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OPTIMAL ALLOCATION OF INVESTMENTS IN A LEONTIEF INPUT-OUTPUT MODEL

By

Amor Farouk Benghezal

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ABSTRACT

OPTIMAL ALLOCATION OF INVESTMENTS IN A LEONTIEF INPUT-OUTPUT MODEL

By

Amor Farouk Benghezal

The purpose of this study was to develop a model based on Leontief input-output relationships that would help economists formulate plans of economic development. A predetermined amount of funds is available for investment in the planning period. Economic planners are faced with the question: 1) in which sectors funds should be allocated; 2) how much should be invested, and 3) when investments should be carried out. A mathematical program was formulated to answer these questions, where the objective function was to maximize value added by the whole economy in the planning period. Three types of constraints were used, namely, demand, capacity, and investment.

The contribution to economic development depends upon the sequence of investments made during the planning period. A theory of economic development recommends to underdeveloped countries to undertake projects in upstream industries (steel, chemicals, etc.). The implementation of the latter ones will provide inputs for downstream industries and thus will favor their development. To take into account the preponderance of foreign trade in underdeveloped economies two types of input-output coefficients are used, namely, domestic and foreign coefficients. In Chapter 2, a model with both types of coefficients is formulated, and a numerical example is solved by dynamic programming. Two elements prevented the author from applying the

dynamic programming solution to the Algerian plan; first, data used to compute both types of input coefficients are not available; second the tediousness of computations required to write a computer program that dealt with all details of the solution procedure. In Chapter 3, a mixed integer program was formulated for the Algerian plan and was solved using the APEX package available on the CDC 6500. In the mixed integer program the input-output coefficients used were those obtained from 1969 input-output table. The program was run first for a four year plan (1970-1973), and second for an eight year plan (1970-1977). The results obtained in the 1970-1973 period disagree with the claim that underdeveloped economies should invest in upstream industries (high capital ratio industries). On the contrary, they showed that investments must be made in downstream industries (low capital ratio and labor intensive sectors). The author thought this theory claim could be valid in the long run. So it was tested in the eight year plan (1970-1977). Here again (section 3.3.3), the results disapproved the implementation of projects in upstream industries in the early years of the plan. However after 1974 high capital ratio projects showed up in the optimal solution. Two elements explained this situation. First, funds available for investment in the 1974-1977 period were two and a half times greater than in the preceeding period. Second, low capital ratio sectors did not need that many funds for investment to satisfy the demand requirements imposed on them. Therefore more funds were available for spending in upstream industries. In Chapter 4, limits to the mixed integer program results were reviewed and some suggestions were made.

To my father Mohammed, who taught me
that education is the best asset one
can have -- 1901-1970

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INTRODUCTION

Economic theory has described underdeveloped countries as non-integrated, dominated, lacking savings for investment purposes, having food shortages, and so on.[1] It has also suggested different economic policies to answer the key question underdeveloped countries face; how to allocate resources to achieve economic development? Each of these policies has its own strength and weakness. It is not this research purpose to examine these policies.

Many investment opportunities exist in underdeveloped countries, because many economic activities are lacking, or, are at a primitive stage. Since funds (available for investment) are in short supply, all of the investment possibilities cannot be funded. At a micro-economic level, they could be ranked according to their rates of return. But at a macro economic level, this would not work, because relationships among activities are ignored. If it were used, it would be likely that most of the funds would be spent in few economic activities belonging to the same sector (the clothing sector is an example)[2].

---

[1] Samir Amin, Accumulation on a world scale; a critique of the theory of underdevelopment (New York: Monthly Review Press, 1974). Charles Bettelheim, Planification et croissance acceleree (Paris: F. Maspero, 1965).

[2] This does not mean that the rate of return method could not be used once the funds allocated to a given sector are known.

However, we would like to suggest a model that could help in answering the question of allocating resources--especially investment to bring about economic growth. The Leontief's input-output analysis provides the framework of the model.[1] Input-output analysis shows the relationships among the different sectors of the economy. Some economists have suggested the use of Leontief's system for the formulation of economic development plans.[2]

The purpose of the model that will be presented is to find the optimal sequence of investments that will achieve the goals economic planners set. The model has been formulated in a mathematical program. Its solution can provide planners with an answer to the following three questions:

1) how much must be invested?

2) when must investments be made?

3) in which sectors must investments be made?

This research contains three chapters:

- the first one presents the Leontief's model, some of the research on resource allocation, and the research goals.

- the second chapter deals with the dynamic programming approach to solving the model. A numerical example is carried out to show how the method of solution works.

- the third one presents another approach to the same model. It

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[1]Wassily W. Leontief, The structure of the American economy: 1919-1939 (2d ed., New York: Oxford University Press, 1951).

[2]Oskar R. Lange, Essays on economic planning (Bombay, New York: Asia Publishing House, 2d ed., Calcuta: Statistical Pub. Society, 1967). Mohamed Dowidar, Schemas de la reproduction et la methodologie de la planification socialiste (Algiers: Editions du Tiers Monde, 1964).

is formulated as a mixed integer program. It is applied to the Algerian planning problem.

CHAPTER I

INPUT-OUTPUT MODELS

Input-output models have received a great deal of attention since the publication of Leontief's work on the American economy.[1] Leontief was interested in presenting a sort of balance sheet of the economy: the output and its distribution among the different sectors of the economy. Leontief prepared these balance sheets in table form for the United States for 1919, 1929, and 1939. The input-output tables are very useful in economic planning. In the following, firstly, the Leontief model is presented; secondly, some research works made in the field of input-output analysis are reviewed; thirdly, the purpose of the present research is discussed.

1.1 The Leontief Model

Leontief model views the economy as being composed of several sectors such as, agriculture, mining, steel, chemicals, etc. Let us suppose there are n sectors in the economy. The main idea of input-output models is to show the general dependence among sectors. This dependence among sectors is quantified and is expressed by the flow of goods from one sector to the others.

Let x_i be the total output of sector i. Let x_{ij} denote that

---

[1] Wassily W. Leontief, The Structure of the American Economy 1919-1939 (2d ed. enl.; New York: Oxford University Press, 1951).

part of the total output retained by sector i for its own use.

In general, let x_{ij} be the value of goods of sector i consumed by sector j for production in sector j and y_i be the value of net output. One has the following equality

$$x_i = \sum_{j=1}^{n} x_{ij} + y_i \qquad i = 1, \ldots n \qquad (1.1)$$

Equality (1.1) says the output of sector i is the sum of the flows of sector i goods to all sectors and the net output.

Viewed differently, to get output x_j, sector j must obtain goods from all sectors i in an amount equal to $\sum_{i=1}^{n} x_{ij}$ and sector j must add salaries and return on capital. Let w_j be the wages paid in sector j and p_j the return on capital.

The following equality is obtained.

$$x_j = \sum_{i=1}^{n} x_{ij} + w_j + p_j \qquad j = 1, \ldots n \qquad (1.2)$$

The sum of w_j and p_j is the value added of sector j. With Equalities (1.1) and (1.2) one is able to construct the Leontief Input-Output Table. Let x be the total output of the economy, in other words, the Gross National Product.

$$x = \sum_{i=1}^{n} x_i = \sum_{j=1}^{n} x_j$$

Using Equalities (1.1) and (1.2) one gets:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij} + \sum_{i=1}^{n} y_i = \sum_{j=1}^{n}\sum_{i=1}^{n} x_{ij} + \sum_{j=1}^{n} w_j + \sum_{j=1}^{n} p_j$$

Since $\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n}\sum_{i=1}^{n} x_{ij}$, one obtains

Table 1.1 The Leontief Input-Output Table

sectors	1	2	---	n	net output	total output
1	x_{11}	x_{12}		x_{1n}	y_1	x_1
2	x_{21}	x_{22}		x_{2n}	y_2	x_2
-						
-						
-						
n	x_{n1}	x_{n2}		x_{nn}	y_n	x_n
Wages	w_1	w_2		w_n		
Return on Capital	p_1	p_2		p_n		
Total Output	x_1	x_2		x_n		x

$$\sum_{i=1}^{n} y_i = \sum_{j=1}^{n} w_j + \sum_{j=1}^{n} p_j \qquad (1.4)$$

Equality (1.4) states the net output of the economy (Σy_i) is equal to the sum of the value added of all sectors.

Now, let us define a_{ij} as the ratio of the value of inputs i consumed in sector j to the output of sector j.

$$a_{ij} = \frac{x_{ij}}{x_j} \qquad i = 1 \ldots n \qquad (1.5)$$

From the above ratio, it results

$$x_{ij} = a_{ij}\, x_j \qquad (1.6)$$

Making use of Equation (1.6) in Equality (1.1) one gets:

$$x_i = \sum_{j=1}^{n} a_{ij}\, x_j + y_i, \qquad i = 1 \ldots n \qquad (1.7)$$

or, in matrix form

$$X = AX + Y, \tag{1.8}$$

where X is an n-component column vector, A is an n x n matrix with a_{ij} as elements, and Y is an n-component column vector.

If a matrix A of input coefficients and a set of values for Y are given, the level of output X will be obtained right away, by solving

$$X = (I - A)^{-1} Y \tag{1.9}$$

This model has n equations and n unknowns (x_i), and has a unique solution if $(I - A)^{-1}$ exists. If the y_i are the n unknowns, one can use Equation (1.8) to find them.

The Leontief model is very useful in planning of economic activities. If the planning authorities decide upon a given level of output for each activity (x_i), they can obtain the levels of net output for all activities (y_i). In the following sections, some of the research works made on the input-output models are presented.

1.2 Maximization of the growth rate of net output

Lange formulated a model where the goal is to maximize the growth rate of net output by the end of the planning period using a Leontief type of model.[1]

His objective was to maximize $Z = \frac{\Delta Y}{Y}$, where ΔY is the increase of net output between the year zero and the ending year of the plan and Y is the net output in year zero. The growth rate of net output can be expressed as

---

[1]Oskar R. Lange, Lecons d'Econometrie (Paris: Gauthier-Villars, 1970).

$$\frac{\Delta Y}{Y} = \frac{\Delta Y}{I}\frac{I}{Y}$$

where I is the total investment that should be made during the planning period.

If one is given values of I and Y, the only way to achieve the highest growth rate of net output is to maximize the ratio $\frac{\Delta Y}{I}$.

The constraints of the maximization problem are: the final consumptions of goods, produced by each sector, should be at least equal to some predetermined level for every period of the plan.

Suppose there are n sectors. Let C_{it} be the final consumption of goods of sector i in period t. Let K_{it} be the minimum level of final consumption of sector i goods in period t. The following must hold for each period:

$$C_{it} \geq K_{it} \qquad i = 1 \ldots n \qquad (1.11)$$

Net output of sector i in period t (Y_{it}), must equal the sum of the final consumption of sector i goods, C_{it}, and the amount invested in sector i goods, I_{it}.

$$Y_{it} = C_{it} + I_{it} \qquad i = 1 \ldots n \qquad (1.12)$$

Inequality (1.11) and Equality (1.12) imply that

$$I_{it} \leq Y_{it} - K_{it} \qquad i = 1 \ldots n \qquad (1.13)$$

Hence, Lange's problem reduces to the following linear program.

$$\max Z = \frac{\Delta Y}{I}$$

$$\text{subject to} \quad I_{it} \leq Y_{it} - K_{it}$$

In this LP (linear program), it is not clear what the decision variables are. Also it does not show any resemblance to Leontief model. In order to arrive at an LP containing the input-output coefficients, Lange developed a new formulation by modifying the objective function and the left hand sides of the constraints inequalities along the following lines.

Prior to discussing these modifications, it is useful to define the concept of investment cycle. It is the time elapsed between the moment the investment is made and the moment an increase of the output is obtained. Let I_{ijst} be the investment needed by sector j, in terms of sector i net output, in period t, and of cycle s years.

ΔX_{jst} is the increase in output of sector j obtained after s years due to an investment made in period t. By definition

$$\Delta X_{jst} = X_{j,t+s} - X_{jt} \tag{1.14}$$

and let

$$b_{ijst} = \frac{I_{ijst}}{\Delta X_{jst}} \tag{1.15}$$

b_{ijst} is the ratio of investment of cycle s made by sector j in sector i net output in period t, to the increase of sector j output after s years.

Let I_t be the total investment made by the whole economy in period t.

Let $I^{(jst)}$ be the total investment of cycle s made in sector j in period t.

Let

$$\lambda_{jst} = \frac{I^{(jst)}}{I_t} \qquad j = 1 \ldots n \qquad (1.16)$$

λ_{jst} is the ratio of sector j investment (of cycle s) in period t, to the total investment of year t. From Ratio (1.15) one gets

$$\sum_{i=1}^{n} I_{ijst} = \sum_{i=1}^{n} b_{ijst} \Delta X_{jst}$$

Since $\sum_{i=1}^{n} I_{ijst} = I^{(jst)}$

one has: $I^{(jst)} = \sum_{i=1}^{n} b_{ijst} \Delta X_{jst}$

or $\Delta X_{jst} = I^{(jst)} \beta_{jst}$, where $\beta'_{jst} = \frac{1}{\sum_{i=1}^{n} b_{ijst}}$ (1.17)

Let a_{jit} be the ratio of the amount of sector j output needed in sector i, to the output of sector i in period t

$$a_{jit} = \frac{X_{ijt}}{X_{it}} \qquad (1.18)$$

Let σ_{jst} be that part of sector j increase of output in s years from year t, used to achieve the increase in output in the other sectors during the same period.

$$\sigma_{jst} = \frac{\sum_{i=1}^{n} a_{ij,t+s} \Delta X_{ist}}{\Delta X_{jst}} \qquad (1.19)$$

Let ΔY_{jst} be the increase in net output of sector j, obtained after s years, due to investments made in year t.

$$\Delta Y_{jst} = \Delta X_{jst} - \sum_{i=1}^{n} a_{ji,t+s} \Delta X_{ist} \qquad (1.20)$$

Using Ratio (1.19), Equality (1.20) becomes

$$\Delta Y_{jst} = \Delta X_{jst} (1 - \sigma_{jst}) \qquad (1.21)$$

Making use of Equality (1.17), the equality above becomes

$$\Delta Y_{jst} = (1 - \sigma_{jst}) \beta_{jst} I^{(jst)} \qquad (1.22)$$

or

$$\Delta Y_{jst} = \beta'_{jst} I^{(jst)}, \text{ where } \beta'_{jst} = (1-\sigma_{jst}) \beta_{jst} \qquad (1.23)$$

After using Ratio (1.16) and Equality (1.23)

$$\Delta Y_{st} = \sum_{j=1}^{n} \Delta Y_{jst} = I_t \sum_{j=1}^{n} \beta'_{jst} \lambda_{jst} \qquad (1.24)$$

To obtain the increase in net output during the planning period, a k year plan of investments is considered. These investments have different cycles s and are made in the various sectors j (j = 1 . . . n) in year t, t = 1, 2, . . . k - 1. In fact, it is assumed that no investment is made in year k, since the increase in net output yielded by investments in year k will only show up in year k + 1. Hence in the first year, only investments having cycles s = 1, 2 . . . k-1, need be considered in the second year, only investments of cycles s = 1, 2 . . . k-2 are taken into account. Likewise for the (k - 1)th year, only investments of cycle s = 1 need be considered.

With the above considerations in mind, the increase of net output,

for the whole economy in the second year of the plan, is $\Delta Y_1 = Y_2 - Y_1$. This is the result of investments of cycle s = 1. Now, using the notation of Equality (1.24), one has

$$\Delta Y_1 = \Delta Y_{11}$$

where s = t = 1.

Following the same reasoning, the increase of net output, in the third year of the plan, is due to investments of cycle s = 1 made in the second year and of cycle s = 2 made in the first year.

$$\Delta Y_2 = \Delta Y_{12} + \Delta Y_{21}$$

In general, the increase of net output: $\Delta Y_t = Y_{t+1} - Y_t$, obtained in year (t+1) is

$$\Delta Y_t = \sum_{s=1}^{t} \Delta Y_{s,t-s+1} \qquad (1.25)$$

The increase of net output, after k years, is the sum of the increases during the different years. Let this increase be

$$\Delta_k Y = Y_k - Y_1$$

it follows,

$$\Delta_k Y = \sum_{t=1}^{k-1} \Delta Y_t \qquad (1.26)$$

After using Equality (1.25), Equality (1.26) becomes

$$\Delta_k Y = \sum_{t=1}^{k-1} \sum_{s=1}^{t} \Delta Y_{s,t-s+1} \qquad (1.27)$$

The former relation shows the increase in net output after k years depends on the increases of net output, due to investments of

different cycles made in the various years of the plan.

Now, introducing Equality (1.24) into Equality (1.27), one obtains

$$\Delta_k Y = \sum_{t=1}^{k-1} \sum_{s=1}^{t} \sum_{j=1}^{n} I_{t-s+1} \lambda_{js,t-s+1} \beta'_{js,t-s+1} \qquad (1.28)$$

$I = \sum_{t=1}^{k-1} I_t$ is the total investment, and also the sum of the investments made up to year k-1.

$$\text{Let } \alpha_{jst} = \frac{I^{(jst)}}{I} \quad j = 1 \ldots n;\ s,t = 1 \ldots k-1 \qquad (1.29)$$

$$\text{with } \sum_{t=1}^{k-1} \sum_{s=1}^{t} \sum_{j=1}^{n} \alpha_{jst} = 1$$

$$\text{since } \lambda_{jst} = \frac{I^{(jst)}}{I_t}, \text{ from Ratio (1.16)}$$

Ratio (1.29) is rewritten as

$$\alpha_{jst} = \frac{\lambda_{jst} I_t}{I} \qquad (1.30)$$

From Ratio (1.30), it follows

$$I_t \lambda_{jst} = \alpha_{jst} I \qquad (1.31)$$

Making use of Equality (1.31) in Equality (1.28), one gets

$$\Delta_k Y = I \sum_{t=1}^{k-1} \sum_{s=1}^{t} \sum_{j=1}^{n} \alpha_{js,t-s+1} \beta'_{js,t-s+1} \qquad (1.32)$$

Since the objective is to maximize $\frac{\Delta Y}{I}$, the objective function becomes

$$\frac{\Delta_k Y}{I} = \sum_{t=1}^{k-1} \sum_{s=1}^{t} \sum_{j=1}^{n} \alpha_{js,t-s+1} \beta'_{js,t-s+1} \qquad (1.33)$$

Next, the constraints expressed by Inequality (1.13) are discussed; these are $I_{it} \leq Y_{it} - K_{it}$.

Since $I_{it} = \sum_{s=1}^{k-1} \sum_{j=1}^{n} I_{ijst}$, the investment, in period t in sector i net output, is equal to the sum of all investments of cycles s = 1 . . . k - 1 made by all sectors j in period t, and in sector i net output.

Using ratio (1.15), this equality becomes

$$I_{it} = \sum_{s=1}^{k-1} \sum_{j=1}^{n} b_{ijst} \Delta X_{jst} \tag{1.34}$$

Substituting for ΔX_{jst} from Equality (1.17), this relationship becomes

$$I_{it} = \sum_{s=1}^{k-1} \sum_{j=1}^{n} b_{ijst} \beta_{jst} I^{(jst)} \tag{1.35}$$

Combining Ratios (1.16) and (1.30), it results

$$I^{(jst)} = I\alpha_{jst} \tag{1.36}$$

Using Equality (1.36) into Equality (1.35), one obtains

$$I_{it} = I \sum_{s=1}^{k-1} \sum_{j=1}^{n} b_{ijst} \beta_{jst} \alpha_{jst} \tag{1.37}$$

Now, the modified linear program that must be solved can be stated as:

$$\text{Max } Z = \sum_{t=1}^{k-1} \sum_{s=1}^{k-1} \sum_{j=1}^{n} \beta'_{js,t-s+1} \alpha_{js,t-s+1}$$

subject to

$$I \sum_{s=1}^{k-1} \sum_{j=1}^{n} b_{ijst}\, \beta_{jst}\, \alpha_{jst} \leq Y_{it} - K_{it}$$

$$i = 1 \ldots n$$

$$t = 1 \ldots k-1$$

$$\sum_{t=1}^{k-1} \sum_{s=1}^{k-1} \sum_{j=1}^{n} \alpha_{jst} = 1$$

$$\alpha_{jst} \geq 0 \qquad j = 1 \ldots n;\ s,t = 1 \ldots k-1$$

α_{jst} are the decision variables of this LP. They correspond to the part of total investment, in the k year plan, made in sector j in period t, and having a cycle of s years. The investment coefficients b_{ijst} appear in the constraints. The input-output coefficients a_{jit} do not appear directly in the LP, but must be used to compute β'_{jst} in the objective function (see Ratio (1.19)).

Once a set of α_{jst} is found that maximizes the objective function, one can determine how much must be invested, in each sector every year, by computing $I^{(jst)} = I\, \alpha_{jst}$ from Ratio (1.29).

Since

$$\beta'_{jst} = \frac{\Delta Y_{jst}}{I^{(jst)}}, \text{ from Equality (1.23)}$$

β'_{jst} can be interpreted as the increase of net output, in sector j, due to an investment of cycle s made in period t. β'_{jst} is defined as the investment efficiency of cycle s in sector j. In fact, the objective is to maximize $\beta' = \frac{\Delta_k Y}{I}$, i.e., the investment efficiency in the whole economy.

In summary, Lange developed a model, using Leontief type of relationships between sectors, that maximizes the investment efficiency in the whole economy subject to the constraints that investments in each sector goods, in each year, should not exceed some predetermined level (i.e. $I_{it} \leq Y_{it} - K_{it}$).

In the next section, a more general type of Leontief model is presented that includes sector capacity.

1.3 Introduction of Capacity Building in a Dynamic Leontief Model

Wagner formulated a Leontief model[1] that included both inventories and capacity of sectors.

Assume, for any period t, there is an n x n matrix of input-output coefficients A, where the j^{th} column of A represents the inputs from each of the n sectors required to produce one unit of the j^{th} sector output. Also, consider an n dimensional square matrix B of capital coefficients, where the j^{th} column of B represents the inputs from each sector needed to build one unit of capacity for the j^{th} sector.

In the following vector notation the subscript t denotes the relevant time period. All vectors are n demensional column vectors:

f_t = final demand

c_1 = initial capacity

x_t = level of production

l_t = level of capacity being built (so Bl_t are the direct capital requirements for the creation of l_t)

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[1] Harvey M. Wagner, "A linear programming solution to dynamic Leontief type model," Management Science, Vol. 3, No. 3, April 1957.

s_t = level of inventories at the end of period t

u_t=level of unused capacity.

The output of a sector and its beginning inventories are equal to the sum of intermediate demand, final demand, investment requirements and ending inventories. In vector notation, this can be expressed by the following set of linear equations.

$$x_t + s_{t-1} = Ax_t + Bl_t + f_t + s_t \quad t = 1 \ldots T \tag{1.38}$$

By definition, unused capacity in a sector is the difference between available capacity and capacity used up for production.

$$U_t = C_t - X_t \qquad t = 1 \ldots T \tag{1.39}$$

In any time period, the capacity level is the sum of the capacity existing in the previous period and any increase in capacity undertaken in the previous period:

$$c_t = c_{t-1} + l_{t-1}$$

or $$c_t = c_1 + \sum_{r=1}^{t-1} l_r \qquad t = 2 \ldots T \tag{1.40}$$

Combining Equalities (1.39) and (1.40), one obtains

$$c_1 = x_t + u_t - \sum_{r=1}^{t-1} l_r \qquad t = 2 \ldots T \tag{1.41}$$

Assuming values for final demand f_t, beginning inventories s_o and initial capacity c_1, the model given by Equalities (1.38) and (1.41) can be represented as follows:

$$f_t = (I - A)\, x_t - Bl_t + s_{t-1} - s_t, \quad t = 1 \ldots T \tag{1.42}$$

$$c_1 = x_t + u_t - \sum_{r=1}^{t-1} l_r \qquad t = 2 \ldots T \tag{1.41}$$

Because of the very nature of the economic activities, it is further required that

$$x_t, l_t, s_t, u_t \geq 0 \tag{1.43}$$

Equations (1.42), (1.41) and restrictions (1.43) in the model satisfy the standard form required for the application of linear programming. The following figure gives a starting simplex tableau.

Decision Variables

	x_1	l_1	s_1	u_1	x_2	l_2	s_2	u_2	x_3	l_3	s_3	u_3	→
$f_1 - s_o$	(I-A)	-B	-I										
c_1	I			I									
f_2			I		(I-A)	-B	-I						
c_1		-I			I			I					
f_3							I		(I-A)	-B	-I		
c_1		-I				-I			I			I	
↓		↓				↓				↓			

Note, each line of symbols in the tableau actually represents n separate linear constraints, one for each sector in period t. Hence there are 2nT constraints. Similarly, each column heading is a vector of n elements, one for each sector in period t. There are 4nT

unknowns. The previous tableau may be regarded as a matrix equation $Y = HZ$ where Y is the entire column on the left of the tableau, H is the body of coefficients and Z is the entire top of column headings.

An inspection of the model reveals that if $c_1 \geq (I-A)^{-1} f_t$, for all t, then feasibility is obviously possible by setting $x_t = (I-A)^{-1} f_t$, because production is undertaken under capacity ($c_1 \geq x_t$). However, if $c_1 < (I-A)^{-1} (f_1-s_0)$ then feasibility is not possible. The reason for this becomes clear by examining Equation (1.42)

$$f_1-s_0 = (I-A)\, x_1 - Bl_1 - s_1 \leq (I-A)\, x_1, \text{ since } l_1 \text{ and } s_1 \geq 0;$$

hence, $x_1 \geq (I-A)^{-1} (f_1-s_0)$.

Since $(I-A)^{-1} (f_1-s_0) > c_1$, it follows that $x_1 > c_1$: the production level in period 1 is above capacity. One is faced with an inconsistency since $x_t \leq c_t$ (see Equality (1.41)).

A real problem is the case in which the first period demand is feasible, but $(I-A)^{-1} f_{t'}$ exceeds initial capacity for some $t' > 1$. It is here that the economy must be able either to stockpile in earlier periods or to build additional capacity to meet future demand. If a feasible solution of this case exists, it may not be unique. Therefore, by defining an appropriate objective function, the optimal solution, with respect to the objective function, can be found.

Let us define an m dimensional vector of constants K, where m is equal to 4nT, and V as a vector of these 4nT decision variables (x_t, l_t, s_t, u_t).

The objective function is: $Z = KV$ (1.44)

K can be specified as a vector of manpower input figures

associated with x_t and l_t.

$$K = (K_{11}, K_{12}, 0, K_{21}, K_{22}, 0, \ldots K_{T1}, K_{T2}, 0),$$

where each K_{ti} is a row vector in n manpower coefficients for period t with either x_t $(i = 1)$ or l_t $(i = 2)$ and each 0 is a 2n dimensional vector of zeros. An optimal solution may be found by

$$\min Z = \sum_{t=1}^{T} (K_{t1}\, x_t + K_{t2}\, l_t)$$

subject to constraints (1.42) and (1.41).

K_{T1} can be thought of as the cost of producing an output level of x_t and K_{T2} as the cost of creating a capacity equal to l_t.

If maximizing the total amount of new capacity in the i^{th} sector by the end of period T is sought, K in (1.44) will become K = (0, 1, 0, 1, . . . 0), where the scalar one appears in all positions corresponding in V to l_{ti}, the i_{th} component of l_t. So the objective function becomes

$$Z = \sum_{t=1}^{T} l_{ti}.$$

The model, represented by equations (1.42) and (1.41) contains 2nT equations and 4nT unknowns. Wagner explored the possibility of reducing the model, to one containing only nT equations in 3nT unknowns and hence, one that is computationally easier to handle than the larger model. For this, Equation (1.41) is multiplied by -(I-a) and added to Equation (1.42);

$$f_t - s_{t-1} - (I-A)\, c_1 = -Bl_t - s_t + \sum_{r=1}^{t-1} (I-A)\, l_r - (I-A)\, u_t \quad (1.45)$$

Thus, x_t is eliminated in the model. Since the model is less constrained than the former one, a larger maximum (or smaller minimum) value for Z should be expected in the abbreviated system than would be in the larger and more restrictive system. If the optimal solution to the abbreviated system, when substituted in Equation (1.41), yields all $x_t \geq o$, then by definition this solution is feasible in the general model. It is also optimal, since the optimal Z in the general model is less than or equal to the optimal Z in the abbreviated system.

In the case, where some components of x_t are negative, say x_{tj}, Wagner showed that the input-output production relationship in sector j is reversed in period t, so, x_{tj}, becomes the input and $A_j x_{tj}$ (A_j being the j^{th} column of A) becomes the output. The author explained the negativity of x_{tj}as follows.

"An input of the j^{th} good implied by the negativity of x_{tj} must come from an existing stockpile of the j^{th} good and not from any production in period t. The stockpile has only one possible origin: it must have been created by production during previous periods in the model. The correctness of the value of Z despite the fact that x_{tj} is negative can be explained as follows. The abbreviated model has used a "short-hand" way of storing raw materials. In period t, the model indicated used the vector of inputs of industry j's product in the amount equal to $A_j x_{tj}$ as raw materials for other industries' production. Instead of storing these raw material inputs in the earlier periods when they became available, the model embodied them in the form of the j^{th} industry's product and in period t reversed the productive process. Since the stockpile used in t was actually produced earlier, one may easily translate the "short-hand" program of the

abbreviated system to not produce the stockpile of the j^{th} industry's good in previous periods but rather to save the inputs for use in t."[1]

Moreover, Wagner discussed a general algorithm, which involved the dual problem for proceeding from an optimal solution in the abbreviated system to an optimal feasible solution in the more restrictive system. He argued the computational savings, due to utilizing the abbreviated system, should more than offset the inconvenience of having to switch at the end to the dual algorithm.

Wagner's model is very general and can accommodate any type of objective function, as long as the vector K in the objective function is properly defined. Next, a model of economic development based on Leontief's type of relationships is presented. It introduces other constraints in the model such as foreign exchange, a limited resource of underdeveloped countries.

1.4 A Model for Resource Allocation

The primary purpose of Chenery and Kretschmer's study[2] is to develop a model, based on mathematical programming and input-output analysis, which will assist in the formulation of economic development programs. The authors introduce imports and exports in their analysis; these are important activities when underdeveloped economies are considered. Their model tries to minimize the investment needed to achieve certain goals in the target year.

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[1] Harvey M. Wagner, "A linear programming solution to dynamic Leontief type model." Management Science, 3, No. 3, 1957, pp. 245-246.

[2] Hollis Chenery and Kenneth Kretschmer, "Resource allocation for economic development," Econometrica, Vol. 24, No. 4, October 1956.

Let n be the number of sectors, and let us use their notation.

Y_i' is the total final demand for sector i in the target year;

Y_i is the net final demand for sector i to be satisfied from increased capacity or imports;

$\overline{X}_i$ is the existing productive capacity in sector 1;

X_i is the increase in production in sector i above existing capacity;

M_i is the level of imports in sector i measured in domestic prices;

E_i is the level of exports in sector i measured in domestic prices;

L* is the labor available for increasing production;

D* is the maximum foreign trade deficit;

d_i is the ratio of increased production to increased production plus imports $d_i = X_i/(X_i + M_i)$;

m_i is the ratio of imports to increased production for sector i ($m_i = M_i/X_i$, $X_i \neq 0$);

$\overline{a}_{ij}$ is the amount of input i currently used in producing one unit of output from sector j (current input coefficient);

a_{ij} is the increase in use of input i required to increase the production of sector j by one unit (marginal input coefficient);

ω_i is the marginal labor input coefficient for sector i;

c_i is the amount of capital needed per unit increase in capacity for domestic use in sector i (marginal capital coefficient);

k_i is the amount of capital needed per unit increase in capacity for export in sector i;

h_i is the price of exports from sector i in foreign currency;

g_i is the ratio of import prices to domestic prices in sector i at the given exchange rate;

It is assumed that the goods and services required in the economy are divided into n commodity groups corresponding to the n sectors of production. The final demands Y_i are taken as given. The net final demand that must be satisfied in each sector from new capacity or imports (after allowing for full operation of existing capacity) is

$$Y_i = Y_i' - (\bar{X}_i - \sum_{j=1}^{n} \bar{a}_{ij} \bar{X}_j), \quad i = 1 \ldots n \qquad (1.46)$$

where $(\bar{X}_i - \sum_{j=1}^{n} \bar{a}_{ij} \bar{X}_j)$ is the amount that could be supplied for final use from existing capacity after deducting intermediate uses $(\sum_j \bar{a}_{ij} \bar{X}_j)$.

The authors made the following assumptions. First, an economic sector can be thought of as an aggregate of productive subsectors corresponding to a single commodity. Second, in each subsector there is a choice between imports and only one domestic activity. Third, the input coefficients, for all domestic activities in a given sector, are identical for each input except capital. So, the only difference between subsectors is due to capital coefficients. The authors argue, for any given set of shadow prices, it is efficient to import those commodities for which the corresponding domestic activity is unprofitable and to produce the rest. However, under the second and third assumption, it is possible to rank subsectors according to increasing capital coefficients alone, since the other coefficients are the same. Therefore, the capital coefficient of a sector can be derived from the

coefficients of the subsectors, taken in a definite order.

In general, the authors conclude, the average capital coefficient is an increasing function of the ratio d_j, of total needs supplied domestically in sector j. For simplicity, this function is assumed linear.

$$c_j = \alpha_j + \beta_j d_j \quad , \quad \alpha_j \geq 0 \quad \beta_j > 0 \tag{1.47}$$

The total supply of a given output $(X_i + M_i)$ less the intermediate uses in domestic and export activities is equal to the net final demand:

$$X_i + M_i - \sum_{j=1}^{n} a_{ij} X_j - \sum_{j=1}^{n} a_{ij} E_j = Y_i \tag{1.48}$$

The last equation is the familiar balance equation of the Leontief system, except that, production for export is considered as a separate sector from domestic production of the same commodity.

The price received for exports is assumed to vary with the quantity exported. This function is also supposed to be linear.

$$h_j = \gamma_j + \rho_j E_j \qquad \gamma_j > 0, \quad \rho_j < 0 \tag{1.49}$$

Furthermore, the authors consider two resource limitations:[1] labor and foreign exchange. The labor constraint is

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[1] The authors stated: "The introduction of more resource limitations such as natural resources can be handled quite easily because they will usually affect only a small number of sectors, often only one. In this latter case the resource limitation can be incorporated in the suboptimization procedure for the sector in question without the necessity of adding another equation." Chenery, Kretschmer, p. 376.

$$\sum_{i=1}^{n} \omega_i X_i + \sum_{i=1}^{n} \omega_i E_i \leq L^* \qquad (1.50)$$

The maximum trade deficit constraint is

$$\sum_{i=1}^{n} g_i M_i - \sum_{i=1}^{n} h_i E_i \leq D^* \qquad (1.51)$$

The purpose of the economic development program is to choose increases in capacity (X_i), import levels (M_i) and export levels (E_i) in such a way the given final demand will be supplied in the target year with a minimum total investment, without exceeding the specified resource limitations. So, the problem consists of

$$\text{Min } Z = \sum_{i=1}^{n} c_i X_i + \sum_{i=1}^{n} k_i E_i = \sum_{i=1}^{n} \left[\alpha_i + \beta_i \left(\frac{X_i}{X_i + M_i}\right)\right] X_i + \sum_{i=1}^{n} k_i E_i \qquad (1.52)$$

subject to Constraints (1.48), (1.50) and (1.51). $X_i \geq 0$, $M_i \geq 0$, $E_i \geq 0$. This is a non-linear programming problem, since the objective function is non-linear.

If a solution exists, as Kuhn and Tucker have shown, there is a set of nonnegative numbers (Lagrange multipliers) which may be interpreted as unit prices of the scarce resources. It is assumed that a value P_j is imputed to the j^{th} product, P_w is imputed to labor and a value of P_f is imputed to foreign exchange. These values are the shadow prices and must satisfy the following properties.

- "They must be nonnegative.
- If the supply of labor or foreign exchange is not fully

utilized for an optimum production plan, then its price should be zero.

- Each commodity should be assigend a value equal to the largest decrease in investment cost which could be obtained if one less unit of that commodity were demanded, labor (or foreign exchange) should be assigned a value equal to the largest decrease in investment cost which could result if one more unit of labor (or foreign exchange) were made available."[1]

The authors assume that shadow prices with the above properties exist and are known. Chenery and Kretschmer determine the values of $\hat{X}_i$, $\hat{M}_i$ and $\hat{E}_i$ minimizing the objective function as follows.

They claim, the optimum export levels $\hat{E}_j$ are obtained at the point where $\hat{E}_j$ maximize the difference between the revenue and the cost of export activities. Let

$$\begin{aligned} R_j(E_j) &= h_j P_f E_j - \left(\sum_i^n a_{ij} P_i + \omega_j P_w + k_j\right) E_j \\ &= (\gamma_j + \rho_j E_j) P_f E_j - \left(\sum_i^n a_{ij} P_i + \omega_j P_w + k_j\right) E_j \end{aligned} \tag{1.53}$$

where $\left(\sum_i^n a_{ij} P_i + \omega_j P_w + k_j\right)$ is the unit cost of exporting one unit of project j.

Since $P_f > 0$, $R_j''(E_j) = 2 \rho_j P_f < 0$. Hence, the unrestricted maximum of $R_j(E_j)$ is attained at

$$r_j = \frac{1}{2 \rho_j P_f} \left(\sum_i^n a_{ij} P_f + \omega_j P_w + k_j - \gamma_j P_f\right) \tag{1.54}$$

for $R_j'(r_j) = 0$. Hence the maximum of $R_j(E_j)$ for $E_j \geq 0$ is attained

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[1] Chenery, Kretschmer, p. 377.

at

$$\hat{E}_j = \begin{cases} 0 & r_j \leq 0 \\ r_j & r_j \geq 0 \end{cases} \qquad (1.55)$$

Next, the authors want to determine the optimal proportion of domestic production $\hat{d}_j$ needed to compute $\hat{X}_i$ and $\hat{M}_i$. For this, the total cost of producing the fraction d_j ($d_j = \frac{X_i}{X_i + M_i}$) domestically and importing the fraction $(1 - d_j)$ is

$$T_j(d_j) = (\sum_i^n a_{ij} P_i + \omega_j P_w + c_j) d_j + g_j P_f (1 - d_j) \qquad (1.56)$$

where $(\sum_i^n a_{ij} P_i + \omega_j P_w + c_j)$ is the cost of producing domestically one unit of product j. Using Function (1.47), Equation (1.56) becomes

$$T_j(d_j) = (\sum_i^n a_{ij} P_i + \omega_j P_w + \alpha_j + \beta_j) d_j + g_j P_f (1 - d_j) \qquad (1.57)$$

Since $T_j''(d_j) = 2\beta_j > 0$, therefore the unrestricted minimum is attained at

$$t_j = \frac{1}{2\beta_j} (g_j P_f - \sum_i^n a_{ij} P_i - \omega_j P_w - \alpha_j) \qquad (1.58)$$

for $T_j'(t_j) = 0$, hence the minimum value of Equation (1.57) in the interval $0 \leq d_j \leq 1$ is attained at

$$\hat{d}_j = \begin{cases} 0 & t_j \leq 0 \\ t_j & 0 \leq t_j \leq 1 \\ 1 & t_j \geq 1 \end{cases} \qquad (1.59)$$

From the knowledge of $\hat{d}_j$ and $\hat{E}_j$, $\hat{X}_i$ and $\hat{M}_i$ follow directly from Equation (1.48). If $\hat{d}_j = 0$ it implies that $\hat{X}_j = 0$. However, if $\hat{d}_j > 0$, then $\hat{m}_i$ the ratio of imports to increase in production of sector i

$(\hat{m}_i = \frac{\hat{M}_i}{\hat{X}_i})$ will be:

$$\hat{m}_i = (1 - \hat{d}_i)/\hat{d}_i \tag{1.60}$$

It also follows:

$$\hat{M}_i = \hat{m}_i \hat{X}_i \tag{1.61}$$

Let us substitute Equality (1.61) into Equation (1.48).

$$\hat{X}_i + \hat{m}_i \hat{X}_i - \sum_{j \varepsilon S_1} a_{ij} \hat{X}_j - \sum_{j}^{n} a_{ij} \hat{E}_j = Y_i, \; i \varepsilon S_1 \tag{1.62}$$

where S_1 is the set of i such that $d_i > 0$. There are as many equations as unknowns (i.e. $\hat{X}_i$) so, $\hat{X}_i$ are uniquely determined. Now, in order to find $\hat{M}_i$, one must use Equality (1.61) for i in S_1, but, for those i not in S_1, $\hat{M}_i$ are given by

$$\hat{M}_i = Y_i + \sum_{j \varepsilon S_1} a_{ij} \hat{X}_j + \sum_{j}^{n} a_{ij} \hat{E}_j \tag{1.63}$$

As the given set of shadow prices has the above properties, it follows that $\sum_{i=1}^{n} (c_i \hat{X}_i + k_i \hat{E}_i)$ is the minimum value of Function (1.52) subject to Constraints (1.48), (1.50) and (1.51). Moreover, if $p_w > 0$, there will be a strict equality in Constraint (1.50) and if $P_f > 0$, there will be a strict equality in Constraint (1.51) too.

Hence, the problem is reduced to finding a set of shadow prices satisfying the above properties.

The next step for the authors is to derive a set of shadow prices. For this, they proved the following results.[1]

"Theorem 1: For each P_w and P_f there exists one and only one solution to the following system of equations.

$$P_f = \begin{cases} g_j P_f & , \ s_j \leq 0 \\ g_j P_f - \frac{1}{4\beta_j^2} s_j^2 & , \ 0 \leq s_j \leq 2\beta_j \quad j = 1 \ldots n \\ g_j P_f - s_j + \beta_j & , \ s_j \geq 2\beta_j \end{cases} \tag{1.64}$$

where $s_j = g_j P_f - \sum_i a_{ij} P_i - \omega_j P_w - \alpha_j \qquad j = 1 \ldots n$

Lemma 1: The following iterative process converges to the solution of (1.64). Assume initial values of $s_1, s_2, \ldots s_n$, say $s_1^{(0)}, s_2^{(0)} \ldots s_n^{(0)}$. Set

$$P_j^{(1)} = \begin{cases} g_j P_f & , \ s_j^{(0)} \leq 0 \\ g_j P_f - \frac{1}{4\beta_j} (s_j^{(0)})^2 & , \ 0 \leq s_j^{(0)} \leq 2\beta_j, j = 1 \ldots n \\ g_j P_f - s_j^{(0)} + \beta_j & , \ s_j^{(0)} \geq 2\beta_j \end{cases} \tag{1.65}$$

Determine subsequent values in the order $s_n^{(k)}$, $P_n^{(k+1)}$, $s_{n-1}^{(k)}$, $P_{n-1}^{(k+1)} \ldots s_1^{(k)}$, $P_1^{(k+1)}$ from the recursive formulas

$$s_j^{(k)} = g_j P_f - \sum_{i=1}^{j} a_{ij} P_i^{(k)} - \sum_{i=j+1}^{n} a_{ij} P_k^{(k+1)} - \omega_j P_w - \alpha_j$$

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[1] Chenery, Kretschmer, pp. 379-380.

$$P_j^{(k+1)} = \begin{cases} g_j P_f & , s_j^{(k)} \leq 0 \\ g_j P_f - \frac{1}{4\beta_j} (s_j^{(k)})^2 & , 0 \leq s_j^{(k)} \leq 2\beta_j \\ g_j P_f - s_j^{(k)} + \beta_j & , s_j^{(k)} \geq 2\beta_j \end{cases} \quad j = 1 \ldots n \tag{1.66}$$

Theorem 2: If P_w and P_f are nonnegative, then the P_j given by (1.64) satisfy

$$\min [\omega_j P_w + \alpha_j; g_j P_f] \leq P_j \leq g_j P_f \tag{1.67}$$

Also, for a fixed nonegative value of P_w, the P_j satisfying (1.64) are nondecreasing functions of P_f.

Theorem 3: Suppose P_w^+ is nonnegative, P_f^+ is positive.

$$P_j^+ = \min_{0 \leq d_j \leq 1} [(\sum_i a_{ij} P_i^+ + \omega_j P_w^+ + \alpha_j + \beta_j d_j) d_j + g_j P_f^+ (1-d_j)]$$

$$j = 1 \ldots n \tag{1.68}$$

and the E_i, X_i, and M_i say E_i^+, X_i^+ and M_i^+ are determined in the manner described above for the optimal set of prices.

$$\text{Then if} \quad L^+ = \sum_i (\omega_i X_i^+ + \omega_i E_i^+) \tag{1.69}$$

$$\text{and} \quad D^+ = \sum_i (g_i M_i^+ - h_i E_i^+)$$

the minimum value of (1.52) subject to (1.48), (1.50) and (1.51) with $L^* = L^+$ in (1.50) and $D^* = D^+$ in (1.51), is $\sum_i (c_i X_i^+ + k_i E_i^+)$."

This completes the method of derivation of the shadow prices.

The authors applied this model and its solution to Southern Italy where an economic development program was carried out.

All of the models looked at can be used for the formulation of economic plans. The first model is concerned with the maximization of

the growth rate of net output subject to investment constraints. The second one is a general model which can be used with any objective function as long as it suits the model. The third one minimizes the total investment required to satisfy the final demand subject to some resource limitations.

The model discussed in this research borrowed elements from the previous models.

Lange's formulation provided the objective function. Eventhough the growth rate of net output is not explicitly maximized in this research, the same results are obtained with maximizing the value added (this research objective function). Recall that the net output and the value added of the whole economy are equal.[1] In the growth rate formula ($\Delta Y/I$), investment I is constant. Maximizing growth rate implies maximizing net output. Lange's model and the model of this research seek the same thing but by different means. His decision variables are the α_{jst}'s (fractions of total investment made in sector j in period t and having a cycle of s years). Production levels (x_{it}) are the decision variables in this research.

Wagner's model provided the framework for capacity and demand constraints. His capacity constraints were modified to suit this study purpose. Increase of sector capacity is thought in terms of projects implementation in this research.[2] Demand constraints are slightly different from Wagner's ones. Input-output coefficients matrix A and capital coefficients matrix B are not distinguished

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[1] cf. Equality (1.4). However, net output and value added of a sector are not equal.

[2] cf. Equality (3.3) in section 3.1, Chapter 3.

as is the case in Wagner's model. Imports and exports variables are added to Wagner's demand constraints to take into account the importance of foreign trade, as in Chenery and Kretschmer's model.

Funds available for investments are a limited resource like labor and foreign exchange are in Chenery and Kretschmer's formulation.

More details on the model are given in the next section which presents the purpose of this research.

1.5 Objectives of This Research

Leontief models are very helpful in designing programs of economic development. In the last two decades many underdeveloped countries implemented such programs. This research proposes a model based on Leontief input-output coefficients that would help in allocating resources to achieve some economic goal.

In general, a plan of economic development must achieve both of the following objectives:

- First, state the goals to be accomplished by the end of the plan. These may be increasing the real Gross National Output or National Income by, say 8 percent per year, developing some sectors more than others, and so on. Certain goals can be quantified while others, such as quality of education, cannot be.

- Second, provide means for such a program. These are resources that must be allocated within the plan. Investment and manpower are examples of resources. The only resource dealt within this model is investment. By investment it is meant all funds available for spending in projects of any types (industrial and agricultural). This is

a major constraint underdeveloped countries face because of its shortage.

Assume economic planners want to maximize the value added in the whole economy over the planning period. The maximization of value added as the economic planners' objective is chosen for the following reasons:

- First, value added is a quantitative economic criterion.
- Second, value added is also the origin of all types of incomes: wages, profits and so on. So maximizing value added is also maximizing National Income.
- Third, increasing the value added created within the whole economy contributes to reduce unemployment which is a big problem underdeveloped countries face.

Now the economic planners' problem can be stated in the following question. How can investment be allocated in a plan of economic development aimed at maximizing value added created by all sectors of the economy?

In the next chapter, a model is proposed that can help answer this question.

A mathematical program is used to formulate the economic planners' problem in a Leontief type of framework. It maximizes value added subject to three types of constraints:

1) constraints of demand imposed on sectors
2) constraints of capacity of sectors
3) constraints of investments.

There is an interrelationship between capacity and investment. When an investment is made, the corresponding capacity will increase.

Moreover, when there is already enough capacity in a sector to satisfy demand, there will be obviously no need to make any investments. Both of these elements are taken into account in the mathematical formulation. But, among competing investments in different sectors, the algorithm ranks them according to their value added.

The solution of this mathematical program leads to an optimal allocation of investment among the sectors of the economy. The planners have an answer to the triple question:

- how much money must be invested
- where the investment must be made
- when it must be invested.

Once these answers are provided, the economic planners will know the sequence of projects that must be carried out to maximize the value added by the end of the plan.

Determining the investments per sector and per year is the purpose of this research. Since investment is in short supply in underdeveloped countries there must be a wise way to use it, in order to get the most out of it. For instance, if a country has a given amount of money to be invested, this investment can be made in any sector or group of sectors with different effects on the total value added created in the whole economy. There is also the problem of investing whether in downstream or upstream industries. There are many economists who argue that investing in upstream industries generates more benefits for the whole economy than in downstream sectors. They claim: investments made in upstream activities provide capacity of producing goods that will be used by downstream industries and thus, favoring their development. For instance, the chemical industries

are examples of upstream activities; their productions constitute inputs to many downstream sectors such as agriculture, food industries and so on. This theory recommends investments in these upstream industries. Underdeveloped countries are heavily dependent on developed countries for equipment and machinery needed to implement industrial projects. It would be logical to develop upstream sectors that would provide other sectors with these inputs. The Algerian plan of economic development has been mainly based on this theory: priority is given to heavy industries. When the model is applied to the Algerian case it will be possible to check the claim of this theory. Is it correct to state that upstream industries lead to the maximization of value added?

The model, presented in the next chapter, uses two types of Leontief input-output coefficients. These are domestic and foreign coefficients. This distinction is made because underdeveloped countries must import many of the inputs and most of the equipment necessary for production pruposes. These coefficients would change from one year to the other. Let us suppose, for instance, an investment made in the steel industry results in excess capacity; in the future the different industries importing steel will reduce their foreign purchases to buy more domestic steel. This situation might result in a change of domestic and foreign input coefficients related to the steel sector. Domestic input coefficients might increase, showing the decrease in foreign purchases of steel. Hence, in the next period, new input coefficients must be used, reflecting the change of situation. This situation occurs each time there exists an opportunity of purchasing more domestic inputs.

Both types of input coefficients are used in the model to take into account the dynamics of the situation.

Data that must be used to compute both of these coefficients are not available. The model of the next chapter is presented from a theoretical point of view. It is also used to solve an example to show the method of solution.

CHAPTER II

THE DYNAMIC PROGRAMMING SOLUTION TO THE DYNAMIC LEONTIEF MODEL FOR AN INVESTMENT ALLOCATION

In this chapter the problem defined previously is formulated in a mathematical program using a Leontief type of framework. The model concerns underdeveloped countries deciding to implement programs of economic development.

The key question it must answer is how investment can be allocated in a plan of economic development in order to maximize value added. As indicated previously, three types of constraints are used, namely, final demand, capacity and investment. Once the model is defined, the method of solution by dynamic programming will be presented and applied to a numerical example.

2.1 Definition of the Model

Suppose there are m sectors in the economy.[1]

Let us define the variables and parameters of the model.

The variables are expressed in monetary units (m u) and the subscript t denotes the relevant time period t.

x_{it} = production of sector i in period t.

f_{it} = final demand for goods of sector i in period t.

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[1] It can be assumed: a sector comprises a group of similar products. See Hollis Chenery and Kenneth Kretschner, "Resource allocation for economic development," Econometrica, volume 24, No. 4, 1956.

s_{it} = inventory of goods in sector i at the end of period t.

e_{it} = exports of goods by sector i in period t.

z_{it} = imports of final goods i by the whole economy in period t.

c_{it} = capacity of sector i in period t.

u_{it} = slack capacity in sector i in period t.

g_{it} = investment required to build an average capacity in sector i period t.

The parameters are:

v_{it} = value added per unit produced in sector i in period t expressed in m u.

a_{ijt} = ratio of the value of domestic input i consumed by sector j to the output of sector j.

d_{ijt} = ratio of the value of foreign input i consumed by sector j to the output of sector j.

α_i = coefficient of conversion of investment into capacity in sector i.

An investment of g_{it} in sector i will increase capacity by $\alpha_i g_{it}$.

β_{it} = number of projects built in sector i in period t, β_{it} is an integer.

The value of sector j output is equal to the sum of the value of inputs (domestic and foreign) consumed by sector j and the value added of sector j.[1]

$$\text{For } j = k,\ x_{kt} = \sum_{i=1}^{m} a_{ikt}x_{kt} + \sum_{i=1}^{m} d_{ikt}x_{kt} + v_{kt}x_{kt}, \qquad (2.0)$$

The sum of sector i output, imports of inputs i, and imports of

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[1] cf. section 1.2.

final goods i must equal the sum of final demand of goods i, intermediate demand of inputs i, exports of final goods i, and change of inventories in sector i.

The balance equations of the model are:

$$x_{it} + \sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ijt}x_{jt} + z_{it} = f_{it} + \sum_{j=1}^{m} a_{ijt}x_{jt} + e_{it} + s_{it}$$

$$- s_{i(t-1)}, \quad i = 1 \ldots m$$

$$x_{it} + u_{it} = c_{it}, \quad i = 1 \ldots m$$

In the first set of balance equations, $\sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ijt}x_{jt}$, is explained by the fact that $d_{iit}x_{it}$ is already included in x_{it} as can be seen from Equation (2.0). To avoid double counting, $j\neq i$ appears under the summation sign.

After rearranging, the balance equations are:

$$x_{it} + \sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ijt}x_{jt} - \sum_{j=1}^{m} a_{ijt}x_{jt} + z_{it} = f_{it} + e_{it} + s_{it} - s_{i(t-1)},$$

$$i = 1 \ldots m \qquad (2.1)$$

$$x_{it} + u_{it} = c_{it} \qquad i = 1 \ldots m$$

Suppose a set of values for f_{it}, e_{it}, s_{it} and c_{it} is provided. Now everything that is on the right hand side of the balance equations is known. The model has 3 m variables, namely, x_{it}, z_{it}, u_{it}, even though z_{it} and u_{it} are residuals, they can be determined right away

once a set of values for x_{it} is found. Moreover, the model has 2 m constraints.

In this model, the balance equations can be considered as the constraints of a linear program once an appropriate objective function is chosen. The economic planners are interested in maximizing the value added by the whole economy. The value added of sector i is the difference between the values of its output (x_i) and of its consumed inputs ($\sum_{j=1}^{m} x_{ij}$). Therefore, for a given year, the value added by the whole economy is:

$$\sum_{i=1}^{m} v_i x_i = \sum_{i=1}^{m} x_i - \sum_{i=1}^{m} \sum_{j=1}^{m} x_{ij}$$

This is also the total net output.[1] So, maximizing the total value added is the same thing as maximizing the net output.

The economic planners' objective, over a T year plan can be expressed by:

$$\max \sum_{t=1}^{T} \sum_{i=1}^{m} v_{it} x_{it}$$

Assume the planners have N m u available for capacity building during a T year plan. Of these N, n_1 m u are available for investment in period 1, n_2 in period 2, and so on.

In other terms:

$$\sum_{t=1}^{T} n_t = N$$

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[1] cf. Equation (1.4) in section 1.1, Chapter 1.

The planner's problem can be expressed as:

$$\max Z = \sum_{t=1}^{k} \sum_{i=1}^{m} v_{it} x_{it}$$

$$\sum_{t=1}^{k} \sum_{i=1}^{m} \beta_{it} g_{it} \leq \sum_{t=1}^{k} n_t, \quad k = 1, 2, \ldots T, \qquad (2.3)$$

(LP_k)

$$x_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{m} d_{ijt} x_{jt} - \sum_{j=1}^{m} a_{ijt} x_{jt} + z_{it} = f_{it} + e_{it} + s_{it} - s_{i(t-1)},$$

$$i = 1 \ldots m, \; t = 1 \ldots k$$

$$x_{it} + u_{it} = c_{it}, \quad i = 1 \ldots m, \quad t = 1 \ldots k$$

$$x_{it} \geq 0, \; u_{it} \geq 0, \; z_{it} \geq 0, \; i = 1 \ldots m, \quad t = 1 \ldots k$$

The difficulty in solving this mathematical program stems from the following reasons.

-- c_{it}, i.e., the capacity of sector i in period to is unknown for t = 2, . . ., T, a priori. However, c_{it} depends on the level of investments made previously, namely $\sum_{p=1}^{t} \beta_{ip} g_{ip}$.

-- An investment g_{it} may change the value of the parameters v_{it}, a_{ijt} and d_{ijt} in subsequent periods. This will be discussed in section 2.2.

-- For each value of k, Inequality (2.3) contains k constraints. The solution of the problem (LP_k) does not guarantee that the set $\beta_{i1} g_{i1}, \ldots, \beta_{ik} g_{ik}$, i = 1 . . . m, is optimal for (LP_{k+1}). A dynamic programming approach will be used to solve this problem in section 2.3.

2.2 Assumptions and Dynamics of the Model:

2.2.1 Assumptions:

The model presented in section 2.1 will be used to find an optimal allocation of investment among the sectors, such that the value added is maximized over the planning period in a non-integrated (underdeveloped) economy.

The initial values of the parameters v_{it}, a_{ijt}, d_{ijt} and α_i must be known in year 1 of the planning period; v_{it}, a_{ijt}, d_{ijt} and d_{ijt} may change in subsequent periods whereas α_i stays constant. The constancy of α_i stems from the fact that technology is assumed constant during the planning period. This is a realistic assumption for a model in which T = 4 or 5 years as is the case in this research.

For convenience, inflation has been ignored: the cost of building the same capacity in a given sector is constant each year of the plan.

The parameters v_{it}, a_{ijt} and α_i are given for an integrated economy. An integrated economy can be defined as an economy able to produce any kind of goods, and in which imports and exports as a percentage of gross national product are less than 10 percent. These parameters are denoted as $\bar{v}_i$ and $\bar{a}_{ij}$ respectively and are known for a given year; only values for one year are used.

For an integrated economy the following equality holds:

$$\sum_{i=1}^{m} \bar{a}_{ij} + \bar{v}_j = 1 \qquad \text{for any } j$$

For a non-integrated economy the relation is:[1]

$$\sum_{i=1}^{m} (a_{ijt} + d_{ijt}) + v_j = 1 \qquad \text{for any } j$$

Finally, the value of α_i is computed for an integrated economy and is used for the non-integrated economy.

2.2.2 Dynamics of the Model

In period 1, the model (LP_k) becomes:

$$\max Z = \sum_{i=1}^{m} v_{i1} x_{i1}$$

$$\sum_{i=1}^{m} \beta_{i1} g_{i1} \leq n_1$$

(LP_1)

$$x_{i1} + \sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ij1} x_{j1} - \sum_{j=1}^{m} a_{ij1} x_{j1} + z_{i1} = f_{i1} + e_{i1} + s_{i1} - s_{i0},$$

$$i = 1 \ldots m$$

$$x_{i1} + u_{i1} = c_{i1}, \; i = 1 \ldots m$$

$$x_{i1} \geq 0, \; u_{i1} \geq 0, \; z_{i1} \geq 0, \quad i = 1 \ldots m$$

Suppose k sectors out of m decide to increase their capacities. Let K be the set of these k sectors. For $i \in K$, sector i invest $\beta_{i1} g_{i1}$

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[1] $\sum_{i=1}^{m} a_{ij} \leq 1, \; a_{ij} \geq 0$ is used by Hollis Chenery and Kenneth Kretschner, "Resource allocation for economic development," Econometrica, volume 24, no. 4, 1956.

m u in period 1. It should be noted that

$$\sum_{i \in K} \beta_{i1} g_{i1} \leq n_1$$

An investment of g_{i1} does not always result in a capacity increase of $\alpha_i g_i$, in the year of investment. This is only possible if 100 percent completion is assumed. This means that if an investment is decided at the beginning of year 1 it will be completed during year 1, so that capacity increases to $c_{i1} + \alpha_i g_{i1}$ during year 1. In the following, a method will be proposed to bring to a 100 percent completion in the year of investment investments whose completion times are actually less than a 100 percent. The reason for this will become clear in section 2.3.

Suppose an investment is completed at h% each year. So it will take $\frac{1}{h\%}$ years to reach a capacity level of $c_{i1} + \alpha_i g_{i1}$ in sector i for an investment made in period 1. Let the life time of an investment g_{i1} be ε years, $\varepsilon \geq \frac{1}{h\%}$. During this life time it will be assumed that sector i operates at full capacity. During these ε years the value added per unit in sector i is v_{i1}. The capacity of sector i is

$c_{i1} + h\% \times \alpha_i g_{i1}$	in year 1	
$c_{i1} + 2h\% \times \alpha_i g_{i1}$	in year 2	
$c_{i1} + kh\% \times \alpha_i g_{i1}$	in year k	$k \leq \frac{1}{h\%}$
$c_{i1} + \alpha_i g_{i1}$	in year $\frac{1}{h\%} + 1, \frac{1}{h\%} + 2, \ldots \varepsilon$	

The value added generated by investment g_{i1} is

v_{i1} (h% x $\alpha_i g_{i1}$)	for year 1

$v_{il}\ (kh\% \times \alpha_i g_{il})$ for year k, $k \leq \frac{1}{h\%}$

$v_{il}\ (\alpha_i g_{il})$ for years $\frac{1}{h\%} + 1$ to ε

The total value added generated by investment g_{il} in ε years is

$$v_{il}\ (h\% \times \alpha_i g_{il}) + \ldots + v_{il}\ (kh\% \times \alpha_i g_{il}) + v_{il}\ (\alpha_i g_{il}) + \ldots + v_{il}\ (\alpha_i g_{il})$$

or

$$P = v_{il}\left[\sum_{k=1}^{\frac{1}{h\%}} k\ h\%\alpha_i g_{il} + (\varepsilon - \frac{1}{h\%})\ \alpha_i g_{il}\right]$$

Investment g_{il} can be compared to an investment w_{il} with 100 percent completion in year 1 and with the same value added v_{il} and the same life time ε. So under the same conditions investment w_{il} generates the following value added each year for ε years.

$$v_{il}\ (\alpha_i w_{il})$$

For ε years the total value added is

$$Q = \varepsilon(v_{il} \times \alpha_i w_{il})$$

The method proposed consists of setting P = Q and solving w_{il}.

$$w_{il} = \frac{P}{\varepsilon v_{il}\alpha_i} = \frac{v_{il}\left[\sum_{k=1}^{\frac{1}{h\%}} kh\%\alpha_i g_{il} + (\varepsilon - \frac{1}{h\%}\ \alpha_i g_{il})\right]}{\varepsilon v_{il}\alpha_i}$$

$$w_{il} = \frac{g_{il}\left[\sum_{k=1}^{\frac{1}{h\%}} kh\% + (\varepsilon - \frac{1}{h\%})\right]}{\varepsilon}$$

$$\sum_{k=1}^{\frac{1}{h\%}} kh\% = h\% \sum_{k=1}^{\frac{1}{h\%}} k = h\%\left[\frac{1}{2} \times \frac{1}{h\%}\left(\frac{1}{h\%} + 1\right)\right] = \frac{1}{2}\left(\frac{1}{h\%} + 1\right)$$

The last two equations give:[1]

$$w_{il} = \frac{g_{il}\left[\frac{1}{2}\left(\frac{1}{h\%} + 1\right) + \varepsilon - \frac{1}{h\%}\right]}{\varepsilon}$$

$$w_{il} = \frac{g_{il}\left(\frac{1}{2} + \varepsilon - \frac{1}{2h\%}\right)}{\varepsilon} \qquad (2.4)$$

In Formula (2.4) $0 < h\% \leq 1$. The case where $h\% = 0$ is ruled out. Also note:

$$\frac{\frac{1}{2} + \varepsilon - \frac{1}{2h\%}}{\varepsilon} \leq 1$$

This implies $w_{il} \leq g_{il}$.

Formula (2.4) does not hold if h% is not constant.

In summary, investing g_{il} in a project completed at h% each year will be equivalent to investing w_{il} with 100 percent completion the first year if g_{il} and w_{il} have the same ε, v_{il} and α_{il}.

For any $i \in K$, such that $\sum_{i\in K} \beta_{il} g_{il} \leq n_l$, the capacity of sector i, during year l, becomes $c_{il} + \alpha_i \beta_{il} w_{il}$. Set K is not unique, there are many. Let us label them:

$$K_s, \quad s = 1 \ldots s_l$$

Now, for a given set K_s, the LP to solve is

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[1]Murray R. Spiegel, Mathematical Handbook (New York: McGraw-Hill Book Comapny, 1968).

$$\text{Max } Z = \sum_{i=1}^{m} v_{i1} x_{i1}$$

$$x_{i1} \sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ij1} x_{j1} - \sum_{j=1}^{m} a_{ij1} x_{j1} + z_{i1} = f_{i1} + e_{i1} + s_{i1} - s_{i0},$$

$$i = 1 \ldots m$$

$$(LP_{K_s}) \qquad x_{i1} + u_{i1} = c_{i1}, \; i = 1 \ldots m$$

$$x_{i1} + u_{i1} = c_{i1} + \alpha_i \beta_{i1} w_{i1}, \quad i = 1 \ldots m \text{ and } i \in K_s$$

$$x_{i1} \geq 0, \; z_{i1} \geq 0, \; u_{i1} \geq 0, \; i = 1 \ldots m$$

Let x^*_{i1}, z^*_{i1} and u^*_{i1} be an optimal solution to (LP_{K_s}). The increase of sector i capacity, $i \in K_s$, might change the structure of the economy and the parameters v_{it}, a_{ijt} and d_{ijt}. This change will occur if some $u^*_{i1} \geq 0$. Recall $u^*_{i1} = c_{i1} + \alpha_i \beta_{i1} w_{i1} - x^*_{i1}$, i.e., unused capacity. Let

$O_{K_s} = \{i, \; u^*_{i1} > 0\}$ denote the set of sectors having slack capacity.

The planners would like to use $u^*_{i1} > 0$ to reduce imports of goods i, $i \in O_{K_s}$, by the whole economy. Each sector j, $j = 1 \ldots m$, used to buy from sector i, $i \in O_{K_s}$, $a_{ij1} x^*_j$. The quantity u^*_{i1}, $i \in O_{K_s}$, can be shared by the different buyers of goods i on a proportional basis.

Let γ_{ij} be the increase of sector j purchase of a given $i \in O_{K_s}$:

$$\gamma_{ij} = \frac{u^*_{i1} \; (a_{ij1} x^*_{j1})}{\sum_{j=1}^{m} a_{ij1} x^*_{j1}}, \quad i \in O_{K_s}, \quad j = 1 \ldots m \qquad (2.5)$$

γ_{ij} must be computed for each $i \in O_{K_s}$,

γ_{ij} represents the decrease of purchase of foreign goods i by

sector j.

The new coefficients a_{ijt} that will be used in period 2 are:

$$a_{ij2} = \frac{a_{ij1}x^*_{j1} + \gamma_{ij}}{x^*_{j1}}, \quad i \in O_{K_s}, \quad j = 1 \ldots m, \text{ but } j \neq i \qquad (2.6)$$

The new coefficients d_{ijt} are

$$d_{ij2} = a_{ij1} + d_{ij1} - a_{ij2}, \quad i \in O_{K_s}, \quad j = 1 \ldots m, \; j \neq i \qquad (2.7)$$

The reason for Formula (2.7) is that technology is assumed constant in the model or in other terms:

$$a_{ijt+1} + d_{ijt+1} = a_{ijt} + d_{ijt}, \quad i \in O_{K_s}, \quad j = 1 \ldots m \text{ but } j \neq i.$$

The validity of Formulas (2.6) and (2.7) is explained as follows. The different sectors j=1 . . . m use the coefficients a_{ij1} and d_{ij1} in period 1 which are those given in period zero. At the end of period 1 when (LP_{K_s}) is solved, the planners have an optimal solution: x^*_{i1}, z^*_{i1} and u^*_{i1}. If some $u^*_{i1} > 0$, they will revise their estimates of a_{ijt} and d_{ijt} and will use Formula (2.6) to get a_{ij2} and Formula (2.7) to get d_{ij2}.

In Formulas (2.6) and (2.7) it was pointed out that $j \neq i$. γ_{ij} is the decrease of foreign purchase of goods i by sector j; this does not have any effect on the value added by sector j, unless j=1. In this case, the decrease of purchase of foreign goods i by sector i contributes to increase the value added by sector i. γ_{ij} will be shared by the two elements on a proportional basis.

-- The first element contributes to increase the value added of sector i. This element is μ_{1i}.

-- The second element contributes to increase the purchase of domestic good i by sector i. This is μ_{2i}.

$$\mu_{1i} = \bar{v}_i \times \gamma_{ii}, \quad i \in 0_{K_s} \tag{2.8}$$

$\bar{v}_i$ is the value added coefficient of an integrated economy, so $\bar{v}_i \gamma_{ii}$ is the increase of value added in sector i due to a decrease of purchase of foreign good i by sector i in an amount of γ_{ii}.

$$\mu_{2i} = \gamma_{ii} - \mu_{1i}, \quad i \in 0_{K_s} \tag{2.9}$$

Now the parameters of sector i, $i \in 0_{K_s}$, become:

$$a_{ii2} = \frac{a_{ii1}x^*_{i1} + \mu_{2i}}{x^*_{i1}}, \quad i \in 0_{K_s} \tag{2.10}$$

$$v_{i2} = \frac{v_{i1}x^*_{i1} + \mu_{1i}}{x^*_{i1}}, \quad i \in 0_{K_s} \tag{2.11}$$

$$d_{ii2} = a_{ii1} + d_{ii1} + v_{i1} - a_{ii2} - v_{i2}, \quad i \in 0_{K_s} \tag{2.12}$$

Formula (2.12) is justified by the assumption of constant technology.

In summary: for a given set K_s, $s = 1 \ldots s_1$, such that $\sum_{i \in K_s} \beta_{i1} g_{i1} \leq n_1$, the planners solve LP_{K_s} and use the unused capacities u^*_{i1}, where $i \in 0_{K_s}$, to find a_{ij2} via Formulas (2.5), (2.6) and (2.10), d_{ij2} via (2.7) and (2.12) and v_{i2} via (2.11) for each $i \in 0_{K_s}$ and repeat the previous steps for all values of s, $s = 1, \ldots, s_1$.

The next section will show the solution of LP_k by Dynamic Programming.

2.3 The Dynamic Programming Solution

The Dynamic Programming solution to the planner's problem requires the following steps:

-- definition of states and stages

-- use of a recursion formula

-- satisfaction of the optimality principle.

The planner's problem LP_k was formulated as

$$\max Z = \sum_{t=1}^{k} \sum_{i=1}^{m} v_{it} x_{it}$$

$$\sum_{t=1}^{k} \sum_{i=1}^{m} \beta_{it} g_{it} \leq \sum_{t=1}^{k} n_t \qquad k = 1, \ldots, T$$

LP_k

$$x_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{m} d_{ijt} x_{jt} - \sum_{j=1}^{m} a_{ijt} x_{jt} + z_{it} = f_{it} + e_{it} + s_{it} - s_{i(t-1)}$$

$$x_{it} + u_{it} = c_{it}$$

$$x_{it} \geq o,\ u_{it} \geq o,\ z_{it} \geq 0, \quad i = 1, \ldots, m,\ t = 1, \ldots, k$$

2.3.1 Stages and States of the Problem

The number of stages in LP_k is mT. There are m stages for each year in a T year plan. Each one corresponds to a given sector i, i = 1 . . . m, in a given year of the T year plan. Let (ik) denote the stage corresponding to sector i in year k, i = 1, . . . , m and k = 1 , . . . , T.

The state of the system in stage (ik) depends on the planners' decision to invest, in sector i period k, an amount equal to y m u when r m u are available for investment in stage (ik). Note $y \leq r$.

This implies that r - y was invested optimally in the previous m(k-1) + i-1 stages.

2.3.2 Recursion Formula

Let $P_{ik}(y)$ denote the immediate value added generated by investing y m u in stage (ik).

Let $Q_{i-1,k}\ (r-y)$ be the optimal value added obtained when r-y m u was invested in the previous m(k-1) + i-1 stages.

The recursion formula is:[1]

$$Q_{ik}(r) = \max_{y \leq r} [P_{ik}(y) + Q_{i-1,k}\ (r-y)], \quad r \leq \sum_{t=1}^{k} n_t \tag{2.13}$$

Equation (2.13) is true for any i≠1. Inequality (2.3), i.e.

$$\sum_{t=1}^{k} \sum_{i=1}^{m} \beta_{it} g_{it} \leq \sum_{t=1}^{k} n_t, \quad k = 1, 2, \ldots, T$$

becomes $\sum_{i=1}^{m} \beta_{i1} g_{i1} \leq n_1$ for k = 1,

and for k = 2 it corresponds to two constraints

$$\sum_{i=1}^{m} \beta_{i1} g_{i1} \leq n_1, \text{ and, } \sum_{t=1}^{2} \sum_{i=1}^{m} \beta_{it} g_{it} \leq n_1 + n_2.$$

Both of these constraints imply that the maximum invested in the economy in year 2 is $n_1 + n_2$ and the least is n_2. If an amount less than n_2 were invested in year 2, the following would occur:

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[1] The subscript k to Wagner's recursion formula was added. Harvey M. Wagner, Principles of Operations Research (2d ed.; Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1975), p. 264.

$$\sum_{i=1}^{m} \beta_{i1}g_{i1} + \sum_{i=1}^{m} \beta_{i2}g_{i2} \leq n_1 + n_2 \text{ , } \sum_{i=1}^{m} \beta_{i2}g_{i2} < n_2$$

this implies

$$\sum_{i=1}^{m} \beta_{i1}g_{i1} \leq n_1 + n_2 - \sum_{i=1}^{m} \beta_{i2}g_{i2}$$

since $n_2 - \sum_{i=1}^{m} \beta_{i2}g_{i2} > 0$ which includes the possibility of $\sum_{i=1}^{m} \beta_{i1}g_{i1} > n_1$ because $\sum_{i=1}^{n} \beta_{i1}g_{i1} \leq n_1$ + something positive; this possibility would not happen if n_2 was the least amount that could be invested in year 2. This includes the case where zero is invested in year 2 in all sectors i except i = 1. So the recursion formula for i = 1 is:

$$Q_{1k}(r) = \max_{y \leq r} [P_{1k}(y) + Q_{m,k-1}(r-y)] \qquad (2.14)$$

where $r \leq \sum_{t=1}^{k} n_t$, and , $r-y \leq \sum_{t=1}^{k-1} n_t$.

2.3.3 Principle of Optimality:

Bellman[1] stated the so-called "principle of optimality" encountered in any recursion formula of a dynamic programming problem as follows:

> An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

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[1]Richard E. Bellman, Dynamic Programming (Princeton: Princeton University Press, 1957), p. 83. Stuart E. Dreyfus and Averill M. Law, The art and theory of dynamic programming (New York: Academic Press, 1977).

The decisions in this research are the investment decisions y. The principle of optimality as stated above applied to the backward type solution to dynamic programming problems.

The recursion formulae (2.13) and (2.14) are of the forward types.[1] The model is known only at the beginning of period 1 of the T year plan. In subsequent years, the state of the model will depend on the previous investment decisions r-y for a given available amount r in these subsequent years.

The principle of optimality for a forward type solution can be restated as:

An optimal policy has the property that whatever the present state and present decision are, the previous decisions must constitute an optimal policy with regard to the state resulting from these decisions.

Formulae (2.13) and (2.14) satisfy this statement.

For a given available investment r in stage (ik), a decision to invest y in this stage determines both the state of the system in stage (ik) and the decisions concerning the previous m(k-1) + i-1 stages. These previous decisions are an optimal policy for entering this state at stage (ik).

2.3.4 The Dynamic Programming Approach

Next, the planners' problem (LP_k) is solved by dynamic programming.

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[1] George F. Hadley, Nonlinear and Dynamic Programming (Reading, Mass.: Addison-Wesley Pub. Co., 1964). Harvey M. Wagner, Principles of Operations Research (2d ed.; Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1975).

For a T year plan, there are mT stages. The states in any stage depend on the combination of r and y.

Starting with the first year (k=1), the corresponding stages are (1,1) to (m,1).

For i=1, the recursion formula (2.14) becomes

$$Q_{11}(r) = P_{11}(r) \quad \text{for } r \leq n_1 \tag{2.15}$$

because by definition $Q_{m,o}(r-r) = 0$

$P_{11}(r)$ is obtained by solving the following LP for a given set of values of f_{i1}, e_{i1}, s_{i1}, and s_{i0}, and for $r = \beta_{11}g_{11}$, $\beta_{11} = 0, 1, 2, \ldots$

$$P_{11}(r) = \max Z$$

$$\max Z = \sum_{i=1}^{m} v_{i1}x_{i1}$$

$$r \leq n_1$$

$$x_{i1} + \sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ij1}x_{j1} - \sum_{j=1}^{m} a_{ij1}x_{j1} + z_{i1} = f_{i1} + e_{i1} + s_{i1} - s_{i0}$$

$$i = 1 \ldots m$$

$$x_{i1} + u_{i1} = c_{i1} \quad \text{for } i\neq 1$$

$$x_{11} + u_{11} = c_{11} + \alpha_1\beta_{11}w_{11}$$

$$x_{i1} \geq 0,\ u_{i1} \geq 0,\ z_{i1} \geq 0$$

This LP has to be solved for each value of r, obtained for a given value of β_{11}. The computation of $P_{11}(r)$ requires solving $\hat{\beta}_{11} = \max [\beta_{11}$, integer such that $\beta_{11}g_{11} \leq n_1]$ LP's.

Now, for i=2, . . . , m, the recursion formula (2.13) is applied

$$Q_{i1}(r) = \max_{y\leq r} [P_{i1}(y) + Q_{i-1,1}(r-y)], \quad r \leq n_1 \tag{2.16}$$

The computations are carried first for i=2 and so on. For each i, the change in the capacity of the first i sectors, for a given pair of (r,y), must be accounted for.

$$x_{ji} + u_{ji} = c_{jl} + \alpha_j \beta_{jl} w_{jl} \qquad j = 1 \ldots i$$

$$x_{jl} + u_{jl} = c_{jl} \quad \text{for } j > i$$

also

$$\beta_{il} g_{il} = y$$

and

$$\sum_{j=1}^{i-1} \beta_{jl} g_{jl} = r-y$$

So the LP that must be solved, to get $[P_{il}(y) + Q_{i-1,l}(r-y)$ for a given pair (r,y), is

$$\max Z = \sum_{i=1}^{m} v_{il} x_{il}$$

$$\beta_{il} g_{il} = y$$

$$\sum_{j=1}^{i-1} \beta_{jl} g_{jl} = r-y$$

$$x_{il} + \sum_{\substack{j=1 \\ j \neq j}}^{m} d_{ijl} x_{jl} - \sum_{j=1}^{m} a_{ijl} x_{jl} + z_{il} = f_{il} + e_{il} + s_{il} - s_{i0},$$

$$i = 1 \ldots m$$

$$x_{jl} + u_{jl} = c_{jl} + \alpha_j \beta_{jl} w_{jl}, \qquad j = 1 \ldots i$$

$$x_{jl} + u_{jl} = c_{jl} \quad \text{for } j > i$$

$$x_{jl} \geq 0,\ z_{il} \geq 0,\ u_{jl} \geq 0, \quad j = 1 \ldots m$$

For each i one LP must be solved for each value of y. The number of

LP's that must be solved, for each i, and for a given value of r, is $\hat{\beta}_{i1}$ where $\hat{\beta}_{i1} = \max [\beta_{i1}$, integer such that $\beta_{i1} g_{i1} = y \leq r]$.

For the case where i = 2, . . . , m and y=0, there is an interesting property of the model that can be used to make the dynamic programming solution appealing. For example, if i=2 and y=0 the previous LP becomes

$$\max Z = \sum_{i=1}^{m} v_{i1} x_{i1}$$

$$\beta_{11} g_{11} = r$$

$$x_{i1} + \sum_{\substack{j=1 \\ j \neq i}}^{m} d_{ij1} x_{j1} - \sum_{j=1}^{m} a_{ij1} x_{j1} + z_{i1} = f_{i1} + e_{i1} + s_{i1} - s_{i0},$$

$$i = 1 \ldots m$$

$$x_{11} + u_{11} = c_{11} + \alpha_1 \beta_{11} w_{11}$$

$$x_{21} + u_{21} = c_{21}$$

$$x_{j1} + u_{j1} = c_{j1} \quad \text{for } j > 2$$

$$x_{j1} \geq 0,\ z_{i1} \geq 0,\ u_{j1} \geq 0, \quad \text{for } j = 1, \ldots, m$$

It can be seen that this LP has been solved (see (2.15)).

Max $Z = Q_{i-1,1}(r) = Q_{1,1}(r)$ when i=2 or

$$[P_{21}(y=0) + Q_{2-1,1}(r)] = Q_{1,1}(r).$$

This implies that $P_{21}(y=0) = 0$.

Moreover, for $i \geq 2$ and y=o, $P_{i1}(y=0) = o$. This is generalized to any k, i.e., $P_{ik}(y=0) = 0$, for $i \geq 2$.

The case of $P_{ik}(y=0)$ when i=1 will be discussed shortly. For k=2 the model deals with year 2. First, there is a new set of values

for f_{i2}, z_{i2}, s_{i2} and s_{i1}. Second, some parameters such as v_{i2}, a_{ij2} and d_{ij2} might have changed because of investment decisions in year 1. Both of these situations must be accounted for.

The case of i=1 is treated separately. For i=1, the recursion formula (2.14) gives

$$Q_{12}(r) = \max_{y \leq r} [P_{12}(y) + Q_{m,1}(r-y)] \qquad (2.17)$$

with

$$r \leq \sum_{t=1}^{2} n_t \quad \text{and } r-y \leq n_1.$$

First of all, $Q_{m,1}(r-y)$ is known by Formula (2.16) for any pair (r,y) satisfying the three conditions above. This will determine the values of v_{i2}, a_{ij2} and d_{ij2} if they are different from those of year 1. Also, the new capacities, resulting from investing r-y in year 1 are known. For y=0 and any r, $r \leq n_1$, $P_{12}(y=0)$ is obtained by solving the following LP.

$$\max Z = \sum_{i=1}^{m} v_{i2}x_{i2}$$

$$x_{i2} + \sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ij2}x_{j2} - \sum_{j=1}^{m} a_{ij2}x_{j2} + z_{i2} = f_{i2} + e_{i2} + s_{i2} - s_{i1},$$

$$i = 1 \ldots m$$

$$x_{i2} + u_{i2} = c_{i2} \qquad i = 1 \ldots m$$

$$x_{i2} \geq 0,\ u_{i2} \geq 0,\ z_{i2} \geq 0, \quad i = 1 \ldots m$$

The optimal solution to this LP is:

$P_{12}(y=0) = \sum_{i=1}^{m} v_{i2}x^*_{i2}$, where x^*_{i2} is an optimal production level in sector i, i = 1, . . . , m.

This implies that $P_{12}(y=0) \geq 0$

For $y > 0$ and a given r, i.e., for $\beta_{12} = 1, 2, \ldots$ such that $y = \beta_{12} g_{12} \leq r$, $P_{12}(y)$ is obtained by solving this LP with a new constraint reflecting the change in capacity of sector 1. More precisely,

$x_{12} + u_{12} = c_{12} + \alpha_1 \beta_{12} w_{12}$.

For $i \geq 2$, the recursion formula (2.13) is applied.

$$Q_{i2}(r) = \max_{y \leq r} [P_{i2}(y) + Q_{i-1,2}(r-y)] \ , \ r \leq n_1 + n_2$$

Again $P_{i2}(y=0) = 0$. For example i=2, $y > 0$ and a given r, i.e., for $\beta_{22} = 1, 2, \ldots$ such that $y = \beta_{22} g_{22} \leq r$, $P_{22}(y)$ is obtained by solving the LP

$$\max Z = \sum_{i=1}^{m} v_{i2} x_{i2}$$

$$x_{i2} + \sum_{\substack{j=1 \\ j \neq i}}^{m} d_{ijt} x_{j2} - \sum_{j=1}^{m} a_{ij2} x_{j2} + z_{i2} = f_{i2} + e_{i2} - s_{i1} + s_{i2},$$

$$i = 1, \ldots, m$$

$$x_{i2} + u_{i2} = c_{i2} \qquad \text{for } i > 2$$

$$x_{12} + u_{12} = c_{12} + \alpha_1 \beta_{12} w_{12}$$

$$x_{22} + u_{22} = c_{22} + \alpha_2 \beta_{22} w_{22}$$

$$x_{i2} \geq 0, \ u_{i2} \geq 0, \ z_{i2} \geq 0 \qquad i = 1, \ldots, m$$

The value of the right hand side of capacity constraints i, $i \neq 2$, is determined by the investment decisions associated with $Q_{1,2}(r-y)$. The same approach is used for i=3, 4, . . . , m. Furthermore, the case of k=3, . . . , T and i=1, . . . , m is similar to the one just discussed k=2.

It is now possible to give the very reason for having adopted

the concept of w_{it}: the equivalent investment with 100 percent completion in the year the money is spent for an investment g_{it}. This reason satisfies the principle of optimality in the model of concern. For a given stage (ik) and a state represented by the pair (r,y), in the quantity $[P_{ik}(y) + Q_{i-1,k}(r-y)]$, the principle of optimality says: $Q_{i-1,k}(r-y)$ is the optimal value added generated in stages m(k-1) + i-1 when r-y m u were invested in these stages to reach state (r,y) at stage (ik). The main difference between w_{it} and g_{it} is that w_{it} integrates two things: the immediate value added (generated in the year of investment) and the future value added (generated by the completion of the project); whereas g_{it} takes into account only the immediate value added. If g_{it} were used instead of w_{it}, it would favor investments having the best immediate value added which would prove to be non-optimal at some stage (ik).

A simple example will clarify things. An amount y=2 is invested in any of the 2 sectors, assume 100 percent completion in year 1 for sector 1 and 50 percent completion for sector 2. Let both investments have the same life time, 5 years. Let the value added by each sector be $v_1 = .2$ and $v_2 = .3$.

Assume for convenience $\alpha_1 = \alpha_2 = 1$, $\beta_{it} = \beta_{2t} = 1$, and that both sectors are operating at full capacity, $c_{11} = 20$, $c_{21} = 15$. The total added value is .2 x 20 + .3 x 15 = 9.5.

Assume that with investment y=2, they will be still operating at full capacity.

If g_{it} is used, the value added, generated by y=2 in sector 1, will be .2 x 2 = .4 and in sector 2 it will be .3 x 1 = .3.

Investment in sector 1 will be favored by the recursion formulae.

(2.15) gives

$$Q_{11}(0) = 9.5$$

$$Q_{11}(2) = 9.5 + .4 = 9.9$$

(2.16) gives

$$Q_{21}(0) = 9.5$$

$$Q_{21}(2) = \max [P_{21}(0) + Q_{11}(2) , P_{21}(2) + Q_{11}(0)]$$

$$= \max [9.9 , 9.8] = 9.9$$

In year 2 it will be found out that investment in sector 2 was actually the best. Under the same conditions as year 1 (f_{i2}, e_{i2}, s_{i2}, s_{i1} are those for period 1 even though the sum tends to increase each year), the total value added in year 1 and year 2, if the investment in sector 1 was carried out, would be: 9.9 + 9.9 = 19.8.

If the same investment was made in sector 2, the total value added would be 9.8 + 10.1 = 19.9.

Using w_{21} Formula (2.4) gives

$$w_{21} = \frac{g_{21}\ (\frac{1}{2} + \varepsilon - \frac{1}{2h\%})}{\varepsilon}$$

$\varepsilon = 5$, $h\% = .5$

$$w_{21} = \frac{2\ (\frac{1}{2} + 5 - 1)}{5} = \frac{9}{5} = 1.8.$$

Now the value added generated by investing 2 m u in sector 2, is

.3 x 1.8 = .54 so Formula (2.16) gives

$Q_{21}(2) = \max [9.9, .54 + 9.5] = 10.04$, i.e., investment in sector 2 is favored.

This concludes the remarks made on w_{it} and g_{it}. The problem

faced by the planners is very tedious. For any stage (ik), the planners have to solve $\hat{\beta}_{ik}$ LP's where

$$\hat{\beta}_{ik} = \max [\beta_{ik} \text{ integer such that } \beta_{ik} g_{ik} = y \leq r]$$

The total number of LP's is $\sum_{k=1}^{T} \sum_{i=1}^{m} \hat{\beta}_{ik}$. This quantity can be very large.

Next a method will be proposed to reduce considerably the number of LP's to solve.

2.4 Insights in the Model

2.4.1 Reduction of Computations by Marginal Analysis

Max Z = VX such that AX = b and $X \geq 0$

V is a 3 m-component vector of constants

X is a 3 m-decision variable vector

A is a 2m x 3m matrix

b is a 2m-component vector of constants

Once this LP problem is solved, the following information can be obtained from the final tableau.

J = set of 2m basic variables

B^{-1} = inverse of the basis matrix B; B is a 2m x 2m matrix

$\bar{b}$ = vector of 2m optimal right hand side constraints

π = vector of 2m dual variables.

Let X_B and V_B denote respectively the vector of 2m optimal basic variables and the vector of 2m basic coefficients of the objective function associated with X_B.

Linear programming theorems give:[1]

$B^{-1} b = \bar{b}$

$V_B B^{-1} = \pi$

$V_B X_B = \text{Max } Z = \pi b$

The right hand side ranging method (abbreviated as r.h.s.r.m.) provides the tool for achieving the reduction of computations.

Let the right hand side of the i^{th} constraint increase by y.

Let $\partial(y)$ be a 2m column vector where all components are zero except the i^{th} one equal to y.

For y=1, the new vector of optimal right hand side becomes

$\bar{b}' = B^{-1} (b + \partial(1)) = \bar{b} + i^{th}$ column of B^{-1}

the new objective function is

$\max Z = \pi(b + \partial(1)) = \pi b + i^{th}$ component of π.

Now, for any y, so long as $B^{-1} (b + \partial(y)) = \bar{b}'$ remains non negative, i.e., $\bar{b}_j \geq 0$, for j=1 , . . . , 2m, the basic set remains optimal (and π the optimal dual vector). If y is large enough, $\bar{b}'$ may become negative in which case the old basic set is no longer feasible.

$$\bar{b}'_j = \bar{b}_j + B^{-1}_{ji} y \qquad j=1 , \ldots , 2m \tag{2.18}$$

where B^{-1}_{ji} is the element of B^{-1} corresponding to the j^{th} row and i^{th} column.

$\bar{b}'_j$ will become zero for some value of y if B^{-1}_{ji} is negative.

For the set J to remain basic, the maximum value of y should be

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[1] George B. Dantzig, Linear Programming and Extensions (Princeton, N.J.: Princeton University Press, 1963). Leon S. Lasdon, Optimization Theory for Large Systems (New York: Macmillan, 1972). Allan W. Spivey and Robert M. Thrall, Linear Optimization (New York: Holt, Rinehart, and Winston, Inc., 1970).

$$y_{max} = \hat{y} = \min \left[\frac{\bar{b}_j}{B^{-1}_{ji}} \quad , \quad B^{-1}_{ji} < 0 \right] \tag{2.19}$$

This means that the right hand side of the i^{th} constraint can increase by $\hat{y}$ leaving the set J basic and π the optimal dual vector.

For $0 < y \leq \hat{y}$ the value of the objective function is

$$\text{Max } Z = \pi(b + \partial(y)) = \pi b + \partial(y) \ (i^{th} \text{ component of } \pi) \tag{2.20}$$

Equation (2.20) will be used to reduce the number of LP's to solve.

2.4.2 Further Insights in the Model

The model of concern has the nice following property. Once the value of $\hat{y}$ is reached, increasing the right hand side of the i^{th} constraint will not improve the value of the objective function. In other words for $y \geq \hat{y}$, Max $Z = \pi b + \partial(y) \times \pi_i$, where π_i is the i^{th} component of π. This is so because of the following reasons.

V is a 3m component vector where all components are zero except the first m ones, those corresponding to x_{it}. Since there are 2m constraints, m for demand constraints (f_{it}, e_{it}, s_{it}, $s_{i(t-1)}$), and, m for capacity constraints, it is expected all x_{it} will be at a positive level in the optimal solution unless some constraints have a zero right hand side. At the optimal solution for any x^*_{it}, one of two cases must happen:

1) $z^*_{it} \geq 0$ and $u^*_{it} = 0$, or,

2) $u^*_{it} \geq 0$ and $z^*_{it} = 0$.

The first case states that the sector i is operating at full capacity. Increasing the capacity of sector i by $y \leq \hat{y}$ will improve the value of the objective function by $\partial(y)\pi_i$ and increase the optimal

value of x_{it}. Now, for $y > \hat{y}$ the result is a surplus capacity and the second case occurs. z^*_{it} will leave the basic set J, u^*_{it} will enter it and x^*_{it} will keep its value. Since u^*_{it} has a zero coefficient in vector V, its contribution to the objective function is zero.

This nice property will further reduce the number of LP's to solve, since, beyond $\hat{y}$ the objective function remains the same even though the basic set changes.

The basic matrix has a special characteristic if one is able to know what the basic variables are and consequently whether or not the slack capacity variables u_{it} are basic.

If none of the u_{it} are basic one does not have to solve an LP to get the optimal solution, i.e., Z* and $\bar{b}$; but this is not enough; one must calculate B^{-1} to perform the r.h.s.r.m. computations. In the following, a method will be proposed to find B^{-1}. In the basic set J,there are 2 m basic variables m for z_{it} and m for x_{it} since the case where none of the slack capacity variables u_{it} are basic is assumed. The method consists in arranging the set J starting with the import variables z_{it} occupying the first m places and the last m places being occupied by the x_{it}'s. J has the following format:

$$J = \{z_{1t}, z_{2t}, \ldots, z_{mt}, x_{1t}, x_{2t}, \ldots, x_{mt}\}.$$

The basic matrix B is also arranged by columns according to the order in set J. Its inverse or B^{-1} is obtained very easily: it is B but where all figures switch sign except those equal to 1. Note that the one's are on the diagonal.

Example: J = {3,4,1,2} i.e., m = 2

$$B = \begin{vmatrix} 1 & 0 & .8 & -.2 \\ 0 & 1 & -.1 & .8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \quad B^{-1} = \begin{vmatrix} 1 & 0 & -.8 & .2 \\ 0 & 1 & .1 & -.8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

This method can be used only when one knows what the basic sequence is and if none of the u_{it} are basic.

There is one instance where this is the case. This is when v_{it}, a_{ijt} and d_{ijt} at stage (k,1) (where $k \geq 2$) are recomputed. This is so because of the following reasons. The parameters are recomputed in state (k,1) when one or more sectors are operating below capacity in stage (k-1,m), i.e., the last stage of the previous year; the slack capacity is used to reduce d_{ijt} and increase v_{it} and a_{ijt}. Once the new parameters are obtained, the sectors that had $u^*_{it} > 0$ will operate at full capacity with the new parameters if $d_{ijt} \geq 0$. The case of $d_{ijt} < 0$ occurs when the slack capacity u_{it} more than offsets the imports $\sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ij(t-1)}x_{j(t-1)}$,

which leads to $d_{ijt} < 0$ in the Formulae (2.7) and (2.12)

$$d_{ijt} = a_{ij(t-1)} + d_{ij(t-1)} - a_{ijt}, \text{ for } j\neq i,$$

and

$$d_{ijt} = a_{ij(t-1)} + v_{j(t-1)} + d_{ij(t-1)} - a_{ijt} - v_{jt}, \qquad \text{for } j=i$$

As long as $d_{ijt} \geq 0$ in the previous formulae, its slack capacity is reduced to zero, which implies it cannot be basic. So, one will have to solve an LP only if d_{ijt} becomes negative implying that d_{ijt} is zero and slack capacity exists. Note also, if one or more slack capacity variables are basic and the basic set J is known, an LP need not be solved. Instead, the inverse basic matrix B^{-1} must be found and

Z^*, $\bar{b}$ and π are computed respectively, which can be easier than solving an LP.

The previous section in conjunction with the right hand side ranging method will make the dynamic programming approach efficient.

2.5 Dynamic Programming Solution in a New Light

In this section, a synthesis is made from both of the last sections to arrive at the final format of the dynamic programming solution.

Year 1: Year 1 corresponds to the first m stages.

*Stage (1,1): recursion formula (2.14) gives

$$Q_{11}(r) = P_{11}(r) \qquad r \leq n_1 \tag{2.21}$$

For r=0, the following LP must be solved:

$$\max Z = \sum_{i=1}^{m} v_{i1}x_{i1}$$

$$x_{i1} + \sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ij1}x_{j1} - \sum_{j=1}^{m} a_{ij1}x_{j1} + z_{i1} = f_{i1} + e_{i1} + s_{i1} - s_{i0},$$

$$i = 1 \ldots m$$

$$x_{i1} + u_{i1} = c_{i1}, \quad i=1 \ldots m$$

$$x_{i1} \geq 0,\ z_{i1} \geq 0,\ u_{i1} \geq 0.$$

The solution gives $P_{11}(0) = Q_{11}(0)$, and the basic set J. For $r > 0$, an amount $r = \beta_{11}g_{11}$ is invested in sector 1, resulting in a capacity increase of $\alpha_1\beta_{11}w_{11}$ in sector 1. Using the r.h.s.r.m., the maximum increase in capacity of sector 1, for the set J to remain basic can be obtained. Let $\hat{R}$ be such a capacity increase.

$$\hat{R} = \max \{\alpha_1 \beta_{11} w_{11}\} \tag{2.22}$$

Since α_1 and w_{11} are fixed, $\hat{R}$ depends only on β_{11}

$$\hat{\beta}_{11} = \frac{\hat{R}}{\alpha_1 w_{11}} \tag{2.23}$$

For any $r = \beta_{11} g_{11}$ such that $\beta_{11} \leq \hat{\beta}_{11}$ (β_{11} integer) it follows

$$Q_{11}(r) = P_{11}(0) + \alpha_1 \beta_{11} w_{11} \pi_{m+1} \tag{2.24}$$

where π_{m+1} corresponds to the dual associated with constraint m+1 (the first capacity constraint).

For any $r = \beta_{11} g_{11}$ such that $\beta_{11} > \hat{\beta}_{11}$ (β_{11} integer) the maximum value added is

$$Q_{11}(r) = P_{11}(0) + \alpha_1 \hat{\beta}_{11} w_{11} \pi_{m+1} \tag{2.25}$$

Note that $\hat{\beta}_{11}$ is not restricted to be integer. At state (1,1) the values of $Q_{11}(r) = P_{11}(r)$ for $r \leq n_1$ and the optimal basic set J and output levels $\bar{b}$ are recorded in Table 2.1.

Table 2.1 Stage (1,1)

r	0	g_{11}	$2g_{11}$	---	$\beta_{11} g_{11}$	---	n_1
$Q_{11}(r)$				---		---	
J				---		---	
$\bar{b}$				---		---	

*Stage (i,1), i=2, . . . , m

Recursion formula (2.13) gives

$$Q_{i1}(r) = \max_{y \leq r} [P_{i1}(y) + Q_{i-1,1}(r-y)], \quad r \leq n_1 \tag{2.16}$$

Table 2.2 for stage (i,1) has the following format:

Table 2.2 Stage (i,1)

r \ y	0	g_{i1}	--------	$\beta_{i1}g_{i1}$	--------	$Q_{i1}(r)$	y	J	$\overline{b}$
0									
n_1									

The shaded area corresponds to $y > r$.

In this table, the only boxes filled are those satisfying $y \leq r$. In other words, the computations of Formula (2.16) are carried out up to the point where r=y in each row of the table.

There is an interesting property in the dynamic programming recursion formula (2.16). $Q_{i-1,1}(r-y)$ is constant for (r-y) fixed. The quantity (r-y) assumes values from zero to r. For each value of (r-y), there is one line crossing the boxes in a diagonal manner; this line crosses only one box in each row and each column. It is along this line that the r.h.s.r.m. is applied. For example, in the case of

line r-y=0, y takes on the value 0, g_{i1}, $2g_{i1}$, etc. $P_{i1}(y) + Q_{i-1,1}(r-y)$ is the quantity that must be computed for each box along this line. Since $Q_{i-1,1}(r-y)$ is constant for all these boxes, the only difference between them is the value of y. The value of y only affects the i^{th} capacity constraint. The application of the r.h.s.r.m. is made possible, because $P_{i1}(y=0)$ and $Q_{i-1,1}(r-y)$, for a given (r-y), have the same basic set J.

Recall that $P_{i1}(y=0) = 0$ for l=2, . . . , m, as was shown previously. Once the r.h.s.r.m. is carried out for all lines (r-y), one is able to find $Q_{i1}(r)$ along each row of the table, to record the value of y, the basic set J and $\overline{b}$.

There is a case where one has to be very cautious. The value of $Q_{i1}(r)$ which is equal to $\max_{y \leq r} [P_{i1}(y) + Q_{i-1,1}(r-y)]$ is also the solution of a linear program where y is invested in sector i, r-y has been optimally invested among the other i-l sectors and zero will be invested in the other i+l to m sectors. There is one case where this relationship does not hold. In other words, $Q_{i1}(r)$ can be less than or equal to the Z* solution of the corresponding LP. This happens if both of the following conditions hold.

<u>1st condition</u>: $Q_{i-1,1}(r-y)$, i.e., the value added generated by optimally investing (r-y) among the (i-l) first sectors, is obtained with

$$P_{i-1,1} = 0 + \alpha_{i-1} \times \hat{\beta}_{i-1,1} \times w_{i-1,1} \times \pi_{m+i-1}$$

$P_{i-1,1}$ is the maximum generated value added along its line in stage (i-l,l) because the optimal investment decision leads to slack

capacity in sector i-1.

2nd condition: The value added obtained by investing y in sector i also results in a slack capacity in sector i, i.e., $P_{i1}(y) = 0 + \alpha_i \times \hat{\beta}_{i1} \times w_{i1} \times \pi_{m+1}$. In both of these conditions, the optimal decision has resulted in slack capacity in two adjoining sectors leading to $\hat{\beta}_{i1}$ and $\hat{\beta}_{i-1,1}$ which are not necessarily integer.

This situation is due to the non-independence of the sectors, situation which is not taken into account by the r.h.s.r.m. This weakness of the r.h.s.r.m. does not affect the recursion in future stages, namely i+1 to m, and, can be remediated by solving the LP corresponding to $Q_{i1}(r)$ and thus getting Z*.[1]

Year k: k=2, . . . , T

All that follows applies for any k=2, . . . , T.

In stage (1,k) $P_{1k}(y=0)$ is not equal to zero, contrary to the others.

*Stage (1,k):

Recursion formula (2.14) gives

$$Q_{1k}(r) = \max_{y \leq r} [P_{ik}(y) + Q_{m,k-1}(r-y)]$$

where

$$r \leq \sum_{t=1}^{k} n_t \quad , \quad \text{and,} \quad r-y \leq \sum_{t=1}^{k-1} n_t$$

The table in stage (1,k) has the following format.

---

[1] In the example solved in section 2.6, this situation occurs in stage (2,2) for r=28 along line r-y = 20.

Table 2.3 Stage (l,k)

r \ y	0	g_{ik}	-------	$\beta_{ik}g_{ik}$	-------	$Q_{ik}(r)$	y	J	$\bar{b}$
0									
n_1									
n_1+n_2									
$\sum_{t=1}^{k-1} n_t$									
$\sum_{t=1}^{k} n_t$									

The upper shaded area of the table is $y > r$.

The lower one is $r-y > \sum_{t=1}^{k-1} n_t$.

For $P_{ik}(y=0)$ and a given value of r, an LP must be solved. This LP has two characteristics:

-- First of all, it has a new right hand side for demand constraints: f_{ik}, e_{ik}, s_{ik}, $s_{i(k-1)}$

-- Second, the parameters v_{ik}, a_{ijk} and d_{ijk} might have changed for some i's, because of investments in the previous k-1 years.

*Stage (i,k): i=2, . . . , m

The same steps used in stage (i,1) are applicable here.

An example will be used to show how the dynamic programming approach works.

2.6 An Example

Assume there are two sectors, m=2. The following data pertain to year 1.

j	a_{ij1} i=1	a_{ij1} i=2	v_j
1	.2	.3	.2
2	.3	.2	.3

j	d_{ij1} i=1	d_{ij1} i=2
1	.1	.2
2	.1	.1

For an integrated economy the data are

j	$\bar{a}_{ij}$ i=1	$\bar{a}_{ij}$ i=2	$\bar{v}_j$
1	.4	.3	.3
2	.4	.3	.3

Assume there is a two year plan with the following demands, and also assume that $e_{it} = s_{it} = 0$.

i \ t	1	2
f_{1t}	15	18
f_{2t}	20	22

The initial capacities are $c_{11} = 20$, $c_{21} = 24$.

The coefficients of conversion are

$$\alpha_1 = 1$$

$$\alpha_2 = .9$$

The percentage of completion of project in sector 1 is 100 percent and in sector 2, 50 percent.

The life times of the projects are:

$\varepsilon = 5$ years in sector 1

$\varepsilon = 10$ years in sector 2.

The investment available in each year is 14 m u, i.e.,

$n_1 = n_2 = 14$.

The costs of a project in sector 1 and sector 2 are, respectively, 2 and 4. The equvalent investments w_{it} obtained by using (2.14) are:

$$w_{1t} = \frac{2\ (\frac{1}{2} + 5 - \frac{1}{2})}{5} = 2, \text{ and}$$

$$w_{2t} = \frac{4\ (\frac{1}{2} + 10 - 1)}{10} = 3.8$$

The solution by dynamic programming will be given in 4 stages, 2 for each year.

$$\text{Max } Z = \sum_{j=1}^{2} v_j x_{ji}$$

$$x_{11} + \sum_{\substack{j=1 \\ j\neq 1}}^{2} d_{1j1} x_{j1} - \sum_{j=1}^{2} a_{1j1} x_{j1} + z_{11} = 15$$

$$x_{21} + \sum_{\substack{j=1 \\ j\neq 2}}^{2} d_{2j1}x_{j1} - \sum_{j=1}^{2} a_{2j1}x_{j1} + z_{21} = 20$$

$$x_{11} + u_{11} = 20$$

$$x_{21} + u_{21} = 24$$

or

$$\text{Max } Z = .2x_{11} + .3x_{21}$$

$$x_{11} + .1x_{21} - .2x_{11} - .3x_{21} + z_{11} = 15$$

$$x_{21} + .2x_{11} - .3x_{11} - .2x_{21} + z_{21} = 20$$

$$x_{11} + u_{11} = 20$$

$$x_{21} + u_{21} = 24$$

or

$$\begin{aligned} \text{Max } Z = {} & .2x_{11} + .3x_{21} \\ & .8x_{11} - .2x_{21} + z_{11} = 15 \\ & -.1x_{11} + .8x_{21} + z_{21} = 20 \\ & x_{11} + u_{11} = 20 \\ & x_{21} + u_{21} = 24 \end{aligned}$$

$$x_{i1} \geq 0,\ z_{i1} \geq 0,\ u_{i1} \geq 0,\ i=1,2$$

The solution of this LP gives the following information.

$$Z^* = 11.2 \qquad \bar{b} = (3.8,\ 2.8,\ 20,\ 24)$$

$$J = \{z_{11},\ z_{21},\ x_{11},\ x_{21}\}\ ,\quad \pi = (0,\ 0,\ .2,.3)$$

$$B^{-1} = \begin{vmatrix} 1 & 0 & -.8 & .2 \\ 0 & 1 & .1 & -.8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

From now on, the basic variables in the set J will be denoted by their column number, i.e.,

$$J = \{z_{11},\ z_{21},\ x_{11},\ x_{21}\} = \{3,\ 4,\ 1,\ 2\}$$

If it was known that both of the input variables z_{11} and z_{21} were basic, B^{-1} could have been found by the method illustrated in section 2.5.2 without having to solve an LP.

Year 1

* Stage (1,1)

$P_{11}(r) = Q_{11}(r)$ $\quad r \leq n_1 = 14$

$P_{11}(r=0) = 11.2$

For $r > 0$, the r.h.s.r.m. will be applied.

Firs, none of the slack capacity variables in are basic, this implies that $P_{11}(r) > 11.2$, for $r > 0$.

Sector 1 capacity corresponds to the third constraint, i.e., i=3, so Formula (2.19) gives

$$\hat{y} = \min_j \left[\frac{\bar{b}_j}{-B^{-1}_{j3}}, \quad B^{-1}_{j3} < 0\right] = \frac{3.8}{.8} = 4.75$$

Formula (2.19) is identical to Formula (2.22)

$\hat{y} = \hat{R} = 4.75$

From Formula (2.21), $\hat{\beta}_{11} = \dfrac{\hat{R}}{\alpha_1 w_{11}} = \dfrac{4.75}{2} = 2.375$

$r = \beta_{11} g_{11}$, β_{11} integer. If $\beta_{11} > 2$, sector 1 will operate under capacity.

The discussion following Formulas (2.22) and (2.23) gives

$r = 2$ $\quad Q_{11}(2) = P_{11}(0) + 1 \times 1 \times 2\ \pi_3$

$= 11.2 + 1 \times 1 \times 2 \times .2$

$= 11.6$

$r = 4$ $\quad Q_{11}(4) = 11.2 + 1 \times 2 \times 2 \times .2$

$= 12$

For $\beta_{11} > 2$ or $r > 4$ Formula (2.24) gives

$$Q_{11}(6) = 11.2 + 1 \times 2.375 \times 2 \times .2 = Q_{11}(8) = Q_{11}(10) = Q_{11}(12)$$
$$= 12.15$$

Table 2.4 contains data on the level of investment r, the corresponding value added ($Q_{11}(\bar{r})$), and the optimal basic set J and output levels $\bar{b}$.

Table 2.4 Results of Stage (1,1)

r	$Q_{11}(r)$	J	$\bar{b}$
0	11.2	3, 4, 1, 2	(3.8, 2.8, 20, 24)
2	11.6	3, 4, 1, 2	(2.2, 3, 22, 24)
4	12	3, 4, 1, 2	(.4, 3.2, 24, 24)
6	12.15	5, 4, 1, 2	(1.25, 3.275, 24.75, 24)
8	12.15	5, 4, 1, 2	(3.25, 3.275, 24.75, 24)
10	12.15	5, 4, 1, 2	(5.25, 3.275, 24.75, 24)
12	12.15	5, 4, 1, 2	(7.25, 3.275, 24.75, 24)
14	12.15	5, 4, 1, 2	(9.25, 3.275, 24.75, 24)

The above table requires some explanations.

For r=0, J and $\bar{b}$ are those obtained by the LP solution found previously.

For r=2 and r=4, the r.h.s.r.m. gives the same basic set J as for r=0. To obtain $\bar{b}$, for r=2 and r=4, Formula (2.18) is used, i.e.,

$\bar{b}_j = \bar{b}_j + B^{-1}_{ji}\, r$;

sor for r=2 $\quad \bar{b}' = \begin{vmatrix} 3.8 \\ 2.8 \\ 20 \\ 24 \end{vmatrix} + 2 \begin{vmatrix} -.8 \\ .1 \\ 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 2.2 \\ 3.0 \\ 22 \\ 24 \end{vmatrix}$

The case of r=4 is obtained similarly.

For r=6, by the results of the r.h.s.r.m. J changes, because sector 1 is operating below capacity. This implies that the import variable z_{11} (or 3) leaves the basis and slack capacity u_{11} (or 5) will enter it. The new basic set is J = {5, 4, 1, 2}. However, an LP need not be solved, since the new basic set is known, the inverse basis matrix associated with it is obtained by the method of section 2.4.2.

$$B^{-1} = \begin{vmatrix} -1.25 & 0 & 1 & -.25 \\ .125 & 1 & 0 & -.775 \\ 1.25 & 0 & 0 & .25 \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad \bar{b} = \begin{vmatrix} 1.25 \\ 3.275 \\ 24.75 \\ 24 \end{vmatrix}$$

π = (.25, 0, 0, .35).

For r=6, $\bar{b}$ is computed as follows

$$\bar{b} = B^{-1}b = B^{-1}\ (15, 20, 26, 24) = (1.25, 3.275, 24.75, 24)$$

For r=8 $\bar{b} = B^{-1}\ (15, 20, 28, 24) = (3.25, 3.275, 24.75, 24)$

For r=10 $\bar{b} = B^{-1}\ (15, 20, 30, 24) = (5.25, 3.275, 24.75, 24)$

For r=12 $\bar{b} = B^{-1}\ (15, 20, 32, 24) = (7.25, 3.275, 24.75, 24)$

For r=14 $\bar{b} = B^{-1}\ (15, 20, 34, 24) = (9.25, 3.275, 24.75, 24)$

* Stage (2,1)

$$Q_{21}(r) = \max_{y \le r} [P_{21}(y) + Q_{11}(r-y)] \qquad r \le 14$$

$P_{21}(y=0) = 0$

$\alpha_2 = .9$

$w_{2t} = 3.8$

$g_{2t} = 4$

In each box of Table 2.5 corresponding to stages (2,1) two values will be recorded: $P_{21}(y)$ and $Q_{11}(r-y)$.

Remember the r.h.s.r.m. is used along the lines r-y.

Table 2.5 Results of Stage (2,1)

x^y	0	4	8	12	$Q_{21}(r)$	y	J	$\bar{b}$
0	0+11.2				11.2	0	3,4,1,2	(3.8,2.8,20,24)
2	0+11.6				11.6	0	3,4,1,2	(2.2,3,22,24)
4	0+12	1.03+11.2			12.23	4	3,4,1,2	(4.48,.06,20,27.42)
6	0+12.15	1.03+11.6			12.63	4	3,4,1,2	(2.88,.26,22,27.42)
8	0+12.15	1.03+12	1.05+11.2		13.03	4	3,4,1,2	(1.08,.46,24,27.42)
10	0+12.15	1.2+12.15	1.13+11.6		13.35	4	5,4,1,2	(.4,.63,25.6,27.42)
12	0+12.15	1.2+12.15	1.2+12	1.05+11.2	13.35	4	5,4,1,2	(2.4,.63,25.6,27.42)
14	0+12.15	1.2+12.15	1.48+12.15	1.13+11.6	13.63	8	5,6,1,2	(.19,2.61,25.81,28.23)

Sector 2 capacity corresponds to the fourth constraint, i.e., i=4.

1) line r-y = 0 $\quad Q_{11}(r-y = 0) = 11.2$ with corresponding

$j = \{3, 4, 1, 2\}$ and $\overline{b} = (3.8, 2.8, 20, 24)$

$$\hat{y} = \hat{R} = \min \left[\frac{\overline{b}_j}{-B^{-1}_{j4}}, \quad B^{-1}_{j4} < 0\right] = \frac{2.8}{.8} = 3.5$$

$$\hat{\beta}_{21} = \frac{3.5}{.9 \times 3.8} = \frac{3.5}{3.42} = 1.0234$$

$$P_{21}(y=0) = 0$$

$$P_{21}(y=4) = P_{21}(y=0) + \alpha_2 \times \beta_{21} \times w_{21} \times \pi_4$$

$$= 0 + .9 \times 1 \times 3.8 \times .3$$

$$= 1.026 \simeq 1.03$$

$$P_{21}(y=8) = P_{21}(y=0) + .9 \times 1.0234 \times 3.8 \times .3 = P_{21}(y=12)$$

$$= 1.05$$

2) line r-y = 2 $\quad Q_{11}(r-y = 2) = 11.6$ with corresponding

$J = \{3, 4, 1, 2\}$ and $\overline{b} = (2.2, 3, 22, 24)$

$$\hat{y} = \hat{R} = \frac{3}{.8} = 3.75$$

$$\hat{\beta}_{21} = \frac{3.75}{3.24} = 1.0965$$

$$P_{21}(y=0)$$

$$P_{21}(y=4) = 0 + .9 \times 1 \times 3.8 \times .3$$

$$= 1.03$$

$$P_{21}(y=8) = 0 + .9 \times 1.0965 \times 3.8 \times .3 = P_{21}(y=12)$$

$$= 1.125 \simeq 1.13$$

3) line r-y=4 $\quad Q_{11}(r-y = 4) = 12$ with

$J = \{3, 4, 1, 2\}$ and $\overline{b} = (.4, 3.2, 24, 24)$

$$\hat{y} = \hat{R} = \frac{3.2}{.8} = 4$$

$$\hat{\beta}_{21} = \frac{4}{3.42} = 1.1696$$

$P_{21}(y=0) = 0$

$P_{21}(y=4) = 0 + .9 \times 1 \times 3.8 \times .3 = 1.03$

$P_{21}(y=8) = 0 + .9 \times 1.1696 \times 3.8 \times .3 = 1.2$

4) line r-y = 6 $\quad Q_{11}(r-y = 6) = 12.15$ with

J = {5, 4, 1, 2} and $\bar{b}$ = (1.25, 3.275, 24.75, 24)

$$\hat{y} = \hat{R} = \min\left[\frac{1.25}{.25}, \frac{3.275}{.775}\right] = 4.2258$$

$$\hat{\beta}_{21} = \frac{4.2258}{3.42} = 1.2356$$

$P_{21}(y=0) = 0$

$P_{21}(y=4) = 0 + \alpha_2 \times \beta_{21} \times w_{21} \times \pi_4 = 0 + .9 \times 1 \times 3.8 \times .35$

$= 1.197 \simeq 1.2$

$P_{21}(y=8) = 0 + .9 \times 1.2356 \times 3.8 \times .35 = 1.48$

5) line r-y=8 $\quad Q_{11}(r-y=8) = 12.15$ with

J = {5, 4, 1, 2} and $\bar{b}$ = (3.25, 3.275, 24.75, 24)

$$\hat{y} = \hat{R} = \min\left[\frac{3.25}{.25}, \frac{3.275}{.775}\right] = 4.2258$$

$$\hat{\beta}_{21} = \frac{4.2258}{3.42} = 1.2356$$

$P_{21}(y=0) = 0$

$P_{21}(y=4) = 0 + .9 \times 1 \times 3.8 \times .35$

$\simeq 1.2$

6) line r-y=10 $\quad Q_{11}(r-y=10) = 12.15$ with

J = {5, 4, 1, 2} and $\bar{b}$ = (5.25, 3.275, 24.75, 24)

$$\hat{y} = \hat{R} = \min\left[\frac{5.25}{.25}, \frac{3.275}{.775}\right] = 4.2258$$

$$\hat{\beta}_{21} = \frac{4.2258}{3.42} = 1.2356$$

$P_{21}(y=0) = 0$

$P_{21}(y=4) = 0 + .9 \times 1 \times 3.8 \times .35 \simeq 1.2$

To complete Table 2.5 one proceeds as follows. Along each row, the value or $Q_{21}(r) = \max_{y \leq r} [P_{21}(y) + Q_{11}(r-y)]$ and the corresponding value of y are recorded. To get J and $\bar{b}$ one proceeds as follows. The value of y is zero for the first two rows, i.e., r=0 and r=2, this implies that J and $\bar{b}$ remain the same as in stage (1,1).

For r=4, $Q_{21}(r=4) = 12.23$ is obtained when 4 are invested in stage (2,1) leading to 1 project, i.e., $\beta_{21} = 1$. Since $\beta_{21} = 1 < \hat{\beta}_{21} = 1.0234$, along line r=y=0, J is the same as the one found for $Q_{11}(r-y=0)$; $\bar{b}$ is obtained by using (2.18).

$$\bar{b}_j' = \bar{b}_j + B_{ji}^{-1}\partial$$

where

$$\partial = .9 \times 3.8 = 3.42$$

$$\bar{b} = \begin{vmatrix} 3.8 \\ 2.8 \\ 20 \\ 24 \end{vmatrix} + \begin{vmatrix} .2 \\ -.8 \\ 0 \\ 1 \end{vmatrix} \times 3.42 = \begin{vmatrix} 4.48 \\ .06 \\ 20 \\ 27.42 \end{vmatrix}$$

For r=6, using the same reasoning, it follows

$$\bar{b} = \begin{vmatrix} 2.2 \\ 3 \\ 22 \\ 24 \end{vmatrix} + \begin{vmatrix} .2 \\ -.8 \\ 0 \\ 1 \end{vmatrix} \times 3.42 = \begin{vmatrix} 2.88 \\ .26 \\ 22 \\ 27.42 \end{vmatrix}$$

For r=8,

$$\bar{b} = \begin{vmatrix} .4 \\ 3.2 \\ 24 \\ 24 \end{vmatrix} + \begin{vmatrix} .2 \\ -.8 \\ 0 \\ 1 \end{vmatrix} \times 3.42 = \begin{vmatrix} 1.08 \\ .46 \\ 24 \\ 27.42 \end{vmatrix}$$

For r=10,

$$\bar{b} = \begin{vmatrix} 1.25 \\ 3.275 \\ 24.75 \\ 24 \end{vmatrix} + \begin{vmatrix} -.25 \\ -.775 \\ .25 \\ 1 \end{vmatrix} \times 3.42 = \begin{vmatrix} .40 \\ .63 \\ 25.6 \\ 27.42 \end{vmatrix}$$

For r=12,

$$\bar{b} = \begin{vmatrix} 3.25 \\ 3.275 \\ 24.75 \\ 24 \end{vmatrix} + \begin{vmatrix} -.25 \\ -.775 \\ .25 \\ 1 \end{vmatrix} \times 3.42 = \begin{vmatrix} 2.4 \\ .63 \\ 25.6 \\ 27.42 \end{vmatrix}$$

For r=14, the best thing to do is to invest 8 in sector 2 which means that 2 projects will be built. However, along line r-y=6, the maximum immediate value added of 1.48 corresponds to 1.2356 projects. This means by investing in two projects sector 2 will operate below capacity. Therefore, the basic set J is {5, 6, 1, 2}

$$\text{with } B^{-1} = \begin{vmatrix} -1.2908 & -.3226 & 1 & 0 \\ -.1613 & -1.2903 & 0 & 1 \\ 1.2903 & .3226 & 0 & 0 \\ .1613 & 1.2903 & 0 & 0 \end{vmatrix}, \quad \pi = (.2, .3, 0, 0)$$

$$\bar{b} = (15, 20, 26, 30.84)\ B^{-1} = (.19, 2.61, 25.81, 28.23)$$

Year 2

In stage (1,k), $k \geq 2$, parameters value might change, due to excess capacity in year 1.

* Stage (1,2)

$$Q_{12}(r) = \max_{y \leq r} [P_{12}(y) + Q_{21}(r-y)]$$

$$r \leq n_1 + n_2 = 14 + 14 = 28$$

$$r - y \leq 14$$

To get the value of $P_{12}(y=0)$ in stage (1,2) an LP must be solved because the right hand side of demand constraints has changed, and, so did the capacity constraints because of previous investments.

1) line r-y=0 $\quad Q_{21}(r-y=0) = 11.2$ with

J = {3,4, 1, 2} and $\overline{b}$ = (3.8, 2.8, 20, 24)

There is no need to solve an LP because none of the slack capacity variables are basic. The increase in the right hand side of demand constraints will not improve the value of Z*, but the import variables z_{12} and z_{22} (i.e., 3 and 4 in J) will increase, respecively, by the same amount as demand, namely, 3 and 2.

$P_{12}(y=0) = 11.2$ and $\overline{b}$ = (6.8, 4.8, 20, 24)

Let us apply the r.h.s.r.m. along line r-y=0. Sector 1 capacity corresponds to the third constraint, i.e., i=3.

$$\hat{y} = \hat{R} = \frac{6.8}{.8} = 8.5$$

$$\hat{\beta}_{21} = \frac{8.5}{1x2} = 4.25$$

$P_{12}(y=2) = 11.2 + 1 \times 1 \times 2 \times .2 = 11.6$

$P_{12}(y=4) = 11.2 + .8 = 12$

$P_{12}(y=6) = 11.2 + 1.2 = 12.4$

$P_{12}(y=8) = 11.2 + 1.6 = 12.8$

$P_{12}(y=10) = 11.2 + 1 \times 4.25 \times 2 \times .2 = 12.9 = P_{12}(y=12) = \ldots$

$= P_{12}(y=28)$

2) line r-y=2 $\quad Q_{21}(r-y=2) = 11.6$ with

J = {3, 4, 1, 2} and $\overline{b}$ = (2.2, 3, 22, 24)

Following the same reasoning as above, it follows

$P_{12}(y=0) = 11.6$ and $\overline{b}$ = (5.2, 5, 22, 24)

$$\hat{y} = \hat{R} = \frac{5.2}{.8} = 6.4$$

$$\hat{\beta}_{12} = \frac{6.5}{2} = 3.25$$

$$P_{12}(y=2) = 11.6 + .4 = 12$$

$$P_{12}(y=4) = 12.4$$

$$P_{12}(y=6) = 12.8$$

$$P_{12}(y=8) = 11.6 + 1 \times 3.25 \times 2 \times .2 = 12.9 = \ldots$$

$$= P_{12}(y=26)$$

For lines r-y=4 up to r-y=8, the computation will not be carried out because they follow the same procedure as above since J does not contain any slack capacity variables. The figures are in Table 2.6.

3) line r-y=10 $\quad Q_{21}(r-y=10) = 13.35$ with

$J = \{5, 4, 1, 2\}$ and $\bar{b} = (.4, .63, 25.6, 27.42)$

Here, since slack capacity is present in sector 1, the parameters v_{12}, a_{1j2} and d_{1j2} will change. The slack capacity $u_1 = .4$ is used to reduce imports of goods 1 by the whole economy.

Equation (2.5) gives

$$\gamma_{11} = \frac{.4 \times .2 \times 25.6}{.2 \times 25.6 + .3 \times 27.42} = \frac{2.0488}{13.346} = .1535$$

$$\gamma_{12} = \frac{.4 \times .3 \times 27.42}{13.346} = .2465$$

Equations (2.6) and (2.7) give

$$a_{122} = \frac{.3 \times 27.42 + .2465}{27.42} = .309$$

$$d_{122} = .3 + .1 - .309 = .091$$

Equations (2.8) to (2.12) result in

$$\mu_{11} = .3 \times .1535 = .04605$$

$$\mu_{21} = .1535 - .04605 = .10745$$

Table 2.6 Results of Stage (1,2)

r^y	0	2	4	6	8	10	12	14	16
0	11.2+11.2								
2	11.6+11.6	11.6 +11.2							
4	12.23+12.23	12 +11.6	12 +11.2						
6	12.63+12.63	12.63+12.23	12.4 +11.6	12.4 +11.2					
8	13.03+13.03	13.03+12.63	13.03+12.23	12.8 +11.6	12.8 +11.2				
10	13.48+13.35	13.43+13.03	13.43+12.63	13.43+12.23	12.9 +11.6	12.9 +11.2			
12	14.13+13.35	13.88+13.35	13.8 +13.03	13.83+12.63	13.83+12.23	12.9 +11.6	12.9 +11.2		
14	14.85+13.63	14.52+13.35	14.29+13.35	13.8 +13.03	14.1 +12.63	14.1 +12.23	12.9 +11.6	12.9 +11.2	
16		15.25+13.63	14.97+13.35	14.31+13.35	13.8 +13.03	14.1 +12.63	14.1 +12.23	12.9 +11.6	12.9 +11.2
18			15.65+13.63	15.37+13.35	14.31+13.35	13.8 +13.03	14.1 +12.63	14.1 +12.23	12.9 +11.6
20				15.77+13.63	15.42+13.35	14.31+13.35	13.8 +13.03	14.1 +12.63	14.1 +12.23
22					15.77+13.63	15.42+13.35	14.31+13.35	13.8 +13.03	14.1 +12.63
24						15.77+13.63	15.42+13.35	14.31+13.35	13.8 +13.03
26							15.77+13.63	15.42+13.35	14.31+13.35
28								15.77+13.63	15.42+13.35

Table 2.6 (Continued)

18	20	22	24	26	28	$Q_{12}(r)$	y	J	$\bar{b}$
						22.4	0	3,4,1,2	(6.8,4.8,20,24)
						23.2	0	3,4,1,2	(5.2,5,22,24)
						24.46	0	3,4,1,2	(7.48,2.06,20,27.42)
						25.26	0	3,4,1,2	(5.88,2.26,22,27.42)
						26.06	0	3,4,1,2	(3.08,2.46,24,27.42)
						26.83	0	3,4,1,2	(3.28,2.66,26, 27.42)
						27.48	0	3,4,1,2	(4.75,2.86,28,27.42)
						28.48	0	3,4,1,2	(3.67,3.83,26,30.84)
						28.88	2	3,4,1,2	(2.07,4.27,28,30.84)
12.9 +11.2						29.28	4	3,4,1,2	(.48,4.7,30,30.84)
12.9 +11.6	12.9+11.2					29.4	6	5,4,1,2	(1.4,4.83,30.6,30.84)
14.1 +12.23	12.9+11.6	12.9+11.2				29.4	8	5,4,1,2	(3.4,4.83,30.6,30.84)
141 +12.63	14.1+12.23	12.9+11.6	12.9+11.2			29.4	10	5,4,1,2	(5.4,4.83,30.6,30.84)
13.8 +13.03	14.1+12.63	14.1+12.23	12.9+11.6	12.9+11.2		29.4	12	5,4,1,2	(7.4,4.83,30.6,30.84)
14.31+13.35	13.8+13.03	14.1+12.63	14.1+12.23	12.9+11.6	12.9+11.2	29.4	14	5,4,1,2	(9.4,4.83,30.6,30.84)

$$a_{112} = \frac{.2 \times 25.6 + .10745}{25.6} = .204$$

$$v_{12} = \frac{.2 \times 25.6 + .04605}{25.6} = .202$$

$$d_{112} = .2 + .1 + .2 - .204 - .202 = .094$$

The LP that must be solved with the new parameters has the following form in period 2.

$$\text{Max } Z = .202\, x_{12} + .3\, x_{22}$$

$$x_{12} + .091\, x_{22} - .204\, x_{12} - .309\, x_{22} + z_{12} = 18$$

$$x_{22} + .2\, x_{12} - .3\, x_{12} - .2\, x_{22} + z_{22} = 22$$

$$x_{12} + u_{12} = 26$$

$$x_{22} + u_{22} = 27.42$$

or

$$\begin{aligned} \text{Max } Z = .202\, x_{12} + .3 x_{22} & \\ .796\, x_{12} - .218 x_{22} + z_{12} &= 18 \\ -.1\, x_{12} + .8 x_{22} + z_{22} &= 22 \\ x_{12} + U_{12} &= 26 \\ x_{22} + u_{uu} &= 27.42 \end{aligned}$$

$x_{i2} \geq 0$, $z_{i2} \geq 0$, $u_{i2} \geq 0$.

From the discussion of section 2.2.2, whenever the parameters for a given sector are recomputed, this sector will operate again at full capacity. This is the case for sector 1; since sector 2 was also operating at full capacity there is no need to solve this LP since the basis will comprise z_{12} and z_{22}, and, B^{-1} can be computed as shown in section 2.5.2.

$$J = \{3, 4, 1, 2\}$$

$$B^{-1} = \begin{vmatrix} 1 & 0 & -.796 & .218 \\ 0 & 1 & .1 & -.8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad \pi = (0, 0, .202, .3)$$

$\overline{b} = B^{-1}b = B^{-1}$ (18, 22, 26, 27.42) = (3.28, 2.66, 26, 27.42)

$Z^* = 13.48$

$P_{12} = 13.48$

The r.h.s.r.m. application provides the following value

$$\hat{y} = \hat{R} = \frac{3.28}{.796} = 4.1206$$

$$\hat{\beta}_{12} = \frac{4.1206}{2} = 2.0603$$

$P_{12}(y=2) = 13.48 + 1 \times 1 \times 2 \times .202$

$= 13.884 \simeq 13.88$

$P_{12}(y=4) = 13.48 + .808$

$\simeq 14.29$

$P_{12}(y=6) = 13.48 + .1 \times 2.0603 \times 2 \times .202 = P_{12}(y=8) = \ldots =$

$= P_{12}(y=18) = 14.31$

4) line r-y=12 $\quad Q_{21}(r-y=12) = 13.35$ with

$J = \{5, 4, 1, 2\}$ and $\overline{b} = (2.4, .63, 25.6, 27.42)$

Since $u_{11} = 2.4$, v_{12}, a_{1j2} and d_{1j2} must be recomputed.

$$\gamma_{11} = \frac{2.4 \times .2 \times 25.6}{.2 \times 25.6 + .3 \times 27.42} = \frac{12.288}{13.346} = .9206$$

$$\gamma_{12} = \frac{2.4 \times .3 \times 27.42}{13.346} = 1.4794$$

$$a_{122} = \frac{.3 \times 27.42 + 1.4794}{27.42} = .354$$

$d_{122} = .3 + .1 - .354 = .046$

$\mu_{11} = .3 \times .9206 = .27618$

$\mu_{21} = .64442$

$$a_{112} = \frac{.2 \times 25.6 + .64442}{25.6} = .225$$

$$v_{12} = \frac{.2 \times 25.6 + .27618}{25.6} = .211$$

$d_{112} = .2 + .1 + .2 - .225 - .211 = .064$

The LP to be solved is

$$\begin{aligned}
\text{Max } Z = .211\,x_{12} + .3\,x_{22} & \\
.775\,x_{12} - .308\,x_{22} + z_{12} &= 18 \\
-.1\,x_{12} + .8\,x_{22} + z_{22} &= 22 \\
x_{12} + u_{12} &= 28 \\
x_{22} + u_{22} &= 27.42
\end{aligned}$$

$x_{i2} \geq 0,\ z_{i2} \geq 0,\ u_{i2} \geq 0$

Here again the basic set is J = {3, 4, 1, 2}, for the same previous reasons.

$$B^{-1} = \begin{vmatrix} 1 & 0 & -.775 & .308 \\ 0 & 1 & .1 & -.8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \quad \pi = (0, 0, .211, .3)$$

$\bar{b} = B^{-1}b = B^{-1}\ (18, 22, 28, 27.42) = (4.75, 2.86, 28, 27.42)$

$Z^* = 14.13$

$P_{12}(y=0) = 14.13$

$\hat{y} = \hat{R} = \frac{4.75}{.775} = 6.129$

$\hat{\beta}_{12} = \frac{6.129}{2} = 3.065$

$P_{12}(y=2) = 14.13 + 1 \times 1 \times 2 \times .211 = 14.52$

$P_{12}(y=4) = 14.97$

$P_{12}(y=6) = 15.37$

$P_{12}(y=8) = 14.13 + 1 \times 3.065 \times 2 \times .211 = \ldots$

$= P_{12}(y=16) = 15.42$

5) line r-y=14 $\quad Q_{21}(r-y=14) = 13.63$ with

J = {5, 6, 1, 2} and $\bar{b}$ = (.19, 2.51, 25.81, 28.23)

Since u_{11} and u_{21} are basic, all parameters must be recomputed.

a) $u_{11} = .19$

$$\gamma_{11} = \frac{.19 \times .2 \times 25.81}{.2 \times 25.81 + .3 \times 28.23} = \frac{.98078}{13.631} = .072$$

$$\gamma_{12} = \frac{.19 \times .3 \times 28.23}{13.631} = .118$$

$$a_{122} = \frac{.3 \times 28.23 + .118}{28.23} = .304$$

$$d_{122} = .3 + .1 - .304 = .096$$

$$\mu_{11} = .3 \times .072 = .0216$$

$$\mu_{21} = .0504$$

$$a_{112} = \frac{.2 \times 25.81 + .0504}{25.81} = .202$$

$$v_{12} = \frac{.2 \times 25.81 + .0216}{25.81} = .201$$

$$d_{112} = .2 + .1 + .2 - .202 - .201 = .097$$

b) $u_{21} = 2.61$

$$\gamma_{21} = \frac{2.61 \times .3 \times 25.81}{.3 \times 25.81 + .2 \times 28.23} = \frac{20.20923}{12.289} = 1.5094$$

$$\gamma_{22} = \frac{2.61 \times .2 \times 28.23}{13.389} = 1.1006$$

$$a_{212} = \frac{.3 \times 25.81 + 1.5094}{25.81} = .359$$

$$d_{212} = .3 + .2 - .359 = .141$$

$$\mu_{12} = .3 \times 1.1006 = .33018$$

$$\mu_{22} = .77042$$

$$a_{222} = \frac{.2 \times 28.23 + .77042}{28.23} = .277$$

$$v_{22} = \frac{.3 \times 28.23 + .33018}{28.23} = .312$$

$d_{222} = .2 + .3 + .1 - .227 - .312 = .061$

The above values are summarized in the following tables.

j \ i	a_{ij2} 1	2	v_{j2}
1	.202	.359	.201
2	.304	.227	.312

j \ i	d_{ij2} 1	2
1	.097	.141
2	.096	.061

Once the parameters new values are included the LP is:

$$\text{Max } Z = .201\, x_{12} + .312\, x_{22}$$

$$.798\, x_{12} - .208\, x_{22} + z_{12} = 18$$

$$-.218\, x_{12} + .773\, x_{22} + z_{22} = 22$$

$$x_{12} + u_{12} = 26$$

$$x_{22} + u_{22} = 30.84$$

The basic set is J = {3, 4, 1, 2}, for the same previous reasons; and, hence

$$B^{-1} = \begin{vmatrix} 1 & 0 & -.798 & .208 \\ 0 & 1 & .218 & -.773 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \quad \pi = (0, 0, .201, .312)$$

$$\bar{b} = B^{-1}b = B^{-1}\,(18, 22, 26, 30.84) = (3.67, 3.83, 26, 30.84)$$

$$Z^* = 14.85 = P_{12}(y=0)$$

$$\hat{y} = \hat{R} = \frac{3.67}{.798} = 4.599$$

$$\hat{\beta}_{12} = \frac{4.599}{2} = 2.299$$

$$P_{12}(y=2) = 14.85 + 1 \times 1 \times 2 \times .201 = 15.25$$

$$P_{12}(y=4) = 15.65$$

$$P_{12}(y=6) = 14.85 + 1 \times 2.299 \times 2 \times .201 = \ldots = P_{12}(y=14) = 15.77$$

To complete Table 2.6 one must record the values of $Q_{12}(r)$, y, J and $\bar{b}$. For r=0 to r=14, y=0; J and $\bar{b}$ are those obtained when the different LP's used to get $P_{12}(y=0)$ were solved.

For r=16 and r=18, the value of $Q_{12}(r)$ was obtained with J = {3, 4, 1, 2} which means that $\bar{b}$, for those values of r, can be found using Formula (2.18).

$$\bar{b} = (3.67,\ 3.83,\ 26,\ 30.84) + (\text{3rd column of } B^{-1})\alpha_1\beta_{12}w_{12}$$

For r=20 up to r=28, $Q_{12}(r)$ attains a maximum value because sector 1 will be operating below capacity. So, the slack variable u_{12} will enter the basis and z_{12} (or 3) will leave the basic set. The new basic set is J = {5, 4, 1, 2} and remains the same from r=20 to r=28. B^{-1} is given below.

$$B^{-1} = \begin{vmatrix} 0 & 0 & .798 & -.208 \\ 0 & 1 & -.218 & .773 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad B^{-1} = \begin{vmatrix} -1.253 & 0 & 1 & -.261 \\ .273 & 1 & 0 & -.716 \\ 1.253 & 0 & 0 & .261 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\pi = v_B\, B^{-1} = (0,\ 0,\ .201,\ .312)\ B^{-1} = (.25,\ 0,\ 0,\ .365)$$

For r=20 $\bar{b} = B^{-1}\ (18,\ 22,\ 32,\ 30.84) = (1.4,\ 4.83,\ 30.6,\ 30.84)$

r=22 $\bar{b} = B^{-1}\ (18,\ 22,\ 34,\ 30.84) = (3.4,\ 4.83,\ 30.6,\ 30.84)$

r=24 $\bar{b} = (5.4,\ 4.83,\ 30.6,\ 30.84)$

r=26 $\bar{b} = (7.4,\ 4.83,\ 30.6,\ 30.84)$

r=28 $\bar{b} = (9.4,\ 4.83,\ 30.6,\ 30.84)$

* Stage (2,2)

It is the last stage of this example.

$$Q_{22}(r) = \max_{y \le r} [P_{22}(y) + Q_{12}(r-y)] \qquad r \le 28$$

Sector 2 capacity corresponds to the fourth constraint, i.e., i=4.

1) line r-y=0 $\quad Q_{12}(r-y=0) = 22.4$ with

J = {3,4, 1, 2} and $\overline{b}$ = (6.8, 4.8, 20, 24)

The r.h.s.r.m. provides the following information

$$\hat{y} = \hat{R} = \min_j \left[\frac{\overline{b}_j}{-B^{-1}_{j4}}, B^{-1}_{j4} < 0\right] = \frac{4.8}{.8} = 6$$

$$\hat{\beta}_{22} = \frac{6}{.9 \times 3.8} = \frac{6}{3.42} = 1.7544$$

$$P_{22}(y=0) = 0$$

$$P_{22}(y=4) = 0 + .9 \times 1 \times 3.8 \times .3 = 1.03$$

$$P_{22}(y=8) = 0 + .9 \times 1.7544 \times 3.8 \times .3 = 1.8 = \ldots = P_{22}(y=28)$$

For lines r-y=2 to r-y=18 the results are obtained in a similar way and are shown in Table 2.7.

2) line r-y=20 $\quad Q_{12}(r-y=20) = 29.4$ with

J = {5, 4, 1, 2} and $\overline{b}$ = (1.4, 4.83, 30.6, 30.84)

Here, if the r.h.s.r.m., is applied a smaller $P_{22}(y)$ will be obtained when y > 0, because sector 1 is already operating under capacity (u_{12} = 1.4). The r.h.s.r.m. does not take into consideration the fact that after building in sector 2, β_{22} projects where $\beta_{22} > \hat{\beta}_{22}$, sector 1 might operate at full capacity, because of the interdependence between both of these sectors. It was pointed out $\beta_{22} > \hat{\beta}_{22}$, i.e., the case where the marginal value added generated by investing in sector 2, $\beta_{22}g_{22}$ does not comprise the marginal value added generated by secoor 1 (which was operating at full capacity) resulting from the investment in sector 2. So to get the whole marginal value added due to an investment of $2g_{22}$ = 8 in sector 2, the following LP

Table 2.7 Results of Stage (2,2)

r^y	0	4	8	12	16	20	24	28	$Q_{22}(r)$	y	J	Б
0	0+22.4								22.4	0	3,4,1,2	(6.8,4.8,20,24)
2	0+23.2								23.2	0	3,4,1,2	(5.2,5,22,24)
4	0+24.46	1.03+22.4							24.46	0	3,4,1,2	(7.48,2.06,20,27.42)
6	0+25.26	1.03+23.2							25.26	0	3,4,1,2	(5.88,2.26,22,27.42)
8	0+26.06	.77+24.46	1.8 +22.4						26.06	0	3,4,1,2	(3.08,2.46,24,27.42)
10	0+26.83	.85+25.26	1.88+23.2						26.83	0	3,4,1,2	(3.28,2.66,26,27.42)
12	0+27.48	.92+26.06	.77+24.46	1.8 +22.4					27.48	0	3,4,1,2	(4.75,2.86,28,27.42)
14	0+28.48	1 +26.83	.85+25.26	1.88+23.2					28.48	0	3,4,1,2	(3.67,3.83,26,30.84)
16	0+28.88	1.03+27.48	.92+26.06	.77+24.46	1.8 +22.4				28.88	0	3,4,1,2	(2.07,4.27,28,30.84)
18	0+29.28	1.07+28.48	1 +26.83	.85+25.26	1.88+23.2				29.55	4	3,4,1,2	(4.38,1.19,26,34.26)
20	0+29.4	1.07+28.88	1.07+27.48	.92+26.06	.77+24.46	1.8 +22.4			29.95	4	3,4,1,2	(2.78,1.63,28,34.26)
22	0+29.4	1.07+29.28	1.55+28.48	1 +28.83	.85+25.26	1.88+23.2			30.35	4	3,4,1,2	(1.19,2.06,30,34.26)
24	0+29.4	1.25+29.4	1.72+28.88	1.07+27.48	.92+26.06	.77+24.46	1.8+22.4		30.65	4	5,4,1,2	(.51,2.38,31.49,34.26)
26	0+29.4	1.25+29.4	1.9 +29.28	1.55+28.48	1+26.83	.85+25.26	1.88+23.2		31.18	8	3,6,1,2	(1.75,.75,30,36.93)
28	0+29.4	1.25+29.4	2.36+29.4	1.72+28.88	1.07+27.48	.92+26.06	.77+24.46	1.8+22.4	31.76	8	3,6,1,2	(.27,.19,32,37.49)

must be solved according to the discussion in section 2.5.

$$\text{Max } Z = .201\, x_{12} + .312\, x_{22}$$

$$.798\, x_{12} + -.208\, x_{22} + z_{12} = 18$$

$$-.218\, x_{12} + .773\, x_{22} + z_{22} = 22$$

$$x_{12} + u_{12} = 32$$

$$x_{22} + u_{22} = 37.68$$

$x_{i2} \geq 0,\ z_{i2} \geq 0,\ u_{i2} \geq 0$

Its solution gives

$Z^* = 18.13$

$J = \{3, 6, 1, 2\}$ $\quad \bar{b} = (.27, .19, 32, 37.49)$

$$B^{-1} = \begin{vmatrix} 1 & .269 & -.739 & 0 \\ 0 & -1.294 & -.282 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1.294 & .282 & 0 \end{vmatrix}$$

The marginal value added due to an investment of $2g_{22} = 8$ is 18.13 - 15.77 = 2.36.

The quantity of 15.77 corresponds to the value added due to investing 6,in stage (1,2) when 20 were available in stage (1,2).

Table 2.8 summarizes the resulting investment decisions.

The total value added in the 2 year plan is 31.76 m u.

The optimal solution requires that 14 m u be invested in each year.

At the end of year 2 the capacities are:

$c_{12} = 32$

$c_{22} = 37.68$

The optimal production schedule in year 1 is:

$x_{11} = 25.81,\ x_{21} = 28.23,\ u_{11} = .19,\ u_{21} = 2.61$

Table 2.8 Investment per Year and Sector Unit: m. u.

Investment	Year 1	Year 2
Sector 1	6	6
Sector 2	8	8
Total	14	14

in year 2:

$x_{12} = 32$, $x_{22} = 37.49$, $z_{12} = .27$, $u_{22} = .19$

The optimal value added in year 1 is 13.63 and in year 2 = 31.76 - 13.63 = 18.13.

The application of dynamic programming to the example, presented above, shows the tediousness of computations, carried out to get the optimal solution. A computer program could be designed to perform all of the steps involved in the solution as indicated in section 2.5. This is not the purpose of this research. A more important reason led the author not to design this computer program. It was the non-applicability to the Algerian case. Data used to compute both of the input coefficients (domestic and foreign) are not available in Algeria. Even though imports are known by sector, there is no way of knowing from which foreign sectors they come from. Moreover, import figures contain both intermediate and final demands.

That is why, the model, presented in the next chapter, will be used for the Algerian application. It uses data as given in the Algerian input-output table. The solution of the economic planners' problem is obtained by mixed integer programming.

CHAPTER III

THE MIXED INTEGER PROGRAMMING SOLUTION TO THE DYNAMIC LEONTIEFF MODEL FOR AN INVESTMENT ALLOCATION

In this chapter, a mixed integer programming solution will be presented. However, the previous Leontieff model will have to be modified to make its solution by MIP (a shorter notation for mixed integer programming), possible.

3.1 The MIP Formulation

The MIP requires that all coefficients of the model are known. The value added coefficients v_{jt} and the input coefficients a_{ijt} must be known.

The variables and parameters of the model, where the subscript t denotes the relevant time period are defined.

Let:

x_{it} = production of sector i in period t expressed in m u.

β_{it} = number of projects built in sector i in period t. β_{it} is integer.

f_{it} = demand of final goods i in period t expressed in m u.

e_{it} = export of final goods i in period t in m u.

z_{it} = imports of final goods i in period t in m u.

s_{it} = inventory of final goods i in m u at the end of period t.

g_i = cost of a project in sector i expressed in m u, g_i is

constant.

c_{it} = capacity of sector i in period t in m u.

u_{it} = slack capacity of sector i in period t in m u.

v_i = value added per unit produced in sector i, v_i is known.

a_{ij} = ratio of the value of input i consumed by sector j to the output of sector j. a_{ij} is known.

α_i = coefficient of conversion of investment into capacity in sector i.

An investment of g_i in sector i will increase capacity by $\alpha_i g_i$.

The model has the following balance equations:

$$x_{it} - \sum_{j=1}^{m} a_{ij}x_{jt} + z_{it} = f_{it} + s_{it} - s_{i(t-1)}, \quad i=1 \ldots m$$

$$x_{it} + u_{it} = c_{it}$$

The difference between this set of balance equations and those presented in the previous chapter is that $\sum_{\substack{j=1 \\ j\neq i}}^{m} d_{ijt}x_{jt}$ is ignored in the present case. The input-output tables published to date provide data concerning only a_{ij} and not d_{ij} for a given year. This is the main reason this model will be used in the Algerian application.

In the following, the MIP formulation of the problem will be given. The concept of equivalent investment w_{it} is used again: the subscript t is dropped because w_{it} is constant.

In period zero, the capacity of sector i is known, i.e., c_{io}.

In period 1, an investment of $\beta_{i1}g_i$ will increase capacity by $\beta_{i1}\alpha_i w_i$. So the following relationships hold:

$$x_{il} + u_{il} = c_{il}$$

$$c_{il} = c_{io} + \beta_{il}\alpha_i w_i$$

Finally,

$$x_{il} + u_{il} = c_{io} + \beta_{il}\alpha_i w_i \quad , \quad \text{or,}$$

$$c_{io} = x_{il} + u_{il} - \beta_{il}\alpha_i w_i \tag{3.1}$$

Equality (3.0) can be generalized to any t

$$c_{io} = x_{it} + u_{it} - \sum_{r=1}^{t} \beta_{ir}\alpha_i w_i, \quad i=1, \ldots m, \; t=1, \ldots T \tag{3.2}$$

It is also assumed that n_1 m u are available for investment in period 1, n_2 in period 2 and so on. Let N be the amount available for all T periods. Investments of period 1 must satisfy

$$\sum_{t=1}^{T} \beta_{i1} g_i \leq n_1 \tag{3.3}$$

Inequality (3.3) is generalized for any t as follows:

$$\sum_{p=1}^{t} \sum_{i=1}^{m} \beta_{ip} g_i \leq \sum_{p=1}^{t} n_p \qquad t=1, \ldots, T \tag{3.4}$$

It is also assumed the planners want to maximize the value added over a T year plan.

The formulation of the planners' problem in MIP form is:

$$\text{Max } Z = \sum_{t=1}^{T} \sum_{i=1}^{m} v_i x_{it}$$

$$x_{it} - \sum_{j=1}^{m} a_{ij} x_{jt} + z_{it} = f_{it} + e_{it} + s_{it} - s_{i(t-1)}, \; i=1 \ldots m$$

$$x_{it} + u_{it} - \sum_{p=1}^{t} \beta_{ip}\alpha_i w_i = c_{io}, \quad i=1 \ldots m$$

$$\sum_{p=1}^{t} \sum_{i=1}^{m} \beta_{ip} g_i \leq \sum_{p=1}^{t} n_p, \qquad t=1 \ldots T$$

$x_{it} \geq 0$, $z_{it} \geq 0$, $u_{it} \geq 0$, $\beta_{ip} \geq 0$ and integer.

Note also that $\sum_{p=1}^{T} n_p = N$.

The previous problem can be presented in a matrix form. This is done in Table 3.1, where capital letters X, U. Z, B, V, F, E, ΔS, C_o, G stand for m-component vectors, (I-A) for an mxm matrix, I for the mxm identity matrix, W for an mxm diagonal matrix. The variables are X, U, Z and B where B must be integer. The right hand sides are F, E, ΔS, C_o and $\sum_{p=1}^{t} n_p$. The coefficients of the objective function are V. ΔS_t is the net inventory in period t.

Mixted integer programming will be used to find the optimal solution for the investment allocation in the Algerian model. Before attempting to do that, the parameters of the model must be estimated.

3.2 Estimation of the Parameters and Data

Eleven sectors are considered in this application. Sectors which are not primary producers, namely services, trade, and transportation, are excluded.

Each of these sectors is labeled from 01 to 11.

Agriculture	01
Food industries	02
Petroleum	03

Table 3.1 The MIP Formulation

Variables	X_1	U_1	Z_1	B_1	X_2	U_2	Z_2	B_2	X_3	U_3	Z_3	B_3	X_4	U_4	Z_4	B_4
obj. fct.	V				V				V				V			
$F_1+E_1+S_1$	(I-A)		I													
C_o	I	I		$-\alpha W$												
n_1				G												
$F_2+E_2+\Delta S_2$					(I-A)		I									
C_o				$-\alpha W$	I	I		$-\alpha W$								
n_1+n_2				G				G								
$F_3+E_3+\Delta S_3$									(I-A)		I					
C_o				$-\alpha W$				$-\alpha W$	I	I		$-\alpha W$				
$n_1+n_2+n_3$				G				G				G				
$F_3+E_3+\Delta S_4$													(I-A)		I	
C_o				$-\alpha W$				$-\alpha W$				$-\alpha W$	I	I		$-\alpha W$
$n_1+n_2+n_3+n_4$				G				G				G				G

Utilities	04
Mining	05
Stone and clay products	06
Mechanical, electrical industries	07
Chemical products	08
Textile and leather	09
Miscellaneous manufacturing	10
Construction industries	11

In what follows, methods are presented to estimate each parameter or constant of the model. Such estimations concern the v_i's, a_{ij}'s, right hand sides of constraints, g_i's, α_i's and n_t's.

3.2.1 Estimation of the v_i's and a_{ij}'s

The 1969 Input-Output table of the Algerian economy provides the necessary data to estimate these parameters since the present study concerns the first four year plan which started January 1st, 1970.[1]

The estimation of the value added coefficients v_i is obtained by dividing the value added of sector i by its production. The estimates are found in the bottom row of Table 3.2.

The input coefficients a_{ij} were defined as the ratio of the value of goods i consumed by sector j to the output of sector j. Their estimates are obtained by dividing the amount purchased by sector j from sector i by sector j production. The a_{ij}'s are also in Table 3.2.

---

[1]The Input-Output Table is at the end of the study: Table A1. The data was obtained from : Secretariat d'Etat au Plan, Tableaux de l'Economie Algerienne (Alger: SNED, 1973, pp. 272-273).

Table 3.2 Estimates of a_{ij}'s and v_j's

i j		01	02	03	04	05	06	07	08	09	10	11
Agric	01	.0772	.5382							.0341	.0789	
Food	02	.0136	.1339						.0693	.0124	.0056	
Petroleum	03	.0183	.0051	.0631	.0395	.0369	.0387	.0156	.0061	.0057	.0068	.0379
Utilities	04	.0104	.0042	.0034	.1086	.0417	.0413	.0066	.0087	.0028	.0114	.0027
Mining	05			.0056					.0274		.0002	
Stone	06	.0009	.0029	.0121		.0022	.2419	.0036	.0032		.0066	.0968
Mec. elec.	07	.0051	.0079	.0548	.0618	.1041	.0506	.4554	.0259	.0094	.0267	.1286
Chemicals	08	.0452	.0033	.0077		.0743	.0016	.0193	.3647	.0326	.0566	.0116
Textiles	09	.0014	.0015					.0012	.0010	.4697	.0072	.0068
Misc.	10	.0054	.0100	.0043	.0070	.0233	.0265	.0116	.0440	.0074	.2858	.0205
Const.	11	.0016	.0017	.0130	.0328	.0054	.0058	.0080	.0089	.0055	.0048	.0098
v_j		.7716	.2796	.7145	.7291	.4851	.4871	.4110	.4071	.3871	.4481	.5660

3.2.2 The Estimation of the Right Hand Side of Demand Constraints

The right hand side of demand constraints comprehends four elements: demand of final goods f_{it}, exports of final goods e_{it}, initial and ending inventory of final goods $s_{i(t-1)}$, and s_{it}.

The 1969 Input-Output Table gives the demand, the exports and the change in inventory of final goods. To get an estimate of the demand and exports of final goods for each year of the four year plan, a constant rate of increase for both of them is assumed.

The value of demand and exports as projected in the four year plan is known for the ending year.[1] The estimate of the growth rate g is obtained from the following formula

$$\text{Value}_{73} = \text{Value}_{69} (1 + g)^4$$

Table 3.3 contains the growth rates of demand for food products and industrial goods.

Table 3.3 Rate of Increase of the Different Types of Consumption

Goods	Increase 1969-73[1] %	Annual Increase %
Agriculture and Food Products	23.5	5.42
Industrial Goods	30	6.78
Overall Mean	24	5.53

[1]Source: Ministry of Information and Culture, The four year plan (Algiers: SNED, 1970, p. 70).

[1]Ministry of Information and Culture; The four year plan (Algiers: SNED, 1970).

Table 3.4 shows the increase rates of exports for four categories: 1) agriculture and food products, 2) mining products, 3) Petroleum products, and 4) industrial goods.

Table 3.4 Value of Exports in 1969 and 1973 (as Projected). Unit: Millions of DA

Goods	1969[1]	1973[1]	Annual Increase %
Agriculture and Food Products	860	950	2.6
Mining Products	160	250	11.8
Petroleum Products	3250	5130	12.1
Industrial Goods	180	450	25.74

[1]Source: Ministry of Information and Culture, The four year plan (Algiers: SNED, 1970, p. 33).

Furthermore, it is assumed that the growth rate of the net change in inventory is equal to the one used for the demand of final goods. This assumption is necessary since there is no data available to make a better estimate.

The right hand side of demand constraints is presented in Table 3.5. Table 3.5 requires some explanations. First, the following growth rates are used in computing the demand and net change in inventory of final goods (see Table 3.3).

-- 5.42% for agriculture and food sectors

-- 5.53% for stone and clay products and construction industries sectors

-- 6.78% for all other sectors

Table 3.5 The Right Hand Side of Demand Constraints. Unit: Millions of DA

Sector	1969	1970	1971	1972	1973
01 Agriculture	2040	2145	2255	2371	2494
02 Food ind.	4791	5030	5280	5543	5821
03 Petroleum	3642	4055	4516	5031	5606
04 Utilities	202	216	230	246	263
05 Mining	175	194	215	239	265
06 Stone and Clay	118	124	131	139	146
07 Mech., Elec.	629	690	760	841	936
08 Chemicals	945	1014	1089	1171	1260
09 Textile-leather	2100	2251	2415	2593	2787
10 Misc. mfc.	709	765	828	898	976
11 Const. ind.	1852	1954	2063	2177	2297

Source: Secretariat d'Etat au Plan, Tableaux de l'Economie Algerienne (Alger: SNED, 1973, p. 273).

Second, the exports are multiplied by the growth rates shown in Table 3.4:

-- 2.6% for the agriculture and food sectors

-- 12.1% for the petroleum sector

-- 11.8% for the mining sector

-- 25.74% for mechanical and electrical, chemicals, textile-leather and miscellaneous manufacturing sectors.[1]

3.2.3 Estimation of the Right Hand Side of Capacity Constraints

The estimation of initial capacity is based on the Gross National Product of 1969, and on the assumption that the economy operated at full capacity in 1969. There are no data available to provide a better estimate for initial capacity. The main reason for doing so, is, that the data found in the Input-Output Table show almost every sectors imported goods. Table 3.6 presents the initial production capacity for the different sectors of the economy.

3.2.4 Estimation of projects costs g_i's

The cost of a project is understood as the average investment made in a given sector to acquire the necessary fixed assets to achieve an average production.

The estimation of g_i is based on American data. The main reason for this choice is that Algeria has to import most of its capital goods from western countries. So, American data can provide a fair estimate of a project cost.

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[1] The stone and clay products sector will not export anything during the four year plan because a strong demand will arise during this period. The exports were very low in 1969 for this sector 1 million of DA.

Table 3.6 The Right Hand Side of Capacity Constraints. Unit: Millions of DA

Sectors	Value
01 Agriculture	3312
02 Food ind.	3565
03 Petroleum	4679
04 Utilities	369
05 Mining	185
06 Stone and clay	310
07 Mech., elec.	1256
08 Chemicals	493
09 Textile-leather	1443
10 Misc. mfc	498
11 Const. ind.	2043

Source: Secretariat d'Etat au Plan, Tableaux de l'Economie Algerienne (Alger: SNED, 1973, pp. 272-273).

The data used in this estimation is in Table A2 in the Appendix. The data concerns the period 1969-1972 according to the different sectors used in this model.

3.2.4.1 General Assumptions

Three assumptions are made:

*1st Assumption.

It is assumed that the life-time of a project is

-- ten years in agriculture, food products, textile-leather and miscellaneous manufacturing.

-- fifteen years in construction industries.

-- twenty years in mining and stone-clay products sectors.

-- thirty years in utilities, petroleum, mechanical-electrical and chemical sectors.

The first assumption is justified because the lifetime of a project increases with the degree of capital intensity.

*2nd Assumption.

The completion time of a project is also assumed to vary with the degree of capital intensity. It is

-- one year in agriculture, food products, textile-leather, miscellaneous manufacturing and construction industries sectors.

-- two years in mining and stone-clay products sectors.

-- three years for all other sectors.

*3rd Assumption.

It is assumed that the fixed assets available in the corresponding American sectors have been in use for half their lifetime. It is

necessary to make this assumption to avoid a low estimate of the value of fixed assets.

3.2.4.2 Method of Estimation

Data obtained for all sectors, with the exception of agriculture and mining, measure the net book value of fixed assets.

To obtain a fair replacement value of fixed assets, two assumptions are made.

*1st Assumption.

It is made in conncection with the depreciation method. Among the three methods generally used: straight line, sum of the year's digits and the double declining balance, the sum of the year's digits will be used.[1] Table 3.7 contains the values of the sum.

Table 3.7 Sum of Years' Digits

$\sum_{1}^{n} t = \frac{n(n+1)}{2}$	
n	Sum
5	15
7	28
10	55
15	120
20	210
30	465

[1]This assumption is ignored for fixed assets having a groos book value figure, namely agriculture and mining.

*2nd Assumption.

To get a fair replacement value of fixed assets, inflation is taken into consideration. It is necessary to make such an assumption to avoid a low replacement value of fixed assets whose life-time is quite high. For instance, a fixed asset with a thirty year life-time, in use for fifteen years, will certainly have a high replacement value compared to its acquisition cost, because of the inflation in the capital goods area.

The rate of inflation for capital goods was in the vicinity of 4 to 7% in the sixties.[1] The rate of inflation chosen is 5%.

The method of estimation proceeds along the following steps.

-- First, the net book value (NBV), of fixed assets per establishment is found.

-- Second, the acquisition cost of the fixed assets is computed. An example will clarify the method of obtaining the acquisition cost. Let us suppose, the net book value is NBV, the average life-time of the fixed assets is ALT and they have been in use for T years: $T = \frac{ALT}{2}$, since the fixed assets age is assumed equal to half their life time. The sum of years' digits method of depreciation gives:

$$\text{acquisition cost} = \frac{NBV \times \sum_{t=1}^{ALT} t}{\sum_{t=1}^{T} t} \quad ; t=1, \ldots, T, \ldots, ALT$$

-- Third, to obtain the replacement value of the fixed assets,

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[1] U.S. Department of Labor, Bureau of Labor Statistics, Chartbook on prices, wages and productivity (Editors James McCall and John Tschetter, Vol. 2, No. 11, May 1976, p. 25).

the inflation rate is taken into account.

replacement value after T years = acquisition cost x $(1 + .05)^T$

All replacement values are computed for 1971, i.e., the second year of the plan and are used for each of the four years.

Cost of projects (based on American data), g_i, are in Table 3.8.

Table 3.8 Cost of Projects: g_i (Based on U.S. Data)

Sectors	g_j ($1,000,000's)	g_i^2 (1,000,000's DA)
01 Agriculture	.097	.452
02 Food	5.419	25.168
03 Petroleum	67.075	311.496
04 Utilities[1]	37.592	174.577
05 Mining	2.367	10.991
06 Stone-clay	12.999	60.367
07 Mech. Elec.	18.320	85.080
08 Chemicals	25.717	119.432
09 Textile-leather	1.898	8.815
10 Miscellaneous mfc	2.252	10.456
11 Const. industries	.179	.830

[1]U.S. Federal Power Commission, Technical Advisory committee on finance, National Survey: The financial outlook for the power industry: The report and recommendations of the Technical Advisory committee on fiannce, Part II (Washington: U.S. Government Printing Office, 1974), p. 3.61.

[2]$1 = 4.644 DA in United Nations, Statistical Office, Statistical Yearbook (New York: UN, 1975).

3.2.4.3 Estimation of a Project Cost in Agriculture

The cost of an agricultural project is restricted to the value of machinery and equipment in an average size farm.

The market value of equipment is $24.285/acre.[1]

The average size of a state owned farm in Algeria is 1154 hectares[2] corresponding to 2851 acres.[3]

The acquisition cost of machinery and equipment in 1969 was $24.285 x 2851 x $(1.05)^5$ = $88,365.

In 1971 it was: $88,365 x $(1.05)^2$ = $97,422.

Its corresponding value in DA (the Algerian currency unit) is in column 2 of Table 3.8.

3.2.4.4 Estimation of the Cost of a Project in the Mining Sector

The gross book value of fixed assets in the mining secotr was 16,432 millions of dollars in 1972.[4] The number of establishments was 10,771. The average life-time of fixed assets in this sector is 20 years.

$$\text{Acquisition cost/establishment, in 1971} = \frac{16,432}{10,771} \times (1.05)^9$$

$$= 2.367 \text{ millions of dollars.}$$

---

[1]U.S. Bureau of the Census: Census of Agriculture, 1969 Volume II General Reports, Chapter 3, Farm management, Farm operations (Washington D.C.: U.S. Government Printing Office, 1973), p. 153.

[2]Secretariat d'Etat au Plan, Tableaux de l'Economie Algerienne (Alger: SNED 1973, p. 121).

[3]1 hectare = 2.471 acres.

[4]U.S. Bureau of the Census, Census of Mineral Industries, 1972, Subject, Industry and Area Statistics (Washington, D.C.: U.S. Government Printing Office, 1976), pp. 1-89-1-92.

3.2.4.5 Estimation of the Cost of a Project in the Construction Industries Sector.

The 1972 census of construction industries gives the following data:[1]

Number of establishments = 437,941

Net value of fixed assets = $12,054 millions

Rental payments for machinery and equipment = $1,972 millions

Now, the cost of a project in this sector can be computed.

$$\text{Net value of fixed assets/establishment} = \frac{12,054}{437,941} = \$.027 \text{ millions}$$

To get the acquisition cost, the rental payments for machinery and equipment are incorporated.

$$\text{acquisition cost of a project} = \frac{.027 \times \sum_{t=1}^{15} t}{\sum_{t=1}^{7} t} \times (1.05)^7 + \frac{1972}{437,941}$$

$$= \$.178 \text{ millions}$$

3.2.4.6 Estimation of Projects Costs for All Other Sectors

The data used to obtain an estimate of the replacement value of the fixed assets are shown in Table A2 in the Appendix.

However, the non existence of the number of establishments in 1971 made necessary the use of the following assumption. The number of production workers per establishment is assumed the same for 1971 and 1972. Since the number of production workers and the number of establishments in 1972, are known, the number of production workers

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[1] U.S. Bureau of Census, Census of Construction Industries, 1972, Vol. 1, Industry Statistics and Special Reports (Washington, D.C.: U.S. Government Printing Office, 1976), pp. 6-7.

per establishment can be obtained. The last figure divided into the number of production workers for 1971 gives the number of establishments in 1971 for a given sector. Now it is possible to get the net book value per establishment in 1971. Using the sum of years' digits method of depreciation[1] and the inflation rate, the replacement value of fixed assets per establishment is (the cost of a project) obtained.

This method is applied to the food sector to show how it is used. From Table A2 one obtains:

$$\text{Number of production workers/establishment} = \frac{1,103,900}{22,423} = 49.23$$

$$\text{Number of establishments in 1971} = \frac{1,048,900}{49.23} = \$21,306 \text{ millions}$$

$$\text{Net book value of fixed assets} = \frac{24,673.4}{21,306} = \$1.158 \text{ millions}$$

$$\text{Acquisition cost} = \frac{1.158 \times 55 \times (1.05)^5}{15} = \$5.419 \text{ millions}$$

Table 3.8 indicates the cost of projects in other sectors.

3.2.5 Estimation of the Capacity Increase Due to Investment

An investment g_i will increase capacity of sector i by $\alpha_i g_i$ cone the project is completed. Recall, w_i is an equivalent investment to g_i, being completed the same year it is started

$$w_i = \frac{g_i \left(\frac{1}{2} + \varepsilon - \frac{1}{2h\%}\right)}{\varepsilon} \quad \text{from Formula (2.4)}$$

Multiplying both sides of the previous formula by α_i, it follows

$$\alpha_i w_i = \frac{\alpha_i g_i \left(\frac{1}{2} + \varepsilon - \frac{1}{2h\%}\right)}{\varepsilon}$$

---

[1] cf. Table 3.7.

The increase in capacity $\alpha_i w_i$ must be computed. It can be determined if $\alpha_i g_i$ is known since ε and h% are given.

From the data in Table A2 the value of shipments per establishment in 1971 can be obtained (provided, the number of establishments in 1971 is computed by the method illustrated in 3.2.4.6). However, the value of shipments does not correspond to full production capacity; so the following assumption was made--the value of shipments corresponds only to a certain percentage of production capacity. Once the value of shipment is divided by the corresponding percentage of production capacity, the $\alpha_i g_i$'s, looked for, are obtained. In fact, there was no need to find the α_i's. Table 3.9 contains the values of $\alpha_i w_i$.

The case of the mechanical-electrical sector will illustrate the application.

The value of shipments per establishment is \$5.492 millions. Since this value corresponds to 80% of production capacity, a project with fixed assets of \$18.320 millions[1] will increase capacity by $\frac{5.432}{.80}$ = \$6.865 millions. This figure is precisely $\alpha_7 g_7$. $\alpha_7 w_7$ is obtained as follows

$$\alpha_7 w_7 = \frac{\alpha_7 g_7 \left(\frac{1}{2} + \varepsilon - \frac{1}{2h\%}\right)}{\varepsilon} = \frac{6.865 \left(\frac{1}{2} + 30 - \frac{3}{2}\right)}{30} = \$6.636 \text{ millions}$$

Table 3.9 shows the figures for each sector.

---

[1] cf. Table 3.8.

Table 3.9 The Value of Capacity Increases: $\alpha_j w_j$ (Based on American Data)

Sectors	Average life of a project (in years)	Project completion time=1/h% (in years)	Value of shipments per establishment ($1,000,000's)	Corresponding percentage of production capacity (in %)	Capacity increases due to g_j $\alpha_j g_j$ ($1,000,000's)	$\alpha_j w_j$ = col 5$(1/2+\epsilon-1/2h\%)$ / ϵ ($1,000,000's)	$\alpha_j w_j$ (1,000,000's DA)
01 Agriculture	10	1	.152[1]	90	.169	.169	.786
02 Food	10	1	4.851	90	5.390	5.390	25.029
03 Petroleum	30	3	13.492	80	16.865	16.302	75.709
04 Utilities	30	3	3.167[2]	80	3.959	3.827	17.772
05 Mining	20	2	1.178[3]	75	1.571	1.532	7.114
06 Stone-clay	20	2	2.573	75	3.431	3.345	15.534
07 Mech. Elec.	30	3	5.492	80	6.865	6.636	30.818
08 Chemicals	30	3	4.624	80	5.779	5.587	25.945
09 Textile-leather	10	1	1.573	90	1.748	1.748	8.116
10 Misc. mfc	10	1	.921	90	1.024	1.024	4.754
11 Const. ind.	15	1	.356[4]	80	.445	.445	2.066

[1]Source: U.S. Bureau of the Census, Census of agriculture, 1969, volume II, general reports, chapter 3, Farm management farm operators (Washingtin, D.C.: U.S. Government Printing Office, 1973), p. 153.

[2]Source: U.S. Federal Power Commission, Technical Advisory on Finance, National Survey: The financial outlook for the electric power industry (Washington, D.C.: U.S. Government Printing Office, 1976), p. 3-61.

[3]U.S. Bureau of the Census, Census of Mineral Industries, 1972, Subject, Industry and Area Statistics (Washington, D.C.: US. Government Printing Office, 1976), pp. 1-89-1-92.

[4]U.S. Bureau of Census, Census of Construction Industries, 1972, Vol. 1, Industry Statistics and Special Reports (Washington, D.C.: U.S. Government Printing Office, 1976), pp. 6-7.

3.2.6 Estimation of the Investment Expenditures

The money available for investment in the four year plan was defined as N.

The value of N is computed by adding the public expenditures for investments to the private ones.

The value of N is equal to 20,200 millions of DA.[1]

Since the data figures available did not specify the amount to be spent each year, it was decided to divide N into four equal parts, i.e., 5050 millions of DA per year. Table 3.10 shows the corresponding values of $\sum_{t=1}^{k} n_t$, k=1, . . . , 4.

Table 3.10 Available Investment in Algeria
Unit: Millions of DA

Year	$\sum_{1}^{k} n_t$
1970, k=1	5,050
1971, k=2	10,100
1972, k=3	15,150
1973, k=4	20,200

Source: Ministry of Information and Culture, The four year plan (Algiers: SNED, 1970).

[1] Ministry of Information and Culture, The four year plan (Algiers: SNED, 1970) Public Investments: 1) all industries: 12,400 millions of DA (p. 32); 2) Agriculture: 4100 millions of DA (p. 52); 3) Housing: 1510 millions of DA (p. 84). Private investments only in industrial sectors: 2190 millions of DA (p. 31).

Now, the planners' problem can be solved since all coefficients and right hand side constraints are known. This is the subject of the next section.

3.3 Solution by MIP

The planners' problem, which is to maximize the value added over the four year plan to get the optimal sequence of investments, is solved by using the mixed integer programming option available in the APEX package.[1]

Before going further, a change in the initial tableau, corresponding to the one depicted in Table 3.1 was made. The columns U_t and Z_t were deleted from the tableau; that make the demand and capacity constraints inequalities of the $\leq$ type. This avoided punching unnecessary cards for these columns.

3.3.1 Introductory Comments

The problem formulation required:

-- 8 m variables, i.e., 2 m per year, m for β_{it} and m for x_{it}. Since m = 11, there were 88 variables.

-- 4 (2m + 1) constraints, there are m constraints for demand and m for capacity, 1 for available investment. There was a total of 92 constraints.

All constraints are of the less than or equal type except those representing the demand constraints of utilities (sector 04). These are strict equalities. Less than or equal constraints for this sector demand would mean the possibility of having imports of utilities.

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[1] The APEX package was run on the CDC 6500.

This is not very realistic even though it is conceivable.

The listing of the input cards is shown in Table A.3 in the Appendix. Before interpreting the computer output, some remarks must be made on the names of variables. The integer variables β_{it} are represented by NBRiYt, number of projects in sector i in year t. The PRODiYt's correspond to production levels of sector i in period t, x_{it}. The demand constraints are denoted by the slack variable associated with each as follows:

-- IMPORTiYt stands for the slack variable z_{it}

-- UNUSEDiYt stands for the unused capacity of sector i in period t, u_{it}

-- MONEYLEFTt stands for the funds left over after investment in period t.

The APEX package requires that bounds be set on integer variables. The bounds were set equal to 16383, maximal value allowed in this package.[1]

3.3.2 Analysis of the MIP Results

The MIP results are presented in Table A.4 in the Appendix. The solution was obtained after 101 seconds of Central Processing Unit time. The value of the objective function equaled 61369.063 millions of DA.

The following facts can be observed:

-- First, the funds, available each year, were almost all exhausted, the value of the slack variables associated with the

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[1] In fact, the problem as formulated in section 3.1 is bounded by the investment constraints.

investment constraints are 0.24, 0.109, 4.511 and 1.291 millions of DA at the end of year 1, 2, 3 and 4 respectively.[1]

-- Second, the previous remark is complemented by the fact that most of the sectors operated at full capacity in the fourth year with the exception of the following three: agriculture, food industries and construction industries. These sectors show positive slack for the capacity constraints (rows 82, 83 and 94 in Table A.4 under CONSTRAINTS).

-- Third, three sectors show a negative row activity[2] in each year of the four year plan: stone-clay products, mechanical-electrical and chemicals (sectors 06, 07 and 08). To understand this situation let us refer to the demand constraints.

$$x_{it} - \sum_{j=1}^{m} a_{ij}x_{jt} + z_{it} = f_{it} + e_{it} + s_{it} - s_{i(t-1)}$$

Omitting z_{it} will transform the above equality into a less than or equal constraint. A negative row activity occurs if $x_{it} - \sum_{j=1}^{m} a_{ij}x_{jt}$ is negative. This implies sector i production is not even able to satisfy the intermediate demand coming from other sectors, the negative figure is equal to this deficit. For instance, the right hand side of the demand constraint for sector 08 products is 1014, the deficit is equal to 166.736 (row 9), the sum of both of these figures, 1014 + 166.736 = 1180.736, is the value of imports of chemicals for

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[1] cf. Table A.4 under the heading CONSTRAINTS: rows 24, 47, 70 and 93.

[2] cf. Table A.4 under CONSTRAINTS, column: ROW ACTIVITY, rows 7 to 9, 30 to 32, 53 to 55 and 76 to 78.

the first year, this figure is the value of the slack variable (row 9).

These three facts support the conclusion that if more funds had been available the MIP would have used at least part of them. More funds must have been allocated to the four year plan to achieve the goals represented by the demand constraints.

The optimal sequence of investments to achieve the maximum sum of value added over the planning period is what the planners look for.

This is in Table A.4 under the heading COLUMNS. the values of the integer variables NBRiYt are found in column COL ACTIVITY. Table 3.11 presents the value of investments in each sector per year. The investments are obtained by multiplying the value of NBRiYt (β_{it}) by the corresponding project, g_i.

The following facts can be observed in Table 3.11:

-- The petroleum sector invested 27.76% of the funds available for investment expenditures in the four year plan, no other sector achieved this percentage. This sector has three characteristics: first, it has a high value added coefficient, .7145; second, it is a sector that is capital intensive; and, third, the petroleum sector is highly related to primary activities such as oil and gas extraction. By the way, the activities usually found in the oil industry were not very developed in 1969, which is the date for which the value added coefficient was computed. These activities have a much lower value added coefficient.

-- The next three highest investments were made in the textile-leather, food and miscellaneous manufacturing sectors. They shared

Table 3.11 Structure of Investments in the Four Year Plan. Unit: Millions of DA

Sector	1 1970	2 1971	3 1972	4 1973	Total	%
01 Agriculture	1499.284	261.256	221.48	201.592	2183.612	10.81
02 Food	2416.128	377.52	327.184	327.184	3448.016	17.07
03 Petroleum			2180.472	3426.456	5606.928	27.76
04 Utilities	523.731	349.154	349.154	349.154	1571.193	7.78
05 Mining	76.937		43.964	43.964	164.865	0.82
06 Stone-clay			60.367		60.367	0.3
07 Mech. Elect.						
08 Chemicals						
09 Textile-leather	511.27	2953.025	379.045	387.86	4231.2	20.95
10 Misc.mfc.		1056.056	1432.472	261.4	2749.928	13.61
11 Const. ind.	22.41	53.12	51.46	55.61	182.6	0.9
Total	5049.76	5050.151	5045.598	5053.22	20198.709	100.0

together 20.95 + 17.07 + 13.61 = 51.63% of the total investment. These activities are in the light industry area, where capital investments are low compared to other sectors such as chemicals, mechanical-electrical, and so forth. The sectors in question are labor intensive and are heavily dependent on agriculture which is a primary sector "par excellence."

-- Agriculture had a high level of investment in the first year, 1500 millions of DA, i.e., 30% of total expenditures of that year. In the following year there was a sharp drop in spending. From 1971 to 1973 the investment levels would have seemed to stabilize between 260 and 200 millions of DA. The reason has to do with production levels. For this the reader should refer to Table A.4 under the heading CONSTRAINTS. Even though there was a huge investment in the first year, 1500 millions of DA, the slack capacity in agriculture was only 0.857 millions of DA (row 13). In later years, investments being much lower (between 260 and 200 millions) the capacity constraints showed a lower slack (rows 36, 59 and 82). One can conclude that the high investment level in 1970 was due to the agricultural output capacity having been far behind the demand imposed on agriculture. The investments made in 1971 up to 1973, which were low, were carried out for the sole purpose of keeping up with the annual increase in demand for the agricultural products. This conclusion is also corroborated by the fact that demand constraints were bound in 1970, 1972 and 1973 (rows 2, 48 and 71) and had low imports in 1971 (row 25) of about 4 millions of DA.

-- Next, the sectors generally capital intensive, such as chemicals and mechanical-electrical, had a zero investment level by

the end of the four year plan, even though they showed relatively high imports, 1686 and 1184 millions of DA in 1973, respectively, (rows 78 and 77). Furthermore, in 1970, both sectors had a total of 1180.736 + 717.177 = 1897.913 millions of DA of imports (row 9 and row 8), sensibly equal to the one obtained for textile-leather and miscellaneous manufacturing sectors for the same year, 1273.243 + 630.925 = 1904.168 millions of DA (rows 10 and 11); however, in the following year, sectors 09 and 10 were the best candidates for investment. 2953.025 + 1056.056 = 4009.081 millions of DA (Table 3.11, column number 2) almost 80% of year 2 spending, even though the value added coefficients are not that different. The reason for this resides in the high cost of projects in sectors 07 and 08.

-- The food sector followed the same pattern as the agriculture sector, huge investment in the first year (2416) and stabilization in the following years, 377 in 1971 and 327 in 1972 and 1973.

-- The investments in the utilities sector were quite steady, 523 millions of DA in 1970 against 349 in the following years. Their presence was due to the equality of demand constraints for this sector[1] which forced MIP to have strictly positive values for the corresponding integer variables.

In summary, the MIP solution favors investments in:

1) sectors of primary activities: petroleum, agriculture and mining.

2) sectors with low capital investments: food, textile-leather and miscellaneous manufacturing.

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[1]The first time MIP was run with less than or equal constraints for the demand of utilities, no investment took place in any of the years concerned and we ended up with imports of utilities.

These facts are in complete concordance with a certain theory of economic development that was formulated in the early 1950's and that is still highly criticized. According to it underdeveloped countries starting economic plans should invest in sectors with low capital investments. The reasons behind this theory are:

1) underdeveloped countries are characterized by a high level of unemployment.

2) a low level of savings that can be invested in different industrial projects.

So it seems logical that these countries undertake projects with low capital ratios and in sectors being more labor intensive than others.[1] This theory is confirmed by the investments obtained in the agriculture, food, textile-leather, miscellaneous manufacturing and construction industries sectors. But, how can the huge investments in the petroleum sectors of the last two years be explained? However, Professor Lewis adds,

> Capital works with natural resource as well as labor, and a poor country's natural resource advantage may be such that it pays to develop some rich but capital intensive natural resources.[2]

This explains why large investments are present in the petroleum sector. The results obtained by MIP in the four year plan confirm this theory. The only way the validity of such a theory, at

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[1]Some of the proponents of this theory are:
Rugmar Nurske: Problems of capital formation in underdeveloped countries (New York: Oxford University Press, 1953).
Gunnar Myrdal, Economic theory and underdeveloped regions (London: G. Duckworth, 1957).

[2]William Arthur Lewis, Development planning: The essentials of economic policy (London: George Allen and Murwin, Ltd., 1966, p. 55).

least in the present case can be tested, is on the time horizon, because a four year plan may not be long enough to do so.

The opponents of the above theory are the advocates of the so-called "unbalanced growth" theory of economic development.[1] This theory claims that economic growth and development can be achieved by investing in some "strategic" industries characterized by:

1) a high capital ratio

2) inducing the development of downstream and upstream industries.

Examples of "strategic" industries are the steel and the petroleum industries. Almost any underdeveloped country wanted to have its own steel industry, hoping that it would bring economic growth and development by some magic formula. As Professor Kindleberger pointed out,[2]

> In dynamic terms, Bruton[3] has suggested that the industries which embody external economies are frequently capital intensive. These capital intensive investments must be undertaken before one can take advantages of opportunities for investment in labor intensive industries.... It is believed that capital intensive industries are more effective producers of savings than labor intensive and hence likely to speed development faster despite a possibly lower static level of output.

To test the validity of these conflicting theories, it was decided to increase the time span of the plan from four years to eight years. This is the subject of the next section.

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[1] Albert O. Hirshman, The strategy of economic development (New Haven: Yale University Press, 1961).

[2] Charles P. Kindleberger, Economic development (New York: The MacGraw Hill Book Company, 1958).

[3] H. J. Brutton, "Growth models and underdeveloped countries," Journal of Political Economy, August 1955.

3.3.3 The MIP Solution to the Eight Year Plan

The mathematical formulation of the eight year plan is identical to that of the four year plan. There are 16 m variables, 8 m for the integer (β_{it}) and 8 m for x_{it}. There are 8 (2m + 1) constraints; m for demand constraints, m for capacity constraints and 1 for investment constraints; that is 2m + 1 constraints per year.

All assumptions, stated in the four year plan case, are used again. The same input-output matrix is used. The rates of increase of final demand and of exports are used to arrive at the right hand side values of demand constraints. This is presented in Table 3.12.

Table 3.12 The Right Hand Side of Demand Constraints for 1974-1977

Unit: Millions of DA

Sectors	1974	1975	1976	1977
01 Agriculture	2622	2758	2901	3051
02 Food	6113	6420	6743	7083
03 Petroleum	6248	6965	7767	8663
04 Utilities	280	299	319	341
05 Mining	294	326	362	401
06 Stone-clay	154	163	172	181
07 Mech.Elect.	1046	1176	1330	1513
08 Chemicals	1357	1465	1584	1715
09 Textile-leather	2998	3229	3484	3764
10 Misc. mfc.	1064	1163	1276	1406
11 Const. ind.	2424	2558	2699	2849

The values of the right hand side of investments constraints are in Table 3.13. The first part of Table 3.13 is the same as the one in Table 3.10. The second part, dealing with year 5 up to year 8 of the plan, requires an explanation.

Table 3.13 Available Investment in the Eight Year Plan. Unit: Millions of DA

Year	$\sum_{t=1}^{k} n_t$
1970, k=1	5050
1971, k=2	10100
1972, k=3	15150
1973, k=4	20200
1974, k=5	33010
1975, k=6	45820
1976, k=7	58630
1977, k=8	71440

Sources: 1970-1973 Ministry of Information and Culture, The four year plan (Algiers: SNED, 1970); 1974-1977, Secretariat d'Etat au Plan, Tableaux de l'Economie Algerienne (Alger: SNED, pp. 280-281).

The second four year plan covering the 1974-1977 period made 64,688 millions of DA available to invest in directly productive sectors.[1] This figure is stated in 1974 DA. To get the corresponding value in

[1] Secretariat d'Etat au Plan, Tableaux de l'Economie Algerienne (Alger: SNED, 1975, pp. 280-281).

1970 DA a discount rate of 6% was used to take into account the decrease of purchasing power in 1974 due to inflation. Its rate in the capital goods area was between around 5 to 7% in the early seventies.[1] The corresponding 1970 value is 51,239 millions of DA. Here again, an equal amount available each year, 12810 millions of DA, is assumed.

The listing of the input cards is shown in Table A.5. The results of MIP are given in Table A.6. Both of these tables are in the Appendix.

Before interpreting the results, it must be pointed out that the funds available for investment in the 1974-1977 plan--51239 millions of DA--are more than two and a half (2.5) times the funds for 1970-1973 plan (20200 millions of DA).

The structure of investments is shown in Tables 3.14 and 3.15. If the part of the eight year plan covering the first four years is compared with the investments made in the four year plan, it will be discovered that they are very close. The percentages of total investments as shown in the first column of Table 3.15 are almost identical to the ones of Table 3.11. The only differences are the non existence of investments in mining and stone-clay products sectors and the presence of investments in mechanical-electrical industries.

Moreover, the distribution of investments in the given years remained the same: huge investments in agriculture and food industries in the first year, to fill the big gap between the production

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[1] U.S. Department of Labor, Bureau of Labor Statistics, Chartbook on prices, wages and productivity (Editors James McCall and John Tschetter, Vol. 2, No. 11, May 1976, p. 25).

Table 3.14 Structure of Investments in the Eight Year Plan. Unit: Millions of DA

Sector	1 1970	2 1971	3 1972	4 1973	Sum 1 to 4	5 1974	6 1975	7 1976	8 1977	Sum 5 to 8	Sum 1 to 8
01 Agriculture	1504.708	268.94	184.416	239.108	2197.172	195.716	247.244	270.296	285.212	998.468	3195.64
02 Food	2416.128	352.352	327.184	327.184	3422.848	352.352	427.856	478.192	478.192	1736.592	5159.44
03 Petroleum			4049.448	1868.976	5918.424	4360.944	3486.456	3737.952	3737.952	15263.304	21181.728
04 Utilities	349.154	349.154	349.154	349.154	1396.616	698.308	698.308	698.308	523.731	2618.655	4015.271
05 Mining						252.793	87.928	98.919	164.865	604.505	604.505
06 Stone-clay						60.367	1811.01	241.468	241.468	2354.313	2354.313
07 Mech.Elect.				340.32	340.32	5955.6	2127.0	1446.36	1701.6	11230.56	11570.88
08 Chemicals							2866.368	4657.848	4180.12	11704.336	11704.336
09 Textile-leather	669.94	2803.17	96.965	678.755	4248.83	440.75	484.825	537.715	590.605	2053.895	6302.725
10 Misc. mfc.	73.192	1223.352		1191.984	2488.528	418.24	564.624	575.08	836.48	2394.424	4882.952
11 Const. ind.	22.41	53.95	53.12	55.61	185.09	66.4	68.89	68.89	78.85	283.03	468.12
Total	5035.532	5050.918	5060.287	5051.091	20197.828	12801.47	12810.509	12811.028	12819.075	51242.082	71439.91

capacity of these sectors and the demand, smaller amounts invested in later years for the sole purpose of catching up with the annual increases in the demand for their products, steady investments in the utilities sector, big investments in the third and fourth year in the petroleum industry.

Logically, such a similarity of investments distribution, in both plans for the first four years, could be expected because the same demands and investments constraints are used. On the contrary, it has been thought that lengthening the planning period from four to eight years may have produced completely different results. Such results could have been the implementation of projects in "strategic" sectors such as mechanical-electrical, chemicals, stone-clay products and petroleum in early years of the plan. If this were true, the proponents of the "unbalanced growth" theory of economic development would have been completely right in arguing that growth could be achieved by investing in industries having a huge potential of inducing the creation of upstream and downstream industries.

From the previous remarks one can draw the following important conclusion--lengthening the time horizon of the plan fron four to eight years does not have any major effect on the structure of investments during the time period common to both plans.

Now let us analyze the results of the second part of the eight year plan, that is the 1974-1977 period. All the funds made available between 1970 and 1977 were exhausted, 71439.91 out of 71440 millions of DA.

The tenth column of Table 3.14 shows the total investments made between 1974 and 1977. These figures are computed in percentages in

column 2 of Table 3.15. The comparison of the first two columns of Table 3.15 shows a pattern of investment totally different between the 1970-1973 and the 1974-1977 periods. With the exception of two sectors, petroleum and utilities had steady percentages, all others showed a big decrease or increase. These changes can be grouped in two categories, labor intensive and capital intensive.

First, for labor intensive sectors such as agriculture, there is a decrease of their shares in total investment from 10.88% in the 1970-1973 period to 1.95% in the subsequent period. The same situation occurs for food industries, textile-leather, miscellaneous manufacturing and construction industries sectors.

Second, for capital intensive sectors such as chemicals there is an increase of their share in funds spent from 0% to 22.84%. The same thing can be said about mining, stone-clay products and mechanical-electrical sectors.

In sumary, investments

- in labor intensive activities decreased from

10.88 + 16.95 + 21.04 + 12.32 + .92 = 62.11% to 1.95 + 3.39 + 4 + 4.67 + 0.5 = 14.51%

- in capital intensive sectors increased from

29.3 + 6.92 + 1.68 = 37.9% to 29.79 + 5.11 + 1.18 + 4.59 + 21.92 + 22.84 = 85.43%

Here again the same reason, used to explain the decline of investments between 1970 and 1973 in labor intensive activities, is given--the projects built after the first year (agriculture and food industries), the second year (textile-leather) and the fourth year (miscellaneous manufacturing) served the sole purpose of catching up

Table 3.15 Structure of Investments in the Eight Year Plan.
Unit: %

Sector	Sum of Years 1 to 4 (1970-73)	Sum of Years 5 to 8 (1974-77)	Sum of Years 1 to 8 (1970-77)
01 Agriculture	10.88	1.95	4.47
02 Food	16.95	3.39	7.22
03 Petroleum	29.3	29.79	29.65
04 Utilities	6.92	5.11	5.62
05 Mining	-	1.18	0.85
06 Stone-clay	-	4.59	3.3
07 Mech. Elect.	1.68	21.92	16.2
08 Chemicals	-	22.84	16.38
09 Textile-leather	21.04	4.0	8.82
10 Misc.mfc.	12.32	4.67	6.84
11 Const. ind.	0.92	0.5	0.66
Total	100.0	100.0	100.0

with the annual increases in demand for their products. Besides this reason, the cost of projects in these sectors was rather low compared to the others;these projects created more capacity for each monetary unit invested. This made these projects more attractive than capital intensive projects with comparable value added coefficients, when funds available for investment expenditure were scarce (20200 millions of DA were available for the 1970-1973 period against 51240 for the subsequent four year period).

The implementation of projects in capital intensive sectors, delayed by MIP until the fifth or the sixth year, can be explained by three elements:

First,the low value added coefficients of these sectors made them unattractive compared to similar projects with low value added coefficients but with smaller cost and greater capacity created per monetary unit invested. This explains their delay.

Second, the funds available for investments in the entire period, extending from 1974 to 1977, were huge compared to the investments opportunities offered by labor intensive industries, since they only required enough investment to satisfy the annual increase of demand for their products.

Third, the situation of non-investment lasting until 1973 created a backlog of unsatisfied demand,for example, rows 77 and 78 (under CONSTRAINTS of Table A.6) shows imports of 1091.86 millions of DA by the mechanical-electrical sector and 1664.61 millions of DA by the chemicals sector.

The results of the eight year plan support the theory of economic development, that is, underdeveloped countries should first

implement projects that are labor intensive even though they should also take advantage of their natural resources requiring more capital investment. The sole development of capital intensive industries is not recommended because the economies of underdeveloped countries are so disintegrated that investing in these "strategic" activities does not improve things, in fact, they create idle capital that could have been employed productively somewhere else. As shown in the eight year plan, investments in capital intensive industries were only undertaken when the funds available for investment exceeded considerably the investments required to take advantage of the opportunities existing in labor intensive activities.

CHAPTER IV

CONCLUSIONS AND SUGGESTIONS

The purpose of this research was to investigate the problem of allocating investments in a plan of economic development. The problem was formulated as a mathematical program where the objective function was to maximize the value added over the planning period subject to constraints of demand and capacity. Leontief input-output coefficients were included in demand constraints. The solution of this mathematical program provided the optimal sequence in which investments must be carried out to get the maximum value added. The optimal sequence of investments was defined as the answer to the following three questions:

- in which sectors should investments be made?
- how much should be invested?
- when should investments be made?

Their answers are important in economic planning because:

- First, economic planners want to know in which sectors investments must be decided so as to differentiate activities by their contribution to economic development. This contribution is perceived through increases in value added by the whole economy.

- Second, economic planners do not want to invest more than what is needed to achieve the goals of the plan because investment is in

short supply (in underdeveloped countries). Furthermore, capacity and demand constraints of the model set limits to the amount invested.

- Third, an investment made in a given sector this year does not contribute to economic development in the same way as if made next year or after. For example, there are more benefits in investing this year in the mining sector to support the production of steel next year than in delaying the investment to next year.

To answer these questions two approaches to solving the same mathematical program were given:

- the dynamic programming approach, and
- the mixed integer programming approach.

The first one was presented in Chapter 2 and was applied to a numerical example. The dynamic programming approach involved two types of input-output coefficients (domestic and foreign coefficients) to reflect the importance of foreign trade in underdeveloped countries. This approach was not used in the application to the Algerian plan because first, data is lacking to compute both types of input-output coefficients and second, a computer program must be written to deal with all details of the solution procedure.

The second approach was presented in Chapter 3 and applied to the Algerian plan. The MIP solution is different from the dynamic programming approach in one crucial point: the constancy of value added and input-output coefficients. MIP cannot be applied if these coefficients are not constant. In fact, the dynamic programming solution is more preferable than the MIP one because the assumption of conatancy is relaxed. MIP answered the triple question of where, how much,and when investment must be carried out?

This approach permitted to test partially the theory of economic development that recommends to underdeveloped countries to undertake projects in "strategic" sectors.[1] According to this theory, investments must be made in upstream industries which will induce the implementation and development of downstream industries. These "strategic" activities have a high capital ratio, meaning that a substantial investment outlay must be made to carry out any project. Examples of these are steel, chemical, petroleum activities and so forth. Characteristics of underdeveloped countries such as high level of unemployment, investments in short supply, high rural population, and shortage of food contradict this theory. This research showed that this theory claims were not verified by the MIP results. However, the MIP solution indicated that investments must be carried out in sectors such as agriculture, food, etc., in other words in activities with low capital ratios. The number of jobs created per monetary unit invested is much greater in activities having low capital ratios than in those with high ones. Investing in these sectors is a way of reducing unemployment. This makes agriculture the "strategic" sector "par excellence," around which other activities should develop for three reasons: first, agriculture is a big user of manpower, second, underdeveloped countries have a high rural population (between 50 and 80%), and third, they have a problem of food shortage.

The MIP solution also showed that investments were carried out in labor intensive industries in the early years of the plan to meet the demand requirements imposed on them. It was only later that high

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[1]Albert O. Hirshman, The strategy of economic development (New Haven: Yale University Press, 1961).

capital ratio projects were implemented. It was also shown (in sections 3.3.2 and 3.3.3) that high capital ratio activities could be developed if there were more funds available for investment than needed to implement low capital ratio projects (to satisfy final demand). The funds available for investment in the 1974-1977 period were two and a half times greater than in the 1970-1973 period. Since labor intensive projects were carried out for the sole purpose of catching up with the annual increases of final demands, more funds were made available for possible investments in capital intensive industries. This explained their implementation after 1974. Labor intensive activities did not need that many funds to satisfy the increase of demand requirements from one year to the other. So the next choice for spending the funds left over was in sectors with high capital ratios and low value added coefficients.

The conclusions reached previously were based on the results obtained with MIP. As it is often said, results are no better than the information upon which they are based. Problems related to this kind of situation are looked at. These include:

- errors in data
-sector aggregation.

1) Errors in Data

Errors in data are a very severe limitation to results. They have different origins.

First,they may come from collecting the data necessary to make input-output tables. Collection of data for such a purpose is not always exhaustive for several reasons:

a) Data were not available although they existed. The most

important state owned firms were created between 1966 and 1969.[1] During that period and the coming years, their main concern was to get things going such as implementing projects. Moreover, there was a lack of skilled people who could organize data to get meaningful information on these firms.

b) Shortage of means and skilled people necessary to collect data. Both of these affect quality of data.

c) Underevaluation and nonexistence of data are characteristics of the Algerian private firms.[2]

Second, errors in data can come from the methods of estimating final demands. This research took data as given.

The following question can be raised, did these figures represent estimates obtained with forecasting methods, or targets aimed at? Both of these may not give the same values of final demands because targets are of political nature.

Third, cost of projects as computed could be questioned. The cost of a project was defined as the average investment made to acquire the necessary fixed assets to achieve an average output level. In this cost,two elements were missing: first, the cost of building the project was not taken into account; second, the cost of shipping equipment[3] were ignored. There was no way of determing both of these costs. Their sum could be quite substantial.

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[1] 1969 is the year the input-output table refers to.

[2] When the author worked in a research agency he found out that interviewees underevaluated or concealed their figures. They feared interviews because they thought they were for tax purposes.

[3] Most of the equipment were imported.

2) Sector aggregation

Eleven sectors were considered in the model. The 1969 input-output table made this choice compulsory. If the sector aggregation had been performed at a much lower level, i.e., eleven sectors had been broken down into a much larger number, the MIP results would have been more meaningful. For instance, the American input-output tables are available in two formats: 70 and 450 sectors. This is an important factor in results analysis, a greater number of sectors will increase the level of homogeneity in each sector. For example, the American sector corresponding to the mechanical-electrical sector comprises 25 groups of industries and each of these contains 6 or 7 categories. When a sector like this is broken down into several industries, the level of homogeneity in each of these will increase tremendously, because they are very different. Two project costs can be far apart in two different industries belonging to the mechanical-electrical sector.

Other sectors such as agriculture, food and petroleum, are more or less homogeneous and would not be affected very much by a higher level of disaggregation. But the mechanical-electrical sector would, and, this may make a big difference in terms of investments in this sector; much lower investments would be shown by MIP. Besides having high capital ratios, the mechanical-electrical industries, to be viable in economic terms, must turn out a high level of output exceeding demand underdeveloped countries have, twice or more than that. That is why an increasing number of economists are urging underdeveloped

countries to share investments to speed up integration.[1]

The demand of many industrial goods is so small in underdeveloped countries, that it is not worth producing them. But if several countries decide to share investments (because the project costs are too high), their total demand will become so meaningful that it will be worth implementing such projects. In this way, production capacity will not remain idle and funds will not be tied up in equipment and machinery. For instance, to be viable an automobile plant must turn out an output of at least one hundred thousand cars per year. For an average population size of an underdeveloped country (twenty million inhabitants) the demand for cars could be one third of this amount (this is an optimistic estimate). The amount that could be sold in foreign markets is very small because of a very strong international competition.

Although there were problems raised on the results provided by the MIP solution the following fact should not be overlooked; the previous analysis gave indications to where investments must be made, and in what sequence they have to be carried out. Before deciding where and when investments should be made, economic planners can get a guideline if they use this model. It might provide them with further insights.

In this research, maximization of value added was assumed to be the economic planners' objective. In section 1.5, three reasons were mentioned for such a choice: first, value added is a quantitative criterion; second, it is also the origin of all types of incomes; and

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[1]Such a study was made on the Arab scale. Abdelhamid Brahimi, Dimensions et perspectives du monde arabe (Paris: Economica, 1977).

third, it contributes to lessen unemployment. Planners may want to achieve other objectives.

Improving quality of education or medical care could be objectives planners would like to pursue. It is not easy to assign a fair quantitative criterion for them.

Being less dependent on foreign countries could be another objective. For instance, if a country decides to manufacture commodities previously imported, it may end up importing equipment and machinery for producing them. So, one form of dependence is replaced by another. Minimizing imports of commodities could express this goal.

In the same line of reasoning, economic planners may want the trade deficit not to exceed a predetermined level. Adding a new constraint (the difference imports-exports should not exceed this trade deficit) to the model accomodates this requirement.

Political leadership might require that some projects be carried out in the plan. For example, if a country had natural resources (like oil,iron, and so forth), downstream activities related to these could be developed. If the political leadership required their implementation, the economic planners would need to add constraints to the model. Before doing so, the planners could have used marginal analysis to find out the consequences of these requirements if they had already had an optimal solution.

They may have more than one objective to optimize. If this is the case, MIP cannot handle this situation; and they must turn to another technique. Goal programming is such a technique that ranks goals within the objective function optimized. Maximizing value added, employment, and exports might be an objective function of a

model solved by goal programming.

It will be interesting to formulate a Leontief model in a goal programming format. This could be a subject for further research: applying goal programming in economic planning is a possibility that remains to be explored.

APPENDIX

Table A.1 Input-Output Table of 1969[1] Unit: Millions of DA

Sectors	01	02	03	04	05	06	07	08	09	10	11	Final Demand	Net change of stock	Export
01 Agriculture	255.8	1917.9							49.2	39.3		1735	100.8	204.2
02 Food	45	477.1						34.2	17.9	2.8		3832.2	202.2	756.8
03 Petroleum	60.5	18.1	295.1	14.6	6.8	12	19.6	3	8.2	3.4	77.4	459	64.4	3119.
04 Utilities	34.3	15.1	16.1	40.1	7.7	13	8.3	4.3	4	5.7	5.6	202		
05 Mining			26					13.5		0.1		10.5	24.9	139.9
06 Stone-clay	3	10.4	56.7		0.4	75	4.5	1.6		3.3	197.7	93.7	23.2	1
07 Mech.,Elec.	17	28.3	256.4	22.8	19.2	15.7	572.1	12.8	22.3	13.3	262.7	451.3	78.5	98.8
08 Chemicals	149.7	11.9	36.2		13.7	0.5	24.2	179.9	47	28.2	23.6	825.3	94.1	25.8
09 Textile-leather	4.5	5.5					1.5	0.5	677.9	3.6	14	1933.9	118.8	47.2
10 Misc. mfc	18	35.7	20.1	2.6	4.3	8.2	14.6	21.7	10.7	142.4	41.8	612.6	50.3	45.7
11 Const. Ind.	5.4	6	60.8	12.1	1	1.8	10	4.4	7.9	2.4	20.1	1852		
Value added	2550	996.5	3342.9	269.2	89.5	151	516.3	200.8	558.7	223.3	1156.3			
GNP	3311.5	3563.4	4678.7	369.2	184.5	310	1256.3	493.3	1443.4	498.3	2043			

Source: Secretariat d'Etat au Plan, Tableaux de l'Economie Algerienne (Alger: SNED, 1975, pp. 272-3).

[1]To get the composition of inputs of a sector, read along the columns; to get the distribution of a sector output, read along the row.

Table A.2 Data Used to Compute Projects Costs[1]

Sector	Year	Number of Establish-ments	Number of Production Workers (1000's)	Value of Shipments ($1,000,000's)	Net Book Value of Fixed Assets ($1,000,000's)
02 Food	1972	22423	1103.9		
	1971		1048.9	103348.2	24673.4
03 Petroleum	1972	2017	98		
	1971		97	26935.1	16619.2
06 Stone-clay	1972	4616	319.4		
	1971		305.1	11345.7	9215
07 Mech. Elec.	1972	43138	4114.7		
	1971		3949.8	227418.8	94171.3
08 Chemicals	1972	8841	413.5		
	1971		404	39937.5	27575.9
09 Textile-leather	1972	31355	2024.9		
	1971		1958.2	47693.2	12298.5
10 Misc.mfc	1972	76226	1682.2		
	1971		1612.2	67298.3	35149.2

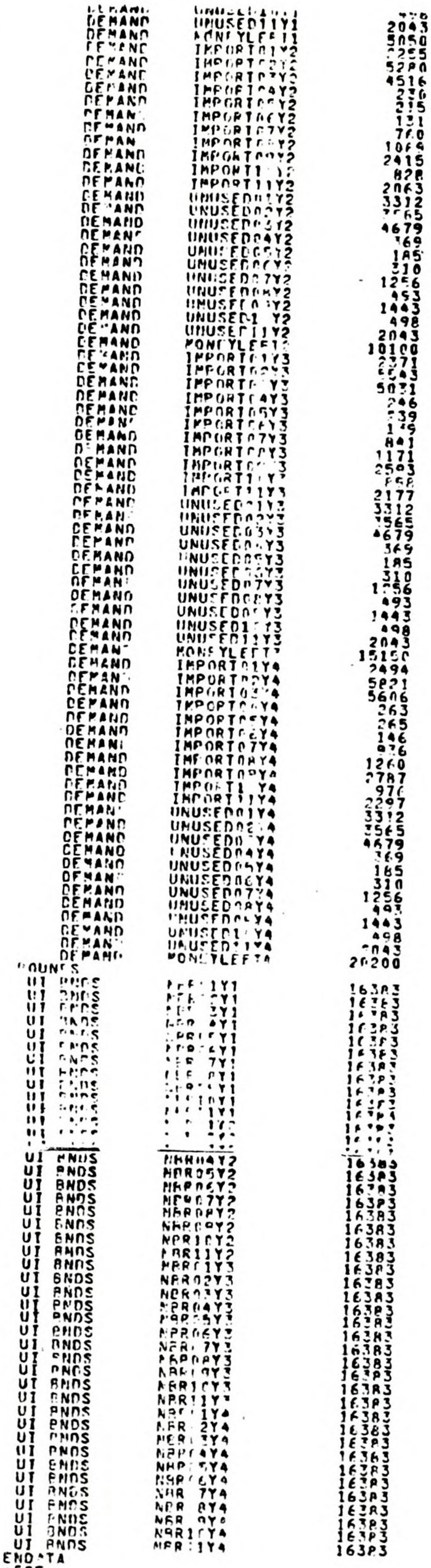
Source: U.S. Bureau of the Census, Census of Manufacturers, 1972 Industries series, volumes 1, 2,3 (Washington D.C.: U.S. Government Printing Office, 1975).

[1]The above data is the aggregate of the data figuring in the 1972 census of manufacturers. The correspondence between these sectors and the Standard Industrial Classification (SIC) used by the 1972 census of manufacture is:

Sector 02 = SIC 20 + 21
Sector 03 = SIC 29
Sector 06 = SIC 32
Sector 07 = SIC 33 + 34 + 35 + 36 + 37
Sector 08 = SIC 28
Sector 09 = SIC 22 + 23 + 31
Sector 10 = SIC 24 + 25 + 26 + 27

Table A3.

```
NAME                  ALLOCATION
ROWS
 N   OBJECTFCT
 L   IMPORT01Y1
 L   IMPORT02Y1
 L   IMPORT03Y1
 L   IMPORT04Y1
 L   IMPORT05Y1
 L   IMPORT06Y1
 L   IMPORT07Y1
 L   IMPORT08Y1
 L   IMPORT09Y1
 L   IMPORT10Y1
 L   IMPORT11Y1
 L   UNUSED01Y1
 L   UNUSED02Y1
 L   UNUSED03Y1
 L   UNUSED04Y1
 L   UNUSED05Y1
 L   UNUSED06Y1
 L   UNUSED07Y1
 L   UNUSED08Y1
 L   UNUSED09Y1
 L   UNUSED10Y1
 L   UNUSED11Y1
 L   MONEYLEFT1
 L   IMPORT01Y2
 L   IMPORT02Y2
 L   IMPORT03Y2
 L   IMPORT04Y2
 L   IMPORT05Y2
 L   IMPORT06Y2
 L   IMPORT07Y2
 L   IMPORT08Y2
 L   IMPORT09Y2
 L   IMPORT10Y2
 L   IMPORT11Y2
 L   UNUSED01Y2
 L   UNUSED02Y2
 L   UNUSED03Y2
 L   UNUSED04Y2
 L   UNUSED05Y2
 L   UNUSED06Y2
 L   UNUSED07Y2
 L   UNUSED08Y2
 L   UNUSED09Y2
 L   UNUSED10Y2
 L   UNUSED11Y2
 L   MONEYLEFT2
 L   IMPORT01Y3
 L   IMPORT02Y3
 L   IMPORT03Y3
 L   IMPORT04Y3
 L   IMPORT05Y3
 L   IMPORT06Y3
 L   IMPORT07Y3
 L   IMPORT08Y3
 L   IMPORT09Y3
 L   IMPORT10Y3
 L   IMPORT11Y3
 L   UNUSED01Y3
 L   UNUSED02Y3
 L   UNUSED03Y3
 L   UNUSED04Y3
 L   UNUSED05Y3
 L   UNUSED06Y3
 L   UNUSED07Y3
 L   UNUSED08Y3
 L   UNUSED09Y3
 L   UNUSED10Y3
 L   UNUSED11Y3
 L   MONEYLEFT3
 L   IMPORT01Y4
 L   IMPORT02Y4
 L   IMPORT03Y4
 L   IMPORT04Y4
 L   IMPORT05Y4
 L   IMPORT06Y4
 L   IMPORT07Y4
 L   IMPORT08Y4
 L   IMPORT09Y4
 L   IMPORT10Y4
 L   IMPORT11Y4
 L   UNUSED01Y4
 L   UNUSED02Y4
 L   UNUSED03Y4
 L   UNUSED04Y4
 L   UNUSED05Y4
 L   UNUSED06Y4
 L   UNUSED07Y4
 L   UNUSED08Y4
 L   UNUSED09Y4
 L   UNUSED10Y4
 L   UNUSED11Y4
 L   MONEYLEFT4
COLUMNS
    DEFUT     #MARKER#                 #INTORG#
    NGR01Y1   UNUSED01Y1       -.786   MONEYLEFT1        .452
    NPR01Y1   UNUSED01Y2       -.786   MONEYLEFT2        .452
    NBR01Y1   UNUSED01Y3       -.786   MONEYLEFT3        .452
    NPR01Y1   UNUSED01Y4       -.786   MONEYLEFT4        .452
    NPR02Y1   UNUSED02Y1    -25.029    MONEYLEFT1      25.168
    NBR02Y1   UNUSED02Y2    -25.029    MONEYLEFT2      25.168
    NBR02Y1   UNUSED02Y3    -25.029    MONEYLEFT3      25.168
    NPR02Y1   UNUSED02Y4    -25.029    MONEYLEFT4      25.168
    NPR03Y1   UNUSED03Y1    -75.709    MONEYLEFT1     311.496
    NBR03Y1   UNUSED03Y2    -75.709    MONEYLEFT2     311.496
    NBR03Y1   UNUSED03Y3    -75.709    MONEYLEFT3     311.496
    NPR03Y1   UNUSED03Y4    -75.709    MONEYLEFT4     311.496
    NPR04Y1   UNUSED04Y1    -17.772    MONEYLEFT1     174.577
    NBR04Y1   UNUSED04Y2    -17.772    MONEYLEFT2     174.577
    NPR04Y1   UNUSED04Y3    -17.772    MONEYLEFT3     174.577
    NPR04Y1   UNUSED04Y4    -17.772    MONEYLEFT4     174.577
    NBR05Y1   UNUSED05Y1     -7.119    MONEYLEFT1      10.591
    NBR05Y1   UNUSED05Y2     -7.119    MONEYLEFT2      10.591
```



```
DATE 08/26/78  TIME 14.42.58        A P E X - I I I   C O N T R O L   P R O G R A M          APEX-III 1.000   FIELD LENGTH 070200 OCTAL

     APEX-III CALL CARD - APEX(SOLVE,MAX,MIP)

                    .LABEL..   .VERB...   .PARAM1.   .PARAM2.   .PARAM3.   .PARAM4.   .PARAM5.

                               INPUT      $
     (GENERATED)               BRANCH     *          MAJERR     MINERR
                               SET        KNDIR      MAX
                               SET        KNOBJ      $
                               SET        KNRHS      $
                               SET        KNBND      $
                               SET        KNRNG      $
                               SELECT
     (GENERATED)               BRANCH     *          MAJERR
                               CRASH
     (GENERATED)               BRANCH     *          MAJERR
                               PRIMAL
                               BRANCH     PRIMEND    MAJERR     *          PROBOUT
                    PERTJK3    SET        RTDELTA    1.0E-5
                    DOPRIM     PRIMAL
                    PRIMEND    BRANCH     *          MAJERR     PROBOUT
                               MIXINT
                               BRANCH     *          MAJERR     PROBSAV    *          PROBSAV
                               OUTPUT
     (GENERATED)               BRANCH     *          MAJERR
                               EXIT
     (GENERATED)               EXIT
     (GENERATED)    MAJERR     MESSAGE    MAJOR ERROR.  APEX TERMINATION
     (GENERATED)               EXIT
     (GENERATED)    MINERR     MESSAGE    MINOR ERROR.  APEX TERMINATION
     (GENERATED)               EXIT
     (GENERATED)    PROBOUT    MESSAGE    PROBLEM OUTPUT.  APEX TERMINATION
     (GENERATED)               BASISOU
     (GENERATED)               OUTPUT     PART
     (GENERATED)               EXIT
     (GENERATED)    PROBSAV    MESSAGE    PROBLEM SAVED.  APEX TERMINATION
     (GENERATED)               SAVE
     (GENERATED)               EXIT
     (GENERATED)    RECOUT     RECREAT
     (GENERATED)               BRANCH     PROBOUT    MAJERR

     INPUT      *                                                  ******** CPU USED =      .189

                                    ***** CARD READ SUMMARY *****

HEADER, CARD NO.       1    (NAME          ALLOCATION                                        )
HEADER, CARD NO.       2    (ROWS                                                            )
                             FREE ROW = OBJECTFCT
HEADER, CARD NO.      96    (COLUMNS                                                         )
HEADER, CARD NO.     449    (RHS                                                             )
                             RHS NAME = DEMAND
HEADER, CARD NO.     542    (BOUNDS                                                          )
                             BOUND SET NAME = BNDS
HEADER, CARD NO.     587    (ENDATA                                                          )
```

********** INPUT SUMMARY STATISTICS **********

***** GENERAL STATISTICS *****

AUTO SELECTION		CONSTRAINTS		VARIABLES		NON-ZEROS		MISC. TOTALS	
NAME =	ALLOCATION	EQ (E) ...	4	COLUMNS ...	88	AIJS (COL) ...	684	MINOR ERRORS ...	0
OBJ =	OBJECTFCT	LE (L) ...	88	RHS ...	1	AIJS (RHS) ...	92	DENSITY 0/0 ...	0.358
RHS =	DEMAND	GE (G) ...	0	TOTAL ...	89	TOTAL ...	776	UNIQUE VALUES ...	184
BND =	BNDS	FR (N) ...	1	MIN FL(8) ...	054100	AVER NZ/COL	7.77	INDIRECT NAMES ...	0
RNG =		TOTAL ...	93	REC FL(8) ...	075000	AVER NZ/ROW	7.35	TOTAL VALUES ...	184
		LC ROWS ...	0	ECS FL(8) ...	0020 K	LC AIJS (COL)..	0	PROBLEM BLOCKS ...	1

***** BOUNDS STATISTICS *****

---- NAME = BNDS					-----
FX	0	ZR	0	NZ	0
PL	44	LO	0	UP	0
MI	0	LO	0	UP	0
FR	0				
BV	0				
I	44				
S1	0	MB	0		
S2	0	MB	0		

***** RANGES STATISTICS *****

NO SETS ACCEPTED

NOTE: NROW NCOL NBND AND NRNG ARE RESIDENT IN CENTRAL MEMORY
TOTAL LENGTH (LKVECNM) IS 228

```
BRANCH      *           MAJERR      MINERR
SET         KNDIR       MAX
SET         KNOBJ       $
SET         KNRHS       $
SET         KNBND       $
SET         KNRNG       $
SELECT                                                  ******** CPU USED =     1.426
```

***** S E L E C T I O N *****

SELECTED VECTORS				MULTIPLIERS			DIRECTION		
FUNCTION	CELL		NAME	CELL		VALUE			
OBJECTIVE FUNCTION	(KNOBJ)	=	OBJECTFCT	(RPSOBJ)	=	-1.000	(KNDIR)	=	MAXIMIZE
CHANGE OBJECTIVE FUNCTION	(KNCHOBJ)	=		(RPCHOBJ)	=	0.000			
RIGHT-HAND SIDE	(KNRHS)	=	DEMAND	(RPSRHS)	=	1.000			
CHANGE RIGHT-HAND SIDE	(KNCHRHS)	=		(RPCHRHS)	=	0.000			
BOUNDS SET	(KNBND)	=	BNDS						
RANGES SET	(KNRNG)	=							

------ SELECTED MIXED INTEGER VARIABLE SUMMARY ------

```
    0 ACTIVE BIVALENT VARIABLES
   44 ACTIVE INTEGER VARIABLES
    0 ACTIVE SPECIAL ORDERED SETS
          0 TYPE 1 SETS WITH TOTAL VARIABLES OF     0
          0 TYPE 2 SETS WITH TOTAL VARIABLES OF     0
          0 MEMBERS IN LARGEST SOS SET

      BRANCH      *           MAJERR

      CRASH                                         ******** CPU USED =     1.535

CRASH RESULTS - REMAINING LOGICALS =    89
                BALANCE STRUCTURALS =    4
                OTHER STRUCTURALS =      0

      BRANCH      *           MAJERR

      PRIMAL                                        ******** CPU USED =     1.732

OPTIMAL SOLUTION,  OBJECTIVE =      61476.541714

      BRANCH      PRIMEND     MAJERR      *           PROBOUT

PRIMEND   BRANCH      *           MAJERR      PROBOUT
```

MIXINT ******** CPU USED = 0.070

NODE NO.	PARENT NODE NO.	ARB. DIR.	STATUS	OBJECTIVE FUNCTION VALUE	SUM OF FRACTIONS	SEPARATION VARIABLE - TYPE	NUMBER	LOWER LIMIT	UPPER LIMIT	RANK	RESOURCE UNITS
1	0		DEVELOPED	61476.541714	6.367					1	0.1
2	1	UP	DEVELOPED	61443.374095	5.939	INTEGER	4	3	16383	2	9.5
3	2	UP	DEVELOPED	61435.391378	5.575	INTEGER	15	2	16383	3	11.8
4	3	UP	DEVELOPED	61431.737612	5.537	INTEGER	26	2	16383	4	12.4
5	4	UP	DEVELOPED	61422.953877	5.298	INTEGER	37	2	16383	5	13.8
6	5	DOWN	DEVELOPED	61419.326195	5.823	INTEGER	25	0	7	6	15.3
7	6	DOWN	DEVELOPED	61418.411861	5.495	INTEGER	2	0	9	7	17.1
8	7	UP	DEVELOPED	61416.137415	4.742	INTEGER	13	14	16383	8	18.4
9	8	UP	DEVELOPED	61416.112125	4.886	INTEGER	2	96	96	9	19.7
10	9	UP	DEVELOPED	61412.752474	4.825	INTEGER	36	11	16383	10	21.0
11	10	UP	DEVELOPED	61411.602175	4.436	INTEGER	24	13	16383	11	22.4
12	11	DOWN	DEVELOPED	61410.620560	5.148	INTEGER	29	0	0	12	23.7
13	12	DOWN	DEVELOPED	61409.514187	4.209	INTEGER	14	0	0	13	24.9
14	13	DOWN	DEVELOPED	61409.455557	4.261	INTEGER	28	0	1	14	26.1
15	14	DOWN	DEVELOPED	61409.297076	4.076	INTEGER	9	0	66	15	27.9
16	15	DOWN	DEVELOPED	61409.297076	3.669	INTEGER	24	13	13	16	29.1
17	16	UP	DEVELOPED	61407.140587	3.545	INTEGER	35	13	16383	17	30.6
18	17	UP	DEVELOPED	61407.140587	3.967	INTEGER	31	43	16383	18	31.8
19	18	UP	DEVELOPED	61407.085989	3.536	INTEGER	27	4	16383	19	33.8
20	19	UP	DEVELOPED	61406.990396	3.723	INTEGER	28	1	1	20	34.2
21	20	UP	DEVELOPED	61406.851331	4.280	INTEGER	5	1	16383	21	35.8
22	21	DOWN	DEVELOPED	61406.747052	4.397	INTEGER	9	0	65	22	36.4
23	22	UP	DEVELOPED	61406.553783	3.572	INTEGER	5	2	16383	23	37.5
24	23	DOWN	DEVELOPED	61406.503708	3.753	INTEGER	9	0	64	24	38.7
25	24	DOWN	DEVELOPED	61405.842064	3.739	INTEGER	32	0	137	25	39.9
26	25	UP	DEVELOPED	61405.696903	3.887	INTEGER	21	101	16383	26	40.6
27	26	UP	DEVELOPED	61405.653260	3.435	INTEGER	32	137	137	27	41.7
28	27	DOWN	DEVELOPED	61405.314208	3.188	INTEGER	38	0	4	28	43.1
29	28	DOWN	DEVELOPED	61404.986336	3.141	INTEGER	27	4	4	29	44.1
30	29	DOWN	DEVELOPED	61404.843098	2.907	INTEGER	16	0	7	30	45.1
31	30	UP	DEVELOPED	61404.835133	2.867	INTEGER	20	329	16383	31	46.0
32	31	DOWN	DEVELOPED	61402.426616	2.437	INTEGER	31	43	43	32	47.1
33	32	UP	DEVELOPED	61402.395603	2.213	INTEGER	11	27	16383	33	48.0
34	33	UP	DEVELOPED	61402.180790	3.327	INTEGER	5	3	16383	34	48.9
35	34	DOWN	DEVELOPED	61402.122042	2.413	INTEGER	16	0	6	35	49.8
36	35	UP	DEVELOPED	61401.926910	2.829	INTEGER	28	331	16383	36	50.8
37	36	DOWN	DEVELOPED	61401.907953	2.579	INTEGER	9	0	62	37	51.7
38	37	UP	DEVELOPED	61401.861758	2.594	INTEGER	5	4	16383	38	52.7
39	38	DOWN	DEVELOPED	61401.824662	2.887	INTEGER	16	0	5	39	53.6
40	39	UP	DEVELOPED	61401.683085	3.143	INTEGER	28	332	16383	40	54.8
41	40	DOWN	DEVELOPED	61401.664128	2.892	INTEGER	9	0	61	41	55.7
42	41	UP	DEVELOPED	61401.576687	3.367	INTEGER	5	5	16383	42	56.7
43	42	DOWN	DEVELOPED	61401.455837	3.798	INTEGER	9	0	60	43	58.1
44	43	DOWN	DEVELOPED	61401.441208	3.335	INTEGER	16	0	4	44	59.0
45	44	UP	DEVELOPED	61401.420303	2.823	INTEGER	28	333	16383	45	59.9
46	45	UP	DEVELOPED	61401.286251	3.341	INTEGER	5	6	16383	46	60.8
47	46	DOWN	DEVELOPED	61401.212012	3.388	INTEGER	9	0	59	47	62.0
48	47	DOWN	DEVELOPED	61401.184338	2.338	INTEGER	16	0	3	48	62.9
49	48	UP	DEVELOPED	61401.176478	2.398	INTEGER	28	334	16383	49	63.8
50	49	UP	DEVELOPED	61400.320507	3.006	INTEGER	5	7	16383	50	65.1
51	50	DOWN	DEVELOPED	61400.038079	3.202	INTEGER	9	0	58	51	66.4
52	51	DOWN	DEVELOPED	61400.038079	3.302	INTEGER	5	7	7	52	67.4
53	52	DOWN	DEVELOPED	61399.806246	2.261	INTEGER	16	0	2	53	68.6
54	53	DOWN	DEVELOPED	61399.754393	2.284	INTEGER	10	0	0	54	69.6

55	54	DOWN	DEVELOPED	61399.262755	2.231	INTEGER	28	334	335	55	70.9
56	55	DOWN	DEVELOPED	61399.215814	2.233	INTEGER	7	0	0	56	71.8
57	56	DOWN	DEVELOPED	61399.197684	2.225	INTEGER	6	0	0	57	72.4
58	57	DOWN	DEVELOPED	61399.157673	2.564	INTEGER	3	0	0	58	73.1
59	58	DOWN	DEVELOPED	61399.136650	2.356	INTEGER	21	101	101	59	74.1
60	59	DOWN	DEVELOPED	61399.061492	2.111	INTEGER	18	0	0	60	75.0
61	60	DOWN	DEVELOPED	61399.059754	2.128	INTEGER	8	0	0	61	75.9
62	61	DOWN	DEVELOPED	61398.917888	2.094	INTEGER	17	0	0	62	76.5
63	62	DOWN	DEVELOPED	61398.795232	1.746	INTEGER	19	0	0	63	77.3
64	63	UP	DEVELOPED	61398.765006	1.760	INTEGER	43	24	16383	64	78.1
65	64	UP	DEVELOPED	61398.765006	2.055	INTEGER	33	61	16383	65	78.8
66	65	UP	DEVELOPED	61398.760586	2.030	INTEGER	38	4	4	66	79.6
67	66	UP	DEVELOPED	61398.749945	2.045	INTEGER	25	7	7	67	80.0
68	67	DOWN	DEVELOPED	61397.676296	2.148	INTEGER	42	0	45	68	80.7
69	68	UP	DEVELOPED	61397.126754	2.201	INTEGER	43	25	16383	69	81.4
70	69	DOWN	DEVELOPED	61396.305464	2.280	INTEGER	42	0	44	70	81.9
71	70	DOWN	DEVELOPED	61396.183983	1.456	INTEGER	43	25	25	71	82.6
72	71	DOWN	DEVELOPED	61395.976924	1.529	INTEGER	30	0	0	72	83.3
73	72	DOWN	DEVELOPED	61395.849114	1.575	INTEGER	36	11	11	73	83.8
74	73	DOWN	DEVELOPED	61395.755101	1.598	INTEGER	40	0	0	74	84.5
75	74	DOWN	DEVELOPED	61395.583822	1.624	INTEGER	39	8	0	75	85.2
76	75	DOWN	DEVELOPED	61395.023277	1.535	INTEGER	41	0	0	76	85.7
77	76	DOWN	DEVELOPED	61395.023277	1.535	INTEGER	26	2	2	77	86.1
78	77	DOWN	DEVELOPED	61395.023277	1.719	INTEGER	37	2	2	78	86.7
79	78	DOWN	DEVELOPED	61395.023277	1.504	INTEGER	35	13	13	79	87.3
80	79	UP	DEVELOPED	61395.023277	1.504	INTEGER	1	3316	16383	80	87.8
81	80	DOWN	DEVELOPED	61395.023277	1.767	INTEGER	1	3316	3317	81	88.1
82	81	DOWN	DEVELOPED	61395.023277	1.439	INTEGER	11	27	27	82	88.5
83	82	DOWN	DEVELOPED	61395.023277	1.435	INTEGER	4	3	3	83	89.0
84	83	UP	DEVELOPED	61395.023277	1.140	INTEGER	22	64	16383	84	89.7
85	84	UP	DEVELOPED	61395.023277	.941	INTEGER	44	67	16383	85	90.0
86	85	DOWN	DEVELOPED	61395.023277	.806	INTEGER	44	67	72	86	90.7
87	86	UP	DEVELOPED	61369.071285	1.113	INTEGER	13	15	16383	87	91.3
88	87	UP	DEVELOPED	61369.212404	.515	INTEGER	20	335	335	88	91.7
89	88	DOWN	DEVELOPED	61369.212404	.515	INTEGER	12	0	579	89	92.4
90	89	UP	DEVELOPED	61369.212404	.475	INTEGER	33	62	16383	90	93.0
91	90	DOWN	DEVELOPED	61369.212404	.475	INTEGER	33	62	64	91	93.3
92	91	DOWN	DEVELOPED	61369.212404	.475	INTEGER	44	67	69	92	93.6
93	92	UP	DEVELOPED	61369.212404	.332	INTEGER	23	499	16383	93	94.3
94	93	UP	DEVELOPED	61369.212404	.332	INTEGER	1	3317	3317	94	94.8
95	94	DOWN	DEVELOPED	61369.175466	.273	INTEGER	12	0	578	95	95.5
96	95	DOWN	DEVELOPED	61369.063479	.264	INTEGER	16	0	0	96	96.1
97	96	DOWN	DEVELOPED	61369.063479	.268	INTEGER	15	2	2	97	96.5
98	97	DOWN	DEVELOPED	61369.063479	.395	INTEGER	13	15	15	98	96.8
99	98	DOWN	DEVELOPED	61369.063479	.150	INTEGER	22	64	64	99	97.3
100	99	UP	DEVELOPED	61369.063479	.150	INTEGER	23	490	16383	100	97.8
101	100	UP	DEVELOPED	61369.063479	.445	INTEGER	34	446	16383	101	98.4
102	101	DOWN	DEVELOPED	61369.063479	.445	INTEGER	33	62	63	102	98.9
103	102	DOWN	INTEGER	61369.063479	0.000	INTEGER	44	67	68	103	99.2

SEARCH FOR OPTIMAL INTEGER SOLUTION TERMINATED DUE TO RPOITCR/RPOPTCA STOPPING CRITERIA
INTEGER SOLUTION FOUND AT NODE 103, WITH FUNCTIONAL VALUE 61369.063479,
IS WITHIN 43.795 UNITS (.07 PER CENT) OF OPTIMUM

BRANCH * MAJERR PROBSAV * PROBSAV

OUTPUT

******** CPU USED = 99.414

PRINT OPTION = COMPLETE OUTPUT
NAME = ALLOCATION OBJ = OBJECTFCT RHS = DEMAND BND = BNDS VALUE OF OBJECTIVE = 61369.06348
DIR = MAXIMIZE COBJ = CRHS = RNG = RPSOBJ = -1.0000 RPSRHS = 1.0000
RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	ROW ACTIVITY	SLACK	RHS LOWER	RHS UPPER	MARGINAL
1	OBJECTFCT	FR	SLACK	61369.06348	-61369.06348	-INF	+INF	.
2	IMPORT01Y1	LE	BINDING	2145.00000	.	-INF	2145.00000	-.86247
3	IMPORT02Y1	LE	SLACK	5027.52485	2.47515	-INF	5030.00000	.
4	IMPORT03Y1	LE	BINDING	4055.00000	.	-INF	4055.00000	-.77748
5	IMPORT04Y1	EQ	BINDING	216.00000	.	216.00000	216.00000	-.87459
6	IMPORT05Y1	LE	BINDING	194.00000	.	-INF	194.00000	-.55352
7	IMPORT06Y1	LE	SLACK	-56.80366	180.80366	-INF	124.00000	.
8	IMPORT07Y1	LE	SLACK	-27.17717	717.17717	-INF	690.00000	.
9	IMPORT08Y1	LE	SLACK	-166.73595	1180.73595	-INF	1014.00000	.
10	IMPORT09Y1	LE	SLACK	977.75683	1273.24317	-INF	2251.00000	.
11	IMPORT10Y1	LE	SLACK	134.07499	630.92501	-INF	765.00000	.
12	IMPORT11Y1	LE	BINDING	1954.00000	.	-INF	1954.00000	-.60374
13	UNUSED01Y1	LE	SLACK	3311.14280	.85720	-INF	3312.00000	.
14	UNUSED02Y1	LE	BINDING	3565.00000	.	-INF	3565.00000	-.75245
15	UNUSED03Y1	LE	SLACK	4639.97466	39.02534	-INF	4679.00000	.
16	UNUSED04Y1	LE	SLACK	362.00274	6.99726	-INF	369.00000	.
17	UNUSED05Y1	LE	SLACK	183.79366	1.20634	-INF	185.00000	.
18	UNUSED06Y1	LE	BINDING	310.00000	.	-INF	310.00000	-.55681
19	UNUSED07Y1	LE	BINDING	1256.00000	.	-INF	1256.00000	-.43373
20	UNUSED08Y1	LE	BINDING	493.00000	.	-INF	493.00000	-.43999
21	UNUSED09Y1	LE	BINDING	1443.00000	.	-INF	1443.00000	-.42671
22	UNUSED10Y1	LE	BINDING	498.00000	.	-INF	498.00000	-.53442
23	UNUSED11Y1	LE	SLACK	2042.75127	.24873	-INF	2043.00000	.
24	MONEYLEFT1	LE	SLACK	5049.76000	.24000	-INF	5050.00000	.
25	IMPORT01Y2	LE	SLACK	2251.07197	3.92803	-INF	2255.00000	.
26	IMPORT02Y2	LE	BINDING	5280.00000	.	-INF	5280.00000	-.32802
27	IMPORT03Y2	LE	SLACK	4056.24174	459.75826	-INF	4516.00000	.
28	IMPORT04Y2	EQ	BINDING	230.00000	.	230.00000	230.00000	-.83904
29	IMPORT05Y2	LE	SLACK	194.89177	20.10823	-INF	215.00000	.
30	IMPORT06Y2	LE	SLACK	-74.48754	205.48754	-INF	131.00000	.
31	IMPORT07Y2	LE	SLACK	-91.96944	851.96944	-INF	760.00000	.
32	IMPORT08Y2	LE	SLACK	-306.15035	1395.15035	-INF	1089.00000	.
33	IMPORT09Y2	LE	SLACK	2414.07482	.92518	-INF	2415.00000	.
34	IMPORT10Y2	LE	SLACK	447.87806	380.12194	-INF	828.00000	.
35	IMPORT11Y2	LE	BINDING	2063.00000	.	-INF	2063.00000	-.57389
36	UNUSED01Y2	LE	BINDING	3312.00000	.	-INF	3312.00000	-.78571
37	UNUSED02Y2	LE	SLACK	3530.25098	34.74902	-INF	3565.00000	.
38	UNUSED03Y2	LE	BINDING	4679.00000	.	-INF	4679.00000	-.72481
39	UNUSED04Y2	LE	SLACK	364.36180	4.63820	-INF	369.00000	.
40	UNUSED05Y2	LE	BINDING	185.00000	.	-INF	185.00000	-.52319
41	UNUSED06Y2	LE	BINDING	310.00000	.	-INF	310.00000	-.52508
42	UNUSED07Y2	LE	BINDING	1256.00000	.	-INF	1256.00000	-.42113
43	UNUSED08Y2	LE	BINDING	493.00000	.	-INF	493.00000	-.44224
44	UNUSED09Y2	LE	BINDING	1443.00000	.	-INF	1443.00000	-.39667
45	UNUSED10Y2	LE	BINDING	498.00000	.	-INF	498.00000	-.46226
46	UNUSED11Y2	LE	SLACK	2041.13014	1.86986	-INF	2043.00000	.
47	MONEYLEFT2	LE	SLACK	10099.89100	.10900	-INF	10100.00000	.
48	IMPORT01Y3	LE	BINDING	2371.00000	.	-INF	2371.00000	-.86395
49	IMPORT02Y3	LE	BINDING	5543.00000	.	-INF	5543.00000	-.87357
50	IMPORT03Y3	LE	SLACK	4529.62755	501.37245	-INF	5031.00000	.

PRINT OPTION = COMPLETE OUTPUT VALUE OF OBJECTIVE = 61369.06348
NAME = ALLOCATION OBJ = OBJECTFCT RHS = DEMAND BND = BNDS RPSOBJ = -1.0000 RPSRHS = 1.0000
DIR = MAXIMIZE COBJ = CRHS = RNG = RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	ROW ACTIVITY	SLACK	RHS LOWER	RHS UPPER	MARGINAL
51	IMPORT04Y3	EQ	BINDING	246.00000	.	246.00000	246.00000	-.84548
52	IMPORT05Y3	LE	SLACK	220.24997	18.75003	-INF	239.00000	.
53	IMPORT06Y3	LE	SLACK	-87.31610	226.31610	-INF	139.00000	.
54	IMPORT07Y3	LE	SLACK	-168.91028	1009.91028	-INF	841.00000	.
55	IMPORT08Y3	LE	SLACK	-380.47318	1551.47318	-INF	1171.00000	.
56	IMPORT09Y3	LE	SLACK	2592.56349	.43651	-INF	2593.00000	.
57	IMPORT10Y3	LE	BINDING	898.00000	.	-INF	898.00000	-.74716
58	IMPORT11Y3	LE	BINDING	2177.00000	.	-INF	2177.00000	-.50938
59	UNUSED01Y3	LE	SLACK	3311.28287	.71713	-INF	3312.00000	.
60	UNUSED02Y3	LE	SLACK	3523.76989	41.23011	-INF	3565.00000	.
61	UNUSED03Y3	LE	BINDING	4679.00000	.	-INF	4679.00000	-.72825
62	UNUSED04Y3	LE	SLACK	366.62961	2.37039	-INF	369.00000	.
63	UNUSED05Y3	LE	BINDING	185.00000	.	-INF	185.00000	-.54095
64	UNUSED06Y3	LE	BINDING	310.00000	.	-INF	310.00000	-.54524
65	UNUSED07Y3	LE	BINDING	1256.00000	.	-INF	1256.00000	-.42996
66	UNUSED08Y3	LE	BINDING	493.00000	.	-INF	493.00000	-.51311
67	UNUSED09Y3	LE	BINDING	1443.00000	.	-INF	1443.00000	-.43853
68	UNUSED10Y3	LE	SLACK	496.72898	1.27102	-INF	498.00000	.
69	UNUSED11Y3	LE	SLACK	2042.88087	.11913	-INF	2043.00000	.
70	MONEYLEFT3	LE	SLACK	15145.48900	4.51100	-INF	15150.00000	.
71	IMPORT01Y4	LE	BINDING	2494.00000	.	-INF	2494.00000	-.85931
72	IMPORT02Y4	LE	BINDING	5821.00000	.	-INF	5821.00000	-.86200
73	IMPORT03Y4	LE	SLACK	5291.39305	314.60695	-INF	5606.00000	.
74	IMPORT04Y4	EQ	BINDING	263.00000	.	263.00000	263.00000	-.83904
75	IMPORT05Y4	LE	SLACK	244.01827	20.98173	-INF	265.00000	.
76	IMPORT06Y4	LE	SLACK	-112.79419	258.79419	-INF	146.00000	.
77	IMPORT07Y4	LE	SLACK	-248.10901	1184.10901	-INF	936.00000	.
78	IMPORT08Y4	LE	SLACK	-425.97442	1685.97442	-INF	1260.00000	.
79	IMPORT09Y4	LE	SLACK	2779.14953	7.85047	-INF	2787.00000	.
80	IMPORT10Y4	LE	SLACK	968.65112	7.34888	-INF	976.00000	.
81	IMPORT11Y4	LE	BINDING	2297.00000	.	-INF	2297.00000	-.57389
82	UNUSED01Y4	LE	SLACK	3311.33199	.66801	-INF	3312.00000	.
83	UNUSED02Y4	LE	SLACK	3530.76675	34.23325	-INF	3565.00000	.
84	UNUSED03Y4	LE	BINDING	4679.00000	.	-INF	4679.00000	-.72481
85	UNUSED04Y4	LE	SLACK	363.39400	5.60600	-INF	369.00000	.
86	UNUSED05Y4	LE	BINDING	185.00000	.	-INF	185.00000	-.52319
87	UNUSED06Y4	LE	BINDING	310.00000	.	-INF	310.00000	-.52508
88	UNUSED07Y4	LE	BINDING	1256.00000	.	-INF	1256.00000	-.42113
89	UNUSED08Y4	LE	BINDING	493.00000	.	-INF	493.00000	-.47924
90	UNUSED09Y4	LE	BINDING	1443.00000	.	-INF	1443.00000	-.43260
91	UNUSED10Y4	LE	BINDING	498.00000	.	-INF	498.00000	-.53305
92	UNUSED11Y4	LE	SLACK	2041.50838	1.49162	-INF	2043.00000	.
93	MONEYLEFT4	LE	SLACK	20198.70900	1.29100	-INF	20200.00000	.

PRINT OPTION = COMPLETE OUTPUT
NAME = ALLOCATION OBJ = OBJECTFCT RHS = DEMAND BND = BNDS
DIR = MAXIMIZE LOBJ = CRHS = RNG =

VALUE OF OBJECTIVE = 61369.86348
RPSOBJ = -1.0000 RPSRHS = 1.0000
RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	COL ACTIVITY	OBJ COEF	BND LOWER	BND UPPER	MARGINAL	
1	NBR01Y1	INT	LOWER	3317.00000	.	.	16383.00000	.61756	ARB INT
2	NBR02Y1	INT	LOWER	96.00000	.	.	16383.00000	18.83301	ARB INT
3	NBR03Y1	INT	LOWER	.	.	.	16383.00000	164.88481	ARB INT
4	NBR04Y1	INT **	LOWER	3.00000	.	.	16383.00000	.	ARB INT
5	NBR05Y1	INT	LOWER	7.00000	.	.	16383.00000	11.29221	ARB INT
6	NBR06Y1	INT	LOWER	.	.	.	16383.00000	33.43243	ARB INT
7	NBR07Y1	INT	LOWER	.	.	.	16383.00000	52.57399	ARB INT
8	NBR08Y1	INT	LOWER	.	.	.	16383.00000	48.63622	ARB INT
9	NBR09Y1	INT	UPPER	58.00000	.	.	16383.00000	13.75266	
10	NBR10Y1	INT	LOWER	.	.	.	16383.00000	7.27228	ARB INT
11	NBR11Y1	INT **	LOWER	27.00000	.	.	16383.00000	.	ARB INT
12	NBR01Y2	INT	UPPER	578.00000	.	.	16383.00000	.61756	
13	NBR02Y2	INT **	LOWER	15.00000	.	.	16383.00000	.	ARB INT
14	NBR03Y2	INT	LOWER	.	.	.	16383.00000	164.88481	ARB INT
15	NBR04Y2	INT **	LOWER	2.00000	.	.	16383.00000	.	ARB INT
16	NBR05Y2	INT	LOWER	.	.	.	16383.00000	11.29221	ARB INT
17	NBR06Y2	INT	LOWER	.	.	.	16383.00000	24.78292	ARB INT
18	NBR07Y2	INT	LOWER	.	.	.	16383.00000	39.29727	ARB INT
19	NBR08Y2	INT	LOWER	.	.	.	16383.00000	37.22065	ARB INT
20	NBR09Y2	INT	LOWER	335.00000	.	.	16383.00000	10.28947	ARB INT
21	NBR10Y2	INT	LOWER	101.00000	.	.	16383.00000	4.73167	ARB INT
22	NBR11Y2	INT **	LOWER	64.00000	.	.	16383.00000	.	ARB INT
23	NBR01Y3	INT **	LOWER	490.00000	.	.	16383.00000	.	
24	NBR02Y3	INT **	LOWER	13.00000	.	.	16383.00000	.	ARB INT
25	NBR03Y3	INT	LOWER	7.00000	.	.	16383.00000	110.00992	ARB INT
26	NBR04Y3	INT **	LOWER	2.00000	.	.	16383.00000	.	ARB INT
27	NBR05Y3	INT	LOWER	4.00000	.	.	16383.00000	7.57026	ARB INT
28	NBR06Y3	INT	LOWER	1.00000	.	.	16383.00000	16.62631	ARB INT
29	NBR07Y3	INT	LOWER	.	.	.	16383.00000	26.22893	ARB INT
30	NBR08Y3	INT	LOWER	.	.	.	16383.00000	25.74675	ARB INT
31	NBR09Y3	INT	LOWER	43.00000	.	.	16383.00000	7.07007	ARB INT
32	NBR10Y3	INT	LOWER	137.00000	.	.	16383.00000	2.53410	ARB INT
33	NBR11Y3	INT **	LOWER	62.00000	.	.	16383.00000	.	
34	NBR01Y4	INT **	LOWER	446.00000	.	.	16383.00000	.	
35	NBR02Y4	INT **	LOWER	13.00000	.	.	16383.00000	.	ARB INT
36	NBR03Y4	INT	LOWER	11.00000	.	.	16383.00000	54.87489	ARB INT
37	NBR04Y4	INT **	LOWER	2.00000	.	.	16383.00000	.	ARB INT
38	NBR05Y4	INT	LOWER	4.00000	.	.	16383.00000	3.72195	ARB INT
39	NBR06Y4	INT	LOWER	.	.	.	16383.00000	8.15661	ARB INT
40	NBR07Y4	INT	LOWER	.	.	.	16383.00000	12.97835	ARB INT
41	NBR08Y4	INT	LOWER	.	.	.	16383.00000	12.43398	ARB INT
42	NBR09Y4	INT	UPPER	44.00000	.	.	16383.00000	3.51096	
43	NBR10Y4	INT	LOWER	25.00000	.	.	16383.00000	2.53410	ARB INT
44	NBR11Y4	INT **	LOWER	67.00000	.	.	16383.00000	.	
45	PROD01Y1	PL	ACTIVE	5918.30480	.77160	.	+INF	.	
46	PROD02Y1	PL	ACTIVE	5967.78403	.27960	.	+INF	.	
47	PROD03Y1	PL	ACTIVE	4639.97466	.71450	.	+INF	.	
48	PROD04Y1	PL	ACTIVE	415.31874	.72910	.	+INF	.	
49	PROD05Y1	PL	ACTIVE	233.59166	.48510	.	+INF	.	
50	PROD06Y1	PL	ACTIVE	310.00000	.48710	.	+INF	.	

PRINT OPTION = COMPLETE OUTPUT VALUE OF OBJECTIVE = 61369.86348
NAME = ALLOCATION OBJ = OBJECTFCT RHS = DEMAND BND = BNDS RPSOBJ = -1.0000 RPSRHS = 1.0000
DIR = MAXIMIZE COBJ = CRHS = RNG = RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	COL ACTIVITY	OBJ COEF	BND LOWER	BND UPPER	MARGINAL
51	PROD07Y1	PL	ACTIVE	1256.00000	.41100	.	+INF	.
52	PROD08Y1	PL	ACTIVE	493.00000	.40710	.	+INF	.
53	PROD09Y1	PL	ACTIVE	1913.72800	.30710	.	+INF	.
54	PROD10Y1	PL	ACTIVE	498.00000	.44010	.	+INF	.
55	PROD11Y1	PL	ACTIVE	2098.53327	.56600	.	+INF	.
56	PROD01Y2	PL	ACTIVE	6373.47000	.77160	.	+INF	.
57	PROD02Y2	PL	ACTIVE	6308.46998	.27960	.	+INF	.
58	PROD03Y2	PL	ACTIVE	4679.00000	.71450	.	+INF	.
59	PROD04Y2	PL	ACTIVE	453.22180	.72910	.	+INF	.
60	PROD05Y2	PL	ACTIVE	234.79800	.40510	.	+INF	.
61	PROD06Y2	PL	ACTIVE	310.00000	.40710	.	+INF	.
62	PROD07Y2	PL	ACTIVE	1256.00000	.41100	.	+INF	.
63	PROD08Y2	PL	ACTIVE	493.00000	.40710	.	+INF	.
64	PROD09Y2	PL	ACTIVE	4632.58800	.30710	.	+INF	.
65	PROD10Y2	PL	ACTIVE	978.15400	.44010	.	+INF	.
66	PROD11Y2	PL	ACTIVE	2229.13614	.56600	.	+INF	.
67	PROD01Y3	PL	ACTIVE	6757.89267	.77160	.	+INF	.
68	PROD02Y3	PL	ACTIVE	6627.36589	.27960	.	+INF	.
69	PROD03Y3	PL	ACTIVE	5208.96301	.71450	.	+INF	.
70	PROD04Y3	PL	ACTIVE	491.03361	.72910	.	+INF	.
71	PROD05Y3	PL	ACTIVE	263.25400	.40510	.	+INF	.
72	PROD06Y3	PL	ACTIVE	325.53400	.40710	.	+INF	.
73	PROD07Y3	PL	ACTIVE	1256.00000	.41100	.	+INF	.
74	PROD08Y3	PL	ACTIVE	493.00000	.40710	.	+INF	.
75	PROD09Y3	PL	ACTIVE	4981.57600	.30710	.	+INF	.
76	PROD10Y3	PL	ACTIVE	1628.18098	.44010	.	+INF	.
77	PROD11Y3	PL	ACTIVE	2358.97887	.56600	.	+INF	.
78	PROD01Y4	PL	ACTIVE	7108.49799	.77160	.	+INF	.
79	PROD02Y4	PL	ACTIVE	6959.73975	.27960	.	+INF	.
80	PROD03Y4	PL	ACTIVE	6041.76201	.71450	.	+INF	.
81	PROD04Y4	PL	ACTIVE	523.34200	.72910	.	+INF	.
82	PROD05Y4	PL	ACTIVE	291.71001	.40510	.	+INF	.
83	PROD06Y4	PL	ACTIVE	325.53400	.40710	.	+INF	.
84	PROD07Y4	PL	ACTIVE	1256.00000	.41100	.	+INF	.
85	PROD08Y4	PL	ACTIVE	493.00000	.40710	.	+INF	.
86	PROD09Y4	PL	ACTIVE	5338.68000	.30710	.	+INF	.
87	PROD10Y4	PL	ACTIVE	1748.30200	.44010	.	+INF	.
88	PROD11Y4	PL	ACTIVE	2496.02838	.56600	.	+INF	.

BRANCH * MAJERR

EXIT

******** CPU USED = 188.914

--- USED WORDS OF DISK (OCTAL) = 000045052

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  08/26/78   MSU MUSTLER 2      LSD 46.35  08/26/78  6588
14.42.40.I883895
14.42.40.NL    17600
14.42.40.RP   00000000000002  000000000174
14.42.41.JOB READ-   08/26/78 .14.42.01.
14.42.41.BENGHEZAL,T600,JC7000,RG1,MT1.
14.42.41.LAST ACCESS- 8.14.29 08/22/78
14.42.42.RUNS- 0041    PN DOLLAR BALANCE $000107.02
14.42.42.000594      CARDS READ   VALUE  30J00000.30
14.42.42.CP-PP SEC.          .035-        .000   $      .00
14.42.42.COPYCF,INPUT,TAPE1.
14.42.42.NL    011300
14.42.42.RP         00000088  000000000000
14.42.45.CP-PP SEC.          .064-        .712   $      .01
14.42.45.AUTORFL,PART.
14.42.45.CP-PP SEC.          .064-        .838   $      .01
14.42.45.RFL,70000.
14.42.45.CP-PP SEC.          .064-        .838   $      .01
14.42.45.NL    014200
14.42.45.RP         00J00011  000000000261
14.42.46.HAL,APEX(SOLVE,MAX,MIP)
14.42.46.ZZZZMPL - CYCLE 01, HAL 4, MPL
14.42.46.FILE ATTACHED
14.42.46.HAL 4.33
14.42.47.APEX      - CYCLE 18, HAL-APEX
14.42.47.FILE ATTACHED
14.42.47.NL    76200
14.42.47.RP   00000000000032  J0J000000517
14.42.48.EXEC BEGUN.14.42.48.
14.42.49.  ********************************
14.42.49.  **       O-O-C APEX-III       **
14.42.49.  **       WITH MIP OPTION      **
14.42.50.  **     WITH REDUCE OPTION     **
14.42.50.  **  WITH PARAMETRICS OPTION    **
14.42.50.  ********************************
14.51.32.       END   APEX
14.51.32.     100.920  CP SECONDS EXECUTION TIME
14.51.32.MAX FILES 0005 MAX PRUS 001100B.
14.51.32.  PP       97.183  SEC.
14.51.32.RP   00000000012UI   0000J0071455
14.51.32.CP USE   101.028 SEC    VALUE$      4.49
14.51.32.PP USE    93.585 SEC    VALUE$       .26
14.51.32.CM USE    2.022 W-H     VALUE$      6.46
14.51.32.TOTAL COMPUTE VALUE AT RG1 $      5.60
14.52.25..000015 PAGES PRINT. 000544 LINES PRINT. FOR $ 000.62 AT RG2.
```

Table A5.

NAME ALLOCATION
ROWS

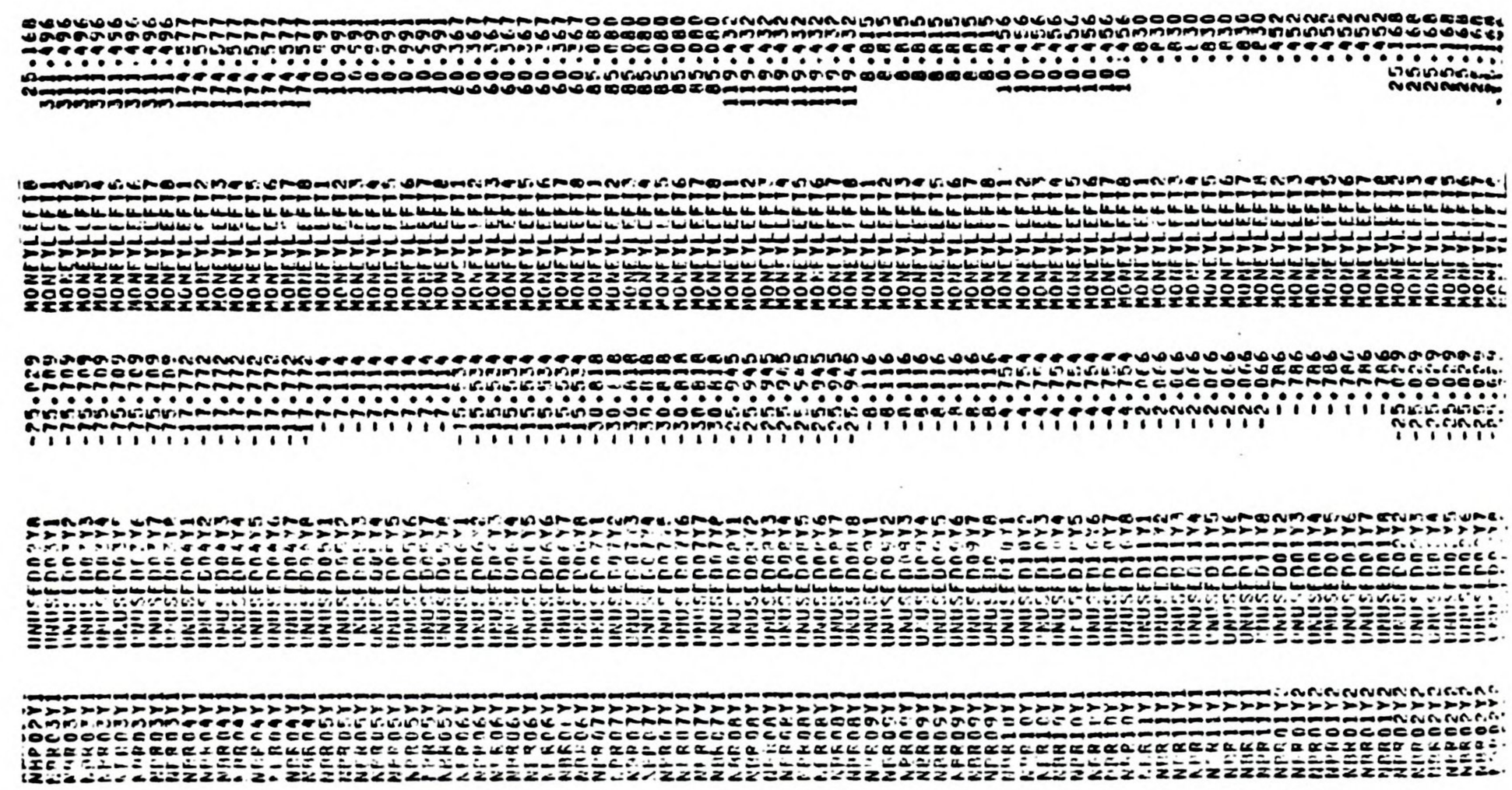
N OBJECTFCT
L IMPORT01Y1
L IMPORT02Y1
L IMPORT03Y1
L IMPORT04Y1
L IMPORT05Y1
L IMPORT06Y1
L IMPORT07Y1
L IMPORT08Y1
L IMPORT09Y1
L IMPORT10Y1
L IMPORT11Y1
L UNUSED01Y1
L UNUSED02Y1
L UNUSED03Y1
L UNUSED04Y1
L UNUSED05Y1
L UNUSED06Y1
L UNUSED07Y1
L UNUSED08Y1
L UNUSED09Y1
L UNUSED10Y1
L UNUSED11Y1
L MONEYLEFT1
L IMPORT01Y2
L IMPORT02Y2
L IMPORT03Y2
L IMPORT04Y2
L IMPORT05Y2
L IMPORT06Y2
L IMPORT07Y2
L IMPORT08Y2
L IMPORT09Y2
L IMPORT10Y2
L IMPORT11Y2
L UNUSED01Y2
L UNUSED02Y2
L UNUSED03Y2
L UNUSED04Y2
L UNUSED05Y2
L UNUSED06Y2
L UNUSED07Y2
L UNUSED08Y2
L UNUSED09Y2
L UNUSED10Y2
L UNUSED11Y2
L MONEYLEFT2
L IMPORT01Y3
L IMPORT02Y3
L IMPORT03Y3
L IMPORT04Y3
L IMPORT05Y3
L IMPORT06Y3
L IMPORT07Y3
L IMPORT08Y3
L IMPORT09Y3
L IMPORT10Y3
L IMPORT11Y3
L UNUSED01Y3
L UNUSED02Y3
L UNUSED03Y3
L UNUSED04Y3
L UNUSED05Y3
L UNUSED06Y3
L UNUSED07Y3
L UNUSED08Y3
L UNUSED09Y3
L UNUSED10Y3
L UNUSED11Y3
L MONEYLEFT3
L IMPORT01Y4
L IMPORT02Y4
L IMPORT03Y4
L IMPORT04Y4
L IMPORT05Y4
L IMPORT06Y4
L IMPORT07Y4
L IMPORT08Y4
L IMPORT09Y4
L IMPORT10Y4
L IMPORT11Y4
L UNUSED01Y4
L UNUSED02Y4
L UNUSED03Y4
L UNUSED04Y4
L UNUSED05Y4
L UNUSED06Y4
L UNUSED07Y4
L UNUSED08Y4
L UNUSED09Y4
L UNUSED10Y4
L UNUSED11Y4
L MONEYLEFT4
L IMPORT01Y5
L IMPORT02Y5
L IMPORT03Y5
L IMPORT04Y5
L IMPORT05Y5
L IMPORT06Y5
L IMPORT07Y5
L IMPORT08Y5
L IMPORT09Y5
L IMPORT10Y5
L IMPORT11Y5
L UNUSED01Y5
L UNUSED02Y5
L UNUSED03Y5
L UNUSED04Y5
L UNUSED05Y5
L UNUSED06Y5
L UNUSED07Y5
L UNUSED08Y5
L UNUSED09Y5

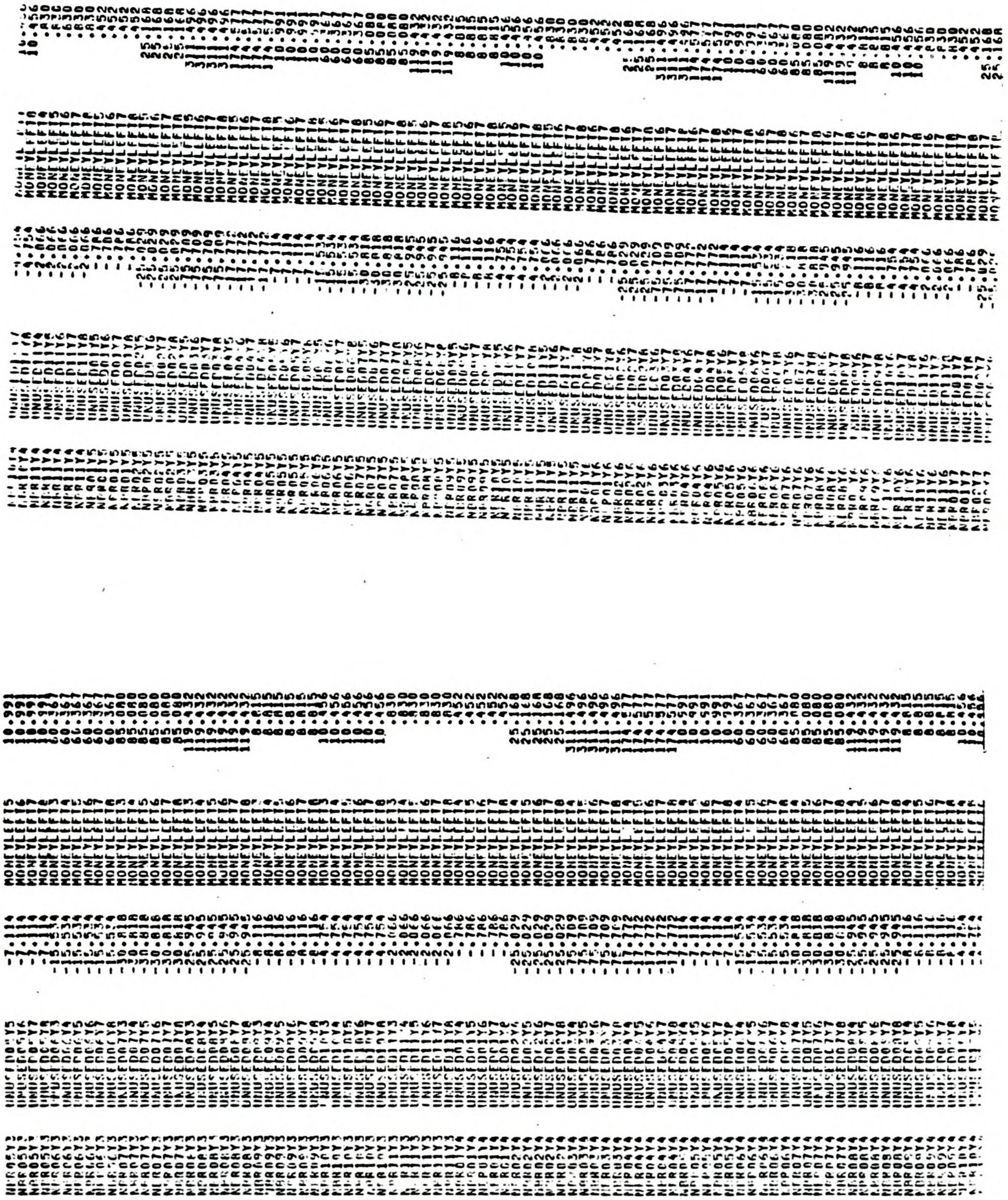
```
$INTORG=1
MONEYLEFT1      .452
MONEYLEFT2      .452
MONEYLEFT3      .452
MONEYLEFT4      .452
MONEYLEFT5      .452
MONEYLEFT6      .452
MONEYLEFT7      .452
MONEYLEFT8      .452
MONEYLEFT1      25.168
MONEYLEFT2      25.168
MONEYLEFT3      25.168
MONEYLEFT4      25.168
MONEYLEFT5      25.168
MONEYLEFT6      25.168
MONEYLEFT7      25.168
```

```
MONEYLEFT1  IMPORT01Y6  IMPORT01Y6  IMPORT02Y6  IMPORT03Y6  IMPORT04Y6  IMPORT05Y6  IMPORT06Y6  IMPORT07Y6  IMPORT08Y6  IMPORT09Y6  IMPORT10Y6  IMPORT11Y6  UNUSED01Y6  UNUSED02Y6  UNUSED03Y6  UNUSED04Y6  UNUSED05Y6  UNUSED06Y6  UNUSED07Y6  UNUSED08Y6  UNUSED09Y6  UNUSED10Y6  UNUSED11Y6  MONEYLEFT1  IMPORT01Y7  IMPORT02Y7  IMPORT03Y7  IMPORT04Y7  IMPORT05Y7  IMPORT06Y7  IMPORT07Y7  IMPORT08Y7  IMPORT09Y7  IMPORT10Y7  IMPORT11Y7  UNUSED01Y7  UNUSED02Y7  UNUSED03Y7  UNUSED04Y7  UNUSED05Y7  UNUSED06Y7  UNUSED07Y7  UNUSED08Y7  UNUSED09Y7  UNUSED10Y7  UNUSED11Y7  MONEYLEFT1  IMPORT01Y8  IMPORT02Y8  IMPORT03Y8  IMPORT04Y8  IMPORT05Y8  IMPORT06Y8  IMPORT07Y8  IMPORT08Y8  IMPORT09Y8  IMPORT10Y8  IMPORT11Y8  UNUSED01Y8  UNUSED02Y8  UNUSED03Y8  UNUSED04Y8  UNUSED05Y8  UNUSED06Y8  UNUSED07Y8  UNUSED08Y8  UNUSED09Y8  UNUSED10Y8  UNUSED11Y8  MONEYLEFT1
COLUMNS
    OF   OUT
    APRR01Y1
    APRR01Y2
    APRR01Y3
    APRR01Y4
    APRR01Y5
    APRR01Y6
    APRR01Y7
    NPRR01Y8
    APRR02Y1
    NPRR02Y1
    APRR02Y2
    NPRR02Y2
    APRR02Y3
    NPRR02Y3
    APRR02Y4
    NPRR02Y4
    APRR02Y5
    NPRR02Y5
    APRR02Y6
    NPRR02Y6
    APRR02Y7
    NPRR02Y7
    APRR02Y8
    NPRR02Y8
    MONEYLEFT1
```

```
    APRR01Y1  -.7R6
    NPRR01Y1  -.7R6
    APRR01Y2  -.7R6
    NPRR01Y2  -.7R6
    APRR01Y3  -.7R6
    NPRR01Y3  -.7R6
    APRR01Y4  -.7R6
    NPRR01Y4  -.7R6
    APRR01Y5  -.7R6
    NPRR01Y5  -.7R6
    APRR01Y6  -.7R6
    NPRR01Y6  -.7R6
    APRR01Y7  -.7R6
    NPRR01Y7  -.7R6
    APRR01Y8  -25.025
    NPRR01Y8  -25.025
    APRR02Y1  -25.029
    NPRR02Y1  -25.029
    APRR02Y2  -25.029
    NPRR02Y2  -25.029
    APRR02Y3  -25.029
    NPRR02Y3  -25.029
    APRR02Y4  -25.029
    NPRR02Y4  -25.029
    APRR02Y5  -25.029
    NPRR02Y5  -25.029
    APRR02Y6  -25.029
    NPRR02Y6  -25.029
    APRR02Y7  -25.029
    NPRR02Y7  -25.029
    APRR02Y8  -25.029
    NPRR02Y8  -25.029
    MONEYLEFT1  -25.029
```







```
IMPORTC  A663
IMPORTC  3341
IMPORTC    4011
IMPORTC     1613
IMPORTC    15715
IMPORTC    17764
IMPORTC    14406
IMPORTC    29462
IMPORTC    25125
IMPORTC    46799
IMPORTC     7766
IMPORTC     1850
IMPORTC     1256
IMPORTC      499
IMPORTC     1443
IMPORTC     1498
UNUSED01    2040
UNUSED01    7140
WORKFILE 163363
DEMAND   163363
DEMAND   163363
DEMAND   
```

Table A6.

```
DATE 08/30/78  TIME 19.07.31        A P E X - I I I   C O N T R O L   P R O G R A M        APEX-III 1.000   FIELD LENGTH 078200 OCTAL

     APEX-III CALL CARD - APEX(SOLVE,MAX,MIP)

                    .LABEL..   .VERB...    .PARAM1.    .PARAM2.    .PARAM3.    .PARAM4.    .PARAM5.

                               INPUT       $
     (GENERATED)               BRANCH      *           MAJERR      MINERR
                               SET         KNDIR       MAX
                               SET         KNOBJ       $
                               SET         KNRHS       $
                               SET         KNBND       $
                               SET         KNRNG       $
                               SELECT
     (GENERATED)               BRANCH      *           MAJERR
                               CRASH
     (GENERATED)               BRANCH      *           MAJERR
                               PRIMAL
                               BRANCH      PRIMEND     MAJERR      *           PROBOUT
                    PERTJKB    SET         RTDELTA     1.0E-5
                    DOPRIML    PRIMAL
                    PRIMEND    BRANCH      *           MAJERR      PROBOUT
                               MIPINT
                               BRANCH      *           MAJERR      PROBSAV     *           PROBSAV
                               OUTPUT
     (GENERATED)               BRANCH      *           MAJERR
                               EXIT
     (GENERATED)               EXIT
     (GENERATED)    MAJERR     MESSAGE     MAJOR ERROR.  APEX TERMINATION
     (GENERATED)               EXIT
     (GENERATED)    MINERR     MESSAGE     MINOR ERROR.  APEX TERMINATION
     (GENERATED)               EXIT
     (GENERATED)    PROBOUT    MESSAGE     PROBLEM OUTPUT.  APEX TERMINATION
     (GENERATED)               BASISOU
     (GENERATED)               OUTPUT      PART
     (GENERATED)               EXIT
     (GENERATED)    PROBSAV    MESSAGE     PROBLEM SAVED.  APEX TERMINATION
     (GENERATED)               SAVE
     (GENERATED)               EXIT
     (GENERATED)    RECOUT     RECREAT
     (GENERATED)               BRANCH      PROBOUT     MAJERR

DATE 08/30/78  TIME 19.07.31        A P E X - I I I   E X E C U T I O N        APEX-III 1.000

          INPUT      $                                                          ******** CPU USED =      .287

                                        ***** CARD READ SUMMARY *****

     HEADER, CARD NO.      1    (NAME          ALLOCATION                                                    )
     HEADER, CARD NO.      2    (ROWS                                                                        )
                                 FREE ROW = OBJECTFCT
     HEADER, CARD NO.    168    (COLUMNS                                                                     )
     HEADER, CARD NO.   1067    (RHS                                                                         )
                                 RHS NAME = DEMAND
     HEADER, CARD NO.   1252    (BOUNDS                                                                      )
                                 BOUND SET NAME = BNDS
     HEADER, CARD NO.   1341    (ENDATA                                                                      )
```

********** I N P U T S U M M A R Y S T A T I S T I C S **********

***** GENERAL STATISTICS *****

AUTO SELECTION	CONSTRAINTS		VARIABLES		NON-ZEROS		MISC. TOTALS	
NAME = ALLOCATION	EQ (E) ...	0	COLUMNS ...	176	AIJS (COL) ...	172)	MINOR ERRORS ...	0
OBJ = OBJECTFCT	LE (L) ...	176	RHS ...	1	AIJS (RHS) ...	184	DENSITY 0/0 ...	5.283
RHS = DEMAND	GE (G) ...	0	TOTAL ...	177	TOTAL ...	1904	UNIQUE VALUES ...	232
BND = BNDS	FR (N) ...	1	MIN FL(8) ...	056100	AVER NZ/COL	9.77	INDIRECT NAMES ...	0
RNG =	TOTAL ...	185	REC FL(8) ...	077100	AVER NZ/ROW	9.30	TOTAL VALUES ...	232
	LC ROWS ...	0	ECS FL(8) ...	0020 K	LC AIJS (COL)..	0	PROBLEM BLOCKS ...	1

***** BOUNDS STATISTICS *****

---- NAME = BNDS				-----
FX	0	ZR	0	NZ 0
PL	88	LO	0	UP 0
MI	0	LO	0	UP 0
FR	0			
BV	0			
I	88			
S1	0	MB	0	
S2	0	MB	0	

***** RANGES STATISTICS *****

NO SETS ACCEPTED

NOTE: NROW NCOL NBIID AND NRNG ARE RESIDENT IN CENTRAL MEMORY
TOTAL LENGTH (LKVE:NM) IS 452

```
BRANCH      *           MAJERR      MINERR
SET         KNDIR       MAX
SET         KNOBJ       1
SET         KNRHS       1
SET         KNBND       1
SET         KNRNG       1
SELECT                                                  ******** CPU USED =     2.834
```

***** S E L E C T I O N *****

SELECTED VECTORS				MULTIPLIERS			DIRECTION	
FUNCTION	CELL		NAME	CELL		VALUE		
OBJECTIVE FUNCTION	(KNOBJ)	=	OBJECTFCT	(RPSOBJ)	=	-1.000	(KNDIR) =	MAXIMIZE
CHANGE OBJECTIVE FUNCTION	(KNCHOBJ)	=		(RPCHOBJ)	=	0.000		
RIGHT-HAND SIDE	(KNRHS)	=	DEMAND	(RPSRHS)	=	1.000		
CHANGE RIGHT-HAND SIDE	(KNCHRHS)	=		(RPCHRHS)	=	0.000		
BOUNDS SET	(KNBND)	=	BNDS					
RANGES SET	(KNRNG)	=						

```
------ SELECTED MIXED INTEGER VARIABLE SUMMARY ------

    0 ACTIVE BIVALENT VARIABLES
   88 ACTIVE INTEGER VARIABLES
    0 ACTIVE SPECIAL ORDERED SETS
          0 TYPE 1 SETS WITH TOTAL VARIABLES OF      0
          0 TYPE 2 SETS WITH TOTAL VARIABLES OF      0
          0 MEMBERS IN LARGEST SOS SET

BRANCH      *           MAJERR

CRASH                                               ******** CPU USED =     2.991

CRASH RESULTS - REMAINING LOGICALS =  177
                BALANCE STRUCTURALS =   0
                OTHER STRUCTURALS =     0

BRANCH      *           MAJERR

PRIMAL                                              ******** CPU USED =     3.348

OPTIMAL SOLUTION.  OBJECTIVE =     155017.906469

BRANCH      PRIMEND     MAJERR      *           PROBOUT

PRIMEND     BRANCH      *           MAJERR      PROBOUT
```

MIXINT

******** CPU USED = 28.685

NODE NO.	PARENT NODE NO.	ARB. DIR.	STATUS	OBJECTIVE FUNCTION VALUE	SUM OF FRACTIONS	SEPARATION VARIABLE - TYPE	NUMBER	LOWER LIMIT	UPPER LIMIT	RANK	RESOURCE UNITS
1	0		DEVELOPED	155017.906489	14.110					1	28.7
2	1	UP	DEVELOPED	155015.978327	14.244	INTEGER	48	3	16383	2	33.9
3	2	DOWN	DEVELOPED	155012.896487	14.633	INTEGER	26	0	2	3	33.1
4	3	UP	DEVELOPED	155005.344955	14.564	INTEGER	37	2	16383	4	36.8
5	4	UP	DEVELOPED	155003.973724	14.839	INTEGER	59	4	16383	5	39.6
6	5	UP	DEVELOPED	155002.383199	14.260	INTEGER	69	12	16383	6	42.5
7	6	UP	DEVELOPED	155001.434991	13.253	INTEGER	58	11	16383	7	47.1
8	7	DOWN	DEVELOPED	155001.459693	13.227	INTEGER	51	0	75	8	52.1
9	8	DOWN	DEVELOPED	155001.435901	13.288	INTEGER	47	0	14	9	57.6
10	9	DOWN	DEVELOPED	155001.426181	13.311	INTEGER	58	11	11	10	61.9
11	10	UP	DEVELOPED	155001.381492	13.301	INTEGER	46	14	16383	11	64.6
12	11	UP	DEVELOPED	155001.330194	13.285	INTEGER	73	17	16383	12	69.1
13	12	UP	DEVELOPED	155001.266216	13.407	INTEGER	62	24	16383	13	73.8
14	13	UP	DEVELOPED	155000.539027	13.281	INTEGER	84	19	16383	14	78.4
15	14	DOWN	DEVELOPED	154999.637953	12.963	INTEGER	2	0	96	15	81.4
16	15	DOWN	DEVELOPED	154999.466842	13.350	INTEGER	9	0	76	16	85.9
17	16	UP	DEVELOPED	154998.912753	13.695	INTEGER	57	17	16383	17	90.4
18	17	DOWN	DEVELOPED	154998.489759	13.408	INTEGER	74	0	40	18	94.8
19	18	UP	DEVELOPED	154984.733356	14.138	INTEGER	70	4	16383	19	99.5
20	19	DOWN	DEVELOPED	154984.428466	13.824	INTEGER	74	0	39	20	104.2
21	20	UP	DEVELOPED	154984.403325	13.664	INTEGER	47	14	14	21	108.8
22	21	DOWN	DEVELOPED	154978.728589	13.663	INTEGER	80	0	13	22	114.5
23	22	UP	DEVELOPED	154978.406054	14.213	INTEGER	26	2	2	23	119.1
24	23	UP	DEVELOPED	154974.280274	12.926	INTEGER	81	3	16383	24	123.3
25	24	DOWN	DEVELOPED	154959.780558	13.473	INTEGER	15	0	2	25	128.5
26	25	UP	DEVELOPED	154956.731380	13.039	INTEGER	25	0	16383	26	132.6
27	26	DOWN	DEVELOPED	154652.658676	13.023	INTEGER	4	0	2	27	140.5
28	27	DOWN	DEVELOPED	154551.344808	13.677	INTEGER	37	2	2	28	146.5
29	28	DOWN	DEVELOPED	154550.952790	14.919	INTEGER	59	4	4	29	150.8
30	29	DOWN	DEVELOPED	154549.634946	14.264	INTEGER	35	0	13	30	153.7
31	30	UP	DEVELOPED	154549.606799	13.708	INTEGER	40	4	16383	31	160.1
32	31	UP	DEVELOPED	154549.549297	14.871	INTEGER	24	13	16383	32	164.5
33	32	UP	DEVELOPED	154525.308022	12.969	INTEGER	48	4	16383	33	171.2
34	33	DOWN	DEVELOPED	154524.554185	13.344	INTEGER	36	0	9	34	175.1
35	34	DOWN	DEVELOPED	154524.412478	13.360	INTEGER	83	0	4	35	178.2
36	35	DOWN	DEVELOPED	154524.356799	13.331	INTEGER	72	0	4	36	181.3
37	36	UP	DEVELOPED	154521.444472	12.953	INTEGER	13	14	16383	37	185.1
38	37	DOWN	DEVELOPED	154521.300191	12.749	INTEGER	69	12	12	38	190.2
39	38	DOWN	DEVELOPED	154520.862766	12.479	INTEGER	79	0	19	39	194.6
40	39	UP	DEVELOPED	154520.577697	12.640	INTEGER	68	19	16383	40	199.3
41	40	UP	DEVELOPED	154520.208460	12.419	INTEGER	79	19	19	41	203.6
42	41	UP	DEVELOPED	154519.402585	12.585	INTEGER	35	13	13	42	207.9
43	42	UP	DEVELOPED	154514.235831	12.253	INTEGER	2	96	96	43	213.9
44	43	DOWN	DEVELOPED	154513.822391	12.002	INTEGER	51	0	70	44	218.2
45	44	UP	DEVELOPED	154513.642391	11.973	INTEGER	49	23	16383	45	221.7
46	45	UP	DEVELOPED	154513.315984	12.005	INTEGER	50	1	16383	46	224.6
47	46	UP	DEVELOPED	154512.996406	11.979	INTEGER	42	46	16383	47	229.2
48	47	UP	DEVELOPED	154512.895964	12.055	INTEGER	72	4	4	48	231.1
49	48	UP	DEVELOPED	154512.516498	12.327	INTEGER	74	39	39	49	234.9
50	49	UP	DEVELOPED	154512.277014	12.085	INTEGER	82	18	16383	50	238.9
51	50	DOWN	DEVELOPED	154507.579002	11.252	INTEGER	85	0	35	51	242.9
52	51	UP	DEVELOPED	154507.123446	10.960	INTEGER	62	25	16383	52	247.1
53	52	UP	DEVELOPED	154501.055930	11.658	INTEGER	51	70	70	53	254.2
54	53	UP	DEVELOPED	154495.109517	11.026	INTEGER	61	30	16383	54	259.2

55	54	DOWN	DEVELOPED	15449 1.700001	11.203	INTEGER	63	0	24
56	55	UP	DEVELOPED	15449 1.710001	10.609	INTEGER	63	0	0
57	56	DOWN	DEVELOPED	15449 0.021346	10.610	INTEGER	52	0	0
58	57	DOWN	DEVELOPED	15449 0.517050	11.054	INTEGER	41	0	0
59	58	DOWN	DEVELOPED	15449 0.467608	10. 696	INTEGER	30	0	0
60	59	DOWN	DEVELOPED	15449 0.467608	10. 955	INTEGER	70	4	4
61	60	DOWN	DEVELOPED	15449 0.467608	10. 710	INTEGER	62	25	25
62	61	DOWN	DEVELOPED	15449 0.467608	10. 776	INTEGER	73	17	17
63	62	DOWN	DEVELOPED	15449 0.467608	11. 021	INTEGER	60	19	19
64	63	DOWN	DEVELOPED	15449 0.467608	10. 966	INTEGER	57	17	17
65	64	DOWN	DEVELOPED	15449 0.467608	10. 991	INTEGER	61	30	30
66	65	UP	DEVELOPED	15449 0.305765	10. 066	INTEGER	60	0	0
67	66	DOWN	DEVELOPED	15449 0.330993	9.990	INTEGER	65	0	0
68	67	DOWN	DEVELOPED	15449 0.339993	10. 075	INTEGER	60	0	0
69	68	DOWN	DEVELOPED	15449 0.290087	9.980	INTEGER	64	0	0
70	69	UP	DEVELOPED	15449 0.290087	0.991	INTEGER	75	61	61
71	70	DOWN	DEVELOPED	15449 0.239087	9.173	INTEGER	82	10	10
72	71	DOWN	DEVELOPED	15449 0.290087	9.063	INTEGER	81	3	3
73	72	UP	DEVELOPED	15449 0.291269	0.071	INTEGER	43	54	54
74	73	DOWN	DEVELOPED	15449 0.291269	0.071	INTEGER	75	61	61
75	74	DOWN	DEVELOPED	15449 0.407156	8.660	INTEGER	10	0	0
76	75	DOWN	DEVELOPED	15449 0.395459	7.932	INTEGER	21	0	0
77	76	DOWN	DEVELOPED	15449 7.777504	7.251	INTEGER	31	0	0
78	77	DOWN	DEVELOPED	15449 7.777504	7.251	INTEGER	3	0	0
79	78	DOWN	DEVELOPED	15449 7.713921	7.260	INTEGER	14	0	0
80	79	DOWN	DEVELOPED	15449 7.671401	7.251	INTEGER	0	0	0
81	80	DOWN	DEVELOPED	15449 7.559801	7.603	INTEGER	19	0	0
82	81	UP	DEVELOPED	15449 7.099504	7.276	INTEGER	25	10	10
83	82	DOWN	DEVELOPED	15449 6.773326	7.819	INTEGER	36	0	0
84	83	UP	DEVELOPED	15449 6.027271	7.770	INTEGER	25	11	11
85	84	DOWN	DEVELOPED	15449 5.701093	8.244	INTEGER	36	0	0
86	85	UP	DEVELOPED	15449 4.955030	7.854	INTEGER	25	12	12
87	86	DOWN	DEVELOPED	15447 5.100929	7.043	INTEGER	36	0	0
88	87	UP	DEVELOPED	15443 7.205601	8.450	INTEGER	25	13	13
89	88	UP	DEVELOPED	15420 3.153541	8.616	INTEGER	36	6	6
90	89	DOWN	DEVELOPED	15425 6.422542	8.209	INTEGER	80	0	0
91	90	DOWN	DEVELOPED	15425 6.422542	7.940	INTEGER	04	19	19
92	91	DOWN	DEVELOPED	15425 6.225066	7.731	INTEGER	71	0	0
93	92	UP	DEVELOPED	15425 6.041972	7.996	INTEGER	76	55	55
94	93	UP	DEVELOPED	15425 5.910606	8.016	INTEGER	63	24	24
95	94	DOWN	DEVELOPED	15425 5.900606	7. 991	INTEGER	20	0	0
96	95	UP	DEVELOPED	15425 5.899466	8.138	INTEGER	64	55	55
97	96	UP	DEVELOPED	15425 5.714105	7.704	INTEGER	71	9	9
98	97	UP	DEVELOPED	15425 5.220904	7.495	INTEGER	65	54	54
99	98	DOWN	DEVELOPED	15425 5.174099	7.260	INTEGER	43	54	54
100	99	UP	DEVELOPED	15425 5.174099	7.263	INTEGER	06	67	67
101	100	UP	DEVELOPED	15425 5.117926	7.014	INTEGER	21	93	93
102	101	UP	DEVELOPED	15425 5.060170	6.940	INTEGER	42	40	40
103	102	DOWN	DEVELOPED	15425 5.029654	6.710	INTEGER	43	54	54
104	103	DOWN	DEVELOPED	15425 5.029654	7.003	INTEGER	31	0	0
105	104	UP	DEVELOPED	15425 4.972801	6.710	INTEGER	31	10	10
106	105	UP	DEVELOPED	15425 4.945663	6.757	INTEGER	21	94	94
107	106	UP	DEVELOPED	15425 4.814009	6.477	INTEGER	42	49	49
108	107	DOWN	DEVELOPED	15425 4.814009	6.512	INTEGER	43	54	54
109	108	DOWN	DEVELOPED	15425 4.804009	6.617	INTEGER	20	0	0
110	109	UP	DEVELOPED	15425 4.027033	6.197	INTEGER	21	95	95
111	110	DOWN	DEVELOPED	15425 4.027033	6. 336	INTEGER	31	10	10
112	111	UP	DEVELOPED	15425 4.027035	6. 400	INTEGER	42	50	50
113	112	DOWN	DEVELOPED	15425 4.023147	7.609	INTEGER	43	54	54
114	113	UP	DEVELOPED	15425 4.730964	7.226	INTEGER	21	96	96
115	114	DOWN	DEVELOPED	15425 4.602790	7.037	INTEGER	31	10	10
116	115								
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116	115	DOWN	JEVELOPED	15425[illegible].31900	6.750	INTERER	53	0	50	116	442.1
117	116	UP	DEVELOPED	15425[illegible].802997	6.094	INTEGER	54	39	16303	117	443.0
118	117	UP	JEVELOPED	15425[illegible].802997	6.411	INTEGER	53	50	5C	118	445.7
119	118	DOWN	DEVELOPED	15425[illegible].760775	6.379	INTEGER	43	96	1336	119	446.7
120	119	UP	JEVELOPED	15425[illegible].712176	6.577	INTEGER	42	52	16303	120	451.3
121	120	DOWN	JEVELOPED	15425[illegible].623730	5.099	INTEGER	43	54	135	121	453.8
122	121	UP	DEVELOPED	15425[illegible].509660	6.572	INTEGER	42	53	16303	122	455.9
123	122	UP	DEVELOPED	15425[illegible].509660	6.326	INTEGER	07	79	16303	123	458.4
124	123	DOWN	DEVELOPED	15425[illegible].470605	5.940	INTEGER	43	54	134	124	461.0
125	124	UP	DEVELOPED	15425[illegible].457145	6.007	INTEGER	42	94	16303	125	462.2
126	125	UP	JEVELOPED	15425[illegible].457145	5.494	INTEGER	20	310	346	126	466.3
127	126	DOWN	DEVELOPED	15425[illegible].457145	5.306	INTEGER	77	0	05	127	468.7
128	127	UP	DEVELOPED	15425[illegible].457145	5.306	INTEGER	77	02	05	120	470.2
129	128	UP	DEVELOPED	15425[illegible].467145	4.700	INTEGER	11	27	16303	129	471.9
130	129	JP	DEVELOPED	15425[illegible].457145	4.726	INTEGER	1	3327	16303	130	473.3
131	130	DOWN	JEVELOPED	15425[illegible].467145	4.047	INTEGER	1	3327	3337	131	475.1
132	131	DOWN	DEVELOPED	15425[illegible].467145	5.290	INTEGER	11	27	32	132	477.7
133	132	UP	DEVELOPED	15425[illegible].367061	4.903	INTEGER	21	99	116	133	479.9
134	133	UP	DEVELOPED	15425[illegible].344629	4.311	INTEGER	42	55	16303	134	482.3
135	134	DOWN	DEVELOPED	15425[illegible].333640	4.488	INTEGER	43	54	133	135	484.3
136	135	UP	JEVELOPED	15425[illegible].222114	4.431	INTEGER	42	56	16303	136	487.1
137	136	DOWN	DEVELOPED	15425[illegible].180594	4.090	INTEGER	43	54	132	137	489.3
138	137	UP	JEVELOPED	15425[illegible].099590	4.909	INTEGER	42	57	16303	138	491.5
139	138	DOWN	DEVELOPED	15425[illegible].043549	4.936	INTEGER	43	54	131	139	493.9
140	139	UP	DEVELOPED	15425[illegible].377083	4.056	INTEGER	42	58	16303	140	496.1
141	140	DOWN	DEVELOPED	15425[illegible].090504	4.507	INTEGER	43	54	130	141	498.6
142	141	UP	DEVELOPED	15425[illegible].355467	4.908	INTEGER	42	59	16303	142	500.6
143	142	DOWN	JEVELOPED	15425[illegible].733459	4.471	INTEGER	43	54	129	143	502.9
144	143	UP	DEVELOPED	15425[illegible].732052	4.538	INTEGER	42	60	16303	144	505.1
145	144	DOWN	DEVELOPED	15425[illegible].608413	4.279	INTEGER	33	0	128	145	507.7
146	145	DOWN	DEVELOPED	15425[illegible].608413	4.502	INTEGER	44	66	16303	147	512.2
147	146	UP	DEVELOPED	15425[illegible].608413	4.502	INTEGER	44	66	16303	147	512.2
148	147	UP	JEVELOPED	15425[illegible].608413	4.043	INTEGER	31	11	37	148	514.2
149	148	UP	DEVELOPED	15425[illegible].600413	4.213	INTEGER	22	59	16303	149	515.4
150	149	UP	DEVELOPED	15425[illegible].437590	4.800	INTEGER	21	105	118	150	516.6
151	150	UP	DEVELOPED	15425[illegible].407021	4.790	INTEGER	42	62	16303	151	518.7
152	151	DOWN	DEVELOPED	15425[illegible].463360	5.005	INTEGER	43	54	127	152	521.3
153	152	UP	JEVELOPED	15425[illegible].364505	4.342	INTEGER	42	63	16303	153	523.[illegible]
154	153	DOWN	DEVELOPED	15425[illegible].310323	5.025	INTEGER	43	54	126	154	525.3
155	154	UP	JEVELOPED	15425[illegible].241909	5.719	INTEGER	42	64	16303	155	527.3
156	155	DOWN	JEVELOPED	15425[illegible].173270	4.007	INTEGER	43	54	125	156	529.3
157	156	UP	DEVELOPED	15425[illegible].119474	4.696	INTEGER	42	65	16303	157	531.4
158	157	DOWN	DEVELOPED	15425[illegible].020232	5.217	INTEGER	43	54	124	158	534.0
159	158	UP	DEVELOPED	15425[illegible].996950	5.697	INTEGER	42	66	16303	159	536.1
160	159	DOWN	DEVELOPED	15425[illegible].807443	4.005	INTEGER	43	54	123	160	539.7
161	160	UP	DEVELOPED	15425[illegible].807443	4.060	INTEGER	42	67	16303	161	540.0
162	161	UP	JEVELOPED	15425[illegible].772364	4.333	INTEGER	21	110	118	162	542.2
163	162	UP	JEVELOPED	15425[illegible].751927	4.631	INTEGER	42	60	16303	163	544.4
164	163	DOWN	DEVELOPED	15425[illegible].730142	4.701	INTEGER	43	54	122	164	547.0
165	164	UP	DEVELOPED	15425[illegible].629412	4.760	INTEGER	42	69	16303	165	549.9
166	165	DOWN	DEVELOPED	15425[illegible].593096	4.730	INTEGER	43	54	121	166	551.2
167	166	UP	DEVELOPED	15425[illegible].516896	4.915	INTEGER	42	70	16303	167	553.8
168	167	DOWN	DEVELOPED	15425[illegible].460051	4.725	INTEGER	43	54	120	160	555.5
169	168	UP	DEVELOPED	15425[illegible].304301	4.465	INTEGER	42	71	16303	169	558.6
170	169	DOWN	DEVELOPED	15425[illegible].333006	4.672	INTEGER	43	54	119	170	561.2
171	170	UP	DEVELOPED	15425[illegible].261065	4.999	INTEGER	42	72	16303	171	563.3
172	171	DOWN	DEVELOPED	15425[illegible].157961	4.518	INTEGER	43	54	118	172	565.1
173	172	UP	DEVELOPED	15425[illegible].139350	4.065	INTEGER	42	73	16303	173	568.3
174	173	DOWN	DEVELOPED	15425[illegible].012915	4.353	INTEGER	43	54	117	174	571.1
175	174	UP	JEVELOPED	15425[illegible].334310	5.175	INTEGER	42	75	16303	175	573.3
176	175	DOWN	JEVELOPED	15425[illegible].067070	5.320	INTEGER	43	54	116	176	575.5

177	176	UP	DEVELOPED	154250.771803	4.646	INTEGER	42	16383	177.0
178	177	DOWN	JEVELOPED	154250.722825	5.101	INTEGER	43	115	179	503.2
179	178	UP	DEVELOPED	154250.684861	4.975	INTEGER	42	16383	179	503.0
180	179	DOWN	DEVELOPED	154249.647190	4.834	INTEGER	43	114	180	505.7
181	180	DOWN	DEVELOPED	154248.636064	5.207	INTEGER	21	117	181	508.7
182	181	DOWN	JEVELOPED	154248.636064	4.700	INTEGER	31	11	182	593.3
183	182	DOWN	JEVELOPED	154248.622827	5.001	INTEGER	20	310	183	592.3
184	183	DOWN	DEVELOPED	154246.554034	4.991	INTEGER	42	77	184	594.2
185	184	UP	DEVELOPED	154247.630902	5.271	INTEGER	54	16383	185	596.3
186	185	UP	DEVELOPED	154246.736078	4.045	INTEGER	43	114	186	597.6
187	186	DOWN	JEVELOPED	154246.530706	4.046	INTEGER	32	0	187	593.2
188	187	DOWN	JEVELOPED	154246.441957	4.671	INTEGER	54	40	188	591.2
189	188	DOWN	JEVELOPED	154246.357735	4.357	INTEGER	7	8	189	602.3
190	189	DOWN	DEVELOPED	154246.356494	3.986	INTEGER	10	0	190	603.0
191	190	DOWN	DEVELOPED	154246.206407	4.040	INTEGER	25	13	191	606.3
192	191	DOWN	JEVELOPED	154246.294607	4.040	INTEGER	27	8	192	609.2
193	192	DOWN	JEVELOPED	154246.284607	4.040	INTEGER	30	0	193	609.9
194	193	DOWN	DEVELOPED	154245.284607	4.406	INTEGER	49	23	194	611.0
195	194	DOWN	DEVELOPED	154246.206407	4.727	INTEGER	16	0	195	613.3
196	195	DOWN	DEVELOPED	154246.275004	3.993	INTEGER	5	8	196	615.2
197	196	DOWN	JEVELOPED	154246.132227	3.992	INTEGER	29	0	197	616.3
198	197	DOWN	DEVELOPED	154246.175765	3.977	INTEGER	40	4	198	617.0
199	198	DOWN	DEVELOPED	154246.102270	4.674	INTEGER	76	55	199	519.7
200	199	DOWN	JEVELOPED	154246.102270	4.360	INTEGER	6	8	200	623.0
201	200	DOWN	DEVELOPED	154246.102270	3.994	INTEGER	17	0	201	622.2
202	201	DOWN	DEVELOPED	154246.101079	3.994	INTEGER	28	0	202	624.0
203	202	DOWN	JEVELOPED	154246.099879	3.994	INTEGER	39	0	203	625.1
204	203	DOWN	JEVELOPED	154246.003027	3.906	INTEGER	50	1	204	626.4
205	204	DOWN	DEVELOPED	154246.003027	4.009	INTEGER	40	4	205	628.1
206	205	DOWN	DEVELOPED	154245.379435	4.376	INTEGER	24	13	206	629.1
207	206	DOWN	DEVELOPED	154245.451330	4.009	INTEGER	13	14	207	630.0
208	207	DOWN	JEVELOPED	154246.003027	3.029	INTEGER	46	14	208	631.8
209	208	UP	DEVELOPED	154246.003027	2.590	INTEGER	33	64	209	632.1
210	209	DOWN	DEVELOPED	154246.003027	2.590	INTEGER	67	0	210	633.7
211	210	DOWN	DEVELOPED	154246.003027	2.590	INTEGER	56	0	211	634.6
212	211	UP	DEVELOPED	154246.003027	2.451	INTEGER	70	554	212	635.4
213	212	UP	DEVELOPED	154246.003027	3.208	INTEGER	44	16383	213	635.2
214	213	DOWN	DEVELOPED	154246.003027	2.374	INTEGER	44	67	214	637.3
215	214	UP	DEVELOPED	154246.003027	2.254	INTEGER	00	91	215	638.0
216	215	UP	DEVELOPED	154246.003027	2.254	INTEGER	66	02	216	639.7
217	216	UP	JEVELOPED	154246.003027	2.616	INTEGER	1	3329	217	641.2
218	217	DOWN	JEVELOPED	154246.003027	2.733	INTEGER	11	27	210	642.0
219	218	UP	DEVELOPED	154246.003027	2.733	INTEGER	33	64	219	643.6
220	219	UP	DEVELOPED	154246.003027	2.075	INTEGER	22	63	220	644.4
221	220	DOWN	DEVELOPED	154246.003027	2.110	INTEGER	22	63	221	645.4
222	221	UP	DEVELOPED	154246.003027	2.009	INTEGER	55	00	222	645.9
223	222	UP	DEVELOPED	154246.003027	2.523	INTEGER	12	505	223	647.5
224	223	DOWN	DEVELOPED	154246.003027	2.523	INTEGER	12	505	224	648.2
225	224	UP	JEVELOPED	154246.003027	2.470	INTEGER	23	374	16303	225	549.3
226	225	UP	JEVELOPED	154246.003027	1.079	INTEGER	12	595	605	226	649.9
227	226	UP	DEVELOPED	154246.003027	2.200	INTEGER	22	64	69	227	651.3
228	227	UP	DEVELOPED	154246.003027	1.079	INTEGER	23	376	16303	228	652.5
229	228	UP	DEVELOPED	154246.003027	1.665	INTEGER	22	65	69	229	654.7
230	229	DOWN	DEVELOPED	154246.003027	1.665	INTEGER	1	3329	3335	230	655.1
231	230	DOWN	JEVELOPED	154246.003027	1.355	INTEGER	12	595	601	231	655.7
232	231	DOWN	DEVELOPED	154246.003027	1.413	INTEGER	56	0	540	232	657.3
233	232	UP	DEVELOPED	154245.451330	2.035	INTEGER	66	03	16303	233	658.5
234	233	UP	DEVELOPED	154245.379435	2.020	INTEGER	21	117	234	659.9
235	234	DOWN	JEVELOPED	154230.804924	2.337	INTEGER	10	0	7	661.7
236	235	UP	JEVELOPED	154230.034924	2.113	INTEGER	07	00	16303	236	663.1
237	236	UP	DEVELOPED	154230.004924	2.473	INTEGER	04	20	20	237	664.0

238	237	UP	DEVELOPED	154230.804924	1.812	INTEGER	82	15	15	238	663.1
239	238	UP	DEVELOPED	154230.804924	1.730	INTEGER	23	394	16383	239	666.5
240	239	UP	DEVELOPED	154230.804924	1.478	INTEGER	23	408	16383	240	667.1
241	240	DOWN	DEVELOPED	154230.804924	1.478	INTEGER	22	65	68	241	668.2
242	241	DOWN	DEVELOPED	154230.804924	1.798	INTEGER	55	80	84	242	669.6
243	242	DOWN	DEVELOPED	154230.804924	1.726	INTEGER	86	67	67	243	670.2
244	243	DOWN	DEVELOPED	154230.804924	1.478	INTEGER	87	80	80	244	670.8
245	244	UP	DEVELOPED	154230.804924	1.478	INTEGER	34	486	16383	245	671.4
246	245	UP	DEVELOPED	154230.804924	1.969	INTEGER	34	529	16383	246	672.4
247	246	DOWN	DEVELOPED	154230.804924	1.969	INTEGER	22	65	67	247	673.0
248	247	DOWN	DEVELOPED	154230.804924	1.478	INTEGER	44	67	69	248	674.4
249	248	DOWN	DEVELOPED	154230.804924	1.478	INTEGER	80	91	95	249	675.8
250	249	UP	DEVELOPED	154230.804924	1.124	INTEGER	45	433	16383	250	675.5
251	250	DOWN	DEVELOPED	154230.814924	.788	INTEGER	45	433	456	251	675.1
252	251	UP	DEVELOPED	154230.804924	.589	INTEGER	56	547	548	252	676.9
253	252	UP	DEVELOPED	154230.804924	.589	INTEGER	67	597	599	253	678.2
254	253	UP	DEVELOPED	154230.804924	.019	INTEGER	67	598	599	254	678.6
255	254	UP	DEVELOPED	154230.804924	.019	INTEGER	88	93	95	255	679.1
256	255	UP	DEVELOPED	154230.814924	.019	INTEGER	77	83	85	256	679.9
257	256	UP	DEVELOPED	154230.791136	.451	INTEGER	88	95	95	257	681.2
258	257	UP	DEVELOPED	154230.733088	.452	INTEGER	85	35	35	258	681.3
259	258	UP	DEVELOPED	154230.646904	.199	INTEGER	80	12	12	259	682.7
260	259	DOWN	DEVELOPED	154230.646904	.199	INTEGER	67	598	598	260	683.9
261	260	DOWN	DEVELOPED	154230.646904	.199	INTEGER	23	408	408	261	684.4
262	261	DOWN	DEVELOPED	154230.646904	.199	INTEGER	34	529	529	262	685.3
263	262	DOWN	DEVELOPED	154230.646904	.199	INTEGER	45	433	433	263	686.5
264	263	DOWN	DEVELOPED	154230.646904	.199	INTEGER	56	547	547	264	687.1
265	264	DOWN	DEVELOPED	154230.646904	.199	INTEGER	78	631	631	265	687.9
266	265	DOWN	DEVELOPED	154230.646904	.199	INTEGER	1	3329	3329	266	689.1
267	266	DOWN	INTEGER	154230.522230	0.000	INTEGER	12	595	595	267	689.5

SEARCH FOR OPTIMAL INTEGER SOLUTION TERMINATED DUE TO RPOPTCR/RPOPTCA STOPPING CRITERIA
INTEGER SOLUTION FOUND AT NODE 267, WITH FUNCTIONAL VALUE 154230.522230,
IS WITHIN 766.930 UNITS (.49 PER CENT) OF OPTIMUM

BRANCH • MAJERR PROBSAV • PROBSAV

OUTPUT

******** CPU USED = 689.994

PRINT OPTION = COMPLETE OUTPUT
NAME = ALLOCATION OBJ = OBJECTFCT RHS = DEMAND BND = BNDS
DIR = MAXIMIZE COBJ = CRHS = RNG =

VALUE OF OBJECTIVE = 154230.52223
RPSOBJ = -1.0000 RPSRHS = 1.0000
RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	ROW ACTIVITY	SLACK	RHS LOWER	RHS UPPER	MARGINAL
1	OBJECTFCT	FR	SLACK	154230.52223	-154230.52223	-INF	+INF	.
2	IMPORT01Y1	LE	BINDING	2145.00000	.	-INF	2145.00000	-.71804
3	IMPORT02Y1	LE	SLACK	5025.41489	4.58511	-INF	5030.00000	.
4	IMPORT03Y1	LE	BINDING	4055.00000	.	-INF	4055.00000	-.72491
5	IMPORT04Y1	EQ	BINDING	216.00000	.	216.00000	216.00000	12.55282
6	IMPORT05Y1	LE	SLACK	-39.54734	233.54734	-INF	194.00000	.
7	IMPORT06Y1	LE	SLACK	-69.25845	193.25845	-INF	124.00000	.
8	IMPORT07Y1	LE	SLACK	-3.02484	693.02484	-INF	690.00000	.
9	IMPORT08Y1	LE	SLACK	-156.29129	1170.29129	-INF	1014.00000	.
10	IMPORT09Y1	LE	SLACK	1054.98203	1196.01797	-INF	2251.00000	.
11	IMPORT10Y1	LE	SLACK	162.74370	602.25630	-INF	765.00000	.
12	IMPORT11Y1	LE	BINDING	1954.00000	.	-INF	1954.00000	-.56512
13	UNUSED01Y1	LE	SLACK	3309.95445	2.04555	-INF	3312.00000	.
14	UNUSED02Y1	LE	BINDING	3565.00000	.	-INF	3565.00000	-.61368
15	UNUSED03Y1	LE	SLACK	4633.87190	48.12810	-INF	4579.00000	.
16	UNUSED04Y1	LE	BINDING	369.00000	.	-INF	369.00000	-11.96586
17	UNUSED05Y1	LE	SLACK	.	185.00000	-INF	185.00000	.
18	UNUSED06Y1	LE	SLACK	292.93704	17.06296	-INF	310.00000	.
19	UNUSED07Y1	LE	BINDING	1256.00000	.	-INF	1256.00000	-.34398
20	UNUSED08Y1	LE	BINDING	493.00000	.	-INF	493.00000	-.30734
21	UNUSED09Y1	LE	BINDING	1443.00000	.	-INF	1443.00000	-.38340
22	UNUSED10Y1	LE	BINDING	498.00000	.	-INF	498.00000	-.36866
23	UNUSED11Y1	LE	SLACK	2041.88710	1.11290	-INF	2043.00000	.
24	MONEYLEFT1	LE	SLACK	5035.53200	14.46800	-INF	5050.00000	.
25	IMPORT01Y2	LE	BINDING	2255.00000	.	-INF	2255.00000	-.71492
26	IMPORT02Y2	LE	BINDING	5200.00000	.	-INF	5200.00000	-.71186
27	IMPORT03Y2	LE	SLACK	4067.56845	448.43155	-INF	4516.00000	.
28	IMPORT04Y2	EQ	BINDING	230.00000	.	230.00000	230.00000	11.87063
29	IMPORT05Y2	LE	SLACK	-39.92810	254.92810	-INF	215.00000	.
30	IMPORT06Y2	LE	SLACK	-135.50141	266.50141	-INF	131.00000	.
31	IMPORT07Y2	LE	SLACK	-65.47029	825.47029	-INF	760.00000	.
32	IMPORT08Y2	LE	SLACK	-295.50158	1384.50158	-INF	1089.00000	.
33	IMPORT09Y2	LE	BINDING	2415.00000	.	-INF	2415.00000	-.73555
34	IMPORT10Y2	LE	SLACK	533.58544	294.41456	-INF	828.00000	.
35	IMPORT11Y2	LE	BINDING	2063.00000	.	-INF	2063.00000	-.54429
36	UNUSED01Y2	LE	SLACK	3303.50189	8.49811	-INF	3312.00000	.
37	UNUSED02Y2	LE	SLACK	3556.25795	8.74205	-INF	3565.00000	.
38	UNUSED03Y2	LE	BINDING	4673.00000	.	-INF	4673.00000	-.68122
39	UNUSED04Y2	LE	BINDING	369.00000	.	-INF	369.00000	-11.32843
40	UNUSED05Y2	LE	SLACK	.	185.00000	-INF	185.00000	.
41	UNUSED06Y2	LE	SLACK	229.60260	80.39740	-INF	310.00000	.
42	UNUSED07Y2	LE	BINDING	1256.00000	.	-INF	1256.00000	-.33789
43	UNUSED08Y2	LE	BINDING	493.00000	.	-INF	493.00000	-.35874
44	UNUSED09Y2	LE	SLACK	1439.13301	4.86699	-INF	1443.00000	.
45	UNUSED10Y2	LE	BINDING	498.00000	.	-INF	498.00000	-.38108
46	UNUSED11Y2	LE	SLACK	2037.45057	5.54943	-INF	2043.00000	.
47	MONEYLEFT2	LE	SLACK	10086.45000	13.55000	-INF	10100.00000	.
48	IMPORT01Y3	LE	BINDING	2371.00000	.	-INF	2371.00000	-.71568
49	IMPORT02Y3	LE	BINDING	5543.00000	.	-INF	5543.00000	-.71187
50	IMPORT03Y3	LE	SLACK	4968.95652	62.04348	-INF	5031.00000	.

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VALUE OF OBJECTIVE = 154230.52223
RPSOBJ = -1.0000 RPSRHS = 1.0000
RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	ROW ACTIVITY	SLACK	RHS LOWER	RHS UPPER	MARGINAL
51	IMPORT04Y3	EQ	BINDING	246.00000	.	246.00000	246.00000	11.70298
52	IMPORT05Y3	LE	SLACK	47.44484	191.55516	-INF	239.00000	.
53	IMPORT06Y3	LE	SLACK	-100.59746	239.59746	-INF	139.00000	.
54	IMPORT07Y3	LE	SLACK	-157.19725	998.19725	-INF	841.00000	.
55	IMPORT08Y3	LE	SLACK	-323.73799	1500.73799	-INF	1171.00000	.
56	IMPORT09Y3	LE	SLACK	2463.12937	129.87063	-INF	2593.00000	.
57	IMPORT10Y3	LE	SLACK	516.63976	381.36024	-INF	898.00000	.
58	IMPORT11Y3	LE	BINDING	2177.00000	.	-INF	2177.00000	-.53969
59	UNUSED01Y3	LE	SLACK	3292.72355	19.27645	-INF	3312.00000	.
60	UNUSED02Y3	LE	SLACK	3540.75527	24.24473	-INF	3565.00000	.
61	UNUSED03Y3	LE	BINDING	4679.00000	.	-INF	4679.00000	-.68173
62	UNUSED04Y3	LE	BINDING	369.00000	.	-INF	369.00000	-11.17884
63	UNUSED05Y3	LE	SLACK	92.88455	92.11545	-INF	185.00000	.
64	UNUSED06Y3	LE	BINDING	310.00000	.	-INF	310.00000	-.00690
65	UNUSED07Y3	LE	BINDING	1255.00000	.	-INF	1256.00000	-.33808
66	UNUSED08Y3	LE	BINDING	493.00000	.	-INF	493.00000	-.35942
67	UNUSED09Y3	LE	BINDING	1443.00000	.	-INF	1443.00000	-.39053
68	UNUSED10Y3	LE	BINDING	498.00000	.	-INF	498.00000	-.37773
69	UNUSED11Y3	LE	SLACK	2036.98680	6.01320	-INF	2043.00000	.
70	MONEYLEFT3	LE	SLACK	15146.73700	3.26300	-INF	15150.00000	.
71	IMPORT01Y4	LE	BINDING	2494.00000	.	-INF	2434.00000	-.71603
72	IMPORT02Y4	LE	BINDING	5021.00000	.	-INF	5321.00000	-.71386
73	IMPORT03Y4	LE	SLACK	5371.12981	234.87019	-INF	5606.00000	.
74	IMPORT04Y4	EQ	BINDING	263.00000	.	263.00000	263.00000	11.70364
75	IMPORT05Y4	LE	SLACK	7.45358	257.54642	-INF	265.00000	.
76	IMPORT06Y4	LE	SLACK	-124.50508	270.50508	-INF	146.00000	.
77	IMPORT07Y4	LE	SLACK	-155.85706	1091.85706	-INF	936.00000	.
78	IMPORT08Y4	LE	SLACK	-404.61350	1664.61350	-INF	1260.00000	.
79	IMPORT09Y4	LE	BINDING	2787.00000	.	-INF	2787.00000	-.73661
80	IMPORT10Y4	LE	SLACK	887.98119	88.01881	-INF	975.00000	.
81	IMPORT11Y4	LE	BINDING	2297.00000	.	-INF	2297.00000	-.54475
82	UNUSED01Y4	LE	SLACK	3277.65833	34.34167	-INF	3312.00000	.
83	UNUSED02Y4	LE	SLACK	3555.06117	9.93883	-INF	3565.00000	.
84	UNUSED03Y4	LE	BINDING	4679.00000	.	-INF	4679.00000	-.68179
85	UNUSED04Y4	LE	BINDING	369.00000	.	-INF	369.00000	-11.17959
86	UNUSED05Y4	LE	SLACK	55.54551	129.45449	-INF	185.00000	.
87	UNUSED06Y4	LE	BINDING	310.00000	.	-INF	310.00000	-.00690
88	UNUSED07Y4	LE	BINDING	1255.00000	.	-INF	1256.00000	-.33900
89	UNUSED08Y4	LE	BINDING	493.00000	.	-INF	493.00000	-.36033
90	UNUSED09Y4	LE	SLACK	1440.20441	2.79559	-INF	1443.00000	.
91	UNUSED10Y4	LE	BINDING	498.00000	.	-INF	498.00000	-.38315
92	UNUSED11Y4	LE	SLACK	2034.99931	8.00069	-INF	2043.00000	.
93	MONEYLEFT4	LE	SLACK	20197.82800	2.17200	-INF	20200.00000	.
94	IMPORT01Y5	LE	BINDING	2622.00000	.	-INF	2522.00000	-.87539
95	IMPORT02Y5	LE	BINDING	6113.00000	.	-INF	5113.00000	-.87681
96	IMPORT03Y5	LE	BINDING	6243.00000	.	-INF	6243.00000	-.77748
97	IMPORT04Y5	EQ	BINDING	280.00000	.	280.00000	280.00000	-.87459
98	IMPORT05Y5	LE	BINDING	294.00000	.	-INF	294.00000	-.55352
99	IMPORT06Y5	LE	SLACK	-152.17767	306.17767	-INF	154.00000	.
100	IMPORT07Y5	LE	SLACK	893.61876	152.38124	-INF	1046.00000	.

PRINT OPTION = COMPLETE OUTPUT VALUE OF OBJECTIVE = 154230.52223
NAME = ALLOCATION OBJ = OBJECTFCT RHS = DEMAND BND = BNDS RPSOBJ = -1.0000 RPSRHS = 1.0000
DIR = MAXIMIZE COBJ = CRHS = RNG = RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	ROW ACTIVITY	SLACK	RHS LOWER	RHS UPPER	MARGINAL
101	IMPORT08Y5	LE	SLACK	-519.82868	1876.82868	-INF	1357.00000	.
102	IMPORT09Y5	LE	SLACK	2997.53509	.46491	-INF	2998.00000	.
103	IMPORT10Y5	LE	SLACK	974.80544	89.19456	-INF	1054.00000	.
104	IMPORT11Y5	LE	BINDING	2424.00000	.	-INF	2424.00000	-.60374
105	UNUSED01Y5	LE	SLACK	3311.57301	.42699	-INF	3312.00000	.
106	UNUSED02Y5	LE	SLACK	3554.75483	10.24517	-INF	3565.00000	.
107	UNUSED03Y5	LE	SLACK	4625.59035	53.40965	-INF	4679.00000	.
108	UNUSED04Y5	LE	SLACK	361.42211	7.57789	-INF	369.00000	.
109	UNUSED05Y5	LE	SLACK	184.14445	.85555	-INF	185.00000	.
110	UNUSED06Y5	LE	BINDING	310.00000	.	-INF	310.00000	-.55681
111	UNUSED07Y5	LE	BINDING	1256.00000	.	-INF	1256.00000	-.43373
112	UNUSED08Y5	LE	BINDING	493.00000	.	-INF	493.00000	-.50075
113	UNUSED09Y5	LE	BINDING	1443.00000	.	-INF	1443.00000	-.43802
114	UNUSED10Y5	LE	BINDING	493.00000	.	-INF	498.00000	-.54034
115	UNUSED11Y5	LE	SLACK	2036.80478	6.19522	-INF	2043.00000	.
116	MONEYLEFT5	LE	SLACK	32999.29800	10.70200	-INF	33010.00000	.
117	IMPORT01Y6	LE	BINDING	2758.00000	.	-INF	2758.00000	-.88313
118	IMPORT02Y6	LE	BINDING	6420.00000	.	-INF	5420.00000	-.89268
119	IMPORT03Y6	LE	BINDING	6965.00000	.	-INF	6965.00000	-.83832
120	IMPORT04Y6	EQ	BINDING	299.00000	.	299.00000	299.00000	-.94005
121	IMPORT05Y6	LE	BINDING	326.00000	.	-INF	326.00000	-.64525
122	IMPORT06Y6	LE	BINDING	163.00000	.	-INF	163.00000	-.79643
123	IMPORT07Y6	LE	BINDING	1176.00000	.	-INF	1176.00000	-.80694
124	IMPORT08Y6	LE	SLACK	-201.94257	1666.94257	-INF	1465.00000	.
125	IMPORT09Y6	LE	SLACK	3228.48707	.51293	-INF	3229.00000	.
126	IMPORT10Y6	LE	SLACK	1091.08882	71.91118	-INF	1163.00000	.
127	IMPORT11Y6	LE	BINDING	2558.00000	.	-INF	2558.00000	-.78891
128	UNUSED01Y6	LE	SLACK	3311.87797	.12203	-INF	3312.00000	.
129	UNUSED02Y6	LE	SLACK	3548.35409	16.64591	-INF	3565.00000	.
130	UNUSED03Y6	LE	SLACK	4621.13585	57.86415	-INF	4679.00000	.
131	UNUSED04Y6	LE	SLACK	362.60933	6.39067	-INF	369.00000	.
132	UNUSED05Y6	LE	SLACK	180.98396	4.01604	-INF	185.00000	.
133	UNUSED06Y6	LE	SLACK	305.21857	4.78143	-INF	310.00000	.
134	UNUSED07Y6	LE	SLACK	1248.54283	7.45717	-INF	1256.00000	.
135	UNUSED08Y6	LE	BINDING	493.00000	.	-INF	493.00000	-.53040
136	UNUSED09Y6	LE	BINDING	1443.00000	.	-INF	1443.00000	-.44762
137	UNUSED10Y6	LE	BINDING	493.00000	.	-INF	498.00000	-.56991
138	UNUSED11Y6	LE	SLACK	2033.81650	9.18350	-INF	2043.00000	.
139	MONEYLEFT6	LE	SLACK	45809.80700	10.19300	-INF	45820.00000	.
140	IMPORT01Y7	LE	BINDING	2901.00000	.	-INF	2901.00000	-.88439
141	IMPORT02Y7	LE	BINDING	6743.00000	.	-INF	5743.00000	-.89494
142	IMPORT03Y7	LE	BINDING	7767.00000	.	-INF	7767.00000	-.83465
143	IMPORT04Y7	EQ	BINDING	319.00000	.	319.00000	319.00000	-.94023
144	IMPORT05Y7	LE	SLACK	361.20999	.79001	-INF	362.00000	.
145	IMPORT06Y7	LE	BINDING	172.00000	.	-INF	172.00000	-.79642
146	IMPORT07Y7	LE	BINDING	1330.00000	.	-INF	1330.00000	-.80879
147	IMPORT08Y7	LE	SLACK	363.02975	1220.97025	-INF	1584.00000	.
148	IMPORT09Y7	LE	BINDING	3484.00000	.	-INF	3484.00000	-.84428
149	IMPORT10Y7	LE	SLACK	1205.29836	70.70164	-INF	1276.00000	.
150	IMPORT11Y7	LE	BINDING	2699.00000	.	-INF	2699.00000	-.79481

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RPSOBJ = -1.0000 RPSRHS = 1.0000
RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	ROW ACTIVITY	SLACK	RHS LOWER	RHS UPPER	MARGINAL
151	UNUSED01Y7	LE	SLACK	3311.52868	.47132	-INF	3312.00000	.
152	UNUSED02Y7	LE	SLACK	3542.82986	22.17014	-INF	3565.00000	.
153	UNUSED03Y7	LE	SLACK	4615.29500	63.70500	-INF	4679.00000	.
154	UNUSED04Y7	LE	SLACK	350.23162	18.76838	-INF	369.00000	.
155	UNUSED05Y7	LE	BINDING	185.00000	.	-INF	185.00000	-.64535
156	UNUSED06Y7	LE	SLACK	303.38328	6.61672	-INF	310.00000	.
157	UNUSED07Y7	LE	SLACK	1245.69743	10.30257	-INF	1256.00000	.
158	UNUSED08Y7	LE	BINDING	493.00000	.	-INF	493.00000	-.51380
159	UNUSED09Y7	LE	SLACK	1441.21200	1.78800	-INF	1443.00000	.
160	UNUSED10Y7	LE	BINDING	498.00000	.	-INF	498.00000	-.57603
161	UNUSED11Y7	LE	SLACK	2038.13148	4.86852	-INF	2043.00000	.
162	MONEYLEFT7	LE	SLACK	58620.83500	9.16500	-INF	58630.00000	.
163	IMPORT01Y8	LE	SLACK	3050.67581	.32419	-INF	3051.00000	.
164	IMPORT02Y8	LE	BINDING	7093.00000	.	-INF	7093.00000	-.34799
165	IMPORT03Y8	LE	SLACK	8631.34093	31.65907	-INF	8663.00000	.
166	IMPORT04Y8	EQ	BINDING	341.00000	.	341.00000	341.00000	-.90721
167	IMPORT05Y8	LE	BINDING	401.00000	.	-INF	401.00000	-.62814
168	IMPORT06Y8	LE	BINDING	181.00000	.	-INF	181.00000	-.77546
169	IMPORT07Y8	LE	BINDING	1513.00000	.	-INF	1513.00000	-.79867
170	IMPORT08Y8	LE	SLACK	850.49543	864.50457	-INF	1715.00000	.
171	IMPORT09Y8	LE	BINDING	3764.00000	.	-INF	3764.00000	-.77476
172	IMPORT10Y8	LE	BINDING	1406.00000	.	-INF	1406.00000	-.69483
173	IMPORT11Y8	LE	BINDING	2849.00000	.	-INF	2849.00000	-.77331
174	UNUSED01Y8	LE	BINDING	3312.00000	.	-INF	3312.00000	-.79661
175	UNUSED02Y8	LE	SLACK	3550.49759	14.50241	-INF	3565.00000	.
176	UNUSED03Y8	LE	BINDING	4679.00000	.	-INF	4679.00000	-.78729
177	UNUSED04Y8	LE	SLACK	359.93406	9.06594	-INF	369.00000	.
178	UNUSED05Y8	LE	SLACK	148.48148	36.51852	-INF	185.00000	.
179	UNUSED06Y8	LE	SLACK	305.01017	4.98983	-INF	310.00000	.
180	UNUSED07Y8	LE	SLACK	1216.49122	39.50878	-INF	1256.00000	.
181	UNUSED08Y8	LE	BINDING	493.00000	.	-INF	493.00000	-.51772
182	UNUSED09Y8	LE	SLACK	1438.69979	4.30021	-INF	1443.00000	.
183	UNUSED10Y8	LE	SLACK	496.78350	1.21650	-INF	498.00000	.
184	UNUSED11Y8	LE	SLACK	2028.33505	14.66495	-INF	2043.00000	.
185	MONEYLEFT8	LE	SLACK	71439.91000	.09000	-INF	71440.00000	.

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RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	COL ACTIVITY	OBJ COEF	BND LOWER	BND UPPER	MARGINAL	
1	NBR01Y1	INT	LOWER	3329.00000	.	.	16383.00000	.62614	ARB INT
2	NBR02Y1	INT	LOWER	96.00000	.	.	16383.00000	15.35972	ARB INT
3	NBR03Y1	INT	LOWER	.	.	.	16383.00000	214.40970	ARB INT
4	NBR04Y1	INT	UPPER	2.00000	.	.	16383.00000	811.34005	
5	NBR05Y1	INT	LOWER	.	.	.	16383.00000	4.59098	ARB INT
6	NBR06Y1	INT	LOWER	.	.	.	16383.00000	8.86382	ARB INT
7	NBR07Y1	INT	LOWER	.	.	.	16383.00000	55.24676	ARB INT
8	NBR08Y1	INT	LOWER	.	.	.	16383.00000	89.47164	ARB INT
9	NBR09Y1	INT	UPPER	75.00000	.	.	16383.00000	13.46915	
10	NBR10Y1	INT	UPPER	7.00000	.	.	16383.00000	15.19810	
11	NBR11Y1	INT **	LOWER	27.00000	.	.	16383.00000	.	
12	NBR01Y2	INT	LOWER	595.00000	.	.	16383.00000	.62614	ARB INT
13	NBR02Y2	INT **	LOWER	14.00000	.	.	16383.00000	.	ARB INT
14	NBR03Y2	INT	LOWER	.	.	.	16383.00000	214.40970	ARB INT
15	NBR04Y2	INT	UPPER	2.00000	.	.	16383.00000	598.68283	
16	NBR05Y2	INT	LOWER	.	.	.	16383.00000	4.59098	ARB INT
17	NBR06Y2	INT	LOWER	.	.	.	16383.00000	8.86382	ARB INT
18	NBR07Y2	INT	LOWER	.	.	.	16383.00000	44.64596	ARB INT
19	NBR08Y2	INT	LOWER	.	.	.	16383.00000	81.49765	ARB INT
20	NBR09Y2	INT	LOWER	318.00000	.	.	16383.00000	10.35744	ARB INT
21	NBR10Y2	INT	LOWER	117.00000	.	.	16383.00000	13.44549	ARB INT
22	NBR11Y2	INT **	LOWER	65.00000	.	.	16383.00000	.	
23	NBR01Y3	INT	LOWER	408.00000	.	.	16383.00000	.62614	ARB INT
24	NBR02Y3	INT **	LOWER	13.00000	.	.	16383.00000	.	ARB INT
25	NBR03Y3	INT	LOWER	13.00000	.	.	16383.00000	162.03555	ARB INT
26	NBR04Y3	INT	LOWER	2.00000	.	.	16383.00000	397.35400	ARB INT
27	NBR05Y3	INT	LOWER	.	.	.	16383.00000	4.59098	ARB INT
28	NBR06Y3	INT	LOWER	.	.	.	16383.00000	8.86382	ARB INT
29	NBR07Y3	INT	LOWER	.	.	.	16383.00000	34.23204	ARB INT
30	NBR08Y3	INT	LOWER	.	.	.	16383.00000	72.19022	ARB INT
31	NBR09Y3	INT	LOWER	11.00000	.	.	16383.00000	10.35744	ARB INT
32	NBR10Y3	INT	LOWER	.	.	.	16383.00000	11.63385	ARB INT
33	NBR11Y3	INT **	LOWER	64.00000	.	.	16383.00000	.	ARB INT
34	NBR01Y4	INT	LOWER	529.00000	.	.	16383.00000	.62614	ARB INT
35	NBR02Y4	INT **	LOWER	13.00000	.	.	16383.00000	.	ARB INT
36	NBR03Y4	INT	LOWER	6.00000	.	.	16383.00000	111.22277	ARB INT
37	NBR04Y4	INT	LOWER	2.00000	.	.	16383.00000	198.68366	ARB INT
38	NBR05Y4	INT	LOWER	.	.	.	16383.00000	4.59098	ARB INT
39	NBR06Y4	INT	LOWER	.	.	.	16383.00000	8.75668	ARB INT
40	NBR07Y4	INT	LOWER	4.00000	.	.	16383.00000	23.81396	ARB INT
41	NBR08Y4	INT	LOWER	.	.	.	16383.00000	62.86508	ARB INT
42	NBR09Y4	INT	LOWER	77.00000	.	.	16383.00000	7.10788	ARB INT
43	NBR10Y4	INT	LOWER	114.00000	.	.	16383.00000	9.83812	ARB INT
44	NBR11Y4	INT **	LOWER	67.00000	.	.	16383.00000	.	
45	NBR01Y5	INT	LOWER	433.00000	.	.	16383.00000	.62614	ARB INT
46	NBR02Y5	INT **	LOWER	14.00000	.	.	16383.00000	.	ARB INT
47	NBR03Y5	INT	LOWER	14.00000	.	.	16383.00000	59.60518	ARB INT
48	NBR04Y5	INT **	LOWER	4.00000	.	.	16383.00000	.	ARB INT
49	NBR05Y5	INT	LOWER	23.00000	.	.	16383.00000	4.59098	ARB INT
50	NBR06Y5	INT	LOWER	1.00000	.	.	16383.00000	8.64950	ARB INT

PRINT OPTION = COMPLETE OUTPUT
NAME = ALLOCATION OBJ = OBJECTIVE RHS = DEMAND BND = BNDS
DIR = MAXIMIZE COBJ = CRHS = RNG =

VALUE OF OBJECTIVE = 154230.52223
RPSOBJ = -1.0000 RPSRHS = 1.0000
RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	COL ACTIVITY	OBJ COEF	BND LOWER	BND UPPER	MARGINAL	
51	NBR07Y5	INT	LOWER	70.00000	.	.	16383.00000	13.36672	ARB INT
52	NBR08Y5	INT	LOWER	.	.	.	16383.00000	53.51622	ARB INT
53	NBR09Y5	INT	LOWER	50.00000	.	.	16383.00000	7.10788	ARB INT
54	NBR10Y5	INT	LOWER	40.00000	.	.	16383.00000	8.01661	ARB INT
55	NBR11Y5	INT	** LOWER	80.00000	.	.	16383.00000	.	
56	NBR01Y6	INT	LOWER	547.00000	.	.	16383.00000	.62614	ARB INT
57	NBR02Y6	INT	** LOWER	17.00000	.	.	16383.00000	.	ARB INT
58	NBR03Y6	INT	LOWER	11.00000	.	.	16383.00000	59.60518	ARB INT
59	NBR04Y6	INT	** LOWER	4.00000	.	.	16383.00000	.	ARB INT
60	NBR05Y6	INT	LOWER	8.00000	.	.	16383.00000	4.59098	ARB INT
61	NBR06Y6	INT	** LOWER	30.00000	.	.	16383.00000	.	ARB INT
62	NBR07Y6	INT	** LOWER	25.00000	.	.	16383.00000	.	ARB INT
63	NBR08Y6	INT	LOWER	24.00000	.	.	16383.00000	40.52415	ARB INT
64	NBR09Y6	INT	LOWER	55.00000	.	.	16383.00000	3.63287	ARB INT
65	NBR10Y6	INT	LOWER	54.00000	.	.	16383.00000	5.44781	ARB INT
66	NBR11Y6	INT	** LOWER	83.00000	.	.	16383.00000	.	
67	NBR01Y7	INT	LOWER	598.00000	.	.	16383.00000	.62614	ARB INT
68	NBR02Y7	INT	** LOWER	19.00000	.	.	16383.00000	.	ARB INT
69	NBR03Y7	INT	LOWER	12.00000	.	.	16383.00000	59.60518	ARB INT
70	NBR04Y7	INT	** LOWER	4.00000	.	.	16383.00000	.	ARB INT
71	NBR05Y7	INT	LOWER	9.00000	.	.	16383.00000	4.59098	ARB INT
72	NBR06Y7	INT	** LOWER	4.00000	.	.	16383.00000	.	ARB INT
73	NBR07Y7	INT	** LOWER	17.00000	.	.	16383.00000	.	ARB INT
74	NBR08Y7	INT	LOWER	39.00000	.	.	16383.00000	26.76261	ARB INT
75	NBR09Y7	INT	** LOWER	61.00000	.	.	16383.00000	.	ARB INT
76	NBR10Y7	INT	LOWER	55.00000	.	.	16383.00000	2.73844	ARB INT
77	NBR11Y7	INT	** LOWER	83.00000	.	.	16383.00000	.	
78	NBR01Y8	INT	LOWER	631.00000	.	.	16383.00000	.62614	ARB INT
79	NBR02Y8	INT	** LOWER	19.00000	.	.	16383.00000	.	ARB INT
80	NBR03Y8	INT	LOWER	12.00000	.	.	16383.00000	59.60518	ARB INT
81	NBR04Y8	INT	** LOWER	3.00000	.	.	16383.00000	.	ARB INT
82	NBR05Y8	INT	** LOWER	15.00000	.	.	16383.00000	.	ARB INT
83	NBR06Y8	INT	** LOWER	4.00000	.	.	16383.00000	.	ARB INT
84	NBR07Y8	INT	** LOWER	20.00000	.	.	16383.00000	.	ARB INT
85	NBR08Y8	INT	LOWER	35.00000	.	.	16383.00000	13.43215	ARB INT
86	NBR09Y8	INT	** LOWER	67.00000	.	.	16383.00000	.	ARB INT
87	NBR10Y8	INT	** LOWER	80.00000	.	.	16383.00000	.	ARB INT
88	NBR11Y8	INT	** LOWER	95.00000	.	.	16383.00000	.	ARB INT
89	PROD01Y1	PL	ACTIVE	5926.54845	.77160	.	+INF	.	
90	PROD02Y1	PL	ACTIVE	5967.78400	.27960	.	+INF	.	
91	PROD03Y1	PL	ACTIVE	4630.87190	.71450	.	+INF	.	
92	PROD04Y1	PL	ACTIVE	404.54400	.72910	.	+INF	.	
93	PROD05Y1	PL	LOWER	.	.48510	.	+INF	-.00855	
94	PROD06Y1	PL	ACTIVE	292.93704	.48710	.	+INF	.	
95	PROD07Y1	PL	ACTIVE	1256.00000	.41100	.	+INF	.	
96	PROD08Y1	PL	ACTIVE	493.00000	.40710	.	+INF	.	
97	PROD09Y1	PL	ACTIVE	2059.81600	.38710	.	+INF	.	
98	PROD10Y1	PL	ACTIVE	531.27800	.44810	.	+INF	.	
99	PROD11Y1	PL	ACTIVE	2097.66910	.56600	.	+INF	.	
100	PROD01Y2	PL	ACTIVE	6387.76589	.77160	.	+INF	.	

PRINT OPTION = COMPLETE OUTPUT
NAME = ALLOCATION OBJ = OBJECTIVE RHS = DEMAND BND = BNDS
DIR = MAXIMIZE COBJ = CRHS = ABND =

VALUE OF OBJECTIVE = 154230.52223
RPSOBJ = -1.0000 RPSRHS = 1.0000
RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	COL ACTIVITY	OBJ COEF	BND LOWER	BND UPPER	MARGINAL
101	PROD02Y2	PL	ACTIVE	6303.44795	.27960	.	+INF	.
102	PROD03Y2	PL	ACTIVE	4679.00006	.71450	.	+INF	.
103	PROD04Y2	PL	ACTIVE	440.00000	.72910	.	+INF	.
104	PROD05Y2	PL	LOWER	.	.48510	.	+INF	-.00697
105	PROD06Y2	PL	ACTIVE	229.60260	.48710	.	+INF	.
106	PROD07Y2	PL	ACTIVE	1255.00000	.41100	.	+INF	.
107	PROD08Y2	PL	ACTIVE	493.00000	.40710	.	+INF	.
108	PROD09Y2	PL	ACTIVE	4635.83701	.38710	.	+INF	.
109	PROD10Y2	PL	ACTIVE	1087.49600	.44810	.	+INF	.
110	PROD11Y2	PL	ACTIVE	2227.52257	.56600	.	+INF	.
111	PROD01Y3	PL	ACTIVE	6697.67555	.77160	.	+INF	.
112	PROD02Y3	PL	ACTIVE	6619.32227	.27960	.	+INF	.
113	PROD03Y3	PL	ACTIVE	5663.21700	.71450	.	+INF	.
114	PROD04Y3	PL	ACTIVE	475.63200	.72910	.	+INF	.
115	PROD05Y3	PL	ACTIVE	92.88455	.48510	.	+INF	.
116	PROD06Y3	PL	ACTIVE	310.00000	.48710	.	+INF	.
117	PROD07Y3	PL	ACTIVE	1256.00000	.41100	.	+INF	.
118	PROD08Y3	PL	ACTIVE	493.00000	.40710	.	+INF	.
119	PROD09Y3	PL	ACTIVE	4729.98000	.38710	.	+INF	.
120	PROD10Y3	PL	ACTIVE	1087.49600	.44810	.	+INF	.
121	PROD11Y3	PL	ACTIVE	2359.28280	.56600	.	+INF	.
122	PROD01Y4	PL	ACTIVE	7098.40433	.77160	.	+INF	.
123	PROD02Y4	PL	ACTIVE	6959.00517	.27960	.	+INF	.
124	PROD03Y4	PL	ACTIVE	6117.47100	.71450	.	+INF	.
125	PROD04Y4	PL	ACTIVE	511.17600	.72910	.	+INF	.
126	PROD05Y4	PL	ACTIVE	55.54551	.48510	.	+INF	.
127	PROD06Y4	PL	ACTIVE	310.00000	.48710	.	+INF	.
128	PROD07Y4	PL	ACTIVE	1379.27200	.41100	.	+INF	.
129	PROD08Y4	PL	ACTIVE	493.00000	.40710	.	+INF	.
130	PROD09Y4	PL	ACTIVE	5352.11641	.38710	.	+INF	.
131	PROD10Y4	PL	ACTIVE	1623.45200	.44810	.	+INF	.
132	PROD11Y4	PL	ACTIVE	2495.71731	.56600	.	+INF	.
133	PROD01Y5	PL	ACTIVE	7472.65701	.77160	.	+INF	.
134	PROD02Y5	PL	ACTIVE	7309.10483	.27960	.	+INF	.
135	PROD03Y5	PL	ACTIVE	7123.98735	.71450	.	+INF	.
136	PROD04Y5	PL	ACTIVE	574.68611	.72910	.	+INF	.
137	PROD05Y5	PL	ACTIVE	347.76645	.48510	.	+INF	.
138	PROD06Y5	PL	ACTIVE	325.53400	.48710	.	+INF	.
139	PROD07Y5	PL	ACTIVE	3536.53200	.41100	.	+INF	.
140	PROD08Y5	PL	ACTIVE	493.00000	.40710	.	+INF	.
141	PROD09Y5	PL	ACTIVE	5760.71200	.38710	.	+INF	.
142	PROD10Y5	PL	ACTIVE	1819.61200	.44810	.	+INF	.
143	PROD11Y5	PL	ACTIVE	2662.80278	.56600	.	+INF	.
144	PROD01Y6	PL	ACTIVE	7902.90397	.77160	.	+INF	.
145	PROD02Y6	PL	ACTIVE	7728.19709	.27960	.	+INF	.
146	PROD03Y6	PL	ACTIVE	7952.33185	.71450	.	+INF	.
147	PROD04Y6	PL	ACTIVE	646.96133	.72910	.	+INF	.
148	PROD05Y6	PL	ACTIVE	401.51796	.48510	.	+INF	.
149	PROD06Y6	PL	ACTIVE	786.77257	.48710	.	+INF	.
150	PROD07Y6	PL	ACTIVE	4299.52483	.41100	.	+INF	.

PRINT OPTION = COMPLETE OUTPUT
NAME = ALLOCATION OBJ = OBJECTFCT RHS = DEMAND BND = BNDS
DIR = MAXIMIZE COBJ = CRHS = RNG =

VALUE OF OBJECTIVE = 154230.52223
RPSOBJ = -1.0000 RPSRHS = 1.0000
RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	COL ACTIVITY	OBJ COEF	BND LOWER	BND UPPER	MARGINAL
151	PROD08Y6	PL	ACTIVE	1115.68000	.40710	.	+INF	.
152	PROD09Y6	PL	ACTIVE	6207.09200	.38710	.	+INF	.
153	PROD10Y6	PL	ACTIVE	2076.32800	.44810	.	+INF	.
154	PROD11Y6	PL	ACTIVE	2031.29250	.56600	.	+INF	.
155	PROD01Y7	PL	ACTIVE	8372.58268	.77160	.	+INF	.
156	PROD02Y7	PL	ACTIVE	8198.22386	.27960	.	+INF	.
157	PROD03Y7	PL	ACTIVE	8654.99900	.71450	.	+INF	.
158	PROD04Y7	PL	ACTIVE	705.67162	.72910	.	+INF	.
159	PROD05Y7	PL	ACTIVE	469.56000	.48510	.	+INF	.
160	PROD06Y7	PL	ACTIVE	647.07328	.48710	.	+INF	.
161	PROD07Y7	PL	ACTIVE	4820.58543	.41100	.	+INF	.
162	PROD08Y7	PL	ACTIVE	2127.53500	.40710	.	+INF	.
163	PROD09Y7	PL	ACTIVE	6703.38000	.38710	.	+INF	.
164	PROD10Y7	PL	ACTIVE	2337.79800	.44810	.	+INF	.
165	PROD11Y7	PL	ACTIVE	3907.08548	.56600	.	+INF	.
166	PROD01Y8	PL	ACTIVE	8869.82000	.77160	.	+INF	.
167	PROD02Y8	PL	ACTIVE	8681.44259	.27960	.	+INF	.
168	PROD03Y8	PL	ACTIVE	9827.21280	.71450	.	+INF	.
169	PROD04Y8	PL	ACTIVE	768.69006	.72910	.	+INF	.
170	PROD05Y8	PL	ACTIVE	539.75148	.48510	.	+INF	.
171	PROD06Y8	PL	ACTIVE	910.83617	.48710	.	+INF	.
172	PROD07Y8	PL	ACTIVE	5407.73922	.41100	.	+INF	.
173	PROD08Y8	PL	ACTIVE	3035.61000	.40710	.	+INF	.
174	PROD09Y8	PL	ACTIVE	7241.63979	.38710	.	+INF	.
175	PROD10Y8	PL	ACTIVE	2716.90150	.44810	.	+INF	.
176	PROD11Y8	PL	ACTIVE	3193.55905	.56600	.	+INF	.

BRANCH * MAJERR

EXIT

******** CPU USED = 693.161

--- USED WORDS OF DISK (OCTAL) - 000056242

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  08/30/78  MSU HUSTLER 2       LSD 46.36  08/26/75  6500
19.07.05.IB86385
19.07.06.NL    17600
19.07.06.RP   000000000002  0000000174
19.07.06.JO3 READ-   08/30/76 .10.33.42.
19.07.06.BENGHEZAL,T800,JC3500,RG1,MT1.
19.07.07.LAST ACCESS- 8 15.11 08/29/78
19.07.07.RUNS- 0047    PN DOLLAR BALANCE  000035.44
19.07.07.031348    CARDS READ   VALUE   $0000000.67
19.07.07.CP-PP SEC.      .036-      .000  $      .00
19.07.07.COPYCF,INPUT,TAPE1.
19.07.07.NL   011300
19.07.07.RP       00000000   0000000000000
19.07.08.CP-PP SEC.      .082-      .902  $      .01
19.07.08.AUTORFL,PART.
19.07.08.CP-PP SEC.      .064-      .992  $      .01
19.07.08.RFL,70000.
19.07.08.CP-PP SEC.      .084-      .992  $      .01
19.07.08.NL   014200
19.07.08.RP       00000015   0000000000471
19.07.09.HAL,APEX(SOLVE,MAX,MIP)
19.07.09.ZZZZMPL - CYCLE 01, HAL 4, MPL
19.07.09.FILE ATTACHED
19.07.10.HAL 4.33
19.07.10.APEX     - CYCLE 18, HAL-APEX
19.07.10.FILE ATTACHED
19.07.29.NL    70200
19.07.29.RP   000000000036  0000000000727
19.07.29.EXEC BEGUN.19.07.29.
19.07.31.  ********************************
19.07.31.  **      0-0-C APEX-III        **
19.07.31.  **      WITH MIP OPTION       **
19.07.31.  **    WITH REDUCE OPTION      **
19.07.31.  ** WITH PARAMETRICS OPTION    **
19.07.31.  ********************************
19.45.39.      END   APEX
19.45.39.    693.164 CP SECONDS EXECUTION TIME
19.45.39.MAX FILES 0005 MAX PRUS 0015008.
19.45.39.  PP    257.434 SEC.
19.45.39.RP   000000004124  0000010271030
19.45.39.CP USE   693.298 SEC   VALUE$    30.81
19.45.39.PP USE   307.944 SEC   VALUE$     .86
19.45.39.CM USE   124.792 M-H   VALUE$    33.54
19.45.39.TOTAL COMPUTE VALUE AT RG1 $     32.61
19.47.04..000022 PAGES PRINT. 000929 LINES PRINT. FOR $ 001.04 AT RG2.
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