INFERRING RISK AVERSION FROM THE PORTFOLIO DECISION

By

Desu Liu

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Economics

2011

ABSTRACT

INFERRING RISK AVERSION FROM THE PORTFOLIO DECISION

By

Desu Liu

This dissertation examines how to infer risk aversion based on observed portfolio decisions. It consists of five chapters. Chapters 1 and 2 are introduction and literature review respectively.

Chapter 3 focuses on the role of uncapitalized future income and investigates the slope of relative risk aversion for consumption, an essential property of utility functions for consumption. The motivation is from the fact that uncapitalized future income is often modeled as a component of current wealth in theory, while it is *not* in most empirical studies. By examining risky asset allocations in multiperiod consumption-investment optimization problems, I analytically show that utility functions for consumption can exhibit either decreasing relative risk aversion (DRRA) or constant relative risk aversion (CRRA), depending on whether uncapitalized future income is introduced to provide another source of consumption. These findings can be used to reinterpret recent empirical evidence at micro level that there is essentially no wealth effect on households' financial asset allocations.

Chapter 4 examines how to infer the magnitude of the Pratt-Arrow measures of risk aversion for wealth, based on a single portfolio choice. Three different procedures are evaluated. First, the existing approach that leads to a point estimate at the initial wealth and estimates risk aversion in the small is discussed. The second approach uses quadratic utility as an approximation to the true utility, and generates an estimate of risk aversion in the large, based only on the mean and

variance of the risky asset return. The third approach directly employs functional forms for utility function or risk aversion to estimate risk aversion in the large. Computed solutions indicate that assuming functional forms for utility or risk aversion performs much better in estimating relative risk aversion over a wide range of the risky return distributions.

Chapter 5 uses theoretical findings in Chapters 3 and 4 to reinterpret empirical evidence on relative risk aversion presented in three important published papers. The first conclusion is that relative risk aversion for liquid financial wealth is probably constant. Second, relative risk aversion for consumption that comes from liquid financial wealth can be decreasing if uncapitalized future income is incorporated into dynamic consumption-investment optimization problems. Third, the opinions on the magnitude of relative risk aversion for Arrow-Pratt wealth are still divergent but at the mean return it usually does not exceed 10 unless for extremely impoverished investors.

Dedicated to my parents and my wife

ACKNOWLEDGMENTS

First and foremost, I thank my advisor, Jack Meyer, for his generous time, energy and commitment. Throughout this dissertation, I have greatly benefited from his persistent encouragement, valuable comments, and openness in sharing his knowledge and insight.

I also express my appreciation to all the committee members: Dr. Carl Davidson, Dr. Antonio Doblas-Madrid and Dr. Robert J. Myers, for their very helpful suggestions that contribute to the improvement of this dissertation.

TABLE OF CONTENTS

LIST OF TABLES vii

LIST OF FIGURES viii

CHAPTER ONE
INTRODUCTION 1

CHAPTER TWO
LITERATURE REVIEW 6
2.1 Theoretical Analysis 6
2.2 Recent Empirical Evidence 11

CHAPTER THREE
UNCAPITALIZED FUTURE INCOME 15
3.1 Introduction 15
3.2 A Two-period Model and the Main Finding 24
3.3 An Infinite Horizon Model 32
3.4 Discussion 37
Appendix 39

CHAPTER FOUR
INFERRING RISK AVERSION USING ONE PORTFOLIO DECISION 48
4.1 Introduction 48
4.2 Inferring Risk Aversion in the Small 52
4.3 Inferring Risk Aversion in the Large 56
4.4 Inferring Risk Aversion in the Large Using Functional Forms for Utility 60
4.5 Numerical Solutions 66
4.6 Conclusion 70

CHAPTER FIVE
REINTERPRETATION OF RECENT EMPIRICAL EVIDENCE 76
5.1 Introduction 76
5.2 Friend and Blume (1975) 80
5.3 Chiappori and Paiella (forthcoming) 86
5.4 Brunnermeier and Nagel (2008) 90
5.5 Summary and Discussion 96

REFERENCES 98

LIST OF TABLES

Table 1: The Magnitudes of Relative Risk Aversion at Four Values for the Risky Asset Return under Three Different Approaches 69

Table 2: Estimates of Relative Risk Aversion at Mean Risky Return for a Representative of Households in Six Net Financial Wealth Categories (Exclusive of Homes and Human Capital)

....85

LIST OF FIGURES

Figure 4.1: relative risk aversion as a function of the risky return for $\alpha^* = .2$ 73

Figure 4.2: relative risk aversion as a function of the risky return for $\alpha^* = .5$ 74

Figure 4.3: relative risk aversion as a function of the risky return for $\alpha^* = .8$ 75

Chapter One

Introduction

This dissertation examines how to infer the Pratt-Arrow measures of risk aversion for an expected utility maximizing decision maker, based on her observed portfolio choice(s). It contains successful attempts from which three different papers have been extracted, as well as some current thinking that may later help in developing other papers. A literature review is provided in Chapter 2. I discuss the intuition and summarize each of the three main chapters of this dissertation in the following.

Chapter 3 mainly studies the role of nonfinancial wealth components when using wealth allocation decisions to infer the slope of relative risk aversion for *consumption*. The literature gives different definitions of wealth but the wealth measures that are frequently used exclude important nonfinancial elements as other sources of consumption. In particular, I focus on the effect of uncapitalized future income, which provides another source of consumption and thus may imply a different slope of relative risk aversion for consumption. This analytical finding is based on the same response of the risky asset share to changes in wealth. The finding has an implication for applied economists who are interested in the current debate on whether constant relative risk aversion (CRRA) power utility or decreasing relative risk aversion (DRRA) habit formation utility is a more appropriate functional form of utility for consumption.

It is often assumed in multi-period models that all future income can be capitalized into current wealth for the portfolio allocation decision; that is, current wealth equals lifetime wealth. As a result, consumption can only comes from the return on wealth. This assumption on the wealth measure, however, does not match what is observed from the real world, where some sources of future income are not a component of current wealth. Examples of uncapitalized future income include labor income, social security benefits, pensions, government transfer, appreciation of housing equity, and other forms. This can happen as long as an agent has sufficient current wealth for consumption and for investment. For instance, a tenured professor has very stable future labor income but he may choose not to capitalize every penny into current wealth. It is also possible that for some reasons, a decision maker "fails" to integrate these sources of income into current wealth. Finally, due to some imperfection, financial frictions or legal restrictions, market does not allow one to fully capitalize various forms of future income.

In a recent study to test the existence of time-varying risk aversion that results from external habit formation utility, Brunnermeier and Nagel (2008) (B-N) find that the share of wealth allocated to the risky assets is essentially not affected by wealth changes across time periods. This may imply that relative risk version for certain measure of wealth is constant for a representative of households from the Panel Study of Income Dynamics (PSID). It looks like this empirical evidence cannot reconcile the positive contemporaneous relationship shown in their testable equation which is derived from the theory. B-N interpret the finding as evidence against the presence of DRRA habit formation utility for consumption at micro level and suggest that CRRA power utility for consumption may prevail.

With the correction for uncapitalized future income, it is demonstrated in my theoretical analysis that the sign of the slope of relative risk aversion for consumption can be totally different from without this correction. First, the study of a two-period model shows that if the comparative static change in the initial wealth has no effect on the risky asset proportion, utility function must exhibit decreasing relative risk aversion (DRRA) for consumption. An infinite horizon model assuming habit formation utility and an exogenous inflow of future income is then examined. If the present value of future income is relatively close to that of future habits, the risky asset share may not respond to changes in wealth over time. These two analytical findings concerning DRRA utility functions for consumption can be used to reinterpret recent empirical micro-level findings, including the one by B-N that there is an absence of wealth effect on households' asset allocation over time.

A workable future direction is to test the existence of habit formation using housing data at micro level. Housing is the largest wealth component for many households, and is also an illiquid asset with risk properties being unclear. For one thing, housing is a durable good and provides constant consumption flow which may be treated as a constant habit. Second, home mortgage helps capitalize one's future income to certain extent, since mortgage loan is usually earmarked and is different from a consumer loan which does not require a specific use. These two features may enable housing to be incorporated into multi-period models in which habit formation utility and a future income stream are assumed.

Chapter 4 examines how to infer the magnitude of the Pratt-Arrow measures of risk aversion for *wealth* using one or more observations on the portfolio allocation decision. While this magnitude is very useful in asset pricing models and in the determination of insurance premium, the literature presents little direct empirical evidence, as Meyer and Meyer (2006) point out. The endeavor in this chapter is in part driven to provide more of such information. More importantly, it is also because the main existing approach to connecting the portfolio decision to risk aversion for wealth infers risk aversion in the small (for small risks), rather than risk aversion in the large (for large risks). This does not make much sense given the fact that portfolio risk is definitely a large risk, often measured in terms of the standard deviation of its returns. For example, during the period of 1890 to 1979, investing $1 in the Standard & Poor 500 Index has an annualized mean return of \$1.07 and a standard deviation of \$0.17; this is compared to the investment of \$1 in the short-term U.S. treasury bills with an annualized return and a standard deviation of \$1.01 and \$0.05 respectively in the same period.

Friend and Blume (1975) (F-B) provide a formula that can be used to infer the measure of risk aversion for wealth in the small at the point of the initial wealth, based on a single observation on the portfolio allocation. This formula is reached using a specific approximation procedure by assuming that time interval is very small. As a consequence, the portfolio risk being evaluated only leads to small wealth variations, and risk aversion for small risks can be inferred at the point of the initial wealth. The same formula is recently utilized by Chiappori and Paiella (forthcoming) (C-P) in the study of Italian household wealth allocation across time periods. A main concern about the Friend and Blume methodology is whether it can be applied to infer or estimate the magnitude of risk aversion for risks whose sizes cannot be assumed to be zero or close to zero.

I study a standard one-period two-asset portfolio allocation model, in which time interval is one year and hence the risks from investing in the risky asset are substantial. Two different methods to infer risk aversion in the large are proposed, assessed and compared with the one used by F-B to infer risk aversion in the small. The first method quadratically approximates the utility function for wealth, and then maximizes the expectation of the approximated utility. This gives rise to an estimate of risk aversion in the large, which only depends on the mean and variance of the risky asset return. The second method directly employs functional forms of utility or risk aversion to infer risk aversion in the large. The procedure involves specifying one or more portfolio choices to identify the same number of unknown parameters in an assumed functional form of utility for wealth. The second method requires complete prior information on the probability distribution function for the risky asset return.

Three functional forms of utility or marginal utility belonging to the family of isoelastic risk preferences recently proposed by Meyer (2010) are considered. These include two commonly used utility: power (CRRA) and exponential (CARA), and one marginal utility chosen to display DRRA. In addition, historical market data of annualized returns on the Standard & Poor 500 Index and on the U.S. treasury bills are borrowed. Using one observed portfolio decision, computed solutions show that picking one of the three functional forms and then inferring relative risk aversion performs much better than assuming a quadratic utility or using the F-B in the small procedure, if the true utility is from the isoelastic risk preferences group. It seems that when the goal is to estimate risk aversion level under regular conditions, choosing a functional form of utility that possesses the property of isoelastic risk preferences (even if it is wrong) to infer risk aversion in the large prevails over the Friend and Blume methodology of inferring risk aversion in the small without restricting functional forms of utility.

Chapter 5 provides a detailed discussion of three published papers: F-B, C-P and B-N. The methodologies used and the empirical evidence presented in these papers have led to the writing of chapters 3 and 4. The theoretical findings in these two chapters are utilized to reinterpret the empirical findings concerning the magnitudes and the slopes of relative risk aversion. There are three tentative conclusions. First, relative risk aversion for liquid financial wealth is probably constant. Second, relative risk aversion for consumption can be decreasing, if uncapitalized future income, an often ignored part of wealth, is assumed to provide another source of consumption. Third, the opinions on the magnitude of relative risk aversion for Arrow-Pratt wealth are still divergent but at the mean return it usually does not exceed 10 unless for extremely impoverished investors. Meanwhile, two major econometric issues that may confound the identification of the effect of wealth changes over time on the risky asset share are indicated.

Chapter Two

Literature Review

This chapter consists of two parts: theoretical analysis and recent empirical evidence. In the first part, I review a portion of literature that studies the demand for risky assets in one-period models by making assumptions on the magnitude and/or the slope of risk aversion for wealth, some literature which focuses on tradeoff between consumption and savings in two-period consumption models by assuming that risk aversion for consumption satisfies certain properties, and several papers that provide analytical solutions for consumption or the risky asset share using multiperiod models in which the functional form of utility for consumption is assumed. Major papers in macroeconomics that use DRRA habit formation utility for consumption to address the equity premium puzzle are also reviewed. In the second part, I review recent literature that uses data on portfolio choice and/or consumption to either deduce or estimate relative risk aversion for wealth and relative risk aversion for consumption. A detailed discussion of three papers that are most important to this dissertation will be presented in chapter 5.

2.1 Theoretical Analysis

Arrow (1965, 1971) and Pratt (1964) introduce the measures of absolute risk aversion and relative risk aversion for wealth. The former refers to risk aversion for risks that cause wealth deviations while the latter concerns risk aversion for risks measured as a proportion of wealth. The original outcome variable is Arrow-Pratt (A-P) wealth, assumed to include only liquid and fully divisible financial assets. One of their major contributions is to propose a theorem on the portfolio allocation decision in a one-period model with one risky asset and one riskless asset.

Specifically, the optimal risky asset share is (strictly) increasing, constant or decreasing in the initial wealth if relative risk aversion for wealth is decreasing, constant or increasing respectively. The theorem can be used to predict how the relative demand for the risky asset varies in response to changes in wealth given assumptions on the sign of the slope of relative risk aversion for wealth. More importantly, it can also be employed to infer the sign of the slope of relative risk aversion for wealth, based on the effect of comparative static changes in wealth on the risky asset proportion.

When a portfolio contains more than two assets, however, the above theorem of wealth effects on portfolio allocation in general does not hold, as Cass and Stiglitz (1972) demonstrate. An exception is to apply the mutual fund theorem proposed by Tobin (1958). As a result, the choice of a multi-asset portfolio can be reduced to the choice of a portfolio including one riskless asset and a mutual fund of all risky assets. Hadar and Seo (1990) study portfolios with a finite number of risky assets. When a portfolio consists of only two risky assets, they provide necessary and sufficient conditions for a dominating shift (first-degree stochastic dominance; mean-preserving contraction; second-degree stochastic dominance) of the distribution of the returns on a risky asset to lead to an increase in the proportion of wealth in that risky asset. If it is further assumed that an investor exhibits constant absolute risk aversion (CARA), the aforementioned conditions also hold for the case of more than two risky assets.

Some literature examines the effect of changes in the riskless return on the fraction of the riskless asset in the standard one-period model with one riskless asset and one risky asset. For example, Fishburn and Porter (1976) show that the share of the safe asset increases as the riskless return increases and the return distribution of the risky asset is fixed, provided that absolute risk aversion for wealth is nondecreasing and relative risk aversion for wealth does not exceed unity. They also give conditions under which a first-degree stochastic dominance shift in the risky asset return results in an increase in the optimal proportion invested in the risky asset.

By making assumptions on relative risk aversion for consumption, a branch of early literature investigates the effects of uncertainty on saving decisions in two-period models where consumption is the only choice variable. Leland (1968) finds that with time *additive* utility, decreasing absolute risk aversion (DARA) is sufficient to ensure that uncertainty of future income has a positive effect on the precautionary demand for savings. Rothschild and Stiglitz (1971) examine the effect of increasing (capital) risk on the savings rate, and find that the effect is positive if relative risk aversion for consumption is non-increasing and greater than unity.[1]

Sandmo (1970) also studies a two-period consumption model but using a time *nonseparable* utility. He defines decreasing temporal risk aversion as that the risk aversion function decreases in the second-period consumption and increases in the first-period consumption, where the risk aversion function refers to minus the ratio of the second derivative of utility function with respect to the second period consumption to the first derivative of utility function with respect to the second period consumption. Sandmo then shows that decreasing temporal risk aversion is a sufficient condition for the increased uncertainty about future income to increase savings. He also demonstrate that the effect of the increased capital risk on savings is ambiguous, since without further assumptions, the increased capital risk has both a substitution effect and an income effect on the demand for savings.

[1] Rothschild and Stiglitz (1971) also show that in a one-period portfolio problem and in a portfolio-savings problem under an infinite horizon, increasing risk in a risky asset return does not necessarily lower the demand for that risky asset and thus improve the savings rate.

A general economic analysis concerning risk aversion is to incorporate more than one choice variable in the same model, say both consumption and investment. Sandmo (1969) studies such a two-period model in which strictly positive and exogenous income exists in the second period. He is mainly interested in the comparative statics of changes in the rates of return, the degree of risk, and capital gains taxation on the optimal consumption and the *amount* of investment in the risky asset in the first period. One of his findings is that with time additive utility for consumption, DARA for consumption is a sufficient condition for an increase in the initial wealth to give rise to an increase in the amount invested in the risky asset.

The optimal consumption and/or risky asset share cannot be derived in many cases. To my best knowledge, the literature using time additive utility for consumption in multiperiod models provides three examples of analytical solutions by making assumptions on the slope of relative risk aversion for consumption. Samuelson (1969) shows that when the utility function takes CRRA power or logarithmic form and the risky asset returns are independently and identically distributed across time periods, the optimal risky asset share is constant over time and the optimal consumption is proportional to wealth in each period. Kimball and Mankiw (1989) provide another explicit solution for consumption as a linear function of wealth, assuming that utility function is of CARA exponential form and that the decision maker receives certain income that is random in each future period. Meyer and Meyer (2005a) present a special habit formation utility which displays DRRA for consumption, and use it to obtain a linear relationship between consumption and wealth in equilibrium. The intercept of the consumption function is just the nonrandom uncapitalized income in each period, which also equals the special habit in the utility function for consumption. Note that consumption is the only choice variable modeled by authors of the latter two papers.

Macroeconomists are also interested in the magnitude of relative risk aversion for consumption. A main reason is that it is concerned with the well-known equity premium puzzle, first presented by Mehra and Prescott (1985). The equity premium puzzle refers to the impossibility of simultaneously explaining the high risk premium and the low risk free rate based on historical market returns, using the consensus level of relative risk aversion in a standard multi-period consumption model assuming that time additively separable utility function is of the CRRA power form. An undesirable property of power utility is that the coefficient of CRRA and the intertemporal elasticity of substitution are governed by the same power parameter and actually are reciprocals of one another. This implies that a high level of relative risk aversion is required to sustain the high risk premium while a low intertemporal elasticity of substitution (plus a low risk free rate) is insufficient to generate a large growth rate in aggregate consumption over time, and vice versa. Numerous subsequent studies have confirmed that the puzzle exists across countries and persists over time.

One way to address this empirical irregularity is to introduce utility functions that do not restrict the relationship between relative risk aversion and the intertemporal elasticity of substitution to behave in the way imposed by CRRA power utility for consumption. Constantinides (1990) find that habit formation utility can help in simultaneously eliminating the equity premium and the risk free rate puzzles in a representative-consumer production economy when time is continuous. Campbell and Cochrane (1999) specifically study a utility function with external habit formation; that is, habit is unrelated to past consumption. They find that a slow-moving external habit can explain not only a high risk premium and a low risk free rate in equilibrium but also other empirical asset pricing phenomena in a representative-consumer endowment economy when time is discrete.

As Meyer and Meyer (2005a) point out, another perspective to understand the usefulness of DRRA utility functions for consumption is to acknowledge that the relationship between the slope of relative risk aversion for consumption and the slope of relative risk aversion for wealth can be different. This relationship depends on how the optimal consumption and wealth are defined, measured and related to each other in equilibrium. Meyer and Meyer provide an example using a multiperiod consumption model, in which the periodic utility function for consumption can exhibit DRRA while the corresponding indirect utility function for wealth can display CRRA, when the equilibrium consumption is linear in wealth with the intercept being a large component of income not included in wealth. This utility function for consumption, together with a marginal utility function that also displays DRRA for consumption, are used to show that the equity premium puzzle can be resolved based on the tests developed by Kocherlakota (1996).

2.2 Recent Empirical Evidence

Direct empirical evidence concerning relative risk aversion for wealth is very limited in the literature. This often comes from examining the portfolio allocation decision.[2] Even so, much of this scarce evidence is about relative risk aversion for a broad measure of wealth, rather than the

[2] An alternative is to study the demand for insurance but it is less frequently seen. In addition, some literature uses asset holdings information in accounts of brokerage firms to estimate the slope of relative risk aversion for wealth, for example: Cohn, Lewellen, Lease and Schlarbaum (1975).

original A-P wealth. As Rabin and Weizsacker (2009) state in their conclusion section, "The currently prevalent approach of measuring, for instance, a coefficient of relative risk aversion over wealth gives the researcher the freedom to choose from a range of possible definitions of wealth (from one-hour experimental earnings to lifetime wealth). This has the undesirable property that the choice of definition changes the measured coefficient by several orders of magnitude."

What Rabin and Weizsacker (2009) point out is just part of a story. In fact, one should also be cautious at making interpretations on the estimated slope of relative risk aversion for different wealth measures used in empirical studies, as well as on the slope of relative risk aversion for consumption. Note that the sign of the slope of relative risk aversion for consumption differs for three commonly used functional forms of utility for consumption in the literature: exponential (IRRA), power (CRRA) and habit formation (DRRA). Chapters 3 and 5 will cover this issue based on a recent study by Brunnermeier and Nagel (2008). Therefore a review on this paper is not included here. The purpose here is to present recent empirical evidence in papers that will not be discussed in detailed in the three main chapters of this dissertation.

Blake (1996) uses wealth composition information of cross sectional households between 1991 and 1992 in the United Kingdom to estimate the magnitude of relative risk aversion for the rate of return on investment portfolio. In a mean-variance model of investment choice, he assumes CRRA power utility with the rate of return being normally distributed. Blake defines financial assets to include three components: interest-bearing accounts, bonds and shares. The estimated magnitude ranges from 7.88 to 47.60 for representatives of households that are grouped into six wealth categories with mid-range wealth from £252 to £100,000. Since Blake does not use A-P wealth as the outcome variable, these estimates are transformed into those of relative risk aversion for A-P wealth by Meyer and Meyer (2005b), who report the adjusted estimates to range from .59 to 16.8.

The latest empirical studies focus on whether or not relative risk aversion for *wealth* is constant. Brunnermeier and Nagel (2008) and Chiappori and Paiella (forthcoming) seem to agree on CRRA for financial wealth measures. One of the major findings by Calvet, Campbell and Sodini (2009), however, suggests DRRA for very liquid financial wealth. Calvet et al. examine portfolio rebalancing behavior using administrative panel data between 1999 and 2002 from all Swedish households. They measure a household's financial wealth as the sum of cash (bank account balances plus money market funds), direct holdings of stock, and risky mutual funds (bonds funds or equity funds). The risky asset share is the ratio of stock and risky mutual funds to financial wealth. Calvet et al. find that an increase in log financial wealth leads to a higher risky asset share. This happens in several specifications including using instrumental variables and replacing changes in log financial wealth with lagged changes in log financial wealth.

A strand of recent work investigates the sign of the slope of relative risk aversion for *consumption* using micro-level panel data. Dynan (2000) tests the presence of DRRA internal habit formation using food consumption data from PSID and finds no such evidence. Brunnermeier and Nagel (2008) also advance to contend that CRRA power utility, rather than DRRA external habit formation utility, better represents households' utility function for consumption based on the absence of response of risky asset allocation to changes in wealth over time. Sahm (2008) examine relative risk aversion measures elicited from responses to hypothetical gamble questions over lifetime income in the 1992-2002 Health and Retirement

Study (HRS) and find some evidence in support of CRRA power utility for consumption.[3] These findings are in contrast with those presented by Ogaki and Zhang (2001), who find supporting evidence of DRRA subsistence utility for food consumption using data from low income Indian and Pakistani households, Lupton (2003), who claims the existence of habit formation by interpreting the negative relationship between past consumption and current risky asset holdings, and Ravina (2007), who uses purchase information in 2,674 U.S. credit card accounts located in California between 1999 and 2002 from the Credit Card Panel (CCP) to discover support for habit formation utility.

The literature review stops here. More relevant papers on the theme of this dissertation, how to infer the level and the slope of relative risk aversion from the portfolio allocation decision, will continue to be mentioned and discussed in the remaining chapters. The next chapter will incorporate uncapitalized future income and build a two-period and an infinite horizon portfolio allocation models to infer the slope of relative risk aversion for consumption, given the observed household wealth allocation behavior.

[3] Barsky et al. (1997) use similar data from HRS, but with a cross section of households in 1992, they find some evidence against CRRA power utility for consumption.

Chapter Three

Uncapitalized Future Income

3.1 Introduction

A standard assumption in multiperiod models where two choice variables, consumption and investment, are simultaneously selected is that, a decision maker starts with a large amount of lifetime wealth. The only source of consumption is thus the return on lifetime wealth which is saved or invested. This assumption captures the essence of an Arrow-Debreu, i.e., complete market economy, in which all the future income can be converted and included in current wealth for the decision making. Of course, it is only when all the future income is *capitalized* in this way, current wealth equals lifetime wealth.[4] It is possible that in a perfect capital market, certain future income is uncapitalized in current wealth as long as an agent has sufficient current wealth for consumption and for investment. When the time period unfolds, uncapitalized income is realized, becomes a component of current wealth, and provides another source of consumption. If this is the case, risk aversion for consumption should exhibit a different pattern and uncapitalized future income has to be separately considered as an important factor in consumption-portfolio allocation decision models under a dynamic context.

[4] In this chapter both current wealth and lifetime wealth are considered to be measured in their net wealth, which is consistent with the wealth measures used by Brunnermeier and Nagel (2008) in their empirical study. This does not preclude the possibility that one can borrow to increase both total wealth and liabilities while keeping net wealth unchanged. In addition, the rate at which future income can be capitalized should be considered as exogenous; that is, the decision making does not affect this rate. This rate need not be the riskless rate and can be heterogeneous across different agents. For the convenience of the analysis, the riskless rate is used later.

Uncapitalized future income in this chapter is specifically referred to as exogenous future income. Examples abound in the real world. These can consist of wage income, social security benefits, pensions, the appreciation of housing equity, and other forms of net income in the future that are not counted as a part of current wealth.[5] In a large body of the empirical literature, wealth is measured in terms of current wealth, which does not contain the part of uncapitalized future income and may consist of a small fraction of lifetime wealth. In contrast, most existing theoretical models assume that wealth includes the value of all future income flow. As a consequence, uncapitalized future income creates an inconsistency between the measure of wealth used in theoretical analysis and the measure of wealth used in empirical studies. The inconsistency needs to be corrected for to make the theory and empirical work match. This chapter finds that the correction changes the common understanding concerning the slope of relative risk aversion for consumption, and therefore alters the conventional wisdom of functional forms of utility for consumption.

When studying the slope of the Pratt-Arrow measure of relative risk aversion for wealth, the share of wealth allocated to the risky asset is a portfolio decision that is frequently examined. The existing literature often resorts to the comparative static finding by Pratt (1964) and Arrow (1965; 1971) who use a standard one-period portfolio allocation model including one riskless asset and one risky asset. Specifically, they independently find that the optimal risky asset share is increasing, constant, or decreasing respectively in the initial wealth, if relative risk aversion for wealth is decreasing, constant, or increasing. Since in this model consumption only comes from

[5] For the ease of illustration, this chapter does not distinguish uncapitalized future income in the case when an agent has enough current wealth for consumption and investment from that in the other case when certain form(s) of market imperfection or legal restrictions keep(s) her from fully capitalizing future income.

the return on wealth, the slope of relative risk aversion for consumption is the same as the slope of relative risk aversion for wealth.

But the slopes of these two relative risk aversion measures can be strikingly different when uncapitalized future income becomes another source of consumption, as Meyer and Meyer (2005a) point out.[6] In a multiperiod model with rate-of-return risk being the only risk, they specify a time-separable periodic utility function for consumption that displays decreasing relative risk aversion (DRRA) for consumption. The utility function takes a power form but the base is the difference between consumption and an exogenous income in each period. Consumption is the *only* choice variable in their model. Using backward induction, Meyer and Meyer (2005a) derive a linear contemporaneous relationship between consumption and wealth, with the exogenous income being the intercept and thus being another source of consumption.[7] This equilibrium condition implies that the indirect utility function for wealth exhibits constant relative risk aversion (CRRA) for wealth.[8] They then argue that the equity premium puzzle can be resolved using such a utility function for consumption.

This chapter takes a further step to explicitly model uncapitalized future income in the theoretical analysis where *two* choice variables: consumption and investment are jointly

---

[6] In Meyer and Meyer (2005a), how these two slopes are related to one another depends on the properties of the optimal consumption as a function of wealth. Unfortunately, this consumption policy has to be assumed or be obtained under very special conditions.

[7] To my best knowledge, the other two special cases to obtain a linear consumption function occur when wealth is measured as lifetime wealth. These include: 1) CARA utility if all of the risk is to labor income; 2) CRRA utility if all of the risk is rate-of-return risk. See Carroll and Kimball (1996) for the discussion on the correctness of a concave consumption function in a more general case.

[8] In an often cited paper using cross sectional data, Friend and Blume (1975) conclude that the assumption of CRRA for wealth is not a bad first approximation.

determined.[9] The slope of relative risk aversion for consumption is then studied, based on the wealth allocation decisions in two models: a two-period model and an infinite horizon model.[10] The focus in the two-period model is to examine changes in the risky asset proportion with respect to the comparative static changes in current wealth. While the main point from the infinite horizon model is to illustrate that with habit formation utility, a special case of the DRRA utility functions for current consumption, the model can be used to derive a reduced form equilibrium relationship between the risky asset share and the wealth level in each period.[11]

A two-period consumption-portfolio decision model is first studied, where it is assumed that some uncapitalized income exists in the second period. The decision maker has to choose a portfolio decision in the first period and one consumption decision in each of the two periods. It is proved that if the risky asset proportion is constant with respect to the comparative static changes in current wealth, the periodic utility function for consumption must exhibit DRRA for consumption, rather than CRRA for consumption as some suggests. Moreover, the periodic utility function for consumption must also exhibit DRRA for consumption if the risky asset share varies positively with the comparative static changes in current wealth.

---

[9] For the convenience of deriving comparative static results and analytical solutions, this paper does not consider other choice variables such as endogenous borrowing and labor supply.

[10] A finite horizon model is not examined here for two reasons. First, optimal consumptions and portfolio decisions are usually solved by applying backward induction, for which some special functional form of utility for consumption has to be assumed. Quadratic utility may help in getting analytical solutions. But it is undesirable in the study of the slope of risk aversion since it exhibits both increasing absolute risk aversion and increasing relative risk aversion. Second, backward induction implies that all other optimal solutions starting from the second period are functions of the optimal consumption and portfolio decision in the first period. In other words, the finite horizon model is reduced to a two-period model.

[11] Whenever a habit is mentioned, it simply means a difference habit. The type of ratio habits introduced by Abel (1990) is not considered because it implies CRRA for consumption.

This comparative static finding based on a single portfolio decision may have an implication for recent empirical work using more than one portfolio decision in multiple periods. A sufficient condition is to assume that in the multiperiod context, each two-period decision making process is independent with one another, the decision maker's risk preference is unchanged, and the riskless return and the risky return distribution are fixed. Then the effect of the comparative static changes in wealth on the risky asset share is similar to the effect of the exogenous wealth fluctuations on changes in the risky asset share over time, and the analytical finding concerning DRRA utility functions for consumption also holds.

The above comparative static finding and others shown in the appendix are consistent with those obtained by Pratt (1964) and Arrow (1965; 1971), who do not have to take into account uncapitalized future income in the standard one-period model. It can be easily shown that the standard one-period portfolio decision model is nested as a special case of the two-period consumption-portfolio decision model used here, when a) all future income is capitalized as a part of current wealth; b) current consumption does not yield any utility for him; and c) the subjective discount factor equals one.[12]

A consumption-portfolio decision model in the discrete infinite horizon is also examined. The procedure extends the one used by Brunnermeier and Nagel (2008), who investigate a slow-moving internal habit that is included in the periodic utility function for consumption. The main difference here is that a generic future income stream is considered. The income stream

---

[12] Sandmo (1969) also studies a two-period consumption-portfolio decision model. But he is mainly interested in the comparative statics on the optimal *amount* of investment in risky assets in the first period. Moreover, his study focuses on the implications of absolute risk aversion (ARA) for consumption in a two-period model. Sandmo does not explain the existence of an exogenous second-period income.

generates some exogenous income in at least one future period which can only be capitalized starting from that period. It is found that relative to the magnitude of current wealth, if the present value of future income stream is close to that of future habit stream, the risky asset share has a slightly positive response to changes in current wealth across time periods. This implies that the response may not be easily identified in empirical studies unless the wealth fluctuations are sufficiently large. This finding better applies to an agent who is young or an agent who is rich in current wealth.

To summarize, in multiperiod consumption-portfolio decision models where uncapitalized future income is introduced, a DRRA periodic utility function for consumption can help reconcile the finding that changes in current wealth have no effect or a positive effect on the risky asset share. Conversely, these models assuming a DRRA periodic utility function for consumption can generate optimal portfolio choices that are consistent with the empirical micro-level findings in some recent papers; that is, the risky asset share is either constant or varies slightly positively in response to changes in liquid financial wealth across time periods (Brunnermeier and Nagel (2008) (B-N henceforth); Calvet, Campbell and Sodini (2009); Chiappori and Paiella (forthcoming)). Particular attention in the following is paid to the measure of wealth in the theory part of B-N, rather than to how well they use panel data from U.S. households to test the existence of micro-foundation of habit formation utility.

B-N do discuss the effect of labor income as part of background wealth in their theory section. But that is only for the purpose of simplifying the process of deriving an empirical estimation equation. Labor income never enters as a part of wealth in any period, let alone other important components of future income. In other words, their wealth measure does not include the part of uncapitalized future income but they implicitly assume it does. To be more specific, the measure of wealth in their theoretical analysis should be better treated as current wealth, rather than lifetime wealth.

Specifically, B-N use habit formation utility as the periodic utility function for consumption, and derive in the theory a simple estimation equation which shows a positive relationship between changes in the risky asset share and changes in log wealth over time. However, the equation may not be the right one unless it can be assumed that current wealth equals lifetime wealth and the portfolio decision is thus based on lifetime wealth. B-N then use data from Panel Study of Income Dynamics (PSID) to test whether there is such a positive relationship, based on the prediction of an infinite horizon model assuming habit formation utility with lifetime wealth. They carefully deal with various econometric issues and empirical specifications. For example, they separately control for the labor income/liquid wealth (or financial wealth) ratio interacted with age, as a proxy for human capital wealth in the regression analysis. [13]

B-N do not find any strong evidence to support that the risky asset share is affected by wealth changes over time, which could mean that relative risk version for some measure of wealth is constant. Although this does not further imply that habit formation is a wrong functional form of utility for consumption, they interpret their finding as it is and later suggest that CRRA power utility function for consumption may prevail. Instead, the finding in this chapter indicates that

[13] Brunnermeier and Nagel (2008) report two measures of wealth: liquid wealth and financial wealth. Liquid wealth is the sum of holdings in stocks and mutual funds (liquid risky assets) and holdings in cash-like assets and bonds (liquid riskless assets), subtracting nonmortgage debt such as credit card debt and consumer loans. Financial wealth is denoted as the sum of liquid wealth, home equity and equity in private business. They calculate two risky asset shares: first, the liquid risky asset share which is the ratio of the liquid risky assets to liquid assets (the sum of liquid risky and riskless assets); second, the financial risky asset share, the sum of liquid risky assets, home equity and equity in private business, divided by financial wealth.

there is nothing wrong with habit formation per se. After adjusting uncapitalized future income in a two-period model and in an infinite horizon model, habit formation is in fact consistent with their empirical finding.

The finding concerning DRRA utility functions for consumption sheds light on recent empirical findings from studying portfolio decisions. It also explains some early empirical evidence on the functional forms of utility for consumption based on the examination of consumption decisions. For example, Barsky, Juster, Kimball and Shapiro (1997) use the responses to survey questions on gambles in the Health and Retirement Study (HRS) to find some evidence *against* the assumption of CRRA utility for consumption;[14] When testing full risk-sharing hypothesis, Ogaki and Zhang (2001) find evidence in support of DRRA subsistence utility for food consumption using panel data from low income Indian and Pakistani households. An exception is by Dynan (2000), who also uses data from PSID but discovers no evidence that household-level food consumption displays the patterns predicted by DRRA habit formation models.

This chapter is in line with a growing number of economic studies in which the class of DRRA utility functions for consumption is proposed and/or used. For instance, in macroeconomics, it is found that habit formation utility can be used to simultaneously eliminate the equity premium and the risk free rate puzzles (Constantinides (1990); Campbell and Cochrane (1999), Meyer and Meyer (2005a)). It is also the case that Stone-Geary, consumption commitment (Chetty and Szeidl (2010), …) or subsistence utility, another form of DRRA utility

---

[14] See the subsection "*Intertemporal Substitution versus Risk Tolerance*" between page 567 and page 568 in Barsky et.al (1997).

function for consumption, is becoming popular in agricultural economics, development economics, labor economics, and public economics.

Of course, the economy facing the decision maker can deviate from a complete market economy in two ways. For one thing, the portfolio studied here consists of only two independent assets: one riskless and one risky, while there are probably much more states of the world. That is, it is highly possible that the number of states of uncertainty exceeds two. Perhaps more importantly, certain future income may not be allowed to be capitalized or fully capitalized due to market imperfection or financial frictions. For instance, one cannot go to a commercial bank and ask for a consumer loan that has exactly the present value of her human capital, or the appreciation of her housing equity in the next thirty years. It is very likely that she has to accept a huge discount since the market does not permit human capital or housing equity to be fully collateralized. Various forms of market imperfection include but are not limited to borrowing constraints, uninsurable stochastic income risk, and transaction or information costs. Modeling these imperfections is beyond the scope of current chapter.

Campbell (2006) points out in the study of household finances, “Until some consensus is reached, normative household finance should emphasize results that are robust to alternative specifications of household utility.” By reinterpreting the recent empirical findings which arise from investigating wealth allocation at micro level, this chapter provides another theoretical support in microeconomics for examining the broader class of DRRA utility functions for consumption. The remainder of this chapter is organized as follows. In the next section the two-period model and the main finding are presented. Section 3.3 studies the infinite horizon model using habit formation utility as the periodic utility function for consumption. The last section concludes and discusses possible extensions.

3.2 A Two-period Model and the Main Finding

Ⅰ. The Model

Assume that a decision maker lives for two periods. He has nonrandom current wealth $W_1 > 0$ in the first period and will receive some nonrandom income $Y_2 > 0$ at the beginning of the second period. Y_2 is not capitalized into W_1 even though a perfect capital market exists. Current wealth includes initial endowment, income in the first period, and the part of second period income that has been capitalized. Current wealth can be used for consumption as well as investment in a riskless asset or a risky asset. Y_2 can include some labor income, social security benefits, pensions and other payments that by assumption are not converted into W_1.

Let $C_1 > 0$ be the amount of consumption in the first period, and α_1 denote the proportion of current wealth minus this consumption being allocated to the risky asset. Given C_1 and α_1, the amount of consumption in the second period is then:

$$C_2 = \left(\alpha_1 r + (1 - \alpha_1)\rho\right)(W_1 - C_1) + Y_2 \tag{1}$$

where $r \geq 0$ is the random return on the risky asset with the cumulative distribution function $F(r)$; $\rho > 0$ is the return on the riskless asset, which is nonrandom; and $E(r) > \rho$ is assumed.[15] The assumption of $r \geq 0$ implies that the worst scenario is that you wake up tomorrow and find that your investment in the risky asset is completely worthless. Note that r is the only source of uncertainty in the model.

[15] In fact, $E(r) > \rho$ implies that F.O.C for α_1 evaluated $\alpha_1 = 0$ is strictly positive. Thus, the optimal $\alpha_1 > 0$.

With a time separable utility function, the decision maker is assumed to maximize the sum of expected utility from consumption in each of the two periods, taking the random return on the risky asset r as given. Since it is assumed that consumption and portfolio decisions in the first period are simultaneously made, the decision maker facing the optimization problem formally chooses C_1 and α_1 to maximize

$$V = u(C_1) + \beta Eu[(\alpha_1 r + (1-\alpha_1)\rho)(W_1 - C_1) + Y_2)] \tag{2}$$

where $u(\cdot)$ is assumed to be increasing, strictly concave and twice continuously differentiable; and $0 < \beta < 1$ is the constant subjective discount factor. It is further assumed that there is a unique interior solution to this optimization problem such that $0 < C_1 < W_1$ and $0 < \alpha_1 < 1$. The first condition requires a sufficiently large W_1, which is often assumed in the literature; the second condition is often imposed to eliminate the possibility of $C_2 < 0$. These two conditions combined guarantee that C_2 is strictly positive. The first-order conditions for C_1 and α_1 are:

$$V_C = u'(C_1) - \beta Eu'(C_2)(\alpha_1 r + (1-\alpha_1)\rho) = 0 \text{ and} \tag{3}$$

$$V_\alpha = \beta Eu'(C_2)(r-\rho)(W_1 - C_1) = 0 \tag{4}$$

Before moving forward to demonstrate the main finding, it is emphasized that even in a perfect capital market where a decision maker is able to capitalize all the future income at the riskless return, he can choose not to do so, and whether he does or does not won't affect optimal C_1 and C_2.[16] As a result, the decision maker is indifferent between these two options. The following proposition is given and the proof is provided in the Appendix.

[16] The value for optimal α_1, however, depends on whether Y_2 is capitalized or not.

Proposition 1: The optimization problem when only part of future income is capitalized is equivalent to the optimization problem when all the future income is capitalized, if the conditions for a unique interior solution, $0 < C_1 < W_1$ and $0 < \alpha_1 < 1$, hold.

Ⅱ. The Main Finding

The sufficient second-order conditions for the optimization problem are:

$$V_{cc} = u''(C_1) + \beta E u''(C_2)\left(\alpha_1 r + (1-\alpha_1)\rho\right)^2 < 0, \tag{5}$$

$$V_{\alpha\alpha} = \beta E u''(C_2)(r-\rho)^2 (W_1 - C_1)^2 < 0, \tag{6}$$

$$V_{c\alpha} = V_{\alpha c} = -\beta E u'(C_2)(r-\rho) - \beta E u''(C_2)(r-\rho)\left(\alpha_1 r + (1-\alpha_1)\rho\right)(W_1 - C_1)$$

$$= -\beta E u''(C_2)(r-\rho)\left(\alpha_1 r + (1-\alpha_1)\rho\right)(W_1 - C_1) \qquad \text{using (4)} \tag{7}$$

$$H = V_{cc}V_{\alpha\alpha} - (V_{c\alpha})^2 > 0 \tag{8}$$

Obviously, conditions (5) and (6) are satisfied if the decision maker is risk averse. As for condition (8), it is satisfied if the matrix $\begin{bmatrix} V_{cc} & V_{c\alpha} \\ V_{\alpha c} & V_{\alpha\alpha} \end{bmatrix}$ is negative definite. This is true because

$$[a\ b]\begin{bmatrix} V_{cc} & V_{c\alpha} \\ V_{\alpha c} & V_{\alpha\alpha} \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = a^2 V_{cc} + 2ab V_{c\alpha} + b^2 V_{\alpha\alpha}$$

$$= a^2\left[u''(C_1) + \beta E u''(C_2)\left(\alpha_1 r + (1-\alpha_1)\rho\right)^2\right]$$

$$-2ab\,\beta E[u''(C_2)(r-\rho)\left(\alpha_1 r + (1-\alpha_1)\rho\right)(W_1 - C_1)]$$

$$+b^2[\beta E u''(C_2)(r-\rho)^2(W_1 - C_1)^2]$$

$$= a^2 u''(C_1) + \beta E\{u''(C_2)[a(\alpha_1 r + (1-\alpha_1)\rho) - b(r-\rho)(W_1 - C_1)]^2\}$$

$$< 0 \qquad \text{for any } \begin{bmatrix} a \\ b \end{bmatrix} \in R^2 \text{ and } \begin{bmatrix} a \\ b \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

By the Implicit Function Theorem (IFT), the effects of changes in current wealth (W_1) on the optimal consumption C_1 and the optimal risky proportion α_1 are:

$$\begin{bmatrix} \partial C_1/\partial W_1 \\ \partial \alpha_1/\partial W_1 \end{bmatrix} = -\begin{bmatrix} V_{cc} & V_{c\alpha} \\ V_{\alpha c} & V_{\alpha\alpha} \end{bmatrix}^{-1} \begin{bmatrix} V_{cw} \\ V_{\alpha w} \end{bmatrix}$$

where $V_{cw} = -\beta E u''(C_2)(\alpha_1 r + (1-\alpha_1)\rho)^2 > 0$ (9)

$$V_{\alpha w} = \beta E u''(C_2)(r-\rho)(\alpha_1 r + (1-\alpha_1)\rho)(W_1 - C_1) \qquad (10)$$

Observe that $V_{\alpha w} = -V_{c\alpha} = -V_{\alpha c}$ (11)

The primary interest is in:

$$[\,\partial \alpha_1/\partial W_1] = -\left[\frac{V_{cc}V_{\alpha w} - V_{\alpha c}V_{cw}}{H}\right]$$

$$= -\frac{V_{\alpha w}(V_{cc} + V_{cw})}{H} \text{ by (11)}$$

$$= -\frac{V_{\alpha w}u''(C_1)}{H} \text{ using (5) and (9).}$$

In other words, the sign of $[\,\partial\alpha_1/\partial W_1]$ is the same as the sign of $V_{\alpha w}$.

But $V_{\alpha w} = \beta E u''(C_2)(r-\rho)(\alpha_1 r + (1-\alpha_1)\rho)(W_1 - C_1)$

$$= -\beta E u'(C_2)[-\frac{u''(C_2)C_2}{u'(C_2)}](r-\rho)\,\frac{C_2-Y_2}{C_2} \quad \text{using (1)}$$

$$= \ -\beta E u'(C_2)(r-\rho)\frac{C_2-Y_2}{C_2} R_u(C_2) \qquad (12)$$

where $R_u(C_2) = \left[-\frac{u''(C_2)C_2}{u'(C_2)}\right] > 0$ denotes the Arrow-Pratt measure of relative risk aversion for consumption. In the case that $Y_2 = 0$, the same conclusion is drawn as that by Pratt (1964) and Arrow (1965; 1971): if relative risk aversion is (strictly) decreasing, constant or increasing for all $C_2 > 0$, then the optimal risky asset proportion is (strictly) increasing, constant or decreasing respectively in current wealth. Furthermore, if the first period consumption does not yield any utility and there is no subjective discount against the second-period utility, this two-period model is reduced to the standard one-period portfolio allocation model.

The important and more general case is when $Y_2 > 0$. Notice that $Eu'(C_2)(r-\rho) = 0$ is given by the first-order condition of (4), while $u'(C_2)(r-\rho)$ changes sign one time from negative to positive at $r = \rho$. Also by the assumptions, both $\frac{C_2-Y_2}{C_2}$ and $R_u(C_2)$ are positive so that $\frac{C_2-Y_2}{C_2} R_u(C_2)$ is positive as well. The partial derivative of $\frac{C_2-Y_2}{C_2} R_u(C_2)$ is then:

$$R_u'(C_2)\frac{C_2-Y_2}{C_2} + R_u(C_2)\,\frac{Y_2}{{C_2}^2} \qquad (13)$$

If $R_u'(C_2)$ is sufficiently negative, (13) becomes zero; that is, $\frac{C_2-Y_2}{C_2} R_u(C_2)$ is then a positive constant. But then (12) turns into zero, which means that $[\partial\alpha_1/\partial W_1] = 0$. Put differently, the optimal proportion of investment in the risky asset α_1 is constant with respect to the comparative static changes in current wealth W_1 if and only if the utility function for consumption displays DRRA for consumption with the elasticity of relative risk aversion being $-\frac{Y_2}{C_2-Y_2} < 0$.

Based on this two-period model, one can also conclude that the indirect utility function for W_1 displays CRRA if $[\partial\alpha_1/\partial W_1] = 0$. That is, a DRRA utility function for consumption is *not* in conflict with a CRRA indirect utility function for wealth. Note that without analyzing the portfolio allocation decision, one has to establish such a relationship between the two slopes by instead *assuming* that the reduced-form solution for the optimal consumption C_2 as a function of current wealth W_1 satisfies certain properties. But this assumption in turn depends on the functional form of utility for consumption and/or on the probability distribution function for the risky asset return, which are usually unknown. An example given by Meyer and Meyer (2005a) has been discussed in the introduction. The following theorem generalizes the above finding.

Theorem 1. Holding future income Y_2 constant, if the optimal risky asset share α_1 is constant or varies positively with respect to changes in current wealth W_1, the utility function for consumption must exhibit DRRA for consumption.

The proof is by contradiction borrowing the results from Theorem 2 in the Appendix, where it is proved that the optimal risky asset share decreases as current wealth increases if the utility function for consumption displays CRRA or increasing relative risk aversion (IRRA) for consumption. The intuition behind the finding of DRRA utility functions for consumption is given. The existence of uncapitalized future income alters the relationship between the optimal proportion invested in the risky asset and current wealth, and thus changes the implied relative risk aversion (of utility functions) for consumption.

The main insight from the comparative static finding in Theorem 1 using a single portfolio decision may be passed to the recent empirical findings by B-N and others, who study more than one portfolio decision across time periods to find either no wealth effect or a positive wealth effect. To do this, it can be assumed that in multiperiod context, each two-period decision making process is independent with one another, the decision maker's risk preference is unchanged, and the riskless return and the risky return distribution are fixed.[17] Then the effect of the comparative static changes in wealth on the risky asset share is similar to the effect of the exogenous wealth changes over time on the risky asset share, and this theoretical finding concerning DRRA utility functions for consumption maintains also.

The major finding from using the two-period model shows that the fact that the theoretical prediction in B-N is not verified by their empirical finding results from the mismatch between the measure of wealth in their theory section and the measure of wealth in their empirical part. Specifically, in the theory they assume that one's lifetime wealth is given at the start of the decision making, and then study the relationship between the optimal proportion invested in the risky asset and the lifetime wealth. While in the empirical part they only observe current wealth,

---

[17] The assumptions on risk preference and returns are often seen in the literature, while the assumption of independent two-period decision making processes is open to question.

a fraction of lifetime wealth that is capitalized, and actually estimate the link between the optimal risky share and current wealth.

B-N derive in the theory a positive relationship between the optimal proportion invested in the risky asset and changes in log lifetime wealth over time, based on the assumption that habit formation utility is the true utility in an infinite horizon model. After correcting for uncapitalized future income and including it as a factor in the two-period decision model, it is shown here that the absence of the comparative static effect of changes in current wealth or a positive effect is a sufficient condition for a DRRA functional form of utility for consumption, which includes the well known habit formation utility as a special case. If each two-period decision making process is repeated over time and is independent with one another, the absence of response to wealth changes may inadvertently validate the existence of habit formation in an infinite horizon model, which is the root of time-varying risk aversion for consumption B-N intend to test.

The simple two-period model is instructive, because no particular form of DRRA utility functions for consumption is specified. This is in contrast with external habit formation utility for consumption used by B-N, and the exogenous income as the habit used by Meyer and Meyer (2005a). To see whether the main theoretical finding concerning the slope of relative risk aversion for consumption also exists for an agent who has to make more than one portfolio decision in a model with more than two periods, an infinite horizon model assuming habit formation utility and a general exogenous income stream is examined in the next section.

3.3 An Infinite Horizon Model

Following Constantinides (1990) in a continuous-time setting and Brunnermeier and Nagel (2008) in a discrete-time context, assume that the non-constant but slow-moving internal habit, x_t, is subject to the difference equation:

$$x_{t+1} - x_t = bC_t - ax_t \tag{14}$$

where C_t is the consumption at time t. The habit grows at a rate of $0 \leq b \leq 1$ and depreciates at a rate of $0 \leq a \leq 1$. For a strictly positive b, the habit is internal since it depends on past consumption. When b equals zero, the habit is reduced to an external habit. And when both a and b are zero, the habit is constant over time and is equivalent to a subsistence level. Note that the value of x_{t+1} is known right after C_t is made at time t.

Define $\tilde{C}_t \equiv (C_t - x_t)\ \frac{\rho+a}{\rho-b+a}$ and $\widetilde{W}_t \equiv W_t + Y_t - (1+\rho)\ \frac{x_t}{\rho-b+a}$, where ρ is the riskless rate of returns, $\tilde{C}_t$ stands for the surplus consumption at time t, and $\widetilde{W}_t$ denotes the surplus of total wealth at time t that is not needed to finance the sum of the present value of future habits. Note that the riskless rate of return, rather than the riskless return, is used here in order to be consistent with the notation in B-N, and so is the risky rate of return that will appear shortly.

Total wealth at time t includes two components: W_t, the wealth level at the beginning of time t, and $Y_t = y_t + \frac{Y_{t+1}}{1+\rho}$, the present value of an exogenous income stream. The period income y_s is positive for all $s \geq t$ and is strictly positive for at least one $s > t$. It is assumed that y_t can only be realized and become a part of current wealth, $W_t + y_t - C_t$, from the beginning of time

t . Also note that $W_t - C_t$ is instead used as the measure of current wealth by B-N.[18] Since x_t is assumed to be slowly moving over time, $\frac{x_t}{\rho-b+a}$ can be treated as an approximation of the sum of the present value of future habits at time $t-1$.

Suppose that the agent invests at time t, a fraction $\tilde{\alpha}_t$ of total wealth minus consumption in excess of $\frac{x_{t+1}}{\rho-b+a}$ in the risky asset, and the rest in the riskless asset. Also invested in the riskless asset is $\frac{x_{t+1}}{\rho-b+a} - \frac{Y_{t+1}}{1+\rho}$, a positive amount that is needed to guarantee the sum of the present value of future habits.[19] The surplus portfolio yields a rate of return $\tilde{R}_t \equiv \tilde{\alpha}_t(r_t - \rho) + \rho$, where r_t is the random rate of return on the risky asset at time t. The dynamic budget constraint then becomes:

$$W_{t+1} = (1+\tilde{R}_t)\left(W_t + y_t + \frac{Y_{t+1}}{1+\rho} - C_t - \frac{x_{t+1}}{\rho-b+a}\right) + (1+\rho)\left(\frac{x_{t+1}}{\rho-b+a} - \frac{Y_{t+1}}{1+\rho}\right) \tag{15}$$

Using this to replace W_{t+1} in the definition of surplus total wealth at time $t+1$, one obtains

$$\widetilde{W}_{t+1} = W_{t+1} + Y_{t+1} - (1+\rho)\frac{x_{t+1}}{\rho-b+a}$$

---

[18] In order to examine the effect of changes in wealth over time, current wealth in this section reflects both the addition of current income and the subtraction of current consumption. This measure of current wealth is different from the one used in the last section for the comparative static analysis, where current wealth is given in the first period.

[19] For a non-positive amount, the dynamic budget constraint could become $W_{t+1} = (1+R_t)(W_t + y_t - C_t)$. It is not clear how to transform this budget constraint into a corresponding one in the optimization problem considered by Samuelson (1969).

$$= \left(1 + \tilde{R}_t\right)\left(W_t + Y_t - C_t - \frac{x_{t+1}}{\rho - b + a}\right)$$

$$= \left(1 + \tilde{R}_t\right)[\left(W_t + Y_t - (1+\rho)\frac{x_t}{\rho - b + a}\right) - \left(C_t + \frac{x_{t+1}}{\rho - b + a} - (1+\rho)\frac{x_t}{\rho - b + a}\right)]$$

The first term in the bracket is just $\widetilde{W}_t$. Replacing x_{t+1} with $(1-a)x_t + bC_t$, the second term in the bracket, $C_t + \frac{x_{t+1}}{\rho-b+a} - \left(1+\rho\right)\frac{x_t}{\rho-b+a}$, can be simplified as $(C_t - x_t)\frac{\rho+a}{\rho-b+a}$ $\equiv \tilde{C}_t$. That is,

$$\widetilde{W}_{t+1} = \left(1 + \tilde{R}_t\right)(\widetilde{W}_t - \tilde{C}_t)$$

The objective function using habit formation utility is then transformed into the objective function using power utility:

$$\max E_0 \sum_{s=0}^{\infty} \frac{\tilde{C}_s{}^{1-\gamma}}{1-\gamma}\left(\frac{\rho - b + a}{\rho + a}\right)^{1-\gamma}$$

s.t. $\widetilde{W}_{t+1} = \left(1 + \tilde{R}_t\right)(\widetilde{W}_t - \tilde{C}_t)$

Samuelson (1969) shows that for the above optimization problem, optimal $\tilde{\alpha}_t$ is constant over time. Or, optimal $\tilde{\alpha}_t = \bar{\alpha}$ for all t. Now let α_t^* be the optimal proportion of current wealth, $(W_t + y_t - C_t)$, allocated to the risky asset at time t. Note that what researchers typically observe is α_t^* rather than $\bar{\alpha}$. As a result, α_t^* can be expressed as

$$\alpha_t^* = \bar{\alpha} \cdot \frac{W_t + y_t + \frac{Y_{t+1}}{1+\rho} - C_t - \frac{x_{t+1}}{\rho - b + a}}{W_t + y_t - C_t}$$

$$= \bar{\alpha} \cdot \frac{(W_t + y_t - C_t) - (\frac{x_{t+1}}{\rho - b + a} - \frac{Y_{t+1}}{1+\rho})}{W_t + y_t - C_t}$$

$$= \bar{\alpha} \cdot (1 - \frac{\frac{x_{t+1}}{\rho - b + a} - \frac{Y_{t+1}}{1+\rho}}{W_t + y_t - C_t}) \qquad (16)$$

This equation is different from the one derived by B-N; that is, income is integrated into the second term in the parenthesis. Specifically, the present value of future income stream enters the numerator as a large minus term, while income in the current period becomes a plus term in the denominator. These two factors combined can substantially dampen the effect of changes in current wealth on the optimal risky asset proportion, α_t^*.

It is observed from equation (16) that relative to the magnitude of current wealth, $(W_t + y_t - C_t)$, if the difference between $\frac{x_{t+1}}{\rho-b+a}$, the approximation of the present value of future habit stream at time t and $\frac{Y_{t+1}}{1+\rho}$, the present value of future income stream at time t, is very small, then α_t^* only responds slightly to changes in current wealth over time.

B-N approximate $\bar{\alpha}$ with one, by assuming that a CRRA investor without habit would invest about 100 percent of the liquid wealth in stocks.[20] Their assumption may or may not result in the same α_t^* as the one derived here, because the integration of uncapitalized income given in

[20] Their assumption is based on the results from some realistically calibrated models of household portfolio choice with CRRA preferences and background wealth such as housing wealth and labor income.

equation (16) can also increase the magnitude of α_t^*. The main perspective from equation (16) is that, when uncapitalized future income is incorporated in the model and separately considered as a factor in studying portfolio decisions, the response of the risky asset share to changes in current wealth over time can be completely different. Therefore, it is too hurried to interpret an empirical finding of lacking such a response as that the risk preferences for consumption are better represented by the CRRA power form of utility.

Equation (16) is better used to explain the optimal portfolio allocation decisions for two groups of people who are more relevant to an infinite horizon model: the young and the wealthy.[21] The basic idea here is that $\left(\frac{x_{t+1}}{\rho-b+a} - \frac{Y_{t+1}}{1+\rho}\right) / (W_t + y_t - C_t)$, the second term in the parenthesis, is probably small, and therefore the risky asset proportion, α_t^*, is approximately constant with respect to changes in current wealth for people in each of these two groups. Young people may have a relatively small amount of W_t, but as they accumulate work experience and build up their future income (especially labor income), future habit is less likely to be a burden. In other words, though the denominator may not be large, the numerator can be much smaller mainly because $\frac{Y_{t+1}}{1+\rho}$ is large. While for rich people, as long as the difference between the present value of future habits and the present value of future income is much smaller than the amount of current wealth, the wealth effect on the risky asset allocations can be hardly detected. The financial behavior of the wealthy people is also interesting due to their disproportionate demand for risky assets.

[21] An infinite horizon model is not appropriate for those who have to plan for the retirement or who have already retired though it seems that the economic situations of these people can also be applied to equation (16).

If on average households choose the optimal risky asset share in the way expressed by equation (16), the empirical finding by B-N in fact suggests that habit formation, which displays DRRA for current consumption, be a potential candidate for the functional form of periodic utility. Consider a special case of external habit when $x_t = y_t$ for all t as in Meyer and Meyer (2005), where $b = 0$. The present value of future habit stream is then reduced to the present value of future income stream, which means that $\alpha_t^* = \bar{\alpha}$ for all t.

3.4 Discussion

The focus in this chapter is to expose uncapitalized future income as an important factor in the study of the slope of relative risk aversion for consumption in multiperiod consumption-portfolio decision models. After adjusting the measure of current wealth and including uncapitalized future income, theoretical analysis shows that models assuming habit formation utility can yield results that are consistent with recent empirical micro-level findings. That is, there is either no wealth effect or a small positive one on the household asset allocation. As a result, there is a micro-foundation for the existence of habit formation, as well as the broader class of DRRA utility functions for consumption. The main comparative static finding using the two-period model is invariant to any specific form of DRRA utility functions for consumption, while the finding in the infinite horizon model is based on the assumption that habit formation utility is the true functional form of utility. Therefore, this chapter calls for searching more flexible forms of DRRA utility functions for consumption to understand the microeconomics of household wealth allocation in multiperiod models.

Uncapitalized future income examined in the infinite horizon model is confined to an exogenous income stream, of which the sources have not been discussed. Various forms of market imperfection can lead to some future income being unable to be borrowed against and integrated into current wealth. First, the existence of exogenous borrowing constraints implies that there is limited ability to borrow future income and thus to reallocate this part of wealth between consumption and investment. Corner solution is a main concern in this line of studies. Second, uninsurable stochastic income risk can also prevent an agent from taking the expected future income to the present. This individual specific income risk belongs to the background wealth risk whose relationship with the risky asset return is not clear. There is a large body of literature on the background wealth risk. Last but not the least, it is possible that there exists some transaction cost or information cost that cannot be ignored and can keep one from having complete access to the credit market. Modeling one or more of these imperfections is an interesting but challenging job. Nonetheless, this can provide more insight into a decision maker's risk preferences for consumption in markets that deviate from the Arrow-Debreu complete economy.

APPENDIX

Proof of Proposition 1

Let $0 < C_1 < W_1$ and $0 < \alpha_1 < 1$ be the unique solution to problem (2) in section 3.2. We need to show that it also solves the following problem which is assumed to have one unique solution as well:

$$\max\ u(C_1) + \beta E u\left(W_1 - C_1 + {}^{Y_2}/_{\rho}\right)\left(\alpha_1 r + (1-\alpha_1)\rho\right) \tag{2*}$$

Suppose that the unique solution to the above problem is C_1^* and α_1^*. This implies it satisfies the following first-order conditions:

$u'(C_1^*) - \beta E u'(C_2^*)\left(\alpha_1^* r + (1-\alpha_1^*)\rho\right) = 0$ and

$$\beta E u'(C_2^*)(r-\rho)\left(W_1 - C_1^* + \frac{Y_2}{\rho}\right) = 0$$

or equivalently

$$u'(C_1^*) = \rho\beta E u'(C_2^*) \text{ and} \tag{3*}$$

$$E u'(C_2^*)(r-\rho) = 0 \tag{4*}$$

We claim that if $\alpha_1 = \frac{W_1 - C_1^* + \frac{Y_2}{\rho}}{W_1 - C_1}\alpha_1^*$, then $C_2 = C_2^*$.

Notice that $C_2^* = \left(W_1 - C_1^* + {}^{Y_2}/_{\rho}\right)\left(\alpha_1^* r + (1-\alpha_1^*)\rho\right)$

$$= \left(W_1 - C_1^* + {}^{Y_2}/_{\rho}\right)\alpha_1^* r + \left(W_1 - C_1^* + {}^{Y_2}/_{\rho}\right)(1-\alpha_1^*)\rho$$

$$= (W_1 - C_1^* + Y_2\ /\rho)\alpha_1^* r + (W_1 - C_1^* + Y_2\ /\rho)[1 - (W_1 - C_1)/(W_1 - C_1^* + Y_2\ /\rho)\alpha_1]\rho$$

$$= (W_1 - C_1^* + Y_2\ /\rho)\alpha_1^* r$$

$$+(W_1 - C_1^* + Y_2\ /\rho)\left[(W_1 - C_1^* + Y_2\ /\rho) - (W_1 - C_1)\alpha_1 \Big/ W_1 - C_1^* + Y_2\ /\rho\right]\rho$$

$$= (W_1 - C_1)\alpha_1 r + (W_1 - C_1)(1 - \alpha_1)\,\rho + Y_2 \text{ if } C_1 = C_1^* \text{ and } \alpha_1 = \frac{W_1 - C_1^* + \frac{Y_2}{\rho}}{W_1 - C_1}\alpha_1^*$$

$$= (\alpha_1 r + (1 - \alpha_1)\rho)(W_1 - C_1) + Y_2 \ = C_2 \text{ for all } r.$$

This means that C_1 and α_1 satisfy (4*) and thus satisfy (3*). ■

Other comparative statics using the two-period model

More comparative statics are provided here concerning the optimal risky asset share α_1 and the optimal current consumption C_1. The goal is to show that when uncapitalized future income is separately considered, the predictions of the two-period model are consistent with those of the standard one-period model in the major body of economic theory. In the following theorems, if current wealth W_1 changes, then uncapitalized future income Y_2 is held fixed, and vice versa. Similar with findings using the one-period model, the effects of changes in the uncertainty of the risky asset return and the effects of changes in the riskless return are far from being clear, and thus are not presented here.

Theorem 2. The optimal risky asset share decreases as current wealth increases if the utility function for consumption displays CRRA or IRRA for consumption.

Proof. If $u(C_2)$ exhibits CRRA, (12) is reduced to $V_{\alpha W} = K \cdot [-\beta E u'(C_2)(r-\rho)\ \frac{C_2 - Y_2}{C_2}]$,

where K is the positive constant RRA for consumption. Let $C_2^* = \rho(W_1 - C_1) + Y_2$, or C_2 evaluated at $r = \rho$. Consider the case when $r > \rho$, which implies that

$$\frac{C_2 - Y_2}{C_2} > \frac{C_2^* - Y_2}{C_2^*} \tag{17}$$

and $u'(C_2)(r-\rho) > 0$ (18)

Multiply both sides of (17) by (18), one have

$$u'(C_2)(r-\rho)\ \frac{C_2 - Y_2}{C_2} > u'(C_2)(r-\rho)\frac{C_2^* - Y_2}{C_2^*} \tag{19}$$

Take the expectation on both sides of (19) to get

$$Eu'(C_2)(r-\rho)\frac{C_2 - Y_2}{C_2} > Eu'(C_2)(r-\rho)\frac{C_2^* - Y_2}{C_2^*} = \frac{C_2^* - Y_2}{C_2^*} Eu'(C_2)(r-\rho) \tag{20}$$

Now consider the other case when $r < \rho$. This implies that

$$\frac{C_2 - Y_2}{C_2} < \frac{C_2^* - Y_2}{C_2^*} \tag{17'}$$

and $u'(C_2)(r-\rho) < 0$ (18')

Multiply both sides of (17') by (18') to have

$$u'(C_2)(r-\rho)\ \frac{C_2 - Y_2}{C_2} > u'(C_2)(r-\rho)\frac{C_2^* - Y_2}{C_2^*} \tag{19'}$$

Take the expectation on both sides of (19') to obtain

$$Eu'(C_2)(r-\rho)\frac{C_2-Y_2}{C_2} > Eu'(C_2)(r-\rho)\frac{C_2^*-Y_2}{C_2^*} = \frac{C_2^*-Y_2}{C_2^*}Eu'(C_2)(r-\rho) \qquad (20')$$

Sum (20) and (20'); use (4) and (12) to have $V_{\alpha W} < 0$; so is $[\,\partial\alpha_1/\partial W_1] < 0$.

If $u(C_2)$ exhibits IRRA, multiply both sides of inequality (19) or (19') by $R_u(C_2)$:

$$\Rightarrow u'(C_2)(r-\rho)\frac{C_2-Y_2}{C_2}R_u(C_2) > u'(C_2)(r-\rho)\frac{C_2^*-Y_2}{C_2^*}R_u(C_2)$$

$$> u'(C_2)(r-\rho)\frac{C_2^*-Y_2}{C_2^*}R_u(C_2^*)$$

$$\Rightarrow Eu'(C_2)(r-\rho)\frac{C_2-Y_2}{C_2}R_u(C_2) > \frac{C_2^*-Y_2}{C_2^*}R_u(C_2^*)Eu'(C_2)(r-\rho) \qquad (21)$$

This is true either for $r > \rho$ or for $r < \rho$. The result then follows by using (4) and (12). ■

Since the sign of $[\,\partial\alpha_1/\partial W_1]$ is the same as the sign of $V_{\alpha W}$, and $-V_{c\alpha} = -V_{\alpha c} = V_{\alpha W}$ by (11), this implies the sign of $V_{\alpha c}$ is just the opposite of the sign of $[\,\partial\alpha_1/\partial W_1]$. But it has just been shown that $[\,\partial\alpha_1/\partial W_1]$ is negative if the utility function for consumption is CRRA or IRRA for consumption. Thus, it is further concluded that optimal C_1 and α_1 are complements in terms of risk preferences if the utility function for consumption exhibits CRRA or IRRA for consumption. That is, consuming more in the first period increases the marginal expected utility of holding relatively more of the risky asset. Alternatively, having relatively more of the risky asset increases the marginal expected utility of first-period consumption.[22]

Theorem 3. The optimal current consumption increases as current wealth increases, regardless of the slope of relative risk aversion for consumption.

Proof. $\left[\partial C_1/\partial W_1\right] = -(V_{\alpha\alpha}V_{cw} - V_{c\alpha}V_{\alpha w})/H$

$$= [V_{\alpha\alpha}(-V_{cw}) - (V_{c\alpha})^2]/H \qquad \text{using (11)}$$

The symmetric matrix $\begin{bmatrix} V_{\alpha\alpha} & V_{c\alpha} \\ V_{\alpha c} & -V_{cw} \end{bmatrix}$ is negative definite since

$$[a\ b]\begin{bmatrix} V_{\alpha\alpha} & V_{c\alpha} \\ V_{\alpha c} & -V_{cw} \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = a^2V_{\alpha\alpha} + 2abV_{c\alpha} - b^2V_{cw}$$

$$= a^2\left[\beta Eu''(C_2)(r-\rho)^2(W_1 - C_1)^2\right]$$

$$-2ab\ \beta E[u''(C_2)(r-\rho)(\alpha_1 r + (1-\alpha_1)\rho)(W_1 - C_1)]$$

$$+b^2[\beta Eu''(C_2)(\alpha_1 r + (1-\alpha_1)\rho)^2]$$

$$= \beta E\{u''(C_2)[a(r-\rho)(W_1 - C_1) - b(\alpha_1 r + (1-\alpha_1)\rho)]^2\}$$

$$< 0 \qquad \text{for any } \begin{bmatrix} a \\ b \end{bmatrix} \in R^2 \text{ and } \begin{bmatrix} a \\ b \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

---

[22] Eeckhoudt, Meyer and Ormiston (1997) investigate the relationship between the demand for the risky asset and the demand for the proportional insurance against portfolio risk in a one-period model.

This is equivalent to $V_{\alpha\alpha}(-V_{cw}) - (V_{c\alpha})^2 > 0$ or $[\partial C_1/\partial W_1] > 0$. ■

In other words, current consumption is a normal good. If $v(W_1)$ is the indirect utility for current wealth, Theorem 3 also confirms that relative risk aversion for current wealth is well defined, since by $u'(C_1) = v'(W_1)$, $R_v(W_1) = R_u(C_1)[\partial C_1/\partial W_1]\,(W_1/C_1) > 0$.

Theorem 4. The optimal current consumption increases as uncapitalized future income increases, regardless of the slope of relative risk aversion for consumption.

Proof. Against by the IFT, the effects of changes in Y_2 on C_1 and α_1 are:

$$\begin{bmatrix} \partial C_1/\partial Y_2 \\ \partial \alpha_1/\partial Y_2 \end{bmatrix} = -\begin{bmatrix} V_{cc} & V_{c\alpha} \\ V_{\alpha c} & V_{\alpha\alpha} \end{bmatrix}^{-1} \begin{bmatrix} V_{cY} \\ V_{\alpha Y} \end{bmatrix}$$

where $V_{cY} = -\beta E u''(C_2)\big(\alpha_1 r + (1-\alpha_1)\rho\big) > 0$ and (22)

$$V_{\alpha Y} = \beta E u''(C_2)(r-\rho)(W_1 - C_1) \tag{23}$$

$$[\,\partial C_1/\partial Y_2\,] = -\frac{V_{\alpha\alpha}V_{cY} - V_{c\alpha}V_{\alpha Y}}{H}$$

$$= \frac{-V_{\alpha\alpha}V_{cY} + V_{c\alpha}V_{\alpha Y}}{H}$$

But $-V_{\alpha\alpha}V_{cY} + V_{c\alpha}V_{\alpha Y}$

$$= \left[\beta E u''(C_2)(r-\rho)^2(W_1 - C_1)^2\right] \cdot [\beta E u''(C_2)\big(\alpha_1 r + (1-\alpha_1)\rho\big)]$$

$$- [\beta E u''(C_2)(r-\rho)\big(\alpha_1 r + (1-\alpha_1)\rho\big)(W_1 - C_1)] \cdot [\beta E u''(C_2)(r-\rho)(W_1 - C_1)]$$

Some calculation and simplification give

$$-V_{\alpha\alpha}V_{cY}+V_{c\alpha}V_{\alpha Y}$$

$$=\beta^2(W_1-C_1)^2\rho\left\{\left[Eu''(C_2)(r-\rho)^2\right]\cdot[Eu''(C_2)]-[Eu''(C_2)(r-\rho)]^2\right\}$$

$$>0$$

This is because the symmetric matrix $\begin{bmatrix} Eu''(C_2)(r-\rho)^2 & Eu''(C_2)(r-\rho) \\ Eu''(C_2)(r-\rho) & Eu''(C_2) \end{bmatrix}$ is negative definite, which implies that $\left\{\left[Eu''(C_2)(r-\rho)^2\right]\cdot[Eu''(C_2)]-[Eu''(C_2)(r-\rho)]^2\right\}>0.$ ■

Theorem 5. The optimal risky asset share increases as uncapitalized future income increases if the utility function for consumption is CRRA or DRRA for consumption.

$$[\,\partial\alpha_1/\partial Y_2\,]=-\frac{V_{CC}V_{\alpha Y}-V_{c\alpha}V_{CY}}{H}$$

$$=\frac{-V_{CC}V_{\alpha Y}+V_{c\alpha}V_{CY}}{H}$$

But $-V_{CC}V_{\alpha Y}+V_{c\alpha}V_{CY}=-\left[u''(C_1)+\beta Eu''(C_2)(\alpha_1 r+(1-\alpha_1)\rho)^2\right]$

$$\cdot[\beta Eu''(C_2)(r-\rho)(W_1-C_1)]$$

$$+[\beta Eu''(C_2)(r-\rho)(\alpha_1 r+(1-\alpha_1)\rho)(W_1-C_1)]$$

$$\cdot[\beta Eu''(C_2)(\alpha_1 r+(1-\alpha_1)\rho)]$$

$$= -u''(C_1) \cdot [\beta E u''(C_2)(r-\rho)(W_1 - C_1)]$$

$$- [\beta E u''(C_2)(\alpha_1 r + (1-\alpha_1)\rho)^2] \cdot [\beta E u''(C_2)(r-\rho)(W_1 - C_1)]$$

$$+[\beta E u''(C_2)(r-\rho)(\alpha_1 r + (1-\alpha_1)\rho)(W_1 - C_1)]$$

$$\cdot [\beta E u''(C_2)(\alpha_1 r + (1-\alpha_1)\rho)] \tag{24}$$

Denote $u''(C_2) = X$, $r - \rho = Y$, and $\alpha_1 r + (1-\alpha_1)\rho = \alpha_1 Y + \rho$. After some calculation and simplification, the last two terms of (24) are reduced to

$$\beta^2 (W_1 - C_1)\alpha_1 \rho \{E\left(XY^2\right) E(X) - [E(XY)]^2\} > 0$$

And the first term of (24) can be transformed into

$$-u''(C_1) \cdot [-\beta(W_1 - C_1) E u'(C_2)(r-\rho) A_u(C_2)] > 0 \text{ if } u(C_2) \text{ displays CRRA or DRRA} \tag{25}$$

where $A_u(C_2) = \left[-\frac{u''(C_2)}{u'(C_2)}\right] > 0$ stands for the Arrow-Pratt measure of absolute risk aversion for consumption.[23]

Proof of (25): First, $R'_u(C_2) = A'_u(C_2) \cdot C_2 + A_u(C_2) \leq 0$ implies that $A'_u(C_2) < 0$. With a positive decreasing $A_u(C_2)$ and the first-order condition for α_1, the conclusion follows. ■

[23] A less restrictive sufficient condition for (25) is simply $u(C_2)$ displays decreasing absolute risk aversion (DARA).

Chapter Four

Inferring Risk Aversion Using One Portfolio Decision

4.1 Introduction

A decision maker's Pratt-Arrow measure of relative risk aversion for wealth is often used to represent his attitude toward risk. This measure is a function of wealth and may or may not be the constant function. The magnitude and slope of this measure have been shown to be determinants of asset prices, insurance premiums and household wealth allocations. Among the decisions that have been examined, perhaps the most heavily studied is the choice of the proportion of wealth to invest in risky assets. This is true both in theoretical analysis and in empirical studies. An important question addressed in this chapter is how to infer the magnitude of a decision maker's measure of relative risk aversion for wealth from a single observation on his wealth allocation between a riskless asset and a risky asset. This is the focus of this research.

The existing literature offers one main method of inferring the magnitude of the measure of risk aversion for wealth, based on a single observed portfolio. This method is presented by Friend and Blume (1975). Friend and Blume assume a continuous Gaussian process for the risky rate of return, and develop an approach that provides a way to infer a risk aversion level locally at the point of the initial wealth. This is all based on observing a single decision, the proportion of wealth invested in the risky asset, when the choice is to allocate wealth between a risky asset and a riskless asset. The procedure suggested by Friend and Blume infers risk aversion in the small at a point. In the small is the terminology of Pratt (1964), and reflects the fact that the approximation used assumes that the risks are small. The Friend and Blume procedure infers the magnitude of risk aversion at the initial wealth and is most accurate when the risks being evaluated involve only small deviations from the initial wealth. The Friend and Blume risk aversion measure is very simple. It is the ratio of the equilibrium market price of risk to the observed risky asset share of wealth.

This chapter also studies how to infer risk aversion from a single observed portfolio decision, but introduces two different alternatives for using this information. Both procedures suggested here estimate risk aversion for wealth in the large, rather than in the small. That is, the suggested procedures make no assumptions limiting the sizes of the risks that the decision maker faces and give a value for risk aversion at all wealth levels not just the initial wealth. The standard portfolio decision model including one riskless asset and one risky asset is used. The first alternative quadratically approximates the utility function for wealth, and then maximizes the expectation of the approximated utility function. This gives rise to a measure of relative risk aversion which is an increasing function of wealth. The measure depends only on the mean and the variance of the risky asset return, and does not require other information about its distribution. Because this is the same information used in the Friend and Blume methodology, the two methods are easily compared. The major difference between this quadratic utility function approach and the approach used by Friend and Blume is that rather than assuming that the time interval in a stochastic process is small so that risk aversion in the small can be determined at the initial wealth, the utility function is quadratically approximated instead and risks of all sizes can be evaluated. Assuming a quadratic utility, however, is a severe restriction.

A second alternative also infers risk aversion in the large from one observed portfolio decision, but no Taylor's series approximation of utility is involved in the procedure. In this approach, a specific functional form for utility or risk aversion is assumed. The portfolio decision then is sufficient to infer one parameter in the functional form. The specific utility or risk aversion function can only have one unknown parameter because this parameter must be determined from the one observed portfolio decision. This second alternative also requires information beyond the mean and variance of the risky asset return; that is, the distribution for the risky asset return has to be specified. Thus the second alternative uses prior information on the functional form of utility and the distribution function of the risky asset return. Several one-parameter functional forms for the utility function are examined. These include the quadratic utility, exponential utility and power utility forms. In addition, a two-parameter marginal utility family recently proposed by Meyer (2010) is also examined. This family is used because it can display decreasing relative risk aversion (DRRA). The existing evidence is used when specifying the distribution function for the risky asset return. Positive skewness and other evidence suggest that log-normality may be an appropriate assumption.

Computed solutions indicate that assuming a functional form for utility or relative risk aversion and inferring relative risk aversion in the large does a better job than approximating utility quadratically or inferring relative risk aversion in the small using the Friend and Blume procedure. Choosing a functional form for utility, even if it is a wrong functional form, still provides a closer estimate of relative risk aversion than does the quadratic approximation approach or the relative risk aversion in the small estimate determined using the Friend and Blume methodology. This may lead to important corrections to the information concerning the measure of relative risk aversion for wealth presented in the existing literature. Estimating relative risk aversion in the small consistently underestimates the true magnitude by 10 to 15% for all the simulations carried out. The quadratic utility approximation approach is most useful when quadratic utility is the true functional form for utility, and is also useful if the focus is relative risk aversion at the mean. This approach has the advantage of not depending on the form for the distribution function of the risky asset return.

By studying the portfolio decision, this work is related to a strand of literature that provides direct empirical evidence concerning the Pratt-Arrow measure of relative risk aversion for wealth. The cross section analysis by Friend and Blume (1975), Blake (1996), and others, has used a number of households each with only one observation on the wealth allocation, to estimate the magnitude of relative risk aversion for wealth for a representative of these households in the same wealth category. Some recent literature using panel data has access to two or more observations on the proportion of wealth invested in risky assets for each household. Brunnermeier and Nagel (2008) assume that multiple observations on the portfolio decision arise in a multiperiod consumption-portfolio model, while Chiappori and Paiella (forthcoming) assume that these observations occur in a one-period portfolio model that is repeated over time. These two papers conclude that the slope of relative risk aversion is very flat for a representative of all the households being studied. The argument of utility function is financial wealth. Another recent work by Calvet, Campbell and Sodini (2009) finds some evidence that households tend to hold a larger portion of wealth in risky assets as they become richer. This evidence is interpreted as a positive link between wealth changes and risk taking, and may imply that relative risk aversion decreases for financial wealth.

The remainder of the chapter is organized as follows. In Section 4.2, how Friend and Blume infer risk aversion in the small, or risk aversion locally about the initial wealth, is presented. Section 4.3 studies inferring risk aversion in the large and describes the quadratic utility approximation approach. Section 4.4 discusses using functional forms for utility or risk aversion to infer risk aversion in the large. Computation results are provided in section 4.5 to evaluate these three approaches for inferring relative risk aversion from the portfolio allocation decision. Finally, section 4.6 concludes and discusses possible extensions.

4.2 Inferring Risk Aversion in the Small

In a portfolio decision model under a continuous time setting, Friend and Blume (1975) derive a measure that can be used to infer the magnitude of risk aversion at the point of the initial wealth, based on a single observed portfolio choice. The measure is a point estimate of risk aversion at the initial wealth for sufficiently small risks. That is, this measure is a local measure of risk aversion, or risk aversion in the small. This measure is based on an approximation procedure that requires a very short time period so that the deviations from the initial wealth are very small. The measure uses information on a decision maker's proportion of the initial wealth allocated to the risky asset, and the equilibrium market price of risk. The equilibrium market price of risk is defined as the ratio of the expected risk premium on the risky rate of return to the variance of the risky rate of return.

Specifically, Friend and Blume (1975) assume that an agent begins with positive wealth at time t, $W_t > 0$, which can be allocated to the safe asset with a rate of return $\bar{\rho} > 0$, or the risky asset with a random rate of return r. They also assume that both assets are liquid and can be traded at no transaction cost and in any quantity. Before moving to show how risk aversion in the small is estimated, it is emphasized that the stochastic process leading to the following wealth conservation equation is assumed to be a continuous Gaussian (Wiener) process.[24] If the time interval is short enough, this continuous stochastic process ensures that risks are small deviations from the initial wealth.

Friend and Blume (1975) indicate that the wealth conservation equation is

$$W_{t+dt} = W_t\{1 + [\bar{\rho} + \alpha(\mu_r - \bar{\rho})]dt + \alpha\sigma_r y(t)\sqrt{dt}\}$$

where α is the proportion of wealth at time t allocated to the risky asset, μ_r is the expected rate of return on the risky asset, σ_r is the standard deviation of the rate of return on the risky asset, and $y(t)$ is a standardized normal random variate. When time is continuous, dt, the time interval, can be chosen to be very small. Friend and Blume (1975) use a Taylor's series approximation to expand $u(W_{t+dt})$ at W_t. They then take the expectation and assume that dt is sufficiently small to allow terms involving dt to the power of 2 or more to be dropped. This implies that

$$Eu\left(W_{t+dt}\right) \approx u(W_t) + u'(W_t)W_t[\bar{\rho} + \alpha(\mu_r - \bar{\rho})]dt + \frac{1}{2}u''(W_t){W_t}^2\alpha^2{\sigma_r}^2 dt \qquad (1)$$

To obtain the optimal value of α, the first-order condition for maximizing $Eu\left(W_{t+dt}\right)$ is:[25]

$$u'(W_t)(\mu_r - \bar{\rho}) + u''(W_t)W_t \cdot \alpha^* \cdot {\sigma_r}^2 = 0$$

---

[24] The Wiener process implies that the risky return, rather than the risky rate of return, is subject to a log-normal distribution, a crucial assumption in the efficient market hypothesis. For the early economic applications, see Mirrlees (1965), Merton (1969) and Ross (1975).

[25] One could instead maximize the expected utility first, and then approximate the first-order condition for the maximization at the point of the initial wealth. The resulting point estimate of relative risk aversion is nearly a constant (or a slightly decreasing function) of the initial wealth, since the initial wealth is typically a large number. Based on the historical market data, this point estimate equals the one derived by Friend and Blume plus a large positive constant.

where α^* is the optimal proportion of wealth to invest in the risky asset. Using the Pratt-Arrow measure of absolute risk aversion, $A_u(W_t) = -\frac{u''(W_t)}{u'(W_t)}$, the above equation can be rewritten as

$$A_u(W_t) = \frac{(\mu_r - \bar{\rho})}{\alpha^* \sigma_r^2 \cdot W_t} \tag{2}$$

where $\frac{(\mu_r - \bar{\rho})}{\sigma_r^2}$ is referred to as the equilibrium market price of risk.

This is a local measure of risk aversion, or absolute risk aversion in the small, because the time interval is sufficiently small such that there are very small variations of the risky asset return, and therefore only small deviations from the initial wealth. Equation (2) indicates that there is an inverse relationship between the measure of absolute risk aversion in the small at the initial wealth and the initial wealth. That is, for two agents who invest the same proportion of the initial wealth in the risky asset, the one with a higher level of the initial wealth has a smaller magnitude of absolute risk aversion. Moreover, for two decision makers with the same initial wealth, the one who invests a lower portion of the initial wealth in the risky asset is more risk averse and has a larger magnitude of absolute risk aversion. These two features are consistent with the concept of absolute risk aversion in the small introduced by Pratt (1964).

Using the Pratt-Arrow measure of relative risk aversion, $R_u(W_t) = -\frac{u''(W_t)W_t}{u'(W_t)}$, equation (2), the expression giving absolute risk aversion, can be written instead as

$$R_u(W_t) = \frac{(\mu_r - \bar{\rho})}{\alpha^* \sigma_r^2} \tag{3}$$

where $R_u(W_t)$ is the decision maker's relative risk aversion measure in the small at the initial wealth. This is the form used by Friend and Blume and others using their methodology. Now equation (3) indicates that there is an inverse relationship between the measure of relative risk aversion in the small at the initial wealth and the share of wealth allocated to the risky asset. In addition, this local measure is invariant to the level of the initial wealth because of the particular approximation procedure used by Friend and Blume. It is emphasized that this measure $R_u(W_t)$ given in equation (3) is a value at a point, W_t, rather than a relative risk aversion function defined at all W_t. The latter, $R_u(W_t)$ for all W_t, is the relative risk aversion measure in the large. Estimating $R_u(W_t)$ for all W_t is the goal of the alternative approaches described in the next two sections.

Another way to make this same point is to note that if dt is not small, the Friend and Blume equation (1) no longer holds. For instance, if dt is one, the wealth conservation equation used by Friend and Blume (1975) becomes

$$W_{t+1} = W_t\{1 + [\bar{\rho} + \alpha(\mu_r - \bar{\rho})] + \alpha\sigma_r y\}$$

$$= W_t[1 + \alpha(\mu_r + \sigma_r y) + (1 - \alpha)\bar{\rho}]$$

where y is a standardized normal random variate.

This is nothing more than the usual discrete time portfolio model with a normally distributed random rate of return, which has a mean μ_r and a standard deviation σ_r. Since time interval is large now, the risks are no longer small and the approximation using equation (1) is no longer valid. This equation with $dt = 1$ is the starting point in the next section. The random rate of return, however, will not be assumed to be normally distributed.

4.3 Inferring Risk Aversion in the Large

Friend and Blume (1975) use equation (3) as the basic specification to estimate the magnitude of relative risk aversion in the small at the initial wealth for a representative of households in the same wealth category, using cross sectional data collected between 1962 and 1963. Recently Chiappori and Paiella (forthcoming) test whether relative risk aversion is constant over time by testing whether the risky asset share varies with respect to changes in wealth over time, using panel data collected every two years. The main perspective is from the one-to-one relationship between relative risk aversion in the small at the initial wealth and the risky asset share given by equation (3). In both cases, the estimate of relative risk aversion in the small at the initial wealth is applied in the discrete analysis.

In reaching the simple relationship between the decision maker's relative risk aversion level at a point and the proportion of wealth allocated to the risky asset, the procedure used by Friend and Blume (1975) requires that the time interval is sufficiently small to ensure that the approximation is valid. When risks are not small and variations about the initial wealth are substantial, it may not be appropriate to use the Friend and Blume procedure for inferring the level of risk aversion from a portfolio decision. If the time period is one year, for instance, as is often assumed when examining the standard portfolio allocation model, the risks arising from the variations of the risky asset return are not small. For example, final wealth from investing the initial wealth of one dollar in the risky asset can deviate from the initial wealth by a positive amount of 24 cents, or a negative amount of 10 cents on average. If the portfolio decision being examined is for such large risks, it may be more useful and accurate to estimate risk aversion in the large for these large risks.

The remainder of this section provides an approximation to utility that allows one to estimate *relative* risk aversion in the large, using only the mean and variance of the random return on wealth. Consider a decision maker in a standard one-period portfolio allocation model. Suppose the agent begins with an initial wealth, $W_0 > 0$, which can be freely allocated to one safe asset with a constant return $\rho > 0$, and one risky asset with a random return $r \geq 0$.[26] The expected return on the risky asset is $\mu_r > \rho$, and the standard deviation of the risky return is σ_r. Assume also that $u'(\cdot) > 0$ and $u''(\cdot) < 0$ so that the decision maker is risk averse. Finally, denote the proportion of the initial wealth allocated to the risky asset as α, which is chosen to maximize

$Eu(W)$, where $W = W(\alpha, r) = (\alpha r + (1-\alpha)\rho)W_0 = [\alpha(r-\rho)+\rho]W_0$

The approximation procedure takes the following two steps:

Step 1: Use the Taylor's series to quadratically approximate $u(W)$ at $W = \widehat{W}$, which is arbitrary at this moment. Doing this yields

$$u(W) \approx u(\widehat{W}) + u'(\widehat{W})(W-\widehat{W}) + \frac{1}{2}u''(\widehat{W})(W-\widehat{W})^2$$

Taking the expectation gives

$$Eu(W) \approx u(\widehat{W}) + u'(\widehat{W})E(W-\widehat{W}) + \frac{1}{2}u''(\widehat{W})E(W-\widehat{W})^2$$

[26] For the convenience of the later assumption of log-normality on the probability distribution, the risky return is used in the following. It is more appropriate in the discrete analysis, though it can be treated as one plus the rate of risky return used by Friend and Blume (1975). To be consistent with this, the riskless return, rather than the riskless rate of return, is also used.

Step 2: Calculate the first-order condition for maximizing the approximated $Eu(W)$ with respect to α and set it equal to zero[27]

$$\Rightarrow u'(\widehat{W})(\mu_r - \rho)W_0 + u''(\widehat{W})E(W - \widehat{W})(r - \rho)W_0 = 0 \tag{4}$$

This equation implicitly defines the optimal risky asset share, α^*. Some rearrangement of the above first-order condition gives

$$R_u(\widehat{W}) = \frac{(\mu_r - \rho)\widehat{W}}{E(W - \widehat{W})(r - \rho)} \tag{5}$$

Choose $\widehat{W} = \widehat{W^*} = [\alpha^*(\hat{r} - \rho) + \rho]W_0$, where α^* is the solution to equation (4) and $\hat{r} \geq 0$ is the risky asset return associated with wealth level $\widehat{W}$ for this α^*. That is, the approximation of wealth is reduced to the approximation of the risky asset return. Doing this and substituting this value of $\widehat{W}$ into (5), and some simplifying gives

$$R_u(\widehat{W^*}) = \frac{(\mu_r - \rho)}{\alpha^*} \cdot \frac{[\alpha^*(\hat{r} - \rho) + \rho]}{\sigma_r^2 + (\mu_r - \rho) \cdot (\mu_r - \hat{r})} \tag{6}$$

Since $\widehat{W}$ and hence $\hat{r}$ can be chosen arbitrarily, this expression gives relative risk aversion in the large for the given α^* whenever the approximation in equation (5) is appropriate. Of course, the approximation is exact when $u(W)$ is indeed quadratic. Equation (6) indicates that $R_u(\widehat{W^*})$ does not depend on the initial wealth, but it is a function of $\hat{r}$. It is easy to show that this

[27] One could also choose to maximize the expected utility first, and then approximate the first order condition for this maximization. The first order condition is $Eu'(W)(r - \rho) = 0$, or $Eu'(W)(W - \rho W_0)/\alpha = 0$. A linear approximation implies that $u'(W)$ is constant and thus the decision maker is risk neutral. A quadratic approximation requires information on $u'''(\cdot)$, but there is little evidence on this information. Kimball (1990) defines $u'''(\cdot) > 0$ as prudence within the expected utility theory.

function is positive and increasing for $\hat{r} < \mu_r + \frac{\sigma_r^2}{(\mu_r - \rho)}$, and is negative and increasing for $\hat{r} > \mu_r + \frac{\sigma_r^2}{(\mu_r - \rho)}$.[28] Since the procedure uses a quadratic approximation to utility, equation (6) is more accurate when the true relative risk aversion is an increasing function of wealth. Observe that only the mean and variance of the risky asset return, together with the riskless asset return and α^* are needed to estimate relative risk aversion in the large for this decision maker.

Equation (6) derived here is a measure of relative risk aversion in the large, while equation (3) derived by Friend and Blume (1975) is a point estimate, or a measure of relative risk aversion in the small at the initial wealth. By varying the chosen value for the risky asset return, or $\hat{r}$ where the approximation occurs, the magnitude of relative risk aversion can be estimated for all $\widehat{W^*}$. Thus these two estimates are directly comparable if a specific value for $\hat{r}$ is chosen for the measure in the large. At the point of the initial wealth, use $\widehat{W^*} = [\alpha^*(\hat{r} - \rho) + \rho]W_0 = W_0$ to obtain $\hat{r} = \frac{1-\rho}{\alpha^*} + \rho < \rho$ if $\rho > 1$. At this value for $\hat{r}$, the measure of relative risk aversion in the large generates a smaller magnitude of relative risk aversion than the measure of relative risk aversion in the small at the initial wealth since

$$\frac{(\mu_r - \rho)}{\alpha^*} \cdot \frac{1}{\sigma_r^2 + (\mu_r - \rho) \cdot (\mu_r - \rho + \frac{\rho - 1}{\alpha^*})} < \frac{(\mu_r - \rho)}{\alpha^* \sigma_r^2}$$

---

[28] $R_u(\widehat{W^*})$ is discontinuous at $\hat{r} = \mu_r + \frac{\sigma_r^2}{(\mu_r - \rho)}$, where the denominator becomes zero. The procedure breaks down at this point because the quadratic utility has negative marginal utility after this point. This implies that the quadratic utility approximation approach is only relevant for $\hat{r} < \mu_r + \frac{\sigma_r^2}{(\mu_r - \rho)}$.

The other two interesting values for $\hat{r}$ are $\hat{r} = \mu_r$ and $\hat{r} = \rho$. At $\hat{r} = \mu_r$, the measure of relative risk aversion in the large is evaluated at a larger magnitude than the measure of relative risk aversion in the small at the initial wealth because

$$\frac{(\mu_r - \rho)}{\alpha^*} \cdot \frac{\alpha^*(\mu_r - \rho) + \rho}{\sigma_r^2} > \frac{(\mu_r - \rho)}{\alpha^* \sigma_r^2}$$

While at $\hat{r} = \rho$, it is ambiguous whether the measure of relative risk aversion in the large at this value for $\hat{r}$ is greater than the point estimate.[29]

$$\frac{(\mu_r - \rho)}{\alpha^*} \cdot \frac{\rho}{\sigma_r^2 + (\mu_r - \rho)^2} \gtreqless \frac{(\mu_r - \rho)}{\alpha^*} \cdot \frac{1}{\sigma_r^2}$$

4.4 Inferring Risk Aversion in the Large Using Functional Forms for Utility

The approach by Friend and Blume (1975) to infer risk aversion in the small, and the quadratic approach to infer risk aversion in the large examined in the last section use different approximation procedures. Despite this, both approaches use a Taylor’s series to expand the expected utility to the second-order term before maximizing the approximated expected utility. If the expected utility is a converging sequence, the approximation using a Taylor’s series is more accurate when more of higher order terms are added in the expansion for the decision making. However, there are two undesirable properties when the Taylor’s series expansion is applied. First, the approximated expected utility can change significantly when higher order terms are included in the expansion. This is true for most commonly used functional forms of utility. One

[29] The former is smaller by about 11% if long-term market data are used.

exception is the combination of exponential utility and a normal probability distribution for the risky asset return. These issues have been discussed by Loistl (1976) and Hlawitschka (1994).

Furthermore, the inclusion of higher order derivatives of utility in the expansion implies that the measure of risk aversion is entangled with and cannot be separated from other risk preferences such as the measure of prudence and the measure of temperance. Of course, more information is also needed on the higher moments of the probability distribution for the risky asset return. Making appropriate assumptions on these terms is not always straightforward and in most cases data are not available to guide the decision. These same issues arise when one chooses to maximize the expected utility first and then uses a Taylor's series to approximate the first-order condition for this maximization. In other words, by expanding the expected utility only to the second-order term, both the Friend and Blume method of inferring risk aversion in the small and the quadratic approximation to infer risk aversion in the large have serious weaknesses. It is also difficult to improve either approach by refining the approximation used.

As a consequence, another approach for inferring risk aversion in the large is used here. This approach assumes a functional form for utility or risk aversion, and the forms chosen have few parameters.[30] That is, in contrast to risk aversion in the small by the Friend and Blume approach and risk aversion in the large using the quadratic utility approximation approach, the procedure of the third approach does not contain any form of a Taylor's series approximation of utility. For

[30] These include the three-parameter hyperbolic absolute risk averse (HARA) utility function by Merton (1971), the two-parameter Expo-power utility function by Saha (1993), the two-parameter Power Risk Aversion utility function by Xie (2000) and the flexible three-parameter utility function by Conniffe (2007). Meyer (2010) provides a comprehensive literature review on such functional forms for utility.

a one-parameter utility function, this approach allows risk aversion in the large to be inferred from the observed portfolio choice by carrying at the following four steps:

1) Choose a functional form among a set of parametric families of utility functions;
2) Use empirical evidence to make a reasonable assumption concerning the probability distribution function for the risky asset return;
3) Specify an optimal risky asset share, α^*, and use this value to numerically solve for the unknown parameter of the functional form, using the first-order condition for choosing α^* for the expectation of that utility function.
4) Infer risk aversion in the large given the assumed functional form of utility and the solved value for the parameter of the utility function.

The basic idea is to use the one observed portfolio choice of allocation to the risky asset to determine one parameter of a pre-specified functional form for utility or risk aversion. Doing this for many different functional forms allows each to provide an estimate of one another. This approach has two nice features. First, risk aversion in the large is expressed in simple terms for the functional forms of utility that are examined. A second desirable property is that, a functional form can be chosen to be one of those commonly used in the literature and inferring risk aversion for this functional form provides a better estimate of risk aversion in the large. Of course, the probability distribution function for the risky asset return must be specified in order to estimate the unknown parameter of the functional form for utility. That is, this approach imposes assumptions on the functional form for utility and also on the probability distribution for the risky asset return.

The remainder of this section discusses several one-parameter functional forms for utility. These one-parameter utility functions are frequently used in the literature. Following this, a two-parameter family of a marginal utility function recently proposed by Meyer (2010) is examined. The one-parameter quadratic utility is first studied, because the quadratic approximation approach discussed in the last section is equivalent to assuming the functional form for utility as quadratic utility studied in this section. That is, if the one-parameter functional form for utility is assumed to be quadratic, no other information regarding the distribution of the risky asset return except for the mean and variance is required. Of course, if the one-parameter quadratic utility is the true utility, the determination of risk aversion in the large is exact at every level of wealth. To see this, suppose that the one-parameter quadratic utility function takes the following form:

$$u(W) = W - c \cdot W^2$$

where c is strictly positive.

Since $u'(W) = 1 - 2c \cdot W$ and $u''(W) = -2c$, relative risk aversion for this one-parameter quadratic utility function is then

$$R_u(W) = \frac{2c \cdot W}{1 - 2c \cdot W} \tag{7}$$

Using the first-order condition for choosing the risky asset share to maximize $Eu(W)$, the parameter c can be solved as a reduced-form function of a specified α^* and the initial wealth, given the information on the mean and variance of the risky asset return, and the riskless return. In particular,

$$c = \frac{(\mu_r - \rho)}{2W_0[\alpha^* \sigma_r^2 + \alpha^*(\mu_r - \rho)^2 + \rho(\mu_r - \rho)]}$$

Plug this value for c into equation (7) and use $\widehat{W^*} = [\alpha^*(\hat{r} - \rho) + \rho]W_0$, the true measure of relative risk aversion in the large, when the true utility function is of this quadratic form, can be shown to be identical with the quadratic functional estimate given by equation (6).

Power utility also has one parameter and it is a common choice in modern macroeconomics and finance. The one-parameter power utility function is

$$u(W) = \frac{W}{1 - \theta}^{1 - \theta}$$

where $\theta > 0$ is called the coefficient of relative risk aversion. Since $R_u(W) = \theta$ is a constant, power utility displays constant relative risk aversion (CRRA) for wealth. Among the frequently used simple forms of utility function in the literature, power utility is best supported by the existing empirical evidence concerning the slope of relative risk aversion for wealth.

Another one-parameter utility function often used is exponential utility, which has a simple form of

$$u(W) = -e^{-\delta W}$$

where δ is strictly positive. Exponential utility exhibits constant absolute risk aversion (CARA) for wealth because $A_u(W) = \delta$, and thus increasing relative risk aversion (IRRA) for wealth since $R_u(W) = A_u(W) \cdot W = \delta W$. Observe that relative risk aversion of exponential utility increases linearly in wealth. That is in contrast with relative risk aversion of the one-parameter quadratic utility, which increases acceleratively as wealth approaches the point where marginal utility turns negative. Therefore, exponential utility is a case between power utility and the one-parameter quadratic utility in terms of the slope of the Pratt-Arrow relative risk aversion measure.

Meyer (2010) proposes a simple set of risk preferences termed isoelastic risk preferences. The marginal utility for these risk preferences takes a specific functional form of:

$$u^{'}(W) = e^{(\frac{\pi}{\lambda W^{\lambda}})}$$

where $\pi > 0$ is a parameter used to adjust the magnitude of the risk aversion measure; λ is a parameter used to indicate the wealth elasticity of the risk aversion measure.

It can be easily shown that the elasticity of absolute risk aversion measure is $-(\lambda + 1)$ while the elasticity of relative risk aversion measure is $-\lambda$. Since only one observation on the risky asset proportion is used when solving, this marginal utility can only have one unknown parameter. For the values for $\lambda = -1$ and $\lambda = 0$, this marginal utility yields exponential utility and power utility respectively. But for other values for λ, the anti-derivative of this marginal utility cannot be expressed in terms of a closed form of utility function. A positive value for λ is chosen so that the marginal utility displays decreasing relative risk aversion (DRRA).[31] For computational convenience, $\lambda = 1$ is picked and the marginal utility is then

$$u^{'}(W) = e^{\frac{\pi}{W}}.$$

Under certain assumptions, the approach discussed here may use multiple observations on the portfolio decision to estimate the same number of unknown parameters of a functional form for

[31] This marginal utility with positive values for λ is used by Meyer and Meyer (2005) to argue that the equity premium puzzle can be resolved.

utility or risk aversion. For instance, it can be assumed that risk preference is unchanged, the riskless return and the distribution function for the risky asset return are fixed, and observations on the portfolio decision over time are independent of one another. In other words, multiple observations for a decision maker arise from the same one-period model that is repeated across time periods.

4.5 Numerical Solutions

In this section, in the standard one riskless asset and one risky asset model, an optimal portfolio decision (α^*) is specified and used to numerically solve for one value for an unknown parameter of a specified functional form of utility or marginal utility, given that the probability distribution of the risky asset return being log-normal. The solved parameter is then used to compute the inferred Pratt-Arrow measure of relative risk aversion in the large for each of the three specified functional forms. Relative risk aversion in the large using the quadratic approximation approach and relative risk aversion in the small using the Friend and Blume methodology are also calculated using the same α^*. These estimates of relative risk aversion are then compared to each other in a graph. To compute these estimates, in the following, the existing evidence in the literature concerning the initial wealth level, the riskless return, and the risky return distribution is borrowed and reported.

When power utility is the true utility, the true measure of relative risk aversion in the large is a constant and is independent of the initial wealth. But this is not the case when exponential utility or the marginal utility with DRRA is the true one. Thus the initial wealth needs to be specified. A natural proxy is the financial assets, which are more likely to satisfy the assumptions on the risky asset in the standard portfolio decision model: be liquid, have no transaction cost and can be traded in any quantity. In the study of household finances, Campbell (2006) reports that the median household has financial assets of $35,000, using data from Survey of Consumer Finances 2001. This number will be used to compute the basic results when exponential utility or the marginal utility with DRRA represents the underlying risk preferences.

Each of the three functional forms is coupled with a log-normal probability distribution for the risky asset return. Both normal and uniform distributions are not picked because there is much evidence that the market return of risky assets is positively skewed, or the standardized third central moment is positive. The distribution of the risky return is chosen so that it has an annualized mean return of 1.07, with a standard deviation of 0.17, while the annualized return on the riskless asset is assumed to be 1.01. Such values are based on studies by Mehra and Prescott (1985) and Kocherlakota (1996) on the annualized real rate of return from investing in the Standard and Poor 500 Index, and from investing in short-term treasury bills respectively, over a long period of 1890-1979. Adding more recent data does not make much difference.

Given a single observed risky asset proportion (0.2, 0.5 and 0.8), each of the three figures (Figures 1 to 3) graphs five measures of relative risk aversion.[32] Three estimates in the large each assuming a specific functional form of utility or marginal utility, are drawn in solid curves. Estimating relative risk aversion in the large using the quadratic approximation approach, and estimating relative risk aversion in the small using the Friend and Blume procedure are drawn in

---

[32] According to Friend and Blume (1975), the median household had a net financial wealth (exclusive of housing) in the range of $10,000 to $100,000. The average holding of riskless plus mixed risky assets (government bonds and long-term corporate bonds) relative to total financial assets was about 44% for households in this wealth category. In addition, using British data, Blake (1996) reports that 75% of the population held at least 80% of their financial assets in the form of relatively safe interest-bearing accounts (IBAs).

dashed curves. Each of the five relative risk aversion measures is drawn for the risky asset return in the range between 0.9 and 1.24, which is one standard deviation from the mean. Equivalently, the risky asset return has a probability of about 70% to fall within this range.[33]

In all three figures, the main finding is that assuming a functional form for utility or risk aversion and estimating relative risk aversion in the large is superior to either approximating utility quadratically or estimating relative risk aversion in the small using the Friend and Blume procedure. Of course, the true utility function must be among the three functional forms for isoelastic risk preferences, which generate measures of relative risk aversion in the large very close to one another. By choosing a functional form for utility, even if it is a wrong functional form, researchers can still provide a closer estimate of relative risk aversion in the large than does the quadratic utility approximation approach or the relative risk aversion in the small estimate determined using the Friend and Blume methodology.

To further compare the precision of each approach for inferring relative risk aversion, four important values for the risky return are chosen: 0.9 and 1.24 which stand for one standard deviation from the mean; 1.01 which equals the riskless return; and 1.07, the mean of the risky return. The computation results are shown in Table 1. The first three columns are for the third approach, directly employing functional forms for utility or marginal utility to infer relative risk aversion; the results from Friend and Blume in the small approach are reported in the fourth column; the last column belongs to the quadratic utility approximation approach.

---

[33] It seems that the estimate of relative risk aversion in the large for the marginal utility with DRRA is a downward-sloping straight line. In fact, it is a downward-sloping curve with a decreasing rate over a wider range of the distribution.

Since power utility for wealth receives most support from the existing empirical evidence and is frequently used in the literature, let us for the sake of illustration suppose that it is the true functional form for utility. Then apparently assuming exponential utility and marginal utility with DRRA provides a lower bound and an upper bound of the magnitude of relative risk aversion, for the observed risky asset shares or for the chosen values for the risky asset return. The Friend and Blume in the small approach consistently underestimates the true magnitude by about 11 to 12%. And estimates using the quadratic utility approximation approach never fall within the bounds created by assuming functional forms for utility or marginal utility.

Table 1: The Magnitudes of Relative Risk Aversion at Four Values for the Risky Asset Return under Three Different Approaches

	Exponential Utility (CARA)	**Power Utility (CRRA)**	**Marginal Utility ($\lambda = 1$) (DRRA)**	**Friend & Blume in the Small**	**Quadratic Approach (IRRA)**
$\alpha^* = 0.2$					
$\hat{r} = 0.9$	11.37	11.691	12.013	10.381	7.581
$\hat{r} = 1.01$	11.623	11.691	11.751	10.381	9.323
$\hat{r} = 1.07$	11.761	11.691	11.613	10.381	10.609
$\hat{r} = 1.24$	12.152	11.691	11.239	10.381	16.941
$\alpha^* = 0.5$					
$\hat{r} = 0.9$	4.395	4.702	5.002	4.152	2.931
$\hat{r} = 1.01$	4.649	4.702	4.73	4.152	3.729
$\hat{r} = 1.07$	4.787	4.702	4.593	4.152	4.318
$\hat{r} = 1.24$	5.178	4.702	4.426	4.152	7.219
$\alpha^* = 0.8$					
$\hat{r} = 0.9$	2.652	2.943	3.217	2.595	1.769
$\hat{r} = 1.01$	2.905	2.943	2.936	2.595	2.331
$\hat{r} = 1.07$	3.043	2.943	2.803	2.595	2.746
$\hat{r} = 1.24$	3.435	2.943	2.484	2.595	4.789

4.6 Conclusion

This chapter finds that assuming functional forms for utility or risk aversion can do a much better job in inferring risk aversion in the large, than either inferring risk aversion in the large using the quadratic utility approximation approach or inferring risk aversion in the small using the Friend and Blume approach. This finding questions the appropriateness of using the approach of inferring risk aversion in the small in applied works to econometrically estimate the magnitude or to test the slope of relative risk aversion for wealth, for example, Friend and Blume (1975) as the former and Chiappori and Paiella (forthcoming) as the latter. Unexpectedly, another finding is that assuming power utility is a safe and nice starting point to infer relative risk aversion in the large even if the true functional form for utility is exponential utility or the marginal utility with DRRA. This may offer some theoretical support for power utility for wealth, which is a standard assumption of the functional form for utility in macroeconomics and finance.

It is also observed that inferring relative risk aversion in the small consistently underestimates the true magnitude by 10 to 15% for each of the three utility or marginal utility functions with isoelastic risk references. Suppose that an investor's preference can be represented by power utility and he holds one half of his wealth in the risky asset. This implies that the investor is in fact willing to pay a proportional risk premium of up to 6.8% of the amount invested in the risky asset to avoid the risk, rather than a 6% premium based on inferring risk aversion in the small using the Friend and Blume approach.[34] In addition, the mean-variance approach, or quadratic

[34] Pratt (1964) uses a formula to approximate the risk premium of a small, actuarially neutral, proportional risk $\tilde{z}$:

$$\pi^{*}(W,\tilde{z}) \approx \frac{1}{2} \cdot \sigma_{z}^{2} \cdot R_{u}(W)$$

where W is the wealth level and σ_{z}^{2} is the variance of the proportional risk $\tilde{z}$.

utility approximation, is less appealing due to its sizable variation in inferring relative risk aversion in the large. This approach is most useful when quadratic utility is or is approximately the true functional form for utility, and it can somewhat be useful if the focus is relative risk aversion at the mean of the risky return.

The robustness of these findings is also checked. Overall the results for both exponential utility and the marginal utility with DRRA are not sensitive to variations in the initial wealth level, when the initial wealth is increased to ten times, $350,000, or is reduced to one tenth, $3,500.[35] Moreover, to see whether major findings in this work are robust to the assumption on the distribution function for the risky asset return, gamma distribution is used as the alternative of log-normal distribution. The results are very similar with those based on the log-normality assumption. Finally, if λ equals .5 for the marginal utility with DRRA, and the computation results are more favorable for the approach of employing functional forms for utility or risk aversion to infer risk aversion in the large. This is obvious because a smaller λ implies a less elastic and thus a flatter measure of relative risk aversion in the large.

Since only one observation on the portfolio decision is used to infer risk aversion in the large for an individual decision maker, a possible extension in the future is to consider how to infer risk aversion in the large for a representative decision maker, given a cross section of individual decision makers each with one observed portfolio choice. For instance, the agent can be a representative of a group of decision makers in the same wealth category and/or with similar demographic characteristics.

[35] The computation is combined with changes in the convergence tolerance.

Another extension of this chapter would be to employ two or more observed portfolio choices for the same decision maker and to infer the magnitude of her risk aversion. With multiple observations, risk aversion in the large can be inferred for a utility function or risk aversion function with multiple unknown parameters. To compare this with risk aversion in the small using the Friend and Blume approach and risk aversion in the large using the quadratic utility approximation approach, however, one may have to consider how to adjust the measurement error since multiple observations allow each of these three approaches to yield more than one risk aversion measure.

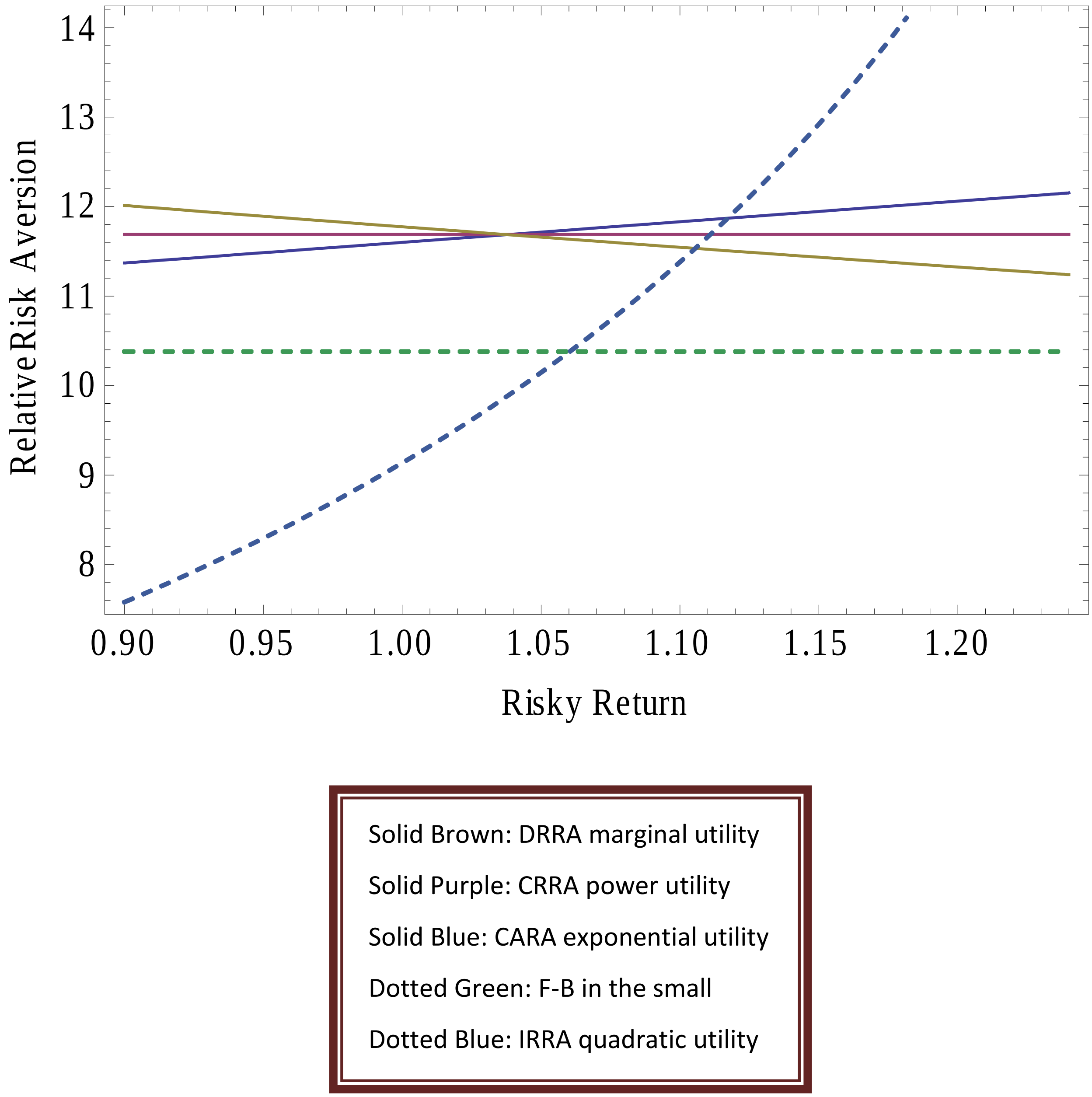
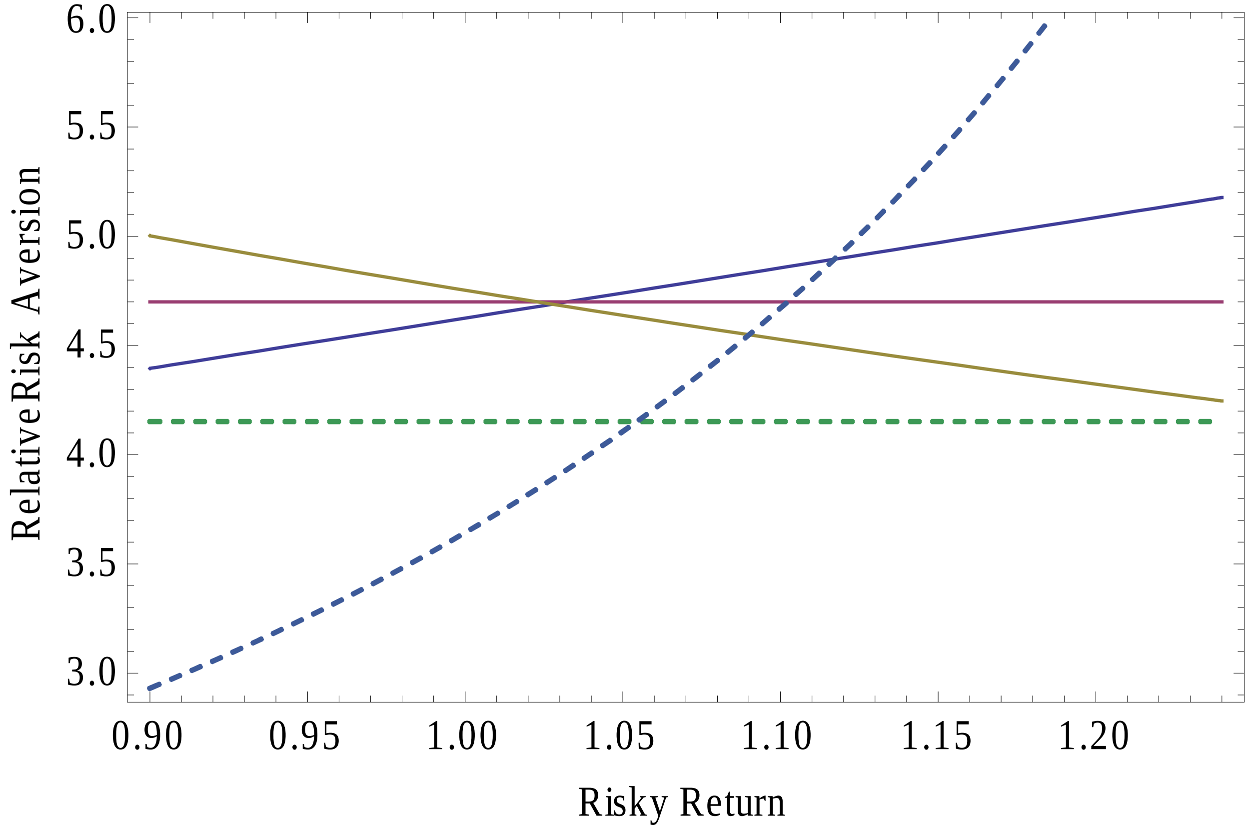


Figure 4.1: relative risk aversion as a function of the risky return for $\alpha^* = .2$

(For interpretation of the references to color in this and all other figures, the reader is referred to the electronic version of this dissertation)



Solid Brown: DRRA marginal utility

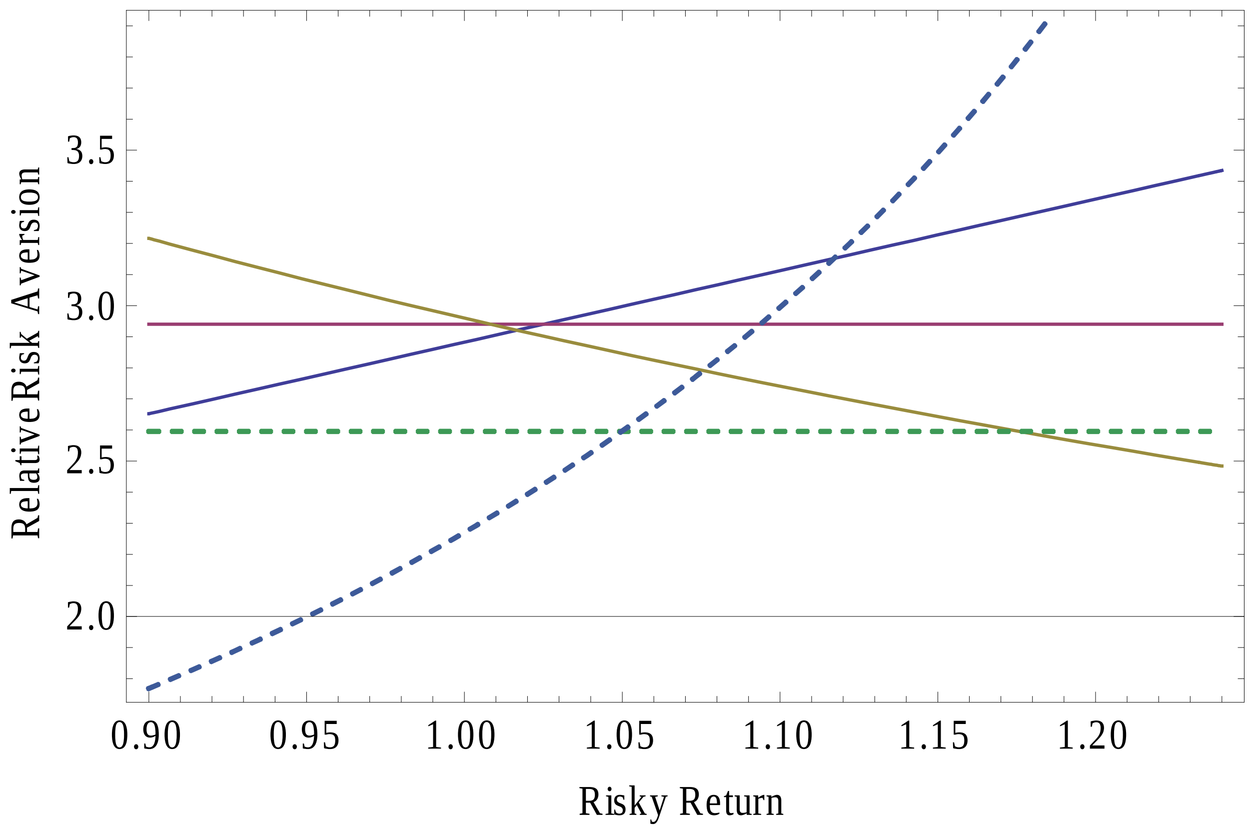
Solid Purple: CRRA power utility

Solid Blue: CARA exponential utility

Dotted Green: F-B in the small

Dotted Blue: IRRA quadratic utility

Figure 4.2: relative risk aversion as a function of the risky return for $\alpha^* = .5$



Solid Brown: DRRA marginal utility

Solid Purple: CRRA power utility

Solid Blue: CARA exponential utility

Dotted Green: F-B in the small

Dotted Blue: IRRA quadratic utility

Figure 4.3: relative risk aversion as a function of the risky return for $\alpha^* = .8$

Chapter Five

Reinterpretation of Recent Empirical Evidence

5.1 Introduction

This chapter discusses in detail several published papers that have led to the writing of my dissertation on how to infer risk aversion from portfolio allocation decisions. It also builds on the theory in chapters 3 and 4 and reinterprets the empirical findings concerning the magnitudes and the slopes of relative risk aversion in these papers.[36] In doing this, major econometric issues that may cloud the identification of the slope or the elasticity of relative risk aversion for wealth are pointed out. I start with a discussion of inferring the slope of relative risk aversion since it is the main question in the most recent work. It is also because the literature has been offering confusing evidence using two different but related arguments of utility: wealth and consumption.

Consider a standard one-period portfolio allocation model. There are two liquid financial assets: one riskless and one risky, and the fraction of wealth to invest in the risky asset is the only choice variable. Then based on the well-known theorem by Pratt (1964) and Arrow (1965), the comparative static changes in the optimal risky asset share in response to changes in the initial wealth can be used to infer whether relative risk aversion is monotonically increasing, constant or decreasing in that measure of wealth. This one time period result is often extended to multiple time periods, when estimating the slope of relative risk aversion for wealth using multiple observations on the risky asset share across time periods. A major point examined in chapter 3 is

[36] Meyer and Meyer (2005a, 2005b and 2006) provide a comprehensive literature review on relative risk aversion for wealth and relative risk aversion for consumption.

that, one should be careful at making different interpretations of this slope based on the estimated relationship, since the wealth measures in empirical studies differ and are often inconsistent with each other. These wealth measures are also not exactly those used in the theoretical models. For example, constant relative risk aversion for liquid financial wealth does not imply that relative risk aversion is also constant for a broader measure of wealth consisting of uncapitalized future income, private business equity, housing equity or human capital.

Now suppose that the study of wealth allocation decisions over time can to some extent allow one to infer the sign of the slope of relative risk aversion for wealth. Can this information then also be used to infer the slope of relative risk aversion for consumption? The answer is probably not. Knowing the latter is very important because it is directly related to the debate on which commonly used functional forms of utility for consumption should prevail in the literature, for instance, CRRA power utility vs. DRRA habit formation utility. The relationship between the slope of relative risk aversion (of utility) for consumption and the slope of relative risk aversion (of indirect utility) for wealth is a derived relationship whose properties depend on how consumption and wealth are defined, measured, and related to one another in equilibrium. To obtain the equilibrium relationship between consumption and wealth, however, in turn requires one to first assume the functional form of utility and hence the slope of relative risk aversion for consumption. Two examples of an explicit relationship between the slopes of these two relative risk aversion measures are given by Meyer and Meyer (2005a and 2005b) who demonstrate that the equilibrium consumption (as a function or policy of wealth) is linear. Note that consumption is the *only* choice variable in their multiperiod models.

Assuming a specific form for the optimal consumption as a function of wealth, however, is a severe restriction.[37] For one thing, it arises only under special theoretical conditions on the functional form of utility and on the risk(s) a decision maker may face (Carroll and Kimball (1996)). Moreover, the analytical relationship between consumption and wealth cannot be derived in many cases and usually is nonlinear (ex: marginal propensity to consume decreases in wealth) when *two* choice variables: consumption and investment have to be simultaneously made in multi-period models. The only known exception in the literature may be the case that under the assumptions: a) CRRA power or logarithm utility; b) lifetime wealth being available at the start of decision making; and c) portfolio risk being the only risk, the consumption function is then proportional in wealth in every period (Samuelson (1969)). Finally, though a linear consumption policy is sometimes assumed and does receive certain support from the aggregate data, there is little such evidence at micro level.

The analysis of portfolio allocation decisions in chapter 3 using multi-period models where consumption and investment are jointly determined suggests that both DRRA habit formation utility and CRRA power utility are acceptable functional forms of utility for consumption. The key is whether or not uncapitalized future income is introduced into these models and thus provides another source of consumption; equivalently, whether or not current wealth is in general smaller than lifetime wealth. As a consequence, in two periods, the same response of the risky asset share to a comparative static change in the initial wealth implies different signs of the slope of relative risk aversion for consumption. Moreover, in a discrete infinite horizon, assuming

[37] It can be shown that for a concave consumption function of the form $C = a + bW + cW^2$, where $a > 0$, $b > 0$ and $c < 0$, the relationship between the slope of relative risk aversion for consumption and the slope of relative risk aversion for wealth becomes less straightforward than the relationship under the linear case, let alone for other nonlinear consumption functions.

DRRA habit formation utility *with* uncapitalized future income may lead to the same contemporaneous relationship between the risky asset proportion and the wealth level in every period as that under the assumption of CRRA power utility *without* uncapitalized future income. Therefore, by examining wealth allocation decisions, one is able to deduce the slope of relative risk aversion for consumption and thus infer the general functional form of utility for consumption with this risk preference. These two analytical findings have a valuable implication for researchers who are undecided in choosing functional forms of utility in multi-period discrete models. The two findings can also be used to reconcile recent empirical micro-level findings on household wealth allocation decisions using panel data (Brunnermeier and Nagel (2008); Calvet, Campbell and Sodini (2009); Chiappori and Paiella (forthcoming)).

This chapter also summarizes the empirical micro evidence concerning the magnitudes of relative risk aversion for wealth presented by Friend and Blume (1975) and by Chiappori and Paiella (forthcoming), who employ the approach for inferring risk aversion in the small. Using the one observed portfolio decision presented by Friend and Blume (1975), more evidence concerning the magnitude of relative risk aversion is then provided based on the two approaches for inferring risk aversion in the large discussed in chapter 4. Note that the major difference between inferring risk aversion in the large and inferring risk aversion in the small is whether a functional form of (indirect) utility for wealth is assumed or not.

In particular, I present new information concerning the magnitude of relative risk aversion for financial wealth, by assuming functional forms of utility for this wealth. This is compared to the information produced by Friend and Blume (1975), who use the approach for inferring risk aversion in the small and micro-level cross sectional data to econometrically estimate the magnitude in a reduced form for a representative agent. Given the statistical advantage of using multiple observations, one may consider a two-step approach to estimating risk aversion in the large: 1) use (overtime or cross section) multiple observations to structurally estimate parameters of more general functional forms of utility or marginal utility for wealth; 2) infer risk aversion in the large for a (representative) decision maker at every wealth level.

Finally, two main econometric issues when estimating the slope of relative risk aversion for wealth are pointed out. First, changes in the risky asset share *over time* and changes in liquid wealth *over time* are serially correlated; that is, the latter can in part result from the former in the past. If this is the case, controlling for past changes in the risky asset share in the regressions becomes necessary. It is also true if the research question is instead whether there exists time-varying relative risk aversion for liquid wealth, provided that (indirect) utility for liquid wealth is time-additive. Second, excluding measures of consumption from the empirical analysis weakens one's statement on whether relative risk aversion for wealth is constant. This happens because consumption is correlated with both the wealth level and the risky asset share. Failure to include it on the right-hand side of a regression equation suffers from the omitted variable bias.

5.2 Friend and Blume (1975)

In an often cited paper, Friend and Blume (1975) (F-B) systematically examine both the level and the slope of relative risk aversion for wealth, using cross-sectional data from the Federal Reserve Board Surveys for approximately 2,100 households. The primary interest here is *not* the slope of relative risk aversion inferred from the information on how the risky asset proportion varies across different wealth categories. Why? Consider a world of CRRA investors with heterogeneous risk preferences. If less risk averse investors tend to have more wealth, then the study on a cross sectional investors automatically generates a positive relationship between the risky asset share and the wealth level, which implies DRRA for a representative agent of these investors. The focus in the following is thus placed on how F-B infer and estimate the level of relative risk aversion for wealth using the information on the ratio of mixed-risky and risky assets to net wealth. This is based on a one-to-one inverse relationship between the magnitude of relative risk aversion for wealth and that ratio for a decision maker. The relationship is derived using the method of inferring risk aversion in the small (discussed in chapter 4) at the point of the initial wealth in a time continuous portfolio allocation model. In particular,

$$\alpha_{it} = \frac{\mu_r - \rho}{\sigma_r^2} \cdot \frac{1}{R_u(W_{it})} \quad (1)$$

where α_{it} is the fraction of the initial wealth for investor i to allocate in risky assets at time t; $\frac{\mu_r - \rho}{\sigma_r^2}$ is the market price of portfolio risk, assumed to be identical for each investor and constant over time; and $R_u(W_{it}) = -\frac{u''(W_{it})W_{it}}{u'(W_{it})}$ is the Pratt-Arrow measure of relative risk aversion for wealth. For the analysis using cross sectional data, the time subscript can be omitted.

F-B present three different measures of relative risk aversion each associated with a different measure of wealth: financial assets, financial assets plus housing equity, and financial assets plus housing equity and human capital.[38] However, none of these three wealth measures is exactly the Arrow-Pratt (A-P) wealth that is often discussed and used as the argument of relative risk aversion measure in the literature. An ideal financial asset included in the A-P wealth should

[38] In addition to common stocks, their risky financial assets also include less liquid assets of preferred stock, equity in UnInc business, investment in real estate assets and miscellaneous assets such as patents.

satisfy at least three conditions: liquid, fully divisible and with no transaction cost. F-B do not employ equation (1) to compute relative risk aversion using the information on α_{it} and the market price of portfolio risk. Instead, they replace the magnitude of relative risk aversion for investor i in equation (1) with that for a representative of investors in the same wealth category, and include other individual controls and an error term to have the basic estimation equation. F-B then run a reduced-form regression and estimate the intercept, which is now the product of market price of portfolio risk and the reciprocal of the magnitude of relative risk aversion for a representative of households in each of six wealth categories.

This empirical strategy has a benefit of using multiple observations to correct for noise in the estimation. Nonetheless, equation (1) comes from the approach for inferring risk aversion in the small which has been shown to underestimate when using annual data on market returns. Specifically, this approach implicitly assumes that time interval is infinitesimal so that the portfolio risks involve only small deviations from the initial wealth and therefore risk aversion in the small for such small risks can be evaluated at the single point of initial wealth. In addition, computation results in chapter 4 indicate that this approach consistently underestimates the true magnitude for the CRRA power utility by about 11 to 12% when only one observed portfolio is used.[39] Thus this section provides additional evidence on the magnitude for *financial assets* which are closest to the A-P wealth, by assuming functional forms of utility or marginal utility to infer risk aversion in the large. To calculate the magnitude of relative risk aversion, I borrow the information in F-B on the average risky asset share in each wealth class, as well as the

[39] CRRA power utility for wealth receives most support in the literature, including recent papers by C-P and B-N that will be discussed in the next two sections.

annualized real market returns on S&P 500 index and on short-term U.S. treasury bills during 1890 and 1979 reported by Mehra and Prescott (1985) and Kocherlakota (1996).

The data set used by F-B provides a representative sample of the entire population and oversamples the wealthy families, who hold the majority of financial assets. It also contains detailed information on the breakdown of wealth into different asset categories, for example: risk free, mixed-risky, and risky assets for each household. Unfortunately, the surveys do not report disaggregated asset holdings within the same asset class and thus is not very useful if one is interested in diversification issues. The surveys are the precursors of the Survey of Consumer Finances (SCF), which has been interviewing a new sample of households every three years since 1989. SCF is now generally considered as the best U.S. survey on household financial wealth allocation.

For each of the six net financial wealth classes in Table 2, both the average ratio of mixed-risky and risky assets to net wealth ($\bar{\alpha}$) and the average share of mixed-risky and risky assets in total assets ($\bar{\alpha}^*$) are reported. But only the latter, $\bar{\alpha}^*$, is used to compute relative risk aversion since it is less than one for all wealth categories and is consistent with the choice variable studied in the standard one-period portfolio model. Note that financial assets reported by F-B include less liquid risky assets of preferred stock, private business equity, patents and others.

Columns 4 to 6 report estimates of the magnitude by assuming three functional forms of utility or marginal utility: exponential, power and the marginal utility of $u'(W) = e^{(\frac{\pi}{\lambda W^{\lambda}})}$ with $\lambda = 1$. For exponential utility and the marginal utility, the risky return is evaluated at the mean (1.07) and the initial wealth level can be any number in the corresponding wealth category.[40] Estimates using F-B in the small approach and the quadratic utility approximation approach are reported in column 7 and 8 respectively. Column 9 is for reduced-form estimates by F-B and the last column is for their adjustments; these last two columns are reported by Meyer and Meyer (2005b).[41]

[40] Within each wealth category, computation results for exponential utility and for the marginal utility with DRRA are not sensitive to changes in the initial wealth. See the results reported in Table 2.

[41] Meyer & Meyer (2005b) provide a detailed review on F-B. They also scale the estimates of the magnitude of relative risk aversion for the three wealth measures reported in F-B by one minus the average tax rate, and convert them into the corresponding ones for the A-P wealth for representatives of six net wealth categories studied by F-B.

Table 2: Estimates of Relative Risk Aversion at Mean Risky Return for a Representative of Households in Six Net Financial Wealth Categories (Exclusive of Homes and Human Capital)

Net Wealth ($1,000)	$\bar{\alpha}$ (net wealth)	$\bar{\alpha}^*$ (financial assets)	Exponential Utility (CARA)	Power Utility (CRRA)	Marginal Utility ($\lambda = 1$) (DRRA)	F-B in the Small	Quadratic Utility (IRRA)	F-B Reduced-form Estimates	M-M Scale F-B by $(1-t)$
1-10	0.313	0.277	8.529-8.529 [0.01]	8.456 [100]	8.371-8.371 [100,000]	7.495	7.695	7.02	6.39
10-100	0.664	0.635	3.798-3.798 [0.01]	3.707 [100]	3.586-3.586 [1,000,000]	3.269	3.427	3.32	3.01
100-200	0.875	0.842	2.898-2.899 [0.01]	2.796 [100]	2.652-2.653 [1,000,000]	2.466	2.615	2.67	2.28
200-500	0.936	0.913	2.683-2.684 [0.001]	2.577 [100]	2.427-2.427 [10,000,000]	2.274	2.421	2.62	2.14
500-1,000	0.941	0.937	2.619-2.619 [0.001]	2.511 [100]	2.358-2.358 [10,000,000]	2.216	2.362	2.95	2.13
1,000+	1.000	0.946	2.595 [0.001]	2.487 [100]	2.332 [10,000,000]	2.195	2.341	3.08	2.00

Convergence tolerance in the bracket.

5.3 Chiappori and Paiella (forthcoming)

Using panel data from the Bank of Italy Survey of Household Income and Wealth (SHIW), Chiappori and Paiella (forthcoming) (C-P) present the latest empirical evidence concerning both the magnitude and the slope of relative risk aversion for *financial wealth*, with the focus on the latter. C-P define risky financial assets as the sum of corporate bonds, investment funds, Italian shares of listed and unlisted companies and partnerships, managed savings, foreign securities and loans to cooperatives, and risk free financial assets as the sum of bank and post office deposits, certificates of deposits, Italian government bills and bonds. The sum of these two types of financial assets gives total financial wealth. The new information on relative risk aversion for financial wealth C-P provide is interesting because this measure of wealth includes only very liquid and divisible financial assets, which is quite close to the original A-P wealth. C-P also give evidence on the slope of relative risk aversion for different wealth measures by successively adding to the financial wealth less liquid assets such as business equity, housing equity and human capital.

C-P use the F-B approximation approach for inferring risk aversion in the small to show the inverse relationship between the risky asset share of financial wealth and an agent's relative risk aversion in the small for that wealth; that is, equation (1). The empirical test on CRRA risk preferences for financial wealth is then based on the insight from this approximated optimal one-to-one relationship. If $R_u(W_{it})$ remains constant when W_{it} changes, so does α_{it}. When testing CRRA preferences, panel data has a major advantage over cross sectional data. That is, one can control for time-invariant unobserved heterogeneity in risk preferences. If there is an inverse relationship between relative risk aversion and the wealth level, this pattern of association can be misinterpreted as DRRA though it could just reflect a group of investors with heterogeneous CRRA's. Specifically, C-P test the econometric specification:

$$\log \alpha_{it} = \beta_0 + \beta_1 \log W_{it} + \beta_2 X_{it} + u_i + v_{it} \tag{2}$$

where α_{it} denotes investor i's risky financial wealth share at time t; W_{it} consists of her riskless and risky financial wealth; X_{it} includes a vector of changes in life-cycle factors or family composition such as age, age squared and family size, and a set of time dummies to capture aggregate shocks to wealth and asset prices; u_i stands for any time invariant individual heterogeneity in preferences that is unobserved, for example: coefficient of constant relative risk aversion; and v_{it} is a random error term uncorrelated with W_{it}, X_{it}, and u_i. The main attention is then placed on its first difference equation:

$$\Delta \log \alpha_{it} = \beta_1 \Delta \log W_{it} + \beta_2 \Delta X_{it} + \Delta v_{it} \tag{3}$$

Provided that the econometric specification using equation (3) is correct, the estimate of β_1 should be zero if the (indirect) utility function for financial wealth is of CRRA power form.

In fact, C-P do not find any significant financial wealth effect in several specifications. These include adding only year dummies, adding both year dummies and changes in life-cycle factors or family size, and using a sample of households who experienced financial wealth changes by at least 25 percent. C-P also investigate a restricted sample which excludes the young because of possible liquidity constraints, the elderly who may exhibit different portfolio behavior, and households with less than 5,000 Euros of wealth or with risky asset share less than 3.5 percent who are more likely to be affected by transaction costs. All of these empirical findings seem to suggest CRRA for financial wealth. In the following however, I argue two potential problems concerning the identification of β_1 using equation (3).

First, when carrying the implication of equation (1) from the one-period model in F-B to a dynamic context using equation (2), consumption decision is not included in X_{it}. Thus in the first-difference equation, change in consumption is not included in ΔX_{it} and is not considered as a critical factor that is related to the log wealth changes and can also affect $\Delta \log \alpha_{it}$. In other words, this causes omitted variable bias when estimating β_1, the elasticity of relative risk aversion for wealth. Suppose that an increase in consumption is associated with a decrease in the share of risky assets. In addition, assume that consumption is a normal good so that an increase in wealth results in a rise in consumption. Then all else equal, suppressing change in consumption in the error term necessarily leads to a downward bias of estimate of β_1 toward zero. In this case, when the estimate of β_1 is not significantly different from zero, CRRA is adopted while DRRA is true.

Second, the effect of past changes in the risky asset share on current changes in wealth, or the reverse causality issue, is ignored. This is important because the choice of the risky portfolio in previous period affects the wealth level in current period, and this feedback effect cannot simply be differenced out. One way to lessen the concern about such a reverse causality is to control for households' financial conditions other than the wealth level in previous period. Reverse causality is different from the inertia of slowly adjusting portfolios in response to wealth changes. For the latter issue, C-P include $\Delta \log W_{i,t-1}$ in some specifications and still do not find statistically significant wealth effects on portfolio allocation.

When net equity in private business is added to the financial wealth, C-P do find some statistically significant and positive estimates of β_1. Note that business equity is in general less liquid and relatively indivisible, it does not satisfy the requirements on the risky financial wealth and hence on relative risk aversion for the A-P wealth measure. C-P interpret this finding either as evidence of DRRA or as an illustration of the business equity puzzle pointed out by Moskowitz and Vissing-Jorgensen (2002). Moreover, when housing equity is also included in the risky wealth measure, the wealth effect on the risky portfolio turns out to be negatively significant. One should be cautious to read this as evidence of IRRA since the risk properties of housing are not clear. Finally, C-P claim that results do not change when a set of variables as proxies for changes in the composition of human and non-human wealth are controlled for as well.

C-P conclude that the hypothesis of CRRA for financial wealth cannot be rejected by the panel data. Based on the assumptions of CRRA preferences and constant market price of risk for all households, they then use equation (1) and the cross sectional distribution of financial risky shares in each round of survey to retrieve the corresponding distribution of CRRA preferences. As I argue in the previous section, the F-B in the small approach is better used to estimate risk aversion for a specific portfolio risk when time interval is sufficiently small to ensure trivial deviations from the initial wealth. This assumption on the portfolio risk is not compatible with the framework studied by C-P where the households are interviewed every two years and the portfolio risk during each two-year period can by no means be treated as a small risk. *It may be better to apply approaches for estimating risk aversion in the large*. In addition, C-P calibrate the expected risk premium at $\mu_r - \rho \approx .04$ and the standard deviation of risky return at $\sigma_r \approx .2$ so that the market price of portfolio risk is calibrated at 1. This market price of risk is much smaller than 2.1, the one calculated using the S&P 500 index as the risky asset and the U.S. treasury bills as the riskless asset during the period of 1890 and 1979. C-P report the median and the mean of relative risk aversion for financial wealth among the population of Italian households at 1.7 and 4.2 respectively.[42]

5.4 Brunnermeier and Nagel (2008)

Recently, Brunnermeier and Nagel (2008) (B-N) examine how households' portfolio allocations change in response to wealth fluctuations, using data from the Panel Study of Income Dynamics (PSID). In the presence of constant habit (or external habit as B-N label) formation at the micro level, wealth changes over time should generate time-varying relative risk aversion for wealth among households and create a positive relationship between changes in the risky asset share and changes in the wealth level over time. This is because habit formation utility implies DRRA for consumption and thus DRRA for wealth if the only source of consumption is wealth. To see this, recall that in a simple discrete portfolio allocation model with an infinite horizon, B-N derive the equilibrium risky asset share of liquid wealth for household i at time t as:

$$\alpha_{it} = \alpha_i^* [1 - \frac{X_i/\rho}{(W_{it} - C_{it})}] \tag{4}$$

where W_{it} and C_{it} are the liquid wealth level and consumption for the household at time t; X_i is the household-specific constant habit; ρ is the riskless rate of return that is assumed to be

[42] C-P do not describe how they obtain the median and the mean of the level of relative risk aversion for financial wealth. They do report the average risky financial asset share in each survey. They also present eight histograms of the distribution of relative risk aversion, each for one single survey.

constant across time periods and identical for all households; α_i^* is the constant risky asset proportion for household i when her DRRA external habit utility and the standard budget constraint are transformed into a CRRA power utility and the corresponding budget constraint.

B-N further assume that α_i^* equals one, borrowing the results from some realistically calibrated models of household portfolio choice with CRRA preferences in the presence of background wealth such as housing wealth and labor income. In other words, equation (4) now becomes:

$$\alpha_{it} = 1 - \frac{X_i/\rho}{(W_{it} - C_{it})}$$

$$= 1 - \exp\,(x_i - w_{it})$$

$$\approx \pi - \theta\,(x_i - w_{it}) \tag{5}$$

where $x_i = \log(X_i/\rho)$, $w_{it} = \log\left(W_{it} - C_{it}\right)$, and the approximate equality comes from the first-order Taylor series approximation with π and θ being constants, and $\theta > 0$. Taking first difference of equation (5), one obtains:

$$\Delta\alpha_{it} = \theta\Delta w_{it} \tag{6}$$

which forms the basis of their empirical tests, with the null hypothesis that the estimate of θ is positive and statistically significant. To be consistent with the notation in equation (6), liquid wealth is measured as the sum of holdings in stocks and mutual funds (liquid risky assets) and holdings in cash-like assets and bonds (liquid riskless assets), subtracting nonmortgage debt such as credit card debt and consumer loans; and liquid risky asset share is defined as the ratio of the liquid risky assets to liquid assets (the sum of liquid risky and riskless assets). B-N subsequently denote financial wealth as the sum of liquid wealth, home equity and equity in private business, and the financial risky asset share as the sum of liquid risky assets, home equity and equity in private business, divided by financial wealth.

In the regressions B-N include a broad set of household characteristics comprising variables that are related to the life cycle or can cause preference shifts, time dummies to eliminate aggregate shocks to asset prices and returns that can create a mechanical wealth effect, and asset composition controls. They also use income growth and inheritance receipt as instruments for the wealth changes to alleviate the concern on measurement errors on wealth. B-N do not find any strong evidence that changes in the proportion of liquid wealth households invest in the risky assets can be explained either by current wealth fluctuations or by past wealth changes, which may instead suggest CRRA for liquid wealth. For their measure of financial wealth, there exists a negative and statistically significant relationship between changes in the financial risky asset share and changes in financial wealth. Housing equity and private business equity have some common features: lack of liquidity, incomplete divisibility but generating stable income flow. However, their risk properties are ambiguous and it is not even clear whether a decision maker assigns these two assets into riskless category or risky category. Therefore, I focus on the findings of CRRA for liquid wealth and its implication for utility functions for consumption. Unlike the critique on C-P that is mainly concerned with the omitted variable bias due to the missing consumption and the reverse causality from past portfolio allocations to current wealth level, the results reported by B-N are less susceptible to these two empirical problems. For the former they use post-consumption liquid wealth, while for the latter they control for financial situation of households in the previous period.

The main argument here is that their measure of liquid wealth and thus its risky asset share in the model do not incorporate uncapitalized future income, though later in the regression analysis they add the labor income/liquid wealth (or financial wealth) ratio interacted with age, as a proxy for human capital wealth. Uncapitalized future income includes but is not limited to labor income, social security benefits and pension funds. The existence of uncapitalized future income can be attributed to at least two factors: a decision maker has enough current wealth for consumption and for investment; or certain financial frictions keep her from fully capitalizing the future income. This type of income is often not included as a part of liquid wealth and its role is not considered when making portfolio decisions in multi-period models. In other words, B-N effectively assume that on average households choose their risky asset share based on current liquid wealth, rather than lifetime liquid wealth. B-N do discuss the diversification effect of labor income as part of background wealth on portfolio allocations, but the purpose is to help simplify the process of deriving the testable equation (6). In sum, B-N fail to integrate uncapitalized future income into their measure of liquid wealth when modeling dynamic portfolio allocation decisions. This negligence creates an ostensible positive relationship in equation (6).

The analysis in chapter 3 finds that after adjusting for the exogenous uncapitalized future income using the same discrete infinite-horizon model, the positive impact of changes in liquid wealth on changes in its risky asset share can be greatly reduced and even become tiny. In particular, the following counterpart of equation (4) is derived:

$$\alpha_{it} = \alpha_i^* [1 - \frac{\frac{X_i}{\rho} - \frac{Y_{i,t+1}}{1+\rho}}{(W_{it} + y_{it} - c_{it})}] \qquad (7)$$

For the second term in the bracket, observe that $\frac{Y_{i,t+1}}{1+\rho}$, the present value of uncapitalized future income, enters the numerator as a large minus term, while y_{it}, income in the current period, becomes a plus term in the denominator. These two factors combined can substantially weaken the positive effect of changes in liquid wealth on the optimal risky asset share. When coupled with some measurement errors, it is not surprising that B-N cannot find any evidence to support the contemporaneous positive relationship expressed in equation (4) or (6), and therefore cannot verify the existence of habit formation at micro level. It also confirms the point by Meyer and Meyer (2005a) that DRRA habit formation utility for consumption can be reconciled with CRRA indirect utility for liquid wealth.

The other major contribution by B-N is to propose the concept of inertia. That is, if capital gains and losses of liquid wealth realize in the risky asset category and households are slow to adjust their portfolios, this can induce a positive contemporaneous relationship between liquid wealth changes and risky asset shares; and if the inflow and outflow of liquid wealth materialize in the riskless asset category and households are also reluctant to rebalance their portfolios, a negative contemporaneous relationship is created. B-N postulate that the costs of paying close attention to the portfolio due to inertia may be more important than the actual transaction costs. Inertia is then used to explain why households in general fail to rebalance their portfolio even after big inflow and outflow in the riskless assets or big capital gains or losses in the risky assets.

B-N test inertia as an alternative explanation of household portfolio allocation and seems to find strong evidence. They do this because there is basically an absence of wealth effects from regressions with rich controls, which cannot explain the seemingly large changes in the liquid risky asset share across time periods that are observed in the data (see their descriptive statistics). B-N do consider the possibility of the systematic underreporting of trades (forgotten trades) in the PSID, but they contend that unless underreporting is extremely common, inertia seems to be a major driver of household portfolio allocation. They also use recent findings of infrequent trade on the asset holdings in one's 401(k) retirement accounts to augment inertia as their explanation. But I would suspect the actual degree of liquidity of one's risky asset position in the retirement accounts because she may have to invest and hold for a long time stocks of the firm she serves for reasons like tax purposes or stability of stock prices.

More importantly, it is possible that the apparent changes in liquid risky asset share at micro level result from the failure to integrate labor income into liquid wealth. Labor income when realized is an important part of current liquid wealth, and the inclusion of labor income has a disproportionate influence on households at lower wealth ranking. It is observed from Table 1 (summary statistics) in B-N that income at the tenth percentile is almost twice as much as liquid wealth at the same percentile, while income at the ninetieth percentile is just one third of liquid wealth level at the ninetieth percentile.[43] Consider the adjustment of including labor income as part of liquid wealth. The inclusion of riskless labor income has two possible effects. It increases liquid wealth level and decreases the magnitude of log liquid wealth fluctuations if labor income is relatively constant over time. It also significantly reduces the risky asset share and its changes

[43] This comparison is useful only when households at certain percentile of income level are the same households at that percentile of liquid wealth level.

over time. The correction may lead to a huge reduction in $\Delta\alpha_{it}$ across households. As a consequence, the need to employ inertia as an account for portfolio allocation can become irrelevant.

5.5 Summary and Discussion

The purpose of this dissertation is to infer the magnitude and the sign of the slope of relative risk aversion for two highly related arguments of utility functions: wealth and consumption, based on the observed portfolio allocation decisions at micro level. Three major conclusions can be drawn from the analysis. First, assuming CRRA for liquid financial wealth is safe given the latest empirical evidence on household wealth allocation decisions over time. Second, in the presence of uncapitalized future income as another source of consumption, the assumption of DRRA for consumption is compatible with such empirical findings. Last but not the least, choosing commonly used functional forms of utility to infer the magnitude of relative risk aversion in the large for financial wealth seems to perform much better than simply inferring the magnitude of relative risk aversion in the small without assuming any specific form of utility function for financial wealth.

Therefore one natural extension of this dissertation is to make the assumption of CRRA and hence DARA for liquid financial wealth as a starting point and study the demand for a risky asset or the demand for insurance on this asset in the presence of *two* risks on liquid financial wealth. One approach to modeling two risks is to assume that both risks cause random gains or losses on the same asset. For instance, Meyer and Meyer (2010) motivate covered losses and excluded losses that an insured asset can incur as two mutually exclusive losses, and use theoretical findings to explain the observed low take-up rate for the insurance coverage. The other approach is more technical and it follows the background wealth literature to assume that the joint probability distribution of the asset risk and the background wealth risk is subject to certain statistical properties. Li (2010) recently provides such an example.

Another extension is to model uncapitalized future labor income as a non-insurable risk or as the outcome of a chosen debt level and examine its effects on puzzling housing market phenomena, for example, the strong correlation between house prices and trading volume. On the theoretical front, Stein (1995) uses a partial equilibrium model with one-shot home trading to study the role of financial constraints, in particular, the down-payment requirement. There are a number of directions to extend the static model by Stein. These include endogenizing the down-payment requirement to reflect debt level, relaxing the assumption that housing stock is divisible, adding market thickness and thus search frictions to the model, making the trading process an intertemporal one, introducing heterogeneity, for example, existing home owners vs. first-time buyers into the owner-occupied housing market and etc.

REFERENCES

REFERENCES

Abel, Andrew B. (1990). Asset prices under habit formation and catching up with the Joneses. American Economic Review (P&P), 80(2), 38-42.

Arrow, Kenneth J. (1965). Aspects of the Theory of Risk Bearing. Yrjö Jahnssonin Säätiö, Helsinki.

Arrow, Kenneth J. (1971). Essays in the Theory of Risk Bearing. Markham. New York.

Barsky, Robert B., Juster, F Thomas., Kimball, Miles S. and Shapiro, Matthew D. (1997). Preference parameters and behavioral heterogeneity: an experimental approach in the health and retirement study. Quarterly Journal of Economics, 112(2), 537-579.

Blake, David. (1996). Efficiency, risk aversion and portfolio insurance: an analysis of financial asset portfolios held by investors in the United Kingdom. Economic Journal, 106(438), 1175-1192.

Brunnermeier, Markus K. and Nagel, Stefan. (2008). Do wealth fluctuations generate time-varying risk aversion? Micro-evidence on individuals' asset allocation. American Economic Review, 98(3), 713-736.

Calvet, Laurent E., Campbell, John Y. and Sodini, Paolo. (2009). Fight or flight? Portfolio rebalancing by individual investors. Quarterly Journal of Economics, 124(1), 301-348.

Campbell, John Y. (2006). Household finance. Journal of Finance, 61(4), 1553-1604.

Campbell, John Y. and Cochrane, John H. (1999). By force of habit: a consumption-based explanation of aggregate stock market behavior. Journal of Political Economy, 107(2), 205-251.

Carroll, Christopher D. and Kimball, Miles S. (1996). On the concavity of the consumption function. Econometrica, 64(4), 981-992.

Cass, David and Stiglitz, Joseph E. (1972). Risk aversion and wealth effects on portfolios with many assets. Review of Economics Studies, 39(3), 331-354.

Chetty, Raj and Szeidl, Adam. (2010). Consumption commitments: a foundation for reference-dependence preferences and habit formation. Working paper.

Chiappori, Pierre-André and Paiella, Monica. Forthcoming. Relative risk aversion is constant: evidence from panel data. Journal of the European Economic Association.

Cohn, Richard A., Lewellen, Wilbur G., Lease, Ronald C. and Schlarbaum, Gary G. (1975). Individual investor risk aversion and investment portfolio composition. Journal of Finance, 30(2), 605-620.

Conniffe, Denis. (2007). The generalised extreme value distribution as utility function. The Economic and Social Review, 38(3), 275-288.

Constantinides, George M. (1990). Habit formation: a resolution of the equity premium puzzle. Journal of Political Economy, 98(3), 519-543.

Dynan, Karen E. (2000). Habit formation in consumer preferences: evidence from panel data. American Economic Review, 90(3), 391-406.

Eeckhoudt, Louis., Meyer, Jack and Ormiston, Michael B. (1997). The interaction between the demands for insurance and insurable assets. Journal of Risk and Uncertainty, 14(1), 25-39.

Fishburn, Peter C. and Porter, R. Burr. (1976). Optimal portfolios with one safe and one risky asset: effects of changes in rate of return and risk. Management Science, 22(10), 1064-1073.

Friend, Irwin and Blume, Marshall E. (1975). The demand for risky assets. American Economic Review, 65(5), 900-922.

Hadar, Josef and Seo, Tae Kun. (1990). The effects of shifts in a return distribution on optimal portfolios. International Economic Review, 31(3), 721-736.

Hlawitschka, Walter. (1994). The empirical nature of Taylor-series approximations to expected utility. American Economic Review, 84(3), 713-719.

Kimball, Miles S. and Mankiw, N. Gregory. (1989). Precautionary saving and the timing of taxes. Journal of Political Economy, 97(4), 863-879.

Kimball, Miles S. (1990). Precautionary saving in the small and in the large. Econometrica, 58(1), 53-73.

Kocherlakota, Narayana R. (1996). The equity premium: it's still a puzzle. Journal of Economic Literature, 34(1), 42-71.

Leland, Hayne E. (1968). Saving and uncertainty: the precautionary demand for saving. Quarterly Journal of Economics, 82(3), 465-473.

Li, Jingyuan. (2010). The demand for a risky asset in the presence of a background risk. Journal of Economic Theory, 146(1), 372-391.

Loistl, Otto. (1976). The erroneous approximation of expected utility by means of a Taylor's series expansion: analytic and computational results. American Economic Review, 66(5), 904-910.

Lupton, Joseph P. (2003). Household portfolio choice and the habit liability: evidence from panel data. Federal Reserve Board working paper.

Mehra, Rajnish and Prescott Edward C. (1985). The equity premium: a puzzle. Journal of Monetary Economics, 15(2), 145-161.

Merton, Robert C. (1969). Lifetime portfolio selection under uncertainty: the continuous-time case. Review of Economics and Statistics, 51(3), 247-257.

Merton, Robert C. (1971). Optimum consumption and portfolio rules in a continuous-time model. Journal of Economic Theory, 3(4), 373-413.

Meyer, Donald J. and Meyer, Jack. (2005a). Risk preferences in multi-period consumption models, the equity premium puzzle, and habit formation utility. Journal of Monetary Economics, 52(8), 1497-1515.

Meyer, Donald J. and Meyer, Jack. (2005b). Relative risk aversion: what do we know? Journal of Risk and Uncertainty, 31(3), 243-262.

Meyer, Donald J. and Meyer, Jack. (2006). Measuring risk aversion. Foundations and Trends in Microeconomics, 2(2), 107-203.

Meyer, Donald J. and Meyer, Jack. (2010). Excluded losses and the demand for insurance. Journal of Risk and Uncertainty, 41(1), 1-18.

Meyer, Jack. (2010). Representing risk preferences in expected utility based decision models. Annals of Operations Research, 176(1), 179-190.

Mirrlees, James A. (1965). Optimum economic policies under uncertainty. Unpublished.

Moskowitz, Tobias J. and Vissing-Jorgensen, Annette. (2002). The returns to entrepreneurial investment: a private equity premium puzzle? American Economics Review, 92(4), 745-778.

Ogaki, Masao and Zhang, Qiang. (2001). Decreasing relative risk aversion and tests of risk sharing. Econometrica, 69(2), 515-526.

Pratt, John W. (1964). Risk aversion in the small and in the large. Econometrica, 32(1/2), 122-136.

Rabin, Matthew and Weizsacker, Georg. (2009). Narrow bracketing and dominated choices. American Economic Review, 99(4), 1508-1543.

Ravina, Enrichetta. (2007). Habit persistence and keeping up with the Jones: evidence from micro data. New York University working paper.

Rothschild, Michael and Stiglitz, Joseph E. (1971). Increasing risk II: its economic consequences. Journal of Economic Theory, 3, 66-84.

Ross, Stephen A. (1975). Uncertainty and the heterogeneous capital good model. Review of Economic Studies, 42(1), 133-146.

Saha, Atanu. (1993). Expo-power utility: a 'flexible' form for absolute and relative risk aversion. American Journal of Agricultural Economics, 75(4), 905-913.

Sahm, Claudia R. (2008). How much does risk tolerance change? Federal Reserve Board working paper.

Samuelson, Paul A. (1969). Lifetime portfolio selection by dynamic stochastic programming. Review of Economics and Statistics, 51(3), 239-246.

Sandmo, Agnar. (1969). Capital risk, consumption, and portfolio choice. Econometrica, 37(4), 586-599.

Sandmo, Agnar. (1970). The effect of uncertainty on saving decisions. Review of Economic Studies, 37(3), 353-360.

Stein, Jeremy C. (1995). Prices and trading volume in the housing market: a model with down-payment effects. Quarterly Journal of Economics, 110(2), 379-406.

Tobin, James. (1958). Liquidity preferences as behavior towards risk. Review of Economic Studies, 25(2), 65-86.

Xie, Danyang. (2000). Power risk aversion utility function. Annals of Economics and Finance, 1, 265-282.