HYDRAULIC DESIGN OF SPRINKLER IRRIGATION SYSTEM COMPONENTS USING THE FINITE ELEMENT METHOD

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ABSTRACT

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The objective of irrigation has been to increase both the quality and quantity of food production by allowing the timely delivery of water to meet crop requirements. To attain maximum utilization of the existing water resources it is necessary to employ irrigation water in the most efficient manner. Proper hydraulic design of irrigation system considering the system components such as elbows, tees, gate valves, sprinklers, and pumps has become the key to irrigation efficiency.

The primary objective of this research was to incorporate the system components in the design of sprinkler irrigation networks. The procedure was based on the solution of the nonlinear algebraic equations derived from the finite element formulation of the system components. The secondary research objective was to assess the applicability of the finite element formulation employed in the design of sprinkler irrigation systems by comparing it to a linear theory model.

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LIST OF SYMBOLS

a	a constant in the friction loss equation
A	total cross-sectional area of the sprinkler nozzel
c	pump coefficient
C	pipe roughness coefficient
C_p	linearized pipe constant
C_{pc}	linearized component constant
C_{pp}	linearized pump constant
C_{sp}	linearized sprinkler constant
C_q	sprinkler discharge coefficient
dh	pressure distribution with respect to length
dH	slope of the energy line
$\Delta x_{(n)}$	correction to the x variable
dz	slope of the ground
D	pipe diameter
D	differential equation constant
D^{-1}	inverse of the Jacobian matrix, D
e	tolerance value
ΔH	total friction loss
ΔH_L	friction loss in the lateral line
ΔH_s	friction loss in the submain
$\Delta Q(n+1)$	vector of newly extimated corrective flows
$\Delta Q(n)$	vector of previously estimated corrective flows

Δy	absolute value of the mean deviation of irrigation depth
f	dimensionless friction factor
$\{f\}$	force vector
$f^{(e)}$	element contribution to the force vector
F	vector of the function of x
F	friction loss correction factor
g	acceleration of gravity
γ	specific weight of water
ρ	density of water
h_c	component head loss
h_f	head loss
h_i	head for a given length ratio
H	total energy ($P/\gamma + z$)
H_{max}	maximum lateral line pressure
H_{min}	minimum lateral line pressure
H_{var}	lateral line pressure variation
H_p	pump head
$H1$	cut off head
ΔH_i	pressure drop at length ratio i
H_o	fixed grade node
H_i	junction pressure at the upstream node
H_j	junction pressure at the downstream node
i	length ratio ℓ/L
k	constant of proportionality for the sprinkler discharge equation
$k^{(e)}$	element contribution to the stiffness matrix
$[K]$	stiffness matrix

K	pipe line constant which is a function of line length, diameter and pipe material
K'	modified pipe line constant
K_c	component coefficient
K_{sp}	sprinkler constant
L	length of pipe
ℓ	length of pipe measured at desired location
m	empirical head loss exponent
m	pump coefficient
N	number of sprinklers
n	iteration number
n	shape function
π	ratio of the circumference of a circle to its diameter
p	static pressure head
ϕ	unknown pressure
$\{\Phi\}$	vector of unknown pressures
q	sprinkler discharge
q_i	discharge at a given length ratio i
q_o	original discharge
Q	flow rate
$\Delta Q_{(n)}$	flow correction
$Q_{(i-1)}$	flow rate from the previous trial
$Q_{(i-2)}$	flow rate from the trial previous to Q(n-1)
$R^{(e)}$	vector of residual equations
R_i	energy drop ratio
$R_i \Delta H$	head loss due to friction
$R_i \Delta H'$	head loss or gain due to slope

S_f energy line

U_c Christiansen uniformity coefficient

U_s statistical uniformity coefficient

V velocity

V^2 velocity head

V_y coefficient of variation for the depth of irrigation water

W weighting function

W friction loss, including minor losses as equivalent length

$x_{(n+1)}$ vector of new extimates of variable, x

$x_{(n)}$ vector of first estimates of variable, x

y mean depth of irrigation

z elevation head

$z_{(n)}$ solution vector of the linear system

I. INTRODUCTION

Irrigation technology has been known and practiced for at least 5,000 years. It is an important agricultural technology, one that has facilitated the development of many civilizations, particularly in arid regions. The primary benefits of irrigation have been to increase both the quantity and quality of food production by allowing the timely delivery of water to meet crop requirements.

It is estimated that the total irrigated area in the world has increased from 95 million hectares in 1950 to approximately 250 million hectares in 1986 (Power, 1986). While irrigated areas only comprise 20 percent of all harvested land, it accounts for 40 percent of total crop output worldwide.

Many different irrigation technologies are employed throughout the world. These include surface, subsurface, sprinkler and drip irrigation. Surface irrigation is the most commonly practiced irrigation method worldwide. However, since World War II, more efficient forms of irrigation such as drip and sprinkler irrigation have increased in popularity. This is primarily due to the introduction of aluminum, galvanized steel and plastic pipes in Europe and the United States. Sprinkler irrigation and, to a greater extent, drip irrigation apply water in very controlled and precise amounts. Therefore, these methods ultimately provide a higher water use efficiency and greater uniformity than surface irrigation. In the United States sprinkler and drip irrigation have expanded to 9 million hectares in the last two decades and presently account for over 35 percent of the irrigated lands (Irrigation Journal, 1986).

In Michigan, sprinkler and drip irrigation account for over 95 percent of the land under irrigation or approximately 175,000 hectares (Irrigation Journal, 1986).

Although Michigan is considered quite humid, the majority of the measured precipitation occurs with winter snow. Short term summer droughts are common, thus irrigation can eliminate the most costly and unpredictable farming hazard; insufficient moisture at critical growth periods. In addition, sprinkler irrigation systems in humid areas such as Michigan have been designed and operated for frost and freeze protection, blossom delay, and in some cases crop cooling (Jensen, 1980).

A common sprinkler irrigation arrangement is a solid set system. These systems may be installed on the surface of the ground using portable aluminum pipe and are usually left in place for the entire growing season.

In a solid set system as shown in Figure 1, the water flows sequentially through the following components: the water supply and pump set, main line, lateral lines, riser assemblies, and sprinkler heads. In addition, elbows, tees, gate valves, and pressure regulating valves form part of the system.

Because expanding needs of industry along with increasing population are making great demands on the limited water supply, conservation of agricultural water resources has become a worldwide priority. To attain maximum utilization of the existing water resources it is necessary to employ irrigation water in the most efficient manner. Proper design of irrigation systems considering pressure losses through irrigation components such as elbows, tees, gate valves, and sprinklers, as well as along laterals, is the key to irrigation efficiency.

A. SCOPE AND OBJECTIVES

The ultimate purpose of irrigation design is to determine the optimal size and arrangement of the system components (pipes, pumps, nozzles, etc.) so that the crop water requirements can be met without violating constraints on water, energy, and investment.

Sprinkler and drip irrigation system design procedures have been presented by several researchers:

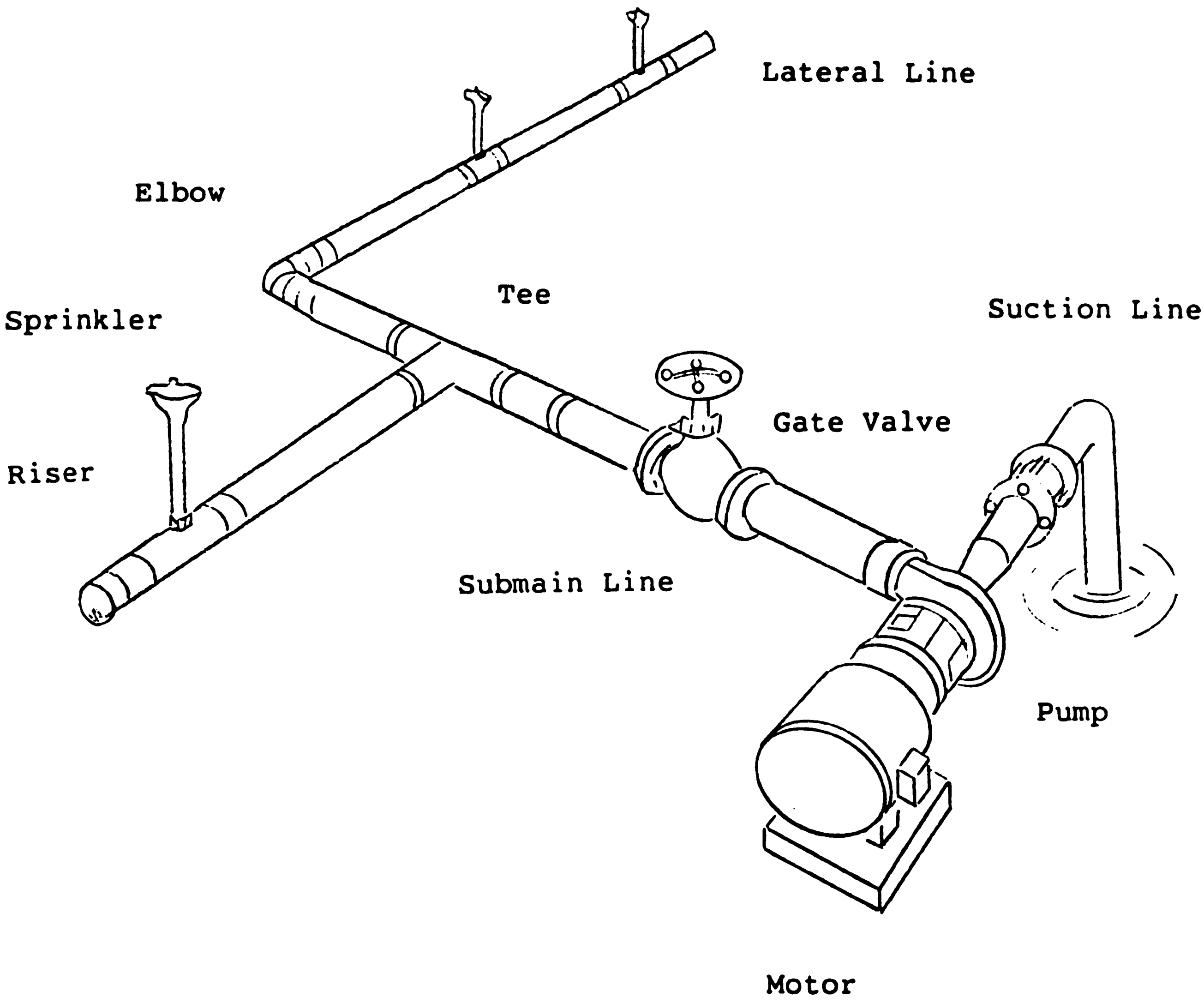

Figure 1. Sprinkler irrigation system and components (Schwab et al., 1981)

Howell and Hiler, 1974; Keller and Karmeli, 1974; Wu and Gitlin, 1974, 1983; Pair, 1975; Perold, 1977; Wood, 1980; Jensen, 1982; and Bralts and Segerlind, 1985.

Lateral length and diameter, orifice spacing, slope, pressure, flow rate, and uniformity are important parameters which have been considered in these design processes. Until now, in considering pressure losses from the water source to the outlet, only the losses occurring in the sprinkler or drip lateral were considered important.

Wood (1980), Howell and Barinas (1980), Finkel (1982) and Haghighi et al. (1987), suggested that, in addition to pressure losses in laterals, the pressure losses across hydraulic components such as elbows, tees and valves in a network of pipes should be considered in the system design procedure.

In most of the literature the pressure losses across hydraulic components have been termed "minor losses", but since a large number of these components may be present in a single sprinkler irrigation system, these minor losses can become significant and alter considerably the hydraulic design. Thus, a pronounced need has arisen for the inclusion of tees, elbows and valves in the irrigation design process.

The design of sprinkler irrigation systems is an intensive and complicated procedure which involves a large number of calculations. The use of microcomputer based iterative techniques removes much of the tedious work associated with these calculations. This results in a more detailed analysis with fewer errors when compared with conventional techniques.

The use of microcomputer based iterative techniques in the design of network irrigation systems has been growing steadily. Edwards and Spencer (1972) presented a computer based design procedure for the analysis of sprinkler irrigation systems utilizing the Hardy Cross method. Other investigators that have employed iterative techniques to solve for flow rates and pressures in drip irrigation systems, based on assumed end line pressures, were Solomon and Keller (1974), Wu and Fangmeier (1974), Howell and Hiler (1974), and Perold (1977). Wood (1980) included sprinkler irrigation design in his hydraulic network analysis using linear theory method.

Bralts and Segerlind (1985) developed an interactive microcomputer program (DESIGNER) which uses the finite element method to assist in the analysis and design of drip irrigation submain units. DESIGNER is based on the iterative solution of a set of linearized flow equations and the result is a symmetric banded matrix which requires minimal computer storage. At this point in time the DESIGNER program only evaluates pressure losses from lateral lines and does not include pumps and other essential components of sprinkler irrigation systems.

The purpose of this study is to incorporate system components such as tees, elbows, gate valves, and pumps in the design of sprinkler irrigation systems. The procedure is based on the solution of nonlinear algebraic equations employing the finite element method.

The specific objectives of this research are :

1. To develop a finite element formulation of essential sprinkler irrigation components such as elbows, tees, gate valves, sprinklers, and pumps.

2. To demonstrate the potential of applying the finite element formulation to the design of sprinkler irrigation systems by comparing it to a linear theory model.

II. REVIEW OF LITERATURE AND THEORY

In order to understand the vadility of the proposed design analysis it is necessary to present a review of important literature and theory. The topics to be developed in this section are a) basic hydraulics of sprinkler irrigation, b) methods for the analysis of pipe networks, and c) analysis of hydraulic components.

A. Basic Hydraulics of Sprinkler Irrigation

A sprinkler irrigation system is composed of a series of pipes sections and of a number of hydraulic components such as tees, elbows, valves, and pumps. The pipes convey the water through the various components and supply it to the sprinklers at the correct pressure head. A diagram of a sprinkler irrigation system is presented in Figure 1.

An important element in sprinkler irrigation design procedure is the determination of pressure losses occurring in the sprinkler lateral. This pressure loss occurs when a fluid is forced through a pipe section increasing friction and causing energy to be consumed. This results in a drop in pressure across the pipe section.

1. Governing Equations

The analysis of fluid flow through networks of pipes is based on Newton's laws of conservation of mass and energy.

Conservation of mass or Continuity states:

$$\Sigma Q = 0 \qquad [1]$$

or that the flow in a network system must be balanced at every junction (node). In essence, the flow into a node must equal the flow out.

Conservation of energy states:

$$\Sigma H = 0 \qquad [2]$$

or that the algebraic sum of the head losses around any closed network must be zero.

Water flowing in a section of pipe contains three basic forms of energy: energy due to elevation, due to pressure, and due to its motion. Jeppson (1982) presents these three forms of energy in equation form as:

$$E = gZ + \frac{P}{\rho} + \frac{V^2}{2} \qquad [3]$$

where E = average kinetic energy per unit mass in Newton-meter per kilogram,

g = acceleration due to gravity in meters per squared second,

Z = elevation above a reference plane in meter,

P = pressure in the pipe in Newton per squared meter, and

ρ = density of the fluid in kilograms per cubic meter.

Equation [3] is also known as the mechanical energy equation. When this equation is applied between two points i and j in a pipe section, a mechanical energy balance can be formed. This is shown in Figure 2 and in equation form

$$g\ Z_i + \frac{P}{\rho} + E_i + W = g\ Z_j + \frac{P}{\rho} + E_j + E_L \qquad [4]$$

where W = work input to the network in Newton per meter per kilogram, and

E = energy loss per unit mass due to friction and minor losses in Newton per meter per kilogram.

For turbulent flow, the velocity profile is negligible and the kinetic energy is simply

$$E = \frac{V^2}{2} \qquad [5]$$

Writing the mechanical energy balance equation, [4], in terms of energy per unit weight and incorporating equation, [5], we find

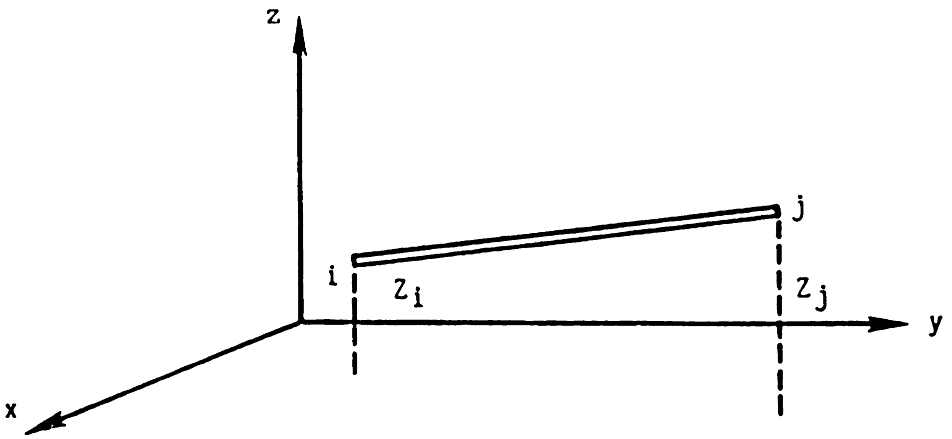

Figure 2. Pipe section (Bralts and Segerlind, 1985)

$$Z_i + H_i + \frac{V^2}{2g} + h_m = Z_j + H_j + \frac{V^2}{2g} + h_f \qquad [6]$$

where $H = \frac{P}{\gamma}$ is the pressure head; γ is the specific weight,

h_m = W/g is the head gain to the network due to external mechanical energy supply, and

h_f = E_L/g is the total head loss due to pipe friction and/or network components.

Equations [1] through [6] form the basis for the analysis of water flow through sprinkler irrigation pipes.

2. Hydraulics of Sprinkler Flow

The flow of water through an irrigation sprinkler follows the same behavior as flow through any orifice. At the nozzle of the sprinkler the pressure head of the water is converted to a velocity head. The water flows out of the nozzle in the form of a jet and breaks down into drops of water which wet the area within a certain diameter.

Pair (1975) and Finkel (1982) have shown that the discharge of a sprinkler depends on the combination of the inlet pressure head and the nozzle cross-sectional area. In equation form

$$Q = CqA\sqrt{2gh} \qquad [7]$$

where Q = discharge in Liters per second,

Cq = discharge coefficient, describing the ratio of actual to theoretical discharge capacities,

A = total cross-sectional area of sprinkler nozzle in squared meters,

h = inlet pressure head in meters, and

g = acceleration due to gravity in meters per squared second.

The discharge coefficient for small nozzles commonly varies between 0.95 and 0.98. Some large diameter nozzles have discharge coefficients as low as 0.80. Normally the larger the nozzle, the lower the coefficient (Schwab et al., 1981).

3. Lateral Line Hydraulics

To characterize the energy losses occurring in sprinkler irrigation laterals, the Darcy-Weisbach and the Hazen-Williams models will be presented, (Keller and Karmeli, 1975; Pair, 1975; Finkel, 1982; Jeppson, 1982).

The Darcy-Weisbach equation is

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad [8]$$

where h_f = head loss due to friction in meters,

f = friction factor,

L = length of pipe in meters,

V = velocity on pipe in meters per second

D = diameter of pipe in meters, and

g = acceleration due to gravity in meters per square seconds.

The second equation used to evaluate the friction head loss is the Hazen-Williams equation, which takes the following form

$$h_f = 10.7 \frac{Q^{1.852}}{C^{1.852}\, D^{4.871}} L \qquad [9]$$

where Q = pipe flow rate in cubic meters per second,

C = pipe roughness coefficient, and all other variables are as previously defined.

Jeppson (1982) found it useful, when analyzing the flow distribution in large pipe networks, to express the frictional head losses by the exponential formulation

$$h_f = K\ Q^m \qquad [10]$$

where h_f = head loss per unit length of pipe in meters,

K = pipe coefficient,

Q = discharge of pipe in cubic meters per second,

m = empirical head loss exponent which has a value of 2 when the Darcy-Weisbach equation is used and 1.852 when the Hazen-Williams equation is used.

The pipe coefficient, K, in equation [10] is a function of pipe roughness, Reynolds Number, length of pipe, and diameter of pipe.

If the Darcy-Weisbach equation is substituted in equation [10], then the pipe coefficient K becomes

$$K = \frac{8f\ L}{g\ \pi^2\ D^5} \qquad [11]$$

If the Hazen-Williams equation is substituted into equation [10], then the pipe coefficient K becomes

$$K = \frac{10.7}{C^{1.852}\ D^{4.871}}\ L \qquad [12]$$

The flow of water in a sprinkler lateral with a number of equally spaced sprinkler outlets will have less friction loss for a given diameter and length of pipe than if the flow was constant for the entire length (Wu and Gitlin, 1975; Pair, 1975). To accurately compute the friction loss in multiple outlet laterals it is necessary to start at the last outlet on the line and work back to the supply line computing the friction loss between each outlet.

Christiansen (1942) simplified this tedious process by introducing an adjustment factor F to correct the friction loss assuming that all the water is carried to the end of the line and that the first sprinkler is one sprinkler riser spacing from the beginning of the lateral. Employing the Hazen-Williams model for friction losses, equation [9].

$$h_f = F\ 10.7\ \frac{Q^{1.852}}{C^{1.852}\ D^{4.871}}\ L \qquad [13]$$

The F value for a finite number of outlets (sprinklers) can be determined by,

$$F = \frac{1}{N^{2.852}} \sum_{1}^{N} n^{1.852} \qquad [14]$$

where N = number of outlets (sprinklers) on the lateral line and

n = number from 1 to N

Jensen and Fratini (1957) modified the above expression for F values to account for the first sprinkler being located one half the sprinkler spacing from the supply line. In equation form,

$$F = \frac{2}{2N - 1} \left(1/2 + \frac{1}{N^{1.852}} \sum_{1}^{N-1} n^{1.852} \right) \qquad [15]$$

Wu and Gitlin (1983) calculated F values for both cases, equations [14] and [15]. Figure 3 shows the F values against the total number of outlets for both cases. Case I for placing the first sprinkler one head spacing (equation [14]) and case II for placing the first sprinkler one half head spacing (equation [15]). The shape of the curves in Figure 3 shows that the change of F values with respect to the number of outlets is relatively large for a small number of sprinklers (less than 10). When the number of sprinklers is 10 or more the change in F values is very small. It is also showed that the difference between cases I and II is insignificant when the number of sprinklers is larger than 10.

It can also be seen in Figure 3 that when the number of sprinklers is from 5 to 50 for both cases, the F values ranges from 0.36 to 0.40. Since this is practically the range of number of sprinklers for most designs, an average F value of 0.38 can be used in equation [9] for calculating total friction drop without causing any significant error.

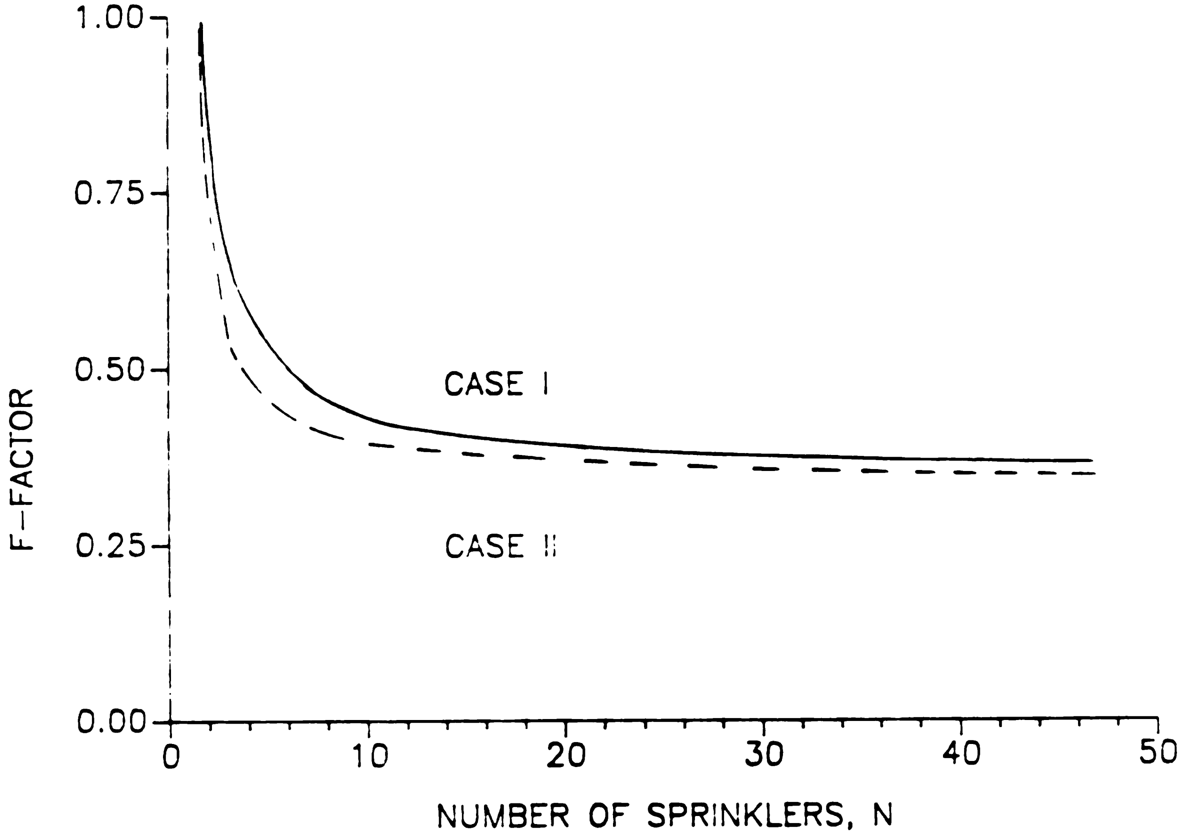

Figure 3. Relationship between F-factor and number of sprinklers on a lateral line for cases I and II (Wu and Gitlin, 1983).

Equation [9] is used to calculate only the total friction drop at the end of the lateral line or submain unit. However, in order to determine the friction drop along the line Myers and Bucks (1972) and Wu and Gitlin (1973) developed an approach to solve for the decreasing flow rate in drip irrigation lateral lines with respect to the length of the line. Bralts (1983) presented a complete development of this approach for the design of drip irrigation submain units. Parts of his work will be presented here, but will be applied to the design of sprinkler irrigation lateral lines.

As the total discharge decreases with respect to the length of the line, the energy gradient will not be a straight line but an exponential curve. Assuming an infinite number of outlets and uniform flow the shape of the energy gradient line can be expressed by a dimensionless gradient pressure as derived by Wu and Gitlin (1975)

$$R_i = \frac{\Delta H_i}{\Delta H} = 1 - (1 - i)^{m+1} \qquad [16]$$

where R_i = pressure drop ratio in meters per meters,

ΔH_i = pressure drop in meters at length ratio i,

ΔH = total pressure due to friction in meters,

i = length ratio, (L/ℓ) in meters per meters,

L = total length of the line in meters,

ℓ = given length measured from the head end of the line in meters, and

m = exponent of the flow rate.

Figure 4, defines the variables of equation [16].

When the Hazen-Williams equation is used to compute the friction loss, the dimensionless energy gradient line equation [16] can be expressed as:

$$R_i = 1 - (1 - i)^{2.852} \qquad [17]$$

The usefulness of the dimensionless energy gradient line resides in its ability to determine the head loss at any point along the lateral line with respect to the original total energy head. The total pressure drop for a sprinkler irrigation lateral line can be determined by using the total discharge as shown by Wu and Gitlin (1975).

$$\Delta H = \frac{-a\ Q^m}{m+1} L \qquad [18]$$

A commonly used form of equation [18] is the modified form of the Hazen-Williams equation, equation [9], where the exponent of the flow rate, m, is equal to 1.852 and the roughness coefficient, C, is equal to 150 for smooth pipes. The resulting equation is

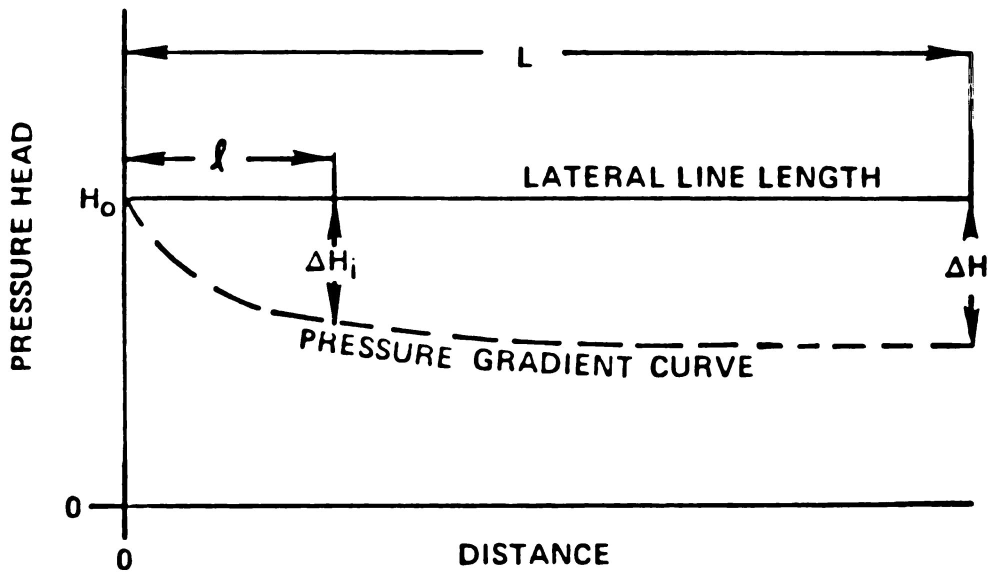

Figure 4. Lateral line notation and dimensionless gradient line variables (Bralts, 1983).

$$H = 3.50\times10^{-4} \frac{Q^{1.852}}{D^{4.871}} L \qquad [19]$$

where all the variables are as previously defined.

Wu and Gitlin (1983) and Bralts (1983) showed that the pressure variation along a sprinkler irrigation line not only contains the head loss due to friction but also must include the head loss or gain due to slope. The total energy at any section of a lateral line can be expressed from the energy equation as

$$H = Z + h + \frac{V^2}{2g} \qquad [20]$$

The loss or gain in pressure is linearly proportional to the slope and length of the line and can be shown as follows

$$\frac{dH}{dL} = \frac{dZ}{dL} + \frac{dh}{dL} + \frac{d\left(\frac{V^2}{2g}\right)}{dL} \qquad [21]$$

where $\frac{dH}{dL}$ = slope of the energy line (-Sf),

$\frac{dZ}{dL}$ = slope of the lateral line (-So),

$\frac{dh}{dL}$ = pressure distribution with respect to length, and

$$\frac{d\left(\frac{V^2}{2g}\right)}{dL}$$ = change of velocity head with respect to length.

The change of velocity head with respect to the length along the a lateral line can be considered small and can be neglected therefore the energy equation can be reduced to

$$\frac{dH}{dL} = \frac{dZ}{dL} + \frac{dh}{dL} \qquad [22]$$

rearranging terms, the pressure distribution along a lateral line was shown by Wu and Gitlin (1974) to be

$$\frac{dh}{dL} = So - Sf$$ for downslope conditions and

$$\frac{dh}{dL} = -So - Sf$$ for upslope conditions

Therefore, the pressure distribution along a lateral line is a linear combination of the line slope and the energy slope.

Wu, et al. (1979) expressed the total pressure variation as a function of the original pressure and to the variation due to energy slope and line slope. In equation form

$$h_i = H_o - R_i \Delta H + R_i' \Delta H' \qquad [23]$$

where h_i = head for a given length ratio in meters

H_o = original pressure head in meters

$R_i \Delta H$ = head loss due to friction in meters, and

$R_i' \Delta H'$ = head loss or gain due to slope in meters

Figures 5 and 6 show the pressure variation along a lateral line for downslope and upslope conditions respectively.

As shown in equation [7], the sprinkler flow is directly proportional to the pressure head therefore the sprinkler flow rate at any point along the lateral line will be equal to a combination of equations [7] and [23] or

$$q_i = K(h_i)^{0.5} = K(H_o - R_i \Delta H + R_i' \Delta H')^{0.5} \qquad [24]$$

If equation [24] is divided by the sprinkler flow equation for the first sprinkler qo, (qo = K $h^{0.5}$) then the resulting equation becomes independent of the coefficient K and takes the following form

$$q_i = q_o \left(1 - R_i \frac{\Delta H}{H_o} + R_i' \frac{\Delta H'}{H_o}\right)^{0.5} \qquad [25]$$

Equation [25] serves to determine the sprinkler flow at any point along the lateral line once the sprinkler flow rate at the original pressure is known.

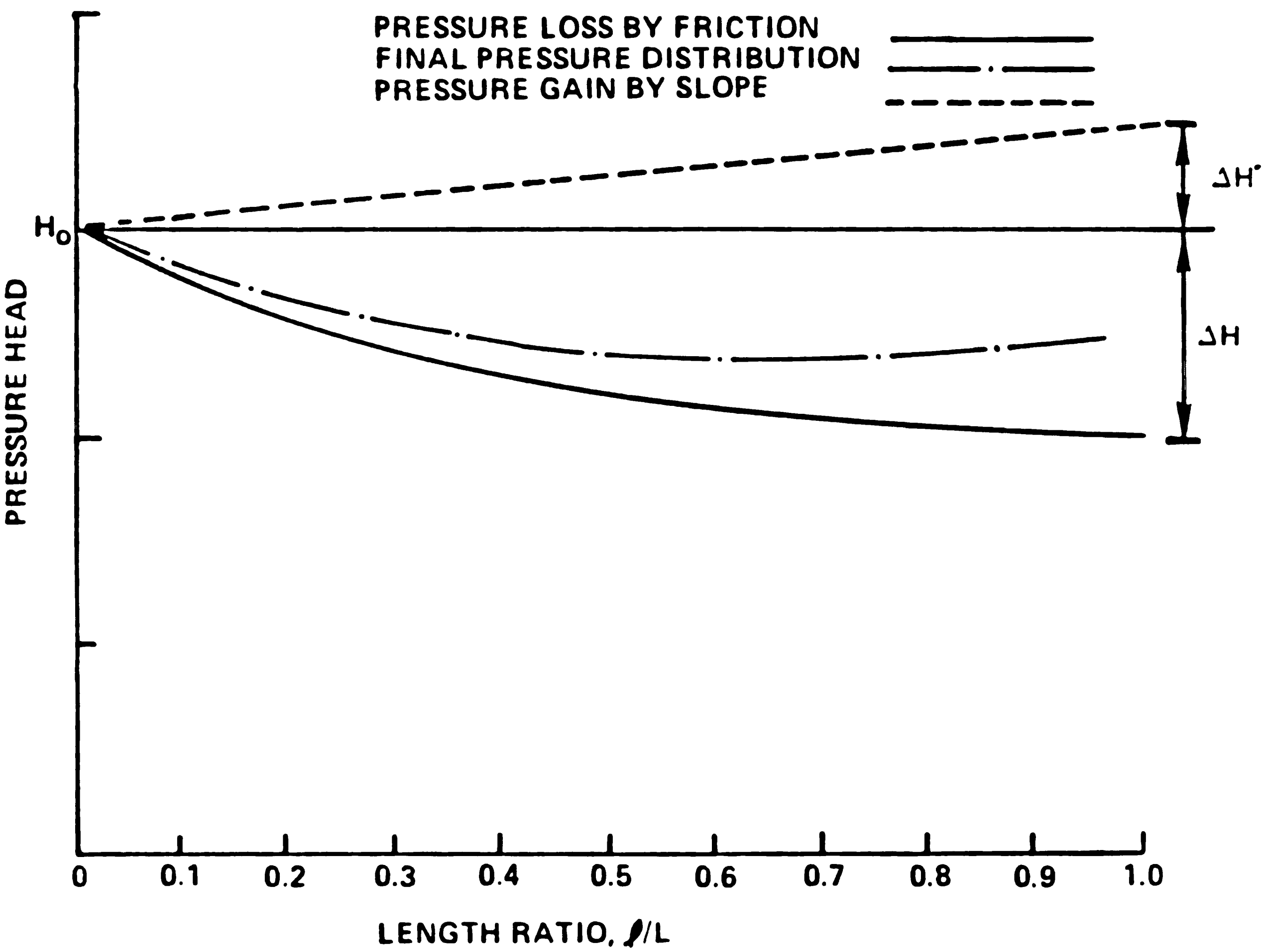

Figure 5. Pressure distribution for downslope conditions (Wu et al., 1979).

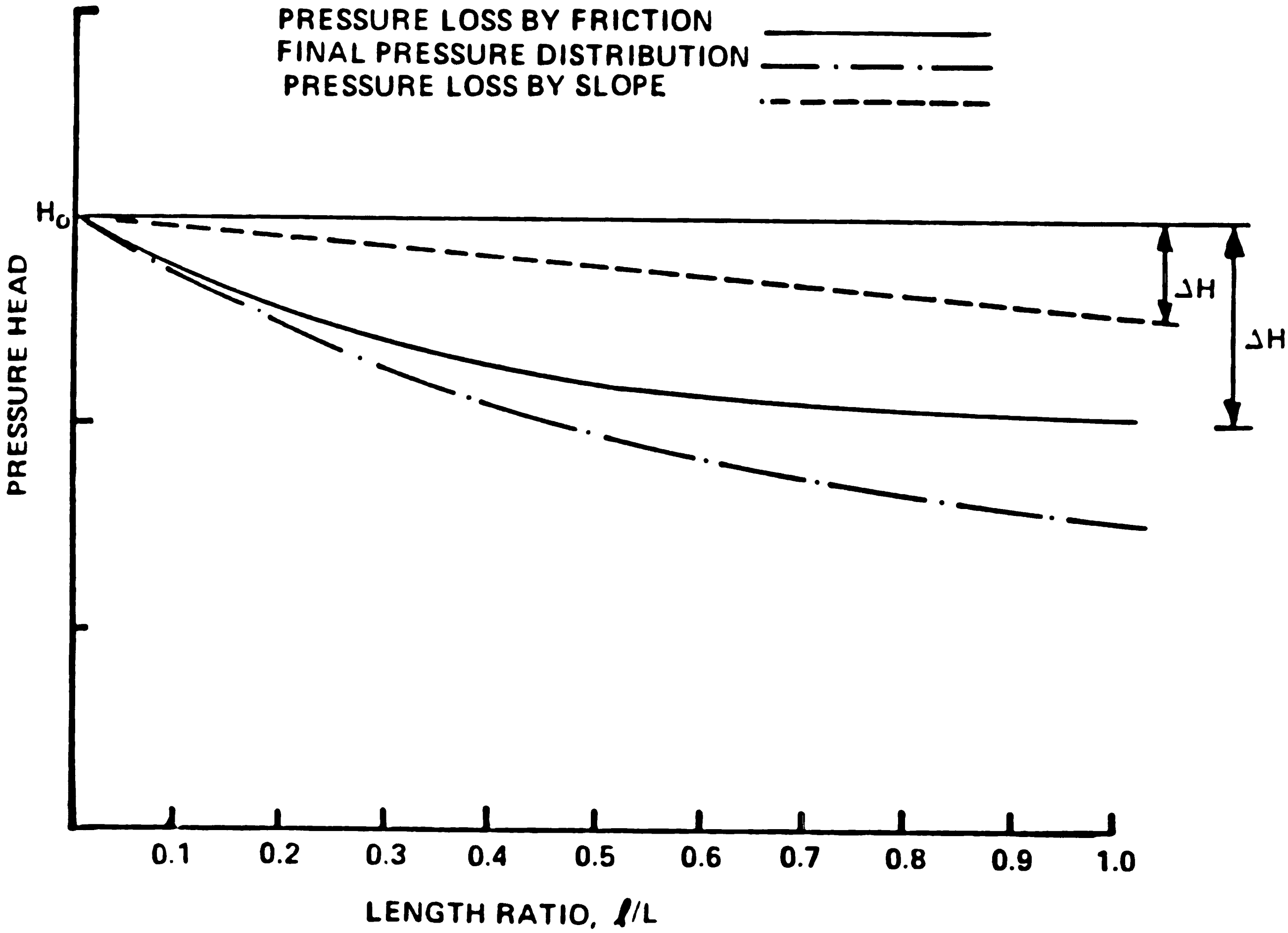

Figure 6. Pressure distribution for upslope conditions (Wu et al., 1979).

4. Design Criteria

The purpose of the sprinkler system is to uniformly distribute the water to the soil surface of the irrigated area. The uniformity coefficient, proposed by Christiansen (1942) is, in general, considered as one of the best design criteria available for sprinkler irrigation design.

Christiansen defined sprinkler irrigation uniformity by the equation

$$U_C = 100 \left(1 - \frac{\Delta y}{y} \right) \qquad [26]$$

where U_C = uniformity coefficient in a percentage basis,

Δy = absolute deviation from the mean of the individual observations, and

y = mean depth of observation.

Most recently, the uniformity of sprinkler irrigation systems has been evaluated employing the statistical uniformity coefficient presented by Wilcox and Swailes (1947). The advantage of this formulation is the use of common statistical parameters such as the coefficient of variation (V_y), which is basically the standard deviation over the mean. In equation form

$$U_S = 100\ (1 - V_y) \qquad [27]$$

where U_s = statistical uniformity coefficient and

V_y = coefficient of variation for the depth of irrigation water, y.

Edwards and Spencer (1972), Pair (1975), and Wu and Gitlin (1983) reported that in order to achieve high uniformity system performance it was essential that the pressure variation along a lateral line be held to 20 % so the discharge variation from all the sprinklers along the lateral line would be maintained to be equal or less than 10 %.

Wu and Gitlin (1983) expressed the pressure variation along a lateral line as

$$H_{var} = \frac{H_{max} - H_{min}}{H_{max}} 100 \qquad [28]$$

where H_{var} = pressure variation expressed as a percentage,

H_{max} = maximum pressure expressed in meters of head, and

H_{min} = minimum pressure expressed in meters of head

The pressure variation H_{var}, can be determined from the maximum and minimum pressures which can be obtained from the pressure profile using equation [23].

In summary, this section has presented the equations that govern the pressure losses due to friction of the pipes and slope of the ground in sprinkler irrigation laterals. The dimensionless gradient line concept allows for the determination of the pressure or flow rate at any point along the lateral line with respect to the original total pressure or flow rate.

B. Methods for the Analysis of Pipe Networks

Numerical methods have been employed in the solution of a wide range of engineering problems. The combined efforts of numerical methods and microcomputers have enhanced the possibility of solving complicated problems which require iterative procedures.

Hydraulic design of sprinkler irrigation systems include the solution of nonlinear algebraic equations, which by their nature, must be solved in an iterative manner. Several algorithms have been proposed for solving the nonlinear equations. These include the Hardy Cross method (Cross, 1936), the Newton-Raphson method, first used in pipe flow analysis by Martin and Peters (1963), and the Linear Theory method (Wood and Charles, 1972). More recently, the finite element method has been proposed by Bralts and Segerlind (1985) and Haghighi et al. (1987) as an appropriate technique for the analysis of networks of irrigation pipes. Each of these methods will be reviewed in this section.

1. Hardy Cross Method

The Hardy Cross method (Cross, 1936) was one of the first methods developed for the solution of pipe networks. This method continues to be very popular and is used today by many practicing engineers. The Hardy Cross method is a simplified version of a method of successive approximations applied to a set of linearized equations (Wiggert, 1986). The method employs a flow corrective technique which uses assumed flow rates, based on continuity, to solve the energy based loop equations of the network system.

Applying the continuity principle for each junction, equation [1], becomes

$$\Sigma Q = \Sigma Q_{in} = \Sigma Q_{out} = 0 \qquad [29]$$

The energy equation in the exponential formulation of equation [10] in terms of the unknown flow rates becomes

$$W = R\ Q^{m} \qquad [30]$$

where W = friction loss for any pipe, including minor losses as equivalent length,

R = Pipe coefficient function of pipe roughness, Reynolds Number, length of pipe, diameter, and all the other variables are as previously defined.

The resulting energy equations are nonlinear and to obtain a solution require the implementation of a linearization process, employing a Taylor series expansion. An example of a Taylor series expansion is presented in Appendix A.

After this process the equations become

$$R_i \, (Q_{oi})^n + nR_i \, (Q_{oi})^{n-1} \, (Q_i - Q_{oi}) = 0 \qquad [31]$$

where Q_i = assumed estimate of discharge and

Q_{oi} = estimate of discharge from previous iteration.

The flow correction is introduced as ΔQ

$$\Delta Q = Q_i - Q_{oi} \qquad [32]$$

Substituing equation [32] into equation [31] results in the following correction term

$$\Delta Q = \frac{- \; R_i \;\; Q_{oi} \;\; n-1 \;\; Q_{oi}}{nR_i \;\; Q_{oi} \;\; n-1} \qquad [33]$$

The deviation from zero of the assumed flow rate is used to determine the correction value. When the correction value is applied to the assumed flow rate, a better approximation of the true flow is obtained. The successive iterations are carried out until the corrective flows are within a specified tolerance.

A possible criterion for convergence could be

$$\frac{Q_i - Q_{oi}}{Q_i} < e \qquad [34]$$

where e = tolerance value.

A second iterative method suggested by Cross (1936) and later developed by Cornish (1939), called the method of balancing heads or nodal method, was described by Barlow and Markland (1969) and by Chenoweth and Crawford (1974). This method solves the nonlinear equations by adjusting the heads at each node until the specified outflows or continuity is obtained. Equation [29] becomes

$$\Sigma Q_{in} - \Sigma Q_{out} = 0 \qquad [35]$$

In terms of the head loss

$$\Sigma\left(\frac{W}{R}\right)_{in}^{1/m} - \Sigma\left(\frac{W}{R}\right)_{out}^{1/m} = 0 \qquad [36]$$

The method assumes a set of heads which is successively corrected at each node. The correction term then becomes

$$\Delta H = \frac{\Sigma\left(\frac{W}{R}\right)^{1/m}}{\Sigma \quad \frac{1}{m}\left(\frac{W}{R}\right)^{1/m - 1}} \qquad [37]$$

The major disadvantage of the Hardy Cross method is the slow convergence characteristic and in some cases no convergence at all. This is due to the fact that it neglects the change in flow at each node, caused by iterative head corrections at all connected nodes, except for the node at which the flow is being calculated. Several measures, such as the ones suggested by Barlow and Markland (1969), Dilligham (1967), and McCormick and Bellamy (1968), have been implemented for improving the convergence characteristics of this method. Another drawback of the Hardy Cross method is that a reasonably good assumption of the flow distribution in the network is necessary for fast convergence. Epp and Fowler (1970), and Jeppson (1982) reported that the Hardy Cross method, either loop or node oriented, is best used for relatively small, simple networks.

2. Newton-Raphson Method

The Newton-Raphson method is an iterative procedure which starts with an estimate of the solution and repeatedly computes better estimates. The Newton-Raphson method is illustrated for a one dimensional case as shown in Figure 7. The goal of this method is to find a solution to an equation of the form F(x) = 0. The procedure starts with an initial estimate which is not too far from the solution, Xm, then, extrapolates along the tangent to its intersection with the X-axis, and takes that as the next approximation. This is continued until either the successive X-values are sufficiently close, or the value of the function is near zero. In equation form

$$X_{(n+1)} = X_{(n)} - \frac{F(X_{(n)})}{\dfrac{df(X_{(n)})}{dx}} \qquad [38]$$

where $X_{(n+1)}$ = new estimate to the solution,

$X_{(n)}$ = initial estimate to the solution, and

$\dfrac{F(X_{(n)})}{\dfrac{df(X_{(n)})}{dx}}$ = correction term

The application of the Newton-Raphson method to solve simultaneous nonlinear algebraic equations presented by Jeppson (1982) is similar to the one dimensional case described above with the following considerations.

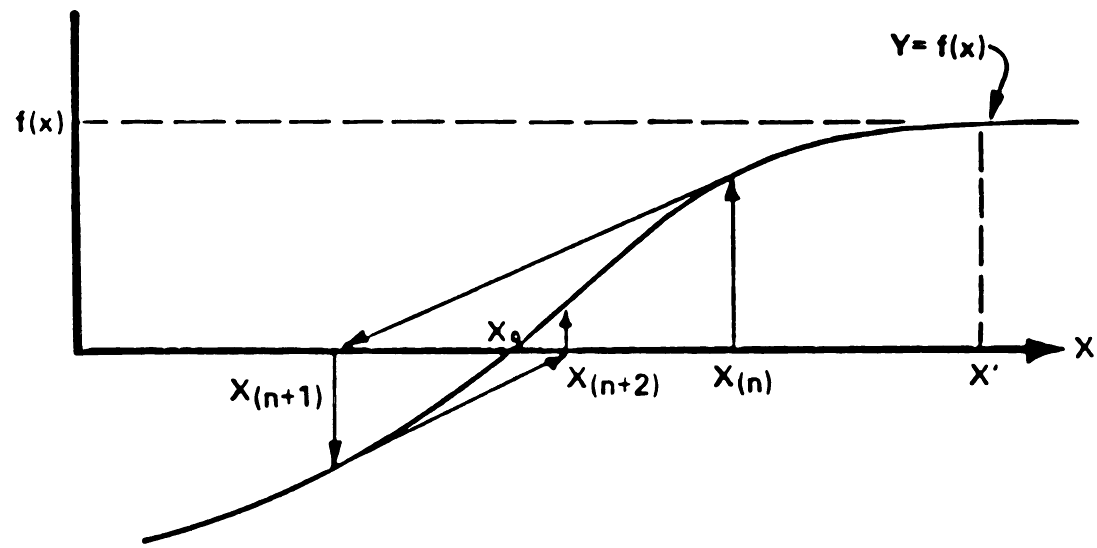

Figure 7. The Newton-Raphson method for the one dimensional case (Shamir and Howard, 1968).

$$X_{(n+1)} = X_{(n)} - D^{-1}\ F(X_{(n)}) \qquad [39]$$

where $X_{(n+1)}$ = vector of new estimates of the solution,

$X_{(n)}$ = vector of the initial estimate of the solution,

D^{-1} = inverse of the Jacobian matrix, and

$F(X_{(n)})$ = vector of the function of x.

The Jacobian matrix D consists of derivative elements, which are derivatives of that particular functional equation.

For the x variable the Jacobian matrix is

$$D = \begin{bmatrix} \frac{\partial F_1}{\partial X_1} & \frac{\partial F_1}{\partial X_2} \cdots\cdots & \frac{\partial F_1}{\partial X_j} \\ \frac{\partial F_2}{\partial X_1} & \frac{\partial F_2}{\partial X_2} \cdots\cdots & \frac{\partial F_2}{\partial X_j} \\ \vdots & \vdots & \vdots \\ \frac{\partial F_j}{\partial X_1} & \frac{\partial F_j}{\partial X_2} \cdots\cdots & \frac{\partial F_j}{\partial X_j} \end{bmatrix}$$

If solving the network with the heads as the unknowns the vector x becomes the vector H such that

$$\Delta H_{(n+1)} = \Delta H_{(n)} - Z_{(n)} \qquad [40]$$

where $\Delta H_{(n+1)}$ = nodal head vector estimate of the solution,

$\Delta H_{(n)}$ = nodal head vector initial estimate of the solution, and

$Z_{(n)}$ = solution vector of the linear system or

$$D_{(n)} \quad Z_{(n)} \quad = \quad F_{(n)} \tag{41}$$

Likewise, if solving the network equations containing the loop equations as unknowns the vector x is replaced by the vector ΔQ such that

$$\Delta Q_{(n+1)} \quad = \quad \Delta Q_{(n)} \quad - \quad Z_{(n)} \tag{42}$$

where $\Delta Q_{(n+1)}$ = vector of new estimate of corrective flow,

$\Delta Q_{(n)}$ = vector of initial estimate of corrective flow, and

$Z_{(n)}$ = solution vector of the linear system or

$$D_{(n)} \quad Z_{(n)} \quad = \quad F_{(n)} \tag{43}$$

The Newton-Raphson method converges quadratically. This means that when compared to linear convergence of other methods it requires fewer iterations to obtain a solution given a certain tolerance value (Jeppson, 1982).

The Newton-Raphson method was first implemented in the solution of pipe networks by Martin and Peters (1963). Subsequently it was used by Giudice (1965), who added a sensitivity analysis and considered more than one fixed head, and by Pitchai (1966), who treated networks with pumps as boundary conditions. Later, Shamir and Howard (1968), also used the Newton-Raphson method to present the basis of the formulation for other types of elements such as pumps and valves. Epp and Fowler (1970) included a method for reducing the storage requirements of the matrix coefficients.

The major difference between the Newton-Raphson and the Hardy Cross methods of solution is that the Newton-Raphson method iterates on a set of equations simultaneously, while the Hardy Cross performs iterations on separate equations, one at a time. Because the Newton-Raphson method adjusts the flow rate in all the loops simultaneously, convergence to a solution is much quicker than that obtained using the Hardy Cross method (Wood, 1972). This convergence characteristic is very important for the analysis of networks involving large numbers of pipes.

There are two major disadvantages of the Newton-Raphson method. First, it requires the evaluation, either analytically or numerically, of the first derivative of each flow equation with respect to each corrective flow (Lam and Wolla, 1972) and (Bralts, 1983). Secondly, the convergence of the Newton-Raphson method is highly dependent on the initial estimate of the flow distribution in the network.

If the initial estimate is sufficiently inaccurate the Newton-Raphson method can lead to slow convergence or in some cases, no convergence at all (Shamir and Howard, 1968; Wood and Charles, 1972; Jeppson, 1982; Bralts, 1983; Wiggert, 1986).

Another minor drawback of the Newton-Raphson method is the fact that large networks of pipes, containing numerous components, can lead to the formulation of large size matrices. This limitation can be overcome by employing a bandwidth reduction or renumbering of network elements and junctions, similar to the procedure proposed by Epp and Fowler (1970) and Grooms (1972). Jeppson and Davis (1976) and Chin et al. (1978) described a method for banding the coefficient, or Jacobian matrix, to increase the efficiency of computation. This procedure is a modification of the loop numbering algorithm proposed by Epp and Fowler (1970).

Gay et al. (1978) also presented a two node reordering procedure for water network analysis. This procedure minimizes the required matrix computations, and a sparse matrix technique requiring only off-diagonal non-zero partial derivative elements of the matrix dramatically reduces storage requirements of the matrix equation.

3. Linear Theory Method

The linear theory method of pipe network analysis was first presented by Wood and Charles (1972). This technique transforms the loop equations by approximating the nonlinear energy equations in the following form.

$$\Sigma h_f = \Sigma K_i \; Q_i^{\,n} = \Sigma K_i \; Q_{io}^{\,n-1} \; Q_i = \Sigma K_i^{'} \; Q_i = 0 \qquad [44]$$

where h_f = head loss in the pipe,

K_i = modified pipe constant,

Q_i = actual discharge, and

Q_{io} = approximate discharge.

The loop equations can then be expressed as linear equations that when combined with the continuity equations yield "n" linear simultaneous network equations, which can be readily solved for the discharge in each line.

An initial approximation of the flow rates can be obtained by assuming that the modified pipe constant is independent of the flow rate such that $K_i = K_i$. The solution obtained in this manner is very similar to one which would be obtained assuming laminar flow distribution in the network, such that the head loss is assumed to vary linearly with flow rate and be dependent on line length and diameter (Wood and Charles, 1972). By far the most common application of the linear theory method is with loop equations, rather than nodal equations.

There are several distinct features of the linear theory method which make it desirable for network analyses. Most importantly, it generally converges in relatively few iterations. This is in part accomplished by averaging the values of the discharge for the previous two iterations and employing the average to calculate the new estimate of flow and the modified pipe constant. This is expressed as:

$$Q_{io} = \frac{Q_{i-1} + Q_{i-2}}{2} \qquad [45]$$

where Q_{i-1} = flow rate obtained from previous trial and
Q_{i-2} = flow rate obtained from trial previous to Q_{i-1}.

Also, the method is notable for its ease in programming and use in optimization analysis. In addition, the linear theory method overcomes the principal disadvantage of the Newton-Raphson method or the Hardy Cross method by not requiring an initial estimate of the flow in the network.

Wood and Charles (1972) compared the linear theory method with the Hardy Cross and Newton Raphson methods in terms of their convergence characteristics. They found that the linear theory method converges very quickly and more accurately in fewer iterations than the other two methods.

Wood and Rayes (1981) reported the reliability of five different algorithms in terms of their convergence and accuracy: PATH, based on the Hardy Cross method for closed loops; SPATH, a modification of the PATH algorithm which calculates all flow adjustments simultaneously (this algorithm is equivalent to the Newton-Raphson method); LINEAR, based on the linear theory method and the linearization of the energy equations; NODE, based on the Hardy Cross method for the solution of nodal equations; SNODE, based on the linear theory method and the linearization of the nodal equations. They found significant convergence problems for PATH, NODE and SNODE methods. Both, the SPATH and LINEAR provided accurate solutions and consistently rapid convergence within a tolerance value of 0.0005 of the exact solution.

One of the drawbacks of the linear theory method is that considerably more computer storage is required than needed by the other methods. If used for loop oriented networks as recommended by Wood and Charles (1972) and Jeppson (1982), it uses more equations to solve the network system than are used by other methods. In comparison, the Jacobian matrix in the Newton-Raphson method requires much less storage, since it is a relatively narrow banded symmetric matrix with as many rows as loops. The linear theory method requires computer storage for NxN (number of pipes) plus 1 coefficient matrix while iteratively solving the linearized system of algebraic equations.

4. Finite Element Method

The equations which model the flow of water in irrigation pipes are nonlinear in nature and require solution by numerical methods. Norrie and deVries (1978), Segerlind (1984) and others have shown the applications of the finite element method of solution to many types of engineering problems governed by equations of this nature. The finite element method can be formulated using two types of elements: discrete and continuous. Discrete element formulation is generally employed in the analysis of structures. Continuous element formulation is employed in the solution of heat transfer, fluid mechanics, and soil mechanics problems (Segerlind, 1984).

In general the finite element method uses an integral formulation to generate the equations describing a problem and employs continuous piecewise equations to approximate the unknown parameters. The advantages of employing the finite element method include: 1) efficient use of computer storage space since the resulting matrix of coefficients is banded and symmetric, and 2) ease of data preparation following a systematic procedure of incorporating the element contributions to the final system of equations.

A number of investigators have applied the finite element method to the analysis of pipe network. Norrie and deVries (1978) presented a procedure for solving networks of pipes under laminar flow conditions. In this case, the friction is a linear function of the flow velocity thus allowing the direct formulation of pipe elements without linearizing the flow equations.

Henriksen (1984) developed a finite element approach for the analysis of networks of pipes. He employed element conductivities matrices using the method of weighted residuals. Sections of pipes were the only element used and results were based on varying the number of elements that described each pipe.

A finite element model of blood flow in arteries including taper, branches and obstructions was developed by Porenta et al. (1986). This model involves the formulation of a set of nonlinear equations which are transformed into a system of algebraic equations that when solved it yield values of pressure and flow as a function of time and arterial position.

Haghighi et al. (1987) formulated tee and wye components based on the linear theory method, nodal head equations, and the application of the finite element method. Bralts (1983) and Bralts and Segerlind (1985) developed a finite element formulation for the analysis of drip irrigation submain units. The method is based on the iterative solution of a set of linearized flow equations. They extended Norrie and deVries formulation to include turbulent flow conditions in drip irrigation hydraulic networks. The Galerkin's method of weighted residual was employed to generate the equations representing the two types of elements used: pipe and emitter elements. The following development of the finite element formulation closely resembles the approach used by Bralts and Segerlind (1985).

The objective of employing the finite element method is to arrive at an approximate solution for the one dimensional differential equation

$$D \frac{d^2\phi}{dx^2} + Q = 0 \qquad [46]$$

Several procedures are available to describe the approximate solution to the differential equation. Among the most popular are: 1) the finite difference method, 2) the variational method, and 3) methods that weight a residual. In this case, the Galerkin's method of weighted residuals has been selected to generate a system of equations by evaluating the integral

$$-\int_0^H W(x)\left(D \frac{d^2\phi}{dx^2} + Q\right) dx = 0 \qquad [47]$$

where ϕ = unknown parameters,

$W(x)$ = weighting function, and

D, Q = differential equation coefficients.

Galerkin's formulation employs the shape functions N_i and N_j to define the weighting function for every node. Using a sequence of nodes r,s,t, and u as shown in Figure 8, the residual integral, equation [47] for node s becomes

$$Rs = Rs^{(e-1)} + Rs^{(e)} \qquad [48]$$

or

$$\int_{X_r}^{X_s} [Ns(D \frac{d^2\phi}{dx^2} + Q)]^{(e-1)}dx \quad - \int_{X_s}^{X_t} [Ns(D \frac{d^2\phi}{dx^2} + Q)]^{(e)}dx \qquad [49]$$

where $Rs^{(e-1)}$ and $Rs^{(e)}$ represent the contribution of elements (e-1) and (e) to node S. Carrying out the integration of equation [49] for elements (e-1) and (e) results in

$$Rs^{(e-1)} = -(D \frac{d\phi}{dx})^{(e-1)}\Big|_{X=X_s} + \frac{D}{L}(-\phi_r + \phi_s) - \frac{QL}{2} \qquad [50]$$

and

$$Rs^{(e)} = (D \frac{d\phi}{dx})^{(e)}\Big|_{X=X_s} + \frac{D}{L}(\phi_s - \phi_t) - \frac{QL}{2} \qquad [51]$$

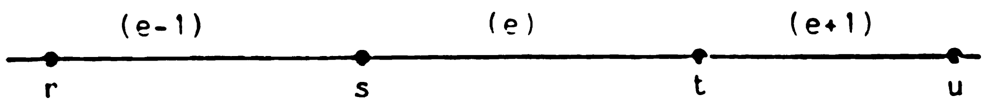

Figure 8. Sequence of nodes and elements (Bralts and Segerlind, 1985).

For the special case of steady state flow through pipes, the first derivative term, $D(d\phi/dx)$, and the constant Q are always equal to zero. Therefore , the residual equation for node s becomes

$$Rs = Rs^{(e-1)} + Rs^{(e)} = \frac{D}{L}(-\phi_r + \phi_s) + \frac{D}{L}(-\phi_s + \phi_t) = 0 \quad [52]$$

For a straight pipe element, as shown in Figure 9, the hydraulic analogy to the residual equation [52] begins by applying the energy equation and the continuity equation.

The energy equation, considering $V_i = V_j$ and $h_m = 0$ since there are no external mechanical energy supply such as a pump, reduces to

$$Z_i + \frac{P}{\gamma} = Z_j + \frac{P}{\gamma} + h_f \quad [53]$$

or

$$Z_i + H_i = Z_j + H_j + K\ Q^m \quad [54]$$

where H_i and H_j are the downstream and upstream static pressures respectively and all the other variables already defined.

Linearizing the exponential friction head loss term:

$$h_f = \Delta H = K\ Q^m \quad [55]$$

results in the following relationship

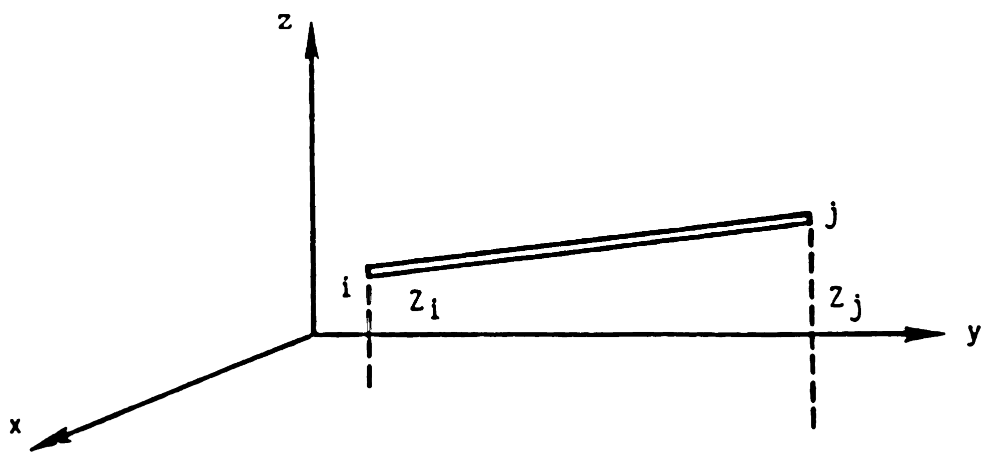

Figure 9. Straight pipe element section (Bralts and Segerlind, 1985).

$$Q = K^{-1/m} \Delta H^{1/m} = C_p \Delta H \qquad [56]$$

equation [54] can be rearranged into

$$K^{1/m}\, Q = [\ (Z_i + H_i) - (Z_j + H_j)\ ]^{1/m} \qquad [57]$$

or

$$Q = C_p\ (H_i - H_j) + C_p\ (Z_i - Z_j) \qquad [58]$$

where $$C_p = \frac{\left|(Z_i + H_i) - (Z_j + H_j)\right|^{1-m/m}}{K^{1/m}}$$

is the linearized coefficient for the straight pipe element and all the other variables are as previously defined.

The contribution to the residual equation [52] considers a sequence of nodes as shown in Figure 8. These nodes are separated by elements (e-1), (e) and (e+1). Assuming flow into a node is negative and flow out from a node is positive, the continuity principle applied to nodes s and t results

$$- Q_s(e-1) + Q_s(e) = 0 \qquad [59]$$

and

$$- Q_t(e) + Q_t(e+1) = 0 \qquad [60]$$

the contribution of element (e) to the residual nodal equation is $Q_s(e)$ and $Q_t(e)$ or

$$Q_s(e) = C_p(e)\ (H_s - H_t) + C_p(e)\ (Z_s - Z_t)$$

and

$$Q_t(e) = -C_p(e)\ (H_s - H_t) - C_p(e)\ (Z_s - Z_t)$$

in matrix notation

$$\begin{Bmatrix} Q_s(e) \\ \\ Q_t(e) \end{Bmatrix} = \begin{bmatrix} C_p & -C_p \\ \\ -C_p & C_p \end{bmatrix} \begin{Bmatrix} H_s \\ \\ H_t \end{Bmatrix} - \begin{Bmatrix} C_p\Delta Z \\ \\ -C_p\Delta Z \end{Bmatrix} \qquad [61]$$

where $\Delta Z = Z_t - Z_s$

The element matrices, shown in equation [61], are assembled employing a direct stiffness procedure which yields a system of equations having the following standard finite element form (Segerlind, 1984):

$$\{R^{(e)}\} = [K^{(e)}]\ \{\phi^{(e)}\} - \{F^{(e)}\} \qquad [62]$$

where $\{R^{(e)}\} = \begin{Bmatrix} Q_s(e) \\ Q_t(e) \end{Bmatrix}$

is the vector of residual equations,

$$[K^{(e)}] = \begin{bmatrix} Cp & -Cp \\ -Cp & Cp \end{bmatrix}$$

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$$\{\phi^{(e)}\} = \begin{Bmatrix} H_s \\ H_t \end{Bmatrix}$$

is the vector of unknown pressures, and

$$\{F^{(e)}\} = \begin{Bmatrix} C_p\Delta Z \\ -C_p\Delta Z \end{Bmatrix} = \begin{Bmatrix} g \\ -g \end{Bmatrix}$$

is the element force vector.

In summary, this section has shown four numerical analysis techniques employed in the design of pipe networks. The finite element method presents a direct formulation of the system of equations which is constructed by calculating the element's contribution and placing the values in the proper position within the final system of equations.

C. Analysis of Hydraulic Components

Sprinkler irrigation systems commonly contain a number of hydraulic components such as valves, elbows and tees that contribute a minor loss to the system. Figure 10 is an illustration of the different components noted above.

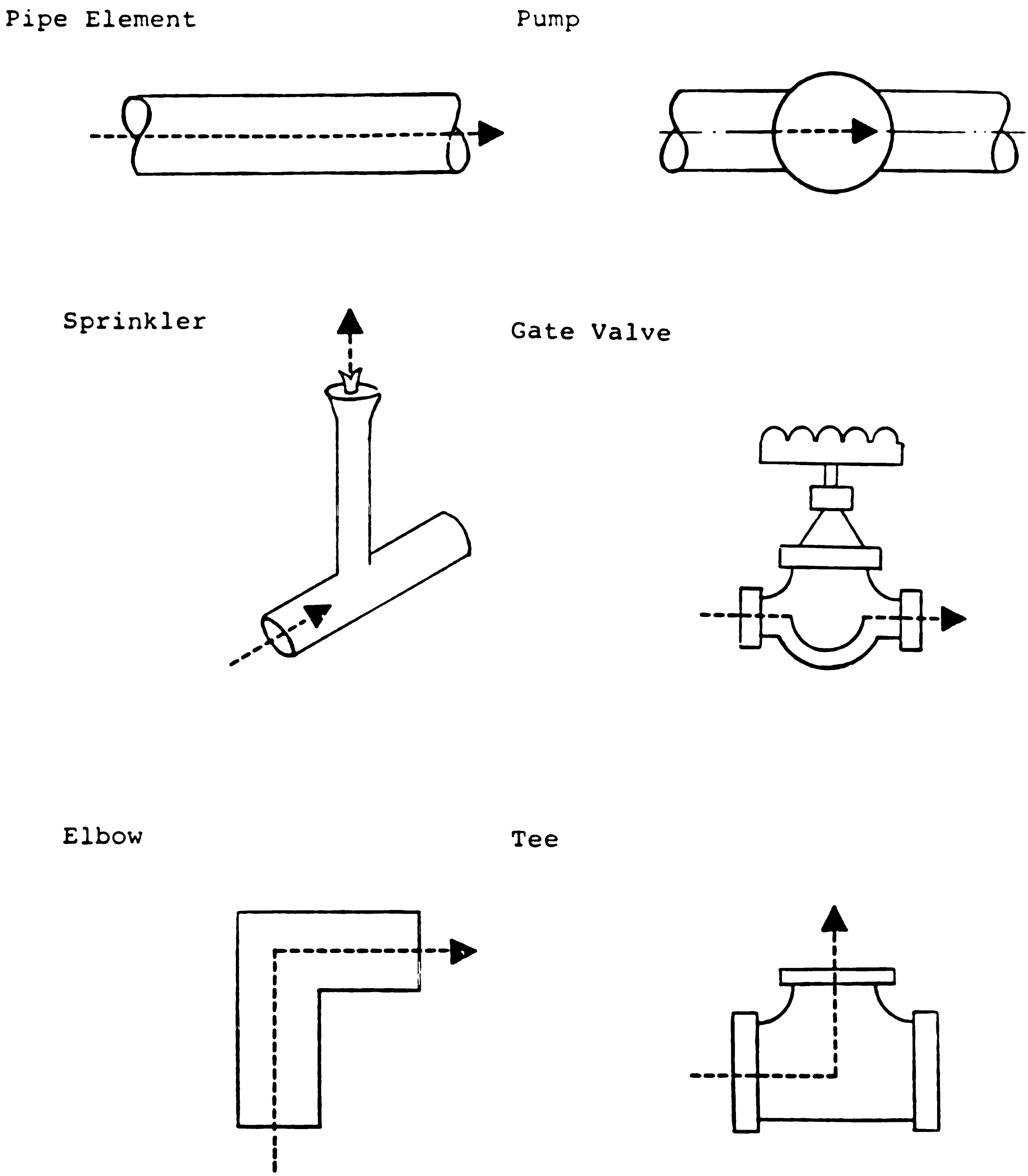

Figure 10. Sprinkler irrigation components.

Hydraulic components alter the flow pattern in the sprinkler irrigation system usually creating turbulence which results in head loss in addition to the normal friction loss occurring in the pipes (Jeppson, 1982). According to Finkel (1982), head loss due to hydraulic components may amount to from 2 to 20% of the total head losses in the sprinkler irrigation system and consequently are not always negligible as is often assumed.

Head losses occurring in hydraulic components can be expressed in several different ways. In trickle irrigation systems it has been customary to assign pressure losses across fittings as some unspecified percentage of the total pressure loss occurring in the system. In water distribution systems, the procedure is to compute the component losses in one of the following manners: employing the equivalent pipe concept or as a function of the velocity head.

The equivalent pipe concept stipulates that head losses occurring in hydraulic components can be determined by forming an equivalent pipe. This equivalent pipe should have the same head loss for any flow rate as the sum of the frictional loss occurring in the pipe and the minor head loss occurring in the component (Jeppson, 1982).

The equivalent pipe is formed by adding a length, L to the actual pipe length such that the frictional head loss in the added length of pipe equals the loss from the component. The calculation of L is slightly different depending upon whether the Darcy-Weisbach or the Hazen-Williams equation is to be used.

Pair, (1975), Miller, (1978) and others have shown the expression for the head loss occurring in the components of the network system as a function of the velocity head as

$$h_c = K_c \frac{V^2}{2g} \qquad [63]$$

where h_c = component head loss in meters,

K_c = component loss coefficient dimensionless, and

$\frac{V^2}{2g}$ = velocity head in meters.

The component loss coefficient, K_c, is a term which multiplies the velocity head to give the concentrated head loss at the component. The component head loss coefficient may vary according to the flow conditions but is usually convenient to consider it constant for each component (Wood, 1982). Some values normally used for common components are given in Table I. Numerically, Kc, has the same value for English and SI units.

Many researchers have investigated the inclusion of hydraulic components in the analysis of networks of pipes. The usual procedure has been to employ one of the three methods of network system analysis namely, the

Table 1. Loss coefficients for common components (Wood, 1980).

Component	K_C
Globe valve, fully open	10.0
Angle valve, fully open	5.0
Swing check valve, fully open	2.5
Gate valve, fully open	0.2
Gate valve, 3/4 open	1.0
Gate valve, 1/2 open	5.6
Gate valve, 1/4 open	24.0
Short-radius elbow	0.9
Medium-radius elbow	0.8
Long-radius elbow	0.6
45 Elbow	0.4
Closed return bend	2.2
Tee, through side outlet	1.8
Tee, straight run	0.3
Coupling	0.3
45 Wye, through side outlet	0.8
45 Wye, straight run	0.3
Entrance	
square	0.5
bell mouth	0.1
re-entrant	1.0
Exit	1.0

Hardy Cross, the Newton Raphson, or the linear theory method in conjunction with node equations or loop equations.

Pitchai (1966) used the Newton-Raphson method in solving a network which included reservoirs and pumps. In the study, the pumps were treated as reservoirs with fixed heads until the network was balanced using the Newton-Raphson method. From this solution, the flow rate at each node connected with a pump is determined. If the flowrate does not satisfy the pump characteristic curve, then the head of the pump is incremented. With this set of new pressure heads at nodes incident to the pumps, the network is rebalanced by the Newton-Raphson method. This process is repeated until the flow rate at each pump node satisfies, within some tolerance, the pump characteristics.

Dilligham (1967) employed pumps and reservoirs using the Hardy Cross method of balancing flows. He incorporated the pumps and reservoirs directly into the analysis by assuming that they were connected to the system at joints.

Shamir and Howard (1968) included valves and pumps using the Newton-Raphson method and nodal head equations. They found that the convergence of this method can not be guaranteed when any of the characteristic functions of the elements (valves and pumps) in the network do not have continuous derivatives. This theoretical difficulty was overcome in practice by starting the solution with a good initial estimate.

Lam and Wolla (1972) considered booster pumps using a modified Newton-Raphson method and linear graph theory. Instead of evaluating and inverting the Jacobian matrix at each iteration, the modified Newton-Raphson method, based on the residue of the functions, uses an iterative equation to update an approximation to the Jacobian and its inverse at each iteration. They compared the modified Newton-Raphson with the standard Newton-Raphson method and found that, when booster pumps were included, the modified method converges faster for larger systems than the standard method.

Lemieux (1972) proposed that booster pumps and other elements could be included in the network analysis by using a modified Newton-Raphson method without altering the form of the Jacobian matrix. He obtained a more efficient algorithm by employing a Gaussian elimination procedure.

Kesavan and Chandrashekar (1972) employed a graph-theoretic model to solve networks that included pumps and reservoirs. Their approach involved the formulation of two continuity equations, node and loop continuity. The nonlinearities associated with the components in the network were treated as an integral part of the formulation procedure. The graph-theoretic approach was compared with the Hardy Cross method showing a faster convergence and ease of including components in the formulation of the equations.

Chenoweth and Crawford (1974) employed the Hardy Cross method of balancing heads in the analysis of networks which included pumps and reservoirs. This method of solution was based on guessing the elevation of the hydraulic grade line at all the nodes in the system. The elevation of the hydraulic grade line at each node, one at a time, is adjusted up or down sufficiently until continuity is satisfied at that node. Calculations then proceed from node to node until all the nodes in the network are covered. To model the pump performance, points from the head-flow characteristic curve are stored on a two dimensional array were they can be conveniently retrieved in each iteration.

Wood and Charles (1973) in the closure paper mentioned that a generalized procedure could be implemented to include pumps, reservoirs, valves and other essential components in the analysis of networks by the linear theory Method. Jeppson and Tavallace (1975) found that implementing the above consideration would slow the convergence of the method and suggested a modified approach for the inclusion of pumps and reservoirs in the analysis which greatly improved the convergence. This involved the addition of a pseudoloop to connect two reservoirs and also the inclusion of a linear equation for each pump in the network.

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Dodge et al. (1978) used the Newton-Raphson method and the nodal formulation of the matrix equation to solve networks that included constant discharge pumps, reservoirs, booster pumps, check valves, pressure regulating valves and sprinklers for fire and agricultural use. The formulation of the continuity and energy equations followed Shamir and Howard (1968) analysis. Since network components were included, a node reordering provision had to be implemented such that it reduced the number of computations required. Also a sparse matrix routine was employed which stored only the non-zero elements.

Jeppson and Davis (1976) and Jeppson (1982) presented an efficient method for including pressure regulating valves in the network analysis. The introduction of a pressure regulating valve in the network had no significant effect in increasing the number of iterations needed or increasing the amount of execution time required for a solution.

Issacs and Mills (1980) employing a Linear Theory method for solving junction heads concluded that minor fittings, pressure regulating valves, and check valves can be readily included in the manner proposed by Jeppson and Davis (1976).

Gofman and Rodeh (1981) described a method for incorporating unknown pipe characteristics such as pressure regulating valves and booster pumps into loop oriented hydraulic network solvers. He found that, when head generators (booster pumps) were introduced, the method either may have no solution or may have many solutions. To obtain a satisfactory solution it required that some conditions be meet, such as that the number of head generators be equal to the number of nodes with fixed pressure.

Chandrashekar (1980) extended his early work on graph theoretic concepts to include pressure regulating valves, check valves, and booster pumps in the analysis of networks. He employed the junction (nodal) head analysis in the formulation of the system of equations. He found that if several pressure regulating valves or check valves, or both, are present, the method may not yield a correct solution. Numerical problems of oscillations or slow convergence, or both, may also occur.

Wood (1981) presented a comprehensive computer program based on his early application of the linear theory method to the analysis of network systems. The computer model is capable of handling any type of pipe system configuration along with any number of components such as booster pumps, pressure regulating valves, check valves and reservoirs.

Ohtmer (1983) formulated different network components such as valves, bows, tees, pump, knee, contraction, expansion and diffusers by taking a very distinct approach. He used the Newton-Raphson iteration procedure to solve the nonlinear system of equations and the graph theory concept to determine the initial flow rate estimate. To model the network components, he employed the analogy between the structural network loaded by moments to the flow in pipe networks. With this procedure, the flow rate is the equivalent variable of the moment as the pressure is to the rotation. This was made possible by applying mesh (force) method to include the components.

D. Summary

In summary, the literature review has shown that the finite element formulation can be employed in the design and analysis of pipe networks. Furthermore, a need has arisen to include the pressure losses occurring in elbows, tees, valves and also the contribution of pumps in the design process. The proposed research will focus on the formulation of equations that model these components, employing the finite element method.

III. METHODOLOGY

A review of the literature indicates the need for better sprinkler irrigation design so that water and energy may be used in the most efficient manner possibly.

In order to achieve maximum efficiency it is necessary to take into consideration the losses occurring in the hydraulic components of sprinkler irrigation systems. The review of the literature also shows the difficulty of incorporating these hydraulic components when employing any of the conventional solution techniques of pipe network analysis. This suggests the need for a better formulation where in a systematic manner such hydraulic components can be readily incorporated in the analysis of sprinkler irrigation systems.

A. Research Approach

Based upon the need to obtain a more accurate design of sprinkler irrigation systems including the effects of losses occurring in the hydraulic components, the following procedures are proposed for attaining the research objectives.

Objective 1. To develop finite element formulation of essential sprinkler irrigation components such as elbows, tees, gate valves, sprinklers and pumps.

To achieve objective 1, various sprinkler irrigation hydraulic components will be formulated employing the finite element methodology. A computer model will be designed based on the finite element formulation of the hydraulic components and on the application of the dimensionless energy concept to sprinkler irrigation design. The purpose of the computer model will be to provide reliable values of pressure and discharge in order to satisfy the sprinkler irrigation system design.

Objective 2. To demonstrate the potential of applying the finite element formulation to the design of sprinkler irrigation systems by comparing it to a linear theory approach.

To attain objective 2, the finite element based computer model will be compared to a commercially available linear theory computer model employed in the solution of pipe networks. Specifically, both models will be compared for reliability and accuracy of solution.

B. Theoretical Development

The theoretical development section is composed of two parts. In the first part, the finite element formulation of a number of sprinkler irrigation components is presented. In the second part, an example implementation is presented.

The following development of the finite element formulation is an extension of the approach outlined in the literature review section.

The analysis is based on the mechanical energy balance equation and the contribution of each of the system components to the nodal equation. Applying the mechanical energy balance between two points in a network system, Figure 11, (neglecting the velocity component) is

$$Z_i + \frac{P_i}{g\rho} + H_p = Z_j + \frac{P_j}{g\rho} + h_f + h_c \qquad [64]$$

where the subscripts i and j denote the downstream and upstream conditions respectively and all other variables are as previously defined.

Equation [64] can be rearranged into the different contributions of each element to the system

$$(Z_i + H_i) - (Z_j + H_j) = KQ^m + \frac{K_c\ Q^2}{2g\ A^2} - H_p \qquad [65]$$

or

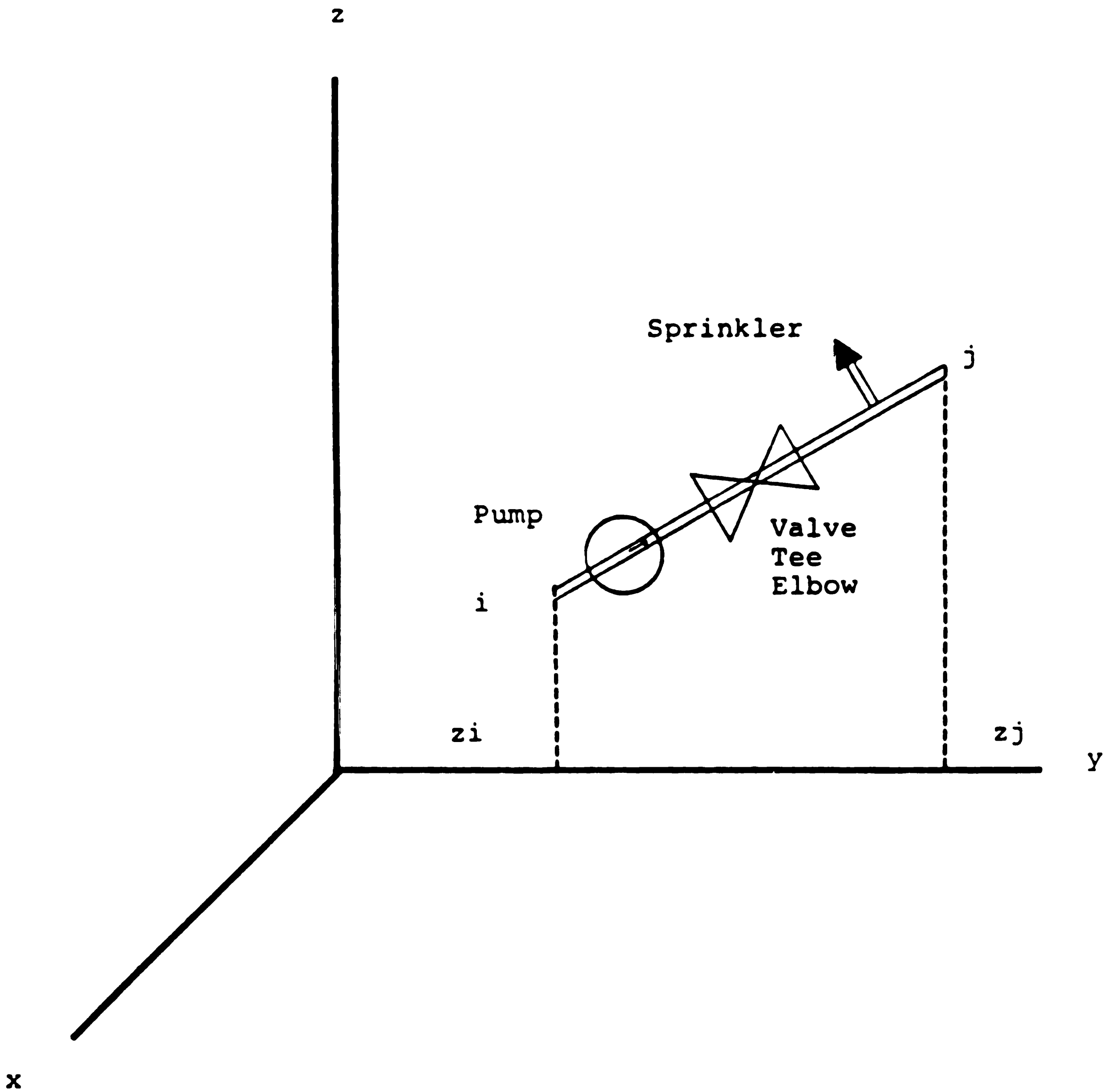

Figure 11. Pipe section and system components

$$\Delta H = KQ^m + \frac{K_c\ Q^2}{2gA^2} - H_p \qquad [66]$$

where A is the area of the element and ΔH is defined as

$$\Delta H = (Z_i + H_i) - (Z_j + H_j)$$

Equation [66] can be separated further into three equations:

$$\Delta H = KQ^m \qquad \text{Pipe Section} \qquad [67]$$

$$\Delta H = \frac{K_c\ Q^2}{2gA^2} \qquad \text{System Components} \qquad [68]$$

$$\Delta H = -H_p \qquad \text{System Pump} \qquad [69]$$

starting with equation [67] it can be rearranged into

$$Q = K^{-1/m}\ \Delta H^{1/m} \qquad [70]$$

equation [70] can be linearized in the following form

$$Q = K^{-1/m}\ \Delta H^{1/m}\ \frac{\Delta H}{\Delta H}$$

$$Q = K^{-1/m}\ \Delta H^{1/m+1-1}$$

$$Q = K^{-1/m}\ \Delta H^{1/m-1}\ \Delta H$$

resulting in the following equation

$$Q = Cp\ \Delta H \qquad [71]$$

equation [71] can then be rearranged into

$$Q = Cp\ (H_i - H_j) + Cp\ (Z_i - Z_j) \qquad [72]$$

where $$Cp = \frac{\left|(Z_i + H_i) - (Z_j + H_j)\right|^{(1-m)/m}}{K^{1/m}} \qquad [73]$$

is the linearized coefficient for the straight pipe element. Equation [68], representing the system component contribution, can be rearranged into

$$\frac{\Sigma K_c Q^2}{2gA^2} = R_c\ Q^2 = \Delta H \qquad [74]$$

or $$R_c{}^{1/2}\ Q = \Delta H^{1/2} \qquad [75]$$

and $$Q = C_{pc}\ (H_i - H_j) + C_{pc}\ (Z_i - Z_j) \qquad [76]$$

where $$C_{pc} = \frac{\Delta H^{1/2}}{R_c{}^{1/2}} \qquad [77]$$

is the linearized coefficient for the system components.

The effects of the pump in equation [69] can be described in three basic forms:

First, the effect of the pump can be specified depending on the useful power it puts into the system (in horsepower or kilowatts). The useful power refers to the actual power which is transformed into an increase in pressure head and kinetic energy of the liquid as it passes through the pump. The useful power can be computed from a typical head-discharge data set using the following equation

$$H_p = -\frac{P}{\rho g Q} \qquad [78]$$

where H_p = operating head in meters,

P = useful power in kilowatts,

Q = discharge in cubic meters per second,

ρ = density in kilograms per cubic meters, and

g = acceleration due to gravity in meters per squared second.

Rearranging equation [78] results into

$$\Delta H = -\frac{P}{\rho g Q} \qquad [79]$$

or

$$Q = C_{pp} (H_i - H_j) + C_{pp} (Z_i - Z_j) \qquad [80]$$

where
$$C_{pp} = - \frac{P}{\Delta H \rho g} \quad [81]$$

is the linearized pump coefficient.

Alternately, a pump can be described by points of operating data. A polynomial or an exponential curve can be fit to this data to obtain a pump characteristic curve describing the pump operation as

$$H_p = H_1 - cQ^m \quad [82]$$

where H_1 = cutoff head in meters,

c and m = coefficients derived from each particular pump application, and all the other variables as previously defined.

Equation [82] can be rearranged into

$$\Delta H = - H_1 + cQ^m \quad [83]$$

or

$$(\Delta H + H_1)^{1/m} = c^{1/m} Q \quad [84]$$

where

$$Q = C_{pp} (H_i - H_j) + C_{pp} (Z_i - Z_j) \quad [85]$$

and

$$Cpp = \frac{|(\Delta H + H_1)|^{(1-m)/m}}{c^{1/m}} \quad [86]$$

is the linearized pump coefficient.

A third method of incorporating the effects of a pump into the system of equations is by specifying the pump discharge pressure. For this application the pump discharge is considered at a fixed grade node, where both the pressure and the elevation are known values. The pump contribution can be written as

$$\Delta H = H_o \Rightarrow \Delta H = \frac{P_o}{\rho} + Z_o \qquad [87]$$

where H_o is the fixed grade node.

The pump contribution in this case is accounted for in the forcing vector as

$$f^{(e)} = C_{pp} H_o \qquad [88]$$

where $$C_{pp} = \frac{\left|(Z_i + H_i) - (Z_j + H_j)\right|^{(1-m)/m}}{K^{1/m}} \qquad [89]$$

is the linearized pipe element coefficient in which the pump is located.

The sprinkler element can be considered as a separate component. Rearranging equation [68] and neglecting the elevation difference across the sprinkler results into

$$(H_i - H_j) = \frac{K c Q^2}{2gA^2} \qquad [90]$$

or

$$(H_i - H_j) = K_{sp} Q^2 \qquad [91]$$

and

$$(H_i - H_j)^{1/2} = K^{1/2}Q \qquad [92]$$

where

$$Q = C_{sp} (H_i - H_j) \qquad [93]$$

and

$$C_{sp} = \frac{(H_i - H_j)^{1/2}}{K_{sp}^{1/2}} \qquad [94]$$

is the linearized coefficient for the sprinkler element.

Combining all the linearized coefficients into one equation results into

$$Q = \Sigma C \ (\Delta H) \qquad [95]$$

or

$$Q = (C_p + C_c + C_{pp} + C_{sp}) \ \Delta H \qquad [96]$$

which gives the linearization of all the elements as shown in Figure 11.

The finite element method utilizes the concept of an element stiffness matrix and an element force vector to construct the system of equations. The element matrices for the pipe element is developed below.

Considering a sequence of nodes as shown in Figure 12. These nodes are separated by elements (e-1), (e) and

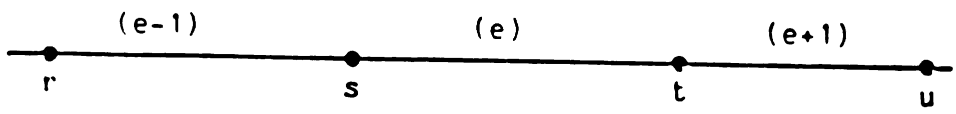

Figure 12. Sequence of nodes and elements (Bralts and Segerlind, 1985).

(e+1). Assuming flow into a node is negative and flow out from a node is positive, the continuity principle applied to nodes s and t results

$$- Q_s(e-1) + Q_s(e) = 0 \qquad [97]$$

and

$$- Q_t(e) + Q_t(e+1) = 0 \qquad [98]$$

the contribution of element (e) to the residual nodal equation is $Q_s(e)$ and $Q_t(e)$ or

$$Q_s(e) = C_p(e)\ (H_s - H_t) + C_p(e)\ (Z_s - Z_t) \qquad [99]$$

and

$$Q_t(e) = -C_p(e)\ (H_s - H_t) + C_p(e)\ (Z_s - Z_t) \qquad [100]$$

in matrix notation

$$\begin{Bmatrix} Q_s(e) \\ \\ Q_t(e) \end{Bmatrix} = \begin{bmatrix} C_p & -C_p \\ \\ -C_p & C_p \end{bmatrix} \begin{Bmatrix} H_s \\ \\ H_t \end{Bmatrix} - \begin{Bmatrix} C_p\ \Delta Z \\ \\ -C_p\ \Delta Z \end{Bmatrix} \qquad [101]$$

where $\Delta Z = Z_t - Z_s$

Likewise, the components' (elbows, tees and valves) contribution to the residual nodal equation is

$$\begin{Bmatrix} Q_s(e) \\ \\ Q_t(e) \end{Bmatrix} = \begin{bmatrix} C_{pc} & -C_{pc} \\ \\ -C_{pc} & C_{pc} \end{bmatrix} \begin{Bmatrix} H_s \\ \\ H_t \end{Bmatrix} - \begin{Bmatrix} C_p \ \Delta Z \\ \\ -C_p \ \Delta Z \end{Bmatrix} \qquad [102]$$

The pump contribution is also defined in the same format as

$$\begin{Bmatrix} Q_s(e) \\ \\ Q_t(e) \end{Bmatrix} = \begin{bmatrix} C_{pp} & -C_{pp} \\ \\ -C_{pp} & C_{pp} \end{bmatrix} \begin{Bmatrix} H_s \\ \\ H_t \end{Bmatrix} - \begin{Bmatrix} C_p \ \Delta Z \\ \\ -C_p \ \Delta Z \end{Bmatrix} \qquad [103]$$

Finally, the sprinkler contribution to the residual nodal equation can be written as

$$\begin{Bmatrix} Q_s(e) \\ \\ Q_t(e) \end{Bmatrix} = \begin{bmatrix} C_{sp} & -C_{sp} \\ \\ -C_{sp} & C_{sp} \end{bmatrix} \begin{Bmatrix} H_s \\ \\ H_t \end{Bmatrix} - \begin{Bmatrix} C_p \ \Delta Z \\ \\ -C_p \ \Delta Z \end{Bmatrix} \qquad [104]$$

A direct stiffness procedure, similar to the one employed in structural analysis, is used to incorporate the element matrices shown in equations [101], [102], [103], and [104] into the final system of equations.

To demonstrate the direct stiffness procedure a sprinkler irrigation system as shown in Figure 13 has been selected. The sprinkler system consists of a submain and eight laterals. The submain contains seven

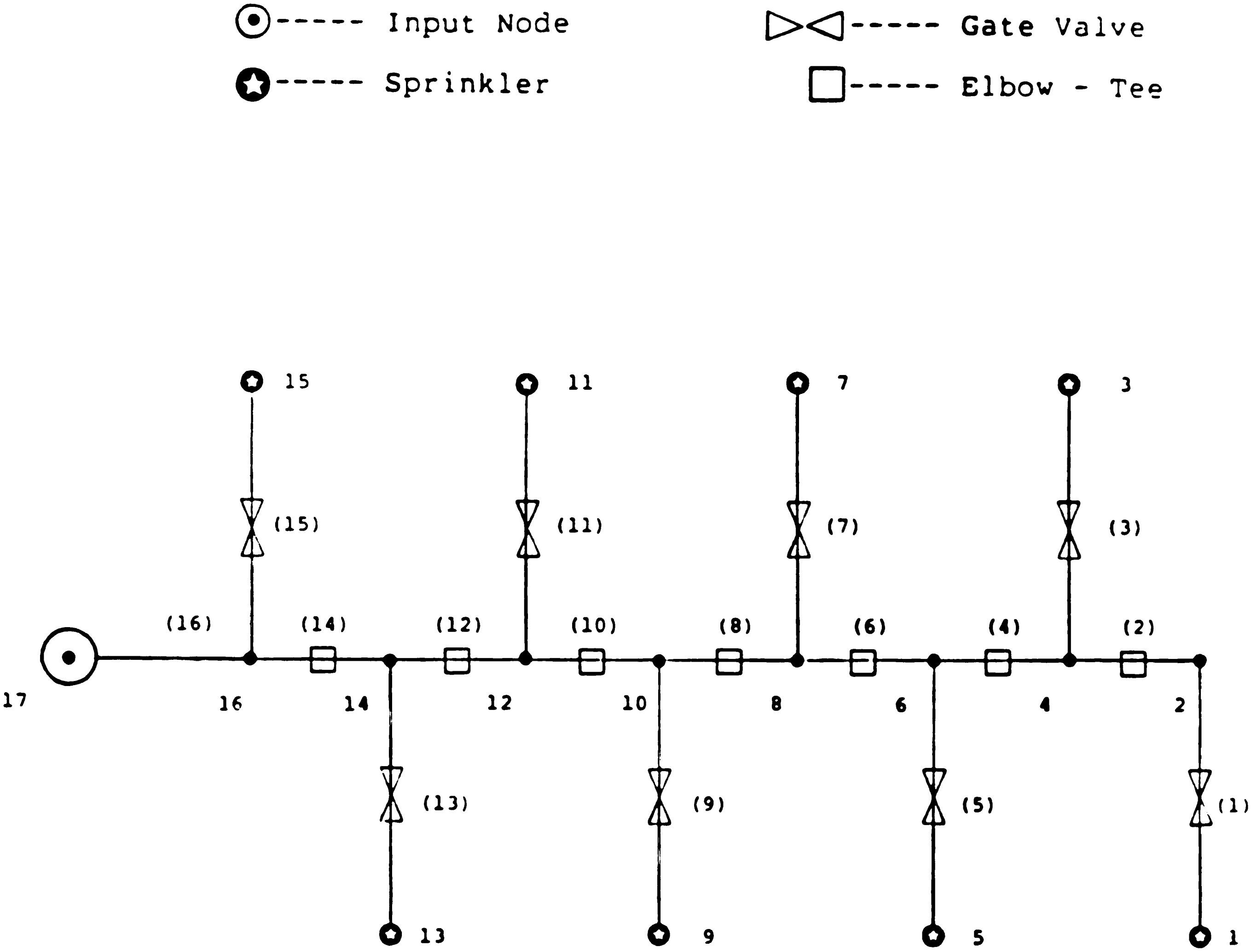

Figure 13. Example sprinkler irrigation system.

tees and one elbow. Each lateral contains one sprinkler and one gate valve. The input has been considered as a fixed grade node where pressure and elevation are known quantities.

The direct stiffness method is very simple to apply and only requires the knowledge of the location of the elements defined by two nodes. The contribution of each element in the system of pipes and components is accordingly added or subtracted at the proper location in the stiffness matrix. An example of the solution matrix is shown in Figure 14. The direct stiffness procedure yields a system of equations which has the general matrix form:

$$[K] \ \{H\} \ - \ \{F\} \ = \ \{0\} \qquad [105]$$

The vector {H} contains the element nodal pressures, the stiffness matrix [K] contains the algebraic summation of each element contribution, and the vector {F} contains the element contribution to the force vector.

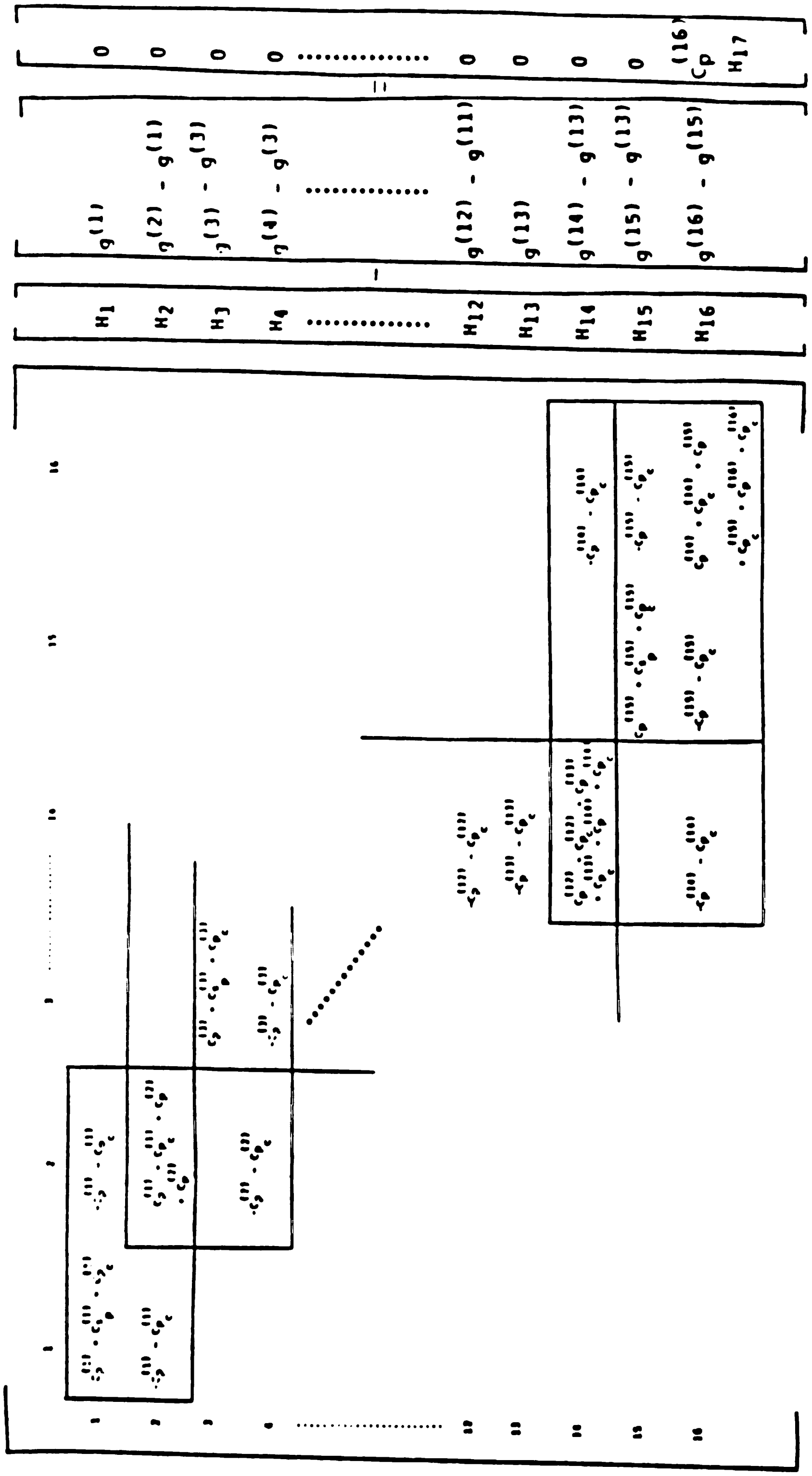

Figure 14. Example solution matrix.

IV. RESULTS AND DISCUSSION

The results and discussion section consists of a comparison of the data generated by both finite elements and linear theory models. Data analysis and representation will be performed using the PLOT-IT statistical package. Linear regression analysis will be performed to determine the correlation between the junction pressures and sprinkler flows for the finite elements and linear theory models. Also, the accuracy and reliability of each model will be assessed in this section.

A. Computer Models

A computer program (DESIGNER) developed by Bralts and Segerlind (1985), which uses the finite element method to assist in the analysis and design of drip irrigation submain units, was modified to incorporate the element matrices derived in the methodology section. The finite element based computer model (SPIRR-FE, see appendix B) was compared with a computer model based on linear theory developed by Wood (1980) (see appendix C).

The finite element based computer program (SPIRR-FE) solves the hydraulic network problem by obtaining values of pressure and discharge, at the junction and sprinkler nodes, and employing them to satisfy the sprinkler irrigation system design. The SPIRR-FE computer program uses the dimensionless grade line concept as presented in the literature review to generate initial estimates of both junction pressures and elevations.

The linear theory model, as presented in the literature review section, utilizes a different approach to approximate the initial estimates of flow. Wood's computer program calculates initial flowrates assuming a modified pipe line constant which is made independent of the flowrate. The solution obtained in this manner is very similar to one which would be obtained assuming laminar flow distribution in the system.

The results of this section will show that both techniques for computing the initial flow distribution in the hydraulic system give very reasonable values as it does approximately compute a laminar flow distribution. This gives results which are feasible estimates of the turbulent flow distribution existing in the sprinkler system.

B. Model Comparisons

Both the finite element and the linear theory computer programs provide empirical solutions. For this reason, comparison of their solution should be made considering a variety of initial conditions. A group of comparisons were conducted which attempted to cover a wide range of possible solid set sprinkler irrigation hydraulic designs.

Five different slope conditions were employed in the comparisons. They ranged from two percent downhill to two percent uphill. Also a zero percent slope was considered. Four different values of component K_c coefficients were also selected. They varied from zero to three. The purpose of this type of evaluation was to examine the impacts of the system components and the sloping ground conditions in the overall sprinkler irrigation system design.

The comparisons were further divided into two general situations. The first evaluation procedure consisted of comparing the results of head and flow generated in a sprinkler irrigation system, considering the input node of the system as a fixed grade node. The second evaluation procedure consisted of comparing the same system, but including a pump as a source of head input into the system.

The sprinkler irrigation system selected for the first evaluation is shown in Figure 15. It consists of a submain and eight laterals. The submain line has a diameter of 203.2 millimeters (8 inches) and a total length of 487.7 meters (1600 feet). The submain line also contains six tees and one elbow. The lateral

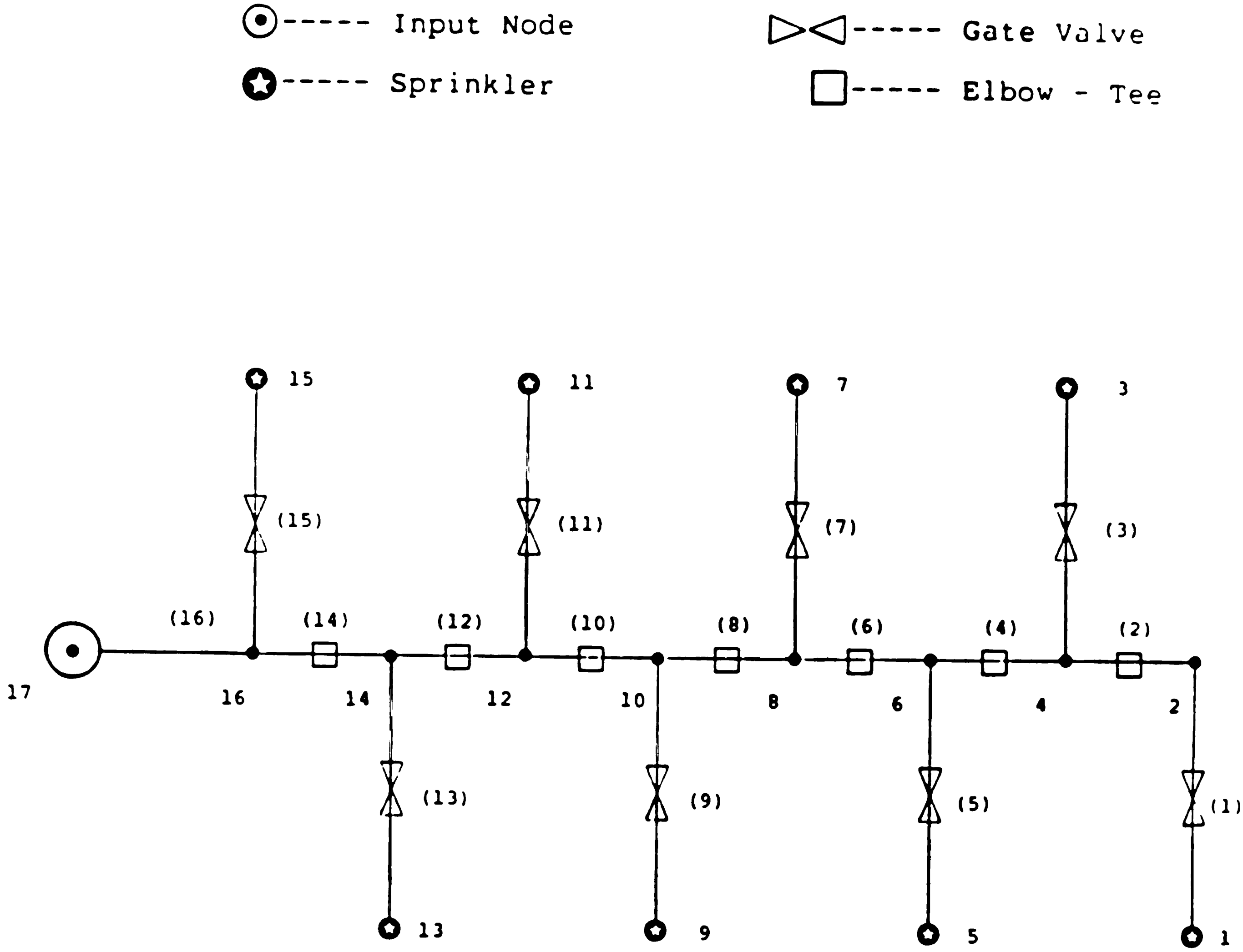

Figure 15. Sprinkler irrigation system layout.

lines have a diameter of 50.8 millimeters (2 inches) and are spaced every 61.0 meters (200 feet) along the submain line. The lateral lines are 30.5 meters (100 feet) long, each containing one sprinkler and one gate valve.

In this first evaluation the input node has been considered a fix grade node, as if the sprinkler system was being fed by a large pressure main at a constant pressure equal to 239.1 Kilopascals and at an elevation equal to 15.2 meters

The Hazen-Williams equation is used for the friction head calculations with a C value equal to 150 for all the pipes. Minor loss coefficients associated with valves, elbows and tees are specified at each of these components. The sprinklers selected for all the simulations have rated conditions of 5.7 liters per second (90 gpm) of discharge at a pressure of 90.5 Kilopascals (30.3 feet) in a 50.8 millimeters line (2 inches). The minor loss coefficient for this type of sprinkler is equal to 23.

An example output of the finite element computer model is shown in Figure 16. An example of the output of the linear theory computer model is shown in Figure 17.

A correlation and linear regression analysis was performed with the linear theory values as the dependent variable and the finite element values as the independent variable. For an exact fit of a model, the intercept and slope of the resulting equations would be 0.0 and 1.0,

```
INPUT PRESSURE (FT) =          80.000
     PUMP HEAD  (FT) =          40.000
INPUT HEIGHT    (FT) =          50.000
NUMBER OF LATERALS   =         8

-----SUBMAIN DATA-----

    DIAMETER   (IN) =           8.00
    CONSTANT  K     =           10.46
H-W FRICTION COEFF. =          150.0
TEES--ELBOWS COEFF. =           2.0
    EXPONENT  M     =          1.852
    SLOPE     %     =          -2

------LATERAL DATA-----

    DIAMETER   (IN) =           2.00
    CONSTANT  K     =           10.46
H-W FIRCTION COEFF. =          150.0
    VALVES COEFF.   =           2.0
    EXPONENT  M     =          1.852
    SLOPE     %     =          0
  LAT SPACING  (FT) =          200.0

-----SPRINKLER DATA----

 SPRINKLER COEFF.    =         16.3820
 SPRINKLER EXPONENT  =         0.5
 SPRINKLER SPAC.(FT) =         100.0
 NO. SPRINKLERS/LAT  =         1


INITIAL HEIGHTS (FT)    INITIAL PRESSURES (FT)
      18.00                65.51
      18.00               136.64
      22.00                63.46
      22.00               132.68
      26.00                61.53      FINAL PRESSURES ADJUSTED W/ HEIGHTS IN
      26.00               128.94         FEET                   KILOPASCALS
      30.00                59.80
      30.00               125.58            89.15                 266.47
      34.00                58.35           134.86                 403.10
      34.00               122.77            86.53                 256.65
      38.00                57.27           130.98                 391.50
      38.00               120.66            84.13                 251.46
      42.00                56.62           127.41                 380.85
      42.00               119.40            82.06                 245.28
      46.00                56.49           124.34                 371.68
      46.00               119.14            80.43                 240.41
      50.00                80.00           121.93                 364.46
          ITERATION  1 .                    79.35                 237.18
          ITERATION  2 .                   120.32                 359.66
          ITERATION  3 .                    78.91                 235.88
          ITERATION  4 .                   119.67                 357.72
                                            79.22                 236.80
                                           120.13                 359.09
                                            80.00                 239.13
                                  SUBMAIN FLOW (GPM)= 1189.90      (l/s)    75.07

                                  SPRINKLER FLOWS
                                   GALLONS PER MINUTE              LITERS PER SECOND
                                          154.68                         9.76

                                          152.39                         9.61

                                          150.26                         9.48

                                          148.40                         9.36

                                          146.92                         9.27

                                          145.93                         9.21

                                          145.53                         9.18

                                          145.81                         9.20
```

Figure 16. Example finite element computer program output.

PIPE NO.	NODE NOS.		LENGTH (FEET)	DIAMETER (INCHES)	ROUGHNESS	MINOR LOSS K	FIXED GRADE
1	0	1	200.0	8.0	150.0	.00	130.00
THERE IS A PUMP IN LINE 1 WITH USEFUL POWER = 12.03							
2	1	0	100.0	2.0	150.0	25.00	46.00
3	1	2	200.0	8.0	150.0	2.00	
4	2	0	100.0	2.0	150.0	25.00	42.00
5	2	3	200.0	8.0	150.0	2.00	
6	3	0	100.0	2.0	150.0	25.00	38.00
7	3	4	200.0	8.0	150.0	2.00	
8	4	0	100.0	2.0	150.0	25.00	34.00
9	4	5	200.0	8.0	150.0	2.00	
10	5	0	100.0	2.0	150.0	25.00	30.00
11	5	6	200.0	8.0	150.0	2.00	
12	6	0	100.0	2.0	150.0	25.00	26.00
13	6	7	200.0	8.0	150.0	2.00	
14	7	0	100.0	2.0	150.0	25.00	22.00
15	7	8	200.0	8.0	150.0	2.00	
16	8	0	100.0	2.0	150.0	25.00	18.00

JUNCTION NUMBER	DEMAND	ELEVATION	CONNECTING PIPES		
1	.00	46.00	1	2	3
2	.00	42.00	3	4	5
3	.00	38.00	5	6	7
4	.00	34.00	7	8	9
5	.00	30.00	9	10	11
6	.00	26.00	11	12	13
7	.00	22.00	13	14	15
8	.00	18.00	15	16	

THE RESULTS ARE OBTAINED AFTER 4 TRIALS WITH AN ACCURACY = .00005

TRIAL NO. 6
8 LATERALS WITH ONE SPR. PER LAT
WITH K=2. COMPONENTS 2% DOWN SLOPE

PIPE NO.	NODE NOS.		FLOWRATE	HEAD LOSS	PUMP HEAD	MINOR LOSS	VELOCITY	HL/1000
1	0	1	1191.00	3.87	39.90	.00	7.60	19.37
2	1	0	145.90	33.92	.00	86.17	14.90	339.21
3	1	2	1045.10	3.04	.00	1.38	6.67	15.21
4	2	0	145.64	33.81	.00	85.86	14.87	338.08
5	2	3	899.46	2.30	.00	1.02	5.74	11.52
6	3	0	146.06	33.99	.00	86.35	14.92	339.88
7	3	4	753.40	1.66	.00	.72	4.81	8.30
8	4	0	147.06	34.42	.00	87.54	15.02	344.21
9	4	5	606.34	1.11	.00	.47	3.87	5.55
10	5	0	148.55	35.07	.00	89.32	15.17	350.68
11	5	6	457.79	.66	.00	.27	2.92	3.30
12	6	0	150.41	35.89	.00	91.57	15.36	358.88
13	6	7	307.38	.32	.00	.12	1.96	1.58
14	7	0	152.54	36.84	.00	94.19	15.58	368.36
15	7	8	154.84	.09	.00	.03	.99	.44
16	8	0	154.84	37.87	.00	97.04	15.81	378.68

JUNCTION NUMBER	DEMAND	GRADE LINE	ELEVATION	PRESSURE
1	.00	166.09	46.00	52.04
2	.00	161.66	42.00	51.85
3	.00	158.34	38.00	52.15
4	.00	155.96	34.00	52.85
5	.00	154.38	30.00	53.90
6	.00	153.46	26.00	55.23
7	.00	153.03	22.00	56.78
8	.00	152.91	18.00	58.46

THE NET SYSTEM DEMAND = .00

SUMMARY OF INFLOWS(+) AND OUTFLOWS(-) FROM FIXED GRADE NODES

PIPE NUMBER	FLOWRATE
1	1191.00
2	-145.90
4	-145.64
6	-146.06
8	-147.06
10	-148.55
12	-150.41
14	-152.54
16	-154.84

THE NET FLOW INTO THE SYSTEM FROM FIXED GRADE NODES = 1191.00
THE NET FLOW OUT OF THE SYSTEM INTO FIXED GRADE NODES = -1191.00

Figure 17. Example linear theory computer program output

respectively. Deviation from these values indicated variations on the accuracy and predictability of the models.

The results of the correlation and linear regression analysis for the junction heads and sprinkler flows, for all the simulations studied under the first evaluation procedure, are presented in Tables 2 and 3 respectively. Figures 18 through 25 show the values of the junction head and sprinkler flow relationships graphically for the four cases selected.

From Tables 2 and 3 and Figures 18 through 25, it can be seen that under the conditions analyzed here there is a high degree of correlation in the junction heads and sprinkler flows between the finite element and linear theory models. The coefficient of determination, R^2, measures the strenght of the linear relationship between the finite element and linear theory models. In this case there was a strong linear association between the models which validated the finite element solution.

The results of the junction heads comparisons between both models is presented in Table 4. This table shows the average percent difference varying from a low of 0.002 percent (for a sprinkler system with a 1 percent up hill slope and with component coefficients equal to 1.0) to a high of 0.097 percent (for a sprinkler system with 2 percent down hill slope and component coefficients equal to 3.0). These results suggest a

Table 2. Results of the correlation and linear regression analysis of the junction heads.

Slope (%)	Component Coeff. K_C	N	Regression Equations	R^2
2	0.0	8	Y = -0.030+1.000*X	1.000
1	0.0	8	Y = -0.010+1.000*X	1.000
0	0.0	8	Y = -0.244+1.001*X	1.000
-1	0.0	8	Y = 0.073+1.000*X	1.000
-2	0.0	8	Y = 0.004+1.000*X	1.000
2	1.0	8	Y = 0.056+1.000*X	1.000
1	1.0	8	Y = 0.132+0.999*X	1.000
0	1.0	8	Y = 0.255+0.999*X	1.000
-1	1.0	8	Y = -0.186+1.001*X	1.000
-2	1.0	8	Y = -0.162+1.001*X	1.000
2	2.0	8	Y = 0.204+0.999*X	1.000
1	2.0	8	Y = 0.390+0.998*X	1.000
0	2.0	8	Y = 1.248+0.995*X	1.000
-1	2.0	8	Y = -1.028+1.005*X	1.000
-2	2.0	8	Y = -0.631+1.003*X	1.000
2	3.0	8	Y = 0.453+0.998*X	1.000
1	3.0	8	Y = 0.902+0.996*X	1.000
0	3.0	8	Y = 3.114+0.986*X	1.000
-1	3.0	8	Y = -2.028+1.009*X	1.000
-2	3.0	8	Y = -1.297+1.006*X	1.000

Table 3. Results of the correlation and linear regression analysis of the sprinkler flows

Slope (%)	Component Coeff. K_c	N	Regression Equations	R^2
2	0.0	8	Y = 0.015+0.999*X	1.000
1	0.0	8	Y = -0.042+1.006*X	1.000
0	0.0	8	Y = 0.124+0.984*X	0.998
-1	0.0	8	Y = -0.173+1.023*X	0.999
-2	0.0	8	Y = 0.028+0.997*X	1.000
2	1.0	8	Y = -0.013+1.002*X	1.000
1	1.0	8	Y = -0.034+1.006*X	1.000
0	1.0	8	Y = 0.115+0.985*X	0.998
-1	1.0	8	Y = -0.170+1.023*X	0.999
-2	1.0	8	Y = -0.097+1.013*X	1.000
2	2.0	8	Y = -0.012+1.003*X	1.000
1	2.0	8	Y = 0.056+0.992*X	1.000
0	2.0	8	Y = 0.085+0.989*X	0.999
-1	2.0	8	Y = -0.121+1.017*X	0.999
-2	2.0	8	Y = 0.010+1.000*X	1.000
2	3.0	8	Y = 0.010+1.000*X	1.000
1	3.0	8	Y = 0.010+1.000*X	1.000
0	3.0	8	Y = 0.100+0.987*X	0.999
-1	3.0	8	Y = 0.093+0.989*X	0.998
-2	3.0	8	Y = -0.062+1.010*X	0.999

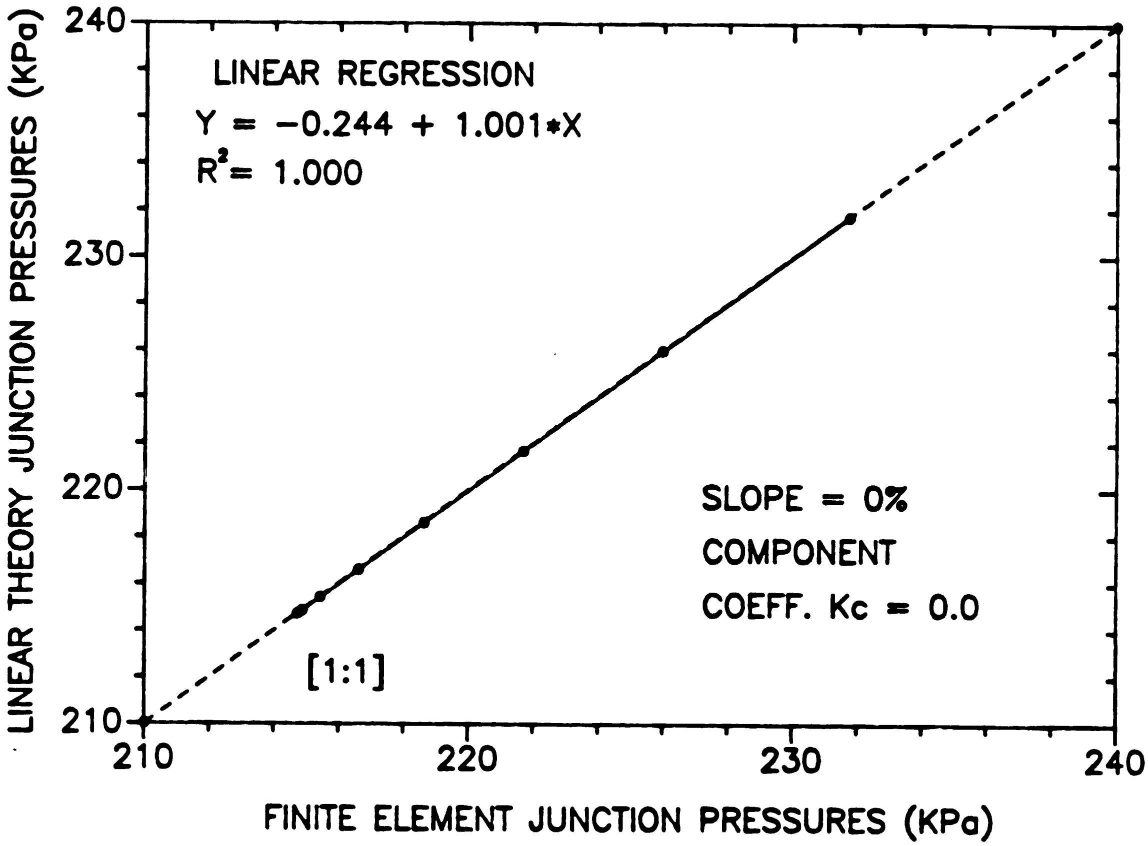

Figure 18. Comparison of finite element and linear theory solution for junction heads with 0 percent slope and Kc equal to 0.0.

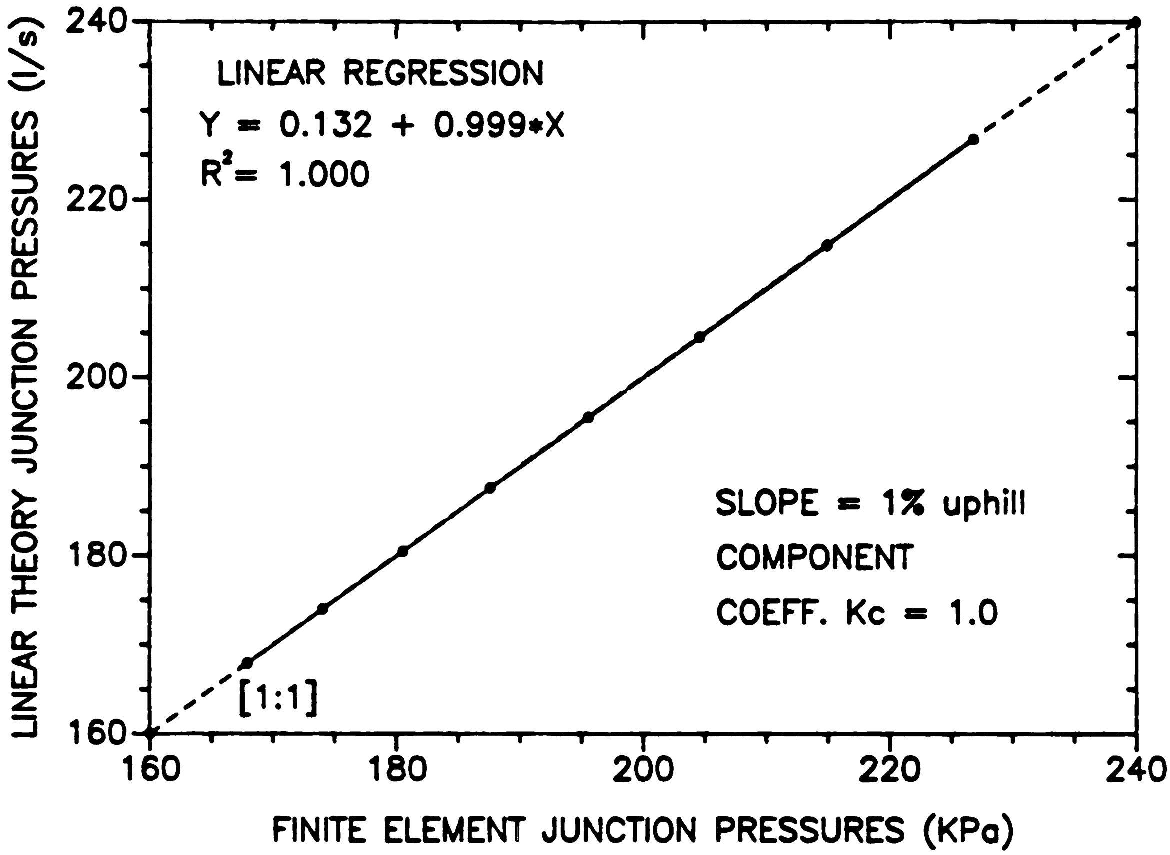

Figure 19. Comparison of finite element and linear theory solution for junction heads with 1 percent upslope and Kc equal to 1.0.

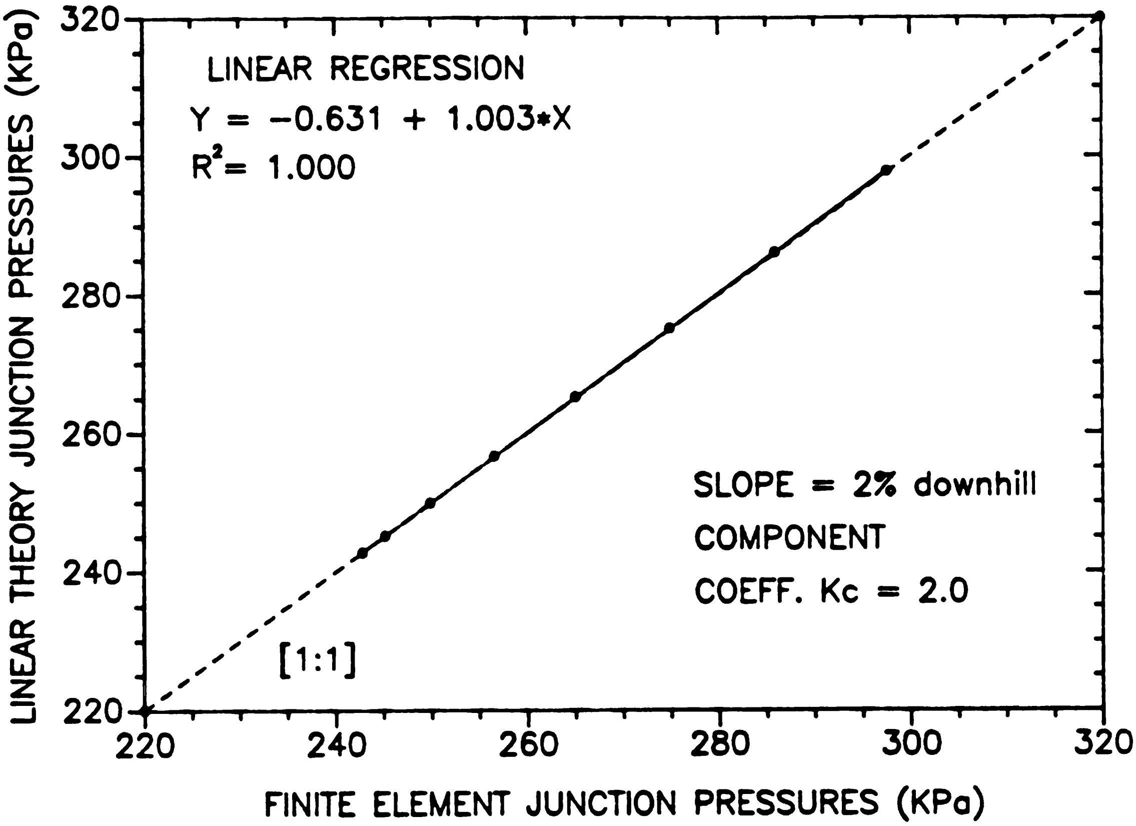

Figure 20. Comparison of finite element and linear theory solution for junction heads with 2 percent downslope and Kc equal to 2.0.

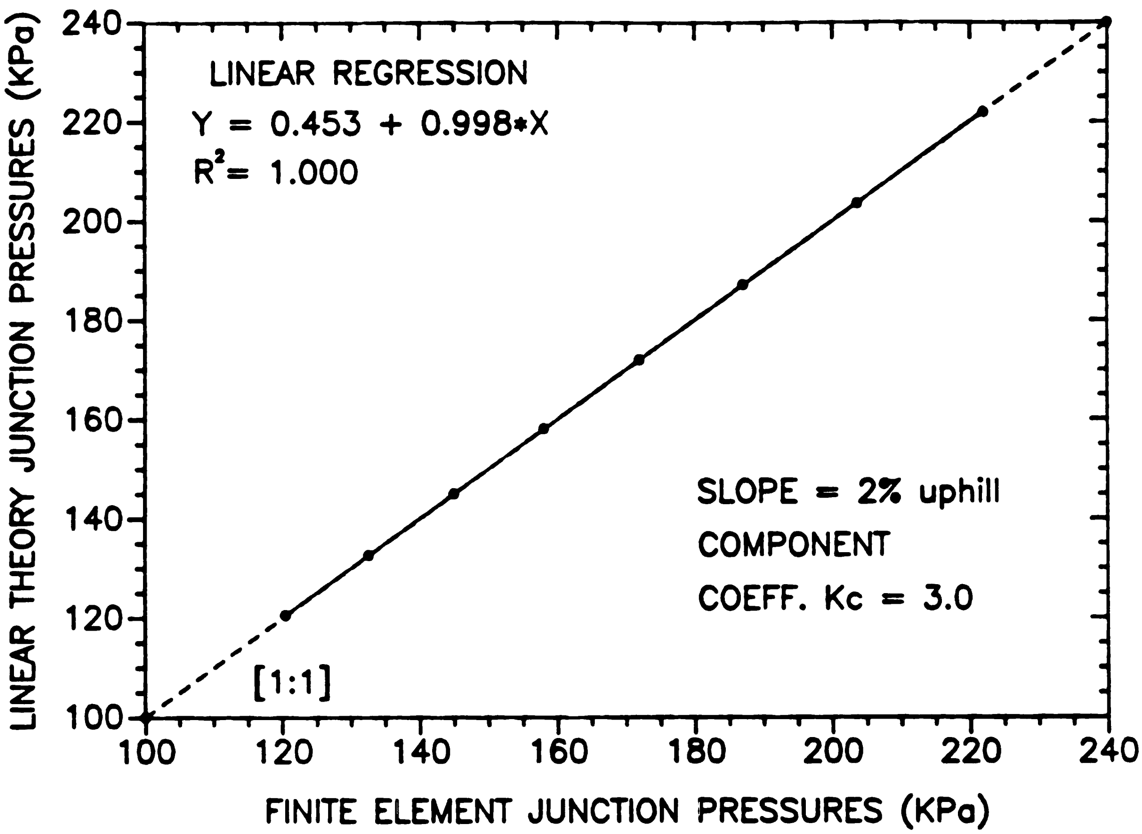

Figure 21. Comparison of finite element and linear theory solution for junction heads with 2 percent upslope and Kc equal to 3.0.

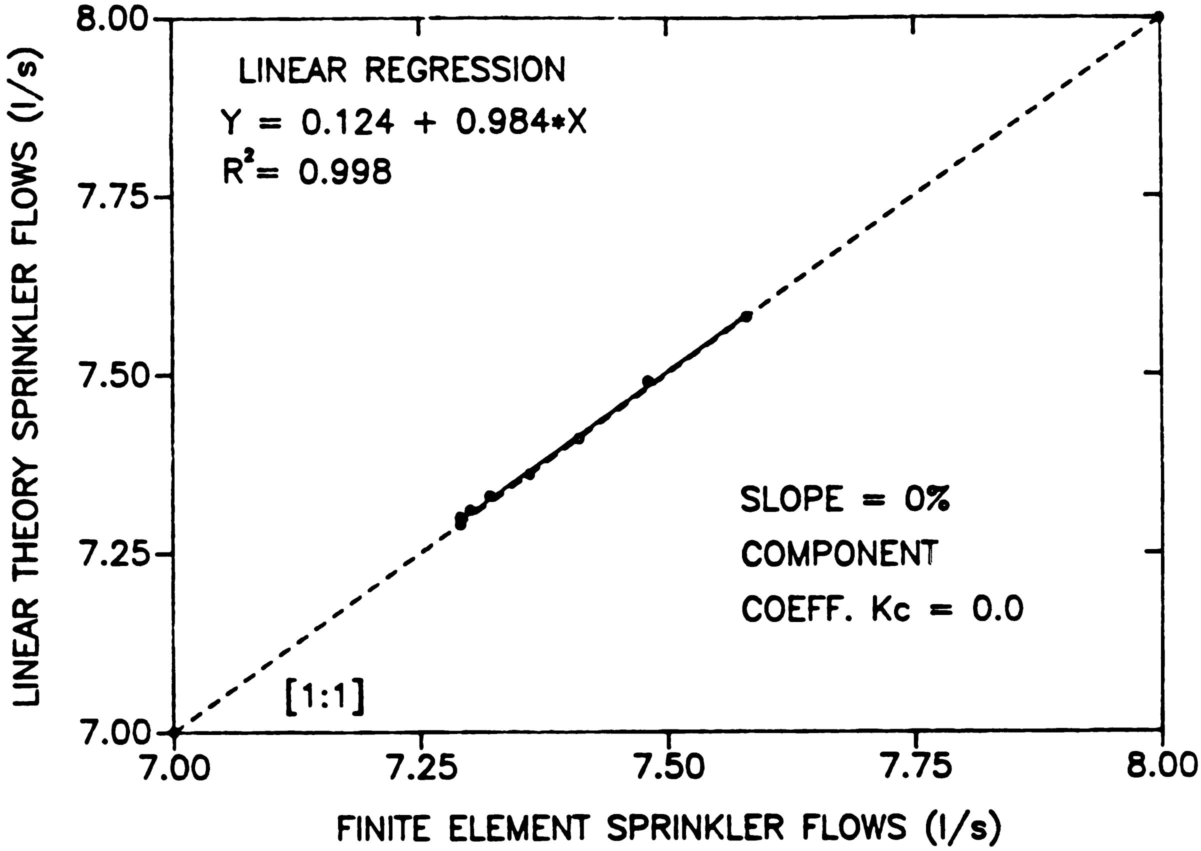

Figure 22. Comparison of finite element and linear theory solution for sprinkler flows with 0 percent slope and Kc equal to 0.0.

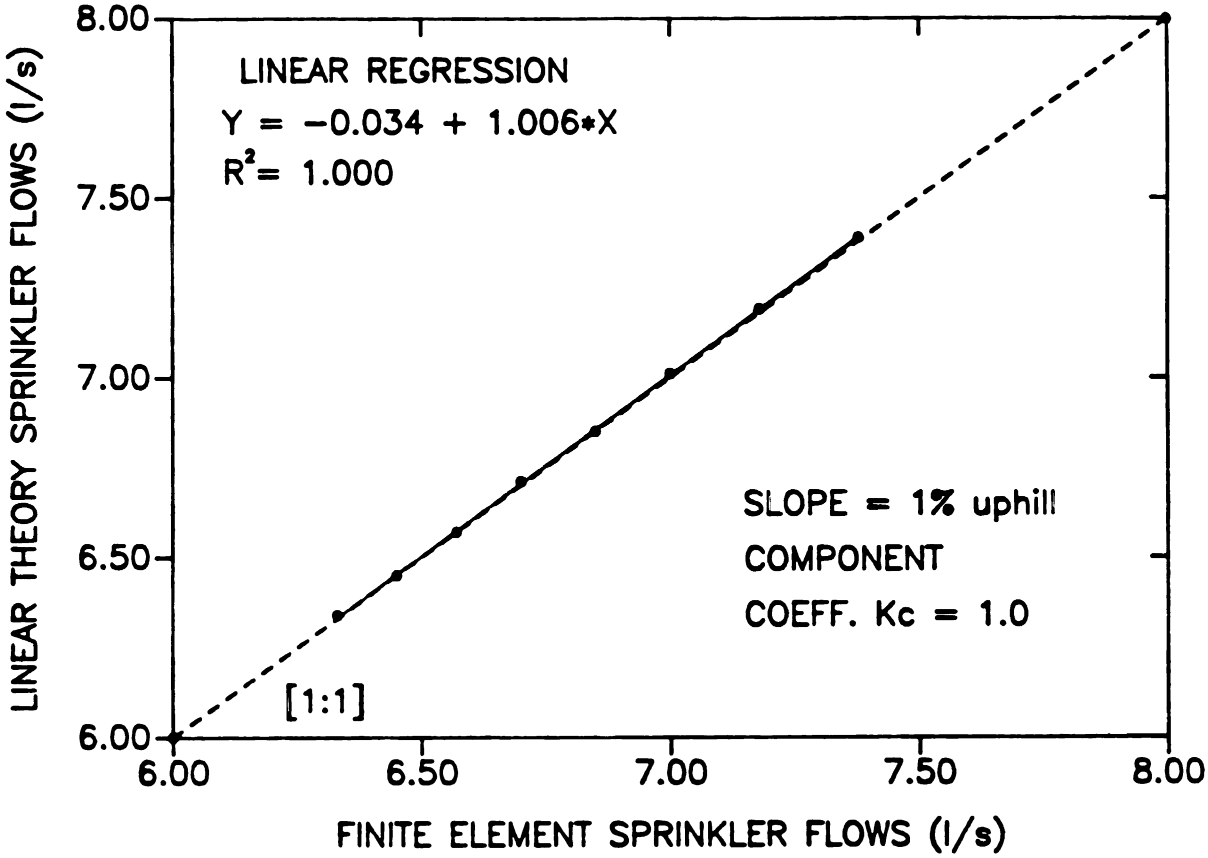

Figure 23. Comparison of finite element and linear theory solution for sprinkler flows with 1 percent upslope and Kc equal to 1.0.

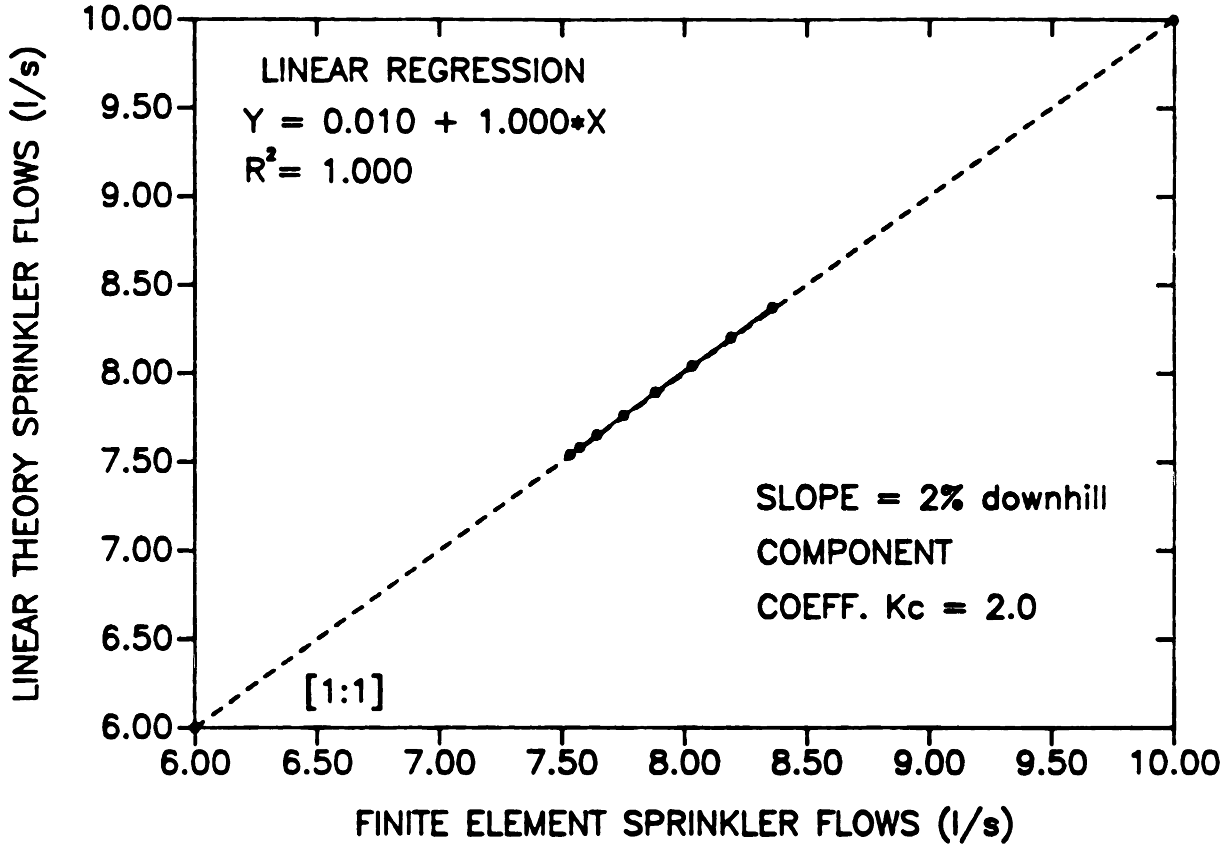

Figure 24. Comparison of finite element and linear theory solution for sprinkler flows with 2 percent downslope and Kc equal to 2.0.

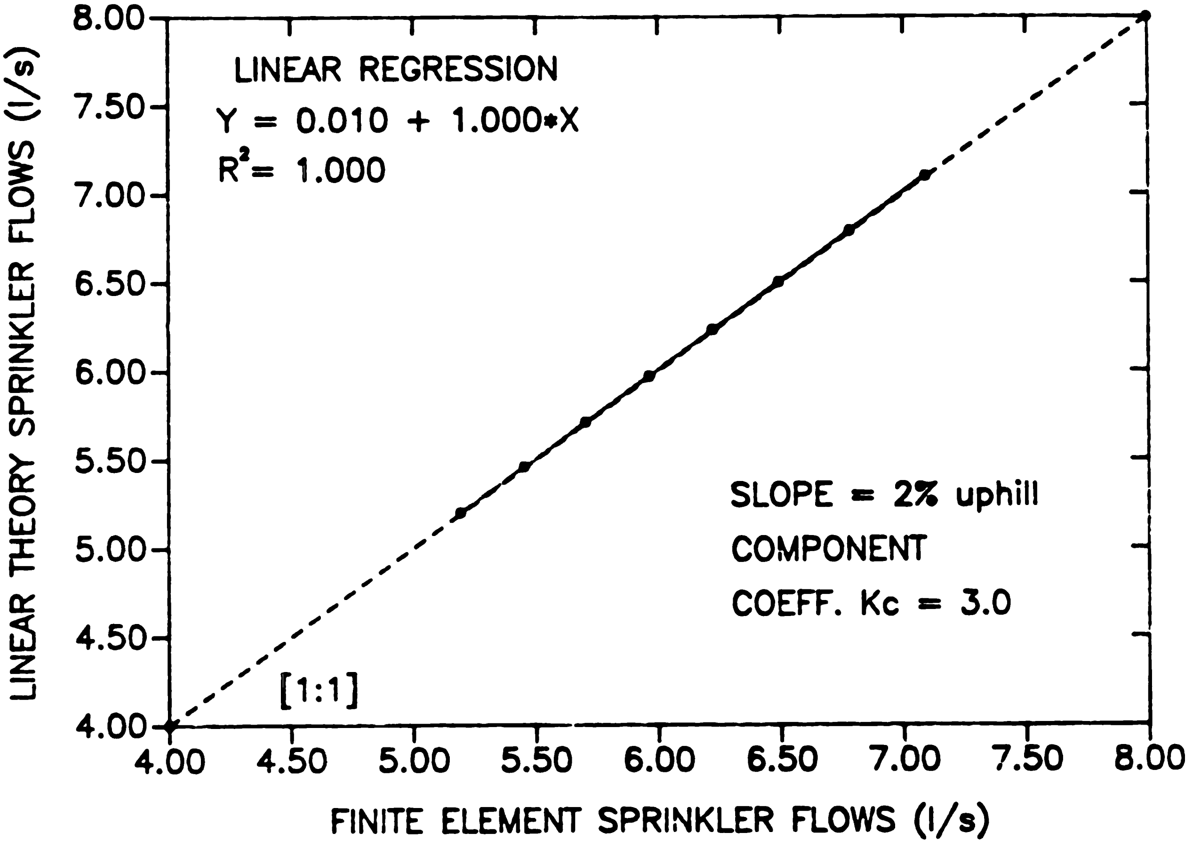

Figure 25. Comparison of finite element and linear theory solution for sprinkler flows with 2 percent upslope and Kc equal to 3.0.

Table 4. Comparison of junction heads between the finite element and linear theory models

SLOPE (%)	K_C	AVERAGE PERCENT DIFFERENCE
2	0.0	0.011
1	0.0	0.004
0	0.0	0.005
-1	0.0	0.006
-2	0.0	0.008
2	1.0	0.004
1	1.0	0.002
0	1.0	0.012
2	1.0	0.004
-2	1.0	0.007
2	2.0	0.035
1	2.0	0.035
0	2.0	0.041
-1	2.0	0.042
-2	2.0	0.047
2	3.0	0.078
1	3.0	0.084
0	3.0	0.087
-1	3.0	0.095
-2	3.0	0.097

maximum percent difference or error for the junction heads in the order of 0.1 percent. This is probably due to the accuracy of the initial estimates and to the convergence criteria employed by each model. Also, this error can be partially attributed to the different methods employed by each model to solve the final system of equations.

The results of the total sprinkler flow comparisons between both models is presented in Table 5. This table shows the percent difference varying from a low of 0.031 percent (for a sprinkler system with a 2 percent down hill slope and with component coefficients equal to 0.0) to a high of 0.194 percent (for a sprinkler system with 2 percent down hill slope and a component coefficient equal to 3.0). These results suggest a minimum degree of variation between the sprinkler discharges in each model.

The results of Table 5 also show the components contribution to the sprinkler system design. As the component coefficient Kc is increased, the total system flow rate decreases. Both models simulate this condition with a high degree of accuracy.

Figures 26 and 27 show the hydraulic grade line for two of the cases studied; up hill and down hill respectively. These figures demonstrate the effects on the hydraulic gradient line due to the main line slope, the pressure on the laterals and the system components.

It is apparent from Figure 26 that the pressure loss

Table 5. Comparison of sprinkler flow rate between the finite element and linear theory models

		SYSTEM FLOW RATE (liters/second)		
SLOPE (%)	K_C	FINITE ELEMENT MODEL	LINEAR THEORY MODEL	PERCENT DIFF.
2	0.0	51.69	51.71	0.039
1	0.0	55.53	55.55	0.036
0	0.0	59.05	59.07	0.034
-1	0.0	62.34	62.36	0.032
-2	0.0	65.43	65.45	0.031
2	1.0	50.71	50.74	0.059
1	1.0	54.47	54.50	0.055
0	1.0	57.92	57.95	0.052
-1	1.0	61.13	61.17	0.065
-2	1.0	64.16	64.19	0.047
2	2.0	49.78	49.83	0.100
1	2.0	53.45	53.51	0.112
0	2.0	56.83	56.89	0.106
-1	2.0	59.98	60.04	0.100
-2	2.0	62.94	63.01	0.111
2	3.0	48.88	48.96	0.164
1	3.0	52.48	52.57	0.171
0	3.0	55.79	55.89	0.179
-1	3.0	58.87	58.98	0.187
-2	3.0	61.77	61.89	0.194

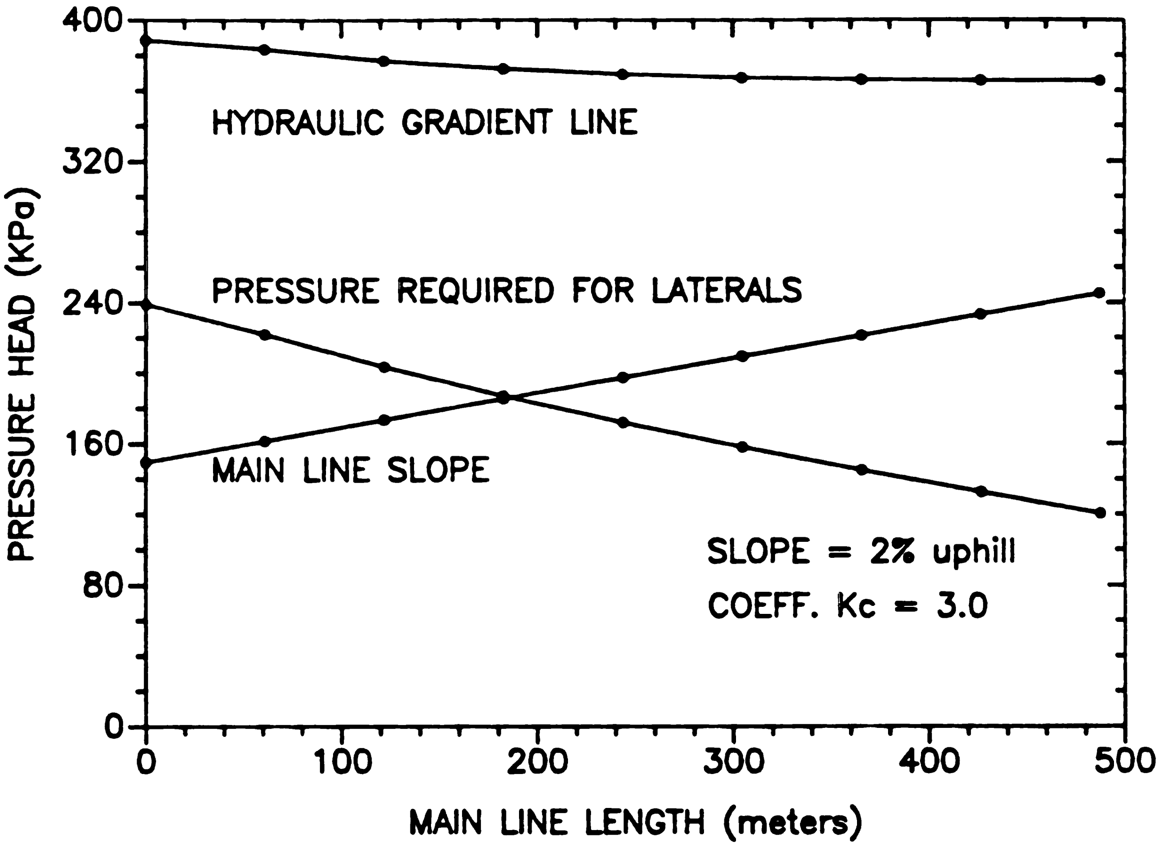

Figure 26. Hydraulic grade line representation for 2 percent uphill and Kc equal to 3.0.

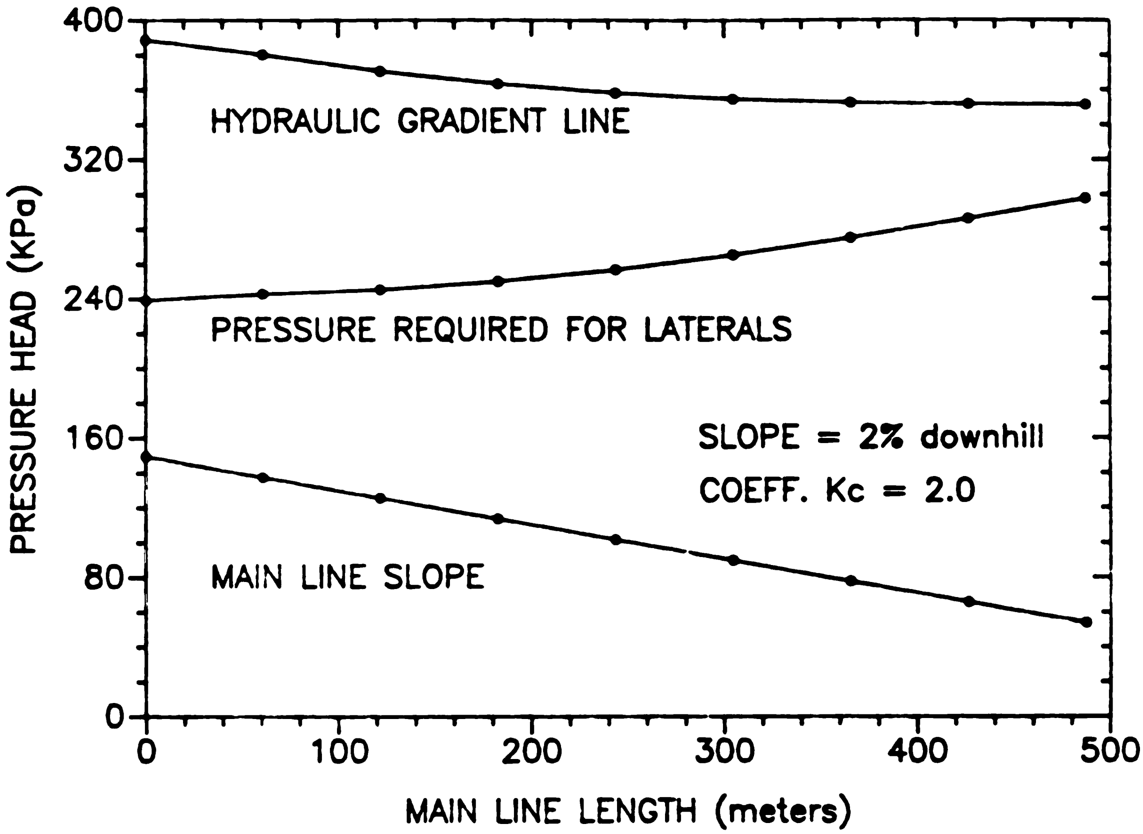

Figure 27. Hydraulic grade line representation for 2 percent slope downhill and Kc equal to 2.0.

due to the combination of the friction in the lines and the system components is less than the gain in pressure due to the slope of the main line. This result suggest a hydraulic gradient line with a gradual slope along the main line length.

From figure 27, it can be seen that the combination of uphill slope and pressure loss due to friction reach an equlibrium point at approximately 180 meters. The hydraulic gradient line in this case is more horizontal than in the previous Figure suggesting larger junction head pressures towards the end of the sprinkler system.

The second evaluation procedure was a comparison of both models using the same system layout but with the addition of a pump in line 1 between nodes 16 and 17. Figure 28 shows the sprinkler irrigation system layout and the components.

The pump selected in this part of the evaluation procedure was capable of providing a pressure head equivalent to 119.6 Kilopascals (40 feet), in addition to the existing conditions of head and elevation in the system.

Tables 6 and 7 show the results of the correlation and linear regression analysis of the junction heads and sprinkler flows respectively, with a pump in the system.

Figures 29 through 32 show this relationships in a graphical form for the four cases selected. From these Tables and Figures it is demonstrated the high degree of

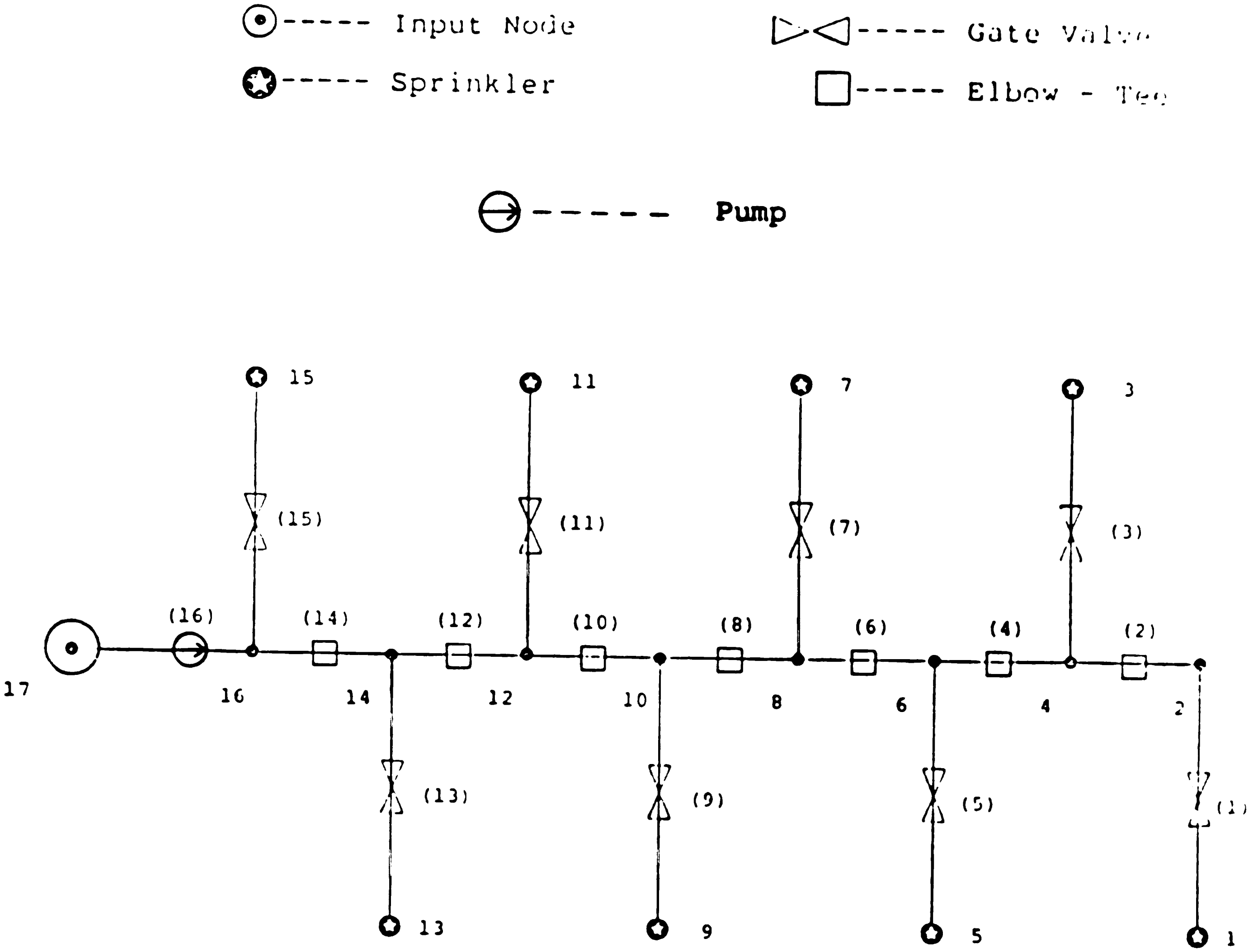

Figure 28. Sprinkler irrigation system layout with a pump.

Table 6. Results of the correlation and linear regression analysis of the junction heads with a pump in the system.

Slope (%)	Component Coeff. K_C	N	Regression Equations	R^2
2	3.0	8	Y = 0.977+0.997*X	1.000
1	1.0	8	Y = -0.792+1.003*X	1.000
0	0.0	8	Y = -0.991+1.003*X	1.000
-2	2.0	8	Y = -1.739+1.005*X	1.000

Table 7. Results of the correlation and linear regression analysis of the sprinkler flows with a pump in the system

Slope (%)	Component Coeff. K_C	N	Regression Equations	R^2
2	3.0	8	Y = 0.045+0.996*X	1.000
1	1.0	8	Y = -0.047+1.006*X	1.000
0	0.0	8	Y = -0.015+1.002*X	1.000
-2	2.0	8	Y = -0.097+1.011*X	1.000

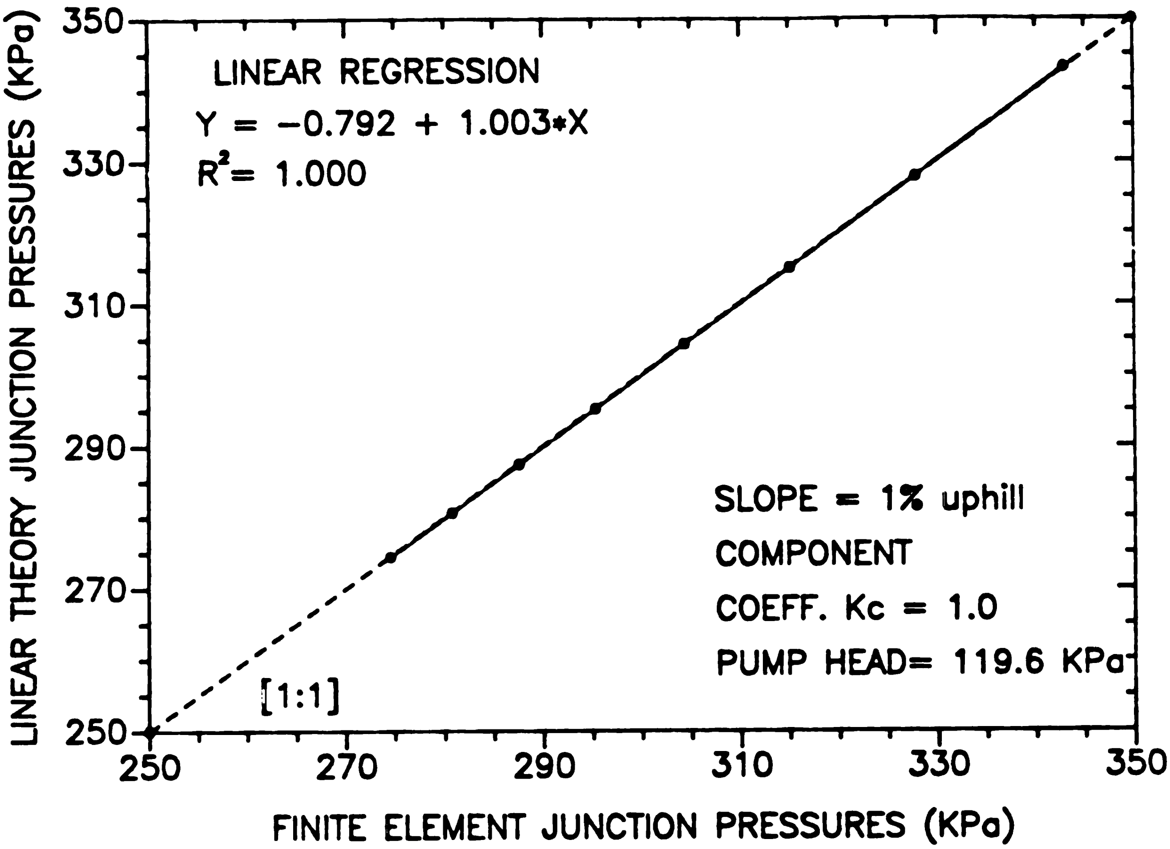

Figure 29. Comparison of finite element and linear theory solution for junction heads with 1 percent upslope and Kc equal to 1.0 with a pump in the system

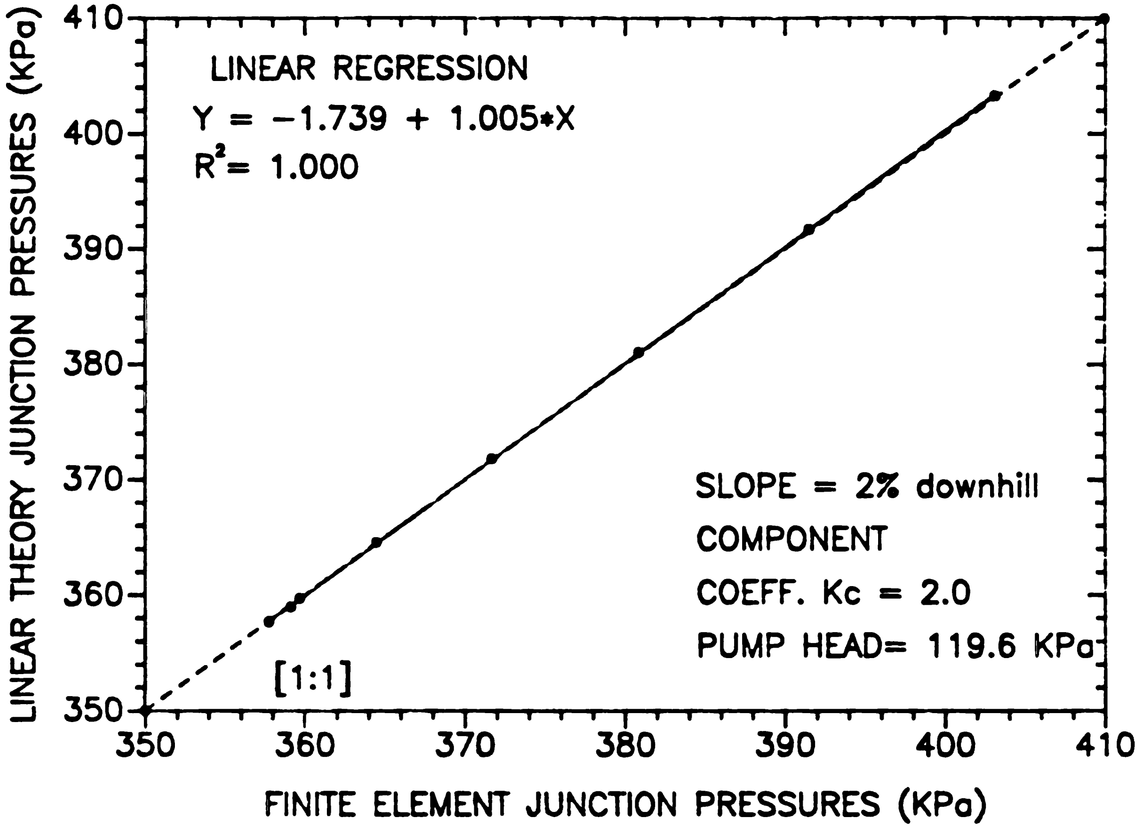

Figure 30. Comparison of finite element and linear theory solution for junction heads with 2 percent downslope and Kc equal to 2.0 with a pump in the system.

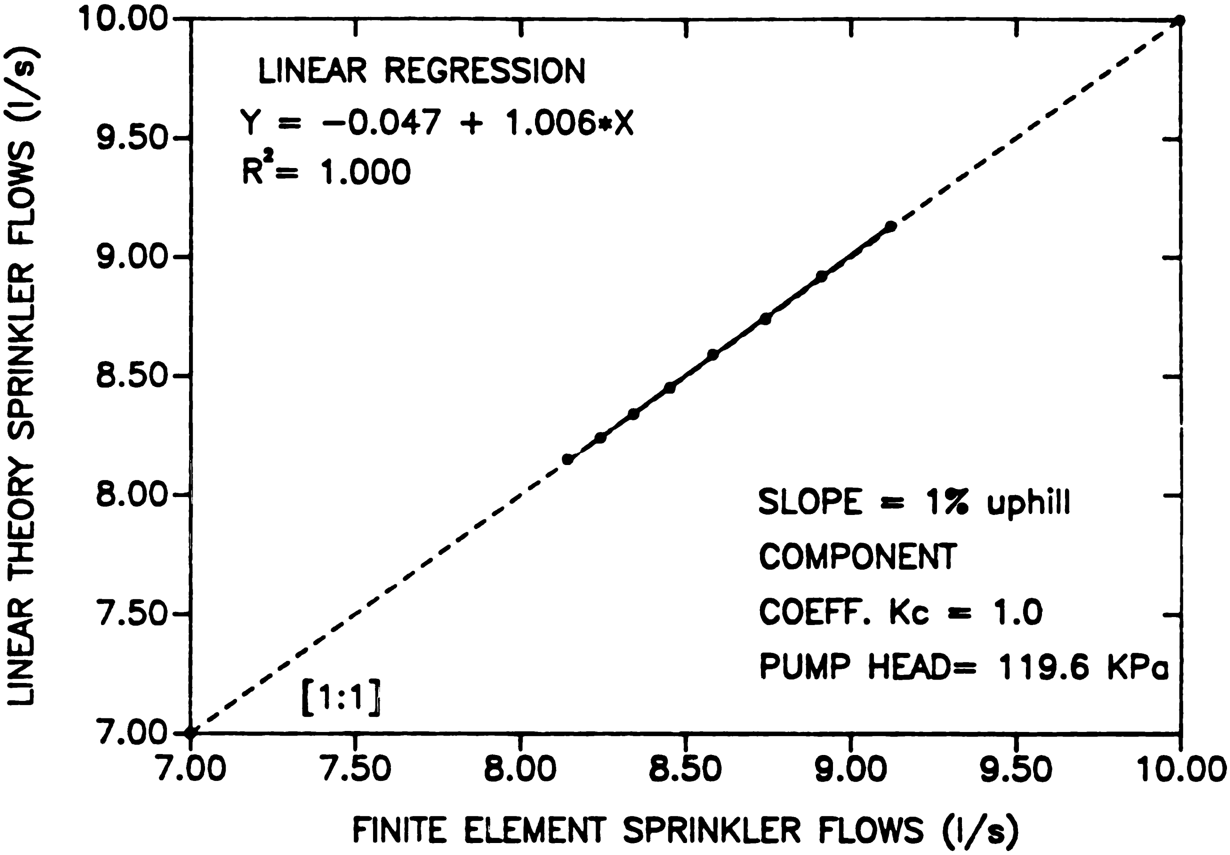

Figure 31. Comparison of finite element and linear theory solution for sprinkler flows with 1 percent upslope and Kc equal to 1.0 with a pump in the system

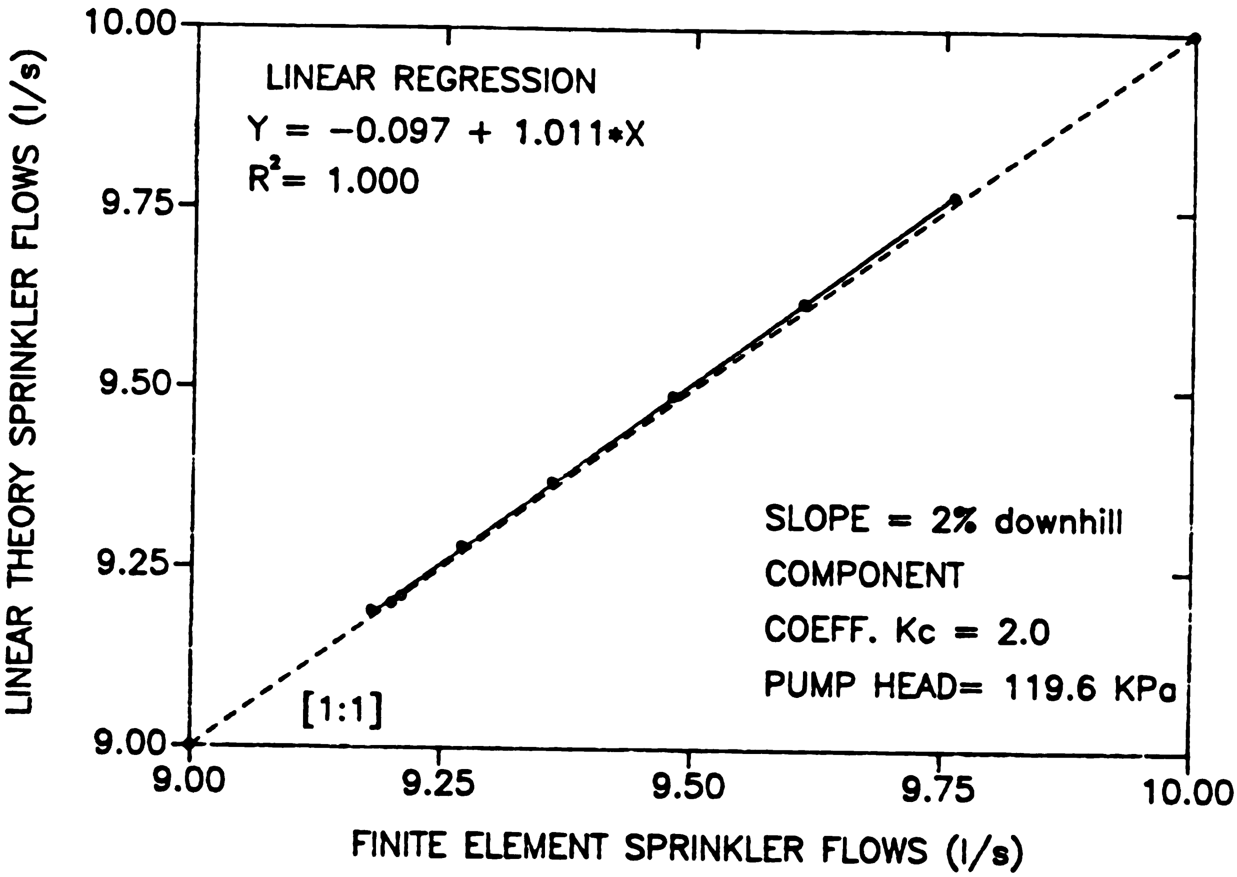

Figure 32. Comparison of finite element and linear theory solution for sprinkler flows with 2 percent downslope and Kc equal to 2.0 with a pump in the system

correlation between the finite element and linear theory solutions.

Table 8 shows the comparison between the junction heads for each model with a pump in the system. The results of this table suggest that, for the four cases selected, the percent difference or error did not exceed 0.022 percent. This corroborates the assumption that a solution can be considered to compare favorably if the percent difference between the junction heads did not exceed 0.1 percent.

Table 9 shows the comparison between the total sprinkler flow for each model with a pump in the system. The highest percent difference or error between the models was found to be equal to 0.142 for a system with a 2 percent up hill slope and with component coefficients equal to 3.0. This value seems to be in line with the assumption that a reasonable solution can be obtained provided the sprinkler flows did not varied by more than 0.2 percent.

Figures 33 and 34 show the hydraulic grade line for the up hill and down hill cases with a pump in the system. These figures demonstrate the effects of the pressure added by the pump to the system. The impacts of the overall increase of pressure in the system is reflected in the higher level attained by the hydraulic grade line.

Table 8. Comparison of junction heads between the finite element and linear theory models with a pump in the system

SLOPE (%)	K_c	AVERAGE PERCENT DIFFERENCE
2	3.0	0.022
1	1.0	0.002
0	0.0	0.003
-2	2.0	0.019

Table 9. Comparison of sprinkler flow rate between the finite element and linear theory models with a pump in the system

		SYSTEM FLOW RATE (liters/second)		
SLOPE (%)	K_c	FINITE ELEMENT MODEL	LINEAR THEORY MODEL	PERCENT DIFF.
2	3.0	63.16	63.25	0.142
1	1.0	68.52	68.56	0.058
0	0.0	72.72	72.75	0.041
-2	2.0	75.07	75.14	0.093

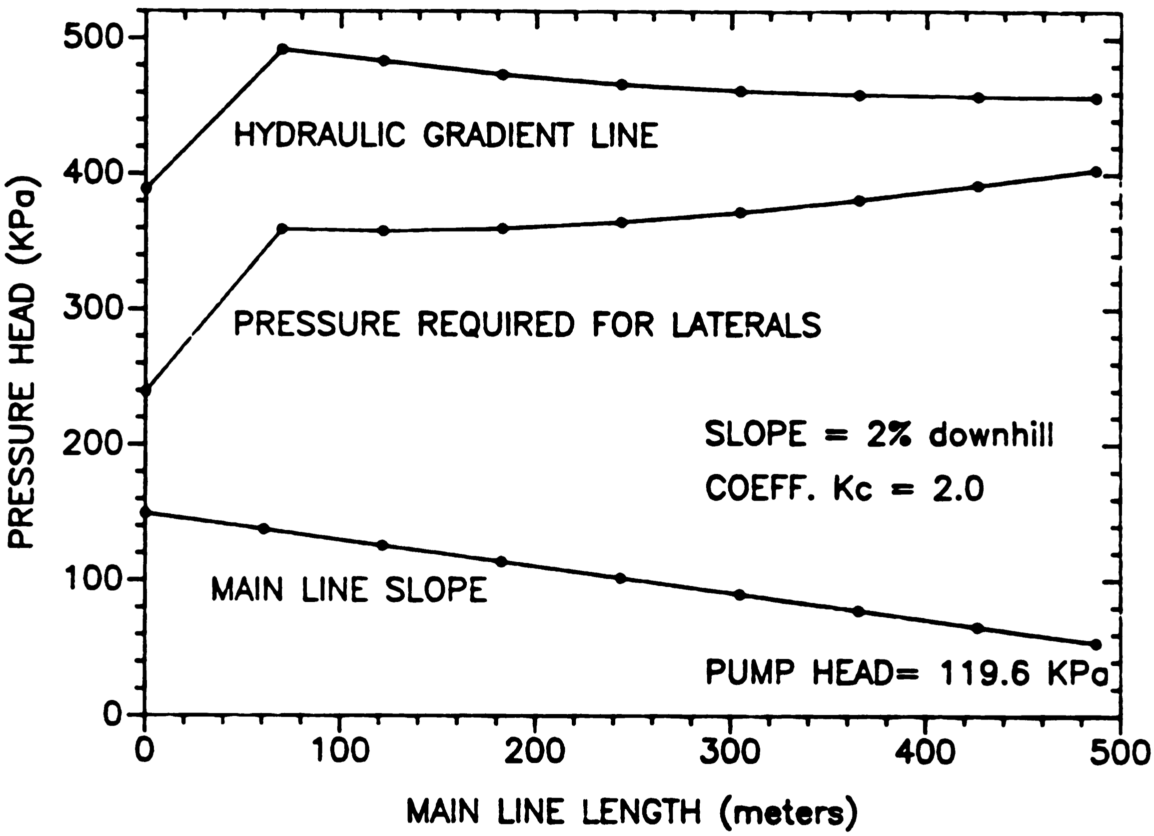

Figure 33. Hydraulic grade line representation for 2 percent downslope with Kc equal to 2.0 with a pump in the system.

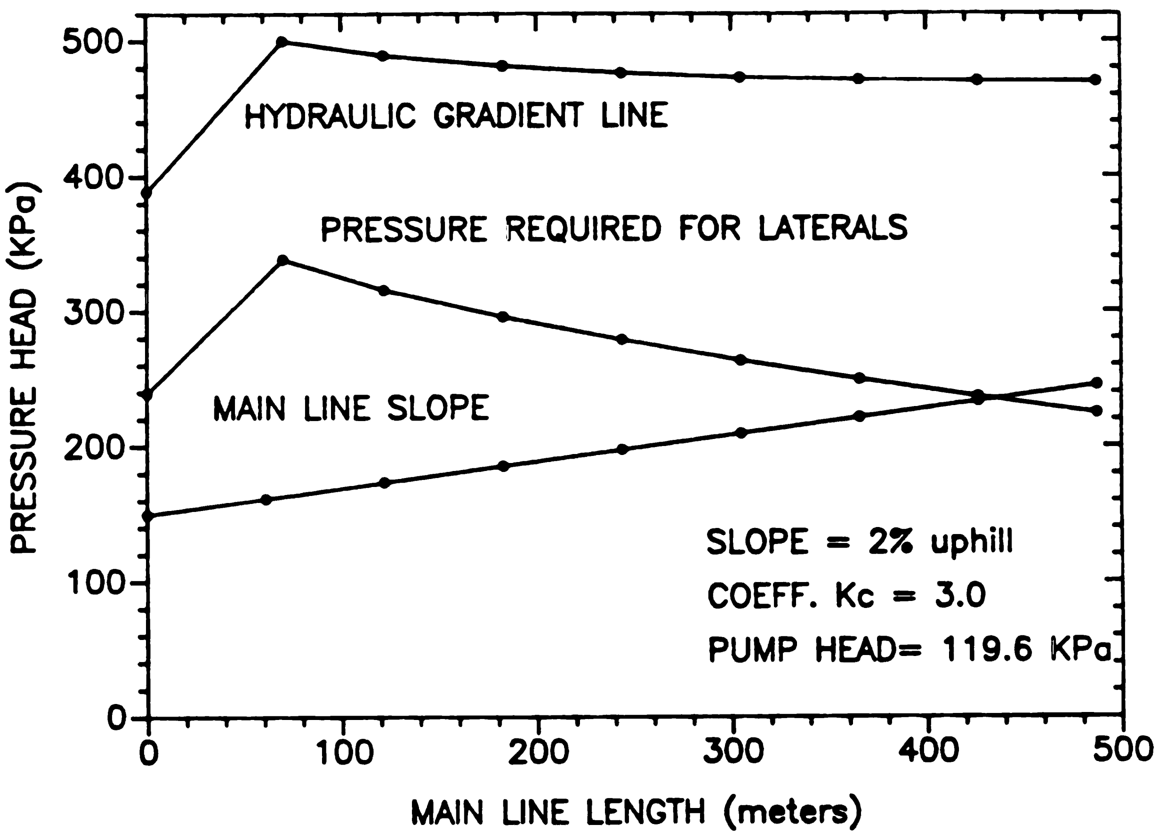

Figure 34. Hydraulic grade line representation for 2 percent upslope with Kc equal to 3.0 with a pump in the system.

B. Accuracy and Reliability

The reliability of both computer models was examined by evaluating a large number of sprinkler irrigation designs employing each algorithm and assessing the consistency of each model in arriving to a solution. Under the 24 simulations studied, an acceptable solution was always attained.

The accuracy of a solution is a measure of the exactness to which the basic hydraulic equations are satisfied for a given irrigation design. The accuracy of the computer model solutions was determined by comparing the sprinkler flows and the junction head. The solutions were considered to compare favorably if the percent difference between sprinkler flow rates did not exceed 0.2 percent and the percent difference between junction heads did not exceed 0.1 percent. This criteria was susscessfully meet in all the simulations studied as shown in Tables 5 and 9 for the sprinkler flows and in Tables 4 and 8 for the junction heads.

The convergence criteria is a measure of the speed at which a relatively stable solution can be obtained. In this particular case, the convergence criteria employed by each computer model was different. The linear theory computer model employed the change in flowrate between successive trials to check for convergence. The specific convergence criteria employed in this method is referred as the relative accuracy. This criteria roughly states that when the average change in flow rates between successive trials is less than 0.5 percent, the calculation cease. Although this criteria is more stringent than others normally applied in practice, it does not however, assure that the flow rates are within 0.5 percent of the correct values.

The finite element method arrives at a solution with a relative accuracy of 0.03 Kilopascals (0.01 feet). The model uses the criteria that the sum of the total pressure variations in all the nodes has changed less than a predetermined relative accuracy, in this case 0.03 Kilopascals.

C. Summary Discussion

The results and discussion section have demonstrated the capabilities of the finite element computer model to simulate a wide variety of solid set sprinkler irrigation systems. The results comparing the finite element computer model (SPIRR-FE) and the linear theory model were very positive. The results strongly support the use of the dimensionless energy gradient line concept for initial estimating the junction pressures and elevations.

The reliability of the algorithm employed in the irrigation system solution is of great importance. Failure to obtain a solution is an inconvenience and the failure to recognize a poor solution may be even a greater problem because this may lead to poor design or management of the irrigation system.

No limitations to the SPIRR-FE computer program were found for the range of analysis reported. The results of the above comparisons clearly show the validity of the finite element solution.

V. CONCLUSIONS AND RECOMENDATIONS

The objectives of the proposed research have been addressed in full. A computer program which uses the finite element method to model essential sprinkler irrigation system components was developed. The computer program was compared to a linear theory model which demonstrated the applicability of the finite element formulation to the design of sprinkler irrigation systems. The computer based finite element solution results in a symmetric banded matrix which greatly reduces the computer storage.

The specific conclusions of the research are:

1. The finite element method can be used to model sprinkler irrigation system components such as elbows, tees, gate valves, sprinklers and pumps.

2. The finite element based computer model, when compared to a linear theory model, showed very good correlation for junction pressures and sprinkler flows.

3. The finite element method converged to reliable solutions provided reasonable initial pressure estimates were obtained by employing the dimensionless gradient line concept.

Recommendations for further research include:

1. Extend the finite element methodology to cover hydraulic network analysis including systems with loops.

2. Apply optimization techniques to the finite element model to obtain the best design possible given a set of initial conditions.

APPENDICES

APPENDIX A

TAYLOR SERIES EXPANSION

TAYLOR SERIES EXPANSION

In the Review of Literature section all the methods presented in the analysis of pipe networks employed some form of linearizing the nonlinearities associated with the energy equations. One of the most widely used linearizing technique employed in these methods is the Taylor series expansion. The mathematical basis of this technique suggests that if a power series $\Sigma x_{(n)} (x-x_{(n)})$ converges to a function f(x) then the coefficients of the power series should be determined by the values of that function and its successive derivatives.

Applying this principle to the Newton-Raphson method suggests that in order to arrive to a solution of f(x) = 0, it is necessary to determine the tangent of the curve at the first approximation $(x_{(n)})$. Mathematically the evaluation of f(x) = 0 can be written as

$$f(x) = f(x_{(n)}) + (x-x_{(n)})f'(x_{(n)} + (x-x_{(n)})^2 \frac{f''(\zeta)}{2} \quad [A.1]$$

where $\zeta_{(n)}$ lies between $x_{(n)}$ and x. Solving for x results in

$$x = x_{(n)} - \frac{f(x_{(n)})}{f'(x_{(n)})} - (x-x_{(n)})^2 \frac{f''(\zeta)}{2f'(x_{(n)})} \quad [A.2]$$

or

$$x = x_{(n+1)} - (x-x_{(n)})^2 \frac{f''(\zeta)}{2f'(x_{(n)})} \qquad [A.3]$$

In this case the error of the (n+1) estimate is proportional to the square of the previous error.

APPENDIX B

SPIRR-FE PROGRAM CODE

```
{*******************************************************}
          PROGRAM SPIRR-FE(INPUT,LSST);

                        { SPIRRE-FE
THIS PROGRAM CALCULATES THE JUNCTION HEADS AND SPRINKLER
FLOW RATES OF A SPRINKLER IRRIGATION SYSTEM EMPLOYING
THE FINITE ELEMENT FORMULATION.  THE COMPUTER PROGRAM WAS
WRITTEN IN TURBO-PASCAL.

                           By

               LUIS ALFONSO SALDIVIA
             MICHIGAN STATE UNIVERSITY
        AGRICULTURAL ENGINEERING DEPARTMENT
                         1988

 {*******************************************************}


          VARIABLES USED IN THE PROGRAM:


A(1400,2)--THE MAIN DIAGONAL OF THE STIFFNESS MATRIX
A(1400,1)--THE OFF DIAGONAL OF THE STIFFNESS MATRIX
DELTAHP(1400) - STORES CHANGES IN PRESSURES AND ELEV.
DIFF(1400) - STORES PRESSURES FROM PREVIOUS ITERATIONS
             TO CHECK FOR CONVERGENCE
EDIFF - UPPER LIMIT CONVERGENCE CRITERIA
HEIGHT--ELEVATION OF THE INPUT NODE
HIN--PRESSURE OF THE INPUT NODE
HINP - PUMP HEAD
ITERDELTA - DIFFERENCE IN PRESSURES BETWEEN ITERATIONS
                COVERGENCE CRITERIA
ITS - NUMBER OF ITERATIONS
KS - SPRINKLER CONSTANT
LATC - LATERAL FRICTION CONSTANT
LATD - LATERAL DIAMETER
LATK - LATERAL CONSTANT
LATM - LATERAL EXPONENT
LATS - LATERAL SPACING
M - SUBMAIN EXPONENT
NEL - NUMBER OF SPRINKLERS PER LATERAL
NL - NUMBER OF LATERALS
P(1400) - STORES THE JUNCTION NODES OF THE SPRINKLER
          IRRIGATION SYSTEM
RHS(700) - STORES THE VALUES OF THE RIGHT HAND SIDE
           OF THE SOLUTION EQUATIONS
Q(1400) - STORES THE CHANGES IN FLOW RATE USED TO
          CALCULATED THE COMPONENT CONTRIBUTION
```

```
SLOPS - SUBMAIN SLOPE
SLOPL - LATERAL SLOPE
SPS - SPRINKLER SPACING
SUBC - SUBMAIN FRICTION CONSTANT
SUBD - SUBMAIN DIAMETER
SUBK - SUBMAIN CONSTANT
UNK - NUMBER OF UNKNOWS
XS - SPRINKLER EXPONENT
Y(1400) - STORES THE SPRINKLER FLOWS

*****************************************************}


            {VARIABLE DECLARATION SECTION}

 CONST ARRAY_SIZE = 700;
       ARRAY_SIZE2 = 1400;
 TYPE  ARRAY_TYPE = ARRAY[1..ARRAY_SIZE2] OF REAL;
 VAR   FILENAME,FILENAM:STRING[20];
       LSST,INFILE:TEXT;
       A:ARRAY[1..ARRAY_SIZE,1..2] OF REAL;
       DELTAHP,DIFF,P,Y,Q:ARRAY_TYPE;
       ITERDELTA,EDIFF:REAL;
       ITS,UNK,UNK1:INTEGER;
       RHS:ARRAY[1..ARRAY_SIZE] OF REAL;
       KS,XS:REAL;
       TK,TX:REAL; {TEMPORARIES}
       KLAT,KSUB,KLAT1,KSUB1:REAL;
       SLOPLL,SPS:REAL;
       LATS,LATC,LATD,LATK,LATM,SLOPL:REAL;
       NL,NEL,NEL1,NSP,NLP:INTEGER;
       TSLOPS,TSLOPLL:INTEGER;
       HEIGHT,HIN,HINP:REAL;
       SLOPS,SUBD,SUBC,SUBK:REAL;
       BIAS,MUNK,ID,MP,LNO,I,J,K,CH:INTEGER;
       PTEMP,CTEMP,CTEMP1,CTEMP3,EE4,CONSTM,M:REAL;
       KCOML,COMKL,DFL,KCOMS,COMKS,DFS:REAL;
       SUML,MEANL,SUBFLOW:REAL;
       NOL,OFFSET:INTEGER;

             {END THE DECLARATION OF VARIABLES}

{****************************************************}

         {PROCEDURE TO OPEN A FILE TO READ DATA FROM}

PROCEDURE OPENINFILE;
VAR OK:CHAR;
    FLAG1:BOOLEAN;

    BEGIN
       FLAG1:=FALSE;
       REPEAT
```

```
        WRITELN('THE FILE TO READ DATA FROM IS ?');
        READLN(FILENAME);
        WRITELN('THE FILE TO READ DATA FROM IS ',FILENAME
        ,'? (Y OR N E=EXIT)');
        READLN(OK);
     CASE OK OF
        'Y':BEGIN
            FLAG1:=TRUE;
            ASSIGN(INFILE,FILENAME);
            RESET(INFILE);
            IF EOF(INFILE) THEN
            BEGIN
            WRITELN('UNABLE TO OPEN FILE',FILENAME);
            FLAG1:=FALSE;
            END;
            END;
        'N':FLAG1:=FALSE;
        'E':BEGIN
            WRITELN('NO FILE OPENED PROCEDURE EXITED');
            FLAG1:=TRUE;
            END;
            ELSE FLAG1:=FALSE;
            END;{CASE}
            UNTIL FLAG1;
            END;  {END OPENINFILE}


{*******************************************************}

{THIS PROCEDURE OPENS A TEXT FILE TO WRITE TO}


PROCEDURE OPENLSST;
VAR FLAG1:BOOLEAN;
OK:CHAR;
BEGIN
FLAG1:=FALSE;
REPEAT
WRITELN('THE NAME OF THE TEXT FILE TO WRITE TO IS ? ');
READLN(FILENAM);
WRITELN('THE TEXT FILE TO BE WRITTEN TO IS ',FILENAM,
' ? (Y OR N E=EXIT)');
READLN(OK);
CASE OK OF
        'Y':BEGIN
            FLAG1:=TRUE;
            ASSIGN(LSST,FILENAM);
            REWRITE(LSST);
            END;
        'N':FLAG1:=FALSE;
        'E':BEGIN
            WRITELN('NO FILE OPENED. PROCEDURE EXITED');
            FLAG1:=TRUE;
            END;
```

```
            ELSE FLAG1:=FALSE;
            END; {CASE}
        UNTIL FLAG1;
        END; {END OPENLSST}

{*******************************************************}

{PROCEDURE SETDATA READS THE SUBMAIN, THE LATERALS AND
 THE SPRINKLERS INFORMATION FROM A DATA FILE}



PROCEDURE SETDATA;

   BEGIN
     READLN(INFILE,HIN,HINP,HEIGHT);
     READLN(INFILE,NL,NEL);
     READLN(INFILE,SUBD,SUBK,SUBC,COMKS,M,TSLOPS);
     READLN(INFILE,LATD,LATK,LATC,LATM,LATS,COMKL,
     TSLOPLL);
     READLN(INFILE,SPS,KS,XS);
     END;
     {END SET DATA}

{*******************************************************}

{PROCEDURE PRINTDATA WRITES THE INPUT DATA TO A FILE}

PROCEDURE PRINTDATA;

BEGIN

WRITELN(LSST,' INPUT PRESSURE (FT) =  ','   ',HIN:7:3);
WRITELN(LSST,'     PUMP HEAD  (FT) =  ','   ',HINP:7:3);
WRITELN(LSST,' INPUT HEIGHT   (FT) =  ','   ',HEIGHT:7:3);
WRITELN(LSST,' NUMBER OF LATERALS  =  ','   ',NL);
WRITELN(LSST,'                        ');
WRITELN(LSST,' -----SUBMAIN DATA----- ');
WRITELN(LSST,'                        ');
WRITELN(LSST,'     DIAMETER   (IN) =  ','   ',SUBD:5:2);
WRITELN(LSST,'     CONSTANT  K     =  ','   ',SUBK:6:2);
WRITELN(LSST,' H-W FRICTION COEFF. =  ','   ',SUBC:5:1);
WRITELN(LSST,' TEES--ELBOWS COEFF. =  ','   ',COMKS:4:1);
WRITELN(LSST,'     EXPONENT  M     =  ','   ',M:5:3);
WRITELN(LSST,'     SLOPE     %     =  ','   ',TSLOPS);
WRITELN(LSST,'                        ');
WRITELN(LSST,'------LATERAL DATA----- ');
WRITELN(LSST,'                       ');
WRITELN(LSST,'     DIAMETER   (IN) =  ','   ',LATD:5:2);
WRITELN(LSST,'     CONSTANT  K     =  ','   ',LATK:6:2);
WRITELN(LSST,' H-W FIRCTION COEFF. =  ','   ',LATC:5:1);
WRITELN(LSST,'     VALVES COEFF.   =  ','   ',COMKL:4:1);
```

```
WRITELN(LSST,'      EXPONENT  M      =  ','   ',LATM:5:3);
WRITELN(LSST,'      SLOPE     %      =  ','   ',TSLOPLL);
WRITELN(LSST,'   LAT SPACING  (FT) =  ','   ',LATS:4:1);
WRITELN(LSST,'                          ');
WRITELN(LSST,'-----SPRINKLER DATA---- ');
WRITELN(LSST,'                          ');
WRITELN(LSST,'  SPRINKLER COEFF.    =  ','   ',KS:7:4);
WRITELN(LSST,'  SPRINKLER EXPONENT =  ','   ',XS:3:1);
WRITELN(LSST,'  SPRINKLER SPAC.(FT)=  ','   ',SPS:4:1);
WRITELN(LSST,'  NO. SPRINKLERS/LAT =  ','   ',NEL);
WRITELN(LSST,'                          ');
WRITELN(LSST,'                          ');

     END; {PRINTDATA}
{*******************************************************}

{FUNCTION POWER RAISES BASE TO THE POWER PWR.}


FUNCTION POWER(BASE,PWR : REAL) : REAL;

  BEGIN
       IF BASE>0.0 THEN
            POWER:=EXP(PWR*LN(BASE))
       ELSE
            POWER:=0.0;
  END; {POWER}



{*******************************************************}

{PROCEDURE INITIALIZATION OBTAINS NEEDED VALUES
 OF CONSTANTS AND INITIALIZES THE ARRAYS}


PROCEDURE INITIALIZATION;

  VAR I,J:INTEGER;

        BEGIN
           SLOPS:=TSLOPS/100.0;
           SLOPLL:=TSLOPLL/100.0;


KLAT:=(LATK*SPS)/(POWER(LATC,M)*POWER(LATD,4.871));

KSUB:=(SUBK*LATS)/(POWER(SUBC,M)*POWER(SUBD,4.871));

DFL:=LATD/12.;

DFS:=SUBD/12.;
```

```
        {INITIALIZATION OF THE ARRAYS}

  FOR I:=1 TO ARRAY_SIZE DO BEGIN
      RHS[I]:=0.0;
      FOR J:=1 TO 2 DO A[I,J]:=0.0;
      END;

  FOR I:=1 TO ARRAY_SIZE2 DO BEGIN
      P[I]:=0.0;
      Y[I]:=0.0;
      DIFF[I]:=0.0;
      END;


ITERDELTA:=0.01;
EDIFF:=0.0;
UNK:=NEL*NL+NL;
UNK1:=UNK+1;
NEL1:=NEL+1;
CONSTM:=(1./M)-1.;
END;

                  {END INITIALIZATION}

{*******************************************************}

   {REAL FUNCTION C CALCULATES THE CONSTANTS NEEDED
    TO CREATE THE DIRECT STIFFNES MATRIX}


FUNCTION C(I,ID : INTEGER) : REAL;

  BEGIN

 {SUBMAIN CONSTANT AND COMPONENT CONSTANT}


 IF ID=1 THEN BEGIN
 KCOMS:=0.00000012498*COMKS*POWER(Q1[I],0.148)
        /POWER9DFS,4);

        KSUB1:=KSUB+KCOMS;

 C:=POWER(ABS(P[I+NEL1]-P[I]),CONSTM)*
                  POWER(KSUB1,-1./M);


 {LATERAL CONSTANT AND COMPONENT CONSTANT}
```

```
IF ID=4 THEN BEGIN
KCOML:=0.00000012498*COMKL*POWER(Q1[I],0.148)
       /POWER(DFL,4);

       KLAT1:=KLAT+KCOML;

C:=POWER(ABS(P[I+1]-P[I]),CONSTM)*
                POWER(KLAT1,-1./M);


{SPRINKLER CONSTANT}

IF ID=5 THEN BEGIN

            C:=POWER(ABS(P[I]-P[I+UNK1]),
               (-1.+XS))*KS;

END; {IF}

END; {FUNCTION C}
{*******************************************************}

{PROCEDURE INITIAL HEIGHTS WILL GENERATE THE HEIGHTS OF
 THE NODES BASED ON SLOPES AND ON THE INITIAL ELEVATION
 OF THE INPUT NODE}


PROCEDURE INITIAL_HEIGHTS;

  VAR
     I,J,OFFSET,BIAS,MP:INTEGER;
     BEGIN {INITIAL HEIGHTS}
     FOR I:=1 TO NL DO BEGIN


{FIRST CALCULATE THE ELEVATION OF THE LATERAL-SUBMAIN
 JUNCTION NODE}


 OFFSET:=UNK-(I-1)*NEL1;
 P[OFFSET+UNK1]:=P[2*UNK1]+(SLOPS*I*LATS);
 BIAS:=OFFSET-1;


{PROCESS EACH EMITTER ON THE LEFT LATERAL}


FOR J:=1 TO NEL DO BEGIN
P[BIAS+UNK1]:=P[OFFSET+UNK1]+(SLOPLL*J*SPS);
BIAS:=BIAS-1
END;{FOR J}
```

```
END;{FOR I}

  END;{INITIAL HEIGHTS}


{*******************************************************}


{PROCEDURE INITIAL PRES WILL MAKE INITIAL ESTIMATES OF
 NODE PRESSURES IN ARRAY P, FROM P[1] THROUGH P[UNK]}


 VARIABLES

{  DHL---CHANGE IN PRESSURE FOR THE LATERAL
   DHS---CHANGE IN PRESSURE FOR THE SUBMAIN
   QL----FLOW IN THE LATERAL
   QS----FLOW IN THE SUBMAIN}


PROCEDURE INITIAL_PRES;

VAR

OFFSET,I,J,MP:INTEGER;
QL,QS,T1,T2,DHS,DHL:REAL;

BEGIN

{CALCULATE THE SUBMAIN FLOW AND DELTA H BASED ON INPUT
 PRESSURE. OR IN THE PUMP HEAD}


QS:=KS*POWER(P[UNK1],XS)*NL*NEL;  {PUMP HEAD}
T1:=3.6679/POWER(SUBC,1.852);
T2:=POWER(QS,1.852)/POWER(SUBD,4.871);
DHS:=T1*T2*LATS*NL;


{CALCULATE ORIGINAL PRESSURES STARTING WITH THE
 LATERAL CLOSEST TO THE NODE CONTAINIG THE INPUT VALUES}


FOR  I:=1 TO NL DO BEGIN
   OFFSET:=UNK-(I-1)*NEL1;
   P[OFFSET]:=P[UNK1]-DHS*(1-POWER(1-I/NL,2.852))
              -SLOPS*I*LATS;

{CALCULATE PRESSURES DOWN THE LATERAL LINE}
```

```
BIAS:=OFFSET-1;
QL:=KS*POWER(P[OFFSET],XS)*(NEL1);
T1:=3.6679/POWER(LATC,1.852);
T2:=POWER(QL,1.852)/POWER(LATD,4.871);
DHL:=T1*T2*SPS*NEL;
FOR  J:=1 TO NEL DO BEGIN
P[BIAS]:=P[OFFSET]-DHL*(1-POWER(1-J/NEL,2.852))
            -SLOPLL*J*SPS;

BIAS:=BIAS-1

END; { END LOOP J}
END; { END LOOP I}

{CHECK FOR NEGATIVE PRESSURE ESTIMATES }

FOR I:=1 TO UNK DO BEGIN
IF P[I] <=0.0 THEN P[I]:=0.00001;
END;


{PRINT THE VECTOR CONTAINIG THE INITIAL HEIGHTS AND
 PRESSURES ESTIMATES}


WRITELN(LSST,'INITIAL HEIGHTS (FT) INITIAL PRESSURES (FT)');

    FOR MP:=1 TO UNK1 DO BEGIN

IF MP=UNK1 THEN
P[UNK1]:=HIN;

    WRITE(LSST,'    ',P[MP+UNK1]:8:2,'      ',P[MP]:8:2);
    WRITELN(LSST);
    END;{PRINTING}

P[UNK1]:=HIN+HINP;


END; {INITIAL_PRES PROCEDURE}


{*********************************************************
                   MAIN PROGRAM
 ********************************************************}


{THIS IS THE MAIN PROGRAM-- IT CALLS SEVERAL PROCEDURES}


BEGIN
```

```
OPENINFILE;
OPENLSST;
SETDATA;
PRINTDATA;
INITIALIZATION;


        {DEFINE CONSTANTS TO BE USED IN THE PROGRAM}

UNK:=NEL*NL+NL;
UNK1:=UNK+1;
NEL1:=NEL+1;
CONSTM:=(1./M)-1.;
P[UNK1]:=HIN+HINP;
P[UNK1*2]:=HEIGHT;
ITERDELTA:=0.01;
NOL:=NEL*NL;
SUML:=0.0;


INITIAL_HEIGHTS;

INITIAL_PRES;


{CALCULATE THE FINAL PRESSURES ESTIMATES.  CALCULATE
 THE CHANGES IN ELEVATIONS AND STORE THEM IN DELTAHP}


FOR MP:=1 TO UNK1 DO BEGIN
PTEMP:=P[UNK1+MP];
DELTAHP[UNK1+MP]:=HEIGHT-PTEMP;
P[MP]:=P[MP]+PTEMP;
DIFF[MP]:=P[MP];
END; {FOR}


{ADD THE CHANGE IN ELEVATION TO THE ORIGINAL PRESSURE
  ESTIMATE}


FOR I:=1 TO UNK DO Y[I]:=P[I]+DELTAHP[I+UNK1];


{ITERATE UNTIL THE SOLUTION CONVERGES (EDIFF<1.0)}

CONSTM:=(1./M)-1.;
EDIFF:=100.;
ITS:=1;

WRITELN('     SPRINKLER IRRIGATION DESIGN
          EMPLOYING FINITE ELEMENT FORMULATION');
```

```
WRITELN(' *******************************************');
WRITELN(' THE PROGRAM IS ITERATING UNTIL SOLUTION
               IS OBTAINED W/IN 0.01 FT.');

WRITELN('                ');


   WHILE EDIFF>ITERDELTA DO BEGIN
   WRITELN('          ITERATION  ',ITS:2,' . ');
   WRITELN(LSST,'          ITERATION ',ITS:2,' . ');


{CREATE THE STIFFNES MATRIX}

      FOR I:=1 TO UNK DO A[I,2]:=0.0;
      FOR I:=1 TO UNK DO A[I,1]:=0.0;
      FOR I:=1 TO UNK DO BEGIN
 IF I=((I DIV NEL1)*NEL1)THEN

    {THIS IS THE FINAL NODE}

 IF I=UNK THEN
   A[I,2]:=A[I,2]+POWER(ABS(P[I+1]-P[I]),CONSTM)*
           POWER(KSUB,-1./M)

{THIS IS A SUBMAIN NODE PLUS A COMPONENT (TEE OR ELBOW)}


   ELSE  BEGIN
   CTEMP:=C(I,1);
   A[I,2]:=CTEMP+A[I,2];
   A[I,1]:=-CTEMP+A[I,1];
   A[I+NEL1,2]:=CTEMP+A[I+NEL1,2];
   END {IF}

{THIS IS A LATERAL W/ A SPRINKLER}

   ELSE BEGIN
   ID:=4;
   CTEMP:=C(I,ID);
   A[I,2]:=CTEMP+C(I,ID+1)+A[I,2];
   A[I,1]:=-CTEMP;
   A[I+1,2]:=CTEMP+A[I+1,2];
   END; {ELSE}

   END; {FOR}


{-------------------------------------------------------
        FINISH CREATING THE STIFFNESS MATRIX
 -------------------------------------------------------}


{CALCULATE THE RIGHT HAND SIDE OF THE SYSTEM OF EQUATIONS}
```

```
   FOR I:=1 TO UNK DO BEGIN
   IF I=((I DIV NEL1)*NEL1) THEN


   {THIS IS A SUBMAIN NODE}


   IF I<>UNK THEN
   RHS[I]:=0.0
   ELSE


   {THIS IS FOR THE FINAL NODE}


   RHS[I]:=P[UNK1]*(POWER(ABS(P[UNK1]-P[UNK]),CONSTM)*
                   POWER(KSUB,-1./M))


   ELSE BEGIN

{THIS IS A LATERAL WITH A SPRINKLER NODE}


   ID:=5;
   RHS[I]:=P[I+UNK1]*C(I,ID);
   END; {IF}
   END; {FOR}


{NOW SOLVE THE EQUATION AX=RHS.  FIRST MAKE THE MATRIX
 INTO UPPER TRIANGULAR FORM}

   NSP:=NEL1+1;

   FOR J:=2 TO UNK DO BEGIN
{EXTENDED BANDWIDHT.  SUBMAIN NODES}


   IF J=NSP THEN BEGIN
   EE4:=ABS(A[J-1,1]);
   A[J+NEL,2]:=A[J+NEL,2]-SQR(EE4)/A[J-1,2];
   RHS[J+NEL]:=RHS[J+NEL]-A[J-1,1]*RHS[J-1]/A[J-1,2];
   NSP:=NSP+NEL1;
   END
   ELSE BEGIN
```

```
{FOR THE LEFT LATERAL}

   EE4:=ABS(A[J-1,1]);
   A[J,2]:=A[J,2]-SQR(EE4)/A[J-1,2];
   RHS[J]:=RHS[J]-A[J-1,1]*RHS[J-1]/A[J-1,2];
   END; {IF}
   END; {FOR}


{NOW DO BACKWARD SUBSTITUTION TO FIND THE SOLUTION
 SET IN VECTOR P[I]}

{********************************************************}

NSP:=UNK-NEL1;
NLP:=UNK-NEL1;
MUNK:=UNK-1;

P[UNK]:=RHS[UNK]/A[UNK,2];
BIAS:=MUNK;
FOR I:=1 TO MUNK DO BEGIN
     IF (BIAS<>NSP) AND (BIAS<>NLP) THEN
       P[BIAS]:=(RHS[BIAS]-A[BIAS,1]*P[BIAS+1])/A[BIAS,2]
ELSE IF BIAS=NSP THEN BEGIN
   P[BIAS]:=(RHS[BIAS]-A[BIAS,1]*P[BIAS+NEL1])/A[BIAS,2];
       NSP:=NSP-NEL1 END
ELSE BEGIN
   P[BIAS]:=(RHS[BIAS]-A[BIAS,1]*P[BIAS+NEL1])/A[BIAS,2];
       NLP:=NLP-NEL1
       END;
          BIAS:=BIAS-1;
          END;

{********************************************************}



{CHECK FOR NEGATIVE PRESSURE ESTIMATES}

 FOR I:=1 TO UNK DO BEGIN
   IF P[I]<=0.0 THEN
   P[I]:=0.00001;
 END;
```

```
{CHECK FOR CONVERGENCE. HERE USE THE CRITERION THAT THE
 SUM OF THE TOTAL PRESSURE VARIATION IN ALL THE NODES
 HAVE CHANGED LESS THAN 0.01 FEET}


   EDIFF:=0.0;
   FOR I:=1 TO UNK DO BEGIN

       EDIF:=ABS(DIFF[I]-P[I]) + EDIFF;

   DIFF[I]:=P[I];
   END;
   ITS:=ITS+1;


CALCULATE THE CP'S AND THE CORRESPONDING ELEMENT FLOW
TO BE USED AS THE NEW ESTIMATES IN THE SUBMAIN AND
LATERAL COMPONENTS}

FOR I:= 1 TO UNK DO BEGIN

IF I=((I DIV NEL1)*NEL1) THEN

{THIS IS FOR THE FINAL NODE}

     IF I = UNK THEN

Q1[I]:=POWER(ABS(P[I+1]-P[I]),CONSTM)
       *POWER(KSUB,-1./M)*(ABS(P[I+1]-P[I])

     ELSE BEGIN

 Q1[I]:=POWER(ABS(P[I+NEL1)-P[I]),CONSTM)
        *POWER(KSUB,-1./M)*(ABS(P[I+NEL1]-P[I]))

      END

      ELSE BEGIN

  Q1[I]:=POWER(ABS(P[I+1]-P[I]),CONSTM)
         *POWER(KLAT,-1./M)*(ABS(P[I+1]-P[I]))

       END; {IF}
       END; {FOR}


       END; {WHILE}



{THE SOLUTION HAS CONVERGED.  READJUST THE PRESSURES
 BY ADDING CHANGES IN HEIGHTS}
```

```
   FOR I:=1 TO UNK1 DO BEGIN
   P[I]:=P[I]-P[UNK1+I];
   IF P[I]<=0.0 THEN P[I]:=0.00001;
   END; {FOR}

{DISPLAY THE FINAL PRESSURE ESTIMATES}


 WRITELN(LSST,'FINAL PRESSURES ADJUSTED WITH HEIGHTS );
 WRITELN(LSST,'  FEET                      KILOPASCALS')

 FOR I:= 1 TO UNK1 DO BEGIN

 IF I=UNK1 THEN
    P[UNK1]:=HIN;
    WRITE(LSST,'      ',P[I]:8:2,'  ',P[I]*2.9891:8:);
    WRITELN(LSST);
    END;
    P[UNK1]:=HIN+HINP;

{CALCULATE THE SPRINKLER DISCHARGES AND PLACE THEM IN
 THE Y MATRIX.  DISPLAY THEM IN GPM}


   FOR  I:=1 TO UNK DO BEGIN
        TK:=KS;
        TX:=XS;
        Y[I]:=TK*POWER(P[I],TX);
        END; {FOR}
{DETERMINE THE SUBMAIN FLOW}

    FOR I:=1 TO UNK DO BEGIN
    IF (I MOD NEL1) <>0 THEN
    SUML:= SUML+Y[I];
    END;

    MEANL:=SUML/NOL;
    SUBFLOW:=NL*(NEL*MEANL);
    SUBFLPS:=SUBFLOW*0.06309;
    WRITELN(LSST,'SUBMAIN FLOW(GPM) =',SUBFLOW:8:2
                 ,'    (L/S)',SUBFLPS:8:2);
    WRITELN(LSST);
    WRITELN(LSST,' SPRINKLER FLOWS');
    WRITELN(LSST,'          GPM             L/S  );
    FOR I:=1 TO UNK DO BEGIN
    IF (I MOD NEL1)<>0 THEN
    WRITE(LSST,'  ',Y[I]:8:2,'   ',Y[I]*0.06309:8:2);

    END;

    CLOSE(LSST);
    END. {END PROGRAM SPIRR-FE}
```

APPENDIX C

LINEAR THEORY COMPUTER MODEL

LINEAR THEORY COMPUTER PROGRAM

1. Introduction

The linear theory model developed by Wood (1980) is a FORTRAN based computer program capable of carrying out regular simulations of steady state pressure and flow in pipe distribution systems. In addition, extended period simulations can be carried out which simulate the operation of the system over a period of time.

The basis of the program is a direct solution to the basic pipe system hydraulic equations using a linearization technique and sparse matrix methods to handle the nonlinear terms in the energy equations.

The linear theory computer program is appropiate for use on most IBM compatible microcomputer with at least 256K of memory and Microsoft DOS operating system.

2. Data Coding Guidelines

The input requirements for the linear theory computer model under the regular simulation mode are contained in 9 files. All of the required parameters are readily available or may be easily prepared. This section will discuss in general each of the files and parameters.

I. System Data

a) type of simulation (regular or EPS)

b) flow units (CFS, GPM, MGD, or lit/sec)

c) number of pipes

d) number of junction nodes

e) number of PRV's

f) program options

II. Label Cards

cards that contain any desired information which will be used as a label for the computer output

III. Pressure Regulating Valve (PRV) Data

a) junction node number for PRV

b) pipe number - downstream from PRV

c) grade set by PRV

IV. Pipeline Data

a) connecting nodes

b) length

c) diameter

d) roughness

e) Σ (minor loss coefficients)

f) pump power (or key to read pump data)

g) grade (if this pipe connects a Fixed Grade Node)

h) pipe number (if nonconsecutive numbering specified)

V. Pump Data

head-discharge data for three operating points

VI. Junction Node Data

a) demand

b) elevation

c) junction node number

VII. Output Option

a) key for full or limited output.

b) number of junction nodes for summary of maximum and minimum pressures

c) Number of pipes for limited output

d) number of junction nodes for limited output

e) additional data keying program options

VIII. Pipe numbers for limited output option

IX. Juction node numbers for limited output option

APPENDIX D

MODELS SIMULATION DATA

Slope = 2% uphill
Comp. Coeff. Kc = 0.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	24.90 (82)	125.45 (41.97)	125.42 (41.96)
4	23.77 (78)	137.50 (46.00)	137.50 (46.00)
6	22.56 (74)	149.81 (50.12)	149.78 (50.11)
8	21.34 (70)	162.54 (54.38)	162.52 (54.37)
10	20.12 (66)	175.86 (58.83)	175.85 (58.83)
12	18.90 (62)	189.97 (63.56)	189.95 (63.55)
14	17.68 (58)	205.07 (68.61)	205.05 (68.60)
16	16.46 (54)	221.38 (74.06)	221.37 (74.06)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	5.54 (87.79)	5.54 (87.82)
3	5.80 (92.01)	5.81 (92.04)
5	6.06 (96.13)	6.07 (96.17)
7	6.32 (100.23)	6.33 (100.27)
9	6.58 (104.35)	6.59 (104.39)
11	6.85 (108.56)	6.85 (108.60)
13	7.12 (112.89)	7.13 (112.94)
15	7.41 (117.40)	7.41 (117.45)

Slope = 1% uphill
Comp. Coeff. Kc = 0.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	20.12 (66)	170.05 (56.89)	170.05 (56.89)
4	19.51 (64)	176.16 (58.93)	176.15 (58.93)
6	18.90 (62)	182.59 (61.08)	182.57 (61.08)
8	18.29 (60)	189.54 (63.41)	189.54 (63.41)
10	17.68 (58)	197.21 (65.98)	197.19 (65.97)
12	17.07 (56)	205.80 (68.85)	205.80 (68.85)
14	16.46 (54)	215.50 (72.10)	215.48 (72.09)
16	15.85 (52)	226.54 (75.79)	226.54 (75.79)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	6.47 (102.58)	6.47 (102.61)
3	6.59 (104.44)	6.59 (104.48)
5	6.71 (106.38)	6.71 (106.42)
7	6.84 (108.43)	6.84 (108.47)
9	6.98 (110.66)	6.98 (110.70)
11	7.14 (113.10)	7.14 (113.14)
13	7.31 (115.79)	7.31 (115.84)
15	7.49 (118.79)	7.50 (118.84)

Slope = 0%
Comp. Coeff. Kc = 0.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	15.24 (50)	214.72 (71.83)	214.70 (71.83)
4	15.24 (50)	214.88 (71.89)	214.85 (71.88)
6	15.24 (50)	215.43 (72.07)	215.42 (72.07)
8	15.24 (50)	216.61 (72.47)	216.59 (72.46)
10	15.24 (50)	218.62 (73.14)	218.59 (73.13)
12	15.24 (50)	221.68 (74.16)	221.67 (74.16)
14	15.24 (50)	225.97 (75.60)	225.97 (75.60)
16	15.24 (50)	231.72 (77.52)	231.71 (77.52)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	7.29 (115.58)	7.29 (115.62)
3	7.29 (115.62)	7.30 (115.66)
5	7.30 (115.77)	7.31 (115.82)
7	7.32 (116.10)	7.33 (116.14)
9	7.36 (116.65)	7.36 (116.69)
11	7.41 (117.48)	7.41 (117.52)
13	7.48 (118.64)	7.49 (118.68)
15	7.58 (120.17)	7.58 (120.22)

Slope = 1% downhill
Comp. Coeff. Kc = 0.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	10.36 (34)	259.44 (86.79)	259.42 (86.79)
4	10.97 (36)	253.54 (84.86)	253.62 (84.85)
6	11.58 (38)	248.32 (83.08)	248.30 (83.07)
8	12.19 (40)	243.72 (81.54)	243.70 (81.53)
10	12.80 (42)	240.08 (80.32)	240.05 (80.31)
12	13.41 (44)	237.59 (79.48)	237.57 (79.48)
14	14.02 (46)	236.47 (79.11)	236.46 (79.11)
16	14.63 (48)	236.91 (79.26)	236.91 (79.26)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	8.03 (127.32)	8.04 (127.37)
3	7.94 (125.86)	7.94 (125.91)
5	7.85 (124.50)	7.86 (124.55)
7	7.78 (123.32)	7.78 (123.36)
9	7.72 (122.37)	7.72 (122.42)
11	7.68 (121.72)	7.68 (121.77)
13	7.66 (121.43)	7.66 (121.47)
15	7.67 (121.54)	7.67 (121.59)

Slope = 2% downhill
Comp. Coeff. Kc = 0.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	5.49 (18)	304.19 (101.77)	304.17 (101.76)
4	6.71 (22)	292.45 (97.84)	292.42 (97.83)
6	7.92 (26)	281.25 (94.09)	281.21 (94.08)
8	9.14 (30)	270.87 (90.62)	270.84 (90.61)
10	10.36 (34)	261.56 (87.50)	261.54 (87.50)
12	11.58 (38)	253.53 (84.82)	253.50 (84.81)
14	12.80 (42)	246.99 (82.63)	246.96 (82.62)
16	14.02 (46)	242.12 (81.00)	242.11 (81.00)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	8.71 (138.12)	8.72 (138.17)
3	8.54 (135.37)	8.54 (135.41)
5	8.37 (132.69)	8.37 (132.74)
7	8.21 (130.16)	8.21 (130.21)
9	8.07 (127.85)	8.07 (127.90)
11	7.94 (125.83)	7.94 (125.88)
13	7.83 (124.16)	7.84 (124.21)
15	7.75 (122.90)	7.76 (122.95)

Slope = 2% uphill
Comp. Coeff. Kc = 1.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	24.99 (82)	123.66 (41.37)	123.69 (41.38)
4	23.77 (78)	135.72 (45.41)	135.73 (45.41)
6	22.56 (74)	148.07 (49.54)	148.08 (49.54)
8	21.34 (70)	160.91 (53.83)	160.93 (53.84)
10	20.12 (66)	174.44 (58.36)	174.44 (58.36)
12	18.90 (62)	188.90 (63.20)	188.91 (63.20)
14	17.68 (58)	204.52 (68.42)	204.51 (68.42)
16	16.46 (54)	221.58 (74.13)	221.58 (74.13)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	5.42 (85.84)	5.42 (85.89)
3	5.68 (90.02)	5.68 (90.07)
5	5.94 (94.12)	5.94 (94.18)
7	6.20 (98.21)	6.20 (98.27)
9	6.46 (102.35)	6.46 (102.44)
11	6.73 (106.61)	6.73 (106.66)
13	7.00 (111.03)	7.01 (111.09)
15	7.30 (115.67)	7.30 (115.73)

Slope = 1% uphill
Comp. Coeff. Kc = 1.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	20.12 (66)	167.87 (56.16)	167.90 (56.17)
4	19.51 (64)	173.98 (58.21)	173.99 (58.21)
6	18.90 (62)	180.47 (60.38)	180.48 (60.38)
8	18.29 (60)	187.57 (62.75)	187.59 (62.76)
10	17.68 (58)	195.51 (65.41)	195.51 (65.41)
12	17.07 (56)	204.51 (68.42)	204.51 (68.42)
14	16.46 (54)	214.85 (71.88)	214.85 (71.88)
16	15.85 (52)	226.77 (75.87)	226.75 (75.86)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	6.33 (100.36)	6.34 (100.42)
3	6.45 (102.21)	6.45 (102.27)
5	6.57 (104.15)	6.57 (104.21)
7	6.70 (106.22)	6.71 (106.28)
9	6.85 (108.50)	6.85 (108.56)
11	7.00 (111.02)	7.01 (111.09)
13	7.18 (113.86)	7.19 (113.92)
15	7.38 (117.05)	7.39 (117.11)

Slope = 0%
Comp. Coeff. Kc = 1.0

Junction Pressures

Junction Node Number	System Elevatio meters (f)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	15.24 (50)	212.14 (70.97)	212.16 (70.98)
4	15.24 (50)	212.31 (71.03)	212.34 (71.04)
6	15.24 (50)	212.94 (71.24)	212.97 (71.25)
8	15.24 (50)	214.30 (71.69)	214.32 (71.70)
10	15.24 (50)	216.63 (72.47)	216.65 (72.48)
12	15.24 (50)	220.18 (73.66)	220.20 (73.67)
14	15.24 (50)	225.21 (75.34)	225.23 (75.35)
16	15.24 (50)	231.98 (77.61)	231.98 (77.61)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	7.14 (113.12)	7.14 (113.19)
3	7.14 (113.17)	7.14 (113.23)
5	7.15 (113.34)	7.16 (113.41)
7	7.17 (113.71)	7.18 (113.77)
9	7.21 (114.34)	7.22 (114.40)
11	7.27 (115.30)	7.28 (115.36)
13	7.36 (116.63)	7.36 (116.70)
15	7.47 (118.41)	7.47 (118.47)

Slope = 1% downhill
Comp. Coeff. Kc = 1.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	10.36 (34)	256.46 (85.80)	256.49 (85.81)
4	10.97 (36)	250.69 (83.87)	250.72 (83.88)
6	11.58 (38)	245.46 (82.12)	245.49 (82.13)
8	12.19 (40)	241.07 (80.65)	241.10 (80.66)
10	12.80 (42)	237.79 (79.55)	237.81 (79.56)
12	13.41 (44)	235.89 (78.92)	235.90 (78.92)
14	14.02 (46)	235.61 (78.82)	235.63 (78.83)
16	14.63 (48)	237.20 (79.36)	237.21 (79.36)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	7.86 (124.64)	7.87 (124.72)
3	7.77 (123.20)	7.78 (123.27)
5	7.69 (121.88)	7.69 (121.95)
7	7.62 (120.76)	7.62 (120.83)
9	7.57 (119.92)	7.57 (119.99)
11	7.53 (119.43)	7.54 (119.50)
13	7.53 (119.36)	7.53 (119.42)
15	7.56 (119.77)	7.56 (119.83)

Slope = 2% downhill
Comp. Coeff. Kc = 1.0

Junction Pressures

Junction Node Number	System Elevations meters (ft	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	5.49 (18)	300.81 (100.64)	300.85 (100.65)
4	6.71 (22)	289.10 (96.72)	289.13 (96.73)
6	7.92 (26)	278.01 (93.01)	278.04 (93.02)
8	9.14 (30)	267.87 (89.62)	267.91 (89.63)
10	10.36 (34)	258.99 (86.64)	259.00 (86.65)
12	11.58 (38)	251.62 (84.18)	251.62 (84.18)
14	12.80 (42)	246.02 (82.31)	246.03 (82.31)
16	14.02 (46)	242.44 (81.11)	242.44 (81.11)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	8.53 (135.23)	8.54 (135.31)
3	8.36 (132.52)	8.37 (132.59)
5	8.19 (129.89)	8.20 (129.97)
7	8.04 (127.45)	8.05 (127.52)
9	7.90 (125.27)	7.91 (125.34)
11	7.79 (123.44)	7.79 (123.50)
13	7.70 (122.03)	7.70 (122.09)
15	7.64 (121.11)	7.64 (121.17)

Slope = 2% uphill
Comp. Coeff. Kc = 2.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	24.90 (82)	121.97 (40.80)	122.04 (40.83)
4	23.77 (78)	134.03 (44.84)	134.12 (44.87)
6	22.56 (74)	146.43 (48.99)	146.52 (49.02)
8	21.34 (70)	159.37 (53.32)	159.44 (53.34)
10	20.12 (66)	173.10 (57.91)	173.16 (57.93)
12	18.90 (62)	187.88 (62.85)	187.92 (62.87)
14	17.68 (58)	204.00 (68.25)	204.03 (68.26)
16	16.46 (54)	221.77 (74.19)	221.76 (74.19)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	5.30 (83.98)	5.30 (84.08)
3	5.56 (88.13)	5.57 (88.23)
5	5.82 (92.20)	5.82 (92.31)
7	6.07 (96.28)	6.08 (96.39)
9	6.34 (100.43)	6.34 (100.54)
11	6.61 (104.73)	6.61 (104.83)
13	6.89 (109.23)	6.90 (109.33)
15	7.19 (113.99)	7.20 (114.50)

Slope = 1% uphill
Comp. Coeff. Kc = 2.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	20.12 (66)	165.79 (55.47)	165.89 (55.50)
4	19.51 (64)	171.92 (57.52)	172.02 (57.55)
6	18.90 (62)	178.47 (59.71)	178.57 (59.74)
8	18.29 (60)	185.70 (62.13)	185.80 (62.16)
10	17.68 (58)	193.88 (64.86)	193.96 (64.89)
12	17.07 (56)	203.29 (68.01)	203.35 (68.03)
14	16.46 (54)	214.22 (71.67)	214.26 (71.68)
16	15.85 (52)	226.99 (75.94)	226.99 (75.94)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	6.20 (98.25)	6.21 (98.36)
3	6.31 (100.09)	6.32 (100.20)
5	6.44 (102.02)	6.44 (102.13)
7	6.57 (104.11)	6.58 (104.22)
9	6.71 (106.43)	6.72 (106.54)
11	6.88 (109.04)	6.89 (109.14)
13	7.07 (111.99)	7.07 (112.10)
15	7.28 (115.35)	7.28 (115.45)

Slope = 0%
Comp. Coeff. Kc = 2.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	15.24 (50)	209.69 (70.15)	209.80 (70.19)
4	15.24 (50)	209.88 (70.21)	210.01 (70.26)
6	15.24 (50)	210.58 (70.45)	210.70 (70.49)
8	15.24 (50)	212.10 (70.96)	212.22 (71.00)
10	15.24 (50)	214.73 (71.84)	214.82 (71.87)
12	15.24 (50)	218.76 (73.19)	218.83 (73.21)
14	15.24 (50)	224.49 (75.10)	224.54 (75.12)
16	15.24 (50)	232.22 (77.69)	232.22 (77.69)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	6.99 (110.78)	7.00 (110.90)
3	6.99 (110.83)	7.00 (110.95)
5	7.00 (111.02)	7.01 (111.14)
7	7.03 (111.43)	7.04 (111.55)
9	7.07 (112.13)	7.08 (112.25)
11	7.14 (113.20)	7.15 (113.31)
13	7.24 (114.70)	7.24 (114.81)
15	7.36 (116.71)	7.37 (116.80)

Slope = 1% downhill
Comp. Coeff. Kc = 2.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressure KPa (ft)	Linear Theory Pressures KPa (ft)
2	10.36 (34)	253.63 (84.85)	253.77 (84.90)
4	10.97 (36)	247.88 (82.93)	248.03 (82.98)
6	11.58 (38)	242.74 (81.21)	242.89 (81.26)
8	12.19 (40)	238.55 (79.81)	238.68 (79.85)
10	12.80 (42)	235.62 (78.83)	235.75 (78.87)
12	13.41 (44)	234.27 (78.37)	234.37 (78.41)
14	14.02 (46)	234.79 (78.55)	234.82 (78.56)
16	14.63 (48)	237.48 (79.45)	237.45 (79.44)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	7.70 (122.08)	7.71 (122.22)
3	7.61 (120.66)	7.62 (120.80)
5	7.53 (119.38)	7.54 (119.51)
7	7.46 (118.32)	7.47 (118.45)
9	7.42 (117.58)	7.43 (117.70)
11	7.40 (117.23)	7.40 (117.35)
13	7.40 (117.36)	7.41 (117.47)
15	7.45 (118.05)	7.45 (118.14)

Slope = 2% downhill
Comp. Coeff. Kc = 2.0

Junction Pressures

Junction Node Number	System Elevations meters (ft	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	5.49 (18)	297.61 (99.57)	297.80 (99.63)
4	6.71 (22)	285.92 (95.65)	286.08 (95.71)
6	7.92 (26)	274.93 (91.98)	275.11 (92.04)
8	9.14 (30)	265.03 (88.67)	265.19 (88.72)
10	10.36 (34)	256.55 (85.83)	256.67 (85.87)
12	11.58 (38)	249.81 (83.57)	249.92 (83.61)
14	12.80 (42)	245.11 (82.00)	245.16 (82.02)
16	14.02 (46)	242.74 (81.21)	242.74 (81.21)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	8.36 (132.48)	8.37 (132.63)
3	8.19 (129.79)	8.20 (129.94)
5	8.03 (127.22)	8.04 (127.37)
7	7.88 (124.86)	7.89 (125.00)
9	7.75 (122.80)	7.76 (122.94)
11	7.64 (121.14)	7.65 (121.27)
13	7.57 (119.97)	7.58 (120.08)
15	7.53 (119.38)	7.54 (119.47)

Slope = 2% uphill
Comp. Coeff. Kc = 3.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	24.90 (82)	120.36 (40.27)	120.55 (40.33)
4	23.77 (78)	132.44 (44.31)	132.62 (44.37)
6	22.56 (74)	144.87 (48.47)	145.06 (48.53)
8	21.34 (70)	157.91 (52.83)	158.09 (52.89)
10	20.12 (66)	171.82 (57.48)	171.96 (57.53)
12	18.90 (62)	186.91 (62.53)	187.03 (62.57)
14	17.68 (58)	203.50 (68.08)	203.56 (68.10)
16	16.46 (54)	221.95 (74.25)	221.94 (74.25)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	5.19 (82.20)	5.20 (82.37)
3	5.45 (86.32)	5.46 (86.49)
5	5.70 (90.37)	5.71 (90.54)
7	5.96 (94.44)	5.97 (94.61)
9	6.22 (98.60)	6.23 (98.77)
11	6.49 (102.93)	6.50 (103.09)
13	6.78 (107.50)	6.79 (107.65)
15	7.09 (112.37)	7.10 (112.51)

Slope = 1% uphill
Comp. Coeff. Kc = 3.0

Junction Pressures

Junction Node Number	System Elevations meters (ft	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	20.12 (66)	163.83 (54.81)	164.07 (54.89)
4	19.51 (64)	169.98 (56.87)	170.20 (56.94)
6	18.90 (62)	176.58 (59.07)	176.80 (59.15)
8	18.29 (60)	183.94 (61.54)	184.16 (61.61)
10	17.68 (58)	192.35 (64.35)	192.53 (64.41)
12	17.07 (56)	202.14 (67.62)	202.27 (67.67)
14	16.46 (54)	213.63 (71.47)	213.72 (71.50)
16	15.85 (52)	227.19 (76.01)	227.17 (76.00)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	6.07 (96.23)	6.08 (96.42)
3	6.19 (98.06)	6.20 (98.25)
5	6.31 (99.98)	6.32 (100.18)
7	6.44 (102.09)	6.45 (102.28)
9	6.59 (104.45)	6.60 (104.63)
11	6.76 (107.13)	6.77 (107.31)
13	6.95 (110.20)	6.96 (110.36)
15	7.17 (113.72)	7.18 (113.86)

Slope = 0%
Comp. Coeff. Kc = 3.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	15.24 (50)	207.37 (69.38)	207.65 (69.47)
4	15.24 (50)	207.58 (69.45)	207.86 (69.54)
6	15.24 (50)	208.35 (69.70)	208.61 (69.79)
8	15.24 (50)	210.03 (70.26)	210.28 (70.35)
10	15.24 (50)	212.94 (71.24)	213.15 (71.31)
12	15.24 (50)	217.42 (72.74)	217.57 (72.79)
14	15.24 (50)	223.81 (74.87)	223.80 (74.90)
16	15.24 (50)	232.46 (77.77)	232.43 (77.76)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	6.85 (108.54)	6.86 (108.75)
3	6.85 (108.59)	6.86 (108.81)
5	6.86 (108.80)	6.88 (109.01)
7	6.89 (109.24)	6.91 (109.45)
9	6.94 (110.02)	6.95 (110.22)
11	7.02 (111.19)	7.03 (111.38)
13	7.12 (112.85)	7.13 (113.02)
15	7.26 (115.05)	7.27 (115.20)

Slope = 1% downhill
Comp. Coeff. Kc = 3.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	10.36 (34)	250.96 (83.96)	251.29 (84.07)
4	10.97 (36)	245.23 (82.04)	245.55 (82.15)
6	11.58 (38)	240.17 (80.35)	240.47 (80.45)
8	12.19 (40)	236.17 (79.01)	236.46 (79.11)
10	12.80 (42)	233.57 (78.14)	233.83 (78.23)
12	13.41 (44)	232.74 (77.86)	232.94 (77.93)
14	14.02 (46)	234.01 (78.29)	234.10 (78.32)
16	14.63 (48)	237.74 (79.53)	237.72 (79.53)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	7.55 (119.64)	7.56 (119.88)
3	7.46 (118.24)	7.47 (118.47)
5	7.38 (116.99)	7.40 (117.22)
7	7.32 (115.99)	7.33 (116.21)
9	7.28 (115.34)	7.29 (115.55)
11	7.26 (115.12)	7.28 (115.32)
13	7.28 (115.45)	7.29 (115.62)
15	7.34 (116.38)	7.35 (116.53)

Slope = 2% downhill
Comp. Coef. Kc = 3.0

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	5.49 (18)	294.59 (98.55)	294.96 (98.68)
4	6.71 (22)	282.92 (94.65)	283.27 (94.77)
6	7.92 (26)	272.03 (91.01)	272.39 (91.13)
8	9.14 (30)	262.34 (87.77)	262.68 (87.88)
10	10.36 (34)	254.24 (85.06)	254.52 (85.15)
12	11.58 (38)	248.09 (83.00)	248.30 (83.07)
14	12.80 (42)	244.24 (81.71)	244.36 (81.75)
16	14.02 (46)	243.03 (81.30)	243.01 (81.30)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	8.19 (129.84)	8.21 (130.10)
3	8.02 (127.19)	8.04 (127.44)
5	7.87 (124.67)	7.88 (124.91)
7	7.72 (122.38)	7.74 (122.62)
9	7.60 (120.44)	7.61 (120.66)
11	7.50 (118.94)	7.52 (119.15)
13	7.44 (117.99)	7.46 (118.18)
15	7.43 (117.70)	7.43 (117.84)

Slope = 2% uphill
Comp. Coeff. Kc = 3.0
Pump Head = 119.6 KPa

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	24.90 (82)	224.42 (75.08)	224.60 (75.14)
4	23.77 (78)	236.60 (79.15)	236.77 (79.21)
6	22.56 (74)	249.41 (83.44)	249.56 (83.49)
8	21.34 (70)	263.27 (88.08)	263.40 (88.12)
10	20.12 (66)	278.61 (93.21)	278.71 (93.24)
12	18.90 (62)	295.90 (98.99)	295.92 (99.00)
14	17.68 (58)	315.63 (105.59)	315.56 (105.57)
16	16.46 (54)	338.34 (113.19)	338.13 (113.12)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	7.13 (113.01)	7.14 (113.20)
3	7.32 (116.14)	7.34 (116.29)
5	7.52 (119.26)	7.54 (119.46)
7	7.74 (122.60)	7.75 (122.80)
9	7.96 (126.20)	7.97 (126.39)
11	8.21 (130.14)	8.22 (130.32)
13	8.49 (134.50)	8.50 (134.66)
15	8.79 (139.35)	8.80 (139.49)

Slope = 1% uphill
Comp. Coeff. Kc = 1.0
Pump Head = 119.6 KPa

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	20.12 (66)	274.51 (91.84)	274.46 (91.82)
4	19.51 (64)	280.71 (93.91)	280.68 (93.90)
6	18.90 (62)	287.50 (96.18)	287.46 (96.17)
8	18.29 (60)	295.25 (98.78)	295.21 (98.76)
10	17.68 (58)	304.31 (101.81)	304.26 (101.79)
12	17.07 (56)	315.03 (105.39)	314.99 (105.38)
14	16.46 (54)	327.78 (109.66)	327.82 (109.67)
16	15.85 (52)	342.95 (114.73)	343.12 (114.79)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	8.14 (129.05)	8.15 (129.11)
3	8.24 (130.54)	8.24 (130.59)
5	8.34 (132.14)	8.34 (132.20)
7	8.45 (133.95)	8.45 (134.01)
9	8.58 (136.03)	8.59 (136.10)
11	8.74 (138.46)	8.74 (138.53)
13	8.91 (141.30)	8.92 (141.38)
15	9.12 (144.60)	9.13 (144.72)

Slope = 0%
Comp. Coeff. Kc = 0.0
Pump Head = 119.6 KPa

Junction Pressures

Junction Node Number	System Elevations meters (ft	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	15.24 (50)	322.79 (107.99)	322.77 (107.98)
4	15.24 (50)	323.02 (108.07)	323.00 (108.06)
6	15.24 (50)	323.84 (108.34)	323.81 (108.33)
8	15.24 (50)	325.57 (108.92)	325.55 (108.91)
10	15.24 (50)	328.53 (109.91)	328.50 (109.90)
12	15.24 (50)	333.02 (111.41)	333.02 (111.41)
14	15.24 (50)	339.34 (113.53)	339.36 (113.53)
16	15.24 (50)	347.79 (116.35)	347.84 (116.37)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	8.98 (142.38)	8.99 (142.43)
3	8.99 (142.43)	8.99 (142.48)
5	9.00 (142.61)	9.00 (142.66)
7	9.02 (143.00)	9.03 (143.05)
9	9.06 (143.66)	9.07 (143.72)
11	9.13 (144.67)	9.13 (144.72)
13	9.22 (146.06)	9.22 (146.13)
15	9.33 (147.91)	9.34 (147.99)

Slope = 2% downhill
Comp. Coeff. Kc = 2.0
Pump Head = 119.6 KPa

Junction Pressures

Junction Node Number	System Elevations meters (ft)	Finite Element Pressures KPa (ft)	Linear Theory Pressures KPa (ft)
2	5.49 (18)	403.10 (134.86)	403.26 (134.91)
4	6.71 (22)	391.50 (130.98)	391.67 (131.03)
6	7.92 (26)	380.85 (127.41)	380.99 (127.46)
8	9.14 (30)	371.68 (124.34)	371.79 (124.38)
10	10.36 (34)	364.46 (121.93)	364.55 (121.96)
12	11.58 (38)	359.66 (120.32)	359.71 (120.34)
14	12.80 (42)	357.72 (119.67)	357.68 (119.66)
16	14.02 (46)	359.09 (120.13)	358.96 (120.09)
17	15.24 (50)	239.13 (80.00)	239.13 (80.00)

Sprinkler Flows

Sprinkler Number	Finite Element Flow liters/sec (gpm)	Linear Theory Flow liters/sec (gpm)
1	9.76 (154.68)	9.77 (154.84)
3	9.61 (152.39)	9.62 (152.54)
5	9.48 (150.26)	9.49 (150.41)
7	9.36 (148.40)	9.37 (148.55)
9	9.27 (146.92)	9.28 (147.06)
11	9.21 (145.93)	9.21 (146.06)
13	9.18 (145.53)	9.19 (145.64)
15	9.20 (145.81)	9.20 (145.90)

LIST OF REFERENCES

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Adams, R.W. 1961. Distribution Analysis by Electronic Computer. Journal of the Institute of Water Engineers 15:415-428.

Barlow, J.F. and E. Markland. 1969. Computer Analysis of Pipe Networks. Proceedings I.C.E. 43:249-259.

Benedict, R.P. 1980. Fundamentals of Pipe Flow, John Wiley and Sons.

Bralts, V.F. 1983. Hydraulic Design and Field Evaluation of Drip Irrigation Submain Units. PhD Thesis (Agricultural Engineering), Michigan State University.

Bralts, V.F. and L.F. Segerlind. 1985. Finite Element Analysis of Drip Irrigation Submain Units. TRANS ASAE 28(3):809-814.

Chandrashekar, M. 1980. Extended Set of Components in Pipe Networks. Journal of the Hydraulics Division, ASCE 106(HY1):133-149.

Chandrashekar, M. and K.H. Stewart. 1975. Sparsity Oriented Analysis of Large Pipe Networks. Journal of the Hydraulics Division ASCE 101(HY4):341-355.

Chaudhry, H. and V. Yevjevich. 1981. Closed Conduit Flow. Water Resources Publications.

Chenoweth, H. and C. Crawford. 1974. Pipe Network Analysis. Journal Anerican Waterworks Association, Jan.

Chin, K.K., R.K. Gay, S.H. Hua, C.H. Chan, and S.Y. Ho. 1978. Solution of Water Networks by Sparse Matrix Methods. International Journal for Numerical Methods in Engineering 12:1261-1277.

Christiansen, J.E. 1942. Irrigation by Sprinkling. California Agricultural Experimental Station Bulletin 670, University of California, Berkeley, California.

Cornish, R.J. 1939. The Analysis of Flow in Network of Pipes. Journal of the Institute of Civil Engineering 13:147-164.

Cross, H. 1936. Analysis of Flow in Networks of Conduits or Conductors. Bulletin 286, University of Illinois, Engineering Experimental Station, Urbana, Illinois.

Demuren, A.O. and F.J. Ideriah. 1986. Pipe Network Analysis by Partial Pivoting Method. Journal of the Hydraulics Division, ASCE 112(5):327-334.

Dillingham, J.H. 1967. Computer Analysis of Water Distribution Systems. Water and Sewage Works, Jan-May, pp 43-45.

Dodge, E.R., H.R. Hoellin, and L. Tetmajer. 1978. The Analysis of Large, Complex Water Networks with Small Computer Systems. Journal of the American Waterworks Association 70(7):336-370.

Edwards, D.M. and B. Spencer. 1972. Design Criteria for Irrigation systems with Complex Pipe Loops. TRANS ASAE 15(1):76-78.

Epp, R. and A.G. Fowler. 1970. Efficient Code for Steady-State Flows in Networks. Journal of the Hydraulics Division, ASCE 96(HY1):43-56.

Everstine, G.C. 1986. Dynamic Analysis of Fluid-filled Piping Systems Using Finite Element Techniques. Journal of Pressure Vessel Technology, ASME 108:57-61.

Finkel, H. 1982. CRC Handbook of Irrigation Technology, Vol 1, Chapter 8, pp. 171-193.

Gay, R.K.L., K.K. Chin, S.H. Chua, C.H. Chan, and S.Y. Ho. 1978. Node Reordering Algorithms for Water Network Analysis. International Journal for Numerical Methods in Engineering 12:1241-1259.

Giudice, J.J. 1965. Analysis of Pipe Network Based on the Newton-Raphson Method. Thesis presented to MIT at Cambridge, Mass., in partial fulfillment of the requirements for the degree of Master of Science.

Gofman, E. and M. Rodeh. 1981. Loop Equations with Unknown Pipe Characteristics. Journal of the Hydraulics Division, ASCE 107(HY9):1047-1060.

Grooms, H.R. 1972. Algorithm for Matrix Bandwidth Reduction. Journal of the Hydraulics Division, ASCE 98(ST1):203-214.

Haghighi, K., V.F. Bralts, and L.J. Segerlind. 1987. Finite Element Formulation of Dividing and Combining Flows in Tee and Wye Components, paper presented at ASAE Summer meeting, Baltimore, Maryland.

Hatfield, F.J. and Wiggert, D.C. Fluid-Structure Interaction in Piping by Component Synthesis. The Fluids Engineering Conference, Boulder, Colorado, June 22-24, 1981.

Hart, W.E. and W.N. Reynolds. 1965. Analytical Design of Sprinkler Systems. TRANS ASAE 8(1):83-85, 89.

Henriksen, M. 1984. A Finite Element Model For a Single and Multi-phase Flow. Pipeline Engineering Symposium, ASME, pp 105-110.

Hoang, L.N. and G. Weinberg. 1957. Pipeline Network Analysis by Electronic Digital Computer. Journal of the American Waterworks Association 49:517-524.

Howell, T.A. and E.A. Hiler. 1974. Trickle Irrigation Lateral Design. TRANS ASAE 17(5):902-908.

Howell, T.A. and F.A. Barinas. 1980. Pressure Losses Across Trickle Irrigation Fittings and Emitters. TRANS ASAE 23(3):928-933.

Irrigation Journal. 1986. Irrigation Survey, Brantwood Publications, Tampa, Florida. Nov/Oct Issue.

Isaacs, L. and K.G. Mills. 1980. Linear Theory Methods for Pipe Network Analysis. Journal of the Hydraulics Division, ASCE 106(HY7):1191-1201.

Jenson, M.C. and A.M. Frantini. 1957. Adjusted F Factors for Sprinkler Lateral Design. Agricultural Engineering 38:247.

Jenson, M.E. 1981. Design and Operation of Farm Irrigation Systems. ASAE, publ. 1-80, St Joseph, Michigan.

Jeppson, R.W. 1982. Analysis of Flow in Pipe Networks. Ann Arbor Science, Ann Arbor, Michigan.

Jeppson, R.W. 1982. Equivalent Hydraulic Pipe for Parallel Pipes. Journal of the Hydraulics Division, ASAE 108(HY1):35-45.

Jeppson, R.W. and A. Tavallence. 1975. Journal of the Hydraulics Division, ASCE 101(HY3):567-580.

Jeppson, R.W. and A.L. Davis. 1976. Journal of the Hydraulics Division, ASCE 102(HY7):987-1001.

Keller, J. and D. Karmeli. 1974. Trickle Irrigation Design Parameters. TRANS ASAE 17(4):678-684.

Keller, J. and D. Karmeli. 1975. Trickle Irrigation Design. Rain Bird Sprinkler Manufacturing Co., Glendora, California.

Kesavan, H.K. and M. Chandrashekar. 1972. Graph-Theoretic Models for Pipe Network Analysis. Journal of the Hydraulics Division, ASAE 98(HY2):345-364.

LaLiberte, F.E., M.N. Shearer and M.J. English. 1983. Design on Energy-Efficient Pipe-Size expansion. Journal of Irrigation and Drainage Engineering, ASCE 109(1):13-28.

Lam, C.F. and M.L. Wolla. 1972. Computer Analysis of Water Distribution Systems: Part I - Formulation of EQT. Journal of the Hydraulics Division, ASCE 98(HY2):335-344.

Lam, C.F. and M.L. Wolla. 1972. Computer Analysis of Water Distribution Systems: Part II- Numerical Solution. Journal of the Hydraulics Division, ASCE 98(HY3): 447-460.

Lemieux, P.F. 1972. Efficient Algorithm for Distribution Networks. Journal of the Hydraulics Division, ASCE 98(HY11):1911-1920.

Marlow, T.A., R.L. Hardison, H. Jacobson, and G.E. Biggs. 1966. Improved Design of Fluid Networks with Computers. Journal of the Hydraulics Division, ASCE 92(HY4):43-61.

Martin, D.W. and G. Peters. 1963. The Application of Newton's Method to Network Analysis by Digital Computers. Journal of the Institute of Water Engineers 17:115-129.

McCormick, M. and C.J. Bellamy. 1968. A Computer Program for the Analysis of Pipes and Pumps. The Journal of the Institution of Engineers, Australia 38(3):51-58.

McPherson, M.B. and J.V. Radzivl. 1958. Water Distribution Disign and the McIlroy Network Analyzer. Journal of the Hydraulic Division HY2

Miller, D.S. 1971. Internal Flow, a Guide to Losses in Pipe and Duct Systems. The British Hydromechanics Research Association, Ranfield, Bedford, England.

Miller, D.S. 1978. Internal Flow Systems. The British Hydromechanics Research Association, Fluid Engineering, Cranfield, Bedford, England.

Myers, L.E. and D.A. Bucks. 1972. Uniform Irrigation with Low Pressure Trickle Systems. Journal of the Irrigation and Drainage Division, ASCE 98(IR3):341-346.

Norrie, D.H. and G. DeVries. 1978. An Introduction to Finite Element Analysis, Academic Press, New York.

Ohtmer, O. 1983. Nonlinear Flow Analysis in Pipe Networks. International Journal for Numerical Methods in Engineering 19:373-392.

Onizuka, K. 1986. System Dynamics Approach to Pipe Network Analysis. Journal of the Hydraulics Division, ASCE 112(8):728-749.

Pair, C.H. 1975. Sprinkler Irrigation. Sprinkler Irrigation Association, 4th edition.

Perold, P.R. 1977. Design of Irrigation Pipe Laterals with Multiple Outlets. Journal of the Irrigation and Drainage Division, ASCE 103(IR2):179-193.

Pitchai, R. 1966. A Model for Designing Water Distribution Pipe Networks. Thesis presented to Harvard University at Cambridge, Mass., in partial fulfillment of the Requirements for the Degree of Doctor of Philosophy.

Power, J.W. 1986. Sharing Irrigation Know-How with Developing Countries. Agricultural Engineering, Oct/Sept 1986, pp.15-18.

Porenta, G., D.F. Young, and T.R. Rogge. 1986. A Finite Element MOdel of Blood Flow in Arteries Including Taper, Branches and Obstruction. Journal of Biomechanical Engineering, ASME 108:161-167.

Russo, E.P. and L.A. Smith. 1986. Amalyzing Piping Systems. ASHRAE 28:82-84.

Schwab, G.O., R.K. Frevert, T.W. Edminster, and K.K. Barnes. 1981. Soil and Water Conservation Engineering. John Wiley and Sons.

Schwirian, R.E. and M.E. Karabin. Use of SPAR Elements to Simulate Fluid-Solid Interaction in the Finite Element Analysis of Piping System Dynamics. Symposium on Fluid Transients and Structural Interactions in Piping Systems, ASME, June 1981m pp. 1-11.

Segerlind, L.J. 1984. Applied Finite Element Analysis. John Wiley and Sons.

Shamir, U. and C.D. Howard. 1968. Water Distribution Systems Analysis. Journal of the Hydraulics Division, ASCE, 94(HY1):219-234

Shamir, U. and C.D. Howard. 1977. Engineering Analysis of Water-Distribution Systems. Journal of the American Waterworks Association 69(9):510-514.

Shimizu, Y., K. Sugino, M. Yasui, Y. Hayakawa, and S. Kuzuhara. 1985. Flow Patterns and Hydraulic Losses in Quasi-Coil Pipes. Bulletin of the Journal of the Society of Mechanical Engineers 28(241):1379-1386.

Solomon, K. and J. Keller. 1974. Trickle Irrigation Uniformity and Efficiency. Paper presented at ASCE Annual Meeting, Kansas City, Missouri.

Walski, T.M. 1983. Using Water Distribution System Models. Journal of the American Waterworks Association 75:59-63.

Wiggert, D.C. 1986. Closed Conduit Hydraulics. Unpublished Course Material.

Wiggert, D.C., R.S. Otwell, and F.J. Hatfield. 1985. The Effect of Elbow Restraint on Pressure Transients. TRANS ASME 107:402-406.

Wilcox, J.C. and G.E. Swailes. 1947. Uniformity of Water Distribution by some Undertree Orchard Sprinklers. Scientific Agriculture 27(11): 565-583.

Wood, D.J. and C.O.A. Charles. 1972. Hydraulic Network Analysis Using Linear Theory. Journal of the Hydraulics Division, ASCE 98(HY7):1157-1170

Wood, D.J. and C.O. Charles. 1973. Closure of Hydraulics Network Analysis Using Linear Theory. Journal of the Hydraulics Division, ASCE 99(HY11):2129.

Wood, D.J. 1980. Users Manual - Computer Analysis of Flow in Pipe Networks Including Extended Period Simulations. Office of Engineering Continuing Education, University of Kentucky, Lexington, KY.

Wood, D.J. and A.G. Rayes. 1981. Reliability of Algorithms for Pipe Network Analysis. Journal of the Hydraulics Division, ASCE 107(HY10):1145-1161.

Wu, I-Pai and D.D. Fangmeier. 1974. Hydraulics Design of Twin Chamber Trickle Irrigation Laterals. Technical Bulletin 216, University of Arizona.

Wu, I-Pai and H.M. Gitlin. 1974. Design of Drip Irrigation Lines. HAES Technical Bulletin 96, University of Hawaii, Honolulu, Hawaii.

Wu, I-Pai and H.M. Gitlin. 1975. Energy Gradient Line for Drip Irrigation Laterals. Journal of the Irrigation and Drainage Division, ASCE 10(IRU):321-326.

Wu, I-Pai and H.M. Gitlin. 1983. Sprinkler Irrigation Design for Uniformity on Slopes. TRANS ASAE 26(6):1698-1703.

Wu, I-Pai, T.A. Howell, and E.A. Hiler. 1979. Hydraulic Design of Drip Irrigation Systems. HAES Technical Bulletin 105, University of Hawaii, Honolulu, Hawaii.