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EFFICIENCY COMPARISONS OF VOTING SYSTEMS WITH STRATEGIC VOTING

By

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ABSTRACT

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The purpose of this dissertation is to investigate the effect of strategic voting on efficiency measures of different multi-candidate voting systems. The voting systems compared include the standard plurality system; the Borda system, a weighted ranking voting system; and approval voting, in which the number of alternatives receiving a vote is a choice variable for the voter.

Efficiency measures have already been developed theoretically and estimated via simulation for these voting systems, assuming voters use sincere strategies. Given this assumption, the Borda system is found to be the most efficient, followed by approval voting, followed by the standard voting system.

However, a set of sincere strategies for the voting population does not always constitute a Nash equilibrium. It is shown that sincere strategies do converge to a Nash equilibrium as the voting population becomes large. Similarly, as the degree of information the voting population is assumed to have decreases, i.e. the standard error of their estimates of alternatives' total votes received increases, sincere strategies converge to a Nash

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equilibrium. Thus, for small, sufficiently knowledgeable voting populations, efficiency measures may change with the assumption of strategic voting as opposed to sincere voting.

A simulation of the voting systems under consideration confirms that efficiency measures do change significantly under these conditions. In addition, the results of the simulation show that strategic voting can alter the ranking of the voting systems. For one of the two efficiency measures used, the standard voting system is found to be most efficient, followed by the Borda system, with approval voting being the least efficient of the three.

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INTRODUCTION

It is often necessary to make decisions which will affect a group of individuals. Arrow [1] in his General Possibility Theorem, proved the impossibility of constructing a social welfare function (without using cardinal utilities) which fulfilled the following conditions: (1) unrestricted domain; (2) consistency with the Pareto principle; (3) independence of irrelevant alternatives; and (4) nondictatorship. Certainly if such a social welfare function could be constructed it could be used to determine which of the possible alternatives to choose. Despite the fact that no such social welfare function exists, the decisions remain to be made. In lieu of using a social welfare function with these characteristics, voting systems are often used.

There are many different voting systems to choose from, and different voting systems may produce different outcomes. The voting systems considered here are the standard plurality system, the Borda system, and the approval voting system. The standard voting system is the one commonly used in the United States, where each voter casts one vote for the alternative of his choice. The Borda system is a weighted ranking system in which alternatives are ranked and

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then assigned points according to their rank. For example, in an election with five alternatives (A, B, C, D, and E) an individual would rank the alternatives from first to last. For simplicity, let the alternatives be ranked in alphabetical order. Then points are assigned as follows:

Alternative	Rank	Points Assigned
A	1	4
B	2	3
C	3	2
D	4	1
E	5	0

The Borda system was presented for the first time to the French Academy in 1784 by Jean-Charles de Borda, and was promptly adopted by the Academy. It remained in use until 1800, when it was challenged by a new member and modified soon afterward. The new member was Napoleon Bonaparte.[1] Currently, a modified Borda system is used as the selection method for the Heisman trophy winner, as well as for several other athletic awards.

In the approval voting system, voters are allowed to vote for as many of the alternatives as they find acceptable or approve of. In the example above, a voter could cast from zero to five votes, although zero and five are equivalent strategies in the sense that neither affects the outcome of the election. Approval voting was first discussed by S. Brams in 1976 [17], and there have been efforts to have this system adopted for use in the Massachusetts primary.

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The Institute of Management Sciences tested the approval voting system against the standard voting system in its 1985 annual elections. 85% of the 1,851 voters, or 1,579 voters returned the test ballot. Members were also asked to rank the candidates, and 82% provided at least some rankings. Three elections were used for comparison. The results of the first election are presented here.

Candidate	Official Vote	Approval Vote
A	166	417
B	827	1038
C	835	908
	1828	2363
		(1,562 voters)

The outcome of the election is C under the standard voting system, while B wins under approval voting. This difference is caused by the pattern of second choices. There is no scope for information about second choices in the standard voting system, but some of this information is used in the approval voting system.

1st choice	2nd vote	
A	B	36%
A	C	23%
B	C	27%
C	B	45%

As shown above, among A's followers, more approve of B than C (36% to 23%), and more of C's followers approve of B (45%) than B's followers do of C (27%). Using the ranking data submitted, Little and Fishburn [87] extrapolated to obtain the result of a hypothetical pairwise race between B and C. Interestingly, the expected outcome of such a race is a tie, wi

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with both B and C obtaining 914 votes. Clearly, the choice of voting system used impacts directly on the outcomes achieved. The question now becomes one of determining which voting system is "best," and the criterion which should be used in making this determination.

A brief outline of the dissertation is presented here. In chapters 1 and 2, the literature on voting systems is reviewed. The literature focuses on three major areas:

1) voting systems as ways of aggregating individual preferences, and their characteristics, e.g. Arrow's General Possibility Theorem, work on incentive compatibility;
2) how voting systems work in terms of individual motivation and equilibrium: voting equilibria, and why individuals vote; and
3) comparisons of voting systems in terms of expected outcomes.

Chapter 1 outlines the historical background of voting system research, while Chapter 2 defines the comparison measures for voting systems and reviews voting system comparisons in terms of expected outcomes. Chapter 3 presents the formal model used for simulation as well as investigating some of the implications of the model, such as equilibria found. Also presented is a discussion of when sincere strategies constitute a Nash equilibrium. Chapter 4 presents the results of simulations run under the complete information assumption and an intermediate level of

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information. A discussion of the results and their policy implications is presented in Chapter 5, along with possible extensions and areas for further research.

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CHAPTER 1

HISTORICAL BACKGROUND

Approaches to the study of voting systems vary widely. The earliest work, beginning with Jean-Charles de Borda in 1781 [11] and continuing through the 19th century, appears for the most part to be a continuing ideological debate on the subject. Later work can be categorized into three major areas. The first of these focuses on voting systems as a means of aggregating individual preferences and the characteristics of the aggregation process. The second looks rather at individual motivation and equilibria in a voting system, usually one specific voting system. The third area, which can be characterized as a strictly modern approach, compares voting systems in terms of outcomes or expected outcomes. A great deal of the literature falls strictly into one class or another, although there is of course some work which crosses these lines.

1.1 Early Work

Jean-Charles de Borda's work [11], the earliest commonly cited on voting and voting systems, begins with an example to show that the "single vote" (the standard voting system), may select the "wrong" candidate. In this example, he makes implicit use of the Condorcet criterion, showing that the standard voting system may select a candidate who can be beaten by another candidate in a pairwise race. Borda then shows that this "defect" can be remedied either by his method of ranking or by pairwise voting. He defines hi

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his method of ranking as giving points to each candidate in accordance with their rank on a preference scale, which is equivalent to assuming a linear utility function for voters.

During the same period, Condorcet [26],[118] discussed the "paradox of voting" and internal consistency of social choices. Condorcet motivated his work as follows: "...it is in the interest of those who dispose of the public power to employ that power only to sustain decisions that conform to the truth, and to give, to the representatives they have charged to decide on their behalf, rules which guarantee the goodness of their decisions."[2] He focuses on how to determine the best rules by applying the laws of probability to the voting process. Condorcet's own description of his work explains much more fully his reasoning:

> "...we shall first suppose assemblies composed of voters possessing equal soundness of mind and equal enlightenment. We shall suppose that none of the voters influences the votes of others and that all express their opinion in good faith. Supposing then that one knows the probability that the opinion of each voter will be in conformity with the truth, the form of the decision, the hypothetical majority and the number of voters, one seeks to discover (1) the probability of not having an decision contrary to the truth; (2) the probability of having a true decision; (3) the probability of having any decision (true or false); (4) the probability that a decision that one knows to have been taken will be true rather than false; and, finally, the probability of this decision when the majority by which it has been taken is known. Such is the subject of the first part of this book."[3]

In the second part of his work, he deals more explicitly with the standard voting system. He uses an example in the same manner as Borda to show that the s

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standard voting system "can result in a decision really contrary to the opinion of the majority."[4]

> "...to have a majority decision that merits confidence, it is absolutely necessary to reduce all opinions in such a way that they represent in a distinct manner the different combinations that can arise from a system of simple propositions and their opposites; ...every complex proposition is reducible to a system of simple propositions, and that all the opinions that can be formed in deliberating upon this proposition are equal in number to the combinations that one can make of these propositions and those contradicting them."[5]

Pairwise comparisons of candidates were to be used to determine a social ranking, and the Condorcet criterion, although used implicitly by de Borda, was made explicit for the first time. The candidate (or alternative) which obtains a majority in a pairwise race with each other candidate (or alternative), now called the Condorcet winner, has the highest social ranking and ought to be chosen. Condorcet showed, however, that pairwise comparisons would not necessarily give a social preference order which was internally consistent, foreshadowing Arrow's work. However, he suggested that the propositions be taken in successive order with the size of the majority, and "as soon as these propositions produce a result, it should be taken as the result, without regard for the less probable decisions that follow."[6]

The third part of his work discusses the probability of obtaining an inconsistent social ordering and represents the first attempt to estimate the frequency of the paradox of voting. Given a set of n candidates, there are n! sets of consistent social rankings. If each candidate is paired with each other candidate, then there are (1/2)n(n-1) pairs, i.e. candidate A vs. candidate B, candidate A vs. candidate C., etc., which is equal to the number of combinations of n things taken two at a time. In each of these pairings, a choice must be made between the two candidates. Therefore, $2^{(1/2)n(n-1)}$ gives the number of possible social `preference profiles´. This minus n!, the set of internally consistent pairings, is the number of inconsistent preference orderings, and the limit of the percentage of inconsistent social orderings,

$$\frac{2^{(1/2)n(n-1)} - n!}{2^{(1/2)n(n-1)}}$$

is equal to one as $n \rightarrow \infty$. Condorcet´s work does not make any obvious assumptions about individual voter preferences, except that given two candidates, any voter is equally likely to vote for either. He does not require that an individual´s vote be consistent with a preference ordering.

In 1795, LaPlace essentially duplicated Borda´s method of ranking using a different line of reasoning. He assumed that the "merit" attributed on average to candidates was linear, similar to Borda, and that the candidate who ought to be elected is the one to whom the most merit is attributed by the entire group of voters. Interestingly, the merit attributed on average to candidates will be linear, as will individual expected utilities for candidates by rank, if all voter utilities are drawn from an identical uniform distribution. The "merit" discussed by LaPlace is the voter's marginal rate of substitution or ratio of exchange of the candidate for money.

Other early work was produced by Hare, Nanson, Galton, and Dodgson, and contained the same types of arguments. The most extensive review of this work is contained in Black [7]. A more rigorous approach did not appear until Hotelling's work.

1.2 Voting Systems as Ways of Aggregating Individual Preferences

1.2.1 Impossibility Theorems

Work in this area has focused on the incompatibility of specific characteristics in an aggregation procedure. The seminal work, Arrow's General Possibility Theorem [1], showed the incompatibility of 1) unrestricted domain on (ordinal) preferences; 2) consistency with the Pareto principle; 3) independence of irrelevant alternatives; and 4) nondictatorship. Zeckhauser's [145] explanation of these conditions is clear and concise. "(1) The procedure must include all logically possible combinations of individuals' orderings. (2) It must lead to Pareto-optimal outcomes. (3) The choice between any two alternatives cannot be influenced by the presence or nonpresence of a third alternative. (4) No individual can always secure his choice regardless of the presence of others."[7] Arrow proved that there is no aggregation procedure (social welfare function) which simultaneously fulfills these conditions. Condition 2 is simply that if all individuals prefer an alternative x to an alternative y, or are indifferent between them, with at least one individual strictly preferring x to y, then x is socially preferred to (Pareto dominates) y. Any alternative y for which an alternative x can be found which fulfills this condition is not an acceptable outcome. Condition 3, independence of irrelevant alternatives, is the requirement that the social ranking between any two alternatives be independent of any other alternative. In essence, this limits us to pairwise comparisons of alternatives, as in Condorcet's method, and implicitly accepts the Condorcet criterion. However, the General Possibility Theorem shows that if we limit ourselves to using pairwise comparisons, then any aggregation procedure which is to be used for all preference profiles (unrestricted domain) is either inconsistent with the Pareto principle (some outcomes will be Pareto-dominated), or dictatorial.

Arrow's work was followed by many attempts at relaxing his requirements in order to find a set of compatible conditions with little success. Expansion and comment (e.g. Sen [124],[125],[127], Plott [107],[108]) provided insight into Arrow's result, but no progress in solving the problem of social choice. To clarify the issue, the problem needed to be stated in a different form. Gibbard [58] did just that: instead of referring to a social welfare function, he looked at the problem in terms of a game form.

A game form, in Gibbard's terms, is "...any scheme which makes an outcome depend on individual actions of some specified sort...strategies. A voting scheme, then, is a game form in which a strategy is a profession of preferences..."[8] He also makes use of the term `straightforward' to mean a game form for which all players, for every preference profile, have a dominant strategy. A strategy is dominant for an individual player if, given any set of strategies of the other players, no other strategy available to the player will produce an outcome preferable to him. Using these definitions, Gibbard proved that every straightforward game form with at least three possible outcomes is dictatorial, and every voting scheme with at least three outcomes is either dictatorial, or can be manipulated by an individual.[9]

Satterthwaite [120] independently made the same contribution, although his terminology differs somewhat. Instead of straightforwardness, he looks at strategy-proofness, which in his work corresponds to Arrow's independence of irrelevant alternatives and Pareto conditions for social welfare functions. He showed that all strategy-proof voting procedures are dictatorial.

Interestingly, these results break down if lotteries over alternatives are allowed as outcomes of a social choice function (Gibbard [59]). However, Gibbard proved that all strategy-proof decision schemes are either random dictatorships, pairwise majority rule over a random pair, or a system which chooses randomly between the first two. Unfortunately, either method violates one of Arrow's co

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conditions, which is where the alteration in terminology, at first appearance eminently useful, comes back to haunt us. When randomness is introduced, strategy-proofness no longer corresponds to Arrow's second and third conditions.

A final work in this area is discussed because of the direct relevance it bears on this work. Postlewaite and Schmeidler [109] considered social choice functions in terms of (first-degree) stochastic dominance. "A person is said to prefer in the stochastic dominance sense one lottery-over-outcomes over another lottery-over-outcomes if the probability of his (at least) first choice being selected in the first lottery is greater than or equal to the analogous probability in the second lottery, the probability of his at least second choice being selected in the first lottery is greater than or equal to the analogous probability in the second lottery, and so on, with at least one strict inequality."[10] Individuals, assumed to know the relative frequency of (ordinal) preference profiles for two social choice functions (which may include an element of randomness) can compare the social choice functions in terms of stochastic dominance. If a social choice function F stochastically dominates a social choice function G for all individuals in a society, F stochastically dominates G socially. This implies ex ante Pareto efficiency of F over G. Postlewaite and Schmeidler comment that "Arrow's Pareto principle, which is ex post, should be implied by a reasonable notion of ex ante efficiency in a model which

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admits such evaluations."[11] Their main result is that for more than 3 voters and alternatives, there does not exist a social choice function which is simultaneously Pareto undominated (ex ante efficient) and straightforward.[12] That is, a social choice function which is ex ante efficient in the stochastic dominance sense will present individuals in the society with situations in which misrepresenting their preferences (as a strategy) dominates their sincere strategy of truthful revelation of preferences.

These major contributions to the social choice literature provide a background for comparisons of voting systems, but do not provide any positive criteria which can be used for comparison because of the incompatibility of desired characteristics. If these characteristics were compatible, a social welfare function could be constructed that would specify the "correct" choice for every social choice situation.

1.2.2 Incentive Compatibility

The concern with strategy-proofness or manipulability has been addressed from another viewpoint, that of incentive compatibility. In this line of research, attempts have been made to construct voting systems which are incentive-compatible: truthful revelation of preferences is a dominant strategy in an incentive-compatible mechanism.

This emphasis on incentive compatibility is due in large part to the `free-rider problem´ which is a consequence of the existence of pure public goods (the classic example is national defense). The main characteristics of a pure public good (Samuelson [119]) are joint consumption and nonexcludability. Joint consumption is the property that all members of the consuming body for this good benefit from its production (although not necessarily equally), without preventing other consumers from benefiting or reducing the benefits available to them. Nonexcludability is just that: individuals cannot be prevented from enjoying these benefits. The problem is to determine the Pareto-optimal level of a pure public good to be produced. The condition for Pareto-optimal production of a good is that marginal benefit be equal to marginal cost. Since marginal benefit is distributed across the consuming body, the marginal benefit for one unit of a pure public good is the sum of marginal benefits for all consumers. The level of the pure public good should be chosen such that the sum of marginal benefits across consumers is equal to the marginal cost of production. The difficulty lies in determining what the sum of marginal benefits across consumers is for different levels of production. Generally, individuals would be asked to provide their marginal benefit curve. However, the method of financing production of the pure public good influences the information provided. If individual marginal cost (the marginal tax rate) is zero over the level of production of the good (total cost is constant), each individual has an incentive to overstate his marginal benefits at each level of the pure public good, which will lead to overproduction of the good and a misallocation of resources. If, on the other hand, individual marginal cost is set to correspond with stated marginal benefit, individuals have an incentive to understate marginal benefits in order to reduce their marginal cost, which leads to underproduction of the good.

Because of this difficulty, attention focused on the formulation of a direct mechanism which would induce truthful revelation as a dominant strategy. Dasgupta, Hammond, and Maskin [33] review the major results of this approach. They discuss general results on incentive compatibility in the implementation of social choice rules. Their discussion involves the use of a "planner" to implement the social choice rule; however, a "planner" is not necessary to their discussion except as a pedagogical tool. The general problem is approached as follows: A social choice mechanism depends on signals from the individual agents to implement the social choice rule. It is assumed that each individual agent sends his own signal. The mechanism is then a rule which specifies a social state for each list of signals sent by the individual agents. It is assumed that each agent knows the precise form of the mechanism being used. Then each agent realizes that he is involved in a game, because the outcome of the mechanism depends on the signals which he and all the other agents send. More precisely, this is a "game form," in which there is a fixed set of strategies, consisting of signals, and in which the outcomes of these strategies are known to all "players." It is then assumed that the players in this game form, who are the individuals in the society, reach some kind of equilibrium which depends on their true characteristics - in particular, their preferences. The mechanism generates a particular social state given these equilibrium signals. "Presumably, one wants this social state to be in the social choice set given the agents' true characteristics - i.e. to be something the planner might have chosen had he known these characteristics right from the start. ...The basic problem, then, is to devise a game form which always has at least one equilibrium, and whose possible outcomes in equilibrium all belong to the appropriate social choice set for the individuals' true characteristics. A mechanism (or game form) with this property is said to <u>implement</u> the social choice rule."[13]

Dasgupta, Hammond and Maskin discuss mechanisms which are individually incentive compatible, both direct and indirect. A direct mechanism is one where the agent's signal is a characteristic: preferences, endowments, etc., relevant to the economic decision to be made. In contrast, with an indirect mechanism, agents' signals "may be quite arbitrary, without any obvious economic significance."[14] Such mechanisms can be and have been found, such as the Clarke tax [24]. However, as Dasgupta, Hammond, and Maskin point out, "the papers which find straightforward mechanisms restrict themselves to rather special economic environments.

Either the preferences are special, (Clarke [24], Green and Laffont [61], Groves and Loeb [65]) or there is a large economy in which no one individual's lie can significantly affect the overall outcome (Hammond [66], Roberts and Postlewaite [116])."[15]

They then present their versions of impossibility theorems, which extend Arrow's work. First, in any "rich economic environment"[16] (e.g. unrestricted domain of ordinal preferences), any Pareto optimal single valued social choice rule which can be truthfully implemented in dominant strategies is dictatorial. Secondly, in any "rich economic environment," any Pareto-optimal single-valued social choice rule which can be implemented in Nash strategies is dictatorial. This follows naturally from their proof that in a rich economic environment, a single-valued social choice function which is implementable in Nash strategies is truthfully implementable in dominant strategies.

However, this does not mean that the task is hopeless. All that this implies is that a non-dictatorial Pareto optimal single-valued social choice rule cannot be `implemented' in Nash strategies. This means that the use of the Nash equilibrium concept implies that all possible outcomes in equilibrium do <u>not</u> belong to the appropriate social choice set for the individuals' true characteristics. However, recall from the previous section that straightforwardness (truthful implementation in dominant strategies) is inconsistent with ex ante efficiency in the stochastic dominance sense. Postlewaite and Schmeidler's result is that without restricting preferences, ex ante efficiency comes at the cost of straightforwardness.

1.3 Voting Systems, Equilibrium, and Individual Motivation

A different approach to voting systems is to look at specific parts of a system. How are voter preferences formed? What are admissible strategies? Finally, what is (are) the equilibrium outcome(s)?

1.3.1 Voting Equilibria

One branch of this literature concerns itself with the equilibrium outcome(s) of specific voting systems. Different assumptions about the restrictions on formation of voter preferences account for the differences in outcomes, but the models are set up in essentially the same way. The most famous of these is the median voter model.

1.3.1.1 Unidimensional Spatial Model (Median Voter Model)

The spatial theory of voting has a long and distinguished history. Black [7] states that "Galton (1907) notices the property of the median optimum when the variable under consideration is measurable (provided the voters' preference curves can be taken as single-peaked)."[17] However, a close reading of his citation from Galton reveals that what Galton noted was the equilibrium property of the median.[18] The impetus to the approach must lie with Hotelling [74] and Smithies [135], who showed the existence of a spatial location equilibrium in a model where producers of goods must choose a location given the existence of posi

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positive transportation costs. Their work, along with Galton's, inspired Black to prove the equilibrium properties of the median position in pairwise majority voting.

Black [6] essentially limited his analysis to pairwise majority voting, although in a related work he includes an extensive discussion of the literature including alternative voting methods. He first defines single-peakedness of preferences. Preferences of a society are single-peaked, if, for some arrangement (order) of alternatives, each voter's utility curve over alternatives "changes its direction at most once, from up to down."[19] In this case, the highest point on an individual's utility curve is his "peak preference." It is important to point out that single-peakedness of preferences does not imply a `satiation point.' The median voter model is ordinarily used in the context of decisions on the production of public goods. In this context, given a method of financing production (tax system), the individual is solving a constrained maximization problem based on his resources (income). This implies an optimal level of consumption of each good available, including the public good on which a decision is to be made. It is not unreasonable that a graph of the individual's total utility as a function of the level of production of the public good would be single peaked (as it would be, for example, if there were a constant marginal tax rate for increments of the public good and the usual assumptions on individual utility were made).

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The median voter is the individual with the median peak preference. Black's main result is that the peak preference of the median voter, in this case, is the pairwise majority voting equilibrium. As Galton deduced, anything less will have a majority in favor of increasing it, and anything more will have a majority in favor of decreasing it. However, preferences must be single-peaked, and the unrestricted domain used by Arrow will cause nonexistence of an equilibrium point for some cases in this model.

Bowen [15] extended Black's result to an economic context. He showed that under certain conditions plurality or simple majority voting would produce a Pareto optimal outcome in equilibrium, when the decision to be made is the level of production of a pure public good. The conditions under which this holds are: (1) There is complete and sincere participation of the voting population; all voters in the voting population do vote, and they vote sincerely, i.e. in correspondence with their true preferences. (2) The cost curves for production of the public good are known. (3) The public good is produced under conditions of (eventually) nondecreasing marginal cost. (4) The cost of the public good is divided equally across the population, or there are equal tax shares. (5) The marginal rate of substitution of the public good for money is normally distributed across the population at any level of the public good. (6) The public good is nonexcludable and equally ava

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available to all voters, corresponding to Samuelson's definition of a pure public good.

Conditions 2, 3, 4, and 5 imply that there are single-peaked preferences for all members of the population for the public good, if it is a normal good with a decreasing marginal rate of substitution for money. Since preferences are single-peaked, each voter has a most preferred level of the public good, and condition 5 implies that the most preferred level then has a continuous normal distribution across the population. The point of maximum density of this distribution would be the simple plurality voting winner, and the outcome under a simple plurality system would be the output of the public good for which this maximum density occurs. Since the most preferred level of the public good has a continuous symmetric distribution, the point of maximum density coincides with both the median and mean most preferred level. Since each voter's most preferred level is that at which his marginal benefit is equal to his marginal cost, this implies that mean marginal benefit equals mean marginal cost, and therefore the sum of the marginal benefits across the population will be equal to the marginal cost of production of the public good. In other words, the equilibrium point of the simple plurality system is Pareto-optimal.

With simple majority voting over increments of the public goods, the outcome will be the same. As shown by Black, the median most preferred level (median peak preference) is the equilibrium point. However, in Bowen's model, the median coincides with the mean, and Pareto-optimality results. Therefore simple plurality voting or simple majority voting will produce the optimal level of the public good if the conditions postulated by Bowen are fulfilled. It should be noted here that any continuous symmetric distribution of peak preferences for which the point of maximum density is both the mean and the median will produce this same result.

1.3.1.2 Multidimensional Spatial Model

The multidimensional spatial model, developed by Enelow and Hinich [40], is a simple extension of Bowen and Black's median voter model. The major difference is that one dimension is no longer thought sufficient to describe how individuals' preferences are formed. An issue may have more than one dimension, and each dimension in this model is a dimension in the "issue space." The justification for this assumption is the prevalence of 'package votes,' such as a decision on the level of two or more public goods at once. The peak preference level of the unidimensional model is described here as a voter's ideal point in the issue space. However, preferences are again assumed to be single-peaked. "The key element of spatial models is the relationship between preference and distance. ...The weighted Euclidean distance between y and z is defined to be $\|y - z\|_a = [a_{11}(y_1-z_1)^2 + 2a_{12}(y_1-z_1)(y_2-z_2) + a_{22}(y_2-z_2)^2]^{1/2}$, where $a_{11}>0$, $a_{22}>0$, and $(a_{12})^2<a_{11}a_{22}$ to ensure that $\|y - z\|_a>0$

for all y≠z. ...Weighted Euclidean distance defines a symmetric preference rule...the closer (in weighted Euclidean distance) an alternative is to his ideal point, the more he prefers it."[20] $a_{12}=0$ implies separability of preferences; that is, the most preferred level in one dimension is independent of the most preferred level in all other dimensions. Given this mechanism for formation of preferences, and again assuming, with Black and Bowen, complete and sincere participation of the voting population, determination of the equilibrium is made. In the classic spatial model, it makes a great deal of difference whether `dimensions´ are voted on sequentially or simultaneously. Unless all voters´ preferences are separable, the equilibrium outcome will differ. Separability of preferences along with sequential voting implies Pareto-optimality of the equilibrium outcome, just as in the unidimensional model. If preferences are not separable, however, sequential voting produces differing outcomes depending on the order in which dimensions are voted on. In essence, this is because in all but the first election, voters take the values of public goods decided on in previous elections as given. In any case, results of `secondary´ elections may be Pareto-optimal given the result of the first election, but the converse does not hold, as shown in Figure 1.1. This in turn implies that the overall results of the system are not Pareto-optimal.

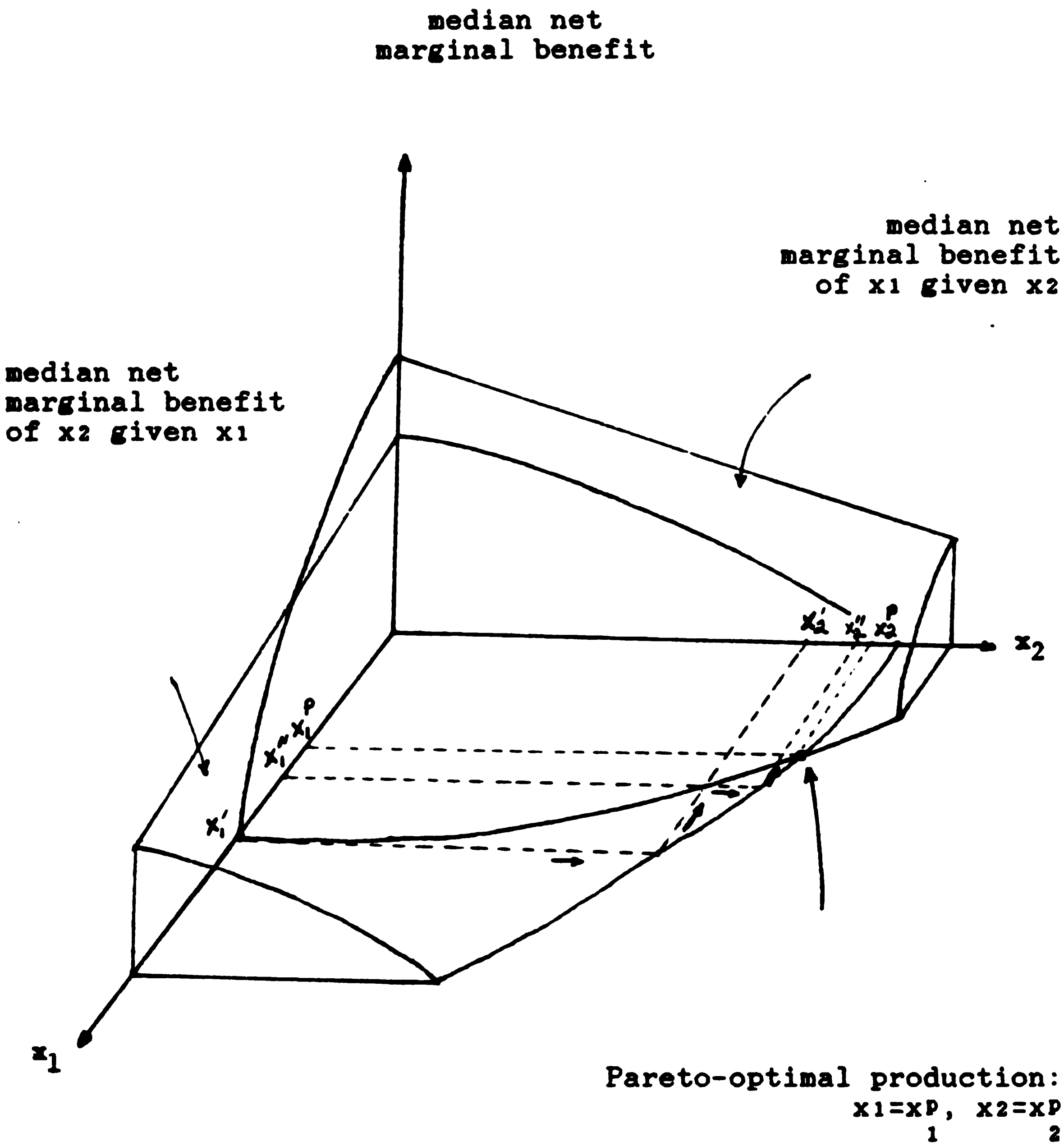

Figure 1.1. Sequential Voting Equilibrium with Reintroduction of Issues.

If voting on x_1 takes place first, the level selected will be x_1'. A vote on x_2 then selects x_2'. If x_1 were reintroduced, the level chosen would be x_1'', etc.

Separability or nonseparability of preferences does not matter with respect to Pareto-optimality of outcomes if a dimension can be re-introduced into the process. In Figure 1.1, reintroduction of the first dimension after the second has been decided on will move the outcome towards the Pareto-optimal point, and if this process is continued, the limiting equilibrium point is indeed Pareto-optimal.

However, if preferences are separable but voting on dimensions is simultaneous, the outcome is not necessarily the peak preference point, which corresponds to the median ideal point on each dimension. "...once both issues are voted on simultaneously,...x_{med} can be beaten in a majority contest, and furthermore there may exist no proposal that cannot be beaten...this result is a general problem for the multidimensional spatial model."[21] A dominant point only exists if there is a point in the multidimensional space which is a median in all directions. "If a dominant point exists, all that we are guaranteed is that no other point can beat it in a pairwise contest. This does not mean that a dominant point beats all others."[22] A dominant point receives at least as many votes as any other point in a pairwise contest. In other words, some other point may tie with the dominant point in a pairwise contest. However, if a point y is closer to x_{med} than z, then y beats z in a majority contest. This suggests that the limiting equilibrium point is the dominant point x_{med}.

In the absence of a dominant point, the outcome of a sequence of pairwise votes depends upon the agenda. "It is possible to reach literally any point in the space through _some_ sequence of votes, pairing each previously winning proposal with some new proposal that a majority prefers until the chosen point is finally reached."[23] Thus the spatial model, in the absence of a dominant point, has no implications for outcomes without a model of agenda control, which is beyond the scope of this work.

1.3.2 Individual Motivation, or Why Vote?

"Much theorizing about the utility of voting concludes that voting is an irrational act in that it usually costs more to vote than one can expect to get in return."[24] This includes the work of Downs [37] and Tullock [141]. If we are to apply a rational choice perspective, the expected return from voting should be at least equal to the cost or expected cost of voting in order to induce voters to participate. The expected return is the difference in utility between the voter's preferred alternative and another alternative, times the probability that the voter is decisive (the probability that his action in voting causes the change in outcome). If expected return exceeds expected cost, it is rational to vote; if not, voting is an irrational act. Since in any election where the voting population is large, as in the U.S., the probability of being decisive is very small (Riker cites 10^{-8}), the difference in utility must be extremely large in order to compensate for a relatively low cost of voting. The general conclusion is that voting is not a rational act.

Some attempts to modify this conclusion have postulated direct benefits from voting as opposed to its expected return. Palfrey and Rosenthal [102] critique this approach, commenting that "...many observations are inconsistent with the proposition that an individual's net cost of voting...is anywhere near constant. The greater turnout in presidential than in off-year elections and the greater turnout in contested than in uncontested elections belie any simple citizen-duty story. Of course, citizen duty could be rescued by arguing that there is a greater sense of duty in presidential and contested elections, but such logic is difficult if not impossible to test."[25]

Another approach is Ferejohn and Fiorina's [42] minimax regret model. They contrast voting as decision-making under risk, which is the conventional analysis, with voting as decision-making under uncertainty. "Under risk, probabilities can be assigned to the states of nature; under uncertainty, state probabilities are unknown or unknowable."[26] They analyze voting under Savage's minimax regret criterion, and come to two interesting conclusions. First, voting for one's second choice is never minimax regret optimal. This implies that strategic voting never occurs, which would make the current work irrelevant if believed. Secondly, minimax regret decision makers find it rational to vote for their most-preferred alternative rather

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than abstain under relatively weak conditions. This model thus avoids the difficulties that the expected utility analysis runs into. Unfortunately, a great deal of empirical evidence indicates that probabilities have a significant effect on voter participation.[27] The minimax regret framework denies that these probabilities are known or knowable.

An alternative approach to the problem of voter participation is suggested by Palfrey and Rosenthal, who model simultaneous determination of participation and the probability of being decisive. "If everyone else votes, p can readily be very small. But if no one else votes, the probability of being decisive would be 1. Clearly, if citizens are rational, the voting probabilities and the turnout decisions are simultaneously determined."[28]

Ledyard [85],[86] modeled simultaneity of voting participation and the probability of being decisive in the spatial model. Each voter knows the size of the voting population, the spatial positions of the alternatives, and his own preferences. His information on other voters' preferences is limited to knowledge of the continuous probability distribution from which they are drawn. Under these conditions, if expected return is sufficiently large relative to the cost of voting, turnout is positive, and he proves existence of a symmetric equilibrium.

Palfrey and Rosenthal take a similar approach, but model only two types ("teams") of citizens, each with identical preferences. Voting in this model is over two fixed alternatives "as in a two-candidate election or in a referendum or initiative vote between a proposal and a status quo."[29] They find the possibility of substantial voter turnout in equilibrium, although depending on the size of the electorate, multiple equilibria are common. Thus for small numbers of voters "there are not strong predictions about the size of voter turnout."[30] For a large voting population, they find only two types of equilibria: one in which turnout approaches zero, and one in which percentage turnout approaches twice the `minority´ side´s percentage of the electorate.

Table 1.1 below presents percentage turnout for the 1972, 1976, and 1980 presidential elections along with the percentage of voters registered under the `minority´ party (Republican or Democrat only).[31] Percentage turnout can only roughly be described as double the minority side´s percentage of the electorate, but Palfrey and Rosenthal´s conditions are not strictly complied with. There are more than two `types´ of citizens, and it is improbable that all citizens of a specified type have identical preferences. Certainly, this can be considered as some support for Palfrey and Rosenthal´s model. However, the important point is that even with a cost of voting, substantial turnout can be an equilibrium outcome for rational voters.

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Table 1.1 Voter Turnout as Percentage of Minority Registration

	Registered Voters (thousands)	Total Votes Cast	% Turnout	% Registered as `Minority´ (R or D only)
1972	92,702	77,719	83.84	37.5
1976	105,837	81,556	77.06	48.0
1980	112,945	86,515	76.60	41.0

1.3.3 Randomness in Voting Models

Several models have introduced randomness via probabilistic voting. These have included Hinich, Ledyard, and Ordeshook [69], and Fishburn and Gehrlein [56],[57]. In these models, there is a probability that an individual will abstain as opposed to voting his (sincere) preferences. However, if we think of sincere voting as one possible strategy and abstention as another, this type of model arbitrarily restricts voters´ possible strategies to these two. Hinich, Ledyard, and Ordeshook model a two-alternative system which makes this plausible, since sincere voting is the unique optimal strategy when there are only two alternatives. However, a social welfare function fulfilling Arrow´s conditions exists for a two-alternative system, casting some doubt on the applicability of this model. Intriligator [79] and Coughlin and Nitzan [31],[32] use a different type of model, in which each voter has a probabilistic density function $f_i(x)$, and for any subset A of the set of feasible social alternatives X, $\int_A f_i(x)$ is the probability that individual i chooses some member of A, given than he can unilaterally determine the social choice.

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An individual's choice probabilities are "proportional to his strength of preferences."[32] Intriligator develops this model to extend standard systems (Borda, majority rule, Pareto rule, etc.) into a probabilistic framework. In contrast, Coughlin and Nitzan assume that each individual's density function is also his differential utility function, and using two candidates, develop a model based on the probability of voting for each candidate. These probabilities are determined by the alternatives each candidate proposes to enact if elected. They then go on to analyze candidate behavior in the sense that electoral equilibrium depends on proposed policies.

The major drawback to Intriligator's model is that if all utilities are positive, all probabilities are positive, implying that in some case an individual would choose his lowest-ranked alternative, given that he could unilaterally determine the social choice. Unless a framework is specified in which the welfare of other individuals enters into an individual's utility function, this doesn't make sense. In the current literature, the only readily understandable context for randomness in voting models is in generating preferences or utilities, or in tie-breaking.

1.3.4 Strategic Voting

Farquaharson [41] was the first author to approach voting systems from a game-theoretic point of view. He discussed only binary procedures; at any point within the procedure, voters have only two choices. This is

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distinguished from pairwise voting because all possible outcomes are not paired with each other. In Congress, a bill may be amended or not, but if it is amended, the decision to be made is to pass or fail the amended bill. The possible outcomes of passage of the amended bill and passage of the original bill are not directly compared in the process. In one-stage binary processes, sincere voting (voting in accordance with one's preferences) is always optimal. In contrast, multistage binary processes are "vulnerable"[33] to strategic voting.

> "A situation is vulnerable if another situation
> i) can be obtained from the first by substituting a strategy of at least one voter;
> ii) is preferred to it by that voter or those voters."[34]

A set of strategies is "invulnerable" if it is a Nash equilibrium, one in which "each voter can say `no other strategy would have given a better outcome.´"[35] As Farquaharson points out, sincere voting may or may not be an equilibrium. In fact, it is certain that sincere voting will not always be an equilibrium strategy in multistage games in which Gibbard's conditions on unrestricted domain, Pareto principle, and nondictatorship are fulfilled. In this case, another strategy will be used by at least one voter for some social preference profile.

Given this, is it reasonable to assume that voters use sincere strategies, or is it possible that voters actually calculate optimal strategies? `Sophisticated´ voting as developed by Farquaharson (his terminology for the use of optimal strategies) received theoretical attention from McKelvey and Niemi [89], focusing on legislative voting games characterized by a finite sequence of two-alternative issues. This theory is examined by Enelow and Koehler [39], who look specifically at two amendment strategies: (1) amend to save a losing bill; (2) amend to "kill a winning bill." In either case, the amendment is voted on first (amended bill ab vs. bill b), followed by the vote on final passage, with each voter voting either yes or no on each. The game tree for this is shown in Figure 1.2.

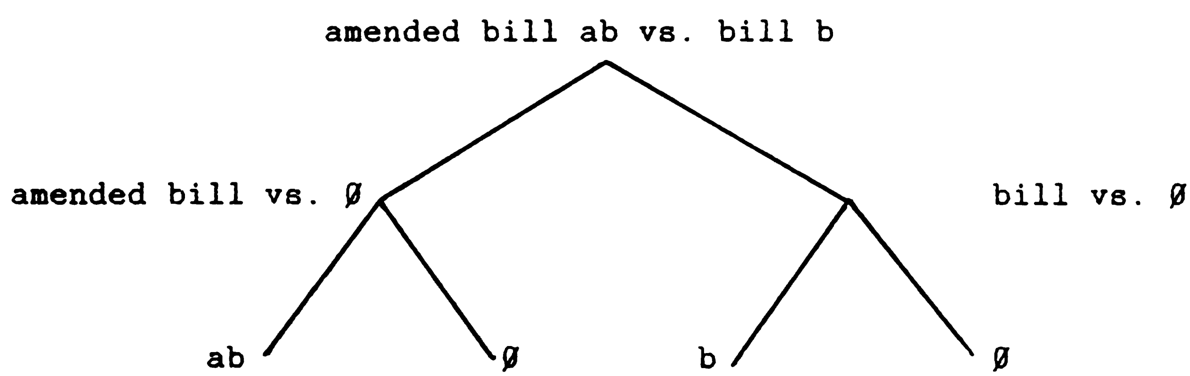

Figure 1.2. The Game Tree for Pairwise Majority Voting with Amendment.

If the first strategy is being employed, then the original bill is expected to lose, and the amended bill is expected to win. Therefore, "...the sophisticated voter realizes that while the nominal contest on the amendment vote is ab vs. b, the expected fate of ab and b, respectively, on final passage indicates that the actual contest on the amendment vote is between ab and Ø. Therefore, the sophisticated voter votes for the amendment if he prefers ab to Ø and against the amendment if he prefers Ø to ab."[36]

Table 1.2. Possible Preference Orders, Sincere Votes, and "Sophisticated" Votes on a Saving Amendment and Final Passage (Amendment Expected to Pass)

preference order	b>ab>Ø	b>Ø>ab	ab>b>Ø	ab>Ø>b	Ø>b>ab	Ø>ab>b
sincere votes	N,Y	N,N	Y,Y	Y,Y	N,N	Y,N
sophisticated	Y,Y	N,N	Y,Y	Y,Y	N,N	N,N

Votes on passage of a "saving amendment" (the Sarasin amendment on House bill 4250) were compared to predicted votes. Actual voting patterns were: Y,Y - 204 or 48.5%; N,N - 177 or 42.0%; Y,N - 40 or 9.5%; N,Y - 0 or 0%. 90.5% of these vote patterns used were predicted by the theory. A more in-depth analysis of how the vote patterns support the theory is presented in the article. An analysis of a killer amendment is also presented. Enelow [38] subsequently extended this paper to conform to an "expected utility theory of sophisticated voting."[37] In this case, comparison of the `lotteries´ described by the left hand and right hand second branches determines voting on the amendment for an individual voter. In order to test this model, "...group rating scores were used to distinguish among congressmen by preference types. It was then shown that the aggregate voting patterns on a well-known example of a saving amendment and a well-known example of a killer amendment were consistent with the predictions of the EUS (expected utility sophisticated) voting model for each preference type."[38] Thus these articles indicate that there is empirical support for the notion of `sophisticated´ (strategic) voting.

CHAPTER 2

LITERATURE REVIEW: EXPECTED OUTCOMES

2.1 Comparison Measures

Given the different outcomes of voting systems, an explanation of the criteria that can be used to compare them is necessary for any comparisons to be meaningful. Two measures have been used in comparing voting systems: Condorcet efficiency, and social utility of voting systems.

2.1.1 Condorcet Efficiency

In order to understand the idea of Condorcet efficiency, it is necessary to define the Condorcet winner. Given a set of alternatives, the Condorcet winner is that alternative which would achieve a majority in a pairwise race with any other alternative. For example, if there are three alternatives A, B, and C, there are three pairwise races possible: A vs. B, A vs. C, and B vs. C. Let A>B indicate that alternative A achieves a majority over B in a pairwise race. Then A is the Condorcet winner if and only if A>B and A>C. Similarly, in a four-alternative election, A is the Condorcet winner if and only if A>B, A>C, and A>D.

Condorcet efficiency is a measure of the extent to which a voting system complies with the Condorcet criterion: "...a candidate who receives a majority as against each other candidate should be elected."[1] As Arrow points out, this criterion implicitly accepts that there should be independence of irrelevant alternatives. Since pairwise majority choice may lead to intransitivity of social preferences, only those cases where a Condorcet winner exists are used in the construction of Condorcet efficiency. Explicitly, Condorcet efficiency is the percentage of Condorcet winners expected to be elected by a voting system, when they exist. By this measure, a voting system which is more likely to elect Condorcet winners (i.e. has a higher expected percentage of Condorcet winners) is judged to be a "better" voting system.

2.1.1.1 Existence of a Condorcet Winner

One difficulty with Condorcet efficiency is that a Condorcet winner may not exist. Existence of a Condorcet winner is not precluded by the presence of majority voting cycles; however, all of the alternatives in any cycle must be beaten in a pairwise contest by another alternative (which is the Condorcet winner) to avoid this problem. What is the frequency of existence of a Condorcet winner? It should be substantial if Condorcet efficiency is to be used as a comparison measure, since it is undesirable to compare voting systems on the basis of a minority of cases. Fortunately, probabilities of a social preference profile with no Condorcet winner have been calculated by Niemi and Weisberg [99] for an infinite voting population where all preference orders are equally likely. The probabilities are shown in Table 2.1 below. These are limiting probabilities for an infinite population; however, for small numbers of voters, probabilities for existence of a Condorcet winner are slightly higher. Until the number of alternatives exceeds ten, the majority of social preference profiles do have a Condorcet winner.

Table 2.1 Probabilities of the Existence of a Condorcet Winner for Various Numbers of Alternatives

# of alternatives	P(no Condorcet winner)	P(Condorcet winner)
2	0	1
3	.0877	.9123
4	.1755	.8245
5	.2513	.7487
6	.3152	.6848
7	.3692	.6308
8	.4151	.5849
9	.4545	.5455
10	.4887	.5113
11	.5187	.4813

2.1.1.2 Condorcet Efficiency and Pairwise Majority Voting

As mentioned previously, the Condorcet winner, when it exists, is the pairwise majority voting equilibrium. This is true regardless of whether voters use sincere or "sophisticated" strategies, since the Condorcet winner is a pairwise majority voting equilibrium in either case. A simple example should make this clear. Suppose there are three alternatives: A, B, and C. The game trees below diagram possible outcomes of a pairwise majority voting game, depending on the agenda.

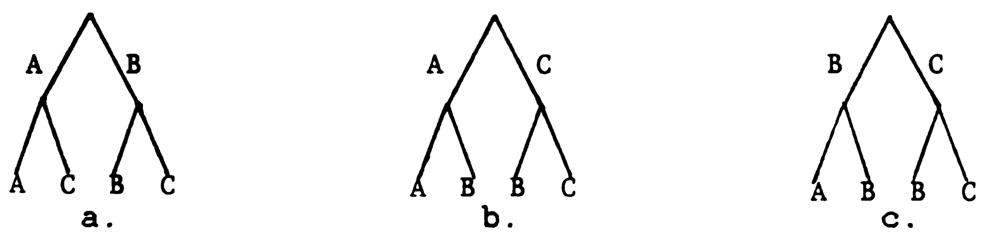

Figure 2.1. Possible Agendas and Outcomes for Pairwise Majority Voting

Let C be the Condorcet winner. Then in Figure 2.1a, at the first branch of the tree, individuals ranking C last cannot prevent C from being considered as an alternative, and at the second branch, cannot prevent it from being chosen since a majority of the voting population sincerely prefers C and has no incentive to vote other than sincerely. In Figure 2.1b and 2.1c, these individuals could prevent the choice of C if they could influence the game by moving down the left branch of the tree. However, again they are working against a majority of the voting population which has no incentive to vote other than sincerely. Clearly, whether voters are assumed to vote sincerely or strategically, the Condorcet winner remains a pairwise majority voting equilibrium.

Current legislative voting systems are characterized by a sequence of pairwise votes. Thus, when a Condorcet winner exists, it is the unique equilibrium outcome. Condorcet efficiency is therefore one measure of how closely different voting systems would correspond to current legislative methods' equilibria in those cases where a Condorcet winner exists.

2.1.2 "Effectiveness" or Social Utility Efficiency

Another way of looking at the problem of comparing voting systems is to use a social welfare function even though we know this cannot fulfill all of Arrow's conditions. Specifically, if individuals have cardinal utilities for the alternatives in the choice set, then a social welfare function of the form

$\{\Sigma_j [u_{ij}]^{\tau}\}^{1/\tau}$ $\tau \leq 1; \tau \neq 0,$

where τ is a constant reflecting society's aversion to inequality, is often used to measure the social utility of each of these alternatives. If $\tau = 1$, one way to interpret this efficiency measure is as the a priori expected utility of the outcome of a voting system, given the stated assumptions about individual utility. It is equally likely than an individual voter will have any of the possible preference orderings. Thus his expected utility for the outcome is 1/n times the expected social utility of the outcome as measured by a utilitarian social welfare function. Given a distribution from which utilities are drawn and a method of determining voters' strategies, an expected value for social utility can be determined for each voting system. As an example, Weber's derivation of "effectiveness" for the standard voting system with two alternatives is reproduced here.[2] In this work, individual utilities are independent identically distributed random variables drawn from a uniform [0,1] distribution. Given no specific information about other voters' strategies and a `large' voting population, an individual voter's optimal strategy in the standard voting system is to cast his vote for his most-preferred alternative.[3] Since the winner is the alternative with the most votes, the expected social utility of the elected alternative is:

$$\Sigma_{k=0}^{n} \quad (1/2^n) \binom{n}{k} (2/3 \max(k, n-k) + 1/3 \min(k, n-k)),$$

where $(1/2^n)\binom{n}{k}$ describes the probability of a certain pattern of votes occurring, max(k, n-k) is the number of votes cast for the winning candidate and min(k, n-k) is the number of votes cast for the losing alternative, and 2/3 and 1/3 are expected values for the utility of an alternative ranked first and second, respectively, since the expected values of the maximum and minimum of two independent [0,1] uniform random variables are 2/3 and 1/3. Using Stirling's factorial approximation, this expression simplifies to $n/2 + \sqrt{n/18\pi}$.

Weber uses a transformation of this to make social utility measures more comprehensible. He defines the effectiveness of a voting system as follows:

$$\frac{E(\text{elected}) - E(\text{random})}{E(\text{maximal}) - E(\text{random})},$$

where E(*) is the expected social utility of the elected, maximal, or random alternative. Values for 'effectiveness' of course will vary according to the scaling factor used, which is E(random) in this transformation, but relative effectiveness of any two systems (in terms of ranking) will remain the same regardless of the scaling factor used. This is a particularly nice transformation since E(random) = n/2 and E(maximal) is asymptotic to $n/2 + \sqrt{n/12}$ $\text{Normmax}(m)$. $\text{Normmax}(m)$ is the expected value of the maximum of m unit normal random variables, and $\text{Normmax}(2) = 1/\sqrt{\pi}$, simplifying the expression considerably. Effectiveness of the two alternative standard voting system is $\sqrt{2/3} = .8165$.

This method can be used to determine the theoretic effectiveness (hereafter referred to as social utility efficiency) for the Borda system. The theoretic social utility efficiency as derived by Weber, of the standard voting system and the Borda voting system for m-alternative elections is:

Standard voting system: $\sqrt{3m}/(m+1)$

Borda system: $\sqrt{m/(m+1)}$

Weber was not able to derive a formula in terms of m for the approval voting system; however, he did derive social utility efficiency for a 3-alternative election: 87.5%.

2.2 Relationship of Comparison Measures

For voting populations which are assumed to use sincere strategies, both comparison measures generally have given the same rankings of voting systems, indicating some overlap in criteria. Indeed, it is easily verified that when individual utilities are i.i.d. random variables of a given distribution, when a Condorcet winner exists, it has maximum expected social utility over all alternatives. Since at least a majority of voters prefer the Condorcet winner to any other alternative, the expected social utility of the Condorcet winner is greater than or equal to $(\text{int}[n/2]+1)(E[\text{dist}_{max}(2)]) + (n-\text{int}[n/2]-1)(E[\text{dist}_{min}(2)])$, where int[n/2] is the largest integer smaller than or equal to n/2 and $E[\text{dist}_{max}(2)]$ and $E[\text{dist}_{min}(2)]$ are the expected values of the maximum and minimum of two independent random variables of the given distribution. For at least a

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majority of the voting population, the utility of the Condorcet winner exceeds that of the other alternative. In contrast, the expected social utility of the other alternative does not exceed

$(n\text{-}int[n/2]-1)(E[dist_{max}(2)]) + (int[n/2]+1)(E[dist_{min}(2)])$.

This implies that a voting system which always chooses the Condorcet winner when it exists maximizes expected social utility in these situations. Therefore, differences in rankings which occur given the two efficiency measures may be due to statistical variation or to the outcomes of the voting systems in cases where the Condorcet winner does not exist. An additional possibility is that a voting system which has a lower Condorcet efficiency but higher social utility efficiency chooses another alternative than the Condorcet winner in precisely those situations in which a smaller than majority group of voters benefit disproportionately. This would be expected to occur in voting systems with greater scope for strategic voting.

For the interested reader, optimality properties of comparison measures are discussed in chapter 5.

2.3 Voting Systems and Expected Outcomes

The more modern approach of comparing voting systems by looking at their expected outcomes was pioneered by Fishburn [45]. Rather than analyzing the characteristics of the process or mechanism, he analyzed the characteristics of expected or mean outcomes.

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Fishburn's approach was designed to fulfill many of the previously discussed conditions on the process. To begin with, he allowed all logically possible preference orderings (unrestricted domain), in keeping with Arrow's justification that "...the decision making process should be applicable to all possible profiles since when we choose it, we don't know to which profiles it will be applied."[4] In addition, all logically possible preferences orderings are taken as equally likely (since termed the `impartial culture' assumption). He assumed there would be complete and sincere participation of the voting population, as in the median voter model, and that other voters' preferences and voting behavior are independent of each other.

Under these conditions, Fishburn analyzed the degree to which the Borda and Copeland extension of Borda give, or fail to give, the same selection. The Copeland extension of Borda is a Condorcet completion method, consisting of pairwise comparisons of all alternatives, so in essence what he was doing was determining the degree to which the Borda method of ranking would produce the Condorcet winner. Since the Condorcet winner is the result (in the absence of cyclical majorities) which is chosen by a pairwise majority voting system such as is used in Congress or in Parliament, this is one way of comparing how closely the Borda system would correspond to equilibrium outcomes of voting systems currently used.

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Additional work by Fishburn and Brams [51],[52] and Fishburn and Gehrlein [53],[54],[55],[56],[57] proceeds along the same lines, comparing voting systems in terms of their likelihood of choosing the Condorcet winner when it exists. In an article summarizing their work, Fishburn and Gehrlein [55] present the findings of their earlier studies. They consider the cases of 3, 4, and 5 alternatives, but restrict their summarization to `large numbers´ of voters. Their summary of `simple majority efficiencies´ (Condorcet efficiencies) for one stage procedures is presented below.

Table 2.2. Condorcet Efficiencies for Various Voting Systems[5] (%)

	Profile Generating Method			
Procedures	random n=101	model 1 power 1 n=101	model 2 power 1 n=101	MAX n=101
vote for 1	77	76	81	78
vote for 2	74	72	73	76
vote for ≤2	75-79			
w=(2,1,0)		91		
vote for 1	66	67	69	63
vote for 2	74	76	72	77
vote for 3	61	61	62	65
vote for ≤2	70-76			
vote for ≤3	64-70			
w=(3,2,1,0)		87		87
w=(2,1,0,0)		82		79
vote for 1	58	58	76	58
vote for 2	70	71	64	68
vote for 3	68	67	54	71
vote for 4	53	50	38	54
vote for ≤2	61-72			
vote for ≤3	63-69			
vote for ≤4	59-64			
w=(4,3,2,1,0)	85	87		87
w=(3,2,1,0,0)		84		
w=(2,1,0,0,0)	73	73		72

Fishburn and Gehrlein used several methods to generate preference profiles. These include (a) random: each of the voters is independently and randomly assigned one of the m! linear orders on the m candidates; (b) model 1, power 1: same as random but recorded differently (power 2 squares the number of voters with each preference order); (c) model 2, power 1: each of the linear orders is selected randomly. Each order is then sequentially assigned voters, with the probability that n_1 voters have this order assigned according to a binomial distribution. The second order is then taken and n_2 voters assigned it, etc., until all voters have been assigned a preference order or until the last preference order is reached, in which case all remaining voters are assigned it; (d) MAX: each of the preference orders is randomly assigned an integer in {1,2,...,101} as the number of voters who have that preference order (the number of voters varies between m! and 101(m!)). All of these methods have the expectation of producing the same number of voters for each preference order, but the variance of methods (b), (c), and (d) differs. The methods used do "tend to generate `close elections´ among the $m \geq 3$ contenders."[6] Fishburn and Gehrlein see this as a drawback because "the efficiency percentages...may represent only a small proportion of relevant multicandidate elections, and the `correct´ efficiency figures could well be much higher than those given in the tables."[7] However, there is no reason to believe that rankings of voting systems would change by adding in `non-close' elections. In these cases, the result is pretty much a foregone conclusion regardless of the voting system used. In fact, the relevant cases for a comparison of voting systems are precisely those in which the outcome would differ depending on the system used.

Table 2.2 clearly shows that the Borda weighted ranking system achieves higher Condorcet efficiency than the standard voting system. The approval voting system (vote for $\leq(m-1)$) generates a range of Condorcet efficiency numbers that in 2 out of 3 cases contains the estimated Condorcet efficiency for the standard voting system and in one case exceeds it. Fishburn and Gehrlein's work produces the following ranking: (1) Borda system; (2) approval voting system; (3) standard voting system.

Although the work assumed sincere voting, Fishburn and Gehrlein do discuss the possible effect of strategic voting on Condorcet efficiencies. They argue that "approval voting is more immune to strategic voting than any of the other =k or $\leq$k procedures...its efficiency estimates may compare more favorably to the efficiencies of other procedures when strategic voting is taken into account."[8] They do not predict the effect of strategic voting on Condorcet efficiency of the Borda system, but do not "count its apparent sensitivity to strategic misrepresentation of preferences in its favor."[9]

Weber [143] also compared voting systems from the point of view of their outcomes or expected outcomes. He did not use Condorcet efficiency as his comparison measure; instead he used social utility efficiency. Social utility can be considered the expected utility of a given alternative to a randomly chosen voter. Weber, assuming equally likely preference orders and complete and sincere participation, performed his analysis to determine the efficiency of a voting system in terms of social utility. Individual utilities were drawn from a uniform [0,1] distribution, and as previously noted, social utility was defined as the sum of individual utilities over all voters. Weber then determined what the expected social utility of the elected candidate for particular voting systems would be, and by comparing this with the expected social utility of the alternative with maximum social utility, scaled by the social utility of a randomly chosen alternative, developed the social utility efficiency measure: the percentage of maximum social utility a voting system is expected to produce. Using statistical tools for expected value, Weber computed the theoretical values of this efficiency measure for an infinite population of voters.

Weber [143] showed that the Borda system, the approval voting system and the standard voting system could be ranked in the order given. The social utility efficiencies of the systems for a 3-alternative race are, respectively, 87.5%, 86.6%, and 75%. Weber also showed that the Borda system increases in efficiency as the number of candidates is increased, whereas the standard voting system decreases in efficiency as the number of alternatives increases. He also proved that sincere voting is an optimal strategy asymptotically, and produces a unique symmetric Nash equilibrium. Sincere strategies are also sophisticated optimal strategies, given no information about the preferences of other voters.

In a subsequent article, Weber [143] first defined essentially equivalent voting systems as voting systems whose weights are positive affine transformations of each other; if a positive affine transformation of an optimal strategy under one system will yield the optimal strategy under the other system, this implies that these voting systems are essentially equivalent. He also showed that every nontrivial voting system is essentially equivalent to a unique minimal 0-1 normalized voting system ; the voting system weights are 0-1 normalized and the voting system is minimal in the sense that for every weight set of the system, there is at least one vector of utilities for which the weight set must be used in the corresponding optimal strategy. Using this definition, it is clear that all two-alternative voting systems are essentially equivalent to the standard voting system, which in the previous article was shown to have a social utility efficiency of $\sqrt{2/3}$ = 81.65%.

Following Weber's analyses, voting systems were compared by Chamberlin and Cohen [22]. Chamberlin and Cohen used the comparison method of the expected percentage of Condorcet winners (Condorcet efficiency), but also compared the multidimensional spatial model with the unidimensional impartial culture model. Spatial theory assumes that there are dimensions to an election corresponding to salient issues, and that every voter has a preferred ideal position in the voting space. The voter is assumed to cast his vote in the standard voting system for the alternative or candidate closest to him in the space that describes the factors that are of concern to the voter. They perceive the use of the spatial model as a generalization of the impartial culture assumption. This is not strictly correct, since, as noted earlier, the classic spatial model, with individual utility being a function of weighted Euclidean distance, gives all voters single-peaked preferences. The standard assumptions on complete and sincere participation continue to apply.

The voting systems which Chamberlin and Cohen compare include the standard voting system, the Borda system, and two multistage systems, the Hare and Coombs voting systems, which will not be discussed here. Their impartial culture results as presented below do not differ significantly from previous results.

Table 2.3. Chamberlin and Cohen: Proportion of Condorcet Winners Selected - Impartial Culture

	21 voters	1000 voters
Borda system	86%	89%
standard system	69%	69%

In contrast, their spatial model simulations produce varying results. All voters are represented by their ideal points in a four-dimensional space. The four numbers are generated as follows: voter j's position on the first dimension is chosen from a standard normal distribution; his position on the second dimension is generated from the first dimension position by perturbing it with normal noise; the third position is produced from the second with fresh noise, and the fourth from the third likewise. All values are then normalized to have variance 1. However, this produces an electorate characterized by the correlation matrix shown in Table 2.4.

Table 2.4. Expected Correlations Among Voter Dimensions

Dimension	1	2	3	4
1	-	.45	.33	.28
2	.45	-	.75	.68
3	.33	.75	-	.83
4	.28	.68	.83	-

Candidate or alternative positions are generated in the same way, but three variances are used: low (.04), medium (1.0), and high (1.5). Given this structure, Chamberlin and Cohen find that the existence of a Condorcet winner is more likely in the spatial model than the impartial culture assumption. Depending on candidate (alternative) dispersion and the number of voters, the probability ranges from 92 to 100%, as opposed to 84-85% for the impartial culture assumption for 4 alternatives. The arbitrary correlation used in assigning utilities to voters may have some influence on this result.

However, as shown below, the ranking of the Borda and standard voting system does not change. Because of the arbitrary nature of dimensional correlation, Chamberlin and Cohen's results do not generalize well for the spatial model.

Table 2.5. Chamberlin and Cohen: Proportion of Condorcet Winners Selected - Spatial Model

	21 voters			1000 voters		
cand variance:	low	med	high	low	med	high
Borda	83	83	92	85	86	97
standard	59	53	77	27	33	70

Following Chamberlin and Cohen, Merrill [93] compared voting systems using both Condorcet efficiency and social utility efficiency. He also varied the candidate dispersion in space in the spatial model relative to voters. Merrill's results for the impartial culture model bear a striking similarity to Fishburn and Gehrlein's. His spatial model results differ from Chamberlin and Cohen's, but he used a multivariate normal distribution to generate voter and candidate positions, with a variety of correlation structures.

Table 2.6. Merrill: Proportion of Condorcet Winners Selected (%)

Impartial Culture (25 voters)

# of candidates:	2	3	4	5	7	10
standard	100	79.1	69.4	62.1	52.0	42.6
approval	100	76.0	69.8	67.1	63.7	61.3
Borda	100	90.8	87.3	86.2	85.3	84.3

Table 2.6 (cont'd.)

Spatial Model (201 voters, 5 candidates)

dispersion	1.0		.5	
# of dimensions	2	4	2	4
standard	61	81	27	42
approval	81	84	75	82
Borda	89	92	86	88
% with Condorcet winner	99+	99+	98	99

Merrill's dispersion is the ratio of standard deviations of the marginal distributions for candidates and voters. Thus if dispersion is greater than 1, there is more variance in candidate positions than in voter positions and vice versa. His results do indicate that as dispersion increases, Condorcet efficiency increases for all voting systems. If candidate dispersion is high relative to voter dispersion, the median has a greater probability of winning, whereas if candidate dispersion is low, extreme candidates or alternatives have a greater probability of winning. Thus there should exist an equilibrium level of relative dispersion under which all distances from the center or median of the voting space are equally attractive to candidates. This nonconvergent equilibrium is in strong contrast to the median voter result of the unidimensional model, but is due to the discrete choice set. The same result occurs in the unidimensional model when a discrete choice set is used. Also, the nonconvergent equilibrium depends on the voting system. For the Borda and approval systems of voting, the advantage of the centrist candidates is little affected by the relative dispersion of voters and candidates because they are systems in which either approximately half or all but one of the candidates receive votes from voters.

Another interesting point is that Chamberlin and Cohen's assertion that existence of a Condorcet winner is more likely under spatial model assumptions is borne out. In their development of the spatial model, Enelow and Hinich show that when more than one dimension is used, the existence of a Condorcet winner, or a median in all directions (dominant point), becomes less and less likely as the number of dimensions is increased. However, any point in the issue space may be introduced as an alternative in their model. They are essentially working with a continuous choice set. In contrast, the discrete choice set may have an equilibrium where the continuous one does not, and based on Merrill's results, an increase in the number of dimensions increases the likelihood of an equilibrium point (Condorcet winner) when the size of the choice set (number of alternatives) remains constant.

Merrill's social utility efficiency results for the impartial culture assumption differ from his Condorcet results only in ranking the approval voting system above the standard voting system for a 3 alternative election. Otherwise all rankings remain the same. His results for the spatial model also parallel his Condorcet results.

Table 2.7. Merrill: Social Utility Efficiency

Impartial Culture (25 voters)

# of candidates	2	3	4	5	7	10
standard	100	83.0	75.0	69.2	62.8	53.3
approval	100	95.4	91.1	89.1	87.8	87.0
Borda	100	94.8	94.1	94.4	95.4	95.9

Spatial Model (201 voters, 5 candidates)

dispersion	1.0		.5	
# of dimensions	2	4	2	4
standard	74	93	22	52
approval	97	98	95	98
Borda	98	99	96	99

Note the close correspondence between social utility efficiency and Condorcet efficiency numbers between Tables 2.6 and 2.7. The distinct relationship between the two efficiency measures as discussed earlier is apparent here.

Merrill's social utility efficiencies for the two and three alternative races are appreciably larger than the asymptotic limits calculated by Weber. He cannot be using the same exact formulation, since Weber calculates that expected social utility of a two alternative election for all voting systems is 81.65%.

A final work using the expected outcomes approach to comparisons was written by Bordley [12]. He used both the spatial model and impartial culture assumptions to simulate the effect of various changes in the model on social utility efficiency. The variables analyzed included the number of alternatives and the number of voters. Generally, regardless of these values, rankings of the systems were (1) Borda; (2) approval voting; and (3) the standard voting

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system. However, social utility efficiency estimates for the approval voting system approached those of the Borda system as the ratio of the number of voters to the number of alternatives increased.

2.4 Varying Other Parameters of the System

2.4.1 Varying Weight Sets

In a 1974 article, Fishburn [49] took a different approach. In this article, he analyzed how many candidates should be voted for, as a parameter of the voting system, in order to maximize the efficiency of a voting system, in terms of agreement with the Condorcet criterion. He looked at both the simple voting system (vote for k of m), and the rank ordering system in which k are rank-ordered of m. He determined that a simple voting system reaches maximum efficiency by this criterion when as close to half of the candidates as possible are voted for. He also determined that weighted ranking systems, such as the Borda system, are most efficient when all candidates are ranked (k=m).

Evidence about the efficiency of various values for the k parameter, which is the number of alternatives about which information is provided, is presented in Table 2.2. For the standard voting system and the approval voting system, voting for as close to half as possible of the alternatives is seen to increase Condorcet efficiency; for the Borda system, ranking less than all alternatives decreases efficiency.

Weber also analyzed how the weight sets used (admissible strategies) affect the efficiency of the approval voting system and the Borda system. First it must be clarified that use of a different weight set may not produce an essentially equivalent system, which would have identical social utility efficiency to the original system. Although the Borda system with weights (m,m-1,...,1) is essentially equivalent to the Borda system with weights (m-1,m-2,...,0), the 3 alternative system with weights (4,3,0) is not essentially equivalent to the one with weights (2,1,0). His analysis does show that alternative weight sets can increase the social utility efficiency of a voting system.

Weber [143] also directly compared three voting systems with different parameter values:

a. vote for k of m voting system, the family in which $k \in (1,...,m-1)$. The standard system with which we are all acquainted has k=1.

b. the weighted ranking voting system with a single weight set $(w_1,...,w_m)$, of which the Borda system is representative with the weight set (m,m-1,...1). This is in contrast to the original Borda system with weight set (m-1,...,0).

c. the vote for-or-against k system, with weight sets $(w_1,...w_m)$ and $(w_1',...,w_m')$, where the first set, with w_1 through $w_k = 1$, w_{k+1} through $w_m = 0$ corresponds to voting for k candidates, and the second set, with w_1'

through $w_{m-k} = 1$ and w_{m-k+1} through $w_m = 0$ corresponds to voting against k candidates. Where m=3, this is the approval voting system with k=1. With more than three candidates, however, the approval voting system does not fit this model because in approval voting k is a choice variable for each voter.

Examining these three systems, Weber determined which k would maximize the effectiveness of each voting system according to his social utility efficiency measure. Looking at system a, in which one votes for k of m, he determined that its efficiency measure was:

$$1/(m+1) \times \left[\frac{3mk(m-k)}{m-1}\right]^{1/2}$$

Taking the derivative of this with respect to k and setting it equal to zero gives the result that when m/2 = k, the social utility efficiency of the vote for k system is maximized. Therefore, in the vote for k system, as close to half as possible of the candidates should be voted for. It is easily verified that social utility efficiency is symmetric about m/2 and that k = m/2 - 1 and k = m/2 + 1 are equivalent. Interestingly, this result corresponds to Fishburn's earlier work showing that Condorcet efficiency is also maximized when k = m/2. It can be shown that if k is set to be m/2, then as the number of candidates increases ($m \to \infty$), then the effectiveness of the vote for k system approaches a constant equal to $\sqrt{3}/2$ (= .866 or 86.6% social utility efficiency). This is in contrast to any fixed k as m increa

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m increases, in which case social utility efficiency decreases and the limiting value is zero.

In the weighted ranking voting system, the social utility efficiency of the voting system is:

$$\sqrt{m/(m+1)}$$

Any weighted ranking voting system has maximum social utility efficiency when all candidates are ranked, as opposed to any $k < m$.

Considering the vote for-or-against k system, Weber showed that vote for k and vote for n-k (i.e., vote against k) are essentially equivalent. The social utility efficiency of the vote for-or-against k system is:

$$w \times \left[\frac{12k(m-k)}{m(m-1)}\right]^{1/2}$$

where w = the expected value of the difference in utility between a_1 and b_1, where [a] is the set of alternatives voted for and [b] is the set of alternatives not voted for. $a_1 - b_1$ is the difference in utility between the most-liked in the set of alternatives which receive votes and the most-liked (or least-hated) in the set of alternatives which do not receive votes. It is easily verified that the vote for-or-against k system has strictly greater social utility efficiency than the vote for k system. This system has maximum social utility efficiency when k = .368m. The limiting social utility efficiency of the vote for-or-against k system is 92.25%. However, the limiting social utility efficiency of the Borda system is 100%.

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Asymptotically, the Borda system has social utility efficiency as great as any voting system possible.

2.4.2 Voter Concordance and Correlation

Fishburn extended his work to allow for the possibility of different `levels of agreement´ in the voting population. In a 1973 article [48], he examined the effect of voter concordance as measured by the Kendall-Smith coefficient of concordance, W, on the existence of a Condorcet winner and the degree to which the Borda system agrees with the Condorcet winner. The Kendall-Smith coefficient of concordance, developed in 1939, is a transformation of the variance of the rank of candidates across the voting population, adjusted for the numbers of voters and candidates. If variance in rank is high, there is little concordance, whereas if variance in rank is low, there is a substantial amount of agreement among voters in the population as to what is a desirable outcome. The analysis showed that there is more agreement between the Borda system and the Condorcet winner when W is extreme. If there is either very little voter concordance or extreme agreement across the voting population, then the systems tend to select the same outcome. That is, the likelihood of the Borda system selecting the Condorcet winner is greater at the extremes of W.

Bordley [12] examined the correlation coefficient r, with a range of -1 to 1. For the correlation coefficient, his assumption was of two equally sized groups in the voting

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population, with the correlation being between utilities in the two groups. When r=-1, there are two diametrically opposed groups in the voting population, whereas when r=1, the two groups are identical. Bordley showed that as r changes, the best voting system will change radically. When r=-1, dictatorship may be a preferable alternative to any voting system, whereas when correlation is perfect, the voting system used is of little importance with the exception of the approval voting system. This is in contrast to Fishburn's results on the Kendall-Smith coefficient of concordance and Condorcet efficiency. Fishburn's results showed an increase in Condorcet efficiency for the Borda system when there was little concordance. Presumably the difference is due to Bordley's assumption of diametrically opposed groups, which would decrease the variance in rank across the voting population. If _all_ preference orders occurred in equal numbers, this variance would increase; the `extreme disagreement' implied by a high Kendall-Smith coefficient is qualitatively different from the extreme disagreement produced in Bordley's model by a correlation coefficient of -1.

2.4.3 Other Parameters

Bordley's work indicates that the effect of altering the standard deviation of utilities is negligible. "Changing the standard deviation only changes the scale of utilities and does not affect results."[10] Normal or uniform distributions for utilities of alternatives to voters were also co

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also compared. Bordley provided evidence to support the idea that whether a normal or uniform utility distribution is used, the results in terms of ranking voting systems do not change.

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CHAPTER 3

A MODEL FOR SIMULATION OF VOTING SYSTEMS WITH STRATEGIC VOTING

A model which investigates the results of strategic voting as opposed to sincere voting should correspond as closely as possible to previous work for the comparisons to be meaningful. Therefore, the standard assumptions of previous work, with the exception of sincere voting should be incorporated into the model. In particular, generation of individual preference profiles is identical to the most commonly used method.

A more detailed explanation of how a voter's preferences are formed, how the possible strategies and structure of the voting system along with these preferences and information about other voters determine the strategy he chooses, and how all voters' strategies determine the outcome of the system is presented here.

3.1 Assumptions of the Model

1. A voter's preference ordering is based on the utility of various alternatives to him. Let u_{ij} be the cardinal utility of alternative i to voter j, where i indexes alternatives 1 through m and j indexes voters 1 through n. All u_{ij} are independently and identically distributed uniformly on the interval [0,1]. This determination of cardinal utilities allows all ordinal preference orderings, and in fact makes them equally likely for any given individual - the `impartial culture' assumption. Arrow does not require equal

probability for preference orderings, just admissibility of all preference orderings, or unrestricted domain.

2. Voters' possible strategies for a voting system include all weight sets $\underline{W} = [w_1,\ldots,w_m]$ which conform to the requirements of the particular voting system.

   a) the Borda system strategy set includes all weight sets $[w_1,\ldots,w_m]$ for which each w_i is an element of $\{0,1,\ldots,m-1\}$, and $w_i \neq w_j$ for all $i \neq j$. Thus voters' possible strategies for the Borda system include all permutations of $[0,1,\ldots,m-1]$.

   b) the standard voting system strategy set includes all weight sets $[w_1,\ldots w_m]$ for which each w_i is an element of $\{0,1\}$ and $\Sigma_i \, w_i = 1$. Therefore standard voting system strategies include exactly one weight of 1, with the remaining weights being 0.

   c) For the approval voting system, the strategy set includes all weight sets $[w_1,\ldots,w_m]$ for which each w_i is an element of $\{0,1\}$. Approval voting system strategies may include from zero to m weights of 1, and correspondingly m to zero weights of 0.

3. Voters' strategies determine the outcome of a voting system. The weights assigned by the n voters are summed over alternatives. The outcome of a voting system is the alternative which receives the greatest

total weight over the voting population, or $\arg\max_i \sum_{j=1}^{n} w_{ij}$. All ties are broken randomly.

4. Voters choose optimal strategies from their possible strategy sets by maximizing their expected utility based on their information about other voters' strategies.

5. Let $E\{u_j(\underline{W}_1,\underline{W}_2,\ldots,\underline{W}_n)\}$ be voter j's expected utility as a function of all voters' strategies including his own. An equilibrium point is a matrix of strategies $(\underline{W}_1,\underline{W}_2,\ldots\underline{W}_n)$ such that for each j=1,2,...,n,

$$E\{u_j(\underline{W}_1,\underline{W}_2,\ldots,\underline{W}_n)\} = \max_{\underline{W}_j} E\{u_j(\underline{W}_1,\underline{W}_2,\ldots,\underline{W}_n)\}$$

Simply put, all outcomes of a voting system must be in the set of Nash equilibria for the associated voting game.

3.2 Voting Strategies

3.2.1 Sincere Strategies

A sincere strategy for an individual voter is the strategy he chooses based only on his own preferences. Thus for the Borda system, the sincere strategy is an assignment of weights $[w_1,\ldots,w_m]$, where w_i = m-rank(i), which corresponds to the voter's true ranking of alternatives. Then if $u_1 \geq u_2 \geq \ldots \geq u_m$, the sincere strategy assigns weights so that $w_1 \geq w_2 \geq \ldots \geq w_m$. For the standard voting system, the sincere strategy is to assign a weight of one to the most preferred alternative ($w_i = 1$ iff $i = \arg\max_i u_i$, otherwise $w_i = 0$.) For the approval voting system, the sincere strategy is to vote for every alternative of greater than average utility [143]. Let $\underline{u} = (\Sigma_i u_i)/m$. Then if (ui-u) >

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$(u_i - \underline{u}) > 0$, $w_i = 1$; otherwise $w_i = 0$. Intuitively, $\underline{u}$ is the expected utility of the election when the voter does not participate, and the individual voter "approves" of any alternative which betters that.

3.2.2 Optimal Strategies

Since voters are maximizing expected utility, the information upon which they base their expectations is an important part of the model. In all cases voters are assumed to know the distribution from which all individual utilities are drawn (or, equivalently, the likelihood of each individual preference profile). They may know the strategies of voters other than themselves. However, the key piece of information that is used to determine an individual's optimal strategy is his estimate of total votes accruing to each of the alternatives, and the confidence level of his estimates. If the voter has full information, his determination of optimal strategy is based on the actual values of total votes accruing to alternatives, $\Sigma_i\ w_{ij}$, and his knowledge of his own strategy. If the voter has less than full information, it is based on his estimates $\hat{W}_i$, and given his confidence level, the probabilities of various outcomes occurring. The voter solves

$$\max_{W_j} \quad \Sigma_i \quad p_i(W_j, \hat{W}^n_{i \neq j}) u_{ij}$$

subject to the constraints of the voting system. The use of maximizing behavior on the part of voters can make a great difference to the performance of a voting system as measured by either Condorcet efficiency or social utility efficiency.

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3.3 An Analytical Example

Suppose that there is a committee of three (voters 1,2, and 3) which has to choose one of three alternatives A, B, and C. The model specified makes all preference orders equally likely. Consider then the problem of a `representative´ voter. This voter will, under the assumption of sincere voting, choose a strategy which corresponds to his true preference ordering. Outcomes of sincere strategies for the standard and Borda voting systems are presented in Table 3.1, along with the Condorcet winner for the given preference profile. Because of the equal likelihood of individual preference orders and symmetry of the system, the Condorcet efficiency of a system given a profile for voter 1 is the same as that for the system. The outcomes shown are used to determine Condorcet efficiencies for the standard voting system and the Borda system with sincere voting, which are 88.24% (30/34) and 95.59% (32.5/34) respectively.

Table 3.1. Outcomes for the Standard and Borda Voting Systems with Sincere Voting

Voter 1: A>B>C

2 \3	A>B>C	A>C>B	B>A>C	B>C>A	C>A>B	C>B>A
A>B>C	a/a/a	a/a/a	a/a/a	a/a,b/a	a/a/a	a/a/a
A>C>B	a/a/a	a/a/a	a/a/a	a/a/a	a/a/a	a/a/a
B>A>C	a/a/a	a/a/a	b/b/b	b/b/b	*/a/a	*/b/b
B>C>A	a/a,b/a	a/a/a	b/b/b	b/b/b	*/*/X	*/b/b
C>A>B	a/a/a	a/a/a	*/a/a	*/*/X	c/a,c/c	c/c/c
C>B>A	a/a/a	a/a/a	*/b/b	*/b/b	c/c/c	c/c/c

Entries are standard outcome/Borda outcome/Condorcet winner. A * indicates that a tie occurs among all alternatives, which is broken randomly. Where two alternatives are listed, only those two are tied.

When instead voters choose optimal strategies, a Nash equilibrium point will determine the outcome. Possible strategies for the standard and Borda systems and outcomes given total votes accrued are presented below.

Table 3.2 Possible Strategies and Outcomes Given Others' Strategies

3.2.1: Borda System

$W^{n}_{i\neq j}$		W_j 2,1,0	2,0,1	1,2,0	1,0,2	0,2,1	0,1,2
4,2,0		<u>a</u>	<u>a</u>	<u>a</u>	<u>a</u>	a,b	<u>a</u>
4,0,2		<u>a</u>	<u>a</u>	<u>a</u>	<u>a</u>	<u>a</u>	a,c
4,1,1		<u>a</u>	<u>a</u>	<u>a</u>	<u>a</u>	<u>a</u>	<u>a</u>
3,3,0		<u>a</u>	<u>a</u>	b	<u>a</u>	b	b
3,0,3		<u>a</u>	<u>a</u>	<u>a</u>	c	c	c
3,2,1		<u>a</u>	<u>a</u>	a,b	<u>a</u>	b	*
3,1,2		<u>a</u>	<u>a</u>	<u>a</u>	a,c	*	c
2,4,0		b	<u>a,b</u>	b	b	b	b
2,0,4		<u>a,c</u>	c	c	c	c	c
2,3,1		a,b	<u>a</u>	b	*	b	b
2,1,3		<u>a</u>	a,c	*	c	c	c
2,2,2		<u>a</u>	<u>a</u>	b	c	b	c
1,4,1		<u>b</u>	<u>b</u>	<u>b</u>	<u>b</u>	<u>b</u>	<u>b</u>
1,1,4		<u>c</u>	<u>c</u>	<u>c</u>	<u>c</u>	<u>c</u>	<u>c</u>
1,3,2	X	<u>b</u>	<u>*</u>	<u>b</u>	c	<u>b</u>	b,c
1,2,3	X	<u>*</u>	c	<u>b</u>	c	b,c	c
0,4,2		<u>b</u>	<u>b</u>	<u>b</u>	<u>b</u>	<u>b</u>	b,c
0,2,4		c	c	<u>b,c</u>	c	c	c
0,3,3		<u>b</u>	c	<u>b</u>	c	<u>b</u>	c

3.2.2: Standard System

$W^{n}_{i\neq j}$		W_j 1,0,0	0,1,0	0,0,1
2,0,0		<u>a</u>	<u>a</u>	<u>a</u>
0,2,0		<u>b</u>	<u>b</u>	<u>b</u>
0,0,2		<u>c</u>	<u>c</u>	<u>c</u>
1,1,0		<u>a</u>	b	*
1,0,1		<u>a</u>	*	c
0,1,1	X	<u>*</u>	<u>b</u>	c

\`Optimal' strategies for a voter with A>B>C are underlined. In the rows marked with an X, where more than one strategy is underlined, determination of the optimal strategy depends on whether the expected utility from a random choice exceeds the individual's utility for his second-ranked alternative. If strategies are equivalent in terms of payoffs, the voter is assumed to maintain the current strategy (usually the sincere strategy).

If a preference profile is given, sincere strategies can be determined and then voters checked individually to see if an increase in expected utility can be obtained by changing strategy. When no voter can unilaterally increase his expected utility given the strategies of others, a Nash equilibrium has been reached.

Given the classic majority cycle profile, either one, two or three Nash equilibria will be found for the standard voting system.

Voter 1: A>B>C
Voter 2: B>C>A
Voter 3: C>A>B

If all voters have expected utility for a random choice (EU(*)) exceeding the utility of their second-ranked alternative (u_2), sincere voting will be the only Nash equilibrium found. If exactly one voter has u_2>EU(*), that voter will vote for his second choice, which will be the Nash equilibrium outcome found. If more than one voter has u_2>EU(*), then the number of voters with this characteristic is the number of equilibria this method of solving can find. The equilibria found are not equally probable for a given social preference profile. However, the probability of a particular equilibrium can be determined using the probability that voters' cardinal utilities have specific characteristics, and the frequency with which this equilibrium is found will reflect this probability.

The equilibrium outcomes of the 3-alternative 3 voter election for the standard and Borda voting systems, assuming voters choose optimal strategies, are shown in Table 3.3.

Table 3.3 Outcomes for the Standard and Borda Voting Systems with Strategic Voting

Voter 1: A>B>C

2 \3	A>B>C	A>C>B	B>A>C	B>C>A	C>A>B	C>B>A
A>B>C	a/a/a	a/a/a	a/a/a	a/a,b/a	a/a/a	a/a/a
A>C>B	a/a/a	a/a/a	a/a/a	a/a/a	a/a/a	a/a/a
B>A>C	a/a/a	a/a/a	b/b/b	b/b/b	I/1/a	II/2/b
B>C>A	a/a,b/a	a/a/a	b/b/b	b/b/b	*/*/X	III/3/b
C>A>B	a/a/a	a/a/a	I/1/a	*/*/X	c/a,c/c	c/c/c
C>B>A	a/a/a	a/a/a	II/2/b	III/3/b	c/c/c	c/c/c

Entries are standard outcome/Borda outcome/Condorcet winner. Numbered outcomes have probabilities for each of the alternatives being an equilibrium outcome:

I: 15/24 a + 8/24 b + 1/24 c
II: 8/24 a + 15/24 b + 1/24 c
III: 1/24 a + 15/24 b + 8/24 c
1: 19/24 a + 4/24 b + 1/24 c
2: 4/24 a + 19/24 b + 1/24 c
3: 1/24 a + 19/24 b + 4/24 c

Condorcet efficiencies are now 93.38% for the standard voting system and 95.22% for the Borda system. Condorcet efficiencies clearly do change as the assumption of sincere voting is dropped. The Borda system suffers a slight decrease, while the standard voting system performs significantly better.

3.4 Information Conditions

Previous work has either assumed that voters use sincere strategies or, alternatively, that voting takes place under zero information conditions. Particularly for committee voting, zero information is not the most realistic assumption to make. Frequently committees have one or more `vocal´ members whose preferences are common knowledge.

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The example in the previous section assumed complete information. Let us see how `incomplete´ information affects Condorcet efficiency under the standard voting system. As before, voters may have any preference profile, but now know only the strategy of voter 3 beside their own. This of course implies that voter 3 knows only his own strategy.

Table 3.4. Possible Strategies and Expected Utility for Voters 1 and 2, Given Voter 3´s Strategy

Voter 3	$EU(W_j)$	W_j
1,0,0	u_A	1,0,0
	$4/9(u_A + u_B)+1/9(u_C)$	0,1,0
	$4/9(u_A + u_C)+1/9(u_B)$	0,0,1
0,1,0	$4/9(u_A + u_B)+1/9(u_C)$	1,0,0
	u_B	0,1,0
	$4/9(u_B + u_C)+1/9(u_A)$	0,0,1
0,0,1	$4/9(u_A + u_C)+1/9(u_B)$	1,0,0
	$4/9(u_B + u_C)+1/9(u_A)$	0,1,0
	u_C	0,0,1

Our representative voter´s optimal strategy again depends on his cardinal utilities. However, where he previously had a 50% chance of preferring the insincere strategy on the basis of his expected utility, he now has only a 20% chance of this occurring ($P[(4u_A+u_C)<5u_B]$). The median of three independent uniform random variables (A>B>C) is distributed uniformly on [C,A]. The conditional probability density function of the median of three independent uniform random variables on the same interval is 1/(A-C). Therefore $P[(4A+C)<5B] = \int_{(4A+C)/5}^{A} 1/(A-C)\, dB = 1/5$. With this specification of information structure, the equilibrium outcomes of the standard voting system are as shown in Table 3.5.

Table 3.5. Equilibrium Outcomes of the Standard Voting System When One Specific Voter's Strategy is Known.

Voter 1: A>B>C

2 \3	A>B>C	A>C>B	B>A>C	B>C>A	C>A>B	C>B>A
A>B>C	a	a	1	1	a	a
A>C>B	a	a	2	2	a	a
B>A>C	a	a	b	b	*	*
B>C>A	a	a	b	b	3	3
C>A>B	a	a	4	4	c	c
C>B>A	a	a	5	5	c	c

Numbered outcomes have probabilities for each of the alternatives being an equilibrium outcome:

1: .64 a + .36 b
2: .6933 a + .2533 b + .0533 c
3: .2667 a + .2667 b + .4667 c
4: .2667 a + .4667 b + .2667 c
5: .2133 a + .5733 b + .2133 c

In this case, Condorcet efficiency for the standard voting system is 85.33%.

If two voters' strategies are known, the results change again. Suppose the strategies of voters 2 and 3 are known. Both of these voters calculate optimal strategies in accordance with Table 3.4, while voter 1 uses Table 3.2.2. Then results are as shown in Table 3.6.

Table 3.6. Equilibrium Outcomes of the Standard Voting System When Two Voters' Strategies are Known.

Voter 1: A>B>C

2 \3	A>B>C	A>C>B	B>A>C	B>C>A	C>A>B	C>B>A
A>B>C	a	a	a	a	a	a
A>C>B	a	a	a	a	a	a
B>A>C	a	a	b	b	1	2
B>C>A	a	a	b	b	3	4
C>A>B	a	a	1	3	c	c
C>B>A	a	a	2	4	c	c

Table 3.6 (cont'd.)

Numbered outcomes have probabilities for each of the alternatives being an equilibrium outcome:

1: .1667 a + .6667 b + .1667 c
2: .1333 a + .7333 b + .1333 c
3: .1333 a + .5333 b + .3333 c
4: .1067 a + .6267 b + .2667 c

Condorcet efficiency in this case is 91.33%. Interestingly, Condorcet efficiency does not follow a predictable pattern given the information level. Condorcet efficiencies when 0, 1, 2, and 3 voters' strategies are known are 88.24%, 85.33%, 91.33%, and 93.38%. This is due in part to the asymmetry of information between voters.

An important point here is that in the 3 voter, 3 alternative standard voting game, it is always in the voter's interest to reveal his strategy. Presented below are the probabilities of first, second, and third choices being chosen by the system if the voter either does or does not reveal his strategy.

Table 3.7. Probabilities of Voters' 1st, 2nd, and 3rd Choices Being Chosen by the Standard Voting System Given the Information Structure of the Game.

	P(first)	P(second)	P(third)
zero information	.6296	.1852	.1852
one voter known			
strategy revealed	.6919	.1541	.1541
strategy unknown	.5784	.2252	.1963
two voters known			
strategy revealed	.6517	.1822	.1661
strategy unknown	.5856	.2533	.1611
three voters known	.6296	.2176	.1528

The first voter to reveal his strategy does so because this policy stochastically dominates that of concealing his strategy (zero information). The same holds true for the second and third voters, who compare their previous strategies of one voter known, strategy unknown, and two voters known, strategy unknown, respectively. Interestingly, this implies that an incomplete information game, at least in this example, is not an equilibrium outcome, because it is in each individual's interest to reveal his strategy. However, when the voting population becomes larger, it may in reality be difficult for each individual voter to communicate his strategy to all other voters unless there is systematic reporting, such as on support for various bills before Congress. An incomplete information game may therefore occur.

3.5 Sincere Strategies and Nash Equilibria

It has been shown [58],[120] that every non-dictatorial voting system with at least three alternatives is manipulable. That is, there is always some social preference profile for which an individual can improve his utility by misrepresenting his preferences. In other words, there is always a case for which sincere strategies do not constitute a Nash equilibrium. Given the stated assumptions about voters' behavior and an infinite voting population, Weber [139] showed that sincere strategies are asymptotically optimal if only the distribution from which cardinal utilities are drawn is known to voters besides their own utilities. To show this, he used the fact that the number of votes cast by one voter for a particular candidate and the probability that this number of votes is critical (changes the outcome of the election) are asymptotically proportional. Then subjective expected gain from a vote vector $[w_1, \ldots w_m]$ is asymptotically proportional to

$\Sigma_{c \neq d}\ (u_c - u_d)\ \max\ \{0,\ w_c - w_d\} = m[\Sigma_c\ w_c(u_c - \underline{u})]$.

An optimal strategy is then an assignment of weights which maximizes $\Sigma_c\ w_c(u_c - \underline{u})$, and Weber demonstrates the optimality of sincere strategies for each voting system, showing that sincere strategies under these conditions produce a unique symmetric Nash equilibrium.

It can be shown that either an infinite voting population or zero information conditions are sufficient for sincere strategies to constitute a Nash equilibrium, and that both are not needed.

3.5.1 An Infinite Voting Population

A set of strategies is not a Nash equilibrium if for any voter j, there exists some strategy $\underline{W}$ for which

$$E\{u_j(\overset{n}{\underline{W}}_{i \neq j}, \underline{W})\} > E\{u_j(\overset{n}{\underline{W}}_{i \neq j}, \underline{W}_j)\},$$

where $\overset{n}{\underline{W}}_{i \neq j}$ is the set of strategies for all other voters. Clearly, an individual must be able to change the outcome of the voting system by altering his strategy for $(\overset{n}{\underline{W}}_{i \neq j}, \underline{W}_j)$ to be excluded from the set of Nash equilibria.

<u>Theorem 1:</u> As the voting population becomes large, i.e. $n \to \infty$, the probability that sincere strategies constitute a Nash equilibrium approaches one.

<u>Proof</u> (standard voting system):

For the individual voter, any w_{ij} is a binomial random variable (either a vote is cast for it or not), with p = 1/m. Then $W_i = \Sigma_j w_{ij}$ is distributed approximately normally with mean np = n/m and variance np(1-p) = $n(m-1)/m^2$, and the W_i have an approximate multivariate normal distribution. In order for an individual voter to change the outcome of the system, there must be some $|W_i - W_k| \leq 1$. That is, the voter's maximum weight assignment of one can cause the ordering of two totals to change. Let $Y = W_i - W_k$. Then Y has a mean $\mu_Y = \mu_{W_i} - \mu_{W_k} = 0$; and variance $\sigma_Y^2 = \sigma_{W_i}^2 + \sigma_{W_k}^2 + 2\sigma_{W_i W_k}$. Because of the relationship between the covariance and correlation coefficient this variance can be computed exactly; the correlation coefficient is -1/(m-1). Intuitively, when one of the W_i is above its mean, the others are expected to be slightly below the mean. Computing this, a variance of $\sigma_Y^2 = 2n(m-2)/m^2$ is obtained. Obviously, as $n \to \infty$, the variance of Y becomes infinite. Therefore $P\{|W_i - W_k| \leq 1\} = P\{-1 \leq Y \leq 1\}$, the probability that Y falls within the specified interval, approaches zero. Thus scope for strategic behavior diminishes asymptotically and the probability that sincere strategies constitute a Nash equilibrium approaches one. An analogous proof can be constructed for the Borda system and the approval voting system (see appendix A).

3.5.2 Information Conditions Again

Recall that voters choose optimal strategies based on their information about other voters' strategies (assumption

4). I will assume that this information is obtained by sampling the voting population and that the information obtained is correct. As an individual voter's sample size becomes smaller, his estimates of the total votes accruing to alternatives become less accurate, and their variances increase. Specifically, let $\hat{W}_i$ be representative voter j's estimate of total votes accruing to alternative i and n_s be the number of voters sampled, with W_i^s being the sample total. Because of the independence of the u_{ij}, the voter's best estimate $\hat{W}_i$ is

$$\hat{W}_i = W_i^s + (n - n_s)\, E(\Sigma_i\, w_{ij})/m,$$

where $E(\Sigma_i\, w_{ij})$ is the expected total weight for an individual voter. For the standard voting system and the Borda system this can be calculated precisely since it is not random, but for the approval voting system it must be designated as an expectation. The variance of $\hat{W}_i$ is $(n - n_s)$ times $\mathrm{var}(w_{ij})$. If $n_s = n$, variance is zero and the voter has complete information. If $n_s = 0$, then $\hat{W}_i = n\, E(\Sigma_j\, w_{ij})/m$, and the variance of $\hat{W}_i$ is n times $\mathrm{var}(w_{ij})$, which is the zero information condition used by Weber.

<u>Theorem 2:</u> As individual voters' estimates of other voters' specific strategies become less accurate, i.e., their sample size becomes smaller, the probability that sincere strategies constitute a Nash equilibrium approaches one.
<u>Proof:</u> As shown above, as sample size diminishes, the limiting condition is the zero information condition used by Weber. It remains to be shown that with zero information,

the sincere strategy is the optimal strategy for an individual voter. Recall that Weber used the asymptotic proportionality of the number of votes cast by one voter for a particular candidate and the probability that this number of votes is critical (p_i). With this, he shows that subjective expected gain from a vote vector $[w_1,\ldots,w_m]$ is asymptotically proportional to $m[\Sigma_i\, w_i(u_i-\underline{u})]$. Asymptotic proportionality of p_i and w_i is a sufficient but not necessary condition for this result. The necessary conditions are that P_i, the probability that outcome i occurs, be a positive function of w_i ($P_i = f(w_i)$), with

$$\delta P_i/\delta w_i > 0; \quad \delta^2 P_i/\delta w_i^2 \geq 0; \quad \Sigma_i\, P_i = 1$$

That is, the probability of a specified alternative i occurring is strictly positively related (increasing at an increasing rate) to the number of votes cast for alternative i, w_i, within the constraints of the voting system. This condition holds for the model employed here.

Under zero information conditions, the probability of occurrence of a specified alternative increases at an increasing rate with w_i, with a strict one-to-one correspondence of w_i and P_i. Therefore, given that the sum of the w_i is a constant, a vote vector which maximizes $\Sigma_i\, w_i(u_i-\underline{u})$ over $\underline{W}$ also maximizes $\Sigma_i\, f(w_i)u_i = \Sigma_i\, p_i u_i$ over $\underline{W}$, or expected utility under the constraints of the voting system, and is an optimal strategy for the voter. However, the vote vector which does this is simply the sincere strategy, as shown by Weber. Therefore, under zero information conditions, sincere strategies constitute a Nash equilibrium.

As an example for the standard voting system, consider the following three-alternative, three voter election. For representative voter j, possible values of $W^n_{i \neq j}$ are shown in the left-hand column, along with their probable occurrence in parentheses. Voter j's possible strategies of voting for alternatives A, B, or C, and the possible outcomes of the strategy are shown in columns 2, 3, and 4.

Table 3.8. Strategies and Possible Outcomes of a Three-Alternative,Three Voter Election.

$W^n_{i \neq j}$	W_j: A = [1,0,0]	B = [0,1,0]	C = [0,0,1]
[1,1,0] (2/9)	a	b	*
[1,0,1] (2/9)	a	*	c
[0,1,1] (2/9)	*	b	c
[2,0,0] (1/9)	a	a	a
[0,2,0] (1/9)	b	b	b
[0,0,2] (1/9)	c	c	c

*A tie occurs which will be broken randomly.

It is easily verified that if voter j votes for alternative A, his expected utility is

$$p_A u_A + p_B u_B + p_C u_C = .6926\ u_A + .1852\ u_B + .1852\ u_C.$$

For the standard and Borda voting systems, given the number of voters and alternatives, any vote vector has a corresponding probability vector, and a permutation of the vote vector corresponds to an analogous permutation of the probability vector. Although probabilities are not a linear function of the weights assigned for small voting populations, there is a strict mapping from vote vectors to probability vectors (which is asymptotically linear). For the approval voting system, there is a strict mapping for any fixed number of total votes. Vote vectors and their corresponding probability vectors are as shown in Table 3.9.

Table 3.9. Vote Vectors and Corresponding Probability Vectors for a 3 Alternative, 3 Voter Election

Vote Vector	Probability Vector (=[P_A,P_B,P_C])
	Standard Voting System
[1,0,0]	[.6926,.1852,.1852]
[0,1,0]	[.1852,.6926,.1852]
[0,0,1]	[.1852,.1852,.6926]
	Borda System
[2,1,0]	[.6162,.2689,.1159]
[2,0,1]	[.6162,.1159,.2689]
[1,2,0]	[.2689,.6162,.1159]
[0,2,1]	[.1159,.6162,.2689]
[1,0,2]	[.2689,.1159,.6162]
[0,1,2]	[.1159,.2689,.6162]
	Approval Voting System
[1,0,0]	[.6574,.1713,.1713]
[0,1,0]	[.1713,.6574,.1713]
[0,0,1]	[.1713,.1713,.6574]
[1,1,0]	[.4491,.4491,.1018]
[1,0,1]	[.4491,.1018,.4491]
[0,1,1]	[.1018,.4491,.4491]

3.6 The Simulation Program and Solving Algorithm

The simulation program is set up in accordance with the model specified. (For specific programs, see appendix B.) A cardinal utility vector is generated for each voter. With these, the Condorcet winner, it if exists, and the alternative with maximum social utility are determined. Given the voting system, the sincere strategy corresponding to the utility vector for each voter is determined. Then for each voter, possible pure strategies are taken one at a time and the voter's expected utility for the possible strategy calculated. With complete information, expected utility is only an expectation in the case of a tie occurring (because the tie will be broken randomly). With incomplete information, expected utility depends on the calculation of probabilities of outcomes, which depend on the strategy chosen, as well as the voter's sample size.

Expected utility is calculated for all possible strategies. It is then compared with expected utility for the voter's sincere strategy. If expected utility from another strategy exceeds that of the voter's sincere strategy (an alternative strategy strictly dominates the sincere strategy), the individual's vote vector is changed accordingly. If more than one alternative strategy has the same (maximum) expected utility, one of these strategies is chosen randomly, and the individual's vote vector changed accordingly. The process continues, checking each possible strategy for an expected utility increase. If expected utility remains constant with a change of strategy, the original vote vector is kept; there is no reason to assume that a strategy will change unless a gain is expected. Each voter is checked in a similar fashion until a Nash equilibrium is reached, or a specified number of iterations checking strategies (40) have been done. If after 40 iterations no equilibrium has been found, voters are randomly reordered and the process repeated. When an equilibrium is found, the outcome of voters' strategies is determined along with its social utility; it is compared with the Condorcet winner when it exists to see if they are the same, and the results are used to estimate the efficiency measures. Efficiency measure estimates are based on 2000 repetitions of the voting system for a given number of voters and alternatives. Numbers of alternatives range from 3 to 6, and the size of the voting population ranges from 3 to 125.

For the incomplete information game, the number of alternatives is set at three, and sample size (the number of voters used in determining total votes for a subset of the population) is taken as 2/3 of the voting population, rounded to the nearest integer ($\alpha = .6667$). The probability of each outcome (given the sample) should be approximately equal for different electorate sizes, given essentially equivalent population profiles, inducing equivalent optimal strategy responses from voters. Differences are due to the reduced likelihood of a tie in the larger voting population, just as in the complete information simulations. Voting populations again range from 3 to 125.

3.7 Restriction to Pure Strategy Equilibria

Voters' possible strategies in the solving algorithm include only pure strategies. Models in which an element of randomness is introduced for voters (see p. 31) have in general introduced a probability of voting as opposed to probabilities for strategies. If we think of sincere voting as one possible strategy and abstention as another, this type of model arbitrarily restricts voters' possible strategies to these two. Also, I question whether it is reasonable to expect voters, even in committee voting, to choose a mixed strategy when an optimal pure strategy response can be found.

Merrill [92] proved that all "potentially uniquely optimal strategies"[1] are pure strategies. A potentially uniquely optimal strategy may be a unique best response to others' strategies. A point W_j in a convex subset S of R^m is called extreme if it is not interior to any line segment contained in S.

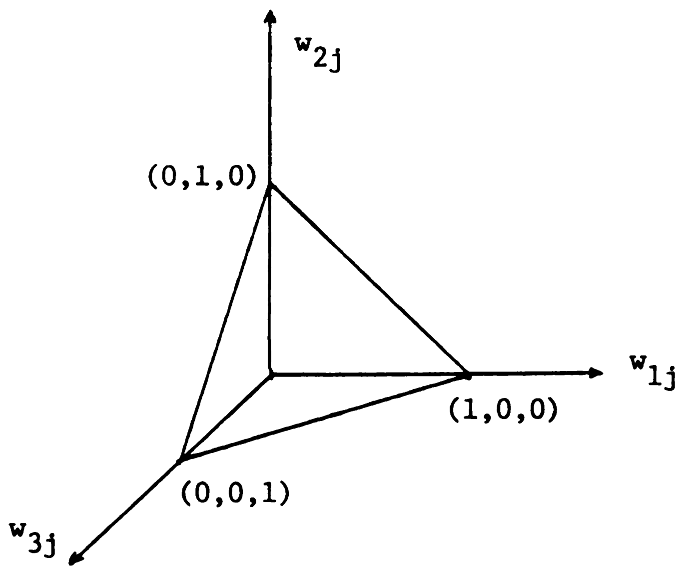


Figure 3.1. Possible Strategies in the Standard Voting System with 3 Alternatives: a Convex Subset of R^m.

If a voting system S is a convex subset of R^m, then "the potentially uniquely optimal strategies are extreme points of S."[2] Let E(i), the 'strategic value' of alternative i, be $\sum_{j=1}^{m} (u_i-u_j)p_{ij}$, where p_{ij} is the probability of being decisive between alternative i and j ($p_{ii}=0$). Merrill's formulation of expected utility is

$EU(W_j) = \sum_{i=1}^{m} E(i)v_i$, where v_i is the number of votes in W_j for alternative i. If W_j is a potentially uniquely optimal strategy, then there exists a total utility function such that $EU(W_j) > EU(W_j')$ for all W_j' in S other than W_j. Because $EU(W_j)$ is a linear combination of the E(i)'s, W_j must be an extreme point.

One significant conclusion can be drawn from Merrill's work. A mixed strategy is a linear combination of pure strategies and therefore interior to a line segment contained in S. Therefore a mixed strategy is never potentially uniquely optimal. In other words, a mixed strategy can never be a unique best response in the game. All unique best responses are pure strategies.

Additionally, in cases where a mixed strategy is a best response, there exists a pure strategy with equal expected utility. A mixed strategy is only optimal if the voter is indifferent between two or more pure strategies which in linear combination produce the mixed strategy. However, if this is the case, he is also indifferent between the pure strategies which produce the mixed strategy and the optimal mixed strategy itself. There is thus always a pure strategy response with equal expected utility to the optimal mixed strategy response.

There are cases for which the solving algorithm does not find a pure strategy equilibrium given a fixed order of voters for checking strategies. An example of such a case for the 3-alternative 5 voter Borda voting system will serve to illustrate the point.

Table 3.10. An Example of Preferences for Which a Pure Strategy Equilibrium is Not Found when Voters are Taken in a Specified Order

Expected Utility Matrix

j \ i	1	2	3
1	4.967650E-002	9.129716E-001	3.133120E-001
2	8.345773E-001	7.409244E-001	7.170978E-001
3	2.628670E-001	6.382484E-003	2.704006E-001
4	6.233332E-001	3.598900E-001	6.352836E-001
5	3.980304E-001	6.974258E-001	6.352836E-001

Sincere Vote Matrix

j \ i	1	2	3
1	0	2	1
2	2	1	0
3	1	0	2
4	1	0	2
5	0	2	1

Preferences
Voter 1: 2 > 2,3 > 1,2 > 1,2,3 > 3 > 1,3 > 1
Voter 2: 1 > 1,2 > 1,3 > 1,2,3 > 2 > 2,3 > 3
Voter 3: 3 > 1,3 > 1 > 1,2,3 > 2,3 > 1,2 > 2
Voter 4: 3 > 1,3 > 1 > 1,2,3 > 2,3 > 1,2 > 2
Voter 5: 2 > 2,3 > 3 > 1,2,3 > 1,2 > 1,3 > 1

a,b denotes a tie which will be broken randomly.

The solving algorithm produces the following sequence of strategy changes:

Table 3.11. Sequence of Strategy Changes Produced by the Solving Algorithm

Individual Strategies and Total Votes

j	1	2	3	4	5	Total
	0,2,1	2,1,0	1,0,2	1,0,2	0,2,1	4,5,6
W_j	1,2,0	2,1,0	1,0,2	1,0,2	0,2,1	5,5,5
	1,2,0	2,1,0	2,0,1	1,0,2	0,2,1	6,5,4
	0,2,1	2,1,0	2,0,1	1,0,2	0,2,1	5,5,5
	0,2,1	2,1,0	1,0,2	1,0,2	0,2,1	4,5,6

The pure strategy equilibria {[2,1,0], [2,1,0], [0,1,2], [1,0,2], [0,1,2]}, {[0,2,1], [2,1,0], [1,0,2], [1,0,2], [1,0,2]}, and {[0,2,1], [2,1,0], [2,0,1], [1,0,2], [0,1,2]} all exist for this preference profile but are not found by the solving algorithm because the voters are taken in a specified order. However, if voters are taken at random to have their strategies checked, there is no way of ensuring that all voters' strategies are checked (verifying the existence of the Nash equilibrium). A random reordering of all voters and repeat of the process solves the problem, and an equilibrium is found in every case.

3.8 The Nature of Equilibria Found

Not all Nash equilibria are found by the solving algorithm. Because of its construction, if sincere strategies constitute a Nash equilibrium, then for that social preference profile the outcome of the voting system is the outcome of sincere voting. Only if sincere strategies do not constitute a Nash equilibrium is strategic voting taken into consideration. In the previous example (Tables 3.10 and 3.11) the strategy [2,1,0] for all voters is a pure strategy Nash equilibrium point; none of the voters can unilaterally increase his expected utility. However, the solving algorithm provides no motivation for individual voters to alter their strategies to reach this equilibrium. In fact, both voters 1 and 5 are strengthening their last-ranked alternative at the expense of their first and second choices. The strength of the solving algorithm lies in the fact that any Nash equilibrium found can be reached via individual strategy changes (motivated by expected utility maximization) from the sincere strategy matrix. In this case, the equilibrium will correspond to a minimal β-coalition of the associated cooperative game.

In cooperative games, the characteristic set V(s) delineates a set of payoff vectors for each possible coalition S which represent the worth or effectiveness of the coalition S. In beta theory, a vector of payoffs is included in the characteristic set V(s) if and only if it is non-preventable by players outside the coalition.[3] In other words, if players outside the coalition have some strategy or set of strategies which could prevent this payoff vector from occurring, it is not included in the beta solution. A simple example using the standard voting system should clarify the idea of the beta solution.

Table 3.12. Expected Utility, Preference Orderings, and Sincere Strategies of Voters Using the Standard Voting System.

	Voter		
Alternative	1	2	3
A	.016	.365	.694
B	.682	.482	.247
C	.793	.218	.413
*	.497	.355	.4513

Expected value of u_{ij}

Preference orderings	Sincere strategies
voter 1: C>B>*>A	[0,0,1]
voter 2: B>A>*>C	[0,1,0]
voter 3: A>*>C>B	[1,0,0]

If voters 1 and 2 form a coalition, they can achieve any of the possible payoff vectors (rows of Table 3.12) for A, B, or C. They cannot guarantee the payoff vector for a random choice (*) because regardless of the strategies they choose, voter 3 has a strategy which can prevent it. If voters use sincere strategies, this final payoff vector is the outcome. Sincere strategies clearly do not constitute a Nash equilibrium in this case. If voter 1 votes for alternative B instead of his most-preferred alternative C, his expected utility increases. Additionally, if voter 1 does this, neither of the other voters can increase their expected utility by altering strategy and this set of strategies is a Nash equilibrium.

1: [0,1,0]
2: [0,1,0]
3: [1,0,0]

However, the set of strategies

1: [0,0,1]
2: [1,0,0]
3: [1,0,0]

is also a Nash equilibrium in this game. If the solving algorithm looks at voter 2 before voter 1, this is the equilibrium that will be found. Because of the randomness of individual utilities, the Monte Carlo techniques employed make it equally likely that the individual utilities will occur in either order, and a sufficient number of repetitions will find each equilibrium; furthermore, they will occur with equal probability (given that exactly 2 voters have $u_2 > EU(*)$).

The equilibria above correspond to minimal β-coalitions because the removal of one player from the coalition causes it to fall apart. If we looked at a standard voting system game with five players, a coalition of 4 would not be minimal because the removal of one player would still leave a decisive coalition of 3. The solving algorithm will not find an equilibrium in which individuals vote strategically corresponding to a non-minimal β-coalition in a game with complete information. Subsequent to the assignment of strategies corresponding to a minimal β-coalition, no voter outside the coalition can increase expected utility by altering his strategy so as to "join the coalition." All equilibria corresponding to non-minimal β-coalitions will be sincere strategy equilibria, and the coalitions will occur with probability determined by the approximate multivariate normal distribution.

In contrast, in an incomplete information game, a non-minimal β-coalition equilibrium with strategic voting may occur because a player may have a positive probability that this minimal coalition does not exist, due to his uncertainty about voters' strategies. Even though a minimal β-coalition already exists, a voter may have preferences such that either joining the coalition or voting strategically against it can increase his expected utility because of this positive probability.

The equilibria found are also perfect equilibria in the sense of Selten [123]. Although his concept of a perfect equilibrium was intended to apply to extensive games, the point of view which looks at "complete rationality as a limiting case of incomplete rationality"[4] is useful in this model because of the difficulty of accepting the concept of the rational voter. Suppose voters are rational in the sense that they can evaluate different alternatives, compare strategies available to them, and estimate the effect of these strategies on the outcome of the system. However, this hypothetical rational voter is not perfect; he may make `mistakes.´ When he has had a `bad day´ with probability ε, he is equally likely to choose any of the strategies available to him, as he is no longer thinking straight. If all this happens to all voters, we have Selten´s perturbed game. The `rational´ part of the voter knows that this happens and uses it in his calculation of optimal strategy as far as he is able. Then if the strategies of the perturbed game approach the strategies of the original game as $\varepsilon \rightarrow 0$, the Nash equilibrium of the original game is `perfect.´ The model as constructed is set up in exactly this way. The zero information game corresponds to a complete information game in which the rational voter assigns $\varepsilon = (Q-1)/Q$ (where Q is the number of admissible strategies) to every other voter and determines his optimal (sincere) strategy on that basis. As the original value of ε gets smaller, the variance of estimates $\hat{W}^{n}_{i \neq j}$ decreases, exactly as if the voter had better information. Complete information (or perfect rationality) is the limiting case.

Conversely, a Nash equilibrium which cannot be reached from the sincere strategy matrix is not perfect, since the sincere strategy matrix is the unique equilibrium of a sufficiently perturbed game. Therefore if the number of repetitions is sufficiently large, the set of equilibria found will correspond to the set of perfect equilibria, and Nash equilibria which are not found will not be perfect equilibria.

CHAPTER 4

RESULTS

The results of the simulations are presented and analyzed here. Some of the questions examined are 1) the relationship between social utility efficiency estimates and Weber's theoretical values; 2) how social utility efficiency estimates compare given the use of sincere strategies as compared to optimal strategies; 3) how Condorcet efficiency estimates compare given sincere and optimal strategies; 4) the effect of strategic voting on rankings of the systems using either social utility or Condorcet efficiency with strategic voting; 5) the relationship between Condorcet efficiency and social utility efficiency; and 6) the effect of the amount of information available to voters on efficiency estimates given optimal strategies.

4.1 Theoretical Values

4.1.1 Sincere Voting

Weber's social utility efficiency values are asymptotic. It is therefore possible that social utility efficiencies may be significantly different for small voting populations. This possibility was investigated, but the differences were found to be insignificant for the most part. Using the student's t distribution, t-tests indicated only three cases, all for the standard voting system, for which the differences were significant at the 90% level or better. In each of these cases, the number of voters differed from the number of alternatives by at most one, and social utility efficiency was significantly greater than the theoretical value. However, in these cases, the standard voting system had appreciably lower social utility efficiency than either of the other systems considered, and rankings were not affected. Again, for 3 alternative elections, rankings according to Weber's asymptotic social utility efficiencies are 1) approval voting system, 87.5%; 2) Borda system, 86.6%; and 3) standard voting system, 75%. The simulations tended to confirm this for 3 alternative elections, although there is difficulty in differentiating the efficiency of the Borda and approval systems. In fact, the Borda system ranked above the approval voting system 12/22 times, but a t-test detects no significant difference in means.

Although Weber did not develop a formula for theoretical values of the approval voting system with more than three alternatives, simulation estimates of social utility efficiency for the approval voting system appear to indicate that asymptotic social utility efficiency is constant at 87.5%, regardless of the number of alternatives. Figures 4.5-4.9 (pages 96-97) show social utility efficiency estimates for the approval voting system and their approach to this limit. Deviations are greater for a smaller number of voters, and the size of the deviation is greater the larger the number of alternatives considered.

For more than 3 alternatives, rankings were, without exception: 1) Borda system; 2) approval voting system; and 3) standard voting system.

4.1.2 Strategic Voting

When voters' use of optimal strategies was incorporated, social utility efficiency estimates diverged markedly from theoretical values for the standard voting system. Differences are predictably greater for small electorates, and given the number of voters, greater for a larger number of alternatives.

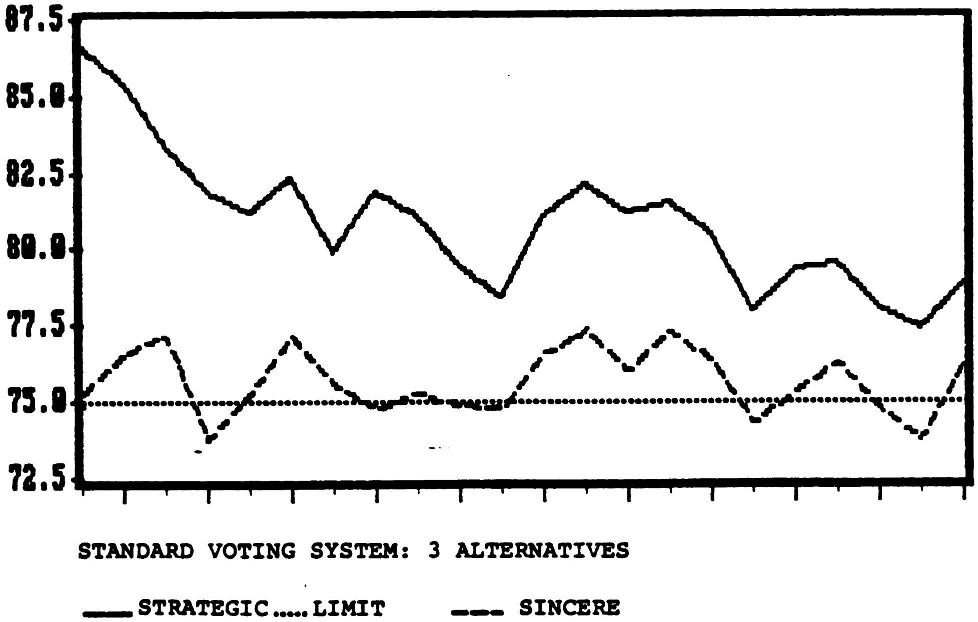


Figure 4.1 Social Utility Efficiency for the Standard Voting System with 3 Alternatives: Strategic, Sincere, and Limit Values

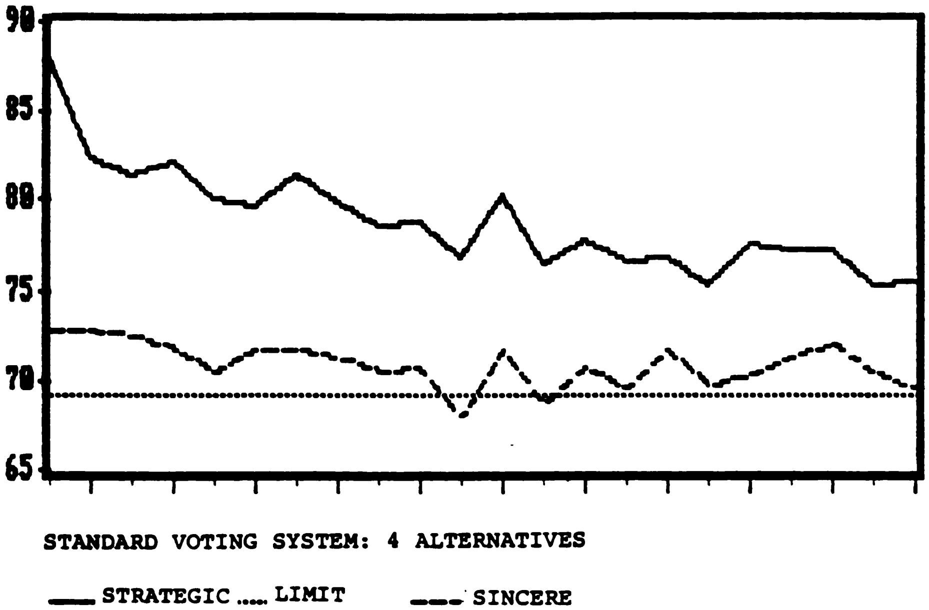


Figure 4.2 Social Utility Efficiency for the Standard Voting System with 4 Alternatives: Strategic, Sincere, and Limit Values

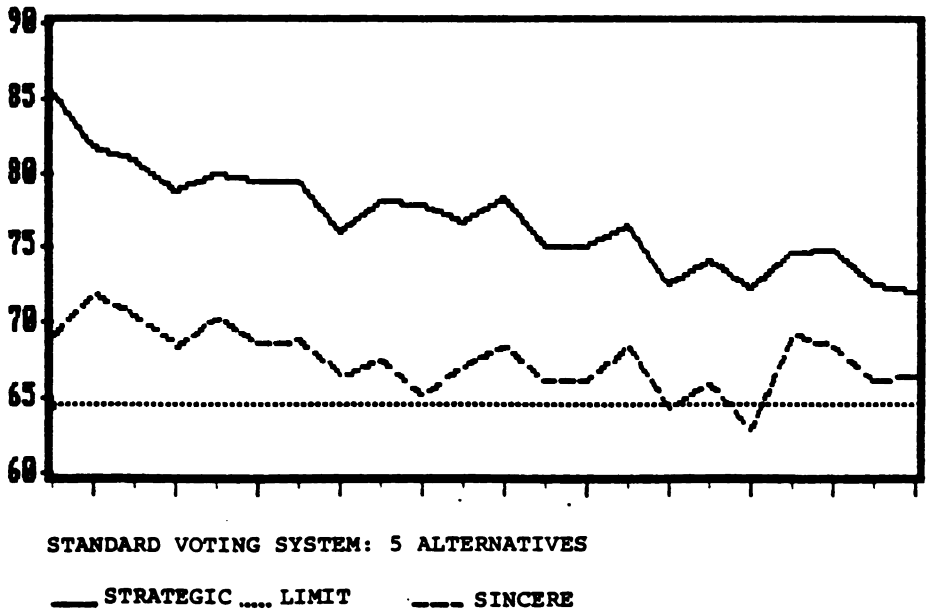


Figure 4.3 Social Utility Efficiency for the Standard Voting System with 5 Alternatives: Strategic, Sincere, and Limit Values

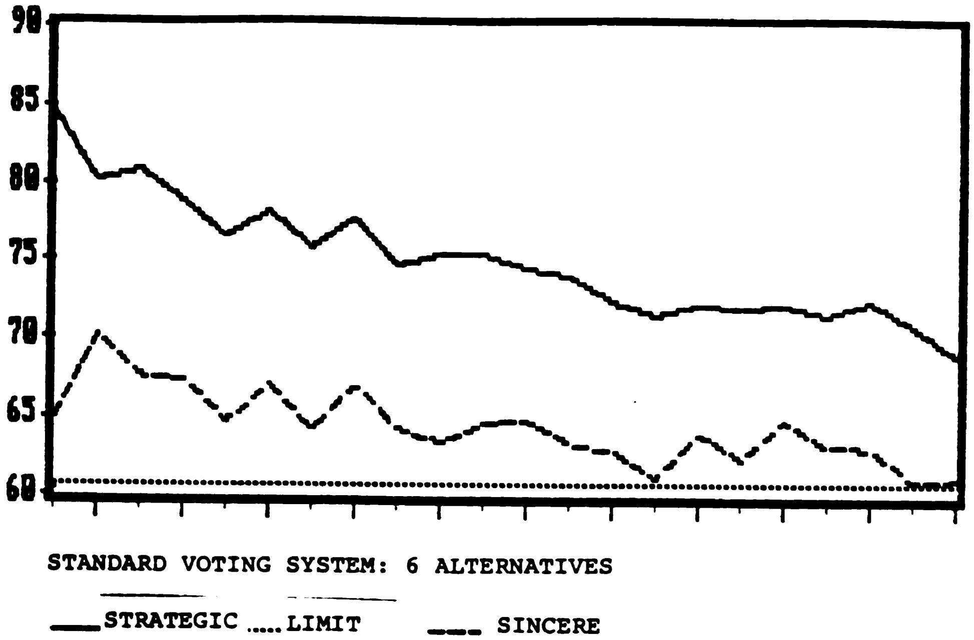


Figure 4.4 Social Utility Efficiency for the Standard Voting System with 6 Alternatives: Strategic, Sincere, and Limit Values

This difference did not occur to such an extent for the approval voting system. In only two cases was the difference great enough to produce a t-statistic significant at the 80% level. However, an interesting pattern to social utility efficiency estimates appeared. For small voting populations, the estimates are very close to 87.5%; they decline as the number of voters increases and after a certain point begin to increase again toward 87.5%. This decline is more marked as the number of alternatives increases, as shown in Figures 4.5-4.8.

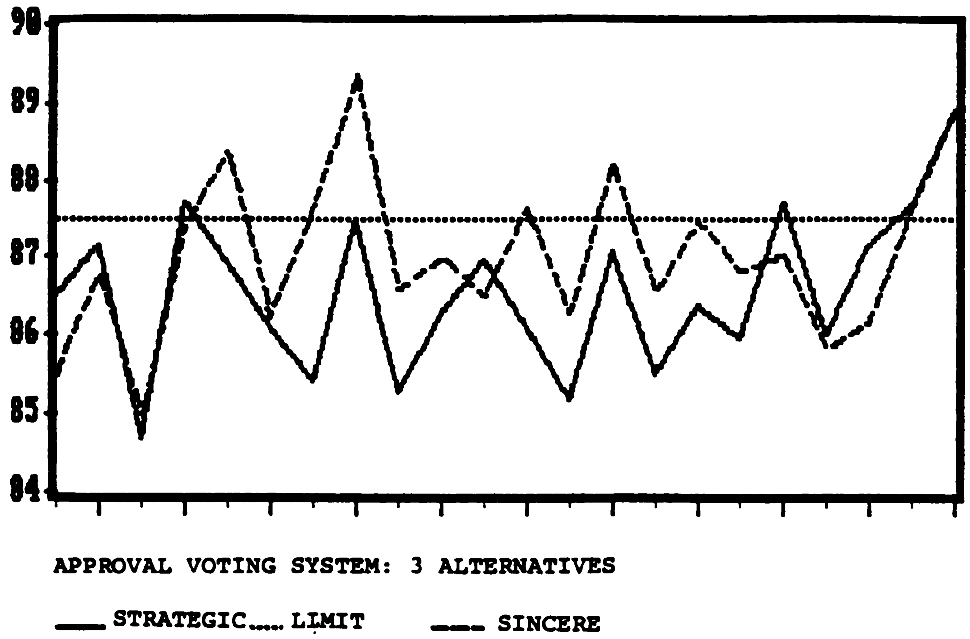


Figure 4.5 Social Utility Efficiency for the Approval Voting System with 3 Alternatives: Strategic, Sincere, and Limit Values

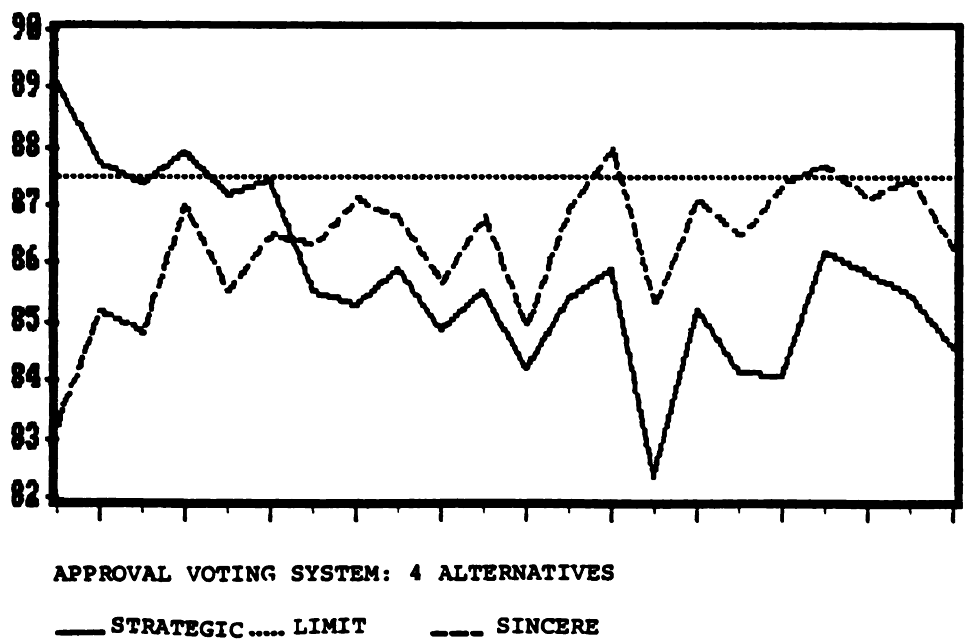


Figure 4.6 Social Utility Efficiency for the Approval Voting System with 4 Alternatives: Strategic, Sincere, and Limit Values

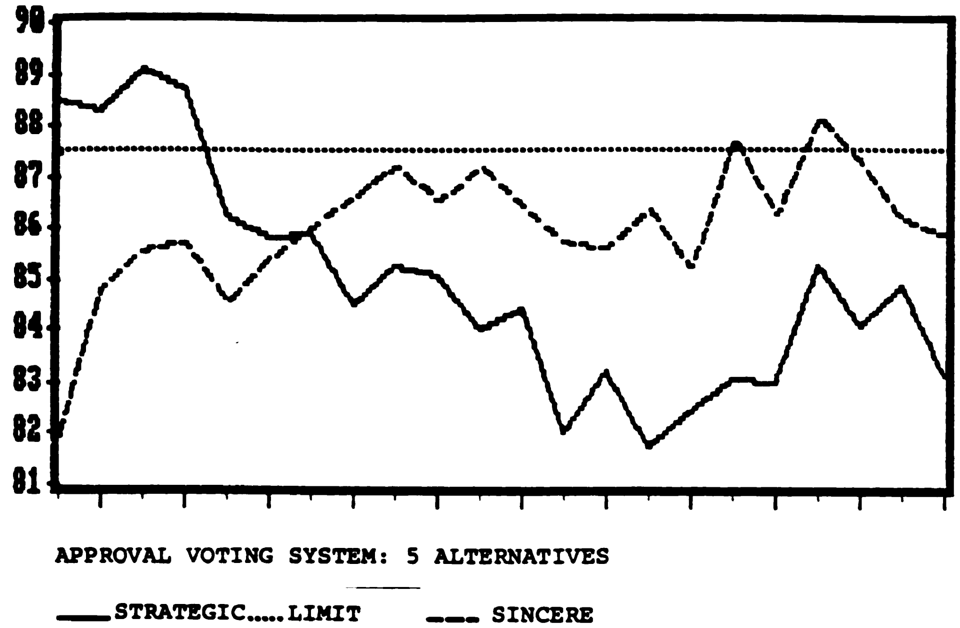


Figure 4.7 Social Utility Efficiency for the Approval Voting System with 5 Alternatives: Strategic, Sincere, and Limit Values

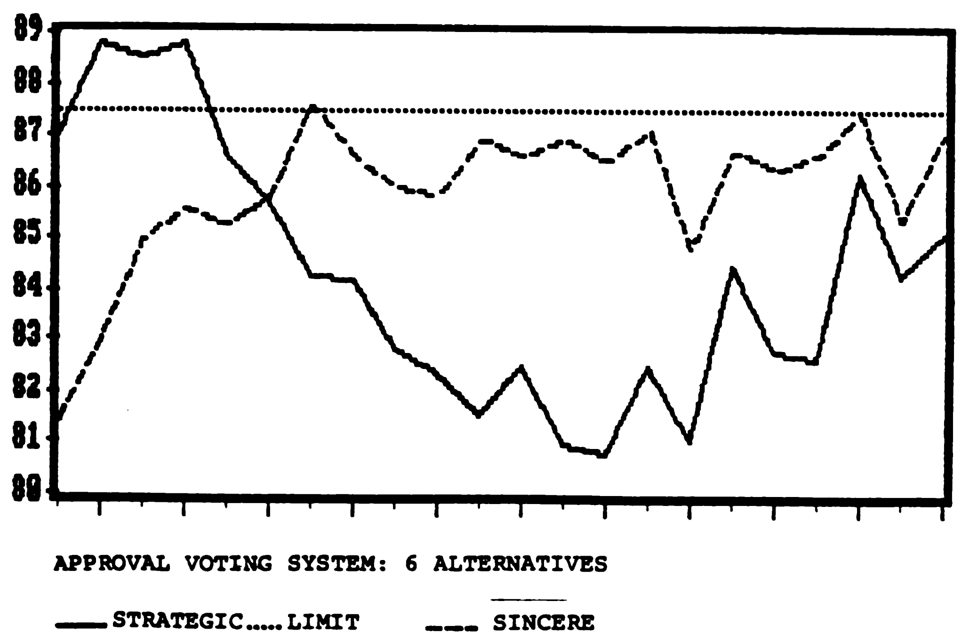


Figure 4.8 Social Utility Efficiency for the Approval Voting System with 6 Alternatives: Strategic, Sincere, and Limit Values

Strategic estimates again diverge for the Borda system; the same pattern is discernable as for the approval system. Once again the effect is greater where there is more scope for strategic voting. Efficiency measures for the 6 alternative system are predicted values using regression coefficients estimated (see page 115).

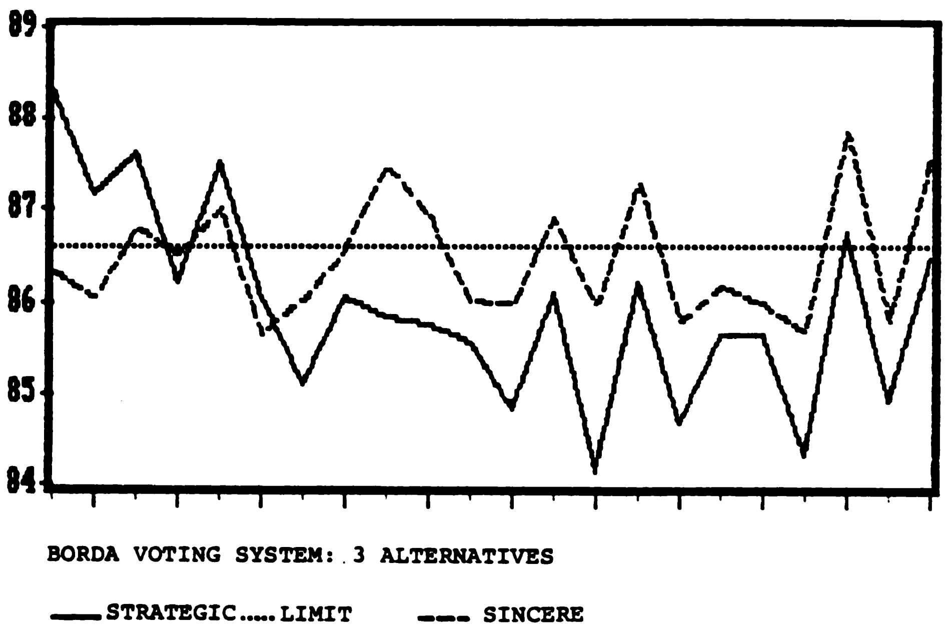


Figure 4.9 Social Utility Efficiency for the Borda Voting System with 3 Alternatives: Strategic, Sincere, and Limit Values

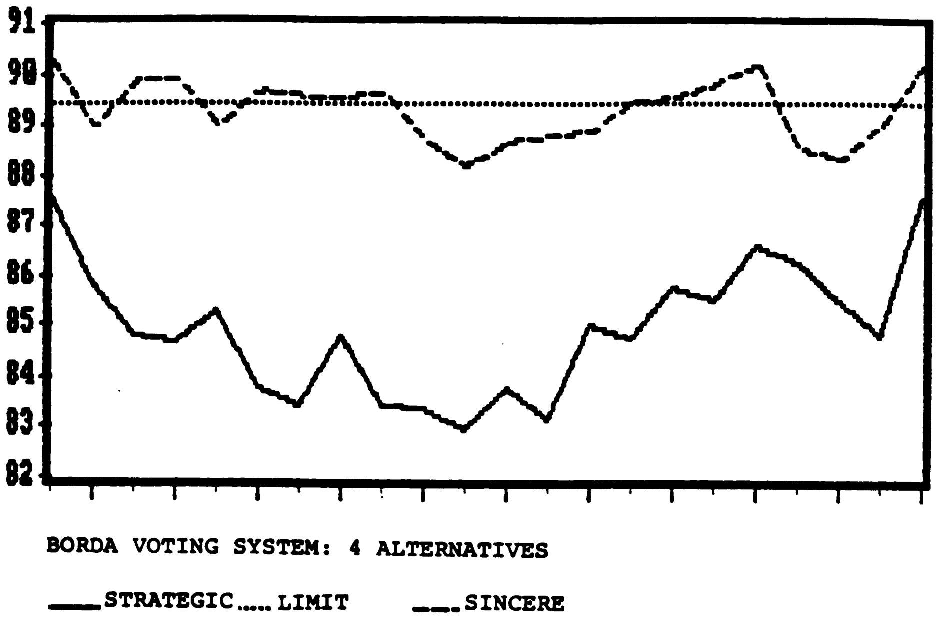


Figure 4.10 Social Utility Efficiency for the Borda Voting System with 4 Alternatives: Strategic, Sincere, and Limit Values

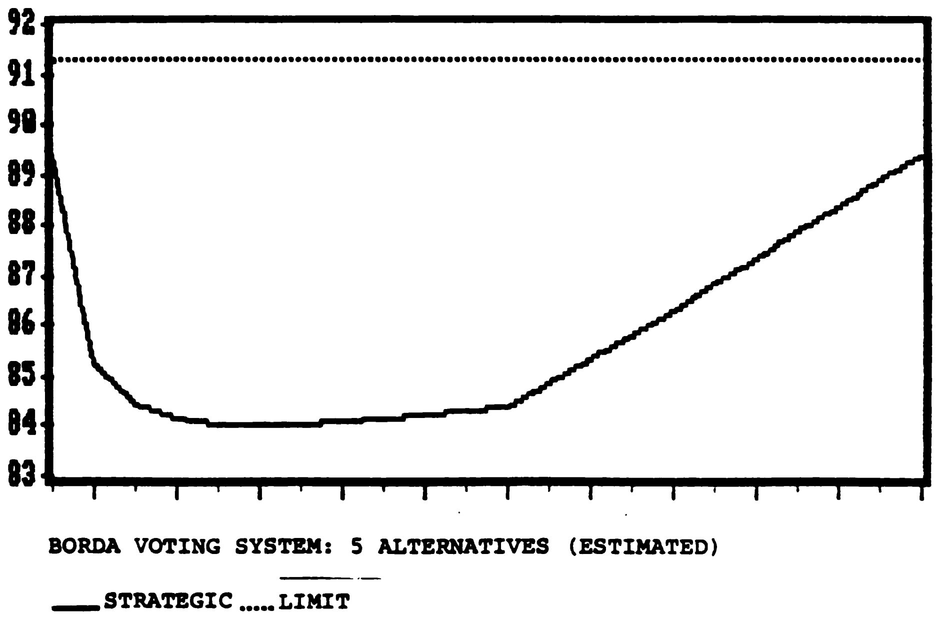


Figure 4.11 Social Utility Efficiency for the Borda Voting System with 5 Alternatives: Strategic and Limit Values

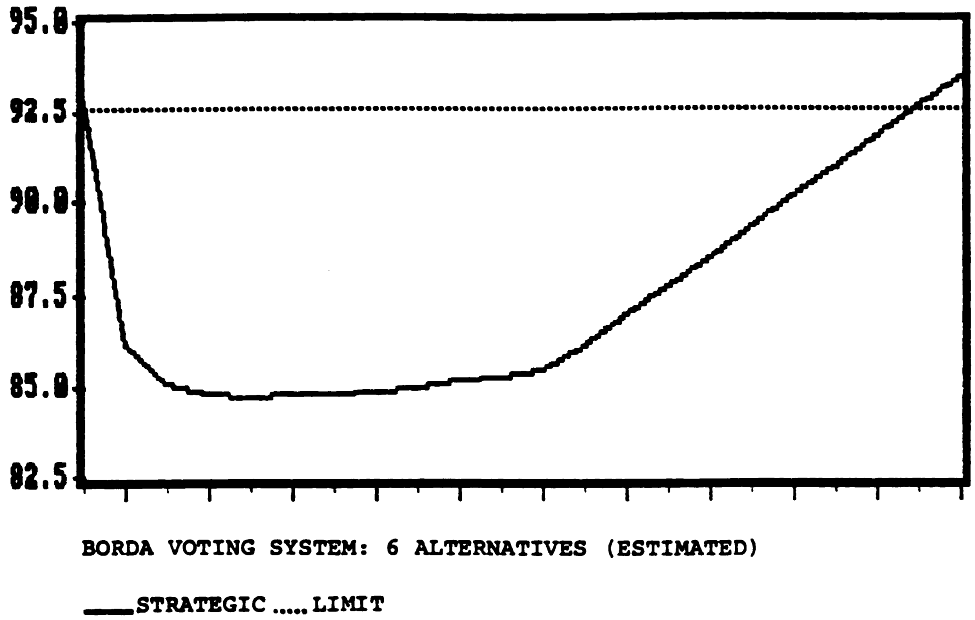


Figure 4.12 Social Utility Efficiency for the Borda Voting System with 6 Alternatives: Strategic and Limit Values

4.2 Social Utility Efficiency Rankings with Sincere and Strategic Voting

Under the assumption of sincere voting, social utility efficiency rankings from the simulation estimates are compatible with the results of previous work. When voters are assumed to use optimal strategies, estimates of social utility efficiency are in many cases significantly different from their sincere voting estimates. Despite this, overall rankings of the systems do not change much. The approval voting system does rank above the Borda system for small electorates given more than 3 alternatives. As the voting population increases, this ranking is reversed. In all cases, the standard voting system is ranked below the other two systems, despite the pronounced increase in social utility efficiency for the standard voting system and decrease for the Borda system. For the approval voting system, small voting population estimates of social utility efficiency are significantly greater than their sincere counterparts, while larger electorates tend to have strategic estimates below the sincere estimates.

Given these changes, for small electorates the approval voting system moves up in ranking while the Borda system moves down to second place. The standard voting system, while having social utility estimates which are roughly comparable (for 4 alternatives and 3 voters, estimates are: approval, 89.1%; Borda, 88.0%; and standard, 87.9%), remains ranked in third place. As the size of the voting population increases, the ranking between the approval and Borda system is reversed, and the estimates for the standard voting system decrease steadily toward their limit.

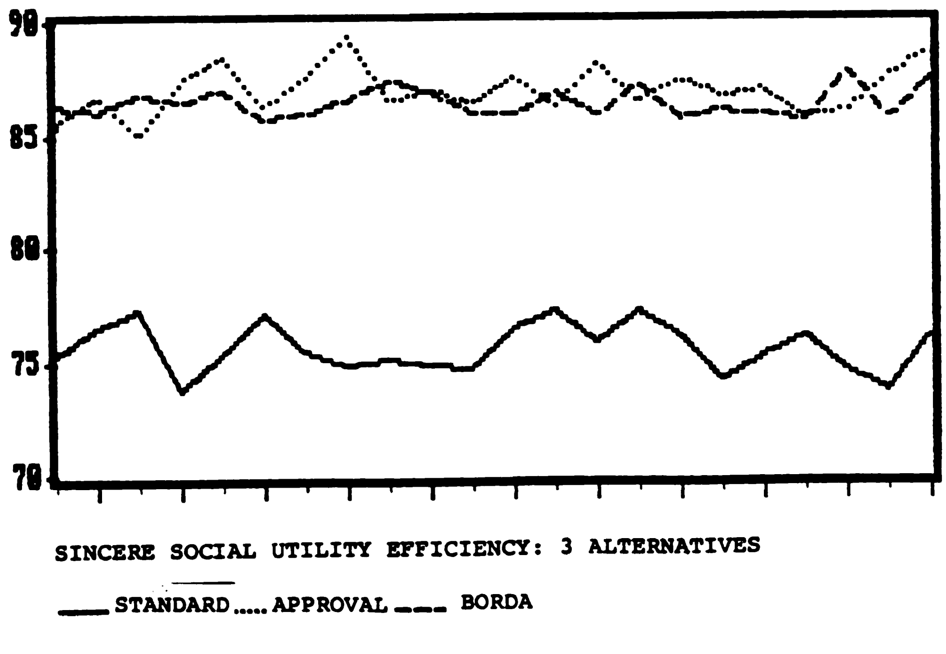


Figure 4.13 Sincere Social Utility Efficiency: 3 Alternatives

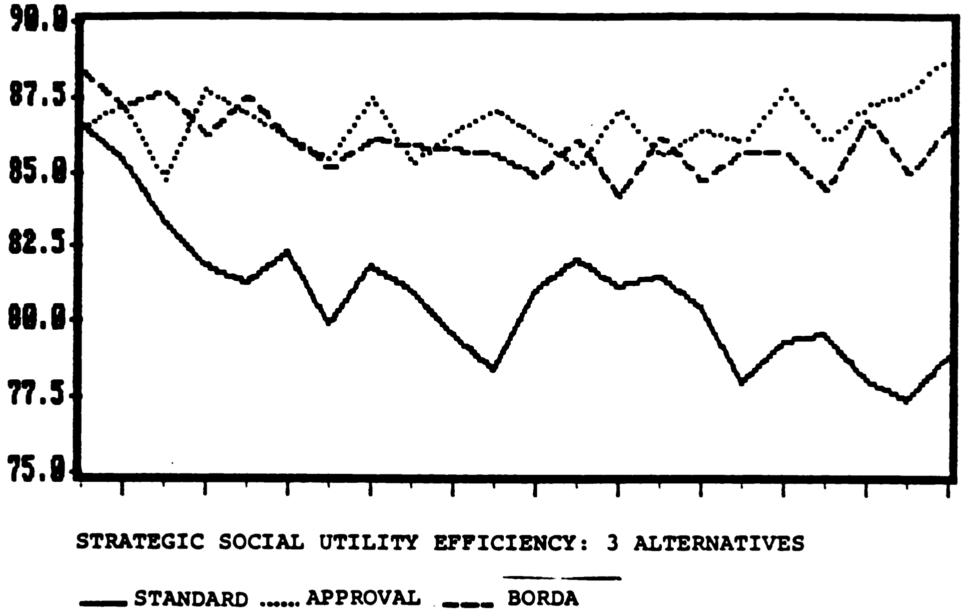


Figure 4.14 Strategic Social Utility Efficiency: 3 Alternatives

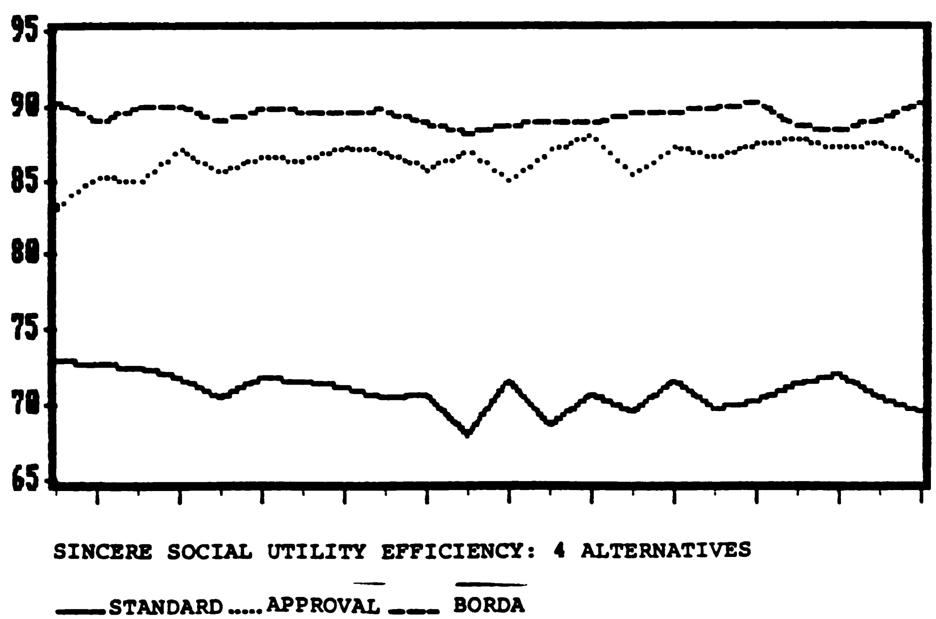
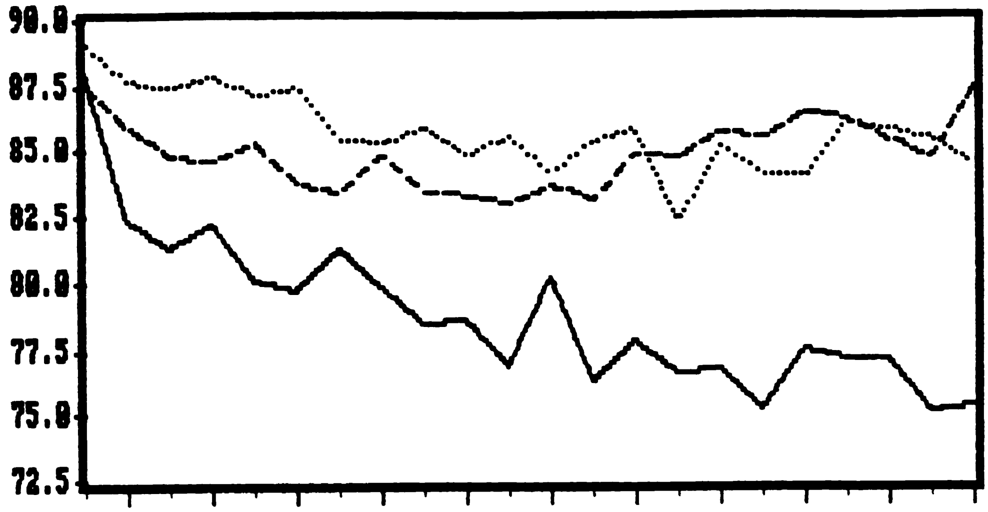


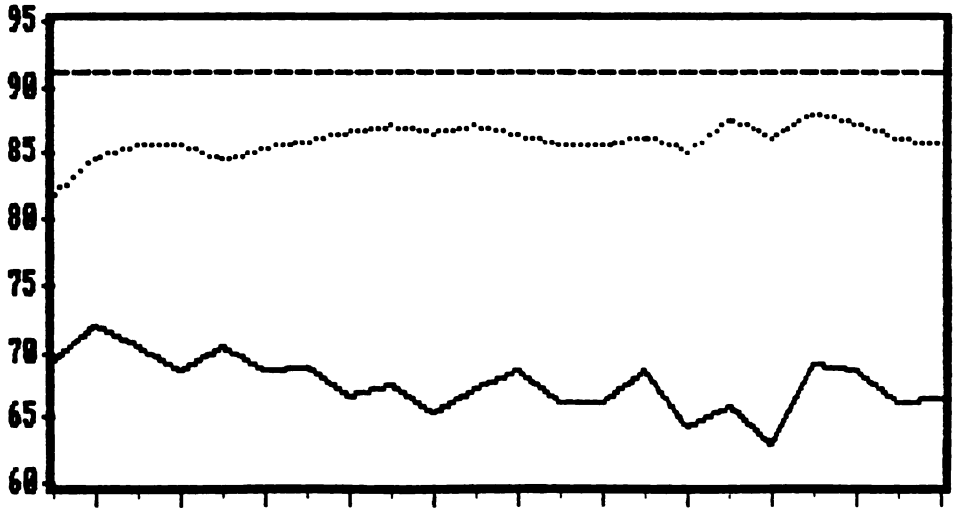
Figure 4.15 Sincere Social Utility Efficiency: 4 Alternatives



STRATEGIC SOCIAL UTILITY EFFICIENCY: 4 ALTERNATIVES

—— STANDARD .... APPROVAL —— BORDA

Figure 4.16 Strategic Social Utility Efficiency: 4 Alternatives



SINCERE SOCIAL UTILITY EFFICIENCY: 5 ALTERNATIVES

—— STANDARD .... APPROVAL —— BORDA (LIMIT)

Figure 4.17 Sincere Social Utility Efficiency: 5 Alternatives

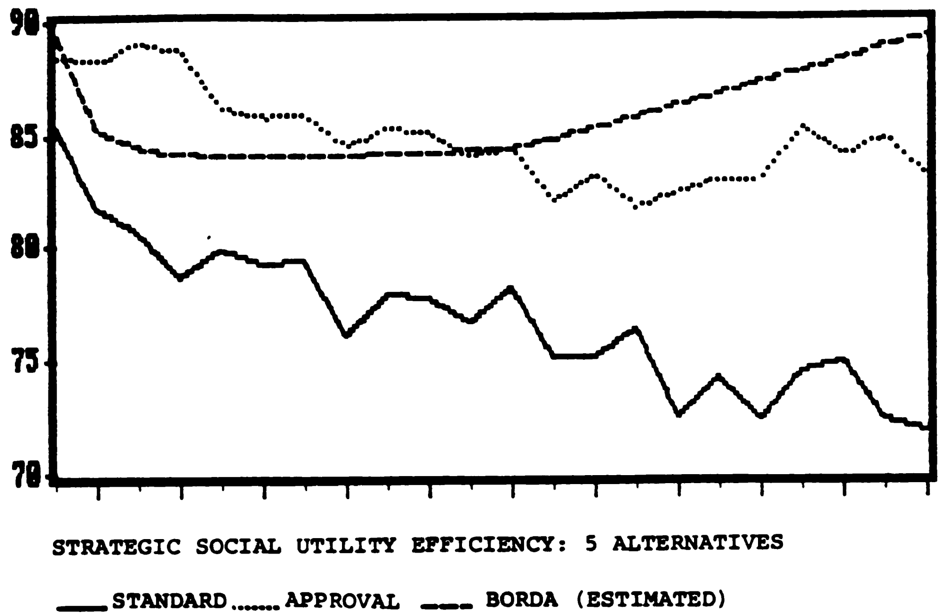


Figure 4.18 Strategic Social Utility Efficiency: 5 Alternatives

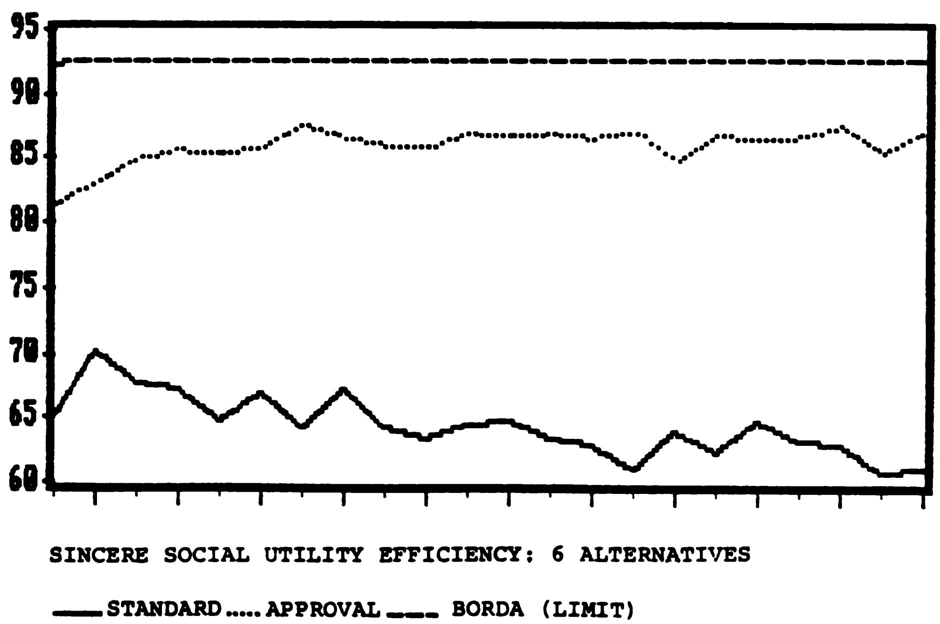


Figure 4.19 Sincere Social Utility Efficiency: 6 Alternatives

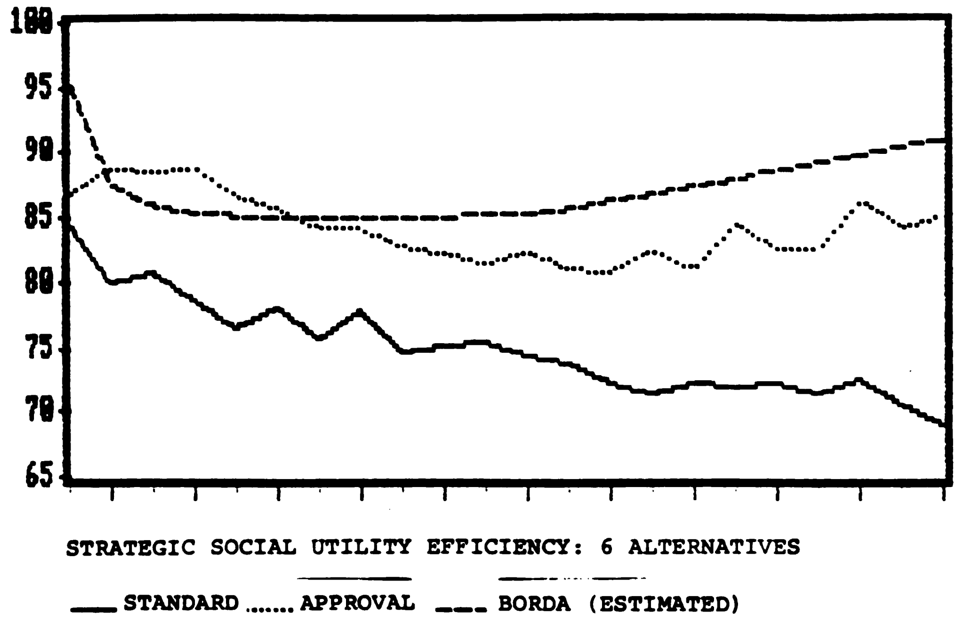


Figure 4.20 Strategic Social Utility Efficiency: 6 Alternatives

4.3 Condorcet Efficiency Rankings

Condorcet efficiency rankings under sincere voting are, for "small" electorates: 1) Borda system; 2) standard voting system; and 3) approval voting system. Given a specified number of alternatives, as the size of the electorate increases, the approval voting system reverses rank with the standard voting system, and as with social utility efficiency, we have the Borda system ranked first, followed by approval voting, followed by the standard voting system.

Strategic voting produces a dramatic change in these rankings. Condorcet efficiency increases significantly for both the approval and standard voting systems, while in all but a few cases it decreases significantly for the Borda system. For most small voting populations (committee size), the standard voting system is ranked first in Condorcet efficiency, followed by the Borda system, with approval voting ranked last.

For any number of alternatives considered (3-6), Condorcet efficiency for the standard voting system with strategic voting peaks when there are five voters and decreases more or less consistently thereafter. In contrast, the approval voting system with strategic voting has maximum Condorcet efficiency with 3 voters and declines thereafter. Within the voting populations used in the simulation, there is no U-shaped curve as found for social utility efficiency; Condorcet efficiency does not reach some approximate minimum and begin to climb towards a limit. Instead Condorcet efficiency begins from a level above its "limiting value" and approaches the value in an approximate logarithmic curve.

For the Borda system, with 3 or 4 alternatives, the U-shaped curve is again apparent. Estimates for Condorcet efficiency with 5 or 6 alternatives follow the same pattern.

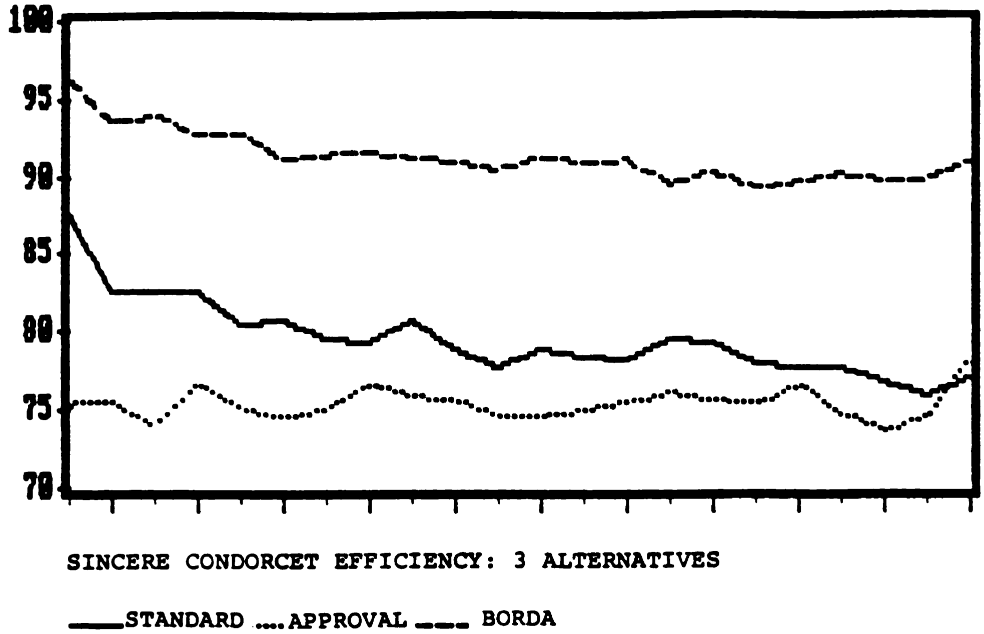


Figure 4.21 Sincere Condorcet Efficiency: 3 Alternatives

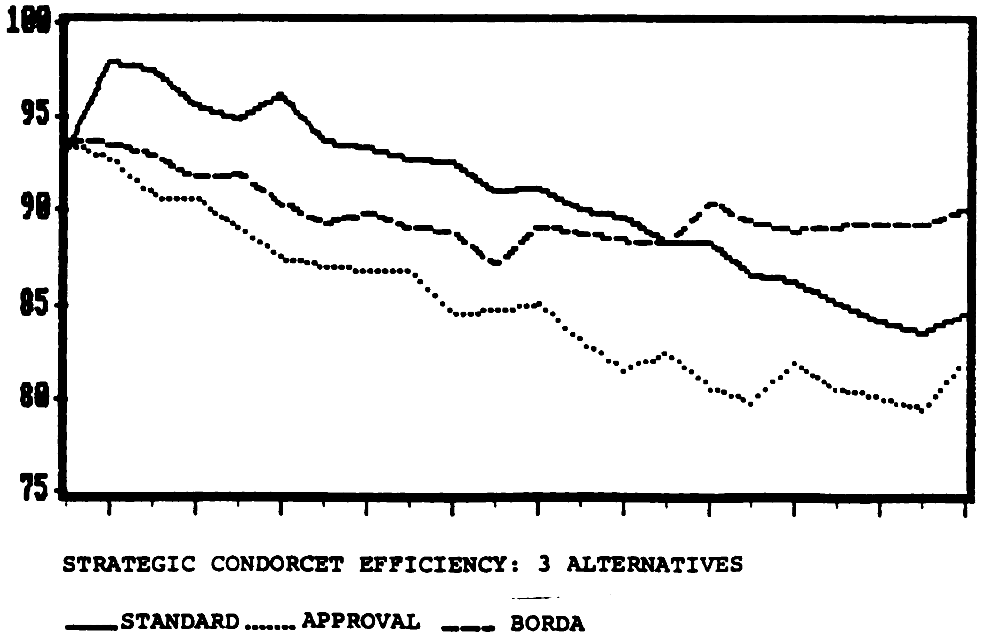


Figure 4.22 Strategic Condorcet Efficiency: 3 Alternatives

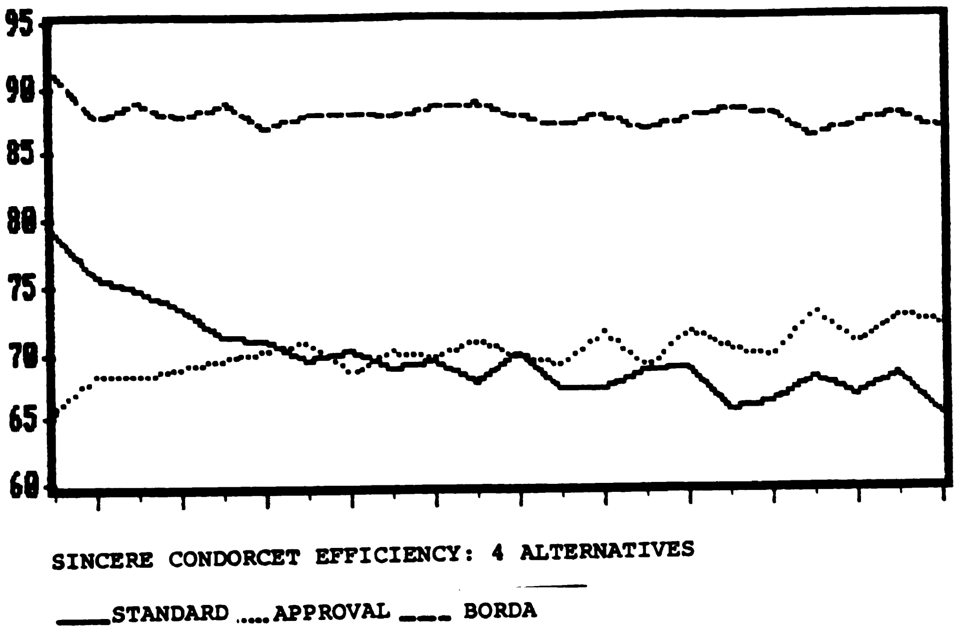


Figure 4.23 Sincere Condorcet Efficiency: 4 Alternatives

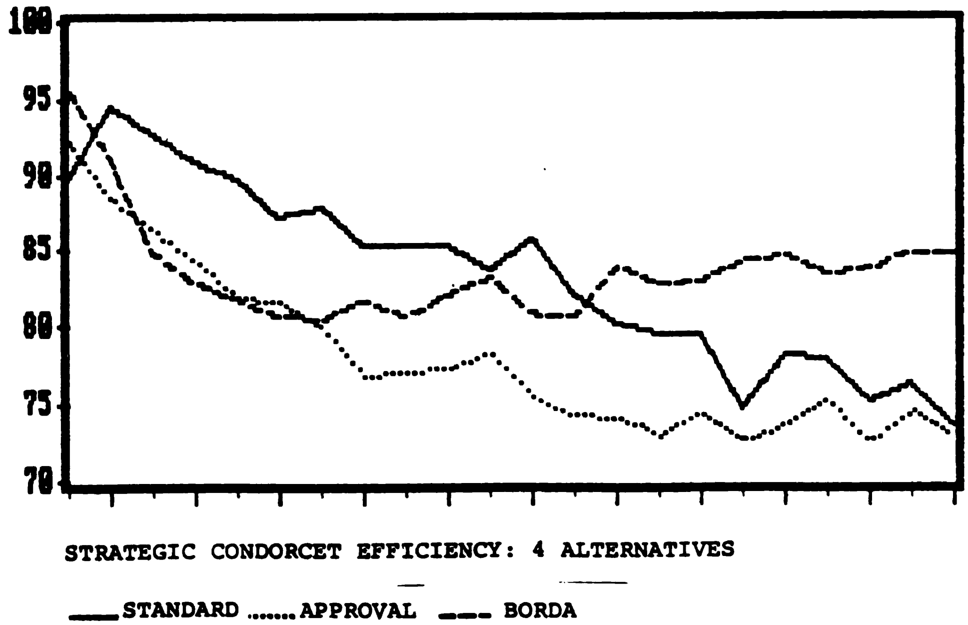


Figure 4.24 Strategic Condorcet Efficiency: 4 Alternatives

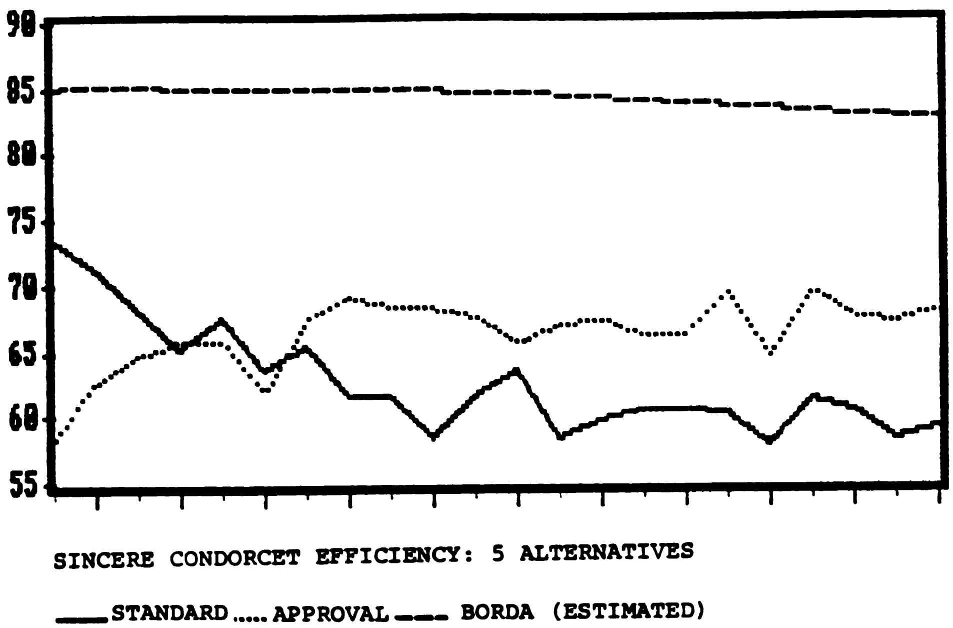


Figure 4.25 Sincere Condorcet Efficiency: 5 Alternatives

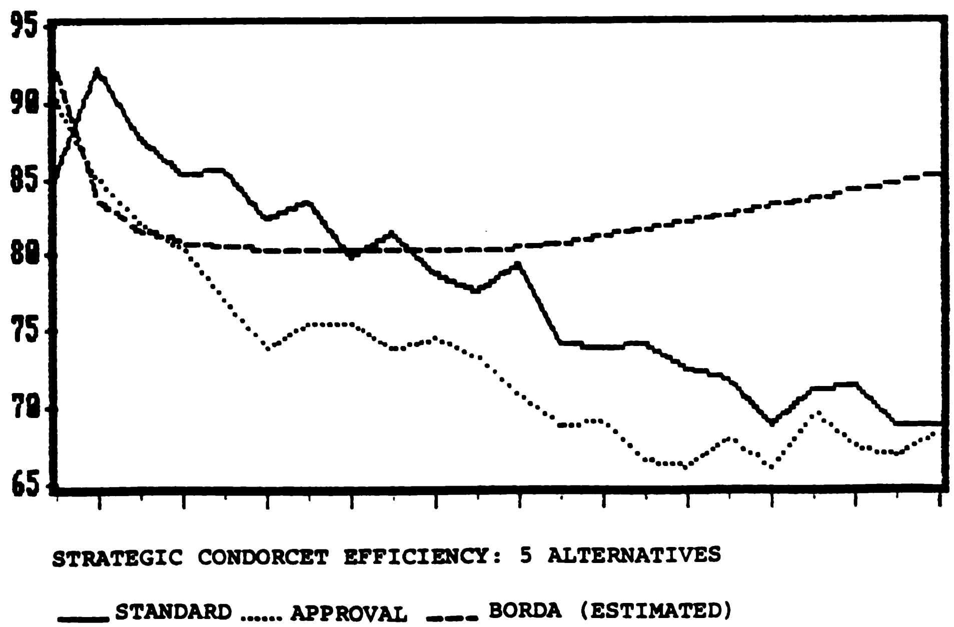


Figure 4.26 Strategic Condorcet Efficiency: 5 Alternatives

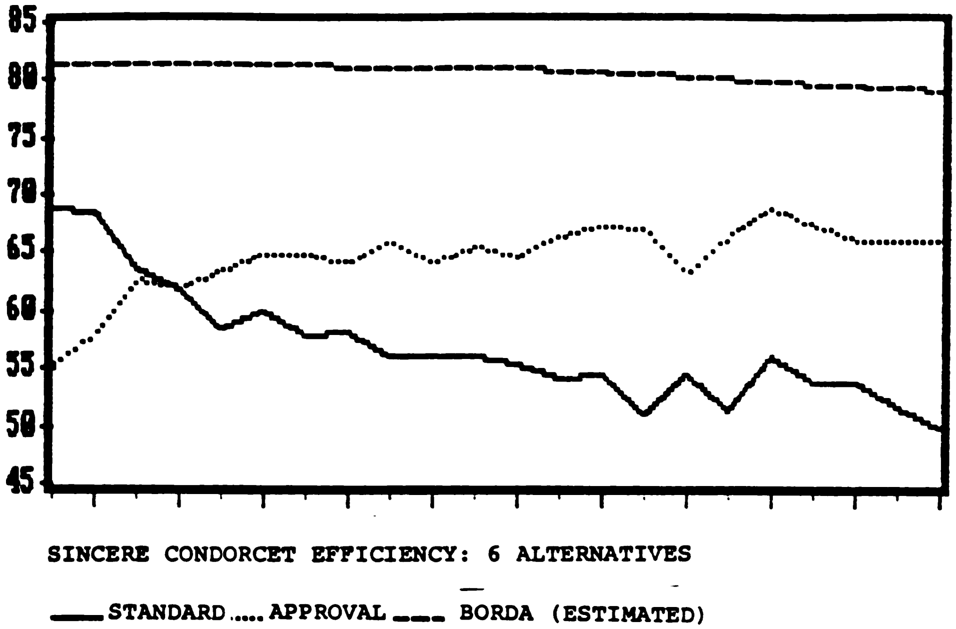


Figure 4.27 Sincere Condorcet Efficiency: 6 Alternatives

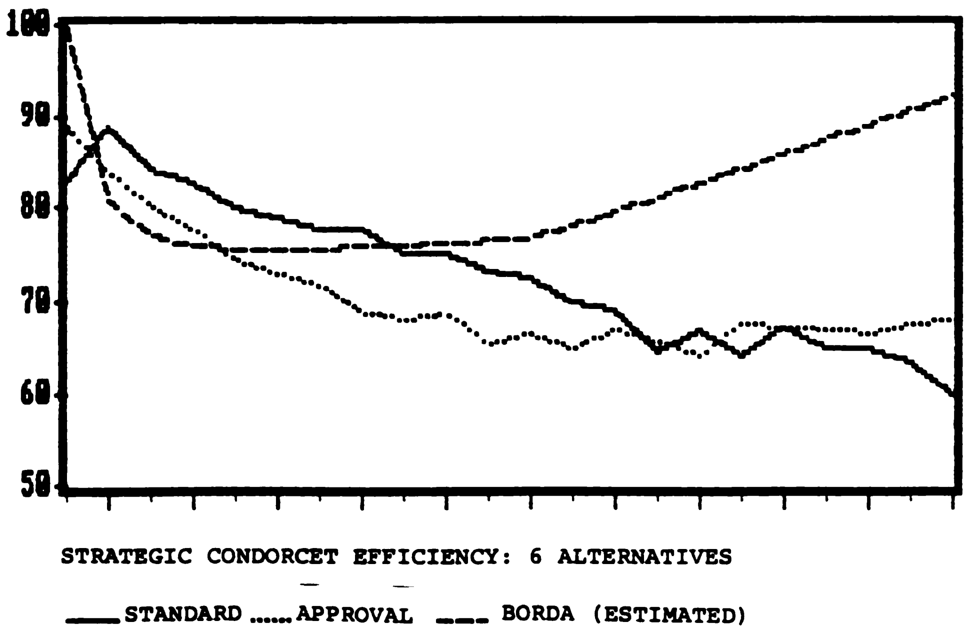


Figure 4.28 Strategic Condorcet Efficiency: 6 Alternatives

4.4 Condorcet Efficiency and Social Utility Efficiency

The relationship between Condorcet efficiency and social utility efficiency was discussed in Chapter 2. An attempt to quantify this relationship more precisely was made by running simple linear regressions (OLS) of the form

SCON = A + B1(SSU) + B2(ALTS) + B3(V),

where SSU is strategic social utility efficiency, SCON is strategic Condorcet efficiency, ALTS is the number of alternatives, and V is the number of voters. This regression was run for each voting system. The results of the regressions are presented below, with Figures 4.29-4.31 showing estimated and actual strategic Condorcet efficiencies.

Table 4.1 Regression Results for Strategic Condorcet Efficiency

Borda System	Dependent Variable: SCON
Mean of Dependent Variable	86.898530
Standard Deviation	4.166679
Sum of Squared Residuals	159.787800
Standard Error of Regression	1.998673
Number of Observations	44
R^2	.785960

Variable	Estimate	Std. Error	T-Statistic
Intercept	-30.9682720	22.8332530	-1.3562795
ALTS	-4.6555093	.6553843	-7.1034801
V	-.0138721	.0078597	-1.7649777
SSU	1.5771413	.2558077	6.1653391

Standard System	Dependent Variable: SCON
Mean of Dependent Variable	81.270320
Standard Deviation	9.219158
Sum of Squared Residuals	472.696268
Standard Error of Regression	2.372201
Number of Observations	88
R^2	.936074

Variable	Estimate	Std. Error	T-Statistic
Intercept	29.9295465	11.3396821	2.6393638
ALTS	-4.0469293	.3360611	-12.0422427
V	-.0876380	.0101957	-8.5955649
SSU	.9422485	.1261226	7.4708959

<u>Approval System</u> Dependent Variable: SCON

Mean of Dependent Variable	76.712010
Standard Deviation	7.978904
Sum of Squared Residuals	699.152623
Standard Error of Regression	2.885004
Number of Observations	88
R^2	.873769

Variable	Estimate	Std. Error	T-Statistic
Intercept	-58.6598397	15.4701347	-3.7918118
ALTS	-3.4314452	.3058458	-11.2195253
V	-.0762712	.0083586	-9.1248403
SSU	1.8056124	.1722115	10.4848536

Note that in each of the regressions, strategic social utility efficiency has a fairly strong positive relationship with strategic Condorcet efficiency. In fact, strategic Condorcet efficiency can be predicted fairly well given the value of strategic social utility efficiency, as will be seen in Figures 4.29-4.31. Even so, values for strategic Condorcet efficiency estimated with regression coefficients are not too far off from the simulation values. Note also that the sign of the coefficients on both ALTS and V is negative in every case, as would be expected. The regressions do support the hypothesis of a strong relationship between the two efficiency measures.

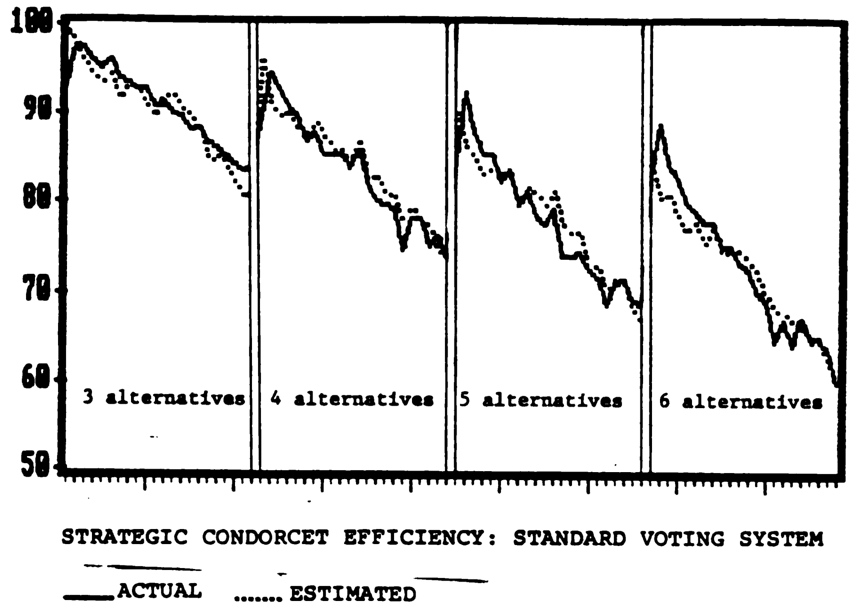


Figure 4.29 Actual and Estimated Condorcet Efficiency: Standard Voting System

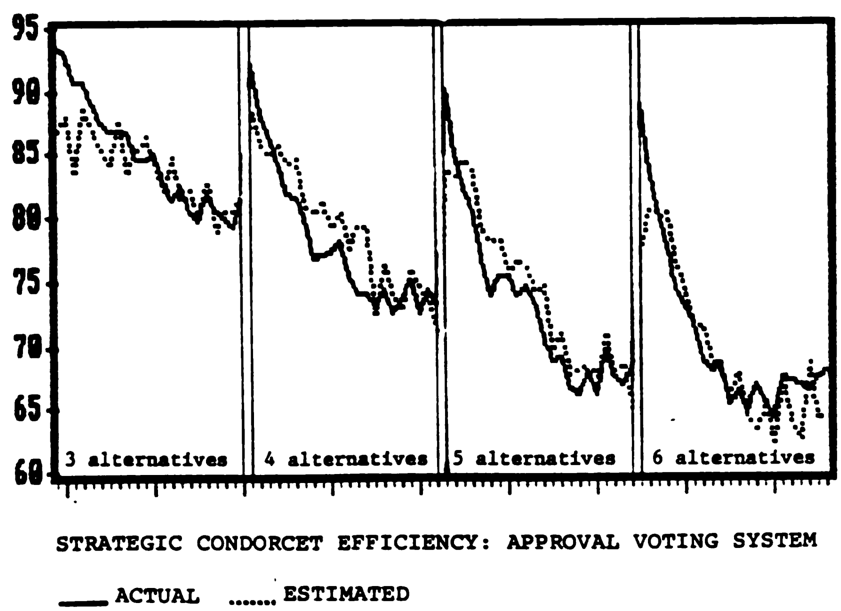


Figure 4.30 Actual and Estimated Condorcet Efficiency: Approval Voting System

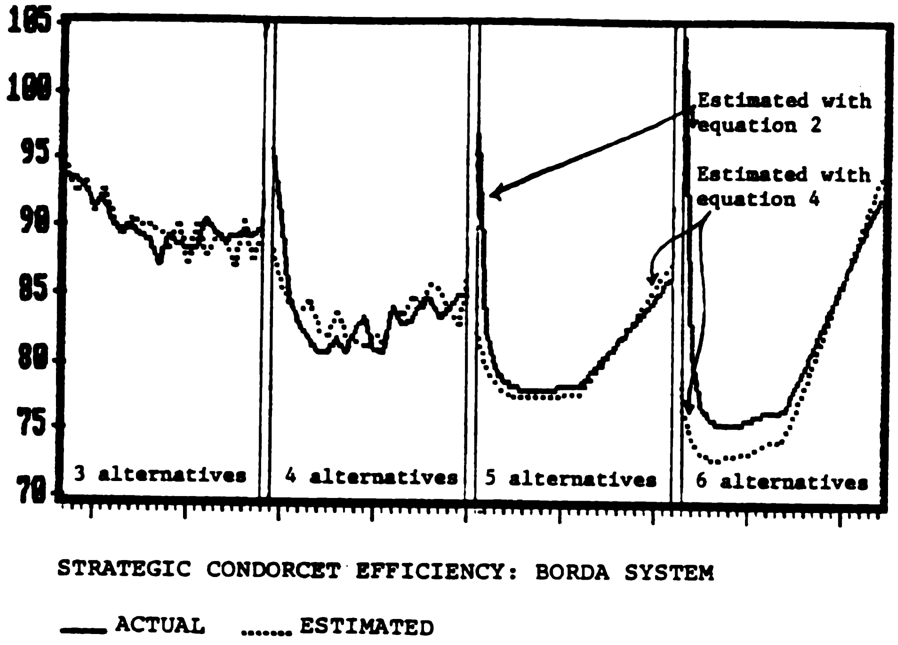


Figure 4.31 Actual and Estimated Condorcet Efficiency: Borda Voting System

4.5 Forecasted Values

In addition, regressions were run to allow prediction of efficiency measures for these systems when the number of alternatives is greater than is feasible to simulate. The variables used for sincere efficiency measures were THEO, DIF, MEAN, and VAR. THEO is the theoretical social utility efficiency value. DIF is a measure of the difference between the actual distribution of total votes and the normal distribution which total votes approach as the number of voters increases. DIF is defined as the difference between the normal distribution standard deviation and the actual standard deviation divided by two times mean votes for the voting system. MEAN and VAR are the mean and variance of total votes for the voting system.

The variables used for strategic efficiency measures were THEO, DIF, 1/P, Q^(ALTS/(2*(ALTS+V))), and four powers of V. The new variables are functions of P, the probability of a tie, and Q, the number of admissible strategies. P is defined as $\Phi(\sqrt{V}K/SD)-\Phi(-\sqrt{V}K/SD)$, where V is the number of voters, SD is the standard deviation of (W_i-W_k), and K is a constant term equal to the maximum weight assignment of the voting system. Q has the above functional form to display the following characteristics: as V gets large, the effect of Q decreases, and as the number of admissible strategies Q increases, the damping effect of V decreases.

The numerical results of these regressions are presented in appendix D. The results were used to forecast values for efficiency measures for 7 alternative elections, which are shown in Figures 4.32-4.35.

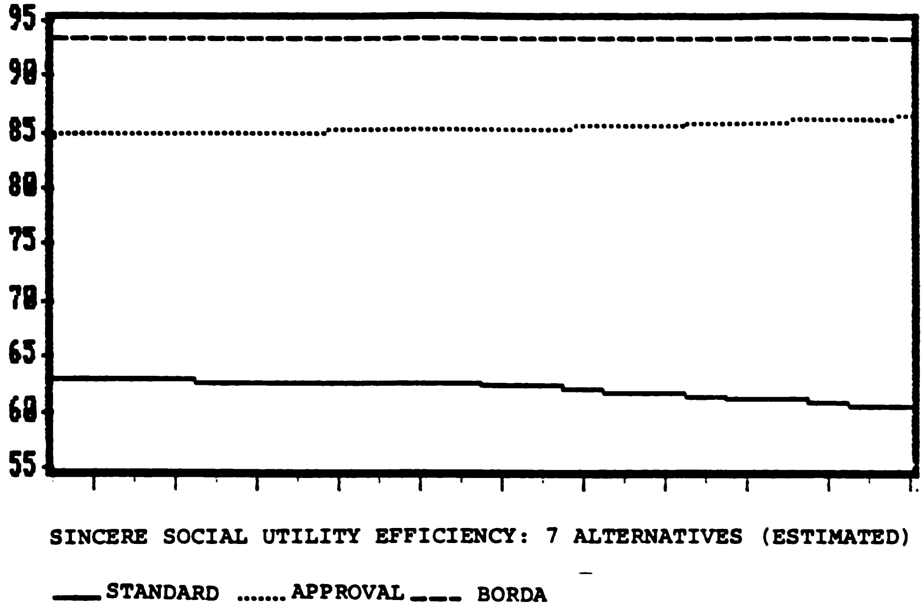


Figure 4.32 Sincere Social Utility Efficiency: 7 Alternatives (Estimated)

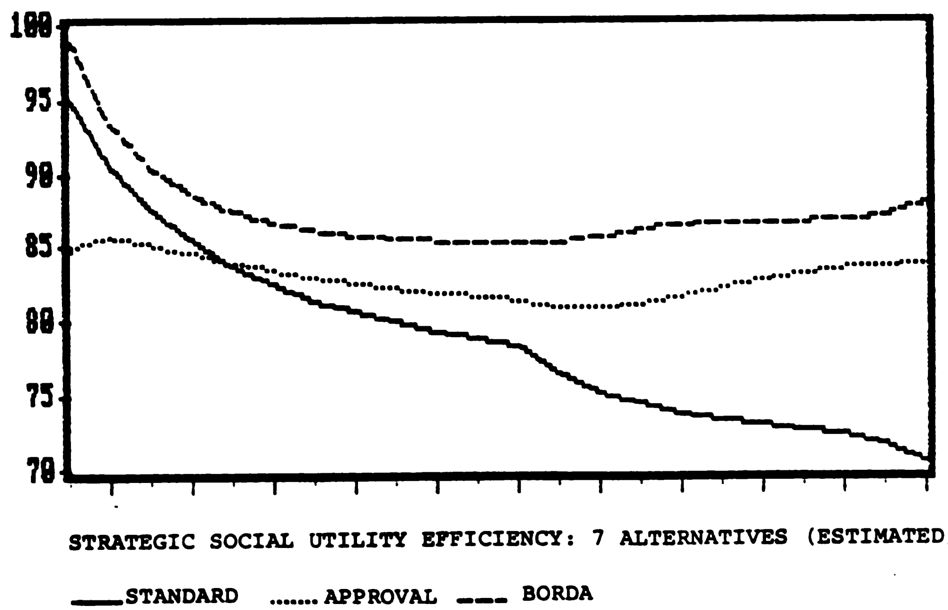


Figure 4.33 Strategic Social Utility Efficiency: 7 Alternatives (Estimated)

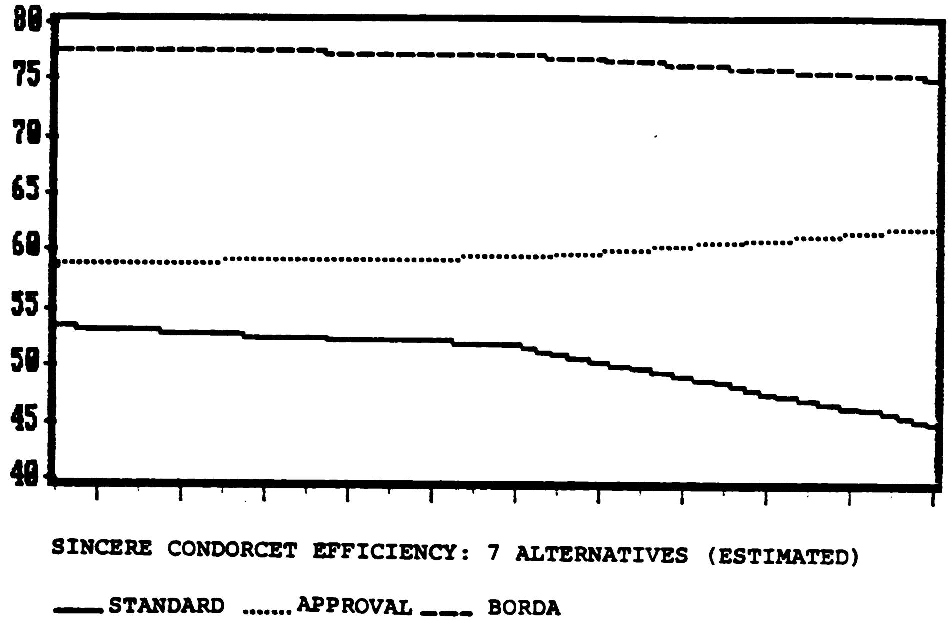


Figure 4.34 Sincere Condorcet Efficiency: 7 Alternatives (Estimated)

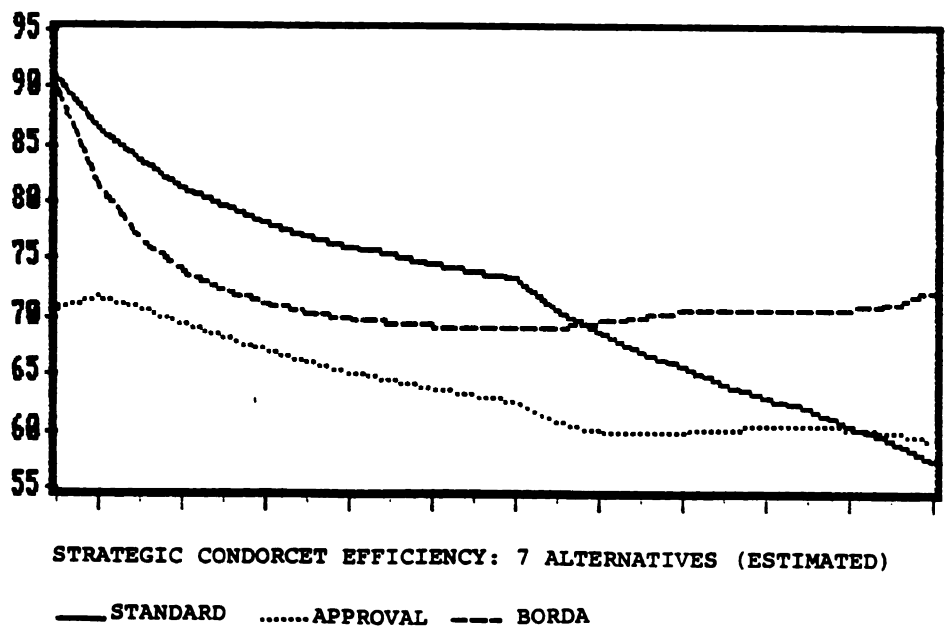


Figure 4.35 Strategic Condorcet Efficiency: 7 Alternatives (Estimated)

4.6 Incomplete Information

A variation of the simulations was run to determine the effect of less than full information on efficiency estimates. It was assumed that the total votes of 2/3 of the voting population (to the nearest integer) were known to all voters, who also knew whether or not they were included in the group. Unfortunately this provided no useful information because with more than approximately 7 voters, efficiency estimates were practically identical to those for sincere voting (the zero information case). This is due primarily to voters' knowledge of the distribution from which individual utilities are drawn. Because of this, voters cannot treat the sample total vote vector as a random sample from population total votes and assign corresponding probabilities or expected values to the unknown votes. The unknown votes continue to have the known (approximately multivariate normal) distribution as under zero information, adjusted for sample size. Therefore, if 3 or more voters are not in the sample, the variance of population total votes is large enough to discourage most strategic voting (as in the analytical example, p. 69).

The results that were obtained are presented here for the sake of completeness, although the simulations were aborted when it was apparent that the level of information was not large enough, given the structure of the model, to provide information on the movement of sincere estimates toward strategic estimates as the information level increases.

Table 4.2 Incomplete Information Efficiency Measures for 3 Alternatives and 3 Voters

<u>Standard Voting System</u>

Voters	SU	SSU	CON	SCON
3	81.28015	85.64013	89.9151	92.43286
5	73.89849	74.64334	81.86399	81.4175
7	77.15541	77.15031	82.56971	82.80641
9	78.36027	79.40004	81.01244	81.60281
11	76.51844	75.74441	80.96372	80.25922
13	76.13150	75.44744	80.24274	80.33026
15	75.09851	75.92546	78.97618	80.12462
17	75.06600	73.67316	80.62215	80.56885
19	76.10229	76.65386	80.56049	80.69643
21	76.26308	75.64381	79.39276	78.56687
23	76.66461	76.86256	78.59663	78.81081
25	75.36212	75.63785	79.0997	79.17628

<u>Approval Voting System</u>

Voters	SU	SSU	CON	SCON
3	84.59656	84.38637	73.80964	74.48530
5	85.49946	84.88809	76.43347	75.29308
7	84.75676	85.13169	75.97089	75.95857
9	86.40394	86.43179	73.26244	73.43390
11	87.68660	87.51424	75.68910	76.54454
13	88.60281	87.75786	74.57857	74.97629

<u>Borda Voting System</u>

Voters	SU	SSU	CON	SCON
3	87.47958	85.14944	97.28929	94.75961
5	86.7208	86.7208	93.3808	93.3808
7	85.8735	85.8735	92.6008	92.6008
9	86.7582	86.7582	92.7946	92.7946
11	86.8360	86.8360	93.3957	93.3957
13	85.4114	85.4114	90.2788	90.2788
15	88.1715	88.1715	91.4930	91.4930

CHAPTER 5

DISCUSSION AND SUGGESTIONS FOR FURTHER RESEARCH

Perhaps the most pertinent question which can be addressed to this research is why it is of any interest to compare multi-alternative voting systems with strategic voting. After all, any voting system has 100% Condorcet efficiency with only two alternatives, regardless of whether sincere or optimal strategies are assumed. Additionally, most of the voting situations in which there are more than two alternatives occur with large electorates, where the possibility of strategic voting is more or less precluded. However, there are two points to keep in mind. First, a series of sequential pairwise votes on the same issue implies more than two alternatives, and this occurs frequently in committee voting. Second, we know that increasing the number of alternatives decreases the likelihood of a Condorcet winner, and as appealing as the Condorcet criterion is, that means that we disregard those cases where extreme conflict occurs (no Condorcet winner exists). We also know that maximum social utility efficiency of a two alternative election is 81.65%. Thus, we must expect social utility efficiency to decrease with every step in a sequence of pairwise votes.

Multi-alternative elections are an option to be compared with a sequence of pairwise votes. Condorcet efficiency is the appropriate comparison measure for this purpose. However, different multi-alternative voting systems can also be compared to each other using both Condorcet efficiency and social utility efficiency. Given this rationale, it is important to differentiate between sincere and strategic efficiency measures. Strategic efficiency measures are more appropriate because they recognize maximizing behavior on the part of individuals.

5.1 Efficiency Measure Changes with Strategic Voting

The striking difference in the way social utility efficiencies change for a given voting system is not very difficult to explain. Recall that with the assumption of strategic voting, standard voting system social utility efficiency increased markedly, while for the approval and Borda systems it decreased, particularly for small electorates. However, in the standard voting system, for strategic voting to occur, some alternative must be ranked first by as large or nearly as large a percentage of the voting population as the winning or tied alternative. The individual who changes the outcome increases his utility by doing so; the voters who had ranked the strategic voter's more preferred alternative as first gain, while those who had ranked his less-preferred alternative as first lose. The other voters' losses and gains essentially balance each other out, with the gain of the strategically voting individual being the predominant effect. In contrast, for both the approval and Borda systems, strategic voting can occur if there is an alternative which is ranked as high or nearly as high _on average_ as the winning or tied alternative.

These characteristics are combined with the fact that you can't "go around in circles" in the standard voting system. Strategic voting is an all or nothing proposition. Suppose two alternatives are vying for first place, and an individual changes his vote from his most preferred alternative to his more preferred of the two vying for first place. At that point, there is nothing more he can do to change the outcome, and he has reduced or eliminated the possibility of strategic voting on his most preferred alternative. In the Borda system he would have the option of `removing' votes from the less preferred alternative, which would increase the total of some 3rd alternative and the possibility of strategic voting on it. In the approval system he can either remove a vote from the less preferred alternative, or add one to the more preferred alternative, but this does not prevent yet another voter from adding or subtracting a vote without affecting _his_ most preferred alternative. In other words, strategic voting in the Borda or approval system may entail changes in total votes which can cause other strategic (insincere) voters to change their minds. In the standard voting system, the total of the 3rd alternative can only decrease.

It is easy to show that expected social utility of the standard voting system should increase and expected social utility of the Borda system should decrease with strategic voting. Let W_k be the maximum of total votes with sincere voting, and W_i be within range of winning. Then $|W_k - W_i| \leq k$, where k is the maximum weight assignment of the system.

Standard Voting System: The value of k is 1. Any strategic voter either makes or breaks a tie, and the adjusted total votes are such that $|W_k - W_i| \leq 1$. Let $N_{i>k}$ be the number of voters who prefer i to k, and $N_{k>i}$ be the number that prefer k to i. There continues to be an incentive for strategic voting until $\min(W_i, W_k) = \min(N_{i>k}, N_{k>i})$ and either $W_i \neq W_k$ or $W_i + W_k = N$. However, for an odd number of voters, this implies that a majority of the voting population prefers the winning alternative after strategic voting to the contending alternative, and the change in expected social utility is positive if the outcome is different after strategic voting, and zero if the outcome remains the same.

Borda Voting System: The change in expected social utility from a change in outcome from i to k with strategic voting is $E(\Sigma_j(u_{ij} - u_{kj}))$. Let $r_j(i)$ be an individual voter's ranking of alternative i, and $\underline{r}(i)$ be the average rank across the voting population of alternative i. Given that individual utilities are i.i.d. uniform [0,1] variables, $E(u_{ij} - u_{ik}) = (r_j(k) - r_j(i))/(m+1)$, and $E(\Sigma_j(u_{ij} - u_{ik})) = n(\underline{r}(i) - \underline{r}(k))/(m+1)$. However, we know that $r_j(i) = (m - w_{ij})$, where w_{ij} is the sincere vote. Using this information, we obtain $E(\Sigma_j(u_{ij} - u_{ik})) = (W_i - W_k)/(m+1)$. But $W_k \geq W_i$, so the change in expected social utility with strategic voting is negative or zero.

5.2 Implications of the Results

The first major implication of the results is that strategic voting can increase efficiency measures of a voting system. Manipulability of a voting system is not necessarily an undesirable characteristic. It should be pointed out that the voting system which is least manipulable (the standard voting system), is the one which showed the most dramatic increase for both Condorcet efficiency and social utility efficiency. However, the fact remains that strategic voting can actually produce a "better" outcome.

Unless a fairly high level of information is available to voters, rankings according to sincere efficiency estimates are correct. Without nearly complete information, the incentives for strategic voting disappear, and estimates approach their sincere counterparts. Similarly, with large electorates (>125 voters) the advantages of strategic voting disappear, although for the standard voting system, strategic Condorcet efficiency can still be significantly greater than sincere Condorcet efficiency.

Second, when optimal strategies are used by voters, differences between voting systems are not as clear-cut for small electorates. For very small voting populations, efficiency estimates for all three voting systems fall within a very small range when the number of alternatives is 4 or less. The advantages of using the approval or Borda system as opposed to the standard voting system are not as large as previous work has indicated for these situations. Again, however, for large electorates or less than nearly complete information, the conclusions of previous work hold.

Third, multi-alternative voting decreases Condorcet efficiency, but fairly high efficiencies are still obtainable if strategic voting is assumed. The cost of repeated (sequential) pairwise votes may be large enough relative to a single multi-alternative election to justify multi-alternative voting in committees.

5.3 Limitations on Nash Equilibria Found

In estimating efficiency measures, the use of the first equilibrium found (sincere voting, if it is a Nash equilibrium) is based on two points. The model is designed to approximate as closely as possible to previous work, which has always assumed the use of sincere strategies by voters. The cases which differentiate the current research from previous work are those in which sincere voting is not a Nash equilibrium. A base vote matrix is necessary in solving for equilibria, and the sincere vote matrix is the simplest and most logical choice. Again, there is no reason to assume that individuals' strategies will change unless a gain in expected utility can be achieved. Therefore, if sincere voting is a Nash equilibrium, it is the equilibrium used.

The algorithm does not go on to find all equilibria after the first both because of the number of equilibria that exist (regardless of the number of voters) and because asymptotically this approach is incapable of differentiating between voting systems. Recall theorem 1, which says that asymptotically sincere strategies are a Nash equilibrium. The theorem implies that asymptotically, <u>any</u> set of strategies is a Nash equilibrium. If one assumes that equilibria are equally likely, then as the voting population increases, the voting system degenerates to a random choice. Some restriction of equilibria is necessary in order to differentiate between voting systems.

For small electorates, efficiency estimates do differ when all equilibria are found. Table 5.1 presents these estimates for the 3 voter 3 alternative case (1000 repetitions). However, it is clear that efficiency estimates increase as the number of strategy profiles which are not equilibria increases.

Table 5.1 Summary Statistics for the 3 Voter 3 Alternative System When All Nash Equilibria are Found

	Approval	Borda	Standard
# possible strategy profiles	216	216	27
mean # equilibrium strategy profiles	18.297	29.544	6.111
% profiles which are equilibria	8.47	13.68	22.63
% profiles which are not equilibria	91.53	86.32	77.37
social utility efficiency (%)	97.7247	91.3310	55.5714
Condorcet efficiency (%)	75.7878	75.1140	58.7851

Finally, the use of sincere voting as the equilibrium each time it is a Nash equilibrium is supported by the concept of bounded rationality as expressed in perfect equilibria [123]. Each voter has some probability ε_i for the breakdown of rationality. When this occurs, he will use each admissible strategy S_i with probability q_{S_i}, and $\sum_{i=1}^{Q} q_{S_i} = 1$.

<u>Theorem 3:</u> If sincere voting is a Nash equilibrium, it is a perfect pure strategy equilibrium.

<u>Proof:</u> Let Q be the number of admissible pure strategies in the voting game. Q^n is the set of admissible strategy profiles. Let S_1 be a representative voter's sincere strategy, with S_2 being any other admissible strategy. $N(S_1,S_2) \in Q^n$ is the subset of admissible strategy profiles for which the expected outcomes of the two strategies differ. For any profile $k \in N$, p_k^{ε} is the probability that this profile occurs in the perturbed game. p_k^{ε} is a function of all voters' ε_i and q vectors, and $\sum_{k\in Q^n} p_k^{\varepsilon} = 1$. Then $EU(S_1) - EU(S_2)$, the difference in expected utility of strategies 1 and 2, is equal to

$$\sum_{k\in N} p_k^{\varepsilon}(u_i - u_j) \qquad (1)$$

As $\varepsilon \to 0$, all p_k^{ε} $(k \in Q^n)$ approach either 0 or 1 (the degenerate distribution of the complete information case). Then there are two possible cases: either p_k^{ε} approaches one for a profile for which the expected outcomes of the two strategies do not differ, or p_k^{ε} approaches one for $k \in N$.

Case 1: $p_k^{\varepsilon} \to 1$, $k \notin N$.

For any sequence of ε in which $q_{Si} = q_{Sj} = 1/Q$ for each voter, the maximand is a positive linear transformation of the zero information game maximand. As shown in Chapter 3, sincere strategies are optimal.

Case 2: $p_k^{\varepsilon} \to 1$, $k \in N$.

(a) $k \ni$: $p_k^{\varepsilon} \to 1$ has $u_i > u_j$: $\Sigma_{k \in N}\ p_k^{\varepsilon}(u_i - u_j) > 0 \Rightarrow$ strategy S_1 is optimal.

(b) $k \ni$: $p_k^{\varepsilon} \to 1$ has $u_i < u_j$: $\Sigma_{k \in N}\ p_k^{\varepsilon}(u_i - u_j) < 0 \Rightarrow$ $\Sigma_{k \in N}\ p_k(u_i - u_j) < 0$. However, this implies that strategy S_1 was not optimal in the original game, and sincere strategies were not a Nash equilibrium, which is a contradiction.

Therefore, if sincere voting is a Nash equilibrium, it is a perfect pure strategy equilibrium. It is also clear that for this sequence of ε, the equilibrium is the only one that will be found.

5.4 Optimality Properties of Comparison Measures

The optimality properties of the comparison measures used depend on the decisions being made by using a voting system. Two classic situations in which voting systems appear to be reasonable methods of choice are a) determination of the level of a pure public good to be produced; and b) choice of an allocation of "resources" along a Pareto-frontier.

5.4.1 Choice of the Level of a Pure Public Good

The condition for Pareto-optimal provision of a pure public good was derived by Samuelson in 1954 [119]. A pure public good has the property that it is consumed simultaneously by all individuals in its entirety. The Samuelson condition is

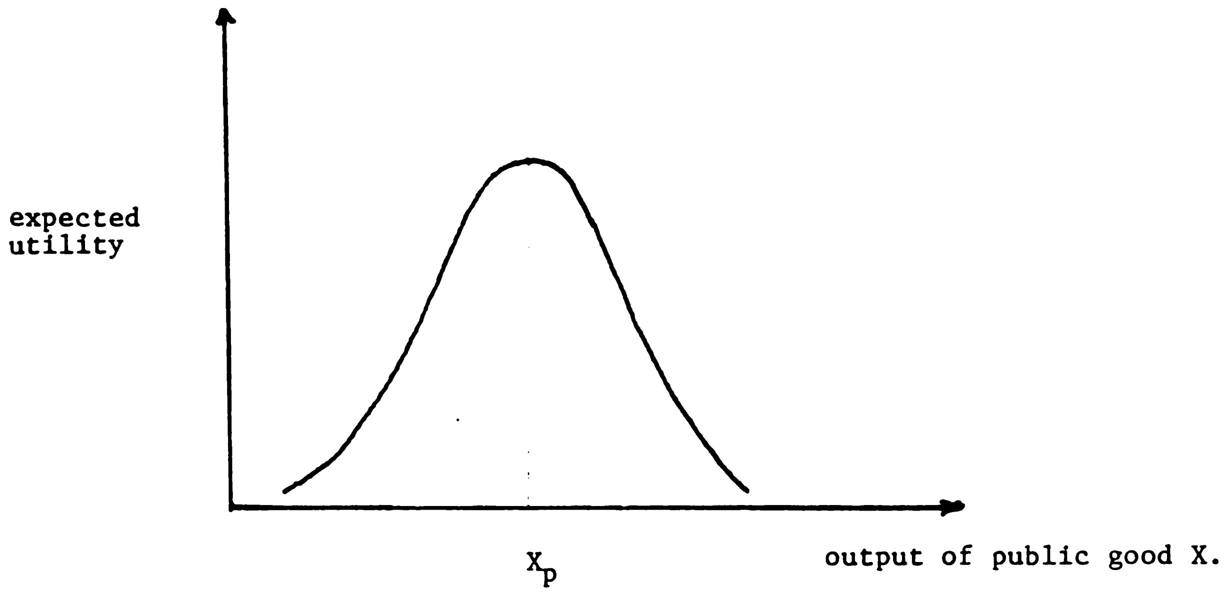
$$\Sigma_i \; MRS^i_{GX} = MRT_{GX},$$

where MRS^i_{GX} is individual i's marginal rate of substitution of the private good X for the public good G, and MRT_{GX} is the marginal rate of transformation of X for G. Intuitively, "...at the optimum, the marginal cost of supplying the last unit of G in terms of X foregone just equals the sum of the marginal benefits that all users of the increment G simultaneously obtain in terms of X."[1] Since individual marginal benefits are equally weighted, this is identical to maximizing social utility in terms of a utilitarian social welfare function.

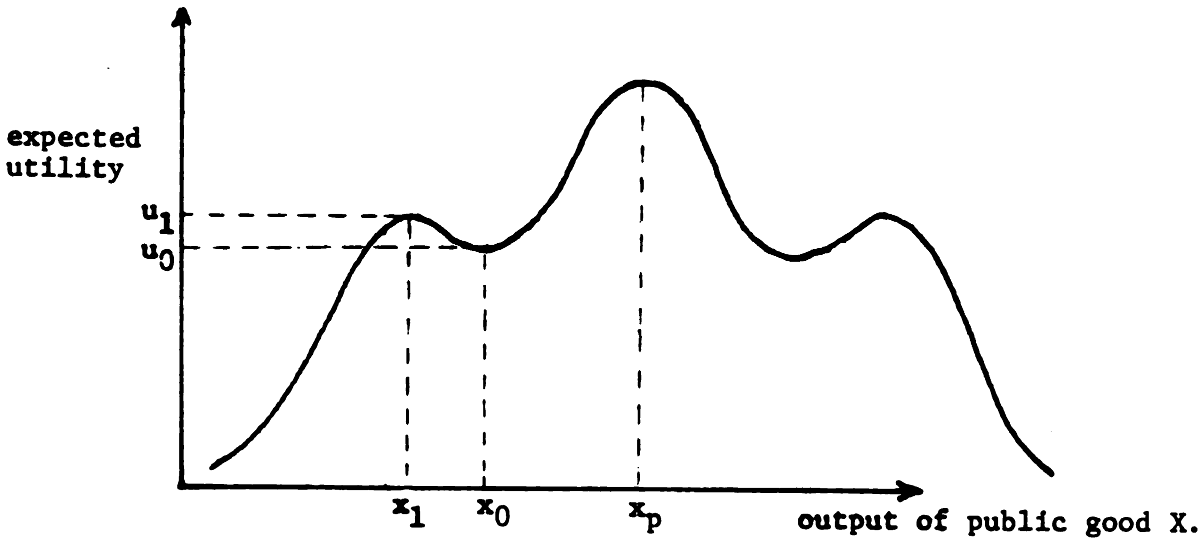
5.4.1.1 Social Utility Efficiency and Optimality in the Provision of Pure Public Goods

A voting system which maximizes social utility in terms of a utilitarian social welfare function will produce Pareto-optimal outcomes when used for decisions about the level of pure public goods to be produced. Social utility efficiency measures the "closeness" of outcomes of a voting system to maximum social utility, and is the ratio of the expected social utility of the outcome to the expected maximum social utility over the alternatives. If this ratio is equal to one, then the voting system being evaluated is expected to produce a Pareto-optimal outcome. Given the same variance, a voting system with lower social utility efficiency will be expected to achieve a Pareto- optimal outcome less frequently. One difficulty is that the variance of social utility efficiency does not remain constant across voting systems. A better measure might be the frequency with which a voting system is expected to attain maximum social utility, but the same problem surfaces that occurs with Condorcet efficiency: there is no differentiation between social-utility outcomes which do not attain the maximum. Given this problem, the social utility efficiency measure used is a reasonable compromise. Because it does reflect to some extent the probability of Pareto-optimal provision of a pure public good, a voting system with greater social utility efficiency than another is in some sense "better."

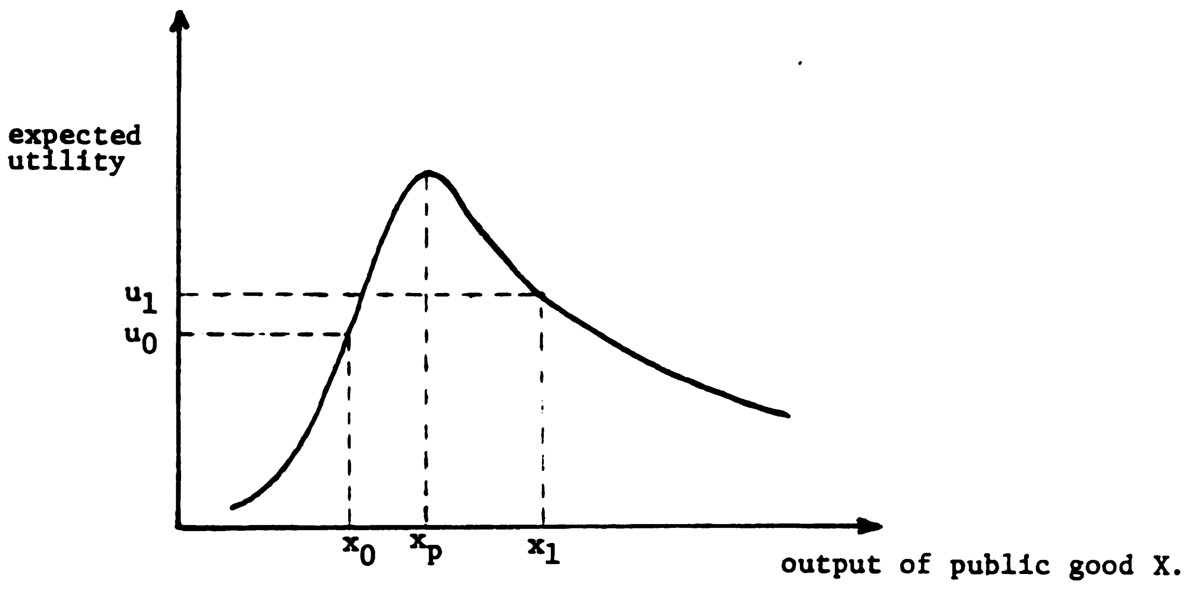
Because social utility efficiency reflects a random individual´s expected utility of a voting system´s outcome, a further insight into the optimality properties of this measure can be gained. Each alternative (level of the public good) X has a corresponding mean utility level across the population, u(x), which is the expected utility of that level to a randomly chosen voter. Conversely, an expected utility of the voting system´s outcome implies one or more expected outcomes (the inverse function is not generally single-valued). If expected utility for the average (mean) voter is single peaked and symmetric about its maximum X_p, the Pareto-optimal level (Figure 5.1a), then as expected utility increases, the level of under- or over-provision of the public good, $|X_p - X|$, decreases, i.e. the level actually produced is closer to the Pareto-optimal level. Both single-peakedness and symmetry are necessary conditions for this conclusion, however. In Figures 5.1b and 5.1c, an increase in expected utility does not necessarily move the level of provision of the public good closer to the Pareto-optimal level.



5.1a. Symmetric and single-peaked mean expected utility



5.1b. Symmetric and non-single-peaked mean expected utility $|X_p - X|$ may increase; $|X_p - X_0| < |X_p - X_1|$.



5.1c. Asymmetric and single-peaked mean expected utility. $|X_p - X|$ may increase; $|X_p - X_0| < |X_p - X_1|$.

Figure 5.1. Mean Expected Utility and Corresponding Levels of the Public Good (G) Produced.

5.4.1.2 Condorcet Efficiency and Optimality in the Provision of Pure Public Goods

Bowen [15] showed that the Condorcet winner (median voter equilibrium) is a Pareto-optimal outcome if the median voter is also the mean voter. Since the median voter is decisive in his model, and the equilibrium point is the median voter's most preferred level of the public good, if the median coincides with the mean then the mean voter also has a utility-maximizing outcome. In algebraic terms,

$$\frac{\Sigma_i MRS^{i}_{G\$}}{N} = t = \frac{MC_G}{N},$$

or the average marginal rate of substitution of money for the public good G is equal to the marginal tax rate, which in his model is an equal share of the marginal cost of production of the public good. Under this condition, then, the Condorcet winner is a Pareto-optimal outcome. However, the existence of a Condorcet winner does not require single-peakedness of preferences, nor if preferences are single peaked does the mean peak preference necessarily coincide with the median. Without these assumptions, the Condorcet winner need not be a Pareto-optimal outcome in choosing the level of provision of a pure public good.

5.4.2 Choice Along a Pareto-Frontier

The second situation in which voting systems are of interest to an economist is the situation of choice along a Pareto frontier. Using lump-sum taxes and transfers, the government can attain alternative points along the grand utility possibility frontier. When choosing an allocation of resources along a Pareto-frontier, the general guideline is that the allocation which maximizes social welfare should be chosen. A widely-used formulation of the social welfare function [140] is

$$\{\Sigma_j [u_{ij}^{\tau}]\}^{1/\tau} \qquad \tau \leq 1;\ \tau \neq 0.$$

If $\tau = 1$, then we are using a utilitarian social welfare function.

5.4.2.1 Social Utility Efficiency and Choice Along a Pareto-Frontier

The social utility efficiency used by Weber and Bordley, among others, is a transformation of a utilitarian social welfare function. If indeed a society has a social welfare function for which $\tau = 1$, a voting system with higher social utility efficiency will be expected to produce outcomes of greater social welfare and will be in some sense a "better" voting system. Additionally, if in fact $\tau \neq 1$, social utility efficiency measures can easily be constructed which use different values of τ. If an estimate of τ can be obtained, then a social utility efficiency measure can be constructed which will rank possible voting systems appropriately.

5.4.2.2. Condorcet Efficiency and Choice Along a Pareto-Frontier

As mentioned previously, when it exists, the Condorcet winner has maximum expected social utility. Therefore a voting system which is expected to choose the Condorcet winner with greater frequency when it exists might also have greater expected social utility. Rankings of voting systems obtained by using Condorcet efficiency have agreed with those obtained with social utility efficiency when voters use sincere strategies. Unfortunately, because the Condorcet efficency measure does not differentiate between outcomes in cases where there is no Condorcet winner, no correspondence between the two measures can be shown unless preferences are restricted so that a Condorcet winner always exists.

5.4.3 Implicit Equity Considerations

Condorcet efficiency does have one implicit equity consideration. If the Condorcet winner is chosen, at least a majority of the voting population prefer it to any other alternative. Also, the Condorcet winner tends to have high social utility. The converse is not true. Social utility efficiency does not imply anything about equity.

5.5 Suggestions for Further Research

5.5.1 Costs of Voting

A cost of voting is not included in the model used for the simulation. Tullock and Downs [141],[37] both concluded that "voting is an irrational act in that it costs more to vote than one can expect to get in return."[2] An estimate of voting costs appropriate to a comparison of voting systems is presented below.

The expected utility of voting is:

$$EU = (u_i - u_j)p_{ij} - c$$

where (u_i-u_j) is the gain in utility to the voter if alternative i defeats alternative j as a result of his vote; p_{ij} is the probability of this occurring, and c is the cost to the individual of voting. Since p_{ij} approaches zero rapidly, and c is generally assummed to be positive, (u_i-u_j) must be of extreme magnitude for voting to be a rational act.

Once again, let p be the probability that an individual voter is decisive. P depends upon the size of the voting population. Now, where a cost of voting is included, complete participation cannot be assumed. Voter participation will depend upon whether the expected utility of voting is positive, which in turn is based on the individual voter's estimate of p. The question of the "rationality" of voting is not therefore as clear-cut as would appear on preliminary examination. Using a model in which p and n are determined simultaneously, Palfrey and Rosenthal find that substantial voter turnout can be consistent with the inclusion of a cost of voting. Their model uses only two alternatives; however, increasing the number of alternatives would, under the assumptions presented at the beginning of this chapter, only increase p, making substantial participation more likely. Thus the inclusion of a cost of voting is consistent with the rationality assumptions employed.

However, because the purpose of this work is to compare voting systems, a determination of the possibly differential costs of voting for different systems is necessary. Voting involves not only a fixed cost of taking the time to go to the polling place and vote, but the cost of determining which strategy (vote vector) to use. Strategy determination costs clearly vary with the level of information the individual has, since as discussed previously, under zero information conditions, sincere voting is the unique optimal strategy. However, even under zero information conditions this cost will vary across voting systems because of the amount of information \`requested' from the voter. The standard voting system asks only for the voter's most-preferred alternative; the approval voting system required identification of all alternatives with above-average utility; and the Borda voting system requires a full ranking of all alternatives. Let the individual cost of voting be approximated by

$$c_i = a_i + f(s[C],\alpha) \qquad 0 \leq \alpha \leq 1$$

where a_i is some fixed cost to the individual voter i of taking the time to go to the polling place, s[C] is the number of possible strategies in the strategy set of the voting system or choice rule C, α is the information level voters are assumed to have, and $f(s[C],\alpha)$ represents the cost of optimal strategy determination. Individual voting costs may differ due to a_i, which may be modelled as a random variable. Given this determination of the individual cost of voting, an equilibrium in p and n can be determined. Palfrey and Rosenthal's model, however, finds multiple equilibria in p and n, and there are no strong predictions about voter turnout. Their model used only two alternatives, so that an extension of this model would be necessary prior to drawing any conclusions about voter turnout. It is also highly likely that such an extension would produce multiple equilibria in p and n for small voting populations. However, the multiple equilibria problem could be handled as it has been here, with Monte Carlo techniques.

At this point a pertinent consideration would be the administrative, or social costs of the voting system. Once individual strategies (including abstention) are determined, even if the equilibrium outcome of the election is known by the modeler, there is still the problem of "counting votes." Again there are differences between voting systems in this regard. The factor which immediately appears significant is the number of elements in the vote vector to be tallied. Let the social cost of the voting system be $c_s(S) = n(S) \times k(S)$,where n(S) is expected participation in the voting system as determined above, i.e. the number of ballots completed, and k(S) is the number of positive elements in an individual vote vector. k(S) would of course be one for the standard voting system, m/2 for the approval voting system, and m-1 for the Borda system. Given this information, appropriate efficiency measures, based on the expected net social cost (= expected social utility of chosen alternative - $\Sigma_i c_i$ - social cost) can be constructed for comparison of these voting systems. Given the difficulties, extending the model to include a cost of voting at this time would probably not produce any useful results.

5.5.2 Other Equilibria: The Competitive Solution

The possibility of modeling voting systems as cooperative games has not been overlooked. "Cooperative game theory for the most part focuses on games with transferable utility, even though...this assumption excludes the possibility of modeling most interesting political coalition processes. For the more general case, though, standard solution concepts are inadequate because they are undefined or they fail to exist, and even if they do exist, they focus on predicting payoffs rather than the coalitions that are likely to form."[3] Thus values such as the Shapley value or the Banzhaf-Coleman index of power, which have been widely used to estimate, for example, the "coalitional" value of states in a U.S. presidential election game, cannot be used to compare different voting systems, as the only information which they can provide is on the "coalitional" value of the players and not on outcomes.

McKelvey, Ordeshook and Winer [90] propose a different solution concept entirely, the competitive solution for games without transferable utility. The solution concept hypothesizes that "potential coalitions must bid for their members in a competitive environment via the proposals they offer. Given that several coalitions are attempting to form simultaneously, each coalition must if possible, bid efficiently by appropriately rewarding its "critical" members."[4]

Let A be the set of feasible outcomes. Then for any coalition C, v(C) = A if C is winning and v(C) = $\emptyset$ if C is losing. Thus if there is a majority voting game and C is a majority coalition, v(C), in a repeated game, is "the set of all possible dispositions of all bills."[5] A coalition's proposal is their policy platform; in their work a coalition's proposal is an ordered pair (u:C) such that u is an element of v(C) and u is an element of v(N). Then given two proposals, the coalition's proposal (u_1:C_1) is viable against the proposal (u_2:C_2) if $u_1 \geq u_2$ for all individuals belonging to both coalitions ($i \in C_1 \cap C_2$). Let K be any set of proposals. (u:C) is viable in K if it is viable against all proposals in K. K is balanced if each coalition can have exactly one proposal, and all proposals in K are viable against each other.

Of course, there may exist many distinct balanced sets of proposals. McKelvey, Ordeshook and Winer focus on the class of proposals in which the coalitions represented "make offers that are as attractive as possible to their respective critical members."[6] A proposal upsets a set of proposals K if it is a viable proposal in K and there is an alternative proposal (u′:C′) in K for which u > u′ for all individuals belonging to both coalitions.

A set of proposals K is a competitive solution if K is balanced and there is no proposal (u:C) that upsets K. This implies that the coalitions represented in K do indeed make offers that are as attractive as possible to their critical members. A stronger definition of "balanced" allows them to exclude coalitions greater than minimal winning size. K is strongly balanced if it is balanced and there are no two proposals (u_1:C_1) and (u_2:C_2) for which $u_1 \geq u_2$, with strict inequality for at least one i, for all individuals belonging to both coalitions. If K is a competitive solution and strongly balanced, the authors refer to it as a "strong competitive solution."

The competitive solution does predict vote trading; in one example the authors show that none of the proposals in the unique competitive solution correspond to the outcome of sincere voting. Additionally, a preliminary test or empirical validity found impressive correspondence between actual outcomes and the competitive solution´s predictions. The predicted coalitions all formed at least once, and no other coalitions formed.

As a solution concept this is very attractive. Not only do the conditions of the solution have intuitive appeal, but they can be placed in the familiar context of committee voting, as for example in Congress. Different voting systems such as the approval and Borda voting system can be analyzed, and the competitive solution predicts different size coalitions with each because of the different requirements for a winning coalition. However, some assumption about the likelihood of coalitions must be made to get any prediction on expected outcomes in order to compare different voting systems.

5.5.3 Social Welfare Functions

Because equity considerations are ignored in the utilitarian social welfare function, it would be useful to see if another formulation ($\tau \neq 1$) would produce any changes in rankings of voting systems. Certainly if equity is important to the choice of a voting system, the utilitarian social welfare function is not the appropriate comparison measure to use.

5.6 Conclusion

It has been shown that the use of optimal strategies by voters as opposed to sincere strategies can significantly change both social utility and Condorcet efficiency estimates for multi-candidate voting systems. Furthermore, the changes in Condorcet efficiency estimates change the rankings of the voting systems when the voting population is small. The standard voting system is seen to achieve the highest Condorcet efficiency, followed by the Borda system, with approval voting ranked last.

APPENDICES

APPENDIX A

SINCERE VOTING AS A NASH EQUILIBRIUM WITH AN INFINITE VOTING POPULATION

Theorem 1: As the voting population becomes large, i.e. $n \to \infty$, the probability that sincere strategies constitute a Nash equilibrium approaches one.

A.1 Proof: Borda System

For the individual voter, any w_{ij} is a random variable with $\mu = (m-1)/2$, $\sigma^2 = \sum_{w=1}^{m-1} w^2/m - [(m-1)/2]^2$. Then $W_i = \sum_j w_{ij}$ is distributed approximately normally with mean $n(m-1)/2$ and variance $n[\sum_{w=1}^{m-1} w^2/m - ((m-1)/2)^2]$, and the W_i have an approximate multivariate normal distribution. In order for an individual voter to change the outcome of the system, there must be some $|W_i - W_k| \leq m-1$. That is, the voter's maximum weight assignment of m-1 can cause the ordering of two totals to change. Let $Y = W_i - W_k$. Then Y has a mean $\mu_Y = \mu_{W_i} - \mu_{W_k} = 0$; and variance $\sigma_Y^2 = \sigma_{W_i}^2 + \sigma_{W_k}^2 + 2\sigma_{W_i W_k}$. Because of the relationship between the covariance and correlation coefficient this variance can be computed exactly; the correlation coefficient is $-1/(m-1)$. Intuitively, when one of the W_i is above its mean, the others are expected to be slightly below the mean.

Computing this, a variance of
$\sigma_Y^2 = 2n([\sum_{w=1}^{m-2} w^2/m] - (m-2)(m-1)^2/2m] - 2/(m-1)$ is obtained.
Obviously, as $n \to \infty$, the variance of Y becomes infinite. Therefore $P\{|W_i - W_k| \leq m-1\} = P\{-m+1 \leq Y \leq m-1\}$, the probability that Y falls within the specified interval,

approaches zero. Thus scope for strategic behavior diminishes asymptotically and the probability that sincere strategies constitute a Nash equilibrium approaches one.

A.2 Proof: Approval System

For the individual voter, any w_{ij} is a binomial random variable (either a vote is cast for it or not), with p = 1/2. Then $W_i = \Sigma_j w_{ij}$ is distributed approximately normally with mean np = n/2 and variance np(1-p) = n/4, and the W_i have an approximate multivariate normal distribution. In order for an individual voter to change the outcome of the system, there must be some $|W_i - W_k| \leq 1$. That is, the voter's maximum weight assignment of one can cause the ordering of two totals to change. Let $Y = W_i - W_k$. Then Y has a mean $\mu_Y = \mu_W - \mu_W = 0$; and variance $\sigma_Y^2 = \sigma_{W_i}^2 + \sigma_{W_k}^2 + 2\sigma_{W_i W_k}$. Because of the relationship between the covariance and correlation coefficient this variance can be computed exactly; the correlation coefficient is -1/(m-1). Intuitively, when one of the W_i is above its mean, the others are expected to be slightly below the mean. Computing this, a variance of $\sigma_Y^2 = [2n(m-1) - 8]/4(m-1)$ is obtained. Obviously, as $n \to \infty$, the variance of Y becomes infinite. Therefore $P\{|W_i - W_k| \leq 1\} = P\{-1 \leq Y \leq 1\}$, the probability that Y falls within the specified interval, approaches zero. Thus scope for strategic behavior diminishes asymptotically and the probability that sincere strategies constitute a Nash equilibrium approaches one.

APPENDIX B

SIMULATION PROGRAMS

VARIABLES USED

<u>INTEGER VARIABLES</u>

ALTS - number of alternatives used
CHOOS - randomly chosen voter for the reordering
COMMON - number of elections for which there is neither a Condorcet winner nor a pure strategy Nash equilibrium (always = 0)
COMP - indices of alternatives within "reach" of the winner; those which need to be compared for strategic voting
CVOTES(6) - Condorcet votes
CWINNE - Condorcet winner
F - indicator of strategic voting
G - number of tied alternatives
G1(720) - vector of strategies with maximum expected utility
G2 - number of tied strategies
H - loop counter
I - loop counter for alternatives
J - loop counter for voters
K - randomly chosen alternative for breaking ties
L - loop counter
LAST - loop counter for sorting by rank
M - loop counter for elections
MONE - loop counter for random reorderings of voters
N - loop counter for determining expected utility of strategies
NCOND - number of elections for which the Condorcet winner is chosen by sincere voting
NONASH - number of elections for which a pure strategy Nash equilibrium is not found (always = 0)
P - loop counter for repetitions of 100 election simulations
PVOTE(6,721) - matrix of admissible strategies within the voting system
Q - number of admissible strategies for the voting system
RANK(6) - the vector contains the index of the alternative in the specified rank for an individual voter
SIN - number of elections for which sincere voting is not manipulable
SNCOND - number of elections for which the Condorcet winner is chosen by strategic voting
STRAT - strategy which maximizes expected utility for a given voter
TEMPR - holding variable for sorting by rank
TIED(6) - the vector contains the indices of the tied alternatives
TOTAL(6) - total number of votes accruing to specified alternatives
TRANK(6) - rank ordering of total votes

TVOTE - holding variable for random reordering of voters
VOTE(6,125) - individual votes
VOTERS - the number of voters
VOTES(6) - total number of votes accruing to specified alternatives
WINNER - the alternative chosen by the voting system
WINS(6) - number of alternatives beaten in pairwise races by the specified alternative
VMAX - maximum number of votes accruing to any alternative
Z - 2,147,483,647: used in random number generation

REAL VARIABLES
CEFFIC - Condorcet efficiency with sincere voting
DSEED - current seed value for the random number generator
EELECT - total social utility of all winners chosen by sincere voting for a voting system
EFFIC - social utility efficiency with sincere voting
EMAX - maximum social utility over alternatives
EU(721) - expected utility of an admissible strategy
NOCC - number of elections without a Condorcet winner
NUM - number of elections with a Condorcet winner
RUTIL(6) - holding variable for sorting by rank
SCEFFI - strategic Condorcet efficiency
SEELEC - total social utility of all winners chosen by strategic voting for a voting system
SEFFIC - strategic social utility efficiency
SOCUT(6) - vector of social utilities of alternatives
TEMPU - holding variable for random reordering of voters
TOTUT - total utility of all alternatives in an election; divided by the number of alternatives, the expected utility of the election if the specified voter does not participate
UTIL(6,125) - matrix of individual utilities
UTMAX - sum over elections of maximum social utility
M1 - mean Condorcet efficiency with sincere voting
M2 - mean strategic Condorcet efficiency
M3 - mean social utility efficiency with sincere voting
M4 - mean strategic social utility efficiency
SD1 - standard deviation of Condorcet efficiency with sincere voting
SD2 - standard deviation of strategic Condorcet efficiency
SD3 - standard deviation of social utility efficiency with sincere voting
SD4 - standard deviation of strategic social utility efficiency
X - multiplier for random number generation
Y - double precision value of Z

VOTING SYSTEM PROGRAMS

```
      PROGRAM STANDARD (BORDA, APPROVAL)
      COMMON/PICK/ALTS,VOTERS,I,J,TOTAL,VOTES,LAST,
     +TVOTE,TEMPR,COMP,WINNER,TIED,G,K,TRANK,
     +SOCUT,VOTE
      INTEGER*2 ALTS,CVOTES(6),CWINNE,F,G,H,I,
```

```
      +J,K,L,LAST,M,N,NCOND,RANK(6),COMP,TOTAL(6),MONE,
      +SNCOND,STRAT,COMMON,P,SIN,Q,PVOTE(6,721),G2,
      +TEMPR,TIED(6),NONASH,TVOTE,G1(720),CHOOS,TRANK(6),
      +VOTE(6,125),VOTERS,VOTES(6),WINNER,WINS(6),VMAX
       REAL CEFFIC,EELECT,EFFIC,EMAX,EU(721),NOCC,NUM,
      +RUTIL(6),SCEFFI,SEELEC,SEFFIC,SOCUT(6),TEMPU,
      +TOTUT,UTIL(6,125),UTMAX,M1,M2,M3,
      +M4,SD1,SD2,SD3,SD4
       INTEGER*4 Z
       REAL*8 DSEED,X,Y
       DATA X/1.6807D4/
       Z=2147483647
       Y=DBLE(Z)
       ALTS=5
       Q=5
```

The value of Q (the number of admissible strategies) depends on the voting system being simulated. For the Borda system, Q=ALTS!, while forthe Approval system, $Q=(\sum_{i=1}^{alts-1} 2^i)$

```
C
C      ADMISSIBLE STRATEGIES ARE DETERMINED
C
       DO 2 N=1,Q
         DO 1 I=1,ALTS
           IF(I.EQ.N)THEN
             PVOTE(I,N)=1
           ELSE
             PVOTE(I,N)=0
           ENDIF
1        CONTINUE
2      CONTINUE
```

For the Borda and Approval Systems, admissible strategies are read from a file. The above lines are replaced with the following:

```
        Open(5,File='STRAA',Status='Old')
        Do 2 N=1,Q
          Do 1 I=1,ALTS
            Read(5,*)PVOTE(I,N)
    1     Continue
    2   Continue
```

```
       OPEN(4,FILE='RESULT',STATUS='OLD')
       OPEN(3,FILE='SEED',STATUS='OLD')
       READ(3,*)DSEED
       DO 295 VOTERS=3,25,2
```

```
C
C     INITIALIZE LOOP VALUES
C
      M1=0.
      M2=0.
      M3=0.
      M4=0.
      SD1=0.
      SD2=0.
      SD3=0.
      SD4=0.
C
C     DO 20 REPETITIONS OF 100 ELECTIONS
C
      DO 294 P=1,20
C
C     INITIALIZE LOOP VALUES
C
      SIN=0
      COMMON=0
      NONASH=0
      NCOND=0
      SNCOND=0
      EELECT=0.
      SEELEC=0.
      EMAX=0.
      NUM=100.
C
C     DO 100 ELECTIONS
C
      DO 286 M=1,100
        UTMAX=0
        DO 4 I=1,ALTS
          SOCUT(I)=0
          WINS(I)=0
C
C     ASSIGN UTILITIES TO VOTERS FOR EACH ALTERNATIVE
C
          DO 3 J=1,VOTERS
            DSEED=DMOD(DSEED*X,Y)
            UTIL(I,J)=SNGL(DSEED/Y)
            SOCUT(I)=SOCUT(I)+UTIL(I,J)
3         CONTINUE
          IF(SOCUT(I).GT.UTMAX)THEN
            UTMAX=SOCUT(I)
          ENDIF
4       CONTINUE
        EMAX=EMAX+UTMAX
C
C     DETERMINE CONDORCET WINNER BY MAKING ALL PAIRWISE
C     COMPARISONS
C
        CWINNE=0
        DO 11 I=1,ALTS
```

```
            DO 10 H=I+1,ALTS
              CVOTES(I)=0
              CVOTES(H)=0
              DO 9 J=1,VOTERS
                IF(UTIL(I,J).GT.UTIL(H,J))THEN
                  CVOTES(I)=CVOTES(I)+1
                ELSE
                  CVOTES(H)=CVOTES(H)+1
                ENDIF
9             CONTINUE
              IF(CVOTES(I).GT.CVOTES(H))THEN
                WINS(I)=WINS(I)+1
              ELSE IF(CVOTES(I).LT.CVOTES(H))THEN
                WINS(H)=WINS(H)+1
              ENDIF
10          CONTINUE
            IF(WINS(I).EQ.ALTS-1)THEN
              CWINNE=I
              GOTO 12
            ENDIF
11        CONTINUE
C
C       NO CONDORCET WINNER: SUBTRACT 1 FROM NUMBER OF
C       ELECTIONS WITH CONDORCET WINNER
C
          NUM=NUM-1.
C
C       ORDER UTIL(I,J) AND RANK(I,J) SO WE HAVE UTILITIES IN
C       ORDER AND CANDIDATES IN ORDER BY RANK
C
12        DO 17 J=1,VOTERS
            DO 13 I=1,ALTS
              RANK(I)=I
              RUTIL(I)=UTIL(I,J)
13          CONTINUE
            DO 15 LAST=ALTS,2,-1
              DO 14 I=1,LAST-1
                IF(RUTIL(I).LT.RUTIL(I+1))THEN
                  TEMPU=RUTIL(I)
                  RUTIL(I)=RUTIL(I+1)
                  RUTIL(I+1)=TEMPU
                  TEMPR=RANK(I)
                  RANK(I)=RANK(I+1)
                  RANK(I+1)=TEMPR
                ENDIF
14            CONTINUE
15          CONTINUE
C
C       CANDIDATES ARE RANKED FROM HIGHEST TO LOWEST.  RANK(I)
C       GIVES NUMBER OF CANDIDATE IN RANK I FOR VOTER J.
C       ASSIGN VOTES (SINCERE) FOR STANDARD SYSTEM.
C
            DO 16 I=1,ALTS
              K=RANK(I)
```

```
          IF(I.EQ.1)THEN
            VOTE(K,J)=1
          ELSE
            VOTE(K,J)=0
          ENDIF
16      CONTINUE
17    CONTINUE
```

For the Borda and Approval systems, assignment of sincere votes differs slightly. The above lines are replaced with the following:

```
Borda:
          Do 16 I=1,ALTS
            K=RANK(I)
            VOTE(K,J)=ALTS-I
16        Continue
17      Continue

Approval:
          TOTUT=0
          Do 16 I=1,ALTS
            TOTUT=TOTUT+UTIL(I,J)
16        Continue
          TOTUT=TOTUT/ALTS
          Do 18 I=1,ALTS
            If(UTIL(I,J).GT.TOTUT)Then
              VOTE(I,J)=1
            Else
              VOTE(I,J)=0
            Endif
18        Continue
17      Continue
```

```
      CALL COUNT
      IF(WINNER.EQ.CWINNE)THEN
        NCOND=NCOND+1
      ENDIF
      EELECT=EELECT+SOCUT(WINNER)
C
C     ASSIGN VOTES (STRATEGIC) FOR STANDARD SYSTEM
C     IF NUMBER OF ALTERNATIVES WITHIN "REACH" OF WINNING
C     IS NOT EQUAL TO 1, THE ELECTION IS MANIPULABLE.
C     OTHERWISE DO NOT NEED TO CHECK STRATEGIES.  SKIP TO
C     LINE 65, P. 159.
C
      IF(COMP.NE.1)THEN
      DO 37 L=1,40
        DO 36 J=1,VOTERS
        N=Q+1
        EU(N)=0
        G=1
        VMAX=MAX(VOTES(1),VOTES(2),VOTES(3),VOTES(4),
```

```
     +VOTES(5))
          DO 26 I=1,ALTS
            IF(VOTES(I).EQ.VMAX)THEN
C
C      ASSIGN INDEX TO TIED ALTERNATIVE
C
             TIED(G)=I
              G=G+1
              EU(N)=EU(N)+UTIL(I,J)
            ENDIF
            VOTES(I)=VOTES(I)-VOTE(I,J)
            PVOTE(I,Q+1)=VOTE(I,J)
26        CONTINUE
          EU(N)=EU(N)/REAL(G-1)
          STRAT=Q+1
          DO 31 N=1,Q
          IF(TOTAL(1).GT.TOTAL(2))THEN
            DO 27 I=1,ALTS
C
C      IF STRATEGY CAN CHANGE OUTCOME OF ELECTION
C
            IF((TOTAL(1)-VOTE(TRANK(1),J)+PVOTE(TRANK(1),N)
     +)-(TOTAL(I)-VOTE(TRANK(I),J)+PVOTE(TRANK(I),N)).LE.0)
     +THEN
              GOTO 30
            ENDIF
27          CONTINUE
C
C      OTHERWISE EXPECTED UTILITY OF STRATEGY IS EQUAL TO
C      EU OF CURRENT STRATEGY.  SKIP TO END OF LOOP AND
C      GO TO NEXT STRATEGY.
C
            EU(N)=EU(Q+1)
            GOTO 31
          ENDIF
C
C      DETERMINE EXPECTED UTILITY OF STRATEGY
C
30          DO 28 I=1,ALTS
              VOTES(I)=VOTES(I)+PVOTE(I,N)
28          CONTINUE
            VMAX=MAX(VOTES(1),VOTES(2),VOTES(3),VOTES(4),
     +VOTES(5))
            EU(N)=0
            G=1
            DO 29 I=1,ALTS
              IF(VOTES(I).EQ.VMAX)THEN
                TIED(G)=I
                G=G+1
                EU(N)=EU(N)+UTIL(I,J)
              ENDIF
              VOTES(I)=VOTES(I)-PVOTE(I,N)
29          CONTINUE
            EU(N)=EU(N)/REAL(G-1)
```

```
C
C     IF EXPECTED UTILITY EXCEEDS EU OF CURRENT STRATEGY,
C     CHANGE STRATEGY
C
            IF(EU(N).GT.EU(STRAT))THEN
              STRAT=N
            ENDIF
31        CONTINUE
          G2=1
          IF(STRAT.NE.Q+1)THEN
            F=F+1
            DO 33 N=1,Q
C
C     IF MORE THAN ONE STRATEGY HAS MAX EU, CHOOSE ONE
C     RANDOMLY
C
              IF(EU(N).EQ.EU(STRAT))THEN
                G1(G2)=N
                G2=G2+1
              ENDIF
33          CONTINUE
            DSEED=DMOD(DSEED*X,Y)
            K=INT(((SNGL(DSEED/Y))*(REAL(G2-1)))+1.)
            STRAT=G1(K)
          ENDIF
C
C     REASSIGN VOTES IN ACCORDANCE WITH CHOSEN STRATEGY
C     DETERMINE NEW TOTALS
C
          DO 34 I=1,ALTS
            VOTE(I,J)=PVOTE(I,STRAT)
            VOTES(I)=VOTES(I)+VOTE(I,J)
34        CONTINUE
36      CONTINUE
C
C     IF NO STRATEGY CHANGES HAVE OCCURRED, NASH EQUILIBRIUM
C     HAS BEEN FOUND.  DETERMINE WINNER AND GO TO
C     CALCULATION OF STATISTICS.
C
        IF(F.EQ.0)THEN
          CALL COUNT
        IF(WINNER.EQ.CWINNE)THEN
          SNCOND=SNCOND+1
        ENDIF
        SEELEC=SEELEC+SOCUT(WINNER)
          GOTO 285
        ENDIF
        F=0
37    CONTINUE
C
C     HERE WE HAVE NOT REACHED THE NASH EQUILIBRIUM
C     WRITE UTILITIES AND VOTES TO A FILE
C
```

```
      REWIND 11
      DO 44 J=1,VOTERS
        DO 13 I=1,ALTS
          RANK(I)=I
          RUTIL(I)=UTIL(I,J)
13      CONTINUE
        DO 15 LAST=ALTS,2,-1
          DO 14 I=1,LAST-1
            IF(RUTIL(I).LT.RUTIL(I+1))THEN
              TEMPU=RUTIL(I)
              RUTIL(I)=RUTIL(I+1)
              RUTIL(I+1)=TEMPU
              TEMPR=RANK(I)
              RANK(I)=RANK(I+1)
              RANK(I+1)=TEMPR
            ENDIF
14        CONTINUE
15      CONTINUE
        DO 43 I=1,ALTS
          K=RANK(I)
          VOTE(K,J)=ALTS-I
43      CONTINUE
        WRITE(11,*)UTIL(1,J),UTIL(2,J),UTIL(3,J),UTIL(4,J)
        WRITE(11,*)VOTE(1,J),VOTE(2,J),VOTE(3,J),VOTE(4,J)
44    CONTINUE
C
C     DO LOOP FOR NUMBER OF REORDERINGS
C
      DO 63 MONE=1,40
      REWIND 11
      DO 46 J=1,VOTERS
        READ(11,*)UTIL(1,J),UTIL(2,J),UTIL(3,J),UTIL(4,J)
        READ(11,*)VOTE(1,J),VOTE(2,J),VOTE(3,J),VOTE(4,J)
46    CONTINUE
C
C     RANDOM REORDERING OF VOTERS
C
      DO 48 J=1,VOTERS
        DSEED=DMOD(DSEED*X,Y)
        CHOOS=INT(SNGL(DSEED/Y)*(VOTERS-J+1))+J
        IF(CHOOS.NE.J)THEN
        DO 47 I=1,ALTS
          TVOTE=VOTE(I,J)
          VOTE(I,J)=VOTE(I,CHOOS)
          VOTE(I,CHOOS)=TVOTE
          TEMPU=UTIL(I,J)
          UTIL(I,J)=UTIL(I,CHOOS)
          UTIL(I,CHOOS)=TEMPU
47      CONTINUE
        ENDIF
48    CONTINUE
      WRITE(*,*)'SEARCH    ',MONE
```

```
C
C     AFTER REORDERING, REPEAT PROCESS OF SEARCHING FOR
C     NASH EQUILIBRIUM
C
        DO 62 L=1,40
          DO 60 J=1,VOTERS
            N=Q+1
            EU(N)=0
            G=1
            VMAX=MAX(VOTES(1),VOTES(2),VOTES(3),VOTES(4),
     +VOTES(5))
            DO 50 I=1,ALTS
              IF(VOTES(I).EQ.VMAX)THEN
                TIED(G)=I
                G=G+1
                EU(N)=EU(N)+UTIL(I,J)
              ENDIF
              VOTES(I)=VOTES(I)-VOTE(I,J)
              PVOTE(I,Q+1)=VOTE(I,J)
50          CONTINUE
            EU(N)=EU(N)/REAL(G-1)
            STRAT=Q+1
            DO 55 N=1,Q
              DO 52 I=1,ALTS
                VOTES(I)=VOTES(I)+PVOTE(I,N)
52            CONTINUE
              VMAX=MAX(VOTES(1),VOTES(2),VOTES(3),VOTES(4),
     +VOTES(5))
              EU(N)=0
              G=1
              DO 53 I=1,ALTS
                IF(VOTES(I).EQ.VMAX)THEN
                  TIED(G)=I
                  G=G+1
                  EU(N)=EU(N)+UTIL(I,J)
                ENDIF
                VOTES(I)=VOTES(I)-PVOTE(I,N)
53            CONTINUE
              EU(N)=EU(N)/REAL(G-1)
              IF(EU(N).GT.EU(STRAT))THEN
                STRAT=N
              ENDIF
55          CONTINUE
            G2=1
            IF(STRAT.NE.Q+1)THEN
              F=F+1
              DO 57 N=1,Q
                IF(EU(N).EQ.EU(STRAT))THEN
                  G1(G2)=N
                  G2=G2+1
                ENDIF
57            CONTINUE
              DSEED=DMOD(DSEED*X,Y)
              K=INT(((SNGL(DSEED/Y))*(REAL(G2-1)))+1.)
```

```
              STRAT=G1(K)
            ENDIF
            DO 58 I=1,ALTS
              VOTE(I,J)=PVOTE(I,STRAT)
              VOTES(I)=VOTES(I)+VOTE(I,J)
58          CONTINUE
60        CONTINUE
          IF(F.EQ.0)THEN
            CALL COUNT
            IF(WINNER.EQ.CWINNE)THEN
              SNCOND=SNCOND+1
            ENDIF
            SEELEC=SEELEC+SOCUT(WINNER)
            GOTO 285
          ENDIF
          F=0
61      CONTINUE
62    CONTINUE
63    CONTINUE
C
C     IF AFTER 40 RANDOM REORDERINGS OF VOTERS, AN
C     EQUILIBRIUM STILL HAS NOT BEEN FOUND,
C
      NONASH=NONASH+1
      IF(CWINNE.EQ.0)THEN
        COMMON=COMMON+1
      ENDIF
      WRITE(10,*)VOTERS,ALTS,P
65    ELSE
C
C     SINCERE VOTING IS A NASH EQUILIBRIUM
C
        SIN=SIN+1
        IF(WINNER.EQ.CWINNE)THEN
          SNCOND=SNCOND+1
        ENDIF
        SEELEC=SEELEC+SOCUT(WINNER)
      ENDIF
285   WRITE(*,*)VOTERS,P,M
286   CONTINUE
C
C     CALCULATE STATISTICS FOR 100 ELECTION SIMULATION AND
C     WRITE TO RESULT FILE
C
      EFFIC=((EELECT/100.)-(VOTERS/2.))/
     +((EMAX/100.)-(VOTERS/2.))
      SEFFIC=((SEELEC/(REAL(100-NONASH)))-(VOTERS/2.))/
     +((EMAX/100.)-(VOTERS/2.))
      CEFFIC=(REAL(NCOND))/NUM
      SCEFFI=(REAL(SNCOND))/(NUM-REAL(NONASH)+REAL(COMMON))
      NOCC=100.-NUM
      WRITE(4,*)ALTS,VOTERS
      WRITE(4,*)NONASH,NOCC,COMMON,SIN
      WRITE(4,*)EFFIC,SEFFIC,CEFFIC,SCEFFI
```

```
      IF(NONASH.GT.0)THEN
        WRITE(12,*)EELECT,SEELEC,EMAX
        WRITE(12,*)NCOND,SNCOND,NUM
      ENDIF
      M1=M1+EFFIC
      M2=M2+SEFFIC
      M3=M3+CEFFIC
      M4=M4+SCEFFI
      SD1=SD1+(EFFIC**2)
      SD2=SD2+(SEFFIC**2)
      SD3=SD3+(CEFFIC**2)
      SD4=SD4+(SCEFFI**2)
294   CONTINUE
C
C     CALCULATE STATISTICS FOR 20 REPETITIONS OF 100
C     ELECTION SIMULATION AND WRITE TO RESULT FILE
      M1=M1/20.
      M2=M2/20.
      M3=M3/20.
      M4=M4/20.
      SD1=((SD1-(20.*(M1**2)))/19.)**0.5
      SD2=((SD2-(20.*(M2**2)))/19.)**0.5
      SD3=((SD3-(20.*(M3**2)))/19.)**0.5
      SD4=((SD4-(20.*(M4**2)))/19.)**0.5
      WRITE(4,*)M1,M2,M3,M4
      WRITE(4,*)SD1,SD2,SD3,SD4
      REWIND 3
      WRITE(3,*)DSEED
295   CONTINUE
298   STOP
      END
      SUBROUTINE COUNT
      COMMON/PICK/ALTS,VOTERS,I,J,TOTAL,VOTES,LAST,
     +TVOTE,TEMPR,COMP,WINNER,TIED,G,K,TRANK,
     +SOCUT,VOTE
      INTEGER*2 ALTS,CVOTES(6),CWINNE,F,G,H,I,
     +J,K,L,LAST,M,N,NCOND,RANK(6),COMP,TOTAL(6),MONE,
     +SNCOND,STRAT,COMMON,P,SIN,Q,PVOTE(6,721),G2,
     +TEMPR,TIED(6),NONASH,TVOTE,G1(720),CHOOS,TRANK(6),
     +VOTE(6,125),VOTERS,VOTES(6),WINNER,WINS(6),VMAX
      REAL CEFFIC,EELECT,EFFIC,EMAX,EU(721),NOCC,NUM,
     +RUTIL(6),SCEFFI,SEELEC,SEFFIC,SOCUT(6),TEMPU,
     +UTIL(6,125),UTMAX,M1,M2,M3,
     +M4,SD1,SD2,SD3,SD4
      INTEGER*4 Z
      REAL*8 DSEED,X,Y
      DATA X/1.6807D4/
      Z=2147483647
      Y=DBLE(Z)
      CALL ADD
      COMP=0
      DO 23 I=2,ALTS
        IF((TOTAL(1)-TOTAL(I)).GT.2)THEN
```

For the Borda system, the difference between totals must be (ALTS-1)x2 for the totals to be comparable.

```
          COMP=I-1
          GOTO 24
        ENDIF
23    CONTINUE
      COMP=ALTS
24    G=1
      WINNER=TRANK(1)
      DO 25 I=1,COMP
        IF(TOTAL(I).EQ.TOTAL(1))THEN
          TIED(G)=TRANK(I)
          G=G+1
        ENDIF
25    CONTINUE
      DSEED=DMOD(DSEED*X,Y)
      K=INT(((SNGL(DSEED/Y))*REAL(G-1))+1.)
      WINNER=TIED(K)
      END
      SUBROUTINE ADD
      COMMON/PICK/ALTS,VOTERS,I,J,TOTAL,VOTES,LAST,
     +TVOTE,TEMPR,COMP,WINNER,TIED,G,K,TRANK,
     +SOCUT,VOTE
      INTEGER*2 ALTS,CVOTES(6),CWINNE,F,G,H,I,
     +J,K,L,LAST,M,N,NCOND,RANK(6),COMP,TOTAL(6),MONE,
     +SNCOND,STRAT,COMMON,P,SIN,Q,PVOTE(6,721),G2,
     +TEMPR,TIED(6),NONASH,TVOTE,G1(720),CHOOS,TRANK(6),
     +VOTE(6,125),VOTERS,VOTES(6),WINNER,WINS(6),VMAX
      REAL CEFFIC,EELECT,EFFIC,EMAX,EU(721),NOCC,NUM,
     +RUTIL(6),SCEFFI,SEELEC,SEFFIC,SOCUT(6),TEMPU,
     +UTIL(6,125),UTMAX,M1,M2,M3,
     +M4,SD1,SD2,SD3,SD4
      INTEGER*4 Z
      REAL*8 DSEED,X,Y
      DO 19 I=1,ALTS
        VOTES(I)=0
        DO 18 J=1,VOTERS
          VOTES(I)=VOTES(I)+VOTE(I,J)
18      CONTINUE
        TOTAL(I)=VOTES(I)
        TRANK(I)=I
19    CONTINUE
C
C     SORT TOTALS FROM HIGHEST TO LOWEST
C
      DO 22 LAST=ALTS,2,-1
        DO 21 I=1,LAST-1
          IF(TOTAL(I).LT.TOTAL(I+1))THEN
            TVOTE=TOTAL(I)
            TOTAL(I)=TOTAL(I+1)
            TOTAL(I+1)=TVOTE
            TEMPR=TRANK(I)
```

```
              TRANK(I)=TRANK(I+1)
              TRANK(I+1)=TEMPR
            ENDIF
 21       CONTINUE
 22     CONTINUE
        END
```

APPENDIX C

NUMERICAL EFFICIENCY ESTIMATES

M = number of alternatives
V = number of voters
SU = sincere social utility efficiency estimate
SSU = strategic social utility efficiency estimate
CON = sincere Condorcet efficiency estimate
SCON = strategic Condorcet efficiency estimate
STAN = standard voting system
APP = Approval voting system
BOR = Borda voting system

Table C.1 Numerical Efficiency Estimates

SYSTEM	M	V	SU	SSU	CON	SCON
STAN	3	3	75.29620	86.58792	87.62981	93.21197
STAN	3	5	76.56103	85.39255	82.70782	97.85348
STAN	3	7	77.25581	83.33194	82.69535	97.51660
STAN	3	9	73.77195	81.90794	82.55657	95.61583
STAN	3	11	75.30593	81.24983	80.56394	94.93969
STAN	3	13	77.16174	82.38519	80.80422	96.09350
STAN	3	15	75.54303	79.91310	79.67271	93.77476
STAN	3	17	74.82475	81.87458	79.45167	93.37818
STAN	3	19	75.25263	81.06884	80.78949	92.77217
STAN	3	21	74.86692	79.54131	78.96233	92.64790
STAN	3	23	74.75631	78.49162	77.83499	90.97667
STAN	3	25	76.57353	81.04321	79.02290	91.19462
STAN	3	35	77.40226	82.07282	78.56488	90.17090
STAN	3	45	75.97288	81.16663	78.21436	89.63732
STAN	3	55	77.33902	81.52497	79.50794	88.26764
STAN	3	65	76.32630	80.55056	79.27800	88.37267
STAN	3	75	74.28504	78.01983	78.06476	86.70417
STAN	3	85	75.34359	79.35842	77.75959	86.19793
STAN	3	95	76.29606	79.59635	77.73936	85.09785
STAN	3	105	74.77132	78.14500	76.90236	84.22219
STAN	3	115	73.76456	77.43318	75.86477	83.64887
STAN	3	125	76.26712	78.93581	77.13104	84.46829
STAN	4	3	72.92103	87.93572	79.24885	89.70616
STAN	4	5	72.72946	82.38886	75.83340	94.59490
STAN	4	7	72.51401	81.32061	74.56514	92.59365
STAN	4	9	71.83081	82.21461	73.29077	90.72868
STAN	4	11	70.39012	80.12018	71.34916	89.64331
STAN	4	13	71.76589	79.64956	70.98848	87.13876
STAN	4	15	71.74075	81.34573	69.41348	87.87754
STAN	4	17	71.24847	79.90104	70.07659	85.42102
STAN	4	19	70.59990	78.53442	68.90776	85.36750
STAN	4	21	70.67474	78.64796	69.48363	85.36599
STAN	4	23	67.91703	76.85156	67.86444	83.83754
STAN	4	25	71.71382	80.27880	69.98096	85.81014

Table C.1 (cont'd.)

SYSTEM	M	V	SU	SSU	CON	SCON
STAN	4	35	68.69625	76.43157	67.31848	82.23264
STAN	4	45	70.75148	77.81714	67.28976	80.31257
STAN	4	55	69.56684	76.58865	68.50451	79.62701
STAN	4	65	71.68784	76.79339	68.73268	79.57056
STAN	4	75	69.76920	75.28825	65.53801	74.82826
STAN	4	85	70.28945	77.55321	66.03845	78.15042
STAN	4	95	71.34871	77.18910	67.95308	78.11021
STAN	4	105	71.99607	77.21820	66.63617	75.37265
STAN	4	115	70.39350	75.21690	68.25966	76.30755
STAN	4	125	69.60443	75.41456	65.07824	73.55512
STAN	5	3	69.30213	85.41085	73.50789	85.04213
STAN	5	5	72.01346	81.82657	71.19073	92.17719
STAN	5	7	70.53767	80.76190	67.97408	87.65211
STAN	5	9	68.56567	78.66927	65.11762	85.37357
STAN	5	11	70.35698	79.95623	67.46361	85.56151
STAN	5	13	68.58490	79.37249	63.63131	82.46771
STAN	5	15	68.84727	79.47679	65.33905	83.63716
STAN	5	17	66.64464	76.05931	61.66896	79.90193
STAN	5	19	67.48163	77.99164	61.80851	81.43187
STAN	5	21	65.22192	77.77954	58.44027	78.62954
STAN	5	23	67.13834	76.69456	61.56055	77.48210
STAN	5	25	68.53179	78.24449	63.45115	79.38334
STAN	5	35	66.07011	75.17092	58.49493	74.21334
STAN	5	45	66.22192	75.15441	59.75353	73.99453
STAN	5	55	68.44178	76.39528	60.68997	74.27016
STAN	5	65	64.36284	72.59757	60.49124	72.53312
STAN	5	75	65.96298	74.23602	60.45285	71.83448
STAN	5	85	62.85065	72.40006	57.94659	68.78442
STAN	5	95	69.14971	74.65492	61.47445	71.10004
STAN	5	105	68.48776	74.97199	60.55389	71.30886
STAN	5	115	66.05291	72.51652	58.55390	68.91547
STAN	5	125	66.53121	72.02755	59.26700	68.81782
STAN	6	3	65.05819	84.54540	68.88606	82.76113
STAN	6	5	70.21628	80.06387	68.45763	89.08249
STAN	6	7	67.70756	80.83061	63.49053	84.24884
STAN	6	9	67.32233	78.70402	61.83347	82.75975
STAN	6	11	64.75424	76.42666	58.51207	80.27396
STAN	6	13	67.02317	78.15276	59.82922	79.05948
STAN	6	15	64.16193	75.80341	57.94467	77.83272
STAN	6	17	67.08984	77.70395	58.05382	77.74515
STAN	6	19	64.30106	74.64041	55.95757	75.27487
STAN	6	21	63.41932	75.27146	55.99650	75.10687
STAN	6	23	64.62094	75.43137	55.87289	73.15285
STAN	6	25	64.84376	74.44135	55.40077	72.46808
STAN	6	35	63.34317	73.88823	54.30782	69.83503
STAN	6	45	62.89053	72.27750	54.37664	69.03371
STAN	6	55	61.16387	71.43884	51.11140	64.65038
STAN	6	65	63.93198	72.13074	54.54405	66.66238
STAN	6	75	62.34725	71.82034	51.38808	63.97619

Table C.1 (cont'd.)

SYSTEM	M	V	SU	SSU	CON	SCON
STAN	6	85	64.83648	72.14029	55.85432	67.21089
STAN	6	95	63.16966	71.39917	53.74177	64.91936
STAN	6	105	63.03431	72.47893	53.86528	64.88135
STAN	6	115	60.85734	70.66135	51.61251	63.32732
STAN	6	125	61.06819	68.93041	49.98654	60.09759
APP	3	3	85.54782	86.50523	75.37545	93.53417
APP	3	5	86.76192	87.16843	75.53414	92.74299
APP	3	7	84.96577	84.70829	74.02934	90.76896
APP	3	9	87.39510	87.71189	76.77046	90.62442
APP	3	11	88.40237	86.92465	75.19446	89.11999
APP	3	13	86.26318	86.12803	74.48354	87.51484
APP	3	15	87.62360	85.42539	75.08113	87.04270
APP	3	17	89.33706	87.48282	76.62137	86.85290
APP	3	19	86.58625	85.28451	75.94226	86.78934
APP	3	21	87.00711	86.29616	75.62419	84.61798
APP	3	23	86.49373	86.97909	74.68584	84.64653
APP	3	25	87.61190	86.09740	74.66165	85.07454
APP	3	35	86.29799	85.18873	75.00669	83.04808
APP	3	45	88.19337	87.06731	75.55133	81.47882
APP	3	55	86.54852	85.50975	76.12923	82.50941
APP	3	65	87.47870	86.38679	75.69059	80.61050
APP	3	75	86.80450	85.96913	75.41066	79.80009
APP	3	85	87.05258	87.70838	76.62486	81.94229
APP	3	95	85.86053	85.99498	74.82373	80.49583
APP	3	105	86.16889	87.17405	73.75183	80.08118
APP	3	115	87.64536	87.66277	74.58713	79.36689
APP	3	125	88.89993	88.88324	78.55407	82.17543
APP	4	3	83.24270	89.08760	65.82455	92.18280
APP	4	5	85.19677	87.71691	68.34278	88.41451
APP	4	7	84.82350	87.40166	68.15905	86.34493
APP	4	9	87.01805	87.89871	68.87648	84.34413
APP	4	11	85.56432	87.17891	69.49039	81.98366
APP	4	13	86.51937	87.44219	70.18739	81.56656
APP	4	15	86.31638	85.52591	70.69696	80.11818
APP	4	17	87.12918	85.30611	68.47785	76.92387
APP	4	19	86.77905	85.92585	70.03425	77.08565
APP	4	21	85.68650	84.89963	69.68154	77.34931
APP	4	23	86.83046	85.53661	70.80437	78.29694
APP	4	25	84.91855	84.20312	69.64778	75.61001
APP	4	35	86.95143	85.40885	69.03116	74.25896
APP	4	45	87.99035	85.91824	71.40127	74.21697
APP	4	55	85.27197	82.39204	68.72196	73.05139
APP	4	65	87.12198	85.21752	71.57727	74.42327
APP	4	75	86.47388	84.13744	70.18837	72.60501
APP	4	85	87.37745	84.05399	69.67994	73.65379
APP	4	95	87.69239	86.21799	72.94270	75.30832
APP	4	105	87.13148	85.84409	70.48587	72.54697
APP	4	115	87.45842	85.50881	72.59964	74.55119
APP	4	125	86.22935	84.55644	72.08245	72.99924

Table C.1 (cont'd.)

SYSTEM	M	V	SU	SSU	CON	SCON
APP	5	3	81.98217	88.48648	58.54050	90.21635
APP	5	5	84.80266	88.25239	62.78744	84.92978
APP	5	7	85.55452	89.08007	64.75148	82.16798
APP	5	9	85.69252	88.67025	65.69541	80.46416
APP	5	11	84.56228	86.20818	65.71094	76.92305
APP	5	13	85.39934	85.78516	61.89796	74.00210
APP	5	15	86.01438	85.86404	67.38301	75.51271
APP	5	17	86.64363	84.50732	69.04124	75.49237
APP	5	19	87.18051	85.28253	68.34642	73.78397
APP	5	21	86.52733	85.05332	68.28285	74.61666
APP	5	23	87.18386	84.04398	67.54831	73.36091
APP	5	25	86.40400	84.41889	65.56187	70.79627
APP	5	35	85.68403	82.01510	66.98200	68.76119
APP	5	45	85.60542	83.22161	67.39540	69.20474
APP	5	55	86.34493	81.76808	66.09430	66.57378
APP	5	65	85.18947	82.42341	66.34728	66.21878
APP	5	75	87.70741	83.03763	69.31528	67.97062
APP	5	85	86.21618	83.02423	64.64047	66.07212
APP	5	95	88.14697	85.23026	69.55197	69.68063
APP	5	105	87.27399	84.10872	67.63228	67.46377
APP	5	115	86.15009	84.84105	67.29317	66.85830
APP	5	125	85.83158	83.15607	68.06752	68.39041
APP	6	3	81.4510	87.0374	55.3689	88.7413
APP	6	5	83.0145	88.7801	57.9033	83.8793
APP	6	7	84.9432	88.5049	62.6600	80.3162
APP	6	9	85.5776	88.8011	61.8705	77.7891
APP	6	11	85.2458	86.5913	63.4915	74.4312
APP	6	13	85.7690	85.6819	64.9445	73.0305
APP	6	15	87.5883	84.2656	64.7796	71.8593
APP	6	17	86.5806	84.1778	64.0682	68.8760
APP	6	19	85.9490	82.8449	65.9134	67.9790
APP	6	21	85.8315	82.3106	64.1945	68.6384
APP	6	23	86.9331	81.4839	65.4576	65.4006
APP	6	25	86.5765	82.4709	64.6519	66.4206
APP	6	35	86.8988	80.9931	66.4443	64.8411
APP	6	45	86.4856	80.7611	67.2754	66.9338
APP	6	55	87.0993	82.4859	67.0288	65.5682
APP	6	65	84.7763	81.0578	63.1868	64.1771
APP	6	75	86.7024	84.4762	66.4729	67.5858
APP	6	85	86.3331	82.7308	68.8211	67.1620
APP	6	95	86.6051	82.5778	67.3545	66.9230
APP	6	105	87.4425	86.2362	66.1508	66.4455
APP	6	115	85.3308	84.2571	65.9759	67.5199
APP	6	125	87.0038	85.0965	65.9769	68.0048
BOR	3	3	86.3415	88.3202	96.1444	93.8449
BOR	3	5	86.0486	87.1573	93.4408	93.5651
BOR	3	7	86.8037	87.6408	93.8945	92.8997
BOR	3	9	86.5210	86.2089	92.5571	91.6947
BOR	3	11	87.0156	87.5185	92.7245	91.9497

Table C.1 (cont'd.)

SYSTEM	M	V	SU	SSU	CON	SCON
BOR	3	13	85.6521	86.0514	91.0840	90.3618
BOR	3	15	86.0413	85.1241	91.4254	89.3947
BOR	3	17	86.5786	86.0729	91.4962	89.9234
BOR	3	19	87.4681	85.8277	91.0791	89.1793
BOR	3	21	86.9231	85.7577	90.8613	88.8679
BOR	3	23	86.0169	85.5753	90.4614	87.0584
BOR	3	25	85.9809	84.8370	91.0876	89.3036
BOR	3	35	86.9052	86.0876	90.8584	88.7467
BOR	3	45	85.9639	84.1891	90.9682	88.4536
BOR	3	55	87.2799	86.2050	89.4846	88.3376
BOR	3	65	85.7878	84.6726	90.2689	90.4917
BOR	3	75	86.1616	85.6402	89.1739	88.4144
BOR	3	85	85.9609	85.6280	89.6652	88.9880
BOR	3	95	85.6847	84.3600	90.0702	89.1871
BOR	3	105	87.8802	86.7592	89.7204	89.3822
BOR	3	115	85.7873	84.9270	89.8063	89.2558
BOR	3	125	87.5652	86.4840	90.8998	90.1502
BOR	4	3	90.2885	87.5803	91.0800	95.5689
BOR	4	5	88.9593	85.8698	87.7640	90.8517
BOR	4	7	89.8753	84.8071	88.8192	84.8081
BOR	4	9	89.9198	84.6571	87.7144	82.8791
BOR	4	11	88.9953	85.2660	88.8147	81.8023
BOR	4	13	89.6839	83.8101	87.0150	80.7521
BOR	4	15	89.5930	83.4478	87.9289	80.5604
BOR	4	17	89.5385	84.8454	88.0877	81.9271
BOR	4	19	89.6547	83.4629	87.7410	80.7406
BOR	4	21	88.7063	83.3710	88.5702	82.1670
BOR	4	23	88.1805	82.9895	88.7625	83.2401
BOR	4	25	88.6720	83.7541	87.7058	80.8639
BOR	4	35	88.7871	83.1956	87.1246	80.7768
BOR	4	45	88.8816	85.0088	87.8488	84.0707
BOR	4	55	89.4796	84.7656	86.7280	82.7086
BOR	4	65	89.5331	85.8105	87.5145	83.1699
BOR	4	75	89.8240	85.5151	88.3134	84.3461
BOR	4	85	90.2245	86.5894	88.0251	84.6901
BOR	4	95	88.5688	86.2847	86.0952	83.4126
BOR	4	105	88.3059	85.4874	87.2052	83.8800
BOR	4	115	89.0266	84.7904	87.8618	84.9852
BOR	4	125	90.2115	87.5790	86.7301	84.9834

Values estimated with regression coefficients:

SYSTEM	M	V	SU	SSU	CON	SCON
BOR	5	3	91.28709	93.62172	90.37112	92.07680
BOR	5	5	91.28709	90.17734	89.51751	86.85212
BOR	5	7	91.28709	88.38428	89.06592	84.12371
BOR	5	9	91.28709	87.30332	88.75550	82.47178
BOR	5	11	91.28709	86.59463	88.51114	81.38258
BOR	5	13	91.28709	86.10698	88.30303	80.62754
BOR	5	15	91.28709	85.76300	88.11699	80.08968

Table C.1 (cont'd.)

SYSTEM	M	V	SU	SSU	CON	SCON
BOR	5	17	91.28709	85.51881	87.94538	79.70265
BOR	5	19	91.28709	85.34747	87.78372	79.42577
BOR	5	21	91.28709	85.23127	87.62923	79.23223
BOR	5	23	91.28709	85.15794	87.48007	79.10351
BOR	5	25	91.28709	85.11855	87.33498	79.02609
BOR	5	35	91.28709	85.23824	86.64645	79.11763
BOR	5	45	91.28709	85.59481	85.98971	79.56725
BOR	5	55	91.28709	85.96695	85.34758	80.04041
BOR	5	65	91.28709	86.24383	84.71337	80.36958
BOR	5	75	91.28709	86.38506	84.08395	80.49369
BOR	5	85	91.28709	86.40879	83.45764	80.44016
BOR	5	95	91.28709	86.38742	82.83347	80.31847
BOR	5	105	91.28709	86.44564	82.21082	80.31709
BOR	5	115	91.28709	86.75959	81.58930	80.70229
BOR	5	125	91.28709	87.55638	80.96865	81.81739
BOR	6	3	92.58201	99.99999	92.66837	96.54270
BOR	6	5	92.58201	98.52258	91.47831	94.29146
BOR	6	7	92.58201	94.43214	90.74345	88.09014
BOR	6	9	92.58201	92.06107	90.17973	84.48798
BOR	6	11	92.58201	90.54598	89.70072	82.17977
BOR	6	13	92.58201	89.51460	89.27052	80.60278
BOR	6	15	92.58201	88.78283	88.87129	79.47870
BOR	6	17	92.58201	88.24993	88.49310	78.65525
BOR	6	19	92.58201	87.85621	88.12994	78.04219
BOR	6	21	92.58201	87.56397	87.77792	77.58254
BOR	6	23	92.58201	87.34811	87.43443	77.23834
BOR	6	25	92.58201	87.19115	87.09765	76.98319
BOR	6	35	92.58201	86.93927	85.47752	76.51305
BOR	6	45	92.58201	87.09959	83.91613	76.66600
BOR	6	55	92.58201	87.35057	82.38425	76.95602
BOR	6	65	92.58201	87.54510	80.86955	77.16069
BOR	6	75	92.58201	87.62660	79.36590	77.19451
BOR	6	85	92.58201	87.60497	77.86980	77.07243
BOR	6	95	92.58201	87.54790	76.37914	76.89675
BOR	6	105	92.58201	87.57726	74.89255	76.85175
BOR	6	115	92.58201	87.86737	73.40907	77.20091
BOR	6	125	92.58201	88.64410	71.92808	78.28568

APPENDIX D

REGRESSION RESULTS

Equation 1: SU = C(1)*THEO + C(2)*DIF + C(3)*MEAN + C(4)*VAR

Table D.1. Sincere Social Utility Efficiency Regression Results

Standard Voting System

	Coefficient	Std. Error	T-Stat.
C(1)	0.9660104	0.0134421	71.8642880
C(2)	12.5755110	2.0373614	6.1724490
C(3)	-0.0908287	0.0509604	-1.7823393
C(4)	0.1744424	0.0523831	3.3301283

R^2	0.862424
Standard Error of Regression	1.717966
Sum of Squared Residuals	247.9183

Approval Voting System

	Coefficient	Std. Error	T-Stat.
C(1)	0.9834290	0.0049852	197.2711100
C(2)	-14.4611890	2.6824025	-5.3911334
C(3)	3.6120629	0.5574343	6.4797997
C(4)	-3.6261399	0.5612308	-6.4610496

R^2	0.432635
Standard Error of Regression	0.995403
Sum of Squared Residuals	83.22952

Borda Voting System

	Coefficient	Std. Error	T-Stat.
C(1)	0.9988894	0.0028917	345.4374500
C(2)	-1.4911846	5.7122250	-0.2610514
C(3)	0.0056901	0.0148756	0.3825100
C(4)	-0.0036556	0.0094704	-0.3860020

R^2	0.834664
Standard Error of Regression	0.663750
Sum of Squared Residuals	17.62255

Equation 2: CC = C(1)*THEO + C(2)*DIF + C(3)*MEAN + C(4)*VAR

Table D.2. Sincere Condorcet Efficiency Regression Results

Standard Voting System

	Coefficient	Std. Error	T-Stat.
C(1)	0.9347468	0.0225973	41.3654810
C(2)	17.4107530	3.4249585	5.0848752
C(3)	0.7782854	0.0856683	9.0848752
C(4)	-0.8640046	0.0880599	-9.8115514

R^2	0.908270
Standard Error of Regression	2.888031
Sum of Squared Residuals	700.6208

Approval Voting System

	Coefficient	Std. Error	T-Stat.
C(1)	0.6636912	0.0092258	71.9385700
C(2)	-18.8665650	4.9641934	-3.8005298
C(3)	20.5012300	1.0316170	19.8729090
C(4)	-20.4961870	1.0386428	-19.7336240

R^2	0.854252
Standard Error of Regression	1.842145
Sum of Squared Residuals	285.0538

Borda Voting System

	Coefficient	Std. Error	T-Stat.
C(1)	0.9702868	0.0032940	294.5577000
C(2)	136.2546000	6.5070940	20.9393930
C(3)	0.1357765	0.0169456	8.0125011
C(4)	-0.0832901	0.0107883	-7.7204378

R^2	0.887074
Standard Error of Regression	0.756112
Sum of Squared Residuals	22.86822

Equation 3: SSU = C(1)*THEO + C(2)*DIF + C(3)/P + C(4)*(Q^(ALTS/(2*(ALTS+V)))) + C(5)*V + C(6)*V^2 + C(7)*V^3 + C(8)*V^4

Table D.3. Strategic Social Utility Efficiency Regression Results

Standard Voting System

	Coefficient	Std. Error	T-Stat.
C(1)	0.5207019	0.0542326	9.6012750
C(2)	18.1723560	17.5191300	1.0372864
C(3)	22.8116840	4.8082186	4.7443110
C(4)	15.1048090	2.0875383	7.2357038
C(5)	0.0750898	0.1612042	0.4658056
C(6)	-0.0026245	0.0039308	-0.6676666
C(7)	3.237D-05	4.058D-05	0.7978145
C(8)	-1.343D-07	1.459D-07	-0.9206889

R^2	0.921995
Standard Error of Regression	1.116703
Sum of Squared Residuals	99.76201

Approval Voting System

	Coefficient	Std. Error	T-Stat.
C(1)	-1.4149151	0.2701843	-5.2368524
C(2)	92.3039750	10.0521490	9.1825119
C(3)	169.3911200	19.1548330	8.8432624
C(4)	1.3284882	0.5041483	2.6351140
C(5)	-0.2029406	0.1278940	-1.5867872
C(6)	0.0051127	0.0033746	1.5154758
C(7)	-3.835D-05	3.624D-05	-1.0583883
C(8)	9.423D-08	1.333D-07	0.7066768

R^2	0.741881
Standard Error of Regression	1.097526
Sum of Squared Residuals	96.36510

Borda Voting System

	Coefficient	Std. Error	T-Stat.
C(1)	0.7174308	0.0998939	7.1819298
C(2)	262.6485400	54.7449520	4.7976760
C(3)	15.0476340	10.2901170	1.4623384
C(4)	2.6690842	1.3929331	1.9161611
C(5)	-0.0878516	0.1415911	-0.6204598
C(6)	0.0044036	0.0036143	1.2183975
C(7)	-5.407D-05	3.806D-05	-1.4209206
C(8)	2.095D-07	1.383D-07	1.5148517

R^2	0.709214
Standard Error of Regression	0.766167
Sum of Squared Residuals	21.13243

Equation 4: SCC = C(1) + C(2)*SSU + C(3)*ALTS + C(4)*V

See Chapter 4, page 112.

APPENDIX E

NOTES TO TEXT

Introduction and Chapter 1

1Black [7], p. 180.
2Condorcet, in Rosenstein [118], p. 36.
3Condorcet, in Rosenstein [118], p. 46-47.
4Condorcet, in Rosenstein [118], p. 53.
5Condorcet, in Rosenstein [118], p. 51.
6Condorcet, in Rosenstein [118], p. 56.
7Zeckhauser [145], p. 935.
8Gibbard [58], p. 587.
9Gibbard [58], p. 595.
10Postlewaite and Schmeidler [109], p. 37.
11Postlewaite and Schmeidler [109], p. 38.
12Postlewaite and Schmeidler [109], p. 37.
13Dasgupta, Hammond, and Maskin [33], p. 186.
14Dasgupta, Hammond, and Maskin [33], p. 186.
15Dasgupta, Hammond, and Maskin [33], p. 188.
16Dasgupta, Hammond, and Maskin [33], p. 189.
17Black [7], p. 188.
18Black [7], p. 188.
19Black [7], p. 7.
20Enelow and Hinich [40], p. 16.
21Enelow and Hinich [40], p. 30.
22Enelow and Hinich [40], p. 30.
23Enelow and Hinich [40], pp. 30-31.
24Riker and Ordeshook [114], pp. 25-26.
25Palfrey and Rosenthal [102], p. 9.
26Ferejohn and Fiorina [42], p. 527.
27Palfrey and Rosenthal [102], p. 9.
28Palfrey and Rosenthal [102], p. 10.
29Palfrey and Rosenthal [102], p. 8.
30Palfrey and Rosenthal [102], p. 8.
31Data are taken from the Statistical Abstract of the United States, 1982-83, 103rd Edition, U.S. Bureau of the Census, 1982.
32Intriligator [79], p. 553.
33Farquaharson [41], p. 24.
34Farquaharson [41], p. 24.
35Farquaharson [41], p. 25.
36Enelow and Koehler [39], p. 399.
37Enelow [38], p. 1062.
38Enelow [38], pp. 1088-1089.

Chapter 2

1Arrow [1], pp. 94-95.
2Weber [143], pp. 7-8.
3Weber [143], pp. 9-11.
4Postlewaite and Schmeidler [109], p. 38.

[5]Fishburn and Gehrlein [55], p. 143.
[6]Fishburn and Gehrlein [55], p. 149.
[7]Fishburn and Gehrlein [55], p. 149.
[8]Fishburn and Gehrlein [55], p. 151.
[9]Fishburn and Gehrlein [55], p. 151.
[10]Bordley [12], p. 129.

Chapter 3

[1]Merrill [92], p. 119.
[2]Merrill [92], p. 119.
[3]Shubik [130], p. 136.
[4]Selten [123], p. 35.

Chapter 5

[1]Boadway, R.W., Public Sector Economics, Little, Brown, and Co., 1979, pp. 71-72.
[2]Riker and Ordeshook [114], pp. 25-26.
[3]McKelvey, Ordeshook, and Winer [90], p. 599.
[4]McKelvey, Ordeshook, and Winer [90], p. 605.
[5]McKelvey, Ordeshook, and Winer [90], p. 602.
[6]McKelvey, Ordeshook, and Winer [90], p. 606.

LIST OF REFERENCES

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[1] Arrow, K.J., Social Choice and Individual Values, 2nd edition, John Wiley & Sons, Inc., 1963.

[2] Arrow, K.J. and H. Raynaud, Social Choice and Multicriterion Decision-Making, MIT Press, 1986.

[3] Barzel, Y. and E. Silberberg, "Is the Act of Voting Rational?" Public Choice 16:51-58 (1973).

[4] Beck, N., "A Note on the Probability of a Tied Election," Public Choice 23:75-79, 1975.

[5] Bensel, R.F. and M.E. Sanders, "The Effect of Electoral Rules on Voting Behavior: The Electoral College and Shift Voting," Public Choice 34:69-85 (1979).

[6] Black, "On the Rationale of Group Decision Making," Journal of Political Economy 56:23-34 (1948).

[7] Black, D., The Theory of Committees and Elections, Cambridge University Press (1963).

[8] Black, G.S., "Conflict in the Community: A Theory of the Effects of Community Size," American Political Science Review 68:1245-1261 (1974).

[9] Black, J.H., "The Probability-Choice Perspective in Voter Decision-Making Models," Public Choice 35:565-574 (1980).

[10] Blair, D. and E. Muller, "Essential Aggregation Procedures on Restricted Domains," Journal of Economic Theory 30:34-53 (1983).

[11] de Borda, "Memoire sur les elections au scrutin," Memoires de l'Academie Royale de Sciences, 1781.

[12] Bordley, R.F., "A Pragmatic Method for Evaluating Election Schemes through Simulation," American Political Science Review 77:123-141 (1983).

[13] Bordley, R.F., "Using Factions to Estimate Preference Intensity: Improving Upon One Person/One Vote," Public Choice 45:257-268 (1985).

[14] Bordley, R.F., "A Precise Method for Evaluating Election Schemes," Public Choice 46:113-123 (1985).

[15] Bowen, H.R., "The Interpretation of Voting in the Allocation of Economic Resources," Quarterly Journal of Economics 58:27-48 (1943).

[16] Brams, S.J., Game Theory and Politics, The Free Press (Macmillan), 1975.

[17] Brams, S.J., "One Man, N Votes," MAA Modules in Applied Mathematics, Mathematical Association of America, 1976.

[18] Brams, S.J. and P.C. Fishburn, Approval Voting, Birkhauser, 1982.

[19] Brown, K.M. and C.E. Zech, "Welfare Effects of Announcing Election Forecasts," Public Choice 21:117-123 (1975).

[20] Buchanan, J.M. and G. Tullock, The Calculus of Consent, University of Michigan Press, 1962.

[21] Chamberlin, J., "Discovering Manipulated Social Choices: The Coincidence of Cycles and Manipulated Outcomes," Public Choice 51:295-313 (1986).

[22] Chamberlin, J. and M. Cohen, "Toward Applicable Social Choice Theory: A Comparison of Social Choice Functions Under Spatial Model Assumptions," American Political Science Review 72(4):1341-1356 (1978).

[23] Champsaur, P. and G. Laroque, "Strategic Behavior in Decentralized Planning Procedures," Econometrica 50(2):325-344 (1982).

[24] Clarke, E., "Multipart Pricing of Public Goods," Public Choice 11:17-33 (1971).

[25] Coleman, J.S., "The Possibility of a Social Welfare Function," American Economic Review 56:1105-1122 (1966).

[26] de Condorcet, "Essai sur l'application de l'analyse a la probabilite de decisions rendues a la pluralite de voix," Paris, 1785.

[27] Coughlin, J., "Special Majority Rules and the Existence of Voting Equilibria," Social Choice and Welfare 3(1):31-36 (1986).

[28] Coughlin, P., "Pareto Optimality of Policy Proposals with Probabilistic Voting," Public Choice 39:427-433 (1982).

[29] Coughlin, P., "Davis-Hinich Conditions and Median

Outcomes in Probabilistic Voting Models," Journal of Economic Theory 34:1-12 (1984).

[30] Coughlin, P. and M. Hinich, "Necessary and Sufficient Conditions for Single-Peakedness in Public Economic Models," Journal of Public Economics 25:161-179 (1984).

[31] Coughlin, P. and S. Nitzan, "Electoral Outcomes with Probabilistic Voting and Nash Social Welfare Maxima," Journal of Public Economics 15:113-121 (1981).

[32] Coughlin, P. and S. Nitzan, "Directional and Local Electoral Equilibria with Probabilistic Voting," Journal of Economic Theory 24:226-239 (1981).

[33] Dasgupta, P., P. Hammond, and E. Maskin, "The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility," Review of Economic Studies 46:185-216 (1979).

[34] Davis, O.A., M.J. Hinich, and P.C. Ordeshook, "An Expository Development of a Mathematical Model of the Electoral Process," American Political Science Review 64:426-448 (1970).

[35] Denzau, A.T. and A. Kats, "Expected Plurality Voting Equilibrium and Social Choice Functions," Review of Economic Studies 44:227-233 (1977).

[36] Denzau, A.T. and R. Mackay, "Structure-Induced Equilibria and Perfect-Foresight Expectations," American Journal of Political Science 25:762-779 (1981).

[37] Downs, A., An Economic Theory of Democracy, Harper & Bros., 1957.

[38] Enelow, J.M., "Saving Amendments, Killer Amendments, and an Expected Utility Theory of Sophisticated Voting," Journal of Politics 43:1062-1089 (1981).

[39] Enelow, J.M. and D.H. Koehler, "The Amendment in Legislative Strategy: Sophisticated Voting in the U.S. Congress," Journal of Politics 42:396-413 (1980).

[40] Enelow, J.M. and M.J. Hinich, The Spatial Theory of Voting, Cambridge University Press, 1984.

[41] Farquaharson, R., Theory of Voting, Yale University Press, 1969.

[42] Ferejohn, J.A. and M.P. Fiorina, "The Paradox of Not Voting: A Decision Theoretic Analysis," American Political Science Review 68:525-536 (1974).

[43] Ferejohn, J.A., D.M. Grether, and R. McKelvey, "Implementation of Democratic Social Choice Functions," Review of Economic Studies 49:439-446 (1982).

[44] Filer, J.E. and L.W. Kenny, "Voter Turnout and the Benefits of Voting," Public Choice 35:575-585 (1980).

[45] Fishburn, P.C., "A Comparative Analysis of Group Decision Methods," Behavioral Science 16:538-544 (1971).

[46] Fishburn, P.C., "Lotteries and Social Choices," Journal of Economic Theory 5:189-207 (1972).

[47] Fishburn, P.C., "Even-Chance Lotteries in Social Choice Theory," Theory and Decision 3:18-40 (1972).

[48] Fishburn, P.C., "Voter Concordance, Simple Majorities, and Group Decision Methods," Behavioral Science 18:364-376 (1973).

[49] Fishburn, P.C., "Simple Voting Systems and Majority Rule," Behavioral Science 19:166-176 (1974).

[50] Fishburn, P.C. and S.J. Brams, "Efficacy, Power, and Equity under Approval Voting," Public Choice 37:425-434 (1981).

[51] Fishburn, P.C. and S.J. Brams, "Expected Utility and Approval Voting," Behavioral Science 26:136-142 (1981).

[52] Fishburn, P.C. and S.J. Brams, "Approval Voting, Condorcet's Principle, and Runoff Elections," Public Choice 36:89-114 (1981).

[53] Fishburn, P.C. and W.V. Gehrlein, "An Analysis of Simple Two-Stage Voting Systems," Behavioral Science 21:1-12 (1976).

[54] Fishburn, P.C. and W.V. Gehrlein, "An Analysis of Voting Procedures with Nonranked Voting," Behavioral Science 22:178-185 (1977).

[55] Fishburn, P.C. and W.V. Gehrlein, "Majority Efficiencies for Simple Voting Procedures: Summary and Interpretation," Theory and Decision 14:141-153 (1982).

[56] Gehrlein, W.V. and P.C. Fishburn, "The Effects of Abstentions on Election Outcomes," Public Choice 33(2):69-82 (1978).

[57] Gehrlein, W.V. and P.C. Fishburn, "Effects of Abstentions on Voting Procedures in Three-Candidate Elections," Behavioral Science 24:346-354 (1979).

[58] Gibbard, A., "Manipulation of Voting Schemes: A General Result," Econometrica 41(4):587-601 (1973).

[59] Gibbard, A., "Manipulation Schemes that Mix Voting with Chance," Econometrica 45:665-681 (1977).

[60] Gibbard, A., "Straightforwardness of Game Forms with Lotteries as Outcomes," Econometrica 46:595-614 (1978).

[61] Green, J.R. and J.-J. Laffont, Incentives in Public Decision-Making, North-Holland, 1977.

[62] Grether, D.M. and C. Plott, "Nonbinary Social Choice: An Impossibility Theorem," Review of Economic Studies 49:143-149 (1982).

[63] Grofman, B., "Models of Voter Turnout: A Brief Idiosyncratic Review," Public Choice 41:55-61 (1983).

[64] Groves, T. and J. Ledyard, "Optimal Allocation of Public Goods: A Solution to the 'Free Rider' Problem," Econometrica 45:783-809 (1977).

[65] Groves, T. and M. Loeb, "Incentives and Public Inputs," Journal of Public Economics 4:211-226 (1975).

[66] Hammond, P.J., "Straightforward Individual Incentive Compatibility in Large Economies," Review of Economic Studies 46:263-282 (1979).

[67] Harsanyi, J.C., Rational Behavior and Bargaining Equilibrium in Games and Social Situations, Cambridge University Press, 1977.

[68] Hinich, M.J., "Equilibrium in Spatial Voting: The Median Voter Result is an Artifact," Journal of Economic Theory 16:208-219 (1977).

[69] Hinich, M.J., J. Ledyard and P.C. Ordeshook, "Nonvoting and the Existence of Equilibrium under Majority Rule," Journal of Economic Theory 4:144-153 (1972).

[70] Hinich, M.J. and P.C. Ordeshook, "Plurality Maximization vs. Vote Maximization: A Spatial Analysis with Variable Participation," American Political Science Review 64:772-791 (1970).

[71] Hoel, Port, and Stone, Introduction to Probability Theory, Houghton Mifflin, 1971.

[72] Hoel, Port, and Stone, Introduction to Statistical Theory, Houghton Mifflin, 1971.

[73] Hogg, R.V. and A.E. Tanis, Probability and Statistical Inference, 2nd edition, Macmillan Publishing Co., 1983.

[74] Hotelling , H., "Stability in Competition," Journal of Political Economy 39:41-57 (1929).

[75] Hume, J.N.P. and R.C. Holt, Fortran 77 for Scientists and Engineers, Reston Publishing Company, 1985.

[76] Hurwicz, L., "On Allocations Attainable Through Nash Equilibria," Journal of Economic Theory 21:140-165 (1979).

[77] Hurwicz, L., "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," Review of Economic Studies 46:217-225 (1979).

[78] Hurwicz, L. and D. Schmeidler, "Construction of Outcome Functions Guaranteeing Existence and Pareto Optimality of Nash Equilibria," Econometrica 46:1447-1474 (1978).

[79] Intriligator, M., "A Probabilistic Model of Social Choice," Review of Economic Studies 40:553-560 (1973).

[80] Kau, J.B. and P.H. Rubin, "The Electoral College and the Rational Vote," Public Choice 27:101-107 (1976).

[81] Kelly, J.S., "Voting Anomalies, the Number of Voters, and the Number of Alternatives," Econometrica 42:239-251 (1974).

[82] Kelly, J.S., "Profile Restrictions and Strategy-Proofness," Social Choice and Welfare 4:63-67 (1987).

[83] Kramer, G.H., "Sophisticated Voting over Multi-Dimensional Choice Spaces," Journal of Mathematical Sociology 2:165-181 (1972).

[84] Kramer, G.H., "On a Class of Equilibrium Conditions for Majority Rule," Econometrica 41:285-297 (1973).

[85] Ledyard, J., "The Paradox of Voting and Candidate Competition: A General Equilibrium Analysis," in G. Horwich and J. Quirk, eds., Essays in Contemporary Fields of Economics, Purdue University Press, 1981.

[86] Ledyard, J., "The Pure Theory of Large Two-Candidate Elections," Public Choice 44:7-41 (1984).

[87] Little and Fishburn, "TIMS Tests Voting Method," in OR/MS Today 13(5):14-15 (1986).

[88] McKelvey, R.D., "Policy Related Voting and Electoral Equilibrium," Econometrica 43:815-843 (1975).

[89] McKelvey, R.D. and R.G. Niemi, "A Multistage Game

Representation of Sophisticated Voting for Binary Procedures," Journal of Economic Theory 18:1-22 (1978).

[90] McKelvey, R.D., P.C. Ordeshook, and M.D. Winer, "The Competitve Solution for N-Person Games Without Transferable Utility, With an Application to Committee Games," American Political Science Review 72:599-615 (1978).

[91] May, K.O., "A Set of Necessary and Sufficient Conditions for Simple Majority Decision," Econometrica 20:680-684 (1952).

[92] Merrill, S. III, "Strategic Decisions Under One-Stage Multi-Candidate Voting Systems," Public Choice 36:115-134 (1981).

[93] Merrill, S. III, "A Comparison of Efficiency of Multicandidate Electoral Systems," American Journal of Political Science 28:23-48 (1984).

[94] Merrill, S. III, "A Statistical Model for Condorcet Efficiency Based on Simulation under Spatial Model Assumptions," Public Choice 47:389-403 (1985).

[95] Morton, R.B., "A Group Majority Voting Model of Public Good Provision," Social Choice and Welfare 4:117-131 (1987).

[96] Mueller, D., "Public Choice: A Survey," Journal of Economic Literature 14:395-433 (1976).

[97] Niemi, R.G., "The Problem of Strategic Behavior under Approval Voting," American Political Science Review 78(4):952-958 (1984).

[98] Niemi, R.G., "Costs of Voting and Nonvoting," Public Choice 8:115-119 (1970).

[99] Niemi, R.G. and H.F. Weisberg, "A Mathematical Solution for the Probability of the Paradox of Voting," Behavioral Science 13:317-323 (1968).

[100] Niemi, R.G. and H.F. Weisberg, eds., Probability Models of Collective Decision Making, Charles E. Merrill Pub. Co., 1972.

[101] Nurmi, H., "Majority Rule: Second Thoughts and Refutations," Quality and Quantity 14:743-765 (1980).

[102] Palfrey, T.R. and H. Rosenthal, "A Strategic Calculus of Voting," Public Choice 41:7-53 (1983).

[103] Palfrey, T.R. and H. Rosenthal, "Participation and Provision of Discrete Public Goods: A Strategic

Analysis," Journal of Public Economics 24:171-193 (1984).

[104] Palfrey, T.R. and H. Rosenthal, "Voter Participation and Strategic Uncertainty," American Political Science Review 79:62-78 (1985).

[105] Payne, J.A., Introduction to Simulation: Programming Techniques and Methods of Analysis, McGraw-Hill, 1982.

[106] Peleg, B., Game Theoretic Analysis of Voting in Committees, Cambridge University Press, 1984.

[107] Plott, C., "A Notion of Equilibrium and Its Possibility under Majority Rule," American Economic Review 57:787-806 (1967).

[108] Plott, C., "Axiomatic Social Choice Theory: An Overview and Interpretation," American Journal of Political Science 20(3):511-596 (1976).

[109] Postlewaite, A. and D. Schmeidler, "Strategic Behaviour and a Notion of Ex Ante Efficiency in a Voting Model," Social Choice and Welfare 3(1):37-50 (1986).

[110] Rae, D.W., "Decision Rules and Individual Values in Constitutional Choice," American Political Science Review 63:40-56 (1969).

[111] Rae, D.W., The Political Consequences of Electoral Laws, Yale University Press, 1967 (1971).

[112] Rawls, J., A Theory of Justice, Harvard University Press, 1971.

[113] Richelson, J., "A Comparative Analysis of Social Choice Functions," Behavioral Science 20:331-337 (1975).

[114] Riker, W.H. and P.C. Ordeshook, "A Theory of the Calculus of Voting," American Political Science Review 62:25-42 (1968).

[115] Riker, W.H. and W.J. Zavoina, "Rational Behavior in Politics: Evidence from a Three Person Game," American Political Science Review 64:48-60 (1970).

[116] Roberts, J. and A. Postlewaite, "The Incentives for Price-Taking Behaviour in Large Exchange Economies," Econometrica 44:115-127 (1976).

[117] Roberts, K.W., "Interpersonal Comparability and Social Choice Theory," Review of Economic Studies 47:421-439 (1980).

[118] Rosenstein, Leonora Cohen, ed., Condorcet Studies I, Universal Press, 1982.

[119] Samuelson, P.A., "The Pure Theory of Public Expenditures," Review of Economic Studies 36:378-389 (1954).

[120] Satterthwaite, M.A., "Strategy-Proofness and Arrow's Conditions: Existence and Corresponding Theorems for Voting Procedures and Social Welfare Functions," Journal of Economic Theory 10:187-217 (1975).

[121] Satterthwaite, M.A. and H. Sonnenschein, "Strategy-Proof Allocation Mechanisms at Differentiable Points," Review of Economic Studies 68:587-597 (1981).

[122] Schofield, N., "Instability of Simple Dynamic Games," Review of Economic Studies 45:575-594 (1978).

[123] Selten, R., "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," International Journal of Game Theory 4:25-55 (1975).

[124] Sen, A., "A Possibility Theorem on Majority Decisions," Econometrica 34:491-499 (1966).

[125] Sen, A., Collective Choice and Social Welfare, Holden-Day, 1969.

[126] Sen, A., "Strategies and Revelation: Informational Constraints in Public Decisions," in Aggregation and Revelation of Preferences, J.-J. Laffont, ed., North-Holland, 1979.

[127] Sen, A. and P. Pattanaik, "Necessary and Sufficient Conditions for Rational Choice under Majority Decision," Journal of Economic Theory 1:178-202 (1969).

[128] Shapiro, M.J., "Rational Political Man: A Synthesis of Economic and Social-Psychological Perspectives," American Political Science Review 63:1106-1119 (1969).

[129] Shepsle, K.A., "The Strategy of Ambiguity: Uncertainty and Electoral Competition," American Political Science Review 66:555-568 (1972).

[130] Shubik, M., Game Theory in the Social Sciences, MIT Press, 1982.

[131] Shubik, M., A Game Theoretic Approach to Political Economy, MIT Press, 1984.

[132] Silberman, J. and G. Durden, "The Rational Behavior Theory of Voter Participation: The Evidence from

Congressional Elections," Public Choice 23:101-108 (1975).

[133] Slutsky, S., "A Voting Model for the Allocation of Public Goods: Existence of Equilibrium," Journal of Economic Theory 14:299-325 (1977).

[134] Smith, J.W., "A Clear Test of Rational Voting," Public Choice 23:55-67 (1975).

[135] Smithies, A., "Optimal Location in Spatial Competition," Journal of Political Economy 49:423-439 (1941).

[136] Stratmann, W.C., "The Calculus of Rational Choice," Public Choice 17:93-105 (1974).

[137] Tideman, T. and G. Tullock, "A New Superior Principle for Collective Choice," Journal of Political Economy 84:1145-1159 (1976).

[138] Tollison, R., M. Crain and P. Pautler, "Information and Voting: An Empirical Note," Public Choice 24:43-49 (1975).

[139] Tollison, R. and T.D. Willett, "Some Simple Economics of Voting and Not Voting," Public Choice 16:59-71 (1973).

[140] Tresch, R.W., Public Finance: A Normative Theory, Business Publications, Inc., 1981.

[141] Tullock, G., Towards a Mathematics of Politics, University of Michigan Press, 1968.

[142] Tullock, G., "Computer Simulation of a Small Voting System," Economic Journal March 1970, pp. 97-104.

[143] Weber, R.J., "Comparison of Voting Systems," "Multiply-Weighted Voting Systems," and "Reproducing Voting Systems," Cowles Foundation Discussion Paper No. 498 (1978).

[144] Zeckhauser, R., "Majority Rule with Lotteries on Alternatives," Quarterly Journal of Economics 83:696-703 (1969).

[145] Zeckhauser. R., "Voting Systems, Honest Preferences, and Pareto Optimality," American Political Science Review 67:934-946 (1973).