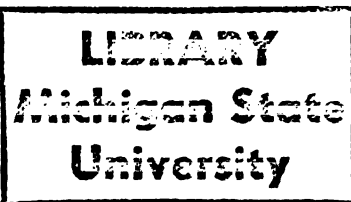




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**ANALYSIS OF PLATE EFFICIENCIES AND ENERGY
REQUIREMENTS FOR BAFFLE TOWERS IN
COUNTERCURRENT AND CONCURRENT
OPERATION**

BY

JANICE LYNN KING

A THESIS

**Submitted to
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1984

ABSTRACT

ANAYLSIS OF PLATE EFFICIENCIES AND ENERGY REQUIREMENTS FOR BAFFLE TOWERS IN COUNTERCURRENT AND CONCURRENT OPERATION

by

Janice Lynn King

Equations are developed leading to a more rigorous mathematical analysis of baffle plate extraction towers than has been previously seen in the open literature. Analytical solutions are used to set bounds upon the amount of mass transfer between fluids in countercurrent contact; and the results seem to correlate well with experimental observations. One result is then used to provide a new extension to the Graetz Problem.

Our derived equation for mass transfer to nondispersed countercurrent flow is

$$N_{SH} = \frac{1.400}{Ne} (N_{RE} N_{SC} B/L)^{1/3} \exp (K_f \ln Ne)$$

where N_{SH} is the local Sherwood number, B and L are plate dimensions; Ne is the Nernst distribution coefficient; and K_f is a constant described in the paper.

It is shown that most of the mass transfer for countercurrent liquid extraction comes from transitory drops located in the spout region of the plate. It is also demonstrated that concurrent flow is preferable to countercurrent in regions of slowly diffusing species, narrow plate spacings, large diameter towers and high throughputs.

ACKNOWLEDGEMENTS

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CHAPTER ONE

INTRODUCTION

The most common types of liquid-liquid extraction devices are mixer settlers and agitated differential columns, which are operated in countercurrent mode. These extractor designs have served admirably for many years. However, they continue to have problems with limited throughputs, inefficient energy use, and inability to process emulsifying streams.

The alternative non-agitated extractors include packed towers, perforated plate towers and spray towers. Countercurrent flow is maintained by gravity operating on the density difference between the two fluids. These devices have several advantages, but they still have limited throughput and an inability to process suspensions or precipitating fluids. Beyond these problems, spray towers also suffer from severe axial dispersion. By contrast, the baffle plate tower (another non-agitated extractor) has low energy cost, low maintenance cost, and low capital cost. Also, baffle towers can often process surfactants, suspensions, or emulsions. The primary disadvantage of the baffle tower is its low stage efficiency, typically less than 10%, Morello and Poffenburger (1950) and Cooper (1984). We choose to analyze the operation of the side-to-side,

countercurrent, baffle tower. We also analyze a novel variant of this tower, which is designed for operation with concurrent flow. These towers are shown in Figure 1.

Our analysis joins a growing literature involving concurrent flows in fluidized beds (Gawronski and Roszak, 1979), packed towers (Hutton and Leung, 1974; Leacock and Churchill, 1961; Specchie, Sicardi and Gianetto, 1974; Rao, Ananth and Varma, 1983), reciprocating plate towers (Noh and Baird, 1984), and pipeline flow (Watkinson and Cavers, 1967). Our theoretical treatment of the baffle tower is more rigorously mathematical than previously seen in the open literature, and our results correlate with published experimental observations (Watkinson and Cavers, 1967; Linton and Sherwood, 1950).

Our analysis of the countercurrent models can provide an extension to the "Graetz Problem" (laminar flow heat transfer in tubes). We consider variations of the boundary conditions at the fluid-fluid interface, and consider the cases of short contact times and small penetration depths. The results, given in Chapter two, are applicable for low rates of mass transfer between countercurrent fluids or for low rates of heat transfer between a fluid and a wall.

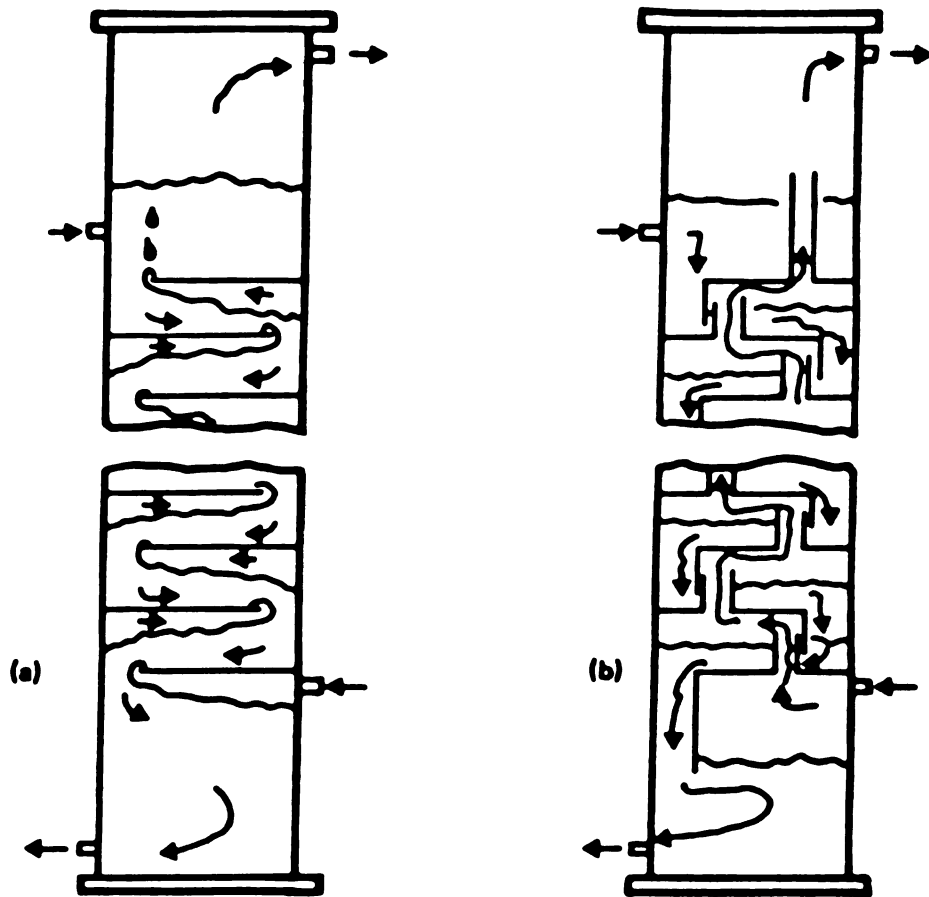


Figure 1. Baffle towers. (a) countercurrent

(b) pseudo concurrent

CHAPTER TWO

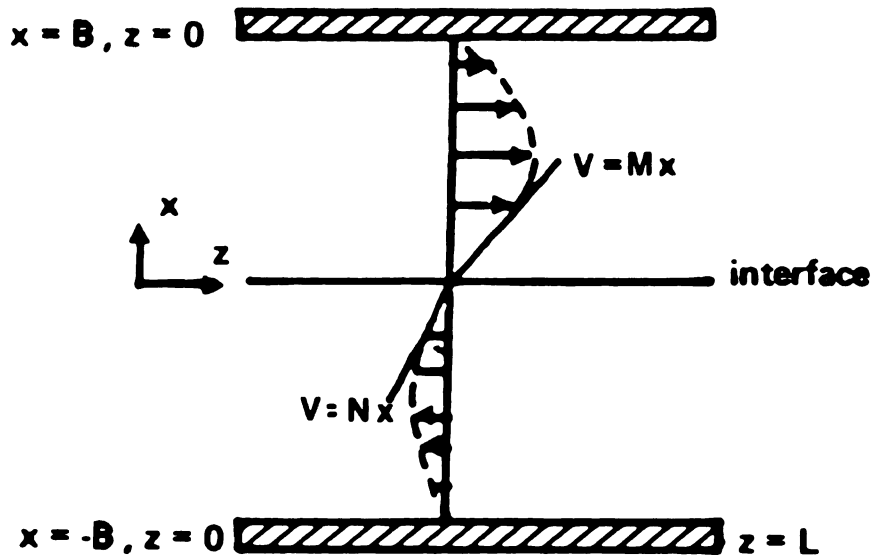
COUNTERCURRENT EXTRACTION

2.1 General Model for Mass Transfer

Consider the mass transfer between two laminar immiscible streams 1 and 2 flowing between two flat plates. The flow fields (countercurrent or concurrent) are fully developed as shown in Figure 2a and b, respectively. Solute "A" diffuses from fluid 2 into fluid 1, which is initially solute-free. Both fluids flow in the Z-direction. This analysis can be restricted to constant velocity profiles because the depth of penetration of "A" into fluid 1 is small in comparison with the film thickness; this is evidenced by the low experimental plate efficiencies and by a computer modelling study described in this chapter.

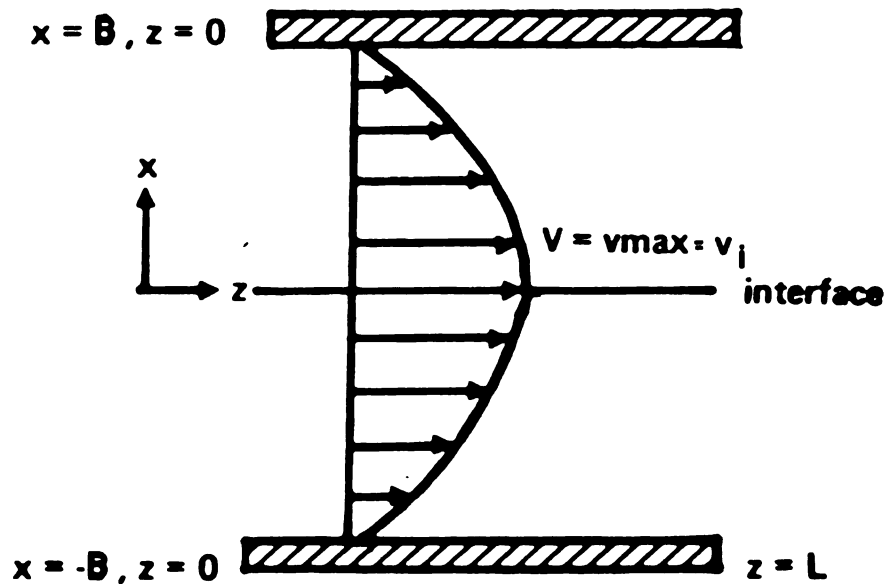
2.2. Countercurrent Model and Computer Model

For countercurrent flow, as shown in Figure 2a, fluids 1 and 2 have velocity gradients "M" and "N" respectively. Steady-state mass balances are written for both fluids with the interface boundary conditions set by equating the fluxes. Fluid 1 is considered to have a normalized concentration of zero at the entrance ($Z=0$), where the normalized concentration, C_1 , equals $(C_{in} - C(x,z))/C_{in}$.



Velocity profile of countercurrent case

(a)



Velocity profile of concurrent case

(b)

Figure 2. Flow fields of model for mass transfer

Similarly, fluid 2 is considered to have a normalized concentration of 1.0 when it enters the plate at $Z = 1$. The equations describing the mass transfer of "A" are as follows:

fluid 1,

$$Mx \frac{\partial C_1}{\partial Z} = D_1 \frac{\partial^2 C_1}{\partial X^2} \quad [2.1]$$

fluid 2,

$$Nx \frac{\partial C_2}{\partial Z} = D_2 \frac{\partial^2 C_2}{\partial X^2} \quad [2.2]$$

The boundary conditions are

at	$Z=0$	$C_1=0$	
at	$X=+d$	$C_1=0$	
at	$X=0$	$C_2=C_1/Ne$	
at	$X=0$	$D_1 \frac{\partial C_1}{\partial X} = D_2 \frac{\partial C_2}{\partial X}$	
at	$X=-d$	$C_2=1$	
at	$X=1$	$C_2=1$	

Here C_1 and C_2 are dimensionless concentrations; d is the laminar film thickness; L is the plate length; and Ne is the Nernst distribution coefficient which is

$$Ne = \frac{C_2}{C_1} \quad [2.3]$$

Here C_1 and C_2 denotes the molar concentrations of component "A" in fluids 1 and 2; and over the length of the plate, Ne is considered independent of the individual values of C_1 and C_2 .

A computer algorithm is used to solve this boundary-value problem for the mass transfer of "A" into fluid 1. We

use a "shooting method" iterative solution (Carnahan, Luther and Wilkes, 1969). A copy of our program is given in Table 2. Unsteady-state mass balances are written for both fluids and the equations describing the time variation of the concentration are:

$$\text{fluid 1,} \quad \frac{\partial C_1}{\partial t} = -MX \frac{\partial C_1}{\partial Z} + D_1 \frac{\partial^2 C_1}{\partial X^2} \quad [2.4]$$

$$\text{fluid 2,} \quad \frac{\partial C_2}{\partial t} = -NX \frac{\partial C_2}{\partial Z} + D_2 \frac{\partial^2 C_2}{\partial X^2} \quad [2.5]$$

These equations are solved simultaneously with initially assumed concentration profiles for C1 and C2 decaying to steady state solution through repeated forward iterations. At steady-state, the computer solution should equal the correct simultaneous solution of Eqs. [2.1] and [2.2]. A copy of the program is given in Table 2.

A check on the validity of this computer solution is that the convective flux at the plate exit must equal the diffusive flux across the interface

$$\int_0^L D_1 \frac{\partial C_1}{\partial X} dZ = \int_0^B M X C_1 (X,1) dX \quad [2.6]$$

Although the computer algorithm can accurately solve a given differential equation, it is found that for fast convergence, the program must be tailored to the particular Nernst distribution coefficient, diffusivities, and velocity

gradients (M or N). The computational procedure thus is not particularly suited towards design. Therefore, we shall consider two simplifications of the countercurrent equations approximating the interfacial concentration profiles of the two fluids; these predict different amounts of mass transfer. Eqs. [2.1] and [2.2] are solved separately, rather than simultaneously, and give analytical solutions, which are better suited for design purposes. The computer solution will be used to check the effect of our simplifications.

2.3. Approximate Solution 1: Upper Bound

The first simplification to Eqs. [2.1] and [2.2] assumes that the concentration profile at the interface is a constant.

For small penetration depths, Eq. [2.1] for the mass transfer of "A" into fluid 1 becomes

$$\beta \frac{\partial C_1}{\partial Z} = \frac{\partial^2 C_1}{\partial X^2} \quad [2.7]$$

where $\beta = \frac{M}{D_1} \quad [2.8]$

and the differential equation for fluid 2 is the same as Eq. [2.2]. The boundary conditions for this model are

at	$Z = 0$	$C_1 = 0$
at	$X = \infty$	$C_1 = 0$
at	$X = -\infty$	$C_2 = 1$
at	$Z = L$	$C_2 = 1$

where at $X = 0$ $C_1 = C_0$ and $C_2 = \frac{C_0}{N_e}$

N_e is the Nernst distribution coefficient; and C_0 is found by equating the fluxes at the interface. This situation is equivalent to a super-diffusing layer of fluid existing at the fluid-fluid interface. The effect of this very permeable layer of fluid is to increase transport in both X and Z directions. Thus, this approximation is expected to predict higher rates of mass transfer than the computer solution.

In order to equate the fluxes between the two fluids, and for further use, we will now derive a relationship between β (which equals M/D_1) and N/D_2 , the equivalent term in the other fluid. At the interface, the momentum transport is continuous between the two fluids:

$$\text{at } X=0 \quad \tau_1 = \tau_2 \quad [2.9]$$

where τ_1 and τ_2 are the interfacial shear stress tensors in fluids 1 and 2 respectively. This means that for Newtonian fluids:

$$\mu_1 \frac{dv_{z1}}{dx} = \mu_2 \frac{dv_{z2}}{dx} \quad [2.10]$$

or

$$\frac{\mu_1}{\mu_2} = \frac{N}{M} \quad [2.11]$$

where μ_1 and μ_2 are the viscosities of fluids 1 and 2 respectively; and V_{z1} and V_{z2} are the fluid velocities in the z-direction for fluids 1 and 2 respectively as from Eq. [2.16]. This allows a particularly nice simplification of Eq. [2.2] which employs a result of the Stokes-Einstein diffusivity correlation:

$$\frac{D_1}{D_2} = \frac{\mu_2}{\mu_1} \quad [2.12]$$

This relationship is considered valid for all non-polymeric fluids with approximately constant activity coefficients, Bidlock, Kett, Kelly and Anderson (1969). Combining Eqs. [2.11] and [2.12] gives

$$\beta = \frac{M}{D_1} = \frac{N}{D_2} \quad [2.13]$$

which suggests that the same value of β can be used for any two fluids in countercurrent contact.

The interfacial velocity gradient "M" is found from the following steady-state force balance. Since the flow rates are chosen such that the interface is a stagnant region, the fluid velocity will be considered zero at this boundary, as in Figure 4. The flow conditions are as follows:

$$W_L \tau_{xz} = W (B/2 - X) (P_0 - P_1) \quad [2.14]$$

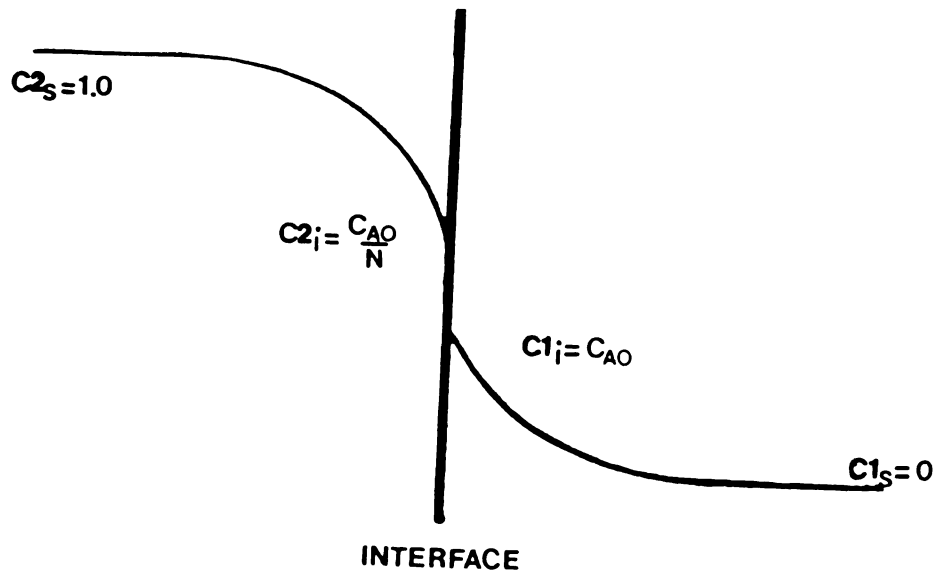


FIGURE 3. CONCENTRATION PROFILE

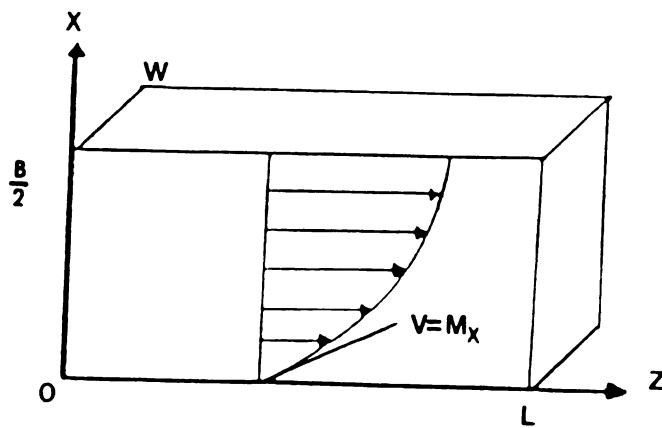


FIGURE 4. COUNTERCURRENT FLUID ELEMENT

where τ_{xz} is the stress tensor in the z-direction; B is the half-plate spacing; $(P_0 - P_1)$ is the head loss across the plate; L is the plate length; and W is the plate width. For a Newtonian fluid, Eq. [2.14] becomes

$$\frac{\mu dv_z}{dx} = \frac{(P_0 - P_1)}{L} (B/2 - x) \quad [2.15]$$

where the boundary condition follows:

$$x=0 \quad v_z=0$$

Here μ is the viscosity of the fluid. This is integrated to yield

$$v_z(x) = \frac{\Delta P B^2}{2L\mu} \left(x/B - (x/B)^2 \right) \quad [2.16]$$

and M is determined by solving for the velocity gradient at the interface.

$$M = \left(\frac{dv_z}{dx} \right)_{x=0} = \frac{\Delta P B}{2\mu L} \quad [2.17]$$

Given a value for β and a constant interfacial concentration, Kramers and Kreyger (1956) have solved Eq. [2.7] for the mass flux at the interface. Defining the averaged mass transfer rate as

$$\overline{Na}(z)_{x=0} = -D_1 \int_0^L \frac{\partial C_1}{\partial x} \bigg|_{x=0} dz \quad [2.18]$$

It is found that the concentration gradient of fluid 1 at the interface is

$$\left(\frac{\partial C_1}{\partial x} \right)_{x=0} = \frac{-C_0}{\Gamma(4/3)} (\beta/9L)^{1/3} \quad [2.19]$$

where C_0 is the constant interfacial concentration of A in fluid 1 at the interface.

The mass flux of "A" across the liquid-liquid interface for fluid 1 is

$$\overline{N_A(z)}_{x=0} = \frac{3 D_1}{2 \Gamma(4/3)} (\beta/9L)^{1/3} C_0 \quad [2.20]$$

where $\Gamma(4/3)$ equals 0.893. Similarly for fluid 2,

$$\overline{N_A(z)}_{x=0} = \frac{3 D_2}{2 \cdot 893} (\beta/9L)^{1/3} (1 - C_0/N) \quad [2.21]$$

From Eq. [2.20], the individual mass transfer coefficient averaged over the plate length, L , is defined by Skelland (1974) as

$$\overline{k_c} = \frac{\overline{N_A(z)}_{x=0}}{C_0} \quad [2.22]$$

where C_0 is found by equating the mass fluxes at the interface. Because β is the same for both fluids and because the mass flux, N_{A1} , is the same for both fluids, Eqs. [2.20] and [2.21] can be equated to show that

$$Co = \frac{Ne}{(NeD_1/D_2+1)} . \quad [2.23]$$

Dimensional analysis suggests that we can write \bar{k}_c in terms of dimensionless groups. Eq. [2.20] becomes

$$N_{SH} = 1.165 (N_{RE} N_{SC} B/L)^{1/3} \quad [2.24]$$

The local Sherwood number for countercurrent flow is defined as

$$N_{SH} = \frac{\bar{k}_c B}{D_1} \quad [2.25]$$

where N_{RE} is $(2B\langle V_1 \rangle \rho / \mu)$; N_{SC} is $(\mu / \rho D_1)$; ρ is the density; and μ is the viscosity.

2.4 Approximate Solution 2: Lower Bound

The second simplification to Eqs. [2.1] and [2.2] assumes that the concentration profile at the interface is linear, where C_1 is zero at the plate entrance ($z=0$) and C_1 is NeC_{20} at the plate exit ($z=L$). The linear interfacial concentration profile is expected to give a more conservative estimate of mass transfer than the constant concentration profile case of approximate solution one. The equations describing the mass transfer of "A" are the same as the previous case, but the boundary conditions are:

$$\begin{aligned}
&\text{at } z=0 & C_1=0 \\
&\text{at } x=\infty & C_1=0 \\
&\text{at } x=0 & C_1 = C_{20} \text{ Ne } z/L \\
&\text{at } x=0 & C_2 = C_{20} - C_{20} z/L \\
&\text{at } x=-\infty & C_2 = C_{20} \\
&\text{at } z=L & C_2 = C_{20}
\end{aligned}$$

Here C_{20} is the concentration of "A" in fluid 2 at its entrance ($z=L$). It is proposed that Eq. [2.7] can be solved by similarity transform; a proposed transform variable, y , is derived by standard techniques. But it is found that the similarity transform fails because this variable cannot transform the interfacial boundary conditions. If, however, Eq. [2.7] is differentiated with respect to z , it and its boundary conditions can be written

$$\beta x \frac{\partial G}{\partial z} = \frac{\partial^2 G}{\partial x^2} \quad [2.26]$$

$$G = \left(\frac{\partial C_1}{\partial z} \right)_x \quad [2.27]$$

where β is defined from Eq. [2.8]

$$\begin{aligned}
&\text{at } x = 0 & G = C_{20} \text{ Ne}/L \\
&\text{at } x = \infty & G = 0 \\
&\text{at } z = 0 & G = 0
\end{aligned}$$

The following similarity transform is now applicable to the equations and their boundary conditions.

$$y = (\beta / (9z))^{1/3}. \quad [2.28]$$

Eq. [2.26] is solved for the concentration profile, yielding

$$C_1 = C_{20} \text{Ne} \int_0^Z F(Y) dZ. \quad [2.29]$$

Where F is equal to $G/(\text{Ne } C_{20})$ and is a function of Y .

After substitution and some manipulations, Eq. [2.26]

becomes

$$\frac{d^2 F}{dY^2} + 3Y^2 \frac{dF}{dY} = 0 \quad [2.30]$$

with the following transformed boundary conditions

$$\begin{aligned} F &= 0 & \text{at } Z=0, & \quad Y = \infty \\ F &= 1/L & \text{at } X=0, & \quad Y=0 \\ F &= 0 & \text{at } X=\infty, & \quad Y = \infty. \end{aligned}$$

Integration of Eq. [2.30] twice gives a gamma function.

$$F = \int_0^Y a \exp(-Y^3) dY + b \quad [2.31]$$

where a and b are integration constants. This is integrated with respect to Z yielding an expansion for C_1 in terms of Z and Y .

$$C_1 = C_{20} \text{Ne} \left[\int_0^Z dZ \int_0^Y a \exp(-Y^3) dY + bZ + c \right] \quad [2.32]$$

$$c = 0$$

$$b = 1/L$$

and

$$a = \frac{-1/L}{\Gamma(4/3)} = \frac{-1/L}{.893}$$

To solve for C_1 , one expands the exponential term, and inserts the result into Eq. [2.32]. Integrating with respect to Y and Z gives

$$\frac{C_1}{N_e C_{20}} = \frac{-1/L}{.893} \int_0^Z dz \left(Y - \frac{Y^4}{4} + \frac{Y^7}{7 \times 2!} - \dots \right) + Z/L \quad [2.33]$$

$$\frac{C_1}{N_e C_{20}} = \frac{-1/L}{.893} \left[(X(\beta/9))^{1/3} \frac{Z^{2/3}}{(2/3)} + X^4 (\beta/9)^{4/3} \frac{Z^{-1/3}}{4(-1/3)} + \dots \right] \quad [2.34]$$

where the mass flux is determined by the first term of the bracketed series. As with the previous simplification, the average rate of mass transfer at the interface is defined by the following equation:

$$\overline{N_A}(z)_{x=0} = -D_1 \int_0^L \frac{\frac{\partial C}{\partial X} \big|_{X=0}}{L} dz \quad [2.35]$$

The local concentration gradient of "A" at the interface is found by differentiating Eq. [2.34] with respect to X .

$$\left(\frac{\partial C_1}{\partial X} \right)_{X=0} = \frac{-3}{2(.893)} (\beta Z^2/9)^{1/3} N_e C_{20} \quad [2.36]$$

The average mass flux of "A" across the liquid-liquid interface is found for fluid 1 by integrating Eq. [2.36]

$$\overline{Na1}(z)_{x=0} = \frac{9 D_1}{10 (.893)} (\beta / 9L)^{1/3} Ne C_{20} \quad [2.37]$$

It is found that the average mass flux of "A" for fluid 2 yields a different result.

$$\overline{Na2}(z)_{x=0} = -\frac{\overline{Na1}(z)_{x=0}}{Ne} \quad [2.38]$$

In magnitude, the calculation for the transfer is $1/Ne$ times the result from Eq. [2.37]. The difference arises because β is the same for both fluids (Eq. [2.13]), and because C_1 is equal to NeC_2 ; thus, the driving force in the second fluid is $1/Ne$ times the driving force of the first. Clearly, however, a different mass flux for fluids 1 and 2 is a physical impossibility. The results of Eqs. [2.37] and [2.38] could not be universally correct unless the Nernst distribution coefficient equals 1. However, the results of Eqs. [2.37] and [2.38] can be viewed as the upper and lower bounds on the countercurrent solution. Furthermore, the result of Eqs. [2.20] and [2.21] can also represent a bound on the correct solution. The use of these approximate solutions as solution bounds is not unreasonable, as is demonstrated by comparison with the computer solution in Table 1.

In a way similar to Eq. [2.22], the averaged mass transfer coefficient for fluid 1 can be expressed

$$\overline{kc} = \frac{\overline{Na(z)}_{x=0}}{Ca_1} \quad [2.39]$$

where $\overline{Ca_1}$ is the averaged interfacial concentration defined by $((Ca_{1in} - Ca_{1out})/2)$; and where \overline{kc} can be written in terms of standard dimensionless groups. The local Sherwood number for countercurrent flow becomes

$$N_{SH} = 1.400 (N_{RE} N_{SC} B/L)^{1/3} \quad [2.40]$$

where N_{SH} , N_{RE} and N_{SC} are defined from Eq. [2.24].

2.5. An Extension of the Graetz Problem

Sellars, Tribus and Klein (1956) present an analysis of the Graetz Problem (laminar-flow heat transfer in tubes) and present eigenvalue solutions to extensions of the Graetz Problem, which include variations of the fluid-wall boundary conditions. Their results for these boundary conditions are:

for constant wall temperature, the Nusselt number is

$$N_{NU} = 1.357 (N_{RE} N_{PR} r_w/X)^{1/3} \quad [2.41]$$

for linear wall temperature, the Nusselt number is

$$N_{NU} = 2.035 (N_{RE} N_{PR} r_w/X)^{1/3} \quad [2.42]$$

where N_{NU} is the local Nusselt number of the fluid at a distance X from the wall; and r_w is the tube radius.

Approximate solutions 1 and 2 analyze the cases of constant concentration at the interface and linear varying concentration at the interface respectively. Since the mechanism of diffusion for mass transfer is analogous to heat transfer, our results from the approximate solutions can provide a similar extension to the Graetz Problem for the cases of short contact times and small penetration depths. The solutions are:

for constant concentration at the interface (a Leveque analogue),

$$N_{SH} = 1.165 (N_{RE} N_{SC} B/L)^{1/3} \quad [2.24]$$

for linear varying concentration at the interface,

$$N_{SH} = 1.400 (N_{RE} N_{SC} B/L)^{1/3} \quad [2.40]$$

where N_{SH} , N_{RE} and N_{SC} are defined from Eq. [2.24]. Eqs. [2.24] and [2.40] are applicable for low rates of mass transfer between countercurrent fluids; and for the case of heat transfer between a fluid and a wall in the entrance region of plate.

It is now worthwhile to consider the mass transfer to the dispersed phase droplets located in the spout region of the baffle plate, shown in Figure 1. We shall show that most of the mass transfer comes from droplets.

2.6. Dispersed Phase

Many correlations for mass transfer from dispersed phase drops are available, see Skelland (1974). The important droplet conditions include direction of mass transfer and internal circulation pattern, if any. The amount of mass transfer from a drop can be determined by the exposure time of the dispersed phase drop in the continuous phase liquid. The rise time, t_r , of a drop is determined by performing an unsteady-state force balance on an accelerating drop:

$$F_w + F_d = ma \quad [2.43]$$

where F_w is the effective weight of the drop; F_d is the drag force on the drop; m is the drop mass; and a is its acceleration. Eq. [2.43] becomes

$$\frac{4}{3} \pi r^3 g \Delta\rho - d_f \mu_1 r \pi V_t = \frac{4}{3} \pi r^3 \rho_d \frac{dV_t}{dt} \quad [2.44]$$

where V_t is the time dependent velocity of the drop; $\Delta\rho$ is the density difference; g is the force of gravity; ρ_d is the density of the drop; μ_1 is the viscosity of the continuous phase; μ_2 is the viscosity of the drop; r is the drop radius; and the drag factor, d_f , is found from the equation

$$d_f = 6 \frac{1 + (2/3) \mu_1/\mu_2}{(1 + \mu_1/\mu_2)} \quad [2.45]$$

It follows that from Eq. [2.44]

$$V_t = \frac{a'}{b} (1 - \exp(-bt)) \quad [2.46]$$

where the vertical velocity of the drop is considered to be zero at fluid interface; and $a' = \frac{\Delta \rho g}{\rho_d}$, cm per sec² and $b = \frac{3 \mu_l d_f}{4 r^2 \rho_d}$, sec⁻¹.

The rise time of the drop, t_r , is found by integrating Eq. [2.46] from $-B$ to 0 , here B is the half-plate spacing with respect to x , the vertical distance from the interface.

$$\int_{-B}^0 dx = \int_0^{t_r} V_t dt \quad [2.47]$$

$$\frac{a}{b^2} (\exp(-bt_r) + bt_r - 1) - 2B = 0 \quad [2.48]$$

Eq. [2.48] is solved for t_r by expanding the exponential term

$$\exp(-bt_r) = 1 - bt_r + \frac{(bt_r)^2}{2!} + \dots \quad [2.49]$$

For most baffle plate spacings, the drop is considered to have a constant acceleration throughout its rise. This is supported by the experimental work of Scheele and Meister (1968), which concludes that for most liquid-liquid systems, the drag on a drop is negligible under low velocity formation. Hence, t_r , is

$$t_r = \frac{2 \rho_d B}{\Delta \rho g}^{1/2} \quad [2.50]$$

The local mass transfer rate for a liquid drop is estimated by assuming that the internal circulation patterns are laminar, via the Higbie drop model, (1935). Drop circulation introduces fresh drop fluid at the top of the drop. This fluid slides along the outside of the drop and enters the well-mixed interior of the drop at its bottom. This model adequately describes the mass transfer rate of a drop for short exposure times and slow diffusion times. The mass flux per surface area during drop rise is

$$\int_0^{t_r} N_a \text{ avg } dt = \int_0^{t_r} \left[\frac{4D_1}{\pi t_{\text{exp}}} \right]^{1/2} C_{A0} dt \quad [2.51]$$

where the continuous phase mass transfer coefficient of a drop is $(4D_1/(\pi t_{\text{exp}}))^{1/2}$; exposure time, is given as $2r/V(t)$; and $V(t)$ the time dependent velocity of a drop rising under constant acceleration is

$$V(t) = a t \quad [2.52]$$

$$= \frac{\Delta \rho g t}{\rho_d} \quad [2.53]$$

C_{A0} is the steady-state concentration of "A" in fluid 1 at the interface and is found by equating the fluxes at the interface. For two fluids in concurrent contact and a

solute diffusing from one to the other, the mass transfer coefficient is proportional to the square root of the exposure times the diffusivity. An equivalent calculation for concurrent flow in heat transfer is demonstrated by Luikov (1980).

At the interface,

$$X=0 \quad \left(\frac{D_2}{\pi \text{ texp}} \right)^{1/2} (1 - \text{CAO}/\text{Ne}) = \left(\frac{D_2}{\pi \text{ texp}} \right)^{1/2} (\text{CAO} - 0) \quad [2.54]$$

where the mass flux is in units of moles "A" transferred per unit area per unit time; and texp is the exposure time for mass transfer. It is seen that

$$\text{CAO} = \frac{\text{Ne}}{1 + \text{Ne} (D_1/D_2)^{1/2}} \quad [2.55]$$

which is contact time independent and which is seen to satisfy the initial assumption of constant concentration at the interface. The mass transfer of "A" across the interface for a drop is

$$\int_0^{t_r} \text{Na avg} \, dt = \left[\frac{2D_1}{\pi r} \frac{\Delta \rho g}{\rho_d} \right]^{1/2} \frac{t_r^{3/2}}{3/2} \text{CAO} \quad [2.56]$$

where the surface area of a drop is $4\pi r^2$.

When typical results from Eq. [2.56] are compared to those from the previous two sections, it is found that in typical tower geometries, the majority of mass transfer and

the majority of exposed surface area comes from the relatively transitory droplets located about the spout of the baffle plate.

2.7. Comparisons Between Computer Solution and Approximate Solutions

A computer program can give exact solutions to any well-posed set of differential equations with arbitrarily difficult boundary conditions. If Eqs. [2.1] and [2.2] accurately describe mass transfer between the non-dispersed regions of countercurrent flow, then a computer program can give an exact solution to mass transfer. Sources of computer error can arise from the derivative approximations, from the integration technique and from inherent algorithm techniques. We believe these errors in our application to be slight. Our computer solution solves Eqs. [2.1] and [2.2] to within an accuracy of 8%, as shown in Table 1. The value of accuracy is calculated by Eq. [2.6], which compares the convective flux at the plate exit to the diffusive flux across the interface.

Approximate solutions one and two (Eqs. [2.20], [2.37] and [2.38]) are seen to parallel the computer solution. Approximate solution 2 (Eqs. [2.37] and [2.38]) predicts the higher rate of mass transfer, which means that the linear varying concentration profile at the interface gives higher rates of mass transfer between countercurrent fluids, than

the constant concentration profile (Eq. [2.20]). Since the driving force of approximate solution 2 is a linear function of the Nernst distribution coefficient, it will give the extremas of the computer solution. On the other hand, the driving force of approximate solution 1 is bounded by the asymptotic behavior of CA_0 , Eq. [2.54]. These approximate solutions are given in Table 1.

One can now use a weighted combination of the upper and lower bounding equations, [2.37] and [2.38], to give an optimal fit to the computer solution. If this fit is calculated where N_{A1} is the predicted mass flux of "A" in fluid 1 from the computer solution; and where N_1 and N_2 are the predicted mass fluxes from Eqs. [2.37] and [2.38] respectively; then K_f can be determined by a least square fit of Eq. [2.57] using the data in Table 1.

$$N_{A1} = N_2 (K_f \ln N_1/N_2) \quad [2.57]$$

$$K_f = 0.37 \quad N_e > 1.0 \quad [2.58]$$

$$K_f = 0.76 \quad N_e < 1.0 \quad [2.59]$$

The resulting correlation is

$$N_{A1} \quad x=0 = \frac{N_1}{N_e} \exp (K_f \ln N_e) \quad [2.60]$$

and

$$N_{SH} = \frac{N_{SH}}{N_e} \exp (K_f \ln N_e) \quad [2.61]$$

where N_1 is found from Eq. [2.37]; and N_{SH} , the Sherwood number is found from Eq. [2.40]; and N_e is the Nernst distribution coefficient. Comparisons between the correlation and the computer solution is given in Table 1.

While this correlation for non-dispersed flow mass transfer may not be particularly accurate, it was shown in section 2.5 that this contact contributes less than 20% of the total mass transfer. Thus, inaccuracies from Eq. [2.60] should not contribute significantly to design errors. The total mass transfer for a baffle plate extractor with countercurrent flow is the sum of the mass transfer from the drops and from the non-dispersed phases. Adding Eqs. [2.56] and [2.60] gives

$$Na = \frac{0.9 D_1 (\beta/9L)^{1/3} NeC20}{.893Ne} \exp (K_f \ln Ne) + \left(\frac{2 D_1 \Delta \rho g}{\pi r \rho_d} \right)^{1/2} \frac{t_r^{3/2}}{3/2} CAO \quad [2.62]$$

It is now worthwhile to consider liquid extraction with concurrent flow.

CHAPTER THREE

LIQUID EXTRACTION WITH CONCURRENT FLOW

We consider a variant of the standard side-to-side baffle tower that can operate with concurrent flow. The design of this baffle plate liquid extractor is shown in Figure 1b. In order to compare the plate efficiencies of countercurrent and concurrent liquid extractors, we shall consider a set of entrance and flow conditions which are similar to those in Chapter Two.

Consider the mass transfer between two immiscible fluids flowing concurrently in the Z-direction. As shown in Figure 2b, the interface is at or near a region of zero velocity gradient. Fluid 1 enters with a normalized concentration of zero at the entrance ($Z=0$); and similarly, fluid 2 enters with a normalized concentration of 1.0 (at $Z=0$). Since the penetration depth is small, solute "A" moves with a constant velocity, V_I , at the interface. The interfacial concentration of each fluid will remain constant over the plate length so that

$$\text{at } X=0; \quad C_1 = C_{A0} \quad \text{and} \quad C_2 = C_{A0}/N_e.$$

Where N_e is the Nernst distribution coefficient. The constant, C_{A0} , is calculated by equating the mass flux of "A" for the two fluids, as in equation [2.55].

$$CAO = \frac{Ne}{1 + Ne(D_1/D_2)^{1/2}} \quad [3.1]$$

We can simplify the equations describing the mass transport between two concurrent fluids to

$$\frac{V_I}{D_1} \frac{\partial C_1}{\partial z} = \frac{\partial^2 C_1}{\partial x^2} \quad [3.2]$$

where V_I is the constant velocity at the interface. This equation can also be written

$$W \frac{\partial C_1}{\partial z} = \frac{\partial^2 C_2}{\partial x^2} \quad [3.3]$$

where W is defined as

$$W = \frac{V_I}{D_1} \quad [3.4]$$

and where, for a Newtonian fluid,

$$V_{z1} = (3/2 \langle V \rangle) (1 - (X/B)^2) = V_I (1 - (X/B)^2) \quad [3.5]$$

where B is the half-plate spacing; and X is the vertical distance from the interface. The boundary conditions to Eq. [3.2]:

at $z=0$	$C_1=0$
at $X=0$	$C_1=CAO$
at $X= \infty$	$C_1=0$

Eq. [3.2] is solved by any of a variety of techniques, e.g. similarity transform; the concentration profile for fluid 1 is

$$\frac{C_1}{C_{A0}} = \operatorname{erfc}(\vartheta) \quad [3.6]$$

where ϑ is the dimensionless variable

$$\vartheta = X (W/(4Z))^{1/2} \quad [3.7]$$

The local concentration gradient of "A" at the interface is found by differentiating Eq. [3.6] with respect to X.

$$\left(\frac{\partial C_1}{\partial X} \right)_{X=0} = -\frac{C_{A0}}{\sqrt{\pi}} (W/Z)^{1/2} \quad [3.8]$$

After integrating D_1 times Eq. [3.8] with respect to Z, one finds the average mass transfer of "A" across the interface.

$$\overline{N_A}(z)_{X=0} = D_1 \left(\frac{4W}{\pi L} \right)^{1/2} C_{A0} \quad [3.9]$$

Where Eqs. [3.4] and [3.1] define W and C_{A0} respectively.

The average mass transfer coefficient for fluid 1 is

$$\overline{k_c} = \left(\frac{\overline{N_A}(z)}{C_{A0}} \right)_{X=0} \quad [3.10]$$

$$\overline{k_c} = D_1 \left(\frac{4W}{\pi L} \right)^{1/2} \quad [3.11]$$

Eqs. [3.10] and [3.11] can be written in terms of standard dimensionless groups:

$$N_{SH} = (3/\pi)^{1/2} (N_{RE} N_{SC} B/L)^{1/2} \quad [3.12]$$

where

$$N_{SH} = \frac{\overline{kc} B}{D_1} . \quad [3.13]$$

Here, N_{RE} equals $(2B\langle V_1 \rangle \rho_1 / \mu_1)$; and N_{SC} equals $(\mu_1 / \rho_1 D_1)$.

CHAPTER FOUR

GENERAL ENERGY RELATIONS

It is now worthwhile to estimate and to compare the energy requirements of countercurrent and concurrent baffle plate extractors. The total hydrodynamic head loss through friction and through interfacial tension effects is called "H". This can be divided into H_c , the head loss in the continuous phase; and H_d , the head loss in the dispersed phase where applicable. As shown in Figure 1a, countercurrent extraction involves dispersed drops, while concurrent extraction does not.

Treybal (1980) approximates the energy loss for the continuous phase flow as the equivalent of 4.5 velocity heads, H_c . This includes

- (1) friction effects (usually negligible)
- (2) contraction and expansion upon entering and leaving the plate (1.5 velocity heads)
- (3) two abrupt changes in direction (3.0 velocity heads)

$$H_c = 4.5 \frac{V_c^2 \rho_c}{2g_c \Delta\rho} \quad [4.1]$$

where V_c is the average velocity of the continuous phase; ρ_c is the density of the continuous phase; $\Delta\rho$ is the density difference; and g_c is the gravitational constant.

Similarly, for the dispersed phase, the energy losses are from the following:

- (1) entrance effects, H_f
- (2) energy losses during drop formation, H_σ .

The latter is usually dominant where V_d , the average velocity of the dispersed phase fluid, is less than 0.3 m/s.

The energy loss for the dispersed phase flow is

$$H_d = H_f + H_\sigma \quad [4.2]$$

where

$$H_f = 4.5 \frac{V_d^2 \rho_d}{2g_c \Delta\rho} \quad [4.3]$$

V_d is the average velocity of the dispersed phase fluid; ρ_d is the density of the dispersed phase fluid; $\Delta\rho$ is the density difference; and g_c is the gravitational constant. The energy due to drop formation is

$$H_\sigma = \frac{6 g_c \sigma}{\Delta\rho g d_p} \quad [4.4]$$

Where σ is the interfacial tension given by the equation

$$\sigma = |\sigma_1 - \sigma_2| \quad [4.5]$$

and where d_p is the drop diameter; d_p may be calculated by the correlation of Klee and Treybal (1956):

$$d_p = 7.25 \left(\frac{g_c \sigma}{g \Delta\rho N_p} \right)^{1/2} \quad [4.6]$$

$$Np = \frac{\rho_c^2 \sigma^3 g_c^3}{\mu_c^4 \Delta \rho g} \quad [4.7]$$

Here ρ_c is the density of the continuous phase; and μ_c is the viscosity of the continuous phase.

At low flow velocities (less than 0.3 m/s), the countercurrent flow energy requirement is largely due to drop formation in the spout region of the plate. Thus, it is fairly independent of plate spacings and plate length. At higher flow velocities, however, the energy requirements of countercurrent flow is not constant and depends on throughput.

The energy loss for concurrent flow occurs solely in the continuous phase:

$$H = \frac{4.5 \bar{V}_A^2 \bar{\rho}}{2g_c \rho} \quad [4.8]$$

where $\bar{\rho}$ is the arithmetic average density, $\bar{\rho} = (\rho_1 + \rho_2)/2$; and \bar{V}_A is the arithmetic average of the average fluid velocities, which is defined from the volumetric flow rates, Q_1 and Q_2 .

$$\bar{V}_A = \frac{(Q_1 / BL) + (Q_2 / BL)}{2} \quad [4.9]$$

The energy utilization for concurrent and counter-current extraction are shown in Figure 5. It is seen that the energy requirement for concurrent flow is always lower

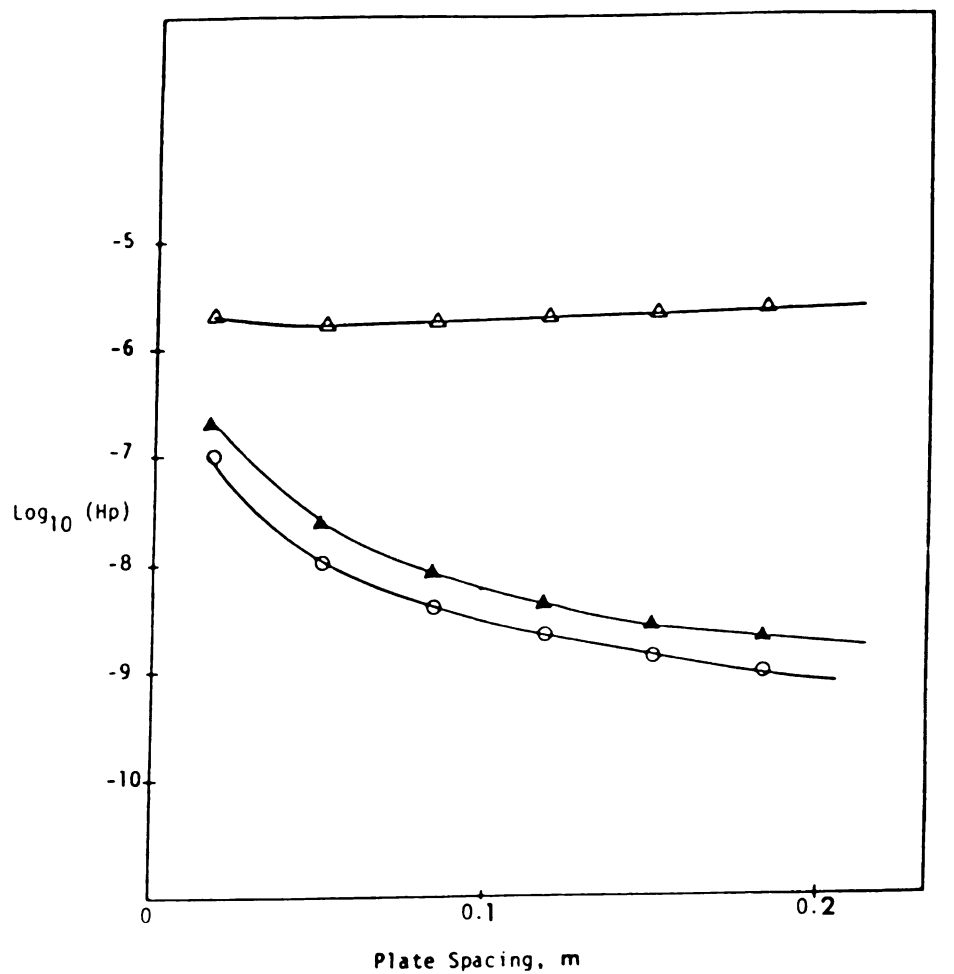


Plate Length = 2.0 m

Re = 92

$D_{ab} = 1.96 \text{ E-5 cm}^2/\text{sec}$

$D_{ac} = 1.24 \text{ E-5 cm}^2/\text{sec}$

System: (a) acetic acid
(b) isopropyl ether
(c) water

○ Concurrent

△ Countercurrent
(with dispersion)

▲ Countercurrent
(without dispersion)

Figure 5. Power Requirements

than that for countercurrent flow. Since both liquids are flowing in the same direction, there is no dispersion contribution for concurrent flow; and the value of H_d is always lower than for countercurrent flow.

CHAPTER FIVE

PLATE EFFICIENCY RELATIONS

The overall plate efficiency is defined in terms of mass transfer coefficients and exposure times of fluids on the plate. The countercurrent plate efficiency considers mass transfer from the continuous and dispersed phases, while the concurrent plate efficiency considers only mass transfer from the continuous phase. The overall plate efficiency becomes

(i) for countercurrent flow,

$$\eta_0 = \eta_{\text{count}} + \eta_{\text{drop}} \quad [5.1]$$

(ii) for concurrent flow,

$$\eta_0 = \eta_{\text{concurrent}} \quad [5.2]$$

The mass flux from the interface to the bulk fluid in the direction of decreasing concentration of solute "A" is

$$N_a = \bar{k}_c (C_a^* - C_a) \quad [5.3]$$

where \bar{k}_c is the local mass transfer coefficient; C_a is the concentration of "A" in bulk phase; and C_a^* is the steady-state concentration of "A" at the interface. For concurrent flow, C_a^* equals C_{AO} , the constant, from Eq. [2.55]; and for countercurrent flow, the interfacial concentration C_a^* equals \bar{C}_{ai} (average), which is found from Eq. [2.39]. The mass flux of "A" can also be defined as

$$N_a = \frac{V}{A} \frac{dC_a}{dt} \quad [5.4]$$

where V is the volume of fluid on the plate; and where A is the surface area of the plate. Eqs. [5.3] and [5.4] are combined to give

$$\frac{dC_a}{(C_a^* - C_a)} = \bar{k}_c \frac{A}{V} dt \quad [5.5]$$

Since $C_a = C_{a0}$ at $t=0$, we obtain

$$\frac{(C_a^* - C_a)}{(C_a^* - C_{a0})} = \exp (-\bar{k}_c A t/V) \quad [5.6]$$

which can be used to calculate the channel efficiency for countercurrent and concurrent extractors. Eq. [5.6] becomes

$$\eta_c = 1 - \exp (-\bar{k}_c A/Q) \quad [5.7]$$

where η_c is the channel efficiency; and Q is the volumetric flow rate.

To calculate the overall tray efficiency for countercurrent and concurrent flow, it is necessary to sum the effects of channel flow and dispersed flow. For dispersed flow, Eq. [5.6] becomes

$$\eta_{\text{drop}} = 1 - \exp (-6k_p t_r/d_p) \quad [5.8]$$

where d_p is the droplet diameter as in Eq. [4.6]; t_r is the rise time of the drop as in Eq. [2.50]; and k_p is the continuous phase mass transfer coefficient of the drop as in Eq. [2.51]. The overall energy relations are:

for countercurrent flow,

$$\eta_0 = \exp(-\overline{k_c}A/Q) \text{ count.} + \exp(-6k_p t_r/d_p) \text{ drop} \quad [5.9]$$

for concurrent flow,

$$\eta_0 = \exp(-\overline{k_c}A/Q) \text{ concurrent} \quad [5.10]$$

Concurrent operation in baffle plate extraction towers can represent an improvement over countercurrent operation for certain tower designs. It appears to be more advantageous than dispersed countercurrent operation in regions where mass transfer from the continuous phase is greater than mass transfer from drops. Concurrent operation produces higher plate efficiencies than countercurrent operation in large diameter towers and in narrow plate spacings. Concurrent operation can also achieve higher throughputs than countercurrent operation, Leacock and Churchill (1961); thus making it possible to achieve higher rates of mass transfer than standard countercurrent operation.

Where average plate spacings are .07 to .1 meters and average plate lengths are 0.9 to 1.8 meters, Treybal (1951), we find that countercurrent operation generally achieves higher plate efficiencies than concurrent operation for most baffle tower designs. Figure 6 presents a plot of plate efficiency vs. plate length, which shows that plate efficiencies are linearly related to the plate lengths. It also shows that most of the mass transfer for countercurrent operation comes from drops.

The effect of mass transfer from drops upon countercurrent operation is also evident in Figure 7, a plot of plate spacing vs. plate efficiency. An increase in plate spacing decreases both concurrent's and non-dispersed countercurrent's efficiency for operation. Yet, the efficiency for dispersed countercurrent operation increases, because the exposure time of the droplets increases. However, this is not a fair comparison, because the cost of a liquid-liquid extraction tower increases with increasing plate spacings. In effect, the large plate spacings make a baffle plate tower into a spray tower. Mass transfer from drops generally helps increase the plate efficiency of countercurrent operation; and it makes countercurrent operation advantageous for most baffle tower designs.

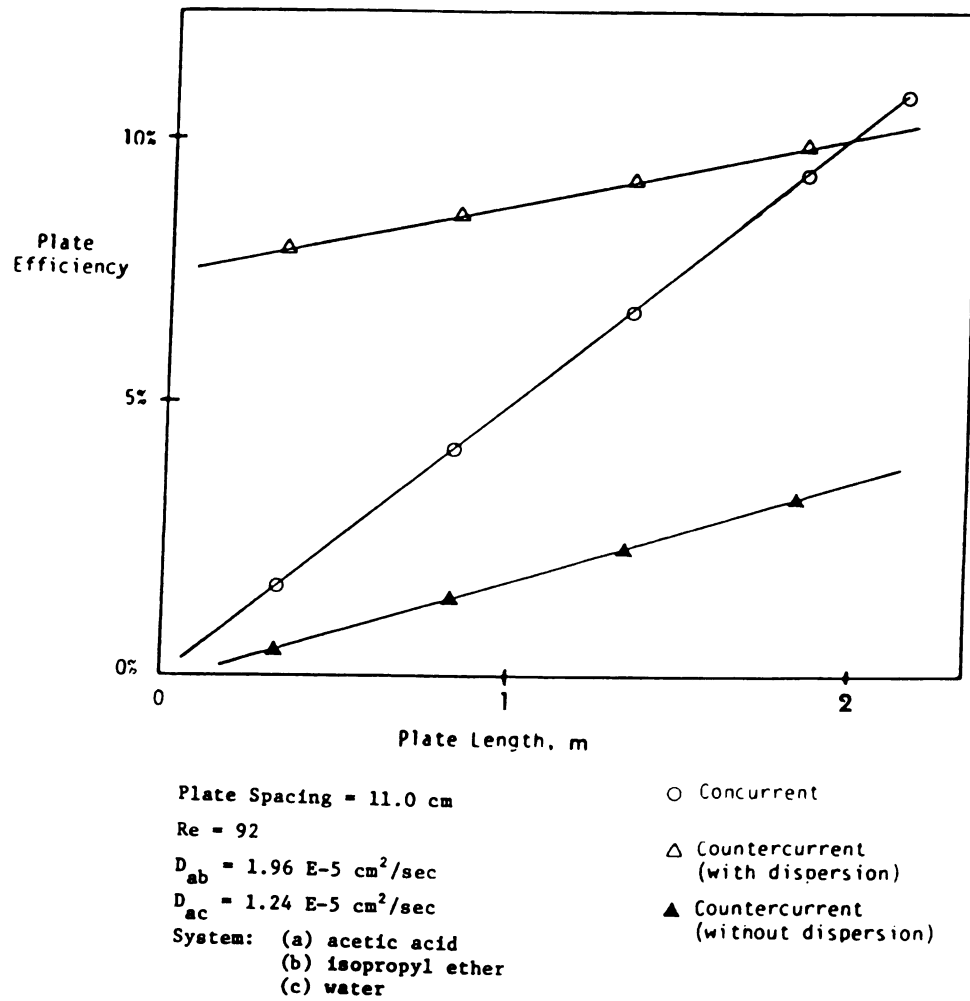


Figure 6. Plate Length vs. Plate Efficiency

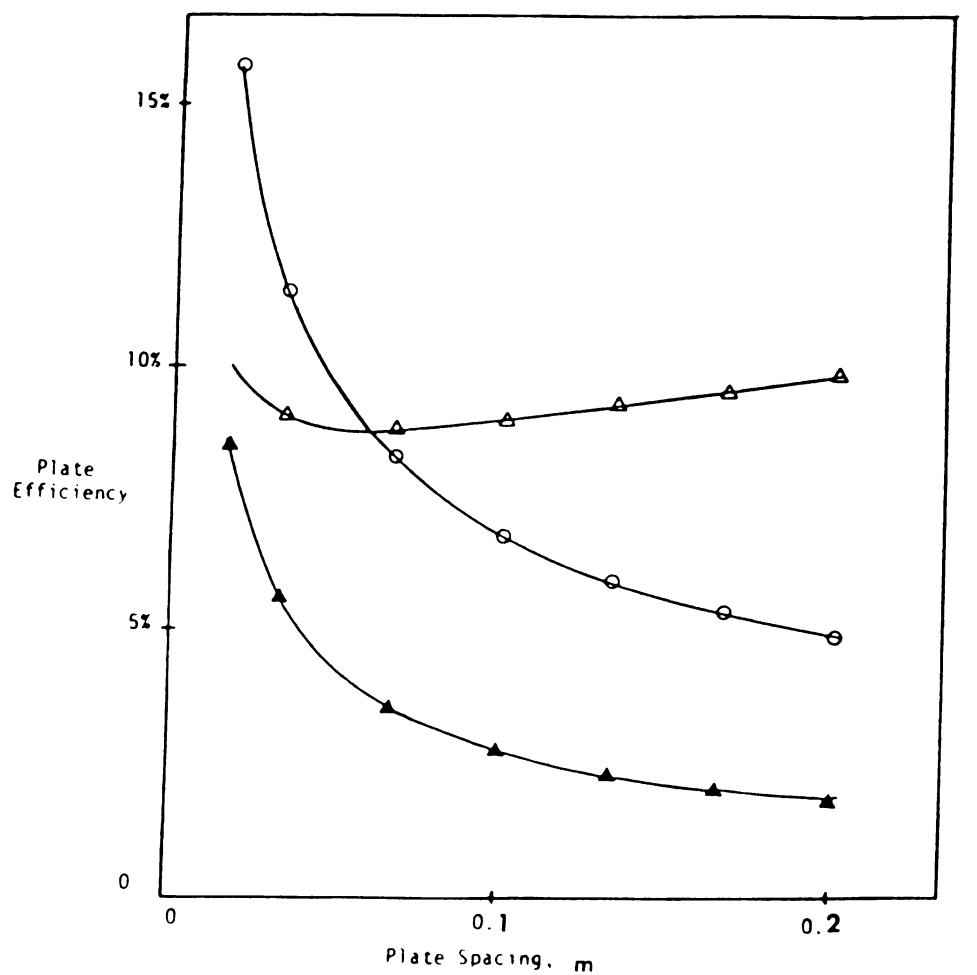


Plate Length = 2.0 m

Re = 92

$D_{ab} = 1.96 \text{ E-5 cm}^2/\text{sec}$

$D_{ac} = 1.24 \text{ E-5 cm}^2/\text{sec}$

System: (a) acetic acid
(b) isopropyl ether
(c) water

○ Concurrent
△ Countercurrent
(with dispersion)
▲ Countercurrent
(without dispersion)

Figure 7 Plate Spacing vs. Plate Efficiency

APPENDIX

TABLE 1
Comparisons of Countercurrent Models

β Equation [2.5]	N_e Equation [2.3]	Method One Equation [2.20] $N_a \times 10^7$	Method Two a. Equation [2.37] b. Equation [2.38] $N_a \times 10^7$	Correlation Equation [2.60] $N_a \times 10^7$	Computer Solution $N_a \times 10^7$	Computer Error (%)	Correlation Error (%)
11,579,630	10	8.116	a. 42.820 b. 4.382	10.272	9.659	3.0	+6.4
14,251,852	7	8.317	a. 32.876 b. 4.696	9.649	9.840	4.0	-1.9
16,000,000	5	8.116	a. 24.406 b. 4.881	8.854	9.110	1.0	-2.8
17,814,816	4	8.073	a. 20.237 b. 5.059	8.450	9.023	2.0	-6.3
23,753,088	2	7.219	a. 11.137 b. 5.569	7.197	7.762	2.0	-7.3
29,691,360	1	5.656	a. 5.999 b. 5.999	5.999	6.045	6.0	-0.7
35,629,632	0.5	3.889	a. 3.187 b. 6.375	3.764	3.773	6.0	-0.2
66,239,078	0.1	1.251	a. 0.784 b. 7.838	1.362	1.308	8.0	+4.1

LIST OF COMPUTER VARIABLES

B = Plate spacing

CHECK = Convective flux at plate exit for fluid 1

CB (I,J) = Concentration profile of fluid 2

CON (I,J) = Concentration profile of fluid 2

DAB = Molecular diffusivity in fluid 1

DT = Time increment

DX = Vertical increment, B/KS

DY = Axial increment, L/LNG

EQN = Nernst distribution coefficient

L = Plate Length

LOOP = Number of iterations over CON (I,J) and CB (I,J)

PNA = Diffusive flux across interface for fluid 1

PK = Velocity gradient/Diffusivity

Q = Volumetric flow rate

RATIO = Diffusivity of fluid 2/Diffusivity of fluid 1

TABLE 2
Computer Program

```

PROGRAM CHE(INPUT, OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
DIMENSION CON(30,30),CB(30,30),F(30)          ,G(30)
REAL L,
C   JANICE KING      FALL"1983
C   PROGRAM CALCULATE CONCENTRATION PROFILES  C=C(X,Y)
C   MASS TRANSFER MECHANISM** CONVECTION IN Y, DIFFUSION IN X.
C   METHOD OF SOLUTION** EXPLICIT METHOD
C   PDE**  $DC/DT = AX(DC/DT) + DAB D/DX(DC/DX)$ , WHERE A AND DAB ARE #
C   L=PLATE LENGTH/ B=PLATE SPACING/ CAO= INITIAL CON(A)
C   EQN= NERNST DIST. COEF.
C   BC** (1)  $CA=CB*EQN$  (2)  $X=+B$   $CA=$  0 (3)  $X=-B$   $CB=1$ 
C   BC** (4)  $Y=0$   $CA=0$  (5)  $Y=L$   $CB=1$ 
C   BC** (6)  $X=0$   $DAB(DCA/DX) = DAC (DCB/DX)$ 

C   *****SET CONCENTRATION PROFILES*****

C   *****READ IN DATA AND COMPUTE CONSTANTS*****
DT+ 1.5325E-6
Q=0.0031
PK= 10E6
EQN=5.
LOOP= 1000
RATIO = 1.2
CBO=1.
B= 4.102564E-3*5.
L= 6.0
DX= B/6.
DY= L/6.
DAB= 7.5347373E-9
LNG = NINT( L/DY)
LNG1 = NINT( L/DY + 1.)
MT=LNG1 +1
KS= NINT( B/DX)
KSP1= NINT( B/DX + 1.)
WRITE(*,33)
WRITE(*,52) DX,DY,DT          ,LOOP
WRITE(*,99)
WRITE(*,100) B,L,Q,EQN,DAB

C   *****COMPUTE SUCCESSIVE CONCENTRATIONS*****
DO 55 MTIMES=1,LOOP
DO 60 J= 2,LNG
DO 60 I=2,KS
CON(I,J)=CON(I,J)+ DT*( -PK*(I,DX) *(( CON(I,J+1) - CON(I,J-1)
*   ))/(2.*DY)) + (( -2.*CON(I,J) + CON(I+1,J) + CON(I-1,J))/
*   ( DX*DX)))
CB(I,J) = CB(I,J) + DT*( PK*(I*DX)*((CB(I,J+1) - CB(I,J-1))/
*   (2.*DY)) + ((-2.*CB(I,J)+CB(I+1,J) + CB(I-1,J))/(DX*DX)))
CON(1,J) = CON(1,J) + DT*((-1.*CON(1,J) + CON(2,J) + RATIO*

```

TABLE 2
Computer Program
(Continued)

```

*   ( CB(2,J) - CB(1,J)))/( DX*DX)          )
      CB(1,J) = CON(1,J)/EQN
DO 54 JJ=1,LNG1,LNG
      CON(1,JJ)=CCN(1,JJ) + DT*((CON(2,JJ)-CON(1,JJ) + RATIO*(CB(2
      ,JJ)-CB(1,JJ)))/(DX*DX))
      CB(1,JJ) = CON(1,JJ)/EQN
54  CONTINUE

      CON(I,LNG1)=CON(I,LNG1)+DT*(-PK*(I*DX)*((CON(I,LNG1)-CON(I,LNG
*   ))/DY) + (( -2.*CON(I,LNG1) + CON(I+1,LNG1) + CON(I-1,LNG1))/
*   ( DX*DX)))
      CB(I,1) = CB(I,1) + DT*( PK*(I*DX)*((CB(I,2)-CB(I,1)      )/DY)
*   + ((-2.*CB(I,1)+CB(I+1,1) + CB(I-1,1))/(DX*DX)))
      CON(KSP1,J) = 0.
      IF ( CON(I,J) .LT. 0. ) CON(I,J)=0.
      IF ( CB(I,J) .LT. 0.) CB(I,J) =1.
      IF ( CB(I,J) .GT. 1.0) CB(I,J) = 1.0
      IF ( CON(I,J) . GT. 10.) CON(I,J)= 0.
60  CONTINUE
55  CONTINUE
      DO 62 I=KSP1,1,-1
62  WRITE(*,88) ( CON(I,J),J=1,LNG1)
      WRITE(*,74)
      DO 89 I=1,KSP1
89  WRITE(*,88) (CB(I,J),J=1,LNG1)
C   *****CALAULATE THE DC/DX at X=0*****
      DO 350 I=1,LNG1
350  F(I)= (2.*CON(2,I))-(CON(3,I)/2.) - 1.5*CON(1,I)
C   *****SIMPSON"S RULE TO EVALUATE INTEGRAND*****
      SUM1= 0.
      DO 450 I=2,LNG,2
          W=4.*F(I)
450  SUM1= SUM1+ W
      SUM2 = 0.
      DO 470 I=3,LNG-1,2
          W2 = 2.*F(I)
470  SUM2 = SUM2+W2

C   *****CALC THE MASS TRANSFER*****
      DER = L*( F(1) + SUM1 + SUM2 + F(LNG1) )/(3. * LNG)
      PNA= -DAB*DER/L
      DO 351 J=1,KSP1
351  G(J)=DAB*PK*(J-1)*CON(J,LNG1) *DX
      PART=0.
      DO 500 J=2,KS,2
          Y=4.*G(J)
500  PART = PART + Y
      PART2=0.

```


TABLE 2
Computer Program
(Continued)

```

      DO 510 J=3,KS-1,2
      Y2 = 2.*G(J)
510    PART2 = PART2 + Y2
      CHECK= (G(1)+PART+PART1+G(KSP1))*DX/(3.*1)
      WRITE(*, 101) CHECK
C    *****FORMATS*****
33    FORMAT(2X,////////////////////)
52    FORMAT(///,5X,"DX=",E9,4,2X,"DY=",E9.4,2X,"DT=",E9.4,2X,"LOOP",I5 )
74    FORMAT(/,20X,"*****")
      * *****")
88    FORMAT(/,1X,13E10.4)
99    FORMAT(///, 12X,"B",10X,"L",13X,"G",13X,"EQN",10X,"DAB")
100   FORMAT(///,5X, 4F12.4,E10.4)
101   FORMAT( ///,5X,"CHECK IS ...",E12.5)
900   FORMAT(///,10X,"MASS TRANSFER AVERAGE IS .....",E15.4)
      END

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