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# CONTRADICTIONS OF A CURRICULUM REFORM: THE LIMITS OF INSTRUMENTAL IDEOLOGY ON MATHEMATICS EDUCATION IN AN URBAN DISTRICT

By

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## ABSTRACT

## CONTRADICTIONS OF A CURRICULUM REFORM: THE LIMITS OF INSTRUMENTAL IDEOLOGY ON MATHEMATICS EDUCATION IN AN URBAN DISTRICT

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This study examines an elementary curriculum reform effort being carried out in a large urban school district. Teachers were charged with implementing city-wide mandated curricula in seven content areas over a three year period. Using the mathematics curriculum as the case, this study explores contextual factors, in and out of schools, that influenced the course of this reform. The study aims to show the relationship between the nature of a district-wide educational reform and the larger social context in which it was conceived. Three questions frame the study. 1) In what ways and to what extent had political, economic and organizational factors shaped the design of this curriculum and mandates for its implementation? 2) What was the nature of the curriculum reform and what was its potential to develop children's mathematical knowledge and power? 3) In what ways did the district prepare teachers to implement the newly-mandated curriculum in their classrooms?

The data collected for this field-based study included: observations of teacher inservice; interviews with district teachers and administrators, community people and experts on the city's economy; district prepared curriculum and instructional materials; documents from the federal desegregation order and the decentralization, then recentralization of the district; figures on school financing, student achievement and drop-out; and economic and demographic statistics for the city. The study's results include analysis of the formal curriculum documents and the inservice opportunities provided for teachers using a framework that poses three orientations to mathematics education: certification of numeracy; comprehension of mathematical ideas; and enabling mathematical inquiry.

Using the concept of genealogy, the reform is located within a set of linked events, ideas, practices and people that shaped the initiative. The study provides a genealogical account of the reform, the antecedants that preceeded the reform and out of which it emerged. The genealogy provides a useful way to investigate what appear to be contradictions between a rhetoric of empowering youngsters and a curriculum and a model of instruction embedded in instrumentalism.

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**To**

**Jack and Michael**

## ACKNOWLEDGMENTS

This dissertation represents the culmination of a half decade of study, teaching and research. A number of people have contributed in multiple ways, as teachers and mentors, as friends and colleagues. As this thesis awaits transport to the printer and binder, I am honored to complete these final pages by acknowledging my appreciation for all who have helped make it possible.

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pleasure in the unfolding of his life as he pursues his own interests. His independence, personal responsibility and support allowed me to be a sometimes absent parent without worry and without much guilt.

I dedicate this work to Jack and Michael as an expression of my deep love and respect.

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# LOCATING THE STUDY IN A BROADER SOCIAL, POLITICAL AND HISTORICAL CONTEXT

## INTRODUCTION

While the educational reform agenda of the eighties has been aimed at improving schooling for all students, those who have benefitted most are college bound youth. It is only recently that the educational community has begun to explicitly address the needs of "at-risk" and non-college bound youth. In a recent report entitled The Forgotten Half: Non-College Youth in America, the W. T. Grant Foundation Commission on Work, Family and Citizenship called attention to the bleak economic future facing many of our young people who finish their education when they leave high school. Although many young people without post-secondary education do succeed, an increasingly larger number are finding it harder than ever to find work that is rewarding, provides steady employment, decent wages and benefits, and offers opportunities for further training and advancement. This is particularly true for minorities in impoverished inner cities. The picture is even more bleak for those who drop out of high school.

The worsening economic situation for many young adults is due in large measure to structural changes in the U.S. economy (Wilson, 1987), a situation over which schools have no control. Yet schools continue to be the targets of criticism for failing to prepare young people with the appropriate skills and attitudes to find and hold steady, decent-paying jobs.

School boards have responded to this growing criticism by increasing the graduation requirements in English, mathematics, social studies and science. Urban districts with large working-class, poor and minority populations have tended to focus their efforts on increasing proficiency in basic communication, reading and mathematics skills. However, minimum competency initiatives alone will not provide the educational experiences to help all young people become critical, flexible, interactive, self-confident problem solvers, arguably the single most important skill for work and citizenship in the 21st century (Giroux, 1984; Jacobs, 1987). Literacy and numeracy must take on new meaning, not in just a limited economic and technical sense, but also in a political and social sense. Classrooms must become communities of critical dialogue, where multiple forms of intellectual inquiry are promoted, where young people collaborate to solve problems they have helped to define, where they share understandings and build upon each others' insights, and where they learn to make reasoned judgments about their own thinking and that of others. Education must provide the knowledge, skills and dispositions for work in an increasingly complex, technological economy. But it must also contribute to a citizenry educated in civic responsibility by developing a commitment and a set of analytic tools to engage in the sociopolitical life of a democratic society (Giroux, 1984; Katz, 1982; Reich, 1988). As Young and Melnick (1988) point out, schools "have a dismal record for having bettered the lives of most of the youthful poor entrusted to their care" (p. 387). Districts with significant poor and minority populations can no longer support policies, programs and practices that continue to ill-serve children and youth who are "'truly' educationally at-risk" (p. 387). These districts must conceive an educational program that empowers young people to become problem posers and problem solvers, able to use the power of disciplinary knowledge to understand, participate in, and **shape** the world around them.

This study examines a curriculum reform effort being carried out in a large urban school district. In 1987, the Superintendent and the Board approved and mandated city-wide curricula in seven content areas--language arts, social studies, mathematics, science, music, art, and health/physical education. Teachers were charged with implementing the newly mandated curricula over a three year period beginning with the 1987-88 school year. Using the mathematics curriculum as the case, this study explores contextual factors, in and out of schools, that influenced the course of this reform. Three questions frame the study. 1) In what ways and to what extent had political, economic and organizational factors shaped the design of this curriculum and mandates for its implementation? 2) What was the nature of the curriculum reform and what was its potential to develop children's mathematical knowledge and power? 3) In what ways did the district prepare teachers to implement the newly-mandated curriculum in their classrooms?

## THE LABORATORY OF THE STUDY: DETROIT PUBLIC SCHOOLS

This study was conducted in the Detroit Public Schools during the 1987-88 school year. The city and its schools provided an extraordinary laboratory in which to examine the curriculum reform efforts of a large urban district faced with a number of educational dilemmas. Some of the dilemmas arose in the context of the schools themselves; others grew out of unequal social conditions in the larger community. To begin, it is necessary to situate the story in a sociopolitical and historical context, to take a look back at events that changed the Detroit schools in fundamental ways and created the conditions for a new curriculum reform initiative.

**Desegregation and Decentralization: Politics and Unintended Consequences**

In 1971, the National Association for the Advancement of Colored People (NAACP) filed suit in Federal Court charging the Detroit School Board with intentional racial segregation in the district. This event set the stage for five years of litigation challenging various plans to desegregate the schools including altering attendance boundaries of high schools and creating voluntary magnet schools. In 1972, Federal District Court Judge Stephen Roth, who heard the case until his death in 1974, selected a metropolitan remedy involving Detroit and 52 suburban school districts in Wayne, Oakland and Macomb counties, involving about 780,000 children. However, in 1974, the United States Supreme Court ruled that, because only Detroit had been found guilty of de jure segregation, the remedy must be confined to the city. In 1975, Judge Robert DeMascio was assigned to the case and in the next year issued a series of orders on student and teacher reassignment and a set of educational components.[1]

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[1]The desegregation orders also created a 55-member monitoring commission to act as an arm of the Court to oversee the order. It was to function as a citizen's committee, auditing the District's efforts and assisting the Court with reports and recommendations regarding implementation of the components of the desegregation plan. For a history of the desegregation of the Detroit schools, see Eleanor P. Wolf's, Trial and Error.

Student reassignment. In January, 1976, bussing for the purposes of student reassignment began. The district had been decentralized by act of the state legislature in 1969, dividing the district into eight regions. The school board was determined not to bus students across regional boundaries and to limit the number of white pupils reassigned to predominantly black schools. In 1961 when data on enrollment by race was first released, 54% of the pupils in the system were white. By 1975, the schools were 75% black.[2] The desegregation order specified that schools 30% or more black would be considered integrated. Those schools that were less than 30% black could be involved in bussing to achieve integration. In January, 1976, nearly 28,000 students (11% of the student enrollment) were reassigned.

The reassignment plan changed the racial balance of 105 of the 300 schools. Sixty-seven of the 80 schools with white majorities received black students. Thirty-eight schools that were at least 80% black received white students. But half the schools in the district remained

[2]Although the departure of white families from the city had begun in earnest in the late 1950s, two events hastened the exodus. Detroit was one of the nation's urban centers where black communities erupted in the summer of 1967. Sparked by a police raid on a "blind pig," the rebellion symbolized the growing resentment of the black community toward a nearly all-white, openly racist police department. The violence on the streets was met by organized state violence (President Johnson ordered Federal troops into the city) leaving hundreds of homes and stores in ruin, and thousands of people homeless. The turmoil in the neighborhoods spilled over into the schools. Militant groups of students, black radicals and white right-wingers, precipitated major confrontations at some of the high schools. On Malcolm X Day in 1969, hundreds of high school students across the city staged black power demonstrations. The growing polarization among black and white students and the inability of school administrators to find a common ground for community dialogue led to the white population's abandonment of the city's schools. By 1970, fewer than 40% of the students were white (see Conot, 1974; Georgakas & Surkin, 1975). The second event that further accelerated the flight of whites from the city was the series of court decisions from 1970 until 1974 that ultimately led to the Detroit school desegregation order.

more than 90% black. Regions 1, 5, and 8, those with the greatest proportion of black pupils (90.3%, 96.7%, 95.2%, respectively) were excluded from the student reassignment plan (United States District Court, Eastern District of Michigan, Southern Division, May 11, 1976). There was considerable fear that the reassignment of white pupils to schools in Regions 1 and 8, inner-city regions with areas of extreme poverty, would exacerbate the problem of "white flight." There seemed to be a further explanation for the exclusion of Regions 1 and 5 from the reassignment plan--political clout of residents in those Regions. Region 1 contained a part of the city -- in an earlier time called "Black Bottom" -- where black residents had a long and distinguished history of community activism. Region 5 contained one of the middle-class enclaves where black and white professionals, city government officials, managers and administrators of social service agencies and educational institutions made their homes.[3] Several sources suggested that organized political pressure from influential members of these communities put sufficient pressure on the school board to exclude these Regions from the student reassignment plan.

In 1978, the NAACP sought to have bussing expanded and the Sixth Court of Appeals ordered such an expansion, even though by then fewer than 15% of the students in the district were white. By 1987, when this study was conducted, less than 10% of the students were white.

[3]City residency for workers was required by a number of employers in the city. Workers employed by the City of Detroit were required to live within the city. Also, members of OSAS, the Organization of School Administrators in the Detroit Public Schools, were required to be city residents. There was no such condition of employment for teachers in the system. The Detroit school board had tried in several contract negotiations to win a residency requirement for teachers in the Detroit Public Schools, a concession they were never able to win from the Detroit Federation of Teachers.

The district reported the following racial/ethnic composition of the student population in 1987: Black, 159,275 (88.45%); White, 15,713 (9.01%); Hispanic, 3,515 (1.65%); Asian, 1,106 (.62%); and American Indian, 497 (.27%). Most white families had abandoned the Detroit schools, either leaving the city altogether, or -- for those who remained and could afford several thousand dollars a year in tuition -- enrolling their children in private or parochial schools.

Educational components. The Court's overview of the district was not limited to assuring racial integration of the schools. A second stipulation required the district to include **educational components** to "eradicate the effects of past discrimination" (United States District Court, Eastern District of Michigan, Southern Division, August 15, 1975). In the first year of the Court's oversight, Judge Robert DeMascio ordered the implementation of eight educational components--reading, testing, bilingual education, inservice training, school-community relations, vocational education, counseling and guidance, and a student code of conduct. The specific set of proposals that created the educational components originated with the central administration of the District and not the Court (with the exception of the Code of Conduct).

At the time of the desegregation order, central administration had limited authority over educational programs. The Michigan legislature had enacted a law in 1969 decentralizing administrative authority in the Detroit schools.[4] The legislation divided the district into eight

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[4]The key architect of the bill was then Senator Coleman A. Young from Detroit. The legislation was intended to give the black community some measure of input to and control over its schools. Young was elected mayor of Detroit in 1973, the first black to serve as the city's chief executive, a position he continues to hold.

Regions, each with its locally elected school board. Each board appointed a regional superintendent. In addition, the district maintained a central administration, with a general superintendent and a central board, some of whose members came from the regional boards, others who were elected in city-wide elections. Regional superintendents and boards had control over curriculum and staffing decisions in their schools. The central administration maintained control over financial matters for the district. However, in many other areas of the day-to-day operations of the schools, there was considerable ambiguity about lines of authority among schools, regions, and the central administration.

The Court considered the decentralized administrative structure a serious impediment to its desegregation efforts. Judge DeMascio's order to the District to include educational components in its remedy opened the door for the general superintendent and his lieutenants to consolidate their power, with the Court's support. As the district proposed programs they sought to initiate, the Judge looked for a rationale to support the implementation, consistent with his order to remedy past discriminatory practices.

The history of implementation of the educational components was a mix of consensus and controversy. Those programs that did not affect local and regional administrative prerogatives and where Region cooperation was solicited were supported. Those components that encroached on local administrative authority were met with resistance or outright opposition by regional and building administrators. Many school principals actively opposed the establishment of school-community relations councils and the new standards imposed by the student code of conduct and bilingual education. They also resisted attempts to redefine the role of guidance counselors. The court-ordered increase in the delivery of guidance services meant that counselors no longer had time to handle administrative tasks -- membership accounting, attendance record-keeping, student discipline -- for which they had been previously responsible. The central administration in its reports to the Court indicated wide support for the inservice training, testing and reading components. While it appeared there was substantial support from regional and local school administrators, teachers were extremely unhappy with the reading component. The Detroit Federation of Teachers filed a number of grievances over the increased paper work that resulted from the reading program.

Recentralizing authority. By the late seventies, there was considerable public displeasure with the inefficiency, administrative chaos and regional board patronage that had come to characterize the decentralized structure. In addition, Detroit had become a majority black city. Many of its leaders were black, including the mayor and the general superintendent. The perceived need for a decentralized system to provide control to the black community had decreased. In 1981, city voters approved a ballot proposal to recentralize the school district under a central administration.

The recentralization of the district did not fully restore complete administrative authority to the central administration. The eight Regions were reconfigured into six geographical Areas. Regional boards were eliminated and a central board was constituted by members elected in both Area and city-wide contests. Area superintendents were appointed by and accountable to the general superintendent and the central board. Textbook adoptions were centrally approved but decisions about which among the approved texts would be used resided at the Area and individual school level. However, the changes in administrative structuring had done little to clarify lines of authority over curriculum and instruction. Teachers received conflicting advisories from different levels in the administrative hierarchy about the learning goals for children and the knowledge and skills they should be teaching in the classroom. There remained considerable confusion among school personnel, particularly teachers, as to who was in control and to whose directives they were bound.

## Structural Changes in the Economy: Problems for the Schools

The Detroit schools curriculum reform occurred in one of the nation's five largest cities. Structural changes in the economy over two decades had resulted in increased unemployment, massive concentrations of poverty, homelessness, crime and welfare dependency. Beginning in the early 1970s, Detroit had lost over half its manufacturing base through economic disinvestment and industrial relocation. The automobile industry left the city as it sought new facilities in the suburbs and sites of cheaper labor in the South and the Third World. The market for automotive supplier firms deteriorated. A city that once boasted of 280,000 manufacturing jobs in 3300 firms by early-1980 employed less than 100,000 in manufacturing in fewer than 1700 firms (Luria & Russell, 1981). The city had lost close to 70% of the high-wage, national market industrial jobs.

Economic disinvestment from the city had an enormous impact on the schools. The district was under tremendous pressure to provide an educational program that responded to the needs and interests of children and youth in a community "where joblessness and the relationship between schooling and postschool employment take on different and defeatest meanings" (Young & Melnick, 1988, p. 387). At the same time, the loss of manufacturing facilities eroded the property tax base on which school millage was levied. The Detroit School Board and its financial officers found it increasingly difficult to raise sufficient revenues to provide a quality education program for its young people.

Economic disinvestment and social dislocation. Between the peak to peak cycle in the economy from 1979 and 1986, the number of chronically jobless in Detroit increased by 26% even as the city's population declined by 12% in the same seven-year period (Detroit Free Press, January 1, 1989). "The city that had once been thought of as being the land of opportunity for the uneducated and undereducated had suddenly turned into a land of unemployment, poverty and crime" (Detroit Public Schools, 1988, p. 3).

The social dislocation of the poor that had begun with urban renewal and freeway expansions in the 1950s and 60s was exacerbated by the dramatic increase in the ranks of Detroit's chronically unemployed in the 1970s and 80s. Between 1970 and 1980, the number of persons living in census tracts with a poverty rate of at least 20 percent increased dramatically.[5] In 1981, one out of three Detroit residents

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[5]Wilson (1987) reports that in the nations five largest cities (New York, Chicago, Los Angeles, Philadelphia, and Detroit) "population living in poverty areas grew by 40 percent overall, by 69 percent in

was receiving some form of public assistance. Thirty-eight percent (38.0%) of the residents of the city lived below the federal poverty level in 1986. In 1980, 55.8% of all black female headed families in Detroit were living in poverty.

There were other indices of the social consequences of joblessness and poverty. The infant mortality rate in the city was among the highest in the nation. Young males, lured by the prospect of fast bucks, were increasingly being recruited as couriers for drug dealers in the city. In 1987, 336 youth under 17 years of age were shot in Detroit, 35 fatally (Detroit Public Schools, 1988). The number of homeless in the city at the time of the study was estimated at over 12,000 with the number of homeless children rapidly growing.[6] There were 10,500 public low-income housing units in 29 developments in the city. While there was a crying need for additional low-cost housing, the vacancy rate in these developments was 30%! For many families, even low-income public housing was too expensive. But the high vacancy rate also reflected the city's inability to adequately maintain the developments. A considerable number of units, in some cases whole buildings, were simply uninhapitable.

The impact on the city's schools of economic disinvestment and the resulting social dislocation cannot be underestimated. It was

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high-poverty areas (i.e., areas with a poverty rate of at least 30 percent), and by a staggering 161 percent in extreme-poverty areas (i.e., areas with a poverty rate of at least 40 percent)" (p. 46).

[6]The Detroit Free Press reported, "That figure (12,000) reflects only people who have obtained help from social service agencies. Groups working with the homeless say the actual number may be as high as 60,000" (February 24, 1989). Peggy Posa, Director of the Coalition of Temporary Shelters (COTS) in Detroit, estimated the number of homeless at 30,000.

estimated that as many as two out of every three children attending Detroit's public schools came from families receiving public assistance. The forced moves of families within the city to secure temporary housing meant that many children might attend several different schools in one year. Students who moved among schools during the year were often at a disadvantage in their new schools if their prior learning experiences were a mismatch with those in the new setting.

Responding with new educational programs. The Detroit economy had been transformed--from unionized, high-wage goods production to non-unionized, low-wage service provision. The steady automation of industrial jobs and the application of new computer technology in manufacturing meant that more was produced with a smaller work force. The manufacturing work that remained in the city required skills beyond those developed by the Detroit schools. These developments had a profound impact on the employment opportunities for young people in the city. Black youth who in the past could anticipate taking high-wage semi-skilled jobs in the plants alongside older family members now faced a very different future.

The Detroit schools attempted to respond to these developments with the creation of five vocational technical education centers. Ordered by the federal court in its 1975 desegregation order, these centers offered a variety of training programs in health care, protective services, cosmetology, and food services as well as technical training for industrial work and the construction trades. But the centers had failed to meet the expectations of students and the community. All centers were undersubscribed with enrollments 70-75% of capacity. The placement rate of vocational center graduates from June, 1983, until March, 1984, was only 68% and of these, 24% were in "non-related" areas. A study by New Detroit Inc. (1986) cited major deficiencies in the vocational programs to develop broader analytical skills as opposed to specific "narrow" technical skills. Of concern to employers of these graduates had been the inability of some to "make good decisions and judgments, their inability to work without constant supervision, and their difficulty in communicating effectively with others and working under pressure" (Monitoring Commission Report, October, 1984; p. B-54). As Jacobs (1987) pointed out, training at the vocational centers had focused on giving students short-term low-level job skills while neglecting the development of long-term basic, analytic and technical skills that could lead to further training and advancement.

The underutilization of vocational centers by Detroit youth was, in part, a function of the organization of the programs. Students enrolled in their neighborhood high school for academic courses. They were bused to the vocational center for their technical courses and then bused back to their high school. Transportation from regular high schools to vocational technical centers was an enduring problem. A common complaint from teachers and students was that buses were late or simply did not appear. The U.S. District Court Monitoring Commission reported that "Detroit Public Schools buses have recently had a failure rate exceeding 88% on State Police bus inspections" resulting in disruption of teaching programs and serious loss of academic time (Monitoring Commission Report, June, 1984).

The growing fiscal crisis. While there was evidence of questionable management practices, the fact was that the costs of operating the schools continued to rise while the revenues available to do the job declined. The impact of a growing fiscal crisis was felt daily in schools and classrooms across the city.

Education in the state of Michigan is financed primarily through local property taxes. Table 1.1 compares Detroit's effort to finance its schools to several neighboring communities. The particular districts were chosen to dramatize the huge disparity in per pupil expenditures that existed among districts in the Detroit metropolitan area.

Although Detroit had one of the largest operating millage rates in the state, it was able to raise only 30% of its revenues through local taxes. The state, through its school aid formula, provided 62% of the operating revenue, the federal government the remaining 8%. While Detroit levied six mills more than Birmingham and nearly nine more than Dearborn, it was able to raise less than one-fourth the per pupil yield of its two neighbors in local tax revenues. Even with the contribution of state and federal aid, Detroit had nearly 40% fewer dollars per pupil than Birmingham and 23% fewer dollars per pupil than Dearborn. Detroit's shrinking property tax base resulted from economic disinvestment, industrial relocation, and residential devaluation.

By the 1980s, the district was in a near constant state of fiscal crisis.[7] Teachers universally complained about not having sufficient materials, particularly textbooks and paper. It was common for a middle school teacher to have one set of textbooks that had to be

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[7]In 1975 the district was $73 million in debt. By the end of the 1988-89 school year, the district was $151 million in debt (Detroit Free Press, June 15, 1989).

Table 1.1 SEV Per Pupil, Operating Millage Rates, Local Tax Yield, State Formula Yield and Operating Expenditures for Selected Michigan School Districts, 1986-1987

| District | SEV per pupil | Millage rate | Local Tax Yield per pupil | State Formula per pupil | Operating Expenditures per pupil[a] |
|---|---|---|---|---|---|
| Detroit | $ 25,375 | 35.90 | $ 911 | $2,028 | $3,374 |
| Birmingham[b] | 175,883 | 29.90 | 5,453 | 0 | 5,453 |
| Dearborn[c] | 154,544 | 26.98 | 4,170 | 0 | 4,366 |
| Inkster[d] | 20,710 | 35.65 | 738 | 2,182 | 2,969 |

Source: Kearney, C. (1987). A primer on Michigan school finance.

[a]The operating expenditures are the sum of local tax revenue, state formula aid, state special and categorical aid, and federal categorical aid.

[b]Virtually the entire tax base of this nearly all-white, high income community is residential or commercial.

[c]Dearborn, a city with a long history of public policies designed to keep out blacks, owes a significant portion of its substantial tax base to the world headquarters of Ford Motor Company and the Rouge Steel complex.

[d]Inkster, like Detroit, is a largely black city that has suffered economic disinvestment and social dislocation.

shared by five classes of students. Reductions in the budget for maintenance resulted in a continual deterioration of the physical plant of the district, particularly its older structures. Downsizing the custodial staff meant schools were cleaned less often. Although the superintendent and the chief financial officer were under increasing attack for not adequately managing what revenues were available, the fact remained that the fiscal problems were primarily linked to a method of school financing in the state that exacerbated the financial crisis facing districts with shrinking property tax bases.

The growing fiscal crisis was but one matter of extreme urgency. The other was the crisis in academic performance of children and youth in the district.

**Educational Crises: Academic Achievement and Drop-outs**

Student achievement and testing. Since the mid-1970s, the district had been under continual pressure to improve student achievement on standardized tests. The test that received most public scrutiny was the Michigan Educational Assessment Program (MEAP), administered in early October to all 4th, 7th and 10th grade students in the state's public schools. MEAP tested sets of objectives in reading and mathematics. The results were given considerable attention in both the print and electronic media.

MEAP scores were presented both as the number of objectives attained and the Category of Achievement. Category 1 indicated mastery of 0-24% of the objectives, Category 2 mastery of 25-49%, Category 3, 50-74% and Category 4, 75-100%. Table 1.1 shows the city-wide achievement results in mathematics for 4th graders for the ten-year period beginning 1978.

Table 1.2 Percentage of 4th Grade Students City-wide in Each Achievement Category in Mathematics, MEAP, 1978-1987

| | Category 1 | Category 2 | Category 3 | Category 4 |
|---|---|---|---|---|
| 1978 | 10.0 | 15.8 | 24.9 | 49.3 |
| 1979 | 4.7 | 10.4 | 21.2 | 63.7 |
| 1980 | 4.0 | 13.1 | 29.2 | 53.7 |
| 1981 | 3.3 | 10.1 | 25.9 | 60.7 |
| 1982 | 2.2 | 7.6 | 24.0 | 66.1 |
| 1983 | 2.0 | 8.0 | 23.1 | 66.9 |
| 1984 | 1.0 | 5.2 | 20.1 | 73.6 |
| 1985 | 1.0 | 5.1 | 19.3 | 74.5 |
| 1986 | 1.2 | 6.0 | 19.9 | 72.9 |
| 1987 | 0.4 | 2.5 | 11.8 | 83.3 |

Source: Annual Reports prepared by the Department of Research and Evaluation, Detroit Public Schools

The steady improvement in achievement -- lowering the percentage of students in Categories 1 and 2, increasing the percentage in Category 4 -- resulted from increasingly focused attention to MEAP objectives, particularly in the later part of Grade 3 and the first four weeks of Grade 4. In 1987, for example, 4th grade teachers were given a set of curricular materials to be used during September to prepare youngsters for the mathematics portion of MEAP. The packet included a schedule of instruction for the first four weeks of school, a pre-test, 16 lessons on place value, addition, subtraction, and multiplication, a post-test to be administered at the conclusion of the lessons, one lesson on test-taking skills, and one lesson for enrichment/review. This set of preparatory lessons may have contributed to the dramatic increase in student achievement in 1987.

While the city-wide scores showed steady improvement, there was considerable variation among schools. The disparity in achievement results made the Board, central and Area administrators, school principals and teachers vulnerable to the charge that there existed considerable inequality in education across schools. Many schools in the poorest neighborhoods of the city had reputations for providing inferior educational opportunities for the children they served in comparison to schools located in the city's middle-class neighborhoods.

Table 1.3 gives the 4th grade MEAP achievement scores in mathematics for two schools in the district. School A consistently outscored School B in the percentage of children achieving in Category 4 (with the exception of 1987). For the nine-year period 1978-1986, School A had, on the average, 20.9% more of its students in Category 4 than School B.

Table 1.3 Percentage of 4th Grade Students from Two Schools in Each Achievement Category in Mathematics, MEAP, 1978-1987

| | Category 1 | | Category 2 | | Category 3 | | Category 4 | |
|---|---|---|---|---|---|---|---|---|
| | A | B | A | B | A | B | A | B |
| 1978 | 1.2 | 15.4 | 3.6 | 20.2 | 10.8 | 27.9 | 84.3 | 36.5 |
| 1979 | 0.8 | 9.0 | 5.0 | 15.7 | 17.4 | 14.6 | 76.9 | 60.7 |
| 1980 | 1.8 | 8.0 | 4.5 | 10.6 | 19.6 | 35.4 | 74.1 | 46.0 |
| 1981 | 7.3 | 2.4 | 12.2 | 7.2 | 19.5 | 38.4 | 61.0 | 52.0 |
| 1982 | 1.4 | 7.4 | 9.9 | 16.8 | 16.9 | 40.9 | 71.8 | 34.9 |
| 1983 | 0.0 | 1.0 | 4.5 | 10.5 | 7.5 | 33.3 | 88.1 | 55.2 |
| 1984 | 1.2 | 0.9 | 11.0 | 10.1 | 18.3 | 27.5 | 69.5 | 61.5 |
| 1985 | 2.5 | 1.0 | 8.9 | 11.3 | 26.6 | 25.8 | 62.0 | 61.9 |
| 1986 | 2.4 | 5.8 | 10.7 | 12.8 | 19.0 | 31.4 | 67.9 | 50.0 |
| 1987 | 2.2 | 0.0 | 4.4 | 2.4 | 25.6 | 7.1 | 67.8 | 90.5 |

Source: Annual Reports prepared by the Department of Research and Evaluation, Detroit Public Schools

School A was located in a middle-class neighborhood. The housing stock, built in the 1920s and 30s, was well-maintained, tudor-style brick, rich in architectural detail inside and out. Neighborhood residents tended to be two-income families of professionals, managers and skilled workers, many of whom were employees of the city or the public schools. Although the neighborhood was racially mixed, most of the youngsters who attended the school were black.[8] School B was located in a neighborhood that contrasted sharply with the financial well-being of the School A community. The signs of poverty were visible at every turn: boarded up and burned out shells of abandoned homes; weed-chocked and trash-strewn lots where the city had bulldozed delapidated and unsafe houses; a commercial strip where rusted security gates protected empty stores and where black men sat hunched over in doorways, wearing glazed-over looks of despair and defeat.

Critics of the schools who charged that a quality education was not available to _all_ children found considerable empirical evidence to support their charge. The demand to improve education for the poor children who were being underserved was voiced in school board meetings, in newspaper editorials, and in reports prepared by civic organizations.

At the secondary level, the district instituted its own minimum competency testing program in the late 1970s. Stopping short of requiring a minimum competency for high school graduation, the district created a system of dual diplomas. Those who passed the Proficiency Exam by the time they were graduating seniors received an "endorsed"

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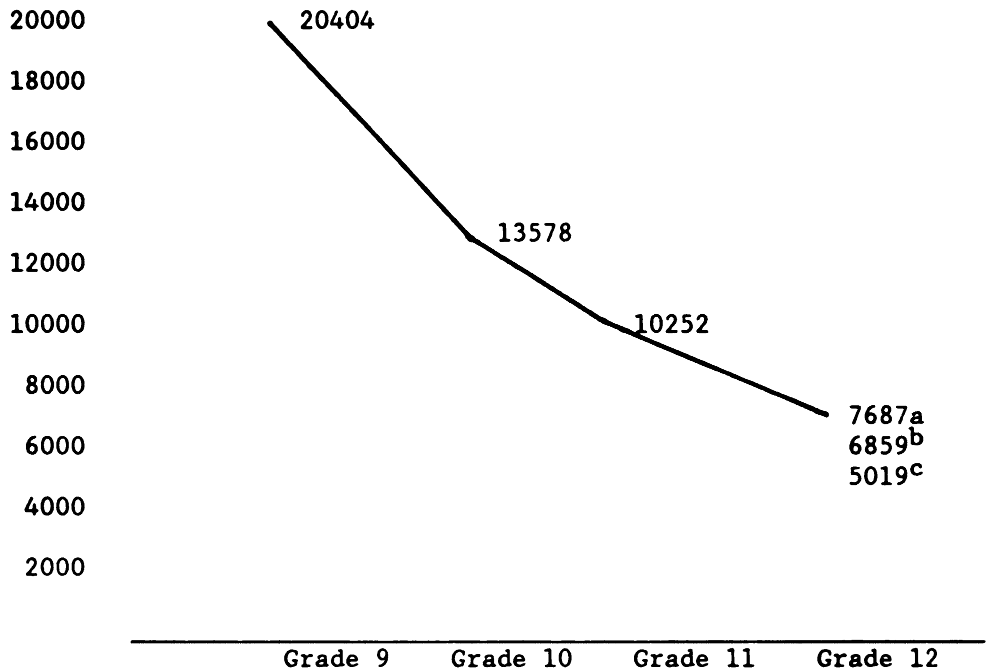
[8]Exact figures were not available, but a number of residents in this neighborhood, black and white, chose to send their children to private and parochial schools.

diploma. Those who failed to pass the Proficiency Exam received a diploma that did not carry the "endorsed" stamp. The Proficiency Exam measured basic reading, writing and mathematics skills at the eighth grade level.

Teachers at the middle and high schools were supplied with curriculum materials that had been created specifically around the test competencies. Teachers were directed to teach to one of the competencies every week and demonstrate that intention in their weekly lesson plans. This directive applied to all teachers regardless of the course level they were teaching. Students took the Proficiency Exam for the first time in tenth grade. They were permitted to retake any portion not passed at least once a year until graduation. Although scores on the Proficiency Exam had improved over the several years of its administration, nearly one-third of the graduates continually failed to meet the requirements for an "endorsed" diploma. The fact that a significant number of students graduated without demonstrating competency at the eighth grade level was a growing problem for school administrators and teachers. Increasingly, there were calls to improve the literacy and numeracy of all children and youth in the Detroit schools.

School drop-outs. The Detroit schools faced a staggering problem of youth who were alienated from school, who saw no payoff to remaining in school or engaging in academic work when they were there. The district experienced an extraordinary loss of students from the regular day school program. Less than one-half the students who entered the ninth grade graduated four years later.




Source: Profiles of Detroit's High Schools; 1975 to 1984. A Monitoring Commission Report, Detroit School District United States District Court Monitoring Commission, October, 1984.

[a]Students who entered Grade 12.

[b]Graduating seniors.

[c]Graduates passing Proficiency Exam.

**Figure 1.1 Student Progression Through the Detroit Public High Schools. Regular Day School Program, 1981-81 9th Grade Class**

Figure 1.1 shows the progression over succeeding years for the cohort of students of the 1984 graduating class, beginning at grade 9 in 1980 and continuing through graduation four years later. Over 6800 students did not progress with their classmates from ninth to tenth grade, a loss of one-third of the cohort. The number of non-promotions accounted for 13.6% of the cohort, leaving nearly 20% - one of every five ninth graders - who did not return to the Detroit Public Schools as tenth graders.

An accurate accounting of dropouts was first undertaken by the district in 1982. The district's Student Information System Data Base "tracked" 13,769 students who were new ninth-graders in 1982 to account for the status of this population four years later.[9] Students were placed into one of three categories: **dropouts**; high school **graduates**; or **active**--students that remained in high school for more than four years. A fourth category, **involuntary losses**, were not included in the percentages.[10] The findings showed that of the 13,769 students who entered the ninth grade for the first time in 1982, four years later 5,111 (41.31%) had dropped out, 5,959 (48.16%) had graduated, and 1,303

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[9]The discrepancy in the size of the cohort "tracked" from 1982 and the size of the ninth grade cohort in Figure 1.1 is explained by that fact that Figure 1.1 begins with all students who were enrolled in the ninth grade in the 1980-81 school year. That is, the enrollment of 20,404 ninth graders includes students who were repeating the ninth grade. The 1982 enrollment figure of 13,769 for 1982-83 includes only those who were enrolled in the ninth grade for the first time.

[10]The district coded the loss of students as follows: **dropouts** - non-returns/20 days absent, moved can't locate, suspended/expelled, employment, left over age, unable to adjust, needed at home, enlisted in armed services, pregnancy/marriage; **involuntary losses** - left to Detroit non-public, Michigan public or non-public, or adult education, left to other state or country, deceased, special release, miscellaneous reasons.

(10.53%) were still attending a Detroit public high school. Involuntary losses accounted for the remaining 1,396 students. A similar study of the 14,946 students who were first time ninth graders in 1983 showed that four years later 5,003 (39.73%) had dropped out, 5,994 (47.60%) had graduated and 1,596 (12.67%) were still attending a Detroit high school. Involuntary losses accounted for the remaining 2,353 students (Detroit Public Schools, May 19, 1988). The drop-out rate had become an educational disgrace and a political problem.

### The District's Response to a Growing Crisis

By the mid-1980s, the Board and the central administration were under seige. The evidence was strong that a "back-to-basics" educational program based on minimum competencies had not reduced the alienation and disengagement of kids who were truly at-risk. A vocational program that emphasized "narrow" technical skills training was not improving the chances of young adults to find work that was challenging and rewarding. Every segment of the community was demanding reform: reduce the number of drop-outs; increase scores in all schools on the MEAP test; provide the kinds of knowledge, skills and attitudes that would prepare young people for a changed Detroit and national economy. The clamor came from the mayor's office, community and grass-roots organizations, organized labor, business and civic organizations, and groups specifically organized around improving the quality of education in the public schools.

In January, 1987, the Board and the superintendent responded with a curriculum reform initiative. Beginning with the 1987-88 school year, elementary teachers were instructed to begin a three-year process to implement standardized curricula in seven content areas--language arts, social studies, mathematics, science, art, music, and health an physical education. This study investigated the nature of the mathematics curriculum reform.

The mathematics curriculum was of particular interest. With the exception of pockets of experimentation with an innovative program in the early 1980s, mathematics education in the Detroit Schools had a basic skills, minimum competency orientation for more than a decade. The newly mandated mathematics curriculum was described by those most intimately involved in its development as "conceptually-based." It was organized around ten mathematical "strands" -- computers and calculators, estimation and approximation, functions and relations, geometry, measurement, numeration, operations, patterns, probability and statistics, and sets and logic -- that were spiraled within and across the grades. The goals for students as learners of mathematics were "to think critically, flexibly, cooperatively and independently...to create and to analyze...to gain understanding and to develop higher order thinking skills...to apply knowledge of mathematics in solving problems" (Detroit Public Schools, January, 1987).

The reform appeared to be a significant departure from the basic skills, computational orientation that had previously informed the district's mathematics program. On the face of it, the reform seemed consistent with calls from leaders in mathematics education to reorganize the curriculum around concept development and problem solving to help students develop their mathematical knowledge and power

(An Agenda for Action, 1980; Commission on Standards for School Mathematics, 1987).

This study inquires about the following: Why did the reform take the shape that it did? What factors or set of conditions supported the initiative? What was the potential of the reform to develop the mathematical knowledge and power of children who were truly educationally at-risk? What support was given to teachers who were mandated to implement the new curricula in their classrooms?

## STRUCTURE OF THE DISSERTATION

Chapter 2 makes the case for considering an investigation of the mathematics curriculum, locating the initiative within the context of the current national agenda to reform the teaching and learning of mathematics. Three orientations to mathematics education are offered as a framework for analysis of the Detroit initiative. Those orientations are teaching for **certification of numeracy**, teaching for **comprehension of mathematical ideas**, and teaching for **enabling mathematical inquiry**. I make the argument for providing all youngsters, and especially those at-risk, with a mathematics education program in which the goal is to empower students through mathematical inquiry--where they actively participate in making sense of mathematical situations; engage in making conjectures, developing arguments, inventing procedures, building abstractions and generalizations; and communicate with others about mathematical ideas. The demands of work and citizenship facing the next generations require a new definition of what it means to be mathematically literate.

Chapter 3 describes the methodology of this fieldwork study. The chapter describes how immersion in the setting led to a recognition that the magnitude of the problem and the questions to be asked about this reform were considerably greater than first formulated. The chapter further describes the data that were collected and how data gathered from different sources, at different points in time and in different contexts were triangulated.

Chapters 4, 5, 6, and 7 are devoted to an analysis of the data to answer the questions that frame the study. Chapter 4 draws on historical documents and interviews with key informants to locate the mathematics reform in a political and organizational context that shaped its conception and mandates for its implementation. Chapters 5 and 6 examine the formal curriculum documents -- the strands that organized the curriculum, the objectives that sequenced the content for instruction -- and the Model Lessons that provided examples of how the objectives were to be met in the classroom. The analytic framework developed in Chapter 3 is used to determine the orientation of this curriculum reform and its potential to address and achieve its purported goals.

Since teachers are the primary agents of curriculum changes in the classroom, the way in which they are prepared to implement new curriculum is fundamental to assessing the potential of a reform to make a difference in the classroom experiences of children. Chapter 7 examines the inservice opportunities that were provided for teachers in the district charged with implementing the new mathematics curriculum. The context and the content of inservice are analyzed for what the inservice provider thought teachers needed to know and how they would learn what they needed to know.

Chapter 8 returns to the context of the reform. Drawing on the concept of history as genealogy, I give a genealogical account of the Detroit reform initiative. I locate the antecendants of the reform in an instrumental ideology that dominates curricular practice, the politics of race and school governance in Detroit, thirty-years of reform agendas in mathematics education, and a recognition of a new economic order and Detroit's position in it. The genealogy reveals both consistency and contradiction. An instrumental ideology cannot inform a curriculum or a practice whose educational goals contravene or extend beyond the practical and utilitarian.

The question then is raised of what would be required to conceive and implement a curriculum aimed at empowering students with mathematical knowledge. I conclude with an attempt to sketch some answers.

# CHAPTER 2

# WHY THE CASE OF MATHEMATICS

## INTRODUCTION

Since the late 1950s, mathematics education in the United States has been under continuous scrutiny. Each of the last several decades has been marked by a new national reform agenda, each with its own slogan that becomes the shorthand for the substance of the reform. In the sixties it was "new math;" in the seventies, "back-to-basics" and "minimum competency;" in the eighties it is "computer literacy," "problem solving," and "conceptual understanding." These reform initiatives have differed in terms of the students to whom they have been directed, the mathematics embedded in them and the outcomes they were intended to achieve. The agendas have reflected a number of influences: 1) changing conceptions of mathematics and the teaching and learning of mathematics derived from research and deliberation among mathematicians, mathematics educators, cognitive scientists and educational psychologists; 2) larger economic and political factors--challenges to U.S. preeminence in scientific and technological advances, military superiority, and world markets; and 3) assumptions about the mathematical learning opportunities that should be made available to diverse constituencies whether distinguished by "academic ability," race, class or gender.

Why is an investigation of the mathematics curriculum reform in Detroit's elementary schools important? The Detroit initiative occurs in the context of a current national agenda to improve the teaching and learning of mathematics for <u>all</u> youngsters. Despite over two decades of attempts to improve the mathematics achievement of students in our nation's schools, the record shows evidence of only slight improvement in what students know about and can do in mathematics. Of increasing concern is the considerable gap in achievement between whites and minorities. In its report, Everybody Counts, the National Research Council (1989) sounds an alarm that "the widening gap between those who are mathematically literate and those who are not coincides, to a frightening degree, with racial and economic categories" (p. 14). What is becoming increasingly clear is that our urban schools are failing to adequately educate minorities and the poor to function as citizens and workers in environments that require increasing quantitative literacy.

The situation is described as a crisis. It might be argued, as Young and Melnick (1989) do, that schools serving minorities and the poor have never accepted full responsibility for providing quality education for these groups of students. The recent "recognition" of the problem facing urban schools may result from the increasing numbers of these students whose educational needs simply can no longer be ignored.

The current reform agenda is informed by concerns that are political and social as well as technical and economic in nature. The growth of technology and its application to virtually every field of work means that the ability to think mathematically is no longer the sole province of scientific workers. The loss of manufacturing jobs that required mid-level skills are being replaced, in part, by jobs that require not only the ability to compute but the ability to see the mathematics in problems and to apply mathematical knowledge to their solution. Responsible citizenship in a democracy requires a citizenry capable of using mathematics to make sense of political and social issues. Being an informed citizen means being able to understand positions on issues of public policy that employ mathematical arguments. Nearly every political issue -- acid rain, abortion, AIDS, nuclear power, crime, poverty, homelessness -- to some degree is framed in terms that are numeric. As the National Research Council points out, "A public afraid or unable to reason with figures is unable to discriminate between rational and reckless claims in public policy" (p. 32). Being mathematically illiterate condemns one to second-class status at work and in the community.

Information about Detroit, its schools, and its children and youth make real the extent of the crisis facing urban districts. At the same time, multi-faceted reform efforts in the district stand as testimony of a history of struggle in the black community to make a difference in the lives of each new generation. The elementary mathematics curriculum reform represents a continuation of that struggle. A question this study attempts to address is "What is the potential of this reform to contribute to the empowerment of the next generation of Detroit youth?"

This chapter provides a framework for examining the nature of this latest reform initiative in the Detroit schools. It begins by exploring the larger context of mathematics reform in which the Detroit project is located. The first section reviews the reforms of the 1960s and 1970s to inquire about intent and outcomes. The second section considers in detail the forces driving the eighties reform rhetoric and how this latest agenda for reform emerged from earlier efforts. Included here is an extensive discussion of three different orientations to mathematics education -- **certification of numeracy**, **comprehension of mathematical ideas**, and **enabling mathematical inquiry** -- and the mathematical and political interests that each embodies. The chapter concludes with an argument that, among the three orientations, only an orientation centered on **enabling mathematical inquiry** is likely to provide Detroit young people with the set of intellectual tools - knowledge, skills, dispositions, critical habits of mind - to understand, participate in, and shape the world around them. And while enabling mathematical inquiry may be a necessary condition to providing Detroit young people with the skills to conceive the world in which they must survive as problem solvers, it is not sufficient. There remain more fundamental issues of race, class and gender that underlie social and economic stratification in American society and that limit the life chances of minorities, women, the poor and the working class.

## POST-SPUTNIK REFORMS IN MATHEMATICS EDUCATION

### "New Math"

The reform initiative of the late 1950s and the 1960s was an undertaking aimed at enriching the content of mathematics for the "academically talented," those most likely to pursue post-secondary education in the sciences. The launching of Sputnik, the world's first unmanned space satellite, by the Soviet Union in 1957 shocked the scientific and political communities in the United States. The fear of losing this country's edge in the creation of new technologies and the implications for world markets and the balance of world power in the nuclear age led the federal government to embark on a program to reform mathematics and science education. The National Science Foundation and the National Defense Education Act (passed by Congress in 1958) provided considerable financial resources for curriculum development and recruitment of "talented" youngsters to the study of science, mathematics and engineering.

The curriculum materials that were developed emphasized the internal structure and the unifying concepts of the discipline of mathematics. They incorporated a broader sample of topics from mathematics including sets and logic, probability, and the structure of the real number system. They advocated the use of physical materials and the discovery and exploration of mathematics by students working independently on their own. Little attention was paid to the ways in which teachers taught. Government sponsored teacher institutes retrained many secondary teachers about "new math" programs. But as Fey (1978) pointed out, the institute programs concentrated on teaching teachers more and deeper mathematics while ignoring questions about how to implement programs in real classrooms. The training available to elementary teachers charged with implementing "new math" programs was extremely limited. As Fey commented,

> In retrospect it is almost comical that National Science Foundation teacher education programs were specifically prohibited from serving elementary teachers. Now nearly every would-be curriculum reformer acknowledges the crucial role of classroom teachers in implementation of new programs (p. 349).

By the beginning of the 1970s, there was growing disenchantment with the "new math" initiative. The biggest concern was the decline in student achievement in mathematics as measured by national standardized tests. In addition, questions were being raised about the appropriateness of the substance of the reform, especially what critics viewed as excesses of abstraction, symbolism and deductive reasoning. A popular critique was that schools were turning out young people who knew about union and intersection of sets and binary numbers but who could not balance a checkbook. One of the most popular books at the time was Morris Kline's Why Johnny Can't Add. Parents complained that the language and ideas of "sets" and "properties of number systems" were unfamiliar; their own limitations with the new content made helping their children difficult at best.

School people, too, became disaffected. Early enthusiasm gave way to the realities of implemention; many teachers, especially at the elementary level, did not have sufficient subject matter knowledge to convey the substance or the spirit of the new curricula. Even "teacher-proof" curricula conceived by experts to be implemented by mathematically "unsophisticated" teachers did not insure faithful presentation of materials as they had been conceived. As Apple (1986) points out,

> ...when the material was introduced into many schools, it was not unusual for the 'new' math and 'new' science to be taught in much the same manner as the old math and old science. It was altered so that it fitted into both the existing regularities of the institution and the prior practices that had proven successful in teaching (p. 37).

By the mid-1970's, "new math" was being described as a failure at improving the teaching and learning of mathematics. Fey (1978) suggested that reformers paid insufficient attention to helping the public and a broader constituency within the mathematics education community understand the "fundamental rationale and substance" of the new programs. But he pointed out another influence, the social context of the seventies.

> Furthermore, the criticism coincides with a period of straitened economic conditions in the schools and for the public at large. Much of the challenge to make school programs more practical and accountable for their effectiveness seems to reflect anxiety about personal economic pressures much more than philosophical disagreement about educational policy (p. 352).

The call was for a return to the basics.

### "Back-to-Basics"

The initiatives of the seventies focused on minimum competencies, the least that could be expected from youngsters studying mathematics. Unlike the earlier reform that had been aimed at the "academically talented," this campaign was aimed at those presumed to be "less able." The National Advisory Committee on Mathematical Education (NACOME) noted the shift:

> Mathematics program improvements of the "new math" in the 1960s were primarily motivated and designed to provide high quality mathematics for college capable students. Today mathematics curriculum development focuses on issues largely ignored in the activity of 1955-1970. Attention has now shifted to programs for less able students, to minimal mathematical competence for effective citizenship, to mathematical application, and to the impact of new computing technology on traditional priorities and methods in mathematics (NACOME, 1975, p. 23).

The "back-to-basics" movement had a profound impact on the development of curriculum materials. The goals of learning were expressed in reductive behavioral objectives that defined specific skill learning with skills narrowly defined. The commercial textbook market jumped on the bandwagon, producing prepackaged sets of curricular materials (Apple, 1983b, 1986; Giroux, 1983; Gitlin, 1983). As Apple noted:

> It is nearly impossible now to walk into an American classroom, for instance, without seeing boxes upon boxes of science, social studies, mathematics and reading materials ("systems," as they are sometimes called) lining the shelves and in use. Here, a school system purchases a total set of standardized material usually, one

> that includes statements of objectives, all of the curricular content and material needed, prespecified teacher actions and appropriate student responses, and diagnostic and achievement tests coordinated with the system. These tests usually reduce the curricular knowledge to "appropriate" behaviors and skills (1983b, p. 149).

The systematic integration of competency-based instruction and prepackaged curricula was supported by initiatives in nearly 40 states to mandate some form of state-wide competency testing programs (Apple, 1986). A number of districts instituted minimum competencies for advancement from one grade to the next (e.g., New York's Promotional Gates Program; see Labaree, 1984); some required demonstration of proficiency at minimum skills in reading, writing and computation for high school graduation. Other districts, like Detroit, fearful of potential legal challenges to minimum competence as a requirement for graduation, devised systems of differential diplomas.

These reform efforts did not just "spring out of nowhere." The instrumental logic that informed the undertaking was not new. As Giroux (1983) has suggested:

> It is instructive to remember that the underlying instrumental logic that infuses educational theory and practice at the present time is not new. It has simply been recycled and repackaged to meet the needs of the existing political and economic crisis. For example, the technological and behaviorist models that have long exercised a powerful influence on the curriculum field were, in part, adapted from the scientific management movement of the 1920s, just as the roots of the competency-based education movement were developed in earlier research work adapted "from the systems engineering procedures of the defense industry" (Franklin, 1976). The issue here is that...the redefinition of the curriculum in watered-down pragmatic and instrumental terms cannot be viewed as problems *solely* due to demographic shifts in the population and short-term recessional tendencies in the economy. Such a position not only abstracts the current crisis from its historical and political roots, it also uses the existing economic crisis to legitimate conservative modes of pedagogy and to silence potential critics (p. 44).

The "back-to'basics" movement did not exert a uniform influence across schools or across districts. Minimum competency programs became the mainstay in districts with increasing numbers of working-class, poor and minority youngsters. Educators and employers talked about the need to provide these youngsters with "survival skills," the presumed knowledge and attitudes needed by the next generation of service and industrial workers. As Anyon (1981, 1983) found in the fifth grade classrooms she observed, curriculum and instruction in mathematics (and other subjects, as well) varied by the social class of the students in those classrooms.

In the working class schools Anyon studied, children learned basic facts and skills. They were taught and expected to master steps in conventional algorithmic procedures. Misapplication of routines by students was met with teacher comments like "You're confusing yourselves. You need more practice." Teachers skipped pages in the text that called for reasoning and inference because they were "too hard" for these children. Learning was mechanical, rote and fragmented. Classroom management was a constant struggle and a lot of time was spent trying to maintain quiet and order.

In contrast, Anyon found a very different learning environment in schools that served affluent communities. Instruction was characterized by individual discovery as well as group work, an emphasis on creativity and understanding, and the development of intellectual tools to aid problem-solving and decision-making. Teachers used questioning techniques that tended to place more responsibility on students to make sense of problem situations. There was less reliance on the text for the curriculum.

Anyon's analysis suggested that differences in learning opportunities were, in part, a function of the beliefs held by the educational community about the assumed abilities and presumed occupational destinations of children from different class backgrounds. Teaching in working-class schools denied the capacity for human creativity. Children learned how to carry out procedures without explanation. They learned rules "unconnected to thought processes or decision making of their own" (1981, p. 8). They were being prepared for the more practical curricula of clerical and vocational training. On the other hand, children from affluent communities were acquiring "cultural capital" -- linguistic, analytic and scientific skills, the ability to analyze, plan and control -- forms of knowledge with an exchange value in business and the professions.[1]

The failure of the "back-to-basics" movement. The return to basics had not met the expectations of its advocates for improving mathematical literacy. Data collected over nearly twenty years showed the sad results of a focus on minimum competencies. While there was some evidence that most students were able to compute, the majority did not understand many basic mathematical concepts and were unable to apply mathematical knowledge and skills to simple problem solving situations (Carpenter et al., 1978; Carpenter et al., 1981; Dossey et al., 1988; Erlwanger, 1973; McKnight et al., 1987; Schoenfeld, 1985).

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[1]Others have also noted the relationship between the educational opportunities available to students from different class backgrounds at the elementary, secondary and post-secondary levels and the reproduction of unequal class structures; e.g. Apple, 1979; Bowles & Gintis, 1976; Karabel, 1972; Karabel & Halsey, 1977; Rosenbaum, 1976; Young, M.F.D., 1974).

Dossey et al. (1988) reported on trends in mathematics achievement as assessed by the National Assessment of Educational Progress (NAEP) in 1972-73, 1977-78, 1981-82 and 1985-86. Their findings showed that there had been modest gains in mathematics achievement but the gains were confined to lower-order skills. Among 9 year-olds, 26% did not have an understanding of "rudimentary skills and concepts." But more alarming was that less than 25% demonstrated an understanding of the operations of arithmetic or the ability to apply them to one-step word problems. They were unable to use charts and graphs to compare information or to analyze logical relations. One-fourth of the 13 year-olds were unable to demonstrate competency at this level as well. While the trend had been for Blacks and Hispanics to show improvement, they continued to achieve considerably below the level of whites. Furthermore, as the age and level of knowledge and skill increased, so did the performance gaps among whites and minorities. The Second International Mathematics Study (SIMS) criticized the repetitive nature of mathematics curricula in the United States and the concentration on computation relative to other important areas of mathematical knowledge (McKnight et al., 1987).

## The Persistence of Traditional Instructional Practices

The reform efforts had little effect on classroom pedagogy. The profile of a typical elementary mathematics class has remained virtually untouched by the efforts (NACMOE, 1975; Stodolsky, 1987; Welch, 1978). The mathematics teacher begins instruction by reading the answers to yesterday's textbook assignment while children check their paper (or someone else's to avoid the "temptation" to "cheat").

If there is a problem that has been particularly troublesome, the teacher (or one of the "brighter" students) works the problem at the board. Then the teacher explains briefly the next piece of material to be learned, typically what is contained on the next two pages of the text. The remainder of the time allowed for mathematics is spent having children work on the next assignment, usually another set of problems from the text or a workbook. Students work individually at seatwork while the teacher moves among the desks, answering questions. As Welch (1978) noted in a National Science Foundation study of mathematics teaching, "The most noticeable thing about math classes was the repetition of this routine" (p. 6).

By the end of the 1970s, there were increasing calls to improve the quality of mathematics instruction. The argument was that if mathematics learning was to be improved, mathematics teaching had to improve. It is in the reform movements of the 1980s that improving the quality of the mathematics curriculum has been joined by the explicit recognition of the need to improve the quality of mathematics instruction.

## REFORMING MATHEMATICS EDUCATION IN THE 1980S

### "Conceptual Understanding" and "Problem Solving"

The rhetoric of mathematics reform in the 1980s has focused on the need to develop students' understanding of mathematical ideas and their problem solving abilities. With its 1980 publication, An Agenda for Action, the National Council of Teachers of Mathematics (NCTM) proposed a new agenda for mathematics education. NCTM recommended that problem solving -- problem situations in forms other than traditional textbook word problems -- become the focus of school mathematics in the 1980s. In addition, NCTM advocated reducing the emphasis on pencil-and-paper practice on computational skills and using calculators and computers in problem solving situations. NCTM was joined by other groups -- National Science Board, Board on Mathematical Sciences, Mathematical Sciences Education Board, Committee on the Mathematical Sciences in the Year 2000, American Association for the Advancement of Science -- in calling for a reshaping of content and focus in the elementary mathematics curriculum.

A lesson learned from the initiatives of the previous two decades was that reforming the content of what is to be taught to children was not sufficient. Improving mathematics education would require attention to changing traditional instructional practices. However, the calls for reform early in the decade did not specify how instruction should be improved. This absence of direction has led to different interpretations of what instructional practices need to be changed and how those changes might be achieved.

There has been a tendency to posit a dichotomy between the traditional approach to teaching mathematics and one focused on developing understanding of mathematical concepts and processes. Within a traditional orientation, mathematics education exhibits these characteristics: the content is static, rule-bounded and linearly-ordered; students are passive receptacles into which bits and pieces of mathematical knowledge are poured; teachers are technicians who dispense knowledge that students must absorb and remember. Teaching for understanding assumes an opposing set of characteristics: mathematics is a dynamic body of knowledge connected by a web of relationships; students are active participants in the learning process; teachers are facilitators of student learning. Unfortunately, this set of binary distinctions masks what has emerged as two quite different meanings attached to "teaching for conceptual understanding." These differences have emerged from diverse theoretical and empirical studies of teaching and learning and have shaped two distinct approaches to mathematics education that differ from the traditional approach. Since this study examines the orientation of the Detroit curriculum initiative, a discussion of these three orientations is warranted.

**Orientations to Mathematics Education**

The framework for analysis used in this study poses three orientations to the elementary mathematics curriculum. I have labeled these orientations teaching for **certification of numeracy**, teaching for **comprehension of mathematical ideas**, and teaching for **enabling mathematical inquiry**.[2] The labels were chosen to represent the intended outcomes for students who encounter mathematics in these ways.

These orientations differ in assumptions about how mathematics is represented, what it means to know mathematics and how mathematics is learned, what should constitute the elementary mathematics curriculum,

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[2]Ball (1988) uses the terms "ordinary mathematics teaching," "conceptual mathematics teaching" and "mathematical pedagogy" to distinguish three approaches to the teaching of mathematics whose characteristics are congruent with the categories I have employed. Madsen-Nason (1988) defines three Levels - 1, 2 and 3 - that roughly correspond to the three orientations. The notion of Levels has also been used to analyze data from a longitudinal research project with colleagues Lappan, Lanier and Schram (see Schram & Wilcox, 1988; Schram et al., 1988, 1989). Our categorization of Levels is also a map of the orientations employed in this dissertation.

the goals for learners of mathematics instruction, and the role of the teacher and students in the mathematics classroom. They are submitted as ideal types with the knowledge that any mathematics program might embody elements of more than one.

Certification of Numeracy. Teaching for certification of numeracy corresponds to what is typically called traditional mathematics teaching. Kliebard (1972) describes this orientation as based on a metaphor of production, where children are viewed as "raw material" to be transformed by "skilled technicians." Children are seen as passive receptacles into which mathematical knowledge is poured and who absorb what they are told. As Romberg and Carpenter (1986) point out, this orientation to teaching mathematics assumes that acquiring knowledge created by others is the end of mathematics instruction. It is an epistemological orientation where the record of knowledge is taken to be knowledge (Dewey, 1916).

Teaching for certification of numeracy has its origins in the behaviorist learning theories of Thorndike (1922) and Skinner (1968) and an instructional practice informed by behaviorism. To facilitate learning, mathematical topics are fragmented into isolated pieces that are linearly and hierarchically arranged. The curriculum is organized by scores of discrete behavioral objectives stated a priori, each of which focuses on a single skill. The implicit assumption is that learning occurs only through repeated drill and practice and appropriate reinforcement.

Mathematics is represented as static, as "discovered" and rule-bound. The emphasis is on the formal language and symbolic representation of mathematics. The discipline is portrayed as a system of symbols to be manipulated and a collection of skills, definitions, rules, procedures and algorithms to be memorized and mastered. Arithmetic -- addition, subtraction, multiplication and division of whole numbers, fractions and decimals -- constitutes the bulk of the topics to be studied in the elementary curriculum. The goal of instruction is computational speed and accuracy, proficiency with conventional algorithms and their application to routine word problems. It is assumed that problem solving requires prior mastery of basic number facts and algorithmic procedures.

The role of the teacher is to show children how to carry out procedures and provide ways for them to remember facts and steps in algorithmic procedures. The teacher acts as a technician who implements a curriculum conceived by "experts." The teacher dispenses knowledge, the student is expected to absorb and remember it. Learning mathematics is an individual, solitary effort where children search their memory for accumulated facts and algorithmic solutions. Teacher and text act as the source of epistemological authority--teacher as the judge, textbook as the standard for judgement (Lampert, in press, a).

Comprehension of mathematical ideas. Teaching for **comprehension of mathematical ideas** aims at developing understanding for mathematical concepts and processes. It has much in common with certification of numeracy. The content continues to be organized around scores of behavioral objectives stated a priori, each of which focuses on a single skill or procedure. The curriculum divides mathematics into "topics, each topic into studies, each study into lessons, and each lesson into specific facts" (Romberg, 1983, pp. 133-134). The curriculum may represent mathematics as a body of related ideas, but it does so within a linear, rule-bounded, hierarchical arrangement. Arithmetic continues to dominate but may be supplemented by topics from geometry, statistics and probability, and number and set theory.

Teaching for **certification of numeracy** and **comprehension of mathematical ideas** differ in the goals for instruction and the assumptions about how students learn and what instructional practices enhance learning. Teaching for **comprehension of mathematical ideas** grows out of an interest in "improving traditional mathematics instruction by making it more efficient and effective" (Romberg & Carpenter, 1986, p. 860). It has been shaped by two research agendas: 1) to demonstrate the efficacy of beginning mathematics instruction at the concrete level, and 2) to identify the teacher behaviors associated with effective teaching.

The goal of instruction is to develop among students not only proficiency with mathematical procedures but also an understanding of why they work. It is believed the instructional route to realizing this goal requires a particular sequence of development. The assumption is that learning occurs by first manipulating concrete materials that model a procedure, then linking concrete models to pictorial representations, and, finally, using pictorial representations as the bridge to abstract or symbolic representations. This developmental sequence derives from several theories of learning, in particular, Piagetian stages of development and individual differences in learning styles. A considerable body of research has been conducted to demonstrate the power of beginning instruction at the concrete level, particularly in developing understanding of algorithmic procedures (see, for example, Dienes, 1970; Fennema, 1972a, 1972b;

Hynes, 1979; Merseth, 1978; Payne & Rathmell, 1975; Prigge, 1978; Stevenson, 1975; Suydam & Higgins, 1977; Wheatley, 1978).

This orientation has also been shaped by more than two decades of "process-product" research that has sought to identify specific teacher behaviors that correlate with student achievement (see Brophy & Good, 1986, for a review of this research). Most of this research has focused on generic teacher instructional behaviors. Examples of teacher behaviors studied include teacher praise, criticism and reinforcement (Flanders, 1970); the pattern of teacher questioning, student response, followed by teacher feedback (Stallings & Kaskowitz, 1974); quantity of instruction--time spent in whole-group, direct instruction (Stallings et al., 1978); clarity of teacher presentations (Rosenshine, 1968; Hiller et al., 1969); "withitness"--techniques for monitoring the class, minimizing time spent on transitions and student conduct (Kounin, 1970; Brophy & Evertson, 1976; Anderson et al, 1979); pace of instruction and time-on-task (McDonald, 1976); expectations of students communicated by the teacher (Berliner & Tikunoff, 1976); the use of teacher questioning (Soar & Soar, 1979). In most of these research efforts, the behaviors of learners and the specific nature of the content being taught have been ignored or considered outside the scope of inquiry. In virtually all of them, the dependent variable has been student achievement scores on standardized tests.

Good and Grouws' work (1975, 1977) -- an elaboration on process-product inquiry -- investigated effective instructional practices of mathematics teachers. Their initial studies in mathematics classrooms found that the quality of the developmental portion of a mathematics lesson correlated with student achievement scores on standardized tests. Using this finding, they developed and tested a teaching model for whole-class instruction. The model prescribes a sequence of instruction that includes daily review, development of the topic of the lesson for about 20 minutes, seatwork, and homework (Good, Grouws, & Ebmeier, 1983). The development portion of the lesson is key. The teacher does not just tell students what they are to learn but incorporates lively explanations and demonstrations. During this portion of the lesson, the teacher assesses student understanding with "product" and "process" questions and controlled practice, providing immediate feedback. Seatwork is assigned where students are held accountable for completing a set of exercises.

Combining results from these areas of research, a picture of "effective instruction" in the mathematics classroom emerges. The role of the teacher is to begin instruction with a review or mental arithmetic activity, tell students the objective for the day's lesson -- what they are expected to learn -- and introduce the lesson with an activity that draws on previous experience or prior knowledge. The teacher develops the topic for the day through direct instruction, showing students what they are to learn by demonstrating to the entire class with the appropriate concrete materials, pictures or symbols. The teacher poses "product" questions (i.e., "What answer did you get?") and "process" questions (i.e., "How did you get that answer?"), evaluating whether student responses are correct. Students are assigned exercises for controlled or guided practice, where the teacher is available to provide immediate feedback and repeat or elaborate on instruction as necessary. Each lesson ends with independent "successful" practice, a homework assignment, and closure--where students tell what they learned about the day's topic.

A distinguishing feature of this orientation to the teaching and learning of mathematics is the assumption that student achievement is directly related to the instructional technique and technical competence of the teacher. An effective teacher maintains a brisk pace of instruction, keeps students on task and holds them accountable to complete assigned work. Effective mathematics teaching is a procedural and managerial matter. The teacher plans for instruction by selecting suitable materials to model a mathematical concept or procedure and developing questions to assess student understanding. Within the structure imposed by the teacher, students pursue teacher defined explorations with concrete, pictorial and symbolic representations, answer teacher questions and complete practice exercises. Authority for knowing rests with the teacher who decides if student answers are correct and explanations sufficient.

Enabling mathematical inquiry. An orientation centered on **enabling mathematical inquiry** derives from recent research and deliberations in mathematics, mathematics education, cognitive science and educational psychology (see Romberg & Carpenter, 1986, for a review of this research). There are several basic assumptions that underlie this orientation and that distinguish it from the others. One of the assumptions is that children come to school with a rich informal knowledge of mathematics and demonstrate a natural capacity for and interest in understanding mathematical ideas (Ginsberg, 1977). Further, children are not passive learners, but actively construct, interpret and put structure on new mathematical learning (Carpenter et al., 1982; Resnick & Ford, 1981; Steffe et al., 1983; Wittrock, 1974).

This orientation to mathematics education has an explicit goal to empower students. This requires an environment where students become active participants in the construction of mathematical knowledge, making sense of mathematical situations and communicating with others about mathematical ideas. It means to engage in the "practice of mathematics, learning what it means to do mathematics" (Ball, 1988, p. 16). A learning community is created in which students learn to make conjectures, develop arguments to validate assertions, invent procedures, build abstractions and generalizations, and apply quantitative and spatial reasoning to real-world situations (Romberg, 1983).

Mathematics is seen as dynamic--the creation of human activity, derived from abstractions of empirical observations in the real world, bounded by culture and history, changing over time. Mathematics is seen as a body of knowledge -- skills, concepts, propositions -- connected by a rich web of relationships.

Teaching and learning take on characteristics quite different from the other two orientations. The traditional emphasis on arithmetic shifts to include a broad range of content. Instruction is "chunked" around key mathematical ideas that are explored over time to develop an understanding of concepts and procedures and the relationships among them. The intention is to develop a set of intellectual tools--mathematical thinking, higher-order reasoning skills, problem solving abilities, and natural and symbolic language to communicate mathematical ideas to others.

The teacher plays a critical role as a facilitator of student learning. The teacher affords opportunities for students to engage collectively and cooperatively in mathematical inquiry, providing problem situations in real-world contexts that exemplify mathematical ideas so that students can experience mathematics as a language and a tool for understanding the world around them. The teacher provides a variety of representations - physical, pictorial, numeric, geometric, graphical, algebraic - and helps students to see the power of each representational form to enrich the understanding of the mathematics embedded in a situation. And the teacher acts as a guide, listening to students, posing questions for further reflection, responding to questions by deciding which ones to pursue and which to put aside for the moment. The locus of epistemological authority shifts from the teacher and text toward the community of "teacher and students as inquirers who have the power to use mathematical tools to decide whether an answer or a procedure is reasonable" (Lampert, in press, a).[3]

The political interest embedded in each orientation. Each of these orientations embodies a political interest as well as a mathematical interest although the former is not often made explicit. A political interest is found in the different power relations that exist in the classroom between teacher and students in relation to the locus of epistemological authority. A political interest inheres also in a moral and ethical sense in the intended outcomes for students--certification, comprehension, or enablement. Gordon (1983) labels

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[3]The recent NCTM publication Curriculum and Evaluation Standards for School Mathematics provides a vision of teaching for mathematical inquiry.

these interests teaching for control, teaching for understanding, and teaching for liberation.[4]

**Certification of numeracy** embodies an interest in control. Learners are dependent on the teacher to create meaning. They lack comprehension of fundamental ideas. Mathematics is offered as "objective, lucid, and unequivocal" (Gordon, p. 361). Students have no sense of connectedness, among mathematical ideas or their application to life outside classrooms. Students do not question; they come to believe that much of mathematics -- the "clear demonstrations, logical relationships, and solutions that are errorless, if not elegant" -- is beyond comprehension (Gordon, p. 361). What students learn is that "mastery of truths has to do with getting the appropriate beliefs" (Scheffler, 1976, p. 205).

**Comprehension of mathematical ideas** embodies an interest in understanding. "The emphasis is on relational knowledge in the sense that the student comes to see why 'things are the way they are'" (Gordon, p. 374). Students no longer just fill in blanks; they solve problems. But the problems have been posed by others; other's formulations are taken as given. The emphasis is on "adjustment" so that one can "function objectively within the paradigm of the status quo" (Gordon, p. 374).

The third orientation, **enabling mathematical inquiry**, embodies an interest in liberation. Students become problem posers. They commit themselves to inquiry and in creating mathematical knowledge, come to

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[4]English (1983) describes three values around which curriculum decisions are made that parallel Gordon's categories. English terms the positions "control," " consensus," and "emancipation."

know more about themselves and others. The language of mathematical inquiry itself can be liberating.

> Consider that all mathematics begin with expressions such as "what if?," "suppose," "let"--suggestive of the way in which mathematics provides for its own liberation, creation and aesthetic excitement...we could replace these expressions as follows: "what if" with "what if I," "let" with "if I let" or "if we let" (Gordon, p. 378).

Power relations shift as authority for knowing moves from teacher and text to the community of teacher and learners. Being a part of a collectivity that has the power and the intellectual tools to create knowledge is emancipatory. It is an act which Gordon describes as

> paramount in providing personal, adequate evidence for knowing, not only evidence for substantiation but, more importantly, evidence that clarifies personal (as versus a weak or surface) understanding and tests the taken-for-granted reality (p. 362).

In arguing for mathematics as liberation, Gordon quotes Paulo Freire.

> Any situation in which some men prevent others from engaging in the process of inquiry is one of violence. The means used are not important; to alienate men from their own decision-making is to change them into objects (p. 361).[5]

It is an orientation that embodies the principles of equality and justice.

## THE IMPERATIVE FOR ENABLING MATHEMATICAL INQUIRY

The Commission on Standards for School Mathematics (1989) argues for the necessity of reforming the teaching and learning of mathematics that is consistent with the aim of **enabling mathematical inquiry**. They assert that the demands of work in the new economy require a degree and kind of mathematical literacy that go well beyond certification of numeracy and comprehension of mathematical ideas. The Commission

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[5]While I find this quote powerful, Freire is guilty of committing another kind of violence - making women invisible.

summarizes the views of Henry Pollak, a noted industrial mathematician, about the needs of the next generations of workers.

* The ability to set up problems with the appropriate operations.
* Knowledge of a variety of techniques to approach and work on problems.
* Understanding of the underlying mathematical features of a problem.
* The ability to work with others on problems.
* The ability to see the applicability of mathematical ideas to common and complex problems.
* Preparation for open problem situations, since most of real problems are not well formulated.[6]
* Belief in the utility and value of mathematics (p. 4).

But there are other important reasons to aim for **enabling** beyond this key practical and utilitarian one, reasons of a social and political nature. Again, the Commission speaks to the issue.

> In a democratic country in which political and social decisions involve increasingly complex technical issues, an educated, informed electorate is critical. Current issues--such as environmental protection, nuclear energy, defense spending, space exploration, and taxation--involve many interrelated questions. Their thoughtful resolution requires technological knowledge and understanding. In particular, citizens must be able to read and interpret complex, and sometimes conflicting, information (pp. 4-5).

For too long, mathematics has played the role of gatekeeper. Those who have managed to penetrate the "finely polished, impersonal mathematics curriculum" (Gordon, p. 362) have secured a key to opportunity. Most who have acquired the key have had to suspend sense-making in favor of mastery of mathematical knowledge as presented. A few have persisted in getting behind the facade. Their efforts have revealed the jumps of intuition, the stumblings and false starts, the "terror and triumph" of constructing mathematical knowledge. Those who

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[6]A colleague has quoted Pollak as admonishing the mathematics education community with the following: "When will teachers understand that in the real world you don't know which page of the text the problem is on!"

have given up trying to make sense, who have concluded that they are incapable of mastering mathematics as given, "try to escape the situation, the feelings, and the knowledge that one does not know" (Gordon, p. 372). Gordon continues,

> These beliefs lead to fear--the fear is not of mathematics qua mathematics, as I understand it; rather, this self-destructive condition derives from the fear of not knowing, believing that one should know, and from the fear of being discovered, of not being able to defend oneself (pp. 373-373).

Escaping the situation diminishes one's options for further study, future employment and full participation in society as an enlightened and mathematically literate citizen.

Mathematics education must have as a fundamental goal the empowerment of all learners to create knowledge and meaning. Orientations to the teaching and learning of mathematics that turn children into anonymous learners and teachers into mere technicians are not liberatory--they enslave. Mathematics education must assume an orientation that promotes mathematical inquiry. As the Commission on Standards for School Mathematics has argued, teachers and administrators can no longer tolerate practices that distort what it means to know mathematics and that delimit the full participation of far too many students in our classes.

Teaching for **enabling mathematical inquiry** holds the potential for what Aronowitz and Giroux (1985) call "creative and reflective discourse and action."

> [It] takes the issues of community and liberation seriously, and in doing so, gives new meaning to the pedagogical and political necessity of creating the conditions for emancipatory forms of self and social empowerment among both educators and students (p. 43).

Teaching for **enabling mathematical inquiry** is a radical agenda, mathematically and politically. It is a struggle worthy of engagement.

# CHAPTER 3

# METHODS OF DATA COLLECTION AND ANALYSIS

## INTRODUCTION

As is often the case with field-based studies that grow out of an ethnographic paradigm, the nature of the questions that initially inform a research study may change with immersion in a setting. The collection and analysis of data proceed jointly throughout the study. As new materials are collected and analyzed, new questions may emerge, conceptual frameworks may prove to be inadequate and, in some cases, a study may be radically redirected (Bogdan & Biklen, 1982; Eisenhart, 1988; Erickson, 1986; Hammersley & Atkinson, 1983). Initial questions may fade in significance, to be replaced by new questions that, given circumstances not anticipated, take on greater importance and even urgency. Such was the case with this study. The study reported here and the questions it considers represent a considerable departure from its initial conceptualization.

This chapter describes the evolution of the study - the emergence of new questions, the recognition of a much larger problem, the need for different conceptual and analytic frameworks. In addition, the methods of data collection and analysis used to carry out this study are described. The intent of the chapter is to report on the methodology and to tell the intellectual autobiography.

# EVOLUTION OF THE STUDY

## The Beginning: Acting to Influence Teacher Knowledge and Beliefs About Conceptually-Oriented Mathematics through Inservice

The project was initially conceived to be an observational case study of Marilyn Miller,[1] an elementary mathematics supervisor in the Detroit schools, as she planned and conducted inservice opportunities for teachers, as she worked with teachers in their classrooms, and as she developed and disseminated curriculum and resource materials. The purpose was to investigate the nature of inservice and instructional support she provided for teachers charged with implementing the newly-mandated, "conceptually-based," elementary mathematics curriculum. The main question with which I began the study was, "In what ways and to what extent does the supervisor act to influence teacher knowledge and beliefs about mathematics, the elementary mathematics curriculum, mathematics teaching and mathematics learning?"

The problem to be studied. Leaders in mathematics education had been calling for a reorganization of the mathematics curriculum around concept development and problem solving since the beginning of the 1980s. At the same time, there was the recognition that implementation of a conceptually-based, problem solving approach to mathematics instruction posed several substantial problems. Chief among them was the limitation of teachers' knowledge about mathematics and the teaching and learning of mathematics. There was a widely held belief among reformers that teaching conceptually-oriented content required teachers to have a conceptual understanding of mathematics, to know why

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[1]Marilyn Miller is a pseudonym. In Chapter 7 I provide background information on Marilyn's history as a mathematics teacher, consultant, and supervisor.

understanding concepts is important, and to know how to help students gain that understanding (Devaney, 1983; Shulman, 1986; Lampert, 1986b; Resnick, 1983). If teachers are to overcome these limitations, it was reasoned, they must have opportunities for high-quality professional development programs.

A number of recommendations were made. School districts should provide inservice on the mathematical content and pedagogy that develop conceptual understanding and problem solving ability. Districts should provide incentives for teachers to invest in inservice. School systems should employ well-qualified mathematics specialists to coordinate inservice efforts. Inservice should provide opportunities to integrate theory and research with applications and practice in classrooms (An Agenda for Action, 1980; NCTM, 1985). These guidelines were useful in planning inservice but they seemed to underestimate or fail to acknowledge the complexity of changing teacher knowledge and beliefs.

The literature on inservice education overwhelmingly pointed to the ineffectiveness of most efforts to facilitate and sustain change in teacher beliefs, attitudes and classroom practices (Wood & Thompson, 1980; Guskey, 1986). In an attempt to map the terrain leading to teacher change, wide-ranging research and development agendas had been mounted. A sampling suggests the diversity: defining models for more effective staff development (Gall & Renchler, 1985; Wood & Thompson, 1980; Rauh, 1978; Jacullo-Noto, 1981); examining the institutional context in which inservice occurs (McLaughlin & Marsh, 1978; Little, 1984; Barth, 1985); attending to teacher concerns (Lieberman, 1985; Swanson-Owens, 1985; Hall & Loucks, 1978); investigating the role of the teacher as learner, advisor and translator of research (Chism, 1985; Greene, 1978; Clark & Lampert, 1985; Little, 1985; Doyle & Ponder, 1977; Day, 1985).

The missing link in this research was the inservice provider and her/his role in the implementation process. Even when developers of distinctive mathematics curriculum materials acknowledged that a skilled and influential local coordinator was essential for training, implementation and management, their research on the effectiveness of innovative programs paid little attention to the specific ways in which the district specialist influenced the course of the implementation efforts.[2]

This study, as originally conceived, aimed to provide insights and knowledge about planning and conducting inservice for teachers charged with implementing an innovative mathematics curriculum. In particular, how did one of Detroit's elementary mathematics supervisors, Marilyn Miller, act to influence teacher knowledge and beliefs about mathematics, teaching mathematics and learning mathematics? What teacher knowledge bases -- subject matter knowledge, pedagogical content knowledge, strategic knowledge -- did Marilyn act to influence and in what ways? In what ways did she attempt to influence teacher beliefs about curriculum content and methods of instruction and what sources of authority - research findings, the "wisdom of practice" (Shulman, 1987), district mandates, equity - did she invoke?

There was also a concern about the ways in which contextual factors might affect implementation efforts. The structure of existing curricular materials (particularly textbooks), the pressure to increase

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[2]See, for example, the evaluation of the Comprehensive School Mathematics Program, CEMREL, Inc., conducted by the CSMP Review Panel for submission to the Joint Dissemination Review Panel, 1982.

test scores, the burdens of size, numbers and space, the movement of children in and out of classrooms (or from school to school), may frustrate a teacher's efforts at implementation. Most of these factors are beyond the control of classroom teachers and staff developers. In fact, some scholars have argued that changes in teacher knowledge, skills and beliefs will not suffice to overcome the environmental constraints (Cuban, 1984; Sarason, 1971; Jackson, 1978). Of concern in this case was the extent to which Marilyn attended to contextual constraints: the degree to which they were acknowledged; the degree to which she was sympathetic to the demands that constitute the worklife of teachers; the examples she provided of instructional practices and curriculum in use under conditions of contextual constraints.

Collecting the data. The first phase of data collection involved interviewing the elementary mathematics supervisor and attending a series of inservice workshops she conducted for teachers in the district. Over a period of four months, I observed nearly 50 hours of inservice, recording the events with fieldnotes and audio-tapes. I accumulated materials that were part of the inservice curriculum--research articles, instructional activities focused on subject matter and cooperative learning, model lessons, a quiz on effective mathematics teaching practices. I also collected samples of teachers' work on various activities. I secured copies of teachers' evaluations of inservice - a written "Workshop Feedback" distributed by Marilyn and completed by participants at the end of the three-day series.

I interviewed nearly a score of classroom teachers representing a broad range of tenure with the district, from newly hired to 30-year veterans. My interest in talking to teachers was three-fold. The first was to inquire about how they were making sense of this new "conceptually-based" mathematics curriculum and the district-wide mandate to implement. Second was to learn what influenced their decisions on whether to attend inservice. The third was to find out from those who had attended inservice their assessment of the sessions. Some of the interviews were tape-recorded; all were recorded with fieldnotes. Some of the teachers with whom I spoke had taken advantage of the inservice opportunities provided by the supervisor, others had not. The interviews were partly structured. There was a set of questions asked of all teachers and an additional set asked of those who attended inservice. At the same time, the interviews were conducted so as to allow for probes to pursue further issues raised by an individual teacher. The Appendix contains the standard interview schedule that formed the core of teacher interviews.

I also collected curriculum materials that had been developed by the district and distributed to teachers and Area mathematics specialists. These materials included: a document describing the district's philosophy regarding mathematics education and the mathematical strands that organized the K-5 curriculum; the sets of objectives that constituted the intended curriculum and sequenced instruction at each grade level; the sets of model lessons that provided examples of how the curriculum was to be enacted in the classroom as teachers taught to the instructional objectives.

<u>The first wave of analysis: Emerging themes</u>. Erickson describes the analytic process in ethnography as "repeated trials at understanding recurrent events" (1986, pp. 143-44). My focus in the first "trial" was on what Marilyn thought teachers needed to know and

be able to do to implement the new curriculum and how she thought they would acquire the necessary knowledge and skills. At this point in the study I had conducted two formal interviews with Marilyn, both prior to any of the inservice offerings. I had observed two different 9-hour workshop series for K-1 teachers and collected artifacts from those workshops. In addition, I had accumulated most of the formal curriculum documents that had been distributed to Area mathematics specialists and classroom teachers.

I transcribed each of the interviews with Marilyn. In analyzing these interviews, I looked for the following: 1) the goals Marilyn had for teachers as she planned inservice; and 2) what she intended to focus on -- teachers' subject matter knowledge, classroom activities, the strands and objectives, instructional strategies, instructional materials, classroom management, research, testing -- and why. I did not do a full transcription of the inservice workshops. Rather, I listened to the audio recordings and elaborated on fieldnotes that I had taken during inservice. I listened to one of the recordings a second time, making a list of activities/events and the number of minutes spent on each one.[3] For each event/activity I noted the following: 1) where the event was centered--Marilyn "lecturing" or giving a demonstration where teachers listened and observed; Marilyn and teachers acting together as one large group; teachers working individually or in small groups on an activity; 2) what the topic of the event was--curriculum strands, research, using manipulatives,

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[3]One of these workshops was conducted on December 7, 8 and 10, the other December 14, 15 and 17, 1987. Both were for teachers in grades K-1. There was only slight variation in the two series and that was caused by extremely bad weather on December15 that resulted in a late beginning and an early dismissal.

grouping for instruction, teaching to an objective, understanding concepts, problem solving; 3) how the event/activity was carried out--lecture, demonstration, reading, "jigsawing," using manipulatives, taking a test, making lists.

I compared what Marilyn had talked about as goals for the inservice with the activities she conducted in the workshops. I wrote an analytic memo (Hammersley & Atkinson, 1983, pp. 164-167), noting several emerging themes: 1) there seemed to be a heavy emphasis on a selected, fairly narrow body of research findings on teacher effectiveness and effective mathematics teaching as a rationale for instructional strategies; 2) there seemed to be no getting away from the emphasis on testing in the district, either in practice or in teachers' minds; 3) contextual factors seemed to be a significant element in the inservice and implementation effort; 4) there seemed to be a strong technical, prescriptive orientation to the inservice design; 5) there seemed to be a contradiction between a rhetoric of "teaching for conceptual understanding" that Marilyn used and the choices she made for the curriculum of inservice.

These last two points became increasingly significant as I tried to make sense of what I was hearing and what I was seeing. At the time, I was assuming the dichotomous traditional/conceptual orientations to mathematics and mathematics teaching and learning that I described in Chapter 2. It appeared to me that a substantial part of inservice was not aimed at any substantive change in teaching practice. Despite the rhetoric of "teaching for conceptual understanding," inservice seemed to be more closely aligned to a traditional approach but aimed at making it more effective and efficient. I puzzled over this apparent contradiction, wrote an analytic memo which I asked several colleagues to comment on, and read (and re-read) some theoretical and empirical works suggested by them (Ball, 1988; Good, Grouws & Ebmeier, 1983; Hiebert & Lefevre, 1986; Lampert, 1986a, 1986b, in press a, in press b; Madsen-Nason, 1988). What I began to see was that the rhetorical goal of developing conceptual understanding of mathematical ideas had become commonplace in the discourse of the mathematics education community. But the meanings contained in this goal and the means to attainment of it were matters of disagreement and even debate. What was becoming apparent was that the framework I had brought to the study in terms of orientations to mathematics education, traditional vs. conceptual, was inadequate. It simply did not reflect two very different meanings attached to "teaching for conceptual understanding." The meaning I attached to "teaching for conceptual understanding" reflected an orientation aligned with **enabling mathematical inquiry**. There had been an assumption, on my part at least, that informant and researcher shared a common meaning for a common language. That assumption was being called into question. Another possible interpretation for the "apparent" contradiction between Marilyn's rhetoric and her practice was that for her, teaching for conceptual understanding was more aligned with an orientation of teaching for **comprehension of mathematical ideas**.

**What Does "Teaching For Conceptual Understanding" Mean in This Setting?**

<u>The second wave of analysis: New categories</u>. The second wave of analysis began with a new framework for thinking about orientations to the teaching and learning of mathematics. The new framework posed

three orientations as ideal types: 1) **certification of numeracy**; 2) **comprehension of mathematical ideas**; and 3) **enabling mathematical inquiry** (see Chapter 2).[4] I returned to the data for a second "trial," shifting the focus to the question, "What does teaching for conceptual understanding mean in this setting?"

I distinguished the three orientations to mathematics education along the following categories: 1) the goals for students as learners of mathematics; 2) the representation of mathematics; 3) the selection of and emphasis on topics to be studied; 4) the verbs that describe student actions; 5) the role of the teacher in instruction; 6) the role of the student in instruction; 7) how learning is portrayed; and 8) the source of authority for knowing.

These categories provided a useful framework for data-source triangulation--the interrogation and comparison of data gathered at different points in time, in different contexts and from different sources (Hammersley & Atkinson, 1983, pp. 198-200). The categories are presented below with brief descriptions for each orientation to illustrate the differences among them.

<u>Goals for Students as Learners of Mathematics</u>

| | |
|---|---|
| Certification of Numeracy | computational speed and accuracy; proficiency with conventional algorithms and their application to routine word problems. |
| Comprehension of Mathematical ideas | proficiency with and understanding of mathematical procedures - knowing how to use them and why they work; understanding basic concepts such as place value and operations. |

[4]These orientations have been described in detail in Chapter 2. I am particularly indebted to Deborah Ball for helping me to think less simplistically about this matter.

| | |
|---|---|
| Enabling Mathematical Inquiry | empowering students - to learn <u>about</u> mathematics and to learn to <u>do</u> mathematics. |

## The Representation of Mathematics

| | |
|---|---|
| Certification of Numeracy | a system of abstract symbols to be manipulated and a collection of skills, definitions, rules and procedures to be memorized and mastered; static, "discovered," and rule-bounded. |
| Comprehension of Mathematical Ideas | a body of related ideas within a linear, rule-bounded hierarchical arrangement; an abstract system of concepts and procedures for which there are appropriate concrete models and pictorial representations. |
| Enabling Mathematical Inquiry | dynamic, the creation of human activity; a body of skills, concepts and propositions connected by a rich web of relationships. |

## Topics To Be Studied

| | |
|---|---|
| Certification of Numeracy | arithmetic - addition, subtraction, multiplication and division of whole numbers, fractions and decimals - constitutes the bulk of the topics to be studied; emphasis is on formal language and symbolic representation. |
| Comprehension of Mathematical Ideas | arithmetic dominates but is supplemented by topics from geometry, statistics and probability, and number and set theory. |
| Enabling Mathematical Inquiry | shift from arithmetic to a broad range of content and processes--numeration and number sense, computation and estimation, concepts of whole number operations, measurement, geometry and spatial sense, statistics and probability, fractions and decimals, patterns and relationships; mathematics as problem solving, communication, reasoning; mathematical connections.[5] |

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[5]See <u>Curriculum and Evaluation Standards for School Mathematics</u>, 1989, from which these were drawn.

**Verbs That Describe Student Actions**[6]

| | |
|---|---|
| Certification of Numeracy | read, identify, name, recall, recognize, add, subtract, multiply, divide, state the rule, demonstrate accuracy. |
| Comprehension Mathematical Ideas | relate, extend, demonstrate, complete, estimate, of measure, compare, apply strategies, determine reasonableness of answers. |
| Enabling Mathematical Inquiry | investigate, explore, predict, invent, generalize. |

**The Role Of The Teacher In Instruction**

| | |
|---|---|
| Certification of Numeracy | shows students how to carry out procedures and provides ways for them to remember facts and steps in algorithmic procedures. |
| Comprehension of Mathematical Ideas | shows students what they are to learn by demonstrating with appropriate concrete materials and letting students practice what they have been shown; monitors student learning during instruction with "product" and "process" questions; evaluates student responses and provides immediate feedback; maintains a brisk pace of instruction, keeps students on task and holds them accountable to complete assigned work. |
| Enabling Mathematical Inquiry | creates a total environment where teacher and students engage with one another in inquiry; poses interesting, challenging and non-routine problem situations that lead to learner inventions; provides a variety of representations; acts as a guide, posing questions for further reflection, responding |

[6]The schema used here represents an alteration of categories of student actions proposed by Avital and Shettleworth (1968) who adapted Bloom's Taxonomy (1965) to describe levels of mathematical performance more precisely. Avital and Shettleworth provide three levels: (1) recall-recognition (knowledge in Bloom); (2) algorithmic thinking (comprehension and application); and (3) open search (analysis/synthesis). The first category calls for recall of facts, definitions and procedures in exactly the way they were presented. The second category requires transfer of knowledge. Most common would be application of algorithms to solve routine story problems. The third category calls for using knowledge in nonroutine situations to discover relationships and make generalizations.

to questions by deciding which to pursue and which to table.

## The Role Of The Student In Instruction

| | |
|---|---|
| Certification of Numeracy | absorbs and remembers the knowledge that a teacher dispenses. |
| Comprehension of Mathematical Ideas | pursues teacher defined explorations, answers teacher questions and completes practice exercises within structure imposed by the teacher. |
| Enabling Mathematical Inquiry | becomes a mathematical risk-taker--makes guesses and pursues hunches, offers hypotheses, accepts challenges to hypotheses and marshalls arguments in support of them, yielding to the force of an argument that is mathematically more reasonable. |

## How Learning Is Portrayed

| | |
|---|---|
| Certification of Numeracy | an individual, solitary effort where students search their memory for accumulated facts and algorithmic solutions; students are passive receptacles into which mathematical knowledge is poured. |
| Comprehension of Mathematical Ideas | requires a particular sequence--concrete to pictorial to abstract/symbolic; is related to the instructional technique and technical competence of the teacher. |
| Enabling Mathematical Inquiry | children are not passive learners; students actively construct, interpret and put structure on new mathematical learning; requires a learning community that supports individual and collective efforts to make conjectures, develop arguments, invent procedures, build abstractions and generalizations, apply quantitative and spatial reasoning. |

## The Source Of Authority For Knowing

| | |
|---|---|
| Certification Numeracy | teacher and text - teacher as judge, text as the of standard for judgment. |
| Comprehension Mathematical Ideas | teacher - decides if student answers are correct of and explanations sufficient. |

| | |
|---|---|
| Enabling Mathematical Inquiry | classroom community of students and teacher who have power to use mathematical tools and standards of the mathematical community to decide about the reasonableness of processes and the results of investigations. |

There were several "settings" to which this framework was applied. The first that I interrogated was the set of formal curriculum documents. These included: a booklet Detroit Public Schools Pre-kindergarten-Grade 12 Curriculum: Strands & Objectives - Mathematics Education that contained the district's philosophy regarding mathematics education and defined the ten mathematical strands and learning objectives for each strand at each grade level; the Instructional Sequence that specified the behavioral objectives for each grade and the sequence in which teachers were to teach to those objectives; and Model Lessons to Promote Thinking, a set of materials that included an elaboration of each of the ten curriculum strands followed by a set of model lessons for specific objectives at each grade level. The purpose in examining these materials was to determine what warranted describing the curriculum as "conceptually-based" and whether it aimed for **comprehension of mathematical ideas** or **enabling mathematical inquiry**. Chapters 5 and 6 are devoted to the analysis of the formal curriculum documents.

The second setting I examined was the set of interviews I had conducted with Marilyn. The third was the setting of inservice workshops. Chapter 7 scrutinizes these settings.

Using the framework to assess the orientation of the reform: New questions. The schema just described provided a useful framework to analyze the nature of this elementary mathematics curriculum reform.

The analysis of the formal curriculum documents revealed a tension between an orientation toward teaching for **certification of numeracy** and an orientation toward teaching for **comprehension of mathematical ideas**. The inservice workshops seemed to be oriented toward teaching for **comprehension of mathematical ideas**. As I began to get a sense of the orientation of this initiative, another set of questions emerged.

How would implementation of this mathematics curriculum make a real difference in the lives of the substantial number of at-risk children in the Detroit schools? The District's publicly expressed goals for students as learners of mathematics were to help them "think critically, flexibly, copperatively and independently," to develop "higher-order thinking skills of analysis, synthesis and evaluation," "to apply their knowledge of mathematics in solving problems," and "to develop, to create, and to analyze." These seemed like goals that required an environment, a curriculum and modes of teaching aimed at **enabling mathematical inquiry**. There appeared to be a new contradiction, a contradiction between these intended goals and the nature of the reform itself and how teachers were being educated for its implementation.

I began to sense that the magnitude of the problem embedded in this study was far greater than simply how the supervisor was using inservice to influence teacher knowledge and beliefs toward implementation of this reform. Why was the reform conceived as it was? What factors shaped the design of the strands and objectives, the instructional sequence, and the set of Model Lessons? What set of conditions supported mandates for its implementation? With these questions, I returned to data already collected, shifting the focus of features to attend to.

## THE CONTEXTS OF EDUCATIONAL POLICYMAKING

An interrogation of the data with these questions as the lens produced new insights. An important element in shaping this reform was a district-created test called the Assessment of Basic Curriculum Skills (ABCS). I set out to learn more about its development and its influence on this reform. A second insight was the frequency of references to "increasing administrative authority." I wanted to learn how this was manifested and to what extent it was a factor in this reform initiative. Put broadly, I wondered about the contexts in which this reform was conceived. The search led me back nearly two decades to events that were to shape this initiative. The inquiry brought into clear focus the historical, sociopolitical, economic and organizational situatedness of this reform effort.

### Historical, Political and Organizational Contexts

A number of sources provided data relative to the contexts that influenced the course of this reform. I interviewed several key figures with a long history of involvement in Detroit school politics and policymaking. One such individual was a veteran, high-ranking and highly-respected central administrator in the district. He provided an account of factors that led to the decision to adopt the city-wide curricula, going back to the decentralization of the district in the early 1970s. He made available his extensive files on the U.S. District Court desegregation orders, including in-house memoranda as well as formal documents prepared for and submitted to the Court and the Monitoring Commission. He provided his perspective on the tendency in the district to centralize authority over curriculum.

A person with a unique vantage point from inside as well as outside the schools was a member of the U.S. District Court Monitoring Commission. The Monitoring Commission was established as an arm of the Court to oversee the federal desegregation order. It was to function as a citizen's committee, auditing the District's efforts and assisting the Court with reports and recommendations regarding implementation of the components of the desegregation plan. He provided a perspective on the central administration's response to the court order to include educational components to wipe out the effects of past discriminatory practices in the district. He also linked the desegregation efforts with changes in the administrative structure of the district and the attempt of the central administration to wrestle control over instructional matters from the decentralized regional boards and superintendents. He provided documents prepared by the Monitoring Commission including press releases, status reports and lengthier Profiles submitted to the District Court, N.A.A.C.P., the Detroit Schools, the State of Michigan, the Detroit Federation of Teachers and made available to the public.

A third key individual whom I interviewed was the education director of a civic leadership organization representing a coalition of corporate, business, labor and community organizations. The organization was conceived following the 1967 rebellion to combat the urban social problems that had contributed to the uprising. Its education director at the time of this study was a man with a long and distinguished history of educational activism at a number of levels -- from teaching in Mississippi Freedom schools to grassroots community efforts in Detroit to the Michigan Department of Education. He, too, provided another perspective on decentralization, the struggle of the black community for control over its schools, and the recent trend to recentralize increasingly more authority over curriculum and instruction at the central administration level.

An interview with an administrator in Research and Evaluation provided a history of the development of ABCS. Interviews with additional personnel in the district -- Area supervisors, district curriculum and instructional specialists, -- added considerably to the unfolding story of the conceptualization of this reform.

This rich set of data was subjected to data reduction out of which emerged a number of categories: school governance; racial politics; equity; testing; curriculum decision-making; goals for learning. I looked for instances of these categories in the various interviews I had conducted and documents I had collected. In some cases these categories intersected. For example, understanding school governance involved understanding the influence of the politics of race in Detroit. Curriculum decision-making intersected with testing and goals for learning. In addition, sub-categories emerged that were important to consider. School governance subsumed several themes: administrative authority over the District--decentralization, then recentralization; administrative authority over curriculum; administrative authority over testing; city-wide mandates. The interrogation of the data helped to reveal in what ways and to what extent these contexts were significant in explaining the nature of this reform. Context also became a lens for further reflection on teacher inservice. A reexamination of inservice data revealed how context -- the structure of the inservice sessions and the settings of participating teachers' practice -- influenced the content of inservice.

**Economic Contexts**

This reform was conceived in a district faced with an incredible set of problems, many of which grew out of conditions in the larger community over which schools had no control. I collected data to assess the impact of economic and social dislocation on the district's ability to provide a quality educational program for its youngsters. Nearly every person in the schools with whom I spoke made some reference to the impact of the increasing poverty and social disorganization on the system and in individual classrooms.

I interviewed experts on the changing structure of the Detroit economy. They provided important demographic and economic data for the city that graphically portrayed the extent of social disorganization that resulted from two decades of economic disinvestment and industrial dislocation. They portrayed a grim future of further impoverishment and reduced life chances for the next generation of its citizens. They also voiced a sense of urgency about the need for the Detroit schools to prepare young people with the knowledge, skills and dispositions to find and hold steady, decent paying jobs in a changed Detroit economy. Additional data were gathered on school financing, student achievement, school drop-out rates, and student mobility within the district.

Coincidentally, The Detroit Free Press, in 1988, dedicated the "op ed" page once a week to its investigations of the state of education in the city's schools. The weekly published reports served as another data collection point, providing valuable information about organizational politics in the schools and the impact on schools of economic and social disorganization in the city.

## A PLAN FOR PRESENTING THE FINDINGS

In the chapters that follow I describe the results of the analysis. Chapter 4 describes the influence of sociopolitical and organizational contexts that ultimately led to and shaped this curriculum reform initiative. Chapters 5 and 6 examine the curriculum reform through its formal documents to determine the orientation of the reform and to inquire about its potential to empower Detroits' youngsters with mathematical knowledge and power. Chapter 7 examines the inservice provided for teachers charged with implementing this reform. The inquiry focuses on what the inservice provider thought teachers needed to know to implement the reform and how she thought they would learn what they needed to know. As in the previous two chapters, the ultimate concern is the extent to which inservice provided teachers with the knowledge, skills and dispositions to develop mathematical literacy among the next generation of youth, particularly minority youth, to participate fully as citizens and workers in a democratic society.

In Chapter 8, I return to the idea of context. Using Foucault's notion of history as genealogy, I provide a "genealogical analysis" (see Noujain, 1987) of the Detroit elementary mathematics curriculum reform. It is a narration of the set of ideas, events, practices and people that combined to form the initiative. The basic antecedants were an instrumental ideology and a technical rationality that dominate American education and the curriculum field; the politics of race and school governance in Detroit; the recognition of a new economic order and Detroit's position in it.

I conclude by arguing for a further transformation of mathematics education in Detroit toward **enabling mathematical inquiry**. To do anything less means that mathematics will continue to be a gatekeeper, denying the possibility of the good life to far too many of our citizens, particularly those of color.

# CHAPTER 4

# THE INFLUENCE OF SOCIOPOLITICAL AND ORGANIZATIONAL CONTEXTS IN SHAPING THE REFORM

## INTRODUCTION

This chapter examines the contextual factors that had a direct influence on the nature of the Detroit mathematics curriculum reform initiative. This reform was shaped and supported, in part, by factors located within the school context. The Federal Court desegregation order of 1975 included a set of educational components that would ultimately lead to this curriculum reform effort. In addition, two decades of changes in organizational and authority structures and recent efforts to recentralize administrative control over curriculum would influence the nature of the mandate for its implementation. This chapter addresses the first question of this study: In what ways and to what extent had political and organizational factors shaped the design of this curriculum and mandates for its implementation?

## THE TESTING COMPONENT OF THE DESEGREGATION ORDER

### The Assessment of Basic Curriculum Skills

The development of ABCS. The testing component of the desegregation order has particular relevance to this study of the elementary mathematics curriculum reform. The testing component was established by orders of the Court on May 11 and June 1, 1976. The orders directed the Detroit Board of Education and the State Board of Education to jointly plan a comprehensive testing program "consistent with the goals of a desegregated system." It specified four elements: 1) review all tests and testing procedures for racial, ethnic and cultural bias; 2) provide inservice training to all staff on non-discriminatory administration of tests and uses of test results; 3) develop an objective-referenced city-wide testing program in reading, writing and mathematics at all levels; 4) establish an evaluation program to assess the effectiveness of instructional programs and use the evaluations for curriculum development and planning and policy determinations. The district's accomplishments in developing a city-wide objective-referenced testing program and using the test results to evaluate the effectiveness of instructional programs ultimately led to the curriculum reform that is the object of this study.

At the time of the Court order, the city-wide testing program at the elementary grades used the Stanford Achievement Test, the Iowa Tests of Basic Skills, and the Cognitive Abilities Test, all norm-referenced standardized tests. In July, 1977, the Detroit Testing Task Force appointed a Test Review Team to conduct an examination of all tests used in the city-wide testing program. At the conclusion of their review, the Team made the following recommendations: "1) the California Achievement Tests (CAT, 1977 edition) be adopted for all pupils in benchmark grades (to be determined by the Central Board of Education); 2) objective-referenced tests (ORT's) be given city-wide in grades 1 through 12 as they are developed; 3) until ORT instruments were ready, the CAT be used to fill any voids in the new city-wide testing program; and 4) no norm-referenced group aptitude tests be administered" (Detroit Public Schools, June, 1983, p. 10). By 1980, CAT had replaced all previously used standardized tests of student achievement in grades 1-11.

The district launched it's objective-referenced test development program in 1977. The first test developed was the Ninth Grade Communication Skills Test, administered as a pre-test and post-test to assess reading, writing, and study skills. The second test was the High School Proficiency Program Examination (see Chapter 1). Both tests were developed in collaboration with a private test development company. The development of a criterion-referenced program for the elementary and middle grades was an in-house effort. From 1977 until 1982, the Research and Evaluation Department, Test Development Team, and Curriculum Development and Services staff worked to develop a program to measure student achievement in reading, writing and mathematics for grades 1-8. The program was called Assessment of Basic Curriculum Skills (ABCS). Teams of curriculum specialists, mathematics supervisors, mathematics instructional specialists, testing specialists and classroom teachers worked on the development of the mathematics portion of ABCS for grades 1-8. They drafted sets of learning objectives, specified mathematical competencies, developed scope and sequence charts of objectives, wrote test specifications and test items for each competency and field tested some of the math items (Detroit Public Schools, June, 1982; Detroit Public Schools, April, 1984).

The mathematics portion of ABCS represented a compromise of contending views about what it means to know mathematics and how that knowledge should be assessed. Mathematics specialists argued that the test should assess student understanding of mathematical concepts and their application in problem solving and that each competency be introduced in a problem solving mode. Teachers, on the other hand, wanted conventional standardized tests emphasizing computational proficiency with some word problems. To reach a compromise, a group of mathematics supervisors, teachers and testing specialists were brought together and charged with designing a test to strike a balance between the two positions. An administrator in Research and Evaluation provided the following account of the conflict surrounding the effort to create the mathematics portion of ABCS.

> The math portion of ABCS was the most difficult. We worked closely with the elementary mathematics supervisor who had been very active in the NCTM movement. She wanted more than just to have students compute. We hired math teachers to develop test items. When the math supervisor and curriculum people saw the first test, they rejected it. It looked too much like a standardized test. Problem solving was not real problem solving, just computational problems using words. So curriculum told the math people to go off and develop their own. So they did. When we saw their test, <u>we</u> said, 'No way!' It was just too difficult - real hard. So curriculum hired a third group of teachers and got the math supervisors to work with us and to supervise the teachers. But there was still daily conflict between the two groups trying to strike a balance. So what you see in ABCS is a balance between traditional math tests and the far extreme problem solving. This is in the middle. When I get comments about how difficult it is I think, 'You should have seen what we had before.'

The test development team identified nine "competency strands" that provided the organizing framework for test objectives and test items--operations, numeration, estimation, patterns, geometry, measurement, sets and logic, functions and relations, and statistics and probability. Drawing on skills selected from textbooks adopted by the district and skills endorsed by the NCTM (<u>An Agenda for Action</u>, 1980), the test-makers created two sub-tests, one assessing computational proficiency, the other conceptual understanding and application. In its final report to the Court on implementation of the testing components, the Board noted that "several revisions of the competencies were considered before agreement was reached...Members of the mathematics department strongly urged that each of the mathematics competencies be introduced as word problems or in problem solving modes and in line with the goals of the National Council of Teachers of Mathematics" (Detroit Public Schools, June, 1983, p. 25). The final form of the Assessment of Basic Curriculum Skills was administered for the first time in spring, 1983.

Using the test to drive the curriculum. In the first administration of ABCS, the results on the mathematics portion were dismal. Table 3.1 shows the percentage of students at each grade level who achieved 60% or above on each competency.

Of the 45 cells in the table, 18 show percentages of 50% or less. These cells indicate that at a particular grade for a specific competency, fewer than half the children were able to answer at least 60% of the items correctly. First and second graders did relatively well. At least half the children in Grades 1 and 2 scored at or above 60% on all nine competencies. But beyond second grade, the level of achievement tended to drop. The lack of achievment was acute at grades 4 and 5. At the fourth and fifth grade, the majority of children did well on only two competencies--operations and measurement at Grade 4, numeration and sets and logic at Grade 5. The poor performance of children on the mathematics portion of ABCS raised a question about the degree to which the test items were correlated with what children were being taught.

Table 3.1 Percent of students at Each Grade Achieving At or Above 60% on Each Mathematics Competency, ABCS, May 1983

| COMPETENCY | GRADE 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Operations | 61 | 84 | 61 | 57 | 48 |
| Numeration | 73 | 62 | 71 | 34 | 66 |
| Estimation | 55 | 56 | 38 | 41 | 50 |
| Patterns | 64 | 50 | 58 | 28 | 39 |
| Geometry | 69 | 83 | 60 | 44 | 24 |
| Measurement | 81 | 88 | 64 | 64 | 47 |
| Sets and Logic | 92 | 78 | 77 | 35 | 58 |
| Functions and Relations | 53 | 75 | 47 | 37 | 30 |
| Statistics and Probability | 80 | 91 | 39 | 44 | 38 |

Source: Report from _______________[1] to Area Superintendents, November 14, 1983.

The Board had informed the Court in its final report on the testing component -- which was submitted before results of the first administration of the ABCS test were available -- that the mathematics portion of ABCS correlated "highly" with the "new curricular emphasis" on problem solving. It is unclear what was used for the correlation study since the district did not have a mathematics curriculum per se. Apparently the scope and sequence of objectives that had been developed for ABCS was distributed to Area offices and may have made its way to some teachers' hands, but it had not been defined as the explicit

[1]The name of the person who released the memo is not included for confidential reasons.

mathematics curriculum. A later study, conducted in 1985 by mathematics specialists from one of the district Area offices, assessed the degree of correlation between the scope and sequence of ABCS objectives and three of the four textbook series adopted by the Board for use in the district. That assessment revealed a significant number of objectives at each grade level that were absent from or inadequately attended to in one or more of the texts.

One element of the testing component stipulated that test results be used to improve the effectiveness of educational programs, inform policy and planning decisions and assist curriculum development. Given the considerable support of mathematics supervisors and testing, evaluation and curriculum personnel for ABCS, the decision was made to reexamine the K-5 mathematics curriculum. A writing team was charged with developing an instructional sequence and a guide that reflected the "new and changing needs with emphasis on higher level thinking skills and future employment" (Detroit Public Schools, 1984).

The administrator in Research and Evaluation confirmed that key personnel involved in the test development effort viewed ABCS as a vehicle to promote changes in the mathematics curriculum.

> There is no question that people were thinking about ways the test could lead to an improvement in the curriculum. In fact, ____________________ (a highly respected veteran administrator in the district who was a moving force in the test development effort) talked about a test driven curriculum. We all recognized the power of tests. If you look at ABCS, the writing program in the city has changed drastically because of testing. The High School Proficiency Exam with its paragraph writing - that made us put it in ABCS.

In January, 1987, the Superintendent and the Board approved a K-5 mathematics curriculum derived from the scope and sequence of objectives that had been developed for the competency strands of ABCS.

The curriculum consisted of a set of one-hundred or more objectives for each grade level. The objectives were sequenced by month and teachers were told to teach the objectives in the order in which they were listed. The Board mandated city-wide implementation of the curriculum over a three year period beginning with the 1987-88 school year.

## THE MANDATE FOR CITY-WIDE IMPLEMENTATION

The decision to adopt a standardized mathematics curriculum district-wide reflected two administrative concerns: 1) improving educational equity for all children; and 2) increasing central administrative control over curriculum matters.

### Improving Educational Equity

The adoption of a city-wide mathematics curriculum was an attempt to equalize learning opportunities across schools, regardless of their size or location within the city. The Detroit School Board noted in a September, 1975, response to the Court's stipulation that the testing program would provide information on the comparability of student achievement across schools in the district. The comparison of MEAP scores showed considerable variation across schools (see Chapter 1). Children attending schools in some of the poorest neighborhoods of the city were scoring substantially below children who attended schools in the city's middle-class neighborhoods. The district was under considerable pressure to improve the quality of mathematics instruction for <u>all</u> children in the district. Adoption of the standardized curriculum announced to the community -- teachers, administrators, students, parents -- that every youngster in the city was entitled to quality instruction in mathematics. As a high level central administrator in the district commented:

> The city-wide curriculum says there are common goals for <u>all</u> students. Teachers can still tend to individual needs and teachers can still be creative...We are going along with a national movement to define a core curriculum for <u>everyone</u>.

The mandate to implement the curriculum city-wide was also a reponse to the problem of multiple transfers within the district. The dramatic increase in the ranks of the city's chronically unemployed and the forced moves of families within the city to secure temporary housing meant that many children might attend several different schools in one year. For many teachers this meant that a significant subset of their students changed over the course of the year. Some teachers reported that as many as half the students in their class might change in one school year. A third grade teacher in a combined elementary/junior high school told me of visiting a sixth grade room in her school that fall.

> I walked into her room and I looked around expecting to see some familiar faces. But there was not a <u>single youngster</u> in that classroom that I had had in my class just three years before. You see, nearly all our children are bussed in and many come from a low-income housing project. These poor kids move around from school to school.

The educational problems associated with a mobile student population was mentioned by every person at every level of the school organization that I interviewed. The hope was that standardization of curricula across the district would reduce the educational disruption for children who moved among schools during the year. In this respect, the district initiative was an attempt to deal with a set of educational problems rooted in social and economic inequalities within the larger community.

An egaliatarian impulse was not the only force driving the standardization of curricula. The move to mandate district-wide implementation of curricula in seven content areas reflected a continuing attempt by the central administration to exert increasing control over curriculum and instruction.

**Increasing Central Administrative Control Over Curriculum**

Over a period of nearly two decades, the district had undergone significant changes in organizational and authority structures. The decentralization of the district in 1970 had shifted considerable administrative authority to regional boards and the superintendents they appointed. Under the decentralized arrangement, regional administrators assumed control over curriculum and staffing decisions in their schools. The Court overseeing the federal desegregation order found the decentralized structure problematic. The member of the Monitoring Commission provided the following account.

> The Court and the Monitoring Commission saw decentralization as an obstacle. The Court was determined to deal with all the effects of segregation including educational programs which were different among Regions. For example, the reading programs were different in the Regions. With bussing imminent, the Court and the Commission felt the district needed to standardize the reading program.

The educational components stipulated in the 1975 federal desegregation order, to which the central board was obliged to respond, restored some administrative authority over educational programs to the central administrative level. Again, the Monitoring Commission member:

> The central administration at the time of the desegregation order didn't have much power. The Court order gave them the opportunity to consolidate power. The stipulation to include educational components gave the central administration the opening to take recommendations from earlier task force reports and propose them to the Court. They said, "We want these programs," and the Judge looked for a rationale to support them that was consistent with the

> intent of the order. Generally they wrote proposals that would get funding from the Feds. Reading, inservice and testing were going to get ESEA funding...Actually what they came in with was not a reading program. It was a reading management program, a testing program. The move to ORT (objective-referenced testing) was supported by the state and they got sufficient funding from ESEA and the state...Those programs that were most successful was where cooperation with Region administrators was solicited.

The desegregation order began the transfer of power over instructional programs from the regional to the central level. The recentralization of the district in 1981 further altered the balance of authority over educational programs but the transfer of power from Areas to central administration was not complete or automatic. There remained considerable ambiguity about what schools should be teaching and who should make those decisions. According to key personnel who led the way in the reform initiative, the decision in 1987 to adopt a standardized curriculum and mandate its implementation city-wide reflected an attempt to address two important issues; 1) providing educational leadership at the administrative level, and 2) defining the essential learnings in mathematics for grades K-5.

Providing educational leadership. One concern of central administrators was the lack of educational leadership exhibited by Area superintendents and school principals. There was the belief that these administrators lacked knowledge about curriculum matters. As a central administrator commented:

> Unfortunately, many or our (Area) superintendents and principals are not very knowledgeable about curriculum. They have seen their responsibility as administration and supervision, not instruction or curriculum. Principals never were expected to be great instructional leaders. We haven't articulated this expectation of principals.

With respect to mathematics, he offered:

> There has been a lot of resistance on the part of principals. They think math is computation. There is a lot of resistance to

> estimation, geometry and problem solving. They want to keep it simple--train' em. They would rather have teachers teach the black box. We have been getting a lot of flack on ABCS.

A district elementary mathematics supervisor put it this way:

> We had the ABCS objectives on paper but when we would go from one Area to another, we saw different things. One Area mandated this, another mandated that. Some Area superintendents had a reputation for insisting that their teachers teach to the tests. They put a heavy emphasis on improving MEAP test scores. Even some of the Area curriculum administrators had their math specialists preparing materials just aimed at the tests. And mostly that was at a computation level...Some teachers say to me, "I have been told to teach the test. You tell me I'm supposed to follow the instructional sequence. I'm told I have to spend one week on this skill, one week on that skill. Practice, practice, practice." Or, "My principal doesn't like the noise that's coming out of my room," when that is indeed good noise. That's a very legitimate complaint.

Key administrators and mathematics supervisors at the central level felt the educational leadership provided by some Area administrators and building principals had been inappropriate or inadequate. In the case of mathematics, they felt the dominant interest in improving test scores resulted in an inordinate emphasis on developing low-level computational skills while ignoring other important mathematical understandings. It was not the case that those promoting the new mathematics reform were not concerned about test scores. Indeed they were. However, their belief was that the new objectives, if followed, would not only better prepare children for the tests but would enhance their understanding of a broader range of mathematical ideas. By mandating implementation of a uniform curriculum city-wide, central administration was stepping in to take control of educational matters where it felt local leadership had failed.

Defining the fundamental learnings of mathematics. In the process of developing the ABCS test, those working on the mathematics portion identified nine "mathematical strands" in which all students should be competent. With this set of mathematical topics as an organizing framework, the team defined specific learning objectives for each strand. These objectives were made available to teachers several years before the curriculum initiative of 1987. Some teachers said they looked at the objectives occasionally as they prepared their children for ABCS but what they taught day-to-day was pretty much defined by what they found in their textbooks.

Textbook adoptions were centrally approved but selections for classroom use resided at the Area and individual school level. Given the considerable differences that might exist across textbook series for a particular subject matter, the city-wide adoption of texts did not insure a common orientation to the teaching of a content area. In the case of mathematics, the district had approved four textbook series for elementary classrooms--Harper & Row, 1982 edition, Holt, 1981 edition, Open Court, 1981 edition, and CEMREL. Both Harper & Row and Holt represented a fairly traditional approach to mathematics instruction. Open Court and CEMREL, however, were rather innovative programs that promoted a more conceptual orientation and emphasized problem solving and higher-order thinking. While the proponents of the curriculum initiative would have preferred to see Open Court or CEMREL used in classrooms, it was estimated that as many as 9 out of 10 teachers used Harper & Row or Holt. The mandate to implement a standardized curricula defined by "mathematical strands" and sets of behavioral objectives was an effort to specify what should be the outcomes of instruction in all schools, regardless of what text an individual Area or school chose to use.

The set of common objectives for all classrooms was also intended to define for teachers the fundamental learnings in mathematics. The high-ranking central administrator commented on teachers' subject matter knowledge:

> A lot of people don't understand the fundamental concepts of the subject matter they are teaching. The strands and objectives tell teachers what are the fundamental ideas that are the basis of the elementary mathematics curriculum. We have identified for them the fundamental learnings for K-5. Our teachers are good at teaching low level skills. But when you look at the failure of our kids on test scores at grades 4 and 5, the instruction may not be good enough. They (teachers) may not understand what they should be teaching. We have tried to help teachers here. Not only have we specified the objectives at each grade level, we have developed some lessons.

The elementary mathematics supervisor spoke of teachers' subject matter knowledge and their assumptions about what is appropriate for children to learn.

> Many teachers admit they don't have the knowledge. At one inservice, one teacher got really angry. She thinks estimation is too hard for first graders. She didn't understand what we were requesting. She didn't understand at what level we wanted to have kids doing estimation. The same goes for functions and relations and statistics and probability. Many teachers see those words and think children can't do that. But really, they have been doing a lot of this. One teacher was surprised to discover that the graphing she had her children doing was statistics. Calculators is another. We even get complaints from parents that their kids won't be able to add, subtract, multiply and divide.

Organizing the mathematics curriculum around "mathematical strands," defining the fundamental learnings for each strand with scores of specific objectives, and mandating the implementation of the instructional sequence in every classroom in the district was perceived as essential to improve the quality of mathematics instruction. The strands refocused the curriculum around a broader set of mathematical ideas. The curriculum objectives defined the outcomes of instruction. The instructional sequence provided the order in which the objectives were to be covered in the classroom. Considering the beliefs of administrators and supervisors about teachers' limited knowledge and understanding of mathematics, the decision to specify a uniform curriculum for all classrooms was understandable. But those who were prime movers in the initiative also expressed some reservations.

**The Dilemmas in Increasing Centralization and Standardization**

The centralization of decision-making about curricular matters created some dilemmas. The high-ranking central administrator worried about how teachers would implement the objectives in the classroom. He acknowledged the potential "risks" inherent in this curriculum reform agenda.

> There is the risk that implementation will just be mechanical and that we are taking away decisions of teachers.

The elementary mathematics supervisor worried about this as well.

> I worry that we are taking away too much of the creative juices of teachers. But I see this curriculum is intended as a support. For teachers who can do differently and are capable and can show me a plan and rationale, fine.

The central administrator expressed a desire for an approach that would provide for greater teacher decision making.

> What I would like to see is a limited number of objectives for each strand. These would not be the whole curriculum but the fundamental curriculum. I would like to see us not prescribe the learning activities. That would be the teacher's choice. They could draw on mastery learning, Madeline Hunter, Socratic questioning, whatever. I would like to see us write some objectives in a more constructivist manner. I would like to keep our function in the district at the level of general objectives - essential learnings - and let them (teachers) figure out how to do that. I would like to see us link this with a program of staff development. We really don't have any staff development agenda in the district.

At the same time he talked about efforts currently under way at the central administrative level to create whole sets of materials for instruction and evaluation for each objective.

> We are working on some other ways to help teachers who don't have the skills or the strategies or don't use them the way they ought to be taught. We are developing some lesson plans, resource materials and test items that are correlated among the curriculum objectives. If we are successful, teachers will be able to pull up on the screen (computer monitor) a specific objective, and with it a set of suggested resources, a lesson plan developed by teachers, and some test items. We know this can become very restrictive and controlling or it can be freeing. It depends on the use that is made of it. But I guess everything worth doing presents a dilemma.

Despite their concerns and perhaps misgivings about the extent to which the reform denied teachers the opportunity to make instructional decisions based on the real needs of their kids, the plan for the next three years was to work toward total implementation in every classroom. As the central administrator summed it up:

> The main thing that led to the decision to adopt a city-wide curriculum was disarray. There were so many different things going on that it was hard to help the schools. Maybe it's a function of size, maybe specialization. But others have had to step in and assume instructional leadership. When things aren't working, when the district is under pressure, there is the tendency to tighten up.

## SUMMARY

The initiative to reform the elementary mathematics curriculum in the Detroit Public Schools was situated in and grew out of a broader context of sociopolitical and organizational factors. Over a period of two decades, structural changes in the city's economy, intervention of the federal court in the schools through its desegregation order, decentralization, and then recentralization, of the administrative organization of the district had interacted to shape this curriculum reform agenda.

The stipulation of the court to include a testing component provided the occasion for key personnel at the central administrative level to influence the mathematics testing program to reflect the recommendations issued by the National Council of Teachers of Mathematics--expanding basic skills beyond computational proficiency and incorporating problem solving. The test then became the means to instigate curriculum change. Under different circumstances, the poor showing of students on the ABCS test might have resulted in abandonment of the test. However, the substantial support it enjoyed at the highest levels of the central administration produced another outcome. An instructional sequence was defined for each grade level to reflect the mathematical objectives tested by ABCS and was mandated for district-wide implementation as the K-5 mathematics curriculum.

The NAACP had filed suit in federal court on April 6, 1971, charging the Detroit schools with intentional and official racial segregation. Sixteen years later, as a direct outcome of the orders issued by the court in Bradley v. Milliken, the Detroit schools initiated a city-wide curriculum reform agenda.

The development and implementation of a standardized district-wide curriculum required a shift in the locus of power over curricular matters away from the decentralized Regions that had been created by state legislative action in 1969. The desegregation order facilitated a partial shift since it was the Central Board and administration that were required to respond to the court's orders creating the educational components. The decision by city voters in 1981 to recentralize the district provided the opportunity for further consolidation of power at the central administration level. With the power over curricular decision-making for the district in its hands, central administration used its authority to respond to a number of educational problems that resided within the system itself.

First, there was the belief that Area administrators and building principals lacked sufficient knowledge about curricular matters to provide proper educational leadership. Central administrators from a variety of departments stepped in and assumed leadership over what they perceived to be "disarray" and "chaos" in educational programs. Second, it was widely assumed that many elementary teachers did not understand the central ideas of the subjects they were teaching, especially mathematics. Mathematics instruction had emphasized computational proficiency and basic skills to improve test scores to the exclusion of teaching a broader range of content. Teachers tended to teach what came next in the text. The city-wide curriculum represented an attempt to make explicit to Area and building staff the fundamental concepts and the essential learnings at each grade level. The instructional sequence was intended to discourage teachers from using the traditional texts that dominated classrooms as the sole determinant of what they would teach and what children would have an opportunity to learn.

The mandate to implement a uniform curriculum in all classrooms in the city was also an attempt to deal with a set of educational problems rooted in social and economic inequalities within the larger community. The social dislocation that resulted from structural changes in the Detroit economy was felt by the schools. It was hoped that the standardization of curricula would equalize learning opportunities across schools, regardless of their location within the city, and would assure that students who transferred during the year could do so with little disruption in learning.

Several administrators expressed some reservations about the effect a reform of this nature would have on teacher creativity and professionalism. But at the same time, the dominant concern seemed to be standardization. And that concern went beyond defining curricular objectives. Teachers had been given an instructional management system in reading. They were required to keep a profile sheet for each student on which they were to record each reading competency mastered. The district was deeply invested in an instructional and classroom management model that specified a set of generic teacher behaviors that, it was claimed, would lead to effective and efficient instruction in any classroom. The district had committed to put every staff member through 30 hours of training on those behaviors and how they should be implemented. The mathematics supervisor promoted a model of "effective mathematics instruction" that incorporated the generic teacher behaviors with a model for "active mathematics teaching."

The explanations given by key personnel for mandating a city-wide elementary mathematics curriculum were certainly reasonable. But there was another possible explanation that merits consideration. Given the extent of efforts to standardize educational practice through mandated curricula, an instructional model, testing programs, and management systems, I would posit that these initiatives were also designed to serve administrative convenience and accountability. For example, the set of teaching behaviors associated with the instructional model in which staff were trained was used regularly by school administrative and supervisory personnel when they observed teachers in their classrooms. The behaviors had become a criteria by which teacher effectiveness was assessed. These criteria had the potential to become powerful regulators of teacher and student behavior during instruction. The instructional sequence that defined the mathematics curriculum held the same potential to regulate teacher behavior and student learning. It appeared that decisions about curriculum and instruction were increasingly made at higher levels in the system hierarchy with little input from the classroom teachers charged with implemention.

The education director of the civic leadership organization, a proponent of investing greater authority for decision-making at the local school level, was highly critical of what he called the administration's "lock-step" approach to education.

> The district has a reponsibility to define goals and objectives for all its children and young people. But these behavioral objectives have nothing to do with the real needs of kids. It has to do with the need to quantify, the public relations need to show numbers like improved test scores. This district has got to trust the profession to make appropriate decisions. Right now, the way things are handed down to teachers, there is no sense of ownership and therefore no sense of responsibility. The administration says, "We know what's in the best interest of our children so we will make the curriculum city-wide." Their effort to put into lock step a uniform curriculum and an instructional practice denies teachers at the local school level the right to decide how they can get to where they need to go.

In the chapters that follow, the inquiry will examine in detail the content of the mathematics curriculum reform and the nature of inservice provided for teachers charged with its implementation.

# CHAPTER 5

# THE FORMAL CURRICULUM DOCUMENTS

## INTRODUCTION

The city-wide elementary mathematics curriculum that had been adopted by the Detroit Public Schools was described by those most intimately involved in its development as "conceptually-based." This chapter examines how teaching for conceptual understanding was represented in the formal curriculum documents that had been adopted by the district and distributed to its teachers. The formal curriculum documents were of particular interest. First they represented a choice of knowledge selected from a much larger universe of knowledge (Apple, 1979). Curriculum materials described what it was that children should learn in the study of mathematics. They also represented the discipline itself through the selection of content and the organization of that content. That is, curriculum documents represented the substance and the nature of the discipline to those who used them. The inquiry undertaken here examines the explicit and implicit goals that were the foundation of the formal curriculum documents, the mathematical strands that broadly defined the curriculum content, and the behavioral objectives that organized and sequenced that content for instruction.

This chapter is divided into three sections. The first section describes the district's goals for mathematics education and the ten mathematical strands that organized the K-5 curriculum as elaborated in the document, <u>Detroit Public Schools Pre-kindergarten-Grade 12</u>

Curriculum: Strands & Objectives - Mathematics Education (Detroit Public Schools, January, 1987).

The second section examines in detail the objectives in the Grade 3 curriculum. This grade level was chosen for several reasons. In Grade 3 children typically study the four basic operations on whole numbers and the use of conventional algorithms to perform computations. Often the content is a recycling of processes introduced in earlier grades but applied to larger numbers. The goals of instruction are usually weighted toward children becoming proficient in symbol manipulation. The introduction of new ideas appears late in the school year (if at all). An examination of Detroit's Grade 3 curriculum reveals the extent to which this district-wide curriculum reform aimed to develop conceptual understanding of whole number operations. Further, a close examination will show the extent to which the curriculum represented a reshaping of content over what had been typical fare. This section is divided into three parts. One interrogation examines a subset of the Grade 3 objectives to explore the extent to which they were linked to enhance conceptual understanding of numbers, numeration and operations with numbers. The second interrogation examines the entire set of objectives, attending to the verbs that described student actions to determine the degree to which skill proficiency, interpretation and application of information, or mathematical inquiry were emphasized. In the third instance, the examination focuses on the relative emphasis placed on each strand at Grade 3 and how the sequence of objectives portrayed the development of new ideas over time.

The third section in this chapter examines the Grade 3 textbook most commonly used in the district to assess the extent to which the text was aligned with and supported the Instructional Sequence.

**The Analytic Framework**

In an earlier chapter I proposed a framework for analyzing the orientation of this curriculum reform. Within an orientation of **certification of numeracy**, arithmetic constitutes the bulk of topics studied. The goal of instruction is computational speed and accuracy and proficiency with conventional algorithms. Mathematics is portrayed as a system of abstract symbols to be manipulated and a collection of skills, definitions, rules and procedures to be memorized and mastered.

An orientation that aims for **comprehension of mathematical ideas** emphasizes understanding procedures -- knowing how to use them and why they work. Mathematics is represented as an abstract system of concepts and procedures for which there are appropriate concrete models and pictorial representations. Arithmetic continues to dominate the curriculum but is supplemented by topics from geometry, statistics and probability, and number and set theory.

A mathematics curriculum centered on **enabling mathematical inquiry** has an explicit goal of empowering students -- to become active participants in the construction of mathematical knowledge, to make sense of mathematical situations, to engage in doing mathematics, to communicate with others about mathematical ideas, and to use the power of mathematics to understand the world around them. The content is "chunked" around key mathematical ideas and problem situations provide the context in which students engage in inquiry.

These orientations are ideal types and any mathematics program is likely to contain elements of more than one. The intent in this chapter is to determine the orientation of this curriculum reform effort as it was represented in the formal curriculum documents.

## THE K-5 MATHEMATICS CURRICULUM

### Goals for Students as Learners of Mathematics

In January, 1987, the Office of Curriculum Development and Services presented to the Superintendent and School Board a document that defined the strands and objectives in mathematics education for all Detroit Public Schools. The booklet, Detroit Public Schools Pre-kindergarten-Grade 12 Curriculum: Strands & Objectives - Mathematics Education, contained a statement of philosophy regarding mathematics education. The statement is presented here in its entirety to demonstrate the ways in which the district viewed mathematics education and the goals for students as learners of mathematics.

> Never before has a generation of Detroit students had a greater need for enjoyable, understandable, and relevant experiences in mathematics. The primary focus of mathematics instruction in Detroit Public Schools is to improve the ability of students to think systematically and apply their knowledge of mathematics in solving problems. The advent of calculators, computers, and other high technological advances is making it necessary for students to develop a facility in mathematics far superior to that possessed by previous generations. Space exploration, international competition and fiscal economics have created new and eliminated old jobs and are challenging the youth of tomorrow to think critically, flexibly, cooperatively, and independently. The quest for new medical breakthroughs complemented with the wonders of microelectronics challenges our youngsters to develop, to create, and to analyze. The rapidly changing routine procedures for everyday experience calls for effective problem solving to lead a successful life.
>
> A mathematics program built on success looks to the future and provides background experiences necessary to prepare children for varied career choices. It prepares students to function effectively, intellectually and independently in all areas of life.

> It provides a multitude of experiences to foster a positive attitude towards mathematics and provides exposure and enrichment in geometry, measurement, estimation and other basic skill areas in mathematics.
>
> The program's strengths lie in providing concrete aids and/or models for young people to gain understanding and to release student potentials by developing higher order thinking skills of analysis, synthesis, and evaluation. The mathematics program spirals concepts and promotes pupil proficiency in all basic skill areas from pre-kindergarten through grade twelve.

This statement described a set of goals for students as learners of mathematics that seemed to be aimed at helping them develop their mathematical knowledge and power. This was evidenced in the following passages: "to think systematically and apply their knowledge of mathematics in solving problems...to think critically, flexibly, cooperatively and independently...to develop, to create, and to analyze...to foster a postive attitude towards mathematics...to gain understanding...by developing higher order thinking skills of analysis, synthesis, and evaluation."

It is of note that these expanded goals for a reformed mathematics curriculum were tied almost exclusively to technological revolution, competition in the international marketplace and structural changes in the U.S. economy. For example, "The advent of calculators, computers and other high technological advances is making it necessary for students to develop a facility in mathematics far superior to that possessed by previous generations...Space exploration, international competition and fiscal economics have created new and eliminated old jobs...The quest for new medical breakthroughs complemented with the wonders of microelectronics challenges our youngsters." The context for reforming mathematics was technological advance and the increased use of quantitative methods in business, industry, and social, biological and physical sciences. This is not to suggest that our information-age, advanced manufacturing economy should not have an influence on the shape of mathematics education in the schools. Indeed it should. Mathematical literacy is becoming essential for a growing number of occupations within the new economy, particularly in the widening sector of knowledge workers. Mathematics is practical and utilitarian to the extent that it trains a workforce. But there are other reasons beyond this key one for transforming the elementary mathematics curriculum and mathematics instruction. The goals were not linked to the explosion of mathematical knowledge that is altering conceptions of what fundamental knowledge should be selected and emphasized for all learners.[1] The goals of engaging in mathematical inquiry, of understanding the historical and cultural evolution of mathematics, of learning to reason and communicate mathematically, of becoming confident in one's ability to _do_ mathematics (see, for example, Commission on Standards for School Mathematics, 1989) were absent from the District's philosophy.

**The Curriculum Strands and Objectives**

The Detroit Public Schools mathematics curriculum was organized around ten mathematical strands that were spiralled within and across the grades. The strands were described in _Detroit Public Schools Pre-kindergarten-Grade 12 Curriculum: Strands & Objectives - Mathematics Education_ (Detroit Public Schools, January, 1987) as "major concept themes whose objectives increase in scope and complexity" from Pre-

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[1] Davis and Hersh (1981) claim that over half of all mathematics has been invented in the last half-century.

kindergarten to Grade 5. The curriculum strands, as defined in this document, are presented below.

CALCULATORS and COMPUTERS enables students to use modern technologies to solve problems at a level of difficulty beyond paper and pencil skills.

ESTIMATION and APPROXIMATION helps students to make intelligent judgmental decisions when given situations where exact answers are not necessary. This includes rounding, checking for reasonableness and using mental arithmetic strategies to formulate estimates.

FUNCTIONS and RELATIONS empowers students to form comparisons of set elements or numerical quantities that have defined relationships.

GEOMETRY permits students to organize, construct, and apply abstract concepts to physical models in the real world.

MEASUREMENT equips students to relate numerical quantities to physical models.

NUMERATION allows students to form understandings of the structures of number systems.

OPERATIONS provides students with facility in computational skills.

PATTERNS simplifies problems enabling students to see relationships between visual and abstract information.

PROBABILITY and STATISTICS helps students read, interpret, make simple tables, charts, graphs and to use mathematics to determine the likelihood of future events.

SETS and LOGIC assists students to organize and interpret information, to use sequential thinking, and to draw conclusions in solving problems that arise in an ever changing society.

The organization of the curriculum around central mathematical themes as well as the inclusion of some of these strands represented a clear departure from the traditional emphases of the elementary mathematics curriculum. The range of content that comprised the newly adopted curriculum -- estimation, operations, measurement, geometry, statistics and probability, patterns, relations and functions, and sets and logic -- was consistent with recommendations from the National

Council of Teachers of Mathematics to reshape the content and refocus the intent of the elementary mathematics program (Commission on Standards for School Mathematics, 1987). However, the language used to describe the strands in this document seemed more symbolic than mathematical. Each strand was elaborated with a statement of a student outcome. In some cases, the outcome was quite narrow. For example, the operations strand was defined as "provid(ing) students with facility in computational skills." Yet, an understanding of operations involves considerably more--knowing properties of and relationships among operations, having a sense of the effect of operating on numbers, and recognizing real-world situations that exemplify an operation. The measurement strand "equips students to relate numerical quantities to physical models." But the study of measurement should also help students to understand attributes that one measures and the process of measuring. The description of each strand did not capture the range of mathematical ideas -- concepts, skills, procedures -- to be explored, the ideas to be emphasized, and the multiple ways in which students might encounter these ideas.

In addition to representing the ten "central themes" rather narrowly, the elaboration of each strand did not suggest the interrelatedness of the themes. For example, estimation is essential to children's understanding of measurement but the statements for these two strands did not indicate this linkage. Geometric ideas contribute to the development of number and measurement concepts but these connections were not made explicit. Patterns -- the regularities that are the essence of mathematics -- are central to the ideas of number, geometry and measurement, but as in the other cases, linkages among these strands were, at best, ambiguous.

Written texts serve varying purposes. These strand statements were both political and symbolic in that they defined for the school community a new framework around which the elementary mathematics curriclum was to be organized. However, they served a limited mathematical purpose. They defined a set of general student outcomes but they provided little sense of the range of mathematical ideas embedded in each strand, how those ideas might be connected, and why those linkages might be important. One place to examine the mathematical content of these curriculum strands is the set of grade level objectives. The next section provides an examination of the City-Wide Instructional Sequence for Grade 3.

## GRADE 3 INSTRUCTIONAL SEQUENCE

### Objectives for student learning about numbers, numeration and operations with whole numbers

Traditionally arithmetic has constituted the bulk of the elementary mathematics curriculum. The central aim of instruction has been proficiency - accuracy and speed - with computational algorithms and their application to routine word problems. Recent calls to reform mathematics education have advocated a revision in teaching the concept of whole number operations in the elementary curriculum. The National Council of Teachers of Mathematics (NCTM) has recommended that considerably less time be spent on drill and practice on paper-and-pencil computations. Instead there should be an expanded emphasis on developing the concepts of addition, subtraction, multiplication and division--the properties and relationships of each, the relationships among them, and problem situations in real-world contexts that exemplify the operations. In addition, considerable time should be spent developing children's informal experiences with the operations, providing opportunities to use concrete materials and invent techniques to solve problem situations before introducing symbolic work. Clearly computation should continue to have a key place in the elementary mathematics curriculum, but instruction in computation should emphasize a variety of algorithms, mental arithmetic strategies, estimation and the use of calculators. The move should be toward conceptual understanding of whole number operations and away from memorizing rules for carrying out the conventional algorithmic procedures (Commission on Standards for School Mathematics, 1987, 1988, 1989). This section examines a subset of the Grade 3 objectives to determine the extent to which the organization and presentation of these objectives was consistent with the NCTM recommendations for reduced attention to paper-and-pencil computations and increased attention to developing children's number and operation sense.

The Grade 3 instructional sequence was composed of a series of behavioral objectives organized in sets under one of the ten strands.[2] Strands were revisited several times over the course of the school year and occasionally a particular objective was revisited. A total of 116 objectives constituted the Grade 3 curriculum. Of these, 55 (47%) were devoted to developing ideas about number, numeration and operations with whole numbers. Figure 5.1 shows the strands devoted to these

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[2]The City-Wide Instructional Sequence of Grade 3 appears in the Appendix.

concepts, an example of an objective from each strand, and the number of objectives in that strand that addressed these ideas.

The objectives in these strands fell into four categories: (1) reading, writing and computing with whole numbers; (2) demonstrating the meaning of operations and the effects of operating on whole numbers; (3) understanding relationships among numbers; and (4) applying operations to solve word problems. The first category, reading, writing and computing with whole numbers, included writing numbers in standard notation, demonstrating place value, and recalling facts and using the conventional algorithms with increasing speed and accuracy. Nearly half the objectives related to developing number and operation ideas (25 of the 55) were concerned with increasing computational proficiency.

The second category, demonstrating the meaning of operations and the effects of operating on numbers, included using properties in computation,[3] relating addition and subtraction as inverses, relating multiplication to repeated addition and division to repeated subtraction, and applying a given rule to a set of numbers to determine the missing input or output. Less than one-third of the objectives devoted to whole numbers (16 of 55) were aimed at developing conceptual understanding of whole number operations. There was, however, some ambiguity about several of these objectives. For

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[3]This included using the property of zero in addition and subtraction and zero and one in multiplication, and applying the commutative and associative laws to addition and multiplication.

| STRAND | EXAMPLE OF AN OBJECTIVE | NUMBER OF OBJECTIVES |
|---|---|---|
| NUMERATION | Demonstrate accuracy in reading and writing whole numerals through 999.<br><br>Identify numbers as multiples of 2, 3, 5 or 10. | 17 |
| FUNCTIONS AND RELATIONS | Interpret and complete addition, subtraction or multiplication number sentences using one of these symbols, $<$, $>$, $=$, $\neq$. | 8 |
| OPERATIONS | Recall the addition and subtraction facts through sums of 18 with or without the use of aids (pictures, number lines).<br><br>Apply addition, subtraction, multiplication or division strategies to solve money or other word problems. | 17 |
| PATTERNS | Interpret and complete a number pattern (using numbers 0-99; starting at any number) in ascending or descending order counting by ones, twos, fives or tens. | 4 |
| ESTIMATION | Round any two-digit number to the nearest multiple of ten.<br><br>Round the individual problem numbers to the nearest multiple of ten or hundred and estimate the sum or difference. | 5 |
| CALCULATORS AND COMPUTERS | Check computation using a calculator.<br><br>Predict number to be displayed and verify its results. | 4 |

Figure 5.1 The Strands and Objectives Devoted to Number, Numeration and Operations with Whole Numbers

example, the objective "apply a given rule to a set of whole numbers and determine the missing whole number output" could be met in a way that provided an opportunity for students to explore the effects of operating on a set of numbers and the patterns and relationships that emerge. On the other hand, this objective could simply provide another occasion for computational drill and practice. How this objective was treated in instruction was, in part, a function of its presentation in textbooks, a matter that will be examined in a later section of this chapter.

The third category, understanding relationships among whole numbers, included identifying rules when given input and output numbers, identifying numbers as odd or even and as multiples, ordering and classifying numbers, completing number patterns, and using mental arithmetic strategies. Ten of the 55 objectives fell in this category. It is of note here that the objective "apply mental arithmetic strategies to solve addition, subtraction, multiplication or division problems" was met only once and that was in late May. One might have expected a consideration of mental arithmetic strategies earlier in the school year to help children develop more efficient ways to recall basic facts and answer fact problems, particularly given the emphasis at this grade on computational proficiency.[4]

The fourth category was composed of one objective that was visited four times during the year: "apply addition, subtraction or multiplication strategies to solve money, time or other word problems

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[4]During inservice conducted by a district mathematics supervisor, teachers were urged to devote the first five minutes of each class to mental arithmetic activities that might or might not be connected to the topic of the day's lesson.

(one or two steps)." Work on word problems was not integrated throughout the curriculum. Instead, word problems tended to come after a series of objectives devoted to computational proficiency. The hierarchical arrangement of the objectives suggested that work with problem solving was appropriate only after children had mastered basic facts.

These objectives reflected a tension between mastering basic facts to enhance computational proficiency and developing conceptual understanding of place value and whole number operations. Two objectives called for relating "the inverse of addition and subtraction" and "the inverse of operations." Yet children were required to learn nearly two-hundred subtraction and division facts by Grade 5. At third grade, children were expected to "state the subtraction facts through sums of 18." At fourth grade, they were expected to "state the 90 basic division facts." By Grade 5 children were expected to recall these facts "with greater accuracy and speed." It appeared that memorizing hundreds of facts for addition, subtraction, multiplication and division took precedence over using the concept of inverse to derive subtraction facts from addition facts and division facts from multiplication facts. Learning conventional algorithmic procedures to enhance computational proficiency seemed to be an end in itself. No objective suggested that students might explore or invent alternative algorithms. Long multiplication -- "three digit times a two digit" -- and long division -- "4 digit dividends by 2 digit divisors" -- were practiced even though the curriculum stressed the importance of the calculator in the elementary mathematics classroom.

The tension was also reflected in the lack of integration of calculator objectives throughout the sequence. Calculators and computers was one of the ten mathematical strands of the curriculum. This suggested that calculators would be used throughout the study of mathematics. There were four objectives devoted to the use of calculators in the Grade 3 sequence. These objectives appeared once, consecutively at the end of January. Three of the objectives had children use the calculator to solve problems, check the accuracy of predicted answers, determine the reasonableness of displayed answers and check paper-and-pencil computation. With the fourth, children used the calculator to predict a number to be displayed. The single appearance of these four objectives was curious, leaving one to wonder about the extent to which calculators were integrated in the curriculum. Only one of these objectives exploited the power of the calculator as an investigative tool to explore number concepts. It might be argued here that what is not explicitly stated is implied. The matter of whether and to what extent calculators were integrated into the curriculum will be revisited in the next chapter when another source of evidence, the Model Lessons, is examined.

Few of the objectives devoted to numbers, numeration and operations with whole numbers were concerned with decision-making. Where decision-making was involved, the process was mechanistic. For example, the exclusive focus of the estimation objectives was having students apply a rule to round numbers to the nearest ten or hundred, use the estimated numbers to calculate sums and differences, and then use that answer as a check of the reasonableness of the "real" answer.[5] Making judgments about whether a situation required an exact answer or if an approximation would be sufficient was not considered. Deciding whether and under what conditions calculations could be performed more efficiently mentally, with paper-and-pencil, with calculators or computers was not addressed in any objective.

The objectives were only statements. They did not assure or even predict the behavior of teachers, the performance of students, or the

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[5]The potential difficulties embedded in this mechanistic, rule-bounded approach were evidenced in a demonstration lesson conducted by a district mathematics specialist in a third grade classroom. The lesson involved rounding individual problem numbers to the nearest multiple of ten and using estimation to check the reasonableness of an answer. The first example the specialist demonstrated at the board was 42 + 59. Children estimated 42 as close to 40 and 59 as close to 60 and said the sum should be close to 100. They computed the sum of 42 and 59 and got 101. The specialist asked, "Do you think 101 is a good answer?" The children answered in unison, "Yes." She asked one youngster to explain. "Because 101 is close to 100," was the child's response. After several more examples, the specialist assigned problems from the textbook with the directions to "find the estimate before you do your problem." Almost immediately, a youngster found a problem that "didn't work" and asked for help. The specialist put the problem, 26 + 27, at the board. Children estimated 26 as close to 30 and 27 as close to 30 and the sum as close to 60. Then they added 26 and 27 getting 53. "Is 53 a good answer?" the specialist asked. The youngsters replied loudly in unison, "NO." Seeming surprised, the specialist said, "Sometimes estimates are not real close. This happens when these numbers (circles 6 and 7) are close to five. Estimation isn't a sure fire thing. I would say, 'This isn't close, maybe I should go back and check.'" She then turned to the classroom teacher and said, "I try to stay away from ones like this but they keep creeping in."

The children had been taught a rule and a procedure and were applying it as they had been shown. But sometimes it "didn't work" like it was supposed to. The specialist's response to the children was simply to say that it wasn't a "sure fire" rule. After the class, I asked her about a process that did not always "work." Her concern was not about how to help children understand the limitations of the process in deciding if answers are "good." Instead, she said to me, "I should have looked more carefully at the problems in their book before I gave them the practice. I wouldn't have given them the ones that didn't work." Her concern was to do a better job of planning so that children would not encounter situations that defied the rule.

actual life of the classroom. The claim here is not that these objectives -- the limited mathematical goal of each one individually, the manner in which they were sequenced, or the absence of objectives that would contribute to further understanding of numbers, numeration and operations with whole numbers -- precluded teachers from pursuing an expanded, linked set of objectives and goals. Rather, the argument is that these objectives represented a view of mathematics and an approach to mathematics teaching that atomized content, isolated computational proficiency from problem solving contexts, and ignored decision-making and judgment.

These objectives are but one source of evidence to bring to the question of the orientation of this curriculum reform initiative and the degree to which it might foster conceptual understanding and higher order thinking in learners of mathematics. I turn next to the entire set of Grade 3 objectives to examine the verbs that described student actions.

**Verbs That Described Student Actions**

The second interrogation of the Grade 3 curriculum involves an examination of the verbs used in the objectives to describe student actions. A mathematics curriculum that aims to develop higher order reasoning and conceptual understanding provides opportunities for students to explore problem situations, make conjectures, validate assertions, and communicate the results to others. Problem situations provide the context in which students build abstractions and generalizations about mathematical properties and relationships. To determine the extent to which the Grade 3 instructional sequence promoted conceptual understanding and higher order reasoning, the entire set of objectives was analyzed.

The schema used here represents an alteration of the categories proposed by Avital and Shettleworth (1968) as described in Chapter 3. Three categories or levels of actions are described. These levels are distinguished by the degree to which skill proficiency, interpretation and application of information, or mathematical investigation and inquiry are emphasized. Low-level objectives aim at skill proficiency and emphasize recall of basic number facts, speed and accuracy in conventional algorithmic procedures, and a reliance on memory to attain the objectives. Examples of verbs that describe student actions at this level would include read, identify, name, recall, recognize, add, subtract, multiply, divide, state the rule, and demonstrate accuracy. Mid-level objectives require the application of mathematical information. Examples of verbs that describe student actions at this level include relate, extend, demonstrate, complete, estimate, measure, compare, apply strategies and determine reasonableness. High-level objectives emphasize mathematical investigations and open searches. At this level student actions would include investigate, explore, predict, invent, and generalize. Figure 5.2 indicates the number of Grade 3 objectives that fell into each category.[6]

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[6]There is some degree of interpretation that has gone into placing these objectives within categories. A single verb was not always the determining factor. For example, "identify a circle graph" and "identify and state the rule which indicates the relationship between whole numbers when given input or output" seemed to be objectives aimed at different levels of thinking, the former a matter of recognition, the latter a matter of application. And as noted earlier, some objectives could be met as simply occasions for additional drill-and-practice of computational algorithms rather than opportunities for application to nonroutine situations. Where there was some ambiguity, I placed the objective in the higher category.

| Level of actions | Number of objectives |
|---|---|
| Low level - Skill proficiency | 58 (50.0%) |
| Mid level - Application | 52 (44.8%) |
| High level - Open searches and investigation | 6 ( 5.2%) |

Figure 5.2 Number of Objectives by Category of Level of Actions

Half the Grade 3 objectives were devoted to memorization and recall of basic number facts, computational proficiency with conventional algorithms and manipulation of symbols. Most of the remaining objectives aimed at application. Here students constructed graphs, classified numbers and objects by their attributes, completed patterns of numbers and objects, estimated measures of length and weight, related and compared units of measure, investigated ways to use a hand-held calculator, applied addition, subtraction, multiplication and division strategies to word problems, demonstrated the relationship between operations and compared fractional parts of numbers and objects. Only six objectives approached fitting the category of investigation. In these instances, students were expected to interpret graphs and interpret and create patterns of numbers and objects.

None of the objectives captured the spirit of open-ended inquiry where students make conjectures, validate assertions, abstract generalizations or invent procedures. Content was fragmented. What was stressed was the individual objective that was to be accomplished in a day, or at most two. Even where several objectives appeared in sequence under one strand, they did not support open searches. For example, the first four objectives in the month of September were related to probability and statistics. They required students to "1) read and interpret a pictograph using ratios of 1:1 and 1:2 using a key; 2) read and interpret a bar graph using calibrations of units of 1, 2, and 5, using a key; 3) construct a bar and pictograph using data with ratios: 1:1 and 1:2; 4) identify a circle graph." None of these objectives involved collecting and organizing data or making decisions about appropriate displays for different types of data. There was no objective devoted to formulating questions and solving problems that would involve collecting and analyzing data. Action verbs such as "investigate, explore, predict, make decisions and discuss findings" were absent from this set of objectives. It was a significant omission because the emphasis on reading and interpreting graphs ignored the importance of using statistical ideas as tools to solve problems and to describe and interpret the world around children.

The absence of objectives that are explicit about opportunities for open-searches does not preclude such explorations in classrooms. How these objectives are met in the classroom is a function, in part, of how these ideas are treated in textbooks and how teachers interpret the range of concepts, skills and procedures embedded in each objective. In a subsequent section I will examine the most commonly used textbook to determine if activities were included that asked students to generate questions about which they might collect data, collect data, make decisions about appropriate displays, and use data to make decisions or predictions.

**The Relative Emphasis Placed On Each Strand And The Development Of New Ideas**

A second way to interrogate the entire set of Grade 3 objectives to determine the orientation of this curriculum is to examine two related notions: 1) the relative emphasis placed on each major theme as represented by the distribution of objectives among the ten strands and 2) the extent to which the curriculum included new content and how these new ideas were developed over time.

The distribution of objectives among the strands. The strands included content that traditionally had not been emphasized in the elementary school mathematics curriculum. The ideas of functions and relations had certainly been embedded in elementary mathematics, but making these concepts one of the central topics of instruction was a departure from the traditional curriculum. Patterns, probability and statistics, and estimation and approximation had been given the status of central themes of the curriculum. Figure 5.3 describes the number of objectives in the entire Grade 3 sequence devoted to each curriculum strand. The interest here is to determine the extent to which this curriculum represented a reshaping of content from the traditional emphasis on arithmetic.

Two strands, numeration and operations, were devoted to developing an understanding of place value and the meaning of operations, to reading, writing and ordering numbers symbolically and to developing computational proficiency. The following objectives were

| Curriculum Strand | Number of Objectives |
|---|---|
| CALCULATORS AND COMPUTERS | 5 ( 4.3%) |
| ESTIMATION | 6 ( 5.2%) |
| FUNCTIONS AND RELATIONS | 11 ( 9.5%) |
| GEOMETRY | 5 ( 4.3%) |
| MEASUREMENT | 22 (18.9%) |
| NUMERATION | 21 (18.1%) |
| OPERATIONS | 24 (20.7%) |
| PATTERNS | 8 ( 6.9%) |
| PROBABILITY AND STATISTICS | 6 ( 5.2%) |
| SETS AND LOGIC | 8 ( 6.9%)[7] |

Figure 5.3 The Distribution of Grade 3 Objective Among the Curriculum Strands

[7]The objectives at the other grade levels showed a somewhat similar distribution.

| Strand | Number of Objectives | | | |
|---|---|---|---|---|
| | Grade 1 | Grade 2 | Grade 4 | Grade 5 |
| Calc. & Comp. | 2 ( 2.1%) | 5 ( 5.6%) | 6 ( 4.5%) | 8 ( 6.2%) |
| Est. & Approx. | 2 ( 2.1%) | 5 ( 5.6%) | 5 ( 3.8%) | 6 ( 4.6%) |
| Func. & Rel. | 8 ( 8.3%) | 10 (11.2%) | 12 ( 9.1%) | 14 (10.8%) |
| Geom. | 5 ( 5.2%) | 7 ( 7.9%) | 6 ( 4.5%) | 7 ( 5.4%) |
| Meas. | 15 (15.6%) | 10 (11.2%) | 18 (13.6%) | 21 (16.2%) |
| Numer. | 34 (35.4%) | 21 (23.6%) | 26 (19.7%) | 20 (15.4%) |
| Oper. | 15 (15.6%) | 12 (13.5%) | 38 (28.8%) | 29 (22.3%) |
| Patt. | 8 ( 8.3%) | 6 ( 6.7%) | 7 ( 5.3%) | 6 ( 4.6%) |
| Prob. & Stat. | 4 ( 4.2%) | 5 ( 5.6%) | 6 ( 4.5%) | 8 ( 6.2%) |
| Sets & Log. | 3 ( 3.1%) | 8 ( 9.0%) | 9 ( 6.8%) | 11 ( 8.5%) |

The large number of objectives for operations at Grade 4 reflected a total focus during September to prepare students for the state-wide MEAP test administered in October.

typical: "demonstrate accuracy in reading and writing whole numerals through 999;" "interpret and/or illustrate the meaning of whole numbers through 999;" "subtract whole numbers through 3 digits (with and without regrouping);" "demonstrate the meaning of multiplication as repeated addition;" "recall the multiplication facts through 9 times 9;" "identify a number that is 10 or 100 more or less than a given number;" "sequence up to 4 counting numbers, least to greatest or greatest to least;" "apply the property of zero to addition and subtraction." Several of the objectives in these strands called for the use of physical aids, concrete models and pictures as ways to represent the mathematical ideas in the lesson: "determine and/or demonstrate place value through 999 using aids;" "recall the addition and subtraction facts through sums of 18 with or without the use of aids (pictures, number line);" "identify positive and negative integers on a number line;" "determine one-half of an even whole number through 18; even multiples of 10 and 100, with or without aids."

In addition, several objectives from other strands seemed to be aimed at computational proficiency. One objective in the measurement strand called upon students to "identify, compare, write and compute money values through $9.99." An objective of the calculator and computer strand called upon students to "check computation using a calculator." Under functions and relations, students were to "apply a given rule to a set of whole numbers and determine the missing whole number output" and "identify the relationship of a pictured sum of money to a given value in a problem involving money through $9.99." Assessing the reasonableness of results of computation was relegated to the mechanical process of rounding off addends and finding their sums:

"round the individual problem numbers to the nearest multiple of ten or hundred and estimate the sum." Two objectives from the patterns strand represented a form of skip-counting: "interpret and complete a number pattern (using numbers 0-999; starting at any number) in ascending or descending order counting by ones through sixes, tens and hundreds." In all, 59 of the 116 objectives (50.8%) were devoted to place value understanding, symbol manipulation and computational proficiency with whole numbers, fractions and decimals.[8]

Geometry was given scant attention in the Grade 3 sequence. The five objectives in this strand focused narrowly on naming, identifying and drawing shapes, knowing their characteristics, and memorizing terms and symbols: "identify geometric figures, symbols and their written words: square corner, point, ray, line segment, line and angle;" "identify and recognize cube, cone, cylinder and sphere by word names and models in the everyday environment;" "draw points, line segments, rays, angles and lines using appropriate tools;" "identify and reproduce lines of symmetry." None of the objectives was designed to foster development of students' spatial sense - how shapes are related, the effects of changes made on shapes as they are rotated and flipped, or as shapes are combined.[9] There were no objectives to help students

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[8]Analysis of the objectives for Grades 4 and 5 revealed a similar concentration: 66 of the 132 Grade 4 objectives (50.0%) and 66 of the 130 Grade 5 objectives (50.8%) were devoted to symbol manipulation and computational proficiency with whole numbers, fractions and decimals.

[9]At Grade 4, 6 of the 132 objectives were devoted to the geometry strand, at Grade 5, 7 of 130 objectives. At these grades, there were a few objectives devoted to developing spatial sense. Grade 4: "complete a drawing applying the concept of symmetry." Grade 5: "use geometric tools to construct circles, angles of a given measure or other known geometric figures;" "identify and relate two dimensional shapes to three dimensional figures;" "construct models of solids;" and "identify two dimensional pictures of three dimensional shapes." No

see how geometrical ideas can contribute to an understanding of measurement and number concepts.

Some objectives in the measurement strand seemed to be aimed at developing an understanding of the attributes of length and weight. Linear measure was introduced by using non-standard units and then progressing to standard units. One objective called for using non-standard units to measure the weight of objects. Several objectives appeared to be aimed at deciding on appropriate units of measure depending on the size of the object to be measured: "relate and compare metric units of measure, centimeter and meter;" "identify the reasonable customary or metric unit to measure the length of an object, height or distance;" "identify the reasonable customary or metric unit to measure weight (mass)." Estimation of quantities was included in several of the measurement objectives: "estimate and determine the length of an object or distance using non-standard units;" "estimate and determine the length of an object using whole metric units." Likewise, measurement ideas were incorporated in some objectives in the estimation strand: "identify and estimate length when given a non-standard unit." This was the only instance of an explicit link between strands.[10]

Of the six objectives in the probability and statistics strand, only one aimed to develop probabilistic ideas: "demonstrate the probability of a simple event with six or fewer outcomes, using aids." The sets and logic strand lacked coherence as evidenced by a jumble of

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objective explored the changes in shapes as they are rotated or combined.

[10]At Grade 4, the link between estimation and measurement included estimating the perimeter of a polygon and at Grade 5, estimating the area of a polygon on a grid.

randomly placed, discrete objectives: "identify the position of objects according to left/right, top/bottom, over/under, above/below;" "identify the correct word statement, picture, place, thing or number from 2 or more statements;" "read, interpret and supply elements in a matrix;" "read and interpret Venn diagrams;" "decide the truth or falsity of a number of statements involving the terms all, every, each, exactly, and no."[11]

The development of new ideas. Of the 116 objectives in the Grade 3 sequence, 30 (25.9%) were review from prior grades, 24 (20.7%) represented extensions of ideas first introduced in an earlier grade,[12] and 62 (53.4%) were presented for the first time. More than half the objectives represented the introduction of new mathematical content. Simple calculation indicated that this schedule of instruction allowed at most two days to consider an objective that introduced a mathematical idea for the first time. To explore the implications of this schedule, consider two measurement ideas that were introduced for the first time at Grade 3.

In January, three objectives related to measurement were introduced for the first time: "determine and record the area of a figure on a grid in square units;" "use a metric or customary ruler to measure the sides of a polygon or pictured object;" "determine and record the perimeter of a polygon or grid following the lines on the grid"

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[11]The single Grade 4 probability objective read "determine the probability of an event and express as a ratio (spinner, die, etc.)." The two probability objectives at Grade 5 were "analyze the probable possibilities of a coin, die, deck of cards and various kinds of spinners" and "determine the probability of a simple event and express it as a ratio (fraction)."

[12]Most of these objectives extended the numeration strand and addition and subtraction to three digit numbers.

(emphasis in curriculum document). Measuring objects was not new. An objective in PK-K called upon children to determine the length of a given object using nonstandard units. At Grade 1 students determined the length or height of an object using a standard unit. However, this was the first appearance of an objective that specifically called for using metric or customary rulers. In addition, this was the first time the concepts perimeter and area were included.

Perimeter and area are key measurement concepts that require considerable exploration with a variety of activities to build student understanding (Commission on Standards for School Mathematics, 1989; Driscoll, 1981; Nelson & Reys, 1976). The two objectives devoted to these concepts employed only one way - figures on grid paper - for students to investigate these powerful ideas. Students, prior to these objectives, had considerable experience taking linear measurements of objects. Extending the idea of measuring length to finding perimeter was a reasonable objective. What was interesting was that the objective did not link this measurement concept to determining the perimeter of common objects. Instead, the medium of instruction was the grid. The notion of perimeter was removed from the real world and abstracted to a piece of paper. The objective for determining and recording area was also problematic. The objective did not speak to the notion of area as covering but as simply counting squares on a grid. As with perimeter, the concept of area was separated from the world of real objects and abstracted to drawings on a piece of paper.

Key mathematical ideas were not "chunked" to explore relationships and make connections. For example, in Grade 2, objectives called for estimating and measuring the lengths of objects with non-standard and standard units. At Grade 3, these objectives were revisited again, twice in November and twice in December. However, the work on perimeter and area to be carried out in January was not conntected to these four objectives on linear measure. The objectives on linear measure, perimeter and area were isolated rather than being a part of a larger unit on measurement where prior learning could be extended by applying length to the concepts of perimeter and area. The organization and placement of objectives related to length, perimeter and area portrayed these key mathematical ideas as distinct from each other rather than part of a rich network of connected ideas.

**SUMMARY**

The Grade 3 instructional sequence embodied a tension between **certification of numeracy** and **comprehension of mathematical ideas**. The tendency toward the **certification of numeracy** was evidenced in a number of ways. The topics to be studied were organized around scores of behavioral objectives, each of which focused on a single skill or procedure. Computational proficiency with whole numbers, fractions and decimals dominated the set of objectives. Although the organization of objectives around ten central themes suggested that mathematics is a body of related ideas, the sometimes disparate objectives that were grouped together within a strand failed to capture the essence of the connections. The hierarchical arrangement of objectives continued to perpetuate the notion that mastery of basic facts and development of skills and procedures must precede work with word problems. And the sheer number of over one-hundred objectives precluded in-depth study over time of fundamental, yet complex, mathematical concepts.

At the same time, there was some evidence of **comprehension of mathematical ideas**. The concept of place value was emphasized, contributing to an understanding of number and the base ten system as well as to computational speed and accuracy with conventional algorithms. Estimation strategies were taught to enhance mental computation and to provide a technique to assess the reasonableness of answers, although there were limitations in the specific strategy of rounding off. The objectives suggested that mathematics is a system of concepts and processes for which there are appropriate concrete models. Over a dozen objectives stated explicitly that physical aids or models were to be used to recall number facts, carry out measurements, compare fractions, identify geometric figures, and interpret and extend patterns. Several objectives were explicit about using pictorial representations.

These formal curriculum documents did not embody attributes of **enabling mathematical inquiry**. Key mathematical ideas were presented as isolated skills and processes to be covered in a day or two rather than topics that demand exploration over time to develop an understanding of concepts and procedures and the relationships among them. The actions that would signify an orientation toward mathematical inquiry -- investigate, explore, predict, formulate problems, make conjectures, develop arguments to validate assertions, invent procedures, build generalizations, discuss findings -- were totally absent from the formulation of these objectives.

In reference to the orientations to teaching mathematics, these objectives moved between **certification of numeracy** and **comprehension of mathematical ideas**. Taken separately, some objectives seemed to fall in the first category while others seemed more appropriately located within the second. Taken together, they seemed to portray an attempt to shift teaching and learning from an exclusive focus on the acquisition of facts and rules and the manipulation of symbols toward understanding key ideas about number, numeration, operations and measurement. While arithmetic tended to dominate the objectives, there was the inclusion of objectives representing a broader range of mathematical topics. Despite the broad goals presented in the district's statement of philosophy to develop higher order reasoning skills and problem solving abilities, these curriculum objectives seemed aimed at a lesser goal--to understand mathematical processes and their application to routine word problems.

## THE TEXTBOOK AND THE INSTRUCTIONAL SEQUENCE

The Board of Education had approved four textbook series for use in elementary mathematics classrooms in the district. The two series most commonly used were Holt Mathematics (1981 edition) and Harper & Row Mathematics (1982 edition). One or the other of these series was used in an estimated 90% or more of the classrooms in the district.[13] This section examines the extent to which the most commonly used Grade 3 textbooks provided an instructional program aligned with the district's Instructional Sequence.

Both textbook series represented a traditional orientation to mathematics education. Computational proficiency with conventional

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[13]Also approved for use in the district were Real Math (Open Court, 1981) and Comprehensive School Mathematics Program (CEMREL, 1979), two textbook series that were significant departures in both content and orientation from traditional textbook series.

algorithms was overwhemingly emphasized. Twelve of the 16 chapters in the Harper & Row Grade 3 text were devoted to operations on whole numbers. Word problems were found at the end of chapters with an occasional set of two or three word problems following a page of computational exercises. The hierarchical arrangement suggested that problem solving was an appropriate activity only after children had mastered basic facts and operational algorithms. Virtually all the word problems were of a routine nature. The method of solution was obvious from the statement of the problem or its placement in the text. Few of the problems involved open searches or provided for multiple answers.

The pages included here from the Harper & Row text were typical. Lesson 3 from Chapter 6 on subtraction with regrouping provided children with examples, a set of computational exercises, and a set of word problems that could be solved by simply following the example. Lesson 9, the last regular lesson in the chapter did ask children to decide if a problem required addition of subtraction to solve. Most chapters in this text had a "challenge" at the end. Some were logic activities, some provided atypical environments in which to work on basic facts, like "magic squares." Several "challenges" incorporated geometric representations of numbers such as "Building Numbers--Rectangular Arrays." One-third of the "challenges" involved problems with money, as in "You be the Clerk!" These problems allowed for

# LESSON 3 More Renaming

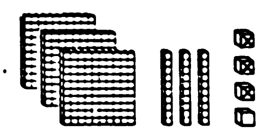
334 students go to the Emerson School.
193 are boys. How many are girls?

$$\begin{array}{r} 334 \\ -\,193 \\ \hline \end{array}$$

**THINK**

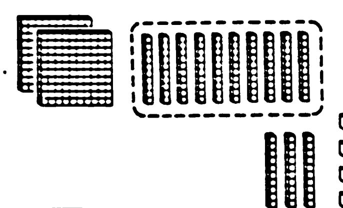
$$\begin{array}{r} 334 \\ -\,193 \\ \hline 1 \end{array}$$

**Step 1**
Can you subtract ones? Yes.
4 − 3 = 1
Write **1** in the ones place.



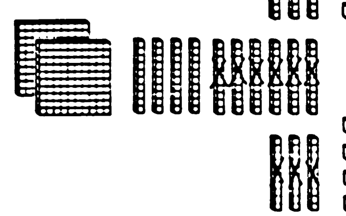
$$\begin{array}{r} {\scriptstyle 2\,13}\phantom{4} \\ \cancel{3}\cancel{3}4 \\ -\,193 \\ \hline 1 \end{array}$$

**Step 2**
Can you subtract tens? No.
Rename a hundred.



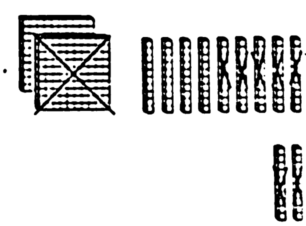
$$\begin{array}{r} {\scriptstyle 2\,13}\phantom{4} \\ \cancel{3}\cancel{3}4 \\ -\,193 \\ \hline 41 \end{array}$$

**Step 3**
Subtract tens.
13 tens − 9 tens = 4 tens
Write **4** in the tens place.



$$\begin{array}{r} {\scriptstyle 2\,13}\phantom{4} \\ \cancel{3}\cancel{3}4 \\ -\,193 \\ \hline 141 \end{array}$$

**Step 4**
Can you subtract hundreds? Yes.
2 hundreds − 1 hundred
= 1 hundred
Write **1** in the hundreds place.



√ **Check.** 193 + 141 = 334
141 girls go to the Emerson School.

**TRYOUT** **Copy and subtract. Check.**

A. $\begin{array}{r} {\scriptstyle 3\,14}\phantom{8} \\ \cancel{4}\cancel{4}8 \\ -\,263 \\ \hline \end{array}$ B. $\begin{array}{r} {\scriptstyle 5\,18}\phantom{0} \\ \cancel{6}\cancel{8}0 \\ -\,190 \\ \hline \end{array}$ C. $\begin{array}{r} 275 \\ -\,185 \\ \hline \end{array}$ D. $\begin{array}{r} 367 \\ -\,176 \\ \hline \end{array}$



Harper & Row Mathematics, 1982.

## Exercises

Copy and subtract.

Check the first row by adding.

$$\begin{array}{r} {\scriptstyle 5\ 15} \\ 1.\ \cancel{6}\cancel{5}9 \\ -\ 185 \\ \hline 474 \end{array} \qquad \begin{array}{r} 185 \\ +\ 474 \\ \hline 659 \end{array}$$

| | | | | |
|---|---|---|---|---|
| 1. $\overset{5\ 15}{\cancel{6}\cancel{5}9}$ − 185 | 2. $\overset{3\ 17}{\cancel{4}\cancel{7}0}$ − 290 | 3. 867 − 387 | 4. 740 − 390 | 5. 345 − 174 |
| 6. 568 − 197 | 7. 458 − 288 | 8. 650 − 390 | 9. 507 − 325 | 10. 806 − 374 |
| 11. 702 − 281 | 12. 608 − 216 | 13. 415 − 283 | 14. 518 − 392 | 15. 614 − 281 |
| 16. 719 − 275 | 17. 632 − 191 | 18. 957 − 394 | 19. 607 − 135 | 20. 818 − 136 |

Solve.

21. 864 seats in the baseball stand.
192 are empty.
people at the game.

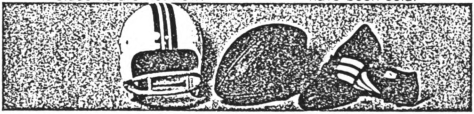
22. 500 people at the game.
260 for the home team.
for the visitors.

23. 338 seats in the gym.
184 people in seats.
seats are empty.

24. 450 seats in the gym.
290 seats are empty.
seats are not empty.

25. 627 students at the Green School.
132 are new this year.
students are not new.

26. 953 tickets for the football game.
162 are left.
have been sold.





Harper & Row Mathematics, 1982

## Exercises

**Write *add* or *subtract*. Then solve the problem.**

1. 900 people are at the beach.
360 came in cars.
☐ did not come in cars.

2. Pam found 362 shells.
Ann found 197 shells.
They found ☐ in all.

3. 520 rafts are for rent.
120 are rented.
☐ are still left.

4. 475 came to the beach.
296 went on rides.
☐ did not go on rides.

5. 544 people came to the beach on Sunday.
325 came on Monday.
☐ came both days.

6. Jane has $5.25.
She needs $8.19 for a beach ball.
She needs ☐ more.

7. Mark has $8.25.
A raft costs $9.15.
He has ☐ too little.

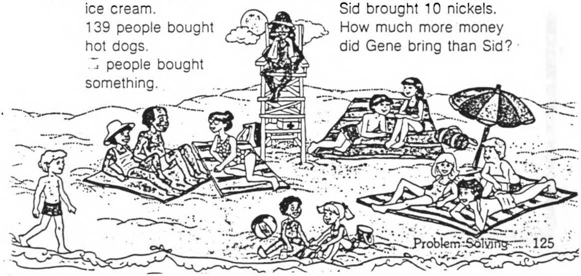
8. A hat costs $6.02.
A beach coat is $9.00.
Together they cost ☐.

9. Tom has $7.25.
He spends $3.00 on ride tickets.
He has ☐ left.

10. Jane had $7.25.
She spent $2.25 on a sand pail.
She has ☐ left.

11. 247 people bought ice cream.
139 people bought hot dogs.
☐ people bought something.

12. Gene brought 8 dimes.
Sid brought 10 nickels.
How much more money did Gene bring than Sid?



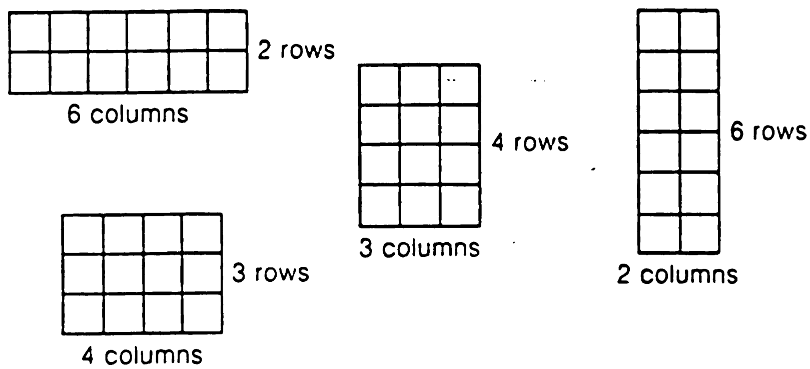


Harper & Row Mathematics, 1982

# CHALLENGE

## Building Numbers— Rectangular Arrays

**Use 12 squares to build these arrays.**

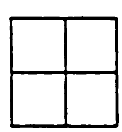
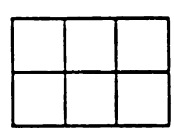
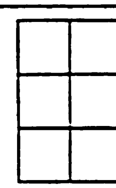



**Use squares to build as many different arrays as you can for each number:**

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.

Each array must have at least 2 rows and 2 columns.

Keep a record like this:

| Number | 4 | 6 | |
|---|---|---|---|
| Arrays |  |  |  |
| Rows | 2 | 2 | 3 |
| Columns | 2 | 3 | 2 |



Harper & Row Mathematics, 1982

multiple answers. However, problems of this sort were not incorporated into regular lessons but were suggested as optional activities for more able students.

Each series had a chapter devoted to geometry located near the end of the text. The focus of these chapters was on naming, identifying and drawing shapes, memorizing terms, adding numbers marked on sides of figures to determine perimeter, and counting squares inside figures to determine area. Harper & Row had one lesson on symmetry. Probability was treated formally in only one of the texts and there it was listed as an optional topic.

The use of calculators in the classroom was deemphasized. Occasional "calculator activities" were included at the end of computational exercises but often were unconnected to the topic at hand. The calculator activity shown here followed the chapter on measurement in Harper & Row. It was one of four calculator activities in the Grade 3 text.

In many cases, neither text provided material responsive to a particular objective in the Instructional sequence. The Harper & Row text provided one lesson on bar graphs but none on pictographs or circle graphs. In that single lesson students were expected to read and interpret a bar graph but not to gather data or construct a bar graph. The Holt text did suggest kinds of data that students might themselves collect and display. But even here, the activity was text or teacher directed. It was not suggested that students might generate questions about which they would collect data or make decisions about how to display that data.

## Calculator

LETTER VALUES

| AEIOU | BCD | FGH | JKLMN | PQRST | VWXYZ |
|---|---|---|---|---|---|
| 100 | 10 | 20 | 30 | 40 | 50 |

**1.** Use the word MATHEMATICS. See how many smaller words you can make from those letters. Find the values of the smaller words.

Samples:

| | | | | |
|---|---|---|---|---|
| T | 40 | | S | 40 |
| H | 20 | | A | 100 |
| E | + 100 | | T | + 40 |
| | 160 | | | 180 |

**2.** A puzzle:

Enter a number in your calculator.

$\boxed{+}$ 17

$\boxed{-}$ 10

$\boxed{+}$ 3

$\boxed{-}$ 5

$\boxed{+}$ 14

$\boxed{-}$ 19

$\boxed{=}$

The display shows the number you first entered.

**3.** Another puzzle:

Enter a number in your calculator.

$\boxed{+}$ 100

$\boxed{-}$ 50

$\boxed{+}$ 25

$\boxed{+}$ Number in display

$\boxed{+}$ 50

$\boxed{-}$ The number you entered first.

$\boxed{-}$ The number you entered first again.

The display shows 200.



Harper & Row Mathematics, 1982

In July, 1985, the Area D office published Enriching the math curriculum: A textbook correlation to the mathematics instructional sequence. Their analysis found that of the 116 Grade 3 objectives, the Harper & Row student text did not include material perinent to 51 of the objectives, Holt did not include material relevant to 36 of the objectives.[14] Neither text provided materials to meet the following objectives from the functions and relations strand: "apply a given rule to a set of whole numbers and determine the missing whole number output and input;" "identify and state the rule which indicates the relationship between whole numbers when given input or output;" and "interpret and complete addition, subtraction or multiplication number sentences using -, -, <, >." These are key objectives in developing an understanding of relationships among whole numbers. Of the eight objectives from the sets and logic strand, only one was covered in either text.

The two most commonly used textbooks in the district supported those objectives in the Grade 3 Instructional Sequence that were aimed at computational speed and accuracy, proficiency with conventional algorithms and their application to routine story problems. They were far less effective in meeting many of the objectives related to understanding the meaning of operations, the effects of operating on whole numbers, the relationships among whole numbers, and the application of whole number operations to problem situations. In some

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[14]Although a math instructional specialist from another Area indicated some disagreement with the textbook correlation, a number of teachers with whom I spoke complained that they could not find material in their textbooks to tach to many of the objectives.

cases, they completely failed to provide material pertinent to objectives in the Instructional Sequence.[15]

The lack of alignment between the content of the most commonly used textbooks and the objectives of the instructional sequence raises some serious questions. The objectives represented the intended city-wide curriculum. But what of the enacted curriculum? Was it driven by the set of instructional objectives, the content in the textbook, teacher decisions or some combination? How were objectives to be met if appropriate material was not contained in the approved textbooks? Which of the three -- textbook, instructional sequence, or teacher decision -- determined the order in which mathematical ideas were studied? How could teachers and students be held accountable for teaching and learning mathematical topics that were inadequately treated in or absent from teacher and student texts?

Given the discrepancy between some of the objectives in the instructional sequence and the available material in the commonly used textbooks, supplemental materials had been provided to teachers. These materials included "Tips for Using the Instructional Sequence," a further elaboration of the range of ideas within each of the ten strands, and a set of model lessons, many of which focused on objectives for which the treatment in the commonly used textbooks was lacking or was inadequate. The next chapter examines the set of model

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[15]The textbook adoptions were made in 1982 at the same time that the objectives for the ABCS test were being developed. I was unable to determine the extent to which these objectives were used as criteria for the selection process. The district had a six-year cycle for textbook adoption. The mathematics texts were eligible for review and new adoptions during the 87-88 school year. I was told by the mathematics supervisor that a decision was made to delay new adoptions, reflecting dissatisfaction with any texts they had reviewed to adequately meet the district's objectives.

lessons that accompanied the instructional sequence for Grade 3. Such an investigation will provide a sense of how the curriculum objectives were to be enacted in the classroom as teachers taught to these objectives.

# CHAPTER 6

# THE MODEL LESSONS

## INTRODUCTION

In 1986, the elementary mathematics supervisors in the district brought together a team of math instructional specialists, classroom teachers, a building principal and a city-wide program coordinator for a research and writing project. The team was charged with the task of creating sets of model lessons for each grade level, drawing on two key areas of research. The first domain concerned findings from research in mathematics education, cognitive science and educational psychology about how children learn mathematics. The particular finding that influenced the team's work was using concrete materials to introduce mathematical ideas, linking concrete models to pictorial representations, and then using pictorial representations as the bridge from the concrete to the abstract/symbolic. The second domain concerned findings from effective teaching research-- those teacher behaviors that seem to be effective in increasing student achievement on standardized tests and that result in more efficient teaching. The lessons were intended to develop the critical thinking skills and problem solving abilities of youngsters. The lessons were also intended to address the mismatch between objectives in the Instructional Sequence and available material in the commonly used textbooks in the District.

Committees of five to seven teachers at each grade level piloted the lessons during the 1986-87 school year. In September, 1987, the Mathematics and Science Department of the District published Model

Lessons to Promote Thinking. This set of materials included an elaboration of each of the ten curriculum strands followed by a set of model lessons for specific objectives at each grade level. The number of model lessons ranged from 22 for Grade P1 to 40 for Grade 4. At each grade, some of the model lessons focused on the use of calculators and were designated "hand-held calculator lessons." The model lessons provided teachers with examples of mathematical ideas that fit within the strands as well as particular instances to meet specific objectives.

In Chapter 4 the argument was made that the goal of developing students' critical thinking and higher order reasoning skills and problem solving abilities by reorganizing the curriculum around ten mathematical strands was in tension with the objectives that defined content and sequenced instruction. The instructional objectives coupled with the commonly used textbooks tended to perpetuate a traditional view of mathematics curriculum and instruction that atomized content, isolated computational proficiency from problem solving contexts, and ignored inquiry, decision-making and judgment. At the same time, there was evidence of an interest in and attempt to develop an understanding of numeration and whole number operations. Place value was emphasized, concrete models to develop number facts were incorporated, relationships between operations were suggested, and estimation strategies to enhance mental computation and assess the reasonableness of answers were included. Yet, while the reorganization around mathematical strands suggested a reshaping of content from the traditional emphasis on arithmetic, less than a quarter of the objectives were devoted to geometry, probability and statistics, patterns, and sets and logic.

But the objectives alone, as simply statements of expected student outcomes, provide scant information about how these objectives were to be met in the classroom as teachers taught to the objectives. This chapter examines the set of model lessons for Grade 3. The model lessons afford the opportunity to investigate how teaching to specific objectives was portrayed to teachers. The model lessons provide another source of evidence about the potential of this curriculum reform to develop youngsters' critical thinking and problem solving abilities.

The chapter is divided into three sections. The first section examines the introduction to Model Lessons to Promote Thinking. The inquiry concerns the goals for student learning as expressed in this document and the explication of the range of ideas encompassed in each of the curriculum strands. The second section examines the content of the Grade 3 model lessons -- the distribution of the lessons among the strands, an example of a model lesson that embodied elements of developing understanding, and an example of a model lesson that did not. The third section examines the form of the model lessons to determine the extent to which curricular form enhanced or constrained teaching for understanding.

## The Analytic Framework

The framework for analysis of the model lessons continues with the three orientations developed in Chapter 2. The analysis inquires about the role of the teacher in instruction, the role of the student in learning mathematics, assumptions -- explicit or implicit -- about how math is learned, what teachers take as proxies for learning, and where authority for knowing rests.

Within **certification of numeracy**, the role of the teacher is to show students how to carry out procedures. The teacher gives directions, works examples and assigns seatwork for students to practice a skill or procedure. The role of the student is to absorb and remember the knowledge that a teacher dispenses. Learning mathematics is an individual, solitary effort where children search their memory for accumulated facts and algorithmic solutions. Task completion, right answers and periodic paper-and-pencil tests for recording a mark serve as proxies for student learning. Teacher and text stand as the source of epistemological authority.

Within **comprehension of mathematical ideas**, learning is portrayed as requiring a particular sequence of development moving from concrete to pictorial to abstract. The explicit/implicit mode of instruction draws on effective teaching research. The role of the teacher is to show students what they are to learn by demonstrating to the entire class with the appropriate materials and then letting students practice. The teacher monitors student learning during instruction by posing "product" and "process" questions, evaluating student responses and giving immediate feedback. Opportunities are provided for students to work in groups on "fun" -- though generally routine -- activities. Within the structure imposed by the teacher, students pursue teacher defined explorations with concrete materials, answer teacher questions and complete assignments on practice exercises. More than task completion and right answers are sought. Students are expected to provide explanations for procedures they have used. However, authority for knowing continues to rest with the teacher who decides whether student answers are correct and explanations sufficient.

An orientation aimed at **enabling mathematical inquiry** makes several important assumptions about learners: they actively construct mathematical knowledge; their prior knowledge and current conceptions affect how they make sense of new knowledge; they are able to construct arguments about why mathematical ideas are true and are able to figure out if strategies and solutions are reasonable. The role of the teacher is to create a total environment where teacher and students can actively engage with one another in inquiry. That requires posing interesting, challenging and non-routine problem situations that are rich in mathematics and that lead to learner inventions. Assessing student understanding is a continual, on-going process in instruction, seeking evidence by listening to students and asking questions to determine what they are <u>coming</u> to know. Students learn to use multiple representations - concrete, numeric, graphic, algebraic, spatial - to describe a mathematical situation. In a supportive environment, they become mathematical risk-takers--making guesses and pursuing hunches, offering hypotheses, accepting challenges to hypotheses and marshalling arguments in support of them, and yielding to the force of a better argument. The locus of epistemological authority shifts from the teacher and text to the classroom community of students and teacher. Together they have the power to use mathematical tools and standards of the mathematical community to decide about the reasonableness of processes and the results of investigations.

This chapter examines the set of model lessons for Grade 3 to determine their orientation.

## MODEL LESSONS TO PROMOTE THINKING

### The Introduction to the Model Lessons

The introduction to the set of model lessons placed emphasis on developing students' ability to reason.

> Skill development of the fourth (r)easoning has been designated by the general superintendent for the students in the Detroit Public School System. The mathematics core curriculum emphasizes the teaching of thinking skills. To help students attain reasoning (thinking) skills, it is of utmost importance that students understand the concepts being taught...Thinking skills have been interwoven into each lesson to develop/improve the problem solving abilities of students...Increasing the emphasis on critical thinking reduces the traditional difficulties students have had with organizing, presenting and interpreting data, estimating results and calculating mentally (Detroit Public Schools, September, 1987; p.ii).

In keeping with the District's philosophy regarding mathematics education, the introduction described developing critical thinking and reasoning skills and problem solving abilities as goals that informed the lesson writing effort. Three pedagogical tools were specified as key to helping students understand mathematical concepts--questioning, using concrete materials, and using hand-held calculators.

> Teaching by means of posing questions is to be valued over expository instruction...Above all, many concrete and/or semi-concrete models have been used to help the learners gain understanding <u>before</u> symbols/abstraction. The most recent research results regarding how a child learns has been incorporated into these lessons...Hand-held calculator lessons using the problem solving approach are included for certain strand objectives. A summary of numerous studies indicates that achievement scores of student (sic) using calculators in the classroom are as high or higher than those of students not using calculators in their instruction (Detroit Public Schools, September, 1987; p. ii).

The introduction also provided an explication of each of the curriculum strands beyond the brief description in the <u>Strands and</u>

Objectives and, in some cases, a rationale for including the strand in the curriculum. For example, the value of estimation in the curriculum was linked to the fact that "Most people use estimation every day, more often than they use pencil and paper computation. Therefore, it is imperative that estimation be given much emphasis..."(p. ix). The range of activities suggested for this strand included rounding, front end estimation, averaging, and compatible numbers as well as estimating with measurement. The development of linear measurement ideas was portrayed as requiring a particular sequence.

> In order to help students develop the concept of a unit, each lesson has as its primary objective the perception of length and then its measure using non-standard units before standard units are introduced...The experience with a non-standard unit help (sic) to develop ideas common to all measurement situations... the process by which a child learns to measure is sequential in development from perception, use of non-standard units, comparison and finally to the use of a standard unit of measure. Children should have continuous experiences with the hands-on activity of measurement throughout the grades (p. xiii).

Numeration was described as "basic to mathematical proficiency" and computational proficiency dependent on a "thorough knowledge of place value" (p. xv). Several concrete materials -- beans and beansticks, Dienes blocks and money -- were suggested as models for place value. It is noteworthy that while each of these materials embodies a different model as a representation of place value, this was not acknowledged nor were teachers given a sense of the mathematical situation for which a particular model might be appropriate.[1] A particular developmental sequence for place value was defined.

> All ideas children internalize fully must be met first in the world of three-dimensional things - the manipulative world. Later, the same ideas can be encouraged and understood in sketches, pictures,

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[1]For a discussion on the "thoughtful use of manipulatives," see Schram, Feiman-Nemser and Ball, 1989.

> and diagrams - <u>the model or representational world</u>. Eventually, children work with these same ideas expressed in symbols - <u>the abstract world</u> (p. xv).[2]

The operation strand emphasized "facility with the basic facts of arithmetic" and "thinking mathematically." More than drill and practice was demanded; posing questions "where the use of memory is just the beginning" was suggested.

> Are there 6 different ways of finding parts or addends that equal 3 in the form ___ + ___ = ___? Are there more? Can you be sure that you have found them all (p. xxvii).

A direct link between operations and another strand, patterns, was made explicit. It was suggested that to answer questions like those posed above, students should make a chart and observe the patterns that emerge.

Sets and logic encompassed organizing and interpreting data, using "sequential steps to analyze and conceptualize problems," and drawing conclusions. The purpose of including probability in the curriculum was "to familiarize students with intuitive ideas and to enrich the mathematics program, as well as provide practice work in whole numbers, graphing, percents and decimals...make predictions and...discover new information" (p. xxxiii). Statistics developed skills in "counting, measuring, recording, ordering, classifying, comparing, displaying and interpreting information" and reinforced "basic arithmetic skills in an interesting setting" (p. xxxiv).[3]

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[2]In a subsequent paragraph, the "model or representational world" was referred to as the pictorial level and that "picturing numeration" was a "prerequisite to long term memory and success at the abstract level" (p. xvii).

[3]There was no elaboration on the geometry strand in this document - presumably an oversight in putting the materials together.

Unlike the brief definition of the curriculum strands in Strands and Objectives (described in Chapter 5), the elaboration of each strand in Model Lessons to Promote Thinking suggested a range of mathematical ideas -- concepts, skills, procedures -- to be studied, ideas to be emphasized, and various ways in which students might encounter them. The elaborations also provided instances in which ideas from one strand would be met in another. Estimation was essential to developing computational proficiency and measurement concepts. Understanding the four arithmetic operations contributed to developing the ability to solve simple equations and find their inverses, ideas from the functions and relations strand. Developing an understanding of place value and operations with whole numbers was linked to discovering patterns and finding relationships.

While the strand definitions in Strands and Objectives appeared to serve a political and symbolic purpose, their elaboration in Lessons to Promote Thinking served a mathematical purpose. The explications provided a sense of a range of mathematical ideas embedded in each strand, how some of those ideas were connected, and why those linkages were important. The elaborations also served an instructional purpose. Teachers were instructed to follow a specific developmental sequence for some ideas. They were alerted to special problems children had in developing some concepts. The importance of "asking questions to promote student understanding and check comprehension of concepts" was emphasized. It was recommended that some lessons be taught and then revisited throughout the school year, "with added insight." Teachers were told that while it was important for youngsters to master basic facts for instant recall, recall alone was insufficient.

> ...it is important to realize that students need more than just basic knowledge. They must begin to internalize the relationships that exist...to develop higher order thinking skills. These higher levels include: analysis, synthesis and evaluation (pp. xxviii-xxix).

It appeared that developing higher order thinking skills and problem solving abilities was as important a goal as developing computational speed and accuracy and proficiency with conventional algorithms. The document seemed to be oriented toward **comprehension of mathematical ideas**.

The next section inquires about the extent to which the model lessons themselves represented a shift from the traditional emphasis on arithmetic by examining the distribution of model lessons among the curriculum strands and special features of lessons that could contribute to developing conceptual understanding. The investigation then moves to a consideration of several specific lessons as examples of the unevenness in which the broad goals of developing understanding, higher order thinking skills and problem solving abilities were embedded in instructional activities.

## The Content of the Grade 3 Model Lessons

Distribution of model lessons among the strands. Table 6.1 shows the distribution of model lessons among the curriculum strands and notes particular features of the lessons.

Seventeen of the lessons focused on objectives for which the treatment in one or both of the commonly used textbooks was lacking or was inadequate, as assessed in Enriching the math curriculum: A

| CURRICULUM STRANDS | MODEL LESSONS | CALCULATOR LESSONS |
|---|---|---|
| Estimation and Approximation | ML 18 [a]<br>ML 19 [f] | HC 8 |
| Functions and Relations | ML 5<br>ML 7 [a,f]<br>ML 8 [f]<br>ML 9 [a,f] | HC 3<br>HC 4 [f] |
| Measurement | ML 14 [a]<br>ML 15 [a]<br>ML 16 [a]<br>ML 17 [a,f] | |
| Numeration | ML 2 [a,b]<br>ML 3 [a,c,e]<br>ML 10 [a,f]<br>ML 11 [a,c,f] | HC 2 [b]<br>HC 5 [b,f]<br>HC 7 [b] |
| Operations | ML 4<br>ML 12 | HC 3 [b,f]<br>HC 6 [b] |
| Patterns | ML 6 [a,b,f]<br>ML 13 [b,e,f] | |
| Probabiity and Statistics | ML 1 [f]<br>ML 22 [a,c,f] | HC 1 |
| Sets and Logic | ML 20 [d,f]<br>ML 21 [a,d,f]<br>ML 23 [a,b,f] | |

[a] uses concrete materials.
[b] provides extensions.
[c] incorporates games.
[d] teacher demonstration only; no student activities.
[e] series of lessons covering 3-6 days.
[f] objective is absent from or inadequately treated in commonly used texts.

Figure 6.1 Distribution of Model Lessons Among Curriculum Strands and Features of the Lessons textbook correlation to the mathematics instructional sequence (see Chapter 5). These objectives included

* students will be able to construct, read and interpret a bar graph/pictograph (ML 1).
* the student will be able to 1) interpret and/or extend a pattern of shapes, pictures or objects using a combination of any three of the following attributes: color, shape, size, thickness, position, texture; 2) state the rule used in forming a given pattern and extend a pattern; 3) create and state the rule of a pictorial pattern (ML 6).
* students will be able to demonstrate their perception of their length/height (ML 17).
* the student will be able to read, interpret, and supply elements in a matrix (ML 20).
* students will demonstrate the probability of simple events with two outcomes (ML 22).

Fifteen of the 31 lessons required the use of concrete materials, supporting a view that learning must begin with concrete experiences. The six model lessons from the numeration strand used concrete, pictorial and symbolic representations. Beansticks, hundreds squares and Dienes blocks were suggested as possible models to demonstrate place value. Pictorial representations included drawing bundles of sticks and coloring centimeter grid paper to illustrate numbers. The symbolic representations included place value charts, expanded notations (e.g., 300 + 20 + 1 and 3 hundreds 2 tens and 1 one) and standard notation (e.g., 321). Other lessons employed counters, dice, attribute blocks, and non-standard measuring devices.

Suggestions for extending an objective were included in 10 lessons. The nature of these extensions included expanding work to larger numbers, broadening the set of operations to be performed, and increasing the complexity of the activities. The use of games as a context to practice computation and regrouping were incorporated into three lessons. The games in Model Lesson 11 required players to be able to represent whole numbers concretely, pictorially and symbolically and tell from each representation the relative sizes of whole numbers.

In several instances a set of consecutive lessons was devoted to the same objective, showing how a topic might be pursued over several days of instruction. Model Lessons 2 and 3 were devoted to having students "determine and/or demonstrate place value through 999 using aids and identify a number 100 more or less than a given number." Together, these two lessons were intended to be covered over a period of at least four days. Model Lessons 7, 8 and 9 had students "identify the missing relationship or operational symbol in an incomplete number sentence." Model Lesson 13 from the patterns strand incorporated a series of lessons to be carried out over a six day period. Hundreds charts were used to discover number patterns. This work was expanded at the abstract level to patterns that arise when counting by hundreds. Patterns were used to help reinforce basic number facts and find sums and differences of two digit numbers.

Interestingly, no model lessons had been created for the geometry strand, a noteworthy absence considering the limited treatment of this topic in the commonly used textbooks. However, two lessons in other strands employed attribute blocks in classification activities and geometric pattern explorations.[4]

The analysis here has taken a broad sweep of the entire set of Grade 3 model lessons, attending to their distribution among the strands and certain features of the lessons. The analysis suggests that the model lessons seemed oriented toward comprehension of

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[4]There were no model lessons for the geometry strand at any of the other grade levels. One model lesson from the sets and logic strand involved classification activities with geometric shapes.

mathematical ideas. Developing an understanding of number, numeration and whole number operations was given particular attention. Mathematics was represented as an abstract system of concepts and processes for which there are appropriate concrete models and pictorial representations. Learning was portrayed as requiring a particular sequence of development moving from concrete to pictorial to abstract. At the same time, numeration and operations dominated the model lessons. Nineteen of the 31 model lessons (61%) were devoted to objectives related to place value, computation and operations. Probability and statistics and sets and logic received scant attention. The geometry strand was ignored.[5] The dominance of lessons devoted to topics on numeration and operations tended to perpetuate a view of elementary mathematics as primarily arithmetic.

This broad sweep has given a sense of the orientation of the model lessons as a whole. There appeared to be a tension between a traditional emphasis on topics of arithmetic and an attempt to develop understanding of mathematical ideas about numeration and operations.

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[5]The chart shows the distribution of model lessons among the curriculum strands for the other grades.

| Strand | Number of Model Lessons | | | |
|---|---|---|---|---|
| | Grade P1 | Grade P2 | Grade 4 | Grade 5 |
| Est. & Approx. | 1 | 3 | 3 | 3 |
| Func. & Rel. | 4 | 5 | 6 | 8 |
| Geom. | 0 | 0 | 0 | 0 |
| Meas. | 4 | 4 | 8 | 3 |
| Numer. | 5 | 4 | 5 | 4 |
| Oper. | 3 | 4 | 7 | 8 |
| Patt. | 2 | 3 | 2 | 2 |
| Prob. & Stat. | 1 | 1 | 5 | 6 |
| Sets & Log. | 2 | 1 | 3 | 1 |

The increase in lessons in the probability and statistics strand represented more attention to displaying and interpreting data. Only one lesson was devoted to probabilistic ideas. Roughly one-third of the Grade 4 and 5 lessons had been introduced at an earlier grade.

But the investigation has examined elements of lessons taken out of context; the investigation has been limited and partial. Further probing at the level of a specific lesson in its entirety is warranted.

The model lesson is another piece of evidence about the nature of this reform. It exemplifies the way in which mathematics and mathematics teaching and learning were represented to both teachers and students. And it provides the occasion for further inquiry about what this reform would help students to learn and know. The inquiry continues with an analysis of several of the Grade 3 model lessons.

The first model lesson was chosen because it embodied a number of elements that could contribute to developing understanding of mathematical ideas. It stood out from all the other lessons as an exemplar in connecting mathematics to the real-world of children, using interesting problem situations and multiple representations, and extending the world of mathematics beyond numbers and computation. The other lessons were selected because they typified the unevenness that existed among the majority of the lessons in terms of conceptualization, mathematical substance, and student activities.

A model lesson to develop understanding. Model Lesson 6 was created for an objective of the patterns strand. The lesson begins with a statement of the objectives.

> **OBJECTIVE**: The student will be able to: 1) interpret and/or extend a pattern of shapes, pictures or objects using a combination of any three of the following attributes - color, shape, size, thickness, position, texture; 2) state the rule used in forming a given pattern and extend the pattern; 3) create and state the rule of a pictorial pattern.

The lesson describes materials the teacher would need to carry out instruction and the vocabulary to be developed.

**MATERIALS**: Attribute blocks (or railroad board or construction paper) for teacher and students; teacher made pattern cards.

**VOCABULARY**: attribute blocks, attributes.

Instruction begins with a motivation activity. In all instructional phases, teachers are provided with examples of what to say during instruction, what questions to ask, and responses to accept.

**MOTIVATION**: "Let's imagine that you were invited to your friend's birthday party last week. One of the games the children played was "Find a Pattern." The rule was to look around your friend's home and see how many patterns you could find and then to write down the pattern and the rule used to make the pattern. Let's imagine that you won the game by finding the greatest number of patterns. We're going to play that game in class today to see how many patterns we can find."

The next component of the lesson is called instructional input, the direct instruction phase of the lesson.

**INSTRUCTIONAL INPUT**:

I. Location of patterns in the environment

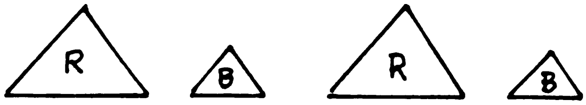
Discuss patterns in the environment and the rule used to form them: placement of windows, cupboards, doors and hardware; flag stripes (alternating red and white stripes); clothing designs; fencing (chain link); black keys on a piano (2 space 3); designs on towels; patterns in furniture; wallpaper designs; brick walls; floor tile; pairs of shoes, mittens.
Compile a list of patterns found and add to it <u>throughout</u> the school year.

II. Pattern formation and extension

Pass out attribute blocks to students and discuss them by asking:
1. "Describe this shape." (note color, shape, size)
2. "What is the same about these two shapes?" (Pick out common attributes; use the term attribute)
3. "What is different about these two shapes?"
4. "How many ways are these two shapes the same?"
5. "How many ways are these two shape different?"
6. "Find another piece that has the same common attribute. What is it?"

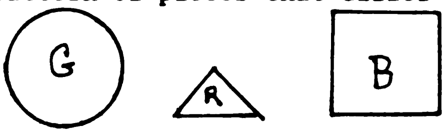
Hold up one attribute block and say:
1. "Hold up a piece that is different from this in two ways?"
2. "How is your piece different, Johnny?"
3. "Let's make a pattern of pieces that are different from each other in two ways."

4. Form the pattern using student input. Discuss how the pieces differ.




State the rule used. (change in color and size)

Form a pattern of pieces that differ by three attributes: e.g.,




"How do these pieces differ?" (Color, shape, size)

The next phase of the lesson, guided practice, provides an opportunity for children to practice what the teacher has taught.

**GUIDED PRACTICE:**
Play "Guess My Rule" with attribute blocks.

A. "I am going to put some attribute blocks on the chalkborad. Each time I put one up, I must follow a rule. See if you can guess what my rule is."

B. Display a pattern. Say: "What piece do you think I could put up next? Why?" Answer by saying: "That piece does follow the rule I am thinking of" or "That piece does not follow my rule."

C. Continue asking students to select blocks following the pattern. When you think most students know the rule, ask someone to verbalize it.

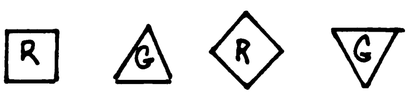
Debugging an incorrect pattern.

A. Provide students with patterns, some of which are correct and some with one incorect piece.

B. Ask questions such as: Is this pattern correct? How do you know? How are these pieces different? What rule was used to form the pattern?

Challenge

Ask: "Can we make a pattern of pieces that differ in four ways?" How?" (Change of position)
e.g.,



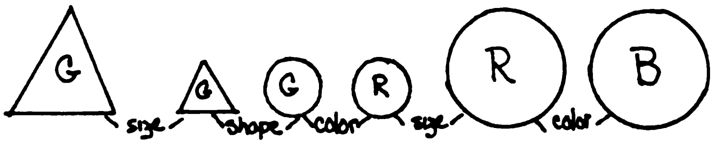

The lesson ends with closure.

**CLOSURE:**
1. Provide the students with an opportunity to develop their own patterns.
2. Other students should guess the rule used to make up the pattern. They may also extend the pattern.

This lesson was one of three in the set that incorporated suggestions for ways in which a teacher might extend the ideas.

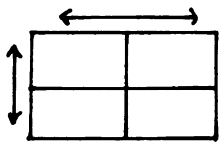
**INSTRUCTIONAL INPUT:**
III. Logic extension of patterning:
A. Review the meaning of attributes. Select two blocks and discuss how they are the same and how they differ from one another.

B. Make "one difference" trains. Each piece must differ from the one next to it by 1 attribute: e.g.,

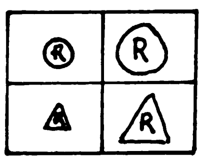


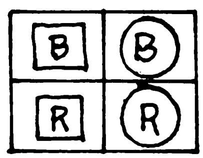

Explain the differences as the pieces are selected.

C. Provide the children with a grid (matrix) such as the following.



The pieces must differ from each other vertically and horizontally by 1 difference: e.g.,



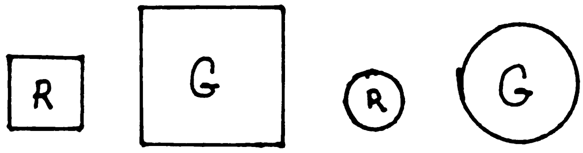

Other answers are possible.

**EXTENSION - INDEPENDENT PRACTICE:**
Tonight for homework play "Find a Pattern" at home. Write down the patterns you find and state the rule used to make the pattern. You may also find patterns somewhere around your school.

The analysis of this lesson focuses on the mathematics children have an opportunity to learn and the intellectual abilities being developed. The lesson begins in the concrete world of children. Finding patterns in their environment could help children see how mathematical ideas are a part of the world in which they live. More than recognition is expected. Children are to describe the "rule" used to form the patterns. In their own language, children could begin to describe the regularity and repetition they find in elements of their surroundings.

The lesson continues at the concrete level with attribute blocks. Teacher questions direct children to attend to the attributes of each piece: color -- red, blue or green; size -- big or little; and shape -- square, circle or triangle. Teacher questions then ask children to compare a block with pieces in their set. Responding correctly to teacher questions requires knowing the ways in which attribute pieces are alike and ways in which they differ. The activity allows for more than one correct response. For example, if the teacher displays a small red triangle, children could select several pieces that differ by one or two attributes. This series of activities helps children develop ways to classify the blocks in their set and see relationships among the various pieces.

The several versions of "Guess My Rule" provide a problem solving context in which children reason about what features seem to be important as a pattern is being created, when a set of blocks appears to repeat, and what the nature of the pattern is. The context also provides an opportunity for patterns within patterns (although this is not mentioned at this point in the scripted lesson). For example, children might see several different patterns in the following arrangement depending on the attribute around which they think the pattern has been created.




One pattern is red, green, red, green. A second pattern is little, big, little, big. A third pattern is square, square, circle, circle. Adding a small red triangle as the next piece in the sequence would invite children to see if their initial idea needs to be revised. The activites provide an interesting context in which creative thinking and logical reasoning are encouraged.

This initial set of activities focuses on the three attributes -- size, shape and color -- that distinguish each block from every other. The "challenge" later in the lesson provides a problem situation in which children consider if it is possible to create a pattern where the pieces differ in four ways. This could lead children to consider the position or orientation in which the pieces are placed on the table--as is suggested in the lesson.

Following the teacher led activities where children discover patterns and the "rules" by which they are created, youngsters are given the opportunity to create their own patterns. As one child creates a pattern, another guesses the "rule" and then extends the pattern. Such an activity could provide the occasion for some very creative pattern-making while helping children develop their ability to classify and to reason mathematically. It provides a context in which children could talk about mathematical ideas. The "rule" by which one child creates a particular pattern might not be unique (see the example above). In seeing another "rule" and justifying it by extending the pattern, youngsters are actively engaged in constructing meaning in a situation where more than one "right" solution is possible.

The extensions -- making "one difference" trains and completing a two-by-two matrix -- provide further problem situations in which to explore relationships. Introducing the matrix affords youngsters another form in which to organize and present information. Whereas previous activities employ a linear organization of the attribute pieces, the matrix suggests a different representation in which relationships can be explored vertically, horizontally and along the diagonals.

This lesson embodies elements of teaching for **enabling mathematical inquiry**. The conceptualization of the lesson, its connection to the real-world of children, the use of interesting problem situations and the use of different forms in which to classify, organize and present information attempts to develop a range of intellectual tools to enhance children's mathematical abilities. The lesson draws on youngster's abilities to reason, to communicate about mathematical ideas, to invent, generalize and create. The lesson extends the child's conception of the world of mathematics beyond numbers and computation to include the patterns and regularities that are part of the naturally geometric world they encounter, both in and out of the classroom.

Unfortunately, few of the model lessons approached this one in terms of its potential to develop children's abilities to reason mathematically. More often than not, the model lessons seemed more focused on instructional technique and "fun" activities than the substance of the mathematics. Model Lessons 8 and 4 and 12 exemplify those model lessons that seemed unlikely to develop a set of intellectual tools that could help youngsters become self-confident problem solvers.

<u>Model lessons unlikely to develop a set of intellectual tools for problem solving</u>. Model Lesson 8 was created for an objective in the functions and relations strand. The lesson begins with the statement of the objective.

> **OBJECTIVE**: The student will be able to identify the missing relationship or operational symbol in an incomplete number sentence.

The vocabulary for the lesson includes the following terms; open, true, number sentence, relative, table, greater than, less than. The lesson begins.

> **MOTIVATION**: Ask the students to tell you what the word "relative" means. Have them give you examples. (mother, aunt, uncle, etc.) Say: "Today we are going to make a table that is 'related' to an open number sentence. (Note: You may have to review the words: table, true and open number sentences from the previous lessons.)
>
> **INSTRUCTIONAL INPUT**:
> Write on the board: $\square + 3 = \triangle$
>
> Ask: "What do the $\square$ and $\triangle$ represent?" (numbers)
>
> Say: "Let's put numbers in the and to change our open sentence to a true sentence. Let's write our true sentences and then make a table to represent our sentences."

$\square + 3 = \triangle$
$\boxed{1} + 3 = 4$
$\boxed{0} + 3 = 3$
$\boxed{7} + 3 = 10$
$\boxed{6} + 3 = 9$
$\boxed{3} + 3 = 6$

TABLE

| $\square$ +3 | $\triangle$ |
|---|---|
| 1 | 4 |
| 0 | 3 |
| 7 | 10 |
| 6 | 9 |
| 3 | 6 |

Do several other sentences with the students, using addition and subtraction.

In the guided practice, inequalities are introduced.

**GUIDED PRACTICE:**
Say: "Let's look at some open sentences that contain the greater than or less than symbol."

Write on the board: $\square + 3 > \triangle$

Ask: "What are some true sentences?" (Examples are given.)

Say: "Let's make a table related to our sentences." (See below)

| $\square$ | $\triangle$ |
|---|---|
| 1 | 3 |
| 3 | 3 |

Divide the class into pairs and ask them to write true number sentences using less than or greater than. They are to write a related table for each as demonstrated.

Select several students to write the true sentences on the board and the related table - sharing various strategies and answers.

Independent practice calls for the following.

**INDEPENDENT PRACTICE:**
Have the students make up three (3) open sentences, giving at least five (5) true sentences and constructing a "related" table. Or use these samples.

$\triangle + \square = 7$
$\triangle + \bigcirc = 5$
$7 - \square = \bigcirc$
$\triangle - \square = 3$
$6 - \square = \triangle$

The lesson ends with closure.

**CLOSURE:**
Ask: "How do these tables differ from a function table?" (The output number is unique in a function table but not in a 'related' table.) "How are they alike?" (input, output, rule and relationship)

Model Lesson 8 is poorly conceived. As in most lessons, the single attempt to link the objective with the world of children is the motivational phase of the lesson. In this part of the lesson, "related to" is linked to youngsters' understanding of the word "relative." The use of the terms "relative" and "related to" in the context of this particular lesson is rather peculiar. It is not clear how calling on children's prior knowledge about family relationships is likely to facilitate their understanding of mathematical relations.[6]

The activities in the lesson do not seem to follow from the stated objective. The objective is to "identify the missing relationship or operational symbol in an incomplete number sentence." But problems for

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[6]The motivational part of the lessons was curious. In most cases it tended to be a teacher statement telling students what they would be doing during the lesson. In some instances it was a story or problem. For example, Model lesson 2, from the numeration strand begins with the following motivation.

> Teacher presents the following problem: "____________(name a student in your class), wanted to buy a Big Mac. She/he had 75 cents. The Big Mac cost 85 cents. Could she/he buy the Big Mac? (wait for yes or no responses) If she/he had 95 cents, could she/he buy it? Today we are going to explore an easy way of solving problems like these.

Model lesson 10 from the numeration strand focuses on writing three digit numbers in expanded notation. For motivation, the teacher is instructed to say,

> Today we are going to expand numbers like a rubber band.

Model lesson 18 on estimating by rounding numbers to the nearest multiple of ten, begins with the following:

> Review by counting by tens. Say: "There was a little town call(sic) Ten-town. No other numbers but tens lived there, or so they thought. One day 38 came to Ten-town. All the tens told him/her to leave because he wasn't a ten. Thirty-eight saw the King of Ten-town and ran over to him. The King whispered something in his ear. Thirty-eight belongs to Ten-town. What was the King's rule? Today we are going to find out what the King's rule was."

It is not clear how these motivational statements might inspire children to engage in the upcoming lesson. Nor is it clear how they provided "hooks" into prior knowledge or experiences. In some cases, there seemed to be no logical link between the motivation and the central idea of the lesson, Model lesson 8 being a case in point.

teacher demonstration and student practice involve supplying missing *numbers* in an open sentence. The lesson is marked by inconsistency in the use of terms "relation," "related to," "related table," and "function table." In the introduction to the functions and relations strand, teachers were advised that the terms functions and relations not be defined "*too precisely mathematically*."

> Function: Special or typical work or purpose or thing or person - to do its work:act, e.g., a table can function as a desk or the function of the brakes is to stop the car or...+5 means to add five to a number you started with...Relations are simply 'connections.' Example: Brother is related to sister (p. xi).

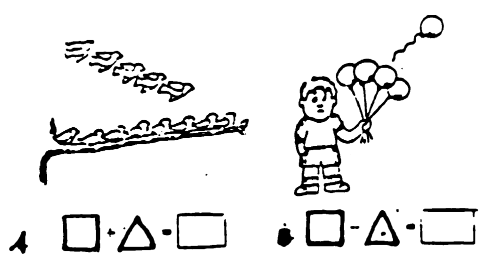
In this lesson, the distiction made between a function and a relation is that a function yields a unique output for a given input whereas a relation may yield more than one output. For example, $x + 5 = y$ is a function because a given value for x gives a unique value for y. The open sentence $x + 5 > y$ is a relation since for a given value of x there are many values for y that make the statement true. The term "related to" is applied only to those tables constructed from inequalities. These tables are distinguished from "function tables," constructed from equalities. The directions for independent practice call for constructing a "related table" for a set of open sentences. *Yet every open sentence in the set is an equation, a function*!

The notion of beginning with the concrete and using pictorial representations as a bridge from the concrete to the abstract seemed to be treated rather simplistically in this and other model lessons. While this lesson represented a mathematical idea solely at the abstract/symbolic level, it followed a lesson that employed concrete materials (counters) and pictorial representations. In Model Lesson 7, devoted to the same objective as Lesson 8, the instructional input phase begins with a story.

> We had five puppies at home. Two ran away. How many puppies are still at home?

The lesson moves from illustrating the story with counters to drawing five dots and crossing out two of them to then writing the problem symbolically, 5 - 2 = 3. The pace with which the lesson moves from one representation to the next suggests that making connections among multiple representations comes easily and naturally for children.

Creating appropriate pictures for stories -- or stories for pictures -- and then translating those forms into the symbolic language of mathematics is a complex set of ideas for children (see Hiebert, 1980). The confusion that youngsters might be expected to encounter, especially in moving from the pictorial to the symbolic, is exacerbated in Model Lesson 7 by the examples themselves and what constitutes "right" answers. The two drawings below are given for teachers to demonstrate. The shapes are to be "replaced by unique numbers" to make each open sentence a true sentence that reflects the mathematical idea in the drawing.

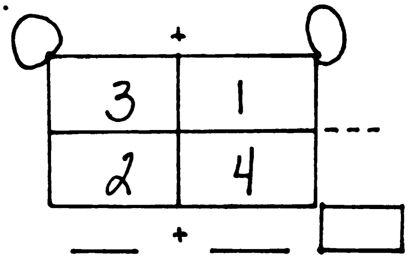


As in most model lessons, "correct" answers are provided for teachers. The "correct" true sentence for drawing A is "5 + 3 = 8." The "correct" true sentence for drawing B is "4 - 1 = 3." It is not clear why those and not others are the "right" answers. For example, 5 + 8 = 13 and 5 - 1 = 4 are interpretations that children might reasonably and logically make given the pictorial representations of the situations. In fact, it is probably likely that children would have considerable difficulty making sense of the lesson's "right answers."

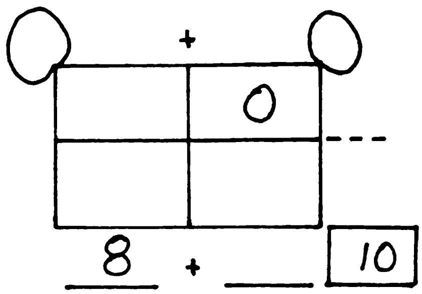
Model Lesson 8 typified most of the other model lessons in that it treated a mathematical idea outside a real-world context. No situations were provided from the real-world of children which would give rise to the mathematical concepts of the lesson. In contrast to Model Lesson 6 where a broad range of intellectual tools were being developed to enhance children's mathematical reasoning abilities, Model Lessons 7 and 8 pursued a more limited set of goals. Getting to the symbolic representation and becoming proficient in symbol manipulation seemed to be the goals that drove the lessons.

Model Lessons 4 and 12 are devoted to recall of addition and multiplication facts. They are included here as examples of missed opportunities to engage in further inquiry about the mathematical ideas embedded in the context. Both lessons use cross number problems as the environment in which children practice recalling addition and multiplication facts. A cross number problem is one in which sums (or products) can be obtained using three different sets of addends (or factors). For example, the following cross number problem in Model

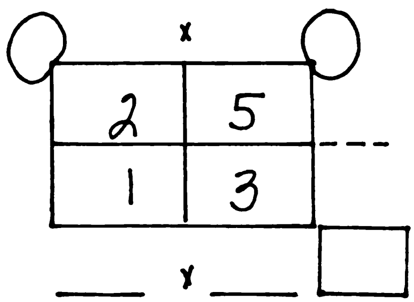
Lesson 4 "works" because adding across gives 4 and 6 whose sum is 10, adding down gives 5 and 5 whose sum is 10, and adding diagonally gives 7 and 3 whose sum is 10.




In some problems, the sum or partial sums are given and children fill in the missing pieces.




A similar approach is used in Model Lesson 12 with multiplication.

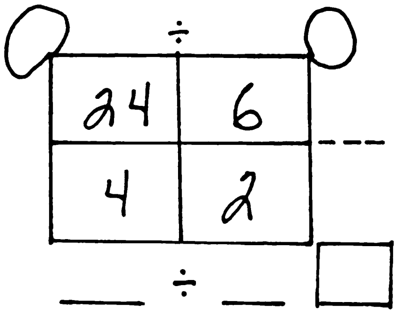



Because the objective of the lessons is simply to recall facts, interesting investigations are not pursued. For example, can a cross number problem with some elements supplied be completed in more than one way? What is the nature of numbers that "work" for cross addition or cross multiplication? How many "clues" are required to complete a cross number problem? Can those clues be provided in any of the eleven places? Is there a systematic way in which to solve these problems?

Cross number problems could have been used to extend children's understanding of addends and factors but the opportunity is missed. Rather than using this as an occasion to extend students' learning about numbers and operations with numbers, becoming more proficient in the recall of basic facts is an end in itself.

Model Lesson 12 incorporated an "challenge" activity for "children proficient in division."

1. "As a challenge, find out if this one works." (no)




2. "Can you make it work?" (It will work if you plan ahead or if you multiply the diagonals to avoid "ear trouble."
24 ÷ 6 = 4, 4 ÷ 2 = 2 [horizontal]
24 ÷ 4 = 6, 6 ÷ 2 = 3 [vertical]
24 x 2 = 48, 6 x 4 = 24 [diagonal]

3. "Let's prove our results."
4 ÷ 2 = 2 [vertical]
6 ÷ 3 = 2 [horizontal]
48 ÷ 24 = 2 [diagonal]

This optional activity underscores the extent to which manipulating symbols took precedence over "sense-making" in some of the lessons. There are no questions posed to inquire about what "plan ahead" might mean or how one would go about it so cross division problems would "work." Nor are there questions about why multiplying along the diagonal might make sense or if that would always "work." Recall of basic facts is removed from the typical format of rows of problems for drill-and-practice. But the narrow focus of the objective results in a missed opportunity to explore the richness of the mathematics embedded in the situation.

The majority of model lessons were limited in terms of the conception of mathematical content and the goals for student learning. While a few were exemplary in their attempt to enhance children's abilities to reason mathematically and to make connections between the world of mathematics and the real-world of children, many were mediocre attempts to wrap computational proficiency in new packaging. My discussion of the limitations of these lessons as manifested in their text does not imply that teachers would not be able to do something other than what the lesson, as text, does. As I stated in Chapter 5, curriculum texts dod not assure or even predict the behavior of teachers or students in the classroom. What I am arguing is that nearly all these model lessons represent a view of mathematics and an approach to mathematics teaching oriented toward certification of numeracy and comprehension of mathematical ideas.

The potential in these lessons to promote higher-order thinking and problem solving abilities was limited and was, in part, a function of the way in which mathematical topics were conceived and objectives and activities selected. But the limitation of these lessons was also embedded in the instructional model that informed the design of the lessons. The next section examines the instructional model to determine the extent to which curricular form enhanced or constrained teaching for understanding.

**The Form of the Model Lessons**

Each model lesson was constructed using the same format; grade, strand, objective, materials, vocabulary, motivation, instructional input, guided practice, independent practice and closure. The curricular form was influenced by research on teaching, in particular, the work of Hunter (1976, 1984) and Good, Grouws and Ebmeier (1983). Hunter's work has aimed at categorizing the acts of teaching and learning -- performing task analyses, sequencing instruction, developing a "professional vocabulary" to describe it all -- and identifying the "scientifically-based" cause and effect relationships that exist between teaching and learning. The aim is to increase the speed and efficiency of teaching and learning (Costa, 1984). To that end, Hunter has turned a select set of findings from effective teaching research into a recipe for "effective lesson design." Hunter's elements of an effectively designed lesson include anticipatory set, objective and purpose, input, modeling, checking for understanding, guided practice and independent practice. Hunter has been the inspiration for a number of educational entrepreneurs who market staff development workshops modeled on her program for lesson planning, instruction and classroom management.

The Detroit schools had invested heavily in a version called Essential Elements of Effective Instruction (EEEI), known locally as "triple E I." In 1983, the district committed to provide 30 hours of inservice for all its administrative, supervisory and instructional personnel. The program was to be conducted over a period of five years. Initially, the inservice providers were staff development personnel from the intermediate school district. Over the years, in-house personnel were trained as trainers and they replaced the original inservice providers. EEEI had become a powerful regulator of teacher behavior. Not only were elements incorporated in these Model lessons, they were also used by supervisory personnel as criteria for assessing teacher competence.[7]

The second influence on the form of these model lessons was the work of Good, Grouws and Ebmeier referred to in Chapter 2. Using results from process/product and experimental studies, these researchers defined a teaching model for mathematics. The elements of an effective lesson from this perspective include beginning a lesson with a mental arithmetic activity, devoting at least 20 minutes to the development portion of the lesson and increasing student participation through guided practice before assigning independent practice. The model calls for active teaching where the teacher gives clear explanations, monitors student understanding with product and process questions, provides immediate and corrective feedback, and allows for distributed and successful practice.

There is a clear compatability between the models of teaching derived from Hunter and Good et al. The first model, grounded in research on teaching not directly related to mathematics, prescribes a set of generic teacher behaviors that are assumed to be applicable to any content area. The second model aims to "improve traditional mathematics teaching by making it more efficient or effective" (Romberg and Carpenter, 1986). Both offer a "technology" of instruction. Both

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[7]Teachers and administrators used the term "triple E I" without explanation--it was a common schoolhouse term in the district. Nearly every teacher with whom I talked could recall without difficulty the teacher behaviors associated with EEEI.

are direct instruction models that aim for efficiency in teaching and learning. Neither model is definitive about the specific content of a lesson. The emphasis is not on what should be taught - content is ignored or is taken for granted. The emphasis is on controlling the pace of instruction, demonstrating what students need to learn, attending to student understanding through questioning, and holding learners accountable for work.

Every model lesson bore the stamp of the merging of elements from these singular approaches to instructional design. Interviews with mathematics specialists in the district who had worked on the model lessons acknowledged the influence of "EEEI" and "Good and Grouws." They felt that Good and Grouws had not given much direction about the development portion of the lesson and therefore the lesson writing team had tried to be specific about what teachers should do during the instructional input phase of the lesson. The instructional input of each lesson instructed teachers how to conduct the lesson: demonstrations and concrete materials to model an idea; problems to use for demonstration purposes; charts and diagrams to write at the board or overhead to facilitate instruction; questions to ask students (along with the correct student response); activities to model and then have students practice. The role of the teacher was to show students what they needed to know by demonstrating to the whole class and then providing exercises for guided and independent practice.

Teachers were to monitor student learning by posing questions during the instructional input, guided practice and closure phases of the lesson. Teachers were told to ask product and process questions. Product questions required a simple number or word response (e.g., What are the sums when we add across? What did we do to 73 to get 83?) Process questions, which tended to be asked at the end of work on a set of problems, required more. Some required the interpretation of data (e.g., How many more/less birthdays are in May than June?). Some allowed for a variety of responses (e.g., How are these numbers alike? How are they different?) A few were intended to elicit a discussion about solutions and strategies (e.g., Where would you start to solve this problem? What did you do first? Why? Did you do it a different way? Tell us about it?) Despite the variety in form, process questions served mostly as another check of correctness.

Lessons ended with closure. In some lessons, teachers were instructed to ask students to tell what they learned from the day's lesson, occasionally sharing their learning with a neighbor. In others, teachers were instructed to discuss solutions and strategies with children. In a few, teachers asked students to compare the topic of the day's lesson with a topic covered earlier. And sometimes the teacher was instructed to review or summarize for the children.

The instructional model assumed that learners absorb what has been taught. The emphasis was on managing instructional delivery -- maintaining a brisk pace of direct instruction, structuring information and presenting it to students, monitoring their performance and providing corrective feedback, keeping them on task and holding them accountable to produce completed exercises. The model supported a view of mathematics as "well-organized and analyzed" knowledge and sets of skills that "can be presented (explained, modeled) systematically and then practiced or applied during activities that call for student performance that can be evaluated" (Brophy and Good, 1986; p. 130). It supported a view of learning mathematical concepts and processes by attending to teacher demonstrations and practicing what has been shown. Questions occasionally asked children to be reflective about the process they had used. But questions were never posed that invited children to make conjectures, pursue further hunches or make convincing arguments.

The Introduction to the Model Lessons described developing critical thinking and reasoning skills and problem solving abilities as goals that informed the lesson writing effort. The foregoing analysis suggests there were several constraints toward achieving those goals. One set of constraints was mathematical: objectives that focused on narrow bits and pieces of mathematics; the superficial treatment of topics rather than extended explorations to capitalize on the richness of the mathematics embedded in a lesson; inconsistencies in content among objective, motivation, instructional input and practice activities within a lesson; a naive faith that the sequence concrete to pictorial to abstract would automatically lead to student understanding of mathematical ideas.

The second set of constraints resided in the management approach to instruction embedded in the instructional model. There is a growing body of research on mathematics teaching and learning that is calling into question the efficacy of the direct instruction model to promote higher-order reasoning and critical thinking (Doyle, 1983; Peterson, 1979; Peterson and Fennema, 1985). Peterson (1988) suggests that

> Higher-order thinking may require a less direct instructional approach that transfers some of the burden for teaching and learning from the teacher to the student and promotes greater student autonomy and independence in the teaching-learning process (pp.5-6).

The model lessons and the instructional delivery system did not facilitate a shift away from the teacher as the sole source of epistemoligical authority. The teacher continued to be the authority for knowing, deciding if student answers were correct and explanations sufficient.

## SUMMARY

The model lessons appeared to grow out of an orientation that moved between **certification of numeracy** and **comprehension of mathematical ideas**. Learning was portrayed as requiring a particular sequence of development moving from concrete to pictorial to symbolic. Concrete and pictorial representations were used with some consistency to model situations and operations. In several lessons where work was primarily at the symbolic level, i.e., writing number sentences, constructing input/output tables, writing numbers in expanded notation, teachers were reminded to return to the use of concrete materials if children were having difficulty.

It was clear from the design of the lessons and the directions for "instructional input" that the role of the teacher was to show students what they were to learn by demonstrating to the whole class and then providing exercises for "guided" practice. Within a structure imposed by the model lessons, students were expected to pursue activities, answer teacher questions and complete assignments for independent practice. In many instances not only were they required to provide correct answers but also explanations for procedures used. A few lessons incorporated games as a fun activity but these were of a fairly routine nature. They did not require logical reasoning or mathematical knowledge to develop game winning strategies. Rather, they were games of chance adapted to include practice in basic skills.

A few lessons attempted to enhance children's abilities to reason mathematically and to see connections between the world of mathematics and the real-world around them. But the majority made scant effort to develop children's higher-order thinking and problem solving abilities. This was a function, in part, of the way lessons were conceived in terms of the mathematical content, the learning objectives, and the instructional activities. But it was also a function of a model for teaching that emphasized management of instruction, that assumed learning required breaking subject matter into discrete pieces for students to master, and that divorced mathematics education from inquiry.

# CHAPTER 7

# INSERVICE FOR TEACHERS CHARGED WITH IMPLEMENTATION

## INTRODUCTION

The previous two chapters focused on the formal curriculum documents--the mathematical strands that organized the curriculum, the objectives that sequenced the content for instruction, and the model lessons that provided examples of how objectives were to be taught in the classroom. The analysis suggested that the reform embodied a tension between teaching for **certification of numeracy** and teaching for **comprehension of mathematical ideas**. The argument was made that the potential to develop children's critical thinking and higher-order reasoning and problem solving abilities was constrained by several factors: scores of objectives that atomized content and focused on narrow bits and pieces of mathematics; a pace of instruction demanded by the need to cover over one-hundred behavioral objectives that limited the time available to explore new and complex ideas; a direct instruction model that embodied a management approach to instruction.

However, curriculum reform remains political and symbolic until consideration is given to what is required for its implementation. The reform represents a commitment to the community to improve the education of its children. If implementation is compromised, the reform exists only as political rhetoric and a curriculum on paper. Since teachers are the primary agents of curriculum change in the classroom, the ways in which they are prepared to implement new curricula are fundamental to assessing the potential of a reform to make a difference in the learning opportunities for children. This chapter examines the inservice opportunities that were provided for elementary teachers in Detroit charged with implementing the newly-mandated mathematics curriculum.

The two elementary mathematics supervisors in the Mathematics and Science Department planned and conducted a series of workshops to help teachers implement the mathematics curriculum. This chapter examines the inservice conducted by Marilyn Miller,[1] one of the supervisors. The investigation takes at look at both the context and content of inservice. The first section introduces Marilyn Miller by briefly describing some of her history as a mathematics educator, how she thought about her role as supervisor, and what ideas were central to how she thought about inservice. The second section examines the context of inservice in terms of the structure of the inservice sessions and the settings of participating teachers' practice. The third section examines the content of inservice to determine how "teaching mathematics for conceptual understanding" was made manifest in the enacted curriculum of inservice. Finally, the inservice programs are analyzed as opportunities for teacher learning. Using a framework developed by Ball and Wilcox (1989), the discussion inquires about (1) assumptions about what teachers needed to know to implement the curriculum; (2) assumptions about how teachers would learn what they needed to know; and 3) the model of changing teachers' practices that underpinned inservice.

## THE PROVIDER OF INSERVICE

Marilyn Miller had worked in the Detroit schools since the mid-1970s as a teacher, a teacher consultant and most recently as an

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[1]Marilyn Miller is a pseudonym.

elementary mathematics supervisor. In the early 1980s, she served as a consultant to elementary teachers in several schools who were implementing an innovative instructional program from CEMREL called the Comprehensive School Mathematics Program (CSMP). CSMP had a number of distinctive features: 1) it included content from probability, set theory, relations and functions; 2) problem solving provided a context for generating topics and developing computation skills; 3) the curriculum was spiraled--a topic was treated briefly for one lesson and then returned to several days later; 4) the program provided teachers with a set of highly detailed, scripted lessons that specified the sequence of tasks and the questioning techniques to carry out whole group instruction. Marilyn trained teachers using CSMP in both content and pedagogy. Her staff development work with CSMP ended when the district lost the resources that had funded the experimental program.

With funding for CSMP teacher consultants no longer available, Marilyn returned to the classroom where she herself used CSMP materials in instruction. In the mid-80s, she took a leave and entered graduate school to pursue doctoral studies in mathematics education and curriculum. She returned to the district in January, 1987, as an elementary mathematics supervisor. Part of her responsibilty was to oversee mathematics instruction in over 80 elementary schools in the district!

During her doctoral studies, Marilyn encountered research on effective teaching and effective mathematics instruction. This was to have an important influence on how she thought about her role as a supervisor.

> I did a lot of reading during the Ph.D on effective instruction. What I don't like about Madeline Hunter is that she really doesn't

talk much about higher-level thinking and actually have people talk about how they are thinking. In this system you are not authentic unless you go through the (EEEI) training so I went through the training. It was exactly what I had read. And the criticisms I had still stuck. Her model doesn't help you where you really need it. It doesn't tell you how to teach the lesson, how to develop the idea of the lesson. Neither can Good and Grouws. This is where the supervisor's job comes in - inservice, working with teachers, showing them the steps in how to develop a lesson.

Sometimes I feel like I'm a salesperson. I'm selling mathematics, selling the proper teaching of mathematics. That's why I want to do inservice. The other supervisor is having Area instructional specialists provide inservice. I think my doing this shows more of a bottom up rather than top down. They (teachers) need to feel some kind of stake in change...I'm new so I need to establish my credibility. So I try to deal with the affective as well as the cognitive domain. I want them to know me as a person, as someone who has this information and is willing to share.

Marilyn was deeply committed to changing the typical practice of teachers in mathematics classrooms. She described Detroit as "ahead of a lot of other systems." She felt the organization of the elementary curriculum around the mathematical strands was innovative. But she viewed the objectives and the instructional sequence as transitional until there were textbooks that fit a more conceptual approach to mathematics teaching and learning.

The objectives and the instructional sequence don't promote conceptual understanding. Teachers can read these and they might not be able to pick up the concepts. Hopefully the model lessons provide a model in each strand. But you have to spend time on a concept. Like in subtraction, maybe two weeks worth of lessons. That's one of the problems with our instructional sequence. That's why I'm not a real stickler. My vision does not fit with our instructional sequence at all. It is a band-aid approach until we get a new book. Then we can revisit the sequence in light of the new text...I'm trying to model in inservice that lesson, what good mathematics teaching is, from top to bottom. I'm trying to model a caring kind of feeling which I think teachers should have to kids so that I realize there are negative forces that exist, but I try to maintain a positive attitude toward them and the kids.

Marilyn considered inservice her strength and a key element in reforming the teaching and learning of mathematics in the district. In the short time she had been a supervisor, she had established a reputation among top administrators as a dynamic, intelligent mathematics educator. One senior administrator said he would pay careful attention to anything she had to say.

## THE CONTEXT OF INSERVICE

### The Structure of the Inservice Sessions

Marilyn and her colleague planned a nine-hour workshop series to be conducted during this first year of the implementation effort. Inservice was organized by Area and by grade level with the following pairings: K-1; 2-3; and 4-5. Each supervisor planned a total of nine workshops among the three Areas for which she was responsible. As the junior supervisor, Marilyn acquiesed to her colleague's suggestion that the nine hours be scheduled in three sessions in a single week--as opposed to being offered over a more extended period of time. Although Marilyn thought such an arrangement would not provide an optimum environment, she had been in her position less than a year and was reluctant to press the matter with a senior member of the department. Each series of workshops was held after school on Monday, Tuesday and Thursday from 4:00 - 7:00 p.m. Inservice for four of the Areas was conducted at an educational center at a local private college. Attendance for many teachers in these Areas required a considerable drive from their schools. Workshops for the remaining two Areas were held in a centrally located middle school.

Teachers self-selected to participate and were paid the contractual stipend of $12.64 per hour. Attendance at each workshop series was limited to roughly 45 participants. Every staff member's request to participate was honored. While the total number of participants preregistered for Marilyn's nine workshops was near 400, the number who attended was less than 300.[2] Several principals and assistant principals also took advantage of these inservice opportunities. One assistant principal explained her participation.

> I come to these workshops to become better acquainted with the curriculum strands and objectives. I think as an administrator I need to be knowledgeable about the new curriculum. By being here I show support for teachers in my building who are trying to implement the instructional sequence. And it's a message to my people that I think this deserves the best efforts of all of us.

At the conclusion of the first series for K-1 teachers, Marilyn expressed extreme dissatisfaction with the schedule to which she had reluctantly agreed.

> This scheduling is bad. These teachers are already tired when they come after a full day at school. And this way doesn't give them time to think about what we have done or try some things and then come back and talk about it. You need more time to pound in this conceptual development. I'm <u>not</u> going to agree to this next time. I'm tired too by the third day. She (her colleague) organized this but she doesn't give the inservice. She gets three specialists to do it. Well, I don't want to do that.

Marilyn was not alone in her assessment of the constraints imposed by the schedule. At each series of workshops, attendance fell off on the third day. Some teachers found it difficult to muster the energy to engage thoughtfully in some of the activities. One instance called for teachers to read two short articles on research in mathematics and then tell a partner what they had learned. At one workshop, a teacher rolled her eyes, stifled a yawn, and commented to a colleague at her table,

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[2]This discrepancy between pre-registration and attendance was partly explained by the fact that two of the workshop series were held in December in the two weeks prior to the holiday break. Teachers who did attend reported that this was an especially "hectic" time in their buildings given the additional activities for observing and celebrating the holidays.

> These articles are just too heavy for this time of day. I don't want to do this. I'm just going through the motions.

The others at her table agreed.

Some teachers who chose not to attend the workshops said they made their decision based on when the inservice was offered. At one school where I met informally at lunch with a group of teachers, not a single faculty member planned to take advantage of the inservice program. As one teacher put it,

> Why are teachers expected to attend these meetings on their own time? If this is so important why don't they provide for this during the school day? They send teachers to EEEI training and give them five full days of released time. This just says to me that they don't think this is all that important.

There was an additional complication for those in this school who might otherwise have been interested in the inservice. The school was on a late schedule to accommodate student bussing. As a consequence, teachers were not released from their buildings until after 3:45, making attendance at a 4:00 workshop held elsewhere difficult, at best.

The structural features of the city-wide workshops were a constraint on Marilyn's ability to provide a set of experiences that engaged and challenged teachers to explore new ideas, reflect on current practice, try new activities and arrangements in their classrooms, and talk with others about attempts at these new efforts. Teachers came to these workshops after a full day's work--as did Marilyn herself. Teacher fatigue was evident in stifled yawns, drooping eyelids and nodding heads, and, in some cases, minimum engagement in some of the workshop activities, especially those that required more intellectual investment.[3]

**The Settings of Participating Teachers' Practice**

This inservice effort must be understood within a broader set of contextual factors that had an impact on the worklife of teachers in the district. Nearly two decades of industrial dislocation and economic disinvestment from the city had significantly reduced the district's ability to provide a quality education program. A universal complaint from teachers was the lack of materials in their classrooms, especially textbooks and paper. The new curriculum embraced the use of calculators in the mathematics classroom, yet few teachers who attended the workshop series had a set of calculators for their students. The mathematics supervisor promoted the use of concrete materials to introduce mathematical ideas, in particular, attribute blocks, base 10 materials, and geoboards. But the district did not allocate funds to schools or individual teachers to purchase these materials. The

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[3]This was in sharp contrast to the level of energy and engagement of the math lab teachers from one Area at their quarterly meetings. These meetings were held in the Area office and were scheduled during the school day. Math lab teachers were given released time from their schools to attend. The Area curriculum director described this group of teachers as often feeling like "the stepchildren in the area," unappreciated for their special efforts with children, not included in professional development activities. He had invited Marilyn to three of their meetings to demonstrate the use of manipulatives. She had chosen attribute blocks, base 10 materials, and geoboards as the concrete materials she thought would be most useful. During the morning sessions, teachers were immersed in explorations with the materials, working with colleagues, asking questions about the multiple uses of each set of materials. At the end of the first workshop, one participant said to me, "Isn't she (Marilyn) wonderful! She makes you feel like you can conquer the world." And with that she rushed down to the materials center and made her own set of base ten materials to take back to her lab.

conditions of teachers' work was further affected by the social dislocation that accompanied economic disinvestment and increased impoverishment of families. Many teachers found themselves trying to teach a classroom of youngsters whose population was transient. Some teachers reported that they had experienced years where only half the class membership remained stable.

Structural changes in the economy were coupled with significant changes in the district's organizational structure. The recent history of decentralization, then recentralization, had left considerable confusion about who was in charge and to whose directive teachers were bound. While the central administration was telling teachers to follow the sequence of instructional objectives that accompanied the new curriculum "in the order listed," some Area and building administrators were ordering teachers to concentrate on a narrow set of computational skills assessed by the various tests administered city-wide.

Teachers had been mandated to implement city-wide curricula in <u>seven</u> content areas--language arts, social studies, mathematics, science, music, art, and health/physical education. This initiative represented not only an attempt to equalize learning opportunities across the district but was a response to considerable pressure to increase student scores on state-administered tests of educational achievement. The mandate also reflected a continuing effort by central administrators to further reduce the authority of Area superintendents, exert increasing control and centralization over curriculum, and fashion curriculum in a way that supported instruction to improve test scores.

The response to the mandate among teachers with whom I spoke was mixed. Some welcomed the reshaping of mathematics content around strands as evidence that the district was moving away from a "back-to-basics, minimum competency, teach-to-the-test" orientation. Others felt the content had not really changed, that the strands were "just fancy names for things we have always taught." Others felt some of the new objectives were "too sophisticated for the kinds of kids we get." Regardless of their opinion about the appropriateness of the new mathematics curriculum, the majority who attended inservice felt overwhelmed by the demand to simultaneously implement newly mandated curricula in <u>all</u> content areas. And some displayed skepticism about the district's long-term commitment to the new curriculum. At one inservice, a teacher questioned Marilyn:

> How long are we going to do this (use the strands and objectives)? Are we going to be doing something else in three years? I'm trying to learn this but if I do are you just going to snatch this away in a couple of years and then we'll have to learn something new?

Many teachers wanted assurances that their efforts to become comfortable with the new curriculum strands and competent in following the instructional sequence would not be met by a new and different initiative a few years hence.

## THE CONTENT OF INSERVICE

The city-wide elementary mathematics curriculum was described by Marilyn as "conceptually-based." The meaning attached to "teaching for conceptual understanding" was evidenced by the ways in which Marilyn talked about mathematical concepts and concept development, what she believed teachers needed to know and do to teach for understanding, and how she modeled teaching for conceptual understanding in workshops.

**What Did Teaching for Conceptual Understanding Mean?**

Marilyn, to use her own words, was "very model oriented." At every point where she talked about "teaching a concept," the notion of modeling was key.

> Teaching for conceptual understanding is the most important process. It is not unique to math but it is the groundstone of learning...I am very model oriented - this is the model I want to see. The second day of the workshop we talk about concept understanding and I model it.

Model here had three meanings. First, Marilyn believed that teachers should introduce a mathematical idea with a manipulative that is a concrete representation - a model - of the idea. As she put it,

> Everything (in math) is a concept and you need that before other processes. Like you need to understand numeration before addition and before the algorithm. For every topic there is a model to develop that concept. Take place value and regrouping. Base ten materials are better than bundles of sticks because you can't take one from the ten strip...Sometimes it's hard to find the model for a particular concept but I know there is one. You need to find the right manipulative, the right material to model a concept. Then you need to go to the pictorial so that when kids are led to the symbolic they have a picture in their memory bank.

Marilyn incorporated a number of concrete materials. With lower elementary teachers, she used plastic objects for classification activities and pattern explorations. She used "squared materials" (a base 10 model) to develop an understanding of addition with and without regrouping. With upper elementary teachers she used attribute blocks to develop an understanding of the critical attributes of geometric shapes and as another environment for pattern explorations. She also used base 10 materials to develop the conventional long multiplication and division algorithms. At every inservice session, teachers used some kind of concrete materials as a mathematical model.

A second meaning Marilyn attached to the term model referred to the process by which she believed students learn concepts. On the second day of inservice, Marilyn introduced teachers to the "Rathmell Model for Mathematics Concept Development." The model specified five steps to concept development requiring multiple opportunities for children to 1) generate the idea and give multiple examples; 2) recognize instances of the idea -- what it is, what it is not; 3) represent the idea in three forms -- a concrete model, oral language and written symbols; 4) represent the idea from one form to another; and 5) learn the properties of the specific concepts.

The third meaning of the term model refered to lesson design and a mode of teaching the lesson. Marilyn described planning a lesson in the following way.

> The key to developing conceptual understanding is lesson development. I want to have teachers appreciate the need for more lesson development when they are planning to teach a concept. Look at the concept. Know what they are getting across. What is the model and what modeling needs to be done and what are the steps I need to take to model a concept. What are the questions I should ask to get at understanding. What materials do they need to practice with...Teachers should have a single objective in mind. It should be clear and stated to the learner in a way so they know what they are expected to learn. The focus of the lesson should be on that one specific piece of information.

Once a lesson had been planned, Marilyn believed there was a particular sequence that instruction should follow. She called it "the model for effective math instruction." The instructional components were 1) **mental arithmetic** -- five minutes of mental computation at the beginning of class; 2) **motivation** -- getting students into a "mathematical mode," recalling previous learning and relating it to the day's lesson; 3) **lesson development/instructional input** -- a minimum of 20-25 minutes where the "<u>teacher is teaching</u>", demonstrating with manipulatives that model the concept or process, asking product and process questions to check for student understanding; 4) **guided**

**practice**; 5) **independent practice**; and 7) **closure** -- students say what they have learned from the day's lesson. Marilyn explained to me how she came to this model of instruction.

> Parts of it came through my reading in research on math education. It comes a lot from those models already existing like Good and Grouws and Madeline Hunter which I know Detroit teachers are being inserviced in, so you have to tap into something that is already being pushed by the system.[4] Those kinds of models which research says makes effective instruction and my own experience with kids regarding manipulatives and the kinds of questions I know get responses from kids.

Marilyn told workshop participants,

> This is a model of instruction based on research. Research has identified what effective teachers do that lead to increased student achievement on standardized tests.

The ability to "teach for conceptual understanding" did not seem to require a sense of what makes a mathematical idea a concept. Nor did it require distinguishing between mathematical concepts, skills and processes. Everything tended to be a concept. Conceptual understanding was not linked to a notion of seeing connections among "chunks" or "nodes" of mathematical ideas. Rather, developing conceptual understanding was a technical and procedural matter: finding a concrete material to model the process; motivating students to engage in the lesson; demonstrating how to use the materials; asking two kinds of questions -- product and process -- to check for understanding; providing guided and independent practice; closing each lesson by having students tell what they did.

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[4]Marilyn did not mean this as a point of criticism of the District's investment in EEEI, its version of Madeline Hunter. In fact, she found most of the techniques of the training compatible with her favored model from Good and Grouws. Her criticism of both was that neither model gave teachers enough direction on exactly how to carry out the direct instruction phase of the lesson.

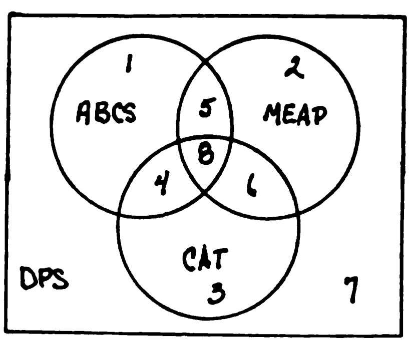
**The Curriculum of Teaching for Conceptual Understanding**

Marilyn had three main objectives for the city-wide inservice.

> First, I want to develop their understanding of what concept understanding is - the use of manipulatives and hands-on activities with lessons. I want them to become familiar with the research on effective math instruction. And I want to introduce them to cooperative learning, give them an opportunity to work in groups to see how that arrangement contributes to concept understanding.

While there were variations in teacher activities among the inservices depending on the grade level, there was a standard format that Marilyn used to organize each day's workshop.

Day one. The first day of the three-day inservice was devoted to two main topics; (1) defining the mathematical strands that organized the newly mandated curriculum and (2) introducing a specific model of cooperative learning. Teachers created their own definitions of each strand -- calculators and computers, estimation and approximation, functions and relations, geometry, measurement, numeration, operations, patterns, probability and statistics, and sets and logic --and then compared with Marilyn's version. A subsequent activity employed a Venn diagram. Teachers were given a handout that identified the mathematics strands that were tested by ABCS, MEAP and CAT along with the mathematical strands of the Detroit curriculum (DPS). Teachers were to locate the strands for each test and the DPS curriculum within the appropriate numbered section of the intersecting circles.



For example, measurement was a strand in ABCS, MEAP, CAT and DPS so it belonged in the section numbered 8. Algebra was a strand only in MEAP so it was correctly placed in section 2. Marilyn's purpose in selecting this activity was for teachers to realize that the new curriculum encompassed all topics assessed on the three major tests administered in the district. The message she wanted to convey to teachers and administrators was that if teachers taught to the objectives of the new curriculum, they would simultaneously and automatically be preparing youngsters for the various tests.

Marilyn produced a handout on different goal structures -- cooperative, competitive and individual -- and then provided a specific model of cooperative learning and the four characteristics of this model. Marilyn told teachers,

> Over 700 research studies support this structure for learning... Cooperative learning leads to higher achievement, more intrinsic motivation, better relationships, better attitudes toward teachers and the school, higher self-esteem, peer group support, more on-task behavior and better collaborative skills.

Interestingly, less than half-an-hour the first day was given to teacher cooperative group activities.

Day two. The content of the second day was a mixture of various activities. Marilyn told me before inservice the focus would be on "How shall we learn mathematics and problem solving?" She began with a review of the curriculum strands and the three learning goal structures. Then she handed out a "cooperative logic activity." Each group of four teachers was given an envelope with a bar graph and four clue cards, one for each group member. The clue cards read, 1) there are a total of 15 pets; 2) 6 pets are dogs; 3) there are 4 less gerbils than dogs; and 4) there are twice the number of cats than gerbils. To solve the puzzle, the group had to make a bar graph to show the number of dogs, cats, fish and gerbils. Members were not allowed to show their cards to each other. Solving the problem led to a discussion about cooperative learning and problem solving strategies.

The next activity was "jigsawing." Marilyn provided copies of six articles from Research Within Reach. The articles focused on estimation and mental arithmetic, problem solving, and manipulatives. Each teacher read two articles and then "taught" others at the table what she/he learned from the research. Then Marilyn produced a transparency that identified "five learning processes: technological applications, algorithmic procedures, mental strategies and estimation, problem solving and concept development." Workshop participants were asked to make connections between these learning processes and what they had just read in the articles on research.

As the final activity, teachers were introduced to the Rathmell Model of Concept Development and Marilyn elaborated the meaning of each step by demonstrating how a teacher could use the model to help a youngster understand the meaning of the number five.

Day three. The final day of inservice was devoted almost exclusively to "effective mathematics instruction." Marilyn spent about 30 minutes telling teachers about effective schools and effective teaching research. The researchers whose work she referenced were Edmonds, Hunter, and Good and Grouws. She indicated that these researchers had conducted their studies by observing teachers in classrooms and recording teacher behaviors. As a result, common teacher behaviors that correlated with increased student scores on standardized tests had been identified. Marilyn did indicate to teachers that she had a problem with using standardized tests as the sole measure of teaching effectiveness.

> I have a problem with scores on standardized tests. I'm not sure standardized tests are the only or the best tool.

While she questioned the measure of teaching effectiveness in the research to which she referred, she embraced the set of "effective" teacher behaviors that derived from it.

After introducing the subject of this body of research, she gave teachers a "Quiz on Effective Math Teaching" (see Appendix). Marilyn gave the correct answers and entertained discussion about each of the 23 items. Then she made a list of the common teacher behaviors associated with effective mathematics instruction.

- assess prerequisite skills
- state the objective
- warm-up - motivation - anticipatory set
- develop concept/idea - discussion, questions
- time-on-task
- systematic review and maintenance
- high teacher expectations
- comprehension/understanding
- practice - multiple ways besides paper-and-pencil
- informative feedback
- independent practice

This listing led to a discussion of what teachers needed to consider when planning a lesson. Teachers offered the following: objective; materials; procedure; time-line; evaluation; why are you teaching something; practice; and review. Marilyn accepted these as important considerations. Then she identified the components of an effective mathematics lesson and demonstrated by teaching a model lesson.

A considerable portion of inservice was spent acquainting teachers with research. Marilyn had found research to be a powerful influence on her own thinking about teaching and learning. She believed the new curriculum was based on a solid foundation of research and she wanted to communicate that to teachers. In one of our many conversations, she told me,

> Some teachers tell me they don't like the curriculum. Well, they don't know the research. Other teachers are going back and saying this is really worthwhile. We're not just making these decisions out of the clear blue because we want to mandate. There is something based on solid ground as to why these decisions are being made - based on research, experience and a true understanding of how children learn. Educators are more involved in collecting that research and it is becoming more believable and being used more in schools.

This matter of the presentation of research findings will be taken up again later in the chapter.

**What Did Teaching For Conceptual Understanding Look Like?**

Marilyn considered it essential that she model effective instruction by demonstrating with a model lesson. She had been critical of the shortcomings of the sets of model lessons distributed with the strands and objectives. She felt the lessons did not give adequate attention to the development/instructional input phase of a lesson or the kinds of process questions that would "get kids to think." She also was interested in demonstrating how a teacher should begin at the concrete level, procede to the pictorial representation, and then move to the symbolic level. This section describes three model lessons that Marilyn taught. These lessons will be examined for their potential to help teachers think about how to teach for understanding and their potential to develop children's higher-order thinking and problem solving abilities.

A lesson on numeration. The objective of the model lesson Marilyn demonstrated for K-1 teachers was, "Given a place value model (base 10 materials), the learner will group ten ones as one ten to rename a number."[5] She began with a mental arithmetic activity called "You are my ten friend." Marilyn stated a number less than ten and then called upon a teacher to give the number that together with hers would make ten. She told teachers they should do this with children until each one had a chance to respond.

She then introduced the lesson with the motivation.

> Suppose you wanted to make a phone call. You reached in your pocket and pulled out thirteen pennies. What would you do?

The teacher responses varied. Marilyn pressed until one teacher gave her the answer she was looking for: "I'd see if I could trade ten pennies for a dime." Marilyn continued;

> Today we are going to use the same idea and practice trading ten ones for one ten.

Each pair of teachers was given a zip-lock bag with a set of plastic materials; one large orange square, 16 orange strips and 20 or more small white squares.[6] Marilyn held up each shape and identified it.

> We are going to call this (a small white square) a unit. This (the orange strip) we will call a long. This one (the large orange square) is going to be called a flat. This is a universal model and we use this language because you may want to use these materials later with decimals.

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[5]This model lesson appeared in a packet of 20 model lessons entitled "Success Strategies for Mathematics." These lessons were designed to be taught to fourth graders during the month of September in preparation for the state-wide MEAP test given in October. The introduction to the set of lessons stated "In developing the lessons, attention was given to research on how students learn and understand mathematical concepts and algorithms. A variety of methods, questions and concrete materials are needed to provide the learner with a rich conceptual background, as well as, provide for the various learning styles of the students."

[6]An orange strip was equivalent to ten small white squares. The large orange square was equivalent to 10 orange strips.

She demonstrated the use of **product** questions to get at student understanding.

> Show me with your materials how many units it takes to make a long. Show on your fingers how many units it takes to make a long. Show me with your materials how many longs it takes to make a flat. Now I want you to figure out how many units it takes to make a flat.

When teachers were ready to answer the last question she posed, she added:

> I want you not only to tell me how many units it takes to make a flat but how you figured it out. This is a **process** question, asking kids how they got an answer, not just what their answer is.

The lesson continued with Marilyn demonstrating the Bankers Game. As she spun a spinner, she set out the number of units corresponding to the number pointed to on the spinner. She told teachers the rule of the game: whenever a player has ten units, those units must be traded for a long. She continued the play, first spinning a 6, then a 5. With the second spin she asked:

> How many units do we have now? Do we have enough for a trade. How many units do we need to trade? How many units are left? How many longs do we have?

For the **guided practice** portion of the lesson, teachers duplicated her moves at their places, raising their hands to indicate when a trade was necessary. For **independent practice**, teachers played the Bankers Game (teachers used a die rather than a spinner) until one player had five longs.

After all groups had completed the game, Marilyn said:

> Every lesson needs to end with **closure**. You need to ask children to tell you in their own words what they did. Tell me what we did.

Teachers answered almost in unison, "We traded ten units for one long."

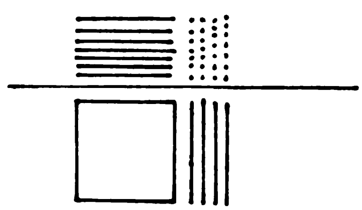
The entire lesson was conducted with concrete materials only. As Marilyn told the participants:

> We did this whole lesson without writing a single abstract symbol. This kind of work is necessary for the addition algorithm. You can also see that there is no reason to teach regrouping separate from non-regrouping.

A lesson on operations. For upper elementary teachers, Marilyn worked with base ten materials in a somewhat different context. Rather than teach a model lesson devoted to a single objective, she demonstrated how these materials could be used to model and teach the four conventional algorithms of arithmetic. As she manipulated the materials for addition and subtraction, making trades, she wrote the problem symbolically saying,

> Writing this with numbers is simply making a record of what we have done with the concrete materials. This is what the algorithm means. You're showing them what this process means. Many times kids don't know what this means. They've heard the rules. Now they know why the regrouping actually took place... Another thing you could do is make a record with pictures. A large square could represent a hundred, a bar could represent a ten, and a small square could represent a one. So if they don't have the concrete materials, they can draw representations of the problem and show the trading.

Marilyn deliberately manipulated the arrangement of the base ten materials to exactly replicate the procedural steps in long multiplication and long division. To model "seventeen times fourteen," teachers were instructed to start with "ten times ten" (which they represented with a hundreds square) and then "build up and out," creating the array shown below. Then they were instructed to draw a horizontal line above the hundred square.



As Marilyn wrote a record of what they had done, she told them,

> Above the line we have seven rows with 14 in each row. We have seven tens and we have seven rows of four. We can trade twenty of the ones for two tens (she drew circles grouping two sets of ten dots) and have eight ones left. So seven fours is twenty eight, we trade for two tens, write the eight and carry the two which is combined with the seven other tens giving us nine tens. Below the line we have ten rows with fourteen in each row, ten times fourteen is one-hundred forty.

This is what she wrote at the overhead.

$$\begin{array}{r} 2\phantom{4} \\ 14 \\ \underline{17} \\ 98 \\ \underline{140} \\ 238 \end{array}$$

Teachers seemed very confused about how to arrange the concrete materials, uncertain which number represented the number of rows and which told the number of elements in each row. Noticing their confusion but aware that little workshop time remained, she said:

> You're going to have to practice this before going to kids. Be sure and plan the <u>exact</u> problems you are going to use with them. You can see I have my cards. I always know exactly what problems I'm going to do whether it's with teachers or kids.

And one teacher responded loudly, "I guess so!!!"

These two lessons exemplified a key element in Marilyn's approach to teaching for understanding--introducing new ideas at the concrete level. Beginning with the concrete meant manipulating physical representations of a mathematical idea or procedure. Marilyn was deeply concerned about developing youngsters' understandings of place value, numeration and operations. She emphasized providing a physical model and manipulating it so there was a clear map from the model to the paper-and-pencil algorithm.

Although teachers, as well as Marilyn, lapsed into using the language of ones and tens rather than units and longs in the first lesson, there was no discussion about how or when to introduce this language with young children. Nor was there discussion about how to connect the concrete representation embodied by the base ten materials with the symbolic representation of numerals. Marilyn expressed concern on several occasions that early elementary teachers were in a rush to get to the symbolic. She wanted to show them that they could develop children's understanding of place value and addition without writing a single number. Teachers physically made trades of ones for tens and tens for ones. They appeared to enjoy using the base 10 materials and talked among themselves about how they could use these with young children. They seemed to be persuaded of the effectiveness of using concrete materials to develop competency with addition problems that involved regrouping.

The teachers who participated in the second lesson also enjoyed using the materials to model addition and subtraction with regrouping. They were shown how to move from manipulating physical materials to making a picture of them. And then they were shown how to "make a written record" of what they had done with the physical model. But there was considerable confusion and skepticism about the use of the materials for long multiplication.

This set of lessons raises a question about the choice of mathematical goals embedded in lessons. Numeration is a fundamental mathematical concept and developing an understanding of place value is a significant goal of instruction. Similarly, the four operations are fundamental processes and developing an understanding of and proficiency with them is a significant goal. In addition, developing an understanding of operating on large numbers and using physical representations to generalize about computational processes is a worthy goal of instruction. But is mastery of the conventional long multiplication algorithm a significant goal?

The second lesson as conducted was not likely to develop higher-order reasoning and problem solving abilities, although such potential existed. For example, there were many ways in which teachers could have shown 14 x 17 with the concrete materials. Various arrangements could have led to different ways to "record the answer" and perhaps alternative algorithms. The objective was not "opening the way to invention"[7] of algorithms or abstractions. Marilyn demonstrated how to manipulate base 10 materials to <u>exactly replicate</u> the paper-and-pencil steps in the long multiplication algorithm. The procedural steps of the conventional algorithm were the ends of instruction and the physical materials were manipulated to that end.

<u>A lesson from measurement</u>. This lesson was taught to upper elementary teachers. The topic was rectangles and areas of rectangles. Marilyn began with a mental arithmetic activity on estimation where teachers judged the reasonableness of answers to a set of computational exercises. Then came the lesson objective.

> My objective, and again I would tell this to kids, the purpose of this lesson is to be able to identify a rectangle as shapes with

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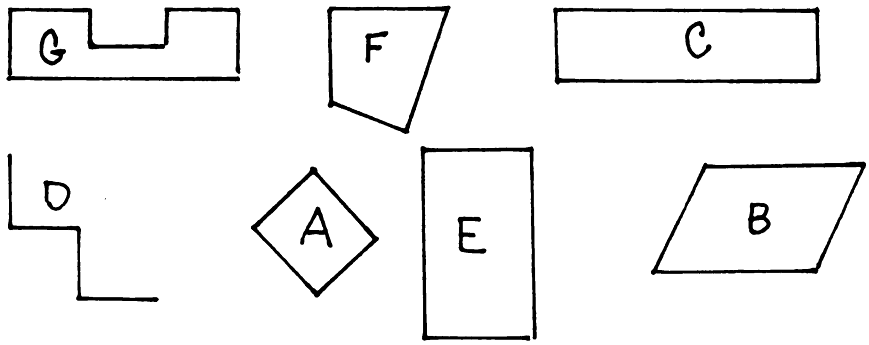
[7]This is a phrase Glenda Lappan used in a conversation about the importance of symbolic development and our shared belief that getting to the "abstraction" is necessary for completeness. But that requires that the abstraction itself be significant.

> four square corners and four straight sides and find many rectangles with the same area.

The lesson took the following course. As the motivation for the lesson, Marilyn showed the teachers how to fold a sheet of paper to make a square corner and then they used it to find examples of square corners around the room. Then she presented at the overhead the figures drawn below and asked which were rectangles and why.




When she pointed to D she realized she had forgotten the additional condition that a rectangle is a closed figure and added that condition. Next she asked teachers to tell which ones had at least two square corners. When there was uncertainty, she had them check it out with their square corners. Then she proceeded to the area of rectangles.

She distributed a sheet of centimeter grid paper with one square darkened in. She explained that this square was one centimeter on a side and "so we say that this square has an area of one square centimeter." She told the teachers to draw on their grid paper a rectangle with an area of 12 square centimeters and said,

> That means it is going to be twelve of these squares. After you have drawn that, I want you to draw as many rectangles as you can that have an area of 12 square centimeters.

One teacher asked, "Do we count it laying down and standing up as the same thing?" Marilyn responded with:

Good question, Cynthia. Sure, for right now it is the same thing. After teachers had a chance to make all the possible rectangles they could, Marilyn indicated that with students she would have them come to the overhead and draw their various rectangles. She then proceeded to draw rectangles that teachers suggested: 1 x 12, 2 x 6, and 3 x 4. One teacher suggested 24 x 1/2 which Marilyn accepted and said:

> That's very good. And some kids might come up with that. That is very good thinking."

After the first three rectangles had been drawn, Marilyn asked the teachers:

> Does anybody know a multiplication sentence that we can give to each rectangle?

One teacher responded with "An array, one times twelve." Marilyn repeated the teacher.

> That's right. One times twelve. Now I would have taught understanding of multiplication already using that so they would have had this knowledge to be able to say that this is one times twelve. And this is two times six (pointing to the 2 by 6 rectangle and writing "2 x 6" at the overhead). And this is three times four (pointing to the 3 by 4 rectangle and writing "3 x 4").

Next Marilyn handed out another sheet of grid paper on which was drawn a 1 x 12 rectangle and a 12 x 1 rectangle, each with the lower left corner at (0,0) and said

> You will see that two of those rectangles have already been graphed. We're going to graph the rest of them and we're going to see something happen. I want you to graph the other ones that we saw, and what they would be if you turned them, and you're always going to start with the left-hand corner of your shape at zero. Then put a dot on the upper right-hand corner of each rectangle.

Marilyn drew the various rectangles at the overhead, including the 1/2 by 24, as teachers worked at their tables.

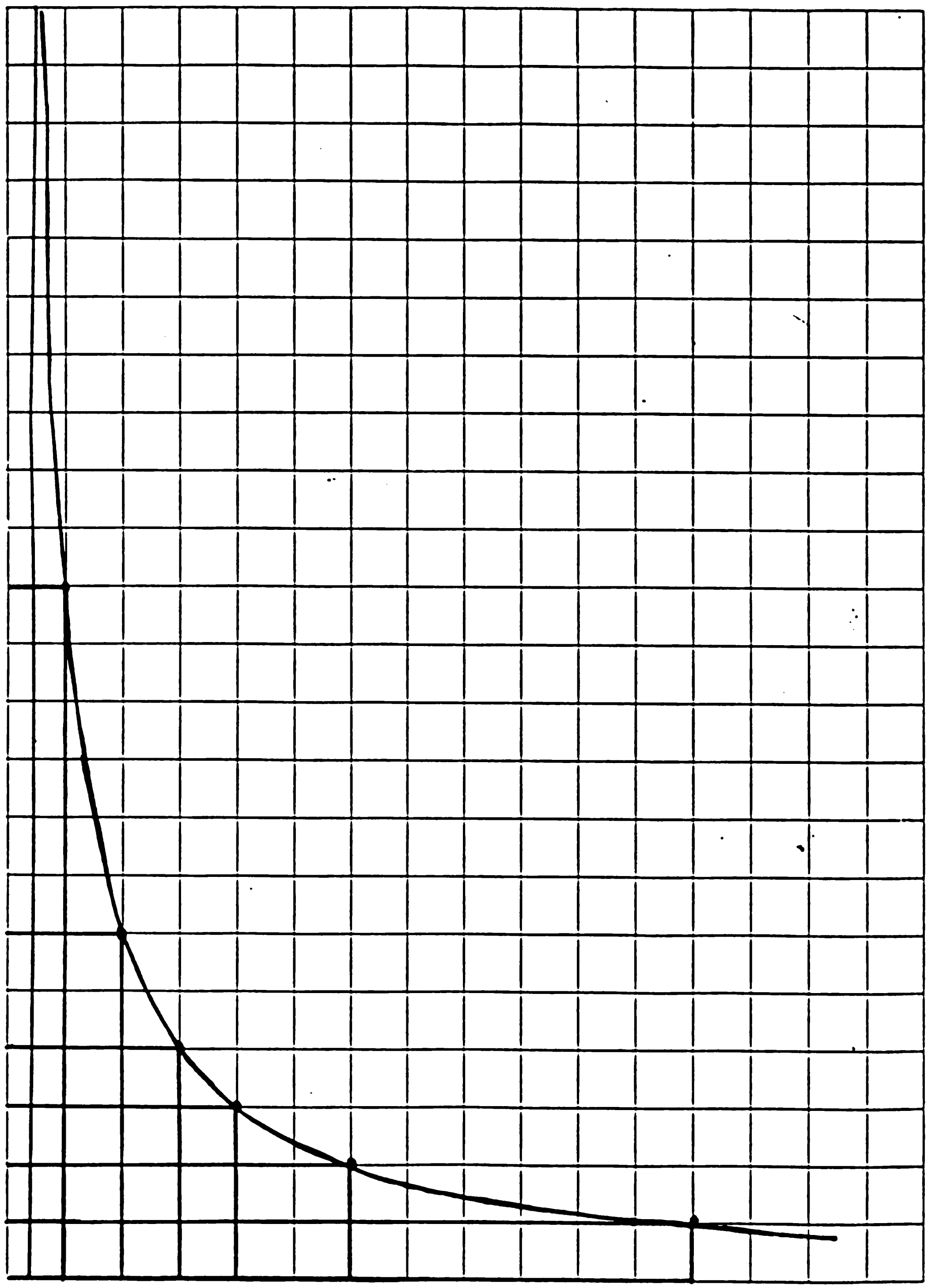


Figure 7.1 Rectangles With Area 12

> Okay, now we're going to draw, very carefully we're going to connect those points. What happens? (See Figure 7.1).

Several teacher answered, "We got a curve."

And Marilyn replied,

> Okay, we made a curve. And I'll tell you that any rectangle that has an area of 12 square centimeters, its right hand corner will be on that curve that we just drew. And that is where I would end the lesson for the kids.

But then she proceeded,

> Okay, so does anybody know what this curve is called. (Short pause, no teacher offers a guess). This is called a hyperbolic curve. But, when you know about the area. This is a graph of this function (writes xy) length times width, x times y. Whatever that would be would give me a graph of that. So when you graph x times y you're going to get this kind of curve. Now we're not talking about this with kids, but we're giving kids experience starting to see some of the patterns so that when they get to that higher level mathematics in high school they say, "I remember that. We drew a picture of that." And there's a lot of calculus when you start talking about the area of the curve and all kinds of other stuff built into this kind of an idea. So, I'd like you to tell your neighbor what a rectangle is.

In unison the teachers told each other that a rectangle is a closed shape with four square corners and four straight sides. At this point the model lesson was concluded.

This lesson was considerably different from a traditional textbook approach to area of a rectangle where youngsters are given the formula, A - l x w, and a set of problems for calculation. In fact, the formula was never introduced. Instead, teachers were asked to draw as many rectanges as possible with an area of 12 squares on a piece of centimeter grid paper. Part of Marilyn's intention was to have teachers see that area involves covering a surface and counting the number of units, in this case centimeter squares, that cover it. But she went beyond a simple draw and count exercise. She had teachers explore all possible rectangles with a given area. This opened the activity to consideration of rectangles whose sides were not whole numbers. She connected the calculation of the number of squares covering a rectangle with prior learning--the array representation of multiplication. And she used a graphical representation to show all the rectangles with an area of 12 square centimeters. The lesson demonstrated an approach to teaching youngsters about rectangles and area of rectangles that was a considerable improvement over typical "plug in the formula, grind out the answer" approaches. At the same time, the lesson could be described as a missed opportunity.

This lesson was rich in mathematical ideas that were not explored. Time was a factor. The lesson was taught at the end of the last day of inservice and time was running short. Teachers were tired after a long week and so was Marilyn. The context of inservice -- the limited amount of time, offered after a full day's work, nine hours compressed into three after school sessions in one week -- was a constraint on opportunities to pursue a topic over a more extended period of time.

At the same time, the missed opportunity reflected a more fundamental issue -- defining objectives for instruction and goals for student learning. The assumption underpinning lesson development, as evidenced in the objectives for the Model Lessons and those that Marilyn taught at inservice, was that a lesson should focus on, to use Marilyn's words, "one specific piece of information." This orientation to defining instructional objectives seemed unlikely to create in teachers a disposition to look for connections among mathematical ideas or help children to see those connections.

Consider some of the linkages that could have been made to other important ideas by posing additional questions for investigation. "How can we be sure we have found all the possible rectangles with a given area if the sides are whole numbers?" Such a question would invite learners to consider ways to systematically account for all possibilities, an important problem solving strategy. It could also provide another context in which to think about factors of whole numbers. "What happens to the perimeter of these rectangles as we change the lengths of the sides?" "Is it possible to find a rectangle that has the smallest perimeter?" "Is it possible to find a rectangle that has the largest perimeter?" These questions would invite learners to look for patterns, to make some guesses and pursue some hunches. Learners could explore the idea of limits at an intuitive level. A complimentary problem could be posed to investigate how area changes when perimeter is held constant.

The ideas in this lesson could have been explored by locating them in a problem situation connected to the real-world of youngsters. Ball (1988) describes a problem she has used in her third grade classroom to explore a range of ideas about area and perimeter.

> Suppose you had 64 meters of fence with which you were going to build a pen for your large dog, Bozo. What are some different pens you can make if you use all the fence? Which is the pen with the most play space for Bozo? Which pen allows him the least play space (p. 4)?

Problems of this kind suggest to children that there is a connection between school mathematics and their world outside the classroom.

It was not at all clear if or how teachers made sense of the curve they constructed by connecting the upper right corners of the rectangles. Marilyn did not ask them to make conjectures about what they thought the curve represented. Instead she told them what it represented. It was not clear how Marilyn's brief "lecture" about hyperbolic curves and their algebraic representation contributed to teacher understanding. Up until teachers were told to connect the corners of their rectangles, there was spirited conversation at the tables. But when Marilyn mentioned "a hyperbolic curve," "function" and "calculus," teachers looked at each other, some shook their heads, and there was not another question asked or comment made.

But the objective had been accomplished - teachers were able to tell what a rectangle is and find several with the same area.

## INSERVICE AS TEACHER EDUCATION

This section examines the inservice programs as opportunities for teacher learning. The analysis uses three frames to examine inservice as teacher education: 1) assumptions about what teachers needed to know to implement the curriculum; 2) assumptions about how teachers would learn what they needed to know; and 3) the underlying model of changing teachers' practices.

### Assumptions About What Teachers Needed To Know

Marilyn believed that teachers lacked essential research knowledge about effective mathematics teaching and the pedagogical skills derived from that research. Her goal was two-fold: 1) to equip teachers with that knowledge by introducing them to a body of research on "effective mathematics teaching," and 2) to demonstrate how to "teach for understanding" by teaching a model lesson. For Marilyn, the essence of conceptual understanding was captured in the model of "effective mathematics instruction."

Learning was portrayed as requiring a specific sequence of development; first manipulating concrete models, linking concrete models to pictorial representations, and then using pictorial representations as the bridge to the abstract. The use of base 10 materials suggested that correctly arranging the materials in the specified configuration, moving to drawings with squares, lines and dots, and then "making a record of what had been done" automatically brought conceptual understanding to procedural routines. However, there is growing research evidence that making meaningful connections among these representations is not automatic. At the concrete level, the ability to manipulate materials may lie not in children's conceptual understanding but in the structure of the materials themselves and the ease with which they can be manipulated to get right answers. Moreover, even when the physical materials are manipulated to exactly map the steps of paper-and-pencil algorithms, linking conceptual knowledge with procedures has not been consistently effective (see, for example, Carpenter, 1986; Driscoll, 1981; Resnick, 1982). As with the sets of model lessons that had been given teachers, there was an assumption that if teachers began instruction with concrete materials then student understanding would be automatic. But as Hiebert (1980) has pointed out,

> ...if concrete materials are going to be useful, frequent, explicit links must be made between the physical and symbolic representation...It is not just the use of concrete materials that improves mathematical understanding, but rather the explicit construction of links between understood actions on the objects and the related symbol procedures (p. 509).

Virtually untouched in the inservice was teacher subject matter knowledge. Although Marilyn acknowledged that many teachers, by their own admission, were uncomfortable with the content of some of the strands, attending to this teacher concern was not an objective. Instructional techniques and technical competence were at the heart of inservice.

**Assumptions About How Teachers Would Learn What They Needed To Know**

Marilyn used the program's content as pedagogy. She modeled in inservice what she expected teachers to do in their classrooms: tell students what they needed to know, demonstrate with concrete materials, give opportunities to practice what they had been shown, ask product and process questions to assess understanding, and have students tell what they learned. At the conclusion of the series of workshops, Marilyn wondered about how successful she had been.

> I'm not sure I help them make the connection between the objective, the concept in the objective and the manipulative to model it. The way you write your objective you would try to represent that concept. I'm not sure I help them make that connection. They may walk away with a manipulative but maybe not an understanding of the concept that it models.

Having teachers teach each other was another pedagogical tool. Marilyn provided six articles on research in mathematics education and had triads of teachers at a table read two of the articles and then teach the other groups. In several of our conversations Marilyn spoke of the importance of having small group interaction, where learners (whether teachers or students) could "bounce ideas off one another."

Marilyn believed that change took place over time. But the limited resources, financial and human, available to her precluded an extended inservice program. The organization of three after-school sessions in a single week produced increasing levels of disengagement among many participants by the last session. The time for beginning a workshop overlapped dismissal time at several schools thereby limiting attendance for those teachers who might otherwise have been interested. The distance teachers had to drive between individual schools and the inservice site reduced potential participation. In this inservice, the context itself was a considerable constraint on teachers' participation and opportunities for teacher learning.

Marilyn told teachers she would be happy to respond to individual requests for classroom visits, school-site workshops, and resource materials. Inservice provided the techniques. What teachers needed was time in their classrooms to practice delivering instruction in the mode prescribed.

**The Underlying Model of Changing Teachers' Practice**

The inservice embodied a technical rationality. Teachers were presented with "systematic knowledge," -- specialized, scientific and standardized -- that "ignore(d) complexity, uncertainty, instability, uniqueness and value-conflicts" (Schon, 1983; p. 39). The instructional model, the research out of which it grew, and the practices it prescribed were presented as objective facts rather than as something to be questioned, analyzed or negotiated. Teachers were not told that there is considerable disagreement about the suitability of the direct instruction model to teach for conceptual understanding and higher-order thinking. They were not told that realizing the potential of manipulatives to increase student learning has been shown to be exceedingly complex and far from automatic. What was portrayed was a consensus model of research. The dichotomy between the expert -- the researcher and the supervisor -- and the practitioner -- the classroom teacher -- was maintained.

In three days of inservice, teachers were taught about the

* 10 strands of the K-5 mathematics curriculum.
* 3 goal structures for learning in the classroom.
* 4 elements of cooperative learning.
* 3 rules for "groups of four" in cooperative learning.
* 6 outcomes of cooperative learning.
* 5 mathematical learning processes.
* 5 steps to concept understanding.
* 4 characteristics of a problem situation.
* 4 steps in problem solving.
* 16 problem solving strategies.
* 11 behaviors that characterize effective teachers.
* 11 components to planning and delivering an effective mathematics lesson.

Teachers were "inserviced" - a term that Marilyn herself used to describe her efforts - on a model for grouping children for instruction, a model specifying a sequence of steps to develop an understanding of mathematical concepts, a model for problem solving, a model for planning and teaching an effective lesson. The emphasis was on "methodological refinement," giving teachers models and sets of instructional procedures to effectively and efficiently implement a newly mandated, pre-determined curriculum. Aronowitz and Giroux have called this management pedagogy, "forms of pedagogy that routinize and standardize classroom instruction," and that reduce teaching to "lock-step" behaviors (1985, p. 28).

Underlying the orientation of this inservice program was what Zeichner has called "a view of teaching as an 'applied science' and a view of the teacher as primarily an 'executor' of the laws and principals of effective teaching" (1983, p. 4). The topics that constituted the curriculum of inservice suggested that it is possible and desirable to quantify effective teaching practice. As Marilyn said, "I'm selling the proper teaching of mathematics."[8]

## SUMMARY

Inservice focused on providing teachers with a set of "research-based" instructional strategies and techniques. As Ball & Wilcox (1989) have argued, there is a logic in selecting a specific focus for inservice.

> Choosing a focus for inservice makes sense; it suggests a strategy of engaging teachers in changing aspects of their practice by targeting a single aspect or ingredient of their teaching. A broader sweep, especially under conditions of limited time, seems likely to do little more than ripple gently the waters of practice. Yet choosing a singular focus has its drawbacks, for the interactions among the threads of teaching are not well-understood. Focusing on techniques of teaching without, for instance, engaging teachers in considering their assumptions about learning, may prove a futile intervention (p. 34).

Take the use of manipulatives as an example. When teachers encounter this as a research-based pedagogical strategy, but do not reexamine their view that operating at the symbolic level is "not only preferable

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[8]Teachers who attended the workshops completed a "Workshop Feedback" at the end of the session. I collected data from 109 of these forms. Teachers indicated they had a better understanding of the mathematics strands and how they were related as a result of the workshop. The components of the workshop they mentioned as liking best were 1) the use of manipulatives -- 35% of the teachers mentioned this specifically, 2) cooperative learning/small group work, and 3) sharing thoughts, ideas with other teachers. Teacher suggestions included having more materials available -- more manipulatives, model lessons for specific objectives on basic facts and regrouping, lessons relating to the tests, more "make-and-take." They also commented on the desire for more inservice on instructional strategies such as cooperative learning and motivating activities. And they voiced their concern about time - the limited time available for inservice and the inconvenient time at which the workshops were scheduled. There was overwhelming praise for Marilyn. These comments were typical: "She makes me feel motivated to change how I teach;" "The instructor knew her material and was enthusiastic'" "...the professional treatment of the presenter, the equalitarian mode."

but also a sign of understanding" (Schram et al., 1989), their use of manipulatives in instruction may be superficial. If teachers are encouraged to use manipulatives without attention to what are appropriate models and what mathematics are embedded in the physical material, the use of concrete materials may contribute to misunderstanding and misconceptions. Without explicitly considering the connection between mathematical ideas and appropriate concrete representations, manipulatives have the potential to be merely "fun things to do" to break the routine/monotony in the mathematics classroom. Or they are made available as "crutches," to be abandoned in favor of symbols as soon as possible.

The choice of focus for this inservice made sense in light of the reform "writ small." As I have argued in earlier chapters, the reform appeared to grow out of an orientation aimed at teaching for comprehension of mathematical ideas. The curriculum for students and the curriculum of inservice emphasized management of instruction, assumed learning required breaking knowledge -- mathematical and instructional -- into discrete pieces to be mastered, and divorced mathematics education from inquiry.

But when considering the reform "writ large," the inservice did not make sense. Simply improving teachers' instructional technique will not provide the knowledge -- about mathematics or mathematics teaching and learning -- that is required if Detroit children are to have opportunities to develop higher-order reasoning, critical thinking and problem solving abilities. These youngsters need environments in which they can engage in mathematical inquiry. And creating such environments is not just a matter of perfecting an algorithmic approach to mathematics teaching. Teachers need a deeper knowledge of mathematical ideas beyond arithmetic. They need opportunities themselves to experience "doing" mathematics, to see mathematics as the creation of human activity. Without a richer understanding of subject matter, their efforts to guide student inquiry -- listening to students, posing questions for further reflection, responding to questions by deciding which to pursue and which to defer to a later time -- will be meager. Like the model lessons that accompanied the formal curriculum documents, inservice provided minimal help to teachers about how to develop children's higher-order thinking and problem solving abilities. Effective mathematics teaching and learning was tied to technical competence and efficient management of instruction.

The matter of the constraints imposed by the context of inservice was serious and contributed to the limitations of these workshops as opportunities for teacher education. At the same time, the technical orientation that dominated the inservice leaves me to wonder how and to what extent inservice might have been different had contextual constraints not been so dominant. In the final chapter, I return to a further look at the context in which this curriculum reform was embedded.

# CHAPTER 8

# THE GENEALOGY OF THE CURRICULUM REFORM

## INTRODUCTION

This final chapter brings me full circle, back to the context in which this curriculum reform was conceived. In the preceeding three chapters, I have argued that despite a rhetoric of conceptual understanding, developing higher-order analytic and critical thinking skills, and applying knowledge to problem solving, this curriculum reform was likely to fall far short of providing young students in Detroit with the mathematical tools they will need as citizens in the 21st century.

One interpretation that could be made is that school people were saying one thing but doing another. The problem with this interpretation is that it is too simplistic. Further, it casts doubt on the intentions of reformers to meet the real needs of Detroit children. To draw this conclusion would be both a mistake and a disservice to the dedicated professionals in this study who have committed many years to improving Detroit's schools. The limitations of this reform do not, I think, represent personal shortcomings or failures. Instead, I will argue that the conception of this reform grew out of the efforts of well-intentioned people who were doing the best they could given a particular context, the dominance of a particular ideology and a particular moment in history.

## A GENEALOGICAL ACCOUNT

### Introduction

Noujain (1987), drawing on the historical and philosophical works of Michel Foucault, argues that "history possesses a genealogical character" (p. 158). The past is a series of intersecting, discontinuous elements that are linked like a chain. The elements may be any identifiable entity--events, ideas, practices, people. In undertaking a genealogical analysis of a particular object, the historian looks for the elements that preceeded the object and out of which it emerged. Noujain stresses that the relationship between elements is not a causal relation but is a relation of succession. However, he goes on to say that once having established the succession, the historian may proceed to investigate causal relations, but then the historian turns into a "social scientist of sorts" (p. 160).

In this final chapter, I assume the role of historian and critical social scientist. I provide a genealogical account of the Detroit mathematics curriculum reform to show how it can be understood when placed in a broader historical context.

Providing a genealogical account of the Detroit curriculum reform has required searching for the antecedents of the reform, those components of events, ideas, institutions, practices, and persons that intersected and combined to form it. Figure 8.1 gives the schematized genealogy. Earlier chapters have been devoted to some of these elements in detail, in particular the politics of race and school governance in Detroit, the reform agendas for mathematics education, and the recognition of a new economic order and Detroit's position in it. What has received less explicit attention is the instrumental

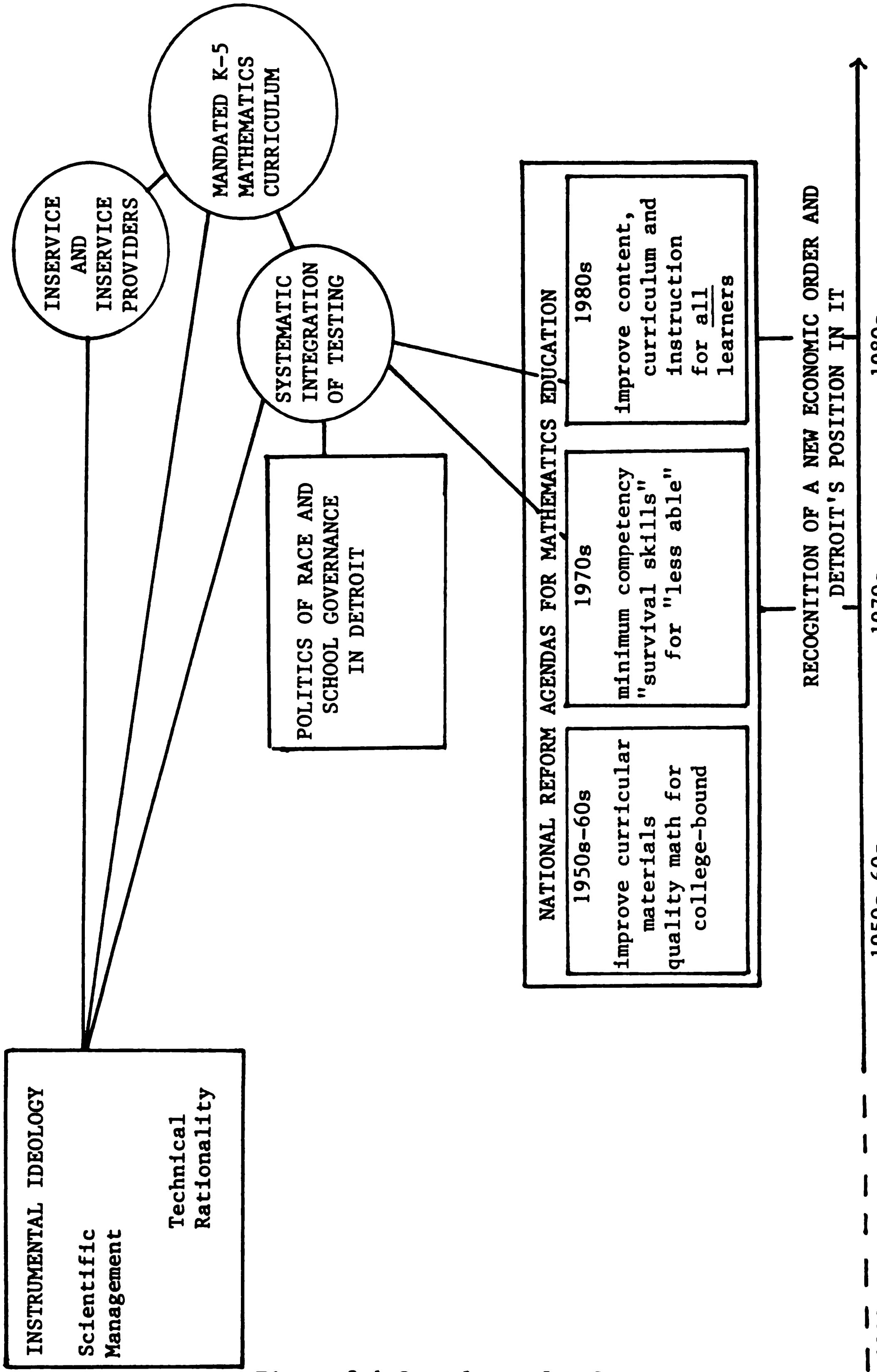


Figure 8.1 Genealogy of Reform

ideology that has dominated American education and the field of curriculum.

I begin this chapter with a discussion of the domination of an instrumental ideology on the field of curriculum development and its extension to other aspects of educational life, for teachers and students. From that, I show the influence of instrumentalism on the Detroit reform initiative. I then revisit the other antecedents of the reform to show the linkages among events, ideas, practices and people that shaped this initiative.

I conclude this study with a look at a new challenge to the dominance of instrumentalism in mathematics education and I suggest changes that would need to take place for Detroit to transform its curriculum toward teaching for **enabling mathematical inquiry**.

## Instrumental Ideology

The domination of an instrumental ideology in curriculum. Giroux, in his essay "Literacy, ideology and the politics of schooling," identifies a number of assumptions that define an instrumental ideology.

> The major premises of instrumental ideology are drawn from the logic and method in inquiry associated with the natural sciences, especially the principles of prediction, efficiency, and technical control....Central to the logic of instrumental ideology and its view of theory is the notion that all social relations shall be subject to quantification....Knowledge in this view is seen as objective, outside of the existence of the knower...neatly severed from the world of values....Since knowledge in this perspective is valued for its utility and practical application, there is little room for questions concerning the ethical nature and consequences of the use of knowledge (1983, pp. 209-210).

Giroux goes on to argue that an instrumental ideology has dominated American education in general and the curriculum field in particular.

> Instrumental ideology, in both historical and contemporary terms, has had a powerful and pervasive influence on American education. Curriculum theory and practice as well as more specific pedagogies of literacy have been largely structured by the values and assumptions inherent in instrumental ideology....The strength and pervasiveness of the logic of instrumental ideology in the curriculum field can be seen historically in the early work of educators like Franklin Bobbitt (1918), W.W. Charters (1923), D.S. Sneeden (1921), and others....That this view is deeply ingrained in the ideology of schooling can be seen in the powerful support given by educators to competency-based systems of instruction, behaviorist models of pedagogy, and the various versions of systems theory approaches to curriculum theory and policy development (p. 211).

The birth of the field of curriculum is generally associated with the publication of Bobbitt's The Curriculum in 1918. It was initially and powerfully dominated by administrative imperatives to organize and manage time and activities according to sound business principles. Almost from the beginning, the curriculum field was influenced by the principles of scientific management promoted by Frederick Winslow Taylor. The basics of scientific management -- efficiency, control, prediction and technical expertise -- were appropriated by those responsible for curricular matters. Kliebard (1975) quotes Elwood Cubberley, an early advocate of the adoption of the tenets of scientific management to school management and curriculum development.

> Every manufacturing establishment that turns out a standard product or a series of products of any kind maintains a force of efficiency experts to study methods of procedure and to measure and test the output of its works. Such men ultimately bring the manufacturing establishment large returns by introducing improvements in process and procedures and in training the workmen to produce larger and better output. Our schools are, in a sense, factories in which the raw products (children) are to be shaped and fashioned into products to meet the various demands of life. The specifications for manufacturing come from the demands of twentieth-century civilization, and it is the business of the school to build its pupils according to the specifications laid down. This demands good tools, specialized machinery, continous measurement of production to see if it works according to specifications, the elimination of waste in manufacture, and a large variety in the output (p. 32).

The ideology and practices of scientific management were highly compatible with behaviorist psychology. The efficient management of curriculum was bolstered by defining the goals of learning with behavioral objectives and observable and measureable competencies. Curriculum development was a practical matter, a systematic attempt at guiding learning in schools. Tyler's (1949) four questions were the quintessence of the dominant tradition.

(1) What educational purposes should the school seek to attain?
(2) How can learning experiences be selected that are likely to be useful in attaining these objectives?
(3) How can learning experiences be organized for effective instruction?
(4) How can the effectiveness of learning experiences be evaluated?

The traditionalists,[1] who have dominated the field, have been overly concerned with practical considerations and classroom applications at the expense of curriculum theory and research. Service to practitioners, guided by principles, has been more important in directing curriculum development and implementation than theory. The Tyler legacy lives on in later curriculum texts by Taba (1962), Saylor and Alexander (1966), Popham (1969), Tanner and Tanner (1980), Zais (1976) and Glatthorn (1984).

During the curriculum reform movements of the 1960s, the atheoretical, unscientific traditional approaches to curriculum development were challenged by a new breed of curriculum worker, conceptual-empricists. Heavily influenced by mainstream social science and behavioral science, they sought more valid principles for curriculum decisions based on theoretical development and research.

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[1]The terms traditionalists, conceptual-empiricists and reconceptualists are used by Pinar (1978) and Giroux et al., (1981).

But as Pinar (1978) and Giroux et al. (1981) have argued, ultimately their differences with traditionalists were less a matter of kind than degree. Their efforts resulted in research-based prescriptions for ways to structure the intended and enacted curricula. The classroom became the site to categorize and investigate problems using input-output schemes, and devise models for improved practice. The efforts resulted in such innovations as mastery teaching (Block, 1980) and information-processing models of teaching (Gagne, 1976). Short (1986), in a brief historical look at curriculum design, acknowledged that the field was dominated by conceptions of a "measured" curriculum, where the key feature is "the reduction of their intended outcomes to prespecified elements, which, when 'taught' and 'learned,' can be measured, and on which a definitive report of 'results' can be made public" (p. 6). The instrumental ideology had been given a legitimacy derived from quantitative research modeled on the natural sciences.

A consequence of a technical and instrumental rationality[2] in education is that curriculum decisions have been removed from the classroom teachers who implement them. The effect has been a deskilling of the nations' teaching force. There has been a separation of the conception of curriculum from its execution. Teachers often are asked to do little more than carry out plans, goals and activities conceived by someone else. At the same time, teachers have been reskilled to be more efficient managers of classroom instruction and record-keeping (Apple, 1983b; Gitlin, 1983). The standardization of

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[2]I use the term rationality as Giroux has defined it; "a specific set of assumptions and social practices that mediate how an individual or group relates to the wider society. Underlying any one mode of rationality is a set of interests that defines and qualifies how one reflects on the world" (1981, p. 8).

curricula, the focus on minimum competencies and scores of behavioral objectives, the integration of testing and teaching to the test makes possible more efficient planning, in the district and in the classroom.

The attraction of integrated systems to efficiency-minded administrators, especially in inner-city schools, is captured in a statement by some Chicago school administrators.

> Providing materials that were centrally developed and successfully field tested would: 1) reduce greatly the time needed to prepare and organize materials; 2) require little inservice time; 3) be economical for schools in Chicago and elsewhere to implement; 4) standardize the definition, sequencing, and quality of instruction necessary for mastery of each objective; 5) reduce greatly the time needed for developing lesson plans; and 6) be easy for substitutes to use (quoted in Aronowitz & Giroux, 1985, p. 29).[3]

But the consequences for teachers and students should not to be applauded. Professional decision-making on matters of curriculum and instruction is removed from the hands of professional teachers. As Apple (1983a) has argued,

> When individuals cease to plan and control their own work, the skills essential to these tasks atrophy and are forgotten. Skills that teachers have built up over decades of hard work -- setting curricular goals, establishing content, designing lessons and instructional strategies, individualizing instruction from an intimate knowledge of each student's desires and needs, and so on -- are lost. In the process, the very things that make teaching a professional activity -- the control of one's expertise and time -- are also dissipated. There is no better formula for alienation and burnout than the loss of control of the job (p. 323).

At the same time that teachers are being deskilled, children are being turned into anonymous learners. Their behavior is preselected before instruction is carried out or learning activities engaged in. That behavior becomes that basis on which effectiveness of teaching and

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[3]M. Katims & B.F. Jones, "Chicago Mastery Learning Reading: Mastery Learning Instruction and Assessment in Inner City Schools." Paper presented at the Annual Meeting of the International Reading Association, New Orleans, 1981, p. 7.

learning is judged (Apple, 1979). The logic of a technical rationality removes teachers from participating in a critical way in curriculum planning and curriculum evaluation. Teacher competence gets defined as refinement of technical skills. Students are not encouraged to generate their own meanings or to participate in evaluating their own classroom experiences. Student competence gets defined as mastery of discrete objectives. The principles of order, control, and certainty preclude other approaches. Teaching for **certification of numeracy** and teaching for **comprehension of mathematical ideas** emerge from an instrumental ideology.

<u>Challenge to an instrumental ideology</u>. A challenge to the dominance of an instrumental ideology in the curriculum field has come from the reconceptualists. Theorists such as James Macdonald, Dwayne Huebner, Herbert Kliebard, Maxine Greene, William Pinar, Michael Apple, Madeleine Grumet, Henry Giroux, and Jean Anyon have played a major role in reconceptualizing the issues, interests and modes of inquiry that inform curriculum theory and practice. The singular theme that has united their various interests is opposition to the instrumental and technical rationality that dominates traditional curriculum theory and practice. Their work has been aimed at offsetting the apolitical, ahistorical and technical orientations that dominate the field.

Drawing on the intellectual traditions of existentialism, phenomenology, psychoanalysis, neo-Marxism and critical theory, they have made the human subject the primary focus of a theory and practice aimed at emancipation from exploitation and oppression. Instead of asking how knowledge can be organized so that teachers can more efficiently deliver it and students more effectively master it, they raise a different set of questions about knowledge. Whose knowledge is it? Whose meanings are attached to it? Who has access to what kind of knowledge? How do forms of evaluation legitimize certain ways of knowing? Questions of this nature expose the ideological underpinnings of curriculum design, implementation and evaluation and reveal the complex relationship between schools and the larger social order.

Despite a growing body of theoretical and empirical literature that has emerged from the work of the reconceptualists, they have not posed a significant threat to the dominance of an instrumental rationality. The effects of their work have yet to be felt in any substantial way in the classrooms of America's schools. As one of their main proponents has pointed out, in the history of the curriculum field from its roots in scientific management to its present domination by systems management schemes, there is an ideological commitment to instrumentalism that seems particularly resistant to challenge, even when it is contradictory to broader goals of learning (Giroux, 1981). One reason for this may be that the work of reconceptualists has tended to focus on theory development and research but not the practical implications of this study for classroom teachers. The critical work of reconceptualists has been to uncover the structures and practices of schooling that deny justice and equality to all learners. They have not devoted their efforts, by and large, to providing a vision of a new school environment except at a fairly abstract level.

## The Inheritance Of An Instrumental Ideology In The Detroit Reform

The Detroit reform contained unmistakable elements of an instrumental ideology. The most obvious manifestation was the set of one-hundred plus behavioral objectives at each grade level that defined what children should learn. The noble goals for Detroit children, as stated in the District's philosophy, were reduced to forms of overt behavior that could be observed, quantified and evaluated. An investigation of the verbs that described student actions showed the dominance of actions that could easily be measured--read, identify, name, recall, state the rule, measure, apply strategies. Verbs that would describe student actions more difficult to quantify -- investigate, explore, invent, predict, generalize -- were absent from the formulation of learning objectives.

An instrumental ideology informed the sets of model lessons. The form of the lessons and the prescribed teacher and student actions embodied a technical and management orientation. They specified what a teacher had to know, say and do. They specified the materials to be used during instruction. They even specified correct student responses. With these lessons in hand, a teacher's limited knowledge of mathematics would not stand in the way of efficient and effective instruction.

The process of decision-making that shaped the reform was centralized among curriculum, testing and mathematics specialists. The teacher representation on committees did not alter the fact that experts conceived the reform. The majority of teachers in this district were asked to do little more than "to execute someone else's goals and plans and to carry out someone else's suggested activities" (Apple, 1983a, p. 323).

The technical rationality embedded in an instrumental ideology was evident in the inservice opportunities. Refining teachers' technical competence in instruction was at the heart of inservice. Teachers were provided with research-based techniques for planning and carrying out instruction and for evaluating student understanding. This was true not only of the inservice conducted specifically around this reform but also the much larger effort to train all district personnel in EEEI. Inservice in this district aimed not to improve teachers' subject matter knowledge or to improve their decision making. Research was not offered as something to be questioned or held up against one's own experience. It was not used to build the reflective capacity of teachers. Research was presented as having revealed the cause-effect relationships that could be turned into skills for teachers to use in improving their practice. Research was being "translated" into practice.

It is almost certain that an instrumental ideology shaped the professional education of the agents of this reform -- those who created ABCS, conceptualized the curriculum, created Model Lessons, provided inservice for teachers charged with its implementation. I am not claiming that they were unwitting servants to the domination of an instrumental ideology, although perhaps they were. It seems to me it is a matter of not questioning the taken-for-granted, of not being reflective about what interests guided their own behavior. An instrumental ideology saturates consciousness and shapes daily behavior. It reifies hierarchical forms of decision-making. It aims to find the best means to deliver knowledge, not to inquire why this knowledge and not others. The curriculum planners, the test makers, the inservice providers were acting reasonably given the dominant ideology. They believed they were on "the cutting edge" of mathematics education reform. What they did not seem to be asking themselves was how and to what extent this initiative would make a real difference in developing children's mathematical literacy.

**The Inheritance of the Politics of Race and School Governance in the Detroit Reform**

There are structural and institutional determinants in school systems that sustain and uphold efficiency-oriented management techniques. But at times there are forces that pull in an opposite direction. In Detroit, the demand for community control by the parents of black children led to the decentralization of adminstrative authority over the schools. Curriculum decisions were decentralized to the local Region. Region Boards were more visible and more likely to consider demands from organized groups of parents, students and teachers in the community.[4] But the politics of race and the politics of school governance were soon in tension. Desegregation and decentralization produced administrative conflicts.

The Judge overseeing the desegregation order was not satisfied that simply ordering the reassignment of students and teachers to achieve racial integration would end the effects of past discriminatory practices. To that end, he ordered the implementation of eight educational components. The stipulation of educational components to

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[4]Although the evidence is anecdotal, the early 1970s appear to have been a time of increased studies in Black history and literature in the Detroit schools. In an earlier study of one of the Detroit high schools, I found that a number of such courses had been taught, including Swahili. The English Department also published a Black Literary Journal of student writing. A veteran teacher in the school told of going with a group of parents and students to the Region Office to petition for these courses and finding a sympathetic Board.

be administered by the Central Board and its administrative staff produced an uneasy mix of Region and central administrative authority. As the Court provided the opportunity for greater centralization of authority and management over educational programs in the District, there was increasing confusion about who had ultimate authority.

At the same time, the city was experiencing a significant downturn in the economy and the schools felt the fiscal effects. In addition, Detroit had become a majority black city. The confluence of tighter economic times, administrative disorder in the schools, and a black majority in the city led to calls to recentralize the schools. The concerns of community control had diminished, to be replaced by demands for efficiency and accountability.

Recentralization did not completely resolve the struggle for control over school administration. But the corner had been turned toward restoring centralized control over the District. City-wide testing had been mandated by the Court. Recentralization provided the opportunity to use the test to develop curriculum, to standardize it for easier management and order its city-wide implementation for easier accountability.

## The Inheritance of Reform Agendas for Mathematics EducationaAnd Changing Economic Realities

The testing component of the desegregation order specified that the District undertake a review of its city-wide testing program and develop an objective-referenced testing program in reading, writing and mathematics. The first efforts at test development began in 1977. At the time, the slogan of the national agenda for mathematics education was "back-to-basics." It was not surprising that one of the first tests developed was the High School Proficiency Exam. The test, created to determine if a graduate would receive an "endorsed" diploma, measured basic reading, writing and mathematics competencies at the eighth grade level.

By the time work began on ABCS (the test from which the new curriculum was derived), the reform agenda for mathematics had changed. The National Council of Teachers of Mathematics was calling for a redefinition of "basics." Computational proficiency was deemed no longer sufficient. Problem solving must become the focus of school mathematics and calculators and computers should be available to all children. The recommendations for reforming mathematics education were a recognition that technology was changing patterns of production. Basic skills would no longer provide the knowledge workers would need in an increasingly complex, technological economy.

The mathematics portion of ABCS represented a compromise between those who held to testing computational proficiency and those who advocated assessing student understanding of ideas and using word problems as the context for computation. Once the test had been developed, the next step was to determine the degree to which the curriculum - in this case, textbooks - were aligned with the test. The curriculum reform that emerged was an effort to assure that what teachers taught and what children learned correlated with the tests.

The desegregation order, a city-wide testing program, a national agenda to reform the mathematics curriculum in recognition of a new economic order had come together to influence the Detroit initiative.

**Summary**

This reform emerged from an ideology, a set of agents and practices, time and space focused events, and events more global and long term. An instrumental ideology was the antecedant for the systematic integration and alignment of testing, curriculum planning, instructional materials development, and inservice opportunities. The politics of race and school governance, and the interaction of school governance with management concerns embedded in instrumentalism were antecedants of city-wide mandates for testing and curriculum implementation. The recognition of a new economic order and the mathematical knowledge required for participation in it influenced national reforms for mathematics education that in turn influenced the Detroit initiative.

This reform was situated in and created out of a set of social, political, economic, organizational and educational contexts. By seeing it in relation to these contexts, the reform becomes understandable. It becomes understandable how well-intentioned professional educators who wanted to make a real difference in the lives of Detroit children conceived a reform of this nature. There is a consistency of practice that emerges from the analysis. But there is also a fundamental contradiction. An instrumental ideology cannot inform a curriculum or a practice that empowers. To paraphrase Giroux (1983) -- who was speaking about the literacy of writing -- an instrumental approach to mathematical literacy does not treat mathematics as "both the medium and product of one's experience in the world." Mathematics is stripped of its "normative and critical dimensions" and is reduced to the mastery of skills and the understanding of concepts and processes (1983, p. 213). Likewise, an instrumental approach to improving teaching practice reduces the knowledge teachers need to utilitarian and practical forms.

The common curriculum Detroit children would encounter as it was organized cannot develop the abilities they will need to succeed when they leave school. Citizens and workers must be able to work with others, defining problems in the real-world that are messy and not well-formulated, seeing the mathematics in them, and bringing their "math sense" to solve problems. It is difficult to see how the Detroit curriculum reform, with its emphasis on covering hundreds of discrete behavioral objectives, could create this kind of learner.

An instrumental ideology is dominant in American education, but it is never total or exclusive. In this reform we have seen struggles over the control of school administration, compromises over the content of ABCS and a shift away from an exclusive focus on minimum computational competencies, an exemplary Model Lesson where children would have the opportunity to investigate a mathematical idea and begin to see the power of multiple representations. The District had approved two distinctive textbook series for use in mathematics classrooms. These texts embodied elements of teaching for **comprehension of mathematical ideas** and teaching for **enabling mathematical inquiry**. Although they were used in few classrooms, their availability suggested an interest in moving beyond certification and comprehension as goals for student learning.

Curriculum is not static. In the past three decades we have seen changes in national agendas for reforming mathematics education. The 1980s have been marked by calls to improve the content, curriculum and instruction for all learners. The recommendations in the early part of this decade called for more problem solving and the use of calculators. But as this decade draws to a close, a very different campaign is being waged. There has emerged a new challenge to the domination of an instrumental rationality.

## A NEW CHALLENGE TO THE DOMINANCE OF INSTRUMENTALISM

### An Agenda For Enabling Mathematical Inquiry

The 1989 publication of the National Council of Teachers of Mathematics, Curriculum and Evaluation Standards for School Mathematics, represents a challenge to the instrumentalism that has dominated mathematics education. The document represents three years of development in which draft versions have been presented to the members of NCTM, classroom teachers across the country, state policy makers, textbook publishers, supervisors of curriculum, supervisors of mathematics instruction, and mathematics educators in teacher preparation programs. The presentations were intended to promote a vision of mathematics education aimed at enabling children and youth with the power of mathematical knowledge, to judge the degree to which these constituencies supported such a vision, and to raise questions about what would be required to implement the vision. The document is in some ways unique. It is not simply a statement of broad goals for reforming mathematics education. Each standard provides an elaboration of mathematical content and mathematical instruction. Each standard emphasizes knowing and doing mathematics. Each standard describes content that is appropriate for all learners. And each standard is accompanied with some activities as examples of what knowing and doing mathematics can look like in the classroom. To provide an example of the vision, portions of the first standard, **mathematics as problem solving**, for grades K-4 are presented.

> In grades K-4, the study of mathematics should emphasize problem solving so that students can--
>
> * use problem-solving approaches to investigate and understand mathematical content;
> * formulate problems from everyday and mathematical situations;
> * develop and apply strategies to solve a wide variety of problems;
> * verify and interpret results with respect to the original problem;
> * acquire confidence in using mathematics meaningfully.
>
> Problem solving is not a distinct topic but a process that sould permeate the entire program and provide the context in which concepts and skills can be learned (p. 23).

Each standard provides examples of activities appropriate to the content of the standard and the vision. These involve activities that children work on individually, in small groups and as a whole class.

> Project problems, which often require several days of class time, provide an opportunity for children to become immersed in problem-solving activity. Some situations allow children to be particularly creative in their formulation of problems. Here is one such situation:
>
> The class is given the opportunity to plan and participate in an all-school "Estimation Day." The children, in pairs or threes, are to design estimation activities to be completed by children in other classes. Each group will supply all the necessary materials and monitor the activities. The activities might include guessing children's heights, the number of candies in a jar, the lengths of various pieces of string, the weight of a bag of potatoes, the length of the room, the number of times they can write their names in a minute, or the length of time required for an ice cube to melt (p. 25).

The vision portrayed in the goals for learning and the types of learning activities stand in stark contrast to what is typically found in traditional textbooks and what existed in the formal curriculum documents of the Detroit reform. It is a vision toward teaching for enabling mathematical inquiry. The question is, what would be required to create this vision in classrooms in the Detroit Public Schools?

**What is Required to Create This Vision in Classrooms?**

My effort here is to sketch out some of the key issues that are are likely to be raised concerning this new reform agenda.

Teacher knowledge. The dominance of a typical routine in mathematics classrooms results from teachers' own experiences, as learners of mathematics, as students in elementary teacher preparation programs, and as practicing classroom teachers. As students in elementary and secondary mathematics classrooms, the "apprenticeship of observation" has provided teachers with a view of how mathematics is taught. Preservice programs typically require a methods course in teaching elementary mathematics that is likely to focus on improving technical competence, to make the typical routine more efficient and effective. Their experiences as students, in elementary, secondary and university mathematics classes, have taught them what it means to know mathematics -- being able to recall appropriate rules and algorithms to apply to problems. As practicing teachers, the curricular materials they may be mandated to use and the instructional practices that are endorsed contribute to the continuation of business as usual.

This is a problem that must be addressed at both the preservice and inservice level. If teachers are to incorporate materials and practices aimed at enabling mathematical inquiry they must have opportunities to deepen their knowledge about mathematics and what it means to know and do mathematics. They must have opportunities to deepen their knowledge of how children learn mathematics. And they must have opportunities to consider what kinds of instructional practices promote mathematical inquiry.

This requires districts to take a hard look at what constitutes inservice in their systems. The continuing professional development of their staffs should become a priority. The one-shot workshop, whether offered after-school by in-house people or at all day sessions provided by nationally recognized, charismatic educational entrepreneurs selling their wares, is notoriously ineffective in changing what goes on in classrooms. The practice needs to be reconsidered. Where offerings are long-term, as Detroit's EEEI training, districts need to look carefully at what these programs are intended to do. Refining teacher techniques at delivering direct instruction will not develop ways of thinking, acting and questioning that promote mathematical inquiry in classrooms. Just as students need knowledge beyond that which is technical and practical, so do professional teachers.

Testing. Districts, especially those in urban settings with large minority and poor populations, have been under intense pressure to improve student scores on achievement tests. It is not likely that will subside. But tests need to change because a curriculum aimed at enabling mathematical inquiry is very different from most curricula and the tests with which they are integrated. Using standardized test scores, whether norm- or criterion-referenced, as the sole indicator of the effectiveness of instructional programs may simplify reporting but it has limited value in telling us what children are coming to know.

The NCTM Standards suggest that increased attention needs to be paid to assessing not only what students know but how they think about mathematics, multiple methods to assess student understanding, and multiple schemes for scoring and recording what they know. They also argue that the purposes of testing need to be reexamined and broadened. Giving periodic paper-and-pencil tests for the purposes of recording a mark and then averaging a set of marks to give a course grade represent the narrowest view of assessment. Giving multiple choice tests but not asking why students answer questions incorrectly ignores the importance of meaning that students attach to situations and how it influences the ways in which they think about and solve problems.

Rethinking testing and evaluation is no small matter. Having a single score to report to the community can make for good public relations. It fits well with administrative concerns about accountability and standardization.[5] The public has a right to be informed about the progess of the students in its schools and the effectiveness of instructional programs. But educational professionals need to help the public realize how little they learn by simply asking, "How much have our scores gone up?"

School administration and organization. Superintendents, district administrators and building principals are often trained in organizational theory and systems management where they learn elaborate language systems, management controls and accountability systems. The ideas embedded in this instrumental and technical approach to school administration led to further centralization of decision making and standardization of instructional programs. If mathematics education is

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[5]Administration of ABCS was discontinued in the Detroit schools after 1987 because students continued to do poorly and because the test results were not reported as a single score. Area superintendents found the public relations value of the test to be negative. They applied sufficient pressure on central administrators that the decision was finally made to abandon the testing program that had been the basis on which the curriculum was conceived.

to be transformed, fundamental changes need to occur in the ways schools are organized and administered.

Teachers need to be freed from lock-step curricula and pedagogies that reduce the work of teaching to managing instruction. If we want to develop a set of intellectual tools in our students, then we need to redefine the work of teachers as intellectual work, not technical work. This means teachers must have a role in shaping the purposes and conditions of mathematics education in their schools and in the district.

Teachers also need to be continual learners; schools should become sites of the continuing professional development of their staffs. That requires rethinking how schools are organized, what time is available for the planning and reflection that intellectual work demands, the norms and structures that shape relations among the staff.

The tendency toward increasing centralization of control is contradictory to the diffusion of decision making that the new vision of mathematics education suggests. The recent interest in site-based management schemes may be the opening to restoring decisions about the specifics of educational programs to local schools.

<u>The value of mathematics</u>. If teaching mathematics for enabling mathematical inquiry is to become a reality in elementary classrooms, we need to confront the extent to which mathematics is seen as having only utilitarian value. In <u>Everybody Counts</u>, the National Research Council calls mathematics "the invisible culture of our age" because it influences in multiple but often hidden ways the practical, civic, professional, leisure and cultural layers of our lives. Mathematics education has a responsibility to reveal the layers and to do it in

interesting ways so that the familiar lament, "Why do I have to know this?" no longer rings in classrooms.

I argued earlier that mathematical literacy needed to take on new meaning, not just in a limited economic and technical sense, but also in a political and social sense. Classrooms must become places where intellectual inquiry is promoted, where children working together engage in purposeful activity, where they learn to make reasoned judgments about their own thinking and that of others. This does not deny the practical and utilitarian value in training a workforce. Rather it recognizes that the mathematical needs of workers in a new economy require precisely these skills. But there are reasons beyond this key one. A new kind of quantitative literacy is required for people to engage thoughtfully and responsibily as citizens in a democratic society. And this requires not only changing the content of mathematics instruction but providing environments where students working together experience the ways in which collective efforts at problem solving enhance understanding.

## A Necessary but Not Sufficient Condition for Liberation

This reform initiative was a response, in part, to problems that were rooted in the larger context of social, political and economic factors. Economic disinvestment in the city of Detroit has meant that black youth face a very different future from earlier generations who were able to find high-wage, semi-skilled work in the plants. Some of these manufacturing jobs lost to the city have been replaced by manufacturing and service work that requires increasing levels of technical expertise and quantitative literacy. But many of the lost jobs have been replaced by low-skill, low-wage work in the service sector. The problems that confront Detroit youth and young adults are not just school-based. Consequently, the solutions are not to be found solely in educational reform. Proposing technical solutions to problems that are, in part, politically and economically rooted ignores the relationship between schools and society. There is a much larger agenda of reform.

Transforming mathematics education in the Detroit schools will not change the structure of the Detroit economy. Transformation of the mathematics curriculum toward **enabling mathematical inquiry** cannot by itself overcome the forces of racism, sexism and class bias that structure inequality in this society. But mathematics education can no longer play a gatekeeping role, limiting the opportunities of many people to function in a world that is demanding increasing mathematical literacy of its citizens. Students who have the opportunity to experience mathematics education in an environment that promotes inquiry, where others' formulations are not taken as given, where the subject matter itself becomes the vehicle to investigate the structured inequality in society, will have developed a set of intellectual tools to participate more critically and fully as workers and citizens.

The transformation of mathematics education toward **enabling mathematical inquiry** is bold and will be expensive and disruptive. This may prove to be a sizeable hurdle for urban districts like Detroit that are already overburdened by contextual factors over which they have no control. But we have come to recognize that our society is becoming increasingly multi-racial, multi-cultural and multi-lingual.

Urban districts that serve increasing numbers of students of color must implement policies and programs that do not leave their students marginalized in society, as workers or as citizens. It is required in the new economic order and it is appropriate to the good society.

# LIST OF REFERENCES

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An agenda for action: Recommendations for school mathematics in the 1980s. (1980). Reston, VA: National Council of Teachers of Mathematics.

Anderson, L., Evertson, C., & Brophy, J. (1979). An experimental study of effective teaching in first-grade reading groups. Elementary School Journal, 79, 193-223.

Anyon, J. (1981). Social class and school knowledge. Curriculum Inquiry, 11, 3-42.

Anyon, J. (1983). Social class and the hidden curriculum of work. In H. Giroux & D. Purpel (Eds.), The hidden curriculum and moral education. Berkeley, CA: McCutchan.

Apple, M. (1979). Ideology and curriculum. London: Routledge & Kegan Paul.

Apple, M. (1983a, January). Curriculum in the year 2000: Tensions and possibilities. Phi Delta Kappan, 321-326.

Apple, M. (1983b). Curricular form and the logic of technical control. In M. Apple & L. Weis (Eds.), Ideology and practice in schooling. Philadelphia, PA: Temple University Press.

Apple, M. (1986). Teachers and texts: A political economy of class & gender relations in education. New York: Routledge & Kegan Paul.

Aronowitz, S., & Giroux, H. (1985). Education under seige. South Hadley, MA: Bergin & Garvey.

Avital, S., & Shettleworth, S. (1968). Objectives for mathematics learning. Toronto: The Ontario Institute for Studies in Education.

Ball, D. (1988). Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education. Unpublished doctoral dissertation, Michigan State University, East Lansing, MI.

Ball, D., & Wilcox, S. (in press). Inservice teacher education in mathematics: Examining the interaction of context and content. (Research Report 89-3). East Lansing, MI: Michigan State University, National Center for Research on Teacher Education.

Barth, R. (1985, April). The principal and the profession of teaching. Paper prepared for the California Commission on the Teaching Profession.

Berliner, D., & Tikunoff, W. (1976). The California Beginning Teacher Evaluation Study: Overview of the ethnographic study. Journal of Teacher Education, 27(1), 24-30.

Block, J. (1980). Promoting excellence through mastery learning. Theory Into Practice, 19(1), 66-74.

Bogdan, R., & Biklin, S. (1982). Qualitative research for education: An introduction to theory and methods. Boston: Allyn & Bacon.

Bowles, S., & Gintis, H. (1976). Schooling in capitalist America. New York: Basic Books.

Brophy, J., & Evertson, C. (1976). Learning from teaching: A developmental perspective. Boston: Allyn and Bacon.

Brophy, J. & Good, T. (1986). Teacher behavior and student achievement. In M. Wittrock, (Ed.), Handbook of research on teaching. New York: Macmillan.

Carpenter, T. (1986). Conceptual knowledge as a foundation for procedural knowledge. In J. Heibert (Ed.), Conceptual and procedural knowledge: The case of mathematics.

Carpenter, T., Coburn, T., Reys, R., & Wilson, J. (1978). Results from the first mathematics assessment of the National Assessment of Educational Progress. Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T., Corbett, M., Kepner, H., Lindquist, M., & Reys, R. (1981). Results from the second mathematics assessment of the National Assessment of Education Progress. Reston, VA: National Countil of Teachers of Mathematics.

Carpenter, T., Moser, J., & Romberg, T. (1982). Addition and subtraction: A cognitive perspective. Hillsdale, NJ: Erlbaum.

Chism, N. (1985, April). The knowledge that teachers value: Findings from a case study. Paper presented to the annual meeting of the American Educational Research Association, Chicago, IL.

Clark, C., & Lampert, M. (1985, April). What knowledge is of most worth to teachers? Insights from studies of teacher thinking. Paper presented to the annual meeting of the American Educational Research Association.

Commission on Standards for School Mathematics. (1977). Curriculum and evaluation standards for school mathematics: A draft. Reston, VA: National Council of Teachers of Mathematics.

Commission on Standards for School Mathematics. (1988). Curriculum and evaluation standards for school mathematics: A draft. Reston, VA: National Council of Teachers of Mathematics.

Commission on Standards for School Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Conot, R. (1974). American odyssey. New York: William Morrow.

Costa, A. (1984). A reaction to Hunter's knowing, teaching and supervising. In P. Hosford (Ed.), Using what we know about teaching. Alexandria, VA: Association for Supervision and Curriculum Development.

Cuban, L. (1984). Transforming the frog into a prince: Effective schools research, policy, and practice at the district level. Harvard Educational Review, 54, 129-150.

Davis, P., & Hersh, R. (1981). The mathematical experience. Boston, MA: Houghton Mifflin.

Day, C. (1985, May). Why teachers change their thinking and behavior: Case studies in professional learning through inservice activity. Paper presented at the conference of The International Study Association on Teacher Thinking, The Netherlands.

Detroit Free Press, (January 1, 1989). No steady job: It's becoming more common in the state. Detroit, MI.

Detroit Free Press, (February 24, 1989). Council considers housing homeless. Detroit, MI.

Detroit Public Schools, (June, 1982). ABCS test specifications, reading, writing and mathematics, P-5. Department of Research and Evaluation, Test Development Section.

Detroit Public Schools. (June, 1983). Testing component: Final report.

Detroit Public Schools. (April, 1984). Summary of ABCS development. Office of Instructional Improvement, Department of Evaluation and Testing.

Detroit Public Schools. (January, 1987). Pre-kindergarten - grade 12 curriculum: Strands and objectives - Mathematics education. Office of Curriculum Development and Services.

Detroit Public Schools. (September, 1987). Model lessons to promote thinking. Mathematics and Science Department.

Detroit Public Schools (May 19, 1988). Dropout, graduation and retention for new ninth graders, 1982 and 1983. Division of Educational Services, Office of Instructional Improvement, Information Systems Department.

Detroit Public Schools. (1988). Project Head Start: Community needs assessment. Department of Early Childhood Education.

Devaney, K. (1983, December). Math beyond computation. Educational Leadership, 58-60.

Dewey, J. (1916). Democracy and education. New York: Macmillan.

Dienes, Z. (1970). Some basic processes involved in mathematics learning. In R. Ashlock and W. Herman (Eds.), Current research in elementary school mathematics. New York: Macmillan.

Dossey, J., Mullis, I., Lindquist, M., & Chambers, D. (1988). The mathematics report card: Are we measuring up? Princeton, NJ:Educational Testing Services.

Doyle, W. (1983). Academic work. Review of Educational Research, 53, 159-200.

Doyle, W., & Ponder, G. (1979). The practicality ethic in teacher decision-making. Interchange, 8(3), 1-12.

Driscoll, J. (1981). Research within reach: Elementary school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Eisenhart, M. (1988). The ethnographic research tradition and mathematics education. Journal for Research in Mathematics Education, 19(2), 99-114.

English, F. (1983). Contemporary curriculum circumstances. In Fundamental curriculum decisions. Alexandria, VA: The Association for Supervision and Curriculum Development.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. Wittrock (Ed.), Handbook of research on teaching. New York: Macmillan.

Erlwanger, S. (1973). Benny's conception of rules and answers in IPI mathematics. Journal of Children's Mathematical Behavior, 1(2), 7-26.

Fennema, E. (1972a). The relative effectiveness of a symbolic and a concrete model in learning a selected mathematical principle. Journal for Research in Mathematics Education, 3, 233-238.

Fennema, E. (1972b). Models and mathematics. Arithmetic Teacher, 19, 635-640.

Fey, J. (1978). Change in mathematics education since the late 1950's - Ideas and realisation, U.S.A. Educational Studies in Mathematics, 9, 339-353.

Flanders, N. (1970). Analyzing teacher behavior. Reading, MA: Addison-Wesley.

Franklin, B. (1976, March). Technological models and the curriculum field. Educational Forum.

Gagne, R. (1976). The learning basis of teaching methods. In The psychology of teaching methods. Seventh-fifth Yearbook of the National Society for the Study of Education. Chicago, IL: University of Chicago Press.

Gall, M., & Renchler, R. (1985). Effective staff development for teachers: A research based model. Eugene, OR: Clearinghouse on Educational Management, University of Oregon.

Georgakas, D., & Surkin, M. (1975). Detroit: I Do Mind Dying. New York: St. Martin's Press.

Ginsberg, H. (1977). Children's arithmetic: The learning process. New York: Van Nostrand.

Giroux, H. (1981). Ideology, culture and the process of schooling. Philadelphia: Temple University Press.

Giroux, H. (1983). Theory and resistance in education: A pedagogy for the opposition. South Hadley, MA: Bergin & Garvey.

Giroux, H. (1984). Public philosophy and the crisis in education. Harvard Educational Review, 54 (2), 186-194.

Giroux, H., Penna, A., & Pinar, W. (1981). Curriculum and instruction. Berkeley, CA: McCutchan.

Gitlin, A. (1983). School structure and teachers' work. In M. Apple & L. Weis (Eds.), Ideology & practice in schooling. Philadelphia, PA: Temple University Press.

Glatthorn, A. (1984). Differentiated supervision. Alexandria, VA: Association for Supervision and Curriculum Development.

Good, T., & Grouws, D. (1975). Process-product relationship in fourth-grade mathematics classrooms (Final Report: National Institute of Education Grant NIE-G-00-3-0123). Columbia: University of Missouri, College of Education.

Good, T., & Grouws, D. (1977). Teaching effects: A process-product study in fourth grade mathematics classrooms. Journal of Teacher Education, 28, 49-54.

Good, T., Grouws, D., & Ebmeier, H. (1983). Active mathematics teaching. New York: Longman.

Gordon, M. (1983). Conflict and liberation: Personal aspects of the mathematics experience. In H. Giroux & D. Purpel (Eds.), The hidden curriculum and moral education. Berkeley, CA: McCutchan.

Grant Foundation Commission on Work, Family and Citizenship. (February, 1988). The forgotten half: Non-college-bound youth in America. Phi Delta Kappan, 409-414.

Greene, M. (1978). Teaching: The question of personal reality. Teachers College Record,80(1), 23-35.

Guskey, T. (1986). Staff development and the process of teacher change. Educational Researcher, 15(5), 5-12.

Hall, G., & Loucks, S. (1978). Teacher concerns as a basis for facilitating and personalizing staff development. Teachers College Record, 80(1), 36-53.

Hammersley, M., & Atkinson, R. (1983). Ethnography: Principles in practice. New York: Tavistock Publications.

Hiebert, J. (1980). Children's thinking. In R. J. Shumway (Ed.), Research in mathematics education. Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), Conceptual and procedural knowledge: The case of mathematics. Hillsdale, NJ: Lawrence Erlbaum.

Hiller, J., Fisher, G., & Kaess, W. (1969). A computer investigation of verbal characteristics of effective classroom lecturing. American Educational Research Journal, 10, 21-37.

Hunter, M. (1976). Rx improved instruction. El Segundo, CA: T.I.P. Publications.

Hunter, M. (1984). Knowing, teaching, and supervising. In P. Hosford (Ed.), Using what we know about teaching. Alexandria, VA: Association for Supervision and Curriculum Development, 169-192.

Hynes, M. (1979). Using manipulative aids to model algorithms in remedial situations. In M. Hynes (Ed.), Topics related to diagnosis in mathematics for classroom teachers. Bowling Green, OH: Research Council for Diagnostic and Prescriptive Mathematics.

Jacobs, J. (1987). Black workers and the new technology: The need for a new urban training policy. Ann Arbor, MI: Industrial Technology Institute.

Jacullo-Noto, J. (1984). Interactive research and development - partners in craft. Teachers College Record, 86(1), 208-222.

Karabel, J. (1972). Community colleges and social stratification. Harvard Educational Review, 42(4), 521-562.

Karabel, J., & Halsey, A. (1977). Power and ideology in education. New York: Oxford University Press.

Katz, M. (1982). Critical literacy: A conception of education as a moral right and social ideal. In R. Everhart (Ed.), The public school monopoly. Cambridge, MA: Ballinger press.

Kearney, C. (1987). A primer on Michigan school finance. Prepared for The Coalition for Equity and Excellence in Michigan Schools, New Detroit, Inc. Ann Arbor, MI: The University of Michigan School of Education, Bureau of Accreditation and School Improvement Studies.

Kliebard, H. (1972). Metaphorical roots of curriculum design. Teachers College Record, 73, 403-404.

Kliebard, H. (1975). Bureaucracy and curriculum theory. In W. Pinar (Ed.), Curriculum theorizing: The reconceptualists. Berkeley, CA: McCutchan.

Kounin, J. (1970). Discipline and group management in classrooms. New York: Holt, Rinehart & Winston.

Labaree, D. (1984). Setting the standard: Alternative policies for student promotion. Harvard Educational Review, 54(1), 67-87.

Lampert, M. (1986a). Teaching multiplication. The Journal of Mathematical Behavior, 5(3), 241-280.

Lampert, M. (1986b). Knowing, doing, and teaching multiplication. Cognition and Instruction, 3, 305-342.

Lampert, M. (in press a). Choosing and using mathematical tools in classroom discourse. In J. Brophy (Ed.), Advances in research on teaching: Teaching for meaningful understanding (Vol. 1). Greenwich, CT: JAI Press.

Lampert, M. (in press b). What can research on teacher education tell us about improving quality in mathematics education? Teaching & Teacher Education.

Lieberman, A. (1985, April). Collaborative research: Working with not on... Paper presented to the annual meeting of the American Education Research Association, Chicago, IL.

Little, J. (1984). Seductive images and organizational realities in professional development. Teachers College Record, 86(1), 84-102.

Little, J. (1985, April). Schools contributions to teaching as a profession. Paper presented to the annual meeting of the American Educational Research Association, Chicago, IL.

Luria, D., & Russell, J. (1981). Rational reindustrialization. Detroit, MI: Widgetripper Press.

Madsen-Nason, A. (1988). A teacher's changing thoughts and practices in ninth grade general mathematics classes: A case study. Unpublished doctoral dissertation, Michigan State University, East Lansing, MI.

McDonald, F. (1976). Report on Phase II of the Beginning Teacher Evaluation Study. Journal of Teacher Education, 72(1), 39-42.

McKnight, C., Crosswhite, F., Dossey, J., Kifer, E., Swafford, J., Travers, K., & Cooney, T. (1987). The underachieving curriculum: Assessing U.S. school mathematics from an international perspective. Champaign, IL: Stripes Publishing.

McLaughlin, M., & Marsh, D. (1978). Staff development and school change. Teachers College Record, 80(1), 69-94.

Merseth, K. (1978). Using materials and activities in teaching addition and subtraction algorithms. In M. Suydam and R. Reys (Eds.), Developing computational skills, 1978 NCTM Yearbook. Reston, VA: National Council of Teachers of Mathematics.

Monitoring Commission Report, (June, 1984). Overview and status of the Detroit desegregation case. Detroit, MI: Detroit School District United States District Court Monitoring Commission.

Monitoring Commission Report, (October, 1984). Profiles of Detroit's high schools: 1975 to 1984. Detroit, MI: Detroit School District United States District Court Monitoring Commission.

National Advisory Commission on Mathematical Education. (1975). Overview and analysis of school mathematics, grades K-12. Washington, DC: Conference Board of Mathematical Sciences.

National Council of Teachers of Mathematics. (1985). A position statement on professional development programs for teachers of mathematics. Reston, VA.

National Research Council. (1989). Everybody counts. Washington D.C.: National Academy Press.

Nelson, D, & Reys, R. (1976). Measurement in school mathematics. 1976 NCTM Yearbook. Reston, VA: National Council of Teachers of Mathematics.

New Detroit, Inc. (1986). Report. Detroit Public Schools Vocational Technical Centers Subcommittee.

Noujain, E. (1987). History as genealogy: An exploration of Foucault's approach to history. In A. Griffith (Ed.), Contemporary French philosophy. London: Cambridge University Press.

Payne, J., & Rathmell, E. (1975). Number and numeration. In J. Payne (Ed.), Mathematics learning in early childhood, 37th NCTM Yearbook. Reston, VA: National Council of Teachers of Mathematics.

Peterson, P. (1979). Direct instruction reconsidered. In P. Peterson & H. Walberg (Eds.), Research on teaching: Concepts, findings and implications. Berkeley, CA: McCutchan.

Peterson, P. (1988). Teaching for higher-order thinking in mathematics: The challenge for the next decade. In D. Grouws, T. Cooney & D. Jones (Eds.), Perspectives on research on effective mathematics teaching. Reston, VA: Lawrence Erlbaum Associates and National Council of Teachers of Mathematics.

Peterson, P. & Fennema, E. (1985). Effective teaching, student engagement in classroom activities, and sex-related differences in learning mathematics. American Educational Research Journal, 22(3), 309-335.

Pinar, W. (1978, September). Notes on the curriculum field 1978. Educational Researcher, 5-12.

Popham, W. (1969). Instructional objectives. Chicago: Rand McNally.

Prigge, G. (1978). The differential effects of the use of manipulative aids on the learning of geometric concepts by elementary school children. Journal for Research in Mathematics Education, 9, 361-367.

Rauh, P. (1978). Helping teacher: A model for staff development. Teachers College Record, 80(1), 157-171.

Reich, R. (1988). Education and the next economy. Washington, D.C.: National Education Association.

Resnick, L. (1982). Syntax and semantics in learning to subtract. In T. Carpenter, P. Moser & T. Romberg (Eds.), Addition and subtraction: A cognitive perspective. Hillsdale, NJ: Erlbaum.

Resnick, L. (1983). Mathematics and science learning: A new conception. Science, 220, 477-478.

Resnick. L., & Ford, W. (1981). The psychology of mathematics for instruction. Hillsdale, NJ: Erlbaum.

Romberg, T. (1983). A common curriculum for mathematics. In G Fenstermacher and J. Goodlad (Eds.), Individual differences and the common curriculum. Eighty-second Yearbook of the National Society for the Study of Education. Chicago, IL: University of Chicago Press.

Romberg, T & Carpenter, T. (1986). Research on teaching and learning mathematics: Two disciplines of scientific inquiry. In M. Wittrock (Ed.), Handbook of research on teaching. New York: Macmillan.

Rosenbaum, J. (1976). Making inequality: The hidden curriculum of high school tracking. New York: Wiley.

Rosenshine, B. (1968). To explain: A review of reseearch. Educational Leadership, 26, 275-280.

Sarason, S. (1971). The culture of the school and the problem of change. Boston, MA: Allyn & Bacon.

Saylor, G., & Alexander, W. (1966). Curriculum planning for modern schools. New York: Holt, Rinehart, & Winston.

Scheffler, I. (1976). Basic mathematical skills: Some philosophical and practical remarks. Teachers College Record, 78(2).

Schoenfeld, A. (1985). When good teaching leads to bad results. The disasters of well taught mathematics courses. In P. Peterson & T. Carpenter (Eds.), Learning through instruction: The study of students' thinking during instruction in mathematics. (Program Report No. 86-9). Madison, WI: University of Wisconsin, School of Education.

Schon, D. (1983). From technical rationality to reflection-in-action. In The reflective practitioner: How professionals think in action. New York: Basic Books.

Schram, P., Feiman-Nemser, S., & Ball, D. (1989, March). Thinking about teaching subtraction with regrouping: A comparison of beginning and experienced teachers' responses to textbooks. Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Schram, P., & Wilcox, S. (1988). Changing preservice teachers' conceptions of mathematics learning. In M. Behr, C. Lacampagne and M. Montague Wheeler (Eds.), Proceedings of the Tenth Annual Meeting, North American Chapter of the International Group for the Psychology of Mathematics Education, DeKalb, IL.

Schram, P., Wilcox, S., Lanier, P., & Lappan, G. (1988). Changing mathematical conceptions of preservice teachers: A content and pedagogical intervention. (Research Report 88-4). East Lansing, MI: National Center for Research on Teacher Education.

Schram, P., Wilcox, S., Lanier, P., & Lappan, G. (1989). Changing preservice teachers' beliefs about mathematics education. Proceedings of the Eleventh Annual Meeting, North American Chapter of the International Group for the Psychology of Mathematics Education, Short, E. (1986). A historical look at curriculum design. Theory Into Practice, 25(1), 3-9.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 15(2), 4-14.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. Harvard Educational Review, 57(1).

Skinner, B. (1968). The technology of teaching. New York: Appleton.

Soar, R.S., & Soar, R.M. (1979). Emotional climate and management. In P. Peterson and H. Walberg (Eds.), Research on teaching: Concepts, findings, and implications. Berkeley, CA: McCutchan.

Stallings, J., Cory, R., Fairweather, J., & Needels, M. (1978). A study of basic reading skills taught in secondary schools. Menlo Park, CA: SRI International.

Stallings, J., & Kaskowitz, D. (1974). Follow Through classroom observation evaluation 1972-1973 (SRI Project URU-7370). Stanford, CA: Stanford Research Institute.

Steffe, L., vonGlasersfeld, E., Richards, J., & Cobb, P. (1983). Children's counting types: Philosophy, theory and application. New York: Praeger.

Stevenson, H. (1975). Learning and cognition. In J. Payne (Ed.), Mathematics learning in early childhood, 37th NCTM Yearbook. Reston, VA: National Council of Teachers of Mathematics.

Stodolsky, S. (1987, May). The subject matters: Instruction in math and social studies. News and News Colloquim Report. East Lansing, MI: Michigan State University.

Suydam, M., & Higgins, J. (1977). Activity-based learning in elementary school mathematics: Recommendations from research. Reston, VA: National Council of Teachers of Mathematics.

Swanson-Owens, D. (1985, April). Identifying natural sources of resistance: A case study analysis of curriculum implementation. Paper presented to the annual meeting of the American Educational Research Association, Chicago, IL.

Taba, H. (1962). Curriculum development. New York: Harcourt, Brace, Javonovich.

Tanner D., & Tanner, L. (1980). Curriculum development, 2nd ed. New York: Macmillan.

Thorndike, E. (1922). The psychology of arithmetic. New York: Macmillan.

Tyler, R. (1949). Basic principles of curriculum and instruction. Chicago, IL: University of Chicago Press.

United States District Court, Eastern District of Michigan, Southern Division, (August 15, 1975). Partial judgment and order. Detroit, MI.

United States District Court, Eastern District of Michigan, Southern Division, (May 11, 1976). Final judgment. Detroit, MI.

Welch, W. (1978). Science education in Urbanville: A case study. In R. Stake & J. Easley (Eds.), Case studies in science education. Urbana, IL: University of Illinois.

Wheatley, G. (1978). Hemispheric specialization and cognitive development: Implications for mathematics education. Journal for Research in Mathematics Education, 9, 19-32.

Wilson, W. (1987). The truly disadvantaged: The inner city, the underclass, and public policy. Chicago: The University of Chicago Press.

Wittrock, M. (1974). A generative model of mathematics learning. Journal for Research in Mathematics Education, 5, 181-196.

Wolf, E. (1981). Trial and error. Detroit, MI: Wayne State University Press.

✓ Wood, F., & Thompson, S. (1980, February). Guidelines for better staff development. Educational Leadership, 374-378.

✓ Young, M.F.D. (1974). Curricula and the social organization of knowledge. In R. Brown (Ed.), Knowledge, education, and cultural change: Papers in the sociology of education. New York: Harper & Row.

Young, L., & Melnick, S. (1988). Forsaken lives, abandoned dreams: What will compel us to act? Harvard Educational Review, 58(3), 380-394.

Zais, R. (1976). Curriculum: Principles and foundations. New York: Thomas Crowell.

Zeichner, K. (1983). Alternative paradigms on teacher education. Journal of Teacher Education, 34(3), 3-9.

# APPENDIX A

**Teacher Interview Schedule**

1. In what ways is the DPS curriculum -- strands and objectives -- different from the mathematics curriculum you previously taught?

2. Are there some mathematical topics you will be teaching for the first time?

3. What would help you to implement the strands and objectives?

4. What materials do you have that you find helpful?

5. What influenced your decision about whether or not to attend inservice?

6. (For those who attended inservice) What did you hope to learn from inservice? How has inservice influenced the way you think about the mathematics curriculum and ways of teaching mathematics?

7. What support, in addition to inservice workshops, has been made available to you?

8. What factors enhance/constrain your efforts to implement the curriculum strands and objectives?

9. What goals do you have for your students as learners of mathematics?

10. Do you think the goals of the district are the same as your?

11. Using manipulatives has received considerable emphasis in inservice. What purpose do you think manipulatives serve? Do you use manipulatives? When, how often, to teach what? With what results?

12. What do you think about when you hear the term "conceptual understanding?"

# APPENDIX B

## CITY-WIDE INSTRUCTIONAL SEQUENCE - GRADE 3

September

PROBABILITY AND STATISTICS

Read and interpret a pictograph using ratios of 1:1 and 1:2, using a key.

Read and interpret a bar graph using calibrations of units of 1, 2 and 5, using a key.

Construct a bar and pictograph using data with ratios of 1:1 and 1:2.

Identify a circle graph.

SETS AND LOGIC

Identify the position of objects according to left/right, top/bottom, over/under, above/below, on/off, first/last, first through tenth, before/between/after, inside/outside, on and both.

MEASUREMENT

Identify the value of dozen(s).

Identify and record time by the hour, half-hour and quarter hour (non-digital).

Name the days of the week and the months of the year in order.

CALCULATORS AND COMPUTERS

Boot and interact with appropriate computer assisted learning software (if available).

NUMERATION

Interpret and/or illustrate the meaning of whole numbers through 999.

Demonstrate accuracy in reading and writing whole numerals through 999.

Determine and/or demonstrate place value through 999 using aids.

FUNCTIONS AND RELATIONS

Interpret and complete addition, subtraction or multiplication number sentences using one of these symbols, $<$, $>$, $=$ or $\neq$.

OPERATIONS

Recall the addition and subtraction facts through sums of 18 with or without the use of aids (pictures, number line).

FUNCTIONS AND RELATIONS
Relate the inverse of addition and subtraction.

PATTERNS
Interpret and/or extend a pattern of shapes, pictures or objects using a combination of any three of the following attributes: color; shape; size; thickness; position; texture.

OPERATIONS
Add whole numbers through two digits with and without regrouping.

Demonstrate the meaning of multiplication as repeated addition within reasonable limits, and vice versa.

<u>October</u>

FUNCTIONS AND RELATIONS
Apply a given rule to a set of whole numbers and determine the missing whole number output.

Apply a given rule to a set of whole numbers to determine the missing input numbers using addition, subtraction or multiplication.

GEOMETRY
Demonstrate ways to partition a circle, square, rectangle or equilateral triangle as appropriate into a given number of congruent parts (2, 3, 4, 6, 8, or 16).

FUNCTIONS AND RELATIONS
Identify and state the rule which indicates the relationship between whole numbers when given input or output.

OPERATIONS
Add and subtract whole numbers through two digits.

NUMERATION
Recognize and write number words through 999.

Identify the value of a given digit up through a 3 digit number.

Identify a number that is 10 or 100 more or less than a given number.

Write numbers in standard notation given in words (orally or written) or expanded notation and vice versa.

Write money amounts using standard money notation, given dollars, dimes or pennies or given the money notation select the monetary amount.

OPERATIONS
Apply addition, subtraction strategies to solve money or other word problems.

MEASUREMENT
Identify, compare, write and compute money values through $9.99.

OPERATIONS
Addition and subtraction of money numbers.

Recall the multiplication facts where both factors are 0-5.

NUMERATION
Identify numbers as :odd or even; two times as much; twice as much; half of a given set through 18.

Use the property of zero and one in multiplication.

SETS AND LOGIC
Identify the correct word statement, picture, place, thing or number from 2 or more statements.

November

NUMERATION
Use the commutative law in addition to solve equations.

Use the associative law for addition to solve equations.

OPERATIONS
Apply addition, subtraction or multiplication strategies to solve money, time or other word problems (one or two steps).

Add and subtract whole numbers through three digits.

NUMERATION
Using place value analyze two numbers through 999 using <, >, or =.

PATTERNS
Interpret and complete a number pattern (using numbers 0-99; starting at any number) in ascending or descending order counting by ones, twos, fives or tens.

MEASUREMENT
Read and interpret thermometers Celsius and Fahrenheit of various calibrations.

OPERATIONS
Multiply whole numbers through 3 digit times 1 digit (no regrouping).

MEASUREMENT

Estimate and determine the length of an object or distance using non-standard units.

Estimate and determine the length of an object using whole metric units.

Relate and compare metric units of measure (cm and m).

December

GEOMETRY

Identify geometric figures, symbols and their written words: square corner; point; ray; line segment; line; angle.

Identify and recognize cube, cone, cylinder and sphere by word names and models in the everyday environment.

Draw points. line segments, rays, angles and lines using appropriate tools.

Identify and reproduce lines of symmetry.

MEASUREMENT

Compare and identify the lengths of objects.

Draw line segments for given whole unit lengths.

PATTERNS

Interpret and complete a number pattern using numbers 0-999 (starting at any number) in ascending or descending order counting by ones through sixes, tens and hundreds. Include multiplication patterns.

ESTIMATION

Identify and estimate length when given a non-standard unit.

OPERATIONS

Subtract whole numbers through three digits (with and without regrouping).

NUMERATION

Identify numbers as multiples of 2, 3, 5 or 10.

January

ESTIMATION

Round any two-digit number to the nearest multiple of ten.

Round any three-digit number to the nearest multiple of hundred.

Round the individual problem numbers to the nearest multiple of ten or hundred and estimate the sum or difference: incomplete number sentence with results of less than a thousand; money problems using pennies, nickels, dimes, dollars or ten dollars; word problems.

Use estimation to check the reasonableness of answers.

MEASUREMENT

Determine and record the area of a figure on a grid in square units.

Identify the reasonable customary or metric unit to measure the length of an object, height or distance.

Use a metric or customary ruler to measure the sides of a polygon or pictured object.

Determine and record the perimeter of a polygon or grid following the lines on the grid.

PATTERNS

State the rule used in forming a given pattern and extend the pattern.

Continue a pattern (objects or pictures) or equivalent ratios by adding to the series.

CALCULATORS AND COMPUTERS

Investigate ways to utilize the hand-held calculator to solve problems: basic facts; two-step problems; check on accuracy of predicted answers; two and three digit computation.

Determine reasonableness of displayed answers.

Check computation using a calculator.

Predict number to be displayed and verify its results.

<u>February</u>

NUMERATION

Classify a number by its attributes.

Sequence up to 4 counting numbers, least to greatest or greatest to least.

FUNCTIONS AND RELATIONS

Determine the appropriate operation needed to solve addition, subtraction or multiplication word problems.

SETS AND LOGIC

Read, interpret and supply elements in a matrix (vertical/horizontal format).

PATTERNS

Interpret and complete a number pattern where two function rules repeat to form a pattern.

Create and state the rule of a pictorial and/or number pattern using whole numbers.

NUMERATION

Identify positive and negative integers on a number line.

OPERATIONS

Apply addition, subtraction, multiplication or division strategies to solve money or other word problems (one or two steps).

Recall the multiplication facts through 9 times 9.

Multiply whole numbers through 3 digit times 1 digit (with or without regrouping).

ESTIMATION

Round the individual problem number to the nearest multiple of ten or hundred and estimate the sum, difference or product: incomplete number sentence with results of less than a thousand; money problems using pennies, nickels, dimes, dollars or ten dollars; word problems.

<u>March</u>

OPERATIONS

Recall the addition and subtraction facts through sums of 18.

FUNCTIONS AND RELATIONS

Identify the relationship of a pictured sum of money to a given value in a problem involving money through $9.99.

PROBABILITY AND STATISTICS

Demonstrate the probability of a simple event with six or fewer outcomes, using aids.

SETS AND LOGIC

Read and interpret Venn diagrams, with 2 or 3 sets.

Construct Venn diagrams.

Use synonyms for comparative statements, i.e., "not large" means "small."

Use negations to describe new sets.

Decide the truth or falsity of a number of statements involving the terms: all;, every; each; exactly; no.

MEASUREMENT

Estimate and determine the length of an object using whole numbers of metric and customary units.

Determine and record the weight of objects using customary and metric units.

Identify the reasonable customary or metric unit to measure weight (mass).

Compare and identify the weight (mass) of objects.

<u>April</u>

NUMERATION

Identify positive and negative integers on the number line.

PROBABILITY AND STATISTICS

Read a pictograph using data of ratios of 1, 2, 5 and 10:1.

NUMERATION

Apply the commutative and associative laws to addition and multiplication (single digit).

OPERATIONS

Demonstrate the meaning of division as repeated subtraction.

Divide whole numbers through 2 digit dividends by 1 digit divisors (limit to division facts).

<u>May</u>

FUNCTIONS AND RELATIONS

Manipulate models to order fractional parts.

OPERATIONS

Demonstrate ways to partition a circle or square into 2, 4, 8 or 16 congruent parts.

Compare halves through sixteenths of various physical wholes.

Identify models of halves through fourths, sixths, eights, twelfths, and sixteenths.

Compare fraction "families" using models (halves, fourths, eighths, sixteenths; thirds, sixths and twelfths).

Add and subtract like fractions with aids.

PATTERNS

Interpret, complete and extend a pattern when time, money or temperature is used to develop the pattern.

FUNCTIONS AND RELATIONS

Order a set of 3 to 5 two and/or three digit numbers.

Order two or three sets of objects or pictures: more to fewer; most to fewest, fewest to most; ascending or descending order.

OPERATIONS

Apply mental arithmetic strategies to solve addition, subtraction, multiplication or division problems.

NUMERATION

Apply the property of zero to addition and subtraction.

FUNCTIONS AND RELATIONS

Relate the operation of addition to multiplication.

June

MEASUREMENT

Investigate the volume of a figure in cubic units using aids.

Investigate and compare the capacity of various containers.

NUMERATION

Read and write Roman numerals using the symbols I, V and X.

MEASUREMENT

Measure and describe the weight of objects using non-standard units.

Relate and compare customary units of measure (in., ft., yd., mi.).

OPERATIONS

Determine one-half of an even whole number through 18; even multiples of 10 and even multiples of 100, with or without aids.

# APPENDIX C

Dr. Douglas Grouws
Univ. of Missouri

## QUIZ ON EFFECTIVE MATH TEACHING

There is data available to support the statement...

T F 1. Teachers make a difference in the amount of learning that takes place in classrooms.

T F 2. Some teachers are more effective than others.

T F 3. Some math teachers are more effective than others.

T F 4. Effective and less effective teachers can be identified.

T F 5. Effective and less effective mathematics teachers have been identified and studied.

T F 6. A teacher's impact on pupil learning is relative stable from year to year.

T F 7. Characteristics of teachers such as number of math courses, number of years of teaching experience, and so on, are highly correlated with how much pupils learn in math classrooms.

T F 8. Teacher behaviors (i.e., what a teacher does in the classroom) are highly correlated to how much pupils learn in math classrooms.

T F 9. Highly effective math teachers outperform their counterparts by getting greater gains from the high ability students in their classes.

T F 10. Time variables correlate highly with effective teaching.

T F 11. Periodic observation of teachers increases student learning in those classrooms.

T F 12. List three teaching behaviors associated with effective math teaching.

__________________________________

__________________________________

__________________________________

T F 13. Similar teaching behaviors are associated with effective math teachers and effective English teachers.

T F 14. Math teachers can be trained to teach differently.

T F 15. Teachers that teach according to prescribed conditions for effectiveness perform little better than their counterparts who teach in their regular ways.

T F 16. When classroom observers rank math teachers according to their effectiveness their rankings are very similar to the rankings one would get from examining standardized achievement test scores.

T F 17. Instructional pace (i.e., rate of movement through the curriculum) is positively correlated to student achievement.

T F 18. Spending more math time on developing ideas than on practice is associated with better student math scores.

T F 19. Frequency of teacher lecturing or explaining to students is positively related to students' perceived usefulness, enjoyment, and importance of mathematics.

T F 20. The average amount of time teachers spend on development in mathematics classrooms is in the 15-30% range.

T F 21. Instruction during mathematical problem solving units is very much like regular instruction.

T F 22. High effective mathematics teachers tend to individualize instruction with regard to pace and content than do less effective teachers.

T F 23. There is a best way of teaching mathematics.

T F 24. A good reference on this topic is the book, Selected Issues in Mathematics Education edited by Mary Lindquist.