ALTERNATIVE METHODS OF ESTIMATION IN THE DEMAND FOR NATURAL GAS

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VENKATA MADHAVA RAO TUMMALA

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ABSTRACT

ALTERNATIVE METHODS OF ESTIMATION
IN THE DEMAND FOR NATURAL GAS

by Venkata Madhava Rao Tummala

The basic purpose of this study is to develop some dynamic models to explain the demand for natural gas in residential, commercial and industrial sectors and estimate them. It is a known fact, among econometricians and economists in general, that there exists competition among energy fuels and so the relevant economic variables are interdependent regarding the consumption of natural gas. These variables belong to a system and hence if any aspect of any one of them is considered, it should be considered from the point of view of the system as a whole. With this in view, a simultaneous equations system model is formulated which is capable of explaining the influence of the burner-tip price of natural gas (along with the consideration of the simultaneous nature of economic relationships among the relevant variables) on the consumption in each of the residential, commercial and industrial sectors.

It is also a known fact that the behavior of consumption cannot be adapted immediately to changes in the

variables which condition it because of possible existence of a "lag" in economic response. The economic reactions, for a given change, are processes in time and these processes explain the nature of adaptive behavior of consumption. In relevance to natural gas demand, the economic response in the consumption of natural gas is subject to certain psychological, technological and institutional factors. In such a case a static demand model is inadequate to represent the decision making process in each of the residential, commercial and industrial sectors. An attempt is made to formulate a distributed lag model to explain the reaction (or adjustment) mechanism in the behavior of consumption of natural gas. This model is developed on the basis of an "expected-income" hypothesis which states that consumption during any period of time is determined by the consumer's expected income, not by his current income. The total consumption expenditures tend to be stable relative to current incomes and a change in current income tends to affect consumption only insofar as it affects consumers' notions of their "expected" incomes.

Several alternative econometric estimation procedures are used to estimate the parameters of the two models described above: Ordinary Least Squares (OLS), Two-Stage Least Squares (2SLS), Unbiased Nagar K-Class (UNK), Limited Information Single Equation (LISE), Three-Stage Least Squares (3SLS). These alternative sets of estimates are presented and compared. The equations of the two models were fit using annual time series data for the State of Michigan for a sample period from 1947 to 1964. All data used were secondary, primarily from the records of federal government departments and other agencies; although some data series were transformed to meet the special needs of the study.

Major findings of this study are as follows: from the results of the simultaneous equations system model, the burner-tip price elasticities of demand in the residential, commercial and industrial sectors are estimated as -0.4386, -0.6048, and -1.3344, respectively. The results also indicate that electricity is providing a keen competition to natural gas. From the results of the distributed lag model, the burner-tip price elasticities in the residential and commercial sectors are estimated as -1.6320 and -1.4378, respectively. Other interesting results such as estimates of income elasticities and cross-elasticities can be found in the text of the study. The coefficient estimate of speed of adjustment is found to be 0.2 in the residential sector and 0.3 in the commercial sector. The corresponding results in the industrial sector are rather not satisfactory.

The two dynamic demand models formulated in this study, thus presents some features that may have shed some new light and complemented the results obtained in the earlier studies. Finally, it is hoped that these models may be applied successfully to the data of other states in this country.

ALTERNATIVE METHODS OF ESTIMATION
IN THE DEMAND FOR NATURAL GAS

By

Venkata Madhava Rao Tummala

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TABLE OF CONTENTS

Table of Contents (continued)

LIST OF TABLES

List of Tables (continued)

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

1.1 The Characteristics of the Natural Gas Industry

The natural gas industry is one of the oldest public utility industries and plays a significant economic role along with other public utilities such as electric, telegraph and telephone industries. It supplies an indispensable service subject to government regulation at the federal and local levels. The origin of the rapid growth of the natural gas industry may be traced back to 1930. From this date on the growth in the gas consumption has been spectacular, particularly in the post-war years. The dramatic growth of usage of natural gas in the post-war period may be attributed either to increased estimated reserves, or to the expansion of pipelines accompanied by vast technological innovations and developments in gas pipe construction and operation. Also certain characteristics of natural gas--economic and psychological--such as reasonable price, dependability of supply, cleanliness, controllability, high efficiency of utilization, encouraged the growth of consumption of natural gas.

The use of natural gas is ordinarily broken down into several end-use categories or classes of service: residential service, commercial service, industrial service, and other services. Residential service applies to customers supplied for residential purposes. The residential service includes use of natural gas mainly for cooking, water heating, kitchen heating, space heating, air conditioning. Similarly, commercial service applies to customers primarily engaged in wholesale or retail trade, agriculture, forestry, fisheries, transportation, communication, sanitary services, finance, insurance, real estate (clubs, hotels, rooming houses, etc.), personal services. These customers use natural gas mainly for space heating, air conditioning and cooking. The industrial service applies to customers engaged in a process which creates or changes raw or unfinished materials into another form or product. This service includes space heating and air conditioning also. Finally, other services include services to municipalities or divisions (agencies) of state and federal governments.[1]

The consumption of gas under each of the end-uses is presented in Table 1.1.1. There is a gradual increase in usage in each of the end-use categories. But this gradual increase in usage is slowed down during the

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[1]See Gas Facts, American Gas Association, Inc. Bureau of Statistics, New York, N.Y., 1966, pp. 242-243.

TABLE 1.1.1 Consumption of Natural Gas in the United States According to Various Uses, 1946-1964

(Millions of MCF)

Year	Residential Use	Commercial Use	Industrial Use	Electric Power Generation Use	Field Use	Carbon Use
1946	661	242	1,427	307	898	478
1947	802	286	1,547	373	934	485
1948	896	323	1,745	478	1,022	481
1949	992	348	1,816	550	1,060	429
1950	1,198	388	2,213	629	1,187	411
1951	1,475	464	2,532	764	1,442	426
1952	1,622	515	2,714	910	1,484	368
1953	1,685	531	2,957	1,034	1,471	301
1954	1,894	585	3,051	1,165	1,457	251
1955	2,124	629	3,411	1,153	1,508	245
1956	2,328	717	3,759	1,239	1,421	243
1957	2,500	776	3,952	1,338	1,480	234
1958	2,714	872	3,987	1,373	1,604	211
1959	2,913	975	4,353	1,627	1,737	215
1960	3,103	1,020	4,683	1,725	1,780	198
1961	3,249	1,077	4,889	1,825	1,881	161
1962	3,479	1,207	5,112	1,966	1,993	133
1963	3,589	1,268	5,442	2,143	2,081	117
1964	3,787	1,375	5,852	2,322	2,093	107

Source: 1946-1961 Exhibit No. 236, FPC, Docket No. AR61-1
1962-1964 Minerals Yearbook, Vol. II, Fuels, Bureau of Mines: U.S. Department of the Interior

successive sub periods from 1940-1950 which can be evidenced from Tables 1.1.1 and 1.1.2. The reason for this is either the saturation of new markets for gas in different states or the competition from other fuels--coal, fuel oil, and electricity. The competitive aspects between fuels and electricity is discussed in later chapters while formulating the necessary models to explain the demand for natural gas in each sector.

TABLE 1.1.2 Percentage Increase in the Consumption of Natural Gas in Different End-Use Categories

Period	Residential Use	Commercial Use	Industrial Use	Electric Power Generation Use
1946-50	81.2	60.3	55.1	104.9
1951-55	77.3	62.1	54.1	83.3
1955-60	46.1	62.2	37.3	49.6
1961-64	22.0	34.8	25.0	34.6

Source: Calculated from Table 1.1.1

1.2 Scope and Nature of the Study

The basic purpose of this study is to develop dynamic models to explain the demand for natural gas in each of the sectors--residential, commercial, industrial, steam electric generating plants--and estimate them. By doing this, one is essentially estimating the relevant price and income elasticities in each of the above sectors.[2] The problem of determining price elasticities in each sector is very important and plays a significant role because the natural gas industry is a regulated industry, being regulated by the Federal Power Commission. The Federal Power Commission sets the wellhead ceiling price for initial contracts for natural gas which is supposed to perform an economic function. This function was clearly stated by Justice Jackson as:

> "Far sighted gas regulation ... will use price as a tool to bring goods to market ... to obtain for the public service the needed amount of gas. Once a price is reached that will do that, there is no legal or economic reason to go higher; and any rate above one that will perform this function is unwarranted ... on the other hand if the ... price is not a sufficient incentive to bring forth the quantity needed ... the price is unwisely low."[3]

Thus from the point of view of the Federal Power Commission, what price level should be adequate to perform such an economic function is an important problem. In order to determine such a price, one must know the relationship between wellmouth price or wellhead ceiling

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[2]If the demand models are assumed in double-log form, then the estimates of coefficients of corresponding price and income variables are the respective elasticities. Otherwise they are the corresponding "slopes." It is in this general sense the term "elasticities" is used.

[3]H. H. Wein, Natural Gas Supply and Demand, Docket No. AR 61-1, Federal Power Commission, Office of Economics, Washington, D.C., p. 7.

price and exploration, and also the relationship between wellmouth price and ultimate consumption. The wellmouth price affects average wellhead field price which in turn affects the ultimate consumption through the burner-tip prices at the points of consumption of natural gas. The average wellhead field price is different from the wellhead ceiling price (for initial contracts) and is discussed elaborately by Prof. Wein in his study.[4] How well the average wellhead field price affects the burner-tip prices was studied extensively by Dr. Wein and established some sound statistical relationships between

(i) the average wellhead field price and residential-commercial burner-tip price,

(ii) the average wellhead field price and the burner-tip price of industrial consumers.[5]

These relationships are useful for the Federal Power Commission to set up price because the Commission should be able to know the impact of wellhead ceiling price of initial contracts (through average wellhead field price) on the burner-tip prices paid by residential, commercial, and industrial users. Once the Commission knows

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[4] *Ibid.*, pp. 80-84.

[5] *Ibid.*, pp. 69-84

the above relationships, then it can evaluate the impact of wellhead ceiling price for initial contracts on the consumption of natural gas in various end-uses through the respective elasticities. Thus the price and income elasticities play a significant role in evaluating and regulating economic policies of the Federal Power Commission. It is in this context that some dynamic demand models to explain the behavior of consumption of natural gas are developed in this study.

It is well accepted by various authors who studied the demand for natural gas in residential commercial and industrial markets that the demand depends jointly on the demand for other fuels, namely, fuel oil and coal, and on the demand for electricity.[6] The nature of substitutability may vary according to the market, but the corresponding prices at the points of consumption would depend upon each other and hence influence the rate of consumption of each other. For example, electricity has become popular for space and water heating and so is providing a keen competition for natural gas. Similarly, coal is an important substitute for natural gas in the case of steam electric generation. Therefore, while formulating the structural

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[6]In Chapter 2, the studies of the respective authors are described and reviewed. No one has attempted to formulate the models which take account of this kind of simultaneous nature among substitutes.

demand model for natural gas, one should consider this kind of simultaneous nature between natural gas and its substitutes. Such models are formulated, in this study, to estimate the concerned elasticities. These models are called, in econometric literature, the "simultaneous equation system" models. Also some alternative models are formulated to estimate the concerned elasticities in order to compare with "simultaneous equation system models." These alternative models are formulated on the basis of the lag in the response of the "effect" variable due to a change in the "cause" variables. The "effect" variable in the demand model is the quantity consumed of natural gas in each of the sectors and the "cause" variables in the demand model are the respective price variables, income variable and other exogeneous variables. The existence of lag in explaining the dynamic behavior of natural gas demand is explained in a later chapter.

Some simple static demand models, along with some generalizations on static models formulated on the basis of economic theory, are formulated and estimated before the dynamic demand models are developed. These simple static demand models may not provide valid results but they may bring out in a specific way the deficiencies and limitations of static theory to explain the demand for natural gas. Or they may provide a way to formulate some dynamic models to explain the demand for natural gas in each sector.

1.3 Methodology

In literature, the estimation methods of structural "simultaneous-equation system" are divided into two parts:

(i) Structural Estimation: Single Equation Methods,

(ii) Structural Estimation: System Methods.[7]

The basic difference between these two categories is that in single equation methods each structural equation of a complete system is estimated in turn as if the equation to be estimated is imbedded in the complete system. In estimating each structural equation, these methods consider only the predetermined variables that are excluded from the equation to be estimated, but not the excluded jointly dependent variables. No use is made of the estimates of the parameters of other structural equations. But in system methods, estimation of structural equations is carried simultaneously and they make use of restrictions on the parameters of the complete system in estimating each structural equation.[8] Since system methods use complete systems of structural equations, one may refer the second category as complete-system methods.

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[7]A. S. Goldberger, Econometric Theory, New York: John Wiley & Sons, Inc., Chapter 7. J. Johnson, Econometric Methods, New York: McGraw-Hill Book Company, Chapter 9.

[8]Complete System is the one which has as many equations as there are jointly dependent variables.

Since the main objective in this study is to study the dynamic behavior of the demand for natural gas, no attempt is made to spedify a complete-system of equations to include the supply-side of the natural gas industry. And so the estimation of structural demand equations of natural gas are carried out by single equation methods. The following are various methods that are approached:

(1) Ordinary Least Squares (OLS)[9]

(2) Two-Stage Least Squares (2SLS)[10]

(3) Limited Information Single Equation (LISE)[11]

(4) Unbiased Nagar K-Class (UNK)[12]

(5) Three-Stage Least Squares (3SLS)[9]

The OLS method is strictly a single equation technique which does not account for a particular equation

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[9]See J. Johnston, op. cit., Chapter 4, or A. S. Goldberger, op. cit., Chapter 4.

[10]H. Theil, Economic Forecasts and Policy, Amster dam: North-Holland Publishing Company, 1961, Chapter 6. J. Johnston, op. cit., pp. 258-260.

[11]T. W. Anderson and H. Rubin, "Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations," Annals of Mathematical Statistics, Vol. 20, 1949. J. C. Koopmans and W. C. Hood, "The Estimation of Simultaneous Linear Economic Relationships," Chapter 7, Studies in Econometric Method, Cowles Commission Monograph, No. 14, W. C. Hood and T. C. Koopmans, editors, New York: John Wiley and Sons, Inc., 1953.

[12]H. Theil, op. cit., pp. 230-232. Chapter 6. A. L. Nagar, "The Bias and Moment Matrix of the General K-Class Estimators of the Parameters in Simultaneous Equations," Econometrica, Vol. 27, 1959.

being embedded in a complete system. There are usually two or more "jointly dependent" variables in each equation and one may not know which "jointly dependent" variable to select as the dependent variable; and no matter which is chosen, the remaining variables will be correlated with the disturbance term in that equation because of the simultaneous nature of the relationships in the model. Thus the OLS estimates will be inconsistent. This fact may not rule out the OLS method because this method is computationally simple and asymptotic bias introduced by the OLS method of estimation may not be the most important property of an estimator but has to be judged in conjunction with the variance.[13] With these considerations in mind, the OLS method is tried in estimating the demand function of natural gas for each sector.

Theil has described a general classification by which each of the above four methods become special cases under certain conditions.[14] It is called a K-class estimation procedure. For K = 0, the K-class becomes OLS method and for K = 1, it will become 2SLS method. The K values for the LISE and UNK have to be determined for each of the single equations to be fitted. For the LISE method

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[13]J. Johnston, op. cit., p. 253.

[14]H. Theil, op. cit., pp. 231-232.

the K values will always be greater than one for over-identified equations and equal to one for just-identified equations. In one context, the K-value is derived in a manner such that it corresponds to a minimum ratio of residual variance--i.e., the residual variance from regressing a linear combination of the "jointly dependent" variables on the predetermined variables in the equation, divided by the residual variance of the same linear combination of the "jointly dependent" variables regressed on all the predetermined variables in the complete system.[15] In another context, the K-value corresponds to a minimum characteristic root of a determinant-equation.[16] Similarly for UNK procedure, Nagar has proposed that the K-value should be calculated as

$$K = 1 + \frac{L - 1}{T}$$

where L is the number of predetermined variables in excess of the number of coefficients to be estimated and where T is the number of observations.[17] He has shown that this K-value can be expected to reduce a small sample bias of 2SLS (to the order of T^{-1}) in most simultaneous equation studies.[18] The last four procedures will give

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[15] J. Johnston, *op. cit.*, p. 256.

[16] *Ibid.*, p. 257.

[17] A. L. Nagar, *op. cit.*

[18] Alternative K-values have been proposed by Nagar, but they have not been included in this study.

consistent estimates.

The alternative models which explain certain "reaction or adjustment" mechanism are estimated by OLS method. The justification in using the "ordinary least squares" method of estimation is discussed in a later chapter where the formulation and the method of estimation of the corresponding models are developed. All these methods were programmed for use on the Michigan State University CDC 3600 Computer System and so the corresponding routines are used for estimating the respective demand functions of natural gas.[19]

Thus the main objective of this study is to investigate the endogenous mechanism to explain the consumption behavior of natural gas by formulating an economic model to represent the simultaneous nature among the concerned economic variables and then estimating the model using simultaneous equations estimation methods. Some alternative economic models to represent the "reaction" or "adjustment" mechanism may be formulated to explain the behavior of demand of natural gas. Ultimately, in a study of this kind, the investigator usually has the purposes of forecasting future values of economic variables and of predicting the consequences of various economic policies. So an attempt will be made to predict future values and the corresponding consequences.

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[19] All these routines are part of the MSU-STAT system and are written with double precision arithmetic.

In order to achieve these purposes, the following sequence of steps is followed in this study:

1. Formulation of economic models which are capable of establishing the "joint" relationships (the endogenous mechanism) among the concerned variables to explain meaningfully the dynamic behavior of the demand for natural gas in each of the end-use categories,
2. To estimate the structural parameters which will contribute to a better understanding of the economic interrelationships of the economic models developed in (1).
3. To employ the alternative simultaneous equations system estimation methods (noted as above), compare estimates from the alternative estimation procedures and ascertain the apparent advantages of any particular estimation method.
4. To postulate alternative economic models estimate the parameters and compare these parameters with those obtained with the simultaneous equations system approach, and finally

5. To utilize the information obtained in (3) and (4) for predicting the changes in various economic variables.

Taking this as a framework, we start the next chapter reviewing the literature previously investigated in the area of demand for natural gas.

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CHAPTER II

THE DEMAND FOR NATURAL GAS: REVIEW OF THE LITERATURE AND DATA CONSIDERATIONS

2.1 Nature and Objectives of the Studies

In this chapter studies concerning the demand for natural gas in various end-use categories are reviewed. There are four studies that have dealt with the demand part of natural gas.[1] Each is studied in a different context and with different objectives. In two studies the consideration of demand for natural gas is only a part of the complete study; while in the other two only the study of demand is considered. Dr. MacAvoy's study focuses mainly on the economic reasons for regulation in the natural gas industry. It was argued that there was

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[1] P. W. MacAvoy, Price Formation in Natural Gas Fields, A Study of Competition, Monopsony, and Regulation, New Haven and London: Yale University, 1962.

H. H. Wein, Natural Gas Supply and Demand. Docket No. AR 61-1, Federal Power Commission, Office of Economics, Washington, D.C., 1963

R. S. Villanueva, "An Econometric Estimation of User's Demand for Natural Gas," Ph.D. Thesis, University of Pittsburgh, 1964.

P. Balestra, "The Demand for Natural Gas in the Residential and Commercial Markets: A Dynamic Approach," Ph.D. Thesis, Stanford University, 1965.

monopoly control of in-ground gas and that the pipeline buyer paid higher-than-competitive prices for restricted amounts of gas and hence such kinds of monopolistic pricing in gas fields should be controlled as monopolistic electric rates or rail freight rates were controlled. There were also statements that regulation was needed to prevent future price increases, since such increases would be derived at least in part from the use of monopoly power. These viewpoints prompted Dr. MacAvoy to study the characteristics of monopoly price formation, and of competitive and monopsony price formation in order to see which corresponds more closely to actual price formation in the 1950's.[2]

The pipeline companies "buy" volumes of natural gas under contracts in order to satisfy residential, commercial and industrial demands over extended periods. Of course, in practice as well as in economic theory of consumer demand, the end-use consumers will seek somewhat less natural gas over the lifetime of their burner equipment if the burner-tip price of natural gas is increased. Less demand for natural gas at the above markets implies lower aggregate demand by the pipeline companies in the producing regions. This is because the demand at the producing region by the pipeline companies is derived from the resale demand of residential, commercial and industrial

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[2] P. W. MacAvoy, op. cit., p. vii.

users. It is (the demand for natural gas at the producing region) also affected by the maintenance costs of pipeline transmission companies and by requirements of interstate regulatory authorities. Therefore, in order to study the pipelines' demand for reserves of natural gas, one should study the demand for natural gas in residential, commercial and industrial markets. It is in this context that Dr. MacAvoy studied the demand for natural gas by home users and industrial users.[3]

Dr. Wein directed his study more toward the economic functions in setting the wellhead price by the Federal Power Commission rather than studying the price formation in natural gas fields. His study reveals "what the economic consequences of setting any wellhead price will be, how much exploratory activity and what level of reserves can be expected at that price, what residential-commercial and industrial consumption will result from it, the reserve/production ratio that will ensue, and the effects of the wellhead price on burner tip prices."[4] In order to study and provide answers for the above questions, one must know the relationship between wellhead price and exploration and also the relationship between wellhead price and ultimate consumption

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[3] Ibid., pp. 33-37, 267-270.

[4] H. H. Wein, op. cit., p. 8.

through burner-tip prices at the points of consumption in each end-use category. Dr. Wein's study also provides a means whereby the Federal Power Commission can keep in touch with changes in the natural gas industry and adjust its regulatory policy from time to time. With these objectives in mind and in order to study the effects of wellhead price on burner-tip prices, and consequently on the consumption of natural gas at the different end-points of consumption, Dr. Wein proposed to study the demand aspects of natural gas in both the residential-commercial and industrial markets. Thus these two studies of Dr. MacAvoy and Dr. Wein are mainly policy oriented, useful in understanding and regulating the economic policies of the Federal Power Commission.

The other two studies mainly revolve around the development of economic models and consequent estimation of the models by various econometric methods. So they are more academic in nature. For instance, Dr. Villanueva has stated that "the purpose of this study is to investigate the application of various econometric techniques to the measurement of demand in markets characterized by utility regulation of prices."[5] His study, therefore, is concerned with the estimation of the parameters of the natural gas demand schedules of the major sectors of the consumer markets and the effect of the discriminatory gas

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[5] R. S. Villanueva, op. cit., p. 1.

utility rate structure on the distribution of the fuel between the consumer groups. Besides this, he is also concerned with the economic rationale for the gas utility rate regulation. With these things as objectives, Dr. Villanueva has studied the demand aspects of natural gas in residential-commercial, industrial and steam electric generating plant markets.

It has long been recognized by economists and econometricians that a static demand equation, in general, is not adequate to represent the consumer behavior in the durable goods market. Rather some kind of dynamic mechanism should be built in to study the consumer behavior. For durable type goods, several studies are made which have incorporated some kind of "stock effect."[6] Although natural gas cannot be properly called durable, its consumption, like that of electricity, fuel oil, is intimately related to the stock of gas appliances which are durable goods, and to a large extent it is governed by the existence of such stocks. Therefore, it is reasonable to incorporate a stock effect and some assumptions about the adjustment of their stocks over time in the demand function to explain the behavior of consumption of natural gas. Dr. Balestra's study is concerned with the development of such a model. He developed a dynamic demand model for natural

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[6] See for example, F. M. Fisher and C. Kaysen, A Study in Econometrics: The Demand for Electricity in the United States, Amsterdam: North-Holland Publishing Co., 1962.

gas whose consumption is technologically related to the stock of appliances.[7] Thus, Balestra's study evolved mainly on theoretical development of model. The model is estimated by the econometric techniques more sophisticated than the econometric techniques used by the other three authors. Now each study is reviewed with respect to the analysis and methods used. Later in the chapter, the reasons why the Simultaneous Equation System model is formulated and why the State of Michigan is chosen for estimating the Simultaneous Equation System model will be discussed.

2.2 Methodologies of the Four Studies

While studying the factors affecting the pipelines' demand for reserves of natural gas, Dr. MacAvoy has considered two forms of demand, namely, city-wide home demand and industrial demand.[8] In forming the model for city-wide home demand he considered the price of gas for the marginal thousand cubic feet consumed per capita (p), price of fuel oil for the average consumption per capita (P_o), temperature degree days (T), population in the city (N), and the median income per resident in the city (Y) as explanatory variables to explain the behavior

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[7] P. Balestra and M. Nerlove, "Pooling Cross-Section and Time-Series Data in the Estimation of a Demand Model: The Demand for Natural Gas." Technical Report No. 8, Institute for Mathematical Studies in the Social Sciences, Stanford, California: Stanford University, p. 1.

[8] P. W. MacAvoy, op. cit. pp. 267-269.

of the consumption of natural gas. He has taken a cross-section sample of 52 United States cities for the year 1951 and used cross-section analysis to estimate the city-wide home demand function.[9] In case of industrial demand, Dr. MacAvoy has studied the demand for natural gas in each industry. He considered average price of natural gas (P_{gas}), average price of fuel oil (P_2), average price of coal (P_3), average price of electricity (P_4), average size of the firm (S), and the number of firms (F) as independent variables to explain the demand for natural gas in a particular industry. In this case also he used cross-section analysis.[10]

Assuming the linear form in both the cases, Dr. MacAvoy estimated the corresponding coefficients by ordinary least squares method of estimation. The conclusions he has arrived at are as follows. In case of city-wide home demand, the demand elasticity varies over a range of relevant prices and "the extent of demand at each price varies from city to city as well, so that demand can be said to be less elastic at some locations than at others."[11]

Similarly, in case of industrial demand, he concluded that price elasticity varies greatly from industry

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[9] Ibid., p. 267.

[10] Ibid., p. 269.

[11] Ibid., p. 35.

to industry. "The demand schedule in the meat processing or bakery product industries seems quite elastic at average price. The demand schedules in the structural clay products industry and iron and steel industry are even less elastic at average prices. But the demand schedules in both motor vehicle and the beverage industries are highly elastic."[12] He further commented that "the industrial demand equations and the home-consumption demand equation all exhibit some decrease in quantities purchased when the price increases. Demand is more elastic in certain industrial gas uses and least elastic at low prices for home use, and there is a great variation in elasticity, even given users that are the same in most respects."[13]

In an attempt to study the behavior of consumption of natural gas, Dr. Wein considered the aspects of consumption both in the residential-commercial market and in the industrial market. The forms of demand equations for each market are described below. Instead of studying the demand for natural gas by steam electric generating plants separately, he has included the consumption of natural gas by the steam electric generating plants in industrial consumption. Therefore, he has incorporated the "number of

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[12]Ibid., p. 37.

[13]Ibid. p. 38.

kilowatt-hours generated" by these steam electric generating plants as a variable in the industrial demand model in order to measure the effect due to steam electric generating plants on the consumption of natural gas in the industrial market. After elaborate discussion, Dr. Wein formulated the models to explain the behavior of natural gas consumption in both the markets as follows:

(1) Residential and commercial market

$$(2.2.1) \text{ ---- } Y_i^R = A\ X_{9i}^{\alpha_1}\ X_{10i}^{\alpha_2}\ X_{11i}^{\alpha_3}\ X_{12i}^{\alpha_4}\ e^{ui}$$

$$i = 1, 2, \ldots\ldots 44$$

where

Y_i^R = consumption of natural gas in the residential and commercial market in state i

X_{9i} = burner-tip price of natural gas in residential and commercial market in state i

X_{10i} = price of fuel oil in residential and commercial market in state i

X_{11i} = number of customers of natural gas in residential and commercial market in state i

X_{12i} = degree days in state i

(2) Industrial Market

$$(2.2.2) \text{ ---- } Y_i^I = B\ X_{13i}^{\beta_1}\ X_{14i}^{\beta_2}\ X_{15i}^{\beta_3}\ X_{16i}^{\beta_4}\ X_{17i}^{\beta_5}\ X_{12i}^{\beta_6}\ e^{vi}$$

$$i = 1, 2, \ldots\ldots 44$$

where

Y_i^I = consumption of natural gas in industrial market in state i

X_{13i} = burner-tip price of natural gas in industrial market in state i

X_{14i} = price of fuel oil in industrial market in state i

X_{15i} = price of coal in industrial market in state i

X_{16i} = industrial employment in state i

X_{17i} = number of kilowatt hours in state i

X_{12i} = degree days in state i[14]

Thus Dr. Wein formulated the demand functions in each of the markets, as functions of respective prices of natural gas, prices of closely related substitutes and the appropriate exogenous variables such as degree days and industrial employment. The data he used in order to estimate the corresponding coefficients in equations (2.2.1) and (2.2.2) are purely cross-section analysis. In order to analyze the stability of the regression coefficients α_1, α_2, α_3, α_4, in equation (2.2.1) and β_1, β_2, β_3, β_4, β_5, and β_6 in equation (2.2.2) over time, he computed estimates of cross-section analyses for each year from 1955 through 1960.[15]

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[14] H. H. Wein, _op_. _cit_., pp. 87-90. The notation was exactly followed as in Dr. Wein's study. Alaska, Hawaii, Maine and Vermont are not included in the analysis. Data for North and South Dakota are combined. Similarly data for Delaware, District of Columbia and Maryland are combined.

[15] _Ibid_., pp. 94-95

After brief analysis, Dr. Wein concluded that the price elasticity of natural gas demand is inelastic (-.795) in residential and commercial market, and elastic (-2.55) in industrial market. The coefficient of burner-tip price variable is highly significant in both equations. These findings are for the year 1961. While examining the stability of the concerned estimated coefficients of equations (2.2.1) and (2.2.2), he found out that the price elasticity of natural gas demand is varied in both markets. For example, in the case of residential and commercial market, it is varied from -0.945 to -0.715 during 1955-1961.[16] And in the case of industrial demand, it is varied from -2.872 to -2.546 during the period 1955-1961.[17]

After discussing the stability of the coefficients of each variable in each of the markets, he concluded that the estimated coefficients of the demand equations (for 1961) specified in equations (2.2.1) and (2.2.2), are sufficiently stable to describe the behavior of consumption of natural gas.

Dr. Villanueva, while studying the user's demand for natural gas, has divided the gas demand into (1) residential and commercial demand, (2) industrial demand, and

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[16] Ibid., p. 95.

[17] Ibid., p. 94.

(3) the demand by thermal electric generating plants.[18] He has also studied the total demand and total supply of natural gas from a system of "simultaneous equations model" point of view, but only to a limited extent. Dr. Villanueva has formulated several models in first differences of logarithms of the concerned variables for each of the above kinds of demand and estimated them by ordinary least squares method of estimation.[19] For instance, while considering the demand for natural gas by residential and commercial consumers, he has considered the price of natural gas, prices of competitive fuels, personal income, and temperature as explanatory variables and used the first differences of logarithms of these variables to explain the behavior of consumption of natural gas which is again expressed as a first difference in logarithms.[20] Similarly, he has formulated another model to explain the demand for gas service in order to separate that portion of the total demand response attributable to the response of gas service alone.[21] To find out whether there is any "adjustment process" or "reaction mechanism" present in the natural gas consumption, he has also formulated Nerlove-type

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[18] Villanueva, op. cit.

[19] Ibid., p. 7; pp. 18-24.

[20] Ibid., p. 18.

[21] Ibid., p. 36.

"lagged adjustment models."[22] Similar models are formulated to explain the demand for natural gas by industrial consumers and by thermal electric generating plants.[23]

The estimating procedures of the concerned models are as follows: Both the time-series and cross-section analysis are used for each type of demand. The period under consideration for the time-series analysis is 1950-1960 and the regression analysis is carried out for each census region. For cross-section analysis, each state is an observation and thirty-five states are included in the analysis.[24] Thus there are 35 observations in cross-section analysis and the time periods considered are 1950 and 1959.[25] In case of industrial demand, the demand models are formulated by individual industry categories

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[22] Ibid., p. 23. Also see M. Nerlove, "Distributed Lags and Demand Analysis for Agricultural and other Commodities," U. S. Department of Agriculture, Agricultural Handbook No. 141, June 1958.

[23] Ibid. Models of demand for natural gas in industrial consumers, pp. 72-75. Models of demand for natural gas by thermal electric generating plants, pp. 101-107.

[24] Ibid., p. 64.

[25] The number of states as observations considered for 1959 cross-section analysis is 46. R. S. Villanueva, op. cit., p. 64. This is for residential and commercial market. For industrial market, the time periods considered for the cross-section analysis are 1951-52 and 1959-60. R. S. Villanueva, op. cit., p. 95. For thermal electric generating plants, the cross-section analysis is performed for the period 1960-61 only. R. S. Villanueva, op. cit., p. 116.

and then the above described time series and cross-section analyses are approached.

The following are the conclusions from his analysis. He has found that there is a significant variation of price elasticites of gas demand among different regions in case of residential and commercial markets and industrial market.[26] The same conclusion is reached in the case of demand for natural gas by thermal electric generating plants. The cross-section analysis, in the case of residential and commercial demand, provides a "significant" evidence of the sensitivity of demand to variations in price and temperature.[27] In the case of industrial demand and the demand for gas by steam electric generating plants, the cross-section analysis provided rather unsatisfactory results. The coefficient of determination, R^2, is not high compared to the R^2 of Dr. Wein's cross-section demand equations.[28] All the adjustments models have exhibited little evidence of lagged adjustment of demand with respect to the corresponding explanatory variables.

Dr. Villanueva also formulated a simple simultaneous equation system" model consisting of total demand

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[26] R. S. Villanueva, op. cit. See the conclusions in the case of residential and commercial markets, pp.70-71; industrial market, pp. 99-100; thermal electric generating plants, p. 120.

[27] Ibid., p. 70-71.

[28] Ibid. p. 95.

equation and total supply equation and estimated the model through the method of two-stage least squares.[29] He has found a demand elasticity of price which varied between -.8 and -3.19 in three of the five regions and a demand elasticity of price of value -0.66 for the total United States.[30]

Dr. Balestra studied the dynamic behavior of natural gas in case of residential and commercial sectors. He did not study the dynamic behavior from the standpoint of residential sector only, nor did he study it from the standpoint of commercial sector. He combined both, as similar to other studies in this area, and focused his attention on the dynamic consumption behavior of natural gas in the residential and commercial market. Balestra has based his demand analysis on two basic aspects, namely,

1. formulation of a demand function for commodities whose consumption is technologically related to the stock of appliances, and
2. estimation of parameters of the demand function when the demand function is cast in dynamic terms and when

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[29]Ibid., pp. 137-152.

[30]Ibid., p. 14. For comparison of these estimates with those of Residential-Commercial market, Industrial Market and Thermal Electric Generating Plants Market, see p. 149.

observations are drawn from a time-series of cross-sections.

Balestra has started with a simple static demand model to explain the consumption behavior in residential and commercial sectors and observed that static demand analysis is inadequate to represent the behavior of the consumer in a market in which consumption is technologically related to the stock of appliances. This observation has led him to the incorporation of a stock effect in the demand function. The basic idea underlying this approach is to consider the "new demand" for gas, i.e., that portion of demand that is free from past commitments. On the basis of this concept of "new demand," Balestra has formulated a dynamic model of demand in which the price variable has an effect primarily on the rate of growth of consumption rather than on its absolute levels. The other variables that appeared upon the starting assumptions in the model are lagged population, change in population (i.e., first-difference in population), lagged income, first-difference in income, lagged consumption of natural gas.[31] And the form of equation is specified as

$$(2.2.4) \text{ ---- } G_{ti} = \alpha_o + \alpha_1 P_{gti} + \alpha_2 N_{t-1\,i} + \alpha_3 \Delta N_{ti} + \alpha_4 Y_{t-1\,i} + \alpha_5 \Delta Y_{ti} + \alpha_6 G_{t-1\,i} + \epsilon_{ti}$$

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[31]P. Balestra, op. cit.,p. 7

where

G_{ti} = consumption of gas in BTU 10^{12} in time period t of the i^{th} state

P_{gti} = price of gas in time period t of the i^{th} state

$N_{t-1\,i}$ = population in time period (t-1) of the i^{th} state

$Y_{t-1\,i}$ = per capita income (in 1961 dollars) of time period (t-1) of the i^{th} state

G_{t-1} = consumption of gas in BTU 10^{12} in time period (t-1) of the i^{th} state

ϵ_{ti} = stochastic disturbance

Δ = first difference operator and corresponds to the time period t of the i^{th} state.

In the case of a dynamic model as specified in equation (2.2.4), the application of ordinary least squares to the pooled sample of observations (a time-series of cross section) produced inconsistent estimates of the coefficients of lagged endogenous variable.[32] As far as equation (2.2.4) is concerned, there is only one such lagged endogenous variable. Under certain restrictive assumptions,[33] the coefficient of this lagged endogenous variable is constrained at the value of one, in which case

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[32]Inconsistent in the sense that the estimated coefficient of lagged endogenous variable is greated than one. J. Johnston, Econometric Methods, New York: McGraw-Hill Book Company, pp. 211-221.

[33]P. Balestra, *op. cit.*, p. 10.

this variable can be shifted to the left hand side of equation (2.2.4) and estimation by ordinary least squares is then appropriate under fairly general conditions. For the more general case, i.e., for the case when the coefficient of lagged endogenous variable is not constrained at the value of one, Balestra has developed two procedures to obtain efficient and consistent estimates of the parameters of the dynamic model. The key assumption made by Dr. Balestra in this general case is the separation of the residuals into two components: a time-invariant regional effect and a remainder.[34] The two estimating procedures he has developed are as follows:

1. the maximum likelihood procedure--which has turned out inapplicable in the case of the natural gas model,[35]
2. procedure to derive BAN (<u>B</u>est <u>A</u>symptotically <u>N</u>ormal) estimates--which gave satisfactory results in the case of natural gas model.

Dr. Balestra, after thorough and painful investigation,

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[34] P. Balestra and M. Nerlove, "Pooling Cross-Section and Time Series data in the estimation of a dynamic model: The Demand for Natural Gas," <u>Technical Report No. 8</u>, Institute for Mathematical Studies in Social Services, Stanford, California; Stanford University Press, p. 14.

[35] <u>Ibid</u>., p. 27.

came to the following main conclusions:[36]

1. "The over-all results seem to support the major hypothesis concerning price elasticity embodied in the dynamic model. It appears that for commodities such as gas, the total demand for the commodity is quite insensitive to price changes, however, the incremental demand for the commodity is more responsive for price changes. The estimated magnitude of the average price elasticity of the incremental demand for gas is less than unity, but it appears to be increasing over time (and is above unity in the last year considered). The approach seems to suggest, in turn, that competition from alternative sources of energy may become stronger in the years ahead.

2. The particular characteristics of the gas market necessitate the separation of the time period under investigation into two sub-periods corresponding to two technologically different stages of development: an innovating stage and a mature stage. The identification of different stages of development has important implications for the future. It seems that for future expansion, gas must rely more heavily on the normal growth of population, household formation, and the like. Spectacular advances in gas consumption are no longer as probable as in the early 1950's except in those states in which gas is still a comparatively new commodity.

3. The application of the estimating procedures developed in this study to the pooled sample of cross-sections and time-series produces results that are plausible on the basis of a priori theoretical reasoning and suggests that these methods may prove successful in similar studies of demand."

2.3 Need for Differentiation of Markets

Thus the four studies are different from each other with respect to the models the authors have considered and

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[36] Ibid. p. 38.

with respect to the starting assumptions they believed play a significant role in explaining the behavior of consumption of natural gas in the respective markets. Also the time factor regarding the availability and advancement in use of electronic computer techniques to estimate the appropriate demand models in a more complex setting may have forced the authors who studied the behavior of demand functions earlier than others to build simpler models. The one common viewpoint from these studies is that they agreed to study the demand of natural gas on the basis of major types of different use of natural gas, namely, residential-commercial, industrial and steam electric generating plants. But there is one exception to this point: Dr. Villanueva formulated the demand equation in his system of simultaneous equation model on the basis of total demand consisting of the consumption by residential-commercial, industrial and steam electric generating plant markets.

The structure of the different types of end-uses of natural gas suggests that it is necessary to distinguish the basic types and classes of natural gas service. There are three types of service, namely

(i) the "firm" service,

(ii) the interruptible service, and

(iii) the "off-peak" and "seasonal" service.

The "firm" service is intended to have assured availability to the customer to meet his load requirements.

The load requirements come mainly from the type of weather (cold winters or hot summers) and the type of service the customers have. This service is ordinarily available to all classes of customers. Space heating, although required only in the winter season, is a firm service. The interruptible service, as the name suggests, is made available under agreements which permit the curtailment or cessation of deliveries. Interruptible customers are almost always the large-volume industrial customers such as steam electric generating plants. The curtailment or cessation of deliveries ordinarily occurs when the gas is needed for "firm" service. Lastly, the "off-peak" and seasonal service is provided for intervals of time specified by the utility. This is a "firm" service. Added to these types of services, there are mainly four classes of natural gas service:

(i) Residential Service,

(ii) Commercial Service,

(iii) Firm Industrial Service, and

(iv) Interruptible Industrial Service.

The differentiation between these classifications comes from the type of load factor they require. The residential service requires a substantial regularity and a high load factor, ranging from 75 to 85 per cent in most localities.[37] The main uses in this service are domestic uses and space heating. The commercial service has the same specifications. The load factor in the firm industrial service varies from industry to industry and generally varies from 60 to 75 per cent. Generally the load factor in this class of service depends on the sensitivity of business conditions.[38] The main uses in this category are, of course, the use of natural gas as inputs and for space heating. The interruptible service is generally considered to have no load factor since the sales in this class of service are made under the condition that service can be interrupted in any degree and at any time. Interruptible sales are depended upon by many gas distributors to fill in the off-peak summer "valley" (or low point) in demand and thereby to maintain the system annual load factor at a high level.[39]

These differences in the types and classes of service warrant a recognition in the study of demand of natural gas. A serious consideration should be given

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[37] P. J. Garfield and W. F. Lovejoy, Public Utility Economics, Englewood Cliffs: Prentice-Hall, Inc., 1964. p. 171

[38] Ibid., p. 171.

[39] Ibid., p. 171.

to these differences because they create the differentiation of prices and a variation in degree of competition from other fuels in each of the four services in their corresponding markets. The gas distributing company is compelled to charge differently to different types of service because of the inherent nature of the service. For instance, the rates for interruptible and off-peak gas "should be something more than the out-of-pocket or incremented cost of providing the service and cannot exceed the prices of competitive fuels or the gas will not be sold."[40] The distributing company must consider the competition from other fuels while making sales to large industrial and steam electric power plants on the basis of interruptible service. Similarly by nature the "firm" service is a premium quality service and is therefore more valuable. In addition, the unit cost of serving numerous small customers may be higher compared to those of large industrial users and interruptible users. This causes the rates for residential use and commercial uses to be relatively higher compared to other uses.

The degree of competition exists among energy fuels mainly because of the following reasons: One type or form of energy fuel is superior to all the others in a certain type of market. This is partially a function

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[40] Ibid., p. 170.

of relative costs, but more important are the unique physical characteristics of the particular fuel. For example, in residential and commercial markets, natural gas has an advantage of physical characteristics such as clean burning, no odor, dependability of service compared to other competing fuels. Yet there is a keen competition among fuels in these markets for heating, cooking, clothes drying, water heating, air conditioning, and refrigeration. Cooking is done primarily with gas and electricity, although coal and range oil are used in some rural and low income areas. The vigorous campaign staged by the electric appliance and electric utility distribution companies has resulted in substantial gains for electric cooking in many areas of the country. Such vigorous campaigns are initiated by gas distribution companies also. Similarly, gas and electricity compete for residential water-heating and clothes drying markets. As Dr. Balestra discussed, stocks of appliances through vigorous campaigns would accelerate the competition between natural gas and electricity. In residential heating market, oil and coal compete vigorously with natural gas through pricing and non price factors.

Secondly, the substitution of one or more fuels for another may play an important role in changing the competitive structure. Physical characteristics are of minor importance here and the use of one fuel compared to another is primarily a function of relative costs of the respective competing fuels. This is more prevalent in steam electric generating plants market where the gas service is based on interruptible basis. Almost all the plants in this market are equipped with machinery that can use competitive fuels like coal, fuel oil, so that they can take advantage of the lowest price among the alternative fuels. Thirdly, location of the production of the competitive fuels vary the degree of competition. For instance, for electricity generation in the Southwest region, natural gas is used as plant fuel almost exclusively. In West Virginia, coal is used, while in Maine and Florida, fuel oil is used. Thus, the above three things can influence the degree of competition in each market.

Recognizing these facts, the four authors, while studying the behavior of consumption of natural gas, have divided the total market into three main submarkets, namely, residential and commercial, industrial, and steam electric generating plants.[41] It would be still better if the above division of markets were further divided as residential, commercial, industrial and steam electric generating plant markets. This would enable the research worker to understand better about the variations in prices and competitive aspects in each of the submarkets, rather

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[41]Dr. MacAvoy studied the city-wide home demand and industrial demand for natural gas. He did not consider the commercial demand.

than understanding them in the above three/way submarket structure. Also there is a possibility of getting a more homogenous data for the concerned variables in a four/way submarket structure rather than in the three/way submarket structure. Therefore, in this study, the total market is divided into four submarkets and an attempt is made to study the behavior of consumption in each market by developing appropriate models.

2.4 Problem of Homogeniety of Data

Another important aspect that is commonly recognized by all four authors is the problem of "homogeneity of data." For example, the prices paid for the use of natural gas in the New England States are far different from the prices paid in the West South Central States, which are, in turn, different from those paid in the Pacific States. Thus they may not be representative in any of the markets. But they are at least homogeneous in each state. If cross-section analysis is used, this kind of "homogeneity problem" due to demographic factors may be reduced because cross-section analysis considers each state as an economic unit at a given time point. By this means, individual state differences can be accounted significantly in estimating the demand functions. The cross-section estimation procedure necessarily assumes that it is possible to specify explicitly the differences among the economic units in such a way that once these characteristics are specified, the economic units will, on the average, react in the same way to any particular stimulus.[42] These reasons probably may have led the above authors to use cross-section analysis.[43] Of course, there are certain drawbacks in using the cross-section estimation procedure that should not be overlooked. Certain "nuisance" variables that may not influence significantly the behavior of consumption individually but may affect it collectively, may not be accounted in the statistical estimation of demand functions.[44] If these variables are not taken into account, one may question the underlying relationship. Some of the problems confronted by the "nuisance" variables may be removed by more elaborate cross-section classification with respect to the "nuisance variables." But by making such elaborate classifications, at the same time, one may likely create more difficulties in getting the required data. Also "cross-section analysis, except under rather heroic assumptions, will not provide information on the influence of prices, which may be of importance in long-term projections."[45] However, these points should

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[42] Y. Grunfeld, "The Interpretation of Cross-Section Estimates in a Dynamic Model," Econometrica, Vol. 29, 1961.

[43] There is an exception to this comment. Dr. Balestra used pooled time series of cross-section samples.

[44] L. R. Klein, An Introduction to Econometrics, Englewood Cliffs; Prentice-Hall, Inc., 1962, pp, 52-60.

[45] H. S. Houthakker and L. D. Taylor, Consumer Demand in the United States, 1929-1970, Cambridge: Harvard University Press, 1960, p. 5.

not negate the use of cross-section analysis to estimate the demand functions becaust it may still provide an "insight into the mathematical shape of the important relationships and into the possible importance of a number of factors that cannot be readily isolated in time series because of their trend like behavior.[46]

Another important aspect of "homogeneity problem," particularly in the natural gas industry, which may destroy the homogeneous nature among observations is the presence of the following set of factors:

1. the degree of urbanization,
2. the weather conditions, and
3. the availability of gas.[47]

These three factors are bound to cause the demand response in each market in a unique way from state to state or even from city to city. This is agreed by all the four authors. According to Dr. Balestra, the last factor, namely, the availability of gas, affects significantly the nature of the results and his analysis has suggested "the separation of the time period under investigation into two technoligically different periods. . . The behavior pattern in the gas market seems to be divided into periods corresponding to two stages of development,

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[46] Ibid., p. 5.

[47] P. Balestra, op. cit., p. 117.

an infant stage and a mature stage."[48] This is justified in his study because the transmission pipelines are not laid uniformly at the same time throughout the states so that all regions are supplied with natural gas. Some regions are provided with pipelines earlier than others. Consequently this caused the above stated differentiation of two stages in the natural gas market. Problems due to the weather conditions and urbanization would be reduced to a minimum if some kind of measure is taken by introducing these variables or their effects into the respective demand functions of natural gas. Drs. MacAvoy, Wein and Villanueva approached this point of view and used temperature as a variable in order to measure the effect due to weather conditions. And to take into account the effect due to urbanization, Dr. Villanueva used both time series and cross-section observations.[50] Drs. MacAvoy and Wein used cross-section analysis to take into account the problem due to urbanization in analyzing the behavior of natural gas consumption. Thus, the above stated factors influence the analysis significantly and should be given serious consideration in studying the behavior of consumption of natural gas. One other variation different

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[48] *Ibid.*, p. 119.

[49] R. S. Villanueva, op. cit., p. 4.

[50] *Ibid.*, p. 100

from the above studies is attempted in this study. That is, only one state is selected and the entire demand analysis is investigated by building pertinent econometric models. By this approach, problems due to urbanization are bound to be reduced to a minimum. In order to reduce the problems due to weather conditions, appropriate variable can be introduced in the demand model. The importance of availability of gas is already noted earlier. If one selects a state in which natural gas was introduced during the 1950's, then he may be left with a fewer number of observations for the analysis and this may change the nature of analysis. Therefore, if one selects a state which has an ample number of observations over a relatively long maturity stage with respect to the availability of gas, the problem posed by Dr. Balestra as well as the problem of a small number of observations may be reduced to a minimum, so more weight must be given to this factor compared to the other two in selecting a state. There may be difficulty in using cross-section analysis because of the difficulty of securing data for the individual regions of a state. Also, one may be left with few observations for the cross-section analysis. Therefore, the analysis of time-series is used in this study. By appropriate specification of the demand model and by using advanced techniques of estimation, one may reduce the problems pointed out by the critics of time series analysis. For these reasons the State of Michigan is selected for the analysis. In this state, natural gas was introduced much earlier and has a relative long period of maturity stage.

2.5 Simultaneous Nature Among Natural Gas and Its Substitutes

Another important point to be noted is that in all the four studies, electricity variable, which is an important substitute for natural gas than other fuels, is not included in either of the studies more explicitly. This can be seen through the simple correlations among natural gas and its substitutes that are presented in Tables 2.5.1, 2.5.2, and 2.5.3.

"Competition between electricity and natural gas is one of the major problems to which managerial attention is devoted. Thirty years ago, gas had the cooking load in the home, electricity the lighting, and there was not much to scrap about in addition. But as each has sought to broaden its share of the energy market, this competition has led to continued improvements in the quality and performance of conventional gas and electric appliances, and to the development of new appliances. Now the crossover to those in the gas business is over. No service gas performs in the home cannot be performed by electricity also--not as well, of course, but at least adequately! Not only cooking, but water heating, clothes drying, air conditioning, space

Table 2.5.1 Simple Correlations Among Natural Gas And Its Substitutes: Residential Sector

	Y_1	Y_2	Y_7	Y_8
Y_1	1.0000			
Y_2	0.7891	1.0000		
Y_7	-0.8438	-0.7636	1.0000	
Y_8	-0.0838	-0.0220	-0.0082	1.0000

Y_1 = consumption of natural gas (millions of mcf)

Y_2 = index of burner-tip price of natural gas in residential sector (¢/mcf)

Y_7 = index of price of electricity in residential sector (¢/kwh)

Y_8 = index of price of fuel oil in residential sector (¢/gal)

Source: Computed from data in Appendix A.1

Table 2.5.2. Simple Correlations Among Natural Gas And Its Substitutes: Commercial Sector

	Y_3	Y_4	Y_9	Y_{10}
Y_3	1.0000			
Y_4	0.5220	1.0000		
Y_9	-0.0340	-0.2971	1.0000	
Y_{10}	0.2347	-0.2086	+0.8598	1.0000

Y_3 = consumption of natural gas in commercial sector (millions of mcf)

Y_4 = index of burner-tip price of natural gas in commercial sector (¢/mcf)

Y_9 = index of price of electricity in commercial sector (¢/kwh)

Y_{10} = index of price of fuel oil in commercial sector (¢/gal)

Source: Computed from data in Appendix A.1

Table 2.5.3 Simple Correlations Among Natural Gas And Its Substitutes: Industrial Sector

Y	Y_5	Y_6	Y_{11}	Y_{12}	Y_{13}
Y_5	1.0000				
Y_6	0.4040	1.0000			
Y_{11}	0.0920	0.4863	1.0000		
Y_{12}	0.2678	0.0507	0.2000	1.0000	
Y_{13}	-0.4556	-0.1088	0.6084	0.2356	1.0000

Y_5 = consumption of natural gas in industrial sector (millions of mcf).

Y_6 = index of burner-tip price of natural gas in industrial sector (¢/mcf)

Y_{11} = index of price of electricity in industrial sector (¢/kwh)

Y_{12} = index of price of fuel oil in industrial sector (¢/gal)

Y_{13} = index of price of coal in industrial sector ($/ton)

Source: Computed from data in Appendix A.1

heating, and garbage disposal are all the daily battlefield."[51] As more applications for natural gas and electrie energy are created, the arena in which competition occurs continues to expand. The vigorous campaign staged by the electric appliances and electric-distribution industries may also have caused the increased use of electricity. The number of dwelling units having appliances equipped either for cooking or for heating to consume either natural gas, fuel oil, coal or electricity, and their percentage, are presented in the following tables for the East North Central Region.[52]

From these tables one can see that in the case of cooking, the number of electric appliances increased from 21.5 per cent of the total in 1950 to 36.4 per cent of the total in 1960. For the same period, the number of natural gas equipment decreased from 65.6 per cent to 62.4 per cent of the total. This indicates the corresponding usage of natural gas and electricity as energy fuels and shows the competition between them. But in

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[51]Marvin Chandler, "The Utilities--An Increasingly Competitive Industry," Public Utilities Fortnightly, October 8, 1964, p. 57.

[52]East North Central Region consists of the following states: Illinois, Indiana, Michigan, Ohio and Wisconsin. One can reasonably assume that the conclusions that can be derived from these tables can hold for the state of Michigan because of geographic proximity and other economic conditions that are similar among the states in the region.

Table 2.5.4 Number of Dwelling Units Having Cooking Appliances and the Corresponding Percentages for Selected Types of Fuels: East North Central Region

Year	Type of Fuel	Number of Dwelling Units	Percentage Number of Dwelling Units
1950	Utility gas[a]	4,936	65.6
	Liquid fuel[b]	326	4.3
	Coal[c]	645	8.6
	Electricity	1,617	21.5
	Total	7,524	
1960	Utility gas[a]	5,746	62.4
	Liquid fuel[b]	48	0.5
	Coal[c]	63	0.7
	Electricity	3,344	36.4
	Total	9,201	

a. Utility gas is piped into the dwelling unit from a central system serving the community. Such gas is supplied by a public utility company, municipal government or similar organization.

b. Liquid fuel includes fuel oil, kerosene, distillate oil, furnace oil, coal oil, stove oil, range oil, lamp oil, gasoline and alcohol.

c. Coal includes coke also.

Source: Col. 2: U.S. Census of Housing, 1950, 1960, U. S. Department of Commerce, Bureau of Census
Col. 3: computed from Col. 2

Table 2.5.5 Number of Dwelling Units Having Heating Appliances and the Corresponding Percentages for Selected Types of Fuels: East North Central Region

Year	Type of Fuel	Number of Dwelling Units	Percentage Number of Dwelling Units
1950	Utility gas[a]	1,544	19.0
	Liquid fuel[b]	1.692	20.8
	Coal[c]	4,891	60.0
	Electricity	18	0.2
	Total	8,145	
1960	Utility gas[a]	4,679	45.6
	Liquid fuel[b]	3,358	32.8
	Coal[c]	2,168	21.2
	Electricity	39	0.4
	Total	10,244	

a. Utility gas is piped into the dwelling unit from a central system serving the community. Such gas is supplied by a public utility company, municipal government or similar organization.

b. Liquid fuel includes fuel oil, kerosene, distillate oil, furnace oil, coal oil, stove oil, range oil, lamp oil, gasoline and alcohol.

c. Coal includes coke also

Source: Col. 2: U.S. Census of Housing, 1950, 1960, U. S. Department of Commerce, Bureau of Census. Col. 3: Computed from Col. 2.

the case of heating, percentage equipment increased for both the fuels--both getting the market from coal. Thus electricity is an important variable to be considered while formulating the demand models for natural gas. Similarly, consideration of other competing fuels, namely fuel oil and coal, more explicitly is desirable.

This makes the demand models more explicit and more accurate because most of the relevant variables are included in the models. By including explicitly all the relevant variables as much as possible, greater homogeneity would be attained.[53] Some kind of consideration is given in all the four studies mentioned earlier, either by introducing directly the variables of substitute commodities or by introducing other variables capable of measuring the effects of the variables of substitute commodities. But no systematic account of the substitute variables is given in any study. A systematic account will be given in this study by developing economic models based on simultaneous nature among the relevant substitute economic variables. Together with economic theory and statistical availability of data, a simultaneous equations system model will be developed and then by using advanced methods of statistical estimation the model will be estimated. In this way one can recognize and discover the interdependent

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[53] Y. Gruenfeld, op. cit., p. 399.

nature among the relevant substitute variables by giving an account for the joint or mutual determination of changes in economic variables.

2.6 Expected Normal Income Hypothesis

It is argued that "if a consumer unit knows that its receipts in any one year are unusually high and if it expects lower receipts subsequently, it will surely tend to adjust its consumption to its 'normal' receipts rather than to its current receipts."[54] In other words, the effects of changes in "expected normal income" are strong compared with changes in current income in understanding the consumer behavior. They are even stronger compared with the effects of durations of "expected" future incomes about "expected normal incomes."[55] As said earlier in the first chapter, alternative economic models are developed on the basis of these ideas. The formulation of the corresponding model and estimation methods are discussed later in this study.

Dr. Balestra formulated his dynamic model to explain the behavior of natural gas consumption by relating the consumption to the stock of appliances. He

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[54]M. Friedman, A Theory of the Consumption Function, A Study by the National Bureau of Economic Research, New York. Princeton: Princeton University Press, 1957, p. 10.

[55]M. Nerlove, "Distributed Lags and Demand Analysis for Agricultural and Other Commodities," Agricultural Marketing Service, U. S. Department of Agriculture, Washington, D.C., 1958, p. 29.

argued that the consumption of natural gas, at least at the household level, is closely related to the stock of appliances in existence and hence to a large extent it is governed by such stock. The concept he incorporated in formulating his dynamic model is appreciable because it can shed certain light to explain the consumption behavior. But how significant the stock variable is in explaining the behavior of natural gas consumption is somewhat questionable. In the first place, the stocks are not made by consumers of natural gas, rather they are made by retail distributors of gas appliances. To that extent they are functions of cost of both gas-using and electricity-using appliances, campaign programs engaged in by distributors, "expected normal income" of consumers, prices of natural gas and its substitutes, average consumption of the fuels by appliances, other things being held constant. With higher "expected normal income" the consumer has means to replace a wearing oil furnace or stove with a gas furnace or stove. Or he may afford setting up air conditioning equipment in his house. This implies higher consumption of natural gas. Similarly, the lower the cost of appliances, the higher demand for appliances (consumption of the respective fuels), which is a dictated fact of economic theory of consumer choice. So it is conceivable to argue that the effects of the stock variable are included in the effects of the above discussed variables. Also for relatively short periods of time horizon, prices of fuels may not have significant effects on the behavior of consumption, particularly in the case of natural gas. Thus either the cost of appliances or the "expected normal income" are relevant variables compared to the stock variables to explain the dynamic behavior of consumption of natural gas. In this study, some alternative economic models are developed only on the basis of "expected normal income" concept. The justification for using this concept is discussed above.

Before we develop the simultaneous equations system model and alternative models based on the concept of "expected normal income," some static models will be developed in the next chapter. These static models are helpful in choosing the variables that are to be incorporated in the dynamic models. They may also serve as a standard of comparison for the dynamic models. But before we proceed to the next chapter, the notation followed on the variables both in the static and the dynamic models is described:

Y_1 = average consumption of natural gas (millions of mcf) in the residential sector

Y_2 = index of burner-tip price of natural gas (¢/mcf) in the residential sector

Y_3 = average consumption of natural gas (millions of mcf) in the commercial sector

Y_4 = index of burner-tip price of natural gas (¢/mcf) in the commercial sector

Y_5 = average consumption of natural gas (millions of mcf) in the industrial sector

Y_6 = index of burner-tip price of natural gas (¢/mcf) in the industrial sector

Y_7 = index of price of electricity (¢/kwh) in the residential sector

Y_8 = index of price of fuel oil (¢/gal) in the residential sector

Y_9 = index of price of electricity (¢/kwh) in the commercial sector

Y_{10} = index of price of fuel oil (¢/gal) in the commercial sector

Y_{11} = index of price of electricity (¢/kwh) in the industrial sector

Y_{12} = index of price of fuel oil (¢/gal) in the industrial sector

Y_{13} = index of price of coal ($/ton) in the industrial sector

Z_0 = constant term

Z_1 = lagged (by one period) disposable personal income in Michigan

Z_2 = lagged (by one period) number of degree days in Michigan

Z_3 = lagged (by one period) consumption of natural gas in residential sector

Z_4 = lagged (by one period) consumption of natural gas in commercial sector

Z_5 = lagged (by one period) consumption of natural gas in industrial sector

Z_6 = lagged (by one period) industrial employment (in thousands) in Michigan

Z_7 = lagged (by one period) index of price of electricity in residential sector

Z_8 = lagged (by one period) index of price of electricity in commercial sector

Z_9 = lagged (by one period) index of price of electricity in industrial sector

Z_{10} = lagged (by one period) index of price of fuel oil in residential sector

Z_{11} = lagged (by one period) index of price of fuel oil in commercial sector

Z_{12} = lagged (by one period) index of price of fuel oil in industrial sector

Z_{13} = lagged (by one period) index of price of coal in industrial sector

Z_{14} = disposable personal income in Michigan

Z_{15} = number of degree days in Michigan

Z_{16} = number of customers of natural gas in commercial sector

Z_{17} = industrial employment in Michigan

The data for all these variables are based on the State of Michigan. The compiled data in the form of tables can be seen in Appendix A.1. The demand models are not formulated for steam electric generating plant markets because in Michigan no use, or very little use, of natural gas is made by steam electric generating plants, so the analysis for this sector is not attempted and the consumption of natural gas in the industrial sector obviously excludes the consumption of natural gas by steam electric generating plants, however small the consumption is. Now we go to the next chapter where some static demand models are developed and analyzed.

CHAPTER III

THE DEMAND FOR NATURAL GAS: THE STATIC APPROACH

3.1 The Simple Static Demand Equation

The purpose of this chapter is to formulate some simple static models of demand for natural gas and estimate the parameters of the corresponding demand models. Static models can be helpful in choosing the variables which should be incorporated in a dynamic model. They also may serve as a standard of comparison for dynamic models. Also, the investigator may know:

1. the limitations and deficiencies of static theory to explain the demand for natural gas,
2. the relevant variables that most usually affect the demand for natural gas, and
3. the variables which are later to be incorporated in dynamic models.

Because of these reasons, one may not neglect formulating (and estimating) some static models even if they give inconsistent results.

Assuming that the consumer maximizes his utility function, the consumer's demand for natural gas may be stated as a function of the burner-tip price and income. That is

(3.1.1) ---- $Q = f\ (P\ M)$

where

Q = quantity of natural gas consumed

P = burner-tip price of natural gas at point of consumption

M = money income

Equation (3.1.1) may also be treated as a short-run demand model, because there is no "reaction mechanism" or "adjustment process" to explain the dynamic behavior of natural gas demand. For a relatively shorter period, the influence of the prices of substitutes--coal, fuel oil, and electricity--is almost negligible, since the stocks of appliances owned by the community are assumed fixed in the short-run. So the price variables of substitutes are not included in equation (3.1.1). It simply explains the behavior of consumption for changes in burner-tip price (P) and money income (M). For these reasons, the static approach may be termed as the short-run approach.

The demand model in equation (3.1.1) is simply stated in an abstract form. It is not useful for estimation purposes unless certain functional form is assumed. The following functionsl forms are assumed for equation (3.1.1):

(3.1.2) ---- $Q_t = \alpha_o + \alpha_1 P_t + \alpha_2 M_t$ Linear form

(3.1.3) ---- $\log Q_t = \beta_o + \beta_t \log P_t + \beta_2 \log M_t$

Double-logarithmic form

Introducing a stochastic disturbance term, equations (3.1.2 and (3.1.3) may be written as:[1]

(3.1.4) ---- $Q_t = \alpha_o + \alpha_1 P_t + \alpha_2 M_t + V_t$ and

(3.1.5) ---- $\log Q_t = \beta_o + \beta_1 \log P_t + \beta_2 \log M_t + U_t$

One should note at the outset about the implicit assumptions in equations (3.1.4) and (3.1.5). The implicit assumptions in the linear form, i.e., in equation (3.1.4), is that all positive elasticities will ultimately tend to one. But the double-logarithmic form implicitly assumes that the elasticity remains constant over the entire range of variables.[2] Regarding the computational ease for obtaining the elasticity coefficients between the two forms, double-logarithmic form is preferred to the linear form for the coefficients of the variables in double-logarithmic form are the corresponding estimated elasticities. In addition to this, the double-logarithmic form leads to comparatively easy mathematical manipulations.

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[1]The justification for including the stochastic disturbance term is explained by J. Johnston in his book Economic Methods, New York: McGraw-Hill Book Company, pp. 4-7.

[2]P. Balestra. "The Demand for Natural Gas in the Residential and Commercial Market: A Dynamic Approach." Ph.D. thesis, Stanford University, 1965.

Evan though the choice between the two forms involves a compromise among several criteria including economic theory, goodness of fit, and simplicity, it is not intended to select one functional form compared to the other.[3] Rather it is in the general interest of showing the results of short-run static models to explain the demand for natural gas. The results of both the functional forms are presented and evaluated in this chapter.

3.2 Estimation of the Simple Static Demand Equations

The method used for estimating the corresponding demand functions of natural gas for residential, commercial and industrial sectors is ordinary least squares based upon time-series data. The time-period considered is 1947-1964 and so there are 18 observations on each variable of the corresponding models. Disposable personal income is used instead of personal income in the residential and commercial demand equations. The

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[3]The goodness of fit can be judged by the coefficient of determination, R^2. One should note two points in comparing the two functional forms: (1) it is the corrected coefficient of determination R^2 which is defined as:

$$\bar{R}^2 = R^2 - \frac{K}{T-K-1}(1-R^2)$$

not R^2 to be used in comparison. (T corresponds to the number of observations and K corresponds to the number of exogenous variables). (2) The corrected coefficients of determination should be calculated, in the case of double-logarithmic functional form, on the basis of antilog of the dependent variable and should then be compared with the corrected coefficient of determination based on linear functional form. See page 217, A. S. Goldberger, Econometric Theory, New York: John Wiley & Sons, Inc.

following are the estimated demand functions in both the linear and double-logarithmic functional forms:[4]

1. Residential Sector

Linear form:

$$Y_{1\,t} = -142.3611 + 0.4223\ Y_{2\,t} + 0.0180\ Z_{1\,t}$$

(1.3143) (0.0022)

(0.75) (0.00)

$R^2 = 0.9300$ $d = 0.9748$

Double-Logarithmic form:

$$\log Y_{1\,t} = -4.6331 - 1.2485 \log Y_{2\,t} + 2.2492 \log Z_{1\,t}$$

(0.9438) (0.1998)

(0.20) (0.00)

$R^2 = 0.9516$ $d = 1.0726$

2. Commercial Sector

Linear form:

$$Y_{5\,t} = -12.8523 - 0.3675\ Y_{6\,t} + 0.0060\ Z_{1\,t}$$

(0.9438) (0.1998)

(0.20) (0.00)

$R^2 = 0.8358$ $d = 0.5575$

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[4]For the explanation of the variables, see Chap. II, p. 56. The number in the brackets under the estimated coefficients are the standard errors of the estimated coefficients and the numbers below the standard errors are the corresponding significant probabilities. For example 1.3143 is the standard error of the estimated coefficient 0.4223 and 0.75 is the significant probability at which 0.4223 is different from zero. 0.00 means some value less than 0.005. R^2 is the coefficient of determination and d is the Durbin-Watson-d statistic for testing the autocorrelation in residuals.

Double Logarithmic form:

$$\log Y_{5\,t} = -7.6172 - 1.5830 \log Y_{6\,t} + 2.9565 \log Z_{1\,t}$$

(1.1919) (0.2878)

(0.20) (0.00)

$R^2 = 0.9178$ $d = 0.7286$

3. Industrial Sector

Linear form:

$$Y_{9\,t} = 138.5042 + 1.7127\, Y_{10\,t} - 0.1961\, Z_{6\,t}$$

(1.0093) (0.0896)

(0.11) (0.05)

$R^2 = 0.3659$ $d = 0.5853$

Double-Logarithmic form:

$$\log Y_{9\,t} = 3.6279 + 3.2189 \log Y_{10\,t} - 2.7028 \log Z_{6\,t}$$

(1.3684) (1.5357)

(0.03) (0.11)

$R^2 = 0.3900$ $d = 0.7097$

From these estimated equations the following interpretations may be made:

1. The signs of all the coefficients except for the price coefficient of the linear form in the residential sector, and the price and industrial employment coefficients of both the forms in the industrial sector, are correct according to economic theory. But the coefficients of the price variable in all forms except double-log form in the

industrial sector are not statistically significant.[5] For the coefficient which turned out to be statistically significant, the sign of the coefficient is wrong. For some forms the standard errors of the coefficients of the price variable are larger than the coefficients themselves. In all forms the standard errors of the coefficients of the price variables are considerably larger.

2. The coefficients of the income variable turned out to be statistically significant in both the residential and commercial sectors.[6] For the industrial sector they are not highly significant and have the wrong signs. For the residential and commercial sector they indicate that the natural gas is a "luxury" commodity because the income elasticities in each of the sectors are greater than unity. This result os rather suspicious. Why should natural gas

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[5]That is, the null hypothesis that the coefficient of the price variable is not different from zero is accepted.

[6]That is, the null hypothesis that the coefficient of the income variable is not different from zero is not accepted.

be such a "luxury" commodity (or service) when it is well known that natural gas is used to satisfy basic needs such as cooking, space heating and water heating. Thus they are misleading. One plausible explanation for higher values for income elasticities is that they may have included other effects in addition to income effect and consequently the values of income elasticities are larger than unity. One other plausible explanation may be that as income increases, people may install a new air conditioning equipment or buy a new gas range or dryer to replace an old one. This would increase gas consumption.

3. According to economic theory, the variable $Z_{6\,t}$, industrial employment, should have a positive sign. It is used in these equations to explain the influence of industrial activity on the consumption of natural gas. So naturally the higher the industrial activity, the higher the consumption of natural gas. That means the coefficient of $Z_{6\,t}$ should be accompanied by a positive sign. Similarly, the price variable should be accompanied by a negative sign. But the signs of the estimated coefficients are in the reverse order for both the variables. This may imply

that these two simple static demand models are inadequate to represent the demand for natural gas in the industrial sector.

4. The coefficient of determination, R^2, is high in each equation except for the industrial sector. This high R^2 may be misleading since the high R^2 may have occurred because of the presence of endogenous variable on the right hand side of each equation. Actually the price variables in each sector are not exogenous, they are endogenous variables in reality as already explained in Chapter II. These endogenous variables are involved in explaining the variation in another endogenous variable, namely, the quantity of natural gas consumed. This causes high R^2.[7]

From these conclusions, one may gather that these two simple static models provide an insufficient explanation of demand for natural gas in each sector.

3.3 The Generalization of the Static Approach

The results of the preceding section indicate that the simple static demand models fail to provide a

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P. Balestra, *op. cit.*, p. 46.

satisfactory explanation of natural gas demand. This may be due to inadequate specification of the models themselves, apart from the static nature of the models themselves. Therefore, in this section some sort of generalized static approach will be approached to explain the natural gas demand. This approach may serve as a bridge between the static approach and the dynamic approach which will be developed later. This approach is developed on the basis of the approach followed by Professor Fisher and Professor Kaysen.[8] Their model states that "the demand for electricity, for household use, is derived from the demand by the community of households for the services of its various stocks of white goods."[9] Hence they formulated the demand function for electricity as:

$$(3.3.1) \text{ ---- } D_t = \sum_{i=1}^{n} K_{it} W_{it} \qquad t = 1, 2, \ldots\ldots T$$

where

D_t = total metered use of electricity in kilowatt hours by all households in the community during the period t

W_{it} = i^{th} white good possessed by the community during the period t

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[8] F. M. Fisher and C. Kaysen, A Study in Econometrics: The Demand for Electricity in the United States. Amsterdam: North-Holland Publishing Company, 1962.

[9] Ibid., p. 10.

K_{it} = parameters representing the intensity of use of the W_{it}[10]

For a relatively short period of time, they have assumed W_{it} to remain the same and K_{it} as a function of the following form:[11]

$$K_{it} = A_i P_t^{\alpha_i} Y_t^{\beta_i}$$

where

P_t = average price of electricity per kilowatt hour to households

Y_t = per capita personal income

A_i, α_i and β_i are constant parameters

In this section an attempt is made to formulate somewhat more generalized demand models than the demand models of the previous section, to explain the natural gas demand in each sector. The corresponding results and conclusions are presented in the next section.

Let S_t be the stock of gas appliances in period t and λ_t be the rate of utilization of this stock of appliances of gas in period t. We may write:

(3.3.3) ---- $Y_{1t} = \lambda_t S_t$

$$t = 1, 2, \ldots\ldots T$$

For a short period, one may assume that S_t to be constant or the period of consideration is such that S_t may be treated as given. But λ_t can be varied according

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[10] Ibid., p. 11.

[11] Ibid., p. 13.

to the price of the gas or temperatore, or income. Therefore, it can be assumed without loss of generality, that

$$(3.3.4) \text{ ---- } \lambda_t = A\, Y_{2\,t}^{\alpha}\; Z_{14\,t}^{\beta}\; Z_{15\,t}^{\gamma}$$

where

$Y_{2\,t}$ = index of burner-tip price of natural gas in the residential sector in period t

$Z_{14\,t}$ = disposable personal income in period t

$Z_{15\,t}$ = degree days in period t

The prices of closely related substitutes may influence λ_t but they do so only in a longer time period. Since the time period is so short in this analysis, one may neglect putting price variables of substitutes in equation (3.3.4)

Substituting equation (3.3.4) in (3.3.3) and adding a stochastic term, V_t

$$(3.3.5) \text{ ---- } Y_{1\,t} = A\, Y_{2\,t}^{\alpha}\; Z_{14\,t}^{\beta}\; Z_{15\,t}^{\gamma}\, S_t\; 10^{V_t}$$

where

$Y_{1\,t}$ = consumption of natural gas in residential sector (millions of mcf) in time period t

$Y_{2\,t}$ = index of burner-tip price of natural gas in residential sector (¢/mcf) in time period t

$Z_{14\,t}$ = disposable personal income in time period t

$Z_{15\,t}$ = degree days in time period t

It is almost impossible to secure data on S_t for each t (t = 1, 2, ... T), and "estimation of similar quantities ... is simply impossible on any reasonable standard of accuracy."[12] The possible way out in such cases is to eliminate S_t by some way and this can be done if one assumes an exponential growth in S_t, although certain price must be paid by making such an assumption. But the estimation procedure turns out very simple by making such an assumption. Moreover, it is not that bad an assumption because the exponential growth in population (demonstration effect) would tend to make the growth in stock of appliances in an exponential fashion.[13] Thus S_t is assumed as

(3.3.6)---- $S_t = (1 + K)\, S_{t-1}$

where

K = rate of growth in the stock of appliances during the period t

Since the growth in S_t is rather smooth, K cannot be expected to change violently and it is not unreasonable to assume K as a constant real number.

Taking the logarithms to both sides of the equation (3.3.5)

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[12] Ibid., p. 27.

[13] Ibid., p. 28. The chief trend in S_t probably comes from the introduction and increasing use of new appliances. To some extent the buying of new appliances depends on the community in the sense of if his neighbor bought an electric or gas appliance, he may tend to buy the same because of the influence of his neighbor. Or if many appliances in that community are gas appliances, he may tend to buy a gas appliance. Such effects are called demonstration effects. In these situations, the number of the appliances sold is proportional to the number already in use.

$$(3.3.7) \text{ ---- } \log Y_{1t} = \log A + \alpha \log Y_{2t} + \beta \log Z_{14t} + \log Z_{15t} + \log S_t + V_t$$

Lagging (3.3.7) by one period and subtracting it from (3.3.7), one may obtain

$$(3.3.8) \text{ ---- } \log Y_{1t} - \log Y_{1t-1} = \alpha(\log Y_{2t} - \log Y_{2t-1}) + \beta(\log Z_{14t} - \log Z_{14t-1}) = (\log Z_{15t} - Z_{15t-1}) + (\log S_t - \log S_{t-1}) + V_t - V_{t-1}$$

Similarly, taking logarithms to both sides of equation (3.3.6)

$$\log S_t = \log (1 + K) + \log S_{t-1}$$

or

$$\log S_t - \log S_{t-1} = \log (1 + K)$$

Substituting this in (3.3.8)

$$(3.3.9) \text{ ---- } \log Y_{1t} - \log Y_{1t-1} = \alpha(\log Y_{2t} - \log Y_{2t-1}) + \beta(\log Z_{14t} - \log Z_{14t-1}) + (\log Z_{15t} - \log Z_{15t-1}) + \log (1 + K) + V_t - V_{t-1}$$

Thus a first-difference equation is obtained to explain the demand for natural gas in the residential sector. By letting

$$Y^*_{1t} = \log Y_{1t} - \log Y_{1t-1}$$

$$Y^*_{2t} = \log Y_{2t} - \log Y_{2t-1}$$

$$Z^*_{14\,t} = \log Z_{14\,t} - \log Z_{14\,t-1}$$

$$Z^*_{15\,t} = \log Z_{15\,t} - \log Z_{15\,t-1}$$

$$V^*_t = V_t - V_{t-1}$$

$$K^* = \log (1 + K)$$

and substituting in (3.3.9) one may obtain

(3.3.10) ---- $Y^*_{1\,t} = K^* + \alpha Y^*_{2\,t} + \beta Z^*_{14\,t} + \quad Z^*_{15\,t} + V^*_t$

One can see that the first differencing method results in making the trend in the growth of stock appliances a constant term. And the variable S_t is eliminated in equation (3.3.10). This equation is used for estimating the demand function of natural gas in the residential sector. This is essentially a static model because of the following reasons:

1. No specific attempt is made to estimate substitution effects since no substitution variables are present.
2. No dynamic element that changes through time is involved in equation (3.3.10).
3. It tells the intensity of gas use relative to the price changes in natural gas, income changes, and temperature changes and changes in S_t.
4. It does not involve in "expected or permanent income" hypothesis which is important in considering the growth factors of stock of gas appliances or any other new

installations of gas-using equipment as argued in Chapter II.

5. Stock effect, a related problem to (4), is not explicitly specified in equation (3.3.10). The related constant (K in $k^* = \log(1 + K)$) is included in constant term K^* which does not explain the behavior of the consumption of natural gas except adding something to "sacle" effect (i.e., K the constant term.)

Equations similar to (3.3.10) corresponding to commercial and industrial sectors can be formulated respectively as:

(3.3.11) ---- $Y^*_{3\,t} = K^* + \alpha_1 Y^*_{4\,t} + \alpha_2 Z^*_{14\,t} + \alpha_3 Z^*_{15\,t} + U^*_t$

where

$$Y^*_{3\,t} = \log Y_{3\,t} - \log Y_{3\,t-1}$$

$$Y^*_{4\,t} = \log Y_{4\,t} - \log Y_{4\,t-1}$$

$$Z^*_{14\,t} = \log Z_{14\,t} - \log Z_{14\,t-1}$$

$$Z^*_{15\,t} = \log Z_{15\,t} - \log Z_{15\,t-1}$$

$$U^*_1 = U_t - U_{t-1}$$

$$K^* = \log(1 + K)$$

and

$$Y^*_{5\,t} = K^* + \beta_1 Y^*_{10\,t} + \beta_2 Z^*_{15\,t} + \beta_3 Z^*_{17\,t} + U^*_t$$

where

$$Y^*_{5\,t} = \log Y_{5\,t} - \log Y_{5\,t-1}$$

$$Y^*_{6\,t} = \log Y_{6\,t} - \log Y_{6\,t-1}$$

$$Z^*_{15\,t} = \log Z_{15\,t} - \log Z_{15\,t-1}$$

$$Z^*_{15\,t} = \log Z_{17\,t} - \log Z_{17\,t-1}$$

$$U^*_t = U_t - U_{t-1}$$

$$K^* = \log\,(1 + K)$$

where

$Y_{3\,t}$ = consumption of natural gas in commercial sector (millions of mcf) in time period t

$Y_{4\,t}$ = index of burner-tip price of natural gas in commercial sector (¢/mcf) in time period t

$Y_{5\,t}$ = consumption of natural gas in industrial sector (millions of mcf) in time period t

$Y_{6\,t}$ = index of burner-tip price of natural gas in industrial sector (¢ mcf) in time period t

$Z_{14\,t}$ = disposable personal income in time period t

$Z_{15\,t}$ = degree days in time period t

$Z_{17\,t}$ = industrial employment in Michigan in time period t

$Y_{i\,t-1}$ = lagged (by one period) variable of $Y_{i\,t}$ (i = 3, 4, 5, 6)

$Z_{i\,t-1}$ = lagged (by one period) variable of $Z_{i\,t}$ (i = 14, 15, 17)

Balestra looked at the demand for natural gas in residential and commercial market from another angle. He has postulated that the "gas consumption is also a function of the availability of gas (or pipeline capacity)."[14] Thus he has formulated a demand function of the form

$$(3.3.13) \text{ ---- } \log G_t = A + \alpha \log P_t + \beta \log Y_t + \gamma \log W_t + U_t$$

where

G_t = consumption of gas in BTU 10^{12} in period t

P_t = price of gas in period t

Y_t = per capita income in period t

W_t = average stock of appliances in period t

U_t = random disturbance

In the short run W_t cannot be expected to change violently and so he assumed an exponential growth in W_t, i.e.,

$$(3.3.14) \text{ ---- } W_t = (1 + K)\, W_{t-1}$$

where K is the given rate of growth in W_t.[15] The variable W_t can be eliminated from equation (3.3.13) as S_t is eliminated from equation (3.3.5) and the resulting equation can be obtained as

$$(3.3.15) \text{ ---- } G_t^* = A^* + \alpha P_t^* + \beta Y_t^* + U_t^*$$

where

G_t^* = $\log G_t - \log G_{t-1}$

P_t^* = $\log P_t - \log P_{t-1}$

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[14] P. Balestra, op. cit.

[15] Ibid.

$Y_t^* = \log Y_t - \log Y_{t-1}$

$U_t^* = U_t - U_{t-1}$

$A^* = \log (1 + K)$

Following Balestra and adding a new variable, namely $Z_{15\,t}$, the number of degree days, one may obtain an equation of the form (3.3.10)

$$Y_{1\,t}^* = K^* + \alpha Y_{2\,t}^* + \beta Z_{14\,t}^* + \gamma Z_{15\,t}^* + U_t^*$$

for the residential sector. But this equation is formulated on the basis of "availability of gas" hypothesis or what Balestra calls "capacity effect" hypothesis.

Thus both the hypotheses, namely the demand for natural gas as a function of stock of appliances and also as a function of availability of gas, lead to the same form of demand equation. In the following section the results of the estimated demand functions, one for each sector, are presented.

3.4 Estimation of the Generalized Static Demand Equations

In the preceding section certain first-difference equations are obtained to represent the demand functions of natural gas. Before presenting the estimated results, the importance and the use of first-difference models in time-series analysis is discussed. It is well known that economic time-series tend to be serially correlated. First-differencing may serve roughly to remove such serial correlation. Also, as already seen in equations (3.3.5)-(31319), first-differencing the logarithms of variables having exponential trend, the trend part may be removed and treated as a constant term.

The method of first-differencing tends to remove serial correlation only when the errors in the original time-series are perfectly positively correlated. Consider, for example, the following scheme in a simple two variable relation:

$$(3.4.1) \text{ ---- } \begin{cases} Y_t = \alpha + \beta Z_t + V_t \\ V_t = \rho V_{t-1} + U_t \end{cases} \qquad |\rho| < 1$$

where U_t satisfying the assumptions

$$(3.4.2) \text{ ---- } E(U_t) = 0$$

$$(3.4.3) \text{ ---- } E(U_t\, U_{t+s}) = \begin{cases} 0 & s \neq 0,\ \forall\, t \\ \sigma^2 & s = 0,\ \forall\, t \end{cases}$$

It can be easily verified that

$$V_t = \sum_{t=0}^{\infty} \rho^{\tau}\, U_{t-\tau}$$

Thus, from (3.4.2) and (3.4.3)

$$E\,(V_t) = 0$$

$$E\,(V_t^2) = E\,(U_t^2) + \rho^2\, E\,(U_{t-1}^2) + \rho^4\, E\,(U_{t-2}^2) + \ldots$$

$$= \sigma^2\,(1 + \rho^2 + \rho^4 + \ldots .)$$

$$= \sigma^2 \frac{1}{(1-\rho^2)}$$

Also

$$\sigma_v^2 = E[V_t - E(V_t)]^2$$

$$= E(V_t^2)$$

Thus

$$(3.4.4) \text{ ---- } \sigma_v^2 = \sigma^2 \frac{1}{(1-\rho^2)}$$

Similarly, it can be shown that

$$(3.4.5) \text{ ---- } E(V_t \; V_{t-s}) = \rho^s \; \sigma^2$$

$$= \rho^s \frac{\sigma^2}{(1-\rho^2)} \qquad (s \neq 0)$$

Suppose we take the first-differences for the original relation

$$Y_t = \alpha + \beta \; Z_t + V_t$$

and let the residuals in the first-differences of V_t by e_t, i.e.,

$$e_t = V_t - V_{t-1}$$

Its variance, σ_e^2, is defined as

$$\sigma_e^2 = E[e_t - E\,e_t]^2$$

Since

$$E\; e_t = E\; V_t - E\; V_{t-1} = 0$$

$$\sigma_e^2 = E\; e_t^2$$

$$= E[(V_t - V_{t-1})^2]$$

$$= E[V_t^2 - 2\; V_t\; V_{t-1} + V_{t-1}^2]$$

$$= 2\sigma_v^2\;(1-\rho)$$

But

$$(3.4.6) \text{ ---- } \sigma_v^2 = \sigma^2 \frac{1}{(1-\rho^2)}$$

$$\sigma_e^2 = 2 \cdot \frac{\sigma^2}{(1-\rho^2)} \cdot (1-\rho)$$

$$= 2\;\sigma^2\;(\frac{1}{1+\rho})$$

Thus

$$\sigma_e^2 = \sigma^2 \text{ if } \rho = 1$$

$$\sigma_e^2 = 2\sigma^2 \text{ if } \rho = 0$$

Consider

(3.4.7) ---- $E(e_t \; e_{t-s}) = E \left[(V_t - V_{t-1})(V_{t-s} - V_{t-s-1}) \right]$

$$= E \left[V_t V_{t-s} - V_t V_{t-s-1} - V_{t-1} V_{t-s} + V_{t-1} V_{t-s-1} \right]$$

$$= \rho^s \sigma_v^2 - \rho^{s+1}\sigma_v^2 - \rho^{s-1}\sigma_v^2 + \rho^s\sigma_v^2$$

$$= \rho^s\sigma_v^2 \left(1 - \rho - \frac{1}{\rho} + 1\right)$$

$$= \rho^{s-1}\sigma_v^2 \left(2\rho - \rho^2 - 1\right)$$

$$= -\rho^{s-1} \frac{\sigma^2}{1-\rho^2} (1 - \rho)^2$$

$$= -\rho^{s-1}\sigma^2 \frac{(1-\rho)}{(1+\rho)}$$

The two expressions in equations (3.4.5) and (3.4.7), namely

$$E(V_t \; V_{t-s}) = \rho^s\sigma_v^2 \qquad (s \neq 0)$$

and

$$E(e_t \; e_{t-s}) = -\rho^{s-1}\sigma_v^2 \frac{(1-\rho)}{(1+\rho)} \qquad (s \neq 0)$$

indicate whether the error terms are serially correlated in original as well as the transformed (by first differences) series or not. For $\rho = 1$ in equation (3.4.7),

$$E(e_t \; e_{t-s}) = 0 \qquad (s \neq 0)$$

which means the assumption of serial independence in the error terms of the transformed series is satisfied. That is, if the error terms in the original series are pervectly serially correlated (which is shown in equation (3.4.5) and if these error terms follow the first-order Markov scheme, as in equation (3.4.1), then taking the first-difference to the original series would completely eliminate the serial correlation provided there is a high and positive autocorrelation, i.e., provided that $\rho = 1$. More general conclusion may be drawn by defining the s - ith autocorrelation in the transformed series as

$$\frac{E\,(e_t\; e_{t-s})}{E(e_t^2)}$$

and from equations (3.4.6) and (3.4.7)

$$\frac{E\,(e_t\; e_{t-s})}{E(e_t^2)} = \rho^{s-1}\sigma^2 \;\frac{(1-\rho)}{(1+\rho)}\;\; \frac{1+\rho}{2\sigma^2}$$

$$= -\rho^{s-1}\;\frac{(1-\rho)}{2}$$

From this one can see that the serial correlation will increase as approaches -1 and will reduce as approaches 1.

Another explanation may also be given in using the first-difference method in time series analysis as follows: Suppose we consider the following scheme in a matrix notation

$$Y = X\,\beta + e$$

where

$Y = N \times 1$ vector of observation on the regressand

$X = N \times K$ matrix of ovservations on the regressors

$\beta = K \times 1$ vector of coefficients, and

e = N x 1 vector of disturbances such that

$E(e) = 0$

$E(ee') = \sigma^2\Sigma$, where Σ is positive definite matrix.

In this case, OLS method fails to give minimum variance unbiased estimates because of the presence of interdependence of disturbances. One possible way out in such cases is to find a linear transformation, say T such that $T(e) = u$, where u satisfying the property

$$E(uu') = \sigma^2 I$$

This property, along with the transformed system, namely,

$$TY = T(X\beta + e)$$
$$= T(X\beta) + T(e) = (TX)\ \beta + U$$

will, by OLS method of estimation, provide minimum variance unbiased estimates.[16] This method is applicable only when the variance-covariance matrix, $\sigma^2\ \Sigma$, is known. Such a transformation can be seen as follows, assuming a simple Markov scheme of the following:

$$e_t = \rho e_{t-1} + V_t$$

$$E(V_t) = 0; \qquad E(V_t\ V_{t+s}) = \begin{cases} 0 & s \neq 0, \forall t \\ \sigma^2 & s = 0, \forall t \end{cases}$$

Then the transformation T assumes a matrix of the form,[17]

$$T = \begin{pmatrix} -\rho & 1 & 0 & \cdots & 0 \\ 0 & -\rho & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & +1 \end{pmatrix}$$

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[16] J. Johnston, Econometric Methods, op. cit., pp. 119-185.

[17] Ibid., p. 186

This transformation matrix defines a new set of variables of the form

$$(3.4.8) \text{ ---- } Y_i^* = Y_i - \rho Y_{i-1} \qquad i = 2, \ldots . N$$

$$Z_i^* = Z_i - \rho Z_{i-1} \qquad i = 2, \ldots . N$$

If ρ is known, then a simple practical estimation procedure is to transform the original variables according to (3.4.8) and apply OLS method to the transformed variables. On the other hand, if ρ is not known, "Some econometricians have taken it as approximately unity, in which case, the appropriate transformation is to take the first-differences of the variables."[18]

The following are the estimated demand equations for each of the sectors:

Residential Sector:

$$Y^*_{1\,t} = 0.0400 - 1.5990\ Y^*_{2\,t} + 0.2841\ Z^*_{14\,t} + 0.1447\ Z^*_{15\,t}$$

(0.5214)	(0.4334)	(0.1767)
(0.01)	(0.43)	(0.53)

$R^2 = 0.4288$ $\qquad d = 1.5589$

Commercial Sector:

$$Y^*_{3\,t} = 0.0761 - 1.4233\ Y^*_{4\,t} - 0.4079\ Z^*_{14\,t} + 0.5310\ Z^*_{15\,t}$$

(0.3896)	(0.4329)	(0.2920)
(0.00)	(0.36)	(0.10)

$R^2 = 0.5278$ $\qquad d = 2.0278$

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[18] *Ibid.*, p. 187

Industrial Sector:

$$Y^*_{5\,t} = 0.0443 - 0.5677\ Y^*_{6\,t} + 0.2290\ Z^*_{15\,t} + 0.1525\ Z^*_{17\,t}$$

$$\begin{array}{ccc} (0.3394) & (0.3620) & (0.1525) \\ (0.11) & (0.54) & (0.70) \end{array}$$

$$R^2 = 0.2355 \qquad d = 2.1499$$

Another attempt is made to estimate the demand functions in each sector. In this attempt the price variables of substitutes of natural gas are included. These variables may show their impact on natural gas consumption and since the realization of the influence of the substitutes is not immediate, these demand equations may be considered as slightly more generalized than the above estimated equations. The appropriate form of the demand equation in this case is

$$(3.4.9) \text{ ---- } Y^*_{1\,t} = K^* + \alpha Y^*_{2\,t} + \beta Y^*_{7\,t} + \gamma Y^*_{8\,t} + \delta Z^*_{14\,t} + \eta Z^*_{15\,t} + V^*_t$$

where $V^*_t = V_t - V_{t-1}$

$Y^*_{7\,t} = \log Y_{7\,t} - \log Y_{7\,t-1}$

$Y^*_{8\,t} = \log Y_{8\,t} - \log Y_{8\,t-1}$

$Y_{7\,t}$ = index of price of electricity in residential sector (¢/kew) in time period t

$Y_{8\,t}$ = index of price of fuel oil in residential sector in time period t

$Y_{i\,t-1}$ = lagged (by one period) variable of $Y_{i\,t}$ $(i = 7, 8)$

V_t = stochastic disturbance term

and $Y^*_{i\,t-1}$, $Y_{2\,t}$, $Z^*_{14\,t}$, $Z^*_{15\,t}$ are defined as before.

This equation corresponds to the residential sector. The corresponding equations for the commercial and industrial sectors are as follows:

Commercial Sector:

(3.4.10) ---- $Y^*_{3\,t} = K^* + \alpha_1 Y^*_{4\,t} + \alpha_2 Y^*_{9\,t} + \alpha_3 Y^*_{10\,t} + \alpha_4 Z^*_{14\,t}$

$$+ {}_5 Z^*_{15\,t} + U^*_t$$

where $U^*_t = U_t - U_{t-1}$

$Y^*_{9\,t} = \log Y_{9\,t} - \log Y_{9\,t-1}$

$Y^*_{10\,t} = \log Y_{10\,t} - \log Y_{10\,t-1}$

$Y_{9\,t}$ = index of price of electricity in commercial sector (¢/kwh) in time period t

$Y_{10\,t}$ = index of price of fuel oil in commercial sector in time period t

$Y_{i\,t-1}$ = lagged (by one period) variable $Y_{i\,t}$ ($i = 9, 10$)

U_t = stochastic disturbance term

and $Y^*_{3\,t}$, $Y^*_{4\,t}$, $Z^*_{14\,t}$, $Z^*_{15\,t}$ are defined as before

Industrial Sector:

(3.4.11) ---- $Y^*_{5\,t} = K^* + \beta_1 Y^*_{6\,t} + \beta_2 Y^*_{11\,t} + \beta_3 Y^*_{12\,t} + \beta_4 Y^*_{13\,t}$

$$+ {}_5 Z^*_{15\,t} + {}_6 Z^*_{17\,t} + V^*_t$$

where $V^*_t = V_t - V_{t-1}$

$Y^*_{11\,t} = \log Y_{11\,t} - \log Y_{11\,t-1}$

$Y^*_{12\,t} = \log Y_{12\,t} - \log Y_{12\,t-1}$

$Y^*_{13\,t} = \log Y_{13\,t} - \log Y_{13\,t-1}$

$Z^*_{17\,t} = \log Z_{17\,t} - \log Z_{17\,t-1}$

$Y_{11\,t}$ = index of price of electricity in industrial sector (¢/kwh) in time period t

$Y_{12\,t}$ = index of price of fuel oil in industrial sector in time period t

$Z_{17\,t}$ = industrial employment in time period t

$Y_{i\,t-1}$ = lagged (by one period) variable of $Y_{i\,t}$ (i = 11, 12, 13)

$Z_{17\,t-1}$ = lagged (by one period) variable of $Z_{17\,t}$

V_t = stochastic disturbance term

and $Y^*_{5\,t}$, $Y^*_{6\,t}$, and $Z^*_{15\,t}$ are defined as before.

The following are the estimated demand equations:

Residential Sector:

$$Y^*_{1\,t} = 0.0439 - \underset{\substack{(0.5672)\\(0.02)}}{1.5812\, Y^*_{2\,t}} + \underset{\substack{(1.8750)\\(0.48)}}{1,3953\, Y^*_{7\,t}} - \underset{\substack{(0.2321)\\(0.00)}}{0.0989\, Y^*_{8\,t}}$$

$$+ \underset{\substack{(0.4619)\\(0.45)}}{0.3623\, Z^*_{14\,t}} - \underset{\substack{(0.2547)\\(0.92)}}{0.0083\, Z^*_{15\,t}}$$

$R^2 = 0.4657$ $\qquad$ $d = 1.5432$

Commercial Sector:

$$Y^*_{3\,t} -0.0742 - \underset{\substack{(0.0465)\\(0.00)}}{1.7205\, Y^*_{4\,t}} + \underset{\substack{(0.9724)\\(0.44)}}{0.7855\, Y^*_{9\,t}} - \underset{\substack{(0.2098)\\(0.53)}}{0.1361\, Y^*_{10\,t}}$$

$$- \underset{\substack{(0.5005)\\(0.70)}}{0.1986\, Z^*_{15\,t}} + \underset{\substack{(0.3124)\\(0.14)}}{0.4882\, Z^*_{15\,t}}$$

$R^2 = .5854$ $\qquad$ $d = .1907$

Industrial Sector:

$$Y^*_{5\,t} = 0.0529 - 0.6119\,Y^*_{6\,t} - 0.3585\,Y^*_{11\,t} - 0.3600\,Y^*_{12\,t}$$

(0.3764) (1.8788) (0.2298)

(0.13) (0.83) (0.14)

$$+ 2.2894\,Y^*_{13\,t} + 0.4777\,Z^*_{15\,t} + 0.4029\,Z^*_{17\,t}$$

(1.3513) (0.3744) (0.7857)

(0.12) (0.23) (0.62)

$R^2 = 0.5162$ $d = 1.9125$

The sign of the price elasticities, i.e., the coefficient estimate of the burner-tip prices of natural gas, is of the right order for all equations and is statistically significant at least in residential and commercial sectors. The significance probability varied from 0.00 to 0.02. This is a significant result compared to the results of the static models of Section 3.2. For the industrial sector, the price elasticity has a right sign but is statistically significant only at 0.13 significance probability at most. Yet this result is better than the corresponding result of the static model formulated in Section 3.2 because the elasticity in the present model possesses a right sign whereas elasticity in the model in Section 3.2 has a wrong sign. Thus the results of the price elasticities in the present models came out better than under the models of Section 3.2. The results of income elasticities, i.e., the results of the coefficient estimates of $Z_{14\,t}$, on the other hand, are rather inconclusive. For the residential sector the standard errors of the coefficients are higher than the coefficients themselves. For the commercial sector the coefficient estimate of $Z^*_{14\,t}$ has the wrong sign. For the industrial sector the coefficients of the industrial employment variable, $Z_{17\,t}$, have the right sign but have higher standard errors. But the results are inconclusive.

The coefficient estimate of $Z_{15\,t}$, the number of degree days, has a right sign but seems to indicate that over all the effect of the number of degree days on natural gas consumption is negligible. The inclusion of price variables of the substitutes in the demand models has shown their influence on the consumption of natural gas in each sector even though the results of the estimate coefficients of the substitute price variables themselves are rather inconclusive. The signs of the coefficient estimates of these substitutes are rather erratic. Most of them have their standard errors higher than the respective coefficients themselves. The DW statistic (Durbin-Watson Statistic) seems to indicate that some serial correlation is reduced. This conclusion can be seen from the DW statistic, d, at the end of each demand equation.

The above results seem to indicate that the gas is a commodity (or service) of "necessity" because the income elasticity is less than unity. This is perhaps the most significant observation to draw from these models in formulating dynamic models to explain the natural gas demand. But here we use *a priori* expectations (knowledge of opinion) to evaluate the reasonableness of a model, which, based upon its purely statistical results, is inconclusive.

Even though the more generalized static models are more reasonable than the simple static approach, they did not provide a satisfactory explanation of the consumption of natural gas. We therefore must explore more dynamic models to see whether they will explain the salient features of natural gas consumption more adequately and conclusively.

CHAPTER IV

THE DEMAND FOR NATURAL GAS: A STRUCTURAL ECONOMETRIC MODEL

4.1 Introduction and Definitions

In the last chapter it was pointed out that the static models are inadequate to represent the demand functions of natural gas, and a need for developing dynamic models which are capable of explaining the behavior of the consumption through time is warranged. An attempt is made in this chapter to develop such a model. The model will be developed essentially on the idea, which is already mentioned in the earlier chapters, that there exists competition among energy fuels and so the price variables are interdependent regarding the consumption of natural gas. Thus they belong to a system and hence if any aspect of any one of them is considered, it should be considered from the point of view of the system as a whole. That is, if the demand for natural gas is considered, then it should be considered in relation to electricity, fuel oil, and coal apart from other exogenous variables that affect the consumption of natural gas. The demand function should be embedded in the total system and the investigator should ask to what extent the simultaneous nature of economic relations within the system explain the behavior of consumption of natural gas for any changed situation. With this in view, a general model should be developed. The prmary interest in constructing such a model, as already stated in Chapter I, is to explain the influence of the burner-tip price of natural gas (along with the consideration of the simultaneous nature of economic relationships among the variables mentioned above) on its consumption in each of the residential, commercial and industrial sectors. But before developing such a model, certain terms which we use in constructing an appropriate econometric model will be defined in this section.

First of all, for constructing a system of simultaneous equations model, one needs to develop a system of simultaneous equations where by the term <u>structure</u> we mean a process by which a set of economic variables is generated. More specifically, we define a structure as a combination of specific numerical sets of relationships connecting the economic variables and a specific, numerical distribution function of the unobservable disturbances. We distinguish a structure from a model by defining a model as the specification of the <u>form</u> of the structural relations (formulated in a structure) and of the form of the distribution function of the disturbance terms. Thus a model can be referred to as a set of structures and a structure can be thought of as a unique representation of the model. This distinction is useful in that empirical analysis is aimed at a single structure including estimates obtained through the statistical estimation of all unknown parameters of the model.

The construction of a system of simultaneous equations model from these viewpoints may constitute what we call a <u>structural econometric model</u>. The construction is carried out in two phases:

(i) economic specification of the model,

(ii) statistical specification of the model.

Through economic specification of the model pertaining relationships among the economic variables to describe the behavior (in our case the behavior of natural gas consumption) under consideration are specified. Economic theory and knowledge of the economic institutions and characteristics pertaining to the sectors being studied are important in this phase of analysis. We may call this phase the development of the economic model. In the second phase, the investigator is required to make some statistical specifications to the economic model. For example, he may specify functional forms to economic relations, or forms of the variables, or the distribution of the stochastic disturbance terms.

The variables of an economic model in the simultaneous equation system framework can be classified into two categories: <u>endogenous or jointly dependent</u> and predetermined. The variables whose values are explained by the structure are called the jointly dependent variables. The predetermined variables are further classified into exogenous and lagged endogenous. The variables whose values are determined outside the structure are called exogenous and the variables whose values are determined by the structure prior to the current period are called lagged endogenous. In other words, the jointly dependent variables are to be explained by the system of structural relationships while the predetermined variables are specified as being determined at the time they enter the structural relationships.

Once the structural econometric model is specified, there remains at least one more critical consideration: namely, the identification problem of the structural parameters. The concept of identification problems is briefly as follows. The structural parameters of a model are said to be identifiable if their values can be deduced from complete knowledge of the distribution function of the observations. If one considers a linear structural model, the parameters are said to be identifiable if their values can be deduced from the complete knowledge of the parameters of the reduced form. Identifiability is usually achieved in linear structural models by imposing *a priori* restrictions on the values of the structural parameters, in particular, by specifying that some parameters are zero. The restrictions may be such that the parameters in a given structural equation are not identified (or under identified), just identified, or over identified. Sometimes just identified and over identified equations are simply called identified. In order to know whether a particular structural equation is identified or not, Koopman, Rubin, and Leipnik have derived a necessary condition.[1] This condition can be stated as follows: Let g be the number of jointly dependent variables appearing in the structural equation; let k be the number of predetermined variables appearing in the structural equation; and let K be the total number of predetermined variables in the system of structural equations. Then the necessary condition for a structural equation to be just identified is:

(4.1.1) ---- $K - k \geq g - 1$

If $K - k < g - 1$, the structural equation is said to be under identified. That is, a structural equation is identifiable if the number of predetermined variables excluded from the equation should be at least equal to the number of jointly dependent variables in the equation less one. Otherwise the equation is under identified. This is known as the order condition of identifiability.

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[1]T. C. Koopman, H. Rubin and R. B. Leipnik, "Measuring the Equation Systems of Dynamic Economics," in Statistical Inference in Dynamic Economic Models, Cowles Commission Monograph 10, New York: John Wiley & Sons, inc., 1950, pp. 53-237.

A necessary and sufficient condition for the identifiability of a structural equation of a linear model, also restricted by the exclusion of certain variables from certain equations, is called the rank condition for identifiability. This condition states that a structural equation is identified if and only if the rank of the matrix, whose rows are equal to the number of jointly dependent variables included in the structural equation and whose columns are equal to the number of predetermined variables in the system but excluded from that equation, is equal to the number of jointly dependent variables included in the structural equation less one.[2] The disadvantages of this rank condition is that one has to cast the model into reduced-form first of all and then examine the rank of the (above mentioned) submatrix of reduced-form coefficients.[3]

4.2 Specification of the Economic Model

As indicated in the previous section, the economic model summarizes the specification of economic relationships made by the investigator. A description of the variables and the structural economic relations of the model will be discussed in this section. The demand for

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[2]For complete discussion on identification problem see any one of the standard econometric textbooks or see footnote 1 of this section.

[3]For more simple and useful statement of the rank condition, see J. Johnston, Econometric Methods, New York: McGraw-Hill Cook Company, Inc., pp. 251-252.

natural gas, as stated in Chapter II, is studied with respect to three sectors, namely, residential, commercial and industrial. It is also mentioned that only five estimation methods are attempted in this study.[4] Therefore, three structural demand equations--one for each sector--are developed. These three structural equations consist of six jointly dependent variables and fourteen predetermined variables which are already defined in Chapter II.[5]

4.2 (a) The Structural Demand Equation - Residential Sector

From the point of view of much of economic theory of consumer demand, the demand for a given commodity is a function of the price of that commodity, prices of closely related goods, either substitutes or complements, prices of all other goods, disposable personal income, and probably some other factors that may be included in the set of predetermined variables. Following this conceptual framework, the structural demand function for natural gas in the residential sector is postulabed in a functional notation as:

(4.2.1) ---- $\left[Y_{1t}, Y_{2t}; Z_{0t}, Z_{1t}, Z_{2t}, Z_{3t}, Z_{7t}, Z_{10t} \right] = U_{1t}$

where

Y_{1t} = consumption of natural gas in residential sector (millions of mcf) in time period t

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[4]See Chapter I, pp. 9-15.

[5]See Chapter II, pp. 57-58.

Y_{2t} = index of burner-tip price of natural gas in residential sector (¢/mcf) in time period t

Z_{0t} = constant term

Z_{1t} = lagged (by one period) disposable personal income in time period t

Z_{2t} = lagged (by one period) number of degree days in time period t

Z_{3t} = lagged (by one period) consumption of natural gas (millions of mcf) in residential sector

Z_{7t} = lagged (by one period) index of price of electricity in residential sector (¢/kwh) in time period t

Z_{10t} = lagged (by one period) index of price of fuel oil in residential sector (¢/gal) in time period t

and

U_{1t} = stochastic disturbance term

In the above expression (4.2.1), a comma should be read as "and" and a semicolon should be read "appear in relation with". The variables to the left of a semicolon, within the bracket, are jointly dependent variables and those on the right are considered predetermined variables within this particular model.

The logic of including the variables in (4.2.1) is explained as follows. Since the quantity of natural gas consumed (Y_{1t}) and the burner-tip price of natural gas (Y_{2t}), influence each other, they are specified as jointly dependent variables. Z_{1t} is included according to the theory of consumer demand and treated as exogenous because it is determined largely by other economic factors that lie outside our system. It is included to reflect the income effect on the consumption of natural gas. Higher income implies that people buy more. They may change their tastes or may want more comforts which means they may install an air conditioning equipment or they may replace their appliance with a new gas-using appliance; or they may buy a new gas-using dryer. For the sake of prestige, they might like to put up yard lighting which uses natural gas. All these points lead to the conjecture that higher disposable income is likely to be associated with a higher consumption of natural gas. On the other hand, gas may be an inferior good. People may switch to all electric homes, eat out more often, use more laundry service, etc., in which case the demand for gas as income increases may decline. Note that Z_{1t} is not the disposable personal income in the current period, it is the disposable personal income in the previous period. The decision to include the lagged variable instead of the current variable is that it is the lagged disposable income, not the current disposable income which may have greater weight in influencing the current period's natural gas consumption. It is reasonable to suppose that the quantity of goods and services consumed in the current period is largely determined by the disposable income in the previous period, at least for the case in which there are no radical changes in income expectations. We might also argue that an increase in disposable income may not bring an instant increase in the consumption of natural gas but brings an increase only after a lapse of time. Consequently, the lagged disposable personal income is more relevant, compared to the current, to explain the behavior of the current period's consumption of natural gas.

Z_{2t}, the number of degree days, is used to measure the effect of temperature on the consumption of natural gas. Since it is determined outside our system, it is assumed as a predetermined variable.

The variable Z_{3t}, being the quantity of natural gas consumed in the previous period, indicates the previous year's consumption in the residential sector, This variable is used to reflect the increase in consumption of natural gas through time. It is a known fact that economic reactions brought by a changed situation may not be realized immediately after the situation is changed, but may take place after a lapse of time. Current decisions are influenced by past events. Changes in the current period can be ascribed to changed situations, in addition to the explicitly mentioned price changes of electricity, fuel oil, and disposable personal income variables, in the previous period. For example, "new" uses may have taken place in the previous period with the installation of natural gas service in the new houses constructed in that period. This kind of increase in the consumption of natural gas is reflected through the coefficient of the variable, $Z_{3\,t}$. Since it is a lagged endogenous variable, it is by definition taken as a predetermined variable. Similarly, the variables $Z_{7\,t}$ and $Z_{10\,t}$, being the lagged variables of indices of price of electricity and fuel oil respectively, are considered as predetermined variables. These variables are included in the expression (4.2.1) to reflect the influence on the gas consumption of changes in the prices of substitute variables. The lagged variables, instead of the variables in the current period are considered for the same reasons as $Z_{1\,t}$ and $Z_{3\,t}$ are included in the demand equation for the residential sector. People will have expectations about the future prices, and guide their decisions on the basis of the information they have about these future prices. Since they may not know specific future prices, they use past prices as their guides to the future prices and make their decisions. It is with this view and assuming that consumers give higher weight to the previous periods' prices compared to the other periods in making up their decisions, the variables $Z_{7\,t}$ and $Z_{10\,t}$ are included. The variable $Z_{10\,t}$ is included in (4.2.1) as a constant term. Finally $U_{1\,t}$, the unobserved random disturbance, is used to represent either the factors that are not accounted for directly in the relation or the unpredictable element of randomness in the consumption of natural gas by the residential customers or the combination of both. Thus, two jointly dependent variables, namely, Y_{1t}, Y_{2t}, and six predetermined variables are included in the structural demand relation to explain the behavior of consumption of natural gas in the residential sector.

4.2 (b) The Structural Demand Equation - Commercial Sector

As in the case of residential sector and according to the theory of consumer demand, the structural demand function of natural gas in the commercial sector is specified in a functional notation as:

(4.2.2) ---- $[Y_{3t}, Y_{4t}; Z_{0t}, Z_{1t}, Z_{2t}, Z_{4t}, Z_{8t}, Z_{11t}] = U_{2t}$

where

Y_{3t} = consumption of natural gas (millions of mcf) in commercial sector in time period t

Y_{4t} = index of burner-tip price of natural gas (¢/mcf) in commercial sector in time period t

Z_{4t} = lagged (by one period) consumption of natural gas (millions of mcf) in commercial sector in time period t

Z_{8t} = lagged (by one period) index of price of electricity (¢/kwh) in commercial sector in time period t

Z_{11t} = lagged (by one period) index of price of fuel oil (¢/gal) in commercial sector in time period t

U_{2t} = stochastic disturbance term

and

$Z_{0\,t}$, $Z_{1\,t}$ and $Z_{2\,t}$ are defined as before.

The variables $Y_{3\,t}$ and $Y_{4\,t}$, being the quantity consumer and index of burner-tip price of natural gas, are jointly dependent variables. $Z_{0\,t}$ is included as a constant term. As in the residential sector, $Z_{2\,t}$ is used to reflect the effect of temperature on the consumption of natural gas. The colder the weather in the previous period, assuming that the commercial consumers expect similar weather conditions in the current period, the higher the consumption of natural gas we can expect because the commercial consumers try to keep their establishments warm and comfortable. This means that there may be a shift towards the right in the demand curve of natural gas. Since the number of degree days is determined outside the system, $Z_{2\,t}$ is taken as a predetermined variable. Commercial consumers would tend to increase the consumption of natural gas if they have higher expectations about their profits in the future. These profits would occur if people have higher disposable personal income so that they can afford to visit and enjoy the services provided by the commercial establishments. It is with this view, the disposable personal income variable is used to summarize the behavior of people which, in turn, induce the owners of commercial establishments with respect to the consumption of natural gas in the commercial sector. Since it is determined outside our system, Z_{1t} is considered as a predetermined variable.

The variables Z_{4t}, Z_{8t}, and Z_{11t}, as lagged endogenous variables of consumption of natural gas, prices of electricity and fuel oil, are included for the same reason as the corresponding variables are included in the structural demand relation in the residential sector. The variable Z_{4t} is included with the idea that there is a lag in explaining the consumption of natural gas to a changed situation and the behavior of consumption is an adaptive process rather than an immediate jump in the consumption when the situation is changed. Similarly, the variables Z_{8t} and Z_{11t} would describe the behavior of prices of electricity and fuel oil in the previous period and guide the owners of commercial establishments with respect to the usage of natural gas. Of course, this is conditioned by the perceived expectations of the owners of commercial establishments. U_{2t} is the stochastic disturbance term and is included in the structural relation (4.2.2) for the similar reasons mentioned as in the case of the residential sector. Thus the jointly dependent variables Y_{5t} and Y_{6t} together with the predetermined variables Z_{0t}, Z_{1t}, Z_{2t}, Z_{4t}, Z_{8t} and Z_{11t} are considered to explain the behavior of consumption of natural gas.

4.2 (c) The Structural Demand Equation - Industrial Sector

Essentially, the demand for natural gas in the industrial sector is the derived demand; derived from various respective final products produced in various industries that use natural gas as an input along with fuel oil, coal and electric power as possible substitutes. Following the same reasoning as in the consideration of structural demand functions of natural gas in residential and commercial sectors, the structural demand function in the industrial sector is specified in the following functional form:

$$(4.2.3) \quad ---- \left[Y_{5\,t},\ Y_{6\,t};\ Z_{0\,t},\ Z_{2\,t},\ Z_{5\,t},\ Z_{6\,t},\ Z_{9\,t},\ Z_{12\,t},\ Z_{13\,t} \right] = U_{3\,t}$$

where

$Y_{5\,t}$ = consumption of natural gas (millions of mcf) in industrial sector in time period t

$Y_{6\,t}$ = index of burner-tip price of natural gas (¢/mcf) in industrial sector in time period t

$Z_{5\,t}$ = lagged (by one period) consumption of natural gas (millions of mcf) in industrial sector in time period t

$Z_{6\,t}$ = lagged (by one period) industrial employment (in thousands) in time period t

$Z_{9\,t}$ = lagged (by one period) index of price of electricity (¢/kwh) in industrial sector in time period t

$Z_{12\,t}$ = lagged (by one period) index of price of fuel oil (¢/gal) in industrial sector in time period t

$Z_{13\,t}$ = lagged (by one period) index of price of coal ($/ton) in industrial sector in time period t

$U_{3\,t}$ = stochastic disturbance term

and

$Z_{0\,t}$, and $Z_{2\,t}$ are defined as before.

Clearly, $Y_{5\,t}$ and $Y_{6\,t}$ as quantity consumed and index of burner-tip price of natural gas in the industrial sector are jointly dependent and are to be "explained" by the system. $Z_{2\,t}$, as in the structural demand equations for residential and commercial sectors, represents the number of degree days and is used to measure the influence of temperature on the use of natural gas by the industrial sector. With the same reason as in the residential and commercial sectors, it is treated as a predetermined variable. Variable $Z_{6\,t}$, which represents the industrial employment, is used to measure the industrial output of a state. Some such measure is necessary because states with more industrial output will have, other things being equal, greater industrial consumption of natural gas than states with smaller amounts of industrial output. The use of industrial output as a factor to influence the consumption of natural gas can also be explained from the point of view of the theory of derived demand; according to which, the demand for natural gas at the industrial sector is a function of the burner-tip price at the industrial sector, quantity of "final" product demanded, and other factors as mentioned in equation (4.2.3). Since the "final" product consists of all the products of the industrial sector whose demand depends mainly on the factors that do not belong to our system, the quantity of the "final" product, namely, industrial output, is considered as a predetermined variable. And since the series corresponding to this variable is not available on a state basis, industrial employment, which is available on a state basis, is used for industrial output. The industrial employment is considered as predetermined variable for the similar reason as given in consideration of industrial output.

The lagged industrial consumption variable, $Z_{5\,t'}$ is included to reflect the behavior and expectations of industrial consumers in the previous period. If industrial consumers expect an increase in the demand for their product and also expect to stay at the new level, then they try to use the existing capacity of gas-using machinery at the maximum (this is the case if they are not operating their plants at the maximum capacity) which, in turn, increases the consumption of natural gas. In the case where the plants are operating at the maximum capacity, then the producers may install new machinery which again causes an increase in industrial consumption. These two effects can be described by including the lagged variable of consumption of natural gas in the industrial sector. Natural gas is supplied to the industrial sector on an interruptible basis, If the residential and part of the commercial sectors would release gas from their "firm" service due to weather conditions, the industrial sector may get natural gas without interruption and this may increase the gas consumption. This kind of effect can also be shown by the variable $Z_{5\,t}$. Also as argued in the case of the residential and commercial sectors, the existence of a lag in economic reactions may justify the inclusion of $Z_{5\,t}$ in the structural demand relation (4.2.3). Similarly, if the prices of competitive fuel energies, namely, fuel oil, coal and electricity, are lower in the previous period and are expected to stay at these levels in the near future, the industrial consumers would tend to switch to these low cost fuels. The adjustment to these new fuels is relatively easier in the industrial sector because most of the industrial plants are equipped with alternative machinery. The industrial plants are so equipped because mainly the natural gas is supplied for industrial use on an "interruptible basis". So the industrial consumers are likely to switch to low cost fuels. In order to show these effects, the $Z_{9\,t}$, $Z_{12\,t}$, and $Z_{13\,t}$ are included in the relation (4,2.3). $U_{3\,t}$ is a random disturbance term and is used to reflect, as argued in the case of the other two sectors, the effects of all variables that are not accounted for in the structural demand relation (4.2.3). Thus the variables

Y_{5t} and Y_{6t} together with the specified predetermined variables Z_{0t}, Z_{2t}, Z_{5t}, Z_{6t}, Z_{9t}, Z_{12t} and Z_{13t} are considered to explain the demand for natural gas in the industrial sector.

The above three structural relations, taken as a system of relations, complete the specification of the economic model. Next the statistical specifications are discussed to complete the discussion of development of the structural econometric model.

4.3 Statistical Specifications of the Economic Model

The statistical specification of the economic model involves, as noted earlier in Section 4.1, the specification of the functional forms of the relationships developed in the economic model, form of the economic variables in the economic model, and specification of the assumptions concerning the stochastic disturbance terms. First of all, the reader should notice that no attempt is made to develop a complete model. This is because this study concerns only the estimation of demand functions of natural gas, and to fulfill this objective, as mentioned in Chapter I, "Structural Estimation - Single Equation Methods" are used to estimate the concerned demand functions.[6] It is with this view, only structural demand relations

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[6] A. S. Goldberger, op. cit., pp. 329-346. Also three-stage estimation procedure is used; see Chapter I.

(4.2.1) - (4.2.3) are developed and "Structural Equations - Estimation Methods" are used to estimate them where appropriate statistical specifications are made to these structural relations. First of all, all of the economic variables, i.e., both the jointly dependent and predetermined variables, are transformed by logarithmic (to the base 10) function and the resulting variables are used in the above structural relations. Secondly, the structural relations are assumed to appear in linear form. That is, we are assuming logarithmic linear structural demand functions in each of the residential, commercial and industrial sectors. Thirdly, the stochastic disturbance terms are assumed to have the following properties:

1. $E[U_{it}] = 0$ $\quad i = 1,2,3;$ $\quad t = 1,2, \ldots . 18$

2. $E[U_{it}U_{jt}] = \sigma_{ij}$ $\quad i, j = 1,2,3;$ $\quad t = 1,2, \ldots . 18$

3. $E[U_{it}U_{jt-\theta}] = 0$ $\quad i, j = 1,2,3;$ $\quad \theta = 0$

4. Z_{jt} are stochastically independent of U_{it}
$\quad j = 0,1, \ldots . 13;$
$\quad i = 1,2,3;$
$\quad t, t' = 1,2, \ldots . 18$

5. It is assumed that the jointly dependent and

predetermined variables are measured without errors of measurement.[7]

With these statistical specifications, each structural demand relation can be represented as follows:

$$(4.3.1) \quad \alpha_{i1} Y_{it} + \alpha_{i2} Y_{2t} + \alpha_{i3} Y_{3t} + \dots \alpha_{i6} Y_{6t} + \beta_{i0} Z_{0t} + \beta_{i1} Z_{1t} + \dots \beta_{i13} Z_{13t} + U_{it} = 0$$

$$i = 1,2,3; \quad t = 1,2, \dots 18.$$

Where α_{ig} and β_{ik} ($i = 1,2,3$; $g = 1,2, \dots 6$; $k = 0,1, \dots 13$) are the structural form coefficients to be estimated, Y_g and Z_k ($g = 1,2, \dots 6$; $k = 0,1, \dots 13$) are the specified observable economic variables and U_{it} ($i = 1,2,3$) is a stochastic disturbance term. From the economic model developed in Section 4.2, the Y_g and Z_k that enter a particular equation are known and for those not specified, the corresponding α_{ig} and β_{ik} are assumed to be zero. The structural econometric model for the demand of natural gas specified in (4.3.1) is summarized in Table 4.3.1.

The structural econometric model is then estimated by using the single equation methods and the results are presented in the next chapter.

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[7]Notice that the assumptions (1) and (4) are in a way "incomplete" in the sense that they form a subset of assumptions of a complete model.

Table 4.3.1 SCHEMATIC SUMMARY OF THE STRUCTURAL ECONOMETRIC MODEL

Structural Demand Equations	Jointly Dependent Variables						Predetermined Variables													
	Y_{1t}	Y_{2t}	Y_{3t}	Y_{4t}	Y_{5t}	Y_{6t}	Z_{0t}	Z_{1t}	Z_{2t}	Z_{3t}	Z_{4t}	Z_{5t}	Z_{6t}	Z_{7t}	Z_{8t}	Z_{9t}	Z_{10t}	Z_{11t}	Z_{12t}	Z_{13t}
1. S.D.E.* Residential Sector	α_{11}	α_{12}					β_{10}	β_{11}	β_{12}	β_{13}				β_{17}			β_{110}			
2. S.D.E. Commercial Sector			α_{23}	α_{24}			β_{20}	β_{21}	β_{22}		β_{24}				β_{28}			β_{211}		
3. S.D.E. Industrial Sector					α_{35}	α_{36}	β_{30}		β_{32}			β_{35}	β_{36}			β_{39}			β_{312}	β_{313}

*S.D.E. means Structural Demand Equation

CHAPTER V

THE STRUCTURAL ECONOMETRIC MODEL: EMPIRICAL RESULTS

5.1 Some Theoretical Comparisons of the Estimators Used in the Study

In the previous chapter, we have discussed the development of the structural econometric model, suitable for the statistical methods of estimation. More specifically, we have specified the demand function for the i-th sector as:

$$\alpha_{i\,1} Y_{1\,t} + \alpha_{i\,2} Y_{2\,t} + \alpha_{i\,3} Y_{3\,t} + \ldots . \alpha_{i\,6} Y_{6\,t}$$
$$+ \beta_{i\,0} Z_{0\,t} + \beta_{i\,1} Z_{1\,t} + \ldots . \beta_{i\,13} Z_{13\,t}$$
$$+ U_{i\,t} = 0$$

$$i = 1,2,3;$$
$$t = 1,2, \ldots . 18.$$

It is a usual procedure in all the estimation methods used in this study to use only one appropriate jointly dependent variable which is used in the left hand side of the equation. For example, if one considers the structural demand equation for the residential sector, $\alpha_{11} = -1$. Similarly, if one considers the structural equation for the commercial sector, $\alpha_{23} = -1$, and so on. And the resulting structural demand equation for the residential sector can be rewritten as:[1]

$$(5.1.1) \text{ ---- } Y_{1t} = \alpha_{12} Y_{2t} + \beta_{10} Z_{0t} + \beta_{11} Z_{1t} + \beta_{12} Z_{2t} + \beta_{13} Z_{3t} + \beta_{17} Z_{7t} + \beta_{110} Z_{10t} + U_{1t}$$

$$t = 1,2, \ldots . 18.$$

The same procedure is applied for the other two equations. With this specification and with the assumptions made in Section 4.3,[2] one can note some of the theoretical differences among the alternative estimators used in this study.

Since each structural demand equation contains two jointly independent variables, and if one selects one of these two variables as the dependent variable, the other one will be correlated with the stochastic disturbance term in that equation because of the simultaneous nature of the relations in the structural model. This makes the estimates of the structural parameters obtained by the method of ordinary least squares (OLS) inconsistent.

The basic idea in the two-stage least squares method of estimation (2SLS) is to replace the right-hand side jointly dependent variable, namely Y_{2t}, with appropriate estimates based on the least squares regression

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[1]Notice that $\alpha_{1j} = 0$ for j = 3,4,5,6 and $\beta_{jt} = 0$ for i = 4,5,6,8,9,11,12,13 because of the specification of the structural demand equation for the residential sector.

[2]See pp. 109-111, Chapter IV of this study.

of Y_{2t} and the predetermined variables in (5.1.1). The estimates of the structural parameters obtained by this procedure are consistent.[3] Theil has shown a general procedure called (K)-class estimation which provides a whole family of estimators of the structural equation (5.1.1), of which two-stage least squares procedure is a special case.[4] The unbiased Nagar K-class (UNK) is one other special case of the above mentioned (K)-class family of estimators. This method was proposed by Nagar and the K-value of the unbiased Nagar K-class estimator has plim (k-1) = 0, and this property establishes, in general, the asymptotic property of consistence for that (K)-class estimator.[5]

The limited information single equation (LISE) was developed prior to the two-stage least squares and

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[3]See H. Theil, Economic Forecasts and Policy, Amsterdam: North-Holland Publishing Company, 1961, pp. 231-237; and R. L. Basmann, "A Generalized Classical Method of Linear Estimation of Coefficients in a Structural Equation," Econometrica, Vol. 25 (January, 1957).

[4]H. Theil, op.cit., pp. 231-237.

[5]A. L. Nagar, "The Bias and Moment Matrix of the General K-Class Estimators of the Parameters in Simultaneous Equations," Econometrica, Vol. 27 (October, 1959).

(K)-class procedures by Anderson and Rubin.[6] Their approach is an application of the maximum likelihood principle under the specification that the structural stochastic disturbances are normally distributed and utilizing only restrictions on the structural equation being estimated. Under the normality assumption, estimates are consistent and are also asymptotically normal and efficient.[7] However, Theil has shown that limited information single equation estimators are members of the (K)-class family, and hence, consistent with or without the normality assumption.[8]

The essential idea in three-stage least squares estimation procedure is to express each equation in such a way that <u>all</u> predetermined variables are involved and then to apply generalized least squares to the whole set of relations to estimate all structural parameters

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[6]T. W. Anderson and H. Rubin, Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations," <u>Annals of Mathematical Statistics</u>, Vol. 20,(October, 1949). See also T. W. Anderson and H. Rubin, "The Asymptotic Properties of the Estimates of the Parameters of a Single Equation in a Complete System of Stochastic Equations," <u>Annals of Mathematical Statistics</u>, Vol. 21 (December, 1950). Also see H. Chernoff and H. Rubin, "Asymptotic Properties of Limited Information Estimates Under Generalized Conditions," <u>Studies in Econometric Method</u>, W. C. Hood and T. C. Koopmans (eds.), <u>op</u>. <u>cit</u>., pp. 200-212.

[7]T. W. Anderson and H. Rubin, <u>op</u>. <u>cit</u>. pp. 570-582.

[8]H. Theil, <u>op</u>. <u>cit</u>., pp. 231-237.

simultaneously.[9] The three-stage least squares estimates are consistent and, in general, they are more asymtotically efficient than the two-stage least squares estimators.[10]

This very brief summary of the properties of the estimators provided by the alternative methods is perhaps sufficient to indicate that the principal theoretical differences are based on large sample properties. We know only a limited amount of information about small sample properties of the alternative estimates as indicated primarily from Monte Carlo studies.[11] Among the methods employed in this study, the three-stage least squares procedure is presumably the most suitable estimation method from the standpoint of asymptotic

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[9] See, for example, J. Johnston, Econometric Methods, op. cit., pp. 266-268.

[10] A. Zellner and H. Theil, "Three Stage Least Squares: Simultaneous Estimation of Simultaneous Equations," Econometrica, Vol. 30 (January, 1962).

[11] Prominent among them are the following: (1) R. Summers, "A Capital-Intensive Approach to the Small Sample Properties of Various Simultaneous Equation Estimations," Econometrica, Vol. 33, 1965. (2) A. L. Nagar, "A Monte-Carlo Study of Alternative Simultaneous Equation Estimators," Econometrica, Vol. 28, 1960. H. Wagner, "A Monte-Carlo Study of Estimates of Simultaneous Linear Structural Equations," Econometrica, Vol. 26, 1958. For a brief description of these studies, and some other studies, see J. Johnston, Econometric Methods, op. cit., Chapter 10, pp. 275-295.

efficiency.[12]

5.2 Empirical Results

We now present the empirical results for the respective structural demand equations obtained by employing the alternative methods of estimation described in the previous section. The variables appear in each equation without any transformations on them. In presenting the results, a standard table format is used throughout; namely, the estimated structural coefficients are listed first, their estimated standard errors are directly beneath, and then the corresponding t-values beneath the estimated standard errors. The corresponding results, one for each structural equation, are presented in Tables 5.2.1, 5.2.2 and 5.2.3.

5.3 Tests of Identifiability of a Structural Demand Equation

As we have pointed out in Chapter IV, the identifiability of the coefficients in a particular equation depends first of all upon the number of jointly dependent and predetermined variablex excluded from that equation.[13] The validity of the *a priori* exclusion of

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[12] J. C. Cragg, "On the Relative Small-Sample Properties of Several Structural Equation Estimators," Econometrica, Vol. 35 (January, 1967) In this paper, Cragg arrived at the conclusion that three stage least squares and full information maximum likelihood estimators were better than two stage least squares, Nagar unbiased K-class and limited information single equation estimators.

[13] See page 95, Chapter IV of this study.

Table 5.2.1

STRUCTURAL DEMAND EQUATION, RESIDENTIAL SECTOR:
COEFFICIENT ESTIMATES AND RELATED STATISTICS*

	$\log Y_{1t}$	$\log Y_{2t}$	Z'_{0t}	$\log Z_{1t}$	$\log Z_{2t}$	$\log Z_{3t}$	$\log Z_{7t}$	$\log Z_{10t}$
(1) OLS	1	-0.4850	-2.7993	0.4265	-0.1776	0.6559	0.8164	0.0634
		0.7722		0.7979	0.3252	0.5536	0.2152	1.3388
		-0.6281		0.5345	-0.5461	1.1848	3.7930	0.0473
(2) 2SLS	1	-1.0983	-11.9071	2.3062	0.8167	0.1036	2.0800	-0.2982
		0.9183		0.7370	0.5520	0.2956	1.1158	0.2230
		-1.1961		3.1291	1.4794	0.3505	1.8641	-1.3375
(3) UNK	1	-1.3575	-10.8982	2.3118	0.7520	0.1119	1.9128	-0.3013
		0.9395		0.7541	0.5648	0.3024	1.1416	0.2281
		-1.4449		3.0657	1.3315	0.3702	1.6756	-1.3209
(4) LISE	1	-5.5679	6.4655	2.4012	-0.2983	0.2479	-0.8023	-0.3518
		2.0393		1.6368	1.2259	0.6563	2.4780	0.4952
		-2.7303		1.4670	-0.2433	0.3777	-0.3238	-0.7105
(5) 3SLS	1	-0.4386	-11.8371	1.8156	0.7976	0.2693	2.1306	-0.2116
		0.6304		0.4289	0.3883	0.1707	0.7306	0.1569
		-0.6958		4.2332	2.0538	1.5774	2.9163	-1.3486

*The coefficient estimates are statistically significant at 5 per cent level if the t-values are greater than 2.201. They are also significant at 1 per cent level if the corresponding t-values are greater than 3.106

Table 5.2.2

STRUCTURAL DEMAND EQUATION, COMMERCIAL SECTOR:
COEFFICIENT ESTIMATES AND RELATED STATISTICS*

	$\log Y_{3t}$	$\log Y_{4t}$	Z'_{0t}	$\log Z_{1t}$	$\log Z_{2t}$	$\log Z_{4t}$	$\log Z_{8t}$	$\log Z_{11t}$
(1) OLS	1	-0.3924	-14.3905	1.5582	2.1480	0.0781	0.8586	0.3200
		1.0749		0.6436	0.2176	0.5410	1.3021	0.3508
		-0.3650		2.4210	9.8729	0.1443	0.6594	0.9123
(2) 2SLS	1	-0.2729	-4.2193	0.8320	0.6789	0.7349	-0.1823	-0.2267
		0.8990		0.6512	1.0082	0.2638	1.3556	0.3308
		-0.3036		1.2776	0.6734	2.7862	-0.1345	-0.5851
(3) UNK	1	-0.2245	-4.4002	0.8344	0.7073	0.7310	-0.1945	-0.2297
		0.8998		0.6518	1.0091	0.2640	1.3568	0.3311
		-0.2495		1.2802	0.7009	2.7689	-0.1433	-0.6938
(4) LISE	1	6.5812	-29.8152	1.1730	4.6844	0.1832	-1.8965	-0.6607
		2.4715		1.7902	2.7717	0.7251	3.7209	0.9095
		2.6628		0.6552	1.6901	0.2526	-0.5089	-0.7265
(5) 3SLS	1	-0.6048	-1.9579	0.5078	0.3026	0.8542	0.0655	0.0363
		0.6298		0.4014	0.6889	0.1602	0.9336	0.2256
		-0.9602		1.2648	0.4392	5.3331	0.0702	0.1611

*See footnote of Table 5.2.1

Table 5.2.2

STRUCTURAL DEMAND EQUATION, COMMERCIAL SECTOR:
COEFFICIENT ESTIMATES AND RELATED STATISTICS*

	$\log Y_{3t}$	$\log Y_{4t}$	Z'_{0t}	$\log Z_{1t}$	$\log Z_{2t}$	$\log Z_{4t}$	$\log Z_{8t}$	$\log Z_{11t}$
(1) OLS	1	-0.3924	-14.3905	1.5582	2.1480	0.0781	0.8586	0.3200
		1.0749		0.6436	0.2176	0.5410	1.3021	0.3508
		-0.3650		2.4210	9.8729	0.1443	0.6594	0.9123
(2) 2SLS	1	-0.2729	-4.2193	0.8320	0.6789	0.7349	-0.1823	-0.2267
		0.8990		0.6512	1.0082	0.2638	1.3556	0.3308
		-0.3036		1.2776	0.6734	2.7862	-0.1345	-0.5851
(3) UNK	1	-0.2245	-4.4002	0.8344	0.7073	0.7310	-0.1945	-0.2297
		0.8998		0.6518	1.0091	0.2640	1.3568	0.3311
		-0.2495		1.2802	0.7009	2.7689	-0.1433	-0.6938
(4) LISE	1	6.5812	-29.8152	1.1730	4.6844	0.1832	-1.8965	-0.6607
		2.4715		1.7902	2.7717	0.7251	3.7209	0.9095
		2.6628		0.6552	1.6901	0.2526	-0.5089	-0.7265
(5) 3SLS	1	-0.6048	-1.9579	0.5078	0.3026	0.8542	0.0655	0.0363
		0.6298		0.4014	0.6889	0.1602	0.9336	0.2256
		-0.9602		1.2648	0.4392	5.3331	0.0702	0.1611

*See footnote of Table 5.2.1

Table 5.2.3

STRUCTURAL DEMAND EQUATION, INDUSTRIAL SECTOR:
COEFFICIENT ESTIMATES AND RELATED STATISTICS*

,	log Y_{5t}	log Y_{6t}	Z'_{0t}	log Z_{2t}	log Z_{5t}	log Z_{6t}	log Z_{9t}	log Z_{12t}	$\log_{13t}$
(1) OLS	1	-0.5713	-0.0256	-0.1464	0.8514	0.0062	3.5167	0.1411	-2.6475
		0.6954		0.6230	0.0920	0.0161	1.6511	0.3607	1.6372
		-0.8214		-0.2349	9.2526	0.3836	2.1299	0.3912	-1.6171
(2) 2SLS	1	-1.0601	-0.1721	-0.5520	0.8890	0.2254	4.5529	0.2529	-2.8216
		0.8932		0.7741	0.0704	0.6649	2.0650	0.3640	1.6887
		-1.1936		-0.7131	11.6381	0.3389	2.2048	0.6948	-1.6709
(3) UNK	1	-1.1591	-0.3508	-0.5505	0.8904	0.2648	4.7017	0.2799	-2.8786
		0.8970		0.7774	0.0767	0.6678	2.0739	0.3655	1.6960
		-1.2921		-0.7080	11.6057	0.3966	2.2670	0.7658	-1.6973
(4) LISE	1	-1.8858	-1.7476	-0.5385	0.9011	0.5732	5.8648	0.4911	-3.3241
		0.9642		0.8356	0.0825	0.7177	2.2292	0.3929	1.8229
		-1.9559		-0.6444	10.9271	0.7986	2.6309	1.2499	-1.8235
(5) 3SLS	1	-1.3344	-0.9433	-0.4465	0.9070	0.3919	4.2673	0.4052	-2.5015
		0.6316		0.5663	0.0554	0.4749	1.4661	0.2568	1.1779
		-2.1128		-0.7886	16.3686	0.8253	2.9107	1.5777	-2.1237

*See footnote of Table 5.2.1

predetermined variables is subject to uncertainty and alternative procedures have been developed to test for the identifiability of the coefficients in a structural equation.

In the following table (Table 5.3.1), the results of a test given in Hood and Koopmans for the limited information single equation estimators are presented.[14] The concerned test statistic is:

$$T \log (\hat{1} \cdot \hat{1}_2)$$

which has an asymptotic Chi-square distribution with K-k-g+2 degrees of freedom, such that if the test statistic is greater than or equal to the corresponding critical value one can conclude that the structural equation in question is identified.[15]

Table 5.3.1 Summary of a Test for the Identifiability of the Structural Demand Equations (S.D.E.)

(Hood and Koopmans procedure using LISE estimators)

S.D.E	D.F.	Value of the Test Statistic	Critical Value at 5 percent LOS	Conclusion
1	8	53.4454	15.5073	identified
2	8	76.4264	15.5073	"
3	7	55.9630	14.0671	"

[14] W. C. Hood and T. C. Koopmans, Studies in Econometric Method, op. cit., pp. 178-181.

[15] Notice that K is the total number of predetermined variables in the system, k is the number of predetermined variables, and g is the number of jointly dependent variables in the structural equation in question.

The reader should note that the above tests are based on an asymptotic distribution of the test statistic and experimental results show that this distribution may not be very closely approximated in finite sample sizes.[16] Some reservation, therefore, is placed on reliability of the results in the Table 5.3.1. Basmann has suggested an alternative test for the identifiability of structural equations which he considers to be more suited for typically small sample sizes. The test statistic in this case is defined as a function of the least variance ratio which corresponds to $\hat{l}_1$ of the Hood and Koopmans procedure and is given as:[17]

$$\frac{T - K}{(K - k) - g + 1} \cdot \hat{\varphi}$$

where

$$\hat{\varphi} = \hat{l}_1 - 1$$

Basmann has shown that this statistic has an F-distribution with $(K - k - g + 1, T - K)$ degree of freedom. The corresponding results are summarized for the limited information single equation estimator in Table 5.3.2.

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[16] For a very brief discussion see J. Johnston, Econometric Methods, op. cit., p. 264.

[17] See R. L. Basmann, "On Finite Sample Distributions of Generalized Classical Linear Identifiability Test Statistics," Journal of The American Statistical Association, Vol. 55 (December, 1960).

Table 5.3.2

SUMMARY OF A TEST FOR IDENTIFIABILITY
OF THE STRUCTURAL DEMAND EQUATIONS

(Basmann procedure using LISE estimators)

S.D.E.	d f	$\hat{l}_1$	$\hat{\phi}=\hat{l}_1-1$	Value of The Test Statistics	Critical Value of 5 per cent LOS	Conclusion
1	(7, 4)	2.4642	1.4642	0.8366	6.0942	"identified"*
2	(7, 4)	5.5202	4.5202	2.5828	6.0942	"
3	(6, 4)	2.7365	1.7365	1.1577	6.1631	"

*The word identified is used in quotes in order to indicate that the particular structural equation is over identified if the hypothesis that the excluded predetermined variable coefficients from that equation were zero. This is the hypothesis formed by Basmann.

As expected, each of the structural demand equations is "identified."

5.4 Conclusions

As expected, the three stage least squares (3SLS) procedure provided estimates that seem consistent with the day to day experience and theoretical reasoning. Day to day experience may force us to think that the price elasticity of demand may be inelastic since an increase in the burner-tip price of natural gas may not force the consumers of natural gas to switch to alternative fuel energies because, in general, the costs associated with replacing natural gas are higher. Similarly, theoretical reasoning reflects the "cross-elasticities" of demand, namely, price elasticities of electricity and fuel oil, with respect to natural gas demand, are expected to have positive signs.[18] The coefficients of $Z_{1\,t}$ and $Z_{2\,t}$, being the lagged (by one period) disposable personal income and the number of degree days, are expected to have positive signs. With minor exceptions (in the sense that the coefficients of the variables are not statistically significant at 5 per cent level for those whose signs of the coefficients are not in the right direction), we base our analysis with respect to the 3SLS method of estimation.

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[18]Actually these are not cross-elasticities by definition because the variables involved are lagged variables.

Residential Sector:

The signs of the coefficients except for the lagged index of price of fuel oil ($Z_{10\ t}$) are in the right direction. Since we have assumed the linear logarithmic structural demand functions, the coefficient of $Y_{2\ t}$, the index of burner-tip price of natural gas, is the price elasticity with respect to the demand of natural gas. As shown in Table 5.2.1, it is estimated as -0.4386. This indicates that a one per cent increase in the burner-tip price of natural gas ia associated with a 0.4386 per cent decrease in the consumption of natural gas, all other things remain constant. The coefficient estimates of $Z_{3\ t}$, lagged consumption of natural gas, and $Z_{7\ t}$, lagged index of price of electricity, as expected are positive. The economic implications of these variables are as follows: A one per cent increase in the average consumption of natural gas in the previous period is associated with a 0.2693 per cent increase in the succeeding year. This increase may have come either from the lag in the behavior of natural gas consumption or from the "new" uses of gas (as explained in Chapter IV)[19] made during that period. Similarly, a one per cent increase in the price of electricity causes 2.1306 per cent of increase in the natural gas consumption. The

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[19]See page 100, Chapter IV of this study.

coefficient is also statistically significant at 5 per cent level. This probably is a significant result obtained from the structural demand relation of the residential sector which signifies the competition between electricity and natural gas. This substantiates the facts that are already noted in Chapter II.[20] Contrary to the theoretical reasoning, the sign of the coefficient of $Z_{10\,t}$, the lagged index of price of fuel oil, is negative but is not significant statistically. This probably is because the price of fuel oil is consistently decreasing (except for the year 1954) over the period of consideration of this study and the burner-tip prices of natural gas are either increasing or remaining relatively the same. Yet the consumption of natural gas increased over the same period. This indicates that fuel oil is losing its market in the residential sector for reasons other than price differentials which have prevailed, and consequently the price variable of fuel oil may not enter into the decision making process of consumers concerning the choice of fuel energy.

Both the coefficient estimates of lagged disposable income, $Z_{1\,t}$, and lagged number of degree days, $Z_{2\,t}$, are positive. The estimate of $Z_{1\,t}$ is statistically significant both at 5 per cent and one per cent levels. The coefficient reflects the fact that the consumption

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[20]See pages   Chapter II of this study.

of natural gas would be increased by 1.8156 per cent for one per cent increase in the previous period's income. This, by definition, is not an "income elasticity" because the "independent" variable is not the current disposable income but the disposable income in the previous period. For our analysis purpose, we refer to it as "indirect elasticity." The high value of the coefficient of $Z_{1\,t}$, namely 1.8156, may be due to the inclusion of the effects of both the lagged disposable income and population in $Z_{1\,t}$. In particular, the effect of the number of customers of natural gas in the residential sector, whose disposable income is included in $Z_{1\,t}$, might have been included in the coefficient of $Z_{1\,t}$. Apart from this, the coefficient estimate reflects the impact of higher disposable income in the previous period that may cause people to have additional comforts or to satisfy their desires (for example, by buying a new gas-using appliance). As one can expect, the colder the weather, the higher the consumption of natural gas and this fact is substianted by the sign and magnitude of the coefficient estimate of $Z_{2\,t}$.

One can see from Table 5.2.1 that 3 SLS estimators differ considerably from the other four procedures. In particular, the LISE estimates differed considerably from 3SLS procedures as well as from the other procedures. The sign of the coefficient estimate of $Z_{7\,t}$ is negative and even the magnitudes of each coefficient differed considerably. This possibly is because of high K-value where K = 2.4642, which is substantially larger than its asymptotic limit of 1. This high value of K or equivalently a higher least variance ratio in LISE procedure, may indicate, in general, the existence of high correlations between the jointly dependent variables that are included and the predetermined variables that are excluded from the structural demand equation. The relatively high value of K in LISE estimation procedure may be due to either (1) mis-specification (of) the model, or (2) a larger loss in degrees of freedom relative to the size of the sample, when "fitting the denominator" of the "least variance ratio," as compared to fitting the numerator.[21] That is, linear combinations of the specified jointly dependent variables, in the structural demand equation, are regressed on not only those predetermined variables specified in that equation in the "numerator" but on all predetermined variables in the system in the "denominator." The ratio of these two then defines the least variance ratio. This suggests that the residual variance in the denominator may be substantially smaller than in the numerator

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[21]See for example S. G. Unger, "Simultaneous Equations System Estimation: An Application in the Cattle-Beef Sector," Ph.D. Thesis, Michigan State University, 1966. He also obtained poor coefficient estimates for LISE compared to other procedures, pp. 90-119. Also see J. Johnston, <u>op</u>. <u>cit</u>., Chapter IV, pp. 288-290.

simply because the number of predetermined variables in the system is larger relative to the sample size. We cannot, however, say, a priori whether a high K-value will result in a substantial influence on the estimators (relative to the K-class estimates) in a particular structural equation. In the case of the commercial and the industrial sectors also, LISE procedure provides estimates which are considerably different from the estimates of the coefficients obtained from 3SLS and other procedures. And the same arguments made above hold in these situations also.

Commercial Sector:

The coefficient estimates of each variable in the structural demand equation for natural gas in the commercial sector has its expected sign (using 3SLS procedures). The estimated coefficient of $Y_{4\,t}$, the index of burner-tip price of natural gas, is -0.6049, which itself is the price elasticity of natural gas demand since the structural demand function is assumed to have a double logarithmic form. This indicates that there will be a 0.6048 per cent increase in the natural gas consumption for one per cent decrease in its burner-tip price. The price elasticity of demand in the commercial sector is slightly elastic compared with that of the residential sector, which is reasonable to expect. The coefficient estimate of $Z_{4\,t}$, the lagged consumption of natural gas in the commercial sector, is 0.8542 and is highly significant.[22] This is the only variable whose coefficient estimate is statistically significant and consequently is important in explaining the consumption of natural gas in the commercial sector. The magnitude of the estimate of the coefficient indicates that one per cent increase in the consumption of natural gas in the previous period would increase the consumption in the succeeding period by 0.8542 per cent. As we have argued in the case of residential demand, this increase may have come from "new" uses that have taken place during the period.

The magnitude of "indirect" cross-elasticities of demand with respect to the prices of electricity and fuel oil are 0.0655 and 0.0363, respectively. The "indirect" income elasticity in this case is estimated as 0.5078 and the coefficient estimate of the lagged number of degree days is 0.3026.

Industrial Sector:

The estimated coefficients of all variables except $Z_{2\,t}$ and $Z_{13\,t}$ have expected signs. The magnitude of the estimated coefficient of $Y_{6\,t}$, the burner-tip price of natural gas in the industrial sector is -1.3344. This also is the price elasticity of natural gas demand in the industrial sector. As the size of the estimator indicates,

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[22]That is, the coefficient estimate is statistically significant at both one per cent and 5 per cent levels.

natural gas demand in industrial demand is elastic, thus indicating a 1.3344 per cent decrease in its consumption for every one per cent increase in the burner-tip price. As in the residential and commercial sectors, the estimated coefficient of the lagged consumption of natural gas is 0.9070 and is highly significant. And consequently this variable has considerable significance in explaining the consumption of natural gas in the industrial sector. The higher consumption in the succeeding period, as indicated by the magnitude of the estimate, may be explained by the intensity of use of natural gas in the industrial sector. This is already argued in Chapter IV, or it may be explained by the release of more gas from the residential and commercial sectors (where the gas service is a "firm" service) to the industrial sector (where the service is based on "interruptible" service).

The coefficient estimate of the lagged index of price of electricity, $Z_{9\,t}$, is 4.2673 and is statistically significant at 5 per cent level. The coefficient estimates of $Z_{12\,t}$ and $Z_{13\,t}$, the indexes of price of fuel oil and coal, are not statistically significant. The estimates of the coefficient of $Z_{13\,t}$ does not have an expected sign. So is the case for $Z_{2\,t}$. And the coefficient estimate of lagged industrial employment, $Z_{6\,t}$, is 0.3919. The negative sign to the coefficient estimate of $Z_{13\,t}$ is rather unexpected. The data on the prices of coal used in this study is taken from that of steam electric power plants data. And the steam electric power plants in Michigan do not use (or consume very little, at least prior to 1964) natural gas as an input to generate electricity. Because there is no other source of data on coal prices, the above source was used. Consequently, the index of the price of coal may not provide a good measure of the effect of coal as a substitute for natural gas in the industrial sector. One would expect that the price of fuel oil in the industrial sector would have more effect than an elasticity of .4. This result, together with the wrong sign of the coal price elasticity, is rather discouraging.

In the next chapter, an alternative demand model is developed and the corresponding results will be presented later in the chapter. The basis for developing such a model is already discussed in Chapter I.

CHAPTER VI

THE DEMAND FOR NATURAL GAS: A DISTRIBUTED LAG MODEL

6.1 Natural Gas Demand and the Relevance of Distributed Lag Models

The dynamic model developed in this chapter is based on the generally accepted idea that current decisions are influenced by past behavior. An economic reaction, say the reaction of the quantity of a certain commodity demanded brought about by a change in its price, is a process in time. This is because the behavior of the consumption cannot be adapted immediately to changes in the variables which condition it. This is well known in economic theory which often distinguishes between short run and long run reactions. Part of the reaction may take place after the lapse of one period, another part of the reaction taking place during the second period, and so on until after a certain number of periods the total reaction is completely realized. Then one may say that the quantity demanded is fully adjusted to the new price. Similarly, reactions to an increase in disposable income, on the consumption of a certain commodity, take their full effect only after some time has passed. In such cases the economic reactions are spread over time and we have a distributed lag where the term "lag" is defined as the lapse of time between a cause and its effect. But in some cases, the lag may be a specific time, say three hours, or three days or three months. In such a situation we have simple lag. Thus the economic reactions described by behavioral relationships between the jointly-dependent or endogenous variables and the predetermined variables may be termed as processes in time. These processes explain the nature of adaptive behavior of endogenous variable(s) to a given change in predetermined variables and can be described by a graph of economic time path. But before that, the nature and dependent relationships between the endogenous and predetermined variables in economic models that cause the distributed lag response or adjustment mechanism will be explained. Of course, particular relevance is given to the behavior of demand for natural gas in each of the residential, commercial and industrial sectors.

There are several factors that reflect the distributed lag response in the consumption of natural gas either for a given rise in the burner-tip price or for an increase in consumer's income, but the following are the main factors:

1. psychological,
2. technological,

3. institutional.[1]

We can reasonably assume that people are slow to change their pattern of consumption or their level of living radically in response to changes in prices or income. This is mainly because of two reasons: (1) presence of psychological inertia in human beings, and (2) imperfect knowledge of the market.

Psychological inertia prevents instantaneous readjustment of the behavior of the consumer to a changed situation. Once a housewife made it a habit to use a gas stove or gas furnace for space heating, it may be hard to convince her to buy an electric range when the gas stove is worn out unless there is a substantial advantage in buying an electric range. Habit is a powerful force and may persist although the reason for it has disappeared. Similarly, the imperfect knowledge of the market may lead to a lagged reaction. A change in the burner-tip price of natural gas may not be known to every potential consumer. In that case, the total effect of the change in burner-tip price will only be realized when every potential consumer is fully aware

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[1] L. M. Koyck, Distributed Lags and Investment Analysis, Amsterdam: North-Holland Publishing Company, 1954, pp. 5-9. M. Nerlove, "Distributed Lags and Demand Analysis for Agricultural and Other Commodities, Agricultural Handbook No. 141, United States Department of Agriculture, Washington, D.C., June, 1958, pp. 1-4.

of the price change. Another set of factors that leads to a lag in the reactions of the consumer is the set of technological factors.

The economic theory of the consumer choice is based on the fact that a consumer maximizes or satisfices a certain level of satisfaction, subject to restraints. We need not argue here the validity of the maximizing or satisficing assumption underlying consumer choice. The existence of either assumption is valid in this case because we are interested in showing the existence of lagged reactions rather than arguing the pros and cons about these assumptions. Just as a firm produces a product with fixed as well as variable factors, so a consumer produces a satisfaction with stocks of durable as well as semi-durable goods. The level of satisfaction he attains is mainly a function of prices of the durable or semi-durable goods, costs of maintenance, apart from other things. For example, in case of natural gas, the level of satisfaction attained is in part a function of the prices of gas ranges, electric ranges, oil furnaces, gas furnaces, electric furnaces, and the prices of these fuel energies apart from the maintenance and other costs. Also the lifetime of each of the above equipments may play a part in accounting for the consumer's level of satisfaction. An increase in the burner-tip price of natural gas may provide an incentive to consumers to substitute other services for it; but they may not do so immediately because it may be too costly to replace the gas-using equipment with others. Even if the consumer wants to replace his equipment, he may require time to buy an appropriate appliance. Apart from this, he may face the restrictions from the distributors of these appliances because the stocks of appliances are maintained by them. The distributors are also susceptible to changes in the prices of the energy fuels. Technological factors making substitutes available also affect the consumption of natural gas. That is, there should be a gas pipeline or appropriate appliance for a consumer to be able to substitute for a given change in the price of, say, electricity. There was an increase of 77 per cent, 62 per cent and 54 per cent in the consumption of natural gas in residential, commercial, and industrial sectors, respectively, during the period 1950-1955.[2] This is mainly due to the vast expansion in transmission and distribution network through technological innovations and developments. The technological innovations made possible pressure welding for joint lengths of pipeline, development of thin-walled pipe which can withstand extremely high pressures, manufacture of larger diameter pipes, development of large capacity high speed compressors for booster stations. And these, in turn, made possible the

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[2]Table 1.1.2, p. 4 , Chapter I of this study.

spread of the transmission and distribution network which made larger amounts of natural gas available. Because of the presence of these points, a complete and immediate adaption to a changed situation is not possible, thus causing a lag in the reaction of consumers of natural gas. The technological reasons play an important role compared to psychological reasons mainly because the consumption of natural gas is related to the stock of appliances rather than the variation in burner-tip prices of natural gas. It is also conceivable to think that disposable income may play a significant role compared to burner-tip prices because the effects in stocks of appliances can better be explained by changes in disposable income rather than the changes in burner-tip prices.

Institutional factors may also produce a certain regidity which leads to a lagged reaction in the consumers of natural gas. These factors influence the reactions through (1) the regulation, and (2) habit. For example, a given increase in the price of electricity or a given increase in disposable income may not bring an increase in the consumption of natural gas unless there is a pipeline connecting to the home. But pipeline construction can only be done through a regulatory process which may be of several years' duration. Similarly, burner-tip prices of natural gas are susceptible to changes in the prices of its substitutes. But gas and electricity prices are regulated, oil and coal are not, so that it may be some time before all fuel prices adjust to their proper competitive relationship, if they do so at all. The lag introduced by habit needs no explanation. Thus the regulatory aspects, together with habit induce lagged reactions, which would show up in the behavior of consumption through the movement along or the shift in the demand curve of natural gas.

The next section is devoted to the description of how or in what form the adjustment mechanism may take place in the consumption for a given change in either the burner-tip prices or the disposable income.

6.2 The Description of the Adjustment Process of an Economic Reaction

For simplicity, let us assume that the demand functions are functions of only one "predetermined" variable, i.e., either burner-tip price or disposable income.[3] Also the economic reaction may not be the same for each individual consumer of natural gas. That is, some individual consumers may react immediately or after a short lag, some may be very slow for one reason or another and will react after a long period, and others may be grouped

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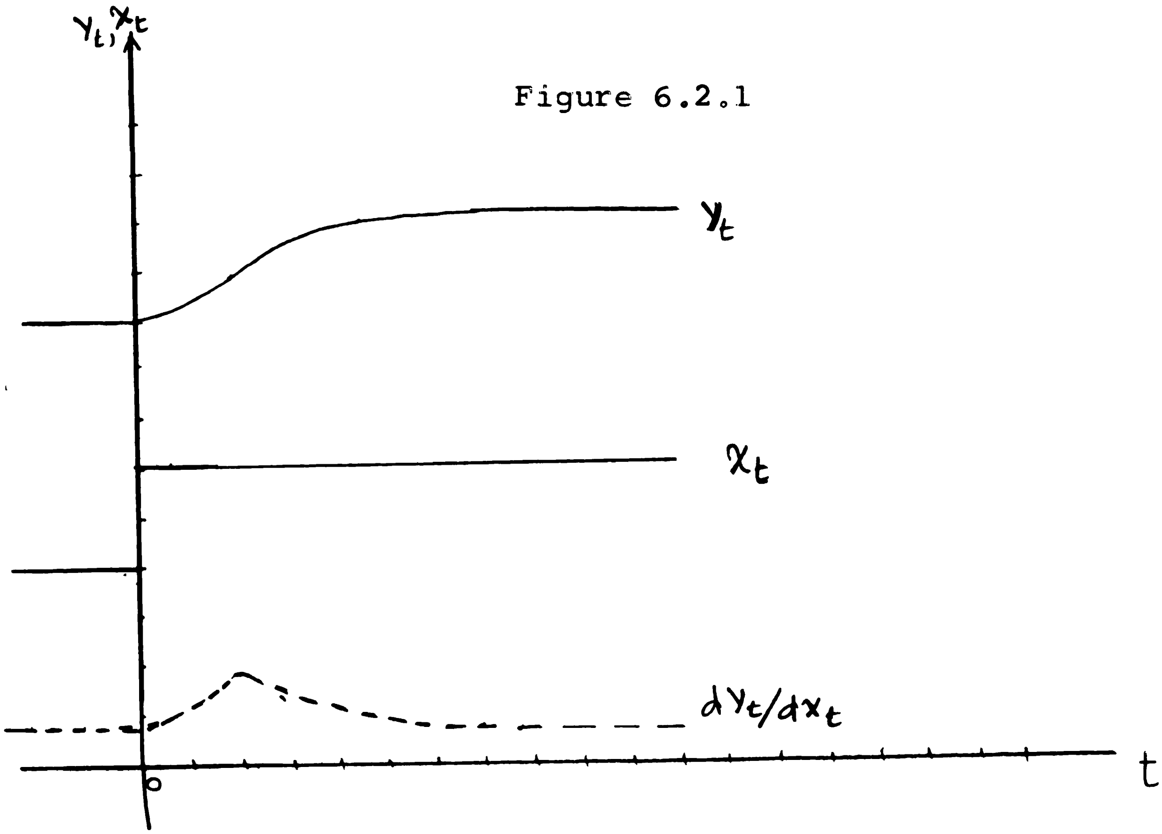
[3]Burner-tip price is strictly not a "predetermined" variable as it is pointed out in Chapter 4. But for our simple explanation of the adjustment process, we assume temporarily that it is a predetermined variable.

between these extremes. So, generally, the lag in the reactions of a number of gas consumers will be distributed over a period of time and we describe the process of reactions in the adjustment process by a continuous curve. One may think that the value of an endogenous variable at time, say, t_1, is the average value of all gas consumers. This value is, of course, subject to the lagged response.

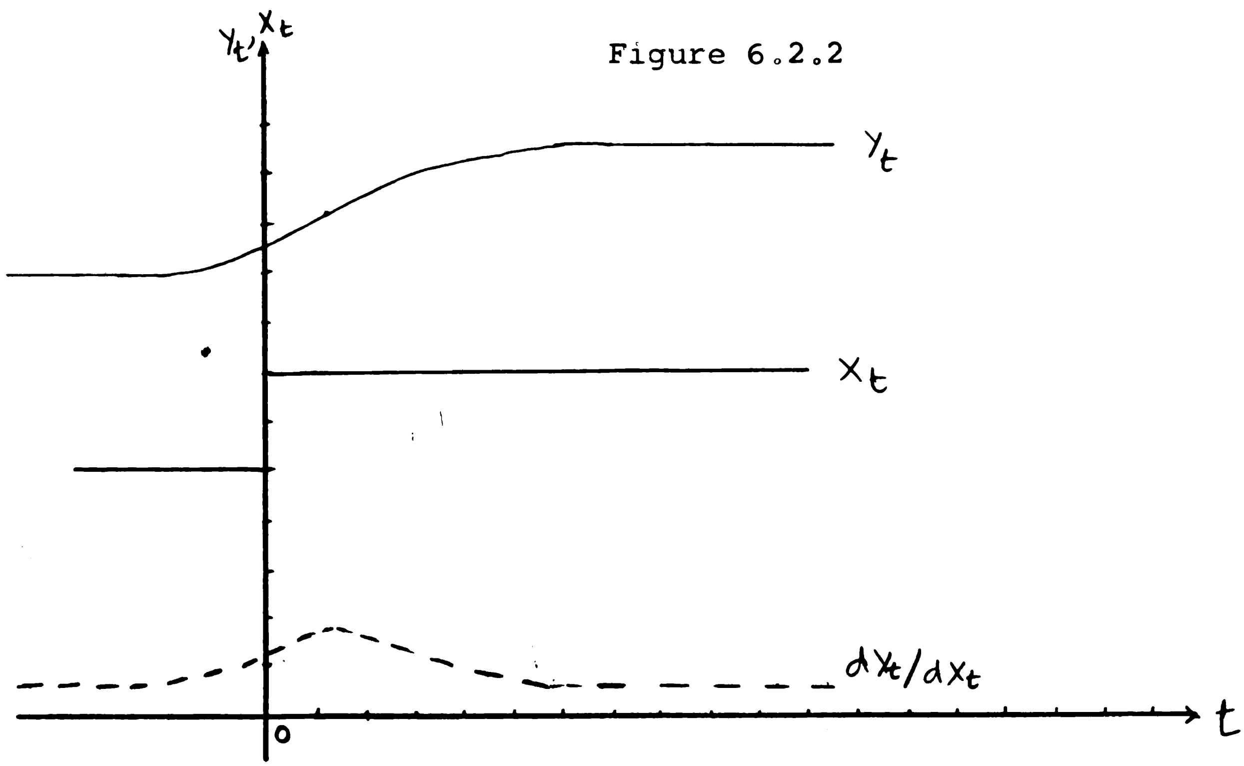
Let Y_t be the quantity of natural gas consumed and let X_t be the disposable income that the community has at time t. Suppose at t = 0, say, an increase in X_t was introduced causing a change in Y_t. The figure 6.2.1 would explain the possible description of adjustment process in the quantity consumed. For a given rise in income and hoping the continuation of the new level,

Figure 6.2.1



the consumer may either replace an electric appliance with a gas using appliance or may have a new gas using air conditioner. This would increase his consumption of natural gas. Thus demand would increase at the beginning periods and then decrease slightly as described by the graph of dY_t/dt. So $\frac{dY_t}{dt}$ essentially describes the time-path of the adjustment process. If the consumers, on the other hand, anticipate an increase in their disposable income, an increase in consumption may start earlier than the time t = 0, in which case the following figure will describe the time-path. Similar arguments can be made with respect to the price variable, i.e. the effects of burner-tip prices on the consumption of natural gas. It should also be pointed out that the time-path or adjustment process of a reaction depends

Figure 6.2.2



on the unit period. That is, the time path of Y_t if Y_t is measured in mcf (millions of cubic feet) per month, is different from the time path of Y_t if Y_t is measured in mcf per year. These explanations are simple and yet are capable of throwing some insights into the mechanism behind the adjustment process of our economic reactions. More complex situations can be thought of and described analagously. One can fit an appropriate mathematical model to describe the situations handled in the figures 6.2.1 and 6.2.2. Based on the economic theory of consumer choice and other a priori assumptions that seem realistic for the study under consideration, one can construct a mathematical model capable of describing the behavior of the consumption of natural gas. This is what is discussed in the next section.

6.3 The Development of the Distributed Lag Model

It was argued in Chapter II that the changes in total expenditures for consumption in response to changes in income may be considered from a different approach.[5] It is reasonable to assume that consumers wish to even out their consumption to a certain extent over their "lifetimes" or at least over the foreseeable future. In such a case, an individual consumer tends to save when his income is temporarily high and to dissave when his income is temporarily low. The total consumption during

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[5]Chapter II, p. 54 of this study.

any period of time is thus determined by the consumer's expected long-range income, not by his current income. Consequently, the total consumption expenditures tend to be stable relative to current incomes and a change in current income tends to affect consumption only insofar as it affects consumers' notions of their "expected" incomes.[6] With these ideas and with the basis of economic theory of consumer choice, the quantity of natural gas demanded can be assumed as a function of the burner-tip price of natural gas, price of electricity, price of fuel oil, "expected" income, and number of degree days. One should note that in this study, the words "expected normal income" and "expected" income are invariably used to represent the same meaning. For simplicity, the word "expected" income will be used in this chapter. If we consider the residential sector, then the data for the above variables can be taken from the residential sector. The prices of electricity and fuel oil enter into the equation as substitutes for natural gas and the number of degree days and expected income variables enter into the equation as exogenous variables. That is, the demand equation can be expressed as:

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[6] M. Nerlove, op. cit., pp. 4-7. Also see pp. 20-37, M. Friedman, A Theory of Consumption Function, Princeton: Princeton University Press, 1957.

(6.3.1) ---- $Y_{1\,t} = F(Y_{2\,t}, Y_{7\,t}, Y_{8\,t}, Z^{*}_{14\,t}, Z_{15\,t})$

where the variables $Y_{1\,t}$, $Y_{2\,t}$, $Y_{7\,t}$, $Y_{8\,t}$, and $Z_{15\,t}$ are defined as in Chapter II[7] and $Z^{*}_{14\,t}$ is expected disposable income in period t. Assuming linear form and adding a disturbance term, (6.3.1) can be written as:

(6.3.2) ---- $Y_{1\,t} = \alpha_0 + \alpha_1 Y_{2\,t} + \alpha_2 Y_{7\,t} + \alpha_3 Y_{8\,t} + \alpha_4 Z^{*}_{14\,t} + \alpha_5 Z_{15\,t} + \varepsilon_t$

where

ε_t = random disturbance

or assuming double logarithmic form and adding an appropriate disturbance term, (6.3.1) becomes

(6.3.3) ---- $\log Y_{1\,t} = \beta_0 + \beta_1 \log Y_{2\,t} + \beta_2 \log Y_{7\,t} + \beta_3 \log Y_{8\,t} + \alpha_4 \log Z^{*}_{14\,t} + \alpha_5 \log Z_{15\,t} + U_t$

where

U_t = random disturbance

Now we have to define $Z^{*}_{14\,t}$. This is so defined that when substituted either in (6.3.2) or in (6.3.3) it should be able to provide us a distributed lag model which is capable of explaining the progressive nature of adaptations in the behavior of consumption of natural gas to a changed situation. In the literature of distributed lag models, the form of distribution of lag is approached in mainly three forms:

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[7]Chapter II, pp. 56-58 of this study.

1. Make no assumptions as to the form of the distribution of lag.

2. Assume a general form for the distribution of the lag and estimate the corresponding form.

3. Develop a specific model based on *a priori* considerations which yield a specific distribution of lag only incidentally.[8]

In developing the distributed lag model for our purpose, the third approach is used. So the expected disposable income is defined as:

$$(6.3.4) \quad Z^{*}_{14\,t} = \sum_{\tau=0} C_{\tau}\, Z_{14\,t-\tau}$$

where

$Z_{14\,t-\tau}$ = disposable income in period $t - \tau$

$\tau = 0, 1, 2, \ldots\ldots;$

$t = 1, 2, \ldots\ldots 18$

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[8]The first approach was used by F.F. Alt and J. Tinbergen. The second approach was used by I Fisher and L. M. Koyck, and the third was used by M. Nerlove, M. Friedman, P. Cagan, and J. F. Muth. All three approaches were discussed in Nerlove's study but the individual studies can be seen as follows: F.F. Alt, "Distributed Lags," *Econometrica*, Vol. 10, 1942. P. Cagan, "The Monetary Dynamics of Hyper-inflations," in M. Friedman (ed.), *Studies in the Quantity Theory of Money*, Chicago: University of Chicago Press, 1957. I. Fisher, "Note on Short-cut Method for Calculating Distributed Lags," *International Statistical Bulletin*, Vol. 29, 1937. M. Friedman, *A Theory of Consumption Function*, National Bureau of Economic Research, Princeton: Princeton University Press, 1957. L. M. Koyck, *loc. cit.* J. F. Muth, "Optimal Properties of Exponentially Weighted Forecasts," *Journal of the American Statistical Association*, Vol. 55, No. 290. 1960. M. Nerlove, *loc. cit.*

Now one has to know the form of the coefficient C_τ in equation (6.3.4). As discussed already, it may be expected that the psychological, technical and institutional factors prevent an immediate adaptation to a changed situation but may cause an increase in the consumption in the earlier periods and tapering off in successive periods after reaching a maximum. Incorporating this idea, one can assume that the sequence $C_0, C_1, C_2, \ldots$ may be increasing in its first terms but must be continuously decreasing once the maximum has been reached. In many cases the coefficient C_0 is the biggest of all and it is valid to assume that

$$C_\tau < C_{\tau-1}$$

for all $\tau \geq 1$. This is precisely the kind of argument that is made in Section 2 of this chapter in explaining the time-path of adjustment process, so it is reasonable to assume a geometric decrease in C_τ, i.e.

(6.3.5) ---- $C_\tau = \lambda^\tau; \quad 0 \leq \lambda < 1; \quad \tau = 0, 1, 2, \ldots$

Similar to this form is also the assumption of Koyck in his ingenious development of distributed lag model.[9] There is another advantage to using (6.3.5). It simplifies greatly the difficulties that arise in statistical estimation of the demand function and certain other

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[9] L. M. Koyck, loc. cit., pp. 19-22. Also a brief discussion on the various forms of C_τ based on assumptions made by several research workers in econometrics is given in E. Malinvaud, Statistical Methods of Economics, Chicago: Rand-McNally & Company, 1966, pp. 479-481.

characteristics which we discuss below. Substituting (6.3.5) in equation (6.3.4):

$$(6.3.6) \text{ ---- } Z^{*}_{14\,t} = \sum_{\tau=0} \lambda^{\tau} Z_{14\,t-\tau}$$

$$= Z_{14\,t} + \lambda Z_{14\,t-1} + \lambda^{2} Z_{14\,t-2} + \ldots$$

$$+ \lambda^{\tau} Z_{14\,t-\tau} + \ldots$$

$$t = 1, 2, \ldots 20;$$

$$0 \leq \lambda < 1$$

The final form of the demand function of natural gas in the residential sector can be obtained by the substitution of (6.3.6) in (6.3.2). By doing so, we have:

$$(6.3.7) \text{ ---- } Y_{1\,t} = \alpha_0 + \alpha_1 Y_{2\,t} + \alpha_2 Y_{7\,t} + \alpha_3 Y_{8\,t}$$

$$+ \alpha_4 \left[\sum_{\tau=0}^{\infty} \lambda^{\tau} Z_{14\,t-\tau} \right]$$

$$+ \alpha_5 Z_{15\,t} + \varepsilon_t$$

This equation can be estimated, but unfortunately it is too difficult for estimation purposes because it involves an infinite number of parameters, namely, α_0, α_1, α_2, α_3, α_4, α_5, $\lambda\alpha_4$, $\lambda^2\alpha_4$ . . . ., to estimate and also because of possible multicollinearity due to the presence of the variables $Z_{14\,t-\tau}$ ($\tau = 0, 1, \ldots$). Fortunately, we can avoid all of these difficulties by differencing the equation (6.3.7). This can be done as follows: Lagging the equation (6.3.7) by one period and multiplying the resulting equation by λ, one obtains:

$$(6.3.8) \text{ ---- } \lambda Y_{1\,t-1} = \lambda\alpha_0 + \lambda\alpha_1 Y_{2\,t-1} + \lambda\alpha_2 Y_{7\,t-1} + \lambda\alpha_3 Y_{8\,t-1} + \lambda\alpha_4 \left[Z_{14\,t-1} + \lambda Z_{14\,t-2} + \lambda^2 Z_{14\,t-1} + \ldots \right] + \lambda\alpha_5 Z_{15\,t-1} + \lambda\varepsilon_{t-1}$$

Subtracting (6.3.8) from (6.3.7) and rearranging the terms

$$(6.3.9) \text{ ---- } Y_{1\,t} - \lambda Y_{1\,t-1} = \alpha_0(1-\lambda) + \alpha_1(Y_{2\,t} - \lambda Y_{2\,t-1}) + \alpha_2(Y_{7\,t} - \lambda Y_{7\,t-1}) + \alpha_3(Y_{8\,t} - \lambda Y_{8\,t-1}) + \alpha_4\, Z_{14\,t} + \alpha_5(Z_{15\,t} - \lambda Z_{15\,t-1}) + (\varepsilon_t - \lambda\varepsilon_{t-1})$$

This equation is suitable for estimation purposes in the sense that it is much simpler compared to (6.3.7). This is because, in the first place, it involves only the parameters α_0, α_1, α_3, α_4, α_5 and λ, subject to certain restrictions on the coefficients of the variables $Y_{i\,t-1}$ $(i = 2, 7, 8)$ and $Z_{15\,t-1}$. In the second place, the multicollinearity problem due to $Z_{14\,t-\tau}$ $(\tau = 0,1, \ldots)$ is completely eliminated in equation (6.3.9). But the parameter λ is now overidentified in the sense that the estimates of it are provided by the ratios of the coefficients of either $Y_{2\,t-1}$, $Y_{2\,t}$ or $Y_{7\,t-1}$, $Y_{7\,t}$ or $Y_{8\,t-1}$, $Y_{8\,t}$ or $Z_{15\,t-1}$, $Z_{15\,t}$. So in order to estimate the corresponding coefficients we assume various values for λ in the admissible interval $[0\ 1)$ and the equation with respect to a value of λ is selected for which SSE, the sum of squares of residuals, is minimum. The method of estimation is described in Section 6.5.

Equation (6.3.9) can be more conveniently arranged by defining the following:

$$(6.3.10) \text{ ---- } \begin{aligned} Y^*_{1\,t} &= Y_{1\,t} - \lambda Y_{1\,t-1} \\ Y^*_{2\,t} &= Y_{2\,t} - \lambda Y_{2\,t-1} \\ Y^*_{7\,t} &= Y_{7\,t} - \lambda Y_{7\,t-1} \\ Y^*_{8\,t} &= Y_{8\,t} - \lambda Y_{8\,t-1} \\ Z^*_{15\,t} &= Z_{15\,t} - \lambda Z_{15\,t-1} \\ \alpha^*_0 &= \alpha_0(1 - \lambda) \\ \varepsilon^*_t &= \varepsilon_t - \lambda \varepsilon_{t-1} \end{aligned}$$

When these new variables are substituted, (6.3.9) can be expressed as:

$$(6.3.11) \text{ ---- } Y^*_1 = \alpha^*_0 + \alpha_1 Y^*_{2\,t} + \alpha_2 Y^*_{7\,t} + \alpha_3 Y^*_{8\,t} + \alpha_4 Z_{14\,t} + \alpha_5 Z^*_{15\,t} + \varepsilon^*_t \quad . \; . \; . \; .$$

This form is used for estimating purposes. The method by which this equation is estimated is discussed in Section 6.5.

Again considering the equation (6.3.3) as the demand function, the corresponding expected disposable income is defined as:

$$(6.3.12) \text{ ---- } \log Z^*_{14\,t} = \sum_{\tau=0}^{\infty} d_\tau \log Z_{14\,t-\tau}$$

where the coefficients d_τ decrease geometrically (τ = 0, 1, ...) as discussed earlier. Then we define

$$d_\tau = \delta^\tau \qquad \tau = 0,1,2, \ldots .$$

and after substituting this in (6.3.12), we have:

$$\log Z^*_{14\,t} = \sum_{\tau=0}^{\infty} \delta^\tau \log Z_{14\,t-\tau}$$

This is substituted in equation (6.3.3) and the resulting equation can be expressed as:

$$(6.3.13) \text{ ---- } \log Y_{1\,t} = \beta_0 + \beta_1 \log Y_{2\,t} + \beta_2 \log Y_{7\,t} + \beta_3 \log Y_{8\,t} + \alpha_4 \Big[\log Z_{14\,t} + \delta \log Z_{14\,t-1} + \delta^2 \log Z_{14\,t-1} \ldots .\Big] + \alpha_5 \log Z_{15\,t} + U_t$$

Lagging this equation by one period, multiplying the resulting equation by δ and subtracting it from (6.3.13), one obtains:

$$(6.3.14) \text{ ---- } \log Y_{1\,t} - \delta \log Y_{1\,t-1} = \beta_0(1 - \delta) + \beta_1 \Big[\log Y_{2\,t} - \delta \log Y_{2\,t-1}\Big] + \beta_2 \Big[\log Y_{7\,t} - \delta \log Y_{7\,t-1}\Big] + \beta_3 \Big[\log Y_{8\,t} - \delta \log Y_{8\,t-1}\Big] + \beta_4 \log Z_{14\,t} + \beta_5 \Big[\log Y_{15\,t} - \delta \log Z_{15\,t-1}\Big] + U_t - \delta U_{t-1}$$

Again defining the following terms as before,

$$\log Y^*_{1\,t} = \log Y_{1\,t} - \delta \log Y_{1\,t-1}$$

$$\log Y^*_{2\,t} = \log Y_{2\,t} - \delta \log Y_{2\,t-1}$$

$$\log Y^*_{7\,t} = \log Y_{7\,t} - \delta \log Y_{7\,t-1}$$

$$\log Y^*_{8\,t} = \log Y_{8\,t} - \delta \log Y_{8\,t-1}$$

$$\log Z^*_{1t\,t} = \log Z_{15\,t} - \delta \log Z_{15\,t-1}$$

and

$$\beta^*_0 = \beta_0(1 - \delta)$$

$$U^*_t = U_t - \delta U_{t-1}$$

and substituting in the equation (6.3.14), we obtain the form suitable for estimation purposes:

$$(6.3.15) \text{ ---- } Y^*_{1\,t} = \beta_0 + \beta_1 \log Y^*_{2\,t} + \beta_2 \log Y^*_{7\,t} + \beta_3 \log Y^*_{8\,t} + \beta_4 \log Y_{14\,t} + \beta_5 \log Z^*_{15\,t} + U^*_t$$

By giving various values to δ in the admissible interval $[0\ 1)$ and by using the method of estimation described in Section 6.5, equation (6.3.15) is estimated. The results are tabulated in Table 6.5.1.

Similar to equations (6.3.11) and (6.3.15), the demand functions of natural gas for commercial and industrial sectors can be developed. Assuming that the form of the demand function in the case of the commercial sector is linear, the demand function of natural gas is defined as:

(6.3.16) ---- $Y_{3t} = \gamma_1 + \gamma_2 Y_{4t} + \gamma_3 Y_{9t} + \gamma_4 Y_{10t}$

$$+ \gamma_5 Z^*_{16t} + \gamma_6 Z_{15t} + V_t$$

where the variables, Y_{3t}, Y_{4t}, Y_{9t}, Y_{10t} and Z_{15t} are defined as before[10] and

Z^*_{16t} = expected number of natural gas customers in the commercial sector in time period t

V_t = random disturbance

The variables Y_{4t}, Y_{9t}, and Y_{10t} are included in the demand function because they are the price variables of the natural gas and its substitutes, electricity and fuel oil. The variable Z_{15t} is included because of the reasons argued as in the case of the residential sector. As before, in order to explain the adjustment mechanism in Y_{3t}, Z^*_{16t} is so defined that it should be capable of providing a distributed lag model. Z^*_{16t}, the expected number of natural gas customers in the commercial sector, is used to reflect the influence of the number of customers of natural gas on the consumption. For example, if two different places, maybe cities or states, were to be compared, in which all factors other than the number of customers of natural gas were the same, it is obvious that the place which had more natural gas customers tends to consume more than the place with fewer natural gas customers. And without loss of generality, we might argue that the total consumption expenditures tend to

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[10]Chapter II, pp.56-58 of this study.

be stable relative to the number of natural gas customers in the current period, and a change in the consumption of natural gas is significantly explained by the "expected number of natural gas customers" in the commercial sector. This is because the "expected number of natural gas customers" in the commercial sector reflects the consumption of natural gas by the new customers, apart from the customers already using natural gas. The new customers, in a given locality, may be brought into use either by replacing old, non-gas using appliances with new gas-using appliances or by laying a new natural gas pipeline to the given locality. After appropriately defining $Z^*_{16\,t}$ and substituting it in the equation (6.3.16) we can obtain:

$$(6.3.17) \text{ ---- } Y^*_{3\,t} = \gamma^*_1 + \gamma_2 Y^*_{4\,t} + \gamma_3 Y^*_{9\,t} + \gamma_4 Y^*_{10\,t} + \gamma_5 Z_{16\,t} + \gamma_6 Z^*_{15\,t} + V^*_t$$

where

$$Y^*_{3\,t} = Y_{3\,t} - \theta Y_{3\,t-1}$$

$$Y^*_{4\,t} = Y_{4\,t} - \theta Y_{4\,t-1}$$

$$Y^*_{9\,t} = Y_{9\,t} - \theta Y_{9\,t-1}$$

$$Y^*_{10\,t} = Y_{10\,t} - \theta Y_{10\,t-1}$$

$$Z^*_{15\,t} = Z_{15\,t} - \theta Z_{15\,t-1}$$

$$\gamma^*_1 = \gamma_1(1 - \theta)$$

$$V^*_t = V_t - \theta V_{t-1}$$

Again assuming double-logarithmic form in the demand function, that is, assuming the demand function as:

$$(6.3.18) \text{ ---- } \log Y_{3\,t} = \eta_1 + \eta_2 \log Y_{4\,t} + \eta_3 \log Y_{9\,t} + \eta_4 \log Y_{10\,t} + \eta_5 \log Z_{15\,t} + \eta_6 \log Z^*_{16\,t} + e_t$$

an equation similar to (6.3.17) can also be obtained in this case as:

$$(6.3.19) \text{ ---- } \log Y^*_{3\,t} = \eta^*_1 + \eta_2 \log Y^*_{4\,t} + \eta_3 \log Y^*_{9\,t} + \eta_4 \log Y^*_{10\,t} + \eta_5 \log Z^*_{15\,t} + \eta_6 \log Z_{16\,t} + e^*_t$$

where

$$\log Y^*_{3\,t} = \log Y_{3\,t} - \omega \log Y_{3\,t-1}$$

$$\log Y^*_{4\,t} = \log Y_{4\,t} - \omega \log Y_{4\,t-1}$$

$$\log Y^*_{9\,t} = \log Y_{9\,t} - \omega \log Y_{9\,t-1}$$

$$\log Y^*_{10\,t} = \log Y_{10\,t} - \omega \log Y_{10\,t-1}$$

$$\text{Log } Z^*_{15\,t} = \log Z_{15\,t} - \omega \log Z_{15\,t-1}$$

$$\eta^*_1 = \eta_1 (1 - \omega)$$

$$e^*_t = e_t - \omega e_{t-1}$$

Equations (6.3.17) and (6.3.19) are used for estimating the respective coefficients and results are tabulated in Table 6.5.2.

As in the commercial sector, the demand function for natural gas in the industrial sector is assumed as:

$$(6.3.20) \text{ ---- } Y_{5\,t} = \nu_1 + \nu_2 Y_{6\,t} + \nu_3 Y_{11\,t} + \nu_4 Y_{12\,t} + \nu_5 Y_{13\,t} + \nu_6 Z_{15\,t} + \nu_7 Z^*_{17\,t} + V_t$$

where $Y_{5\,t}$, $Y_{6\,t}$, $Y_{11\,t}$, $Y_{12\,t}$, $Y_{13\,t}$, $Z_{15\,t}$ and $Z_{17\,t}$ are defined as before[11]

and

$Z^*_{17\,t}$ = expected industrial employment in industrial sector in time period t

V_t = random disturbance.

Being the burner-tip price of natural gas and the prices of electricity, fuel oil and coal as substitutes for natural gas, the variables $Y_{6\,t}$, $Y_{11\,t}$, $Y_{12\,t}$, and $Y_{13\,t}$ are included in the equation (6.3.20). The variable $Z_{15\,t}$ is included to measure the effect on the consumption of natural gas in the industrial sector due to changes in temperature. The reason for including the industrial employment variable in the demand function of natural gas for the industrial sector as already mentioned in Chapter IV is mainly to show the effect of industrial output on the consumption of natural gas. Just as argued in including "expected income" in the demand function of natural gas for the residential sector, we argue here that the consumption of natural gas in the industrial sector depends on "expected industrial

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[11]See pp.56-58, Chapter II of this study.

output" in the sense that the industrial consumers expand their production activities on the basis of what they feel about the future and consequently the total of consumption of natural gas tends to be stable relative to current industrial output and a change in the current industrial output tends to affect consumption only insofar as it affects consumers' notions about the "expected industrial output." Since we are using industrial employment series to measure the industrial output series, we include "expected industrial employment" to explain the consumption of natural gas. Defining the "expected industrial employment" as

(6.3.21) ---- $Z^*_{17\,t} = \sum_{\tau=0} \psi^{\tau} Z_{17\,t-\tau} \qquad t = 1,2,\ldots$

and substituting in (6.3.20) one obtains the following function:

(6.3.22) ---- $Y^*_{5\,t} = \nu^*_1 + \nu_2 Y^*_{6\,t} + \nu_3 Y^*_{11\,t} + \nu_4 Y^*_{12\,t} + \nu_5 Y^*_{13\,t} + \nu_6 Z^*_{15\,t} + \nu_7 Z_{17\,t} + V^*_t$

where

$$Y^*_{5\,t} = Y_{5\,t} - \psi Y_{5\,t-1}$$

$$Y^*_{6\,t} = Y_{6\,t} - \psi Y_{6\,t-1}$$

$$Y^*_{11\,t} = Y_{11\,t} - \psi Y_{11\,t-1}$$

$$Y^*_{12\,t} = Y_{12\,t} - \psi Y_{12\,t-1}$$

$$Y^*_{13\,t} = Y_{13\,t} - \psi Y_{13\,t-1}$$

$$Z^*_{15\,t} = Z_{15\,t} - \psi Z_{15\,t-1}$$

$$\nu^*_1 = \nu_1(1 - \psi)$$

$$V^*_t = V_t - \psi V_{t-1}$$

By using the estimation method described in Section 6.5, the coefficients ν^*_1, ν_2, ν_3, ν_4, ν_5, ν_6, and ν_7 can be estimated.

Assuming the demand function in log-linear form, the appropriate demand function can be derived as:

$$(6.3.23) \text{ ---- } \log Y^*_{5\,t} = \mu_1 + \mu_2 \log Y^*_{6\,t} + \mu_3 \log Y^*_{11\,t} + \mu_4 \log Y^*_{12\,t} + \mu_5 \log Y^*_{13\,t} + \mu_6 \log Z^*_{15\,t} + \mu_7 \log Z_{17\,t} + S^*_t$$

where

$$\log Y^*_{5\,t} = \log Y_{5\,t} - \varphi \log Y_{5\,t-1}$$

$$\log Y^*_{6\,t} = \log Y_{6\,t} - \varphi \log Y_{6\,t-1}$$

$$\log Y^*_{11\,t} = \log Y_{11\,t} - \varphi \log Y_{11\,t-1}$$

$$\log Y^*_{12\,t} = \log Y_{12\,t} - \varphi \log Y_{12\,t-1}$$

$$\log Y^*_{13\,t} = \log Y_{13\,t} - \varphi \log Y_{13\,t-1}$$

$$\log Z^*_{15\,t} = \log Z_{15\,t} - \varphi \log Z_{15\,t-1}$$

$$\mu^*_1 = \mu_1(1 - \varphi)$$

$$S^*_t = S_t - \varphi S_{t-1}$$

Equation (6.3.23) is used to estimate the respective coefficients and the corresponding results are tabulated in Table 6.5.3 (see Section 6.5).

6.4 Statistical Properties of the Model

In this section some of the important characteristics of the model are discussed. It is already noted that the distributed lags arise in theory when any economic cause realizes its effect only after some lag in time, so that the effect is not felt immediately at a single point of time but rather is distributed over a period of time. This is what makes an economic reaction a process which describes the progressive nature of adaptations in behavior of endogenous variables. Therefore, any formulations of economic relationships that may give rise to distributed lag models is related to the following two problems:

1. estimation of the speed of adjustment, i.e., the estimation of a parameter that indicates the time period that has to elapse for the long-term reaction to take place, and
2. estimation of demand elasticities of price and income.[12]

Now by considering any one of the demand functions developed in the previous section, it will be shown that certain coefficients of the demand function reasonably indicate the measure of the speed of adjustment and the price and income elasticities of the demand for natural

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[12] L. M. Koyck, *loc. cit.*, p. 19.

gas. Consider the equation (6.3.14) in a slightly different form

$$(6.4.1) \text{----} \quad \Delta Y_{1\,t} = \beta_0(1 - \delta) + \beta_1 \log Y_{2\,t} - \beta_1\delta \log Y_{2\,t-1} + \beta_2 \log Y_{7\,t} - \delta\beta_2 \log Y_{7\,t-1} + \beta_3 \log Y_{8\,t} - \delta\beta_3 \log Y_{8\,t-1} + \beta_4 \log Z_{14\,t} + \beta_5 \log Z_{15\,t} - \delta\beta_5 \log Z_{15\,t-1} - (1 - \delta) \log Y_{1\,t-1} + U_t - \delta U_{t-1}$$

where

$$\Delta Y_t = \log Y_{1\,t} - \log Y_{1\,t-1}$$

when an equilibrium is attained, $\Delta Y_{1\,t} = 0$ and calling the new equilibrium $\overline{\log Y_{1\,t}}$ it follows from the above equation that[13]

$$(6.4.2) \text{----} \quad \overline{\log Y_{1\,t}} = \frac{1}{(1 - \delta)} \left[\beta_0(1 - \delta) + \beta_1 \log Y_{2\,t} - \beta_1\delta \log Y_{2\,t-1} + \beta_2 \log Y_{7\,t} - \delta\beta_2 \log Y_{7\,t-1} + \beta_3 \log Y_{8\,t} - \delta\beta_3 \log Y_{8\,t-1} + \beta_4 \log Z_{14\,t} + \beta_5 \log Z_{15\,t} - \delta\beta_5 \log Z_{15\,t-1} + U_t - \delta U_{t-1}\right]$$

Substituting $\overline{\log Y_{1\,t}} \cdot (1 - \delta)$ for the corresponding terms in (6.4.1) it follows that

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[13] L. M. Koyck, loc. cit., p. 23.

(6.4.3) ---- $\Delta Y_{1\,t} = (1 - \delta) \cdot \overline{\log Y_{1\,t}} - (1 - \delta) \log Y_{1\,t-1}$

i.e.,

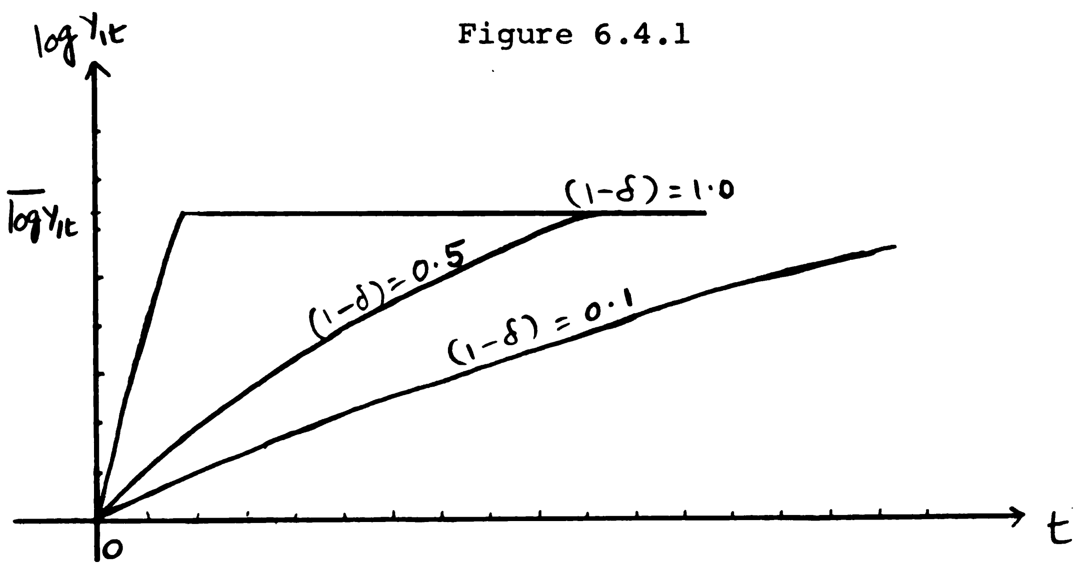
$$\log Y_{1\,t} - \log Y_{1\,t-1} = (1 - \delta) \left[\overline{\log Y_{1\,t}} - \log Y_{1\,t-1}\right]$$

This can be interpreted that the actual change in the consumption of natural gas in period t is the proportion of the change that would be necessary to attain equilibrium in that period. This would indicate reasonably about the progressive nature of the adaptive behavior in the consumption of natural gas. And so this coefficient can be taken to indicate the speed of adjustment. The maximum value that the speed of adjustment can take is 1 since $0 \leq \delta < 1$. The higher the value of $(1 - \delta)$, the higher the speed of adjustment, and conversely. At one extreme $(1 - \delta)$ can be equal to 1 in which case

$$\log Y_{1\,t} = \overline{\log Y_{1\,t}}$$

that is, the speed of adjustment is so great that the consumption reaches its equilibrium in the unit period of time for a given change in the predetermined variables.

Figure 6.4.1



At the other extreme, $(1 - \delta)$ can equal zero in which case $\overline{\log Y_{1t}}$ is reached to a changed situation only after a long period of time. This period can be the longest of the periods of adjustment. The corresponding graphs are described in Figure 6.4.1. Similar arguments can be made for the rest of the demand models developed in the previous section.

Again, if we consider the double-logarithmic demand function specified as in equation (6.3.14), the coefficient estimates of $\log Y_{2t}$ and $\log Z_{14t}$ are respectively the price and income elasticity estimates of natural gas demand. Thus with the models developed in Section 6.3, we may estimate the coefficient of the speed of adjustment and the price and income elasticities of natural gas demand.

Another main characteristic of the models developed in the previous section is that the method of estimation described in Section 6.5 provides us the consistent estimates for the coefficients of the respective variables. One possible difficulty of estimating the demand models is already noted in Section 6.3. This difficulty is avoided by assuming the values to the coefficient of the lagged variable of the consumption in $[0, 1)$. It was argued by Professor Koyck that if the random disturbances, say U_t in (6.3.14) are all correlated as:

(6.4.4) ---- $U_t = \rho U_{t-1} + V_t$

then it is possible to obtain consistent estimates of the coefficients provided ρ is equal to the coefficient of the lagged variable of the consumption, i.e., provided that $\rho = \delta$.[14] He further argued that "there is empirical evidence that (6.4.4) is not contradictory to quite a large body of economic data."[15] Thus if we assume that the random disturbance terms follow a first-order Markov scheme, then following Professors Koyck and Hildreth, it is possible to obtain consistent estimates.[16] Thus, these models are capable of explaining the adjustment mechanism in the behavior of consumption by providing us the consistent estimates of the speed of adjustment and the burner-tip price and income elasticities. And at the same time, these models should provide an alternative means of looking at the demand situation other than the demand from the simultaneous equations system model. In the following section, the results of the models are presented.

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[14]L. M. Koyck, loc. cit., pp. 32-37.

[15]Ibid., p. 34.

[16]C. Hildreth and J. Y. Lu, "Demand Relations with Autocorrelated Disturbances," Technical Bulletin 276, Agricultural Experiment Station, Michigan State University, November 1960, p. 14.

6.5 Results of the Model

In this section, the results of the demand models developed in Section 6.3 are presented. Only the log-linear demand models are estimated. In that section, we have noted the possible difficulties in estimating the parameters of the respective models. These difficulties can be seen as follows:

Consider the demand model

$$(6.5.1) \text{ ---- } Y^*_{1t} = \alpha^*_0 + \alpha_1 Y^*_{2t} + \alpha_2 Y^*_{7t} + \alpha_3 Y^*_{8t} + \alpha_5 Z_{14t} + \alpha_6 Z^*_{15t} + \varepsilon^*_t$$

$$t = 1, 2, \ldots . 18$$

where the variables are defined as in Section 6.3. This equation can be expressed in matrix notation as

$$(6.5.2) \text{ ---- } Y^* = Z^* \alpha + \varepsilon^*$$

where

$$Y^* = \begin{pmatrix} Y^*_{1\,1} \\ Y^*_{1\,2} \\ \vdots \\ Y^*_{1,18} \end{pmatrix}; \qquad \alpha = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_6 \end{pmatrix}$$

$$Z^* = \begin{pmatrix} 1-\lambda & Y^*_{2\,1} & Y^*_{7\,1} & Y^*_{8\,1} & Z_{14\,1} & Z^*_{15\,1} \\ 1-\lambda & Y^*_{2\,2} & Y^*_{7\,2} & Y^*_{8\,2} & Z_{14\,2} & Z^*_{15\,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1-\lambda & Y^*_{2,18} & Y^*_{7,18} & Y^*_{8,18} & Z_{14,18} & Z^*_{15,18} \end{pmatrix};$$

and

$$\varepsilon^* = \begin{pmatrix} \varepsilon_1^* \\ \varepsilon_2^* \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{18}^* \end{pmatrix}$$

let us assume that ε_t^* are normally and independently distributed with zero mean and same variance σ^2.[17]

Then the likelihood function of ε^* can be expressed as

$$(6.5.3) \text{ ---- } F(\varepsilon^*;\ \lambda,\ \alpha,\ \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{18/2} \text{Exp} \left\{ \frac{1}{2\sigma^2}\ \varepsilon^{*\prime}\ \varepsilon^* \right\}$$

But from (6.5.2)

$$\varepsilon^* = Y^* - Z^*\ \alpha$$

$$(6.5.4) \text{ ---- } F(Y^*;\ \lambda,\ \alpha,\ \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{18/2} \text{Exp} \left\{ \frac{1}{2\sigma^2}\ (Y^* - Z^*\ \alpha)'\ (Y^* - Z^*\ \alpha) \right\}$$

As denoted, this likelihood function depends on the parameters λ, α and σ^2. The dependence of $F(\cdot)$ on λ can be seen through the definitions of Y^* and Z. More specifically, it can be seen as follows:

---

[17]The results are presented for double-logarithmic demand functions because they came out better statistically compared to linear demand functions. But the method of estimation is described for linear form because of the obvious simplicity in the notation. The analysis is exactly the same for the double-logarithmic demand functions.

$$Y^* = \begin{pmatrix} Y^*_{1,1} \\ Y^*_{1,2} \\ \vdots \\ Y^*_{1,18} \end{pmatrix} \begin{pmatrix} Y_{1,1} - \lambda Y_{1,0} \\ Y_{1,2} - \lambda Y_{1,1} \\ \vdots \\ Y_{1,18} - \lambda Y_{1,17} \end{pmatrix} = \begin{pmatrix} Y_{1,1} \\ Y_{1,2} \\ \vdots \\ Y_{1,18} \end{pmatrix} - \lambda \begin{pmatrix} Y_{1,0} \\ Y_{1,1} \\ \vdots \\ Y_{1,17} \end{pmatrix}$$

$$= Y - \lambda \overline{Y}$$

Similarly, Z^* can be expressed

$$Z^* = \begin{pmatrix} 1 - Y_{2\,1} \cdots Z_{15\,1} \\ 1 - Y_{2\,2} \cdots Z_{15\,2} \\ \vdots \\ 1 - Y_{2,18} \cdots Z_{15,18} \end{pmatrix} - \lambda \begin{pmatrix} 1\ Y_{2\,0} \cdots,\ 0, Z_{15\,0} \\ 1\ Y_{2\,1} \cdots,\ 0, Z_{15\,1} \\ \vdots \\ 1\ Y_{2,17} \cdots,\ 0, Z_{15,18} \end{pmatrix}$$

$$= Z - \lambda \overline{Z}$$

and $\quad Y^* - Z^*\alpha = (Y - \lambda\overline{Y}) - (Z - \lambda\overline{Z})\,\alpha$

Substituting this in (6.5.4)

$$F(Y;\ \lambda,\ \alpha,\ \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{18/2} \operatorname{Exp}\left\{ -\frac{1}{2\sigma^2} \left[(Y-\lambda\overline{Y}) - (Z-\lambda\overline{Z})\,\alpha\right]' \left[(Y-\lambda\overline{Y}) - (Z-\lambda\overline{Z})\,\alpha\right] \right\}$$

Now the parameters λ, α, σ^2 are estimated by maximizing this likelihood function. To do that, we take the logarithm to the above equation and the relevant part of the logarithm of the likelihood function is given by

$$(6.5.5) \text{ ---- } L(Y;\ \lambda,\ \alpha,\ \sigma^2) = -\frac{18}{2}\log\sigma^2 - \frac{1}{2\sigma^2} \left[(Y-\lambda\overline{Y}) - (Z-\lambda\overline{Z})\,\alpha\right]' \left[(Y-\lambda\overline{Y}) - (Z-\lambda\overline{Z})\,\alpha\right] \Big\}$$

Following Dr. Hildreth, we consider the sum of squares of residuals,

$$S\ (Y;\ \lambda,\ \alpha) = \left[(Y - \lambda\overline{Y}) - (Z - \lambda\overline{Z})\ \alpha\right]' \left[(Y - \lambda\overline{Y}) - (Z - \lambda\overline{Z})\ \alpha\right]$$

and the values of λ and α that minimize S are those which maximize $L(Y;\ \lambda,\ \alpha,\ \sigma^2)$[18] So to find estimates of the parameters λ and α, we minimize S. But direct methods of finding minimum appear quite cumbersome and hence certain interative procedures are to be adopted.[19] The iterative procedure to minimize S followed in this study is quite similar to that followed by Dr. Hildreth. Various values for λ, equally spaced at intervals of length 0.1 in the admissible interval of λ, $[0, 1)$ are given and then S is minimized with respect to α. The corresponding results for the demand equations (6.3.15), (6.3.19), and (6.3.23) are now presented in Tables 6.5.1, 6.5.2, and 6.5.3.

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[18] C. Hildreth and J. Y. Lu, loc. cit., pp. 11-12.

[19] Ibid., pp. 11-14.

Table 6.5.1

COEFFICIENT ESTIMATES AND RELATED STATISTICE FOR EQUATION 6.3.15: RESIDENTIAL SECTOR

δ	$\log Y^*_{1\,t}$	β^*_0	$\log Y^*_{2\,t}$	$\log Y^*_{7\,t}$	$\log Y^*_{8\,t}$	$\log Z_{14\,t}$	$\log Z^*_{15\,t}$	SSE
0.1	$\log Y^*_{1\,t}$	117.6955	-1.1999 1.1295 0.31	0.4345 1.4937 0.77	0.1364 0.4788 0.77	2.0597 0.2507 0.00	-0.0109 0.3608 0.93	.0440
0.2	$\log Y^*_{1\,t}$	53.9516	-1.1997 1.0231 0.26	0.7800 1.5039 0.62	0.0833 0.5012 0.85	1.8558 0.2273 0.00	-0.0498 0.3387 0.86	.0366
0.3	$\log Y^*_{1\,t}$	32.0741	-1.2170 0.9071 0.20	1.1729 1.5483 0.47	0.0160 0.5262 0.93	1.6465 0.2117 0.00	-0.0943 0.3106 0.76	.0300
0.4	$\log Y^*_{1\,t}$	20.5881	-1.2544 0.7887 0.13	1.5951 1.6645 0.36	-0.0606 0.5544 0.88	1.4275 0.2071 0.00	-0.1439 0.2819 0.62	.0242
0.5	$\log Y^*_{1\,t}$	13.2260	-1.3116 0.6782 0.08	2.0014 1.9089 0.32	-0.1398 0.5861 0.80	1.1936 0.2155 0.00	-0.1943 0.2528 0.46	.0194
0.6	$\log Y^*_{1\,t}$	7.6230	-1.3875 0.5894 0.04	2.2153 2.2990 0.30	-0.2401 0.6109 0.70	0.9307 0.2309 0.00	-0.2379 0.2267 0.32	.0159
0.7	$\log Y^*_{1\,t}$	2.6642	-1.4973 0.5320 0.02	1.7265 2.5811 0.52	-0.4300 0.5758 0.48	0.6069 0.2249 0.02	-0.2540 0.2120 0.25	.0139
0.8	$\log Y^*_{1\,t}$	-0.3000	-1.6320 0.4969 0.01	0.6403 2.3922 0.78	-0.6179 0.4585 0.20	0.2624 0.1799 0.17	-0.2170 0.2094 0.32	.0133
0.9	$\log Y^*_{1\,t}$	-0.9578	-1.7342 0.4796 0.00	-0.1744 2.0283 0.89	-0.6613 0.3549 0.09	-0.0289 0.1369 0.82	0.1495 0.2113 0.50	.0138

Table 6.5.2

COEFFICIENT ESTIMATES AND RELATED STATISTICS FOR EQUATION 6.3.19: COMMERCIAL SECTOR

ω	$\log Y^*_{3t}$	η^*_1	$\log Y^*_{4t}$	$\log Y^*_{9t}$	$\log Y^*_{10t}$	$\log Z_{16t}$	$\log Z^*_{15t}$	SSE
0.1	$\log Y^*_{3t}$	-46.1555	-0.3636	-1.1845	-0.3835	1.8657	-0.2758	.0307
			0.6396	0.9697	0.3737	0.1509	0.5812	
			0.59	0.24	0.33	0.00	0.65	
0.2	$\log Y^*_{3t}$	-10.5463	-0.5243	-0.9025	-0.3123	1.6850	-0.1090	.0277
			0.6295	1.0097	0.3651	0.1356	0.5302	
			0.43	0.39	0.41	0.00	0.82	
0.3	$\log Y^*_{3t}$	-0.9881	-0.7081	-0.5715	-9.2463	1.5063	0.0392	.0246
			0.6110	1.0466	0.3500	0.1212	0.4778	
			0.27	0.60	0.50	0.00	0.90	
0.4	$\log Y^*_{3t}$	2.2134	-0.9033	-0.2007	-0.1890	1.3256	0.1667	.0215
			0.5826	1.0752	0.3290	0.1083	0.4255	
			0.14	0.83	0.58	0.00	0.70	
0.5	$\log Y^*_{3t}$	2.9997	-1.0957	0.1886	-0.1423	1.1388	0.2721	.0186
			0.5456	1.0895	0.3034	0.0973	0.3763	
			0.07	0.84	0.05	0.00	0.49	
0.6	$\log Y^*_{3t}$	2.0891	-1.2733	0.5579	-0.1086	0.9417	0.3548	.0162
			0.5053	1.0880	0.2760	0.0882	0.3340	
			0.03	0.62	0.70	0.00	0.31	
0.7	$\log Y^*_{3t}$	1.8986	-1.4278	0.8521	-0.0917	0.7313	0.4144	.01479
			0.4711	1.0790	0.2510	0.0813	0.3036	
			0.01	0.45	0.72	0.00	0.20	
0.8	$\log Y^*_{3t}$	1.0286	-1.5538	1.0214	-0.0930	0.5066	0.4516	.01483
			0.4530	1.0800	0.2333	0.0774	0.2889	
			0.00	0.37	0.70	0.00	0.14	
0.9	$\log Y^*_{3t}$	0.3479	-1.6458	1.0515	-0.1089	0.2704	0.4688	.0166
			0.4556	1.1042	0.2248	0.0770	0.2908	
			0.00	0.36	0.64	0.00	0.13	

Table 6.5.3

COEFFICIENT ESTIMATES AND RELATED STATISTICS FOR EQUATION 6.3.23: INDUSTRIAL SECTOR

ϕ	$\log Y^*_{5\,t}$	μ^*_1	$\log Y^*_{6\,t}$	$\log Y^*_{11\,t}$	$\log Y^*_{12\,t}$	$\log Y^*_{13\,t}$	$\log Z^*_{15\,t}$	$\log Z_{17\,t}$	SSE
0.1	$\log Y^*_{5\,t}$	-291.3135	2.7083	-0.3973	1.7706	-7.4396	0.3769	-2.2892	0.4491
			2.0650	8.9429	1.3877	7.1188	2.4917	2.4046	
			0.21	0.92	0.23	0.32	0.85	0.36	
0.2	$\log Y^*_{5\,t}$	-140.0903	2.5398	-1.4390	1.3871	-5.9144	0.1630	-2.3386	0.4188
			1.9045	8.0467	1.3457	6.6931	2.3147	2.1804	
			0.21	0.84	0.33	0.40	0.90	0.31	
0.3	$\log Y^*_{5\,t}$	-85.5433	2.1190	-2.4632	0.8510	-3.9954	0.0318	-2.3368	0.3793
			1.7609	7.0921	1.2679	6.2368	2.1045	1.9404	
			0.25	0.73	0.52	0.54	0.94	0.25	
0.4	$\log Y^*_{5\,t}$	-55.1268	1.4336	-3.3935	0.2323	-1.7026	-0.0240	-2.2519	0.3202
			1.5990	6.0111	1.1310	5.6323	1.8385	1.6668	
			0.39	0.59	0.82	0.76	0.94	0.20	
0.5	$\log Y^*_{5\,t}$	-34.4758	0.6016	-4.0044	-0.3252	0.6340	-0.0085	-2.0407	0.2395
			1.3732	4.7791	0.9252	4.7757	1.5065	1.3523	
			0.67	0.42	0.73	0.87	0.94	0.16	
0.6	$\log Y^*_{5\,t}$	-19.7703	-0.1417	-4.0791	-0.6722	2.4811	0.0664	-1.6935	0.1507
			1.0702	3.4773	0.6751	3.6830	1.1294	1.0139	
			0.87	0.27	0.34	0.52	0.91	0.12	
0.7	$\log Y^*_{5\,t}$	-10.0004	-0.0042	-3.6251	-0.7628	3.4861	0.1722	-1.2597	0.0757
			0.7322	2.2630	0.4313	2.5138	0.7557	0.6839	
			0.43	0.13	0.12	0.19	0.81	0.10	
0.8	$\log Y^*_{5\,t}$	-4.2488	-0.7596	-2.8557	-0.6661	3.6817	0.2758	-0.8090	0.0294
			0.4331	1.2980	0.2398	1.4949	0.4437	0.4134	
			0.12	0.05	0.02	0.03	0.55	0.08	
0.9	$\log Y^*_{5\,t}$	-1.2734	-0.6983	-1.9994	-0.4824	3.3191	0.3504	-0.3853	0.0138
			0.2790	0.8230	0.1467	0.9726	0.2864	0.2764	
			0.03	0.03	0.01	0.01	0.24	0.19	

Table 6.5.3 (continued)

ϕ	$\log Y^*_{5\,t}$	μ^*_1	$\log Y^*_{6\,t}$	$\log Y^*_{11\,t}$	$\log Y^*_{12\,t}$	$\log Y^*_{13\,t}$	$\log Z^*_{15\,t}$	$\log Z_{17\,t}$	SSE
0.91	$\log Y^*_{5\,t}$	1.1327	-0.6846	-1.9151	-0.4623	3.2624	0.3628	-0.3451	0.0138
			0.2770	0.8163	0.1450	0.9666	0.2844	0.2756	
			0.03	0.04	0.01	0.01	0.23	0.24	
0.92	$\log Y^*_{5\,t}$	0.9967	-0.6698	-1.8315	-0.4421	3.2030	0.3690	-0.3053	0.0140
			0.2776	0.8171	0.1447	0.9696	0.2851	0.2774	
			0.03	0.05	0.01	0.01	0.22	0.30	
0.93	$\log Y^*_{5\,t}$	0.8652	-0.6541	-1.7486	-0.4219	3.1409	0.3749	-0.2658	0.0145
			0.2805	0.8251	0.1456	0.9809	0.2883	0.2817	
			0.04	0.06	0.01	0.01	0.22	0.37	
0.94	$\log Y^*_{5\,t}$	0.7380	-0.6377	-1.6665	-0.4017	3.0765	0.3805	-0.2268	0.0152
			0.2856	0.8394	0.1476	0.9997	0.2937	0.2881	
			0.05	0.07	0.02	0.01	0.22	0.45	
0.95	$\log Y^*_{5\,t}$	0.6149	-0.6204	-1.5852	-0.3815	3.0099	0.3858	-0.1881	0.0162
			0.2926	0.8594	0.1506	1.0253	0.3011	0.2966	
			0.06	0.09	0.03	0.01	0.23	0.54	
0.96	$\log Y^*_{5\,t}$	0.4956	-0.6025	-1.5049	-0.3614	2,9412	0.3908	-0.1498	0.0174
			0.3012	0.8842	0.1544	1.0567	0.3102	0.3069	
			0.07	0.12	0.04	0.02	0.23	0.64	
0.97	$\log Y^*_{5\,t}$	0.3801	0.5840	-1.4254	-0.3414	2.8706	0.3955	-0.1119	0.0188
			0.3112	0.9132	0.1589	1.0931	0.3207	0.3187	
			0.09	0.15	0.06	0.02	0.24	0.73	
0.98	$\log Y^*_{5\,t}$	0.2681	-0.5650	-1.3470	-0.3215	2.7984	0.4000	-0.0143	0.0204
			0.3224	0.9455	0.1640	1.1335	0.3324	0.3319	
			0.11	0.18	0.08	0.03	0.25	0.87	
0.99	$\log Y^*_{5\,t}$	0.1594	-0.5455	-1.2695	-0.3018	2.7245	0.4042	-0.0371	0.0223
			0.3345	0.9806	0.1695	1.1773	0.3451	0.3461	
			0.13	0.22	0.10	0.04	0.27	0.92	

In presenting the results, a standard form as in Chapter V, is used. First of all, the coefficient estimates are presented, beneath them the standard errors of estimate are presented. The significance probabilities are tabulated directly under the standard errors of the estimates. It is already noted that the parameters of the concerned demand equations are estimated if, and only if, the sum of squares of residuals $S(y;\ \cdot,\ \alpha)$ is minimum. According to this criterion, the demand functions in each of the sectors are estimated as follows:

Residential Sector:

(6.5.6) ---- $\log Y^*_{1\,t} = -3.000 - 1.6320 \log Y^*_{2\,t}$

	0.4969
	0.01

$+ 0.6303 \log Y^*_{7\,t} - 0.6179 \log Y^*_{8\,t}$

2.3922	0.4585
0.78	0.20

$+ 0.2624 \log Z_{14\,t} - 0.2170 \log Z^*_{15\,t}$

0.1799	0.2094
0.17	0.25

$R^2 = 0.7520$ $\qquad$ $\delta = 0.8$

Commercial Sector:

(6.5.7) ---- $\log Y^*_{3\,t} = 1.8986 - 1.4278 \log Y^*_{4\,t}$

	0.4711
	0.01

$+ \; 0.8521 \log Y^*_{9\,t} - 0.0917 \log Y^*_{10\,t}$

1.0790 0.2510

0.45 0.72

$+ \; 0.7313 \log Z_{16\,t} + 0.4144 \log Z^*_{15\,t}$

0.0813 0.3036

0.00 0.20

$R^2 = 0.9346$ $\omega = 0.7$

Industrial Sector:

(6.5.8) ---- $\log Y^*_{5\,t} = 1.2734 - 0.6983 \log Y^*_{6\,t}$

0.2790

0.03

$- \; 1.9994 \log Y^*_{11\,t} - 0.4824 \log Y^*_{12\,t}$

0.8230 0.1467

0.03 0.01

$+ \; 3.3191 \log Y^*_{13\,t} + 0.3504 \log Z^*_{15\,t}$

0.9726 0.2864

0.01 0.24

$- \; 0.3853 \log Z_{17\,t}$

0.2764

0.19

$R^2 = 0.7354$ $\varphi = 0.9$

When the appropriate substitutions on variables, depending upon the definitions, are made, the above equations can be expressed as:

Residential Sector:[20]

$$(6.5.9) \text{ ---- } \log Y_{1\,t} = -3.0000 - 1.6320 \log Y_{2\,t} + 1.3056 \log Y_{2\,t-1} + 0.6303 \log Y_{7\,t} - 0.5042 \log Z_{7\,t} - 0.6179 \log Y_{8\,t} + 0.4943 \log Z_{10\,t} + 0.2624 \log Z_{14\,t} - 0.2170 \log Z_{15\,t} + 0.1736 \log Z_{2\,t} + 0.8 \log Z_{3\,t}$$

Commercial Sector:

$$(6.5.10) \text{ ---- } \log Y_{3\,t} = 1.8986 - 1.4278 \log Y_{4\,t} + 0.9995 \log Y_{4\,t-1} + 0.8521 \log Y_{9\,t} - 0.5965 \log Z_{8\,t} - 0.0917 \log Y_{10\,t} + 0.0642 \log Z_{11\,t} + 0.7313 \log Z_{16\,t} + 0.4144 \log Z_{15\,t} - 0.2901\, Z_{2\,t} + 0.7 \log Z_{4\,t}$$

Industrial Sector:

$$(6.5.11) \text{ ---- } \log Y_{5\,t} = -1.2734 - 0.6983 \log Y_{6\,t} + 0.6285 \log Y_{6\,t-1} - 1.9994 \log Y_{11\,t} + 1.7995 \log Z_{9\,t} - 0.4824 \log Y_{12\,t} + 0.4342 \log Z_{12\,t} + 3.3191 \log Y_{13\,t} - 2.9872 \log Z_{13\,t} + 0.3504 \log Z_{15\,t} - 0.3154 \log Z_{2\,t} + 0.9 \log Z_{5\,t} - 0.3854 \log Z_{17\,t}$$

From these equations the following conclusions may be drawn:

1. In the case of the residential sector, only the

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[20]For the definitions of the variables, see pp.56-58, Chapter II,

burner-tip price variable $Y_{2\,t}$ is statistically significant at both the 5 per cent and 1 per cent levels. Since the demand functions are assumed as double logarithmic the coefficient estimate of log $Y_{2\,t}$ is the price elasticity of natural gas demand. It is estimated as -1.6320 which indicates a 1.6320 per cent decrease in natural gas consumption for 1 per cent increase in the burner-tip price. Similarly, income elasticity, the coefficient of log $Z_{14\,t}$, is estimated as 0.2624. The cross-elasticity of natural gas demand with respect to the price of electricity is 0.6303. The cross-elasticity with respect to the price of fuel oil is -0.6179 and has an unexpected sign as to the theoretical reasoning. Both are not statistically significant. The negative sign for the coefficient estimate of fuel oil may be due to the reasons that are already mentioned in Chapter V, Section 4. The coefficient estimate of log $Z_{15\,t}$ also has an unexpected sign.

2. One of the important statistics we hoped to estimate by developing the distributed lag models in this chapter is the coefficient of speed of adjustment. It was pointed out in Section 4 of this chapter that the speed of adjustment can be measured by one minus the coefficient estimate of the lagged consumption of natural gas.[21] In the case of the residential

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[21]See page 161 of this study.

sector, the estimate of the speed of adjustment is $1 - 0.8 = 0.2$. This indicates that the adaptive behavior in the consumption of natural gas for any changes in the explanatory variables is rather slow.

3. In the case of the commercial sector, the burner-tip price variable is highly significant. The magnitude of the coefficient estimate of log $Y_{4\,t}$ is -1.4278 which indicates that for every 1 per cent increase in the burner-tip price of natural gas there will be a 1.4278 per cent decrease in the consumption. All other coefficient estimates of the variables, except that of log $Y_{10\,t}$, are in the right direction as to the theoretical reasoning. The coefficients of log $Y_{9\,t}$ and log $Y_{10\,t}$ are estimated as 0.8521 and -0.0917 respectively and are not statistically significant at 5 per cent level.

4. The coefficient estimate of the number of customers of natural gas in the commercial sector, $Z_{16\,t}$, is 0.7313 and has an expected sign. This estimate is statistically significant at both the 5 per cent and 1 per cent levels. Originally the demand model in the commercial sector was developed on the basis of "expected disposable personal income" but the minimum of $S(Y; \lambda, \alpha)$ is not attained in this case. So the demand model is developed on the basis of "expected number of customers" of natural gas in the commercial sector. The minimum of $S(Y; \lambda, \alpha)$ is

attained and the results of the estimated demand model are also reasonable. The coefficient of $Z_{15\ t}$ is estimated as 0.4144 and has an expected sign.

5. The coefficient of the speed of adjustment in the commercial sector is estimated as 0.3(=1.0 - 0.7). This indicates that the adjustment mechanism in the consumption of natural gas to any changed situation of the explanatory variables is slow. But it is slightly faster than in the residential sector because the estimated coefficient of speed of adjustment in the commercial sector is greater than that of the residential sector.

6. According to the sign of the estimated coefficients of the variables log $Y_{11\ t}$, log $Y_{12\ t}$, and log $Z_{17\ t}$, and according to the nature of the statistical significance of the coefficient estimates, the results of the estimated demand function of natural gas in the industrial sector are rather unsatisfactory. Of course, the coefficient estimates of log $Y_{6\ t}$ and log $Y_{13\ t}$ have expected signs and their magnitudes are -0.6983 and 3.3191. One can notice that $S(Y;\ \varphi,\ \alpha)$ has reached the minimum for both $\varphi = 0.9$ and $\varphi = 0.91$. But the results in equation (6.5.8) (and hence of (6.5.11)) are presented for the value $\varphi = .9$ because R^2, the coefficient of determination, is higher for $\varphi = 0.9$. The magnitude of the coefficient estimate of log $Y_{6\ t}$ indicates that the price elasticity in the

industrial sector is inelastic; a result not in accord with economic reasoning. Finally, the coefficient of speed of adjustment is estimated as 0.1(=1.0 - 0.9) which is very low compared to those of the residential and commercial sectors. This reflects a slow change in the process of adaptive behavior of consumption of natural gas for any changes in any one of the explanatory variables. This probably would be the case if all the industries in Michigan must use natural gas as an input because of the "quality" requirements in the final product and hence cannot change to other fuel energies even when the "other" fuel energies are economically cheaper to use. If this is the case, we may also justify the inelastic demand in the industrial sector. But this seems not to be the case in the industrial sector in Michigan in which steel and automotive manufacturing are dominant. Both of these industries have used fuel oil and are equipped in many plants to shift--particularly those industrial consumers who have interruptible contracts. The low elasticity, together with the extremely low value of the speed of adjustment, tends to cast doubt on the results of the distributed lag model in the industrial sector.

CHAPTER VII

SUMMARY AND CONCLUSIONS

This study focuses on mainly two aspects of demand analysis. The first one is the formulation of demand models to explain the behavior of consumption of natural gas, on the basis of the endogenous mechanism involved in the consumption of natural gas. The second aspect is the formulation of demand models to explain the adaptive behavior of the consumption of natural gas because the changes in the consumption of natural gas, to any changed situation, are realized only after a lapse of time. In order to study the behavior of consumption from the point of view of endogenous mechanism, the total demand of natural gas is divided with respect to three sectors, namely, residential, commercial and industrial. And then a structural econometric model is developed in order to fulfill this purpose. A schematic summary of the structural econometric model is presented in Table 4.3.1.

Several alternative estimating procedures are employed, namely, ordinary least-squares (OLS), two stage least-squares (2SLS), unbiased Nagar K-class (UNK), limited information single-equation (LISE), and three stage least-squares (3SLS). The estimated structural demand equations in the form of coefficient estimates and related statistics are presented in Tables 5.2.1, 5.2.2 and 5.2.3. In general, 3SLS coefficient estimates appeared theoretically plausible and some of the coefficient estimates were found statistically significant. The levels of significance to test the coefficient estimates are not uniform from estimate to estimate. Typically, the OLS, 2SLS, UNK methods yielded coefficient estimates where were comparable with respect to the magnitude and statistical significance probabilities. But the results provided by LISE are erratic. No rigorous attempt is made to ascertain the advantages of a given estimator, but the empirical results for 3SLS were generally plausible and different compared to the estimates provided by the other procedures. Theoretically, the 3SLS estimator has preferred asymptotic properties, compared to the alternative estimation procedures employed in this study. Due to this and due to the nature of the empirical results as described above, the 3SLS coefficient estimates are interpreted in greater detail.

The price elasticities in both the residential and commercial sectors are found less than unity; the price elasticity in the commercial sector is slightly greater than that in the residential sector. The price elasticity of demand in the industrial sector is found elastic. In the residential and industrial sectors, the coefficient estimate of the lagged index of price of electricity, i.e., the indirect cross-elasticity, is statistically significant. This indicates that electricity is providing a keen competition to natural gas. Similarly, the indirect income elasticity in the residential sector is greater than unity. This higher value may be due to the fact that the lagged disposable personal income may be explaining the combined effects of both the disposable personal income and population. In the commercial sector, the indirect income elasticity is less than unity. The lagged consumption of natural gas significantly explained the behavior of consumption of natural gas in all the three sectors. This is also an important observation made from the models and shows that the models developed in some of the earlier studies are relatively simple, even though they are based on a national level.

We cannot, strictly, compare these results with the results obtained in the studies mentioned in Chapter II, mainly because in this study the analysis is based on the State of Michigan, whereas in the earlier studies the analysis is based on a national level.

In order to explain the adaptive behavior of the consumption of natural gas, a distributed lag demand model, one for each sector is developed. In the case of the residential sector, it is formulated on the basis of "expected" disposable personal income hypothesis. This is because of the fact that consumers wish to even out their consumption to a certain extent over their "lifetimes" or at least over the foreseeable future and the total consumption during any period of time is determined by the consumers' expected disposable personal income, not by their current income. Consequently, the total consumption expenditures on natural gas tend to be stable relative to current disposable incomes and a change in disposable income tends to affect natural gas consumption only insofar as it affects consumers' notions of their "expected" disposable personal incomes. Since the "expected" disposable personal income hypothesis did not give a solution in the commercial sector (Section 6.5, page 164), the distributed lag demand model is developed with respect to the hypothesis of the "expected number of customers of natural gas in the commercial sector.

Similarly, the corresponding distributed lag demand model in the industrial sector is formulated on the basis of "expected" industrial employment hypothesis. All of the three models are estimated by OLS method.

The price elasticities in both the residential and commercial sectors indicate that the natural gas demand is elastic in both sectors. In both sectors, the estimates of price elasticities are statistically significant. The coefficient estimate of the number of customers of natural gas in the commercial sector is found statistically significant both at the five and one per cent levels. The results for the industrial sector are found rather unsatisfactory.

Another important estimate provided by the distributed lag demand models is the estimate of speed of adjustment. It is found as 0.2 in the residential sector and 0.3 in the commercial sector; and, although unsatisfactory, 0.1 in the industrial sector. These estimates indicate that the process of adjustment in the consumption of natural gas to any changed situation is slow in all three sectors, even though it is somewhat faster in the commercial sector compared to the residential and industrial sectors.

Thus the two approaches to the demand analysis of natural gas provided two different estimates of price and income elasticities. Of course, one should note that these two approaches are based with two different sets of assumptions. Since the demand models are estimated for the state (State of Michigan) data and since these models have provided plausible results, at least for the residential and commercial sectors in Michigan, it is hoped that they can be applied successfully to the data of other states.

Also, the results provide valuable information for the Federal Power Commission in fixing the wellhead ceiling price for initial contracts for natural gas because, given a mathematical relationship between the wellhead ceiling price and the burner-tip price, the Commission may evaluate the impact of changing the wellhead ceiling price on the ultimate consumption of natural gas in each sector.

APPENDIX A

Appendix A.0

In this appendix, all the economic variables that are used in the development of the demand models are defined. They are as follows:

Y_1 = average consumption of natural gas (millions of mcf) in the residential sector

Y_2 = index of burner-tip price of natural gas (¢/mcf) in the residential sector

Y_3 = average consumption of natural gas (millions of mcf) in the commercial sector

Y_4 = index of burner-tip price of natural gas (¢/mcf) in the commercial sector

Y_5 = average consumption of natural gas (millions of mcf) in the industrial sector

Y_6 = index of burner-tip price of natural gas (¢/mcf) in the industrial sector

Y_7 = index of price of electricity (¢/kwh) in the residential sector

Y_8 = index of price of fuel oil (¢/gal) in the residential sector

Y_9 = index of price of electricity (¢/kwh) in the commercial sector

Y_{10} = index of price of fuel oil (¢/gal) in the commercial sector

Y_{11} = index of price of electricity (¢/kwh) in the industrial sector

Y_{12} = index of price of fuel oil (¢/gal) in the industrial sector

Y_{13} = index of price of coal ($/ton) in the industrial sector

Z_0 = constant term

Z_1 = lagged (by one period) disposable personal income in Michigan

Z_2 = lagged (by one period) number of degree days in Michigan

Z_3 = lagged (by one period) consumption of natural gas in residential sector

Z_4 = lagged (by one period) consumption of natural gas in commercial sector

Z_5 = lagged (by one period) consumption of natural gas in industrial sector

Z_6 = lagged (by one period) industrial employment (in thousands) in Michigan

Z_7 = lagged (by one period) index of price of electricity in residential sector

Z_8 = lagged (by one period) index of price of electricity in commercial sector

Z_9 = lagged (by one period) index of price of electricity in industrial sector

Z_{11} = lagged (by one period) index or price of fuel oil in commercial sector

Z_{12} = lagged (by one period) index of price of fuel oil in industrial sector

Z_{13} = lagged (by one period) index of price of coal in industrial sector

Z_{14} = disposable personal income in Michigan

Z_{15} = number of degree days in Michigan

Z_{16} = number of customers of natural gas in commercial sector

Z_{17} = industrial employment in Michigan

Z_{18} = lagged (by one period) index of burner-tip price of natural gas in residential sector ($Y_{2\ t-1}$)

Z_{19} = lagged (by one period) index of burner-tip price of natural gas in commercial sector ($Y_{4\ t-1}$)

Z_{20} = lagged (by one period) index of burner-tip price of natural gas in industrial sector ($Y_{6\ t-1}$)

In Appendix A.1, the data concerning these variables are presented.

Appendix A.1

TABLE A.1.1 Degree Days, Michigan, 1946-1964

(1) Year	(2) Detroit WBAP City	(3) Lansing WB Airport	(4) Average No. of Degree Days
1946	6405	6987	6696
1947	6595	7133	6964
1948	5832	6407	6120
1949	6333	6937	6635
1950	6383	6984	6684
1951	6277	7009	6643
1952	5861	6512	6187
1953	5801	6462	6132
1954	5835	6160	5998
1955	6560	6937	6749
1956	6078	6316	6197
1957	6233	6564	6399
1958	6466	7344	6905
1959	6373	7200	6787
1960	6306	7070	6688
1961	6202	7072	6637
1962	6202	7072	6637
1963	5699	6394	6047
1964	6331	7277	6804

Source: 1945-1950, Climate of Michigan by Stations, Michigan Weather Service and Weather Bureau, U. S. Department of Commerce.
1950-1964, Climatological Data, U. S. Department of Commerce.
Data in Col. (4) is obtained as a simple average cf Cols. (2) and (3).

TABLE A.1.2 PRICE AND AVERAGE NUMBER OF CUSTOMERS OF ELECTRICITY BY TYPE OF SERVICE, AREA IN MICHIGAN SERVED BY DETROIT EDISON COMPANY, 1946-1964

Year	Residential Service*		Commercial Service		Industrial Service	
	Average No. of Customers	Price (¢/kwh)	Average No. of Customers	Price (¢/kwh)	Average No. of Customers	Price (¢/kwh
1946	735,025	2.80	86,541	2.50	531	1.23
1947	757,903	2.80	91.208	2.51	494	1.19
1948	787,032	2.81	94.544	2.55	485	1.24
1949	817,835	3.09	97,077	2.87	491	1.27
1950	851,470	3.08	98.895	2.87	513	1.21
1951	892,601	3.02	100.269	2.83	552	1.20
1952	924,729	2.97	101.476	2.81	577	1.21
1953	956,851	2.91	103,127	2.78	608	1.18
1954	990.819	2.87	105,401	2.77	630	1.21
1955	1,030,198	2.82	107,903	2.69	654	1.14
1956	1,004,778	2.79	108.911	2.71	690	1.17
1957	1,100,248	2.75	110,729	2.68	707	1.19
1958	1,124,718	2.72	112,388	2.68	707	1.24
1959	1,143,957	2.69	113,158	2.65	718	1.20
1960	1,162,644	2.67	113,865	2.64	743	1.18
1961	1,173,805	2.65	113,951	2.64	760	1.19
1962	1,184,236	2.64	114,163	2.61	773	1.15
1963	1,197,875	2.62	114,621	2.58	814	1.12
1964	1,220,279	2.60	115,867	2.56	874	1.10

*Residential Service is classified as Domestic Service

Source: The Detroit Edison Company Statistics, 1964, pp. 74-83.

TABLE A.1.3 PRICE AND AVERAGE NUMBER OF CUSTOMERS OF ELECTRICITY BY TYPE OF SERVICE, AREA IN MICHIGAN SERVED BY CONSUMER POWER COMPANY, 1946-1964

Year	Residential Service*		Commercial Service		Industrial Service	
	Average No. of Customers	Price (¢/kwh)	Average No. of Customers	Price (¢/kwh)	Average No. of Customers	Price (¢/kwh)
1946	469,128	2.59	64,610	2.99	731	1.20
1947	492,300	2.51	68,696	2.68	814	1.19
1948	517,769	2.45	71,840	2.71	906	1.26
1949	538,810	2.40	74,237	2.81	933	1.26
1950	565,214	2.56	78,730	2.84	937	1.23
1951	602,509	2.55	83,780	2.82	1,030	1.26
1952	619,627	2.49	85,106	2.78	1,095	1.28
1953	638,303	2.44	87,373	2.74	1,167	1.25
1954	660,303	2.38	88,195	2.70	1,251	1.25
1955	683,890	2.34	89,843	2.64	1,277	1.17
1956	707,308	2.29	91,624	2.61	1,419	1.24
1957	725,774	2.25	93,201	2.59	1,492	1.27
1958	740,513	2.23	89,366**	2.39	6,353**	1.36
1959	755,513	2.29	90,030	2.37	6,322	1.31
1960	770,217	2.38	89,966	2.39	6,300	1.29
1961	782,518	2.36	91.713	2.35	6,248	1.31
1962	794,618	2.35	91,868	2.31	6,223	1.25
1963	805,305	2.34	92,584	2.28	6,300	1.23
1964	821,042	2.33	93,920	2.24	6,433	1.21

* Residential Service is classified as Domestic Service.
** Customer Reclassification

Source: Obtained directly from the Consumer Power Company. For each type of service, weighted average price of electricity, weights are given with respect to the average number of customers, is computed and is used, respectively in Tables A.1.4, A.1.5 and A.1.6.

TABLE A.1.4 NATURAL GAS CONSUMPTION AND DEMAND FACTORS, RESIDENTIAL SECTOR, MICHIGAN, 1946-1947

(1) Year	(2) Cons. of Natural Gas (mcf)	(3) Burner-tip price (¢/mcf)	(4) Price of electricity (¢/kwh)	(5) Price of fuel oil (¢/gal)
1946	37.74	89.5	2.72	6.06
1947	44.07	85.6	2.69	8.60
1948	43.81	86.9	2.67	11.26
1949	47.58	92.9	2.82	10.40
1950	75.71	85.4	2.87	10.24
1951	99.31	83.8	2.83	10.65
1952	103.04	85.0	2.78	10.61
1953	103.88	97.1	2.72	10.57
1954	118.48	95.2	2.67	11.82
1955	128.17	93.1	2.63	11.32
1956	155.14	94.1	2.59	11.62
1957	170.00	92.8	2.55	12.08
1958	173.21	94.4	2.53	11.20
1959	192.96	95.1	2.53	10.82
1960	201.98	95.4	2.55	10.34
1961	218.08	99.4	2.53	10.01
1962	234.48	99.5	2.52	10.08
1963	242.89	99.4	2.51	10.11
1964	251.97	99.3	2.49	9.24

Source: Cols. (2)and (3) Minerals Yearbook, Bureau of Mines, U. S. Department of Interior, Washington, D.C., editions for 1948-1964. Col. (4) compiled from Tables A.1.2 and A.1.3 (See Source of Table A.1.3). Col. (5) Refinery and terminal prices for No. 2 fuel oil, Detroit Area, Platt's Oil Price Handbook and Almanac, editions for 1955-1963; 1964, Petroleum Facts and Figures, 1965 edition.

TABLE A.1.5 NATURAL GAS CONSUMPTION AND DEMAND FACTORS, COMMERCIAL SECTOR, MICHIGAN, 1946-1964

(1) Year	(2) Cons. of Natural Gas (mcf)	(3) Burner-tip price (¢/mcf)	(4) Price of electricity (¢/kwh)	(5) Price of fuel oil (¢/gal)
1946	6.16	74.3	2.70	5.23
1947	7.04	72.0	2.58	7.76
1948	6.88	72.5	2.62	10.27
1949	7.32	83.4	2.84	6.98
1950	10.75	77.2	2.86	7.74
1951	13.27	75.9	2.83	8.65
1952	13.87	77.5	2.80	7.99
1953	14.01	86.7	2.76	7.73
1954	15.34	80.2	2.74	7.84
1955	16.52	80.5	2.67	8.81
1956	20.31	86.1	2.67	9.60
1957	26.62	79.5	2.64	10.02
1958	32.02	79.6	2.55	8.86
1959	38.46	80.4	2.53	9.07
1960	43.00	79.8	2.53	9.10
1961	42.84	89.8	2.51	9.10
1962	64.50	88.1	2.48	8.95
1963	68.68	86.4	2.45	8.60
1964	74.37	86.0	2.42	8.60

Source: Cols.(2) and (3), see Source of Table A.1.4.
Col. (4) see Source of Table A.1.4.
Col. (5), Refinery and Terminal Prices, No. 5 Fuel Oil, Detroit Area: Platt's Oil Price Handbook and Almanac. See Source of Table A.1.4.

TABLE A.1.6 NATURAL GAS, CONSUMPTION AND DEMAND FACTORS, INDUSTRIAL SECTOR* MICHIGAN, 1946-1964

(1) Year	(2) Cons. of Natural Gas (mcf)	(3) Burner-tip price (¢/mcf)	(4) Price of electricity (¢/kwh)	(5) Price of fuel oil (¢/gal)	(6) Price of coal ($/ton)
1946	23.11	43.9	1.20	4.76	5.99
1947	27.62	38.5	1.19	7.84	7.80
1948	23.26	48.8	1.26	9.47	7.96
1949	27.47	55.0	1.26	6.74	7.61
1950	39.20	50.3	1.23	7.49	7.83
1951	39.96	53.2	1.25	8.40	7.71
1952	45.51	51.0	1.27	7.74	7.77
1953	59.04	54.1	1.24	7.11	7.82
1954	53.23	54.8	1.25	7.09	7.43
1955	60.30	53.3	1.17	8.06	7.17
1956	65.80	54.8	1.23	8.61	7.63
1957	73.55	53.6	1.26	9.02	7.94
1958	90.25	54.9	1.35	8.19	7.98
1959	100.17	61.3	1.30	8.57	7.88
1960	122.02	55.2	1.28	8.60	7.71
1961	144.21	55.4	1.30	8.60	7.65
1962	141.32	53.1	1.24	8.45	7.36
1963	149.91	53.5	1.22	8.10	7.33
1964	168.21	53.1	1.20	8.10	7.28

*This sector excludes the consumption of natural gas by steam electric plants.

Source: Cols. (2) and (3), see Source of Table A.1.4. Col. (5), Refinery and Terminal Prices, No. 6 Fuel Oil, Detroit Area: Platt's Oil Price Handbook and Almanac. See Source of Table A.1.4. Col. (6), "Steam-Electric Plant Factors, An Annual Study by the Department of Economics and Transportation," National Coal Association, Washington, D.C.

TABLE A.1.7 NATURAL GAS DEMAND FACTORS, RESIDENTIAL SECTOR, MICHIGAN, 1947-1964

(1) Year	(2) Y_1	(3) Y_2	(4) Y_7	(5) Y_8	(6) Z_{14}	(7) Z_{15}
1947	44.07	96.76	98.53	85.23	7,873	6,864
1948	43.81	98.23	97.80	111.60	8,584	6,120
1949	47.58	105.01	103.30	103.07	8,663	6,635
1950	75.71	96.53	105.13	101.49	9,776	6,684
1951	99.31	94.72	103.66	105.55	10,595	6,643
1952	103.04	96.08	101.83	105.15	11,074	6,187
1953	103.88	109.75	99.63	104.76	12,540	6,132
1954	118.48	107.61	97.80	117.15	12,639	5,998
1955	128.17	105.23	96.34	112.19	13,873	6,749
1956	155.14	106.36	94.87	115.16	14,843	6,197
1957	170.00	104.89	93.41	119.72	14,781	6,399
1958	173.21	106.70	92.67	111.00	14,648	6,905
1959	192.96	107.49	92.67	107.23	15,383	6,787
1960	201.98	107.83	93.41	102.48	15,852	6,688
1961	218.08	112.34	92.67	99.21	15,873	6,637
1962	234.48	112.47	92.31	99.90	16,741	7,179
1963	242.89	112.35	91.94	100.20	17,939	6,047
1964	251.97	112.24	91.21	91.58	21,557	6,804

Source: Cols. (2)-(5), (7), derived from Tables A.1.4 and A.1.1. The base period 1947-1949 is taken for computing the index.
Col. (6), Michigan Statistical Abstracts, Sixth Edition, 1966

TABLE A.1.8 NATURAL GAS DEMAND FACTORS, RESIDENTIAL SECTOR, MICHIGAN, 1947-1964

(1) Year	(2) Z_1	(3) Z_2	(4) Z_3	(5) Z_7	(6) Z_{10}	(7) Z_{18}
1947	6,938	6,696	37.74	99.63	60.06	101.16
1948	7,873	6,864	44.07	98.53	82.23	96.76
1949	8,584	6,120	43.81	97.80	111.60	98.23
1950	8,663	6,635	47.58	103.30	103.07	105.01
1951	9,776	6,684	75.71	105.13	101.49	96.53
1952	10,595	6,643	99.31	103.66	105.55	94.72
1953	11,074	6,187	103.02	101.83	105.15	96.08
1954	12,540	6,132	103.88	99.63	104.76	109.75
1955	12,639	5,998	118.48	97.80	117.15	107.61
1956	13,873	6,749	128.17	96.34	112.19	105.23
1957	14,843	6,197	155.14	94.87	115.16	106.36
1958	14,781	6,399	170.00	93.41	119.72	104.89
1959	14,648	6,905	173.21	92.67	111.00	106.70
1960	15,383	6,787	192.96	92.67	107.23	107.49
1961	15,852	6,688	201.98	93.41	102.48	107.83
1962	15,873	6,637	218.08	92.67	99.21	112.83
1963	16,741	7,179	234.48	92.31	99.90	112.47
1964	17,939	6,047	242.89	91.94	100.20	112.35

Source: Derived from Tables A.1.7 and A.1.4.

TABLE A.1.9 NATURAL GAS DEMAND FACTORS, COMMERCIAL SECTOR, MICHIGAN, 1947-1964

(1) Year	(2) Y_3	(3) Y_4	(4) Y_9	(5) Y_{10}	(6) Z_4	(7) Z_8	(8) Z_{11}	(9) Z_{16}	(10) Z_{19}
1947	7.05	94.77	96.27	93.05	6.16	100.75	62.71	433	97.80
1948	6.88	95.43	97.76	123.14	7.05	96.27	93.05	448	94.77
1949	7.32	109.78	105.97	83.69	6.88	97.76	123.14	477	95.43
1950	10.75	101.62	106.72	92.81	7.32	105.97	83.69	564	109.78
1951	13.27	99.91	105.60	103.72	10.75	106.72	92.81	601	101.62
1952	13.87	102.01	104.48	95.80	13.27	105.60	103.72	624	99.91
1953	14.01	114.12	102.99	92.69	13.87	104.48	95.80	637	102.01
1954	15.34	113.47	102.24	94.00	14.01	102.99	92.69	648	114.12
1955	16.52	113.86	99.63	105.64	15.34	102.24	94.00	710	113.47
1956	20.31	113.33	99.63	115.11	16.52	99.63	105.64	790	113.86
1957	26.62	104.65	98.51	120.14	20.31	99.63	115.11	850	113.33
1958	32.02	104.78	95.15	106.24	26.62	98.51	120.14	1,008	104.65
1959	38.46	105.83	94.40	108.75	32.02	95.15	106.24	1,069	104.78
1960	43.00	105.04	94.40	109.11	38.46	94.40	108.75	1,135	105.83
1961	42.84	118.20	93.66	109.11	43.00	94.40	109.11	1,173	105.04
1962	64.50	115.97	92.54	107.31	42.84	93.66	109.11	1,188	118.20
1963	68.68	113.73	91.42	103.12	64.50	92.54	107.31	1,226	115.97
1964	74.37	113.20	90.30	103.12	68.68	91.42	103.12	1,286	113.73

Source: Cols. (2)-(8), (10), Derived from Table A.1.5. See Source of Table A.1.7. Col. (9), See Source of Table A.1.4.

TABLE A.1.10 NATURAL GAS DEMAND FACTORS, INDUSTRIAL SECTOR, MICHIGAN, 1947-1964

(1) Year	(2) Y_5	(3) Y_6	(4) Y_{11}	(5) Y_{12}	(6) Y_{13}	(7) Z_{17}
1947	27.62	81.17	95.97	97.76	100.13	1,251.6
1948	23.26	102.89	101.61	118.08	102.18	1,292.4
1949	27.47	115.96	101.61	84.04	97.69	1,204.7
1950	39.20	106.05	99.19	93.39	100.51	1,304.9
1951	39.96	112.17	100.81	104.74	98.97	1,373.3
1952	45.51	107.53	102.42	96.51	99.74	1,361.8
1953	59.04	114.06	104.00	86.65	100.39	1,496.2
1954	53.23	115.54	100.81	88.40	95.38	1,337.8
1955	60.30	112.38	94.35	100.50	92.04	1,446.6
1956	65.80	115.54	99.19	107.36	97.75	1,375.7
1957	73.55	113.01	101.61	112.47	101.93	1,344.6
1958	90.25	115.75	108.87	102.12	102.44	1,132.3
1959	100.17	129.24	104.84	106.86	101.16	1,199.5
1960	122.02	116.38	103.23	107.23	98.97	1,220.7
1961	144.21	116.30	104.80	107.23	98.20	1,112.4
1962	141.32	111.95	100.00	105.36	94.48	1,174.3
1963	149.91	112.80	98.39	101.00	94.09	1,215.9
1964	168.21	111.95	96.77	101.00	93.45	1,264.8

Source: Cols. (5) and (6), derived from Table A.1.6.
See Source of Table A.1.7.
Col.(7), Michigan Statistical Abstracts, Sixth Edition, 1966.

TABLE A.1.11 NATURAL GAS DEMAND FACTORS, INDUSTRIAL SECTOR, MICHIGAN, 1947-1964

(1) Year	(2) Z_5	(3) Z_6	(4) Z_9	(5) Z_{12}	(6) Z_{13}	(7) Z_{20}
1947	23.11	1,132.9	96.77	59.35	100.00	92.56
1948	27.62	1,251.6	95.97	97.76	100.13	81.17
1949	23.26	1,292.4	101.61	118.08	102.18	102.89
1950	27.47	1,204.7	101.61	84.04	97.69	115.96
1951	39.20	1,304.9	99.19	93.39	100.51	106.05
1952	39.96	1,373.3	100.81	104.74	98.97	112.17
1953	45.51	1,361.8	102.42	96.51	99.74	107.53
1954	59.04	1,496.2	100.00	88.65	100.39	114.06
1955	53.23	1,337.8	100.81	88.40	95.38	115.54
1956	60.30	1,446.6	94.35	100.50	92.04	112.38
1957	65.80	1,375.7	99.19	107.36	97.75	115.54
1958	73.55	1,344.6	101.61	112.47	101.93	113.01
1959	90.25	1,132.3	108.87	102.12	102.44	115.75
1960	100.17	1,199.5	104.84	106.86	101.16	129.24
1961	122.02	1,220.7	103.23	107.23	98.97	116.38
1962	144.21	1,112.4	104.84	107.23	98.20	116.30
1963	141.32	1,174.3	100.00	105.36	94.48	111.95
1964	149.91	1,215.9	98.39	101.00	94.09	112.80

Source: Col. (3), Michigan Statistical Abstracts, Sixth Edition, 1966.
Cols. (2), (4)-(7), derived from Table A.1.9.

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