THE DEMAND FOR MAJOR HOUSEHOLD APPLIANCES: AN ECONOMETRIC ANALYSIS

Thesis for the Degree of Ph. D.
MICHIGAN STATE UNIVERSITY
WILLIAM RICHARD CRON
1972

This is to certify that the

thesis entitled

The Demand for Major Household

Appliances: An Econometric Analysis

presented by

William Richard Cron

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Economics

Major professor

Date December 21, 1972

O-7639

ABSTRACT

THE DEMAND FOR MAJOR HOUSEHOLD

APPLIANCES: AN ECONOMETRIC ANALYSIS

By

William R. Cron

This study attempts to examine alternative approaches to specifying a demand function for several major household appliances. Of the existing approaches, a stock adjustment and a habit persistence model are examined in detail. Demand analysis in the area of durable goods seems to have developed independently of the great amount of work performed on the estimation of demand functions based upon maximization of a utility function subject to a budget constraint. In this paper an analysis of the existing approaches to the study of durable demand are examined to determine if they have a tie to the classical utility maximization framework.

Two alternative approaches to the development of demand function are then attempted. The first approach accepts the constant elasticity of demand model as a good first order approximation to the demand functions in question and proceeds to impose demand restrictions (notably symmetry) derived from the utility maximization framework as a set of Lagrangean constraints. The second approach specifies a particular utility function and proceeds to derive the demand function by maximizing this function subject to an assumed budget constraint.

The stochastic specifications for each of the models are examined and estimation methods which are consistent with the various

stochastic formulations are presented. Each model is then estimated using aggregate U.S. data for the period 1950 to 1970. An approach to expanding the data base upon which the demand functions are estimated is presented in the form of a method for combining time series and cross-sectional observations. The pooling method, which is applied to the existing "ad hoc" models, allows for observations which are heteroskedastic and cross-sectionally correlated, as well as timewise autoregressive.

All of the models performed very well if judged by their resultant R^2's. In addition, the use of a two-stage generalized linear regression method for pooling time series and cross-section data caused significant improvements in the efficiency of the regression estimates. An examination of the linear expenditure system of demand equations revealed that this functional form is compatible with the habit persistence type of model, provided the income variable is interpreted as a type of "supernumerary income." The constant elasticity of demand model is shown to offer a plausible alternative to specification of a functional form for the demand equations, which the imposition of demand restrictions as side conditions is shown to be relevant and to offer an improvement in the efficiency of the estimates obtained.

THE DEMAND FOR MAJOR HOUSEHOLD APPLIANCES: AN ECONOMETRIC ANALYSIS

By

William Richard Cron

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Economics

1972

G86289

To: My wife and family

ACKNOWLEDGEMENTS

I am forever in the debt of Dr. Jan Kmenta for the invaluable assistance he has rendered to me during my course of study at Michigan State University. He suggested the topic of this dissertation to me, guided me in each stage of its preparation, and, when the future appeared bleak, he never ceased to be a beacon of encouragement.

I would like to thank Dr. Anthony C. Koo and Dr. Robert L. Guftason for agreeing to serve on my committee, and Dr. James Ramsey for his helpful comments on an earlier draft.

Acknowledgement must be made to Wayne State University for providing the necessary hours of computer time on the University's IBM 360 computer. A special thanks is also extended to Mr. David Johnstone for his invaluable programming assistance and his incisive comments which caused me to rethink many areas.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF APPENDICES

Chapter 1

Introduction

Expenditures on durable commodities represent a significant proportion of annual national expenditure. In light of this significance, it would be generally felt that the literature on the demand for these products would be extensively documented. The expectation would also be that the amount of research performed would be distributed over the various types of durables roughly in relation to the importance the item assumes in the consumer budget. A review of the literature does not confirm our expectations. Instead, we find a relatively small number of studies, with those that do exist being concentrated on either aggregate durable expenditure, or, if applied to a specific commodity, being applied primarily to automobiles.

The durable commodity presents a unique problem. Durables by their nature provide services over a period of time, thus implying that the impact of the existing stock on the current purchase decision must be considered. The classical theory of demand contained as arguments consumption of commodies, which for a non-durable good coincided with its purchase. However, for a durable commodity consumption does not occur simultaneously with purchase, and its availability, or the stock of a durable good, affects the amount consumed in a given period. Since durables were not conformable to the standard utility framework, the recourse was to neglect the idea of utility maximization in favor of models based on generalizations as to how an individual might be expected to formulate implicitly his purchase plans.

The lack of readily available data contributed to making studies of household appliance demand scarce. Available figures on retail sales of durables other than automobiles are subject to a considerable amount of measurement error and are not extensively published. Data on the stocks of these durables are practically non-existent. In contrast, the automobile industry has large amounts of reliable data available to the researcher. Automobile companies have developed their reporting practices to the extent that franchised dealers report sales summaries by ten-day periods, with some daily information developed by means of warranty card counts.

This study is concerned with the demand for a few of the major household appliances. Specifically, three appliance groupings representing the functional purposes of cold storage, food heating, and laundry service were selected for analysis.

To place the analysis in historical perspective, some of the past contributions to our knowledge of durable demand are reviewed in Chapter 2. Chapter 3 then proceeds to develop some of the existing models and additional models based on a utility maximization concept for adaptation to household appliance demand. The objective is to develop models which are plausible in terms of observable behavior and to discover if the classical utility maximization framework provides a plausible background against which new models may be developed or existing models connected. In Chapter 4 alternative stochastic formulations of the models presented in the previous chapter are examined and estimation methods consistent with these specifications are developed. Additionally, a procedure for increasing the amount of available data by an appropriate combination or regional data for a number of years is developed.

Before we can proceed to turn our attention to the estimation of demand functions, we must be certain the relationship being estimated is truly a demand function.[1] By virtue of the Cournot-Walras theory of economic equilibrium there is a tendency for the market totals of demand to equal the market totals of supply, so that the observed quantity variables might determine a sort of hybrid between the true demand and supply functions.

A general approach to the problem is to specify both a supply and demand model, and to estimate both functions simultaneously. An alternative is to specify assumptions for the functions, which, if correct, will allow a demand function to be estimated by single equation methods. In this paper we will proceed with the later approach, and assume the supply function is of the recursive type. A simplified example of a supply and demand model, where the supply function is assumed to be of the recursive type, is given by

$$d_t = D(p_t)$$

$$s_t = S(p_{t-1})$$

$$p_t = p_{t-1} + b(d_{t-1} - s_{t-1}),$$

where d_t, d_{t-1}, s_t, and s_{t-1} refer to demand and supply at periods t and t-1, and p_t and p_{t-1} refer to the prices at periods t and t-1. This model explains the price during period t as an adjustment in the price from period t-1. If b is specified to be positive, the price will rise with excess demand and fall with excess supply, as would be expected by the classical

[1]An elementary discussion of the problem can be found in Lawrence R. Klein, An Introduction to Econometrics, New York: Prentice-Hall (1962), pp. 10-19. A more complete discussion of the problem is contained in H. Wold and L. Jureen, Demand Analysis, New York: Richard D. Irwin (1953).

theory of economic markets. Starting from some initial values of d, s, and p, the above model allows a series for each of the three variables to be recursively calculated. Single equation estimates of the demand equation in this system will produce estimates which can be identified as pertaining to the demand function, and not a combined market equilibrium function.

The household appliance industry is felt to be typical of a market where the quantity supplied is not instantaneously adjusted to changing market conditions, but instead one in which the quantities are adjusted as the result of a deliberate policy consideration with some lag in implementation. Given this condition the recursive system appears to be a reasonable working hypothesis for this industry.

Chapter 2

A Review of Durable Demand Models

Our intent in this chapter is to present various models which could be adapted to household appliance demand. Accordingly, the models selected have often been developed in connection with durables other than appliances. Studies that attempted to measure the impact of various explanatory variables through existing models are not discussed. Following the review of the models, a brief section to illustrate the application of demand models to household appliances is included. The studies specifically treated are those of Roos and von Szeliski, Farrell, Chow, Stone, Rowe and Nerlove, Houthakker and Taylor, and Suits. In the section dealing with household appliance studies the contributions of Burnstein, Miller and Wu are presented. The symbols used in the discussion will be those of the original authors.

2.1 Roos and von Szeliski

One of the earliest attempts to develop a consistent theoretical framework for household durables was supplied by C. F. Roos and Victor von Szeliski[1] in a study of U.S. automobile demand. In this study demand, taken in the sense of total retail passenger sales of automobiles, is considered to consist of two basic components, new owner sales and replacement sales. The factors determining the individual components are then explored.

---

[1] Roos, C. F. and von Szeliski, Victor, "Factors Governing Changes in Domestic Automobile Demand", in The Dynamics of Automobile Demand, General Motors Corporation, New York, N.Y., (1939).

New owner sales are assumed to be a product of the number of potential new owners and the probability that an individual selected at random from the potential new owner group will purchase a car. The potential new owners are assumed to be given by the difference between an upper limit of the car maintaining ability of the country and the number of cars in operation. This upper limit is a product of f(t), the number of families at time t, and m(t), which is a measure of the maximum ownership level per family. At this point the authors introduce an innovation by assuming the maximum ownership level m(t) is continuously changing in response to other factors. Functionally this was given by

$$m(t) = B(t)\, p^{-\delta}\, I^{\varepsilon}\, d^{\delta},$$

where p is the price of cars, I is per capita income, d is average car life, B(t) represents all neglected factors, and ε and δ are the elasticities of m(t) with respect to I and d/p respectively.

The probability a potential owner will purchase a new car is assumed to vary as the product becomes known and wanted and facilities for use built up. It was felt that this influence could be measured by the cars in operation (C). Other factors affecting the probability of purchase are thought to be income (I), price of new cars (p), and the trade in ratio of used car allowance to new car price (T). This is given by

$$A_0 = A_3(t) p^{-\alpha}\, T^{\beta}\, I^{\gamma}\, C,$$

where A_0 is the probability of purchase, p, I, T and C are as defined above, $A_3(t)$ represents neglected factors,[2] and α, β, and γ are parameters to be determined.

---

[2]Although $A_3(t)$ and B(t) are specified to include neglected factors, they are not necessarily the same since they measure the effect of these factors on different variables.

The complete specification for new owner sales is given by

$$S_N = A_0 \left[f(t) \cdot m(t) - C\right]$$
$$= A_3(t)p^{-\alpha} T^{\beta} I^{\gamma} C \left[f(t) \cdot B(t) \cdot (p/d)^{-\delta} I^{\varepsilon} - C\right],$$

where all of the terms are as previously defined.

Replacement sales are assumed to be proportional to the theoretical scrapping based on survival tables for cars in operation. Theoretical scrapping was recognized as only one of the many factors which contribute to replacement sales. In keeping with an earlier paper by de Wolff[3] the factor of proportionality is assumed to be a function of the price of a new unit, the trade-in price ratio, and per capita income. Using $A_4(t)$ to measure neglected factors, retaining the other symbols from the new owner sales, and using X for theoretical scrapping produces an equation for replacement sales as follows:

$$S_R = A_4(t)\, p^{-\alpha} T^{\beta} I^{\gamma} X.$$

Combining new car sales and replacement sales, and adding an assumption that new car sales should be reduced by some fraction of replacement sales to allow for interdependent effects produces an equation for estimation of retail sales,

$$S = A_3(t)\, p^{-\alpha} T^{\beta} I^{\gamma} \left[C(f \cdot B \cdot (p/d)^{-\delta} I^{\varepsilon} - C) + A_6 X + G\right],$$

where G is a measure of the interdependent effect of replacement sales, A_6 has now replaced $A_4(t)$ because the exponents for p, T, and I have been combined in the equation for retail sales, and all other factors are as previously defined. However, the equation fitted by the authors

[3]P. de Wolff, "The Demand for Passenger Cars in the United States", Econometrica, 6, (1938), pp. 113-129.

represents a slight modification of this form. The result obtained is[4]

$$S = j_t^{1.20} \, P_t^{-.65} \left[.0254\, C_t (M_{3_t} - C_t) + .65\, X_{2_t} \right],$$

where j_t represents supernumerary money income[5], M_{3_t} is the maximum ownership level at time t, X_{2_t} is a measure of replacement pressure[6] and other variables are as previously defined. The price and income elasticites are -.65 and +1.20 respectively.

The remainder of the study deals with the implications of other variables that might be significant. In addition, the authors experimented with different concepts of the variables included and fitted the equations to different time periods.

2.2 Farrell

A study of the demand for automobiles in the United States which attempted to incorporate a utility maximization concept was made by M. J. Farrell.[7] The automobile market is viewed as being made up of a series of interrelated markets corresponding to model age groups. For each market a demand function of the form

$$(1) \qquad X_i = f_i\,(y, P, t)$$

is specified, where X_i represents the demand for ownership in the i'th

---

[4]Roos and von Szeliski, op. cit., equation 16, p. 60.

[5]The concept of income used is that of supernumerary money income which is defined as the excess of disposable income over subsistence expenditures. The symbol (j_t) has been used to represent this variable as opposed to (I) which was utilized previously in the derivation of the final form.

[6]The variable X_{2_t} is a measure of replacement pressure at time t obtained by applying a shifting mortality table to the age distribution of passenger cars.

[7]M. J. Farrell, "The Demand for Motor Cars in the United States," Royal Statistical Society Journal, 87, (Part II, 1954), pp. 171-200.

group, y represents income, P represents a vector of prices of the current and prior model year groups, and t is assumed to represent individual taste for various age group cars. An assumption that the supply function for each group, other than new cars, is perfectly inelastic with respect to prices and income allowed Farrell to express the demand for new cars as a function of its own price (P), income (y), tastes (t), and the known quantities ($X_2 . . ., X_n$).

Several additional assumptions were made in order to specify the form of the demand function. Using U_{ik} to represent the highest price the k'th individual would pay for an i year old car if no other car were available to him, the assumptions are given by: (1) No individual owns more than one car; (2) For all k, $U_{ik} > U_{i+1,k}$ ($i = 1,....,n-1$), or that an individual will place a higher value on a newer car than an older one; (3) For all i, k, U_{ik} is a function of η_k, where $\eta_k = \mu_k y_k$ with y_k being the k'th individual's income and μ_k is a constant representing his tastes; (4) $\partial U_i / \partial \eta > 0$ ($i = 1,....,n$); (5) $\partial(U_i - U_j)/\partial \eta > 0$ ($i = 1,......,n-1$; $j = i+1,......,n$); (6) The distribution of individual incomes is uniquely determined by the national or community income (Y), and may be represented by the frequency function F(Y,y); (7) u is a random variable, distributed independently of y, with frequency function f(u) and may be interpreted as representing the taste variable, including many factors, such as family size or geographical location, which might not normally be implied by the word "tastes".

Based on the above assumptions Farrell derived an aggregate demand

function of the form

(2) $$X_i = \int_0^\infty F(Y,y)\, dy \int_{(1/y)G_i(P_i)}^{(1/y)G_{i-1}(P_{i-1})} f(u)\, du \qquad (i = 1,\ldots\ldots,n),$$

where X_i is the demand for i year old automobiles, $F(Y,y)$ and $f(u)$ are the income distribution and taste functions mentioned above and $G_i(P_i)$ and $G_{i-1}(P_{i-1})$ are functions defining the upper and lower limit of incomes respectively between which an i year old car would be purchased. For estimation it was more convenient to express the above function as follows:

(3) $$x_i = \int_0^\infty F(Y,y)\, dy \int_{(1/y)G_i(P_i)}^{\infty} f(u)\, du,$$

where $x_i = \sum_{j=1}^{i} X_j$.

To apply equation (3) the form of the functions $F(Y,y)$, $f(u)$ and $G_i(P_i)$ must be specified. As a general method cross-section data were used to estimate $F(Y,y)$ and $f(u)$. These estimates were then regarded as known and a time series regression was used to determine $G_i(P_i)$. The second integral in equation (3) may be represented by Q_i and $q_i = \sum_{j=1}^{i} Q_j$ $(i = 1,\ldots,n)$. It is assumed that $\log(u)$ is normally distributed which gives a function for q_i as

(4) $$q_i = \int_{\lambda_i}^{\infty} N\, dx \qquad (i = 1,\ldots\ldots,n),$$

where

$$N = (1/\sqrt{2\pi}\,)\, e^{-(x^2/2)}$$

and

(5) $$\lambda_i = (1/\sigma) \log(G_i) - (1/\sigma) \log (Y) \qquad (i = 1,\ldots\ldots,n).$$

The proportion of families owning a car not more than i years old (q_i), and average group income (y) was calculated from budget studies. Values of λ_i corresponding to the q_i obtained from the budget data were obtained by reference to a table of the normal integral. OLS method was then applied to obtain estimates of G_i (the highest price at which an i year old car would be purchased) and $1/\sigma$ for each age group. In performing the estimation, equation (5) was fitted to observations which were weighted by an estimate of their sampling variance. For new cars this procedure produced an estimate of G_i of 7,530 and of σ of .387. Estimates were also obtained for older age groups.

To obtain estimates of F(Y,y) information as to income distribution in 1941 and the assumption[8]

$$\lambda F(\lambda Y, \lambda y) = F(Y,y)$$

were used. Utilizing this assumption allowed equation (3) to be written as

$$(6) \qquad x_i = \int_0^\infty F_o(Y,y)dy \qquad \int_{(1/\sigma)(\log G_i(P_i) - \log y - \log Y_t)}^\infty N \, du,$$

where Y_t is the ratio of national income in year t to that in 1941, $F_o(Y)$ is the distribution of income given by the 1941 data and the other symbols are as previously defined. It is then possible to calculate a value of k_{it} corresponding to each observed value of x_{it} in the time

---

[8]Farrell points out that this condition is satisfied if the incomes of all families vary proportionately.

series, where k_t has been substituted for Y_t such that equation (6) is satisfied.

Farrell ends up with a demand equation

$$(7) \qquad X_i = \int_0^{\infty} F_o(y)dy \int_{(1/\sigma)\log(H_i/y)}^{(1/\sigma)\log(H_{i-1}/y)} N\,du,$$

where

$$H_i = \left[\frac{G_i}{b_i Y_t}\right] (P_{it} - P_{i+1,t} - P_{ot} (a + c_i t))$$

with the G_i and σ_i having been obtained from cross section data as explained, and $F_o(y)$ from the 1941 income distribution study. The a_i, b_i, and c_i are parameters that have been estimated from calculated values for k_{it} and an assumed linear relationship for G_i (P_i). The following values of a_i, b_i, and c_i for new cars were estimated : $a_i = .7257$; $b_i = 1.02$; $c_i = -.045$. An additional facet explored in the study was the complication introduced to the cross section data because of difference in habits between urban, rural non-farm and farm families.

2.3 Chow

Gregory C. Chow[9] has provided a study of the U.S. automobile market in which he attempts to develop demand functions for new automobile ownership consistent with the utility maximization framework. Using data for the years 1921 to 1953, he presents two models to explain automobile ownership. The first model, referred to as the "existing theory", explains the per capita stock of automobiles in the United States as a

---

[9]Gregory C. Chow, Demand for Automobiles in the United States, Amsterdam : North Holland Publishing Company, 1957.

function of its relative price and income. The specific function is of the form

$$X = Jp^a I^b u,$$

where X is the per capita stock of automobiles, p is the relative price per unit of the durable, I is the income variable in per capita terms, u is the random disturbance term, a, b, and J are parameters to be estimated. For estimation the equation is transformed to log linear terms and the logarithm of price regressed against the other variables. Using expected per capita income for the income variable and including a trend term produced the best fit,

$$\ln(p) = -5.854 - \underset{(.065)}{1.048} \ln X + \underset{(.256)}{2.007} \ln I_e + \underset{(.00433)}{.00238t} \qquad R^2 = .949.$$

The second model of automobile ownership, referred to as the "alternative theory", explains automobile ownership by introducing a hypothesis on the desired asset structure of the individual into the existing theory. Specifically, it is assumed the individual desires to maintain a constant ratio between his durables and other assets, which includes his stock of money and securities. Following this approach, Chow proposes a function of the form

$$X = Jp^a I^b M_a^c u,$$

where X, p, and I, are as previously defined and M_a is a monetary variable.[10] The estimated equation is found to be

$$\ln(p) = -5.420 - \underset{(.087)}{.975} \ln(X) + \underset{(.356)}{1.701} \ln(I_e) + \underset{(.184)}{.237} \ln(M_a)$$
$$R^2 = .951.$$

---

[10]This variable is defined as the per capita stock of currency, demand deposits and time deposits in commercial banks, in 1937 dollars held by all sectors of the American economy, except the banking sector.

In testing both the existing theory and the alternative theory, Chow experimented with two income variables, per capita disposable income and a longer run expected per capita income. In the context of the existing theory the inclusion of the expected income term outperformed the disposable income concept. Including expected income as a variable in a demand equation, which excluded the trend variable, improved the R^2 from .898 with the disposable income concept to .948, while the coefficient of the income term was significant for both concepts. Employing the expected income variable in the alternative theory improved the R^2 from .920 with the disposable income concept to .951, while causing the monetary variable to become insignificantly different from zero.

Based on these results, the alternative theory is shown to exhibit a slightly larger R^2 than the existing theory, no matter which concept of income is utilized. However, two possible arguments which affect the interpretation of the results should be considered. The alternative theory may have given better results than the existing theory when disposable personal income was used only because the stock of money is a better approximation to an appropriate concept of income than is disposable personal income. A second argument advanced is that the existing theory has come out as well as it has relative to the alternative theory, when "expected income" is used in both, simply because "expected income" is a better measure of the equilibrium stock of money than the empirical definitions employed in the regressions.

After applying the models to explain automobile ownership, Chow proceeds to examine three purchase models. The first is derived by noting new purchases in period t equals the difference between the total stock at the end of t and the depreciated stock left over from t-1. The second and third models respectively are derived by interpreting the arguments in the utility function as purchases and then including a variable representing the relative price of old cars which are substitutes for new cars. The best result is obtained from the third model when the price variable used in the regression is the price index of the total car stock. Choosing this price variable as the dependent variable the result is

$$p = -12.492 + \underset{(.0294)}{.4685}\, I_e - \underset{(3.909)}{17.072}\, X' - \underset{(1.412)}{12.420}\, X_{t-1} \qquad R^2 = .924,$$

where p is the price variable, I_e is expected income, X' is the purchases of period t, and X_{t-1} is the stock of passenger cars at the end of period t-1.

2.4 Stone and Rowe-Nerlove

Stone and Rowe have proposed a model of durable demand that has made a considerable contribution to empirical research.[11] Their model, which is based on the concept of a stock adjustment mechanism, has the desirable feature that no data on stocks of appliances are required. However, some estimation problems were originally encountered, but these were later handled with some modifications suggested by Nerlove.[12] The model with the Nerlove modifications is one of the models to be tested in the current paper and so only a brief heuristic discussion of its methodology and a presentation of its results in empirical application will be given here.

---

[11] Richard Stone and D.A. Rowe, "The Market Demand for Durable Goods," Econometrica, 25, (July, 1957), pp. 423-43.

[12] Marc Nerlove, "The Market Demand for Durable Goods: A Comment," Econometrica, 28, (February, 1960), pp. 132-142.

A basic premise is that net investment (*i.e.*, the difference between beginning of period and end of period stocks) occurs in fixed proportion to the difference between desired and actual stocks. New purchases are simply equal to the sum of depreciation, which is assumed to be determined by declining balance method, and net investment. By a series of successive substitutions, a reduced form for the model which contains only observable magnitudes is obtained, but some of the variables in the reduced form depend on an arbitrary selection of the asset's life. In this context one of the Nerlove modifications was helpful for he pointed out that estimating the reduced form based on various values for the asset's life and selecting the life which gives the highest R^2 will produce maximum likelihood estimates.

The modified model was applied by Stone and Rowe in estimating the demand for two groups of durables, furniture and hardware.[13] Using annual data for the period 1922-38, the estimate of the reduced form for the furniture group produced the best results as follows:

$$\Delta q = \underset{(1.21)}{-4.54} + \underset{(.0074)}{.0139}\,\Omega(\mu/p) + \underset{(2.50)}{1.92}\,\Omega(\pi/p) - \underset{(.06)}{.20}\,E^{-1}q \qquad R^2 = .781,$$

where n = 2 was the life used, Δq is the change in the expenditure on furniture, $E^{-1}q$ is expenditure on furniture lagged one period, μ/p is income deflated by its own price, π/p is an index of all other prices divided by its own price, and Ω is an operator dependent on n. A second application of the model was made to quarterly data for a comparable (but slightly narrower) commodity grouping in the postwar period 1953-58.

---

[13] Richard Stone and D. A. Rowe, "The Durability of Consumers' Durable Goods," Econometrica, 28, (April, 1960), pp. 407-416.

2.5 Houthakker and Taylor

Houthakker and Taylor have used a habit persistence type model in studying the demand for several types of products.[14] The details are given in the next chapter and therefore will not be presented here. In essence, the authors have assumed that purchases of a durable good occur in response to a "state" or stock variable. This method, as in the stock adjustment model of Stone and Rowe-Nerlove, does not require data on the stock, but rather eliminates the stock through a series of algebraic substitutions and manipulations.

The findings relevant to the current study are those obtained when the model was applied to kitchen and other household appliance expenditures. The results are

$$q_t = \underset{(4.126)}{-29.11} - \underset{(.1504)}{.1715}\, q_{t-1} + \underset{(.0059)}{.0411}\, \Delta X_t + \underset{(.0057)}{.0418}\, X_{t-1} + \underset{(.2208)}{.6830}\, Z_t \qquad R^2 = .988,$$

where q_t is a measure of purchases at time t, X_t is an income measure equal to total per capita personal consumption expenditure in year t, $\Delta X_t = X_t - X_{t-1}$, and Z_t is a variable introduced on the third pass in the three-pass least squares[15] method of estimation utilized.

2.6 Suits

An analysis of the U.S. automobile market incorporating a complete

---

[14] H. Houthakker and L. Taylor, Consumer Demand in the U.S., 1929-1970, Cambridge, Mass.: Harvard University Press, (1966).

[15] The three-pass least squares method of estimation was utilized because the equation estimated contains a lagged dependent variable and an autocorrelated error term. The composite error term in this model is assumed to be of the form $u_t = \lambda u_{t-1} + E_t$. The Z_t is intended to be a consistent estimate of u_{t-1}. Details of the three-pass least squares method of estimation can be found in L. Taylor and T. Wilson, "Three-Pass Least Squares: A Method for Estimating Models with a Lagged Dependent Variable," Review of Economics and Statistics, XLVI, (November, 1964), pp. 329-346.

supply and demand model was formulated by Daniel B. Suits.[16] The analysis covered the period 1929 to 1956. The model contained four basic equations for a) the demand for new cars by the public (R_d); b) the supply of new cars by retail dealers (R_s); c) the supply of used cars by retail dealers (R_s'); and d) the demand for used cars by the public (R_d'). They are given by

a) $R_d = a_1 \frac{(P-U)}{M} + a_2 Y + a_3 \Delta Y + a_0 + u_1$,

b) $R_s = b_1 P + b_2 W + b_3 T + b_0 + u_2$,

c) $R_s' = c_1 R + C_0 + u_3$,

d) $R_d' = d_1 (U/M) + d_2 Y d_3 \Delta Y + d_4 S + d_0 + u_4$,

where the symbols (other than the R's) may be interpreted as: P, the price of a new car, Y, disposable income, U, the average real price of used cars, M, the number of months the average automobile installment contract runs, S, the stock of used cars, W, the real wholesale price of new cars, T, retailer operating costs, u_1, u_2, u_3, u_4, random disturbances to measure the impact of omitted factors, and the a's, b's, c's, and d's are coefficients to be determined.

These equations are then subjected to a series of substitutions to eliminate several of the variables for which data were unavailable. The reduced form for the model is given by

$$R = C_1 (P/M) + C_2 Y + C_3 \Delta Y + C_4 S + C_0 + u_5 ,$$

---

[16] D. B. Suits, "The Demand for New Automobiles in the United States, 1929 to 1956," The Review of Economics and Statistics, 40, (August, 1958), pp. 273, 280.

where u_5 is now a linear combination of u_1, u_2, u_3, and u_4. The regression coefficients were estimated by applying OLS to the first differences of the equation. Differencing the equation was done in an attempt to minimize the autocorrelation effects of time series data.[17] The result when this equation was fitted to annual data for the period 1929 to 1956 was

$$\Delta R = \underset{(.011)}{.106\Delta Y} - \underset{(.088)}{.234}\ (P/M) - \underset{(.086)}{.507\Delta S} - \underset{(.261)}{.827\Delta X} + .115 \qquad R^2 = .93,$$

where the Δ represents a "change", ΔX is a dummy shift variable, ($\Delta X = 0$ for all years except 1941 when $\Delta X = +1$ and 1952 when $\Delta X = -1$), and all other terms are as previously defined.

The demand equations for new and used cars include a variable to account for the influence of credit conditions. In the demand equation for new cars this variable is given by (P-U)/M while in the demand equation for used cars it is U/M. The symbols are: P, the price of new cars, U, the price of used cars, and M, the number of months an average installment contract runs. The interpretation of these two credit variables is that they are a measure of the average monthly payment. In testing his model without the inclusion of a term to measure credit conditions, the R^2 was only .80, while the coefficient for price had the wrong sign and was no longer significant.

---

[17] The use of first differences will remove the autocorrelation only if the residuals were autocorrelated according to a first order autoregressive scheme and the autocorrelation parameter was equal to +1, an unlikely circumstance. For a proof and discussion of this statement, see J. Kmenta, <u>Elements of Econometrics</u>, New York: Macmillan Company, (1971), pp. 289-292.

A later article by Suits extended this model in several respects.[18] The income concept was reformulated to be supernumerary income which was defined as disposable income less a subsistence level of income. To select the subsistence income level, various levels were tried and the one which gave the highest R^2 selected. This turned out to be $1500. Upon incorporation of the new income concept the R^2 was improved from .782 to .851. Account was also taken of the possible influence of the age composition of the stock of used cars has on the new car market. This was accomplished by introducing lagged sales into the regression, but it was shown that, although the coefficient was of the right sign, it was not statistically significant. A third experiment was to isolate separate price and credit responses. Upon separation it is shown that the demand elasticities with respect to wholesale price was higher than elasticity with respect to retail price. Originally the model was fitted to both a prewar and a postwar period, and a test whether the relations were the same in both periods was conducted. In this test it was discovered that the relations held for both periods but the impact of the stock of cars on the road was significantly different.

2.7 Household Appliance Studies

M. L. Burstein[19] has attempted to focus information on the computation of price and income elasticities for household refrigeration in the

---

[18] Daniel B. Suits, "Exploring Alternative Formulations of Automobile Demand," Review of Economics and Statistics, 43, (Feb. 1961), pp. 66-69.

[19] M. L. Burstein, "The Demand for Household Refrigeration in the United States," in A.C. Harberger, The Demand for Durable Goods, University of Chicago Press, Chicago (1960), pp. 99-145.

United States. The model used for estimation is

$$\log S^* = a + B_1 \log (P^*) + B_2 (\log Y) + B_3 T + u,$$

where S* is a measure of per capita consumption of services,[20] P* is a measure of the real price of refrigerators, Y is per capita real income, T is a trend variable, and u is a random disturbance term.

The author experimented with two alternative income concepts - per capita disposable income and per capita expected income. Using the disposable income concept and a depreciation rate of .10, the best results obtained were as follows:

$$S^* = a - \underset{(0.195)}{1.172} P^*_1 + \underset{(.212)}{.825} Y_e + \underset{(0.380)}{1.246} T + u \qquad R^2 = .997.$$

Some interesting aspects of this study include the calculation of a price index adjusted for quality changes and the utilization of the concept of a unit of refrigeration service for the aggregation of appliance stocks.

H. Laurence Miller[21] has provided another empirical study of refrigerator demand. The Roos and von Szeliski model forms the basis for his study, but it is never applied with the completeness contained in the authors' original study of automobile expenditures. Miller's study attempts to estimate several linear regressions in which the stock of refrigerators per household at the end of year t is the dependent variable. The independent variables (examined individually) include average personal income, the proportion of households having electricity available, and the proportion of owner-occupied households at the end of year t. The regressions are applied to cross sectional State data for a number of years, but no attempt

---

[20] The measure of consumption must be based on knowledge of the stock and on assumptions regarding the depreciation rate. Since the stock is unknown, it must be estimated by using sales figures and mortality tables. It is this requirement of the knowledge of the stock which makes this model unsuitable for our present investigation.

[21] H. Laurence Miller, Jr., "The Demand for Refrigerators: A Statistical Study," Review of Economics and Statistics, 42, (May, 1960), pp. 196-202.

to pool the State data for a number of years is attempted. Estimates of new refrigerator sales are obtained by combining information as to the demand for the stock, the existing stock and expected scrapping. This approach is not useful for the present study in that the data on stocks are not directly available and must be estimated based on sales and mortality tables.

A third study for an aggregate of household durable goods is provided by De-Min Wu[22] who examines the purchases of household durable goods in response to various input and status variables. Specific features of this study include attempts to: 1) disaggregate the income variable into different observable components that are heterogeneous in their effect on the durable purchase; 2) study the effects of lagged input and lagged status variables; 3) consider the effect of interdependence of decisions and different family characteristics on the purchase decision. The model used is a combination of a two stage decision model in which the individual first decides whether or not to purchase and then, given the decision to purchase, decides on the amount of purchase. Although the model is very fruitful for the study of cross sectional family budget data it does not appear to be applicable to the aggregate time series data which are the concern of the present study.

2.8 Summary

Of the studies presented only the Suits and the Farrell studies attempted to construct complete supply and demand models. However, as pointed out in the first chapter, the circumstances in a particular industry could justify a procedure in which only demand equations are

---

[22] De-Min Wu, An Empirical Analysis of Household Durable Goods Expenditure, Unpublished Ph.D. dissertation, University of Wisconsin, (1963).

specified. Both the Chow and Farrell studies attempted to provide a utility basis for their model. In this regard the Farrell model appeared to be very promising but its application in our present study is limited. This is because the model utilizes cross sectional data rather than time series observations, and it assumes the existence of a fairly well developed second hand market which does not exist for household appliances. None of the previous studies attempted to incorporate an intertemporal utility basis.

The Stone and Rowe-Nerlove and the Houthakker and Taylor studies were selected for inclusion in the present investigation. These studies were selected because they offered the dual advantage of not requiring data on stocks and at the same time were in concert with generally accepted notions as to how consumers formulate their plans.

Chapter 3

Durable Demand Models

3.1 Introduction

Durable demand studies, as demonstrated by our review of past work, have historically been of the "ad hoc" variety. That is, they have proceeded by utilizing a peculiar mixture of armchair theorizing and economic theory to explain the quantities of durable goods demanded. The two primary models of this type are the "stock adjustment" model and the "habit persistence" type model. In comparison with the armchair theorizing of the durable demands studies, the estimation of complete demand systems based on the concept of utility maximization has received much attention.[1] The primary objective has been an explanation of the allocation of budget resources on competing commodities by a representative typical consumer. Part of the contribution of classical demand theory has been the development of theoretical restriction which should be exhibited by demand functions. However, in a large portion of the work on the demand for specific commodities, no attempt has been made to incorporate these restrictions. This is particularly in evidence in the area of durable demand studies where there is a noticeable void in the development of demand equations based on utility theory. One reason for this neglect has been that classical theory is not well-developed in the area of intertemporal problems which the demand for durables would entail. In the current chapter we will attempt to develop demand models that give attention to the inherent durable stock

[1]For a summary of the work that has been done on the specification and estimation of a complete set of demand equations, see A. Zaman, Formulation and Estimation of a Complete System of Demand Equations, (Unpublished Ph.D. Thesis, Michigan State University, 1970).

problems and incorporate the classical restrictions. As part of the investigation, it will be shown that, subject to a slight reinterpretation of the variables, the habit persistence model is consistent with utility maximization. In particular, it can be shown to be derivable from the linear expenditure system of section 3.4.2. Before proceeding to consider durable demand models of both the "ad hoc" and utility based types, a brief review of consumer demand theory will be undertaken to provide a background against which our models will be developed.

3.2 Review of Consumer Demand Theory

The typical approach to demand theory is based on the concept of a consumer with given income and tastes maximizing his utility subject to his limited resources. Starting with an assumed utility function and budget constraint, the derivation of demand functions, utilizing classical calculus thechniques, had been extensively discussed by many authors.[2] Uzawa[3] has investigated the axiomatic foundation of demand theory and a brief partial summary of his findings will be given here.

The basic axiom, which may be referred to as the axiom of choice, provides the explanation as to how the consumer selects his particular consumption bundle among competing alternatives. The axiom states that an individual with given prices and a fixed expenditure level, which may not be exceeded, will select that combination of affordable goods

---

[2]For example, see P. A. Samuelson, Foundations of Economic Analysis, Cambridge, Mass.: Harvard University Press (1947); J. R. Hicks, Value and Capital, Oxford: Clarendon Press (1939); J. R. Hicks and R. G. D. Allen, "A Reconsideration of the Theory of Value," Economica, 1, (1934), pp. 52-75 and 196-219.

[3]H. Uzawa, "Preference and Rational Choice in the Theory of Consumption," in Proceedings of a Symposium on Mathematical Methods in the Social Sciences, 1959, (K. J. Arrow, S. Karlin, and P. Suppes, eds.).

which is highest on his preference scale. This axiom in turn implies the existence of a preference relation over the set of all conceivable commodity bundles. A commodity bundle will be assumed to consist of an n dimensional vector x or y whose elements x_i or y_i are assumed to represent the quantity of the i th commodity in the bundle. In addition, the collection of all positive bundles of which x and y are elements will be denoted by Ω. A complete statement of the existence of a preference relation is: "There exists a dichotomous binary preference relation P defined on Ω." The assumption of a dichotomous, binary relation implies it is possible to make pairwise comparisons over all bundles $x \in \Omega$, while a "preference" relation may be defined as a relation P on the set of all conceivable commodity bundles possessing the properties of irreflexivity, transitivity, monotonicity, convexity, and continuity.

The properties of a preference relation may then be expanded as follows:

I. Irreflexivity: For any $x \in \Omega$, $x \overline{P} x$ where $\overline{P}$ means the negation of P. This implies that each bundle is as good as itself.

II. Transitivity: For any $x, y, z \in \Omega$, the relation xPy and yPz xPz.

III. Monotonicity: For any $x, y \in \Omega$ such that $x_i \geq y_i$, $i = 1, \ldots n$, and $x_i > y_i$, for some i, then xPy. The implication of of this statement is that given two bundles with the first containing more than the second for some of the goods and at least the same amount of the other goods in the bundle then the first bundle will be preferred.

IV. Convexity: For any $x, y \in \Omega$ such that $x \neq y$ and $x\overline{P}y$, then $[(1-\lambda)x+\lambda y]$ Px for all $0 < \lambda < 1$. The implication of this property is that on any budget hypersurface there will exist a unique point preferred to all others on the surface.

Stanford, California: Stanford University Press, (1959), Chapter 9.

V. Continuity: For any $x^0 \in \Omega$, the set $\{x: x \in \Omega, x^0 P x\}$ is an open set in Ω.

Based on the existence of a preference relation with the properties as stated, Uzawa[4] has shown by applying Debreu's[5] Theorem there exists a continuous function U(x) defined on Ω such that

For any $x, y \in \Omega$, xPy iff $U(x) > U(y)$.

In addition, properties III and IV imply that U(x) is monotone and strictly quasi-concave.

In addition to possessing a utility function, the consumer is faced with the existence of a budget constraint which separates the commodity space Ω into attainable and unattainable regions. Formally the attainable space is given by $\{x: p'x \leq Y, x > \Omega\}$ where x and Ω are as previously defined, p' is nxl vector of prices, and Y is a scalar representing the predetermined amount of expenditure that must not be exceeded.

Samuelson[6] has then stated the general problem as "an individual confronted with given prices and confined to a given expenditure selects that combination of goods which is highest on his preference scale." Utilizing a utility function derived previously, which preserves a preference ordering, the problem may be stated mathematically as

$$\text{maximize} \quad U(x) \quad \text{subject to} \quad Y = p'x \; .$$

---

[4]Ibid., p. 135.

[5]G. Debreu, "Representation of a Preference Ordering by a Numerical Function," in Decision Processes, (R. M. Thrall, C. H. Coombs, and R. L. Davis, eds.), New York: John Wiley & Sons, (1954), pp. 159-165.

[6]P. A. Samuelson, op. cit. p. 97.

A further assumption, not provided for previously by the existence of a continuous function, is that U(x) is at least twice differentiable. This addition provides for the utilization of the classical calculus techniques in the derivation of demand functions and their restrictions.

Maximization of a function subject to a constraint is achieved by utilizing the Lagrangean technique. In this case, the Lagrangean function is

1) $$L = U(x) + \lambda\ (Y - p'x)\ ,$$

where λ is the undetermined Lagrangean multiplier. Differentiating this function with respect to the n x's and λ and setting each derivative equal to 0 gives the n+1 equations which are the first order conditions necessary for maximization.

2) $$U_i - \lambda\ p_i = 0 \qquad (i = 1, 2, \ldots, n)$$

and $$Y - p'x = 0\ ,$$

where U_i is the derivative of u with respect to the i th quantity and p_1 is the i th price.

The n+1 conditions involve 2(n+1) variables ($-\lambda$, X_1, X_2, ..., X_n, p_1, p_2, ..., p_n Y). Demand equations are derived by solving n+1 of the variable in terms of the remaining n+1 as follows:

3) $$X_1 = h^i\ (p_1, \ldots, p_n, Y) \quad (i = 1, \ldots, n)$$

$$-\lambda = f\ (p_1, \ldots, p_n, Y)\ .$$

The h^i represents the demand functions which are the object of our investigation. It is legitimate to solve for n+1 unknowns in terms of the other n+1 variables by virtue of the negative definiteness of the Hessian matrix U, defined as follows:

$$4)\qquad U = \begin{bmatrix} U_{11} & \cdots\cdots & U_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ U_{n1} & \cdots\cdots & U_{nn} \end{bmatrix},$$

where the elements of $U(U_{ij})$ represent the second order derivatives of the utility function with respect to x_i and x_j respectively. A single subscripted U (i.e., U_i) will similarly indicate the first derivative of U with respect to x_i. A property which follows from the assumption of a twice differentiable utility function is

$$U_{ij} = U_{ji}.$$

This property will be of considerable importance in the development of one of the restrictions on the demand functions generated.

The demand functions generated are subject to various restrictions. This is confirmed by Samuelson's frequently quoted statement[7] "... utility analysis is meaningful only to the extent that it places hypothetical restrictions upon these demand functions." Before proceeding with their derivation, however, several additional terms must be defined and explained, as many of the restrictions are framed in elasticity terms. The price elasticity of the i good with respect to the j th price, defined as the percentage change in quantity of the i'th good in response to a percentage change in the j th price is

$$5)\qquad e_{ij} = \frac{\partial(\log X_i)}{\partial(\log p_j)} = (p_j/X_i)\ (\partial X_i/\partial p_j)\quad (i, j = 1, \ldots, n),$$

while the price elasticity of the marginal utility of income is

$$6)\qquad e_{\lambda j} = \frac{\partial(\log \lambda)}{\partial(\log p_j)} = (p_j/\lambda)\ (\partial\lambda/\partial p_j).$$

---

[7] Ibid., p. 97.

The income elasticities are

$$7) \qquad E_i = \frac{\partial(\log X_i)}{\partial(\log Y)} = (Y/X_i) \quad (\partial X_i/\partial Y)$$

and

$$8) \qquad E_\lambda = \frac{\partial(\log \lambda)}{\partial(\log Y)} = (Y/\lambda) \quad (\partial\lambda/\lambda Y)$$

respectively, where the E_i and E_λ are defined as the percentage change in X_i and λ per percentage change in income.

The change in quantity resulting from a given price change may be decomposed into substitution and income effects. This may be seen by examining the total differential of the n+1 first order conditions. The n+1 differentials are

$$
9) \qquad
\begin{array}{l}
U_{11}dX_1 + U_{12}dX_2 \cdots\cdots + U_{1n}dX_n + p_1 d\lambda = (-\lambda)dp \\
U_{21}dX_1 + U_{22}dX_2 \cdots\cdots + U_{2n}dX_n + p_2 d = (-\lambda)dp_2 \\
\quad\vdots \hspace{12em} \vdots \\
U_{n1}dX_1 + U_{n2}dX_2 \cdots\cdots + U_{nn}dX_n + p_n d\lambda = (-\lambda)dp_n \\
-p_1dX_1 - p_2dX_2 \cdots\cdots - p_n dX_n = \\
\qquad\qquad dY - X_1dp_1 - X_2dp_2 \cdots\cdots - X_n dp_n \, .
\end{array}
$$

These may be regarded as n+1 equations in n+1 unknowns (n dX_i's and $d(-\lambda)$). Solving utilizing Cramer's rule gives

$$10) \quad dX_j = \left[\frac{\sum_{i=1}^{n} (-\lambda)\, D_{ij}\, dp_i + (dY - \sum_{k=1}^{n} X_k\, dp_k)\, D_{n+1,\, j}}{D} \right] \quad (j=1, \ldots, n)$$

and

$$11) \quad d(-\lambda) = \left[\frac{\sum_{i=1}^{n} (-\lambda)\, D_{i,\, n+1} dp_i + (dY - \sum_{k=1}^{n} X_k dp_k\; D_{n+1,\, n+1}}{D} \right],$$

where D is the determinant

$$D = \begin{vmatrix} U_{11} & U_{12} & \cdots\cdots & U_{1n} & p_1 \\ U_{21} & U_{22} & \cdots\cdots & U_{2n} & p_2 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ U_{n1} & U_{n2} & \cdots\cdots & U_{nn} & p_n \\ p_1 & p_2 & \cdots\cdots & p_n & 0 \end{vmatrix},$$

while D_{ij} is the cofactor of the element in the i th row and j th column. Dividing both sides of equation (10) by dp_i (i = 1, ..., n) while holding the remaining n-1 prices and Y constant gives

12) $$\partial X_j/\partial p_i = \frac{(-\lambda)\ D_{ij} - X_i\ D_{n+1,j}}{D} \qquad (i = 1, \ldots, n),$$

while dividing both sides by dY with n prices constant gives

13) $$\partial X_j/\partial Y = \frac{D_{n+1,j}}{D}\ .$$

These results are brought together in Slutsky's relation (K_{ji}) which is a measure of the income compensated effect of a price change as follows:

14) $$K_{ji} = (\partial X_j/\partial p_i) + X_i\ (\partial X_j/\partial Y) = (-\lambda)\ (D_{ij}/D)\ .$$

An alternative price elasticity measure, which incorporates the Slutsky term above, will be referred to as the "Slutsky" price elasticity of demand (S_{ij}). It is given as follows:

15) $$S_{ij} = (p_i/X_i)K_{ij}\ .$$

Finally, the budget share of the i th good (W_i) may be written as

16) $$W_i = (p_iX_i)/Y\ .$$

The first set of restrictions come from a consideration of the n+1 equation of the first order conditions. A simple examination of this "budget constraint" reveals

$$17)\qquad \text{I} \qquad \sum_{i=1}^{n} p_i X_i = Y \qquad \text{(Adding up restriction).}$$

If the budget equation is differentiated and then both sides multiplied and then in turn divided by Y/X, the second condition becomes

$$18)\qquad \text{II} \qquad \sum_{i=1}^{n} W_i E_i = 1 \qquad \text{(Engel Aggregation).}$$

The third set of restrictions derived from the budget equation is obtained by differentiating the budget equation with respect to price (p_j), multiplying both sides by p_j/Y, and converting the derivatives to elasticity measures. The restriction is then

$$19)\qquad \text{III} \qquad \sum_{i=1}^{n} W_i e_{ij} = -W_j \quad (j = 1, \ldots, n) \qquad \text{(Cournot Aggregation).}$$

Samuelson[8] has downgraded the importance of these restrictions since they are direct consequences of the definition of the budget equation. He states, "At best, they could but reveal that we have not applied our defined operations with numerical accuracy."

Inspection of the Determinant D shows that it is symmetrical since

$$U_{ij} = U_{ji} .$$

From this it follows that

$$20)\qquad K_{ji} = \frac{((-\lambda)\ D_{ij}}{D} = \frac{(-\lambda)\ D_{ji}}{D} = K_{ij} ,$$

[8] Ibid., p. 106.

or in more familiar terms our fourth restriction is

21) IV $\partial X_j/\partial p_i + X_i(\partial X_j/\partial Y) = \partial X_i/\partial p_j + X_j(\partial X_i/\partial Y)$ $(i,j=1,\ldots,n)$

(Symmetry Relation).

An alternative expression of the same restriction, utilizing the definition of Slutsky's price elasticity, may be written as

22) IV(a) $W_i S_{ij} = W_j S_{ji}$ $(i,\ j=1,\ \ldots,\ n)$.

Finally, it is noted the demand functions are homogeneous of order zero. The validity of this statement is demonstrated by an examination of the first order conditions which are seen to be unaffected by a proportional increase in all prices and income. By virtue of Euler's theorem, we have

23) V $(\partial X_i/\partial p_i)\ p_1 + (\partial X_i/\partial p_2)\ p_2 + \ldots + (\partial X_i/\partial Y)\ Y=0$ $(i=1,\ldots,n)$

(Homogeneity Restriction),

or in elasticity terms

24) V(a) $e_{i1} + e_{i2} + \ldots + e_{in} + E_i = 0$.

The five conditions stated form the background upon which meaningful demand functions should be estimated. In particular, conditions IV and V provide meaningful restrictions which should be maintained.

3.3 "Ad Hoc" Models

As a prelude to an examination of "ad hoc" models of demand for durable goods, it is necessary to specify the units in which demand is measured. Two possibilities are: (1) a simple aggregate of units, where distinctions between various sizes and models are neglected in the aggregation; and (2) the total dollar value of units, where each unit is weighted

by its unit value, whose representation is assumed to be its price. The specification of measurement units is necessary because the theoretical underpinnings of various models are often more conformable to a particular mode of measurement.

In this section it is assumed that consumers desire a stock level sufficient to cover their expected use levels. This assumption can be justified by the indivisible nature of appliances. An appliance may be considered as a store of potential services which, in the normal case, are released over some time span often referred to as the expected life. In an individual case the life may be lengthened or shortened depending on the intensity of use, but, in the aggregate, it is reasonable to assume that the level of services attainable is a monotonically increasing function of the level of stocks.

Conversely, this assumption implies that it is necessary for an individual to have a stock of appliances if he wishes to utilize their services. Caution must be taken in drawing direct conclusions from observation of individual use of appliances, for the stock itself can be owned by the consuming unit or leased from an owner unit. At the present time there is no well developed rental market for appliances in which transactions, and hence prices, can be observed. The leasing arrangements that do take place occur primarily in connection with the rental of a living unit, such as an apartment, where the only price that can be ascribed to the rental service is the differential between an equipped and unequipped unit. Washers and dryers do have one additional type rental outlet with coin-op laundries where a direct rental price can be observed. Rental units must be considered for they present the possibility that the demand model should incorporate one explanation for owner units and a separate explanation for rental units.

3.3.1 A Stock Adjustment Model

The basic model used in this study[9] is attributable to Stone and Rowe.[10] Implicit in this formulation is the concept of a desired stock level which varies over time as a result of changing conditions. The desired stock level is assumed initially to be a function of per capita income and price. An individual utility function might exist, but it is never formally stated. One might envision that each consumer possesses a utility function, constant over time, in which stocks of appliances are one of the arguments. Prices and income go to make up the budget constraint, and, hence, desired stock levels would shift with these parameters. The desired stock equation is written as

I-1 $$S^* = a + b\rho + c\ (P/\pi),$$

where S^* represents a desired stock, ρ represents the income variable, P/π represents relative price, and a, b, and c are coefficients to be determined.

New purchases, q, are defined as the sum of depreciation, u, and net investment, v,

I-2 $$q = u + v\ ,$$

where v is simply the increment in stock levels from one period to the next,

I-3 $$v = ES - S$$

with S the stock at the beginning of the period and where E is an operator such that,

I-4 $$E^{\theta}\ x(t) = x(t + \theta).$$

---

[9]The model will be presented in its non-stochastic form only in this section. A discussion of its stochastic form is found in Chapter 4.

[10]R. Stone and D. A. Rowe, op.cit, pp. 423-43.

The portion of the stock consumed as depreciation in each whole period is assumed to be a proportion 1/n of the stock at the beginning of the period, where n is a measure of the asset's life in years. New purchases will have a smaller depreciation rate than an identical asset held from the beginning of the period, simply because there has been less time to utilize its services. For this reason, depreciation in the year of purchase is assumed to be 1/m of the purchases during the period where 1/n>1/m and

I-5 $$1/m = 1 - \frac{1}{n \log_e [n/(n-1)]} .$$ [11]

The combined depreciation is then

I-6 $$u = S/n + q/m .$$

This equation may also be expressed in terms of the opening stock and net investment as follows:

I-7 $$u = \left[\frac{m}{n(m-1)}\right] S + \left[\frac{1}{m-1}\right] v.$$

During a given period there may exist a discrepancy between the desired and actual stock which the consumer tries to rectify, but it is not assumed that the entire discrepancy will be closed within one period. Rather, it is assumed that only a fixed proportion r of this difference will be made up in any given period. This implies that net investment v is an increasing function of the size of the gap, as follows:

I-8 $$v = r\ (S^* - S).$$

When this assumption is applied to total durable purchases of an individual family, the behavioral implications appear reasonable, since durable acquisition frequently requires significant expenditures. A typical family is

---

[11] Ibid., equation 32, p. 430.

constrained by their income in relation to the cost when it comes to acquiring durables, so a choice as to which durables should be acquired in a given period must inevitably be made.

When applied to individual appliances, the rationale is not as clear cut. For many families the purchase decision is an "all or nothing" decision. They will either tend to purchase the unit this period or postpone the decision until a future period, such that r will have a value equal to 1 or 0, depending on whether or not the item in question was purchased. In an aggregate setting, r would tend to have a value between 0 and 1 depending on the frequency of purchase in relation to the total discrepancy between desired and actual stock.

The value of r is assumed to be constant from period to period and location to location in our formulations. However, a more realistic assumption would make r a variable which assumes different values for each year and location in the analysis. This variable (r) may then be considered to be influenced by causal factors such as family income and prices of the product.

In developing their model, Stone and Rowe encountered some difficulties which were later solved by Nerlove.[12] In particular, it was necessary to assume a value for n a-priori and, secondly, it was necessary to use up degrees of freedom in computing values of S and u for use in the model. The solution for the second problem just raised was contained in a substitution procedure which eliminated the need for direct computation of a series for S and u. The first problem of an arbitrary choice for n and consequently m was resolved by computing the regression under various

---

[12] M. Nerlove, op. cit.

specifications for n and m, where m was computed from an approximation based on n, and then selecting that value which yielded the highest R^2. This procedure yielded results which approximated maximum likelihood estimates.[13] In deference to these problems the model developed here follows the Nerlove adaptation closely. In fact, the portion of the model developed to this point incorporates some of the Nerlove influence, for the desired stock equation (I-1) is linear in contrast to a logarithmic formulation used by Stone and Rowe.

An equation for the end of period stock, ES, may be obtained from the definition of net investment (I-3)

I-9 $$ES = S + v,$$

which may be rewritten in terms of beginning of period stock and purchases by incorporating the equation for total purchases (I-2), and depreciation (I-6),

I-10
$$\begin{aligned} ES &= S + q - u \\ &= S + q - S/n - q/m \\ &= \left[\frac{n-1}{n}\right] S + \left[\frac{m-1}{m}\right] q. \end{aligned}$$

The net investment equation may be written as

I-11
$$\begin{aligned} ES - S &= r\,(S* - S) \\ ES &= r\,S* + (1-r)S, \end{aligned}$$

and this may be further simplified by incorporating the definition of ending stock (I-10) above.

I-12
$$\begin{aligned} \left[\frac{n-1}{n}\right] S + \left[\frac{m-1}{m}\right] q &= rS* + (1-r)S \\ q &= \left[\frac{m\ r}{(m-1)}\right] S* + \left[\frac{m(1 - r\ n)}{n(m-1)}\right] S \end{aligned}$$

---

[13] Ibid.

This equation could not be applied to empirical data as S* is not an observable magnitude, and data on stocks of appliances, which is currently unattainable, are required. If knowledge of purchases for a sufficiently long period of time were available, and the life (n) were known, a series for stocks could be computed from the approximation for m and the ending stock equation (I-10), which is a first order difference equation, expressible for the beginning stock after successive substitution, as follows:

I-13 $$S = \left[\frac{n\ (m-1)}{m\ (n-1)}\right] \sum_{\theta=1} \left[\frac{n-1}{n}\right]^{\theta} E^{-\theta} q$$

An easier alternative to direct computation of the stocks from this equation is substitution of this equation in the purchase equation (I-13) giving

I-14 $$q = \left[\frac{m\ r}{(m-1)}\right] S^* + \left[\frac{1-r\ n}{(n-1)}\right] \sum_{\theta=1} \left(\frac{n-1}{n}\right)^{\theta} E^{-\theta} q.$$

This yields after applying a Koyck transformation to simplify.

I-15 $$q = \left[\frac{m\ r}{(m-1)}\right]\left[S^* - \left(\frac{n-1}{n}\right) E^{-1}\ S^*\right] + (1-r)\ E^{-1} q.$$

The second problem of a non-observable S* is solved by substitution of our desired stock equation (I-1). This yields the final non-stochastic form for the model.

I-16 $$q = a' + b'\left[\rho - \left(\frac{n-1}{n}\right)E^{-1}\rho\right] + c'\left[(P/\pi) - \left(\frac{n-1}{n}\right)E^{-1}\ (P/\pi)\right] + r'\ E^{-1} q$$

where

$$a' = \left[\frac{m\ r}{m-1}\right]\left[a - \left(\frac{n-1}{n}\right) a\right]$$

$$b' = \left[\frac{m\ r}{m-1}\right] b$$

$$c' = \left[\frac{m\ r}{m-1}\right] c$$

$$r' = 1-r$$

3.3.2 Modifications of the Standard Model

A basic framework for a model has been developed, which must now be explored for possible modifications to improve its results when applied to the demand for household appliances. These modifications expand the model in two basic ways: (1) by introducing additional explanatory variables; and (2) by changing certain formulations within the model.

Of particular relevance for household appliances are the influences of liquid assets and other monetary variables, such as interest rates, on household appliance demand. Suits and Sparks,[14] in studying the influence of liquid assets on five categories (automobiles, other durables, food, other non-durables, and services), found that this variable[15] was a significant factor in explaining expenditures on the "other durable" category. An exact rationale as to why it was significant was not given, but it was strongly suspected that liquid assets were an indicator of the longer run status of households. This is in contrast to current income which is felt to exert its influence on the short run spending decision.

Evidence offered in support of this rationale includes Friedman's "permanent income" hypothesis[16] and Modigliani, Brumberg and Ando's "life cycle" hypothesis.[17] Although these theories were applied to explaining total consumption expenditures, their implications are felt to be relevant to appliance expenditures.

---

[14] D. Suits and G. Sparks, "Consumption Regressions with Quarterly Data," in The Brookings Quarterly Econometric Model of the United States, (J. Duesenberry, G. Fromm, L. Klein, and E. Kuh, eds.), Chicago: Rand McNally Company (1965).

[15] Liquid assets were defined as the sum of currency, demand deposits, and fixed value redeemable claims as estimated in the Federal Reserve Board's "Flow of Funds."

[16] M. Friedman, The Theory of the Consumption Function, Princeton; Princeton University Press (1967).

Further support for liquid assets as a determinant of the demand for household appliances was provided in a study by Klein.[18] Assets in liquid form were felt to be available for the purchase of durables or at least to provide the means for a down payment. Both this rationale and the previous one are felt to be sufficiently strong so as to make liquid assets a candidate to be tested in the models.

A further study of the influence of monetary variables on consumer durable expenditures was made by M. J. Hamberger.[19] As part of his study, the author attempted to estimate the role of liquid assets, which were separated into two categories: those reflecting actions taken by the monetary authorities, and those measuring consumer liquidity. Of particular interest is the latter category which included:

M_{c1}: the consumer stock of demand deposits plus currency

M_{c3}: M_{c1} plus consumer holdings of time and savings accounts at commercial banks, mutual savings banks, and savings and loan associations.

M_{c3} was found to be significant and accordingly will be the variable to be used in the current study.

There are two basic mechanisms through which liquid assets can act:

1) Directly, by influencing net investment,

2) Indirectly, by influencing the desired stock level.

---

[17] A. Ando and F. Modigliani, "The Life Cycle Hypothesis of Savings," American Economic Review, 53 (March, 1953), pp. 50-84.

[18] L. Klein, "Major Consumer Expenditures and Ownership of Durable Goods," Bulletin Oxford University Institute of Statistics, 15, (November, 1955), pp. 387-414. The first rationale was also used by Klein, "A Postwar Quarterly Model: Description and Applications," in Models of Income Determination, Princeton University Press, 1964, where liquid assets were considered as a proxy for total wealth.

[19] M. J. Hamberger, "Interest Rates and the Demand for Consumer Durable Goods, American Economic Review, 57, (December, 1967), p. 1144.

If L is assumed to represent the liquid asset in question at time t, the assumptions are expressable as

I-17(a) $$v = r(S^* - S) + dL$$

and

I-17(b) $$S^* = a + b + c\ (P/\pi) + dL$$

respectively.

After incorporation of I-17(a) in Equation I-16 of the model, the reduced form for estimation under this assumption becomes

I-18 $$q = a' + b'\left[\rho - \left(\frac{n-1}{n}\right)E^{-1}\ \rho\right] + c'\left[(P/\pi) - \left(\frac{n-1}{n}\right)E^{-1}(P/\pi)\right] + d'\left[L - \left(\frac{n-1}{n}\right)E^{-1}\ L\right] + (1-r)\ E^{-1}\ q,$$

where a', b', and c' are as initially defined and

$$d' = \frac{m\ d}{m-1}.$$

Under assumption I-17(b), the result is as follows:

I-19 $$q = a' + b'\left[\rho - \left(\frac{n-1}{n}\right)E^{-1}\ \rho\right] + c'\left[(P/\pi) - \left(\frac{n-1}{n}\right)E^{-1}(P/\pi)\right] + d'\left[L - \left(\frac{n-1}{n}\right)E^{-1}\ L\right] + (1-r)\ E^{-1}\ q,$$

where again a', b', and c' are as previously defined and

$$d' = \frac{m\ r}{m-1}\ d.$$

Although the definition of d' varies under the alternative assumptions, the equations estimated are the same under both assumptions and will be used in the present study. The difference between the two assumptions as point out by Hamberger is the interpretation attached to the parameters and the feasibility of the implied behavior.

One of the main purposes of the Hamberger study was to test the direct influence of interest rates on consumer durable expenditures. To test his hypothesis Hamberger introduced two basic interest rate measures: i_{Aaa}, the yield on Aaa rated long term corporate bonds, and i_{sv}, the yield on savings accounts. Both measures were found to be significant in determining expenditures on consumer durables other than automobiles with the yield on corporate bonds taking between four and six quarters for its maximum impact to be felt. The implication of these results is to suggest that this rate acts primarily as a proxy for rates charged on consumer credit rather than a measure of the substitutability of marketable financial assets and consumer durable goods.

The yield on savings accounts i_{sv} was significant, with no lag, indicating savings accounts were providing a substitute for durables other than automobiles. The same conclusion was reached via a more direct route by using the stock of liquid assets in other studies. Since liquid assets have been considered previously, this study will only test the influence of i_{Aaa} on household appliance demand. When i_{Aaa} (denoted simply by i) is inserted in the desired stock expression, the equation becomes

I-20 $$S^* = a + b\rho + c(P/\pi) + h\,i,$$

which yields a form for estimation as follows:

I-21 $$\begin{aligned} q = a' &+ b'\left[\rho - \left(\frac{n-1}{n}\right) E^{-1}\rho\right] + c'\left[(P/\pi) - \left(\frac{n-1}{n}\right) E^{-1}(P/\pi)\right] \\ &+ h'\left[i - \left(\frac{n-1}{n}\right) E^{-1}\, i\right] + (1-r)\, E^{-1} q, \end{aligned}$$

where a', b', and c' are again as defined by the initial S-R derivation and

$$h' = \left[\frac{m\ r}{m-1}\right] h.$$

An internal modification of this dynamic theory will allow for its application to demand in units and give a better explanation of replacement sales. Replacement sales play a significant role in the demand for the appliances as evidenced by figures from the 1969 Merchandising Week,[20] which showed estimated replacement sales for refrigerators, electric ranges, automatic washers and dryers to amount to 73%, 59%, 72%, and 43% respectively, based on a survey of retail outlets sampled.

The model, as initially presented, allowed for replacement sales directly by assuming new purchases were the sum of consumption, or depreciation of the existing stock, and net investment. Replacement sales were also considered indirectly in the net investment term, where net investment was assumed to be proportional to the discrepancy between desired and actual stock levels. Therefore, an individual who purchased a larger or newer appliance than he already had contributed to replacement sales.

As long as sales figures were expressed in dollar terms, the model as first presented could seek its justification by recourse to individual behavior alone. It was possible under these circumstances for a discrepancy between desired and actual stock to exist for a current appliance holder, to the extent that a larger or more expensive appliance than already held was desired. This discrepancy also may not have been fully closed by the purchase of a new appliance in that the purchaser may have had to settle for less than desired due to bedgetary constraints. If the sales were expressed in units, then the form of the model carries the implication that stock measures are correspondingly expressed. To see the rationale for this statement, it is noted that the final form for estimation contains current and lagged purchases. This has resulted from

---

[20] Merchandising Week, (February, 1970), pp. 32-33

the procedure of expressing the original variable, stocks, for which no direct data were available, as an infinite sum of past purchases minus depreciation, which was eventually reduced to its final form by a suitable transformation. However, because of the lagging procedure in the transformation, current purchases remain and, since these purchases went to build up the stock figure, are in the same unit of measurement as the stock.

The interpretation that must be given to the model when the data are in terms of units varies from the initial formulation. The basic change in interpretation is that the partial adjustment in stocks results from a constant proportion of those out of equilibrium purchasing new models to satisfy their discrepancy, rather than a single individual satisfying his discrepancy by a purchase. Since stocks are implied in units, it is no longer possible within the confines of the initial model to allow for partial adaptive behavior by an individual alone. In addition, the model becomes dichotomized between replacement sales and new purchases, as replacement sales are no longer indirectly considered. A discrepancy between desired and actual stocks would be allowed for only where there is a desire for an increase in the number of units.

Since the replacement sales are relegated to a direct role, when the sales are in units, a modified approach will be developed to give the model a more realistic interpretation. This modified approach recognizes a single average life span used in a declining balance depreciation formula is deficient as the sole explanation of replacement sales. It is acknowledged these sales do have as a base the average life of the asset, but in any given period household units may be expected to adjust the timing of their replacement sales in response to changing circumstances.

It is assumed that total sales now have a third component, which measures discretionary replacement sales, thus:

II-1 $$q = u + v + w,$$

where q, u, and v are as defined previously, and w is a measure of discretionary sales. These discretionary sales are not directly observable, but this problem is of no consequence since we will initially assume that they result from a short term adjustment based on current conditions, which are represented by the level of current disposable income ρ. The equation for w can then be written as

II-2 $$w = \alpha + \beta\rho.$$

The equations for v and u remain as formulated in (I-9) and (I-5). From equations (II-1), (I-2), and (I-3) the end of period stock may be written as follows:

II-3 $$ES = S + q - u - w,$$

which after substitution and algebraic rearrangement becomes

II-4 $$ES = \left[\frac{n-1}{n}\right] S + \left[\frac{m-1}{m}\right] q - w.$$

When this modified equation is substituted in equation (I-8) for net investment and the terms rearranged, the equation becomes

II-5 $$q = \left[\frac{m\ r}{m-1}\right] S* + \left[\frac{m(1-r\ n)}{n(m-1)}\right] S + \left[\frac{m}{m-1}\right] w.$$

The expression for S may again be replaced by an infinite weighted sum of past purchases, which, after transformation, yields an equation for q in terms of weighted first differences of desired stocks, and discretionary replacements as well as lagged sales.

II-6 $$q = \left[\frac{m\ r}{m-1}\right] \left[S* - \left(\frac{n-1}{n}\right) E^{-1}S*\right] + \left[\frac{m}{m-1}\right]\left[w - \left(\frac{n-1}{n}\right) E^{-1} w\right]$$
$$+ (1-r)\ E^{-1} q,$$

which in final form after substitution for S* and w and simplification becomes

$$\text{II-7} \qquad q = A' + B'\left[\rho - \left(\frac{n-1}{n}\right) E^{-1}\,\rho\right] + C'\left[(P/\pi) - \left(\frac{n-1}{n}\right) E^{-1}\,(P/\pi)\right] + (1-r)\,E^{-1}\,q,$$

where

$$A' = \left(\frac{m}{m-1}\right)(\alpha + r\ a) + \left(\frac{n-1}{n}\right)(\alpha + r\ a),$$

$$B' = \left(\frac{m}{m-1}\right)(\beta + r\ b),$$

$$C' = \left(\frac{m\ r}{m-1}\right).$$

The reduced form (II-7) is identical to the form under the initial assumption, but the definition of A', B', and C' is now different.

3.3.3 Habit Persistence: An Alternative Model

In making demand projections for the U.S., Houthakker and Taylor[21] have developed an alternative dynamic model. A major feature which differentiates this model is an assumption that the current level of demand is directly influenced by past behavior, while the stock adjustment models have assumed net investment takes place in response to a discrepancy between desired and actual stocks. In this habit persistence model, past behavior is assumed to be represented by a "state" variable S(t) which may encompass not only the stock in physical or value terms, but also psychological stocks which may have been built up over past periods. A prime example of the influence of psychological stocks is furnished by observation of new car purchasers. A-priori it would be expected that the level of stocks would exert a negative influence on the current level

[21] H. Houthakker and L. Taylor, Op. cit.

of demand, but recent studies have indicated just the opposite; new car sales are strongly influenced by buyers who trade every year. In its application to household appliances, this allows a more flexible approach than the stock adjustment models, for when the immeasurable "state" variable is removed by substitution, no implications as regards the units of measure in which demand is expressed remain. The demand equation which incorporates this new approach is

III-1 $$q(t) = a + b\,S(t) + c\,\rho(t),$$

where q (t) is the current level of demand, S(t) the "state" variable as previously defined, and $\rho(t)$ the current level of income which is initially assumed to be the only other independent variable. In empirical application other explanatory variables can be incorporated.

In the development of the reduced form of the model the unobservable "state" variable will be eliminated by utilizing appropriate substitutions. The substitution procedure requires the introduction of some definitions and assumptions. An initial definition required is

III-2(a) $$\dot{S}(t) = q(t) - u(t),$$

where $\dot{S}(t)$ is the change in the state variable at time t, q(t) purchases and u(t) depreciation. According to this formulation the positive influence on S(t) is confined to the "amount" of current purchases. A more flexible assumption, which is consistent with the possibility of psychological stocks, recognizes the stock may be adjusted by more or less than current purchases. This is given by

III-2(b) $$\dot{S}(t) = f \cdot q(t) - u(t) \qquad f>0,$$

where the factors tending to cause an increase in the state variable are assumed to be a fixed proportion f of current purchases.

The depreciation term represents the using up of the stock at time t, and is assumed to follow a declining balance method of depreciation. That is, depreciation for the t th time period is assumed to be a fixed proportion 1/n of the stock at time t, where n represents the asset's life in years.

$$\text{III-3} \qquad u(t) = (1/n)\ S(t)$$

This expression makes some concessions to reduce the complexity of the problem in that depreciation on current purchases is ignored. Theoretically, this would be justified only if all purchases were concentrated at the end of period. However, its omission will have only a minor influence on the empirical results obtained. It is noted that this assumption contrasts with the Stone-Rowe-Nerlove depreciation formulation of section 3.3.1 equation I-6 and their assumed uniform distribution of purchases.

The objective of the remaining calculations is to obtain a reduced form which will contain only measurable magnitudes. Equation (III-3) for depreciation is substituted in equation III-2(b) and S(t) is eliminated by using III-1 giving

$$\text{III-4} \qquad \dot{S}(t) = f \cdot q(t) - \left(\frac{1}{nb}\right) \left[q(t) - a - c\ \rho(t) \right].$$

Equation III-1 is assumed to be continuous and differentiable with respect to "t", so that upon differentiating we obtain

$$\text{III-5} \qquad \dot{q}(t) = b\ \dot{S}(t) + c\ \dot{\rho}(t),$$

where the dot above each symbol is to be interpreted as the derivative with respect to time. This equation is solved for $\dot{S}$ in explicit form and then used in equation III-4 yielding the following expression after simplification,

III-6 $$q(t) = a/n + (b\ f - (1/n))q(t) + c\ \dot{\rho}\ (t) + c/n\ \rho(t),$$

which is a first order differential equation involving only observable magnitudes.

The preceding derivation has been carried out by assuming the equations were continuous in nature. Since empirical data is in discrete form a suitable approximation must be made. Exact magnitudes for a given period r (which may be later defined as a year) are given by the following integrals.

$$\bar{q}_{t_o} = \int_{t_o}^{t_{o+r}} q(t)\ dt,$$

III-7 $$\bar{\rho}_{t_o} = \int_{t_o}^{t_{o+r}} \rho(t)\ dt,$$

$$\bar{S}_{t_o} = \int_{t_o}^{t_{o+r}} S(t)\ dt.$$

Since S(t) refers to a stock level which is not cumulative over time, $\bar{S}_{t_o}$ must be interpreted as an average stock level over the n th period.

Equation III-6 is a continuous function, so a discrete approximation for it must be obtained. The conversion is accomplished by integrating first for period t_o to t_{o+r} and then for period t_{o+r} to t_{o+2r}. Upon subtraction, the difference in equation III-1 between discrete periods is given by

III-8 $$\bar{q}_{t_{o+r}} - \bar{q}_{t_o} = b(\bar{S}_{t_{o+r}} - \bar{S}_{t_o}) + c(\bar{\rho}_{t_{o+r}} - \bar{\rho}_{t_o}).$$

The remaining portion of the calculations parallels the continuous case with the added assumptions that the between period change in the average stock level may be approximated by

III-9 $$\bar{S}_{t_{o+r}} - \bar{S}_{t_o} = (r/2)\ (\Delta{*}S_{t_{o+r}} + \Delta\ {*}S_{t_o}),$$

where ${}^{*}S_{t_{o+r}}$ and ${}^{*}S_{t_o}$ are the changes in the stock during the t_o and t_{o+r} periods respectively. This may be recognized as a linear approximation to the change in the average stock level between periods t_o and t_{o+r}. If the average stock (S_t) is changing according to some exact linear function, the approximation would produce a perfect fit. If this approximation is accepted and utilized in equation III-8, the result can be written as

$$\text{III-10} \qquad \bar{q}_{t_{o+r}} - \bar{q}_{t_o} = (r/2)\, b\, (\Delta^{*}S_{t_{o+r}} + \Delta^{*}S_{t_o}) + c(\bar{\rho}_{t_{o+r}} - \bar{\rho}_{t_o}),$$

which after substitution for $\Delta^{*}S_{t_{o+r}}$ and $\Delta^{*}S_{t_o}$ becomes

$$\text{III-11} \qquad \bar{q}_{t_{o+r}} - \bar{q}_{t_o} = (r/2)\, b \Big[f\, \bar{q}_{t_{o+r}} - (1/(nb))(\bar{q}_{t_{o+r}} - ar - c\bar{\rho}_{t_{o+r}}) + f\, \bar{q}_{t_o} - (1/(nb))(\bar{q}_{t_o} - ar - c\bar{\rho}_{t_o}) \Big] + c(\bar{\rho}_{t_{o+r}} - \bar{\rho}_{t_o}).$$

If r is specified to represent 1 on a time scale, equation III-1 becomes after simplification

$$\text{III-12} \qquad \bar{q}_{t_{o+r}} = \left[\frac{(a/n)}{1 - (1/2)[b\ f - (1/n)]}\right] + \left[\frac{1 + (1/2)[b\ f + (1/n)]}{1 - (1/2)[b\ f - (1/n)]}\right] \bar{q}_{t_o} + \left[\frac{c[1 + (1/2n)]}{1 - (1/2)[b\ f - (1/n)]}\right] \bar{\rho}_{t_{o+r}} + \left[\frac{c[1 - (1/2n)]}{1 - (1/2)[b\ f - (1/n)]}\right]$$

Finally, a variable with a subscript t_{o+r} is specified as pertaining to the t th period, and with subscript t_o to the t-1 period. If A_0, A_1, A_2, and A_3 are written for the coefficients above, the form for estimation becomes

$$\text{III-13} \qquad q_t = A_0 + A_1\, q_{t-1} + A_2\, \rho_t + A_3\, \rho_{t-1}.$$

In the original formulation Houthakker and Taylor wrote ρ_t as equal to $(\rho_t - \rho_{t-1}) + \rho_{t-1} = \rho_t + \rho_{t-1}$ to reduce the incidence of multi-collinearity. If this algebraic substitution is utilized, the form for

estimation becomes

III-14 $$q_t = A_0 + A_1 q_{t-1} + A_2 \Delta\rho_t + A_3 \rho_{t-1},$$

where now A_3 is specified to be

$$\frac{c/n}{1 - (1/2)\ (b\ f\ -\ (1/n))}\ .$$

3.4 An Intertemporal Utility Model[22]

Historically durable demand studies based on a concept of utility maximization have received little attention. Instead, application of demand systems have been concentrated primarily in the area of non-durables commodities. A motivating force behind this avoidance has been a desire to avoid the intertemporal problems inherent in analyzing commodities which are not fully consumed in the period of acquisition. However, this reasoning had some merit, for it allowed the author to bring into clearer focus the particular model under scrutiny.

In the investigation of the demand for household appliances a plausible rationale for the inherent intertemporal problems must be provided. Specifically, the existing stock, the life span of the durable, and expected future incomes and prices should be considered as well as the conventional variables of current income and prices. The general approach will be to impose restrictions which will reduce the demand equations thereby generated to estimatible form.

---

[22]The intertemporal model considered here will not attempt to describe the allocation of consumption over time, but rather it will try to incorporate the implication of stocks on current behavior. For a discussion of the broader problem see R. H. Strotz, "Myopia and Inconsistency in Dynamic Utility Maximization," Review of Economic Studies, 23, (1956), pp. 165-180. (This article attempts to describe the allocation process

The typical utility function may be written as

1) $$u = u(x_1, x_2, \ldots, x_n),$$

where x_i is defined as the consumption of the i th commodity. For durables, consumption has been defined as the fraction of the existing stock used, typically referred to as depreciation of the stock. The object of the demand equations then became consumption as opposed to the purchase of the durable good. As part of this investigation, a reformulation of the problem to that of durable purchases will be made. The first step will be to modify the utility function by redefining its arguments in terms of the "expected utility from purchases." As is readily seen, this approach ignores the utility derivable from the existing stock, while the budget constraint must also be reinterpreted to be consistent with this modification.

The conventional budget constraint, which may be written as

2) $$Y = \sum_{i=1}^{n} p_i x_i \qquad (i=1, \ldots, n),$$

deals with a consumer allocating a fixed money income (Y) among competing alternatives (x_i), subject to a fixed price (p_i). For a durable good the budget constraint as written implies that its entire price is paid for out of current income. This implication makes the assumption of a budget constraint in the form of equation 2 questionable. An alternative approach recognizes that only a portion of the price of the durable good must be outlayed in any given period. Using this alternative assumption, the revised budget constraint may be written as

---

through the use of the calculus of variations.) A summary of the classical Lagrangean analysis of the problem is contained in J. M. Henderson and R. E. Quandt, Microeconomic Theory, New York: McGraw-Hill Company, (1971), Chapter 8, pp. 293-333.

$$3)\qquad Y = \sum_{i=1}^{m} h\, p_i x_i + \sum_{j=m+1}^{n} p_j x_j ,$$

where x_i and x_j represent the quantities purchased of durable and non-durable goods respectively, p_i and p_j represent their prices and d represents the proportion of the total price of durables purchased that is paid out during the current period.[23] It is assumed h is a constant in our revised budget constraint (equation 3), but a broader interpretation, that will not be pursued here, could consider d as a variable dependent on other factors. The rate of interest should be a prime consideration as a factor which might have significant effect on d, as it is a measure of the cost of borrowing against future income.

The term "expected utility from purchase" requires a more concrete interpretation. For this, it is assumed the consumer under consideration has in mind a definite consumption pattern or use plan, consistent with the physical composition of the asset which will generate services in each year of asset ownership. The consumer then evaluates this service stream in terms of its utility worth today which will be referred to as $\hat{X}_i$. It is further assumed that $\hat{X}_i$ is proportional to the amount of current period purchases so that the utility function may be written as

$$4)\qquad U = u\,(\hat{X}_i,\, x_j) \quad (i=1,\, \ldots,\, m),\ (j=m+1,\, \ldots,\, n),$$

or upon substitution of the proportionality assumption as

$$U = u\,(\rho x_i,\, x_j) \quad (i=1,\, \ldots,\, m),\ (j=m+1,\, \ldots,\, n),$$

---

[23]This approach is an adaptation of an approach developed by Vernon Smith in an analysis of investment expenditures. In his approach he attempts to isolate the cost on current account for both indestructible and fixed life capital goods. See V. L. Smith, Investment and Production, Cambridge, Mass.: Harvard University Press, (1961), pp. 68-70 and 109-111.

where U is the redefined utility which we suppose the consumer to be maximizing, x_i and x_j are quantities purchased of durable and non-durable goods respectively, and ρ is the factor of proportionality.

The proportionality assumption can be supported in two alternative ways. On one hand, it can be supported if it is assumed the discount rate is zero and the asset services are released according to a declining balance method of depreciation. This may be demonstrated as follows:

a) It is assumed η is the depreciation rate and x_i and $\hat{X}_i$ are as previously defined.

b) With zero discount rate, we may write the "expected utility from purchase" as the sum of the service generated in each period the durable is expected to serve as follows:

5) $$\hat{X}_i = \eta \left[x_i + (1 - \eta)x_i + (1 - \eta)^2 x_i \ldots + (1 - \eta)^{n-1} x_i\right],$$

which becomes after collecting terms and simplifying

$$\hat{X}_i = \left[1 - (1-\eta)^n\right] x_i .$$

c) It is readily seen that $(1 - (1-\eta)^n)$ is a constant, and hence the proportionality assumption is maintained.

A second approach would be to assume some positive rate of discount, but with services being released in uniform amounts each period. Using the symbols as defined above, but using r for the discount rate and ϕ for the straight line depreciation rate, we can write

6) $$\hat{X}_i = (\phi x_i) \left[\frac{1}{(1+r)} + \frac{1}{(1+r)^2} \cdots + \frac{1}{(1+r)^n}\right].$$

Again, the above expression can be factored into a constant times x_i to obtain the desired result.

The assumption of a constant depreciation rate (η or ϕ) for all assets regardless of life, whether applied in a declining balance or straight line

manner, is a little hard to accept. A more palatable assumption is that the depreciation rate is a decreasing function of the life of the asset. Using ρ_i for the depreciation rate of the i th asset, this assumption is given by

7) $$\rho_i = \rho(t),$$

where

$$\partial\rho_i/\partial t < 0,$$

and where t is considered the life of the asset in years. A further assumption is that future income and prices will be regarded as known with certainty.

With these amendments to the standard utility maximization model, the problem may be stated as

maximize $U = u(\rho_i x_i, x_j)$ (i=1, ..., m; j=m+1, ..., n)

subject to $Y = \sum_{i=1}^{m} h\, p_i x_i + \sum_{j=m+1}^{n} p_j x_j.$

Forming the Lagrangean expression L and maximizing with respect to x_i, x_j and the Lagrangean multiplier λ gives the first order conditions

$$\partial L/\partial x_i = \rho_i \partial u/\partial x_i - \lambda h\, p_i x_i = 0 \qquad (i=1, \ldots, m),$$

$$\partial L/\partial x_j = \partial u/\partial x_j - \lambda p_j x_j = 0 \qquad (j=m+1, \ldots, n),$$

$$\partial L/\partial \lambda = Y - \left[\sum_{i=1}^{m} h\, p_i x_i + \sum_{j=m+1}^{n} p_j x_j\right] = 0.$$

It is readily seen that the properties of the revised demand equations will not be affected by the above changes. Specifically, the additivity restrictions can be assured as they were simply derived from a budget constraint which was unchanged under the proportionality assumption.

Further, the utility matrix

$$U = \begin{bmatrix} U_{11} & \cdots\cdots\cdots & U_{1n} \\ \vdots & & \vdots \\ U_{n1} & \cdots\cdots\cdots & U_{nn} \end{bmatrix}$$

will retain its symmetric nature under the modified version, so its inverse will also be symmetrical which in turn implies the symmetry relations (restriction IV - Section 3.2) will hold.

3.4.1 Constant Elasticity of Demand Systems

A choice must now be made as to the functional form for the demand equations. The simplest system to be considered falls under the heading of a constant elasticity of demand (CED) system. This system has in the past been one of the more commonly utilized forms, deriving its popularity from its ease of application and straightforward interpretation of the parameters as elasticity coefficients. The theoretical justification for the system lies in application of Taylor's Theorem, for this theorem states it is possible to approximate an elementary function by a Taylor series expansion. The constant elasticity formulation in essence assumes a first order approximation with the remainder terms being subsumed under the random error term in estimation. The system may be represented as

1) $$x_i = a_i y^{E_i} (\Pi_j \, p_j^{e_{ij}}) \quad (i, j = 1, \ldots, n)$$

in its non-linear form, or upon transformation by logarithms,

2) $$\log x_i = \log a_i + E_i \,(\log y) + \sum_j e_{ij} \,(\log p_j),$$

where x_i is the quantity of the i'th good, p_j, the price of the j'th good, y, total expenditure and E_i and e_{ij} are the Engel and Cournot elasticities respectively, which are assumed constant.

It is precisely this constancy assumption which has, on one hand, built the simplicity in the model and, on the other, rendered the form incompatible with traditional demand restrictions. Wold and Jureen[24] have stated in connection with demand relations that if the system if of the constant elasticity type then "unless all elasticities are equal to unity, such a function cannot satisfy the balance relation in the whole range of the variables involved." This was given in more explicit form by Zaman[25] as follows:

The budget share of the i'th commodity is defined as

3) $$w_i = \frac{p_i x_i}{y} ,$$

or in logs

4) $$\log w_i = \log p_i + \log x_i - \log y .$$

If prices are assumed constant and we consider the total differential of the function, we get

5) $$\begin{aligned} d\,(\log w_i) &= d\,(\log x_i) - d\,(\log y) \\ &= (E_i - 1)\ d(\log y), \end{aligned}$$

where the E_i is the income elasticity and its definition as $\frac{d(\log x_i)}{d(\log y)}$ is utilized. Carrying out the calculations we have

6) $$dw_i/w_i = (E_i - 1)\ dy/y.$$

---

[24]H. Wold and L. Jureen, op. cit., p. 106.

[25]A. Zaman, op. cit., pp. 98-99.

Now starting from an initial position of income y^0, prices p^0, and the i'th good budget share w_i^0, we can state that the budget share of any commodity must be bounded by 0 and 1, thus for the i'th commodity we have

$$0 < (w_i^0 + dw_i) < 1,$$

or or

$$-w_i < dw_i < 1 - w_i^0,$$

$$\text{Iff} \quad [(E_i - 1) - 1]y^0 < (E_i - 1)(y^0 + dy) < [(E - 1) + \frac{(1 - w_i^0)}{w_i^0}]\, y^0.$$

The sign of E_i then determines the limits within which the income, (y - dy), must remain if its budget share is to be a positive finite magnitude. Only if $E_i = 1$ will income lie between plus and minus infinity. Thus, if income elasticities are constant and are not all equal to unity, the budget constraint will be violated.

In this study we will try to salvage this system by considering only a subsystem of equations and not a complete system. This possibility was recognized by Wold and Jureen[26] who pointed out that for part of the field it would be perfectly possible to yield a demand function of the constant elasticity type. Actually, there are two alternatives which can be recognized here. On one level we can think of the elasticities and their values as a particular solution to the set of differential equations defining the demand functions. This approach was exploited by Zaman[27] in deriving two alternative systems which were consistent with utility maximization. The second approach which will be utilized

---

[26] H. Wold and L. Jureen, op. cit.

[27] A. Zaman, op. cit.

here is attributable to Court.[28] In this paper the author recognizes that additivity restrictions which are relevant in complete systems may be effectively ignored in subsystems. In a complete system this implies

$$7)\qquad \sum_i p_i x_i = y,$$

where p_i, x_i and y are as previously defined, or alternatively, upon replacing x_i by its demand function d_i, we have the more general specification

$$8)\qquad \sum_i p_i d_i = y.$$

The interpretation of this equation is that the true demand functions in a complete system must "add up" to fulfill the budget equation. This possibility under a constant elasticity specification was seen to prevail only under very unlikely circumstances. There is no reason, however, to assume that all demand equations are of the same form so that in studying a subset of the complete system it is plausible to assume that the other equations are of such a form so as to satisfy the additivity conditions without explicitly considering their particular formulation.

Additionally, since the CED system is an approximation to the true form, the system must be constructed so as to exhibit the other properties of homogeneity and symmetry. The homogeneity property is handled in traditional fashion by selecting a price from the n possibilities and deflating all other prices and income in the sybsystem by dividing through by the selected price. Symmetry in the system is then imposed by incorporating a set of exact linear restrictions.

---

[28]R. H. Court, "Utility Maximization and the Demand for New Zealand Meats," Econometrica, 35 (July-October, 1967), p. 424-446.

The elasticity of substitution between the i'th and j'th good is defined as

9) $$\sigma_{ij} = \frac{s_{ij}}{w_j},$$

where s_{ij} is the Slutsky income compensated elasticity of demand and w_j is the j'th good budget share. The symmetry conditions require

10) $$\sigma_{ij} = \sigma_{ji}\ .$$

By utilizing the Slutsky relation

11) $$s_{ij} = e_{ij} + w_j E_i$$

and the definitions of σ_{ij} the symmetry conditions can be expressed in Cournot elasticity terms, which has the advantage of being directly calculable. This gives

12) $$e_{ij}/w_j + E_i = e_{ji}/w_i + E_j.$$

The complete statement of the symmetry condition can then be made by forming a matrix

13) $$\underline{\bar{S}} = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ & \sigma_{22} & \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix},$$

which is symmetrical to satisfy restriction IV of Section 3.3, and negative semi-definite to insure maximization and homogeneity respectively.[29]

In application the symmetry conditions are not exact. Alternatives are then to use an exact formulation which would cause the restrictions to be of non-linear form or force a linear form by calculating the elasticities of substitution at given values of the budget proportions. The most

---

[29] P. A. Samuelson, op. cit., p. 113.

appropriate value being the mean of the budget proportion. The latter alternative appears to be the most promising and it was thus selected by Court and will be utilized here. The symmetry conditions may then be written as

$$k_j e_{ij} + E_i = k_i e_{ji} + E_j, \tag{14}$$

where the k_i and k_j are now calculated constants. The estimation problem is then to estimate n equations of the form

$$\log x_i = e_{i0} + \sum_{j=1}^{n} e_{ij} \log(p_j/p) + E_i \log(y/p) + \varepsilon_i \qquad (i = 1, \ldots, n), \tag{15}$$

where ε_i is the random error term, subject to the $(n/2)\cdot(n-1)$ restrictions of the form of equation (14). Details of these computations are given in the next chapter.

3.4.2 Linear Expenditure System

A popular form for demand functions has been the linear expenditure system, which is derivable from a particular form for the underlying utility function. It is considered here because it exemplifies the approach of deriving demand equations from specific utility functions, and the behavior implied by its form appears reasonable in the context of household appliances. In addition, this system aggregates perfectly over both individuals and commodities due to the linearity of the Engel curves. The system itself was proposed by Klein and Rubin,[30] while its inherent utility basis was proveded by Samuelson[31] and Geary.[32]

---

[30] L. R. Klein and H. Rubin, "A Constant-Utility Index of the Cost of Living," Review of Economic Studies, 15, (1947), p. 84-87.

Specifically, the demand equations of the linear expenditure system may be represented in their non-stochastic form as

1) $$x_i = c_i - \left[a_i/p_i\right] \sum_{k=1}^{n} p_k c_k + \left[a_i/p_i\right] Y,$$

while the resulting expenditure equations are

2) $$p_i x_i = p_i c_i + a_i \left(Y - \sum_k p_k c_k\right) \quad (i=1, \ldots, n),$$

where p_i represents prices, Y is income, x_i the quantity of the i'th good demanded, while the c_i represent what is referred to as the subsistence bundle of the i'th good. The consumer may be thought of as first allocating his income to the subsistence bundle and then determining the "additional" expenditure based on his remaining income.

The utility function underlying this form is of the type

3) $$U(x) = \sum_{i=1}^{n} a_i \log (x_i - c_i) \qquad a_i > 0;$$

$$\sum_{i=1}^{n} a_i = 1; \qquad x_i - c_i > 0.$$

Since the demand functions are directly derivable from the above utility function we may be sure that these demand functions exhibit the desirable properties of homogeneity, additivity, and symmetry. The only parametric restriction required is

4) $$\sum_{i=1}^{n} a_i = 1$$

to satisfy the Engel aggregation condition, and this is applicable to complete systems only.

---

[31] P. A. Samuelson, "Some Implications of Linearity," Review of Economic Studies, 15, (1947), pp. 88-90.

[32] R. C. Geary, "A Note on 'A Constant-Utility Index of the Cost of Living'," Review of Economic Studies, 18, (1949), pp. 65-66.

The utility function as stated is of the directly additive variety, thus limiting its scope of application. This is pointed out by Houthakker who states that additivity reduces the scope of substitution and complimentarity to the barest minimum.[33] Under direct additivity the substitution effect between the i'th and k'th commodity may be written as

$$K_{ik} = (-\gamma)\ (\partial x_i/\partial Y)\ (\partial x_k/\partial Y), \tag{5}$$

where K_{ik} is the substitution effect, and γ is the money flexibility parameter, while its own-price substitution effect is

$$p_i K_{ii} = \gamma(\partial x_i/\partial Y)\ (1 - p_i \cdot \partial x_i/\partial Y). \tag{6}$$

In the first case, the cross price substitution effect is seen to be proportional to the income derivatives of both commodities and to the income flexibility, while in the second case its own-price substitution effect is proportional to the money flexibility parameter, income derivative, and marginal propensity to spend on commodities other than the i'th. From these conditions Houthakker has noted that inferior goods and complements are ruled out while the substitution effect is relegated to the more general type in the sense of competing for a place in the consumer's budget.

It has been felt that by choosing broad aggregate classifications the above conditions could be more readily met. Appliances, however, can be seen to possess many of the desirable characteristics. No previous studies have found appliances to be an inferior type good, nor would we a-priori expect inferiority to be the case. In considering appliances in terms of three categories, refrigerators-freezers, ranges and ovens, and laundry products, which reflect their service functions, the desired

---

[33]H. S. Houthakker, "Additive Preferences," Econometrica, 27, (1960), pp. 244-257.

characteristics of substitution and complementarity may be met. The groups are not complements in the technical sense of function performed nor are they directly substitutable in terms of their natural function. In a given year we would expect each group to compete for a place in the consumer's budget as required by the general substitution idea.

The original empirical implementation of the linear expenditure system by Stone[34] suffered from some deficiencies which were later pointed out by Parks[35] and Pollak and Wales.[36] These concerned the stochastic properties of the system which will be considered in the next chapter, and the lack of a satisfactory explanation for the "subsistence bundle" of purchases. Stone recognized the problem and attempted to incorporate a time trend in explanation,

7) $$c_{it} = c_i^* + c_i t.$$

Pollak and Wales have suggested an alternative which appears more promising. The effect of habit formation can be incorporated by basing current period subsistence purchases on last period's purchases,

8) $$c_{it} = c_i^* + c_i x_{i,t-1},$$

or as a modification of this form

9) $$c_{it} = c_i^* + c_i z_{i,t-1},$$

where $z_{i,t-1}$ represents a variable such as average consumption of the i'th commodity over a number of years or the highest attained level of

---

[34] J. N. R. Stone, "Linear Expenditure Systems and Demand Analysis," Economic Journal, 64, (1954), pp. 511-527.

[35] R. W. Parks, "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms," Econometrica, 37, (1969), pp. 629-650.

[36] R. A. Pollak and T. J. Wales, "Estimation of the Linear Expenditure System," Econometrica, 37, (1969, pp. 611-628.

consumption during an appropriately selected lag period.

The expenditure equation for appliances provides the opportunity to incorporate a habit persistence mechanism of the Houthakker and Taylor type considered in Section 3.3.3. The subsistence bundle, which could be interpreted as a type of replacement sales, can be written as a linear function

10) $$c_{it} = c_i^* + c_i S_i(t),$$

where c_i^* is a constant, while $S_i(t)$ is a state or stock variable as before with coefficient c_i. Upon substituting this equation in the basic demand equation of the linear expenditure system we have

11) $$x_i = c_i^* + c_i S_i(t) + b_i[(Y_t - \sum_k (p_{kt} c_{kt})/p_{it}].$$

This equation corresponds very closely to equation III-1 of section 3.3.3 presented earlier, with the exception of income which is now interpreted as a type of "supernumerary income" deflated by current price. The corresponding expenditure equation becomes

12) $$p_{it} x_{it} = c_i^* + c_i p_{it} S_i(t) + b_i \left[Y_t - \sum_k p_{kt} c_{kt}\right].$$

Utilizing the definitions and eliminations presented earlier and defining $\rho_i = \frac{Y_t - \sum_k p_{kt} \cdot c_{kt}}{p_{it}}$, the equation for estimation in non-stochastic form becomes

13) $$x_{it} = A_0 + A_i x_{i,\, t-1} + A_2 \rho_t + A_3 \rho_{t-1},$$

which is identical in form to equation III-13 of Section 3.3.3.

3.5 Summary

In summary, it is observed that the "ad hoc" and utility maximization models have some relationship. A clear example was given by the "ad hoc" models of the H & T habit persistence type which were found to have their implicit roots in utility maximization, as was evidenced by their link-up with the linear expenditure system. However, it must be pointed out that those models which rely on an explicit formulation of the utility function have a built-in drawback in the arbitrariness that marks any choice of utility function. As is often the case, the empirical nature of the hypothesis restricts testing to proving only whether the implied behavior is plausible. The CED system, as opposed to the linear expenditure system, presents no claim to justification from an explicit utility function. Its only claim is that of an approximation which is consistent with utility maximization. The final choice as to which is the "true" demand function must remain unanswered as no one can know the truth with certainty. All that can be done in this paper is to present the theoretical merits of each and their empirical results so an appropriate choice can be made.

Chapter 4

Stochatic Specification and Estimation Methods

4.1 Introduction

Each of the models considered to this point in time has been presented in only a non-stochastic form. Of these models we see that both "ad hoc" models have attempted to incorporate a "stock" effect for durables into their computations, while the utility based models have more or less accepted the stock as given and attempted to explain the consumer expenditure allocation in terms of utility maximization behavior. The "ad hoc" models have been extensively utilized in many demand studies for particular goods, while of the utility based models, only the CED model has had considerable application for this purpose. In addition, the CED applications have been of limited extent, for demand restrictions, notably symmetry, have not been generally included.[1] The linear expenditure system model has been applied primarily to the estimation of complete demand systems.

The original Stone and Rowe model was applied to the determination of the demand for clothing and household durable goods in the United Kingdom. Fairly good results were obtained in that the regressions accounted for around 90% of the observed variance in each case. In a later study utilizing the amendments as suggested by Nerlove, Stone

---

[1]A significant exception to this was the application to a "non-durable" (meats) in New Zealand by Robin Court, op. cit. It is this article which has suggested the possibility of its extension to durable commodities.

and Rowe again found a good fit when applying their model to British data for various categories of household durables. Houthakker and Taylor likewise have found reasonably good results in application of their model to demand projections for the U.S.. One of the many constant elasticity formulations, which is mentioned here due to its popularity, is attributable to G. C. Chow, who studied the demand for automobiles in the U.S. and also found a "good fit" for his model.

In view of the seemingly good results which each of the models have enjoyed, the choice as to which is the "best" demand model must obviously be made on grounds other than simple comparisons of R^2s or other singular statistic derived from the data. All that can be expected from an application is affirmation that the model is a logical candidate for consideration. The final choice must then rest upon a comparative consideration of the theoretical underpinnings of the models, the plausibility of their stochastic specifications, and their ability to make accurate forecasts. The theoretical considerations have been the topic of the previous chapter. This leaves the task of formulating the various stochastic specifications to which the model will be subjected, and the development of estimation methods compatible with these specifications, to the present chapter.

The "ad hoc" models will be estimated using both aggregate U.S. data for a twenty-one year period, and data by region for an eleven year period, while the utility based models will be estimated using only the aggregate U.S. data. Our first consideration will be to specify stochastic assumptions for the "ad hoc" models, which are applicable to the aggregate data, and to suggest some estimation methods which consider these specifications.

Following this summary, the additional stochastic specifications involved, when the combined time-series and cross-sectional data are utilized, are presented, and a procedure for incorporating these specifications into the estimation method is developed.

In regard to the "utility based" models, we will attempt to specify stochastic specifications and estimation methods compatible with the particular model and data under consideration. Specifically, stochastic specifications for the CED model, when it is to be applied to aggregate U.S. data, are examined. An estimation procedure which incorporates these specifications, and the symmetry restrictions given in Chapter 3, is then developed. Some additional problems in the estimation of the linear expenditure system are also explored.

4.2 "Ad Hoc" Models

4.2.1 Standard Specifications

The "ad hoc" models derived in equations I-16 and III-13 in Chapter 3 are repeated here in their reduced form as follows:

1a) Stone and Rowe - Nerlove

$$q = A' + B'\left[\rho - \left(\frac{n-1}{n}\right)E^{-1}\rho\right] + C'\left[(p/\pi) - \left(\frac{n-1}{n}\right)E^{-1}(p/\pi)\right] + r'E^{-1}q\,,$$

1b) Houthakker and Taylor

$$q = A_0 + A_1\rho + A_2E^{-1}\rho + A_3(p/\pi) + A_4E^{-1}(p/\pi) + A_5E^{-1}q\,, \quad (2)$$

where q represents the quantity purchased, ρ represents income, and p/π represents relative price.

---

[2]The Houthakker and Taylor model shown here includes relative price (p/π) as an explanatory variable.

If the simplest stochastic specification concerning these forms is made, the assumption of a random error term measuring the cumulative effect of all remaining influences is attached additively to each reduced form giving

2a) Stone and Rowe — Nerlove

$$q = A' + B'\left[\rho - \left(\frac{n-1}{n}\right) E^{-1}\rho\right] + C'\left[(p/\pi) - \left(\frac{n-1}{n}\right)E^{-1}(p/\pi)\right] + r'E^{-1}q + \varepsilon_t,$$

2b) Houthakker and Taylor

$$q = A_0 + A_1\rho + A_2E^{-1}\rho + A_3(p/\pi) + A_4E^{-1}(p/\pi) + A_5E^{-1}q + \varepsilon_t.$$

The standard assumptions concerning ε_t is that it is a normally distributed random variable with the following specifications:

1.	$E(\varepsilon_t) = 0$	Zero mean
2.	$E(\varepsilon_t\varepsilon_s) = 0 \quad (t \neq s)$	No autocorrelation
3.	$E(\varepsilon_t^2) = \sigma^2$	Homoskedasticity

Estimation can be carried out utilizing ordinary least squares (OLS), or, if desired, the lack of autocorrelation specification may be dropped implying that the estimation procedure should be amended to include an adjustment for removal of the time effect before application of OLS.

4.2.2 Alternative Stochastic Specifications

The annexing of a random error term to the reduced form renders these equations suitable for estimation, but their interpretation from a behavioralistic point of view is somewhat questionable. To alleviate this situation an error structure must be incorporated to recognize that

the final reduced structure is a combination of behavioralistic and purely definitional equations. The consequences for estimation of the reduced form must then be investigated for some of the standard stochastic specifications (e.g., homoskedasticity, etc.) may be lost by the manipulations.

As an initial step, consider the desired stock equation III-1 of 3.3.1

3) $$S^*_t = a + b\rho_t + c(p/\pi)_t.$$

This equation is intended to define some desired level to which the individual is striving. An error term u_{1t}, appended additively, may be interpreted as the cumulative effect of factors other than those explicitly considered. This gives

4) $$S^*_t = a + b\rho_t + c(p/\pi)_t + u_{1t}.$$

If the rest of the equation of the S-R-N model is considered to hold without modification, the reduced form associated with this specification may be written as

5) $$q_t = a' + b'\left[\rho_t - \frac{n-1}{n}\rho_{t-1}\right] + c'\left[(p/\pi)_t - \left(\frac{n-1}{n}\right)(p/\pi)_{t-1}\right] + r'q_{t-1} + \left[\frac{mr}{m-1}\right]\left[u_{1t} - \left(\frac{n-1}{n}\right)u_{1,t-1}\right].$$

Since m, r, and n are considered as fixed parameters of the model, the composite error term

$$\varepsilon_{1t} = \left[\frac{mr}{m-1}\right]\left[u_{1t} - \left(\frac{n-1}{n}\right)u_{1,t-1}\right]$$

may be looked upon as a linear combination of normally distributed random variables which is also a normally distributed random variable.

A second alternative in the S-R-N model is to assume the adjustment of actual to desired stock as given by equation I-8 of 3.3.1 includes an error term u_{2t} which is intended to measure the random influence in the adjustment process.

6) $$v = r(s^* - s) + u_{2t}$$

The reduced form associated with this specification is

7) $$q_t = a' + b'\left[\rho_t - \left(\frac{n-1}{n}\right)\rho_{t-1}\right] + c'\left[(P/\pi)_t - \left(\frac{n-1}{n}\right)(P/\pi)_{t-1}\right] + r'q_{t-1} + \left[\frac{m}{m-1}\right]\left[u_{2t} - \left(\frac{n-1}{n}\right)u_{2,t-1}\right],$$

where the composite error term is given by

$$\varepsilon_{2t} = \left[\frac{m}{m-1}\right]\left[u_{2t} - \left(\frac{n-1}{n}\right)u_{2,t-1}\right].$$

Combining both specification simultaneously will produce a combined error term

8) $$\varepsilon_t = \left[\frac{m}{m-1}\right]\left[(u_{2t} + ru_{1t}) - \left(\frac{n-1}{n}\right)(u_{2,t-1} + ru_{1,t-1})\right].$$

The primary error specification in the H&T habit persistence model involves attaching the error to equation III-1 of 3.3.3 which describes the current level of demand in terms of its explanatory variables. The error term can be considered as encompassing other influences not explicitly included in the formulation. This equation, including the error term ε_t, may be written as

9) $$q(t) = a + b\ S(t) + c\ \rho(t) + \varepsilon(t).$$

Defining the discrete analog of $\varepsilon(t)$ for the <u>rth</u> period as

10) $$\bar{\varepsilon}_{to} = \int_{t_o}^{t_{o+n}} \varepsilon(t)dt$$

and utilizing this definition in the derivation of the reduced form produces

11) $$q_t = A_0 + A_1 q_{t-1} + A_2 \Delta\rho_t + A_3 \rho_{t-1} + Z_{1t},$$

where Z_{1t} is the composite error term given as

$$Z_{1t} = \left[1 - 1/(2n)\right] \varepsilon_t - \left[1 + 1/(2n)\right] \varepsilon_{t-1}.$$

An alternative specification would identify the random error term as occurring in equation III-2b of 3.3.3 which describes the change in the state variable during period t. This modified version can be written as

12) $$\dot{S}(t) = fq(t) - u(t) + \varepsilon_t,$$

where ε_t is the disturbance term which was annexed. The interpretation of this equation is that adjustment in the state variable occurs in relation to the net purchases of the period, while the ε_t measures the random influence in the adjustment from period to period. The reduced form will be the same as equation III-14 except for the disturbance term which becomes

13) $$Z_{2t} = (1/2)(\varepsilon_t + \varepsilon_{t-1}).$$

If both specifications hold simultaneously, the composite error would then become

14) $$Z_{3t} = Z_{1t} + Z_{2t} = \left[1 + 1/2 - 1/(2n)\right] \varepsilon_t - \left[1 + 1/2 + 1/(2n)\right] \varepsilon_{t-1}.$$

All of the previous specifications have resulted in reduced forms of the general type

15) $$Y_t = \alpha + \beta X_t - \lambda Y_{t-1} + u_t - \lambda u_{t-1},$$

where Y_t is the endogenous variable to be explained and X_t and Y_{t-1} are the explanatory variables. The equation may be recognized as the reduced form resulting from the application of a Koyck transformation to a geometrically distributed lag equation. Three assumptions regarding the composite disturbance

16) $$\varepsilon_t = u_t - \lambda u_{t-1}$$

have been recognized in the literature.[3] The simplest assumption would assume the u_t follow a first order autoregressive process

17) $$u_t = \rho u_{t-1} + Z_t$$

with $\rho = \lambda$, where the Z_t are assumed to be independent, normally distributed random variables with zero mean and constant variance.

$$Z_t \sim N(0,\sigma_z{}^2)$$

The composite error term would then become

18) $$\varepsilon_t = Z_t$$

making OLS the appropriate method of estimation.

---

[3]See A. Zellner and M. Geisel, "Analysis of Distributed Lag Models with Applications to Consumption Function Estimation," Econometrica, 38 (Nov., 1970), pp. 865-889, for a discussion of these assumptions and their implications for estimation. This section will rely heavily on their work. An excellent summary of these specifications and estimation of distributed lag equations is contained in J. Kmenta, Elements of Econometrics, Macmillan, 1971.

The second assumption regarding the error term is that the u_t's are normally and independently distributed variables with zero mean, constant variance, and no correlation over time. This implies certain desirable properties of u_t carry over to ε_t as follows:

1. Normality: Since ε_i is a linear combination of normally distributed random variables, it too is normally distributed.
2. Unbiasedness: $E(\varepsilon_t) = E(u_t) - \lambda E(u_{t-1}) = 0.$
3. Homoskedasticity:

$$E(u_t - \lambda u_{t-1}) = E(u_t^2) + \lambda^2 E(u_{t-1}^2) - 2\lambda E(u_t u_{t-1})$$
$$= \sigma^2 + \lambda^2\sigma^2 = \text{a constant.}$$

However, it is no longer appropriate to claim a complete lack of autocorrelation as demonstrated by the relationship between ε_t and ε_{t-1}.

$$\text{19)}\quad E(\varepsilon_t\varepsilon_{t-1}) = E(u_t u_{t-1}) + \lambda^2 E(u_{t-1}u_{t-2}) - \lambda E(u_t u_{t-2}) - \lambda E(u_{t-1}^2)$$
$$= -\lambda\sigma^2.$$

In addition, ε_t is no longer independent of the regressors as may be shown by the relationship between ε_t and Y_{t-1}.

$$\text{20)}\quad E(\varepsilon_t Y_{t-1}) = E(u_t - \lambda u_{t-1})(\alpha + \beta X_{t-1} + \lambda Y_{t-2} + u_{t-1} - \lambda u_{t-2}) = -\lambda\sigma^2.$$

The lack of independence renders the OLS estimates inconsistent. For this reason alternative methods of estimation must be sought.

The last specification to be considered follows closely the lead of the first in that it is assumed the u_t's are $\sim N(0,\sigma^2)$, and follow a first order autoregressive process. However, it is not assumed the autoregression parameter ρ is equal to λ. As with the second case, normalcy, unbiasedness,

and homoskedasticity carries over to ε_t, while the ε_t are now autocorrelated over time and not independent of the regressors.

In this study we will assume that the first specification presented ($\rho=\lambda$) is applicable. This choice will avoid the problem of having to specify the elements of the variance-covariance matrix of disturbances and make it possible to use OLS to estimate the "ad hoc" models from the aggregate data. In addition, previous demand studies, utilizing the "ad hoc" models, have proceeded on the basis of this implicit assumption.[4]

4.2.3 A Procedure for Combining Cross-Section and Time Series Observations

As part of our investigation the possibility of extending the model by expanding the data base must be explored. If the first part of this paper is thought of as building a model which is theoretically consistent, then this part must be considered as an attempt to refine the estimates by supplying additional data to which the model may be applied. Initial attempts at estimating regression parameters from cross-section and time series observations proceeded by using cross-section data, such as those for states, firms, or households, to derive some estimates, and then following up by holding these estimates constant at the computed values while estimating the remaining parameters from time series observations.[5] An alternative procedure which we shall follow involves simultaneous estimation of all regression coefficients from pooled time series and cross-section data. Specifically it is our intention to utilize the data for

---

[4]See for example R. Stone and D. A. Rowe, op. cit. and M. Hamberger, op. cit..

[5]See for example H. Staehle, "Relative Prices and Postwar Markets for Animal Food Prices," Quarterly Journal of Economics, 59 (1944-45), pp. 237-279. A good summary of the procedure can be found in L. Klein, A Textbook of Econometrics, (Evanston, Illinois: Rowe Peterson & Co., 1953).

individual selected regions for a number of years. To this end we will develop a procedure that leads to the application of the generalized linear regression method. In the remainder of this paper this method will be referred to as the "Two Stage Generalized Linear Regression" method (TSGLR).

The earliest attempt at providing estimates from pooled data used an "analysis of covariance" technique.[6] The essence of this technique is to provide dummy variables for the firm and time effects. This is shown as

$$Y_{ij} = Z_i + W_j + \sum_{r=1}^{k} \beta_r X_{r,ij} + U_{ij} , \tag{21}$$

where Z_i is intended to be a dummy variable representing the firm effects, W_j a dummy representing the time effect and $X_{r,ij}$ a variable which varies over both firm and time dimensions. This technique will produce estimates which are both unbiased and efficient. However, there are some drawbacks of the method, as pointed out by Maddala.[7] The use of an extremely large number of "dummy" variables may tend to eliminate large portions of the variation between the dependent and explanatory variables. This would be especially true if the between firm or time period variation is large. In addition, there is a loss of degrees of freedom due to the large number of independent variables, and the interpretation of the dummy variables is awkward.

---

[6]See for example I. Hoch, "Estimation of Production Function Parameters Combining Time-Series and Cross-Section Data", Econometrica, 30 (January, 1962), pp. 34-53; P. R. Johnson, "Some Aspects of Estimating Statistical Cost Functions", Journal of Farm Economics, 46 (February, 1964), pp. 179-187; Y. Mundlak, "Estimation of Production and Behavioral Functions From A Combination of Cross-Section and Time-Series Data", in Measurement and Economics, (C. F. Christ, Ed.), Palo Alto, California: Stanford University Press, (1963), pp. 138-166.

[7]G. S. Maddala, "The Use of Variance Components Models in Pooling Cross-Section and Time-Series Data", Econometrica, 39 (March, 1971), pp. 341-359.

An alternative approach is known as the "error component method."[8] This method assumes that the regression equation can be written as

22) $$Y_{it} = \alpha + \sum_{r=1}^{k} \beta_r X_{r,it} + \varepsilon_{it} ,$$

where α and β_r $(r = 1, \ldots, k)$ are the intercept and slope parameters respectively, Y_{it} represents the independent variable, $X_{r,it}$ represents the explanatory variables, and the error term ε_{it} is now composed of three parts as follows:

23) $$\varepsilon_{it} = U_i + V_t + W_{it} \quad (i = 1, \ldots, n_j),(t = 1, 2, \ldots, T) ,$$

where the parts are assumed to represent random components with U_i representing the firm effects, V_t representing the time effect, and W_{it} representing a component varying over both dimensions. Since the components are random variables, we make the further assumption that each component is normally distributed with 0 mean and constant variance σ_u^2, σ_v^2, and σ_w^2 respectively. In addition, the following inter-relationship between components is specified:

24) $$E(U_iV_t) = E(U_iW_{it}) = E(V_tW_{it}) = 0,$$
$$E(U_iU_j) = 0 \text{ for } i \neq j,$$
$$E(V_tV_s) = 0 \text{ for } t \neq s,$$
$$E(W_{it}W_{is}) = E(W_{it}W_{jt}) = E(W_{it}W_{js}) = 0.$$

If these assumptions are investigated for their implications it may be shown that the variance of the composite error term ε_{it} is a constant.

---

[8] T. D. Wallace and A. Hussain, "The Use of Error Components Models in Combining Cross Section with Time Series Data", Econometrica, 34 (July, 1966), pp. 585-612.

This is given as follows:

$$25)\qquad E(\varepsilon_{it}^2) = E(U_i + V_t + W_{it})^2$$
$$= \sigma_u^2 + \sigma_v^2 + \sigma_w^2.$$

The covariance between cross sectional units is

$$26)\qquad E(\varepsilon_{it}\varepsilon_{jt}) = E(U_i + V_t + W_{it})(U_j + V_t + W_{it})$$
$$= \sigma_v \quad \text{for } (i \neq j),$$

and the covariance over time is given by

$$27)\qquad E(\varepsilon_{it}\varepsilon_{is}) = E(U_i + V_t + W_{it})(U_i + V_s + W_{is})$$
$$= \sigma_u \quad \text{for } (t \neq s).$$

According to (27), the covariance over time of the error term (ε_{it}) is the same for any two time periods. The implication of constant autocorrelation effect holds no matter how far the periods were separated in time. This is in contrast to the general consensus that regards the autocorrelation effect as diminishing as the distance in time of the errors increases, as would be exemplified by a first-order autoregressive scheme. The second objection involves the assumption of homoskedasticity. Error component model assumptions were seen to generate a variance of $\sigma_u^2 + \sigma_v^2 + \sigma_w^2$. The assumption of the same variance of the errors for two distinct regions again is hard to accept, especially when utilizing data by regions with extreme geographic and economic differences. A third problem arises from the constant covariance between cross sectional units. In the 'error component' approach this was shown to be σ_v^2 for two distinct cross sectional units. A much more flexible assumption

would allow the interdependence between cross sectional units to vary depending on the two units under consideration.

A method, suggested by Kmenta,[9] will be utilized in this study. The advantage of this approach is that it avoids the three unacceptable implications referred to above. This method uses the generalized linear regression method, where the estimates of the variance-covariance matrix of disturbances are based on residuals from OLS estimated equations. The estimates of the regression coefficients given by this method will be both consistent and asymptotically efficient.

The variance-covariance matrix of residual error terms (Ω) can be written as

$$\Omega = \begin{bmatrix}
E(\varepsilon_{11}^2) & E(\varepsilon_{11}\varepsilon_{12}) & \cdots & E(\varepsilon_{11}\varepsilon_{1T}) & E(\varepsilon_{11}\varepsilon_{21}) & E(\varepsilon_{11}\varepsilon_{22}) & \cdots & E(\varepsilon_{11}\varepsilon_{NT}) \\
E(\varepsilon_{12}\varepsilon_{11}) & E(\varepsilon_{12}^2) & \cdots & E(\varepsilon_{12}\varepsilon_{1T}) & E(\varepsilon_{12}\varepsilon_{21}) & E(\varepsilon_{12}\varepsilon_{22}) & \cdots & E(\varepsilon_{12}\varepsilon_{NT}) \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
E(\varepsilon_{1T}\varepsilon_{11}) & E(\varepsilon_{1T}\varepsilon_{12}) & \cdots & E(\varepsilon_{1T}^2) & E(\varepsilon_{1T\ 21}) & E(\varepsilon_{1T\ 22}) & \cdots & E(\varepsilon_{1T}\varepsilon_{NT}) \\
E(\varepsilon_{21}\varepsilon_{11}) & E(\varepsilon_{21}\varepsilon_{12}) & \cdots & E(\varepsilon_{21}\varepsilon_{1T}) & E(\varepsilon_{21}^2) & E(\varepsilon_{21}\varepsilon_{22}) & \cdots & E(\varepsilon_{21}\varepsilon_{NT}) \\
E(\varepsilon_{22}\varepsilon_{11}) & E(\varepsilon_{22}\varepsilon_{12}) & \cdots & E(\varepsilon_{21}\varepsilon_{1T}) & E(\varepsilon_{22}\varepsilon_{21}) & E(\varepsilon_{22}^2) & \cdots & E(\varepsilon_{22}\varepsilon_{NT}) \\
 & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
E(\varepsilon_{NT}\varepsilon_{11}) & E(\varepsilon_{NT}\varepsilon_{12}) & \cdots & E(\varepsilon_{NT}\varepsilon_{12}) & E(\varepsilon_{NT}\varepsilon_{21}) & E(\varepsilon_{NT}\varepsilon_{22}) & \cdots & E(\varepsilon_{NT}^2)
\end{bmatrix},$$

where the ε is a NT x 1 vector of residuals with element ε_{it}, where the first subscript i refers to the cross sectional unit and the second t,

---

[9] J. Kmenta, Elements of Econometrics, Macmillan, 1970, Chapter 12, pp. 508-517.

to the time period. If the elements of this matrix were known a-priori then we could utilize this information in the Aitken formulas,

$$\tilde{\beta} = (X' \Omega^{-1} X)^{-1} (X' \Omega^{-1} y) \tag{29}$$

with variance-covariance matrix

$$\text{Var-Cov } (\tilde{\beta}) = (X' \Omega^{-1} X)^{-1}. \tag{30}$$

Since this information is not known, we must provide consistent estimates of the elements of Ω. In providing these estimates, stochastic specifications will be imposed which are more realistic than those of the preceding models.

The most complete specification assumes the disturbances are "cross-sectionally correlated and time-wise autoregressive." Since the data represent observations drawn from nine regions, we assume that there is heteroskedasticity between regions, and that the disturbances are not mutually independent over geographically defined boundaries. Independence would in fact be more a condition of the economic configuration of the regions than their geographical boundaries. The specification of this model is given by

$$E(\varepsilon_{it})^2 = \sigma_i^2 \qquad \text{(heteroskedasticity)},$$

$$E(\varepsilon_{it}\varepsilon_{jt}) = \sigma_{ij}^2 \qquad \text{(mutual correlation)},$$

$$\varepsilon_{it} = \rho_i \varepsilon_{i,t-1} + u_{it} \qquad \text{(autoregression)}$$

with

$$u_{it} \sim N(0, \phi_{ji}),$$

$$E(\varepsilon_{i,t-1} u_{it}) = 0,$$

$$E(u_{it} u_{jt}) = \phi_{ij},$$

$$E(u_{it} u_{js}) = 0 \quad (t \neq s;\ i,\ j = 1,\ 2, \ldots,\ N)$$

and initial values of ε_i with the following properties:

$$\varepsilon_{i0} \sim N(0, \frac{\phi_{ii}}{1 - \rho_i^2}),$$

$$E(\varepsilon_{io}\varepsilon_{jo}) = \frac{\phi_{ij}}{1 - \rho_i\rho_j}.$$

These specifications give rise to the following variance-covariance matrix of disturbance terms:

$$31) \qquad \Omega = \begin{bmatrix} \sigma_{11}P_{11} & \sigma_{12}P_{12} & \cdots & \sigma_{1N}P_{1N} \\ \sigma_{21}P_{21} & \sigma_{22}P_{22} & \cdots & \sigma_{2N}P_{2N} \\ \vdots & & & \\ \sigma_{N1}P_{N1} & \sigma_{N2}P_{N2} & \cdots & \sigma_{NN}P_{NN} \end{bmatrix},$$

where P_{ij} is

$$P_{ij} = \begin{bmatrix} 1 & \rho_j & \rho_j^2 \cdots & \rho_j^{T-1} \\ \rho_i & 1 & \rho_j \cdots & \rho_j^{T-2} \\ \rho_i^2 & \rho_i & 1 \cdots & \rho_j^{T-3} \\ \vdots & \vdots & & \vdots \\ \rho_i^{T-1} & \rho_i^{T-2} & \rho_i^{T-3} \cdots & 1 \end{bmatrix}.$$

To eliminate autocorrelation OLS is initially applied to the combined time series and cross-section data to estimate ρ_i by the formula

$$32) \qquad \rho_i = \sum_{t=2}^{T} \varepsilon_{it}\varepsilon_{i,t-1} \Big/ \sum_{t=2}^{T} \varepsilon_{i,t-1}^2 .$$

The estimates of ρ_i can then be utilized to transform the observations s follows:

$$\text{3)} \qquad Y^*_{it} = \sum_{n=1}^{k} \beta_n X^*_{it,n} + \varepsilon^*_{it} ,$$

here

$$Y^*_{it} = Y_{it} - \hat{\rho}_i Y_{i,t-1} ,$$

$$X^*_{it,k} = X_{it,k} - \hat{\rho}_i X_{i,t-1,k} \ (k=1,\ldots,K) ,$$

$$\varepsilon^*_{it} = \varepsilon_{it} - \hat{\rho}_i \varepsilon_{i,t-1} ,$$

$$t = 2, 3, \ldots, T ,$$

$$i = 1, 2, \ldots, N .$$

the calculated value equals the true value (i.e., $\hat{\rho}_i = \rho_i$), then $= u_{it}$ and the transformed data would be without the autocorrelation fects. As it turns out, the $\hat{\rho}_i$ can be shown to be a consistent estimate ρ_i and hence variance and covariance estimates derived from the trans- med data will also be consistent.

After transforming the data to remove the autocorrelation effects, elements (ϕ_{ij}) of the variance-covariance matrix of disturbances are imated by the formula

$$\hat{\phi}_{ij} = \left[\frac{1}{T - K - 2}\right] \sum_{t=2}^{T} u^*_{ij} u^*_{it} \qquad (i,j = 1, \ldots, N) ,$$

e the u^*_{it} have been obtained as residuals from equations estimated LS applied to the transformed data. If $\hat{\Omega}^*$ is used to represent the mated variance-covariance matrix of disturbances from the transformed tions, the matrix representation of $\hat{\Omega}^*$ will be given by [10]

---

[10] Ω^* may be written as

$$\hat{\Omega}^* = \begin{bmatrix} \hat{\phi}_{11} & \cdots & \hat{\phi}_{N1} \\ \cdot & \hat{\phi}_{22} & \cdot \\ \cdot & \cdot & \cdot \\ \hat{\phi}_{1N} & \cdots & \hat{\phi}_{NN} \end{bmatrix} \otimes I_{T-1} = \hat{\phi} \otimes I_{T-1} ,$$

$$\hat{\Omega}^* = \begin{bmatrix} \hat{\phi}_{11} I_{T-1} & \hat{\phi}_{12} I_{T-1} & \cdots & \hat{\phi}_{1N} I_{T-1} \\ \hat{\phi}_{21} I_{T-1} & \hat{\phi}_{22} I_{T-1} & \cdots & \hat{\phi}_{2N} I_{T-1} \\ \vdots & \vdots & & \vdots \\ \hat{\phi}_{N1} I_{T-1} & \hat{\phi}_{N2} I_{T-1} & & \hat{\phi}_{NN} I_{T-1} \end{bmatrix},$$

e the I_{T-1} is a T-1 x T-1 identity matrix and the $\hat{\phi}_{ij}$ are as ned above. When the $\hat{\Omega}^*$ matrix, and the transformed observation and $X^*_{it,k}$) are used in conjunction with the Aitken formulas, nates of β which possess all desirable asymptotic properties are ned.

Constant Elasticity of Demand Models

The problem stated in Chapter 2 is to estimate n equations of the

$$\text{og } x_{it} = e_{io} + \sum_{j-1}^{n} e_{ij} \log(p_{jt}/P) + E_i \log(Y_t/P) + E_{it}$$
$$(i = 1, \ldots, n),$$

t to (n/2)(n-1) independent exact linear restrictions,

$$_j e_{ij} + E_i - k_i e_{ji} - E_j = 0 \quad (i = 2, \ldots, n;\ j = 1, \ldots, i\text{-}1)$$

:o impose the symmetry conditions. It is noted that the stochastic is introduced by attaching additively the random disturbance term he stochastic specifications for the CED model are very similar to

---

$\otimes$ represents a Kronecker product and $\hat{\phi}$ is an N x N matrix with s $\hat{\phi}_{ij}$ as estimated in equation (49). This procedure involves a rable saving of computer time for $\hat{\Omega}^*$ may be inverted as

$$\hat{\Omega}^{*-1} = \hat{\phi}^{-1} \otimes I_{T-1}.$$

ns that it is necessary to invert only a N x N rather than a x N(T-1) matrix; a considerable savings in terms of core space.

e specifications for the method used to combine cross-section and time-ries data. However, a major difference is that the disturbances are w applicable to equations for product groups rather than cross-sectional its. The disturbances are again specified to be heteroskedastic and tually correlated across equations. This specification appears reason-e for our study, in that the equations represent demand for various mpeting household appliances. The equations are to be estimated from e series data, so it is additionally assumed the disturbances for each ation follow a first order auto-regressive process.

Because of the presence of the symmetry restrictions, which are ng imposed, the generalized linear regression method, which was used to bine the time-series and cross-sectional data, cannot be utilized.[11] tead we will use a modified type of maximum likelihood estimation, e the symmetry restrictions are imposed as a set of Lagrangean traints.[12]

The first step in the estimation procedure is to remove the auto-elation. This is accomplished by transforming the variables as in tion (33) of Section 4.2.3 with the ρ_i used in the transformation g obtained from OLS residuals. A second application of OLS to the sformed variables will produce residuals which can be used in ion (34) to estimate the variance-covariance matrix of disturbances sponding to these transformed variables.

---

[11]Each equation in the CED system contain the same explanatory bles, so that without the restrictions, OLS would have been a factory method of estimation.

[12]See R. H. Court, op. cit. The estimation procedure described is based on the procedure developed by Court.

After elimination of the autocorrelation the system of n demand equations can be written as

$$38)\quad \begin{bmatrix} x_1^* \\ x_2^* \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_n^* \end{bmatrix} = \begin{bmatrix} Z_1^* & \emptyset & \cdots\cdots & \emptyset \\ \emptyset & Z_2^* & \cdots\cdots & \emptyset \\ \cdot & \cdot & \cdots\cdots & \emptyset \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \emptyset & \emptyset & \cdots\cdots & Z_n^* \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{bmatrix} + \begin{bmatrix} u_1^* \\ u_2^* \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_n^* \end{bmatrix},$$

where the x_i^* represents a (T-1) x 1 vector of purchases of the ith product and Z_i^* represents a (T-1) x (n+2) matrix of explanatory variables, and both variables are after the autocorrelation transformation. Other symbols above are e_i, which represents a (n+2) x 1 vector of elasticities, u_i^*, which represents a (T-1) x 1 vector of disturbances, and $\emptyset$ which represents a null matrix of order (T-1) x (n+2).

In matrix form the restrictions are written as

$$39)\qquad Re = \Psi,$$

where R is a (n/2)(n-1) x n(n+2) matrix of known elements $\pm k_i$, $\pm k_j$, ± 1, or 0 representing the coefficients in the restriction equations, is a n(n+2) x 1 vector of elasticities, and Ψ is a (n/2)(n-1) x 1 ector of zeros. If we let the first n variables represent prices, the 1 variable represent income, and the n+2 variable represent a dummy riable for the constant, the coefficients in the first restriction ation of the symmetry conditions can be written as

$$e_{12} e_{13} \cdots e_{1,n+1} e_{1,n+2} e_{21} e_{22} \cdots e_{2n} e_{2,n+1} e_{2,n+2} e_{31} \cdots e_{n,n+1} e_{n,n+2}$$

$$k_j \quad 0 \quad \cdots \quad 1 \quad 0 \quad -k_i \quad 0 \quad \cdots \quad 0 \quad -1 \quad 0 \qquad 0 \quad \cdots \quad 0 \quad 0,$$

re the e_{ij} has been written above the row of R to indicate which sticity measures correspond to the values given. As pointed out earlier, re will be $(n/2)(n-1)$ rows in R representing the corresponding restric- equations.

The joint density function for each period can be written as

$$f(u_{1t}^*, u_{2t}^*, \ldots, u_{nt}^*) = (2\pi)^{-(n/2)} V^{-(1/2)} e^{-(1/2)u_t'V^{-1}u_t},$$

e V represents the true variance-covariance matrix of residuals d on the transformed data.[13] The function actually maximized is n by the logarithm of the joint density function for all periods ined. This is given by

$$\log L = \sum_{t=2}^{T} \log f(u_{1t}, u_{2t}, \ldots, u_{nt}),$$

after irrelevant constants are excluded, becomes

$$\log L = -T \log \left| V \right| - \sum_{t=2}^{T} u_t'V^{-1}u_t$$

$$= \log \left| \Omega^{*-1} \right| - u'(\Omega^{*-1})u,$$

Ω is the assumed true variance-covariance matrix of disturbances removal of autocorrelation, u is a $n(T-1) \times 1$ vector of disturbances, ne brackets $| \, |$ are to be interpreted as a determinant.

Maximization of a function subject to restrictions is performed gh the technique of Lagrangean multipliers. The complete Lagrangean on to be maximized may be written as

---

[13]The matrix V differs from the matrix $\hat{\Omega}^*$ given by equation (35) in * is for all time periods under consideration, while V is for a time period. In addition, the elements of $\hat{\Omega}^*$ are estimated values. write Ω^* for the matrix containing the true values, the relationship n Ω^* and V is given by

$$\Omega^* = V \otimes I_{T-1}.$$

$$L(e,\Omega^*,\lambda) = \log \left|\Omega^{*-1}\right| - u'\Omega^{*-1}u - 2\lambda'Re,$$

e λ is a n x 1 vector of Lagrangean multipliers and the other symbols as defined previously. This function is seen to depend on the para- rs Ω^{*-1}, e and λ, so that by setting the partial derivatives with ect to the parameters equal to zero maximum likelihood estimates of elasticities may be obtained. Differentiating first with respect to elements of Ω^{*-1} gives the result

$$\hat{\Omega}^* = 1/(T-1)\sum_{t=2}^{T}(u_t u'_t),$$

e the $\hat{\Omega}^*$ represents the variance-covariance matrix of calculated duals from the equation for which the restrictions hold. If $\hat{\Omega}^*$ is rded as a fixed, known matrix the problem is reduced to maximizing ction

$$L(e,\lambda) = -u'(\hat{\Omega}^{*-1})u - 2\lambda Re$$

respect to e and λ to obtain estimates ($\hat{e}$ and $\hat{\lambda}$) of these parameters. noted that these estimates are conditional on the initially assumed s for $\hat{\Omega}^*$. Initial consistent estimates of $\hat{\Omega}^*$, are provided by using ormula given by equation (44), where the residuals are obtained from pplied to the transformed data, without incorporation of the symmetry ictions.

Differentiating equation (45) with respect to the remaining parameters d λ), and setting these derivatives equal to zero gives the following order conditions:

$$\frac{\partial L}{\partial e} = 2\eta'\hat{\Omega}^{*-1}X^* - 2\eta'\hat{\Omega}^{*-1}\eta e - 2R'\lambda = 0,$$

$$\frac{\partial L}{\partial \lambda} = -2Re = 0,$$

or after simplification the set of linear equations

$$47) \qquad \eta'\hat{\Omega}^{*-1}\eta e + R'\lambda = \eta\hat{\Omega}^{*-1}X^{*},$$

and

$$Re = 0,$$

where X^{*} represents a n(T-1) x 1 vector of dependent variables, η is given by the matrix

$$48) \qquad \eta = \begin{bmatrix} Z_1^{*} & \emptyset & \cdots & \emptyset \\ \emptyset & Z_2^{*} & \cdots & \emptyset \\ \vdots & \vdots & & \vdots \\ \emptyset & \emptyset & \cdots & Z_n^{*} \end{bmatrix},$$

and Z_i^{*} and e are as previously defined. Also, the equivalent expression $X^{*} - \eta e$ was used in place of the residual vector u in equation (46).

The solution for these equations is given by the following expression

$$49) \qquad \begin{bmatrix} \hat{e} \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} \eta'\Omega^{*-1}\eta & R' \\ R & \Phi \end{bmatrix}^{-1} \begin{bmatrix} \eta'\Omega^{*-1}X^{*} \\ \Psi \end{bmatrix},$$

where Φ is a (n/2)(n-1) x (n/2)(n-1) null matrix and Ψ is a (n/2)(n-1) x 1 null vector defined earlier. Since the $\hat{e}$ and $\hat{\lambda}$ are conditional on the assumed values of $\hat{\Omega}^{*-1}$, second round estimates of Ω^{*-1} $(\hat{\hat{\Omega}}^{*-1})$ may be obtained by utilizing residuals from the estimated demand equations based on e. The new estimates are then used in equation (49) to obtain revised estimates of e and λ. This iteration procedure will be repeated until convergence

is obtained.[14] The final estimates will be consistent and asymptotically efficient. The symptotic variances and covariances of e and λ are given by the appropriate elements of the matrix

$$50) \quad \begin{bmatrix} \eta'\Omega^{*-1}\eta & R' \\ R & \Phi \end{bmatrix}^{-1} .$$

4.4 Linear Expenditure System

The linear expenditure system, applicable to our study, is not a complete demand system; instead, it is a subsystem, which can be applied to groups of household appliances. As for restrictions of the demand equations required to maintain their desirable properties, it is noted they either result automatically from the nature of the budget constraint (as would be the case with additivity properties), or are imposed in estimation (as with homogeneity), or are implicit in the system itself (as would be the case with symmetry conditions). This last statement can be demonstrated by first deriving the Engel and Cournot elasticities by directly applying their definitions to the demand equations. This gives

$$51) \quad E_i = \frac{b_i}{w_i},$$

and

$$52) \quad e_{ij} = \frac{-b_i c_j p_j}{p_i x_i} \qquad \text{for } i \neq j .$$

---

[14] R. P. Byron has suggested that estimates of e and λ which are consistent and,asymptotically efficient will be obtained after the first round. See R. P. Byron, "The Restricted Aitken Estimation of Sets of Demand Relations", Econometrica, 37 (November, 1970), pp. 816-831. In this paper the iterative procedure was used. However, the second and subsequent rounds had only minor effects on the first round estimates.

Upon substitution of these results in the Slutsky relation, and after algebraic rearrangement, we have

53) $$s_{ij} = [b_i(1 - c_j/x_j)w_j]/w_i \qquad \text{for } i \neq j\,.$$

Rearrangement of the terms in the demand equation gives the additional result

54) $$1 - c_i/x_i = (b_i/p_ix_i)(Y - \sum_k p_kc_k) = \phi E_i\,,$$

where

$$\phi = \frac{Y - \sum_k p_kc_k}{Y}.$$

Using these results gives

55) $$s_{ij} = (-b_i\phi)(b_j/w_j)(w_j/w_i) = (-b_ib_j)(\phi/w_i)\,,$$

so that

56) $$w_is_{ij} = -b_ib_j\phi = w_js_{ji}.$$

The linear expenditure system model was shown to be consistent with the Houthakker and Taylor model in Chapter 3. Therefore, the stochastic specifications and estimation method, applicable to the Houthakker and Taylor model, carry over to the linear expenditure system model. However, an estimate of the subsistence income level is required for estimation. When the linear expenditure system was applied to a complete system, the subsistence amount of each good consumed was estimated along with the other parameters.[15] Since we are estimating only a single equation, there will

[15] See J. R. N. Stone, A. Brown, and D. A. Rowe, "Demand Analysis and Projections for Britain, 1900-1970: A Study in Method," in Europe's Future Consumption, (Sande, ed.), Amsterdam: North-Holland Publishing Company,

not be a sufficient number of equations to determine the subsistence bundle. For this reason, "outside" estimates of subsistence income must be used.

(1964). Also see E. Malinvaud, Statistical Methods in Econometrics, Chicago: Rand McNally Company, (1966), pp. 310-314.

Chapter 5

Empirical Tests of the Models

5.1 Introduction

The previous sections have presented the theoretical foundations of several models. Alternative stochastic specifications were then examined, and their impact on the estimation procudure noted. This chapter will apply the "ad hoc" models to both aggregate U.S. data and combined cross-sectional and time series data, and also will apply the CED model to aggregate U.S. data. Following these applications an analysis of the results obtained will be made.

Two limitations must be kept in mind when examining the results of this chapter. First, as pointed out in the introductory chapter, the amount of statistical data on household appliances is limited and may be subject to considerable measurement error. A principal source of data for the appliance industry is "Association of Home Appliance Manufacturers." However, not all of the association's data are released to the public. Those data that are released are reported in the annual statistical issue of "Merchandising Week," a weekly industry publication. In addition to the association, the statistical departments of the major appliance manufacturers maintain considerable data files. These data may have been privately accumulated or have been reported to them by the association.

A brief consideration of the development of the sales series will reveal the possibility of a large measurement error. Manufacturers' sales of association members are reported to the association who accumulates

these data. Although the association members account for a very high percentage of the total sales, there still exists a volume of non-members' sales that must be estimated. This is accomplished by dividing reported sales by an estimated percentage representing the percentage of total sales actually reported. Information on retail sales are subject to an even greater measurement error in that they must rely on estimates of changes in inventory levels from one period to the next, as there is no direct reporting of retail sales.

The second limitation has also been previously mentioned in the paper. This limitation concerned conclusions that could be drawn from a comparison of test results. As has been pointed out, inferences as to which model is superior based on small differences in statistics are of doubtful validity. The principal contribution of the tests are to determine whether a particular model should be accepted or rejected, and to test which variables are significant within the confines of a given model.

5.2 The Data Base

Sales. At the outset it was necessary to make a choice as to a point in the chain of distribution of product at which to measure the volume of sales. Conceptually, retail sales would be the best measure to use in testing the demand models under investigation; however, retail sales are probably subject to the largest measurement error of the possible sales measures. In addition, a retail sales series is not readily available. For these reasons the bulk of the testing was accomplished by utilizing a series for manufacturers' sales. This choice was not without precedent, for this is the same concept of sales used by

Burstein[1] and by Miller[2] in two studies of the demand for household refrigerators. In addition, a series for retail sales for an eleven year period was obtained and the retail sales series regressed against the manufacturers' sales series. The resultant R^2 was approximately 99% lending support to the use of manufacturers' sales as a good proxy for retail sales.

A slightly different sales series was utilized when combined cross-section and time-series data was used, since it was necessary to have sales by regions. A series for sales (in units) by distributors to retail outlets for the fifty states and the District of Columbia covering the period 1959-1969 inclusive was obtained.[3] Using information as to the distribution of sales from the state report, the series for retail sales, and making the assumption that the distribution of retail sales and sales by distributors were the same, a retail sales series by states was constructed. The series for the states was then aggregated into nine regions[4] which corresponded to the geographical breakdown used by the Department of Commerce in presenting their economic statistics.

---

[1]M. L. Burstein, op. cit.

[2]H. L. Miller, op. cit.

[3]Some of the earlier reports only indicated the percent of total sales that each state represented so it was necessary to convert the percentages given to unit sales before proceeding.

[4]Initially an attempt to apply the TSGLR method to the 51 States for an eleven year period, but this attempt failed because the required inversion of a 51 x 51 matrix exceeded the capacity of the machine. This situation was further aggravated by the extreme differences in the size of the residuals between densely populated and sparcely populated States.

A major problem faced by the researcher is the units of measure in which to express the dependent variable. A simple solution would be to ignore size and quality in each appliance grouping. This approach may fail to take account of significant shifts in expenditure patterns. As an example, consider the shifts that have occurred in refrigerator sales where there has been a pronounced trend to larger models with additional features. A second drawback to this approach was pointed out in Chapter 3 where it was noted that certain models are more amenable to measurement of demand in dollars rather than a simple aggregate of units.

The use of the sales in dollars would surmount these problems, but a data series for sales in dollars is not readily available. It was then decided to use the retail value of sales as the dependent variable. It might be pointed out that the retail value is a measure of the suggested price as opposed to a measure of the actual price paid. However, its use did avoid the two previous drawbacks mentioned, and it was felt to be closely correlated with actual prices paid.

A series for the retail value of factory sales is readily available, but a corresponding magnitude by states could not be obtained. It was decided to compute an average retail value for each component on which data was available. This average retail value was then assumed to hold across states. By multiplying the sales in units by this average value, a dollar value series was obtained.

Income. The income concept used in the study is basically that of disposable personal income. A major exception to the use of disposable personal income occurs when combining time series and cross-section data. A disposable income series by states was not available, so a series for personal income was used in its place.

<u>Prices</u>. It was necessary to select a price series which was reflective of the actual prices paid in contrast to the suggested price series used in the calculation of the retail value series. The major hurdle faced was the effect of shifting model mix and quality changes on the retail value. It was decided to utilize the CPI index for refrigerators, electric ranges, and automatic washers as being representative of the prices paid for our three commodity grouping. Conceptually, the use of a price index which would have allowed for gas ranges for the range category and electric and gas dryers for the laundry product category would have been preferable, but it was not available. Likewise, there was a lack of price index data for the geographical regions used in the pooling method. The data that were available reflected information for selected standard metropolitan areas. It was decided that the best alternative under the data constraint was to utilize the same index for each area under the assumption that prices charged were approximately uniform over the regions considered.

<u>Monetary Variables.</u> Two monetary variables were selected to be included as additional explanatory variables; these included liquid assets as defined in Chapter 3, section 3.3.2, and interest rates on Aaa rated corporate bonds. Only the interest rate variable was tested in connection with the TSGLR pooling method, and the assumption that the interest rate was uniform across regions was made.

<u>Other Variables</u>. The only other variable selected for testing was a measure of the overall consumer price index. This variable was assumed to be uniform across regions, when the combined time series and cross-section data was utilized. This produced a result which was harder to justify

on theoretical grounds.

5.3 "Ad hoc" Model Results

The results of estimating the Stone-Rowe-Nerlove model by OLS applied to aggregate U.S. data, is presented in Table 5-1. Each of the three product groupings of refrigerators, ranges, and laundry products were estimated using 21 raw observations for the period 1950 through 1970 inclusive. Income and price variables were initially selected as the independent variables for each of the groups above. Three other independent variables (interest rates, liquid assets, and the general price level index) were then each alternatively included. The R^2's indicated that the S-R-N model was adequate in explaining the sample variation. The t statistic indicated the coefficient of price was not significantly different from 0 in all cases, while income and generally lagged sales were indicated as being significant. The inclusion of the three additional variables caused little or no improvement in the R^2 and their coefficients were generally insignificant.

By virtue of the lagging procedure to remove the immeasureable stock variable, the reduced form estimated was a first order difference equation with constant coefficients. The values obtained for the coefficient of lagged sales (positive and less than one in all cases) indicated the adjustment path of sales to changes in the explanatory variables is a convergent series.

The Stone-Rowe-Nerlove model requires an estimate of the durable good's life. Nerlove has suggested that, by estimating the model under various life specifications and selecting that life which produced the highest R^2, maximum likelihood results would be obtained. Each of the three product groups used in this study were estimated with life values

TABLE 5-1

RESULTS OF OLS ESTIMATES OF STONE-ROWE-NERLOVE MODEL

(Dependent Variable - Sales in Dollars)

	Constant	Income*	Price*	Lagged Sales	Other		R^2	$\bar{R}^2$
					Variable	Coefficient		
Refrigerators								
1	269.8	866.3 (292.7)	641.3 (654.1)	.3938 (.1741)			.587	.504
2	488.1	580.5 (267.1)	362.3 (592.8)	.2264 (.1900)	Interest Rates*	.1603 (.6908)	.732	.655
3	262.1	897.8 (369.1)	699.0 (717.9)	.4342 (.1961)	Liquid Assets*	-.4084 (1.6737)	.601	.488
4	333.4	840.4 (752.5)	423.9 (752.2)	.2717 (.3014)	General Price Level*	11.11 (17.15)	.618	.508
Ranges								
1	-29.61	586.9 (248.1)	1.649 (1.797)	.7025 (.1812)			.794	.753
2	53.47	589.1 (247.6)	.954 (2.073)	.5721 (.2498)	Interest Rates*	.6158 (.7019)	.818	.766
3	10.60	416.7 (294.2)	1626.5 (1904.6)	.6165 (.2113)	Liquid Assets*	1.515 (1.435)	.828	.779

TABLE 5-1 (cont.)

RESULTS OF OLS ESTIMATES OF STONE-ROWE-NERLOVE MODEL

(Dependent Variable - Sales in Dollars)

	Constant	Income*	Price*	Lagged Sales	Other		R^2	$\bar{R}^2$
					Variable	Coefficient		
Ranges								
4	24.57	790.0 (256.3)	1.272 (1.679)	.2548 (.3121)	General Price Level*	.0251 (.0149)	.822	.772
Laundry Products								
1	-88.64	778.3 (261.2)	3088.3 (2554.7)	.6489 (.1507)			.764	.717
2	211.3	813.8 (314.4)	3659.9 (2867.1)	.7193 (.2135)	Interest Rates*	-.2393 (.9782)	.794	.735
3	-165.6	985.9 (385.8)	3056.8 (2588.1)	.7053 (.1539)	Liquid Assets*	1.280 (1.707)	.796	.738
4	-184.8	833.8 (262.9)	3360.4 (2498.3)	.8195 (.1827)	General Price Level*	13.704 (13.395)	.822	.772

*These variables represent a weighted first difference where the current observation is given a weight of 1 and the variable lagged one period is given a weight of $\frac{n-1}{n}$. This result was obtained because of the transformation utilized to eliminate the immeasurable stock variable.

OLS estimates are after transformation for removal of autocorrelation effects.

varying between 0 and 30, but the attempt failed to produce results as anticipated. The maximum value of R^2 occurred at either the initial or terminal life value. This method was then rejected in favor of "outside" estimates obtained from the statistical department of one of the large appliance manufacturers. Half-life estimates[5] for both refrigerators and ranges are set at approximately 16 years, while laundry product half-life estimates are about 10 years.[6]

The results of estimating the Houthakker and Taylor model by OLS is given in Table 5.2. The resultant R^2's were slightly higher than those obtained for the Stone-Rowe-Nerlove model, with the same variables included. The only variable that consistently appeared significant was the "change in income" variable. Lagged sales were significant only for the laundry product group. It might be noted that the seemingly large standard error of the coefficients might be caused by the limited number of observations in relation to the parameters to be estimated. The inclusion of interest rates and liquid assets as two additional variable add little improvement to the R^2's and their values were not significantly different from zero. The Houthakker and Taylor study of U.S. Demand Projections considered a coefficient as significant if its value were simply as large as its standard error and its sign were in line with a priori expectations. In other studies, application of the Houthakker and Taylor model indicated the income variable coefficients were generally positive, while the price variable coefficients were generally negative as would be expected.

---

[5]Half-life refers to the median point on the distribution of scrappage of appliances of a given model year.

[6]The estimate for the laundry product group has been holding stable over a number of years, but the figures for ranges and refrigerators have been subject to a declining trend as a result of the introduction of product innovations such as frost-free refrigeration and self-cleaning ovens. This

TABLE 5-2

OLS ESTIMATES OF HOUTHAKKER & TAYLOR MODEL

(Dependent Variable - Sales in Dollars)

Dependent Variable	Constant	Income		Prices		Lagged Sales	Other			R^2	$\bar{R}^2$
							Variable	Variable	Variable_{t-1}		
Refrigerators											
1	642.54	7.877 (4.646)	.339 (.972)	-2.386 (10.092)	1.057 (1.712)	.055 (.356)				.711	.599
2	845.58	9.144 (.555)	.742 (4.132)	-4.651 (10.958)	.768 (1.818)	.221 (.493)	Liquid Assets	-3.635 (3.445)	1.366 (3.100)	.769	.622
3	628.13	2.880 (5.472)	1.433 (1.260)	5.207 (12.150)	1.404 (1.863)	.145 (.389)	Interest Rates	1.159 (1.003)	-1.183 (.962)	.753	.597
Ranges											
1	1271.10	3.163 (3.120)	.720 (.505)	-1.267 (13.026)	-9.954 (7.838)	-.029 (.277)				.856	.801
2	1230.34	3.159 (4.236)	-1.360 (2.681)	3.118 (13.061)	-10.999 (8.414)	-.160 (.333)	Liquid Assets	-1.242 (2.326)	1.969 (2.270)	.858	.767
3	1264.26	1.915 (4.524)	.888 (1.400)	-4.267 (10.010)	-10.471 (12.696)	-.048 (.303)	Interest Rates	.429 (1.069)	-.094 (1.041)	.849	.753

TABLE 5-2 (cont.)

OLS ESTIMATES OF HOUTHAKKER & TAYLOR MODEL

(Dependent Variable - Sales in Dollars)

Dependent Variable	Constant	Income		Prices		Lagged Sales	Other			R^2	$\bar{\bar{R}}^2$
		$\Delta\rho_t$	ρ_{t-1}	ΔP_t	P_{t-1}		Variable	Δ Variable	Variable $_{t-1}$		
Laundry Products											
1	698.04	9.636 (4.984)	-.772 (.796)	-17.420 (27.024)	-3.209 (7.457)	.663 (.209)				.748	.651
2	1429.27	-.555 (6.382)	-10.256 (3.784)	-39.640 (19.513)	9.516 (7.546)	1.167 (.226)	Liquid Assets	7.790 (3.997)	8.563 (3.390)	.894	.827
3	479.94	.328 (5.863)	3.464 (1.534)	-40.542 (20.740)	7.849 (7.822)	.687 (.184)	Interest Rates	.623 (1.377)	-2.835 (.956)	.890	.820

OLS estimates are after transformation for removal of autocorrelation effects.

5.3.1 Two Stage Generalized Linear Regression Method Results

In applying the "ad hoc" models to the combined time-series and cross-section data, OLS, adjusted for first order autocorrelation effects, was initially used to estimate the parameters from a combined eleven year, nine region sample. The results of this procedure are given in Table 5-3 for the Stone-Rowe-Nerlove Model and in Table 5-5 for the Houthakker and Taylor Model.[7] As is readily seen, the R^2 is extremely high for all product groups in both models. A disturbing result in the S-R-N case was the extremely high value obtained for the coefficient of lagged sales. This coefficient is theoretically equal to 1-r where "r" represents the proportion of the difference between desired and actual stock which is made up each period. In each case the value of this coefficient was not significantly different from unity, which in turn implied a value of "r" equal to 0. This is in contrast to the results obtained from the twenty-year U.S. aggregate sample.

This same problem was encountered by Balestra and Nerlove,[8] who found the coefficient of the lagged endogenous variable was not significantly different from zero, which in their case implied an implausible value for the depreciation rate on gas appliances. To surmount the problem the use of a dummy shift variable for time effects was tried, but this approach proved unsuccessful. This led Balestra and Nerlove to conclude

---

suggests that future studies might be directed toward treating depreciation of the existing stock as a subjective phenomenon rather than simply a constant proportion of the existing stock. These life estimates are used in this study.

[7]The TSGLR method was also applied to the models when the additional explanatory variables of interest rates, liquid assets, and general price level were included, but the results were generally inconclusive, so they are not presented.

[8]P. Balestra and M. Nerlove, "Pooling Cross-Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas," Econometrica, 34 (July, 1966), p. 590.

TABLE 5-3

OLS ESTIMATES OF STONE-ROWE-NERLOVE MODEL
USING POOLED DATA BY REGIONS

(Dependent Variable - Sales in Dollars)

Product	Constant	Income	Prices	Lagged Sales	R^2-s[1]
1. Refrigerators	6.44	21.236	-202.324	1.008	.976
	(3.28)	(11.557)	(108.112)	(.019)	9.738
2. Ranges	-1.07	14.708	55.089	1.002	.970
	(1.71)	(5.259)	(44.926)	(.0219)	4.720
3. Laundry Products	-1.43	14.091	104.976	1.010	.980
	(5.25)	(9.083)	(99.110)	(.018)	7.281

OLS Estimates are after transformation for removal of autocorrelation effects.

[1] Standard error of the regression.

TABLE 5-4

TWO STAGE GENERALIZED LINEAR REGRESSION ESTIMATES OF STONE-ROWE-NERLOVE MODEL USING POOLED DATA BY REGIONS

(Dependent Variable - Sales in Dollars)

Product	Constant	Income	Prices	Lagged Sales	R^2-s \1
1. Refrigerators	6.79 (.50)	18.302 (3.920)	-220.073 (19.814)	1.010 (.004)	.976 9.745
2. Ranges	-1.39 (.78)	14.142 (1.316)	44.096 (12.791)	1.001 (.007)	.970 4.790
3. Laundry Products	-1.39 (.58)	12.463 (2.986)	88.447 (19.870)	1.018 (.007)	.980 7.296

TSGLR estimates are after transformation for removal of autocorrelation effects.

\1 Standard error of the regression

TABLE 5-5

OLS ESTIMATES OF HOUTHAKKER AND TAYLOR MODEL
USING POOLED DATA BY REGIONS

(Dependent Variable - Sales in Dollars)

Product	Constant	△Income	Lagged Income	△Prices	Lagged Prices	Lagged Sales	R^2-s \1
Refrigerators	2.64 (2.78)	26.652 (14.039)	11.314 (15.702)	-135.998 (114.445)	-6.023 (6.125)	.972 .073	.983 10.095
Ranges	2.32 (1.91)	15.300 (5.129)	1.046 (1.760)	168.140 (58.900)	1.550 (2.421)	1.022 (.023)	.984 4.695
Laundry Products	-.25 (2.72)	24.801 (9.648)	30.902 (12.135)	634.810 (153.195)	31.063 (8.614)	.863 (.054)	.987 6.812

OLS Estimates are after transformation for removal of autocorrelation effects.

\1 Standard error of the regression.

TABLE 5-6

TWO STAGE GENERALIZED LINEAR REGRESSION ESTIMATES OF HOUTHAKKER AND TAYLOR MODEL USING POOLED DATA BY REGIONS

(Dependent Variable - Sales in Dollars)

Product	Constant	ΔIncome	Lagged Income	ΔPrices	Lagged Prices	Lagged Sales	R^2 - s \1
Refrigerators	2.105 (.755)	23.587 (6.313)	13.287 (3.638)	-145.135 (11.209)	-6.101 (2.362)	.954 (.015)	.982 10.256
Ranges	1.464 (2.165)	13.826 (1.584)	.736 (.565)	158.315 (45.520)	2.406 (2.007)	1.023 (.017)	.983 4.711
Laundry Products	-1.035 (1.388)	22.925 (9.371)	30.731 (12.012)	602.802 (191.321)	28.803 (7.982)	.869 (.067)	.987 6.897

TSGLR Estimates are after transformation for removal of autocorrelation effects.

\1 Standard error of the regression.

that "time invariant but perhaps unobservable state effects were responsible for biasing the coefficient of lagged gas consumption in the demand equation." It is not inconceivable that these same effects are at work in the case of major household appliances. This would be especially true in light of the differing results obtained between the pooled and aggregate U.S. data. The Houthakker and Taylor model divorces the coefficient of the lagged endogenous variable from any interpretation of its value as a depreciation rate or an adjustment parameter. However, any inference drawn as to why the value of this coefficient was so seemingly large would also be applicable to the Houthakker and Taylor model.

In using the pooled data, the objective was to improve the efficiency of the results over those obtained from the annual U.S. data. This improvement in efficiency should result from an increased number of observations (i.e., 99 vs. 20). As can be observed from Tables 5-3 and 5-5, OLS applied to the pooled data has slightly improved our tests of significance of the coefficients as shown by the value of the 't' statistics. Our a priori expectation was that the use of the TSGLR method developed in Chapter 4 would further improve the efficiency of the estimates and this was confirmed by the results shown in Tables 5-4 and 5-6 vs. those in Tables 5-3 and 5-5. By utilizing the TSLGR method, the 't' test has been sharpened to the extent that all variables are now significant for both "ad hoc" models. The sign of the income variable coefficients were positive for both cases as expected, but as regards the price variable, only the coefficient of prices for refrigerators in the S-R-N model, and the change in prices for refrigerators in the H & T model, were negative as would normally be expected.

5.4 Constant Elasticity of Demand Model Results

The constant elasticity of demand model was estimated using data for the period 1950 to 1970. The three product categories estimated reflect the use function of the appliance, and are the same as were estimated for both the Stone-Rowe-Nerlove and Houthakker and Taylor models. The dependent variable chosen represents the dollar value of sales, as opposed to a simple summation of units in order to avoid the aggregation problems inherent in appliances groupings. All prices and income have been deflated by the retail price index so as to satisfy the homogenity conditions.

The results of OLS applied to each equation separately are given in Table 5-7. The results shown are after a first-order autocorrelation transformation has been applied to the data. Although the Durbin-Watson statistic generally falls around the upper limit at the 5% level of significance, it was decided to carry out the transformation to reduce any inconclusiveness. The R^2's obtained indicate the model was successful in explaining a significant amount of the variation in the sample data. The income elasticity for each product group was positive and fell in the interval between +2 and +4.

The only results that contradicted our a priori expectations were the signs of the price elasticities obtained. In particular our expectations were that the "own" price elasticity would be less than +1 for each group.[9]

---

[9]The demand function for the ith product may be written as

$$\log (p_i q_i) = e_{io} + \sum_{j=1, j\neq i}^{n} e_{ij}\log(p_j) + e_{ii}\log(p_i) + E_i\log(Y) ,$$

where $p_i q_i$ has been written for x_i, with q_i, referring to the quantity demanded, and the other symbols are as previously defined. Upon algebraic rearrangement, we have

TABLE 5-7

UNRESTRICTED OLS ESTIMATES OF CED MODEL

(Dependent Variable - Sales in Dollars)

Dependent Variable[1]	Constant	Income[1] (y)	Prices[1]			R^2	S[2]	Durbin-Watson Statistic
			Refrigerators P_1	Ranges P_2	Laundry Products P_3			
Refrigerators	-43.5746	2.5097 (.6487)	.3619 (.2163)	.5969 (1.3959)	1.5618 (.9879)	.786	.0605	2.1724
Ranges	-37.8369	2.3631 (.8403)	.1547 (.2928)	3.4774 (1.7897)	-1.0560 (1.3162)	.875	.0769	1.8697
Laundry Products	-54.3602	3.4123 (.6740)	-.9060 (.2399)	1.0229 (1.4285)	4.1675 (1.0693)	.879	.0613	1.6143

[1] All variables have been converted to logarithms before estimation.

[2] Standard error of the regression.

TABLE 5-8

RESTRICTED AITKEN ESTIMATES OF CED MODEL

(Dependent Variable - Sales in Dollars)

Dependent Variable [1]	Constant	Income [1] (y)	Prices [1]			R^2	S [2]	Durbin-Watson Statistic
			Refrigerator P_1	Ranges P_2	Laundry Products P_3			
Refrigerators	-18.6384	1.2531 (.4255)	.7996 (.1781)	.3957 (.1418)	-.5813 (.2103)	.647	.0777	2.1724
Ranges	-9.2481	.7983 (.6369)	.6342 (.2266)	1.7905 (.6728)	-2.5450 (.5400)	.816	.0933	1.8697
Laundry Products	-31.3064	2.1025 (.5823)	-.5571 (.2009)	-1.5210 (.3221)	3.8525 (.5793)	.813	.0760	1.6143

[1] All Variables have been converted to logarithms before estimation.

[2] Standard error of the regression.

Instead the "own" price elasticity was positive for all cases, and significantly greater than +1, at the 95% confidence level, for ranges and laundry products. These results suggest two possible avenues for future exploration to account for the apparent abnormalities. First, the price series may need improvement. As pointed out in our discussion of the data actually chosen, the use of a price index may fail to adequately reflect the actual prices paid for the models purchased. Secondly, there may have been a number of variables omitted with cumulative effect of causing the irregularity. In particular these omitted variables could be a measure of intertemporal considerations as to income and prices. Finally, the results may in fact represent reality, in that the consumers reaction to a higher price may have been, "Well, if I'm going to spend this much for an appliance, I might as well get a deluxe model."

The results when the equations are subjected to restrictions stated below are given in Table 5-8. The R^2's are not as large as those obtained for the unrestricted case, but nonetheless they account for a significant part of the sample variation. The income elasticities obtained with the restrictions imposed are smaller than those obtained from the unrestricted case, but the positive sign has been maintained. The imposition of the restrictions has also improved the efficiency of all income and price elasticities obtained. As regards the price elasticities, the symmetry restriction has caused the cross price elasticities of refrigerators with respect to the price of laundry products, and laundry products with respect to the price of ranges to become negative.

$$\log(q_i) = e_{io} + \sum_{j=1, j\neq i}^{n} e_{ij}\log(p_j) + (e_i-1)\log(p_i) + E_i\log(Y) .$$

In this form we would expect that (e_i-1) would be negative. This implies the e_{ii} estimated must be less +1 if (e_i-1) is to be negative.

The restrictions imposed were

$$502.35\ e_{12} + E_1 - 314.14\ e_{21} - E_2 = 0$$

$$299.72\ e_{12} + E_2 - 502.36\ e_{32} - E_3 = 0$$

$$314.14\ e_{31} + E_3 - 299.72\ e_{13} - E_1 = 0,$$

where 314.14, 502.36, 299.72 are means of budget proportion reciprocals for refrigerators, ranges and laundry products respectively, and the e_{ij} and E_i are price and income elasticities as pointed out in Chapter 3. Convergence was obtained quickly, usually by the third or fourth iteration.[10] The elasticites of substitution were obtained from our elasticity estimates, the budget proportion reciprocals, and the formula

$$\sigma_{ij} = k_i\ e_i + E_i,$$

where $_{ij}$ is the elasticity of substitution between the i'th and j'th good. These values are as follows:

$$(\sigma_{ij}) = \begin{matrix} 252.4 & 200.0 & -172.9 \\ 200.0 & 900.2 & -762.0 \\ -172.9 & -762.0 & 1156.8 \end{matrix} .$$

The restrictions were tested[11] by first forming the null hypothesis that "the variance-covariance matricies of error terms for the restricted

---

[10] R. P. Byron, in examining the problem of restricted Aitken estimation, concluded that the approximation bias introduced in estimating the elements of Ω by OLS are sufficiently small so that a single iteration will be sufficient. See R. P. Byron, "The Restricted Aitken Estimation of Sets of Demand Relations," Econometrica, 38 (1970), p. 819.

[11] An alternative test of the restrictions is based on the Wald test statistic which uses the unrestricted parameter to test if overall the prior and sample information are compatible. See Wald, A., "Tests of Statistical Hypothesis Concerning Several Parameters When the Number of Observations is Large," Transactions of the American Mathematical Society, 54 (1943), pp. 426-483.

and unrestricted cases are equal." The test statistic

$$-T \log_e \frac{\det \Omega^0}{\det \Omega^*} = -21 \log_e \frac{.0526}{.1169}$$

$$= 10.25,$$

where Ω^0 and Ω^* refer to the variance-covariance matrix for the unrestricted and restricted case respectively,was then formed. This statistic is asymptotically distributed as χ^2 with three degrees of freedom. Its value of 10.25 is significantly greater than the 5 percent significance level of 7.81 and just slightly smaller than the 1 percent level of 11.34. Accordingly, we reject the null hypothesis of equality of error variance-covariance matricies and conclude the imposition of the restrictions have a significant effect on the elasticities calculated.

5.5 Summary and Conclusion

The Stone-Rowe-Nerlove and the Houthakker and Taylor models both performed very well for all product groupings if the R^2 value is used as criteria for evaluation. In fairness it should be pointed out that this might have been expected because of the presence of the lagged sales value as an independent variable. Although its presence has legitimately been occasioned by the algebraic manipulation of the behavioral and definitional equations of the model, the "good" results do little for us in delineating which model is the appropriate description as to how consumers formulate their durable purchase decisions. As to the contributions of the financial variables of interest rates or liquid assets, these variables have made little improvement in the results obtained. In addition, the price variable was generally found to be insignificant, and its coefficient was frequently positive in contrast to our a priori expectations of a negative value.

The results obtained when demand restrictions, derived from maximization of a utility function subject to an income constraint, were applied to the constant elasticity of demand model suggested that this approach has considerable merit if we desire to tie our results to the utility maximization framework; however, the price variable again proved troublesome.

As pointed out above, the coefficient of the price variable generally was not in accord with our expectations. Several possible suggestions can be offered to improve these estimates. A significant improvement in the results may be obtained by considering the impact of quality changes. The distribution of appliance purchases has generally shifted toward larger models with additional features, but the price index for appliances has been adjusted for only those quality changes which are capable of being priced separately. The quality changes have also made their impact felt in other ways, such as by increasing replacement sales, by their mere availability. An approach suggested by Z Griliches[12] attempts to adjust for quality changes by calculating an index of quality change, which can be used to deflate the "observed price" index. The information used in constructing the quality index is obtained from cross-sectional observations on the prices and features of different models. Some problems encountered in applying this method were the reliance on list price data rather than actual prices paid, and the weight to assign to changing model mixes over the years. In spite of these limitations, this approach does offer promise as a procedure for incorporating quality changes.

---

[12]Griliches, Z., "Hedonic Price Indexes for Automobiles: An Econometric Analysis of Quality Change," in Government Price Statistics, Hearings (U.S. Congress, Joint Economic Committee, January 24, 1961),U.S. Government Printing Office, (1961), pp. 173-196.

A second reason for the poor showing of the price coefficient estimates may be due to the assumption of a recursive supply function. If the recursive supply function assumption was invalid, it is conceivable that what is being measured is not a demand function, but some "mongrel" function, which is approximating a supply function. The appropriate procedure in this case is the use of simultaneous equation estimates.

Our examination of the linear expenditure system model has led us to conclude that in estimation its form is compatible with the Houthakker and Taylor model provided that the income variable for the Houthakker and Taylor model is defined as "supernumerary income." This reasoning is of considerable importance, for although the H & T model was developed by recourse to armchair theorizing as to formulation of consumer durable purchase decisions, the model in fact has a basis in the more conventional demand framework of economic theory. Future work in this area will have to be directed to the development of adequate measures of the "supernumerary income" variable.

The procedure for combining time series and cross-section data proved to be a very successful development and is shown to be a very powerful and useful tool for future work. Originally, it was our expectation that the use of the TSGLR pooling method would make some minor improvement in the size of the standard errors of the regression coefficients. The results more than realized our expectations, as significant improvements in efficiency were made in all of the coefficient estimates.

Although the estimation results were unsatisfactory for direct application to forecasting appliance demand, we were able to identify some approaches (particularly the Houthakker and Taylor and CED models) as offering considerable promise for future research. As pointed out above,

the primary area where future work can make a significant contribution is in the development of a procedure for handling quality changes, and shifting demand patterns as a result of the quality change. A last suggestion for future work, which may lead to more reliable results, is an improved data series for appliance sales, especially at the retail level.

BIBLIOGRAPHY

SELECTED BIBLIOGRAPHY

Aitken, A. C. "On Least Squares and Linear Combination of Observations," Proceedings of the Royal Society of Edinburgh, LV (1934-35), 42-48.

Allen, R. G. D. Mathematical Economics. 2nd ed. London: MacMillan and Co., Ltd., 1965.

American Home Appliance Association of the U.S., Unpublished State Sales Reports, (1950-1971).

Ando, A. and Modigliani, F. "The Life Cycle Hypothesis of Savings," American Economic Review, LIII (March, 1953), 50-84.

Arrow, K. J., Karlin, S. and Suppes, P. (eds.). Proceedings of a Symposium on Mathematical Methods in the Social Sciences, 1959. Stanford, California: Stanford University Press, 1959.

Balestra, P. and Nerlove, M. "Pooling Cross-Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas," Econometrica, XXXIV (July, 1966), 585-612.

Baumol, W. J. Economic Dynamics. 2nd ed. New York: MacMillan Co., 1959.

Billboard Publications, Inc. Merchandising Week. New York, 1950-71.

Brown, T. M. "Habit Persistence and Lags in Consumer Behavior," Econometrica, XX (January, 1952).

Byron, R. P. "The Restricted Aitken Estimation of Sets of Demand Relations," Econometrica, XXXVII (November, 1970), 816-831.

Chow, G. C. Demand for Automobiles in the United States: A Study in Consumer Durables. Amsterdam: North-Holland Publishing Co., 1959.

Christ, C. F. Econometric Models and Methods. New York: J. Wiley & Sons, 1966.

Cockrane, D. and Orcutt, G. H. "Applications of Least Squares Regression to Relationships Containing Autocorrelated Error Terms," Journal of the American Statistical Association, XLIV (March, 1949), 32-61.

Court, R. H. "Utility Maximization and the Demand for New Zealand Meats," Econometrica, XXXV (July, 1967), 424-446.

Debrau, G. "Representation of a Preference Ordering by a Numerical Function," Decision Processes. (ed. R. M. Thrall), New York: John Wiley & Sons, 159-165.

Durbin, J. and Watson, G. S. "Testing for Serial Correlation in Least Squares Regressions I," Biometrica, XXXVII (December, 1951), 409-28.

__________. "Testing for Serial Correlation in Least Squares Regressions II," Biometrica, XXXVIII (June, 1952), 1959-78.

Farrell, M. J. "The Demand for Motor Cars in the United States," Journal of the Royal Statistical Society, CKVII(Part II, 1954), 171-98.

Friedman, M. The Theory of the Consumption Function. Princeton, New Jersey: Princeton University Press, 1957.

Fuller, W. A. and Morton, J. E. "The Effects of Autocorrelated Errors on the Statistical Estimation of Distributed Lag Models," Journal of Farm Economics, XLIII (February, 1969), 71-82.

Geary, R. C. " A Note on 'A Constant-Utility Index of the Cost of Living'," Review of Economic Studies, XVIII (1949) 65-66.

Goldberger, A. S. Economic Theory. New York: John Wiley & Sons, Inc., 1964.

Griliches, Z. "Distributed Lags: A Survey," Econometrica, XXXV (January, 1967), 16-49.

__________. "Hedonic Price Indexes for Automobiles: An Econometric Analysis of Quality Change," in Government Price Statistics, Hearings, (U.S. Congress Joint Economic Committee,January 24, 1961), U.S. Government Printing Office, (1961), 173-196.

Hamberger, M. J. "Interest Rates and the Demand for Consumer Durable Goods," American Economic Review, LVII (December, 1967), 1144.

Harberger, Arnold C. (ed.). The Demand for Durable Goods. Chicago: University of Chicago Press, 1960.

Henderson, C. R. Jr. "Comment on 'The Use of Error Components Models in Combining Cross Section With Time Series Data'," Econometrica, XXXIX (March, 1971), 397-402.

Henderson, J. M. and Quandt, R. E. Microeconomic Theory: A Mathematical Approach. New York: McGraw-Hill Book Co., 1958.

Hicks, J. R. A Revision of Demand Theory. Oxford: Clarendon Press, 1956.

__________. Value and Capital. Oxford: Clarendon Press, 1939.

__________. and Allen, R. G. D. "A Reconsideration of the Theory of Value," Econometrica, I, (1934), 52-75, 196-219.

Hoch, I. "Estimation of Production Function Parameters Combining Time-Series and Cross-Section Data," Econometrica, XXX (January, 1962), 34-53.

Houthakker, H. S. "Additive Preferences," Econometrica, XXVII (1960), 244-257.

__________. and Taylor, L. Consumer Demand in the U.S., 1929-1970. Cambridge, Mass.: Harvard University Press, 1966.

Huang, D. S. "Initial Stock and Consumer Investment in Automobiles," Journal of the American Statistical Association, LVIII (September, 1963), 789-98.

__________. "Discrete Stock Adjustment: The Case of Demand for Automobiles," International Economics Review, V (1964), 46-62.

Johnson, P. R. "Some Aspects of Estimating Statistical Cost Functions," Journal of Farm Economics, XLVI (February, 1964), 179-187.

Johnston, J. Econometric Methods. New York: McGraw Hill Book Co., 1963.

Klein, L. R. An Introduction to Econometrics, Englewood Cliffs, New Jersey: Prentice Hall Inc., 1962.

__________. "A Postwar Quarterly Model: Description and Applications," Models of Income Determination. Princeton, New Jersey: Princeton University Press, 1964.

__________. A Textbook of Econometrics. Evanston, Illinois: Row Peterson and Co., 1953.

__________. "Major Consumer Expenditures and Ownership of Durable Goods," Bulletin Oxford University Institute of Statistics, XV (November, 1955), 387-414.

__________. and Rubin, H. "A Constant-Utility Index of the Cost of Living," Review of Economic Studies, XV (1947), 84-87.

Kmenta, J. Elements of Econometrics. New York: MacMillan Co., 1971.

Kmenta, J. and Gilbert, R. F. "Small Sample Properties of Alternative Estimators of Seemingly Unrelated Regressions," Journal of the American Statistical Association, LXIII (December, 1968), 1180-1200.

Koyck, L. M. Distributed Lags and Investment Analysis. Amsterdam: North-Holland Publishing Co., 1954.

Liviatan, N. "Consistent Estimation of Distributed Lags," International Economic Review, IV (January, 1963), 44-52.

Maddala, G. S. "The Use of Variance Components in Pooling Cross Section and Time Series Data," Econometrica, XXXIX (March, 1971), 341-359.

____________. "Generalized Least Squares with an Estimated Variance Covariance Matrix," Econometrica, XXXIX (January, 1971), 23-25.

Malinvaud, E. Statistical Methods of Econometrics. Chicago: Rand McNally Co., 1966.

Merchandising Week, New York, 1950-1971.

Miller, H. L. Jr. The Demand for Refrigerators, Unpublished Ph.D. dissertation, Harvard University, 1956.

____________. "The Demand for Refrigerators: A Statistical Study," Review of Economics and Statistics, XLII (May, 1960), 196-202.

Mundlak, Y. "Estimation of Production and Behavioral Functions from a Combination of Cross-Section and Time Series Data," in Measurement in Economics, (C. F. Christ, ed.), Palo Alto, California: Stanford University Press, (1963), 138-166.

Nerlove, M. "A Note on Error Components Models," Econometrica, XXXIX (March, 1971), 383-396.

__________. "A Note on Long-Run Automobile Demand," Journal of Marketing, XXII (July, 1957), 54-64.

__________. "Further Evidence on the Estimation of Dynamic Economic Relations from a Time Series of Cross Sections," Econometrica, XXXIX (March, 1971), 359-383.

__________. "On Automobile Demand: A Reply," Journal of Marketing, XXIII (April, 1958), 415-16.

Nerlove, M. "Professor Suits on Automobile Demand," Review of Economics and Statistics, XLII (February, 1960), 102-105.

__________. "The Market Demand for Durable Goods: A Comment," Econometrica, XXXVIII (February, 1960), 132-142.

Parks, R. W. "Efficient Estimation of a System of Regression Equations When Distributions are Both Serially and Contemporaneously Correlated, " Journal of the American Statistical Association, LXII (June, 1967), 500-509.

__________. "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms," Econometrica, XXXVII (1969) 629-650.

Pennock, J. L. and Jaeger, C. M. "Estimating the Service Life of Household Goods by Actuarial Methods," Journal of the American Statistical Association, LII (1957), 175-185.

Pollak, R. A. and Wales, T. V. "Estimation of the Linear Expenditure System," Econometrica, XXXVII (1969), 611-628.

Prais, S. J. and Houthakker, H. S. The Analysis of Family Budgets. Cambridge: University Press, 1955.

Richardson, D. H. "The Asymptotic Unbiasedness of Two-Stage Least Squares," Econometrica, XXXVIII (September, 1970), 772.

Roos, C. F. and von Szeliski, V. "Factors Governing Changes in Automobile Demand," in Dynamics of Automobile Demand. New York: General Motors Corp., 1939, 21-95.

Samuelson, P. A. Foundations of Economic Analysis. Cambridge, Mass.: Harvard University Press, 1966.

__________. "Some Implications of Linearity," Review of Economic Studies, XV (1947), 88-90.

Smith, V. L. Investment and Production, Cambridge, Mass.: Harvard University Press, 1961.

Staehle, H. "Relative Prices and Postwar Markets for Animal Food Products," The Quarterly Journal of Economics, LIX (1944-45) 237-279.

Stone, J. R. N. "Linear Expenditure Systems and Demand Analysis," Economic Journal, LXIV (1954), 511-527.

__________., Brown, A. and Rowe, D. A. "Demand Analysis and Projections for Britain, 1900-1970: A Study in Method," in Europe's Future Consumption, (Sande, J., ed.), Amsterdam: North-Holland Publishing Co., 1964.

__________. and Rowe, D. A. "Dynamics Demand Functions: Some Econometric Results," Economic Journal, LXVIII (June, 1958), 256-270.

Stone, R. D. and Rowe, D. A. "The Durability of Consumers' Durable Goods," Econometrica, XXVIII (April, 1960), 407-416.

__________. "The Market Demand for Durable Goods," Econometrica, XXV (July, 1957), 423-43.

Strotz, R. H. "Myopia and Inconsistency in Dynamic Utility Maximization," Review of Economic Studies, XXIII (1956), 165-180.

Suits, D. B. "Exploring Alternative Formulation of Automobile Demand," Review of Economics and Statistics, XLIII (February, 1961), 66-69.

__________. "The Demand for New Automobiles in the United States, 1929-56," Review of Economics and Statistics, XL (August, 1958), 273-80.

__________. and Sparks, G. "Consumption Regressions with Quarterly Data," in The Brookings Quarterly Econometric Model of the United States, (G. Fromm, L. Klein, and E Kuk, eds.), Chicago: Rand McNally, 1965.

Taylor, L. D. and Wilson, T. A. "Three-Pass Least Squares: A Method For Estimating Models with a Lagged Dependent Variable," Review of Economics and Statistics, XLVI (November, 1964), 329-346.

U.S. Board of Governors of the Federal Reserve System. Federal Reserve Bulletin, 1950-71.

U.S. Department of Commerce. Current Population Reports. 1950-71.

__________. Survey of Current Business, 1950-71.

U.S. Department of Labor, Bureau of Labor Statistics, Handbook of Labor Statistics, 1971.

Wald, A. "Tests of Statistical Hypothesis Concerning Several Parameters When the Number of Observations is Large," Transactions of the American Mathematical Society, LIV (1943), 426-483.

Wallace, T. D. and Hussain, A. "The Use of Error Components Models in Combining Cross-Section with Time-Series Data," Econometrica, XXXVII (January, 1969), 55-72.

Wold H. and Jurein, L. Demand Analysis. New York: John Wiley & Sons, Inc., 1953.

Wu, D. An Empirical Analysis of Household Durable Goods Expenditure. Unpublished Ph.D. dissertation, University of Wisconsin, 1963.

Zaman, A. Formulation and Estimation of A Complete System of Demand Equations, Unpublished Ph.D. dissertation, Michigan State University, 1970.

Zellner, A. "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," <u>Journal of the American Statistical Association</u>, LVII (June, 1962), 348-68.

_________. and Geisel, M. S. "Analysis of Distributed Lag Models with Applications to Consumption Function Estimation," <u>Econometrica</u>, XXXVII (November, 1970), 865-889.

APPENDICES

APPENDIX A

DATA USED IN THE EMPIRICAL TESTS

TABLE A-1

RETAIL VALUE OF MANUFACTURERS' SALES
(AGGREGATE U.S. DATA)
1950 - 1970

YEAR	REFRIGERATORS	RANGES	LAUNDRY PRODUCTS
1950	1602266000	709203590	855013000
1951	1120625000	587696840	836342500
1952	1017450000	487482658	871676300
1953	1076750000	541188300	985266300
1954	1095865000	544325300	1049187800
1955	1323000000	727233000	1341487000
1956	1202500000	706594000	1489185000
1957	1072000000	627456000	1326796000
1958	997344000	619264000	1301295000
1959	1271760000	751981000	1409675000
1960	1129375000	683565000	1147942000
1961	1026600000	680275000	1100805500
1962	1083425000	746311200	1183050000
1963	1146750000	866531000	1254778000
1964	1172610000	831838600	1313927000
1965	1281800000	880772400	1398425000
1966	1328058000	867819800	1457929000
1967	1286649000	838745500	1510733000
1968	1442280000	1001863000	1611708000
1969	1466992000	1071716000	1612199000
1970	1448364000	1005523000	1402405000

TABLE A-2

INDEPENDENT VARIABLES USED IN REGRESSIONS
(AGGREGATE U.S. DATA)
1950 - 1970

YEAR	DISPOSABLE INCOME (BILLIONS)	LIQUID ASSETS (BILLIONS)	INTEREST RATE ON AAA BONDS	CPI FOR REFRIG.	CPI FOR RANGES	CPI LAUNDRY PRODUCTS	CPI ALL PRODUCTS
1950	206.9	271.4	2.62	151.1	93.3	102.3	83.8
1951	226.6	281.0	2.86	159.1	103.9	109.3	90.5
1952	238.3	296.0	2.96	153.1	102.8	109.7	92.5
1953	252.6	311.5	3.20	147.3	103.2	108.1	93.2
1954	257.4	320.3	2.90	137.3	101.6	105.5	93.6
1955	275.3	332.5	3.06	129.0	99.2	102.6	93.3
1956	293.2	343.2	3.36	111.3	98.6	100.2	94.7
1957	308.5	356.0	3.89	102.4	100.5	101.4	98.0
1958	318.8	373.1	3.79	98.9	99.7	100.3	100.7
1959	337.3	393.9	4.38	98.6	99.9	98.4	101.5
1960	350.0	399.2	4.41	96.6	98.8	95.9	103.1
1961	364.4	424.6	4.35	95.3	96.8	93.0	104.2
1962	385.3	459.0	4.33	93.0	96.4	90.5	105.4
1963	404.6	495.4	4.26	90.6	95.7	89.2	106.7
1964	438.1	530.5	4.40	88.8	95.0	88.0	108.1
1965	473.2	573.1	4.49	86.2	93.7	86.8	109.9
1966	511.9	601.5	5.13	82.9	92.0	86.3	113.1
1967	546.5	650.4	5.51	82.7	92.7	86.6	116.3
1968	590.0	709.6	6.18	83.8	95.2	88.8	121.2
1969	629.6	731.6	7.03	85.3	97.7	90.6	127.7
1970	684.8	787.9	8.04	87.5	100.6	92.9	135.3

TABLE A-3

ESTIMATED DOLLAR VALUE OF RETAIL SALES BY REGION
1959 - 1969

YEAR	REGION	REFRIGERATORS	RANGES	LAUNDRY PRODUCTS
1959	1	65054304	27557774	76534773
	2	234911712	52838106	230119233
	3	180093648	100553996	147539031
	4	234720864	76585953	258352605
	5	62668368	32400494	58329152
	6	113392608	30479996	110529124
	7	85637328	38151684	101530230
	8	43599360	26822761	51825096
	9	173149200	73653370	179785774
1960	1	59041775	26313555	61310846
	2	216087300	47233543	196977029
	3	158379000	65671493	123906884
	4	203645000	72346471	214569352
	5	56456075	30885854	49097854
	6	99039850	27018283	96319683
	7	77154350	33782273	82250991
	8	37018150	23947266	42818088
	9	152995050	62643114	155497786

TABLE A-3 (CONTINUED)

ESTIMATED DOLLAR VALUE OF RETAIL SALES BY REGION
1959 - 1969

YEAR	REGION	REFRIGERATORS	RANGES	LAUNDRY PRODUCTS
1961	1	56483355	26615808	64436663
	2	204958330	45222324	193664963
	3	141755465	82524660	119662070
	4	172973545	66834117	189932695
	5	46685815	30552603	46759206
	6	91260610	28647591	95995195
	7	68747390	32806221	77268071
	8	35677890	24374562	40967473
	9	136354015	63595461	148162969
1962	1	55109453	26704180	65179902
	2	204113826	44851090	193045424
	3	155125796	88889270	135349414
	4	191866388	69352010	209246275
	5	53068883	32002060	52052641
	6	101035480	30463350	99151174
	7	71438605	34313510	85214219
	8	38780014	26949860	44337628
	9	150022936	70087490	162686474

TABLE A-3 (CONTINUED)

ESTIMATED DOLLAR VALUE OF RETAIL SALES BY REGION
1959 - 1969

YEAR	REGION	REFRIGERATORS	RANGES	LAUNDRY PRODUCTS
1963	1	57524594	27514947	68301537
	2	210928052	47874543	202763502
	3	160858862	91205431	145173928
	4	200274258	75261276	226031006
	5	54328984	38512093	57761292
	6	102267582	33562157	104839340
	7	76699644	37218044	89467764
	8	39414562	27943408	47301089
	9	162988898	78413075	173735215
1964	1	61512876	26906334	70936161
	2	223685484	47832316	206540260
	3	173355618	86493904	158666817
	4	221448882	71774444	241014550
	5	58157844	34491352	63285952
	6	105132162	29219792	110576501
	7	77170896	33740644	97567193
	8	35788986	21859992	50657697
	9	162172092	69599838	178838813

TABLE A-3 (CONTINUED)

ESTIMATED DOLLAR VALUE OF RETAIL SALES BY REGION
1959 - 1969

YEAR	REGION	REFRIGERATORS	RANGES	LAUNDRY PRODUCTS
1965	1	69327440	31873893	74504461
	2	234741520	51981974	219734972
	3	201900400	95338870	181700689
	4	243254960	77572800	266235407
	5	69327700	36987100	75043138
	6	117978640	28891074	122536345
	7	85138820	36682648	108137806
	8	37704160	19027922	53173208
	9	156899340	58732074	180544262
1966	1	74001720	31413204	79873161
	2	233160153	55023612	230474078
	3	208164414	94587180	192913244
	4	254613069	81443892	279263361
	5	72970833	37328844	80904177
	6	121191567	28460862	129772288
	7	92505888	37438752	115514170
	8	38323044	17734620	55446253
	9	155957904	52973418	182815540

TABLE A-3 (CONTINUED)

ESTIMATED DOLLAR VALUE OF RETAIL SALES BY REGION
1959 - 1969

YEAR	REGION	REFRIGERATORS	RANGES	LAUNDRY PRODUCTS
1967	1	73844585	29989690	86347717
	2	232336377	53421200	247307319
	3	207401103	91956100	203525887
	4	248419353	79391600	286809664
	5	73989825	35883750	86444577
	6	122789121	28539530	136934024
	7	90604332	36651680	120180703
	8	39673998	17974320	56495830
	9	160181385	51521640	189484864
1968	1	80971800	35879370	90026577
	2	250397560	61168755	261853846
	3	241818360	115144220	233334412
	4	285040000	92538110	309536024
	5	83806800	41795175	94948989
	6	140329280	38423255	154132809
	7	102567920	45273585	129029910
	8	45836280	23029820	64640158
	9	183503040	66716415	205829180

TABLE A-3 (CONTINUED)

ESTIMATED DOLLAR VALUE OF RETAIL SALES BY REGION
1959 - 1969

YEAR	REGION	REFRIGERATORS	RANGES	LAUNDRY PRODUCTS
1969	1	82284789	36883546	90257726
	2	247333023	62318210	261652490
	3	257615540	122012486	235422002
	4	281388511	91281770	302247052
	5	91980343	43666702	90942589
	6	150145911	44705200	146866059
	7	102872260	44272194	126310581
	8	48808508	25595024	65556832
	9	188212913	69107018	204154654

TABLE A-4

INDEPENDENT VARIABLES USED IN REGRESSIONS BY REGION
1959 - 1969

YEAR	REGION	PERSONAL INCOME (MILLIONS)	INTEREST RATE ON AAA BONDS	CPI FOR REFRIG.	CPI FOR RANGES	CPI LAUNDRY PRODUCTS	CPI ALL PRODUCTS
1959	1	24786	4.38	98.6	99.9	98.4	101.5
	2	85185	4.38	98.6	99.9	98.4	101.5
	3	46064	4.38	98.6	99.9	98.4	101.5
	4	83065	4.38	98.6	99.9	98.4	101.5
	5	17007	4.38	98.6	99.9	98.4	101.5
	6	29655	4.38	98.6	99.9	98.4	101.5
	7	30372	4.38	98.6	99.9	98.4	101.5
	8	13474	4.38	98.6	99.9	98.4	101.5
	9	52975	4.38	98.6	99.9	98.4	101.5
1960	1	25500	4.41	96.6	98.8	95.9	103.1
	2	88200	4.41	96.6	98.8	95.9	103.1
	3	47700	4.41	96.6	98.8	95.9	103.1
	4	86400	4.41	96.6	98.8	95.9	103.1
	5	17800	4.41	96.6	98.8	95.9	103.1
	6	30800	4.41	96.6	98.8	95.9	103.1
	7	31800	4.41	96.6	98.8	95.9	103.1
	8	14400	4.41	96.6	98.8	95.9	103.1
	9	55800	4.41	96.6	98.8	95.9	103.1

TABLE A-4 (CONTINUED)

INDEPENDENT VARIABLES USED IN REGRESSIONS BY REGION
1959 - 1969

YEAR	REGION	PERSONAL INCOME (MILLIONS)	INTEREST RATE ON AAA BONDS	CPI FOR REFRIG.	CPI FOR RANGES	CPI LAUNDRY PRODUCTS	CPI ALL PRODUCTS
1961	1	26987	4.35	95.3	96.8	93.0	104.2
	2	91602	4.35	95.3	96.8	93.0	104.2
	3	50602	4.35	95.3	96.8	93.0	104.2
	4	88015	4.35	95.3	96.8	93.0	104.2
	5	18543	4.35	95.3	96.8	93.0	104.2
	6	31978	4.35	95.3	96.8	93.0	104.2
	7	33153	4.35	95.3	96.8	93.0	104.2
	8	15110	4.35	95.3	96.8	93.0	104.2
	9	58964	4.35	95.3	96.8	93.0	104.2
1962	1	28526	4.33	93.0	96.4	90.5	105.4
	2	96331	4.33	93.0	96.4	90.5	105.4
	3	54469	4.33	93.0	96.4	90.5	105.4
	4	92706	4.33	93.0	96.4	90.5	105.4
	5	19537	4.33	93.0	96.4	90.5	105.4
	6	33586	4.33	93.0	96.4	90.5	105.4
	7	35262	4.33	93.0	96.4	90.5	105.4
	8	16509	4.33	93.0	96.4	90.5	105.4
	9	62871	4.33	93.0	96.4	90.5	105.4

TABLE A-4 (CONTINUED)

INDEPENDENT VARIABLES USED IN REGRESSIONS BY REGION
1959 - 1969

YEAR	REGION	PERSONAL INCOME (MILLIONS)	INTEREST RATE ON AAA BONDS	CPI FOR REFRIG.	CPI FOR RANGES	CPI LAUNDRY PRODUCTS	CPI ALL PRODUCTS
1963	1	29348	4.26	90.6	95.7	89.2	106.7
	2	99562	4.26	90.6	95.7	89.2	106.7
	3	57577	4.26	90.6	95.7	89.2	106.7
	4	97636	4.26	90.6	95.7	89.2	106.7
	5	21335	4.26	90.6	95.7	89.2	106.7
	6	35935	4.26	90.6	95.7	89.2	106.7
	7	36391	4.26	90.6	95.7	89.2	106.7
	8	17325	4.26	90.6	95.7	89.2	106.7
	9	67227	4.26	90.6	95.7	89.2	106.7
1964	1	31300	4.40	88.8	95.0	88.0	108.1
	2	106200	4.40	88.8	95.0	88.0	108.1
	3	62400	4.40	88.8	95.0	88.0	108.1
	4	104400	4.40	88.8	95.0	88.0	108.1
	5	22600	4.40	88.8	95.0	88.0	108.1
	6	38400	4.40	88.8	95.0	88.0	108.1
	7	37900	4.40	88.8	95.0	88.0	108.1
	8	18100	4.40	88.8	95.0	88.0	108.1
	9	72100	4.40	88.8	95.0	88.0	108.1

TABLE A-4 (CONTINUED)

INDEPENDENT VARIABLES USED IN REGRESSIONS BY REGION
1959 - 1969

YEAR	REGION	PERSONAL INCOME (MILLIONS)	INTEREST RATE ON AAA BONDS	CPI FOR REFRIG.	CPI FOR RANGES	CPI LAUNDRY PRODUCTS	CPI ALL PRODUCTS
1965	1	33800	4.49	86.2	93.7	86.8	109.9
	2	113700	4.49	86.2	93.7	86.8	109.9
	3	68300	4.49	86.2	93.7	86.8	109.9
	4	115100	4.49	86.2	93.7	86.8	109.9
	5	24800	4.49	86.2	93.7	86.8	109.9
	6	41600	4.49	86.2	93.7	86.8	109.9
	7	42000	4.49	86.2	93.7	86.8	109.9
	8	19500	4.49	86.2	93.7	86.8	109.9
	9	77200	4.49	86.2	93.7	86.8	109.9
1966	1	36400	5.13	82.9	92.0	86.3	113.1
	2	121900	5.13	82.9	92.0	86.3	113.1
	3	74700	5.13	82.9	92.0	86.3	113.1
	4	125100	5.13	82.9	92.0	86.3	113.1
	5	27200	5.13	82.9	92.0	86.3	113.1
	6	45500	5.13	82.9	92.0	86.3	113.1
	7	45400	5.13	82.9	92.0	86.3	113.1
	8	20600	5.13	82.9	92.0	86.3	113.1
	9	83600	5.13	82.9	92.0	86.3	113.1

TABLE A-4 (CONTINUED)

INDEPENDENT VARIABLES USED IN REGRESSIONS BY REGION
1959 - 1969

YEAR	REGION	PERSONAL INCOME (MILLIONS)	INTEREST RATE ON AAA BONDS	CPI FOR REFRIG.	CPI FOR RANGES	CPI LAUNDRY PRODUCTS	CPI ALL PRODUCTS
1967	1	39700	5.51	82.7	92.7	86.6	116.3
	2	131700	5.51	82.7	92.7	86.6	116.3
	3	81400	5.51	82.7	92.7	86.6	116.3
	4	132900	5.51	82.7	92.7	86.6	116.3
	5	29200	5.51	82.7	92.7	86.6	116.3
	6	49500	5.51	82.7	92.7	86.6	116.3
	7	48300	5.51	82.7	92.7	86.6	116.3
	8	22000	5.51	82.7	92.7	86.6	116.3
	9	26500	5.51	82.7	92.7	86.6	116.3
1968	1	42900	6.18	83.8	95.2	88.8	121.2
	2	142400	6.18	83.8	95.2	88.8	121.2
	3	89300	6.18	83.8	95.2	88.8	121.2
	4	144600	6.18	83.8	95.2	88.8	121.2
	5	31600	6.18	83.8	95.2	88.8	121.2
	6	54600	6.18	83.8	95.2	88.8	121.2
	7	52300	6.18	83.8	95.2	88.8	121.2
	8	24200	6.18	83.8	95.2	88.8	121.2
	9	99900	6.18	83.8	95.2	88.8	121.2

TABLE A-4 (CONTINUED)

INDEPENDENT VARIABLES USED IN REGRESSIONS BY REGION
1959 - 1969

YEAR	REGION	PERSONAL INCOME (MILLIONS)	INTEREST RATE ON AAA BONDS	CPI FOR REFRIG.	CPI FOR RANGES	CPI LAUNDRY PRODUCTS	CPI ALL PRODUCTS
1969	1	46500	7.03	85.3	97.7	90.6	127.7
	2	154800	7.03	85.3	97.7	90.6	127.7
	3	99400	7.03	85.3	97.7	90.6	127.7
	4	157100	7.03	85.3	97.7	90.6	127.7
	5	34700	7.03	85.3	97.7	90.6	127.7
	6	59700	7.03	85.3	97.7	90.6	127.7
	7	56700	7.03	85.3	97.7	90.6	127.7
	8	26500	7.03	85.3	97.7	90.6	127.7
	9	107600	7.03	85.3	97.7	90.6	127.7

NOTES TO APPENDIX A:

1. The range product group is a combination of electric free standing ranges and built-in ranges.

2. The laundry product group is a combination of automatic washers, electric and gas dryers, combination washer-dryer units, and wringer and other washers.

3. The raw observations were scaled internally within the computer and the results presented are based on the scaled values. The variables were scaled as follows:

   a) Sales for each product group are expressed in millions of dollars.

   b) The income variable and liquid assets are expressed in billions of dollars.

Appendix B

States Included in Regions
Used for Combining Time Series and Cross-Section Data

I New England

Maine
New Hampshire
Vermont
Massachusetts
Rhode Island
Connecticut

II Middle Atlantic

New York
New Jersey
Pennsylvania

III South Atlantic

Delaware
Maryland
District of Columbia
Virginia
West Virginia
North Carolina
South Carolina
Georgia
Florida

IV East North Central

Ohio
Indiana
Illinois
Michigan
Wisconsin

V East South Central

Kentucky
Tennessee
Alabama
Mississippi

VI West South Central

Arkansas
Louisiana
Oklahoma
Texas

VII West North Central

Minnesota
Iowa
Missouri
North Dakota
South Dakota
Nebraska
Kansas

VIII Mountain

Montana
Idaho
Wyoming
Colorado
New Mexico
Arizona
Utah
Nevada

IX Pacific

Washington
Oregon
California
Hawaii
Alaska