuring the nutritional contribution of food. Cornfield[26] in 1941 and Stigler[27] in 1945 mathematically expressed the problem of obtaining the minimum cost diet satisfying the recommended allowances for the essential nutrients. V. E. Smith[28] utilized this mathematical method and also presented a bibliography on the subject. However, as I pointed out in Chapter I, there are conditions under which the linear programming method cannot be applied. If the resources available do not permit the purchase of the least-cost diet obtained by linear programming, some modification of the latter method is required, if there is a lack of skilled

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[26]The reference to Cornfield is found in V. E. Smith, op. cit., p. 12.

[27]G. J. Stigler, "The Cost of Subsistence," Journal of Farm Economics, 1945, Vol. 27, pp. 303-314.

[28]V. E. Smith, op. cit.

professional personnel and computers that linear programming requires, then in such cases measuring the nutritional contribution of food is an alternative method to be employed.

## CHAPTER III

## NEW MEASURES OF THE NUTRITIONAL CONTRIBUTION OF FOOD

When an individual has a food consumption pattern such that the intake of one or more nutrients is smaller than its allowance, to improve his nutritional status there is need to know the marginal nutritional contributions of the different foods. In this thesis, the objective is to bring the intake of the deficient nutrients to their recommended levels.

The empirical part of this thesis is based on four studies done by nutritionists of the Brazilian Ministry of Health (see Chapter V), who employed the allowances recommended by the United States' National Research Council in 1958. Table 1 shows an example of such allowances.

The nutritional contribution of an additional amount of a food includes all the essential nutrients that this food supplies. The nutrients are measured in different units, but expressing them as percentages of the allowances permits their aggregation. Because nutrients are different, they may differ in relative importance, so their aggregation must reflect this by attaching weights to each nutrient.

TABLE III-1. Daily allowances for a male 45 years old

| Nutrient | Allowance |
|---|---|
| Calories | 3000 Cal. |
| Protein | 70 gm. |
| Calcium | .8 gm. |
| Iron | 10 mg. |
| Vitamin A | 5000 I. U. |
| Thiamin | 1.5 mg. |
| Riboflavin | 20 mg. |
| Ascorbic Acid | 75 mg. |

Source: National Research Council, Food and Nutrition Board, Recommended Dietary Allowances, National Research Council Publication No. 589, National Academy of Sciences, Washington, D. C., 1958, p. 18.

If $N_{ij}$ is the amount of nutrient i in one unit of food j, $R_i$ is the recommended allowance of nutrient i and $W_i$ is the weight attached to nutrient i, then

$$F_j = \sum_{i=1}^{m} W_i \frac{N_{ij}}{R_i} (100), \; j = 1...q,$$

where $F_j$ is the weighted nutritional contribution of the given unit of food j.

The determination of appropriate weights is the main objective of this chapter.

## A. Available Alternatives

The intrinsic nutritional importance of the nutrients provides several sets of weights.

If nutritionists were to say that all nutrients are equally important, this information would suggest that the weights of the different nutrients should be equal. Assuming that $I_1$ is the importance of nutrient 1 and $I_1 = 1$, it follows that $I_i = 1$ for $i = 2...m$. Then the weights should be:

$$W_i = I_i = 1 \text{ for } i = 1......m. \qquad (1)$$

If nutritionists were to say that nutrient i is $I_i$ times as important as nutrient 1, this information would suggest that the weight of nutrient i should be $I_i$ times the weight of nutrient 1. Assuming that $I_1$ is the importance of nutrient 1 and $I_1 = 1$, the weights should be:

$$W_1 = I_1 = i \text{ and } W_i = I_i \text{ for } i = 2....m. \qquad (2)$$

Christensen,[1] Mighell and Christensen,[2] and Zobler[3] assumed that all nutrients are equally important. Sherman and Gillet[4] arbitrarily considered calories 3 or 6 times as important as protein, calcium, phosphorus and iron. Davis' method[5] implied that either calories or protein are eight

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[1] R. P. Christensen, Using Resources to Meet Food Needs, Department of Agriculture, Bureau of Agricultural Economics, U. S. Government Printing Office, Washington, D. C., 1943.

[2] R. L. Mighell and R. P. Christensen, "Measuring Maximum Contributions to Food Needs by Producing Areas," Journal of Farm Economics, 1944, Vol. 26, pp. 181-195.

[3] L. Zobler, "A New Measure of Food Production Efficiency," Geographical Review, 1963, Vol. 51, pp. 549-569.

[4] R. C. Sherman and L. R. Gillet, The Adequacy and The Economy of Some City Dietaries, New York Association for Improving the Condition of the Poor, Publication 121, New York, 1917.

[5] J. G. Davis, "The Nutritional Index and the Economical Nutritional Index of Foods," Dairy Industries, 1965, Vol. 30, No. 3, pp. 193-197.

times as important as any vitamin or mineral. Armstrong's method[6] presupposed that either calories or protein are seven times as important as any vitamin or mineral. I showed in Chapter II that none of these weighting systems is appropriate when the objective is to bring the intake of the deficient nutrients to the recommended allowances in an efficient manner. These methods may distort the choice of the efficient food, because they consider as important, nutrients that are not deficient.

Giving a value of zero to the intrinsic nutritional importance of the nutrients that are not deficient and taking the intrinsic importance as the weights of the nutrients that are deficient provides new sets of weights that avoids the distortion referred to above.

If nutritionists were to say that all nutrients are equally important and if the importance of nutrient 1 is set at $I_1 = 1$ and the h first nutrients are the deficient ones, then the weights should be:

$$W_i = I_i = 1 \text{ for } i = 1...h \text{ and } W_i = 0 \text{ for } i = h + 1,..m. \quad (3)$$

If nutritionists were to say that nutrient i is $I_i$ times as important as nutrient 1 and if the importance of nutrient 1 is set at $I_1 - 1$ and the h first nutrients are the deficient ones, then the weights should be:

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[6] J. G. Armstrong, "An Economical-Nutritional Index of Foods," Canadian Nutrition Notes, 1966, Vol. 22, pp. 25-39.

$W_1 = I_1 = 1$, $W_i = I_i$ for $i = 2....h$ and $W_i = 0$ for
$i = h + 1...m$. (4)

Although the two last set of weights consider only the deficient nutrients, their use may still distort the choice of the most efficient food, when the objective is to bring the intake of the deficient nutrients to the recommended allowances, in an efficient manner. The amount of a nutritional insufficiency is also important, because the greater the deficiency the more serious the physical consequences from undernutrition.

Combining the intrinsic nutritional importance with the magnitude of the deficiency of the nutritional intake, expressed as a percentage of the allowance, forms new sets of weights. (As before, nutrients that are not deficient receive a weight of zero.)

If $N_i$ is the intake of nutrient i and $R_i$ is the recommended allowance of nutrient i, then

$D_i = \left(\frac{R_i - N_i}{R_i}\right)100$, for all i such that $N_i$ is smaller than $R_i$,

and

$D_i = 0$, for all i such that $N_i$ is equal to or greater than $R_i$,

where $D_i$ is the deficiency of nutrient i.

If nutritionists were to say that all nutrients are equally important and if the importance of nutrient 1 is set at $I_1 = 1$ and the first h nutrients are the deficient ones, then the weights that combine the magnitudes of the deficiencies with the intrinsic importance of the nutrients should be:

$$W_i = I_i D_i = D_i \text{ for } i = 1...h \text{ and } W_i = 0 \text{ for } i = h + 1...m. \qquad (5)$$

If nutritionists were to say that nutrient i is $I_i$ times as important as nutrient 1, and if the importance of nutrient 1 is set at $I_1 = 1$ and the first h nutrients are the deficient ones, then the weights that combine the magnitudes of the deficiencies and the intrinsic importance of the nutrients should be:

$$W_1 = I_1 D_1 = D_1,\ W_i = I_i D_i \text{ for } i = 2...h \text{ and } W_i = 0 \text{ for } i = h + 1...m. \qquad (6)$$

The two last set of weights are appropriate for finding the most economical food when the objective is to bring the deficient nutritional intake to the recommended allowance. They combine the intrinsic nutritional importance and the amount of insufficiency of the nutritional intake to form the weights attached to the nutrients that a food supplies.

Nutritionists recommend allowances for only a limited number of nutrients. This restricts the application of the measures of the marginal nutritional contribution of food proposed in this thesis.

The new measures can be utilized to find the most efficient food to use in improving the nutritional status or the population of a region or of a country where the nutritional intake is deficient.

## B. Geometrical and Analytical Measures of the Nutritional Contribution of Food

This thesis tries to answer the following question: "On what food should a consumer spend an additional dollar, if he wants to improve his nutritional status as much as possible?"

The nutritional status of an individual depends upon his intake of the essential nutrients. This implies the existence of a nutritional status function. In general, to say that a food is better than another in its nutritional aspect also implies the existence of a nutritional status function. I shall consider a few such functions explicitly.

Let $N_i$ be the quantity of nutrient i ($N_i$ International Units of vitamin A, for example) and let $R_i$ be the recommended allowance of nutrient i, then

$$n_i = \frac{N_i}{R_i}(100)$$

is the quantity of nutrient i expressed as a percentage of its recommended allowance.

The properties of the nutritional status function S are:

$$S = S(n_1 \ldots\ldots n_h, n_{n+1} \ldots\ldots n_m), \text{ with}$$

$$\frac{\partial S}{\partial n_i} > 0, \text{ for } i = 1 \ldots\ldots h \text{ and}$$

$$\frac{\partial S}{\partial n_i} = 0, \text{ for } i = h+1 \ldots\ldots m,$$

where h is the number of deficient nutrients.

To allow a representation in two dimensions, I assume that there are only two deficient nutrients, 1 and 2. In the figures of this chapter, I shall suppose that $n_1^o$ and $n_2^o$ are the quantities of the nutrients 1 and 2 consumed at the initial position $n^o$, expressed as percentages of the allowances ($n_1^o$, $n_2^o$ < 100).

If the consumer spends one dollar on food A, he gets $A_1$ and $A_2$ of nutrients 1 and 2. If he spends one dollar on food B, he gets $B_1$ and $B_2$ of nutrients 1 and 2. Let me write:

$$a_1 = \frac{A_1}{R_1}(100) \text{ and } a_2 = \frac{A_2}{R_2}(100) \text{ and}$$

$$b_1 = \frac{B_1}{R_1}(100) \text{ and } b_2 = \frac{B_2}{R_2}(100).$$

To choose the food on which the consumer is going to spend the additional dollar, he needs a measure of the nutritional contribution of each food. I shall suggest four simple measures: the sum of the nutrients; the sum of the nutrients, weighted by their intrinsic nutritional importance; the sum of the nutrients, weighted by the deficiencies, and the sum of the nutrients, weighted by both the nutritional importance and the deficiency. I shall present the nutritional aspect of the measure first, and then the problem as it appears in its diagrammatic form. Finally, I shall generalize the measure for an anlytical treatment.

## Measure One--The Sum of the Nutrients

The first measure considers the nutritional contribution of the food as the sum of the quantities of the deficient nutrients that a food furnishes.

a. Nutritional Aspect: The sum of the deficient nutrients supplied by a food is the same measure as the one proposed by the set of weights (3): all nutrients are equally important, the weight of the first deficient nutrient is assumed to be 1 and only deficient nutrients enter into the measure (p. 24). Increasing the consumption of one deficient nutrient by one percent of its allowance is as good as increasing the consumption of any other deficient nutrient by one percent of its allowance. The proportion in which the nutrients are combined in a particular food is of no importance.

If the sums are equal for two foods, then the consumer is indifferent between the foods. This means that he can substitute one food for the other, leaving his nutritional status unchanged, because he increases the intake of nutrient 1 and diminishes the intake of nutrient 2 by the same quantities.

The nutritional contributions of foods A and B, according to the sum of their nutrient, are:

$F_a = a_1 + a_2$ and

$F_b = b_1 + b_2.$

The food with the largest sum is the one with the largest nutritional contribution.

<u>b. Geometrical Aspect</u>: In Figure 1, if the additional consumption consists of food A, this brings the intake to $n_1^a$ and $n_2^a$ of nutrients 1 and 2. If the additional consumption consists of food B, this brings the intake to $n_1^b$ and $n_2^b$ of nutrients 1 and 2. The differences between the two possible intake levels and the original one are: $n_1^a - n_1^o = a_1$ and $n_2^a - n_2^o = a_2$, if food A is consumed, and

$n_1^b - n_1^o = b_1$ and $n_2^b - n_2^o = b_2$, if food B is consumed.

In Figure 1, the nutritional contributions of foods A and B, measured by the sums of their nutrients, are:

$$F_a = a_1 + a_2 = a_1 + a_2^{'} = \left| n^{a'} - n^o \right| \text{ and}$$

$$F_b = b_1 + b_2 = b_1 + b_2^{'} = \left| n^{b'} - n^o \right|,$$

where $a_2^{'} = a_2$ and $b_2^{'} = b_2$. Here $a_2^{'}$ and $b_2^{'}$ are the vertical representations of the horizontal distances $a_2$ and $b_2$. Adding $a_2^{'}$ to $a_1$ and $b_2^{'}$ to $b_1$ gives the distances $|n^{a'} - n^o|$ and $|n^{b'} - n^o|$. Linking $n^a$ to $n^{a'}$ and $n^b$ to $n^{b'}$ by straight lines forms right triangles. From elementary trigonometry, it follows that:

$$\frac{a_2}{a_2^{'}} = \frac{b_2}{b_2^{'}} = 1 = \tan \Omega,$$

where $\Omega$ is the angle that the straight lines from $n^a$ and $n^{a'}$ and from $n^b$ to $n^{b'}$ make with the vertical line drawn through $n^o$. As $\tan \Omega = 1$, this implies that the angle $\Omega = 45°$.

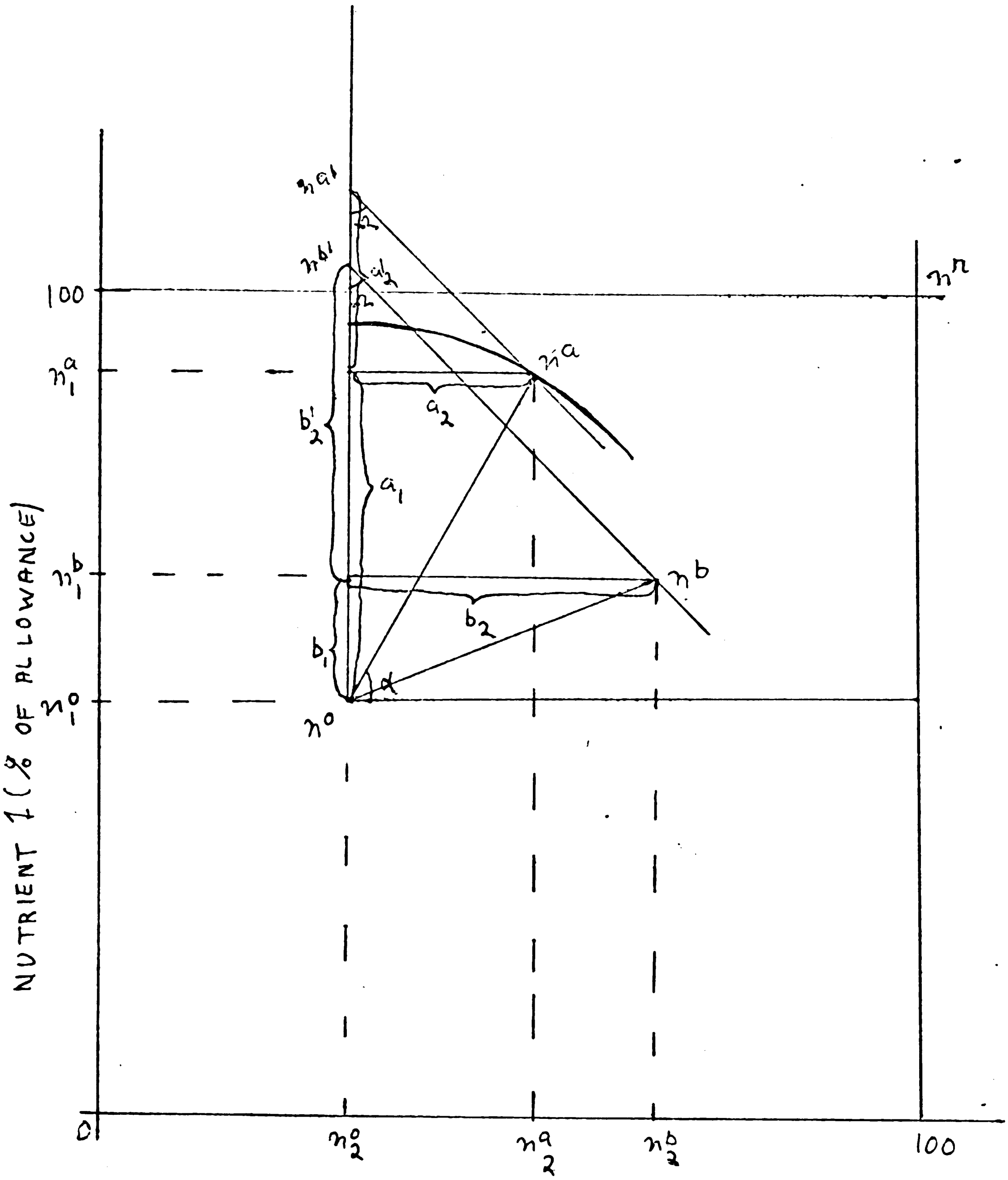


FIGURE III-1

MEASURE ONE – THE SUM OF THE NUTRIENTS

The food j for which $F_j$ (or $|n^{j'} - n^o|$) is the largest is the one with the largest nutritional contribution according to this measure. Alternatively, the food j that is on the line nearest to $n^r$ that makes a 45° angle with the vertical line is the one with the largest nutritional contribution, as measured by the sum of the nutrients.

The consumer will be indifferent between spending one dollar on food A or on any other food that brings his intake of the two nutrients the straight line passing through $n^a$ that makes a 45° angle with the vertical line drawn through $n^o$.

c. Analytical Aspect: The change in the nutritional status of a consumer, when he considers the sum of the deficient nutrients as the measure of this change, and has only two deficiencies, is given by:

$$dS = dn_1 + dn_2,$$

where dS, $dn_1$ and $dn_2$ are the changes in the nutritional status and in the consumption of the deficient nutrients 1 and 2.

Solving this differential equation gives:

$$S - n_1 + n_2 + C,$$

where S is the nutritional status function implied by the measure based on the sum of the deficient nutrients, $n_1$ and $n_2$ are the total amounts consumed of nutrients 1 and 2 and C is a constant of integration. Take

$$C = (m - 2)100,$$

where m is the total number of nutrients and 100 indicates that the consumption is equal to or greater than the recommended allowance. Then C represents the value that the consumer gives to the intake of non-deficient nutrients. It follows that:

$$S = n_1 + n_2 + (m - 2)100.$$

Generalizing this result for h deficient nutrients, I get:

$$S = n_1 + n_2 + \ldots + n_h + (m - h)100, \qquad \text{(I)}$$

I can write

$$n_1 = \sum_{j=1}^{q} n_{1j}x_j, \; n_2 = \sum_{j=1}^{q} n_{2j}x_j, \ldots\ldots, \; n_h = \sum_{j=1}^{q} n_{hj}x_j,$$

where q is the number of foods considered in the study, $n_{ij}$ ($i = 1 \ldots h$, $j = 1 \ldots q$) is the amount of nutrient i in one unit of food j, expressed as a percentage of its allowance, and $x_j$ is the amount of food j consumed. (The unit of food j is arbitrary and can be the amount of food j that one dollar can buy.)

The ratio between the increase in the value of function (I) and an infinitesimal increase in the consumption of food j is the partial derivative of S with relation to $x_j$:

$$\frac{\partial S}{\partial x_j} = n_{ij} + n_{2j} + \ldots + n_{hj}.$$

This is equal to the sum of the deficient nutrients supplied by one unit of food j. The $\max_j \frac{\partial S}{\partial x_j}$ gives the food with the largest marginal nutritional contribution. The consumer should choose this food, if he wants to improve his nutritional status as much as possible, using the sum of the nutrients as the criterion.

d. Other Aspects: The weakness of this measure is that nutrients of which the deficiency is very small are considered as important as nutrients of which the deficiency is very large. For example, the consumer may have a 10% deficiency of vitamin A and a 90% deficiency of ascorbic acid (vitamin C). He has one dollar and wants to spend it on either of two foods. If he spends the dollar on the first food, he will get 10% of the recommended allowance of vitamin A. If he spends the dollar on the second food, he will get 10% of the recommended allowance of ascorbic acid. According to the sum of the nutrients measure, the consumer would be indifferent in choosing between the two foods, although the second food may be much more important for his health. While the small deficiency of vitamin A may have no serious effects, he may suffer from scurvy because of the very small intake of vitamin C.

Geometrically, it may be attractive to consider as the nutritional contribution of a food the distance between the previous intake and the new intake resulting from the consumption of this food. In Figure 1, for food A this is given by $|n^a - n^o|$, the distance between $n^a$ and $n^o$. With this measure the consumer would be indifferent between spending an additional dollar on food A or on any other food that would bring the new intake point to the circle with center at $n^o$ and passing through $n^a$. The consumer's nutritional status would be regarded as unchanged if he substituted food A for another food that would bring his intake point to where this circle cuts the vertical line drawn from $n^o$. Such a substitution, however, would bring a very small increase in the intake of nutrient 1 and would take away the whole amount of nutrient 2 that food A supplies. This shows that the distance measure favors foods that furnish only one nutrient. As in the previous measure, the sum of the nutrients, the distance measure considers a nutrient with a very small deficiency to be as important as one with a very large deficiency.

## Measure Two--The Sum of the Nutrients, Weighted by the Intrinsic Nutritional Importance

This measure considers the nutritional contribution of the food as the weighted sum of the deficient nutrients that a food furnishes. The weights indicate the intrinsic nutritional importance of the nutrients. The consumer or nutritionists may consider that one nutrient is more important than another, independent of the intake.

a. Nutritional Aspect: This weighted sum of the deficient nutrients supplied by a food is the same measure as the one proposed by the set of weights (4): nutrients i is $I_i$ times as important as nutrient 1, the intrinsic importance of the first nutrient is assumed to be 1 and only deficient nutrients enter into the measure (p. 25). Increasing the intake of deficient nutrient i by one percent of its allowance is as good as increasing the intake of nutrient 1 by $I_i$ percent of its allowance. If nutrient 2 is 5 times as important as nutrient 1, increasing the intake of nutrient 2 by 1% of its allowance is as good as increasing the intake of nutrient 1 by 5% of its allowance.

If the weighted sums are equal for two foods, then the consumer is indifferent between the foods. This means that he can substitute one food for the other leaving his nutritional status unchanged, when the proportion in which he increases the intake of nutrient 1 and diminishes the intake of nutrient 2 is equal to $I_2$, the intrinsic importance of nutrient 2.

Suppose nutrient 2 is 5 times as important as nutrient 1. Assume a food that furnishes 5% and 2% of the recommended allowances of nutrients 1 and 2 respectively. By the weighted measure, the consumer should be indifferent between this food and another one that supplies 10% and 1% of the allowances of nutrients 1 and 2 respectively.

The nutritional contributions of foods A and B, according to this weighted sum, are:

$F_a = a_1 + I_2 a_2$ and

$F_b = b_1 + I_2 b_2$.

The food with the largest weighted sum is the one with the largest nutritional contribution.

<u>b. Geometrical Aspect</u>: In Figure 2, if the additional consumption consists of food A, this brings the intake to $n_1^a$ and $n_2^a$ of nutrients 1 and 2. If the additional consumption consists of food B, this bring the intake to $n_1^b$ and $n_2^b$ of nutrients 1 and 2. The differences between the possible intake levels and the original are:

$n_1^a - n_1^o = a_1$ and $n_2^a - n_2^1 = a_2$, if food A is consumed, and

$n_1^b - n_1^o = b_1$ and $n_2^b - n_2^o = b_2$, if food B is consumed.

In Figure 2, the nutritional contributions of foods A and B, according to this weighted sum of their nutrients, are:

$F_a = a_1 + I_2 a_2 = a_1 + a_2' = |n^{a'} = n^o|$ and

$F_b = b_1 + I_2 b_2 = b_1 + b_2' = |n^{b'} = n^o|$,

where $a_2' = I_2 a_2$ and $b_2' = I_2 b_2$. Here $a_2'$ and $b_2'$ represent the equivalent of $a_2$ and $b_2$ in terms of nutrient 1. Adding $a_2'$ and $a_1$ and $b_2'$ to $b_1$ on the vertical line drawn through $n^o$ gives the distances $|n^{a'} - n^o|$ and $|n^{b'} - n^o|$. Linking $n^a$ to $n^{a'}$ and $n^b$ to $n^{b'}$ by straight lines forms right triangles.

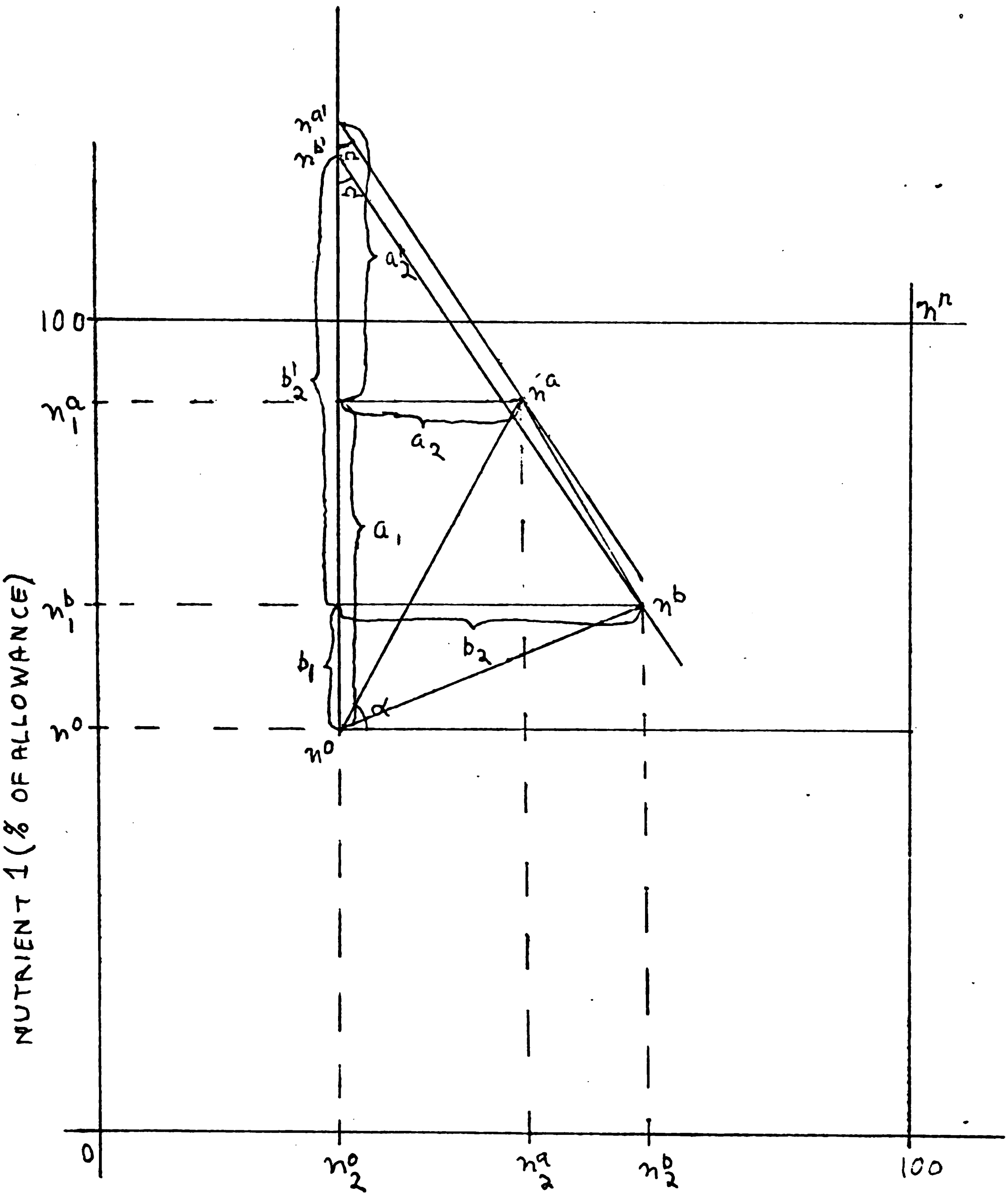


FIGURE III -2

MEASURE TWO - THE SUM OF THE NUTRIENTS, WEIGHTED BY THE INTRINSIC NUTRITIONAL IMPORTANCE

From elementary trigonometry, it follows that:

$$\frac{a_2'}{a_2} = \frac{b_2'}{b_2} = \frac{1}{I_2} = \tan \Omega,$$

where $\Omega$ is the angle that the straight lines from $n^a$ to $n^{a'}$ and from $n^b$ to $n^{b'}$ make with the vertical line drawn through $n^o$.

The food j for which $F_j$ (or $|n^{j'} - n^o|$) is the largest is the one with the largest nutritional contribution according to this measure. Alternatively, the food j that is on the line nearest to $n^r$ that makes an angle $\Omega$ with the vertical line is the one with the largest nutritional contribution, by the weighted sum of the nutrients.

The consumer will be indifferent between spending one dollar on food A or on any other food that would bring his intake of the two nutrients to the straight line passing through $n^a$ that makes an angle $\Omega$ with the vertical line drawn through $n^o$.

c. Analytical Aspect: The change of the nutritional status of a consumer, when he considers this weighted sum of the nutrients as measure of the change and he has only two deficiencies, is given by:

$$dS = dn_1 + I_2 dn_2,$$

where dS, $dn_1$ and $dn_2$ are the changes in the nutritional status and in the intake of the deficient nutrients 1 and 2.

Solving this differential equation gives:

$$S = n_1 + I_2 n_2 + C,$$

where S is the nutritional status function implied by this weighted sum of the nutrients, $n_1$ and $n_2$ are the total amounts consumed of the nutrients 1 and 2 and C is a constant of integration. Take

$$C = (m - 2)100,$$

where m is the total number of nutrients and 100 indicates an intake that is equal to or greater than the recommended allowance. Then C represents the value that the consumer gives to the intake of non-deficient nutrients. I obtain:

$$S = n_1 + I_2 n_2 + (m - 2)100.$$

Generalizing this result for h deficient nutrients, I get:

$$S = n_1 + I_2 n_2 + \dots + I_h n_h + (m - h)100, \qquad \text{(II)}$$

I can write:

$$n_1 = \sum_{j=1}^{q} n_{ij} x_j, \; n_2 = \sum_{j=1}^{q} n_{2j} x_j, \; \dots, \; n_h = \sum_{j=1}^{q} n_{hj} x_j.$$

where q is the number of foods considered in the study, $n_{ij}$ ($i = 1 \dots h$, $j = 1 \dots q$) is the amount of nutrient i in one unit of food j, expressed as percentage of the recommended allowance, and $x_j$ is the amount of food j consumed.

The ratio between the increase in the value of function (II) and an infinitesimal increase in the consumption of food j is the partial derivative of S with relation to $x_j$:

$$\frac{\partial S}{\partial x_j} = n_{ij} + I_2 n_{2j} + \ldots + I_h n_{hj}.$$

This is equal to the weighted sum of the nutrients supplied by one unit of food j, when the weights are the intrinsic nutritional importance of the nutrients. The $\max_j \frac{\partial S}{\partial x_j}$ gives the food with the largest nutritional contribution. The consumer should choose this food, if he wants to improve his nutritional status as much as possible, using this weighted sum of the nutrients as the criterion.

d. Other Aspects: The weakness of this measure is that it may consider a nutrient with a very small deficiency more important than a nutrient with a very large deficiency. For example, suppose the consumer considers vitamin A twice as important as ascorbic acid (vitamin C) and he has deficiencies of 10% and 90% of vitamins A and C respectively. He has one dollar and wants to spend it on either of two foods. If he spends the dollar on the first food, he will get 10% of the recommended allowance of vitamin A. If he spends the dollar on the second food, he will get 10% of the recommended allowance of ascorbic acid. The consumer (according to this weighted measure) would choose the first food, although the second food may be more important for his health.

These first two measures, because they do not take into account the amounts of the deficiencies, will lead the consumer always to choose the same food, for a given set of deficient nutrients, although the levels of the deficiencies change. This means that changing the original intake $n^o$ in Figures 1 and 2 does not alter the choice of food A, as long as the consumption of either nutrient 1 or nutrient 2 does not attain the recommended allowance.

In Figure 2, consider the straight lines passing through $n^a$ and $n^b$, making an angle $\Omega$ with the vertical line drawn through $n^o$, as iso-nutrient curves. To maximize the nutritional status as given by these iso-nutrient curves. subject to budget constraint (one dollar, for example), is equivalent to a linear programming problem:

$$\max.\ S' = n_1 + I_2 n_2 - e_1 - I_2 e_2,$$

$$\text{subject to } n_1 = a_1 x_a + b_1 x_b + n_1^o,$$

$$n_2 = a_2 x_a + b_2 x_b + n_2^o,$$

$$n_1 - e_1 \leq 100,$$

$$n_2 - e_2 \leq 100,$$

and

$$p_a x_a + p_b x_b \leq 1,$$

$$x_a,\ x_b,\ n_1,\ n_2,\ e_1,\ e_2 \geq 0,$$

where $p_a$ and $x_a$ are the price and quantity of food A, $p_b$ and $x_b$ are the price and quantity of food B, and $e_1$ and $e_2$ are excess variables for nutrients 1 and 2 respectively, so that the amounts of a nutrient exceeding its recommended allowance (100 percent) is not counted in the objective function $S'$.

In Figure 2, if $n^a$ and $n^b$ represent the expenditure of one dollar on food A and of one dollar on food B, the budget constraint is the straight line from $n^a$ to $n^b$, in terms of the deficient nutrients. The consumer can maximize his nutritional status, as represented by straight lines like the ones from $n^a$ to $n^{a'}$ and from $n^b$ to $n^{b'}$, by attaining the straight line nearest to $n^r$, still being on the budget constraint. Thus, at $n^a$ he achieves this objective. This is a corner solution (he only buys food A). With straight lines as iso-nutrient curves, corner solutions (one food) are obtained from this linear programming problem. (If the straight lines iso-nutrient curves are parallel to the budget constraint line, $n^a n^b$, then any combination of foods A and B, including the corner solutions $n^a$ and $n^b$, maximize the nutritional status of the consumer.)

Measure One presented earlier is equivalent to a similar linear programming problem, only without $I_2$ in $S'$.

Measure Three--The Sum of the Nutrients, Weighted by the Deficiency Level

This measure considers the nutritional contribution of the food as the weighted sum of the deficient nutrients that a food furnishes, where the weights are the deficiencies of the nutrients. The consumer or nutritionists may consider that the larger the amount of the deficiency the more important is the nutrient.

<u>a. Nutritional Aspect</u>: This weighted sum of the deficient nutrients furnished by a food is the same measure as the one proposed by the set of weights (5): all nutrients are equally important and the magnitude of the deficiencies give the weights of the nutrients (p. 26). Increasing the intake of a deficient nutrient by one percent of its recommended allowance is as good as increasing the intake of the other nutrient in an amount given by the ratio of the deficiency of the first over the deficiency of the other nutrient. That is, if the intake of a consumer is deficient by 20% and by 60% of the allowances of nutrients 1 and 2 respectively, then to increase the intake of nutrient 1 by 1% is as good as increasing the intake of nutrient 2 by 1/3%.

If the weighted sums are equal for two foods, the consumer would be indifferent between the two foods. This means that he can substitute one food for the other, leaving his nutritional status unchanged, if the proportion in which he increases the intake of nutrient 1 and diminishes the intake of nutrient 2 is equal to the ratio of the deficiencies: $\frac{\text{deficiency of nutrient 2}}{\text{deficiency of nutrient 1}}$.

Suppose the intake of a consumer is deficient by 20% and by 60% of the allowances of nutrients 1 and 2 respectively. A food furnishes 5% and 2% of the allowances of nutrients 1 and 2 respectively. By this weighted measure, the consumer should be indifferent between this food and another one that supplies 8% and 1% of the allowances of nutrients 1 and 2 respectively.

The nutritional contributions of foods A and B, according to this weighted sum of the nutrients, are:

$$F_a = (100 - n_1)a_1 + (100 - n_2)a_2 \text{ and}$$

$$F_b = (100 - n_1)b_1 + (100 - n_2)b_2,$$

where $(100 - n_1)$ and $(100 - n_2)$ are the magnitudes of the deficiencies, $n_1$ and $n_2$ represent the intake of nutrients 1 and 2, and all four magnitudes are measured as percentages of the recommended allowances. In application, the deficiency weights are based on the intake of the deficient nutrients before the unit of the food is added to the diet. The food with the largest weighted sum is the one with the largest nutritional contribution.

b. Geometrical Aspect: In Figure 3, if the additional consumption consists of food A, this brings the intake to $n_1^a$ and $n_2^a$ of nutrients 1 and 2. If the additional consumption consists of food B, this brings the intake to $n_1^b$ and $n_2^b$ of nutrients 1 and 2. The differences between the new intake levels and the original one are:

$n_1^a - n_1^o = a_1$ and $n_2^a - n_2^o = a_2$, if food A is consumed, and

$n_1^b - n_1^o = b_1$ and $n_2^b - n_2^o = b_2$, if food B is consumed.

In Figure 3, the nutritional contributions of foods A and B, according to this deficiency-weighted sum of the nutrients, are:

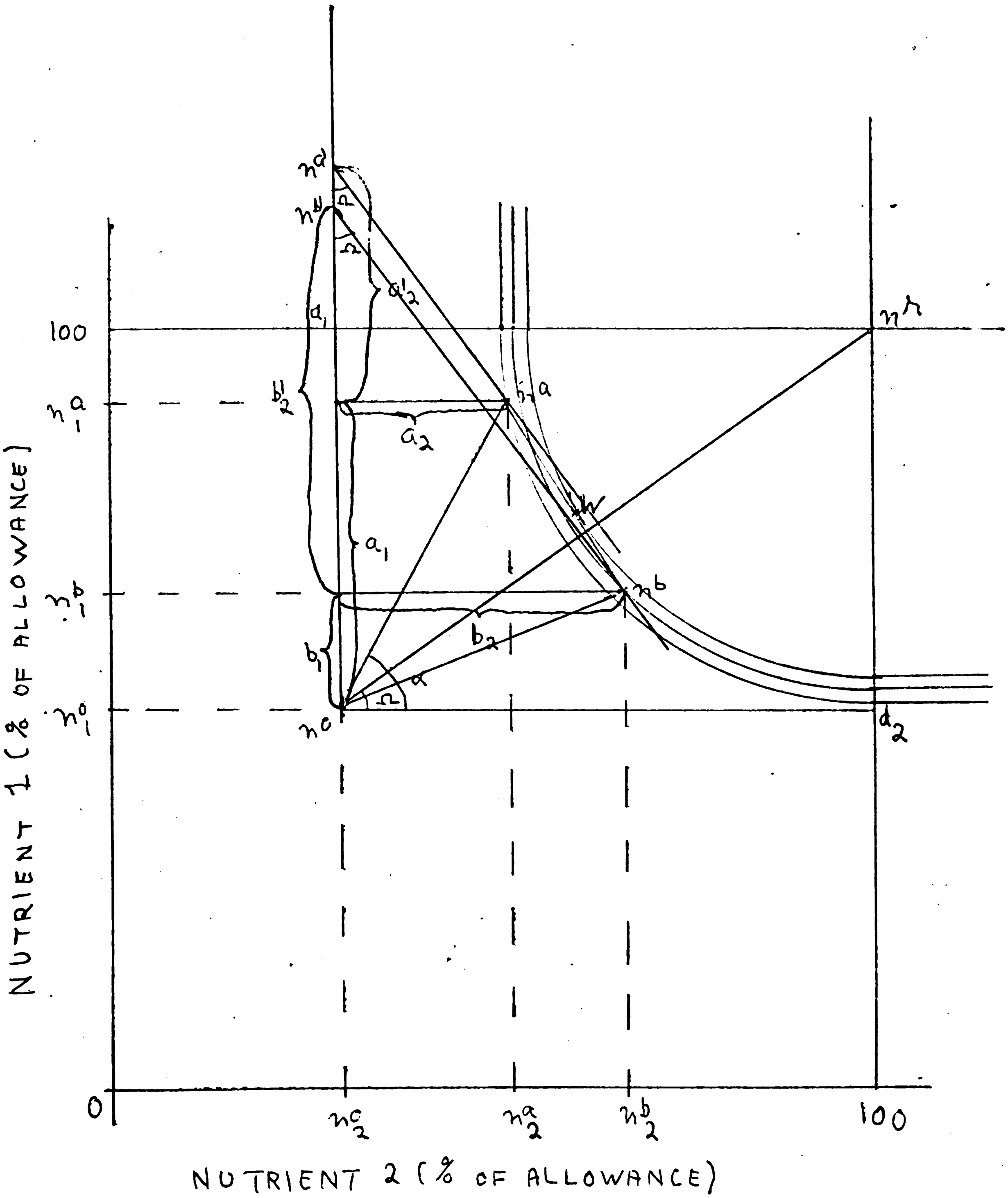


FIGURE III-3

MEASURE THREE – THE SUM OF THE NUTRIENTS, WEIGHTED BY THE DEFICIENCY LEVEL

$$F_a = (100 - n_1^o)a_1 + (100 - n_2^o)a_2 = (100 - n_1^o)\left(a_1 + \frac{100 - n_2^o}{100 - n_1^o}a_2\right)$$

$$= (100 - n_1^o)(a_1 + a_2^{'}) = (100 - n_1^o)|n^{a'} - n^o|, \text{ and}$$

$$F_a = (100 - n_1^o)b_1 + (100 - n_2^o)b_2 = (100 - n_1^o)\left(b_1 + \frac{100 - n_2^o}{100 - n_1^o}b_2\right)$$

$$= (100 - n_1^o)(b_1 + b_2^{'}) = (100 - n_1^o)|n^{b'} - n^o|,$$

where

$$a_2^{'} = \frac{100 - n_2^o}{100 - n_1^o}a_2 \text{ and } b_2^{'} = \frac{100 - n_2^o}{100 - n_1^o}b_2.$$

Here $a_2^{'}$ and $b_2^{'}$ represent the equivalent of $a_2$ and $b_2$ in terms of nutrient 1. Adding $a_2^{'}$ to $a_1$ and $b_1^{'}$ gives the distances $|n^{a'} - n^o|$ and $|n^{b'} - n^o|$. Linking $n^{a'}$ to $n^a$ and $n^{b'}$ to $n^b$ by straight lines forms right triangles. From elementary trigonometry, it follows that:

$$\frac{a_2}{a_2^{'}} = \frac{b_2}{b_2^{'}} = \frac{100 - n_1^o}{100 - n_2^o} = \tan \Omega,$$

where $\Omega$ is the angle that the straight lines from $n^a$ to $n^{a'}$ and from $n^b$ to $n^{b'}$ make with the vertical line drawn through $n^o$.

The line from $n^o$ to $n^r$, which I call the deficiency line, makes the same angle $\Omega$ with the <u>horizontal</u> line drawn through $n^o$. Consider the right triangle $n^o n^r d_2$. From elementary trigonometry, it follows that:

$$\frac{n^r d_2}{n^o d_2} = \frac{100 - n_1^o}{100 - n_2^o} = \tan \Omega,$$

where $\Omega$ is the angle that the deficiency line makes with the horizontal line $n^o d_2$. Thus the deficiency line is perpendicular to the lines that pass through $n^a$ and $n^b$ making the same angle $\Omega$ with the vertical line. Therefore, to draw the lines from $n^a$ to $n^{a'}$ and from $n^b$ to $n^{b'}$, it is enough to take the perpendiculars to the deficiency line passing through $n^a$ and $n^b$.

The food j for which $F_j$ (or $|n^{j'} - n^o|$) is the largest is the one with the largest nutritional contribution according to this measure. Alternatively, the food j that is on the line nearest to $n^r$ that makes an angle $\Omega$ with the vertical line is the one with the largest nutritional contribution by the deficiency-weighted sum of the nutrients.

The consumer will be indifferent between spending one dollar on food A or on any other food that would bring his intake of the two nutrients to the straight line passing through $n^a$ that makes an angle $\Omega$ with the vertical line drawn through $n^o$.

Note that when the original intake of the nutrients changes, the ratio of the deficiencies may change; if so, these measures of the nutritional contributions of the foods also change.

c. Analytical Aspect: The change of the nutritional status of a consumer, when he considers this weighted sum of the nutrients as the measure of the change and he has only two deficiencies, is given by:

$$dS = (100 - n_1)dn_1 + (100 - n_2)dn_2,$$

where dS, $dn_1$ and $dn_2$ are the changes in the nutritional status and in the intake of nutrients 1 and 2.

Solving this differential equation gives:

$$S - 100n_1 - \frac{1}{2}n_1^2 + 100n_2 = \frac{1}{2}n_2^2 + C,$$

where S is the nutritional status function implied by this measure and C is a constant of integration. Take

$$C = (m - 2)5000,$$

where m is the total number of nutrients and 5000 indicates an intake that is equal to or greater than the recommended allowance ($100 \times 100 - \frac{1}{2}100^2 = 5000$). Then C represents the value that the consumer gives to the intake of non-deficient nutrients. I obtain:

$$S = 100n - \frac{1}{2}n + 100 - \frac{1}{2}n + (m - 2)5000.$$

Generalizing this result for h deficient nutrients, I get[7]:

$$S = (100n_1 - \frac{1}{2}n_1^2) + (100n_2 - \frac{1}{2}n_2^2) +...+ (100n_h - \frac{1}{2}n_h^2)+(m-h)5000, \quad \text{(III)}$$

[7]P. A. Samuelson, Foundations of Economic Analysis, Harvard University Press, Cambridge, 1955, in p. 93, credits Gossen, as early as 1854, with the following utility function:

$$U = K + (A_1x_1 - b_1x_1^2) + (a_2x_2 - b_2x_2^2) + ....$$

where the x's are the amounts of commodities consumed.

I can write:

$$n_1 = \sum_{j=1}^{q} n_{ij}x_j, \; n_2 = \sum_{j=1}^{q} n_{2j}x_j, \; \ldots., \; n_h = \sum_{j=1}^{q} n_{hj}x_j,$$

where q is the number of foods considered in the study, $n_{ij}$ ($i = 1 \ldots h$, $j = 1 \ldots q$) is the amount of nutrient i in one unit of food j, expressed as percentage of the recommended allowance, and $x_j$ is the amount of food j consumed.

The ratio between the increase in the value of the function (III) and an infinitesimal increase in the consumption of food j is the partial derivative of S with relation to $x_j$:

$$\frac{\partial S}{\partial x_j} = (100 - n_1)n_{1j} + (100 - n_2)n_{2j} + \ldots + (100 - n_h)n_{hj}.$$

This is equal to the weighted sum of the nutrients supplied by one unit of food j, when the weights are the magnitudes of the deficiencies. The $\max_j \frac{\partial S}{\partial x_j}$ gives the food with the largest nutritional contribution. The consumer should choose this food, if he wants to improve his nutritional status as much as possible, using this weighted sum of the nutrients as the criterion.

<u>d. Other Aspects</u>: Nutritional status function (III), in the space of the deficient nutrients (see Figure 3), gives iso-nutrient curves that are circles with their center at the point where the intake is equal to the recommended

allowance.[8] The closer is the circle to the center, the greater the value of the nutritional status function. The iso-nutrient curves are the portions of the circles contained in the rectangle given by $n^1 d_1 n^r d_2$. Outside the sides $d_1 n^r$ they become horizontal and vertical lines. At $n^r$ the iso-nutrient curve is the right angle curve.

Because the tangent to a circle is perpendicular to the radius and the straight lines making an angle $\Omega$ with the vertical line drawn through $n^o$ are perpendicular to the deficiency line (p. 48). These straight lines are tangent to the circles representing the indifference curves at the intersections of the circles with the deficiency line. Changing the origin ($n^o$) of the deficiency line may change the

---

[8]The nutritional status function (III) is closely related to the circles that I considered as iso-nutrient curves. Take R as the radius and the center at the point where the intake of the deficient nutrients is equal to the recommended allowances, expressed as percentages of the allowances. Then

$$R^2 = (100 - n_1)^2 + \dots. + (100 - n_2)^2$$

$$= h(10000) - (200n_1 - n_1^2) - \dots - (200n_h - n_h^2)$$

$$\frac{h(10000) - R^2}{2} = (100n_1 - \tfrac{1}{2}n_1^2) + \dots. + (100n_h - \tfrac{1}{2}n_h^2).$$

Note that

$$\frac{h(10000) - R^2}{2} = S,$$

the nutritional status function (III) associated with the deficiency-weighted measure.

slope of the deficiency line. If so, the tangent lines are also changed.

Consider the tangent lines as linear approximations to the iso-nutrient curves. To maximize the nutritional status as given by those straight lines, subject to a budget constraint (one dollar, for example) is equivalent to the following linear programming problem:

$$\max.\ S' = (100-n_1^o)n_1 + (100-n_2^o)n_2 - (100-n_1^o)e_1 - (100-n_2^o)e_2,$$

subject to $n_1 = a_1x_a + b_1x_b + n_1^o,$

$$n_2 = a_2x_a + b_2x_b + n_2^o,$$

$$n_1 - e_1 \leqslant 100$$

$$n_2 - e_2 \leqslant 100$$

and

$$p_ax_a + p_bx_b \leqslant 1,$$

$$x_a,\ x_b,\ b_k,\ b_2,\ e_1,\ e_2 \geqslant 0.$$

In Figure 3, the solution of this problem is $n^a$. The consumer should spend his dollar on food A only.

However, when the deficiency weights are allowed to vary continuously, the nutritional status function is quadratic, leading to iso-nutrient curves that are circles with center at $n^r$. The consumer is now faced with the following quadratic programming problem:

$$\max.\ S = 100(n_1-e_1) - \frac{1}{2}(n_1-e_1)^2 + 100(n_2-e_2) - \frac{1}{2}(n_2-e_2)^2,$$

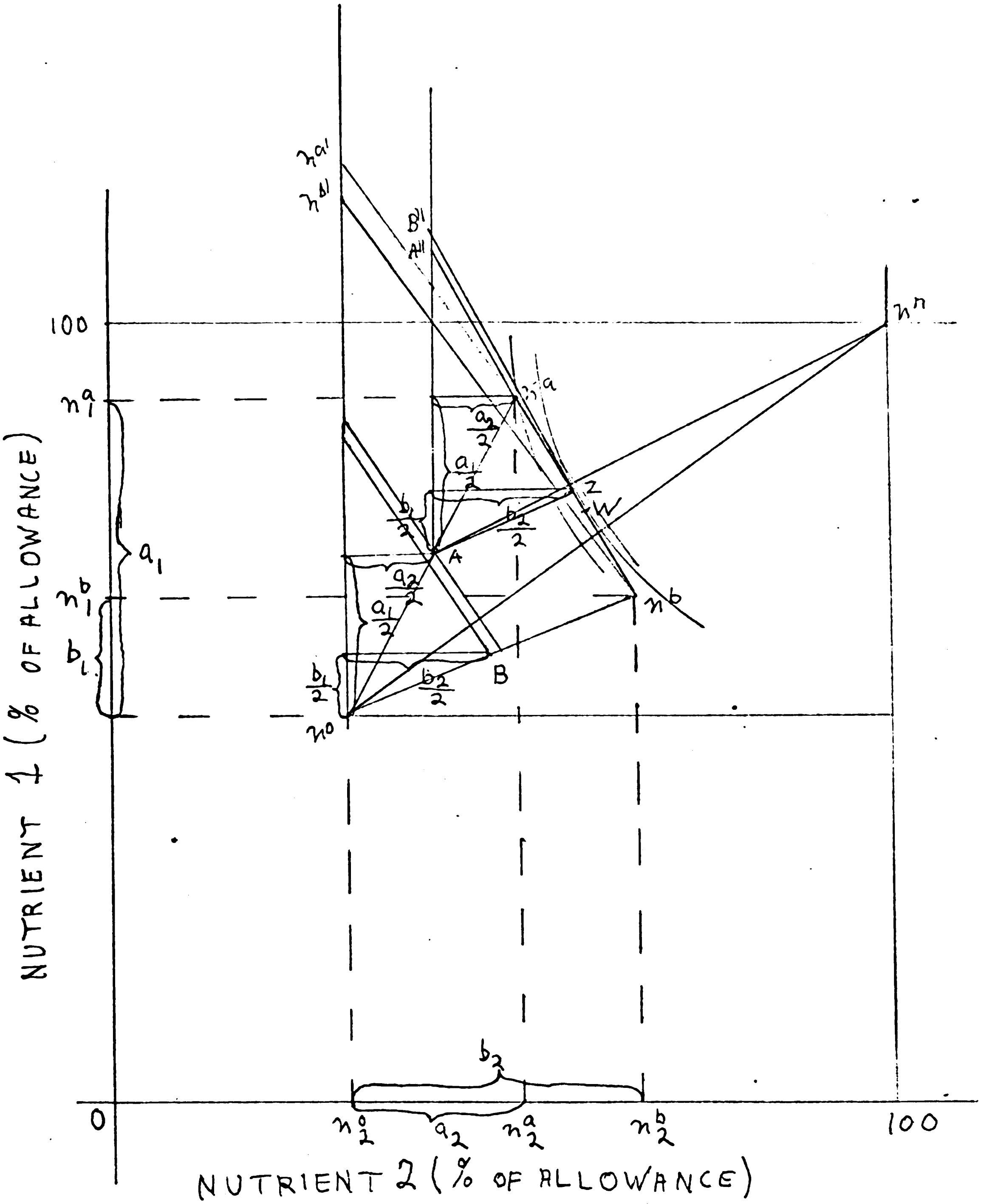


FIGURE III-3A

New Method for Selecting Economical Supplementary Diet subject to $n_1 = a_1x_a + b_1x_b + n_1^o$,

$$n_2 = a_2x_a + b_2x_b + n_2^o,$$

$$n_1 - e_1 \leqslant 100,$$

$$n_2 - e_2 \leqslant 100,$$

and

$$p_ax_a + p_bx_b \leqslant 1,$$

$$x_a,\ x_b,\ n_1,\ n_2,\ e_1,\ e_2 \geqslant 0.$$

(in both problems the variables are the same as in the linear programming problem presented earlier on pp. 41, 42 and 43.)

In Figure 3, the solution of the quadratic problem is W. The consumer should spend his dollar on foods A and B.

In the linear programming problem the deficiency weights are considered as constant, while in the quadratic programming problem, they are continuously varying with the current intake. This accounts for the two different problems. However, when the original intake is allowed to vary in finite steps, the linear programming problem can be used in a new method that gives the solution of the quadratic problem.

In Figure 3A, suppose that the consumer decides to spend fifty cents, and then the remaining fifty cents. By the linear programming problem, the consumer should spend his first fifty cents of food A, bringing his intake to A.

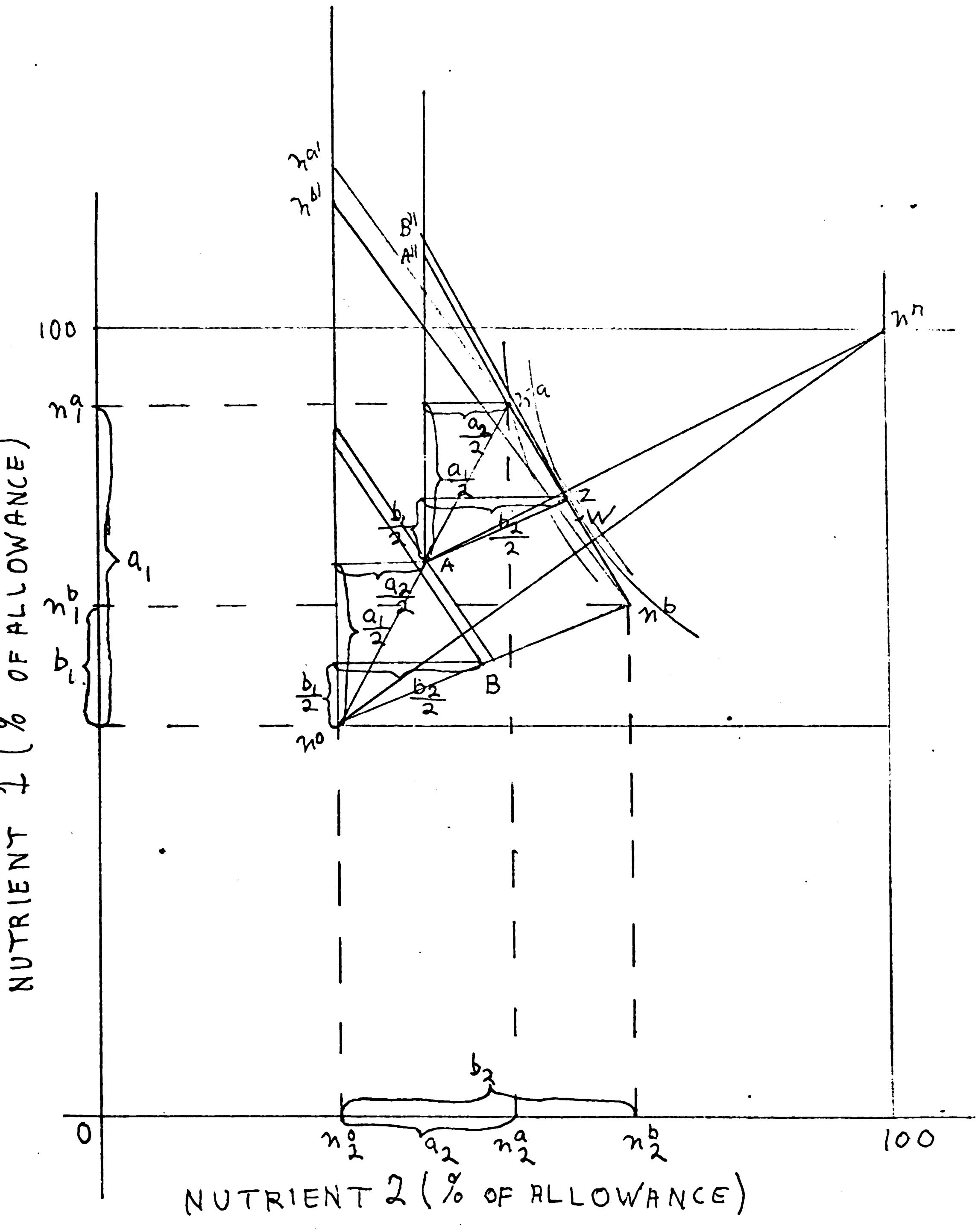


FIGURE III-3A

New Method for Selecting Economical Supplementary Diet

This changes his existing intake of the two nutrients from $n^o$ to A. Because of the new deficiency weights at A, the linear approximations of the iso-nutrient curves are now the lines $n^aA''$ and $ZB''$. The consumer should spend the remaining fifty cents on food B, bringing his intake to Z. (The distances from A to Z and from $n^o$ to B are equal and AZ is parallel to $n^oB$.) If the consumer were to spend his dollar in only one incremental step, the solution would be $n^a$. The two step solution Z is closer to W, the solution when the circles are the iso-nutrient curves, than the one step solution $n^a$. The iso-nutrient curve (circle) passing through Z is just outside the one passing through W (at this point, the iso-nutrient curve is tangent to the budget constraint) while the one passing through $n^a$ is considerably farther removed.[9] In Chapter V, I shall fully develop this new method.

## Measure Four--The Sum of the Nutrients, Weighted by the Intrinsic Importance and the Deficiency Level

This measure combines the weights used for the two previous measures to form new weights for the nutrients that a food furnishes.

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[9] If a portion of the line $n^an^b$ lies in the Northeast quadrant with origin at $n^r$, then linear programming gives the minimum cost diet that eliminates the nutritional deficiencies. Using the method suggested in this thesis an economical diet can be selected that eliminates the nutritional deficiencies (see Chapter V).

a. Nutritional Aspect: This weighted sum of the nutrients that a food supplies is the same measure as the one proposed by the set of weights (6): nutrient i is $I_i$ times as important as nutrient 1, the importance of nutrient 1 is assumed to be 1 and the magnitudes of the deficiencies also count (p. 26). Increasing the intake of deficient nutrient i by one percent of its recommended allowance is as good as increasing the intake of nutrient 1 by the ratio:

$$\frac{\text{intrinsic importance x deficiency of nutrient i}}{\text{deficiency of nutrient 1}}.$$

If the weighted sums are equal for two foods, the consumer should be indifferent between the two foods. This means that he can substitute one food for the other leaving his nutritional status unchanged, if the proportion in which he increases the intake of nutrient 1 and diminishes the intake of nutrient 2 is equal to:

$\frac{I_2 \text{ x deficiency of nutrient 2}}{\text{deficiency of nutrient 1}}$, where $I_2$ is the intrinsic nutritional importance of nutrient 2.

Suppose nutrient 2 is 5 times as important as nutrient 1. The intake of the consumer is deficient by 20% and by 60% of nutrients 1 and 2 respectively. A food furnishes 5% and 2% of the recommended allowances of nutrients 1 and 2 respectively. By this weighted sum of the nutrients, the consumer should be indifferent between choosing this food and another one that supplies 20% and 1% of the allowances of nutrients 1 and 2.

The nutritional contributions of foods A and B, according to this weighted sum of the nutrients, are:

$$F_a = (100 - n_1)a_1 + I_2(100 - n_2)a_2 \text{ and}$$

$$F_b = (100 - n_1)b_1 + I_2(100 - n_2)b_2,$$

where $(100 - n_1)$ and $(100 - n_2)$ are the amounts of the deficiencies, $n_1$ and $n_2$ are the intake levels of nutrients 1 and 2, and the four magnitudes are expressed as percentages of the recommended allowances. In application, the deficiency weights are based on the intake of the deficient nutrient before the unit of food is added to the diet. The food with the largest weighted sum is the one with the largest nutritional contribution.

b. Geometrical Aspect: In Figure 4, if the additional consumption consists of food A, this brings the intake to $n_1^a$ and $n_2^a$ of nutrients 1 and 2. If the additional consumption consists of food B, this brings the intake to $n_1^b$ and $n_2^b$ of nutrients 1 and 2. The differences between the potential intake and the original one are:

$n_1^a - n_1^b = a_1$ and $n_2^a - n_2^o = a_2$, if food A is consumed, and

$n_1^b - n_1^o = b_1$ and $n_2^b - n_2^o = b_2$, if food B is consumed.

In Figure 4, the nutritional contributions of foods A and B, according to this weighted sum of the nutrients, are:

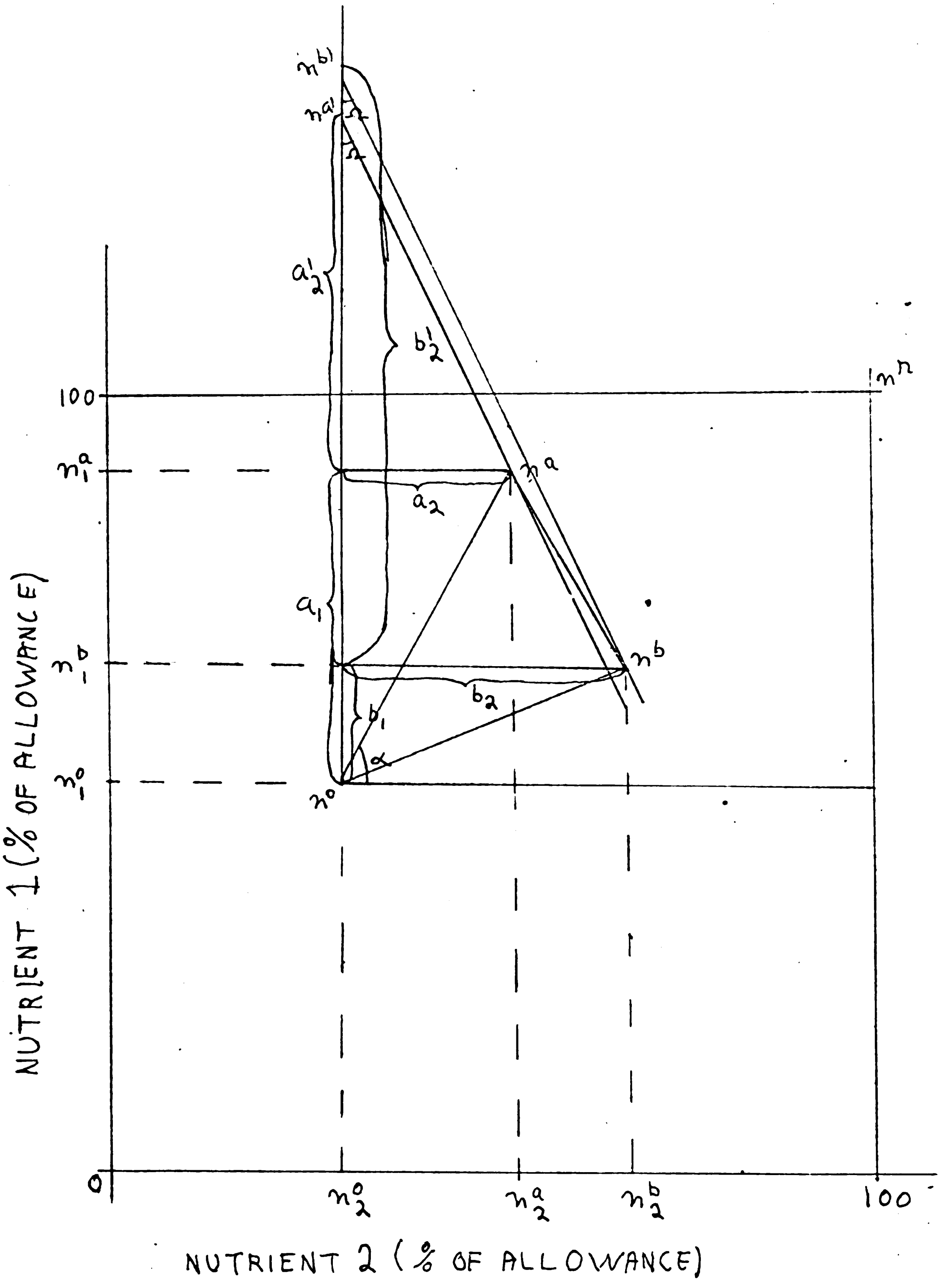


FIGURE III-4

MEASURE FOUR – THE SUM OF NUTRIENTS, WEIGHTED BY THE INTRINSIC IMPORTANCE AND BY THE DEFICIENCY LEVEL

$$F_a = (100-n_1^o)a_1 + I_2(100-n_2^o)a_2 = (100-n_1^o)\left(a_1+\frac{I_2(100-n_2^o)}{100-n_1^o}a_2\right)$$

$$= (100 - n_1^o)(a_1 + a_2') = (100 - n_1^o)\,|n^{a'} - n^a| \text{ and}$$

$$F_b = (100-n_1^o)b_1 + I_2(100-n_2^o)b_2 = (100-n_1^o)\left(b_1+\frac{I_2(100-n_2^o)}{100-n_1^o}b_2\right)$$

$$= (100 - n_1^o)(b_1 + b_2') = (100 - n_1^o)\,|n^{b'} - n^o|,$$

where

$$a_2' = \frac{I_2(100 - n_2^o)}{100 - n_1^o}a_2 \text{ and } b_2' = \frac{I_2(100 - n_2^o)}{100 - n_1^o}b_2.$$

Here $a_2'$ and $b_2'$ represents the equivalent of $a_2$ and $b_2$ in terms of nutrient 1. Adding $a_2'$ to $a_1$ and $b_2'$ to $b_1$ gives the distances $|n^{a'} - n^o|$ and $|n^{b'} - n^o|$. Linking $n^a$ to $n^{a'}$ and $n^b$ to $n^{b'}$ forms right triangles. From elementary trigonometry, it follows that:

$$\frac{a_2}{a_2'} = \frac{b_2}{b_2'} = \frac{100 - n_1^o}{I_2(100 - n_2^o)} = \tan\Omega,$$

where $\Omega$ is the angle that the straight lines from $n^a$ to $n^{a'}$ and from $n^b$ to $n^{b'}$ make with the vertical line drawn through $n^o$. For a given $I_2$ the angle $\Omega$ varies with the ratio of the deficiencies.

The food j for which $F_j$ (or $|n^{j'} - n^o|$) is the largest is the one with the largest nutritional contribution according to this measure. Alternatively, the food j that is on the line nearest to $n^r$ that makes an angle $\Omega$ with the largest nutritional contribution by this weighted sum of the nutrients.

The consumer will be indifferent between spending one dollar on food A or on any other food that would bring his intake of the two nutrients to the straight line passing through $n^a$ that making an angle $\Omega$ with the vertical line drawn through $n^o$.

The diagrammatic presentation of this fourth measure would be the same as that of Measure Three, if the distance on the horizontal axis that measures one unit of nutrient 2 (one percent of its allowance) is $I_2$ times as large as the distance on the vertical axis that measures one unit of nutrient 1 (one percent of its allowance).

c. Analytical Aspect: The change of the nutritional status of a consumer, when he considers this weighted sum of the nutrients as the measure of the change and he has only two deficiencies, is given by:

$$dS = (100 - n_1)dn_1 + I_2(100 - n_2)dn_2,$$

where $dS$, $dn_1$ and $dn_2$ are the changes in the nutritional status and in the levels of intake of nutrients 1 and 2.

Solving this differential equation gives:

$$S = (100n_1 - \tfrac{1}{2}n_1^2) + I_2(100n_2 - \tfrac{1}{2}n_2^2) + C,$$

where S is the nutritional status function implied by this measure and C is a constant of integration. Take

$$C - (m - 2)5000,$$

where m is the total number of nutrients and 5000 indicates an intake that is equal to or greater than the recommended allowance ($100 \times 100 - \frac{1}{2}100^2 = 5000$). Then C represents the value that the consumer gives to the intake of non-deficient nutrients. I obtain:

$$S = (100n_1 - \frac{1}{2}n_1^2) + I_2(100n_2 - \frac{1}{2}n_2^2) + (m - 2)5000.$$

Generalizing this result for h deficient nutrients, I get:

$$S = (100n_1-\frac{1}{2}n_1^2) + I_2(100n_2-\frac{1}{2}n_2^2) +\ldots+ I_h(100n_h-\frac{1}{2}n_h^2) + (m-h)5000, \tag{IV}$$

I can write:

$$n_1 = \sum_{j=1}^{q} n_{1j}x_j,\ n_2 = \sum_{j=1}^{q} n_{2j}x_j, \ldots,\ n_h = \sum_{j=1}^{q} n_{hj}x_j,$$

where q is the number of foods considered in the study, $n_{ij}$ ($i = 1 \ldots h$, $j = 1 \ldots q$) is the amount of nutrient i in one unit of food j, expressed as percentage of the recommended allowance, and $x_j$ is the amount of food j consumed.

The ratio between an increase in the value of the function (IV) and an infinitesimal increase in the consumption of food j is the partial derivative of S with relation to $x_j$:

$$\frac{\partial S}{\partial x_j} = (100 - n_1)n_{1j} + I_2(100 - n_2)n_{2j} +\ldots+ I_h(100 - n_h)n_{hj}.$$

This is equal to the weighted sum of the nutrients supplied by one unit of food j, when there is one weight for the intrinsic importance and the magnitude of the deficiency serves as another weight. The $\max_j \frac{\partial S}{\partial x_j}$ gives the food with the largest nutritional contribution. The consumer should choose this food if he wants to improve his nutritional status as much as possible, using this weighted sum of the nutrients as the criterion.

d. Other Aspects: This measure should be applied if the consumer of nutritionists think that one nutrient is more important than the other, independent of the intake levels of these nutrients.

There are also linear and quadratic programming problems that express this fourth measure. The only difference between this case and the one in Measure Three (p. 52-54) is that here a constant $I_2$ is included in the objective functions.

## C. Directional Derivatives[10]

There is an alternative way to derive the measures proposed. Using the same nutritional status functions, I can obtain the directional derivatives of these functions and relate them to the proposed measures.

Take food A in Figures 1, 2, 3 and 4. Consumption of food A represents moving in the direction given by angle

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[10]I thank Prof. T. R. Saving for suggesting to me the relationship of these measures to the directional derivatives. See W. Kaplan, Advanced Calculus, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1959, pp. 107-110 for the mathematical background of the directional derivative.

$\alpha$ that the line showing the additional consumption of food A makes with the horizontal line drawn through $n^o$. The directional derivative gives the ratio between the change of the value of the function and the change in the independent variables (nutrients 1 and 2).

The directional derivative of the function $S = S(n_i, n_2)$, in the direction given by the consumption of food A, is:

$$V_a S = \frac{\partial S}{\partial n_1} \cos \alpha + \frac{\partial S}{\partial n_2} \sin \alpha.$$

For the four specific functions, when there are only two deficiencies, the directional derivatives in the direction given by the consumption of food A, are:

$$(I') \quad V_a S = \frac{a_1}{\sqrt{a_1^2 + a_2^2}} + \frac{a_2}{\sqrt{a_1^2 + a_2^2}}$$

$$\left(\sqrt{a_1^2 + a_2^2}\right) V_a S = a_1 + a_2 = F_a,$$ according to Measure One;

$$(II') \quad V_a S = \frac{a_1}{\sqrt{a_1^2 + a_2^2}} + I_2 \frac{a_2}{\sqrt{a_1^2 + a_2^2}}$$

$$\left(\sqrt{a_1^2 + a_2^2}\right) V_a S = a_1 + I_2 a_2 + F_a,$$ according to Measure Two;

$$(III') \quad V_a S = \frac{(100 - n_1) a_1}{\sqrt{a_1^2 + a_2^2}} + \frac{(100 - n_2) a_2}{\sqrt{a_1^2 + a_2^2}}$$

$$\left(\sqrt{a_1^2 + a_2^2}\right) V_a S = (100 - n_1)a_1 + (100 - n_2)a_2 = F_a,$$

according to Measure Three, and

$$(IV') \quad V_a S = \frac{(100 - n_1)a_1}{\sqrt{a_1^2 + a_2^2}} + I_2 \frac{(100 - n_2)a_2}{\sqrt{a_1^2 + a_2^2}}$$

$$\left(\sqrt{a_1^2 + a_2^2}\right) V_a S = (100 - n_1)a_1 + I_2(100 - n_2)a_2 = F_a,$$

according to Measure Four.

The square root term $\sqrt{a_1^2 + a_2^2}$ measures the distance between the original intake and the intake after consuming one unit of food A. The larger is the unit of food A, the greater is the square root, hence, the distance. The directional derivative evaluates the proportion in which the food supplies the essential nutrients. This is independent of the unit by which the foods are measured.

For h deficient nutrients and the general nutritional status function $S = S(n_1 \ldots . n_h)$, the directional derivative in the direction v, is:

$$V_v S = \frac{\partial S}{\partial n_1} \cos \alpha + \ldots . + \frac{\partial S}{\partial n_h} \cos \eta,$$

where $\alpha \ldots \eta$ are the angles that the direction v makes with the $n_1 \ldots . n_h$ axes. If the consumption of food j represents moving in the direction given by these angles, then the directional derivative becomes:

$$V_j S = \frac{\partial S}{\partial n_1} \frac{n_{ij}}{\sqrt{n_{1j}^2 + \ldots + n_{hj}^2}} + \ldots + \frac{\partial S}{\partial n_h} \frac{n_{hj}}{\sqrt{n_{1j}^2 + \ldots + n_{hj}^2}}.$$

The general measure of the nutritional contribution of food j, $F_j$, is:

$$\left(\sqrt{n_{1j}^2 + \ldots + n_{hj}^2}\right) V_j S = \frac{\partial S}{\partial n_1} n_{1j} + \ldots + \frac{\partial S}{\partial n_h} n_{hj} = F_j .$$

Consider the general nutritional status function $S = S(n_1 \ldots n_h)$, where

$$n_1 = \sum_{j=1}^{q} n_{1j} x_j , \; \ldots . , \; n_h = \sum_{j=1}^{q} n_{hj} x_j . \qquad \text{(A)}$$

Taking the partial derivative of S with relation to $x_j$ gives:

$$\frac{\partial S}{\partial x_j} = \frac{\partial S}{\partial n_1} n_{1j} + \ldots + \frac{\partial S}{\partial n_h} n_{hj} = F_j ,$$

the same measure as the one above.

If condition (A) does not hold, but

$$n_1 = n_1(x_1 \ldots x_q), \; \ldots, \; n_h = n_h(x_1 \ldots x_q), \qquad \text{(B)}$$

then the total intake of a nutrient is not necessarily equal to the sum of the amounts of the nutrients furnished by the foods consumed. In this more general case, the measures derived from the general nutritional status function $S = S(n_1 \ldots . n_h)$, either by using the directional derivative or by using the partial derivative, are equal if the angles that form the direction given by the consumption of food j are:

$$\cos\alpha = \frac{\frac{\partial n_1}{\partial x_j}}{\sqrt{\left(\frac{\partial n_1}{\partial x_j}\right)^2 + \ldots + \left(\frac{\partial n_h}{\partial x_j}\right)^2}}, \ldots, \cos\eta = \frac{\frac{\partial n_h}{\partial x_j}}{\sqrt{\left(\frac{\partial n_1}{\partial x_j}\right)^2 + \ldots + \left(\frac{\partial n_h}{\partial x_j}\right)^2}}$$

Thus it is possible to write:

$$\left(\sqrt{\left(\frac{\partial n_1}{\partial x_j}\right)^2 + \ldots + \left(\frac{\partial n_h}{\partial x_j}\right)^2}\right) V_j S = \frac{\partial S}{\partial n_1}\frac{\partial n_1}{\partial x_j} + \ldots + \frac{\partial S}{\partial n_h}\frac{\partial n_h}{\partial x_j} = F_j$$

$$\frac{\partial S}{\partial x_j} = \frac{\partial S}{\partial n_1}\frac{\partial n_1}{\partial x_j} + \ldots + \frac{\partial S}{\partial n_h}\frac{\partial n_h}{\partial x_j} = F_j.$$

These two last expressions can represent any complicated relationship among the intake of the nutrients. Although, the analysis in this thesis uses the simple model given in p. 22 (a model widely employed in practical work), it is known that much more complicated relationships exist. For example, the requirement for thiamin depende upon the intake of calories. Likewise, a negative $\frac{\partial n_i}{\partial x_j}$ exists for calcium with respect to the consumption of spinach, because the consumption of this food has a negative effect on the utilization of calcium by the human body.

## CHAPTER IV

## RECENT DEVELOPMENT IN CONSUMPTION THEORY AND THE NUTRITIONAL CONTRIBUTION OF FOOD

In 1960[1] and in 1964[2] V. E. Smith presented a new development in consumption theory. He based his studies on the application of linear programming to human diets. Using the same tools as Smith, Lancaster,[3] in 1966, did a theoretical investigation of consumer behavior. I shall relate their works to the nutritional status functions proposed in Chapter III.

I shall base my presentation of Smith's works on his latter study (1964). I shall adopt the nutritional status function (III) derived from the deficiency-weighted measure (Measure Three) in this Chapter, because the two

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[1]V. E. Smith, "Measurement of Product Attributes Recognized by Consumers," in Center for Agricultural and Economic Adjustment, College of Agriculture, Iowa State University of Science and Technology, Seminar on Consumer Preferences and Market Development for Farm Products, CAEA Report 5, Ames, Iowa, 1960, pp. 1-27.

[2]V. E. Smith, Electronic Computation of Human Diets, MSU Business Studies, Bureau of Business and Economic Research, Graduate School of Business Administration, Michigan State University, East Lansing, 1964.

[3]K. J. Lancaster, "A New Approach to Consumer Theory," Journal of Political Economy, Vol. LXXIV, April, 1966, No. 2, pp. 132-157.

first functions are very simple (linear) and the fourth one can be digrammatically treated as function (III), as I pointed out in Chapter III (p.   ).

A. V. E. Smith

Smith (Chapter VIII, pp. 136-143) says that the conventional economic theory assumes that the utility obtained by a household depends upon the amounts of commodities consumed. If U* is the index of utility and $x_1,\ldots,x_j,\ldots,x_q$ are the quantities of commodities $1\ldots j\ldots q$ consumed by the household, then

$$U^* = U^*(x_1\ldots x_j\ldots x_q).$$

With the budget constraint this is the conventional treatment of consumption theory using indifference curves.

In his new approach Smith considers that consumption goods are "bundles of separate goal-satisfying attributes" (p. 137). Then "the index of utility U is a single-valued function of the levels of attainment of a set of goals or objectives.

$$(1) \quad U = f(b_1\ldots b_i\ldots b_m),$$

where the $b_i$ are variables representing the levels of attainment of the i goals which are important for the individual." (p. 139)

The levels of the attributes depend upon the quantities of commodities consumed:

$$(2) \quad b_i = g_i(x_1\ldots x_j\ldots x_q).$$

Substituting (2) into (1) he gets the traditional utility function,

(3) $U = w(x_1 \ldots . x_j \ldots . x_q)$.

Specifying a certain level of utility of (1), $U = U_k$, where

$U_k = f(b_1^k \ldots . b_j^k \ldots . b_m^k)$

makes the $b_i$ variables to be constant. "Each such set of $b_i$ corresponds to a particular level of utility. (Several such sets may correspond to the same utility level. If $b_i^k$, for instance, provides satiation of the $i^{th}$ goal, increases in its value will not alter the utility level as long as ever-attainment involves no loss of satisfactions.)" (p. 139)

The problem is to economize within the indifference region where $U = U_k$. I shall quote the procedure and the figure (pp. 140-141):

> The diet problem in linear programming is exactly this kind of analysis of the problem of minimizing the cost of attaining specified levels of the various objectives, that is, a specified level of preference. Consider a case in which there are two commodities and three nutritional goals to be attained at specified levels. In Figure 1 [Figure 2 in the original], OA represents the quantity of commodity I needed if the specific caloric level is to be attained by the consumption of I alone: OB is the quantity of commodity II required for this purpose if only II is to be consumed. If less of I is taken, the deficiency in calories may be made up by adding II, at a constant rate determined by the ratio of the caloric content of commodity II to that of commodity I. The slope of the line AB measures this substitution ratio. Any combination of foods I and II plotted along line AB will satisfy the caloric requirement. Similarly, the

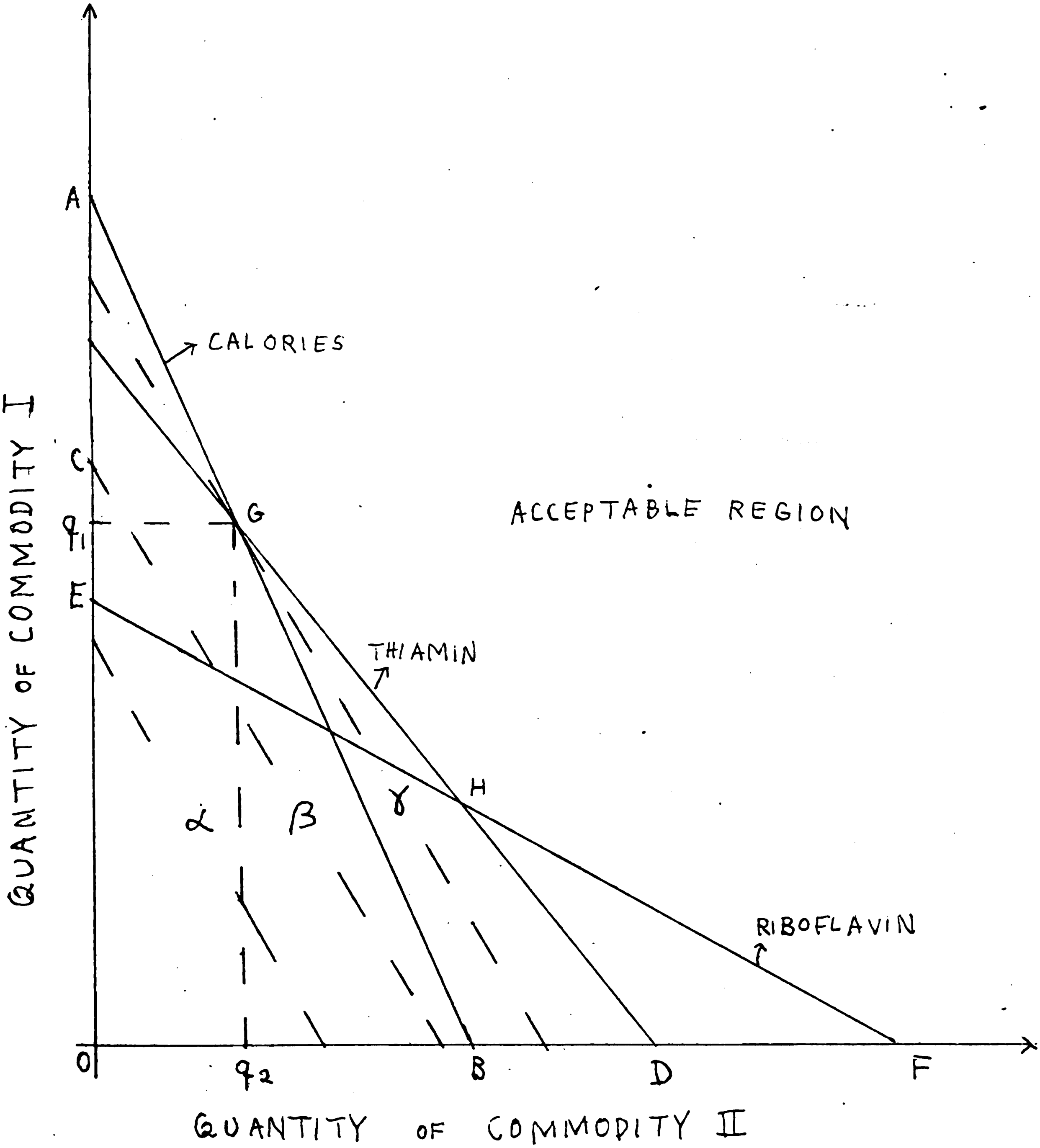


FIGURE IV-1

LINEAR PROGRAMMING ANALYSIS

thiamin level desired ($b_2^k$) can be attained by any combination of foods along line CD, and the riboflavin level ($b_3^k$) by any combination of foods along line EF.

If the utility obtained from one nutritional element is independent of the quantities of the other elements present, any point along AB represents a given level of utility obtained from calories, any point along CD represents a given level of utility from thiamin, and any point along EF represents a given level of utility from riboflavin. The conventional programming requirement that each of these three levels be equalled or exceeded rules out any possibility of substituting one goal for another--that is, of replacing deficiencies in one nutrient by excesses of another. Thus the only acceptable points taking all three goals into account, are those on or above AGHF, in the shaded region. Any such point provides a level of utility at least equal to $U_k = U(b_1^k) + U(b_2^k) + U(b_3^k)$. The combinations of foods along AG provide thiamin and riboflavin in excess of the $b_i^k$ levels, but the conventional linear programming procedure regards any such excesses as irrelevant (possessing zero marginal utilities). Thus curve AGHF can be regarded as the lower boundary of an indifferent region within which any combination of commodities is an equally satisfactory means of attaining the specified $b_i^k$ levels of these goals.

The programming problem is to choose the acceptable commodity set which requires the least expenditure. For positive prices of both commodities, the least-cost combination must lie on the curve AGHF, for all other acceptable points involve larger quantities of at least one commodity. The dotted lines $\alpha$, $\beta$ and $\gamma$ in Figure 1 [Figure 2 in the original] show the combinations of commodities I and II which can be purchased for dollar expenditures of $\alpha$, $\beta$ and $\gamma$, respectively. It is obvious to the eye that $0q_1$ of commodity I and $0q_2$ of commodity II constitute the least-cost combination. Changes in the prices of the foods, by changing the slopes of $\alpha$, $\beta$ and $\gamma$, may alter the optimal combinations of foods.

In more recent work (not yet published) Smith uses linear programming to find the least-cost diet that supplements the deficient nutritional intake of Colombian families. (The right hand side values of the constraints of the least-cost diet are the amounts of the deficiencies.) Nevertheless, this assumes that the families can afford the added expenditure and this may not necessarily happen.

## B. K. Lancaster

In 1966, Lancaster proposed a similar analytical apparatus to explain consumer bahavior. I shall base my argument on his "simplified model" (p. 136). For Lancaster the consumer tries to maximize a utility index that is a function of the characteristics of commodities, subject to certain constraints. Lancaster's commodity characteristics are equivalent to Smith's attributes of consumer goods. Lancaster's theory states:

$$\begin{aligned} &\text{max.} && U(z) \\ &\text{subject to} && px \leq k \\ &\text{with} && z = Bx \\ &\text{and} && z, x \geq 0, \end{aligned}$$

where U(z) gives the utility index as a function of the characteristics, $z = (z_1 \ldots . z_i \ldots . z_m)$ is a vector that shows the amounts of characteristics consumed, $z_i$ is the same as $b_i$ in Smith's notation, $p = (p_1 \ldots p_j \ldots p_q)$ is a vector with the prices of the q commodities, finally, B is the matrix that presents the "consumption technology," such that any element $b_{ij}$ represents the amount of characteristic i in one unity of commodity j.

For this thesis, only foods enter into the list of consumption goods and only deficient nutrients enter into the list of characteristics provided by the foods. This means that $b_{ij} = n_{ij}$ or the amount of nutrient i in one unit of food j, expressed as a percentage of the recommended allowance. As a matter of fact, the number of foods is larger than the number of nutrients for which nutrietionists suggest allowances. This means that q is larger than h, the number of deficient nutrients. In consequence, I shall use part 3 of Lancaster's section "The Structure of the Consumption Technology" (pp. 139-140.)

I shall quote his analysis (p. 139):

> Here, the consumption technology, $z = Bx$, has fewer equations than variables so that, for every characteristics vector there is more than one goods vector. For every point in his characteristics-space, the consumer has a choice between different goods vectors. Given a price vector, this choice is a pure efficiency choice, so that for every characteristics vector the consumer will choose the most efficient combination of goods to achieve that collection of characteristics, and the efficiency criterion will be minimum cost.
>
> The efficiency choice for a characteristics vector $z^*$ will be the solution of the canonical linear program
>
> Minimize $px$
> subject to $Bx = z^*$
> $x \geq 0$.
>
> Since this is a linear program, once we have the solution $x^*$ for some $z^*$, with value $k^*$, we can apply a scalar multiple to fit the solution to any budget value k and characteristics vector $(k/k^*)z^*$. By varying $z^*$, the consumer, given a budget constraint $px = k$, can determine a characteristics frontier consisting of all z such that the value of the above program is just equal to k. There will be a determinate goods vector associated with each point of the characteristics frontier.

In Figure 2, I show how the efficiency frontier is obtained the consumer has one dollar to spend on commodites A, B and C. The three commodities supply him with two chracteristics 1 and 2, in different proportions. This is shown by lines 0a, 0b and 0c. Points a, b and c are given by the levels of characteristics 1 and 2 that he can get if he spends his dollar on either A or B or C. The lines ab and bd show the amounts of characteristics 1 and 2 that he can get if he spends his dollar on various combinations of commodities A and B or B and C. Any combination of commodities A and C given on the line ac is dominated by some other combinations. This means that the consumer can get the same amounts of characteristics 1 and 2 by combining commodities A and B or B and C or B alone, while spending a smaller amount of money. Thus, the efficiency frontier is 0abc0.

Usually it is possible to obtain a more economical solution for the linear programming problem above when, instead of equalities, inequalities are used, making $Bx \geq z^*$. With the inequalities the consumer can obtain a combination of characteristics such that at least one characteristic is exactly achieved and all others are provided in excess. In Figure 2, the point F now can be

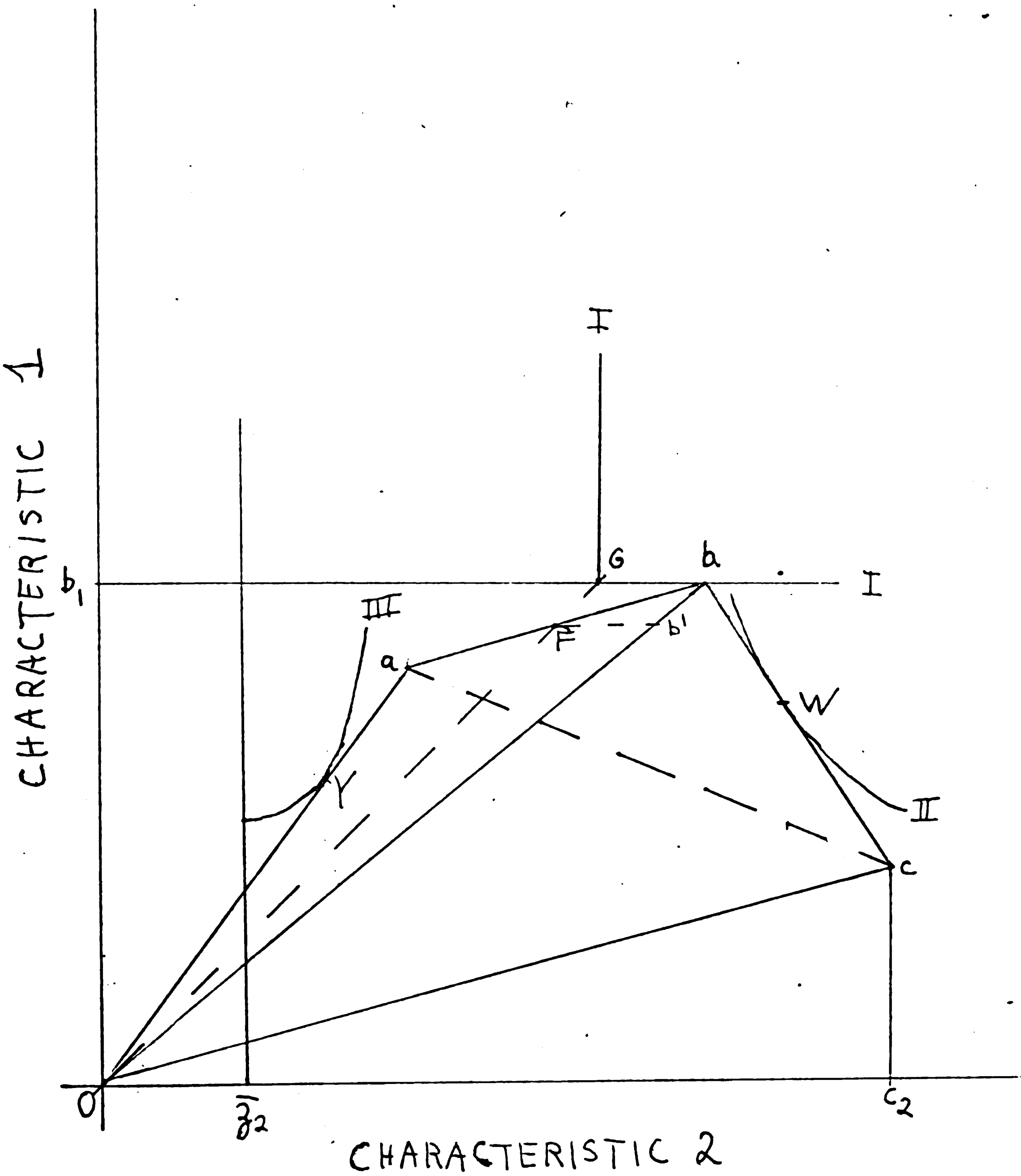


FIGURE IV-2

LANCASTER'S MODEL

achieved by buying only 0b' of commodity B. Characteristic 1 is supplied in the exact amount demanded, while characteristic 2 has an excess equal to Fb'. If the consumer buys only 0b' of commodity B, he spends less than one dollar. If he buys commodities A and B in the proportions that give him exactly F, he must spend the whole dollar. When excesses are permitted, then, the equivalent of any combination that includes food A can be achieved more cheaply by buying only commodity B.

Suppose the consumer wants to spend a maximum of one dollar. Because of the inequalities $Bx \geq z^*$, that allow the consumer to have excesses, he can reach points that he could not attain when equalities were used. Points in the area $0abb_10$ can be reached by buying commodity B, which gives an excess of characteristic 2. For example, the consumer can reach G by spending one dollar on commodity B (0b). He gets the exact amount of characteristic 1 demanded and an excess equal to Gb of characteristic 2. Points in the area $0cc_20$ can be attained by buying commodity C, which provides an excess of characteristic 1. Thus the efficiency frontier is expanded to $0b_1bcc_20$.

Using a linear programming problem in which $Bx \geq z^*$ implies the existence of a special indifference curve, in which the marginal utilities of excesses are equal to zero. In Figure 2, suppose the consumer wants to obtain the characteristics in the proportion given at F, while spending exactly one dollar. Then he can attain the point G by buying Ob of commodity B, that gives him an excess of characteristic 2 equal to Gb. As excesses of characteristics have zero marginal utility, the indifference curve is the right angle curve I, passing through G: an excess of any one of the characteristics does not put the consumer on a different indifference curve.

The decision that the marginal utility of any excess is equal to zero is a subjective decision that must be made by the consumer. Lancaster used the efficiency frontier derived from the equalities, $BX = z^*$, because he wanted to separate the objective problem of finding the efficiency frontier from the subjective problem of choosing a point on the efficiency frontier. That is, he does not want to make any assumption concerning the form of the indifference curve. The following passage (p. 139) states his purpose:

A consumer's complete choice subject to a budget constraint $px \leq k$ can be considered as consisting of two parts:

a) An efficiency choice, determining the characteristics frontier and the associated efficient goods collection.

b) A private choice, determining which point on the characteristics frontier is preferred by him.

In Figure 2, Lancaster's efficiency choices for a maximum expenditure of one dollar are shown on the line OabcO. The private choice will be determined by the consumer. If his indifference curve is II, he chooses W.

Earlier Smith developed a similar procedure, as the following passage shows (pp. 141-142):

> Although the programming procedure itself does not provide for comparisons among goals, an experiment can be devised that will accomplish this. Compute minimum cost diets for a comprehensive group of alternative sets of goal attainment levels [characteristics levels] and ask a consumer to choose among these diets, telling him what the cost of each diet will be. When the marginal cost of raising an attainment level [characteristic level] is positive, choice among diets involving different sets of attainment levels [characteristics levels] may involve changes in his expenditure. The diet he chooses will embody that set of attainment levels [characteristics levels] which he finds just worth the extra expenditure.

Once the subjective choice is regarded as a choice among characteristics rather than among commodities, a new problem arises. Because characteristics are provided in fixed proportions, it may happen that one characteristic has a negative marginal utility. In Figure 2, let the consumer desire to acquire $O\bar{z}_2$ of characteristic 2. Beyond this amount the marginal utility of characteristic 2 becomes negative. The consumer has a positive marginal utility for characteristic 1. His indifference curve looks like III. Commodities A, B and C furnish characteristics 1 and 2 in fixed proprotions given by lines Oa, Ob and Oc, respectively. The consumer maximized his utility at Y, buying only OY of commodity A. Substituting vitamin A and calories for characteristics 1 and 2 and foods for commodities, this is the behavior of a consumer that is on a diet to loose weight. Because of the joint supply of calories and vitamin A, if vitamin A has a positive marginal utility, he will choose an intake of calories larger than the one that would otherwise have been desired. This is so, because he has to take calories to obtain vitamin A.

## C. The Purchase of Nutrition

The problem of choosing economical foods is one in which commodities are to be chosen for their nutritional characteristics. Moreover, these characteristics are measurable, so it is a natural choice for an empirical application of Lancaster's model.[4] (The work on this thesis started before Lancaster's article appeared.)

The nutritional status functions of Chapter III correspond to Lancaster's utility function U(z), where the only characteristics considered are the nutritional ones. Here Lancaster's consumption technolocy matrix

---

[4]The results of this thesis do not depend upon the analysis developed by Lancaster.

gives the amounts of the deficient nutrients in one unit of each food. Only foods appear on the commodity and in the price vectors. Lancaster's budget constraint k is the amount of money reserved for food expenditure. In the present application, there is an added constraint that determines the excesses of nutrients over the recommended allowance levels. These excesses enter in the nutritional status function such that the value of the function changes as long as the intake of a nutrient is smaller than the allowance. Thus the constraint has relevance only to the subjective aspect of the problem. In Lancaster's model, the consumer wants to maximize the utility function subject to constraints. In this application the consumer wishes to maximize the nutritional status function subject to constraints.

Taking nutritional status function (III) and making

$z = (z_1 \ldots Z_i \ldots Z_h) = (n_1 \ldots n_i \ldots n_h) = n,$

$p = (p_1 \ldots p_j \ldots p_q),\ x = (x_1 \ldots x_j \ldots x_q),$

$B = b_{ij} = n_{ij} = N,$ where $i = 1 \ldots h$ and $j = 1 \ldots q,$

$e = (e_1 \ldots e_i \ldots e_h)$ and $r = (100 \ldots\ldots 100),$

where the vector e represents the excesses over the allowance levels and r is a (lxh) vector, I can write the following application of Lancaster's model:

$$\text{max. } S = 100(n_1 - e_1) - 1/2(n_1 - e_1)^2 + \ldots$$
$$+ 100(n_h - e_h) - 1/2(n_h - e_h)^2 + (m - h)5000,$$

subject to $n = Nx,$

$n - e \leqslant r,$

$px \leqslant \kappa$

and $n, x, e \geqslant 0.$

This is the same quadritic programming proglem presented in Chapter III.

Lancaster's theoretical analysis is designed to separate the problem of choice into its technical and subjective components. He does this by first determining an efficiency frontier that depends only on the costs of providing specified combinations of characteristics and then allowing subjective preferences to play their part in making the choice among the various combinations of characteristics available at a given cost. The subjective portion of the problem is represented in the model by the objective function to be maximized (the utility function); the technical aspects of the problem are represented by the characteristics matrix which relates quantities of goods to the bundles of characteristics provided by them.

First calculate the efficiency frontier from the linear programming problem:

min. $px$,

subject to $n^* = Nx$

and $x \geqslant 0$,

where $n^*$ is a specified set of nutritional levels. After obtaining the cost $k^*$ for this problem calculate $n$, the amount of nutrients provided in the proportions of $n^*$, that can be purchased by the food budget k; that is $n = (k/k^*)n^*$. This must be done for $n^*$ ranging over all possible sets of proportions. (The restriction $n - e \leq r$ is ignored because it relates to the subjective aspect of this problem.) Once the efficiency frontier is obtained, find the point in this frontier that is tangent to the indifference curves given by the nutritional status function (III).

This analytical procedure could also be used as a computing procedure, but it would be extremely inefficient. Imagine the number of linear programming problems necessary to express all the possible intake combinations of the deficient nutrients. Of course, parametric linear programming can reduce these computations. However, I am sure that for computational purposes, Lancaster would suggest the use of ordinary quadratic programming methods, even though these blur the distinction between technical and subjective aspects of the problem.

In Figure 3, I present a graphical analysis of Lancaster's analytical solution when there are only two foods and two deficient nutrients. The amounts of nutrients are expressed as percentages of their allowances. The points $n^a$ and $n^b$ represent the quantities of nutrients 1 and 2 obtained by spending k dollars on either food A or on food B. The efficiency frontier is $On^a n^b O$. In Chapter III, I pointed out that the isonutrient curves given by

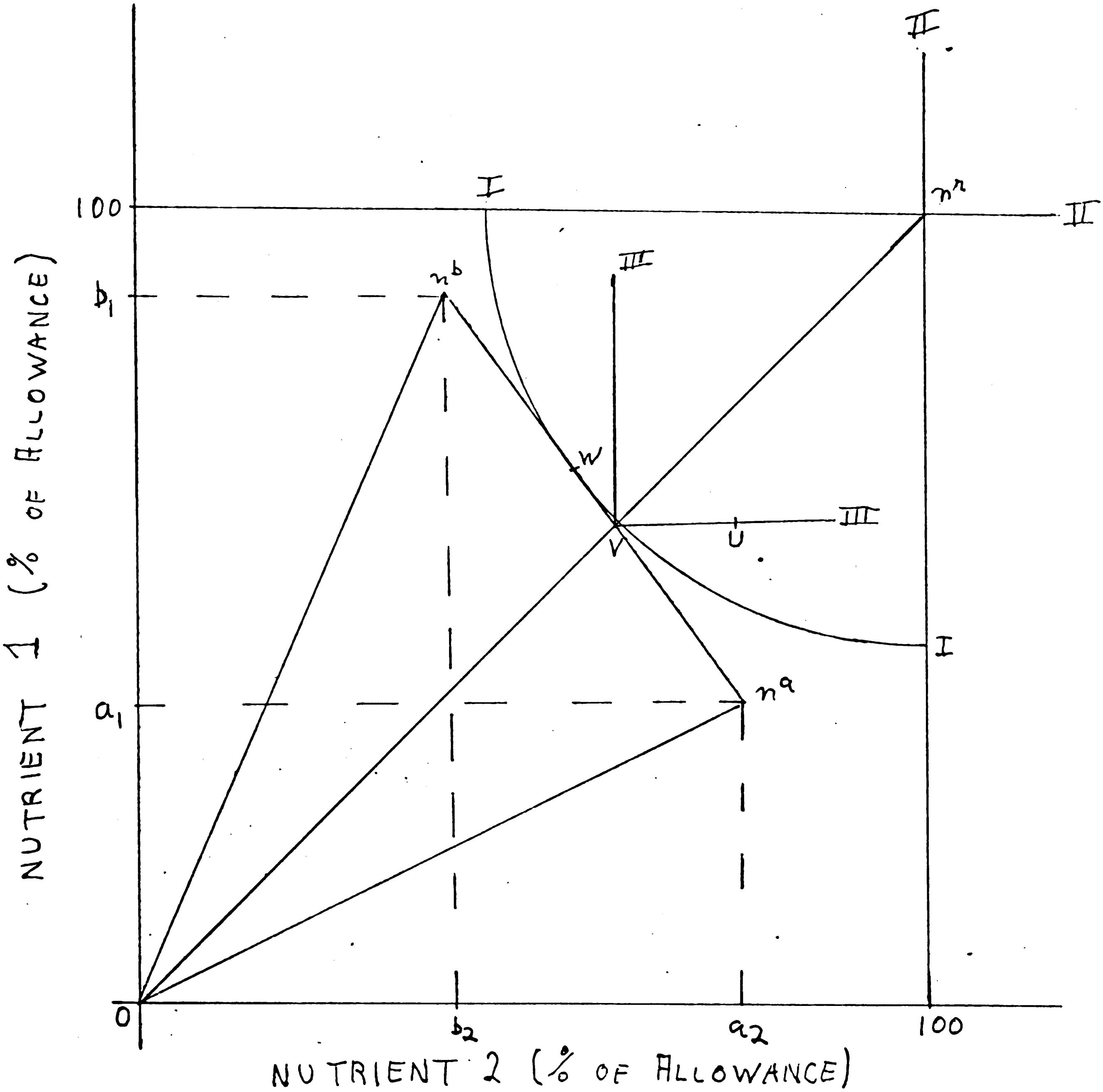


FIGURE IV-3

THE PURCHASE OF NUTRITION (I)

function (III) are portions of circles with their center at the point where the recommended allowances of deficient nutrients are exactly fulfilled. Therefore, the consumer maximizes his nutritional status at W, where the iso-nutrient curve I is tangent to the efficiency frontier. This is the solution of the quadratic programming problem stated above. It is also the solution approached by the new method suggested in this thesis (see Chapter V).

The consumer does not necessarily maximize his utility at a tangency point. He may be in a point inside the efficiency frontier. When the utility function is the nutritional status function (III), the maximum nutritional status attainable is when the intake of all nutrients are equal to their allowances; that is $n^r$ in Figure 3. Therefore, if the budget constraint were greater than the minimum cost diet that provides all the recommended allowances, this means that a portion of the efficiency frontier is in the northeast quadrant with origin at $n^r$, the consumer would choose $n^r$, purchasing the minimum cost diet obtained by the standard linear programming problem. Such a problem makes no explicit reference to a nutritional status function.

The absence of an explicit nutritional status function in the standard linear programming formulation of the least-cost diet problem becomes significant when the budget constraint does not allow all the deficiencies to be removed.

A natural procedure derived from the standard least-cost diet approach would be to select as large a fraction of the diet at $n^r$ as the budget allows. Let me call this solution the fractional least-cost diet. If the least-cost diet that provides $n^* \geq r$ and costs $t^*$ dollars, and the budget constraint is $t < t^*$, then the fractional least-cost diet will supply $n = (t/t^*)n^*$. The fractional least-cost diet, however, does not in general lead to the solution given by the quadratic problem above. In Figure 3, V is the solution obtained from the fractional least-cost diet. It is different than W.

The fractional least-cost diet leads to iso-nutrient curves that are right angle curves, with the angle on the deficiency line. These iso-nutrient curves are rather peculiar. They show that the consumer is indifferent between U and V in Figure 3. This means that for the same level of nutrient 1, the consumer is not better off when he has a greater amount of nutrient 2 in U than in V, although the intake of nutrient 2 is deficient. So, the fractional least-cost diet is not the best solution, if nutritional benefit can be received from an increase in the quantity of one deficient nutrient alone. On the other hand, if a nutrient can be utilized only in fized proportions to the other nutrients, the fractional least-cost diet points to the proper solution (as land as excesses of individual nutrients have no nutritional significance).

When the solution of the quadratic programming problem presented above is such that at least one deficient nutrient is provided in excess of its allowance, this solution is also a solution of the fractional least-cost diet problem. In the fractional least-cost diet excesses over the proportions given by the deficiency ratios do not count and in the nutritional status function (III) the excesses over the allowances do not incluence the value of the function. The slopes of the iso-nutrient curves become the same in both problems, when there is an excess over the allowance. However, it is not true that all fractional least-cost diets are solutions of the quadratic programming problem, when the solution of the latter presents at least one excess. In Figure 4 this is illustrated.

In Figure 4, the Lancaster efficiency frontier is OABCO. Suppore that the point where the intake is equal to the allowances is at $n_I^r$ . The fractional least-cost diet solutions are the combinations of foods A and B represented by points on line AB between $V_I$ and B. The points between $V_I$ and B cost no more than $V_I$ and provide the same amount of nutrient 1 and larger amounts of nutrient 2. (However, the excesses of nutrient 2 yield neither benefit nor loss.) The quadratic programming solutions are the combinations between $W_I$ and B. It is clear from Figure 4 that the $W_I B$ set is smaller than the $V_I B$ set. Solutions like those occur when $n^r$ is in area I. This area is

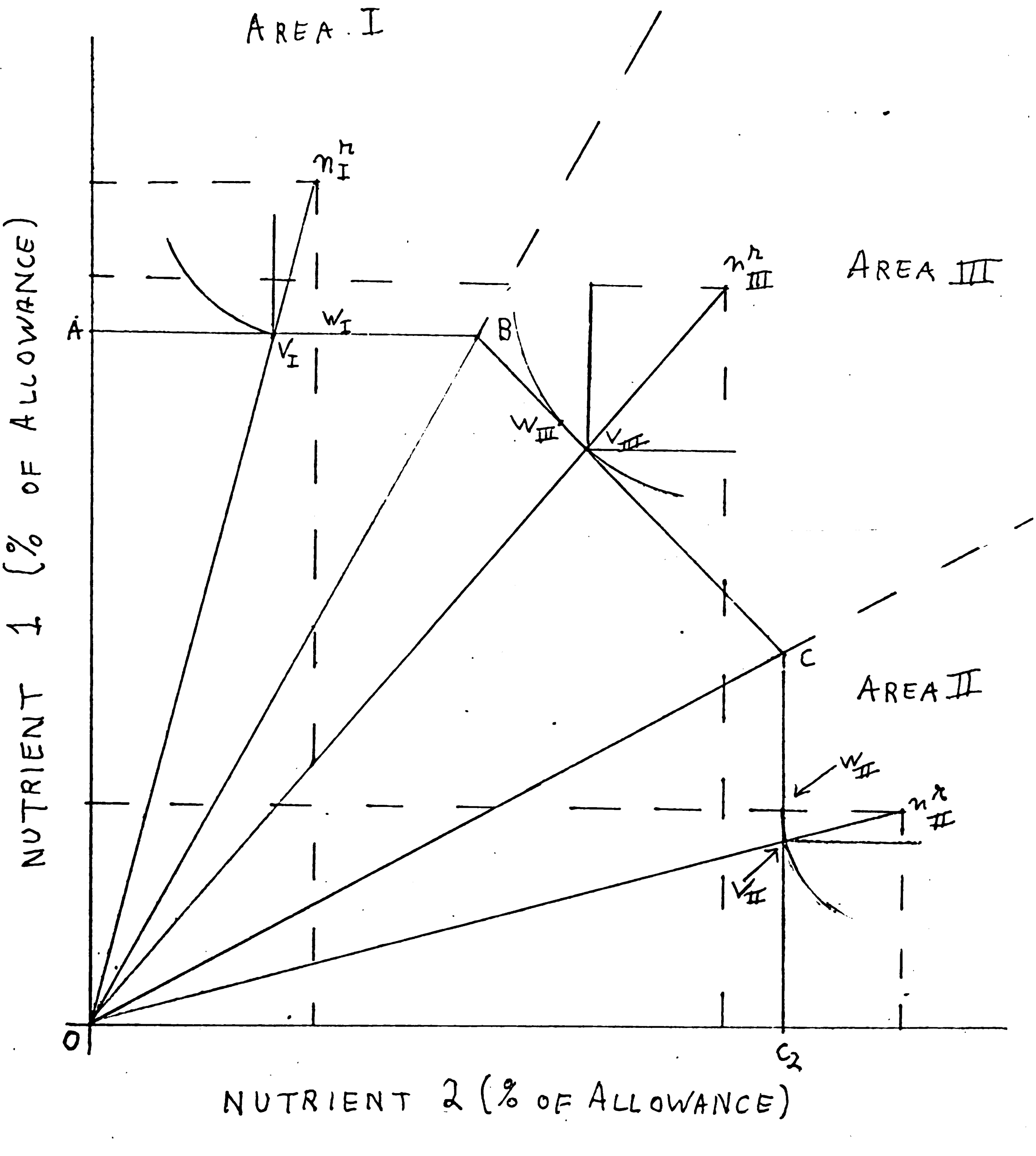


FIGURE IV-4

THE PURCHASE OF NUTRITION (II)

bounded by the vertical axis, line AB and the extension from B of the line OB.

Suppose that $n^r$ is at $n^r_{II}$. The two solutions are the same: only food C should be purchased. Solutions like that occur when $n^r$ is in area II. This area is bounded by the horizontal axis, line $CC_2$ and the extension from C of the line OC.

Suppose that $n^r$ is at $n^r_{III}$. This is equivalent to the solutions of Figure 3. The solutions, in general, will be different. Solutions like that occur when $n^r$ is in area III. This area is bounded by extensions from B and C of the lines OB and OC, respectively, and the line BC.

This chapter showed that the problem studied in this thesis is a natural application of Lancaster's theory. It also showed that when the resources are not sufficient to buy the least-cost diet providing all the recommended allowances, a fractional least-cost diet may not be the best diet in nutritional terms.

# CHAPTER V

## A NEW METHOD FOR COMPUTING ECONOMICAL SUPPLEMENTARY DIETS-- APPLICATION TO THE NORTHEAST BRAZIL

This chapter develops a method for finding economical supplementary diets for families whose nutritional intake is deficient. Rules formulated later give the basic operations of the method. There are established methods for the computer solutions of linear and quadratic programming problems. It is not within the scope of this thesis to discuss the details of these computational procedures.

I applied this new method to obtain economical supplementary diets for families with deficient nutritional intake in the Northeastern region of Brazil. The existing consumption pattern was assumed constant; only the additional consumption of foods constitutes the supplementary diet.

Nutritionists of the Brazilian Ministry of Health have surveyed the nutritional conditions of families living in four villages of the state of Rio Grande do Norte: Santo Antônio in November 1959, Boacica in December 1960, Currais in August 1961, and São Paulo do Potengi in September 1961.[1]

These investigators studied several families living in these four villages. They classified the families into four categories according to their wealth: very poor, poor, well to do and very well to do. However, they do not present any data that permits the reader to know the amount of wealth or income of the families.

They compared the actual nutritional intake, as calculated from tables of food composition, with the recommended allowances, suggested by the National Research Council of the U. S. in 1958, on a per capita basis for the families samples.[2] From these results the deficiencies of the different nutrients can be obtained.

---

[1]a) Brazil, Ministério da Saúde, Comissão Nacional de Alimentação, Inquérito de Alimentação, Realizado em Santo Antônio, Estado do Rio Grande do Norte (Brasil), Novembro de 1959.

b) Brazil, Ministério da Saúde, Comissão Nacional de Alimentação, Estudo do Consumo de Alimentos e das Condições Socio-Econômicas nas Familias Representativas do Povoado de Boacica, Município de Touros, Rio Grande do Norte, Brasil, Dezembro de 1960.

c) Brazil, Ministério da Saúde, Comissão Nacional de Alimentação, Estudo da Alimentação e das Condições Econômico-Sociais Realizado no Povoado de Currais, Município de Nisia Floresta - Rio Grande do Norte (Brasil), Agôsto de 1961.

d) Brazil, Ministério da Saúde, Comissâo Nacional de Alimentação, Inquérito sôbre Hábitos e Recursos Alimentares, Sâo Paulo do Potengi, Rio Grande do Norte, Brasil.

[2]In Ibid., Santo Antônio, Tables 46 and 47. In Ibid., Boacica, Tables 45 and 46. In Ibid., Currais, Tables 51 and 54. To be consistent with the entries in

They listed the prices of some twenty foods for each village and computed the amounts of the essential nutrients that could be obtained by the expenditure of Cr $10.00 (ten Brazilian cruzeiros) on these foods.[3]

I shall compare the economical supplementary diets computed by the new method with the minimum cost supplementary diets calculated by linear programming. Mainly intended for small problems, the new method needs only a desk calculator, but the process quickly becomes very time consuming.

In poor countries or in poor regions of a country, where the nutritional deficiencies are more acute, the new method becomes even more important, because such areas lack the computers and skilled personnel that linear programming demands.

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these tables, the recommended allowance of riboflavin for the poor families would have to be 3.11 mg. per capita. This is much too large, suggesting that there was an error in the computations concerning this nutrient. According to the U. S. Interdepartmental Committee on Nutrition for National Development, Northeast Brazil, Nutrition Survey, March-May 1963, Washington, D. C., May 1965, p. 288, an acceptable intake of riboflavin is between 1.2 mg. and 1.4 mg. per day. Therefore I did not compute the supplementary diet for this family.
In Ibid., São Paulo do Potengi, Tables 41 (p. 60) and 42 (p. 61).

[3]In Ibid., Santo Antônio, Table 30. In Ibid., Boacia, Table 44. In Ibid., Currais, Table 50. The entries for banana "anã" are obviously in error. As the price per unit is the same in 1d, Table 42 (p. 53), I use the amounts of the nutrients from that table. In Ibid., São Paulo do Potengi, Table 42 (p. 53).

## A. Selection Procedure

The basic procedure of the new method is to answer the following questions.

Question 1: "What is the food that gives the largest nutritional contribution from a given unit of money spent on food?"

The unit of money can only be spent on one food. Using a measure of the nutritional contribution of food an answer to Question 1 is obtained. The food given by answering Question 1 is included in the supplementary diet. This changes the nutritional intake and, hence, the deficiencies.

Question 2: "Do nutritional deficiencies still exist?"

If the answer is "yes," then go back to Question 1. Recompute the nutritional contribution of food and continue introducing new foods into the supplementary diet until the answer is "no."

Questions 1 and 2 constitute the selection procedure of the new method.

Nutritionists are not able to say that any essential nutrient is intrinsically more important than any other. All the essential nutrients are equally important for a healthy human life. Among the simple measures proposed in Chapter III the choice is between Measure One and Measure Three: the sum of the deficient nutrients and the deficiency-weighted sum of the nutrients. Between these measures, the latter one takes into account the sizes of the deficiencies. Chapter III identifies certain difficulties that arose from the use of Measure One (p. 29), but are avoided by Measure Three. Therefore, Measure Three should be employed to evaluate the nutritional contribution of one unit of money spent on food.

Now it is possible to formulate a set of rules that gives an economical supplementary diet that eliminates the nutritional deficiencies.

Rule 1: Calculate the nutritional contribution of spending one unit of money on each food by multiplying the amount of each nutrient furnished by the food by its deficiency and adding these products. (The amount of nutrient and the deficiency are measured as percentages of the recommended allowances.) The food with the largest nutritional contribution should be introduced into the supplementary diet.

Rule 2: In computing the nutritional contribution, when the amount of a nutrient furnished in spending one unit of money on a food is larger than the deficiency, consider only that amount of the nutrient which is equal to the deficiency. (The reason for this rule is that when a deficiency is eliminated its weight becomes zero.)

Rule 3: After introducing the food with the largest nutritional contribution, recompute the deficiencies and go back to Rules 1 and 2, until all deficiencies are eliminated.

I applied these rules to select economical supplementary diets for the families surveyed by the nutritionists of the Brazilian Ministry of Health. Because the number and the amount of deficiencies are not very large in the well to do and very well to do families, I computed supplementary diets only for the poor and very poor families studied.

I illustrate the selection procedure of the new method by showing part of the computations and the development of the supplementary diet for the very poor families of Boacica. I discuss this case because it involves the smallest number of deficient nutrients. Table 1 presents the beginning of the calculations. For simplicity, I just show the five foods that at the beginning had the largest nutritional contributions.

The first five lines of Table 1 give the amounts of the deficient nutrients that a consumer can obtain by spending Cr $ .10 (ten Brazilian "centavos") on each of the foods showed in Table 1. The next five lines present the nutrient contents as percentages of the respective recommended allowances. Lines (11) to (15) are obtained by multiplying the nutrient contents as percentages of the allowances by the deficiencies (also expressed as percentages of the allowances). These are the weighted nutrient

Table V-1. Nutritional Contribution of Cr $ .10 Spent on Food -- Very Poor Families of Boacica.

Measure: Deficiency-Weighted Sum of the Nutrients

| | | Sweet Potato 1 | "Macassa" Beans 2 | Mango 3 | Banana 4 | Fresh Milk 5 |
|---|---|---|---|---|---|---|
| Nutrient Content | (1) Protein (gm.) | .24 | 1.325 | .054 | .131 | .17 |
| | (2) Calcium (mg.) | 6.2 | 8.19 | 1.62 | 1.7 | 5.78 |
| | (3) Vitamin A (I. U.) | 92. | 1.79 | 105.03 | 34.06 | 6.19 |
| | (4) Thiamin (mg.) | .018 | .0322 | .0054 | .0052 | .0015 |
| | (5) Riboflavin (mg.) | .01 | .0107 | .0054 | .0066 | .0083 |
| Nutrient Content as % of Per Capita Allowance | (6) (1)x100 / Allowance (52.2 gm.) | .46 | 2.54 | .1 | .25 | .33 |
| | (7) (2)x100 / Allowance (900 mg.) | .69 | .91 | .18 | .19 | .64 |
| | (8) (3)x100 / Allowance (3847 I. U.) | 2.34 | .05 | 2.72 | .88 | .16 |
| | (9) (4)x100 / Allowance (.81 mg.) | 2.22 | 3.98 | .67 | .64 | .19 |
| | (10) (5)x100 / Allowance (1.31 mg.) | .76 | .82 | .41 | .5 | .63 |

Table V-1. Continued.

| | | Sweet Potato 1 | "Macassa" Beans 2 | Mango 3 | Banana 4 | Fresh Milk 5 |
|---|---|---|---|---|---|---|
| Weighted Nutrient Contribution | (11) (6)xDeficiency (12%) | 6 | 30 | 1 | 3 | 4 |
| | (12) (7)xDeficiency (72%) | 50 | 66 | 13 | 14 | 46 |
| | (13) (8)xDeficiency (28%) | 66 | 1 | 76 | 25 | 4 |
| | (14) (9)xDeficiency (5%) | 11 | 20 | 3 | 3 | 1 |
| | (15) (10)xDeficiency (55%) | 41 | 45 | 23 | 28 | 35 |
| (16) Total Nutritional Contribution (11)+(12)+(13)+(14)+(15) | | 174 | 162 | 116 | 73 | 90 |

Source: The basic data used in these calculations came from Tables 44, 45 and 46 of the study done by nutritionists and published in: Brazil, Ministério da Saúde, Comissai Nacional de Alimentação, Estudo do Consumo de Alimentos e das Condições Socio-Econômicas nas Familias Representativas do Povoado de Boacica, Município de Touros, Rio Grande do Norte, Brasil, Dezembro de 1960.

contributions. Finally, the last line is the sum of the weighted nutrients contributions, that is, the deficiency-weighted sum of the nutrients. The food with the largest sum should be introduced in the supplementary diet.

The first step introduces Cr $ .10 of sweet potato in the supplementary diet, because this food has the largest deficiency-weighted sum, at the beginning. To find the food that should be introduced by spending the next Cr $ .10, recalculate lines (11) to (15) using the new deficiencies. The new deficiencies are the old ones minus the amounts of nutrients furnished by sweet potato in lines (6) to (10) respectively. Continue this process, paying attention to Rule 2, until all deficiencies are eliminated. The reader can note that this procedure can be easily executed on a desk calculator.

For this supplementary diet, the sequence of choices is: Cr $ .50 of sweet potato, Cr $ .10 of "macassa" beans, Cr $ .10 of sweet potato, Cr $ .10 of "macassa" beans, Cr $ .20 of sweet potato, Cr $ .10 of "macassa" beans, Cr $ .20 of sweet potato, Cr $4.10 of "macassa" beans, Cr $ .10 of sweet potato and finally Cr $2.70 of "macassa" beans.

Tables 2, 3, 4, 5, 6, 7 and 8 present the economical diets computed by employing the rules of the selection procedure. These tables also show the minimum cost supplementary diets calculated by linear programming, using the amounts of deficiencies as the quantities of nutrients to be provided.

Note that the new method selected diets that are very similar to the diets found by linear programming. (The quantities given are for weights as purchased.) For the families of São Paulo do Potengi the suggested daily consumption of sweet potato (1889 gm.) seems to be too large to be acceptable (more than 10 times the existing consumption).[4] Any diversification in the supplementary diet, however, would imply an increase in its cost.[5]

Only four foods: sweet potato, "macassa" beans, pumpkin and fresh milk, qualified for inclusion in any of the diets. Most often the diet consisted of sweet potato plus one other food. Nutritionists would expect "macassa" beans or even fresh milk to be economical; however, they might find it strange that sweet potato and pumpkin appear in all these diets. This happens because sweet potato and pumpkin are very inexpensive in these villages.

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[4] In Ibid., São Paulo do Potengi, Table 41 (p. 50).

[5] In this case, it is possible to establish a maximum acceptable level for the consumption of sweet potato. When this maximum is achieved the selection rules choose the food with the second largest nutritional contribution. Suppose the maximum acceptable consumption of sweet potato is 1000 gm.. This means that after spending Cr $5.40 on sweet potato, another food should be introduced. This food is "macassa" beans. The new economical diet that eliminates the deficiencies is: 1000 gm. of sweet potato at Cr $5.40 and 221 gm. of "macassa" beans at Cr $4.90. The new diet costs Cr $10.30, a very small increase over the one that includes only sweet potato.

Table V-2. Daily Supplementary Diet, Per Capita--Very Poor Families of Boacica.

| Food | Quantity | Expenditure |
|---|---|---|
| I - New Method: Economical Supplementary Diet (E.S. Diet) Selection Procedure Only | | |
| Sweet Potato | 220 gm. | Cr $1.10 |
| "Macassa" Beans | 424 gm. | Cr $7.10 |
| | | Cr $8.20 |
| II - Linear Programming: Minimum Cost Supplementary Diet (M.C.S. Diet) | | |
| Sweet Potato | 206 gm. | Cr $1.032 |
| "Macassa" Beans | 426 gm. | Cr $7.131 |
| | | Cr $8.163 |

Error of New Method: .45%

Table V-2. Continued.

| Deficient Nutrients | Allowance | Deficiency | Provided by E.S. Diet | Percent of Allowance: Deficiency | Percent of Allowance: Provided by E.S. Diet |
|---|---|---|---|---|---|
| III - Nutritional Data | | | | | |
| Protein (gm.) | 52.2 | 6.3 | 96.715 | 12 | 185 |
| Calcium (mg.) | 900. | 648. | 649.69 | 72 | 72 |
| Vitamin A (I. U.) | 3847. | 1077. | 1139.9 | 28 | 29 |
| Thiamin (mg.) | .81 | .04 | 2.4842 | 5 | 306 |
| Riboflavin (mg.) | 1.31 | .72 | .8697 | 55 | 66 |

Source:

The basic data used in the computations of these diets came from Tables 44, 45 and 46 of the study done by nutritionists and published in: Brasil, Ministério da Saúde, Comissão Nacional de Alimentação, _Estudo do Consumo de Alimentos e das Condições Socio-Econômicas nas Familias Representativas do Povoado de Boacica, Município de Touros, Rio Grande do Norte, Brasil, Dezembro de 1960_.

Table V-3. Daily Supplementary Diet, Per Capita--Poor Families of Boacica.

| Food | Quantity | Expenditure |
|---|---|---|
| I - New Method: Economical Supplementary Diet (E.S. Diet) Selection Procedure Only | | |
| Sweet Potato | 100 gm. | Cr $ .50 |
| "Macassa" Beans | 394 gm. | Cr $6.60 |
| | | Cr $7.10 |
| II - Linear Programming: Minimum Cost Supplementary Diet (M.C.S. Diet) | | |
| Sweet Potato | 85 gm. | Cr $ .427 |
| "Macassa" Beans | 393 gm. | Cr $6.576 |
| | | Cr $7.003 |

Error of New Method: 1.39%

Table V-3. Continued.

| Deficient Nutrients | Allowance | Deficiency | Provided by E.S. Diet | Percent of Allowance Deficiency | Percent of Allowance Provided by E.S. Diet |
|---|---|---|---|---|---|
| Protein (gm.) | 55.1 | 3.9 | 88.65 | 7 | 160 |
| Calcium (mg.) | 897. | 565. | 571.54 | 63 | 63 |
| Vitamin A (I. U.) | 4251. | 510. | 578.14 | 12 | 13 |
| Thiamin (mg.) | .99 | .01 | 2.2152 | 1 | 223 |
| Riboflavin (mg.) | 1.35 | .62 | .7562 | 46 | 56 |

Source:

The basic data used in the computations of these diets came from Tables 44, 45 and 46 of the study done by nutritionists and published in: Brasil, Ministério da Saúde, Comissão Nacional de Alimentação, Estudo do Consumo de Alimentos e das Condições Socio-Econômicas nas Familias Representativas do Povoado de Boacica, Munícipio de Touros, Rio Grade do Norte, Barsil, Dezembro de 1960.

Table V-4. Daily Supplementary Diet, Per Capita--Very Poor Families of Santo Antônia.

| Food | Quantity | Expenditure |
|---|---|---|
| I - New Method: Economical Supplemenrary Diet (E.S. Diet) Selection Procedure Only | | |
| Sweet Potato | 324 gm. | Cr $1.20 |
| Fresh Milk | 397 gm. | Cr $3.90 |
| | | Cr $5.10 |
| II - Linear Programming: Minimum Cost Supplementary Diet (M.C.S. Diet) | | |
| Sweet Potato | 299 gm. | Cr $1.109 |
| Fresh Milk | 397 gm. | Cr $3.895 |
| | | Cr $5.004 |

Error of New Method: 1.92%

Table V-4. Continued.

| Deficient Nutrients | Allowance | Deficiency | Provided by E.S. Diet | Percent of Allowance: Deficiency | Percent of Allowance: Provided by E.S. Diet |
|---|---|---|---|---|---|
| III - Nutritional Data | | | | | |
| Calories (Cal.) | 1895. | 360. | 467.6 | 19 | 24 |
| Protein (gm.) | 52. | 10.9 | 16.98 | 21 | 32 |
| Calcium (mg.) | 826. | 536.9 | 545.19 | 65 | 66 |
| Iron (mg.) | 10.8 | .86 | 3.306 | 8 | 30 |
| Vitamin A (I. U.) | 3828. | 1071.8 | 1964.61 | 28 | 51 |
| Thiamin (mg.) | .91 | .39 | .4115 | 43 | 45 |
| Riboflavin (mg.) | 1.25 | .7 | .7941 | 56 | 63 |
| Niacin | 9.3 | 1.3 | 2.334 | 14 | 25 |

Source:

The basic data used in the computations of these diets came from Tables 30, 46 and 47 of the study done by nutritionists and published in: Brasil, Ministério da Saúde, Comissão Nacional de Alimentação, Inquérito de Alimentação, Realizado em Santo Antônio, Estado do Rio Grande do Norte, Brasil, Novembro de 1959.

Table V-5. Daily Supplementary Diet, Per Capita--Poor Families of Santo Antônio.

| Food | Quantity | Expenditure |
|---|---|---|
| I - New Method: Economical Supplementary Diet (E.S. Diet) Selection Procedure Only | | |
| Sweet Potato | 172 gm. | Cr $ .60 |
| Fresh Milk | 285 gm. | Cr $2.80 |
| | | Cr $3.40 |
| II - Linear Programming: Minimum Cost Supplementary Diet (M.C.S. Diet) | | |
| Sweet Potato | 136 gm. | Cr $ .507 |
| Fresh Milk | 286 gm. | Cr $2.802 |
| | | Cr $3.309 |

Error of New Method: 2.75%

Table V-5. Continued.

| Deficient Nutrients | Allowance | Deficiency | Provided by E.S. Diet | Percent of Allowance: Deficiency | Percent of Allowance: Provided by E.S. Diet |
|---|---|---|---|---|---|
| III - Nutritional Data | | | | | |
| Calories (Cal.) | 1840. | 92. | 344.74 | 5 | 18 |
| Protein (gm.) | 52.2 | 7.3 | 11.346 | 14 | 21 |
| Calcium (mg.) | 842. | 362. | 369.58 | 43 | 43 |
| Vitamin A (I. U.) | 4077. | 775. | 1086.26 | 19 | 26 |
| Thiamin (mg.) | .89 | .21 | .2326 | 24 | 26 |
| Riboflavin (mg.) | 1.27 | .48 | .5356 | 38 | 42 |

Source:

The basic data used in the conputations of these diets came from Tables 30, 46 and 47 of the study done by nutritionists and published in: Brasil, Ministério da Saúde, Comissão Nacional de Alimentação, Inquérito de Alimentação, Realizado em Santo Antônio, Estado do Rio Grande do Norte, Brasil, Novembro de 1959.

Table V-6. Daily Supplementary Diet Per Capita--Very Poor Families of São Paulo do Potengi.

| Food | Quantity | Expenditure |
|---|---|---|
| I - New Method: Economical Supplementary Diet (E.S. Diet) Selection Procedure Only | | |
| Sweet Potato | 1889 gm. | Cr $10.20 |
| II - Linear Programming: Minimum Cost Supplementary Diet (M.C.S. Diet) | | |
| Sweet Potato | 1875 gm. | Cr $10.122 |

Error of New Method: .77%.

Table V-6. Continued.

| Deficient Nutrients | Allowance | Deficiency | Provided by E.S. Diet | Percent of Allowance: Deficiency | Percent of Allowance: Provided by E.S. Diet |
|---|---|---|---|---|---|
| III - Nutritional Data | | | | | |
| Calories (Cal.) | 1848. | 261. | 2040. | 14 | 110 |
| Protein (gm.) | 55.4 | 4.4 | 22.644 | 8 | 40 |
| Calcium (mg.) | 908. | 581. | 585.48 | 64 | 64 |
| Vitamin A (I.U.) | 3912. | 391. | 8689.38 | 10 | 222 |
| Thiamin (mg.) | 1.15 | .21 | 1.7034 | 18 | 148 |
| Riboflavin (mg.) | 1.33 | .61 | .9486 | 46 | 71 |
| Niacin (mg.) | 10.7 | 1.8 | 11.322 | 17 | 105 |
| Ascorbic Acid (mg.) | 56. | 9. | 398.78 | 16 | 712 |

Source: The basic data used in the computations of these diets came from Tables 42 (p. 53), 41 (p. 60) and 42 (p. 61) of the study done by nutritionists and published in: Brazil, Ministério da Saúde, Comissão Nacional de Alimentação, Inquérito sôbre Hábitos e Recursos Alimentares, São Paulo do Potengi, Rio Grande do Norte, Brasil.

Table V-7. Daily Supplementary Diet, Per Capita -- Poor Families of Sao Paulo do Potengi.

| Food | Quantity | Expenditure |
|---|---|---|
| I - New Method: | Economical Supplementary Diet (E.S. Diet) Selection Procedure | |
| Sweet Potato | 1889 gm. | Cr $10.20 |
| Pumpkin | 49 gm. | Cr $ .50 |
| | | Cr $10.70 |
| | Substitution Procedure: | Cr $ .10 of Sweet Potato for Cr $ .50 of Pumpkin |
| Sweet Potato | 1908 gm. | Cr $10.30 |
| II - Linear Programming: | Minimum Cost Supplementary Diet (M.C.S. Diet) | |
| Sweet Potato | 1907 gm. | Cr $10.297 |

Error of New Method: .03%.

Table V-7. Continued.

| | | | Provided by E.S. Diet | | | Provided by E.S. Diet | |
|---|---|---|---|---|---|---|---|
| Deficient Nutrients | Allowance | Deficiency | Selection Procedure | Substitution Procedure | Precent of Allowance Deficiency | Selection Procedure | Substitution Procedure |
| Calories (Cal.) | 2145. | 21. | 2045.5 | 2060. | 1 | 95 | 96 |
| Protein (gm.) | 61.7 | 3.1 | 22.839 | 22.866 | 5 | 37 | 37 |
| Calcium (mg.) | 969. | 591. | 592.33 | 591.22 | 61 | 61 | 61 |
| Vitamin A (I. U.) | 4419. | 1414. | 9729.98 | 8774.6 | 32 | 220 | 198 |
| Riboflavin (mg.) | 1.54 | .8 | .9876 | .9597 | 52 | 64 | 62 |
| Niacin (mg.) | 10.7 | .1 | 11.437 | 11.433 | 1 | 106 | 106 |

Source: The basic data used in the computations of these diets came from Tables 42 (p. 53), 41 (p. 60) and 42 (p. 61) of the study done by nutritionists and published in: Brazil, Ministério da Saúde, Comissão Nacional de Alimentação, Inquérito Sôbre Hábitos e Recursos Alimentares, São Paulo do Potengi, Rio Grande do Norte, Brasil.

Table V-8. Daily Supplementary Diet, Per Capita -- Very Poor Families of Currais.

| Food | Quality | Expenditure |
|---|---|---|
| I - New Method: Economical Supplementary Diet (E.S. Diet) Selection Procedure | | |
| "Macassa" Beans | 54 gm. | Cr $1.30 |
| Pumpkin | 360 gm. | Cr $1.80 |
| Fresh Milk | 359 gm. | Cr $6.60 |
| | | Cr $9.70 |
| Substitution Procedure: a) Cr $ .70 of Fresh Milk for Cr $1.50 of Pumpkin | | |
| "Macassa" Beans | 54 gm. | Cr $1.30 |
| Pumpkin | 60 gm. | Cr $ .30 |
| Fresh Milk | 397 gm. | Cr $7.30 |
| | | Cr $8.90 |

Table V-8. Continued.

| Food | Quantity | Expenditure |
|---|---|---|
| Substitution Procedure: b) Cr $ .80 of Fresh Milk for Cr $ .80 of "Macassa" Beans | | |
| "Macassa" Beans | 21 gm. | Cr $ .50 |
| Pumpkin | 60 gm. | Cr $ .30 |
| Fresh Milk | 431 gm. | Cr $8.10 |
| | | Cr $8.90 |
| II - Linear Programming: Minimum Cost Supplementary Diet (M.C.S. Diet) | | |
| "Macassa" Beans | 20 gm. | Cr $ .479 |
| Pumpkin | 56 gm. | Cr $ .276 |
| Fresh Milk | 429 gm. | Cr $8.045 |
| | | Cr $8.800 |

Error of New Method: 1.14%.

Table V-8. Continued.

III - Nutritional Data

| Deficient Nutrient | Allowance | Deficiency | Provided by E. S. Diet: Selection Procedure | Provided by E. S. Diet: Substitution Procedure a) | Provided by E. S. Diet: Substitution Procedure b) |
|---|---|---|---|---|---|
| Calories (Cal.) | 1770. | 252. | 432.83 | 431.65 | 333.16 |
| Protein (gm.) | 55.3 | 14.4 | 25.35 | 25.418 | 19.442 |
| Calcium (mg.) | 890. | 525. | 526.57 | 527.2 | 530.24 |
| Vitamin A (I. U.) | 3682. | 663. | 8108.03 | 1769.74 | 1812.08 |
| Thiamin (mg.) | .94 | .11 | .5781 | .4393 | .1716 |
| Riboflavin (mg.) | 1.32 | .78 | .9597 | .7806 | .7902 |
| Niacin | 8.9 | 1.1 | 3.214 | 1.809 | 1.145 |
| **% of Allowance** | | | | | |
| Calories | | 14 | 24 | 24 | 18 |
| Protein | | 26 | 45 | 45 | 35 |
| Calcium | | 59 | 59 | 59 | 59 |
| Vitamin A | | 18 | 220 | 48 | 49 |
| Thiamin | | 12 | 61 | 46 | 18 |
| Riboflavin | | 59 | 72 | 59 | 59 |
| Niacin | | 12 | 36 | 20 | 12 |

Source: The basic data used in the computations of these diets came from Tables 50, 51 and 54 of the study done by nutritionists and published in: Brazil, Ministério da Saúde, Comissão Nacional de Alimentacão, Estudo da Alimentacão e das Condicões Econômico-Sociais Realizado no Povoado de Currais, Município de Nísia Floresta - Rio Grande do Norte, Brasil, Agôsto de 1961.

The reader should be aware that the surveys covered a period of only one week. They did not take into account seasonal variations in the prices and quantities of foods available for consumption. At other times of the year, for instance, sweet potato might not be an economical food.

Because of the finite steps used in the selection procedure there are generally excesses in the amounts of all the nutrients provided by the economical supplementary diets. For the very poor families of Boacica (Table 2), the excess of protein is more than 15 times as large as the amount of its deficiency, while there is almost no excess of calcium.

Let me define as the scarcest nutrient the one that has the smallest of the following ratios:

$$\frac{\text{amount of nutrient provided by the supplementary diet.}}{\text{amount of original deficiency of the same nutrient}}$$

For the very poor families of Boacica, calcium is the scarcest nutrient. In Tables 2, 3, 4, 5, 6, 7 and 8 the amounts provided of the scarcest nutrients are encircled.

## B. Substitution Procedure

The economical supplementary diets for the poor families of São Paulo do Potengi and for the very poor families of Currais can still be made more inexpensive by changing the proportions of the foods in these diets. The economy will not be larger than 10% of the minimum cost supplementary diet. To accomplish this, the following question must be answered:

Question 3: "Is it possible to substitute a food for another in the economical supplementary diet, so that the cost of this diet is reduced?"

To avoid large number of computations, the substitution among foods is limited to the ones in the economical supplementary diet.

To answer Question 3, it is necessary to determine whether it is possible to reduce the cost of the supplementary diet obtained in the selection procedure by substitution among the foods contained in this diet. When this is possible, it is necessary to know how the substitution takes place.

If, by spending the same amount of money for the supplementary diet, it is possible to increase the level of the scarcest nutrient, while the amounts of the other nutrients do not become smaller than their allowances, then substitution among these foods can provide the same amount of the scarcest nutrient, with a diet that is less expensive. This leads to the following rule.

Rule 4: Substituting one unit of a food for one unit of another food (the unit is in money terms, Cr $ .10 for example), both in the supplementary diet, see if there is an increase in the level of the scarcest nutrient, while the amounts of the other nutrients do not become smaller than their recommended allowances.

In the supplementary diets for Boacica (Tables 1 and 2), when trying to substitute Cr $ .10 of "macassa" beans for Cr $ .10 of sweet potato, the level of calcium increases while the level of vitamin A becomes smaller than its allowance. For the very poor families, the amount of calcium increases from 649.69 mg. to 651.68 mg., and the amount of vitamin A decreases from 1139.9 I. U. to 949.69 I. U., which is smaller than the deficiency: 1077 I. U.. For the poor families the amount of calcium increases from 571.14 mg. to 573.73 mg., and the amount of Vitamin A decreases from 578.14 I. U. to 478.33 I. U., which is smaller than the deficiency: 565 I. U.. To reproduce these calculations use the data in lines (2) and (3) and columns 1 and 2 of Table 1. In the supplementary diets for Santo Antônio (Tables 4 and 5), when trying to substitute Cr $ .10 of fresh milk for Cr $ .10 of sweet potato, the level of calcium increases, but the amount of thiamin becomes smaller than its allowance. There is only one food in the supplementary diet of the very poor families of São Paulo do Potengi (Table 6). In none of the above five supplementary diets, therefore, is substitution among foods possible.

Substitutions are possible in the cases shown in Tables 7 and 8. These show the economical supplementary diets for the poor families of São Paulo do Potengi and for the very poor families of Currais. They also present the minimum cost supplementary diets computed by linear programming, using the amounts of the deficiencies as the quantities of nutrients to be provided.

For the poor families of São Paulo do Potengi, Table 7, the scarcest nutrient is calcium. At the prices prevalent in this village it is possible to obtain either 5.74 mg. or 1.37 mg. of calcium by spending Cr $ .10 on sweet potato or on pumpkin, respectively. Then, by substituting Cr $ .10 worth of sweet potato for Cr $ .10 worth of pumpkin, the level of calcium in the supplementary diet increases. As the amounts of the other nutrients do not become smaller than their allowances, such substitution is permissible. If made, it allows one to reduce the cost of this diet.

For the very poor families of Currais, Table 8, the scarcest nutrient is calcium. It is possible to obtain either 5.71 mg., 2.8 mg. or 6.09 mg. of calcium by spending Cr $ .10 on "macassa" beans, on pumpkin or on fresh milk, respectively. Then, by substituting Cr $ .10 worth of fresh milk for Cr $ .10 worth of "macassa" beans or pumpkin or Cr $ .10 worth of "macassa" beans for Cr $ .10 worth of pumpkin, the level of calcium in the supplementary diet increases. As the amounts of the other nutrients do not become smaller than their allowances, such substitutions are permissible. If made, they would reduce the cost of this diet.

Keeping the level of the scarcest nutrient unchanged allows one to withdraw an amount of food that is greater in monetary cost than the amount being added. Thus the cost of the diet is reduced. The substitution goes on until the food being withdrawn is completely excluded from the diet or the amount of some other nutrient becomes smaller than its allowance. This leads to the following rule:

Rule 5: The rate of substitution between two foods in the supplementary diet is given by:

$$\frac{\text{amount of food to be withdrawn}}{\text{amount of food to be added}} =$$

$$= \frac{\text{amount of scarcest nutrient in one value unit of food being added}}{\text{amount of scarcest nutrient in one value unit of food being withdrawn}} .$$

The substitution takes place until the food being withdrawn is totally excluded from the supplementary diet or the amount of some other nutrient is reduced to the level of its allowance. Then go back to Rule 4.

For the poor families of São Paulo do Potengi, the rate of substitution of sweet potato for pumpkin is 4.19, given by the ratio of the amounts of calcium supplied by Cr $ .10 worth of sweet potato and pumpkin (5.74/1.37). For simplicity in calculation use the largest multiple of Cr $ .10 that is smaller than the rate of substitution. Thus, it is possible to increase the expenditure on sweet potato by Cr $ .10 and decrease the expenditure on pumpkin by Cr $ .40, without reducing the level of calcium in the supplementary diet. As there is an excess of calcium from the selection phase to start with, and the rate of substitution is not fully used, one can then take out another Cr $ .10 worth of pumpkin from the supplementary diet, without reducing the level of any nutrient to an amount smaller than the allowance. This excludes pumpkin from the diet. The cost of the supplementary diet is reduced from Cr $10.70 to Cr $10.30. Rule 4 cannot be applied again, because there is now only one food in this diet.

For the very poor families of Currais, the rates of substitution of fresh milk for pumpkin or "macassa" beans and of "macassa" beans for pumpkin are 2.17, 1.06 and 2.03, given by the ratios of the amounts of calcium supplied by Cr $ .10 worth of fresh milk and pumpkin (6.09/2.8), fresh milk and "macassa" beans (6.09/5.71) and "macassa" beans and pumpkin (5.71/2.8), respectively. Start the substitution process with fresh milk and pumpkin, because this gives the largest rate of substitution and thus the largest saving for each substitution. For simplicity in calculation use the largest multiple of Cr $ .10 that is smaller than the rate of substitution. Thus it is possible to increase the expenditure on fresh milk by Cr $ .10 and decrease the expenditure on pumpkin by Cr $ .20, without reducing the level of calcium in the supplementary diet. At the eighth substitution the level of riboflavin becomes smaller than its allowance. So only seven substitutions are possible. As there is an excess of calcium from the selection procedure to start with and the rate of substitution is not fully used, one can take out still another Cr $ .10 worth of pumpkin from the supplementary diet without reducing the level of any nutrient to an amount smaller than the allowance. Then the cost of this diet is reduced from Cr $9.70 to Cr $8.90.

The scarcest nutrient now becomes riboflavin. It is possible to obtain .0075 mg., .016 mg. or .0087 mg. of riboflavin by spending Cr $ .10 on "macassa" beans, pumpkin or fresh milk, respectively. By substituting Cr $.10 worth of pumpkin for Cr $ .10 worth of "macassa" beans or fresh milk, the level of riboflavin increases, but the amount of calcium in the diet becomes smaller than its allowance. These substitutions are not possible. By substituting Cr $ .10 worth of fresh milk for Cr $ .10 worth of "macassa" beans, the level of riboflavin increases, without making the level of any other nutrient smaller than the allowance. This substitution is permissible. If made, it would reduce the cost of this diet.

The rate of substitution of fresh milk for "macassa" beans is 1.16, given by the ratio of the amounts of riboflavin furnished by Cr $ .10 worth of fresh milk and "macassa" beans (.0087/.0075). For simplicity in calculation, use the largest multiple of Cr $ .10 that is smaller than the substitution ratio. It is possible to increase the expenditure on fresh milk by Cr $ .10 and decrease the expenditure on "macassa" beans by Cr $ .10 without reducing the level of riboflavin.[6] At the ninth substitution the amount of niacin becomes smaller than its allowance, only eight substitutions are possible. (The reader can check this result knowing that by spending Cr $ .10 on "macassa" beans, pumpkin or fresh milk, it is possible to obtain .088 mg., .1 mg. or .005 mg. of niacin, respectively.) As it is no longer possible to reduce the cost of the diet by at least one full step (Cr $ .10), return to Rule 4.

Other substitutions are still possible, but none of them allows the cost of the diet to be reduced by a full step. Therefore, I leave this economical supplementary diet as it is, after the last substitution.

Note that the number of scarcest nutrients can be greater than one. This makes this method much more difficult to operate, because it is necessary to pay attention to all the scarcest nutrients at the same time.

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[6]It is obvious that substituting Cr $ .10 of fresh milk for Cr $ .10 of "macassa" beans does not reduce the cost of the supplementary diet. However, such substitutions may build up excesses that allow one to withdraw one value unit of the food being withdrawn without adding one value unit of the food being added. This would reduce the cost of the diet by one value unit.

Instead of using steps of Cr $ .10, if one had used smaller steps, Cr $ .01 for example, the excess nutrients provided in the selection phase would be smaller. The smaller steps would also allow more precise results in the substitution procedure.

## C. The Expenditure on Food

When food habits are not taken into account, still more economical diets can be obtained by drastically changing the proportions in which the existing foods are being consumed. Some foods may even be excluded from the diet entirely. By employing linear programming, it would be possible to obtain the least-cost diet that provides the recommended allowances of all the essential nutrients. I suppose that the families would not accept such least-cost diets. They might accept the supplementary diets, however, because these diets do not require them to give up any existing consumption. The foods in the economical supplementary diets, suggested in this chapter, are chosen from some twenty eligible foods that are widely consumed in the villages, so, these supplementary diets do not disregard the food habits of the families.

It is important to know how much is the cost of each supplementary diet in comparison with the cost of each existing diet. Table 9 shows the daily cost of the existing diet on a per capita basis and compares it with

TABLE V-9.--Cost of Daily Diet per Capita (Cr $).

| Village | Actual Diet | M.C.S. Diet | Supplementary Diet as % of Actual Diet |
|---|---|---|---|
| Poor | | | |
| Santo Antônio | 14.33 | 3.309 | 23 |
| Boacica | 23.98 | 7.003 | 29 |
| São Paulo do Potengi | 31.08 | 10.297 | 33 |
| Very Poor | | | |
| Santo Antônio | 15.19 | 5.004 | 32 |
| Boacica | 18. | 8.163 | 45 |
| São Paulo do Potengi | 24.46 | 10.122 | 41 |
| Currais | 18.30 | 8.8 | 48 |

Source: Cost of Actual Diet.
For Santo Antônio see Brazil, Ministério da Saúde, Comissão Nacional de Alimentação, Inquérito de Alimentação Realizado em Santo Antônio, Estado do Rio Grande do Norte (Brasil), Novembro de 1959, Table 33.
For Boacica see Brazil, Ministério da Saúde, Comissão Nacional de Alimentação, Estudo do Consumo de Alimentos e das Condições Socio-Econômicas nas Familias Representativas de Boacica, Município de Touros, Rio Grande do Norte, Brasil, Dezembro de 1960, Table 38.
For São Paulo do Potengi see Brazil, Ministério da Saúde, Comissão Nacional do Alimentação, Inquérito sôbre Hábitos e Recorses Alimentares, São Paulo do Potengi, Rio Grande do Norte, Brasil, Table 37.
For Currais see Brazil, Ministério da Saúde, Comissão Nacional de Alimentação, Estudo da Alimentação e Condições Econômico-Sociais Realizado no Povoado de Currais, Município de Nisia Floresta - Rio Grande do Norte (Brasil), Agôsto de 1961, Table 45.

the per capita minimum cost supplementary diet (obtained by linear programming) for each of the families studied. The supplementary diet that represents the smallest increase in the existing expenditure on foods is for the poor families of Santo Antônio: 23% of the actual cost. Others would cost as much as 48% of the actual cost. Considering that the families studied were poor and very poor of an underdeveloped area, it is hardly conceivable that they could afford such additions to their food budgets.

If the families decide to spend an additional amount of money that is smaller than the minimum cost supplementary diet, but hope to improve their nutritional status as much as possible, this can be done with a small modification of the method developed in this chapter.

Considering that all the nutrients have equal intrinsic importance, assume that function (III) represents the nutritional status function:

$$\text{(III)}\quad S = (100n_1 - 1/2n_1^2) + \ldots + (100n_h - 1/2n_h^2) + \\ + (m - h)5000,$$

when $n_i > 100$, take $n_i = 100$.

The new modified method uses Rules 1 and 2 of the selection procedure. Rule 3 can still be used until all the available money is depleted. However, in this case it is not possible to eliminate all the deficiencies.

After the selection procedure, see whether substituting one value unit of a food for one value unit of another food will increase the value of the function (III). This substitution keeps the amount of money unchanged. This leads to the following rules.

Rule 4A: Calculate the nutritional contributions of foods at the end of the selection procedure. See whether at this point, all the contributions of the foods in the supplementary diet are equal (or approximately equal). See whether the nutritional contributions of the foods not selected are smaller than the ones in the diet. If this occurs the nutritional status function (III) is at the attainable maximum (or close to it).

Rule 5A: When the condition in Rule 4A is not satisfied, substitute one value unit (Cr $ .10, for example) of the food with the largest nutritional contribution for one value unit of the food in the diet with the smallest nutritional contribution. Continue this substitution process until the condition in Rule 4A is met.

Rule 4A is a paraphrase of a very familiar conclusion of economic analysis. The consumer maximizes his nutritional status function (III) (utility function), subject to a budget constraint, when the nutritional contributions per unit of money (marginal utilities per dollar) of the foods in the diets are equal. The nutritional contributions per unit of money (marginal utilities per dollar) of the foods not contained in the diet are smaller than those for the foods in the supplementary diet.

This chapter has developed a new method for selecting economical supplementary diets, without the need of linear programming. This method can also be applied to find an economical diet when the intake of all nutrients is considered equal to zero. Then the starting point of the calculations considers that each nutrient has a deficiency equal to 100. When the consumer has an amount of money to spend on food that is smaller than the one required by the minimum cost diet given by linear programming, the new modified method, employing Rules 4A and 5A, gives a diet that approximately maximizes his nutritional status as represented by function (III).

# CHAPTER VI

# CONCLUSION

Considering that nutritionists are not able to say that one nutrient is intrinsically more important than another one, Measure Three, the deficiency-weighted sum of nutrients, is the most appropriate of the simple measures proposed to evaluate the nutritional contributions of foods. This measure can only be used when the intake is smaller than the recommended allowance of at least one nutrient.

In countries or regions of a country in which deficiencies exist, the resources available are generally very scarce. These resources must be efficiently applied, so that the state of poverty can disappear as quickly as possible.

Suppose that there is a certain amount of resources designated for research and development of foods. The research and development can be for the production of foods, for example. Having a measure of the nutritional contributions of foods, the research and development can be concentrated on a few foods that have the largest nutritional contributions.

Suppose that the government decides to start a campaign to introduce new foods into the actual consumption pattern of the population. This measure will select the foods with the largest nutritional contributions.[1] The introduction of these foods will yield the largest improvement in the nutritional status of the population for a dollar spent on such a campaign.

When additional resources become available for expanding the production of foods, having the production functions (or approximations) for the foods, the measure will indicate the foods that give the largest nutritional contirubitons per unit of additional resources employed.

If the countries or regions of a country in which deficiencies prevail want to import foods to improve the nutritional status of their populations, then the measure can be used to indicate the foods with the largest nutritional contributions per unit of foreign exchange.

For countries like the U. S. that provide food aid to underdeveloped nations, the measure points out which foods have the largest nutritional contributions for a specific underdeveloped country. The measure can also be employed to select the country where a particular food can be of most nutritional benefit.

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[1]The government should select only foods that can be acceptable to the population, according to their food habits. For instance, beef is not acceptable for most Indians. A campaign for its introduction into the food consumption pattern of the Indian population is bound to fail.

Many other applications and policies can be formulated for a measure of the nutritional contributions of foods.

This thesis is an empirical application of the new consumer theory that Lancaster is the best expositor, although this study started before Lancaster's work was published. (See Chapter IV.)

I hope that this thesis will lead to an improvement in the nutritional status of the populations of poor regions. The new method (Chapter V) to obtain economical diets may be very useful, because it does not need skilled personnel for its computation. With the measures proposed, such regions may become more efficient when employing their scarce resources on activities related to feeding human beings.

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