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AN INQUIRY INTO CONSTRUCTION OF ACCOUNTING PREDICTORS
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David Richard Lee Gabhart

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Gardner M. Jones
Major professor

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AN INQUIRY INTO CONSTRUCTION OF ACCOUNTING PREDICTORS OF FUTURE MUNICIPAL FINANCIAL INSOLVENCY

By

David Richard Lee Gabhart

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ABSTRACT

AN INQUIRY INTO CONSTRUCTION OF ACCOUNTING PREDICTORS OF FUTURE MUNICIPAL FINANCIAL INSOLVENCY

By

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The primary objective of this research effort was to find, in municipal financial statements, reliable predictors of future financial insolvency of municipalities in the State of Michigan. The constrained financial condition of some municipalities is of great concern to state and local officials, and therefore an "early warning" system is desired to head off conditions of insolvency before they have a chance to develop. Since under federal bankruptcy law, cities cannot feasibly declare bankruptcy, a surrogate for bankruptcy is used; that is, technical financial insolvency, which in turn is defined, for measurement purposes, as having a June 30 cash balance which is less than 10% of total General Fund assets.

The predictors for this study were drawn from accounting data only, presented in the audited financial statements of Michigan municipalities, prepared under uniform accounting procedures prescribed in Public Act 2, 1968. Data used were for the period of 1971 through 1975. Data were from the General Fund only, reflecting the revenues

and expenses of the general operation of the municipality, the administration to, protection of, and service to the public. Data were gathered for 86 municipalities; missing data and year ends other than June 30 reduced the eligible cities to 60.

Some 32 potential independent variables for each year were drawn from the financial statements which might serve as prior-year indicators of subsequent-year insolvency. By judgmental review, then successive application of factor analysis, multiple linear regression, and quantal response (a methodology similar to multiple discriminant analysis) the 32 potential indicators were reduced to sixteen which appeared to be significant in ex ante classification of cities into one of three classes: insolvent (cash-poor), cash-adequate, and cash-rich for a subsequent year.

When these predictors were applied, using 1971-2-3-4 data to classify cities according to their June 30, 1975 cash position, 87.5% of the 60 cities were properly classified. In the combination of independent predictor variables which gave the highest (87.5%) rate of correct classifications, earlier-year expenditures for Administration and for Parks and Recreation contributed the most to correct classification. In other high-yielding predictor variable combinations, Property Tax Revenues, Interfund Borrowings Due to the General Fund, and Past Due Taxes Receivable were significant measures for classifying cities into subsequent-year Insolvent, Cash-Adequate, and Cash-Rich categories.

Various ratios and balances were tried as indicators, using both Balance Sheet and Operating Statement numbers, with disappointing results.

The universe size was too small to have a hold-out group for validation of the findings. However, subsequent validation can be tested by using the predictors found here, to apply to 1972-3-4-5 data to classify cities _ex ante_ as of June 30, 1976; then to compare the predictions with the actual cash status of those cities at June 30, 1976. The pragmatic usefulness of our research may be tested by finding out whether these predictors _ex ante_ classify cities more or less reliably than do the more intuitive means relied upon by state supervisory officials, and on which they act.

The conclusion was drawn that the investigation partially achieved its goals in that it is possible through this methodology to classify certain Michigan municipalities as to the percentage of cash they will hold. Greater time spans and other significant statistical characteristics of the accounting information of municipalities should be investigated.

PREFACE

This preface allows the inclusion of three items in this paper which are quite unrelated, but will, nevertheless give the reader some insight into the study. The three items are, (1) the means by which the study originated; (2) some observations noted in the course of gathering information, and (3) the acknowledgments to those who contributed to the paper.

The idea for this study originated while attending a gathering for a couple who was moving out to the West coast. During the idle chatter, someone made the chance inquiry as to how they had picked the town where they were going to live. As one might guess, the conversation then turned to the subject of how one should properly evaluate a municipality, whether for choice of a home or other reasons.

Somewhat spontaneously, we offered the thought that the financial statements of a municipality contained one item that might disclose three significant generalizations about the city. It was the amount of money a city spent on the library, of course relative to amounts spent on other factors and relative to amounts spent in previous years on the library. Those three conclusions which might be reached were that expenditures on the library show:

1. The amount of future planning that a city does, for the expenditures on a library are certainly of a long-term nature as opposed to expenditures for something like a swimming pool.

2. The educational aspirations of the community, for even though the number of books on the shelves of the library do not directly influence the intellectuality of the citizens, they certainly do reflect the interests.

3. The financial solvency of the city, to a degree, for the library will probably be one of the first of discretionary expenditures to suffer in a financial "crunch".

The line of reasoning that was then taken was that the activities which are supported (or not) are assumed to be reflective of the needs and wants of the standing citizenry and those activities are reflected in the financial data of the municipality. If such data was readily available, it might serve as a mirror of the community. From this, it was felt that the financial data could be a portent of the future of the municipality. Thus, the motivation for the study.

The second point to be covered in this preface is that during the span of the effort of gathering data, two peculiarities regarding municipal solvency were noted. Although they are not verifiable at this point, they were sufficiently interesting to warrant mention. First, it became apparent that a city which is financially solvent has a readily identifiable, positive personality. The inverse is equally apparent--those cities which are insolvent lack a distinct personality or may even have a negative one. Detroit's one-time positive image as the automobile capital of the world is quickly slipping into oblivion, for operative producing plants in Detroit are few. Other less commendable characteristics of the city have become more prominent in the public's mind. Similarly, few refer to New York City any longer as the "Big Apple" or "Fun City". At the same time, Atlanta and Dallas retain positive images of stability and progress--"nice places to live."

Second, it seemed that those cities with apparent financial difficulties frequently filed their financial statements with the appropriate regulatory agencies in a manner that was not timely. In many cases, an insolvent condition could be anticipated if financial statements were "not in file" at the regulatory agency.

Finally, a portion of papers such as this is usually given to some degree of reflection. We do not wish to be an exception to such a tradition and therefore offer the following thoughts. The students entering doctoral programs bring a wide variety of skills and knowledge, but one necessary characteristic for all is that of commitment. This is a recognition that full accomplishment of one's objectives is possible only through sacrifice of time and effort in varying degrees. As with his other educational experiences, the student usually realizes that the rewards of the entire experience are surely as pleasurable as the achieving of the end in view. However, certain portions of an advanced degree program differ significantly from the other educational experiences which the candidate might have had. Here, the candidate depends upon the effort of many in the earning of his degree. Consequently, the sense achievement falls upon all who lent their knowledge to the task.

Perhaps it is this unique characteristic which impelled the poet and song writer Percy French to memorialize the Ennis and West Clare Railroad in song. The road climbed seemingly ever upward through extremely mountainous country in Ireland to the town of Ennistymon and then down to West Clare. The railroad had a long history of never making its schedule, seldom completing its run, and at all times imperiling its passengers. But the customers did not abandon

this institution. Even when the locomotive quit completely, the passengers joined the crew in seeing that the train continued to move.

As French recorded it:

> Uphill the old engine is climbin,
> While the passenger pushed with a will.
> You're in luck when you reach Ennistymon
> For all the way home is downhill.

To those who have made this journey with me, I want to express grateful appreciation. Faculty, fellow candidates and friends will all find evidence of their contributions. The list reciting their names and ways of participation would probably be longer than this paper. Nevertheless, recognition of specific contributions is a pleasure afforded writers of papers such as this and is not to be denied here.

The Chairman of the Committee, Dr. Gardner M. Jones, is to be thanked for his magnificent patience, the rewriting of bits and pieces of thoughts so that the thread of logic was somewhat visible, his sage guidance to a goal of practicality and most of all, for his constant friendship throughout the entirety of the experience. Dr. William Schmidt provided the statistical model and guidance through the maelstrom of the "numbers". His spirit of accommodation was always present in spite of a most pressing schedule. Dr. Robert Anderson of the Institute for Community Development is to be thanked for his encouragement and criticisms and that ever-present reminder that the products of our research are best if they make the world a better place in which to live.

Programming for the Quantal Response model was done by Rita Grant. Her hours were evidence of a friendship beyond belief. Our

wishes are that her success in this vein might be realized quickly. In addition, we wish to acknowledge all the help of Mr. Douglas Arnold, Local Audit Bureau, State Treasury Department. His assistance made the data gathering an education experience in and of itself. Without his cooperation and guidance, the project would have been impossible. Finally, and perhaps most important of all, Mrs. Jo McKenzie deserves applause loud and long. In her role of typist, she assumed the function of editor, counselor, the one who not only gently reminded me of deadlines but prodded me into meeting them and both she and I know the extent of her effort.

With apologies to John Crecine, a graduate student at the University of Michigan who made a similar conclusion to the preface of his dissertation, we add this postscript. We acknowledge for the last time all those who asked innumerable times, "How's it going Dave?" To them we say:

"now we have reached Ennistymon--all the way home is downhill."

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CHAPTER I

INTRODUCTION

Statement of purpose . . .

This dissertation is about an inquiry into construction of an accounting predictor of future municipal financial insolvency. Primary objectives of the study were:

1. That the prediction model to be established will identify municipalities of population in excess of 10,000 in the State of Michigan which are likely to be financially insolvent; and to predict the probability thereof.

2. That the model will set forth the characteristic differences between that group of municipalities which are likely to become insolvent and that group which are not likely to do so.

3. That, based on the above findings, inferences may be made and probabilities expressed regarding the future financial solvency of specific municipalities.

4. That a financial "profile of a healthy city" and a corresponding financial "profile of a weak city" will be generated.

In addition, the accumulation of this body of data will be useful to both the Treasury Department and the Municipal Finance Commission in their monitoring of the fiscal health of Michigan's cities. Plans are being made to extend this study into one of national scope.

Statement of the problem . . .

For sundry reasons, numerous cities in the State of Michigan (and other states for that matter) have encountered seemingly insoluable

financial problems recently. Within a relatively short span of years, the cities of Ann Arbor, Ecorse, River Rouge and others have faced financial insolvency. In 1974 the City of Hamtramck was so short of cash it was unable to make pension payments to retirees; workers scheduled for retirement refused to leave their jobs because of fear of being without income. The fiscal problems of the City of Detroit have existed for some time and are now chronic. Currently, the plight is so bad that continuous services are now being funded with discontinuous revenues. The present dilemma could be amplified significantly in the future for it is predicted that Detroit's unfunded accrued pension liability will exceed the legal limit for bonded debt and that every dollar of tax revenue could be obligated to that liability.[1]

With increased severity of the situation in many cities has gone an increased awareness of that severity. In fact, ". . . it is now fashionable among American intellectuals to express tender concern for the city's future, to hope that its decay may be arrested, and to offer plans for its revitalization."[2] Often such plans call for significant disruption of existing living patterns, reallocation of expenditures, or more likely, a heavy infusion of federal monies such as Detroit's Mayor Coleman Young's request for $2.6 billion.[3] These and other proposed solutions could be described most aptly as "major

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[1]After revision of actuarial methods, it is now estimated that the unfunded accrued pension liability of the city amounts to $1,166,000,000; See: Financial Statements, City of Detroit, June 30, 1975.

[2]Morton White and Lucia White, The Intellectual Versus the City (Boston: The President and Fellows of Harvard College and the Massachusetts Institute of Technology, 1972), p. 13.

[3]Detroit Free Press (May 1, 1975), p. 1.

corrective surgery"--what in fact usually takes place is a "mercurochrome and band-aid" treatment. The most significant characteristic of such a palliative effort is that they are ex post facto (which perhaps translates colloquially as "too little, too late").

The State of Michigan has recognized a responsibility to local units of government. That recognition is pronounced in the preamble to the Final Report Part 1, State Supervision of Local Finances,[4] prepared by the Sub-Committee on Fiscal Powers of Local Government which begins:

> While the state has the responsibility to provide local units of government with the necessary fiscal authority to finance essential local public services, the state has a concurrent responsibility to insure that local units maintain sound and proper financial practices. As the state becomes increasingly involved in the financing of local units of government, state exercise of its responsibility to insure sound local government fiscal practices becomes even more important.[5]

The analysis performed in this research effort was an attempt to assess financial insolvency in a qualitative manner. It was desired that this be done in a timely and meaningful context. Here the word "timely" should connote that the assessment can occur before the incidence of financial insolvency; "meaningful" should connote that statistical meaningfulness is present, but more importantly, inferential meaningfulness is present.

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[4]Sub-Committee on Fiscal Powers of Local Government, Governor's Special Commission on Local Government, Final Report, Part 1, State Supervision of Local Finance (Lansing, Michigan: 1971).

[5]Ibid., p. 1.

Present means of evaluating municipal performance and status . . .

Currently, two means of evaluating the financial performance and status of a municipality are employed. The first method--of which there are unnumerable variations--could be described as an intuitive process. Such a method is utilized in the development of municipal bond ratings by agencies such as Standard & Poors or Moody's. The second method--equally arbitrary but less subjective--includes the exercise (in some states) of supervisory control analogous to receivorship.

Bond ratings . . .

Mr. James Marling, former Deputy Treasurer of the State of Michigan, detailed the development of a bond rating for a city or an issue previously unrated. He stated that the city would provide the following to the rating agency:

Budgets, current and for two or three preceding years;
Audited (or unaudited) financial statements for prior years;
Information on the tax base for the past five years
    This would be broken down by industrial, commercial and residential classifications. It would include an array of the ten to fifteen largest taxpayers and copies of the 1960 and 1970 censuses.
Present indebtedness--bond issues and contracts outstanding;
Brief 'public relations' type descritpions of the community discussing history, stability, etc.;
Brief biographical sketch of local politicians, councilmen, and civic leaders;
Details on anticipated bond issues discussing security, revenues, etc.; and,
Overlapping debt. (Overlapping debt is that debt for which the residents of the city are obligated via another governmental body such as the county. The opposite is called underlying.)

In addition there are discussions between the rating agent and the city manager, the municipal finance director, the auditors, etc. In the case of a larger issue, the rating agent might visit the community. The questionnaire of one of the large rating agencies is shown as Appendix A.

The eventual rating is simply the sum of value judgments--subjective considerations by learned people of what are deemed to be significant variables. Recommendations have been made to develop models which would employ quantifiable objective factors while still retaining the opportunity for expression of those subjective conclusions. "Under such an approach, ratings would clearly be a product of both objectively measurable and subjective impressionistic considerations."[6]

Other individuals with whom we have met have expressed preference for different indicators of a municipality's financial difficulty. Mr. William Carter of the Citizens' Research Council (Detroit) prefers a singular discrete measure: an unbalanced budget (i.e., forecast expenditures exceed forecast revenues).[7] Carter believes that this antedates innumerable other measures in signalling financial problems. Likewise he summarized financial difficulty per se quite succinctly: a deficit cash position.

Mr. Phillip Dearborn, formerly of the Advisory Commission on Intergovernmental Relations (ACIR) and now with D. C. Municipal

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[6] John E. Petersen, The Rating Game (New York: The Twentieth Century Fund, Inc., 1974), p. 149.

[7] Carter cites one potential difficulty of using this criterion. Presently the State of Michigan does not require cities to budget, but budgets must be balanced. In order to avoid an unbalanced budget the City of Detroit simply did not submit a budget. Ergo, they did not violate the state law requiring that budgets be balanced.

Research Bureau, posed several alternatives which might indicate financial difficulty:

> General operating fund deficit for two consecutive years;
> A ratio less than a certain percentage of cash to total assets;
> The ratio of floating (current) debt to total revenues;
> The ratio of short-term borrowing to property tax revenue;
> That ratio of interfund borrowing to total revenue;
> An accumulated deficit in excess of total revenue;
> The ratio of total taxes to total revenue;
> The ratio of total pension expenditure to total revenue;
> The ratio of unfunded pension liability to total revenue;
> The amount of delinquent taxes; and,
> The percentage of increase in total annual expenditures.

Dearborn's ultimate test is comparable to that of Carter: " . . . the degree of likelihood that a government will have the cash available in the future to pay debt service commitments when due. The key element by this definition is cash."[8]

Legal measures . . .

The second method, used by some states in a regulatory manner, is dependent upon the occurrence of one or more specific conditions before the state imposes direct management. Review of the literature discloses that there is no single event or measure which the states have adopted as a universal measure for evaluating performance and status. Rather, there appears to be diversity in regulatory measures used, varying from some states in which numerous standards are used to other states which have no measures of financial performance or status of a municipality.

An example of the latter is found in the State of Michigan

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[8] Municipal Finance Officers' Association, Proceedings of the Municipal Credit Information and Credit Quality User/Research Seminar (Washington, D.C.: October, 1974), p. 3.

which has no legal criteria for determination of insolvency of a municipality. Rather, insolvency is determined through judicial proceedings. As an example of the other extreme, the State of New Jersey assumes fiscal control of a city upon occurrence of one or more of the five following conditions:

1. Municipal default of debt principal or interest;
2. Over-due payments of taxes to state or other agencies;
3. A budget deficit for two years in excess of 5 percent of the tax levy;
4. Excessive floating debt, measured as a percent of budget; and,
5. Excessive tax delinquency measured as a percent of taxes levied.[9]

The State of Maine uses a tripartite measure:

> When a municipality becomes one year and six months in arrears in the payment of its taxes to the state in full or in part or defaults on any bond issue or payment of interest thereon or refuses or neglects to pay school or other salaries due . . .[10]

Both of these examples might be described as weak laws for determining the point at which state control should be exercised in that a condition of financial difficulty probably existed for some time prior to the occurrence of any of the criterion events cited. Consequently the problem is more complex and demanding than it should be.

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[9] Advisory Commission on Intragovernmental Relations, City Financial Emergencies (Washington, D.C.: Government Printing Office, 1973), p. 155.

[10] Ibid., p. 155.

Other studies . . .

Bibliographic research[11] disclosed no study similar in goal or technique to this effort, specifically, predicting future municipal financial insolvency from accounting data only. On the periphery to this study, there is a plethora of theoretical and empirical works. The literature regarding urban problems is not only vast but also longwithstanding; the recent fiscal crises in local units of government have given impetus to publishing in this topical area. It has occurred frequently as a topic in widely read publications such as Business Week and the New York Times. In my search for background for this study, due attention was devoted to the various levels of the related literature.

Relevant writings . . .

Any attempt to develop and present an exhaustive listing of relevant works might be just that bo both researcher and reader--exhausting. An extensive and detailed bibliography of related materials is found in the Index of Economic Journals.[12] Although neither as contemporary or comprehensive as the "Index," an excellent guide to the specific area of Public Finance is found in Mitchell and Walter's State and Local Finance.[13]

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[11]The search began with employment of "Datrix II," and the adjunct publications: Comprehensive Dissertation Index, Business and Economics, Supplements for 1973, 1974 and 1975 (Ann Arbor: Xerox University Microfilms), The "Datrix II" disclosed no similar dissertations.

[12]Index of Economic Journals (Homewood, Ill., Richard D. Irwin, Inc.).

[13]William E. Mitchell and Ingo Walter (ed.), State and Local Finance (New York: The Ronald Press Company, 1970).

A logical beginning point for an investigation of municipal insolvency is with a general viewing of the public finance literature. Although these works do not constitute a reasonable description of the positive model, they do constitute a foundation of information. Musgrave and Musgrave[14] provide an established text which analyzes the empirical and theoretical material of the public sector. Local govnermental units are not treated specifically, but rather throughout the text. Equally acceptable as an alternative is Herber's elementary text.[15]

Moving from introductory texts to those dealing specifically with urban problems, the work of Mills[16] must be recognized as outstanding. He provides a unique analysis of urban phenomena and problems. The short work is generally non-quantitative in its approach (Chapter 5 uses calculus), but perceptive in insight. Two readings books are stimulating enough to warrant mention. The first, edited by Schreiber et al,[17] though somewhat out of date, presents the readings in such a way that they are complemented through their

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[14]Richard A. Musgrave and Peggy B. Musgrave, Public Finance in Theory and Practice, 2nd ed. (New York: McGraw-Hill Book Company, 1976).

[15]Bernard P. Herber, Modern Public Finance: The Study of Public Sector Economics, 3rd ed. (Homewood, Ill.: Richard D. Irwin, Inc., 1975).

[16]Edwin S. Mills, Urban Economics (Glenview, Ill.: Scott, Foresman and Company, 1972).

[17]Arthur F. Schreiber, Paul K. Gatons, and Richard B. Chamber (ed.) Economics of Urban Problems Selected Readings (Boston: Houghton Mifflin Company, 1971).

interrelationships. Hochman's collection of readings[18] includes some of the most provocative ranging from the highly illuminating "The Cost of Disease of the Personal Services and the Quality of Life" by Baumol and Oates to an excerpt from The Unheavenly City[19] in which Banfield states that there is no solution to urban problems!

Two works concentrating on the fiscal aspects of the urban economy are outstanding. The first, by Hirsch et al[20] looks first at the broad question of fiscal health and then specific problems of commuters, nonwhites and overlapping governments. Unfortunately, the populations included in the statistical studies (cities) are not comparable making conclusions less convincing than they might have been. The work of Greene et al[21] is more comprehensive in its viewing of the aspects of the fiscal problems. However, the study dealt with only one city--Washington D.C.--and therefore the validity of inferential generalizations are constrained by the reader's appraisal of the quality of the authors' measurements and logic. Innumerable other economic works propose innumerable explanations and solutions, but few authors approached the subject with the candor of Pettengill and Uppal.[22]

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[18]Harold M. Hochman (ed.), The Urban Economy (New York: W. W. Norton & Company, Inc., 1976).

[19]Edward C. Banfield, The Unheavenly City (Boston: Little, Brown and Company, 1970).

[20]Ertnrt Z. Hirsch, Phillip E. Vincent, Henry S. Terrell, Donald C. Shoup and Arthur Rosett, Fiscal Pressures on the Central City (New York: Praeger Publishers, 1971).

[21]Kenneth V. Greene, William B. Neenan, Claudia D. Scott, Fiscal Interactions in a Metropolitan Area (Lexington, Mass.: D.C. Heath and Company, 1974).

[22]Robert B. Pettengill and Jogindar S. Uppal, Can Cities Survive? The Fiscal Plight of American Cities (New York: St. Martin's Press, Inc., 1974).

> The foregoing description and analysis show that the question should really be 'How Can Cities Survive?' That they do survive, that they have survived many fiscal crises is clear. But cities are basically people and people have their discontents. They are not satisfied with this or that aspect of their lives. One group of dissatisfactions centers around the political-economic-social-geographic entities we call cities.
>
> The urban situations that people don't like and want to improve range all the way from too high taxes to too few services. Some people deplore this aspect, and some deplore that. The dissatisfied people range from the powerful to the impotent, from the wealthy and the incumbents to the poor and the leaders of the party out of power. At any moment each sees the survival of his city somewhat differently. Each seeks to improve a different facet of the situation. Struggling against opposition and inertia, each may wonder whether his city as he sees it can survive the apathy, the greed, the shortsightedness, the tax burdens, the neglect, the deprivation, the whatever, that are the particular objects of his despair.
>
> So, too, the methods differ that people follow in trying to ensure the healthier survival of their city, healthier when measured by their value scales. A 'solution' from one point of view may be seen as a serious problem from another. One man's meat is another man's poison. Therefore no universal prescription is possible.
>
> The authors have tried to show that situations differ as well as goals and that all benefits have cost. Each concerned citizen should be aware of the manifold alternatives and, weighing each carefully, push that one which seems best in the long run for himself, for his group, for his city as he sees it. Then in the give and take of the political process, the pulls and pressures of contending forces, our cities will move from one crises to another, continuing to survive while forever changing and being changed.[23]

The publications of the Advisory Commission on Intergovernmental Relations have been well received. City Financial Emergencies: The Intergovernmental Dimension, mentioned earlier,[24] is colloquially captioned "The Bible" by those involved with municipal fiscal problems. Seventeen other works of ACIR are listed in the bibliography.

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[23]Ibid., pp. 147-148.

[24]See Supra, fn. 9.

One important horizon of current research is the application of data processing to the budgetary process of the municipality. The viewing of the budget as a problem in resource allocation with solutions subject to a series of constraints was posed in a significant manner by Wildavsky.[25] Since then there have been forecasting and budgeting efforts in Cleveland,[26] Detroit,[27] Los Angeles,[28] Mobile,[29] New Haven,[30] Philadelphia,[31] and Pittsburgh.[32] These models are in part an attempt to avert financial difficulties through sound budgeting and forecasting. As additional capability is gained such efforts will surely become more successful. Then there should be greater employment of sophisticated techniques in the maximization of available resources.

Further considerations . . .

Certain extensions of the findings are properly excluded from consideration. These are by necessity, considering the data involved.

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[25] Aaron Wildavsky, The Politics of the Budgetary Process (Boston: Little, Brown and Company, 1964).

[26] John P. Crecine, Governmental Problem-Solving (Chicago: Rand McNally & Company, 1969).

[27] Ibid.

[28] Hall and Licari, Journal of Regional Science, 1974.

[29] Semoon Chang, "Forecasting Revenues to Municipal Government: The Case of Mobile, Alabama," Governmental Finance (February, 1976), pp. 16-20.

[30] Claudia Devita Scott, Forecasting Local Government Spending (Washington, D.C.: The Urban Institute), 1972.

[31] Glickman, Journal of Regional Science, 1971.

[32] Crecine, Problem-Solving.

The first, and perhaps most significant of these exclusions, is found in the objective[33] in that the study refers only to cities in Michigan having population greater than 10,000. Therefore in absence of further research efforts generalizations to cities in other states could be justified only by the reader "bridging" from this study to another population. Furthermore, certain cities (having populations greater than 10,000) were considered unacceptable for the project, in that data was missing or a different year-end was used.

Missing from the study is any attempt to explain the variance in the amount of cash a city would hold. No investigation in the causal link has been investigated, as this effort was viewed in the eyes of the researcher as being the first approach to the problem of urban financial strain.

Other variables to be considered in future studies might be concerned with the type of city government and management, the effect of political parties, the location of the municipality, its size, its sympathetic relationship to an adjunct, its sources of revenues, composition of standing citizenry, base valuation of property, and other innumerable factors. In the interest of eventually bringing this study to a conclusion, these will be deferred to the future.

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[33] See supra, p. 1.

CHAPTER II

DATA--SOURCE, NATURE AND COLLECTION

Source of data . . .

The data to be analyzed were drawn from the financial reports of municipalities in the State of Michigan. These municipal reports differ significantly from the more familiar financial statements of profit-seeking ventures or those of numerous other governmental units in that the form of the financial report is prescribed by law. In 1968 the State of Michigan enacted legislation[1] which had as its primary objective the providing of ". . . a means for the accumulation of financial information which will be uniform for all local units[2] and of similar size."[3] The uniformity decreed in Act 2 has three general aspects; statements are to be:

1. Prepared in accord with a designated chart of accounts[4]

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[1] Act 2, Public Acts of 1968, State of Michigan. (This law is hereafter referred to as Act 2).

[2] In reality, the uniform accounting legislated in Act 2 covers all lower governmental units (e.g., counties, municipalities, drain districts, villages, etc.); for our purposes, the term "local unit" will be construed to mean "municipality."

[3] Systems and Procedures Staff, Local Audit Division, Department of Treasury, Uniform Accounting and Procedures Manual for Local Units of Government (Lansing, Michigan: State of Michigan, 1975), p. 1.

[4] Ibid., pp. F-1 and F-26.

2. Audited by certified public acountants[5]

3. Submitted annually to the Local Audit Division of the State Treasury Department.

Filings were to have begun with the fiscal year ending in 1968; however, the fiscal year 1970 is generally regarded as the first year for which filings are complete. As of this writing reports have been filed for nine years. The study originally focused on the financial statements of 86 cities in the State of Michigan each having a population of over 10,000--an arbitrarily set lower limit.[6]

Some consideration was given to the possibility that (for research purposes) the time period of six years might be viewed as being insufficient. An alternative population was available; it consisted of those cities which had had audits for years prior to 1968. In such a case, the audited statement of that city would have been prepared on a basis (modified accrual, which is explained later) which is generally held to be comparable to the statement prepared in accord with the requirements of Act 2.

The adversities of such a choice were clear. First, because less than half of the 86 cities had annual audits before 1968 by certified public accountants, such a choice would have resulted in a significant dimunition of the population. Even more important was a constraint of generalizability of findings. Conclusions could then

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[5]The City of Detroit enjoys de jure relief from the filing of annual financial statements which have been audited by certified public accountants; unaudited statements are acceptable. This is because cities of over 1,000,000 population are required to file such statements only every fifth year.

[6]Appendix B lists those 86 cities.

be imputed only to those cities in the State of Michigan having population over 10,000 which previously prepared audited financial statements on the modified accrual basis. In addition to that applicability considerations, there was the practical aspect of laxity in filings and inaccessibility of those statements as they are already stored in archives.

The conclusion was reached that the benefits to be gained--if any--from the extra years were not worth the constraint upon the conclusions. Although it is not directly related to an understanding of the accounting prescribed for municipalities in Act 2, the following discussion should lend itself to an understanding of the nature of the relevant data variables.

Fund accounting . . .

Although there are several similarities in the accounting of profit-seeking ventures and that of municipalities, there are also several differences. The most salient difference is that the accounting for the latter is organized around funds. One definition of a fund is:

> An independent fiscal and accounting entity with a self-balancing set of accounts recording cash and/or other resources together with all related liabilities, obligation reserves, and equities which are segregated for the purpose of carrying on specific activities or attaining certain objectives in accordance with specual regulations, restrictions or limitations.[7]

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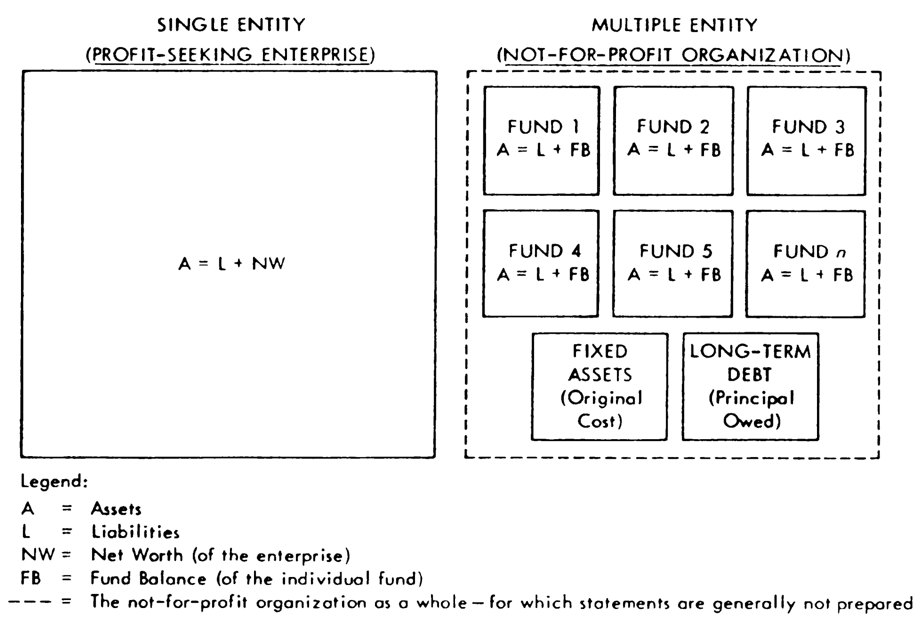
[7]National Council on Governmental Accounting, Governmental Accounting, Auditing and Financial Reporting (Chicago: Municipal Finance Officers Association, 1968), pp. 161-62. In subsequent citations this work will be referred to by its acronym, GAAFR.

The word "fund" should have a connotation greater than cash alone. A fund could consist of assets, liabilities and an equity balance. Those assets could be cash, receivables, inventories, fixed assets, and prepaid items at times; liabilities would consist of accounts and notes payable.

In summary, each fund may be viewed as a separate self-contained reporting entity, with the total of all the funds seldom presented in municipal reporting. In contrast, profit-seeking ventures seek to present the economy entity in full. This contrast between the accounting of the profit-seeking venture and the municipality is emphasized in the following figure:

FIGURE 1*

SINGLE ENTITY VERSUS MULTIPLE ENTITY ACCOUNTING



*After Lynn and Freeman, Fund Accounting, p. 9.

The commercial enterprise will have a single unified set of accounting records which will summarize all of the financial transactions while the city will have a set of self-balancing accounts for each fund that has been set up. Each fund will have its own budget; usually the financial statements of that fund will display with particular emphasis the comparison between budgeted and actual revenues and expenses.

General types of funds . . .

The two general types of funds reflect one way of classifying the types of municipal operations. The primary criterion for this categorization is whether the ". . . resources of the fund may be expended or are to be maintained on a self-sustaining basis."[8] These two types of funds are frequently captioned "Expendable" and "Non-Expendable." The former would be utilized in accounting for recurring operations which supply basic services to the general populace. Such a fund is usually under stringent budget control. Resources are expendable. The budget is prepared under the assumption that resources will be replenished and expended each year. The latter covers those self-sustaining functions of a municipality which are operated as an entity on a basis similar to that of a commercial enterprise. In such a case, resources of the fund are not expendable.

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[8]Edward S. Lynn and Robert Freeman, Fund Accounting Theory and Practice (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1974), p. 32.

Specific types of funds . . .

The National Council on Government Accounting recommends eight different types of funds:

(1) The General Fund to account for all financial transactions not properly accounted for in another fund;[9]

(2) Special Revenue Funds to account for the proceeds of specific revenue sources (other than special assessments) or to finance specific activities required by law or administrative regulation;

(3) Debt Service Funds to account for the payment of interest and principal on long term debt other than special assessment and revenue bonds;

(4) Capital Projects Funds to account for the receipt and disbursement of moneys used for the acquisition of capital facilities other than those financed by special assessment and enterprise funds;

(5) Enterprise Funds to account for the financing of services to the general public where all or most of the costs involved are paid for in the form of charges by users of such services;

(6) Trust and Agency Funds to account for assets held by a governmental unit as trustee or agent for individuals, private organizations and other governmental units;

(7) Intragovernmental Service Funds to account for the financing of special activities and services performed by a designated organization unit within a governmental jurisdiction for other organization units within the same governmental jurisdiction;

(8) Special Assessment Funds to account for special assessments levied to finance public improvements or services deemed to benefit the properties against which the assessments are levied.[10]

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[9] Such a negative, global definition lacks informational content. The General Fund will be explained in a positive manner on page

[10] National Council, GAAFR, pp. 161-62. (?)

Those funds which are generally considered as Expendable Funds are 1,2,3,4, and 8; those funds which would be classified as Non-Expendable Funds would be the Enterprise (5) and Intragovernmental Services (7) Funds. Trust and Agency Funds (6) is often classified as either expendable or non-expendable depending on the intended use of resources; in the State of Michigan, it is expendable. As a further elaboration on the particulars of these funds, it should be noted that there is only one General Fund but there can be many of the seven other types of funds. Each fund would have its own accounts necessary to reflect the transactions, assets, liabilities, revenues and expenditures. Each fund would be self-balancing.

Capital assets . . .

A further distinction between the accounting used by municipalities and commercial enterprises exists in regard to the accounting for expenditures for capital assets by municipalities. Capital assets of the expendable funds are not recorded in those funds, but rather in an account entitled "General Fixed Assets."[11]

The accounting treatment for expenditures for capital assets to be used in the operations of the Enterprise Funds and Intragovernmental Service Funds is similar to the treatment of such expenditures in the accounting records of the commercial enterprise in that they are capitalized at historical cost. Depreciation--using the straight line method--is recorded on such assets by municipalities.

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[11]Therefore, there exists a third type of accounts. These are called the "Non-Fund Group of Accounts" and two primary groups are required in Michigan statements. They are the General Fixed Assets and General Long Term Debt. Although they are self-balancing they function more as a schedule than as an account.

Basis of accounting . . .

The third and final difference, not as significant as the other two, is found in the basis of accounting for certain of the funds of a municipality. Generally speaking, the accounts of the Enterprise Funds, Intragovernmental Service Funds, Special Assessment Funds and the Trust and Agency Funds are maintained and reported on an accrual basis comparable to the basis employed by most commercial enterprises. The difference is found not in those accounts but rather in the General Fund, the Special Revenue Funds, and the Debt Service Funds which are maintained and reported on the Modified Accrual Basis.

Modified accrual basis . . .

Under the Modified Accrual Basis of accounting those revenues which are measurable and available are accrued because they are resources which may then be appropriated. Those revenues not susceptible to accurate estimation are recorded when they are received. Revenues from fees for services and income tax would be two examples of revenues which do not meet a test of reasonable certainty.

Expenditures under the Modified Accrual Basis are recorded when the goods or services are received excepting for interest on general obligation long-term debt which is accrued. An additional modification to the Modified Accrual Basis of accounting can be made in the form of an encumbrance; this will be explained in the paragraph on the format of the Act 2 statements. Having cited the differences between the accounting system for a municipality and that for a commercial enterprise, we present a summary view of how the accounting system is related to the general operations of the municipality. We shall also explain the general transactions to be found in the various funds.

The municipal accounting system . . .

This overview of the typical municipal accounting system follows the order given earlier in the recommendations of the National Council on Governmental Accounting. The first of the funds is the General Fund, the prime focus of this research effort. In this discussion it should be remembered that the General Fund is singular in number and that it is expendable, that is, the resources are expended and replenished annually.

> In this category the general fund are funds established to account for resources devoted to financing the general services which the governmental unit performs for its citizens. These include general administration, protection of life and property, sanitation and similar broad services.[12]

In the State of Michigan, the typical sources of revenue for the General Fund are principally the real and personal property tax, state shared revenues and to a lesser degree, fees for services rendered by various departments. State and Federal grants are often present. In some municipalities another major source of revenue has been attained through the levying of a personal income tax.

Expenditures of the General Fund are primarily concerned with the operation of the municipality: administration, police, fire, parks and recreation, public works and several minor activities. The

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[12] R. M. Mikesell and Leon E. Hay, Governmental Accounting (Homewood, Ill.: Richard D. Irwin, Inc., 1969), p. 4.

General Fund also functions as a clearing account for the distribution of revenues (e.g. property taxes) to other funds or governmental units.

Special Revenue Funds . . .

Revenues of a Special Revenue Fund are similar to those of the General Fund except that they are self-imposed for specific purposes or so to speak are "earmarked." The expenditures for this type of fund are for specific purposes required by law or contract and are of a continuing nature, usually. Titles of some special revenue funds which can be established by municipalities will aid an understanding of this type of fund:

Cemetery Fund
Ambulance Fund
Drain Fund
Street Lighting Fund
Parking Meter Fund
Comprehensive Employment and Training Act Fund[13]

If any capital assets are acquired, the expenditure from this fund is treated as an expenditure for any other expense, such as that for wages or materials; the capital asset acquired would be carried as the asset of some other entity within the governmental unit or would be shown in the non-fund account, General Fixed Assets.

Debt Service Fund . . .

This fund is rigidly controlled; it is used ". . . to account for the payment of interest and principal on long term debt other than special assessment and revenue bonds."[14] Furthermore, the Debt Service

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[13] Local Audit Division, Uniform Accounting Manual, pp. 5-39.

[14] National Council, GAAFR, p. 161.

Fund ". . . is necessary to maintain the separate identity and character of general debt operations by governmental units and to permit the proper disclosure thereof in financial statements and reports."[15]

The expenditures of this fund are for principal, interest and any service charges adjunct to the general long-term debt. Revenues are derived from numerous sources related to the General Fund revenues, typically property tax. In the case of a refunding of an outstanding issue with another issue, the Debt Service Fund would be used as a clearing account for the transaction.

Capital Projects Funds . . .

The revenues[16] of the Capital Projects Funds come from transfers from the General Fund, other governmental units (such as county, state or federal) and the sale of certain bonds; in the case of bonds, the proceeds would be designated for a capital project and the liability would subsequently be transferred to the Debt Service Funds. The Capital Projects Funds account for the purchase of new facilities and equipment or the undertaking of such projects as street paving, building additions or improvements, etc. It does not include the acquisition of capital facilities financed by a Special Assessment or Enterprise Fund.

As examples, the State of Michigan provides the following fund titles:

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[15]Ibid., p. 37.

[16]Some accountants question the use of the term "Revenues" in connection with Capital Project Funds. Such "revenues" are inflows to the unit as a whole but rather transfers and appropriations from other funds. See: Lynn and Freeman, Fund Accounting, p. 282.

Hospital Building Fund
Industrial Complex Construction Fund
Animal Shelter Construction Fund
Mental Health Construction Fund
Library Building Construction Fund
Medical Care Facility Building Fund
Park System Construction Fund
MVH Act 175 Major Street Construction Fund
MVH Act 175 Local Street Construction Fund
Airport Fund
Sewage Disposal Fund[17]

In addition to these titles listed above (in order of their number in the prescribed chart of accounts) other titles could be set up as needed in the "open" numbers provided. For each major project there would be a fund and the accounts in that fund would be used in the recording of expenditures related to that project and the assets retained. The individual funds are presented on a summary statement, classified as to whether they are completed or not.

The expenditures would typically consist of land, building materials, labor and other related costs for the project for which the fund was established. Such expenditures could even include indirect costs if the municipality is acting as general contractor; overhead is usually not included because of some rather notorious instances of fraud in the past; but if it is included it will be for well-defined items. The fund could be abolished upon completion of the project or it might be of an on-going nature such as street paving.

Enterprise Funds . . .

The charges to the users for the operations and sale of products or services are the principal revenues of an Enterprise Fund.

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[17] Local Audit Division, Uniform Accounting Manual, pp. 59-75.

A familiar example of such a fund would be a municipally owned and operated electric power company or a water system. The City of Detroit's water system has both bulk and individual customers and revenues would be so classified on the Statement of Revenues and Expenditures.

It is to the revenues that one looks for determination of whether a fund should be classified as an Enterprise Fund or not:

> . . .If a substantial amount of the revenues used to finance an activity or series of related activities in a single fund is derived from user charges, the fund can be appropriately classified and accounted for as an Enterprise Fund.[18]

Expenditures of an Enterprise Fund are quite comparable to those of a profit-seeking venture. Included are all of the expenses of producing the product or rendering the service. Contrary to the traditional not-for-profit accounting, depreciation may be recognized as an expense by an Enterprise Fund. From time to time some municipalities have an Enterprise Fund make remittances to the General Fund. This action is another departure from the usual governmental accounting, for these distributions are treated in a manner analogous to the dividends of a commercial enterprise to its shareholders.

Some of the titles of Enterprise Funds could be:

Abstract Fund
Ambulance Fund
Mobile Home Park System Fund
Markets Fund
Fair Board Fund
Airport Fund
Golf Course Fund
Auto Ferry Fund
Civic Auditorium Fund
Marina Fund[19]

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[18]National Council, GAAFR, p. 50.

[19]Local Audit Division, Uniform Accounting Manual, pp. 77-107.

Trust and Agency Funds . . .

Trust and Agency Funds are used for moneys which are being held by the municipality (as a trustee or agent) to be distributed later. Because of the custodial nature of such funds, assets will exactly equal liabilities and there will be no equity balance. Revenues are supplied by tax collections, payroll deductions, transfers and shared revenues from other governmental units. Expenditures are in accord with the prescribed purpose of the particular fund.

Some of the titles suggested by the State of Michigan illustrate uses of Trust and Agency Funds:

Emergency Employment Act Program
Cemetery Trust Fund
Employees Death Benefit Fund
Employees Sick Pay Fund
Police and Fire Retirement System Fund
Urban Renewal Escrow Fund[20]

Trust and Agency Funds in the State of Michigan are expendable; this type of fund is explained in the subsequent section.

Expendable/nonexpendable Funds . . .

In theory, the classification of a fund as expendable or nonexpendable is determined by whether the resources of that fund are expendable or not. The constraints (or lack of constraints) upon consumption of those resources could emanate from law, contract or perhaps action of an administrative or regulatory body. While most funds are expendable, those which are nonexpendable might be constrained in total or in part (e.g., being able to spend only the interest from certain investments). By nature then, it follows that an expendable fund

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[20]Ibid., pp. 145-66.

would hold only "liquid" assets--those which may be expended--consisting of cash, receivables and short-term investments (and perhaps limited supplies); assets which would not be held would include long-term investments, property, plant and equipment. By definition, one would conclude that assets such as these are not readily expendable.

The "other side of the coin" is then found in the nature of the capital of the expendable fund. It would be of a non-permanent nature and, in general, would rise and fall in sympathy to the assets of that fund. In recognition of the fact that resources could be dissipated, expendable funds come under budgetary control. This element of budgetary control, stated simply, is another of the characteristics of the expendable fund. Although some non-expendable funds might also be subject to budgetary control, most expendable funds are annually budgeted. Finally, it remains true only in theory (and seldom in practice) that the resources of the expendable fund should be expended completely within a twelve month period.

Intragovernmental Service Funds . . .

The Intragovernmental Service Funds are:

> . . . used to account for the financing of special activities and services performed by a designated activity or department within a governmental jurisdiction for other units or departments within the same governmental jurisdiction.[21]

These funds are operated as self-supporting enterprises. They derive their revenues from billing other departments for the services to those departments. Expenditures might include salaries and wages, interest on long-term debt, supplies, administrative fees, etc. Here

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[21] National Council, GAAFR, p. 162.

again, depreciation is recorded as an expense. Gains or losses on sale of equipment are recognized. These funds do have equity balances. Both fixed assets and long-term liabilities are recognized on the balance sheet of an Intragovernmental Service Fund. Some of the typical serviçes for which intragovernmental charges could be made would be building and grounds maintenance, data processing, mailing, radio communications, vehicles, office equipment, electricity, gas and water.

Special Assessment Funds . . .

The final type of fund is the Special Assessment Funds. These are used to account for construction or improvement projects which benefit only a particular group of real property owners rather than the general populace. Examples of such projects would be streets, roads, sidewalks, lighting, sewers and watermains. The owners who benefitted from the improvements are charged pro-rata shares of the cost of the improvements.

These funds are a hybrid of two other funds, Capital Projects and Debt Service Funds. Special Assessment Funds combine the functions of both but only as related to special assessments. Revenues are from General Fund appropriations, sale of bonds, special assessments against the property owners who benefitted from the improvement, interest received from investing excess cash, and in some cases, from sale of delinquent special assessment receivables. Expenditures, in general, are for payment of contracts, bond principal and interest, and any direct and/or indirect costs relating to the construction of the project. The eight types of funds are summarized in Figure 2 on page 30 and Resource Flows are illustrated in Figure 3, page 30.

FIGURE 2 **

TYPES OF STATE AND LOCAL GOVERNMENT FUNDS AND ACCOUNT GROUPS (NCGA RECOMMENDATIONS)

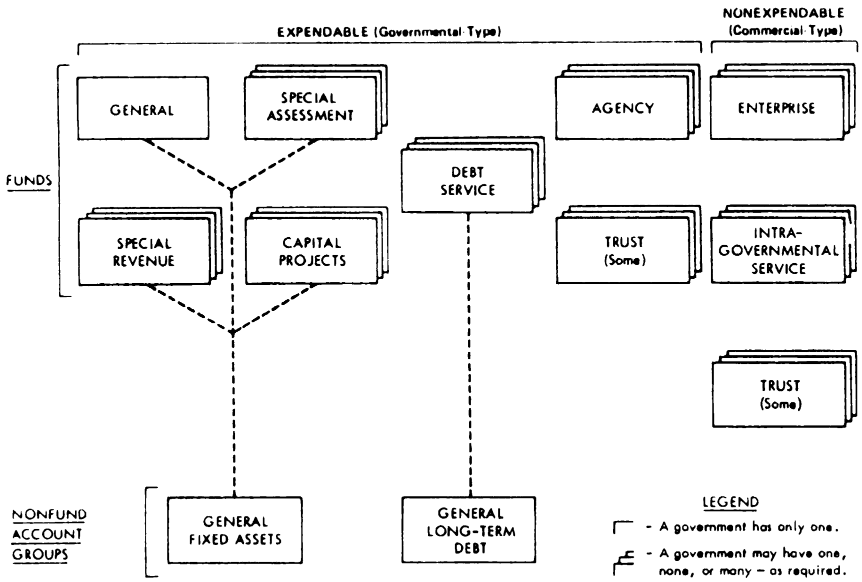
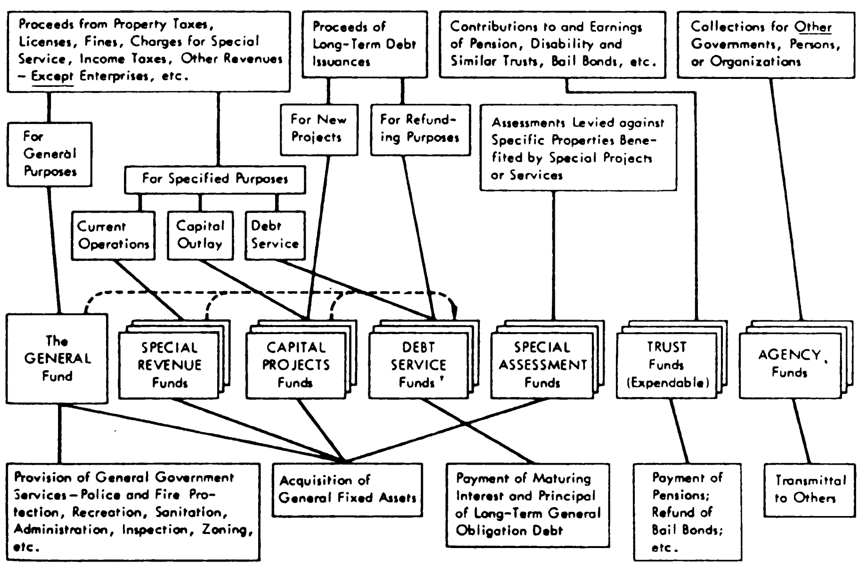


FIGURE 3**

TYPICAL RESOURCE FLOW PATTERN—EXPENDABLE FUNDS*



* Flows to and from nonexpendable funds are excluded.

† As indicated by ----, resources may be transferred to the Debt Service Fund from other funds; also, General Long-term Debt may be serviced directly from other expendable funds.

**After Lynn and Freeman, Fund Accounting, pp. 32-33.

Format of Act 2 financial statements . . .

The minimum financial statements and schedules to be furnished by each city (regardless of size) to the state in accord with the format of uniform financial reporting under Act 2 are:

GENERAL FUND (With or Without Encumbrances)
- Balance Sheet
- Analysis of Changes in Fund Balance
- Statement of Revenues--Estimated and Actual
- Statement of Appropriations and Expenditures

MAJOR STREET FUND (Without Encumbrances)
- Balance Sheet
- Analysis of Changes in Fund Balance
- Statement of Revenues--Estimated and Actual
- Statement of Appropriations and Expenditures

DEBT SERVICE FUND
- Balance Sheet
- Statement of Revenues, Expenditures and Fund Balance

CAPITAL PROJECTS FUND
- Balance Sheet
- Analysis of Changes in Fund Balance

WATER AND SEWER FUND
- Balance Sheet
- Statement of Retained Earnings
- Statement of Income
- Analysis of Income Available for Debt Retirement
- Schedule of Operating Statistics

INTRAGOVERNMENTAL SERVICE FUND
- Balance Sheet
- Analysis of Changes in Contributions
- Statement of Retained Earnings
- Statement of Operations

EMPLOYEES RETIREMENT SYSTEM
- Balance Sheet
- Analysis of Changes in Reserves

TRUST AND AGENCY FUNDS
- Balance Sheet
- Statement of Revenues, Expenditures and Fund Balance

GENERAL FIXED ASSET GROUP OF ACCOUNTS
- Schedule of Changes in Fixed Assets

LONG-TERM DEBT GROUP OF ACCOUNTS
Statement of General and Special Revenue Long-Term Debt
Schedule of Indebtedness[22]

Other statements, schedules and statistics are frequently furnished.

An encumbrance is defined as

> Obligations in the form of purchase orders, contracts or salary commitments which are chargeable to an appropriation and for which a part of the appropriation is reserved. They cease to be an encumbrance when paid or when the actual liability is set up.[23]

The choice then is between mutually exclusive alternatives, the General Fund Without Encumbrances or the General Fund With Encumbrances. In the opinion of Mr. James Marling, former Deputy Treasurer of the State of Michigan, the difference between these two types of General Fund is not significant enough to make data from the two systems non-comparable; therefore, we include cities having both reporting formats.

The General Fund . . .

Additional comments regarding the General Fund are in order, particularly because it is the focus of our attention. The importance of this fund is found in the fact that the accounting for most of the current activities and operations of a municipality are contained within this fund. The city's main budget is prepared and administered through this fund. The important economic factors and the financial status of a municipality should be reflected in the revenues of this fund. The level of the choices of the various expenditures should not only indicate the needs of the city, but also to a degree, its

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[22]Department of Treasury, Uniform Reporting Format for Financial Statements for Local Government Units in Michigan (Lansing, Mich.: State of Michigan, 1971), pp. 1-2.

[23]Lynn and Freeman, Fund Accounting, p. 985.

desires. In addition to being encompassing, the revenues and expenditures are also recurring on a regular basis. Where some funds could span several years or a period as short as a month, the General Fund is distinctly related to a single year. It is this fund from which the majority of the observations are drawn.

Observations . . .

The following data points were to be gathered for each city for each year:

Cash and Certificates of Deposit in the General Fund
Unpaid delinquent taxes receivable
Interfund borrowing due to the General Fund
Total of the General Fund
Total current liabilities of the General Fund (often captioned "Floating Debt") not including encumbrances or bonded debt
Unencumbered, unappropriated balance (equity) of the General Fund
Total revenues applicable to the General Fund
Revenues from current property taxes applicable to the General Fund
Revenues from local income tax applicable to the General Fund
Revenues from state shared revenues applicable to the General Fund
Revenue from Federal Revenue Sharing Fund included in the total General Fund revenue
Total expenditures applicable to the General Fund
Administrative expenditures applicable to the General Fund
Police expenditures applicable to the General Fund
Fire expenditures applicable to the General Fund
Park and recreation expenditures applicable to the General Fund
General obligation bonds outstanding
Unfunded accrued pension liability
Cash and Certificates of Deposit in Federal Revenue Sharing Fund
Total of Federal Revenue Sharing Fund
Capital Expenditures of Federal Revenue Sharing Fund
Other expenditures of Federal Revenue Sharing Fund

In general, most observations were readily available. In some instances, however, items were missing. For example, the City of

Farmington combines police and fire protection costs in a single figure captioned "Public Safety." The figure most frequently missed was that of unfunded accrued pension liability. Where figures were missing either that data point was then omitted from the analysis (as in the case of the unfunded accrued pension liability) or the particular city was considered ineligible (as in the case of Farmington).

Another cause of disqualification of a particular municipality from study was that of having a year-end other than June 30, the typical fiscal year-end for not-for-profit organizations, and especially governmental units.[24] Such cities have been marked with an asterisk in Appendix B. Revenues and expenditures would have been comparable regardless of the year-end. The difference occurs with the balance sheet observations--particularly cash. The primary revenue of most municipalities is the property tax; this flows in at a specific time unlike other revenues.

Nature of the observed variables . . .

In this section we intend to comment regarding two aspects of the data points. First, we shall view the accounting content of some of these variables and then we shall comment on the predictive ability hypothesized for some of the variables at the time of selection.

The first is Cash and Certificates of Deposit in the General Fund. This observation serves as both criterion and predictor variable, depending upon the year-end balance used. As discussed in Chapter I, the balance of cash at the end of the most recent year will

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[24]Currently there is a movement underway for legislation that would standardize all municipalities to the same year-end, June 30.

serve as a surrogate for liquidity (or its counterpart, financial insolvency). The cash balance of the other funds was not considered relevant because it was not readily available for satisfying operations or liabilities of the General Fund.

The Unpaid Delinquent Property Taxes Receivable are for both real and personal property taxes from prior years which remain unpaid. They are traditionally shown net of an allowance for uncollectibles. This treatment is comparable to that found in the accounting of most commercial enterprises. In a few instances, the balance was totally reserved. Upon the advice of Mr. James Bolthouse, then Deputy Treasurer of the State of Michigan, the balance of the outstanding delinquent taxes was added back, increasing that account, and the total of the General Fund, and the Unencumbered, Unappropriated (equity) balance of the General Fund. It was postulated that this observation would serve as an excellent predictor of insolvency, reflecting perhaps unemployment, a declining average income, abandoned property, etc. The next variable, Interfund Borrowing Due to the General Fund, should be considered relative to the account Interfund Borrowing Due From the General Fund. These two accounts are considered to be so relevant that if a consolidated balance sheet for all the funds of the city is prepared, these balances cannot be eliminated in consolidation but (by law) must be shown. The former account, an asset of the General Fund, arises from the lending of moneys by the General Fund to other funds, the rendering of services, moneys due to be remitted by some other fund to the General Fund, etc. The latter account, of course, would arise from an opposite transaction taking place. In particular though, there can be borrowings of cash from other funds if the revenues of the General Fund are insufficient to keep the city solvent. Such borrowings are generally held to be indicative of overwhelming financial problems which might be accompanied by deceptive budgeting and possibly poor fiscal management.

An example of insufficient revenues is found in the case of the City of Royal Oak. The financial statements for the year 1973 were held up for over twelve months while the auditors deliberated with the Local Audit Division of the State Treasury Department over the disposition of long-outstanding interfund borrowings by the General Fund from the Water System Fund. The one million dollars had been outstanding for several years. The State maintained that repayment was not probable and therefore taxes were imposed to repay the amount within a reasonably short time. The unwritten rule of thumb among financial managers was found to be that neither asset nor liability should be outstanding for a long period of time nor should the liability exceed the asset by a significant amount.

It was assumed that the presence of revenues from a local income tax applicable to the General Fund would be an excellent indicator of financial problems. If a municipality levied such a tax, they had then used up a significant portion of their "revenue capacity." A similar reasoning was applied in the case of revenues from the Federal Revenues Sharing Fund included in the revenue of the General Fund. It was presumed that this was a use of non-continuous revenues (assuming that Federal Revenue Sharing had a stated expiration and might not be renewed) to meet continuing expenses. As a corollary to that, it was concluded on an intuitive basis that the more solvent municipality could use its Federal Revenue Sharing Funds for capital expenditures rather than traditional expenses.

Of the four expenditures considered all were presumed to be strong indicators of financial solvency. It was hypothesized that as a municipality became financially troubled, it would be less efficient and that administrative costs would rise relative to total expenditures. It was assumed that the need for police expenditures could be highly related to financial difficulties. The fire expenditures were thought to be reflective of the decline in the value of the property, property having poor wiring, being used beyond its capacity--that is, because of deterioration of the community. The fourth expenditure, parks and recreation, was considered to be related to financial insolvency in a negative manner. That is that the municipality with insufficient cash would reduce its park and recreation expenditures.

Data gathering . . .

Data were gathered directly from the annual financial statements of each municipality. Data were recorded on the ten digit, 124 field form shown as Appendix C. To minimize errors and employ consistent interpretation the data were gathered only by this researcher. Upon receipt of the statement, a review was made of the auditor's opinion and relevant footnotes. Scoresheets were then marked, optically scanned and cards produced. Cards were occasionally verified to statements on a rather random basis.

CHAPTER III

STATISTICAL METHODOLOGY

Dichotomous situation--a basis for the design. . .

Much of the statistical analysis in business is concerned with observations that take one of two possible mutually exclusive outcomes such as:

A borrower pays his loan or defaults
A purchase order is deemed to be properly prepared
or is not
The balance of an account receivable is confirmed
by the debtor or is not
An interviewee is hired or is not
A part ordered from a supplier meets specifications
or it is found to be defective.

At times these "two outcome" situations and the subsequent observations are described as "zero-one" with the zero (0) utilized to record a failure or a "miss" while the one (1) represents a success or a "hit." An equally descriptive caption is "all or nothing at all." Such observations are also called "binary."

Throughout the field of biological sciences (which is responsible for much of the methodology employed in analysis of binary data) an older and somewhat more obscure term, quantal, is frequently used to describe these mutually exclusive observations. This term emanates from the dosal or quantal response to a measured dose in the controlled treatment experiment. When such a dose is administered to a subject a binary outcome can be the result--alive or dead, cured or still ill, etc. This term, quantal, now connotes a meaning beyond merely binary observation in that methods under this label--such as the one utilized in this research--can be applied to the polychotomous outcome.

Discriminant Analysis . . .

The statistical design originally proposed for this study was to utilize two methods of multivariate analysis, factor analysis and Discriminant Analysis (DA). Although it is an interruption at this point, comments regarding DA are enlightening to the discussion in view of the large amount of research which is allied to this project which has employed DA.

Discriminant Analysis can be used either in a descriptive or a predictive manner. In the latter situation, ". . . we seek linear combinations of a set of variables that best differentiate among several (two or more) groups."[1] Specifically in view of the data at hand, DA could be used to find linear combinations of those sundry predictor variables which would show significant differences between those municipalities which will experience financial difficulties and those which will not experience financial difficulties with the smallest possible proportion of misclassification.

Since first developed by R. A. Fisher, this technique, which was originally applied in the biological sciences, has seen use in numerous areas. Specifically, in the analysis of problems related to

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[1] Maurice M. Tatsuoka, Multivariate Analysis (New York: John Wiley & Sons, Inc., 1971), p. 5.

finance and business, DA was first used by Durand[2] in 1941 to differentiate between "good" and "bad" consumer loan applications, by Walter[3] in 1959 in classifying firms into high and low price-earnings ratio groups, in 1963 as one of several methods in developing a numerical credit evaluation system by Myers and Forgy,[4] by Smith[5] in 1965 in classifying firms into standard investment categories, in 1968 by Altman[6] in predicting corporate bankruptcy and again in 1971[7] and by Pinches and Mingo[8] in 1973 in evaluating industrial bond ratings.

The 1968 study of Altman was considered a "landmark" article in the field of finance, but criticisms were pronounced in that the data suffered from the malady of non-homogeneity. Altman accepted the criticisms as valid and corrected the deficiency by employing data

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[2] D. D. Durand, "Risk Elements in Consumer Installment Financing," Studies in Consumer Installment Financing (New York: National Bureau of Economic Research, 1941), pp. 105-42.

[3] J. E. Walter, "A Discriminate Function for Earnings Price Ratios of Large Industrial Corporations," Review of Economic and Statistics, Vol. XLI (February, 1959), pp. 44-52.

[4] H. Myers and E. W. Forgy, "Development of Numerical Credit Evaluation Systems," Journal of American Statistical Association, Vol. 50 (September, 1963), pp. 797-806.

[5] K. V. Smith, Classification of Investment Securities Using MDA, Institute Paper #101 (Lafayette, Indiana: Purdue University, Institute for Research in the Behavioral, Economic and Management Sciences, 1965).

[6] Edward Altman, "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," Journal of Finance, Vol. XXIII (September, 1968), pp. 589-609.

[7] Edward Altman, "Railroad Bankruptcy Propensity," Journal of Finance, Vol. XXVI (May, 1971), pp. 333-45.

[8] George E. Pinches and Kent A. Mingo, "A Multivariate Analysis of Industrial Bond Ratings," Journal of Finance, Vol. XXVII (March, 1973), pp. 1-15

related to only one industry, railroads. It is to that earlier study that we look to find an acceptable and easily understood explanation of Multiple Discriminant Analysis (MDA):

> MDA is a statistical technique used to classify an observation into one of several a priori groupings dependent upon the observation's individual characteristics. It is used primarily to classify and/or make predictions in problems where the dependent variable appears in a qualitative form, e.g., male or female, bankrupt or non-bankrupt. Therefore the first step is to establish explicit group classifications. The number of original groups can be two or more.
>
> After the groups are established, data are collected for the objects in the groups; MDA then attempts to derive a linear combination of these characteristics which "best" discriminates between the groups. If a particular object, for instance a corporation, has characteristics (financial ratios) which can be quantified for all of the companies in the analysis, the MDA determines a set of discriminant coefficients. When these coefficients are applied to the actual ratio, a basis for classifications into one of the mutually exclusive groupings exists. The MDA technique has the advantage of considering an entire profile of characteristics common to the relevant firms, as well as the interaction of these properties. A univariate study, on the other hand, can consider the measurements used for group assignment only one at a time.
>
> Another advantage of MDA is the reduction of the analyst's space dimensionality, i.e., from the number of different independent variables to g - 1 dimension(s), where g equals the number of original a priori groups. This paper is concerned with two groups, consisting of bankrupt firms on the one hand, and of non-bankrupt firms on the other. Therefore, the analysis is transformed into its smallest form: one dimension. The discriminant function of the form $Z = v_1x_1 + v_2x_2 + \ldots + v_nx_n$ transforms individual variables into a single discriminant score or Z value which is then used to classify the object where $v_1, v_2, \ldots, v_n$ = discriminant coefficients and $x_1, x_2, \ldots, x_n$ = independent variables. The MDA computes the discriminant coefficients, v_j while the independent variables, x_j, are the actual values where $j = 1, 2, \ldots, n$.
>
> When utilizing a comprehensive list of financial ratios in assessing a firm's bankruptcy potential there is reason to believe that some of the measurements will have a high degree of correlation or collinearity with each other. While this aspect necessitates careful selection of predictive variables (ratios), it also has the advantage of yielding a model with a relatively small number of selected measurements which has the potential of conveying a great deal of information. This information might very well indicate

> differences between groups but whether or not these differences are significant and meaningful is a more important aspect of the analysis. To be sure, there are differences between bankrupt firms and healthy ones; but are these differences of a magnitude to facilitate the development of an accurate prediction model?
>
> Perhaps the primary advantage of MDA in dealing with classification problems is the potential of analyzing the entire profile of the object simultaneously rather than sequentially examining its individual characteristics. Just as linear and integer programming have improved upon traditional techniques in capital budgeting the MDA approach to traditional ratio analysis has the potential to reformulate the problem correctly. Specifically, combinations of ratios can be analyzed together in order to remove the possible ambiguities and misclassifications observed in earlier traditional studies.[9]

Then the advantages of MDA over other methods are clear; all characteristics common to the subjects can be considered concommitantly while at the same time viewing interaction thereby avoiding redundancy and reducing the researcher's scope of view. Careful selection of variables can eliminate those which are highly correlated; this means that the researcher's view is reduced even more.

What Altman does fail to discuss is the prime inherent deficiency of MDA. That principal shortcoming is the need for relatively equal size classificatory groups (e.g., the number of bankrupt firms, in Altman's study should have qualled the number of non-bankrupt firms). Morrison[10] states:

> In summary, when one group is much larger than the other, almost all individuals are classified as the larger group. This means several will automatically be correctly classified. When we allow the posterior odds to classify the individuals--see 5--we usually get even fewer classified in the smaller group than actually belong to it. There is

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[9] Altman, "Corporate Bankruptcy," pp. 591-93.

[10] Donald G. Morrison, "On the Interpretation of Discriminant Analysis," Journal of Marketing Research (May, 1969), pp. 156-63.

> often more interest in the smaller group and classification tables like the preceding two are not the best way to assess the discrimination power of the independent variables.[11]

There does exist, however, an analogue to MDA which not only offers all the advantages claimed for MDA but overcomes the noted deficiency. In addition, it is also possible to gain the advantage of parameter estimation and calculation of probability of correct classification in either the dichotomous or polychotomous situation. For these reasons, a more applicable and versatile technique under the Maximum Likelihood concept has been chosen.

Maximum Likelihood . . .

The concept of Maximum Likelihood provides a means of determining estimations of population parameters which have to a substantial degree, those desired characteristics of efficiency, consistency and sufficiency.[12] Essentially the Maximum Likelihood concept may be summarized as being the estimation of a population parameter, Θ, from the data of the actual samples. The concept can be stated in a more formal manner: If a population parameter, Θ, is a variable with many possible values, Maximum Likelihood methods will lead to the choice of that one Θ, if in fact it does exist, which renders the likelihood (i.e., the probability of occurrence) of randomly obtaining the observed sample outcome as great as possible.

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[11] Ibid., pp. 160-61.

[12] Properties of estimators are treated in numerous texts on statistics. For example, see: Gene V. Glass and Julian C. Stanley, Statistical Methods in Education and Psychology (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1970), pp. 250-56.

Here the logic of the Maximum Likelihood concept is inverse to the logic employed in the more familiar research situation in that assignment of likelihood is not to the parameter, but rather to the sample in estimating the probability density of that parameter. Simply stated, the logic of this inversion can be summarized as: "Choose as the estimate the value which the parameter must have in order to maximize the likelihood to the observations obtained."[13]

In particular, there is a certain enhancement in the application of Maximum Likelihood to the data at hand. Without previous research to guide, estimation of parameters through this technique seems intuitively attractive. To reach a more encompassing conclusion based upon such a fixed sample could be considered a fault in the design.

Development of the concept . . .

The concept of Maximum Likelihood is intimately woven into the fabric of the history and development of statistics. A point of orientation to the origination of this concept is furnished by Ashton:[14] "One might begin by recalling that Gauss, in a letter to Bessel, specifically repudiated the principle of Maximum Likelihood in favor of least squares."[15,16] The concept remained dormant from the

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[13]Ann Hughes and Dennis Grawoig, Statistics: A Foundation for Analysis (Reading, Mass.: Addison Wesley Publishing Company, 1971), p. 167.

[14]Winifred D. Ashton, The Logit Transformation with Special Reference to Its Uses in Bioassay (New York: Hafner Publishing Company, 1972).

[15]Ibid., p. 34.

[16]Ashton does not cite the date of the letter, but a general interval of time is found in the fact that Karl Gauss was born in 1777 and died in 1855.

nineteenth century until 1922 when Fisher[17] revived interest in the concept. At that time he was the first to explicate a method of Maximum Likelihood. Fisher,[18] in an earlier work that same year had startled the "world of statistics" by challenging concepts regarding χ^2 which had been laid down by the "master," Karl Pearson. It is not only interesting to consider Fisher's thoughts in view of the description of Maximum Likelihood given above but also to note their relation to the data at hand:

> Any opinion put forward by Professor Pearson is worthy of respect, but it is impossible to agree with his statement that 'This result cannot be taken as obvious, as the size of the array in the sample varies.' The fact, however, Pearson has verified for large samples as far as the third order of approximation. The difference in principle is of some importance, since the simplicity of many of the results here obtained is a consequence of the fact that we have not attempted to eliminate known quantities, given by the sample studied, but only the unknown quantities--parameters of the population from which the sample is drawn. . . .[19] (Emphasis added.)

Since that time, several specific methods dependent upon the Maximum Likelihood concept have been developed. The one which is most familiar is linear regression. Numerous other methods have been developed, primarily in the field of biological sciences. The common characteristic of these methods is that they permit the researcher to

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[17] R. A. Fisher, "The Goodness of Fit of Regression Formulae and the Distribution of Regression Coefficients," Journal of the Royal Statistical Society, Vol. LXXXV (1922), pp. 597-612. Reprinted in: R. A. Fisher, Contributions to Mathematical Statitics, ed. Walter Shewhart (New York: John Wiley & Sons, Inc., 1950).

[18] R. A. Fisher, "On the Interpretation of Chi Square from Contingency Tables and the Calculation of P," Journal of the Royal Statistical Society, Vol. LXXXV (1922), pp. 87-94. Reprinted in: R. A. Fisher, Contributions to Mathematical Statistics, ed. Walter Shewart (New York: John Wiley & Sons, Inc., 1950).

[19] Fisher, "Goodness of Fit," p. 598.

estimate parameters which fit various functions. Attention is directed to two methods utilizing curvilinear functions.

Probits and logits . . .

The first of two methods under the Maximum Likelihood concept is the method of probits.[20] This method is attributed to J. H. Gaddum and C. I. Bliss and it is based on the integrated Normal curve. Gaddum and Bliss' work was summarized and extended by D.J. Finney,[21] who is considered by many to be the prime developer of probit analysis. The second method, that of "logits,"[22] was initiated in 1944 by Joseph Berkson who continued his work through the 1950's. This method is based on the logistic or dosal curve conceived by Pearl and Reed[23] in 1920 to describe the population growth in the United States of America beginning in 1790. Although both curves are frequently mentioned in the field of bioassay, the method we have chosen best utilizes the logistic curve.

In comparing the two functions, it can be said that the integrated Normal curve and the logistic curve are congruent throughout much of their range. Ashton illustrates the two curves with the following considerations having been made. Set $\alpha = 0$, so that the

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[20] The term "probit" is a contraction of the words "probability unit" and is used to express the deviation of a unit.

[21] D. J. Finney, Probit Analysis (Cambridge: Cambridge University Press, 1947).

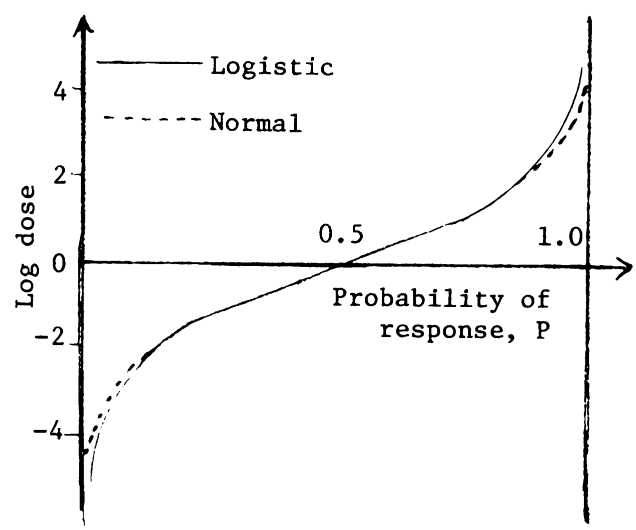
[22] An Analogy to Bliss' "probit"; a contraction of the words, logistic unit."

[23] R. Pearl and L. J. Reed, "On the Rate of Growth of the Population of the U. S. since 1790 and its Mathematical Representation," Proceedings of the National Academy of Science, Vol. 6 (1920).

curves are skew-symmetric about the lines x = 0 and P = 1/2. To complete the traditional linear function, $\alpha + \beta x$, β's are chosen so that the curves agree at some point; values chosen were 0.6 for the integrated Normal curve and 0.988 for the logistic curve. The agreement of the curves is shown as Figure 1 below:

FIGURE 4*

COMPARISON OF CURVES



*After Ashton: The Logit, p. 12.

Although the curves are congruent throughout much of their range, the point of inflection away from the limit occurs earlier in the logistic curve than in the integrated Normal curve. The general shape of these curves being quite similar makes it necessary to look once again to Ashton; she notes:

> The logistic estimates--both those obtained by the method of Maximum Likelihood and those obtained by Minimum Logit χ^2--are sufficient as well as asymptotically efficient. Those obtained by using Maximum Likelihood and the integrated Normal curve are not sufficient. General agreement

has it that 'sufficient statistics' are the best that can be had.[24]

The form of the logistic curve is

$$P = \frac{1}{1 + e^{-(\alpha + \beta x)}}, \quad -\infty < x > \infty$$

Once again, the utilization of Maximum Likelihood permits the choosing of those parameters, α and β, so that the given sample has the highest probability of occurring. The view is now directed to parameter estimation, first in a general sense and then specifically as concerns Maximum Likelihood.

Parameter estimation . . .

The statistician frequently employs inferential reasoning, moving from that which he observes (his sample) to that which he cannot or chooses not to observe (the population). A traditional means of reasoning from the particular to the general is by utilizing estimating procedures. Specifically, it is that "A statistic computed on a sample can be regarded as estimating a parameter in a population."[25] It is well known that the population parameter μ is best estimated with the sample mean, $\bar{X}$, and that r, the correlation coefficient for a sample having two variables, is wisely used to estimate ρ, the correlation between two variables having a bivariate normal distribution in the population, and that σ^2 is best estimated by s^2.

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[24]Ashton, The Logit, p. 75.

[25]Glass and Stanley, Statistical Methods, p. 242.

Kirk[26] provides an elementary but useful exposition on parameter estimation:

> Associated with every experimental design is a mathematical model that purports to include all sources of variability affecting individual scores. To the extent that the model accurately represents these sources of variability, the experimenter can evaluate the effects of a treatment. The linear model for a completely randomized design is:
>
> $$X_{ij} = \mu + \beta_j + \varepsilon_{ij}$$
>
> According to this model, an individual score is equal to the population mean μ, plus a treatment effect β_j, plus an error effect ε_{ij}, which is unique for each individual subject. In a particular experiment, the parameters μ, β_j, and ε_{ij} are unknown, but sample estimates of these parameters are given by $\hat{\mu}$, $\hat{\beta}_j$, and $\hat{\varepsilon}_{ij}$, respectively. It can be shown by maximum likelihood methods that unbiased estimates of the required parameters are provided by the statistics
>
> $$\hat{\mu} = \bar{X}.. \longrightarrow \mu$$
>
> $$\hat{\beta}_j = (\bar{X}._j - \bar{X}..) \longrightarrow \beta_j$$
>
> $$\varepsilon_{ij} = (X_{ij} - \bar{X}._j) \longrightarrow \varepsilon_{ij}$$
>
> The symbol $\longrightarrow$ indicates that the term on the left is an estimator of the term on the right. According to the maximum-likelihood method, the best estimate is one that gives the highest probability of obtaining the observed data. It should be noted that a maximum-likelihood estimator is not necessarily unbiased, although the center of its distribution is generally close to the value of the parameter estimated.[27]

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[26]Roger E. Kirk, <u>Experimental Design: Procedures for the Behavioral Sciences</u> (Belmont, California: Brooks/Cole Publishing Company, 1968).

[27]Kirk, <u>Design</u>, pp. 13-14.

Parameters under Maximum Likelihood . . .

Before proceeding further, two matters of notation must be attended to; as above let Θ represent the parameter to be estimated. Further, as in traditional matrix notation, let the underlining of $\underline{\theta}$ indicate a vector of parameters to be estimated and the addition of a caret, $\underline{\hat{\theta}}$, shall signify a vector of statistics to estimate those parameters.

The goal of the researcher can be stated as a logical question "What is the best $\underline{\hat{\theta}}$?" That $\underline{\hat{\theta}}$ shall be deemed best which when substituted for Θ maximizes the likelihood of the joint density of the sample obtained, or, maximizes the likelihood function. Therefore, the desired outcome is that $\underline{\hat{\theta}}$ which gives the maximum likelihood of the sample's occurrence. The basic principle may be stated in functional form

$$L \; (\underline{\Theta}) \;=\; \max_{\underline{\Theta}} \; L(\Theta)$$

or, take that estimate which is the Maximum Likelihood of the parameter of interest.

It is to be understood that the Likelihood function, $L(f)$, is the joint density (probability, Π, of the observations, where observations are taken as given and the parameters are considered as mathematical variables. If the observations are independent and if $f(y)$ represent the density function from which all observations were taken, then the joint density of the sample may be expressed as

$$\prod_{i=1}^{N} f \; (y)$$

which states that the joint density of a sample may be achieved by taking the joint product of the probability of the observations. The Likelihood function is a simple restatement of this joint density statement

$$L = \prod_{i=1}^{N} (y)$$

The Maximum Likelihood parameters are estimated mathematically from the function which has been defined above. However, the task of seeking a Maximum Likelihood parameter is simplified through the utilization of the natural logarithm of the Likelihood function for $\log_e L$ and L are monotonically related and will achieve their maximum at the same point.

To obtain the estimates of α and β, one first obtains the logarithm of the expression for the probability of all the observations occurring jointly or the joint density statement shown above. Then partial derivatives with respect to the desired parameters are computed. Then if $\log_e L$ is differentiable, the maximum point will be that point at which the partials equal zero, or

$$\frac{\partial \ \log_e L}{\partial \ \underline{\Theta}} = 0$$

These are the solutions to the Maximum Likelihood equations (in a general sense) and provide the desired estimates.

In many situations, solutions to Maximum Likelihood equations are familiar and obvious. For example, the Maximum Likelihood estimate of μ is $\bar{X}$. In other cases, the Maximum Likelihood estimates do not result in simple solutions. This situation could be the case, to use the previous example of graphic portrayal, where a maximum point and a similar but only near maximum point exist on a surface in two or three dimensional space. Here an iterative program (such as LISREL) is appropriate.

Likelihood ratio test . . .

The discussion of Maximum Likelihood concept is concluded with a further elaboration of the concept, the introduction of a Likelihood ratio test. This test provides a test of fit showing consistency between the data and the model to which it is being fitted, or more specifically the fit of the data to the logistic curve in this case. A concomitantly realized benefit is that this is also a test against constraints. That is, if the model has fewer parameters than the true data, there are constraints imposed by the model and the Likelihood ratio test provides a convenient test of these constraints.

In the case of Maximum Likelihood testing, the null hypothesis is (usually) stated in some expression indicating that the "model does not fit" or approximate the logistic functional curve. The alternative, therefore, would be that the "model fits." As is true in the more familiar hypothesis testing situation, to reject the null hypothesis in the Maximum Likelihood test does not prove the alternative hypothesis. Rather, it only proves, in the case of the Maximum Likelihood test, that the observed data are consistent with the model--they fit the logistic functional curve--and that it may be assumed reasonable only within the confines of that model. Nor should it be forgotten that other models equally reasonable could exist.

The null hypothesis could be stated in notational form

$$H_o: \ \underline{\Theta} \ \varepsilon\omega$$

In words, this states that the vector of parameters is in some restricted space ω. This then sets the form for the alternative hypothesis

$$H_1: \ \Theta \ \varepsilon\Omega$$

or, in words, the alternative hypothesis is that the vector of parameters is in some larger (and therefore less restricted) space.

The Maximum Likelihood ratio test is then, with a slight modification, the relationship of these two hypotheses. It results in the ratio which we shall designate λ,

$$\lambda = \frac{\overset{\max}{\underline{\Theta} \ \varepsilon\omega} \ L}{\overset{\max}{\underline{\Theta} \ \varepsilon\Omega} \ L}$$

which is the ratio of two Likelihood functions, each for their own maximum. The numerator indicates the Maximum Value the Likelihood can produce in a restricted space (the maximum being designated by the "max" superscripted above the $\underline{\Theta}$. The space in the denominator is the less restricted space. The ratio λ indicates whether the constraining of the function to a subspace had much effect or not.

The distribution of $\chi^2 = -\ 2\log\ \lambda$ is asymptotically χ^2. In this Likelihood ratio test, the denominator should be larger than the numerator for it should be easier to obtain a higher likelihood without restrictions than with restrictions. If $\lambda \simeq 1$, then the constraint of a restricted space did not reduce the likelihood significantly and $\chi^2 = -\ 2\log\ (1) = 0$. Finally, the degrees of freedom (df) are equal to the number of parameters free to vary in unrestricted space less the number of parameters estimated to the model.

It was not until 1935 that an exact method for solving the estimation problem was developed by Fisher.[28] His solution was with the integrated Normal curve in the context of the probit method. Fisher's solution has since been extended to the logistic function; one of those solutions is now presented.

Quantal response techniques . . .

In a paper entitled "Quantal Response Techniques for Random Predictor Variables"[29] McSweeney and Schmidt develop and present a versatile alternative to the oft-used multiple linear regression and the previously discussed Multiple Discriminant Analysis, specifically extensions of Quantal Response techniques.[30] Earlier in this paper, the term "quantal" was described as the dosal or quantal response measurement so familiar in the biological sciences.

The quantal response technique is one of numerous bioassay techniques for the analysis of the relationship between one or more quantitative predictor variables and the quantal response which takes the form of a qualitative criterion (usually referred to as the

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[28]R. A. Fisher, "The Case of Zero Survivors," Appendix to C. I. Bliss, "The Calculation of the Dosage-Mortality Curve," Annals of Applied Biology, Vol. 22 (1935), pp. 164-67.

[29]Maryellen McSweeney and William H. Schmidt, "Qualtal Response Techniques for Random Predictor Variables," (Paper presented at the Annual Convention of American Educational Research Association, 1974).

[30]The thoughts expressed here draw heavily upon the work of McSweeney and Schmidt as summarized in their paper and reflected in their computer programs. Appreciation is expressed for their generosity.

"criterion variable" but also recognizable as the "dependent variable"). The predictors may be captioned as treatments (at various levels); the outcomes may be dichotomous (alive or dead) or polychotomous (alive, moribund, or dead). Recently, the usage of Quantal Response has not been limited to research in the field of biological for

> In the social sciences, quantal response techniques have been used primarily in calibration of subjective estimates of weight, pitch and loudness in psychophysics, in latent trait analysis in psychometrics and in the determination of receiver-operating characteristic curves in signal detection.[31]

The sustentacular relationship of the Maximum Likelihood concept is implicit in the discussion of the four Quantal Response models.[32]

<u>Two models for two situations . . .</u>

Although many familiar statistical models pose no constraint as to the nature of predictor variables, such a distinction is inherent in Quantal Response techniques. The two general Quantal Response models are classified as to whether predictor variables are mathematical or stochastic. The contrast between the situations and the variables is forthright. In one situation the researcher will have discretionary choice as to the amount of treatment (or doses) to be administered to the subject. Here the quantitative classification variable is considered to be mathematical. In the second situation the researcher has no control over the predictor variables.

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[31] McSweeney and Schmidt, "Quantal Response," p. 1.

[32] See supra, pp. 7-16.

Probability of the first model . . .

The resultant probability estimates elaborate the nature of the first model. Cornfield et al.[33] comment

> For the first model (A) we consider an experiment situation in which the probability that an observational unit is characterized by a dose between X and X + dX is $f(X)dX$, where f may depend on one or more parameters. The conditional probability that the quantal variate for a unit will will take on the value one, given that it is exposed to dose X is P(X). The function P may also depend on one or more unknown parameters, say Θ_1, but it is also assumed that
>
> $$(2.1) \quad \partial f(X)/\partial\Theta = 0$$
>
> As a situation to which this model might apply, consider a population of animals exposed to food containing a poison. The actual dose, X, ingested depends on the amount of food eaten and is a random variable with p. d. f. $f(X)$. Having eaten amount X, the probability that the animal will die is $p(X,\Theta_1\ldots)$. The Θ_i are determined by physiological characteristics of the animal and the assumption (2.1) is not unreasonable for such a situation.[34]

From this, four relevant thoughts may be arrayed:

1. The quantitative classification variable can be used as a stratification dimension in the model.
2. "The researcher can determine a priori how many subjects will be exposed to amounts (X_1, X_2 . . .X_k) of the K quantitative predictors;. . ."[35]
3. An alternative design would permit the researcher to determine how many of the subjects at each level, (X_1, X_2, . . .X_k) of the quantitative predictor variable will be chosen for his study.
4. Independence between subjects is a necessary assumption.

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[33]Jerome Cornfield, Tavia Gordon, and Willie W. Smith, "Quantal Response Curves for Experimentally Uncontrolled Variables," Bulletin of the International Statistical Institute, Vol. 38 (1960), pp. 97-115.

[34]Cornfield, et al., "Quantal Curves," p. 98.

[35]McSweeney and Schmidt, "Quantal Response," pp. 1-2.

Description of the second situation . . .

If the first situation is classified as a controlled case, the second involving stochastic variables may be viewed as the uncontrolled case. "The researcher records and classifies according to the observed values of the predictors but does not have the liberty of determining the sample composition with respect to values for those variables."[36] Stated simply, the particular observed element is not under the control of the researcher and it is therefore properly considered to be a random variable.

Probability of the second model . . .

Once again, the statement of probability extends the description of the model applicable to the second situation. Cornfield et al. state

> Consider now Model B. We have a population, each element of which is characterized by values for two sets of variables, a quantal variate and a quantitative classification variable, X. We shall refer to units for which the quantal variate takes value one or zero as responding or non-responding units respectively. Denote the probability that the quantal variate takes on the value one by p. Denote the conditional probability density functions with respect to X of responding and non-responding units by $f_1(X)$ and $f_0(X)$ respectively. The probability that an element selected at random has a dose between X and X + dX is then
>
> $$(2.2) \qquad [qf_o\ (X) + pf_1\ (X)]$$
>
> where q = 1 - p. The joint probability that an element selected at random has a dose between X and X + dX and a response of one is
>
> $$(2.3) \qquad pf_1\ (X)dX$$

[36]McSweeney and Schmidt, "Quantal Response," p. 2.

The conditional probability that an element with dose between X and X + dX will have a response of one is then the ratio of (2.3) to (2.2). Thus,

$$(2.4) \qquad p(X) = 1 \,/\, [1 + q f_0(X) \,/\, p f_1(X)].$$

This yields a dose-response curve whose functional form is dependent upon $-_0(X)$. Furthermore, if (2.4) depends upon parameters Θ, so does (2.2). Then assumption (2.1) cannot be made; the distribution of the number of observations by dose level is dependent upon the parameters of the dose-response curve, and the traditional estimation procedures do not apply. As an example of a situation which might be appropriately described in this way, let X be the squared difference between two measures on a pair of twins and let p be the probability that a pair selected at random is monozygotic. Then $f_1(X)$ and $f_0(X)$ might be chi-quared distributions each with one degree of freedom, but with E(X) much larger for dizygotics. The probability that a twin pair with squared difference X is monozygotic is then given by P(X).[37]

Further issues . . .

Thus far the appropriate model to apply seems to be dictated in to by consideration of whether the predictor variables are mathematical or stochastic. This criterion leads to the logical conclusion that in most cases models MA1[38] and MA3 are appropriate when the relationship of the predictor variable is close to the criterion variable could be considered to be a link in a causal chain.

McSweeney and Schmidt maintain that the use of models MA1 and MA3 are less clear in two situations. First, in that situation in which the predictor variable is not manipulated but is used as a

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[37] Cornfield et al. "Quantal Curves," pp. 98-99.

[38] As is often true, the models assume different captions from different authors. We shall follow the McSweeney-Schmidt mode of letting "A" and "B" indicate mathematical and stochastic respectively; for each model, the lower number indicates dichotomous criterion and the higher number polychotomous criterion.

stratification device; second, that situation in which the researcher employs a classification procedure based upon the observed values of the predictor and the criterion variables of a sample randomly drawn from a particular population. Here both predictor and criterion variables would be random with joint distribution and correlation coefficients determinable. The applicability of the MA models is dependent upon the directness of the linkage between predictor and criterion variables. If such linkage is direct, then MA models are appropriate and the characteristic of the stochastic variable is ignored.

Direct causal linkage . . .

McSweeney and Schmidt represent the direct and indirect causal linkage between variables graphically in the following manner. Letting X_i represent the predictor variables, Y the criterion variables and Z, a factor which is causally related to the predictors X_i, then

FIGURE 5

DIRECT CAUSAL LINKAGE

$$\begin{array}{lcl} (Z_1, Z_2. \, . \, . \; Z_L) & \searrow & \\ & & P(Y|X_1, X_2. \, . \, . \; X_k) \\ (X_1, X_2. \, . \, . \; X_K) & \nearrow & \end{array}$$

Here the linkage is direct and the predictors are not influenced by the Z factors and the functional relationship is reasonable. The factors $(Z_1, Z_2, \ldots, Z_L)$ may be thought of as latent or unobserved variables not disturbing the functional relationship between predictor and criterion variables.

Indirect causal linkage . . .

The indirect causal linkage is contrasted with the direct in this manner

FIGURE 6

INDIRECT CAUSAL LINKAGE

$$(Z_1, Z_2, \ldots, Z_L) \longrightarrow (X_1, X_2, \ldots, X_K) \longrightarrow P(Y|X_1, X_2 \ldots X_K)$$

This is described by McSweeney and Schmidt as "Indirect causal linkage to Y mediated through $(X_1, X_2, \ldots, X_K)$."[39] In the above figure, the $P(Y|X_1, X_2, \ldots, X_K)$ is another of the desired results of the analysis through Quantal Response--the model of the probability of the ". . .occurrence of each level of the criterion to the predictor variables."[40]

The linkage between predictor and criterion variables is direct but the factors $(Z_1, Z_2, \ldots, Z_L)$ are not only direct but also indirect. Therefore influence will be direct but also mediated through the predictors. McSweeney and Schmidt state that the "parameters governing $P(Y|X_1, X_2, \ldots, X_K)$ are not influenced by the same factors that the parameters of $f(X_1, X_2, \ldots X_K)$ would be."[41] Consequently, the MB models would be considered applicable in that this relationship is not functional but rather predictive.

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[39]McSweeney and Schmidt, "Quantal Response," p. 4.

[40]Ibid., p. 4.

[41]Ibid., p. 4.

Mathematical distinction between models . . .

Although the design of this research focused on a model consisting of multiple random predictor variables and polychotomous criterion variables, distinctions can be advantageously drawn between this model and others. Specifically these distinctions are related to the particulars of estimation of parameters, Maximum Likelihood estimators, probabilities, etc. Beginning with the simplest, the presentation takes the following form

Single predictor, dichotomous criterion
Multiple predictors, dichotomous criterion
Multiple predictors, polychotomous criterion

for both the MA and MB models where applicable.

Single predictor, dichotomous criterion . . .

Let X represent the quantitative predictor variable and Y the criterion variable. For this model the predictor is single and the totality of the outcome of is represented in Y = 0 or 1. Then P, the probability of the occurrence of the desired outcome given a specified amount of treatment, may be represented as $P = (\text{Prob}\ (Y = 1|X)$. Further, the Likelihood takes the form of the familiar binomial probability function, that is, the probability of obtaining exactly x successes in n independent trials. If N_i represents the number of subjects receiving the specified treatment x_i, if n_i represents that number having the desired outcome and if Q is computed in the traditional manner as 1 - P and signifies the probability of the outcome which is not desired, then Likelihood is computed

$$L = \prod_i \binom{N_i}{n_i} P^{n_i} Q^{N_i - n_i}$$

The second model is viewed in the context of random predictor variables. The conditional probability is still represented as P, while p signifies the unconditional probability of the outcome of interest. Then let n_o signify those respondents showing the alternative outcome and n_1 those having the desired outcome. The denominator will be simply the sum of the outcomes, or those outcomes which will appear and the numerator is the outcome of interest, in this case, then

$$P = \frac{p \mathcal{f}\ (X|Y = 1)}{p \mathcal{f}\ (X|Y = 1) + q \mathcal{f}\ (X|Y = 0)}$$

letting $\mathcal{f}\ (X|Y = 1)$ and $\mathcal{f}\ (X|Y = 0)$ represent the conditional distributions of the predictors of the two outcomes.[44] In words, this formula states that the conditional probability, P, is equal to the unconditional probability of the function of X given that Y = 1, the outcome of interest, divided by the sum of unconditional probability of the function of X given that Y = 1 plus the unconditional probability of the function of X given that Y = 0, the latter part of the denominator being that outcome which is the alternative to the outcome of interest.

The model is extended by Cornfield et al.:

> In many problems it is natural to assume that $\mathcal{f}_0$ and $\mathcal{f}_1$ are both normal density functions, but with differing means and variances, i.e., that

$$\mathcal{f}_0(X) = \frac{1}{\sigma_0 \sqrt{2\ \pi}} \quad \exp - \frac{1}{2\ \sigma_0^2} \quad (X - \mu_0)^2$$

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[44]For a discussion of Bayes Theorem with regards to this application, see: Samuel A. Schmitt, Measuring Uncertainty an Elementary Introduction to Bayesian Statistics (Reading, Mass: Addison-Wesley Publishing Company, 1969.

(2.5)
$$f_1(X) = \frac{1}{\sigma_1 \sqrt{2\pi}} \quad \exp - \frac{1}{2\sigma_1^2} \quad (X - \mu_1)^2$$

for this case (2.4) becomes[45]

$$(2.6) \quad P = 1 \;/\; \{1 + q\sigma_1 \;/\; p\sigma_0 \;\exp - \frac{1}{2} \quad \frac{X - \mu_0}{\sigma_0}^2 - \frac{X - \mu_1}{\sigma_1}^2 \}$$

This is a dose-response curve which for $\sigma_0 \neq \sigma_1$ reverses directions at

$$(2.7) \quad X = \quad \mu_0/\sigma_0^2 - \mu_1/\sigma_1^2 \qquad (1/\sigma_0^2) - (1/\sigma_1^2)$$ [46]

Certain assumptions simplify Cornfield's formula (2.7) to a degree. In many cases the dose-response curve can be assumed to be monotonic. This in turn leads to the simplifying assumption that the conditional distributions have the characteristic of equal variance i.e. $\sigma_1^2 = \sigma_0^2$, (not an unusual assumption in statistical analysis), which then implies the additional assumption $\sigma_1 = \sigma_0$ (standard deviations are equal) and then the conditional probability expressed in Cornfield's (2.6) reduces to

$$P = \frac{1}{1 + \frac{q}{p} \;\exp \; - \;\frac{\mu_1 - \mu_0}{\sigma^2} \quad X - \frac{\mu_0 + \mu_1}{2}}$$

This states that the conditional probability is equal to one divided by the sum of one plus the fraction of the unconditional probability of the alternative outcome over the unconditional probability of outcome of interest (hereafter referred to as the "ratio of probability

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[45] Formula (2.4) was presented on p.

[46] Cornfield et al., "Quantal Curves," p. 90.

of outcomes") times the exponent less the product of the mean differences divided by the variance times the observation less the average of the two means.

This is the logistic dose-response curve of the form:

$$P = 1 \ / \ \{1 + \exp - ^{(\alpha+\beta x)}\}$$

with the parameters

$$\beta = (\mu_1 - \mu_0)/\sigma^2 \quad , \qquad \alpha = \ln\left(\frac{q}{p}\right) - \beta \, \frac{(\mu_0 + \mu_1)}{2}$$

In words this states that the conditional probability is equal to one over the sum of one plus the exponent to the negative power of alpha plus beta x; beta is equal to the difference between the means divided by the variance, while alpha is equal to the difference between one times the number of observations times the ratio of the unconditional probabilities of outcomes and beta (as previously defined) times the average of the two means. McSweeney and Schmidt present the Likelihood function for the dichotomous case as:

$$L = \binom{n_0 + n_1}{n_1} \quad p^{n_1} \, (q^{n_0}) \prod_{i=1}^{n_1} f\,(X_i | Y = 1) \prod_{i=1}^{n_0} f\,(X_i | Y = 0)$$

and the effective part of the logarithm of the Likelihood is

$$\ell nL' = n_1 \ \ell np + n_0 \ \ell n \ (q) - \frac{n_0 - n_1}{2} \ \ell n\sigma^2 - \frac{1}{2} \sum_{i=1}^{n_1} \frac{(X_i - \mu_1)^2}{\sigma^2}$$

$$- \frac{1}{2} \sum_{i=1}^{n_1} \frac{(X_i - \mu_0)^2}{\sigma^2} \quad [47]$$

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[47]McSweeney and Schmidt, "Quantal Response," p. 8.

That Likelihood function formula may be expressed in narrative form as the product of the combination of the number of observations taken in the number of the outcomes of interest times the unconditional probability of the outcome of interest raised to the power of the number of the outcomes of interest times the unconditional probability of the alternative outcome raised to the power of the number of alternative outcomes times the Likelihood of the function of X given that Y equals one times the Likelihood of the function of X given that Y equals zero. Then, as is traditional, ". . .the parameters of the dosage-response curve are estimated by substituting sample proportions, means and variances for the corresponding population parameters."[48]

$$\hat{p} = n_1/(n_0 + n_1) \qquad \hat{\mu}_0 = \Sigma\, X_0/n_0 = \bar{X}_0 \qquad \hat{\mu}_i = \Sigma\, X_1/n_1 = \bar{X}_1$$

$$\hat{\sigma}^2 = \frac{1}{n_0 + n_1} \quad \Sigma\ (X_0 - \bar{X}_0)^2 \ + \ \Sigma\ (X_1 - \bar{X}_1)^2 \quad = s^2$$

In the same sequence as above, these state that the estimated unconditional probability of the outcome of interest is equal to the ratio of the number of the occurrences of such observations in the sample to the number of all observations in the sample; the estimated mean of the alternative and the desired outcomes are computed in traditional manner, i.e., the sum of the observations of that type of outcome divided by the number of those outcomes; the estimate of variance is equal to one over the number of observations times the sum of squares. Use of the unbiased estimator, s^2 leads to,

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[48]Cornfield, et al., "Quantal Curves," p. 101.

$$\hat{\beta} = (\bar{X}_1 - \bar{X}_0) / s^2, \text{ and,}$$

$$\hat{\alpha} = - \ln \frac{\hat{q}}{\hat{p}} \quad \hat{\beta} \quad \frac{\hat{\mu}_0 + \hat{\mu}_1}{2}$$

Although the point has been established earlier, it seems worthy of repetition in view that some might consider linear regression to be an analogue to Quantal Response in the context of the MB models.

> Cornfield et al. note that approximate $(1 - \alpha)\%$ confidence intervals for β can be constructed by referring to the t-distribution with $n_1 + n_2 - 2$ degrees of freedom. These intervals differ from traditional linear regression intervals for the slope because the quantitative variable is a random variable under model two assumptions, but a mathematical variable under model one or traditional linear regression assumptions.[49]

<u>Multiple predictors, dichotomous criterion . . .</u>

The previously given model having single predictor and dichotomous criterion is easily amplified for inclusion of multiple predictors by stating the model in matrix form. The conditional distribution of the multiple predictors having outcomes Y = 1 and Y = 0 are indicated as $f\ (\underset{\sim}{X}_i |\ Y = 1$ and $f\ (\underset{\sim}{X}_i |\ Y = 0)$. Further, the vector of K random predictor variables $(X_1, X_2, \ldots, X_K)$ shall be indicated as $\underset{\sim}{X}^T$ where the small superscript "T" is construed to mean a transpose. Necessary assumptions are that the conditional distributions of the predictor variables, $f\ (\underset{\sim}{X}_i | Y)$:

1. Are multi-variate normal
2. Have identical variance-covariance matrices (Σ)
3. Have mean vectors μ_1 and μ_0 respectively.

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[49]McSweeney and Schmidt, "Quantal Response," p. 9.

McSweeney and Schmidt state that ". . . the conditional probability of occurrence of response Y = 1 can be expressed as a cumulative compound logistic distribution function:

$$P = 1/\left[1 + \frac{q}{p} \exp - 1/2 \quad [(\underset{\sim}{X} - u_0)^{\tau}\ \Sigma^{-1} \cdot (\underset{\sim}{X} - \underset{\sim}{\mu_1})^{\tau}\ \Sigma^{-1}\ (\underset{\sim}{X} \quad \underset{\sim}{\mu_1})]\right]$$."[50]

where Σ^{-1} (sigma inverse) is understood to be the inverse of the "grand" variance covariance matrix. The formula may be stated in this manner: the conditional probability is equal to one over one plus the ratio of unconditional probabilities of outcomes times the exponent less one-half of the difference between the product of the transpose of the matrix of mean differences of alternative outcomes times the inverse of the variance-covariance matrix times the matrix of mean differences of alternative outcomes and the product of a similar term for the desired outcomes.

In this case of multiple predictor variables, P is in form similar to that presented for the single predictor[51] excepting for the expression of β and X as vectors:

$$P = 1/(1 + \exp - (\alpha \quad \underset{\sim}{\beta}^{\tau}\ \underset{\sim}{X}).$$

X being in one class and not the other. The parameters α and β are found to be:

$$\alpha = \ell\ n(q/p) - 1/2\ (\underset{\sim}{\mu}_1^{\tau}\ \Sigma^{-1}\ \underset{\sim}{\mu}1 - \underset{\sim}{\mu}0 \quad \Sigma^{-1}\ \mu_0) \quad \text{and}$$

$$\beta = \Sigma^{-1}\ (\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_0)$$

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[50]Ibid., p. 10.

[51]See supra, p. 26.

the latter bearing distinct similarity to the formula presented in the case of the single predictor variable excepting of course that the variance-covariance matrix is now substituted for σ^2 and the expression of the means μ_1 and μ_0 as vectors. These parameters are conditioned on class zero. Appropriate changes to the Likelihood function are the expression of predictor variables in vector form:

$$L = \binom{n_0 + n_1}{n_1} \quad p^{n_1} \quad q^{n_0} \prod_{i=1}^{n_1} f\,(X_i \; Y = 1) \prod_{i=1}^{n_0} f\,(X_i \; Y = 0).$$

The effective part of the logarithm of the Likelihood is:

$$\ell nL^{\tau} = n_1 \; \ell np + n_0 \; \ell n(q) + \frac{(n_0 + n_1)}{2} \ell n \; |\Sigma^{-1}| - 1/2 \sum_{i=1}^{n_0} (\underset{\sim}{X}_{i0} - \mu_0)^{\tau}$$

$$\Sigma^{-1} \; (\underset{\sim}{X}_{i0} - \mu_0) - 1/2 \sum_{i=1}^{n_0} (X_{i1} - \underset{\sim}{\mu}_1)^{\tau} \; \Sigma^{-1} \; (\underset{\sim}{X}_{i1} - \underset{\sim}{\mu}_1).$$

<u>Multiple Predictors and Polychotomous Criteria . . .</u>

Because the research effort utilized three and at times four and five criterion variables and multiple predictors it was necessary to make certain modifications to the original McSweeney-Schmidt program. Furthermore, it seemed more logical to present the findings of the maximum Likelihood estimators in this section rather than the preceding discussion.

Once again, $\underset{\sim}{X}^{\tau} = (X_1, X_2, \ldots, X_K)$ is taken as a vector of K random predictor variables, but now $f(\underset{\sim}{X}_i | y_j)$ shall denote the conditional distribution of those predictor variables having outcome y_j, and such conditional distributions are assumed to be multi-variate

normal with identical covariance matrices having mean vectors u_j.

In the theoretical solution of the probability of occurrence of response y_k it is necessary to condition on one of the categories. In the practical application of the program, conditioning is always on category 1. Letting subscript "j" represent the category being conditioned upon and subscript "k" denote the category being worked with, the necessary constraint arises that $j \neq k$ as subsequently noted beneath the summation sigma. Then, the computational formula for the conditional probability of the occurrence of outcome y_k becomes:

$$P_k = 1/ \quad 1 + \sum_{\substack{j=1 \\ j \neq k}}^{J} \frac{P_j}{P_k} \exp - 1/2 \quad (\underset{\sim}{X} - \underset{\sim}{\mu}_j)^T \Sigma^{-1} (\underset{\sim}{X} - \underset{\sim}{\mu}_j) -$$

$$(\underset{\sim}{X} - \underset{\sim}{\mu}_k)^T \quad \Sigma^{-1} (\underset{\sim}{X} - \underset{\sim}{\mu}_k)$$

The earlier expressed Likelihood function is modified even more:

$$L = N!/ \prod_{j=1}^{J-1} n! \ (n - \sum_{j=1}^{J-1} n_j)! \ \prod_{j=1}^{J-1} P_j^{n_j} \ (1 = \sum_{j=1}^{J-1} P_j)^{n - \sum_{j=1}^{J-1} n_j} \frac{|\Sigma|^{-\frac{n}{2}}}{(2\pi)^{\frac{nk}{2}}}$$

$$\exp - 1/2 \sum_{j=1}^{n_j} (\underset{\sim}{X}_i - \underset{\sim}{\mu}_j)^T \quad \Sigma^{-1} (\underset{\sim}{X}_i - \underset{\sim}{\mu}_j)$$

where once again the vertical lines enclosing the variance-covariance matrix Σ indicate not the absolute value but rather the determinant of that matrix. The effective part of the logarithm of the Likelihood is:

$$\ell n L^T = \sum_{j=1}^{J} n_j \ell n \ p_j + (n - \sum_{j=1}^{J-1} n_j) \ell n \quad (1 - \sum_{j=1}^{J-1} P_j) + \frac{N}{2} \ell n |\Sigma^{-1}| -$$

$$1/2 \sum_{j=1}^{J-1} \sum_{i=1}^{n_j} (\underset{\sim}{X}_i - \underset{\sim}{\mu}_j)^T \; \Sigma^{-1} \; (\underset{\sim}{X}_i - \underset{\sim}{\mu}_j)$$

when we "let $\bar{X}_j$ denote the vector of means on the predictors for subjects whose responses fall in category y_j, let S_j^{KxK} denote the corresponding sample matrix of sums of squares and cross-product deviations about the respective means. Let $\psi = \Sigma^{-1}$."[52]

The cumulative logistic distribution function is represented as

$$P_h = 1/(1 + \exp - (\alpha + \underset{\sim}{\beta}^T s))$$

The parameters then become:

$$\alpha = \ln P_{j}/P_{k} - 1/2 \; [\underset{\sim}{\mu}_k^T \; \Sigma^{-1} \; \underset{\sim}{\mu}_k - \underset{\sim}{\mu}_j^T \; \Sigma^{-1} \; \underset{\sim}{\mu}_j]$$

where J is being conditioned upon, and

$$\beta = \Sigma^{-1} \; (\underset{\sim}{\mu}_k - \underset{\sim}{\mu}_j)$$

The difficulties encountered in expanding the program to permit expression of a greater number of β was paramount to the success of the research. As Cornfield, et al. note:

> For applications in which model B applies, such as those of sections 8 and 9, one wishes to know whether the classificatory variable is associated with the quantal variate, and, if so, the magnitude of the association. For such a question, β, and the confidence limits (4.7) are of more interest than α or confidence limits about it.[53]

Letting $(\underset{\sim}{X} - \underset{\sim}{X}_j)$ represent the input vector and remembering that the constraint of $j \neq k$ will be met by "skipping the computation"

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[52]McSweeney and Schmidt, "Quantal Response," p. 13.

[53]Cornfield et al., "Quantal Curves," p. 102.

if j = k, then sample estimates are:

$$\hat{P}_k = 1/[1 + \sum_{\substack{j=1 \\ j \neq k}}^{J} \hat{P}_j/\hat{P}_k \exp - [(\underset{\sim}{x} - \bar{\underset{\sim}{x}}_j)^T S^{-1} (\underset{\sim}{x} - \underset{\sim}{x}_j) - (\underset{\sim}{x} - \underset{\sim}{x}_k)^T S^{-1} (\underset{\sim}{x} - \underset{\sim}{x}_k)]]$$

$$\alpha = -\ell n\ \hat{P}_j/\hat{P}_k - 1/2\ [\underset{\sim}{X}_k^{-T}\ S^{-1}\ \bar{\underset{\sim}{X}}_k - \bar{\underset{\sim}{X}}_j^T\ S^{-1}\ \bar{\underset{\sim}{X}}_j\ ] \quad \text{and}$$

$$\beta = S^{-1} (\bar{\underset{\sim}{X}}_k - \bar{\underset{\sim}{X}}_j)$$

where S is substituted for the biased maximum likelihood estimator Σ and $\bar{X}$ is an unbiased linear estimator of μ.

<u>Assumptions and effects of departure from those assumptions . . .</u>

The brevity of this section is dictated by lack of established consideration of these aspects. Two assumptions previously cited were that the conditional distributions of the predictor variables are multi-variate normal and that those predictor variables have identical covariances. Without substantiation it has been held by some that the method of Quantal Response is robust to violation of these assumptions. An additional assumption, that of correct classification, has been investigated by Cornfield:

> The previous section has established that if model B is applicable and model A assumptions are made, there may be considerable loss in precision. This arises from disregarding the additional information contained in the pdfϕ(X). The estimates of the parameters of the dose-response are clearly consistent, however, so long as the dose-response curve has the form assumed.
>
> If model A is applicable, however, but model B assumptions are made, the estimates obtained are inconsistent.[54]

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[54]Ibid., p. 102.

Cornfield investigates the effect of incorrect classification by viewing

> how far the P(X) obtained by assuming f_0 (X) and f_1 (X) to be normal departs from the P(X) which, in fact applies when they are not. We have considered the following case:
>
> 6.1 $f_0(X) = X^{n-1} \exp - X/\Gamma(n)$, $f_1(X) = f_1(X + c)$
>
> and have contrasted the P(X) implied by (6.1) with that by assuming f_1 and f_0 normal, with the same means and variances as those given by (6.1). We contrast in Table 1 numerical values for P(X) obtained from the normal assumption with the true ones for p = 1/5 and the two cases n = 4, c = 1, and n = 9, c = 2. In the first case the means are 4 and 5 respectively, with standard deviations of 2 in both cases. In the second case the means are 9 and 11, with standard deviations of 3. It will be seen that in the interval mean ± two sigma, true and approximate curves are in reasonably good agreement in both cases. The agreement becomes poorer farther out in the tails, where of course, there are many fewer observations. More extreme departures from normality than those assumed would, of course, entail larger discrepancies, although appropriate transformations might in some cases prevent this from becoming a major problem.[55]

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[55]Ibid., p. 102.

CHAPTER IV

FINDINGS AND CONCLUSIONS

Introduction . . .

The purpose of this chapter is to report the findings of this research project and from those findings draw appropriate conclusions. The presentation of findings should be preceded with somewhat divergent comments about the dilemma of completeness of problem description versus simplicity of model specification. The brevity and non-technical approach should not be construed to be an intended dimunition of the subject; rather the thoughts are presented in this manner so that they may serve as a fitting caveat.

Specification of the model . . .

Generally, the researcher approaches his investigation with two desires which might be at times diametrically opposed. The desired model will be:

1. As realistic as possible
2. As mathematically simple as possible.

The first of these two desired characteristics relates to the number of variables to be included in the analysis. The difficulty inherent to the determination of the proper number is of itself paradoxical; if the researcher were to attempt to capture every nuance of the observed event in his research model, he would have replicated the happening which he seeks to explain. This could result in such a complex model that it might be uninterpretable if not impossible to manipulate. Accordingly the researcher may well approach his investigation by specifying a model that will not fully explain the facet of life he views, sacrificing completeness for operability. Wisely, the researcher accepts less than a complete description, by settling for those predictor variables which may be considered as strongly associated as possible with the dependent variable(s). Another way of describing this--our goal--is to build the model in as parsimonious fashion as possible, but still representative of the observed events. With these thoughts in hand we return to the findings.

Quantal response . . .

The computer program for the Logistic Polychotomous Technique was written in fortran IV for the Control Data 6500 computer. By modifying the original McSweeney-Schmidt program it was possible to "pack" the matrix thus permitting the employment of as many as fifteen predictor variables (drawn from the financial data for the years 1971-74) in an analysis. This afforded computation of probabilities of classification of each of the 60 cities into three groups utilizing as criterion variable the amount of cash in the general fund June 30, 1975 expressed as a percentage of total general fund assets, June 30, 1975. Because it seemed reasonable to assume that other underlying variables in the causal chain may be jointly influential on that amount of cash held by a municipality, Model B Quantal Response Technique was utilized.[1]

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[1]For the means by which linkage effects model choice, see Supra, pp. 59-60.

<u>Range of analysis--criterion variable . . .</u>

Various analyses were made to enhance the classificatory ability of the model because of the lack of prior information regarding criterion and predictor variables.

Recalling that the criterion (dependent) variable is that which will be the basis of classification of the municipality into one actual specific group or another, it should be noted that the Quantal Response technique provides the researcher with great flexibility and ease of actual classification of observations. Specifically, an observation is designated as <u>actually</u> belonging to one group or another by simply changing a "header" card which precedes the data deck for that observation from one group's designation to another. The ultimate impact of such a change is in reality a change in the dimensions or ranges of the groups. An example will serve to clarify the discussion.

Suppose that a researcher was investigating test results which were expressed in the form of a decimal ratio having potential outcomes between zero and one. Using four groups he might logically choose the dimensions for classification as:

	Range of Groups		
Group 1	0		.250
Group 2	.251	to	.500
Group 3	.501	to	.750
Group 4	.751	to	1.000

Suppose now that the researcher decided that the analysis should reflect a traditional grading scale, A = 90 to 100%, etc., and that he decided that he would still keep four groups calling his last, "D and lower." He could change the dimensions of the groups by redefining certain levels of the criterion variables:

	Range of Groups		
Group 1 (D's and lower)	0	to	69%
Group 2 (C's)	70	to	79%
Group 3 (B's)	80	to	89%
Group 4 (A's)	90	to	100%

In this particular example, the dimensions would change significantly. The first two groups and part of the third in the quartile classification above would be collapsed into the Group 1 on the grading scale (immediately above).

We can amplify the process and make it more analogous to the situation at hand by looking at an additional example. Assume that a researcher was investigating the various "causes" of why a city would lose population, retain its existing levels of population or increase its population, a three group model. Clearly the researcher can change the range of the classificatory variable for a particular group--in this case, some percentage change of population from previous years--by simply redefining what he means when he says "losing population," etc. To extend this reasoning to our problem, we varied our definitions of our three groups by varying the level of the classificatory variable, the amount of cash, 1975, expressed as a percentage of total general fund assets, 1975 as shown on the financial statements. The ranges investigated were arbitrarily chosen.

For example, the ranges investigated began with:

	Range of Cash[2]		
Group 1 (Cash Poor)	0	to	25%
Group 2 (Cash Satisfactory)	26	to	75%
Group 3 (Cash Rich)	75	to	100%

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[2]The variable, cash, is computed in this way: \$ Cash, 1975/ Total General fund assets, 1975.

Group 1, as discussed shortly, became 0 to 10%, for at this level the greatest classificatory ability was found. Obviously, the change in the level of the classificatory variable causes a shift in the number of cities in a particular group. For example, lowering the upper limit of Group 1 (above) from 25% to say 10% would shift approximately ten cities from Group 1 to Group 2.

Criterion variables--final runs . . .

The majority of runs were based on a triad of levels of the criterion variable, percentage of cash, for classification. Once again, the implication of a variation in the percentage level chosen as criterion is a variation in the classification of specific municipalities, ergo, variation in the number in a specific group.

The levels chosen for final analysis were:

Category 1 (captioned "Cash Poor" or "Cash Short")
This category included ten municipalities having cash ranging from zero percent to ten percent of total fund assets, on the Balance Sheet, June 30, 1975.

Category 2 (captioned "Cash Satisfactory")
This category included 41 municipalities having cash ranging from eleven percent to seventy-four percent of total general fund assets on the Balance Sheet, June 30, 1975.

Category 3 (captioned "Cash Rich" or "Cash Excess")
This category included nine municipalities having cash ranging from seventy-four percent to one hundred percent of total general fund assets on the Balance Sheet, June 30, 1975.

Reiteration of already established points regarding our choice of cash as our criterion variable, the focus of our attention, seems apropos at this point. Earlier, we reasoned that although over 200 cities in this country have declared bankruptcy, there have been no recent declarations nor has there been any of significant size. Such an action seem unlikely if at all feasible because of the constraining requirement of Chapter IX of the Federal Bankruptcy Act necessitating that the plan of settlement have approval of over 50 percent of the creditors.[3] Although declaration of bankruptcy provides a critical event for research in private sector solvency, it is essentially a vestigial in studies of public sector solvency.

It became necessary, therefore, to establish another item as an acceptable surrogate for bankruptcy. Although various states have governing measures of insolvency or poor financial condition, we felt that a readily discernible measure would be desirable. After extensive conversations with knowledgeable individuals, it was concluded that the amount of cash a municipality held would be an acceptable substitute for insolvency.

Range of analysis--predictor variables . . .

In addition to varying the levels of the criterion variables for the three groups, enhancement of the model was achieved through variations in the predictor variables. Specifically, three aspects of the predictor variables were changed: the combinations of the variables used, the metric of the variables used, and the time frame of the variables used. There is little practicality in discussing all of the

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[3]In the recent fiscal crises of New York City, the individual who was aware of the implications of Chapter IX understood that although the bonds might go "flat" bankruptcy was remote and highly improbable. One need only imagine the vast amount and number of bearer bonds issued by the City and held throughout the crises and he becomes relatively certain that consensus among such lenders--to a plan of arrangement--if their identity could ever even be determined--might be a greater task than resolving the fiscal dilemma.

variations, so we shall describe them narratively. Others are discussed on later pages dealing with the findings.

The first variation is found in the combinations of variables utilized. First, variables were segregated by the statement on which they appeared, the Balance Sheet or the Income Statement. Each of the elements (in "raw" dollar amounts) from both statements was analyzed for its classificatory power. Some of these, including the element "Cash," will be discussed on later pages. Greater classificatory ability was realized through two approaches. First, the using of totals, such as Total General Fund Assets, Total Revenues, Total expenditures, etc. improved the predictive ability of the model over that of one made up of an intuitively appealing combination of elements. Additional classificatory ability was realized through the utilization of totals and the elements composing that total. For example, one of the more successful runs had as predictor variables the Total Expenditures and the four subsets of expenditures, Administration, Police, Fire and Parks and Recreation Expenditures for various years. Another utilized Total Revenues and its subsets for various years.

The second variation is in the metric utilized. Initial runs employed predictor variables expressed in "raw" dollars. Later runs were based on the development of ratios, such as

$$\frac{\text{Administration Expenditures}}{\text{Total Expenditures}}$$

It was felt that this would minimize any potential distortion because of disparity in size of the units observed. A probable insufficient investigation was made into trends through the computation of the percentage change in some of the elements. It is stated that this is probably insufficient because there is the feeling that the data possesses time lags worthy of exploration.

Finally, the predictor variables were drawn from different years. This is also discussed on the following pages. The base year of the classificatory variable, 1975--that to which the prediction is made--remained the same throughout the study. It can be generally concluded that the model improved with a lag of at least one year. That is, in general, greater classificatory ability was realized utilizing predictor variables from the years 1971 through 1973 than from 1971 through 1974 or 1972 through 1974. As additional years of data are accumulated, this phenomenonological behavior can be explored further.

Data output . . .

Output for Model B Quantal Response Technique presents the estimated (percentage) probabilities of classification of a particular municipality into each of the three groups, that is an estimated probability that the city is "cash poor," that it is "cash satisfactory" and that it is "cash rich." These three estimations can be considered as a probabilistic profile of each city, given a particular set of predictor variables. Obviously, the category having the highest estimated probability associated with it is the category into which that city would be classified. Correct classification (a "hit") should connote that through fitting of the logistic polychotomous model and proper viewing of the estimated probabilities, a municipality was classified in the same category as the experienced actual percentage of cash on June 30, 1975 did actually place it. That is predicted percentage cash level 1975 was the same as actual percentage cash level, 1975.

Runs of data . . .

The original list of predictor variables was reduced through numerous computer runs to the following optimal combination of 15 variables:

Total Expenditures	1973, 1972, 1971
Administrative Expenses	1973, 1972, 1971
Police Expense	1973, 1972, 1971
Fire Expense	1973, 1972, 1971
Park and Recreation Expense	1973, 1972, 1971

Items are arranged solely in order of appearance in the data bank and such an arraying should not be construed as to imply significance either as related to the variables or to the factor of time. Furthermore, it is only the classificatory ability of the entire combination which is evaluated--although we do offer calculations of the weights of individual variables. In addition, we make no claim that the resultant logistic function (format 11) is the most optimal; rather, it is the most optimal we evaluated.

Empirical results . . .

A total of 50 correct classifications of cities and ten incorrect classifications was realized. This results in a "hit" ratio of 87.5%. These classifications can be viewed in Table 1. These tables should be interpreted in this manner. The analytical results are shown in vertical columns. The actual classifications are shown horizontally. The diagonal running from the upper left to the lower right should be interpreted as the number of correct classifications. The total number in the group is the sum of the (horizontal) row.

TABLE 1

CLASSIFICATIONS FROM FORMAT 11

		Classification as predicted from the fitted model		
		Poor	Sat.*	Rich
ACTUAL CLASSIFICATION	Poor	6	3	1
	Sat.*	1	39	1
	Rich	0	4	5

*Satisfactory

The estimated probabilities for format 11 are shown in Table 2:

TABLE 2

ESTIMATED PROBABILITIES OF CLASSIFICATION UNDER FORMAT 11

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
		First Category	
1	.4431	.5189	.0380
2	.1968	.0594	.7438
3	.7342	.2510	.0148
4	.9964	.0006	.0029
5	.7680	.2310	.0010
6	1.0000	.0000	.0000
7	.1540	.7985	.0474
8	.0829	.8541	.0630
9	.5513	.4475	.0012
10	.7675	.2154	.0171
		Second Category	
11	.1007	.8177	.0816
12	.0479	.8962	.0289

TABLE 2--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
13	.4098	.4868	.1034
14	.0209	.8792	.0999
15	.0454	.9052	.0494
16	.1235	.8288	.0477
17	.0698	.9259	.0043
18	.1155	.8256	.0589
19	.0995	.6418	.2587
20	.0205	.9708	.0087
21	.0820	.8867	.0312
22	.0733	.3774	.5493
23	.0909	.9053	.0038
24	.2376	.7421	.0202
25	.1295	.8524	.0181
26	.0965	.8731	.0304
27	.0229	.9637	.0134
28	.0227	.9617	.0156
29	.1869	.5424	.2707
30	.1655	.7648	.0696
31	.0244	.9413	.0343
32	.0352	.9470	.0178
33	.0636	.8781	.0583
34	.0228	.8892	.0880
35	.0322	.9526	.0152
36	.2644	.6771	.0585
37	.1281	.8563	.0156
38	.1918	.7080	.1002
39	.0304	.5736	.3960
40	.0044	.9912	.0044
41	.0850	.8740	.0409
42	.1074	.7986	.0940

TABLE 2--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
43	.0332	.9424	.0244
44	.3168	.6761	.0071
45	.0842	.7248	.1910
46	.2537	.7277	.0186
47	.0938	.8489	.0572
48	.1122	.8825	.0053
49	.4085	.5722	.0193
50	.6599	.3363	.0038
51	.0295	.9415	.0290
		Third Category	
52	.0002	.0059	.9939
53	.0952	.6906	.2142
54	.1283	.0734	.7983
55	.0807	.8801	.3312
56	.0320	.7837	.1843
57	.0110	.0539	.9361
58	.1204	.3231	.5565
59	.0384	.0255	.9361
60	.1400	.7934	.0666

As is true in all probability functions, the sum of the estimated probabilities for a particular city must equal one:

$$\hat{P}_1 + \hat{P}_2 + \hat{P}_3 = 1$$

The occasional rounding error evidences itself in $\Sigma P = .9999$.

Additional insight into the optimality of a particular run of Quantal Response can be gained by observing the distance of groups.

TABLE 3

INCORRECT CLASSIFICATIONS
UNDER FORMAT 11

City Number	Actual Classified in Group	Classified as From the Fitted Model
1	1	2
2	1	3
7	1	2
8	1	2
22	2	3
51	2	1
53	3	2
55	3	2
56	3	2
60	3	2

In only one case (City Number 2) was the category into which the city was classified incorrectly further than the adjacent category further inspection of the data of Table 1 shows that the misclassification errors had associated conditional probabilities in the range $.03 < \hat{P} < .45$ or a range of 42 percent.

In reality, this "importance" of the predictor variables is reflected as variability of the criterion variable. This relationship may be compared to the simple linear regression model, which is expressed as:

$$Y_1 = \alpha + \beta x_i$$

That expression of a simple linear model may be expanded to recognize that variability can be partitioned into two distinct parts, systematic and random (error):

$$Y_1 = \alpha + \beta x_u + \mu_i$$

In this case, the α, as is traditional, represents the intercept of the regression line and the Y axis; the β represents the systematic variability (the slope of the regression line); the μ represents the random variability or that which will not be explained by the model. The less the size of the random error the greater the explanation of total variability as systematic variability.

In practice, it is most obvious that the outcome of any particular Y is dependent upon several factors. Liquidity preference as evidenced by the amount of cash held by a municipality is the function of innumerable input variables such as revenues, expenditures, the partitioning of those revenues and expenditures, demands of the community and higher levels of governments, etc. The description of the linear model having a dependent variable Y which is related to two or more independent variables X_2 and X_3 is shown in the following model:

$$Y_i = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \ldots + \beta_n X_n + \mu_i$$

In this case, the β_2, β_3 ... β_n still denote the slope of the regression line. They are usually captioned "partial regression coefficients."

Having dealt with the connotation of the various elements in both a simple and multiple linear model, there is no difficulty of a theoretical nature in extending our analysis to the Quantal Response model. Recalling that the formula for the estimation of probability under Quantal Response is

$$P = 1/[1 + \exp^{-(\alpha+\beta x)}]$$

with the meaning of the β comparable to that stated in the discussion of the linear model, excepting that the appraisal of the particular variable in classification is seen as an indirect relation to the size of $\hat{\beta}$, that is, the smaller the estimation of β, the greater the probability of correct classification. Thus the means of estimating the relative importance of predictor variables for Model B is to view the vectors of estimations of β ($\hat{\beta}$) for the categories (as shown in Table 4, for example). With one degree of freedom and, as stated above, estimated probabilities for the three categories summing to 1, $\hat{\underset{\sim}{\beta}}$ is given for only two categories. The estimate of β for the remaining category would be obtainable by conditioning on a different category. However, as structured, the program conditions on the first category and consequently vectors of the estimations are for β_2 and β_3. It is quite relevant to realize that similar probability estimations were obtained regardless of which category was conditioned upon. The vectors obtained are shown below as Table 4.

The variables relevant to the array of estimators would be in reverse chronological order, 1973 through 1971. This ordering possesses no significance other than that of the arbitrary programming of the computer, putting the most recent first. The following order of appearance in each year prevails.

Total Expenditures
Administrative Expenditures
Police Expenditures
Fire Expenditures
Park and Recreation Expenditures

The relative weight of the $\hat{\underset{\sim}{\beta}}$'s is the product of those estimated parameters and the arithmetical average of each of the fifteen observations

TABLE 4

ESTIMATIONS OF VECTORS FOR β_2 and β_3
UNDER FORMAT 11

$\hat{\beta}_2$	Variables	$\hat{\beta}_3$
.00074		.00051
-.00070		-.00082
-.00534		-.00624
.00368		.00781
.00600		.00569
-.00083		.00018
.00022		.00452
.01496		.01266
-.02104		-.03456
-.00362		.00017
.00035		-.00117
-.00031		-.00107
-.01334		-.00478
.02139		.02292
.00009		-.00002

(the predictor variables) over the sixty cities. These are arrayed in Table 5 for category 2. Similar computations for category 3 are shown in Table 6.

TABLE 5

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES
OF CATEGORY TWO UNDER FORMAT 11

Predictor Variable	$\hat{\beta}_2$	x	$\bar{X}_i$	=	$\hat{\beta}_2\bar{X}_i$
Tot Exp 73	.00074		16835.80		12.45849
Adm Exp 73	-.00070		1604.13		- 1.12289
Pol Exp 73	-.00534		4365.08		-23.30953
Fire Exp 73	.00368		1883.18		6.93010
P&R Exp 73	.00600		873.98		5.24388
Tot Exp 72	-.00083		15210.45		-12.62467
Adm Exp 72	.00022		1367.20		.30078
Pol Exp 72	.01496		3867.65		57.86004
Fire Exp 72	-.02104		1628.30		-34.25943
P&R Exp 72	-.00362		837.45		- 3.03157
Tot Exp 71	.00035		14744.88		5.16071
Adm Exp 71	-.00031		1257.43		- .38980
Pol Exp 71	-.01334		3288.15		-43.86390
Fire Exp 71	.02139		1447.68		30.96590
P&R Exp 71	.00009		1104.38		.09939

TABLE 6

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES
OF CATEGORY THREE UNDER FORMAT 11

Predictor Variable	$\hat{\underset{\sim}{\beta}}_3$	x	$\bar{X}_i$	=	$\hat{\underset{\sim}{\beta}}_3 \bar{X}_i$
Tot Exp 73	.00051		7132.00		3.63732
Adm Exp 73	-.00082		1370.44		-1.12376
Pol Exp 73	-.00624		1274.11		-7.95045
Fire Exp 73	.00781		744.56		5.81501
P&R Exp 73	.00569		534.67		3.04227
Tot Exp 72	.00018		6566.56		1.18198
Adm Exp 72	.00452		1301.78		5.88404
Pol Exp 72	.01266		1164.89		14.74751
Fire Exp 72	-.03456		699.11		-24.16124
P&R Exp 72	.00017		603.56		1.02605
Tot Exp 71	-.00117		5477.67		-6.40887
Adm Exp 71	-.00107		1148.89		-1.22931
Pol Exp 71	-.00478		1066.33		-5.09706
Fire Exp 71	.02292		643.22		14.74260
P&R Exp 71	-.0002		620.22		.01240

Although these arrays display the relative weight, a logical extension of the analysis is found in displaying the estimated $\hat{\beta}_2$ in order of their effect. In order of absolute size, beginning with the smallest (that one having the greater effect).

TABLE 7

RANKING OF $\underset{\sim}{\hat{\beta}}\ \bar{X}$
UNDER FORMAT 11

$\underset{\sim}{\hat{\beta}}_2$	$\underset{\sim}{\hat{\beta}}_3$
P&R Exp 71	P&R Exp 71
Adm Exp 72	P&R Exp 72
Adm Exp 71	Adm Exp 73
Adm Exp 73	Tot Exp 72
P&R Exp 72	Adm Exp 71
Tot Exp 71	P&R Exp 73
P&R Exp 73	Tot Exp 73
Fire Exp 73	Pol Exp 71
Tot Exp 73	Fire Exp 73
Tot Exp 72	Adm Exp 72
Pol Exp 73	Tot Exp 71
Fire Exp 71	Pol Exp 73
Fire Exp 72	Fire Exp 71
Pol Exp 71	Pol Exp 72
Pol Exp 72	Fire Exp 72

Because of the relationship of the four expenditures to the total expenditures and the total relationship of the financial items it seems reasonable to conclude that multicollinearity could be present. Consequently, it is logical to look at the application of the entire analysis rather than the parts. Nevertheless insight into the analysis can be gained in another way, looking at the relative importance of the variables:

TABLE 8

RANKING OF VARIABLES UNDER FORMAT 11

	$\hat{\beta}_2$	$\hat{\beta}_3$
Tot Exp 73	9	7
Adm Exp 73	4	3
Pol Exp 73	11	12
Fire Exp 73	8	9
P&R Exp 73	7	6
Tot Exp 72	10	4
Adm Exp 72	2	10
Pol Exp 72	15	14
Fire Exp 72	13	15
P&R Exp	5	2
Tot Exp	6	11
Adm Exp	3	5
Pol Exp 71	14	8
Fire Exp	12	13
P&R Exp 71	1	1

Viewing Table 7 once again discloses that of the first five variables appearing in each columns, four are similar. In addition, it is observed, on an intuitive basis, that the expenses having the greatest impact in classification were those which might be described as more discretionary, Administrative and Parks and Recreation; the Police and Fire are more closely aligned with the concept of a fixed cost because of the usual union contracts.

An additional casual impression can be realized by observing the rankings of Table 8. By way of emphasis we create Table 9 which is simply the extraction of Administration and Parks and Recreation Expenditures from Table 8 to amplify the importance of the earlier years:

TABLE 9

RANKING OF ADMINISTRATIVE, AND PARKS AND RECREATION VARIABLES

	$\hat{\utilde{\beta}}_2$	$\hat{\utilde{\beta}}_3$
Adm Exp 73	4	3
P&R Exp 73	7	6
Adm Exp 72	2	10
P&R Exp 72	5	2
Adm Exp 71	3	5
P&R Exp 71	1	1

The implication which may be drawn is that there is, simply stated, a time lag. Not only is this lag significant here but the variables of Total Expenditures, Administrative Expenditures, Police Expenditures, Fire Expenditures and Park and Recreation Expenditures for the three years 1972-1974, format 10, one year later than format 11 had a "hit" ratio of only 77 percent. In addition other runs evidenced similar findings.

Format 13 utilized the same variables from similar periods as format 11 excepting that Total Expenditures was omitted. That omission resulted in a reduction in the "hit" ratio to 75 percent. More critical was the weakness in classification in Category 1 as illustrated in Table 10.

TABLE 10

CLASSIFICATIONS FROM FORMAT 13

		Classification as predicted from the fitted model		
		Poor	Sat.	Rich
ACTUAL CLASSIFICATION	Poor	1	7	2
	Sat.	0	41	0
	Rich	0	6	3

A similar description of findings will be presented for the other elements in the operating statement. Because of the evidenced weakness in the classificatory ability in the Poor category, further discussion of the results will not be entered into.

<u>Format 10 . . .</u>

Affirmation of the belief of time lag is found in the analysis of the results of Format 10 runs. The combination of predictor variables is similar to that of Format 11, excepting that Format 10 covers a period one year later:

Total Expenditures	1974, 1973, 1972
Administrative Expenses	1974, 1973, 1972
Police Expense	1974, 1973, 1972
Fire Expense	1974, 1973, 1972
Park and Recreation Expense	1974, 1973, 1972

The criterion variable, as in all other runs, was the percentage of cash to total assets in the general fund, June 30, 1975. Correct classifications decreased from the fifty of Format 11 to forty-four, with the most significant declination in the Poor category. Results are summarized in Table 11:

TABLE 11

CLASSIFICATIONS FROM FORMAT 10

		Classification as predicted from the fitted model		
		Poor	Sat.	Rich
ACTUAL CLASSIFICATION	Poor	0	3	6
	Sat.	0	40	1
	Rich	0	5	4

Another way of viewing the results is to form a "fraction" expressing the "hit" ratio for each class. This involves utilizing the number of "hits" as the numerator and the actual total in the class as the denominator. For Format 10, the diagonal fractions of the matrix read:

0/10, 40/41, 4/9, : 44/60, or,

a "hit" ratio of 73.3 percent. The estimated probabilities are shown as Table 12.

TABLE 12

ESTIMATED PROBABILITIES OF CLASSIFICATION
UNDER FORMAT 10

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
		First Category	
1	.0000	.0000	1.0000
2	.0001	.0013	.9986
3	.0000	.0000	1.0000
4	.0000	.0000	1.0000
5	.0000	.0000	1.0000
6	.0142	.1366	.8492
7	.0768	.7491	.1741
8	.2595	.7404	.0000
9	.1354	.8524	.0122
10	.0777	.9011	.0212
		Second Category	
11	.1297	.8544	.0159
12	.0741	.8680	.0579
13	.0334	.8575	.1091
14	.1625	.8371	.0005
15	.0947	.9008	.0045
16	.1848	.7923	.0229
17	.0418	.9519	.0063
18	.0944	.8567	.0490
19	.0816	.5353	.3830
20	.0273	.9696	.0031
21	.1181	.8805	.0014
22	.1034	.6811	.2155
23	.1036	.8955	.0010
24	.1352	.8585	.0063
25	.1457	.8490	.0052
26	.1167	.8583	.0250
27	.1175	.8786	.0039

TABLE 12--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
28	.1560	.8325	.0115
29	.0541	.4287	.5172
30	.2390	.7520	.0090
31	.0659	.8107	.1234
32	.1280	.8574	.0147
33	.0573	.7660	.1767
34	.2948	.7046	.0006
35	.0842	.9111	.0046
36	.0837	.9098	.0065
37	.1900	.8050	.0050
38	.2799	.7186	.0015
39	.1438	.8248	.0314
40	.0531	.9468	.0001
41	.1150	.8456	.0394
42	.2015	.7850	.0135
43	.1005	.8956	.0039
44	.3049	.6875	.0076
45	.0378	.9598	.0023
46	.1416	.8450	.0134
47	.0542	.6771	.2686
48	.3576	.6414	.0010
49	.1498	.8334	.0167
50	.0046	.9415	.0538
51	.0310	.9465	.0225
		Third Category	
52	.0000	.0021	.9979
53	.0704	.8261	.1035
54	.0889	.8195	.0916
55	.0717	.7784	.1499
56	.0599	.5880	.3521

TABLE 12--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
57	.0003	.0025	.9972
58	.0143	.0327	.9530
59	.0007	.0072	.9921
60	.1126	.8437	.0437

Then, as previously done, we show an array of the estimated vectors:

TABLE 13

ESTIMATIONS OF VECTORS FOR β_2 AND β_3 UNDER FORMAT 10

$\hat{\underset{\sim}{\beta}}_2$	Variables	$\hat{\underset{\sim}{\beta}}_3$
-.00032		-.00500
.00086		.00309
.00075		.00424
-.00106		-.00982
.00313		.01087
.00033		.00142
.00069		-.00169
-.00498		-.00317
.01055		.03535
.00201		-.00079
.00014		.00458
-.00191		.00321
.00408		.00222
-.00136		-.04150
.00229		.00479

TABLE 14

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES
OF CATEGORY TWO UNDER FORMAT 10

Predictor Variable	$\hat{\beta}_2$	x	$\bar{X}_i$	=	$\hat{\beta}_2\bar{X}_i$
Tot Exp 74	-.00032		18063.65		-5.78018
Adm Exp 74	.00086		1832.93		1.57632
Pol Exp 74	.00075		2434.88		1.82616
Fire Exp 74	-.00106		2017.95		-2.13903
P&R Exp 74	.00313		1071.66		3.35430
Tot Exp 73	.00033		16500.61		5.44520
Adm Exp 73	.00069		1540.46		1.06292
Pol Exp 73	-.00498		4269.63		-21.26276
Fire Exp 73	.01055		1846.00		19.47530
P&R Exp 73	.00201		856.12		1.72080
Tot Exp 72	.00014		14891.61		2.08483
Adm Exp 72	-.00191		1340.44		-2.56024
Pol Exp 72	.00408		3783.41		15.43631
Fire Exp 72	-.01136		1597.22		-18.14442
P&R Exp 72	.00229		822.46		1.88343

TABLE 15

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES OF CATEGORY THREE UNDER FORMAT 10

Predictor Variable	$\hat{\underset{\sim}{\beta}}_3$	x	$\bar{X}_i$	=	$\hat{\underset{\sim}{\beta}}_3\bar{X}_i$
Tot Exp 74	-.00500		7406.67		-37.03335
Adm Exp 74	.00309		1524.78		4.71157
Pol Exp 74	.00424		1453.67		6.16356
Fire Exp 74	-.00982		799.44		-7.85050
P&R Exp 74	.01087		481.44		5.23325
Tot Exp 73	.00142		7132.00		10.12744
Adm Exp 73	-.00169		1370.44		-2.31604
Pol Exp 73	-.00317		1274.11		-4.03893
Fire Exp 73	.03535		744.56		26.32020
P&R Exp 73	-.00079		534.67		-0.42239
Tot Exp 72	.00458		6566.56		30.07484
Adm Exp 72	.00321		1301.78		4.17871
Pol Exp 72	.00222		1164.89		2.58606
Fire Exp 72	-.04150		699.11		-29.01307
P&R Exp 72	.00479		603.56		2.89105

TABLE 16

RANKING OF $\underset{\sim}{\hat{\beta}}\,\bar{X}$
UNDER FORMAT 10

$\underset{\sim}{\hat{\beta}}_2\bar{X}_i$	$\underset{\sim}{\hat{\beta}}_3\bar{X}_i$
Adm Exp 73	P&R Exp 73
Adm Exp 74	Adm Exp 73
P&R Exp 73	Pol Exp 72
Pol Exp 74	P&R Exp 72
P&R Exp 72	Adm Exp 72
Tot Exp 72	Pol Exp 73
Fire Exp 74	Adm Exp 74
Adm Exp 72	P&R Exp 74
P&R Exp 74	Pol Exp 74
Tot Exp 73	Fire Exp 74
Tot Exp 74	Tot Exp 73
Pol Exp 72	Fire Exp 72
Fire Exp 72	Fire Exp 73
Fire Exp 73	Tot Exp 72
Pol Exp 73	Tot Exp 74

Once again it is possible to take this data and present them in a format comparable to Table 8. However this presentation will involve an additional computation, the averaging of the two rankings. This is shown in Table 17. Because of lack of recognition of differences in group sizes, it should be realized that this is not a weighted average.

TABLE 17

RANKING OF VARIABLES UNDER FORMAT 10

	$\hat{\beta}_2\bar{X}_i$	$\hat{\beta}_3\bar{X}_i$	Average
Tot Exp 74	11	15	13.0
Adm Exp 74	2	7	4.5
Pol Exp 74	4	9	6.5
Fire Exp 74	7	10	8.5
P&R Exp 74	9	8	8.5
Tot Exp 73	10	11	10.5
Adm Exp 73	1	2	1.5
Pol Exp 73	15	6	10.5
Fire Exp 73	14	12	13.0
P&R Exp 73	3	1	2.0
Tot Exp 72	6	14	10.0
Adm Exp 72	8	5	6.5
Pol Exp 72	12	3	7.5
Fire Exp 72	13	13	13.0
P&R Exp 72	5	4	4.5

Recognizing that the variables having the lower numbers contribute most to the correct classification, a visual review of Table 17 leads to the appraisal that it seldom matters how much is spent (Total Expenditures) rather, the discretionary amounts (Administration and

Parks and Recreation) have greater impact on the statistical outcome than the expenditures which approach fixed costs by nature of union contracts (Police and Fire). The lag effect, discussed as it was recognized in analysis of format 11 results, is present in format 10 although not as pronounced. An interesting combination of the ranking is shown in Table 18 which presents the averages of the four year span of formats 10 and 11. It should be stated that although both had the same criterion variable, results are sensitive to the combination of predictor variables. Table 18 is shown on page 104.

Formats 8 and 9 . . .

The two primary sources of revenue for the General Fund of a municipality in the State of Michigan are Property Tax Income and those items categorized under the general caption of State Shared Revenues. As commented on previously, this latter amount will consist of those tax revenue items collected by the state and then distributed on the basis of a "sharing" formula. These items consist of, but are not limited to, sales and use tax, state income tax, liquor licenses, and intangibles tax. The two items generally comprise approximately 60 to 70 percent of total revenues, however if a personal income tax is present, the percentage drops significantly to 40 to 50 percent or less. Property Tax Income, derived from local taxes on both real and personal property, is to some extent a measure of the base value of taxable property in that it is standardized to a degree through State Equalized Values. Essentially then, both property tax income and State Shared Revenues are somewhat beyond the immediate discretion of the administration of the municipality.

TABLE 18

AVERAGE OF RANKINGS OF VARIABLES
OF FORMATS 10 AND 11

	Format 10 Average	Format 11 Average
Total Exp 74	13.0	-
Adm Exp 74	4.5	-
Pol Exp 74	6.5	-
Fire Exp 74	8.5	-
P&R Exp 74	8.5	-
Tot Exp 73	10.5	8.0
Adm Exp 73	1.5	3.5
Pol Exp 73	10.5	11.5
Fire Exp 73	13.0	8.5
P&R Exp 73	2.0	6.5
Total Exp 72	10.0	7.0
Adm Exp 72	6.5	6.0
Pol Exp 72	7.5	14.5
Fire Exp 72	13.0	14.0
P&R Exp 72	4.5	3.5
Tot Exp 71	-	8.5
Adm Exp 71	-	4.0
Pol Exp 71	-	11.0
Fire Exp 71	-	12.5
P&R Exp 71	-	1.0

Formats 8 and 9 investigated these two particular revenues combined with the total revenues of the municipality. In summary, two things could be stated. First, the lag effect was present once again but not as strong as in the expenditures analysis. Second, there was a slight deterioration in correct classification in comparison to the analysis of expenditures under formats 10 and 11. The erosion which was most noticed is found in the Poor category. Format 8 utilized the nine variables listed below:

Total Revenues	1973, 1972, 1971
Property Tax Income	1973, 1972, 1971
State Shared Revenues	1973, 1972, 1971

There is no way to evaluate the effect of six less variables other than the classification table.

The classification results for format 8 are presented as Table 19:

TABLE 19

CLASSIFICATIONS FROM FORMAT 8

		Classification as predicted from the fitted model		
		Poor	Sat.	Rich
ACTUAL CLASSIFICATION	Poor	4	2	1
	Sat.	6	39	6
	Rich	0	0	2

The "hit" rates can be derived from this table:

4/10, 39/41, 2/9 : 45/60

which is then stated as a 75% correct classification. Probability estimations are arrayed in Table 20.

TABLE 20

ESTIMATED PROBABILITIES OF CLASSIFICATION
UNDER FORMAT 8

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
		First Category	
1	.4439	.5448	.0213
2	.1652	.7503	.0845
3	.8294	.1533	.0173
4	.3217	.5631	.1152
5	.4804	.4508	.0688
6	.0925	.8208	.0867
7	.0544	.8193	.1263
8	.0741	.7790	.1469
9	.8673	.1162	.0165
10	.4790	.4568	.0642
		Second Category	
11	.0855	.7709	.1436
12	.0756	.7985	.1259
13	.0282	.9387	.0331
14	.0541	.8678	.0781
15	.1397	.7787	.0816
16	.0644	.8467	.0889
17	.1139	.7450	.1411
18	.0537	.8396	.1067
19	.0747	.8270	.0982
20	.0250	.9427	.0323
21	.0809	.8374	.0816
22	.0499	.7677	.1824
23	.0555	.8309	.1137
24	.0346	.8734	.0920
25	.0408	.8210	.1382
26	.0654	.7932	.1413
27	.1699	.7340	.0961

TABLE 20--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
28	.1830	.7567	.0603
29	.0931	.7479	.1590
30	.2895	.6141	.0963
31	.0808	.8316	.0876
32	.1793	.7001	.1206
33	.0525	.8015	.1460
34	.0380	.6348	.3272
35	.0630	.8850	.0520
36	.1071	.8014	.0915
37	.0590	.8118	.1292
38	.5431	.8366	.1091
39	.2780	.6514	.0706
40	.9362	.0566	.0072
41	.0261	.7909	.1831
42	.0401	.8230	.1369
43	.4256	.4772	.0972
44	.0076	.7392	.2532
45	.0210	.8931	.0859
46	.0372	.7924	.1704
47	.0230	.6369	.3401
48	.1477	.8260	.0263
49	.0711	.8341	.0948
50	.5709	.3194	.1097
51	.0977	.7233	.1790
		Third Category	
52	.0006	.0241	.9753
53	.6071	.3373	.0555
54	.3193	.8770	.0910
55	.0615	.8167	.1218
56	.1223	.7779	.0998

TABLE 20--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
57	.0185	.1897	.7919
58	.0784	.8442	.0774
59	.0706	.8421	.0873
60	.0372	.8542	.1086

Once again, observation of misclassifications shows only 15 misses were recorded; of those 15, only one was not in the next group.

TABLE 21

INCORRECT CLASSIFICATIONS
UNDER FORMAT 8

City Number	Actual Classified in Group	Classified as From the Fitted Model
1	1	2
2	1	2
4	1	2
6	1	2
7	1	2
8	1	2
38	2	1
40	2	1
53	3	1
54	3	2
55	3	2
56	3	2
58	3	2
59	3	2
60	3	2

The estimation of the $\underset{\sim}{\beta}$'s for the nine variables is shown in Table 22.

TABLE 22

ESTIMATIONS OF VECTORS FOR β_2 AND β_3
UNDER FORMAT 8

$\hat{\underset{\sim}{\beta}}_2$	Variables	$\hat{\underset{\sim}{\beta}}_3$
.00010		-.00094
-.00053		.00005
-.00996		-.00749
.00118		.00140
-.00017		.00023
-.00209		-.00528
-.00078		.00045
.00041		-.00071
.01020		.01012

The means of the variables of the first category when multiplied by the appropriate vectors produce the relative weight of each variable.

TABLE 23

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES
OF CATEGORY TWO UNDER FORMAT 8

Predictor Variable	$\hat{\beta}_2$	x	$\bar{X}_i$	=	$\hat{\beta}_2\bar{X}_i$
Tot Rev 73	.00010		17325.07		1.73251
Pry Tax 73	-.00053		4950.12		-2.62356
SS Rev 73	-.00996		2330.49		-23.21168
Tot Rev 72	.00118		17184.37		20.27756
Pry Tax 72	-.00017		5880.39		0.99967
SS Rev 72	-.00209		2475.73		5.17428
Tot Rev 71	-.00078		14551.76		11.35037
Pry Tax 71	.00041		5678.46		2.32817
SS Rev 71	.01020		1772.12		18.07562

Table 24 presents similar computations for Category Three.

TABLE 24

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES
OF CATEGORY THREE UNDER FORMAT 8

Predictor Variable	$\hat{\beta}_3$	x	$\bar{X}_i$	=	$\hat{\beta}_3\bar{X}_i$
Tot Rev 73	-.00094		7806.44		-7.33805
Pry Tax 73	.00005		4061.00		0.20305
SS Rev 73	-.00749		1121.22		-8.39738
Tot Rev 72	.00140		7518.78		10.52629
Pry Tax 72	.00023		4067.56		0.93554
SS Rev 72	-.00528		1032.22		-5.45012
Tot Rev 71	.00048		6852.67		3.28928
Pry Tax 71	-.00071		3781.11		-2.68459
SS Rev 71	.01012		928.56		9.39703

These rankings are arrayed in Table 25:

TABLE 25

RANKING OF VARIABLES UNDER FORMAT 8

	$\hat{\underset{\sim}{\beta}}_2\bar{X}_i$	$\hat{\underset{\sim}{\beta}}_3\bar{X}_i$	Average
Tot Rev 73	2	6	4.0
Pry Tax 73	4	1	2.5
SS Rev 73	9	7	8.0
Tot Rev 72	8	9	8.5
Pry Tax 72	1	2	1.5
SS Rev 72	5	5	5.0
Tot Rev 71	6	4	5.0
Pry Tax 71	3	3	3.0
SS Rev 71	7	8	7.5

By reformulating the ranking we can emphasize the variables contributing the most to the classification beginning with the smallest, that which contributes most.

TABLE 26

RANKING OF $\hat{\underset{\sim}{\beta}}\bar{X}_i$ UNDER FORMAT 8

$\hat{\underset{\sim}{\beta}}_2\bar{X}_i$	$\hat{\underset{\sim}{\beta}}_3\bar{X}_i$
Pry Tax 72	Pry Tax 73
Tot Rev 73	Pry Tax 72
Pry Tax 71	Pry Tax 71
Pry Tax 73	Tot Rev 71
SS Rev 72	SS Rev 72
Tot Rev 71	Tot Rev 73
SS Rev 71	SS Rev 73
Tot Rev 72	SS Rev 71
SS Rev 73	Tot Rev 72

Comments will be reserved until following format 9. In format 9 nine similar variables are used with an advance of one year:

Total Revenues	1974, 1973, 1972
Property Tax Income	1974, 1973, 1972
State Shared Revenues	1974, 1973, 1972

The presentation of classifications is in Table 27:

TABLE 27

CLASSIFICATIONS FROM FORMAT 9

		Classification as predicted from the fitted model		
		Poor	Sat.	Rich
ACTUAL CLASSIFICATION	Poor	3	7	
	Sat.	2	39	0
	Rich	0	8	1

The 43 "hits" (for a percentage of 71.7 correct classification) can be summarized in fractions:

3/10, 39/41, 1/9, : 43/60

However two of the probabilities were sufficiently close so as to warrant mention. For the first city in Category One the classification was:

$$\hat{P}_1 = .4330, \quad \hat{P}_2 = .4668$$

for the sixth city in Category Three,

$$\hat{P}_1 = .4879 \text{ and } \hat{P}_2 = .4698$$

Correct classification would have improved both "tails", the areas of difficult classification.

In accord with previous presentations estimated β's are shown:

TABLE 28

ESTIMATION OF VECTORS FOR β_2 AND β_3
UNDER FORMAT 9

$\underset{\sim}{\hat{\beta}}_2$	Variables	$\underset{\sim}{\hat{\beta}}_3$
-.00026		-.00051
-.00056		-.00009
-.00793		-.00420
.00136		.00224
-.00175		-.00116
.00897		.00550
-.00050		-.00105
.00286		.00083
-.00569		-.00601

The estimated vectors are multiplied against the mean of the variable to produce the relative weight:

TABLE 29

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES OF CATEGORY TWO UNDER FORMAT 9

Predictor Variable	$\hat{\beta}_2$	x	$\bar{X}_i$	=	$\hat{\beta}_2\bar{X}_i$
Tot Rev 74	-.00026		17184.37		-4.46794
Pry Tax 74	-.00056		5880.39		-3.29302
SS Rev 74	-.00793		2475.73		-19.63254
Tot Rev 73	.00136		14551.76		19.79039
Pry Tax 73	-.00175		5678.46		-9.93731
SS Rev 73	.00897		1772.12		15.89592
Tot Rev 72	-.00050		14852.80		-7.42640
Pry Tax 72	.00286		5352.34		15.30769
SS Rev 72	-.00569				-6.39198

The approach is now applied to Category Three:

TABLE 30

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES OF CATEGORY THREE UNDER FORMAT 9

Predictor Variable	$\hat{\beta}_3$	x	$\bar{X}_i$	=	$\hat{\beta}_3\bar{X}_i$
Tot Rev 74	-.00051		7518.78		-3.83458
Pry Tax 74	-.00009		4067.56		-0.36608
SS Rev 74	-.00420		1032.22		-4.33532
Tot Rev 73	.00224		6852.67		15.35000
Pry Tax 73	-.00116		3781.11		-4.38609
SS Rev 73	.00550		928.56		5.10708
Tot Rev 72	-.00105		5461.78		-5.70337
Pry Tax 72	.00083		2659.00		2.20697
SS Rev 72	-.00601		774.78		-4.65643

The combination of these two rankins in Table 31 will assist in formulation of general statements:

TABLE 31

RANKING OF VARIABLES UNDER FORMAT 9

	$\hat{\underset{\sim}{\beta}}_2 \bar{X}_i$	$\hat{\underset{\sim}{\beta}}_3 \bar{X}_i$	Average
Tot Rev 74	2	3	2.5
Pry Tax 74	1	1	1.0
SS Rev 74	8	4	6.0
Tot Rev 73	9	9	9.0
Pry Tax 73	5	5	5.0
SS Rev 73	7	7	7.0
Tot Rev 72	4	8	6.0
Pry Tax 72	6	2	4.0
SS Rev 72	3	6	4.5

Because of the smaller group, the averages will not have as great a visual impact. However, as might be anticipated, those variables of a wider ranking will become less influential in the classification. The comparison for the four-year period is achieved through the combining of formats 8 and 9 in Table 32.

TABLE 32

AVERAGE OF RANKINGS OF VARIABLES
UNDER FORMATS 8 AND 9

	Format 9 Average	Format 8 Average
Tot Rev 74	2.5	-
Pry Tax 74	1.0	-
SS Rev 74	6.	-
Tot Rev 73	9.0	4.0
Pry Tax 73	5.0	2.5
SS Rev 73	7.0	8.0
Tot Rev 72	6.0	8.5
Pry Tax 72	4.0	1.5
SS Rev 72	4.5	5.0
Tot Rev 71	-	5.0
Pry Tax 71	-	3.0
SS Rev 71	-	7.5

If the variables are partitioned into years, it may be noted that the Property Tax is consistently the lowest in each year, thus contributing the most to the proper classification of observations. One interpretation of this prominence of Property Tax Revenue might be that the Total Revenues, on a comparative basis, are not as important for estimation as are the sources of those revenues, particularly the amount generated by property tax.

One difficulty in perfecting the analysis to any considerable degree is found in the lack of information in the financial reports as to whether tax revenues are from residential property, or non-residential property and their proportion to the total base value of the property in the municipality. Trends in the relative amounts contributed by each category of property might also potentially improve the analysis. Finally, it should be noted that although the previous analysis dealt with expenditures which were discretionary to a degree, revenues are much less controllable by the city management.

Formats 1 and 2 . . .

The first two runs investigated the classificatory ability of the Total General Fund and the Assets, Cash, Past Due Taxes Receivable and the receivable, Interfund Borrowings Due from (other funds). Format 1 utilized twelve variables:

Total General Fund	1974, 1973, 1972
Cash	1974, 1973, 1972
Past Due Taxes Receivable	1974, 1973, 1972
Interfund Borrowing Due from	1974, 1973, 1972

The program assigned the municipalities correctly in 76.7 percent of the cases:

TABLE 33

CLASSIFICATIONS FROM FORMAT 1

		Classification as predicted from the fitted model		
		Poor	Sat.	Rich
ACTUAL CLASSIFICATION	Poor	4	6	0
	Sat.	1	39	1
	Rich	0	6	3

These results are expressed in fractional from:

4/10, 30/41, 3/9 : 46/60

The weakness in tail classification appears moderate. Estimations of probabilities are shown in Table 34 so that the reader may observe the classification strength.

TABLE 34

ESTIMATED PROBABILITIES OF CLASSIFICATION UNDER FORMAT 1

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
		First Category	
1	.1444	.7868	.0688
2	.0413	.8115	.1472
3	.0190	.6759	.3051
4	.9992	.0007	.0000
5	.9944	.0056	.0000
6	.0716	.8484	.0800
7	.0519	.8319	.1162
8	.8535	.1390	.0075
9	.1914	.8071	.0015
10	.5390	.4595	.0015
		Second Category	
11	.0192	.8544	.1264
12	.1098	.7618	.1284
13	.4483	.4051	.1466
14	.0441	.9496	.0063
15	.0288	.8990	.0722
16	.0356	.8736	.0908
17	.0704	.8485	.0811
18	.1457	.8304	.0239
19	.0480	.9128	.0392
20	.0133	.9759	.0108
21	.0143	.9330	.0527
22	.0075	.9653	.0272
23	.1372	.7676	.0952

TABLE 34--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
24	.1054	.8615	.0331
25	.0443	.8907	.0650
26	.0362	.8489	.1149
27	.0276	.9330	.0394
28	.0502	.8980	.0519
29	.1053	.8452	.0494
30	.1191	.8505	.0304
31	.0265	.9727	.0008
32	.0726	.8788	.0486
33	.0731	.9198	.0071
34	.0021	.7274	.2705
35	.0501	.9269	.0230
36	.0848	.8110	.1042
37	.1083	.8777	.0140
38	.0166	.9758	.0076
39	.0298	.7488	.2215
40	.0103	.9738	.0160
41	.0286	.8714	.1000
42	.0252	.8026	.1722
43	.2377	.7410	.0213
44	.0095	.4342	.5564
45	.0729	.9024	.0247
46	.0425	.9175	.0401
47	.0272	.8811	.0917
48	.0521	.7674	.1805
49	.0802	.8256	.0942
50	.1531	.8235	.0235
51	.3287	.6073	.0640
		Third Category	
52	.0001	.0046	.9953

TABLE 34--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
53	.0463	.8818	.0719
54	.0362	.8854	.0784
55	.0733	.7982	.1285
56	.0904	.8217	.0880
57	.0002	.0219	.9779
58	.0571	.7680	.1749
59	.0005	.1595	.8399
60	.0300	.8292	.1412

Of those sixty classifications, three were sufficiently narrow to warrant citation:

City 10: $\hat{P}_1 = .5390$, $\hat{P}_2 = .4595$
City 13: $\hat{P}_1 = .4483$, $\hat{P}_2 = .4051$
City 44: $\hat{P}_2 = .4342$, $\hat{P}_3 = .5564$

Of these three, City 13 is most likely the only one for which meaningful classification could be difficult, or questionable.

The vectors were estimated and are shown in the following table.

TABLE 35

ESTIMATION OF VECTORS FOR β_2 AND β_3
UNDER FORMAT 1

$\hat{\underset{\sim}{\beta}}$	Variables	$\hat{\underset{\sim}{\beta}}$
-.00078		-.00208
.00866		.02205
.00135		.00423
-.12096		.00462
-.01155		-.02665
-.00048		.00219
.25494		.16672
-.00246		-.00287
-.00290		-.00100
-.00367		-.00470
-.12044		-.06293

The relative weight of the asset predictor variables for format 1 can be calculated in this manner:

TABLE 36

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES OF CATEGORY TWO UNDER FORMAT 1

Predictor Variable	$\hat{\beta}_2$	x	$\bar{X}_i$	=	$\hat{\beta}_2\bar{X}_i$
Tot GF 74	-.00078		1537.95		-1.19960
Cash 74	.00866		389.61		3.37402
PD Tax 74	.00135		909.12		1.22731
IFBDF 74	-.12096		46.07		-5.57262
Tot GF 73	.00310		1104.49		3.42392
Cash 73	-.01155		378.59		-4.37271
PD Tax 73	-.00048		770.17		-0.36968
IFBDF 73	.25494		45.61		11.62781
Tot GF 72	-.00246		806.44		-1.98384
Cash 72	.00290		331.56		0.96152
PD Tax 72	-.00367		415.32		-1.52422
IFBDF 72	-.12044		33.88		-4.08051

Similar computations are now made for the third category.

TABLE 37

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES
OF CATEGORY THREE UNDER FORMAT 1

Predictor Variable	$\hat{\beta}_3$	x	$\bar{X}_i$	=	$\hat{\beta}_3\bar{X}_i$
Tot GF 74	-.00208		2261.67		4.70427
Cash 74	.02205		117.56		2.59220
PD Tax 74	.00423		102.33		0.43286
IFBDF 74	-.00351		30.78		0.10804
Tot GF 73	.00462		2207.67		10.19944
Cash 73	-.02665		103.00		2.74495
PD Tax 73	.00219		113.78		0.24918
IFBDF 73	.16672		26.22		4.37140
Tot GF 72	-.00287		1794.89		5.15133
Cash 72	-.00100		115.44		0.11544
PD Tax 72	-.00470		114.33		0.53735
IFBDF 72	-.06293		21.78		1.37062

These rankings result in Table 38 which presents the variables listed in order of most important through least important.

TABLE 38

RANKING OF $\hat{\underset{\sim}{\beta}}\bar{X}_i$ UNDER FORMAT 1

$\hat{\underset{\sim}{\beta}}_2\bar{X}_i$	$\hat{\underset{\sim}{\beta}}_3\bar{X}_i$
PD Tax 73	IFBDF 74
Cash 72	Cash 72
Tot Gf 74	PD Tax 73
PD Tax 74	PD Tax 74
PD Tax 72	PD Tax 72
Tot GF 72	IFBDF 72
Cash 74	Cash 74
Tot GF 73	Cash 73
IFBDF 72	IFBDF 73
Cash 73	Tot GF 74
IFBDF 74	Tot GF 72
IFBDF 73	Tot GF 73

This array can be presented as in previous viewings. Table 39 shows the ordering and averages of that ordering.

TABLE 39

RANKING OF VARIABLES UNDER FORMAT 1

	$\underset{\sim}{\hat{\beta}}_2\bar{X}_i$	$\underset{\sim}{\hat{\beta}}_3\bar{X}_i$	Average
Tot GF 73	3	10	6.5
Cash	7	7	7.0
PD Tax 74	4	4	4.0
IFBDF 74	11	1	6.0
Tot GF 73	8	12	10.0
Cash 73	10	8	9.0
PD Tax 73	1	3	2.0
IFBDF 73	12	9	10.5
Tot GF 72	6	11	8.5
Cash 72	2	2	2.0
PD Tax 72	5	5	5.0
IFBDF 72	9	6	7.5

The consistently high ranking of the three Past Due Taxes Receivable as a classifying variable is in accord with perceptions prior to the analysis. It was assumed that the amount of taxes which were past due would provide a good predictor for intuitively there should be pronounced differences. Those residents in the cash rich cities will be less subject to the situations in which taxes will become past due.

The introduction of format 2 finds the time lag missing. Table 40 shows the classifications:

TABLE 40

CLASSIFICATIONS FROM FORMAT 2

ACTUAL CLASSIFICATION	Classification as predicted from the fitted model: Poor	Sat.	Rich
Poor	3	7	0
Sat.	0	39	3
Rich	0	7	2

The fractions then are:

3/10, 39/41, 2/9 : 44/60

which results in a "hit" ratio of 73.3 percent.

The probabilities comprising those fractions are:

TABLE 41

ESTIMATED PROBABILITIES OF CLASSIFICATION UNDER FORMAT 2

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
		First Category	
1	.1214	.8275	.0511
2	.2518	.7278	.0004
3	.2832	.3827	.3341
4	.9864	.0136	.0001
5	.9669	.0326	.0004
6	.0543	.8006	.1451
7	.0710	.8685	.0605
8	.5351	.4610	.0031
9	.3874	.6116	.0010
10	.1625	.8321	.0054

TABLE 41--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
		Second Category	
11	.0442	.9061	.0498
12	.9757	.8021	.1222
13	.0446	.9214	.0340
14	.4254	.5499	.0247
15	.9787	.8577	.0636
16	.9551	.8805	.0644
17	.0652	.6563	.2786
18	.1155	.8202	.0643
19	.0785	.8751	.0464
20	.0282	.9680	.0112
21	.0458	.9173	.0369
22	.1965	.7857	.0178
23	.1384	.7647	.0970
24	.0712	.8876	.0412
25	.0606	.8767	.0628
26	.0565	.8826	.0609
27	.0613	.9130	.0258
28	.0728	.8338	.0934
29	.2695	.7183	.0122
30	.1061	.8804	.0135
31	.1953	.7972	.0075
32	.0854	.8167	.0979
33	.1339	.8494	.0167
34	.0153	.5332	.4514
35	.1130	.8668	.0202
36	.0997	.8900	.0103
37	.0959	.8827	.0214
38	.1716	.8173	.0111
39	.3766	.4202	.5421
40	.0803	.8501	.0696

TABLE 41--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
41	.0419	.9147	.0434
42	.0431	.8879	.0690
43	.2974	.6920	.0106
44	.0010	.4513	.5477
45	.4370	.4529	.1100
46	.0604	.9212	.0184
47	.0614	.8178	.1209
48	.0435	.7746	.1819
49	.0698	.8374	.0928
50	.4205	.5713	.0082
51	.2013	.7110	.0877
		Third Category	
52	.0001	.0023	.9977
53	.0471	.8616	.0913
54	.0479	.8732	.0788
55	.0159	.6250	.3591
56	.0569	.7973	.1458
57	.0004	.0762	.9233
58	.0399	.7553	.2049
59	.0697	.6619	.2684
60	.0433	.7899	.1668

Review of Table 41 indicates predominately precise classification excepting for the city numbered 3 which is spread over the three categories:

$$\hat{P}_1 = .2832, \quad \hat{P}_2 = .3827, \quad \hat{P}_3 = .3341$$

In addition the city numbered 45 did not have precise classification:

$$\hat{P}_1 = .4370, \qquad \hat{P}_2 = .4529$$

The presentation of the $\hat{\underset{\sim}{\beta}}$'s is made below:

TABLE 42

ESTIMATION OF VECTORS FOR β_2 AND β_3 UNDER FORMAT 2

$\hat{\underset{\sim}{\beta}}_2$	Variables	$\hat{\underset{\sim}{\beta}}_3$
.00197		.00170
.00043		.00780
.00103		.00019
-.07085		-.20574
-.00268		.00060
-.00981		-.02197
-.00530		-.00661
.13378		.11832
.00003		-.00205
.00746		.00205
-.00018		-.00400
.01224		.32215

As in prior analyses, the relative weights are calculated:

TABLE 43

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES OF CATEGORY TWO UNDER FORMAT 2

Predictor Variables	$\hat{\beta}_2$	x	$\bar{X}_i$	=	$\hat{\beta}_2\bar{X}_i$
Tot GF 73	.00197		1104.49		2.17584
Cash 73	.00043		378.59		1.62794
PD Tax 73	.00103		770.17		0.79328
IFBDF 73	-.07085		45.61		-3.23147
Tot GF	-.00268		806.44		-2.16126
Cash 72	-.00981		331.56		-3.25260
PD Tax	-.00530		415.32		-2.20120
IFBDF 72	.13378		33.88		4.53247
Tot GF 71	.00003		591.80		0.01775
Cash 71	.00746		296.41		2.21122
PD Tax 71	-.00018		290.85		-0.05235
IFBDF 71	.01224		32.93		0.40306

Relative weights are shown for Category Three in Table 44:

TABLE 44

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES
OF CATEGORY THREE UNDER FORMAT 2

Predictor Variables	$\hat{\underset{\sim}{\beta}}_3\bar{X}_i$	x	$\bar{X}_i$	=	$\hat{\underset{\sim}{\beta}}_3\bar{X}_i$
Tot GF 73	.00170		2207.67		3.75303
Cash 73	.00780		103.00		0.80340
PD Tax 73	.00019		113.78		0.02162
IFBDF 73	-.20574		26.22		-5.39450
Tot GF 72	.00060		1794.89		1.07693
Cash 72	-.02197		115.44		-2.53622
PD Tax 72	-.00661		114.33		-0.75572
IFBDF 72	.11832		21.78		2.57701
Tot GF 71	-.00205		1713.67		-3.51302
Cash 71	.00205		138.00		0.28290
PD Tax 71	-.00400		163.78		-0.65512
IFBDF 71	.32215		20.89		6.72971

These computations may be ranked for the following order:

TABLE 45

RANKING OF $\hat{\underset{\sim}{\beta}}\bar{X}_i$ UNDER FORMAT 2

$\hat{\underset{\sim}{\beta}}_2\bar{X}_i$	$\hat{\underset{\sim}{\beta}}_3\bar{X}_i$
Tot GF 71	PD Tax 73
PD Tax 71	Cash 71
IFBDF 71	PD Tax 73
PD Tax 73	PD Tax 72
Cash 73	Cash 73
Tot GF 72	Tot GF 72
Tot GF 73	Cash 72
PD Tax 72	IFBDF 72
Cash 71	Tot GF 71
IFBDF 73	Tot GF 73
Cash 72	IFBDF 73
IFBDF 72	IFBDF 71

From this ranking the following additional array can be devised as a preliminary to intuitive comments:

TABLE 46

RANKING OF VARIABLES UNDER FORMAT 2

	$\hat{\beta}_2\bar{X}_i$	$\hat{\beta}_3\bar{X}_i$	Average
Tot GF 73	6	10	8.5
Cash 73	5	5	5.0
PD Tax	4	1	2.5
IFBDF 73	10	11	10.5
Tot GF	6	6	6.0
Cash 72	11	7	8.5
PD Tax 72	8	4	6.0
IFBDF 72	12	8	10.0
Tot GF 71	1	9	5.0
Cash 71	9	2	5.5
PD Tax 71	2	3	2.5
IFBDF 71	3	12	7.5

From this listing two variables appear to have consistent strength, Past Due Taxes Receivable 1973 and 1971. Table 39 shows the 1973 Past Due Taxes Receivable of a like consistent strength, but comparable in average to Cash 72.

Formats 16 and 17 . . .

The final runs to be presented are an inquiry into the utilization of ratios in the classification of municipalities. Ratios were drawn from combinations of elements from both statements. For format 16 three ratios spanned two years while the other two covered four years; this total of fourteen was one less than the capacity of the model. The variables utilized were:

Cash/Total Liabilities	1974, 1973
Cash/Total Revenues	1974, 1973, 1972, 1971
Total Liabilities/Total General Fund	1974, 1973
Interfund Borrowing Due From/Total General Fund	1974, 1973, 1972, 1971
Interfund Borrowing Due To/Total General Fund	1974, 1973

The ratios utilized numerator and denominator drawn from the year's data indicated in the caption. For example, the first ratio was Cash at the end of 1974 divided by total liabilities at the end of 1974. The other variables in order were Cash and Total Revenue, Total Liabilities as a percentage of the Total General Fund, Interfund Borrowing due from the other funds as a percentage of the Total General Fund and Interfund Borrowing (by the General Fund) due to other funds as a percentage of the Total General Fund. Selection of these variables was based upon results of runs preceding format 16.

The analysis resulted in the correct classification of 49 municipalities:

TABLE 47

CLASSIFICATIONS FROM FORMAT 16

		Classification as predicted from the fitted model		
		Poor	Sat.	Rich
ACTUAL CLASSIFICATION	Poor	6	4	0
	Sat.	0	38	3
	Rich	0	4	5

The hit ratios derived from this table are:

6/10, 38/41, 5/9 : 49/60

The probability estimation from application of these ratios.

TABLE 48

ESTIMATED PROBABILITIES OF CLASSIFICATION
UNDER FORMAT 10

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
		First Category	
1	.8579	.1420	.0001
2	.9236	.0757	.0007
3	.3479	.4412	.2109
4	.9954	.0044	.0000
5	.6138	.3776	.0086
6	.2466	.7376	.0157
7	.0607	.9361	.0032
8	.9875	.0125	.0000
9	.9837	.0163	.0000
10	.2916	.7003	.0080
		Second Category	
11	.0079	.9715	.0207
12	.0053	.7587	.2360
13	.1480	.8405	.0115
14	.0008	.9259	.0733
15	.0324	.8762	.0914
16	.0045	.9001	.0953
17	.0110	.8880	.1010
18	.3148	.6849	.0003
19	.3639	.6297	.0064
20	.0924	.8982	.0093
21	.0002	.9956	.0042
22	.0278	.9694	.0028
23	.0744	.8623	.0633
24	.0749	.9079	.0172
25	.0836	.8875	.0288

TABLE 48--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
26	.2246	.7332	.0421
27	.0001	.9969	.0030
28	.0023	.9849	.0127
29	.0373	.9509	.0118
30	.1388	.8562	.0050
31	.0007	.9991	.0001
32	.3154	.6777	.0069
33	.0060	.9930	.0010
34	.0001	.1905	.8094
35	.1492	.8325	.0184
36	.0256	.9708	.0036
37	.0035	.9725	.0239
38	.0006	.9975	.0019
39	.0027	.4884	.5090
40	.0030	.8914	.1056
41	.0275	.9295	.0430
42	.0814	.7664	.2254
43	.0071	.6773	.3157
44	.0002	.6703	.3295
45	.1462	.8475	.0062
46	.0324	.8553	.1123
47	.2721	.7243	.0037
48	.0209	.7033	.2759
49	.1776	.8220	.0004
50	.1304	.7599	.1097
51	.0002	.0238	.9759
		Third Category	
52	.0039	.6406	.3554
53	.0287	.9302	.0411
54	.0294	.4965	.5005

TABLE 48--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
55	.0014	.4034	.5953
56	.0016	.5876	.4108
57	.0000	.0060	.9940
58	.0692	.2047	.7261
59	.0001	.0504	.9496
60	.0020	.5143	.4837

Of these cities, three classifications were with less than clear definition.

City Number	$\hat{P}_2$	$\hat{P}_3$
39	.4884	.5090
54	.4965	.5005
60	.5143	.4837

Of these three, one (60) was incorrectly classified by the model. Classification was always into the next adjacent class if wrong.

Vectors were estimated as follows:

TABLE 49

ESTIMATION OF VECTORS FOR β_2 AND β_3
UNDER FORMAT 16

$\underset{\sim}{\beta}_2$	Variables	$\underset{\sim}{\beta}_3$
.35843		.43800
.15805		.80548
2.62740		2.99873
19.94017		18.76468
-12.77597		19.65129
-3.81666		3.53225
3.48423		4.31339
-3.82480		-5.93604
4.25447		-1.51457
5.23904		9.20041
-3.85439		-9.21081
-4.53639		-5.31218
-12.08583		-9.76234
.52254		2.74759

These vectors, when multiplied against the means of Category Two and Three give the relative weight of the variables.

TABLE 50

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES OF CATEGORY TWO UNDER FORMAT 16

Predictor Variables	$\hat{\beta}_2$	x	$\bar{X}_i$	=	$\hat{\beta}_2\bar{X}_i$
Cash/Tot Liab 74	.35843		1.79		.64159
Cash/Tot Liab 73	.15805		1.93		.30504
Cash/Tot Rev 74	2.62740		.13		.34156
Cash/Tot Rev 73	19.94017		.14		2.79162
Cash/Tot Rev 72	-12.77597		.14		-1.78864
Cash/Tot Rev 71	-3.81666		.13		- .49617
Tot Liab/Tot Gen Fund 74	3.48423		.44		1.53306
Tot Liab/Tot Gen Fund 73	-3.82480		.39		-1.49167
IB from/Tot Gen Fund 74	4.25447		.30		1.27634
IB from/Tot Gen Fund 73	5.23904		.24		1.25737
IB from/Tot Gen Fund 72	-3.85439		.23		-1.15632
IB from/Tot Gen Fund 71	-4.53639		.17		.77119
IB to/Tot Gen Fund 74	-12.08583		.09		1.08773
IB to/Tot Gen Fund 73	.52254		.08		.04180

The computation is then repeated for Category Three.

TABLE 51

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES
OF CATEGORY THREE UNDER FORMAT 16

Predictor Variables	$\underset{\sim}{\hat{\beta}}_3$	x	$\bar{X}_i$	=	$\underset{\sim}{\hat{\beta}}_3 \bar{X}_i$
Cash/Tot Liab 74	.43830		4.35		1.90530
Cash/Tot Liab 73	.80548		4.41		3.55217
Cash/Tot Rev 74	2.99873		.33		.98958
Cash/Tot Rev 73	18.76468		.29		5.44176
Cash/Tot Rev 72	19.65129		.27		5.30584
Cash/Tot Rev 71	3.53225		.26		.91839
Tot Liab/Tot Gen Fund 74	4.31339		.21		.90581
Tot Liab/Tot Gen Fund 73	-5.93604		.25		1.48401
IB from/Tot Gen Fund 74	-1.51457		.02		.03029
IB from/Tot Gen Fund 73	9.20041		.06		.55202
IB from/Tot Gen Fund 72	-9.21081		.04		.36843
IB from/Tot Gen Fund 71	-5.31218		.05		.26561
IB to/Tot Gen Fund 74	-9.76234		.03		.29297
IB to/Tot Gen Fund 73	2.74759		.03		.08243

These weighted variables can then be arrayed to emphasize their ranking:

TABLE 52

RANKING OF $\hat{\underset{\sim}{\beta}}\bar{X}_i$ UNDER FORMAT 16

$\hat{\underset{\sim}{\beta}}_2\bar{X}_i$	$\hat{\underset{\sim}{\beta}}_3\bar{X}_i$
IB to/Tot Gen Fund 73	IB from/Tot Gen Fund 74
Cash/Tot Liab 73	IB to/Tot Gen Fund 73
Cash/Tot Rev 74	IB from/Tot Gen Fund 71
Cash/Tot Rev 71	IB to/Tot Gen Fund 74
Cash/Tot Liab 74	IB from/Tot Gen Fund 72
IB from/Tot Gen Fund 71	IB from/Tot Gen Fund 73
IB to/Tot Gen Fund 74	Tot Liab/Tot Gen Fund 74
IB from/Tot Gen Fund 72	Cash/Tot Rev 71
IB from/Tot Gen Fund 73	Cash/Tot Rev 74
IB from/Tot Gen Fund 74	Tot Liab/Tot Gen Fund 73
Tot Liab/Tot Gen Fund 73	Cash/Tot Liab 74
Tot Liab/Tot Gen Fund 74	Cash/Tot Liab 73
Cash/Tot Rev 72	Cash/Tot Rev 72
Cash/Tot Rev 73	Cash/Tot Rev 73

TABLE 53

RANKING OF VARIABLES UNDER FORMAT 16

	$\hat{\beta}_2 \bar{X}_i$	$\hat{\beta}_3 \bar{X}_i$	Average
Cash/Tot Liab 74	5	11	8.0
Cash/Tot Liab 73	2	12	7.0
Cash/Tot Rev 74	3	9	6.0
Cash/Tot Rev 73	14	14	14.0
Cash/Tot Rev 72	13	13	13.0
Cash/Tot Rev 71	4	8	6.0
Tot Liab/Tot Gen Fund 74	12	7	8.5
Tot Liab/Tot Gen Fund 73	11	10	10.5
IB from/Tot Gen Fund 74	10	1	5.5
IB from/Tot Gen Fund 73	9	6	7.5
IB from/Tot Gen Fund 72	8	5	6.5
IB from/Tot Gen Fund 71	6	3	4.5
IB to/Tot Gen Fund 74	7	4	5.5
IB to/Tot Gen Fund 73	1	2	1.5

Here, no clear pattern may be discerned. Those predictor variables of a low ranking for Category Two ranked high for Category Three, and vice versa. In part, the inability to critically evaluate mixed results may be due to the lack of a rule of thumb or test of significance for this methodology applied to this kind of data at this point. Therefore, we look to our final run of ratios, format 17.

Format 17 was a somewhat intuitive combination of ratios, based like format 16 on prior analyses. The unique characteristic of this analysis is that although it evidenced a slight decrease in total classificatory ability, it displayed a significant increase the lower class classification:

8/10, 35/41, 5/9 : 48/60

for a "hit" ratio of 80%. The fourteen predictor variables utilized in format 17 were:

Cash/Tot Liab	1974,	1973		
Cash/Tot Rev	1974,	1973,	1972,	1971
Tot Liab/Tot Gen Fund	1974,	1973,	1972,	1971
IF from/Tot Gen Fund	1974,	1973,	1972,	1971

The classification results are shown in

TABLE 54

CLASSIFICATIONS FROM FORMAT 17

ACTUAL CLASSIFICATION	Classification as predicted from the fitted model		
	Poor	Sat.	Rich
Poor	8	2	0
Sat.	3	35	3
Rich	0	4	5

For an explanation of the ratios and caption see: Supra, p. 134.

TABLE 55

ESTIMATED PROBABILITIES OF CLASSIFICATION
UNDER FORMAT 17

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
		First Category	
1	.9907	.0093	.0000
2	.1782	.8191	.0027
3	.4303	.3439	.2258
4	.9950	.0050	.0000
5	.8981	.0980	.0039
6	.5819	.4017	.0164
7	.8982	.1006	.0012
8	.9244	.0754	.0002
9	.2546	.7451	.0003
10	.7728	.2232	.0040
		Second Category	
11	.0204	.9508	.2887
12	.0070	.7550	.2379
13	.0722	.9094	.0184
14	.0004	.9224	.0772
15	.0293	.9023	.0685
16	.0650	.7822	.1527
17	.0131	.8706	.1163
18	.5733	.4264	.0003
19	.1708	.8238	.0054
20	.4627	.5279	.0094
21	.0016	.9898	.0085
22	.0140	.9842	.0018
23	.0323	.8968	.0690
24	.1094	.8718	.0189
25	.0394	.9278	.0329
26	.0822	.8892	.0286
27	.0006	.9934	.0060

TABLE 55--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
28	.0002	.9915	.0082
29	.0279	.9569	.0152
30	.2161	.7793	.0046
31	.0007	.9992	.0001
32	.0009	.9974	.0017
33	.0924	.9892	.0015
34	.0000	.2588	.7412
35	.0729	.9161	.0110
36	.6270	.3683	.0047
37	.0025	.9700	.0275
38	.0067	.9887	.0046
39	.0112	.3934	.5953
40	.0077	.8609	.1315
41	.0175	.9237	.0588
42	.0028	.8387	.1585
43	.0019	.6953	.3028
44	.0006	.5375	.4619
45	.0002	.9984	.0014
46	.0002	.9710	.0288
47	.1617	.8345	.0038
48	.0316	.6824	.2861
49	.9003	.0995	.0002
50	.0036	.9512	.0452
51	.0002	.0256	.9743
		Third Category	
52	.0030	.6335	.3634
53	.0369	.9210	.0421
54	.0059	.4085	.5856
55	.0045	.2815	.7140
56	.0016	.5755	.4228

TABLE 55--Continued

City Number	$\hat{P}_1$	$\hat{P}_2$	$\hat{P}_3$
57	.0000	.0059	.9941
58	.0123	.3900	.5977
59	.0000	.0543	.9457
60	.0012	.5365	.4623

Vectors were then estimated for β_2 and β_3.

TABLE 56

ESTIMATIONS OF VECTORS FOR β_2 AND β_3 UNDER FORMAT 17

$\hat{\underset{\sim}{\beta}}_2$	Variables	$\hat{\underset{\sim}{\beta}}_3$
.41779		.49773
.32045		.90799
-6.64581		-.37541
28.17662		24.53208
-13.338990		-18.79221
-5.82788		1.78895
-1.32388		.43239
4.98293		4.53155
-12.87480		-12.72850
6.41366		4.42183
.50460		-4.22338
7.59569		10.67230
-1.30348		-6.59703
-3.07429		-4.08173

The relative weights are attained by the multiplication of the estimated vector by the mean of the variable:

TABLE 57

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES OF CATEGORY TWO UNDER FORMAT 17

Predictor Variable	$\hat{\beta}_{\sim 2}$	x	$\bar{X}_i$	=	$\hat{\beta}_{\sim 2}\bar{X}_i$
Cash/Tot Liab 74	.41779		1.79		.07478
Cash/Tot Liab 73	.32045		1.93		.61847
Cash/Tot Rev 74	-6.64581		.13		-.86395
Cash/Tot Rev 73	28.17662		.14		3.94473
Cash/Tot Rev 72	-13.33890		.14		-1.86745
Cash/Tot Rev 71	-5.82788		.13		-.75762
Tot Liab/Tot Gen Fund 74	-1.32388		.44		.58251
Tot Liab/Tot Gen Fund 73	4.98293		.36		1.79385
Tot Liab/Tot Gen Fund 72	-12.87480		.39		-5.02117
Tot Liab/Tot Gen Fund 71	6.41366		.49		3.14269
IB from/Tot Gen Fund 74	.50460		.30		.15138
IB from/Tot Gen Fund 73	7.59569		.24		1.82297
IB from/Tot Gen Fund 72	-1.30348		.23		-.29980
IB from/Tot Gen Fund 71	-3.07429		.17		-.52263

This algorithm is then performed for Category 3:

TABLE 58

COMPUTATION OF RELATIVE WEIGHTS OF PREDICTOR VARIABLES OF CATEGORY THREE UNDER FORMAT 17

Predictor Variable	$\hat{\beta}_2$	x	$\bar{X}_i$	=	$\hat{\beta}_2\bar{X}_i$
Cash/Tot Liab 74	.39773		4.35		2.16531
Cash/Tot Liab 73	.90799		4.41		4.00424
Cash/Tot Rev 74	-.37541		.33		-.01239
Cash/Tot Rev 73	24.53208		.29		4.90642
Cash/Tot Rev 72	-18.79221		.27		-5.07390
Cash/Tot Rev 71	1.78895		.26		.46513
Tot Liab/Tot Gen Fund 74	.43239		.21		.09080
Tot Liab/Tot Gen Fund 73	4.53155		.19		.86099
Tot Liab/Tot Gen Fund 72	-12.72850		.25		-3.18213
Tot Liab/Tot Gen Fund 71	4.42183		.25		1.10546
IB from/Tot Gen Fund 74	-4.22338		.02		-.08447
IB from/Tot Gen Fund 73	10.67230		.06		.64034
IB from/Tot Gen Fund 72	-6.59703		.04		-.26239
IB from/Tot Gen Fund 71	-4.08173		.05		-.20409

These predictor variables were then ranked:

TABLE 59

RANKING OF $\underset{\sim}{\hat{\beta}}\bar{X}_i$ UNDER FORMAT 17

$\underset{\sim}{\hat{\beta}}_2\bar{X}_i$	$\underset{\sim}{\hat{\beta}}_3\bar{X}_i$
Cash/Tot Liab 74	Cash/Tot Rev 74
IB from/Tot Gen Fund 74	IB from/Tot Gen Fund 74
IB from/Tot Gen Fund 72	Tot Liab/Tot Gen Fund 74
IB from/Tot Gen Fund 71	IB from/Tot Gen Fund 71
Tot Liab/Tot Gen Fund 74	Cash/Tot Liab 74
Cash/Tot Liab 73	IB from/Tot Gen Fund 72
Cash/Tot Rev 71	Cash/Tot Rev 71
Cash/Tot Rev 74	IB from/Tot Gen Fund 73
Tot Liab/Tot Gen Fund 73	Cash/Tot Rev 71
IB from/Tot Gen Fund 73	Tot Liab/Tot Gen Fund 73
Cash/Tot Rev 72	Tot Liab/Tot Gen Fund 72
Tot Liab/Tot Gen Fund 71	Cash/Tot Liab 73
Cash/Tot Rev 73	Cash/Tot Rev 73
Tot Liab/Tot Gen Fund 14	Cash/Tot Rev 72

By applying ordinal ranking we can appraise the variables.

TABLE 60

RANKING OF VARIABLES UNDER FORMAT 17

	$\underset{\sim}{\hat{\beta}}_2\bar{X}_i$	$\underset{\sim}{\hat{\beta}}_3\bar{X}_i$	Average
Cash/Tot Liab 74	1	5	3.0
Cash/Tot Liab 73	6	12	9.0
Cash/Tot Rev 74	8	1	4.5
Cash/Tot Rev 73	13	13	13.0
Cash/Tot Rev 72	11	14	12.5
Cash/Tot Rev 71	7	7	7.0
Tot Liab/Tot Gen Fund 74	5	3	4.0
Tot Liab/Tot Gen Fund 73	9	9	9.0
Tot Liab/Tot Gen Fund 72	14	11	12.5
Tot Liab/Tot Gen Fund 71	12	10	11.0
IB from/Tot Gen Fund 74	2	2	2.0
IB from/Tot Gen Fund 73	10	8	9.0
IB from/Tot Gen Fund 72	3	6	4.5
IB from/Tot Gen Fund 71	4	4	4.0

In contrast to the immediately preceding analysis, the ratios of format 17 display consistency in certain aspects, particularly the ratios of Interfund Borrowing due from Other Funds/Total General Fund. The relatively low and consistent ranking possibly holds this ratio forth in this combination as one of the more discerning characteristics of municipalities. Furthermore, the deletion of the ratio for Interfund Borrowing Due to/Total General Funds resulted in significant improvement in classification of municipalities with low levels of cash. The conclusion may also be drawn in regard to ratio analysis that it resulted in less correct classifications overall than other formats but greater strength in correctly classifying the "Cash Poor" municipalities.

Common dollar statements . . .

Another of the objectives of this effort was to prepare financial statements representative of the "Cash Poor" and "Cash Rich" municipalities. These statements were prepared by averaging raw dollar balances of accounts for both categories of municipalities:

TABLE 61

COMMON DOLLAR BALANCE SHEETS OF MUNICIPALITIES

	Cash Poor	Cash Rich
	%	%
Assets		
Cash	3.1	86.1
Past Due Taxes Receivable	9.6	2.7
Interfund Borrowing Due from	41.1	2.7
Other Assets	46.2	8.5
Total Assets	100.0	100.0
Liabilities and Fund Balance		
Interfund Borrowing Due to	26.7	6.6
Other Current Liabilities	36.2	14.8
Total Current Liabilities	62.9	21.4
Fund Balances		
Reserves	14.9	28.5
Unappropriated Fund Balance	22.2	50.1
	100.0	100.0

The Operating Statement for the Cash Poor and Cash Rich municipalities is presented next as Table 62.

TABLE 62

COMMON DOLLAR OPERATING STATEMENT OF MUNICIPALITIES
1975

	Cash Poor	Cash Rich
	%	%
Revenues		
Property Tax Income	47.7	49.7
Personal Income Tax Income	3.2	3.4
State Shared Revenues	22.5	20.7
Other Revenues	26.6	26.2
Total Revenues	100.0	100.0
Expenditures (as a % of Total Revenues)		
Administration	19.9	19.4
Police	26.4	24.0
Fire	16.3	8.8
Parks & Recreation	6.0	7.2
Other	30.7	35.9
Total Expenditures	99.3	95.3

Summary and conclusions . . .

In summary, the primary objective of this research effort was to make an inquiry into the development of a predictor of future municipal financial insolvency and to attempt to set forth differences in those cities which display indications of financial insolvency and those cities which do not. The predictor was based upon information that is readily available without great cost, accounting information taken from the uniform financial reports which are filed annually by the municipalities with the appropriate regulatory agencies of the State of Michigan.

The reader should be aware of particular factors underlying those conclusions and his inferences from those conclusions should be so savored. First, the cities investigated were in the State of Michigan. Consequently, these cities are affected by laws and conditions peculiar to that state alone. To the extent that other states have similar laws and conditions, inferences might be drawn, if the reader so desires. Second, the accounting information was taken from financial statements prepared under the uniform accounting procedures prescribed in Public Act 2, 1968. Reporting requirements likewise are prescribed. To the extent that accounting and reporting practices of other states do not differ significantly, the statements of those municipalities might be considered similar and inferences could be drawn to the degree desired.

Finally, it was decided at the beginning of the study to not make use of validation techniques such as a holdout group or a random iterative process such as a "Midas Two-Way" test. Opinions expressed by Dr. McSweeney were that the group of sixty units was too small to partition into sample body and holdout group. Validation would have been (and will be) best achieved by extension of the study to other areas and other time frames.

The model utilized a classification technique similar to that of multiple discriminant analysis. Several runs were made utilizing various combinations of predictor variables to classify cities into the three groups, Cash Poor, Cash Satisfactory and Cash Rich. The criterion variable that was found to be most effective was the amount of cash of the General Fund at the end of 1975 expressed as a percentage of the total assets of the General Fund at the end of 1975.

The numerous runs resulted in ranges of ability to classify

the cities correctly. In the analysis displaying the most correct classifications, (87.5%), the predictor variables making the greatest contribution to those results were the Administrative Expenditures and the Park and Recreation Expenditures. It was inferred that these discretionary expenses would exhibit the the following trends: that as the Cash Poor city was able to retain less cash it would also find Administrative Expenditures increasing as the city's governing body attempted to serve the changing body of standing citizenry. Furthermore, the Parks and Recreation Expenditures would be contracted in a direct relationship to the amount of cash held.

Three other analyses displayed a significant number of correct classifications and supportive strength in the Cash Poor class. The prominent variables taken from those runs were:

Past Due Taxes Receivable (Formats 1 and 2)
Property Tax Revenues (Format 8)
Interfund Borrowings Due to the General Fund as a percentage of the General Fund Total Assets (Format 17)

A proper extension of the study should involve devlopment of coefficients and expression of them in formula form.

Other extensions of the study would involve investigation of the prominent lag factor. This lag factor was deduced from runs using the same variables over different time periods; those runs of the earlier years gnerally exhibited greater strentgh than the subsequent years. In addition, a greater time span will be possible as additional data becomes available. In these subsequent investigations it is felt that the real value of this study will be realized in that a longer time horizon will permit the state and other interested parties the opportunity for corrective action, if possible.

APPENDIX A

STANDARD & POOR'S CORPORATION

MUNICIPAL DATA BANK QUESTIONNAIRE

Official Government Unit Name ______________________________

Fiscal Year Ends ____________

State ____________

Have any areas been annexed since 1970? If yes -

Population		Source of Data	A.V.	Pop.	Area (Sq. Mi.
1974					
1973					
1972					
1971					
1970					
1969					

Fiscal Year Ending	Assessed Valuation of Taxable Property (Real & Personal) in $	Basis of Assessed Value (% of A.V. to Mkt. Value)
1974		
1973		
1972		
1971		
1970		
1969		

Date of last revaluation ____________
Dollar ($) loss to A.V. due to exemptions (current year) ____________
Is exemption reimbursed by State? ____________

Percent of Property Tax Collected

	Current Levy	Total Collected
1974	%	%
1973	%	%
1972	%	%
1971	%	%
1970	%	%
1969	%	%

Percentage Composition of A.V. (by type of property)
Latest Year ____________

Residential	%
Commercial	%
Industrial	%
Tax-Exempt	%
Other	%

Tax Rate ($1,000 of A.V.)

1974 ____________________
1973 ____________________
1972 ____________________
1971 ____________________
1970 ____________________
1969 ____________________

Tax Rate Limit

Operations Only ________________
Operations & Debt Svc. __________
Debt Svc. Only ________________

Debt Limit ____________________

List other taxing units overlapping your unit: ____________________
____
____
____

Debt Information

	Long-Term G.O. Debt Outstanding at Fiscal Year End	G.O. Debt Supported by Other Than Property Taxes
1974	____________	____________
1973	____________	____________
1972	____________	____________
1971	____________	____________
1970	____________	____________
1969	____________	____________

G.O. debt presently authorized but unissued: ____________________
____
____
____
____

Note any change in State or Federal Air programs affecting your unit: ____
____
____
____
____
____
____
____
____

List the Ten Top Taxpayers

	Name of Taxpayer	Type of Business	Assessed Value	Employment
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				

Municipal Official to Contact:

Name ______________________ **Title** ______________

Address ______________________ ______________

______________________ **Telephone** ______________

______________________ ______________

APPENDIX B

Alphabetical Listing of Cities in the State of Michigan Having Populations Exceeding 10,000 in 1970

Source: U.S. Census, 1970

Adrian	20,382
Albion*	12,112
Allen Park	40,747
Alpena	13,805
Battle Creek	38,931
Bay City	49,449
Benton Harbor	16,481
Berkley	21,878
Big Rapids	11,995
Birmingham	26,170
Burton	32,540
Centerline	10,379
Clawson	17,617
Dearborn	104,199
Dearborn Heights	80,069
Detroit	1,511,482
East Detroit	45,920
East Grand Rapids	12,565
East Lansing	47,540
Ecorse	17,515
Escanaba	15,368
Farmington	10,329
Ferndale	30,850
Flint	193,371
Fraser	11,868
Garden City	41,864
Grand Haven	11,844
Grand Rapids	197,649
Grandville	10,764
Grosse Pointe Farms	11,701
Grosse Pointe Park	15,585
Grosse Pointe Woods	21,878
Hamtramck	27,245
Harper Woods*	20,186
Hazel Park	23,784
Highland Park	35,444
Holland	26,337
Inkster	38,595
Jackson	45,484
Kalamazoo*	85,555

APPENDIX B--Continued

Kentwood	20,310
Lansing	131,546
Lincoln Park	52,984
Livonia*	110,109
Madison Heights	38,599
Marquette	21,967
Melvindale*	13,862
Menominee*	10,748
Midland	35,176
Monroe	23,894
Mt. Clemens	20,476
Mt. Pleasant*	20,504
Muskegon*	44,631
Muskegon Heights	17,304
Niles*	12,988
Norton Shores	22,271
Oak Park	36,762
Owosso	17,179
Plymouth	11,758
Pontiac*	85,279
Portage	33,590
Port Huron	35,794
River Rouge	15,947
Riverview	11,342
Romulus	22,879
Roseville	60,529
Royal Oak	86,238
Saginaw	91,849
St. Clair Shores	88,093
St. Joseph	11,042
Sault Ste. Marie	15,136
Southfield	69,285
Southgate*	33,909
Sterling Heights	61,365
Taylor	70,020
Traverse City	18,048
Trenton	24,127
Troy	39,419
Walker	11,492
Warren	179,260
Wayne	21,054
Westland	86,749
Wyandotte	41,061
Wyoming	56,560
Ypsilanti	29,538

*Has year-end other than June 30; data being collected only for benefit of State Treasury Department.

APPENDIX C

DATA GATHERING FORM

CITY	CITY	YEAR	Aud. Opinion
1 0 1 2 3 4 5 6 7 8 9	2 0 1 2 3 4 5 6 7 8 9	3 0 1 2 3 4 5 6 7 8 9	4 0 1 2 3 4 5 6 7 8 9
Cash	**Unaprop. Fund Bal.**	**Cur. Liabilities**	**Past Due Taxes Rcvble**
5 0 1 2 3 4 5 6 7 8 9	6 0 1 2 3 4 5 6 7 8 9	7 0 1 2 3 4 5 6 7 8 9	8 0 1 2 3 4 5 6 7 8 9
9 0 1 2 3 4 5 6 7 8 9	10 0 1 2 3 4 5 6 7 8 9	11 0 1 2 3 4 5 6 7 8 9	12 0 1 2 3 4 5 6 7 8 9
13 0 1 2 3 4 5 6 7 8 9	14 0 1 2 3 4 5 6 7 8 9	15 0 1 2 3 4 5 6 7 8 9	16 0 1 2 3 4 5 6 7 8 9
17 0 1 2 3 4 5 6 7 8 9	18 0 1 2 3 4 5 6 7 8 9	19 0 1 2 3 4 5 6 7 8 9	20 0 1 2 3 4 5 6 7 8 9
Interfund Bor. D. From	**Interfund Bor. D. To**	**Tot. Gen Fund**	**Total Revenue**
21 0 1 2 3 4 5 6 7 8 9	22 0 1 2 3 4 5 6 7 8 9	23 0 1 2 3 4 5 6 7 8 9	24 0 1 2 3 4 5 6 7 8 9
25 0 1 2 3 4 5 6 7 8 9	26 0 1 2 3 4 5 6 7 8 9	27 0 1 2 3 4 5 6 7 8 9	28 0 1 2 3 4 5 6 7 8 9
29 0 1 2 3 4 5 6 7 8 9	30 0 1 2 3 4 5 6 7 8 9	31 0 1 2 3 4 5 6 7 8 9	32 0 1 2 3 4 5 6 7 8 9
33 0 1 2 3 4 5 6 7 8 9	34 0 1 2 3 4 5 6 7 8 9	35 0 1 2 3 4 5 6 7 8 9	36 0 1 2 3 4 5 6 7 8 9
37 0 1 2 3 4 5 6 7 8 9	38 0 1 2 3 4 5 6 7 8 9	39 0 1 2 3 4 5 6 7 8 9	40 0 1 2 3 4 5 6 7 8 9
41 0 1 2 3 4 5 6 7 8 9	42 0 1 2 3 4 5 6 7 8 9	43 0 1 2 3 4 5 6 7 8 9	44 0 1 2 3 4 5 6 7 8 9
Prpty Tax Income	**Pers. Income Tax Rev**	**Fed Shrd Rev, G.F.**	**State Shared Rev.**
45 0 1 2 3 4 5 6 7 8 9	46 0 1 2 3 4 5 6 7 8 9	47 0 1 2 3 4 5 6 7 8 9	48 0 1 2 3 4 5 6 7 8 9
49 0 1 2 3 4 5 6 7 8 9	50 0 1 2 3 4 5 6 7 8 9	51 0 1 2 3 4 5 6 7 8 9	52 0 1 2 3 4 5 6 7 8 9
53 0 1 2 3 4 5 6 7 8 9	54 0 1 2 3 4 5 6 7 8 9	55 0 1 2 3 4 5 6 7 8 9	56 0 1 2 3 4 5 6 7 8 9
57 0 1 2 3 4 5 6 7 8 9	58 0 1 2 3 4 5 6 7 8 9	59 0 1 2 3 4 5 6 7 8 9	60 0 1 2 3 4 5 6 7 8 9
61 0 1 2 3 4 5 6 7 8 9	62 0 1 2 3 4 5 6 7 8 9	63 0 1 2 3 4 5 6 7 8 9	64 0 1 2 3 4 5 6 7 8 9
Total Expenditures	**Adminstrative Expend**	**Police Expend.**	**Fire Expend.**
65 0 1 2 3 4 5 6 7 8 9	66 0 1 2 3 4 5 6 7 8 9	67 0 1 2 3 4 5 6 7 8 9	68 0 1 2 3 4 5 6 7 8 9
69 0 1 2 3 4 5 6 7 8 9	70 0 1 2 3 4 5 6 7 8 9	71 0 1 2 3 4 5 6 7 8 9	72 0 1 2 3 4 5 6 7 8 9
73 0 1 2 3 4 5 6 7 8 9	74 0 1 2 3 4 5 6 7 8 9	75 0 1 2 3 4 5 6 7 8 9	76 0 1 2 3 4 5 6 7 8 9
77 0 1 2 3 4 5 6 7 8 9	78 0 1 2 3 4 5 6 7 8 9	79 0 1 2 3 4 5 6 7 8 9	80 0 1 2 3 4 5 6 7 8 9
81 0 1 2 3 4 5 6 7 8 9	82 0 1 2 3 4 5 6 7 8 9	83 0 1 2 3 4 5 6 7 8 9	84 0 1 2 3 4 5 6 7 8 9
Parks & Rec. Expend	**Rev.; Fed Rev Sh Fund**	**Expend Capitalized**	**Expend. Other FRSF**
85 0 1 2 3 4 5 6 7 8 9	86 0 1 2 3 4 5 6 7 8 9	87 0 1 2 3 4 5 6 7 8 9	88 0 1 2 3 4 5 6 7 8 9
89 0 1 2 3 4 5 6 7 8 9	90 0 1 2 3 4 5 6 7 8 9	91 0 1 2 3 4 5 6 7 8 9	92 0 1 2 3 4 5 6 7 8 9
93 0 1 2 3 4 5 6 7 8 9	94 0 1 2 3 4 5 6 7 8 9	95 0 1 2 3 4 5 6 7 8 9	96 0 1 2 3 4 5 6 7 8 9
97 0 1 2 3 4 5 6 7 8 9	98 0 1 2 3 4 5 6 7 8 9	99 0 1 2 3 4 5 6 7 8 9	100 0 1 2 3 4 5 6 7 8 9
101 0 1 2 3 4 5 6 7 8 9	102 0 1 2 3 4 5 6 7 8 9	103 0 1 2 3 4 5 6 7 8 9	104 0 1 2 3 4 5 6 7 8 9
Cash, FRSF	**Total Balance FRSF**	**Gen Oblig Bond**	**Unfunded Accrd Pension**
105 0 1 2 3 4 5 6 7 8 9	106 0 1 2 3 4 5 6 7 8 9	107 0 1 2 3 4 5 6 7 8 9	108 0 1 2 3 4 5 6 7 8 9
109 0 1 2 3 4 5 6 7 8 9	110 0 1 2 3 4 5 6 7 8 9	111 0 1 2 3 4 5 6 7 8 9	112 0 1 2 3 4 5 6 7 8 9
113 0 1 2 3 4 5 6 7 8 9	114 0 1 2 3 4 5 6 7 8 9	115 0 1 2 3 4 5 6 7 8 9	116 0 1 2 3 4 5 6 7 8 9
117 0 1 2 3 4 5 6 7 8 9	118 0 1 2 3 4 5 6 7 8 9	119 0 1 2 3 4 5 6 7 8 9	120 0 1 2 3 4 5 6 7 8 9
121 0 1 2 3 4 5 6 7 8 9	122 0 1 2 3 4 5 6 7 8 9	123 0 1 2 3 4 5 6 7 8 9	124 0 1 2 3 4 5 6 7 8 9

BIBLIOGRAPHY

Books

Ashton, Winifred D. The Logit Transformation with Special Reference to its Uses in Bioassay. New York: Hafner Publishing Company, 1972.

Clark, Charles E. and Schadke, Lawrence L. Statistical Analysis for Administrative Decisions. Cincinnati, Ohio: South-Western Publishing Co., 1974.

Cooley, William W. "Techniques for Considering Multiple Measurements," Educational Measurement. Edited by Robert L. Thorndike. 2nd ed. Washington, D.C.: American Council on Education, 1971.

Cox, D. R. Analysis of Binary Data. London: Methuen & Co. Ltd., 1970.

Cullen, Charles G. Matrices and Linear Transformations. 2nd ed. Reading, Mass.: Addison-Wesley Publishing Company, 1972.

DeGroot, Morris H. Optimal Statistical Decisions. New York: McGraw-Hill Book Company, 1970.

Finn, Jeremy D. A General Model for Multivariate Analysis. New York: Holt, Rinehart and Winston, Inc., 1974.

Finney, D. J. Probit Analysis. Cambridge, England: Cambridge University Press, 1947.

Fisher, R. A. "The Goodness of Fit of Regression Formulae and the Distribution of Regression Coefficients," in Contributions to Mathematical Statistics Edited by Walter Shewhart. New York: John Wiley & Sons, Inc., 1950.

________. "On the Interpretation of Chi Square from Contingency Tables and the Calculation of P," in Contributions to Mathematical Statistics. Edited by Walter Shewhart. New York: John Wiley & Sons, Inc. 1950.

________. Statistical Methods and Scientific Inference. New York: Hafner Publishing Company, 1956.

Glass, Gene V. and Stanley, Julian C. Statistical Methods in

Education and Psychology. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1970.

Goldberger, Arthur S. and Duncan, Otis Dudley, ed. Structural Equation and Psychology. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1970.

Harris, Richard J. A Primer of Multivariate Statistics. New York: Academic Press, 1975.

Hughes, Ann and Grawoig, Dennis. Statistics: A Foundation for Analysis. Reading, Mass.: Addison-Wesley Publishing Company, 1971.

Kirk, Roger E. Experimental Design: Procedures for the Behavioral Sciences. Belmont, Cal.: Brooks/Cole Publishing Company, 1968.

Mills, Frederick C. Statistical Methods. 3rd ed. New York: Holt, Rinehart and Winston.

Mood, Alexander J., Graybill, Franklin A. and Boes, Duane C. Introduction to the Theory of Statistics. 3rd ed. New York: McGraw-Hill Book Company, 1974.

Schmitt, Samuel A. Measuring Uncertainty. Reading, Mass.: Addison-Wesley Publishing Company, 1969.

Tatsuoka, Maurice M. Multivariate Analysis. New York: John Wiley & Sons, Inc., 1971.

Theil, Henri. Applied Economic Forecasting. Amsterdam: North-Holland Publishing Company, 1966.

Magazines

Altman, Edward. "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy." Journal of Finance, September, 1968, pp. 589-609.

________. "Railroad Bankruptcy Propensity," Journal of Finance, May, 1971, pp. 333-45.

Armitage, P. Discussion on the Paper by Dr. Buck and Mr. Wicken. "Models for Use in Investigating the Risk of Mortality from Lung Cancer and Bronchitis," Applied Statistics, 1967, 16, pp. 185-210.

Ball, Ray and Brown, Phillip. "An Empirical Evaluation of Accounting Income Numbers," Journal of Accounting Research, Autumn, 1968, pp. 159-78.

Beaver, William H. "Alternative Accounting Measures as Predictors of Failure," The Accounting Review, January, 1968, pp. 113-122.

________. "Market Prices, Financial Ratios, and the Prediction of Failure," Journal of Accounting Research, pp. 179-92.

Buck, S. F. and Wicken, A. J. "Models for Use in Investigating the Risk of Mortality from Lung Cander and Bronchitis," Applied Statistics, pp. 185-203.

Cornfield, Jerome, Gordon, Tavia and Smith, Wille W. "Quantal Response Curves for Experimentally Uncontrolled Variables," Bulletin de L'Institute International de Statistique, 1960, pp. 97-115.

Demski, Joel S. "Predictive Ability of Alternative Performance Measurement Models," Journal of Accounting Research, pp. 96-115.

Durand, D. D. "Risk Elements in Consumer Installment Financing," Studies in Consumer Installment Financing. New York: National Bureau of Economic Research, 1941, pp. 105-42.

Goldberg, Lewis R. "The Search for Configural Relationships in Personality Assessment: The Diagnosis of Psychosis vs. Neurosis from the MMPI," Multivariate Behavioral Research. October, 1969, pp. 523-86.

McSweeney, Maryellen and Schmidt, William H. "Quantal Response Techniques for Random Predictor Variables," Paper presented at Annual Convention of American Educational Research Association, 1974.

Meehl, P. E. "A Comparison of Clinicians with Five Statistical Methods of Identifying Psychotic Profiles," Journal of Counseling Psychology, 1959, Vol. 6, pp. 102-9.

Morrison, Donald G. "On the Interpretation of Discriminant Analysis," Journal of Marketing Research, May, 1969, pp. 156-63.

Myers, H. and Forgy, E. W. "Development of Numerical Credit Evaluation Systems," Journal of American Statistical Association. September, 1963, pp. 797-806.

Pearl, R. and Reed, L. J. "On the Rate of Growth of the Population of the U.S. since 1790 and its Mathematical Representation," Proceedings of the National Academy of Science, Vol. 6, 1920.

Pinches, George E. and Mingo, Kent A. "A Multivariate Analysis of Industrial Bond Ratings," Journal of Finance, March, 1973, pp. 1-15.

Revsine, Lawrence. "Predictive Ability, Market Prices, and Operating Flows," The Accounting Review, July, 1971, pp. 480-89.

Smith, K. V. "Classification of Investment Securities Using MDA," Institute Paper #101 Purdue University Institute for Research in the Behavioral Economic and Management Sciences, 1965.

Walter, J. E. "A Discriminant Function for Earnings Price Ratios of Large Industrial Corporations," Review of Economics and Statistics, February, 1959, pp. 44-52.

Wiggins, Nancy and Hoffman, Paul J. "Three Models of Clinical Judgment," Journal of Abnormal Psychology, Vol. 73, No. 1, pp. 70-77.

Zellner, Arnold and Lee, Tong Hun. "Joint Estimation of Relationships Involving Discrete Random Variables," Econometrica, April, 1965, pp. 382-94.