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Capital Budgeting Formulas For Determining Land Values With Programs For Hand-Held Calculators

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CAPITAL BUDGETING FORMULAS FOR DETERMINING LAND VALUES WITH PROGRAMS FOR HAND-HELD CALCULATORS

By

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ABSTRACT

CAPITAL BUDGETING FORMULAS FOR DETERMINING LAND VALUES WITH PROGRAMS FOR HAND-HELD CALCULATORS

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In this thesis capital budgeting models of increasing complexity, are built to explain the maximum bid price for land. The study begins with the Basic Capital Budgeting (BCB) model,which equates land value with net return to land divided by the opportunity cost of capital. However, this model which ignores such factors as inflation, land productivity changes and risk is shown to be inadequate. Large discrepancies were found between the actual and the predicted values of land using the BCB model.

Inflation rate, productivity and tax rates are then included in the model. The resulting predictions of land values using the revised model are superior to those obtained using BCB model.

The Maximum Bid Price Model, developed by Lee and Rask, which incorporates financial terms such as down payment, interest rates on mortgage loans and the length of the amortization period along with inflation and productivity changes and tax rates, was used to estimate

land values. In some respects this more complicated model failed to explain land prices as well as the simplier model. It was useful, however, in examining the relationship between financial terms and land values.

The thesis then discussed three methods for adjusting the Capital Budgeting Model for risk. They included adjusting the discount rate, reducing income to its certainty equivalent using multiplicative coefficient of risk and finally subtracting the cost of risk. The thesis concluded that the best means of adjusting the capital budgeting formula for risk was to subtract the cost of risk.

Programs for the hand-held programmable calculator were developed for the Lee-Rask model, and to measure the cost of risk assuming the decision maker can supply information used to construct a triangular distribution function.

In general, it is concluded that inclusion of terms of financing such as down payment required on loan, interest rate on loan, the length of the loan repayment period, the rate of expected inflation on land prices have an important influence on the price of farmland.

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CHAPTER I

INTRODUCTION

A REVIEW OF CHANGES IN LAND VALUES

The price of farm products rose from an index of 100 in 1914 to a peak of 225 in early 1920. They then started to decline to an index of 124 in 1921. High farm product prices during World War I, along with savings accumulated by farmers in the form of Liberty Bonds and other liquid assets, started a land boom in 1916 which lasted through 1920 and raised land prices by 56 percent during that period. After the four year boom, a decline began that continued until 1933.

With the advent of World War II, agricultural commodity prices again increased dramatically. Then, like the period from 1916 to 1920, they were followed by increases in land values. During the years 1946, 1947 and 1948 land values increased by 13, 12 and 8 percent respectively (U.S. Bureau of Census, U.S.D.A. a).

Two inflationary influences were largely responsible for land price increases. The first was high levels of domestic employment and income. The second was an abnormally large foreign demand for United States agricultural commodities, because of shortages brought about by World War II and devastating droughts in the Southern Hemisphere in 1945. These pressures on food production along with large amounts of liquid funds accumulated in the hands of farmers and non-farmers were translated into increased demand for farmland.

It is interesting to note the similarity of conditions that pushed up land values from 1916 to 1920 and in the 1940's. In both cases, increased farm commodity prices and liquid funds in the hands of prospective purchasers led to increased demand for land.

After the second major boom in land prices, which peaked in 1949, income began to decline and land buyers became more cautious. Land values continued to rise, but at slower rates through the 1950's, except for a brief period from 1953 to 1954.

During the early 1950's there were two important developments which greatly influenced the farmland market during the 1960's and 1970's. A revised price support program was instituted which assured farmers minimum prices for their products and reduced the risk of loss due to low commodity prices. The second development was farm enlargements. Modern agricultural technology created situations in which existing farm operations were deficient in land, in relation to other farm inputs such as labor and capital. To take advantage of economies of scale created by the new technology, farm sizes had to increase. As a result, most land purchases were for expansion purposes which, with price supports, led to continued strong demand for farmland and higher land prices. Beginning in 1972, land prices increased more rapidly than in earlier periods. Because; there was a large increase in foreign demand for american food products; deficit spending by the federal government led to inflation reflected in rising land values.

Figure 1 illustrates the historical pattern.

Factors Affecting the Price of Farmland

In this section several important variables which affect the supply and demand of farmland are discussed. First, the factors affecting supply will be considered.

The price at which farmland owners offer land for sale

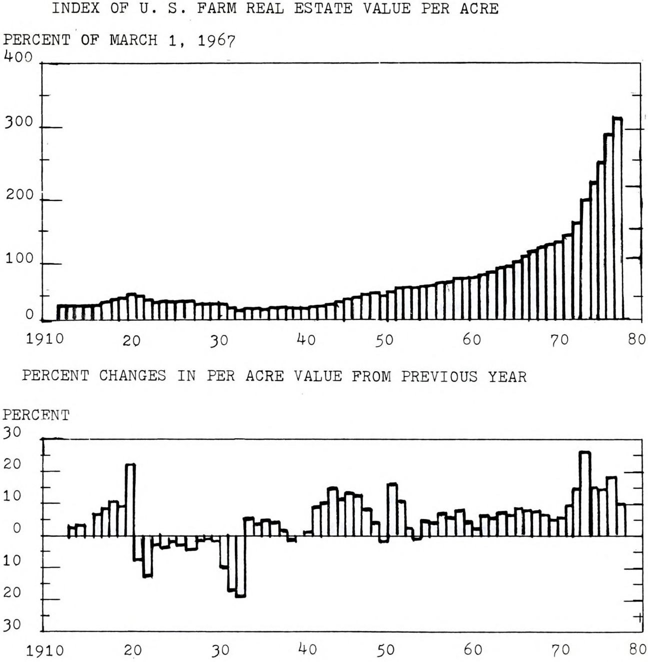

Figure 1. Historical index of farm real estate value per acre and percent change in per acre value from previous year.

Source: Farm Real Estate Market Development E. R. S., U. S. Department of Agriculture, 1978.

is influenced by such seller characteristics as his occupation, age and income. Colyer found that sellers who are farmers, laborers, or retirees at the time of the land sale tended to receive more per acre than those in other occupations.

Colyer also found a positive relationship between seller's income and the price per acre of land, possibly because sellers with greater income have fewer economic pressures and can bargain for a higher price per acre than those who sell because they need the money.

On the demand side, the expansion buyer has been responsible for an increasingly large percentage of farm sales. In 1977, these buyers purchased about 63 percent of all farms transferred (U.S.D.A. b). In most cases, an expansion buyer is receiving above average net rent on acreage he already owns. Therefore, he can often afford to pay a higher than average price for additional farmland. When technology creates a situation in which a farm operation becomes land deficient in relation to other inputs such as labor and machinery, the farmer needs to increase the land input. Thus, it may be economically feasible for him, if necessary, to pay a higher than average price for land to increase the total land input of the farm and spread his fixed costs over a larger land area, while increasing only certain variable inputs, such as fertilizer and pesticides.

Government programs also influence the demand for land and hence land prices. Commodity price support programs insure farm owners a certain minimum price for their crop (at least in the short run), even if production increases. Therefore, it becomes profitable to increase production and the net income to land. Acreage allotments, which restrict output in order to increase total revenue, may increase land prices for those fortunate enough to have an allotment. Expectations about the continuance of government programs may also affect land prices. With acreage allotments it is essential that the owner be relatively certain the program will be continued, if he is to pay the higher prices for farmland with nontransferable allotments.

Other government programs which influence land prices are those which attempt to increase agricultural productivity. Such programs include agricultural conservation programs that provide cost-sharing arrangements for approved practices and improvements, technical assistance programs provided by soil bank conservation services, research and development by agricultural experiment stations and research by land grant colleges and agriculture extension services. All of these programs, which make land more productive, tend to increase its market price.

Tax laws may contribute to higher land prices. Since tax rates on capital gains are lower than those on income, land may be purchased for tax purposes. Capital appreciation of land over time will increase the net worth position of the owner. If he sells the land at some future date for more than he paid, the difference between the original price and his selling price is called capital gain. People in high personal income tax brackets can purchase farmland and manage it in such a way as to show net losses and reduce their total tax liability. Part of this loss can arise from investment in items to improve the value of farmland such as drainage, leveling and capital improvements. Thus the investor can add to net capital worth of the farmland investment, which will be taxed when he later sells it at the capital gains rate. At the same time, he can show a net loss from farming operations, allowing him a lower personal income tax liability. This type of tax advantage may make farmland a very desirable form of investment for some investors (Rossmiller).

Availability and cost of credit also influence land values. As credit becomes easier to obtain, the number of potential buyers for a tract of land increases, and as the demand for land increases, so does its price. The cost of the credit adds to the cost of the land purchase; therefore, if he pays less for the cost of credit, he may be willing to pay more for the farmland.

The Constant Nature of Agricultural Land Supply

The price of land at a given point in time can be thought of as an intersection between demand and supply curves. Historical data indicate that the amount of land used for agriculture purposes has changed very slowly over time. Moreover, year to year changes in the quantity of land for sale is small; only 2 to 3 percent of farms turn over each year. Thus, it is reasonable to conclude that the supply of agricultural land is reasonably inelastic at a given point in time and changes only slowly over time.

However, it should be noted that, because land is a heterogeneous input, the supply of land can be flexible due to improvement of low grade land to high quality. Government actions also add flexibility and affect the supply of land. For example, government can take land out of the market, add land to the market and influence owners to sell or hold land.

Although factors determining the amount of land offered for sale are not well-understood, primarily land sales occur because an owner retires or dies and his land becomes available. Thus, with an almost constant supply of land, it may be assumed that price changes are caused by shifts in demand for land.

Statement of the Problem

The question of how to determine the price of land has received considerable attention (Boxley, Harris and Nehring, Lee and Rask). Much of the interest in assessing the value of land has been for taxation and mortgage purposes. The old method for calculating the price of land, called the capitalization formula, suggested that land prices could be determined by dividing the average net after-tax income per acre by the discount rate or return from the next best alternative. For example, if after-tax net return from land is $50.00 per acre and the rate of return on the best alternative is 10 percent, dividing $50.00 by .10 would price the land at $500.00 an acre.

The formula, unfortunately, has become hopelessly outdated, yet it continues in use for lack of accurate and easy to apply alternatives. It is outdated because it assumes constant income from land and that land will be valued the same after some planning period as it was at the beginning. Historical data indicates that neither land values nor income to land have remained the same.

During the last ten years land values have increased at an average rate of 9.5 percent while income to land has increased at an average annual rate of 7 percent. Therefore, if the capitalization formula were to be used to estimate land values using today's earnings, they would likely fall below current land prices.

Purpose of the Study

It is the intent of this study to find more accurate capitalization formulas to determine land values which include changes in income attributed to land, capital gains, variability of income, decision maker's attitudes toward risk, available financing, and tax rates. In addition to determining improved capitalization formulas, this study also adapts the formulas for use with hand-held calculators. That is, all of the models to be developed will be programmed for use with the Texas Instruments (TI-59) hand-held computer.

Objectives

More specifically, the objectives of this study are as follows:

1. To develop an improved capital budgeting formula to include such important variables as inflation rates, time preference rates, productivity changes of farmland, cost of

capital, marginal tax rates of annual net income to land, capital gains income tax rates, planning period for investment, variations in annual net returns to land, and the risk attitudes of decision makers.

2. To develop simple programs that can be used with the new hand-held programmable calculators to determine land values.
3. To develop cash flow statements associated with land purchase models.
4. To study the effects of financial variables (downpayment, interest rate on mortgage loans, and the loan amortization period).
5. To develop a program to estimate the cost of risk for owning farmland, assuming the return to farmland is a random variable, that can be described by a triangular probability distribution.
6. Compare predicted land values using alternative capitalized formulas with actual land values to determine the usefulness of each model.

Overview

In order to attain the above objectives, in an orderly manner, this study is organized as follows:

In Chapter II basic capital budgeting formulas are introduced, their limiting assumptions are examined and the land values derived by these formulas are compared with actual prices of land in Michigan.

Chapter III will be used to introduce inflation and land productivity changes into the capital budgeting model. Each formula derived will be used to estimate land values and these estimated values will then be compared with actual values for Michigan to determine the predictive ability of the formulas.

The effect of financial arrangements on land values will be discussed in Chapter IV. The procedure already outlined above will be used to compare the accuracy of estimated land values with actual values.

Analysis of the impact of risk on land prices will be undertaken in Chapter V. Chapter VI will then explain how the cost of risk can be estimated and incorporated into the capital budgeting model. The predictive values of the various formulas will be discussed in Chapter VII along with this study's conclusions.

CHAPTER II

CAPITAL BUDGETING AS A METHOD FOR DETERMINING LAND PRICES

An investment in real estate requires capital outlays when the investment is made, while return on the investment will be received over time. Financially the investment produces a set of changes in the investor's monetary position at the time the investment is made, and in subsequent years.

The notion of discounting is common to most methods of investment appraisal. Discounting methods take into consideration the time value of receipts and costs of an investment. For example, one dollar today is worth more to an investor than one dollar a year from now. However, its value to a particular individual depends on the objectives of the individual concerned and the opportunities that are open to him. If an investor invests one dollar today at the interest rate of 10 percent per annum, then the original dollar becomes $1.10 a year later. Thus the guaranteed receipt of one dollar in a year, has a current value of $1/1.1 = \$.91$. By the same reasoning, one dollar received two years from now has a present value of $1/(1.1)^2 = \$.83$. In general, if r is the discount rate, in n years from now $1 is equal to $\$1/(1+r)^n$.

The technique of discounting is used to evaluate investments which generate a stream of income over time. That is, for an investment to be made, the value paid for such investment should be less than or equal to the present value of the stream of income generated from the investment in the future.

The purpose of this chapter is to review formulas and methods for valuing assets that generate returns over time.

Valuing Assets Which Produce Returns Over Time

An investment project, such as the purchase of farmland, generates a stream of income over future time periods. The value of these future returns may be converted to current dollar values through discounting, so that the future income stream can be compared to the cost in current dollars of the investment. Thus the first step is to estimate the income from the investment in each period and convert this to current dollar equivalents. Let R_i denote net income in the i-th time period. Three approaches can be used to estimate the income attributed to an investment in land. These three approaches are the landlord method, the residual method, and the pro-rata share method.

The landlord method involves an estimation of the income stream (R) to farmland based on the net rental payments received by the landlord for the use of his farmland. Where land is rented and the rental fee is known, as well as the costs associated with land ownership (such as taxes), the net income stream to the landlord is also the return on land.

For the residual approach, consider Table 1 which illustrates net income for a typical corn grain farm which yields an average of 85 bushels an acre. The income from the land is the income earned from the sale of the corn grain or its equivalent value if the grain is used on the farm. From this gross income, we subtract all the operating expenses associated with growing the corn, including seed, fertilizer, fuel for machines, labor, interest charged on short-term debt, herbicides and insecticides and taxes. The difference between the gross income and farm operating expenses equals net income--the income expected from the land purchase (Huff).

TABLE 1: Enterprise Budget for One Acre of Medium-Yield Corn Grain

GROSS INCOME (85 bu. X $2.25)		$191.25
EXPENSES:		
Labor (6.1 hrs. X $5.50)	$ 33.44	
Repairs and Maintenance	9.80	
Seeds	11.33	
Fertilizer	38.25	
Insecticides and Herbicides	12.40	
Fuel	6.00	
Utilities	2.30	
Harvesting, Trucking	6.20	
Corn Drying	14.00	
Other Expenses (including interest on operating debt)	$ 7.53	
	$141.25	
NET INCOME (Gross Income - Expenses)		$ 50.00

Source: Lindon Robison, "The Effect of Financial Arrangements on the Maximum Bid Price for Land". Paper presented at the Department of Agricultural Economics, Michigan State University, Oct. 24, 1979

The pro-rata share method involves estimating the marginal value productivity for farmland, labor and capital. This method requires that the total output be apportioned among the inputs based on their productivity assuming the capital as the total expense.

The difficulty, of course, is in determining the marginal product of land. Euler's theorem directs that, for homogeneous production functions of degree one, output can be apportioned among the inputs by multiplying the marginal product of each by the amounts of the input used in the production process.

After having determined the income to land, we next find its present value. To do so, we discount by the opportunity cost of capital.

Consider an investment (an asset purchase) that generates returns R in each of n future time periods. Futhermore, assume that the opportunity cost of capital is r and denote the asset's beginning and terminal value as V. The present value of the investment can be written:

(II.1) $V = R/(1+r) + \ldots + R/(1+r)^n + V/(1+r)^n.$

That is, the assets' beginning value is equal to the discounted present value of the income it produces, plus its discounted salvage value. The value of the discounted income alone can be written as:

(II.2) $S = R/(1+r) + \ldots + R/(1+r)^n$

and after multiplying by (1+r) and sutracting from the results , it can be written:

(II.3) $S = R(1-(1+r)^{-n})/r.$

Next, substituting into (II.1) for the discounted income gives:

(II.4) $V = R(1-(1+r)^{-n})/r + V/(1+r)^{n}.$

Then solving for V, by subtracting $V/(1+r)^{n}$ from both sides of (II.4), we can write:

(II.5) $V = R/r.$

That is, V depends only on the discount rate r and net income R.

Consider now the effect of taxes on land values. Taxes affect both income and the cost of capital. Tax rates appear both in the numerator and in the denominator of the capital budgeting formula. If we let t be the tax rate on income, the land price V can be determined as the following:

(II.6) $V = R(1-t)/(1+r(1-t)) + \ldots. + R(1-t)/(1+r(1-t))^{n} + V/(1+r(1-t))^{n}.$

Using the same method as was used to obtain (II.3), the discounted income accounting for taxes can be written as:

(II.7) $S = R\ [1-(1+r(1-t))^{-n}\ ]/r$

Next, substituting into (II.6) for the discounted income and solving for V by subtracting $V/(1+r(1-t))^{n}$ from both sides of the equation, we can write:

(II.8) $V = R/r.$

This model which is the same one obtained in (II.5), we refer to as the Basic Capital Budgeting (BCB) model.

This formula provides an accurate estimation only if the following conditions are met:

1) The investment is expected to produce the same annual net rent over time; that is, no inflation.

2) The capitalization rate used to discount future net rent remains constant.

This model is examined using the net income to land and the interest rate on mortgage loans (capitalization rate for Michigan). Table 2 compares actual land prices with those estimated using equation (II.8). Column 4 represents the estimated land values determined by R/r. This estimate will be compared with the actual price of land shown in the 5th column of the table. The differences between columns 4 and 5 given in column 6 indicates the accuracy of the BCB model.

The average error E was estimated by summing the absolute differences between the predicted price of land in the t-th year denoted P(t) and the actual value of land in the t-th year denoted A(t), all divided by the number of observations. That is:

(II.9) $E = \sum_{t=1}^{n} \left| P(t) - A(t) \right| / n.$

Where n is the number of observations.

The average error for this model was 91.50. This model apparently underestimates the price of land in periods with high inflation rates. However, it performs better in years with low inflation rates. That is, with inflation, one assumption underlying the BCB model no longer holds and as a result its accuracy is reduced.

Table 2 shows that neither net income to farmland, nor the discount rate, which is approximated by the intrest rate charged by the Federal Land Bank, are constant. Thus violating both assumptions underlying the BCB model.

To demonstrate that returns to farmland (R/r) is not a constant r, the actual rate of return of the opportunity cost of capital was calculated. Table 3 summarizes the results. The estimated capitalization rate indicates that the assumption of a fixed discount rate does not hold.

In the following chapter a more realistic model will be derived to estimate land values which includes inflation and productivity changes of farmland. This model will also be tested for its ability to predict land values.

Table 2: Estimated Land Prices Obtained Using the BCB model

Year	Fed. Land Bank Int. Rate 1/ %	Net Income to Land 2/	Estimated Price of Land $/ac.	Actual Price of Land $/ac. 3/	Estimated Minus Actual Price
1960	6.0	12.62	210.33	197.49	12.84
1961	5.6	12.46	222.50	207.73	14.77
1962	5.6	12.95	231.25	213.97	17.97
1963	5.6	13.04	232.85	209.85	23.00
1964	5.5	13.27	241.27	220.36	20.91
1965	5.5	13.88	252.36	230.36	22.00
1966	5.8	15.01	258.62	257.04	1.58
1967	6.0	17.16	286.00	273.59	12.41
1968	6.7	18.04	268.66	330.10	-61.44
1969	7.7	18.48	240.00	315.35	-75.35
1970	8.7	15.58	179.08	290.07	-110.99
1971	7.9	19.90	251.90	319.36	-67.46
1972	7.4	19.61	265.00	344.84	-79.84
1973	7.5	20.17	268.93	416.62	-147.69
1974	8.1	25.89	319.63	486.00	-166.37
1975	8.7	28.03	322.18	552.00	-229.82
1976	8.7	30.72	353.10	617.00	-263.90
1977	8.4	36.81	438.21	757.00	-318.79

Average Error, E = 91.50

1. Source: Lindon Robison and David J. Leatham, "Intrest Rates Charged and Amounts Loaned by Major Farm Real Estate Lenders", Agricultural Economics Research, vol. 30, no. 2, April, 1978, Table 5.

2. Source: Ralph Hepp, Michigan Agricultural Data, Department of Agricultural Economics, M.S.U.

3. Source: Michigan Department of Agriculture, Michigan Agricultural Statistics, 1978.

TABLE 3: Estimated Opportunity Cost of Capital Obtained by Dividing Net Return to Land by Actual Land Values (R/V) 1/

Year	Opportunity Cost %	Year	Opportunity Cost %
1960	6.40	1969	5.90
1961	6.00	1970	5.40
1962	6.00	1971	6.30
1963	6.20	1972	5.70
1964	6.00	1973	4.80
1965	6.00	1974	5.30
1966	5.80	1975	5.00
1967	6.30	1976	5.00
1968	5.45	1977	4.80

The Average Opportunity Cost for the 18 year Period is 5.67%

1/ Net returns to land and actual land values are those reported in Table 2.

CHAPTER III

THE EFFECT OF INFLATION AND PRODUCTIVITY CHANGES ON LAND PRICES

The BCB model, explained in Chapter II, shows that the price of land can be estimated by dividing net returns to land R by the discount rate r. However, if r is the opportunity cost of capital, then it has been changing. To understand why it is changing, consider it being composed of two parts: the time preference rate denoted r*, and the inflation rate denoted i.

The rate of return required to induce savers to postpone consumption, assuming constant prices, is the time preference rate which is sometimes referred to as the real rate of return. This is usually assumed to be constant over time. For the BCB model to be valid, the opportunity cost must equal the constant time preference rate. The second part is the inflation rate, a rate of return that savers must receive in addition to the time preference rate to compensate them for losses of purchasing power due to increased prices. Inflation means that the purchasing power of the dollar declines. Dollars received one year from now are not as valuable as current dollar because prices will have increased and dollars at the end of a year cannot buy as much as the dollar expended today. Thus savers demand higher returns, and if inflation changes, so will the oppertunity cost on investments, i.e. r^* will change.

Inflation may be introduced into the BCB model by assuming that expected net returns to land and the discount rate increase by the same inflation rate. If the net returns to land increase by the rate of inflation then the net income to land in the first period becomes $R(1+i)$ where i is the inflation rate, and in the n^{-th} period equals $R(1+i)^n$. Meanwhile, the discount rate which also is increased by the inflation rate equals $(1+r^*)$ $(1+i)$ in the first period and in the n^{-th} period equals $(1+r^*)^n$ $(1+i)^n$. Thus, the value of a capital asset V with inflation rate i can be written as:

(III 1.) $$V = R(1+i)/(1+i)(1+r^*) + \ldots + R(1+i)^n/(1+i)^n(1+r^*)^n + V(1+i)n/(1+r^*)^n(1+i)^n$$

Note that the inflationary impact on income and on the discount rate cancel so that we can write:

(III 2.) $$V = R/(1+r^*) + \ldots + R/(1+r^*)^n + V_t/(1+r^*)^n$$

This, however, is the BCB model except in place of the opportunity cost of capital r, we have r^* the time preference rate. Hence, the value of an asset under inflation is the current year's income divided by the time preference rate r^*:

(III 3.) $$V = R/r^*$$

It is obvious that inflation affects the purchase price of land only as it affects current income. The formula R/r^* is used to estimate land value V given actual values for net income to land R and the rate of pure time preference r*. Moreover, returns R divided by V should equal r*. Thus, Table 3 was, in essence an estimate of r*, equal to an average of 5.67 percent and if the marginal tax rate were .25, r*(1-t) equaled 4.25.

In Table 4 actual land prices are compared with those estimated using equation III.3 with r* = 5.67. Column 2 reports the estimated land values determined by the formula R/r*. This estimated value compares with the actual value of land for the period 1960-1977 reported in column 3.

Table 4: Estimated Land Prices Obtained Using the Model with r* Equal to 5.67%.

Year	Estimated Value of Land $/ac.	Actual Value of Land $/ac. 1/	Estimated Minus Actual Price $/ac.
1960	222.96	197.49	25.47
1961	220.14	207.73	12.41
1962	228.80	213.97	14.83
1963	230.39	209.85	20.54
1964	234.03	220.91	13.12
1965	244.80	230.36	14.44
1966	264.73	257.04	7.69
1967	302.64	273.59	29.05
1968	318.17	330.10	-11.93
1969	325.93	315.35	10.58
1970	274.78	290.07	-15.29
1971	350.97	319.36	31.61
1972	345.86	344.84	1.02
1973	355.73	416.62	-60.89
1974	456.61	482.00	-29.39
1975	494.37	552.00	-57.63
1976	541.80	617.00	-73.20
1977	650.00	757.00	-107.00

Average Error, E = 29.90

1. Source: From Table 2.

We now introduce productivity changes into the BCB model. The BCB model ignores the effects of productivity changes in land which alter income streams and land values. If land becomes more productive by the use of technologies such as new seed varieties, insecticides and fertilizers, they can be expected to increase the net returns to land. Therefore the productivity of farmland will affect land prices. An increase in the productivity of land will also explain why farmers accept a lower rate of return on farm real estate investments, than on nonagricultural investments.

The question now is how does changes in productivity of land affect the purchase price of land. Changes in the productivity of land can be positive, zero, or negative. If we assume positive productivity changes in land over time then the net income to land will increase by the rate of the productivity increase. That is, the net income to land in the first period will be $R(1+g)$, where g is the productivity change, and in the n-th period net income will equal $R(1+g)^n$. Assuming the price of land also increases at rate g, we can write the value of land V_g with productivity changes as:

(III. 4) $V_g = R(1+g)/(1+r^*) + \ldots + R(1+g)^n/(1+r^*)^n + V_g\,(1+g)^n/(1+r^*)^n$

and solving for V_g as it was done before gives:

(III. 5) $V_g = R(1+g)/(r^*-g)$

For purposes of comparison, the above model was used to estimate the land values given the actual data for net income to land, length of planning period, and productivity changes of farm land. Productivity changes were obtained from U. S. D. A. compiled statistics (see U. S. D. A., c, 1978) Table 5 compares actual values of farmland with those estimated by equation (III. 5) given an estimate of g equal to the average productivity changes during the previous three years. Given g, net returns R and land value V, a new average for r* was obtained equal to 7.5 percent. Then using (III. 5) estimated values of V_g obtained with an average error of 105.00.

Table 5: Estimated Price of Land by Equation (III.5).

Year	Productivity Changes % 1/ 2/	Estimated Price of Land $/ac.	Actual Price of Land 3/ $/ac.	Estimated Price Minus Actual Price $
1960	3.90	364.00	197.49	166.51
1961	5.07	538.75	207.73	331.02
1962	2.30	255.00	213.97	41.03
1963	3.80	365.82	209.85	155.97
1964	2.50	272.00	220.91	51.10
1965	1.10	219.00	230.36	-11.36
1966	1.77	266.60	257.04	9.56
1967	0.07	231.00	273.59	-42.59
1968	1.73	318.00	330.10	-12.10
1969	1.67	322.00	315.35	6.65
1970	3.00	356.60	290.07	66.53
1971	1.40	330.80	319.36	11.44
1972	2.27	383.00	344.84	38.16
1973	2.83	444.00	416.62	27.38
1974	3.80	726.00	486.00	240.00
1975	-2.10	285.85	552.00	-266.15
1976	-0.43	386.00	617.00	-231.00
1977	1.06	577.50	757.00	-179.50

Average Error, E = 105.00

1. Source: U. S. D. A., "Changes in Farm Production and Efficiency 1977", Economic Statistics, and Cooperative Service. Statistical Bulletine no. 612.

2. Productivity changes for each period as reported in Column 2 of this table is estimated as equal to the average productivity changes for the previous three years.

3. Source: From Table 2.

The average error of Table 5 is greater than the average error without productivity gains suggests that either (1) the model incorrectly incorporates gains into the model or (2) that the gains series reported by U. S. D. A. does not match the actual gain incorporated by decision makers into their land pricing model.

The later explanation seems more reasonable, because the productivity changes series were fluctuating. Looking at the net income to land series, it is obvious that the growth in income was positive throughout the series with the exception of 1970. To demonstrate that the productivity change used in the study may not explain the growth rate in income, a new productivity change were estimated given r* of 5.67, from Table 3. This was estimated by solving the equation for g in (III. 5):

(III. 6) $g = (r^{*}.V-R)/(R+V)$

Table 6 summarizes the results of equation (III. 6).

Table 6: Estimated Productivity Changes Using Equation $(r^*.V-R) / (R+V)$.

Year	Actual Land Values \$/ac. 1/	Actual Net Income To Land 2/	Estimated Productivity Changes
1960	197.49	12.62	-.007
1961	207.73	12.46	-.003
1962	213.97	12.95	-.0036
1963	209.85	13.04	-.005
1964	220.91	13.27	-.003
1965	230.36	13.88	-.003
1966	257.04	15.01	-.001
1967	273.59	17.16	-.006
1968	330.10	18.04	.002
1969	315.35	18.48	-.002
1970	290.07	15.58	.003
1971	319.36	19.90	-.005
1972	344.84	19.61	.000
1973	416.62	20.17	.008
1974	486.00	25.89	.003
1975	552.00	28.03	.006
1976	617.00	30.72	.007
1977	757.00	36.81	.008

1/ From Table 2

2/ From Table 2

The results of Table 6 indicate that productivity changes are small, given a rate of time preference equal to 5.67 percent. It is obvious from Table 6 that the productivity changes estimated are significantly different from those used in estimating land vlaues reported in Table 5. In fact the results reported in Table 6 indicate that productivity changes are unimportant in estimating land values.

Chapter Summary

In this chapter, two factors beleived to be affecting land prices were discussed. Those two factors were inflation rate and productivity changes. Starting out with the BCB model, changes were made to allow for inflation and productivity changes. The final model (III. 5) showed that changes in productivity affect land prices but did not improve our ability to explain changes in land values.

CHAPTER IV

THE LEE-RASK MAXIMUM BID PRICE MODEL

The Importance of the Maximum Bid Price for Land:

Determining the maximum bid price is an important task for a real estate buyer and his lender. It is important because the opportunity to purchase a particular parcel of land occurs infrequently and the number of farms being sold has declined in recent years. Thus, if the buyer's bid price is not close to the seller's asking price, another buyer may acquire the real estate. On the other hand, a bid price above what the real estate can repay may mean financial difficulties for the buyer that may finally result in liquidation of his entire assets.

The Effect of Financial Arrangments on the Maximum Bid Price:

Land purchases are usually financed with borrowed money. A downpayment from 10 to 50 percent of the purchase price is usually required; with the remaining amount paid over a number of years. Financial arrangements such as interest rates, downpayments, and the length of the loan amortization period, should be considered in evaluating the agricultural land values, along with the marginal tax rate of income, a factor which is often overlooked. Expected costs and returns will be reduced by the amount of taxes. Historical data also indicate that land prices continue to increase in the U.S. and that a capital gain will be realized when the land is sold at the end of the investment period.

Nevertheless, even when all of the above mentioned factors are included in determining the maximum bid price for land, a purchase should only be made if the asking price is not above the maximum bid price and the buyer can meet cash flow requirements. These cash flow requirements are determined by constructing a cash flow statement.

Cash Flow Statements:

The major cash inflow associated with an investment in agricultural land consists of annual net return to the land (usually estimated from the rental rates on comparable land in the area) and returns from selling land at the end of the investment period. The cash outflow associated with an investment consists of the required downpayment, principal and interest payments on the mortgage loan, and income taxes.

Using the cash flow statement, the present value of the projected after-tax net cash inflow from added land can be compared to the initial cost outflow for purchase. In this case, since the cash outflow for purchase is spread over several years because of financing terms, the present value of this cash flow must be measured and compared to the present value of cash inflow arising from the land purchase.

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In purchasing land by a loan there are at least two alternative loan amortization methods.

(1) Constant payment, where the total payment in each period remains constant over the term of the loan, with varing proportions allocated to interest and principal as payments are made.

(2) Constant payment on the principal, in which an equal payment in each period is made on the principal plus a varying amount of interest. In both cases, interest is calculated on the remaining balance.

The pattern of annual payments on a loan of $757 amortized over 25 years will be compared using the constant payment and the constant payment on principal methods of repayment. To complete the example assume the loan is repaid over 25 years, the borrower is in the 25 percent tax bracket, income in the first year is $36.81 which increases at a rate of 5 percent and that capital gains accure at 6 percent per year.

Notice that the total payments (principal and interest) during the early years of the loan are lower for the constant payment method. Beginning with the 9th year, however, payments become lower in the constant payment on the principal method. Figure 2 shows the curvilinear and linear relationships. The comparative results of the constant payment method and the constant payment on principal method are shown in Tables 7 and 8, respectively.

Although the length of time required to amortize the loan is the same under both methods, the outstanding loan balance for any particular year will be greater for the constant payment method, as illustrated in Figure 2. Thus, the total interest paid during the life of the loan will be greater for the constant payment method. This method may seem less attractive to cost-conscious farmers who are operating under severe capital constraints. However, the higher payment obligation during the early years using the constant payment on the principal method reduces the amount of cash flow available for other uses. It especially reduces the cash available for servicing non real estate loand, and thus tends to reduce the availability of such loans. a reduction in non realestate credit may severely limit growth for expansion minded farmers.

The total interest will be less for shorter term loans. However, the cash required each year to repay principal and interest on the loan will be greater, thus adversely affecting non realestate credit.

Table 7, column 9 shows the present value of net cash flow for each period. The total present value of the net cash flow over the amortization period is $29.66. Table 8, column 9 shows the present value of net cash flow for each period during the constant payment on the principal payment method. The total present value of net cash flow over the amortization period is $9.00. In both cases, the present value of net cash flows are greater than zero, thus the land purchase is profitable.

Table 7: Cash flow Statement for a $757 loan repaid in equal payments after a downpayment of 25 percent.

Year	Amortization Payment $/Year	Loan Balance $	Interest $	Net Income to Land	Taxable Income	Tax	Net Cash Flow	Present Value of Net Cash Flow
0	189.25	567.75	-	-	-	-	-189.25	-189.25
1	62.54	561.97	56.775	38.65	-18.12	-4.53	- 19.36	- 18.00
2	62.54	555.63	56.20	40.58	-15.61	-3.90	- 18.06	- 15.63
3	62.54	548.64	55.56	42.61	-12.95	-3.24	- 16.70	- 13.44
4	62.54	540.96	54.86	44.75	-10.12	-2.53	- 15.27	- 11.43
5	62.54	532.50	54.09	46.98	- 7.12	-1.78	- 13.79	- 9.60
6	62.54	523.21	53.25	49.33	- 3.92	-0.98	- 12.24	- 7.93
7	62.54	512.98	52.32	51.79	- 0.53	-0.13	- 10.62	- 6.40
8	62.54	501.73	51.30	54.39	3.09	0.77	- 8.93	- 5.00
9	62.54	489.36	50.17	57.10	6.93	1.73	- 7.18	- 3.74
10	62.54	475.74	48.93	59.96	11.02	2.76	- 5.34	- 2.59
11	62.54	460.77	47.57	62.96	15.38	3.84	- 3.44	- 1.55
12	62.54	444.30	46.08	66.10	20.03	5.07	- 1.44	- 0.60
13	62.54	426.18	44.43	69.41	24.98	6.24	0.62	0.24
14	62.54	406.25	42.62	72.88	30.26	7.57	2.77	1.00
15	62.54	384.33	40.63	76.53	35.90	8.97	5.00	1.68
16	62.54	360.21	38.43	80.35	41.92	10.48	7.32	2.30
17	62.54	333.69	36.02	84.37	48.35	12.09	9.73	2.86
18	62.54	304.51	33.37	88.59	55.22	13.80	12.23	3.33
19	62.54	272.41	30.45	93.01	62.56	15.64	14.83	3.73
20	62.54	237.10	27.24	97.67	70.43	17.61	17.51	4.12
21	62.54	198.30	23.71	102.55	78.84	19.71	20.33	4.44
22	62.54	155.55	19.82	107.68	87.85	21.96	23.17	4.72
23	62.54	108.55	15.55	113.06	97.50	24.38	26.18	4.96
24	62.54	62.54	10.85	118.72	107.86	26.97	29.60	5.15
25	62.54	-	5.68	3373.50	2611.00	341.50	2969.50	274.00

Table 8: Cash Flow Statement for a $757 Loan Repaid with Equal Payments on the Principal after a 25 Percent Downpayment.

Year	Amortization Payment $	Loan Balance $	Amount of Interest $	Net Income	Taxable Income	Tax	Net Cash Flow	Present Value of Net Cash Flow
0	189.25	567.75	-	-	-	-	-189.25	-189.25
1	79.49	545.04	56.78	36.81	-19.97	-4.99	-37.68	-35.06
2	77.21	522.29	54.50	38.65	-15.85	-3.96	-34.60	-29.94
3	74.94	499.59	52.23	40.58	-11.65	-2.91	-31.45	-25.30
4	72.67	476.89	49.96	42.61	- 7.35	-1.84	-28.22	-21.13
5	70.40	454.25	47.69	44.72	- 2.95	-0.74	-24.92	-17.35
6	68.13	431.55	45.42	46.98	1.56	0.38	-22.54	-13.96
7	65.86	408.85	43.15	49.33	6.18	1.54	-18.07	-10.89
8	63.59	386.07	40.88	51.79	10.92	2.73	-14.52	- 8.15
9	61.32	363.36	38.61	54.39	15.78	3.94	-10.87	- 5.67
10	59.05	340.65	36.34	57.10	20.77	5.19	- 7.13	- 3.46
11	56.78	317.94	34.07	59.96	25.89	6.47	- 3.29	- 1.48
12	54.50	295.23	31.79	62.96	31.16	7.79	0.66	0.28
13	52.23	272.53	29.52	66.10	36.58	9.14	4.72	1.84
14	49.96	249.81	27.25	69.11	42.16	10.54	8.91	3.24
15	47.69	227.10	24.98	72.88	47.90	11.97	13.21	4.46
16	45.42	204.39	22.71	76.52	53.82	13.45	17.65	5.54
17	43.15	181.58	20.44	80.35	59.91	14.98	22.22	6.50
18	40.88	158.97	18.17	84.37	66.20	16.55	26.94	7.33
19	38.61	136.26	15.90	88.59	72.69	18.17	31.81	8.05
20	36.34	113.55	13.63	93.02	79.39	19.84	36.83	8.67
21	34.06	90.84	11.35	97.67	86.31	21.58	42.02	9.20
22	31.79	68.13	9.08	102.55	93.47	23.37	47.39	9.65
23	29.52	45.42	6.81	107.68	100.87	25.22	52.93	10.03
24	27.25	22.71	4.54	113.06	108.52	27.13	58.68	10.34
25	24.98	-	2.27	3367.62	2608.44	340.60	3002.00	277.00

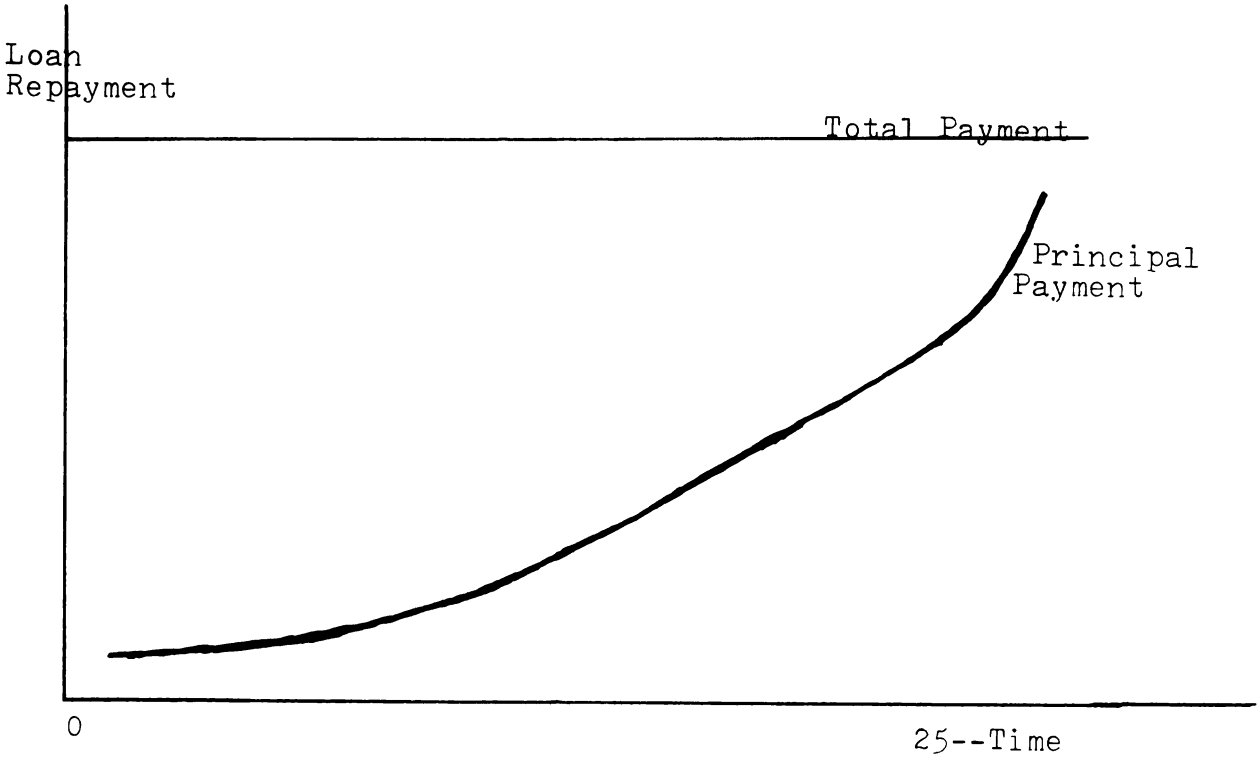

Figure 2-a Constant Payment Plan.

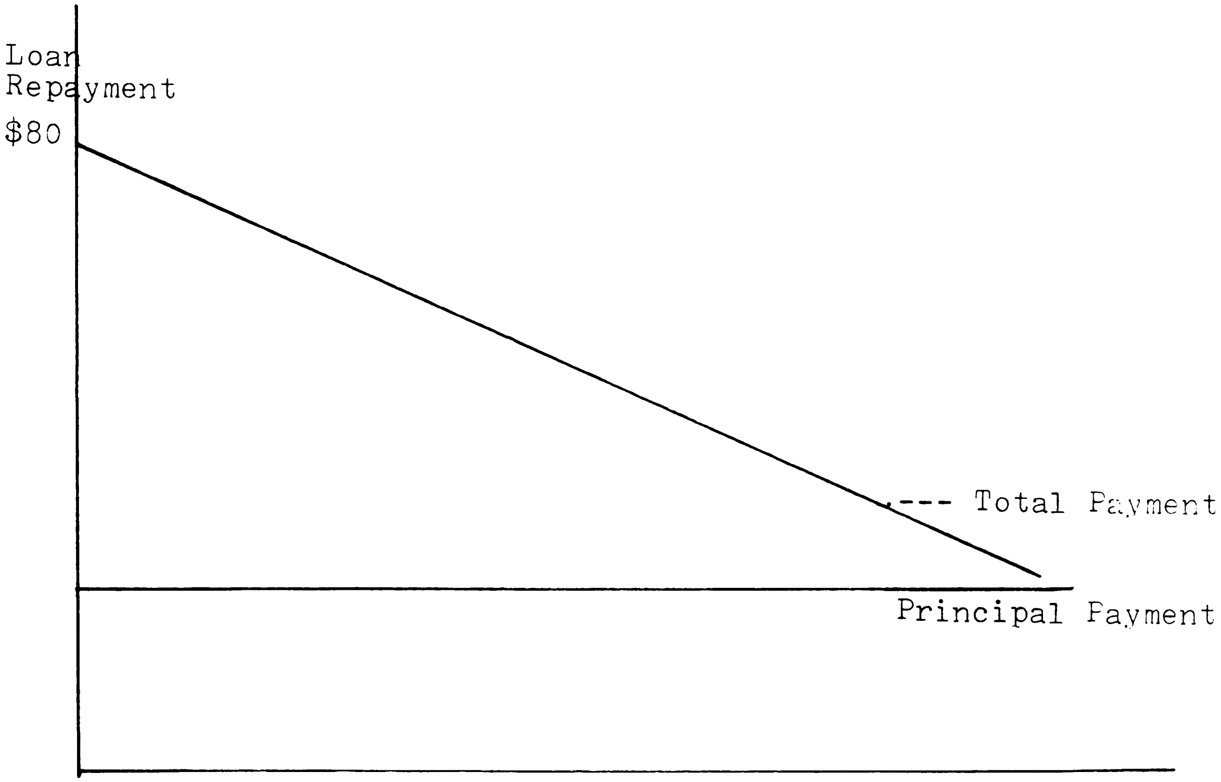

Figure 2-b Constant Payments on Principal Plan.

<u>Effect of Capital Gains</u>:

Equation (III. 3) demonstrates that inflation has no effect on the discounted after-tax income associated with land; however, the capital gains associated with farmland will be affected by the rate of inflation. Capital gain in farmland accrues over time but is realized only when the asset is sold.

The question is how does inflation affect capital gains and therefore the price of land. If the price of land in the first period is V(o) and it increases by the rate of inflation, then n periods later equals $V(o)(1+i)^n$. If n is the planning horizon, after-tax capital gain realized at the end of the planning period is $(V(o)(1+i)^n - V(o))(1-t^*)$, the difference between the beginning and ending value of land where t^* is the tax rate applied to capital gains income. This value should be discounted by the after-tax discount rate $[1+(i+r+ir)(i-t)]$ to determine its present value. Only if the income tax rate was equal to the capital gains tax rate, and the rate of increase in land prices equalled the rate of increase in income from farmland, then equation (III.3) correctly estimates land prices.

<u>A Summary List of Factors Affecting Land Prices Include</u>:

The factors influencing or determining the maximum bid price for land include the following:

(1) the average price per acre of recent sales of comparable parcels in the area.

(2) the after-tax opportunity cost of capital.

(3) the expected annual net cash income per acre before taxes.

(4) the expected annual rate of growth in annual net cash income per acre.

(5) the buyer's marginal income tax rate.

(6) downpayment (the proportion of the purchase price paid down).

(7) the rate of interest charged on the mortgage loan.

(8) the amortization period on the loan.

(9) the expected annual rate of inflation in land values.

(10) planning horizon, years.

(11) the capital gains tax rate.

The first step in determining the maximum bid price for land is to estimate the income to be earned from the land. As mentioned earlier, there are at least two methods for determining the net income to land. One is the rental method where the annual net income to land equals the net rent received by the landlord minus his share of production costs. In another approach, the residual method, net income to land is calculated as a residual after subtracting the operating expenses from the gross income of one acre of land. The subtracted operating expenses include seed, fertilizer,

fuel for machines, labor, interest charged on short-term loans, and herbicides and insecticides. As an example, a sample budget is described in Table 1. In that budget after subtracting operating expenses, net income to land equaled \$50.

The next step in determining the maximum bid price for land is setting the planning horizon. The horizon should at least equal the life of the mortgage loan and is important because, as the length of the planning horizon increases, the importance of income from the land also increases relative to capital gains. Since the capital gains are realized at the end of the planning period, the further away the end of that period, the less important are the capital gains to be realized. The converse is also true; the shorter the planning period is, the more important are the capital gains.

The after-tax opportunity cost of capital is used to convert future income to its equivalent value in the present. The before-tax opportunity cost of capital is made up of two parts, the rate of the pure time value of money and the expected rate of inflation. The rate of pure time preference is the cost of postponing consumption. However, if prices are increasing, as they are now, the saver must be compensated, not only for his time preference rate, but also for the loss of purchasing power. Thus income earned in all future time periods is divided by or discounted by the discount rate which

includes both time preferences and inflation.

Next, the present value of future after-tax income should be summed together with the present value of after-tax capital gains income. The result is the return to land. A maximum bid price is determined by equating the returns to land to the cost of purchasing the land. The cost of the land purchase is the downpayment plus the present value of the mortgage loan payment minus the tax deductible interest payments. Thus, the cost of acquiring land is: the downpayment plus the remaining repayment due after the downpayment minus the interest savings attributable to the interest deducted against income, respectively.

The remaining portion of the model consists of two parts: the discounted after-tax income and the capital gains which are equated to the return. Lee and Rask have constructed a maximum bid price model which includes all the factors discussed so far. This model was used to predict the maximum bid price for land in the period 1960-1977 given a downpayment required of 25 percent of the cost; a marginal tax rate on income of 25 percent, and a capital gain tax rate of 12.5 percent, a planning horizon and amortization loan period of 20 years while before tax opportunity cost of capital is assumed to be equal to the interest rate on the mortgage loan.

Table 9: Summarizes the result of this model:

Table 9: Estimated the Maximun Bid Price for Land Using the Lee and Rask Model: Assuming a 20 year planning horizon and loan amortization period, a 25 percent marginal tax rate, income from land and price of comparable tracts reported in Table 2. Interest rate of loan also reported in Table 2.

Year	Growth Rate of Income g and in Land Values 1/ %	Opportunity Cost of Capital 2/ %	Estimated Land Values $/ac.	Actual Value of Land $/ac.	Estimated Value Minus Actual Value
1963	1.00	5.60	231.43	208.89	21.54
1964	1.23	5.50	249.50	220.36	29.14
1965	2.17	5.50	295.82	230.36	65.45
1966	5.17	5.80	472.43	257.04	214.39
1967	7.60	6.00	719.10	273.59	445.51
1968	11.77	6.70	1222.00	330.10	891.90
1969	9.13	7.70	786.50	315.35	471.15
1970	7.26	15.58	466.22	290.07	176.22
1971	-2.70	7.90	177.18	319.36	-142.19
1972	4.90	7.40	486.10	344.84	141.26
1973	3.60	7.50	444.62	416.62	28.04
1974	9.80	8.10	1208.10	486.00	722.10
1975	9.90	8.70	1215.00	552.00	663.00
1976	13.00	8.70	2052.70	617.00	1435.70
1977	15.00	8.40	3839.70	757.00	3082.70
				Average Error, E =	569.00

1. It is assumed that average growth rate in income equals to average inflation rate in land values. The inflation rate reported in this column is calculated as the average growth rate in the previous three years net income to land for each period.

2. Source: From Table 2, Column 2.

To calculate land values using the Lee and Rask model, this author wrote a program for the Texas Instruments (TI-59) programmable calculator (for details refer to Appendix A, Table A:1). This program estimates the maximum bid price for land, annual loan payment, unpaid balance remaining on loan in any year, net cash flow in any period, market value of the land and equity, given the variables discussed earlier. To test the sensitivity of the program, a sample problem was first solved with input data equal to:

(1) Income growth rate of 8 percent.

(2) Before-tax opportunity cost of capital of 11 percent.

(3) Annual net income to land, 36.81 $/Ac.

(4) Marginal tax rate of income, 25 percent.

(5) Expected rate of inflation on land values of 6.5 percent.

(6) The market value of land 757 $/Ac.

(7) The capital gain income tax, 12.5 percent.

(8) Downpayment, 25 percent.

(9) Interest rate on mortgage loan, 10 percent annum.

(10) Planning horizon, years, 20 years.

(11) Amortization period on the loan, years, 20 years.

The resulting maximum bid price = 1091.00

Table 10 summarizes the cash flows per acre for the basic case with a maximum bid price of $879.00.

Table 10: Summary of Cash Flow Per Acre for the Sample Case with a Maximum Bid Price of 879.00.

Year	Total Loan Payment	Unpaid Balance	Taxable Income	Income Tax	Net Cash Flow	Market Value	Equity
0	272.75	818.25	-	-	-272.75	757.00	-61.25
1	96.10	803.96	-40.65	-10.15	-46.20	806.21	2.24
2	96.10	788.25	-35.90	- 8.97	-44.21	858.60	70.35
3	96.10	770.96	-30.74	- 7.70	-42.06	914.42	143.45
4	96.10	751.95	-25.13	- 6.30	-39.76	973.85	221.90
5	96.10	731.00	-19.00	- 4.75	-37.30	1037.15	306.13
6	96.10	708.00	-12.40	- 3.10	-34.60	1104.60	396.55
7	96.10	682.70	- 5.20	- 1.30	-31.74	1176.36	493.65
8	96.10	654.90	2.63	0.66	-28.65	1252.83	597.95
9	96.10	624.25	11.14	2.78	-25.33	1334.26	710.00
10	96.10	590.56	20.40	5.10	-21.76	1421.00	830.43
11	96.10	553.50	30.45	7.60	-17.92	1513.35	959.85
12	96.10	512.75	41.40	10.35	-13.80	1611.73	1099.00
13	96.10	468.00	53.30	13.30	- 9.35	1716.50	1248.60
14	96.10	418.60	66.23	16.56	- 4.60	1828.00	1409.50
15	96.10	364.34	80.30	20.00	0.55	1946.90	1582.55
16	96.10	304.66	95.60	23.90	6.06	2073.43	1768.77
17	96.10	239.00	112.26	28.00	12.00	2208.20	1969.20
18	96.10	166.80	130.37	32.60	18.35	2351.74	2185.00
19	96.10	87.37	150.08	37.52	25.18	2504.60	2417.22
20	87.37	0.00	171.52	42.88	32.53	2667.40	2667.40

Sensitivity Analysis:

The solution for the base case will serve as the point of departure to examine the sensitivity of the maximum bid price to changes in the input variables. The sensitivity of the maximum bid price was tested by altering the input variables one at a time. Each variable was examined over a range. In every case the values for all variables, other than the one being tested, were fixed as specified in the original case.

The results of the sensitivity analysis from the base case are summarized below:

(1) An increase in the mortgage loan interest rate from 10.00 to 14.00 percent, reduces the maximum bid price for land by $140.00.

(2) Increasing the precent of loan paid as a down-payment from 10.00 to 50.00 percent, decreases the maximum bid price for land by $21.00.

(3) An increase in the before-tax opportunity cost of capital from 10.00 to 14.00 percent, reduces the maximum bid price by $217.00.

(4) An increase in average price of comparable tract of land from $700.00 to $800.00, increases the maximum bid price for land by $68.00.

(5) Increase in the expecede rate of inflation from 5.00 to 10.00 percent, increases the maximum bid

price for land by $593.20.

(6) If the expected net income to land increases from $30.00 to 50.00 , the maximum bid price for land increases by $199.00.

(7) Income growth rate of 6.00 percent instead of 2.00 percent, increases the maximum bid price by $142.75.

(8) An increase in the income tax rate from 20.00 to 40.00 percent and capital gains tax rate from 10.00 to 20.00 percent, increase the maximum bid price for land by $156.65. This result occurs because reduction in the expected annual net income per acre, due to income taxes is more than offset by the tax deductible interest payments and the decrease in after tax opportunity cost of capital.

(9) An increase in loan amortization and planning horizon from 20 to 30 years, increase the maximum bid price for land by $11.00.

The accuracy of the maximum bid price developed by Lee and Rask and reported in Table 9 has an average error of 569.00 making it the least accurate of all the models examined thus far.

Chapter Summary:

In this chapter the effect of financing and taxes on the price of land were demonstrated. The chapter began with two cash flow statements and ended by demonstrating the effects of financing arrangements and taxes on land values, using the maximum bid price developed by Lee and Rask.

The two cash flow methods examined were: (1) constant payment where the total payment in each period remains constant over the terms of the loan; (2) constant payment on the principal where an equal payment in each period is made on principal, plus varying amounts of interest.

Finally, the maximum bid price for land was discussed using a model incorporating eleven variables. Those variables were the average price per acre in recent sales of comparable tracts of land, the after-tax opportunity cost of capital, the expected annual net income to land, the expected annual rate of growth in annual income, the buyer's marginal income tax rate, downpayment, the rate of interest on mortgage loan, the amortization period of the loan, inflation rate in land values, planning period, and the capital gains tax rate. This model demonstrated the sensitivity of land prices to changes in any of the variables discussed above (refer to sensitivity analysis for more detail).

CHAPTER V

THE EFFECT OF RISK ON FARMLAND VALUES

Thus far, all the models used to estimate land prices assumed perfect knowledge. The future net incomes to land in each period have been assumed known with certainty. However, the value of the future net income which determine land prices is rarely known with certainty.

There are several different opinions as to what constitutes risk and uncertainty (Robison, 1979,a). In this thesis, risk and uncertainty will be used interchangeably. Robison describes risk as "actions with more than one possible outcome; where the likelihood of all possible outcomes is described by a probability density function".

The net return to land is a function of the price of the agricultural output and the cost of agricultural input. The market environment provides risk in price and in other terms of trade. The organized activities of the government and other institutions add uncertainty to the market expectations. Commodity price support programs are an example of the factors affecting the price of inputs and agricultural products. Weather and biological environment introduce risk and uncertainty in the production output of crops and livestock. The effect of uncertainty is more pronounced in investment decisions because of the long term nature of real

estate investments; the effects of a wrong decision may be felt for many years. One means of reducing risk is to improve the planning information of the decision maker through effective capital budgeting.

In the following sections ways of handling risk in the capital budgeting analysis will be discussed.

Adjustment of Discount rate:

A simple method of accounting for risk in capital budgeting models is to vary the discount rate. For investments which involve greater risk, the cost of capital is higher than those investments which involve less risk. That is, let the cost of capital consists of three parts:

(V.1) $cc = r^{*}+i+\gamma.$

where: r^{*} = pure time value of money

i = the inflation rate

γ = the risk factor

Investment with a higher degree of risk leads to greater cost of capital and this reduces the value of an acre of land. The weakness of this approach lies in determining the appropriate discount rate for an investment. This determination is rather subjective and arbitrary. The difficult question becomes: How much should the discount rate be changed for an investment which involves a greater risk? This is a hard question because the present value of a sum to be received in the future may be dramatically changed by a few percentage point changes in the discount rate.

Another weakness of this approach is that this method conceptually adjusts the wrong element. It is the future income of an investment which is subject to risk, not the cost of capital. Yet this method adjusts the discount rate and does not adjust the variable income. Finally, this approach does not use all the information available from the probability distribution of an investment. This model in its simplist form is:

$$(V.2) \quad V_{\gamma} = R/(1+r^{*}+\gamma) + \ldots + R/(1+r^{*}+\gamma)^{n} + V_{\gamma}/(1+r^{*}+\gamma)$$

and after summing geometrically it gives:

$$(V.3) \quad V_{\gamma} = R/(r^{*}+\gamma)$$

where: V_{γ} = the value of one acre of farmland estimated by (V.3).

R = the annual net income to farmland.

r^{*} = the time preference of money rate.

γ = the risk factor.

Multiplicative Risk coefficient:

A second method is to reduce income to its certainty equivalent by multiplying income by some coefficient α. The advocates of this method argue that any adjustment of risk should occur in the numerator of the present value equation in the form of a coefficient with a value which varies between zero and one according to the degree of risk. This model can be written as:

(V.4) $V_\alpha = \alpha R/(1+r^*) + \ldots + \alpha R/(1+r^*)^n + V_\alpha/(1+r^*)^n$

After summing geometrically it gives:

(V.5) $V_\alpha = \alpha R/r^*$.

The coefficient α is called the risk coefficient. This coeficient leads the investor to regard the expected annual net return to investment as equal to a certain return. For example, consider an expected net return of $1,000.00 from a specific investment. The investor must choose a certain income, which he would accept in lieu of this expected income. If the investor chooses $1,000.00, then α equals 1.00 and the investment is called riskless. If he chooses $800.00, then α equals 0.80. The smaller the value of α, the larger is the risk associated with the income.

The weakness of this method is that the risk measurement α is still subjective and arbitrary. The multiplicative risk coefficient avoids the problem that investors cannot ignore: the investor's attitude toward risk and uncertainty. Clearly the investor's attitude toward risk and uncertainty must be considered in decision making. In the following section a procedure that takes the investor's attitude into account will be discussed.

Subtracting the Cost of Risk:

Another way to adjust income R to its certainty equivalent is to subtract the cost of risk. This method is based on the expected utility hypothesis. Consider an individual

with asset X and utility function U. The risk premium π is such that the decision maker is indifferent between a random variable Z with expected value 0 and variance σ^2 and the certain income $X-\pi$. Then, the utility of a certain $U(X-\pi)$ equals the expected utility of the random variable $EU(X+Z)$:

(V.6) $U(X-\pi) = EU(X+Z)$.

By taking the Taylor expansion around both $U(X-\pi)$ and $EU(X+Z)$, Pratt has shown that:

(V.7) $\pi = \sigma^2 U''(X)/2U'(X)$.

Where $-U''(X)/U'(X)$ equals the absolute risk aversion coefficient which most economists argue decreases with income X.

Another commonly used risk measure is the equilibrium trade-Off between expected returns and variance on an Expected Value - Variance (EV) efficient set (see figure 3).

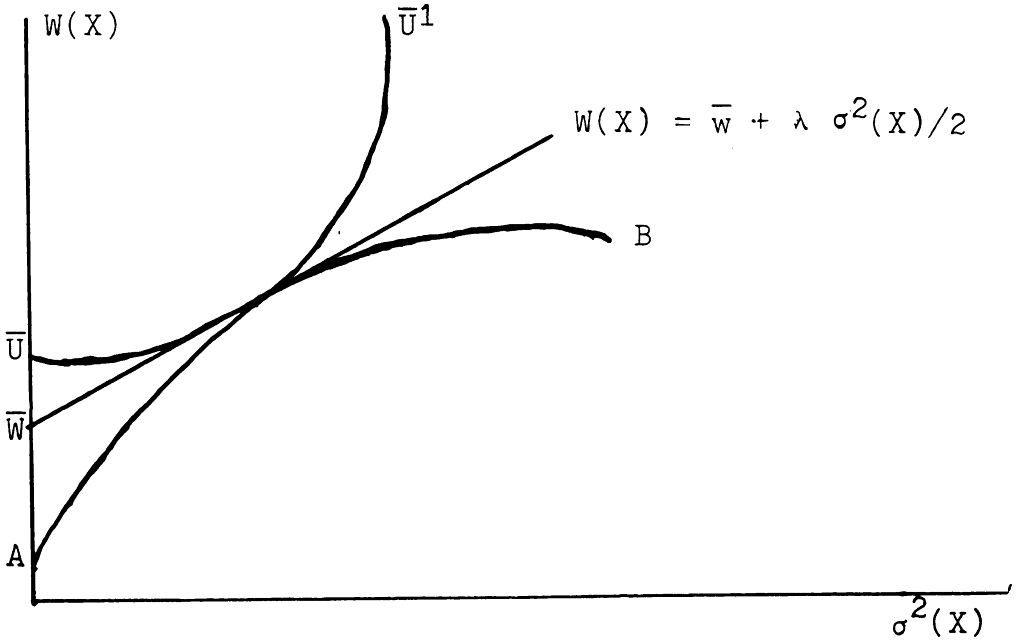

Figure 3. Equilibrium between an EV efficient set and a decision maker's isoexpected utility function.

Let AB be the EV set, $\overline{U}$ $\overline{U}^1$ be its isoexpected utility lines and let:

(V.8) $W(X) = \overline{W} + \lambda\ \sigma^2(X)/2$

be the linear tangent drawn to the equilibrium point with slope λ.

Rearranging (V.8), gives:

(V.9) $W(X) - \overline{W} = \lambda\ \sigma^2(X)/2.$

The amount $W(X)-\overline{W}$ by definition is the risk premium π. If $U''(X)/U'(X)$ is constant, then (V.9) is equivalent to (V.7) and the equilibrium is the amount of risk aversion.

Let the expected value of land, E(V) equals to R/r^*, then the certainty equivalent of land value, CE(V) is:

(V.10) $CE(V) = R/r^* - \pi.$

Where CE(V) is the certainty equivalent value of one acre of land.

To determine the risk premium π, first the total variance Var(V) must be estimated.

The variance of the sum $U = aX$, is $\sigma 2 = a^2\ \sigma^2(X)$, where a is constant and X is a random variable. Using the above concept, the variance of land values Var(V), is derived as:

(V.11) $Var(V) = Var(R)/(r^*)^2.$

Where Var(R) is the variance of annual net income to land and r^* is the rate of pure time preference which equals 5.67 percent as calculated earlier in the thesis. However, this is not the end of the story, because Var(R) also needs to be estimated before π can be calculated. Empirically, the variance of net income to land, Var(R) is estimated first by regressing the previous three year values of net income against time. Then, the variance is estimated by squaring and summing the difference between the observed value and those predicted by the regression equation. If Y(t) is Observed and $\hat{Y}(t)$ is Predicted by the regression equation; then, the variance estimate is $\sum_{t=1}^{3}(Y(t) - \hat{Y}(t))^2/3$.

Finally, the certainty equivalent of land value CE(V) is:

(V.12) $CE(V) = R/r^* - \lambda\, Var(R)/2(r^*)^2$.

The model was examined given the data for cash rent to land as reported in table 2, variance of net income to land as reported in table 11, risk aversion coefficient of 0.003, and the rate of pure time preference of 5.67 percent. The results are given in table 11.

Table 11: Estimated Land Prices Using Equation (V.12).

Year	Variance of Income Var(R). 1/	Estimated Price of Land, $/ac.	Actual Price of Land, $/ac.	Estimated minus Actual Price ,$
1963	0.023	230.00	209.85	20.15
1964	0.005	234.00	220.36	13.64
1965	0.004	244.80	230.36	14.44
1966	0.006	264.55	257.04	7.51
1967	0.015	302.60	273.59	29.01
1968	0.014	317.45	330.10	-12.65
1969	0.026	325.90	315.35	10.55
1970	0.130	274.72	290.07	-15.35
1971	0.620	350.70	319.36	31.34
1972	0.800	345.30	344.84	0.46
1973	0.700	355.40	416.62	-61.22
1974	0.500	456.40	486.00	-29.60
1975	0.400	493.60	552.00	-58.40
1976	0.400	541.60	617.00	-75.40
1977	0.270	645.00	757.00	-112.00

Average Error = 32.80

1- Those series were estimated by regressing net income against time (for detail refer to page 57).

The results as given in table 11 indicate that inclusion of risk in the model did not improve the predictibility of the model. The average error of the estimated values of land from the actual values is 32.80, while the average error of the simple model R/r^* was 29.90.

<u>Sensitivity Analysis</u>:

The sensitivity of land prices to change in most of the variables was discussed in Chapter IV. However, in this chapter the effects of changes in risk coefficient λ, variation in income, and net income on land values will be demonstrated.

Mathematically to demonstrate such effects the partial derivatives of land prices (V.12) with respect to expected net returns to land R, risk aversion coefficient λ, and the variation in income Var(R) were obtained.

The partial derivative with respect to net income to land equals:

(V.13) $\partial V/\partial R = 1/r^* > 0$.

The above derivative is greater than zero, since r^* is greater than zero.

The derivative of land prices with respect to variation in income equals:

(V.14) $\partial V/\partial Var(R) = -\lambda/2(r^*)^2 < 0$.

That is, the effect of variation in income to land prices is negative, since both λ and r^* are positive.

The derivative with respect to the risk aversion coefficient λ equals:

(V.15) $\partial V/\partial\lambda = -Var(R)/2(r^*)^2 < 0.$

Which is also less than zero, because the cost of risk increases with risk aversion. To examine the effect of net income to land, variation in income and risk aversion coefficient on land value, equation (V.12) was used. Table 12 summarizes the results of the sensitivity analysis.

Table 12: A Sensitivity Analysis of Land Values with respect to Changes in Income, Variation in Income and Risk Aversion.

A: The sensitivity of land values to net income: given rate of pure time preference r^* equal to 5.67 percent, λ equal to 0.003 and Var(R) equals 5 .

Net Income	Land Values,$/ac.
10.00	174.00
20.00	350.00
40.00	703.00
80.00	1408.60
160.00	2819.50

B: The sensitivity of land values to variation in income: given rate of pure time preference r^* equals 5.67 percent, λ equals 0.003 and net income to land R equals $10.

Variance of Income, Var(R)	Land Values, $/ac.
5.00	174.00
10.00	171.70
20.00	167.00
40.00	157.70
80.00	139.00

Table 13 continued:

C: The sensitivity of land values to risk aversion coefficient λ : given rate of pure time preference r^* equals 5.67 percent, net income R equals to $10 and the variation in income equals 5.

Risk Aversion Coefficient, λ	Land Values, $/ac.
0.003	174.00
0.006	171.70
0.012	167.00
0.024	157.70
0.048	139.00

Thus, while changing R, has a positive effect and changing Var(R), λ have negative effect, land prices appear most sensitive to changes in income.

Chapter Summary:

In this chapter discussion centered on the effect of risk on land prices. The model discussed was (V.12). Sensitivity analysis demonstrated that any increase in variation in annual net income would decrease the amount the buyer is willing to pay. Increasing the risk aversion coefficient λ, decreases the maximum bid price for land.

CHAPTER VI

ABSOLUTE RISK AVERSION OBTAINED USING TRIANGULAR DISTRIBUTION FUNCTION

The risk was included in the BCB model in Chapter V. Using Pratt's formula which defined the cost of risk to be equal to a function of: a risk aversion coefficient, the variation in income and the mean of the probability distribution of returns. Often, reliable data on probability distribution is not available; requiring instead a subjectively determined triangular probability distribution. The triangular distribution (figure 4) can be determined by setting three values: the pessimistic value X_1, the most likely value X_2 and the optimistic value X_3. Then the expected value of X and the variance can be calculated. It is of course, recognized that the sum of the area in figure 4, must equal one--the sum of probabilities for an event must equal one.

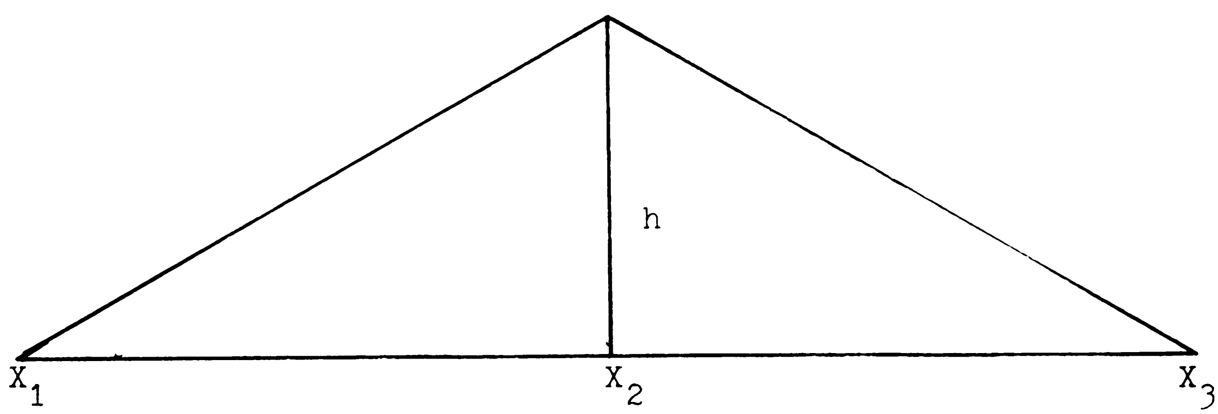

Figure 4. Triangular Distribution Function.

To calculate the area of the triangle is the height h multiply by one-half of the base. Setting the total area of the triangle equal to one (since probability must sum to one) and solving for height, h, gives:

(VI.1) $h = 2/(X_3 - X_1)$.

Then the slope, m of the triangle over the range X_1 to X_2 is simply the height, h, divided by the distance $(X_2 - X_1)$.

$$m = h/(X_2 - X_1)$$

or

(VI.2) $m = 2/(X_3 - X_1)(X_2 - X_1)$.

Thus, the height at any point along the $X_1 - X_2$ range is determined by multiplying the slope, m, and the distance of the random variable X from the point, X_1. That is:

(VI.3) $f(X)_1 = 2(X - X_1)/(X_2 - X_1)(X_3 - X_1)$. for $X_1 \leq X \leq X_2$

Using the same procedure, the height or the probability of occurrence over the range X_2 to X_3 was determined as:

(VI.4) $f(X)_2 = 2/(X_3 - X_1) - 2(X - X_2)/(X_3 - X_1)(X_3 - X_2)$
for $X_2 \leq X \leq X_3$.

Next the expected value for a random variable X is determined by taking the integral of the product X.f(X) over the range of $X_1 - X_3$. That is:

(VI.5) $E(X) = \int_X X\, f(X)dX$ for $X_1 \leq X \leq X_3$.

Where E(X) is the expected value of the random variable X, f(X) is the probability that X will occur. Thus the expected value was found equal to:

$$(VI.6)\quad E(X) = (2/(X_3-X_1)(X_2-X_1))(X_2^3/3 - X_1X_2^2/2 + X_1^3/6)$$
$$+ (1/(X_3-X_1))(X_3^2 - X_2^2) - 2(X_3^3/3 - X_2X_3^2/2$$
$$+ X_2^3/6)/(X_3 - X_1)(X_3 - X_2).$$

To calculate variance, we solve for the expression;

$$(VI.7)\quad \sigma^2 = E(X^2) - (E(X))^2.$$

The first part of which equals:

$$(VI.8)\quad E(X^2) = (2/(X_3- X_1)(X_2- X_1))(X_2^4/4 - X_1X_2^3/3 + X_1^4/12)$$
$$+ (2/(X_3- X_1))(X_3^3/3 - X_2^3/3) - (2/(X_3 - X_1)$$
$$\times (X_3 - X_2))(X_3^4/4 - X_2X_3^3/3 + X_2^4/12).$$

Then the variance is calculated as the difference between (VI.8) and the square of the expected value.

The cumulative density function, that is the probability that X less than or equal to some given value over the ranges $X_1 - X_2$ and $X_2 - X_3$ are determined by taking the integrals of $f(X)_1$, $f(X)_2$, respectively. That is:

$$(VI.9)\quad F(X)_1 = 2(X_2^2/2 - X_1X_2 + X_1^2/2)/(X_3 - X_1)(X_2 - X_1)$$

Where X_1 is the lowest value, X_2 is the most likely, and X_3 is the highest value given.

For $X_2 \leq X \leq X_3$, the cumulative density function is:

$$(VI.10)\quad F(X)_2 = 1 - (X^2 - 2X_3X + X_3^2) / (X_3 - X_1)(X_3 - X_2)$$

Knowing the expected value, variance, and the certainty equivalent of income derived from a risky investment, the absolute risk aversion for a particular investor can be determined using Pratt's formula as:

(VI.11) $\lambda = 2(E(V) - CE(V))/\sigma^2$.

Where λ is the absolute risk aversion, E(V) is the expected value, CE(V) is the certainty equivalent, and σ^2 is the variance.

A program has been provided to estimate the expected market price of land (or other investments), variance, absolute risk aversion, and the probability that the market price, P, is going to be less than or equal to a given price. The probability that the market price is going to be greater than a given value can also be determined. Thus, this program enable the buyer of land or other investments to estimate probability density functions by specifying the lowest, most likely, and highest price of land. Then, if they specify the random variable, certainty equivalent, their average risk aversion can be calculated. For example, if the lowest, most likely and highest price and their certainty equivalent were \$10,000.00, \$30,000.00, \$100,000.00 and \$29,000.00, respectively, the result using the program would be the following

1. Expected Value = \$46,666.70
2. Variance = $\$3.70 \times 10^8$
3. Standard Error = \$19,293.00
4. Absolute Risk Aversion = 0.0009

The probability that the value of investment is less than or equal to some value in the range of X, can be determined by pressing the value for the input X and then D. Table 13 summarizes the results of that probability for different values of X.

Table 13: Summary of the Cumulative Density Function: that is, the probability that $X \leq x$.

$X	Cumulative Density Function
9,000.00	0.0000
15,000.00	0.0139
20,000.00	0.0550
25,000.00	0.1250
30,000.00	0.2220
35,000.00	0.3300
40,000.00	0.4286
45,000.00	0.5190
50,000.00	0.6000
55,000.00	0.6785
60,000.00	0.7460
65,000.00	0.8050
70,000.00	0.8570
75,000.00	0.9000
80,000.00	0.9360
85,000.00	0.9600
90,000.00	0.9800
95,000.00	0.9960
100,000.00	1.0000

Note: The program to estimate the above table is given in table A:2 in the appendix.

Chapter Summary:

This Chapter introduced one way of estimating the probability distribution of returns -- the subjective triangular probability distribution. This is necessary to estimate the risk premium of an investment.

CHAPTER VII

PREDICTABILITY COMPARISON AND CONCLUSION

1. Comparing the predictability of the Models

All the models discussed throughout the thesis are maximum bid price models. Maximum bid price models for land are determined by equating returns from land to the costs. The maximum bid price should be correlated with the actual price of land. So, to determine which model was the best predictor, the actual price of land V was regressed against the estimated price of land $\hat{V}$ using the simple linear regression described below:

(VII.1) $V = \alpha + \beta\hat{V}$

where α and β are estimated parameters of the simple regression model. If the actual price was equal to the estimated price β would equal one and α would equal zero. Instead, for the BCB model we obtained the following estimates:

(VII.2) $V = -710.00 + 3.47\hat{V}$

The correlation coefficient R^2 is 0.67 indicating that 67 percent of the variation in land prices is explained by the linear relationship with independent variable $\hat{V}$.

For the second model, which expresses land values as net income to land in current R, over the rate of pure time of money, r^*, the following regression was obtained:

(VII.3) $V = -85.95 + 1.28 \hat{V}$

The correlative coefficient determination R^2 is 0.98. This regression indicates that 98 percent of the variation in the actual values of land is explained by the variation in the model which depends on net income and the preference rate.

The third model examined includes productivity changes (see equation III.5). The regression estimated using the data for the independent variable $\hat{V}$ and the dependent variable V was:

(VII.4) $V = 128 + 0.59 \hat{V}$.

The correlation coefficient R^2 is 0.68 .

The fourth model is the maximum bid price model. The following regression was obtained, using estimated values, by this model:

(VII.5) $V = 235 + 0.15 \hat{V}$.

The correlation coefficient R^2 in this model is 0.80.

The fifth model is equation (V.12) which was adjusted for risk. The certainty equivalent method was used to determine the risk premium and obtain the following regression for this model:

(VII.6) $V = -92.60 + 1.30 \hat{V}$.

The correlation coefficient R^2 was 0.98. The above regression does not show good predictability of the actual price of land. This can be explained by fluctuation in productivity changes, especially low and sometimes negative during the recent years.

2. Summary and Conclusion:

The objective of this thesis has been to build models of increasing complexity, in order to help explain the maximum bid price for land. Factors such as the time preference rate, marginal tax rates, capital gains tax rates, and risk are included.

The process began with the Basic Capital Budgeting model; one which expressed land values as equal to net return to land, divided by the opportunity cost of capital. According to the formula, the net return to land and the opportunity cost of capital were the only tow factors affecting the land value. It ignored such factors as the inflation rate, productivity of land and risk. Consequently, results with this formula showed large discrepancies between predicted value of land and the actual value of land.

The second model expressed land value as net returns to land over the rate of pure time preference. This model was the best predictor among all the models.

The third model, which included such factors as the income to land, the rate of time preference, the inflation rate, the productivity changes of land. The average error of this model is significantly higher than the second model.

It is concluded that this model is not a good predictor given the productivity changes used in the study.

The Maximum Bid Price Model, discussed in Chapter IV incorporated such financial arrangements as downpayments, interest rates on mortgage loans, and the length of the investment period. This model showed that an increase in the downpayment decreases the maximum bid price, and an increase in amortization period of the loan increases the maximum bid price of land. This model also yielded a large discrepancy between the predicted and the actual value of land. The major contribution of this study was to adopt this program for use on TI-59 hand held calculator.

Tow kind of loan payments were discussed; the Constant Payment Method in which the total payment each period remains constant over the term of the loan, and the Constant Payment on Principals Method in which an equal payment in each period is made on the principal, plus a varying amount of interest. The total payments(interest and principal), during the early years of the loan, are lower in the Constant Payment Method. They increase during the second half of the term of the loan. Total interest paid over the life of the loan was greater using the constant payment method.

The Maximum Bid Price Model demonstrated that mortgage loan interest rates have considerable impact on land prices. An increase in interest rate decreases the value of land.

Downpayment size had an inverse impact on land prices. The length of amortization period had a direct positive effect on land prices.

Adjustment of the discount rate for risk was discussed and it was concluded that this method did not provide any objective way of estimating the risk factor. The adjustment for risk cost was subjective and arbitrary. The second method of adjusting the Capital Budgeting Formula for risk, the Certainty Equivalent Multiplicative Coefficient of risk, was also found to be arbitrary and subjective. Both adjustment of the discount rate for risk and the multiplicative coefficient of risk avoid the problem that investors can not ignore, the investor's attitude toward risk.

A third method of adjusting the Capital Budgeting Formula for risk is the Certainty Equivalent Method, based on the expected utility hypothesis. The results to this model indicate that inclusion of risk does not improve the predictability of the model. Variation in income to land was shown to have a negative impact on land values. That is, as variation in income increases, land value decreases. Absolute risk aversion coefficient also has a negative impact on land values. There is a positive relationship between the expected income to land and the value of land.

The Maximum Bid Price model first developed by Lee and Rask at Ohio State University, was adapted here. The model was adjusted for risk by subtracting the cost of risk from the expected maximum bid price. This demonstrated that there is a negative relationship between the variation in income to land determined by variance and the maximum bid price of land. Investors with greater absolute risk aversion indicated lower maximum bid price than those who had less absolute risk aversion. Variation in market price of land had a small negative effect on the maximum bid price for land.

In this thesis, it was made available a program for the hand-held programmable calculator to estimate the expected value of land, variance, absolute risk aversion of the investor, and the probability that market price is going to be greater than the maximum bid price, given lowest market price, most likely price, the highest market price, and the one that is their certainty equivalent. Program to estimate market price and maximum bid price for different models are discussed. In addition, a program was provided, not only to estimate the maximum bid price, but the buyer's annual loan payments, loan balance remaining in each period,taxable income, net cash flow, market value of land, equity, and after-tax capital gains income.

In general, it is concluded that terms of financing such as downpayment required, interest rate and length of the loan repayment period and expected inflation rate of land price are very imporant in determining the price of farmland. However, inclusion of risk in the capital budgeting model does not improve land price predictability, because the cost of risk is too small given the value of the risk aversion coefficient.

The effect of increases in the cost of factors of production such as fertilizer, labor, capital and fuel on land prices are currently the subject of the auther's Ph. D. thesis.

APPENDIX A

PROGRAMMING THE HAND-HELD CALCULATOR

The models and the mathematical operations involved throughout the thesis can be complicated, and determination of land values using any of the models discussed may be difficult and time-consuming. Recent developments in computer technology led to the development of the hand-held computer which has provided a powerful compilation capacity that can solve problems that formerly could be solved only by large computers. These programmable calculators are currently available at reasonable prices which seem to be decreasing as technology advances.

The hand-held programmable calculator, like any computer, can carry out the following:

1. Read in both data and instructions.
2. Store the data and instructions in a memory.
3. Perform calculations in manner prescribed by the instructions.
4. Read out the results.
5. Control all aspects involved in getting an answer.

The advantages of these hand-held programmable calculators to a large number of decision-makers and professionals are clear-cut. Its use helps speed up business decisions and eliminates manual calculations.

The TI-59:

Many of the principles of programming are common to large computers and programmable calculators of all manufacturers. However, each manufacturer's equipment requires the user to follow some specific rules and conventions that are unique to that particular line. Since the Texas Instruments-59 line of programmable calculator was used to solve for land values throughout the thesis, some of their features will be discussed briefly.

The TI-59 is one of the recent programmable calculators made by Texas Instruments and capable of handling problems that formerly could be solved only by large computer. The most striking feature of the TI-59 is the use of removable solid-state modules for the storage and execution of library programs.

Program steps are entered into the memory of the calculator by pressing keys on the keyboard. The program will be stored in the memory and can be used repeatedly with different data. If a given program is to used only once, it can be erased from the program memory when the power is turned off. However, if needed again, the same program can be saved by recording it on a magnetic card.

Then when it is needed the card containing the program can be read into the calculator memory and the program reused.

The Tables in this Appendix:

Table A:1. Lists the Maximum Bid Price and Cash Flow program.

Table A:2. Lists the procedure to follow once the program (table A:1) is read into the computer.

Table A:3. List the program for the Subjective Probability Distribution.

Table A:4. Lists the procedure to follow once the program (table A:3) is read into the computer.

Table A. 1: Maximum Bid Price and Cash Flow Program.

1. is Line No. 2. is Key Code 3. is Key

1	2	3	1	2	3	1	2	3
000	91	R/S	039	43	RCL	078	19	19
001	76	LBL	040	19	19	079	54	)
002	11	A	041	54	)	080	55	÷
003	43	RCL	042	65	×	081	53	(
004	20	20	043	43	RCL	082	53	(
005	42	STO	044	15	15	083	01	1
006	00	00	045	65	×	084	85	+
007	43	RCL	046	53	(	085	43	RCL
008	11	11	047	01	1	086	01	01
009	65	×	048	75	-	087	54	)
010	53	(	049	43	RCL	088	45	Y^X
011	01	1	050	16	16	089	43	RCL
012	75	-	051	54	)	090	19	19
013	43	RCL	052	95	=	091	54	)
014	13	13	053	42	STO	092	54	)
015	54	)	054	02	02	093	54	)
016	95	=	055	43	RCL	094	65	×
017	42	STO	056	12	12	095	53	(
018	01	01	057	65	×	096	53	(
019	53	(	058	53	(	097	01	1
020	53	(	059	01	1	098	85	+
021	01	1	060	75	-	099	43	RCL
022	85	+	061	43	RCL	100	10	10
023	43	RCL	062	13	13	101	54	)
024	14	14	063	54	)	102	55	÷
025	54	)	064	65	×	103	53	(
026	45	Y^X	065	53	(	104	43	RCL
027	43	RCL	066	01	1	105	01	01
028	19	19	067	75	-	106	75	-
029	54	)	068	53	(	107	43	RCL
030	55	÷	069	53	(	108	10	10
031	53	(	070	53	(	109	54	)
032	53	(	071	01	1	110	54	)
033	01	1	072	85	+	111	95	=
034	85	+	073	43	RCL	112	42	STO
035	43	RCL	074	10	10	113	03	03
036	01	01	075	54	)	114	43	RCL
037	54	)	076	45	Y^X	115	17	17
038	45	Y^X	077	43	RCL	116	85	+

Table A. 1 Cont.

1	2	3	1	2	3	1	2	3
117	53	(	156	54	)	195	95	=
118	53	(	157	54	)	196	42	STO
119	01	1	158	54	)	197	04	04
120	75	-	159	65	×	198	53	(
121	43	RCL	160	53	(	199	01	1
122	17	17	161	53	(	200	75	-
123	54	)	162	43	RCL	201	43	RCL
124	65	×	163	18	18	202	17	17
125	53	(	164	65	×	203	54	)
126	53	(	165	53	(	204	65	×
127	53	(	166	53	(	205	43	RCL
128	53	(	167	01	1	206	13	13
129	01	1	168	85	+	207	65	×
130	85	+	169	43	RCL	208	43	RCL
131	43	RCL	170	18	18	209	18	18
132	01	01	171	54	)	210	65	×
133	54	)	172	45	Y^x	211	53	(
134	45	Y^x	173	43	RCL	212	53	(
135	43	RCL	174	20	20	213	43	RCL
136	20	20	175	54	)	214	18	18
137	54	)	176	54	)	215	65	×
138	75	-	177	55	÷	216	53	(
139	01	1	178	53	(	217	01	1
140	54	)	179	53	(	218	85	+
141	55	÷	180	53	(	219	43	RCL
142	53	(	181	01	1	220	18	18
143	43	RCL	182	85	+	221	54	)
144	01	01	183	43	RCL	222	45	Y^x
145	65	×	184	18	18	223	43	RCL
146	53	(	185	54	)	224	20	20
147	53	(	186	45	Y^x	225	54	)
148	01	1	187	43	RCL	226	55	÷
149	85	+	188	20	20	227	53	(
150	43	RCL	189	54	)	228	53	(
151	01	01	190	75	-	229	53	(
152	54	)	191	01	1	230	01	1
153	45	Y^x	192	54	)	231	85	+
154	43	RCL	193	54	)	232	43	RCL
155	20	20	194	54	)	233	18	18

Table A. 1 Cont.

1	2	3	1	2	3	1	2	3
234	54	)	273	53	(	312	65	×
235	45	Y^x	274	01	1	313	53	(
236	43	RCL	275	85	+	314	01	1
237	20	20	276	43	RCL	315	85	+
238	54	)	277	01	01	316	43	RCL
239	75	-	278	54	)	317	18	18
240	01	1	279	45	Y^x	318	54	)
241	54	)	280	43	RCL	319	45	Y^x
242	54	)	281	00	00	320	53	(
243	95	=	282	54	)	321	43	RCL
244	42	STD	283	54	)	322	20	20
245	05	05	284	65	×	323	75	-
246	43	RCL	285	53	(	324	43	RCL
247	16	16	286	53	(	325	00	00
248	55	÷	287	53	(	326	85	+
249	53	(	288	53	(	327	01	1
250	53	(	289	01	1	328	54	)
251	01	1	290	85	+	329	54	)
252	85	+	291	43	RCL	330	54	)
253	43	RCL	292	18	18	331	95	=
254	01	01	293	54	)	332	44	SUM
255	54	)	294	45	Y^x	333	07	07
256	45	Y^x	295	53	(	334	97	DSZ
257	43	RCL	296	43	RCL	335	00	00
258	19	19	297	20	20	336	10	E'
259	54	)	298	75	-	337	53	(
260	95	=	299	43	RCL	338	43	RCL
261	42	STD	300	00	00	339	02	02
262	06	06	301	85	+	340	85	+
263	25	CLR	302	01	1	341	43	RCL
264	00	0	303	54	)	342	03	03
265	42	STD	304	54	)	343	54	)
266	07	07	305	75	-	344	55	÷
267	76	LBL	306	01	1	345	53	(
268	10	E'	307	54	)	346	43	RCL
269	53	(	308	55	÷	347	04	04
270	01	1	309	53	(	348	75	-
271	55	÷	310	43	RCL	349	53	(
272	53	(	311	18	18	350	43	RCL

Table A. 1 Cont.

1	2	3	1	2	3	1	2	3
351	05	05	390	54	)	429	53	(
352	65	×	391	55	÷	430	01	1
353	43	RCL	392	53	(	431	85	+
354	07	07	393	53	(	432	43	RCL
355	54	)	394	53	(	433	18	18
356	75	-	395	01	1	434	54	)
357	43	RCL	396	85	+	435	45	Y^x
358	06	06	397	43	RCL	436	43	RCL
359	54	)	398	18	18	437	20	20
360	95	=	399	54	)	438	54	)
361	42	STO	400	45	Y^x	439	55	÷
362	08	08	401	43	RCL	440	53	(
363	91	R/S	402	20	20	441	53	(
364	76	LBL	403	54	)	442	53	(
365	12	B	404	75	-	443	01	1
366	53	(	405	01	1	444	85	+
367	01	1	406	54	)	445	43	RCL
368	75	-	407	54	)	446	18	18
369	43	RCL	408	95	=	447	54	)
370	17	17	409	42	STO	448	45	Y^x
371	54	)	410	26	26	449	43	RCL
372	65	×	411	91	R/S	450	20	20
373	43	RCL	412	76	LBL	451	54	)
374	21	21	413	13	C	452	75	-
375	65	×	414	53	(	453	01	1
376	53	(	415	01	1	454	54	)
377	43	RCL	416	75	-	455	54	)
378	18	18	417	43	RCL	456	65	×
379	65	×	418	17	17	457	53	(
380	53	(	419	54	)	458	53	(
381	53	(	420	65	×	459	53	(
382	01	1	421	43	RCL	460	53	(
383	85	+	422	21	21	461	01	1
384	43	RCL	423	65	×	462	85	+
385	18	18	424	53	(	463	43	RCL
386	54	)	425	43	RCL	464	18	18
387	45	Y^x	426	18	18	465	54	)
388	43	RCL	427	65	×	466	45	Y^x
389	20	20	428	53	(	467	53	(

Table A. 1 Cont.

1	2	3	1	2	3	1	2	3
468	43	RCL	507	53	(	546	76	LBL
469	20	20	508	43	RCL	547	16	A'
470	75	-	509	12	12	548	43	RCL
471	43	RCL	510	65	×	549	12	12
472	22	22	511	53	(	550	65	×
473	54	)	512	53	(	551	53	(
474	54	)	513	01	1	552	53	(
475	75	-	514	85	+	553	01	1
476	01	1	515	43	RCL	554	85	+
477	54	)	516	10	10	555	43	RCL
478	55	÷	517	54	)	556	10	10
479	53	(	518	45	Y^x	557	54	)
480	43	RCL	519	43	RCL	558	45	Y^x
481	18	18	520	22	22	559	43	RCL
482	65	×	521	54	)	560	22	22
483	53	(	522	54	)	561	54	)
484	53	(	523	75	-	562	75	-
485	01	1	524	53	(	563	43	RCL
486	85	+	525	43	RCL	564	26	26
487	43	RCL	526	18	18	565	75	-
488	18	18	527	65	×	566	43	RCL
489	54	)	528	43	RCL	567	29	29
490	45	Y^x	529	27	27	568	95	=
491	53	(	530	54	)	569	42	STO
492	43	RCL	531	95	=	570	30	30
493	20	20	532	42	STO	571	91	R/S
494	75	-	533	28	28	572	76	LBL
495	43	RCL	534	91	R/S	573	17	B'
496	22	22	535	76	LBL	574	43	RCL
497	54	)	536	15	E	575	15	15
498	54	)	537	43	RCL	576	65	×
499	54	)	538	28	28	577	53	(
500	54	)	539	65	×	578	53	(
501	95	=	540	43	RCL	579	01	1
502	42	STO	541	13	13	580	85	+
503	27	27	542	95	=	581	43	RCL
504	91	R/S	543	42	STO	582	14	14
505	76	LBL	544	29	29	583	54	)
506	14	D	545	91	R/S	584	45	Y^x

Table A. 1 Cont.

1	2	3	1	2	3
585	43	RCL	624	65	×
586	22	22	625	53	(
587	54	)	626	01	1
588	95	=	627	75	-
589	42	STO	628	43	RCL
590	31	31	629	16	16
591	91	R/S	630	54	)
592	76	LBL	631	95	=
593	18	C'	632	42	STO
594	43	RCL	633	33	33
595	31	31	634	91	R/S
596	75	-			
597	43	RCL			
598	27	27			
599	95	=			
600	42	STO			
601	32	32			
602	91	R/S			
603	76	LBL			
604	19	D'			
605	53	(			
606	43	RCL			
607	21	21			
608	65	×			
609	53	(			
610	53	(			
611	01	1			
612	85	+			
613	43	RCL			
614	14	14			
615	54	)			
616	45	Y^x			
617	43	RCL			
618	20	20			
619	54	)			
620	75	-			
621	43	RCL			
622	21	21			
623	54	)			

Table A:2. Procedure to follow for the Maximum Bid Price model.

Objective: To determine: (1) The maximum amount one can afford to pay for one acre of land, (2) Annual loan payment, (3) Unpaid balance remaining on loan at year j; (4) Netcash flow at period j, (5) Market price of land at period j, (6) Equity at year j; j=1 . . . . m, where m is the amortization period of the loan.

STEP	INPUT DESCRIPTION	INPUT VALUE	PRESS
1.	Turn calculator off, and back on, to clear program and memory.		
2.	Partition memory (Note 639.39 should appear on the screen. If not, return to step 1.)		(4) (2nd) (op) (17)
3.	Clear Display		(CLR)
4.	Insert side 1 of the card containing the program (A:1). If the calculator has read the card successfully, a "1" will appear and remain stationary. If a flashing "0" appears, repeat step 3 and 4.		
5.	Clear Display		(CLR)
6.	Insert side 2 of the card. If the calculator reads side 2 successfully, a "2" will		

Table A:2 continue.

STEP	INPUT DESCRIPTION	INPUT VALUE	PRESS
	appear and remain stationary. If a "0" appear, repeat steps 5 and 6.		
7.	Clear Display		(CLR)
8.	Insert side 3 of the card containing the program. If the calculator has read the card successfully, a "3" will appear and remain stationary. If a "0" appears, repeat steps 7 and 8.		
9.	Clear Display		(CLR)
10.	Insert side 4 of cards contining the program. If the calculator has read the card successfully, a "4" should appear and remain stationary. If "0" flashes on the display after the card has been read, steps 9 and 10 should be repeated.		
11.	Clear Display		(CLR)
12.	Growth rate of annual net income to land, % annum	________	(STO) 10
13.	Before tax opporunity cost of capital, % annum	________	(STO) 11
14.	Annual Net Income to land; $ per acre.	________	(STO) 12
15.	Marginal tax rate on annual income, %.	________	(STO) 13
16.	Expected rate of inflation	________	(STO) 14

Table A:2 continue.

STEP	INPUT DESCRIPTION	INPUT VALUE	PRESS
17.	Price of comarable tract, $ per acre.	__________	(STO) 15
18.	Capital gain tax rate, %	__________	(STO) 16
19.	Down payment, %	__________	(STO) 17
20.	Interest rate, % annum.	__________	(STO) 18
21.	Planning horizon, years.	__________	(STO) 19
22.	Amortization period, years.	__________	(STO) 20

OUTPUT

STEP	OUTPUT DESCRIPTION	PRESS	VALUE	RESULTS
1.	The maximum bid price $/ac.	A	_____	_______
	INPUT DESCRIPTION			
2.	Enter the price $/acre that will be used in the cash flow analysis.	(STO) 21	_____	
	OUTPUT DESCRIPTION			
3.	Annual loan payment (principal and interest).	B	_____	

CASH FLOW ANALYSIS

Note: To prepare an annual cash flow chart, enter the year you want to examine in (STO) 22. Then press (C) to get the unpaid balance at the end of that year. Press (D) and you will see the taxable income. Press (E) for income tax paid and (2nd) A for the net cash flow that year. Press (2nd) B for the inflated investment (market price) and press (2nd) for the equity (cost less principal paid plus inflation) use the chart as shown in the next page to record your data.

Table A:2 continue.

CASH FLOW CHART

Year	Unpaid Balance	Taxable Income.	Income Tax.	Net Cash Flow	Market Price	Equity

Table A. 3: The Program for the Subjective Probability Distribution.

"1" is Line No. "2" is Key Code "3" is Key

1	2	3	1	2	3	1	2	3
000	91	R/S	039	03	3	078	54	)
001	76	LBL	040	54	)	079	54	)
002	10	E'	041	75	-	080	65	×
003	47	CMS	042	53	(	081	53	(
004	25	CLR	043	53	(	082	53	(
005	91	R/S	044	43	RCL	083	43	RCL
006	76	LBL	045	01	01	084	03	03
007	11	A	046	65	×	085	33	X^2
008	53	(	047	53	(	086	54	)
009	02	2	048	43	RCL	087	75	-
010	55	÷	049	02	02	088	53	(
011	53	(	050	33	X^2	089	43	RCL
012	53	(	051	54	)	090	02	02
013	43	RCL	052	54	)	091	33	X^2
014	03	03	053	55	÷	092	54	)
015	75	-	054	02	2	093	54	)
016	43	RCL	055	54	)	094	75	-
017	01	01	056	85	+	095	53	(
018	54	)	057	53	(	096	02	2
019	65	×	058	53	(	097	55	÷
020	53	(	059	43	RCL	098	53	(
021	43	RCL	060	01	01	099	53	(
022	02	02	061	45	Y^X	100	43	RCL
023	75	-	062	03	3	101	03	03
024	43	RCL	063	54	)	102	75	-
025	01	01	064	55	÷	103	43	RCL
026	54	)	065	06	6	104	01	01
027	54	)	066	54	)	105	54	)
028	54	)	067	54	)	106	65	×
029	65	×	068	85	+	107	53	(
030	53	(	069	53	(	108	43	RCL
031	53	(	070	01	1	109	03	03
032	53	(	071	55	÷	110	75	-
033	43	RCL	072	53	(	111	43	RCL
034	02	02	073	43	RCL	112	02	02
035	45	Y^X	074	03	03	113	54	)
036	03	3	075	75	-	114	54	)
037	54	)	076	43	RCL	115	54	)
038	55	÷	077	01	01	116	65	×

Table A. 3: (cont'd)

1	2	3	1	2	3	1	2	3
117	53	(	156	42	STO	195	53	(
118	53	(	157	05	05	196	53	(
119	53	(	158	91	R/S	197	43	RCL
120	43	RCL	159	76	LBL	198	01	01
121	03	03	160	12	B	199	65	×
122	45	Y^X	161	53	(	200	53	(
123	03	3	162	02	2	201	43	RCL
124	54	)	163	55	÷	202	02	02
125	55	÷	164	53	(	203	45	Y^X
126	03	3	165	53	(	204	03	3
127	54	)	166	43	RCL	205	54	)
128	75	-	167	03	03	206	54	)
129	53	(	168	75	-	207	55	÷
130	53	(	169	43	RCL	208	03	3
131	43	RCL	170	01	01	209	54	)
132	02	02	171	54	)	210	85	+
133	65	×	172	65	×	211	53	(
134	53	(	173	53	(	212	53	(
135	43	RCL	174	43	RCL	213	43	RCL
136	03	03	175	02	02	214	01	01
137	33	X²	176	75	-	215	45	Y^X
138	54	)	177	43	RCL	216	04	4
139	54	)	178	01	01	217	54	)
140	55	÷	179	54	)	218	55	÷
141	02	2	180	54	)	219	01	1
142	54	)	181	54	)	220	02	2
143	85	+	182	65	×	221	54	)
144	53	(	183	53	(	222	54	)
145	53	(	184	53	(	223	85	+
146	43	RCL	185	53	(	224	53	[illegible]
147	02	02	186	43	RCL	225	02	2
148	45	Y^X	187	02	02	226	55	÷
149	03	3	188	45	Y^X	227	53	(
150	54	)	189	04	4	228	43	RCL
151	55	÷	190	54	)	229	03	03
152	06	6	191	55	÷	230	75	-
153	54	)	192	04	4	231	43	RCL
154	54	)	193	54	)	232	01	01
155	95	=	194	75	-	233	54	)

Table A. 3: (cont'd)

1	2	3	1	2	3	1	2	3
234	54	)	273	43	RCL	312	43	RCL
235	65	×	274	03	03	313	02	02
236	53	(	275	75	-	314	45	Y^X
237	53	(	276	43	RCL	315	04	4
238	53	(	277	02	02	316	54	)
239	43	RCL	278	54	)	317	55	÷
240	03	03	279	54	)	318	01	1
241	45	Y^X	280	54	)	319	02	2
242	03	3	281	65	×	320	54	)
243	54	)	282	53	(	321	54	)
244	55	÷	283	53	(	322	95	=
245	03	3	284	53	(	323	42	STO
246	54	)	285	43	RCL	324	06	06
247	75	-	286	03	03	325	43	RCL
248	53	(	287	45	Y^X	326	06	06
249	53	(	288	04	4	327	75	-
250	43	RCL	289	54	)	328	53	(
251	02	02	290	55	÷	329	43	RCL
252	45	Y^X	291	04	4	330	05	05
253	03	3	292	54	)	331	33	X²
254	54	)	293	75	-	332	54	)
255	55	÷	294	53	(	333	95	=
256	03	3	295	53	(	334	42	STO
257	54	)	296	43	RCL	335	07	07
258	54	)	297	02	02	336	91	R/S
259	75	-	298	65	×	337	76	LBL
260	53	(	299	53	(	338	13	C
261	02	2	300	43	RCL	339	53	(
262	55	÷	301	03	03	340	02	2
263	53	(	302	45	Y^X	341	65	×
264	53	(	303	03	3	342	53	(
265	43	RCL	304	54	)	343	43	RCL
266	03	03	305	54	)	344	05	05
267	75	-	306	55	÷	345	75	-
268	43	RCL	307	03	3	346	43	RCL
269	01	01	308	54	)	347	04	04
270	54	)	309	85	+	348	54	)
271	65	×	310	53	(	349	54	)
272	53	(	311	53	(	350	55	÷

Table A. 3: (cont'd)

1	2	3	1	2	3	1	2	3
351	43	RCL	390	91	R/S	429	65	×
352	07	07	391	76	LBL	430	43	RCL
353	95	=	392	16	A'	431	03	03
354	42	STO	393	01	1	432	65	×
355	08	08	394	42	STO	433	43	RCL
356	91	R/S	395	12	12	434	10	10
357	76	LBL	396	91	R/S	435	85	+
358	14	D	397	76	LBL	436	43	RCL
359	42	STO	398	17	B'	437	03	03
360	10	10	399	01	1	438	33	X²
361	43	RCL	400	75	-	439	54	)
362	03	03	401	53	(	440	95	=
363	32	X!T	402	01	1	441	42	STO
364	43	RCL	403	55	÷	442	13	13
365	10	10	404	53	(	443	91	R/S
366	77	GE	405	53	(	444	76	LBL
367	16	A'	406	43	RCL	445	18	C'
368	43	RCL	407	03	03	446	53	(
369	02	02	408	75	-	447	01	1
370	32	X!T	409	43	RCL	448	55	÷
371	43	RCL	410	01	01	449	53	(
372	10	10	411	54	)	450	53	(
373	77	GE	412	65	×	451	43	RCL
374	17	B'	413	53	(	452	03	03
375	43	RCL	414	43	RCL	453	75	-
376	01	01	415	03	03	454	43	RCL
377	32	X!T	416	75	-	455	01	01
378	43	RCL	417	43	RCL	456	54	)
379	10	10	418	02	02	457	65	×
380	77	GE	419	54	)	458	53	(
381	18	C'	420	54	)	459	43	RCL
382	76	LBL	421	54	)	460	02	02
383	15	E	422	65	×	461	75	-
384	00	0	423	53	(	462	43	RCL
385	42	STO	424	43	RCL	463	01	01
386	11	11	425	10	10	464	54	)
387	91	R/S	426	33	X²	465	54	)
388	43	RCL	427	75	-	466	54	)
389	11	11	428	02	2	467	65	×

Table A. 3: (cont'd)

1	2	3
468	53	(
469	43	RCL
470	10	10
471	33	X²
472	75	-
473	02	2
474	65	×
475	43	RCL
476	01	01
477	65	×
478	43	RCL
479	10	10
480	85	+
481	43	RCL
482	01	01
483	33	X²
484	54	)
485	95	=
486	42	STO
487	14	14
488	91	R/S

Table A. 3: (cont'd)

1	2	3
468	53	(
469	43	RCL
470	10	10
471	33	X^2
472	75	-
473	02	2
474	65	×
475	43	RCL
476	01	01
477	65	×
478	43	RCL
479	10	10
480	85	+
481	43	RCL
482	01	01
483	33	X^2
484	54	)
485	95	=
486	42	STO
487	14	14
488	91	R/S

Table A:4. Procedure to follow for the Subjective Probability Distribution.

Objective: To determine expected market price of land, variance, risk aversion coefficient of the investor and the probability that the market price is less than or equal to a given price.

STEP	INPUT DESCRIPTION	INPUT VALUE	PRESS
1.	Turn calculator off, and back on, to clear program and memory.		
2.	Partitioning memory (Note 639.39 should appear on the screen. If not , return to step 1.)		(4)(2nd) (OP)(17)
3.	Clear Display		(CLR)
4.	Insert side 1 of the cards containing the program (A:3). If the calculator has read the card successfully, a "1" will appear and remain stationary. If a flashing "0" appear repeat steps 3 and 4.		
5.	Clear Display		(CLR)
6.	Insert side 2 of the cards containing the program. If the calculator reads side 2 successfully, a "2" will appear and remain stationary. If a "0" appears, repeat steps 5 and 6.		

Table A:4 continue.

STEP	INPUT DESCRIPTION	INPUT VALUE	PRESS
7.	Clear Display		(CLR)
8.	Insert side 3 of the cards. If the calculator has read the card successfully, a "3" will appear and remain stationary. If a "0" flashing repeat steps 7 and 8.		
9.	Clear Display		(CLR)
10.	Insert side 4 of cards . If the calculator read the card successfully, a "4" should appear and remain stationary. If a "0" flashes on the display steps 9 and 10 should be repeated.		
11.	Clear Display		(CLR)
12.	Lowest price of one acre of land, $ per acre.	____________	(STO) 01
13.	Most likely price of one acre of land, $ per acre.	____________	(STO) 02
14.	Highest price of one acre of land, $ per acre.	____________	(STO) 03
15.	What the investor would like to pay for one acre of land, $ per acre.	____________	(STO) 04

Table A:4 continue.

RESULTS

STEP	OUTPUT DESCRIPTION	PRESS	RESULTS
1.	Expected Value of one acre of land, $ per acre.	A	________
2.	Variance of price of land	B	________
3.	Risk aversion coefficient	C	________
4.	Probability that the market price is less than or equal to some value $ X.	D	________

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