THE DEMAND FOR "FREE" GOODS

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ABSTRACT

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A significant amount of government goods and services are provided "free" to their recipients. However, the demand in many cases has not equaled the expectation of agency forecasts. This thesis examines the determinants of the demand for such "free" goods offered to clients or recipients at an explicit cost of zero.

The model considers the consumption of goods by an individual to be an activity produced by combining physical and time inputs. Therefore, all "free" goods have at least an implicit cost in the form of consumption time which must be allocated from alternative uses. In addition, all forms of forfeited income, such as welfare or unemployment compensation, are included as costs to the individual. Thus, in the specification of the model, all parameters influencing the choice of the individual recipient are included.

The model was then used to establish explicitly the individual's demand relationship for "free" vocational rehabilitation services and his supply relationship of time. These relationships were then estimated using cross sectional data from the Division of Vocational Rehabilitation of the Michigan Department of Education. The statistical results generated by the application suggest that the variables included in the model are important.

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This thesis is dedicated
to my parents William and Bertha Reifel
for the opportunities they have given me.

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I assume full responsibility for any errors or omissions.

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CHAPTER I

INTRODUCTION

The decade of the 1960's witnessed a significant growth in the amount of government goods and services provided "free" to their recipients. Rapid funding increases for established social welfare programs such as public assistance, education, food stamps, and vocational rehabilitation, combined with the inception of new antipoverty programs such as job training, community action, and VISTA, propelled the percentage of all government spending allocated for social welfare programs to 47% by 1970.[1]

However, demand for "free" goods in many cases has not equalled the expectation of agency forecasts.[2] Typically, the cause of these discrepancies is the manner

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[1]Alfred M. Skolnik and Sophie R. Dales, "Social Welfare Expenditures, 1969-70," Social Security Bulletin, XXXIII (December, 1970), 8-11.

[2]See, for example, Marian Gornick, "Use of Medical Services as Demanded by the Urban Poor," American Journal of Public Health, LIX (August, 1969), 1304; Mark Lepper, "Approaches to Meeting Health Needs of Large Poverty Populations," American Journal of Public Health, LVII (July, 1967), 1154-55; and U.S., Department of Labor, Manpower Administration, Manpower Report of the President--January 1969 (Washington: Government Printing Office, 1969), p. 201.

in which agency forecasts are made, usually being set at the agency's estimation of the population's "need" for such goods and services. "Need" is defined as the difference between the population's present consumption and some arbitrary standard deemed optimal by the agency but having no economic basis.[3]

The consequence of this procedure for forecasting "demand" is that forecasts rarely coincide with effective demand. The "need" for the goods and services supplied by a government agency is economically relevant only if expressed by effective demand. In the case of government programs where effective demand is less than agency forecasts, the implication for resource allocation is that, if agencies are funded to the level of expected demand and such demand does not materialize, resources will be wasted.

To overcome this difficulty, it is necessary to introduce economic considerations into the procedure for forecasting demand. Consumption of "free" goods and services, though not requiring explicit money expenditures by the recipient, will require use of his scarce time that

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[3]Bibliographies of journal articles estimating the "need" for health service programs are found in Abstracts of Hospital Management Studies (Ann Arbor: The Cooperative Information Center for Hospital Management Studies) indexed under the heading "Health Services: Use, Need, and Demand" and in Hospital Literature Index (Chicago: American Hospital Association) under "Hospital and Health Facilities: Need." Estimates of "needs" for manpower training programs can be found in Manpower Report of the President (Washington: Government Printing Office).

could be allocated to other activities. This generates an implicit opportunity cost to the recipient and demand considerations immediately arise. Any forecast procedure that ignores these economic considerations will be handicapped in achieving results that approximate effective demand. Therefore, construction of a rational basis for forecasting demand, consistent with economic theory, is required.

The purpose of this dissertation is to facilitate the inclusion of economic considerations in demand forecasts by developing a comprehensive model of consumer behavior incorporating "free" goods and services. The specification of the model will include all parameters influencing the individual's choices. The model will then be employed to estimate the demand for the "free" good vocational rehabilitation services supplied by the Department of Education in Michigan.

Several benefit-cost analyses of the "free" good vocational rehabilitation services have already been made, but the optimal social allocation of resources approach has always been taken.[4] This dissertation will view the

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[4]See, for example, Ronald W. Conley, "A Benefit-Cost Analysis of the Vocational Rehabilitation Program," The Journal of Human Resources, IV (Spring, 1969), 226-52; Ronald W. Conley, The Economics of Vocational Rehabilitation (Baltimore: Johns Hopkins Press, 1965); U.S. Department of Health, Education, and Welfare, An Exploratory Cost-Benefits Analysis of Vocational Rehabilitation

rehabilitation process from both the client's demand for services viewpoint and from the client's supply of time viewpoint.

Knowledge of the magnitude and significance of the parameters in the model, combined with a knowledge of the characteristics of the population to be served, will allow development of "free" good demand forecasts having a rational economic basis. Also, knowledge about the model parameters will suggest ways by which agency officials can choose magnitudes of discretionary variables so as to either increase or decrease effective demand.

Preliminary considerations and the development of the comprehensive model of consumer equilibrium will be handled in Chapter II. In Chapter III, the model will be applied to the "free" good vocational rehabilitation services and estimated using data from the Department of Education of the State of Michigan. Conclusions and implications of the analysis will be made in Chapter IV.

(Washington: Government Printing Office, 1967); and U.S. Department of Health, Education, and Welfare, Human Investment Programs: Vocational Rehabilitation (Washington: Government Printing Office, 1967).

CHAPTER II

THE ANALYTICAL FRAMEWORK

Preliminary Formal Considerations

"Free" goods are goods or services supplied by a dispensing agency to the consumer at a zero money price. The spectrum of such "free" goods being considered ranges from "free" money (welfare payments) to "free" services (inoculations, dental care, birth control clinics, vocational rehabilitation). Some "free" goods, such as food and welfare payments, are used up in the immediate market period and thus generate benefits only in that period. Other "free" goods, such as inoculations and vocational rehabilitation services, are human capital-enhancing and hence generate benefits for longer periods of time.

Because of their nature, analysis of the demand for "free" goods necessitates the use of a more comprehensive model than the traditional static consumer equilibrium model which envisions the individual allocating a fixed income to consume that combination of market goods and services that maximizes his utility. In this simple model, the individual is constrained in his acquisition of goods solely by his budget (his total money income),

which is assumed constant. This is reflected in the first-order equilibrium conditions for optimal consumption which state that the individual's equilibrium consumption is such that the marginal utility of each good consumed is equal to the product of its marginal cost and the marginal utility of money.[1] The application of this analysis to "free" goods is inappropriate because their explicit money price is zero and the income constraint is irrelevant. The equilibrium consumption of a zero priced good is independent of the income constraint.

While the individual's time constraint is not considered in the traditional static model, consumption of a "free" good, or any good, requires the use of a portion of the individual's limited time. In consuming a "free" good, the individual pays an opportunity cost in terms of the time required which must be allocated from alternative uses. Therefore, considerations of "free" good consumption require the development of a fully comprehensive model of consumer choice which includes the crucial implicit cost of time and the total time constraint.

Furthermore, some "free" goods are human capital-enhancing commodities, such as scholarships or vocational rehabilitation. That is, their consumption is an

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[1]Paul A. Samuelson, Foundations of Economic Analysis, Harvard Economic Series, Vol. LXXX (Cambridge: Harvard University Press, 1947), pp. 90-100.

investment which generates a stream of benefits that extends into future periods. In addition, their consumption may require more than one period. Since such human capital-enhancing commodities introduce intertemporal considerations, the consumer framework must be general enough to encompass the investment aspects of consumer behavior.

Such a comprehensive framework for consumer demand was developed by Becker, who views consumption as a production activity and specifically incorporates time as one of the inputs of the production process.[2] In this model, the individual is envisioned to produce consumed goods by combining market goods and time subject to his production functions. The individual produces that combination of commodities, subject to income and time constraints, that maximizes his utility.

In viewing consumption as a production activity, all of the inputs--both physical and time--required for the production of a commodity are explicitly included. Thus, the opportunity cost of producing a given bundle of commodities is made explicit. This exposes the crucial nature of the time constraint. Time is needed both to produce and consume commodities and to earn income in the market with which to purchase the market good inputs.

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[2]Gary S. Becker, "A Theory of the Allocation of Time," Economic Journal, LXXV (September, 1965), 493-517.

Thus, Becker's model is essentially a model of the allocation of time. While Becker makes no attempt to include either commodities with an intertemporal nature or "free" goods, his model does expose the crucial nature of the time constraint, as well as the underlying input-output relations.

Successful incorporation of "free" goods into a consumer equilibrium model necessitates the development of a comprehensive theory of consumer choice which incorporates the possibility of zero priced inputs, consumption of commodities with intertemporal consequences, and all of the relevant constraints (production possibilities, time, and budget) the individual faces. Since the role of producer is being attributed to the individual, it seems plausible that his actions should be describable within a production theory framework. In the world of the consumer, the production theory framework can be used to describe how the individual combines physical inputs and time, subject to his production functions and to his time and budget constraints, to produce vectors of present and future commodities that maximize his utility.

In the remainder of this chapter a model of static utility maximization under certainty will be developed. This model will specify all of the important decision variables relevant to an individual's utility

maximizing consumption decisions. From this model a demand function for "free" goods will be derived.

Static Choice

In any market period, an individual is assumed to engage in a set of activities, combining market purchased goods and time to produce a set of commodities Z_i, $i = 1,\ldots, m$.[3] For example, if the commodity being consumed is a meal (Z_e), its production requires inputs of the different food items eaten as well as the time required to purchase, prepare, and eat the food.

The production function for a commodity Z_i may be written, in general, as

$$Z_i = f^i(x_1^i, x_2^i,\ldots, x_n^i, \tau^i) \qquad (1)$$

where Z_i is the amount of commodity i produced by the individual and x_j^i, $j = 1,\ldots, n$, are the explicit quantities used of the n physical inputs available for use by the individual in producing commodity i. τ^i is the individual's time input in producing commodity i. f^i is this individual's production function for combining the physical and time inputs (x_j^i's and τ^i) into the consumed commodity Z_i. The individual's production technology for commodity Z_i is embodied in f^i. That is,

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[3] Ibid., p. 495.

f^i determines which market goods (x_j's) may be used in the production activity and how they may be combined with time to produce Z_i. f^i is unique for this individual and depends on the circumstances or characteristics of the individual at a particular point in time.

Generally, the individual's production possibilities are specified by the set of m economic production functions

$$Z_i = f^i(x_1^i, x_2^i, \ldots, x_n^i, \tau^i), \quad i = 1, \ldots, m, \qquad (2)$$

one for each of the m commodities the individual may produce. For a given rate of final output $Z_i = \bar{Z}_i$, f^i, $i = 1, \ldots, m$, defines an isoquant in the input space giving the technical range of substitution of the x_j and τ in production of Z_i. However, Z_i represents only the quantitative dimension of commodity i. If factor proportions are continuously variable along the isoquant, the qualitative dimension of the commodity may be assumed to be determined by the factor proportions selected. To the extent that the production function allows substitution between factors, therefore, the individual can alter the qualitative nature of the commodity by changing the relative proportions of the factors used.[4] For example,

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[4]This method of characterizing the commodity is analogous to that suggested by Sven Dano, Industrial Production Models (New York: Springer Verlag New York, 1966), pp. 132-47.

if a housewife is preparing an evening meal (Z_e), she could choose between many different combinations of food items and her time. With the use of alternative food item combinations, the meal will vary qualitatively on the basis of nutritional value, taste, aroma, etc. Also, since many food items are available in semiprepared states, the housewife can use more or less time in preparing the meal. In general, the quantitative and qualitative properties of a commodity are completely determined by its input vector ($x_1^i, x_2^i, \ldots, x_n^i, \tau^i$) and its production function.

The individual's production set G, the activities (consumed commodities) technically possible for the individual, is defined by combining the time constraint ($\sum_i \tau^i = T$) with his production possibilities (2). That is, since the individual's total time T available in the market period is fixed, this time constraint must be incorporated with his production functions (each of which requires a time input τ^i) to generate his production set (G). This defines

$$G = \{g \mid g = (f^1, f^2, \ldots, f^m, x^1, x^1, \ldots, x_n^m, \tau^1, \ldots, \tau^m);\ x_j^i, \tau^i \geq 0;\ \sum_i \tau^i = T\} \qquad (3)$$

Since the individual is considered to be a producer of the commodities he consumes, our theory must provide solutions for the quantitative final commodity vector $Z = [Z_1,\ldots, Z_m]$ produced and for the qualitative choice of production methods (input proportions) used. This is facilitated by assuming that the individual's preferences are defined on the entire production set G, i.e., that the individual has ordered preferences for both the quantities of commodities produced (consumed) and for the production techniques (factor proportions) used to produce them. Thus, the individual's utility function is of the form

$$U = U(x_1^1,\ldots, x_n^1, \tau^1;\ldots; x_1^m,\ldots, x_n^m, \tau^m). \quad (4)$$

The individual's choice of preferred $g \varepsilon G$ is restricted by a budget constraint, i.e., expenditures per period must not exceed receipts. One of the activities the individual may engage in is work for an employer. The rate of remuneration W received may be assumed to depend on the individual's labor skills. Total work remuneration is therefore $W\tau^w$ where τ^w is the amount of time spent on the job. Also, the individual may be the recipient of non-wage income V such as interest, dividends, social security, pensions, disability income, and unemployment compensation. His total income Y is then defined as

$W\tau^w + V$, wage income plus non-wage income. To acquire the market goods (x_j's) used as inputs in the production of commodities, the individual must pay the market prices of the goods used. Therefore, given the vector of market good prices P, his obtainable wage rate W, and his non-wage income V, the individual's choice is constrained to those productions for which $Px \leq W\tau^w + V$. The set of attainable productions K for a given vector [P,W,V] can now be defined as

$$K(p,w,V) = \{g \varepsilon G \mid Px \leq W\tau^w + V\} \qquad (5)$$

For a given [P,W,V], the individual chooses that production $g \varepsilon K$ which maximizes his utility as determined by his preference function. This is his equilibrium consumption relative to [P,W,V]. The individual's demand correspondence associating equilibrium consumptions with given parameter values in [P,W,V] is therefore obtained by varying K over G to obtain the individual's choice response to changes in P, W, and V. In this way one obtains the individual's demand correspondence

$$Z = [x_1^1, x_2^1, \ldots, x_n^1, \tau^1; \ldots; x_1^m, x_2^m, \ldots, x_n^m,$$

$$\tau^m; \tau^w] = \phi(P,W,V \mid \{f\}). \qquad (6)$$

Investment Activities

Investment activities are consumptions resulting in an alteration of one or more of the individual's already available production functions f^i and/or in an expansion of the set of attainable productions available to the individual in present and future periods. These alterations of the production functions generate an expansion of the individual's production possibilities set G. Usually, investment activities require more than one period. Examples of such investment activities that are human capital-enhancing include educational training, exercise programs, and vocational rehabilitation.

Investment activities can be incorporated into the previous analysis by recognizing that each $g \varepsilon G$ defines a specific set of present consumptions and therefore a specific set of attainable productions in future periods, depending on which, if any, investment activities are included in g. That is, each g has an intertemporal significance in that it defines a future set of G's which can be expressed as

$$G_g = \{G_g^t\}, \; t = 1, \ldots, N, \tag{7}$$

where G_g^t is the individual's production possibility set in period t if he selects g in the present period, and where N is the last period in the individual's time horizon. If g includes investment activities then the

G_g^t, $t = 1,\ldots, N$, will be different, expressing the altered set of production possibilities that become available through time. If g contains no investment activities then all G_g^t, $t = 1,\ldots, N$ can be assumed identical.

The motivation for an individual to select a $g \varepsilon G$ containing one or more investment activities is that he prefers some commodity set which will become available in the expanded production possibility sets resulting in future periods. Consumption of multiperiod investment activities that expand an individual's production possibility sets in future periods requires the individual to allocate some resources from the production of utility yielding commodities during the investment period. This implies that the individual's time stream of consumption $Z^0,\ldots, Z^H$--where $Z^t = (Z_1^t,\ldots, Z_m^t)$ and H is the number of periods required for consumption of the investment activity--will be such that if he chooses a multiperiod investment activity, then the quantity and/or quality of commodities consumed must be reduced for the duration of the investment period so that resources can be allocated to the investment activity.

It is assumed that the individual's preference function is defined over all alternative time streams of consumptions that are possible as a result of possible

investment consumptions. To arrive at a price in the present period for a multiperiod investment, it is necessary to discount the multiperiod costs of the investment to the present. In other words, it is assumed that the individual must, in the present period, set aside enough money to pay for both the inputs in the present period and, with interest collected on the remainder that is put aside, the costs for the entire investment period when they come due. Since the individual must pay for his multiperiod investment in the present period, the interest rate--denoted by d--used to discount the costs becomes a crucial parameter and must be considered along with P, W, and V as part of the individual's economic environment.

Therefore, investments--except for the multiperiod nature of their benefits and costs--are not different from other types of consumptions. For this reason, they are also included in the individual's production set G and are routinely considered by the individual for consumption. That is, the individual's demand correspondence associating equilibrium consumptions with given parameter values is obtained, in the intertermporal case, by varying the price vector, the wage rate, non-earned income, and the interest rate and noting the individual's choice response to changes in these

parameters. In this way the present period's equilibrium consumption (Z^O) is obtained as

$$Z^O = [Z_1^O, \ldots, Z_m^O] = \Phi(P,W,V,d|\{f\}) \qquad (8)$$

"Free" Goods

The last objectives of this chapter are to incorporate so called "free" goods into the analytical framework and to explicitly obtain a demand correspondence for such "free" goods. As explained in the introduction, a "free" good is a good or service (i.e., an x_j^i) that is offered by a dispensing agency at an explicit money price of zero. The good may have a single or a multiperiod nature. The individual combines these "free" inputs with his time to produce consumptions which maximize his utility. Some "free" goods, such as food and welfare payments, are used up in the immediate market period. Other "free" goods, such as human capital-enhancing services, are inputs in investment activities that alter the individual's set of production functions and thereby change the individual's future production possibilities. The benefits of such "free" goods have a multiperiod nature.

From (2) it is known that the individual combines market inputs and a time input to produce a consumed commodity. In the production process, the individual demands the market inputs used and supplies the time

input. The demand relationship for a market input (an x_j^i) is obtained from relation (8) as

$$x_j^i = \Phi_j^i(P,W,V,d|\{f\}). \tag{9}$$

Likewise, the supply relationship of time by the individual is obtained from relation (8) as

$$\tau^i = \Phi^i(P,W,V,d|\{f\}). \tag{10}$$

When an input or set of inputs is "free," the individual's crucial consideration becomes the opportunity cost of time allocated to consume the "free" good. The quantity demanded, therefore, of a "free" good will vary with the time cost of consuming it. In order to discuss the demand curve of a "free" good x_j^i in the price-quantity plane, we define the price or time cost as

$$W\left(\frac{\tau^i}{x_j^i}\right) \tag{11}$$

where τ^i/x_j^i is the time required per unit of x_j^i consumed, and W is the individual's opportunity wage rate. An agency may initiate a change in the time cost by changing the rate at which an individual may consume x_j^i per τ^i. The individual's demand response to such a change $d(\tau^i/x_j^i) < 0$ will be characterized by the usual income and substitution effects. The substitution effect will

be negative as usual. Since the time cost per unit of x_j^i consumed has fallen, x_j^i will be relatively less expensive to consume and the individual will substitute into it $dx_j^i > 0$. The income effect will be indeterminate because for a given T, an individual can consume more of everything, so $dx_j^i \lesseqgtr 0$.

Agencies supplying "free" goods view the process differently. In supplying physical input "free" to an individual, they demand time from him for production of the consumption. The individual's time supply response to an increase $d(x_j^i/\tau^i) > 0$ in the price per unit of time supplied can also be analyzed in terms of the implied income and substitution effects. Since this price per unit of time supplied is really a negative price or a benefit--like a wage rate--the substitution effect will be positive $d\tau^i > 0$. The income effect will be indeterminate because the larger price may induce him to supply more time, supply the same amount of time, or supply less time.

CHAPTER III

THE EMPIRICAL APPLICATION

In the previous chapter a theoretical model of static utility maximizing consumer equilibrium under certainty, including "free" goods (zero priced market inputs used by individuals in the production of certain commodities), was developed. This model was constructed from the individual's viewpoint. It was in essence a model describing how the individual allocates his time to produce consumptions so as to maximize his utility. Since the model considered the individual to be a producer of the consumed commodities (by combining the physical and time inputs required for their production), the individual demands the market inputs and supplies the time input needed for the production activity. The chapter concluded with the establishment of demand relationships for the physical inputs used and of a supply relationship of time.

The purpose for developing the consumer equilibrium model was to derive the individual's demand relationship for a "free" market input. This is the relationship that is of paramount importance to the suppliers of such "free" goods. The primary purpose of this chapter is to

establish explicitly the individual's demand relationship for "free" vocational rehabilitation services. That is, the model developed in the previous chapter will now be used to obtain a demand function for a particular "free" good which then will be estimated. Before deriving the particular demand function, it will be necessary to examine the operation of the vocational rehabilitation program in Michigan and the individual's vocational rehabilitation decision.

The Supply of Vocational Rehabilitation in Michigan

The vocational rehabilitation program in Michigan is organized for the purpose of rehabilitating any individual who for some physical or mental reason is unable to participate in the activities of life to the same extent as other individuals in his age and sex group.[1]

The State of Michigan is divided into five regional offices which are further subdivided into 25 district offices. The vocational rehabilitation program operates as follows. First, a potential recipient arranges an interview at one of the district offices. He may have decided to do this as a first step on his own initiative or he may have been referred there by some other agency

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[1]Michigan Department of Education, A State Plan for Vocational Rehabilitation Services, Volume 2 Technical Appendices (Lansing: Department of Education, 1970), p. 55.

which he first contacted, such as the Department of Social Services or the Department of Health.

During this initial interview, the client is assigned a particular counselor who records a series of data about the individual, including the disability that the client feels he is suffering from.

Next, the client is sent to a medical doctor for a general examination and to investigate the reported disability to see if, in fact, such a disability exists, and, if it exists, to determine exactly how severe it is. The medical examiner then sends the counselor his findings. On the basis of this information a decision is made about accepting the client. If the client is accepted, another interview is arranged with the counselor and another series of data items are recorded that describe the client's socio-economic situation before vocational rehabilitation services are begun.

If the client is not accepted, it may be due to any of a number of reasons, including the following: unable to contact client, client moved, client died, client failed to cooperate, client found to have no disabling condition, client found to have no vocational handicap, and client refused services. The primary reason why a client would be refused services is that the district does not have enough funds allocated from the

state to handle all the cases of this type of disability that appear for treatment. The percentage of potential recipients having a particular disability that are refused services because of a lack of funds depends on the particular disability they have. This is the case because the state allocates vocational rehabilitation funds on the basis of disability type, not in lump sums. The district office then allocates its funds for each specific disability type on a first-come, first-served basis. When the allocation for a particular disability type in any district becomes fully committed for the year, then that district must refuse to accept any more clients with that disability type for the remainder of that fiscal year.

If the client is accepted for vocational rehabilitation services, the counselor and the client then together formulate the rehabilitation plan necessary for the solution of the client's problems. Services available include diagnostic procedures (medical, psychological, social, and vocational), physical restoration services (surgery, prosthetic appliances, and hospital care), and training (remedial, literacy, vocational, academic, on-the-job). Through a bargaining process, the client attempts to procure his equilibrium set of vocational rehabilitation services. After some negotiations, a

mutually agreeable program of vocational rehabilitation is decided upon. Then, the plan is implemented.

Once services are begun, they continue until the client is rehabilitated and a new job is found or until he drops out of the program. The rehabilitation process involves two stages. First, medical treatment is used to correct the client's disability. Second, the client is then given some vocational training which will enable him to pursue a new career. When the client either becomes rehabilitated and finds a new job or else drops out of the program, his case is closed. Closure of the client's case includes the collection of a final set of data. A total dollar value of services supplied is recorded. Unfortunately, dollar quantities of each specific service rendered are not recorded. However, available records do indicate the broad categories (diagnostic, restoration, training) from which the client's equilibrium set of services come.

The Individual's Demand for Vocational Rehabilitation Services

Vocational rehabilitation is a "free" good whose consumption alters the individual's production functions and expands the set of activities available to the individual. For example, acquisition of an artificial leg by an amputee enables him to change the manner in which he

produces many commodities by reducing the need for outside help. Also, it opens up a whole new range of activities which were previously denied him such as walking or driving an automobile.

Since alterations of the production set result from consumption of vocational rehabilitation services, such consumption is an investment activity. When the individual surveys his production possibilities set G in the present period (t = 0), one of the commodities he contemplates is vocational rehabilitation. The benefits in the form of an altered production set that would accrue to the individual from consumption of vocational rehabilitation depend on the severity of the disability and therefore on how restricted his production function set presently is. If he is not disabled, consumption of vocational rehabilitation will allow only minimal alterations of his production functions and therefore only a marginal expansion of his production possibilities. Conversely, if the individual is greatly disabled, many alterations in his production functions are possible and therefore a large expansion of his production possibilities is feasible.

Also, since vocational rehabilitation is a human capital-enhancing commodity, its consumption will facilitate not only an expansion of the production set G, but, if labor skills are improved, the individual's

obtainable wage rate will rise and this will enable him in the future to consume larger commodity bundles for any given work effort (τ^W). That is, since his wage rate rises, for any given work effort, his budget constraint will be expanded. This implies that his equilibrium consumptions in future periods $Z^t = (Z_1^t, \ldots, Z_m^t)$, $t > H$ (where H is the final period in which the investment commodity vocational rehabilitation must be consumed) can be greater. Thus, the benefits to be derived from consumption of vocational rehabilitation are in the form of an expanded production possibility set (in proportion to how badly disabled the individual is) and of expanded equilibrium consumptions in future periods due to an increased wage rate facilitating an enlarged budget constraint.

Since consumption of most vocational rehabilitation services requires more than one market period, the costs, as well as the benefits, have a multiperiod nature. To arrive at a price in the present period for a multiperiod vocational rehabilitation investment, it is necessary to discount the multiperiod costs to the present using the interest rate d. Thus, it is assumed that the individual must, in the present period, set aside enough money to pay for both the inputs in the present period and, with interest collected on the remainder that is put aside, the costs for the entire investment period when they come due. However, since vocational rehabilitation

services are "free," the individual does not have to allocate income to pay for such inputs. Nevertheless, the individual is still faced with a two component cost of consuming vocational rehabilitation.

The first component consists of the time input required for a vocational rehabilitation investment. Since time is not transferable through time, it is impossible for the individual to set aside time today which can draw interest and be used in future periods when it is needed. Rather, the individual can assess the time cost by assuming that any amount of time spent in consuming vocational rehabilitation could alternatively be spent working and earning income at his present wage rate. Thus, a magnitude for the time cost can be arrived at by discounting, using d, the future time inputs valued at the individual's present market wage rate.

The second component of the cost of consuming vocational rehabilitation stems from the institutional arrangements whereby agencies dispensing disability and public assistance payments reduce the recipient's welfare payments upon his completion of a vocational rehabilitation program. The individual can place a present value on the fall of such non-earned income by discounting to the present, using d, the fall experienced in such benefits. Therefore, combining the time lost component and the

forfeited public assistance component gives a cost for consumption of vocational rehabilitation by the individual.

Formulation of the Hypotheses

In the preceeding section it was shown that the benefits to be derived from consumption of vocational rehabilitation are in the form of an expanded production possibility set and of expanded equilibrium consumptions in future periods due to an increased wage facilitating an enlarged budget constraint. The magnitude of this second benefit can be estimated by discounting to the present the yearly increase in earnings attributable to the vocational rehabilitation investment for the remainder of the individual's working life.

It was also shown in the preceeding section that the costs incurred in consuming vocational rehabilitation consist of time that could be allocated to alternative uses and disability payments that are forfeited upon completion of the program. The time cost can be determined by obtaining the product of the individual's monthly earnings and an index of time supplied (in months). The forfeited disability payment cost can be determined by discounting to the present the yearly fall in disability payments for the remainder of the individual's life.

In contemplating the relative desirability of consuming vocational rehabilitation, the individual will weigh

the costs and benefits, and his demand for services and his supply of time will depend on them. This leads to the empirical hypothesis: "An individual's demand for 'free' vocational rehabilitation services is a function of the costs and of the benefits generated from such a demand." This also leads to the empirical hypothesis: "An individual's supply of time to the vocational rehabilitation process is a function of the costs and of the benefits generated from such a supply."

Choice of Sample and Variables

Data collected at each of the different stages of the vocational rehabilitation process for all clients in Michigan has been recorded on computer tapes which are kept in Lansing. It is this supply process that generates the data on vocational rehabilitation that will be used to estimate the demand function for services and the supply function of time. Recently closed cases (third quarter of fiscal year 1970) will be used for estimating the model because the recorded responses from these cases are more accurate and more observations are complete than on cases from earlier years.

Data recorded on services consumed are observations of individual's equilibrium consumptions. They observe where supply equals demand for consumption of vocational rehabilitation services. That is, the vector of vocational

rehabilitation services that is observed and recorded as being consumed is the vector agreed upon by the counselor and the client. It is the equilibrium output determined by the client's demand and the agency's supply.

In the present analysis it is possible to abstract from simultaneous equation problems concerning the individual's equilibrium consumption of vocational rehabilitation services by choosing for examination that type of disability for which there are minimal supply limitations, i.e., no rationing is necessary. A direct regression of the amount of vocational rehabilitation services demanded onto the independent variables is justified if, in fact, the values of all the independent variables used in the regressions are generated independently of the dependent variable.

However, if supply limitations exist, then the dispensing agency must ration the limited services according to some criterion. If the agency has control over the magnitude of some parameter, such as cost, that appears in the individual's demand equation, then by varying its magnitude, it can influence the quantity of services demanded by the client. Hence, the magnitude of this parameter would not be generated independently of the dependent variable and it would not be truly exogenous. This would call for a simultaneous equation model. The

observed (equilibrium) values of the dependent variable and the agency determined parameter should be considered as being determined simultaneously.

By choosing for examination that disability type which has minimal rejections due to supply limitations, it is possible to abstract from the simultaneous equation problem. As was explained earlier, the state allocates vocational rehabilitation funds to its district offices on the basis of disability type, not in lump sums. The district office then allocates its funds for each specific disability type on a first-come, first-served basis until the allocation for that particular disability type in that district becomes fully committed for the year. Then that district must refuse to accept any more clients with that disability type for the remainder of that fiscal year. The size of the allocation each district receives for each disability type depends on how the state allocates its limited budget. The state's vocational rehabilitation case priorities are revealed by the relative sizes of its allocations for different disability types. Examinations of the ratios of acceptances to rejections for each disability type reveals which disability types are well funded (not subject to budget or supply limitations) and which are not. Choosing that disability type having the highest acceptance to rejection ratio minimizes the

possibility of simultaneous equation problems. Analysis of the more than 6,000 cases closed in the third quarter of fiscal year 1970 shows that the type of disability in which individuals have lost one or both major lower extremities (feet or legs) has the highest acceptance to rejection ratio (9.7 to 1). This 55 case subset will be the sample used.

Construction of empirically testable formulations of the hypotheses requires observations on all the parameters in the hypotheses. The hypotheses state that both the services demanded and the time supplied depend on the costs and the benefits generated by the demanding and the supplying.

On the demand for services side, the benefits will be calculated as a return per unit of service received. The basic unit of service will be a dollar's expenditure by the agency. The return will be the discounted present value of the yearly increase in earnings (B) attributed to the vocational rehabilitation investment for the remainder of the individual's working life. The return per unit of service is then computed as B/Q where Q is the dollar amount of services the agency supplies. The client is made aware of the approximate magnitude of B/Q at the time he and his counselor negotiate his vocational rehabilitation program.

Since data has been kept on the individual's weekly earnings both before vocational rehabilitation services were begun (w_b) and after they were concluded (w_e), B can readily be determined as

$$B = \sum_{a=\pi}^{65} \frac{(w_e - w_b)52}{(1+d)^a} \qquad (12)$$

where π is the number of months the client is in the program divided by 12. Use of π is necessary because the individual's higher wage rate does not begin until he completes the program.

Though 17 individuals in the sample had positive weekly earnings both before and after making the vocational rehabilitation investment, 38 individuals did not have a job before making the investment, 11 did not have a job after making the investment, and 11 did not have a job either before or after making the investment. Several possibilities exist to explain why an individual did not have a job before making the vocational rehabilitation investment. First, he could have been so disabled that he could not obtain a job. Second, he may be a young man just coming onto the labor market but who has not yet found his first job. Third, he may be able to get a job, but the pay is so low that he values higher the domestic non-market work that he can do by staying at home.

Since estimation of the significance of the increased earnings benefit from vocational rehabilitation requires data on earnings both before and after vocational rehabilitation, it will be necessary to estimate the opportunity earnings of those individuals who did not have earnings before, after, or both before and after consuming vocational rehabilitation. This can easily be accomplished using a prediction equation. Since data has been collected on most of the items likely to affect earnings, it is possible to regress weekly earnings before vocational rehabilitation, for all individuals who had such earnings, on the set of explanatory variables. Similarly, it is possible to regress weekly earnings after vocational rehabilitation, for all individuals who had such earnings, on the set of explanatory variables. Using the coefficients that result from this procedure, it is then possible to construct two prediction equations--one for weekly earnings before vocational rehabilitation and one for weekly earnings after vocational rehabilitation. These two prediction equations will generate values for weekly earnings before and after vocational rehabilitation for those individuals who do not have them.

The explanatory variables to be included in the prediction equation for weekly earnings and the reasons for including them will now be examined. First, a continuous variable indicating the number of years of

school completed by the individual will be used. Numerous studies have shown the significance of education in explaining earnings differentials between individuals.

Next, binary variables for race and sex will be used. Studies by Thurow, Weiss, and Welch show that education has lower financial value to blacks than to whites, even if differences in the quality of schooling are accounted for.[2] Thurow's work suggests that blacks fail to receive on-the-job training which is necessary, in addition to education, to raise black incomes.[3] Studies by Holtman, Bayer, , and White revealed that employers were paying women less than men for the same work in a wide range of occupations.[4] A study by Sanborn found discrimination against women by customers

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[2]Lester C. Thurow, "The Occupational Distribution of the Returns to Education and Experience for Whites and Negroes," American Statistical Association, Proceedings of the Social Statistics Section 1967, X (1967), 233-44; R. D. Weiss, "The Effect of Education on the Earnings of Blacks and Whites," Review of Economics and Statistics, LII (May, 1970), 150-59; and Finis Welch, "Labor-Market Discrimination: An Interpretation of Income Differences in the Rural South," Journal of Political Economy, LXXV (June, 1967), 225-40.

[3]Thurow, op. cit., p. 241.

[4]A. G. Holtman and A. E. Bayer, "Determinants of Professional Income Among Recent Recipients of Natural Science Doctorates," Journal of Business, XLIII (October, 1970), 410-18; Donald J. McNulty, "Differences in Pay Between Men and Women Workers," Monthly Labor Review, XC (December, 1967), 40-43; James White, "Women in the Law," Michigan Law Review, LXV (April, 1967), 1051-1122.

and fellow employees.[5] These empirical studies support the contention that individuals who are members of racial minorities or who are women can expect lower earnings for the same quantity and quality of work, ceteris paribus.

A continuous variable indicating the client's age will also be included in quadratic form, the assumption being that the individual's earnings will increase with age, as he acquires more experience, but an age will eventually come after which decreased physical productivity will be reflected in falling earnings.

So, the weekly wage prediction equation for clients, all of whom had the same disability category and were not employed at the time they entered the vocational rehabilitation program, becomes

$$\hat{w}_b = \gamma_0 + \gamma_1 E + \gamma_2 R + \gamma_3 S + \gamma_4 A + \gamma_5 A^2 + \varepsilon \qquad (13)$$

where the γ's are determined from the data on individuals who were employed at the time of entrance into the vocational rehabilitation program.

The weekly wage prediction equation for clients who did not take a job at the end of their vocational rehabilitation program becomes

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[5] Henry Sanborn, "Pay Differences Between Men and Women," Industrial and Labor Relations Review, XVII (July, 1964), 534-50.

$$\hat{w}_e = \delta_0 + \delta_1 E + \delta_2 R + \delta_3 S + \delta_4 A + \delta_5 A^2 + \mu \qquad (14)$$

where the δ's are determined from the data on individuals who acquired employment at the conclusion of their vocational rehabilitation program.

In the demand for services formulation, costs will be calculated as a time price per unit of service demanded and as forfeited disability payments per unit of service demanded. The time price will be calculated by dividing the monetary value of the time supplied C_1 by Q. The monetary value of the time supplied by the individual must be estimated from the available data. An estimated time input will be transformed into months of time supplied and then multiplied by the individual's monthly earnings before vocational rehabilitation services were begun to get a dollar value for his time input. In estimating amount of time supplied by the individual, data is available on the type of services received (diagnostic, restoration, on-the-job training, formal schooling such as vocational or college, and maintenance) and for how many months these services were received. By estimating how time intensive reception of these different services was, it is possible to construct an index of time inputs whose value depends on the services received and the length of time in the program. Once this quantity of time supplied in terms of months is determined, it can be multiplied by the

individual's prevocational rehabilitation monthly earnings to get a dollar value of time supplied. This opportunity time cost C_1 can be expressed as

$$C_1 = \left(\sum_{\substack{\text{all services} \\ \text{requiring a time} \\ \text{input}}} \begin{matrix}\text{months of time supplied} \\ \text{for each service}\end{matrix} \right) \times \begin{pmatrix}\text{monthly earned income} \\ \text{before vocational} \\ \text{rehabilitation}\end{pmatrix} \quad (15)$$

Only those services that required use of the individual's time were included in the index. Those services included and their assigned weights are as follows. First, all clients received diagnostic services in the form of a thorough medical check-up. The time assigned for diagnostic services is one day. Next, restoration services were provided. Since the particular type of disability selected for examination consists of individuals who have lost one or both major lower extremities (feet or legs), restoration services included provision of and adaptation to artificial limbs. The time assigned for these services is two weeks plus one day per month for the number of months the individual stayed in the program. These additional days represent medical check-ups.

Academic training services, such as vocational school or college, were the only other services provided that were considered time consuming. They were weighted as requiring full usage of the individual's time. Other

services provided such as maintenance payments do not require use of the client's time and are therefore not included in the time index.

The forfeited disability payments per unit of service demanded price is calculated by dividing the discounted present value of forfeited yearly disability income payments C_2 by Q. Symbolically

$$C_2 = \sum_{a=\pi}^{E(A)} \frac{(\Delta V \cdot 12)}{(1+d)^a} \tag{16}$$

represents the forfeited disability income cost where ΔV is the fall in monthly disability income payments and E(A) is the expected number of years of life the individual has remaining, as determined by an expected life table.

On the basis of this information, the demand relation to be estimated is:

$$Q = \alpha_0 + \alpha_1 (\frac{C_1}{Q}) + \alpha_2 (\frac{B}{Q}) + \alpha_3 (\frac{C_2}{Q}) + \zeta \tag{17}$$

where

Q = the dollar amount of vocational rehabilitation expenditures demanded by (supplied to) the client

C_1/Q = the time price per unit of service demanded

B/Q = the discounted present value of the return per unit of service demanded

C_2/Q = the discounted present value of forfeited disability payments per unit of service demanded.

It is expected that α_0 will be positive in all cases because it is expected that most individuals would demand some of the services to increase their information about the program. Also, available data do not record the benefits generated by an expanded production possibility set. This source of benefits is likely to be important and will lead to positive α_0's. It is expected that α_1 will be negative because α_1 is the coefficient of the time price of the services. At higher prices, less will be demanded. It is expected that α_2 will be positive because the larger the return per unit of service, the more services will be desired. The sign of α_3 in (17) is expected to be negative because if an individual's disability benefits decrease, he will be expected to demand less services. Of the 55 cases in the sample, only four had changes in disability benefits. Consequently, in estimating (17), α_3 will be restricted to zero because of the lack of explanatory power of the four observations in this category.

On the supply side, time supplied will be measured by the total number of calendar months M the individual continues in his agreed upon vocational rehabilitation program, supplying whatever amount of time per month that his program requires. M will be regressed on a time price per month in the program, on a services provided per

month supplied price, and on a measure of financial benefits generated per unit of time supplied. The time price is C_1 divided by M. This is a measure of the time cost for being in the program one month. It will be less than the individual's potential monthly earnings unless his rehabilitation program requires the full use of his time. The services provided price per month in the program is Q divided by M. This price, or benefit rate, tells how intensively the agency is supplying services per unit of time supplied by the client. The financial benefit per month of time spent in the program will be measured as B divided by M. This is a measure of the discounted present value of the increased earnings generated per month spent in the rehabilitation program.

This formulation of the supply of time relation then becomes

$$M = \beta_0 + \beta_1 (\frac{C_1}{M}) + \beta_2 (\frac{Q}{M}) + \beta_3 (\frac{B}{M}) + \psi \qquad (18)$$

where

M = the total number of calendar months the individual supplies to the rehabilitation program

C_1/M = the time price per month supplied

Q/M = the services provided per month supplied price

B/M = the discounted present value of the increased earnings per month of time supplied.

It is expected that β_0 will be positive because individuals are likely to be willing to supply some time to gain more information about the program. Also, β_0 will be positive because it will reflect the non-pecuniary benefit of the expanded production possibility set. Since the magnitude of this benefit is not observable, its effect will appear in the constant term.

The sign of β_1 is unpredictable because C_1 is the product of the client's time index and his monthly opportunity earnings. If we have two clients, alike in all respects except that the second individual has a larger monthly opportunity earnings, we might expect the second individual to supply less time since his price of time per month would be higher or we might expect him to supply more time because he could acquire his needed consumptions by working fewer hours and therefore have more time remaining to supply to a rehabilitation program. Either possibility follows from the model in Chapter II. If the substitution effect generated by larger C_1/M dominates, he will supply less time and β_1 will be negative. If the income effect dominates, he will supply more time and β_1 will be positive. Which effect dominates is an empirical question.

The sign of β_2 is unpredictable. As shown in Chapter II, the substitution effect generated by increased Q/M will be positive since Q/M is actually a benefit rate. Which effect dominates is an empirical question.

It is expected that β_3 will be positive because the larger the financial benefit per month spent in the program, the more time the individual would be likely to supply to the program.

It should be noted that implicit in the above demand and supply formulations is the assumption that marginal cost is equal to average cost and marginal benefit is equal to average benefit over the domain of C and B.

These relations will be estimated with several variations. First, two different interest rates--the triple A bond rate and the bank savings rate--will be used. In 1968, the time at which these vocational rehabilitation decisions were made, the triple A bond rate was 6.18%[6] and the bank savings rate was 4%. Then, because there is some evidence that individuals intentionally understate their earnings during the initial interview in order to establish greater need,[7] the relations will be estimated using before and after earnings figures which are the maximum of reported earnings and predicted earnings as given by the prediction equations.

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[6] Moody's Industrial Manual 1971, p. a18.

[7] U.S. Department of Health, Education, and Welfare, Human Investment Programs: Vocational Rehabilitation (Washington: Government Printing Office, 1967), p. 24.

Empirical Results

The Wage Prediction Equations

Prevocational rehabilitation.--The prevocational rehabilitation wage prediction equation (13) estimated for the 17 individuals who had jobs before beginning vocational rehabilitation is as follows:

$$\hat{w}_e = \underset{(232.86)}{71.147} + \underset{(8.05)}{1.584E} + \underset{(46.96)}{38.483R} + \underset{(64.23)}{55.745S} - \underset{(9.05)}{2.178A} + \underset{(0.09)}{.018A^2} \qquad (19)$$

Standard errors are given in parentheses below the coefficients. R^2 was .2114 and $\bar{R}^2$ was -.1471. There were 11 degrees of freedom. $F_{(5,11)}$ was .5897 and its significance was .708. The signs of E, R, and S were all positive as predicted. However, the signs of A and A^2 were (-) and (+), the opposite of what was expected. None of the coefficients were significantly different from zero.

These less than encouraging results prompted several experiments which failed. First, a race-sex interaction term was introduced on the assumption that the income differential between a male minority worker and a male non-minority worker is greater than the differential between a female minority worker and a female non-minority worker. This effort failed because there was no black female with a wage before vocational rehabilitation.

This made the matrix of independent variables (the X^TX matrix) singular because the RS term was linearly dependent on the constant term, R, and S.

Second, a rural-urban index was introduced to reflect wage differentials between urban and rural areas, the assumption being that if the cost of living is higher in urban areas, ceteris paribus, then urban wage rates will be higher than rural wage rates. Since each client's county of residence was known, census data was used to separate all Michigan counties into either rural or urban classifications and a rural-urban binary explanatory variable was introduced. The problem that arose here was that all 17 observations were from urban counties.

Postvocational rehabilitation.--The postvocational rehabilitation wage prediction equation (14) estimated for the 44 individuals who had jobs after concluding their vocational rehabilitation services is as follows:

$$\hat{w}_e = \underset{(53.14)}{-76.132} + \underset{(3.66)}{5.178E} + \underset{(19.21)}{18.938R} + \underset{(19.09)}{64.320S} + \underset{(2.15)}{4.430A} - \underset{(0.03)}{0.056A^2} \qquad (20)$$

Standard errors are given in parentheses below the coefficients. R^2 was .3851 and $\bar{R}^2$ was .3042. There were 38 degrees of freedom. $F_{(5,38)}$ was 4.7600 with significance .002. The signs of E, R, S, and A are all positive

as predicted while the sign of A^2 is negative as predicted. The estimated coefficients of S, A, and A^2 were highly significant.

Again, experiments of including first a race-sex interaction term, then a rural-urban index, and then both together were performed. In all three cases, observations were sufficient to permit execution of the regressions (the matrix of independent variables was non-singular). The regressions run with a race-sex interaction term, a rural-urban index, and both all showed a slight increase in R^2, but $\bar{R}^2$ fell considerably. The significance levels of the added regressors was low in all cases. Also, in all three cases, the F statistic fell, but remained highly significant.

Since the regression results obtained in (20) gave a highly significant F test, I am led to conclude that the approach used was satisfactory and that the poor results obtained in (19) were due to the limited number of observations available. In estimating (13), only 17 observations were available while there were 44 observations available for estimating (14). So, (19) and (20) will be used as the prediction equations for weekly earnings for individuals who did not have them before receiving vocational rehabilitation services and after receiving them.

The Demand for Vocational Rehabilitation Services

The estimation results for the demand for services relation using both measures of before and after earnings figures--actual reported earnings when a job was held (w_b and w_e) and predicted values ($\hat{w}_b$ and $\hat{w}_e$) when not and maximum of (w_b, $\hat{w}_b$) and (w_e, $\hat{w}_e$)--and both interest rates are given in Table 1. Results are based on 54 observations in each case.

In all four regressions α_0 is positive (as expected), large, and significant at the .005 level. α_1 is negative (as expected) in all cases, but not significant at the .10 level. α_2 is negative (not expected) in all cases, but significant at the .05 level only in the formulations using maximum (w_b, $\hat{w}_b$) and maximum (w_e, $\hat{w}_e$). I attribute the negative value of α_2 to the poor pre-vocational rehabilitation wage prediction equation because a poor estimate here will give a poor estimate for B and therefore for B/C. Overall, significant F tests resulted only when opportunity wages were estimated using maximum (w_b, $\hat{w}_b$) and maximum (w_e, $\hat{w}_e$). The triple A bond rate gives slightly more significant F statistics and R^2's than the bank savings rate, but the improvements are marginal.

TABLE 1.--Demand for vocational rehabilitation regression results.

Regression	Wage	d	α_0	α_1	α_2	R^2	$\bar{R}^2$	F
17a	$\hat{w}_b$ unless $w_b \neq 0$ and	6.18%	662.691	-0.772	-0.145	0.0311	-0.0069	$F_{(2,51)} = 0.8193$
	$\hat{w}_e$ unless $w_e \neq 0$		(90.130)	(0.771)	(0.186)			sig.: 0.446
17b	$\hat{w}_b$ unless $w_b \neq 0$ and	4.0	661.111	-0.771	-0.100	0.0291	-0.0089	$F_{(2,51)} = 0.7656$
	$\hat{w}_e$ unless $w_e \neq 0$		(90.089)	(0.772)	(0.141)			sig.: 0.470
17c	maximum (w_b, $\hat{w}_b$) and	6.18	712.802	-0.783	-0.480	0.1004	0.0651	$F_{(2,51)} = 2.8456$
	maximum (w_e, $\hat{w}_e$)		(90.456)	(0.746)	(0.225)			sig.: 0.067
17d	maximum (w_b, $\hat{w}_b$) and	4.0	710.341	-0.777	-0.359	0.0977	0.0623	$F_{(2,51)} = 2.7602$
	maximum (w_e, $\hat{w}_e$)		(90.383)	(0.747)	(0.172)			sig.: 0.073

The Supply of Time

The estimation results for the supply of time relation using both ways to measure before and after earnings--actual reported earnings when a job was held (w_b and w_e) and predicted values ($\hat{w}_b$ and $\hat{w}_e$) when not and maximum of (w_b, $\hat{w}_b$) and (w_e, $\hat{w}_e$)--and both interest rates are given in Table 2. Results are based on 55 observations in each case.

In all four regressions β_0 is positive (as expected) and significant at the .0005 level. The sign of β_1 is positive and significant at the .01 level in all four regressions. Apparently the income effect of larger opportunity earnings dominates the substitution effect and the client supplies more time. β_2 is negative and significant at the .10 level in all four regressions, especially the two where the wages used are actual reported earnings when a job was held (w_b and w_e) and predicted values ($\hat{w}_b$ and $\hat{w}_e$) when not. Apparently the income effect is negative and large enough to swamp the positive substitution effect because less time is supplied as Q/M increases. This implies a backward bending supply curve of time to the vocational rehabilitation process. β_3 is negative (not expected) in all four regressions but significant at the .10 level only in those formulations where wages are taken as the maximum of (w_b, $\hat{w}_b$) and (w_e, $\hat{w}_e$). I attribute

TABLE 2.--Supply of time to vocational rehabilitation regression results.

Regression	Wage	d	β_0	β_1	β_2	β_3	R^2	$\bar{R}^2$	F
18a	$\hat{w}_b$ unless $w_b \neq 0$ and	6.18%	26.824	0.047	-0.047	-0.008	0.1877	0.1399	$F_{(3,51)} = 3.9281$
	$\hat{w}_e$ unless $w_e \neq 0$		(3.335)	(0.017)	(0.025)	(0.013)			sig.: 0.013
18b	$\hat{w}_b$ unless $w_b \neq 0$ and	4.0	26.784	0.047	-0.049	-0.006	0.1861	0.1383	$F_{(3,51)} = 3.8880$
	$\hat{w}_e$ unless $w_e \neq 0$		(3.336)	(0.017)	(0.025)	(0.010)			sig.: 0.014
18c	maximum (w_b, $\hat{w}_b$) and	6.18	27.948	0.048	-0.038	-0.020	0.2430	0.1985	$F_{(3,51)} = 5.4568$
	maximum (w_e, $\hat{w}_e$)		(3.306)	(0.017)	(0.023)	(0.010)			sig.: 0.002
18d	maximum (w_b, $\hat{w}_b$) and	4.0	27.994	0.047	-0.039	-0.016	0.2445	0.2001	$F_{(3,51)} = 5.5020$
	maximum (w_e, $\hat{w}_e$)		(3.306)	(0.017)	(0.023)	(0.008)			sig.: 0.002

the negative values of β_3 to the poor prevocational rehabilitation wage prediction equation because a poor estimate here will give a poor estimate for B and therefore for B/M. Overall, all four F ratios were very significant. The regressions using maximum (w_b, $\hat{w}_b$) and maximum (w_e, $\hat{w}_e$) for opportunity wages have larger R^2's and more significant F statistics. The results obtained from both interest rates are not significantly different.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The empirical results support the stated hypotheses. In the demand for "free" services regressions, the constant term (α_0) is always positive (as expected) and significant. It reflects the benefit of expanded production possibilities. The coefficient of the time price (α_1) for services is negative (as expected) in all cases, but never more significant than the .30 level. The coefficient of the benefit per unit of service received (α_2) is negative (not expected) in all cases and significant in half the cases (the half using maximum (w_b, $\hat{w}_b$) and maximum (w_e, $\hat{w}_e$) for opportunity wages). These unexpected results are attributable to the poor prevocational rehabilitation wage prediction equation because a poor estimate here will give a poor estimate for B and therefore for B/C. Overall, significant F statistics and higher R^2's result for the formulations using maximum (w_b, $\hat{w}_b$) and maximum (w_e, $\hat{w}_e$) for opportunity wages. The triple A bond rate gives slightly more significant F statistics and R^2's than the bank savings rate, but the improvements are marginal.

In the supply of time regressions, the constant term (β_0) is positive (as expected) and highly significant in all cases. β_0 reflects the clients' willingness to supply time to gain more information about the program

and the non-pecuniary expanded production possibilities benefit. The coefficient (β_1) of the time cost for being in the program is positive and highly significant in all four regressions. Apparently the income effect of larger opportunity earnings dominates the substitution effect and the client supplies more time if his time price for being in the program is larger. The coefficient (β_2) of the services provided price per month supplied is negative and significant in all four regressions, especially the two where the wages used are actual reported earnings when a job was held (w_b and w_e) and predicted values ($\hat{w}_b$ and $\hat{w}_e$) when not. Apparently the income effect is negative and large enough to swamp the positive substitution effect because less time is supplied as Q/M increases. This implies a backward bending supply curve of time to the vocational rehabilitation process. Though our observations give a backward bending supply curve of time, it would not be advisable to generalize that the entire supply curve of time is negatively sloped. Since a client will consume vocational rehabilitation services until he has consumed that quantity (a target amount) required to rehabilitate him and since there is some variability in proportions available for combining services and time, the observed backward bending supply curve of time probably reflects the fact that if the client can receive his target amount of services more intensively (larger Q/M), then he will

not have to supply as much time. The technology of rehabilitation would exclude the possibility that the client could become rehabilitated by supplying a very small M if Q/M were very large. The coefficient (β_3) of the increased earnings benefit per month spent in the program is negative (not expected) in all four regressions but significant only in those formulations where wages are taken as the maximum of (w_b, $\hat{w}_b$) and (w_e, $\hat{w}_e$). I attribute the negative values of β_3 to the poor pre-vocational rehabilitation wage prediction equation because a poor estimate here will give a poor estimate for B and therefore for B/M. Overall, all four F ratios are highly significant. The regressions using maximum (w_b, $\hat{w}_b$) and maximum (w_e, $\hat{w}_e$) for opportunity wages have larger R^2's and more significant F statistics. The results obtained from both interest rates are not significantly different.

The implications of these empirical results is that "free" vocational rehabilitation services are not "free" to the recipient because of the time cost. As expected, clients' demand functions are inversely related to the time price. Using α_1 (-.783) in 17c (the most significant demand regression) and, from the transformed data, average Q ($637.31) and average C_1/Q ($21.46), the elasticity of demand for "free" services with respect to their time price can be computed as .013. Using the elasticity of demand formula, it can be calculated that if

the time price were zero, clients, on the average, would demand $16.07 more of services, thus making the average total demand $653.38. This would be a marginal increase in the quantity of services demanded.

The implications of the empirical results for supply of time considerations are that, since β_1's are positive, the individual will continue to stay in the program additional months if his time cost for being in the program is larger. Since all the clients in the sample had similar disabilities and therefore similar prescribed treatments, differences in time cost per month in the program reflect differences in opportunity earnings. Those individuals who have low opportunity earnings must spend their time realizing those earnings to support themselves. Those individuals who have larger opportunity earnings can acquire the earnings needed to purchase required goods and services by working a smaller number of hours than individuals with low opportunity earnings and therefore have more hours available to supply to rehabilitation activities. That is, if an individual's opportunity earnings should rise, it would make consumption of vocational rehabilitation more expensive because of the more costly time input. Thus, the individual would be motivated to substitute out of rehabilitation. But, the income effect generated by the larger opportunity earnings allows

the individual to acquire needed goods and services for fewer hours of work and therefore release hours from necessary work which may be used to consume vocational rehabilitation.

Using β_1 (.047) in 18d (the most significant supply regression) and, from the transformed data, average M (28.18) and average C_1/M ($87.92), the clients' elasticity of supply of time with respect to its time price can be computed as .079. Using the elasticity of supply formula, it can be calculated that if the price of supplying time were zero, clients, on the average, would remain in the program 4.11 fewer months, thus making the average total amount of time supplied 24.07 months. This would be a marginal decrease in the number of months supplied. The quite inelastic time supplied elasticity suggests that attempts to keep clients in rehabilitation programs additional months by attempting to increase their opportunity earnings would have only marginal effects. For example, a wage supplement program depending on hours worked, would have limited success in increasing the time period clients would stay with their rehabilitation programs.

Another implication of the empirical results for supply of time considerations is that the elasticity of time supplied with respect to the services provided per month supplied price is low. For example, computation of this

elasticity for regression 18d gives .031. Thus, if vocational rehabilitation agencies desire to increase the quantity of time clients remain in their programs, they should lower Q/M. But such decreases will generate only marginal increases in time supplied. It would be expected that at lower levels of Q/M, the negative income effect would cease to dominate the positive substitution effect and supply of time would be positively sloped (not backward bending).

In summary, the purpose of this dissertation was to facilitate the inclusion of economic considerations in "free" good demand forecasts by developing a comprehensive model of consumer behavior incorporating "free" goods and services. Such a model was developed and then used to establish explicitly the individual's demand relationship for "free" vocational rehabilitation services and his supply relationship of time. The statistical results generated by the application suggest that the variables included in the model are important.

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APPENDIX

VOCATIONAL REHABILITATION DATA

VOCATIONAL REHABILITATION DATA

In order to keep case materials confidential, the Vocational Rehabilitation Division of the Michigan Department of Education required me to sign a contract in which I agreed to not disclose or reproduce the raw data which they provided. So, to give the reader some feel for the data, some statistics for the raw data and for the transformed data are given in Table 3.

TABLE 3.--Statistics on raw and transformed vocational rehabilitation data.

Variable	Mean	Standard Deviation	Minimum Value	Maximum Value
Education (E)	10.77[a]	2.29[a]	4.00[a]	16.00[a]
Race (R)	0.82[a]	0.39[a]	0.00[a]	1.00[a]
Age (A)	36.23[a]	14.90[a]	0.00[a]	67.00[a]
Services Demanded (Q)	$637.31	$643.84	$ 15.00	$3,980.00
Time Price of Services (C_1/Q)	$ 21.46[b]	$114.69[b]	$ 0.01[b]	$ 837.01[b]
Service Benefit Rate (B/Q)	$122.37[b]	$380.71[b]	-$676.47[b]	$1,632.90[b]
Time Supplied (M)	28.18	22.79	1.00	99.00
Time Price of Time (C_1/M)	$ 87.92[c]	$166.17[c]	$ 0.56[c]	$ 641.32[c]
Services Price of Time (Q/M)	$ 47.48	$128.50	$ 0.00	$ 937.00
Time Benefit Rate (B/M)	$131.34[c]	$361.14[c]	-$449.41[c]	$2,178.98[c]

[a]These results are for the 44 individuals who acquired employment after consuming vocational rehabilitation services.

[b]These results are for regression 17c, the most significant demand regression.

[c]These results are for regression 18d, the most significant supply regression.