This is to certify that the

thesis entitled

SYMBOLIC LINEAR ANALYSIS PROGRAM

presented by

VIVEK JOSHI

has been accepted towards fulfillment
of the requirements for

M.S. degree in Elec. Engr.

Gregory M. Wierzba
Major professor

Date 8/21/87

SLAP: Symbolic Linear Analysis Program

By

Vivek Joshi

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE

Department of Electrical Engineering

1987

ABSTRACT

SLAP: Symbolic Linear Analysis Program

by

VIVEK JOSHI

A symbolic linear circuit analysis program is developed. The program incorporates four modules. The first module writes the nodal equations of a circuit defined to it in a particular format. The second module computes the transfer function of the circuit from the matrix nodal equations. The third module identifies any filter functions present. The filter parameters are also identified if a filter function is present. The fourth module helps in reducing the error due to finite gain-bandwidth-product (GBWP) of an operational amplifier.

DEDICATION

I would like to thank my parents for putting up with me during my teenage years and college years

ACKNOWLEDGMENTS

I would like to thank Dr. Wierzba for his guidance and help during my research.

I would like to thank Dr. D.P. Joshi and his family for providing me with help whenever I needed it during my college years.

I would like to acknowldge the Michigan REED grant which allowed me complete the project on time.

Table of Contents

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

Preview of Results and Background Information

1.1 INTRODUCTION

Circuit analysis is an integral part of circuit design. It gives the designer room to experiment with different components to realize a particular circuit. Unfortunately as circuit realizations get more complicated it becomes very tedious to perform circuit analysis by hand. As an aid computer analysis packages, such as ECAP and SPICE were developed in the 1960's and 1970's. These packages perform complicated circuit analysis on a computer in a fraction of time of what it would take to do by hand.

There is always a new or better program that is being developed that eases circuit analysis or improves on speed and accuracy. For example, PSPICE developed by Microsim Corporation is a version of SPICE which runs on a personal computer. In addition, to the standard features of SPICE, PSPICE has graphics, current sensing and Monte Carlo analysis. PSPICE as well as other programs place emphasis on numeric analysis, that is the elements have numeric values that the program uses to compute the circuit variables. The problem in using numeric circuit analysis is that the element values are combined in the final answer. This makes it difficult to evaluate the effect an element has on the total circuit response.

The ultimate solution to this problem is to obtain a symbolic analysis. Unfortunately symbolic analysis is notorious for generating large amounts of results. However, a symbolic analysis package written for a particular class of problems can be useful since many complicated circuits have relatively simple design equations. Since the software is designed for a specific application, it is important for the user to have a thorough understanding of that application. Furthermore, since there are usually an infinite number of component selections for a given circuit, the output of a symbolic program will not be the final step and the user must complete the design.

1.2 Problem Statement

Since the late 1960's, the use of active elements in filter design has become more and more involved. As chip fabrication and design procedures improved, the cost of realizing an op-amp on a chip decreased and the use of op-amps in filter design has increased. Op-amps have been used extensively in hybrid active filters where the op-amp is connected to a ceramic substrate containing passive components. Op-amps have also been used for realizing switched capacitor active filters where the entire filter is fabricated on a single chip.

An ideal op-amp has infinite gain and infinite bandwidth. The IC op-amp, of course, has finite gain and finite bandwidth. In a resistive feedback configuration, the bandwidth of an op-amp circuit can be extended by trading off the gain. Essentially the product of gain and bandwidth remains constant and is referred to as the gain-bandwidth-product (GBP in Hz.). One way to get more bandwidth is to cascade duplicate circuits. The cost of this approach lies in the increased number of circuit elements.

In active filters, the effect of GBP is that the design poles (and zeroes) are shifted. Besides this, additional poles are introduced which can cause stability problems. Active filter transfer functions are usually expressed as the product of second order equations. There are two terms in the denominator of a second order transfer function, f_0 and Q_0. Since the non-ideal op-amp shifts the desired poles, this results in a new value of f_0, $\hat{f}_0$ and a new value of Q_0, $\hat{Q}_0$.

It has been shown that for the Multiple-Input-Biquad of Fleischer and Tow [9] using matched op-amps.

$$\frac{\Delta f_0}{f_0} = -\frac{f_0}{GBP} \tag{1}$$

$$\frac{\Delta Q_0}{Q_0} = +\frac{4Q_0 f_0}{GBP} \tag{2}$$

Since $\Delta f_0 = \hat{f}_0 - f_0$ and $\Delta Q_0 = \hat{Q}_0 - Q_0$, therefore solving for $\hat{f}_0$ and $\hat{Q}_0$ with the appropriate approximations yields:

$$\hat{f}_0 = \frac{f_0}{1 + \dfrac{f_0}{GBP}} \tag{3}$$

$$\hat{Q}_0 = \frac{Q_0}{1 - \dfrac{4Q_0 f_0}{GBP}} \tag{4}$$

Equation 3 is plotted in Figure 1.1 to show how the actual frequency changes with respect to the desired frequency. Equation 2 is plotted in Figures 1.2 and 1.3 to show the dependence of Q_0 on the selectivity factor and the natural frequency of the circuit. These figures were obtained by using the GBP = 1.00E+06 (a typical value for a 741 op-amp).

The Multiple-Input-Biquad is a three op-amp active filter and the errors found with equations 1-4 are small compared to one and two op-amp active filters. As in the case of resistive feedback amplifiers, adding more op-amps results in a wider range of performance. Hence the product of f_0 and Q_0 must be traded off in order to keep the circuit stable, since it can be noted from equation 4 that $\hat{Q}_0$ approaches infinity as $4Q_0 f_0$ approaches GBP.

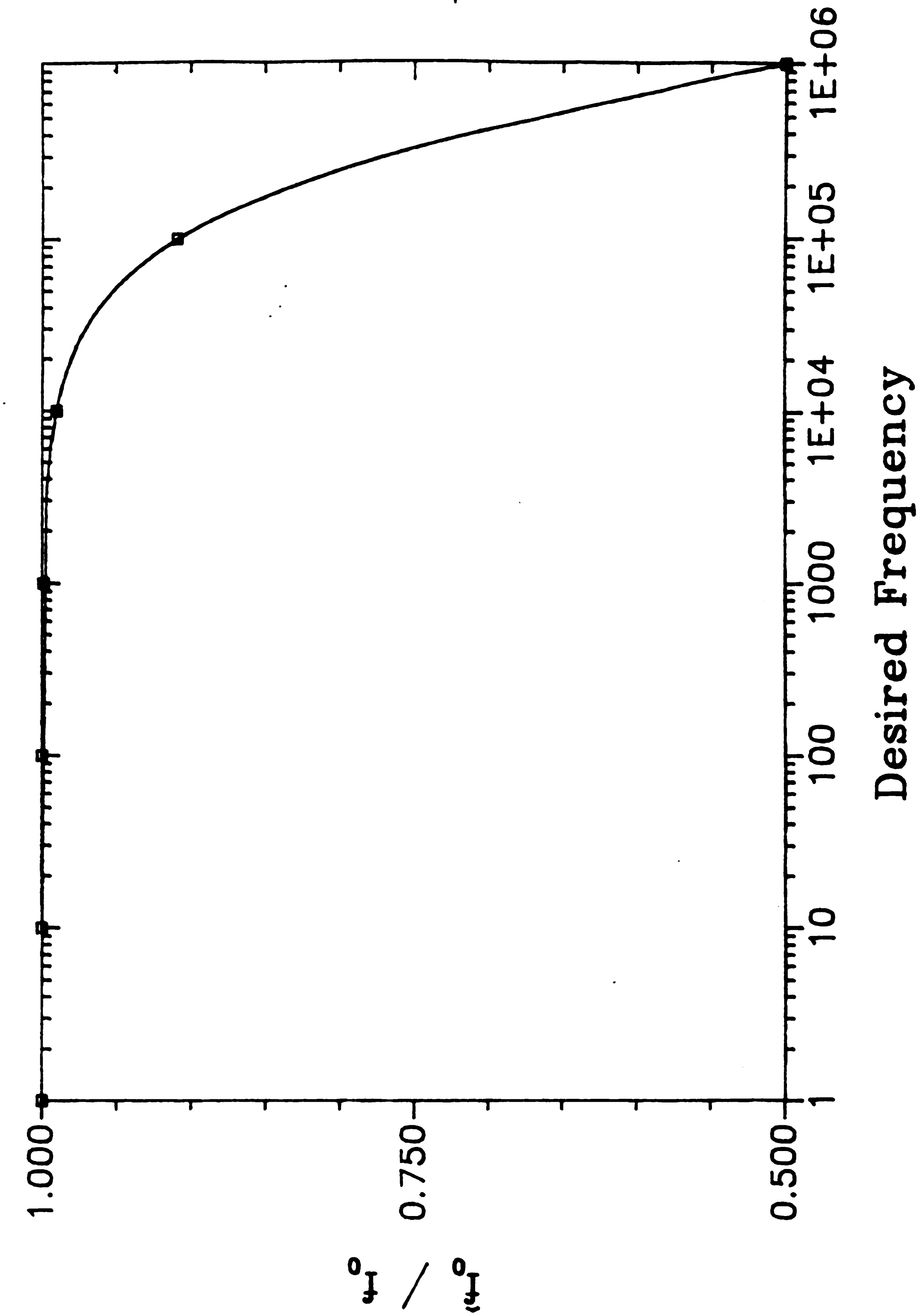


Figure 1.1: Error in natural frequency as a function of design frequency

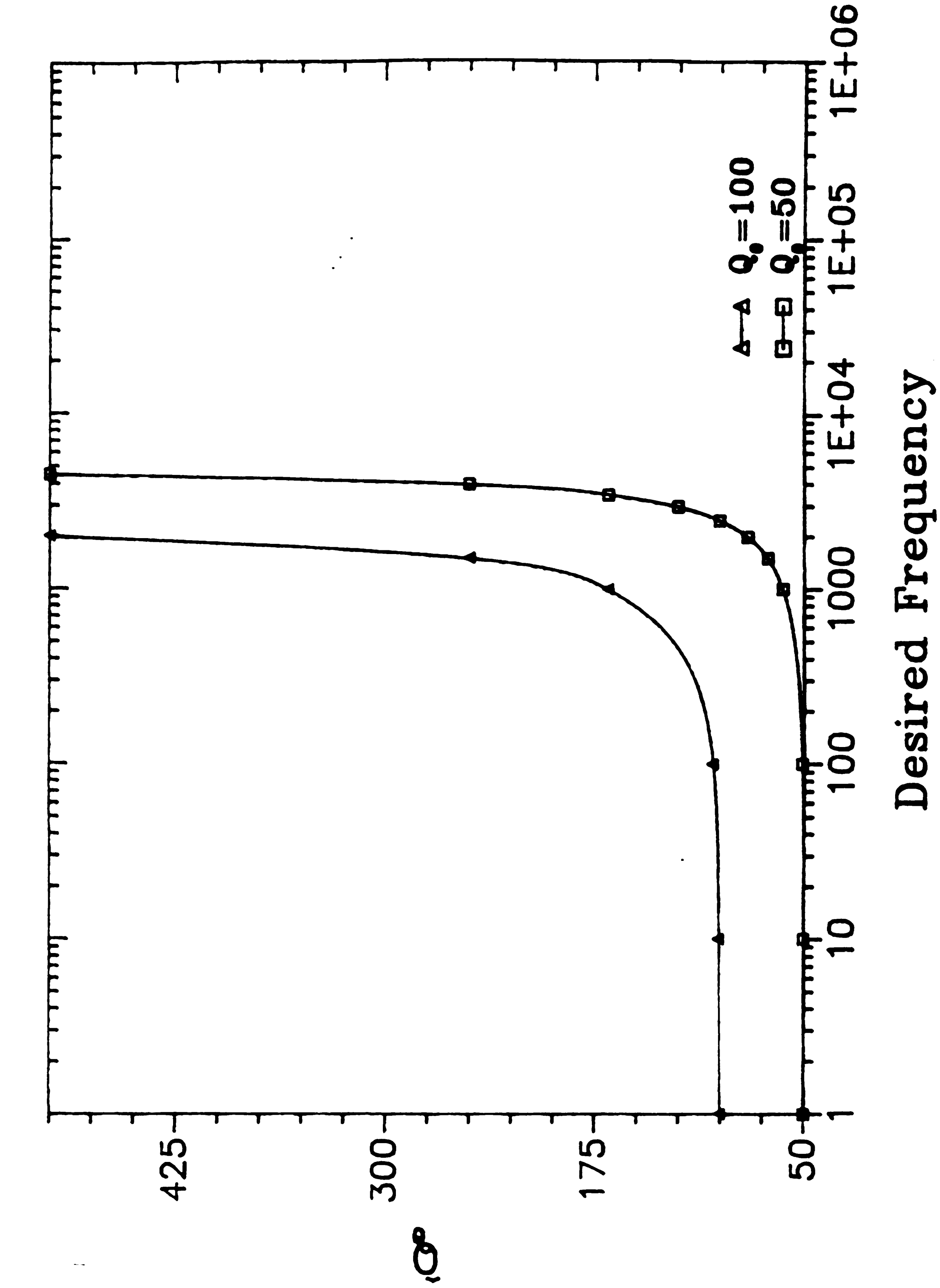


Figure 1.2: Error in the selectivity factor as a function of design frequency

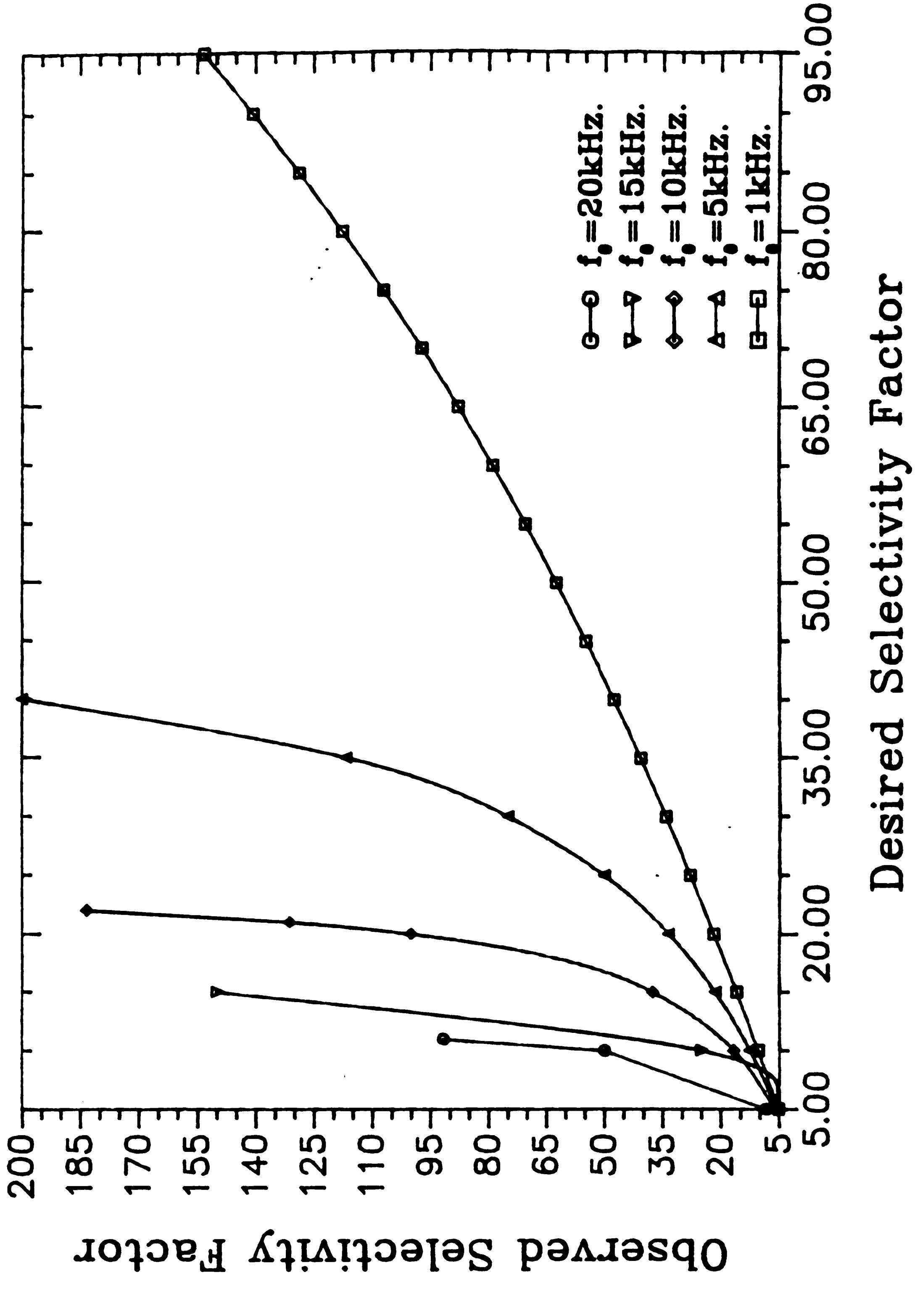


Figure 1.3: Error in the selectivity factor as a function of selectivity factor

Recently, a systematic procedure [24] has been proposed for generating active filter circuits which uses an existing seed circuit. From a single seed circuit, thousands of new circuits can be generated. For each of these circuits equations 1-4 needs to be determined again. The time required to compute these equations manually is on the order of four hours. Clearly, with thousands of circuits this is a hard task.

In the Multiple-Input-Biquad, counting the op-amps and the passive components there are eleven symbolic elements. In other biquad circuit structures, there may be as many as fifteen to twenty symbolic elements. At present there are five symbolic circuit analysis programs, SNAP [16], VLACH [13], MECA [17],CORNAP [19] and SLIC available for this purpose. CORNAP does not really allow for any symbols. It just produces the transfer function using the symbol s for it's frequency. The number of variables allowed is too few to be effectively used for the scope of this problem. For example VLACH [13] allows for only ten voltage sources, five symbolic elements and forty elements.

1.3 Solution Approach

The purpose of this research is to develop a Symbolic Linear Analysis Program (SLAP) which would provide the analysis to generate formulas like those obtained in Equations 1-4.

Symbolic manipulation can be performed for multiplication, addition and subtraction with concatenation of strings. Then term reduction is done by comparison. Division is far more complicated especially for term reduction because of the need for a common factor. Also, a large amount of storage is wasted since at least twice the amount of memory has to be set aside. Therefore, division must be avoided. Without division, the inverse of a symbolic matrix can not be obtained. Thus matrix techniques which require an inverse such as LU decomposition will not be helpful.

In SLAP, the nodal admittance equations of a circuit are first formed. This involves forming a matrix using addition and subtraction. A recursive technique is then used to compute the determinant of the admittance matrix. In order to determine the transfer function Cramer's Rule is used. Error analysis is performed by solving for the general form of the answer and searching for the appropriate terms in the determinant. In a similar manner, transfer function identification is performed to find the ideal design equations.

The programming language chosen was C. The reason being that C is an extremely versatile language and C has the advantage of being perhaps the most portable language. FORTRAN was rejected because it does not have a flexible method for handling the string manipulations involved and FORTRAN does not support recursive calling of a function. PASCAL allows for strings manipulation and recursive calling, but its portability is a very serious drawback. For example, a program written in PASCAL for the VMS operating system can not be compiled on the UNIX operating system. The only limitations of the program developed are the amount of computer memory and computer speed.

1.5 Organization of Thesis

The remainder of the thesis is organized in the following manner. In Chapter 2 the writing of nodal equations and the formulation of the transfer function are discussed. The circuit elements used in the program are identified in Appendix A. An example is given showing the input and output format alongwith a sample run of the program.

Chapter 3 describes the filter identification program. Also presented are the various filter types available and an example is worked out using the program. The program of Chapter 2 can be used to write the file which is the input to this program.

Chapter 4 discusses the error analysis according to the Wilson-Bedri-Bowron Approximation. A new derivation is worked out completely where op-amp matching is not assumed. The various error terms and their significance are discussed. An example is done to show the working of the program. Chapter 5 states the conclusions and discusses further research topics for extending the program.

CHAPTER II

Linear Circuit Analysis Using Nullators and Norators

2.1 INTRODUCTION

Given a specific circuit with element values, then it may be tested by using SPICE or other numeric programs. However, a circuit designer needs to select the component values to meet some design criteria. In order to do this the formulas for the design parameters must be found. This is a tedious process and requires some skill in writing and selecting the equations.

This chapter describes a technique for systematically formulating the nodal equations of any linear active circuit.

2.2 Nullator and Norator

A short circuit has a voltage which is zero and a current which is arbitrary. By arbitrary it is meant that the value is determined the circuit in which it is used. An open circuit has zero current and a voltage which is arbitrary. Combining both of the properties of zero voltage and zero current results in a new circuit element called a nullator [25]. The circuit symbol for a nullator is presented in Figure 2.1a.

A voltage source has a specified voltage and an arbitrary current. A current source has a specified current and an arbitrary voltage. The combination of both arbitrary current and arbitrary voltage is also a new circuit element and is referred to as a norator [25]. In Figure 2.1b, the circuit symbol for a norator is shown.

An ideal op-amp can be modelled using a grounded voltage-controlled- voltage-source with a gain of A, where A approaches infinity. The controlled source draws zero input current and has an arbitrary output current. In a stable closed loop circuit, the output voltage of the controlled source is finite and determined by the circuit in which it is used. Since the output voltage is the product of the sensing voltage and infinity, then for a finite arbitrary product the sensing voltage must be zero. Thus the ideal op-amp has zero-voltage-zero-current at the input terminals and arbitrary current and arbitrary voltage at the output terminal with respect to ground. This is modelled with a nullator and a grounded norator.

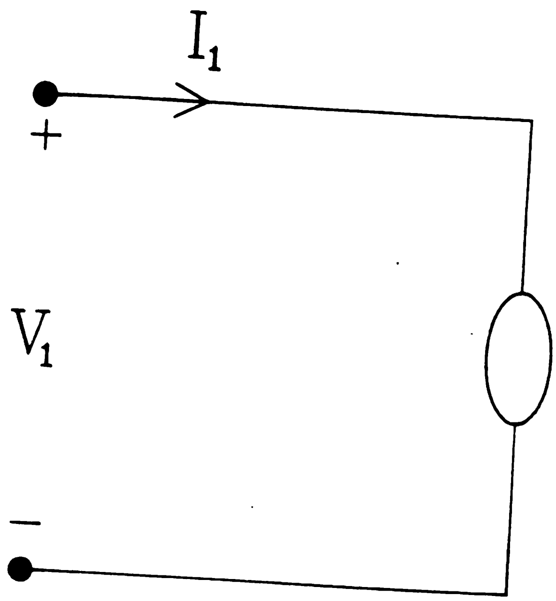
2.3 Modelling of Dependent Sources using Nullors

Besides modelling of an ideal op-amp, nullators and norators can also be used to model controlled sources [25].

Voltage-Controlled-Current-Source (VCCS):

A VCCS is given in Figure 2.2 along with a nullator-norator-conductor model. The current entering the input terminals of Figure 2.2b is zero because of the nullators. With a voltage drop of zero across each nullator, the drop across the conductor is V_1 and thus the current through this conductor is $g_m V_1$. With no current going through the nullators, the current in both norators is forced to be $g_m V_1$. Therefore, $I_2 = g_m V_1$. The voltage V_2 is the sum of the voltages across the norators and the conductor. Since the drop across a norator is arbitrary, this sum is also arbitrary.

Voltage-Controlled-Voltage-Source (VCVS)



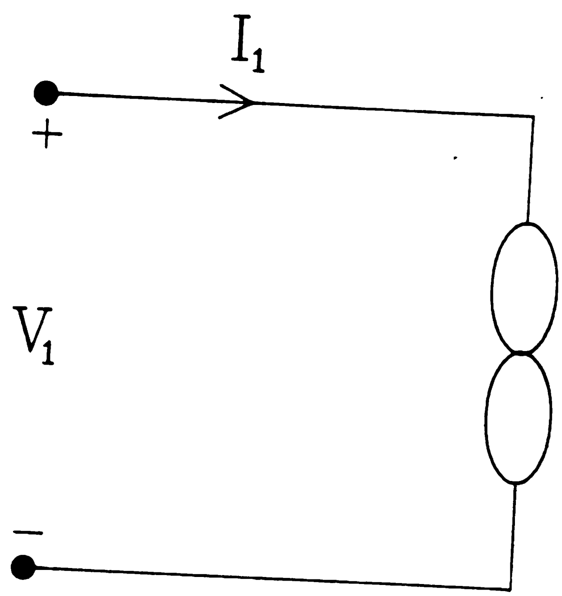


Figure 2.2 a) Nullator Symbol

b) Norator Symbol

A VCVS and a nullator-norator-conductor model are presented in Figure 2.3. For Figure 2.3b, the input current is zero due to the nullators. The current in the one mho conductor is V_1*1. This current is forced to flow through B because the nullators prevent any current from entering the leaving the loop which contains the two conductors. Thus the voltage across B is V_1/B. This voltage is transferred to V_2 with the nullator connected to B. The current I_2 is the current in the norator across the output terminals. Thus I_2 is arbitrary.

Current-Controlled-Current-Source (CCCS)

In Figure 2.4, a CCCS and a nullator-norator-conductor model are shown. For Figure 2.4b, the input voltage is across a nullator, and this by definition is zero. The current I_1 flows into the one mho conductor and the voltage is transferred across a. Thus $I_2 = (I_1 * 1)/a = aI_1$. The output voltage V_2 is the sum of the norator voltage and the voltage across a. Therefore V_2 is arbitrary.

Current-Controlled-Voltage-Source (CCVS)

A CCVS is shown in Figure 2.5 along with a nullator-norator-conductor model. The input voltage in Figure 2.5b is zero and the current I_1 flows through g creating a voltage drop of I_1/g. This voltage is transferred to V_2 by using the two nullators. The current I_2 is the current of a norator and is therefore arbitrary.

1.2 Nodal Equations of Passive Circuits

In order to be able to analyze a circuit with a computer it essential that the circuit be described in an equation form. Given a passive circuit with independent current

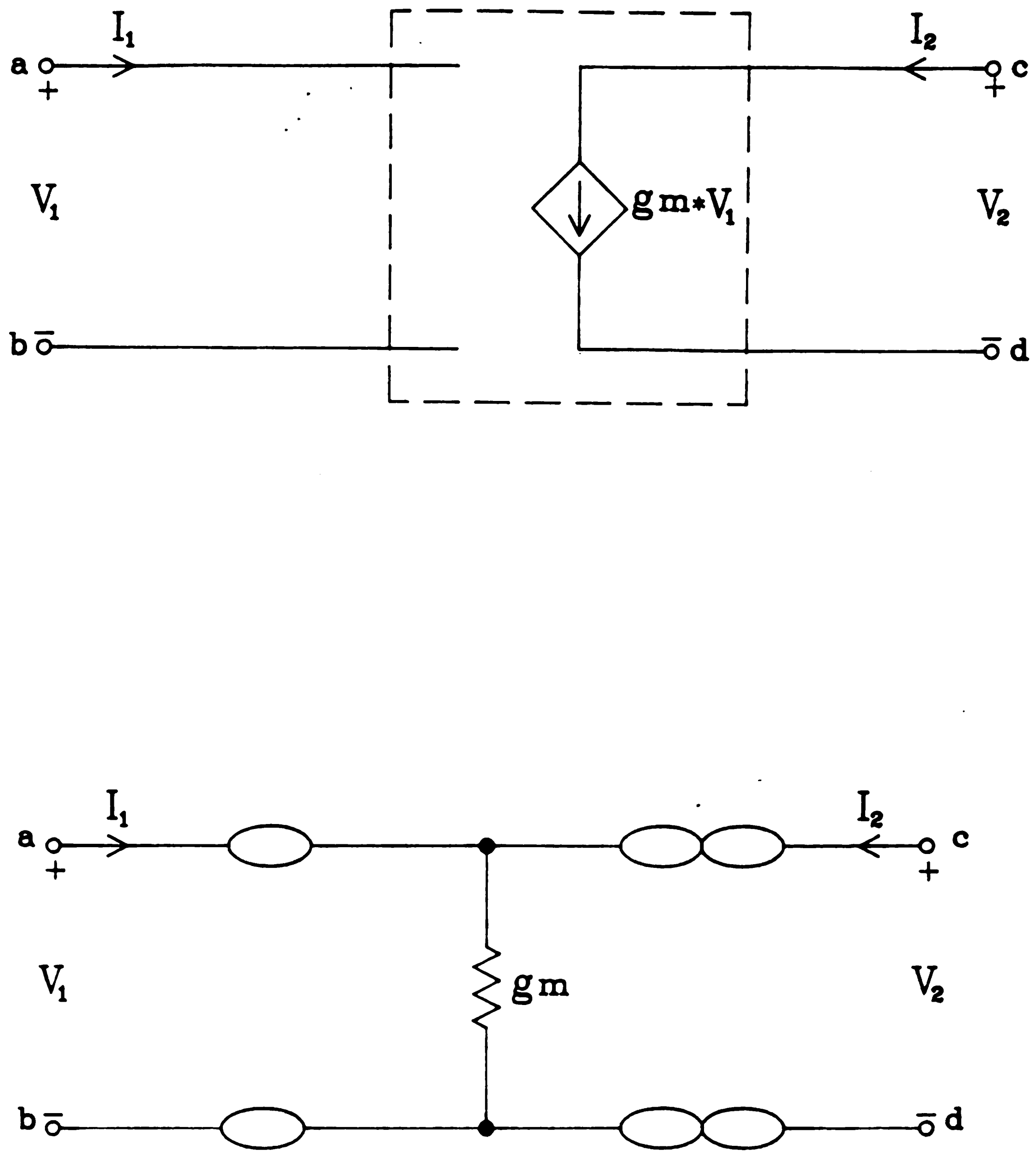
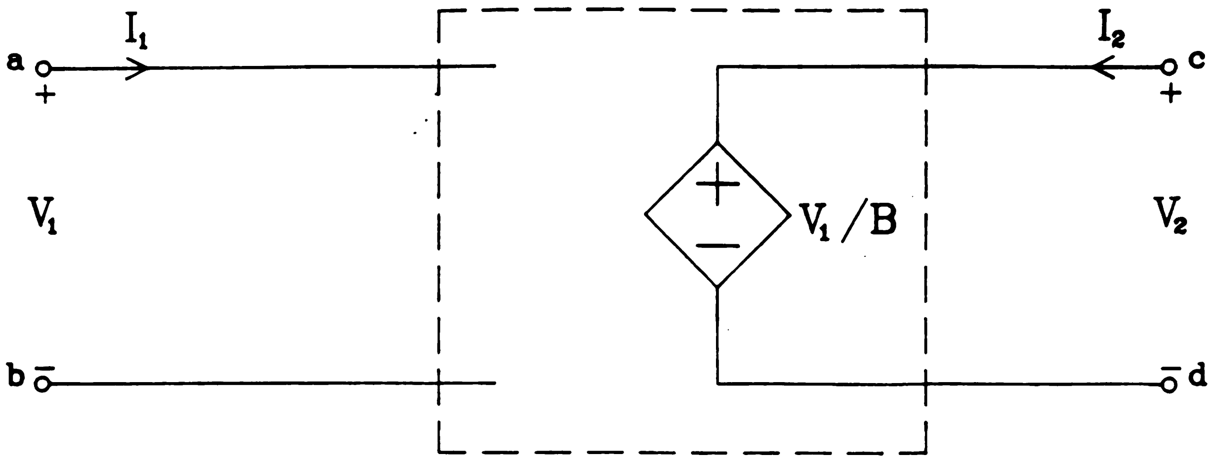


Figure 2.2 a) Voltage Controlled Current Source

b) Nullor model of a VCCS



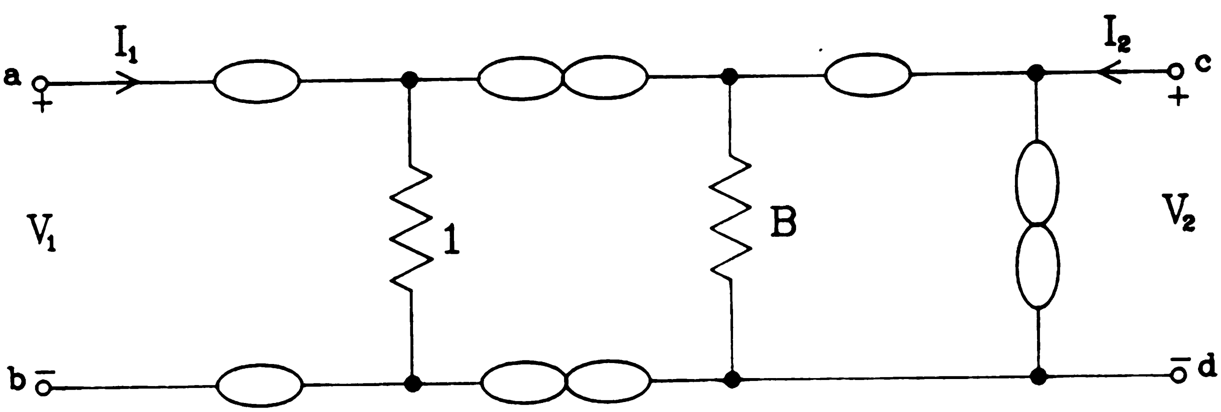
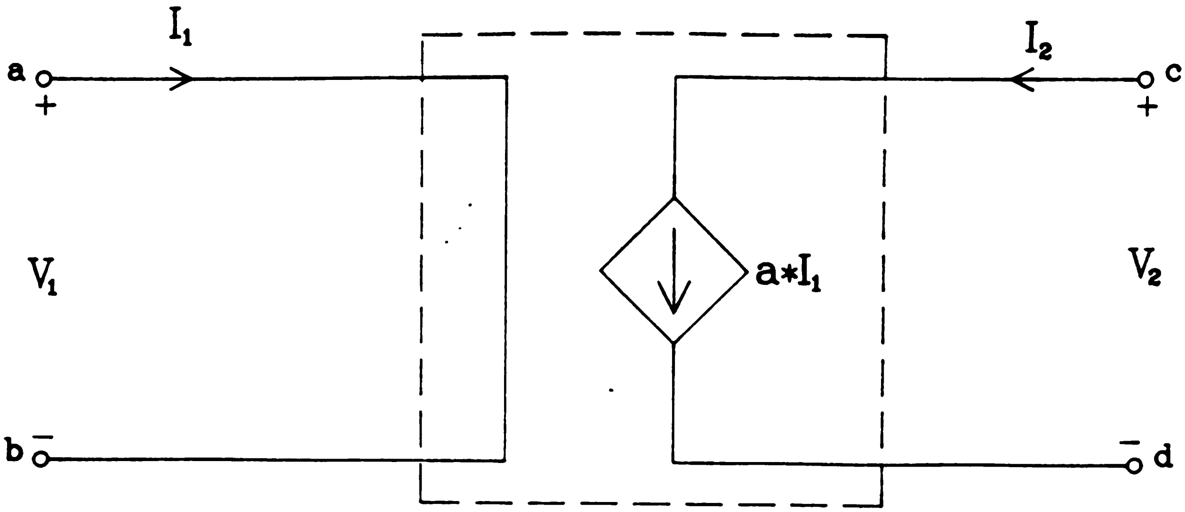


Figure 2.3 a) Voltage Controlled Voltage Source

b) Nullor model of a VCVS



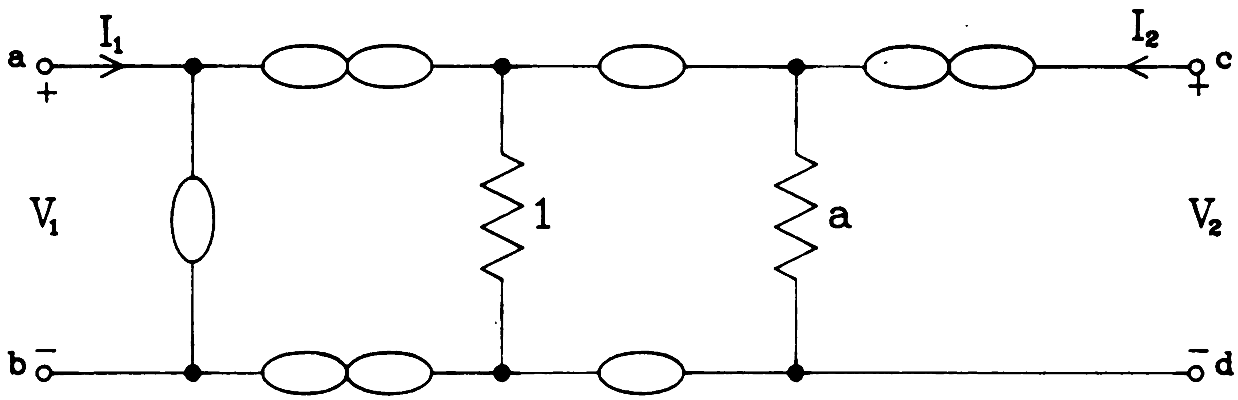
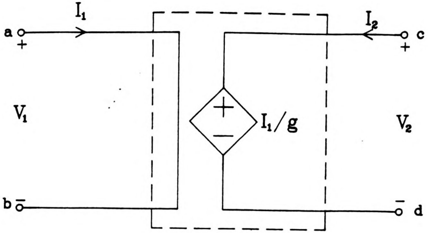


Figure 2.4 a) Current Controlled Current Source

b) Nullor model of a CCCS



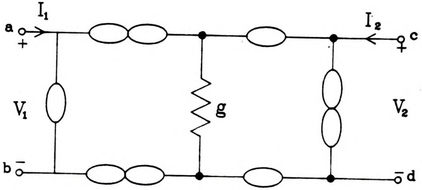


Figure 2.5 a) Current controlled Voltage Source

b) Nullor model of a CCVS

sources, it is possible to form the circuit equations by inspection. This technique uses simple summation of terms and does not require an understanding of circuit equations. This can be programmed and is as follows:

ALGORITHM 1 [25]:

If an (n+1) node network is composed of RLC elements and independent currents sources, then the following steps may be used to form the nodal equations for the circuit.

1) A reference node is selected and labeled as node 0.

2) All other nodes are labelled sequentially from 1 to n.

3) The node equations contain a current vector I of dimension $n \times 1$:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

where the i^{th}component, i_i is defined as the sum of the currents flowing into the i^{th} node from the independent current sources.

4) The nodal admittance matrix $Y_{n \times n}=\left[y_{ij} \right]$ has dimension $n \times n$ and may be written by inspection:

4a. y_{ii} is the sum of the admittances connected to node i.

4b. y_{ij} is the negative sum of the admittances connected between nodes i and j.

5) Therefore the nodal equations of the network in matrix form are:

$$I = Y_{nxn} \times V \qquad 2.1$$

where V is a column vector of unknown node voltages, of dimension $n \times 1$ and is represented as :

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

2.4 Nodal Equations of Nullator-Norator Circuits

In the last section, equations were formed for passive circuits by inspection. If controlled sources or ideal op-amps are included in the circuit then decisions on which equations to write have to be made. This is extremely difficult to program. However, an algorithm for the formulation of circuit equations by inspection for norators containing nullators and networks does exist. Since controlled sources and op-amps can be modelled with nullators, norators and conductors, then this algorithm would allow for the formulation of equations for all linear circuits.

ALGORITHM 2 [25]:

If an $(n + 1)$ node network is composed of RLC elements, independent current sources and m-nullators-m-norators, then the following steps may be used to form the nodal equations for the circuit.

1) With all of the nullators and norators open circuited, form the nodal equations using Algorithm 1. The result is of the form:

$$I = Y_{n \times n} V$$

2) For a nullator between nodes d and j, add column j to column d of the matrix $Y_{n \times n}$. Delete column j from $Y_{n \times n}$ and delete v_j from the voltage vector V.

Search through the remaining nullator node connections and replace every occurrence of node j with d.

3) For a nullator between nodes e and ground, delete column e from the admittance matrix $Y_{n \times n}$ and delete v_e from the voltage vector V. Search through the remaining nullator node connections and replace every occurrence of node e with 0 which signifies the ground connection.

4) For a norator between nodes f and h, add row h to row f of the admittance matrix $Y_{n \times n}$ and the current vector I. Delete row h from the matrix $Y_{n \times n}$ and the current vector I. Search through the remaining norator node connections and replace every occurrence of node h with f.

5) For a norator between nodes g and ground, delete row g from the admittance matrix $Y_{n \times n}$ and the current vector I. Search through the remaining norator node connections and replace every occurrence of node g with 0 which signifies the ground connection.

6) Repeat steps 2-5 for the remaining $(m - 1)$ nullators and norators.

For each nullator the number of columns is reduced by one. Similarly for the case of the norator the number of rows is reduced by one. Therefore, for m-nullators and m-norators, the number of columns and rows is reduced by m. The nodal equations for the circuit therefore reduce to:

$$I = Y_{(n-m) \times (n-m)} V$$

Proof:

Replace the nullators and norators in the circuit with open circuits. Consider two nodes of the circuit and the corresponding nodal equations.

$$\begin{bmatrix} \vdots \\ \sum i_d \\ \vdots \\ \sum i_j \\ \vdots \end{bmatrix} = \begin{array}{c} \begin{array}{ccccc} \quad & d & \quad & j & \quad \end{array} \\ \left[\begin{array}{c|c|c|c|c} \quad & \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad & \quad \end{array}\right]_{n \times n} \end{array} \begin{bmatrix} \vdots \\ v_d \\ \vdots \\ v_j \\ \vdots \end{bmatrix}_{n \times 1}$$

1) Let a nullator be inserted between nodes d and j with a voltage $v = 0$ and $i = 0$. Since $v = v_d - v_j = 0$ and $i = 0$, then the entries are modified as follows:

$$\begin{bmatrix} \vdots \\ \sum i_d - 0 \\ \vdots \\ \sum i_j + 0 \\ \vdots \end{bmatrix} = \begin{array}{c} \begin{array}{ccccc} \quad & d & \quad & j & \quad \end{array} \\ \left[\begin{array}{c|c|c|c|c} \quad & \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad & \quad \end{array}\right]_{n \times n} \end{array} \begin{bmatrix} \vdots \\ v_d \\ \vdots \\ v_d \\ \vdots \end{bmatrix}_{n \times 1}$$

This reduces to:

$$\begin{bmatrix} \vdots \\ \sum i_d \\ \vdots \\ \sum i_j \\ \vdots \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} \quad & d & \quad \end{array} \\ \left[\begin{array}{c|c|c} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{array}\right]_{n \times (n-1)} \end{array} \begin{bmatrix} \vdots \\ v_d \\ \vdots \end{bmatrix}_{(n-1) \times 1}$$

2) If a nullator is inserted between nodes e and ground, then by a similar argument as above $v_e = 0$. This results in each entry in column e of $Y_{n \times n}$ being multiplied by zero. Thus column e of $Y_{n \times n}$ is deleted and v_e is removed from v.

3) Let a norator be inserted between nodes f and h with a voltage $v = \text{arbitrary}$ and $i = \text{arbitrary}$. Then the entries for rows f and h are modified as follows:

$$\begin{bmatrix} \vdots \\ \sum i_f - i \\ \vdots \\ \sum i_h + i \\ \vdots \end{bmatrix}_{n \times 1} = \begin{array}{c} \\ f \\ \\ h \\ \\ \end{array} \begin{bmatrix} \\ - - - - - - - - - \\ - - - - - - - - - \\ \\ - - - - - - - - - \\ - - - - - - - - - \\ \\ \end{bmatrix}_{n \times n} \begin{bmatrix} \vdots \\ v_f \\ \vdots \\ v_h \\ \vdots \end{bmatrix}$$

If row h is added to row f, then the above matrix equation reduces to:

$$\begin{bmatrix} \vdots \\ \sum i_f + \sum i_h \\ \vdots \\ \vdots \end{bmatrix}_{(n-1) \times 1} = \begin{array}{c} \\ f + h \\ \\ \\ \end{array} \begin{bmatrix} \\ - - - - - - - - - \\ - - - - - - - - - \\ \\ \\ \end{bmatrix}_{(n-1) \times n} \begin{bmatrix} \vdots \\ v_f \\ \vdots \\ v_h \\ \vdots \end{bmatrix}$$

Hence the arbitrary (unknown) value of i is eliminated from the equations by adding rows f and h. The value of v may be neglected since it does not directly enter into the node equations.

4) If a norator is inserted between nodes g and ground, then i will appear only in row g of I. Since i is arbitrary this equation has an additional

unknown. Furthermore, for each norator there is a nullator and this nullator has reduced the number of columns by one. Hence, this row is not needed to solve for the remaining unknowns. Therefore, row g can be neglected.

2.5 Modeling of Voltage Source and Inductor:

A voltage source is not included as an element in Algorithm 2. This is due to the fact that currents are summed at every node and the current through a voltage source is an additional unknown. However, most circuits are driven by a voltage source and it would be convenient not to do a source transformation, where a voltage source and a conductor connected in series are converted to a current source with a conductor in parallel.

An independent voltage source can be modelled as shown in Figure 2.6a. The idea of a current source feeding a 1Ω conductor is used to develop the neccessary potential. The nullator has zero current through its nodes and zero voltage across its terminals. Therefore, the voltage developed across the conductor is of the value v and is also developed across the norator. During programming of this element the value of the voltage source is set to i internally in the program. The program can be used to find the transfer function as the program sets the value of this voltage source to unity internally. If a different value is desired the program can be easily modified to handle different values. The symbol for the ideal voltage source is given in Appendix A1.2 while its equivalent nullor model is given in Figure 2.6a.

An inductor is also an element that has to be uniquely handled. Since admittance is used to form the nodal equations, then an inductor can not be used as such without introducing division. Using a special symbol for $1/s$ is an alternative but this would cause very long terms to appear in the final answer since at this time the program is unable to cancel the $1 / s * s$ terms.

A capacitor has an admittance which is a string of terms and an inductor can be simulated using a capacitor, conductors and nullators-norators. Thus using a simulated inductor as a model for an inductor is a convenient way of avoiding performing division. The simplest model for a simulated inductor is given in Figure 2.6b.

2.6 Example

This technique for reduction and forming of the nodal equations is used in one of the modules of SLAP. The nodal equations are written by the software and the simplifications due to nullators and norators are then performed automatically for the user. Another module of SLAP finds the symbolic determinants of the numerator and denominator of the transfer functions of the circuit. This program also gives the user an option to write out files for use with other modules of SLAP. An example is presented which illustrates the concepts described in this chapter. The user can use the programs at any step if the files are prepared in the proper order for that module.

The circuit given in Figure 2.7 was obtained by op-amp relocation[23]. First we need to obtain the transfer functions of this circuit using ideal op-amps.

The input file for Module 1 of SLAP is shown in Table 2.1 (see Appendix A1.2 on help in constructing this file). This module formulates the node

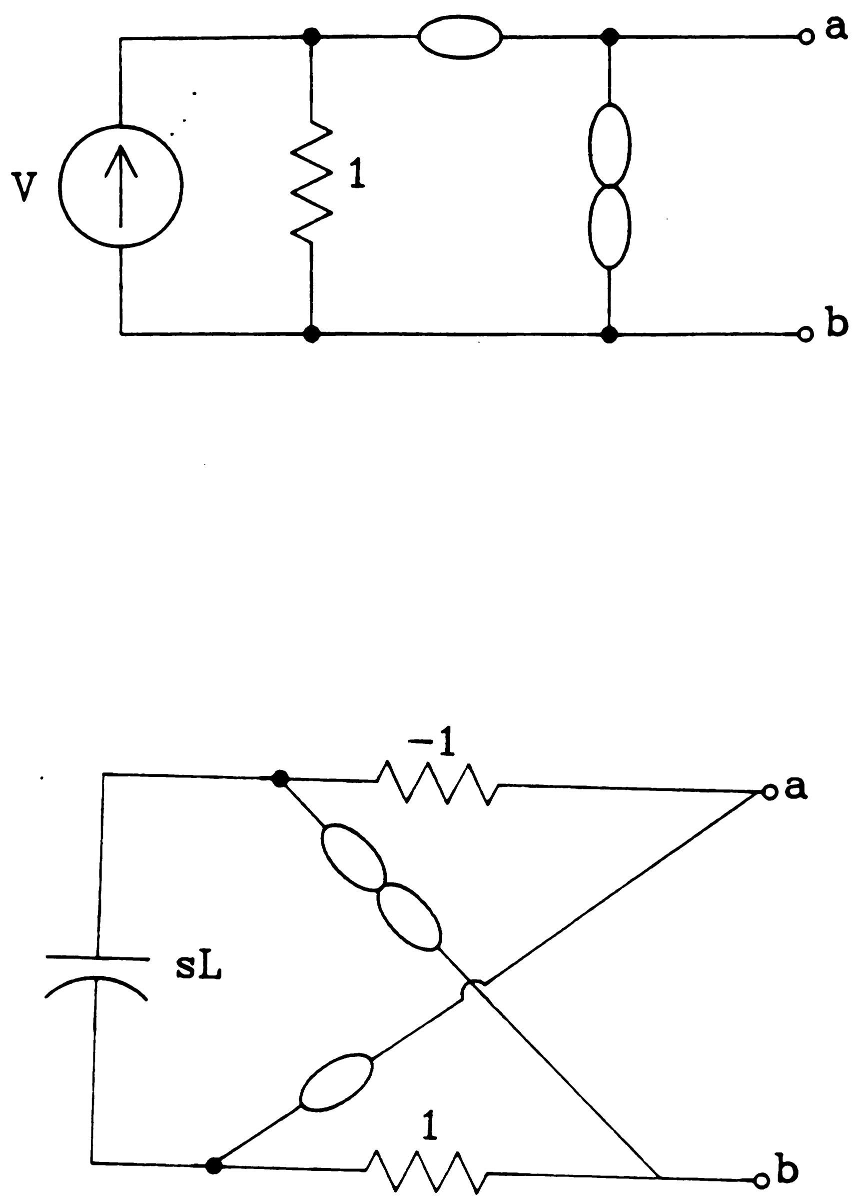


Figure 2.6 a) Voltage Source Nullor Model

b) Inductor Nullor Model

equations:

$$I = Y_{nxn} \times V$$

and prepares an output file which is used by module 2. The prompts and responses for the example are given in Table 2.2. The output file, "ideal.2" is created and the file contents are listed in Table 2.3 (see Appendix A1.3 on how to write this file).

Module 2 uses the file "ideal.2" as an input and has a variety of outputs. The prompts and responses for this example are presented in Table 2.4. The output file, "ideal.3" is listed in Table 2.5.

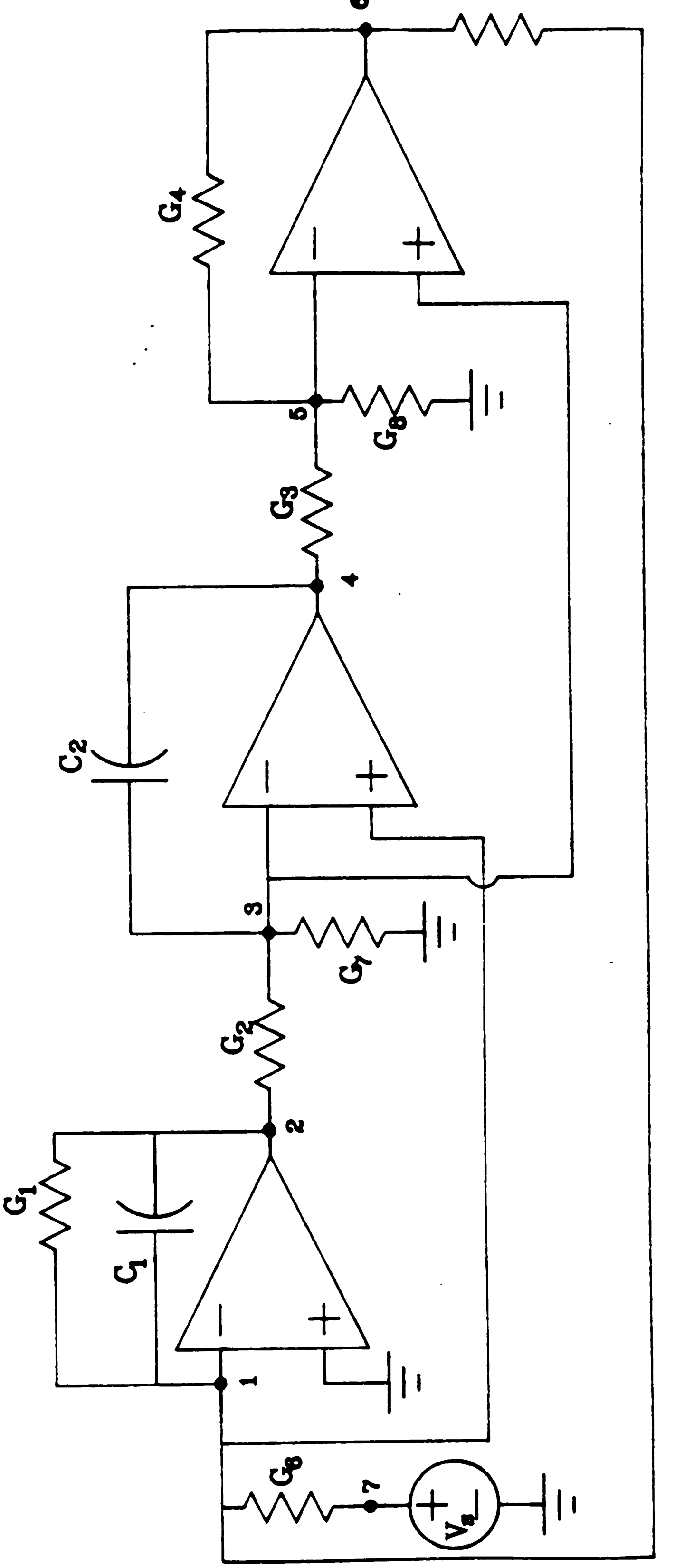


Figure 2.7 Circuit Generated from Tow-Thomas Biquad

```
7
Vs  7  0
G6  1  7
G1  1  2
C1  1  2
OA1  0  1  2
G2  2  3
G7  3  0
C2  3  4
OA2  1  3  4
G3  4  5
G8  5  0
G4  5  6
OA3  3  5  6
G5  6  1
.END
```

Table 2.1 Input to the matrix writing program, Module 1

```
% a.out
****  INFORMATION ABOUT THE INPUT FILE *****
Please input the file name you would like to use
ideal.1
You have chosen to use file : ideal.1
Please answer Y or N to proceed
y
****  INFORMATION ABOUT THE OUTPUT FILE ****
Please input the file name you would like to use
ideal.2
You have chosen to use file : ideal.2
Please answer Y or N to proceed
y
```

Table 2.2 Prompts and Responses for Module 1

```
4
0
0
0
+1
-G1-sC1
0
-G5
-G6
-G2
-sC2
0
0
0
-G3
-G4
0
0
0
0
+1
V2
V4
V6
V7
```

Table 2.3 Output of Module 1

```
% a.out
****  INFORMATION ABOUT THE INPUT FILE *****
Please input the file name you would like to use
ideal.2
You have chosen to use file : ideal.2
Please answer Y or N to proceed
y
****  INFORMATION ABOUT THE OUTPUT FILE ****
Please input the file name you would like to use
ideal.3
You have chosen to use file : ideal.3
Please answer Y or N to proceed
y
Does the circuit contain any non-ideal operational amplifiers?
Please answer y or n only
n
Would you like to see the numerator terms?
Please answer y or n only
y
Would you like to prepare a file to check the transfer
function(s) for any valid filter functions?
y
Please input the file name you would like to use
ideal.4
You have chosen to use file : ideal.4
Please answer Y or N to proceed
y
```

Table 2.4 Prompts and Responses for Module 2

0	-G1-sC1	0	-G5	-G6	V2
0	-G2	-sC2	0	0	V4
0	0	-G3	-G4	0	V6
1	0	0	0	1	V7

```
DENOMINATOR IS :

-sC2*sC1*G4
-sC2*G4*G1-G5*G3*G2

numerator for V2 is :

sC2*G6*G4

numerator for V4 is :

-G6*G4*G2

numerator for V6 is :

G6*G3*G2

numerator for V7 is :

-sC2*sC1*G4
-sC2*G4*G1-G5*G3*G2
```

Table 2.5 Output from Module 2

CHAPTER III

Filter Function Identification

3.1 INTRODUCTION

The purpose of a filter is to pass a band of signals and block the undesired signals. Filters are described in the frequency domain by unique transfer functions in s. These transfer functions can be further simplified into a product of second order or biquadratic denominators.

This chapter describes the basic biquad transfer functions for low pass, high pass, band pass, band reject and all pass filter. Module 3 of SLAP identifies these transfer functions and extracts variables such as f_0, Q_0 and H_0 in terms of the circuit elements.

3.2 Low Pass Filter:

The transfer function obtained for the case of a low pass filters is of the form:

$$H(s) = \frac{H_0\omega_0^2}{s^2 + (\omega_0/Q_0)s + \omega_0^2} \qquad 3.1$$

It is observed that as the quantity s tends to zero the denominator tends to ω_0^2 and the transfer function tends to H_0. On the other hand if the quantity s tends to infinity the denominator tends to infinity and the gain of the transfer function tends to zero. The magnitude of the transfer function is expressed as:

$$|H(j\omega)| = G(\omega) = \left[\frac{H_0^2\omega_0^4}{(\omega_0^2 - \omega^2)^2 + (\omega\omega_0/Q_0)^2}\right]^{\frac{1}{2}} \qquad 3.2$$

The low and high frequency response can be approximated as:

$$|H(j\omega)| \approx \begin{cases} H_0 & \text{for } \omega \ll \omega_0 \\ H_0\omega_0^2/\omega^2 & \text{for } \omega \gg \omega_0 \end{cases} \qquad 3.3$$

It is seen from equation 3.2 that at the cut-off frequency the value of gain $|H(j\omega)| = H_0Q_0$. As is shown in Figure 1.1 and 1.2, the finite GBP affects the observed ω_0 and Q_0. The increase in Q_0 will cause larger gains at the cut-off frequency. This results in amplification instead of attentuation at high frequencies and might even saturate the op-amp leading to undesireable results.

3.3 High Pass Filter:

The transfer function obtained for the case of a high pass filter is of the form:

$$H(s) = \frac{H_0s^2}{s^2 + (\omega_0/Q_0)s + \omega_0^2} \qquad 3.4$$

It can be observe that as the quantity s tends to infinity the gain of the transfer function tends to H_0 and as s approaches zero the transfer function tends to zero. The magnitude of the transfer function is written as:

$$|H(j\omega)| = G(\omega) = \left[\frac{H_0^2\omega^4}{(\omega_0^2 - \omega^2)^2 + (\omega\omega_0/Q_0)^2}\right]^{\frac{1}{2}} \qquad 3.5$$

The circuit response at high and low frequencies can be approximated as:

$$|H(j\omega)| \approx \begin{cases} H_0\omega^2/\omega_0^2 & \text{for } \omega \ll \omega_0 \\ H_0 & \text{for } \omega \gg \omega_0 \end{cases} \tag{3.6}$$

It is observed from equation 3.5 that at the cut-off frequency the value of gain $|H(j\omega)| = H_0Q_0$. As was observed in the case of low pass filter, the errors due to finite GBP may lead to an unsuitable filter at the cutoff frequency.

3.4 Band Pass Filter:

The transfer function of the band pass filter is of the form:

$$H(s) = \frac{H_0(\omega_0/Q_0)s}{s^2 + (\omega_0/Q_0)s + \omega_0^2} \tag{3.7}$$

The magnitude of the transfer function can be written as:

$$|H(j\omega)| = G(\omega) = \left[\frac{H_0^2\omega(\omega_0^2/Q_0^2)}{(\omega_0^2 - \omega^2)^2 + (\omega\omega_0/Q_0)^2}\right]^{\frac{1}{2}} \tag{3.8}$$

The low frequency response and the high frequency response can be approximated as:

$$|H(j\omega)| \approx \begin{cases} H_0\omega/(Q_0\omega_0) & \text{for } \omega \ll \omega_0 \\ H_0\omega_0/(Q_0\omega) & \text{for } \omega \gg \omega_0 \end{cases} \tag{3.9}$$

It is observed from equation 3.8 that at the center frequency, ω_0, the value of the gain $|H(j\omega)| = H_0$. The selectivity factor Q_0 can be written as:

$$Q_0 = \frac{\omega_0}{\omega_2 - \omega_1} \tag{3.10}$$

where ω_2 and ω_1 are the frequencies at which the magnitude response is 3dB down from H_0.

The errors in Q_0 and ω_0 can play havoc with the design. A very large value of observed Q_0 obtained alongwith a shifted design frequency would mean that the circuit could completely miss selecting the signal it was designed to pass.

3.5 Band Stop Filter:

The second order transfer function of a band-stop filter can be written as:

$$H(s) = \frac{H_0\left[s^2 + w_z^2\right]}{s^2 + (\omega_0/Q_0) + \omega_0^2} \qquad 3.11$$

The gain of this transfer function is:

$$|H(j\omega)| = G(\omega) = \left[\frac{H_0^2\left[\omega_z^2 - \omega^2\right]^2}{\left[\omega_0^2 - \omega^2\right]^2 + \left[\dfrac{\omega\omega_0}{Q_0}\right]^2}\right]^{\frac{1}{2}} \qquad 3.12$$

Depending on the values of ω_z and ω_0, there are three possible cases:

Case 1.:

Rewriting the above equation with $\omega_0 = \omega_z$ for low and high frequencies we see that:

$$|H(j\omega)| \approx \begin{cases} H_0 & \text{for } \omega \ll \omega_0 \\ H_0 & \text{for } \omega \gg \omega_0 \end{cases} \qquad 3.13$$

It is observed that at $\omega = \omega_0 = \omega_z$, $|H(j\omega)| = 0$.

Case 2.:

Rewriting the above equation with $\omega_0 < \omega_z$ for low and high frequencies we see that:

$$|H(j\omega)| \approx \begin{cases} H_0\omega_z^2/\omega_0^2 & \text{for } \omega \ll \omega_0 \\ H_0 & \text{for } \omega \gg \omega_z \end{cases} \qquad 3.14$$

Since $\omega_0 < \omega_z$, the value of $|H(j\omega)|$ is greater at low frequencies than at high frequencies. At ω_z and ω_0:

$$|H(j\omega)| = \begin{cases} 0 & \text{for } \omega = \omega_z \\ H_0 Q_0 \left[\omega_z^2 - \omega_0^2\right] / \omega_0^2 & \text{for } \omega = \omega_0 \end{cases} \qquad 3.15$$

Since $\omega_0 < \omega_z$, we note that $|H(j\omega)| \approx H_0 Q_0 \omega_z^2/\omega_0^2$ at $\omega = \omega_0$. This is an error of $20\log_{10} Q_0$ *dB* from the low frequency asymptote. Thus H(s) has some properties of a low pass filter and a notch filter and is referred to as a low pass notch filter.

Case 3.:

Rewriting the above equation with $\omega_0 > \omega_z$ for low and high frequencies we see that:

$$|H(j\omega)| \approx \begin{cases} H_0 \omega_z^2/\omega_0^2 & \text{for } \omega \ll \omega_z \\ H_0 & \text{for } \omega \gg \omega_0 \end{cases} \qquad 3.16$$

Since $\omega_0 > \omega_z$, the value of $|H(j\omega)|$ is greater at low frequencies than at high frequencies. At ω_z and ω_0:

$$|H(j\omega)| = \begin{cases} 0 & \text{for } \omega = \omega_z \\ H_0 Q_0 \dfrac{\left[\omega_0^2 - \omega_z^2\right]}{\omega_0^2} & \text{for } \omega = \omega_0 \end{cases} \qquad 3.17$$

Since $\omega_0 > \omega_z$, we note that $|H(j\omega)| \approx H_0 Q_0$ at $\omega = \omega_0$. This is an error of $20\log_{10} Q_0$ *dB* from the high frequency asymptote. Thus for $\omega_0 > \omega_z$, H(s) has some properties of a high pass filter and a notch filter and is referred to as a high pass notch filter.

As in the case of the band pass filters, the errors in ω_0 and Q_0 can greatly alter the desired frequency ω_z.

3.6 All Pass Filter:

All pass filters ideally pass a signal from a frequency range of zero to infinity. The phase of the signal is shifted in accordance with the phase characterstic of the filter. The transfer function of an all pass filter is of the form:

$$H(s) = H_0 \frac{s^2 - (\omega_0/Q_0)s + \omega_0^2}{s^2 + (\omega_0/Q_0)s + \omega_0^2} \qquad 3.18$$

The magnitude of the transfer function can be expressed as follows:

$$|H(j\omega)| = G(\omega) = H_0 \qquad 3.19$$

The gain is therefore a constant over the whole frequency spectrum. The phase on the other hand is:

$$\phi(\omega) = -\ 2*\arctan\left[\frac{\omega_0\omega/Q_0}{\omega_0^2 - \omega^2}\right] \qquad 3.20$$

In this case the errors in Q_0 and ω_0 will alter the desired phase response.

3.8 Example with Filter Function Identification

Module 3 of SLAP is used to determine some of the symbolic quantities of a second order transfer function. Continuing with the example of Section 2.6, module 2 is run again but this time answering "y" to a check for valid filter functions. The prompts and responses are listed in Table 3.4. The output for module 2 for this case is listed in Figure 3.5a. Module 3 is run with the output file from module 2, "ideal.4".

This file is listed in Table 3.5b. The prompts and responses for module 3 are listed in Table 3.7. The output of this module is listed in Table 3.8.

Using the information obtained we can now obtain the ideal design equations for the bandpass filter.

$$\omega_0 = \left[\frac{G_5G_3G_2}{G_4C_2C_1}\right]^{\frac{1}{2}} = \left[\frac{R_4}{R_5R_3R_2C_2C_1}\right]^{\frac{1}{2}}$$

$$\frac{\omega_0}{Q_0} = \frac{G_4G_1C_2}{G_4C_2C_1} = \frac{G_1}{C_1}$$

$$H_0\frac{\omega_0}{Q_0} = \frac{G_6G_4C_2}{G_4C_2C_1} = \frac{G_6}{C_1}$$

Solving for H_0 we find that

$$H_0 = \frac{G_6}{G_1} = \frac{R_1}{R_6}$$

Solving for Q_0, we find that

$$Q_0 = R_1\left[\frac{C_2R_5R_3R_2}{C_1R_4}\right]^{\frac{1}{2}}$$

```
7
Vs  7  0
G6  1  7
G1  1  2
C1  1  2
OA1  0  1  2
G2  2  3
G7  3  0
C2  3  4
OA2  1  3  4
G3  4  5
G8  5  0
G4  5  6
OA3  3  5  6
G5  6  1
.END
```

Table 3.1 Input to module 1

```
4
0
0
0
+1
-G1-sC1
0
-G5
-G6
-G2
-sC2
0
0
0
-G3
-G4
0
0
0
0
+1
V2
V4
V6
V7
```

Table 3.2 Output from module 1

```
4
0
0
0
+1
-G1-sC1
0
-G5
-G6
-G2
-sC2
0
0
0
-G3
-G4
0
0
0
0
+1
V2
V4
V6
V7
```

Table 3.3 Input to module 2

```
% a.out
****  INFORMATION ABOUT THE INPUT FILE *****
Please input the file name you would like to use
ideal.2
You have chosen to use file : ideal.2
Please answer Y or N to proceed
y
****  INFORMATION ABOUT THE OUTPUT FILE ****
Please input the file name you would like to use
ideal.3
You have chosen to use file : ideal.3
Please answer Y or N to proceed
y
Does the circuit contain any non-ideal operational amplifiers?
Please answer y or n only
n
Would you like to see the numerator terms?
Please answer y or n only
y
Would you like to prepare a file to check the transfer
function(s) for any valid filter functions?
y
Please input the file name you would like to use
ideal.4
You have chosen to use file : ideal.4
Please answer Y or N to proceed
y
```

Table 3.4 Prompt and Responses for module 2

```
% a.out
****  INFORMATION ABOUT THE INPUT FILE *****
Please input the file name you would like to use
ideal.4
You have chosen to use file : ideal.4
Please result Y or N to proceed
y
****  INFORMATION ABOUT THE OUTPUT FILE ****
Please input the file name you would like to use
ideal.5
You have chosen to use file : ideal.5
Please result Y or N to proceed
y
```

Table 3.5a Output from module 2

```
Denominator:
3
-sC2*sC1*G4
-sC2*G4*G1
-G5*G3*G2
V2:
1
+sC2*G6*G4
V4:
1
-G6*G4*G2
V6:
1
+G6*G3*G2
V7:
3
-sC2*sC1*G4
-sC2*G4*G1
-G5*G3*G2
.END
```

Table 3.5b Output for module 3

```
There exists a band pass filter at : V2
The value (H0*omega0/Q0) is :
        The numerator is :

G6*G4*C2
        The denominator is :

-G4*C2*C1

*****************************************************
The value of (omega0)**2 is :
        The numerator is :

-G5*G3*G2
        The denominator is :

-G4*C2*C1

*****************************************************
The value (omega0/Q0) is :
        The numerator is :

-G4*G1*C2
        The denominator is :

-G4*C2*C1

*****************************************************


There exists a low pass filter at : V4
The value (H0*(omega0)**2) is :
        The numerator is :

-G6*G4*G2
        The denominator is :

-G4*C2*C1

*****************************************************
The value of (omega0)**2 is :
        The numerator is :

-G5*G3*G2
        The denominator is :

-G4*C2*C1

*****************************************************
The value (omega0/Q0) is :
        The numerator is :

-G4*G1*C2
        The denominator is :

-G4*C2*C1
```

Table 3.8 Output from module 3

```
****************************************************

There exists a low pass filter at : V6
The value (H0*(omega0)**2) is :
        The numerator is :

G6*G3*G2
        The denominator is :

-G4*C2*C1

****************************************************
The value of (omega0)**2 is :
        The numerator is :

-G5*G3*G2
        The denominator is :

-G4*C2*C1

****************************************************
The value (omega0/Q0) is :
        The numerator is :

-G4*G1*C2
        The denominator is :

-G4*C2*C1

****************************************************

There exists no filter function at : V7
```

Table 3.8(contd) Output from module 3

CHAPTER IV

Error Analysis due to GBWP

4.1 INTRODUCTION

In this chapter, a correlation is made between errors in active filters and the circuit component values. A design example is given showing the use of the software to essentially eliminate these errors.

4.2 Methodologies of Error Reduction:

Even though the multiple op-amp biquads tended to perform better than their single op-amp counterparts still the errors due to the non-ideal op-amp cannot be reduced. It has been demonstrated in [5] that an important parameter in this respect is the gain-bandwidth product ($GBWP = 2\pi GBP$) and when excessive demands are made on it, high Q_0 performance will be limited to low frequencies and high frequency performance will be limited to low Q_0's.

To overcome this dependency on GBWP, a parameter the deigner has no control over, the following methods have been proposed [7]:

1. Placing an external trimming capacitor across a resistor [3].
2. Using identical op-amps having matched GBWPs [4-6].
3. Using matched resistors in such a manner that matched op-amps are not required [7-9].

With the advent in current VLSI technology it is possible to realize a complete circuit on a single chip instead of an op-amp on a chip. The short-comings of the first method are:

a. Capacitors of very small capacit ance can only be realized on a chip and even then a large amount of chip area is used.

b. This method also does not provide temperature compensation since the trimming capacitance and the GBWP's differ in their temperature coefficients.

The second and third methods presented above have been gaining popularity. It is easier to match op-amp GBWP's to an extent if they are realized on the same chip. If on the other hand the designer has to use different op-amps(i.e. those not realized on the same chip) then the third method is definitely attractive. In general the third method is more attractive since matching resistors is easier than trying to match GBWP of op-amp's. Also laser trimming of resistors can be performed to match resistors leading to the realized circuit having better performance even at high Q's and ω_0.

Before we can use any of these methods, we need to get some handle on why the GBWP's cause errors. Using ideal op-amps the form of the denominator of a biquad is:

$$s^2 + \frac{\omega_0}{Q_0}s + \omega_0^2 \qquad 4.1$$

The roots of this equation are the poles of the transfer function. If each op-amp is approximated by a first order equation in s then the realized transfer function is of order two plus the number of op-amps . In finding the roots of this equation one finds that the original roots are shifted and the remaining roots are located very far away in the left half plane. In the following section a method is presented for calculating the change in ω_0 and Q_0 due to the element values and GBWP.

4.3 Generalized Error Analysis using the Wilson-Bedri-Bowron Approximation[1,2,6]

For an arbitrary configuration realizing a second-order function the, denominator polynomial can be expressed as:

$$D(s) = s^2\, \alpha\, f_2(A) + s\, \beta\, f_1(A) + \gamma\, f_0(A) \tag{4.2}$$

where α, β and γ are constants determined by the passive components in the circuit. $f_0(A)$, $f_1(A)$ and $f_2(A)$ are functions of the open loop gain A of the operational amplifiers. These functions would tend to value of 1 if the op-amp has infinite gain. For the case of ideal op-amps the design equations may be defined by the natural frequency:

$$\omega_0 = \frac{1}{T_0} = \left[\frac{\gamma\, f_0(\infty)}{\alpha\, f_2(\infty)}\right]^{\frac{1}{2}} \tag{4.3}$$

and the selectivity factor, Q_0 is defined as:

$$Q_0 = \frac{\left[\alpha\, \gamma\, f_0(\infty) f_2(\infty)\right]^{\frac{1}{2}}}{\beta f_1(\infty)} \tag{4.4}$$

The gain dependent functions may be expressed as:

$$f_i(A) = f_i(\infty)\left[\,1 + R_i(A)\,\right] \tag{4.5}$$

where $R_i(A)$ is the i^{th} remainder function. Since for the ideal case we know that $R_i(\infty) = 0$, the remainder function can be written as:

$$R_i(A) = \sum_{j=1}^{N} \frac{n_{ij}}{A_j} + \sum_{k=1}^{N}\sum_{j=1}^{N} \frac{n_{ijk}}{A_j A_k} + \dots + \frac{n_{i12,\dots,N}}{A_1 A_2,\dots,A_N} \tag{4.6}$$

where N is the total number of amplifiers employed in the circuit design. The bilinear nature of the network requires that all the repeated-suffix coefficients $(n_{ijj}, n_{jkk}, \ldots)$ are zero. Since for a practical operational amplifier the open loop gain is much greater than one at the operating frequency,(for example the 741 op-amp has an open loop gain of 200,000 and its gain-bandwidth-product is 1MHz) then to a first approximation all the higher order terms in $R_i(A)$ can be neglected. In order to simplify the notation let:

$$\frac{n_i}{A} = \sum_{j=1}^{N} \frac{n_{ij}}{A_j} \tag{4.7}$$

Then $D(s) = \gamma D_1(s)$, where

$$D_1(s) = s^2 \frac{1}{\omega_0^2} \left[1 + \frac{n_2}{A}\right] + s \frac{1}{\omega_0 Q_0} \left[1 + \frac{n_1}{A}\right] + \left[1 + \frac{n_0}{A}\right] \tag{4.8}$$

Adopting a one-pole representation for an operational amplifier gain:

$$A_j = \frac{A_{0j}\omega_{0j}}{s + \omega_{0j}} \tag{4.9}$$

where A_{0j} is the open loop d.c. gain, ω_{0j} is the open-loop $3dB$ bandwidth and $A_{0j}\omega_{0j} = GBWP_j$ is the gain-bandwidth-product.

Therefore substituting for A_j in equation 4.10 we note:

$$\frac{n_i}{A} = \sum_{j=1}^{N} \frac{n_{ij}(s + \omega_{0j})}{A_{0j}\omega_{0j}} \tag{4.10}$$

$$\frac{n_i}{A} = s\sum_{j=1}^{N} \frac{n_{ij}}{A_{0j}\omega_{0j}} + \sum_{j=1}^{N} \frac{n_{ij}}{A_{ij}} \tag{4.11}$$

Let

$$\frac{n_i}{A} \equiv s\frac{n_i}{GBWP} + \frac{n_i}{A} \tag{4.12}$$

Note that $\dfrac{n_i}{GBWP}, \dfrac{n_i}{A_0} \ll 1$. Thus

$$D_1(s) = s^2\frac{1}{\omega_0^2}\left[1 + s\frac{n_2}{GBWP} + \frac{n_2}{A_0}\right] + s\frac{1}{\omega_0 Q_0}\left[1 + s\frac{n_1}{GBWP} + \frac{n_1}{A_0}\right]$$

$$+ \left[1 + s\frac{n_0}{GBWP} + \frac{n_0}{A_0}\right] \qquad 4.13$$

Multiplying and collecting the terms, we have:

$$D_1(s) = s^3\left[\frac{1}{\omega_0^2}\right]\frac{n_2}{GBWP} + s^2\left[\frac{1}{\omega_0^2}\right]\left[1 + \frac{n_2}{A_0} + \frac{\omega_0}{Q_0}\frac{n_1}{GBWP}\right]$$

$$+ s\left[\frac{1}{\omega_0}\right]\left[\frac{1}{Q_0} + \frac{n_1}{Q_0 A_0} + \omega_0\frac{n_0}{GBWP}\right] + \left[1 + \frac{n_0}{A_0}\right] \qquad 4.14$$

Multiplying equation 4.17 throughout by $\dfrac{1}{1 + \dfrac{n_0}{A_0}} \approx 1 - \dfrac{n_0}{A_0}$ transforms equation 4.17

to:

$$D_1(s) = \left[1 + \frac{n_0}{A_0}\right]\left\{s^3\left[\frac{1}{\omega_0^2}\right]\left[1 - \frac{n_0}{A_0}\right]\frac{n_2}{GBWP} + s^2\left[\frac{1}{\omega_0^2}\right]\left[1 - \frac{n_0}{A_0}\right]\left[1 + \frac{n_2}{A_0} + \frac{\omega_0}{Q_0}\frac{n_1}{GBWP}\right]\right\}$$

$$+ \left[1 + \frac{n_0}{A_0}\right]\left\{s\left[\frac{1}{\omega_0}\right]\left[1 - \frac{n_0}{A_0}\right]\left[\frac{1}{Q_0} + \frac{n_0}{Q_0 A_0} + \omega_0\frac{n_0}{GBWP}\right] + 1\right\} \qquad 4.15$$

Since the open loop gain of an operational amplifier is usually very high, then the quantities $\frac{1}{A_0} \approx 10^{-5}$ and $\frac{1}{A_0^2} \approx 10^{-10}$ can be neglected. If we let the term in the square brackets equal $D_2(s)$, then with the above approximation in mind we have:

$$D_2(s) = s^3\left[\frac{1}{\omega_0^2}\right]\left[\frac{n_2}{GBWP}\right] + s^2\left[\frac{1}{\omega_0^2}\right]\left[1 + \frac{n_2}{A_0} + \frac{\omega_0}{Q_0}\frac{n_1}{GBWP} - \frac{n_0}{A_0}\right]$$

$$+ s\left[\frac{1}{\omega_0}\right]\left[\frac{1}{Q_0} + \frac{n_0}{Q_0 A_0} + \omega_0\frac{n_0}{GBWP} - \frac{n_0}{Q_0 A_0}\right] + 1 \qquad 4.16$$

Equation 4.16 can be factored as:

$$D_2(s) = \left[s\tau + 1\right]\left[s^2\frac{1}{\hat{\omega}_0^2} + s\frac{1}{\hat{\omega}_0\hat{Q}_0} + 1\right] \qquad 4.17$$

where τ is the auxiliary time constant, $\hat{\omega}_0$ is the realized natural frequency and $\hat{Q}_0$ is the realized selectivity factor. Equation 4.17 can be rewritten as: as:

$$D_2(s) = \left[s\tau + 1\right]\left[s^2\hat{T}_0^2 + s\frac{\hat{T}_0}{\hat{Q}_0} + 1\right] \qquad 4.18$$

where $\hat{T}_0$ is the realized natural period. We will assume that $\hat{T}_0 = T_0(1 + \Delta t)$ and $\frac{1}{\hat{Q}_0} = \frac{1}{Q_0}(1 + \Delta q)$, where Δt and Δq are the fractional shifts in the design values of the frequency and selectivity factor due to finite GBWP, i.e., $\Delta t < 1$ and $\Delta q < 1$. Multiplying the terms in the equation 4.18 yields:

$$D_2(s) = s^3\tau\hat{T}_0^2 + s^2\left[\frac{\tau\hat{T}_0}{\hat{Q}_0} + \hat{T}_0^2\right] + s\left[\tau + \frac{\hat{T}_0}{\hat{Q}_0}\right] + 1 \qquad 4.19$$

Substituting for the terms $\hat{T}_0$ and $\hat{Q}_0$ in the equation 4.19 yields:

$$D_2(s) = s^3\tau T^2(1 + 2\Delta t + \Delta^2 t) + s^2\left\{\frac{\tau T_0}{Q_0}(1 + \Delta t)(1 + \Delta q) + T_0^2(1 + 2\Delta t + \Delta^2 t)\right\}$$

$$+ s\left\{\tau + \frac{T_0}{Q_0}(1 + \Delta t)(1 + \Delta q)\right\} + 1 \qquad 4.20$$

$D_2(s)$ is simplified by using the approximation that Δt and Δq are less than one.

$$D_2(s) \approx s^3\tau T^2(1 + 2\Delta t) + s^2\left\{\frac{\tau T_0}{Q_0}(1 + \Delta t + \Delta q) + T_0^2(1 + 2\Delta t)\right\}$$

$$+ s\left\{\tau + \frac{T_0}{Q_0}(1 + \Delta t + \Delta q)\right\} + 1 \qquad 4.21$$

Comparing the coefficients of s^3 in equation 4.19 and equation 4.24 yields the following equality:

$$\tau T_0^2(1 + 2\Delta t) = T_0^2\left[\frac{n_2}{GBWP}\right] \qquad 4.22$$

or $\tau \approx \dfrac{n_2}{GBWP}$. Comparing the coefficients of s^2 in equation 4.19 and equation 4.24 yields the following equality:

$$\tau\frac{T_0}{Q_0}(1 + \Delta t + \Delta q) + T_0^2(1 + 2\Delta t) = T_0^2\left\{1 + \frac{n_2}{A_0} + \frac{1}{T_0Q_0}\frac{n_1}{GBWP} - \frac{n_0}{A_0}\right\} \qquad 4.23$$

Dividing the equation by T_0^2 yields:

$$\frac{\tau}{T_0Q_0}(1 + \Delta t + \Delta q) + (1 + 2\Delta t) = \left\{1 + \frac{n_2}{A_0} + \frac{1}{T_0Q_0}\frac{n_1}{GBWP} - \frac{n_0}{A_0}\right\} \qquad 4.24$$

Neglecting the quantity $(\Delta t + \Delta q)$ in the left hand side of equation 4.24. This yields:

$$2\Delta t \approx \left\{\frac{n_2}{A_0} + \frac{1}{T_0Q_0}\frac{n_1}{GBWP} - \frac{n_0}{A_0} - \frac{\tau}{T_0Q_0}\right\} \qquad 4.25$$

Replacing τ by its value found, equation 4.25 yields:

$$\Delta t = \frac{1}{2}\left[\frac{n_2}{A_0} + \frac{1}{T_0Q_0}\frac{n_1}{GBWP} - \frac{n_0}{A_0} - \frac{n_2}{GBWP}\frac{1}{T_0Q_0}\right]$$

$$\Delta t = \frac{1}{2}\left\{\frac{n_2}{A_0} - \frac{n_0}{A_0}\right\} + \frac{\omega_0}{2Q_0}\left\{\frac{n_1}{GBWP} - \frac{n_2}{GBWP}\right\} \qquad 4.26$$

Comparing the coefficients of s in equation 4.19 and equation 4.24 yields the following equality:

$$\tau + \frac{T_0}{Q_0}(1 + \Delta t + \Delta q) = T_0\left[\frac{1}{Q_0} + \frac{n_1}{Q_0 A_0} + \frac{1}{T_0}\frac{n_0}{GBWP} - \frac{n_0}{Q_0 A_0}\right] \quad 4.27$$

$$\frac{Q_0}{T_0}\tau + 1 + \Delta t + \Delta q = 1 + \frac{n_1}{A_0} + \frac{Q_0}{T_0}\frac{n_0}{GBWP} - \frac{n_0}{A_0} \quad 4.28$$

$$\Delta q = \frac{n_1}{A_0} + \frac{Q_0}{T_0}\frac{n_0}{GBWP} - \frac{n_0}{A_0} - \frac{Q_0}{T_0}\tau - \Delta t \quad 4.29$$

Expanding the known quantities in equation 4.32 and rewriting the equation yields:

$$\Delta q = \frac{n_1}{A_0} + \frac{Q_0}{T_0}\frac{n_0}{GBWP} - \frac{n_0}{A_0} - \frac{Q_0}{T_0}\frac{n_2}{GBWP} - \frac{1}{2}\left\{\frac{n_2}{A_0} - \frac{n_0}{A_0}\right\}$$
$$- \frac{\omega_0}{2Q_0}\left\{\frac{n_1}{GBWP} - \frac{n_2}{GBWP}\right\} \quad 4.30$$

$$\Delta q = \frac{n_1}{A_0} - \frac{1}{2}\frac{n_0}{A_0} - \frac{1}{2}\frac{n_2}{A_0} + \frac{Q_0}{T_0}\left[\frac{n_0}{GBWP} - \frac{n_2}{GBWP}\right]$$
$$- \frac{1}{2Q_0T_0}\left[\frac{n_1}{GBWP} - \frac{n_2}{GBWP}\right] \quad 4.31$$

and finally the error in Q_0 due to non-ideal effects in the operational amplifier:

$$\Delta q = \frac{n_1}{A_0} - \frac{1}{2}\frac{n_0}{A_0} - \frac{1}{2}\frac{n_2}{A_0} + \omega_0 Q_0\left[\frac{n_0}{GBWP} - \frac{n_2}{GBWP}\right]$$
$$- \frac{\omega_0}{2Q_0}\left[\frac{n_1}{GBWP} - \frac{n_2}{GBWP}\right] \quad 4.32$$

From equations 4.29 and 4.35 we can observe that both Δt and Δq have terms dependent upon the gain-bandwidth-product of the amplifiers. It is also observed from equations 4.29 and 4.35 that Δq is dependent on the error in natural period Δt.

Since the quantity $\omega_0 Q_0$ is usually a large number, equation 4.35 is dominated by the term:

$$\omega_0 Q_0 \left[\frac{n_0}{GBWP} - \frac{n_2}{GBWP} \right] \qquad 4.33$$

If it is possible to select the components such that $\frac{n_0}{GBWP} = \frac{n_2}{GBWP}$ then the error in Q_0 is greatly reduced. If this term were eliminated then the term $\frac{\omega_0}{Q_0}\left[\frac{n_1}{GBWP} - \frac{n_2}{GBWP} \right]$ would dominate the error. However this is the term that dominates the error in Δt. Thus if it is possible to select $\frac{n_1}{GBWP} = \frac{n_2}{GBWP}$ then both Δt and Δq would be very small. In the above discussion the term GBWP appears. Recall from equation 4.15 that $\frac{n_i}{GBWP}$ is a summation of terms. If the op-amps are matched then the individual GBWP's appear as a common factor and the matching described could then be performed.

The final terms in equations 4.29 and 4.34 have a $1/A_0$ term. Since A_0 is of the order of a hundred thousand, these terms have no measurable effect.

4.4 Example

To illustrate how the Wilson-Bedri-Bowron approximation can be used to design a low error circuit, consider the example of Section 2.6. This time we need to perform the analysis with non-ideal operational amplifiers modelled as a voltage controlled voltage source of gain A_i. Since $1 / A_i$ appears in the answers if the node equations are formed, $1 / A_i$ was defined as B_i for the VCVS given in Appendix A1.2.

The source file for the circuit of Figure 4.1 must be prepared with ideal op-amps replaced by the non-ideal op-amps. This file is listed in table 4.1. Module 1 is run again and its output file is listed in table 4.2. The prompts and responses for module 2 are listed in table 4.3 while the relevant output from this module is listed in table 4.4. The output format is the determinant found, the number of terms, one term per line with the ".END" statement as the last line. The determinant output of module 2 is presented in table 4.5. Equation 4.11 uses the terms in the determinant which have the order of B_i less than or equal to one. Higher order terms of B_i are neglected.

From table 4.5 it is possible to factor and group terms to match equation 4.11. The results are as follows:

$$D(s) = \gamma\, D_2(s) = s^2(C_2C_1G_4)(1 + N_2) + (C_2G_4G_1)(1 + N_1) + G_5G_3G_2(1 + N_0) \quad 4.34$$

$$N_2 = \frac{G_8}{G_4A_3} + \frac{1}{A_3} + \frac{1}{A_2} + \frac{1}{A_1} + \frac{G_3}{G_4A_3} \quad 4.35$$

$$N_1 = -\frac{G_8G_5}{G_4G_1A_1} + \frac{G_8}{G_4A_3} + \frac{G_6}{G_1A_1} + \frac{1}{A_1}$$

$$+ \frac{1}{A_2} + \frac{1}{A_3} + \frac{G_3}{G_4A_3} + \frac{C_1G_7}{C_2G_1A_2} + \frac{C_1G_2}{C_2G_1A_2} \quad 4.36$$

$$N_0 = \frac{G_8}{G_3A_2} + \frac{G_7}{G_2A_1} + \frac{G_7G_4G_1}{G_5G_3G_2A_2} + \frac{G_4}{G_3A_2} + \frac{1}{A_2} + \frac{1}{A_1} + \frac{G_4G_1}{G_5G_3A_2} \quad 4.37$$

Replacing A_i by $GBWP_i/s$, then

$$\Delta t = \left\{N_1 - N_2\right\}\frac{f_0}{2Q_0} \quad 4.38$$

$$\Delta q = \left\{N_1 - N_2\right\}f_0Q_0 - \Delta t \quad 4.39$$

Thus $\Delta t = 0$ and $\Delta q = 0$ when $N_1 - N_2 = 0$ and $N_0 - N_2 = 0$

$$N_1 - N_2 = -\frac{G_8G_5}{G_4Gsub1GBWP_1} + \frac{G_6}{G_1GBWP_1} + \frac{C_1G_7}{C_2G_1GBWP_2} + \frac{C_1G_2}{C_2G_1GBWP_2} \quad 4.40$$

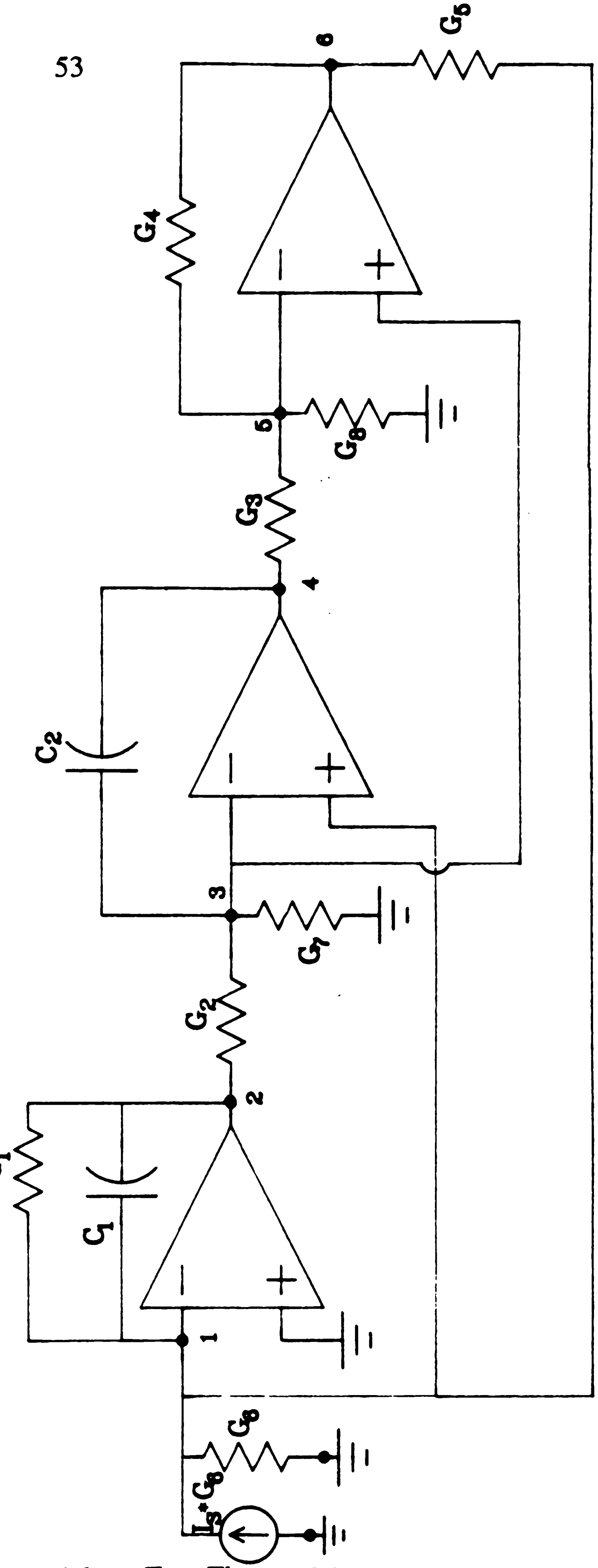


Figure 4.1 Circuit Generated from Tow-Thomas Biquad

```
6
Is*G6  1  0
G6  1  0
G1  1  2
C1  1  2
NOA1  0  1  2
G2  2  3
G7  3  0
C2  3  4
NOA2  1  3  4
G3  4  5
G8  5  0
G4  5  6
NOA3  3  5  6
G5  6  1
.END
```

Table 4.1 Input to module 1

```
6
+G6
0
0
0
0
0
+G6+G1+sC1+G5
-G1-sC1
0
0
0
-G5
0
-G2
+G2+G7+sC2
-sC2
0
0
0
0
0
-G3
+G3+G8+G4
-G4
+1
+B1
0
0
0
0
-1
0
+1
+B2
0
0
0
0
-1
0
+1
+B3
V1
V2
V3
V4
V5
V6
```

Table 4.2 Output from module 1

```
% a.out
****  INFORMATION ABOUT THE INPUT FILE *****
Please input the file name you would like to use
thesis.2
You have chosen to use file : thesis.2
Please answer Y or N to proceed
y
****  INFORMATION ABOUT THE OUTPUT FILE ****
Please input the file name you would like to use
thesis.3
You have chosen to use file : thesis.3
Please answer Y or N to proceed
y
Does the circuit contain any non-ideal operational amplifiers?
Please answer y or n only
y
Would you like the B**2 and higher order terms removed?
Please answer y or n only
y
Would you like to prepare a file to run the error analysis?
Please answer y or n only
y
Would you like to see the numerator terms?
Please answer y or n only
n
Please input the file name you would like to use
thesis.4
You have chosen to use file : thesis.4
Please answer Y or N to proceed
y
```

Table 4.3 Prompt and Responses for module 2

```
Denominator:
24
-sC2*sC1*G8*B3
-sC2*sC1*G4*B3
-sC2*sC1*G4*B2
-sC2*sC1*G4*B1
-sC2*sC1*G4
-sC2*sC1*G3*B3
+sC2*G8*G5*B1
-sC2*G8*G1*B3
-sC2*G6*G4*B1
-sC2*G4*G1*B3
-sC2*G4*G1*B2
-sC2*G4*G1*B1
-sC2*G4*G1
-sC2*G3*G1*B3
-sC1*G7*G4*B2
-sC1*G4*G2*B2
-G8*G5*G2*B2
-G7*G5*G3*B1
-G7*G4*G1*B2
-G5*G4*G2*B2
-G5*G3*G2*B2
-G5*G3*G2*B1
-G5*G3*G2
-G4*G2*G1*B2
.END
```

Table 4.4 Output of module 2 used as input to module 4

G6	LENGTH ERROR	-G1-sC1	0	0	0	-G5	V1
0	0	-G2	G2+G7+sC2	-sC2	0	0	V2
0	0	0	0	-G3	G3+G8+G4	-G4	V3
0	1	B1	0	0	0	0	V4
0	-1	0	1	B2	0	0	V5
0	0	0	-1	0	1	B3	V6

DENOMINATOR IS :

```
-sC2*sC1*G8*B3
-sC2*sC1*G4*B3-sC2*sC1*G4*B2-sC2*sC1*G4*B1-sC2*sC1*G4
-sC2*sC1*G3*B3+sC2*G8*G5*B1-sC2*G8*G1*B3-sC2*G6*G4*B1-sC2*G4*G1*B3
-sC2*G4*G1*B2-sC2*G4*G1*B1-sC2*G4*G1-sC2*G3*G1*B3-sC1*G7*G4*B2
-sC1*G4*G2*B2-G8*G5*G2*B2-G7*G5*G3*B1-G7*G4*G1*B2-G5*G4*G2*B2
-G5*G3*G2*B2-G5*G3*G2*B1-G5*G3*G2-G4*G2*G1*B2
```

Table 4.5 Determinant with B^2 and higher order terms suppressed

$$N_0 - N_2 = \frac{G_8}{G_3 GBWP_2} + \frac{G_7}{G_2 GBWP_1} + \frac{G_7 G_4 G_1}{G_5 G_3 G_2 GBWP_2} + \frac{G_4}{G_3 GBWP_2}$$

$$+ \frac{G_4 G_1}{G_5 G_3 GBWP_2} - \frac{G_8}{G_4 GBWP_3} - \frac{1}{GBWP_3} - \frac{G_3}{G_4 GBWP_3} \qquad 4.41$$

If $C_1 = C_2$ and $GBWP_1 = GBWP_2$ then $\Delta t = 0$, if $G_8 = G_4(G_2 + G_6 + G_7)/G_5$. If $C_1 = C_2$, $G_3 = G_4$ and $GBWP_1 = GBWP_2 = GBWP_3$ then $\Delta q = 0$ if

$$G_7 = \frac{G_2(G_5 - G_1)}{G_5 + G_1} \qquad 4.42$$

Expressing these results in terms of resistors we have the following:

Let $C_1 = C_2$, $R_3 = R_4 = R_5$ thus

$$\omega_0 = \frac{1}{C_1\sqrt{R_3 R_2}} \qquad 4.43$$

$$Q_0 = \frac{R_1}{\sqrt{R_3 R_2}} \qquad 4.44$$

$$H_0 = \frac{R_1}{R_6} \qquad 4.45$$

Select C_1, R_3, R_2, R_1 and R_6. Then select $R_7 = \dfrac{R_2(R_5 + R_1)}{R_1 - R_5}$ and $R_8 = R_2 \| R_6 \| R_7$.

If we assume matched op-amps then $GBWP_i$ factors out and it is possible to automate this last task. For example to find Δt from the determinant, we have some $X(s)(1 + N_1)$ and $Y(s)(1 + N_2)$ where $X(s)$ and $Y(s)$ are quantities that contain no B_i terms. Finding the term $Z(s) = X(s) \cap Y(s)$. Therefore

$$N_1 - N_2 = \frac{Z(s)(1 + N_1) - Z(s)(1 + N_{2)}}{Z(s)} \qquad 4.46$$

Module 4 finds the common denominator, multiplies the appropriate terms of $D(s)$, performs the subtraction with B_i set equal to one and cancels the like terms. This is shown in tables 4.6 and 4.7. From this output it is easy to see if any opposite signs exist so as to allow $\Delta t = 0$. A similar process is used to find Δq as illustrated in table 4.8 and 4.9.

```
Aug 19 21:47 1987  ideal Page 1

% a.out
****  INFORMATION ABOUT THE INPUT FILE *****
Please input the file name you would like to use
ideal.4
You have chosen to use file : ideal.4
Please result Y or N to proceed
y
****  INFORMATION ABOUT THE OUTPUT FILE ****
Please input the file name you would like to use
ideal.5
You have chosen to use file : ideal.5
Please result Y or N to proceed
y
```

Table 4.7 Prompt and Responses for module 4 to find Δt

```
Jul  2 02:19 1987  thesis.5 Page 1


sC2*sC1*G8*G5*B1
-sC2*sC1*G6*G4*B1-sC1*sC1*G7*G4*B2-sC1*sC1*G4*G2*B2

The common denominator is: -sC2*sC1*G4*G1
You have subtracted the  2 from the  1 order terms
```

Table 4.8 Error in Δt

```
% a.out
****  INFORMATION ABOUT THE INPUT FILE *****
Please input the file name you would like to use
thesis.4
You have chosen to use file : thesis.4
Please answer Y or N to proceed
y
****  INFORMATION ABOUT THE OUTPUT FILE ****
Please input the file name you would like to use
thesis.6
You have chosen to use file : thesis.6
Please answer Y or N to proceed
y
The highest orders power of 's' in the terms input is: 2
Which two coefficients would you like subtracted
0
2
```

Table 4.8 Prompt and Responses for module 4 to find Δq

```
Jul 28 21:24 1987  thesis.6 Page 1


-sC2*sC1*G8*G5*G4*G2*B2
+sC2*sC1*G8*G5*G3*G2*B3-sC2*sC1*G7*G5*G4*G3*B1
-sC2*sC1*G7*G4*G4*G1*B2-sC2*sC1*G5*G4*G4*G2*B2+sC2*sC1*G5*G4*G3*G2*B3
+sC2*sC1*G5*G3*G3*G2*B3-sC2*sC1*G4*G4*G2*G1*B2

The common denominator is: -sC2*sC1*G5*G4*G3*G2
You have subtracted the  2 from the  0 order terms
```

Table 4.9 Part of error in Δq

CHAPTER 5

Future Research

5.1 CONCLUSION

The four modules can be effectively used for the identification of filter function, writing transfer functions and reduction of errors using the Wilson-Bedri-Bowron approximation. The program was first developed in VMS PASCAL. Due to lack of portability of that language as well as the long run-times an alternative was sought. The new version was written in C. Dynamic memory allocation is used in the program to avoid wasting memory space during peak memory use. The run times have also improved considerably. The modules behave as expected and can be used to reduce dependency of ω_0 and Q_0.

5.2 Future Research and Topic of Interest

There are some very interesting problems that can be studied. The following problems can have a definite impact on the design of active filters and their properties.

1. The band reject or notch filter has a finite notch depth. This notch depth is also a function of the gain-bandwidth-product. Also the numerator term needs to be studied to remove the effect of parasitic zeroes introduced.
2. The high frequency one-pole model introduces parasitic roots. The value of these roots can tell a lot about the circuit. If these roots are real then the circuit is 2-pole stable, else it is 2-pole unstable[10]. The high frequency poles can be

studied symbolically by shorting out the capacitors. This can be used to study the high frequency response of the circuit.

3. A very important class of circuits needs to be looked at. Cascaded circuits are widely used to obtain desired circuit response. At present the program can only handle a biquad, extending the idea for cascaded circuits would make the program more complete.

4. MOSFET-C circuits can be similarly analysed. Unfortunately these circuits have fourth order denominators and have a common second order numerator and denominator. If these common roots can be symbolically determined then this program can be used to analyze this new form of circuits.

APPENDIX

Appendix A

Writing Input Files for SLAP Modules

A1.1 INTRODUCTION

The program consists of four modules which can be run independently. The user can prepare files for using any particular module. Each program prepares files to be used by the next module if the user answers the prompts properly as to what he wants to be done.

A1.2 File Format for Module 1

For any linear active circuit, label the ground node "0" and number the remaining nodes from 1 to n consecutively. The first line in the file must be n, the number of nodes in the circuit (not counting the ground node). The remaining lines must identify the elements to the program as follows. Always the last statement in the file should be ".END". The apostrophes are not to be included.

Resistor (see Figure A1.1a)

Form: Gxxx a b

Examples: G1 1 2

G37 4 7

Ga*2 3 4

G is the conductance of the resistance connected between nodes *a* and *b*. xxx is any alphanumeric string that can be used to uniquely identify this resistor. This alphanumeric string can be anything except the character "B", since it is a special symbol used by the second module of the program.

Capacitor (see Figure A1.1b)

Form:Cxxx a b

Examples: C1 1 2

C37 4 7

Ca*2 3 4

C denotes a capacitor connected between nodes *a* and *b*. xxx is any alphanumeric string that can be used to uniquely identify this capacitor. Similar constraints as that for the conductor are applicable in the choice of the alphanumeric string.

Inductor (see Figure A1.1.1c)

Form:Lxxxx a b

Examples: L 1 2

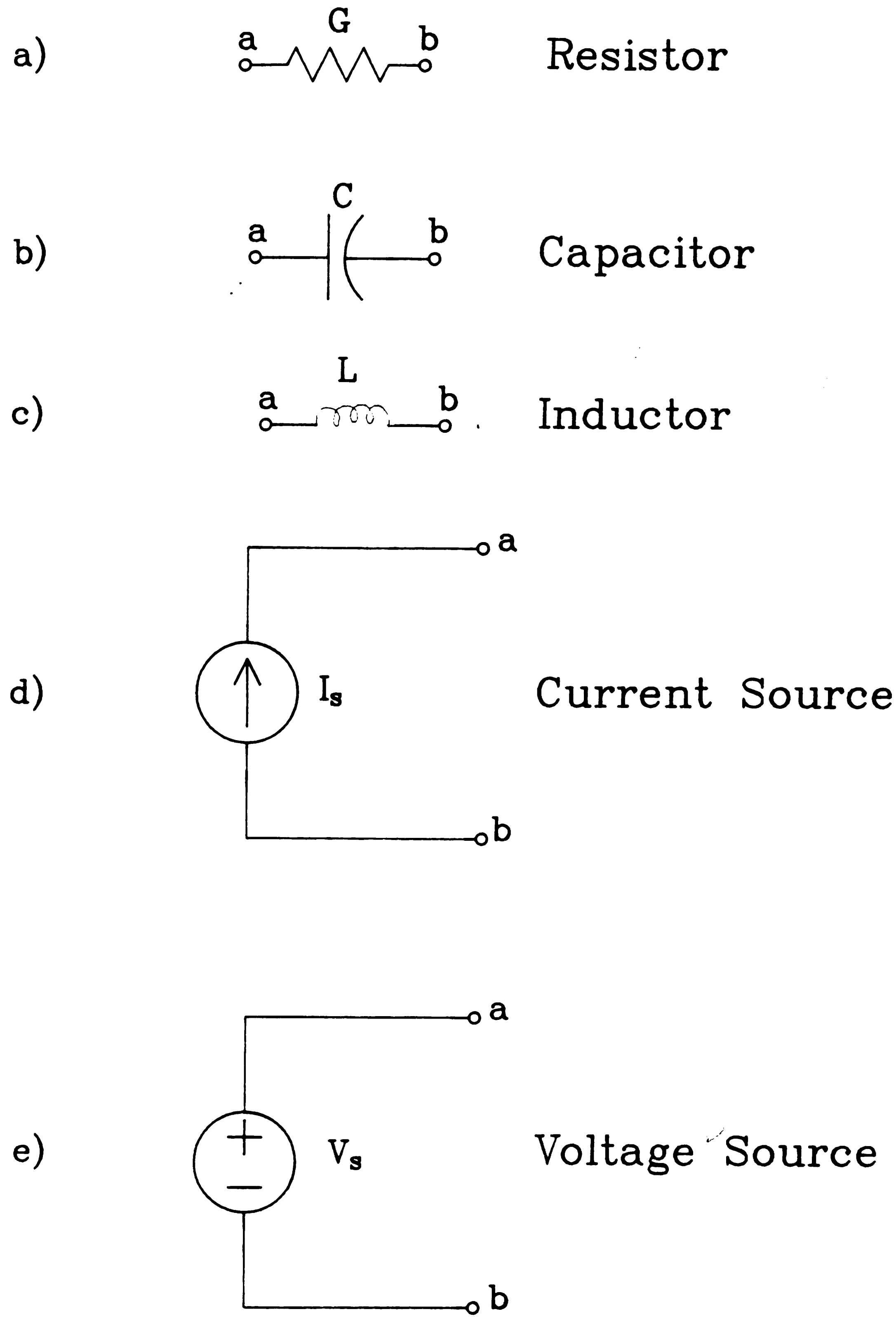


Figure A1.1 Circuit Symbols

L is the inductance connected between nodes *a* and *b*. xxx is any alphanumeric string that can be used to uniquely identify this inductor. Similar constraints as that for the conductor are applicable in the choice of the alphanumeric string. The model used for the inductor creates an extra node. This node will be greater than the total number of nodes. Any results with regard to this node should be neglected.

Current Source (see Figure A1.1d)

Form: Ixxxx a b

Examples: I1 1 2

Is*G6 4 7

Ia*2 3 4

I is the current source connected between nodes *a* and *b*. xxx is any alphanumeric string that can be used to uniquely identify this current source. There are no constraints on the alphanumeric string for this element. If an asterisk is present in the string then the program uses the value of the current source the as string to the right of the asterisk, otherwise it uses the constant one (1).

Voltage Source (see Figure A1.1e)

Form: Vxxxx a b

Examples: V 1 2

V12 4 7

Va*s 3 4

V is the voltage source connected between nodes *a* and *b*. xxx is any alphanumeric string that can be used to identify this element. There are no constraints on the alphanumeric string for this element. The program internally replaces the voltage source by a unity. This in effect increase the matrices order by one. None of the voltage sources are uniquely identified.

Voltage Controlled Current Source (see Figure A1.2a)

VCCxx a b c d

VCC is the element identifier for a voltage controlled current source. The four nodes are labeled as shown in the figure A1.2a. The model that is inserted for this element is shown in Chapter 2. The program does not care if the identifiers xx are unique or not as it supplies those identifiers itself by sequentially numbering the VCC's.

Current Controlled Voltage Source (see Figure A1.2b)

CCVxx a b c d

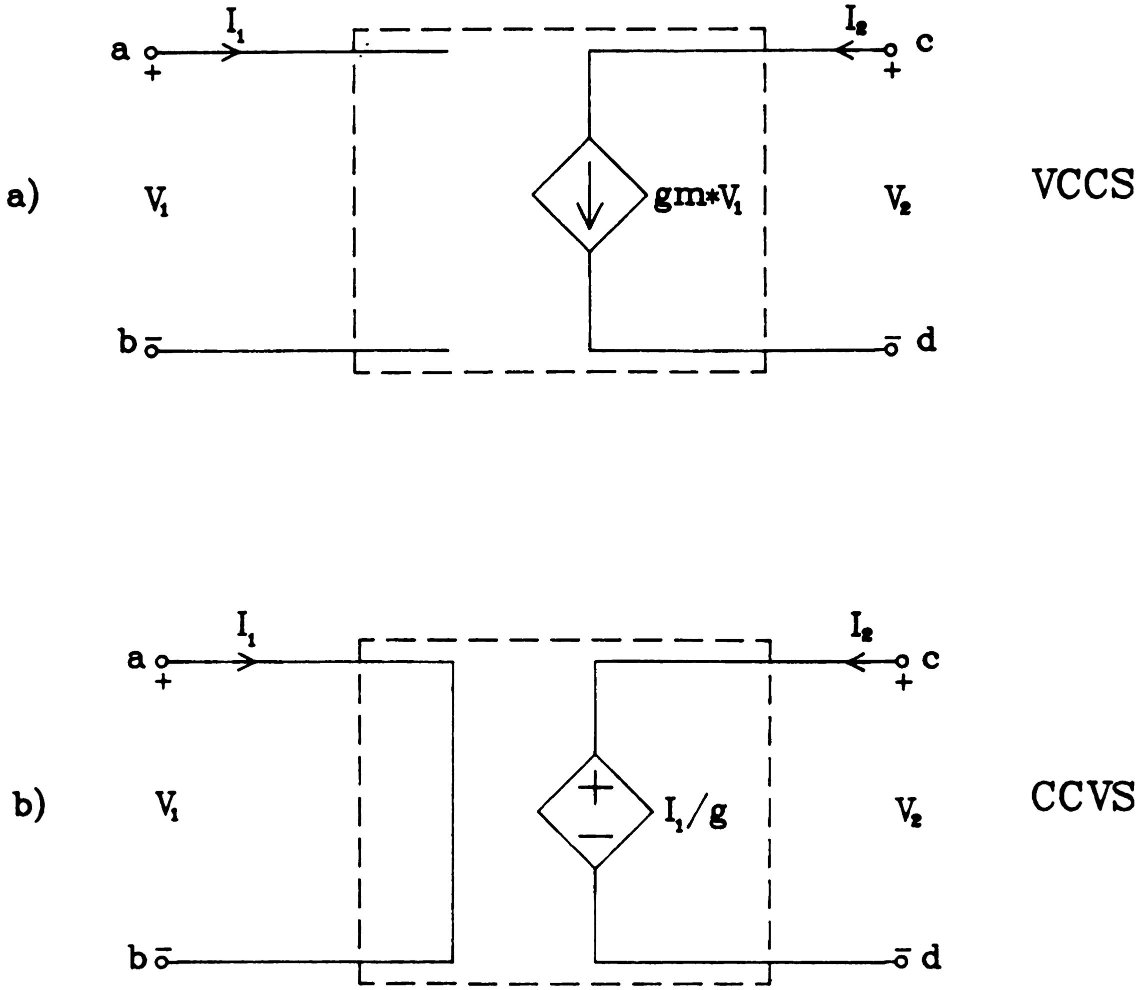


Figure A1.2 Circuit Symbols

CCV is the element identifier for a current controlled voltage source. The four nodes are labeled as shown in the figure A1.2b. The model that is inserted for this element is shown in Chapter 2. The program does not care if the identifiers xx are unique or not as it supplies those identifiers itself by sequentially numbering the CCV's.

Voltage Controlled Voltage Source (see Figure A1.3a)

VCVxx a b c d

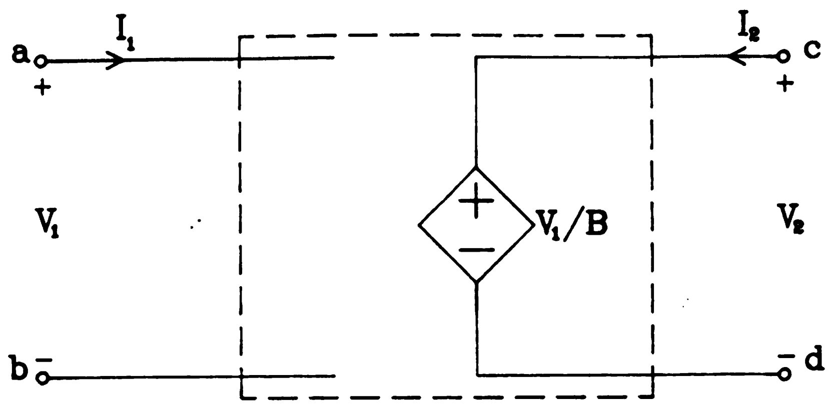
VCV is the element identifier for a voltage controlled voltage source. The four nodes are labeled as shown in the figure A1.2c. The model that is inserted for this element is shown in Chapter 2. The program does not care if the identifiers xx are unique or not as it supplies those identifiers itself by sequentially numbering the VCV's.

Current Controlled Current Source (see Figure A1.3b)

CCCxx a b c d

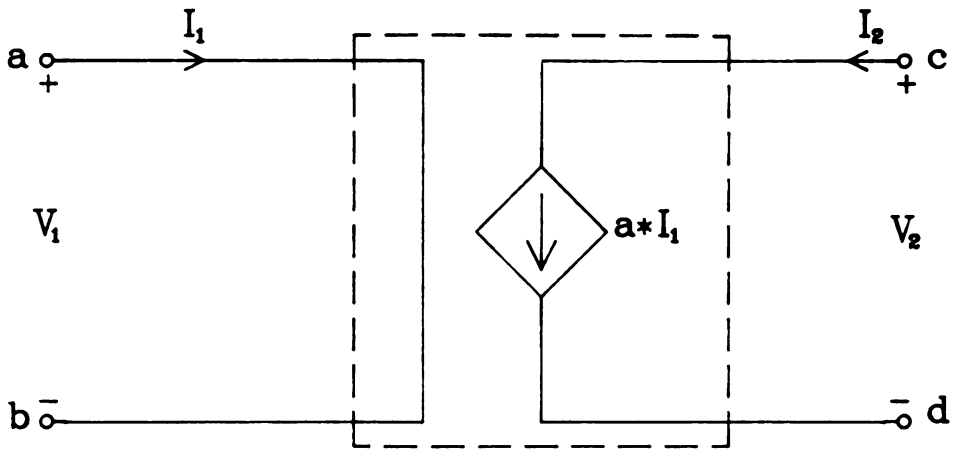
CCC is the element identifier for a current controlled current source. The four nodes are labeled as shown in the figure A1.3a. The model that is inserted for this element is shown in Chapter 2. The program does not care if the identifiers xx are unique or not as it supplies those identifiers itself by sequentially numbering the CCC's.

a)



VCVS

b)



CCCS

Figure A1.3 Circuit Symbols

Non ideal Operational Amplifier

NOAxx a b c

NOA is the element identifier for a non-ideal amplifier. The nodes are numbered as shown in the figure 2c. This element is modelled as a voltage controlled voltage source with node *d* set to 0 therefore, the user has to supply only three nodes. Node *a* is the non-inverting input, node *b* is the inverting input and node *c* is the output. A "Bxx" is inserted as an element that is actually the inverse of the ideal gain of the amplifier. The program supplies the xx identifers as well as the B identifier.

Nullor Combination (see Figure A1.4a)

NNxx a b c d

NN is the element identifier for a nullor combination in which all four the nodes are floating. Figure A1.3b shows the connections of this element. xx is any alphanumeric string. The program ignores xx.

Ideal Operational Amplifier (see Figure A1.4c)

OAxx a b c

OA is the element identifier for an ideal op-amp. The first two nodes are the input nodes and the last node is the output node. Node *b* is the inverting terminal, node *a* is the non-inverting terminal and node *c* is the output terminal. The fourth node is set to zero internally in the program. The model is the same as of Figure A1.3b with the differences talked about above.

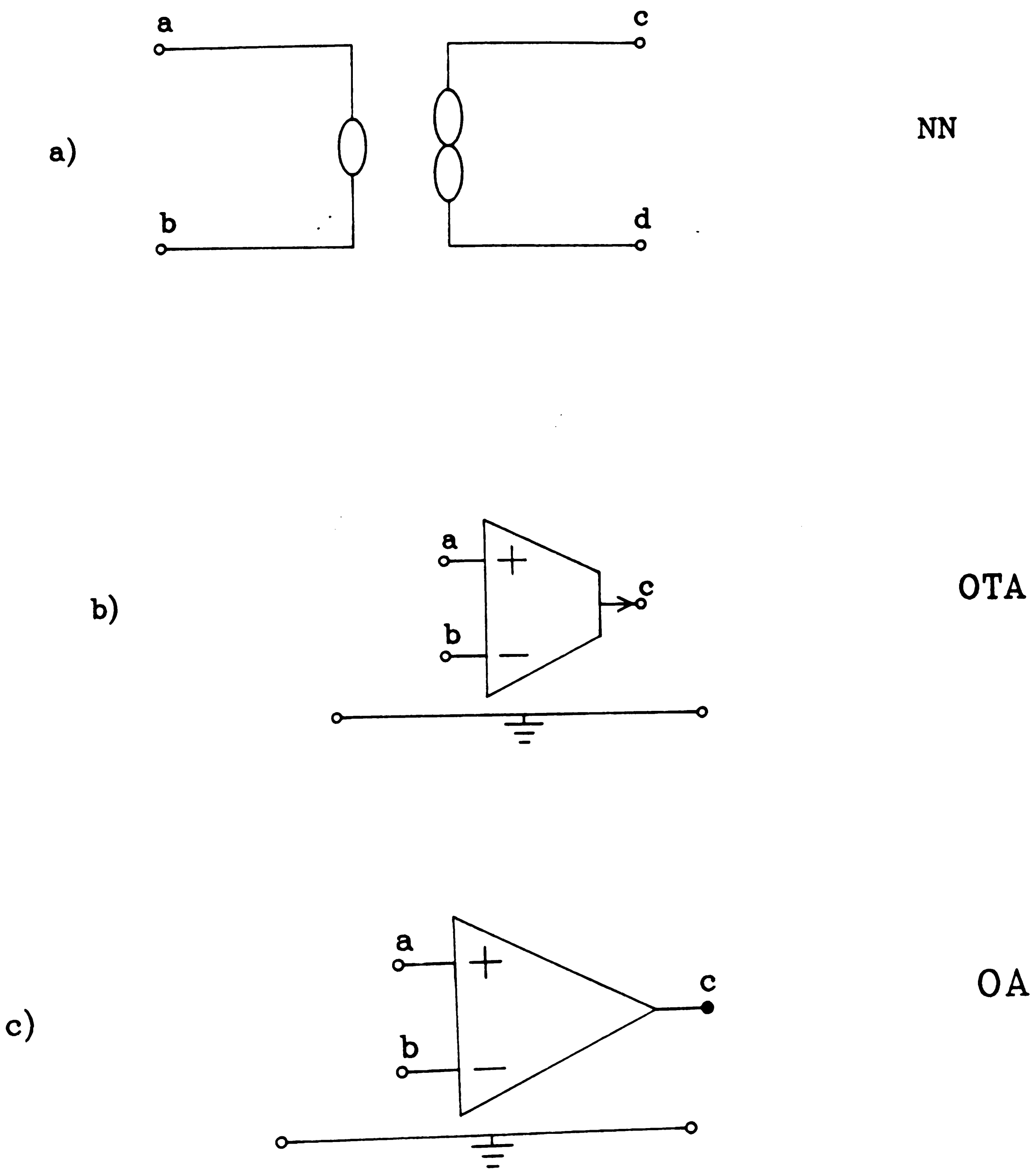


Figure A1.4 Circuit Symbols

Operational Transconductance Amplifier (see Fig A1.4b)

OTAxx a b c

OTA is the element identifier for an operational transconductance amplifier. It has been modelled as a voltage controlled current source inside the program. Ths model is shown in Chapter 2. Node connections are the same as that of a VCCS, with the following exceptions. Node *a* is the invering input, node *b* is the non-inverting input and node *c* is the output. Node *d* for the case of a VCCS is set to zero within the program. Also the xx identifier is rejected by the program and it gives the same symbol as that for a VCCS counting up from one.

General Information

For all devices when writing their connections the convention followed is that the very first node is the "positive" node, followed by the "negative" node. For four terminal devices the third node is the "positive" node which is then followed by the "negative" node. The current sources have the first node as the one into which current is flowing and the second node is the node from where the current is flowing from.

Although no checking is done but it is recommended that the user reserve a ground node as "0". Also the user should number all nodes consecutively in the circuit. If this is not done then the user will get a big matrix of symbolic entries which he/she can do without. In order to use the determinant program it is important that the the matrix does not have a row or column of zeroes as that would lead to the determinant being zero. If the user desires to increase the length of the individual strings for each element, the varibale TERML should be set to the desired length accordingly.

A1.3 File Format for Module 2

Module 1 creates a file which can be used as the input to module 2. The user can create his/her own file to use module 2 (the determinant module) to solve the system of equations:

$$I = Y_{nxn}\ V$$

The first line is the number of rows, n of the square matrix, Y_{nxn}. The next n lines are the input vector I. The matrix Y_{nxn} is listed by rows, i.e., the next n lines are row one, the following n lines are row 2 and so on and so forth. The final n lines are elements of the unknown vector V.

A1.4 File Format for Module 3

The second module creates a file that is compatible with this program. If the user answers with a "y" to the question:

"Do you want to see the numerator terms?"

the program generates an output file for this program. The file can be created independently of any program. The first line should be a term identifying the next string entries. The second line should be the number of terms in the string. The rest of the lines should contain the terms specified above. Repeat this procedure till all the terms have been entered. The last line should be ".END"

A1.5 File Format for Module 4

The file format is similar to that for module 3. The second module can be used to create a file compatible with this program. The circuit description in the second module must contain non-ideal op amp definitions. The user should then answer properly the questions in the second module an a file for this module would be automatically created.

If the user wants to create the file personally the following instructions should be followed. The first line should be a descriptive term.(the program neglects this term) The second line should contain the number of terms in the string followed by the string written term by term. The last line should be ".END".

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] G. Wilson, Y. Bedri and P. Bowron, "RC-Active networks with reduced sensitivity to amplifier gain-bandwidth-product," *IEEE Trans. on Circuits and Systems, CAS 21,* pp. 618-626, September, 1974.

[2] P. Padukone, J. Mulawka and M.S. Ghausi, "An Active-RC biquadratic section with reduced sensitivty to operational amplifier imperfections," *Journal of The Franklin Institute, Vol. 310, No. 1,* pp. 27-40, July, 1980.

[3] L.C. Thomas, "The biquad-I. Some practical design considerations," *IEEE Trans. Circuit Theory, Vol. CT-18,* pp. 350-357, 1971.

[4] D. Akerberg and K. Mosberg, "A versatile RC building block with inherent compensation for the finite bandwidth of the amplifier," *IEEE Trans. Circuit Syst., Vol. CAS-21,* pp. 75-78, 1974.

[5] W.B. Mikhael and B.B. Bhattacharyya, "A practical design for insensitive RC-active filters," *IEEE Trans. Circuit Syst., Vol. CAS-22,* pp. 407-415, 1975.

[6] M. Reddy, "A insensitive active RC filter for high Q and high frequencies," *IEEE Trans. Circuit Syst., Vol. CAS-23,* pp. 830-845, 1976.

[7] L.T. Bruton, "Multiple-amplifier RC-active filter design with emphasis on GIC realizations," *IEEE Trans. Circuit Syst., Vol. CAS-25,* pp. 830-845, 1978.

[8] R. Tarmy and M.S. Ghausi, "Very high-Q insensitive active RC networks," *IEEE Trans. Circuit Theory, Vol. CT-17,* pp. 358-366, 1970.

[9] P.E. Fleischer and J. Tow, "Design formulas for biquad active filters using three op-amps," *Proc. IEEE, Vol. 61, No. 5,* pp. 662-663, May, 1973.

[10] R.L. Geiger, "Parasitic pole approximation techniques for active filter design," *IEEE Trans. on Circuits and Systems, CAS-27,* pp. 793-799, September 1980.

[11] J.A. Svoboda, G.M. Wierzba and T.Reynolds, "Op-amp relocation, the complementary transformation and stability," *Proc.* 29th *Midwest Symp. on Circuits and*

Systems, August, 1986.

[12] G.M. Wierzba and J.A. Svoboda, "An op-amp relocated bandpass filter with zero center frequency sensitivity to the gain-bandwidth-product," *Proc. 29th Midwest Symp. on Circuits and Systems,* pp. 28-32, August, 1986.

[13] J. Vlach and K. Singhal, *Computer methods for circuit analysis and design,* van Nostrand Reinhold Electrical/Computer Science and Engineering Series, 1983.

[14] L.O. Chen and P.M. Lin, *Computer aided analysis of electronics circuits,* Prentice Hall, New Jersey, 1975.

[15] L.W. Nagel and D.O. Pederson, *SPICE(Simulation Program with Integrated Circuit Emphasis),* University of Califomia, Berkeley, Memorandum ERL-M382, April, 1973.

[16] P.M. Lin and G.E. Alderson, *SNAP-A Computer Program for Generating Symbolic Networks Functions,* Memorandum TR-EE70-16, Purdue University, Indiana, August 1970.

[17] L.O. Chua and P.A. Medlock, *MECA-A User Oriented Computer Program for Analyzing Resistive Nonlinear Networks,* Memorandum TR-EE69-7, Purdue University, Indiana, April, 1969.

[18] T.E. Idleman, F.S. Jenkins, W.J. McCalla and D.O. Pederson, "SLIC-A Simulator for Linear Integrated Circuits," *IEEE J. Solid State Circuits,* (Special Issue on Computer-Aided Circuit Analysis and Device Modelling), Vol. SC-6, pp. 188-203, August, 1971.

[19] C. Pottle, *CORNAP User Manual,* Ithaca, N.Y.:Cornell University, School of Engineering, 1968.

[20] W.D. Stanley, *Digital Signal Processing,* Reston Publishing Company, 1975.

[21] J.V. Wait, L.P. Huelsman and G.A. Kom, *Introduction to Operational Amplifier Theory and Applications,* McGraw-Hill Book Company, 1975.

[22] C.S. Lindquist, *Active Network Design with Siganl Filtering Applications,* Steward and Sons, 1977.

[23] G.M. Wierzba, "Op-amp relocation: A topological active network synthesis," *IEEE Trans. on Circuits and Systems, CAS-33*, pp. 469-475, May, 1986.

[24] G.M. Wierzba and J.A.Svoboda, "A comparison of circuits generated by op-amp relocation," *Proc. 28th Midwest Symp. on Circuits and Systems,* pp. 800-804, August, 1980.

[25] L.T. Bruton, *RC-Active Circuits Theory and Design* Prentice Hall, 1980.