COMPLEX MODAL ANALYSIS OF CARANGIFORM SWIMMING KINEMATICS

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ABSTRACT

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The overall purpose of this study is to do a complex modal analysis of the kinematic data of carangiform swimming. The research considers the midline motion in itself, leaving out the causes to which the initiation of motion is ascribed. Today, motion analysis is commonly based on the FFT (fast Fourier transform). Instead, this research examines a more recent technique, COD (complex orthogonal decomposition), to perform complex modal analysis. This analysis describes the main modes of the lateral displacement, and the associated frequency and wavelength. Body center lines were used for the kinematic analysis of the swimming motion. We acquired raw data of midline lateral movements from the resources available in the literature for three carangiform fish: whiting, mackerel, and saithe. The midline motion data for these fish was originally available in photo form, so it had to be digitized in order to be used as input to COD. Using COD, the total motion was decomposed into the main modes (eigenvectors) with the corresponding contribution (eigenvalues). Each mode, in addition, was decomposed to its standing and traveling parts. COD was used also to do accurate measurement of amplitude, wavelength and frequency (as functions of location or time). The main focus of this analysis is to compare the traveling wave model with the true motion (raw data). In this regard, a traveling wave model was also built based on amplitude profile, average wavelength, and frequency of raw data, and the same analysis that was applied to the raw data was applied to this model. The results for both cases (raw data and the traveling wave model) were

then compared. The results show that in both cases, there is only one main mode which incorporates more than 99% of the energy. The other finding is that if we consider the main mode of carangiform motion in the context of $amplitude(x) \times cos(\omega t - cx)$, then the coefficients ω and c in the argument of harmonic function are not constant, but functions of time and location, respectively. Also, this analysis shows that the standing part for both cases has non-zero amplitude, which challenges the notion of a pure traveling wave suggested in nearly all literature for the kinematics of fish swimming. COD can be performed with the goal of isolating a single mode of interest, and then representing the mode by the real and imaginary parts of the extracted complex mode, quantifying the motion parameters based on the complex mode and modal coordinate, enabling visualization and computation of additional quantities of interest, and "purifying" or isolating the motion in terms of the extracted mode. I dedicate this thesis to my husband, Vahid for supporting me with love and sympathy.

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KEY TO ABBREVIATIONS

- COD: Complex Orthogonal Decomposition
- COV: Complex Orthogonal Values
- COM: Complex Orthogonal Modes
- COC: Complex Orthogonal Coordinates
- BCF: Body and Caudal Fin

CHAPTER I

Introduction

In this research, we analyzed the kinematics of the carangiform swimming midline. Kinematics is a subdivision of classical mechanics concerned with the motion of moving bodies. It considers the various possible types of motion in themselves, leaving out the causes to which the initiation of motion may be ascribed. In comparison, analytical dynamics is that branch of knowledge in which the motions of material bodies are considered as due to the mutual interactions of the bodies (*Whittaker* (1988)). Fish kinematic analysis is the process of measuring the kinematic quantities used to describe swimming motion without any regards to the forces and masses. In comparison, in dynamic analysis of fish swimming, the motion is regarded by taking into account all force interactions between the body and external environment, to investigate the time evolution and spatial distribution of displacement field and the vibration of continua.

Carangiform swimming is one of aquatic propulsion modes that generates lateral undulatory non-synchronous waves propagating down the body and through the caudal fin. Because of the parts contributing in the propulsion, it is count as a BCF (body and caudal fin) modes. The same trend is observed in some other BCF aquatic locomotion types such as anguilliform and sub-carangiform. What discretize the various BCF modes is the amplitude envelope representing a body part's extend of participation in the swimming. Specifically in carangiform mode, the vast majority of movement is concentrated in the very rear of the body and tail, leading to appearance of a rapidly oscillating tails (*Breder* (1926)). This study aims to show that other than the amplitude envelope, there are other kinematic features that are common among the species in a same category. We are going to evaluate this for three carangiform fish species: whiting (Gadus merlangus or Merlangius merlangus), mackerel (Scomber scombrus, fam. Scombridae), and saithe (Pollachius virens, fam. Gadidae). The evaluation is based on the wave parameters like modal shape, amplitude, frequency, wavenumber, and traveling index discussed later. However, we cannot judge if these features are unique to this category until we do the same study on further carangiform species and on the other locomotion types and compare the results. Among all the particles of the body, we chose the midline points for the kinematic analysis. This selection is because according to Shadwick et al. (1998), mackerel fish body acts like a simple thin beam in bending when swimming at low to moderate speeds. On this basis, displacement of other points can be expressed in terms of displacement of midline points. Regarding the whiting and saithe, we do not know if the pure bending is governing the lateral motion or at least there is not such an evaluation in the literature. However, we consider the midline in these cases, because we have access to only midline data.

The swimming kinematics is a necessary part of the investigation of any dynamical study on the subject of swimming. Swimming dynamics is a puzzle with the pieces such as locomotor kinematics, muscle dynamics, neuromuscular activation, and fluid dynamics. The kinematic model is often used as input to computational models of the fluid flow around the fish. The kinematics of swimming fish is an important topic to investigate since it can be the underlying scientific support for nature-inspired aquatic robot construction. Kinematic analysis may be used to find the range of movement for a fish, and working in reverse, to design a robot for a desired range of motion. Recently, there has been increasing interest in the study of aquatic robots with applications like exploration in challenging environment. A good reason to inspire from the natural motions is the high efficiency of propulsion (in terms of energy consumption), and maneuverability, especially in BCF swimming. BCF swimming which is known to be inherently stable, has good acceleration performance and good cruising performance (*Blake* (2004)). With a known kinematics, robots can be constructed that emulate natural fish, and so establish a high propulsive efficiency or a high maneuverability (in terms of mission requirements).

In the following sections, we aim to trace the intellectual progression of the field of swimming kinematics and the application of the only model proposed for it in hydrodynamic and dynamic studies and in robot fish design.

1.1 Background on fish kinematic analysis

The current understanding of fish swimming kinematics is indebted to the triple research paper of *Gray* (1933a,b,c). These were the first scientific studies on kinematics of fish swimming. He recorded photographically the top view of body and fin (if any) movements of various swimming fish species. These photographs are still being used as a source of experimental data for current research. He developed an accurate inter-frame timing circuit to ensure having precise knowledge of the time between each film frame for the final purpose of calculating velocities of points on the fish body. He found out that every point on the body follows a wave track in space in the swimming direction at an average speed equal to the whole fish. He also measured several characteristic features like wavelength and period.

Following Gray's work, *Bainbridge* (1963), in an attempt to gain a much more precise knowledge of the extent and variability of the lateral movements of the body and the relationship of these to the speed of forward movement, made observations on bream, goldfish and dace swimming by means of the 'Fish Wheel' apparatus. He measured several kinematic quantities such as complex changes in curvature of the caudal fin during different phases of the locomotory cycle and the associated angles of attack. Based on these measurements, he determined the extent of the lateral propulsive movements in some parts of the body, the wavelength of this movement and the rate of progression of the wave down the body.

Gray's three papers on aquatic animal locomotion continue to be a foundation to many later research activities in this field even after 85 years. The idea of using the traveling wave in describing the kinematics of swimming has flourished after Gray and has been validated by many scientific experiments. As a prominent example, *Videler and Hess* (1984) developed a method of describing the kinematics of lateral displacements and body curvature in straight forward swimming of saithe and mackerel. They tried to decompose the total lateral motion of each point along the midline to its main modes. Through the fast Fourier transform (FFT), they found three Fourier main terms for the transverse midline displacement, y(x, t), as

$$y(x,t) = \sum_{j=1,3,5} h_j(x) \cos(j\omega[t - \tau_j(x)])$$

where, $h_j(x)$ is the amplitude at point x belonging to frequency j, and $\tau_j(x)$ is the phase function; In their model, the phase was considered as a function of x, not necessarily a linear one. The parameters of the harmonic function (amplitude, frequency and phase) were estimated using least square algorithms to minimize the error between the actual and predicted motions in time.

Use of a traveling wave in the kinematic model was not limited to straight swimming; *Akanyeti and Liao* (2013) adopted a similar approach as *Videler and Hess* (1984) for analyzing the kinematics of Kármán gaiting midlines using the FFT. They developed their model of Kármán gaiting based on the traveling wave equation and decomposed the midlines into four fundamental motion components, concluding that the Kármán gaiting is a superposition of undulatory swimming with translational and rotational motion. They computed the FFT of the lateral motion for each point along the midline and found out that the lateral motion of all midline points could be represented by the same fundamental frequency and analyzed the changes in amplitude and phase as a function of position along the body. They obtained a linear phaseposition relationship and confirmed that a traveling wave was present during Kármán gaiting like the case of free-stream swimming kinematics in work of *Videler and Hess* (1984). However, for each behavior, the amplitude, wavelength and frequency values of the traveling wave equation are substantially different.

In two above works, like it is common in applied physics, the analysis of the signals typically relies on Fourier analysis. When processing signals, Fourier analysis can isolate individual components of a compound waveform, concentrating them for easier detection or removal. However, a new method proposed by *Feeny* (2008) called complex orthogonal decomposition (COD) can also decompose a total oscillation to its main modes and even further dissect a wave mode into its traveling and standing parts. Like a complex singular value decomposition, the decomposition is based on the complex eigen-solution of a complex correlation matrix formed from a wave's sampling data. Generally, the application of this method is in the analysis of the waves from the vibrational point of view. This can be used to find the relationship between the complex modes and wave motion. From the complex modes and modal coordinates, the frequencies, wavelengths, amplitude envelope, and characteristic wave speeds can be obtained. As mentioned before, one of the deterministic factors in accuracy of a CFD computation is the accuracy of the kinematic model input to it. Since the affecting factors on efficiency of the kinematic model are the amplitude envelope, the harmonic function and its argument used in the model, COD can give the accurate data on the wave parameters. Another side benefit of this method can be the filtration of experimental data from noise. This application is shown in the work of *Feeny and* Feeny (2013). They performed a complex modal analysis of the kinematics of the transverse motion of a swimming fish by applying COD to the digitized data of midline extracted from the cinematographic images of a swimming whiting from the

work of Gray (1933c). They extracted the dominant modal coordinates and used it to estimate the frequency, wavelength, and wave speed.

The literature mentioned so far conducted a kinematical analysis of midline displacement data. However, a more comprehensive kinematics would consider all the body points motion. In this regard, Shadwick et al. (1998) investigated the kinematics of several reference points in different depths and lengths. They related the body kinematics, muscle contractile properties, strain, and patterns of muscle activation that produce the transmission of muscular waves along the body. They presented kinematic data for mackerel obtained by videoradiography (an x-ray technique) for the first time to study skeletal muscle in fish. They visualized motion of radio-opaque markers placed within the muscle mass of a swimming fish and subsequently determined muscle deformation. From the lateral motion of markers at several depths, they recognized the traveling wave of deformation on the body, characterized by a progressive delay in peaks and increase in amplitude from anterior to posterior sites. They observed at each axial location all markers moved in synchrony with each other and with the body midline, regardless of depth. They concluded that these observations to a first approximation, support the idea that the fish body can be modeled as a homogeneous bending beam during slow swimming¹. So treating the fish body as a homogeneous beam, they calculated the local midline (as neutral axis) curvature (or an approximation of it) for swimming mackerel, and the lateral distance from this axis, to predict local muscle strain. A similar study has been done on saithe (Videler and Hess (1984)).

Shadwick et al. (1998) also used electromyographic (EMG) recordings in vivo to measure neuronal activation patterns. They observed for each tail beat cycle, the electrical activity proceeds like a wave from anterior to posterior, with the two sides of body side showing alternating EMG activity. By a coupled analysis of electromyo-

¹slow swimming occurs when red muscle is active and white muscle is passively deformed

graphic and videoradiographic data, they identified that the temporal relationship between muscle strain and neuronal activation varies with position along the body and suggested that in most cases it appears that the EMG signal progresses along the body faster than the wave of body deformation. From the analysis of electromyographic data collected and their measurement of the strain distribution, they quantified muscle contractile properties of cyclic contractions in vitro. They also calculated work done by the red muscles based on muscle forces and displacement rates and concluded that the neuronal activation patterns are consistent with red muscle performing net positive work at all axial positions. They measured the cross-section of red muscle along much of the body and observed that it is relatively constant, so suggesting that positive power for swimming is generated fairly uniformly along the length of the fish.

1.2 The role of kinematics in fluid mechanics studies

A group of researchers has made kinematic measurements for the interest of hydrodynamics. Kinematic measurements of this kind indeed enables mathematician or hydrodynamicist to embark on the subsequent calculations. Gray (1933c), from his quantitative analysis made on the successive film frames, recognized a body wave and a propulsive wave and linked this kinematics to the forces that propel the fish forward. Based on identification of wave crests and velocity calculations, he concluded that the wave speed of bending that travels down the body length must always be greater than the forward velocity of the fish through the water in order to produce forward thrust, an idea also proposed by *Breder* (1926). Gray recognized this feature as being involved in the production of forward thrust in steady swimming. Based on the kinematic quantities measured, he concluded that the magnitude of the forward thrust depends on speed and amplitude of the undulatory waves generated during locomotion, as well as on the transverse velocity and orientation of body segments along the fish while swimming.

Although Gray's general conclusions greatly enhanced our understanding of how undulatory movements of fish lead to forward movement during swimming, there remained a lack of detailed quantitative kinematic analyses regarding undulatory locomotion with technical advances since the 1930s. In this regard, *Gillis* (1996) examined the generality of Gray's conclusions by analyzing data from his own highspeed videos of swimming eels and salamanders. Based on his measurements, he concluded some facts about the kinematics of swimming that were in contrast to Gray's general conclusions. As an example of one of these discrepancies, the relative velocity of the traveling waves is not necessarily faster the swimming velocity but depends greatly upon swimming speed and species, despite what Gray and others previous to him had described that the velocity of these traveling waves is faster than the velocity of the swimming animal.

Among the first scientific fluid models, the work of *Taylor* (1952) can be mentioned. He presented laboratory measurements of forces acting on a long straight circular cylinder set obliquely to a stream of fluid. He examined these aerodynamic data and formulated the lateral and longitudinal components of forces acting on straight smooth and rough cylinders, and tried to generalize this for the flexible cylinder down which waves of bending of constant amplitude were being propagated, and compared this geometry with measurements of Gray's photographs of smooth animals swimming. He calculated the energy required for propulsion of smooth and rough animals and found the amplitudes of the waves which drive them fastest for a given output of energy is found.

Taylor's drag model was used by some swimming dynamics researchers like *McMillen* and Holmes (2006) and Bhalla et al. (2013). *McMillen and Holmes* (2006) decomposed the drag force into normal and tangential components for smooth oblique cylinders based upon Taylor's fitting of drag coefficients and used it in the development of

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a continuum mechanical model for anguilliform (eel-like) swimming. They analyzed the periodic traveling waves in uniform rods in terms of the effects of bending stiffness, body geometry, and activation patterns on swimming speed, turning behavior, and acceleration to steady swimming. They considered the feedforward problem of seeking body traveling wave solution motions in response to imposed activation in the form of a time-dependent preferred curvature in a coupled elastic-fluid system. In their study the fish body was modeled by an elastic rod actuated via time-dependent intrinsic curvature and subject to hydrodynamic drag forces. They employed a geometrically exact theory and discretized the resulting nonlinear partial differential evolution to perform numerical simulations.

Bhalla et al. (2013) used Taylor's resistive model for the hydrodynamic forces. They applied a forced damped oscillation framework to a chain-link model for undulatory swimming to understand how the forcing leads to deformation and movement. They showed that the forcing triggers the first few deformation modes of the body, which in turn causes the translational motion. They also showed that relatively simple forcing patterns can trigger seemingly complex deformation kinematics that lead to movement. They proposed an approach to analyze qualitatively the optimal deformation kinematics for fast swimming and confirmed their results by a computational fluid dynamics (CFD) simulations.

Taylor's model, although based on experimental data, was precise only for the straight rod and in the case of curved flexible rods, it only could give an approximation. This model may be called a 'resistive' theory in that the forces water exerts on a small section of the animal was regarded a resistive force depending on the instantaneous value of the relative velocity of that section. A newer inviscid-flow theory was proposed by *Lighthill* (1960) for the forces produced by swimming movements that may be called a 'reactive' theory in that the forces from the water on parts of animal's surface in contact with it were regarded as reactive forces. Comparing to the

resistive models, the reactive forces were considered proportional to rate of change of the relative velocity of animal surface. These forces were neglected in the resistive theories.

In development of the theory described in Lighthill (1960), Lighthill followed the idea of the travelling wave introduced by Gray and utilized it as a simplifying assumption in hydrodynamic force modeling in that he considered the whole motion of surface as superposition of a forward motion and an oscillation in the form of traveling wave. He evaluated reactive forces between body and the water surrounding it and regarded the flow as compounded of the steady flow around the stretched straight body and the flow due to the displacements h(x,t). Then he sought a kind of transverse oscillatory movement that generates an efficient thrust. He concluded that the simple movement of a standing wave cannot satisfy his criteria for efficient thrust while the form of a traveling wave which moves down the fish's body can be a satisfactory shape for the displacements and can generate an efficient thrust. This travelling wave can be written as

$$h(x,t) = f(x)g(t - \frac{x}{c})$$
 (1.1)

where h(x,t) is transverse displacement, f(x) is wave amplitude envelope that may vary with position along the fish, and g is an oscillatory function like sinusoid providing velocity c > U where U is the swimming speed. It should be mentioned that the validity of Lighthill's theory is restricted to lateral oscillations of the fish body with an amplitude small in comparison to the body length. It is based on a perturbation expansion in powers of an amplitude parameter ϵ and derives thrust and energy consumption as multiples of ϵ^2 with terms of order ϵ^4 neglected. The inadequacy of such an expansion in powers of amplitude led him to concentrate upon the possible improvement of elongated-body theory. He claimed (*Lighthill* (1971)) that depending on the mode of swimming, one of reactive forces or resistive forces dominates over the other. He combined the resistive-reactive theory for motions of large amplitude that finally led to development of "large-amplitude elongated-body" theory. The new theory could be applied to interpret observational data on animal movements of arbitrary large amplitude, regular or irregular.

Lighthill's large amplitude elongated body theory was used by *Videler and Wardle* (1977) in their kinematic analysis. They developed a practical method of accurate measurement of the parameters of the body wave like speed, wavelength, wave period and parameters of the sinusoidal track in space of different points of the body of a swimming fish, like speed, wavelength, wave period and amplitude. These characteristic wave parameters were measured from high speed cine recordings of straight forward swimming behavior of one cod.

Using the kinematic data obtained in the analysis of Videler and Hess (1984), Hess and Videler (1984) continued with a dynamic analysis of the straigth forward swimming of saithe. They used models for the hydrodynamic forces between fish and water, the bending moments generated inside a fish and calculated the mechanical work done by the fish body on water during swimming. They treated the fish as a flexible elongated body and used Lighthill's elongated-body theory. They used the assumption of linearity to imply the total lateral hydrodynamic force distribution is obtained by summing the force distributions belonging to each Fourier term. The lateral bending moments inside the fish were analyzed similarly. Their major result was that the bending moment does not travel as a running wave from head to tail like the lateral body curvature does, but behaves as a standing wave.

Following in Lighthill's footsteps, many people contributed to modeling the hydrodynamics of fish swimming. For example, Wu (1961) treated a simplified model of two-dimensional potential flow over a waving flexible and thin plate of finite chord. The plate was capable of performing the motion which consists of a traveling wave. He solved the problem by applying the general theory for oscillating deformable airfoils and calculated the thrust, power required, and energy imparted to the wake, and also evaluated propulsive efficiency.

In 1964, *Uldrick and Siekmann* (1964) developed a theory for a flexible thick body undergoing preassigned undulations on the basis of the complex velocity potential method. They formulated the flow-field around a plate or hydrofoil undergoing a perturbation motion of small amplitude in the transverse direction and simulated the propulsion of the fish. The problem was linearized in that they assumed the unsteady perturbation theory. The plate was a flexible solid of constant depth and of infinite length (two-dimensional) with arbitrary finite thickness immersed in an inviscid incompressible fluid in an uniform flow of constant velocity in the direction of swimming. The thrust was assumed to be generated by displacements forming a train of travelling waves of small amplitude which pass down the body of the fish from the head to the tail with the envelope of these waves varying arbitrarily along the length of the plate. They took the amplitude of these displacement waves to be a harmonic function of time and concluded the magnitude of the thrust depends on the propagation velocity of these waves.

Later a moderate generalization of Lighthill's slender body theory was proposed by Wu (1971) by shedding an oscillating vortex sheet to trail the body in swimming. He considered the same problem of finding an optimal shape of transverse oscillatory movements that produces a prescribed thrust. He found the solution to be a traveling wave with phase speed c greater than the desired swimming speed U, with an amplitude nearly uniform from the maximum span section to the tail.

Contributions by Lighthill and Wu strongly influenced later developments of inviscid hydromechanics of fish-like propulsion. *Cheng et al.* (1991) developed the threedimensional waving plate theory with application in hydrodynamics of undulatory propulsion in anguilliform and carangiform modes. They used the vortex ring panel method to calculate the unsteady potential flow over model rectangular and triangular flexible plates performing a motion as sum of traveling waves with variable amplitudes in the form $h(x, y, t) = \sum_{n=-M}^{M} a_n x^{n-l} e^{i(\omega t - kx)}$, with a_n a complex constant. Based on this motion, the rest of the calculations were done leading to a formulation of propulsive effectiveness.

The 3D waving plate theory was used by *Cheng et al.* (1998) as the model for the unsteady fluid force acting on the swimming body in their development of continuous dynamic beam model for the saithe. They followed the work of *Hess and Videler* (1984) in showing how the transverse hydrodynamic forces together with fish body inertia combine to require the generation of a certain distribution of bending moment by the fish. They assumed a linear visco-elastic passive behavior for the internal tissues, skin and backbone. The input to their model was the muscle activation patterns and a single frequency oscillation as input to their linear dynamic model similar to the work of *Hess and Videler* (1984). They analyzed the dynamic response of the system in steady swimming as output and calculated muscle bending moment distribution due to the various components and from the whole dynamic system acting together.

Cheng and Blickhan (1994) in their dynamic analysis of body undulation for the saithe and eel used as their hydrodynamic model two theories: the three-dimensional unsteady waving plate theory and the slender body theory. For the kinematic waving pattern, they chose a traveling wave motion format. They quantified the distribution of bending moment along the body and compared them with electromyographic records with respect to amplitude and phase. At the end, they concluded a better momentum balance is resulted with waving plate theory.

Mchenry et al. (1995) studied the mechanical control of speed in steady undulatory swimming with the controlling variables as flexural stiffness, driving frequency and driving amplitude. They swam their pumpkinseed sunfish models by powering them via the input of an oscillating sinusoidal bending couple at the posterior margin of head in a flow tank. To simulate the hydrodynamic conditions of steady swimming, drag and thrust acting on the model were balanced by adjusting flow speed. To calculate wave speed, Froude efficiency and the relative mechanical power, they measured the kinematic response variables from the digitized midlines of the videotaped models and interpreted them as traveling waves. They predicted that a live sunfish double the flexural stiffness of their bodies in order to swim at fast speeds.

Through the evolution of swimming hydrodynamics studies from resistive to reactive theories and waving plate theory, several investigators turned to experimental visualization of the flow. *Anderson* (1996) employed flow visualization techniques to demonstrate the mechanism of vorticity origination near fluid boundary. She showed that with the body motion having the form of a traveling wave along the fish, vortices are generated that are characterized by spatially traveling waves of body-bound curved trajectories.

Alben (2009) used the periodic traveling-wave model for body motions to formulate a new theoretical model for swimming using a vortex street. He solved optimization problems in finding body wave which maximizes efficiency for a given output power. This work is limited to small amplitude of oscillation.

Meanwhile, several researchers were interested in the boundary layer flow field since it can lead to computation of hydrodynamic forces and the hydrodynamic forces are highly dependent on the surface motions since they affect the boundary layer quantities. There is no escaping the fact that the boundary layer flow field can be described fully by the boundary layer equations that are special cases of Navier-Stokes equation. However, the analytical solutions for the boundary layer equations exist only for simple cases with stationary surfaces. This is why the boundary layer equations of a moving surface like the swimming fish body seems impossible. This is firstly because the exact motion is not identified in form of a well-behaved mathematical model. Secondly, even if the motion was known, the high complexity of the problem makes the analytical solution impossible. Therefore, a possible approach was to employ flow-visualization techniques to measure the flow around fish body, like in the work of *Wolfgang et al.* (1999). They gathered experimental data from sequences of images taken over three giant danio straight-line swimming cycles using digita particle tracking velocimetry (DPTV). On the other hand, they developed a computational tool for investigating the velocity and vorticity fields around a fish model performing the same motions to clarify the basic propulsive mechanisms used by the fish. They imposed a transverse motion from the nose to tail having the form of traveling wave. The parameters were derived by applying curve fitting to the experimental data to be purely sinusoidal and to consist of a smooth amplitude-modulated traveling wave along the body length with constant phase speed and constant swimming speed. They compared the unsteady two-dimensional velocity fields from the experimental flow-visualization with their analytical results showing a good agreement.

Techet (2001) in her thesis performed experiments on the near boundary flow about fish-like swimming bodies up to Reynolds number 10^6 using laser Doppler velocimetry and particle imaging techniques. She created a traveling wave motion down the fish-like body and investigated the mechanism of control of both the turbulence production and the boundary layer development by motion of the boundary.

Anderson et al. (2001) determined the tangential and normal velocity profiles of the boundary layer surrounding live swimming fish by DPTV. They measured velocity components at several locations over the surfaces of two species in carangiform and anguilliform categories throughout complete undulatory cycles of their propulsive motions. They concluded that the local friction coefficients, boundary layer thickness and fluid velocities at the edge of the boundary layer have oscillatory behavior with relative position along the fish. This suggests that the streamwise distributions of these variables can be represented as traveling waves moving in the same direction as the fish body wave and also are suggestive of local oscillatory and mean streamwise acceleration of the boundary layer. In addition, the clockwise procession of maximum values in the phase plots reveals an ever-increasing downstream shift in the streamwise distributions of these variables with respect to the phase of the body travelling wave. Also, the regular periodic behavior of these variables at fixed positions on the fish reveals that these 'distribution waves' and the body travelling wave have the same frequency. They concluded that the increasing streamwise phase shift of the variable distributions with respect to the body wave is therefore due to the distribution waves having a longer wavelength and higher wave speed than the body travelling wave.

Assuming a traveling wave model for the kinematics of motion, $M\ddot{u}ller \ et \ al.$ (2002) studied modulation of thrust production in mullet by changing the ratio between its swimming speed U and the phase speed v (the speed with which the body wave travels down the body). They investigated the wake of the fish in which the variation in thrust is reflected. For this purpose, they conducted subimage cross-correlations on pairs of consecutive images to obtain a velocity vector field from the particle displacement and analyzed some areas in the flow using particle tracking velocimetry (PTV) to track the particle centroids manually.

1.3 The role of kinematics in CFD studies and robotics

The classical fluid mechanics studies mainly used the principle of superposition to solve the Navier-Stokes equations with the boundary conditions at the fish body surface (see the work of *Dong and Lu* (2007)). However even with a simple traveling wave model at the fluid-structure boundary, the mathematics was still complex. With the emergence of computational fluid dynamics (CFD) a detailed investigation of the flow was made possible, which allows for a more thorough analysis of the propulsion mechanisms (*Liu et al.* (1996); *Lamas et al.* (2011)).

In general, research shows that undulating fish movement has high propulsive efficiency more and less in all fish categories. On the other hand, the differences between
body shapes and the types of motion produced a large range of achievable velocities, power and thrust production in various fish species. This fact attracted the attention of several researchers. *Borazjani and Sotiropoulos* (2010) carried out fluid-structure interaction simulations of virtual swimmers to investigate the effects of body shape and swimming kinematics on the hydrodynamics of undulatory swimming. They used the hybrid Cartesian immersed boundary (HCIB) method to numerically solve the coupled fluid-structure system of equations with boundary conditions applied to complex 3-D flexible body moving with prescribed kinematics of the form of a traveling wave. Consequently, they computed the components of the hydrodynamic force by integrating the pressure and viscous forces acting on the body to calculate ultimately the hydrodynamic forces and Froude propulsive efficiency based on the thrust force for constant speed swimming.

The benefit of a numerical approach is to create a more complete time-dependent data-set for the entire flow field, which allows for a more thorough analysis of the propulsion mechanisms. So the accuracy of CDF studies and other numerical or analytical methods to model the hydrodynamics depends on the accuracy of the traveling wave model in describing the real kinematics.

It should be noted that although CFD is a very powerful tool, it is exposed to computational errors and inaccurate modeling of physical phenomena. For this reason, it should be accompanied by an experimental verification. Under these circumstances, the emergence of biomimetics can make it possible to construct fish-like mechanisms, which provides additional means for studying undulating fish movement in the laboratory. Another benefit is that a higher variety of motion parameters can be applied to the robot mechanism. Thus it can make up for the limitations in the laboratory data for real fish that are confined to individual fish and individual modes of motion.

Recently, many researchers have used the traveling wave model originally proposed by Lighthill to develop their robot fish. Among the first works in the field of robofish is the work of *Barrett et al.* (1999). In his thesis, he presented the design and construction of a robotuna, in which he used the traveling wave model of Lighthill as the kinematic model of flexible body swimming motion. He used the genetic algorithm to find the optimizing value of parameters in traveling wave model that he guessed they affect the swimming performance. Then he fully tested the instrumented tuna-like vehicle to substitute a lacking analytic description of the flow about this complex three dimensional body propelling with oscillating foil at high speed.

Barrett (1996) performed experimental measurements of force and power on a fish-like robotic mechanism covered with a flexible skin and equipped with a tail with induced lateral motion of the body in the form of a traveling wave with constant wavelength and varying amplitude along the length, smoothly increasing from the front to the tail end. They demonstrated that the power required to propel an actively swimming, streamlined, fish-like body is significantly smaller than the power needed to tow the body straight and rigid at the same speed.

Yu et al. (2005) presented a simplified kinematics propulsive model for carangiform propulsion in straight swimming. They modeled the fish by a serial multi-joint oscillating mechanism and an oscillating foil. For the kinematics of the carangiform motion, they used a traveling body-wave originally suggested by Lighthill. They used a set of seven key parameters for the kinematics model of a RoboTuna in the literature and used a genetic algorithm to guide the search for an optimal swimming efficiency. For simplification, their model consisted of two parts; a time-independent spline curve sequences and a time-dependent oscillation frequency controlled by changing time interval between displacements due to each sequence and its next sequence.

A same kinematic model was used by Liu and Hu (2004) for the Joint kinematics model in the simulator except that virtual servo motors are used instead of real ones. They presented a 3D simulator to develop and test autonomous navigation algorithms and motion control algorithms for a four-joint robot fish that is able to swim like a real fish and realize autonomous navigation. Their objective was to realize and mimic real fish behavior such as decelerating/accelerating swimming, constant swimming, turning and hovering. The complex hydrodynamic model for the fish was simplified to allow real-time computation in the simulator. They used limited joints to approximate the traveling wave of a real fish.

Yeo et al. (2014) designed a biomimetic robotic fish and in their model, used the traveling wave model of Lighthill along the fish body as the fundamental fish swimming kinematics. They investigated the effects of parameters like the swimming speed, tail beat amplitude and Strouhal-Number on their kinematic model and compared the real Blacktip shark and simulated biomimetic robot fish. Results showed that Blacktip shark produce a more elongated body displacement shape as compared to the biomimetic robotic fish. Doubling the swimming speed or doubling the tail beat frequency caused no change in body motion. The effect of doubling the tail beat frequency had also shown similar behavior as doubling the swimming speed.

Chowdhury et al. (2014) formulated the biological fish propulsion mechanism by using the kinematic model of Lighthill for a bio-inspired robotic fish. They investigated the ability of a machine mimicking real fish behavior in maintaining a good balance of speed and maneuverability. Different mathematical propulsive waveforms are combined with an inverse kinematics-based approach for generating fish body motion. The simulation and experimental plots are compared and found to be similar to the kinematic behavior study of the biological yellowfin tuna.

Coral Cuéllar (2015) presented the development of bio-inspired robotic fish including development of mathematical models for the kinematics, dynamics and hydrodynamics of swimming. He modeled a robotic fish with a stiff head, oscillating fins and a flexible rear body consisting of multi-link mechanism with oscillating hinge joints. The transverse displacements of multi-link joints was considered to match to an approximate traveling wave.

1.4 Effect of kinematical parameters on the propulsive performance

There is no escaping of the fact that there is a relationship between the midline motions of the fish body and swimming performance. That is why the experimental data of midline whether it is from a real or robot fish, have been used as input to the CFD algorithms. This has been done already in the form of a traveling wave, which is the simplest and closest description of real swimming kinematics so far, with parameters identified from the real data by curve fitting. However, it is worthwhile to change the parameters and see the effect on the final CFD calculations. *Lighthill* (1960), in his work on finding the motion that can optimize the swimming efficiency, tried standing waves and traveling waves. He concluded a pure standing wave cannot produce propulsion efficiently, just the opposite of a traveling wave. However, it is not certainly clear that if there is a combination of these two forms that can optimize the propulsion.

In this regard, *Cui et al.* (2017) studied in their CFD model the effect of changing the traveling index (introduced by *Feeny* (2008)) in the traveling wave model on the swimming performance. They found that the forward speed is closely related to the traveling index and tail-beat frequency and the swimming efficiency depends on the tail-beat frequency and amplitude coefficient.

1.5 Thesis overview and contribution

The result of the later research supports the hypothesis behind our research, that is there can be some small details in the kinematics, disregarded by the traveling wave model, that can affect swimming efficiency and propulsion to some degree.

The research conducted in this thesis aims to provide a more realistic and precise kinematic model comparing the traveling wave model. For this purpose, the more recent technique COD proposed by *Feeny* (2008) is used to reveal some detailed aspects of kinematics of swimming. One of COD's applications is modal analysis of a signal ensemble which is in this research the midline motion data as the response of the fish body plus fluid to the neural excitation. Midline data is the result of measurements already done by other researchers in work of Gray (1933c) and Videler and Hess (1984). Data belonging to the former was already digitized in the work of Feeny and Feeny (2013). However, we had to digitize data in latter. In Chapter II, the methods used to digitize data in the format of a photo, and to process the data, such as interpolations, are described. In Chapter II, the methodologies used to examine the research problem is explained. In this chapter, we applied COD to both the real data of swimming midline transverse motions and to the traveling wave model presented for the real motion. Chapter III outlines the potential outcomes of this study. The major findings as a result of this analysis are that in the dominant complex mode related to real data, the phase change rate with respect to axial coordinate, x, and with respect to time is not fixed; in other words, the phase speed and frequency (and therefore, wavelength) are not constant through one cycle of motion. This means that the traveling wave model that is commonly proposed for the swimming motion could consider a variable phase speed and frequency for higher fidelity.

CHAPTER II

Methodology

This research consists of a complex modal analysis on the time histories of midline points' lateral motion for three swimming carangiform fish. In the following, the procedures or techniques used in this research to collect data and the techniques used to process and analyze the data are discussed. A bulk of the analysis done is related to decomposition of the total oscillation. To decompose a total harmonic motion to its main modes, a common method used in the literature is to apply the FFT. However, in this research a more recent technique COD is used to decompose the total motion to its main modes. The rest of the chapter emphasizes the methods used to calculate amplitude and wavelength as functions of location, and the frequency as a function of time, based on the results of complex modal decomposition.

2.1 Data collection

2.1.1 Raw data

Modern experimental modal analysis systems are composed of 1) sensors that can be contact or non contact; 2) a data acquisition system and an analog-to-digital converter, and 3) host personal computer for viewing the data and a technique for data analysis. This study did not do the first and second part of data gathering; rather it used the midline data already gathered by two sources. The average characteristics of three fish used in this research is summerized in table 2.1 with data compiled from *Feeny and Feeny* (2013); *Gray* (1933c); *Videler and Hess* (1984).

	Length	speed	cycle period	Angular frequency	sampling rate
whiting	12 in	3.95 in/s	0.6 s	5.2 m ~rad/s	20 Hz
mackerel	0.32 m	2.08 m/s	$0.153 \mathrm{\ s}$	41 rad/s	34.4 1/m
saithe	0.37 m	1.26 m/s	$0.278 \ { m s}$	$22.60 \ rad/s$	$29.7 \ 1/m$

Table 2.1 Average characteristics of three fish used in this research. Data compiled from *Feeny and Feeny* (2013); *Gray* (1933c); *Videler and Hess* (1984).

2.1.1.1 Whiting fish

For whiting, we were provided with the digitized data by the author of *Feeny and Feeny* (2013), who in turn had digitized data from the successive cinematographic images of top views of swimming fish in the work of *Gray* (1933c). Figure 2.1 shows the original successive top view images from the work of *Gray* (1933c). Each photo in the figure shows a moment of swimming and it represent an individual fish, rather than the average of all whitings. *Feeny and Feeny* (2013) have processed these photos in order to determine the midline and done the other adjustments to finally digitize the curves of midlines for 49 virtual body markers along the midline. Gray's photographic data included one half cycle, and so symmetry was applied to obtain one cycle of

swimming data. Data were generated in two coordinate systems: an inertial system and an anterior-body fixed system. the latter produces data relative to an undeformed but possibly rotated and translated midline. For details of this process, the reader is referred to *Feeny and Feeny* (2013). For both frames, the whiting has 49 body markers and 12 time samples.

Using the digitized data from whiting raw data in an anterior-body fixed coordinates (body frame), Figure 2.2 is produced showing superposed snapshots for one period of motion, wherein, each curve shows the mid-line at specific instance of time. The color of the curves are adjusted from red to blue based on time sequence. Compare this figure with Figure 2.3 that is produced using the digitized data from whiting raw data in an inertial coordinates (Newtonian frame), showing superposed snapshots for one period of motion. Again, each curve shows the mid-line at specific instance of time. The color of the curves are adjusted from red to blue based on time sequence.

Figure 2.4 shows time histories of oscillation for each body marker produced using the digitized data from whiting raw data in an anterior-body fixed coordinates (body frame). For better clarity, curves are shown for every five body markers. It is because there were 49 time history curves that made the plot crowded and hard to distinguish. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail. Compare this figure with Figure 2.5 that is produced using the digitized data from whiting raw data in an inertial coordinates (Newtonian frame), showing time histories of oscillation for each body marker. For better clarity, curves are shown for every five body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.



Figure 2.1 Successive cinematographic images of top views of swimming whiting. Figure from Gray (1933c).



Figure 2.2 Superposed snapshots for one period of motion from whiting (body frame) raw data. The color of the curves are adjusted from red to blue based on time sequence.



Figure 2.3 Superposed snapshots for one period of motion from whiting (Newtonian frame) raw data. The color of the curves are adjusted from red to blue based on time sequence.



Figure 2.4 Time history curves for body markers from whiting (body frame) raw data. The curves are shown for every five body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail



Figure 2.5 Time history curves for body markers from whiting (Newtonian frame) raw data. For better clarity, curves are shown for every five body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail

2.1.1.2 Mackerel and saithe fish

For mackerel and saithe, we used a set of plots related to the lateral temporal oscillation in time of some midline reference points from the work of *Videler and Hess* (1984). Figure 2.6 shows these plots from the work of *Videler and Hess* (1984)). Each plot is pertaining to a single reference point on the midline. These reference points include the nose and the tail. The plots were generated by high-speed filming of the top views of several swimming fish in the laboratory; However, the plots represent the average saithe and the average mackerel. The position of the midline for each image was determined by digitizing the outline of the fish image on each frame with an HP 9874A digitizer and using a programmed applications within an image-analysis environment.

They smoothed the plots by splines using 20 segments along the body. This resolution turns out to be quite sufficient, considering the noise in the data. The resulting, smoothed function y(x,t) can also be written as sum of Fourier terms up to the fifth frequency such as

$$y(x,t) = \sum_{j=1,3,5} h_j(x) \cos(j\omega[t - \tau_j(x)])$$

It took into account the differences in h(x,t) for time points separated by T/20and for body points separated for distances down to L/20. Figure 2.7 shows the superimposed generated outlines of fish in different imaging times, from *Videler and Hess* (1984). The reader is referred to method section of *Videler and Hess* (1984) for details of these processes.

While producing their plots, *Videler and Hess* (1984) used a Newtonian frame of reference whose x-axis is the mean path of motion, and the z-axis is perpendicular to the x-axis. The x-axis is calculated by using the coordinates of the reference points. Figure 2.8 shows the coordinate system used in generation of the above



Figure 2.6 Lateral deflection (drawn curves) for 'average' saithe (A) and 'average' mackerel (B), at 11 equidistant points including nose and tail points. Numbers at left indicate position along body (in units of body length, L) from nose. Vertical subdivisions: 0.02 L for drawn curves. The dashed curves and stippled curves are not of this research's interest and represent the lateral velocity and local angle of incidence, respectively. Figure from *Videler and Hess* (1984).



Figure 2.7 Digitized superimposed outlines of a saithe with computed centre lines. Figure from *Videler and Hess* (1984).



Figure 2.8 Coordinate system x, z. The water moves with velocity U in the x-direction. Figure from *Videler and Hess* (1984).

mentioned plots in the work of *Videler and Hess* (1984). This figure implies that in the coordinate system which moves with the fish at speed U, the body points move in a lateral (z) direction only. The x-component of the motion is ignored. The numerical data underlying these curves were extracted in this study using the software "WebPlotDigitizer". This software is a semi-automated tool that has a precision of 5 digits after the point.

2.1.2 Interpolation

Since the number of body markers was low in the digitized data of mackerel and saithe (M = 11), so imposing a low resolution, it was preferred to increase the spatial resolution for visual interpretations. For this reason, interpolation in terms of cubic splines is made along the length to increase M from 11 to 41. It means that the fish's centre line is divided into 40 segments by 41 equidistant points including nose and tail. As a result, 41 body markers and 20 time samples are considered for the mackerel, and 41 body markers and 51 time samples are considered for the saithe.

Using the digitized data from raw data of the average mackerel, Figure 2.9 was produced showing superposed snapshots for one period of motion, wherein, each curve shows the mid-line at specific instance of time. For better clarity, curves are shown for every other sampling times, because there were 21 snapshots making the plot crowded and hard to interpret. The curves are adjusted based on time sequence in the colour spectrum from red to blue. Figure 2.10 shows time histories of oscillation for every four body marker from mackerel raw data. For better clarity, curves are shown for every four body markers, because there were 41 time history curves that made the plot crowded and hard to discern. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.

Using the digitized data from raw data of average saithe, Figure 2.11 is produced showing superposed snapshots for one period of motion, wherein, each curve shows the mid-line at specific instance of time. For better clarity, curves are shown for every four sampling times, because there were 41 snapshots making the plot crowded. The curves are adjusted based on time sequence in the colour spectrum from red to blue. Figure 2.12 shows time histories of oscillation for each body marker from saithe raw data. For better clarity, curves are shown for every four body markers, because there were 41 time history curves that made the plot crowded. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.



Figure 2.9 The superposed snapshots for one period of motion, from mackerel raw data. Each curve shows the mid-line at a specific instance of time. For better clarity, curves are shown for every other sampling times. The curves are adjusted based on time sequence in the colour spectrum from red to blue (nose to tail).



Figure 2.10 Time history curves for body markers from mackerel raw data (nose to tail). For better clarity, curves are shown for every four body markers. The curves are adjusted based on body-marker sequence in the colour spectrum from red to blue.



Figure 2.11 The superposed snapshots for one period of motion, from saithe raw data. Each curve shows the mid-line at a specific instance of time. For better clarity, curves are shown for every five sampling times. The curves are adjusted based on time sequence in the colour spectrum from red to blue.



Figure 2.12 Time history curves for body markers from saithe raw data. For better clarity, curves are shown for every four body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.

2.1.3 Traveling wave model data

In addition to the raw data, data was built for a kind of model used in much of the literature for the kinematics of fish. This model is based on a traveling wave of the format $a(x)\sin(\gamma x - \omega t + \phi)$, where a(x) is amplitude as function of location and ω and γ are considered to be constant values.

In order to generate sampling data, for each fish, the same reference body markers and same sampling times are considered in the total length or a complete time cycle. Thus, simply the location of these body markers or the sampling times was put in the traveling wave model. As a result, for the traveling wave models, 49 body markers and 12 time samples are considered for the whiting, 41 body markers and 20 time samples for the mackerel, and 41 body markers and 51 time samples for the saithe. The amplitude is calculated based on the digitized data in section 2.1.1. a(x) is also determined at each body marker from the maximum displacement of the point in digitized data. A curve was fitted also to these maximum values to obtain the amplitude as function of x. The parameter γ was adjusted to produce the same phase difference between the nose and tail in the traveling wave model as in the raw data. The parameter ω was found using the relation $\omega = \frac{2\pi}{T}$ with T based on calculations of *Feeny and Feeny* (2013) for whiting and *Videler and Hess* (1984) for mackerel and saithe. Table 2.2 summarizes the resulting parameter settings.

	Amplitude, $a(x)$	ω	γ
whiting (body	$4.2124 \times 10^{-8} x^{10} - 2.2139 \times 10^{-6} x^9 + 4.8896 \times 10^{-6} x^{10} + 4.8896 \times 10^{-6} \times $	$4.836 \frac{rad}{sec}$	$0.4 \frac{rad}{in}$
frame)	$10^{-5}x^8 - 0.0006x^7 + 0.0044x^6 - 0.0210x^5 +$		
	$0.069x^4 - 0.15736x^3 + 0.22897x^2 - 0.1691x +$		
	0.05868		
whiting (Newto-	$0.0010677x^3 - 0.004265x^2 - 0.03544x +$	$4.736 \ \frac{rad}{sec}$	$0.62 \ \frac{rad}{in}$
nian frame)	0.22853		
mackerel	$8.3972 \times 10^5 x^9 - 1.1544 \times 10^6 x^8 + 6.519 \times$	41.06 $\frac{rad}{sec}$	$20 \frac{rad}{m}$
	$10^5 x^7 + -1.9472 \times 10^5 x^6 + 32897 x^5 -$		
	$3079.2x^4 + 138.72x^3 - 1.035x^2 - 0.1154x +$		
	0.0063153		
saithe	$-168.01x^6 + 124.08x^5 - 25.275x^4 +$	$22.2 \frac{rad}{sec}$	17.8 $\frac{rad}{m}$
	$0.064692x^3 + 0.68674x^2 - 0.077994x +$		
	0.006046		

Table 2.2 Average characteristics of three fish used in this research. Data compiled from *Feeny and Feeny* (2013); *Gray* (1933c); *Videler and Hess* (1984).

Time period used in this research is based on:

- for whiting, *Feeny and Feeny* (2013) used the fact that the seventh sample in Figure 2.1 would represent the start of the second half cycle, and would serve as the cyclic opposite of the first sample, regardless of the phase of the oscillation, meaning that the sampling is nearly commensurate with the oscillation period.
- 2. for mackerel and saithe, Videler and Hess (1984) calculated the time period using two methods. In the first method, the time intervals between successive extreme lateral positions were estimated for half of the time period T. In the second method, the lateral position was approximated as a function of time.

Using the digitized data from traveling wave model data of the whiting in an anteriorbody fixed coordinates, a traveling wave model is built based on data in Table 2.2. Figure 2.13 is produced for the traveling wave model (body frame) showing superposed snapshots for one period of motion. Each curve shows the mid-line at specific instance of time. The color of the curves are adjusted from red to blue based on time sequence.

Using the digitized data from traveling wave model data of the whiting in an inertial coordinate system, a traveling wave model is built based on data in Table 2.2. Figure 2.14 is produced for the traveling wave model (Newtonian frame) showing superposed snapshots for one period of motion. Each curve shows the mid-line at specific instance of time. The color of the curves are adjusted from red to blue based on time sequence.

Figure 2.15 shows time histories of body markers' oscillation from whiting (body frame). For better clarity, curves are shown for every five body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail. Figure 2.16 shows time histories of body markers' oscillation from whiting (Newtonian frame). For better clarity, curves are shown for every five body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from whiting (Newtonian frame). For better clarity, curves are shown for every five body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.

Using the digitized data from traveling wave model data of the average mackerel in a Newtonian frame, Figure 2.17 is produced showing superposed snapshots for one period of motion, wherein, each curve shows the mid-line at specific instance of time. For better clarity, curves are shown for every other sampling times. It is because there was 21 snapshots making the plot crowded and hard to discern. The curves are adjusted based on time sequence in the colour spectrum from red to blue. Also, Figure 2.18 shows time histories of oscillation for each body marker from mackerel traveling wave model data in a Newtonian frame. For better clarity, curves are shown for every four body markers. The color of the curves are adjusted from red to blue,



Figure 2.13 Superposed snapshots for one period of motion from whiting traveling wave model data in an anterior-body fixed frame. The color of the curves are adjusted from red to blue based on time sequence.



Figure 2.14 Superposed snapshots for one period of motion from whiting (Newtonian frame) traveling wave model. The color of the curves are adjusted from red to blue based on time sequence.



Figure 2.15 Time history curves for every five body markers from whiting (body frame) traveling wave model. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.



Figure 2.16 Time history curves for every five body markers from whiting (Newtonian frame) traveling wave model. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.



Figure 2.17 The superposed snapshots for one period of motion, from mackerel traveling wave model data in a Newtonian frame. Each curve shows the mid-line at specific instance of time. For better clarity, curves are shown for every other sampling time. The curves are adjusted based on time sequence in the colour spectrum from red to blue.

based on body-marker sequence from nose to tail.

Using the digitized data from traveling wave model data of average saithe in a Newtonian frame, Figure 2.19 is produced showing superposed snapshots for one period of motion of the saithe. For better clarity, curves are shown for every five sample times. The curves are adjusted based on time sequence in the colour spectrum from red to blue. Figure 2.20 shows time histories of oscillation for each body marker from saithe traveling wave model data in a Newtonian frame. For better clarity, curves are shown for every four body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.



Figure 2.18 Time history curves for body markers from mackerel traveling wave model data in a Newtonian frame. For better clarity, curves are shown for every four body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.



Figure 2.19 The superposed snapshots for one period of motion, from saithe traveling wave model data in a Newtonian frame. Each curve shows the mid-line at specific instance of time. For better clarity, curves are shown for every five sampling time. The curves are adjusted based on time sequence in the colour spectrum from red to blue.



Figure 2.20 Time history curves for body markers from saithe traveling wave model data in a Newtonian frame. For better clarity, curves are shown for every four body markers. The color of the curves are adjusted from red to blue, based on body-marker sequence from nose to tail.

2.2 Preparation of COD inputs

This study performs a complex model analysis by means of the technique COD. COD can receive as input, the signals of lateral oscillation from various reference points on the midline. We considered signals are coming from virtual markers distributed along the midline of the fish. The position of these virtual body markers is the same as:

- for whiting, the locations *Feeny and Feeny* (2013) digitized from the analogue data from *Gray* (1933c)
- for mackerel and saithe, the locations that each lateral oscillation plot in Videler and Hess (1984) belongs to.

Since for each fish, there are M virtual markers (or sensors) distributed on the specimen, there will be M real oscillatory signals, y_j , j = 1, ..., M. The vectors $\mathbf{y}_j = [y_j(t_1), ..., y_j(t_N)]^T$ are obtained by sampling at times t_1 through t_N , where N is the number of equidistant points in time, which are the sampling times. The sampling rate is N/T samples per second (the sampling interval is T/N), where T is the principal period. We omitted the time t = T when processing the data, so the periodic point is not doubled. By periodicity, the signal values are equal at t = 0 and t = T. That way, an $M \times N$ real ensemble matrix is built so that $\mathbf{Y} = [\mathbf{y}_1 \dots \mathbf{y}_M]^T$. As a result, for both the real motion and the traveling wave model, digitized data of 49×12 was produced for whiting, 41×20 for mackerel, and 41×51 for saithe.

2.2.1 Complexification of the real oscillatory signals

In order to apply COD, the real signals are first converted to complex signals. The process of complexification is done by the half-spectrum inversion method or Hilbert transform (*Wolfgang et al.* (1999)). To be more specific, if y(t) is a real signal, and z(t) is its complexfied signal, then y(t) = Re(z(t)) and then the Hilbert transform of y is $y_H(t) = Im(z(t))$ (*Feeny and Feeny* (2013)). Thus the complex analytic signal is $z(t) = y(t) + iy_H(t)$. On this basis, the real vectors \mathbf{y}_j , that are already generated, can be complexified to \mathbf{z}_j , and from these, an $M \times N$ complex ensemble matrix can be built so that $\mathbf{Z} = [\mathbf{z}_1 \dots \mathbf{z}_M]^T$.

2.3 Application of COD

2.3.1 Complex modal decomposition

The complex "correlation" matrix is then constructed as $\mathbf{R} = \mathbf{Z}\overline{\mathbf{Z}}^T/N$, where the overbar indicates complex conjugation. The matrix $\mathbf{R} \in \mathbb{C}^{M \times M}$, where \mathbb{C} is the set of complex numbers. We used COD to extract the eigenvalues, α_i , and eigenvectors, \mathbf{w}_i , i = 1, ..., M, through the eigenvalue problem associated with \mathbf{R} . Since \mathbf{R} is Hermitian ($\overline{\mathbf{R}}^T = \mathbf{R}$, the α_i are real and \mathbf{w}_i are normalized ($\overline{\mathbf{w}}_i^T \mathbf{w}_j = \delta_{ij}$). The eigenvectors or COMs (complex orthogonal modes) represent the principal shapes of complex modal waveforms. A modal motion is associated with one COM and the total motion is a sum of modal motions. The eigenvalues or COVs (complex orthogonal values) represent the mean squared amplitude of modulations corresponding to each COM (*Feeny* (2008)). The total motion, \mathbf{Z} , then can be expressed by the following matrix multiplication,

$$\mathbf{Z} = \mathbf{W}\mathbf{Q} \tag{2.1}$$

where \mathbf{W} is the matrix whose columns are COMs and the matrix matrix \mathbf{Q} is the complex modal coordinate ensemble. If the modes in \mathbf{W} are normalized, then by complex orthogonality, the matrix \mathbf{Q} becomes

$$\mathbf{Q} = \mathbf{W}^{-1}\mathbf{Z} = \bar{\mathbf{W}}^T\mathbf{Z}$$

This is a complex modal coordinate (COC) ensemble matrix, the rows of which are the samples of each modal coordinate, $q_j(t)$, sampled at $t = t_1$, ..., t_N (*Feeny* (2008)).

2.3.2 Modally reduced motion

2.3.2.1 Single-mode motion from dominant COC

In our case study, fewer than N meaningful modes were contributing to the total motion while the other modes were noise dominated and insignificant. This partly has to do with the rank of matrix \mathbf{R} that is bounded by the minimum of N and M. It means that at most, N meaningful modes can be expected (*Feeny* (2008)). It turns out that in each case, only one mode was dominant over other modes, and it captured more than 99 percent of the energy for both raw data and for traveling wave model. Reduced modal motion can be constructed by building the $M \times N$ modally reduced motion ensemble, \mathbf{Z}_r ,

$$\mathbf{Z}_r = \mathbf{W}_r \mathbf{W}_r^T \mathbf{Z} \tag{2.2}$$

where $\mathbf{W}_r = [\mathbf{w}_1; \ldots; \mathbf{w}_r]$, is $M \times r$, where r < M. Also, the matrix \mathbf{Z}_r can be expressed in the equivalent form,

$$\mathbf{Z}_r = \mathbf{W}_r \mathbf{Q}_r \tag{2.3}$$

where \mathbf{Q}_r can be obtained by the following equation:

$$\mathbf{Q}_r = \mathbf{W}_r^T \mathbf{Z} \tag{2.4}$$

The rows of \mathbf{Q}_r are samplings of the associated modal coordinates $\mathbf{q}_j(t), j = 1, \ldots, r$. In r = 1, and the only dominant COC, \mathbf{Q}_1 , and \mathbf{Z}_1 are needed in the re-animation of single (dominant) modal motion. In this case, \mathbf{Q}_1 is $1 \times N$, and the dominant mode \mathbf{W}_1 is $M \times 1$.

2.3.2.2 Single-mode motion from harmonic modulation

A harmonic-based re-animation of single mode motion is made as $\mathbf{Z}_1 = \mathbf{W}_1 \mathbf{Q}_h$, where \mathbf{W}_1 is the dominant mode $(M \times 1)$, and the $1 \times N$ vector \mathbf{Q}_h is a harmonic function of time defined by $\mathbf{Q}_{h_k} = e^{i41t_k}$ for k = 1, ..., N using for example a frequency of $\omega = 41$ rad/sec from the work of *Videler and Hess* (1984). The single-mode motion from harmonic modulation can be obtained $\mathbf{Y}_1 = real(\mathbf{Z}_1)$.

2.3.3 Dissection of main modes into traveling and standing addends

The COMs contain information about the degree that the motion is traveling (Feeny (2008) and Feeny and Feeny (2013)). COD was also used to dissect the dominant COM into traveling part and standing part for raw data and the traveling wave motion, such that

$$\mathbf{w} = \mathbf{w}_t + \mathbf{w}_s \tag{2.5}$$

where \mathbf{w}_t is the traveling part and $\mathbf{w}s$ is the standing part and both are in general complex. Then $\mathbf{w}_s = \mathbf{c}_s + i\mathbf{d}_s$, etc. *Feeny* (2008). In this regard, in order to evaluate the contribution of each traveling and standing parts in the total motion, a traveling index is proposed in *Feeny* (2008). This is the reciprocal of the condition number of the matrix whose two columns are the real and imaginary components of the complex mode.

2.3.4 COD calculation of wavelength, amplitude, frequencies, and wave speeds

The complex modes and modal coordinates can obtain many information like wavelength, amplitude, frequencies, and wave speeds *Feeny* (2008). The wavelength, designated by λ is the spatial period of a periodic wave, or in other words, the distance over which the wave's shape or the wave's phase repeats. Wavelength is a characteristic of both traveling waves and standing waves. So, based on this definition, in the traveling wave of this research's interest, $a(x) \cos(\omega t - \frac{x}{\lambda})$, one way to calculate λ is by considering the snapshots of swimming midline and seeking for the distance between two consecutive crests, troughs, or zero crossings (since the argument of cosine function is the same for each of these couples).

Another way is to use COD and calculate the phase corresponding to each COM element, since the i-th element have the phase and amplitude information related to the i-th body marker (*Feeny and Feeny* (2013)). Since COMs and COCs are complex vectors, each of their elements can be expressed as $\rho_i e^{\phi(x_i)}$, where ρ is the magnitude and ϕ is the phase. For COMs, this magnitude is proportional to the oscillation amplitude of corresponding body marker. The ϕ 's show the relative phase of body markers. The COM thus can be expressed as vector of exponential functions of body markers location,

$$\mathbf{w}_{i} = \begin{bmatrix} \rho(x_{1})e^{\phi(x_{1})}\\ \rho(x_{2})e^{\phi(x_{2})}\\ \vdots\\ \rho(x_{M})e^{\phi(x_{M})} \end{bmatrix}$$

This shows that the amplitude and phase of every main mode at *i*-th body marker can be calculated from $\rho(x_i)$ and $\phi(x_i)$. In turn, the computations based on the phase $\phi(x_i)$ can be made to estimate modal wave number. The wave number or the spatial frequency of a wave, γ is defined as $\gamma = \frac{2\pi}{\lambda}$ with the unit of radians per unit distance. Differentiation of ϕ with respect to x will determine γ ($\frac{\partial \phi}{\partial x} = \gamma$). Since we had discrete data at sampling locations, the central finite difference scheme was used to approximate the rate $\phi(x)$ varies with x as follows,

$$\frac{\partial \phi}{\partial x} = \frac{\Delta \phi_j}{\Delta x} = \gamma_j \tag{2.6}$$

The more number of body markers, the more precise will be the calculation of γ . COCs also are complex quantities. Each COC element corresponds to one of the sampling time. One basic way to obtain ω is to consider the time histories related to each body marker (constant location) in the traveling wave model, $a(x) \cos(\omega t - \frac{x}{\lambda})$, and calculate the time distance between two times that generate a same argument in the cosine function.

Another method is to use COD to generate the complex modal coordinates, COC (rows in ensemble \mathbf{Q}), and calculate phase corresponding to each COC element. The frequency information can be calculated by finding two sampling times giving the same argument. The phase speed is calculated based on ω and γ . The wave speed c of a traveling wave is given by

$$c = \frac{\omega}{\gamma}$$

CHAPTER III

Results

This research applied a complex modal analysis to signals of lateral oscillation sampled from a number of reference points on midline of three swimming carangiform fish. In the following, the findings of this study based upon the methodology applied is described for the forward swimming motion of three fish: a whiting, a mackerel, and a saithe.

3.1 Whiting fish - body frame

3.1.1 Complex modes from swimming midline data

3.1.1.1 Eigenvalues (COVs) and the number of dominant modes

Figures 3.1 and 3.2 show the plots of the COVs in descending order in the linear scale and logarithmic scale, respectively. It can be seen in Figure 3.1 that only one mode (the first) captures most (more than 99 percent) of the energy and the other modes are insignificant compared to the first mode.



Figure 3.2 Plots of the COVs in descending order in a logarithmic scale for whiting (body frame) raw data (blue) and the traveling wave model (red).



Figure 3.1 Plots of the COVs in descending order in a linear scale for the whiting (body frame) raw data (blue) and the traveling wave model (red) for whiting (body frame). Only one mode (the first) captures more than 99 percent of the energy.

It was observed that the values of COVs in the traveling wave model (especially

the first COV) are dependent on the amplitude function used in building the traveling wave model. Thus, a more precise curve fitting (of the amplitude function) produces a better traveling wave model. The precision of the amplitude function depends on the number of body markers. The higher the number of body markers, the more precise the amplitude function will be.

3.1.1.2 Dominant COM

Since the first mode is the dominant mode, the focus of the study will be on the first COM. Figure 3.3 shows the plots of the dominant COM in the complex plane for the whiting (body frame) raw data (blue) and the traveling wave model (red). Based on chapter II, COMs contain the information of the phase and amplitude of oscillation of each body marker. Figure 3.4 shows the plots of COM elements phase (ϕ) vs. body marker for the whiting (body frame) raw data (blue) and the traveling wave model (red). It can be seen that there are two jumps in phase plot in the head region body markers (first 17 body markers). These jumps are related to the very small magnitude of oscillation, in which the phase is less meaningful and less clearly calculated and the associated quick phase change between body markers 4 and 5 (the first jump) and between body markers 16 and 17 (the second jump). This can be better illustrated in Figure 3.5, which shows the COM elements belonging to the head region for the whiting (body frame) raw data, wherein the COM wraps quickly around the origin (first 17 body markers). On the other hand, the slope of the phase plot is nearly constant for the whiting (body frame) traveling wave model but variable in the raw data. It can be seen that the absolute values of the slopes in the head regions between the two jumps are similar $(0.17 \ rad/in$ and $0.11 \ rad/in)$, and these are less than the slope of the plot in the posterior region (0.4 rad/in). It should be noted that in building the traveling wave model, the value of phase plot slope in the posterior region is used. If using an average value of $\gamma = 0.32 \ rad/in$ in the traveling wave model, the resulting COM is plotted in Figure 3.6. Comparing the Figures 3.3 and 3.6, it can be concluded that a value of $\gamma = 0.4 \ rad/in$ obtains a closer resulting COM between the traveling wave model and the raw data, so this value is chosen for the traveling wave model for whitings all plots (body frame).



Figure 3.3 Dominant COM in raw data (blue) and traveling wave model (red) for the whiting (body frame) ($\gamma = 0.4 \ rad/in$).


Figure 3.4 Plot of the COM elements' phases for the whiting (body-frame) raw data (blue) and the traveling wave model (red).



Figure 3.5 Dominant COM elements belonging to the head region (first 17 body markers) wrap quickly around the origin in whiting (body-frame) raw data.



Figure 3.6 Dominant COM in raw data (blue) and traveling wave model (red) for the whiting (body frame) ($\gamma = 0.32 \ rad/in$).

COM elements' magnitude is representative of oscillation amplitude of each body marker. Figure 3.7 is the plot of COM elements' magnitudes vs. body marker for raw data and the traveling wave model. The small differences can be due to the amplitude function chosen in building the traveling wave model. The real part and imaginary part of COM is shown vs. body marker in Figure 3.8. Depending on the time, the midline shape is in transition between the shapes of real part and the imaginary part. The phase difference between the real and imaginary parts shows that the wave is not pure standing wave.



Figure 3.7 Plot of the COM elements' magnitude in the raw data (blue) and the traveling wave model (red) for the whiting.



Figure 3.8 Plot of the COM's real and imaginary parts for the whiting (body frame) raw data (blue) and traveling wave model (red).

3.1.1.3 Dominant COC

Figure 3.9 shows a plot of the dominant modal coordinate, COC, in the complex plane, for the whiting (body frame) raw data and the traveling wave model. As discussed in chapter II, phase change rates with respect to time is calculated by computing the dominant COC and its elements' phase. Figure 3.10 shows the phase changes with respect to time. Figure 3.11 shows plots of the real and imaginary parts of dominant COC, in different colors for the whiting (body frame) raw data and the traveling wave model.



Figure 3.9 Plot of dominant COC (complex orthogonal coordinate) in complex plane in the raw data (blue) and the traveling wave model (red) for the whiting (body frame).



Figure 3.10 Plot of the COC elements' phase, θ , vs. sampling time for the whiting (body frame) raw data (blue) and traveling wave model (red).



Figure 3.11 Plots of the real and imaginary parts of dominant COC, for the whiting (body frame) raw data (blue) and the traveling wave model (red).

3.1.2 Single mode harmonic motion from dominant COC

Using the dominant COC, the dominant modal motion is reanimated. Figures 3.12 and 3.13 show the snapshots of single mode motion (Y_1) for the whiting (body frame) raw data and the traveling wave (the top subfigure). For comparison, the difference between the single mode motion and total motion $(Y - Y_1)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. The colors of the curves are adjusted from red to blue based on the time sequence. From Figures 3.12 and 3.13, it can be seen that $Y - Y_1$ in the middle subfigure is not random-looking, suggesting that although the dominant mode is significantly dominant, the second mode is not just noise.

Figures 3.14 and 3.15 show the time histories of single mode harmonic motion (Y_1) for the whiting (body frame) raw data and the traveling wave (the top subfigure). For comparison, the difference between the single mode motion and total motion $(Y - Y_1)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. For better clarity, the colors of the curves are adjusted based on the body-marker sequence from red to blue. In all cases, the axis scale shows that $Y - Y_1$ is very small, consistent with mode one be strongly dominant (99%).



Figure 3.12 Superposed snapshots of single mode motion Y_1 (top), the difference between the single mode motion and the total motion $Y - Y_1$ (middle), and the total motion Y (bottom), for the whiting (body frame) raw data. The colors of the colors of the curves are adjusted from red to blue based on the time sequence.

3.1.2.1 Single mode motion from harmonic modulation

Based on chapter II, the modal coordinate from harmonic modulation are built into Q_h and the dominant modal motion is reanimated. Figure 3.16 shows the dominant



Figure 3.13 Superposed snapshots of single mode motion Y_1 (top), the difference between the single mode motion and the single mode motion $Y - Y_1$ (middle), and the total motion Y (bottom), for the whiting (body frame) traveling wave model. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.14 Superposed time histories of single mode harmonic motion Y_1 (top), the difference between the single mode harmonic motion and the total motion $Y - Y_1$ (middle), and the total motion Y (bottom), for the whiting (body frame) raw data. For better clarity, the curves are shown for every five body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.15 Superposed time histories of single mode harmonic Y_1 (top), the difference between the single mode and total motion $Y - Y_1$ (middle), and the total motion Y (bottom), for whiting (body frame) traveling wave model. For better clarity, the curves are shown for every five body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.

COC for the whiting (body frame) raw data (blue) and the traveling wave model (red) and Q_h (purple). It can be seen that COC of the traveling wave model is quite close to Q_h . However, COC of the raw data periodically approaches Q_h and gets behind it, suggesting that in the former, the rate of change of its elements' phases is not constant over time (based on how Q_h is constructed, the rate of change of its elements' phases is constant).



Figure 3.16 Dominant COC for the whiting (body frame) raw data (blue) and the traveling wave model (red) and Q_h (green).

Figures 3.17 and 3.18 show the snapshots of single mode harmonic motion (Y_h) for the whiting (body frame) raw data and the traveling wave (the top subfigure) from harmonic modulation. For comparison, the difference between the single mode harmonic motion and the single mode motion $(Y_1 - Y_h)$, and the single mode motion (Y_1) are shown in the middle and bottom subfigures, respectively. The colors of the curves are adjusted from red to blue based on the time sequence.

Figures 3.19 and 3.20 show the time histories of single mode harmonic motion (Y_h) for the whiting (body frame) raw data and the traveling wave (the top subfigure) from harmonic modulation. For comparison, the difference between the single mode harmonic motion and the single mode motion $(Y_1 - Y_h)$, and the single mode motion (Y_1) are shown in the middle and bottom subfigures, respectively. For better clarity, the colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.17 Superposed snapshots of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and the single mode motion $Y_1 - Y_h$ (middle), and the single mode motion Y_1 (bottom) for the whiting (body-frame) raw data. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.18 Superposed snapshots of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and the single mode motion $Y_1 - Y_h$ (middle), and the single mode motion Y_1 (bottom) for the whiting (body-frame) traveling wave model. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.19 Superposed time histories of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and the single mode motion $Y_1 - Y_h$ (middle), and the single mode motion Y_1 (bottom) for the whiting (body-frame) raw data. For better clarity, curves are shown for every five body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.20 Superposed time histories of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and the single mode motion $Y_1 - Y_h$ (middle), and the single mode motion Y_1 (bottom) for the whiting (body-frame) traveling wave model. For better clarity, curves are shown for every five body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.

3.1.3 Dissection of COM into traveling and standing parts

Based on chapter II, the dominant COM was dissected into its traveling and standing parts. The traveling index of 0.4832 was obtained for the whiting (body frame) raw data, while for the traveling wave model, the traveling index was calculated as 0.3869. Figure 3.21 shows the traveling parts of dominant modes of the raw data and traveling part of dominant mode of the traveling wave model. Figure 3.22 shows the standing part of dominant mode for the whiting (body frame) raw data and standing part of dominant mode for the whiting (body frame) traveling wave model. It can be seen that the plots are straight lines, meaning that in the standing part of the motion, all the body markers have the same phase during oscillation. It can be observed also that the angle of red and blue plots are different, meaning at the same time instant, the standing part of the traveling wave model is at a higher angle comparing to the standing part of the raw data. The closeness of these angles can be chosen as a criteria for closeness of the traveling wave model to the real motion and can be adjusted by tuning some parameters in the traveling wave model. However, it may be impossible to have all criteria 100 % complied and we considered an optimum final model wherein all criteria are to some extent satisfied. Figures 3.23 and 3.24 show the dominant COM with its standing and traveling parts for the whiting (body frame) raw data and the traveling wave model. Note that forr each body marker, the vector sum of the standing and traveling parts equals to the dominant COM.



Figure 3.22 The standing part of dominant mode for the whiting (bodyframe) raw data (blue) and the traveling wave model (red).



Figure 3.21 The traveling parts of dominant modes for the whiting (bodyframe) raw data (blue) and the traveling wave model (red).

Based on calculation of the traveling and standing parts of COM, the total motion was dissected into its traveling and standing parts. Figure 3.25 shows the snapshots of traveling (top) and standing (bottom) parts of single mode motion for the whiting



Figure 3.23 Dominant COM (red) with its traveling (blue) and standing (purple) parts parts in complex plane for the whiting (body-frame) raw data.



Figure 3.24 Dominant COM (red) with its traveling (blue) and standing (purple) parts in complex plane for the whiting (body-frame) the traveling wave model.

(body frame) raw data. Figure 3.26 shows the time histories of traveling (top) and standing (bottom) parts of single mode motion for the whiting (body frame) raw data.



Figure 3.25 Superposed snapshots for the traveling (top) and standing (bottom) parts of single mode motion for the whiting (body-frame) raw data. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.26 Superposed time histories for the traveling (top) and standing (bottom) parts of single mode motion for the whiting (body-frame) traveling wave model. For better clarity, curves are shown for every five body markers. The colors of the curves are adjusted based on body markers sequence in the colour spectrum, from red to blue.

3.1.4 Wave number, frequency and wave speed

The modal wave speed through the fish can be investigated via the complex mode for both the raw data and the traveling wave model. From chapter II, the information for the oscillation phase ¹, ϕ , as a function of x can be derived from the dominant mode. Previously in Figure 3.4, ϕ vs. x was plotted for the whiting (body frame) raw data and the traveling wave model. Based on chapter II, the wave number, γ , can be calculated by differentiation of ϕ with respect to x. However, since these data are discrete, the wave number was calculated by using the central finite difference scheme. For the first and last body marker, the forward and backward finite difference is used respectively.

Figure 3.27 shows the plot of γ vs. x. The traveling wave model, based on how it is constructed, will have a constant wave number. The calculations show also the the traveling wave's dominant COM has a constant phase change. The jumps in the figure are due to the jumps in ϕ vs. body marker plot between body marker 4 and 5, and between body markers 16 and 17. After body marker 17, there is a oscillation in γ plot around the value $-0.4 \ rad/in$ which is the γ used in building the traveling wave model.

Previously in Figure 3.10, ϕ vs. time, t was plotted for the whiting (body frame) raw data and the traveling wave model. Based on chapter II, the frequency, ω , can be calculated by differentiation of ϕ with respect to t. However, since these data are discrete, the frequency is calculated by using the central finite difference scheme. For the first and last sampling time, the forward and backward finite difference is used respectively. The result is shown in Figure 3.28.

Based on the approximation of γ and the parameter ω used in building the traveling wave model (the slope of the red line in Figure 3.10, the wave speed can be

¹Hereinafter we use ϕ to indicate the phase of oscillation that can be function of time and space. For example, according to this definition, in the traveling wave model equation $y(x,t) = a(x) \cos(\omega t + \gamma x)$, the argument of cos function is count as ϕ



Figure 3.27 Wave number vs. body marker for the whiting (body-frame) raw data (blue) and traveling wave model (red), calculated by using finite difference scheme applied to the ϕ plot.



Figure 3.28 Plot of frequency, ω , vs. sampling time for the whiting (body frame) raw data (blue) and traveling wave model (red).



Figure 3.29 Plot of wave speed based on γ calculated from COM phase for the whiting (body frame) raw data (blue) and traveling wave model (red).

calculated from chapter II. Figure 3.29 shows the plot of wave speed for the whiting (body frame) raw data based on γ calculated from COM phase, and the traveling wave model. Again, the jumps in the figure are due to the jumps in *phi* vs. body marker plot between body marker 4 and 5, and between body markers 16 and 17. After body marker 17, there are oscillations in γ plot around the value $-26 \ rad/in$ which is the γ used in building the traveling wave model.

3.2 Whiting fish - Newtonian frame

3.2.1 Complex modes from swimming midline data

3.2.1.1 Eigenvalues (COVs) and the number of dominant modes

Figures 3.30 and 3.31 show the plots of the COVs in descending order in the linear scale and logarithmic scale, respectively. It can be seen in Figure 3.30 that only one mode (the first) captures most (more than 99 percent) of the energy and the other modes are insignificant compared to the first mode.

It was observed that the value of COVs in the traveling wave model (especially the first COV) is dependent on the amplitude function used in building the traveling wave model. Thus, a more precise curve fitting (to amplitude function) produces a better traveling wave model.

3.2.1.2 Dominant COM

Since the first mode is the dominant mode, the focus of the study will be on the first COM. Figure 3.32 shows the plots of the main COM in the complex plane for the whiting (Newtonian frame) raw data (blue) and the traveling wave model (red). Based on chapter II, COMs contain the information of the phase and amplitude of oscillation of each body marker. Figure 3.33 shows the plots of COM elements phase (ϕ) vs. body marker for the whiting (Newtonian frame) raw data (blue) and the traveling wave model (red).

Plots of COM elements' magnitude vs. body marker is shown in Figure 3.34. The small differences can be due to the amplitude function chosen in building the traveling wave model. The optimum amplitude function (table 2.2 is obtained by trial and error in fitting curves, with the optimality criteria being the closeness of COVs and COMs (between the two data sets). In order to compare the real and imaginary parts of the two data sets, see Figure 3.35. It can be seen that the undulation of the



Figure 3.30 Plots of the COVs in descending order in a linear scale for the whiting (Newtonian frame) raw data (blue) and the traveling wave model (red) for the whiting (body frame). Only one mode (the first) captures more than 99 percent of the energy.



Figure 3.31 Plots of the COVs in descending order in a logarithmic scale for the whiting (Newtonian frame) raw data (blue) and the traveling wave model (red) for the whiting (body frame).



Figure 3.32 Dominant COM in raw data (blue) and traveling wave model (red) for whiting (Newtonian frame) ($\gamma = 0.4 \ rad/in$).



Figure 3.33 Plot of the COM elements' phases in the raw data (blue) and the traveling wave model (red) for whiting (Newtonian frame).



Figure 3.34 Plot of the COM elements' magnitude for the whiting (Newtonian frame) raw data and the traveling wave model.

real parts lag the imaginary parts by about 90° .



Figure 3.35 Plot of the COM's real and imaginary parts in the raw data (blue) and the traveling wave model (red) for whiting (Newtonian frame).

3.2.1.3 Dominant COC

Figure 3.36 shows a plot of the dominant modal coordinate, COC, in the complex plane, for the whiting (Newtonian frame) raw data and the traveling wave model. Based on calculation of COC elements' phase, Figure 3.37 is produced. The figure shows the COC elements' phase, ϕ , vs. sampling time for the whiting (Newtonian frame) raw data (blue) and traveling wave model (red). Figure 3.38 shows plots of the real and imaginary parts of dominant COC, in different colors for the whiting (Newtonian frame) raw data and the traveling wave model.



Figure 3.36 Plot of dominant COC (complex orthogonal coordinate) in complex plane in the raw data (blue) and the traveling wave model (red) for whiting (Newtonian frame).



Figure 3.37 Plot of the COC elements' phase vs. sampling time for the whiting (Newtonian frame) raw data (blue) and traveling wave model (red).



Figure 3.38 Plots of the real and imaginary parts of dominant COC, for whiting (Newtonian frame) raw data (blue) and the traveling wave model (red).

3.2.2 Single mode motion from dominant COC

Using the dominant COC, the dominant modal motion is reanimated. Figures 3.39 and 3.40 show the snapshots of single mode motion (Y_1) for the whiting (Newtonian frame) raw data and the traveling wave (the top subfigures). For comparison, the difference between the single mode motion and total motion $(Y - Y_1)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. The colors of the curves are adjusted from red to blue based on the time sequence. From Figures 3.39 and 3.40, it can be seen that $Y - Y_1$ in the middle subfigure is not random-looking, suggesting that although the dominant mode is significantly dominant, however the second mode is not just noise.

Figures 3.41 and 3.42 show the time histories of single mode motion (Y_1) for the whiting (Newtonian frame) raw data and the traveling wave (the top subfigure). For comparison, the difference between the single mode motion and total motion $(Y - Y_1)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. For better clarity, the colors of the curves are adjusted based on the body-marker sequence from red to blue. In all cases, the axis scale shows that $Y - Y_1$ is very small, consistent with mode one be strongly dominant (99%).



Figure 3.39 Superposed snapshots of single mode motion Y_1 (top), the difference between the single mode motion and the total motion $Y - Y_1$ (middle), and the total motion Y, (bottom) for the whiting (Newtonian frame) raw data. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.40 Superposed snapshots of single mode motion Y_1 (top), the difference between the single mode motion and the total motion $Y - Y_1$ (middle), and the total motion Y (bottom), for whiting (Newtonian frame) traveling wave model. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.41 Superposed time histories of single mode motion Y_1 (top), the difference between the single mode motion and the total motion $Y - Y_1$ (middle), and the total motion Y, (bottom) for the whiting (Newtonian frame) raw data. For better clarity, curves are shown for every five body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.


Figure 3.42 Superposed time histories of single mode motion Y_1 (top), the difference between the single mode and total $Y - Y_1$ (middle), and the total motion Y (bottom), for the whiting (Newtonian frame) traveling wave model. For better clarity, curves are shown for every five body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.

3.2.2.1 Single mode harmonic motion from harmonic modulation

Based on chapter II, the modal coordinate time histories from harmonic modulation are built into Q_h and the dominant modal motion is reanimated. Figure 3.43 shows the dominant COC for the whiting (Newtonian frame) raw data (blue) and the traveling wave model (red) and Q_h (purple). It can be seen that COC of the traveling wave model is quite close to Q_h . However, COC of the raw data periodically approaches Q_h and gets behind it, suggesting that in the former, the rate of change of its elements' phase is not constant over time (based on how Q_h is constructed, the rate of change of its elements' phase is constant).



Figure 3.43 Dominant COC for the whiting (Newtonian frame) raw data (blue), the traveling wave model (red) and Q_h (green).

Figures 3.44 and 3.45 show the snapshots of single mode harmonic motion (Y_h) for the whiting (Newtonian frame) raw data and the traveling wave (the top subfigure) from harmonic modulation. For comparison, the difference between the single mode harmonic motion and total motion $(Y - Y_h)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. The colors of the curves are adjusted from red to blue based on the time sequence.

Figures 3.46 and 3.47 show the time histories of single mode harmonic motion (Y_h) for the whiting (Newtonian frame) raw data and the traveling wave (the top subfigure) from harmonic modulation. For comparison, the difference between the single mode harmonic motion and total motion $(Y - Y_h)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. For better clarity, the colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.44 Superposed snapshots of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and the single mode motion $Y_1 - Y_h$ (middle), and the total motion Y (bottom), for whiting (Newtonian frame) raw data. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.45 Superposed snapshots of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and the single mode motion $Y_1 - Y_h$ (middle), and the single mode motion Y_1 (bottom) for whiting (Newtonian frame) traveling wave model. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.46 Superposed time histories of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and the single mode motion $Y_1 - Y_h$ (middle), and the single mode motion Y_1 (bottom) for whiting (Newtonian frame) raw data. For better clarity, curves are shown for every five body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.47 Superposed time histories of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and the single mode $Y_1 - Y_h$ (middle), and the single mode motion Y_1 (bottom) for whiting (Newtonian frame) traveling wave model. For better clarity, curves are shown for every five body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.48 The traveling parts of dominant modes for whiting (Newtonian frame) raw data (blue) and the traveling wave model (red).

3.2.3 Dissection of COM into traveling and standing parts

Based on chapter II, the dominant COM was dissected into its traveling and standing parts. The traveling index of 0.5205 was obtained for the whiting (Newtonian frame) raw data, while for the traveling wave model, the traveling index was calculates 0.5634. Figure 3.48 shows the traveling parts of dominant modes of the raw data and traveling part of dominant mode of the traveling wave model. Figure 3.49 shows the standing part of dominant mode for whiting (Newtonian frame) raw data and standing part of dominant mode for whiting (Newtonian frame) raw data and standing part of dominant mode for whiting (Newtonian frame) traveling wave model. Figures 3.50 and 3.51 show the dominant COM with its standing and traveling parts for whiting (Newtonian frame) raw data and the traveling wave model.



Figure 3.50 Dominant COM (red) with its traveling (blue) and standing (purple) parts parts in complex plane for whiting (Newtonian frame) raw data.



Figure 3.49 The standing part of dominant mode for whiting (Newtonian frame) raw data (blue) and the traveling wave model (red).

Based on calculation of COM's traveling and standing parts, the total motion



Figure 3.51 Dominant COM (red) with its traveling (blue) and standing (purple) parts in complex plane for whiting (Newtonian frame) the traveling wave model.

was dissected into its traveling and standing parts. Figure 3.52 shows the snapshots of traveling (top) and standing (bottom) parts of single mode motion for whiting (Newtonian frame) raw data. Figure 3.53 shows the time histories of traveling (top) and standing (bottom) parts of single mode motion for the whiting (Newtonian frame) raw data.



Figure 3.52 Superposed snapshots for the traveling (top) and standing (bottom) parts of single mode motion for whiting (Newtonian frame) raw data. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.53 Superposed time histories for the traveling (top) and standing (bottom) parts of single mode motion for whiting (Newtonian frame) raw data. For better clarity, curves are shown for every five body markers. The colors of the curves are adjusted based on body markers sequence from red to blue.



Figure 3.54 Plot of Wave number, γ vs. body marker for the whiting (Newtonian frame) raw data (blue) and traveling wave model (red), calculated by using finite difference scheme applied to the ϕ plot.

3.2.4 Wave number, frequency and wave speed

The modal wave speed through the fish can be investigated via the complex mode for both the raw data and the traveling wave model. From chapter II, the information for the phase, ϕ , as a function of x can be derived from the dominant mode. Previously in Figure 3.33, ϕ vs. x was plotted for the whiting (Newtonian frame) raw data and the traveling wave model. Based on chapter II, the wave number, γ , can be calculated by differentiation of ϕ with respect to x. However, since these data are discrete, the wave number was calculated by using the central finite difference scheme. For the first and last body marker, the forward and backward finite difference is used respectively. Figure 3.54 shows the plot of γ vs. x. The traveling wave model, based on how it is constructed, will have a constant wave number. The calculations show also the the traveling wave's dominant COM has a constant phase change. Based on this approximation of γ , the wavelength and wave speed can be calculated from chapter II.



Figure 3.55 Plot of frequency, ω , vs. sampling time for the whiting (Newtonian frame) raw data (blue) and traveling wave model (red).

Previously in Figure 3.37, ϕ vs. time, t was plotted for the whiting (Newtonian frame) raw data and the traveling wave model. ω is calculated by differentiation of ϕ with respect to t. Since these data are discrete, the frequency is calculated by using the central finite difference scheme. For the first and last sampling time, the forward and backward finite difference is used respectively. The result is shown in Figure 3.55. Figure 3.56 shows the plot of wave speed for the whiting (Newtonian frame) raw data based on γ calculated from COM phase (Figure 3.37, and the average ω (see chapter II).



Figure 3.56 Plot of wave speed based on γ calculated from COM phase for the whiting (Newtonian frame) raw data (blue) and traveling wave model (red).

3.3 Mackerel fish

3.3.1 Complex modes from swimming midline data

3.3.1.1 Eigenvalues, COVs, and the number of dominant modes

Figures 3.57 and 3.58 show the plots of the COVs in descending order in the linear scale and logarithmic scale, respectively. It can be seen in Figure 3.57 that only one mode (the first) captures most (more than 99 percent) of the energy and the other modes are insignificant compared to the first mode.

It was observed that the value of COVs in the traveling wave model (especially the first COV) is dependent on the amplitude function used in building the traveling wave model. Thus, a more precise curve fitting (to amplitude function) produces a better traveling wave model. The precision of the amplitude function depends on the number of body markers.

3.3.1.2 Dominant COM

Since the first mode is the dominant mode, the focus of the study will be on the first COM. Figures 3.59 shows the plots of main COM in complex plane for the mackerel raw data and the traveling wave model. For comparison, COMs are shown in different colors for the mackerel raw data and the traveling wave model. Plots of COM elements' phases and COM elements' magnitude vs. body marker are shown in Figures 3.60 and 3.61 for the mackerel raw data and the traveling wave model. It can be seen that the slope of the phase plot is constant for the traveling wave model but variable in the raw data. Figure 3.62 shows the real and imaginary parts of the COMs for both data sets.



Figure 3.57 Plots of the COVs in descending order in a linear scale for the whiting (Newtonian frame) raw data (blue) and the traveling wave model (red). Only one mode (the first) captures more than 99 percent of the energy.



Figure 3.58 Plots of the COVs in descending order in a logarithmic scale for the mackerel raw data (blue) and the traveling wave model (red).



Figure 3.59 Plot of dominant COM in complex plane for the mackerel raw data (blue) and traveling wave model (red).



Figure 3.60 Plot of the COM elements' phases for the mackerel raw data (blue) and the traveling wave model (red).



Figure 3.61 Plot of the COM elements' magnitude for the mackerel raw data (blue) and the traveling wave model (red).



Figure 3.62 Plot of the COMs' real and imaginary parts for the mackerel raw data (blue) and the traveling wave model (red).

3.3.1.3 Dominant COC

Figure 3.63 shows plot of the dominant modal coordinate, COC, in the complex plane, for the mackerel raw data and traveling wave model. Based on calculation of COC elements' phase, Figure 3.64 is produced. The figure shows the COC elements' phase, ϕ , vs. sampling time for the mackerel raw data (blue) and traveling wave model (red). Figure 3.65 shows plots of the real and imaginary parts of dominant COC, in blue for the mackerel raw data and red for the traveling wave model. The real and imaginary parts of COCs provide the two time-modulation components of the complex wave, representing the time modulations of the 90°-phased components of the wave.



Figure 3.63 Plot of dominant COC (complex orthogonal coordinate) in complex plane for the mackerel raw data (blue) and the traveling wave model (red).



Figure 3.64 Plot of the COC elements' phase, θ , vs. sampling time for the mackerel raw data (blue) and traveling wave model (red).



Figure 3.65 Plots of the real and imaginary parts of dominant COC, for the mackerel raw data (blue) and the traveling wave model (red).

3.3.2 Single mode motion from dominant COC

Using the dominant COC, the dominant modal motion is reanimated. Figures 3.66 and 3.67 show the snapshots of single mode motion (Y_1) for the mackerel raw data and the traveling wave (the top subfigure). For comparison, the difference between the single mode motion and total motion $(Y - Y_1)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. The colors of the curves are adjusted from red to blue based on the time sequence.

Figures 3.68 and 3.69 show the time histories of single mode motion (Y_1) for the mackerel raw data and the traveling wave (the top subfigure). For comparison, the difference between the single mode motion and total motion $(Y - Y_1)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. For better clarity, the colors of the curves are adjusted based on the body-marker sequence from red to blue. In all cases, the axis scale shows that $Y - Y_1$ is very small, consistent with mode one be strongly dominant (99%).



Figure 3.66 Superposed snapshots of single mode motion Y_1 (top), the difference between the single mode and total $Y - Y_1$ (middle), and the total motion Y (bottom), for the mackerel raw data. For better clarity, curves are shown for every other sampling time. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.67 Superposed snapshots of single mode motion Y_1 (top), the difference between the single mode and total $Y - Y_1$ (middle), and the total motion Y (bottom), for the mackerel traveling wave model. For better clarity, curves are shown for every other sampling time. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.68 Superposed time histories of single mode motion Y_1 (top), the difference between the single mode and total $Y - Y_1$ (middle), and the total motion Y (bottom), for the mackerel raw data. For better clarity, curves are shown for every four body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.69 Superposed time histories of single mode harmonic motion Y_1 (top), the difference between the single mode and total motion $Y - Y_1$ (middle), and the total motion Y (bottom), for the mackerel traveling wave model. For better clarity, curves are shown for every four body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.

3.3.2.1 Single mode motion from harmonic modulation

The modal motion is reanimated using harmonic modulation in place of the modal COC. Figure 3.70 shows the harmonic commplex coordinate (green) with the traveling wave model COC (red) and raw data COC (blue) for mackerel. It can be observed

that the green and red points are quite close all the time. Figures 3.71 and 3.72 show the snapshots of single mode harmonic motion (Y_h) for the mackerel raw data and traveling wave (the top subfigure). For comparison, the difference between the single mode harmonic motion and the total motion $(Y - Y_h)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. The colors of the curves are adjusted from red to blue based on the time sequence.

Figures 3.73 and 3.74 show the time histories of single mode harmonic motion (Y_h) for the mackerel raw data and the traveling wave (top subfigure). For comparison, the difference between the single mode harmonic motion and total motion $(Y - Y_h)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. For better clarity, the colors of the colors of the curves are adjusted from red to blue based on the body-marker sequence.



Figure 3.70 The harmonic complex coordinate (green) with the traveling wave model COC (red) and raw data COC (blue) for the mackerel.



Figure 3.71 Superposed snapshots of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and the total motion $Y - Y_h$ (middle), and the total motion Y (bottom), for the mackerel raw data. For better clarity, curves are shown for every other sampling time. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.72 Superposed snapshots of single mode harmonic motion Y_h (top), the difference between the single mode and total motion $Y - Y_h$ (middle), and the total motion Y (bottom), for the mackerel traveling wave model. For better clarity, curves are shown for every other sampling time. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.73 Superposed time histories of single mode harmonic motion Y_h (top), the difference between the single mode and total $Y - Y_h$ (middle), and the total motion Y (bottom), for the mackerel raw data. For better clarity, curves are shown for every four body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.74 Superposed time histories of single mode harmonic motion Y_h (top), the difference between the single mode and total motion $Y - Y_h$ (middle), and the total motion Y (bottom), for the mackerel traveling wave model. For better clarity, curves are shown for every four body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.75 The traveling parts of dominant modes of the raw data (blue) and the traveling wave model (red) for the mackerel.

3.3.3 Dissection of COM into traveling and standing parts

Based on chapter II, the dominant COM was dissected into its traveling and standing parts. The traveling index of 0.4904 was obtained for the mackerel raw data, while for the traveling wave model, the traveling index was calculates 0.5651. Figure 3.75 shows the traveling part of dominant mode of the raw data and traveling part of dominant mode of the traveling wave model. Figure 3.76 shows the standing part of dominant mode of the raw data and standing part of dominant mode of the traveling wave model. Figures 3.77 and 3.78 show the dominant COM with its standing and traveling parts of the raw data and the traveling wave model. In both cases, the sum of the traveling and standing produced the full mode.

Based on calculation of the traveling and standing parts of COM, the total motion was dissected into its traveling and standing parts. Figure 3.79 shows the snapshots of traveling (top) and standing (bottom) parts of single mode motion for the mackerel raw data. Figure 3.80 shows the time histories of traveling (top) and standing



Figure 3.76 The standing part of dominant mode of the raw data (blue) and the traveling wave model (red) for the mackerel.



Figure 3.77 Dominant COM (red) with its traveling (blue) and standing (purple) parts for the mackerel raw data.



Figure 3.78 Dominant COM (red) with its traveling (blue) and standing (purple) parts for the mackerel traveling wave model.

(bottom) parts of single mode motion for the mackerel raw data.



Figure 3.79 Superposed snapshots of the traveling (top) and standing (bottom) parts of single mode motion for the mackerel raw data. For better clarity, curves are shown for every other sampling time. The colors of the colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.80 Superposed time histories for the traveling (top) and standing (bottom) parts of single mode motion for the mackerel traveling wave model. For better clarity, curves are shown for every four body markers. The colors of the colors of the curves are adjusted based on body markers sequence in the colour spectrum, from red to blue.


Figure 3.81 Wave number vs. body marker for the mackerel raw data (blue) and traveling wave model (red), calculated by using finite difference scheme applied to the ϕ plot.

3.3.4 Wave number, frequency and wave speed

The modal wave speed through the fish can be investigated via the complex mode for both the raw data and the traveling wave model. From chapter II, the information for the phase, ϕ , as a function of x can be derived from the dominant mode. Previously in Figure 3.60, ϕ vs. x was plotted for the mackerel raw data and the traveling wave model. Based on chapter II, the wave number, γ , can be calculated by differentiation of ϕ with respect to x.

Figure 3.81 shows the plot of γ vs. x. The traveling wave model, based on how it is constructed, will have a constant wave number. The calculations show also the the traveling wave's dominant COM has a constant phase change. Based on this approximation of γ , the wavelength and wave speed can be calculated from chapter II.

Previously in Figure 3.64, ϕ vs. t was plotted for the mackerel raw data and traveling wave model. ω is calculated by differentiation of ϕ with respect to t. Since



Figure 3.82 Plot of frequency, ω , vs. sampling time for the mackerel raw data (blue) and traveling wave model (red). It can be seen that the red plot is not fully constant.

these data are discrete, the frequency is calculated by using the central finite difference scheme. For the first and last sampling time, the forward and backward finite difference is used respectively. The result is shown in Figure 3.82. Figure 3.56 shows the plot of wave speed for the mackerel raw data based on γ calculated from COM phase, and the traveling wave model. Figure 3.83 shows the plot of wave speed for the mackerel raw data based on γ calculated from COM phase, and the traveling wave model.



Figure 3.83 Plot of wave speed based on γ calculated from COM phase for the mackerel raw data (blue) and traveling wave model (red).

3.4 Saithe fish

3.4.1 Complex modes from swimming midline data

3.4.1.1 Eigenvalues, COVs, and the number of dominant modes

Figures 3.84 and 3.85 show the plots of the COVs in descending order in a linear scale and logarithmic scale, respectively. It can be seen in Figure 3.84 that only one mode (the first) captures most (more than 99 percent) of the energy and the other modes are insignificant comparing to the first mode.



Figure 3.84 Plots of the COVs in descending order in a linear scale for the saithe raw data (blue) and the traveling wave model (red). Only one mode (the first) captures more than 99 percent of the energy.



Figure 3.85 Plots of the COVs in descending order in a logarithmic scale for the saithe raw data (blue) and the traveling wave model (red).

3.4.1.2 Dominant COM

Since the first mode is the dominant mode, the focus of the study will be on the first COM. Figures 3.86 shows the plots of main COM in complex plane for the saithe raw data and the traveling wave model. For comparison, COMs are shown in blue for the raw data and in red for the traveling wave model. Figure shows that for the saithe, the dominant COMs of the raw data and the traveling wave model are very similar.

Plots of COM elements' phases and COM elements' magnitudes vs. body marker is shown in Figures 3.87 and 3.88 for the saithe raw data and the traveling wave model. It can be seen that the slope of phase plot is constant for the traveling wave model but variable in raw data. Figure 3.89 shows the real and imaginary parts of the COMs for both data sets.



Figure 3.86 Dominant COM for the saithe raw data (red) and traveling wave model (blue).



Figure 3.87 Plot of the COM elements' phases for the saithe raw data (blue) and the traveling wave model (red).



Figure 3.88 Plot of the COM elements' magnitudes for the saithe raw data (blue) and the traveling wave model (red).



Figure 3.89 Plot of the COMs' real and imaginary parts for the saithe raw data (blue) and the traveling wave model (red).

3.4.1.3 Dominant COC

Figure 3.90 shows plot of the dominant modal coordinate, COC, in the complex plane, for the saithe raw data and traveling wave model. Based on calculation of COC elements' phase, Figure 3.91 is produced. The figure shows the COC elements' phase, ϕ , vs. sampling time for the saithe raw data (blue) and traveling wave model (red). Figure 3.92 shows plots of the real and imaginary parts of dominant COC, in different colors for the saithe raw data and traveling wave model. The real and imaginary parts of COCs provide the two time-modulation components of the complex wave, representing the time modulations of the 90°-phased components of the wave.



Figure 3.90 Plot of dominant COC (complex orthogonal coordinate) for the saithe raw data (blue) and the traveling wave model (red).



Figure 3.91 Plot of the COC elements' phase, ϕ , vs. sampling time for the saithe raw data (blue) and traveling wave model (red).



Figure 3.92 Plots of the real and imaginary parts of dominant COC, for the saithe raw data (blue) and the traveling wave model (red).

3.4.2 Single mode motion from dominant COC

Using the dominant COC, the dominant modal motion is reanimated. Figures 3.93 and 3.94 show the snapshots of single mode motion (Y_1) for the saithe raw data and traveling wave (the top subfigure). For comparison, the difference between the single mode motion and total motion $(Y - Y_1)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. For better clarity, curves are shown for every five sampling times. The colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.93 Superposed snapshots of single mode motion Y_h (top), the difference between the single mode and total $Y - Y_h$ (middle), and the total motion Y (bottom), for the saith raw data. For better clarity, curves are shown for every five sampling times. The colors of the colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.94 Superposed snapshots of single mode motion Y_1 (top), the difference between the single mode and total motion $Y - Y_1$ (middle), and the total motion Y (bottom), for the saithe traveling wave model. For better clarity, curves are shown for every five sampling times. The colors of the curves are adjusted from red to blue based on the time sequence.

Figures 3.95 and 3.96 show the time histories of single mode motion (Y_1) for the saithe raw data and the traveling wave (the top subfigure). For comparison, the difference between the single mode motion and total motion $(Y - Y_1)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. For better clarity, curves are shown for every five body markers. The colors of the colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.95 Superposed time histories of single mode motion Y_1 (top), the difference between the single mode and total $Y - Y_1$ (middle), and the total motion Y (bottom), for the saith raw data. For better clarity, curves are shown for every four body markers. The colors of the colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.96 Superposed time histories of single mode motion Y_1 (top), the difference between the single mode and total $Y - Y_1$ (middle), and the total motion Y (bottom), for the saithe traveling wave model. For better clarity, curves are shown for every four body markers. The colors of the curves are adjusted based on the body-marker sequence from red to blue.

In all cases, the axis scale shows that $Y - Y_1$ is very small, consistent with mode one be strongly dominant (99%).

3.4.2.1 Single mode motion from harmonic modulation

The modal motion is reanimated using harmonic modulation in place of the modal COC. Figure 3.97 shows the harmonic commplex coordinate (green) with the traveling wave model COC and raw data COC for saithe. It can be observed that the green and red points are quite close all the time. The single mode harmonic motion is reanimated from harmonic modulation. Figures 3.98 and 3.99 show the snapshots of single mode harmonic motion (Y_h) for the saithe raw data and the traveling wave (the top subfigure). For comparison, the difference between the single mode harmonic motion and total motion $(Y - Y_h)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. For better clarity, curves are shown for every five sampling times. The colors of the colors of the curves are adjusted from red to blue based on the time sequence.

Figures 3.100 and 3.101 show the time histories of single mode harmonic motion (Y_h) for the saithe raw data and the traveling wave (the top subfigure). For comparison, the difference between the single mode harmonic motion and total motion $(Y - Y_h)$, and the total motion (Y), are shown in the middle and bottom subfigures, respectively. For better clarity, the colors of the colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.97 The harmonic complex coordinate (green) with the traveling wave model COC (red) and raw data COC (blue) for the saithe.



Figure 3.98 Superposed snapshots of harmonic modulation-based single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and total motion $Y - Y_h$ (middle), and the total motion Y (bottom), for the saithe raw data. For better clarity, curves are shown for every five time samples. the colors of the colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.99 Superposed snapshots of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and total motion $Y - Y_h$ (middle), and the total motion Y (bottom), for the saithe traveling wave model. For better clarity, curves are shown for every five sampling times. The colors of the colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.100 Superposed time histories of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and total motion $Y - Y_h$ (middle), and the total motion Y (bottom), for the saithe raw data. For better clarity, curves are shown for every four body markers. The colors of the colors of the curves are adjusted based on the body-marker sequence from red to blue.



Figure 3.101 Superposed time histories of single mode harmonic motion Y_h (top), the difference between the single mode harmonic motion and total motion $Y - Y_h$ (middle), and the total motion Y (bottom), for the saithe traveling wave model. For better clarity, curves are shown for every four body markers. The colors of the colors of the curves are adjusted based on the body-marker sequence from red to blue.

3.4.3 Dissection of COM into traveling and standing parts

Based on chapter II, the dominant COM was dissected into its traveling and standing parts. The traveling index of 0.5993 was obtained for the saithe raw data, while for the traveling wave model, the traveling index was calculates 0.6017. Figure 3.102 shows the traveling parts of dominant modes of the raw data and traveling part of dominant mode of the traveling wave model. Figure 3.103 shows the standing part of dominant mode of the raw data and standing part of dominant mode of the traveling wave model. Figures 3.104 and 3.105 show the dominant COM with its standing and traveling parts of the raw data and the traveling wave model.



Figure 3.102 The traveling parts of dominant modes for the saithe raw data (blue) and the traveling wave model (red).



Figure 3.103 The standing part of dominant mode for the saithe raw data (blue) and the traveling wave model (red).



Figure 3.104 Dominant COM (red) with its traveling (blue) and standing (purple) parts for the saithe raw data.



Figure 3.105 Dominant COM (red) with its traveling (blue) and standing (purple) parts for the saithe traveling wave model.

Based on calculation of the traveling and standing parts of COM, the total motion was dissected into its traveling and standing parts. Figure 3.106 shows the snapshots of traveling (top) and standing (bottom) parts of single mode motion for the saithe raw data. Figure 3.107 shows the time histories of traveling (top) and standing (bottom) parts of single mode motion for the saithe raw data.



Figure 3.106 Superposed snapshots for the traveling (top) and standing (bottom) parts of single mode motion for the saithe raw data. For better clarity, curves are shown for every five sampling times. The colors of the colors of the curves are adjusted from red to blue based on the time sequence.



Figure 3.107 Superposed time histories for the traveling (top) and standing (bottom) parts of single mode motion for the saithe raw data. For better clarity, curves are shown for every four body markers. The colors of the colors of the curves are adjusted based on body markers sequence in the colour spectrum, from red to blue.

3.4.4 Wave number, frequency and wave speed

The modal wave speed through the fish can be investigated via the complex mode for both the raw data and the traveling wave model. From chapter II, the information for the phase, ϕ , as a function of x can be derived from the dominant mode. Previously in Figure 3.87, ϕ vs. x was plotted for the saithe raw data and the traveling wave model. Based on chapter II, the wave number, γ , can be calculated by differentiation of ϕ with respect to x. However, since these data are discrete, the wave number was calculated by using the central finite difference scheme. For the first and last body marker, the forward and backward finite difference is used respectively.

Figure 3.108 shows the plot of γ vs. x. The traveling wave model, based on how it is constructed, will have a constant wave number. The calculations show also the the traveling wave's dominant COM has a constant phase change.



Figure 3.108 Plot of wave number vs. body marker for the saithe raw data (blue) and traveling wave model (red), calculated by using finite difference scheme applied to the ϕ plot.

Previously in Figure 3.91, ϕ vs. t was plotted for the mackerel raw data and traveling wave model. ω is calculated by differentiation of ϕ with respect to t. Since



Figure 3.109 Plot of frequency, ω , vs. sampling time for the saithe raw data (blue) and traveling wave model (red).

these data are discrete, the frequency is calculated by using the central finite difference scheme. For the first and last sampling time, the forward and backward finite difference is used respectively. The result is shown in Figure 3.82.

Based on this approximation of γ , the wavelength and wave speed can be calculated from chapter II. Figure 3.110 shows the plot of wave speed for the saithe raw data based on γ calculated from COM phase, and the traveling wave model.



Figure 3.110 Plot of wave speed based on γ calculated from COM phase for the saithe raw data (blue) and traveling wave model (red).

CHAPTER IV

Discussion

We have extracted some of the kinematical features of carangiform fish swimming overlooked by simple existing models. It is possible that fine details of the fish may have a significant impact on achieving a high efficiency in its propulsion. The small details in the kinematics of fish swimming may have significant role in low drag or low energy consumption per unit of forward translation.

The technique used, COD decomposes a total motion, that is sampled in space and time into its characteristic complex modes. This is done by solving an eigenvalue problem described in chapter II. After decomposing the motion into its characteristic complex coordinates modes, the frequency and wave number information can be extracted from the complex modal coordinates and modes. Since it does not make any preselected assumption about the motion parameters (frequencies), it can reveal an underlying relationship between the motion phase and time or location (frequency and wave number). So it can lead to estimations of parameters related to the motion phase and modal amplitudes. Another application is that it can be used to compute a traveling index for the periodic motions to quantify the relative degree of traveling and standing in a waveform. COD can also serve as a filtering tool for the signal data that are accompanied by noise. Altogether, this study concludes that COD can propose a more accurate description of swimming kinematics. In the following, the above-mentioned aspects of this analysis is discussed in the context of comparison done between the modes from raw data and the traveling wave model.

4.1 The main mode shapes and the number of main modes

As a result of this analysis, we found that for all three fish, both the raw data and the traveling wave model have a single dominant mode that captured more than 99 percent of the energy contain in each mode. The other modes tend to have insignificant participation and are noise dominated. The modal amplitudes are proportional to the square root of the eigenvalues. Unlike in the linear scale, the figures that show the eigenvalues in logarithmic scale can make a better distinction between the second mode and the others after it, showing that the second mode, although negligible comparing to the first mode, can contain meaningful information. Finally, this is the mode shape that can be used to judge if the other modes are something other than noise.

This is interesting that the same fact about the eigenvalues (only one dominant mode) holds for the traveling wave model as for the raw data. When building the traveling wave model, the parameters like amplitude function, γ , and ω were adjusted by trial and error to give COV values and COM shapes similar to those in raw data case. The amplitude function was extracted from the raw data and using curve fitting. We observed that the eigenvalues' quantities (especially the first one) were sensitive and dependent on the amplitude function used in the traveling wave model, so that even small changes affected the dominant eigenvalue. Thus, it can be concluded that the amplitude function is an affecting parameter on COVs. On the other hand, the carangiform categorization is made based on the amplitude variation along the fish length. We observed that for these three species, the nature of their amplitude (vs. body marker) beside other possible factors, led to having one dominant mode, either for the raw data or for the traveling wave model. We have neither examined other species in carangiform category nor in other categories (like anguilliform) to comment on the number of dominant COVs as a function of category as a rule. Thus, further analysis on other fish in this or other categories is needed to make a more general conclusion about the number of main modes in carangiform swimming.

4.1.1 Whiting fish

For the whiting considered in body frame, Figures 3.1, 3.2 clearly shows that there is only one dominant mode. In a Newtonian frame, Figures 3.30 and 3.31 show a similar result, however, the dominant eigenvalues in the Newtonian frame (for the raw data and the traveling wave model) are about 2 in^2 , compared to that in the body frame of 4 in^2 . For both frames raw data, there is a possibility that the second and the third modes are meaningful for raw data. For the traveling wave model, the second mode may be meaningful only due to discretization effects of the continuous fish body.

4.1.2 Mackerel fish

Figures 3.57 for mackerel shows that the dominant eigenvalue is about $4 \times 10^{-3} m^2$ for both the raw data and the traveling wave model. Figure 3.58 shows that other than the first dominant mode, the second to tenth modes may be meaningful, although they have far less participation in energy contain comparing to the first mode.

4.1.3 Saithe fish

For the saithe fish, Figure 3.84, shows that the first eigenvalue is far dominant over the other modes and is about $8 \times 10^{-3} m^2$ for both the raw data and the traveling wave model. Figure 3.85 shows that the eigenvalues second to tenth do not have absolute zero participation and may be meaningful.

This is interesting that all the three species have one dominant mode of motion. Actually, it simplifies the models that can be proposed for the swimming kinematics. However, not to mention that the conclusion should not be generalized to all carangiform swimmers. Even more, the data used belong to 'average' fish for mackerel and saithe. It means that different fish in one species can perform different motions and in this research we just considered an average motion that leads to an average forward speed.

For the mackerel and saithe fish, these results can be compared to the results of *Videler and Hess* (1984). They used the FFT to decompose the total motion and found three considerable modes (the first, third, and fifth frequencies) that among them the first one is dominant (Figure 4.1). Their interpretation of the results in this figure is that the first frequency accounts for most of the lateral motion, the third frequency contributes something in the posterior part of the fish and the fifth frequency contributions can hardly be distinguished from noise.



Figure 4.1 Lateral displacement for 'average' saithe and 'average' mackerel. (A) Amplitude and phase functions for 'average' saithe: first frequency (drawn curves), third frequency (dashed), fifth frequency (stippled) contribution. (B) The same for 'average' mackerel. Figure from *Videler and Hess* (1984).

4.2 Dominant COMs and their information about the modal amplitude and phase

The complex orthogonal modes, COMs, resulting from COD technique, contain the information for amplitude distribution among the modes. They also contain information of phase change with body markers (location). To some extent, the traveling wave is not a very high accurate model since it is not expressive of small details about the amplitude distribution and phase change along the fish length. The factors affecting the traveling wave COM shapes are: the amplitude function used, the wave number γ , and the frequency ω . We used trial and error to find a combination of these parameters that obtain the closest COMs to the raw data COMs. Our criteria were: coincidence of the dominant eigenvalue (can be seen in Figures 3.1, 3.30, 3.57, and 3.84), visual closeness of COM shapes between the raw data and the traveling wave model (can be seen in Figures 3.3, 3.3, 3.59, and 3.86), closeness of the phase plots vs. body marker with respect to the slope and location of the corresponding data points (can be seen in Figures 3.4, 3.33, 3.60, and 3.87), and closeness of the phase plots vs. time with respect to the slope and location of the corresponding data points (can be seen in Figures 3.10, 3.37, 3.64, and 3.91). An exact or even match between the COMs in raw data and the traveling wave model was not possible.

As an example, variation of COM (traveling wave model) in complex plane with value changes of γ is shown for mackerel in Figure 4.2. In this figure, we tried to tune the value of γ in the traveling wave model to see the effect. The figure shows the resultant COMs corresponding to $\gamma = 21, 21, 22.3, 24, 18 \ rad/m$ and COM corresponding to raw data. For the rest of the study, the value $20 \ rad/m$ was selected because it produced the same phase difference in body length as in raw data.



Figure 4.2 Effect of γ on dominant COM in traveling wave model.

4.2.1 Whiting fish

Figures 3.3 and 3.32 show the dominant COMs in complex plane for the raw data and the traveling wave model, respectively for the whiting body frame and Newtonian frame.

In both figures, slight different in COMs' shape between the blue and the red plot is seen, especially in the anterior region. This is expressive of small difference between the real motion and the traveling wave model. The differences can be described as follows; each point in the plot is an element of COM vector $(1 \times M)$. The points closer to the center of the spiral belong to anterior regions of fish body, and the outer points belong to posterior regions. Each element in complex plane is expressed as $\rho_i \phi(x_i)$, where ρ is the magnitude and ϕ is the phase. The magnitude is equivalent to the oscillation amplitude of the corresponding body marker. The ϕ s show the relative phase of body markers. The COM thus can be expressed as vector of exponential functions of body markers location,

$$\mathbf{w}_{i} = \begin{bmatrix} \rho_{1}e^{\phi_{1}(x_{1})} \\ \rho_{2}e^{\phi_{2}(x_{2})} \\ \dots \\ \rho_{M}e^{\phi_{M}(x_{M})} \end{bmatrix}$$

Comparing the red and blue curves in these figures reveals that the corresponding points in complex plane for raw data and traveling wave model differ in their placement in the complex plane. In Figures 3.4 and 3.33 that are produced based on the placement of data points, it can be seen that the slope of phase vs. body marker plot is not constant for raw data. An interesting observation about the traveling wave model is that the slope of COM phase vs. body marker calculated to be constant.

In Figures 3.7 and 3.34 it can be seen that the slope of phase vs. body marker plot is not constant for raw data, unlike the traveling wave model. Figures 3.35 and 3.35 show the real and imaginary parts of the COMs vs. body marker. These figures give information about amplitude and phase for each body marker. First of all, a couple of through and crest can be seen for each plot, indicating that the wave length is less than the fish length. Second, the real parts and the imaginary parts differ in phase for less than 90 degrees. Finally, in Figure 3.35, the imaginary parts differ in amplitude in the interior region, while the real parts are close. In 3.8 there is a nearly good match for the phase and amplitude of rel/imaginary parts between the blue and red plots. The two last observations may indicate that for the body frame data we could find a better matched traveling wave.
4.2.2 Mackerel fish

Figure 3.59 shows that the dominant COMs of the raw data and the traveling wave model does not match in amplitude of data points. Also, with respect to the phase, the data points in blue plot get behind and then ahead of the red data points, showing the wave number is not constant in the raw data. This result is confirmed in Figure 3.60 by comparing the slope of the red and blue plots. An interesting observation about the traveling wave model is that the slope of COM phase vs. body marker calculated to be constant.

With respect to the amplitude, however, it can be seen in Figure 3.61 that a nearly good match exist between the blue and red plots. Again, these plots are dependent on the amplitude function, γ , and ω chosen for the traveling wave model. Thus, the optimality of the traveling wave may depend on the raw data of each fish that is representing the nature of the fish motion. In figure 3.62, a nearly good match between the phase and amplitude of real/imaginary parts can be seen between the blue and red plots. Also, in the imaginary part plots, it can be seen that there is more than a complete cycle (a through and a crest are included in the plot) indicating that the wavelength is less than the fish length.

4.2.3 Saithe fish

COMs of raw data and the traveling wave model are quite close in Figure 3.86. Again, for the saithe too, the wave number is not constant in the raw data; however it is closer to constant value comparing saithe with mackerel and whiting. This result is confirmed in Figure 3.87 by comparing the slope of the red and blue plots. In the case of saithe fish also, the slope of traveling wave model's COM phase vs. body marker is observed to be constant.

With respect to the amplitude, there is a good match too in Figure 3.88. Altogether, these results show that for the saithe we could find easier a good model parameters (amplitude function, γ , and ω for the traveling wave model) comparing to the other fish. A good match also exist in 3.89 between the real/imaginary plots of raw data and the traveling wave model with respect to the phase and amplitude. Also, in the imaginary part plots, it can be seen that there is more than a complete cycle (a through and a crest are included in the plot) indicating that the wavelength is less than the fish length.

Videler and Hess (1984) obtained a similar result regarding the phase change over body length for the dominant mode. In Figure 4.1, the right plots show the phase change over body length for average mackerel and saith for three modes of motion. It can be seen that for the first (dominant) mode, the phase change trend is close to a straight line but not exactly straight; a result quite similar to the results of this research. For the second mode, there is a standing wave in the anterior body (zero slope) and positive varying slope for the posterior body. For the third mode, the phase plot has a zero slope, showing a standing wave all over the body length. The modal amplitude is far more significant for the first mode, showing a typical trend in carangiform swimmers.

4.3 COCs and their information about phase change attributes in time

The complex orthogonal coordinates, COCs, resulting from COD technique, contain the information of phase change in time. To some extent, the traveling wave is not a very high accurate model since it is not expressive of small details about the phase change with time in raw data.

4.3.1 Whiting fish

Figure 3.9 and 3.36, show respectively for the body frame and Newtonian frame that the phase difference between the consecutive red points is uniform, while the blue points get ahead and behind of the corresponding red points periodically. The same conclusion can be made from Figures 3.10 and 3.37. It can be seen that in the traveling wave, plot of COC elements' phase vs. time has nearly constant slope, ω , but in raw data, the rate of phase change in time, is not constant over the time. Figures 3.28 and 3.55 show the same fact about the frequency ω .

4.3.2 Mackerel fish

During the digitization of mackerel raw data, we used 20 sampling times. Figure 3.63 shows that the blue points (raw data COC) get ahead and behind the red points (traveling wave model COC) periodically with respect to phase. Figure 3.64 shows a good match between the phase plots of raw data and the traveling wave model. Figure 3.82 shows better the fluctuations in the frequency ω in time. Figure 3.65 shows a good match between the imaginary parts of both data sets and between the real parts of both data sets. It can be seen that the real and imaginary parts are exactly 90° apart.

4.3.3 Saithe fish

For the saithe fish we used 50 sampling times, so a better resolution in time comparing to the mackerel case was examined. Figure 3.90 shows a nearly good match between the blue and red data points. Figure 3.91 shows a good match between the phase plots of raw data and the traveling wave model. Figure 3.109 shows better the fluctuations in the frequency ω in time. Figure 3.92 shows a good match between the imaginary parts of both data sets and between the real parts of both data sets. It can be seen that the real and imaginary parts are exactly 90° apart.

4.4 Extraction of the phase speed information

As a result of applying COD to raw data, a variable phase speed is calculated over the body length. This can be observed in Figures 3.29, 3.56, 3.83, and 3.110 (the blue curves). This result was predictable since we saw that γ is variable over the body length, and the phase speed is given in terms of the wave's angular frequency ω and wave number γ , as $c = \frac{\omega}{\gamma}$. It should be noted that in this formula, a constant average value of ω was used to plot the phase speed. For the whiting (Newtonian frame), mackerel, and saithe, the phase speed is higher in the anterior region and correspondingly the wave number and wavelength are lower and higher respectively in this region (the absolute values). It may be related to this fact that muscular activation does not occur at head region, or the anterior region is probably stiffer. In the posterior region, there is lower phase speed, lower wave number, and lower wavelength (the absolute values). It may be because there is nonzero continuous muscular activation in the posterior region.

4.5 Reanimation of the motion based on the dominant COMs and COCs

We reanimated the motion based on only the dominant mode for both the raw data and the traveling wave model. A typical pattern observed was that the difference between the total motion and the single mode-based motion, $Y - Y_1$, always is within less than 10% of the total motion. This observation confirms that the dominant mode has more than 99% of the total energy ($0.99 = 1 - 0.1^2$). In some of the cases, this difference seems to have a non-random shape, meaning that the difference may be not just a noise. This leads us to the conclusion that the modes other than the dominant one may be not just a noise recorded in data collection process.

4.5.1 Whiting fish

4.5.1.1 Body frame

Figures 3.12 and 3.14 show the single-mode motion in comparison to the total motion for whiting (body frame) raw data. The snapshots show that the difference between the single-mode motion and the total motion, $Y - Y_1$, does not look random, so maybe it is not just noise. Thus, the modes other than the dominant one can be meaningful. Also, $Y - Y_1$ looks like a standing wave in which at all the body markers, the oscillation has nearly the same phase. For example in Figure 3.12, the red plot has the highest amplitude at all body markers, meaning that all body markers reached their highest displacement at the same time. This seems to be true for curves at other snapshots (other colors). Also, $Y - Y_1$ has higher amplitude in the body markers from 25 to 40 and right at the tail. The time histories for $Y - Y_1$ confirm this.

Figures 3.13 and 3.15 shows the single-mode motion in comparison to the total motion for the whiting (body frame) traveling wave model. First of all, comparing $Y - Y_1$ for raw data and the traveling wave model, shows that application of COD

resulted in different patterns for these two data sets (raw data and the traveling wave model). Second, the snapshot plots show that the dominant mode simulates very well the motion of body markers 1 to 20, but for the tail body markers, $Y - Y_1$ reaches to 0.1 of the total motion. This is confirmed by the time-histories. Again for the traveling wave model, $Y - Y_1$ nearly looks like a standing wave with the same reasoning.

4.5.1.2 Newtonian frame

Figures 3.39 and 3.14 show the single-mode motion in comparison to the total motion for whiting (body frame) raw data. The snapshot plot shows that $Y - Y_1$ looks like a standing wave without randomness. However, the time history plot shows that a group of body markers are in-phase and the rest are out-of-phase with the first group.

Figures 3.40 and 3.42 show the single-mode motion in comparison to the total motion for the whiting (body frame) traveling wave model. The snapshots and time histories of $Y - Y_1$ show neither a fully random trend nor a special trend.

4.5.2 Mackerel fish

Figure 3.66 shows the snapshots of single-mode motion in comparison with the total motion for mackerel raw data. The snapshots of $Y - Y_1$ do not look random. It can be seen that the amplitude of $Y - Y_1$ oscillation gets smaller from the head to the tail. No special trend is found in the time histories of $Y - Y_1$ in Figure 3.68 for raw data. Also, no special trend is observed for the snapshots and time histories in Figures 3.67 and 3.69.

4.5.3 Saithe fish

In Figures 3.93 and 3.95, which show the single-mode motion in comparison to the total motion for saithe raw data, $Y - Y_1$ plots do not look fully random nor is a special trend seen in them. This is while, for the traveling wave model, the corresponding $Y - Y_1$ plots show a far smaller amplitude (Figures 3.94 and 3.96, middle subfigures).

4.6 Reanimation of the motion based on harmonic modulation

The complex coordinates that are built by using harmonic modulation are closer to the COC in traveling wave model than the COC in the raw data. The reason may be the way the traveling wave model is constructed and harmonic modulation is done (see Figures 3.16, 3.43, 3.70, and 3.97).

4.6.1 Whiting fish

For the whiting (body frame), the single mode harmonic motion is compared with the single mode based on the COC of the raw data in Figures 3.17 and 3.19. These figures are compared with their corresponding figures that compare the single mode harmonic motion with the single mode based on the COC of the traveling wave (Figures 3.18 and 3.20), the observation is that the single mode harmonic motion is far closer to the single mode based on the COC of the traveling wave.

For the whiting (Newtonian frame), the single mode harmonic motion is compared with the single mode based on the COC of the raw data in Figures 3.44 and 3.46. If these figures are compared with their corresponding figures that compare the single mode harmonic motion with the single mode based on the COC of the traveling wave (Figures 3.45 and 3.47, the observation is that the single mode harmonic motion is far closer to the single mode based on the COC of the traveling wave. Actually, by noting to the $Y_1 - Y_h$ plot, it seems that the difference can be reducible to zero by adjustment of phase difference between Q_1 and the COC, so that the two latter are coincident.

4.6.2 Mackerel fish

For the mackerel, the single mode harmonic motion is compared with the total motion in the traveling wave model in Figures 3.71 and 3.73. If these figures are compared with their corresponding figures that compare the single mode harmonic motion with the total motion in traveling wave model (Figures 3.72 and 3.74), the observation is that there is no accuracy difference between them. The only difference is in time histories where $Y - Y_h$ is not smooth in Figure 3.73, but they are smooth in Figure 3.74.

4.6.3 Saithe fish

The single mode harmonic motion is compared with the total motion (traveling wave model) for the saithe in Figures 3.98 and 3.100. If these figures are compared with their corresponding figures that compare the single mode harmonic motion with the total motion in traveling wave model (Figures 3.99 and 3.101), the observation is that the single mode harmonic motion is far closer to the motion in traveling wave model. accuracy difference between them. Also, the time histories of $Y - Y_h$ are not smooth in Figure 3.100, but they are smooth in Figure 3.101.

4.7 Dissection of the total motion based on dissection of COM into its traveling and standing parts

An important quantity regarding complex mode is the traveling index. It quantifies the relative degree of traveling and standing in a waveform. A traveling index of "1" means a pure traveling wave, whereas a traveling index of "0" means a pure standing wave. A value of traveling index between 0 and 1 means a combination of traveling and standing waves. This parameter can be an important parameter affecting the propulsive efficiency. As a reminder from chapter I, *Lighthill* (1960) tried to find the optimal motion with regard to swimming efficiency. Trying both standing waves and traveling waves, he concluded that a pure standing wave cannot produce considerable propulsion efficiently, unlike a traveling wave. As mentioned in chapter I, the effect of the traveling index on the fluid forces and propulsive efficiency was investigated by *Cui et al.* (2017). Their conclusion was that the traveling index is one of the factors affecting the forward speed and the swimming efficiency.

The swimming mechanisms in nature proved to be efficient. In this regard, we chose three species in the carangiform category and calculated their traveling indices. As a result, we observed that all three have a traveling index between 0 and 1. More specifically, the traveling indices 0.4832, 0.5205, 0.4904, and 0.5993 were obtained for whiting (body frame), whiting (Newtonian frame), mackerel, and saith, respectively, for raw data.

An interesting fact is that the traveling index of the traveling wave models we built were calculated to be some value between 0 and 1. More specifically, traveling indices 0.3869, 0.5634, 0.5651, and 0.6017 were obtained for whiting (body frame), whiting (Newtonian frame), mackerel, and saith traveling wave model, respectively.

A first conclusion that can be made is that the traveling index is different among different carangiform species. In other words, the difference between the traveling indices of whiting, mackerel and saithe is expressive of a difference in motions they use. It is not known yet however, if there is significant variation within a species, or within an individual. Different motions may be related to different neuromuscular activation in each species or different muscle properties (for example elasticity coefficient, and cross-sectional muscle distribution through the body length). Not to mention that the traveling index can be different between various fish in one species. As a reminder, we got the raw data for one specific whiting, average mackerel, and average saithe. This subject can be further investigated in future research. For example one probable research can be applying COD on electromyographic (EMG) recordings in vivo and dissecting it to traveling and standing parts. Furthermore, studies can be done on other species in the carangiform category or other categories.

The second conclusion that can be made is that the traveling wave model used in most of the literature indeed has a standing wave component. In other words, the traveling wave model that was intuitively thought to be purely traveling turns out to contain both a standing and a traveling part. This can be explained theoretically.

Third, comparing the traveling index for whiting in body frame and Newtonian frame, we observe that the former is less than the latter. This fact could be anticipated, because in the body frame, all the distances and angles of rotation are measured with respect to one reference point. Thus, comparing to the Newtonian frame, the rotation of the reference point is added to the rotation of other body markers and this would affect the contribution of traveling index for the total motion.

As it is seen in regarding the standing part of COM in Figures 3.22, 3.49, 3.76, and 3.103, the plots are straight lines passing through the origin. It is expressive of the fact that all points in standing wave are either in-phase or 180° out-of-phase.

The phase vs. body marker however is changing as can be seen in Figures 3.21, 3.48, 3.75, and 3.102. The plot of phase vs. body marker for the traveling part for whiting (Newtonian frame) is shown in Figure 4.3. Comparing this figure with Figure 3.33, it can be seen that the wave number of the traveling wave model's COM is not constant in Figure 4.3. The phase difference between the corresponding plots in these two Figures is because the total COM can be regarded as the vector sum of the traveling and standing parts in complex plane (see Figures 3.50 and 3.51 for example).

Based on the snapshots and time-histories of dissected parts of total motion (Fig-



Figure 4.3 Plot of phase vs. body marker for dominant COM's traveling part for raw data (blue) and traveling wave model (red).

ures 3.25, 3.52, 3.79, 3.106, 3.26, 3.53, 3.80, and 3.107), the following information can be obtained:

- The location of nodes in standing part of motion may be used as side information in studies done on EMG recordings to relate it maybe to the muscle activity over the body length.
- The amplitude of the traveling and standing parts. An interesting observation made by comparing the total motion amplitude with amplitude of traveling part and standing part is that at every body marker location, the amplitude of total motion is approximately equal to scalar sum of standing part amplitude and traveling part amplitude.

4.8 Potential limitations

The limitation encountered in this research was mainly due to raw data, since specially for whiting, the midline data was available for only 12 sampling times. Also, in the case of mackerel and saithe, time histories were available for only 11 body markers. This certainly affects the precision and accuracy of COD results. For example, according to *Feeny* (2008), as the number of sampling times, N, increases, the matrix $\mathbf{R}_Q = \frac{1}{N} \mathbf{Q} \mathbf{\bar{Q}}^T$ gets more diagonal leading to a more accuracy of the results.

A low number of body markers (and consequently a ow number of signals sampled from body markers) does not lead to an accurate curve fitting for amplitude. This is while the accuracy of amplitude as function of body marker locations highly affects the accuracy of the traveling wave model. In this study, we used polynomial of order 10 produced by MATLAB. However, using the same order, various processors gave different coefficients.

The other limitation affecting the digitization accuracy is the software used. As mentioned previously in chapter II, we used "WebPlotDigitizer". To work with this software, the photo should be uploaded and the axis should be determined. Then the data points on the plot should be located manually. Although the precision of this software to digitize the selected data points is 5 digits after the point, manually loading the data points decreased the accuracy. Actually the software is equipped with automatic mode for data point selection, but the quality of some photos were so low that it affected badly the accuracy of the automatic mode.

4.9 Summary

This research provides some measures of evaluation of traveling wave models compared to true motion (raw data). Traveling wave model is quite good (single mode, general shape, traveling index). But fine details are missed (the wave speed variations c(x), $\gamma(x)$, and $\omega(x)$). These fine details were hypothesized to affect the thrust and propulsive efficiency (to be studied in the future). To improve the motion model, either can incorporate $\gamma(x)$ and $\omega(x)$ into the traveling wave model, or use COMs, perhaps with the harmonic modulation.

Also, we believe that COD can lead to more efficient modal analysis of fish midline data rather than FFT, since it does not assume a preselected frequency prior to decomposition. As a recommendations for further research, we consider producing a more set of signals with more advanced filming technology today. Also, we can use a more digitizer and use a software that computes more accurately the midline in the photos. The last recommendation is to apply COD to the EMG recorded data for mackerel in the literature to decompose it into traveling and standing parts, since It may help to increase information about the neuromuscular activation.

CHAPTER V

Conclusion

The purpose of our study was to decompose the lateral motion of midline points into its main modes, and use the modes to identify parameters of the motion, including oscillation amplitude and phase as functions of time or position. In a few other works, this has been done using a two dimensional FFT. Both the FFT and COD are suitable for this problem since the measurements are from sensors in practice. However, the FFT assumes that the harmonics' arguments have a constant derivative with respect to time. In contrast, COD is not constrained by such an assumption. It should be noted that as a result of applying the COD, time and position are automatically decoupled, which may be not true in real motion, since various points may not follow an independent trend in time. In addition, COD is a linear matrix decomposition method and the swimming motion may contain non-linearities to an extent that depends on the species.

5.1 Summary of the results

The results of this study show that for all of the three carangiform swimmers analyzed, the dominant mode shape and phase rates follow a similar trend. However for the other modes the trends were not meaningful, leading to this conclusion that the other modes are noise. Our findings support the results of *Videler and Hess* (1984), in that the phase rate is not constant with respect to position. However, we also observe a small fluctuations about a constant phase rate with respect to time. This may be because of using discrete data or because of the nature of the real motion. To certainly judge about this and whether it is significant, we should use a better temporal and spacial resolution, and also consider additional species.

5.2 Contributions of this research

As a result of applying COD, common features were observed in COMs of three carangiform swimmers. For all the three fish, the COM plot had the shape of a spiral, meaning that at the body anterior regions, the oscillation amplitude was lower and at the posterior regions it gets large. In all fish, $\gamma(x)$ shows lower at the anterior regions with a jump to a maximum at regions nearly behind the head. Then it undergoes fluctuations around the average value. Notice that γ is always negative, meaning the phase is strictly decreasing moving from head to tail. $\omega(t)$ shows fluctuations around an average frequency. The amplitude of fluctuations is at maximum 20 percent of the average value. The wave speed c(x) has a trend opposite to the wave number. It is higher at the anterior regions and then drops to lower values at the posterior regions where it fluctuates around an average. Another contribution of this study is the traveling index, which seems to be a significant factor affecting the propulsion. The traveling index for these three fish was around 0.6 on average, so there is a noticeable standing wave part in the total motion referring to the fact that the motions in nature, which are assumed to be optimal, are a combination of both traveling waves and standing waves. Furthermore, the results show that there may be a relationship between the swimming speed, traveling index and frequency since the fish in order of higher swimming speed (mackerel > saithe > whiting), had lower traveling indices (mackerel < saithe < whiting) and higher frequencies (mackerel > saithe > whiting).

It is worthwhile also noticing the differences between the three fish. Considering the COM plots in the Newtonian frame, whiting is observed to have a wavelength smaller than the body length, while in mackerel and Saithe, the wavelength almost equals the body length.

A comparison of the results for the whiting body fixed frame vs. the Newtonian frame is also useful. In the anterior regions of fish body, γ has more fluctuations in the body frame compared to the Newtonian frame. The average γ , wave speed and ω are higher for the body frame. It is expressive of the fact that the frame which is selected to evaluate the motion affects the phase difference between two body markers at a time instant, or phase difference at a specific body marker between two different time instants. A larger traveling index is calculated for the Newtonian frame, meaning that the traveling part is more prominent in the Newtonian frame compared to the traveling part in the body frame.

Comparing the raw data vs. the traveling model would reveal the deficiencies of this model which is used in almost all studies done on fish hydrodynamics. Clearly, γ vs. body marker, ω vs. sampling time, and wave speed vs. body marker are not constant in the raw data while they are chosen constant in the traveling wave model. An interesting observation is that in the Newtonian frame, the traveling index is higher for the traveling wave model compared to the real motion. However, in the body frame, the traveling index is much lower for the traveling wave model compared to the real motion.

In conclusion, the method COD fits back very well to vibration studies specially where there is a continuum structure undergoing oscillation in position and time. Comparing COD to manual analysis based on the FFT, it is easier to apply and needs less effort for the task of modal decomposition. In total, we believe that COD obtains accurate wave characteristics at the sampling positions and times, and yields more realistic model parameters that would certainly provide a better kinematic model of the swimming motion. The kinematic models are usually served as input to hydrodynamic studies of swimming and in fish robot design. In the traveling wave model used in almost all of the literature, the phase change rates are constant, which is in contrast to the real motion. Based on the results of this research, the traveling wave model can be rectified by considering variations in the frequency and wave number. On the other hand, based on Taylor, Lighthill, and Gray, the hydrodynamic forces depend on \dot{y} and y', and y'', where $y = a(x)f(\omega(t)t - \gamma(x)x))$ models the lateral oscillations. Thus, $\dot{\omega}(t)$ and $\gamma'(x)$, which are not zero in real motion, may have a considerable effect on the hydrodynamic forces. Consequently, ignoring them may lead to significant calculation errors, and can lead to sub-optimal propulsion efficiency in robot designs that emulate natural swimmers.

5.3 Future works

To make sure about the observed relationships between the swimming speed, traveling index and frequency, one recommendation for future research is to evaluate more carangiform species, as well as various swimming speeds among species and even individuals. Also, similarities between the various parameter trends in these three fish suggest that these may be characteristics of all carangiform swimmers. Confirming this requires gathering enough information necessary to get a general conclusion about the carangiform mode. Furthermore, quantitative distinction of carangiform swimming would also involve the evaluation of anguiliform or subcarangiform species to compare the parameter trends in order to distinguish unique properties of each mode. Opportunities for future research can be also include a separate evaluation of the head and tail body markers to confirm Gray's idea that a traveling wave is initiated at some intermediate point, then going forward toward head and backward toward tail.

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