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NUMERICAL SOLUTIONS OF JEFFREY-HAMEL FLOW
AT FIXED FLOW RATES

Thesis for the Degree of Ph. D.
MICHIGAN STATE UNIVERSITY
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This is to certify that the

thesis entitled

NUMERICAL SOLUTIONS OF JEFFREY-HAMEL

FLOW AT FIXED FLOW RATES

presented by

Floyd Ernest LeCureux

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of the requirements for

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David H. S. Yen
Major professor

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ABSTRACT

NUMERICAL SOLUTIONS OF JEFFREY-HAMEL
FLOW AT FIXED FLOW RATES

By

Floyd Ernest LeCureux

The radial flow of viscous incompressible fluid between non-parallel plane walls governed by the Navier-Stokes equations has been previously investigated. However, the known solutions include transcendental equations containing elliptic functions. Therefore, even though some solutions are known, there are no explicit equations or methods available for determining the shape of the velocity profile for specified boundary conditions that do not require an iterative or graphical method of solution. Further it is not readily apparent which profiles are possible for specified boundary conditions.

The major objective of this work is to develop a procedure for determining the shape of the velocity profile that may be assumed by the fluid for specified values of flow rate and angle of inclination of the walls. Further, the limits on each type of flow profile are investigated and comparisons are made between the various profiles.

The basic types of flow considered are symmetrical diverging where the centerline velocity is outward, symmetrical converging where the centerline velocity is inward, nonsymmetrical with one interior zero, and non-symmetrical with three interior zeroes. For each case the transcendental equations resulting from application of the boundary conditions are presented in the form of a flow parameter graph. Each graph has coordinates of flow rate and angle of

inclination and has two intersecting families of curves representing areas where solutions are possible. In each case the boundaries are defined and relations between different profiles are noted. These flow parameter graphs are then used to plot several examples of velocity profiles by reading the flow parameters for use in the velocity distribution function from the graph for particular values of flow rate and angle of inclination.

A minor objective of this work is the verification of a modified perturbation technique as applied to this problem. The modification of the normal perturbation procedure allows the determination of velocity distributions for small flow rates at a critical angle of inclination where standard perturbation equations are not defined. The velocity profiles obtained by use of the flow parameter graphs are used to verify the modified perturbation method and to analyze the accuracy of this method for various boundary conditions.

NUMERICAL SOLUTIONS OF JEFFREY-HAMEL

FLOW AT FIXED FLOW RATES

By

Floyd Ernest LeCureux

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To Jeanne - for her patience and understanding

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I. INTRODUCTION

1.1 Background

One of the classic problems of incompressible, viscous fluid dynamics is that of flow between parallel plane walls. For this problem the non-linear Navier-Stokes equations may be reduced to a relatively simple form which can be integrated to yield an exact solution for the familiar parabolic velocity distribution. A slightly less known but equally interesting problem is that of flow between non-parallel plane walls. For this case, although the non-linear terms in the Navier-Stokes equations do not vanish, an exact solution is still possible in terms of elliptic functions as first derived independently by Hamel [5] and Jeffery [7]. Since these first papers a number of authors have further investigated specialized aspects of the problem [3, 11, 12, 13].

Two of the more comprehensive studies are by Rosenblatt [12] and Rosenhead [13]. Rosenhead's work, especially, contains a thorough study of the possible solutions, the effect of increasing Reynolds Number, and the definition of areas where particular solutions may or may not be mathematically possible. However, as stated in a more recent paper by Millsaps and Pohlhausen [9], the unavailability of an extensive table of elliptic functions appears to be one of the principal reasons why numerically calculated velocity profiles for assigned Reynolds Numbers have been published for only a few selected values. Millsaps and Pohlhausen developed several solutions in their study of thermal distributions between non-parallel plane walls.

Their solutions are in terms of the more familiar Jacobian elliptic functions rather than the Weierstrass elliptic functions as used by some previous authors. Their results, however, still contain a transcendental equation which requires either a graphical solution or some iterative procedure.

Thus, the basic solutions for the problem of purely radial two-dimensional flow of viscous fluid between non-parallel walls are well known, and a few velocity profiles, dependent on the solution of a transcendental equation, have been calculated; and, these works have in fact been referred to in several textbooks, for example, [3, 8, and 11]. On the other hand, it should be mentioned that there is no method that can easily be used to determine the shape of the mathematically possible velocity distributions for a fixed flow rate and a particular angle of inclination of the walls. More specifically, the following questions remain unanswered. First, will a particular set of values of flow rate and angle of inclination allow purely inward converging flow or purely outward diverging flow, or flow with areas of both converging and diverging flows? These various possible solutions may all exist for a given flow rate and angle of inclination. Second, what is the exact shape of the velocity profile for this particular set of conditions? Several authors, [11] for example, have obtained expressions describing the conditions necessary for purely outward flow. Rosenhead [13] has included a series of graphs which define the limits of the various types of flow. However, the determination of the shape of the velocity profile for a particular set of boundary conditions is left to the fairly complex procedure of solving a system of transcendental equations. It is difficult as well as time consuming to use the "apparently available" exact solution for comparison with other investigations or for an indication of how the profiles change between different regions.

1.2 Objectives

The present availability of computerized techniques of solutions not only make the handling of elliptic functions more amenable to calculations but also allow more straightforward investigation of the limitations of the various possible solutions. The major objective of this work is to apply computer solutions to aid in the theoretical analysis of the problem of two-dimensional, viscous flow between non-parallel walls to determine "flow parameter graphs." These graphs can then be used to readily determine which particular velocity profile is possible for a certain specification of flow rate and angle of inclination, what the flow parameters are for this profile to allow easy determination of the velocity distribution function, and what the limits are for each type of velocity profile.

As indicated by Rosenhead [13], for every specification of flow rate and angle of inclination the number of mathematically possible velocity profiles of purely radial motion is infinite. For example, a particular set of conditions might have, mathematically speaking, possible symmetrical profiles with no interior zeroes, two interior zeroes, four interior zeroes, etc., as well as non-symmetrical profiles with 1,3,5,--- etc. interior zeroes. In this investigation only a few of these profiles are considered - in particular, symmetrical flows with no interior zeroes and two interior zeroes, and non-symmetrical flows with one interior zero and three interior zeroes.

Other solutions are indicated and one case of symmetrical flow with four interior zeroes will be noted. The determination of which of the "mathematically possible" flows would actually be assumed by the fluid when a flow rate and angle of inclination are specified can only be made after investigating stability considerations of the flow. Only the relatively simple profiles are to be considered in this study.

A further minor objective of this investigation is the application of the flow parameter graphs to the verification of a modified perturbation technique. This perturbation procedure is used to predict velocity distributions for small flow rates near a critical angle of inclination where standard perturbation techniques are not applicable.

1.3 Procedure

The governing equations of mass and momentum conservation reduce to a single equation which, for each of the different velocity profiles can be solved in terms of elliptic functions. For this investigation each solution for the velocity distribution function is expressed in terms of one or more of the Jacobian elliptic functions. The velocity distribution so expressed is dependent on two "flow parameters," referred to as k and m. These flow parameters are then determined from the boundary conditions by specifying the flow rate and the angle of inclination. The resulting two equations for the flow parameters k and m are transcendental equations in terms of the flow rate and the angle of inclination. Explicit solutions for k and m from these equations in terms of flow rate and angle of inclination are difficult. An alternative procedure used by past authors is to solve graphically or by iterative techniques for one of the variables and then solve explicitly for the other. The method used in this investigation is to relate the Jacobian elliptic functions to standard trigonometric functions. Then explicit solutions can be found for k and the flow rate or for m and the flow rate in terms of the trigonometric parameter and m, or k, respectively. Finally the angle of inclination that has been used, but not explicitly specified, can be determined in terms of the inverse Jacobian elliptic function. As a result graphs of constant m and constant k can be plotted on the flow parameter graph with flow rate and angle of inclination as coordinates and this graph can then be used with reasonable accuracy to determine the values of k and m for any point within the family of graphs.

The above procedure is used and corresponding flow parameter graphs are plotted for each type of flow. The limits of each region are shown graphically and in many cases explicit expressions for the boundaries are derived. Several typical velocity profiles are then plotted from the values read from the flow parameter graphs.

One of the values chosen for plotting a velocity distribution is for small negative flow rate and an angle of inclination of about 128 degrees. For this particular angle the standard perturbation technique for determining the velocity profile breaks down as the unperturbed or linear solution approaches infinity. A general modified perturbation technique developed by Yen and Tang [15 and 16] can be applied to this case. In order to assess the accuracy of the new perturbation technique, exact solutions for the velocity profile determined by use of the flow parameter graph at this critical angle of about 128^{o} can now be used.

1.4 Order of Presentation

Chapter II contains the mathematical development of the general problem. The basic equations and assumptions are presented first. Then the basic equations are reduced to a single second order non-linear differential equation. This equation is then solved in a manner similar to previous work to obtain general solutions for the three basic types of flow: symmetrical flow with outward flow on the centerline, symmetrical flow with inward flow on the centerline, and non-symmetrical flow. The resulting velocity profile functions are discussed and the boundary conditions are applied. The resulting sets of transcendental equations then serve as the basis for the flow parameter charts to be presented in Chapter III.

The sets of transcendental equations derived in Chapter II are used in Chapter III to obtain explicit equations for calculating constant k and constant m curves for the three types of velocity profiles - symmetrical profile with outward flow on the centerline, symmetrical profile with inward flow on the centerline, and non-symmetrical profile. In each case the characteristics of the flow parameter graphs and the limits of each type of flow are discussed. Also, several velocity profiles from various regions of each flow parameter graph are shown.

In Chapter IV an example of the application of the flow parameter graphs developed in Chapter III is presented. The example provides the verification of a modified perturbation technique. The standard perturbation method is reviewed and its limitations of application near certain critical values are discussed. Then a modified method developed by Yen and Tang and applied to other similar types of problems [15 and 16] is applied to this problem of velocity profile for radial flow between non-parallel planes. This modified method gives perturbation solutions for small flow rates near the critical value of the angle of inclination. The velocity distribution functions and flow parameter graphs developed in Chapter III are then used to assess the accuracy of these results.

A summary of the results of this study and suggested alternatives for further study are presented in Chapter V.

II. FLOW MODEL

2.1 Introduction

The governing equations for viscous flows are the familiar Navier-Stokes' equations discussed at length in Schlichting [14]. These equations, even simplified for the conditions specified for this study, still have relatively few exact solutions. However, purely radial flow between non-parallel plates is one case where an exact solution has been developed by Hamel [5], and Jeffery [7]. The solution, expressed in different forms by various authors [5,7,9, and 13], is still very unwieldy since it involves transcendental equations containing various forms of elliptic functions. This difficulty seems to be the primary reason why even though the basic solution is well known the presentation of the solution in a form that would be readily applicable to a specific problem has not been available. In this study the basic derivation of the equations given by Millsaps and Pohlhausen [9] is followed. Then by relating the elliptic functions to standard trigonometric functions the transcendental equations take on a more explicit form. Finally, the application of computer solutions provides flow parameter graphs that can be used to easily determine the exact solutions based on specified values of flow rate and angle of inclination.

This Chapter describes the various velocity profiles to be considered, gives the basic assumptions and governing equations, derives the dimensionless velocity profiles for each type of flow, and applies the boundary conditions to obtain specific solutions.

2.2 Type of Fluid Flow and Coordinate System

The following assumptions are made for this investigation.

1. The fluid is incompressible.
2. The flow is independent of time.
3. Gravitational and other body forces are negligible.
4. The flow is two-dimensional.
5. The fluid is linearly viscous, isotropic, and homogeneous.
6. The reference frame is inertial.

A polar coordinate system (r,θ) is used, as shown in Figure 2.1 where the walls are set at $\theta = \pm\alpha$. The velocity components in the (r,θ) coordinates will be denoted by (u,v). To effect the flow conditions, necessary sources or sinks are assumed at the apex. Examples of pure inward flow towards the apex and pure outward flow away from the apex are shown in Figure 2.2. The dotted lines indicate that r is not allowed to approach zero.

The terms converging and diverging flows were not used in Figure 2.2 as they can be misleading. It is possible to have both converging flow and diverging flow in the same velocity profile, and the difference between purely converging, purely diverging, or part converging and part diverging flows can be a small change in the boundary conditions. To avoid confusion and to abbreviate the names of frequently used descriptive titles the system of identification shown in Figure 2.3 will be used. The first letter, N or S, describes the velocity profile as non-symmetrical or symmetrical, respectively. For symmetrical profiles, a second letter, C or D, denotes that flow on the center line is either converging or diverging, respectively. Also, a number, 0, 1, 2, 3, ---, is used in each name to specify the number of interior zeroes.

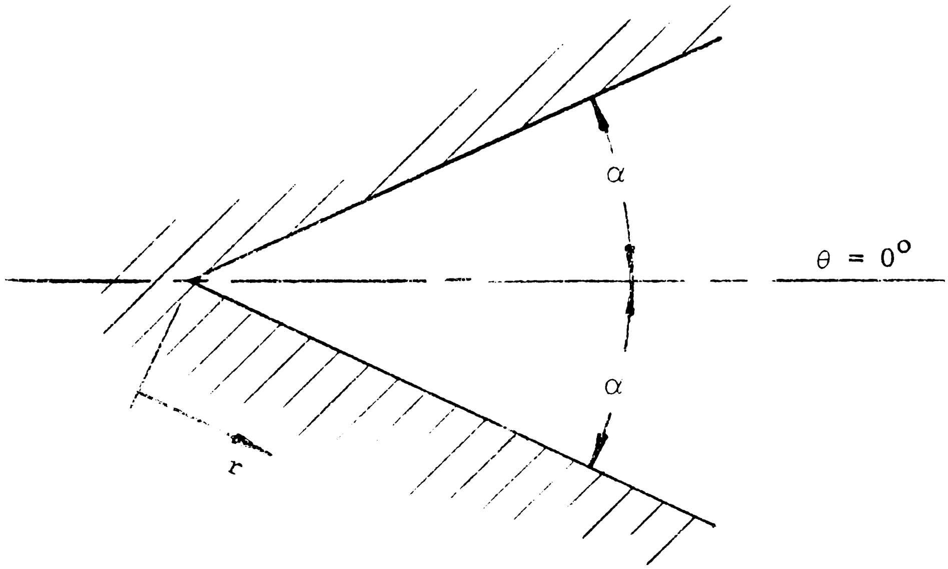

FIGURE 2.1 POLAR COORDINATE SYSTEM

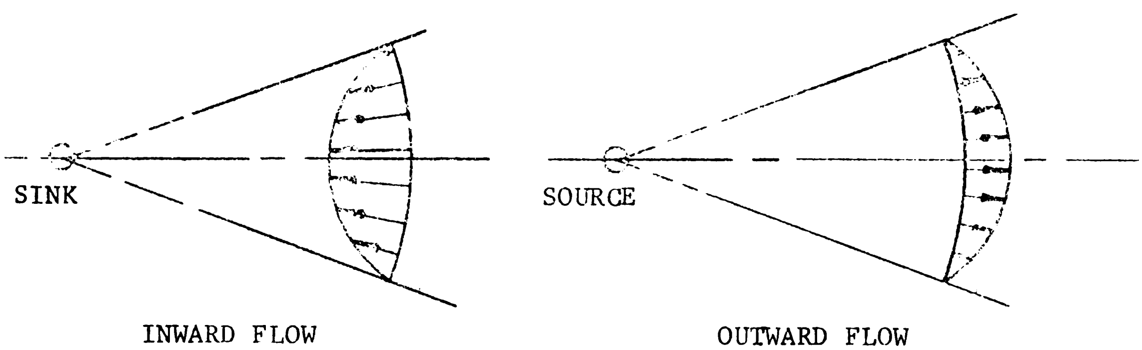

FIGURE 2.2 SCHEMATIC DIAGRAM OF INWARD AND OUTWARD FLOW

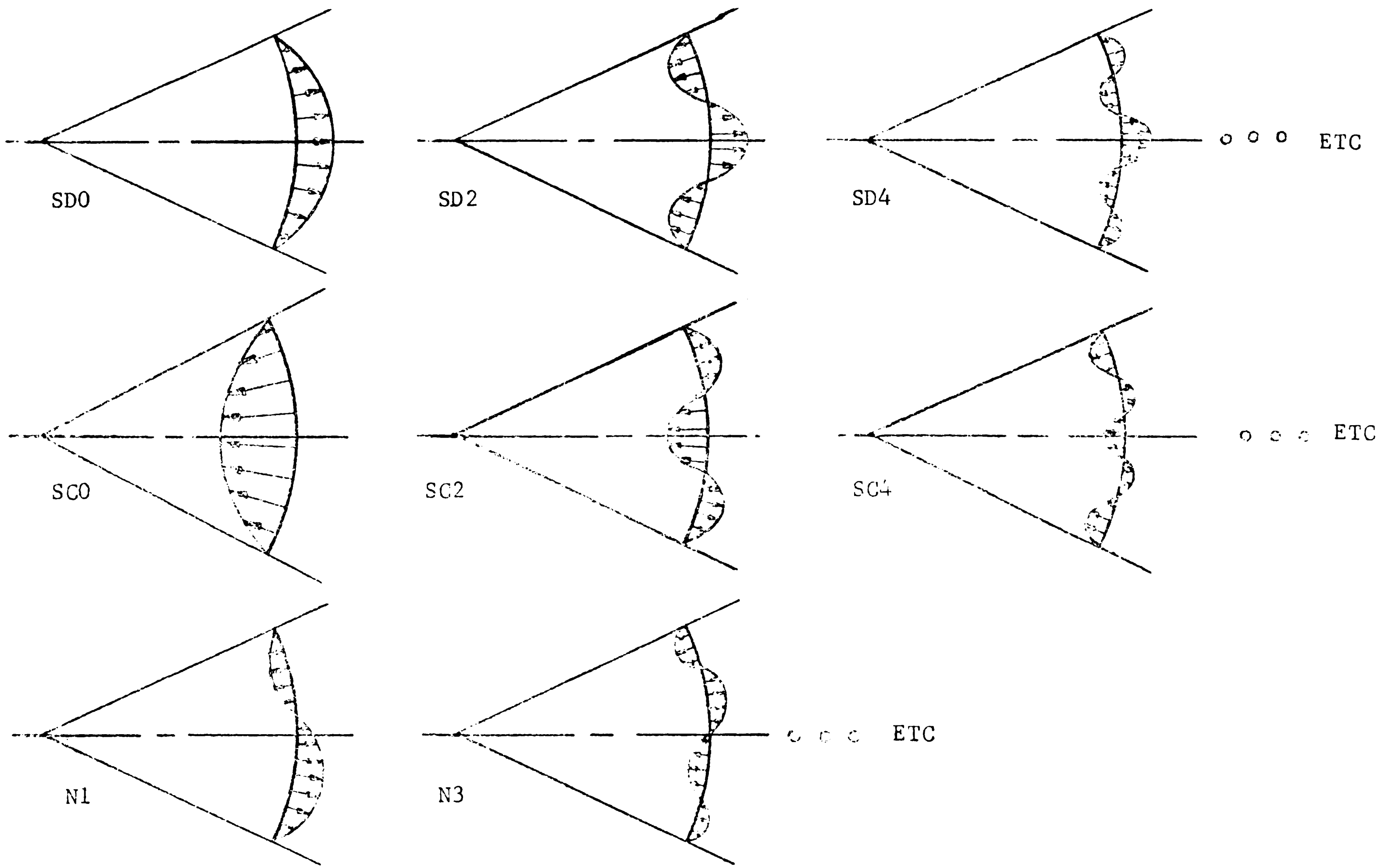

FIGURE 2.3 VELOCITY PROFILE NOTATION

2.3 Fundamental Equations

In the polar coordinate system for the flows described in Section 2.2, the governing equations are presented by Schlichting [14].

Conservation of Mass (Continuity)

$$\frac{\partial(ru)}{\partial r} + \frac{\partial v}{\partial \theta} = 0 \qquad (2.1)$$

Conservation of Momentum

$$u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u - \frac{2}{r^2}\frac{\partial v}{\partial \theta} - \frac{u}{r^2}\right) \qquad (2.2)$$

$$u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{uv}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \nu\left(\nabla^2 v + \frac{2}{r^2}\frac{\partial u}{\partial \theta} - \frac{v}{r^2}\right) \qquad (2.3)$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$ and ρ = density, p = pressure, and ν = kinematic viscosity. The reduction of these equations to a single equation is well-known (for example, see [9]).

Since the flow is purely radial $v \equiv 0$ and equation (2.2.1) then implies

$$\frac{\partial(ru)}{\partial r} = 0 \qquad (2.4)$$

Thus, put $u = \nu F(\theta)/r$ where F is a function of θ only and is the dimensionless velocity profile. Equations (2.2) and (2.3) can now be written as:

$$-\frac{\nu^2 F^2}{r^3} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\nu^2 F''}{r^3} \qquad (2.5)$$

$$0 = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{2\nu^2 F'}{r^3} \qquad (2.6)$$

where the prime denotes differentiation with respect to θ.

Multiplying (2.5) by $\frac{r^3}{\nu^2}$ gives

$$\frac{r^3}{\rho\nu^2}\frac{\partial p}{\partial r} = F^2 + F'' \tag{2.7}$$

Integrating (2.6) with respect to θ gives

$$p = \frac{2\nu^2\rho F}{r^2} + S(r) \tag{2.8}$$

where $S(r)$ is a function of r. Substituting (2.8) into (2.7) gives

$$\frac{r^3}{\rho\nu^2}\frac{dS}{dr} = 4F + F^2 + F'' \tag{2.9}$$

The left hand side of equation (2.9) is a function of r only and the right hand side is a function of θ only. Therefore, each must be equal to a constant. Let this constant be $-J$. Then

$$4F + F^2 + F'' = -J \tag{2.10}$$

and $S(r)$ may be chosen as

$$S(r) = -\int \frac{\rho\nu^2 J dr}{r^3} = \frac{\rho\nu^2 J}{2r^2} \tag{2.11}$$

Therefore, the conservation of mass and the conservation of momentum equations lead to the single second order non-linear ordinary differential equation for F in (2.10). The boundary conditions are specified by the fact that velocity at the wall is zero, and that the total flow between the planes remains constant and can be expressed as an integral of the velocity function, i.e.,

1. $u(\pm\alpha) = 0$, or since $u = \frac{\nu F}{r}$,

$$F(\pm\alpha) = 0 \tag{2.12}$$

2. $\int_{-\alpha}^{\alpha} u(\theta) r d\theta = \text{constant} \equiv Q$ where Q denotes the net volume of fluid passing between the plates per unit of depth in unit time. Expressing the integral in terms of F,

$$\nu \int_{-\alpha}^{\alpha} F(\theta) d\theta = Q$$

or (2.13)

$$\int_{-\alpha}^{\alpha} F(\theta) d\theta = \frac{Q}{\nu} \equiv \epsilon$$

where ϵ^* is a dimensionless flow rate. Equations (2.10) and boundary conditions (2.12) and (2.13) describe completely the mathematical problem.

2.4 Dimensionless Velocity Function

The solutions of equation (2.10) for the dimensionless velocity F are described in [9] and the properties of the various solutions are well illustrated in [11] and [13]. The following is based on these works.

Equation (2.10) may be integrated once after multiplying by $6F'$. This results in

$$2F^3 + 12F^2 + 3(F')^2 + 6JF - 2H = 0 \tag{2.14}$$

where H is a constant of integration. Solving for F' which is $\frac{\partial F}{\partial \theta}$ gives

$$\frac{\partial F}{\partial \theta} = \pm\left(\frac{2}{3}\right)^{\frac{1}{2}}(H - 3JF - 6F^2 - F^3)^{\frac{1}{2}} \tag{2.15}$$

---

* Most previous publications have defined flow rate in terms of Reynolds' Number. For example [9] defines $R_0 = F_0 = u_0 r/\nu$ where u_0 is the centerline velocity. However, when considering flow with interior zeroes u_0 may not be a representative velocity which requires a re-definition of R_0. Therefore, all calculations in this study are made with respect to ϵ as defined above.

It then follows that

$$\theta = \pm \left(\frac{3}{2} \right)^{\frac{1}{2}} \int_{F_1}^{F_2} \frac{dF}{[C(F)]^{\frac{1}{2}}} \tag{2.16}$$

where

$$C(F) \equiv H - 3JF - 6F^2 - F^3 \tag{2.17}$$

The choice of the limits of integration for (2.16) depends on the boundary conditions. Clearly θ is given by an elliptic integral. From (2.12) if $F = 0$ when $\theta = \pm\alpha$ then $C(F)$ in equation (2.16) must be real when $F = 0$. Therefore

$$C(0) \geqslant 0 \tag{2.18}$$

An approximate plot of equation (2.17) can be obtained since $C(0) \geqslant 0$, $C(F) < 0$ for large values of F, and $C(F) > 0$ for large negative values of F. Therefore, there must be at least one positive root for $C(F) = 0$ and there may be three. However, assuming three roots for (2.17), say e_1, e_2, and e_3, then

$$C(F) = H-3JF-6F^2-F^3 = (e_1-F)(e_2-F)(e_3-F) \tag{2.19}$$

$$= e_1e_2e_3+F(-e_1e_2-e_1e_3-e_2e_3)+F^2(e_1+e_2+e_3)-F^3 \tag{2.20}$$

and equating powers of F it shows that

$$H = e_1e_2e_3 \tag{2.21}$$

$$3J = e_1e_2 + e_1e_3 + e_2e_3 \tag{2.22}$$

$$-6 = e_1 + e_2 + e_3 \tag{2.23}$$

Therefore, from (2.23) the three roots must add to -6, so there can only be one real positive root, say e_1. The other roots are both negative or complex conjugates. The possible solutions are sketched in Figure 2.4. The solid curve represents 3 real roots - one positive

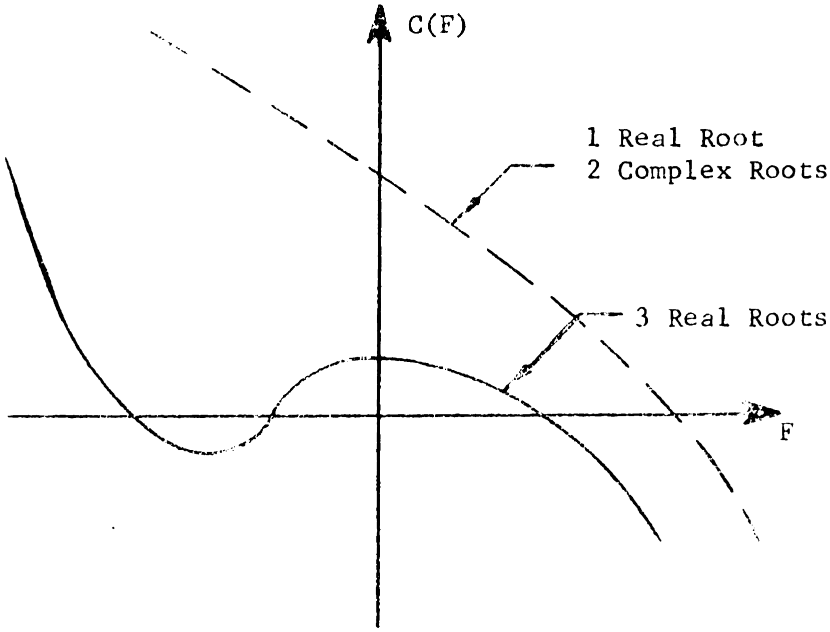

FIGURE 2.4 POSSIBLE SOLUTIONS OF EQUATION (2.17)

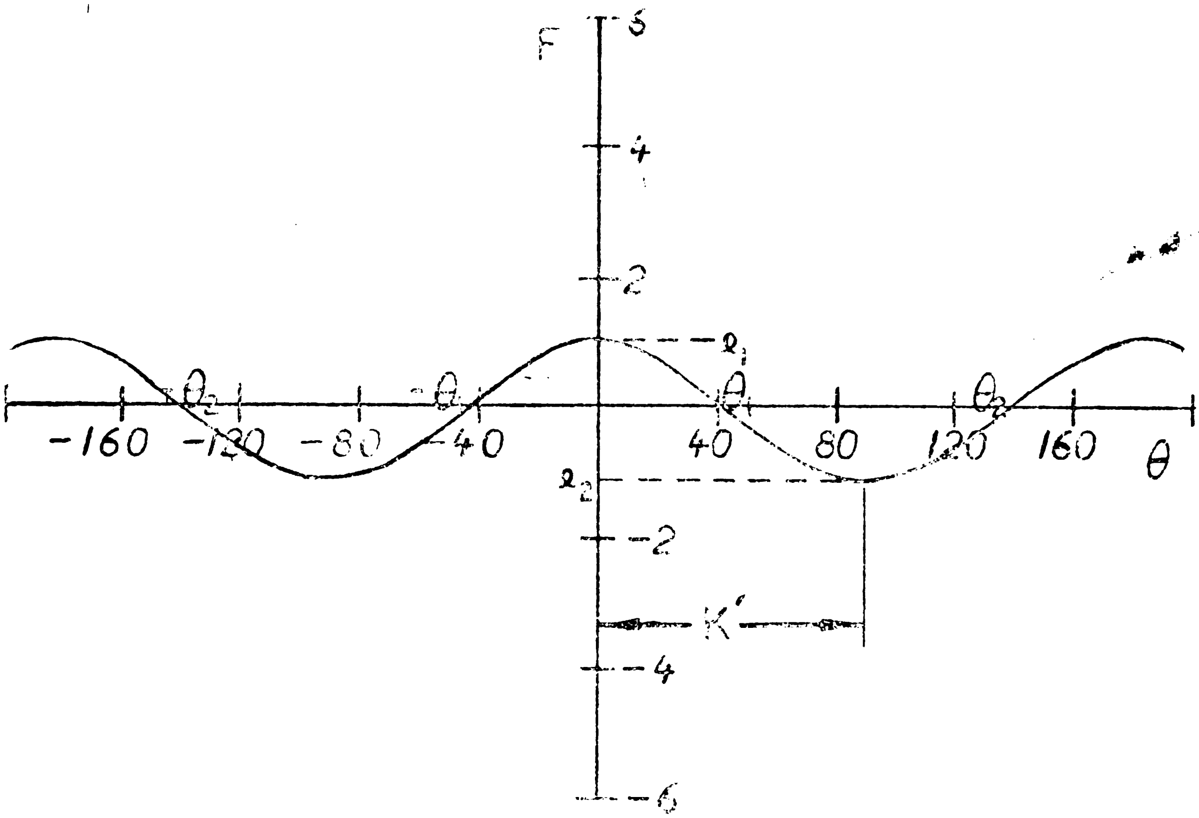

FIGURE 2.5 PLOT OF F, SD FLOW, FOR m_1 = .85, k = .55

and two negative - while the dash line indicates one positive real root and two complex roots. Each curve will of course result in different solutions for F.

The following subsections will consider the solution of equation (2.16) where C(F) has three real roots for SD, SC, and N flows, and then the solution of (2.16) where C(F) has one real and two complex roots for SDO flow. The velocity profile obtained when C(F) has one real and two complex roots will be referred to as SDCO flow.

2.4.1 Dimensionless Velocity Function for SD Profiles. For outward flow on the centerline, which applies to SD0, SD2, SD4, --- etc. profiles, the velocity and therefore the dimensionless velocity function F must be positive for θ equal zero. Therefore, from equation (2.19) C(F) is ≥ 0 only if $F < e_1$ since (e_2-F) and (e_3-F) are both < 0. So

$$0 \leq F \leq e_1 \tag{2.24}$$

Using these limits and equation (2.19), equation (2.16) can be written as*

$$\theta = \left(\frac{3}{2}\right)^{\frac{1}{2}} \int_F^{e_1} \frac{dF}{[(e_1-F)(e_2-F)(e_3-F)]^{\frac{1}{2}}} \tag{2.25}$$

This elliptic integral can be solved in terms of a Jacobian elliptic function by transformation to a standard form. For example, Page 36 of [4] gives

* For convenience the $\pm$ sign in equation 2.16 is not shown, but it will be used later to indicate θ may be positive or negative.

$$\int_F^{e_1} \frac{dF}{[-(F-e_1)(F-e_2)(F-e_3)]^{\frac{1}{2}}} = \frac{2}{(e_1-e_3)^{\frac{1}{2}}} \, sn^{-1}\left[\left(\frac{e_1-F}{e_1-e_2}\right)^{\frac{1}{2}}, k\right] \qquad (2.26)$$

where

$$k^2 = \frac{e_1-e_2}{e_1-e_3} \qquad (2.27)$$

and sn is one of the Jacobian elliptic functions. Using these equations one obtains the solution to (2.25) as

$$\theta = \left(\frac{6}{e_1-e_3}\right)^{\frac{1}{2}} sn^{-1}\left[\left(\frac{e_1-F}{e_1-e_2}\right)^{\frac{1}{2}}, k\right] \qquad (2.28)$$

Let

$$m_1 = \frac{6}{e_1-e_3} \qquad (2.29)$$

solving for F yields

$$F = e_1 + (e_2-e_1)\; sn^2\left(\frac{\theta}{\sqrt{m_1}}, k\right) \qquad (2.30)$$

Equations (2.23), (2.27), and (2.29) can be used to express e_1, e_2, & e_3 in terms of m_1 and k.

$$e_1 = \frac{2(1+k^2)}{m_1} - 2 \qquad (2.31)$$

$$e_2 = \frac{2(1-2k^2)}{m_1} - 2 \qquad (2.32)$$

$$e_3 = \frac{2(k^2-2)}{m_1} - 2 \qquad (2.33)$$

Substituting into (2.30) one obtains

$$F = -\frac{6k^2}{m_1}\left[sn^2\left(\frac{\theta}{\sqrt{m_1}}, k\right) + \frac{1}{3}\left(\frac{m_1-1}{k^2} - 1\right)\right] \qquad (2.34)$$

A plot of F is shown in Figure (2.5) for $m_1 = .65$ and $k = .55$. Items of interest are:

1. $F(0) = e_1$. From equation (2.30).
2. $F(K') = e_2$. From equation (2.30) where K' is the real half period of sn. i.e., $sn(K',k) = 1$.
3. For these values of k and m_1, if α (the inclination of the walls) is equal to θ_1 the flow is similar to the SD0 profile sketched in Figure 2.3. If α is equal to θ_2 the flow is similar to the SD2 profile sketched in Figure 2.3. If α is equal to $(\theta_1 + \theta_2)$ the flow is similar to the SD4 profile in Figure 2.3, and so forth.

Figure 2.6 is a plot of F for several values of m_1 with k equal to a constant. The effect of a larger value of m_1 is to lengthen the cycle, and to cause the entire curve to shift down on the axis. From this plot it is noted that if m_1 is much smaller than .45 or greater than 1.25 the curve shifts above or below the axis, respectively, and there are no practical solutions as F is not zero for any value of θ.

Figure 2.7 is a plot of F for several values of k with m_1 equal to a constant. The effect of a larger value of k is again to lengthen the cycle and also to increase the size of the curve. For this value of m_1, if k is much smaller than .5 there will be no solution as F will not be zero for any value of θ. For $k = 1.0$, the function sn in (2.34) becomes TANH and F reduces to:

$$F = \frac{4}{m_1} - 2 - \frac{6}{\sqrt{m_1}} \tanh \frac{\theta}{m_1} = 5.27 - 10.9 \tanh\left(\frac{\theta}{.74}\right) \quad (2.35)$$

then for large θ, $\tanh \to 1.0$, $F \to -5.63$ and there is only one possible

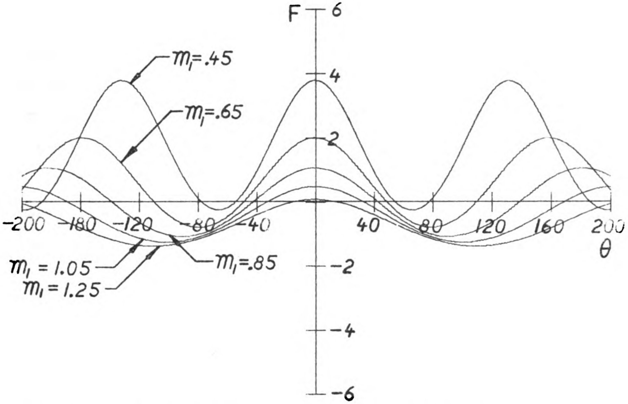

FIGURE 2.6 PLOT OF F FOR SD PROFILES FOR k = .55, m_1 = VARIOUS

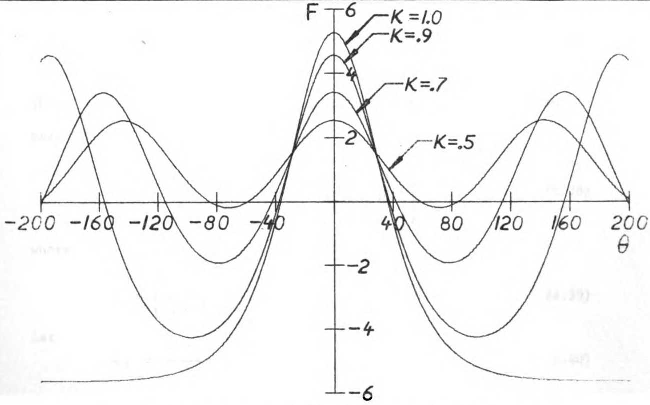

FIGURE 2.7 PLOT OF F FOR SD PROFILES, WITH m_1 = .55, k = VARIOUS

solution for α which has no interior zeroes. This value of $k = 1$ represents the limit for equation (2.34) and is the dividing line between SDO flow being considered here and SDCO flow to be considered in Section 2.4.4. These graphs point out the cyclic nature of F and the limitations on the flow parameters. The same features characterize each of the dimensionless velocity functions.

2.4.2 Dimensionless Velocity Function for SC Profiles. Again for converging flow on the centerline, equations (2.19) and (2.23) can be used to determine the integration limits for (2.16). The only difference is that $F(0) \leqslant 0$. As a result

$$e_2 \leqslant F \leqslant 0 \tag{2.36}$$

With these limits equation (2.16) can be written as

$$\theta = \left(\frac{3}{2}\right)^{\frac{1}{2}} \int_{e_2}^{F} \frac{dF}{[(e_1-F)(e_2-F)(e_3-F)]^{\frac{1}{2}}} \tag{2.37}$$

The integral can be written in the standard elliptic form given on Page 29 of [10] and the solution is

$$\theta = \left(\frac{3}{2}\right)^{\frac{1}{2}} \frac{2}{(e_1-e_3)^{\frac{1}{2}}} \, dn^{-1}\left[\left(\frac{e_2-e_3}{F-e_3}\right)^{\frac{1}{2}}, k\right] \tag{2.38}$$

where

$$k = \left(\frac{e_1-e_2}{e_1-e_3}\right)^{\frac{1}{2}} \tag{2.39}$$

Let

$$m_3 = \frac{e_2-e_3}{6} \tag{2.40}$$

and solve for F.

$$F = e_3 + \frac{e_2-e_3}{dn^2(m_3\theta, k)} \tag{2.41}$$

Using (2.23), (2.39), and (2.40) one obtains

$$e_1 = 2(k^2m_3^{\,2} + m_3^{\,2} - 1) \tag{2.42}$$

$$e_2 = 2(m_3^{\,2} - 2k^2m_3^{\,2} - 1) \tag{2.43}$$

$$e_3 = 2(k^2m_3^{\,2} - 2m_3^{\,2} - 1) \tag{2.44}$$

Substituting into (2.41) yields

$$F = 2[m_3^{\,2}(k^2 - 2) - 1] + \frac{6m_3^{\,2}(1 - k^2)}{dn^2(m_3\theta, k)} \tag{2.45}$$

A plot of equation (2.45) for several values of k and m_3 is shown in Figure 2.8. A comparison of these curves with those in Figures 2.6 and 2.7 indicates the same general shape. In fact, the curves are identical for corresponding values of k, m, and θ. With respect to Figure 2.5, the solution for SD flow assumes flow from $-\theta_1$ to $+\theta_1$, and the solution for SC flow derived above assumes flow from $+\theta_1$ to $+\theta_2$ with the vertical axis shifted to the right by a factor of K'. Therefore, if the following substitutions are made in (2.45), the dimensionless velocity function for SD flow given by equation (2.34) is obtained.

$$m_3^{\,2} = \frac{1}{m_1} \tag{2.46}$$

$$\frac{\theta}{\sqrt{m_1}} = \left(\frac{\theta}{\sqrt{m_1}} + K'\right) \tag{2.47}$$

where

$$sn^2\left(\frac{\theta}{\sqrt{m_1}} + K'\right) = \frac{1 - sn^2\left(\frac{\theta}{\sqrt{m_1}}\right)}{1 - k^2 sn^2\left(\frac{\theta}{\sqrt{m_1}}\right)} \tag{2.48}$$

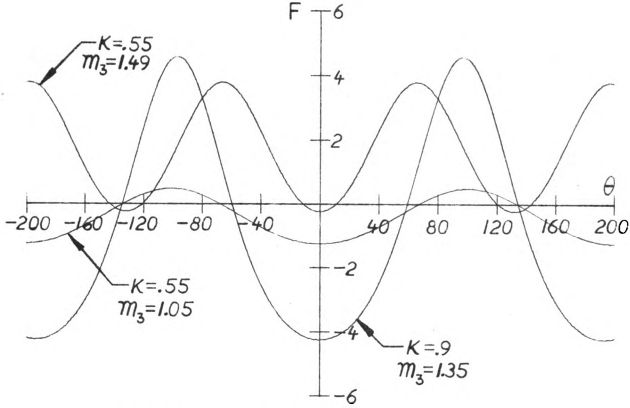

FIGURE 2.8 PLOT OF F FOR SC PROFILES; VARIOUS k AND m_3

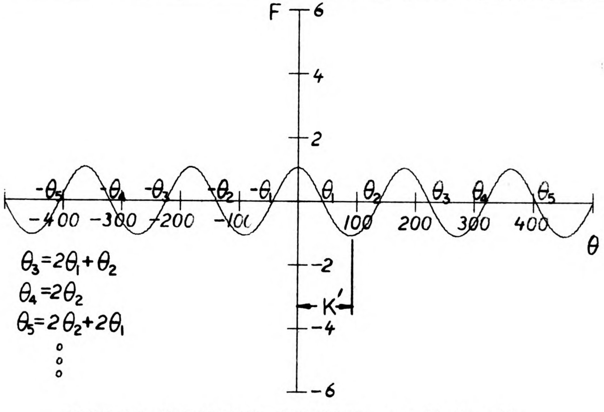

FIGURE 2.9 PLOT OF F FOR SD PROFILES, m_1 = .85, k = .55

2.4.3 Dimensionless Velocity Function for Non-Symmetrical Profiles. The boundary conditions given by equations (2.12) and (2.13) specify the flow rate and require that the velocity be zero at the boundary of $\pm\alpha$. However, there is no requirement that the flow must be symmetrical. Mathematically either symmetrical or non-symmetrical velocity profiles are possible. The plot of F for SD profiles shown in Figure 2.5 is sketched in Figure 2.9 for several cycles.

In Section 2.4.1 it was noted that if $\pm\alpha$ corresponds to $\pm\theta_1$ then flow profile SDO is obtained. In 2.4.2 it was noted that if $\pm\alpha$ corresponds to θ_1 and θ_2, with the F axis shifted by K' then flow profile SCO is obtained. However, since the only requirement is that $F(\pm\alpha) = 0$, then positive and negative α can be any point where $F(\theta) = 0$. For example if $\pm\alpha$ is taken as $-\theta_1$ and $+\theta_2$ then the flow profile appears as in Figure 2.10 (a). Several other possibilities are also shown in Figure 2.10.

Therefore, any of the profiles shown in Figure 2.10 and an infinite number of other possibilities are mathematically feasible. The areas of their applicability will, of course, depend on the boundary conditions. However, the dimensionless velocity function F as given by equation (2.34) for SD flow is used for each case. The m flow parameter for non-symmetrical flow will be designated as m_4 although it is actually the same as m_1 defined by equation (2.29). For this study only the profiles shown in parts (a) and (b) of Figure 2.10 are discussed.

2.4.4 Dimensionless Velocity Function for SDCO Flow. Repeating the equations for θ from Section 2.4,

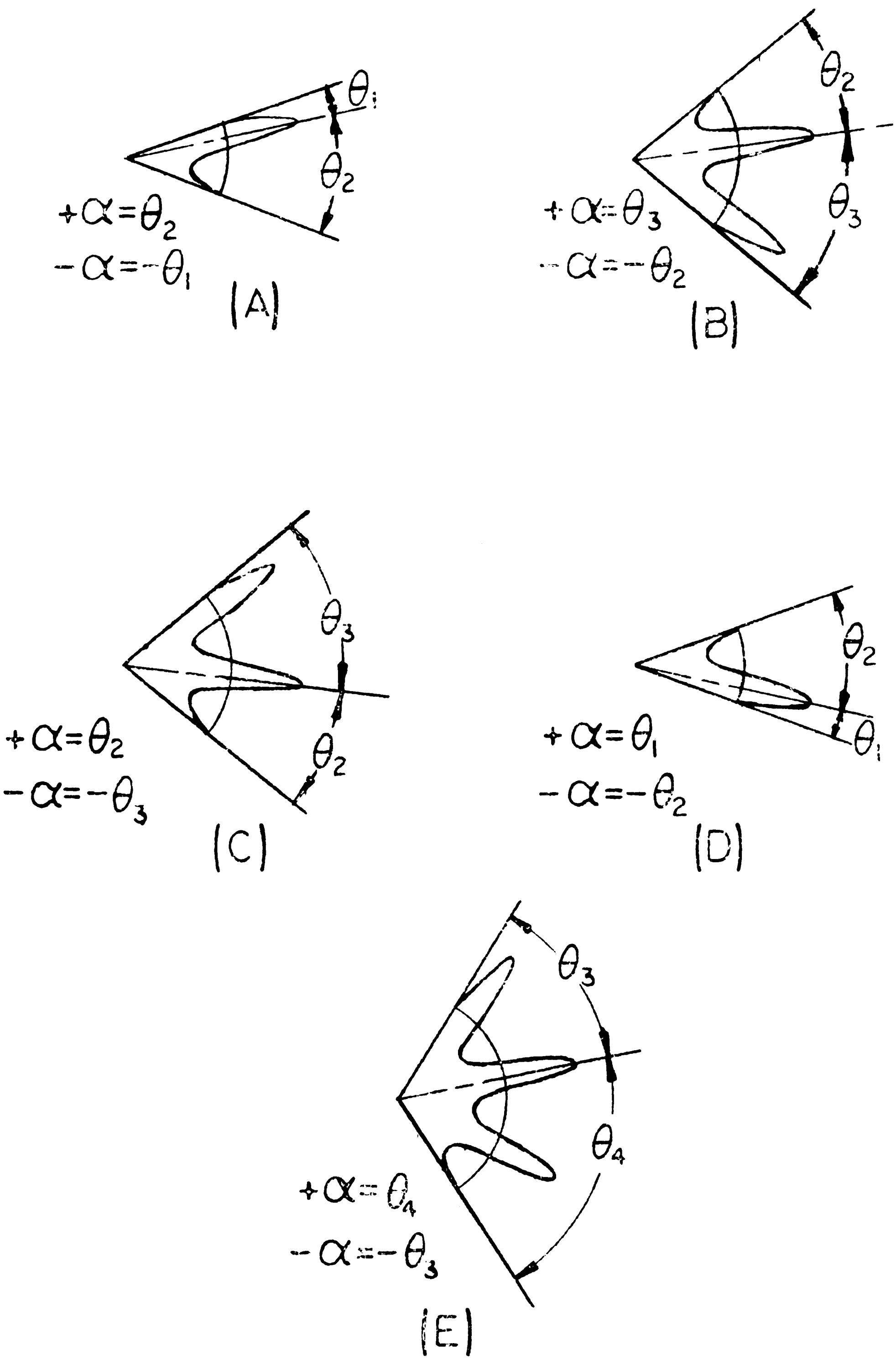

FIGURE 2.10 VARIOUS NON-SYMMETRICAL VELOCITY PROFILES

$$\theta = \left(\frac{3}{2}\right)^{\frac{1}{2}} \int \frac{dF}{[C(F)]^{\frac{1}{2}}} \qquad (2.16)$$

$$C(F) = H - 3JF - 6F^2 - F^3 \qquad (2.17)$$

Now assume one real root $e_1 \geqslant 0$ and two complex roots e_2 and e_3. From (2.23) since $e_1 + e_2 + e_3$ is real, then e_2 and e_3 are complex conjugates.

$$e_2 = a + bi \qquad (2.49)$$

$$e_3 = a - bi \qquad (2.50)$$

Substituting into (2.17) one obtains

$$\begin{aligned} C(F) &= H - 3JF - 6F^2 - F^3 = (e_1-F)(e_2-F)(e_3-F) \\ &= (e_1-F)(F^2+a^2+b^2-2aF) \end{aligned} \qquad (2.51)$$

The real part of e_2 and e_3 is determined from (2.23).

$$a = -(3 + \frac{e_1}{2}) \qquad (2.52)$$

Where $0 \leqslant F \leqslant e_1$, and using (2.51) in (2.16) gives:

$$\theta = \left(\frac{3}{2}\right)^{\frac{1}{2}} \int_F^{e_1} \frac{dF}{[(e_1-F)(F^2+a^2+b^2-2aF)]^{\frac{1}{2}}} \qquad (2.53)$$

The solution of this elliptic integral is given on page 30 of [10] and for this set of parameters is:

$$\theta = \left(\frac{3}{2}\right)^{\frac{1}{2}} \frac{1}{(m_2)^{\frac{1}{2}}} \mathrm{cn}^{-1}\left(\frac{m_2-e_1+F}{m_2+e_1-F}, k\right) \qquad (2.54)$$

where

$$m_2 = (e_1{}^2 - 2ae_1 + a^2 + b^2)^{\frac{1}{2}} \tag{2.55}$$

$$k = \left(\frac{3 + m_2 + \frac{3}{2}e_1}{2m_2}\right)^{\frac{1}{2}} \tag{2.56}$$

Solving (2.54) for F gives

$$F = e_1 - m_2 \left\{ \frac{1-\mathrm{cn}\left[\left(\frac{2m_2}{3}\right)^{\frac{1}{2}}\theta, k\right]}{1+\mathrm{cn}\left[\left(\frac{2m_2}{3}\right)^{\frac{1}{2}}\theta, k\right]} \right\} \tag{2.57}$$

where

$$e_1 = \frac{2}{3}(2m_2k^2 - 3 - m_2) \tag{2.58}$$

$$a = -(3 + \frac{e_1}{2}) \tag{2.59}$$

$$b = (m_2{}^2 - e_1{}^2 + 2ae_1 - a^2)^{\frac{1}{2}} \tag{2.60}$$

Since $e_1 > 0$ there are limits on the values of k and m_2. From equation (2.58)

$$m_2(2k^2 - 1) \gtrsim 3 \tag{2.61}$$

Therefore

$$k > \frac{1}{2} \tag{2.62}$$

and

$$m_2 \gtrsim \frac{3}{2k^2-1} \tag{2.63}$$

A plot of F for some typical values of k and m_2 is shown in Figure 2.11.

The effect of changing values of k and m_2 on F is similar to the case of three real roots discussed in Section 2.4.1. Items of interest are:

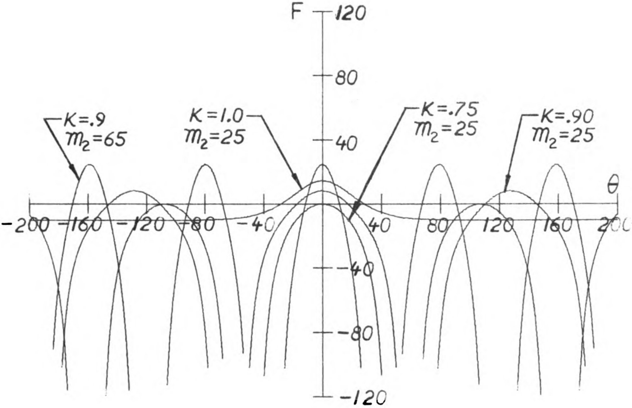

FIGURE 2.11 PLOT OF F FOR SDCO FLOW, FOR VARIOUS m_2 AND k

1. For $k = 1$, $cn(\phi,k) = sech(\phi)$ and the curve becomes asymptotic to a constant value (i.e., for $m = 25$, $F \rightarrow 10.33$). This again defines the dividing line between the solution with 3 real roots and the solution of 1 real root and 2 complex roots.

2. For $\sqrt{\frac{1}{2}} < k < 1$ there are also solutions with 2, 4, or more interior zeroes. However, the lower portion of the curves, which is not shown, reaches values of 1000 to 2000 and the resulting flow profiles with 2 or more interior zeroes have large negative flow rates. Therefore, only the primary solution (i.e., no interior zeroes) was considered. Again the function F is periodic and k and m_2 have similar roles for stretching and adjusting the shape of the curve as with previous cases.

2.5 Application of Boundary Conditions

In Section 2.4 the velocity functions were derived for each type of profile being considered. In each case F is a function of the flow parameters k and m_i ($i = 1,2,3$, or 4) which determines the shape of the velocity profile. The values of k and m_i used in Section 2.4 were arbitrary values chosen for purposes of illustration. However, normally a problem would set specific boundary conditions of α and ϵ as given in Section 2.3.

$$F(\pm\alpha) = 0 \tag{2.12}$$

$$\int_{-\alpha}^{\alpha} F(\theta)\,d\theta = \epsilon \tag{2.13}$$

Application of these boundary conditions to each dimensionless velocity function results in two expressions relating α and ϵ to k and m_i. These expressions can then be used to determine necessary values of k and m_i to affect specified values of α and ϵ.

For each case the application of (2.12) is a straightforward substitution and the application of (2.13) requires an integration of the dimensionless velocity profile between the limits discussed in Section 2.4. The resulting expressions for each flow profile are summarized below:

SD Flow

$$\mathrm{sn}^2\left(\frac{\alpha}{\sqrt{m_1}}, k\right) = \frac{1}{3}\left(1 - \frac{m_1 - 1}{k^2}\right) \tag{2.64}$$

$$\epsilon = \frac{4\alpha}{m_1}\left[3\frac{\sqrt{m_1}}{\alpha}\, E\left(\frac{\alpha}{\sqrt{m_1}}, k\right) - m_1 + k^2 - 2\right] \tag{2.65}$$

Where E is the elliptic integral of the second kind, i.e.,

$$E\,(\phi, k) = \int_0^{\phi} (1 - k^2 \sin^2\phi)\, d\phi \tag{2.66}$$

SDC Flow

$$\mathrm{cn}\left[\left(\frac{2m_2}{3}\right)^{\frac{1}{2}}\theta, k\right] = \frac{m_2 - e_1}{m_2 + e_1} \tag{2.67}$$

$$\frac{\epsilon}{2} = \alpha(e_1 - m_2) + 2(6m_2)^{\frac{1}{2}}\, E\left[\alpha\left(\frac{m_2}{6}\right)^{\frac{1}{2}}, k\right]$$
$$- (6m_2)^{\frac{1}{2}}\left\{\frac{\mathrm{sn}\left[\alpha\left(\frac{m_2}{6}\right)^{\frac{1}{2}}, k\right]\mathrm{dn}\left[\alpha\left(\frac{m_2}{6}\right)^{\frac{1}{2}}, k\right]}{\mathrm{cn}\left[\alpha\left(\frac{m_2}{6}\right)^{\frac{1}{2}}, k\right]}\right\} \tag{2.68}$$

where

$$e_1 = \frac{2}{3}(2m_2k^2 - 3 - m_2) \qquad (2.58)$$

SC Flow

$$dn^2(m_3\alpha, k) = \frac{3m_3^2(1 + k^2)}{1 - m_3^2(k^2 - 2)} \qquad (2.69)$$

$$\epsilon = 4\alpha[m_3^2(k^2-2)-1] + 12m_3\left[E(m_3\alpha, k) - \frac{k^2 sn(m_3\alpha, k)\, cn(m_3\alpha, k)}{dn(m_3\alpha, k)}\right] \qquad (2.70)$$

N1 Flow

$$\alpha = K'\sqrt{m_4} \qquad (2.71)$$

where K' = complete elliptic integral of the first kind (also the half period of sn as noted in Section 2.4.1), i.e.,

$$K'(k) = \int_0^{\pi/2} \frac{d\phi}{(1-k^2\sin^2\phi)^{\frac{1}{2}}} \qquad (2.72)$$

$$\epsilon = \frac{4}{\sqrt{m_4}}\,[(k^2 - m_4 - 2)K' + 3E'] \qquad (2.73)$$

where E' = complete elliptic integral of the second kind, i.e.,

$$E' = \int_0^{\pi/2} (1 - k^2\sin^2\phi)^{\frac{1}{2}} d\phi \qquad (2.74)$$

Also, the values of θ_1 and θ_2 shown in Figure 2.10-(a) can be determined as:

$$\theta_1 = \sqrt{m_4}\, K\left\{\sin^{-1}\left[\frac{1}{3}\left(1 - \frac{m_4 - 1}{k^2}\right)\right]^{\frac{1}{2}}, k\right\} \qquad (2.75)$$

$$\theta_2 = \sqrt{m_4}\, 2K' - \theta_1 \qquad (2.76)$$

where K = elliptic integral of the first kind, i.e.,

$$K(\phi,k) = \int_o^{\phi} \frac{d\phi}{(1-k^2\sin^2\phi)^{\frac{1}{2}}} \qquad (2.77)$$

N3 Flow

$$\alpha = 2K'\sqrt{m_4} \qquad (2.78)$$

$$\epsilon = \frac{8}{\sqrt{m_4}}[(k^2 - m_4 - 2)K' + 3E'] \qquad (2.79)$$

So, α and ϵ for N3 flow are simple twice α and ϵ for N1 flow. θ_1 and θ_2 as shown in Figure (2.10-b) are given by equations (2.75) and (2.76) respectively.

2.6 Summary

The resulting expressions for each flow condition presented in Section 2.5 represent a set of transcendental equations relating desired values of α and ϵ to the flow parameters k and m_i. Previous authors, for example, [9] and [13] have developed similar expressions for the symmetrical flows in terms of Reynolds number. The non-symmetrical profiles have not been discussed as widely although the limits were discussed by Rosenhead [13]. However, in all cases previous authors have stopped at this point and used the equations in a form similar to that shown in Section 2.5 for calculations. Therefore, either a graphical or an iterative method of solution is required. As a result the so-called "known exact solutions" are very difficult to use for determining a velocity profile for set values of α and ϵ. Also, unless one is quite familiar with the equations, their limits are not readily apparent and it is difficult to determine which flow profiles are mathematically possible for particular values of α and ϵ. The representation of these solutions in Chapter III will solve this problem.

III. FLOW PARAMETER GRAPHS

3.1 Introduction

The application of the boundary conditions to each of the velocity distribution functions in Chapter II results in two expressions for each profile relating α and ϵ to the flow parameters k and m_i, where i equals one, two, three, or four. However, since these equations require a graphical or an iterative method of solution to determine values of k and m_i for given values of α and ϵ, their direct application is difficult. It is necessary to determine, for a particular set of α and ϵ, which solutions are possible and what the values of k and m_i are for these solutions so the velocity profile may be determined. Also, it is not readily apparent where dividing lines exist between two types of flow; e.g., where an SD0 flow changes to an SD2 flow.

The above difficulties can be reduced by plotting curves of constant k and m_i on an α-ϵ grid to be called a flow parameter graph. Then for any value of α and ϵ the graph is read to determine corresponding values of k and m_i. Since only curves that represent solutions are plotted, the boundaries of each type of solution are readily apparent. In some cases these boundaries are defined by an analytic expression, and for others the plotted curves serve as the defined boundaries.

For each of the symmetrical profiles to be considered the set of equations derived in Section 2.5 contains a transcendental equation. This indeterminacy arises since for each case one of the equations involves an elliptic function sn, cn, or dn which is a function of two parameters. By relating these elliptic functions to trigonometric functions, which are dependent on one parameter, explicit expressions can be developed for α and ϵ in terms of k and m_i allowing for straightforward calculation for the constant k and constant m curves. Once the curves have been plotted for each profile on a set of coordinate axes, then the graph can be used to determine the boundaries of each type of flow and the values of k and m_i for any set of α-ϵ coordinates within these boundaries. The values of k and m_i can then easily be used to plot the velocity profiles.

The flow parameter graphs are developed and the boundaries of the solutions for each type of flow profile are discussed in this chapter. In each case the transition from an elliptic to a trigonometric function is described and the resulting expressions and limitations are discussed. Then the computer program used for calculating and plotting the curves is discussed and a symbolic flow chart of the program is shown. These flow charts do not include printing or the so-called "book-keeping" operations such as that required to plot dotted rather than solid lines. (Dotted lines are used for all constant m curves, and solid lines are used for all constant k curves.) However, all the basic logic used in solving the expressions is included. Then several typical points are read from each graph and the resulting velocity profiles are shown.

3.2 SD Flow

The case of symmetrical flow with diverging, or outward, flow on the centerline is complicated by the fact that there are two types of solutions - one assuming three real roots in the cubic expression for C(F) as covered in Section 2.4.1, and one assuming one real and two complex roots in the cubic expression for C(F) as covered in Section 2.4.4. These two cases will be considered separately.

3.2.1 Solution where C(F) has Three Real Roots. The expressions developed in Section 2.5 by application of the boundary conditions to the dimensionless velocity function are

$$\mathrm{sn}^2\left(\frac{\alpha}{\sqrt{m_1}},k\right) = \frac{1}{3}\left(1 - \frac{m_1-1}{k^2}\right) \qquad (2.64)$$

$$\epsilon = \frac{4\alpha}{m_1}\left[3\frac{\sqrt{m_1}}{\alpha}\,E\left(\frac{\alpha}{\sqrt{m_1}},k\right) - m_1 + k^2 - 2\right] \qquad (2.65)$$

As pointed out in the Introduction the difficulty in working with these expressions is that the elliptic function sn is dependent on two parameters, e.g., $\frac{\alpha}{\sqrt{m_1}}$ and k. However, the function is really quite similar to the trigonometric function sine. In fact, for $k = 0$, $\mathrm{sn}(\phi,0) = \sin(\phi)$. Although the parameter k has the effect of stretching out the period, $\mathrm{sn}(\phi,k)$ still only has values between plus and minus one. Finally, in the limit, $\mathrm{sn}(\phi,1) = \tanh(\phi)$ and the period is infinite. Because of these relationships sn can be defined in terms of sine. For this case consider

$$\sin(A) = \mathrm{sn}\left(\frac{\alpha}{\sqrt{m_1}},k\right) \qquad (3.1)$$

where

$$A = \sin^{-1}\left[\mathrm{sn}\left(\frac{\alpha}{\sqrt{m_1}},k\right)\right] \qquad (3.2)$$

Or

$$\alpha = \sqrt{m_1}\;\mathrm{sn}^{-1}[\sin(A),k] \qquad (3.3)$$

Further

$$\mathrm{sn}^{-1}[\sin(A),k] = K(A,k) \qquad (3.4)$$

where K is the incomplete elliptic integral of the first kind. Therefore,

$$\alpha = \sqrt{m_1}\ K(A,k) \tag{3.5}$$

With this substitution equation (2.64) can be solved for m_1 to give

$$m_1 = 1 + k^2[1-3\ \sin^2(A)] \tag{3.6}$$

The system of equations does not lend itself to explicit equations for k and m_1 which could then be used in calculating the velocity profile. However, equations (2.65), (3.5) and (3.6) can now be used to calculate directly values of m_1 and ϵ for various values of k and A (with corresponding values of α).

A series of curves for k = constant and a series of curves for m_1 = constant will be combined to produce the flow parameter graph. For the present consider only the constant k curves. The flow chart in Figure 3.1 represents the subroutine used to plot the constant k curves for $0^o < A < 90^o$ from the above equations. It will be noted later that when $A = 90^o$, where $\sin(A)$ and, hence, $\mathrm{sn}(\frac{\alpha}{\sqrt{m_1}},k)$ is a maximum, SD0 flow changes to SD2 flow. The constant k curves as plotted by the subroutine represented in Figure 3.1 are shown in Figure 3.2.

Items of particular interest are:

1. As k approaches zero the curves approach the $\epsilon = 0$ axis.
2. As k approaches 1.0 the curves approach an area on the left side of the graph which will be covered later with the solution for SDC0 flow. Therefore, $k = 1.0$ is the boundary..
3. The area within the plotted lines $k = 0.0$ and $k = 1.0$ represents the only area for solutions of these equations for symmetrical diverging flow with no interior zeroes (except SDC0 flow).

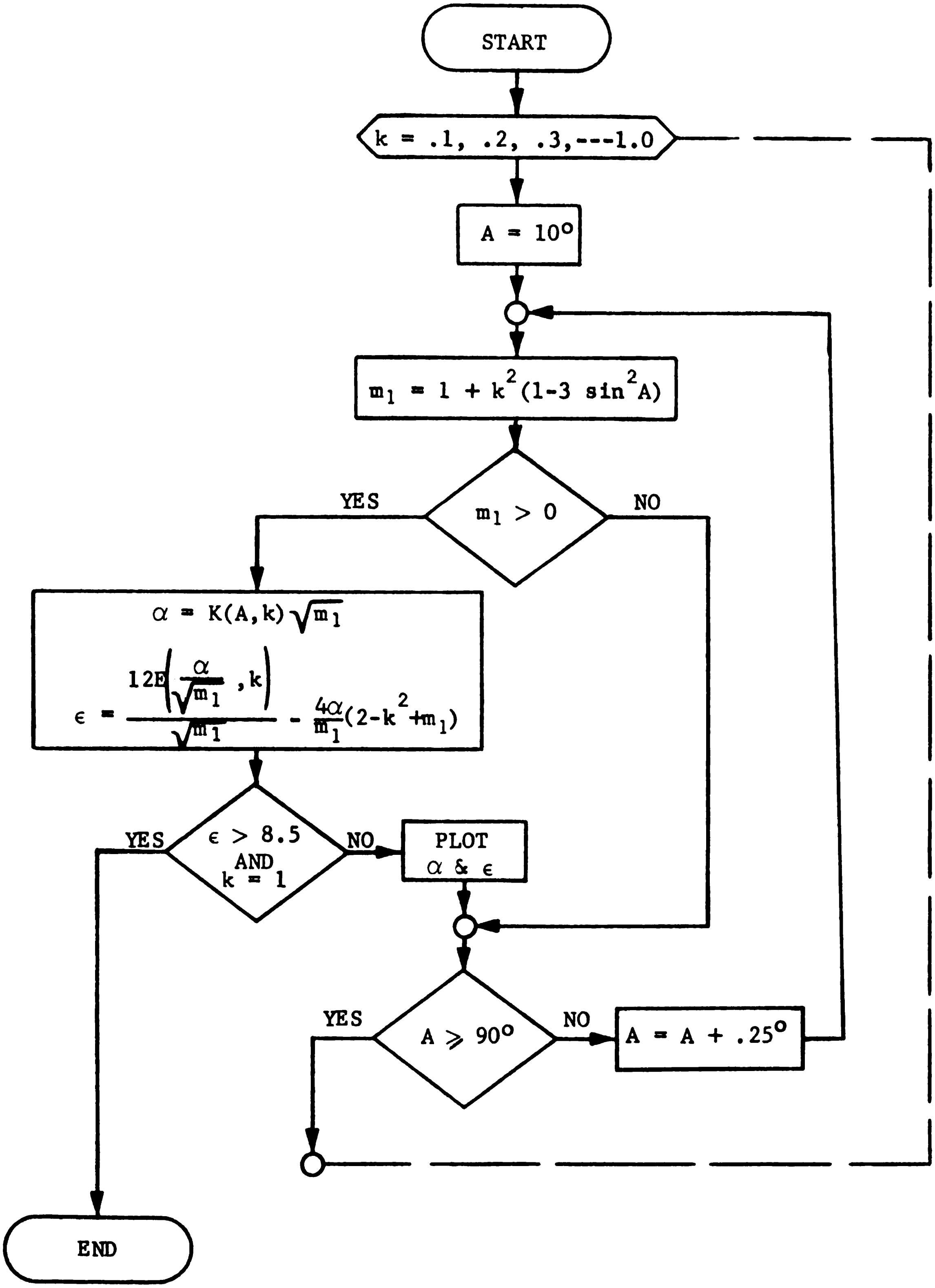

FIGURE 3.1 SYMBOLIC FLOW CHART FOR CONSTANT k CURVES, SDO FLOW

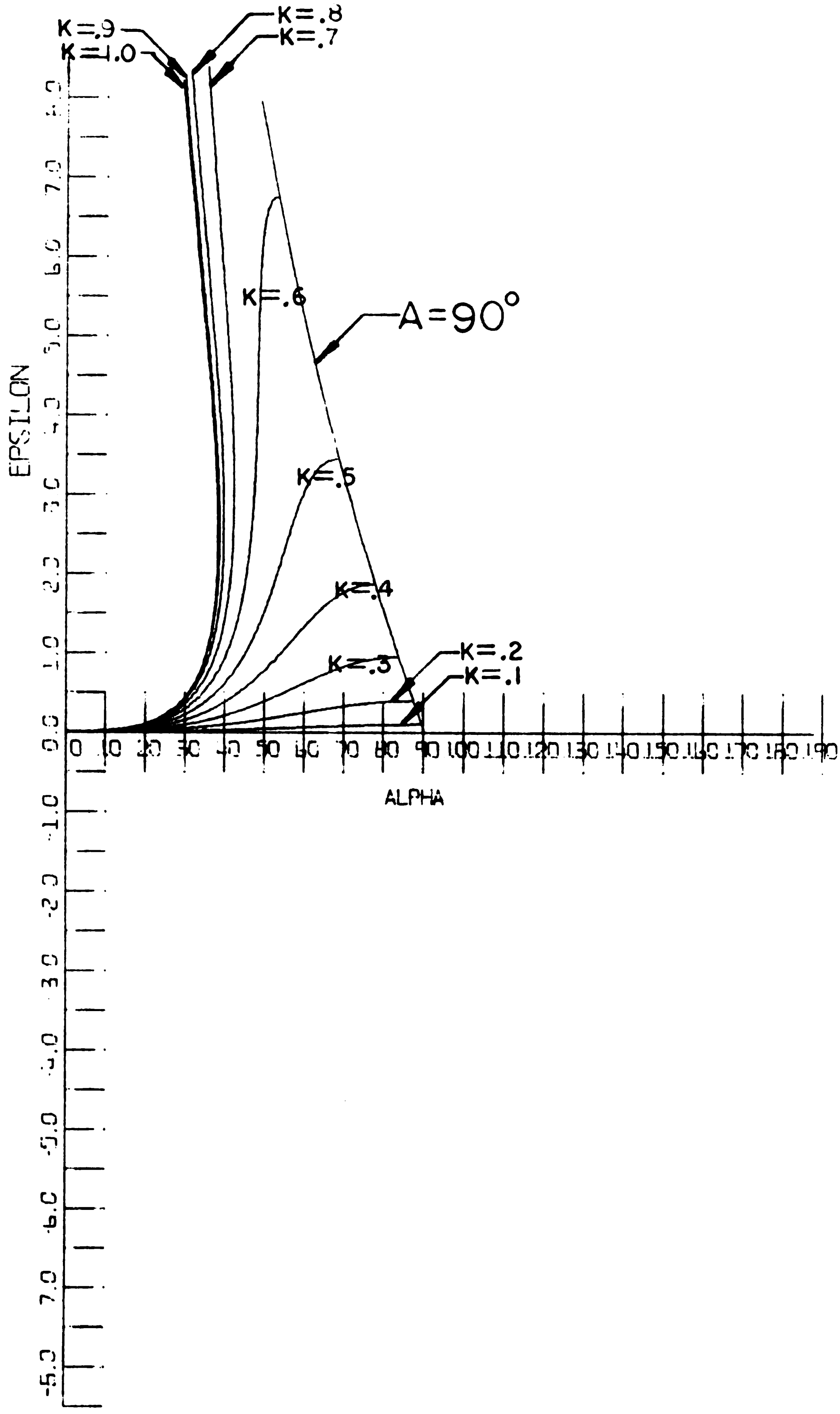

FIGURE 3.2 CONSTANT k CURVES, SDO FLOW

4. Note, for large ϵ all curves approach the $\alpha = 0$ axis. Therefore, for smaller values of ϵ a wider range of angles, α, can be used. For very large values of ϵ the angle α must be very small to insure purely divergent flow.

5. The curves are a maximum for a value of α that corresponds to $A = 90^{o}$, e.g., for $k = 0.0$, the elliptic function sn is equal to the trigonometric sine function, and therefore, $\alpha = 90^{o}$ is this maximum.

   The equation for this boundary can be obtained from equations (2.65), (3.5), and (3.6) by setting $A = 90^{o}$.

$$\mathrm{sn}\,\frac{\alpha}{\sqrt{m_1}},k \quad = \quad \sin 90^{o} = 1.0$$

   Therefore

$$k^2 \quad = \quad \frac{1}{2}\,(1 - m_1) \tag{3.7}$$

$$\epsilon \quad = \quad \frac{12}{\sqrt{1-2k^2}}\left\{E[\mathrm{sn}^{-1}(1),k] - (1 - k^2)\,\mathrm{sn}^{-1}(1)\right\} \tag{3.8}$$

   or

$$\epsilon \quad = \quad \frac{12}{\sqrt{1-2k^2}}\,[E(\pi/2,k) - (1 - k^2)K(\pi/2,k)] \tag{3.9}$$

   where $K(\pi/2,k) \equiv K'$ = Complete elliptic integral of the first kind.

   $E(\pi/2,k) \equiv E'$ = Complete elliptic integral of the second kind.

   This curve is also plotted on Figure 3.2 and labeled as $A = 90^{o}$. One may then state that <u>to the right of this curve any solution must encompass some backflow</u>.

6. To have a real solution for α from equation (3.3), the value of m_1 must be positive. From equation (3.6), m_1 is

always positive if $k \lesssim \sqrt{\frac{1}{2}}$. If $k > \sqrt{\frac{1}{2}}$ then sin A must be less than $\sqrt{\frac{1}{3} + \frac{1}{3k^2}}$. For example, if k = .9, then sin (A) < $\sqrt{\frac{1.81}{2.43}}$, i.e., A < ≈ 59.7° or A > ≈ 120.3°. Then α will approach zero for any value of $k > \sqrt{\frac{1}{2}}$ as ϵ increases since $\alpha = \sqrt{m_1}\, K(A,k)$ and $m_1 \to 0$ while K remains bounded.

Figure 3.3 shows the same constant k curves for $0^\circ < A < 180^\circ$. The part of the curves that correspond to $A > 90^\circ$ represents SD2 solutions with backflow shown in Figure 2.3 Items of interest for this graph are:

1. For space conservation the curves were stopped at $|\epsilon| \approx 8.5$. For the curves with larger values of k (i.e., curves with k > .60) the curve is discontinuous. (For $k > \sqrt{\frac{1}{2}}$ this discontinuity is required by conditions on m_1 as pointed out in item 6 above.)

2. For any particular value of k the curve becomes tangent to the boundary on the right side of the graph then continues to the left of this boundary. The result is that a second solution with two interior zeroes is superimposed over both the SD0 region to the left of the $A = 90^\circ$ line and the SD2 region to the right of the $A = 90^\circ$ line. This secondary solution actually has solutions all the way to the $\alpha = 0^\circ$ axis even though only a few lines have been plotted to reduce confusion due to the over-lapping families of curves. Therefore, to the left of the $A = 90^\circ$ line, for example, $\alpha = 50^\circ$ and $\epsilon = 2.0$, there are two possible solutions - the primary with no interior zeroes and a secondary with two interior zeroes. To the right of the $A = 90^\circ$ line, for example, $\alpha = 85^\circ$ and

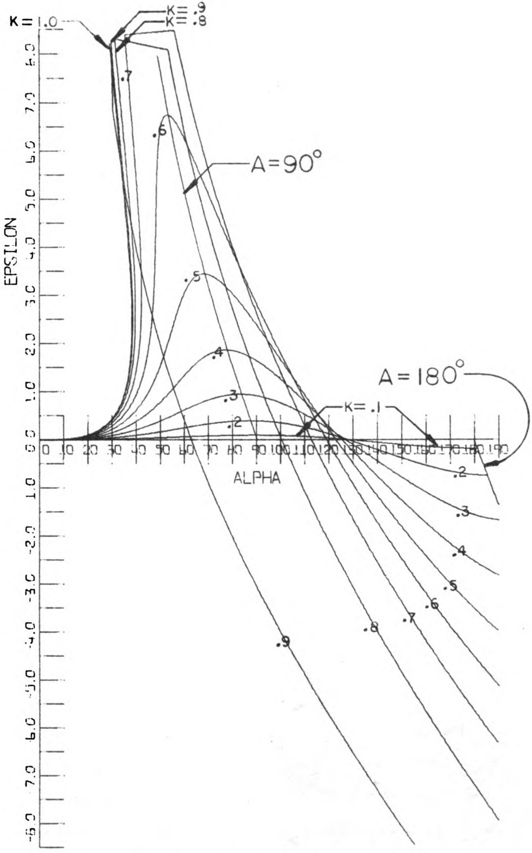

FIGURE 3.3 CONSTANT k CURVES, SDO & SD2 FLOW

$\epsilon = 2.0$, there are two possible solutions each with two interior zeroes.

The existence of the second solution with two interior zeroes simply implies that there are two combinations of k and m_1 which will both cause F, the velocity profile function, to be zero at $\pm\alpha$. Referring back to Figure 2.5, SD2 flow occurs if $\alpha = \pm\theta_2$. Also, referring to Figures 2.6 and 2.7 it is easy to see that two different curves could have the same values of θ_2. For example, the $m_1 = 1.05$ and $k = .55$ curve on Figure 2.6 and the $k = .9$ and $m_1 = .55$ curve on Figure 2.7 both have values of θ_2 very near 160^{o} even though the shape of the curves and, therefore, the shape of the velocity profile is quite different.

3. As the curves continue for larger values of A they drop below the $\epsilon = 0$ axis and in this region represent solutions that are symmetrical, have outward flow on the centerline, have two interior zeroes, and have a net flow back toward the apex. Therefore, even though the equations were derived for so-called diverging flow, physically they can actually represent negative or inward flow. This condition can be easily visualized by referring first to Figure 2.5. It was noted in Section 2.4.1 that if α is chosen to be θ_2 then there will be two interior zeroes. Therefore, choose $\alpha = \theta_2$ for each case and refer to Figure 2.6 or Figure 2.7, where the area between the curve and the axis represents the amount of fluid flow. Then if the positive area is greater than the negative area the net flow will be outward: e.g., $k = .5$ on

Figure 2.6. If there is more negative than positive area then the net flow will be inward: e.g., k = .9 on Figure 2.6. Also, a value of k can be found so that the negative and positive areas just cancel so there is no net flow. For example k = .7 on Figure 2.6 is close to this condition.

4. A portion of the curve for $A = 180^{\circ}$ is also shown on Figure 3.3. This curve represents the second time that sine and hence sn is a maximum. The equation for ϵ obtained from equation (2.65) by setting $A = 180^{\circ}$ is

$$\epsilon = \frac{24}{\sqrt{m_1}} [E(\pi/2,k) - K(\pi/2,k)] = \frac{24}{\sqrt{m_1}} (E'-K') \qquad (3.10)$$

Area to the right of this curve represents solutions with four interior zeroes, and the solutions could be carried to six interior zeroes, etc. However, any area to the right of $\alpha = 180^{\circ}$ is not physically possible and will not be considered.*

5. For positive ϵ regions to the right of the curves (at $\epsilon = 0$ this is about 128°), there is no solution. For example, at $\alpha = 130^{\circ}$ it is not possible to obtain a solution of SD0 or SD2 flow.**

For the constant m curves the same set of equations, (2.65), (3.5), and (3.6) are used, but (3.6) is solved for k giving:

---

* The fact that secondary solutions, which are symmetrical, have four, six, or more interior zeroes, and have positive flow at the centerline, would exist over much of the region described in Figure 3.3 was not considered in this investigation.

** Other velocity profiles to be considered in later sections will have solutions in this region.

$$k = \sqrt{\frac{m_1 - 1}{1 - 3\sin^2 A}} \tag{3.11}$$

The negative root is not meaningful. Also, the value of A is limited by the fact that

$$\text{if } m_1 > 1, \text{ then } \sin A < \sqrt{\frac{1}{3}} \tag{3.12}$$

or, for the range of 0 to 180°: $A < 35.2^\circ$ or $A > 144.8^\circ$

if $m_1 < 1$, for the range of 0 to 180°: $35.2^\circ < A < 144.8^\circ$

if $m_1 = 1$, then $k = 0$, therefore, $\epsilon = 0$ and there is no meaningful flow.

With these considerations, the flow chart on Figure 3.4 represents the computer program used to obtain the constant m curves. The curves for $0^\circ < A < 90^\circ$ are shown in Figure 3.5. Items of interest are:

1. There are two separate families of curves, one for $m_1 < 1.0$ and one for $m_1 > 1.0$.
2. The curves are drawn as dotted lines so they would appear different from the constant k curves.
3. As $m_1 \rightarrow 1.0$, the curves approach the $\epsilon = 0$ axis.
4. As $m_1 \rightarrow 0.0$, the allowable values of $\alpha \rightarrow 0.0$.

Figure 3.6 shows the same constant m_1 curves for $0^\circ < A < 180^\circ$. (Except that the curves for $m_1 > 1$ and $A > 144.8^\circ$ are not included.) Items of interest are:

1. As with the constant k curves there is a corresponding second solution which represents flow with two interior zeroes overlapping the graph..
2. The procedure used in the computer program to produce the

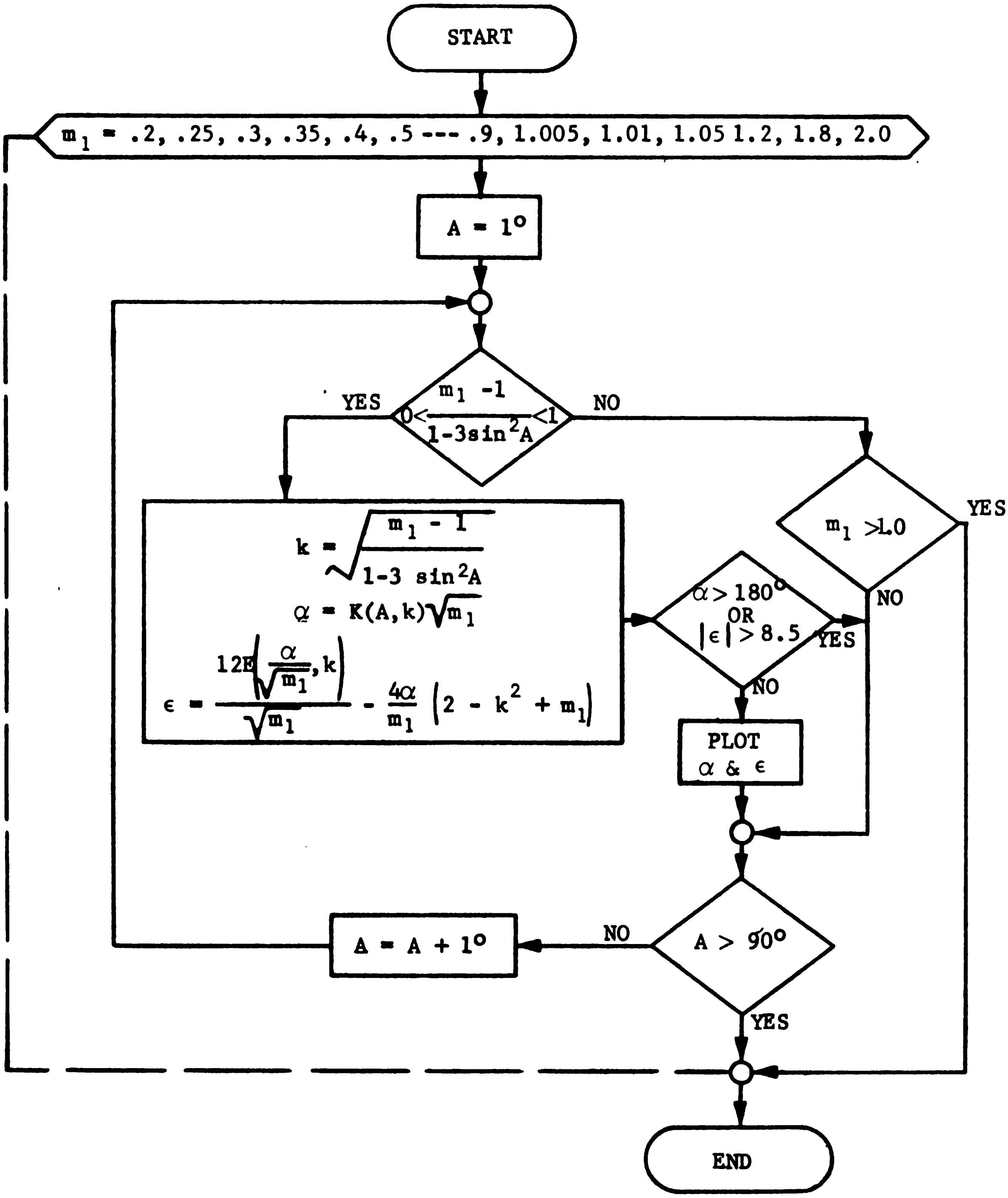

FIGURE 3.4 SYMBOLIC FLOW CHART FOR CONSTANT m CURVES, SDO FLOW

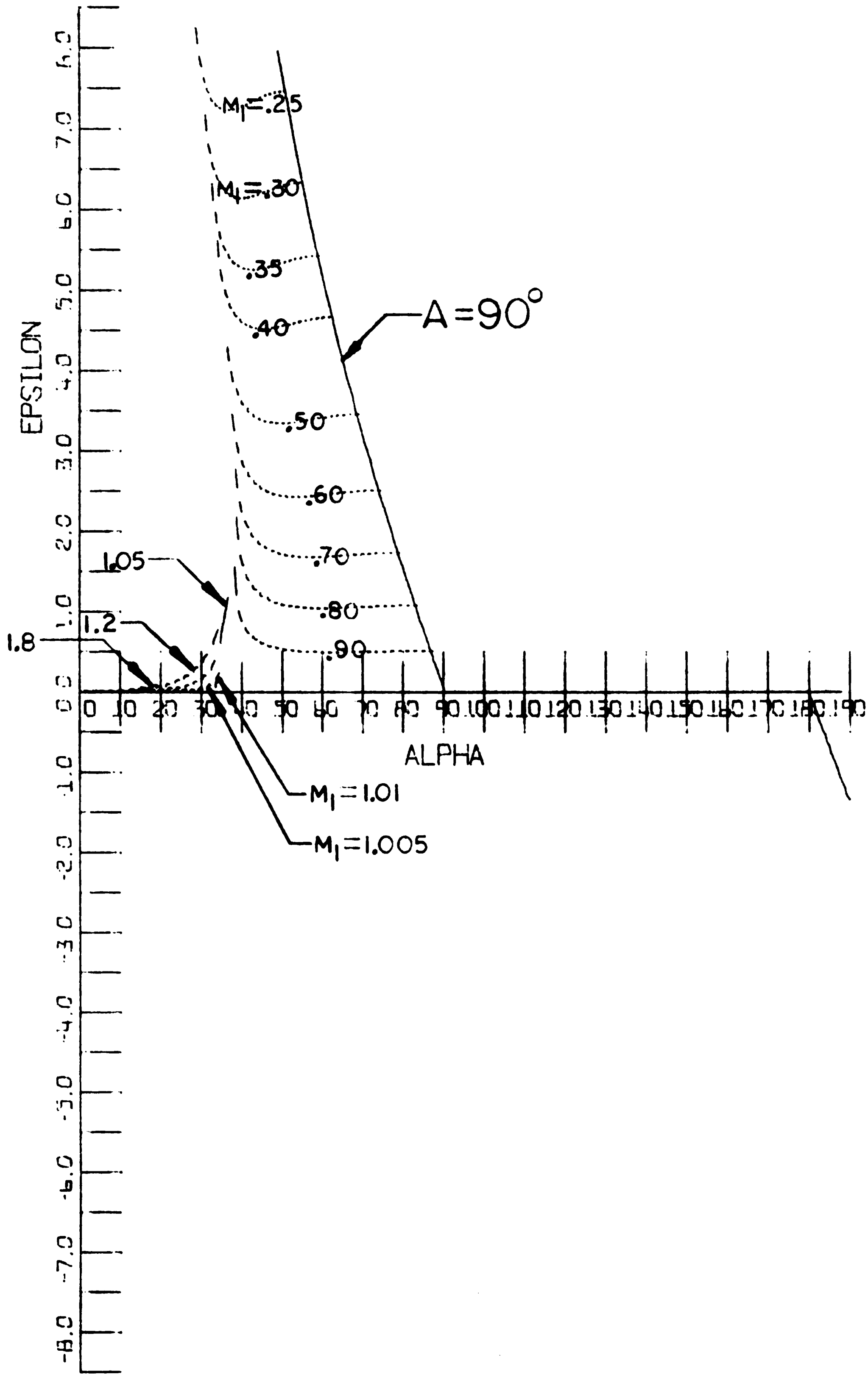

FIGURE 3.5 CONSTANT m CURVES, SDO FLOW

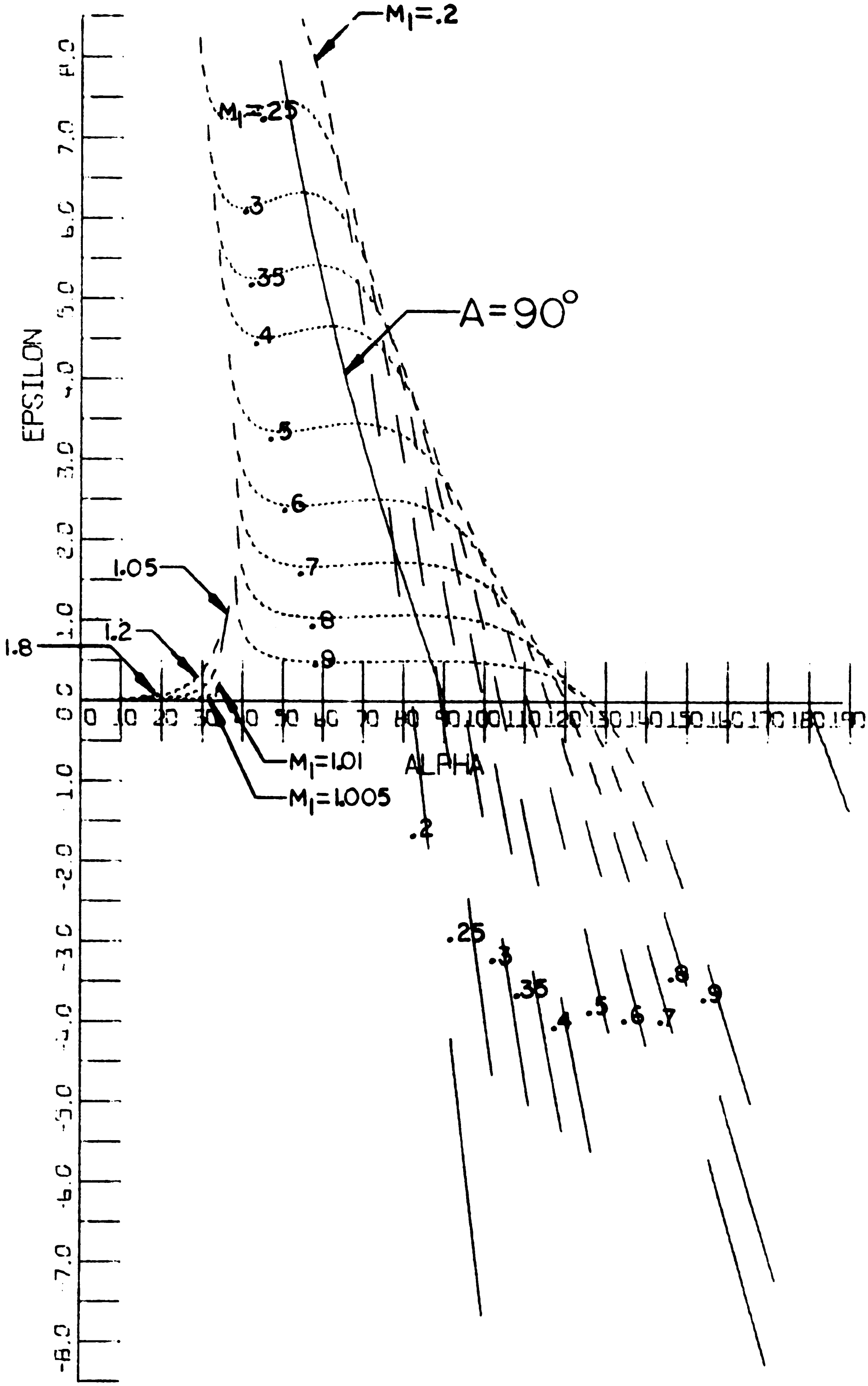

FIGURE 3.6 CONSTANT m CURVES, SDO AND SD2 FLOWS

dotted line effect is related to the increment of A. Therefore, the length of the dashes is not constant.

Figure 3.7 is the combination of the constant k and the constant m curves to give the complete flow parameter graph for SD0 and SD2 flow. This graph includes the lines for the second family of solutions with two interior zeroes. Figure 3.8 is the same as 3.7 except that the second family of solutions is not included and the resulting graph is clearer and easier to use for determining the primary solution which is usually the one desired. The computer routine used to obtain Figure 3.7 is called subroutine DIVERGE and is shown with some sample printout of calculated points in Appendix A. The routine is in FORTRAN and is written from the flow charts given on Figures 3.1 and 3.4. In addition there is a graph subroutine for the axis and two subroutines to determine the elliptic integrals and functions. The graph and function subroutines are used with each flow parameter subroutine and therefore appears in the back of Appendix A.

Figure 3.8 can now be used to determine the flow parameters (m_1 and k) for any value of α and ϵ* within the range of solutions and thereby determine the velocity profile. If the point is to the left of the A = 90° line the flow will be SD0. If the point is to the right the flow will be SD2. If the point is on the A = 90° line, then the corresponding value of ϵ represents the maximum flow rate that can be attained with SD0 flow for the corresponding value of α. Further increasing ϵ for this value of α is only attained by allowing

* If an α-ϵ point is between the constant k lines or the constant m lines then extrapolation must be used for intermediate use. In actual application more intermediate lines could be shown in the area of interest. Fewer lines were used here simply to reduce computer time.

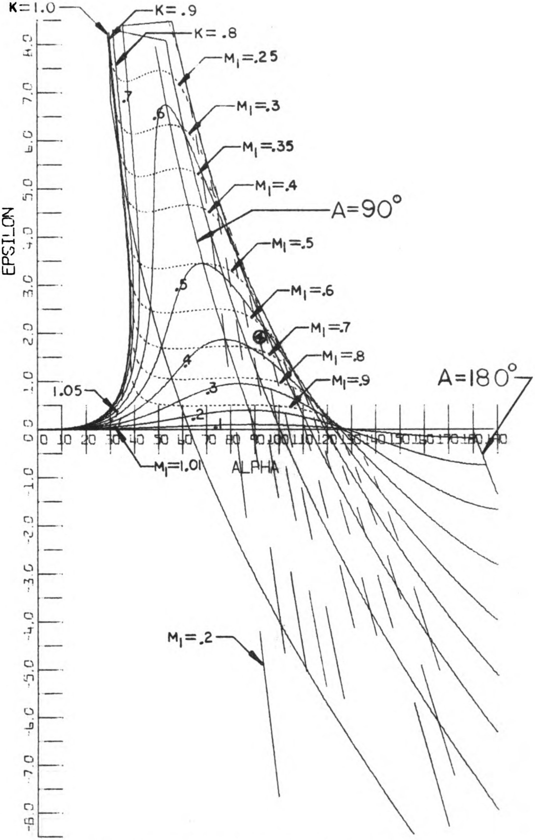

FIGURE 3.7 FLOW PARAMETER GRAPH, SDO AND SD2 FLOWS

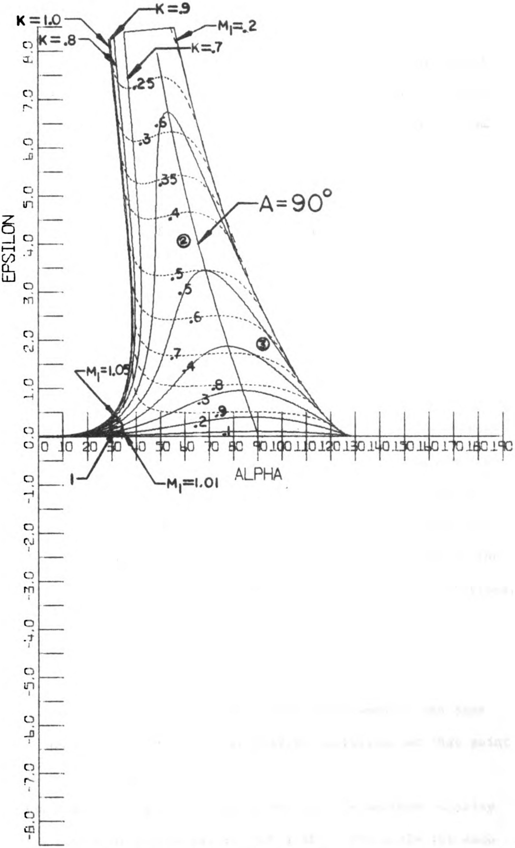

FIGURE 3.8 FLOW PARAMETER GRAPH SD0 AND SD2 FLOWS (PRIMARY SOLUTIONS)

backflow as in SD2 flow. Three points with the above characteristics are selected and shown on Figure 3.8. The values of k and m_1 read from the graph for these points are approximate values as scaled from the graph.

1. $\alpha = 31^o$ $\quad k = .45$

   $\epsilon = .2$ $\quad m_1 = 1.05$

2. $\alpha = 60^o$ $\quad k = .525$

   $\epsilon = 4.1$ $\quad m_1 = .445$

3. $\alpha = 92^o$ $\quad k = .45$

   $\epsilon = 1.95$ $\quad m_1 = .65$

These values are then used in equation (2.34) to determine the corresponding velocity profiles. The profiles are plotted with the computer program shown in Appendix B and the resulting profiles are shown in Figure 3.9. A fourth profile is also shown in Figure 3.9. This profile is obtained by plotting point 3 on Figure 3.7 and determining the following values of k and m_1 from the family of secondary solutions.

4. $\alpha = 92^o$ $\quad k = .75$

   $\epsilon = 1.95$ $\quad m_1 = .35$

It should be noted that profiles 3 and 4 both correspond to the same value of α and ϵ and represent two possible solutions at that point. Items of interest on Figure 3.9 are:

1. Each velocity profile is scaled so that its maximum velocity at a radius of 1.0 inches is one inch. The scale for each plot is as shown.
2. Each velocity profile is calculated and shown at two radii -

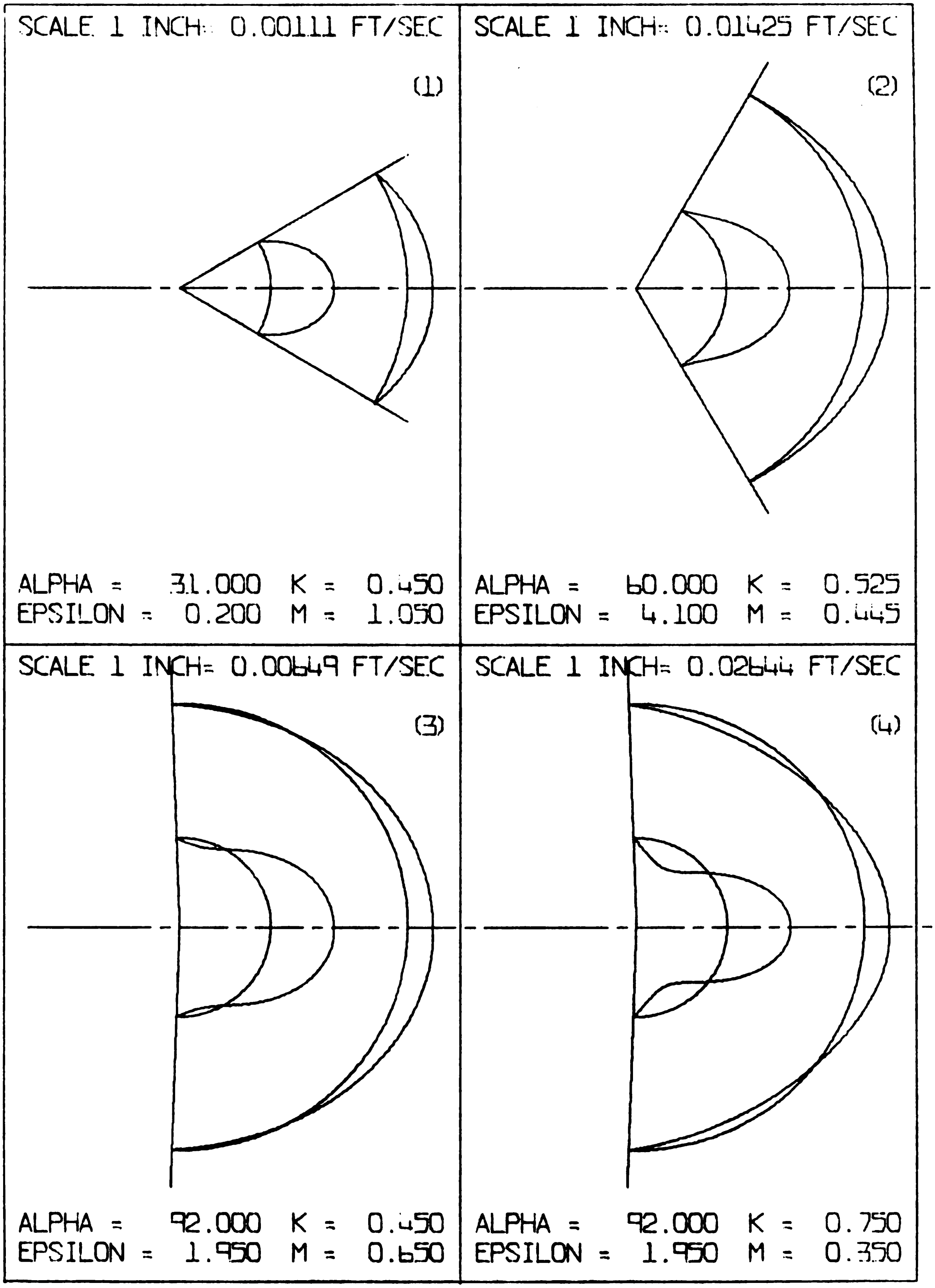

FIGURE 3.9 TYPICAL VELOCITY PROFILES, SD FLOW

1.2 inches and 3.0 inches. Then to fit in a reasonable space the profiles were plotted half size.

3. The plotted values represent a velocity profile where ν was assumed to be .000159 ft^2/sec. However, the plot can also be considered to represent the shape of the dimensionless velocity profile F.

<u>3.2.2 Solution for SDCO Flow</u>. The expressions developed in Section 2.5 by application of the boundary conditions to the dimensionless velocity function are

$$\frac{m_2-e_1}{m_2+e_1} = cn\left[\left(\frac{2m_2}{3}\right)^{\frac{1}{2}}, k\right] \tag{2.67}$$

$$\frac{\epsilon}{2} = \alpha(e_1-m_2) + 2(6m_2)^{\frac{1}{2}} E\left[\alpha\left(\frac{m_2}{6}\right)^{\frac{1}{2}}, k\right] - (6m_2)^{\frac{1}{2}}\left\{\frac{sn\left[\alpha\left(\frac{m_2}{6}\right)^{\frac{1}{2}}, k\right] dn\left[\alpha\left(\frac{m_2}{6}\right)^{\frac{1}{2}}, k\right]}{cn\left[\alpha\left(\frac{m_2}{6}\right)^{\frac{1}{2}}, k\right]}\right\} \tag{2.68}$$

where

$$e_1 = \frac{2}{3}(2m_2k^2 - 3 - m_2) \tag{2.60}$$

To obtain equations for the flow parameter curves consider

$$\sin A = sn\left[\left(\frac{2m_2}{3}\right)^{\frac{1}{2}} \alpha, k\right] \tag{3.13}$$

As cn and sn are related as the cosine and sine functions one has

$$\cos A = cn\left[\left(\frac{2m_2}{3}\right)^{\frac{1}{2}} \alpha, k\right] \tag{3.14}$$

Using this relationship in (2.67) one can solve for m_2 or k to give

$$m_2 = \frac{6(1 + \cos A)}{4k^2(1 + \cos A) + \cos A - 5} \qquad (3.15)$$

or

$$k = \left\{\frac{1}{4(1 + \cos A)} [\frac{6}{m_2} (1 + \cos A) + 5 - \cos A]\right\}^{\frac{1}{2}} \qquad (3.16)$$

Also, by using the relations between cn, sn and dn, equation (2.68) can be reduced to

$$\epsilon = 2\left\{\alpha(e_1 - m_2) + 2(6m_2)^{\frac{1}{2}} E\left[\alpha\left(\frac{m_2}{6}\right)^{\frac{1}{2}}, k\right] - (6e_1)^{\frac{1}{2}}\right\} \qquad (3.17)$$

Also, from (3.13)

$$\alpha = (\frac{3}{2m_2})^{\frac{1}{2}} K(A,k) \qquad (3.18)$$

These equations may now be used to calculate constant k and constant m curves for the flow parameter graphs. The symbolic flow charts for the constant k and constant m curves are shown on Figures 3.10 and 3.11, respectively. The corresponding flow parameter graph for constant values of m_2 and k is shown in Figure 3.12. Items of interest are:

1. For small m_2 the curves approach the $\alpha = 0$ axis. For example, equation (3.15) shows that

$$m_2 \to \infty \text{ as } A \to \cos^{-1}\left(\frac{5-4k^2}{1+4k^2}\right) \qquad (3.19)$$

Therefore, since $\alpha = (\frac{3}{2m_2})^{\frac{1}{2}} K(A,k)$ from (3.18), then $\alpha \to 0$ as $m_2 \to \infty$, since $K(A,k)$ is bounded. For example, if $k = .75$

$$A < \cos^{-1}\left[\frac{5-4(.75)^2}{1+4(.75)^2}\right] \approx 32.1^{\circ} \qquad (3.20)$$

as $A \to 32.1$, $m_2 \to \infty$ and $\alpha \to 0^{\circ}$.

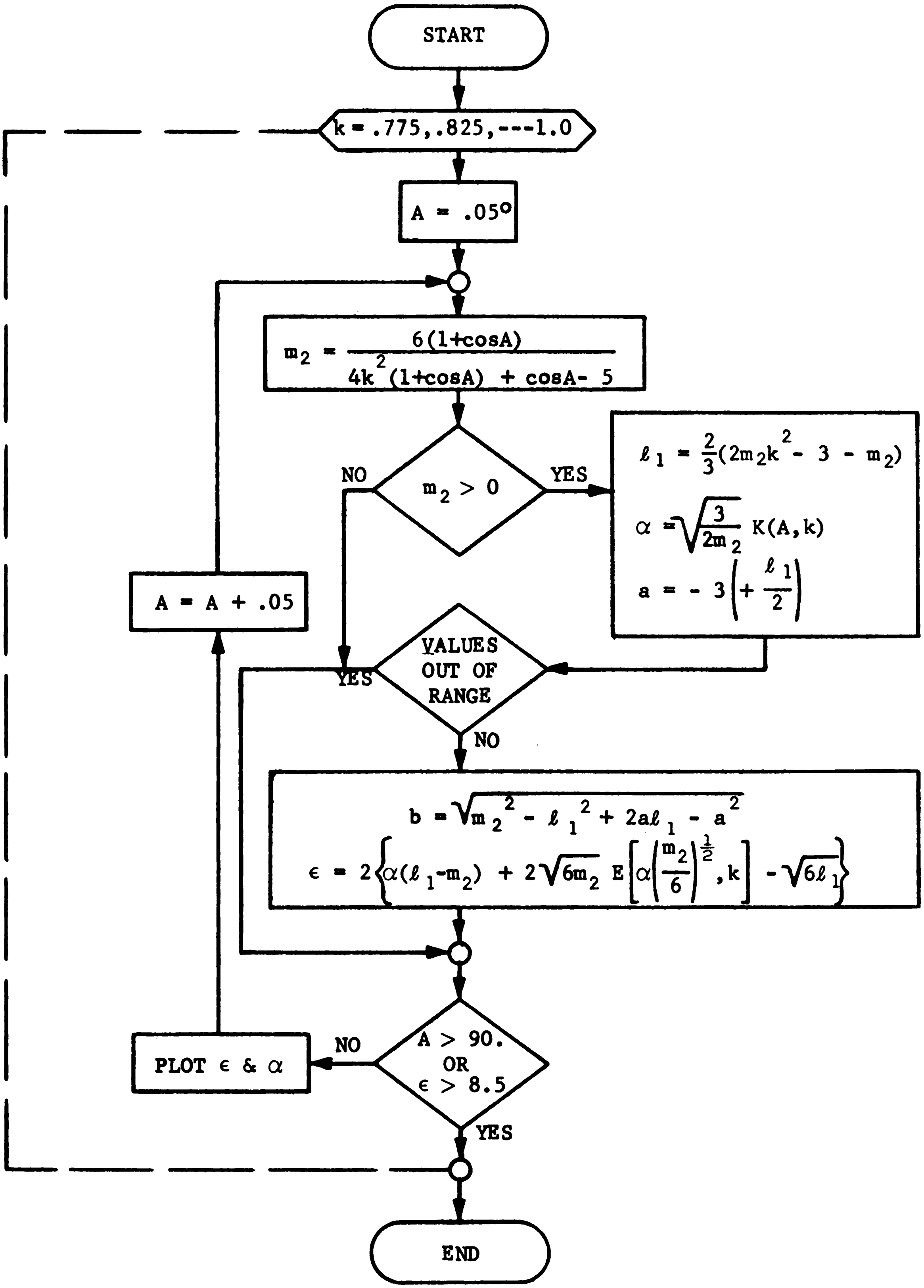

FIGURE 3.10 SYMBOLIC FLOW CHART FOR CONSTANT k CURVES, SDCO FLOW

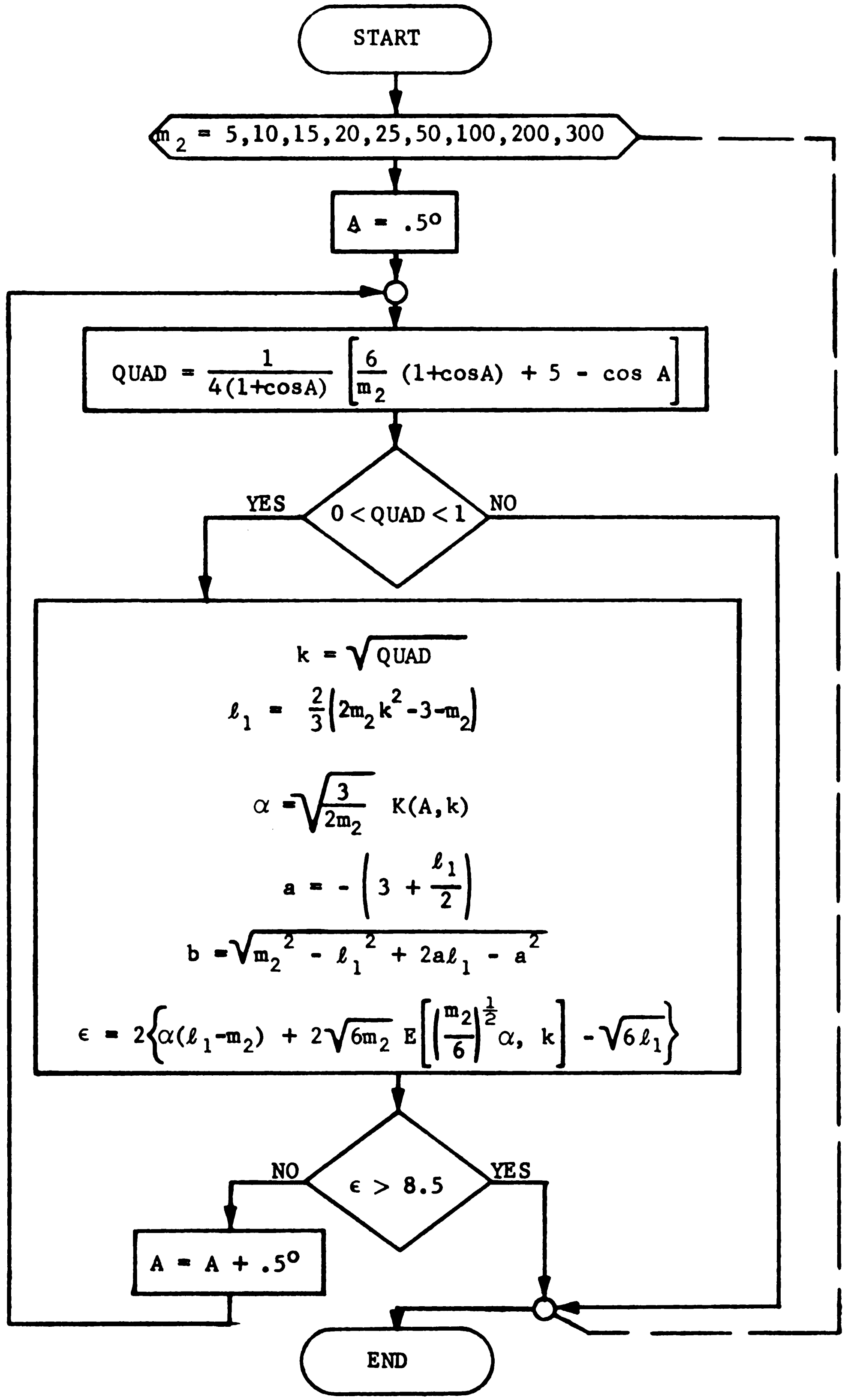

FIGURE 3.11 SYMBOLIC FLOW CHART FOR CONSTANT m CURVES, SDCO FLOW

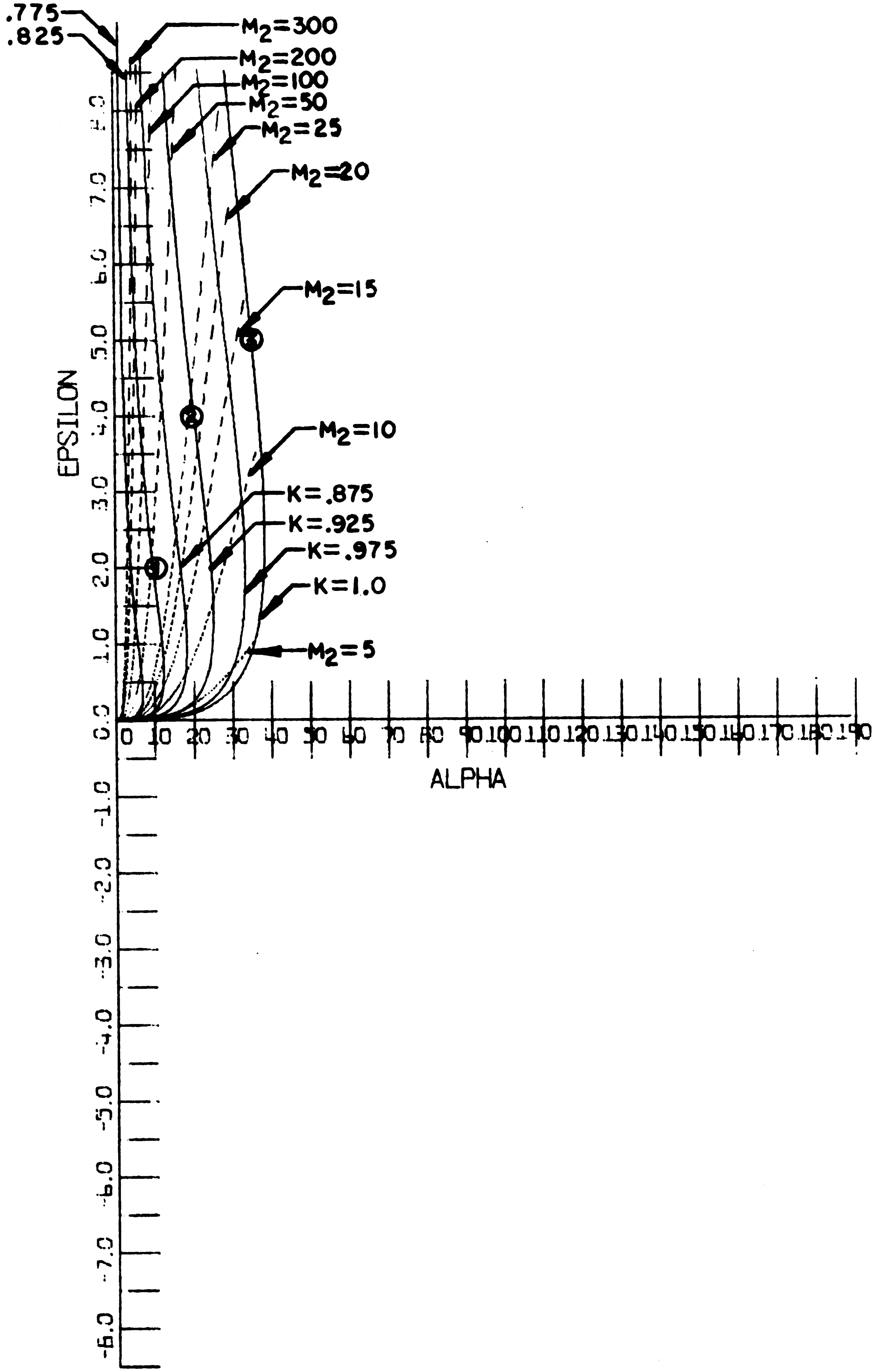

FIGURE 3.12 FLOW PARAMETER GRAPH-SDCO FLOW

2. As noted earlier $k > \sqrt{\frac{1}{2}}$.

3. For k = 1, the curve exactly coincides with k = 1 curve for three real roots. For k = 1 equations (2.58), (2.59), and (2.60) can be used to show b, which is the imaginary part of e_2 and e_3, is zero. Therefore, along the k = 1 curve the complex roots become real roots and on this curve are equal, i.e.,

$$e_2 = e_3 = a = -\left(2 + \frac{m_2}{3}\right) \qquad (3.21)$$

The computer routine to calculate and plot the constant k and constant m curves is called "Subroutine Complex" and is shown in Appendix A with some sample print-out of the calculated points. The programming represents the flow charts on Figures 3.10 and 3.11.

Three points are chosen on Figure 3.12, the values of k and m_2 are read from the graph, and the values are then used in equation (2.57) to determine the velocity profiles shown in Figure 3.13. The coordinates and values of k and m_2 are shown below each velocity profile where the scaling is the same as described in Section 3.2.1. The subroutine used for Figure 3.12 is shown in Appendix B.

The flow parameter graph for the SD solutions shown in Figure 3.8 and the graph for the SDC solution can be combined and the resulting graph is shown in Figure 3.14. This graph represents SD0 and SDC0 profiles on the left of the A = 90° line and SD2 profiles on the right of the A = 90° line. Other families of curves for solutions with 2,4,6,--- interior zeroes have not been included.

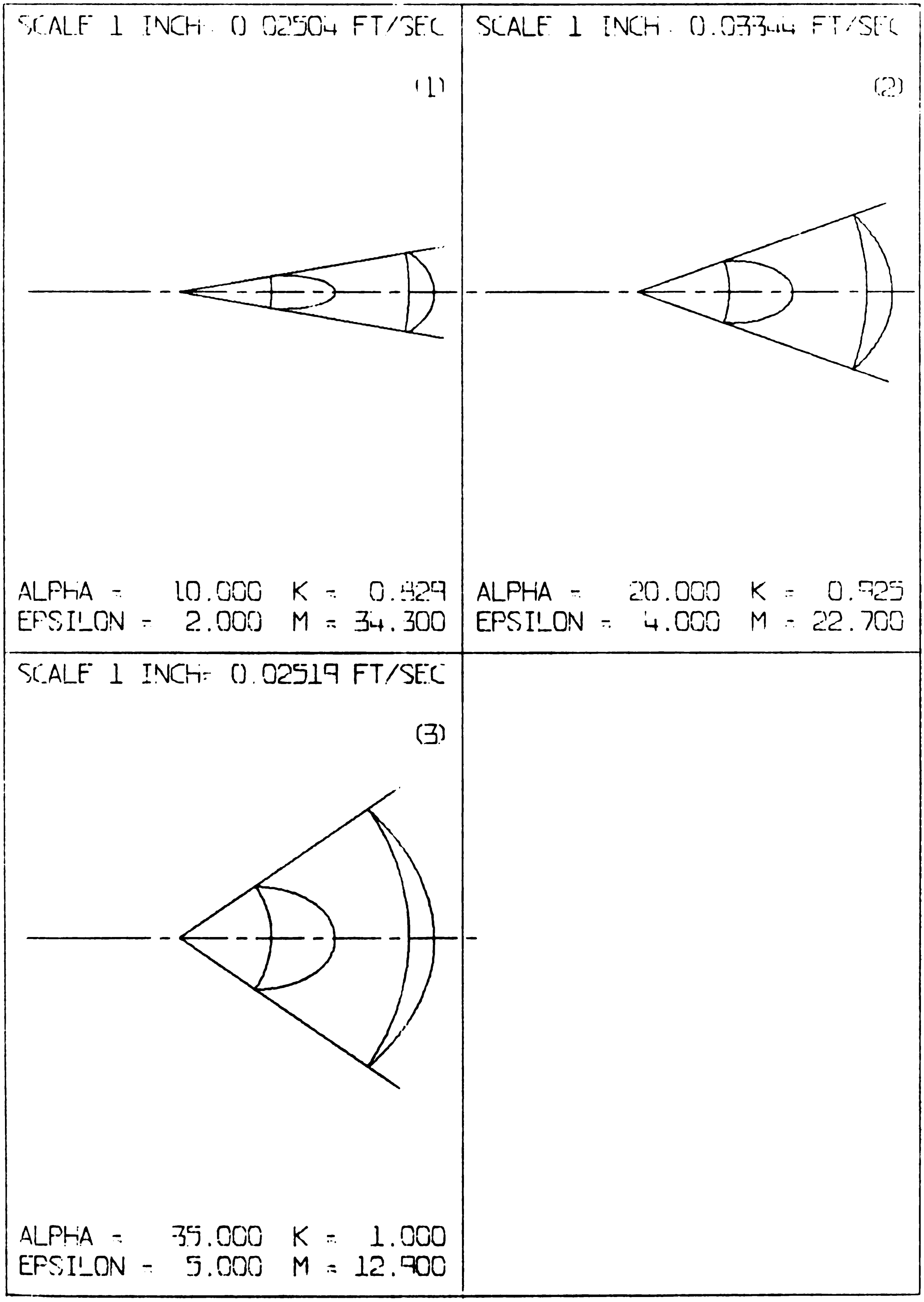

FIGURE 3.13 TYPICAL VELOCITY PROFILES-SDCO FLOW

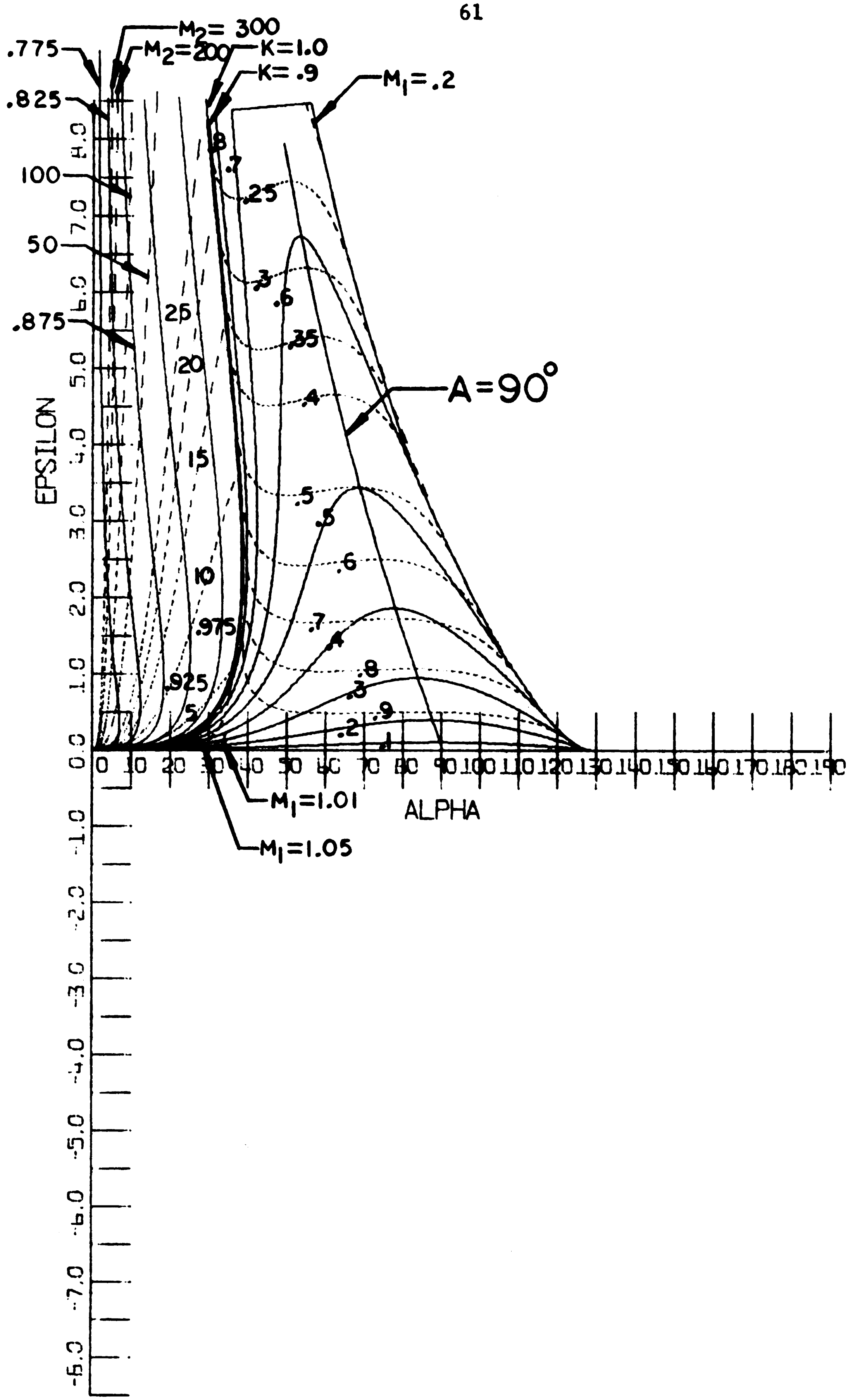

FIGURE 3.14 FLOW PARAMETER GRAPH FOR DIVERGING FLOW

3.3 SC Flow

The expressions developed in Section 2.5 by application of the boundary conditions to the dimensionless velocity function are

$$\mathrm{dn}^2(m_3\alpha,k) = \frac{3m_3^2(1+k^2)}{1-m_3^2(k^2-2)} \qquad (2.69)$$

$$\epsilon = 4\alpha[m_3^2(k^2-2)-1] + 12m_3\left[E(m_3\alpha,k) - \frac{k^2\mathrm{sn}(m_3\alpha,k)\,\mathrm{cn}(m_3\alpha,k)}{\mathrm{dn}(m_3\alpha,k)}\right] \qquad (2.70)$$

To obtain equations for plotting the flow parameter curves again relate the elliptic functions to trigonometric functions but first note

$$\mathrm{dn}^2(\phi,k) = 1 - k^2\mathrm{sn}^2(\phi,k) \qquad (3.22)$$

Then let

$$\sin A = \mathrm{sn}(m_3\alpha,k) \qquad (3.23)$$

Substituting into (2.69) and solving for m_3 one obtains

$$m_3 = \sqrt{\frac{-1+k^2\sin^2 A}{(k^2-2)k^2\sin^2 A - 1 + 2k^2}} \qquad (3.24)$$

where

$$\alpha = \frac{1}{m_3}\,\mathrm{sn}^{-1}(\sin A,k) = \frac{1}{m_3}\,K(A,k) \qquad (3.25)$$

Then equations (2.70), (3.24) and (3.25) can be used to obtain values of α, m_3, and ϵ for various constant values of k and A. For curves with constant values of m_3 equation (2.69) can be solved for k, (again using (3.22) and (3.23)) to obtain

$$k = \left\{ \frac{-B \pm [B^2 - 4m_3{}^2\sin^2 A(1-m_3{}^2)]^{\frac{1}{2}}}{2m_3{}^2\sin A} \right\}^{\frac{1}{2}} \tag{3.26}$$

where

$$B = 2m_3{}^2 - \sin^2 A(1 + 2m_3{}^2) \tag{3.27}$$

Then (2.70), (3.25), (3.26), and (3.27) can be used to obtain values of α, k and ϵ for various constant values of m_3 and A. The limitations on these solutions are extensive and they will be discussed after presentation of the flow parameter graphs.

The symbolic flow charts of the computer programs using these equations to plot the constant k and constant m curves are shown in Figures 3.15 and 3.16, respectively. The flow parameter graph with the curves for $0^\circ < A < 90^\circ$ is shown in Figure 3.17. These curves represent SCO solutions (i.e., all flow is toward the apex). Items of interest with respect to these curves and the governing equations are:

1. As k approaches 0.0 the curves approach the $\epsilon = 0$ axis, and as k approaches 1.0, the curves approach negative infinity.
2. The line labeled $A = 90^\circ$ indicates the maximum value of α for any particular ϵ where purely convergent flow may exist. To the right of this boundary the solutions have two or more interior zeroes with back flow. (Actually this so-called "backflow" is outward, away from the apex.) The equation for this boundary can be obtained from the above equations for $A = 90^\circ$.

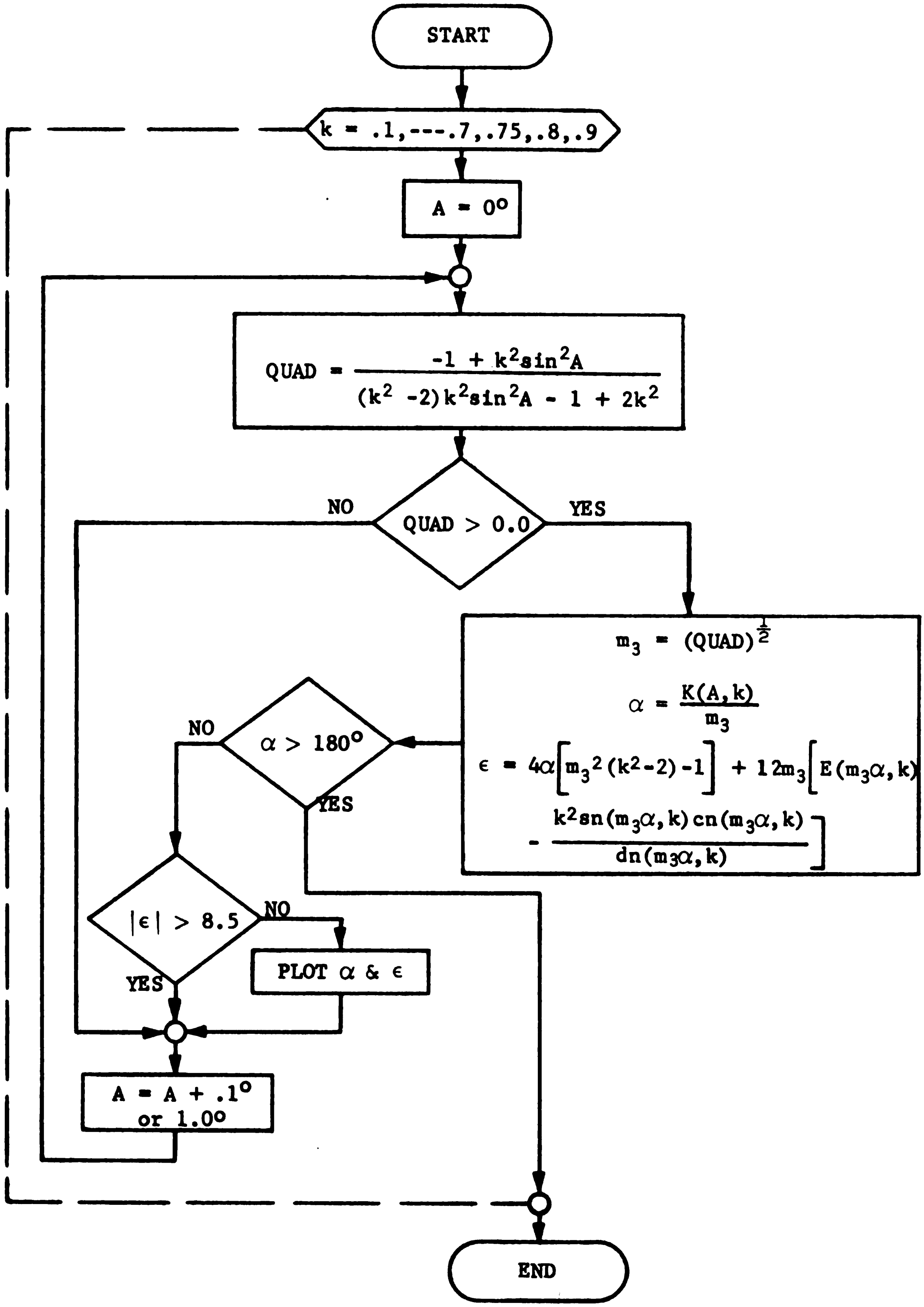

FIGURE 3.15 SYMBOLIC FLOW CHART FOR CONSTANT k CURVES, SC FLOW

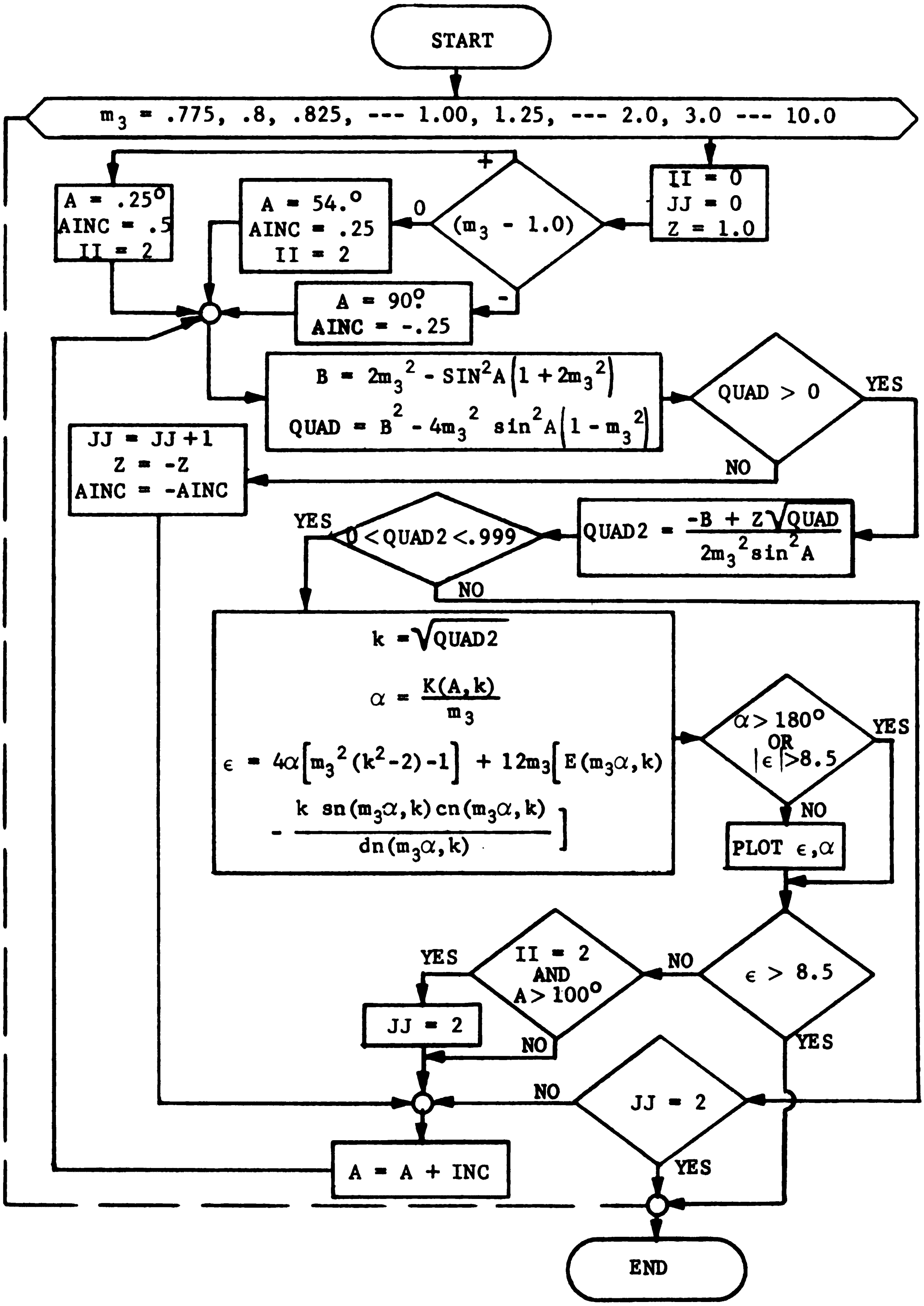

FIGURE 3.16 SYMBOLIC FLOW CHART FOR CONSTANT m CURVES, SC FLOW

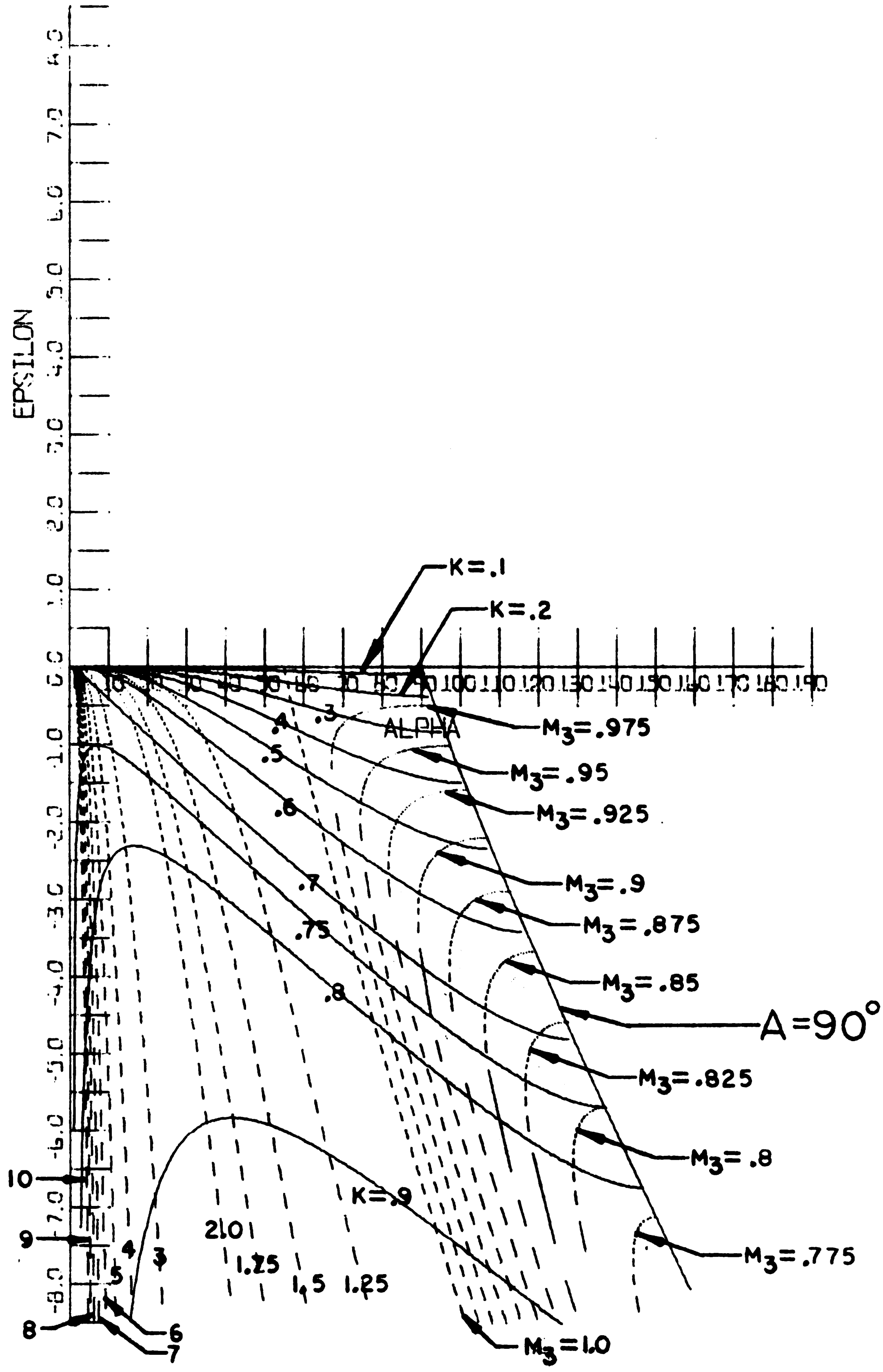

FIGURE 3.17 FLOW PARAMETER GRAPH, SCO FLOW

Thus

$$\mathrm{sn}(\alpha m_3) = \sin 90^{\circ} = 1.0$$

and from (3.24)

$$m_3 = \left(\frac{k^2-1}{k^4-1}\right)^{\frac{1}{2}} \tag{3.28}$$

Then from (2.70)

$$\epsilon = 4K'\left[\left(\frac{k^2-1}{k^4+1}\right)^{\frac{1}{2}}(k^2-2) - \left(\frac{k^4-1}{k^2-1}\right)^{\frac{1}{2}} + 12\left(\frac{k^2-1}{k^4-1}\right)E'\right] \tag{3.29}$$

and

$$\alpha = \frac{K'}{\left(\frac{k^2-1}{k^4-1}\right)^{\frac{1}{2}}} \tag{3.30}$$

where K' and E' are the complete elliptic integrals of the first and second kinds, respectively. The values of ϵ and α can then be plotted for various values of k.

3. For the constant k curves using (3.24) the expression under the radical must be positive. If $A = 0^{\circ}$ then from (3.24)

$$m_3 = \sqrt{\frac{-1}{-1+2k^2}} \tag{3.31}$$

and k must be less than $\sqrt{\frac{1}{2}}$, i.e.,

$$k < \sqrt{\frac{1}{2}} \tag{3.32}$$

On the other hand if one specifies k then A is restricted. For example if $k \geqslant \sqrt{\frac{1}{2}}$, then m_3 is not defined for $A = 0^{\circ}$. For example, if $k = .9$ then

$$A > \approx 53.5^{\circ} \tag{3.33}$$

The angle α as given by (3.25) still approaches zero since

$\alpha = \frac{K(A,k)}{m_3}$ and $m_3 \rightarrow 0.0$ as A decreases to 53.5^o and K is bounded. Therefore, the constant k curves for $k < \sqrt{\frac{1}{2}}$ begin at $\alpha = 0.0$ and $\epsilon = 0.0$, but the curves for $k > \sqrt{\frac{1}{2}}$ are asymptotic to the $\alpha = 0.0$ axis.

4. For the constant m curves equations (3.26) and (3.27) are used to determine allowable values of A and k. For $m_3 = 1.0$, since k must be positive, one has $A > \sin^{-1}\left(\sqrt{\frac{2}{3}}\right) \approx 54.5^o$, and $A < 90^o$. The plus sign for the square root term in (3.26) is used to give real values. For $m_3 > 1.0$, $A \geqslant 0$ and $A \neq 90^o$, since at 90^o $k = 1$, and $\alpha =$ infinity. For practical purposes of representing values on the computer the value of A is allowed to approach 90^o until $k > 0.999$. The plus sign in equation (3.26) is used to give real values. For $m_3 < 1.0$ both the plus and minus signs give acceptable values for certain ranges of A. An analysis of equation (3.26) for $m_3 < 1.0$ shows there are two critical values of A, to be denoted by A_c and A'_c. These are the values of A when the inner quadratic of equation (3.26) is zero, i.e., when

$$B^2 = 4m_3^2\sin^2A(1-m_3^2) \tag{3.34}$$

For $A_c < A < 90^o$ both the positive and the negative roots in equation (3.26) give solutions. The value of A'_c becomes significant on Figure 3.19 where the curves include regions where $A > 90^o$. Figure 3.18 is a sketch of the $m_3 = .950$ curve from Figure 3.17 with an indication of the values of A for each range of the curve. This duplication of solutions

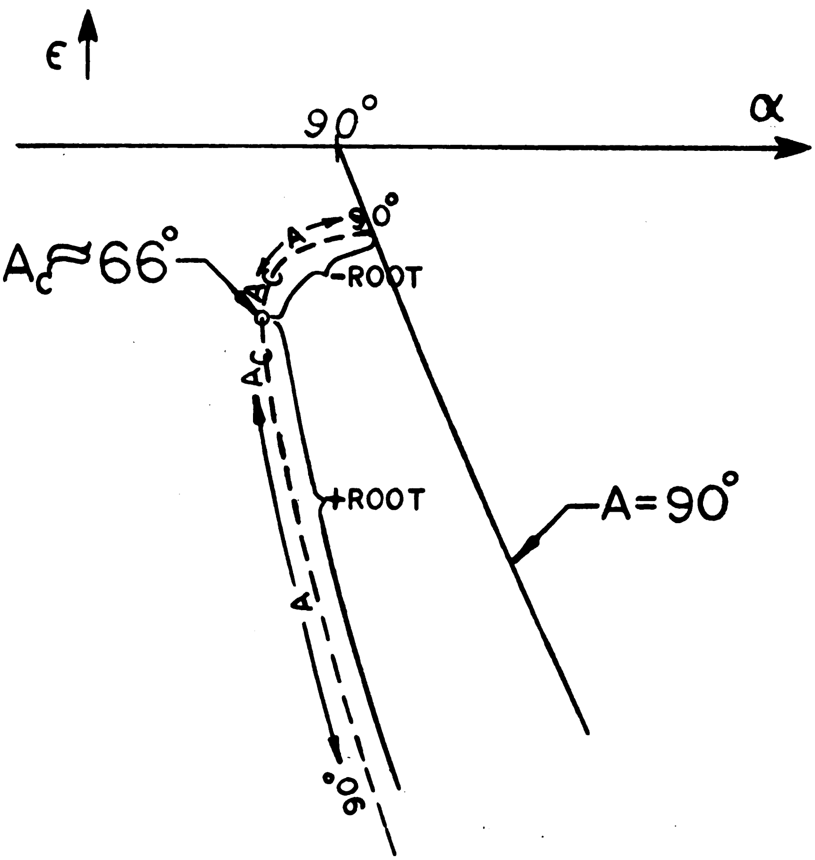

FIGURE 3.18 PLOT OF m_3 = .95, SCO FLOW

causes the computer program to be considerable more complex, as reflected by the flow chart in Figure 3.16. Rather than solving (3.34) for A_c the computer program is written to increase A from 90^o toward A_c using the positive sign in (3.26) until the inner quadratic in (3.26) is negative, indicating that A is now greater than A_c. Then the increment is reversed and A is decreased from A_c toward 90^o this time using the negative sign in (3.26). Thus, A_c and further on A'_c are never explicitly calculated nor do they need to be in order to plot the desired constant m curves. With respect to computer time this procedure is just as efficient if not more so than explicitly solving equation (3.27) for A_c and then increasing A to this known value.*

The Flow parameter graph on Figure 3.19 includes solutions for SC2 and SC4 profiles. For this graph A has been allowed to increase until α is greater than 180^o or $|\epsilon|$ is greater than 8.5. The complete subroutine called CONVERGE and some sample printout of calculated values is shown in Appendix A. Items of interest with respect to these curves and the governing equations are:

1. The $A = 90^o$ line derived earlier divides SC0 flow from SC2 flow.

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* This general procedure is used several times in the accompanying computer programs. Even though explicit solutions for the limits are derived, very often it takes less computer time to simply increment until a resulting expression is out of bounds. Also at times the limits, when they are determined explicitly, are exact values which a digital computer can only represent approximately and the difference between the exact value and the approximation may be infinite.

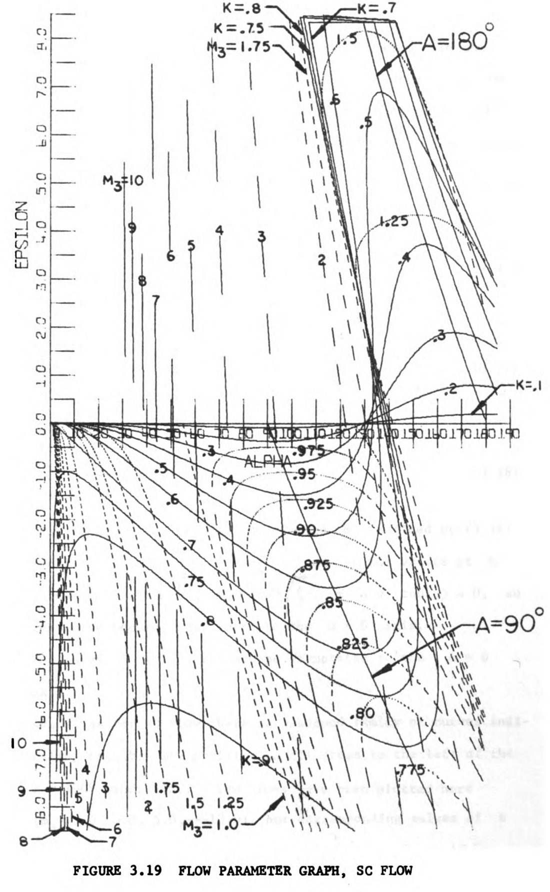

FIGURE 3.19 FLOW PARAMETER GRAPH, SC FLOW

2. The $A = 180^{\circ}$ line divides SC2 flow from SC4 flow. To the right of this line solutions have four interior zeroes. The equation for this boundary is obtained by setting $A = 180^{\circ}$ in equations (3.24), (2.70), (3.25), and (3.23). From (3.23)

$$\sin (180^{\circ}) = \mathrm{sn}(\alpha m_3) = 0 \tag{3.35}$$

From (3.24)

$$m_3 = \sqrt{\frac{-1}{3k^2-1}} \tag{3.36}$$

From (3.25)

$$\alpha = \frac{2K'}{m_3} \tag{3.37}$$

From (2.70)

$$\epsilon = \frac{8K'}{m_3}\left[m_3^2(k^2-2)-1\right] + 24m_3E' \tag{3.38}$$

Then equations (3.37) and (3.38) with m_3 defined by (3.36) can be used to plot values of α and ϵ for values of k between 0.0 and $\sqrt{\frac{1}{2}}$. As $k \to \sqrt{\frac{1}{2}}$, $m_3 \to 0$ and $\alpha \to 0$, so the curve becomes asymptotic to the $\alpha = 0$ axis.

3. For large ϵ all curves become asymptotic to the $\alpha = 0$ axis.

4. Again, as for SD flow there is a second family of curves indicating possible SC2 profiles in all areas to the left of the $A = 180^{\circ}$ line. Only a few lines have been plotted here ($m_3 = 3.0, 4.0, 5.0, \text{---} 10.0$), but corresponding values of k curves may also be plotted. (In this region k would be greater than the $k = .8$ curve plotted and less than 1.0.)

Similarly, a secondary family of curves of SC4 may be plotted that covers all parts of the graph to the left of the envelope of curves on the right side of the graph. (Both SC2 and SC4 solutions would cover all values of α from zero to 180^o for $\epsilon < 0$.) Other families of curves for SC6, SC8, etc. types of flow may also be developed from these basic equations.

5. For the constant m curves, equations (3.26) and (3.27) again give an indication of the allowable values of A and k. For $m_3 = 1.0$ for k to be positive $A < \sin^{-1}(\frac{2}{3}) \approx 125.5^o$. The positive sign in (3.26) is used to give real values for k. So for the entire $m_3 = 1.0$ curve $54.5^o \approx < A < 90^o$ and $90^o < A < 125.5^o$. For $m_3 > 1.0$ the plus sign in (3.26) gives real values for k if $A \neq 90^o$. For computational purposes A has been allowed to approach 90^o until $k > .999$. For $m_3 < 1.0$ similar conditions exist as in the SC0 flow. That is, the solutions of (3.34) determine two critical values of A, called A_c and A_c'. For $90^o < A < A_c'$ both the plus and the negative roots in equation (3.26) give a solution. Figure 3.20 is a sketch of the complete $m_3 = .95$ curve from Figure 3.19 with an indication of the values of A for each portion of the curve.

Figure 3.21 is another plot of SC flow without the secondary solutions. This allows for easier use of the graph to determine values for primary solutions. Four typical data points are selected from Figure 3.21 and the values of k and m_3 are read and used in equation (2.45) to plot the velocity profiles shown in Figure 3.22. The coordinates and values of k and m_3 for each profile are shown on Figure

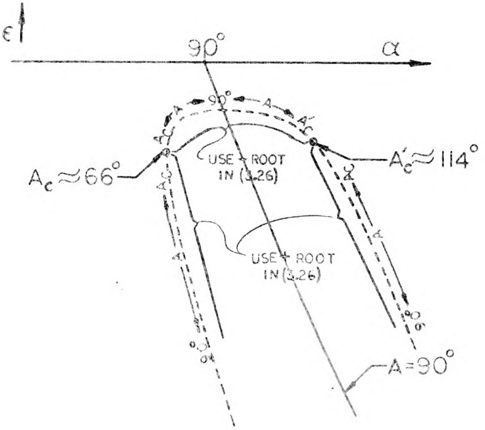

FIGURE 3.20 PLOT OF m_3 = .95, SC FLOW

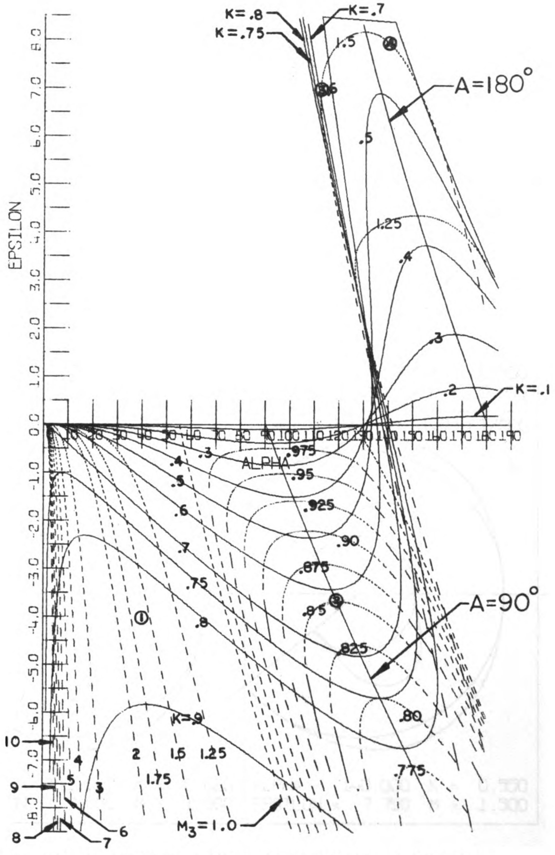

FIGURE 3.21 FLOW PARAMETER GRAPH, SC FLOW (PRIMARY SOLUTIONS)

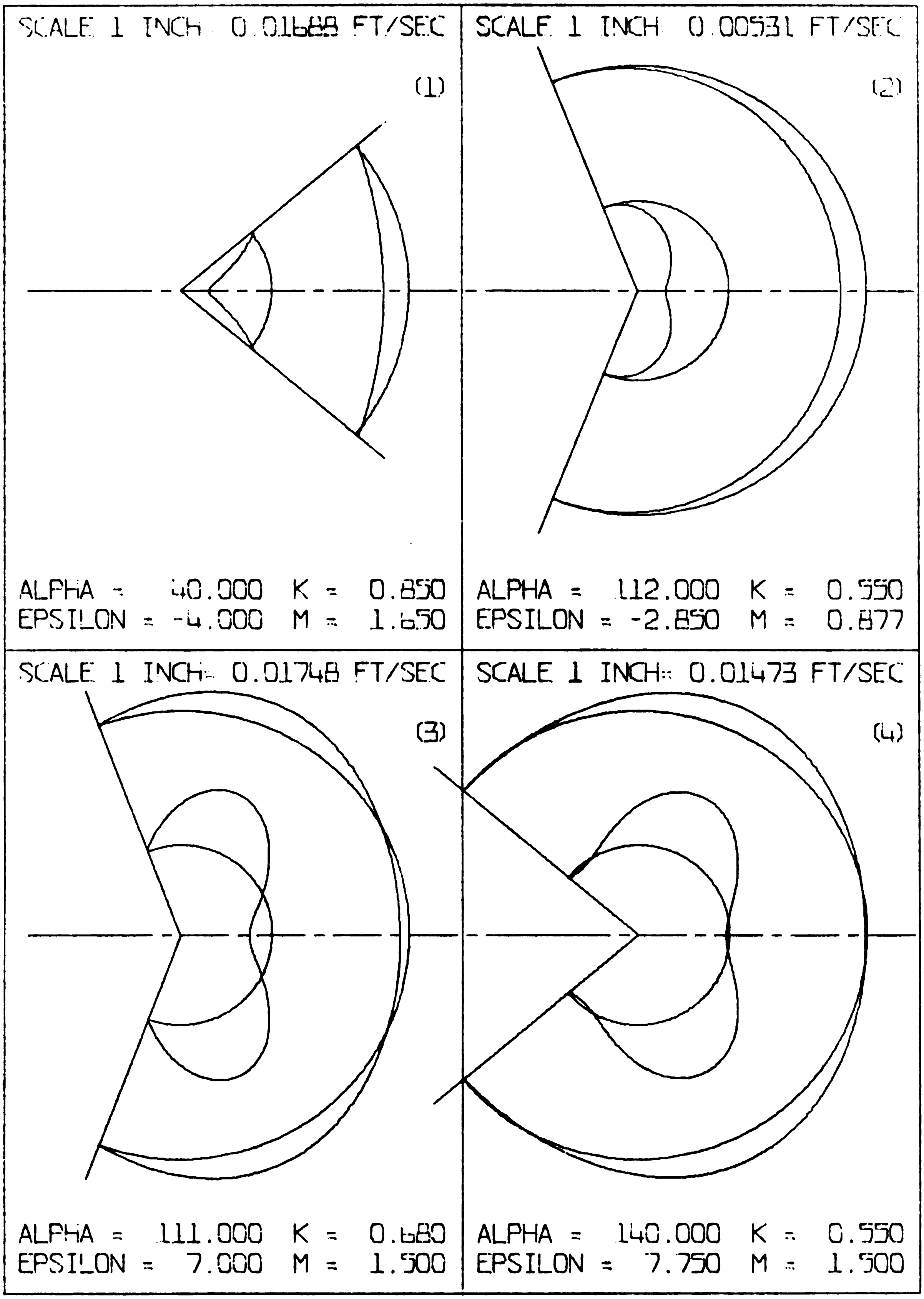

FIGURE 3.22 TYPICAL VELOCITY PROFILES, SC FLOW

3.22. The computer routine used for this plot called subroutine PRO SC is shown in Appendix B.

3.4 Non-Symmetrical Flow

Non-symmetrical flow (N flow) with one and three interior zeroes as sketched in Figure 2.10 parts (a) and (b) are now considered. As noted in Section 2.4.3 there are an infinite number of other possible non-symmetrical profiles. The N1 and N3 profiles are similar enough so that they are considered simultaneously. From section 2.5 the equations for the two types of flow are

for N1 Flow

$$\alpha = K'\sqrt{m_4} \tag{2.71}$$

$$\epsilon = \frac{4}{\sqrt{m_4}}\left[(k^2 - m_4 - 2)K' + 3E'\right] \tag{2.72}$$

for N3 Flow

$$\alpha = 2K'\sqrt{m_4} \tag{2.78}$$

$$\epsilon = \frac{8}{\sqrt{m_4}}\left[(k^2 - m_4 - 2)\ K' + 3E'\right] \tag{2.79}$$

where for both cases the values of θ_1 and θ_2 for $F = 0$ can be determined as

$$\theta_1 = \sqrt{m_4}\ K\left\{\sin^{-1}\left[\frac{1}{3}\left(1 - \frac{m_4-1}{k^2}\right)\right]^{\frac{1}{2}}, k\right\} \tag{2.75}$$

$$\theta_2 = \sqrt{m_4}\ 2K' - \theta_1 \tag{2.76}$$

Also, for N3 flow

$$\theta_3 = \theta_2 + 2\theta_1 = \theta_1 + 2K'\sqrt{m_4} \tag{3.39}$$

For non-symmetrical flows the equations are actually simpler than those for symmetrical flows. The same dimensionless velocity profile as given by equation (2.34) derived for SD flows is used for non-symmetrical flows. The application of the boundary condition

$$\int_{-\theta_1}^{\theta_2} F(\theta)\, d\theta = \epsilon \quad \text{for the N1 flow}$$

and

$$\int_{-\theta_2}^{\theta_3} F(\theta)\, d\theta = \epsilon \quad \text{for the N3 flow}$$

results in expressions for ϵ as given in equation (2.73) for N1 flow and (2.79) for N3 flow. Similarly, the application of the boundary condition $F(\pm\alpha) = 0$ results in expressions involving elliptic functions which are equation (2.75) for θ_1 and (2.76) for θ_2. But, the angle which describes the inclination of the walls is $\alpha = (\theta_1 + \theta_2)/2$ for N1 flows and $\alpha = (\theta_2 + \theta_3)/2$ for N3 flows. From (2.76) it is shown that

for N1 Flow

$$\alpha = \frac{\theta_1 + \theta_2}{2} = \frac{\theta_1 + \sqrt{m_4}\, 2K' - \theta_1}{2} = \sqrt{m_4}\, K' \tag{3.40}$$

for N3 Flow

$$\begin{aligned} \alpha &= \frac{\theta_2 + \theta_3}{2} = \frac{\theta_2 + (2\theta_1 + \theta_2)}{2} = \theta_1 + \theta_2 \\ &= \theta_1 + \sqrt{m_4}\, 2K' - \theta_1 = 2\sqrt{m_4}\, K' \end{aligned} \tag{3.41}$$

As θ_1 and θ_2 cancel, equations (3.40) and (3.41) involve only the complete elliptic integral which does not depend on α. (For previous cases of symmetrical flow the corresponding expressions involved either sn, cn, dn, E, or F, which are functions of both α and k.) As a result there is no necessity to relate the elliptic functions with trigonometric functions as done in Sections 3.1 and 3.2.

The general procedure for determining constant k curves for the N1 flow parameter graph and the N3 flow parameter graph is to assume a value of k. Then vary α within the allowable limits given by equation (2.75) and calculate m_4 from (2.71) or (2.78) and ϵ from (2.73) or (2.79). Then (2.75) and (2.76) can be used to determine θ_1 and θ_2 if desired. (They are not required for plotting the solution although the existence of θ_1 as determined by (2.75) is used to determine the allowable limits of α.) This general procedure is illustrated more clearly in the constant k symbolic flow chart for N1 flow shown on Figure 3.23.*

The constant m symbolic flow chart for N1 flow is shown on Figure 3.24. Here the procedure is not quite as simple because an algorithim is not available for the inverse of the complete elliptic integral K', even though it is a function of the single parameter k. Rather than write the necessary inverse algorithim a simple interation procedure is used and is shown on Figure 3.24. In this way equations (2.71) for N1 flow and (2.78) for N3 flow can be used to determine k for assumed values of α and m_4.

The complete flow parameter graphs for N1 and N3 flows are shown on Figures 3.25 and 3.26, respectively. Items of interest are:

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* The equations and procedures for N3 flow are similar enough to those for N1 flow so that the flow charts for N3 flow are not shown.

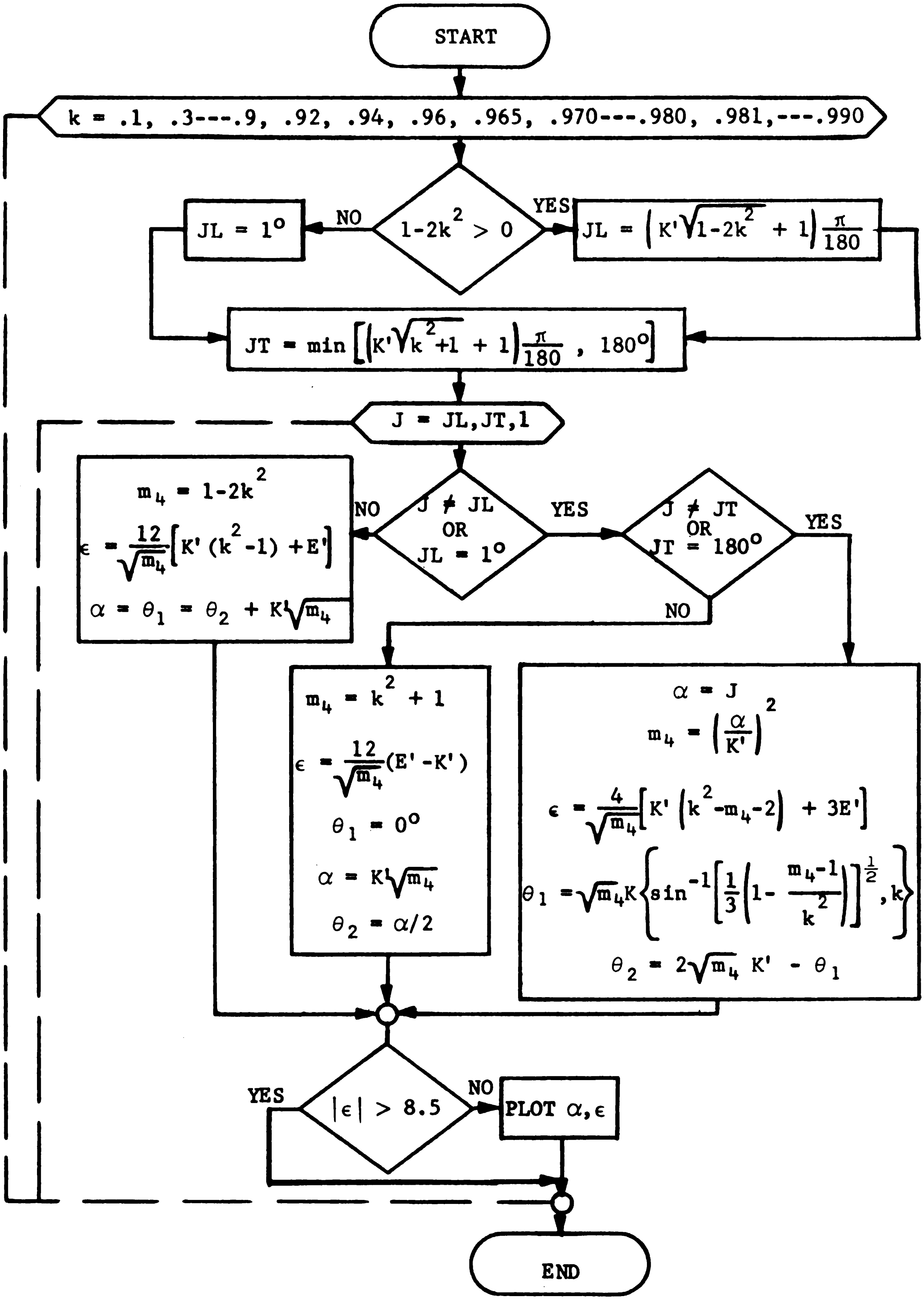

FIGURE 3.23 SYMBOLIC FLOW CHART FOR CONSTANT k CURVES, N1 FLOW

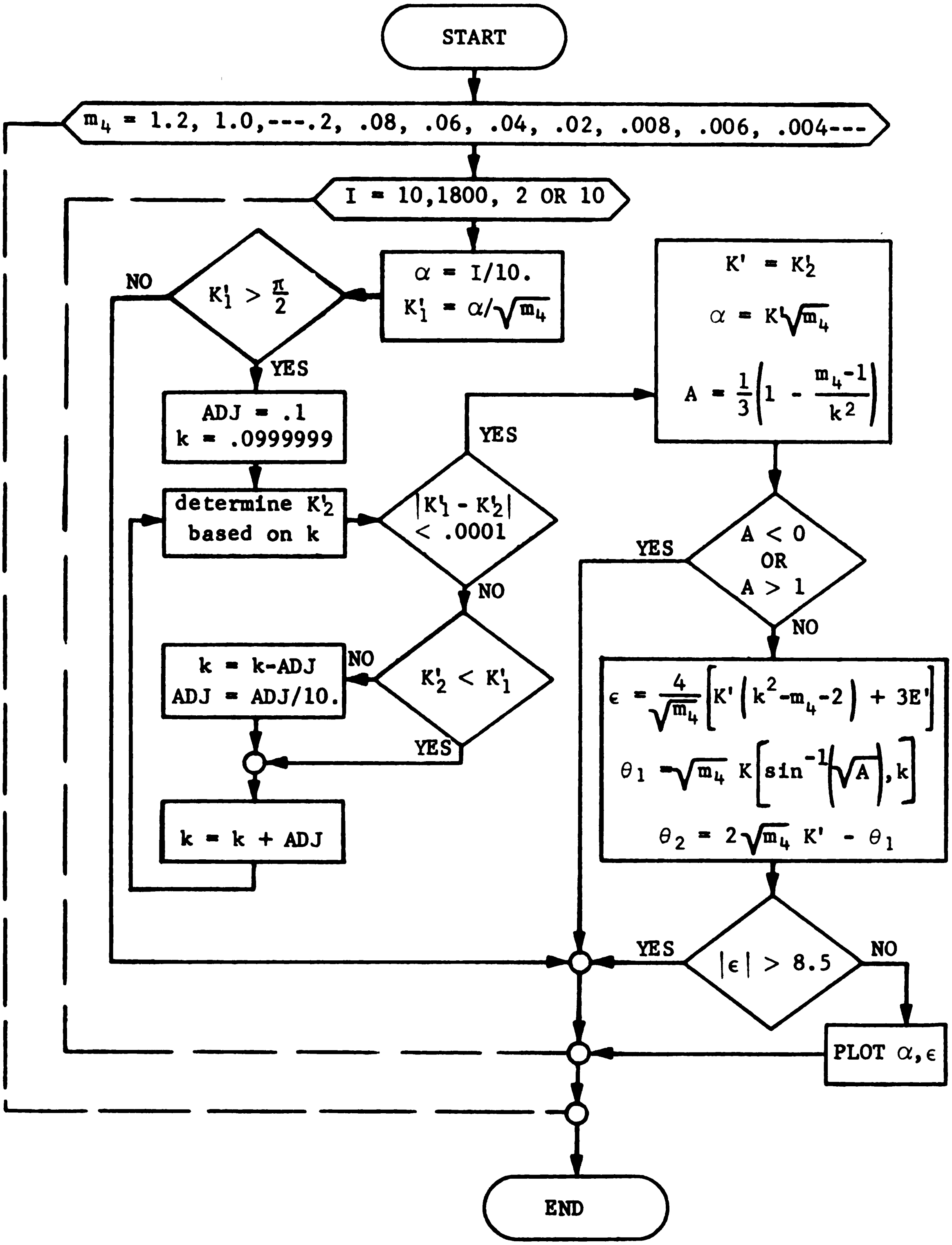

FIGURE 3.24 SYMBOLIC FLOW CHART FOR CONSTANT m CURVES, N1 FLOW

1. Both graphs show that either "net inflow" or "net outflow" may be represented equally as well.
2. For large $+\epsilon$ all curves become asymptotic to the $\alpha = 0$ axis.
3. For $\epsilon = 0$ equations (2.73) and (2.79) show that, assuming $m_4 \neq 0$

$$m_4 = \frac{3E'}{K'} + k^2 - 2 \tag{3.42}$$

Since $m_4 > 0$ for meaningful flow then

$$\frac{3E'}{K'} + k^2 > 2 \tag{3.43}$$

This limit is determined graphically to be between $k = .980$ and .981. Therefore, in both Figures 3.25 and 3.26 the $k = .980$ curve crosses the $\epsilon = 0$ axis and is asymptotic to the positive ϵ axis at $\alpha = 0^o$ while the $k = .981$ curve does not have a solution for $\epsilon = 0$ and as shown is asymptotic to the negative ϵ axis at $\alpha = 0^o$.

4. For $k = 0$ equations (2.73) and (2.79) show that $\epsilon = 0$. For all other constant k curves, and accordingly constant m curves, the upper and lower limits of the curves are restricted by the fact that if the flow is to be non-symmetrical as described, then θ_1 must be $> 0^o$. The equation for θ_1 is

$$\theta_1 = \sqrt{m_4}\ K\left\{\sin^{-1}\left[\frac{1}{3}\left(1 - \frac{m_4 - 1}{k^2}\right)\right]^{\frac{1}{2}}, k\right\} \tag{2.75}$$

For $\theta_1 > 0$ the expression within the inverse sine function

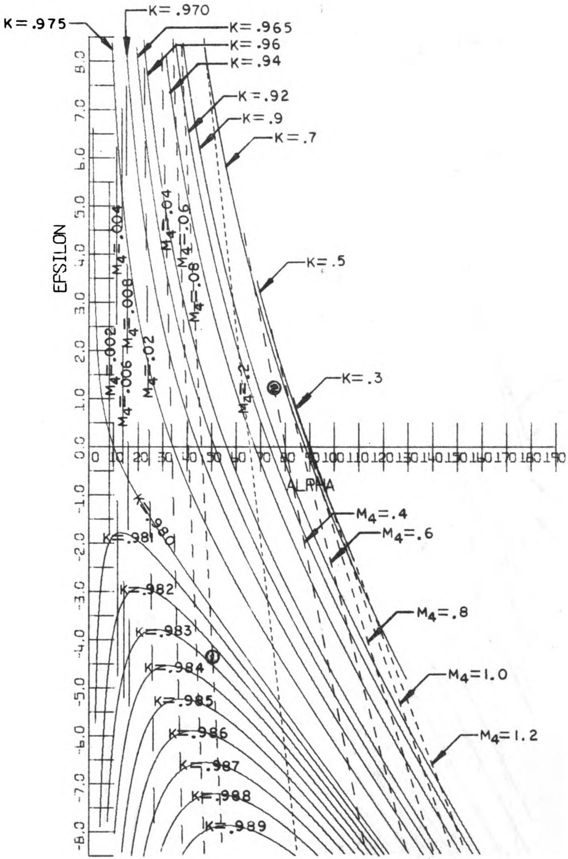

FIGURE 3.25 FLOW PARAMETER GRAPH, N1 FLOW

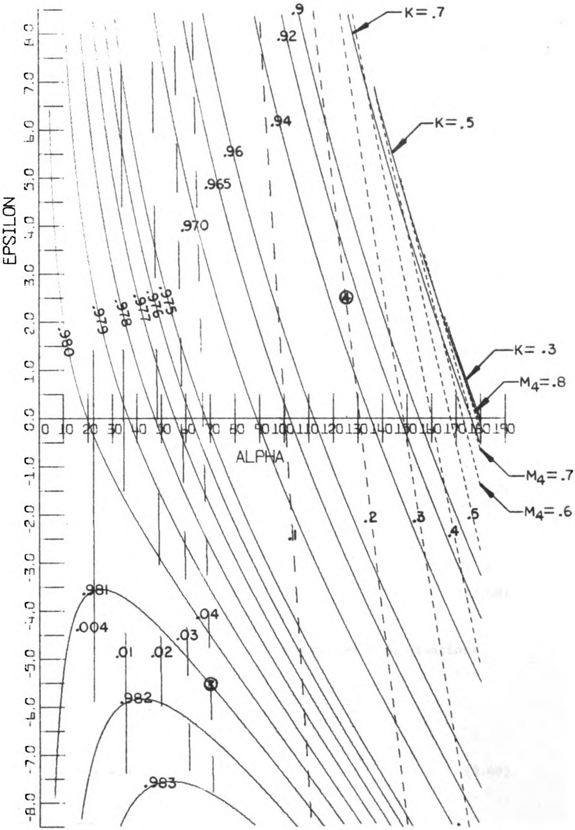

FIGURE 3.26 FLOW PARAMETER GRAPH, N3 FLOW

must have a value between 0.0 and 1.0. Define this quantity as T

$$T \equiv \frac{1}{3}\left(1 - \frac{m_4 - 1}{k^2}\right) \tag{3.44}$$

For the upper limit where θ_1 will be maximum and $\epsilon > 0$, $T = 1.0$. Therefore

$$\frac{1}{3}\left(1 - \frac{m_4 - 1}{k^2}\right) = 1.0 \tag{3.45}$$

or

$$m_4 = 1 - 2k^2 \tag{3.46}$$

Substituting the above into (2.73) and (2.79) one obtains

<u>for N1 Flow</u>

$$\epsilon = \frac{12}{\sqrt{1-2k^2}} [K'(k^2-1) + E'] \tag{3.47}$$

<u>for N3 Flow</u>

$$\epsilon = \frac{24}{\sqrt{1-2k^2}} [K'(k^2-1) + E'] \tag{3.48}$$

Corresponding values of α are determined from equations (2.71) and (2.78)

<u>for N1 Flow</u>

$$\alpha = K'(1-2k^2)^{\frac{1}{2}} \tag{3.49}$$

where

$$k \leq \sqrt{\frac{1}{2}} \tag{3.50}$$

at $k = \sqrt{\frac{1}{2}}$, $\alpha = 0^o$, and $\epsilon = \infty$.

<u>for N3 Flow</u>

$$\alpha \;=\; 2K'(1-2k^2)^{\frac{1}{2}} \tag{3.51}$$

where (3.50) also applies.

Equations (3.47) and (3.49) for N1 flows and (3.48) and (3.51) for N3 flows can be used to determine the right most boundary of possible flow for $\epsilon > 0$. These lines are not plotted as they would interfere with the constant k lines which are very close together in this region. However, they are used to allow plotting of the constant k and m curves right to the limiting values.

For the lower limit where θ_1 will be a minimum and $\epsilon < 0$, $T = 0.0$, therefore

$$\frac{1}{3}\left(1 - \frac{m_4 - 1}{k^2}\right) \;=\; 0.0 \tag{3.52}$$

or

$$m_4 \;=\; k^2 + 1 \tag{3.53}$$

Substituting equation (3.53) into (2.73) and (2.79) and using (2.71) and (2.78) one obtains

<u>for N1 Flow</u>

$$\epsilon \;=\; \frac{12(E' - K')}{\sqrt{k^2 + 1}} \tag{3.54}$$

$$\alpha \;=\; K'\sqrt{k^2 + 1} \tag{3.55}$$

for N3 Flow

$$\epsilon = \frac{24(E' - K')}{\sqrt{k^2 + 1}} \tag{3.56}$$

$$\alpha = 2K'\sqrt{k^2 + 1} \tag{3.57}$$

Equations (3.54) and (3.55) for N1 flows and (3.56) and (3.57) for N3 flows can be used to determine the right-most boundary of possible flow for $\epsilon < 0$. Again, the curves are not plotted but are used in the computer programs for limits on the constant k and constant m curves. Figure 3.27 is a sketch of the dimensionless velocity function for typical values of k and m_4 with the limiting cases described above. The curves are not meant to depict actual curves for fixed k and m_4 but merely to show what occurs as the limiting values are approached. In addition to the previous discussion of ϵ and α it can also be shown from equations (2.75), (2.76), and (3.39) that

(a) for upper limit ($T = 1.0$, $\epsilon > 0$)

$$\theta_1 = \theta_2 = K'(1 - 2k^2)^{\frac{1}{2}} \tag{3.58}$$

$$\theta_3 = \theta_1 + \theta_2 = 2K'(1 - 2k^2)^{\frac{1}{2}} \tag{3.59}$$

(b) for lower limit ($T = 0.0$, $\epsilon < 0$)

$$\theta_1 = 0^\circ \tag{3.60}$$

$$\theta_3 = \theta_2 = 2K'(k^2 + 1)^{\frac{1}{2}} \tag{3.61}$$

One further point of interest is that on the upper limit the ϵ equation for N1 flows given by (3.47) is identical to the $A = 90^\circ$ curve dividing SD0 flow from SD2 flow

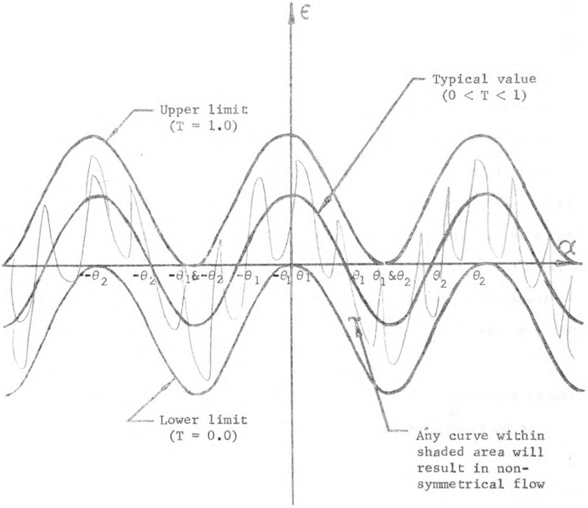

FIGURE 3.27 SKETCH OF F FOR N1 FLOWS SHOWING LIMITING CASES

derived in Section 3.2.1 and given as equation (3.9). This is to be expected as this line denotes the limiting case when $\theta_1 = \theta_2$ and the non-symmetrical flow becomes symmetrical. Also, the equation for the upper limit for ϵ and N3 flows given by (3.48) can be rearranged to become identical to equation (3.38) derived in Section 3.3 as the dividing line between SC2 and SC4 flow. Again this is to be expected as the N3 flow in the limit becomes symmetrical with two or four interior zeroes depending on the boundary conditions. Further, equation (3.30), which was derived in Section 3.3 as the dividing line between SC0 and SC2 flow, can be simplified to be identical to equation (3.54), which is the lower limit for N1 flow. Again this is expected as the lower limit of N1 flow is also the boundary between SC0 and SC2 flow.

5. With respect to the allowable range of α, equations (3.49), (3.50), (3.51), (3.55), and (3.57) can be summarized as follows

<u>for N1 Flow</u>

$$\mathrm{Re}[K'\sqrt{1 - 2k^2}] \leq \alpha \leq \min[180^\circ, K'\sqrt{1 + k^2}] \tag{3.62}$$

<u>for N3 Flow</u>

$$\mathrm{Re}[2k'\sqrt{1 - 2k^2}] \leq \alpha \leq \min[180^\circ, 2K'\sqrt{1 + k^2}] \tag{3.63}$$

The computer routines for the N1 and N3 flow parameter graphs are called NONSYM1 and NONSYM3, respectively, and are shown with some print-out of the calculations in Appendix A. These programs basically follow the flow charts in Figures 3.23 and 3.24.

As illustrations two arbitrary data points are selected and marked on each flow parameter graph in Figures 3.25 and 3.26. The resulting values of k and m_4 are read from the graph and used in the dimensionless velocity function given by equation (2.45) to plot the corresponding velocity profiles. These profiles and their corresponding values of α, ϵ, k, and m_4 are shown on Figure 3.28. Scaling is similar to that used for previous velocity profiles as discussed in Section 3.2.1. The computer routine used for this plot is shown in Appendix B.

3.5 Summary

The flow parameter graphs presented in this chapter summarize the derivations and calculations. These graphs show the limits of the principal solutions for each type of velocity profile. The graphs can now be used to determine the flow parameters m_i and k for particular values of α and ϵ. These parameters as used in the dimensionless velocity functions derived in Chapter II can then be used to quickly and easily determine the exact velocity distribution. The various flow parameter graphs can be superimposed on one graph, but the large number of lines would be difficult to interpret. However, one combination that is useful is the combination of Figures 3.14 and 3.21 which represent the primary solutions for SD and SC flow. The resulting graph is shown on Figure 3.29. This figure shows an area for positive ϵ between the SD2 solution and SC2 solution where there is apparently no possible flow. However, referring back to Section 3.3 and Figure 3.19 one may note that this region (as well as others) is covered by the "secondary" family of curves for SC2,SC4,--- etc. flow profiles.

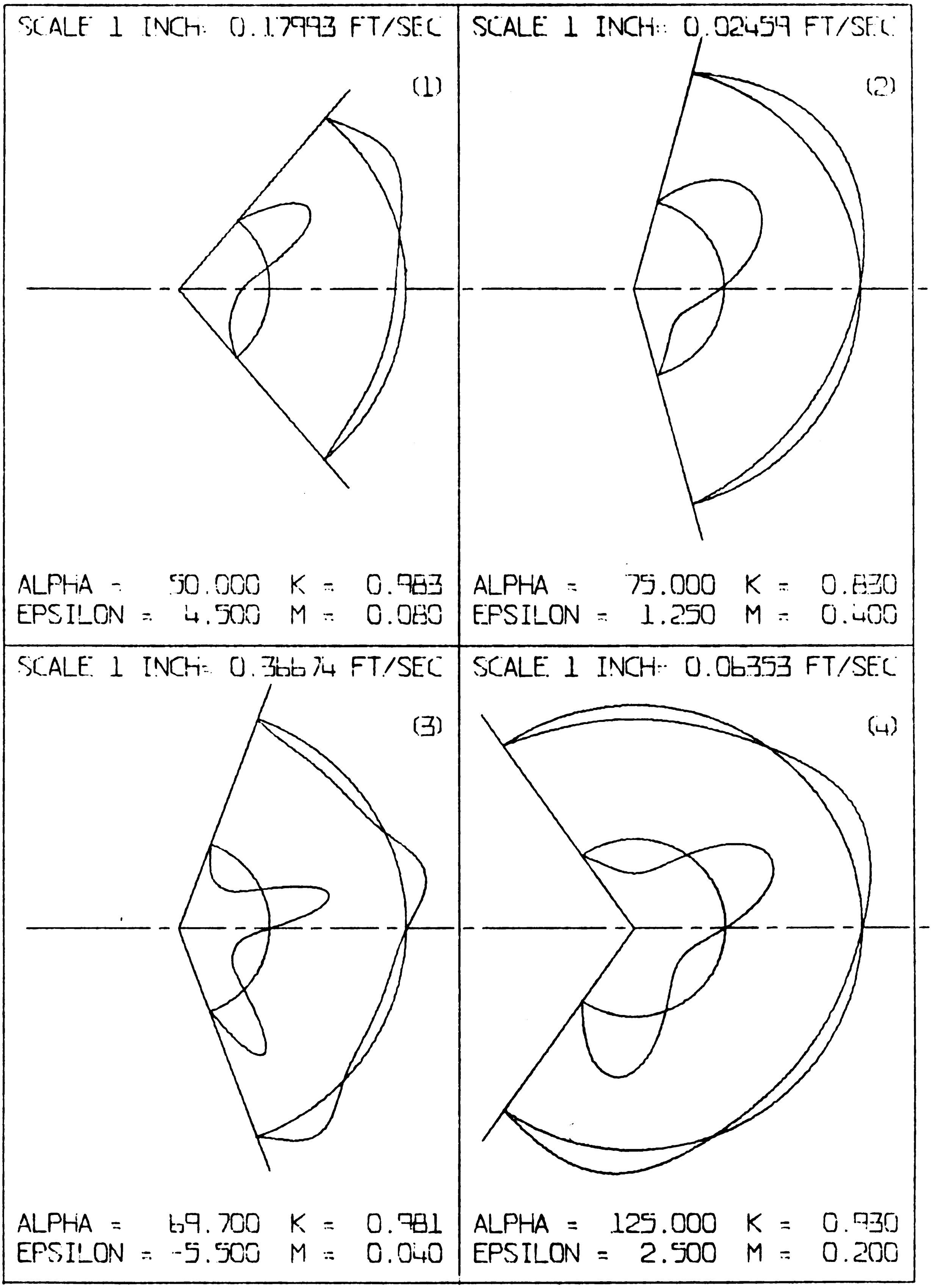

FIGURE 3.28 TYPICAL VELOCITY PROFILES, N1 AND N3 FLOW

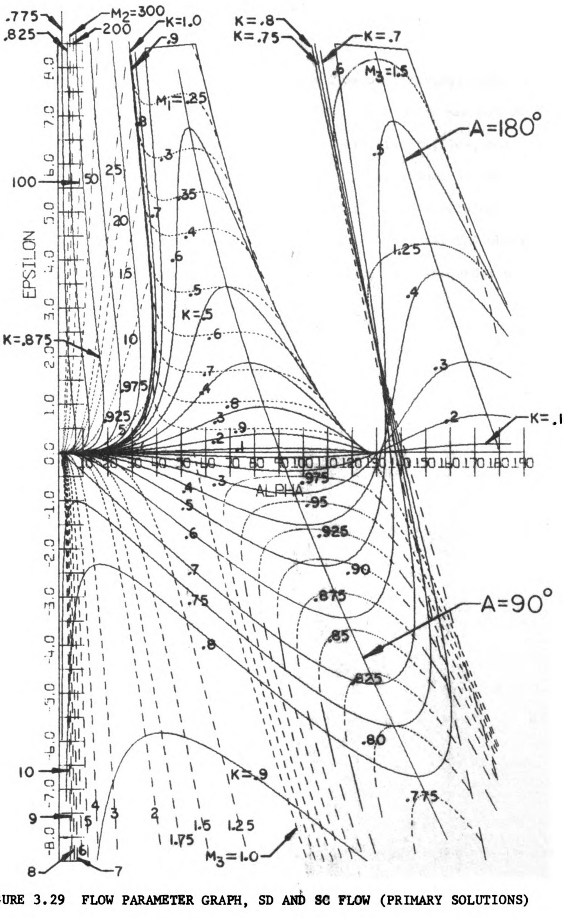

FIGURE 3.29 FLOW PARAMETER GRAPH, SD AND SC FLOW (PRIMARY SOLUTIONS)

A further summary of the possible flow profiles available in any particular region is shown in Figure 3.30. This chart can not be used to determine the flow parameters or the velocity profiles, but does indicate further the range of each type of flow. Some of the values of ϵ are read from the graphs, and some are interpolated between calculated points so these values (particularly for SC4 where $\alpha < 80^{o}$) are approximate. However, the general run and the order of magnitude of the limits are as shown.

	SD0	SD2	SC0	SC2	SC4	N1	N3
α	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
0^{o}	0.0 to $+\infty$	$-\infty$ to $+\infty$	$-\infty$ to 0.0	$-\infty$ to $+\infty$	$-\infty$ to $+\infty$	$-\infty$ to $+\infty$	$-\infty$ to $+\infty$
45^{o}	0.0 to 8.5	$-\infty$ to 11.6	$-\infty$ to 0.0	$-\infty$ to 45.0	$-\infty$ to ≈ 60	$-\infty$ to 8.5	$-\infty$ to 45.0
60^{o}	0.0 to 5.1	$-\infty$ to 7.6	$-\infty$ to 0.0	$-\infty$ to 32.2	$-\infty$ to ≈ 46	$-\infty$ to 5.1	$-\infty$ to 32.0
80^{o}	0.0 to 1.5	$-\infty$ to 4.1	$-\infty$ to 0.0	$-\infty$ to 21.4	$-\infty$ to ≈ 27	$-\infty$ to 1.5	$-\infty$ to 21.4
90^{o}	0.0 to 0.0	$-\infty$ to 2.8	$-\infty$ to 0.0	$-\infty$ to 17.8	$-\infty$ to 20.2	$-\infty$ to 0.0	$-\infty$ to 17.8
120^{o}	0.0 to 0.0	$-\infty$ to $.3$	$-\infty$ to -3.8	$-\infty$ to 10.1	$-\infty$ to 12.8	$-\infty$ to -3.8	$-\infty$ to 10.1
150^{o}	0.0 to 0.0	$-\infty$ to 0.0	$-\infty$ to -7.1	$-\infty$ to 4.5	$-\infty$ to 7.4	$-\infty$ to -7.1	$-\infty$ to 4.5
180^{o}	0.0 to 0.0	$-\infty$ to 0.0	$-\infty$ to -10.3	$-\infty$ to 0.0	$-\infty$ to 3.4	$-\infty$ to -10.3	$-\infty$ to 0.0

FIGURE 3.30 CHART OF THE RANGE OF POSSIBLE VELOCITY PROFILES*

* Most of the discrepancies between these values and the values determined by Rosenhead, [13], seem to be due to the more accurate computer computations used in this research. However, there is a definite difference between the limiting line for SD2 flow. Rosenhead's work shows possible solutions for small ϵ up to an α of 180^{o}.

IV. VERIFICATION OF A MODIFIED PERTURBATION TECHNIQUE

4.1 Introduction

The exact solutions of non-linear differential equations, such as those describing the flow of an incompressible viscous fluid, usually require such complex and tedious mathematical procedures as to be impractical. Two possible alternatives are: (i) find exact solutions for a relatively few problems to obtain a qualitative understanding of the nature of other flow problems where solutions are not obtainable; or (ii) develop simple approximate solutions suitable for practical computations. In the study of fluid flow both methods are useful and tend to complement each other, as the few exact solutions can be used to check the accuracy and reliability of the approximate methods. One of the very important approximate methods of solution is the perturbation technique where the deviation from linearized theory is considered to be small. The flow pattern then can be thought of as a combination of a simple velocity profile given by the linear theory on which is superimposed small perturbation velocities resulting from the non-linearities. The advantage of such an assumption is that the equations for the motion now become linear and can more readily be solved.

This general perturbation method is readily applicable to the case of incompressible viscous flow between non-parallel walls and provides a means of investigating important flow properties such as the stability of the various "mathematically possible" exact solutions. However, since the non-linear terms of the governing equations are ignored or approximated, the standard perturbation techniques break down in areas where the non-linear terms would become the dominant factors in describing the flow properties. In particular, for this case of flow between non-parallel planes at an angle of inclination (α) near the critical value of $\alpha_c = 128.7266988$---* degrees the standard perturbation technique does not give valid results.

A modification of the standard perturbation technique which provides solutions for all α including α_c has been developed by Yen and Tang [15 and 16]. This modified method has been applied with success to problems in other areas where the standard perturbation method breaks down at similar critical values. However, the verification of results obtained by applying this method to non-parallel flow has been difficult, since prior to this investigation each exact solution for a particular value of α and ϵ required an iterative method of solution. The flow parameter graphs developed in Chapter III for the exact velocity distribution can now be used for desired α - ϵ values.

In this Chapter the standard perturbation technique as applied to the non-parallel flow problem will be presented. The modified technique will then be discussed and the results for various α near α_c will be compared to the exact solutions as determined from the flow parameter graphs.

---

* α_c is the root of the transcendental equation (4.28) to be derived in Section 4.2.

4.2 Standard Perturbation Method

The governing equations as derived in Chapter II are

$$F^2 + 4F + F'' = -J \tag{2.10}$$

B.C 1. $F(\pm\alpha) = 0$ (2.12)

$$2. \quad \int_{-\alpha}^{\alpha} F(\theta)\,d\theta = \epsilon \tag{2.13}$$

First it is convenient to normalize the equations by letting

$$f = \frac{F}{\epsilon} \tag{4.1}$$

Therefore, equations (2.10), (2.12) and (2.13) become

$$f'' + 4f + N = -\epsilon f^2 \tag{4.2}$$

$$f(\pm\alpha) = 0 \tag{4.3}$$

$$\int_{-\alpha}^{\alpha} f(\theta)\,d\theta = 1 \tag{4.4}$$

where $N \equiv \frac{J}{\epsilon}$.

Now, define N and f in terms of infinite series of ϵ.

$$N = \sum_{i=0}^{\infty} \epsilon^i N_i = N_0 + \epsilon N_1 + \cdots \tag{4.5}$$

$$f = \sum_{i=0}^{\infty} \epsilon^i f_i = f_0 + \epsilon f_1 + \cdots \tag{4.6}$$

Then

$$f' = f_0' + \epsilon f_1' + \epsilon^2 f_2' + \cdots \tag{4.7}$$

$$f'' = f_0'' + \epsilon f_1'' + \epsilon^2 f_2'' + \cdots \tag{4.8}$$

Using (4.5), (4.6) and (4.8) one may rewrite equation (4.2) as

$$f''_0 + \epsilon f''_1 + \epsilon^2 f''_2 + \text{---} + 4(f_0 + \epsilon f_1 + \text{---}) + (N_0 + \epsilon N_1 + \text{---})$$

$$= -\epsilon[f_0^2 + 2\epsilon f_0 f_1 + \epsilon^2(f_1^2 + 2f_0 f_1) + \text{---}] \qquad (4.9)$$

Since (4.9) must hold for all values of ϵ it follows that

$$f''_0 + 4f_0 + N_0 = 0 \qquad (4.10)$$

$$f''_1 + 4f_1 + N_1 = -f_0^2 \qquad (4.11)$$

$$f''_2 + 4f_2 + N_2 = -2f_0 f_1 \qquad (4.12)$$

$$f''_3 + 4f_3 + N_3 = f_1^2 + 2f_0 f_2 \qquad (4.13)$$

$$\vdots$$

etc.

Also, using (4.6) in the boundary conditions (4.3) and (4.4) and equating powers of ϵ as above gives

$$f_i(\pm\alpha) = 0 \ , \quad i = 1, 2, \text{---} \qquad (4.14)$$

and

$$\int_{-\alpha}^{\alpha} f_0 \, d\theta = 1 \qquad (4.15)$$

$$\int_{-\alpha}^{\alpha} f_i \, d\theta = 0 \ , \quad i = 1, 2, \text{---} \qquad (4.16)$$

So the non-linear equation (4.2) with boundary conditions (4.3) and (4.4) has been replaced by an infinite set of linear equations (4.10), (4.11), (4.12), (4.13) ... etc., and an infinite number of boundary conditions given by (4.14), (4.15) and (4.16). These linear equations can easily be solved in a recursive manner.

The solution of (4.10) is

$$f_0 = C_1 \cos 2\theta + C_2 \sin 2\theta - \frac{N_0}{4} \tag{4.17}$$

applying boundary condition (4.14) at $\theta = \pm\alpha$ gives

$$C_1 \cos 2\alpha + C_2 \sin 2\alpha - \frac{N_0}{4} = 0 \tag{4.18}$$

$$C_1 \cos(-2\alpha) + C_2 \sin(-2\alpha) - \frac{N_0}{4} = 0 \tag{4.19}$$

Solving for $N_0/4$

$$\frac{N_0}{4} = C_1 \cos(2\alpha) \tag{4.20}$$

Substituting equation (4.20) into (4.17) gives

$$f_0 = C_1[\cos(2\theta) - \cos(2\alpha)] \tag{4.21}$$

Applying boundary condition (4.15) gives

$$\begin{aligned} 1 &= \int_{-\alpha}^{\alpha} f_0(\theta)d\theta = \int_{-\alpha}^{\alpha} C_1[\cos(2\theta) - \cos(2\alpha)]\, d\theta \\ &= C_1[\sin(2\alpha) - 2\alpha \cos(2\alpha)] \end{aligned} \tag{4.22}$$

or

$$C_1 = \frac{1}{\sin(2\alpha) - 2\alpha \cos(2\alpha)} \tag{4.23}$$

where (4.23) defines C_1 if $[\sin(2\alpha) - 2\alpha \cos(2\alpha)]$ does not equal zero. Substituting (4.23) into (4.21) one obtains

$$f_0(\theta) = \frac{\cos(2\theta) - \cos(2\alpha)}{\sin(2\alpha) - 2\alpha \cos(2\alpha)} \tag{4.24}$$

Similar procedures can be used to determine solutions for f_1 from (4.11), f_2 from (4.12), etc. For each equation the solution is a function of the previous solution. Then the dimensionless velocity profile can be determined for desired values of α and ϵ from equations (4.6) and (4.1). The solution is then represented by an infinite series and the number of terms required for the solution is dependent on the desired accuracy and the size of ϵ. Of course ϵ must be less than 1.0 to insure the convergence of equation (4.6).

It is noted that f_0 is not defined if

$$\sin(2\alpha) - 2\alpha\cos(2\alpha) = 0 \tag{4.25}$$

Rearranging (4.25) and defining this critical value as α_c gives the following transcendental equation for α_c.

$$\tan(2\alpha_c) = 2\alpha_c \tag{4.26}$$

Solving (4.26) gives a value of $\alpha_c = 128.7266988$-----degrees. Therefore, as $\alpha \rightarrow \alpha_c$, $f_0(\theta) \rightarrow \infty$ and the solution determined by this standard perturbation technique is completely erroneous.

Thus, the general linearization technique of approximating the non-linear equations by a finite number of linear equations gives acceptable results except near the critical value of α_c where the non-linear terms would normally be the dominant factor.

4.3 Modified Perturbation Method

In principle what is desired is a method of determining the velocity function at α_c, the critical angle, based on the solutions of α near α_c. The method developed by Yen and Tang for similar types of problems is to express the boundary specification, in this case α, by an infinite series, i.e.,

$$\alpha = \sum_{i=0}^{\infty} \alpha_i \epsilon^i = \alpha_0 + \alpha_1\epsilon + \alpha_2\epsilon^2 + - - - \tag{4.27}$$

For relatively small values of ϵ reasonable accuracy can be obtained if only the first few terms are used, such as

$$\alpha = \alpha_0 + \epsilon\alpha_1 \tag{4.28}$$

This relation can now be used to relate α to α_0 and α_1 so that α could approach α_c and be determined as a function of α_0 and α_1.

The derivation of the equations using this modification is similar to the derivation for the standard equations as shown in Section 4.2. The basic equations as given in Section 4.2 are

$$f'' + 4f + N = -\epsilon f^2 \tag{4.2}$$

$$f(\pm\alpha) = 0 \tag{4.3}$$

$$\int_{-\alpha}^{\alpha} f(\theta)\, d\theta = 1 \tag{4.4}$$

The following expansions are used

$$f = \sum_{i=0}^{\infty} \frac{\epsilon^i}{i!} f_i = f_0 + \epsilon f_1 + \frac{\epsilon^2}{2!} f + \cdots \tag{4.29}$$

$$N = \sum_{i=0}^{\infty} \frac{\epsilon^i}{i!} N_i = N_0 + \epsilon N_1 + \frac{\epsilon^2}{2!} N_2 + \cdots \tag{4.30}$$

$$\alpha = \sum_{i=0}^{\infty} \frac{\epsilon^i}{i!} \alpha_i = \alpha_0 + \epsilon\alpha_1 + \frac{\epsilon^2}{2!} \alpha_2 + \cdots \tag{4.31}$$

where

$$f'' = f_0'' + \epsilon f_1'' + \frac{\epsilon^2}{2} f_2'' + \cdots \tag{4.32}$$

Substituting (4.30) and (4.32) into (4.2) and equating like powers of ϵ as in Section 4.2 gives

$$f_0'' + 4f_0 + N_0 = 0 \tag{4.33}$$

$$f_1'' + 4f_1 + N_1 = -f_0^2 \tag{4.34}$$

$$f_2'' + 4f_2 + N_2 = -4f_0f_1 \tag{4.35}$$

$$\vdots$$

etc.

where, except for the constants these equations are the same as those derived in Section 4.2. The difference will be in the boundary conditions where α appears. From equation (4.3)

$$f(\alpha) = \epsilon f_0(\alpha) + \epsilon f_1(\alpha) + \frac{\epsilon^2}{2!} f_2(\alpha) + - - - - = 0 \tag{4.36}$$

To substitute for α first expand $f(\alpha)$ in a Taylors' Series about α_0

$$f(\alpha) = f(\alpha_0) + f'(\alpha_0)\frac{(\alpha-\alpha_0)}{1} + f''(\alpha_0)\frac{(\alpha-\alpha_0)^2}{2} + f'''(\alpha_0)\frac{(\alpha-\alpha_0)^3}{3!} + --- \tag{4.37}$$

Then from 4.33

$$(\alpha - \alpha_0) = \epsilon(\alpha_1 + \frac{\epsilon\alpha_2}{2} + - - -) \tag{4.38}$$

Substituting (4.38) into (4.37) gives

$$f(\alpha) = f(\alpha_0) + \epsilon\alpha_1 f'(\alpha_0) + \epsilon^2\left[\frac{\alpha_0}{2} f'(\alpha_0) + \frac{\alpha_1^2}{2} f''(\alpha_0)\right] + --- \tag{4.39}$$

Using (4.39) to express $f_0(\alpha)$, $\epsilon F_1(\alpha)$ and $\frac{\epsilon^2}{2} f_2(\alpha)$, equation (4.36) can be written as

$$f(\alpha) = f_0(\alpha_0) + \epsilon[f_1(\alpha_0) + \alpha_1 f_0'(\alpha_0)]$$

$$+ \epsilon^2\left[\frac{1}{2} f_2(\alpha_0) + \alpha_1 f_1'(\alpha_0) + \frac{\alpha_2}{2} f_0'(\alpha_0) + \frac{\alpha_1^2}{2} f_0''(\alpha_0)\right] + --- \tag{4.40}$$

Where by (4.3) $f(\pm\alpha) = 0$, and therefore,

$$f_0(\alpha_0) = 0 \tag{4.41}$$

$$f_1(\alpha_0) + \alpha_1 f'_0(\alpha_0) = 0 \tag{4.42}$$

$$\frac{1}{2} f_2(\alpha_0) + \alpha_1 f'_1(\alpha_0) + \frac{\alpha_2}{2} f'_0(\alpha_0) + \frac{\alpha_1{}^2}{2} f''_0(\alpha_0) = 0 \tag{4.43}$$

$$\vdots$$

Expressing the second boundary condition specified by (4.4) in terms of the expansion for α gives

$$\int_{-\alpha_0}^{\alpha_0} f_0(\theta)\, d\theta = 1 \tag{4.44}$$

and

$$\int_{-\alpha_0}^{\alpha_0} f_i(\theta)\, d\theta = 0 \qquad i = 1, 2, \text{----} \tag{4.45}$$

So the problem is now represented by the linear equations (4.33), (4.34) and (4.35) with boundary conditions specified by (4.41), (4.42), (4.43), (4.44) and (4.45).

The first equation for f_0 is

$$f''_0 + 4f_0 + N_0 = 0 \tag{4.33}$$

The solution is

$$f_0(\theta) = C_1 \cos(2\theta) - \frac{N_0}{4} \tag{4.46}$$

Applying equation (4.41) - B.C. # 1

$$f_0(\alpha_0) = 0 = C_1 \cos(2\alpha_0) - \frac{N_0}{4} \tag{4.47}$$

Therefore

$$\frac{N_0}{4} = C_1 \cos 2\alpha_0 \tag{4.48}$$

and

$$f_0(\theta) = C_1[\cos(2\theta) - \cos(2\alpha_0)] \tag{4.49}$$

Applying equation (4.44) - B.C. # 2

$$C_1 \int_{-\alpha_0}^{\alpha_0} [\cos(2\theta) - \cos(2\alpha_0)]\, d\theta = 1 \tag{4.50}$$

Integrating and solving for C_1

$$C_1 = \frac{1}{\sin(2\alpha_0) - 2\alpha_0 \cos(2\alpha_0)} \tag{4.51}$$

where $\alpha_0 \neq \alpha_c$. Substituting (4.51) into (4.49)

$$f_0(\theta) = \frac{\cos(2\theta) - \cos(2\alpha_0)}{\sin(2\alpha_0) - 2\alpha_0 \cos(2\alpha_0)} \tag{4.52}$$

Comparing this solution for $f_0(\theta)$ with that obtained by the standard perturbation method (equation (4.24)), it is the same except that (4.52) is in terms of θ and α_0 where (4.24) is in terms of θ and α. Therefore, by the standard perturbation method described in Section 4.2 α could not equal α_c, but by representing α as a series, now it is α_0 that cannot equal α_c. Therefore, α which is equal to $\alpha_0 + \epsilon\alpha_1 + \frac{\epsilon^2}{2}\alpha_2 - - -$ can take on values approaching α_c as long as α_0 does not approach α_c.

Using equation (4.52) in equation (4.34) for $f_1(\theta)$ gives

$$f_1'' + 4f_1 + N_1 = C_1^2[\cos^2(2\theta) - 2\cos(2\theta)\cos(2\alpha_0) + \cos^2(2\alpha_0)] \tag{4.53}$$

A solution is

$$f_1(\theta) = -\frac{1}{4}[N_1 + \frac{C_1^2}{2} + C_1^2\cos^2(2\alpha_0)] + \frac{C_1^2}{24}\cos(4\theta) + \frac{C_1^2}{2}\cos(2\alpha_0)\theta \sin(2\theta) \tag{4.54}$$

Equation (4.42) -B.C. #1 is

$$f_1(\alpha_0) + \alpha_1 f_0'(\alpha_0) = 0 \qquad (4.42)$$

Substituting from (4.52) and (4.54) into (4.42) gives

$$\alpha_1 = -\frac{1}{8C_1}\left[\frac{N_1 + \frac{C_1{}^2}{2} + C_1{}^2\cos^2(2\alpha_0)}{\sin(2\alpha_0)}\right] + \frac{C_1}{48}\frac{\cos(4\alpha_0)}{\sin(2\alpha_0)} + \frac{C_1}{4}\alpha_0\cos(2\alpha_0) \qquad (4.55)$$

Also, applying (4.45) -B.C. # 2 gives, after integration and rearranging

$$N_1 = \frac{2}{\alpha_0}\left[\frac{7C_1{}^2}{48}\sin(4\alpha_0) - C_1{}^2\alpha_0\cos^2(2\alpha_0)\right] - \frac{C_1{}^2}{2} \qquad (4.56)$$

This procedure may be continued to obtain solutions for $f_2(\theta)$, $f_3(\theta)$, etc., and the appropriate boundary conditions may then be used to evaluate the constants. However, even though the theory is straightforward, it is obvious from the preceding derivations that the solutions soon become quite unwieldy. For the purpose of this investigation only these first two solutions for f_0 and f_1 are used, since sufficient accuracy is obtained to determine if the general theory is correct. Therefore, equations (4.29), (4.30) and (4.31) for f, N, and α are approximated as

$$f = f_0 + \epsilon f_1 \qquad (4.57)$$

$$N = N_0 + \epsilon N_1 \qquad (4.58)$$

$$\alpha = \alpha_0 + \epsilon\alpha_1 \qquad (4.59)$$

The equations derived in this section and the definitions in Section 4.2 can now be used to determine approximations of the velocity distribution function for values of ϵ less than one, and hopefully, for values of α approaching α_c. The general procedure with the given equations is to specify a value of α_0 and ϵ where $\alpha_0 \neq \alpha_c$ and calculate α, f_0, f_1, and f. Then the velocity distribution is for the specified value of ϵ and the calculated value of α. Therefore, guesses of α_0 have to be made until α happens to be near α_c to determine if the theory does indeed give acceptable answers near α_c. This guessing procedure is undesirable, and the equations are very difficult to solve in terms of α since it requires that equation (4.55) be solved for α_0. The computer solution suggested here for the velocity distribution has an iterative loop in it so the user can specify ϵ and α. Then within the iteration a value of α_0 is found to agree with the specified value of α within reasonable accuracy.

The symbolic flow chart in Figure 4.1 summarizes the general procedure, with the iterative loop, as used in the computer program for plotting velocity distributions for radii of 1.2 and 3.0 inches and for values of θ between $\pm\alpha$.

4.4 Comparison with Exact Solution

The critical value where the standard perturbation technique is no longer valid is shown in Section 4.2 to be the solution of equation (4.28) which gives

$$\alpha_c = 128.7266988 - - - \text{ degrees}$$

Since the perturbation method - both standard and modified versions - has been derived for symmetrical flow, then the graph combining SD flow and SC flow on Figure 3.29 can be used. From this

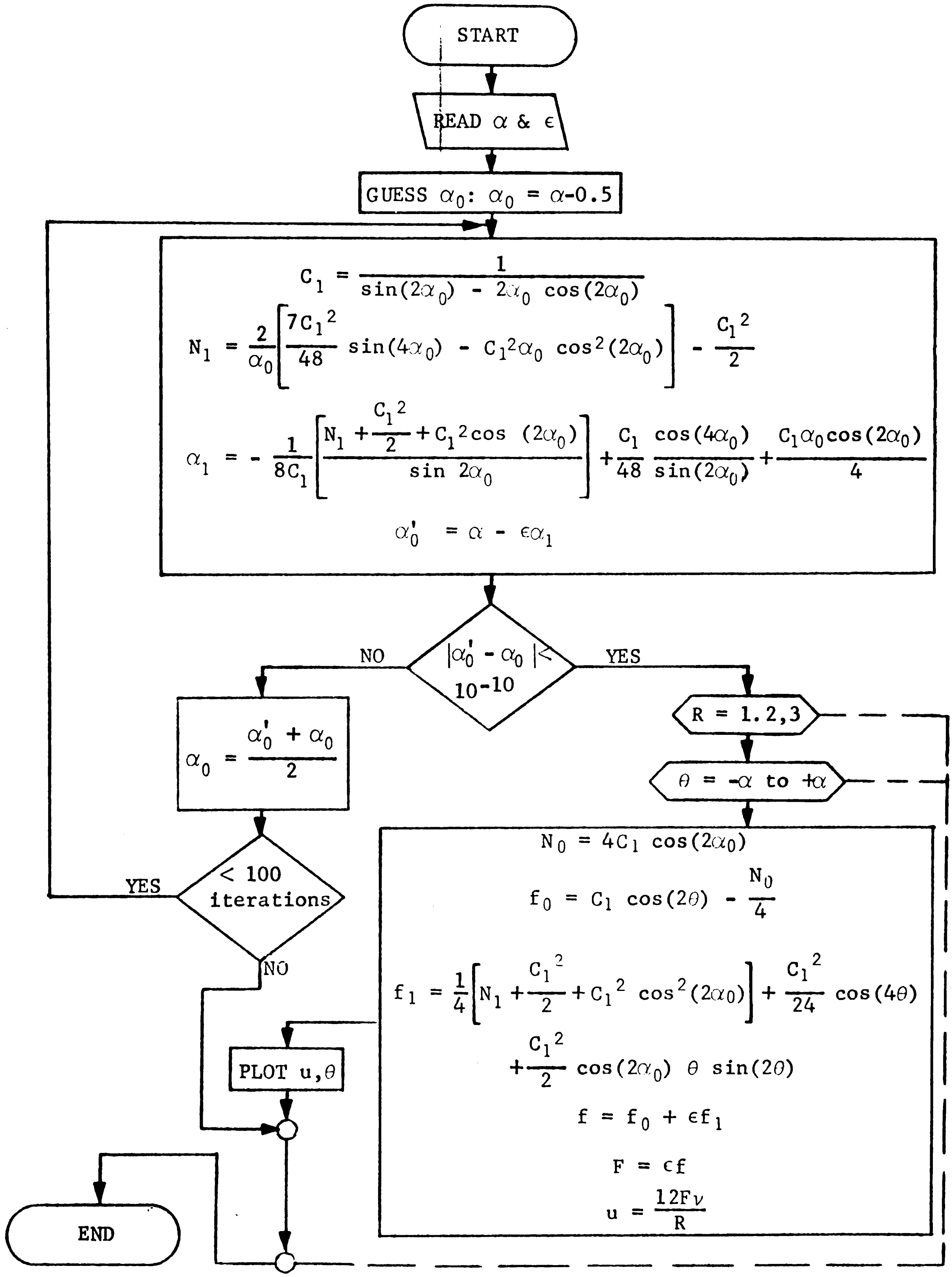

FIGURE 4.1 SYMBOLIC FLOW CHART FOR MODIFIED PERTURBATION METHOD

graph it is obvious there is no primary solution for $\epsilon > 0$ at α_c. In fact, referring back to Figure 3.7 it is clear that even including primary and secondary solutions there is no solution for SD0 or SD2 flow at $\alpha = \alpha_c$ and $\epsilon > 0$. In particular, the primary solution is for SC2 flow. From Figure 3.19 it is also obvious there is a secondary solution for SC2 flow in the same area, however, as will be verified later, the primary solution as can be determined from Figure 3.29 is the significant solution.

With the aid of Figure 3.29, and the accompanying print-out of results where more accuracy is desired, four data points near α_c are selected for comparing the dimensionless velocity distribution function, F, for the modified perturbation method with the exact solution for SC2 flow. The velocity distributions obtained from the modified perturbation method using the flow chart on Figure 4.1 are shown on Figure 4.2 with corresponding values printed for α, ϵ, α_0, and α_1. The velocity distributions obtained from the exact solution, using Figure 2.29 to determine k and m_3 and equation (2.45) to determine the velocity distribution function, are shown on Figure 4.3 with corresponding values of α, ϵ, k, and m_3. Subroutine PRO CON used for the exact solution and subroutine PERT which follows the flow chart in Figure 4.1 are both shown with some of the calculated print-out in Appendix C.

Figures 4.2 and 4.3 clearly verify the general procedure of the modified perturbation technique as the flow profiles on the two graphs are nearly identical. Exact comparison to determine the effect of larger values of ϵ is difficult to obtain from Figures 4.2 and 4.3 due to the relatively small scale. Therefore, selected data values

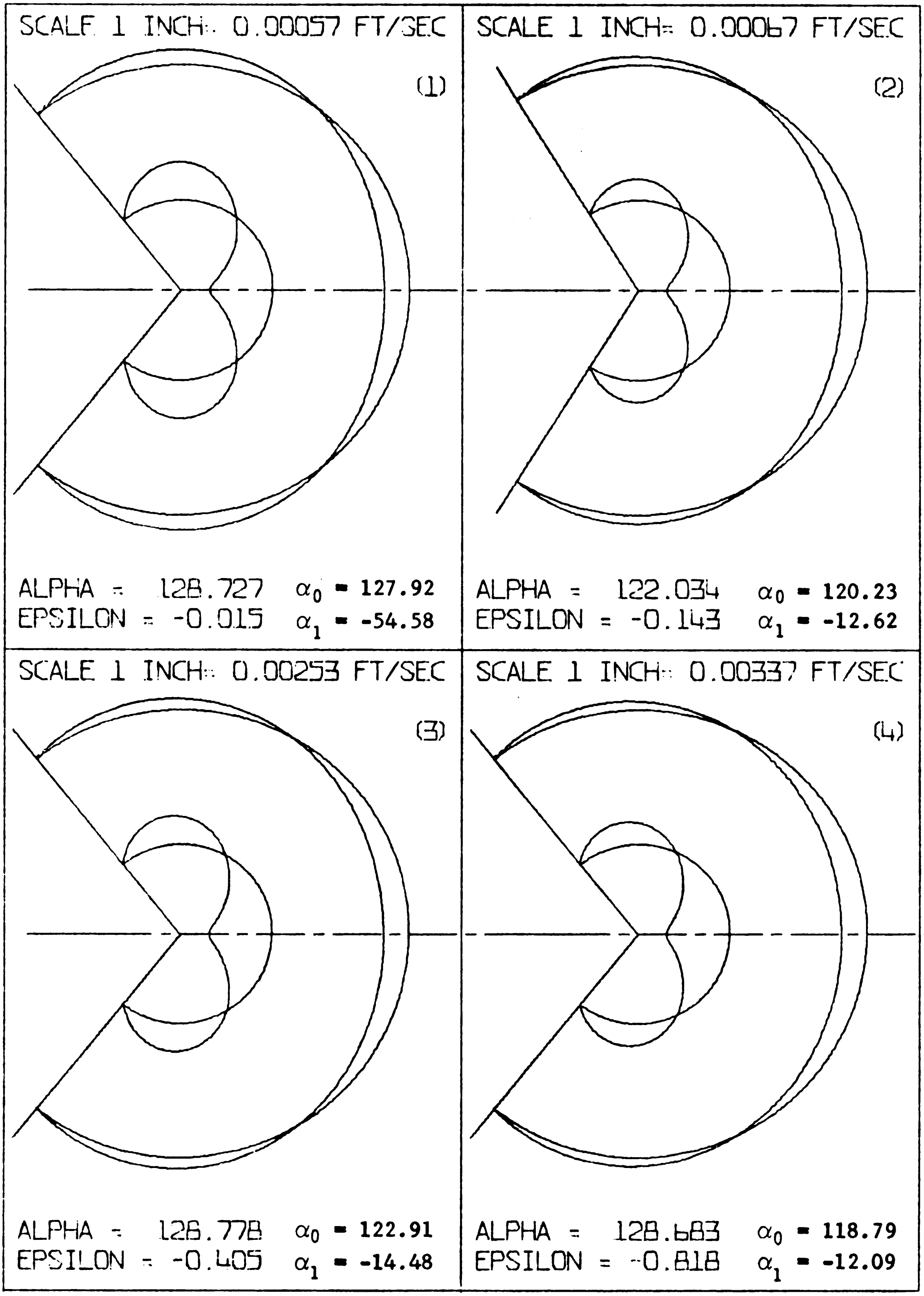

FIGURE 4.2 VELOCITY DISTRIBUTIONS OBTAINED BY THE MODIFIED PERTURBATION METHOD

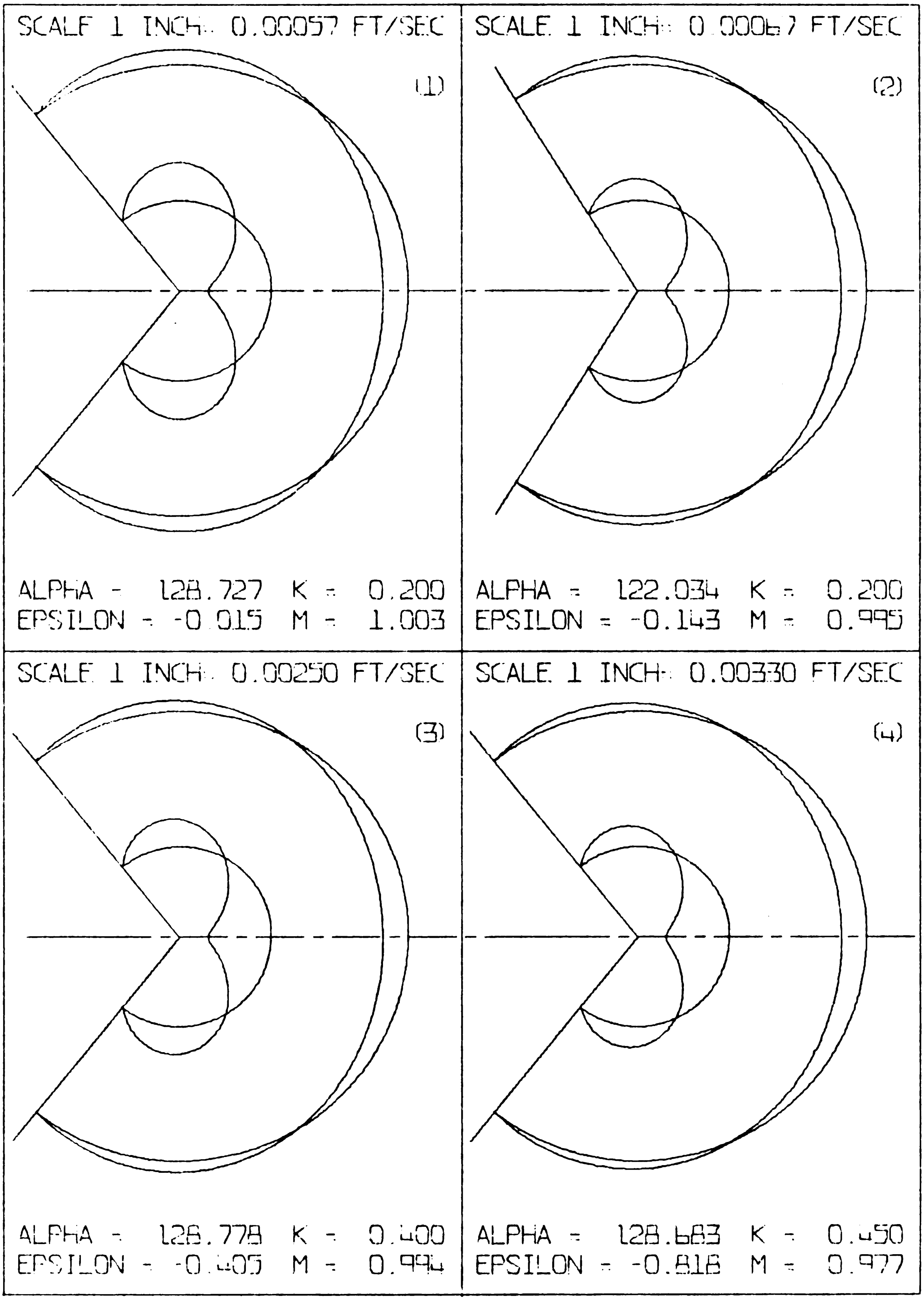

FIGURE 4.3 VELOCITY DISTRIBUTIONS, SC2 FLOW

are obtained from the computer print-out and summarized in Figure 4.4. The approximate percentage of error on Figure 4.4 was calculated as the difference between the two solutions divided by the maximum velocity for that profile. Analysis of this data indicates as expected that as ϵ is increased toward 1.0 the accuracy of the overall profile is reduced. For data point two α was chosen a significant distance from α_c, but the error is comparable to that for data point one where α is very near α_c.

Overall, the exact solutions clearly verify the correctness of the theory of the modified perturbation technique as applied to this problem. The use of one or two additional terms in the infinite series for α, f, and N would surely improve the accuracy, but the general procedure seems to be well founded.

The possibility that the modified perturbation method might indeed predict solutions at $\alpha = \alpha_c$ for $\epsilon > 0$ has also been investigated by calculating values of α for values of α_0 as $\alpha_0 \rightarrow \alpha_c$ and for $\epsilon > 0$. However, in each case as $\alpha_0 \rightarrow \alpha_c$, the resulting values of α are away from α_c. This effect is illustrated by the results plotted in Figure 4.5, which is a plot of α vs. ϵ and can be used to determine values of α for fixed values of α_0 and ϵ. Each line represents a value of α_0 and it is obvious that as $\alpha_0 \rightarrow \alpha_c$ from either direction the slope approaches zero and the value of α for a constant ϵ becomes further and further from α_c. Therefore, as none of the lines cross the α_c axis for $\epsilon > 0$, no solutions are predicted for positive ϵ when $\alpha = \alpha_c$ which agrees with the exact solution. However, all the lines eventually cross the α_c axis for $\epsilon < 0$, so there are definitely solutions in this region, again verifying the exact solution prediction.

Plot	α	ϵ	θ	Velocities at r = 1.2 inches times 10^5		% error
				SC2 Exact Soln.	Mod. Pert. Soln.	
1	128.727	-.015	120°	5.3592	5.3539	0.0223
			90°	14.5837	14.5660	0.0744
			60°	4.5890	4.5851	0.0164
			30°	-14.4651	-14.4473	0.0748
			0°	-23.7867	-23.7552	0.1324
2	122.034	-.143	120°	1.1400	1.1640	0.0855
			90°	9.7264	9.7629	0.1300
			60°	-.3621	-.3475	0.0520
			30°	-19.0067	-19.0270	0.0723
			0°	-28.0703	-28.0999	0.1054
3	128.778	-.405	120°	19.2354	20.3168	1.0372
			90°	45.6877	47.1445	1.3972
			60°	-.5874	-.0453	0.5199
			30°	-72.0406	-72.8874	0.8122
			0°	-104.2626	-105.3270	1.0209
4	128.683	-.818	120°	21.0358	24.8678	2.7863
			90°	43.7890	48.3188	3.2937
			60°	-17.2405	-15.9384	0.9468
			30°	-101.1347	-103.4543	1.6866
			0°	-137.5295	-140.3150	2.0254

FIGURE 4.4 THE DIMENSIONLESS DISTRIBUTION FUNCTION FOR COMPARISON OF SC2 FLOW AND MODIFIED PERTURBATION METHOD

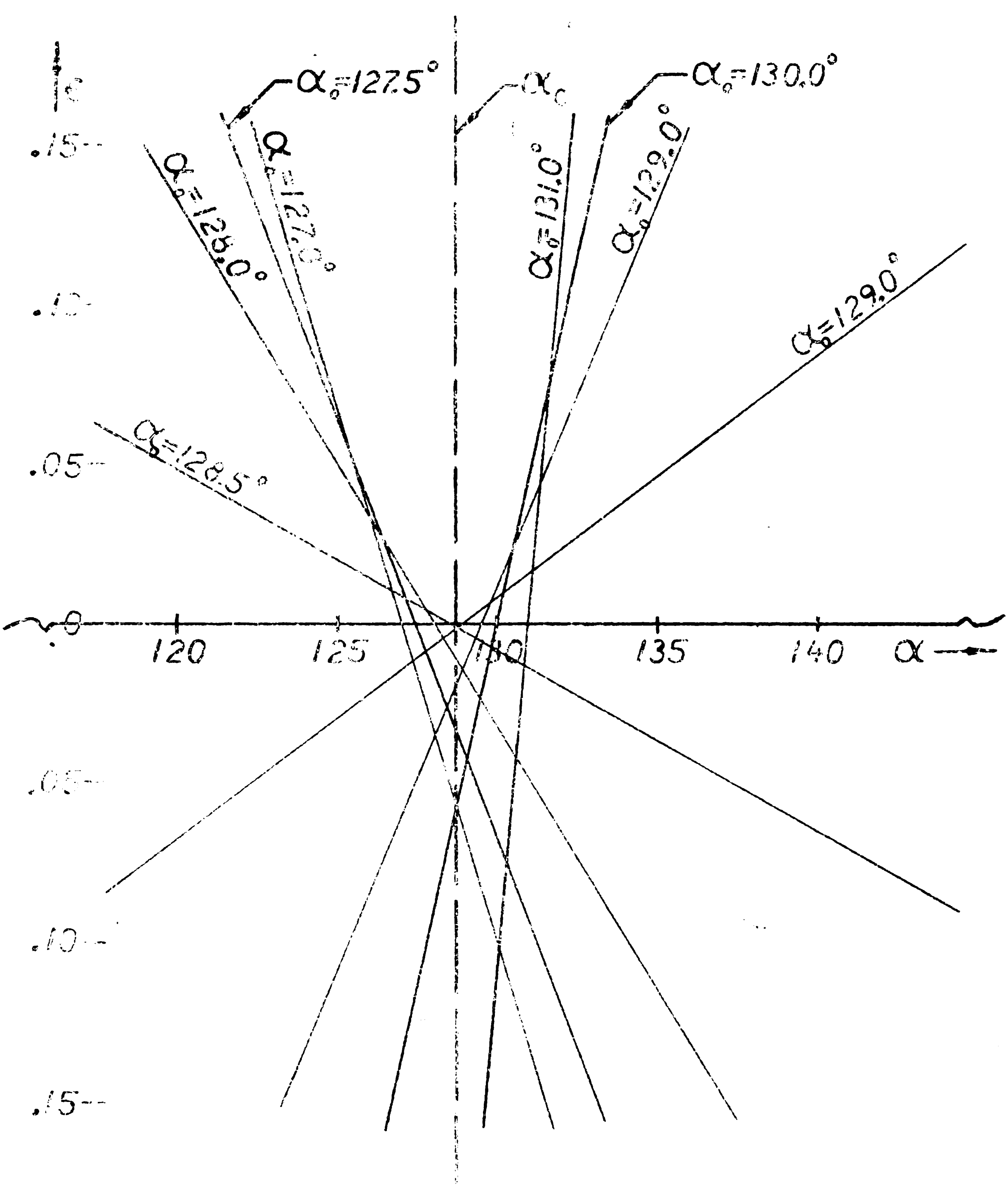

FIGURE 4.5 RELATION BETWEEN α AND α_0 NEAR α_c FROM MODIFIED PERTURBATION METHOD

4.5 Summary

An example of the application of the flow parameter graphs developed in Chapter III is presented in this chapter. However, the verification of the usefulness of the modified perturbation methods in determining velocity distributions for values of α near α_c is but one possible example of the general usefulness of the graphs. For most applications it is advantageous to have the flow parameter graphs plotted to a larger scale for more accurate determination of the flow parameters, and in fact this has been done to determine the values used in this chapter. However, for purposes of presentation in this report each graph had to be reduced in size to fit on a standard size page.

V. SUMMARY

A method has been presented in this dissertation of using the exact solutions for the problem of radial two-dimensional viscous flow between non-parallel plane walls to determine the exact shape of the velocity profile for a specified flow rate (ϵ) and angle of inclination (α). The problem and the governing equations were described in Chapter II with reference to the forms of solutions that have been obtained by previous authors. Also, several velocity profiles were described, an expression for each velocity profile was obtained, and the boundary conditions of flow rate and angle of inclination were applied.

In Chapter III it was pointed out that due to the transcendental form of equations obtained in Chapter II, explicit solutions for the velocity profile in terms of the flow conditions are difficult. However, by relating the elliptic functions in the transcendental equations to trigonometric functions the equations were solved in terms of two flow parameters (k and m), and the results were then plotted on $\alpha - \epsilon$ coordinates referred to as flow parameter graphs. The flow parameter graphs contain curves of constant k and constant m and were developed for each velocity profile that was considered. These curves describe the limits of each velocity profile and can be used to determine the flow parameters for calculating the velocity distribution for any $\alpha - \epsilon$ within the family of curves. Further, in Chapter III the limits of each velocity profile and the relations between the different profiles were investigated.

In Chapter IV an example of the application of the flow parameter graphs developed in Chapter III was presented. The example involved a modified perturbation technique used for calculating velocity profiles for small flow rates near a critical angle of inclination where standard perturbation techniques do not apply. The velocity profiles as obtained by use of the flow parameter graphs verified the modified perturbation theory as applied to this problem and provided a ready means of evaluating the accuracy of the solutions obtained for different boundary conditions.

The flow parameter graphs presented in Chapter III best summarize the results of this study. Several points that are shown on these graphs, however, merit further review here.

1. For every set of $\alpha - \epsilon$ values the number of possible velocity profiles of radial motion is infinite. [13]
2. If $\pi < \alpha < \pi/2$ SDO Flow is impossible. [13]
3. For $-10.3 \approx < \epsilon < 0$ there are regions where SCO Flow is not possible. [13]
4. When increasing positive ϵ for a fixed $\alpha < \pi/2$, the flow becomes more and more concentrated in the center of the channel until finally regions of inflow occur near the walls giving SD2 flow. Further increasing ϵ results in SD4 flow, SD6 flow ... etc., progressively excluding the simpler types of flow. [13]
5. Increasing negative ϵ for a fixed α does not exclude any of the converging flow profiles. The SCO profile exhibits all the well-known characteristics of boundary layers near the walls, and an approximately constant velocity across the rest of the flow. [13]

6. The dividing lines between SD0 and SD2 flows and between SC0 and SC2 flows are the boundary for N1 flow.
7. The dividing line between SC2 and SC4 flows is the boundary for N3 flow.
8. SD2, SD4 ... profiles and SC2, SC4,... profiles may have both positive and negative flow rates.
9. Two possible solutions were shown for SD2 and SC2 flows. Similar duplicity must also exist for SC4, SD4,.... flows.

Also of particular significance is the power of the digital computer used as a tool for this investigation. The complexity of the solutions and their dependence on elliptic functions would have made this investigation very difficult without the high speed of the computer and the graphical output of results obtained by the Calcomp plotter. In many instances the characteristics of a velocity profile and the limits of its applicability were determined by a step-wise process. Theories were developed, and then the resulting equations were evaluated and extended by computations and plotting until further limitations were encountered requiring more theoretical analysis.

The example of the applicability of the flow parameter graphs presented in Chapter IV illustrates the usefulness of the graphs in determining which velocity profiles may exist and what is their exact shape for various boundary conditions. However, there is no indication as to the type of profile that will actually be assumed for specified $\alpha - \epsilon$ conditions. The simpler symmetrical profiles with the smallest number of interior zeroes would seem more reasonable. But, variations from this, such as the non-symmetrical profiles which actually exist in bistable fluidic amplifiers and plane wall diffusers indicate the importance of investigating the stability of the various profiles. These investigations should be aided by the availability of the flow parameter graphs for evaluating results. Further, the author suggests that stability of a particular profile for given $\alpha - \epsilon$ conditions may be evaluated in terms of the variations of the flow parameters. In many instances a slight change in one of the flow parameters has a large effect on the resultant profile.

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APPENDICES

APPENDIX A

SUBROUTINE DIVERGE

for SD flow parameter graph

```
      SUBROUTINE DIVERGE
C START OF K= CONSTANT
      PI=3.1415926536
      PRINT33
   33 FORMAT(*1START OF K= CONSTANT VALUES*)
      PRINT 34
      DO7 K=10,100,10
      EK=K/100.
      KK=1
      J=10
   12 A=J/4.*PI/180.
      CALL ELLI(A,EK,F,E)
      EM=1.+EK*EK*(1.-3.*SIN(A)*SIN(A))
      IF(EM.GT.0.) GO TO 9
      IF(KK.EQ.1) PRINT10
   10 FORMAT(* EM IS NEGATIVE-- NO SOLUTION-- CONTINUE INCREMENTING A UN
     1TIL EM IS POSITIVE*)
      KK=2
      GO TO27
    9 AL=F*SQRT(EM)
      EPS=      12.*E/SQRT(EM)-4.*AL*(2.-EK*EK*EM)/EM
      AMP=6.*EK*EK/(EPS*EM)
      A=A*180./PI
      AL=AL*180./PI
      IF(J/20*20.EQ.J) PRINT11,K,J,EK,A,AL,EPS,EM,AMP
      X=AL/5.
      Y=EPS/.5
      IF(Y.GT.17..AND.K.EQ.100) GO TO 7
      IF(X.GT.38..OR.Y.GT.17..OR.Y.LT.-17.) GO TO 27
      CALL PLOT(Y,X,1)
   27 IF(A .GE.180.) GO TO 7
C  27 IF(A.GE.90.) GO TO 7
      J=J+1
      GO TO 12
    7 CALL PLOT(0.,0.,2)
C END OF K= CONSTANT
C START OF M=CONSTANT
   11 FORMAT(2I10,6F16.8    )
   34 FORMAT( 27X,*K*,15X*A*13X*ALPHA*12X*EPS*14X*M*14X*AMP*)
      PRINT35
   35 FORMAT(*1START OF M=CONSTANT VALUES*)
      PRINT 34
      AL=0.
      NZ=1
      DO28 J=4,32,NZ
      IF(J.EQ.8) NZ=2
      IF(J.EQ.20) GO TO 28
      EM =J/20.
      IF(J.EQ.22) EM=1.005
      IF(J.EQ.24) EM=1.01
      IF(J.EQ.26) EM=1.05
      IF(J.EQ.28) EM=1.2
      IF(J.EQ.30) EM=1.8
      IF(J.EQ.32) EM=2.0
      NNN=2
      NN=-1
```

```
      I=1
      KK=1
   29 A=I*PI/180.
      EK1=(EM-1.)/(1.-3.*SIN(A)*SIN(A))
      IF(EK1.GT.0..AND.EK1.LE.1.) GO TO 17
   18 IF(KK.EQ.1)PRINT 19,EK1,EM,I
   19 FORMAT(2E10.2,I10,
     1                *FOR THIS M AND A, NO VALUE OF K IS SATISFACTORY-- T
     1RY MORE A*S UNTIL ONE IS FOUND*)
      KK=2
      GO TO 30
   17 EK=SQRT(EK1)
   23 CALL ELLI(A,EK,F,E)
      AL=F*SQRT(EM)
      EPS=12.*F/SQRT(EM)-4.*AL*(2.-EK*EK*EM)/EM
      AL=AL*180./PI
      A =A *180./PI
      PRINT 25,J,I,EK,A,AL,EPS,EM
   25 FORMAT(2I10,7F15.8,I10)
      Y=EPS/.5
      X=A /5.
      IF(X.GT.38.             .OR.Y.LT.-17.) GO TO 28
      IF(            Y.GT.17.               ) GO TO 13
      CALL PLOT(Y,X,NNN)
      NNN=NNN+NN
      NN=NN*(-1)
   13 IF(A  .GT.180.) GO TO 28
      I=I+1
      GO TO 29
   30 IF(EM  .GT.1.) GO TO 28
      GO TO 13
   28 AL=0.
C SECTION FOR MAX LINE OF A= 90 DEGREES
      PRINT 36
   36 FORMAT(*1START OF CALCULATION FOR A=90 DEGREES LINE*)
      PRINT34
      Y=0.
      X=90./5.
      CALL PLOT(Y,X,2)
      DO 31 J=1,71
      EK=J/100.
      EM=1.-2.*EK*EK
      IF(EM.LT.0.) GO TO 32
      CALL CELLI(PHI,EK,F,E)
      EPS=12./SQRT(EM)*(E-F*(1.-EK*EK))
      AL=SQRT(EM)*F
      B=AL/SQRT(EM)
      B=SN(B,EK)
      IF(ABS(B-1.).LT..0001) B=1.
      A=ASIN(B)*180./PI
      AL=AL*180./PI
      Y=EPS/.5
      X=A /5.
      IF(Y.GT.17.) GO TO 32
      CALL PLOT(Y,X,1)
   31 PRINT11,J,J,EK,A,AL,EPS,EM
```

```
 32 CALL PLOT(0.,0.,2)
C SECTION FOR MAX LINE OF A= 180 DEGREES
    PRINT39
 39 FORMAT(*1START OF CALCULATION FOR A=180 LINE*)
    PRINT34
    Y=0.
    X=180./5.
    CALL PLOT (Y,X,2)
    A=180.
    DO 37 J=1,71
    EK=J/100.
    EM=EK*EK+1.
    CALL CELLI(PHI,EK,F,E)
    EPS=24./SQRT(EM)*(E-F)
    AL=SQRT(EM)*2.*F*180./PI
    Y=EPS/.5
    X=A./5.
    IF(Y.LT.-22..OR.X.GT.38.) GO TO 38
    CALL PLOT(Y,X,1)
 37 PRINT 11,J,J,EK,A,AL,EPS,EM
 38 CALL PLOT(0.,0.,2)
    END
```

START OF K= CONSTANT VALUES

		K	A	ALPHA	EPS	M	AMP
10	20	0.10000000	5,00000000	5,02443457	0,00005275	1,00977212	1126,48918322
10	40	0.10000000	10,00000000	10,04588098	0,00041828	1,00909539	142,15127577
10	60	0.10000000	15,00000000	15,06150601	0,00139105	1,00799038	42,79088140
10	80	0.10000000	20,00000000	20,06878036	0,00322955	1,00649067	18,45862176
10	100	0.10000000	25,00000000	25,06561554	0,00614008	1,00464181	9,72670538
10	120	0.10000000	30,00000000	30,05048318	0,01026263	1,00250000	5,83187444
10	140	0.10000000	35,00000000	35,02251166	0,01565945	1,00013030	3,83105386
10	160	0.10000000	40,00000000	39,98155531	0,02230695	0,99760472	2,69620291
10	180	0.10000000	45,00000000	44,92823269	0,03009144	0,99500000	2,00394190
10	200	0.10000000	50,00000000	49,86393108	0,03880906	0,99239528	1,55787817
10	220	0.10000000	55,00000000	54,79077587	0,04817009	0,98986970	1,25833339
10	240	0.10000000	60,00000000	59,71156457	0,05780797	0,98750000	1,05105746
10	260	0.10000000	65,00000000	64,62966703	0,06729243	0,98535819	0,90487989
10	280	0.10000000	70,00000000	69,54889477	0,07614683	0,98350933	0,80116306
10	300	0.10000000	75,00000000	74,47334410	0,08386876	0,98200962	0,72850959
10	320	0.10000000	80,00000000	79,40721898	0,08995316	0,98090461	0,67999864
10	340	0.10000000	85,00000000	84,35464100	0,09391697	0,98022788	0,65174860
10	360	0.10000000	90,00000000	89,31945474	0,09532409	0,98000000	0,64227721
10	380	0.10000000	95,00000000	94,30503711	0,09360936	0,98022788	0,65249623
10	400	0.10000000	100,00000000	99,31411978	0,08910044	0,98090461	0,68650645
10	420	0.10000000	105,00000000	104,34863287	0,08103628	0,98200962	0,75397339
10	440	0.10000000	110,00000000	109,40957760	0,06958135	0,98350933	0,87675836
10	460	0.10000000	115,00000000	114,49693413	0,05483479	0,98535819	1,11045502
10	480	0.10000000	120,00000000	119,60960930	0,03703399	0,98750000	1,64064116
10	500	0.10000000	125,00000000	124,74542705	0,01655254	0,98986970	3,66191869
10	520	0.10000000	130,00000000	129,90116262	-0,00610767	0,99239528	-9,89898837
10	540	0.10000000	135,00000000	135,07261958	-0,03032944	0,99500000	-1,98821690
10	560	0.10000000	140,00000000	140,25474714	-0,05539821	0,99760472	-1,08566799
10	580	0.10000000	145,00000000	145,44179360	-0,08052398	1,00013030	-0,74502253
10	600	0.10000000	150,00000000	150,62749067	-0,10486658	1,00250000	-0,57072875
10	620	0.10000000	155,00000000	155,80526214	-0,12756323	1,00464181	-0,46818176
10	640	0.10000000	160,00000000	160,96845006	-0,14775768	1,00649067	-0,40345160
10	660	0.10000000	165,00000000	166,11055084	-0,16462994	1,00799038	-0,36156472
10	680	0.10000000	170,00000000	171,22545350	-0,17742574	1,00909539	-0,33512158
10	700	0.10000000	175,00000000	176,30767247	-0,18548491	1,00977212	-0,32034598
10	720	0.10000000	180,00000000	181,35256726	-0,18826787	1,01000000	-0,31553945
20	20	0.20000000	5,00000000	5,09704284	0,00020800	1,03908847	1110,46039790
20	40	0.20000000	10,00000000	10,18233865	0,00165110	1,03638156	140,25478880
20	60	0.20000000	15,00000000	15,24470212	0,00558032	1,03196152	42,28240516
20	80	0.20000000	20,00000000	20,27405661	0,01279959	1,02596267	18,27610269
20	100	0.20000000	25,00000000	25,26195062	0,02440526	1,01856726	9,65468541
20	120	0.20000000	30,00000000	30,20202778	0,04092960	1,01000000	5,80567074
20	140	0.20000000	35,00000000	35,09043249	0,06269038	1,00052121	3,82634399
20	160	0.20000000	40,00000000	39,92613233	0,08966903	0,99041889	2,70240153
20	180	0.20000000	45,00000000	44,71113881	0,12148020	0,98000000	2,01594956
20	200	0.20000000	50,00000000	49,45060546	0,15735634	0,96958111	1,57305119
20	220	0.20000000	55,00000000	54,15278774	0,19615027	0,95947879	1,27522539
20	240	0.20000000	60,00000000	58,82885007	0,23635916	0,95000000	1,06884616
20	260	0.20000000	65,00000000	63,49251289	0,27617180	0,94143274	0,92308696
20	280	0.20000000	70,00000000	68,15954115	0,31353995	0,93403733	0,81950973
20	300	0.20000000	75,00000000	72,84708625	0,34627229	0,92803848	0,74683990
20	320	0.20000000	80,00000000	77,57290591	0,37214696	0,92361844	0,69823911
20	340	0.20000000	85,00000000	82,35449827	0,38903637	0,92091153	0,66988943
20	360	0.20000000	90,00000000	87,20819682	0,39503572	0,92000000	0,66036956
20	380	0.20000000	95,00000000	92,14827975	0,38858541	0,92091153	0,67066685
20	400	0.20000000	100,00000000	97,18614881	0,36857773	0,92361844	0,70500071
20	420	0.20000000	105,00000000	102,32962950	0,33443938	0,92803848	0,77326410
20	440	0.20000000	110,00000000	107,58243535	0,28618390	0,93403733	0,89784591
20	460	0.20000000	115,00000000	112,94382637	0,22443130	0,94143274	1,13589587

START OF M=CONSTANT VALUES

=8,01=0A1 2.00=001 1FOR THIS M AND A, NO VALUE OF K IS SATISFACTORY== TRY MORE A=S UNTIL ONE IS FOUND

		K	A	ALPHA	EPS	M	AMP
4	51	0.99266431	51.00000000	26.51458491	9.66235905	0.20000000	
4	52	0.96287315	52.00000000	26.86846241	9.47446377	0.20000000	
4	53	0.93583906	53.00000000	27.24316867	9.31794588	0.20000000	
4	54	0.91119932	54.00000000	27.63459714	9.18734631	0.20000000	
4	55	0.88865624	55.00000000	28.03973070	9.07838462	0.20000000	
4	56	0.86796290	56.00000000	28.45630310	8.98765277	0.20000000	
4	57	0.84891258	57.00000000	28.88258065	8.91240020	0.20000000	
4	58	0.83133077	58.00000000	29.31721678	8.85037982	0.20000000	
4	59	0.81506897	59.00000000	29.75915239	8.79973545	0.20000000	
4	60	0.80000000	60.00000000	30.20754597	8.75891819	0.20000000	
4	61	0.78601425	61.00000000	30.66172354	8.72662338	0.20000000	
4	62	0.77301669	62.00000000	31.12114214	8.70174238	0.20000000	
4	63	0.76092454	63.00000000	31.58536276	8.68332530	0.20000000	
4	64	0.74966536	64.00000000	32.05403002	8.67055174	0.20000000	
4	65	0.73917547	65.00000000	32.52685672	8.66270775	0.20000000	
4	66	0.72939870	66.00000000	33.00361197	8.65916737	0.20000000	
4	67	0.72028531	67.00000000	33.48411194	8.65937774	0.20000000	
4	68	0.71179117	68.00000000	33.96821274	8.66284707	0.20000000	
4	69	0.70387696	69.00000000	34.45580475	8.66913463	0.20000000	
4	70	0.69650764	70.00000000	34.94680818	8.67784263	0.20000000	
4	71	0.68965187	71.00000000	35.44116963	8.68860934	0.20000000	
4	72	0.68328157	72.00000000	35.93885931	8.70110337	0.20000000	
4	73	0.67737162	73.00000000	36.43986896	8.71501876	0.20000000	
4	74	0.67189946	74.00000000	36.94421021	8.73007082	0.20000000	
4	75	0.66684487	75.00000000	37.45191339	8.74599254	0.20000000	
4	76	0.66218969	76.00000000	37.96302670	8.76253141	0.20000000	
4	77	0.65791768	77.00000000	38.47761561	8.77944669	0.20000000	
4	78	0.65401430	78.00000000	38.99576270	8.79650689	0.20000000	
4	79	0.65046656	79.00000000	39.51756755	8.81348754	0.20000000	
4	80	0.64726293	80.00000000	40.04314702	8.83016912	0.20000000	
4	81	0.64439317	81.00000000	40.57263567	8.84633514	0.20000000	
4	82	0.64184830	82.00000000	41.10618644	8.86177034	0.20000000	
4	83	0.63962046	83.00000000	41.64397154	8.87625888	0.20000000	
4	84	0.63770287	84.00000000	42.18618356	8.88958267	0.20000000	
4	85	0.63608978	85.00000000	42.73303687	8.90151961	0.20000000	
4	86	0.63477637	86.00000000	43.28476924	8.91184185	0.20000000	
4	87	0.63375880	87.00000000	43.84164376	8.92031399	0.20000000	
4	88	0.63303406	88.00000000	44.40395115	8.92669116	0.20000000	
4	89	0.63260006	89.00000000	44.97201233	8.93071701	0.20000000	
4	90	0.63245553	90.00000000	45.54618152	8.93212155	0.20000000	
4	91	0.63260006	91.00000000	46.12684973	8.93061874	0.20000000	
4	92	0.63303406	92.00000000	46.71444887	8.92590388	0.20000000	
4	93	0.63375880	93.00000000	47.30945647	8.91765072	0.20000000	
4	94	0.63477637	94.00000000	47.91240118	8.90550811	0.20000000	
4	95	0.63608978	95.00000000	48.52386912	8.88909635	0.20000000	
4	96	0.63770287	96.00000000	49.14451138	8.86800292	0.20000000	
4	97	0.63962046	97.00000000	49.77505264	8.84177764	0.20000000	
4	98	0.64184830	98.00000000	50.41630146	8.80992701	0.20000000	
4	99	0.64439317	99.00000000	51.06916225	8.77190773	0.20000000	
4	100	0.64726293	100.00000000	51.73464949	8.72711898	0.20000000	
4	101	0.65046656	101.00000000	52.41390471	8.67489349	0.20000000	
4	102	0.65401430	102.00000000	53.10821664	8.61448683	0.20000000	
4	103	0.65791768	103.00000000	53.81904552	8.54506473	0.20000000	
4	104	0.66218969	104.00000000	54.54805250	8.46568782	0.20000000	
4	105	0.66684487	105.00000000	55.29713528	8.37529323	0.20000000	
4	106	0.67189946	106.00000000	56.06847197	8.27267217	0.20000000	
4	107	0.67737162	107.00000000	56.86457508	8.15644247	0.20000000	
4	108	0.68328157	108.00000000	57.68835879	8.02501461	0.20000000	

4	109	0.68965187	109,00000000	58,54322339	7,87654937	0.20000000
4	110	0.69650764	110,00000000	59,43316248	7,70890441	0.20000000
4	111	0.70387696	111,00000000	60,36290019	7,51956643	0.20000000
4	112	0.71179117	112,00000000	61,33806936	7,30556357	0.20000000
4	113	0.72028531	113,00000000	62,36544548	7,06335138	0.20000000
4	114	0.72939870	114,00000000	63,45325872	6,78866175	0.20000000
4	115	0.73917547	115,00000000	64,61161651	6,47629992	0.20000000
4	116	0.74966536	116,00000000	65,85308665	6,11986657	0.20000000
4	117	0.76092454	117,00000000	67,19351867	5,71136951	0.20000000
4	118	0.77301669	118,00000000	68,65322834	5,24066836	0.20000000
4	119	0.78601425	119,00000000	70,25875356	4,69465841	0.20000000
4	120	0.80000000	120,00000000	72,04554218	4,05603205	0.20000000
4	121	0.81506897	121,00000000	74,06222664	3,30132676	0.20000000
4	122	0.83133077	122,00000000	76,37774416	2,39770283	0.20000000
4	123	0.84891258	123,00000000	79,09389566	1,29731182	0.20000000
4	124	0.86796290	124,00000000	82,36917191	-0,07328938	0.20000000
4	125	0.88865624	125,00000000	86,46852147	-1,83504734	0.20000000
4	126	0.91119932	126,00000000	91,88220610	-4,20825731	0.20000000
4	127	0.93583906	127,00000000	99,67330293	-7,66276670	0.20000000
4	128	0.96287315	128,00000000	112,92760820	-13,53886724	0.20000000
-7.51-001	2,50-001	1FOR THIS M AND A, NO VALUE OF K IS SATISFACTORY-- TRY MORE A*S UNTIL ONE IS FOUND				
5	50	0.99309076	50,00000000	28,87145303	8,23632889	0.25000000
5	51	0.96114308	51,00000000	29,25915816	8,05076375	0.25000000
5	52	0.93229792	52,00000000	29,67164308	7,89661706	0.25000000
5	53	0.90612228	53,00000000	30,10375351	7,76815355	0.25000000
5	54	0.88226495	54,00000000	30,55175482	7,66093946	0.25000000
5	55	0.86043770	55,00000000	31,01287405	7,57148890	0.25000000
5	56	0.84040146	56,00000000	31,48501034	7,49702021	0.25000000
5	57	0.82195607	57,00000000	31,96654496	7,43528419	0.25000000
5	58	0.80493255	58,00000000	32,45621314	7,38444038	0.25000000
5	59	0.78918713	59,00000000	32,95301512	7,34296640	0.25000000
5	60	0.77459667	60,00000000	33,45615324	7,30959035	0.25000000
5	61	0.76105502	61,00000000	33,96498641	7,28323971	0.25000000
5	62	0.74847019	62,00000000	34,47899671	7,26300226	0.25000000
5	63	0.73676202	63,00000000	34,99776447	7,24809568	0.25000000
5	64	0.72586037	64,00000000	35,52094940	7,23784378	0.25000000
5	65	0.71570358	65,00000000	36,04827623	7,23165767	0.25000000
5	66	0.70623725	66,00000000	36,57952361	7,22902068	0.25000000
5	67	0.69741325	67,00000000	37,11451541	7,22947620	0.25000000
5	68	0.68918883	68,00000000	37,65311402	7,23261780	0.25000000
5	69	0.68152594	69,00000000	38,19521499	7,23808109	0.25000000
5	70	0.67439063	70,00000000	38,74074283	7,24553697	0.25000000
5	71	0.66775255	71,00000000	39,28964770	7,25468606	0.25000000
5	72	0.66158454	72,00000000	39,84190286	7,26525398	0.25000000
5	73	0.65586225	73,00000000	40,39750258	7,27698728	0.25000000
5	74	0.65055386	74,00000000	40,95646067	7,28965011	0.25000000
5	75	0.64566977	75,00000000	41,51880927	7,30302115	0.25000000
5	76	0.64116241	76,00000000	42,08459810	7,31689112	0.25000000
5	77	0.63702606	77,00000000	42,65389392	7,33106042	0.25000000
5	78	0.63324662	78,00000000	43,22678029	7,34533713	0.25000000
5	79	0.62981154	79,00000000	43,80335759	7,35953517	0.25000000
5	80	0.62670964	80,00000000	44,38374317	7,37347258	0.25000000
5	81	0.62393101	81,00000000	44,96807188	7,38696996	0.25000000
5	82	0.62146695	82,00000000	45,55649665	7,39984897	0.25000000
5	83	0.61930985	83,00000000	46,14918936	7,41193089	0.25000000
5	84	0.61745315	84,00000000	46,74634196	7,42303524	0.25000000
5	85	0.61589128	85,00000000	47,34816777	7,43297831	0.25000000
5	86	0.61461958	86,00000000	47,95490307	7,44157180	0.25000000
5	87	0.61363432	87,00000000	48,56680895	7,44862134	0.25000000
5	88	0.61293259	88,00000000	49,18417348	7,45392493	0.25000000
5	89	0.61251237	89,00000000	49,80731424	7,45727130	0.25000000

START OF CALCULATION FOR A=90 DEGREES LINE

		K	A	ALPHA	EPS	M	AMP
1	1	0.01000000	90.00000000	89,99324945	0,00094258	0,99980000	
2	2	0.02000000	90.00000000	89,97299122	0,00377161	0,99920000	
3	3	0.03000000	90.00000000	89,93920553	0,00849090	0,99820000	
4	4	0.04000000	90.00000000	89,89185935	0,01510685	0,99680000	
5	5	0.05000000	90.00000000	89,83090628	0,02362846	0,99500000	
6	6	0.06000000	90.00000000	89,75628641	0,03406735	0,99280000	
7	7	0.07000000	90.00000000	89,66792609	0,04643786	0,99020000	
8	8	0.08000000	90.00000000	89,56573771	0,06075705	0,98720000	
9	9	0.09000000	90.00000000	89,44961942	0,07704484	0,98380000	
10	10	0.10000000	90.00000000	89,31945474	0,09532409	0,98000000	
11	11	0.11000000	90.00000000	89,17511217	0,11562066	0,97580000	
12	12	0.12000000	90.00000000	89,01644476	0,13796361	0,97120000	
13	13	0.13000000	90.00000000	88,84328956	0,16238528	0,96620000	
14	14	0.14000000	90.00000000	88,65546706	0,18892146	0,96080000	
15	15	0.15000000	90.00000000	88,45278050	0,21761163	0,95500000	
16	16	0.16000000	90.00000000	88,23501521	0,24849906	0,94880000	
17	17	0.17000000	90.00000000	88,00193772	0,28163115	0,94220000	
18	18	0.18000000	90.00000000	87,75329494	0,31705958	0,93520000	
19	19	0.19000000	90.00000000	87,48981314	0,35484068	0,92780000	
20	20	0.20000000	90.00000000	87,20819682	0,39503572	0,92000000	
21	21	0.21000000	90.00000000	86,91112758	0,43771124	0,91180000	
22	22	0.22000000	90.00000000	86,59726272	0,48293949	0,90320000	
23	23	0.23000000	90.00000000	86,26623381	0,53079887	0,89420000	
24	24	0.24000000	90.00000000	85,91764504	0,58137441	0,88480000	
25	25	0.25000000	90.00000000	85,55107145	0,63475836	0,87500000	
26	26	0.26000000	90.00000000	85,16605691	0,69105081	0,86480000	
27	27	0.27000000	90.00000000	84,76211197	0,75036042	0,85420000	
28	28	0.28000000	90.00000000	84,33871138	0,81280519	0,84320000	
29	29	0.29000000	90.00000000	83,89529142	0,87851342	0,83180000	
30	30	0.30000000	90.00000000	83,43124690	0,94762469	0,82000000	
31	31	0.31000000	90.00000000	82,94592779	1,02029106	0,80780000	
32	32	0.32000000	90.00000000	82,43863552	1,09607842	0,79520000	
33	33	0.33000000	90.00000000	81,90861879	1,17696799	0,78220000	
34	34	0.34000000	90.00000000	81,35506893	1,26135805	0,76880000	
35	35	0.35000000	90.00000000	80,77711463	1,35006602	0,75500000	
36	36	0.36000000	90.00000000	80,17381602	1,44333070	0,74080000	
37	37	0.37000000	90.00000000	79,54415802	1,54141503	0,72620000	
38	38	0.38000000	90.00000000	78,88704277	1,64460918	0,71120000	
39	39	0.39000000	90.00000000	78,20128103	1,75323423	0,69580000	
40	40	0.40000000	90.00000000	77,48558234	1,86764643	0,68000000	
41	41	0.41000000	90.00000000	76,73854381	1,98824233	0,66380000	
42	42	0.42000000	90.00000000	75,95863722	2,11546475	0,64720000	
43	43	0.43000000	90.00000000	75,14419404	2,24980997	0,63020000	
44	44	0.44000000	90.00000000	74,29338813	2,39183631	0,61280000	
45	45	0.45000000	90.00000000	73,40421553	2,54217459	0,59500000	
46	46	0.46000000	90.00000000	72,47447077	2,70154071	0,57680000	
47	47	0.47000000	90.00000000	71,50171691	2,87075122	0,55820000	
48	48	0.48000000	90.00000000	70.48326230	3,05074241	0,53920000	
49	49	0.49000000	90.00000000	69,41610097	3,24259425	0,51980000	
50	50	0.50000000	90.00000000	68,29688472	3,44756037	0,50000000	
51	51	0.51000000	90.00000000	67,12185512	3,66710629	0,47980000	
52	52	0.52000000	90.00000000	65,88677425	3,90295852	0,45920000	
53	53	0.53000000	90.00000000	64,58683060	4,15716851	0,43820000	
54	54	0.54000000	90.00000000	63,21655870	4,43219709	0,41680000	
55	55	0.55000000	90.00000000	61,76963930	4,73102758	0,39500000	
56	56	0.56000000	90.00000000	60,23877970	5,05732015	0,37280000	
57	57	0.57000000	90.00000000	58,61544935	5,41502615	0,35020000	
58	58	0.58000000	90.00000000	56,88957455	5,81169273	0,32720000	
59	59	0.59000000	90.00000000	55,04911680	6,25290607	0,30380000	

START OF CALCULATION FOR A=180 LINE

		K	A	ALPHA	EPS	M	AMP
1	1	0,01000000	180,00000000	180,01350025	-0,00188493	1,00010000	
2	2	0,02000000	180,00000000	180,05400404	-0,00753945	1,00040000	
3	3	0,03000000	180,00000000	180,12152053	-0,01696270	1,00090000	
4	4	0,04000000	180,00000000	180,21606494	-0,03015329	1,00160000	
5	5	0,05000000	180,00000000	180,33765877	-0,04710929	1,00250000	
6	6	0,06000000	180,00000000	180,48632973	-0,06782824	1,00360000	
7	7	0,07000000	180,00000000	180,66211198	-0,09230719	1,00490000	
8	8	0,08000000	180,00000000	180,86504623	-0,12054273	1,00640000	
9	9	0,09000000	180,00000000	181,09517988	-0,15253103	1,00810000	
10	10	0,10000000	180,00000000	181,35256727	-0,18826787	1,01000000	
11	11	0,11000000	180,00000000	181,63726984	-0,22774870	1,01210000	
12	12	0,12000000	180,00000000	181,94935653	-0,27096868	1,01440000	
13	13	0,13000000	180,00000000	182,28890382	-0,31792275	1,01690000	
14	14	0,14000000	180,00000000	182,65599631	-0,36860570	1,01960000	
15	15	0,15000000	180,00000000	183,05072683	-0,42301223	1,02250000	
16	16	0,16000000	180,00000000	183,47319702	-0,48113699	1,02560000	
17	17	0,17000000	180,00000000	183,92351759	-0,54297475	1,02890000	
18	18	0,18000000	180,00000000	184,40180882	-0,60852039	1,03240000	
19	19	0,19000000	180,00000000	184,90820105	-0,67776902	1,03610000	
20	20	0,20000000	180,00000000	185,44283516	-0,75071612	1,04000000	
21	21	0,21000000	180,00000000	186,00586319	-0,82735758	1,04410000	
22	22	0,22000000	180,00000000	186,59744886	-0,90768982	1,04840000	
23	23	0,23000000	180,00000000	187,21776825	-0,99170992	1,05290000	
24	24	0,24000000	180,00000000	187,86701046	-1,07941574	1,05760000	
25	25	0,25000000	180,00000000	188,54537842	-1,17080598	1,06250000	
26	26	0,26000000	180,00000000	189,25308958	-1,26588041	1,06760000	
27	27	0,27000000	180,00000000	189,99037677	-1,36463989	1,07290000	

```
EXECUTE |     0 HOURS   1 MINUTES   01,027 SECONDS
SETUP   |     0 HOURS   0 MINUTES   05.712 SECONDS
FTN     |     0 HOURS   0 MINUTES   28,987 SECONDS
LOADER  |     0 HOURS   0 MINUTES   04,277 SECONDS
*OTHER  |     0 HOURS   0 MINUTES   00,000 SECONDS

ELAPSED TIME|     0 HOURS   1 MINUTES   40,003 SECONDS
 1560 WRITE OPERATIONS ON LU 61 WERE DONE DURING EXECUTION,
TOTAL COMPUTE COST AT REDUCED RATE  $   5,10
SEQUENCE NUMBER 022487:0000 FINISHED AT 164755              DATE  04/03/72
```

SUBROUTINE COMPLEX

for SDCO flow parameter graph

```
      SUBROUTINE COMPLEX
      EQUIVALENCE(C,EM)
      C1(A)=A*PI/180.
      C2(A)=A*180./PI
      PI=3.1415926536
      PRINT 29
   29 FORMAT(*1START OF CALCULATION FOR 1 REAL ROOT AND TWO COMPLEX ROOT
     1S*/* START OF K= CONSTANT*)
      PRINT1
    1 FORMAT(4X* K *8X* A *8X* AL*8X*EPS*8X* M *8X       *E1 *8X*E2R*8X*
     1E2C*8X*ATIL*7X*F FIRST*4X*E SECOND*)
      K=29
    9 K=K+2
      IF(K.GT.40) K=40
      EPS=0.
      MM=0
      EK=K/40.
      IF(K.EQ.41) EK=1.
C NOTE FOR SQRT(1-EK**2) NEGATIVE THERE IS NO SOLUTION- SO LIMITS K
      A=0.
      JJ=1
    8 A=A+.05
      JJ=JJ+1
      AR=C1(A)
      CALL ELLI(AR,EK,F,E)
      Q=6.*(1.+COS(AR))/(4.*EK*EK*(1.+COS(AR))+COS(AR)-5.)
      IF(Q.GT.0.) GO TO 2
      IF(MM.NE.0) GO TO 4
      MM=1
      PRINT3,EK,A,Q
    3 FORMAT(* EK=*F10.3*FEE=*F10.3* Q=*E10.3*TRY NEW VALUE OF A*)
      GO TO 4
    2 E1=2./3.*(2.*Q*EK*EK-3.-Q)
      ALR=F*SQRT(3./2./Q)
      AL=C2(ALR)
      E2R=-(3.+E1/2.)
      IF(K.EQ.40) E2C=0.
      IF(K.EQ.40) GO TO 11
      Z=3*Q-E1*E1+2.*E2R*E1-E2R*E2R
      IF(Z.GT.0.) GO TO 5
                         PRINT6,A,Q,E1,EM,AL,E2R,Z
    6 FORMAT(7E11.3* TROUBLE, CANNOT GET B FOR E2 AND E3- TRY NEXT A*)
      GO TO 4
    5 E2C=SQRT(Z)
   11 ATR=ASIN(SN(F/2.,EK))
      AT=C2(ATR)
      CALLELLI(ATR,EK,F2,E2)
      EPS=2.*(ALR*(E1-Q)+2.*SQRT(6.*Q)*E2-SQRT(6.*E1))
      IF(JJ/10*10.EQ.JJ)PRINT7,EK,A,AL,EPS,    Q,E1,E2R,E2C,AT,F,E2
      Y=EPS/.5
      X=A/V5.
      CALL PLOT(Y,X,1)
    4 IF(A.GT.90..OR.EPS.GT.8.5) GO TO 10
      GO TO 8
   10 CALL PLOT(0.,0.,2)
      IF(EK.LT.1.) GO TO 9
```

```
C START OF G= CONSTANT
      PRINT 30
   30 FORMAT(*1START OF G= CONSTANT*)
      PRINT1
      MM=5
      DO24 J=5,300,MM
      IF(J.EQ.25) MM=25
      IF(J.EQ.50) MM=50
      IF(J.EQ.100) MM=100
      NR=1
      NNN=2
      NN=-1
      EM=J
      I=1
   25 A=I/2.
      AR=D1(A)
      AZ=COS(AR)
      EK1=1./4./(1.+AZ)*(6./EM*(1.+AZ)+5.-AZ)
      IF(EK1.GT.0..AND.EK1.LT.1.) GO TO 27
      PRINT 19,A,EK1,EM
   19 FORMAT(* EK OUT OF RANGE* 3E15.7)
      GO TO 24
   27 EK=SQRT(EK1)
   20 CALL ELLI(AR,EK,F,E)
      E1=2./3.*(2.*Q*EK*EK-3.-Q)
      ALR=F*SQRT(3./2./Q)
      AL=D2(ALR)
      E2R=-(3.+E1/2.)
      IF(EK.EQ.1.) E2C=0.
      IF(EK.EQ.1.) GO TO 23
      Z=Q*C-E1*E1+2.*E2R*E1-E2R*E2R
      E2C=SQRT(Z)
   23 ATR=ASIN(SN(F/2.,EK))
      AT=D2(ATR)
      CALL ELLI(ATR,EK,F2,E2)
      EPS=2.*(ALR*(E1-Q)+2.*SQRT(6.*Q)*E2-SQRT(6.*E1))
      NR=NR+1
      IF(NR.LT.4) GO TO 31
      PRINT7,EK,A,AL,EPS,EM,  E1,E2R,E2C,AT,F,E2
    7 FORMAT(12E11.3,I4)
      NR=1
   31 Y=EPS/.5
      X=AL/5.
      CALL PLOT(Y,X,NNN)
      NNN=NNN+NN
      NN=NN*(-1)
   22 IF(A.GT.99..OR.EPS.GT.8.5) GO TO 24
      I=I+1
      GO TO 25
   24 CALL PLOT(0.,0.,2)
      END
```

START OF CALCULATION FOR 1 REAL ROOT AND TWO COMPLEX ROOTS
START OF K= CONSTANT

K	A	AL	EPS	M	E1	E2R	E2C	ATIL	F FIRST	E SECOND
7.750-001	4.500-001	1.427-001	-1.151-005	1.491+001	2.299-004	-3.000+000	1.460+001	2.250-001	7.854-003	3.927-003
7.750-001	9.500-001	3.013-001	5.379-006	1.491+001	1.025-003	-3.001+000	1.461+001	4.750-001	1.658-002	8.290-003
7.750-001	1.450+000	4.597-001	2.281-005	1.492+001	2.390-003	-3.001+000	1.462+001	7.250-001	2.531-002	1.265-002
7.750-001	1.950+000	6.180-001	5.853-005	1.494+001	4.327-003	-3.002+000	1.463+001	9.751-001	3.404-002	1.702-002
7.750-001	2.450+000	7.760-001	1.189-004	1.496+001	6.840-003	-3.003+000	1.465+001	1.225+000	4.277-002	2.138-002
7.750-001	2.950+000	9.337-001	2.103-004	1.498+001	9.933-003	-3.005+000	1.467+001	1.475+000	5.150-002	2.575-002
7.750-001	3.450+000	1.091+000	3.391-004	1.501+001	1.361-002	-3.007+000	1.470+001	1.725+000	6.024-002	3.011-002
7.750-001	3.950+000	1.248+000	5.120-004	1.504+001	1.788-002	-3.009+000	1.473+001	1.976+000	6.897-002	3.448-002
7.750-001	4.450+000	1.404+000	7.355-004	1.508+001	2.276-002	-3.011+000	1.477+001	2.226+000	7.771-002	3.885-002
7.750-001	4.950+000	1.560+000	1.016-003	1.512+001	2.824-002	-3.014+000	1.481+001	2.476+000	8.646-002	4.321-002
7.750-001	5.450+000	1.716+000	1.361-003	1.516+001	3.435-002	-3.017+000	1.485+001	2.727+000	9.521-002	4.758-002
7.750-001	5.950+000	1.870+000	1.777-003	1.521+001	4.109-002	-3.021+000	1.490+001	2.977+000	1.040-001	5.195-002
7.750-001	6.450+000	2.024+000	2.271-003	1.527+001	4.848-002	-3.024+000	1.496+001	3.228+000	1.127-001	5.632-002
7.750-001	6.950+000	2.177+000	2.863-003	1.533+001	5.652-002	-3.028+000	1.501+001	3.479+000	1.215-001	6.069-002
7.750-001	7.450+000	2.330+000	3.536-003	1.539+001	6.525-002	-3.033+000	1.508+001	3.730+000	1.302-001	6.507-002
7.750-001	7.950+000	2.481+000	4.309-003	1.546+001	7.467-002	-3.037+000	1.515+001	3.981+000	1.390-001	6.944-002
7.750-001	8.450+000	2.631+000	5.191-003	1.554+001	8.480-002	-3.042+000	1.522+001	4.232+000	1.478-001	7.382-002
7.750-001	8.950+000	2.780+000	6.188-003	1.562+001	9.567-002	-3.048+000	1.530+001	4.483+000	1.566-001	7.820-002
7.750-001	9.450+000	2.928+000	7.310-003	1.571+001	1.073-001	-3.054+000	1.539+001	4.735+000	1.654-001	8.258-002
7.750-001	9.950+000	3.075+000	8.564-003	1.580+001	1.197-001	-3.060+000	1.548+001	4.986+000	1.742-001	8.696-002
7.750-001	1.045+001	3.221+000	9.961-003	1.590+001	1.329-001	-3.066+000	1.557+001	5.238+000	1.830-001	9.134-002
7.750-001	1.095+001	3.365+000	1.151-002	1.600+001	1.470-001	-3.074+000	1.568+001	5.490+000	1.918-001	9.573-002
7.750-001	1.145+001	3.507+000	1.321-002	1.611+001	1.620-001	-3.081+000	1.578+001	5.742+000	2.006-001	1.001-001
7.750-001	1.195+001	3.649+000	1.509-002	1.623+001	1.778-001	-3.089+000	1.590+001	5.995+000	2.095-001	1.045-001
7.750-001	1.245+001	3.788+000	1.715-002	1.636+001	1.946-001	-3.097+000	1.602+001	6.247+000	2.183-001	1.089-001
7.750-001	1.295+001	3.926+000	1.940-002	1.649+001	2.124-001	-3.106+000	1.615+001	6.500+000	2.272-001	1.133-001
7.750-001	1.345+001	4.062+000	2.185-002	1.663+001	2.312-001	-3.116+000	1.629+001	6.753+000	2.360-001	1.177-001
7.750-001	1.395+001	4.196+000	2.451-002	1.678+001	2.511-001	-3.126+000	1.644+001	7.006+000	2.449-001	1.221-001
7.750-001	1.445+001	4.328+000	2.740-002	1.694+001	2.722-001	-3.136+000	1.659+001	7.260+000	2.538-001	1.265-001
7.750-001	1.495+001	4.458+000	3.053-002	1.710+001	2.944-001	-3.147+000	1.675+001	7.513+000	2.627-001	1.309-001
7.750-001	1.545+001	4.586+000	3.391-002	1.728+001	3.179-001	-3.159+000	1.692+001	7.767+000	2.716-001	1.353-001
7.750-001	1.595+001	4.711+000	3.755-002	1.746+001	3.427-001	-3.171+000	1.710+001	8.021+000	2.806-001	1.397-001
7.750-001	1.645+001	4.834+000	4.148-002	1.766+001	3.689-001	-3.184+000	1.730+001	8.276+000	2.895-001	1.441-001
7.750-001	1.695+001	4.955+000	4.570-002	1.786+001	3.966-001	-3.198+000	1.750+001	8.531+000	2.984-001	1.486-001
7.750-001	1.745+001	5.073+000	5.024-002	1.808+001	4.258-001	-3.213+000	1.771+001	8.786+000	3.074-001	1.530-001
7.750-001	1.795+001	5.188+000	5.511-002	1.831+001	4.568-001	-3.228+000	1.794+001	9.041+000	3.164-001	1.574-001
7.750-001	1.845+001	5.301+000	6.033-002	1.855+001	4.894-001	-3.245+000	1.818+001	9.297+000	3.254-001	1.618-001
7.750-001	1.895+001	5.410+000	6.592-002	1.881+001	5.240-001	-3.262+000	1.843+001	9.553+000	3.344-001	1.663-001
7.750-001	1.945+001	5.516+000	7.190-002	1.908+001	5.606-001	-3.280+000	1.869+001	9.809+000	3.434-001	1.707-001
7.750-001	1.995+001	5.619+000	7.830-002	1.937+001	5.993-001	-3.300+000	1.898+001	1.007+001	3.525-001	1.751-001
7.750-001	2.045+001	5.719+000	8.514-002	1.968+001	6.403-001	-3.320+000	1.928+001	1.032+001	3.615-001	1.796-001
7.750-001	2.095+001	5.814+000	9.244-002	2.000+001	6.838-001	-3.342+000	1.959+001	1.058+001	3.706-001	1.840-001
7.750-001	2.145+001	5.905+000	1.002-001	2.035+001	7.299-001	-3.365+000	1.993+001	1.084+001	3.797-001	1.885-001
7.750-001	2.195+001	5.995+000	1.086-001	2.071+001	7.790-001	-3.389+000	2.029+001	1.110+001	3.888-001	1.929-001
7.750-001	2.245+001	6.079+000	1.174-001	2.110+001	8.311-001	-3.416+000	2.067+001	1.135+001	3.979-001	1.974-001
7.750-001	2.295+001	6.158+000	1.269-001	2.151+001	8.866-001	-3.443+000	2.107+001	1.161+001	4.071-001	2.019-001
7.750-001	2.345+001	6.233+000	1.370-001	2.196+001	9.458-001	-3.473+000	2.151+001	1.187+001	4.162-001	2.063-001
7.750-001	2.395+001	6.304+000	1.478-001	2.243+001	1.009+000	-3.504+000	2.197+001	1.213+001	4.254-001	2.108-001
7.750-001	2.445+001	6.369+000	1.594-001	2.293+001	1.076+000	-3.538+000	2.246+001	1.239+001	4.346-001	2.153-001
7.750-001	2.495+001	6.429+000	1.716-001	2.347+001	1.149+000	-3.574+000	2.299+001	1.265+001	4.438-001	2.198-001
7.750-001	2.545+001	6.484+000	1.848-001	2.405+001	1.226+000	-3.613+000	2.355+001	1.291+001	4.531-001	2.242-001
7.750-001	2.595+001	6.532+000	1.988-001	2.467+001	1.310+000	-3.655+000	2.416+001	1.318+001	4.623-001	2.287-001
7.750-001	2.645+001	6.575+000	2.138-001	2.534+001	1.399+000	-3.700+000	2.482+001	1.344+001	4.716-001	2.332-001
7.750-001	2.695+001	6.611+000	2.299-001	2.606+001	1.496+000	-3.748+000	2.553+001	1.370+001	4.809-001	2.377-001
7.750-001	2.745+001	6.641+000	2.471-001	2.684+001	1.601+000	-3.801+000	2.629+001	1.396+001	4.903-001	2.422-001
7.750-001	2.795+001	6.663+000	2.655-001	2.769+001	1.715+000	-3.857+000	2.712+001	1.423+001	4.996-001	2.468-001
7.750-001	2.845+001	6.678+000	2.853-001	2.861+001	1.839+000	-3.919+000	2.803+001	1.449+001	5.090-001	2.513-001
7.750-001	2.895+001	6.684+000	3.066-001	2.962+001	1.974+000	-3.987+000	2.901+001	1.475+001	5.184-001	2.558-001

7,750-001	2,945+001	6,682+000	3,295-001	3,072+001	2,122+000	-4,061+000	3,009+001	1,502+001	5,278-001	2,603-001
7,750-001	2,995+001	6,671+000	3,542-001	3,194+001	2,285+000	-4,143+000	3,129+001	1,528+001	5,373-001	2,648-001
7,750-001	3,045+001	6,651+000	3,809-001	3,328+001	2,465+000	-4,233+000	3,260+001	1,555+001	5,468-001	2,694-001
7,750-001	3,095+001	6,620+000	4,098-001	3,477+001	2,665+000	-4,333+000	3,406+001	1,581+001	5,563-001	2,739-001
7,750-001	3,145+001	6,578+000	4,413-001	3,644+001	2,888+000	-4,444+000	3,569+001	1,608+001	5,658-001	2,785-001
7,750-001	3,195+001	6,523+000	4,757-001	3,830+001	3,139+000	-4,570+000	3,752+001	1,635+001	5,753-001	2,830-001
7,750-001	3,245+001	6,456+000	5,132-001	4,042+001	3,422+000	-4,711+000	3,959+001	1,662+001	5,849-001	2,876-001
7,750-001	3,295+001	6,375+000	5,546-001	4,282+001	3,745+000	-4,873+000	4,195+001	1,688+001	5,945-001	2,922-001
7,750-001	3,345+001	6,279+000	6,003-001	4,559+001	4,116+000	-5,058+000	4,465+001	1,715+001	6,042-001	2,967-001
7,750-001	3,395+001	6,166+000	6,511-001	4,880+001	4,547+000	-5,273+000	4,780+001	1,742+001	6,138-001	3,013-001
7,750-001	3,445+001	6,035+000	7,081-001	5,257+001	5,052+000	-5,526+000	5,149+001	1,769+001	6,235-001	3,059-001
7,750-001	3,495+001	5,883+000	7,725-001	5,705+001	5,655+000	-5,827+000	5,588+001	1,796+001	6,333-001	3,105-001
7,750-001	3,545+001	5,708+000	8,461-001	6,248+001	6,383+000	-6,192+000	6,120+001	1,823+001	6,430-001	3,151-001
7,750-001	3,595+001	5,507+000	9,313-001	6,919+001	7,283+000	-6,641+000	6,777+001	1,851+001	6,528-001	3,197-001
7,750-001	3,645+001	5,276+000	1,032+000	7,767+001	8,421+000	-7,210+000	7,608+001	1,878+001	6,626-001	3,243-001
7,750-001	3,695+001	5,009+000	1,152+000	8,874+001	9,906+000	-7,953+000	8,693+001	1,905+001	6,725-001	3,289-001
7,750-001	3,745+001	4,700+000	1,301+000	1,038+002	1,193+001	-8,963+000	1,017+002	1,933+001	6,823-001	3,335-001
7,750-001	3,795+001	4,337+000	1,493+000	1,255+002	1,483+001	-1,042+001	1,229+002	1,960+001	6,922-001	3,381-001
7,750-001	3,845+001	3,904+000	1,756+000	1,593+002	1,937+001	-1,268+001	1,560+002	1,987+001	7,022-001	3,428-001
7,750-001	3,895+001	3,374+000	2,149+000	2,194+002	2,744+001	-1,672+001	2,149+002	2,015+001	7,122-001	3,474-001
7,750-001	3,945+001	2,685+000	2,855+000	3,563+002	4,581+001	-2,590+001	3,490+002	2,043+001	7,222-001	3,520-001
7,750-001	3,995+001	1,646+000	4,919+000	9,748+002	1,288+002	-6,739+001	9,548+002	2,070+001	7,322-001	3,567-001
8,250-001	4,500-001	1,912-001	-1,270-006	8,305+000	1,281-004	-3,000+000	7,744+000	2,250-001	7,854-003	3,927-003
8,250-001	9,500-001	4,037-001	1,539-006	8,307+000	5,710-004	-3,000+000	7,746+000	4,750-001	1,658-002	8,290-003
8,250-001	1,450+000	6,161-001	1,327-005	8,310+000	1,331-003	-3,001+000	7,749+000	7,250-001	2,531-002	1,265-002
8,250-001	1,950+000	8,284-001	3,861-005	8,314+000	2,408-003	-3,001+000	7,753+000	9,751-001	3,404-002	1,702-002
8,250-001	2,450+000	1,040+000	8,233-005	8,320+000	3,805-003	-3,002+000	7,758+000	1,225+000	4,277-002	2,138-002
8,250-001	2,950+000	1,252+000	1,599-004	8,327+000	5,521-003	-3,003+000	7,765+000	1,475+000	5,150-002	2,575-002
8,250-001	3,450+000	1,464+000	2,564-004	8,336+000	7,560-003	-3,004+000	7,773+000	1,726+000	6,024-002	3,011-002
8,250-001	3,950+000	1,676+000	3,856-004	8,346+000	9,924-003	-3,005+000	7,782+000	1,976+000	6,898-002	3,448-002
8,250-001	4,450+000	1,887+000	5,524-004	8,357+000	1,262-002	-3,006+000	7,793+000	2,226+000	7,772-002	3,885-002
8,250-001	4,950+000	2,097+000	7,615-004	8,369+000	1,564-002	-3,008+000	7,804+000	2,477+000	8,647-002	4,322-002
8,250-001	5,450+000	2,308+000	1,018-003	8,383+000	1,899-002	-3,009+000	7,817+000	2,727+000	9,522-002	4,758-002
8,250-001	5,950+000	2,518+000	1,327-003	8,399+000	2,268-002	-3,011+000	7,832+000	2,978+000	1,040-001	5,196-002
8,250-001	6,450+000	2,727+000	1,693-003	8,415+000	2,672-002	-3,013+000	7,847+000	3,228+000	1,127-001	5,633-002
8,250-001	6,950+000	2,936+000	2,122-003	8,434+000	3,110-002	-3,016+000	7,864+000	3,479+000	1,215-001	6,070-002
8,250-001	7,450+000	3,144+000	2,619-003	8,453+000	3,583-002	-3,018+000	7,882+000	3,730+000	1,303-001	6,508-002
8,250-001	7,950+000	3,352+000	3,189-003	8,474+000	4,092-002	-3,020+000	7,902+000	3,982+000	1,391-001	6,945-002
8,250-001	8,450+000	3,559+000	3,837-003	8,497+000	4,637-002	-3,023+000	7,923+000	4,233+000	1,478-001	7,383-002
8,250-001	8,950+000	3,765+000	4,569-003	8,521+000	5,219-002	-3,026+000	7,946+000	4,484+000	1,566-001	7,821-002
8,250-001	9,450+000	3,971+000	5,391-003	8,547+000	5,839-002	-3,029+000	7,970+000	4,736+000	1,654-001	8,259-002
8,250-001	9,950+000	4,176+000	6,309-003	8,574+000	6,497-002	-3,032+000	7,995+000	4,988+000	1,743-001	8,698-002
8,250-001	1,045+001	4,380+000	7,327-003	8,603+000	7,195-002	-3,036+000	8,022+000	5,240+000	1,831-001	9,137-002
8,250-001	1,095+001	4,583+000	8,453-003	8,634+000	7,932-002	-3,040+000	8,051+000	5,492+000	1,919-001	9,575-002
8,250-001	1,145+001	4,785+000	9,692-003	8,666+000	8,710-002	-3,044+000	8,081+000	5,744+000	2,007-001	1,001-001
8,250-001	1,195+001	4,986+000	1,105-002	8,700+000	9,531-002	-3,048+000	8,113+000	5,997+000	2,096-001	1,045-001
8,250-001	1,245+001	5,187+000	1,253-002	8,736+000	1,039-001	-3,052+000	8,146+000	6,250+000	2,185-001	1,089-001
8,250-001	1,295+001	5,386+000	1,415-002	8,774+000	1,130-001	-3,057+000	8,181+000	6,503+000	2,273-001	1,133-001
8,250-001	1,345+001	5,584+000	1,591-002	8,813+000	1,225-001	-3,061+000	8,218+000	6,757+000	2,362-001	1,177-001
8,250-001	1,395+001	5,780+000	1,781-002	8,855+000	1,325-001	-3,066+000	8,257+000	7,010+000	2,451-001	1,221-001
8,250-001	1,445+001	5,976+000	1,987-002	8,898+000	1,430-001	-3,072+000	8,297+000	7,264+000	2,540-001	1,266-001
8,250-001	1,495+001	6,170+000	2,209-002	8,944+000	1,540-001	-3,077+000	8,340+000	7,518+000	2,630-001	1,310-001
8,250-001	1,545+001	6,363+000	2,447-002	8,991+000	1,655-001	-3,083+000	8,384+000	7,773+000	2,719-001	1,354-001
8,250-001	1,595+001	6,554+000	2,704-002	9,041+000	1,775-001	-3,089+000	8,431+000	8,028+000	2,808-001	1,398-001
8,250-001	1,645+001	6,744+000	2,979-002	9,093+000	1,900-001	-3,095+000	8,479+000	8,283+000	2,898-001	1,442-001
8,250-001	1,695+001	6,932+000	3,273-002	9,148+000	2,031-001	-3,102+000	8,530+000	8,538+000	2,988-001	1,486-001
8,250-001	1,745+001	7,119+000	3,587-002	9,205+000	2,168-001	-3,108+000	8,583+000	8,794+000	3,078-001	1,531-001
8,250-001	1,795+001	7,304+000	3,923-002	9,264+000	2,311-001	-3,116+000	8,638+000	9,050+000	3,168-001	1,575-001
8,250-001	1,845+001	7,487+000	4,281-002	9,326+000	2,460-001	-3,123+000	8,696+000	9,307+000	3,258-001	1,619-001
8,250-001	1,895+001	7,669+000	4,661-002	9,391+000	2,616-001	-3,131+000	8,756+000	9,564+000	3,349-001	1,664-001
8,250-001	1,945+001	7,848+000	5,066-002	9,458+000	2,778-001	-3,139+000	8,819+000	9,821+000	3,440-001	1,708-001

START OF G= CONSTANT

K	A	AL	EPS	M	E1	E2R	E2C	ATIL	F FIRST	E SECOND
8,945-001	1,500+000	8,217-001	1,364-005	5,000+000	8,568-004	-3,000+000	3,999+000	7,500-001	2,618-002	1,309-002
8,947-001	3,000+000	1,644+000	1,257-004	5,000+000	3,429-003	-3,002+000	3,996+000	1,500+000	5,238-002	2,618-002
8,951-001	4,500+000	2,467+000	4,348-004	5,000+000	7,719-003	-3,004+000	3,991+000	2,251+000	7,860-002	3,929-002
8,956-001	6,000+000	3,291+000	1,050-003	5,000+000	1,373-002	-3,007+000	3,984+000	3,003+000	1,049-001	5,240-002
8,962-001	7,500+000	4,117+000	2,056-003	5,000+000	2,148-002	-3,011+000	3,976+000	3,756+000	1,312-001	6,552-002
8,970-001	9,000+000	4,946+000	3,561-003	5,000+000	3,097-002	-3,015+000	3,965+000	4,511+000	1,576-001	7,867-002
8,980-001	1,050+001	5,777+000	5,670-003	5,000+000	4,222-002	-3,021+000	3,952+000	5,268+000	1,841-001	9,184-002
8,990-001	1,200+001	6,612+000	8,488-003	5,000+000	5,523-002	-3,028+000	3,936+000	6,027+000	2,107-001	1,050-001
9,003-001	1,350+001	7,450+000	1,213-002	5,000+000	7,004-002	-3,035+000	3,919+000	6,788+000	2,374-001	1,183-001
9,017-001	1,500+001	8,293+000	1,670-002	5,000+000	8,666-002	-3,043+000	3,899+000	7,553+000	2,643-001	1,315-001
9,032-001	1,650+001	9,141+000	2,232-002	5,000+000	1,051-001	-3,053+000	3,877+000	8,320+000	2,913-001	1,448-001
9,049-001	1,800+001	9,994+000	2,911-002	5,000+000	1,254-001	-3,063+000	3,852+000	9,092+000	3,185-001	1,581-001
9,067-001	1,950+001	1,085+001	3,719-002	5,000+000	1,476-001	-3,074+000	3,824+000	9,868+000	3,458-001	1,715-001
9,087-001	2,100+001	1,172+001	4,670-002	5,000+000	1,718-001	-3,086+000	3,793+000	1,065+001	3,735-001	1,850-001
9,109-001	2,250+001	1,259+001	5,777-002	5,000+000	1,978-001	-3,099+000	3,759+000	1,143+001	4,013-001	1,984-001
9,132-001	2,400+001	1,348+001	7,055-002	5,000+000	2,259-001	-3,113+000	3,722+000	1,222+001	4,294-001	2,120-001
9,156-001	2,550+001	1,437+001	8,520-002	5,000+000	2,560-001	-3,128+000	3,681+000	1,302+001	4,578-001	2,256-001
9,183-001	2,700+001	1,527+001	1,019-001	5,000+000	2,882-001	-3,144+000	3,636+000	1,382+001	4,866-001	2,393-001
9,211-001	2,850+001	1,618+001	1,207-001	5,000+000	3,225-001	-3,161+000	3,587+000	1,464+001	5,156-001	2,531-001
9,240-001	3,000+001	1,711+001	1,419-001	5,000+000	3,590-001	-3,179+000	3,533+000	1,545+001	5,451-001	2,669-001
9,272-001	3,150+001	1,804+001	1,657-001	5,000+000	3,977-001	-3,199+000	3,473+000	1,628+001	5,750-001	2,809-001
9,305-001	3,300+001	1,900+001	1,923-001	5,000+000	4,387-001	-3,219+000	3,409+000	1,712+001	6,054-001	2,949-001
9,340-001	3,450+001	1,997+001	2,218-001	5,000+000	4,821-001	-3,241+000	3,337+000	1,797+001	6,363-001	3,091-001
9,376-001	3,600+001	2,095+001	2,546-001	5,000+000	5,279-001	-3,264+000	3,259+000	1,882+001	6,677-001	3,234-001
9,415-001	3,750+001	2,196+001	2,909-001	5,000+000	5,761-001	-3,288+000	3,173+000	1,969+001	6,998-001	3,378-001
9,455-001	3,900+001	2,299+001	3,309-001	5,000+000	6,270-001	-3,313+000	3,078+000	2,058+001	7,325-001	3,523-001
9,498-001	4,050+001	2,404+001	3,750-001	5,000+000	6,805-001	-3,340+000	2,972+000	2,148+001	7,661-001	3,670-001
9,542-001	4,200+001	2,512+001	4,235-001	5,000+000	7,368-001	-3,368+000	2,854+000	2,239+001	8,004-001	3,819-001
9,588-001	4,350+001	2,623+001	4,767-001	5,000+000	7,958-001	-3,398+000	2,723+000	2,333+001	8,358-001	3,969-001
9,637-001	4,500+001	2,737+001	5,351-001	5,000+000	8,579-001	-3,429+000	2,574+000	2,428+001	8,722-001	4,122-001
9,687-001	4,650+001	2,855+001	5,991-001	5,000+000	9,229-001	-3,461+000	2,404+000	2,526+001	9,097-001	4,276-001
9,740-001	4,800+001	2,977+001	6,691-001	5,000+000	9,911-001	-3,496+000	2,207+000	2,626+001	9,487-001	4,433-001
9,795-001	4,950+001	3,104+001	7,458-001	5,000+000	1,063+000	-3,531+000	1,974+000	2,729+001	9,892-001	4,593-001
9,852-001	5,100+001	3,237+001	8,297-001	5,000+000	1,138+000	-3,569+000	1,688+000	2,836+001	1,031+000	4,756-001
9,912-001	5,250+001	3,376+001	9,215-001	5,000+000	1,216+000	-3,608+000	1,315+000	2,946+001	1,076+000	4,922-001
9,974-001	5,400+001	3,523+001	1,023+000	5,000+000	1,298+000	-3,649+000	7,253-001	3,060+001	1,123+000	5,092-001

EK OUT OF RANGE 5,5000000+001 1,0032425+000 5,0000000+000

K	A	AL	EPS	M	E1	E2R	E2C	ATIL	F FIRST	E SECOND
8,063-001	1,500+000	5,810-001	1,873-005	1,000+001	1,714-003	-3,001+000	9,539+000	7,500-001	2,618-002	1,309-002
8,065-001	3,000+000	1,162+000	1,766-004	1,000+001	6,857-003	-3,003+000	9,536+000	1,500+000	5,238-002	2,618-002
8,069-001	4,500+000	1,744+000	6,255-004	1,000+001	1,544-002	-3,008+000	9,532+000	2,251+000	7,859-002	3,928-002
8,075-001	6,000+000	2,327+000	1,486-003	1,000+001	2,747-002	-3,014+000	9,526+000	3,003+000	1,048-001	5,239-002
8,082-001	7,500+000	2,910+000	2,907-003	1,000+001	4,296-002	-3,021+000	9,519+000	3,755+000	1,311-001	6,551-002
8,091-001	9,000+000	3,495+000	5,035-003	1,000+001	6,194-002	-3,031+000	9,510+000	4,509+000	1,575-001	7,865-002
8,101-001	1,050+001	4,082+000	8,015-003	1,000+001	8,443-002	-3,042+000	9,499+000	5,264+000	1,839-001	9,180-002
8,113-001	1,200+001	4,670+000	1,200-002	1,000+001	1,105-001	-3,055+000	9,486+000	6,022+000	2,105-001	1,050-001
8,127-001	1,350+001	5,261+000	1,714-002	1,000+001	1,401-001	-3,070+000	9,471+000	6,781+000	2,371-001	1,182-001
8,142-001	1,500+001	5,854+000	2,359-002	1,000+001	1,733-001	-3,087+000	9,454+000	7,543+000	2,638-001	1,314-001
8,159-001	1,650+001	6,450+000	3,152-002	1,000+001	2,102-001	-3,105+000	9,434+000	8,307+000	2,907-001	1,446-001
8,178-001	1,800+001	7,049+000	4,110-002	1,000+001	2,509-001	-3,125+000	9,413+000	9,075+000	3,176-001	1,579-001
8,198-001	1,950+001	7,651+000	5,250-002	1,000+001	2,953-001	-3,148+000	9,389+000	9,845+000	3,448-001	1,713-001
8,220-001	2,100+001	8,258+000	6,591-002	1,000+001	3,435-001	-3,172+000	9,362+000	1,062+001	3,721-001	1,846-001
8,244-001	2,250+001	8,869+000	8,151-002	1,000+001	3,957-001	-3,198+000	9,332+000	1,140+001	3,997-001	1,981-001
8,270-001	2,400+001	9,485+000	9,950-002	1,000+001	4,518-001	-3,226+000	9,299+000	1,218+001	4,274-001	2,115-001
8,297-001	2,550+001	1,011+001	1,201-001	1,000+001	5,120-001	-3,256+000	9,263+000	1,297+001	4,554-001	2,250-001
8,326-001	2,700+001	1,073+001	1,435-001	1,000+001	5,764-001	-3,288+000	9,223+000	1,376+001	4,836-001	2,386-001
8,357-001	2,850+001	1,136+001	1,700-001	1,000+001	6,450-001	-3,322+000	9,179+000	1,456+001	5,121-001	2,523-001
8,390-001	3,000+001	1,200+001	1,998-001	1,000+001	7,180-001	-3,359+000	9,131+000	1,537+001	5,410-001	2,660-001
8,424-001	3,150+001	1,265+001	2,332-001	1,000+001	7,954-001	-3,398+000	9,078+000	1,618+001	5,701-001	2,797-001
8,461-001	3,300+001	1,331+001	2,704-001	1,000+001	8,774-001	-3,439+000	9,021+000	1,700+001	5,997-001	2,936-001

SUBROUTINE CONVERG

for SC flow parameter graph

```
      SUBROUTINE CONVERG
      PI=3.1415926536
C  CONSTANT K
      PRINT6
    6 FORMAT(*1START OF CONVERGING FLOW*/* K=CONSTANT*)
   15 FORMAT(20X,F16.8,16X,3F16.8)
      PRINT34
   34 FORMAT( 27X,*K*,15X*A*13X*ALPHA*12X*EPS*14X*M*)
      JJ=20
      DO1 K=10,100,10
      EK=K/100.
      IF(K.EQ.80) EK=.75
      IF(K.GE.90) EK=EK-.1
      L=0
      KK=1
      LL=2
      IF(EK.GT..7) JJ=2
C     DO 2 I=L,1800,JJ
      DO 2 I=L,6000,JJ
      IF(I.EQ.1800) GO TO 2
      A=I/20.
      AR=A*PI/180.
      CALL ELLI(AR,EK,F,E)
      T=(-1.+EK*EK*SIN(AR)*SIN(AR))/((EK*EK-2.)*EK*EK*SIN(AR)*SIN(AR)-1.
     1+2.*EK*EK)
      IF(T.GE.0.) GO TO 3
      IF(KK.EQ.2) GO TO 2
      PRINT 4,EK,A,T
      KK=2
    4 FORMAT(3E12.4,* NO SOLUTION FOR EM*)
      GO TO 2
    3 EM=SQRT(T)
      ALR=1./EM*F
      AL=ALR*180./PI
      B=ALR*EM
      EPS=4.*ALR*(EM*EM*(EK*EK-2.)-1.)+12.*EM*          (E-EK*EK*SN(B,E
     1K)*CN(B,EK)/DN(B,EK))
      IF(AL.GT.105.) GO TO 1
      IF(EPS.LT.-8.5                 .OR.EPS.GT.8.5) GO TO 7
      Y=EPS/.5
      X=AL/5.
      CALL PLOT(Y,X,LL)
      LL=1
    7 IF(I/100*100.EQ.I)                                PRINT5,K,I,EK,A,AL,
     1EPS,EM
    5 FORMAT(2I10,6F16.8)
    2 CONTINUE
    1 CONTINUE
  111 CONTINUE
      CALL MCONST
C SECTION FOR PLOTTING MAX LINE FOR A= 90 DEGREES.
      PRINT8
    8 FORMAT(*1CALCULATIONS FOR MAX LINE AT A= 90 DEGREES.*)
      PRINT 34
      L=0
      LL=2
```

```
      DO 14 K=L,95,5
      EK=K/100.
      S=EK*EK
      EM=SQRT((S-1.)/(S*S-1.))
      CALL CELLI(A,EK,F,E)
      AL=F/EM
      A=AL*180./PI
      EPS=4.*F*(EM*(S-2.)-1./EM)+12.*EM*E
      Y=EPS/.5
      X=A/5.
      IF(EPS.LT.-8.5) GO TO 19
      CALL PLOT(Y,X,LL)
      LL=1
   14 PRINT15,EK,A,EPS,EM
   19 CONTINUE
C SECTION FOR A= 180 DEGREES
      CALL PLOT(0.,36.,2)
      PRINT17
   17 FORMAT(*1START OF PRINT FOR A = 180. DEGREES*)
      PRINT 34
      N=0
      DO 16 K=N,71
      EK=K/100.
      EM=SQRT(-1./(2.*EK*EK-1.))
      CALL CELLI(PHI,EK,F,E)
      ALR=2.*F/EM
      AL=ALR*180./3.1415926536
      EPS=8.*F/EM*(EM*EM*(EK*EK-2.)-1.)+24.*EM*E
      IF(EPS.GT.8.5) GO TO 18
      Y=EPS/.5
      X=AL/5.
      CALL PLOT(Y,X,1)
   16 PRINT15,EK,AL,EPS,EM
   18 CONTINUE
      END
```

```
      SUBROUTINE MCONST
C START OF M=CONSTANT
      PRINT35
   35 FORMAT(*1START OF M=CONSTANT VALUES*)
      PRINT 34
   34 FORMAT(  7X,*K*,15X*A*13X*ALPHA*12X*EPS*14X*M*)
      PI=3.1415926536
      N=25
      II=1
      IA=1
      DO1 M=775,10000,N
      Z=1
      NNN=2
      NN=-1
      IF(M.EQ.1000) N=250
      IF(M.EQ.2000) N=1000
      EM=M/1000.
      JJ=0
      KK=1
      IF(M-1000)5,6,11
    6 II=2
      A=54.
      AINC=.25
      GO TO 12
   11 A=.25
      AINC=.5
      II=2
      GO TO 12
    5 A=90.
      AINC=-.25
   12 T=EM*EM
   10 AA=A*PI/180.
      AQ=(SIN(AA)*EM)**2
      BQ=2.*T-(SIN(AA))**2*(1.+2.*T)
      CQ=1.-T
      R=BQ*BQ-4.*AQ*CQ
      IF(R.GE.0.) GO TO 3
      IF(KK.EQ.1)PRINT4,EM,A,AQ,BQ,CQ,R
      KK=2
    4 FORMAT(6F15.9,* NO SOLUTION FOR K*)
      JJ=JJ+1
      Z=-Z
      AINC=-AINC
      GO TO 2
    3 EKS1=(-BQ+Z*SQRT(R))/2./AQ
      IF(EKS1.GT.0..AND.EKS1.LT..999) GO TO14
      PRINT9,EKS1,A
    9 FORMAT(* TROUBLE EK OUT OF RANGE *2E15.7)
      IF(JJ.EQ.2) GO TO 1
      GO TO 2
   14 EK =SQRT(EKS1)
      CALL ELLI(AA,EK ,F,E)
      AL =F/EM
      S=EK*EK
      ALA =AL *180./PI
      W=EM*AL
```

```
      EPS =4.*AL*(T*(S-2.)-1.)+12.*EM*(E-S*SN(W,EK)*CN(W,EK)/DN(W,EK))
    7 Y=EPS/.5
      X=A_A/3.
      IF(II.EQ.2.AND.EPS.LT.-8.5) NNN=2
      IF(II.EQ.2.AND.EPS.LT.-8.5) NN=-1
      IF(ALA.GT.180..OR.EPS.LT.-8.5.OR.EPS.GT.8.5) GO TO 13
      CALL PLOT(Y,X,NNN)
      NNN=NNN+NN
      NN=NN*(-1)
   13 IA=IA+1
      IF(IA.LT.5) GO TO 15
      PRINT8,EK,A,ALA,EPS,EM,AQ,BQ,CQ,R
      IA=1
   15 IF(EPS.GT.8.5) GO TO 1
    8 FORMAT(6F16.8,4F10.3)
      IF(II.EQ.2.AND.A.GT.100.) JJ=2
    2 KK=1
      A=A+AINC
      GO TO 10
    1 CONTINUE
      END
```

START OF CONVERGING FLOW
K=CONSTANT

		K	A	ALPHA	EPS	M
10	0	0.10000000	0.00000000	0,00000000	0,00000000	1,01015254
10	100	0.10000000	5.00000000	4,95037996	-0,00005301	1,01003628
10	200	0.10000000	10.00000000	9,90451839	-0,00042016	1,00969116
10	300	0.10000000	15.00000000	14,86599313	-0,00139638	1,00912805
10	400	0.10000000	20.00000000	19,83802791	-0,00323900	1,00836464
10	500	0.10000000	25.00000000	24,82333321	-0,00615124	1,00742483
10	600	0.10000000	30.00000000	29,82396771	-0,01026813	1,00633794
10	700	0.10000000	35.00000000	34,84122643	-0,01564571	1,00513770
10	800	0.10000000	40.00000000	39,87556026	-0,02225392	1,00386113
10	900	0.10000000	45.00000000	44,92653087	-0,02997358	1,00254739
10	1000	0.10000000	50.00000000	49,99280335	-0,03859764	1,00123651
10	1100	0.10000000	55.00000000	55,07217793	-0,04783667	0,99996819
10	1200	0.10000000	60.00000000	60,16166025	-0,05732849	0,99878057
10	1300	0.10000000	65.00000000	65,25755838	-0,06665172	0,99770918
10	1400	0.10000000	70.00000000	70,35567284	-0,07534211	0,99678586
10	1500	0.10000000	75.00000000	75,45136503	-0,08291232	0,99603794
10	1600	0.10000000	80.00000000	80,53984746	-0,08867302	0,99548746
10	1700	0.10000000	85.00000000	85,61633858	-0,09275541	0,99515059
10	1900	0.10000000	95.00000000	95,71556338	-0,09264833	0,99515059
10	2000	0.10000000	100.00000000	100,73069128	-0,08802411	0,99548746
10	2100	0.10000000	105.00000000	105,71899093	-0,08009061	0,99603794
10	2200	0.10000000	110.00000000	110,67874582	-0,06879566	0,99678586
10	2300	0.10000000	115.00000000	115,60931460	-0,05421634	0,99770918
10	2400	0.10000000	120.00000000	120,51120633	-0,03656502	0,99878057
10	2500	0.10000000	125.00000000	125,38611188	-0,01619010	0,99996819
10	2600	0.10000000	130.00000000	130,23688940	0,00642807	1,00123651
10	2700	0.10000000	135.00000000	135,06750323	0,03068759	1,00254739
10	2800	0.10000000	140.00000000	139,88291797	0,05568022	1,00386113
10	2900	0.10000000	145.00000000	144,68895069	0,08121188	1,00513770
10	3000	0.10000000	150.00000000	149,49208604	0,10582771	1,00633794
10	3100	0.10000000	155.00000000	154,29926034	0,12884076	1,00742483
10	3200	0.10000000	160.00000000	159,11762184	0,14936360	1,00836464
10	3300	0.10000000	165.00000000	163,95427553	0,16654152	1,00912805
10	3400	0.10000000	170.00000000	168,81602088	0,17958620	1,00969116
10	3500	0.10000000	175.00000000	173,70909205	0,18780833	1,01003628
10	3600	0.10000000	180.00000000	178,63890948	0,19064818	1,01015254
10	3700	0.10000000	185.00000000	183,60985196	0,18770232	1,01003628
20	0	0.20000000	0.00000000	0,00000000	0,00000000	1,04257207
20	100	0.20000000	5.00000000	4,79835575	-0,00021218	1,04207644
20	200	0.20000000	10.00000000	9,61171649	-0,00168159	1,04060701
20	300	0.20000000	15.00000000	14,45439058	-0,00558699	1,03821525
20	400	0.20000000	20.00000000	19,33931795	-0,01295438	1,03498421
20	500	0.20000000	25.00000000	24,27744730	-0,02459057	1,03102455
20	600	0.20000000	30.00000000	29,27718537	-0,04102803	1,02646960
20	700	0.20000000	35.00000000	34,34393978	-0,06248329	1,02146963
20	800	0.20000000	40.00000000	39,47977483	-0,08883071	1,01618579
20	900	0.20000000	45.00000000	44,68319726	-0,11959234	1,01078422
20	1000	0.20000000	50.00000000	49,94908574	-0,15394417	1,00543037
20	1100	0.20000000	55.00000000	55,26877411	-0,19073839	1,00028394
20	1200	0.20000000	60.00000000	60,63029340	-0,22854070	0,99549455
20	1300	0.20000000	65.00000000	66,01977185	-0,26568161	0,99119802
20	1400	0.20000000	70.00000000	71,41698476	-0,30032002	0,98751347
20	1500	0.20000000	75.00000000	76,80603809	-0,33051713	0,98454106
20	1600	0.20000000	80.00000000	82,16616165	-0,35431821	0,98236021
20	1700	0.20000000	85.00000000	87,47757931	-0,36983963	0,98102847
20	1900	0.20000000	95.00000000	97,88060907	-0,36939770	0,98102847
20	2000	0.20000000	100.00000000	102,94074611	-0,35081342	0,98236021
20	2100	0.20000000	105.00000000	107,89084130	-0,31886559	0,98454106

```
START OF M=CONSTANT VALUES
     K               A               ALPHA                EPS                 M
TROUBLE EK OUT OF RANGE    1.0000000+000  9.0000000+001
TROUBLE EK OUT OF RANGE    9.9984852-001  8.9750000+001
TROUBLE EK OUT OF RANGE    9.9939346-001  8.9500000+001
    0.99723514     88.50000000     269.43775440     -23.87127675      0.77500000      0.60021343  -0.99849   0.39938   0.03814
    0.99210883     87.50000000     231.47630915     -19.56070756      0.77500000      0.59948222  -0.99581   0.39938   0.03397
    0.98381228     86.50000000     206.11164026     -16.62236244      0.77500000      0.59838652  -0.99180   0.39938   0.02774
    0.97119091     85.50000000     186.52031327     -14.28333351      0.77500000      0.59692765  -0.98645   0.39938   0.01949
    0.95086230     84.50000000     169.46798120     -12.14148415      0.77500000      0.59510741  -0.97978   0.39938   0.00928
  0.775000000   83.500000000    0.592928010   -0.971791053    0.399375000   -0.002824646 NO SOLUTION FOR K
    0.89673930     83.75000000     149.50106356      -9.23856602      0.77500000      0.59350639  -0.97391   0.39938   0.00038
    0.85557302     84.75000000     144.09871008      -7.98196963      0.77500000      0.59559624  -0.98157   0.39938   0.01201
    0.83876532     85.75000000     143.76873618      -7.58346589      0.77500000      0.59732633  -0.98791   0.39938   0.02174
    0.82815159     86.75000000     144.45304292      -7.35728565      0.77500000      0.59869455  -0.99293   0.39938   0.02949
    0.82126911     87.75000000     145.68539120      -7.22019505      0.77500000      0.59969924  -0.99661   0.39938   0.03521
    0.81719104     88.75000000     147.31349952      -7.14247135      0.77500000      0.60033917  -0.99895   0.39938   0.03886
    0.81550338     89.75000000     149.28637166      -7.11134319      0.77500000      0.60061357  -0.99996   0.39938   0.04044
    0.81606174     90.75000000     151.60633825      -7.12124973      0.77500000      0.60052209  -0.99962   0.39938   0.03991
    0.81891261     91.75000000     154.32113074      -7.17364962      0.77500000      0.60006486  -0.99795   0.39938   0.03729
    0.82431933     92.75000000     157.53916957      -7.27490889      0.77500000      0.59924242  -0.99493   0.39938   0.03260
    0.83269811     93.75000000     161.48397588      -7.44212049      0.77500000      0.59805578  -0.99058   0.39938   0.02586
    0.84610087     94.75000000     166.67967387      -7.71835473      0.77500000      0.59650639  -0.98491   0.39938   0.01712
    0.86884753     95.75000000     174.99664344      -8.26281155      0.77500000      0.59459614  -0.97790   0.39938   0.00643
  0.775000000   96.500000000    0.592928010   -0.971791053    0.399375000   -0.002824646 NO SOLUTION FOR K
    0.93323912     96.00000000     203.06469560     -10.68642537      0.77500000      0.59406245  -0.97595   0.39938   0.00346
    0.96242671     95.00000000     224.65690642     -12.87652260      0.77500000      0.59606258  -0.98328   0.39938   0.01463
    0.97816209     94.00000000     244.63576049     -15.01204437      0.77500000      0.59770238  -0.98929   0.39938   0.02386
    0.98840641     93.00000000     267.85122402     -17.55520362      0.77500000      0.59897986  -0.99397   0.39938   0.03111
    0.99502745     92.00000000     298.92245112     -21.00216403      0.77500000      0.59989345  -0.99732   0.39938   0.03632
    0.99878809     91.00000000     350.69259299     -26.77981213      0.77500000      0.60044206  -0.99933   0.39938   0.03945
TROUBLE EK OUT OF RANGE    9.9939346-001  9.0500000+001
TROUBLE EK OUT OF RANGE    1.0000000+000  9.0000000+001
TROUBLE EK OUT OF RANGE    9.9988847-001  8.9750000+001
TROUBLE EK OUT OF RANGE    9.9955363-001  8.9500000+001
    0.99659876     88.75000000     281.19203478     -26.23922785      0.80000000      0.63969543  -0.99891   0.36000   0.07667
    0.99549484     87.75000000     239.14151518     -21.38045171      0.80000000      0.63901355  -0.99649   0.36000   0.07280
    0.99021963     86.75000000     212.79087070     -18.30043967      0.80000000      0.63794299  -0.99267   0.36000   0.06676
    0.98277726     85.75000000     193.45729140     -16.00332125      0.80000000      0.63648508  -0.98748   0.36000   0.05857
    0.97260316     84.75000000     178.03415043     -14.12927010      0.80000000      0.63464157  -0.98091   0.36000   0.04830
    0.95879196     83.75000000     164.97804268     -12.49314895      0.80000000      0.63241472  -0.97298   0.36000   0.03601
    0.93926020     82.75000000     153.23776232     -10.95377790      0.80000000      0.62980724  -0.96369   0.36000   0.02177
    0.90575415     81.75000000     141.20511070      -9.23874189      0.80000000      0.62682232  -0.95305   0.36000   0.00569
  0.800000000   81.250000000    0.625189424   -0.947237324    0.360000000   -0.003014223 NO SOLUTION FOR K
    0.82602634     82.00000000     129.20631754      -6.93713829      0.80000000      0.62760374  -0.95584   0.36000   0.00988
    0.79969742     83.00000000     128.35092055      -6.45526532      0.80000000      0.63049463  -0.96614   0.36000   0.02551
    0.78337842     84.00000000     128.67429161      -6.16374044      0.80000000      0.63300723  -0.97509   0.36000   0.03927
    0.77161979     85.00000000     129.48590977      -5.98734690      0.80000000      0.63513848  -0.98268   0.36000   0.05106
    0.76338096     86.00000000     130.59128298      -5.86637526      0.80000000      0.63688578  -0.98891   0.36000   0.06082
    0.75730390     87.00000000     131.90949783      -5.78327857      0.80000000      0.63824701  -0.99375   0.36000   0.06847
    0.75318163     88.00000000     133.40283580      -5.72889284      0.80000000      0.63922050  -0.99722   0.36000   0.07398
    0.75078627     89.00000000     135.05526561      -5.69811245      0.80000000      0.63980506  -0.99931   0.36000   0.07729
    0.75000000     90.00000000     136.86456141      -5.68970424      0.80000000      0.64000000  -1.00000   0.36000   0.07840
    0.75078627     91.00000000     138.83976968      -5.69796332      0.80000000      0.63980506  -0.99931   0.36000   0.07729
    0.75318163     92.00000000     141.00176303      -5.72780305      0.80000000      0.63922050  -0.99722   0.36000   0.07398
    0.75730390     93.00000000     143.38680718      -5.77951287      0.80000000      0.63824701  -0.99375   0.36000   0.06847
    0.76338096     94.00000000     146.05489955      -5.85705654      0.80000000      0.63688578  -0.98891   0.36000   0.06082
    0.77161979     95.00000000     149.10837620      -5.96797615      0.80000000      0.63513848  -0.98268   0.36000   0.05106
    0.78337842     96.00000000     152.73852496      -6.12716586      0.80000000      0.63300723  -0.97509   0.36000   0.03927
    0.79969742     97.00000000     157.37124555      -6.36896334      0.80000000      0.63049463  -0.96614   0.36000   0.02551
    0.82602634     98.00000000     164.44833250      -6.81221969      0.80000000      0.62760374  -0.95584   0.36000   0.00988
```

```
0.800000000   98.750000000   0.625189424   -0.947237324   0,360000000   -0,003014223 NO SOLUTION FOR K
  0,90575415    98,25000000    189,13728966    -8,87544394     0,80000000     0,62682232  -0,95305   0,36000   0,00569
  0,93926020    97,25000000    205,04929880   -10,46834782     0,80000000     0,62980724  -0,96369   0,36000   0,02177
  0,95879196    96,25000000    218,80508915   -11,93134150     0,80000000     0,63241472  -0,97298   0,36000   0,03601
  0,97260316    95,25000000    233,19593530   -13,51102948     0,80000000     0,63464157  -0,98091   0,36000   0,04830
  0,98277726    94,25000000    249,55953628   -15,34258921     0,80000000     0,63648508  -0,98748   0,36000   0,05857
  0,99021963    93,25000000    269,55981603   -17,60833492     0,80000000     0,63794299  -0,99267   0,36000   0,06676
  0,99540484    92,25000000    296,36492475   -20,66567375     0,80000000     0,63901355  -0,99649   0,36000   0,07280
  0,99859876    91,25000000    338,69142950   -25,50809196     0,80000000     0,63969543  -0,99891   0,36000   0,07667
TROUBLE EK OUT OF RANGE   9,9955363-001  9,0500000+001
TROUBLE EK OUT OF RANGE   1,0000000+000  9,0000000+001
TROUBLE EK OUT OF RANGE   9,9991142-001  8,9750000+001
TROUBLE EK OUT OF RANGE   9,9964556-001  8,9500000+001
TROUBLE EK OUT OF RANGE   9,9920207-001  8,9250000+001
  0,99888923    88,75000000    277,47500425   -26,80350112     0,82500000     0,68030110  -0,99888   0,31937   0,12867
  0,99637426    87,75000000    236,75101652   -21,99065294     0,82500000     0,67957593  -0,99636   0,31937   0,12458
  0,99234372    86,75000000    211,30495576   -18,95722875     0,82500000     0,67843742  -0,99241   0,31937   0,11818
  0,98668127    85,75000000    192,73875404   -16,71685243     0,82500000     0,67688696  -0,98703   0,31937   0,10951
  0,97920234    84,75000000    178,07851714   -14,92020247     0,82500000     0,67492644  -0,98023   0,31937   0,09863
  0,96961733    83,75000000    165,90846032   -13,39912034     0,82500000     0,67255823  -0,97201   0,31937   0,08562
  0,95745486    82,75000000    155,42361541   -12,05564290     0,82500000     0,66978524  -0,96239   0,31937   0,07055
  0,94187951    81,75000000    146,08811650   -10,81986843     0,82500000     0,66661084  -0,95138   0,31937   0,05353
  0,92114452    80,75000000    137,44706811    -9,62275842     0,82500000     0,66303889  -0,93899   0,31937   0,03467
  0,88992855    79,75000000    128,80282159    -8,33266017     0,82500000     0,65907375  -0,92523   0,31937   0,01409
0,825000000   79,000000000   0,655844756   -0,914031438   0,319375000   -0,002388205 NO SOLUTION FOR K
  0,79336655    79,50000000    116,98216567    -5,99026827     0,82500000     0,65802159  -0,92158   0,31937   0,00869
  0,75928328    80,50000000    116,10094351    -5,45569020     0,82500000     0,66208429  -0,93568   0,31937   0,02968
  0,73896119    81,50000000    116,39491678    -5,18084173     0,82500000     0,66575496  -0,94841   0,31937   0,04898
  0,72429091    82,50000000    117,10818617    -5,00000323     0,82500000     0,66902913  -0,95977   0,31937   0,06648
  0,71308524    83,50000000    118,05530393    -4,87108790     0,82500000     0,67190281  -0,96974   0,31937   0,08204
  0,70436533    84,50000000    119,16001334    -4,77610631     0,82500000     0,67437250  -0,97831   0,31937   0,09558
  0,69759733    85,50000000    120,38430402    -4,70560107     0,82500000     0,67643519  -0,98546   0,31937   0,10699
  0,69246138    86,50000000    121,70770348    -4,65401459     0,82500000     0,67808836  -0,99120   0,31937   0,11622
  0,68875288    87,50000000    123,11929292    -4,61787336     0,82500000     0,67933001  -0,99551   0,31937   0,12319
  0,68634359    88,50000000    124,61420268    -4,59495275     0,82500000     0,68015861  -0,99838   0,31937   0,12786
  0,68515689    89,50000000    126,19200377    -4,58383293     0,82500000     0,68057317  -0,99982   0,31937   0,13021
  0,68515689    90,50000000    127,85608263    -4,58395684     0,82500000     0,68057317  -0,99982   0,31937   0,13021
  0,68634359    91,50000000    129,61366179    -4,59470673     0,82500000     0,68015861  -0,99838   0,31937   0,12786
  0,68875288    92,50000000    131,47640229    -4,61653723     0,82500000     0,67933001  -0,99551   0,31937   0,12319
  0,69246138    93,50000000    133,46172681    -4,65021129     0,82500000     0,67808836  -0,99120   0,31937   0,11622
  0,69759783    94,50000000    135,59527260    -4,69723496     0,82500000     0,67643519  -0,98546   0,31937   0,10699
  0,70436533    95,50000000    137,91541404    -4,76016361     0,82500000     0,67437250  -0,97831   0,31937   0,09558
  0,71308524    96,50000000    140,48208470    -4,84327623     0,82500000     0,67190281  -0,96974   0,31937   0,08204
  0,72427091    97,50000000    143,39584429    -4,95406742     0,82500000     0,66902913  -0,95977   0,31937   0,06648
  0,73896119    98,50000000    146,84634748    -5,10708785     0,82500000     0,66575496  -0,94841   0,31937   0,04898
  0,75928328    99,50000000    151,27484032    -5,33693942     0,82500000     0,66208429  -0,93568   0,31937   0,02968
  0,79336655   100,50000000    158,41132547    -5,78206592     0,82500000     0,65802159  -0,92158   0,31937   0,00869
0,825000000  101,000000000   0,655844756   -0,914031438   0,319375000   -0,002388205 NO SOLUTION FOR K
  0,88992855   100,25000000    182,23968200    -7,79582812     0,82500000     0,65907375  -0,92523   0,31937   0,01409
  0,92114452    99,25000000    193,74958352    -8,95969421     0,82500000     0,66303889  -0,93899   0,31937   0,03467
  0,94187951    98,25000000    204,11755702   -10,06816204     0,82500000     0,66661084  -0,95138   0,31937   0,05353
  0,95745480    97,25000000    214,67162107   -11,23488427     0,82500000     0,66978524  -0,96239   0,31937   0,07055
  0,96961733    96,25000000    226,06607462   -12,52292108     0,82500000     0,67255823  -0,97201   0,31937   0,08562
  0,97920234    95,25000000    238,92913970   -13,99939932     0,82500000     0,67492644  -0,98023   0,31937   0,09863
  0,98668127    94,25000000    254,11642312   -15,76089562     0,82500000     0,67688696  -0,98703   0,31937   0,10951
  0,99234372    93,25000000    273,07406326   -17,97416032     0,82500000     0,67843742  -0,99241   0,31937   0,11818
  0,99637426    92,25000000    298,79490412   -20,98838292     0,82500000     0,67957593  -0,99636   0,31937   0,12458
  0,99888923    91,25000000    339,68877041   -25,78659092     0,82500000     0,68030110  -0,99888   0,31937   0,12867
TROUBLE EK OUT OF RANGE   9,9920207-001  9,0750000+001
TROUBLE EK OUT OF RANGE   1,0000000+000  9,0000000+001
TROUBLE EK OUT OF RANGE   9,9992530-001  8,9750000+001
```

CALCULATIONS FOR MAX LINE AT A= 90 DEGREES.

K	A	ALPHA	EPS	M
0.00000000		90.00000000	0.00000000	1,00000000
0.05000000		90.16882939	-0,02355464	0,99875234
0.10000000		90,67628363	-0,09413393	0,99503719
0.15000000		91,52536342	-0,21150611	0,98893635
0.20000000		92,72141758	-0,37535806	0,98058068
0.25000000		94,27268921	-0,58540299	0,97014250
0.30000000		96,19113674	-0,84152880	0,95782629
0.35000000		98,49360536	-1,14398843	0,94385836
0.40000000		101,20347838	-1,49363956	0,92847669
0.45000000		104,35302942	-1,89225134	0,91192151
0.50000000		107,98685641	-2,34291423	0,89442719
0.55000000		112,16707206	-2,85062133	0,87621591
0.60000000		116,98149436	-3,42315026	0,85749293
0.65000000		122,55725807	-4,07249917	0,83844362
0.70000000		129,08490712	-4,81740676	0,81923192
0.75000000		136,86456141	-5,68816966	0,80000000
0.80000000		146,40414193	-6,73689413	0,78086881
0.85000000		158,66145049	-8,06278536	0,76193932

START OF PRINT FOR A = 160. DEGREES

K	A	ALPHA	EPS	M
0.00000000		160.00000000	0.00000000	1.00000000
0.01000000		179.98649890	0.00188517	1.00010001
0.02000000		179.94598245	0.00754322	1.00040024
0.03000000		179.87841107	0.01698180	1.00090122
0.04000000		179.78371871	0.03021371	1.00160385
0.05000000		179.66181257	0.04725692	1.00250941
0.06000000		179.51257282	0.06813471	1.00361956
0.07000000		179.33565218	0.09287571	1.00493631
0.08000000		179.13147543	0.12151409	1.00646210
0.09000000		178.89923884	0.15408969	1.00819976
0.10000000		178.63890946	0.19064818	1.01015254
0.11000000		178.35022434	0.23124133	1.01232414
0.12000000		178.03288953	0.27592722	1.01471870
0.13000000		177.68657913	0.32477055	1.01734085
0.14000000		177.31093412	0.37784293	1.02019573
0.15000000		176.90556100	0.43522326	1.02328902
0.16000000		176.47003041	0.49699613	1.02662695
0.17000000		176.00387544	0.56326229	1.03021638
0.18000000		175.50658989	0.63411916	1.03406479
0.19000000		174.97762628	0.70968137	1.03818038
0.20000000		174.41639364	0.79007144	1.04257207
0.21000000		173.82225515	0.87542248	1.04724961
0.22000000		173.19452543	0.96587898	1.05222359
0.23000000		172.53246762	1.06159774	1.05750558
0.24000000		171.83529008	1.16274882	1.06310814
0.25000000		171.10214290	1.26951671	1.06904497
0.26000000		170.33211382	1.38210162	1.07533098
0.27000000		169.52422394	1.50072084	1.08198245
0.28000000		168.67742276	1.62561039	1.08901710
0.29000000		167.79058284	1.75702684	1.09645432
0.30000000		166.86249380	1.89524937	1.10431526
0.31000000		165.89165558	2.04058212	1.11262311
0.32000000		164.87727103	2.19335685	1.12140326
0.33000000		163.81723757	2.35393597	1.13068360
0.34000000		162.71013787	2.52271610	1.14049481
0.35000000		161.55422927	2.70013204	1.15087068
0.36000000		160.34763204	2.88666140	1.16184853
0.37000000		159.08831605	3.08283007	1.17346969
0.38000000		157.77408556	3.26921837	1.18578001
0.39000000		156.40256206	3.50646845	1.19883051
0.40000000		154.97116467	3.73529286	1.21267813
0.41000000		153.47708763	3.97648467	1.22738660
0.42000000		151.91727445	4.23092951	1.24302753
0.43000000		150.28838808	4.49961993	1.25968164
0.44000000		148.58677626	4.78367262	1.27744033
0.45000000		146.80843106	5.08434917	1.29640745
0.46000000		144.94894154	5.40308142	1.31670163
0.47000000		143.00343782	5.74150244	1.33845904
0.48000000		140.96652460	6.10148483	1.36183678
0.49000000		138.83220193	6.46518651	1.38701725
0.50000000		136.59376945	6.89512075	1.41421356
0.51000000		134.24371024	7.33421258	1.44367647
0.52000000		131.77354851	7.80591704	1.47570334
0.53000000		129.17367321	8.31433703	1.51064986

SUBROUTINE NONSYMI

for N1 flow parameter graph

```
      SUBROUTINE NONSYM1
      PI=3.1415926536
      PRINT7
    7 FORMAT(*1START OF CALCULATIONS FOR NON-SYMMETRICAL FLOW*)
      PRINT8
    8 FORMAT(*0*                    3X*AL1*8X*AL2*8X*AL TOTAL/2 *
     1          *EPS TOTAL  M *       9X*K*)
      MM=200
      DO1 K=100,990,MM
      IF(K.EQ.900) MM=20
      IF(K.EQ.960) MM=5
      IF(K.EQ.980) MM=1
      EK=K/1000.
      S=EK*EK
      L=2
      CALLICELLI(A,EK,FK,EE)
      A=1.-2.*S
      IF(A.GT.0.) GO TO 10
      JL=1
      GO TO 11
   10 JL=SQRT(FK*FK*A)*180./PI+1.
   11 JT=SQRT(FK*FK*(S+1.))*180./PI+1
      IF(JT.GT.180) JT=180
      DO2 J=JL,JT,1
C FOR UPPER LIMITS
      IF(J.NE.JL.OR.JL.EQ.1) GO TO 19
      EPS=12./SQRT(A)*(FK*(S-1.)+EE)
      EM=A
      AP=A1=A2=FK*SQRT(EM)
      APA=AP*180./PI
      GO TO 18
C FOR LOWER LIMITS
   19 IF(J.NE.JT.OR.JT.EQ.180) GO TO 17
      AP=SQRT(FK*FK*(S+1))
      EPS=12.*(EE-FK)/SQRT(S+1.)
      EM=S+1.
      A1=0.
      A2=AP/2.
      APA=AP*180./PI
      GO TO 18
   17 AP=J*PI/180.
      EM=(AP/FK)**2
      EPS=4./SQRT(EM)*((EK*EK-EM-2.)*FK+3.*EE)
      A=ASIN(SQRT((1.-(EM-1.)/S)/3.))
      CALLIELLI(A,EK,F,E)
      A1=SQRT(EM)*F
      A2=SQRT(EM)*2.*FK-A1
      APA=J
   18 A1A=A1*180./PI
      A2A=A2*180./PI
      IF(EPS.GT.8.5.OR.EPS.LT.-8.5) GO TO 12
      Y=EPS/.5
      X=APA/3.
      CALLIPLOT(Y,X,L)
      L=1
   12 IF(J/5*5.EQ.J.OR.J.EQ.JT.OR.J.EQ.JL) PRINT6,A1A,A2A,APA,EPS,EM,EK,
```

```
      1JL,JT
    6 FORMAT(* *3F11.4,2E11.4,F11.4,2I5)
    2 CONTINUE
    1 CONTINUE
C PROGRAM SEGMENT FOR M=CONSTANT CURVES
      PRINT20
   20 FORMAT(*1START OF CONSTANT M CALCULATIONS*)
      PRINT8
      T=.2
      EM=1.4
      KZ=10
      DO3 M=1,19
      EM=EM-T
      IF(EM.LT..2) KZ=2
      MA=0
      NNN=2
      NN=-1
      DO4 I=10,1800,KZ
      AL=I
      AR=I*PI/1800.
      FK=SQRT(AR*AR/EM)
      IF(FK.GT.1.57079) GO TO 14
   16 IF(MA.NE.0) GO TO 4
      PRINT15,FK,EM,AL
      MA=1
   15 FORMAT (* NO SOLUTION FOR THIS ALPHA, GO TO NEXT ALPHA *3E12.5)
      GO TO 4
   14 ADJ=.1
      FK=.0999999
      JJ=1
   13 CALL CELLI(A,EK,F1,EE)
      JJ=JJ+1
      IF(ABS(FK-F1).LT..0001 ) GO TO 5
      IF(F1.LT.FK) GO TO 9
      EK=EK-ADJ
      ADJ=ADJ/10.
    9 EK=EK+ADJ
      GO TO 13
    5 FK=F1
      AP=SQRT(EM*FK*FK)
      A=(1.-(EM-1.)/EK/EK)/3.
      IF(A.LT.0..OR.A.GT.1.) GO TO 16
      S=EK*EK
      EPS=4./SQRT(EM)*(FK*(S-EM-2.)+3.*EE)
      A=ASIN(SQRT(A))
      CALL ELLI(A,EK,F,E)
      A1=SQRT(EM)*F
      A2=SQRT(EM)*2.*FK-A1
      A1A=A1*180./PI
      A2A=A2*180./PI
      APA=AP*180./PI
      PRINT6,A1A,A2A,APA,EPS,EM,EK,JJ
      IF(EPS.LT.-8.5) GO TO 21
      IF(EPS.GT.8.5) GO TO 4
      Y=EPS/.5
      X=APA/3.
```

```
      CALL PLOT(Y,X,NNN)
      NNN=NNN+NN
      NN=NN*(-1)
    4 CONTINUE
   21 IF(ABS(EM-T).GT..00001) GO TO 3
      EM=EM-T/2.
      T=T/10.
    3 CONTINUE
      END
```

START OF CALCULATIONS FOR NON-SYMMETRICAL FLOW

AL1	AL2	AL TOTAL/2	EPS TOTAL	M	K		
89.3195	89.3195	89.3195	9.5324-002	9.8000-001	0.1000	90	91
0.0000	45.3381	90.6755	-9.4134-002	1.0150+000	0.1000	90	91
83.4312	83.4312	83.4312	9.4762-001	8.2000-001	0.3000	84	97
65.8411	104.1589	85.0000	7.1312-001	8.5113-001	0.3000	84	97
44.4935	135.5065	90.0000	-5.2394-003	9.5420-001	0.3000	84	97
18.9742	171.0258	95.0000	-6.9472-001	1.0632+000	0.3000	84	97
0.0000	45.0956	96.1911	-8.4153-001	1.0900+000	0.3000	84	97
68.2969	68.2969	68.2969	3.4475+000	5.0000-001	0.5000	69	108
61.1690	78.8310	70.0000	3.1283+000	5.2525-001	0.5000	69	108
55.6489	94.3511	75.0000	2.2453+000	6.0296-001	0.5000	69	108
51.8251	108.1749	80.0000	1.4287+000	6.8604-001	0.5000	69	108
47.9447	122.0553	85.0000	6.6701-001	7.7447-001	0.5000	69	108
43.4568	136.5432	90.0000	-4.8755-002	8.6827-001	0.5000	69	108
37.8938	152.1062	95.0000	-7.2595-001	9.6742-001	0.5000	69	108
30.5092	169.4908	100.0000	-1.3703+000	1.0719+000	0.5000	69	108
19.1525	190.8475	105.0000	-1.9866+000	1.1818+000	0.5000	69	108
0.0000	55.9734	107.9869	-2.3429+000	1.2500+000	0.5000	69	108
14.9554	14.9554	14.9554	3.5159+001	2.0000-002	0.7000	15	130
18.4284	21.5716	20.0000	2.5676+001	3.5766-002	0.7000	15	130
22.0358	27.9642	25.0000	1.9912+001	5.5838-002	0.7000	15	130
25.3815	34.6185	30.0000	1.5953+001	8.0478-002	0.7000	15	130
28.4416	41.5584	35.0000	1.3026+001	1.0954-001	0.7000	15	130
31.2284	48.7716	40.0000	1.0743+001	1.4307-001	0.7000	15	130
33.6775	56.3225	45.0000	8.8903+000	1.8198-001	0.7000	15	130
35.8451	64.1549	50.0000	7.3381+000	2.2355-001	0.7000	15	130
37.7068	72.2932	55.0000	6.0046+000	2.7050-001	0.7000	15	130
39.2569	80.7431	60.0000	4.8351+000	3.2191-001	0.7000	15	130
40.4884	89.5116	65.0000	3.7919+000	3.7780-001	0.7000	15	130
41.3919	98.6081	70.0000	2.8479+000	4.3816-001	0.7000	15	130
41.9549	108.0451	75.0000	1.9832+000	5.0299-001	0.7000	15	130
42.1612	117.8388	80.0000	1.1827+000	5.7229-001	0.7000	15	130
41.9890	128.0110	85.0000	4.3572-001	6.4606-001	0.7000	15	130
41.4694	138.5306	90.0000	-2.6723-001	7.2430-001	0.7000	15	130
40.3824	149.6176	95.0000	-9.3292-001	8.0702-001	0.7000	15	130
38.8519	161.1481	100.0000	-1.5670+000	8.9420-001	0.7000	15	130
36.7354	173.2646	105.0000	-2.1738+000	9.8586-001	0.7000	15	130
33.9048	186.0952	110.0000	-2.7573+000	1.0820+000	0.7000	15	130
30.1407	199.8593	115.0000	-3.3204+000	1.1826+000	0.7000	15	130
25.0052	214.9948	120.0000	-3.8656+000	1.2876+000	0.7000	15	130
17.2924	232.7076	125.0000	-4.3952+000	1.3972+000	0.7000	15	130
0.0000	54.5425	129.0849	-4.8174+000	1.4900+000	0.7000	15	130
0.5367	1.4533	1.0000	4.1871+002	5.8570-005	0.9000	1	176
2.6812	7.3188	5.0000	8.3406+001	1.4643-003	0.9000	1	176
5.3480	14.6520	10.0000	4.1180+001	5.8570-003	0.9000	1	176
7.9863	22.0137	15.0000	2.6871+001	1.3178-002	0.9000	1	176
10.5822	29.4178	20.0000	1.9543+001	2.3428-002	0.9000	1	176
13.1224	36.8776	25.0000	1.5005+001	3.6606-002	0.9000	1	176
15.5944	44.4056	30.0000	1.1865+001	5.2713-002	0.9000	1	176
17.9864	52.0136	35.0000	9.5216+000	7.1748-002	0.9000	1	176
20.2875	59.7125	40.0000	7.6769+000	9.3712-002	0.9000	1	176
22.4875	67.5125	45.0000	6.1646+000	1.1860-001	0.9000	1	176
24.5773	75.4227	50.0000	4.8849+000	1.4643-001	0.9000	1	176
26.5483	83.4517	55.0000	3.7744+000	1.7717-001	0.9000	1	176
28.3931	91.6069	60.0000	2.7908+000	2.1085-001	0.9000	1	176
30.1045	99.8955	65.0000	1.9049+000	2.4746-001	0.9000	1	176
31.6760	108.3240	70.0000	1.0956+000	2.8699-001	0.9000	1	176
33.1017	116.8983	75.0000	3.4771-001	3.2946-001	0.9000	1	176
34.3756	125.6244	80.0000	-3.5033-001	3.7485-001	0.9000	1	176

START OF CONSTANT M CALCULATIONS

AL1	AL2	AL TOTAL/2	EPS TOTAL	M	K	
NO SOLUTION FOR THIS ALPHA. GO TO NEXT ALPHA				1.59326-002	1.20000+000	1.00000+001
12.3287	197.6394	104.9941	-1.9771+000	1.2000+000	0.4751	21
17.1572	194.8306	105.9939	-2.1082+000	1.2000+000	0.5055	19
20.1506	193.8382	106.9944	-2.2398+000	1.2000+000	0.5330	14
22.2763	193.7163	107.9963	-2.3718+000	1.2000+000	0.5581	23
23.8661	194.1082	108.9971	-2.5039+000	1.2000+000	0.5811	19
25.1587	194.8350	109.9973	-2.6367+000	1.2000+000	0.6023	15
26.1989	195.8026	111.0018	-2.7692+000	1.2000+000	0.6220	13
27.0605	196.9352	111.9978	-2.9015+000	1.2000+000	0.6402	16
27.7927	198.1957	112.9962	-3.0342+000	1.2000+000	0.6572	24
28.4230	199.5660	113.9945	-3.1671+000	1.2000+000	0.6731	21
28.9755	201.0229	114.9992	-3.3009+000	1.2000+000	0.6881	27
29.4584	202.5366	115.9975	-3.4341+000	1.2000+000	0.7021	14
29.8878	204.1073	116.9976	-3.5676+000	1.2000+000	0.7153	20
30.2704	205.7175	117.9940	-3.7007+000	1.2000+000	0.7277	27
30.6175	207.3790	118.9982	-3.8349+000	1.2000+000	0.7395	28
30.9300	209.0650	119.9975	-3.9686+000	1.2000+000	0.7506	22
31.2142	210.7780	120.9961	-4.1023+000	1.2000+000	0.7611	19
31.4751	212.5240	121.9996	-4.2366+000	1.2000+000	0.7711	20
31.7124	214.2753	122.9976	-4.3698+000	1.2000+000	0.7805	24
31.9327	215.0597	123.9962	-4.5041+000	1.2000+000	0.7895	33
32.1373	217.8710	125.0042	-4.6392+000	1.2000+000	0.7961	29
32.3251	219.6797	126.0024	-4.7730+000	1.2000+000	0.8062	20
32.4994	221.4987	126.9991	-4.9066+000	1.2000+000	0.8139	25
32.6632	223.3459	128.0045	-5.0414+000	1.2000+000	0.8213	18
32.8149	225.1897	129.0023	-5.1751+000	1.2000+000	0.8283	25
32.9573	227.0495	130.0034	-5.3093+000	1.2000+000	0.8350	19
33.0910	228.9197	131.0054	-5.4436+000	1.2000+000	0.8414	21
33.2163	230.7945	132.0054	-5.5776+000	1.2000+000	0.8475	28
33.3336	232.6572	133.0004	-5.7110+000	1.2000+000	0.8533	23
33.4451	234.5653	134.0052	-5.8456+000	1.2000+000	0.8589	34
33.5493	236.4497	134.9995	-5.9788+000	1.2000+000	0.8642	24
33.6483	238.3503	135.9993	-6.1126+000	1.2000+000	0.8693	30
33.7422	240.2533	137.0028	-6.2470+000	1.2000+000	0.8742	25
33.8302	242.1595	137.9949	-6.3797+000	1.2000+000	0.8788	40
33.9143	244.0748	138.9945	-6.5135+000	1.2000+000	0.8833	29
33.9941	245.9972	139.9957	-6.6474+000	1.2000+000	0.8876	36
34.0699	247.9224	140.9962	-6.7811+000	1.2000+000	0.8917	34
34.1422	249.8358	141.9950	-6.9151+000	1.2000+000	0.8957	33
34.2109	251.7933	143.0021	-7.0491+000	1.2000+000	0.8995	35
34.2757	253.7144	143.9950	-7.1817+000	1.2000+000	0.9031	19
34.3378	255.6519	144.9948	-7.3151+000	1.2000+000	0.9066	25
34.3971	257.5915	145.9943	-7.4484+000	1.2000+000	0.9099	37
34.4542	259.5537	147.0039	-7.5831+000	1.2000+000	0.9132	19
34.5078	261.4556	147.9967	-7.7154+000	1.2000+000	0.9163	30
34.5593	263.4326	148.9959	-7.8485+000	1.2000+000	0.9192	30
34.6087	265.3854	149.9970	-7.9818+000	1.2000+000	0.9221	18
34.6557	267.3334	150.9945	-8.1145+000	1.2000+000	0.9248	32
34.7011	269.3021	152.0016	-8.2484+000	1.2000+000	0.9275	27
34.7441	271.2309	152.9975	-8.3808+000	1.2000+000	0.9300	20
34.7854	273.2161	153.9958	-8.5135+000	1.2000+000	0.9325	30
NO SOLUTION FOR THIS ALPHA. GO TO NEXT ALPHA				1.74533-002	1.00000+000	1.00000+001
35.2646	144.7399	90.0023	-3.1417-004	1.0000+000	0.0100	3
35.3543	146.6415	90.9979	-1.3979-001	1.0000+000	0.2080	13
35.4415	148.5621	92.0018	-2.8097-001	1.0000+000	0.2910	15
35.5251	150.4762	93.0006	-4.2218-001	1.0000+000	0.3520	13
35.6154	152.3802	93.9958	-5.6347-001	1.0000+000	0.4014	13
35.6873	154.3106	94.9906	-7.1611-001	1.0000+000	0.4436	21

SUBROUTINE NONSYM3

for N3 flow parameter graph

```
      SUBROUTINE NCNSYM3
      PI=3.1415926536
      PRINT7
    7 FORMAT(*1START OF CALCULATIONS FOR NON-SYMMETRICAL FLOW*)
      PRINT8
    8 FORMAT(*0*                          3X*AL1*8X*AL2*8X*AL TOTAL/2 *
     1          *EPS TOTAL   M *        9X*K*)
      MM=200
      DO1 K=100,984,MM
      IF(K.EQ.900) MM=20
      IF(K.EQ.960) MM=5
      IF(K.EQ.975) MM=1
      FK=K/1000.
      S=EK*EK
      L=2
      CALL CELLI(A,EK,FK,EE)
      A=1.-2.*S
      IF(A.GT.0.) GO TO 10
      JL=1
      GO TO 11
   10 JL=2.*SQRT(FK*FK*A)*180./PI+1.
   11 JT=2.*SQRT(FK*FK*(S+1.))*180./PI+1
      IF(JT.GT.180) JT=180
      DO2 J=JL,JT,1
C FOR UPPER LIMITS
      IF(J.NE.JL.OR.JL.EQ.1) GO TO 19
      EPS=24./SQRT(A)*(FK*(S-1.)+EE)
      EM=A
         A1=A2=FK*SQRT(EM)
      A3=2.*A1+A2
      AP=A1+A2
      APA=AP*180./PI
      GO TO 18
C FOR LOWER LIMITS
   19 IF(J.NE.JT.OR.JT.EQ.180) GO TO 17
      AP=SQRT(FK*FK*(S+1))*2.
      EPS=24.*(EE-FK)/SQRT(S+1.)
      EM=S+1.
      A1=0.
      A2=AP
      APA=AP*180./PI
      A3=A2
      GO TO 18
   17 AP=J*PI/180.
      EM=(AP/FK/2.)**2
      EPS=8./SQRT(EM)*((EK*EK-EM-2.)*FK+3.*EE)
      A=ASIN(SQRT((1.-(EM-1.)/S)/3.))
      CALL ELLI(A,EK,F,E)
      A1=SQRT(EM)*F
      A2=SQRT(EM)*2.*FK-A1
      A3=2.*A1+A2
      APA=J
   18 A1A=A1*180./PI
      A2A=A2*180./PI
      A3A=A3*180./PI
      IF(EPS.GT.8.5.OR.EPS.LT.-8.5) GO TO 12
```

```
      Y=EPS/.5
      X=APA/5.
      CALL PLOT(Y,X,L)
      L=1
   12 IF(J/5*5.EQ.J.OR.J.EQ.JT.OR.J.EQ.JL) PRINT6,A1A,A2A,APA,EPS,EM,EK,
     1JL,JT,A3A
    6 FORMAT(* *3F11.4,2E11.4,F11.4,2I5,F11.4)
    2 CONTINUE
    1 CONTINUE
C PROGRAM SEGMENT FOR M=CONSTANT CURVES
      PRINT20
   20 FORMAT(*1START OF CONSTANT M CALCULATIONS*)
      PRINT8
      EM=.9
      T=.1
      DO3 M=1,19
      EM=EM-T
      MA=0
      NNN=2
      NN=1
      KZ=0
      DO4 I=1,1800,2
      AL=I
      AR=I*PI/1800.
      FK=AR/2./SQRT(EM)
      FK=SQRT(AR*AR/EM)
      IF(FK.GT.1.57079) GO TO 14
   16 IF(MA.NE.0) GO TO 4
      PRINT15,FK,EM,AL
      MA=1
   15 FORMAT (* NO SOLUTION FOR THIS ALPHA. GO TO NEXT ALPHA *3E12.5)
      GO TO 4
   14 ADJ=.1
      EK=.0999999
      JJ=1
   13 CALL CELLI(A,EK,F1,EE)
      JJ=JJ+1
      IF(ABS(FK-F1).LT..0001 ) GO TO 5
      IF(F1.LT.FK) GO TO 9
      EK=EK-ADJ
      ADJ=ADJ/10.
    9 EK=EK+ADJ
      GO TO 13
    5 FK=F1
      AP=SQRT(EM)*FK*2.
      A=(1.-(EM-1.)/EK/EK)/3.
      IF(A.LT.0..OR.A.GT.1.) GO TO 16
      S=EK*EK
      EPS=P./SQRT(EM)*(FK*(S-EM-2.)+3.*EE)
      A=ASIN(SQRT(A))
      CALL ELLI(A,EK,F,E)
      A1=SQRT(EM)*F
      A2=SQRT(EM)*2.*FK-A1
      A3=2.*A1+A2
      A1A=A1*180./PI
      A2A=A2*180./PI
```

```
      A3A=A3*180./PI
      APA=AP*180./PI
      KZ=KZ+1
      IF(KZ.LT.10) GO TO 22
      PRINT6,A1A,A2A,APA,EPS,EM,EK,J,JJ,A3A
      KZ=0
   22 IF(EPS.LT.-12.) GO TO 21
      IF(EPS.GT.8.5) GO TO 4
      Y=EPS/.5
      X=APA/3.
      IF(EPS.GT.8.5.OR.EPS.LT.-8.5.OR.APA.GT.180.) GO TO 23
      CALL PLOT(Y,X,NNN)
   23 CONTINUE
      NNN=NNN+NN
      NN=NN*(-1)
    4 CONTINUE
   21 IF(ABS(EM-T).GT..00001) GO TO 3
      EM=EM-T/2.
      T=T/10.
    3 CONTINUE
      END
```

START OF CALCULATIONS FOR NON-SYMMETRICAL FLOW

AL1	AL2	AL TOTAL/2	EPS TOTAL	M	K			
89.3195	89.3195	178.6389	1.9065-001	9.8000-001	0.1000	179	180	267.9584
44.9460	135.0540	180.0000	-1.1900-004	9.9499-001	0.1000	179	180	224.9460
83.4312	83.4312	166.8623	1.8952+000	8.2000-001	0.3000	167	180	250.2937
65.8411	104.1589	170.0000	1.4262+000	8.5113-001	0.3000	167	180	235.8411
54.3949	120.6051	175.0000	6.9733-001	9.0193-001	0.3000	167	180	229.3949
44.4935	135.5065	180.0000	-1.0479-002	9.5420-001	0.3000	167	180	224.4935
68.2969	68.2969	136.5938	6.8951+000	5.0000-001	0.5000	137	180	204.8907
61.1690	78.8310	140.0000	6.2575+000	5.2525-001	0.5000	137	180	201.1690
57.9143	87.0857	145.0000	5.3557+000	5.6344-001	0.5000	137	180	202.9143
55.6489	94.3511	150.0000	4.4907+000	6.0296-001	0.5000	137	180	205.6489
53.6933	101.3067	155.0000	3.6589+000	6.4383-001	0.5000	137	180	208.6933
51.8251	108.1749	160.0000	2.8574+000	6.8604-001	0.5000	137	180	211.8251
49.9328	115.0672	165.0000	2.0832+000	7.2959-001	0.5000	137	180	214.9328
47.9447	122.0553	170.0000	1.3341+000	7.7447-001	0.5000	137	180	217.9447
45.8039	129.1961	175.0000	6.0781-001	8.2070-001	0.5000	137	180	220.8039
43.4568	136.5432	180.0000	-9.7511-002	8.6827-001	0.5000	137	180	223.4568
14.9554	14.9554	29.9108	7.0319+001	2.0000-002	0.7000	30	180	44.8661
16.5589	18.4411	35.0000	5.9435+001	2.7385-002	0.7000	30	180	51.5589
18.4284	21.5716	40.0000	5.1351+001	3.5768-002	0.7000	30	180	58.4284
20.2616	24.7384	45.0000	4.4986+001	4.5269-002	0.7000	30	180	65.2616
22.0358	27.9642	50.0000	3.9824+001	5.5888-002	0.7000	30	180	72.0358
23.7435	31.2565	55.0000	3.5537+001	6.7624-002	0.7000	30	180	78.7435
25.3815	34.6185	60.0000	3.1907+001	8.0478-002	0.7000	30	180	85.3815
26.9478	38.0522	65.0000	2.8781+001	9.4450-002	0.7000	30	180	91.9478
28.4416	41.5584	70.0000	2.6052+001	1.0954-001	0.7000	30	180	98.4416
29.8619	45.1381	75.0000	2.3641+001	1.2575-001	0.7000	30	180	104.8619
31.2084	48.7916	80.0000	2.1487+001	1.4307-001	0.7000	30	180	111.2084
32.4804	52.5196	85.0000	1.9545+001	1.6152-001	0.7000	30	180	117.4804
33.6775	56.3225	90.0000	1.7781+001	1.8108-001	0.7000	30	180	123.6775
34.7993	60.2007	95.0000	1.6165+001	2.0175-001	0.7000	30	180	129.7993
35.8451	64.1549	100.0000	1.4676+001	2.2355-001	0.7000	30	180	135.8451
36.8145	68.1855	105.0000	1.3296+001	2.4640-001	0.7000	30	180	141.8145
37.7068	72.2932	110.0000	1.2009+001	2.7050-001	0.7000	30	180	147.7068
38.5212	76.4788	115.0000	1.0804+001	2.9565-001	0.7000	30	180	153.5212
39.2569	80.7431	120.0000	9.6703+000	3.2191-001	0.7000	30	180	159.2569
39.9130	85.0870	125.0000	8.5993+000	3.4930-001	0.7000	30	180	164.9130
40.4884	89.5116	130.0000	7.5839+000	3.7780-001	0.7000	30	180	170.4884
40.9818	94.0182	135.0000	6.6178+000	4.0742-001	0.7000	30	180	175.9818
41.3919	98.6081	140.0000	5.6958+000	4.3816-001	0.7000	30	180	181.3919
41.7168	103.2832	145.0000	4.8133+000	4.7001-001	0.7000	30	180	186.7168
41.9549	108.0451	150.0000	3.9663+000	5.0299-001	0.7000	30	180	191.9549
42.1038	112.8962	155.0000	3.1515+000	5.3708-001	0.7000	30	180	197.1038
42.1612	117.8388	160.0000	2.3658+000	5.7229-001	0.7000	30	180	202.1612
42.1240	122.8760	165.0000	1.6066+000	6.0862-001	0.7000	30	180	207.1240
41.9890	128.0110	170.0000	8.7144-001	6.4606-001	0.7000	30	180	211.9890
41.7524	133.2476	175.0000	1.5838-001	6.8462-001	0.7000	30	180	216.7524
41.4094	138.5906	180.0000	-5.3445-001	7.2430-001	0.7000	30	180	221.4094
0.2684	0.7316	1.0000	1.6750+003	1.4643-005	0.9000	1	180	1.2684
1.3415	3.6585	5.0000	3.3467+002	3.6606-004	0.9000	1	180	6.3415
2.6812	7.3188	10.0000	1.6681+002	1.4643-003	0.9000	1	180	12.6812
4.0173	10.9827	15.0000	1.1063+002	3.2946-003	0.9000	1	180	19.0173
5.3480	14.6520	20.0000	8.2359+001	5.8570-003	0.9000	1	180	25.3480
6.6716	18.3284	25.0000	6.5259+001	9.1516-003	0.9000	1	180	31.6716
7.9863	22.0137	30.0000	5.3743+001	1.3178-002	0.9000	1	180	37.9863
9.2904	25.7096	35.0000	4.5417+001	1.7937-002	0.9000	1	180	44.2904
10.5822	29.4178	40.0000	3.9085+001	2.3428-002	0.9000	1	180	50.5822
11.8600	33.1400	45.0000	3.4083+001	2.9651-002	0.9000	1	180	56.8600
13.1224	36.8776	50.0000	3.0012+001	3.6606-002	0.9000	1	180	63.1224

```
START OF CONSTANT M CALCULATIONS

   AL1        AL2        AL TOTAL/2 EPS TOTAL  M              K
NO SOLUTION FOR THIS ALPHA, GO TO NEXT ALPHA  1.95134-003 8.00000-001 1.00000+000
   52.5400   116.4498   168.9898 1.5391+000 8.0000-001    0.4221  181   13   221.5299
   46.5616   126.4410   173.0026 8.9150-001 8.0000-001    0.5040  181   12   219.5643
   43.8006   133.1942   176.9947 2.4245-001 8.0000-001    0.5673  181   25   220.7953
   42.2106   139.7953   181.0060-4.1363-001 8.0000-001    0.6190  181   19   223.2166
   41.2000   143.8529   185.0029-1.0705+000 8.0000-001    0.6620  181   17   226.2029
   40.5111   148.4932   188.9943-1.7208+000 8.0000-001    0.6985  181   32   229.5054
   40.0179   152.9779   192.9957-2.3906+000 8.0000-001    0.7300  181   12   233.0136
   39.6536   157.3474   197.0010-3.0544+000 8.0000-001    0.7574  181   27   236.6546
   39.3790   161.6111   200.9902-3.7164+000 8.0000-001    0.7813  181   23   240.3692
   39.1657   165.8408   205.0064-4.3836+000 8.0000-001    0.8025  181   19   244.1721
   38.9990   170.0188   209.0078-5.0485+000 8.0000-001    0.8212  181   17   248.0068
   38.8668   174.1344   213.0012-5.7120+000 8.0000-001    0.8378  181   30   251.8680
   38.7606   178.2309   216.9915-6.3749+000 8.0000-001    0.8526  181   27   255.7521
   38.6741   182.3187   220.9928-7.0392+000 8.0000-001    0.8659  181   34   259.6669
   38.6033   186.3898   224.9932-7.7028+000 8.0000-001    0.8778  181   40   263.5965
   38.5450   190.4494   228.9944-8.3659+000 8.0000-001    0.8886  181   34   267.5394
   38.4966   194.5002   232.9968-9.0285+000 8.0000-001    0.8983  181   32   271.4933
   38.4562   198.5348   236.9910-9.6889+000 8.0000-001    0.9070  181   26   275.4472
   38.4222   202.5770   240.9991-1.0351+001 8.0000-001    0.9150  181   18   279.4213
   38.3935   206.6189   245.0024-1.1011+001 8.0000-001    0.9222  181   19   283.3960
   38.3693   210.6247   248.9940-1.1668+001 8.0000-001    0.9287  181   33   287.3633
O SOLUTION FOR THIS ALPHA. GO TO NEXT ALPHA  2.08607-003 7.00000-001 1.00000+000
   53.1782   107.4141   160.5923 2.7834+000 7.0000-001    0.4795  181   29   213.7704
   47.4548   117.1402   164.5950 2.0811+000 7.0000-001    0.5522  181   18   212.0497
   44.5754   124.0338   168.6092 1.3717+000 7.0000-001    0.6100  181    9   213.1846
   42.8561   129.7391   172.5952 6.6336-001 7.0000-001    0.6571  181   23   215.4514
   41.7133   134.8875   176.6008-5.1519-002 7.0000-001    0.6968  181   33   218.3141
   40.9136   139.6813   180.5949-7.6663-001 7.0000-001    0.7305  181   19   221.5085
   40.3266   144.2749   184.6015-1.4856+000 7.0000-001    0.7596  181   31   224.9281
   39.8848   148.7131   188.5979-2.2038+000 7.0000-001    0.7848  181   31   228.4827
   39.5422   153.0550   192.6072-2.9250+000 7.0000-001    0.8069  181   27   232.1494
   39.2739   157.3177   196.5916-3.6420+000 7.0000-001    0.8262  181   24   235.8655
   39.0575   161.5497   200.6072-4.3645+000 7.0000-001    0.8434  181   23   239.6647
   38.8833   165.7595   204.5928-5.0812+000 7.0000-001    0.8585  181   34   243.4761
   38.7398   169.8514   208.5912-5.7998+000 7.0000-001    0.8720  181   26   247.3311
   38.6209   173.9729   212.5938-6.5183+000 7.0000-001    0.8841  181   27   251.2147
   38.5216   178.0708   216.5924-7.2353+000 7.0000-001    0.8949  181   34   255.1140
   38.4378   182.1527   220.6005-7.9529+000 7.0000-001    0.9046  181   23   259.0383
   38.3669   186.2337   224.6007-8.6681+000 7.0000-001    0.9133  181   20   262.9676
   38.3066   190.2869   228.5934-9.3808+000 7.0000-001    0.9211  181   20   266.9000
   38.2546   194.3468   232.6014-1.0095+001 7.0000-001    0.9282  181   25   270.8559
   38.2099   198.3834   236.5934-1.0805+001 7.0000-001    0.9346  181   33   274.8033
   38.1711   202.4266   240.5977-1.1516+001 7.0000-001    0.9403  181   26   278.7688
O SOLUTION FOR THIS ALPHA. GO TO NEXT ALPHA  2.25321-003 6.00000-001 1.00000+000
   52.4488    98.5436   150.9925 4.2906+000 6.0000-001    0.5270  181   17   203.4413
   46.9993   107.9933   154.9927 3.5142+000 6.0000-001    0.5940  181   21   201.9920
   44.1679   114.8237   158.9916 2.7327+000 6.0000-001    0.6476  181   27   203.1596
   42.4214   120.5768   162.9982 1.9457+000 6.0000-001    0.6918  181   28   205.4196
   41.2485   125.7492   166.9977 1.1570+000 6.0000-001    0.7288  181   29   208.2462
   40.4122   130.5917   171.0039 3.6493-001 6.0000-001    0.7603  181   20   211.4162
   39.7941   135.2197   175.0038-4.2726-001 6.0000-001    0.7873  181   29   214.7978
   39.3238   139.6575   178.9913-1.2177+000 6.0000-001    0.8106  181   19   218.3151
   38.9549   144.0445   182.9994-2.0125+000 6.0000-001    0.8310  181   15   221.9543
   38.6627   148.3311   186.9937-2.8044+000 6.0000-001    0.8488  181   32   225.6564
   38.4261   152.5735   190.9996-3.5981+000 6.0000-001    0.8645  181   27   229.4258
   38.2333   156.7506   194.9940-4.3886+000 6.0000-001    0.8783  181   33   233.2273
   38.0735   160.9278   199.0012-5.1807+000 6.0000-001    0.8906  181   27   237.0747
```

SUBROUTINES GRAPH, ELLI, & SN

GRAPH - grid coordinates for flow parameter graph

ELLI - for calculating complete and incomplete elliptic integrals of the first and second kind. [6]

SN - for calculating sn, cn or dn elliptic functions

```
      SUBROUTINE GRAPH
      WD=4.73  $HT=8.5       $LGD=5HSHORT
      MAX=HT+4.
      CALL PLOT(MAX,0.,3)
      CALL PLOT (80 ,0.,3)
      CALL PLOT (0.,0.,0,200.,200.)
      CALL PLOT (0.,-40.,2)
      CALL PLOT (0.,0.,0)
      CALL PLOT(HT/2.+2.,3.,2)
      CALL PLOT (0.,0.,0)
      YT=HT/2.
      L=0
      DO1 K=L,18,2
      X=WD/19.*K
      CALL PLOT (YT,X,2)
      CALL PLOT (-YT,X,1)
      IF(LGD.EQ.5HSHORT) YT=.25
      CALL PLOT (-YT,X+WD/19.,2)
    1 CALL PLOT (YT,X+WD/19.,1)
      YT=HT/2.
      XT=.25
      IF(LGD.EQ.4HFULL)XT=WD
      Y=-YT
      DO2 K=1,17
      CALL PLOT (Y,0.,2)
      CALL PLOT (Y,XT,1)
      IF(K.EQ.9.AND.LGD.EQ.5HSHORT)XT=WD
      Y=Y+YT/17.
      CALL PLOT(Y,XT,2)
      CALL PLOT (Y,0.,1)
      IF(K.EQ.9.AND.LGD.EQ.5HSHORT)XT=.25
    2 Y=Y+YT/17.
      CALL PLOT (Y,0.,2)
      CALL PLOT (Y,XT,1)
      DO 3 K=L,190,10
      X=WD/19.*K/10.-.12
      ENCODE(8,4,KK)K
    4 FORMAT(I3,5X)
    3 CALL CHAR(-.15,X,KK,3,0.,.18,.12)
      CALL CHAR(-.45,WD/2.-.36,5HALPHA,5 ,0.,.24,.16)
      N=-80
      DO5 K=N,80,10
      A=K/10.
      Y=A/2.-.19
      ENCODE(8,6,KK)A
    6 FORMAT(F4.1,4X)
    5 CALL CHAR(Y,-.06,KK,4,90.,.18,.12)
      CALL CHAR(YT/2.-.51,-.25,7HEPSILON,7,90.,.24,.16)
      CALL PLOT(0.,0.,2)
      SWD=200./38.*WD
      SHT=200./34.*HT
      CALL PLOT(0.,0.,2,SHT,SWD)
      END
```

```
      SUBROUTINE ELLI(CHI,CAY,F,E)
      DATA(EPS=1.E-10),(EPS2=2.E+10)
      DATA(PI2=1.5707963267 9)
      DATA(TOP=.636619772368)
      DATA(PI=3.14159265358979)
      PHI = CHI
      IND = 1
    5 F = PHI
      E=F
      PSI = ABS(PHI)
       IF ((PSI.LT.EPS) .OR. (CAY.EQ.0.))  RETURN
      S1 = CAY * CAY
      RAD = 1. - S1
      IF(RAD)35,30,10
   10 ALPHA=1.
      BETA=SQRT(RAD)
      S2=0.
      PR2=1.
      PWR2=1.
      FINT = PSI * TOP
      NQUAD = INT(FINT)
      FINT = FINT - FLOAT(NQUAD)
      IF(AMIN1(FINT,1.-FINT) .LT. .5E-10) GO TO 20
      TANPSI=TANF(PSI)
      NQUAD=NQUAD+1
   12 IF(ABS(ALPHA-BETA).LE. EPS  ) GO TO 23
      PWR2 = 2. * PWR2
      PR2=PWR2
      DENOM=ALPHA-BETA*TANPSI**2
      TOP=(ALPHA+BETA)*TANPSI
      CN=.5*(ALPHA-BETA)
      S1=S1+PWR2*CN*CN
      TEMP=SQRT(ALPHA*BETA)
      ALPHA=(ALPHA+BETA)*.5
      BETA=TEMP
      IF(DENOM.EQ.0.)GO TO 15
      TANPSI=TOP/DENOM
      IF(ABS(TANPSI).GE.  EPS2 ) GO TO 15
      NQUAD=2*NQUAD
      IF(TANPSI.GT.0.) NQUAD=NQUAD-1
      IF(ABS(TANPSI).LE. EPS  ) GO TO 18
      MQUAD=MOD(NQUAD,4)
      SINP=TANPSI/SQRT(1.+TANPSI*TANPSI)
      IF((MQUAD.EQ.3).OR.(MQUAD.EQ.2))SINP=-SINP
      S2=S2+CN*SINP
      GO TO 12
   15 FINT = 2 * NQUAD - 1
      PSI=FINT*PI2
      SINP=1.
      IF(AMOD( FINT,4.).GT.2.)SINP=-1.
      S2=S2+CN*SINP
      GO TO 20
   18 FINT=NQUAD/2
      PSI=FINT*PI
   20 IF(ABS(ALPHA-BETA).LT. EPS  ) GO TO 25
      PR2=2.*PR2
```

```
      CN=.5*(ALPHA-BETA)
      S1=S1+PR2*CN*CN
      TEMP=SQRT(ALPHA*BETA)
      ALPHA=(ALPHA+BETA)*.5
      BETA=TEMP
      GO TO 20
   23 FINT=NQUAD/2
      PSI=ATAN(TANPSI)+FINT*PI
   25 F=PSI/(ALPHA*PWR2)
      S1=1.-.5*S1
      E=S2+S1*F
      F = SIGN(F,PHI)
      E = SIGN(E,PHI)
      RETURN
   30 FINT =AINT(TOP*PSI)
      E = FINT + ABS(SIN(PSI)-SIN(FINT*P12))
      E = SIGN(E,PHI)
      IF(PSI.GE.P12) GO TO 40
      DENOM=1.-SIN(PHI)
      IF((DENOM.EQ. 0.) .OR. (DENOM.EQ. 2.))  GO TO 40
      F=.5*ALOG((2.-DENOM)/DENOM)
      RETURN
      ENTRY CELLI
      PHI = P12
      IND = 5
      GO TO 5
   35 CONTINUE
   40 PRINT50
   50 FORMAT(* TROUBLE*)
      END
```

```
      FUNCTION SN(U,CAY)
      L=0
    4 IF(CAY.NE.1.) GO TO14
      IF(L.EQ.0) SN=TANH(U)
      IF(L.EQ.1.OR.L.EQ.2) SN=SQRT(1.-TANH(U)*TANH(U))
      RETURN
   14 CALL CELLI(PHI,CAY,CK,E)
      SN=2.*3.1415926536/CK/CAY
      CAYP=0.
      IF(1.-CAY*CAY.LT..0000001    ) GO TO 6
      CAYP=SQRT(1.-CAY*CAY)
    6 CALL CELLI(PHI,CAYP,CKP,E)
      Q=EXP(-3.1415926536*CKP/CK)
      TT=S=T2=0.
    3 T1=Q**(S+.5)/(1.-Q**(2.*S+1.))*SIN((2.*S+1.)*3.1415926536*U/2./CK)
      IF(ABS(T1-T2).LT..000001    ) GO TO 2
      T2=T1
      TT=TT+T2
      S=S+1
      GO TO 3
    2 SN=SN*TT
      IF(L.NE.1) GO TO 9
      IF(SN.GT.1..AND.SN.LT.1.001) SN=.999999999999
      SN=SQRT(1.-SN*SN)
      R=ABS(U)
      DO 7 J=1,200
      N=J/2
      R=R-CK
      IF(R.LT.0.) GO TO 8
    7 CONTINUE
    8 IF (N/2*2.EQ.N) GO TO 9
      SN=-SN
    9 IF(L.EQ.2)SN=SQRT(1.-CAY*CAY*SN*SN)
      RETURN
      ENTRY CN
      L=1
      GO TO 4
      ENTRY DN
      L=2
      GO TO 4
      END
RUN,15.0,5000
```

APPENDIX B

```
      SUBROUTINE PRO SD    (AL,EPS,EK,EM)
      DIMENSION XB(400),Y3(400)
      C1(A)=A*3.1415926536/180.
      Z=1.59E-4
   13 ALR=C1(AL)
      R=1.
      AR=0.
      F=-5.*EK*EK/EM*((SN(AR/SQRT(EM),EK))**2+((EM-1.)/EK/EK-1.)/3.)
      UM=Z/R*F*12.
      CALL CHAR(3.4,-2.1,13HSCALE 1 INCH=,13,0.,.24,.16)
      UMM=UM*2.
      ENCODE(3,11,JJ)UMM
   11 FORMAT(F3.5)
      CALL CHAR(3.1,.5,JJ,8)
      CALL CHAR(3.4,2.1,7H FT/SEC,7)
      CALL PLOT(0.,-2.,2)
      DO  K=1,5
      X=K-1.375
      CALL PLOT (0.,X,1)
      CALL PLOT (0.,X+.125,2)
      CALL PLOT (0.,X+.25,1)
    8 CALL PLOT (0.,X+.375,2)
      X=3.5*COS(ALR)
      Y=3.5*SIN(ALR)
      CALL PLOT(Y,X,2)
      CALL PLOT( 0.,0.,1)
      CALL PLOT( -Y,X,1)
      PRINT2,AL,EPS,EK,EM,Z
    2 FORMAT(*1VELOCITY PROFILE FOR FLOW BETWEEN NONPARALLEL PLANE WALLS
     1 WITH 3 REAL ROOTS.*/* ANGLE OF WALLS = *F9.5/* EPSILON = *F9.5/*
     2 K = *F9.5/* M = *F9.5* */* KINEMATIC VISCOSITY = *E11.4* FT/SEC2*
     3)
      DO  I=1,3,2
      R=I
      IF(I.EQ.1) R=1.2
      XX=R*COS(ALR)
      YY=R*SIN(ALR)
      CALL PLOT (-YY,XX,2)
      M=A[illegible]
      N=2*M+1
      DO K=1,N
      T=-M+K-1
      T=C1(T)
      X=R*COS(T)
      Y=R*SIN(T)
    9 CALL PLOT (Y,X,1)
      CALL PLOT (YY,XX,1)
      A=A[illegible]
      J=1
    6 AR=A*3.1415926536/180.
      F=-5.*EK*EK/EM*((SN(AR/SQRT(EM),EK))**2+((EM-1.)/EK/EK-1.)/3.)
      U=Z/R*F*12.
      XB(J)=X=(R+U/UM)*COS(AR)
      Y=(R+U/UM)*SIN(AR)
C     IF(J/5*5.EQ.J) PRINT4,R,A,F,U,X,Y
                     PRINT1,R,A,F,U,X,Y
```

```
      YB(J)=-Y
      CALL PLOT (Y,X,1)
      IF(A-.5.LT.-0.) GO TO 5
      IF(J.NE.1) GO TO112
      L=A
      A=_
  112 A=A-.5
      J=J+1
      GO TO 5
    5 DO 10 K=1,J
      N=J-K+1
   10 CALL PLOT(YB(N),XB(N),1)
    3 CONTINUE
    4 FORMAT(* R= *F4.1* IN.    ANGLE= *F6.2* DEGREES    FUNCTION= *E11.4
     1*    VELOCITY, U = *E11.4* FT/SEC*,2E10.2)
   12 END
```

```
VELOCITY PROFILE FOR FLOW BETWEEN NONPARALLEL PLANE WALLS WITH 3 REAL ROOTS,
ANGLE OF WALLS =  31,00000
EPSILON =    0,20000
 K =    0,45000
M =   1,05000
KINEMATIC VISCOSITY =  1,5930-004 FT/SEC2
R=  1,2 IN.   ANGLE=  31,00 DEGREES   FUNCTION=  1,4652-003   VELOCITY, U =  2,3297-006 FT/SEC  1,03+000  6,20-001
R=  1,2 IN.   ANGLE=  30,50 DEGREES   FUNCTION=  9,7412-003   VELOCITY, U =  1,5489-005 FT/SEC  1,06+000  6,23-001
R=  1,2 IN.   ANGLE=  30,00 DEGREES   FUNCTION=  1,7941-002   VELOCITY, U =  2,8527-005 FT/SEC  1,08+000  6,26-001
R=  1,2 IN.   ANGLE=  29,50 DEGREES   FUNCTION=  2,6063-002   VELOCITY, U =  4,1440-005 FT/SEC  1,11+000  6,28-001
R=  1,2 IN.   ANGLE=  29,00 DEGREES   FUNCTION=  3,4104-002   VELOCITY, U =  5,4225-005 FT/SEC  1,14+000  6,29-001
R=  1,2 IN.   ANGLE=  28,50 DEGREES   FUNCTION=  4,2061-002   VELOCITY, U =  6,6877-005 FT/SEC  1,16+000  6,30-001
R=  1,2 IN.   ANGLE=  28,00 DEGREES   FUNCTION=  4,9932-002   VELOCITY, U =  7,9392-005 FT/SEC  1,19+000  6,31-001
R=  1,2 IN.   ANGLE=  27,50 DEGREES   FUNCTION=  5,7715-002   VELOCITY, U =  9,1767-005 FT/SEC  1,21+000  6,31-001
R=  1,2 IN.   ANGLE=  27,00 DEGREES   FUNCTION=  6,5408-002   VELOCITY, U =  1,0400-004 FT/SEC  1,24+000  6,30-001
R=  1,2 IN.   ANGLE=  26,50 DEGREES   FUNCTION=  7,3007-002   VELOCITY, U =  1,1608-004 FT/SEC  1,26+000  6,29-001
R=  1,2 IN.   ANGLE=  26,00 DEGREES   FUNCTION=  8,0510-002   VELOCITY, U =  1,2801-004 FT/SEC  1,29+000  6,27-001
R=  1,2 IN.   ANGLE=  25,50 DEGREES   FUNCTION=  8,7916-002   VELOCITY, U =  1,3979-004 FT/SEC  1,31+000  6,25-001
R=  1,2 IN.   ANGLE=  25,00 DEGREES   FUNCTION=  9,5221-002   VELOCITY, U =  1,5140-004 FT/SEC  1,34+000  6,23-001
R=  1,2 IN.   ANGLE=  24,50 DEGREES   FUNCTION=  1,0242-001   VELOCITY, U =  1,6285-004 FT/SEC  1,36+000  6,19-001
R=  1,2 IN.   ANGLE=  24,00 DEGREES   FUNCTION=  1,0952-001   VELOCITY, U =  1,7414-004 FT/SEC  1,38+000  6,16-001
R=  1,2 IN.   ANGLE=  23,50 DEGREES   FUNCTION=  1,1651-001   VELOCITY, U =  1,8525-004 FT/SEC  1,41+000  6,12-001
R=  1,2 IN.   ANGLE=  23,00 DEGREES   FUNCTION=  1,2339-001   VELOCITY, U =  1,9619-004 FT/SEC  1,43+000  6,07-001
R=  1,2 IN.   ANGLE=  22,50 DEGREES   FUNCTION=  1,3016-001   VELOCITY, U =  2,0696-004 FT/SEC  1,45+000  6,02-001
R=  1,2 IN.   ANGLE=  22,00 DEGREES   FUNCTION=  1,3682-001   VELOCITY, U =  2,1754-004 FT/SEC  1,48+000  5,97-001
R=  1,2 IN.   ANGLE=  21,50 DEGREES   FUNCTION=  1,4336-001   VELOCITY, U =  2,2794-004 FT/SEC  1,50+000  5,91-001
R=  1,2 IN.   ANGLE=  21,00 DEGREES   FUNCTION=  1,4978-001   VELOCITY, U =  2,3815-004 FT/SEC  1,52+000  5,84-001
R=  1,2 IN.   ANGLE=  20,50 DEGREES   FUNCTION=  1,5608-001   VELOCITY, U =  2,4817-004 FT/SEC  1,54+000  5,77-001
R=  1,2 IN.   ANGLE=  20,00 DEGREES   FUNCTION=  1,6226-001   VELOCITY, U =  2,5799-004 FT/SEC  1,57+000  5,70-001
R=  1,2 IN.   ANGLE=  19,50 DEGREES   FUNCTION=  1,6831-001   VELOCITY, U =  2,6762-004 FT/SEC  1,59+000  5,62-001
R=  1,2 IN.   ANGLE=  19,00 DEGREES   FUNCTION=  1,7424-001   VELOCITY, U =  2,7704-004 FT/SEC  1,61+000  5,53-001
R=  1,2 IN.   ANGLE=  18,50 DEGREES   FUNCTION=  1,8004-001   VELOCITY, U =  2,8626-004 FT/SEC  1,63+000  5,45-001
R=  1,2 IN.   ANGLE=  18,00 DEGREES   FUNCTION=  1,8571-001   VELOCITY, U =  2,9528-004 FT/SEC  1,65+000  5,35-001
R=  1,2 IN.   ANGLE=  17,50 DEGREES   FUNCTION=  1,9125-001   VELOCITY, U =  3,0408-004 FT/SEC  1,67+000  5,26-001
R=  1,2 IN.   ANGLE=  17,00 DEGREES   FUNCTION=  1,9665-001   VELOCITY, U =  3,1267-004 FT/SEC  1,69+000  5,16-001
R=  1,2 IN.   ANGLE=  16,50 DEGREES   FUNCTION=  2,0192-001   VELOCITY, U =  3,2105-004 FT/SEC  1,71+000  5,05-001
R=  1,2 IN.   ANGLE=  16,00 DEGREES   FUNCTION=  2,0705-001   VELOCITY, U =  3,2920-004 FT/SEC  1,72+000  4,94-001
R=  1,2 IN.   ANGLE=  15,50 DEGREES   FUNCTION=  2,1204-001   VELOCITY, U =  3,3714-004 FT/SEC  1,74+000  4,83-001
R=  1,2 IN.   ANGLE=  15,00 DEGREES   FUNCTION=  2,1689-001   VELOCITY, U =  3,4485-004 FT/SEC  1,76+000  4,72-001
R=  1,2 IN.   ANGLE=  14,50 DEGREES   FUNCTION=  2,2159-001   VELOCITY, U =  3,5233-004 FT/SEC  1,78+000  4,60-001
R=  1,2 IN.   ANGLE=  14,00 DEGREES   FUNCTION=  2,2616-001   VELOCITY, U =  3,5959-004 FT/SEC  1,79+000  4,47-001
R=  1,2 IN.   ANGLE=  13,50 DEGREES   FUNCTION=  2,3057-001   VELOCITY, U =  3,6661-004 FT/SEC  1,81+000  4,35-001
R=  1,2 IN.   ANGLE=  13,00 DEGREES   FUNCTION=  2,3484-001   VELOCITY, U =  3,7340-004 FT/SEC  1,83+000  4,21-001
R=  1,2 IN.   ANGLE=  12,50 DEGREES   FUNCTION=  2,3896-001   VELOCITY, U =  3,7995-004 FT/SEC  1,84+000  4,08-001
R=  1,2 IN.   ANGLE=  12,00 DEGREES   FUNCTION=  2,4294-001   VELOCITY, U =  3,8627-004 FT/SEC  1,86+000  3,94-001
R=  1,2 IN.   ANGLE=  11,50 DEGREES   FUNCTION=  2,4676-001   VELOCITY, U =  3,9234-004 FT/SEC  1,87+000  3,80-001
R=  1,2 IN.   ANGLE=  11,00 DEGREES   FUNCTION=  2,5042-001   VELOCITY, U =  3,9817-004 FT/SEC  1,88+000  3,66-001
R=  1,2 IN.   ANGLE=  10,50 DEGREES   FUNCTION=  2,5394-001   VELOCITY, U =  4,0376-004 FT/SEC  1,90+000  3,51-001
R=  1,2 IN.   ANGLE=  10,00 DEGREES   FUNCTION=  2,5729-001   VELOCITY, U =  4,0910-004 FT/SEC  1,91+000  3,37-001
R=  1,2 IN.   ANGLE=   9,50 DEGREES   FUNCTION=  2,6050-001   VELOCITY, U =  4,1419-004 FT/SEC  1,92+000  3,21-001
R=  1,2 IN.   ANGLE=   9,00 DEGREES   FUNCTION=  2,6354-001   VELOCITY, U =  4,1903-004 FT/SEC  1,93+000  3,06-001
R=  1,2 IN.   ANGLE=   8,50 DEGREES   FUNCTION=  2,6642-001   VELOCITY, U =  4,2361-004 FT/SEC  1,94+000  2,90-001
R=  1,2 IN.   ANGLE=   8,00 DEGREES   FUNCTION=  2,6915-001   VELOCITY, U =  4,2795-004 FT/SEC  1,95+000  2,74-001
R=  1,2 IN.   ANGLE=   7,50 DEGREES   FUNCTION=  2,7172-001   VELOCITY, U =  4,3203-004 FT/SEC  1,96+000  2,58-001
R=  1,2 IN.   ANGLE=   7,00 DEGREES   FUNCTION=  2,7412-001   VELOCITY, U =  4,3585-004 FT/SEC  1,97+000  2,42-001
R=  1,2 IN.   ANGLE=   6,50 DEGREES   FUNCTION=  2,7636-001   VELOCITY, U =  4,3942-004 FT/SEC  1,98+000  2,26-001
R=  1,2 IN.   ANGLE=   6,00 DEGREES   FUNCTION=  2,7844-001   VELOCITY, U =  4,4272-004 FT/SEC  1,99+000  2,09-001
R=  1,2 IN.   ANGLE=   5,50 DEGREES   FUNCTION=  2,8036-001   VELOCITY, U =  4,4577-004 FT/SEC  2,00+000  1,92-001
R=  1,2 IN.   ANGLE=   5,00 DEGREES   FUNCTION=  2,8211-001   VELOCITY, U =  4,4855-004 FT/SEC  2,00+000  1,75-001
R=  1,2 IN.   ANGLE=   4,50 DEGREES   FUNCTION=  2,8369-001   VELOCITY, U =  4,5107-004 FT/SEC  2,01+000  1,58-001
R=  1,2 IN.   ANGLE=   4,00 DEGREES   FUNCTION=  2,8512-001   VELOCITY, U =  4,5333-004 FT/SEC  2,01+000  1,41-001
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R= 1.2 IN.	ANGLE= 3,50 DEGREES	FUNCTION= 2,8637-001	VELOCITY, U = 4,5533-004 FT/SEC	2,02+000	1,23-001
R= 1.2 IN.	ANGLE= 3,00 DEGREES	FUNCTION= 2,8746-001	VELOCITY, U = 4,5706-004 FT/SEC	2,02+000	1,06-001
R= 1.2 IN.	ANGLE= 2,50 DEGREES	FUNCTION= 2,8638-001	VELOCITY, U = 4,5852-004 FT/SEC	2,03+000	8,84-002
R= 1.2 IN.	ANGLE= 2,00 DEGREES	FUNCTION= 2,8913-001	VELOCITY, U = 4,5972-004 FT/SEC	2,03+000	7,08-002
R= 1.2 IN.	ANGLE= 1,50 DEGREES	FUNCTION= 2,8972-001	VELOCITY, U = 4,6066-004 FT/SEC	2,03+000	5,32-002
R= 1.2 IN.	ANGLE= 1,00 DEGREES	FUNCTION= 2,9014-001	VELOCITY, U = 4,6132-004 FT/SEC	2,03+000	3,55-002
R= 1.2 IN.	ANGLE= 0,50 DEGREES	FUNCTION= 2,9039-001	VELOCITY, U = 4,6172-004 FT/SEC	2,03+000	1,77-002
R= 1.2 IN.	ANGLE= 0,00 DEGREES	FUNCTION= 2,9048-001	VELOCITY, U = 4,6186-004 FT/SEC	2,03+000	0,00+000
R= 3.0 IN.	ANGLE= 31,00 DEGREES	FUNCTION= 1,4652-003	VELOCITY, U = 9,3190-007 FT/SEC	2,57+000	1,55+000
R= 3.0 IN.	ANGLE= 30,50 DEGREES	FUNCTION= 9,7412-003	VELOCITY, U = 6,1954-006 FT/SEC	2,59+000	1,53+000
R= 3.0 IN.	ANGLE= 30,00 DEGREES	FUNCTION= 1,7941-002	VELOCITY, U = 1,1411-005 FT/SEC	2,62+000	1,51+000
R= 3.0 IN.	ANGLE= 29,50 DEGREES	FUNCTION= 2,6063-002	VELOCITY, U = 1,6576-005 FT/SEC	2,64+000	1,49+000
R= 3.0 IN.	ANGLE= 29,00 DEGREES	FUNCTION= 3,4104-002	VELOCITY, U = 2,1690-005 FT/SEC	2,66+000	1,47+000
R= 3.0 IN.	ANGLE= 28,50 DEGREES	FUNCTION= 4,2061-002	VELOCITY, U = 2,6751-005 FT/SEC	2,68+000	1,45+000
R= 3.0 IN.	ANGLE= 28,00 DEGREES	FUNCTION= 4,9932-002	VELOCITY, U = 3,1757-005 FT/SEC	2,70+000	1,44+000
R= 3.0 IN.	ANGLE= 27,50 DEGREES	FUNCTION= 5,7715-002	VELOCITY, U = 3,6707-005 FT/SEC	2,72+000	1,42+000
R= 3.0 IN.	ANGLE= 27,00 DEGREES	FUNCTION= 6,5408-002	VELOCITY, U = 4,1599-005 FT/SEC	2,74+000	1,40+000
R= 3.0 IN.	ANGLE= 26,50 DEGREES	FUNCTION= 7,3007-002	VELOCITY, U = 4,6432-005 FT/SEC	2,76+000	1,38+000
R= 3.0 IN.	ANGLE= 26,00 DEGREES	FUNCTION= 8,0510-002	VELOCITY, U = 5,1205-005 FT/SEC	2,78+000	1,36+000
R= 3.0 IN.	ANGLE= 25,50 DEGREES	FUNCTION= 8,7916-002	VELOCITY, U = 5,5914-005 FT/SEC	2,80+000	1,33+000
R= 3.0 IN.	ANGLE= 25,00 DEGREES	FUNCTION= 9,5221-002	VELOCITY, U = 6,0561-005 FT/SEC	2,82+000	1,31+000
R= 3.0 IN.	ANGLE= 24,50 DEGREES	FUNCTION= 1,0242-001	VELOCITY, U = 6,5141-005 FT/SEC	2,84+000	1,29+000
R= 3.0 IN.	ANGLE= 24,00 DEGREES	FUNCTION= 1,0952-001	VELOCITY, U = 6,9656-005 FT/SEC	2,86+000	1,27+000
R= 3.0 IN.	ANGLE= 23,50 DEGREES	FUNCTION= 1,1651-001	VELOCITY, U = 7,4102-005 FT/SEC	2,87+000	1,25+000
R= 3.0 IN.	ANGLE= 23,00 DEGREES	FUNCTION= 1,2339-001	VELOCITY, U = 7,8478-005 FT/SEC	2,89+000	1,23+000
R= 3.0 IN.	ANGLE= 22,50 DEGREES	FUNCTION= 1,3016-001	VELOCITY, U = 8,2783-005 FT/SEC	2,91+000	1,21+000
R= 3.0 IN.	ANGLE= 22,00 DEGREES	FUNCTION= 1,3682-001	VELOCITY, U = 8,7016-005 FT/SEC	2,93+000	1,18+000
R= 3.0 IN.	ANGLE= 21,50 DEGREES	FUNCTION= 1,4336-001	VELOCITY, U = 9,1175-005 FT/SEC	2,94+000	1,16+000
R= 3.0 IN.	ANGLE= 21,00 DEGREES	FUNCTION= 1,4978-001	VELOCITY, U = 9,5259-005 FT/SEC	2,96+000	1,14+000
R= 3.0 IN.	ANGLE= 20,50 DEGREES	FUNCTION= 1,5608-001	VELOCITY, U = 9,9266-005 FT/SEC	2,98+000	1,11+000
R= 3.0 IN.	ANGLE= 20,00 DEGREES	FUNCTION= 1,6226-001	VELOCITY, U = 1,0320-004 FT/SEC	2,99+000	1,09+000
R= 3.0 IN.	ANGLE= 19,50 DEGREES	FUNCTION= 1,6831-001	VELOCITY, U = 1,0705-004 FT/SEC	3,01+000	1,07+000
R= 3.0 IN.	ANGLE= 19,00 DEGREES	FUNCTION= 1,7424-001	VELOCITY, U = 1,1082-004 FT/SEC	3,03+000	1,04+000
R= 3.0 IN.	ANGLE= 18,50 DEGREES	FUNCTION= 1,8004-001	VELOCITY, U = 1,1451-004 FT/SEC	3,04+000	1,02+000
R= 3.0 IN.	ANGLE= 18,00 DEGREES	FUNCTION= 1,8571-001	VELOCITY, U = 1,1811-004 FT/SEC	3,06+000	9,93-001
R= 3.0 IN.	ANGLE= 17,50 DEGREES	FUNCTION= 1,9125-001	VELOCITY, U = 1,2163-004 FT/SEC	3,07+000	9,68-001
R= 3.0 IN.	ANGLE= 17,00 DEGREES	FUNCTION= 1,9665-001	VELOCITY, U = 1,2507-004 FT/SEC	3,08+000	9,43-001
R= 3.0 IN.	ANGLE= 16,50 DEGREES	FUNCTION= 2,0192-001	VELOCITY, U = 1,2842-004 FT/SEC	3,10+000	9,18-001
R= 3.0 IN.	ANGLE= 16,00 DEGREES	FUNCTION= 2,0705-001	VELOCITY, U = 1,3168-004 FT/SEC	3,11+000	8,92-001
R= 3.0 IN.	ANGLE= 15,50 DEGREES	FUNCTION= 2,1204-001	VELOCITY, U = 1,3486-004 FT/SEC	3,13+000	8,67-001
R= 3.0 IN.	ANGLE= 15,00 DEGREES	FUNCTION= 2,1689-001	VELOCITY, U = 1,3794-004 FT/SEC	3,14+000	8,41-001
R= 3.0 IN.	ANGLE= 14,50 DEGREES	FUNCTION= 2,2159-001	VELOCITY, U = 1,4093-004 FT/SEC	3,15+000	8,15-001
R= 3.0 IN.	ANGLE= 14,00 DEGREES	FUNCTION= 2,2616-001	VELOCITY, U = 1,4384-004 FT/SEC	3,16+000	7,89-001
R= 3.0 IN.	ANGLE= 13,50 DEGREES	FUNCTION= 2,3057-001	VELOCITY, U = 1,4664-004 FT/SEC	3,17+000	7,62-001
R= 3.0 IN.	ANGLE= 13,00 DEGREES	FUNCTION= 2,3484-001	VELOCITY, U = 1,4936-004 FT/SEC	3,19+000	7,35-001
R= 3.0 IN.	ANGLE= 12,50 DEGREES	FUNCTION= 2,3896-001	VELOCITY, U = 1,5198-004 FT/SEC	3,20+000	7,09-001
R= 3.0 IN.	ANGLE= 12,00 DEGREES	FUNCTION= 2,4294-001	VELOCITY, U = 1,5451-004 FT/SEC	3,21+000	6,82-001
R= 3.0 IN.	ANGLE= 11,50 DEGREES	FUNCTION= 2,4676-001	VELOCITY, U = 1,5694-004 FT/SEC	3,22+000	6,55-001
R= 3.0 IN.	ANGLE= 11,00 DEGREES	FUNCTION= 2,5042-001	VELOCITY, U = 1,5927-004 FT/SEC	3,23+000	6,27-001
R= 3.0 IN.	ANGLE= 10,50 DEGREES	FUNCTION= 2,5394-001	VELOCITY, U = 1,6150-004 FT/SEC	3,24+000	6,00-001
R= 3.0 IN.	ANGLE= 10,00 DEGREES	FUNCTION= 2,5729-001	VELOCITY, U = 1,6364-004 FT/SEC	3,25+000	5,72-001
R= 3.0 IN.	ANGLE= 9,50 DEGREES	FUNCTION= 2,6050-001	VELOCITY, U = 1,6567-004 FT/SEC	3,25+000	5,44-001
R= 3.0 IN.	ANGLE= 9,00 DEGREES	FUNCTION= 2,6354-001	VELOCITY, U = 1,6761-004 FT/SEC	3,26+000	5,17-001
R= 3.0 IN.	ANGLE= 8,50 DEGREES	FUNCTION= 2,6642-001	VELOCITY, U = 1,6945-004 FT/SEC	3,27+000	4,89-001
R= 3.0 IN.	ANGLE= 8,00 DEGREES	FUNCTION= 2,6915-001	VELOCITY, U = 1,7118-004 FT/SEC	3,28+000	4,61-001
R= 3.0 IN.	ANGLE= 7,50 DEGREES	FUNCTION= 2,7172-001	VELOCITY, U = 1,7281-004 FT/SEC	3,28+000	4,32-001
R= 3.0 IN.	ANGLE= 7,00 DEGREES	FUNCTION= 2,7412-001	VELOCITY, U = 1,7434-004 FT/SEC	3,29+000	4,04-001
R= 3.0 IN.	ANGLE= 6,50 DEGREES	FUNCTION= 2,7636-001	VELOCITY, U = 1,7577-004 FT/SEC	3,30+000	3,76-001
R= 3.0 IN.	ANGLE= 6,00 DEGREES	FUNCTION= 2,7844-001	VELOCITY, U = 1,7709-004 FT/SEC	3,30+000	3,47-001
R= 3.0 IN.	ANGLE= 5,50 DEGREES	FUNCTION= 2,8036-001	VELOCITY, U = 1,7831-004 FT/SEC	3,31+000	3,18-001
R= 3.0 IN.	ANGLE= 5,00 DEGREES	FUNCTION= 2,8211-001	VELOCITY, U = 1,7942-004 FT/SEC	3,31+000	2,90-001

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R=  3.0 IN.   ANGLE=  4.50 DEGREES   FUNCTION=  2.8369-001   VELOCITY, U =  1.8043-004 FT/SEC   3.32+000   2.61-001
R=  3.0 IN.   ANGLE=  4.00 DEGREES   FUNCTION=  2.8512-001   VELOCITY, U =  1.8133-004 FT/SEC   3.32+000   2.32-001
R=  3.0 IN.   ANGLE=  3.50 DEGREES   FUNCTION=  2.8637-001   VELOCITY, U =  1.8213-004 FT/SEC   3.32+000   2.03-001
R=  3.0 IN.   ANGLE=  3.00 DEGREES   FUNCTION=  2.8746-001   VELOCITY, U =  1.8282-004 FT/SEC   3.33+000   1.74-001
R=  3.0 IN.   ANGLE=  2.50 DEGREES   FUNCTION=  2.8836-001   VELOCITY, U =  1.8341-004 FT/SEC   3.33+000   1.45-001
R=  3.0 IN.   ANGLE=  2.00 DEGREES   FUNCTION=  2.8915-001   VELOCITY, U =  1.8389-004 FT/SEC   3.33+000   1.16-001
R=  3.0 IN.   ANGLE=  1.50 DEGREES   FUNCTION=  2.8972-001   VELOCITY, U =  1.8426-004 FT/SEC   3.33+000   8.72-002
R=  3.0 IN.   ANGLE=  1.00 DEGREES   FUNCTION=  2.9014-001   VELOCITY, U =  1.8453-004 FT/SEC   3.33+000   5.82-002
R=  3.0 IN.   ANGLE=  0.50 DEGREES   FUNCTION=  2.9039-001   VELOCITY, U =  1.8469-004 FT/SEC   3.33+000   2.91-002
R=  3.0 IN.   ANGLE=  0.00 DEGREES   FUNCTION=  2.9046-001   VELOCITY, U =  1.8474-004 FT/SEC   3.33+000   0.00+000
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SUBROUTINE PRO SDCC

for SDCO velocity profiles

```
      SUBROUTINE PRO SDCC (AL,EPS,EK,EM)
      DIMENSION XH(400),Y3(400)
      C1(A)=A*3.1415926536/180.
      Z=1.59E-4
   13 ALR=C1(AL)
      R=1.
      AR=0.
      E1=2./3.*(2.*EM*EK*EK-3.-EM)
      F=E1-EM*((1.-CN(SQRT(2.*EM/3.)*AR,EK))/(1.+CN(SQRT(2.*EM/3.)*AR,EK))
     1)))
      UM=Z/R*F*12.
      CALL CHAR(3.4,-2.1,13HSCALE 1 INCH=,13,0.,.24,.16)
      UMM=UM*2.
      ENCODE(8,11,JJ)UMM
   11 FORMAT(F8.5)
      CALL CHAR(3.4,.5,JJ,8)
      CALL CHAR(3.4,2.1,7H FT/SEC,7)
      CALL PLOT(0.,-2.,2)
      DO 8 K=1,5
      X=K-1.375
      CALL PLOT (0.,X,1)
      CALL PLOT (0.,X+.125,2)
      CALL PLOT (0.,X+.25,1)
    8 CALL PLOT (0.,X+.375,2)
      X=3.5*COS(ALR)
      Y=3.5*SIN(ALR)
      CALL PLOT(Y,X,2)
      CALL PLOT( 0.,0.,1)
      CALL PLOT( -Y,X,1)
      PRINT2,AL,EPS,EK,EM,Z
    2 FORMAT(*1VELOCITY PROFILE FOR FLOW BETWEEN NONPARALLEL PLANE WALLS
     1 WITH 1 REAL ROOT AND 2 COMPLEX ROOTS*
     1                        /* ANGLE OF WALLS = *F9.5/* EPSILON = *F9.5/*
     2 K = *F9.5/* M = *F9.5* */* KINEMATIC VISCOSITY = *E11.4* FT/SEC2*
     3)
      DO3 I=1,3,2
      R=I
      IF(I.EQ.1) R=1.2
      XX=R*COS(ALR)
      YY=R*SIN(ALR)
      CALL PLOT (-YY,XX,2)
      M=AL
      N=2*M+1
      DO9K=1,N
      T=-M+K-1
      T=C1(T)
      X=R*COS(T)
      Y=R*SIN(T)
    9 CALL PLOT (Y,X,1)
      CALL PLOT (YY,XX,1)
      A=AL
      J=1
    6 AR=A*3.1415926536/180.
      E1=2./3.*(2.*EM*EK*EK-3.-EM)
      F=E1-EM*((1.-CN(SQRT(2.*EM/3.)*AR,EK))/(1.+CN(SQRT(2.*EM/3.)*AR,EK
     1)))
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```
      U=Z/R*F*12.
      XB(J)=X=(R+U/UM)*COS(AR)
      Y=(R+U/UM)*SIN(AR)
      IF(J/5*5.EQ.J) PRINT4,R,A,F,U,X,Y
      YB(J)=-Y
      CALL PLOT (Y,X,1)
      IF(A-.5.LT.-0.) GO TO 5
      A=A-.5
      J=J+1
      GO TO 5
    5 DO 10 K=1,J
      N=J-K+1
   10 CALL PLOT(YB(N),XB(N),1)
    3 CONTINUE
    4 FORMAT(* R= *F4.1* IN.   ANGLE= *F6.2* DEGREES   FUNCTION= *E11.4
     1*   VELOCITY, U = *E11.4* FT/SEC*,2E10.2)
   12 END
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VELOCITY PROFILE FOR FLOW BETWEEN NONPARALLEL PLANE WALLS WITH 1 REAL ROOT AND 2 COMPLEX ROOTS
ANGLE OF WALLS =  33.00000
EPSILON =    5.00000
 K =    1.00000
M =  12.90000
KINEMATIC VISCOSITY =  1.5900-004 FT/SEC2
R=  1.2 IN.   ANGLE=  33.00 DEGREES   FUNCTION=  4.9038-001   VELOCITY, U =  7.7970-004 FT/SEC  1.06+000  6.87-001
R=  1.2 IN.   ANGLE=  30.50 DEGREES   FUNCTION=  1.0990+000   VELOCITY, U =  1.7474-003 FT/SEC  1.15+000  6.79-001
R=  1.2 IN.   ANGLE=  28.00 DEGREES   FUNCTION=  1.7244+000   VELOCITY, U =  2.7419-003 FT/SEC  1.25+000  6.66-001
R=  1.2 IN.   ANGLE=  25.50 DEGREES   FUNCTION=  2.3585+000   VELOCITY, U =  3.7501-003 FT/SEC  1.35+000  6.45-001
R=  1.2 IN.   ANGLE=  23.00 DEGREES   FUNCTION=  2.9915+000   VELOCITY, U =  4.7565-003 FT/SEC  1.45+000  6.16-001
R=  1.2 IN.   ANGLE=  20.50 DEGREES   FUNCTION=  3.6122+000   VELOCITY, U =  5.7434-003 FT/SEC  1.55+000  5.80-001
R=  1.2 IN.   ANGLE=  18.00 DEGREES   FUNCTION=  4.2080+000   VELOCITY, U =  6.6907-003 FT/SEC  1.65+000  5.35-001
R=  1.2 IN.   ANGLE=  15.50 DEGREES   FUNCTION=  4.7656+000   VELOCITY, U =  7.5773-003 FT/SEC  1.74+000  4.81-001
R=  1.2 IN.   ANGLE=  13.00 DEGREES   FUNCTION=  5.2713+000   VELOCITY, U =  8.3814-003 FT/SEC  1.82+000  4.20-001
R=  1.2 IN.   ANGLE=  10.50 DEGREES   FUNCTION=  5.7116+000   VELOCITY, U =  9.0815-003 FT/SEC  1.89+000  3.50-001
R=  1.2 IN.   ANGLE=   8.00 DEGREES   FUNCTION=  6.0741+000   VELOCITY, U =  9.6577-003 FT/SEC  1.95+000  2.74-001
R=  1.2 IN.   ANGLE=   5.50 DEGREES   FUNCTION=  6.3478+000   VELOCITY, U =  1.0093-002 FT/SEC  1.99+000  1.92-001
R=  1.2 IN.   ANGLE=   3.00 DEGREES   FUNCTION=  6.5243+000   VELOCITY, U =  1.0374-002 FT/SEC  2.02+000  1.06-001
R=  1.2 IN.   ANGLE=   0.50 DEGREES   FUNCTION=  6.5979+000   VELOCITY, U =  1.0491-002 FT/SEC  2.03+000  1.77-002
R=  3.0 IN.   ANGLE=  33.00 DEGREES   FUNCTION=  4.9038-001   VELOCITY, U =  3.1188-004 FT/SEC  2.54+000  1.65+000
R=  3.0 IN.   ANGLE=  30.50 DEGREES   FUNCTION=  1.0990+000   VELOCITY, U =  6.9898-004 FT/SEC  2.63+000  1.55+000
R=  3.0 IN.   ANGLE=  28.00 DEGREES   FUNCTION=  1.7244+000   VELOCITY, U =  1.0967-003 FT/SEC  2.73+000  1.45+000
R=  3.0 IN.   ANGLE=  25.50 DEGREES   FUNCTION=  2.3585+000   VELOCITY, U =  1.5000-003 FT/SEC  2.82+000  1.34+000
R=  3.0 IN.   ANGLE=  23.00 DEGREES   FUNCTION=  2.9915+000   VELOCITY, U =  1.9026-003 FT/SEC  2.90+000  1.23+000
R=  3.0 IN.   ANGLE=  20.50 DEGREES   FUNCTION=  3.6122+000   VELOCITY, U =  2.2973-003 FT/SEC  2.98+000  1.11+000
R=  3.0 IN.   ANGLE=  18.00 DEGREES   FUNCTION=  4.2080+000   VELOCITY, U =  2.6763-003 FT/SEC  3.06+000  9.93-001
R=  3.0 IN.   ANGLE=  15.50 DEGREES   FUNCTION=  4.7656+000   VELOCITY, U =  3.0309-003 FT/SEC  3.12+000  8.66-001
R=  3.0 IN.   ANGLE=  13.00 DEGREES   FUNCTION=  5.2713+000   VELOCITY, U =  3.3525-003 FT/SEC  3.18+000  7.35-001
R=  3.0 IN.   ANGLE=  10.50 DEGREES   FUNCTION=  5.7116+000   VELOCITY, U =  3.6326-003 FT/SEC  3.23+000  5.99-001
R=  3.0 IN.   ANGLE=   8.00 DEGREES   FUNCTION=  6.0741+000   VELOCITY, U =  3.8631-003 FT/SEC  3.27+000  4.60-001
R=  3.0 IN.   ANGLE=   5.50 DEGREES   FUNCTION=  6.3478+000   VELOCITY, U =  4.0372-003 FT/SEC  3.31+000  3.18-001
R=  3.0 IN.   ANGLE=   3.00 DEGREES   FUNCTION=  6.5243+000   VELOCITY, U =  4.1494-003 FT/SEC  3.32+000  1.74-001
R=  3.0 IN.   ANGLE=   0.50 DEGREES   FUNCTION=  6.5979+000   VELOCITY, U =  4.1963-003 FT/SEC  3.33+000  2.91-002


      EXECUTE :      0 HOURS   0 MINUTES   12.480 SECONDS
      SETUP   :      0 HOURS   0 MINUTES   05.221 SECONDS
      FOR     :      0 HOURS   0 MINUTES   07.944 SECONDS
      LOADER  :      0 HOURS   0 MINUTES   03.844 SECONDS
      *OTHER  :      0 HOURS   0 MINUTES   00.000 SECONDS

   ELAPSED TIME:     0 HOURS   0 MINUTES   29.489 SECONDS
     63 WRITE OPERATIONS ON LU 61 WERE DONE DURING EXECUTION.
  TOTAL COMPUTE COST AT REGULAR RATE  $   2.00
SEQUENCE NUMBER 325439:1000 FINISHED AT 110207               DATE  04/25/72
```

SUBROUTINE PRO CON

for SC velocity profiles

```
      SUBROUTINE PRO CON (AL,EPS,EK,EM)
      DIMENSION XB(400),YB(400)
      C1(A)=A*3.1415926535/180.
      Z=1.59E-4
   13 ALR=C1(AL)
      R=1.
      F1=0.
      A =0.
   15 AR=C1(A)
      F=2.*(EM*EM*(EK*EK-2.)-1.)+6.*EM*EM*(1.-EK*EK)/(DN(AR*EM,EK))**2
      IF(ABS(F).GT.ABS(F1)) F1=F
      A =A +1.
      IF(A .GT.AL) GO TO 4
      GO TO15
    4 UM=ABS(Z/R*F1*12.)
      CALL CHAR(3.4,-2.1,13HSCALE 1 INCH=,13,0.,.24,.16)
      UMM=UM*2.
      ENCODE(8,11,JJ)UMM
   11 FORMAT(F8.5)
      CALL CHAR(3.4,.5,JJ,8)
      CALL CHAR(3.4,2.1,7H FT/SEC,7)
      CALL PLOT(0.,-2.,2)
      DO 8 K=1,5
      X=K-1.375
      CALL PLOT (0.,X,1)
      CALL PLOT (0.,X+.125,2)
      CALL PLOT (0.,X+.25,1)
    8 CALL PLOT (0.,X+.375,2)
      X=3.5*COS(ALR)
      Y=3.5*SIN(ALR)
      CALL PLOT(Y,X,2)
      CALL PLOT( 0.,0.,1)
      CALL PLOT( -Y,X,1)
      PRINT2,AL,EPS,EK,EM,Z
    2 FORMAT(*1VELOCITY PROFILE FOR *
     1*CONVERGING *
     1                              *FLOW BETWEEN NONPARALLEL PLANE WALLS
     1 WITH 3 REAL ROOTS.*/* ANGLE OF WALLS = *F9.5/* EPSILON = *F9.5/*
     2 K = *F9.5/* M = *F9.5* */* KINEMATIC VISCOSITY = *E11.4* FT/SEC2*
     3)
      DO3 I=1,3,2
      R=I
      IF(I.EQ.1) R=1.2
      XX=R*COS(ALR)
      YY=R*SIN(ALR)
      CALL PLOT (-YY,XX,2)
      M=AL
      N=2*M+1
      DO9K=1,N
      T=-M+K-1
      T=C1(T)
      X=R*COS(T)
      Y=R*SIN(T)
    9 CALL PLOT (Y,X,1)
      CALL PLOT (YY,XX,1)
      A=AL
```

```
      J=1
    6 AR=A*3.1415926536/180.
      F=2.*(EM*EM*(EK*EK-2.)-1.)+6.*EM*EM*(1.-EK*EK)/(DN(AR*EM,EK))**2
      U=Z/R*F*12.
      XB(J)=X=(R+U/UM)*COS(AR)
      Y=(R+U/UM)*SIN(AR)
                   PRINT4,R,A,F,U,X,Y
      YB(J)=-Y
      CALL PLOT (Y,X,1)
      IF(A-.5.LT.-0.) GO TO 5
      IF(J.GT.1) GO TO 14
      NN=A
      A=NN
   14 A=A-1.
      J=J+1
      GO TO 5
    5 DO 10 K=1,J
      N=J-K+1
   10 CALL PLOT(YB(N),XB(N),1)
    3 CONTINUE
    4 FORMAT(* R= *F4.1* IN.   ANGLE= *F6.2* DEGREES   FUNCTION= *E11.4
     1*   VELOCITY, U = *F13.9* FT/SEC*,2E10.2)
   12 END
```

```
VELOCITY PROFILE FOR CONVERGING FLOW BETWEEN NONPARALLEL PLANE WALLS WITH 3 REAL ROOTS,
ANGLE OF WALLS =  40,00000
EPSILON =  -4,00000
 K =    0,85000
M =    1,65000
KINEMATIC VISCOSITY =  1,5900-004 FT/SEC2
R=  1,2 IN,   ANGLE=  40,00 DEGREES   FUNCTION=  4,7308-001   VELOCITY, U =   0,000752197 FT/SEC  9,88-001  8,29-001
R=  1,2 IN,   ANGLE=  39,00 DEGREES   FUNCTION=  2,2137-001   VELOCITY, U =   0,000351979 FT/SEC  9,65-001  7,81-001
R=  1,2 IN,   ANGLE=  38,00 DEGREES   FUNCTION= -2,4655-002   VELOCITY, U =  -0,000039202 FT/SEC  9,42-001  7,36-001
R=  1,2 IN,   ANGLE=  37,00 DEGREES   FUNCTION= -2,6463-001   VELOCITY, U =  -0,000420767 FT/SEC  9,19-001  6,92-001
R=  1,2 IN,   ANGLE=  36,00 DEGREES   FUNCTION= -4,9831-001   VELOCITY, U =  -0,000792311 FT/SEC  8,95-001  6,50-001
R=  1,2 IN,   ANGLE=  35,00 DEGREES   FUNCTION= -7,2545-001   VELOCITY, U =  -0,001153465 FT/SEC  8,71-001  6,10-001
R=  1,2 IN,   ANGLE=  34,00 DEGREES   FUNCTION= -9,4587-001   VELOCITY, U =  -0,001503926 FT/SEC  8,47-001  5,71-001
R=  1,2 IN,   ANGLE=  33,00 DEGREES   FUNCTION= -1,1594+000   VELOCITY, U =  -0,001843444 FT/SEC  8,23-001  5,35-001
R=  1,2 IN,   ANGLE=  32,00 DEGREES   FUNCTION= -1,3659+000   VELOCITY, U =  -0,002171323 FT/SEC  7,99-001  5,00-001
R=  1,2 IN,   ANGLE=  31,00 DEGREES   FUNCTION= -1,5654+000   VELOCITY, U =  -0,002488917 FT/SEC  7,76-001  4,66-001
R=  1,2 IN,   ANGLE=  30,00 DEGREES   FUNCTION= -1,7517+000   VELOCITY, U =  -0,002785156 FT/SEC  7,53-001  4,35-001
R=  1,2 IN,   ANGLE=  29,00 DEGREES   FUNCTION= -1,9427+000   VELOCITY, U =  -0,003088880 FT/SEC  7,29-001  4,04-001
R=  1,2 IN,   ANGLE=  28,00 DEGREES   FUNCTION= -2,1205+000   VELOCITY, U =  -0,003371659 FT/SEC  7,07-001  3,76-001
R=  1,2 IN,   ANGLE=  27,00 DEGREES   FUNCTION= -2,2912+000   VELOCITY, U =  -0,003642968 FT/SEC  6,85-001  3,49-001
R=  1,2 IN,   ANGLE=  26,00 DEGREES   FUNCTION= -2,4546+000   VELOCITY, U =  -0,003902847 FT/SEC  6,63-001  3,23-001
R=  1,2 IN,   ANGLE=  25,00 DEGREES   FUNCTION= -2,6109+000   VELOCITY, U =  -0,004151351 FT/SEC  6,42-001  2,99-001
R=  1,2 IN,   ANGLE=  24,00 DEGREES   FUNCTION= -2,7601+000   VELOCITY, U =  -0,004388557 FT/SEC  6,21-001  2,77-001
R=  1,2 IN,   ANGLE=  23,00 DEGREES   FUNCTION= -2,9022+000   VELOCITY, U =  -0,004614562 FT/SEC  6,01-001  2,55-001
R=  1,2 IN,   ANGLE=  22,00 DEGREES   FUNCTION= -3,0374+000   VELOCITY, U =  -0,004829455 FT/SEC  5,82-001  2,35-001
R=  1,2 IN,   ANGLE=  21,00 DEGREES   FUNCTION= -3,1655+000   VELOCITY, U =  -0,005033127 FT/SEC  5,64-001  2,16-001
R=  1,2 IN,   ANGLE=  20,00 DEGREES   FUNCTION= -3,2871+000   VELOCITY, U =  -0,005226558 FT/SEC  5,46-001  1,99-001
R=  1,2 IN,   ANGLE=  19,00 DEGREES   FUNCTION= -3,4019+000   VELOCITY, U =  -0,005409024 FT/SEC  5,29-001  1,82-001
R=  1,2 IN,   ANGLE=  18,00 DEGREES   FUNCTION= -3,5100+000   VELOCITY, U =  -0,005580962 FT/SEC  5,12-001  1,66-001
R=  1,2 IN,   ANGLE=  17,00 DEGREES   FUNCTION= -3,6117+000   VELOCITY, U =  -0,005742524 FT/SEC  4,97-001  1,52-001
R=  1,2 IN,   ANGLE=  16,00 DEGREES   FUNCTION= -3,7068+000   VELOCITY, U =  -0,005893864 FT/SEC  4,82-001  1,38-001
R=  1,2 IN,   ANGLE=  15,00 DEGREES   FUNCTION= -3,7957+000   VELOCITY, U =  -0,006035134 FT/SEC  4,68-001  1,25-001
R=  1,2 IN,   ANGLE=  14,00 DEGREES   FUNCTION= -3,8783+000   VELOCITY, U =  -0,006166485 FT/SEC  4,55-001  1,14-001
R=  1,2 IN,   ANGLE=  13,00 DEGREES   FUNCTION= -3,9548+000   VELOCITY, U =  -0,006288063 FT/SEC  4,43-001  1,02-001
R=  1,2 IN,   ANGLE=  12,00 DEGREES   FUNCTION= -4,0252+000   VELOCITY, U =  -0,006400009 FT/SEC  4,32-001  9,18-002
R=  1,2 IN,   ANGLE=  11,00 DEGREES   FUNCTION= -4,0896+000   VELOCITY, U =  -0,006502460 FT/SEC  4,22-001  8,20-002
R=  1,2 IN,   ANGLE=  10,00 DEGREES   FUNCTION= -4,1481+000   VELOCITY, U =  -0,006595543 FT/SEC  4,12-001  7,27-002
R=  1,2 IN,   ANGLE=   9,00 DEGREES   FUNCTION= -4,2009+000   VELOCITY, U =  -0,006679371 FT/SEC  4,03-001  6,39-002
R=  1,2 IN,   ANGLE=   8,00 DEGREES   FUNCTION= -4,2478+000   VELOCITY, U =  -0,006754071 FT/SEC  3,96-001  5,56-002
R=  1,2 IN,   ANGLE=   7,00 DEGREES   FUNCTION= -4,2891+000   VELOCITY, U =  -0,006819736 FT/SEC  3,89-001  4,78-002
R=  1,2 IN,   ANGLE=   6,00 DEGREES   FUNCTION= -4,3248+000   VELOCITY, U =  -0,006876463 FT/SEC  3,83-001  4,03-002
R=  1,2 IN,   ANGLE=   5,00 DEGREES   FUNCTION= -4,3549+000   VELOCITY, U =  -0,006924320 FT/SEC  3,78-001  3,31-002
R=  1,2 IN,   ANGLE=   4,00 DEGREES   FUNCTION= -4,3795+000   VELOCITY, U =  -0,006963380 FT/SEC  3,74-001  2,61-002
R=  1,2 IN,   ANGLE=   3,00 DEGREES   FUNCTION= -4,3986+000   VELOCITY, U =  -0,006993702 FT/SEC  3,71-001  1,94-002
R=  1,2 IN,   ANGLE=   2,00 DEGREES   FUNCTION= -4,4122+000   VELOCITY, U =  -0,007015328 FT/SEC  3,68-001  1,29-002
R=  1,2 IN,   ANGLE=   1,00 DEGREES   FUNCTION= -4,4203+000   VELOCITY, U =  -0,007028291 FT/SEC  3,67-001  6,41-003
R=  1,2 IN,   ANGLE=   0,00 DEGREES   FUNCTION= -4,4230+000   VELOCITY, U =  -0,007032610 FT/SEC  3,67-001  0,00+000
R=  3,0 IN,   ANGLE=  40,00 DEGREES   FUNCTION=  4,7308-001   VELOCITY, U =   0,000300879 FT/SEC  2,33+000  1,95+000
R=  3,0 IN,   ANGLE=  39,00 DEGREES   FUNCTION=  2,2137-001   VELOCITY, U =   0,000140792 FT/SEC  2,34+000  1,90+000
R=  3,0 IN,   ANGLE=  38,00 DEGREES   FUNCTION= -2,4655-002   VELOCITY, U =  -0,000015681 FT/SEC  2,36+000  1,85+000
R=  3,0 IN,   ANGLE=  37,00 DEGREES   FUNCTION= -2,6463-001   VELOCITY, U =  -0,000168307 FT/SEC  2,38+000  1,79+000
R=  3,0 IN,   ANGLE=  36,00 DEGREES   FUNCTION= -4,9831-001   VELOCITY, U =  -0,000316924 FT/SEC  2,40+000  1,74+000
R=  3,0 IN,   ANGLE=  35,00 DEGREES   FUNCTION= -7,2545-001   VELOCITY, U =  -0,000461386 FT/SEC  2,41+000  1,69+000
R=  3,0 IN,   ANGLE=  34,00 DEGREES   FUNCTION= -9,4587-001   VELOCITY, U =  -0,000601570 FT/SEC  2,43+000  1,64+000
R=  3,0 IN,   ANGLE=  33,00 DEGREES   FUNCTION= -1,1594+000   VELOCITY, U =  -0,000737377 FT/SEC  2,44+000  1,59+000
R=  3,0 IN,   ANGLE=  32,00 DEGREES   FUNCTION= -1,3659+000   VELOCITY, U =  -0,000868729 FT/SEC  2,46+000  1,54+000
R=  3,0 IN,   ANGLE=  31,00 DEGREES   FUNCTION= -1,5654+000   VELOCITY, U =  -0,000995567 FT/SEC  2,47+000  1,48+000
R=  3,0 IN,   ANGLE=  30,00 DEGREES   FUNCTION= -1,7517+000   VELOCITY, U =  -0,001114062 FT/SEC  2,48+000  1,43+000
R=  3,0 IN,   ANGLE=  29,00 DEGREES   FUNCTION= -1,9427+000   VELOCITY, U =  -0,001235552 FT/SEC  2,50+000  1,38+000
R=  3,0 IN,   ANGLE=  28,00 DEGREES   FUNCTION= -2,1205+000   VELOCITY, U =  -0,001348664 FT/SEC  2,51+000  1,33+000
R=  3,0 IN,   ANGLE=  27,00 DEGREES   FUNCTION= -2,2912+000   VELOCITY, U =  -0,001457187 FT/SEC  2,52+000  1,28+000
```

R=	3.0 IN.	ANGLE=	26.00 DEGREES	FUNCTION=	-2.4546+000	VELOCITY, U =	-0.001561139	FT/SEC	2.53+000	1.23+000
R=	3.0 IN.	ANGLE=	25.00 DEGREES	FUNCTION=	-2.6109+000	VELOCITY, U =	-0.001660540	FT/SEC	2.54+000	1.18+000
R=	3.0 IN.	ANGLE=	24.00 DEGREES	FUNCTION=	-2.7601+000	VELOCITY, U =	-0.001755423	FT/SEC	2.55+000	1.14+000
R=	3.0 IN.	ANGLE=	23.00 DEGREES	FUNCTION=	-2.9022+000	VELOCITY, U =	-0.001845825	FT/SEC	2.56+000	1.09+000
R=	3.0 IN.	ANGLE=	22.00 DEGREES	FUNCTION=	-3.0374+000	VELOCITY, U =	-0.001931782	FT/SEC	2.57+000	1.04+000
R=	3.0 IN.	ANGLE=	21.00 DEGREES	FUNCTION=	-3.1655+000	VELOCITY, U =	-0.002013251	FT/SEC	2.58+000	9.90-001
R=	3.0 IN.	ANGLE=	20.00 DEGREES	FUNCTION=	-3.2871+000	VELOCITY, U =	-0.002090623	FT/SEC	2.59+000	9.41-001
R=	3.0 IN.	ANGLE=	19.00 DEGREES	FUNCTION=	-3.4019+000	VELOCITY, U =	-0.002163609	FT/SEC	2.59+000	8.93-001
R=	3.0 IN.	ANGLE=	18.00 DEGREES	FUNCTION=	-3.5100+000	VELOCITY, U =	-0.002232385	FT/SEC	2.60+000	8.45-001
R=	3.0 IN.	ANGLE=	17.00 DEGREES	FUNCTION=	-3.6117+000	VELOCITY, U =	-0.002297010	FT/SEC	2.61+000	7.98-001
R=	3.0 IN.	ANGLE=	16.00 DEGREES	FUNCTION=	-3.7068+000	VELOCITY, U =	-0.002357546	FT/SEC	2.62+000	7.50-001
R=	3.0 IN.	ANGLE=	15.00 DEGREES	FUNCTION=	-3.7957+000	VELOCITY, U =	-0.002414054	FT/SEC	2.62+000	7.02-001
R=	3.0 IN.	ANGLE=	14.00 DEGREES	FUNCTION=	-3.8783+000	VELOCITY, U =	-0.002466594	FT/SEC	2.63+000	6.55-001
R=	3.0 IN.	ANGLE=	13.00 DEGREES	FUNCTION=	-3.9548+000	VELOCITY, U =	-0.002515225	FT/SEC	2.63+000	6.08-001
R=	3.0 IN.	ANGLE=	12.00 DEGREES	FUNCTION=	-4.0252+000	VELOCITY, U =	-0.002560004	FT/SEC	2.64+000	5.61-001
R=	3.0 IN.	ANGLE=	11.00 DEGREES	FUNCTION=	-4.0896+000	VELOCITY, U =	-0.002600984	FT/SEC	2.64+000	5.14-001
R=	3.0 IN.	ANGLE=	10.00 DEGREES	FUNCTION=	-4.1481+000	VELOCITY, U =	-0.002638217	FT/SEC	2.65+000	4.67-001
R=	3.0 IN.	ANGLE=	9.00 DEGREES	FUNCTION=	-4.2009+000	VELOCITY, U =	-0.002671748	FT/SEC	2.65+000	4.20-001
R=	3.0 IN.	ANGLE=	8.00 DEGREES	FUNCTION=	-4.2478+000	VELOCITY, U =	-0.002701628	FT/SEC	2.65+000	3.73-001
R=	3.0 IN.	ANGLE=	7.00 DEGREES	FUNCTION=	-4.2891+000	VELOCITY, U =	-0.002727894	FT/SEC	2.66+000	3.26-001
R=	3.0 IN.	ANGLE=	6.00 DEGREES	FUNCTION=	-4.3248+000	VELOCITY, U =	-0.002750585	FT/SEC	2.66+000	2.80-001
R=	3.0 IN.	ANGLE=	5.00 DEGREES	FUNCTION=	-4.3549+000	VELOCITY, U =	-0.002769728	FT/SEC	2.66+000	2.33-001
R=	3.0 IN.	ANGLE=	4.00 DEGREES	FUNCTION=	-4.3795+000	VELOCITY, U =	-0.002785352	FT/SEC	2.66+000	1.86-001
R=	3.0 IN.	ANGLE=	3.00 DEGREES	FUNCTION=	-4.3986+000	VELOCITY, U =	-0.002797481	FT/SEC	2.66+000	1.40-001
R=	3.0 IN.	ANGLE=	2.00 DEGREES	FUNCTION=	-4.4122+000	VELOCITY, U =	-0.002806131	FT/SEC	2.67+000	9.31-002
R=	3.0 IN.	ANGLE=	1.00 DEGREES	FUNCTION=	-4.4203+000	VELOCITY, U =	-0.002811316	FT/SEC	2.67+000	4.65-002
R=	3.0 IN.	ANGLE=	0.00 DEGREES	FUNCTION=	-4.4230+000	VELOCITY, U =	-0.002813044	FT/SEC	2.67+000	0.00+000

SUBROUTINE PRO N1

for N1 velocity profiles

(Subroutine PRO N3 for N3 velocity profiles is not included as it is very similar to PRO N1)

```
      SUBROUTINE PRO N1 (AL,EPS,EK,EM)
      C1(A)=A*3.1415926536/180.
      C2(A)=A*180./3.1415926536
      Z=1.59E-4
   13 ALR=C1(AL)
      UM=0.
      R=1.
      AR=ALR
   14 F=-5.*EK*EK/EM*((SN(AR/SQRT(EM),EK))**2+((EM-1.)/EK/EK-1.)/3.)
      U=ABS(Z/R*F*12.)
      IF(U.GT.UM) UM=U
      AR=AR-.01
      IF(AR.GT.-ALR) GO TO 14
      CALL CHAR(3,4,-2,1,13HSCALE 1 INCH=,13,0.,.24,.16)
      UMM=UM*2.
      ENCODE(8,11,JJ)UMM
   11 FORMAT(F8.5)
      CALL CHAR(3,4,.5,JJ,8)
      CALL CHAR(3,4,2,1,7H FT/SEC,7)
      CALL PLOT(0.,-2.,2)
      DO 8 K=1,5
      X=K-1.375
      CALL PLOT (0.,X,1)
      CALL PLOT (0.,X+.125,2)
      CALL PLOT (0.,X+.25,1)
    8 CALL PLOT (0.,X+.375,2)
      X=3.5*COS(ALR)
      Y=3.5*SIN(ALR)
      CALL PLOT(Y,X,2)
      CALL PLOT( 0.,0.,1)
      CALL PLOT( -Y,X,1)
      CALL CELLI(A,EK,FK,EE)
      A=(1.-(EM-1.)/EK/EK)/3.
      A=ASIN(SQRT(A))
      CALL ELLI(A,EK,F,E)
      A1=SQRT(EM)*F
      A2=SQRT(EM)*2.*FK-A1
      AL1=C2(A1)
      AL2=C2(A2)
      PRINT2,AL,EPS,EK,EM,Z,AL1,AL2
    2 FORMAT(*1NON-SYMMETRICAL *
     1         *VELOCITY PROFILE FOR FLOW BETWEEN NONPARALLEL PLANE WALLS
     1 WITH 3 REAL ROOTS.*/* ANGLE OF WALLS = *F9.5/* EPSILON = *F9.5/*
     2 K = *F9.5/* M = *F9.5* */* KINEMATIC VISCOSITY = *E11.4* FT/SEC2*
     3/* AL1=*F11.5*  AL2=*F11.5 )
      DO3 I=1,3,2
      R=I
      IF(I.EQ.1) R=1.2
      XX=R*COS(ALR)
      YY=R*SIN(ALR)
      CALL PLOT (-YY,XX,2)
      M=AL
      N=2*M+1
      DO3K=1,N
      T=-M+K-1
      T=C1(T)
```

```
      X=R*COS(T)
      Y=R*SIN(T)
    9 CALL PLOT (Y,X,1)
      CALL PLOT (YY,XX,1)
      A=AL
      AP=AL
      J=1
    6 AR=C1(A)
      AS=C2(AP)
      F=-5.*EK*EK/EM*((SN(AR/SQRT(EM),EK))**2+((EM-1.)/EK/EK-1.)/3.)
      U=Z/R*F*12.
            X=(R+U/UM)*COS(AS)
      Y=(R+U/UM)*SIN(AS)
C     IF(J/5*5.EQ.J) PRINT4,R,A,F,U,X,Y
      CALL PLOT (Y,X,1)
      IF(A.LE.-AL2) GO TO 3
      A=A-.5
      AP=AP-.5
      IF(A.LT.-AL2) A=-AL2
      IF(A.LT.-AL2)AP=-AL
      J=J+1
      GO TO 5
    3 CONTINUE
    4 FORMAT(* R= *F4.1* IN.   ANGLE= *F7.2* DEGREES   FUNCTION= *E11.4
     1*   VELOCITY, U = *E11.4* FT/SEC*,2E10.2)
   12 END
```

```
NON-SYMMETRICAL VELOCITY PROFILE FOR FLOW BETWEEN NONPARALLEL PLANE WALLS WITH 3 REAL ROOTS.
ANGLE OF WALLS =  69.70000
EPSILON =  -5.50000
 K =   0.98100
M =   0.04000
KINEMATIC VISCOSITY =  1,5900-004 FT/SEC2
AL1=   12,98184  AL2=   56,01910   AL3=      82,7818
R=  1,2 IN.   ANGLE=   54.82 DEGREES   FUNCTION= -1,2787+001   VELOCITY, U = -2,0332-002 FT/SEC  4,13-001  1,01+000
R=  1,2 IN.   ANGLE=   52.32 DEGREES   FUNCTION= -2,4910+001   VELOCITY, U = -3,9608-002 FT/SEC  4,13-001  8,93-001
R=  1,2 IN.   ANGLE=   49.82 DEGREES   FUNCTION= -3,3454+001   VELOCITY, U = -5,3193-002 FT/SEC  4,17-001  8,09-001
R=  1,2 IN.   ANGLE=   47.32 DEGREES   FUNCTION= -3,9284+001   VELOCITY, U = -6,2462-002 FT/SEC  4,27-001  7,46-001
R=  1,2 IN.   ANGLE=   44.82 DEGREES   FUNCTION= -4,3157+001   VELOCITY, U = -6,8619-002 FT/SEC  4,41-001  6,98-001
R=  1,2 IN.   ANGLE=   42.32 DEGREES   FUNCTION= -4,5653+001   VELOCITY, U = -7,2589-002 FT/SEC  4,59-001  6,60-001
R=  1,2 IN.   ANGLE=   39.82 DEGREES   FUNCTION= -4,7178+001   VELOCITY, U = -7,5013-002 FT/SEC  4,79-001  6,29-001
R=  1,2 IN.   ANGLE=   37.32 DEGREES   FUNCTION= -4,7991+001   VELOCITY, U = -7,6305-002 FT/SEC  5,02-001  6,02-001
R=  1,2 IN.   ANGLE=   34.82 DEGREES   FUNCTION= -4,8235+001   VELOCITY, U = -7,6694-002 FT/SEC  5,26-001  5,78-001
R=  1,2 IN.   ANGLE=   32.32 DEGREES   FUNCTION= -4,7956+001   VELOCITY, U = -7,6249-002 FT/SEC  5,53-001  5,56-001
R=  1,2 IN.   ANGLE=   29.82 DEGREES   FUNCTION= -4,7102+001   VELOCITY, U = -7,4892-002 FT/SEC  5,82-001  5,37-001
R=  1,2 IN.   ANGLE=   27.32 DEGREES   FUNCTION= -4,5523+001   VELOCITY, U = -7,2382-002 FT/SEC  6,15-001  5,20-001
R=  1,2 IN.   ANGLE=   24.82 DEGREES   FUNCTION= -4,2951+001   VELOCITY, U = -6,8291-002 FT/SEC  6,55-001  5,06-001
R=  1,2 IN.   ANGLE=   22.32 DEGREES   FUNCTION= -3,8968+001   VELOCITY, U = -6,1959-002 FT/SEC  7,04-001  4,97-001
R=  1,2 IN.   ANGLE=   19.82 DEGREES   FUNCTION= -3,2987+001   VELOCITY, U = -5,2449-002 FT/SEC  7,69-001  4,94-001
R=  1,2 IN.   ANGLE=   17.32 DEGREES   FUNCTION= -2,4235+001   VELOCITY, U = -3,8534-002 FT/SEC  8,56-001  4,98-001
R=  1,2 IN.   ANGLE=   14.82 DEGREES   FUNCTION= -1,1849+001   VELOCITY, U = -1,8839-002 FT/SEC  9,72-001  5,10-001
R=  1,2 IN.   ANGLE=   12.32 DEGREES   FUNCTION=  4,8832+000   VELOCITY, U =  7,7723-003 FT/SEC  1,12+000  5,29-001
R=  1,2 IN.   ANGLE=    9.82 DEGREES   FUNCTION=  2,5998+001   VELOCITY, U =  4,1336-002 FT/SEC  1,32+000  5,50-001
R=  1,2 IN.   ANGLE=    7.32 DEGREES   FUNCTION=  5,0008+001   VELOCITY, U =  7,9513-002 FT/SEC  1,53+000  5,64-001
R=  1,2 IN.   ANGLE=    4.82 DEGREES   FUNCTION=  7,3283+001   VELOCITY, U =  1,1652-001 FT/SEC  1,75+000  5,58-001
R=  1,2 IN.   ANGLE=    2.32 DEGREES   FUNCTION=  9,0365+001   VELOCITY, U =  1,4368-001 FT/SEC  1,91+000  5,20-001
R=  1,2 IN.   ANGLE=   -0.18 DEGREES   FUNCTION=  9,6082+001   VELOCITY, U =  1,5277-001 FT/SEC  1,98+000  4,47-001
R=  1,2 IN.   ANGLE=   -2.68 DEGREES   FUNCTION=  8,8486+001   VELOCITY, U =  1,4069-001 FT/SEC  1,94+000  3,48-001
R=  1,2 IN.   ANGLE=   -5.18 DEGREES   FUNCTION=  7,0141+001   VELOCITY, U =  1,1152-001 FT/SEC  1,79+000  2,42-001
R=  1,2 IN.   ANGLE=   -7.68 DEGREES   FUNCTION=  4,6449+001   VELOCITY, U =  7,3853-002 FT/SEC  1,60+000  1,45-001
R=  1,2 IN.   ANGLE=  -10.18 DEGREES   FUNCTION=  2,2689+001   VELOCITY, U =  3,6075-002 FT/SEC  1,40+000  6,58-002
R=  1,2 IN.   ANGLE=  -12.68 DEGREES   FUNCTION=  2,1691+000   VELOCITY, U =  3,4489-003 FT/SEC  1,22+000  4,25-003
R=  1,2 IN.   ANGLE=  -15.18 DEGREES   FUNCTION= -1,3909+001   VELOCITY, U = -2,2115-002 FT/SEC  1,08+000 -4,33-002
R=  1,2 IN.   ANGLE=  -17.68 DEGREES   FUNCTION= -2,5714+001   VELOCITY, U = -4,0886-002 FT/SEC  9,74-001 -8,18-002
R=  1,2 IN.   ANGLE=  -20.18 DEGREES   FUNCTION= -3,4010+001   VELOCITY, U = -5,4076-002 FT/SEC  8,98-001 -1,15-001
R=  1,2 IN.   ANGLE=  -22.68 DEGREES   FUNCTION= -3,9657+001   VELOCITY, U = -6,3055-002 FT/SEC  8,44-001 -1,46-001
R=  1,2 IN.   ANGLE=  -25.18 DEGREES   FUNCTION= -4,3401+001   VELOCITY, U = -6,9007-002 FT/SEC  8,05-001 -1,75-001
R=  1,2 IN.   ANGLE=  -27.68 DEGREES   FUNCTION= -4,5806+001   VELOCITY, U = -7,2832-002 FT/SEC  7,76-001 -2,05-001
R=  1,2 IN.   ANGLE=  -30.18 DEGREES   FUNCTION= -4,7267+001   VELOCITY, U = -7,5154-002 FT/SEC  7,54-001 -2,35-001
R=  1,2 IN.   ANGLE=  -32.68 DEGREES   FUNCTION= -4,8030+001   VELOCITY, U = -7,6368-002 FT/SEC  7,37-001 -2,65-001
R=  1,2 IN.   ANGLE=  -35.18 DEGREES   FUNCTION= -4,8232+001   VELOCITY, U = -7,6689-002 FT/SEC  7,23-001 -2,97-001
R=  1,2 IN.   ANGLE=  -37.68 DEGREES   FUNCTION= -4,7909+001   VELOCITY, U = -7,6176-002 FT/SEC  7,12-001 -3,29-001
R=  1,2 IN.   ANGLE=  -40.18 DEGREES   FUNCTION= -4,7005+001   VELOCITY, U = -7,4738-002 FT/SEC  7,04-001 -3,63-001
R=  1,2 IN.   ANGLE=  -42.68 DEGREES   FUNCTION= -4,5359+001   VELOCITY, U = -7,2121-002 FT/SEC  7,00-001 -4,01-001
R=  1,2 IN.   ANGLE=  -45.18 DEGREES   FUNCTION= -4,2691+001   VELOCITY, U = -6,7878-002 FT/SEC  7,01-001 -4,43-001
R=  1,2 IN.   ANGLE=  -47.68 DEGREES   FUNCTION= -3,8572+001   VELOCITY, U = -6,1329-002 FT/SEC  7,11-001 -4,94-001
R=  1,2 IN.   ANGLE=  -50.18 DEGREES   FUNCTION= -3,2400+001   VELOCITY, U = -5,1516-002 FT/SEC  7,31-001 -5,57-001
R=  1,2 IN.   ANGLE=  -52.68 DEGREES   FUNCTION= -2,3392+001   VELOCITY, U = -3,7193-002 FT/SEC  7,66-001 -6,38-001
R=  1,2 IN.   ANGLE=  -55.18 DEGREES   FUNCTION= -1,0676+001   VELOCITY, U = -1,6975-002 FT/SEC  8,19-001 -7,45-001
R=  1,2 IN.   ANGLE=  -57.68 DEGREES   FUNCTION=  6,4241+000   VELOCITY, U =  1,0214-002 FT/SEC  8,91-001 -8,85-001
R=  1,2 IN.   ANGLE=  -60.18 DEGREES   FUNCTION=  2,7845+001   VELOCITY, U =  4,4274-002 FT/SEC  9,78-001 -1,06+000
R=  1,2 IN.   ANGLE=  -62.68 DEGREES   FUNCTION=  5,1962+001   VELOCITY, U =  8,2619-002 FT/SEC  1,07+000 -1,26+000
R=  1,2 IN.   ANGLE=  -65.18 DEGREES   FUNCTION=  7,4956+001   VELOCITY, U =  1,1918-001 FT/SEC  1,13+000 -1,46+000
R=  1,2 IN.   ANGLE=  -67.68 DEGREES   FUNCTION=  9,1295+001   VELOCITY, U =  1,4516-001 FT/SEC  1,15+000 -1,63+000
R=  1,2 IN.   ANGLE=  -70.18 DEGREES   FUNCTION=  9,5958+001   VELOCITY, U =  1,5257-001 FT/SEC  1,10+000 -1,71+000
R=  1,2 IN.   ANGLE=  -72.68 DEGREES   FUNCTION=  8,7353+001   VELOCITY, U =  1,3889-001 FT/SEC  9,85-001 -1,69+000
R=  1,2 IN.   ANGLE=  -75.18 DEGREES   FUNCTION=  6,6363+001   VELOCITY, U =  1,0870-001 FT/SEC  8,33-001 -1,59+000
R=  1,2 IN.   ANGLE=  -77.68 DEGREES   FUNCTION=  [illegible]  VELOCITY, U =  7,0741-002 FT/SEC  6,75-001 -1,43+000
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R= 1.2 IN.	ANGLE= -80.18 DEGREES	FUNCTION= 2.0901+001	VELOCITY, U = 3.3232-002 FT/SEC	5.33-001	-1.27+000
R= 1.2 IN.	ANGLE= -82.68 DEGREES	FUNCTION= 7.1490-001	VELOCITY, U = 1.1367-003 FT/SEC	4.16-001	-1.13+000
R= 3.0 IN.	ANGLE= 54.82 DEGREES	FUNCTION= -1.2787+001	VELOCITY, U = -8.1328-003 FT/SEC	1.12+000	2.73+000
R= 3.0 IN.	ANGLE= 52.32 DEGREES	FUNCTION= -2.4910+001	VELOCITY, U = -1.5843-002 FT/SEC	1.22+000	2.64+000
R= 3.0 IN.	ANGLE= 49.82 DEGREES	FUNCTION= -3.3454+001	VELOCITY, U = -2.1277-002 FT/SEC	1.32+000	2.56+000
R= 3.0 IN.	ANGLE= 47.32 DEGREES	FUNCTION= -3.9284+001	VELOCITY, U = -2.4985-002 FT/SEC	1.42+000	2.49+000
R= 3.0 IN.	ANGLE= 44.82 DEGREES	FUNCTION= -4.3157+001	VELOCITY, U = -2.7448-002 FT/SEC	1.52+000	2.41+000
R= 3.0 IN.	ANGLE= 42.32 DEGREES	FUNCTION= -4.5653+001	VELOCITY, U = -2.9035-002 FT/SEC	1.62+000	2.33+000
R= 3.0 IN.	ANGLE= 39.82 DEGREES	FUNCTION= -4.7178+001	VELOCITY, U = -3.0005-002 FT/SEC	1.72+000	2.26+000
R= 3.0 IN.	ANGLE= 37.32 DEGREES	FUNCTION= -4.7991+001	VELOCITY, U = -3.0522-002 FT/SEC	1.81+000	2.18+000
R= 3.0 IN.	ANGLE= 34.82 DEGREES	FUNCTION= -4.8233+001	VELOCITY, U = -3.0678-002 FT/SEC	1.91+000	2.10+000
R= 3.0 IN.	ANGLE= 32.32 DEGREES	FUNCTION= -4.7956+001	VELOCITY, U = -3.0500-002 FT/SEC	2.00+000	2.01+000
R= 3.0 IN.	ANGLE= 29.82 DEGREES	FUNCTION= -4.7102+001	VELOCITY, U = -2.9957-002 FT/SEC	2.08+000	1.92+000
R= 3.0 IN.	ANGLE= 27.32 DEGREES	FUNCTION= -4.5523+001	VELOCITY, U = -2.8953-002 FT/SEC	2.17+000	1.83+000
R= 3.0 IN.	ANGLE= 24.82 DEGREES	FUNCTION= -4.2951+001	VELOCITY, U = -2.7317-002 FT/SEC	2.26+000	1.74+000
R= 3.0 IN.	ANGLE= 22.32 DEGREES	FUNCTION= -3.8963+001	VELOCITY, U = -2.4784-002 FT/SEC	2.34+000	1.65+000
R= 3.0 IN.	ANGLE= 19.82 DEGREES	FUNCTION= -3.2987+001	VELOCITY, U = -2.0980-002 FT/SEC	2.43+000	1.56+000
R= 3.0 IN.	ANGLE= 17.32 DEGREES	FUNCTION= -2.4235+001	VELOCITY, U = -1.5414-002 FT/SEC	2.52+000	1.47+000
R= 3.0 IN.	ANGLE= 14.82 DEGREES	FUNCTION= -1.1849+001	VELOCITY, U = -7.5357-003 FT/SEC	2.62+000	1.38+000
R= 3.0 IN.	ANGLE= 12.32 DEGREES	FUNCTION= 4.8882+000	VELOCITY, U = 3.1089-003 FT/SEC	2.73+000	1.28+000
R= 3.0 IN.	ANGLE= 9.82 DEGREES	FUNCTION= 2.5999+001	VELOCITY, U = 1.6534-002 FT/SEC	2.85+000	1.19+000
R= 3.0 IN.	ANGLE= 7.32 DEGREES	FUNCTION= 5.0003+001	VELOCITY, U = 3.1805-002 FT/SEC	2.98+000	1.10+000
R= 3.0 IN.	ANGLE= 4.82 DEGREES	FUNCTION= 7.3263+001	VELOCITY, U = 4.6608-002 FT/SEC	3.10+000	9.89-001
R= 3.0 IN.	ANGLE= 2.32 DEGREES	FUNCTION= 9.0365+001	VELOCITY, U = 5.7472-002 FT/SEC	3.20+000	8.69-001
R= 3.0 IN.	ANGLE= -0.18 DEGREES	FUNCTION= 9.6082+001	VELOCITY, U = 6.1108-002 FT/SEC	3.25+000	7.33-001
R= 3.0 IN.	ANGLE= -2.68 DEGREES	FUNCTION= 8.8486+001	VELOCITY, U = 5.6277-002 FT/SEC	3.25+000	5.86-001
R= 3.0 IN.	ANGLE= -5.18 DEGREES	FUNCTION= 7.0141+001	VELOCITY, U = 4.4610-002 FT/SEC	3.21+000	4.35-001
R= 3.0 IN.	ANGLE= -7.68 DEGREES	FUNCTION= 4.6449+001	VELOCITY, U = 2.9541-002 FT/SEC	3.15+000	2.86-001
R= 3.0 IN.	ANGLE= -10.18 DEGREES	FUNCTION= 2.2689+001	VELOCITY, U = 1.4430-002 FT/SEC	3.08+000	1.45-001
R= 3.0 IN.	ANGLE= -12.68 DEGREES	FUNCTION= 2.1691+000	VELOCITY, U = 1.3796-003 FT/SEC	3.01+000	1.05-002
R= 3.0 IN.	ANGLE= -15.18 DEGREES	FUNCTION= -1.3909+001	VELOCITY, U = -8.8461-003 FT/SEC	2.95+000	-1.18-001
R= 3.0 IN.	ANGLE= -17.68 DEGREES	FUNCTION= -2.5714+001	VELOCITY, U = -1.6354-002 FT/SEC	2.90+000	-2.44-001
R= 3.0 IN.	ANGLE= -20.18 DEGREES	FUNCTION= -3.4010+001	VELOCITY, U = -2.1630-002 FT/SEC	2.86+000	-3.66-001
R= 3.0 IN.	ANGLE= -22.68 DEGREES	FUNCTION= -3.9657+001	VELOCITY, U = -2.5222-002 FT/SEC	2.82+000	-4.87-001
R= 3.0 IN.	ANGLE= -25.18 DEGREES	FUNCTION= -4.3401+001	VELOCITY, U = -2.7603-002 FT/SEC	2.78+000	-6.07-001
R= 3.0 IN.	ANGLE= -27.68 DEGREES	FUNCTION= -4.5806+001	VELOCITY, U = -2.9133-002 FT/SEC	2.75+000	-7.26-001
R= 3.0 IN.	ANGLE= -30.18 DEGREES	FUNCTION= -4.7267+001	VELOCITY, U = -3.0062-002 FT/SEC	2.71+000	-8.43-001
R= 3.0 IN.	ANGLE= -32.68 DEGREES	FUNCTION= -4.8030+001	VELOCITY, U = -3.0547-002 FT/SEC	2.67+000	-9.60-001
R= 3.0 IN.	ANGLE= -35.18 DEGREES	FUNCTION= -4.8232+001	VELOCITY, U = -3.0676-002 FT/SEC	2.62+000	-1.07+000
R= 3.0 IN.	ANGLE= -37.68 DEGREES	FUNCTION= -4.7909+001	VELOCITY, U = -3.0470-002 FT/SEC	2.57+000	-1.19+000
R= 3.0 IN.	ANGLE= -40.18 DEGREES	FUNCTION= -4.7005+001	VELOCITY, U = -2.9895-002 FT/SEC	2.52+000	-1.30+000
R= 3.0 IN.	ANGLE= -42.68 DEGREES	FUNCTION= -4.5359+001	VELOCITY, U = -2.8848-002 FT/SEC	2.47+000	-1.41+000
R= 3.0 IN.	ANGLE= -45.18 DEGREES	FUNCTION= -4.2691+001	VELOCITY, U = -2.7151-002 FT/SEC	2.41+000	-1.52+000
R= 3.0 IN.	ANGLE= -47.68 DEGREES	FUNCTION= -3.8572+001	VELOCITY, U = -2.4532-002 FT/SEC	2.35+000	-1.64+000
R= 3.0 IN.	ANGLE= -50.18 DEGREES	FUNCTION= -3.2400+001	VELOCITY, U = -2.0606-002 FT/SEC	2.30+000	-1.75+000
R= 3.0 IN.	ANGLE= -52.68 DEGREES	FUNCTION= -2.3392+001	VELOCITY, U = -1.4877-002 FT/SEC	2.24+000	-1.87+000
R= 3.0 IN.	ANGLE= -55.18 DEGREES	FUNCTION= -1.0676+001	VELOCITY, U = -6.7899-003 FT/SEC	2.19+000	-1.99+000
R= 3.0 IN.	ANGLE= -57.68 DEGREES	FUNCTION= 6.4241+000	VELOCITY, U = 4.0857-003 FT/SEC	2.14+000	-2.13+000
R= 3.0 IN.	ANGLE= -60.18 DEGREES	FUNCTION= 2.7845+001	VELOCITY, U = 1.7710-002 FT/SEC	2.10+000	-2.28+000
R= 3.0 IN.	ANGLE= -62.68 DEGREES	FUNCTION= 5.1962+001	VELOCITY, U = 3.3048-002 FT/SEC	2.05+000	-2.43+000
R= 3.0 IN.	ANGLE= -65.18 DEGREES	FUNCTION= 7.4956+001	VELOCITY, U = 4.7672-002 FT/SEC	1.99+000	-2.58+000
R= 3.0 IN.	ANGLE= -67.68 DEGREES	FUNCTION= 9.1295+001	VELOCITY, U = 5.8063-002 FT/SEC	1.91+000	-2.71+000
R= 3.0 IN.	ANGLE= -70.18 DEGREES	FUNCTION= 9.5958+001	VELOCITY, U = 6.1029-002 FT/SEC	1.80+000	-2.80+000
R= 3.0 IN.	ANGLE= -72.68 DEGREES	FUNCTION= 8.7353+001	VELOCITY, U = 5.5556-002 FT/SEC	1.66+000	-2.85+000
R= 3.0 IN.	ANGLE= -75.18 DEGREES	FUNCTION= 6.8363+001	VELOCITY, U = 4.3479-002 FT/SEC	1.50+000	-2.87+000
R= 3.0 IN.	ANGLE= -77.68 DEGREES	FUNCTION= 4.4491+001	VELOCITY, U = 2.8296-002 FT/SEC	1.34+000	-2.85+000
R= 3.0 IN.	ANGLE= -80.18 DEGREES	FUNCTION= 2.0901+001	VELOCITY, U = 1.3293-002 FT/SEC	1.19+000	-2.83+000
R= 3.0 IN.	ANGLE= -82.68 DEGREES	FUNCTION= 7.1490-001	VELOCITY, U = 4.5467-004 FT/SEC	1.04+000	-2.82+000

APPENDIX C

SUBROUTINE PRO CON

for SC velocity profiles near α_c

(For comparing to the modified perturbation method)

```
      SUBROUTINE PRO CON (AL,EPS,EK,EM)
      DIMENSION XB(400),YB(400)
      C1(A)=A*3.1415926536/180.
      Z=1.59E-4
   13 ALR=C1(AL)
      R=1.
      F1=0.
      A =0.
   15 AR=C1(A)
      F=2.*(EM*EM*(EK*EK-2.)-1.)+6.*EM*EM*(1.-EK*EK)/(DN(AR*EM,EK))**2
      IF(ABS(F).GT.ABS(F1)) F1=F
      A =A +1.
      IF(A .GT.AL) GO TO 4
      GO TO15
    4 UM=ABS(Z/R*F1*12.)
      CALL CHAR(3.4,-2.1,13HSCALE 1 INCH=,13,0.,.24,.16)
      UMM=UM*2.
      ENCODE(8,11,JJ)UMM
   11 FORMAT(F8.5)
      CALL CHAR(3.4,.5,JJ,8)
      CALL CHAR(3.4,2.1,7H FT/SEC,7)
      CALL PLOT(0.,-2.,2)
      DO 8 K=1,5
      X=K-1.375
      CALL PLOT (0.,X,1)
      CALL PLOT (0.,X+.125,2)
      CALL PLOT (0.,X+.25,1)
    8 CALL PLOT (0.,X+.375,2)
      X=3.5*COS(ALR)
      Y=3.5*SIN(ALR)
      CALL PLOT(Y,X,2)
      CALL PLOT( 0.,0.,1)
      CALL PLOT( -Y,X,1)
      PRINT2,AL,EPS,EK,EM,Z
    2 FORMAT(*1VELCCITY PROFILE FOR *
     1*CONVERGING *
     1                                        *FLOW BETWEEN NONPARALLEL PLANE WALLS
     1 WITH 3 REAL ROOTS.*/* ANGLE OF WALLS = *F9.5/* EPSILON = *F9.5/*
     2 K = *F9.5/* M = *F9.5* */* KINEMATIC VISCOSITY = *E11.4* FT/SEC2*
     3)
      DO3 I=1,3,2
      R=I
      IF(I.EQ.1) R=1.2
      XX=R*COS(ALR)
      YY=R*SIN(ALR)
      CALL PLOT (-YY,XX,2)
      M=A_I
      N=2*M+1
      DO9K=1,N
      T=-M+K-1
      T=C1(T)
      X=R*COS(T)
      Y=R*SIN(T)
    9 CALL PLOT (Y,X,1)
      CALL PLOT (YY,XX,1)
      A=A_I
```

```
      J=1
    6 AR=A*3.1415926536/180.
      F=2.*(EM*EM*(EK*EK-2.)-1.)+6.*EM*EM*(1.-EK*EK)/(DN(AR*EM,EK))**2
      U=Z/R*F*12.
      XB(J)=X=(R+U/UM)*COS(AR)
      Y=(R+U/UM)*SIN(AR)
                  PRINT4,R,A,F,U,X,Y
      YB(J)=-Y
      CALL PLOT (Y,X,1)
      IF(A-.5.LT.-0.) GO TO 5
      IF(J.GT.1) GC TO 14
      NN=A
      A=NN
   14 A=A-1.
      J=J+1
      GO TO 5
    5 DO 10 K=1,J
      N=J-K+1
   10 CALL PLOT(YB(N),XB(N),1)
    3 CONTINUE
    4 FORMAT(* R= *F4.1* IN.   ANGLE= *F6.2* DEGREES   FUNCTION= *E11.4
     1*   VELOCITY, U = *F13.9* FT/SEC*,2E10.2)
   12 END
```

```
VELOCITY PROFILE FOR CONVERGING FLOW BETWEEN NONPARALLEL PLANE WALLS WITH 3 REAL ROOTS,
ANGLE OF WALLS = 128,72714
EPSILON =  -0,01485
 K =    0,20000
M =    1,00282
KINEMATIC VISCOSITY =  1,5900-004 FT/SEC2
R=  1,2 IN,   ANGLE= 128,73 DEGREES   FUNCTION= -6,5193-009   VELOCITY, U =  -0,000000000 FT/SEC -7,51-001  9,36-001
R=  1,2 IN,   ANGLE= 127,00 DEGREES   FUNCTION=  6,9751-003   VELOCITY, U =   0,000011090 FT/SEC -7,46-001  9,89-001
R=  1,2 IN,   ANGLE= 126,00 DEGREES   FUNCTION=  1,0958-002   VELOCITY, U =   0,000017423 FT/SEC -7,41-001  1,02+000
R=  1,2 IN,   ANGLE= 125,00 DEGREES   FUNCTION=  1,4894-002   VELOCITY, U =   0,000023681 FT/SEC -7,36-001  1,05+000
R=  1,2 IN,   ANGLE= 124,00 DEGREES   FUNCTION=  1,8778-002   VELOCITY, U =   0,000029857 FT/SEC -7,30-001  1,08+000
R=  1,2 IN,   ANGLE= 123,00 DEGREES   FUNCTION=  2,2606-002   VELOCITY, U =   0,000035943 FT/SEC -7,22-001  1,11+000
R=  1,2 IN,   ANGLE= 122,00 DEGREES   FUNCTION=  2,6375-002   VELOCITY, U =   0,000041936 FT/SEC -7,14-001  1,14+000
R=  1,2 IN,   ANGLE= 121,00 DEGREES   FUNCTION=  3,0076-002   VELOCITY, U =   0,000047820 FT/SEC -7,04-001  1,17+000
R=  1,2 IN,   ANGLE= 120,00 DEGREES   FUNCTION=  3,3706-002   VELOCITY, U =   0,000053592 FT/SEC -6,94-001  1,20+000
R=  1,2 IN,   ANGLE= 119,00 DEGREES   FUNCTION=  3,7259-002   VELOCITY, U =   0,000059241 FT/SEC -6,82-001  1,23+000
R=  1,2 IN,   ANGLE= 118,00 DEGREES   FUNCTION=  4,0735-002   VELOCITY, U =   0,000064768 FT/SEC -6,70-001  1,26+000
R=  1,2 IN,   ANGLE= 117,00 DEGREES   FUNCTION=  4,4127-002   VELOCITY, U =   0,000070162 FT/SEC -6,56-001  1,29+000
R=  1,2 IN,   ANGLE= 116,00 DEGREES   FUNCTION=  4,7432-002   VELOCITY, U =   0,000075416 FT/SEC -6,42-001  1,32+000
R=  1,2 IN,   ANGLE= 115,00 DEGREES   FUNCTION=  5,0644-002   VELOCITY, U =   0,000080524 FT/SEC -6,26-001  1,34+000
R=  1,2 IN,   ANGLE= 114,00 DEGREES   FUNCTION=  5,3761-002   VELOCITY, U =   0,000085480 FT/SEC -6,10-001  1,37+000
R=  1,2 IN,   ANGLE= 113,00 DEGREES   FUNCTION=  5,6777-002   VELOCITY, U =   0,000090276 FT/SEC -5,92-001  1,40+000
R=  1,2 IN,   ANGLE= 112,00 DEGREES   FUNCTION=  5,9690-002   VELOCITY, U =   0,000094908 FT/SEC -5,74-001  1,42+000
R=  1,2 IN,   ANGLE= 111,00 DEGREES   FUNCTION=  6,2496-002   VELOCITY, U =   0,000099369 FT/SEC -5,55-001  1,45+000
R=  1,2 IN,   ANGLE= 110,00 DEGREES   FUNCTION=  6,5191-002   VELOCITY, U =   0,000103654 FT/SEC -5,35-001  1,47+000
R=  1,2 IN,   ANGLE= 109,00 DEGREES   FUNCTION=  6,7772-002   VELOCITY, U =   0,000107757 FT/SEC -5,14-001  1,49+000
R=  1,2 IN,   ANGLE= 108,00 DEGREES   FUNCTION=  7,0235-002   VELOCITY, U =   0,000111673 FT/SEC -4,92-001  1,51+000
R=  1,2 IN,   ANGLE= 107,00 DEGREES   FUNCTION=  7,2577-002   VELOCITY, U =   0,000115398 FT/SEC -4,69-001  1,53+000
R=  1,2 IN,   ANGLE= 106,00 DEGREES   FUNCTION=  7,4796-002   VELOCITY, U =   0,000118926 FT/SEC -4,46-001  1,55+000
R=  1,2 IN,   ANGLE= 105,00 DEGREES   FUNCTION=  7,6889-002   VELOCITY, U =   0,000122254 FT/SEC -4,21-001  1,57+000
R=  1,2 IN,   ANGLE= 104,00 DEGREES   FUNCTION=  7,8853-002   VELOCITY, U =   0,000125376 FT/SEC -3,97-001  1,59+000
R=  1,2 IN,   ANGLE= 103,00 DEGREES   FUNCTION=  8,0685-002   VELOCITY, U =   0,000128289 FT/SEC -3,71-001  1,61+000
R=  1,2 IN,   ANGLE= 102,00 DEGREES   FUNCTION=  8,2384-002   VELOCITY, U =   0,000130990 FT/SEC -3,45-001  1,62+000
R=  1,2 IN,   ANGLE= 101,00 DEGREES   FUNCTION=  8,3946-002   VELOCITY, U =   0,000133474 FT/SEC -3,18-001  1,64+000
R=  1,2 IN,   ANGLE= 100,00 DEGREES   FUNCTION=  8,5371-002   VELOCITY, U =   0,000135739 FT/SEC -2,91-001  1,65+000
R=  1,2 IN,   ANGLE=  99,00 DEGREES   FUNCTION=  8,6656-002   VELOCITY, U =   0,000137782 FT/SEC -2,63-001  1,66+000
R=  1,2 IN,   ANGLE=  98,00 DEGREES   FUNCTION=  8,7799-002   VELOCITY, U =   0,000139601 FT/SEC -2,35-001  1,67+000
R=  1,2 IN,   ANGLE=  97,00 DEGREES   FUNCTION=  8,8800-002   VELOCITY, U =   0,000141192 FT/SEC -2,07-001  1,68+000
R=  1,2 IN,   ANGLE=  96,00 DEGREES   FUNCTION=  8,9656-002   VELOCITY, U =   0,000142554 FT/SEC -1,78-001  1,69+000
R=  1,2 IN,   ANGLE=  95,00 DEGREES   FUNCTION=  9,0368-002   VELOCITY, U =   0,000143685 FT/SEC -1,48-001  1,70+000
R=  1,2 IN,   ANGLE=  94,00 DEGREES   FUNCTION=  9,0933-002   VELOCITY, U =   0,000144583 FT/SEC -1,19-001  1,70+000
R=  1,2 IN,   ANGLE=  93,00 DEGREES   FUNCTION=  9,1351-002   VELOCITY, U =   0,000145249 FT/SEC -8,94-002  1,71+000
R=  1,2 IN,   ANGLE=  92,00 DEGREES   FUNCTION=  9,1623-002   VELOCITY, U =   0,000145680 FT/SEC -5,97-002  1,71+000
R=  1,2 IN,   ANGLE=  91,00 DEGREES   FUNCTION=  9,1746-002   VELOCITY, U =   0,000145876 FT/SEC -2,99-002  1,71+000
R=  1,2 IN,   ANGLE=  90,00 DEGREES   FUNCTION=  9,1722-002   VELOCITY, U =   0,000145837 FT/SEC  4,39-012  1,71+000
R=  1,2 IN,   ANGLE=  89,00 DEGREES   FUNCTION=  9,1549-002   VELOCITY, U =   0,000145564 FT/SEC  2,98-002  1,71+000
R=  1,2 IN,   ANGLE=  88,00 DEGREES   FUNCTION=  9,1230-002   VELOCITY, U =   0,000145055 FT/SEC  5,96-002  1,71+000
R=  1,2 IN,   ANGLE=  87,00 DEGREES   FUNCTION=  9,0763-002   VELOCITY, U =   0,000144313 FT/SEC  8,93-002  1,70+000
R=  1,2 IN,   ANGLE=  86,00 DEGREES   FUNCTION=  9,0149-002   VELOCITY, U =   0,000143337 FT/SEC  1,19-001  1,70+000
R=  1,2 IN,   ANGLE=  85,00 DEGREES   FUNCTION=  8,9390-002   VELOCITY, U =   0,000142130 FT/SEC  1,48-001  1,69+000
R=  1,2 IN,   ANGLE=  84,00 DEGREES   FUNCTION=  8,8485-002   VELOCITY, U =   0,000140692 FT/SEC  1,77-001  1,68+000
R=  1,2 IN,   ANGLE=  83,00 DEGREES   FUNCTION=  8,7437-002   VELOCITY, U =   0,000139026 FT/SEC  2,06-001  1,67+000
R=  1,2 IN,   ANGLE=  82,00 DEGREES   FUNCTION=  8,6247-002   VELOCITY, U =   0,000137133 FT/SEC  2,34-001  1,66+000
R=  1,2 IN,   ANGLE=  81,00 DEGREES   FUNCTION=  8,4916-002   VELOCITY, U =   0,000135016 FT/SEC  2,62-001  1,65+000
R=  1,2 IN,   ANGLE=  80,00 DEGREES   FUNCTION=  8,3446-002   VELOCITY, U =   0,000132679 FT/SEC  2,89-001  1,64+000
R=  1,2 IN,   ANGLE=  79,00 DEGREES   FUNCTION=  8,1838-002   VELOCITY, U =   0,000130122 FT/SEC  3,16-001  1,63+000
R=  1,2 IN,   ANGLE=  78,00 DEGREES   FUNCTION=  8,0095-002   VELOCITY, U =   0,000127351 FT/SEC  3,42-001  1,61+000
R=  1,2 IN,   ANGLE=  77,00 DEGREES   FUNCTION=  7,8219-002   VELOCITY, U =   0,000124369 FT/SEC  3,68-001  1,59+000
R=  1,2 IN,   ANGLE=  76,00 DEGREES   FUNCTION=  7,6213-002   VELOCITY, U =   0,000121178 FT/SEC  3,93-001  1,58+000
R=  1,2 IN,   ANGLE=  75,00 DEGREES   FUNCTION=  7,4078-002   VELOCITY, U =   0,000117784 FT/SEC  4,17-001  1,56+000
R=  1,2 IN,   ANGLE=  74,00 DEGREES   FUNCTION=  7,1818-002   VELOCITY, U =   0,000114190 FT/SEC  4,41-001  1,54+000
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R= 1,2 IN, ANGLE= 73,00 DEGREES FUNCTION= 6,9435-002 VELOCITY, U = 0,000110402 FT/SEC 4,64-001 1,52+000
R= 1,2 IN, ANGLE= 72,00 DEGREES FUNCTION= 6,6933-002 VELOCITY, U = 0,000106423 FT/SEC 4,86-001 1,50+000
R= 1,2 IN, ANGLE= 71,00 DEGREES FUNCTION= 6,4314-002 VELOCITY, U = 0,000102259 FT/SEC 5,07-001 1,47+000
R= 1,2 IN, ANGLE= 70,00 DEGREES FUNCTION= 6,1582-002 VELOCITY, U = 0,000097916 FT/SEC 5,28-001 1,45+000
R= 1,2 IN, ANGLE= 69,00 DEGREES FUNCTION= 5,8741-002 VELOCITY, U = 0,000093398 FT/SEC 5,47-001 1,43+000
R= 1,2 IN, ANGLE= 68,00 DEGREES FUNCTION= 5,5793-002 VELOCITY, U = 0,000088711 FT/SEC 5,66-001 1,40+000
R= 1,2 IN, ANGLE= 67,00 DEGREES FUNCTION= 5,2743-002 VELOCITY, U = 0,000083862 FT/SEC 5,84-001 1,38+000
R= 1,2 IN, ANGLE= 66,00 DEGREES FUNCTION= 4,9594-002 VELOCITY, U = 0,000078855 FT/SEC 6,00-001 1,35+000
R= 1,2 IN, ANGLE= 65,00 DEGREES FUNCTION= 4,6351-002 VELOCITY, U = 0,000073698 FT/SEC 6,16-001 1,32+000
R= 1,2 IN, ANGLE= 64,00 DEGREES FUNCTION= 4,3017-002 VELOCITY, U = 0,000068397 FT/SEC 6,31-001 1,29+000
R= 1,2 IN, ANGLE= 63,00 DEGREES FUNCTION= 3,9597-002 VELOCITY, U = 0,000062959 FT/SEC 6,45-001 1,27+000
R= 1,2 IN, ANGLE= 62,00 DEGREES FUNCTION= 3,6094-002 VELOCITY, U = 0,000057389 FT/SEC 6,58-001 1,24+000
R= 1,2 IN, ANGLE= 61,00 DEGREES FUNCTION= 3,2516-002 VELOCITY, U = 0,000051700 FT/SEC 6,70-001 1,21+000
R= 1,2 IN, ANGLE= 60,00 DEGREES FUNCTION= 2,8862-002 VELOCITY, U = 0,000045890 FT/SEC 6,80-001 1,18+000
R= 1,2 IN, ANGLE= 59,00 DEGREES FUNCTION= 2,5139-002 VELOCITY, U = 0,000039971 FT/SEC 6,90-001 1,15+000
R= 1,2 IN, ANGLE= 58,00 DEGREES FUNCTION= 2,1349-002 VELOCITY, U = 0,000033945 FT/SEC 6,99-001 1,12+000
R= 1,2 IN, ANGLE= 57,00 DEGREES FUNCTION= 1,7502-002 VELOCITY, U = 0,000027829 FT/SEC 7,07-001 1,09+000
R= 1,2 IN, ANGLE= 56,00 DEGREES FUNCTION= 1,3600-002 VELOCITY, U = 0,000021625 FT/SEC 7,13-001 1,06+000
R= 1,2 IN, ANGLE= 55,00 DEGREES FUNCTION= 9,6485-003 VELOCITY, U = 0,000015341 FT/SEC 7,19-001 1,03+000
R= 1,2 IN, ANGLE= 54,00 DEGREES FUNCTION= 5,6512-003 VELOCITY, U = 0,000008985 FT/SEC 7,24-001 9,96-001
R= 1,2 IN, ANGLE= 53,00 DEGREES FUNCTION= 1,6135-003 VELOCITY, U = 0,000002566 FT/SEC 7,28-001 9,66-001
R= 1,2 IN, ANGLE= 52,00 DEGREES FUNCTION= -2,4596-003 VELOCITY, U = -0,000003911 FT/SEC 7,30-001 9,35-001
R= 1,2 IN, ANGLE= 51,00 DEGREES FUNCTION= -6,5632-003 VELOCITY, U = -0,000010436 FT/SEC 7,32-001 9,04-001
R= 1,2 IN, ANGLE= 50,00 DEGREES FUNCTION= -1,0692-002 VELOCITY, U = -0,000017001 FT/SEC 7,33-001 8,74-001
R= 1,2 IN, ANGLE= 49,00 DEGREES FUNCTION= -1,4842-002 VELOCITY, U = -0,000023599 FT/SEC 7,33-001 8,43-001
R= 1,2 IN, ANGLE= 48,00 DEGREES FUNCTION= -1,9007-002 VELOCITY, U = -0,000030222 FT/SEC 7,32-001 8,13-001
R= 1,2 IN, ANGLE= 47,00 DEGREES FUNCTION= -2,3183-002 VELOCITY, U = -0,000036861 FT/SEC 7,30-001 7,83-001
R= 1,2 IN, ANGLE= 46,00 DEGREES FUNCTION= -2,7364-002 VELOCITY, U = -0,000043509 FT/SEC 7,28-001 7,54-001
R= 1,2 IN, ANGLE= 45,00 DEGREES FUNCTION= -3,1545-002 VELOCITY, U = -0,000050157 FT/SEC 7,24-001 7,24-001
R= 1,2 IN, ANGLE= 44,00 DEGREES FUNCTION= -3,5722-002 VELOCITY, U = -0,000056798 FT/SEC 7,20-001 6,95-001
R= 1,2 IN, ANGLE= 43,00 DEGREES FUNCTION= -3,9889-002 VELOCITY, U = -0,000063424 FT/SEC 7,15-001 6,67-001
R= 1,2 IN, ANGLE= 42,00 DEGREES FUNCTION= -4,4042-002 VELOCITY, U = -0,000070027 FT/SEC 7,09-001 6,39-001
R= 1,2 IN, ANGLE= 41,00 DEGREES FUNCTION= -4,8175-002 VELOCITY, U = -0,000076598 FT/SEC 7,03-001 6,11-001
R= 1,2 IN, ANGLE= 40,00 DEGREES FUNCTION= -5,2284-002 VELOCITY, U = -0,000083131 FT/SEC 6,96-001 5,84-001
R= 1,2 IN, ANGLE= 39,00 DEGREES FUNCTION= -5,6363-002 VELOCITY, U = -0,000089617 FT/SEC 6,89-001 5,58-001
R= 1,2 IN, ANGLE= 38,00 DEGREES FUNCTION= -6,0408-002 VELOCITY, U = -0,000096048 FT/SEC 6,80-001 5,32-001
R= 1,2 IN, ANGLE= 37,00 DEGREES FUNCTION= -6,4414-002 VELOCITY, U = -0,000102418 FT/SEC 6,72-001 5,06-001
R= 1,2 IN, ANGLE= 36,00 DEGREES FUNCTION= -6,8376-002 VELOCITY, U = -0,000108718 FT/SEC 6,63-001 4,81-001
R= 1,2 IN, ANGLE= 35,00 DEGREES FUNCTION= -7,2290-002 VELOCITY, U = -0,000114941 FT/SEC 6,53-001 4,57-001
R= 1,2 IN, ANGLE= 34,00 DEGREES FUNCTION= -7,6151-002 VELOCITY, U = -0,000121080 FT/SEC 6,43-001 4,34-001
R= 1,2 IN, ANGLE= 33,00 DEGREES FUNCTION= -7,9954-002 VELOCITY, U = -0,000127128 FT/SEC 6,33-001 4,11-001
R= 1,2 IN, ANGLE= 32,00 DEGREES FUNCTION= -8,3696-002 VELOCITY, U = -0,000133077 FT/SEC 6,22-001 3,89-001
R= 1,2 IN, ANGLE= 31,00 DEGREES FUNCTION= -8,7371-002 VELOCITY, U = -0,000138920 FT/SEC 6,11-001 3,67-001
R= 1,2 IN, ANGLE= 30,00 DEGREES FUNCTION= -9,0976-002 VELOCITY, U = -0,000144651 FT/SEC 6,00-001 3,47-001
R= 1,2 IN, ANGLE= 29,00 DEGREES FUNCTION= -9,4505-002 VELOCITY, U = -0,000150264 FT/SEC 5,89-001 3,27-001
R= 1,2 IN, ANGLE= 28,00 DEGREES FUNCTION= -9,7956-002 VELOCITY, U = -0,000155750 FT/SEC 5,78-001 3,07-001
R= 1,2 IN, ANGLE= 27,00 DEGREES FUNCTION= -1,0132-001 VELOCITY, U = -0,000161105 FT/SEC 5,66-001 2,89-001
R= 1,2 IN, ANGLE= 26,00 DEGREES FUNCTION= -1,0461-001 VELOCITY, U = -0,000166322 FT/SEC 5,55-001 2,71-001
R= 1,2 IN, ANGLE= 25,00 DEGREES FUNCTION= -1,0780-001 VELOCITY, U = -0,000171395 FT/SEC 5,43-001 2,53-001
R= 1,2 IN, ANGLE= 24,00 DEGREES FUNCTION= -1,1089-001 VELOCITY, U = -0,000176318 FT/SEC 5,32-001 2,37-001
R= 1,2 IN, ANGLE= 23,00 DEGREES FUNCTION= -1,1389-001 VELOCITY, U = -0,000181085 FT/SEC 5,21-001 2,21-001
R= 1,2 IN, ANGLE= 22,00 DEGREES FUNCTION= -1,1679-001 VELOCITY, U = -0,000185692 FT/SEC 5,09-001 2,06-001
R= 1,2 IN, ANGLE= 21,00 DEGREES FUNCTION= -1,1958-001 VELOCITY, U = -0,000190132 FT/SEC 4,98-001 1,91-001
R= 1,2 IN, ANGLE= 20,00 DEGREES FUNCTION= -1,2226-001 VELOCITY, U = -0,000194400 FT/SEC 4,88-001 1,77-001
R= 1,2 IN, ANGLE= 19,00 DEGREES FUNCTION= -1,2484-001 VELOCITY, U = -0,000198492 FT/SEC 4,77-001 1,64-001
R= 1,2 IN, ANGLE= 18,00 DEGREES FUNCTION= -1,2730-001 VELOCITY, U = -0,000202403 FT/SEC 4,67-001 1,52-001
R= 1,2 IN, ANGLE= 17,00 DEGREES FUNCTION= -1,2964-001 VELOCITY, U = -0,000206129 FT/SEC 4,57-001 1,40-001
R= 1,2 IN, ANGLE= 16,00 DEGREES FUNCTION= -1,3186-001 VELOCITY, U = -0,000209665 FT/SEC 4,47-001 1,28-001
R= 1,2 IN, ANGLE= 15,00 DEGREES FUNCTION= -1,3397-001 VELOCITY, U = -0,000213007 FT/SEC 4,38-001 1,17-001
R= 1,2 IN, ANGLE= 14,00 DEGREES FUNCTION= -1,3594-001 VELOCITY, U = -0,000216152 FT/SEC 4,30-001 1,07-001
R= 1,2 IN, ANGLE= 13,00 DEGREES FUNCTION= -1,3760-001 VELOCITY, U = -0,000219005 FT/SEC 4,21-001 9,73-002
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R= 1,2 IN,	ANGLE= 12,00 DEGREES	FUNCTION= -1,3952-001	VELOCITY, U = -0,000221834 FT/SEC	4,14-001	8,79-002
R= 1,2 IN,	ANGLE= 11,00 DEGREES	FUNCTION= -1,4111-001	VELOCITY, U = -0,000224366 FT/SEC	4,06-001	7,90-002
R= 1,2 IN,	ANGLE= 10,00 DEGREES	FUNCTION= -1,4257-001	VELOCITY, U = -0,000226687 FT/SEC	4,00-001	7,05-002
R= 1,2 IN,	ANGLE= 9,00 DEGREES	FUNCTION= -1,4390-001	VELOCITY, U = -0,000228795 FT/SEC	3,94-001	6,23-002
R= 1,2 IN,	ANGLE= 8,00 DEGREES	FUNCTION= -1,4509-001	VELOCITY, U = -0,000230688 FT/SEC	3,88-001	5,45-002
R= 1,2 IN,	ANGLE= 7,00 DEGREES	FUNCTION= -1,4614-001	VELOCITY, U = -0,000232362 FT/SEC	3,83-001	4,70-002
R= 1,2 IN,	ANGLE= 6,00 DEGREES	FUNCTION= -1,4706-001	VELOCITY, U = -0,000233818 FT/SEC	3,79-001	3,98-002
R= 1,2 IN,	ANGLE= 5,00 DEGREES	FUNCTION= -1,4783-001	VELOCITY, U = -0,000235052 FT/SEC	3,75-001	3,28-002
R= 1,2 IN,	ANGLE= 4,00 DEGREES	FUNCTION= -1,4847-001	VELOCITY, U = -0,000236064 FT/SEC	3,72-001	2,60-002
R= 1,2 IN,	ANGLE= 3,00 DEGREES	FUNCTION= -1,4896-001	VELOCITY, U = -0,000236852 FT/SEC	3,70-001	1,94-002
R= 1,2 IN,	ANGLE= 2,00 DEGREES	FUNCTION= -1,4932-001	VELOCITY, U = -0,000237416 FT/SEC	3,68-001	1,29-002
R= 1,2 IN,	ANGLE= 1,00 DEGREES	FUNCTION= -1,4953-001	VELOCITY, U = -0,000237754 FT/SEC	3,67-001	6,41-003
R= 1,2 IN,	ANGLE= 0,00 DEGREES	FUNCTION= -1,4960-001	VELOCITY, U = -0,000237867 FT/SEC	3,67-001	0,00+000
R= 3,0 IN,	ANGLE= 128,73 DEGREES	FUNCTION= -6,5193-009	VELOCITY, U = -0,000000000 FT/SEC	-1,88+000	2,34+000
R= 3,0 IN,	ANGLE= 127,00 DEGREES	FUNCTION= 6,9751-003	VELOCITY, U = 0,000004436 FT/SEC	-1,81+000	2,41+000
R= 3,0 IN,	ANGLE= 126,00 DEGREES	FUNCTION= 1,0958-002	VELOCITY, U = 0,000006969 FT/SEC	-1,78+000	2,45+000
R= 3,0 IN,	ANGLE= 125,00 DEGREES	FUNCTION= 1,4894-002	VELOCITY, U = 0,000009473 FT/SEC	-1,74+000	2,48+000
R= 3,0 IN,	ANGLE= 124,00 DEGREES	FUNCTION= 1,8778-002	VELOCITY, U = 0,000011943 FT/SEC	-1,70+000	2,52+000
R= 3,0 IN,	ANGLE= 123,00 DEGREES	FUNCTION= 2,2606-002	VELOCITY, U = 0,000014377 FT/SEC	-1,66+000	2,56+000
R= 3,0 IN,	ANGLE= 122,00 DEGREES	FUNCTION= 2,6375-002	VELOCITY, U = 0,000016774 FT/SEC	-1,62+000	2,59+000
R= 3,0 IN,	ANGLE= 121,00 DEGREES	FUNCTION= 3,0076-002	VELOCITY, U = 0,000019128 FT/SEC	-1,58+000	2,63+000
R= 3,0 IN,	ANGLE= 120,00 DEGREES	FUNCTION= 3,3706-002	VELOCITY, U = 0,000021437 FT/SEC	-1,54+000	2,66+000
R= 3,0 IN,	ANGLE= 119,00 DEGREES	FUNCTION= 3,7259-002	VELOCITY, U = 0,000023697 FT/SEC	-1,49+000	2,70+000
R= 3,0 IN,	ANGLE= 118,00 DEGREES	FUNCTION= 4,0735-002	VELOCITY, U = 0,000025907 FT/SEC	-1,45+000	2,73+000
R= 3,0 IN,	ANGLE= 117,00 DEGREES	FUNCTION= 4,4127-002	VELOCITY, U = 0,000028065 FT/SEC	-1,41+000	2,76+000
R= 3,0 IN,	ANGLE= 116,00 DEGREES	FUNCTION= 4,7432-002	VELOCITY, U = 0,000030167 FT/SEC	-1,36+000	2,79+000
R= 3,0 IN,	ANGLE= 115,00 DEGREES	FUNCTION= 5,0644-002	VELOCITY, U = 0,000032210 FT/SEC	-1,32+000	2,82+000
R= 3,0 IN,	ANGLE= 114,00 DEGREES	FUNCTION= 5,3761-002	VELOCITY, U = 0,000034192 FT/SEC	-1,27+000	2,85+000
R= 3,0 IN,	ANGLE= 113,00 DEGREES	FUNCTION= 5,6777-002	VELOCITY, U = 0,000036110 FT/SEC	-1,22+000	2,88+000
R= 3,0 IN,	ANGLE= 112,00 DEGREES	FUNCTION= 5,9690-002	VELOCITY, U = 0,000037963 FT/SEC	-1,17+000	2,90+000
R= 3,0 IN,	ANGLE= 111,00 DEGREES	FUNCTION= 6,2496-002	VELOCITY, U = 0,000039748 FT/SEC	-1,13+000	2,93+000
R= 3,0 IN,	ANGLE= 110,00 DEGREES	FUNCTION= 6,5191-002	VELOCITY, U = 0,000041461 FT/SEC	-1,08+000	2,96+000
R= 3,0 IN,	ANGLE= 109,00 DEGREES	FUNCTION= 6,7772-002	VELOCITY, U = 0,000043103 FT/SEC	-1,03+000	2,98+000
R= 3,0 IN,	ANGLE= 108,00 DEGREES	FUNCTION= 7,0235-002	VELOCITY, U = 0,000044669 FT/SEC	-9,75-001	3,00+000
R= 3,0 IN,	ANGLE= 107,00 DEGREES	FUNCTION= 7,2577-002	VELOCITY, U = 0,000046159 FT/SEC	-9,24-001	3,02+000
R= 3,0 IN,	ANGLE= 106,00 DEGREES	FUNCTION= 7,4796-002	VELOCITY, U = 0,000047571 FT/SEC	-8,73-001	3,04+000
R= 3,0 IN,	ANGLE= 105,00 DEGREES	FUNCTION= 7,6889-002	VELOCITY, U = 0,000048902 FT/SEC	-8,21-001	3,06+000
R= 3,0 IN,	ANGLE= 104,00 DEGREES	FUNCTION= 7,8853-002	VELOCITY, U = 0,000050150 FT/SEC	-7,68-001	3,08+000
R= 3,0 IN,	ANGLE= 103,00 DEGREES	FUNCTION= 8,0685-002	VELOCITY, U = 0,000051316 FT/SEC	-7,15-001	3,10+000
R= 3,0 IN,	ANGLE= 102,00 DEGREES	FUNCTION= 8,2384-002	VELOCITY, U = 0,000052396 FT/SEC	-6,62-001	3,11+000
R= 3,0 IN,	ANGLE= 101,00 DEGREES	FUNCTION= 8,3946-002	VELOCITY, U = 0,000053390 FT/SEC	-6,08-001	3,13+000
R= 3,0 IN,	ANGLE= 100,00 DEGREES	FUNCTION= 8,5371-002	VELOCITY, U = 0,000054296 FT/SEC	-5,54-001	3,14+000
R= 3,0 IN,	ANGLE= 99,00 DEGREES	FUNCTION= 8,6656-002	VELOCITY, U = 0,000055113 FT/SEC	-5,00-001	3,15+000
R= 3,0 IN,	ANGLE= 98,00 DEGREES	FUNCTION= 8,7799-002	VELOCITY, U = 0,000055840 FT/SEC	-4,45-001	3,16+000
R= 3,0 IN,	ANGLE= 97,00 DEGREES	FUNCTION= 8,8800-002	VELOCITY, U = 0,000056477 FT/SEC	-3,90-001	3,17+000
R= 3,0 IN,	ANGLE= 96,00 DEGREES	FUNCTION= 8,9656-002	VELOCITY, U = 0,000057021 FT/SEC	-3,34-001	3,18+000
R= 3,0 IN,	ANGLE= 95,00 DEGREES	FUNCTION= 9,0368-002	VELOCITY, U = 0,000057474 FT/SEC	-2,79-001	3,19+000
R= 3,0 IN,	ANGLE= 94,00 DEGREES	FUNCTION= 9,0933-002	VELOCITY, U = 0,000057833 FT/SEC	-2,23-001	3,19+000
R= 3,0 IN,	ANGLE= 93,00 DEGREES	FUNCTION= 9,1351-002	VELOCITY, U = 0,000058100 FT/SEC	-1,68-001	3,20+000
R= 3,0 IN,	ANGLE= 92,00 DEGREES	FUNCTION= 9,1623-002	VELOCITY, U = 0,000058272 FT/SEC	-1,12-001	3,20+000
R= 3,0 IN,	ANGLE= 91,00 DEGREES	FUNCTION= 9,1746-002	VELOCITY, U = 0,000058351 FT/SEC	-5,59-002	3,20+000
R= 3,0 IN,	ANGLE= 90,00 DEGREES	FUNCTION= 9,1722-002	VELOCITY, U = 0,000058335 FT/SEC	8,21-012	3,20+000
R= 3,0 IN,	ANGLE= 89,00 DEGREES	FUNCTION= 9,1549-002	VELOCITY, U = 0,000058225 FT/SEC	5,59-002	3,20+000
R= 3,0 IN,	ANGLE= 88,00 DEGREES	FUNCTION= 9,1230-002	VELOCITY, U = 0,000058022 FT/SEC	1,12-001	3,20+000
R= 3,0 IN,	ANGLE= 87,00 DEGREES	FUNCTION= 9,0763-002	VELOCITY, U = 0,000057725 FT/SEC	1,68-001	3,20+000
R= 3,0 IN,	ANGLE= 86,00 DEGREES	FUNCTION= 9,0149-002	VELOCITY, U = 0,000057335 FT/SEC	2,23-001	3,19+000
R= 3,0 IN,	ANGLE= 85,00 DEGREES	FUNCTION= 8,9390-002	VELOCITY, U = 0,000056852 FT/SEC	2,79-001	3,19+000
R= 3,0 IN,	ANGLE= 84,00 DEGREES	FUNCTION= 8,8485-002	VELOCITY, U = 0,000056277 FT/SEC	3,34-001	3,18+000
R= 3,0 IN,	ANGLE= 83,00 DEGREES	FUNCTION= 8,7437-002	VELOCITY, U = 0,000055610 FT/SEC	3,89-001	3,17+000
R= 3,0 IN,	ANGLE= 82,00 DEGREES	FUNCTION= 8,6247-002	VELOCITY, U = 0,000054853 FT/SEC	4,44-001	3,16+000
R= 3,0 IN,	ANGLE= 81,00 DEGREES	FUNCTION= 8,4916-002	VELOCITY, U = 0,000054007 FT/SEC	4,99-001	3,15+000

R= 3.0 IN.	ANGLE= 80.00 DEGREES	FUNCTION= 8.3446-002	VELOCITY, U = 0.000053071 FT/SEC	5.53-001	3.14+000
R= 3.0 IN.	ANGLE= 79.00 DEGREES	FUNCTION= 8.1838-002	VELOCITY, U = 0.000052049 FT/SEC	6.07-001	3.12+000
R= 3.0 IN.	ANGLE= 78.00 DEGREES	FUNCTION= 8.0095-002	VELOCITY, U = 0.000050941 FT/SEC	6.61-001	3.11+000
R= 3.0 IN.	ANGLE= 77.00 DEGREES	FUNCTION= 7.8219-002	VELOCITY, U = 0.000049747 FT/SEC	7.14-001	3.09+000
R= 3.0 IN.	ANGLE= 76.00 DEGREES	FUNCTION= 7.6213-002	VELOCITY, U = 0.000048471 FT/SEC	7.67-001	3.08+000
R= 3.0 IN.	ANGLE= 75.00 DEGREES	FUNCTION= 7.4078-002	VELOCITY, U = 0.000047114 FT/SEC	8.19-001	3.06+000
R= 3.0 IN.	ANGLE= 74.00 DEGREES	FUNCTION= 7.1818-002	VELOCITY, U = 0.000045676 FT/SEC	8.71-001	3.04+000
R= 3.0 IN.	ANGLE= 73.00 DEGREES	FUNCTION= 6.9435-002	VELOCITY, U = 0.000044161 FT/SEC	9.22-001	3.02+000
R= 3.0 IN.	ANGLE= 72.00 DEGREES	FUNCTION= 6.6933-002	VELOCITY, U = 0.000042569 FT/SEC	9.73-001	3.00+000
R= 3.0 IN.	ANGLE= 71.00 DEGREES	FUNCTION= 6.4314-002	VELOCITY, U = 0.000040904 FT/SEC	1.02+000	2.97+000
R= 3.0 IN.	ANGLE= 70.00 DEGREES	FUNCTION= 6.1582-002	VELOCITY, U = 0.000039166 FT/SEC	1.07+000	2.95+000
R= 3.0 IN.	ANGLE= 69.00 DEGREES	FUNCTION= 5.8741-002	VELOCITY, U = 0.000037359 FT/SEC	1.12+000	2.92+000
R= 3.0 IN.	ANGLE= 68.00 DEGREES	FUNCTION= 5.5793-002	VELOCITY, U = 0.000035484 FT/SEC	1.17+000	2.90+000
R= 3.0 IN.	ANGLE= 67.00 DEGREES	FUNCTION= 5.2743-002	VELOCITY, U = 0.000033545 FT/SEC	1.22+000	2.87+000
R= 3.0 IN.	ANGLE= 66.00 DEGREES	FUNCTION= 4.9594-002	VELOCITY, U = 0.000031542 FT/SEC	1.27+000	2.84+000
R= 3.0 IN.	ANGLE= 65.00 DEGREES	FUNCTION= 4.6351-002	VELOCITY, U = 0.000029479 FT/SEC	1.31+000	2.81+000
R= 3.0 IN.	ANGLE= 64.00 DEGREES	FUNCTION= 4.3017-002	VELOCITY, U = 0.000027359 FT/SEC	1.36+000	2.78+000
R= 3.0 IN.	ANGLE= 63.00 DEGREES	FUNCTION= 3.9597-002	VELOCITY, U = 0.000025183 FT/SEC	1.40+000	2.75+000
R= 3.0 IN.	ANGLE= 62.00 DEGREES	FUNCTION= 3.6094-002	VELOCITY, U = 0.000022956 FT/SEC	1.45+000	2.72+000
R= 3.0 IN.	ANGLE= 61.00 DEGREES	FUNCTION= 3.2516-002	VELOCITY, U = 0.000020680 FT/SEC	1.49+000	2.69+000
R= 3.0 IN.	ANGLE= 60.00 DEGREES	FUNCTION= 2.8862-002	VELOCITY, U = 0.000018356 FT/SEC	1.53+000	2.65+000
R= 3.0 IN.	ANGLE= 59.00 DEGREES	FUNCTION= 2.5139-002	VELOCITY, U = 0.000015988 FT/SEC	1.57+000	2.62+000
R= 3.0 IN.	ANGLE= 58.00 DEGREES	FUNCTION= 2.1349-002	VELOCITY, U = 0.000013578 FT/SEC	1.61+000	2.58+000
R= 3.0 IN.	ANGLE= 57.00 DEGREES	FUNCTION= 1.7502-002	VELOCITY, U = 0.000011131 FT/SEC	1.66+000	2.55+000
R= 3.0 IN.	ANGLE= 56.00 DEGREES	FUNCTION= 1.3600-002	VELOCITY, U = 0.000008650 FT/SEC	1.69+000	2.51+000
R= 3.0 IN.	ANGLE= 55.00 DEGREES	FUNCTION= 9.6485-003	VELOCITY, U = 0.000006136 FT/SEC	1.73+000	2.48+000
R= 3.0 IN.	ANGLE= 54.00 DEGREES	FUNCTION= 5.6512-003	VELOCITY, U = 0.000003594 FT/SEC	1.77+000	2.44+000
R= 3.0 IN.	ANGLE= 53.00 DEGREES	FUNCTION= 1.6135-003	VELOCITY, U = 0.000001026 FT/SEC	1.81+000	2.40+000
R= 3.0 IN.	ANGLE= 52.00 DEGREES	FUNCTION= -2.4596-003	VELOCITY, U = -0.000001564 FT/SEC	1.84+000	2.36+000
R= 3.0 IN.	ANGLE= 51.00 DEGREES	FUNCTION= -6.5632-003	VELOCITY, U = -0.000004174 FT/SEC	1.88+000	2.32+000
R= 3.0 IN.	ANGLE= 50.00 DEGREES	FUNCTION= -1.0692-002	VELOCITY, U = -0.000006800 FT/SEC	1.91+000	2.28+000
R= 3.0 IN.	ANGLE= 49.00 DEGREES	FUNCTION= -1.4842-002	VELOCITY, U = -0.000009440 FT/SEC	1.95+000	2.24+000
R= 3.0 IN.	ANGLE= 48.00 DEGREES	FUNCTION= -1.9007-002	VELOCITY, U = -0.000012089 FT/SEC	1.98+000	2.20+000
R= 3.0 IN.	ANGLE= 47.00 DEGREES	FUNCTION= -2.3183-002	VELOCITY, U = -0.000014744 FT/SEC	2.01+000	2.16+000
R= 3.0 IN.	ANGLE= 46.00 DEGREES	FUNCTION= -2.7364-002	VELOCITY, U = -0.000017403 FT/SEC	2.04+000	2.11+000
R= 3.0 IN.	ANGLE= 45.00 DEGREES	FUNCTION= -3.1545-002	VELOCITY, U = -0.000020063 FT/SEC	2.07+000	2.07+000
R= 3.0 IN.	ANGLE= 44.00 DEGREES	FUNCTION= -3.5722-002	VELOCITY, U = -0.000022719 FT/SEC	2.10+000	2.03+000
R= 3.0 IN.	ANGLE= 43.00 DEGREES	FUNCTION= -3.9889-002	VELOCITY, U = -0.000025370 FT/SEC	2.13+000	1.99+000
R= 3.0 IN.	ANGLE= 42.00 DEGREES	FUNCTION= -4.4042-002	VELOCITY, U = -0.000028011 FT/SEC	2.16+000	1.94+000
R= 3.0 IN.	ANGLE= 41.00 DEGREES	FUNCTION= -4.8175-002	VELOCITY, U = -0.000030639 FT/SEC	2.18+000	1.90+000
R= 3.0 IN.	ANGLE= 40.00 DEGREES	FUNCTION= -5.2284-002	VELOCITY, U = -0.000033252 FT/SEC	2.21+000	1.85+000
R= 3.0 IN.	ANGLE= 39.00 DEGREES	FUNCTION= -5.6363-002	VELOCITY, U = -0.000035847 FT/SEC	2.23+000	1.81+000
R= 3.0 IN.	ANGLE= 38.00 DEGREES	FUNCTION= -6.0408-002	VELOCITY, U = -0.000038419 FT/SEC	2.26+000	1.76+000
R= 3.0 IN.	ANGLE= 37.00 DEGREES	FUNCTION= -6.4414-002	VELOCITY, U = -0.000040967 FT/SEC	2.28+000	1.72+000
R= 3.0 IN.	ANGLE= 36.00 DEGREES	FUNCTION= -6.8376-002	VELOCITY, U = -0.000043487 FT/SEC	2.30+000	1.67+000
R= 3.0 IN.	ANGLE= 35.00 DEGREES	FUNCTION= -7.2290-002	VELOCITY, U = -0.000045976 FT/SEC	2.33+000	1.63+000
R= 3.0 IN.	ANGLE= 34.00 DEGREES	FUNCTION= -7.6151-002	VELOCITY, U = -0.000048432 FT/SEC	2.35+000	1.58+000
R= 3.0 IN.	ANGLE= 33.00 DEGREES	FUNCTION= -7.9954-002	VELOCITY, U = -0.000050851 FT/SEC	2.37+000	1.54+000
R= 3.0 IN.	ANGLE= 32.00 DEGREES	FUNCTION= -8.3696-002	VELOCITY, U = -0.000053231 FT/SEC	2.39+000	1.49+000
R= 3.0 IN.	ANGLE= 31.00 DEGREES	FUNCTION= -8.7371-002	VELOCITY, U = -0.000055568 FT/SEC	2.40+000	1.44+000
R= 3.0 IN.	ANGLE= 30.00 DEGREES	FUNCTION= -9.0976-002	VELOCITY, U = -0.000057861 FT/SEC	2.42+000	1.40+000
R= 3.0 IN.	ANGLE= 29.00 DEGREES	FUNCTION= -9.4505-002	VELOCITY, U = -0.000060105 FT/SEC	2.44+000	1.35+000
R= 3.0 IN.	ANGLE= 28.00 DEGREES	FUNCTION= -9.7956-002	VELOCITY, U = -0.000062300 FT/SEC	2.46+000	1.31+000
R= 3.0 IN.	ANGLE= 27.00 DEGREES	FUNCTION= -1.0132-001	VELOCITY, U = -0.000064442 FT/SEC	2.47+000	1.26+000
R= 3.0 IN.	ANGLE= 26.00 DEGREES	FUNCTION= -1.0461-001	VELOCITY, U = -0.000066529 FT/SEC	2.49+000	1.21+000
R= 3.0 IN.	ANGLE= 25.00 DEGREES	FUNCTION= -1.0780-001	VELOCITY, U = -0.000068558 FT/SEC	2.50+000	1.17+000
R= 3.0 IN.	ANGLE= 24.00 DEGREES	FUNCTION= -1.1089-001	VELOCITY, U = -0.000070527 FT/SEC	2.51+000	1.12+000
R= 3.0 IN.	ANGLE= 23.00 DEGREES	FUNCTION= -1.1389-001	VELOCITY, U = -0.000072434 FT/SEC	2.53+000	1.07+000
R= 3.0 IN.	ANGLE= 22.00 DEGREES	FUNCTION= -1.1679-001	VELOCITY, U = -0.000074277 FT/SEC	2.54+000	1.03+000
R= 3.0 IN.	ANGLE= 21.00 DEGREES	FUNCTION= -1.1958-001	VELOCITY, U = -0.000076053 FT/SEC	2.55+000	9.80-001
R= 3.0 IN.	ANGLE= 20.00 DEGREES	FUNCTION= -1.2226-001	VELOCITY, U = -0.000077760 FT/SEC	2.56+000	9.33-001

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R=  3,0 IN.   ANGLE=  19,00 DEGREES   FUNCTION= -1,2484-001   VELOCITY, U =  -0,000079397 FT/SEC  2,57+000  8,86-001
R=  3,0 IN.   ANGLE=  18,00 DEGREES   FUNCTION= -1,2730-001   VELOCITY, U =  -0,000080961 FT/SEC  2,58+000  8,39-001
R=  3,0 IN.   ANGLE=  17,00 DEGREES   FUNCTION= -1,2964-001   VELOCITY, U =  -0,000082452 FT/SEC  2,59+000  7,93-001
R=  3,0 IN.   ANGLE=  16,00 DEGREES   FUNCTION= -1,3186-001   VELOCITY, U =  -0,000083866 FT/SEC  2,60+000  7,46-001
R=  3,0 IN.   ANGLE=  15,00 DEGREES   FUNCTION= -1,3397-001   VELOCITY, U =  -0,000085203 FT/SEC  2,61+000  6,99-001
R=  3,0 IN.   ANGLE=  14,00 DEGREES   FUNCTION= -1,3594-001   VELOCITY, U =  -0,000086461 FT/SEC  2,62+000  6,52-001
R=  3,0 IN.   ANGLE=  13,00 DEGREES   FUNCTION= -1,3780-001   VELOCITY, U =  -0,000087638 FT/SEC  2,62+000  6,06-001
R=  3,0 IN.   ANGLE=  12,00 DEGREES   FUNCTION= -1,3952-001   VELOCITY, U =  -0,000088734 FT/SEC  2,63+000  5,59-001
R=  3,0 IN.   ANGLE=  11,00 DEGREES   FUNCTION= -1,4111-001   VELOCITY, U =  -0,000089746 FT/SEC  2,64+000  5,12-001
R=  3,0 IN.   ANGLE=  10,00 DEGREES   FUNCTION= -1,4257-001   VELOCITY, U =  -0,000090675 FT/SEC  2,64+000  4,66-001
R=  3,0 IN.   ANGLE=   9,00 DEGREES   FUNCTION= -1,4390-001   VELOCITY, U =  -0,000091518 FT/SEC  2,65+000  4,19-001
R=  3,0 IN.   ANGLE=   8,00 DEGREES   FUNCTION= -1,4509-001   VELOCITY, U =  -0,000092275 FT/SEC  2,65+000  3,73-001
R=  3,0 IN.   ANGLE=   7,00 DEGREES   FUNCTION= -1,4614-001   VELOCITY, U =  -0,000092945 FT/SEC  2,65+000  3,26-001
R=  3,0 IN.   ANGLE=   6,00 DEGREES   FUNCTION= -1,4706-001   VELOCITY, U =  -0,000093527 FT/SEC  2,66+000  2,79-001
R=  3,0 IN.   ANGLE=   5,00 DEGREES   FUNCTION= -1,4783-001   VELOCITY, U =  -0,000094021 FT/SEC  2,66+000  2,33-001
R=  3,0 IN.   ANGLE=   4,00 DEGREES   FUNCTION= -1,4847-001   VELOCITY, U =  -0,000094426 FT/SEC  2,66+000  1,86-001
R=  3,0 IN.   ANGLE=   3,00 DEGREES   FUNCTION= -1,4896-001   VELOCITY, U =  -0,000094741 FT/SEC  2,66+000  1,40-001
R=  3,0 IN.   ANGLE=   2,00 DEGREES   FUNCTION= -1,4932-001   VELOCITY, U =  -0,000094966 FT/SEC  2,67+000  9,31-002
R=  3,0 IN.   ANGLE=   1,00 DEGREES   FUNCTION= -1,4953-001   VELOCITY, U =  -0,000095102 FT/SEC  2,67+000  4,65-002
R=  3,0 IN.   ANGLE=   0,00 DEGREES   FUNCTION= -1,4960-001   VELOCITY, U =  -0,000095147 FT/SEC  2,67+000  0,00+000
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SUBROUTINE PERT

for velocity profiles as obtained

by the modified perturbation method

```
      SUBROUTINE PERT(AL,EPS)
      TYPE REAL KO,K1
      DIMENSION XB(400),YB(400)
      C1(A)=A*3.1415926536/180.
      PI=3.1415926536
      Z=1.59E-4
    2 ALO=AL*.5
      K=1
      ALOR=ALO*PI/180.
   10 ALOR2=ALOR*2.
      SALO=SIN(ALOR2)
      CALO=COS(ALOR2)
      A=1./(SALO-ALOR2*CALO)
      ALR=AL*PI/180.
      K1=2./ALOR*(7.*A*A/48.*SIN(4.*ALOR)-A*A*ALOR*CALO*CALO)-A*A/2.
      AL1R=-1./8./A*(K1+A*A/2.        +A*A*CALO*CALO)/SALO+A/48.*COS(4.*A
     1LOR)/SALO+A*ALOR*CALO/4.
      ALORP=ALR-EPS*AL1R
      IF(ABS(ALORP-ALOR).LT.1.E-10) GO TO 9
      K=K+1
      ALOR=(ALORP+ALOR)/2.
      IF(K.LT.100) GO TO 10
      PRINT6,ALOR,ALORP,AL,EPS,K
    6 FORMAT(* NO SOLUTION FOR ALO IN 100 ITERATIONS * 4F20.10,I10)
    9 KO=4.*A*CALO
      J=AL+1.
      ALO=ALOR*180./PI
      AL1=AL1R*180./PI
      UM=0.
      R=1.
      X=0.
    4 XR=C1(X)
      FO=A*COS(2.*XR)-KO/4.
      F1=                 -(K1+A*A/2.+A*A*CALO*CALO)/4.+A*A/24.*COS(4.*XR)+
     1A*A/2.*CALO*XR*SIN(2.*XR)
      F=FO+EPS*F1
      CAPF=EPS*F
      U=Z/R*CAPF*12.
      IF(ABS(U).GT.ABS(UM)) UM=ABS(U)
      X=X+1.
      IF(X.LT.AL) GO TO 4
      CALL CHAR(3.4,-2.1,13HSCALE 1 INCH=,13,0.,.24,.16)
      UMM=UM*2.
      ENCODE(8,11,JJ)UMM
   11 FORMAT(F8.5)
      CALL CHAR(3.4,.5,JJ,8)
      CALL CHAR(3.4,2.1,7H FT/SEC,7)
      CALL PLOT(0.,-2.,2)
      DO18L=1,5
      X=L-1.375
      CALL PLOT (0.,X,1)
      CALL PLOT (0.,X+.125,2)
      CALL PLOT (0.,X+.25,1)
   18 CALL PLOT (0.,X+.375,2)
      X=3.5*COS(ALR)
      Y=3.5*SIN(ALR)
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      CALLIPLOT(Y,X,2)
      CALLIPLOT( 0.,0.,1)
      CALLIPLOT( -Y,X,1)
      PRINT20,AL,EPS,ALO,AL1
   20 FORMAT(*1VELOCITY PROFILE AS OBTAINED BY USING A MODIFIED PERTUBATION METH
     1ION METHOD FOR FLOW BETWEEN NONPARALLEL PLANE WALLS*/* ANGLE OF WA
     2LLS = *F9.5/* EPSILON = *F9.5/* ALO = *F15.9,*  AL1 = *F15.9)
      DO 13 I=1,3,2
      R=I
      IF(I.EQ.1) R=1.2
      XX=R*COS(ALR)
      YY=R*SIN(ALR)
      CALLIPLOT (-YY,XX,2)
      M=AL
      N=2*M+1
      DO 19 K=1,N
      T=-M+K-1
      T=C1(T)
      X=R*COS(T)
      Y=R*SIN(T)
   19 CALLIPLOT (Y,X,1)
      CALLIPLOT (YY,XX,1)
      X=AL
      J=1
   16 XR=C1(X)
      FO=A*COS(2.*XR)-KO/4.
      F1=                -(K1+A*A/2.+A*A*CALO*CALO)/4.+A*A/24.*COS(4.*XR)+
     1A*A/2.*CALO*XR*SIN(2.*XR)
      F=FO+EPS*F1
      CAPF=EPS*F
      U=Z/R*CAPF*12.
      XB(J)=X1=(R+U/UM)*COS(XR)
      Y=(R+U/UM)*SIN(XR)
      PRINT7,R,X,FC,F1,F,J
      YB(J)=-Y
      CALLIPLOT (Y,X1,1)
      IF(X-.3.LT.-0.) GO TO15
      IF(J.GT.1) GO TO 14
      NN=X
      X=NN
   14 X=X-1.
      J=J+1
      GO TO 16
   15 DO 100 K=1,J
      N=J-K+1
  100 CALLIPLOT(YB(N),XB(N),1)
   13 CONTINUE
    7 FORMAT(* R= *F4.1* IN.   ANGLE= *F8.2* DEGREES   FO= *,F10.4,*  F1
     1= *F10.4,* F= *F10.4,
     1*   VELOCITY, U = *F13.9* FT/SEC*)
      END
RUN,3,0,4000

EXECUTION STARTED AT  .2102 -57
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VELOCITY PROFILE AS OBTAINED BY USING A MODIFIED PERTUBATION METHOD FOR FLOW BETWEEN NONPARALLEL PLANE WALLS
ANGLE OF WALLS = 129.72714
EPSILON =  -0.01485
ALO =   127.916391630   AL1 =    -54.580069711
R=  1.2 IN.   ANGLE=  128.73 DEGREES   FO=    0.2234  F1=   15.1450 F=   -0.0016   VELOCITY, U =   0.000000038 FT/SEC
R=  1.2 IN.   ANGLE=  127.00 DEGREES   FO=   -0.2506  F1=   14.8007 F=   -0.4704   VELOCITY, U =   0.000011110 FT/SEC
R=  1.2 IN.   ANGLE=  126.00 DEGREES   FO=   -0.5213  F1=   14.5918 F=   -0.7381   VELOCITY, U =   0.000017433 FT/SEC
R=  1.2 IN.   ANGLE=  125.00 DEGREES   FO=   -0.7891  F1=   14.3763 F=   -1.0026   VELOCITY, U =   0.000023681 FT/SEC
R=  1.2 IN.   ANGLE=  124.00 DEGREES   FO=   -1.0534  F1=   14.1542 F=   -1.2637   VELOCITY, U =   0.000029847 FT/SEC
R=  1.2 IN.   ANGLE=  123.00 DEGREES   FO=   -1.3141  F1=   13.9258 F=   -1.5210   VELOCITY, U =   0.000035923 FT/SEC
R=  1.2 IN.   ANGLE=  122.00 DEGREES   FO=   -1.5707  F1=   13.6910 F=   -1.7741   VELOCITY, U =   0.000041902 FT/SEC
R=  1.2 IN.   ANGLE=  121.00 DEGREES   FO=   -1.8230  F1=   13.4500 F=   -2.0228   VELOCITY, U =   0.000047776 FT/SEC
R=  1.2 IN.   ANGLE=  120.00 DEGREES   FO=   -2.0707  F1=   13.2027 F=   -2.2668   VELOCITY, U =   0.000053539 FT/SEC
R=  1.2 IN.   ANGLE=  119.00 DEGREES   FO=   -2.3134  F1=   12.9493 F=   -2.5058   VELOCITY, U =   0.000059183 FT/SEC
R=  1.2 IN.   ANGLE=  118.00 DEGREES   FO=   -2.5509  F1=   12.6896 F=   -2.7394   VELOCITY, U =   0.000064701 FT/SEC
R=  1.2 IN.   ANGLE=  117.00 DEGREES   FO=   -2.7829  F1=   12.4237 F=   -2.9674   VELOCITY, U =   0.000070086 FT/SEC
R=  1.2 IN.   ANGLE=  116.00 DEGREES   FO=   -3.0090  F1=   12.1514 F=   -3.1895   VELOCITY, U =   0.000075331 FT/SEC
R=  1.2 IN.   ANGLE=  115.00 DEGREES   FO=   -3.2291  F1=   11.8727 F=   -3.4054   VELOCITY, U =   0.000080431 FT/SEC
R=  1.2 IN.   ANGLE=  114.00 DEGREES   FO=   -3.4428  F1=   11.5876 F=   -3.6149   VELOCITY, U =   0.000085379 FT/SEC
R=  1.2 IN.   ANGLE=  113.00 DEGREES   FO=   -3.6499  F1=   11.2960 F=   -3.8177   VELOCITY, U =   0.000090168 FT/SEC
R=  1.2 IN.   ANGLE=  112.00 DEGREES   FO=   -3.8501  F1=   10.9977 F=   -4.0135   VELOCITY, U =   0.000094792 FT/SEC
R=  1.2 IN.   ANGLE=  111.00 DEGREES   FO=   -4.0432  F1=   10.6927 F=   -4.2021   VELOCITY, U =   0.000099246 FT/SEC
R=  1.2 IN.   ANGLE=  110.00 DEGREES   FO=   -4.2290  F1=   10.3808 F=   -4.3832   VELOCITY, U =   0.000103524 FT/SEC
R=  1.2 IN.   ANGLE=  109.00 DEGREES   FO=   -4.4072  F1=   10.0621 F=   -4.5567   VELOCITY, U =   0.000107622 FT/SEC
R=  1.2 IN.   ANGLE=  108.00 DEGREES   FO=   -4.5776  F1=    9.7363 F=   -4.7223   VELOCITY, U =   0.000111532 FT/SEC
R=  1.2 IN.   ANGLE=  107.00 DEGREES   FO=   -4.7401  F1=    9.4035 F=   -4.8797   VELOCITY, U =   0.000115251 FT/SEC
R=  1.2 IN.   ANGLE=  106.00 DEGREES   FO=   -4.8943  F1=    9.0636 F=   -5.0289   VELOCITY, U =   0.000118775 FT/SEC
R=  1.2 IN.   ANGLE=  105.00 DEGREES   FO=   -5.0401  F1=    8.7165 F=   -5.1696   VELOCITY, U =   0.000122098 FT/SEC
R=  1.2 IN.   ANGLE=  104.00 DEGREES   FO=   -5.1774  F1=    8.3622 F=   -5.3016   VELOCITY, U =   0.000125216 FT/SEC
R=  1.2 IN.   ANGLE=  103.00 DEGREES   FO=   -5.3060  F1=    8.0008 F=   -5.4248   VELOCITY, U =   0.000128125 FT/SEC
R=  1.2 IN.   ANGLE=  102.00 DEGREES   FO=   -5.4256  F1=    7.6322 F=   -5.5390   VELOCITY, U =   0.000130822 FT/SEC
R=  1.2 IN.   ANGLE=  101.00 DEGREES   FO=   -5.5363  F1=    7.2565 F=   -5.6441   VELOCITY, U =   0.000133304 FT/SEC
R=  1.2 IN.   ANGLE=  100.00 DEGREES   FO=   -5.6377  F1=    6.8739 F=   -5.7399   VELOCITY, U =   0.000135566 FT/SEC
R=  1.2 IN.   ANGLE=   99.00 DEGREES   FO=   -5.7299  F1=    6.4844 F=   -5.8263   VELOCITY, U =   0.000137607 FT/SEC
R=  1.2 IN.   ANGLE=   98.00 DEGREES   FO=   -5.8127  F1=    6.0881 F=   -5.9032   VELOCITY, U =   0.000139423 FT/SEC
R=  1.2 IN.   ANGLE=   97.00 DEGREES   FO=   -5.8860  F1=    5.6854 F=   -5.9705   VELOCITY, U =   0.000141013 FT/SEC
R=  1.2 IN.   ANGLE=   96.00 DEGREES   FO=   -5.9497  F1=    5.2763 F=   -6.0281   VELOCITY, U =   0.000142374 FT/SEC
R=  1.2 IN.   ANGLE=   95.00 DEGREES   FO=   -6.0037  F1=    4.8612 F=   -6.0760   VELOCITY, U =   0.000143504 FT/SEC
R=  1.2 IN.   ANGLE=   94.00 DEGREES   FO=   -6.0480  F1=    4.4404 F=   -6.1140   VELOCITY, U =   0.000144403 FT/SEC
R=  1.2 IN.   ANGLE=   93.00 DEGREES   FO=   -6.0826  F1=    4.0142 F=   -6.1422   VELOCITY, U =   0.000145068 FT/SEC
R=  1.2 IN.   ANGLE=   92.00 DEGREES   FO=   -6.1072  F1=    3.5831 F=   -6.1605   VELOCITY, U =   0.000145500 FT/SEC
R=  1.2 IN.   ANGLE=   91.00 DEGREES   FO=   -6.1221  F1=    3.1474 F=   -6.1688   VELOCITY, U =   0.000145697 FT/SEC
R=  1.2 IN.   ANGLE=   90.00 DEGREES   FO=   -6.1270  F1=    2.7077 F=   -6.1672   VELOCITY, U =   0.000145660 FT/SEC
R=  1.2 IN.   ANGLE=   89.00 DEGREES   FO=   -6.1221  F1=    2.2644 F=   -6.1557   VELOCITY, U =   0.000145387 FT/SEC
R=  1.2 IN.   ANGLE=   88.00 DEGREES   FO=   -6.1072  F1=    1.8181 F=   -6.1342   VELOCITY, U =   0.000144881 FT/SEC
R=  1.2 IN.   ANGLE=   87.00 DEGREES   FO=   -6.0826  F1=    1.3694 F=   -6.1029   VELOCITY, U =   0.000144141 FT/SEC
R=  1.2 IN.   ANGLE=   86.00 DEGREES   FO=   -6.0480  F1=    0.9190 F=   -6.0617   VELOCITY, U =   0.000143167 FT/SEC
R=  1.2 IN.   ANGLE=   85.00 DEGREES   FO=   -6.0037  F1=    0.4675 F=   -6.0107   VELOCITY, U =   0.000141963 FT/SEC
R=  1.2 IN.   ANGLE=   84.00 DEGREES   FO=   -5.9497  F1=    0.0156 F=   -5.9500   VELOCITY, U =   0.000140528 FT/SEC
R=  1.2 IN.   ANGLE=   83.00 DEGREES   FO=   -5.8860  F1=   -0.4359 F=   -5.8795   VELOCITY, U =   0.000138865 FT/SEC
R=  1.2 IN.   ANGLE=   82.00 DEGREES   FO=   -5.8127  F1=   -0.8862 F=   -5.7996   VELOCITY, U =   0.000136976 FT/SEC
R=  1.2 IN.   ANGLE=   81.00 DEGREES   FO=   -5.7299  F1=   -1.3346 F=   -5.7101   VELOCITY, U =   0.000134864 FT/SEC
R=  1.2 IN.   ANGLE=   80.00 DEGREES   FO=   -5.6377  F1=   -1.7801 F=   -5.6113   VELOCITY, U =   0.000132530 FT/SEC
R=  1.2 IN.   ANGLE=   79.00 DEGREES   FO=   -5.5363  F1=   -2.2220 F=   -5.5033   VELOCITY, U =   0.000129978 FT/SEC
R=  1.2 IN.   ANGLE=   78.00 DEGREES   FO=   -5.4256  F1=   -2.6593 F=   -5.3861   VELOCITY, U =   0.000127212 FT/SEC
R=  1.2 IN.   ANGLE=   77.00 DEGREES   FO=   -5.3060  F1=   -3.0911 F=   -5.2600   VELOCITY, U =   0.000124234 FT/SEC
R=  1.2 IN.   ANGLE=   76.00 DEGREES   FO=   -5.1774  F1=   -3.5166 F=   -5.1252   VELOCITY, U =   0.000121048 FT/SEC
R=  1.2 IN.   ANGLE=   75.00 DEGREES   FO=   -5.0401  F1=   -3.9348 F=   -4.9817   VELOCITY, U =   0.000117659 FT/SEC
R=  1.2 IN.   ANGLE=   74.00 DEGREES   FO=   -4.8943  F1=   -4.3448 F=   -4.8297   VELOCITY, U =   0.000114071 FT/SEC
R=  1.2 IN.   ANGLE=   73.00 DEGREES   FO=   -4.7401  F1=   -4.7455 F=   -4.6696   VELOCITY, U =   0.000110267 FT/SEC
R=  1.2 IN.   ANGLE=   72.00 DEGREES   FO=   -4.5776  F1=   -5.1362 F=   -4.5013   VELOCITY, U =   0.000106314 FT/SEC
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R=	1,2 IN.	ANGLE=	71,00 DEGREES	FO=	-4,4072	F1=	-5,5157	F=	-4,3253	VELOCITY, U =	0,000102156 FT/SEC
R=	1,2 IN.	ANGLE=	70,00 DEGREES	FO=	-4,2290	F1=	-5,8833	F=	-4,1416	VELOCITY, U =	0,000097818 FT/SEC
R=	1,2 IN.	ANGLE=	69,00 DEGREES	FO=	-4,0432	F1=	-6,2380	F=	-3,9506	VELOCITY, U =	0,000093306 FT/SEC
R=	1,2 IN.	ANGLE=	68,00 DEGREES	FO=	-3,8501	F1=	-6,5789	F=	-3,7524	VELOCITY, U =	0,000088626 FT/SEC
R=	1,2 IN.	ANGLE=	67,00 DEGREES	FO=	-3,6499	F1=	-6,9051	F=	-3,5473	VELOCITY, U =	0,000083782 FT/SEC
R=	1,2 IN.	ANGLE=	66,00 DEGREES	FO=	-3,4428	F1=	-7,2158	F=	-3,3356	VELOCITY, U =	0,000078782 FT/SEC
R=	1,2 IN.	ANGLE=	65,00 DEGREES	FO=	-3,2291	F1=	-7,5101	F=	-3,1175	VELOCITY, U =	0,000073631 FT/SEC
R=	1,2 IN.	ANGLE=	64,00 DEGREES	FO=	-3,0090	F1=	-7,7873	F=	-2,8933	VELOCITY, U =	0,000068336 FT/SEC
R=	1,2 IN.	ANGLE=	63,00 DEGREES	FO=	-2,7829	F1=	-8,0465	F=	-2,6634	VELOCITY, U =	0,000062904 FT/SEC
R=	1,2 IN.	ANGLE=	62,00 DEGREES	FO=	-2,5509	F1=	-8,2872	F=	-2,4278	VELOCITY, U =	0,000057341 FT/SEC
R=	1,2 IN.	ANGLE=	61,00 DEGREES	FO=	-2,3134	F1=	-8,5085	F=	-2,1870	VELOCITY, U =	0,000051655 FT/SEC
R=	1,2 IN.	ANGLE=	60,00 DEGREES	FO=	-2,0707	F1=	-8,7099	F=	-1,9413	VELOCITY, U =	0,000045851 FT/SEC
R=	1,2 IN.	ANGLE=	59,00 DEGREES	FO=	-1,8230	F1=	-8,8908	F=	-1,6910	VELOCITY, U =	0,000039938 FT/SEC
R=	1,2 IN.	ANGLE=	58,00 DEGREES	FO=	-1,5707	F1=	-9,0508	F=	-1,4363	VELOCITY, U =	0,000033923 FT/SEC
R=	1,2 IN.	ANGLE=	57,00 DEGREES	FO=	-1,3141	F1=	-9,1892	F=	-1,1776	VELOCITY, U =	0,000027813 FT/SEC
R=	1,2 IN.	ANGLE=	56,00 DEGREES	FO=	-1,0534	F1=	-9,3059	F=	-0,9152	VELOCITY, U =	0,000021616 FT/SEC
R=	1,2 IN.	ANGLE=	55,00 DEGREES	FO=	-0,7891	F1=	-9,4003	F=	-0,6495	VELOCITY, U =	0,000015339 FT/SEC
R=	1,2 IN.	ANGLE=	54,00 DEGREES	FO=	-0,5213	F1=	-9,4723	F=	-0,3806	VELOCITY, U =	0,000008990 FT/SEC
R=	1,2 IN.	ANGLE=	53,00 DEGREES	FO=	-0,2506	F1=	-9,5217	F=	-0,1091	VELOCITY, U =	0,000002577 FT/SEC
R=	1,2 IN.	ANGLE=	52,00 DEGREES	FO=	0,0230	F1=	-9,5483	F=	0,1648	VELOCITY, U =	-0,000003892 FT/SEC
R=	1,2 IN.	ANGLE=	51,00 DEGREES	FO=	0,2989	F1=	-9,5521	F=	0,4408	VELOCITY, U =	-0,000010410 FT/SEC
R=	1,2 IN.	ANGLE=	50,00 DEGREES	FO=	0,5768	F1=	-9,5331	F=	0,7184	VELOCITY, U =	-0,000016969 FT/SEC
R=	1,2 IN.	ANGLE=	49,00 DEGREES	FO=	0,8565	F1=	-9,4915	F=	0,9975	VELOCITY, U =	-0,000023560 FT/SEC
R=	1,2 IN.	ANGLE=	48,00 DEGREES	FO=	1,1376	F1=	-9,4273	F=	1,2776	VELOCITY, U =	-0,000030175 FT/SEC
R=	1,2 IN.	ANGLE=	47,00 DEGREES	FO=	1,4197	F1=	-9,3408	F=	1,5584	VELOCITY, U =	-0,000036807 FT/SEC
R=	1,2 IN.	ANGLE=	46,00 DEGREES	FO=	1,7024	F1=	-9,2324	F=	1,8396	VELOCITY, U =	-0,000043448 FT/SEC
R=	1,2 IN.	ANGLE=	45,00 DEGREES	FO=	1,9856	F1=	-9,1024	F=	2,1208	VELOCITY, U =	-0,000050089 FT/SEC
R=	1,2 IN.	ANGLE=	44,00 DEGREES	FO=	2,2687	F1=	-8,9514	F=	2,4017	VELOCITY, U =	-0,000056723 FT/SEC
R=	1,2 IN.	ANGLE=	43,00 DEGREES	FO=	2,5515	F1=	-8,7799	F=	2,6819	VELOCITY, U =	-0,000063342 FT/SEC
R=	1,2 IN.	ANGLE=	42,00 DEGREES	FO=	2,8336	F1=	-8,5885	F=	2,9611	VELOCITY, U =	-0,000069937 FT/SEC
R=	1,2 IN.	ANGLE=	41,00 DEGREES	FO=	3,1146	F1=	-8,3779	F=	3,2391	VELOCITY, U =	-0,000076502 FT/SEC
R=	1,2 IN.	ANGLE=	40,00 DEGREES	FO=	3,3943	F1=	-8,1488	F=	3,5154	VELOCITY, U =	-0,000083027 FT/SEC
R=	1,2 IN.	ANGLE=	39,00 DEGREES	FO=	3,6723	F1=	-7,9022	F=	3,7897	VELOCITY, U =	-0,000089505 FT/SEC
R=	1,2 IN.	ANGLE=	38,00 DEGREES	FO=	3,9482	F1=	-7,6388	F=	4,0617	VELOCITY, U =	-0,000095930 FT/SEC
R=	1,2 IN.	ANGLE=	37,00 DEGREES	FO=	4,2217	F1=	-7,3597	F=	4,3310	VELOCITY, U =	-0,000102292 FT/SEC
R=	1,2 IN.	ANGLE=	36,00 DEGREES	FO=	4,4925	F1=	-7,0659	F=	4,5975	VELOCITY, U =	-0,000108584 FT/SEC
R=	1,2 IN.	ANGLE=	35,00 DEGREES	FO=	4,7602	F1=	-6,7585	F=	4,8606	VELOCITY, U =	-0,000114800 FT/SEC
R=	1,2 IN.	ANGLE=	34,00 DEGREES	FO=	5,0246	F1=	-6,4385	F=	5,1202	VELOCITY, U =	-0,000120932 FT/SEC
R=	1,2 IN.	ANGLE=	33,00 DEGREES	FO=	5,2853	F1=	-6,1072	F=	5,3760	VELOCITY, U =	-0,000126972 FT/SEC
R=	1,2 IN.	ANGLE=	32,00 DEGREES	FO=	5,5419	F1=	-5,7658	F=	5,6275	VELOCITY, U =	-0,000132913 FT/SEC
R=	1,2 IN.	ANGLE=	31,00 DEGREES	FO=	5,7942	F1=	-5,4156	F=	5,8746	VELOCITY, U =	-0,000138749 FT/SEC
R=	1,2 IN.	ANGLE=	30,00 DEGREES	FO=	6,0419	F1=	-5,0578	F=	6,1170	VELOCITY, U =	-0,000144473 FT/SEC
R=	1,2 IN.	ANGLE=	29,00 DEGREES	FO=	6,2846	F1=	-4,6938	F=	6,3543	VELOCITY, U =	-0,000150078 FT/SEC
R=	1,2 IN.	ANGLE=	28,00 DEGREES	FO=	6,5221	F1=	-4,3249	F=	6,5863	VELOCITY, U =	-0,000155558 FT/SEC
R=	1,2 IN.	ANGLE=	27,00 DEGREES	FO=	6,7540	F1=	-3,9525	F=	6,8127	VELOCITY, U =	-0,000160906 FT/SEC
R=	1,2 IN.	ANGLE=	26,00 DEGREES	FO=	6,9802	F1=	-3,5780	F=	7,0333	VELOCITY, U =	-0,000166116 FT/SEC
R=	1,2 IN.	ANGLE=	25,00 DEGREES	FO=	7,2002	F1=	-3,2028	F=	7,2478	VELOCITY, U =	-0,000171182 FT/SEC
R=	1,2 IN.	ANGLE=	24,00 DEGREES	FO=	7,4139	F1=	-2,8283	F=	7,4560	VELOCITY, U =	-0,000176098 FT/SEC
R=	1,2 IN.	ANGLE=	23,00 DEGREES	FO=	7,6210	F1=	-2,4560	F=	7,6575	VELOCITY, U =	-0,000180858 FT/SEC
R=	1,2 IN.	ANGLE=	22,00 DEGREES	FO=	7,8213	F1=	-2,0871	F=	7,8523	VELOCITY, U =	-0,000185458 FT/SEC
R=	1,2 IN.	ANGLE=	21,00 DEGREES	FO=	8,0144	F1=	-1,7232	F=	8,0400	VELOCITY, U =	-0,000189891 FT/SEC
R=	1,2 IN.	ANGLE=	20,00 DEGREES	FO=	8,2002	F1=	-1,3655	F=	8,2204	VELOCITY, U =	-0,000194154 FT/SEC
R=	1,2 IN.	ANGLE=	19,00 DEGREES	FO=	8,3784	F1=	-1,0155	F=	8,3935	VELOCITY, U =	-0,000198240 FT/SEC
R=	1,2 IN.	ANGLE=	18,00 DEGREES	FO=	8,5488	F1=	-0,6744	F=	8,5588	VELOCITY, U =	-0,000202145 FT/SEC
R=	1,2 IN.	ANGLE=	17,00 DEGREES	FO=	8,7112	F1=	-0,3436	F=	8,7163	VELOCITY, U =	-0,000205865 FT/SEC
R=	1,2 IN.	ANGLE=	16,00 DEGREES	FO=	8,8654	F1=	-0,0243	F=	8,8658	VELOCITY, U =	-0,000209396 FT/SEC
R=	1,2 IN.	ANGLE=	15,00 DEGREES	FO=	9,0113	F1=	0,2823	F=	9,0071	VELOCITY, U =	-0,000212732 FT/SEC
R=	1,2 IN.	ANGLE=	14,00 DEGREES	FO=	9,1485	F1=	0,5750	F=	9,1400	VELOCITY, U =	-0,000215872 FT/SEC
R=	1,2 IN.	ANGLE=	13,00 DEGREES	FO=	9,2771	F1=	0,8526	F=	9,2644	VELOCITY, U =	-0,000218811 FT/SEC
R=	1,2 IN.	ANGLE=	12,00 DEGREES	FO=	9,3968	F1=	1,1142	F=	9,3802	VELOCITY, U =	-0,000221546 FT/SEC
R=	1,2 IN.	ANGLE=	11,00 DEGREES	FO=	9,5074	F1=	1,3588	F=	9,4872	VELOCITY, U =	-0,000224073 FT/SEC

R (IN.)	ANGLE (DEGREES)	F0	F1	F	VELOCITY, U (FT/SEC)
1,2	10,00	9,6089	1,5853	9,5853	-0,000226390
1,2	9,00	9,7011	1,7930	9,6745	-0,000228495
1,2	8,00	9,7839	1,9810	9,7544	-0,000230384
1,2	7,00	9,8572	2,1486	9,8252	-0,000232057
1,2	6,00	9,9209	2,2952	9,8868	-0,000233510
1,2	5,00	9,9749	2,4202	9,9389	-0,000234742
1,2	4,00	10,0192	2,5232	9,9817	-0,000235752
1,2	3,00	10,0537	2,6037	10,0150	-0,000236539
1,2	2,00	10,0784	2,6614	10,0388	-0,000237101
1,2	1,00	10,0932	2,6961	10,0532	-0,000237439
1,2	0,00	10,0981	2,7077	10,0579	-0,000237552
3,0	128,73	0,2234	15,1450	-0,0016	0,000000015
3,0	127,00	-0,2506	14,8007	-0,4704	0,000004444
3,0	126,00	-0,5213	14,5918	-0,7381	0,000006973
3,0	125,00	-0,7891	14,3763	-1,0026	0,000009472
3,0	124,00	-1,0534	14,1542	-1,2637	0,000011939
3,0	123,00	-1,3141	13,9258	-1,5210	0,000014369
3,0	122,00	-1,5707	13,6910	-1,7741	0,000016761
3,0	121,00	-1,8230	13,4500	-2,0228	0,000019110
3,0	120,00	-2,0707	13,2027	-2,2668	0,000021416
3,0	119,00	-2,3134	12,9493	-2,5058	0,000023673
3,0	118,00	-2,5509	12,6896	-2,7394	0,000025880
3,0	117,00	-2,7829	12,4237	-2,9674	0,000028034
3,0	116,00	-3,0090	12,1514	-3,1895	0,000030133
3,0	115,00	-3,2291	11,8727	-3,4054	0,000032172
3,0	114,00	-3,4428	11,5876	-3,6149	0,000034151
3,0	113,00	-3,6499	11,2960	-3,8177	0,000036067
3,0	112,00	-3,8501	10,9977	-4,0135	0,000037917
3,0	111,00	-4,0432	10,6927	-4,2021	0,000039698
3,0	110,00	-4,2290	10,3808	-4,3832	0,000041410
3,0	109,00	-4,4072	10,0621	-4,5567	0,000043049
3,0	108,00	-4,5776	9,7363	-4,7223	0,000044613
3,0	107,00	-4,7401	9,4035	-4,8797	0,000046101
3,0	106,00	-4,8943	9,0636	-5,0289	0,000047510
3,0	105,00	-5,0401	8,7165	-5,1696	0,000048839
3,0	104,00	-5,1774	8,3622	-5,3016	0,000050066
3,0	103,00	-5,3060	8,0008	-5,4248	0,000051250
3,0	102,00	-5,4256	7,6322	-5,5390	0,000052329
3,0	101,00	-5,5363	7,2565	-5,6441	0,000053321
3,0	100,00	-5,6377	6,8739	-5,7399	0,000054226
3,0	99,00	-5,7299	6,4844	-5,8263	0,000055043
3,0	98,00	-5,8127	6,0881	-5,9032	0,000055769
3,0	97,00	-5,8860	5,6854	-5,9705	0,000056405
3,0	96,00	-5,9497	5,2763	-6,0281	0,000056950
3,0	95,00	-6,0037	4,8612	-6,0760	0,000057402
3,0	94,00	-6,0480	4,4404	-6,1140	0,000057761
3,0	93,00	-6,0826	4,0142	-6,1422	0,000058027
3,0	92,00	-6,1072	3,5831	-6,1605	0,000058200
3,0	91,00	-6,1221	3,1474	-6,1688	0,000058279
3,0	90,00	-6,1270	2,7077	-6,1672	0,000058264
3,0	89,00	-6,1221	2,2644	-6,1557	0,000058155
3,0	88,00	-6,1072	1,8181	-6,1342	0,000057952
3,0	87,00	-6,0826	1,3694	-6,1029	0,000057656
3,0	86,00	-6,0480	0,9190	-6,0617	0,000057267
3,0	85,00	-6,0037	0,4675	-6,0107	0,000056785
3,0	84,00	-5,9497	0,0156	-5,9500	0,000056211
3,0	83,00	-5,8860	-0,4359	-5,8795	0,000055546
3,0	82,00	-5,8127	-0,8862	-5,7996	0,000054791
3,0	81,00	-5,7299	-1,3346	-5,7101	0,000053945
3,0	80,00	-5,6377	-1,7801	-5,6113	0,000053012
3,0	79,00	-5,5363	-2,2220	-5,5033	0,000051991

R=	3,0 IN.	ANGLE=	78,00 DEGREES	FO=	-5,4256	F1=	-2,6593	F=	-5,3861	VELOCITY, U =	0,000050885 FT/SEC
R=	3,0 IN.	ANGLE=	77,00 DEGREES	FO=	-5,3060	F1=	-3,0911	F=	-5,2600	VELOCITY, U =	0,000049693 FT/SEC
R=	3,0 IN.	ANGLE=	76,00 DEGREES	FO=	-5,1774	F1=	-3,5166	F=	-5,1252	VELOCITY, U =	0,000048419 FT/SEC
R=	3,0 IN.	ANGLE=	75,00 DEGREES	FO=	-5,0401	F1=	-3,9348	F=	-4,9817	VELOCITY, U =	0,000047064 FT/SEC
R=	3,0 IN.	ANGLE=	74,00 DEGREES	FO=	-4,8943	F1=	-4,3448	F=	-4,8297	VELOCITY, U =	0,000045628 FT/SEC
R=	3,0 IN.	ANGLE=	73,00 DEGREES	FO=	-4,7401	F1=	-4,7455	F=	-4,6696	VELOCITY, U =	0,000044115 FT/SEC
R=	3,0 IN.	ANGLE=	72,00 DEGREES	FO=	-4,5776	F1=	-5,1362	F=	-4,5013	VELOCITY, U =	0,000042526 FT/SEC
R=	3,0 IN.	ANGLE=	71,00 DEGREES	FO=	-4,4072	F1=	-5,5157	F=	-4,3253	VELOCITY, U =	0,000040863 FT/SEC
R=	3,0 IN.	ANGLE=	70,00 DEGREES	FO=	-4,2290	F1=	-5,8833	F=	-4,1416	VELOCITY, U =	0,000039127 FT/SEC
R=	3,0 IN.	ANGLE=	69,00 DEGREES	FO=	-4,0432	F1=	-6,2380	F=	-3,9506	VELOCITY, U =	0,000037323 FT/SEC
R=	3,0 IN.	ANGLE=	68,00 DEGREES	FO=	-3,8501	F1=	-6,5789	F=	-3,7524	VELOCITY, U =	0,000035450 FT/SEC
R=	3,0 IN.	ANGLE=	67,00 DEGREES	FO=	-3,6499	F1=	-6,9051	F=	-3,5473	VELOCITY, U =	0,000033513 FT/SEC
R=	3,0 IN.	ANGLE=	66,00 DEGREES	FO=	-3,4428	F1=	-7,2158	F=	-3,3356	VELOCITY, U =	0,000031513 FT/SEC
R=	3,0 IN.	ANGLE=	65,00 DEGREES	FO=	-3,2291	F1=	-7,5101	F=	-3,1175	VELOCITY, U =	0,000029452 FT/SEC
R=	3,0 IN.	ANGLE=	64,00 DEGREES	FO=	-3,0090	F1=	-7,7873	F=	-2,8933	VELOCITY, U =	0,000027334 FT/SEC
R=	3,0 IN.	ANGLE=	63,00 DEGREES	FO=	-2,7829	F1=	-8,0465	F=	-2,6634	VELOCITY, U =	0,000025162 FT/SEC
R=	3,0 IN.	ANGLE=	62,00 DEGREES	FO=	-2,5509	F1=	-8,2872	F=	-2,4278	VELOCITY, U =	0,000022936 FT/SEC
R=	3,0 IN.	ANGLE=	61,00 DEGREES	FO=	-2,3134	F1=	-8,5085	F=	-2,1870	VELOCITY, U =	0,000020662 FT/SEC
R=	3,0 IN.	ANGLE=	60,00 DEGREES	FO=	-2,0707	F1=	-8,7099	F=	-1,9413	VELOCITY, U =	0,000018340 FT/SEC
R=	3,0 IN.	ANGLE=	59,00 DEGREES	FO=	-1,8230	F1=	-8,8908	F=	-1,6910	VELOCITY, U =	0,000015975 FT/SEC
R=	3,0 IN.	ANGLE=	58,00 DEGREES	FO=	-1,5707	F1=	-9,0508	F=	-1,4363	VELOCITY, U =	0,000013569 FT/SEC
R=	3,0 IN.	ANGLE=	57,00 DEGREES	FO=	-1,3141	F1=	-9,1892	F=	-1,1776	VELOCITY, U =	0,000011125 FT/SEC
R=	3,0 IN.	ANGLE=	56,00 DEGREES	FO=	-1,0534	F1=	-9,3059	F=	-0,9152	VELOCITY, U =	0,000008646 FT/SEC
R=	3,0 IN.	ANGLE=	55,00 DEGREES	FO=	-0,7891	F1=	-9,4003	F=	-0,6495	VELOCITY, U =	0,000006136 FT/SEC
R=	3,0 IN.	ANGLE=	54,00 DEGREES	FO=	-0,5213	F1=	-9,4723	F=	-0,3806	VELOCITY, U =	0,000003596 FT/SEC
R=	3,0 IN.	ANGLE=	53,00 DEGREES	FO=	-0,2506	F1=	-9,5217	F=	-0,1091	VELOCITY, U =	0,000001031 FT/SEC
R=	3,0 IN.	ANGLE=	52,00 DEGREES	FO=	0,0230	F1=	-9,5483	F=	0,1648	VELOCITY, U =	-0,000001557 FT/SEC
R=	3,0 IN.	ANGLE=	51,00 DEGREES	FO=	0,2989	F1=	-9,5521	F=	0,4408	VELOCITY, U =	-0,000004164 FT/SEC
R=	3,0 IN.	ANGLE=	50,00 DEGREES	FO=	0,5768	F1=	-9,5331	F=	0,7184	VELOCITY, U =	-0,000006787 FT/SEC
R=	3,0 IN.	ANGLE=	49,00 DEGREES	FO=	0,8565	F1=	-9,4915	F=	0,9975	VELOCITY, U =	-0,000009424 FT/SEC
R=	3,0 IN.	ANGLE=	48,00 DEGREES	FO=	1,1376	F1=	-9,4273	F=	1,2776	VELOCITY, U =	-0,000012070 FT/SEC
R=	3,0 IN.	ANGLE=	47,00 DEGREES	FO=	1,4197	F1=	-9,3408	F=	1,5584	VELOCITY, U =	-0,000014723 FT/SEC
R=	3,0 IN.	ANGLE=	46,00 DEGREES	FO=	1,7024	F1=	-9,2324	F=	1,8396	VELOCITY, U =	-0,000017379 FT/SEC
R=	3,0 IN.	ANGLE=	45,00 DEGREES	FO=	1,9856	F1=	-9,1024	F=	2,1208	VELOCITY, U =	-0,000020036 FT/SEC
R=	3,0 IN.	ANGLE=	44,00 DEGREES	FO=	2,2687	F1=	-8,9514	F=	2,4017	VELOCITY, U =	-0,000022689 FT/SEC
R=	3,0 IN.	ANGLE=	43,00 DEGREES	FO=	2,5515	F1=	-8,7799	F=	2,6819	VELOCITY, U =	-0,000025337 FT/SEC
R=	3,0 IN.	ANGLE=	42,00 DEGREES	FO=	2,8336	F1=	-8,5885	F=	2,9611	VELOCITY, U =	-0,000027975 FT/SEC
R=	3,0 IN.	ANGLE=	41,00 DEGREES	FO=	3,1146	F1=	-8,3779	F=	3,2391	VELOCITY, U =	-0,000030601 FT/SEC
R=	3,0 IN.	ANGLE=	40,00 DEGREES	FO=	3,3943	F1=	-8,1488	F=	3,5154	VELOCITY, U =	-0,000033211 FT/SEC
R=	3,0 IN.	ANGLE=	39,00 DEGREES	FO=	3,6723	F1=	-7,9022	F=	3,7897	VELOCITY, U =	-0,000035802 FT/SEC
R=	3,0 IN.	ANGLE=	38,00 DEGREES	FO=	3,9482	F1=	-7,6388	F=	4,0617	VELOCITY, U =	-0,000038372 FT/SEC
R=	3,0 IN.	ANGLE=	37,00 DEGREES	FO=	4,2217	F1=	-7,3597	F=	4,3310	VELOCITY, U =	-0,000040917 FT/SEC
R=	3,0 IN.	ANGLE=	36,00 DEGREES	FO=	4,4925	F1=	-7,0659	F=	4,5975	VELOCITY, U =	-0,000043434 FT/SEC
R=	3,0 IN.	ANGLE=	35,00 DEGREES	FO=	4,7602	F1=	-6,7585	F=	4,8606	VELOCITY, U =	-0,000045920 FT/SEC
R=	3,0 IN.	ANGLE=	34,00 DEGREES	FO=	5,0246	F1=	-6,4385	F=	5,1202	VELOCITY, U =	-0,000048373 FT/SEC
R=	3,0 IN.	ANGLE=	33,00 DEGREES	FO=	5,2853	F1=	-6,1072	F=	5,3760	VELOCITY, U =	-0,000050789 FT/SEC
R=	3,0 IN.	ANGLE=	32,00 DEGREES	FO=	5,5419	F1=	-5,7658	F=	5,6275	VELOCITY, U =	-0,000053165 FT/SEC
R=	3,0 IN.	ANGLE=	31,00 DEGREES	FO=	5,7942	F1=	-5,4156	F=	5,8746	VELOCITY, U =	-0,000055500 FT/SEC
R=	3,0 IN.	ANGLE=	30,00 DEGREES	FO=	6,0419	F1=	-5,0578	F=	6,1170	VELOCITY, U =	-0,000057789 FT/SEC
R=	3,0 IN.	ANGLE=	29,00 DEGREES	FO=	6,2846	F1=	-4,6938	F=	6,3543	VELOCITY, U =	-0,000060031 FT/SEC
R=	3,0 IN.	ANGLE=	28,00 DEGREES	FO=	6,5221	F1=	-4,3249	F=	6,5863	VELOCITY, U =	-0,000062223 FT/SEC
R=	3,0 IN.	ANGLE=	27,00 DEGREES	FO=	6,7540	F1=	-3,9525	F=	6,8127	VELOCITY, U =	-0,000064362 FT/SEC
R=	3,0 IN.	ANGLE=	26,00 DEGREES	FO=	6,9802	F1=	-3,5780	F=	7,0333	VELOCITY, U =	-0,000066446 FT/SEC
R=	3,0 IN.	ANGLE=	25,00 DEGREES	FO=	7,2002	F1=	-3,2028	F=	7,2478	VELOCITY, U =	-0,000068473 FT/SEC
R=	3,0 IN.	ANGLE=	24,00 DEGREES	FO=	7,4139	F1=	-2,8283	F=	7,4560	VELOCITY, U =	-0,000070439 FT/SEC
R=	3,0 IN.	ANGLE=	23,00 DEGREES	FO=	7,6210	F1=	-2,4560	F=	7,6575	VELOCITY, U =	-0,000072343 FT/SEC
R=	3,0 IN.	ANGLE=	22,00 DEGREES	FO=	7,8213	F1=	-2,0871	F=	7,8523	VELOCITY, U =	-0,000074183 FT/SEC
R=	3,0 IN.	ANGLE=	21,00 DEGREES	FO=	8,0144	F1=	-1,7232	F=	8,0400	VELOCITY, U =	-0,000075957 FT/SEC
R=	3,0 IN.	ANGLE=	20,00 DEGREES	FO=	8,2002	F1=	-1,3655	F=	8,2204	VELOCITY, U =	-0,000077661 FT/SEC
R=	3,0 IN.	ANGLE=	19,00 DEGREES	FO=	8,3784	F1=	-1,0155	F=	8,3935	VELOCITY, U =	-0,000079296 FT/SEC
R=	3,0 IN.	ANGLE=	18,00 DEGREES	FO=	8,5488	F1=	-0,6744	F=	8,5588	VELOCITY, U =	[illegible] FT/SEC

R= 3,0 IN,	ANGLE= 17,00 DEGREES	F0=	8,7112	F1= -0,3436	F= 8,7163	VELOCITY, U =	-0,000082346 FT/SEC
R= 3,0 IN,	ANGLE= 16,00 DEGREES	F0=	8,8654	F1= -0,0243	F= 8,8658	VELOCITY, U =	-0,000083758 FT/SEC
R= 3,0 IN,	ANGLE= 15,00 DEGREES	F0=	9,0113	F1= 0,2823	F= 9,0071	VELOCITY, U =	-0,000085093 FT/SEC
R= 3,0 IN,	ANGLE= 14,00 DEGREES	F0=	9,1485	F1= 0,5750	F= 9,1400	VELOCITY, U =	-0,000086349 FT/SEC
R= 3,0 IN,	ANGLE= 13,00 DEGREES	F0=	9,2771	F1= 0,8526	F= 9,2644	VELOCITY, U =	-0,000087524 FT/SEC
R= 3,0 IN,	ANGLE= 12,00 DEGREES	F0=	9,3968	F1= 1,1142	F= 9,3802	VELOCITY, U =	-0,000088618 FT/SEC
R= 3,0 IN,	ANGLE= 11,00 DEGREES	F0=	9,5074	F1= 1,3588	F= 9,4872	VELOCITY, U =	-0,000089629 FT/SEC
R= 3,0 IN,	ANGLE= 10,00 DEGREES	F0=	9,6089	F1= 1,5853	F= 9,5853	VELOCITY, U =	-0,000090556 FT/SEC
R= 3,0 IN,	ANGLE= 9,00 DEGREES	F0=	9,7011	F1= 1,7930	F= 9,6745	VELOCITY, U =	-0,000091398 FT/SEC
R= 3,0 IN,	ANGLE= 8,00 DEGREES	F0=	9,7839	F1= 1,9810	F= 9,7544	VELOCITY, U =	-0,000092154 FT/SEC
R= 3,0 IN,	ANGLE= 7,00 DEGREES	F0=	9,8572	F1= 2,1486	F= 9,8252	VELOCITY, U =	-0,000092823 FT/SEC
R= 3,0 IN,	ANGLE= 6,00 DEGREES	F0=	9,9209	F1= 2,2952	F= 9,8868	VELOCITY, U =	-0,000093404 FT/SEC
R= 3,0 IN,	ANGLE= 5,00 DEGREES	F0=	9,9749	F1= 2,4202	F= 9,9389	VELOCITY, U =	-0,000093897 FT/SEC
R= 3,0 IN,	ANGLE= 4,00 DEGREES	F0=	10,0192	F1= 2,5232	F= 9,9817	VELOCITY, U =	-0,000094301 FT/SEC
R= 3,0 IN,	ANGLE= 3,00 DEGREES	F0=	10,0537	F1= 2,6037	F= 10,0150	VELOCITY, U =	-0,000094615 FT/SEC
R= 3,0 IN,	ANGLE= 2,00 DEGREES	F0=	10,0784	F1= 2,6614	F= 10,0388	VELOCITY, U =	-0,000094841 FT/SEC
R= 3,0 IN,	ANGLE= 1,00 DEGREES	F0=	10,0932	F1= 2,6961	F= 10,0532	VELOCITY, U =	-0,000094976 FT/SEC
R= 3,0 IN,	ANGLE= 0,00 DEGREES	F0=	10,0981	F1= 2,7077	F= 10,0579	VELOCITY, U =	-0,000095021 FT/SEC