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A TEACHER'S CHANGING THOUGHTS AND PRACTICES
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A CASE STUDY

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A TEACHER'S CHANGING THOUGHTS AND PRACTICES IN NINTH GRADE GENERAL MATHEMATICS CLASSES: A CASE STUDY

By

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ABSTRACT

A TEACHER'S CHANGING THOUGHTS AND PRACTICES IN NINTH GRADE GENERAL MATHEMATICS CLASSES: A CASE STUDY

By

Anne Lee Madsen-Nason

Recent educational reforms recommend changing teacher practice to improve student learning. In response to their recommendations, intervention projects were developed and implemented to change teachers' practices. The results of these efforts have often been less than satisfactory--either teacher practice was not significantly improved, or the changes that were made were not sustained. However, a three year intervention project begun seven years ago did change significantly teachers' practices. The purpose of this study was twofold: First, to investigate the nature of this intervention project; and second, to examine one teacher's thoughts and practices before, during and after the intervention project.

In order to study the nature and impact of the intervention project on teacher change, this dissertation study examined the data that had been collected on one participating teacher. This data consisted of classroom observations, interviews and project-related documents. Additional observational and interview data was collected for two years after the intervention to ascertain whether or not the teacher's changes were sustained.

The data was examined and the teacher's thoughts (via interviews) and practices (via observations) were compared to one another prior to, during and after the intervention. In addition, a coding structure and procedure was created to enable the observational and interview data to be analyzed in such a way as to provide a longitudinal view of the teacher's change and to assess the impact of the intervention on that change.

The following are the findings of the study which relate to the nature of the intervention project. First, a successful intervention project to change teachers' thoughts and practices should include opportunities for teaches to: (a) acquire new knowledge about teaching and learning; (b) form collegial relationships in discussion and deliberation sessions; and (c) receive in-class consultation and support as they implement changes in their practice. Second, an intervention project requires a sustained effort. One year interventions do not sufficient to change teachers' practices and sustain those changes.

The following findings focus on teacher change. First, changing a teacher's thoughts about instruction and learning precedes changing practice. New knowledge about learning and instruction provides the opportunity for changing previous thoughts and prepares the teacher for changing practices. Collegial relationships and classroom consultation furnish the means through which the teacher's changes in thoughts are translated into practice. Second, changes in a teacher's practice are sustained when they become habituated, and they become habituated through reflection over time.

This case study is dedicated to Pamela Kaye.

ACKNOWLEDGEMENTS

This study was conceived, written and produced through the efforts of many talented, generous and loving people. I would like to acknowledge my sincerest appreciation to them for their assistance, understanding and support.

To Sarah Beth and Steve: As my daughter and husband, I want to thank you for your patience, love and continual support throughout this endeavor.

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To Michael Sedlak, Susan Melnick and Glenda Lappan: As my committee members, my deepest gratitude for your guidance and assistance. Each of you has made a unique and valuable contribution to this case study.

To Perry Lanier: As my friend and my mentor, words cannot express the very special thanks you deserve. The contributions you have made to this study are reflected in every page. The contributions you have made to my education are innumerable. You will always have my deepest respect, admiration and gratitude.

TABLE OF CONTENTS

APPENDICES

LIST OF TABLES

LIST OF FIGURES

Chapter I

LOOKING BACK

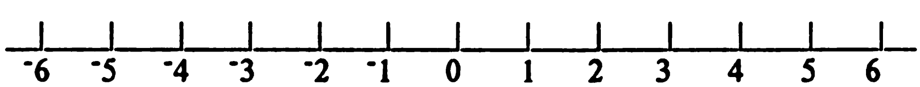
This study begins by looking back--comparing two general mathematics classes taught by Pamela Kaye in the Spring of 1982 and in the Fall of 1985, after an extended, sustained intervention designed to strengthen her instruction. The students in both classes were similar in terms of their social and academic ability. This chapter is separated into four sections. The first, "Pamela Kaye's General Mathematics Class," presents her general mathematics class during the 1981-1982 school year. The second, "A Typical General Mathematics Class," discusses Pamela Kaye's mode of instruction with respect to the characteristics of other General Mathematics classes identified in previous studies and in the literature. The third, "Pamela Kaye's General Mathematics Class Revisited," reconstructs her general mathematics class 3 years later. Lastly, "Two General Mathematics Classes: A Case of Contrasts," assesses the two classes and raises the questions which will frame this study.

Pamela Kaye's General Mathematics Class

Tuesday: 3/16/82 Arborville High School, Room 23

The students enter the general mathematics classroom socializing with one another. Ms. Kaye hands out paper and pencils to students needing them.

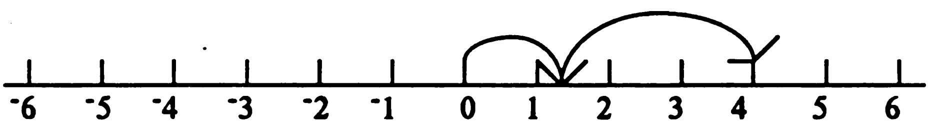
8:03
Ms. Kaye draws a numberline on the chalkboard and begins to review the addition of integers, a technique that was illustrated in the textbook.



Ms. Kaye asks, "What would you have if you had the following?" As she asks this question she writes the problem on the chalkboard.

$$^{+}1\ 1/2 + {}^{+}2\ 1/2$$

Without waiting for the students to respond she tells them the result would be positive four and shows the solution by drawing arrows on the numberline.



Ms. Kaye writes the next example on the chalkboard:

$$^{-}2\ 1/2 + {}^{-}1/2 =$$

Two students tell her quickly the answer is negative three. Ms. Kaye writes the next problem on the chalkboard.

$$^{+}1\ 1/2 + {}^{+}7/8 =$$

She demonstrates how to work the problem using the numberline. One student tells her the answer is +1 7/8 and another student says the answer is +1 1/4. Ms. Kaye tells the students the easiest thing to do is to find out how many eighths there are and then to add the numbers together.

Ms. Kaye asks, "What do you have?" One student replies, "One and eleven-eighths." She asks, "What's wrong with the problem?" The class replies in unison, "It's top-heavy!" Ms. Kaye changes the answer to 2 3/8.

8:12
Ms. Kaye asks, "Some of you have already learned the rule for adding positive and negative numbers, who knows?" None of the students respond. She continues, "If I add positive numbers this way (She walks to the right of the numberline) will I ever end up with a negative number?" The students tell her, "No." She continues,"If I add negative numbers all the time will I ever end up with a positive number?" The students tell her, "No."

One student says, "If you start out with a positive number then the answer will be positive. I mean the bigger number will be what the answer is." (He means when the larger number is positive the answer will be positive.) Ms. Kaye agrees.

Ms. Kaye says, "Now what about the number itself? If I go to the right am I adding?" The students tell her, "Yes." She continues, "I want you to copy this on your papers." Ms. Kaye writes the rules on the chalkboard she wants the students to copy.

ADDING SIGNED NUMBERS:

If the signs are the same, add and put on the sign.

She tells the students, "Signed numbers are the plusses and the minuses, the positives and negatives. So if I add plus three and plus one what do I have?" A student says, "Plus four."

Ms. Kaye continues, "If I add a minus three and a minus one what do I have?" A student says, "I know what it is!" She tells him,"I know you are way ahead of me so copy your answer down." He tells her he doesn't have a pencil.

8:20
Ms. Kaye writes the following on the chalkboard:

If the signs are different, find their difference (subtract) and put the sign of the "largest" (the most).

Ms. Kaye asks the students to copy the following:

$$^{+}4 + {}^{-}1 =$$
$$^{+}2 + {}^{-}3 =$$

She tells the students, "If the signs are different you go so many to the left or right and then go back. If you have a positive four and a minus one what do you do?" A student says, "Five ... no, plus three." Another student says, "Because you go four to the right." Ms. Kaye tells him, "Yes, but I want it in the way of a rule. What do you have with plus two and minus three?" A student tells her, "Minus one." Ms. Kaye tells the class, "I think he seems to get it now!" She writes the following on the board:

1 1/2 + 7/8 =

Ms. Kaye demonstrates how to solve the problem using the rule.

```
  1 1/2  =  1 4/8
+   7/8  =    7/8
          1 11/8  =  2 3/8
```

Ms. Kaye continues, "What happens when you add plus seven point two ($^{+}7.2$) and a minus three point eight ($^{-}3.8$)?" Without waiting for the students to respond she tells them, " You would have plus three point four."

8:23
The students have watched for over 20 minutes. Ms. Kaye gives them their assignment. She tells them, "What I want you to do today is in your text on page 183. You will do rows one through eleven and just the a and b parts."

The students open their books and start working. Several students move their desks next to each other enabling them to work together in groups of three and four.

A student asks if they have to copy the problems and Ms. Kaye tells them, "No."

Ms. Kaye circulates around the room checking the students to make sure they are on task and working the problems correctly.

8:35
One group of students is finished with the assignment and starts playing a card game.

8:39
Most of the students have finished the assignment. Ms. Kaye walks over to the card playing group and asks them to be a little quieter because they are disturbing other students.

8:45
A student enters the room with a handful of pink slips of paper. These are requests from the principal or assistant principal to have certain students sent to the office.

Six students receive pink slips and go to the office. Ms. Kaye sends two students to the school office to use the copier. One student left the room to talk to a school counselor.

8:50
There are eleven students left in the room. Ms. Kaye says,"Needless to say folks, with the number of people gone we will correct these papers later." She starts collecting the assignment as the card group continues playing their card game.

9:00
The bell rings and the students leave the room.

Discussion

A typical day in this general mathematics class begins with Pamela spending several minutes supplying students with materials, loaning textbooks and taking care of bookkeeping chores. Unlike her algebra students, these students do not come to class ready and eager to do mathematics, discussing the homework assignment or speculating on what's ahead. After the students are prepared and the chores completed, Pamela begins her direct instruction of the lesson. This consists of the demonstration of an algorithmic procedure needed for the daily arithmetic topic. The demonstration of problems continues until Pamela believes most of the students can begin the assignment on their own. She will provide individual help during the lesson assignment time to those students still having trouble. When the lesson assignment (or seatwork) starts, many students form groups to collaborate on the problems. This work strategy enables them to finish the task quickly and thereby provides enough time to play card games or read magazines. A few minutes before the end of the class period Pamela usually reads the answers to the assignment and has the students score and hand in their work. With about one-fourth of her students called out of the room Pamela saves the checking until a later date.

Three of the mathematical content characteristics are evident in this example of Pamela Kaye's typical lesson: (a) the content is a review of a topic the students had covered previously in the seventh and eighth grades; (b) since most assigned problems were identical to those demonstrated, this task was clearly one of drill and practice; and (c) both the students and the teacher seemed to accept this daily routine of the drill and practice of arithmetic reviews as the substance of general mathematics. The students would probably admit that there was nothing in this lesson's content that was new, interesting, or challenging.

Just as the class' content consisted of repetitive drill, the lesson was conducted with minimal superficial communication: (a) Pamela told the students how to work the problems in the textbook and gave them the daily assignment; (b) the students did not ask questions related to the mathematical content, instead their questions were procedural--related to the number, location and evaluation of the problems they would be assigned; and (c) both Pamela Kaye and the students used nonmathematical language in their interactions (e.g., "Top-Heavy" for improper fractions). Communcation about the content of the lesson was sparse and nonmathematically oriented.

Finally, the social organization of the class was characterized by: (a) students using every opportunity to socialize(e.g., before class, during the lesson assignment, and after their work was finished); (b) a teacher unable to plan mathematical lessons appropriate for whole-group instruction because of the students' wide range of abilities; and (c) a class organized for doing arithmetic rather than for learning mathematics.

A Typical General Mathematics Class

"Are most general mathematics classes like Pamela's?" Recent scholarship indicates that her lesson was typical and representative of general mathematics classes elsewhere. Usiskin (1985) recently characterized the nature of general mathematics classes in a familiar but slightly different way:

> Students in remedial courses are taught by teachers who, in the main, would prefer to teach more advanced content. There is less time to learn the material than there was in the grade level where it was originally taught. The classroom is filled with an aura of failure and frustration. The students are often taking the class unwillingy, only because it is required or is a prerequisite for another course they would like to take. (p. 10).

Students enrolled in ninth grade general mathematics classes are generally described as less able and less motivated to achieve in mathematics. They generally have a history of poor mathematics achievement and express little interest in pursuing mathematical study beyond the minimal number of courses required for graduation. Many have poor school habits (e.g., tardiness, truancy, irresponsibility).

Goodlad's (1984) extensive study of schooling described students' and teachers' behaviors in low-tracked classes such as general mathematics. His description is paraphrased below:

1. Students in lower tracks were the least likely to experience the types of instruction most highly associated with achievement.

2. There were distinct differences favoring upper tracks in regard to teachers' clarity, organization, and enthusiasm.

3. Students in the lower track classes saw their teachers as more punitive and less concerned about them than did other students.

4. Teachers in lower track classes spent more class time than did any of the teachers on student behavior and discipline.

5. Students in low-track classes agreed the most strongly that other students were unfriendly to them and that they felt left out of class activities.

6. Students in low-track classes reported the lowest levels of peer esteem and the highest level of discord in their classes.

7. Teachers of low-track classes devoted a much larger share of instructional time to rote learning and the application of knowledge and skill.

8. The mathematical content of texts used in low-track classes emphasized "survival skills" - writing checks, balancing a checking account, making deposits, preparing an income tax form, borrowing, insurance, household finances. (pp. 154-55, 208)

Using the Goodlad (1984) project data, Oakes (1988) in a review of tracking suggested that differences between the upper- and lower-tracked classes existed because they are the products of the ways in which students and teachers interact. With respect to teachers of general mathematics classes, Oakes noted that:

> Decisions about what and how to teach are conditioned by traditions and by expectations about what is appropriate for students of different backgrounds and abilities. Obviously, teachers are also greatly influenced by what they think will work--perceptions that are influenced by how the students themselves respond in class. (p. 43)

According to Oakes, students also influenced what goes on in these different classes.

> On the student side, day-to-day classroom behaviors are affected by students' own beliefs about their abilities and their perceptions of their prospects for academic success. Their willingness to make an effort is largely a result of these factors. As students experience success or failure in school, their self-perceptions and attitudes become more or less conducive to high achievement. Students who expect themselves to be successful respond with effort and achievement. Those who expect to fail are often unwilling to try. In low-track classes, especially in the upper grades, it is difficult to conceive of even the most dedicated and skillful teacher not feeling discouraged by a whole classful of students pulling away from academic achievement. (p. 43)

The evidence from A Study of Schooling Project, prepared by Oakes and Goodlad (1984) suggests that Pamela's first general mathematics class was typical. Data from less extensive studies confirms this interpretation. Cain (1985) described a typical general mathematics class as one that held as its goal

> competence in basic mathematics skills, especially computational skills. The level of mathematics content is low, as are the cognitive expectations of the students. The contribution of the instructor to the learning of the content is very high. The role of the instructor is to "show and tell" and to diagnose errors and prescribe remediation for them. The role of the student is to replicate that which they have been shown and told. (p. 24)

Similarly, Driscoll (1983) noted that by the time they reach their first general math class high school students

> have behind them enough years of mathematics to have strong and hardened impressions about the subject: what it is about, how it works, why it is taught, and how it should be learned. For many students, those impressions are far from conducive to good mathematics learning.
>
> Thus, the mistakes that require remediation in secondary school mathematics are often the outgrowths of impoverished impressions of mathematics. Students who are slow in learning mathematics are probably stuck with poor mathematical learning skills. (p. 48)

Both Cain (1985) and Driscoll (1983) described the characteristics and nature of typical general mathematics classes and students. Goodlad (1984) adds to this portrait of the general mathematics class by reporting on teacher's instructional patterns. He found that barely 5% of instructional time in schools was spent on direct questioning and less than 1% was devoted to open-ended questioning which called for higher-level student skills beyond memory. He also found that the kinds of instruction which fostered student learning did not exist in most secondary classrooms (pp. 138-39).

When algebra and general mathematics are compared the problems inherent in the latter are even clearer. Students elect to take an algebra class. They see it as the stepping stone to the study of higher mathematics. They have a history of success in learning mathematics, they are interested in studying mathematics, and often they are a school's "model" students. Secondary mathematics teachers, like their algebra students, are interested in learning mathematics themselves, have a history of success in mathematics and school, and enjoy others who share their interest in mathematics. The content of algebra is new, challenging, and interesting. Students and teachers in algebra classes are attracted to one another and both are attracted to mathematics.

In contrast, students select general mathematics class because they do not want to--or are denied the opportunities to--take algebra. Their interest in and desire to achieve in mathematics is minimal. In fact, most general mathematics students will openly admit they "hate" mathematics. Teachers view their general mathematics classes as problematic, unrewarding, and discouraging. They believe it is extremely difficult if not impossible to arouse any mathematical interest in these students. The content of general mathematics, in contrast to algebra, is unchallenging and uninteresting. It consists of the same drill and practice of arithmetic algorithms the students have encountered for 7 years. The evidence from research studies of high school classes (Lanier, 1981) indicates that the problems which exist in these low-tracked classes are chronic and extremely complex.

Pamela Kaye's General Mathematics Class Revisited

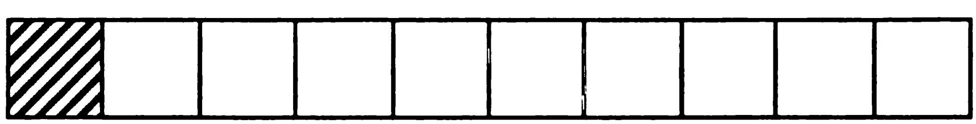
Friday: 11/8/85 Arborville High School, Room 23

9:00
The students enter the classroom, pick up a worksheet from Ms. Kaye's desk, take their seats and start answering the review questions on the page.

9:05
Ms. Kaye hands back corrected assignments to students as they work on their assignment. She tells them, "Those of you who have homework need to turn it in."
A student asks, "Do you have our grades for this marking period done yet?" Ms. Kaye tells him, "No, not yet."

As Ms. Kaye finishes taking attendance the students continue to work. One student asks her, "Is this right?" He shows her the following problem:

> This bar represents the number of people that live in the United States and the shaded part represents the number of people that live in California. What fraction of the population lives in California?

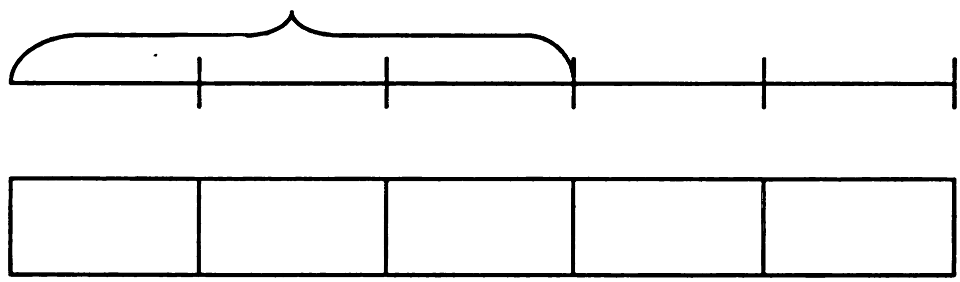


Ms. Kaye asks the student, "What you have is a strip with ten equal parts, so how many parts do you have shaded?"

The student replies, "One." Ms. Kaye asks, "And you are counting 1, 2, 3, 4, 5, to 10 pieces altogether. How many pieces are there altogether?" The student replies, "Ten... oh, that means there is one-tenth."

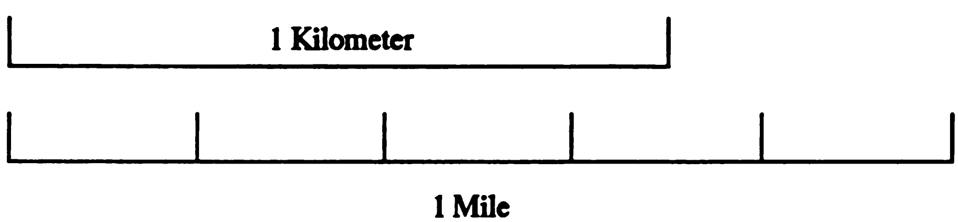
9:10
Ms. Kaye continues to hand back papers and answers questions students have on the begining of class activity sheet. Several students work together on the assignment.

9:11
Ms. Kaye draws the following on the chalkboard:



She tells the students, "If you will look at problem number seven for a minute, some of you are making the same mistake." The problem:

> Congress has passed a resolution accepting the metric system for the United States. The kilometer is used in the metric system. The drawing shows a kilometer compared to a mile. A kilometer is what fraction of a mile?



Ms. Kaye continues, "This line is just like your fraction strips--only it isn't a bar. Some of you are counting the lines and not the spaces--you have five spaces in both." This explanation clarifies the problem for the students.

9:13
Ms. Kaye calls on students to read the answers to the eight word problems. She then asks, "Are there any concerns?" A student says, "For number four I had seven forty-firsts or seven forty-eighths...I wasn't sure which one was right." The problem is the following:

> In one hour Karen and Joan counted 7 trucks and 41 cars passing their school. What is the ratio of this number of trucks to the number of cars?

Ms. Kaye responds, "Why?" The student tells her, "Well, they (the author) said seven trucks and forty-one cars...and I wasn't sure whether it was all of them or not." Ms. Kaye says, "Right, if they ask you for all the things that passed your school then it would be seven forty-eighths." The student says, "Oh, I see, then it was seven forty-firsts because they wanted the ratio of trucks to cars."

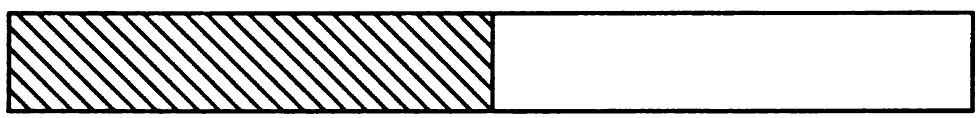
Ms. Kaye continues, "Any more questions?" A student asks a question about the answer to problem number eight:

The metric system is used in many countries.
The drawing shows a centimeter compared to an inch.
A centimeter is what fraction of an inch?

Ms. Kaye asks the student, "What was your answer?" The student tells her, "Two-fifths." Ms. Kaye tells him he was correct.

9:15
Ms. Kaye gives the students the following directions, "This is lesson number eight. I want you to draw a fraction strip that represents one-half." The students draw a fraction strip on their papers.

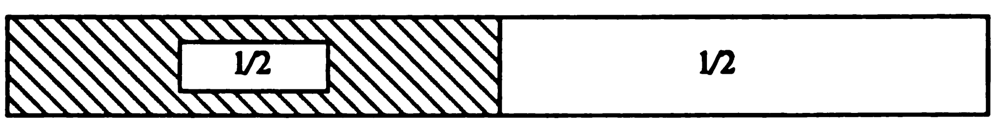
A student asks Ms. Kaye whether or not they were supposed to shade in the one-half strip. She says, "If you want to represent one-half then you had better shade it in, Tom. Why don't you come up here and draw it for us." Tom goes to the chalkboard and draws the following one-half fraction strip:



Ms. Kaye tells the class, "Now, skip a line on your papers and draw another one-half fraction strip just like the first.

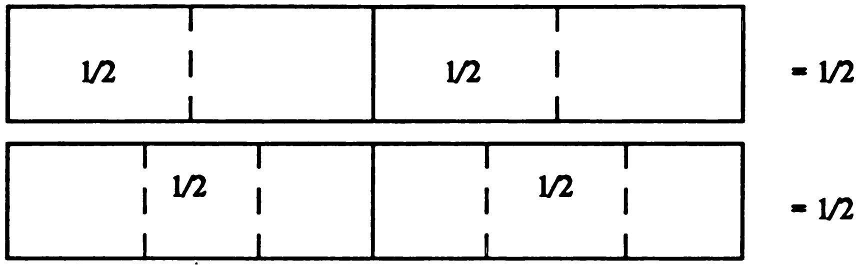
A student asks if they are supposed to shade this one.
Ms. Kaye replies, "You can, but don't make it too dark. Here are your envelopes with your fraction strips in them (She hands envelopes to the students)." She continues, "Now, take out a one-half strip."

The students take out a green construction paper strip which has been folded into halves.



9:18
Ms. Kaye holds up her green one-half strip.
A student says, "Hey! I didn't get my envelope!"
Ms. Kaye hands his envelope to him.
She continues, "I want you to fold your one-half strip into one-half more."
The students fold their one-half strip into halves.

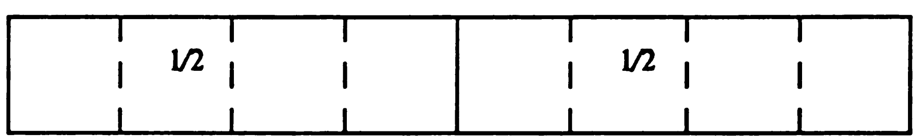
Ms. Kaye asks, "Now, what do you have?"
A student tells her, "I got fourths."
She tells the students, "I want you to dot these folds on the drawing you made on your papers."
The students write the following:



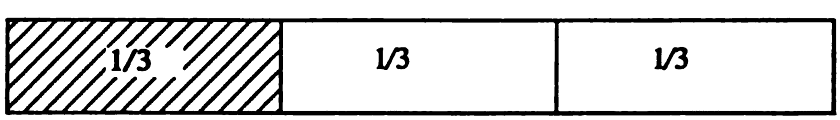
Ms. Kaye tells the students, "Now, take another one-half strip and fold each half into thirds." A student asks, "How do you do that?" Ms. Kaye replies, "Well, kind of like this (She shows him how to fold the half into thirds). Now, how big is each piece?" The students tell her, "One-sixth."

Ms. Kaye says, "I wanted to show you this one-half strip folded into three equal pieces so you could see that one-half was equal to three-sixths." One student says, "That's neat!" Ms. Kaye responds, "I just learned it. Moving right along, take your one-half strip and fold it into four equal pieces on each side."
(She folds her one-half strip as the students fold theirs.)

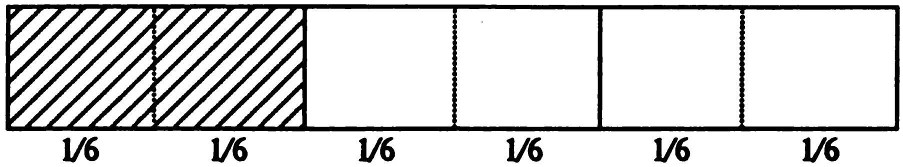
Ms. Kaye continues, "I want you to draw a third one-half strip on your paper (the students draw a one-half strip on their papers)." She says, "Take your one-half strip and fold it into four equal parts on each side." One student asks, "How do you fold it into eighths?" Ms. Kaye says, "How do you do that?" Another student says, "You fold each one-half into fourths." She tells him, "That's right. Now, put the dots in your drawing to show what you just did." The students write the following on their one-half strips:



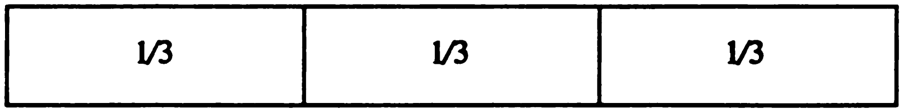
One student asks, "Ms. Kaye, what's the point of all this?" Before she can respond another student says, "I'd rather do this than fifty problems!" Ms. Kaye (responding to the first student) says, "If you will wait a little you will see what the point of all this is. Now, draw me a fraction strip that is divided into thirds and shade in one-third." (She draws a fraction strip on the chalkboard.)



Ms. Kaye tells the students, "Now, pull out a one-third strip from your envelope and fold each third into half."
A student says, "I got sixths!"
Ms. Kaye says, "By folding each of those into half what do you have now?"
(She puts dots in her chalkboard drawing to indicate the folds.)

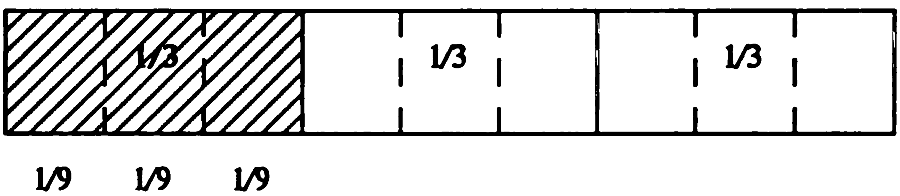


The students tell her, "Sixths." She writes one-sixth under each part. Ms. Kaye draws the following fraction strip on the chalkboard:



A student says, "Oh, I know what you are trying to do here." Ms. Kaye asks, "What?" The student tells her, "You are trying to show us how to reduce these!" Another student says, "Why didn't you just tell us like everybody else has?" Ms. Kaye explains, "Because people have been telling you since third grade." Another student says, "Yes, but I get it now."

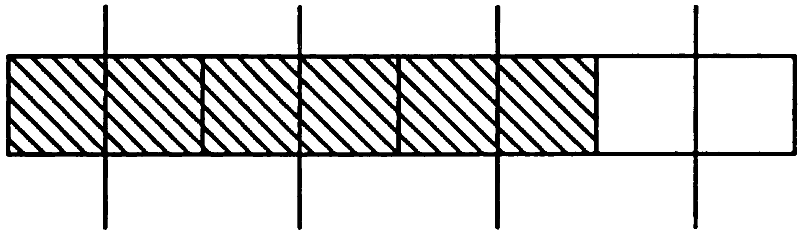
9:28
Ms. Kaye continues, "All right Cindy, when you folded each part into three equal parts what did you get?" Cindy says, "Ninths." Ms. Kaye draws the dots in the drawing to indicate the folds on the fraction strip:



Ms. Kaye asks, "What do you have in the shaded part? How many ninths do you have?" The students tell her they have three-ninths. She continues, "If you go back to the one you folded into sixths (she holds up the one-half fraction strip that had been divided into sixths) then I want you to fold each sixth into half. What do you call each piece?" The students fold their sixths into half and tell her they have twelfths. Ms. Kaye asks, "What would the shaded part be?" The students tell her, "Four-twelfths."

9:31
Ms. Kaye hands out worksheets to the students and reads the directions.

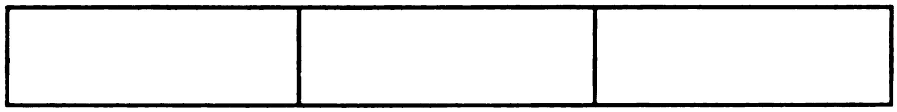
> Each part of the three-fourths fraction bar has been split into two equal parts. Now there are eight parts and six parts are shaded, giving the fraction six-eighths.



3/4 = 6/8

Split all the parts of each bar into two equal parts and complete the equations.

Ms. Kaye tells the students to draw a line through each part to show the parts being cut in half. One student says, "We are just doubling it." The students start working. One student asks, "How do you do this one?"



0/3 =

Ms. Kaye replies, "You cut each part in half. How many do you have?" The student says, "Zero-sixths." Another student tells Ms. Kaye, "I know an easy way...you just double them." Another student says, "You could multiply them by two." Another student comments, "That's what doubling is!"

9:38
Ms. Kaye selects a student to read his answers to the worksheet. After he finishes she says to the students, "Some of you did this assignment without writing anything down or using your pieces. If we had a fraction strip and you folded each part in half you would end up doubling the number of parts. When you take the third strip and you double the number of pieces in the whole strip you also double the number of pieces in the part. So, if you double the number of pieces in the denominator then you double the number of pieces in the numerator." Ms. Kaye writes the following on the chalkboard:

5/12 (2/2)   5/12 = 10/24

9:40
Ms. Kaye continues, "Did you ever wonder why when you multiplied the denominator by two you multiplied the numerator by two?" The students respond, "Yes."

9:42
Ms. Kaye asks, "If you multiplied by two-halves would you get the same answer as if you added by two-halves?"
She writes the following on the chalkboard:

(2/2) x 5/12 = 10/24

(2/2) + 5/12 = 7/14

Ms. Kaye asks, "Does seven-fourteenths equal ten twenty-fourths?"
The students tell her, "Yes."
[Ms. Kaye is surprised the students give her this wrong answer. She tries a new strategy of helping the students understand that any number multiplied by 1 remains the same.]

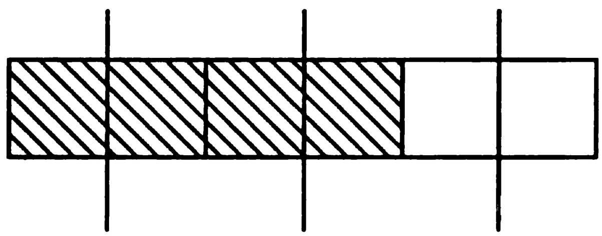
Ms. Kaye says, "Let's see, if you multiplied by two-halves, then you actually multiplied by one."

She writes the following on the chalkboard:

$$1 \times 6 = 6$$
$$1 \times 17 = 17$$
$$1 \times 5/12 = 5/12$$

9:45
Ms. Kaye hands out another worksheet to the students. She tells them, "You may either use your fraction bars or you may draw a picture or you may remember what we just talked about as you work these problems." She reads the directions:

By splitting all the parts of a fraction bar into two equal parts, both the number of parts and the number of shaded parts are doubled.



2/3 = 4/6

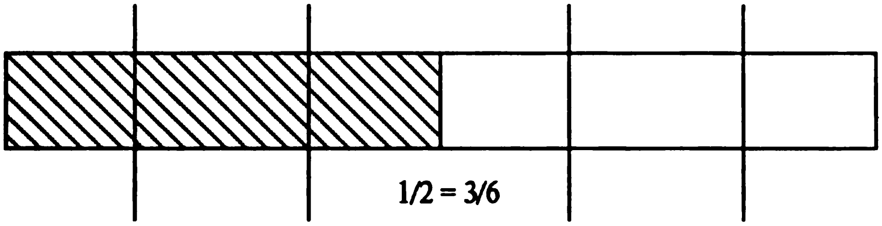
Both the numerator and denominator of the fraction have been multiplied by two.

$$2/3 = (2 \times 2)/(3 \times 2) = 4/6$$

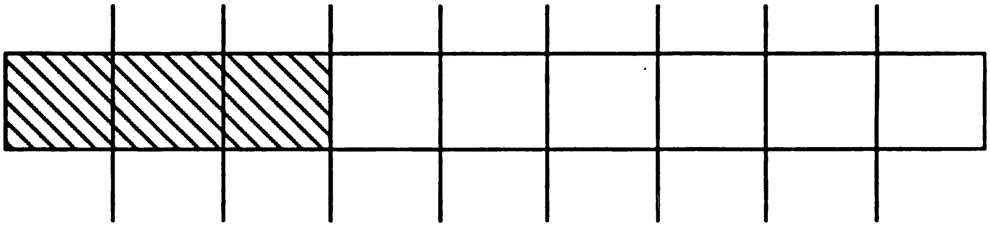
Complete the following equations by multiplying the numerator and denominator of each fraction by two.

The students start working. One student asks, "If you got six-sixths should you write it down as one whole?" Ms. Kaye responds, "I would just as soon have you leave it six-sixths for now." Another student says, "You can't double zero-zeros." A student replies, "Yes you can. Two zeros is just zero!"

9:46
Ms. Kaye asks, "Is everybody just about finished? James, would you read your answers?" James reads his answers and the students check their papers.
Ms. Kaye draws the following on the chalkboard:



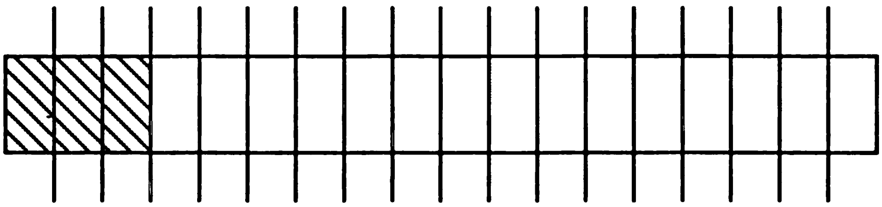
1/2 = 3/6



1/3 = 3/9
(1/3 into 3rds)

Ms. Kaye hands out another worksheet to the students and reads the directions:

> By splitting all the parts of a fraction bar into three equal parts, both the number of parts and the number of shaded parts are tripled.



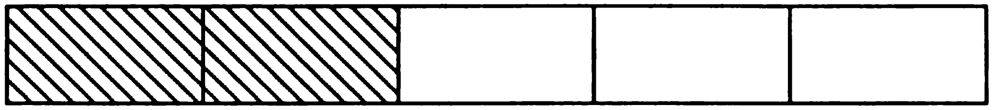
> 1/6 = 3/18
>
> Both the numerator and denominator have been multiplied by three.
>
> 1/6 = (1 X 3)/(6 X 3) = 3/18
>
> Complete the following equations by multiplying the numerator and denominator of each fraction by three.

9:48
The students start working on their worksheet.

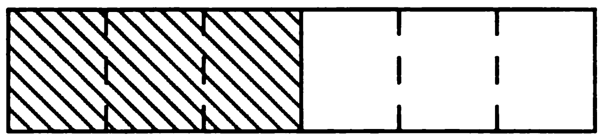
9:49
Ms. Kaye asks a student to read his answers and the students check their papers.

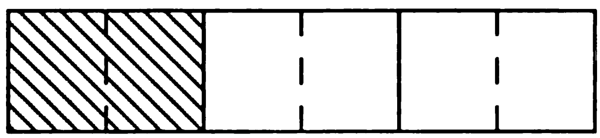
9:50
Ms. Kaye tells the students, "Someone asked what was the point of all this. Well, remember on the second day of this unit when we talked about being able to add one-half and one-third and someone told us the answer was two-fifths? Is the sum of one-half and one-third more or less than one half?" (She holds up a one-half strip together with a one-third strip.) The students tell her the sum would be <u>more</u>.

Ms. Kaye asks, "Is two-fifths more or less than one-half?"
(She holds up a two-fifths strip.)



The students tell her that two-fifths is <u>less</u> than one-half. She replies, "Then something is wrong here. If we took the one-half strip and divided it into three equal pieces and took the one-third strip and divided the pieces into two equal pieces we would have three-sixths and two-sixths which would give you a sum of what?" (She draws the following on the chalkboard.)

1/2 = 3/6 

1/3 = 2/6 

3/6 + 2/6 = 5/6

9:55
Ms. Kaye tells the students, "I want you to look at something. We had halves, fourths, sixths, eighths, and tenths. If we did thirds and then sixths, ninths and twelfths---what would we have?" Ms. Kaye writes the following on the chalkboard:

2, 4, (6,) 8, 10

3, (6,) 9, 12, 15

The students reply, "Multiples!." She continues, "If we had five-twelfths and seven-eighths." (She writes this on the chalkboard.)

8/8 X 5/12 = 40/96
12/12 X 7/8 = 84/96

She continues, "We could multiply twelve by eight and get ninety-six, and we would have to multiply five by eight and get forty, and seven by twelve to get eighty-four. We could also use <u>multiples</u> of twelve and eight. (She writes on the chalkboard)."

12, (24,) 36, 48
8, (24,) 32, 40, 48

Ms. Kaye, "What is the first common multiple?" The students reply, "Twenty-four." Ms. Kaye tells them, "So you have a common denominator of twenty-four. Then you could multiply five-twelfths by two-halves, and seven-eighths by three-thirds, and you would have ten twenty-fourths, and twenty-one twenty-fourths, and their sum would be thirty-one twenty-fourths. What I wanted you to see was that all this splitting up of fraction strips showed you equivalent fractions."

10:00
Ms. Kaye collects the worksheets and class notes from the students while several students go to the bulletin board to check a computer printout of their accumulated math points to date. The bell rings and Ms. Kaye dismisses the class.

Discussion

A typical day in this class now begins with the students working on a short review sheet of mathematical problems as soon as they enter the classroom. While they are doing this activity Pamela takes attendance and hands back corrected papers to the students. When most of the students are finished, Pamela asks a student to read the answers to the review problems. Questions regarding any problem are discussed. Pamela uses a controlled practice mode in her direct instruction of the daily lesson. She begins by asking the students to work along with her using their manipulatives and solving problems at their desks. As they work together as a whole group Pamela asks questions and engages the students in discussions of the mathematical concepts they are learning. The lesson assignment (seatwork) period consists of cycles having the following routine:

1. Pamela hands out a worksheet to the students and reads the directions.
2. She makes certain the students understand what they are to do and then gives them a few minutes to complete the work.
3. As the students work Pamela walks around the room checking to see that the students are working the problems correctly.
4. When most of the students are finished Pamela asks a student to read the answers.
5. Any questions are discussed before moving on to another set of problems.

The cycle is repeated several times during the lesson assignment period. The students are permitted to work together on these assignments if they wish. The last 10 minutes of class is used to discuss and review the mathematical content covered in class during that day.

The mathematical content of this lesson was not new to the students--they have worked with fractions for many years. What was new for the students was the conceptual orientation of the lesson presentation and the lesson assignment. The students used sets of fraction strips they made and they drew sketches of the results of their manipulations with the strips which enabled them to understand equivalent fractions. The number of assigned problems was limited and distributed intermittently throughout the lesson. The word problems at the start of the class and the fraction problems that were part of the daily lesson interested and challenged the students. The mathematical topics covered in one lesson were linked to topics the students had studied previously (e.g., Pamela's reference to multiples as she discussed equivalent fractions and common denominators).

The patterns of communication that were established in the class provided students with the opportunity to better understand the mathematical concepts they were studying. Pamela encouraged the students to interact with her, to tell her their thinking about a problem or a solution, and to discuss their conceptions and misconceptions about the content. While some of the questions the students asked were procedural, most of their questions were related to the mathematical concepts they were working. Pamela and the students used mathematically precise language as they engaged in discussions and dialogues about the mathematical ideas and topics they were studying.

This general mathematics class was organized for the learning of mathematics. When the students entered the room they started working on a math task. When Pamela was instructing, the students worked along with her at their desks. When students were working on their assignments Pamela monitored them. She encouraged the students to work together on solving the word problems and on the lesson assignment task. The students were engaged in activities that promoted the learning of mathematics from the moment they entered the room until they were dismissed at the end of the class period.

Two General Mathematics Classes: A Case of Contrasts

The three areas of focus in the discussion of each observation, the mathematical content/tasks, the communication patterns, and the social organization of instruction, will be used to frame this chapter's final section which explores the contrasts in teaching and learning between the 1982 and 1985 observations of Pamela Kaye's general mathematics classes.

The Mathematical Content and Task Selection

Pamela's general mathematics class was computationally oriented in 1982. This distinctive orientation was apparent in both Pamela's lesson presentation and during the associated assignment. For example, consider Pamela's explanation of the addition of integers: Pamela asks the students to copy the following:

$$^{+}4 + {}^{-}1 =$$
$$^{+}2 + {}^{-}3 =$$

> She tells the students, "If the signs are different you go so many to the left or right and then go back. If you have a positive four and a minus one what do you do?" A student says, "Five ... no, plus three." Another student says, "Because you go four to the right." Ms. Kaye tells him, "Yes, but I want it in the way of a rule."

The students were given no reason for the procedure but were told how to obtain the answer. This contrasts with the conceptual orientation of the 1985 class when Pamela worked with the students on equivalent fractions. Pamela told the class that one of the reasons they came to understand this concept was because of their manipulative work.

> A student says, "Oh, I know what you are trying to do here." Ms. Kaye asks, "What?" The student tells her, "You are trying to show us how to reduce these!" Another student says, "Why didn't you just tell us like everybody else has?"
>
> Ms. Kaye explains, "Because people have been telling you since third grade." Another student says, "Yes, but I get it now."

Similarly, Pamela's method of content presentation in 1982 was limited to showing the students the textbook's suggestion of using a number line to illustrate the addition of integers. Her methods of presentation had changed considerably by 1985 when she used manipulatives, pictures, and symbols to help the students understand the content of the lesson. The students folded their fraction strips, drew pictures of the results of their work and related both the manipulatives and pictures to the numerical symbols.

The mathematical topics of both observations were familiar to the students, they had worked with fractions and integers before. The difference was that in the 1982 observation the topic was treated in the same manner as it had been in previous years. In contrast, the content of the 1985 lesson was treated in a new and unfamiliar fashion. Not only was the topic treated differently but Pamela helped the students to understand the linkages between this and the other work they had done with factors and multiples of numbers. The mathematical task in the first observation was routine and mundane. The second observation contained a variety of tasks from problem solving to paper folding to mini-worksheets. The students in the first observation worked mechanically through the problems with little interest, while the students in the second observation were attentive and interested in all the activities.

The orientation, content presentation, mathematical topic and tasks of the 1985 class differed from the 1982 lesson. Pamela had transformed her general mathematics class into a place where students learned and enjoyed mathematics.

The Patterns of Communication

There was very little interaction between Pamela and the students in 1982. On several occasions she asked a question and answered it herself without giving the students the opportunity to respond. On other occasions when she asked a question she settled for a single word response (usually the numerical answer) from the students. In contrast students actively questioned, responded and interacted with Pamela throughout the class period in 1985. The two examples below illustrate the difference in the quality of communication as compared to the previous observation.

In addition to the changes that were evident in the interactions between Pamela and the students, there were changes in the kind of language that was used in the classes. Mathematical language was not used in 1982.

Nor was there class discussion in 1982. Just 2 1/2 years later, however, Pamela and the students talked vividly about the mathematical content. Pamela uses student questions to discuss with the class a previously covered topic in which many of them shared the same misconception regarding the addition of fractions.

Clearly, the patterns of communication which had been established in the 1985 class were very different from those observed in 1982. The students and Pamela spent much more of their time talking about mathematical ideas and concepts.

Lesson Structure

Pamela organized her 1982 classroom for the computation of arithmetic. Pamela spent 20 minutes in lesson development activities where the procedures for the task were demonstrated. The lesson assignment took the students less than 16 minutes to complete. The remaining 24 minutes of the class period were spent with students socializing or on other nonmathematical activities. The 1985 classroom was organized for the learning of mathematical concepts. The lesson presentation time totalled 32 minutes and was interspersed with the lesson assignments which took the remaining 24 minutes of class time. There was no opportunity for the students to become engaged in off-task nonmathematical socializing.

In addition to the differences in the amount of time students spent on mathematics in the 1982 and 1985 classes, the amount of time Pamela spent in lesson preparation was also different. She had not been prepared for the students finishing early in the first observation and had prepared nothing else for them to work on for that day. In contrast, in 1985 the students and Pamela worked until the end of the period in the second observation reviewing the content they had covered on that day and relating it to previously learned concepts.

Regardless of the aspect of the lesson, the contrasts between the two classes are striking. In fact, it is difficult to believe it was the same teacher teaching the same subject to the the same kind of students.

A Consideration of the Contrasts

The remainder of this study examines the protracted intervention that led directly to these dramatic instructional changes. The 1982 observation was typical of most general mathematics (or lower-tracked) classes as reported in the literature. This observation, however, did not capture Pamela's thoughts about teaching general mathematics classes, the content and orientation, or her general mathematics students. What were her beliefs regarding these? Were these beliefs different for her higher-tracked students in the Algebra I or II classes? For more than 2 years, from April 1982 until June 1984, Pamela participated as a teacher collaborator in a research study to improve learning and instruction in general mathematics classes[1]. How did Pamela's participation in the project contribute to the changes in her thoughts about and practices in teaching general mathematics? Were some components more influential than others in initiating and supporting the instructional changes? How did the instructional changes implemented during the 2-year intervention strengthen Pamela's teaching and improve the students' learning?

The 1985 observation took place 1 1/2 years after Pamela finished participating in the General Mathematics Project. It appears as though the instructional changes she made were being sustained. How were these intervention-initiated changes sustained? How did Pamela's thoughts change over the course of the intervention with respect to teaching general mathematics classes, the content and orientation, and her general mathematics students? What further changes did she make in her instructional practice after the intervention?

These are the questions that shaped this study. Their answers can contribute to our understanding of the process of pedagogical and curriculum reform in the American high school.

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[1]The Ninth Grade General Mathematics Project. Institute for Research on Teaching, College of Education, Michigan State University. East Lansing. Perry E. Lanier: Director.

Chapter II

THE INTERVENTION AND ITS HISTORICAL ROOTS

To study the nature of the impact of an intervention project on a teacher's practice, an understanding of the intervention itself is needed. Part 2 of this chapter presents the structure, components, and chronology of the intervention project with respect to the participants in general and to Pamela in particular. But before examining the intervention, it is appropriate to review the tradition of innovations in mathematics education that provided the context for this initiative. Part 1, therefore, will reconstruct historical and recent efforts to change high school mathematics, secondary education, and teachers' practices in order to provide the background and set the stage for the discussion of the intervention project.

To understand the impact of an instructional intervention on changing a teacher's beliefs and practices relative to general mathematics, the researcher needs to consider such questions as:

1. How did general mathematics classes come to be unpleasant, unrewarding, uneducative, and unchallenging places as portrayed in Pamela's 1982 observation?

2. What were the contributions of previous educational reform efforts in creating and maintaining the "way things are" in these classes?"

3. Why have past educational reforms failed to improve learning and instruction in these classes?

4. How did the recommendations from the 1980s reforms further our knowledge about the kinds of educational changes that would be needed to improve learning and instruction?

5. What attempts have been made to improve the instructional practices of teachers?

PART I

Historical Reform Efforts in Secondary Education and High School Mathematics

A review of reforms during this century reveals a number of efforts to improve education and mathematics. The general reform initiatives shifted focus from improving education via the curriculum to strengthening teaching via instruction.

1890-1920

The authors of the first major national report on secondary education reform, the NEA's Report of the Committee of Secondary School Studies (1893), intended to bring order to the high school curriculum and to standardize students' preparation for higher education. Eliot (NEA, 1893), committee chairperson, regarded secondary education as simply a gap between the elementary school and the college. The high school at this time was viewed principally as a place where academic intelligence was refined.

In 1918 a second NEA sponsored report, the Cardinal Principles of Secondary Education was published. The authors of this report were concerned chiefly with using secondary education as an instrument for transforming the lives of citizens in a democratic society. The recommendations for curricular reform reflected the ideals of this progressive era and the hope that schooling would cure all social ills. They argued that the secondary school should prepare students to become healthy, ethical and worthy citizens. The writers gained public support for their liberal, expansive view of the purpose of the high school as an institution for social adjustment and vocational preparation.

Efforts to reform high school mathematics began with recommendations from the Committee of Ten (1894), the International Commission on the Teaching of Mathematics (1911-18), and the National Committee on Mathematical Requirements (1916-23). The mathematics subcommittee of the Committee of Seconday School Studies (1893) was the first national group to consider the goals and curriculum for mathematics education (Bidwell, 1968). The recommendations from this subcommittee were similar to those of its parent committee in that they emphasized improvements in the secondary mathematics curriculum in order to better prepare students for college work. In the report of the National Committee on Mathematical Requirements (1923), the first ninth grade general mathematics curriculum was recommended as a part of the new junior high school curriculum. Topics in the general mathematics curriculum included: arithmetic, algebra, intuitive geometry, numerical trigonometry, graphs, and descriptive statistics. At the same time Reeve (1920) advocated the creation of a high school general mathematics program that would "give the student...a course in mathematics in the high school which [would] be the equivalent of what is ordinarily done in the high school plus one or one-and-a-half years of college work" (p. 262).

By the mid 1920s, however, the goals of the general mathematics curriculum were beginning to change to reflect the ideals of progressivism. Schloring (1926) wrote that

> [t]he aim of general mathematics is close to that expressed in the document "Cardinal Principles of Secondary Education." The movement, started about ten years ago, represented an effort to get a course in the ninth year which would more nearly meet the needs of pupils, particularly those of low ability and poor background and those who would leave school early before graduating. (pp.100-101)

Schloring's (1926) statement contradicted Reeve's (1920) recommendations for a substantial general mathematics program. However, even Reeve himself came to advocate the "progressivist" general mathematics curriculum which was less rigorous than the one he had supported previously. Regarding students who were excused from taking certain mathematics courses, Reeve (1924) said, "if they are to take mathematics at all, [a] modified form of general mathematics would meet the needs and interests of such pupils" (p. 452).

The reforms in high school mathematics and general mathematics from 1900-1920, like their more comprehensive counterparts in secondary education, were attempts to reform education through a restructuring of the curriculum. Their efforts shifted from broadening and deepening math instruction for all students to designing a more limited, practical basic curriculum for noncollege bound students.

1920-1945

Between the world wars, national reform activities continued to focus on curricular improvements. In a discussion of educational reform of this time, Powell, Farrar and Cohen (1985) noted, "throughout the century's early decades they [reformers] wrote as if blind to the enormous barriers to improved teaching, as if they believed that new books and courses would do the trick" (p. 273). They concluded that changes in secondary education were,

> not one of decline, but of opportunities forgone. For the bargains that America made in order to extend secondary education to all simply assumed that most students neither wanted nor needed much education. Little effort was made to offer quality education to everyone. Reformers instead tried to invent education that would be useful. As they made education relevant, reformers often made it ridiculous. (p. 278)

The reforms of the 1930s continued to emphasize the high school curricula's social utilitarian value.

Throughout the twenties and thirties the mathematics education community became defensive as a result of: (a) the increasing rate of failure and decline in enrollment in traditional mathematics classes; (b) a growing concern for individual differences; and (c) the criticism of instruction that was dominated by drill and practice. At this time a report of the National Committee on Mathematical Requirements (1922, 1923) attempted to define and defend the purpose of secondary mathematics. This committee discarded "mental discipline" as the basis for the curriculum, replacing it with the idea of "transfer of learning." The report included the content requirements for college entrance agreed to by the College Entrance Examination Board (CEEB) and offered a model secondary curriculum.

Two reports on reforms in high school mathematics were published in 1940. The first was a report of the Progressive Education Association (PEA, begun in 1932) Committee, Mathematics in General Education, and the second proposed reform was the product of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics (NCTM begun in 1934). The goal of the PEA (1940) was to examine "the study and teaching of mathematics for its value in relation to the whole process of general education" (p. vi). The committee selected broad categories of mathematical behavior they believed were applicable to problem solving in life and related those categories to appropriate teacher practices. In contrast to the PEA report, the Joint Commission's recommendations were formulated in terms of specific content areas. The difference between the recommendations of the PEA and those of the Joint Commission was that the first focused on needs of the student for his/her life while the second remained grounded in the content of mathematics.

Reforms of the secondary schools and high school mathematics continued to be curricular efforts. According to Coxford in A History of Mathematics Education in the United States and Canada (1970), although there were efforts to reform mathematics teaching in the years between 1920 and 1945 they failed due to: (a) the resistance to change on the part of teachers and administrators; (b) the depression which reduced the amount of money available for teaching experimentation and curriculum improvement; and (c) the outbreak of World War II which distracted attention from educational and mathematical reforms.

1945-1980

New educational reforms--grounded in a commitment to the education of all children--were launched after World War II. To meet the varied needs of the diverse American population, the range of curricular choices expanded sharply. As a result of these efforts, ninth grade general mathematics classes became the noncollege alternative to high school algebra for college-track students.

Post war efforts to improve secondary education were accelerated by sharp criticisms of high schools from several influential people from the academic community. Bestor (1952) attacked schools, state departments of education, and professors of education. He argued that achievement of intellectual pursuits in schools was no longer of high priority, rather, it was social and personal adjustment goals that prevailed. With the advent of Sputnik, educational reformers began to link the need for educational improvement with national defense. The Office of Education (OE) and the National Science Foundation (NSF) sponsored plans to build a more rigorous curriculum for secondary schools. Congress passed the National Defense Education Act in 1958 and the NSF and OE used its funds to recruit students to math, science, and language studies. These agencies also sponsored efforts to improve teachers' content knowledge by subsidizing various educational programs.

Although the reforms helped recruit talented teachers to mathematics, the most visible effect of this effort was the production of new curricular materials designed to be "teacher-proof." Like their predecessors, these reforms attempted to improve secondary education by changing the curriculum. During the 1950s and 1960s money became available for these recommendations to become translated into curricular materials. In spite of initial enthusiasm from the reformers, most of these new materials never achieved wide use in classrooms. According to Powell, Farrah and Cohen (1985), the new materials had only a modest impact "because most teachers continued to work in traditional ways" (p. 283). The teaching method employed by teachers included homework/ discussion/ new homework. Thus the potential positive effects of any new innovative instructional approaches contained in textbooks were overwhelmed by traditional teaching methods.

It was during this period that the ninth grade general mathematics curriculum evolved into the arithmetic drill and practice program it is today, as it was portrayed in the first observation of Pamela's math class. The authors of the Second Report of The Commission on Post-War Plans recommended "the large high school should provide in grade 9 a double track in mathematics, algebra for some and general mathematics for the rest" (Schloring, 1945, p. 209). The main purpose of a general mathematics course in the ninth grade was to provide experiences that would insure the growth in understanding of basic mathematical concepts and improvement in the necessary skills. The Commission believed that this general mathematics course would be parallel to and yet more flexible than its algebra counterpart. It would be a worthwhile course for those students having needs and goals which could not be achieved by taking algebra. Further, the Commission emphasized, the general mathematics course would be organized differently than algebra, it would include a greater variety of topics, and would attend more directly to application. The Commission stressed that good work in general mathematics demanded as much time and exertion as algebra for both the teacher and the students. They noted the selection of students for general mathematics or algebra classes should be primarily based on differences of educational/vocational goals--not differences in mathematical ability. Finally, they believed that the real hazard of general mathematics was its undesireable image in the minds of traditionally-trained mathematics teachers. The Commission's recommendations provided a noble attempt to establish and clarify the goals of the ninth grade general mathematics curriculum.

In an effort to respond to the Commission's recommendations, general mathematics textbooks were written. The preface of one typical ninth grade general mathematics textbook published at this time contained a statement about the goals and nature of a general mathematics course:

> The Second World War helped to focus our attention to the fact that the common man, as well as the scientist and the engineer, needs a thorough knowledge of simple mathematics to live successfully in our complex civilization. The traditional type of mathematics course is planned chiefly to provide the required background for entrance to college. It was necessary that a different type of course should be developed to give training in the kind of mathematics that is needed in the shop, the store, and the kitchen, and in ordinary business affairs. This second type of course--usually named general mathematics--has received wide acceptance during the last decade and is found in the curriculums of many schools throughout the country. Mathematics to Use has been planned to meet the needs of the 75% of the students who do not require a mastery of traditional mathematics to prepare them for a scientific or technical career. (Potter, 1950, p. iii)

This statement differed from the grandiose intentions of the Commission on Post-War Plans (Schloring, 1945). This text's preface identified the general mathematics course as less challenging, less technical and less mathematically demanding. As did other general mathematics textbooks of this period, this book reflected an arithmetic drill-and-practice emphasis as the mathematical focus.

Reform movements in secondary education during the 1950s were similar to the Committee of Ten six decades earlier with greater emphasis on the science, mathematics, foreign languages, and traditional liberal arts curriculum. In his report on high schools, Conant (1959) argued to maintain high school curricula while at the same time sort students out for different vocations. He argued that high schools should be retained but improved and urged educators to enlarge high schools to serve two functions--educating the many and preparing a smaller number for higher education. Conant, in the tradition of Eliot (NEA, 1893), also specified curricular standards by which high schools should be judged. Throughout this period solutions to problems of secondary education and high school mathematics remained grounded in curricular change.

Kline (1961) suggested the problem in mathematics education was not in the curriculum, but rather it was rooted in the presentation of materials: there was little motivation, little intuitive development before generalization, no inclusion of applications, and little active participation on the part of the student. For Kline, the solution was obvious: build a better corps of teachers who

> must not only be better informed in mathematics, but they must also acquire a far better idea of why mathematics is important, why particular topics in mathematics are taken up, and what values mathematics offers to our civilization and culture. The primary value of mathematics...is that it is the language and essential element of science. (p. 3)

He recognized the importance of preparing teachers to teach mathematics better and began to depart from those in the past who advocated simply teaching more mathematics.

The reforms of the 1950s and 1960s failed to improve learning and instruction. McLaughlin (1978) wrote of this failure,

> one explanation for these disappointments focuses on the type of innovations undertaken and points out that until recently few educators have elected to initiate innovations that require change in the traditional roles, behavior, and structures that exist within the school organization or the classroom. Instead, most innovative efforts have focused primarily on technological change, not organizational change. Many argue that without changes in the structure of the institutional setting, or the culture of the school, new practices are simply "more of the same" and are unlikely to lead to much significant change in what happens to students. (p. 19)

The intensive and well-funded efforts to change the content of the curriculum during the 1950s and 1960s failed to have any significant impact on students and teachers. It became increasingly clear that curricular and textual change (technological change) was not sufficient to attain the improvement goals of the reforms or to solve the problems of secondary schools.

Massive changes in both society and education took place in the decades of the 1960s and 1970s. In response to these changes a common theme of many reforms became one of looking beyond the high school to society--to form linkages between the high school, community life, and the world of work. The Panel of Youth of the

President's Science Advisory Committee in 1974 (James & Tyack, 1983), recommended specialized schools, urged more students to work, and suggested a lifelong education voucher system. In similar fashion, the Kettering Foundation report (James & Tyack, 1983) reinforced the panel's critique of the comprehensive high school in outlining alternatives to current high school programs. These reform efforts focused on changing the structure of the schools and recommended less schooling rather than more, and intermittent rather than continuous instruction. High school reforms of the sixties and seventies focused on two issues: first, the issue of individual differences became crucial as more students aspired to go to college while many others of lower ability were not being served by the new (or modern) mathematics curriculum. Second, regarding methodology, discovery teaching captured the interest of teachers and researchers alike. As time went on, interest in the first issue flourished, interest in the second died.

In 1960, the NCTM and NSF sponsored eight Regional Orientation Conferences in Mathematics. The purpose of these meetings was to inform school administrators and mathematics supervisors of the new curriculum efforts to improve school mathematics. Allen (NCTM, 1961) commented on the value of these "new math" programs and the recommendations for improving school mathematics.

> The new programs in school mathematics will serve to increase the nation's supply of technicians, engineers, scientists, and mathematicians. They will also help, in some degree, to bridge the terrifying gap that now exists between mathematics instruction and the outrushing frontiers of mathematics and science. Indeed, they are typical of the programs that must be established in all fields if the members of the next generation are to have the knowledge necessary to operate the complex civilization they inherit. (p. 85)

In the conference's proceedings, Price (NCTM, 1961) concluded,

> I must emphasize that the elementary school teacher, the junior high school teacher, and the senior high school teacher are absolutely essential to the success of our program to provide better mathematics; for these teachers must teach mathematics; and these teachers must teach with enthusiasm so that their students continue the study of mathematics. (p. 14)

The conference proceedings highlighted the importance of mathematics instruction as an essential component of improving mathematics learning. Fifteen years later the National Advisory Committee on Mathematical Education (NACOME, 1975) surveyed and analyzed instructional practices and curricular issues in mathematics education from kindergarten through senior high school. In their report, Overview and Analysis of School Mathematics Grades K-12 (NACOME, 1975), described the shift in emphases of the current programs from the reforms of the early 1960s.

> Mathematics program improvements of the "new math" in the 1960s were primarily motivated and designed to provide high quality mathematics for college capable students. Today mathematics curriculum development focuses on issues largely ignored in the activity of 1955-1970. Attention has now shifted to programs for less able students, to minimal mathematical competence for effective citizenship, to mathematical application, and to the impact of new computing technology on traditional priorities and methods in mathematics. (p. 23)

The NACOME report presented recommendations for the future course of mathematics including a commitment to a full comprehensive mathematics education program for all children; a mathematics curriculum that is ever-changing and "responsive to the human and technological lessons of the past, concerns for the present, and hopes for the future;" and the preparation of "mathematics teachers knowledgable in mathematics, aware of, oriented to, and practiced in a multitude of teaching styles and materials and philosophically prepared to make decisions about the best means to facilitate the contemporary, comprehensive mathematics education of their students" (pp. 137-39). In spite of their recognition of the importance of improving instruction in secondary education and mathematics during this period, nevertheless reform efforts continued to be curricular.

The Educational Reforms of the 1980s

Since 1980, educational reform efforts have more regularly coupled the need to improve the quality of instruction with their recommendations for improving the school curriculum. During the 1980s efforts to reform mathematics education began to seriously consider recommendations to improve the quality of mathematics teaching. In April 1980, the NCTM presented its Agenda for Action: Recommendations for School Mathematics of the 1980s. Their recommendations reflected a concern for the need to improve the mathematics content, curriculum, and mathematics instruction. The authors emphasized that what students learn, how long they retain it, and how readily students apply what they learn depends on the kind of learning experiences in which they engage. They stressed that teachers should employ the most effective and efficient teaching techniques and also apportion instructional time wisely (NCTM, 1980, p. 1). In addition, recommendations were made for all educators to develop, define, and enforce professional standards in terms of highly competent performance. The NCTM Agenda, like the Holmes Group (Holmes Group, 1986) and Carnegie Task Force (1986) reports, recognized the critical need for the inclusion of instructional improvements as an important dimension of educational reform. Although reform efforts emphasized the need to improve the quality of both the curriculum and instruction in school mathematics as early as 1980, little consideration was given to specific instructional improvements. In most classrooms at every grade level mathematics instruction is insufficient to adequately equip children with the mathematical concepts and skills needed for their future. The results from the National Assessment of Educational Progress in mathematics (Carpenter, Corbitt, Kepner, Lindquist & Reyes, 1981) showed that while most students were able to compute, the majority did not understand many basic mathematical concepts and were unable to apply the skills they learned to simple problem solving situations. A concern of many recent reports was that students did not have the opportunity to learn the mathematics required for college or employment because of insufficient and poor instruction.

The Second International Mathematics Study (SIMS) gathered data on what mathematics is taught, how it is taught and what is learned in schools of the United States and 19 other countries in the 8th and 12th grades. In a report of the findings The Underachieving Curriculum, (McKnight, et al., 1987), noted,

> from an international perspective our eighth grade curriculum resembles much more the end of elementary school than the beginning of secondary school.
>
> The mathematics curriculum, furthermore, fails to fairly distribute opportunities to learn to children. As early as the junior high school grades, tremendous differences are created in what mathematics U.S. children have the opportunity to learn and, therefore, in what they are able to achieve.
>
> These differences in opportunity set boundaries on the degree to which individual students are able to reach their fullest potential, boundaries that leave less to reward individual efforts than in any of the other countries for which data were available. (p. 11)

Results from the SIMS (McKnight et. al., 1987) data on the content and instruction of 8th grade mathematics indicated that in fractions, decimals, ratio, proportion and percent, teachers emphasized computational competency. In addition, the teaching of integers and equations was rule oriented and focused on symbol manipulation. Similarly, geometry instruction focused on a statement of the definition and properties rather than on informal explorations that would develop students' intuition.

In addition to these findings, the SIMS data indicated the textbook was used by most teachers to determine the curricular content of mathematics. "The textbook defined boundaries for mathematics taught by teachers" (McKnight et. al., 1987, p. vii). In fact, of the teachers surveyed, 90% reported using the textbook as the primary instructional resource; in contrast, laboratory materials (manipulatives) were used only as additional instructional resources by a mere 10% of the teachers surveyed. The limited use of instructional resources greatly reduced students' learning opportunities in mathematics.

Student's opportunities to learn were limited by the ways in which they spent their time in mathematics class. SIMS (McKnight et. al., 1987) reported that the majority of student time in mathematics classes in the USA was spent listening to teacher presentations, doing seatwork or taking tests. Little time was spent in small group work (McKnight et. al., 1987). In addition, the SIMS findings indicated that the US 8th grade curriculum was fragmented. A frequent pattern of instruction showed that a large proportion of teachers devoted only a single lesson to a mathematics topic. The result was an inadequate and superficial coverage of many topics in the 8th grade curriculum.

As a consequence of these studies, consideration for the improvement of instruction in mathematics became viewed by many as a critical component in achieving excellence in mathematics education. Yet, even as late as 1986, Romberg and Carpenter (1986) commented that the typical mode of "drill and practice" instruction continued to prevail. They identified three problems inherent in this method of instruction. First, emphasis on teaching fragmented concepts and skills resulted in the loss of the essential characteristics of mathematics such as abstracting, inventing, proving, and applying. Second, students spent time absorbing what other people did instead of creating their own mathematical experiences. Romberg and Carpenter noted that "current research indicates that acquired knowledge is not simply a collection of concepts and procedural skills, it is structured by individuals in meaningful ways, which grow and change over time" (p. 13). Third, the teacher's role in this traditional classroom was managerial--making assignments, starting and stopping lessons according to some schedule, explaining/demonstrating rules or procedures, monitoring students, and maintaining order and control.

Chambers (1986), Supervisor of Mathematics for the Wisconsin Department of Education, cited two problems facing reform efforts to improve mathematics education: teachers' underpreparedness and their lack of access to new knowledge.

> At the elementary school level most teachers have an inadequate mathematical background. It is unreasonable to expect that teachers who have no knowledge or preparation in mathematics or the teaching and learning of mathematics will perform adequately in classrooms.
>
> Now growing numbers of teachers at the secondary school level are also underprepared. To meet current shortages, many teachers are now being licensed with minimal preparation.
>
> Futhermore, teachers tend to be isolated in their own classrooms. They have little opportunity to share information with other staff members and little access to new knowledge. Unless the proposed reforms become recognized, understood, and owned by teachers and teachers understand and are valued for their role in the reform movement, real change is unlikely to occur. (p. 213)

Efforts to Improve the Quality of Instruction

Educational reforms during more conservative times were typically concerned with programs that focused on the academically talented, dealt with greater emphasis of the basics, put greater importance on academics, and had a deep concern for the coherence of the curriculum and academic discipline. In contrast, reform efforts of more liberal periods typically dealt with concerns for the "disadvantaged," broadening the functions of schooling, and were less concerned with curricula consistency. Regardless of the times in which reform efforts took place and regardless of the forces which spawned such efforts (political, social, economic), the major thrust of all the secondary school and mathematics education reforms until 1980 remained curricular. However, since 1980 many national educational reform efforts have emphasized the need for instructional reform.

While the problem of the discrepancy between educational reality and educational possibility has yet to be resolved, the current reform efforts made a significant contribution toward recognizing the importance of improving the quality of instruction if educational change is to occur. The Holmes Group (Holme's Group, 1986) emphasized that excellence in education will only be realized through excellence in teaching.

Although attention focused on curricular innovations, and few substantial reforms were important during the 1970s, several influential research projects were undertaken to improve teachers' instruction/practices. This research emphasized the prominent role that teacher practice played in student achievement. Some research studies focused on changing teacher practice in order to facilitate the implementation of an instructional innovation (Berman & McLaughlin, 1978; Hall & Loucks, 1979). Other studies involved a more generalized approach to improving teachers' instruction by increasing their instructional skills (Joyce & Weil, 1986). Still others have involved direct intervention to facilitate instructional changes (Oja, 1980).

The body of research on improving the instructional practice of teachers continued to grow and to take on the appearance of a distinct discipline. By 1980, Joyce and Showers analyzed over 200 studies in which researchers had investigated various efforts to induce change through different teacher training programs. Utilizing the educational research on teacher behavior from the 1970s, Madeline Hunter and her colleagues developed an extensive inservice program with the goal of helping teachers improve instructional practice (Cummings, 1980). While most research studies initially began to change teacher practice, the long-term results of substantial and sustained change over time were not realized during this period.

Leaders of the NCTM recognized that integrating curricular and instructional changes with teachers' current classroom practices depends on the teachers' knowledge, motivation, and commitment to continual professional growth. In addition, they emphasized that instructional changes and improvement, well-planned and conducted, require high-quality professional development programs. Joyce's studies indicate that teachers' classroom instruction can be improved and consequently significant gains in student achievement can be made when intensive professional development programs are implemented successfully (Joyce & Weil, 1986).

The training programs Joyce (Joyce & Weil, 1986) found the most effective combined five elements: theory, modeling, practice, feedback and coaching to application. McLaughlin and Marsh (1978) reported similar findings of the Rand Corporation's Change Agent study. They reported staff development strategies that fostered staff learning and change had two complimentary elements: (a) staff-training activities (Joyce's elements 1, 2, 3); and (b) training-support activities (Joyce's elements 4 & 5). When combined, the study found these activities fostered teacher change, promoted student gains, and enhanced the continuation of the project. Staff-training activities were skill specific--for example, instruction in how to carry out a new mathematics program or how to use new mathematics materials. According to McLaughlin and Marsh, "skill-specific training" had only a small and not significant effect on teacher change and on the continuation of project methods and materials (p. 76). Staff-support activities supported teachers' assimilation of the skills and information delivered in training sessions. These activities included: classroom assistance, project meetings, and teacher participation in project decisions. McLaughlin and Marsh noted, "taken together as a support strategy, these activities had strong positive and direct effects on teacher change and continuation of project methods and materials" (p. 77).

Research and implementation of interventions designed to change the instructional practices of teachers have grown from the needs identified by these recent reform efforts.

A Summary

Social, political and economic forces identified a plethora of problems in secondary education across the last century. In seeking resolution to these problems, reformers centered their recommendations on improvements in two educational arenas; the curriculum and the structure/organization of the school. Such changes did not resolve the problems. Recently, reform efforts have considered a third arena, the instructional practices of teachers. Efforts to make improvements in this arena are extremely difficult. Cuban (1979), in an examination of various forces that shaped the curriculum during the past century, concluded that reforms initiated new curriculum theories which, in turn, influenced professional ideologies and vocabularies, school coursework, and occasionally textbook content. However, he found no evidence that these reforms had changed teacher practice significantly. In a later study of teacher practice, Cuban (1984) mentioned the lack of success of many reforms at the secondary level:

> this study demonstrates how impervious high school classrooms were to such [reform] efforts for nearly a century. What few changes occurred in curricular content, classroom talk, and the formal recitation were overshadowed by the persistent continuity of teaching practices extending back decades into the shadows of a previous century. (p. 260)

In addressing the need to improve teachers' instructional practices, Cuban suggested several strategies, including the retraining of existing teachers through staff development efforts. He saw a two-pronged effort to staff development: first, it should improve teacher knowledge and skills; and second, it should reduce isolation and establish collegiality. According to Cuban, staff development must involve teachers throughout its duration if practice is to be improved. He noted, "where classroom changes occurred teachers seem to have been active collaborators in the process," (1984, p. 265). If instructional excellence is to be realized, then classroom practice must be changed.

PART 2

The Instructional Improvement Intervention Project

Background

The intervention project this study will examine was the second General Mathematics Project (GMP) of the Institute for Research on Teaching at Michigan State University. The major issues and problems that guided this study were grounded in the research results from its predecessor, the first General Mathematics Project, and the recommendations of the recent educational reports. These concerns included:

1. increased societal concern over the need for a mathematically literate populace;
2. tracking, a policy commonly used for placing students within ninth-grade math (whether into algebra [college track] or general mathematics [vocational track]) that was highly correlated with students' junior high school mathematics records;
3. the inability of most secondary teachers to understand the serious problems many general mathematics students had in learning mathematics;
4. teachers felt unsuccessful in teaching general mathematics classes;
5. teachers' general mathematics instruction was less effective than their algebra instruction;
6. low incidence of success and high incidence of frustration encountered by both teachers and learners in general mathematics classes created unpleasant classroom settings;
7. many general mathematics students had great difficulty mastering even minimal mathematical skills such as arithmetic computation;
8. students placed in general mathematics classes appeared to be different from students placed in algebra classes in terms of learning difficulties, memory, reasoning, social interactions, and study habits; and
9. there appeared to be no well established teaching practices that would alleviate the troublesome and problematic nature of teaching and learning general mathematics.[2]

Mathematics instruction for students in general mathematics classes at the secondary level continued to be an area of educational concern which had not been explored by research. Two major questions which the GMP set out to answer included: (a) What new forms can mathematics instruction take if it is to offer all students the chance to learn basic math skills? and, (b) What changes would this mean in teacher thought and action, and in the "social order" of the classroom? Although these questions were general and broad they led to more specific, field-grounded questions in real classroom settings.

Methodologically, the GMP was characterized by a heavy collaboration with teachers and a commitment to fieldwork techniques of data collection. Collaboration with teachers was grounded in the project's focus on "what can be;" it was believed improvement projects conducted with teacher participation in both design and implementation of the interventions had a significantly better chance to affect classroom learning than projects which did not. The use of fieldwork methods was based on the conviction that an intensive detailed study capturing teachers' thoughts and actions in the design and implementation of interventions would generate the kind of knowledge necessary to understand how "what could be" would come to be.

As the GMP focused on the improvement of "what was" in general mathematics classes, new knowledge was sought about the teachers as they attempted to change attitudes, modes of instruction and ways of dealing with students. The strategies for acquiring this knowledge involved the close collaboration with teachers as they read, reacted to, and engaged in acts of instructional modification. The research questions which drove the study and guided the instructional interventions were:

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[2]The background information and intervention description of the General Mathematics Project was obtained from project-related documents. These included the original General Mathematics Study proposal, the continuation proposal and Madsen-Nason's AERA paper "Methods Used in Studying Ninth Grade General Mathematics Classes."

1. What do teachers see as the central problems in teaching general mathematics? What approaches have they used in dealing with the problems, and what effect do they perceive they have had?

2. How do teachers alter their views about general mathematics as a result of (a) exposure to literature, and (b) systematic trial of new approaches to teaching based on that exposure?

3. What concepts, strategies and research results from the literature do teachers see as applicable to the task of improving their general mathematics classes? Through what processes do they make use of new insights and skills?

4. What happens in classrooms when teachers systematically alter their approach to general mathematics? What evidence of student improvement can be found?

The research and collaborative activities of the GMP were organized into four phases. Each phase had its own unique purpose in the overall schema of the study and in answering the study's questions.

Phase I: Pre-Intervention Period
(September 1981-May 1982)

Phase II: Early Intervention Period
(March 1982-July 1982)

Phase III: Initial Intervention Period
(July 1982-June 1983)

Phase IV: Final Intervention Period
(July 1983-June 1984)

These phases serve as four sections of this second part of the chapter. The discussion of each phase includes a description of the project's activities with the teachers and the data that was collected. The last part of each section focuses on Pamela's specific activities during that period.

Phase I: Pre-Intervention Period (September 1981-May 1982)

The GMP staff recruited four teacher collaborators, assessed their individual perspectives toward and practices for the teaching and learning of general mathematics, began an interdisciplinary literature review, and initiated plans for implementing instructional improvements in each teacher's general mathematics class during the 1982-1983 school year. The teachers were employed by two suburban schools (a junior high school and a senior high school), one urban junior high school, and one rural/small town senior high school. Figure 1 contains the project's Phase I activities.

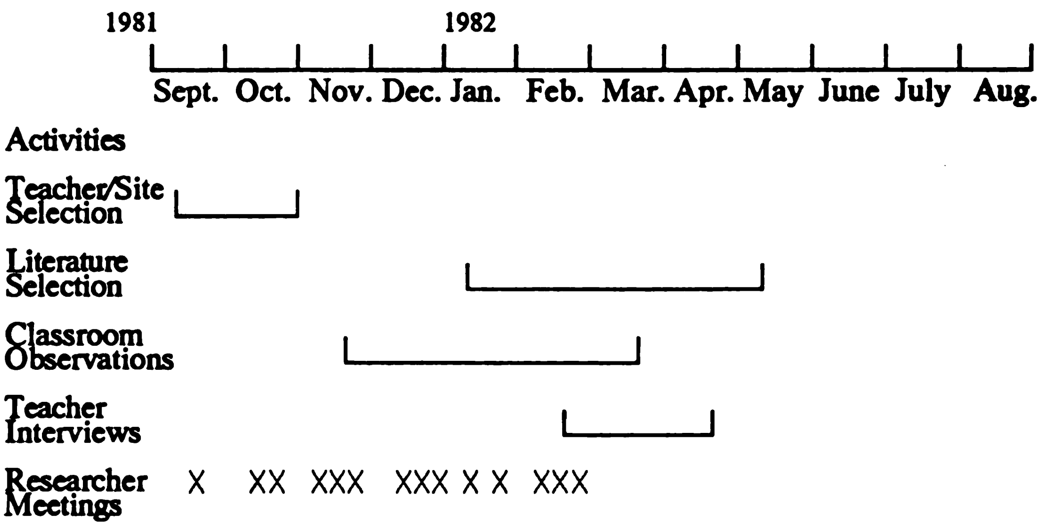

Figure 1

PHASE I: PRE-INTERVENTION PERIOD
(SEPTEMBER 1981 TO MAY 1982)

The assessment of each teacher's perspectives and practices was based on a series of in depth interviews and classroom observations. Four interviews with each teacher were conducted by a staff researcher (Madsen-Nason) and the primary classroom observer (a researcher assigned to a particular teacher). The interviews included an open-ended interview of the teacher's perception of general mathematics classes, two structured interviews emphasizing the four commonplaces of teaching (content, learner, teacher, and milieu) in the context of a general mathematics class, and a projective interview in which the teacher was requested to speculate on his/her perception of the ideal general mathematics class. The interview questions are included in Appendix A.

In addition to the interviews, classroom observations were made on days when each teacher introduced a new topic, when two or three lessons involved practice, and during an evaluation lesson (test or quiz) or review. These observations were made by the primary observer, who was accompanied on at least one occasion by a second staff member in order to verify the observational data. One observation was also made of each teacher in a class other than general mathematics. The purpose of the observations was to capture the teacher's practices in a range of lessons prior to the introduction of any instructional modifications.

From the observations and interviews the project researchers ascertained the four teachers' classes were representative of general mathematics classes. According to the project director, Lanier, "each class, in terms of its operational mode and outcomes, resulted in a generally unrewarding and less than successful experience for the students and the teacher." Further, the observations of the four teachers in classes other than general mathematics showed they each used a distinctively different mode of instruction and teacher-student interaction patterns than they did in their general mathematics classes. These observations matched those of the earlier General Mathematics Study where one teacher was observed teaching two different classes, a general mathematics class and algebra. Finally, the teachers' thoughts about general mathematics content, the students, classroom milieu, and their perceived ability to manipulate these were also comparable with the findings of the first General Mathematics Project.

Pamela had been teaching general mathematics classes for 12 years. She had the same number of years of teaching experience as did the other teachers in the study.

From her responses to the interview questions and the observations made of her general mathematics class, Pamela was considered by the project researchers to be similar to the other teacher collaborators with respect to her thoughts and practices regarding general mathematics.

Phase II: Early Intervention Period (March 1982-July 1982)

The results from the first GMP and Phase I provided a conceptual framework that guided the project's activities during Phase II. The two primary tasks the project researchers faced during this second phase were the (a) collection and selection, and translation of knowledge from literature, and (b) documentation of the ways in which teachers reacted to the literature and based new classroom actions upon it (i.e., changed their instructional mode). Project researchers believed the activities they used to accomplish these two tasks would likely affect changes in teacher attitudes, modes of instruction, and ways of dealing with students. Figure 2 illustrates the project's activities during Phase II.

An analysis of the pre-intervention classroom observations and teacher interviews and the results from the previous General Math Study led researchers to identify three strategic categories of instruction that seemed likely avenues for successful resolution of the difficulties connected with general mathematics. These strategic categories for instructional improvement included:

1. modifying the mathematics content and task selection;
2. increasing the quality and quantity of communication about mathematics; and
3. using the social organization of the class to facilitate mathematics learning and instruction.

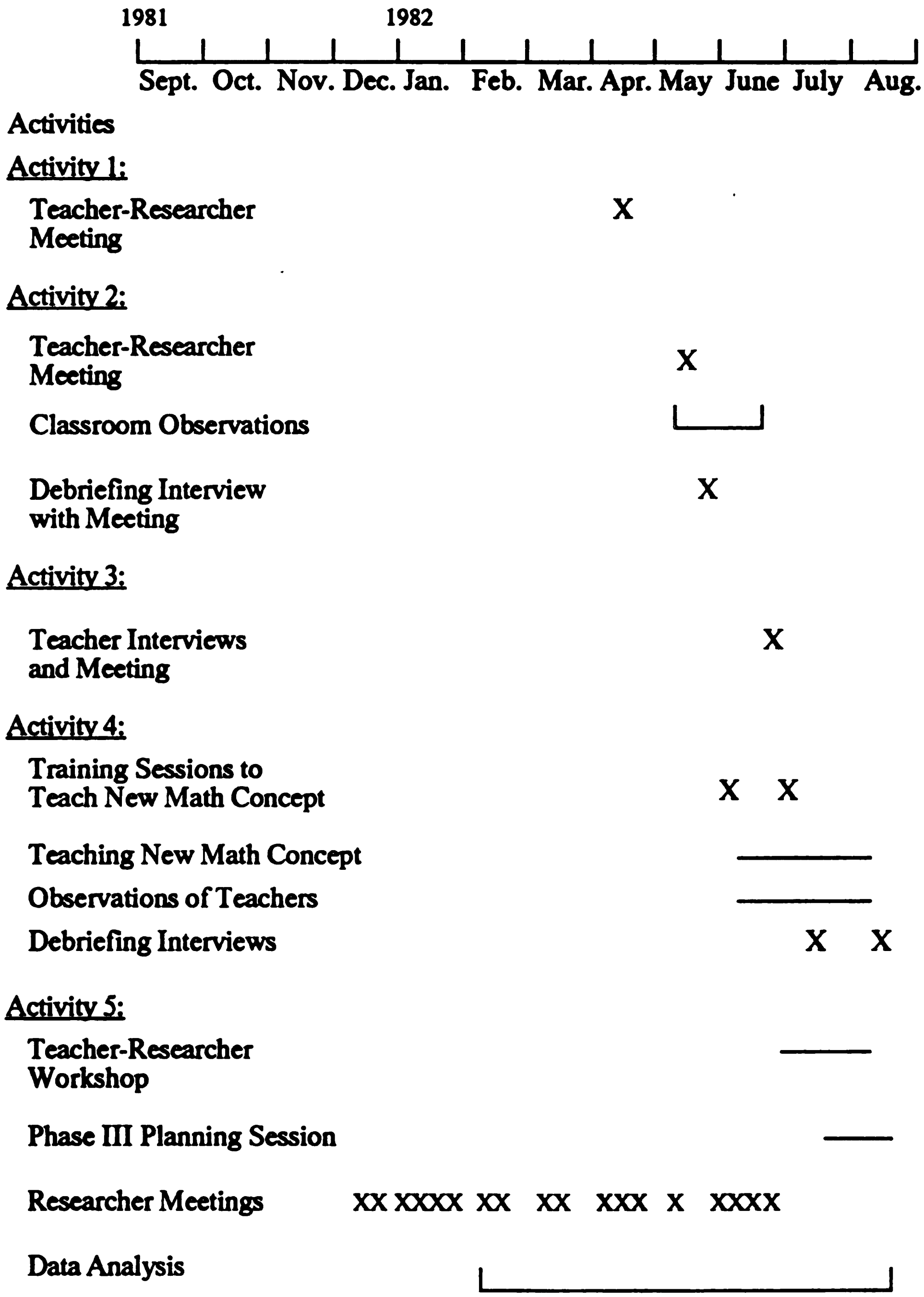

Figure 2

PHASE II: EARLY INTERVENTION PERIOD
(MARCH 1982 TO JULY 1982)

Of the three field-grounded instructional improvement categories above, two were identified as matters of instructional pedagogy and one dealt with the mathematical content. These served as the basis for the selection and organization of literature given to the teachers, and as a major focus for classroom consultation, collaborative deliberation, and ongoing data collection.

The instructional improvement categories were used to frame the project researchers first task: the collection and selection and the translation of knowledge from the literature. The staff and teacher collaborators discussed the literature related to each of the three categories during the Teacher-Researcher meetings held five times during this period. The literature selected for the teachers to read and discuss during this phase is listed according to each instructional improvement category in the Appendix B.

The reasons for the selection of this literature were field grounded: first, they related directly to the three instructional improvement categories which had been identified through observational and interview data as strategic places where improvements could be implemented successfully. Second, they suggested that improvements in the stated categories would likely lead to positive changes in the teachers' general mathematics classes.

The second task faced by the researchers during this phase was to document the ways in which the teachers reacted to the literature and changed their instruction. Although one criteria for teacher selection was their willingness and desire to change their teaching of general mathematics, two factors suggested to the researchers that the teachers needed to be engaged in multiple activities as they implemented any instructional intervention. The first factor was the human tendency to teach as one always taught. The second was the researchers' evaluation that significant change in the outcomes of a general mathematics class would require a complex or multifaceted intervention that was wide in scope. As a result of these two factors, researchers initiated a series of five activities to prepare the teachers cognitively and affectively for designing and implementing instructional interventions to change general mathematics.

Activity 1: The first activity oriented the teachers to fieldwork research in general and its application to general mathematics in particular. They were given Cusick's book, Everhart's article, and Davis' chapter (Appendix C), along with summary statements from the psychometric clinical interview and the classroom observation components of the first GMP study. These were presented as examples of methods of data collection and analysis which would be used in the second GMP study. The primary aim of this activity was to get the teachers to become familiar with the methods and nature of the findings from fieldwork.

Activity 2: The second activity focused on process/product research on teaching. Specifically, the teachers studied Brophy's synopsis, Good and Grouws' article, and the Levin and Long chapter (Appendix C). The teachers selected aspects of the Good and Grouws instructional model to implement in their classrooms. That is, they were asked to try a new instructional pedagogy (e.g., controlled-practice) in their general mathematics classes for the weeks following this activity.

Activity 3: Third, the group discussed a chapter from Thelen's book (Appendix C) in order to make the teachers aware of the noninstructional aspects of the classroom, notably its social organization, and realize that future (next year's) instructional improvement interventions would need to include interventions which focused on improving the social organization of the class.

Activity 4: The teachers next taught two prepared units (Probability and Spatial Visualization) from the Middle Grades Mathematics Project (MGMP) to high school students attending a special 6-week summer school program at Michigan State University. The summer activity occurred in place of having the teachers experiment with new or modified content in their own general mathematics classes at this time. The evidence was convincing that some part of the general mathematics problem was attributable to uninteresting content and the project researchers used the summer program as a means for promoting among the teachers an instructional experience in a content area with which they were not readily and thoroughly familiar. The units offered novelty in their content and used a new approach to the development of the mathematical concepts. Teaching the units required the teachers to use concrete manipulatives (cubes, dice, coins, and pictorial materials) and an activity-based instructional approach. The units also used an instructional model and a suggested script for the teachers. The MGMP directors, Lappan and Fitzgerald,[3] provided a training session for the GMP teachers prior to their instruction.

Activity 5: The fifth activity was a 2-week summer workshop where the teachers and researchers met daily for 3 hours. The teachers were assigned overnight readings focused on one of the three instructional improvement categories. They were asked to react to the readings individually on the following day. After their individual reactions, a general discussion ensued among the teachers and researchers. During the last 3 days of the workshop each teacher (a) outlined a scope of work (mathematical content to be covered) for the coming school year; (b) embellished the outline for the first quarter of the year; (c) explicitly planned the first 3 days of school; (d) identified one or two units of new content and modified significantly one or two units of content usually taught; and (e) selected one or two specific strategies (e.g., wait-time and controlled practice) to implement as part of their intervention plan.

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[3]Lappan, G. & Fitzgerald, W. are professors in the Department of Mathematics, Michigan State University. They acted as consultants for the GMP.

During Phase II the teachers tried new instructional pedagogies and new mathematical content (outside the context of their own classroom). As a result they became aware of the implications from the literature for improving general mathematics. Further, after the classroom trial of the improvement strategies they became convinced that a comprehensive improvement plan would be necessary if significant change in the success level of general mathematics was to occur. This plan would include instructional modifications in the three improvement categories of content, communication and the social organization.

Pamela participated in all five Phase II activities with the other teacher collaborators. She implemented the homework and controlled practice process strategies from the Good and Grouws Instructional Model (Appendix C) and Rowe's (Appendix C) "wait time" strategy in her classroom as well. She taught the MGMP Probability Unit with one of the other GMP teachers to the MSU high school summer students. She selected a set of pedagogical and content strategies to implement for the coming school year.

Phase III: Initial Intervention Period (July 1982-June 1983)

The primary project activities during Phase III included the following:

1. the teachers' implementation and continued development of the multifaceted Instructional Improvement Plan in the target general mathematics class;
2. classroom consultation with the teachers by the project consultant, Lanier;
3. the documentation of each teacher's intervention implementation of their Instructional Improvement Plan, with attention paid to the consequences for teachers and learners;
4. the completion of the literature selection and review by the project researchers;
5. the ongoing organization and analysis of collected data; and
6. the revision of the Instructional Improvement Plan for implementation during Phase IV. Figure 3 lists the activities during Phase III.

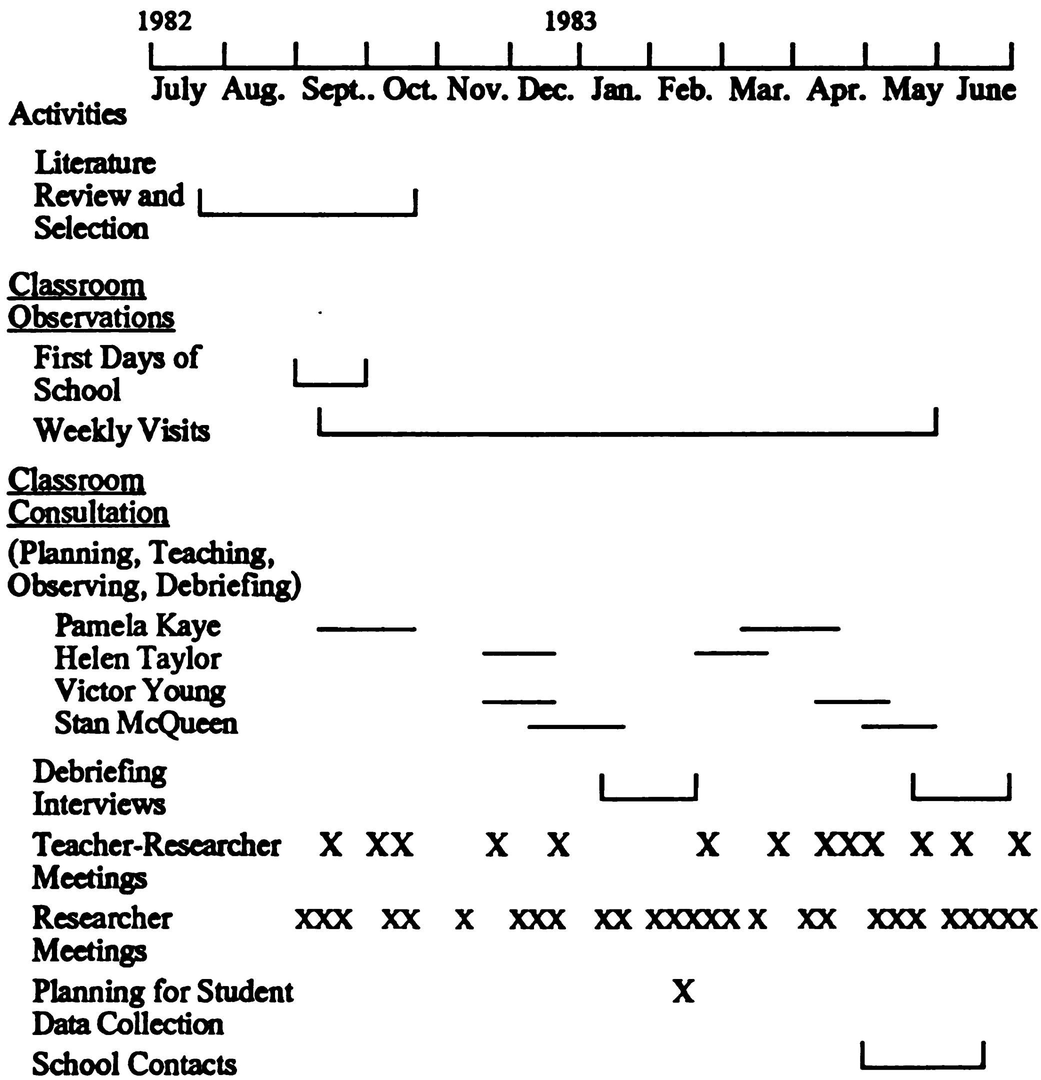

Figure 3

PHASE III: INITIAL INVERVENTION PERIOD (JULY 1982 TO JUNE 1983)

Project researchers began to select a new body of literature during August and September, 1982. The staff chose selections related to each of the three instructional improvement areas. Particular articles were presented at semi-monthly teacher-researcher meetings held during the school year. The literature read and discussed during Phase III is included in Appendix D.

The teachers were given a reading selection prior to the Teacher-Researcher Meeting when it would be discussed. Each meeting was devoted to one of the three Instructional Improvement Categories. The first agenda item was a one-on-one interview of the teacher with their primary classroom observer. The purpose of this interview was to determine the teacher's reaction to the selected reading under consideration. The interview questions remained the same for all the readings. The second agenda item involved the teachers giving an update of classroom activities and the progress he/she had made in their class at the time of the meeting. The last agenda item included a discussion between researchers and teachers regarding their reactions to the literature. The third agenda item consisted of a brief preview of the literature related to the instructional improvement area to be discussed at the next meeting.

Documentation activities began with classroom observations of the first 3 days of class in each teacher's room. This was followed by at least one or more observations by the end of the second week of school. These observations focused on each teacher's implementation of his/her instructional improvement plan. Observers reported on their perceptions of the first days of school.

Classroom consultation activities included the planning and implementation of special units of new or modified mathematical content by the respective teacher in collaboration with the project's classroom consultant (Lanier). Four major considerations were given to the set of decisions that went into this multifaceted intensive activity. They were:

1. Consideration of the possible deterrents to success--for teachers and learners--identified by the first GMP. These deterrents were:

    (a) a history of poor math achievement and attitude;
    (b) a repertoire of fragmented mathematical concepts, algorithmic skills and problem-solving strategies;
    (c) student-teacher interaction problems;
    (d) perception of mathematics as irrelevant to the present or the future;
    (e) school habits (attendance, study, etc.);
    (f) resistance to instruction, particularly that which is somewhat familiar;
    (g) the clamor for seatwork (mundane assignments).

2. Identification of the instructional improvement categories that would likely lead to greater success for the general mathematics teacher and learner.

3. Perusal of the available literature related to the deterrents or the instructional improvement categories.

4. Review and discussion of the classroom observations and interview data collected for the teachers during 1981-1982.

The conceptual construct guiding the planning and implementation of the activities was the change model attributed to Lewin (Blanchard & Zigarmi, 1982)--unfreezing, changing, refreezing. The project, in this sense, was serving as a change agent for the teachers. In this role the researchers participated with the teachers in activities designed to create a motivation to change, thus continuing the "unfreezing process."

Planning and implementing the special mathematics units began with an open-ended planning session between the classroom consultant and the teacher, documented by the observer. This meeting considered the goals (e.g., desired student outcomes) for the instructional unit. In the process of establishing goals the teacher and consultant assessed the level of knowledge and skill possessed by the students in the class. Following these decisions and judgements, the planning then focused on strategies for introducing and carrying out the unit. This led to the development of an overall scheme with specific detail for the first day of instruction while the detailed planning of subsequent days was left to be determined after each day's lesson.

Once the unit was underway, the classroom consultant was present for all or most of the lessons, often participating in some aspect of the instruction. The primary observer was present each day taking field notes. Following every lesson or prior to the next lesson the teacher, consultant, and primary observer met to assess the lesson and set goals, determine strategies, and design/select tasks for the next lesson. These procedures represented the only common aspects of the classroom consultation sessions across the four teachers and classrooms. The teachers planned with the consultant and implemented one mathematical unit each semester during Phase III. At the end of every consultation period the teachers were interviewed (Appendix F) to enable researchers to obtain their perceptions of the unit and the contribution of classroom consultation.

Observations were made daily in the teacher's classroom during the classroom consultation periods and at least once a week during periods of regular instruction. The researchers taking the field notes focused particular attention on changes relative to the instructional improvement categories and any apparent consequences of these changes that could be observed.

At the end of Phase III the teachers were interviewed using a selection of 12 questions from the four Phase I Teacher-Collaborator Interviews. These questions are included in Appendix A.

The new mathematical units Pamela and the classroom consultant developed included one on fraction concepts and one on problem-solving strategies. In addition, she also implemented other new math units during the periods of regular instruction without classroom consultation. She participated in all project-related activities and interviews. During Phase III she selected and administered a computational test three times (September, January, June) in her general mathematics classes in order to measure her students' mathematical/computational improvement across the year.

Phase IV: Final Intervention Period (July 1983-June 1984)

Prior to Phase IV the project staff examined the outcomes of the previous 2 years work with the teachers. Researchers believed the teachers had moved from Lewin's (Blanchard & Zigarmi, 1982) "unfreezing" (or the preparation for change) through the "changing" process. The goal the researchers had for the Phase IV activities was to have the teachers "refreezing" in their improved instructional activities.

In Phase IV the data and experiences previously gathered guided teachers and researchers in their final intervention and data collection activities. The primary project activities during Phase IV included the following:

1. A GMP summer workshop and collaborative planning session where the teachers and researchers worked on activities in preparation for Phase IV.
2. A GMP teacher-researcher planning session in September to finalize the preparations for the first semester's implementation of a unit on fraction concepts, to be taught with the classroom consultant present.
3. The teachers' implementation and continued development of the multifaceted Instructional Improvement Plan for implementation in their target general mathematics class across the school year.
4. Classroom consultation with the teachers on all aspects of a classroom consultation unit implemented each semester.
5. Documentation by project researchers of the teacher's implementation of their Instructional Improvement Plan.
6. Literature selection and review by the GMP teachers and researchers.
7. On going organization and analysis of collected data.
8. Monthly GMP teacher-researcher meetings.

Figure 4 illustrates the project's activities during Phase IV.

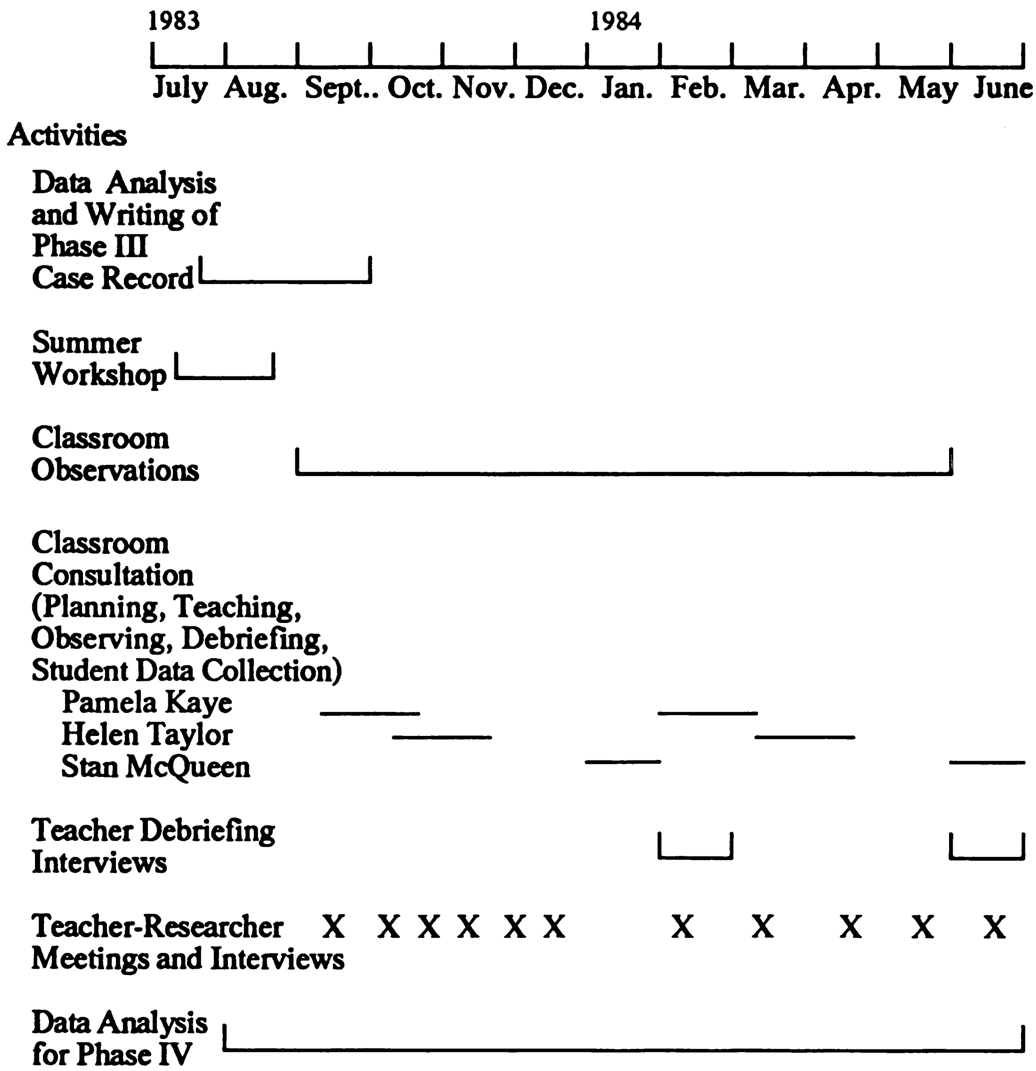

Figure 4

PHASE IV: FINAL INVERVENTION PERIOD
(JULY 1983 TO JUNE 1984)

The GMP summer workshop and collaborative planning session provided the occasion for the teachers to work together to design a unit of math content for implementation during their first classroom consultation period in Phase IV. In addition, the teachers and researchers selected the psychometric instruments that were used to collect student data across the year in their general mathematics classes and in the respective general mathematics classes in their schools. During the summer workshop each teacher reviewed his/her initial instructional improvement plan implemented during Phase III and developed a second Plan for Phase IV. Finally, each teacher met with the classroom consultant individually to reflect on the instructional changes he/she made during Phase III and to determine the instructional improvement categories in which further consultant assistance might be needed.

The GMP collaborative planning session in September provided the opportunity for the teachers to finalize their plans for implementation of the unit on Fraction Concepts.

The teachers implemented one special math unit during the classroom consultation period once each semester in their general mathematics classes during Phase IV. Although the classroom consultant was present on a regular basis as in Phase III, his role was modified. He now expected that each teacher would assume the responsibility for the planning and instruction of the units. His work with the teachers became focused on giving them feedback on their instruction during the classroom consultation period and discussing with them their progress on implementing their Instructional Improvement Plan.

In the GMP teacher-researcher meetings held monthly throughout the year, major attention was focused on identifying consistently effective instructional improvement techniques as well as those classroom problems that persisted and still needed to be resolved.

The literature for Phase IV was individually selected for each teacher based on his/her instructional improvement plan. If one teacher wanted to work on ways to improve the social organization of the classroom, he/she was given readings related to that instructional improvement category. There were only three readings[4] the GMP teachers and researchers read and discussed collectively. The authors and titles of these are:

Dillon, J. T. Teaching and the art of questioning.

Finkel, D. L., & Monk, G. S. "Teachers and learning groups: Dissolution of the atlas complex."

Manning, B. H. "A self-communication structure for learning mathematics."

The sources of data collected during Phase IV included: (a) documentation of the summer GMP planning workshop; (b) classroom observations (daily during the intensive interventions and weekly during the extensive interventions); (c) teacher interviews (formal and informal); (d) documentation of the classroom consultation sessions between the teacher and researcher Lanier; (e) transcriptions from the teacher-researcher monthly GMP meetings; and (f) data on student achievement. Researchers continued their fieldwork activities in the target classes and maintained other data-gathering activities. Having already collected substantial information on the teachers, the researchers now concentrated on documenting any new teacher behaviors or strategies and student reactions that were observed. Student psychometric data was collected during Phase IV in order to answer part of the project's fourth research question about the outcomes of the instructional interventions for general mathematics students.

[4]The complete reference for each reading is contained in the GMP Reading List in Appendix A.

The literature Pamela read during this period included three chapters given to her by the classroom consultant. These chapters focused on the instructional improvement category social organization of her class. These readings[5] included:

Moos, R. H. "Framework for evaluating environments."

Moos, R. H. "Classroom environment scale scoring key."

Moos, R. H. "Social environments of secondary school classes."

Summary of the Instructional Improvement Intervention Project

The GMP was a longitudinal instructional intervention project that utilized field-grounded methods in addition to depending heavily on the intensive interaction between researchers and collaborating teachers. As a team they engaged in indepth study and discussion of the literature related to the instructional improvement categories and worked together in the preparation, implementation and evaluation of the Instructional Interventions undertaken during Phases II, III, and IV. This close relationship continued for the duration of the 3-year study.

The GMP included intervention strategies that reflected Cuban's (1984) suggestions for improving teachers' practices. The first of his two-pronged approach to changing teachers' practices included the improvement of teacher knowledge and skills. His suggestion was accomplished in the GMP by the teachers' literature review/study and classroom trial of planned instructional interventions. The second involved the reduction of teacher isolation in the classroom and the establishment of collegial relationships. His suggestion was realized in the GMP through classroom consultation and the teacher-researcher discussion/deliberation meetings.

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[5]The complete reference for each reading is contained in the GMP Reading List in Appendix A.

Chapter III

CAPTURING CHANGE

The chapter is divided into three sections. The first section, "Pamela Kaye and Arborville High School," is a portrait of the subject and the setting of the case study. The second, a "Historical-Comparative Methodology," is a discussion of the research method used to select the case study data (case data) from the corpus of data which was collected. The last section, "The Instrument Development," describes the creation of the instrument used to examine the teacher's thoughts and practices prior to, and after the instructional intervention and to capture the nature of instructional change across time.

The body of data was collected during the GMP and for 2 years after it concluded. The field study or ethnographic method employed by researchers in the GMP was also used to collect the data during the post-GMP period. This method provided data of the quality necessary in order to address the study's driving questions. Field methods provided data that gave a comprehensive portrait of the teacher's thoughts and practices prior to, during, and after the instructional intervention. In addition, the method provided a longitudinal perspective which could not have been captured sufficiently by other research methodologies.

The task undertaken in this study was two-fold: first, to revisit the corpus of data and select data samples known as case data for this study; and, second, to find a way of looking at this data that would capture teacher change. The instrument used to examine the case data in this fashion was developed after studying the primary and secondary data sources and other related historical documents which had been selected for this case study.

Pamela Kaye and Arborville High School

Pamela teaches mathematics at Arborville High School[6], a rural/small town school district in the midwest. The high school building is a single story structure nearly 30 years old. The 600 to 700 students attending the ninth through twelfth grades at Arborville High School are from socioeconomic backgrounds ranging from upper-lower class to upper-middle class. The district enrolls only a few minority students.

The mathematics curriculum at Arborville High School is separated into two tracks, a college-preparation track and a noncollege, or general track. The students in the college-preparation track are required to take Algebra I (first year algebra) followed by Geometry and then Algebra II (algebra/trigonometry). The college-bound students are encouraged to take a fourth year of precalculus. Students in the noncollege preparation track take 1 year of general mathematics and a year of consumer mathematics. There is a foundations of mathematics course which provides an alternative for students whose low mathematical abilities prohibit them from being successful in general mathematics classes. About half the foundations class are in a special education program.

There are usually four or five sections of general mathematics classes at Arborville High School each year. Three teachers share the responsibility for teaching general math. All the teachers have secondary teaching certificates in mathematics. Pamela, who majored in mathematics, teaches math classes exclusively. Although they use the same textbook, Stein's (1974) Refresher Mathematics, they do not follow a prescribed general mathematics curriculum. Yet, the content of the general mathematics classes is similar across classes: a year long review of basic arithmetic computation. During an interview, prior to the intervention, Pamela described her general math curriculum as

---

[6]Arborville High School is a pseudonym used for Pamela's high school in the General Mathematics Project. It will continue to be used in this study.

"addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals--with some work in percents and integers."

Pamela's classroom, captured in Figure 5, is typical of most high school classrooms.

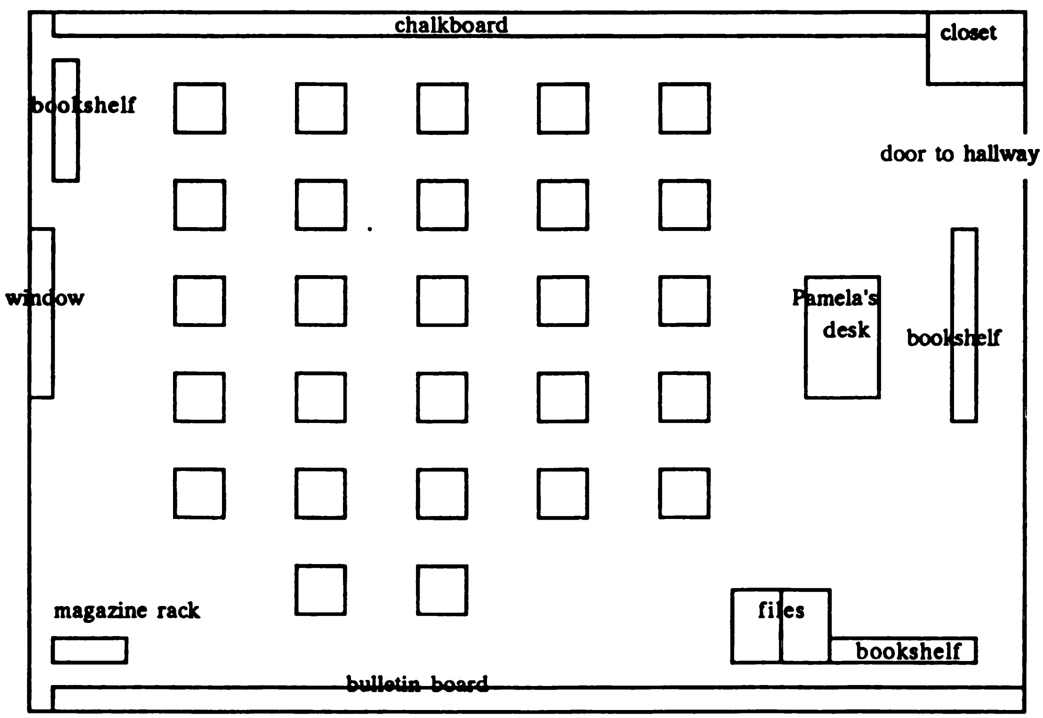


Figure 5

PAMELA KAYE'S MATHEMATICS CLASSROOM
AT ARBORVILLE HIGH SCHOOL

There are posters of Peanuts cartoon characters and school announcements on the bulletin board at the front of the room. The bookshelf behind Pamela's desk contains games and playing cards for the students to use when they have finished their daily assignments. The magazine rack holds a variety of magazines for the students to look at when they have extra time. The students' desks face the chalkboard at the front of the room and Pamela's desk is at the side of the room near the hallway door. The far left-hand side of the chalkboard contains the assignments for the marking period for Pamela's two Algebra II classes.

The students in Pamela's general mathematics classes were judged by the GMP researchers as typical of the students in the other general mathematics classes studied in the project. Results from the previous GMP supported by in depth teacher and student interviews and classroom observations identified characteristics distinctive of most general mathematics students: (a) a history of poor mathematical achievement and attitudes; (b) a repertoire of fragmented mathematical concepts; (c) student-teacher and student-student interaction problems; (d) the perception of mathematics as irrelevant to their present or future needs; (e) poor school habits (attendance, study, etc.); (f) the resistance to instruction, particularly that which is somewhat familiar; and (g) a clamor for seatwork (or mundane, routine assignments). Goodlad (1984) identified the same characteristics with respect to students in low-tracked classes.

Prior to the start of the GMP instructional intervention, Pamela described her general mathematics students in the following manner:

> Skill-wise, they are students who are still having trouble with basic operations. Basically, I think they don't have enough computational skills to survive in the algebra class.
>
> In terms of them socially, they are the lower echelon of kids as far as being accepted--they are not the "IN" kids in the school. As a whole group they don't have any confidence in their math ability whatsoever. They know they can't learn math and it is a real job to convince them that they can.

> Probably the biggest characteristic of them would be that they are totally convinced they can't do math. Nobody has been able to teach them up to this point, so they think there is no way they can learn it now.

Pamela thought her general mathematics students were poor in math because they lacked the self-confidence needed to be successful. Her goal as a general mathematics teacher was to help the students gain some degree of overall self-confidence with one outcome being that they would be successful in mathematics.

Most of the students in Pamela's general mathematics classes were freshmen, although there were several sophomores, juniors, and seniors. These older students were those who had not taken a general mathematics class previously or who were repeating it because they had failed it before. During the 5 years of this study, Pamela's general mathematics classes ranged in size from 25 to 35 students.

Pamela was 33 years old at the start of the GMP. She had a secondary teaching certificate in mathematics and had been teaching mathematics at the high school level for 12 years. Pamela said in an interview that although she taught mathematics, it was not something she had initially intended to do. Her first interest had been in guidance and counseling. In a preproject interview she talked about her perception of herself as a general mathematics teacher.

> Pamela: I think I am pretty good most of the time. I feel pretty good about my ability to teach general mathematics--the basic computation things. The thing that I would like to know more about and that I do not have a strong background in is the specific applications of mathematics. I would like to give my students more examples of math applications they could use right now in their lives.
>
> Interviewer: Do you think your role as a teacher is different from class to class?
>
> Pamela: Yes. The two classes I teach are general mathematics and Algebra II and it is hard for me to think of them in the same context. They are totally different situations. In the general mathematics class I am a lot more concerned about where the individual student is and bringing him through whatever it is that is going on in his life right now and get the math along the way. With the Algebra II students, they are a lot more stable, a lot more settled, so I am a lot more math oriented with them.

Although Pamela thought she was a good general mathematics teacher prior to the start of the intervention project she believed she needed more knowledge about applied mathematics. Her view at this time was that it was more important to help her general math students with their problems than it was to teach them mathematics. Pamela joined the GMP in 1981 as a teacher collaborator to find, as she explained to project researchers, ways to improve her general mathematics classes at Arborville High School.

A Historical-Comparative Methodology

A study of the impact of a longitudinal intervention on changing a teacher's thoughts and practices requires a methodology that has historical and comparative aspects. To answer the questions guiding this study, Pamela's instructional thoughts and practices regarding general mathematics needed to be examined from 1981 to 1987 (historically) and their changes compared (comparative). The overarching questions and its three midrange questions that guided this study are:

> How are the beliefs and practices of a teacher changed as a result of a longitudinal intervention?
>
> > What were the teacher's beliefs and practices prior to the intervention?
> >
> > What were the characteristics of the teacher's thoughts and practices during the intervention?
> >
> > What changes, initiated by the intervention, were sustained?

This historical-comparative case study used both primary source and secondary source data. The primary source data included field notes of direct observations, respondent interviews, and artifacts collected during and after the GMP. Secondary or interpretive source data included reflective case records, documents, papers and articles from the GMP. Each data source is discussed in this section in terms of the methods and rationale for its collection. In addition, the rationale for and methods of the selection of the case data is presented. Case data is the term given to the data selected from the total data collected and used for this case study.

Direct Observations

The field notes of Pamela's general mathematics classes during the 5 years of the study constituted the source of the observational data. The classroom observations provided descriptions of her instructional practices prior to active involvement in the GMP interventions, during the implementation of the interventions, and after the project ended. In general, the field notes taken in the GMP teachers' general mathematics classes included the entire class period from the beginning until the end. Since the teacher was the focus of the observations, efforts were made to document his/her (a) presentations of the mathematics lessons; (b) interactions with students about the mathematics content; and (c) interactions with students during the lesson assignment period. At the end of each observation the observer wrote a reflective document that included his/her impressions, reactions, or thoughts about the lesson. This was appended to the field notes and treated as a secondary data source. This document allowed the researcher to comment on or conjecture about any observed changes in the teachers' practices or students' behavior that might or might not be attributed to project-related activities.

The direct observations captured life in a general mathematics class for the teacher and the students. The observations covered a variety of activities including regular days of instruction, review days, testing days, and days when a new topic was being presented. The observations revealed the general activity flow of the lesson and the student-teacher and student-student relationship and interaction patterns. On four days during Phase I a second GMP researcher was paired with the primary observer to observe the same lesson. The two sets of field notes were compared in order to ascertain the reliability of the observations. An example of the one instructional segment taken from the field notes of a pair of observers in Pamela's class and a brief discussion of the results is included in Appendix E. Figure 6 contains the dates of observations in Pamela's general mathematics class.

During the two intervention years of the GMP there were three periods when the teachers worked in their classes unassisted by the GMP classroom consultant; these were identified as periods of regular instruction. There were two periods during the school year when the teacher worked with the classroom consultant to plan and teach a new or modified unit of mathematical content. During these classroom consultation periods the classroom consultant observed the teacher, provided feedback and gave suggestions for further improvement. During these 2 years all of the direct observations became more focused as the observers captured the ways in which Pamela implemented the instructional improvement strategies in her classes during periods of classroom consultation and periods of regular instruction.

The classroom observational data selected as case data were observations of regular or "typical" mathematics instruction. Excluded from the case data were the math observations of days of testing or other nonmathematical activities such as days when the students scheduled their classes for the next semester or year and when groups of students were called out of class for picture-taking. The selected observations for the case study were grouped in sets of five for the pre-intervention and 2 post-project years. During the two GMP intervention years the groupings of observations were organized according to periods of regular instruction and periods of classroom consultation. In the intervention years, four periods (2 regular and 2 consultation) each consisted of 5 observations and the fifth period (regular instruction) consisted of 2 observations. Figure 7 summarizes the observation dates and topics of lessons selected for the case study.

	Dates of Observations			
Phase I				
	10/30/81	02/18/82	03/16/82	03/18/82
	01/27/82	03/15/82	03/17/82	03/19/82
				03/22/82
Phase II				
	05/14/82	05/21/82	05/27/82	
	05/20/82	05/25/82	06/04/82	
Phase III				
Regular	09/01/82	09/03/82	09/08/82	10/05/82
Instruction	09/02/82	09/07/82	09/20/82	10/07/82
Classroom	10/12/82	10/22/82	10/29/82	11/05/82
Consultation	10/15/82	10/26/82	11/02/82	11/12/82
	10/21/82	10/28/82	11/04/82	
Regular	11/18/82	01/10/83	02/08/83	03/08/83
Instruction	01/05/83	01/24/83	02/17/83	03/22/83
	01/07/83	02/01/83	02/23/83	03/29/83
Classroom	04/18/83	04/21/83	04/24/83	04/27/83
Consultation	04/19/83	04/22/83	04/26/83	04/28/83
	04/20/83			
Regular	05/04/83			
Instruction	05/17/83			
Phase IV				
Regular	08/31/83	09/06/83	09/12/83	09/19/83
Instruction	09/01/83	09/07/83	09/15/83	
Classroom	09/26/83	10/03/83	10/07/83	10/12/83
Consultation	09/27/83	10/05/83	10/10/83	
	09/28/83	10/06/83	10/11/83	
Regular	10/19/83	11/17/83	01/23/84	02/09/84
Instruction	10/27/83	11/22/83	01/30/84	02/16/84
	11/02/83	12/09/83	02/01/84	02/23/84
	11/07/83	12/12/83	02/06/84	03/01/84
				03/08/84

Figure 6

DATES OF OBSERVATIONS OF PAMELA KAYE'S
GENERAL MATHEMATICS CLASS

Dates of Observations (cont.)

Classroom	03/12/84	03/16/84	03/26/84	04/09/84
Consultation	03/13/84	03/19/84	03/27/84	04/10/84
	03/14/84	03/20/84	03/28/84	04/11/84
	03/15/84	03/23/84	03/30/84	
Regular	04/19/84	05/11/84	05/21/84	05/30/84
Instruction	05/02/84	05/16/84	05/23/84	05/31/84
	05/10/84	05/18/84	05/29/84	06/01/84
Post Project	09/12/84	11/08/84	12/20/84	03/19/85
Year 1	10/04/84	11/15/84	01/11/85	04/04/85
	10/09/84	11/20/84	02/01/85	05/01/85
	10/16/84	11/29/84	02/22/85	05/02/85
	10/30/84	12/06/84	02/28/85	05/08/85
	11/01/84	12/10/84	03/01/85	05/22/85
	11/05/84	12/12/84	03/15/85	05/30/85
Year 2	09/13/85	09/25/85	11/08/85	12/12/85
	09/20/85	10/21/85	12/11/85	12/16/85

Figure 6 (con't.)

Period	Observation Dates /Topics				
Phase I: 1981-1982					
Regular Instruction	03/15/82 Integers	03/16/82 Integers	03/17/82 Integers	03/18/82 Integers	03/19/82 Integers
Phase III: 1982-1983					
Regular Instruction	09/02/82 Pentominoes	09/07/82 Similarity	09/08/82 Similarity	09/20/82 Prob. Solv.	10/05/82 Geometry
Classroom Consultation	10/12/82 Fractions	10/15/82 Fractions	10/21/82 Fractions	10/22/82 Fractions	10/29/82 Fractions
Regular Instruction	01/05/83 Percents	01/10/83 Percents	02/08/83 Statistics	02/17/83 Probability	02/23/83 Probability
Classroom Consultation	04/18/83 Prob. Solv.	04/20/83 Prob. Solv.	04/21/83 Prob. Solv.	04/26/83 Prob. Solv.	04/27/83 Prob. Solv.
Regular Instruction	05/04/83 Prob. Solv.	05/17/83 Integers			
Phase IV: 1983-1984					
Regular Instruction	09/01/83 Tangrams	09/06/83 Pentominoes	09/07/83 Pentominoes	09/12/83 Symmetry	09/15/83 Geometry
Classroom Consultation	09/27/83 Fractions	10/05/83 Fractions	10/10/83 Fractions	10/11/83 Fractions	10/12/83 Fractions
Regular Instruction	11/07/83 Factors	11/17/83 GCF/LCM	12/12/83 Decimals	01/30/84 Patterns	02/06/84 Probability
Classroom Consultation	03/12/84 Similarity	03/13/84 Similarity	03/14/84 Similarity	03/15/84 Similarity	03/16/84 Similarity
Regular Instruction	05/11/84 Fractions	05/29/84 Percent			
Post Project: 1984-1985 and 1985-86					
Regular Instruction	10/09/84 Factors	11/15/84 Fractions	11/29/84 Fractions	02/22/85 Probability	05/22/85 Pre-Algebra
Regular Instruction	09/25/85 Geometry	11/08/85 Fractions	12/11/85 Fractions	12/12/85 Prob. Solv.	12/16/85 Fractions

Figure 7

OBSERVATION DATES AND LESSON TOPICS
OF THE CASE DATA OBSERVATIONS
OF PAMELA KAYE'S GENERAL MATHEMATICS CLASS

Teacher Interviews

Classroom observations provided case data that captured changes in Pamela's instructional practices. However, this data did not capture the changes in her thoughts or beliefs regarding teaching and learning in general mathematics. The data needed to accomplish this would have to come from interviews. A series of discussions with Pamela were conducted during the pre-intervention, intervention, and post project years. The interviews will be used in the same manner as described by Stenhouse (1978)--to compare her instructional practices with her thoughts about teaching. In addition, Pamela's responses to the interview questions will be compared with one another across time to ascertain the changes in her thoughts about teaching.

Interviews (some formal and others informal) were conducted with all of the GMP teachers throughout the 3-year project. The formal interviews were those in which the teachers responded to a set of prepared questions. These interviews were audio-recorded and transcribed. In the informal interviews the teachers were not given a set of prepared questions. Instead, the teacher and primary observer discussed project-related activities. Some informal interviews were audio-recorded and transcribed (e.g., the teacher-researcher meetings) while others were simply summarized by the researcher as a secondary data source. The interviews provided the research staff with information regarding the teacher's reactions to and perceptions of the numerous project-related activities in which they participated. In addition, they provided information on the teachers' thoughts about general mathematics students, the nature of the mathematical content, and instruction.

The GMP Teacher Collaborator Interviews were conducted at the beginning of the project, after the first intervention year (Phase III), at the end of the second intervention year (Phase IV), and at the end of the second post-project year. The teacher collaborator interview contained questions concerning four areas of general mathematics. This data provided a longitudinal view of the changes in the teachers' thoughts across the duration of the project and for 2 years after the project ended. The interview questions are contained in Appendix A. The four GMP teacher collaborator interviews with Pamela were selected as the primary source of interview data for the case study.

After the teachers completed the units they taught with classroom consultation (in Phases III and IV), they responded to debriefing interview questions in which they reacted to the content and instruction of the unit. The questions for the Debriefing Interviews are contained in Appendix F. Pamela's interviews were selected for the case data because, in part, they focused on the role of the classroom consultant in changing teacher practice. The more globally-oriented set of Teacher Collaborator Interviews had not elicited Pamela's thoughts regarding this role.

Interviews Not Selected as Case Data

A third set of interviews conducted during Phases III and IV of the project captured Pamela's responses to the literature she read. Pamela's interviews regarding her reactions to the literature were not selected as part of the case data as they did not contribute any additional useful information needed to answer the questions guiding this study. During Phases III and IV the teachers and researchers met to discuss project-related activities. These meetings were audio-recorded and transcribed and provided the GMP staff with additional informal interview data on the teacher collaborators. These transcribed sessions with Pamela were not selected as part of the case data because they did not provide additional information needed to answer the research questions of this study. Two informal interviews between the teacher collaborators and their primary observers took place at the beginning of Phase III and Phase IV. The teachers talked about the instructional improvement strategies they planned to implement during the coming school year. These informal interviews were not selected as case data. However, copies of Pamela's strategies for Phases II, III and IV were chosen as a part of the case study documents.

Document and Artifact Data

Project-related documents and artifacts contributed information that could not be obtained through direct observation or respondent interviews. These data will be divided into three categories: teacher-related, student-related, and project-related.

Teacher Data

The teacher-related data included the instructional improvement plans Pamela wrote prior to Phases II, III and IV, copies of her general mathematics curriculum for each year of this study, and a transcription of her presentation to a group of middle school teachers during the fifth year of this study when she talked about the changes she made in her classes. These data were selected as case data because they documented the nature of changes in Pamela's thoughts and practices and the impact of the intervention project on her instruction.

Other data which related to Pamela included: the "Phase I and II Case Record of Pamela Kaye," written at the end of the GMP's first year; the "Phase III Interim Case Record of Pamela Kaye," written at the end of Phase III; the "Phase IV Final Case Record of Pamela Kaye," written at the end of Phase IV; and, the final case study, Pamela Kaye's General Math Class: From a Computational to a Conceptual Orientation. The difference between case data, case record and case study is that case records are the compilations and reductions of the case data. In contrast, the case study is the interpretation of the case data in relation to some topic, issue, or problem--for example, the evolution of Pamela's concept-oriented instruction.

Student Data

The student-related data accumulated during the GMP and for 2 years after the project is included in Appendix G. Student data collected during the GMP was used by researchers to examine the effects of a teacher's changed instructional mode on the attitudes and mathematics achievement of general mathematics students. The only student data selected for this case study was data from a computational test., the Shaw-Hiehle Computation Test (1972), (Appendix H).

Project Data

Project-related documents from the GMP were the third source of documents and artifacts from which case data was selected. These documents are listed chronologically.

1. General Mathematics Project: Phase I and Phase II

    The NIE/IRT General Mathematics Project Proposal
    The IRT General Mathematics Project's Progress Report
    10/1981 to 3/1982
    The IRT General Mathematics Project's Progress Report
    3/1982 to 10/1982

2. The General Mathematics Project: Phase III

    The IRT General Mathematics Project's Progress Report
    10/1982 to 3/1983
    The IRT General Mathematics Project's Progress Report
    3/1983 to 10/1/83
    American Educational Research Association Annual Meeting Papers, 1983, Montreal, Quebec:
    A. Madsen-Nason, "Methods Used in Studying Ninth Grade General Mathematics Classes"
    J. Bushman, "The Social Organization of the Classroom as a Mediating Factor in Teacher Thought and Action"
    P. E. Lanier, "Teacher Behavior Change and Its Relationship to Instructional Intervention in Ninth Grade General Mathematics Classes"
    N. Brubaker, "A Ninth Grade General Mathematics Teacher's Perception of Her Changing Beliefs About Students, Instruction, and Teacher Role"

3. The General Mathematics Project: Phase IV

The IRT General Mathematics Project's Scope of Work - 1983-1984
The IRT Continuation Proposal for the General Mathematics Project
High School Demographic Data from the State of Michigan
Annotated Literature Review of the General Mathematics Project

The GMP documents selected for the case study included the GMP NIE/IRT proposal, the GMP/IRT Continuation Proposed for the GMP and the AERA paper on the project's methodology. These documents provided data which summarized both the GMP's intervention activities and the data collected during each of the project's four phases. In addition, these documents were selected because they contained information that addressed the questions of this study.

Summary

The direct observations allowed a longitudinal description of Pamela's instructional practice. The interviews provided a similar longitudinal perspective of her thoughts about her practice. The four case records contained information, such as descriptions of Pamela's general mathematics curriculum, that helped describe the impact of the GMP intervention on changing her instructional patterns. Other project documents such as the instructional improvement plans provided historical and comparative information on Pamela's changing thoughts about teaching general mathematics. Figure 8 is a summary of the case data.

Case Data	Phase I	Phase II	Phase III	Phase IV	Post Project
Direct Observations					
Regular Instruction	5	0	12	12	10
Classroom Consultation	0	0	10	10	0
Respondent Interviews					
Teacher Collaborator	1		1	1	1
Consultation Debriefing			2	2	
Other Interviews (informal)					
Presentation to MGMP Teachers					1
Documents/Artifacts					
Case Records	1	1	1	1	
Case Study				1	
Instructional Improvement Plan		1	1	1	

Other Documents:
GMP/NIE Proposal
AERA Paper, "Methods Used in Studying General Mathematics Classes"
GMP/IRT Continuation Proposal

Figure 8

THE CASE DATA USED IN THE STUDY OF PAMELA KAYE'S INSTRUCTIONAL CHANGE

The Instrument Development

After these data were selected, a method of comparison needed to be developed. In other words, a way needed to be found to make sense of and to find connections between these data in order to portray changes which occurred in her thoughts and practices over time. The development of such an instrument to compare the data is described in the third section of this chapter.

The "raw" data which needed to be organized consisted of the selected classroom observations, teacher interviews and project documents. Because no categories in previous studies could be employed to organize these data, a classification system had to be developed. The development of such a system began with the GMP's instructional improvement categories (math content/tasks, communication patterns, and the social organization). These categories were created after the analysis of the pre-intervention direct observations and teacher interviews and were believed to be strategic areas where improvements could be successfully made. These categories provided a schema for looking back into the data because they were used in the planning of all the GMP's intervention activities--including the selection of literature, the discussions during the teacher-researcher meetings, and the classroom consultant's planning feedback sessions. Since the GMP researchers used these categories for implementing instructional changes, it seemed likely that instructional change would be observed in one or more of these three categories. Thus the categories guided the development of a classification system that would be used to analyze the field notes of observations and the transcriptions of the interviews. The instructional improvement categories were used to focus the first examination of the observation and the interview data. During this early analyses of the data, four <u>topics</u> within each category emerged. One example from a classroom observation and an interview segment will be presented in the discussion of the first topic in each category. This will illustrate how the topic was identified in the case data.

Within each topic three levels of behavior/thought were derived. Level 1 represented the typical/traditional practice as noted in all the GMP teachers' instruction prior to the intervention. Level 3 included the instructional changes for improvements in this topic suggested in the literature, during meetings or by the consultant. Level 2 indicated a practice which is improved from level 1 but not yet optimal (as in level 3). These coded levels enabled topic comparisons within a given observation and interview as well as across a set of observations and interviews. The levels are included after the discussion of the respective topics in a category.

The Mathematical Content

The first category centered on the nature of the mathematical content and task selection of general mathematics. A preliminary examination of the case data generated four topics within this category. The first topic, the Mathematical Orientation, considered the mathematical focus of the lesson. Some lessons were computationally oriented--dealing with the development of a mathematical skill or procedure. Other lessons were conceptually-oriented--dealing with mathematical concepts/ideas. This topic surfaced in the interviews when Pamela Kaye talked about her preference for and selection of one orientation over another. The following examples from a classroom observation and a teacher interview during the pre-intervention year indicated a computationally oriented lesson and the teacher's thoughts regarding the general mathematics curriculum.

> Ms. Kaye tells the students, "I have a lengthy assignment for you and it will probably take two days. We are moving into something that will be kind of hard and that is why I want you to do all of these problems. Please copy down these problems I have written on the chalkboard."

1. $^{+}.6$ $^{-}.09$	2. $^{+}.7$ $^{-}.04$	3. $^{+}.5$ $^{-}.20$

4. $^{+}.05$ $^{-}.6$	5. $^{+}.45$ $^{-}.2$	6. $^{+}.25$ $^{-}.6$

The students copy the problems on their papers.

Ms. Kaye explains to the students that the problems they were having on the assignment were mistakes made because they were unable to tell which number was bigger.

She tells them, "I want you to put a check mark on the number in each pair that you think is the biggest."

As the students do this Ms. Kaye walks around the room checking the students work.

Ms. Kaye says, "Are you all finished? Now, I want you to go and fill in all the empty parts with zeros."

She puts zeros in the problems on the chalkboard.

1. $^{+}.60$ $^{-}.09$	2. $^{+}.70$ $^{-}.04$	3. $^{+}.50$ $^{-}.20$
4. $^{+}.05$ $^{-}.60$	5. $^{+}.45$ $^{-}.20$	6. $^{+}.25$ $^{-}.60$

Ms. Kaye tells the students, "The easiest way to tell which number is bigger is to fill in the same number of zeros and take the decimal out and then compare the numbers. I think it is easier to compare them by pulling the decimals off and and putting on the zeros. Then the only problem will be in telling what sign you will have. We have done positives and negatives and we have subtracted decimals so what we're just doing is combining them."

The second topic to emerge was the Mathematical Presentation of the lesson: the ways Pamela presented the mathematical topic during the lesson development (or direct instruction) portion of the math period. Some presentations were demonstration oriented while others were more teacher guided and emphasized a student discovery approach.

The third topic was the Mathematical Topic of the daily lesson: integers, fraction operations, decimals, percents, problem solving, probability, geometry, and measurement. Documentation of Pamela's curriculum in the case records for Phases I, III, IV and during the post-project years provided additional evidence of the mathematical topics she covered.

The last topic in this category to emerge was the nature of the Mathematical Task: the kind of mathematical activity selected for the students' lesson assignment (seatwork). The tasks Pamela selected for the assignment ranged from those which were routine and mundane to those that were interesting, engaging and challenging.

The levels of each topic in this category are described below:

The Levels of Mathematical Content

The Mathematical Orientation

Level 3 Conceptually oriented with linkages to other mathematical content/topics.

Level 2 Conceptually oriented without linkages to other content/topics.

Level 1 Computationally oriented.

The Mathematical Presentation

Level 3 The teacher uses a variety of modes (concrete/manipulative, pictorial/ graphical, symbols), a guided-discovery approach and student participation.

Level 2 The teacher's presentation is predominately demonstration-oriented with some student participation limited to the teacher's invitation.

Level 1 The teacher's content presentation is demonstration-oriented without student participation being solicited.

The Mathematical Topic

Level 3 A new mathematical topic that is unfamiliar to the students.

Level 2 A familiar mathematical topic cast in a new context (e.g., fractions in the Similarity unit).

Level 1 Familiar mathematical topic presented in a familiar context (e.g., whole numbers in a drill format).

The Mathematical Task

Level 3 The task is interesting, challenging and nonroutine. It is selected and planned to provide students with experiences that reinforce and enhance the mathematical concepts of the lesson.

Level 2 The task is interesting, although routine. It is selected to provide experiences that reinforce the mathematical concepts of the lesson.

Level 1 The task is routine, mundane and uninteresting. It is selected to give the students practice in a skill or procedure.

Communication Patterns

The second category was called Communication Patterns. It included the patterns of verbal communication during the lesson development or assignment periods. After a study of the observational and interview data regarding communication patterns, four topics emerged as refinements of this category. The first topic, Instructional Interactions, included the interactions between the teacher and the students during the class period: (a) the ways Pamela questioned the students; (b) her responses to students' questions or comments; and (c) the manner in which Pamela explained mathematical content or ideas to the students. The following is an example from a classroom observation that described Pamela's thoughts and practices regarding the topic of instructional interactions.

> Ms. Kaye and the students are checking the answers to review problems that were written on the chalkboard. Ms. Kaye asks the students, "For number 9, what is the answer?"

$$+\frac{1}{6} = +\frac{1}{6}$$

$$+ \quad -\frac{2}{3} = -\frac{4}{6}$$

> Ms. Kaye asks Tim, "Tim, which one is the larger number?"
> Tim replies, "The four-sixths."
> Ms. Kaye tells him, "Yes, it is easier if you change it [meaning to use equivalent fractions] then look at the size of the numbers." She solves the problem (-3/6).

Tammy asks, "Isn't the answer one-half?"
Ms. Kaye tells her, "Yes, if you reduce it down--but what is the sign?"
Tammy says, "Negative."
Ms. Kaye solves the next problem for the students:

$$+\frac{2}{3} = +\frac{8}{12}$$

$$+ -\frac{3}{4} = -\frac{9}{12}$$

Ms. Kaye tells the students their answer should by negative one-twelfth.
Mary frowns. Ms. Kaye says, "Mary, you are frowning, what is the trouble?"
Mary says, "I don't get it."
Ms. Kaye repeats the rule for her, "When you have unlike signs you subtract. When you have like signs you add."

The instructional interactions between Pamela and the students in this observation included questioning, responding, and explaining in an instructional vignette focused on the learning of a computational procedure.

In the following interview segment, Pamela describes the differences between her first and fourth hour general math classes. For Pamela a class period "goes well" when there is little or no interaction. Pamela uses the facial expression of her students to judge their understanding.

Pamela: The first hour class runs well. I don't have to spend a lot of time corralling people. I am not sure they are any more attentive than my fourth hour group which is usually bouncing off the walls. Now that I think about it I am not sure that they are any more attentive--I think that was a misconception on my part earlier when I thought the class was really going well. Now, I am not so sure. I think I was misreading their apparent attention because I wasn't getting a lot of expression. They were being quiet and they appeared to be listening but I wasn't getting the other signs I look for like facial expressions.

A second topic was Direction-Giving: the way the teacher communicated the task the students were expected to do. These could include directions for the daily assignment or directions during a controlled practice portion of the lesson development.

The third topic was Discussion: the extended conversations that occurred between the teacher and her students regarding the mathematical content or ideas of the lesson. While nearly every observation had some questioning, responding, and explaining, this topic in particular considered the conversations which occurred. Some discussions between Pamela and the students focused on an elaboration of a computational procedure or skill and others were oriented towards an understanding of a mathematical idea or concept. The primary characteristic of this topic was its extended nature. Discussions (conversations) took more time than did instructional interactions (questioning, responding, explaining).

The fourth topic was Feedback and Expectations: Pamela's acknowledgement to the students of their accomplishment or effort and her anticipation of their success with the content they were learning or the task they were assigned. Sometimes Pamela was observed telling the students their assignment would be difficult or long. Other times she told students she wasn't surprised they were unsuccessful in learning a particular mathematical topic because she thought it was something that was difficult for them to learn. The levels of the topics identified in this category are described below.

The Levels of Communication Patterns

Instruction Interactions

Level 3 The teacher's questions are open-ended and require students to provide an explanation. The teacher's responses are made in a variety of modes (concrete, pictorial, symbolic) and refer the students to the mathematical concepts of the lesson or to other concepts studied previously. The teacher's explanations embellish and enrich a mathematical idea/concept. Mathematically-precise language is used in questioning, responding and explaining.

Level 2 Questions are routine but require students to provide an explanation. Responses to the students are clearly stated. Explanations focus on the mathematical ideas/concepts. Mathematical language is used in questioning, responding and explaining.

Level 1 The teacher's questions require one-word responses. Responses to students are limited to a few words and lack clarity and precision. The explanations focus on mathematical procedures or computations. Mathematical language is not used.

Direction-Giving

Level 3 Directions are stated clearly and enable students to begin the task without further teacher assistance.

Level 2 The directions require some students to seek further teacher assistance before starting to work.

Level 1 The directions are stated poorly and the teacher has to repeat/reexplain the task to the whole class.

Discussion

Level 3 The discussion is meaningful, interesting and focused on developing mathematical ideas. Students attend, initiate and participate in the discussion.

Level 2 The discussion is focused on mathematical concepts, but students do not initiate thoughts or ideas.

Level 1 The discussion is focused on the further development or refinement of a computational skill or procedure.

Feedback and Expectations

Level 3 Feedback is related to the students' math achievement or understanding of the mathematical concepts. It is genuine, meaningful and specific. There are high expectations for student success in learning the content of the lesson or the mathematical concepts being studied.

Level 2 Feedback is math-related but insincere. The teacher's expectations for student achievement and success in math is moderate.

Level 1 Feedback is nonspecific and used to keep students on task. Teacher expectations for student learning and success in mathematics are low.

The Lesson Structure

The first category, the mathematical content, considered the nature of the subject matter of general mathematics. The second category, communication patterns, dealt with the quality and quantity of communication about the content in the general mathematics class. The third, the lesson structure, focused on the organization of the general mathematics class period. This category was the same as the GMP instructional improvement category, the social organization. Lesson structure represented the format of the class period in which the content and the communication about the content were embedded. Typically all class periods, regardless of the subject matter, have a kind of structure or format: some class periods are well organized while others may be less structured.

The first topic, the Start of the Class, was characterized as the time during the class period before the lesson development or lesson assignment activities took place. This was the time when the students entered the room and got ready for the class to begin. Sometimes this time was used by the students to socialize with one another while other times the students were engaged in a mathematical review activity prior to the start of the daily lesson. The following vignette illustrated the first seven minutes at the beginning of a class period after the students entered the classroom.

> Ms. Kaye is in the classroom by her desk. She is handing out lined paper and selling pencils to students. [Ms. Kaye's "pencil policy" is to sell pencils she buys herself to students who come to class without one.] As she does this she talks with the students. The students are entering the room and socializing with one another and are taking their seats slowly.
>
> Ms. Kaye is at her desk talking to a woman who came in the room. As she takes attendance she chats with the woman.
>
> The students are in their seats and are chatting quietly with one another. Most of the students seem ready to begin the class.

In a teacher interview Pamela discussed her perception of how a typical general math class began. A view which reflected the observation.

> Pamela: A typical day in general math class usually starts out by my handing out pencils to anywhere from two to five students. Also, trying to make sure that everybody has paper because they usually don't remember to bring that. Frequently, I have to give out a loaner book because someone usually forgets to bring their book. Then I take attendance.

The second topic was the Lesson Development: the time when the mathematical content of the daily lesson was presented to the students. It was the direct instruction portion of the math period. Sometimes the lesson development was well-planned and organized and contained the effective use of controlled-practice activities. At other times it lacked these features.

The third topic, the Lesson Assignment, included the daily math assignment portion of the math period--the seatwork assigned to the students. The students might work in groups or individually on the assignment. The assignment might be well-planned and prepared in advance or it might be unorganized with students off-task.

The last topic in this category was the End of the Class, after the lesson development and lesson assignment ended, when students prepared to leave the classroom. Some classroom observations revealed extensive student socializing; others disclosed students and the teacher engaged in a summary of the mathematical concepts. Sometimes the students and the teachers followed an established routine, while at other times no routines were evident.

The levels of the topics in this category are included below:

The Levels of Lesson Structure

The Start of the Class

Level 3 The start of the class has a mathematical orientation, is well planned and includes established routines or procedures.

Level 2 The start of the class is organized and well-planned but lacks a mathematical focus.

Level 1 The start of the class is characterized by off-task socializing activities.

The Lesson Development

Level 3 The lesson is well-planned and includes the effective use of controlled-practice activities. The students are on-task and engaged in mathematical explorations with the teacher.

Level 2 A planned lesson, but controlled-practice is used ineffectively.

Level 1 The lesson is not well organized and lacks controlled practice.

The Lesson Assignment

Level 3 The activities are well-planned, materials are organized and prepared, the students are grouped purposefully for the task and all of the students are engaged in the activity throughout this period.

Level 2 The activity is planned and the materials are prepared. The students are not grouped purposefully for the task. Most students are on-task during this time.

Level 1 The activity is not well-planned or organized. A number of students are not engaged in the task during this period.

The End of Class

Level 3 The end of the period is used to summarize the mathematical concepts, objectives or tasks. This time is also used to review the lesson or preview the next one. A set of routines/procedures are in place.

Level 2 There is no summary, review or preview of mathematical content. Routines are established. Included in this Level are instances when the teacher and the students work on the lesson development or assignment until the dismissal bell sounds.

Level 1 The end of the class is not well organized. There is no evidence of routines/procedures. Socializing and off-task behaviors are observed.

Instructional Improvement Categories and the Questions of This Study

The three categories and their respective topics and levels will be used to examine further the case data in order to answer the questions guiding this study--questions regarding teacher change. A summary of the instructional improvement categories and the related topics is shown in Figure 9. These categories and topics served as the structure used to code the selected classroom observations and teacher interviews.

In Chapter IV, results of coding the observations and interviews to determine the levels of the topics show a set of data that compares the topics within each category, between categories, and across the 5 years of the case study. To complement this quantitative description of teacher change, Chapters V, VI and VII present the qualitative reconstruction of the change process.

<u>Category</u>:	Mathematical Content	
	<u>Topics</u>	<u>Abbreviations</u>
	Mathematical Orientation	<u>(or)</u>
	Mathematical Presentation	<u>(pr)</u>
	Mathematical Topic	<u>(to)</u>
	Mathematical Task	<u>(ta)</u>
<u>Category</u>:	Communication Patterns	
	<u>Topics</u>	<u>Abbreviations</u>
	Instructional Interactions	<u>(in)</u>
	Directions	<u>(dr)</u>
	Discussion	<u>(di)</u>
	Feedback/Expectations	<u>(f/e)</u>
<u>Category</u>:	Lesson Structure	
	<u>Topics</u>	<u>Abbreviations</u>
	Start of Class	<u>(sc)</u>
	Lesson Development	<u>(ld)</u>
	Lesson Assignment	<u>(la)</u>
	End of Class	<u>(ec)</u>

Figure 9

THE INSTRUCTIONAL IMPROVEMENT CATEGORIES AND TOPICS
OF THE CODING STRUCTURE
FOR THE OBSERVATIONAL AND INTERVIEW DATA

Chapter IV

PATTERNS OF TEACHER CHANGE

The purpose of this chapter is to provide evidence of teacher change by analyzing the case data via the coding structure I developed and described in Chapter III. This chapter is in two parts: Part 1, "Measuring Change", describes the procedure used for analyzing the observational and interview data; Part 2, "An Analysis of Change", is separated into sections that are focused on findings from this analysis. These findings include: (a) changes in instructional practice occurred over time, (b) during teacher change instructional practices were inconsistent, and, (c) changes in thoughts about instruction preceded changes in practice. The last section relates these findings to the questions of this study and discusses the procedures used in the qualitative analysis of the case data in Chapters V, VI, and VII.

PART 1

Measuring Teacher Change

In this section the procedures for measuring teacher change using the coding structure developed in Chapter III are discussed. The coding procedure was an analysis of the observations and interviews that included multiple readings of each and then assigning a level of 1, 2, 3 (0 if not applicable) to each of the twelve topics. The unit of analysis for the coding procedure was the classroom observation or teacher interview. In other words, levels of the topics were generalized for one entire observation or interview.

second focused on assessing the level of mathematical content and its topics; from the third reading, topics were coded in the category of communication patterns; and from the fourth reading topics were coded for the lesson structure. After all the data were coded, a fifth reading confirmed the reliability of the first coding. In this final reading the levels of the topics for the observation/interview were again coded and the results compared with coding previously obtained from the second, third and fourth readings. If a discrepancy occurred between the levels of the topics of the codings, the field notes were reread and a level was determined.

The coding procedure yielded a set of numbers (0,1,2,3) which indicated the level of the topics for every observation and interview. A level of 0 for a topic indicated the thought/behavior was not observed. The level of 0 which appeared in the interview and observational data were not included in the analysis of the data when average levels were calculated. The rationale for excluding 0s in the calculation of the average levels was that the absence of a thought/behavior from an observation/interview was not an indication the thought/behavior was at a level of less than 1--it simply did not exist.

The data set that was coded included the 59 classroom observations, 4 general interviews, 2 consultation interviews and Pamela's presentation to a group of teachers. Although the transcribed presentation was not a formal interview, it was coded as an interview because it captured her thoughts about the changes in her instructional practices. Figure 10 includes the data that was coded. The complete set of coded data is contained in Appendix I. In addition to the observational and interview data, other data used in this chapter to support the findings include student test results and an analysis of the amount of time spent in the classroom on instructional/noninstructional activities. The results of coding are discussed in the analysis of change part of this chapter.

Observation Period		Observations	Interviews
Pre-Intervention		5	1 (general)
Intervention - Year 1			
Regular Instruction	(Fall 1982)	5	
Classroom Consultation	(Fall 1982)	5	1 (consultation)
Regular Instruction	(Winter 1983)	5	
Classroom Consultation	(Spring 1983)	5	1 (consultation)
Regular Instruction	(Spring 1983)	2	
			1 (general)
Intervention - Year 2			
Regular Instruction	(Fall 1983)	5	
Classroom Consultation	(Fall 1983)	5	
Regular Instruction	(Winter 1984)	5	
Classroom Consultation	(Spring 1984)	5	
Regular Instruction	(Spring 1984)	2	
			1 (general)
Post Project - Year 1		5	
Post Project - Year 2		5	1 (general) 1 (presentation)

Figure 10

OBSERVATIONS AND INTERVIEWS CODED

PART 2

An Analysis of Change

Changes and Consequences

From the quantitative analysis it is clear that Pamela's practices had changed. Two subsets of data reveal changes in her practices. The first set includes five coded field notes selected from each year of the study. The second set includes the averages of the coded field notes of the daily observations for the 13 observational periods. Following the presentation of charges, two measurable outcomes of changed practice for general mathematics students and for the amount of class time spent in mathematical activity are discussed.

Changes in Practice

To look at changes in Pamela's practice, five observations of regular instruction were selected during each year of the study (see Table 1). The observations of the second period of regular instruction during each of the two intervention years were selected because they occurred in the middle of the year and followed the first period of classroom consultation. Observations in the first period of regular instruction during each intervention year were not selected because at the start of the year routines were not yet established. The last period of regular instruction each year was not selected because these periods contained two observations. Finally, the classroom consultation periods were not selected because Pamela was not making instructional decisions independent of the consultant. Thus, the observations during the second regular instruction period were chosen for the observational data.

Table 1

LEVELS OF TOPICS OF OBSERVATIONS FOR FIVE YEARS

	Instructional Improvement Categories											
	Mathematical Content				Communication Patterns				Lesson Structure			
Topics	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Pre-Intervention Year 1												
3/15/82	1	2	1	1	1	1	0	1	1	1	1	1
3/16/82	1	2	1	1	1	1	0	1	1	1	1	1
3/17/82	1	1	1	1	1	1	0	1	1	1	1	1
3/18/82	1	1	1	1	1	1	0	1	1	1	1	1
3/19/82	1	1	1	1	1	1	0	1	1	1	1	1
Intervention Year 1 Regular Instruction Period - 2												
1/5/83	2	2	1	1	2	2	2	0	2	2	2	2
1/10/83	2	2	2	2	1	1	1	0	1	2	2	2
2/8/83	2	2	3	2	2	1	2	2	2	2	2	2
2/17/83	2	1	3	2	2	1	2	2	2	2	2	1
2/23/83	2	2	2	2	2	1	2	0	1	2	2	2
Intervention Year 2 Regular Instruction Period - 2												
11/7/83	1	1	2	2	2	2	2	0	1	3	2	1
11/17/83	3	2	2	1	2	2	2	0	3	3	1	2
2/12/83	3	2	2	3	3	2	2	3	3	2	2	2
1/30/84	2	1	3	2	2	1	2	2	2	2	2	1
2/6/84	3	3	3	3	3	3	3	0	1	3	3	2

Table 1 (cont.)

	Instructional Improvement Categories											
	Mathematical Content				Communication Patterns				Lesson Structure			
Topics	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Post Project Year 1												
10/9/84	3	3	2	3	3	3	3	0	2	3	3	2
11/15/84	2	3	2	3	3	3	3	0	3	3	3	2
11/29/84	3	3	3	3	3	3	3	0	3	3	3	2
2/22/85	3	3	3	3	3	3	3	0	3	3	3	2
5/22/85	2	3	3	3	2	3	3	0	3	2	2	2
Post Project Year 2												
9/25/85	2	3	3	3	3	3	3	0	3	3	3	3
11/8/85	3	3	2	3	3	3	3	0	3	3	3	3
12/11/85	3	3	2	3	3	3	3	0	2	3	3	2
12/12/85	2	3	2	3	3	3	3	0	3	3	3	2
12/16/85	2	3	2	2	2	3	2	0	3	2	2	3

Changes in the levels across the 5 years indicate Pamela had improved her practice. However, the changes were not immediate, and the change occurred irregularly over the years. For example, under lesson assignment, Pamela's level 1 practices in the pre-intervention year changed to level 2 in the second regular instruction period of intervention year-1. There was, however, virtually no difference between intervention year-1 and intervention year-2. In the two post-project years the level had almost reached level 3. The gradual movement in the levels over the years indicates improvement in Pamela's practices took time.

Were the changes identified in the analysis of the 5 sets of observations representative of the changes made across all the observations? To answer this question an average level for the topics in each period was obtained (see Table 2). The average levels of the topics across the periods supports the previous finding from the daily data--that Pamela's practices had improved. She had moved from a traditional mode of instruction which was computationally oriented to one that was nontraditional and conceptually oriented. This change in Pamela's practice was not immediate, but occurred gradually over 2 years in most topics.

These measures showing change in Pamela's practice across time are accompanied by two measurable outcomes which suggest improvement in the quality of instruction and learning. One outcome was greater growth in academic achievement and effort on computational tasks. The other was a dramatic increase in the time and the nature of time used on academic tasks during class. The following sections describe these outcomes in detail.

Table 2

AVERAGE LEVELS OF TOPICS
FOR THE OBSERVATIONAL PERIODS OF THE STUDY

	Instructional Improvement Categories											
	Mathematical Content				Communication				Lesson Structure			
	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Pre-Intervention Year												
	1.0	1.5	1.0	1.0	1.0	1.0	0.0	1.0	1.0	1.0	1.0	1.0
Intervention Year-1												
Reg. Inst.	1.2	1.5	1.4	1.8	1.4	1.0	1.4	1.0	1.2	1.0	1.6	1.4
Class. Con.	1.6	1.8	1.6	1.6	1.0	1.2	1.0	1.0	1.0	1.2	1.8	1.0
Reg. Inst.	2.0	1.8	2.0	1.8	1.8	1.2	1.8	2.0	1.6	2.0	2.0	1.8
Class. Con.	1.6	2.0	3.0	2.6	2.2	2.4	2.4	3.0	2.0	2.4	2.6	2.2
Reg. Inst.	2.0	3.0	3.0	2.0	2.5	3.0	3.0	0.0	3.0	3.0	3.0	2.5
Intervention Year -2												
Reg. Inst.	3.0	3.0	2.6	2.8	3.0	3.0	3.0	0.0	2.2	3.0	2.4	2.6
Class. Con.	3.0	3.0	2.0	2.8	3.0	3.0	3.0	0.0	2.2	3.0	2.2	2.0
Reg. Inst.	2.2	1.8	2.4	2.4	2.4	2.4	2.2	3.0	2.0	2.6	2.2	1.8
Class. Con.	2.6	2.6	3.0	3.0	2.8	3.0	2.6	0.0	1.6	2.8	3.0	2.0
Reg. Inst.	2.0	2.0	1.5	2.5	2.5	3.0	2.5	0.0	2.0	3.0	2.5	2.0
Post Project Year-1												
	2.6	3.0	2.4	3.0	2.8	3.0	3.0	0.0	2.8	2.8	2.8	2.0
Post Project Year-2												
	2.1	3.0	2.2	2.8	2.8	3.0	2.8	0.0	2.8	2.8	2.8	2.6

Student Outcomes From a Teacher's Improved Practice

The purpose of the GMP intervention was to improve learning and teaching in general mathematics classes. Evidence from the coded field notes indicated that Pamela's teaching had improved. Student data provided evidence of improved learning and mathematics attitudes.

Improved Student Learning. During the intervention and post-project years, Pamela gave a computational test to her general mathematics students at the start of the year, at the beginning of the second semester, and at the end of the year. She used the results as an indication of her students' improvement in computational skills. Even though her instruction emphasized mathematical concepts, she still believed an important goal for general mathematics was computational competency.

Pamela selected the Shaw-Hiehle Computation Test (1972) to use in her general mathematics classes. The test is included in Appendix H. The class results for 4 years of this study (the intervention and post-project years) are in Appendix J. She did not use this test during her pre-intervention, computationally oriented instruction. Due to the lack of test results on students in her pre-intervention classes, comparisons could not be made between their computational ability and the computational ability of classes taught in later years by a conceptual orientation. Fortunately, during the 1983-1984 school year, both Pamela and one of her colleagues taught two general mathematics classes each. She used a conceptual approach and the other teacher used a computational approach. This teacher agreed to give the Shaw-Hiehle Computation Test to his students during the year. Pamela and the GMP staff agreed that this teacher was using an instructional approach very similar to the one she used the previous year. The students in these four general mathematics classes were comparable.

The results show the students in Pamela's general mathematics classes out performed the students in the other teacher's classes on the computational test. In addition, her students put forth more effort in answering the test items than did the students in the other classes. Figure 11 compares the average percent of correct responses of the classes on the total test. Her students continued to gain in computational skills across the year, whereas the other teacher's classes lost most of the gains made in the first semester during the second semester. Although the pretest results showed the classes were nearly matched, the posttest results indicated Pamela's classes scored about 10% higher than the classes of the other teacher. These results suggest that understanding mathematical concepts promotes computational competency.

For Pamela, these results were significant. She selected and used the Shaw-Hiehle Test (1972) to convince herself that changing her instruction would not be detrimental to her students' ability to compute. She gave her students the Shaw-Hiehle Test in September 1982 when she implemented instructional changes. Even though she continued to teach math concepts during first semester, she remained skeptical of the benefit of these changes for her students. Although she thought learning mathematical concepts was important, her first instructional priority was that students compute competently. At the end of the first semester, she again gave the Shaw-Hiehle Test again to her students and was very pleased with the results. To her surprise, the students performed better than she anticipated. By her own admission, if her students had not done as well as they did, then she would not have continued to implement a conceptually oriented instructional approach. It is possible she would not have continued her participation in the intervention project.

Pamela continued to use this test during and after the intervention for two reasons: first, to inform her of the level of student computational achievement; and second, to inform her students of their progress. She shared with them the results of their pretest,

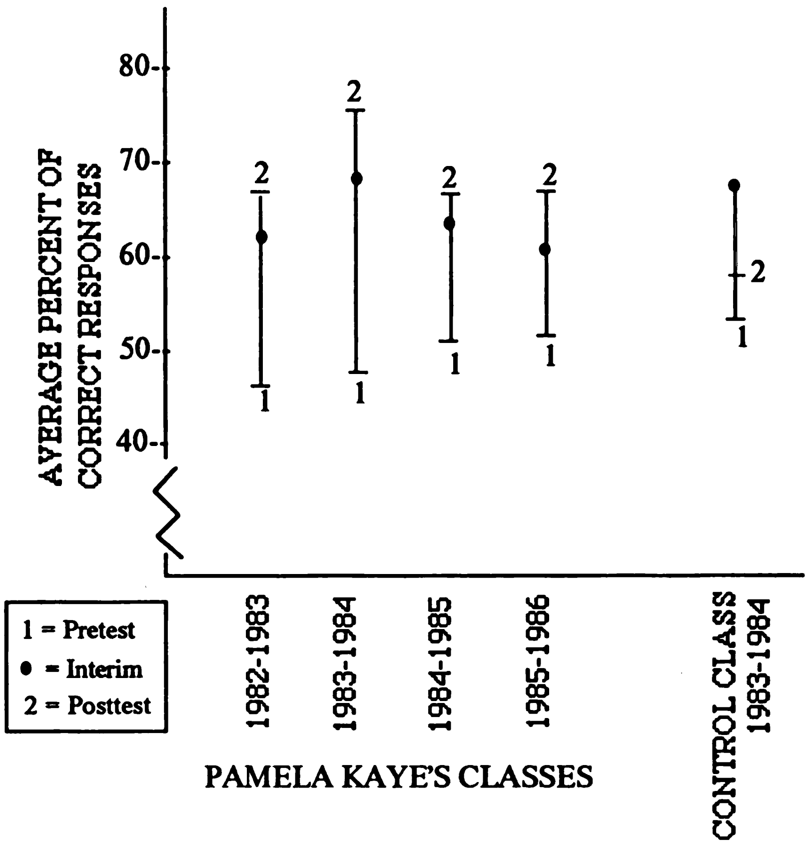


Figure 11

CLASS AVERAGES OF CORRECT RESPONSES ON THE SHAW-HIEHLE COMPUTATION TEST FOR THE TOTAL TEST interim and posttest and indicated the amount of gain (in grade level equivalents) that was made during the semester and across the year. She noted that when she showed students how much they had gained, there was a change in their attitudes towards mathematics. She attributes much of the change over the year to their belief that they could be successful in mathematics because they were successful on the test.

The results of the fraction subtest (Figure 12) reflect Pamela's instructional change in teaching fractions. Her classes gained more during the first semester than did the other teacher's classes and they continued to make gains during the second semester. The results of the percent subtest were particularly interesting. Figure 13 displays the class results of this subtest and shows the gains made by Pamela's students. The results of the subtest by the other teacher's classes indicated that the small gain they made from the pretest to the interim test disappeared in the posttest. The students in this teacher's class performed at the <u>same level at the end of the year as they did at the start of the year,</u> in spite of the fact that they spent a lot of class time on drill excercies computing with percents. The results of this subtest are also interesting because Pamela did not begin to use a conceptual orientation in her percent unit until 1983-1984 and did not teach the percent unit until the second semester. The gains her students made in the second semester suggest that their success in this subtest is attributed to their understanding of percent concepts and the variety of strategies they used in solving these problems.

Conceptually oriented instruction changed the ways Pamela's students thought about mathematics and worked mathematical problems. Specifically, they had nonalgorithmic strategies they could use with computational problems. For example, when given the problem "50% of 36 is ?," the students could solve it by drawing a picture or using a percent stick, a strategy she used in the percent unit (see Figure 14). Her students did not have to rely on the procedure of changing 50% to a decimal fraction

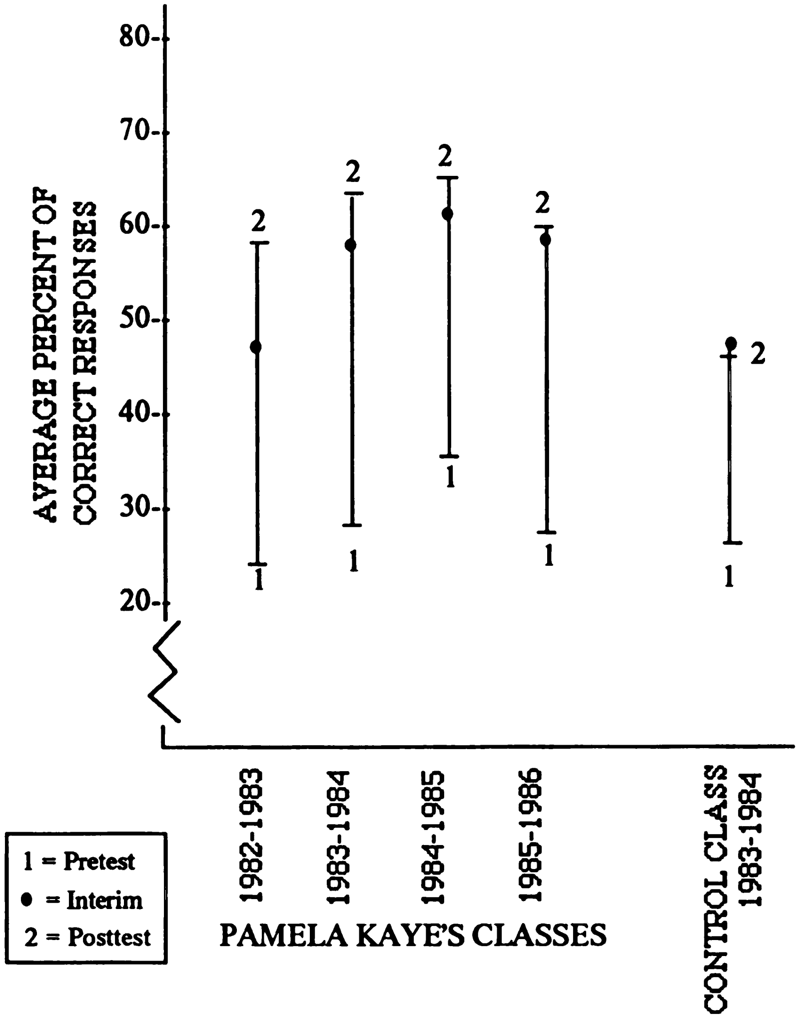


Figure 12

CLASS AVERAGES OF CORRECT RESPONSES ON THE SHAW-HIEHLE COMPUTATION TEST FOR THE FRACTION SUBTEST

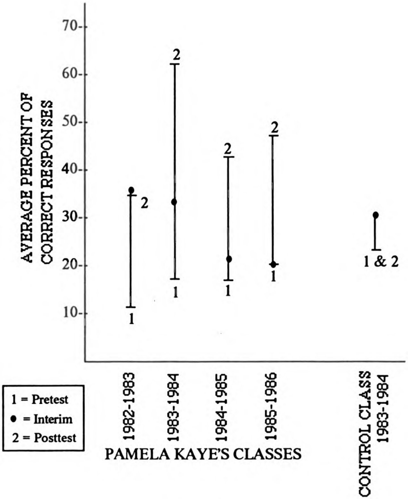


Figure 13

CLASS AVERAGES OF CORRECT RESPONSES ON THE SHAW-HIEHLE COMPUTATION TEST FOR THE PERCENT SUBTEST and then multiplying by 36. Students' work on the test indicated they often used the nonalgorithmic strategies they learned in the class.

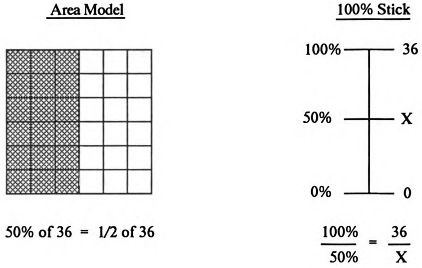


Figure 14

TWO MODELS USED BY PAMELA KAYE'S STUDENTS
TO SOLVE PERCENT PROBLEMS

Students' Attitudes Improved. The attitudes of the students in Pamela's classes changed across the year in each of the intervention and post-project years. They were observed working longer and with more intensity on mathematical tasks as the year progressed. Their increased effort was demonstrated in the number of problems they attempted to answer on the Shaw-Hiehle Test (1972). A problem was attempted if students either wrote an answer or tried to solve the problem. Items were considered not attempted if there was either no answer or no written work that would indicate they attempted to calculate a solution to the problem. During 1983-1984 the average number of test items attempted by the students in both Pamela's and the other teacher's classes were compared.

The results (see Table 3) indicate that the students in both Pamela's classes and the other teacher's classes attempted about the same number of problems on the pretest. However on the posttest, her students tried nearly every problem on the test while the other teacher's students tried about the same number as they had on the pretest. The fractions, percents and practical problems subtests showed the greatest differences between the two data sets. I believe the students tried more problems on the posttest for three reasons. First, they understood the mathematical concepts underlying the computational procedures, this enabled them to understand the procedures for working the problems. Second, the students thought they could be successful in answering the problem correctly because of increased confidence in their mathematical ability. Having increased confidence and feelings of success enabled or encouraged persistence in working the problems. Finally, they had more strategies for working the problems, they could solve a problem using a model, a picture, or algorithmic procedure.

The increase in the computational skills and the changes in math attitudes of Pamela's general mathematics students are attributed to the instructional improvements made during the intervention and post-project years.

Instructional Outcomes of a Teacher's Changed Practice

The instructional changes Pamela implemented altered the flow of activity in her general mathematics classes. More time was being spent on developing mathematics concepts and working on mathematical tasks. Less time was spent on nonmathematical activities.

A comparison of the average amount of time spent in various activities during the pre-intervention, intervention and post-project years captures in a unique way the results of the instructional changes Pamela implemented. The field notes of observations were analyzed and the amount of time spent per class period in each of the following activities

Table 3

AVERAGE NUMBER OF ITEMS ATTEMPTED
ON THE SHAW-HIEHLE COMPUTATION TEST

	Subtests					TOTAL TEST
	Whole Numbers (20)	Fractions (10)	Decimals (10)	Percents (10)	Practical Problems (10)	(60)
Pamela Kaye's Classes						
Pretest	96.7	86.3	94.6	51.6	75.2	83.7
Posttest	99.9	98.2	99.2	94.5	96.6	98.1
Another Teacher's Classes						
Pretest	96.8	89.2	93.5	50.0	71.1	82.9
Posttest	95.8	84.9	92.2	52.2	71.6	82.1

was examined: (a) Direct Instruction, (b) Seatwork; and (c) Nonmathematical. The Direct Instruction activities were defined as those activities in which Pamela and the students interacted verbally with one another about mathematics. Included as direct instruction activities were: (a) oral reviews of previously covered content (Review); (b) lesson presentation (Lesson Development); and (c) oral reviews of the results on the daily assignment (Checking). The effects of the strategies implemented to increase communication patterns are seen in the changes in the flow of activity in Direct Instruction. Seatwork activities were defined as those activities in which students are engaged independently of the teacher. These include the reviews of previously learned content (Review), lesson assignments (Practice), and evaluations of student learning (Tests/Quizzes). The results of the analysis of the Seatwork activities reflects instructional changes implemented to improve student learning and math attitudes. The Nonmathematical activities were those in which neither teacher nor students were engaged in mathematics. There were times when students were allowed to chat with one another before or after the lesson (Socializing). There were also times when the teacher took care of classroom chores, prepared materials for the lesson, or distributed papers/materials to the students (Managing). Table 4 presents the average amount of time spent in these various activities over the years of the study. The flow of classroom activity changed with the greatest changes taking place between the first and second year.

In Direct Instruction in the pre-intervention year, there were no oral reviews of previously learned content. Lesson Development consisted of demonstrations of how to work problems in the lesson assignment. Checking included the teacher's rapid reading of the answers to the daily assignment. By the end of the second intervention year, there were oral reviews including discussions, questions and explanations of previously learned content. Lesson Development included student-teacher dialogues, questions and explanations. Pamela used student volunteers to work problems at the chalkboard and controlled practice activities to increase communication. Checking activities included discussions of common errors and extensions of the lesson to other mathematical concepts/ideas.

Seatwork activities during the pre-intervention year consisted of numerous drill and practice reviews interspersed with unit tests. By the end of the second intervention year, these activities included review problems the students worked at the start of the class. The daily assignments involved activities in which the students integrated the use of manipulatives, illustrations and models. Group work tasks were incorporated in many lessons. Pretests and posttests were used to assess students' achievement and as feedback on their mathematical accomplishments.

One fifth of the class time during the pre-intervention year was spent on nonmathematical activities. These activities included maintaining class records, collecting and distributing materials and preparing materials for the daily lesson. Pamela

Table 4

THE FLOW OF CLASSROOM ACTIVITY
IN PAMELA KAYE'S GENERAL MATHEMATICS CLASSES

	Percent of Time Per Class Period				
	Pre-Intervention Year	Intervention Years		Post-Project Years	
		1	2	1	2
Direct Instruction	16.7	39.0	44.8	45.3	45.6
Review	00.0	04.6	05.5	06.4	02.7
Lesson	14.2	21.4	25.9	26.3	30.6
Checking	02.5	13.0	13.4	12.5	12.3
Seatwork	63.3	48.4	45.0	50.7	49.3
Review	00.0	08.1	12.2	17.0	18.5
Practice	60.2	32.8	24.2	32.8	30.8
Testing	03.1	07.5	08.6	09.9	00.0
Nonmathematical	20.0	12.7	10.2	03.8	07.1
Managing	09.2	10.8	09.1	03.7	06.9
Socializing	10.8	01.9	01.1	00.1	00.2

allowed the students to socialize with one another as soon as they finished their assignment. By the end of the second intervention year, Pamela prepared materials prior to the start of the class period and handled recordkeeping while the students worked on written review problems at the start of the class. Students were now expected to work on mathematics during the entire class period and seldom had time to socialize with one another at the end of the period.

Pamela's practice changed across the years and these changes were reflected in the amount of time that was spent on class activities. By the post-project years she had tripled the amount of time spent in communication and had cut in half the amount of time students spent in practice excercises. As a result of these changes, the amount of time the students spent on mathematics had increased dramatically.

Inconsistency Was a Characteristic of Pamela's Changing Practices

Before the GMP Pamela practiced consistently at level 1, which indicated she was teaching in a typical and traditional manner with a computational orientation. After the project ended her practice had improved sharply and was consistently rated at level 3, reflecting a nontraditional approach to instruction and a conceptual orientation.

Inconsistency in Pamela's Practices

Although her practice improved generally, it did so inconsistently. Inconsistency in practice during an observational period is indicated by one or both of the following: (a) a change of levels within a topic across five observations; and, (b) the inclusion of all three levels or levels 1 and 3 in a topic for the five observations. An examination of the coded sets of observations of the second regular instruction period during the two intervention years indicates these were periods when Pamela's practice was inconsistent (Table 1).

In looking across the levels of topics in each observation of the first intervention year, two of the five observations (2/8/83 and 2/17/83) contained all three levels. Similarly, in looking across the topics of each observation for the second intervention year, four of the five observations (11/7/83, 11/17/83, 1/30/84, 2/6/84) contained all three levels. There was more inconsistency in Pamela's practices during the second intervention year than during the same period in the first intervention year. There was also more movement toward level 3 during the second intervention year. During this year every topic, except the end of the class, contained at least one observation at level 3. This is contrasted to the previous intervention year when only one topic (mathematical topic) reached level 3.

Table 2 provides further evidence of the inconsistency in Pamela's practices during the intervention period. When compared to the average levels of topics during the pre-intervention and post-project years, the average levels of the topics during the two intervention years shows more inconsistency. The average levels moved up and down between periods during each intervention year. At times Pamela's practices reflected a traditional orientation, at other times a nontraditional approach. Even looking across topics for one period, one might be a level 3.0 another in that same period would show a level 1.5.

The average levels of topics of the observations during the intervention years shows inconsistency in Pamela's practices. This inconsistency in practice occurred when Pamela was trying many new instructional approaches and was unable to implement them habitually and with regularity.

An Examination of the Consistencies and Inconsistencies in Three Topics

Looking at the changes in the levels of three topics, one from each of the instructional improvement categories Content Communication and Structure over the years provides an illustration of the consistencies and inconsistencies found in Pamela's practices. Three topics, mathematical task, instructional interactions and start of class, are used. In this analysis the average levels of topics in Table 2 are used and the changes in these levels are discussed.

Mathematical Task. The mathematical activities Pamela selected for the students during the lesson assignment period ranged from routine drill and practice at level 1 to activities that were interesting, challenging, nonroutine and reinforced and enhanced the mathematical concept of the lesson at level 3. Table 5 contains the average levels of the observations for the topic, mathematical task for each of the 13 periods.

The average level during the pre-intervention period indicates Pamela consistently selected computationally oriented drill and practice tasks. These included observations, textbooks reviews and worksheets of review problems.

During the first intervention year, Pamela's tasks moved from an average level of 1.6 to 2.6 across the year. This change of levels indicates she was able to choose tasks that, although still routine, provided students with experiences which reinforced the

Table 5

AVERAGE LEVELS OF CODED OBSERVATIONS
FOR THE TOPIC, MATHEMATICAL TASK

Topic: Mathematical Task

Pre-Intervention	Intervention										Post Project	
	Year 1					Year 2					1	2
1.0	1.8	1.6	1.8	2.6	2.0	2.8	2.8	2.4	3.0	2.5	3.0	2.8

mathematical concepts of the lesson. In the second intervention year, the average level of the mathematical tasks ranged from 2.4 to 3.0. Although changes between levels still occurred, this time the levels were higher. Pamela now included more tasks that were challenging, interesting and nonroutine, while still including some that were routine. The changes in the levels reflects Pamela's attempt to find, develop and implement mathematical tasks at level 3.

By the post-project years, almost all the tasks Pamela selected for the students enhanced mathematical concepts and were interesting, challenging and nonroutine. The average level was between 2.8 and 3.0, which indicates she achieved consistency in her practices in this topic.

Instructional Interactions. The ways Pamela questioned the students, responded to their questions/comments, and explained the mathematics to them changed over the years. Table 6 includes the average levels of the instructional interactions topic during the 13 observational periods of the study.

The average level 1 for the pre-intervention observations indicated consistency in the instructional interactions. Her questions to students required from them single-word responses and her responses to their questions/comments lacked clarity and mathematical language. Her explanations focused on telling the students a mathematical procedure or

Table 6

AVERAGE LEVELS OF CODED OBSERVATIONS
FOR THE TOPIC, INSTRUCTIONAL INTERACTIONS

Topic: Instructional Interactions

Pre-Intervention	Intervention										Post Project	
	Year 1					Year 2					1	2
1.0	1.4	1.0	1.8	2.2	2.5	3.0	3.0	2.4	2.8	2.5	2.8	2.8

a computational result. The focus of the interactions was on how to obtain an answer to a computationally-oriented assignment.

During intervention year-1 the range in the levels of instructional interactions was from 1.0 to 2.5. Pamela was trying to implement strategies in her class that would increase the quality and quantity of communication about mathematics. Sometimes she was successful and other times she reverted to her previous practices. Although she still asked routine questions, she did require students to provide explanations for their responses. She worked on using more mathematical language when she communicated with the students. Finally, Pamela's explanations began to center on the mathematical ideas underlying the lesson. Although there was inconsistency in the average levels during this year, the data shows a continued increase in the levels from the second to the fifth observational period.

The average levels during the second intervention year ranged from 2.4 to 3.0. At this time Pamela was trying to include more open-ended questions in her lessons that required students to think about the responses. She was also working on increasing the number of ways she responded to students questions, i.e., in concrete, pictorial and symbolic modes. Lastly, she began to include in her explanations more linkages to other mathematical concepts studied previously. Although during the first two periods Pamela's practice averaged level 3, she ended the year with an average level of 2.5.

Consistency in Pamela's practice was achieved during the post-project years when both years averaged level 2.8 for the topic. By this time she acquired a repertoire of questions that encouraged students to think about the mathematical concepts, and she could readily provide explanations and examples of mathematical ideas and concepts in a variety of modes. Her responses to students' questions included linkages to other mathematical content. During the post-project years Pamela's communication with her students was consistently established at the higher levels.

Start of Class. The changes in Pamela Kaye's practices regarding this topic show the same consistency-inconsistency-consistency pattern seen in the two previous topics. Table 7 contains the average levels of this topic.

The average level for the pre-intervention period indicates consistency in Pamela's practices at the start of class. This time was characterized by students socializing and engaging in various off-task activities.

Table 7

AVERAGE LEVELS OF CODED OBSERVATIONS
FOR THE TOPIC, START OF THE CLASS

Topic: Start of Class

Pre-Intervention	Intervention										Post Project	
	Year 1					Year 2					1	2
1.0	1.2	1.0	1.6	2.0	3.0	2.2	2.2	2.0	1.6	2.0	2.8	2.8

In the first intervention year, the range in the average levels of this topic were from 1.0 to 3.0. Pamela attempted to implement activities at the start of the class that would engage the students in some kind of mathematical task. There were times during the year when the start of the class was organized and planned, but lacked a mathematical focus. There were times when off-task socializing prevailed; but there were also a few times when the start of class had a mathematical focus with students engaged in a math-related activity. During the second intervention year, the average levels remained inconsistent, ranging from 1.6 to 2.2. Pamela tried to focus on implementing strategies that would better organize the students at the start of the class period; however, these strategies did not engage them in mathematical activities.

By the post-project years, consistency in Pamela's practices was reached when both the average levels of the periods was 2.8. In these years, the observations of the start of the class indicated established routines and procedures that engaged students in mathematical activities.

Summary: Consistency-Inconsistency-Consistency in Practice

The analysis of the average levels of the coded observations across the periods indicated there were periods where Pamela's practices were consistent. These periods occurred prior to and after the intervention years. During the intervention years, her practices were inconsistent. The findings indicate that: (a) practices prior to the intervention consistently characterized a traditional, computationally-oriented mode of instruction; (b) practices after the intervention were consistent with one another and characterized a nontraditional, conceptually-oriented mode of instruction; and, (c) during the intervention years, practices were inconsistent within and across every topic.

Changes in Thoughts Precede Changes in Practice

This section contains a discussion of the finding that instructional change involves both a change in thinking and a change in practice. There is a lag between the time thoughts change and the time when those changes are enacted in practice. The purpose of this section is to examine a teacher's change in thoughts and practices across the years. In the first part, the changes in the levels of the topics for the interviews are compared with the changes in the levels for the observations. The second part compares the changes in the yearly levels of observations and interviews in order to present a more global view of teacher change in thought and practice.

Changes in the Average Levels of the Topics

The changes in Pamela's thoughts and practices across the years are examined by comparing the average levels of the topics in the observations with the levels of topics in the interviews. There were three questions to answer: (a) Do the coded interviews show the same inconsistencies in the topics during the intervention period as was found in the observational data? (b) Do Pamela's thoughts match her practices with the topics prior to, during or after the intervention period? (c) Is there a relationship between her thoughts regarding a topic and her practices? Table 8 contains the average yearly levels of the topics for the coded observations and interviews. Examining these data provides answers to these questions.

The average level of the topics for each year shows a close match between Pamela's thoughts and practices during the pre-intervention year and the two post-project years. In other words, Pamela's thoughts about teaching and learning in general mathematics were enacted in her practice. For example, Pamela believed general mathematics students needed to know how to compute with accuracy before they could be taught the mathematical concepts underlying the computational procedures. This belief was coded in the interview by a level of 1 for the orientation (or) and task (ta). The average levels of 1.0 in the observations for these two topics indicated her practice during this year reflected her belief.

Similarly, during the second post-project year, Pamela believed her students would understand the mathematical concepts when they became engaged in interesting and challenging mathematical activities. This belief was coded in the interview by a level of 3 for the mathematical task (ta). The average level for the observations during the same year was 2.8, which indicates Pamela chose mathematical activities for the students that were interesting, challenging, nonroutine and focused on the development of mathematical concepts.

However, during the two intervention years there was a difference between the observation and interview levels. Her beliefs were not reflected in her practices. For example, during both intervention years for mathematical orientation Pamela believed the focus of the mathematical lessons should be on the mathematical concepts/ideas and that these should form linkages across various content areas. The interview data was coded at level 3 for the two intervention years for the topic which reflected this belief. However, in the observations, levels of the coded data were 1.6 and 2.6 for the two years respectively. Although Pamela believed in a conceptual orientation, her practice reflected a computational orientation.

In most cases, a comparison of the levels of topics of the coded observational and interview data during the intervention years show a difference between Pamela's beliefs and her practices. Also, the data shows that the differences between the levels for the observations and interviews were greater in the first year of the intervention than in the second. The differences between the levels of the observations and interviews indicated Pamela had changed her thoughts about learning and instruction before she was able to change her practices.

Changes in Thoughts and Practices Across the Years

This final analysis is a comparison of Pamela's thoughts and practices across the years. The levels of the 12 topics in the observations and interviews are averaged for each year. The average yearly levels (see Table 9) indicate the level of instructional thought or practice during one year. Level 1 represents traditional/typical instructional approach practiced by most general mathematics teachers. Level 3 is a nontraditional conceptually oriented instructional approach with active student involvement and participation in the learning process. Level 2, although concept oriented, is characterized by limited student participation and engagement in the lesson and mathematical task.

Table 8

AVERAGE YEARLY LEVEL OF THE TOPICS FOR THE OBSERVATIONS AND THE LEVEL OF TOPICS FOR THE INTERVIEWS

	Instructional Improvement Categories											
	Mathematical Content				Communication Patterns				Lesson Structure			
	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Pre-Intervention												
Observation	1.0	1.5	1.0	1.0	1.0	1.0	0.0	1.0	1.0	1.0	1.0	1.0
Interview	1	1	1	1	1	0	1	1	1	1	1	1
Intervention Year 1												
Observation	1.6	2.0	2.1	2.0	1.7	1.6	1.9	1.8	1.6	1.8	2.2	1.7
Interview	3	0	3	3	2	0	0	3	3	0	2	0
Intervention Year 2												
Observation	2.6	2.5	2.4	2.7	2.8	2.9	2.7	3.0	2.1	2.9	2.6	2.1
Interview	3	3	3	3	3	0	3	3	3	3	2	2
Post Project Year 1												
Observation	2.6	3.0	2.4	3.0	2.8	3.0	3.0	0.0	2.8	2.8	2.8	2.0
Post Project Year 2												
Observation	2.4	3.0	2.2	2.8	2.8	3.0	2.8	0.0	2.8	2.8	2.8	2.6
Interview	3	3	2	3	3	0	3	3	3	3	3	0

Table 9

AVERAGE LEVEL OF OBSERVATIONS AND INTERVIEWS PER YEAR

	Average Level Per Year	
	Observations	Interviews
Pre-Intervention	1.05	1.00
	(Δ 0.86)	(Δ 1.71)
Intervention-Year 1	1.91	2.71
	(Δ 0.62)	(Δ 0.11)
Intervention-Year 2	2.53	2.82
	(Δ 0.22)	
Post Project-Year 1	2.75	
	(Δ 0.05)	(Δ 0.08)
Post Project-Year 2	2.70	2.90

The changes in levels from the pre-intervention to the first intervention year observations and interview indicate Pamela's thoughts about instruction changed twice as much as her practice. This was the time when the greatest change took place in her thinking about general mathematics. Although she also changed her instruction during this time, it took the two intervention years for her to make the same amount of change in her practice as she did in her thoughts about teaching. In the intervention years, the greatest change in Pamela's practice took place between the first year and the second year. There was a change of only 0.22 of a level between the second year and first post-project year. In the post-project years, Pamela's practices remained consistent and her thoughts about teaching general mathematics nearly matched her practice.

The data shows changes in Pamela's thoughts preceded changes in her practices. In addition, the data shows a consistency between her thoughts and practices during the pre-intervention and two post-project years and inconsistency during the intervention years. Also, there was more inconsistency between her practices during the intervention years than there was during the post-project years. These findings were also noted in previous analyses of the data.

A Summary of "Patterns of Change" and the Research Questions

In this chapter the development of a coding structure to analyze qualitative data quantitatively was described and was applied in the analysis of the observational and interview data used in the study. In addition to the changes shown by this analysis other quantitative data were presented to show some of the consequences of the changes in practice One purpose of this analysis was to provide a source of data to address the questions of this study. A second purpose was to capture quantitatively the process of teacher change. In this section the findings from this analysis are related to the research questions.

What Were Pamela Kaye's Beliefs and Practices Prior to the Intervention?

The findings from the analysis of the data indicated Pamela's beliefs and practices reflected a typical/traditional instructional mode. What she believed about teaching and learning in general mathematics were reflected in the content selected and in the ways she presented that content to the students.

What are the Characteristics of Teacher Change During the Intervention?

During the intervention, Pamela's practices were inconsistent. One day a lesson would be conceptually oriented, the next day it would be computationaly oriented. This inconsistency occurred throughout the intervention period when instructional changes were being implemented.

The data also showed that during the intervention period, Pamela's thoughts remained consistent with one another but were not consistent with her practices. The rate of change in her practice across the 2 years of the intervention was slower than the change made in her thinking at the start of the intervention. It took 2 years before the level of her practice caught up with the level of her thoughts.

Were the Changes Initiated by the Intervention Sustained?

The analysis of the observational and interview data during the post-project years indicates that teacher change was sustained when it became habituated. Sustained practice occured when practice was judged to be consistent across observations and when thoughts and practices were consistent. Many changes Pamela implemented did not become habituated until the post-project years.

Not only does a teacher's practice need to become habituated, but the teacher's thoughts need to reflect that practice and vice versa. The findings indicated that teacher change takes time, and that over time, change can be sustained in thought and practice.

A Final Look at Teacher Change from a Quantitative Perspective

After examining the data across the intervention years, the results showed, that Pamela was still changing her practice by the end of the second intervention year. The GMP researchers believed that changes in Pamela's instruction would be habituated by the second year of the intervention. This was not the case. It wasn't until the post-project years that this occurred. This finding is illustrated by the data obtained from the coded field notes of observations. Table 10 shows the process of a teacher's change in practice, which is related to the the overarching question of this study.

One way to identify the process of changing teacher practice is to consider the change in the average levels of topics across the years. In Table 10, the frequency of the average levels of the topics in each year are displayed in intervals. During the pre-intervention year, 11 of the 12 average levels of topics were in the interval 1.0 - 1.2 or 0.0. At this time, Pamela had not started to change her practice. In the first intervention year, Pamela began changing her practice which was reflected in the increases in the levels of the topics. At this time, all 12 average levels were in three intervals from 1.6 - 1.8 to 2.2 - 2.4. During the second intervention year, nine levels were in intervals over 2.2 - 2.4. The increase in the average levels across the intervention years indicated Pamela continued changing her practice. The data in the post-project years in Table 10 indicates her practices were becoming habituated. The levels during this time remained almost the same. In the post-project years, two-thirds of the levels were in the interval 2.8-3.0.

Procedures for Studying the Patterns of Teacher Change

Teacher change has been examined in this chapter through a quantitative approach. Analysis of the data led to the identification of three patterns of teacher change. Qualitative analysis of the case data provides further descriptions of the patterns of teacher change. Chapters V, VI and VII use the case data to depict descriptively the kinds of changes that took place in Pamela's thoughts and practices. Whereas the quantitative analysis of this chapter provided evidence that change occurred and illustrated patterns of teacher change, the next three chapters present the findings from the qualitative analysis that describe the nature of these changes. In Chapter V, the analysis of the changes Pamela made in the mathematical content are discussed. In Chapter VI, the changes in the patterns of communication about the mathematics are presented. Chapter VII depicts the changes in the lesson structure of the class.

Table 10

THE FREQUENCY OF THE AVERAGE LEVELS OF TOPICS PER YEAR

	Pre-Intervention	Intervention		Post Project	
	Year	Year-1	Year-2	Year-1	Year-2
Intervals of the Average Levels					
2.8 - 3.0			4	8	8
2.5 - 2.7			5	1	1
2.2 - 2.4		1	1	1	2
1.9 - 2.1		4	2		
1.6 - 1.8		7		1	
1.3 - 1.5	1				
1.0 - 1.2	10				
0.0	1			1	1

Procedures for Studying the Patterns of Teacher Change

The procedures for analysis of the data included the selection of observational vignettes, interview segments and documents from the case data representative of Pamela's thoughts and practices. Samples of the coded observations and interviews are included in Appendix K. The vignettes, segments and documents were organized according to the year in which they occurred and by the category and task to which they belonged (Figure 15). This organization enabled the data to be examined within a category and to be compared across the years. Examples of observational vignettes, interview segments and documents were selected from these data for inclusion in Chapters V, VI and VII.

		Instructional Improvement Category											
		Mathematical Content				Communication Patterns				Lesson Structure			
	Data	or	pr	to	ta	in	dr	di	e/f	sc	ld	la	ec
Pre-Interveniton Year	Observation Vignettes												
	Interview Segments												
	Documents												
Intervention Year 1 & 2	Observation Vignettes												
	Interview Segments												
	Documents												
Post Project Year 1 & 2	Observation Vignettes												
	Interview Segments												
	Documents												

Figure 15

ORGANIZATION SCHEME
FOR THE OBSERVATIONAL AND INTERVIEW DATA
FOR CHAPTERS V, VI, AND VII

Chapter V

PATTERNS OF CHANGE IN THE MATHEMATICAL CONTENT

This chapter examines qualitatively the changes in the content of Pamela Kaye's general mathematics classes across 5 years. Analysis of the observational vignettes, interview segments and case documents revealed that the mathematical content improved in Pamela's classes across the years. Three findings are: (a) the mathematical orientation improved; (b) the assignments and tasks improved; and, (c) the content of the curriculum improved. Each of these findings is discussed in the following sections and are related to the patterns of teacher change identified in Chapter IV: (a) changes in instructional practice occurred over time; (b) improvements in practice are inconsistent during teacher change; and (c) changes in thoughts about instruction precede changes in practice.

From Arithmetic Computations to Mathematical Concepts

The first finding from the analysis of the case data indicated the mathematical orientation of the class had changed. The analysis of the observational vignettes and interview segments during the pre-intervention year indicated Pamela's thoughts and practices reflected a computational orientation. This orientation was depicted in every observation. Pamela demonstrated the procedures used for working the problems that would be on the assignment. This was followed by the assignment of numerous similar problems. Even though Pamela demonstrated the procedure, most students were familiar with the arithmetic procedures and did not require a review or a demonstration.

The following observational vignette from a lesson on addition of integers illustrates Pamela's typical instructional mode.

> Ms. Kaye writes the following problems on the chalkboard:
>
> $^{-}4 + {}^{+}2 =$ $\qquad$ $^{-}1 + {}^{+}6 =$
>
> The students copy the problems on their papers.
>
> Ms. Kaye explains to the students that all they have to do to get the answer is to subtract the two numbers and use the sign of the larger number.
>
> She tells the students, "All right, I want you to do a whole bunch of these problems on pages 179 and 180 in your books."
>
> Ms. Kaye reads the following directions in the book, "'In each of the above problems you see that the absolute value of the sum is the sum of the absolute value of the two addends and the sum of two positive integers is a positive integer.' "
>
> She tells the students, "Well, just ignore what they tell you and do the problems like we did them on the chalkboard."

Pamela demonstrated the procedure for calculating the answers to two sample problems, gave the students a rule for calculating the answers on the assignment and told the students to ignore the directions in the textbook and work the problems using the method she demonstrated. For Pamela, this instructional pattern allowed students to start working on the daily assignment in a minimal amount of time. With most students working, she could circulate around the room assisting individual students. Pamela assigned numerous problems to keep those students who knew how to work the problems busy long enough to let her assist other students needing help. This practice reflected her beliefs that general mathematics students needed to be computationally competent and that a daily routine of drill and practice excercises promoted that competency.

During the first intervention year, Pamela changed her thoughts about the orientation of the general mathematics content. She began to talk about the importance of focusing instruction on mathematical concepts/ideas and finding ways to connect various mathematical ideas. Although she thought differently about the mathematical content, she still believed the goal for general mathematics students was to achieve computational competency. The following interview segment illustrates her changed thoughts about the mathematical orientation and her unchanging mathematical goals.

> General math is different than it was in the past. You still add, subtract, multiply, and divide whole numbers, decimals, and percents, but it's with a conceptual understanding rather than with a computational focus.
>
> Everything I've done in the project led me to believe the major problem students are having is that they really don't have any understanding of what is going on.

In the intervention years, Pamela used the strategies of drawing pictorial representations or making concrete models to help her present mathematical concepts to the students. She started using this strategy in a unit on fractions and continued using it through the decimal unit. In the unit on decimals, Pamela had the students draw 100-square grids to represent decimal fractions. This strategy allowed them to develop the concept of the part-to-whole interpretation of decimals and helped them establish linkages between fractions and decimals. The following vignette occurred in the decimal unit when the students were asked to draw graphs (pictures) for the problems written on the chalkboard. The students were used to drawing pictures for every decimal problem they worked and in this assignment assumed they had to draw pictures for each problem.

Ms. Kaye has written an assignment on the board:

Show on graph paper	Add
1) .3	6) .6 + .3
2) .03	7) 1.3 + .05
3) 4/10	8) .15 + .20
4) .40	9) 1.6 + 1.5
5) .7 + .1	10) 2.13 + 3.4

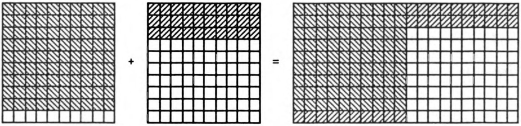
Randy asks, "Do we have to write those as fractions and decimals?"
Ms. Kaye says, "Not today you don't."

> Jennifer asks Ms. Kaye, "Ms. Kaye, on problems six to ten do we have to draw those pictures for them?"
> Ms. Kaye responds again, "No, not today."
>
> Holly, "Do we have to draw those pictures for six to ten?"
> Ms. Kaye says, "No."

When the class period ended, Pamela discussed her reflections regarding this strategy.

The following segment is from the observer's report after the observation.

> Pamela said yesterday was the first day the students started working with addition of decimals and she was having them draw pictures of two decimals and put the two pictures together to come up with the result.
>
> She represented nine-tenths as ninety of the hundred-squares shaded. When she wanted the students to look at three-tenths she used the same size square and shade in thirty of the hundred squares. When the students combined ninety squares plus the thirty squares they realized they had one full hundred square and another part of one.



.90 + .30 = 1.20

> Pamela thought this was a good strategy to get the students to see the sum of nine-tenths and three-tenths did not equal twelve-hundredths ($.9 + .3 \neq .12$). Which is what she said was a common error her students made in previous years.
>
> Pamela noted that if any of her students believed that before, she was convinced many of them now saw that it did not make sense and were able to correct their answers to one and two-tenths.

During the first intervention year, Pamela used the verbal-symbol-picture strategy in the fraction unit and started using it in a limited way in the decimal unit. She thought about using the strategy in other units but did not implement it consistently across the year. By the second intervention year, most of her units included lesson presentations and assignments in which she did use this strategy. When she saw the student test results at the end of the first intervention year, she was convinced the computational gains they made were attributed to this strategy. Pamela believed that their understanding of mathematical concepts enabled them to compute with greater competency. Reflecting on her instructional changes, Pamela talked about how this strategy helped her alter her thinking about this content.

> The changes I made were in the verbal, symbol and picture links. We had done verbal usage before and used pictures but there was an intent to directly teach this and try to link those together and I think that was really a big clarification for me and the students.
>
> The pretest and posttest results showed students the effectiveness of the unit in their learning.

Pamela wanted students to understand mathematical concepts; however, she did not want to trade computational competency for conceptual understanding. After all, she believed students needed to learn to compute if they were to survive in the world outside the classroom. When she saw the gains in the computational ability made by her students' during the first intervention year, she became convinced that a conceptual approach enhanced computational competency.

Pamela did not begin to link units of content together using mathematical ideas during the first intervention year. However, during the second year she used the part-to-whole idea to connect the fractions, decimals and percents units together. In the first intervention year she focused on a few mathematical units and developed the concepts related to the unit. Whereas in the second intervention year, she transferred the strategies used in the units in the first year to other mathematical units. In the post-project years, Pamela's instruction focused on the development of mathematical concepts and the establishment of linkages across units of content. She now used a conceptual orientation for introducing mathematical topics and for helping students understand the meaning of computations. She did not focus attention on the meanings of computations prior to this period. This was a change in her practice from what was observed during the first and second intervention year. After working with a conceptual approach for 3 years, Pamela was able to refine and transfer the strategies she used in fractions and decimals to other units of content. These strategies were used in the post-project years to teach the understanding of computational procedures. The following observation illustrates Pamela's conceptual approach as she illustrates the meaning of subtraction of mixed numbers.

Ms. Kaye, "Turn your papers over and do these three problems for me."
She writes the following problems on the chalkboard.

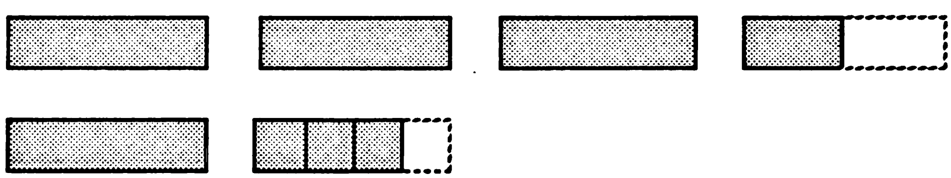
3 1/2 - 1 3/4 =　　　　5 - 2 1/4 =　　　　3 7/8 - 2 =

Ms. Kaye waits for the students to solve the problems.
Ms. Kaye, "I noticed many of you did this." She rewrites the first problem on the chalkboard:

$$\begin{array}{r} 3\frac{1}{2} \\ -\ 1\frac{3}{4} \\ \hline \end{array}$$

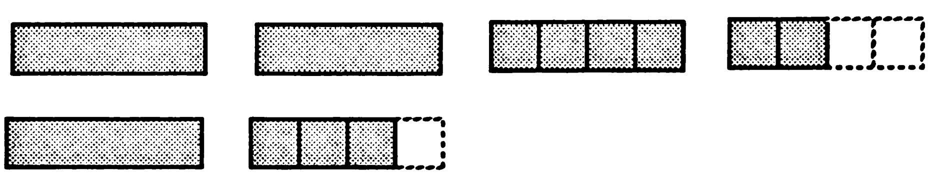
Ms. Kaye, "What could you do to solve the problem?"
A student, "With pictures you could do the same thing."

Ms. Kaye draws the following:



Ms. Kaye, "You can't subtract three-fourths from one-half so you have to change the one-half to fourths. Then you can subtract one and three-fourths from that."

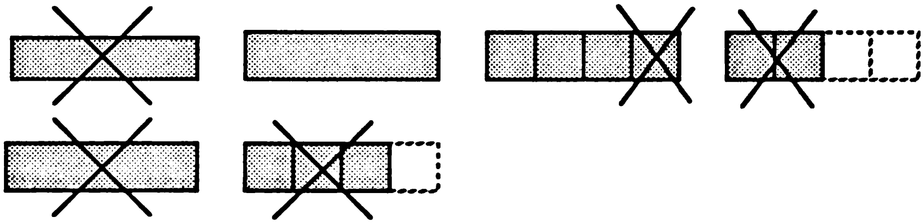
She draws the following:



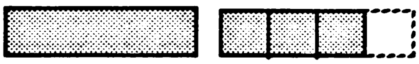
Ms. Kaye asks the students to solve the three problems again using pictures.
She waits for the students to finish.
Ms. Kaye, "The first answer to the first problem looked like this."

She draws the following:

3 1/2 - 1 3/4 = 1 3/4

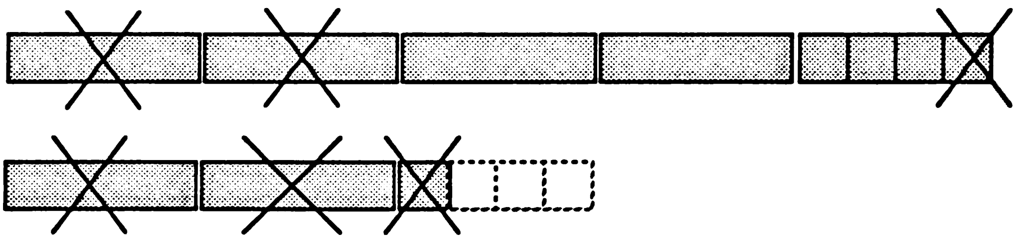


The answer is:

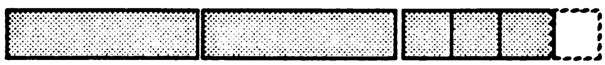


Ms. Kaye, "The answer to the second problem looked like this."

5 - 2 1/4 = 2 3/4

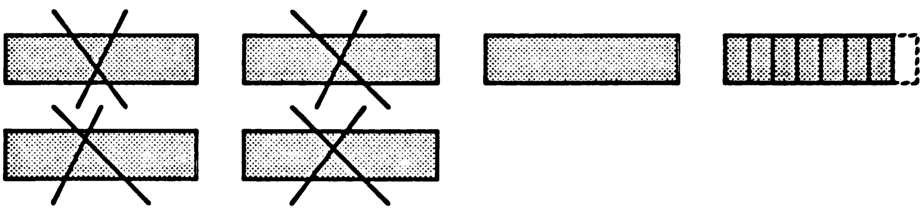


The answer is:

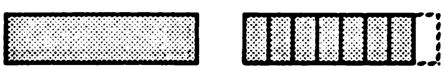


Ms. Kaye continues, "The answer to the last problem looks like this."

3 7/8 - 2 = 1 7/8



The answer is:



Pamela used the strategy of asking the students to represent computational operations with pictures for the fraction, decimal and percent units. She found using pictures and symbols together and the part/whole idea of rational numbers helped the students establish conceptual linkages between these units. Although Pamela thought about using this strategy for teaching other content during the second intervention year, it did not become integrated into each unit/topic until the post-project years.

The mathematical orientation of Pamela's general mathematics classes improved. In the first year, Pamela changed the orientation of only a few units (fractions and decimals). In the second intervention year, she changed the orientation of additional units (e.g., percents). By the post-project years she began linking these conceptually oriented units together with common mathematical ideas. She changed the orientation of her instruction in a limited way at first and then used those changes as the foundation for additional improvements in following years.

The classroom consultant worked with Pamela four times during the intervention years to help her change the mathematical content. The assistance he provided significantly helped change her instructional orientation. He planned with her conceptually oriented lessons and provided her with feedback. Appendix L contains a description of a consultation session between Pamela and the consultant in which the focus was on improving the mathematical content and the development of fraction concepts. Included in this appendix are segments from the planning sessions, observational vignettes and the consultant's feedback to Pamela after the lesson ended.

From Drill and Practice to Nonroutine Assignments

The kinds of tasks Pamela selected for the students improved over the years. During the pre-intervention year, drill and practice excercises were assigned daily from the textbook (Stein, 1974). By the second intervention year, Pamela chose tasks from a variety of sources (e.g., supplemental workbooks, activity books, problem-solving materials) and no longer assigned a textbook. These assignments reinforced a mathematical concept in a nonroutine manner.

One of Pamela's improvement goals for the first intervention year was to find ways to encourage more student involvement in the mathematical tasks by incorporating different materials and using manipulatives. The following observational vignette from this period illustrates the kind of task she selected.

> Ms. Kaye tells them, "Ladies and gentlemen, I want you to take your fraction pieces out and take one of the halves and see if you can make another half by combining two other pieces. Use two different pieces and see if you can make a half. See how many different combinations you can get to make one-half."
>
> Ms. Kaye writes on the chalkboard:
>
> combine: 1 1 + 1 1 1 = 5
>
> combine: 1/3 + 1/6 = 1/2
>
> Ms. Kaye tells the students they may make other fractions than one-half (such as fractional equivalents for one-fourth or one-third).
>
> Ms. Kaye tells several students to go to the chalkboard and record their results. The students write their answers and sign their names:

Bob	1/6 + 1/8 + 1/8 + 1/12 = 1/2
Sue	1/12 + 1/6 + 1/8 + 1/6 = 1/2
Nancy	1/3 + 1/8 + 1/12 = 1/2
Fred	1/12 + 1/12 + 1/12 + 1/12 + 1/12 + 1/12 = 1/2
Steve	1/3 + 1/6 = 1/2
Randy	1/12 + 1/12 + 1/12 + 1/12 + 1/6 = 1/2
Ken	1/4 + 1/6 + 1/12 = 1/2
Dawn	1/3 + 1/8 + 1/12 = 1/2

The students used their fraction pieces and worked on this task for several minutes. This task was clearly different from those observed during the pre-intervention year. First, the use of manipulatives to solve the problem helped the students understand the concept of equivalent fractions and addition of fractions. Second, the open-endedness of the task allowed students to obtain many different results. Third, the task was interesting enough to capture the attention of all the students. Finally, the students were given a single problem instead of many similar problems to solve.

At the start of the GMP intervention Pamela thought her students would resent the use of manipulatives as childish. After teaching the fraction unit in the first intervention year with manipulatives (fraction circles cut into pieces), she found the students choosing to use the pieces throughout the unit on every assignment. Because of the success the students had on the computational test, Pamela decided to include more tasks using manipulatives in other mathematical units.

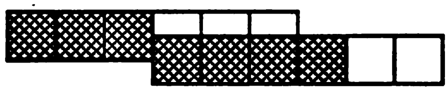
In addition to using of manipulatives, Pamela began to think differently about the kinds of tasks the students were expected to do. In the following interview segment she reflected on a worksheet from the fraction unit she assigned.

> The application worksheet was a specific thing that I changed. I think that it did keep the students engaged. They really began to talk to one another about that. It was not just a worksheet of thirty problems to do.
>
> And for me as a teacher, it was fun to find something that was a good application that worked out well and gave me a new way of looking at how to teach fractions.

By the end of the second intervention year, Pamela began to look for materials to use in her general mathematics classes that she found interesting, challenging and conceptually oriented. She also modfied many of her previously used worksheets by reducing the number of problems and asking students to illustrate their results along with writing the numerical answer.

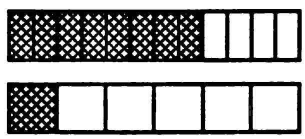
During the post-project years, Pamela found and used commercially prepared fraction and decimal materials (Fraction Bars and Decimal Squares, Seymour Publications, 1987). She felt these materials represented the kinds of tasks she wanted her students to do. The materials included work with manipulatives (fraction bars and 100-square grids) and activities that developed the understanding of fractions and decimals and the arithmetic operations. Examples of fraction bar activities are included below. These commercially prepared materials gave the students practice with familiar content in a different way.

> (p. 53) Write the missing fraction for each pair of bars, and then write the sums using mixed numbers.

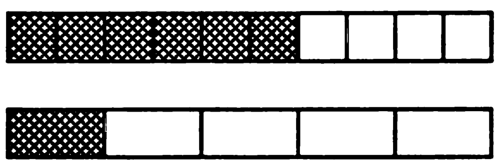


$$\frac{3}{6} + \ldots\ldots = \square$$

> (p. 104) The fraction bars show that 1/6 "fits into" 8/12, 4 times.



> Use the fraction bars to compute the quotients.

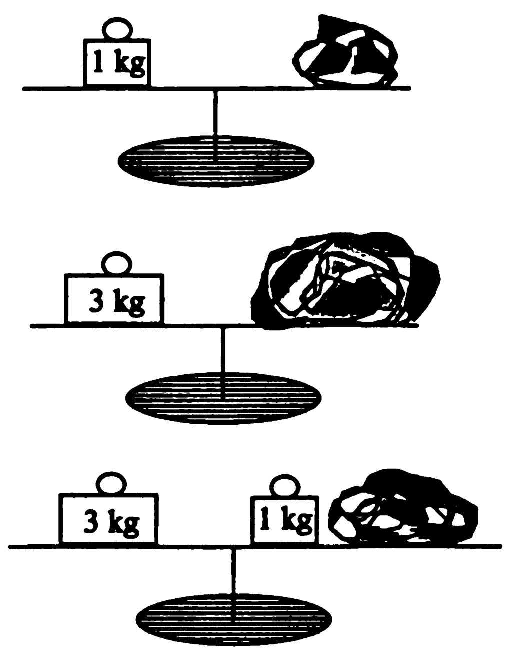


$$\frac{6}{10} \div \frac{1}{5} =$$

The students worked with manipulatives, drew pictures and related them to the symbols to represent fraction, decimal and percent ideas. In addition, Pamela used three activity-based and concept-oriented units, Probability, Similarity and Equivalent Fractions and Factors and Multiples, from the Middle Grades Mathematics Project (Lappan, G., Fitzgerald, W., Winter, M.J., & Phillips, E., 1986), to provide students with computational practice in whole numbers, fractions, decimals and percents. She was convinced the students were getting more practice in computing than ever before, but in a way that was challenging and enjoyable.

Pamela included tasks in new content areas such as problem-solving that were interesting for the students. Problem solving was a new unit introduced at the end of the first intervention year during the second classroom consultation period. At that time, the GMP staff provided her with a teacher's guide to problem-solving strategies (Dolan & Williamson, 1983) with student worksheets. She used this material systematically during each of the post-project years. The observational vignette below depicts one problem from this unit (p. 264).

> Ms. Kaye tells the students, "Your last question for the day--you have a balance scale and you are trying to weigh this meat. You have only four weights to work with--a 1 kilogram, a 3 kilogram, a 9 kilogram, and a 27 kilogram weight. You have bundles of meat ranging from 1 to 40 kilograms. You have to weigh them with just these four weights."
>
> Ms. Kaye demonstrates for the students how to weigh a 1kg., a 3kg., and a 2kg. bundle of meat.
>
> She writes on the chalkboard:



The tasks Pamela selected during the post-project years had changed from those in the pre-intervention year. They were nonroutine and concept oriented. Classroom observations during the post-project years indicated the students were interested in the tasks and working on their assignments.

During the pre-intervention year, Pamela used grades to motivate students to complete their assignments. The students were given 5 points for just completing their assignments. By the end of the post-project period, it was clear the students were self-motivated by the mathematical tasks themselves. Pamela noted in an interview,

> In the problem-solving situation they get interested in the tasks and say, "Gee, I want to really stick with this until I get it." They do tend to perservere and, as you know, perserverence is not one of the general math students' main traits.

By the end of the post-project years, Pamela believed one way to improve general mathematics classes was to change the mathematical tasks. She found tasks that encouraged students to work longer and with more involvement than activities she selected in previous years.

From Arithmetic Reviews to Mathematical Explorations

Pamela's curriculum for general mathematics improved over the years. As with the changes in the mathematical orientation and tasks, this change was gradual also. The curricular changes made in the first year of the intervention served as the foundation for changes made in following years. The listing of mathematics topics she covered in her general mathematics classes for the pre-intervention year, two intervention years, and the last post-project year indicated the changes in mathematical orientation and in the content discussed previously (Appendix M). This section examines Pamela's curriculum over the years of the study and describes the nature of the changes.

During the pre-intervention year, Pamela's general mathematics curriculum consisted of computational reviews. This was the mathematics content she believed was important for her students. The following is a list of the content taught during this year.

<u>General Mathematics Curriculum for the Pre-Intervention Year</u>

Review of Whole Numbers:
Add, Subtract, Multiply, Divide

Review of Decimals:
Add, Subtract, Multiply, Divide

Review of Fractions:
Add, Subtract, Multiply, Divide

Percents:
Computation

Geometry:
Measurement, Shapes, Area Formulas

The students spent the year practicing the same arithmetic computations they had in previous years. Decimals were taught before fractions because, as Pamela maintained, decimals were easier to teach than fractions, "...they are just like whole numbers." All she had to do was teach the students the rules for placing the decimal point in the answers and then they were just like the whole numbers. Although Pamela included a unit on percents, she usually gave it up after the second day. She believed it was too hard to get the students to compute successfully with percents. Pamela said she tried to include some work in geometry if time allowed at the end of the school year.

Pamela changed her general mathematics curriculum during the first intervention year by adding new units of content and modifying other units taught previously. However, the changes she made reflected her intention at that time to simply add more topics to her curriculum. There was no indication Pamela restructured the curriculum to establish linkages between various units. The following is a list of topics Pamela taught during the first intervention year.

General Mathematics Curriculum for the First Intervention Year

First Semester

1 wk. Orientation
Consumer Math
1 wk. Measurement
3 wks. Using Formulas
2 wks. Decimals
3 wks. Calculators
5 wks. Fractions
1 wk. Percents
Concepts
Computation
Problem Solving
3 wks. Coordinate Graphing
Semester Exams

Second Semester

1 wk. Orientation
2 wks. Coordinate Graphing
Fractions
Decimals
3 wks. Probability
4 wks. Introduction to Algebra
3 wks. Statistics, Graphs, Charts
5 wks. Consumer Use of
Mathematics
2 wks. Review and Testing

During the first intervention year, Pamela eliminated whole number review from her curriculum. New units on consumer math and formulas were added during the second semester. Decimals were still taught before fractions. However a unit on calculators was added between the units on decimals and fractions. The calculator unit was used to summarize the decimal work the students had just completed. During this year, Pamela taught several new units of content, but the focus was not on mathematical concepts. Fractions, taught with the consultant, was the only concept-oriented unit. Pamela thought she taught percent concepts in her curriculum. However, the focus was still more computational. The curriculum of the first intervention year was a collection of various unrelated mathematical topics, but it was improved over previous years.

> I will also take a strong look at putting fractions, decimals, and percents together. There is no way students can do percents adequately until they have a grasp of both fractions and decimals.
>
> Probably the biggest thing that needs to be done is to do an overall plan and then try and figure out where things slide in from there--fractions, decimals, and percents really do belong all together in one big unit.
>
> I think fraction concepts are very important, I use them in the Probability and Simlarity units.
>
> I still think computation is important, but not as important as before. The concepts are real important because they will take you into decimals, percents, and so forth.

At the end of the first intervention year, Pamela believed it was important to find ways to link fractions, decimals and percents together in one unit instead of three. However, she wasn't able to come up with a way to accomplish that task. Much of Pamela's curriculum during the first intervention year remained computationally oriented even though she thought the teaching of mathematical concepts was important. While she thought about finding ways to link units of content together, her curriculum still reflected the teaching of mathematical topics separately.

Pamela's curriculum in the second intervention year contained additional units of new content and modified units of content taught previously. In addition, she included two concept-oriented units in the second semester, one on probability and the other on similarity. There are interesting contrasts between the general mathematics curriculum of the first and second intervention years. For example, in the first intervention year Pamela spent only 1 week on percents and included concepts, computations and problem solving. In the second year, she spent 1 week on percent concepts alone and did not teach percent computations. In the first intervention year she spent 5 weeks on consumer use of mathematics and in the second year this unit was dropped. In Pamela's curriculum the second intervention year, fractions were taught before decimals and the concepts of rational numbers developed in this fraction unit were carried over into her decimal unit. The decimal unit was modified to reflect a conceptual orientation using 100 square grids in the same way as fraction bars in the fraction unit. Pamela began the year having students make manipulatives and work with concrete objects.

During the second intervention year, Pamela's general mathematics curriculum reflected more closely her beliefs about teaching general mathematics. There was more evidence of a conceptual orientation throughout the curriculum, and she attempted to organize units of content around a few big mathematical ideas. For instance, the part-to-whole theme was used to link the fraction, decimal, percent and probability units.

<u>General Mathematics Curriculum for the Second Intervention Year</u>

<u>First Semester</u>		<u>Second Semester</u>	
1 wk.	Pretesting	1 wk.	Review
	Tangrams		Fractions
	Pentominoes		Problem Solving Strategy
1 wk.	Areas-- Concept of Formulas		Making Tables
1 wk.	Review Fractions	3 wks.	Fractions
	Concepts and Relationships	5 wks.	Probability Unit (MGMP)
3 wks.	Problem Solving Strategies	4 wks.	Similarity Unit (MGMP)
	Guess & Check	2 wks.	Graphing
	Tables		Coordinate Graphs
2 wks.	Factors & Multiples	2 wks.	Integers
1 wk.	Review Fractions	2 wks.	Algebra
1 wk.	Fractions		Introduction
	Addition and Multiplication	2 wks.	Reviews
4 wks.	Decimals		Concepts of Fractions
	Concepts		Concepts of Decimals
	Relation to Fractions		Concept of Percents
1 wk.	Percents		Problem Solving
	Introduction to Concept		Estimation
1 wk.	Review and Testing		

When Pamela's post-project curriculum is compared to her curriculum of the intervention years, there is strong evidence she continued changing her practice. She allowed more time for the teaching of mathematical topics during this year. She spent 9 weeks on fractions compared to only 5 weeks on this content during the intervention years. She also spent 7 weeks in activity-based, concept oriented units in geometry, problem-solving and number theory. In addition, the Probability and Similarity units were included because they provided new content for the students and additional reviews of fractions, decimals and percents.

During the first post-project year, Pamela found and used prepared materials (<u>Fraction Bars</u> and <u>Decimal Squares</u>, Seymour Publications, 1987) for teaching fractions, decimals and percents. She modified her fraction and decimal units from the second intervention year to include selected activities from these materials. Interestingly, Pamela increased from 2 to 4 weeks the algebra unit she taught at the end of the second

General Mathematics Curriculum for the Second Post Project Year

First Semester		Second Semester	
1 wk.	Surface Area and Volume Making Manipulatives Tangrams	4 wks.	Percents Part-to-Whole Relationship
3 wks.	Problem Solving Strategies Guess and Check Making Tables Finding Patterns	4 wks.	Probability unit (MGMP)
3 wks.	Factors and Multiples (MGMP)	4 wks.	Similarity unit (MGMP)
1 wk.	Fraction Bars Manipulatives Fraction Concepts	4 wks.	Algebra Integers Operations Symbols Formulas
5 wks.	Fraction Concepts Part-to-Whole Relationship	1 wk.	Review Examinations
3 wks.	Decimal Concepts Part-to-Whole Relationship		
1 wk.	Review Examinations		

intervention year. Pamela reflected on the changes she made in her curriculum during the post-project years in the following interview segment.

> I used the probability and similarity units this year because they tied into fractions and decimals and percents.
>
> When I think about the integer and algebra units I recall the success my students had with those.
>
> I never thought of an algebra unit with these students before--all I thought about was add, subtract, multiply, and divide the whole numbers, fractions, and decimals. I had a smattering of everything.
>
> Maybe it does have to do with the concepts, but I think it was just that I had no idea of what was valuable and the fact that I could couch those same skills within some other content.
>
> New math units give me a different way of approaching the content, doing the problem solving unit, doing the probabilty unit, doing the similarity unit. To me, these units provide a different approach that will help students look at mathematics in a way they had not thought of before.

The mathematical content of Pamela's general mathematics class had changed from the beginning of this study. Many new units were added and units taught previously were revised. New mathematical content served a two-fold purpose: (a) it provided the students with computational practice in a new context; and, (b) it was so interesting and challenging for them they worked harder and longer on their mathematics assignments.

The curriculum in Pamela Kaye's general mathematics class at this time was more conceptually oriented and unified than it had been in the past. She now had a yearly plan for teaching general math and a rationale for where particular units belonged in the curriculum. This was not observed in her curriculum plans prior to the post-project period.

Changing the Mathematical Content: A Summary

The patterns of change identified in Chapter IV were discussed in this chapter with respect to the improvements made in the mathematical orientation, tasks and curriculum over the 5 years of the study. The qualitative data reveals that changes in all three areas did occur and continued to occur over the years. In addition, during the intervention years the first changes that were made provided a foundation for further changes as Pamela reflected on their outcomes and success. Finally, the evidence from the qualitative data shows Pamela thought about changing these areas before she was able to do so in practice. By the post-project years, the mathematical content of the general mathematics class was concept oriented; the mathematical tasks were activity-based, interesting and engaging; and the curriculum was well planned and linked together by unifying mathematical ideas.

The improvements in the mathematical content were reflected in improved mathematical achievement and attitudes of students. These results were reported in Chapter IV. In addition to this data, comments written by Pamela's students reflected their awareness of increased mathematics learning during the year. This data was collected during the second intervention year (1983-1983) of the GMP. These comments [sic] were in response to the following question asked by Pamela, "Tell me what (if anything) new you learned this year."

> Last year I didn't understand fractions but now I understand them perfictly. I understand everything else very good also.
>
> This year I learned how to add, subtract, multiply and divide fraction. I never really understood them before.
>
> I think some of the things I understood a few years ago I had forgotten and now I remember them more easily.
>
> Well I didn't quite understand percents but I learned a little. More than I did before.
>
> I learned how to ÷ and x fractions. I understand everything a hole lot better.
>
> I learned how to do the adding and subtracting of fractions better & didn't know how last year. Decimals are a breeze now. Angles & area are easier. I can do them better than last year.
>
> I did learn more about fractions decmels and other things. I thought you were the best teacher who taught Math that I had over the years. I learned more in your class then I did in any one elses math class. I did more things in your class. In 7th and 8th grade we did not do enough math.

The comments by Pamela's students indicated they thought they could do mathematics better this year and they finally understood what they were doing. None of the students attributed their understanding of or ability to do mathematics to their increased understanding of the mathematical concepts, but that is essentially what was at the center of their success. It was the changes in the orientation, tasks and curriculum that led to her students understanding of mathematics.

At the start of the GMP, Pamela believed her general mathematics students were different fom her algebra students. She thought general mathematics students had neither the desire nor the ability to understand mathematical concepts. At the end of the study she thought differently. She found her general mathematics students wanted to learn and wanted to be successful in mathematics. She now knew they were as capable as her algebra students and she now knew how to teach them.

Chapter VI

PATTERNS OF CHANGE IN COMMUNICATION

Communication in Pamela Kaye's classes improved as a result of her instructional changes. This chapter examines qualitatively the ways Pamela changed mathematical communication in her general mathematics classes. Two findings from the analysis of the data are used to frame this discussion. These findings are: the quality of mathematical communication improved and the quantity of that communication increased. These findings are supported by the patterns of teacher change identified in Chapter IV: (a) instructional improvements occurred over time; (b) at the beginning of teacher change, instructional improvements were implemented inconsistently; and (c) changes in thoughts about improving instruction occurred before changes in practice.

The Quality of Mathematical Communication Improved

Communication about mathematics between Pamela and her students about mathematics included three components: (a) interactions regarding the mathematical content; (b) discussions about the content; and, (c) the nature of the language used during these interactions and discussions. This section discusses the changes in these components before, during and after the GMP intervention. Each period is discussed in a separate subsection.

The Quality of Communication During the Pre-Intervention Year

The observational data during the pre-intervention year showed few, if any, interactions and discussions between Pamela and the students. The observed communication was minimal and lacked mathematically precise language. Pamela did not ask students questions that required thoughtful answers nor did she push their thinking toward mathematical concepts. Pamela's lessons consisted of telling her students their assignment for the day and then showing them how to work the problems. She did not communicate with the general mathematics students in the same manner she did with her algebra students. Pamela and her algebra students discussed mathematical ideas as well as the problems they were assigned. She did not do this with the general mathematical students, believing they were not interested in learning mathematical ideas.

Pamela not only assumed general math students just wanted to know what to do, but also that they were rather limited in their ability to understand mathematical concepts since they still had trouble computing. Regardless of the reasons, Pamela chose to limit the amount of communication in her general mathematics classes. The observational vignette below depicts the typical pattern of communication between Pamela and her students during this pre-intervention period.

> Ms. Kaye, "We will continue working the problems on pages 189 and 190 today. I would caution you, when you get to three fractions it would be easier to work with two fractions first and then with the third one later."
>
> She writes the following example using whole numbers to illustrate the addition of 3 integers on the chalkboard:

$$\begin{array}{r} {}^{+}4 \\ +\ {}^{-}3 \\ \underline{{}^{+}7} \end{array}$$

> Ms. Kaye, "Combine the plus [+] 4 and the plus [+] 7 first to get plus [+] 11. After you do that then you add the minus [-] 3."
>
> She shows the students on the chalkboard:

$$\begin{array}{r} {}^{+}11 \\ +{}^{-}\underline{\ 3} \end{array}$$

> Ms. Kaye, "I don't want to do anymore with you here. I will be around to you and work with you individually."

Pamela hurried through an explanation of how to add 3 integers and told the students she would work with them individually. She used this strategy in most lessons to limit the amount of time she worked with the whole group because Pamela believed students who had problems with the task would understand the procedure better if she worked with them alone. The students who already knew how to do the problems didn't need further explanations.

The lack of precise mathematical language in this vignette was typical of Pamela's communication with the general mathematics students. The integers $^{+}4$, $^{+}7$, and $^{-}3$ were not referred to as "positive 4," "positive 7" and "negative 3". They were called "plus 4," "plus 7" and "minus 3." As a consequence, many students were unable to think of integers as directed numbers because they confused integers with the operation signs of addition or subtraction. The lack of mathematically precise language limited the students ability to communicate effectively about mathematics with one another and with Pamela. For example, a student who is told .05 is "point zero five" might think of this number as consisting of three distinct and unrelated parts: a decimal point, a zero and a five. On the other hand, a student who is told .05 is "five hundredths" is likely to relate this to the fractional equivalent 5/100 and to the idea that .05 represents 5 parts out of 100. In the pre-intervention year, Pamela avoided mathematically precise language with the general mathematics students because she believed they would not understand the mathematical terms. She also felt they would have little use for these terms after they got out of school. Pamela thought it was easeir to explain in common language how to work the problems on the assignment.

When Pamela questioned the students, she typically asked a question in the form of an unfinished sentence. In other words, she asked a question omitting the last word (a kind of "fill in the blank" question) and then waited for a student to complete the sentence with the missing word. For example, Pamela asked, "Tim, the larger fraction is ___?" Tim responded with, "three-fourths." Students were never required to give their answers in complete sentences. Tim was never asked to respond with, "The larger fraction is three-fourths." This questioning strategy encouraged students to think of mathematics as a set of fragmented, single-word answers instead of as complete thoughts.

Another questioning technique Pamela used was to ask questions and answer them before students could respond. She frequently used this during direct instruction when she demonstrated computational procedures. Even when she called on particular students to answer questions, Pamela ignored student responses and answered the questions herself. She thought asking questions promoted student interaction and participation. Questioning students could have been a successful strategy if Pamela allowed the students to respond and then listened to their answers. However, this was not observed. Students chose to let Pamela answer her own questions instead of volunteering answers themselves. Rather than encouraging interaction, as she thought, this strategy discouraged it. Why should students attempt to answer questions when they knew Pamela would answer the questions herself or ignore their answers?

During the pre-intervention year, Pamela did not think of communication as an interaction or a discussion with the students. Communication meant that she demonstrated a procedure. She did not expect nor did she encourage students to interact with her, sharing their thoughts about demonstrated procedures and practice problems. Students did not expect questions requiring them to think about mathematical ideas nor were ideas taught in Pamela's general mathematics classes--only procedures. The computational nature of the general math curriculum undoubtedly contributed to the lack of communication between Pamela and her students. The content wasn't very interesting or challenging. There was nothing new to be learned or discussed--there were just procedures to be followed.

It is obvious then Pamela did not expect or encourage students to initiate questions or discussions. She judged her students' understanding of the content by their facial expressions. When they frowned, she knew they did not understand; when students were watching, she assumed they understood.

The pre-intervention general mathematics class was characterized best by a profound lack of mathematical communication between Pamela and her students.

Changing Mathematical Communication During the Intervention Years

During the intervention years, Pamela implemented strategies to encourage more interaction between herself and the students. She asked better questions and listened to student responses. As a result, her intervention students responded more than her pre-intervention students. In the first intervention year, while many of Pamela's questions still required single-word answers, they were more focused on mathematical ideas. In this year, Pamela did not systematically try to use mathematically precise language with her students. The following vignette illustrates the typical instructional interactions between Pamela and her students in the first intervention year. In this vignette, the students are interacting with Pamela and participating in the lesson more than the students did in her pre-intervention class.

> Ms. Kaye tells the students, "Let's say you had a test at the end of this hour and I gave you twelve problems. Let's say you missed one of those problems. What percent would you have missed?"
>
> Without waiting for the students to try to solve the problem Ms. Kaye says, "I want you to figure out the decimal and the percent for one-twelfth."
>
> The class starts working on the problem.
>
> Ms. Kaye walks to the front of the class and asks, "What do you have to do to solve this problem?"
>
> Steve raises his hand.
> Ms. Kaye says, "All right, Steve."
> Steve responds, "You would divide the twelve into the one."

Ms. Kaye says, "The fraction one-twelfth means one divided by twelve. Now Steve, give the rest of the class a chance to figure the answer out."

Steve says, "Yes, but I know it is going to be eight point three percent."
Ms. Kaye, disappointed at Steve's giving away the answer so quickly, says to the students, "Steve says this is the number that I missed on the test, eight point three percent. The number I got right would be three point seven percent. He says that when you add them up you get ten percent."

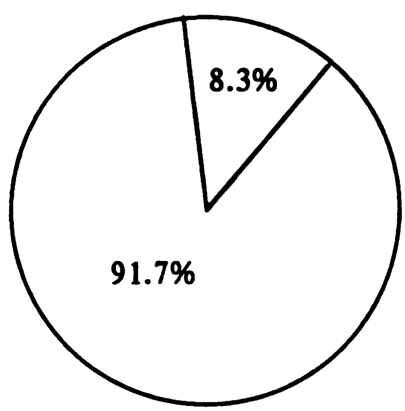
Ms. Kaye gave the students the wrong answer on purpose to get them to think about the difference between 10% and 100%.

Steve tells Ms. Kaye, "No, I didn't. The answer would have to be 92.7% because you have to get to one-hundred."

Steve subtracted 8% from 100% and then subtracted the .3% from 1% to get .7%. He then added the .3% to the 92% to obtain the wrong answer of 92.7%
Ms. Kaye does not respond to Steve's wrong answer.

Ms. Kaye asks, "So then how many percent did you get right?"
Mary, "The rest of the pie."

Ms. Kaye draws a circle picture of Mary's description on the chalkboard:



Ms. Kaye continues, "Mary, am I going to put ninety-one point seven in my grade book?"
Mary tells her, "No, you will put ninety-two percent in your book."
Steve interjects, "I was right, 92%."

Pamela's interactions with the students in this vignette obviously were different from those observed in the pre-intervention year. She asked questions that made students think about answers and she pushed them to give her more complete responses, although she did not require them to use mathematical language. She used a wrong answer to get the students to think about why 10% was incorrect and 100% correct. However, Pamela's questions only encouraged students to give her responses she wanted. For example, instead of asking, "Mary, am I going to put 91.7 in my grade book?" Pamela could have asked, "What grade would I put in my grade book?"

By the end of the year, the nature of the interactions between Pamela and her students was different. Pamela's questions were more open-ended and students were more willing to share their conjectures and thoughts about how to solve problems. The consultant suggested Pamela improve communication by: (a) encouraging students to talk more about how they solved problems; (b) using student groups to work on mathematical tasks; and (c) presenting interesting and challenging content and tasks. All three suggestions resulted in improving the quality of communication. The next observation illustrates how Pamela implemented the consultant's suggestions. Appendix N contains an additional example of a consultation session focused on improving mathematical communication on a fraction unit.

The students are asked to solve the following problem in their groups:

> It takes 12 minutes to cut a log into 3 pieces.
> How long does it take to cut a log into 4 pieces?

After the groups worked on the problem a few minutes Pamela initiated a discussion of the problem.

Ms. Kaye, "Okay let's try this third problem: It takes 12 minutes to cut a log into three pieces. How long does it take to cut a log into four pieces?"

Jeff says quickly, "Sixteen." Jeff divided 12 minutes by 3 pieces to get 4 and then multiplied 4 by 4 to get 16. Many students have made the same mistake as Jeff.

They should have divided 12 minutes by 2 cuts to get 6 minutes per cut and then multiplied 6 by 3 to get 18.

Ms. Kaye, realizing Jeff's wrong response, asks a question to help him correct his mistake."Is making a model going to help you?"
Alice, Mary, and Jeff all tell her, "No."

Sue asks, "Are there 3 pieces or 3 cuts?"
Ms. Kaye tells the class, "Sue just raised a question, do we make three pieces or do we have three cuts?"

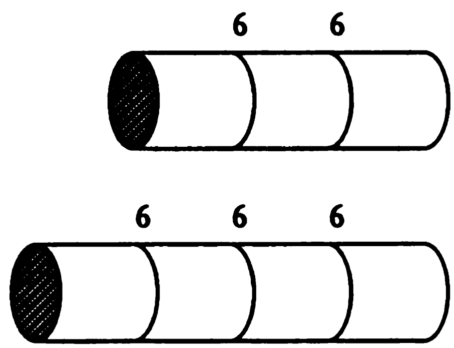
Alice draws a picture of a log on her paper and made two cuts into it realizing that there were only two cuts needed to give three pieces.

Don says, "Well I got the answer that three goes into 12 four times so 4 is the answer."

Jeff (from the other side of the room) responds to Don saying,
"Look, Am I going to have to do it for you?"
Jeff jumps out of his seat, goes across the room to Don and says loudly, "I'll draw a picture for you!"
Jeff takes Don's pencil and draws on his paper a picture of a log and makes two cuts in the log resulting in three pieces.
He then puts Don's pencil down and says, "Boy! Are you dumb! God gave a goose more brains than you got!"

Ms. Kaye says to Jeff, "Well, show us up here what you did."

Jeff goes to the front of the room and draws a log on the chalkboard, makes two cuts in it and above each cut puts a six. Then Jeff adds another piece and Ms. Kaye asks the class, "What did these guys do to solve the problem?"



Don says loudly, "Guess and check!"
Ms. Kaye frowns at him and he realizes he made the wrong response...then he says, "I found a pattern!!"
Ms. Kaye frowns again and says to the class, "What did Jeff do to show you?"
Don says, "He drew a picture."

When Pamela asked for responses to the problem, several students gave the wrong answer. She waited a moment and another student asked her for clarification of the question asking, "Do we make three pieces or do we have three cuts?" This caused most of the students to recalculate their answer--except for Don, who still couldn't understand the difference between "cuts" and "pieces." When Don explained his wrong answer, Jeff became frustrated at Don's inability to understand. He jumped out of his chair, leaped over two rows of desks to Don's desk, took Don's pencil out of his hand, and drew a picture of a log cut into three pieces. When Jeff returned to his desk Pamela asked him to go to the chalkboard and illustrate his answer for the students.

This vignette represents the nature of the interactions by the end of the first intervention year. The kinds of communication patterns in this class were considerably different from those observed a year earlier. In the pre-intervention class, the students did not ask questions about the content, they did not actively participate in class discussions, and they did not appear nearly as interested in the tasks or problems as the students did in the above vignette. These students were on task, interested in the problem, and involved in a discussion of the solution. Those who did not have the correct answer wanted to know how and why they made a mistake.

The students in Pamela's first intervention year class changed over the course of the year. When they entered Pamela's general math class in the fall they resembled the general mathematics students in Pamela's pre-intervention class. They did not interact with each other or with Pamela about mathematics. They expected Pamela to tell them what to do and then give them the assignment. They did not expect to spend time talking about mathematical ideas and problems. Over the year their behavior changed. This change is the result of the strategies Pamela used to improve the patterns of communication. She ased better questions, listened and responded to students' statements, and provided students with better explanations.

At the end of the first intervention year, Pamela reflected on the changes that occurred in her interactions with the students across the year. She was able to identify strategies that promoted interactions and was aware of the effect of these changes on her role as a teacher.

> By the time we got into the third week or so of the problem-solving unit I would say, "Oh, now that's an interesting way of looking at that problem. Tell me more about what you're doing."

> It made it even more interesting for me as a teacher to sit down and talk with a group of students about how they were solving a problem.
>
> My role as a teacher has changed considerably. We could actually discuss the problems together. Before this year I was just telling them. Now they were explaining to me how they solved the problems and now I could respond to them.

The new content and the use of manipulatives during the year contributed to improving the mathematical communication. The content (e.g., problem solving and probability) provided opportunities for students to work together in groups actively solving interesting and challenging problems. Pamela's conceptual orientation in units such as fractions gave students the chance to discuss the concepts underlying familiar mathematical content.

Pamela continued to work on improving communication with her students throughout the second intervention year. During this year, she continued to work on ways to improve her questioning techniques, and implemented strategies to promote more discussions and the use of mathematical vocabulary.

At the start of the second intervention year, her general mathematics students came to the class with the same expectations, attitudes and abilities typical of her general math students during the pre-intervention years. However, by the end of the second intervention year, they changed more than her students in the first intervention year. They initiated discussions and questions and appeared to have more confidence in their ability to understand and to work mathematical problems. At the end, Pamela reflected on the changes in communication that occurred.

> One of the things I'm doing is stressing vocabulary much more this year than I ever have before. When I did the problem-solving unit, one of the biggest difficulties the students had was trying to understand what the problem was about because they didn't know the vocabulary. So we spent a lot of time talking about the meaning of the terms.
>
> The students have begun to interact with me and each other. There was a lot of communication even though it was all in a very fun kind of way. There was a lot of discussion going on.

> It's a fun class. I'm enjoying it very much. I'm pleased with them. There's much more conversation going on.

Pamela believed her efforts to improve communication in the classroom were successful when she saw the progress her students made since the start of school in the quality and quantity of their communication about the mathematical content. She was pleased that students in general mathematics classes could actually enjoy mathematical interactions with one another and with her. In the pre-intervention year, she did not think they were capable of this level of communication or had any interest in mathematics.

Interestingly, an outcome of the improvements Pamela made in questioning, responding and explaining was that these strategies provided students with a model for communication which they started to mimic by the second semester. The students were now observed questioning and responding to her in ways that reflected her style. For example, Pamela started the year by asking students such questions as, (a) "Why do you think this procedure works?" (b) "Will this procedure work for all examples?" (c) "Can you show me another way of thinking about this problem?" (d) "Why is this true?" By the end of the year, her students were asking Pamela these same questions and asking/pushing her to explain an idea/procedure further or in a different way.

By the second semester, students in Pamela's general mathematics classes expected to be asked a lot of questions and they were expected to give thoughtful answers. The effect of the changes Pamela made in communication was observed on the first day of the second semester. There were 10 new students to the class. Pamela started to review fraction concepts using the mode of questioning she established in the first semester. A new student looked puzzled when Pamela asked, "How do you think about the sum of one-half and one-third?" and a student from the first semester responded, "I would draw a one-half fraction circle and a one-third fraction circle and then put them together." The new student turned to another student and asked, "What is going on?" The student shrugged his shoulders and said, "Oh, that's just the way she does things in here--you'll get used to it." The students from the first semester knew their class was different from other mathematics classes they had taken in the past. In this class their teacher expected them to think, to ask questions and to share their ideas and conjectures. Pamela noticed the difference between her first semester students and the new students.

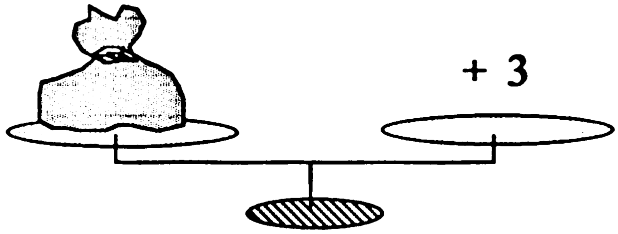
> I think it is the most important thing we can do--to get students talking about mathematics. I found it is real easy to fall back into telling them or asking them right/wrong questions. It got to the point that as I tried to do more discussion my students were anticipating my "How did you get that?" questions after a while.
>
> I noticed that particularly at the start of the second semester when the group of new students came in to the class. As I began to ask the new students those questions they just looked at me with an expression that said, "What do you mean. how did I get that?" I found my old [first semester] students responding almost snobbily with, "I got it this way..." and then going through their routine of explaining their answers.

Pamela mentioned how her students from the first semester responded "snobbily" as they showed the new students how to discuss mathematics. The first semester students seemed to be "showing-off" for the new students, informing them that things were different in this class. This difference seemed to be a source of pride for the first semester students.

During the second intervention year, Pamela combined the use of mathematical vocabulary and illustrations to communicate mathematical ideas more effectively. This strategy enhanced their understanding of the mathematical content because it provided them with an alternative learning strategy. The following vignette describes how Pamela explained the solution to an algebraic problem.

> Ms. Kaye, "All right, there are two ways of approaching this problem. Here is my sack with an unknown r in them. If I take six out then what do I have to do to balance my scale again?"

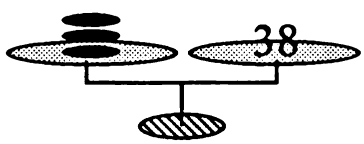
$$r - 6 = {}^{-}3$$



The students, "You have to add six."
Ms. Kaye, "The problem, r minus six equals negative three, is about the hardest kind of problem to do. If I had, X minus a negative four equals negative two, then I would have to take four out of my sack--but then I would have to put four back into both sides to balance it, so it is a very hard problem."

Ms. Kaye, "So if I give you seven X plus three equals thirty-eight what you are to do is to solve for X."

$$7x + 3 = 38$$



Randy, "Boy! Am I ever lost! Is the X an X or does it mean multiply?"

Ms. Kaye, "That's a good question -- one of the things that we have done all along is that when we start algebra problems we don't use an X we use a what?"
Randy, "I remember, a dot. I forgot about that dot when I asked the question."

Ms. Kaye refers to the drawing she has made on the chalkboard, "I took seven Xs all together and I added three extra chips and I kept adding chips on the other side until I got thirty-eight and it balanced."

Ms. Kaye continues, "Subtract three from both sides and what do you have left?"
The students, "Seven X."
She asks, "What's on the right?"
Students, "Thirty-five."
Ms. Kaye, "So, Miss Dyer what's the answer?"
Mary, "Five."

Ms. Kaye, "So if we put it back we then have seven times five plus three is thirty-eight."

Randy, "I know these now! I must be getting smarter!"

Ms. Kaye tells the class, "I'll tell you something you have done in two days what it usually takes me weeks to do with the algebra students!"

Pamela Kaye used balance scales to help the students understand that equations consists of two equal algebraic sentences and that in solving equations that equality (or balance) needed to be maintained. Algebra was new content for the students and a new unit for her to teach in general mathematics. Her explanations included pictures and models to help illustrate the problems and to provide a better understanding of the topic. The strategies Pamela used for getting students to explain their answers, to talk about their thinking, and to ask questions encouraged more interactions and greater participation. In addition, these strateges encouraged students to ask Pamela questions if she did not explain or answer questions to their satisfaction.

At the end of the second intervention year, Pamela asked her general mathematics students to write what they thought about the class and what changes they thought she should make for the next year. Many students wrote comments that indicated their awareness of how much they had learned over the year and attributed this to the kind of communication that had been established in the class during the year. Some comments written by the students [sic] are included as follows:

> Just about everything you taught us I have done before but out of a book. A book just tells you how to get the answer but not why. You should start more classes like this one but in younger grade levels.
>
> I leard a little more about fraction and reduseing them. And you explane things other teachers give you a book and a page number and tell you to read the directions.
>
> I new most all of the things we did But neve understood it real well. You made things clear and helped me all of the time i learned more this semester than i did all last year
> yours turley
> guess
>
> I was always lost on decimalls but in this year they all been explained to me in a language that I can understand.
>
> I learned what GCF & LCM were. How to read decimals. How to change decimals into fractions and %. How to +, ÷, x, -, fractions. I neve understood any of this, that was why I hated math. our teacher would give us page numbers and say good luck. It is more if someone explanes it to you. The only suggestions that I have are to make more teachers do it this way. But don't have

> them start doing it in the 9th grade Start in lower grades like 5th & 6th so ya know what your doing.
>
> Ms. Kaye is good with explaining and she is a good teacher
>
> Keep on teaching the way you have been And every one will learn alot. You explain things so well!
>
> everything I use to do bad in math because my teacher would just say do yur work and he wouldnt explane Ms. Kaye explanes

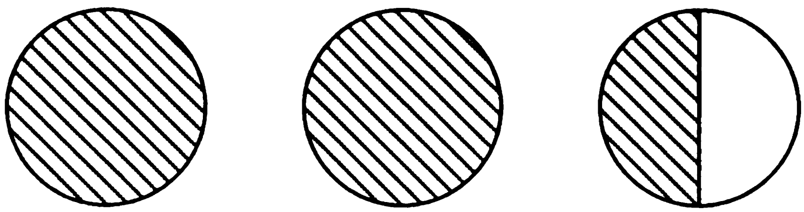
The students comments reflected the improvements in communication that took place in the classroom. It is interesting to note that the students attributed their learning and changed attitudes about themselves and mathematics to the ways in which the mathematical content was presented. They did not attribute their learning to different materials or new content, but rather the ways in which Pamela explained the mathematical ideas to them. They did not attribute their improved learning to the fact that they were talking more and asking more questions. For the students, it was the different ways this teacher taught them mathematics.

Communication in the Post-Project Years

The changes in communication that took place during the intervention years were sustained in the post-project years. In addition, Pamela continued to increase the amount of discussions about mathematics that took place. There was a difference between the quality of communication in the post-project years and tintervention years. During the intervention years, Pamela continually searched for and implemented techniques/ strategies to improve communication; however, she did so inconsistently. She would ask students thought-provoking questions one day, then revert to a "telling" mode on the following day. She would focus on using mathematical language in one unit and not in the next. In some units she would include discussions and explanations, in others she would not. It wasn't until the end of the second intervention year that Pamela implemented systematically these strategies to improve communication. In the post-project years, Pamela knew what communication patterns worked for her and began to add to and refine those strategies. The following vignette illustrates a discussion between Pamela and the students that was observed in most lessons.

Ms. Kaye, "Let's look over the next problem. How many halves is that altogether?"

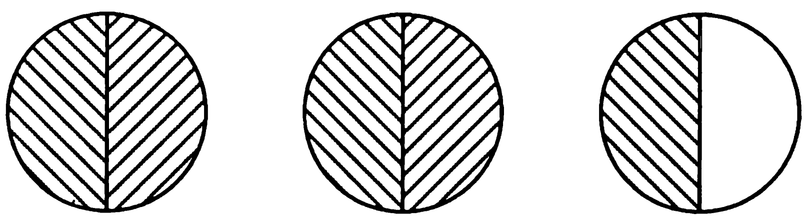
The picture is drawn on the chalkboard:



Frank tells her, "Six."
Joan says, "Five."
Ms. Kaye asks Joan, "How did you get five?"
Joan says, "I counted them."
Ms. Kaye, "All right."

She then draws the following on the board:

2 1/2 = 5/2

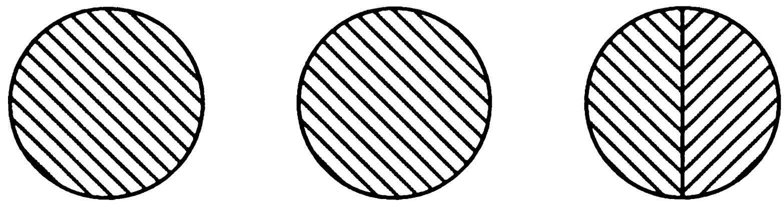


Jack says, "Ms. Kaye, you made a mistake. You were supposed to have five-sixths instead because you have six halves."
Ms. Kaye asks him, "All right, how many should we have?"
Jack replies, "Six." Jack is looking at the six halves as separate pieces.
Frank tells him,"Only two. Look, there are six here, right? But the bottom number is the number of pieces in only one of them."

Richard tells Ms. Kaye, "I was all right until Jack confused me."

Ms. Kaye tells the students, "If I have two and one-half and if I shade in the other empty half would I have six?"

Ms. Kaye draws the following on the chalkboard:



Some of the students start arguing whether or not the result is three or six.

Ms. Kaye says, "Frank, these people are confused. Can you help them out? Can you explain it to them?"
Frank says, "It is because one thing is divided into six parts."

Ms. Kaye says, "You have one whole and it is divided into how many parts?"
Frank tells her, "Two."
She continues, "So you have two-halves, two-halves, and one-half more. So that would be what?"
The students tell her, "Five-halves."

Ms. Kaye tells the students, "Jack brought up a very important concept in fractions, Dave, can you tell me why he said six?"
Dave, "Because you had six halves."

In the vignette Pamela and the students discussed the meaning of two and a half and five halves. The students shared their thoughts and let Pamela know when they were confused or had questions. Discussions and interactions with students were strategies Pamela implemented on the first day of each post-project year and carried through in every lesson throughout the year. Classroom observations during the post-project years indicated Pamela consistently used these questions: (a) "How do you know that?" (b) "Can you say that another way?" (c) "Will you give me another example?" and (d) "Why?" These questions encourage more discussions between the students and Pamela. The communication about mathematics which took place during this period was meaningful, interesting and focused on concepts and ideas.

Pamela talked about the kinds of interactions that occurred in her class in a presentation she made to a group of middle school mathematics teachers. She shared the outcomes for her and the students which resulted from improved mathematical communication.

> My students don't tell me anymore, "Five-fourths is a top-heavy fraction so it will likely fall over and we have to do something to it and so you divide four into five." I don't give that rule anymore.
>
> I ask the students, "Is five-fourths as simple as you can get?"
> They tell me, "No."
> I then ask, "What is the matter with it?"
> The students say, "It is more than a whole."
> Then I ask, "If it is more than a whole how much more is it?"
>
> We go on through a way to find it. They see that a way to find it would be to find out how many fours there are in five--they see there is one whole and another one part--which is one-fourth. I talk about it as an improper fraction.
>
> I discovered one of the things I was doing was avoiding language such as "denominator," "numerator," and "improper fraction" because I felt it was getting in the way of the general math students' learning. Actually, it was the fact that they did not know these words that was getting in the way of their learning. They couldn't communicate with me about the things they didn't understand.
>
> I now try to stick with the mathematical terms so the students and I can communicate. If they want me to draw a parallelogram instead of saying, "Draw one of those smushed-down boxes that got tipped over on one of it's corners." They must use the definitions.
>
> I would have never believed that with general math students. I never believed that this was important.
>
> That may not be a big deal to you, but to me I really get excited when general math students can talk to me and to each other about what was happening. That just didn't happen much before.

The improvements Pamela made in communication during the intervention years were sustained during the post-project years and even improved further. Through constant implementation of questioning, responding and explaining, improved practices in communication became habituated and integrated into Pamela's instructional repertoire.

The Quantity of Mathematical Communication Increased

The discussion in the previous section focused on the finding that the quality of communication about mathematics improved over the years. This section considers the finding that the amount of communication increased. In Chapter IV, the data depicting the flow of classroom activity (Table 4) was presented as evidence that change occurred in Pamela's classroom over the years. In this section a part of that data is reviewed and discussed in more detail as evidence that the amount of time Pamela and the students spent in talking about mathematics had increased dramatically. Table 11 presents the amount of time spent in direct instruction and shows that the amount time increased over the years.

Analysis of the observations indicate that the amount of time spent in direct instruction activities nearly tripled from the pre-intervention to the post-project years. Pamela and the students were indeed talking more about mathematics. There was an increase in the time they spent reviewing the mathematical ideas, in lesson development and in checking the daily assignments.

During the intervention years, Pamela included more occasions when she and the students orally reviewed content. This was not observed during the pre-intervention year. During the intervention and post-project years she and the students frequently began the math lesson with a short discussion regarding the content covered in the previous lesson. This review provided the students with the opportunity to reflect on what they learned the previous day and link that to what they would be studying.

The increase in the time spent in lesson development from the pre-intervention year to the last post-project year doubled. Through controlled-practice activities and discussions Pamela was able to extend considerably the lesson development time. The gradual increases in the lesson development time from the first intervention year to the last post-project year indicates Pamela's continued efforts to improve and habituate controlled-practice activities. At first she was reluctant to teach the students in a whole group because of the great differences in their mathematical abilities and their desire to get started on their daily assignment. Over the years she became more comfortable with teaching the whole class and was able to increase the time spent in lesson development.

Table 11

THE FLOW OF CLASSROOM ACTIVITY
IN DIRECT INSTRUCTION
IN PAMELA KAYE'S GENERAL MATHEMATICS CLASSES

	Percent of Time Per Class Period				
	Pre-Intervention Year	Intervention Years		Post Project Years	
		1	2	1	2
Direct Instruction	16.7	39.0	44.8	45.3	45.6
Review	00.0	04.6	05.5	06.4	02.7
Lesson	14.2	21.4	25.9	26.3	30.6
Checking	02.5	13.0	13.4	12.5	12.3

The time spent in checking assignments increased from 2.5% in the pre-intervention year to between 12% - 13% in the following years. This change was attributed to Pamela's use of error analysis as a way to discuss with the students their mistakes on the assignment or test. Error analysis helped the students understand the reasons why they made mistakes. Pamela implemented this strategy during the first intervention year and continued to use it for the remaining 3 years.

Changing Communication: A Summary

The quality and quantity of communication increased and improved over the years. This took place gradually as Pamela first implemented and then refined strategies such as questioning, responding, explaining and discussing. By the end of the study, the communication patterns Pamela had established over the years became habituated and sustained as a part of her instructional practice.

One result of the changes that were made in the patterns of communication was that by the second semester for each intervention and post-project year, Pamela's students began to imitate her practices and started to ask her the "why?" and "how come?" questions. They began mimicking her methods of explanation using pictures and models, and they started to expect responses from her that were similar to those she expected of them. The students attributed their learning of mathematics during the year to the ways in which Pamela explained the content to them. They did not seem aware that their learning was attributed also to the fact that they communicated more in discussions and interactions which promoted their understanding of mathematics.

More time was now being spent in interactions and discussions about mathematical ideas and concepts than had been spent in the past. Pamela's general mathematics students were given more opportunities to learn and to understand mathematics. In the past, the only opportunity they had to learn mathematics was to copy and practice a demonstrated procedure without thinking and reasoning.

Chapter VII

PATTERNS OF CHANGE IN THE LESSON STRUCTURE

Chapters V and VI depicted the changes in the mathematical content and communication that took place in Pamela's general mathematics classes over the years. This chapter discusses other changes in the ways the students, the class period and the content were organized. These changes are part of the lesson structure category. Analysis of the qualitative data indicated Pamela improved the lesson structure of the general mathematics class. The findings related to these improvements include: (a) students were organized in ways that promoted the learning of mathematical concepts; (b) more class time was allocated for mathematical activities; and (c) there was more long- and short-term planning for general mathematics. These findings are discussed in the following three sections. The last section of the chapter relates these findings to the patterns of teacher change identified in Chapter IV.

Organizing Students in Ways That Promoted Conceptual Learning of Mathematics

When Pamela changed the mathematical orientation of her class from computational to conceptual, she also changed the ways in which students were organized. To learn mathematical concepts, students needed to interact more with Pamela, and with one another in discussions and dialogues. Whole-group instruction and small group work were strategies Pamela used to encourage student involvement and interaction. During the lesson development, controlled-practice activities promoted student participation and attention. In the lesson assignment period, student groups promoted interaction and involvement in the mathematical task/assignment. These two strategies, controlled practice and student groups, changed the ways students were organized so new mathematical content learning was conceptually oriented. These are discussed in the subsections that follow.

Controlled Practice Increased Student Participation in the Mathematics Lesson

In the pre-intervention year, Pamela did not encourage students to participate during the lesson development period. The computational nature of the lesson presentation did not require much student involvement . There was nothing to discuss, only procedures to follow. The amount of time spent in whole-group instruction was limited so students could spend more time on seatwork assignments. Since most students knew the algorithmic procedures, a lengthy presentation was not required. Those students who didn't know what to do received individual help. Pamela thought the students would not pay attention for an extended length of time for whole-group instruction. She also thought there was little in the mathematical content that needed discussion or warranted interaction. As a result, students did not participate in the lesson-development period. The only opportunity students had for interacting with Pamela was when she worked with them individually. Some students who requested help did nothing for the remainder of the period because Pamela did not have time for them.

In the intervention years, controlled-practice excercises were included in the lesson development period to encourage whole-group participation in the learning of mathematical concepts. For example, Pamela gave the students a problem and then waited for them to work on it. When most students were finished, the solution was discussed. When Pamela first started using controlled practice, most of the problems did not encourage discussion. Although many problems focused on the development of mathematical concepts, many others still focused on computational procedures. This changed over the intervention years. By the end of the second intervention year, the controlled-practice problems focused on the development of mathematical concepts and Pamela's questions encouraged much more student participation. She asked students to draw pictures along with her or go to the chalkboard and show the solution . The use of controlled practice extended the amount of time spent in lesson development and encouraged more student participation. Pamela recognized the benefits of using controlled practice: (a) it made her more aware of how students were thinking about the content; and (b) the students were learning more by listening to one another. When students explained how they worked problems the others listened, learned, and asked more questions. Pamela commented on the use of controlled practice as a strategy to increase student involvement in the learning of mathematics.

> Questioning students' responses in large group situations is one of the bigger changes I have made. Trying to find out from the students how they came about the answer has been the most helpful for me and the students.
>
> It helps me to know where their misconceptions are and it helps the students to know that other people in the class have the same or different thoughts.

During the intervention years, the consultant worked with Pamela to implement strategies that increased student participation in the lesson-development period. His suggestions included controlled practice problems. Appendix O contains a planning session, observational vignette and feedback session between Pamela and the consultant which that focused on improving the lesson structure. He helped Pamela think about and plan controlled-practice activities for her class.

Controlled practice continued as part of lesson development during the post-project years and was integrated into Pamela's instructional mode. The following observational vignette illustrates how controlled practice was used during the post-project years to help students understand the concept of multiplication of fractions. In this vignette, Pamela told the students the procedure for multiplying fractions when a student indicated that he didn't understand. She used a controlled-practice excercise with the students that enabled them to see for themselves why the procedure worked.

Ms. Kaye is showing the students how to multiply fractions.
She tells them, "So, the final answer would be one-fourth."
She has written the following on the chalkboard:

$$\frac{2}{3} \quad \frac{\not{X}\text{---}}{\not{X}\text{---}} \quad \frac{3}{8} = \frac{6}{24} \div \frac{6}{6} = \frac{1}{4}$$

Jeff asks, "Why did you do it that way? How come? I thought you were just supposed to multiply the tops. Why didn't you get a common denominator?"

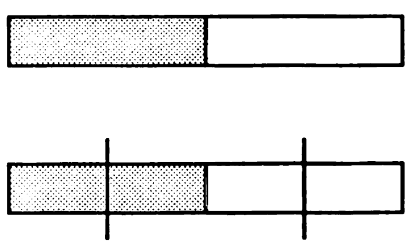
Ms. Kaye seeing Jeff's error in thinking uses another example, she says, "Let's try this one."

She writes the following on the chalkboard:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

This doesn't help Jeff understand the problem.

Ms. Kaye uses another strategy, "All right, Jeff, the reason for how come we have to do that is that 3 times 4 <u>means</u> 3 sets of 4--so, 1/2 times 1/2 <u>means</u> the <u>same</u> thing. It means I want to take 1/2 and cut it in half."
She draws the following picture:



Ms. Kaye tells the students, "Draw a picture of 1/2 and cut it in half."

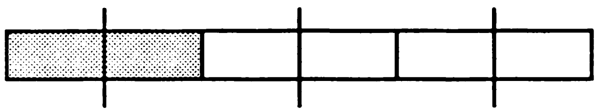
The students draw their illustrations in the same way as Pamela.

A student looks at his picture and tells her, "They turn into fourths!"
Ms. Kaye asks, "How much is half the shaded part?"
A student tells her, "1/4."
Ms. Kaye continues, "Do this, draw a 1/3 fraction bar."

The students follow Ms. Kaye's directions and draw the same illustration as she does on the chalkboard.

Ms. Kaye continues, "Now, I want you to take 1/2 of the 1/3."
The students cut their one-third fraction bar in half.

Ms. Kaye draws the following on the chalkboard:

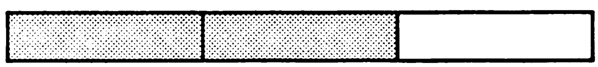


1/3 X 1/2 =

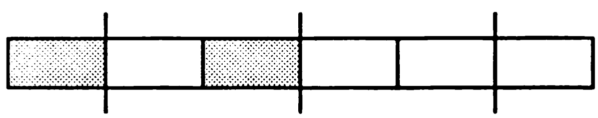
A student tells her, "So, what you get is 1/6, half of the shaded part."

Ms. Kaye asks the students, "Will you draw a 2/3 fraction bar?"

The students draw a 2/3 fraction bar as follows:



Ms. Kaye draws a fraction bar on the chalkboard and then asks the students to take 1/2 of the 2/3. She draws on the chalkboard as the students draw on their papers:



A student says, "If I take 1/2 of the 2/3 I get 2/6 and that reduces to 1/3."
Jeff says, "I see it now."

When the students worked along with Pamela as she directed, they were engaged in an activity that helped them learn the mathematical concept. In the post-project years, Pamela frequently asked the students to work along with her as she worked at the chalkboard. As new units of content (e.g., the Factors and Multiples unit) were implemented, controlled practice encouraged student involvement and interest in the content and served to model the kinds of tasks they would be doing alone or in groups. Controlled-practice excercises helped students understand mathematics. This enabled them to begin assignments quickly and without questions.

Teaching the whole group of students via controlled practice was not an instructional strategy Pamela used prior to the intervention. She believed the diversity of mathematical abilities in her class prohibited her from doing so. She thought students who were slower would feel self-conscious and those who were faster wouldn't want to listen. She also thought students would not learn from listening to one another. Her beliefs regarding whole-group instruction changed during the intervention years. In addition, as she continued to utilize the whole group to teach mathematical concepts, she became more convinced of its effectiveness as a strategy to promote mathematical learning.

Group Work Increased Student Involvement in the Lesson Assignment

The ways Pamela organized the students for working on their assignments changed over the years. In the pre-intervention year, she expected students to work by themselves on math assignments. As students worked alone, they chatted with one another about nonmathematical topics; such as what they did on weekends. On one occasion, two girls worked independently on an assignment and at the same time talked about a movie they saw recently. A student sitting next to one of the girls said to her, "You had better get busy working." She told him, "We are working, see? It just helps to talk while we work." The task they were doing was so routine they could socialize and still complete it. Talking, as the girl said, alleviated the tediousness of the task.

One strategy Pamela used to encourage students to work on assignments in class was to allow "free time" when they finished. In other words, when students completed their work, they had the remainder of the period to do such activities as play cards, games, or read a magazine. There were several card playing groups in the class. Frequently these students worked together on assignments in order to finish early and continued an on going card game. The following vignette depicts one student's desire for his card-playing partner to hurry and complete his work so they could get started with their card game.

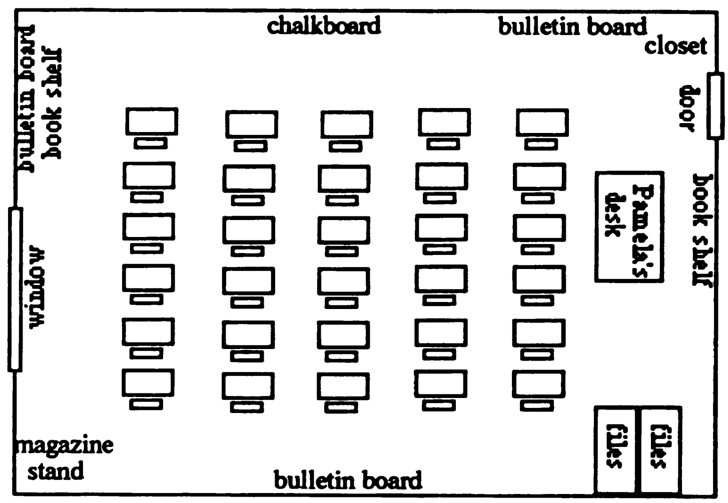
> The students are working on the assignment and are not interacting with one another.
>
> Randy asks her, "How do you do this one? See, I added this and this, is that right?"
> Ms. Kaye tells him, "Yes, that's correct."
> He tells her, "Is that all? That's all I have to do? That's easy!"
> Steve looks at Randy and says, "If everybody was done we could play cards."

The students frequently divided the number of problems on the assignment among themselves and then shared their answers. Instead of working all the problems individually, the students only had to work a few problems each. This strategy enabled them to finish assignments quickly so they could have free time. Even when given the chance to work together, students worked alone to get the task done.

During the intervention years, Pamela grouped the students for lesson-assignment activities. This was to promote interaction among students about mathematical concepts. Students were allowed to select their own groups and were permitted to remain together as long as they stayed on task. Pamela changed the seating arrangement of the room to accommodate group work. Figure 16 depicts changes in the seating arrangement of her classroom during the pre-intervention year and the first intervention year. When Pamela began grouping students she indicated how difficult and frustrating it was to find assignments appropriate for groups. She did not want to give students an assignment they would simply work on individually while in their groups. During a problem-solving unit in the first intervention year Pamela found that students working together in groups helped them persist longer on problems. In an interview, Pamela discussed her thoughts about the value of group work.

> I did utilize groups a little more. Not in the way that I had intended to at the beginning of the year, but I did allow more use of group work to do lesson assignments.
>
> In the process I found the students were discussing among themselves more. Much of the work the students did was in a team situation. Instead of giving them an assignment of certain problems and having each student work alone, they worked on them together. That's probably the biggest single difference in my class, students working cooperatively on problems.

Pre-Intervention Year



Intervention Year-1

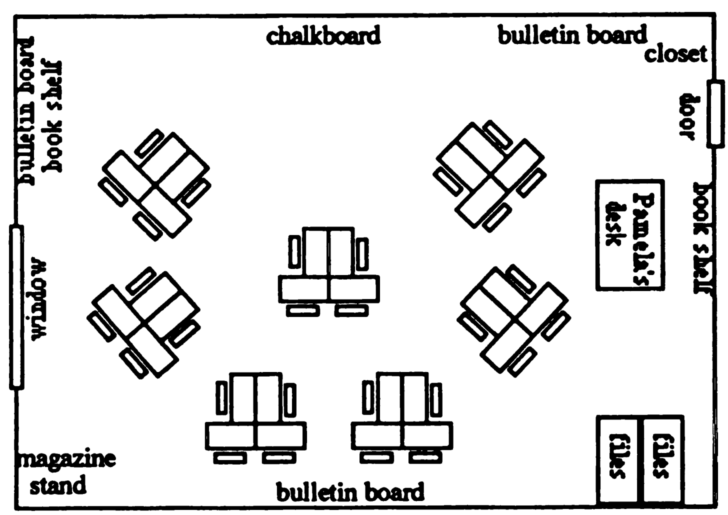


Figure 16

THE SEATING ARRANGEMENT IN PAMELA KAYE'S PRE-INTERVENTION AND INTERVENTION YEAR-1 GENERAL MATHEMATICS CLASSES

> The students were talking back and forth to one another. In the problem solving unit they had something concrete to work with and I found it improved the quality of their questioning one another.
>
> Had they not been in groups I don't think I would have seen the same kinds of test results because I think they would have given up quickly. If I had used the same exact material and said, "All right, you've got to sit in your rows and work on this stuff," I think I would have gotten about five minutes work out of most of them--they would have just quit.

Pamela attributed many improvements in student interactions to the use of group-work activities. In addition, she felt they put more effort into solving problems because they worked together. The classroom observations supported her conjectures. By the second intervention year, Pamela not only had students working together on controlled-practice problems during the lesson-development period, but her students also worked together on assignments. Pamela continued to use group work in the same manner in post-project years in most of the units she taught.

Increasing student participation in whole-group instruction through controlled-practice activities promoted learning and encouraged students to communicate about mathematics. Improving student interactions during the lesson-assignment period through group-work enabled students to remain engaged in mathematical activities longer than in the pre-intervention years.

Pamela's role as a teacher changed as a result of organizational strategies she implemented for learning mathematical concepts. Prior to the intervention, she taught mathematical computations efficiently with few, if any, interactions with the students. Students began working on drill and practice problems in a minimum amount of time. By the end of the post-project years her role was to provide opportunities for students to learn mathematics through a whole-group or small group activities and experiences. This role change was a difficult one for Pamela. She had to learn how to let students guide her lesson presentation and how to help them learn to work together in groups.

Allocating More Class Time for Mathematical Activity

Over the years, Pamela changed the way she organized the class period to allow more time for mathematical activities. The amount of time students spent engaged in mathematical tasks during the lesson presentation and lesson-assignment period increased because of controlled practice and group-work activities. In addition, the amount of time students were engaged in mathematics before and after the lesson also increased. In the next section, changes in the lesson structure at the start and end of class are discussed.

Mathematical Activities Replaced Socializing at the Start of the Class

In the pre-intervention year, Pamela spent the first 5 to 8 minutes of class preparing the daily lesson and completing record keeping chores while students socialized. Students did not expect to work on any mathematical activities during this time. The picture was different in her algebra classes. Students entered the room and began checking their homework assignment without being directed by Pamela to do so. In the algebra classes, the focus of the students at the start of the period was on mathematics; whereas, in the general mathematics class, the focus was on socializing.

During the first intervention year, the classroom consultant brought to Pamela's attention the amount of time students spent socializing at the beginning of class. He suggested students be given math activities when they entered the class to get them working on mathematics. At first, Pamela tried this strategy but did not implement it regularly. Over the course of the intervention years, however, it did became a part of the daily routine. In the second intervention year, Pamela explained the purpose of the start-of-class activities to new students at the beginning of the second semester.

> Ms. Kaye, "When you come to class the first thing you do is you pick up a piece of paper at my desk. If there are supplies that you're supposed to use for the hour they will be next to the paper and you are to take the supplies to your desk.

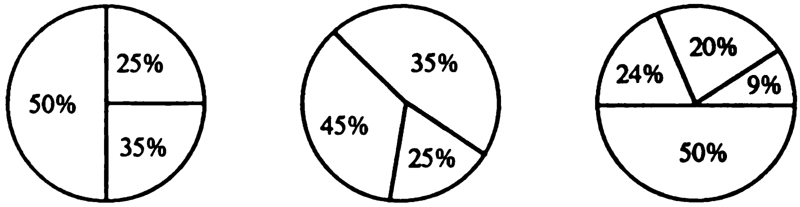
> The first part of the hour we will usually do a review of fractions and decimals and percents. I think that this daily review will last the semester. You will start out with the board review while I take attendance. That gives me time to get my bookwork done and it gets you focused on a math activity for the day. Sometimes we will check them and sometimes I will just collect them."

This daily routine was well established by the second semester of the second intervention year. The students expected math activities when they entered the room, and they expected Pamela to either review the answers or collect the work within 5 minutes. One outcome of this activity was that students were on task at the start of the class period. A second outcome was that it provided students with a review of the mathematical content covered on previously. This continual review at the start of the class helped students think about what they had learned over time. For students returning from absences, the start-of-class reviews provided opportunities to work with other students who knew the content that was missed. Pamela usually allowed students to work together on these assignments and this encouraged them to begin the period talking about mathematics. A typical start-of-class activity, depicted in Figure 17, was given to students on the second day of a percent unit. In this activity the students reviewed the concept of percent as part of a whole and the idea that the whole represented 100%. These problems reviewed the previous days lesson and were easily answered by the students. These activities also allowed Pamela to complete her record keeping and organizational chores without interruptions by her students.

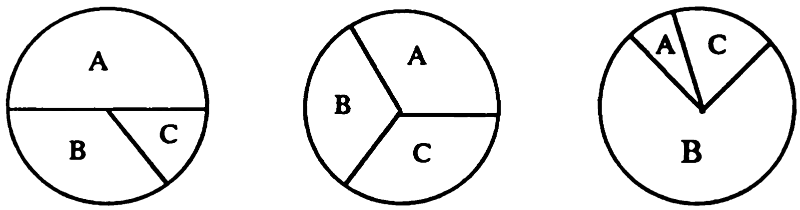
During the post-project years Pamela continued to use start-of-class activities regularly in every lesson. She attributed part of her students' achievement in mathematics to these reviews. "Reviewing at the beginning of the class period worked really well. In fact, I think that was probably the key to the high scores they had in computation." During the post-project years these activities were used not only to start the new lesson but to link units of content together. For example, during the unit on decimals, students might be asked to rename decimals such as .25 as fractions and

1. DO THESE CIRCLES SEEM REASONABLE?

Explain why or why not.



2. GUESS WHAT PERCENT OF EACH CIRCLE IS LABELED A, B, C.



A = _____	A = _____	A = _____
B = _____	B = _____	B = _____
C = _____	C = _____	C = _____

START OF CLASS ACTIVITY

Figure 17

then to draw an illustration of the value. This kind of activity helped students connect decimals with fractions by drawing illustrations.

Thus, the way in which Pamela's general mathematics classes began changed over the years. The students in the post-project years entered the room expecting to work on mathematical review problems. They saw review problems on the chalkboard and knew what to do. In the pre-intervention year, the students did not review mathematical content once it was covered. But by the post-project years, about 5% of the class time was spent reviewing content at the start of the class period.

Mathematical Activities Replaced Socializing at the End of the Class Period

In addition to changes in the amount of time spent in mathematics at the start of the class, the amount of time spent in mathematical activities at the end of the class changed over the years also. In the pre-intervention year the students were given "free time" at the end of the period if they finished their seatwork assignment. Pamela allowed students to do whatever they wished as long as they finished their work first and did not bother anyone still working. Frequently the students finished their work early with 10 minutes left for "free time." Free time was used as a reward for completing the assigned task. In an interview Pamela talked about the end of class.

> I have some alternatives in my room for students like playing checkers, backgammon, cards, that sort of thing. That is definitely intentional on my part and that is interwoven with the social part of general math that I think is important.
>
> In fact, I often find my students are trying to strike deals with me, they'll say, "If we take this home and promise to bring it back tomorrow can we play cards?"
>
> I have received substantial criticism from the administration for that, but I fully believe this is important for these students. They know I won't hassle them about talking with one another if they are done with their assignments.
>
> I have made the statement any number of times, that I don't care if they're talking as long as they're done with their work and that the person they are talking with is also done.

Pamela thought it was as important for general mathematics students to socialize as it was for them to work on their math assignment. Socializing, she said, helped them gain more confidence in themselves. She thought when they gained more confidence they would work harder in the class. So not only was socializing permitted, but it was encouraged. Unfortunately, the amount of time allowed for socializing communicated to the students that (a) the learning of mathematics was not valued as much as was the completion of the task and (b) the teacher did not expect them to spend all their class time working on mathematics.

During the intervention years Pamela changed her thoughts about giving the students extra time to socialize at the end of the period. The end of the class period was now used to check answers, review the day's lesson or discuss a problem students had on the assignment. In the first intervention year, for example, Pamela noticed many students had problems drawing probability trees to determine the outcome of tossing three coins. She decided to review this idea with them at the end of the class period. When the students finished the daily assignment she began the following activity.

> Ms. Kaye says to the students, "Would you turn your worksheets over, please? I would like you to do this instead of what I had planned for the rest of the hour. I would like you to do another probability tree, only this time we are going to be using different colored chips instead of coins. The first chip will be blue on one side and red on the other. The second chip will be yellow on one side and green on the other. The third chip will be white on one side and pink on the other."
>
> Ms. Kaye says, "Now, Freda is going to flip one chip. What do you think she could get? I want you to tell me the outcomes of those colors."
>
> Ms. Kaye writes in the chalkboard:

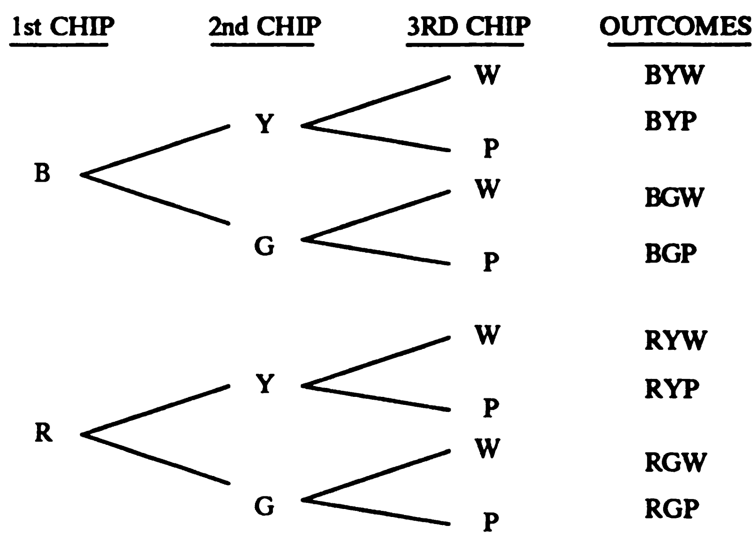
Blue/Red	Yellow/Green	White/Pink	OUTCOMES
CHIP 1	CHIP 2	CHIP 3	

> Ms. Kaye asks, "If Freda flips the first chip, what could she get?"
> The students tell her, "Blue or red."
> Ms. Kaye says, "All right, if Tom flips the second chip what could he get?"
> The students tell her, "Yellow or green."

Ms. Kaye continues, "If Sue flips the third chip, what could she get?"
The students tell her, "Pink or white."
Ms. Kaye says, "Then I want you to make a probability tree for the outcomes of the tosses of the three chips."

The students start working.
After a few moments Ms. Kaye goes to the chalkboard and shows them what the tree should look like:

This review activity helped clarify problems several students had on their assignment and gave the other students a review of the content.

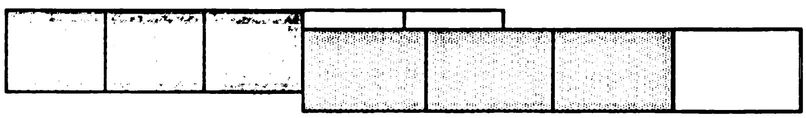


The strategy of discussing common errors or difficulties extended the amount of time students remained on-task during the end of the class period. As the students checked their answers to the daily assignment, Pamela frequently had them discuss wrong answers and asked them to tell her why those answers were wrong. On one assignment the students were adding unlike fractions. As they worked Pamela walked around and wrote down several answers they had for each problem. At the end of the period she wrote the problems and the answers on the chalkboard. For the problem, 3/5 + 3/4 = ?, the following answers (a through d) were obtained:

$$\begin{array}{r} \frac{3}{5} \\ +\ \frac{3}{4} \\ \hline \end{array}$$

(a) $\frac{6}{9} = \frac{2}{3}$ (b) $\frac{3}{9} = \frac{1}{3}$ (c) $\frac{6}{20} = \frac{3}{10}$ (d) $\frac{27}{20} = 1\frac{7}{20}$

Pamela asked the students to tell her how they thought the answers were obtained. After they explained each answer she asked them to decide the correct one. The students disagreed about what was the correct answer, so Pamela asked them to draw a picture of 3/5 and 3/4. When they did, she told them to add the pictures together and decide whether the answer was more or less than 1. They drew the following fraction bars.



They told her the answer was more than 1 and that 1 7/20 had to be the correct answer, since it was more than 1. Discussing common errors on assignments extended the time students remained on task at the end of the period.

During the post-project years Pamela continued to use the time at the end of the class period to review the day's lesson and to discuss the answers to the assignment. The difference in the flow of activity in the classroom is the result of Pamela's efforts to increase the amount of time students remained on task. In the pre-intervention year only 2.5% of class time was spent in checking the lesson assignment at the end of the period. In the intervention and post-project years this time increased to between 12% and 13%. (See Table 4, Chapter IV).

The beginning and the end of the class period in the pre-intervention year were times when the general mathematics students socialized. At least 11% of their time was spent in nonmathematical activities. By implementating reviews at the start of the class and discussions of errors at the end of the class, Pamela reduced the amount of class time spent socializing to 0.2% by the end of the post-project years. The students now entered math class prepared to work on mathematics and usually remained on task until the dismissal bell rang. They knew Pamela expected them to spend their time learning mathematics.

Organizing the Content Through Long- and Short-Term Planning

An important part of the lesson structure is the way in which the content is organized. Deciding what content to teach, when it should be taught and why involves both long and short-term planning. Planning for instruction in the pre-intervention year was problematic for Pamela. General math lessons were not planned in advance because she never knew how far she would get with the students in a single lesson. She attributed her planning problems to the wide range of ability levels of students. On many occasions she misjudged their ability to do an assignment and found she had to reteach the same lesson the next day. On the other hand, Pamela planned lessons and assignments for the algebra classes. In fact, posted on the chalkboard was the schedule of algebra assignments and tests for an entire 9-week grading period. Pamela and her algebra students knew exactly what they would be doing and when On the other hand, she and her general mathematics students did not know from one day to the next what the general math classes would be covering.

Pamela's pre-intervention general mathematics classes lacked content organization and structure. Certainly the computational orientation of the pre-intervention class contributed to her lack of planning. When the tasks were simple arithmetic reviews, there was no need to think about developing concepts and interrelating them in a cohesive manner. Frequently, the next page in the textbook determined the next day's lesson.

During the intervention years she sought ways to think about and plan for general mathematics. When she decided to teach mathematical concepts she needed to know where they fit into her curriculum. Planning for and organizing the teaching of general mathematics became one of the most difficult things for her to accomplish. In an interview, she expressed the desire to develop an instructional plan for general mathematics.

> I do need to have an overall plan. That is something I am not comfortable with yet, but it is important to know where these things come in. I need to do more with organizational and management kinds of things.

During the first intervention year, Pamela tried new units of content in general mathematics, but she did not attempt to organize them in any meaningful or purposeful way. New units were taught anywhere during the year. After planning the units with the classroom consultant she became convinced she needed to work improving her long-term and daily planning in general mathematics. She thought it helped to find ways to link units of content together. However, it wasn't until the end of the second intervention year that she began to think about formulating a yearly plan in general mathematics. In spite of her thinking about a yearly plan, she still acknowledged difficulty in doing this.

> It has always been difficult for me to figure out what I am going to do next Friday. I need to go on a day to day basis because I have a hard time judging how long it is going to take a kid to do a particular thing.

It wasn't until the post-project years that Pamela developed daily and long-term plans in general mathematics. At this time, the observations showed she consistently kept a 2-week calendar on the chalkboard of topics that were covered and would be covered in the class. In addition, she wrote a daily schedule of the activities so students entering the room knew what they would be working on for the day. Pamela knew from the start of the year the content she planned to cover by the end of the first semester and the content she planned to teach the second semester. She said, "In terms of my planning, I now have a pretty good map in my head of where I am going, I know I did not have that at the beginning of the project." She had a rationale for where content belonged and why. For example, she now taught fractions, decimals and percents as one large unit because they were related to the concept of part-to-whole. Probability and similarity units were taught in the second semester because they provided the students with the opportunity to review fractions, decimals, and percents using new and challenging content. An algebra unit was included at the end of the year to introduce general mathematics students to algebraic content. Daily and long-term planning helped Pamela organize the kinds of mathematical experiences she wanted her general mathematics students to have.

One reason Pamela found planning to be one of the more difficult strategies to implement was because she needed time to reflect on the outcomes of the new units of content she included each year. As she tried new units of content she thought about why they were or were not successful and about the changes she would have to make in the unit for the next year. It wasn't until she had been through the intervention and post-project years that she finally had a pretty good idea of where she was going in general mathematics and of where things fit in.

Improving the Lesson Structure: A Summary

The lesson structure consists of the organization of the students, the class time and the content for the learning of mathematics. The decisions a teacher makes regarding the organization of these three elements provides the framework for the learning experiences of the students. Over the years of this study, changes took place in the lesson structure of the general mathematics class. These changes reflected a move away from a computationally-oriented class to one that focused on the development of mathematical concepts.

One aspect of the lesson structure was the way students were organized. By the post-project years the students were taught as a whole group and worked together on assignments instead of alone. These changes increased the amount of interaction between the students and the teacher. The changes encouraged the students to exchange thoughts and ideas and they improved the opportunities for students to learn mathematics. During the intervention years, Pamela found using whole group instruction (such as controlled practice) easier to implement than finding good tasks/activities for students to do when they worked in groups. Finding good tasks for student groups remained a concern for her across the years.

A second aspect of the lesson structure consists of the amount of time allocated to examination of mathematical concepts instead of practicing algorithms. Obviously, students who spend more time engaged in activities designed to enhance their understanding of mathematical concepts have more opportunities to learn. Optimizing learning opportunities, therefore, means increasing the amount of time the students spend on such activities. Pamela implemented activities at the start and end of class to keep student attention focused on mathematical ideas. During the first intervention year these strategies were inconsistently implemented; however, by the post-project years they became a part of Pamela's lesson plans. She always included a review at the start of the class and an activity/review/summary at the end. When the students entered the class they were expected to work on mathematics from the beginning to the end of the period.

Organizing the content via instructional planning on a daily and a long-term basis was a desire of Pamela's long before she enacted it successfully. In the first intervention year she believed planning was needed in general mathematics but did not know how. Perhaps her inability to plan effectively stemmed from the fact that she did not have any mathematical goals or objectives for her students aside from computational competency. Perhaps her weakness in planning was due to her lack of experience in teaching mathematical concept. Perhaps, she simply did not want to take time away from preparing lessons for the algebra classes to plan for general mathematics. During the intervention project, she changed her goals and objectives in general mathematics to include learning mathematical concepts. To accomplish these goals, new/modified content units needed to be added to the curriculum in ways that enhanced the development of mathematical concepts. Throughout the intervention years she taught new mathematical units and became aware of their value and where they should be placed in the curriculum. With experience and reflection, Pamela found herself planning for both daily and long-term general mathematics successfully.

The way in which the general mathematics lesson was structured changed. As with the changes in the mathematical content and the communication patterns, the changes in the lesson structure were gradual. Changing the ways in which the students, class time, and the lessons were organized also took time. However, with constant implementation of the strategies to improve the lesson structure, the changes that were made became a part of Pamela's practices and were sustained over time.

Chapter VIII

FROM TRADITIONAL TO NONTRADITIONAL INSTRUCTION

This is a case study of how the thoughts and practices of a general mathematics teacher, Pamela Kaye, were changed. The findings presented in the previous chapters focused on the patterns of teacher change from both quantitative and qualitative perspectives. In this chapter, the findings are summarized by contrasting traditional and nontraditional beliefs, instructional decisions, and practices. The first section discusses the teacher's beliefs about learning and teaching, how they influenced her instructional decisions in general mathematics, and how they changed. The second section presents the teacher's practices which reflected the instructional decisions and considers how her practice changed. The third section discusses the consequences of traditional and nontraditional approaches for both the teacher and students. The last section contains suggestions for further research and studies of teacher change.

The Influence of a Teacher's Beliefs on Instructional Decisions

What teachers believe about students, teaching and learning influences the kinds of decisions they make regarding general mathematics instruction. These decisions include: (a) choosing the mathematical content and tasks; (b) determing the ways in which that content is communicated; and (c) organizing the students and lesson for the learning of mathematics. Whether teachers teach traditionally or nontraditionally, instructional decisions are made for every lesson. The beliefs that guided Pamela's instructional decisions are discussed in the following subsections.

Traditional Beliefs About Teaching General Mathematics

Pamela's beliefs regarding teaching and learning in general mathematics typified a traditional orientation and were similar to the beliefs ofmost teachers lower-track classes (Goodlad, 1984, pp. 207-210). Central to these beliefs were her thoughts about general mathematics students, whom she characterized as troubled individuals. It was Pamela's perceptions of her students that had the greatest influence on her instructional decisions in general mathematics. She believed her students would work harder if they saw her as a teacher who was concerned about them and their personal problems.

Pamela believed the personal and school problems of general mathematics students contributed to their lack of interest, motivation and achievement in mathematics and to their resulting negative attitudes towards mathematics. She identified poor self-concept, lack of confidence as learners, and low mathematics achievement as predominant problems of general mathematics students, with the lack of confidence being the main factor in their history of failure. Pamela believed, therefore, that improving their confidence would encourage them to put forth more effort, and that this increased effort would produce more mathematical success. She believed her one-on-one interaction with them was the most effective means of improving their self-confidence. Her primary goal was to help students feel good about themselves. Her secondary goal was to "get some mathematics in along the way." She believed her students would work harder if they saw her as a teacher who was concerned about them as persons with problems first and mathematics students second.

Pamela's instructional decisions for teaching general mathematics reflected her goals. The mathematical curriculum and the tasks selected were based on the knowledge that this was the last opportunity for general mathematics students to learn basic computational skills, skills, she believed, Skills they would need to survive in the world. She thought limiting the curriculum content to basic arithmetic reviews would not discourage them from trying some of the problems. She also thought this was content which all students could be successful. Finally, Pamela believed her students could not learn mathematical concepts until they mastered computing. Although she wanted students to learn mathematical concepts, she felt students needed to compute with competency first.

Since the content of the general math curriculum reviewed arithmetic algorithms, Pamela thought routine drill and practice assignments were the most appropriate tasks for the students. She believed drills allowed algorithmic procedures to become automatic and thus lead to computational competency.

After the mathematical content was determined, the next instructional decision was how should to communicate it to students. These computational procedures and drill and practice assignments were already familiar to the students and required only minimal explanation/demonstration. Pamela demonstrated procedures quickly to have most students working on the assignment as soon as possible. She believed general mathematics students understood the procedures better if she eliminated mathematical language and unnecessary discussions from the instruction. Her objective was to get students working quickly so she could work with individual students having difficulty. In addition, she believed this strategy allowed her the extra time necessary to counsel students who needed personal attention with problems.

Another instructional decision involved the organization of students for the task. Pamela believed students should do their assignments individually and not bother others who were working. She believed it was important for them to interact with one another socially and allowed them time to do so after they completed assignments. She believed the more students interacted socially, the better they would feel about themselves and the more self-confidence they would have.

The instructional decisions Pamela made for general mathematics then were based on her beliefs about teaching and learning and the goals she had for students in general mathematics. These beliefs were developed over the years and were grounded in her knowledge of and experiences in teaching.

In a review of research on teacher thinking, Peterson (1983) found that teachers' thinking played an important part in the instructional decisions they made. She also found that teachers plan for instruction and their plans have consequences in the classroom. Further she noted that teachers have belief systems which influence their perceptions, plans, and actions in classrooms. She commented, "In sum, the research on teachers' thought processes to date substantiates a professional view of the teacher as a reflective thoughtful individual " (p. 102).

Pamela was reflective and thoughtful. Her beliefs about learning, teaching and general mathematics students enabled her to formulate instructional decisions regarding the content and how to present it. She thought about lessons she taught and made further instructional decisions based on these reflections. Her traditional beliefs fostered instructional decisions that led to the creation of the traditionally-oriented general mathematics class.

Nontraditional Thoughts About Teaching General Mathematics

Pamela's thoughts about teaching and learning in general mathematics changed over the years and no longer reflected a traditional orientation. Still at the center of her instructional decisions were her perceptions of general mathematics students, but her perceptions had also changed.

She still felt that general mathematics students had problems and that they came to her class lacking both interest and motivation in. Thus producing low levels of achievement. However, she no longer attributed negative attitudes towards mathematics to a lack of self-confidence and low achievement levels to limited mathematical ability. She now knew general mathematics students were as capable of learning mathematics as algebra students, if a different instructional approach was used to help them learn. Her awareness that students could learn mathematics developed as she taught mathematical concepts in her general mathematics classes through a manipulative-pictorial-abstract approach.

Previously, Pamela thought if she helped students gain confidence, their attitude and achievement in mathematics would improve. Unfortunately, this never happened. Her students never gained confidence in themselves and never felt they could be successful in mathematics. However, during the intervention when Pamela showed her students how much progress in mathematics they made during the semester, she noticed their attitudes and efforts in mathematics changed remarkably. She now knew it was their success in learning mathematics that improved their attitudes, which in turn increased their confidence. Pamela's primary goal now was to have students learn and understand mathematics, contrasted with her previous primary goal to help students feel good about themselves.

With mathematical learning as the goal for students, her instructional decisions changed also. The mathematical curriculum and tasks that were selected reflected her belief that students needed a variety of mathematical topics, a conceptual-orientation, and experiences in mathematical explorations. She believed general mathematics students were quite capable of learning mathematical concepts; and through understanding of the mathematical concepts, they would achieve computational competency. She believed the mathematics content should challenge students and increase their interest and participation in learning.

The decision to change the tasks in general mathematics resulted from her new goals and conceptual orientation of the curriculum. Drill and practice tasks were no longer used to teach mathematical concepts. Mathematical tasks now reflected her belief that students needed to become involved in activities to understand mathematical concepts. She saw how interested and involved her students were when she presented them with new tasks and decided it was important for students to be challenged and engaged in interesting mathematical tasks.

The instructional decisions regarding how the mathematical content and tasks were communicated changed. After implementing strategies to encourage more interaction between students, Pamela became aware of how much they learned by talking about mathematics. She now believed mathematical concepts were best learned through interactions and discussions. She decided lesson development and lesson assignment periods should be times when students were expected and encouraged to talk about mathematics. In addition, she decided to use more mathematical language, because she believed it gave students a way of communicating their thoughts.

Pamela also changed the organization of students for learning mathematics. She no longer believed students should work individually, but now encouraged them to work in groups. She believed group work helped them explore mathematical ideas. She realized that as students interacted on mathematical tasks, they became more confident. She also believed the socializing she allowed in the past did not help students gain confidence.

The instructional decisions Pamela made were influenced by her changed beliefs regarding learning and teaching general mathematics. She continued to be a reflective and thoughtful teacher, but now she reflected on her students' learning and understanding of mathematical ideas and on the ways she could promote that learning. Her nontraditional beliefs about learning and teaching general mathematics were based on new goals she had for her students. Her first goal was for students to understand mathematical concepts. Her perspective is captured by Erlwanger (1973) in his statement of the aims of mathematics teaching.

> The aim in teaching mathematics should be to free the pupil to think for himself. He should be provided with opportunities to discover patterns in numerical relationships.
>
> He should realize that he has to reason, seek relationships, make generalizations and verify his discoveries by independent means. Mathematics should be a subject in which rules are generalizations derived from mathematical concepts and principles.
>
> He has to realize that problems can be solved in different ways; that some problems may have more than one answer, and that some may have no answer at all.
>
> He can learn to enjoy mathematics and to appreciate its power and beauty if he shares his thoughts and ideas with others. At the same time, he has to feel that his teacher is there to encourage and assist him in learning how to inquire, and to find answers to questions in mathematics. (p. 22)

Pamela's beliefs about teaching and learning and her goals for general mathematics students were reflected in the instructional decisions she made regarding the mathematical content and task selection of the class. In addition, decisions on the ways the content was communicated and how the students were organized for learning were a result of her changed thoughts about the goals of general mathematics. These nontraditional goals provided a new perspective on instruction that resulted in improved decisions relating to task, communication and organization.

How Do Instructional Beliefs Become Changed?

A teacher's beliefs influence the instructional decisions that are made in general mathematics. Pamela's beliefs, coupled with her goals for general mathematics students, created a traditional general mathematics classroom characterized as an unpleasant and unrewarding place for both teacher and learners. How were these beliefs changed?

By reading selected literature and through discussions with the GMP staff and teachers during the intervention, Pamela became aware that concept-oriented instruction held some potential for improving learning and instruction in general mathematics. However, knowledge and collaboration were not sufficient to change her beliefs about learning and instruction. It wasn't until she saw her students' test results at the end of the first semester of the intervention that she became convinced their achievement was related to their learning of mathematical concepts. Once she believed that concept-oriented instruction improved computational competence, she continued to change her thinking about instruction. As she changed her goals for students regarding the learning of mathematics, other changes in beliefs occurred. She couldn't believe that conceptual understanding was important and still believe the way to achieve that was through drill and practice. She couldn't believe students needed to interact with one another and with her about mathematical concepts and still believe that demonstration lessons followed by independent work accomplished this.

Over the years, Pamela restructured her thinking. Her first changes were in the instructional goals for her students. While she still valued computational competence, her goal now was to have students achieve conceptual understanding. Her primary goal was the learning and understanding of mathematics. It was this goal that directed her instructional decisions.

The Influence of Instructional Decisions on Teacher's Practices

What teachers believe about teaching and learning is reflected in the instructional decisions they make regarding what content is taught and how it is taught. Just as beliefs influence instructional decisions, so do instructional decisions influence practice. Traditional and nontraditional practices are the result of uniquely different sets of instructional decisions. These practices are discussed briefly in the following subsections.

Instructional Decisions and Traditional Teaching Practices

Pamela's instructional decisions were reflected in her traditional practices. The portrait of Pamela's pre-intervention class in Chapter I characterized traditional instruction. Since her instructional goal for the students was computational competency, she selected mathematical content that focused on computing. The textbook she chose contained numerous drill and practice excercises to assure sufficient practice in computing. She decided students needed to start their assignment as soon as possible, so she limited instruction to the demonstration of one sample problem. Students were then assigned enough problems to keep them busy for most of the period. Since she wanted to work one-on-one with students having problems, she spent the remainder of the class period in individual instruction. Teaching general mathematics meant managing instruction and student activity. Romberg (1983) captured teacher practices such as Pamela's in his description of traditionally-oriented mathematics classrooms.

> The role of teachers in the traditional classroom is managerial or procedural in that "their job is to assign lessons to their classes of students, start and stop the lessons according to some schedule, explain the rules and procedures of each lesson, judge the actions of the students during the lesson, and maintain order and control throughout". Furthermore, the individual lessons are provided for teachers via curriculum guide, a syllabus, or most often a textbook. (p. 33)

Pamela's practice was typical of that described by Romberg. This traditional practice was developed from the instructional decisions she made. These instructional decisions evolved from her traditional beliefs about learning, teaching and students.

Instructional Decisions and Nontraditional Teaching Practices

Nontraditional practices are a result of instructional decisions that differ from those that lead to traditional practices. The instructional decisions that lead to nontraditional practice have as their goal for students learning and understanding of mathematical concepts through active constructive processes. Such practice encourages students to actively interact with one another and to share their thoughts and conjectures regarding mathematical princples and ideas. These practices include students working in cooperative learning groups on tasks.

The decisions which represent a nontraditional instructional approach force many changes in practice, including the teacher's control of the class. Teachers cannot control the amount of discussion or interaction that occurs, the questions that students ask, or the direction of every lesson. The teacher's practices must be responsive to the mathematical needs of the students and sensitive to their learning and understanding. Nontraditional practices require teachers to teach differently, in ways similar to those described in the portrait of Pamela's post-project class in Chapter I. Romberg (1988) presents a description of the practices of nontraditional teachers, like Pamela.

> We recognize that teachers who abandon traditional practices take on a large agenda: they and their students must abandon the safety of rote learning. Instead they must instruct these students in framing and testing hypotheses, build a climate for tolerance of other's ideas, and foster curiosity. Teachers who take this path must work harder, concentrate more, and have deep understanding of mathematics. (p. 7)

The instructional decisions that were made centered around the goal of having students understand mathematical ideas/concepts. These decisions forced Pamela to make changes in her instructional practices. None of the practices she used in the past could be used to promote the new goals she had for general mathematics students. Pamela frequently complained that teaching general mathematics this way was much harder--it took her a lot longer to plan, prepare and teach each lesson. She also noted it was worth the effort when she saw how much her students learned.

How Do Instructional Practices Become Changed?

Instructional decisions are based on what teachers believe about learning and teaching and the learning goals they have for the students. These decisions influence teachers' practice--the content that is selected and the ways in which that content is taught. Traditional beliefs foster instructional decisions that are implemented as traditional practices. Nontraditional beliefs foster instructional decisions that require the use of nontraditional practices. How were traditional practices changed to nontraditional ones?

Change in teaching practice was preceded by a change in belief. Pamela had to believe that the most important learning goal for students was understanding mathematical concepts and ideas. Once she believed this goal was of primary importance, she then had to believe that traditional practices would not help her attain that goal. In fact, she had to realize that traditional practices actually prevented the attainment of that goal. The discrepancy between her traditional practices and nontraditional thoughts created dissonance. She saw the need to change her practices, but had yet to construct new nontraditional modes that would lead students toward this goal, thus resolving the dissonance.

Changing teacher's practices requires time. Once convinced of the need to change her practice, Pamela had to identify the areas that needed to be changed. Then she had to gradually go about changing them. Knowledge of what and how to change was obtained through literature and collaborative discussions with the GMP staff and teachers. More importantly, classroom consultation provided her with knowledge and feedback regarding specific instructional areas to change.

At the end of each intervention and post-project year, Pamela reflected on student learning goals and the instructional changes she had made. She considered these as she planned the curriculum and activities for the next year. The amount and quality of her reflection increased, contributing to the changes made in her practice over the years.

Outcomes of Traditional and Nontraditional Instruction for Teaching and Learning

This section considers the outcomes or consequences of traditional and nontraditional instuction for Pamela and the students.

The Consequences of Traditional Instruction

Pamela's traditional instruction was guided by a set of beliefs formulated knowledge of learning and instruction, of curriculum, and of general math students. She used these beliefs to make instructional decisions which promoted traditionally-oriented classes. Pamela believed teachers should: (a) guide students to computational competence; (b) keep students on-task and productive; (c) teach students to be cooperative; and (d) make students feel comfortable in the mathematics class. The consequences for students as a result of these decisions included: (a) most of their time was spent working on computations similar to those practiced in previous years, (b) procedures were practiced in isolation, but mathematics was never learned, and (c) there was no opportunity to experience new or interesting mathematical topics such as probability and statistics.

Because Pamela believed students should be on-task and productive, she assigned countless computational problems. Both Pamela Kaye and the students knew these computational assignments were "the same old stuff," yet it was the unstated "common understanding" between them that this was the work of general mathematics students. The students did not expect to learn anything new, and Pamela did not think they should.

Since Pamela valued student cooperation and interaction, she allowed students to play games when their assignments were completed. Although this strategy fostered student cooperation, it did not promote learning of mathematics. In their drive to complete the work, students shared or copied answers. Allowing students to socialize resulted in a considerable amount of Pamela's time being spent quieting students, keeping them on task, and monitoring students copying from each other.

Pamela believed her students would be successful in an environment that was nonthreatening and comfortable. She established an environment in her classroom that emphasized nonacademic instruction, encouraged low performance levels, and did not hold students accountable for learning the mathematical content. Students responded to Pamela's low standards and expectations with minimal performance and little interest in mathematics.

Pamela's traditional instructional decisions had several consequences. First, her emphasis on practicing basic computational skills allowed little time in the curriculum for the development of substantive mathematical concepts, experiences, or understandings. Second, her belief that general mathematics students were limited in their ability to be successful in mathematics, were not interested in learning anything new in mathematics, and were not excited by mathematics, caused her not to invest the time in planning or developing a mathematical curriculum that was meaningful and challenging. Furthermore, since she felt students would not take any higher level mathematics classes such as algebra, they would not have to learn mathematics. Third, Pamela's emphasis on establishing a nonthreatening and comfortable environment to increase student confidence contributed to the perpetuation of failure for her students. They did not learn mathematics, and they did not feel better about themselves.

The students were not given the opportunity to learn and study mathematical ideas and concepts in this traditional class. Results from the recent National Assessment of Educational Progress (NAEP) (Educational Testing Service, 1988), suggest traditional teaching practices prevail and continue to hinder students' learning of mathematics.

> Instruction in mathematics classes is characterized by teachers explaining material, working problems on the board, and having students work mathematics problems

> on their own--a characterization that has not changed across the eight-year period from 1978 to 1986. Considering the prevalence of research suggesting that there may be better ways for students to learn mathematics than by listening to their teachers and then practicing what they have heard in rote fashion, the rarity of innovative instructional approaches is a matter of true concern.
>
> Students need to apply their newly acquired mathematics skills by involvement in investigative situations, and their responses indicate very few opportunities to engage in such activities. (p. 76)

In a traditional mathematics classroom, mathematical topics are taught in isolation. Procedures are segmented and never integrated in ways that enable students to make linkages across mathematical units. Students spend their time practicing what others have done and never have their own mathematical experiences. These students are not being prepared for their future.

The Consequences of Nontraditional Instruction

Nontraditional instruction, like traditional instruction, has consequences for the students and the teacher. Pamela now believed teachers should: (a) provide opportunities for students to understand mathematical concepts; (b) engage students in activities that are interesting and challenging and that reinforce concept development; (c) help students learn to work productively in groups; and (d) give students successful experiences in mathematics. Her instructional decisions regarding the content and tasks, communication, and the organization of the students for learning reflected these beliefs and were observed in her practice. The consequences of this approach for the students and herself were different.

Nontraditional instruction is rewarding for the teacher and the students. The students are involved in learning mathematical concepts through mathematical experiences with interesting and challenging content. As they work together to solve mathematical problems, they begin to construct a new understanding and meaning of mathematics. Students experience success in mathematics and often understand mathematics for the first time. As one student said, "I new most all of the things we did But neve understood it real well. I learned more this semester than I did all last year [sic]." Her students success in mathematics was an outcome that was valued by Pamela. As a result of nontraditional instruction, about one-third of her general mathematics students elected to take algebra after general mathematics and were successful in it.

Nontraditional mathematics classes are intrinsically motivating for the students. Mathematical problems are pursued for their own sake, not for the purpose of receiving a letter grade or mathematics credit. It is rewarding for the teacher to see students interested and achieving in mathematics (Mitchell, 1982). Romberg (1988), discussed the goals for students in nontraditional classes.

> The goals for students also reflect the notion of becoming mathematically literate. To acomplish this we see classrooms as places where interesting problems are regularly explored using important mathematical ideas.
>
> Our premise is that what a student learns is dependent on how he or she has learned it. For example, one could expect to see students recording measurements of real objects, collecting information and describing their properties using statistics, and exploring the properties of a function by examining its graph. This vision sees students studying much of the same mathematics currently taught but with quite a different emphasis; it also sees some mathematics being taught which in the past has received little emphasis in schools. (p. 4)

There are countless positive consequences of nontraditional teaching, from the outcomes for teachers to those for students. Perhaps the most significant outcome was that general mathematics students were successful in mathematics, learned to compute with competency and enjoyed the subject.

Further Research and Studies on Changing and Improving Mathematics Practice

There are contributions this study can make to teaching and teacher education. There are also questions raised by this study that remain unanswered. These questions should be considered for further research and studies of teacher practice. These contributions and unanswered questions are presented and discussed.

Studies of Teachers and Teaching

Did Pamela have certain dispositions/propensities that enabled her to be successful in changing her instruction? Can all veteran general mathematics teachers implement changes in their traditional instructional modes successfully? Pamela volunteered and was selected from a field of candidates to participate in the intervention project. Does the fact that she volunteered indicate certain dispositions/propensities existed that made her more likely and willing to implement changes in her instruction? There needs to be further studies of the dispositions/propensities of teachers and the impact of these on changing their beliefs and practices.

The finding from this study indicate that teacher practice can change and that this change takes time. Pamela continues to implement instructional changes today. The study showed that after 2 years, the changes in her practices were sustained. Do they continue to change? What new forms have the changes she implemented taken? For example, how has group work evolved? How has controlled practice changed? Follow-up studies of a teacher who has changed her practice are needed to learn how the changes are further altered.

In addition, other studies would also show us about the long-term effects of the intervention on teacher change. What literature does Pamela read or refer to now that relates to improving mathematics instruction? How does she use and interpret what she has read? How does she interact with other teachers? To whom does she look for feedback in the absence of the classroom consultant?

There should be studies of Pamela's practices in classes other than general mathematics to learn the impact of intervention on practices in other classes. For example, in what ways did Pamela's instruction in her algebra classes change as a result of the changes she made in general mathematics? Does she think differently about the algebra students as a result of her changed perceptions of general mathematics students?

The Students

The changes in Pamela's thoughts and practices were captured in this study. However, with the exception of some student test data, there was no systematic study of the changes that took place in the beliefs or behavior of her students over the year. There should be more qualitative studies of the impact of nontraditional instruction on the students. In what ways were her students changed by nontraditional instruction? How do they think about mathematics as a result of concept-oriented instruction? Are the changes that they made over the year sustained? Are the changes transferred to their learning of content other than mathematics? For example, do they continue to use a manipulative, pictorial, symbolic approach in problem solving in general? I suggest qualitative studies to consider the effects of teachers' changed practices because the use of observational and interview data would capture the impact of the change more completely than quantitative methods.

The Intervention

This is a study of the nature of teacher change. Pamela changed her thoughts and practices because of her participation in aa three-component intervention that included literature review, collaboration and consultation. The data that was collected could not be studied to judge the impact of each intervention component separately, and so it is unclear which one had the greatest impact on changing her instruction. Perhaps there were times when one component was more influential than the others in changing Pamela's thoughts and practices. Further studies of teacher change as the result of an intervention should examine the impact of each component of the intervention and how the impact changes over time.

The School Setting

The GMP worked with four teachers in isolated schools. The administrators did not actively participate in the intervention in any manner. Other teachers in the school were not encouraged to participate in the intervention activities. If Pamela had been expected to work with the teachers in her school, would the outcomes change? If the administrator in her building participated more actively, would the outcomes change?

Could Pamela be prepared to work with the mathematics teachers in her high school in the same way as the classroom consultant worked with her? Would she resist sharing the knowledge from the intervention with her colleagues? What would it take to get her to work with other teachers in her high school? How much time would be needed to change other teachers practices?

Preservice and Inservice Teacher Education

What are the implications of this study for preservice teacher education? A finding from this study indicated that a teacher's practices can be changed, and these changes take time. Can preservice teachers begin to change practices they have not yet tried? Could this case study serve as a model to start them thinking about nontraditional instruction?

Another finding of the study indicated that teachers' thoughts about instruction changed before practice changed. What about preservice teachers? Can their thoughts about instruction be changed before they become practitioners? What will it take for preservice teachers to value and have as their goal, their students' understanding of mathematical concepts? Case studies with preservice teachers need to be conducted that focus on changing their thoughts and practices over time.

What contributions does this study make to inservice teacher education programs? This study's message is that changing traditional practice takes time, a great deal of effort, and much support both inside and outside the classroom. Workshops will not change practice and short-lived programs will not sustain teacher change. Inservice education programs need to focus on improving the mathematical content and tasks and the ways content is communicated and students are organized. Questions remain unanswered. For example, how much in-class support is needed to help teachers change their practice? How do other teachers in the same school contribute to or prevent teachers from changing their practice? This case study can serve as an example of what one teacher can do working alone in a school to improve her thoughts and practices.

Contribution of the Study to Research on Teaching

A message from this study of teacher change is that teachers can change their practices in ways that are rewarding to them and to their students. It is rewarding for teachers to know they can positively influence at-risk students to change their attitudes and achievement in mathematics. It is rewarding for students in the increased confidence and knowledge that they are capable of learning, understanding and being successful in mathematics. This study captured both the nature and the character of change which occurred in the classroom over time. The fact that both qualitative and quantitative methods were used makes this study a unique contribution to research on teaching and generates the need for future research of this type and intensity.

Romberg (1988) calls for such research studies, "Instead of dealing solely with the careful study of 'what is' happening currently in the teaching and assessment of mathematics instruction, research will need to deal more broadly with 'what ought to be' (p. 5)." Other educators and researchers have called for studies of exemplary teaching practices and teacher change (Cohen, 1987; Wilson & Shulman, 1987). It is through studies of teachers, like Pamela, who struggled to improve her teaching that we can become smarter about the nature of teacher change.

APPENDICES

APPENDIX A

GENERAL INTERVIEW QUESTIONS FOR PRE-INTERVENTION AND POST-INTERVENTION YEARS

Appendix A

General Interview Questions for Pre-Intervention, Intervention and Post-Intervention Years

1. When you hear the term "General Math" what is the first thing that comes to mind?

2. Would you describe, as a group, the students who usually make up general math class?

3. How do the students relate to one another and to you in the general math class?

4. When you think of general math what mathematics do you think of?

5. What is your perception of yourself as a general math teacher?

6. Of the different teaching techniques that you have tried were there some that were more successful than others or were they about the same?

7. Using a scale from 1 to 5 (low to high), how would you rate the level of fuctioning of your general math students in terms of the following: memory, skills, concepts, problem solving, applications, and generalization. Do you think these ratings reflect their capabilty?

8. What factors are motivating your general math students in class?
   What motivates them to learn the content?
   What motivates your students to complete tasks?

9. Would you comment on the attitudes of the general mathematics students toward: school, learning, mathematics and achievement.

10. What are some topics you typically cover in general mathematics?

11. How important is drill and practice in general mathematics?

12. Can you think of any changes that might improve general mathematics?

APPENDIX B

AUTHORS AND TITLES OF LITERATURE SELECTED FOR THE GMP EARLY INTERVENITON PERIOD (PHASE II)

Appendix B

Authors and Titles of Literature Selected for the GMP Early Intervention Period (Phase II)*

1. Modifying the Mathematics Content and Task Selection

    Bruner, J. The relevance of education.

    Thelen, H. A. The classroom society.

    Butts, T. "Posing Problems Properly."

2. Increasing the Quality and Quantity of Communication About the Mathematical Content

    Anderson, L. "Short-term student responses to classroom instruction."

    Carpenter, T. "N.A.E.P. Note: Problem Solving."

    Copeland, W. D. "Teaching-learning behaviors and the demands of the classroom environment."

    Good, T., & Grouws, D. "The Missouri Mathematics Effectiveness Project: An experimental study on fourth grade classrooms."

    Jencks, S. M. "Why blame the kids? We teach mistakes!"

    Levin, T., & Long, R. "Feedback and corrective procedures."

    Rowe, M. B. "Wait, wait, wait."

    Rudnitsky, A. "Talking mathematics with children."

3. Using the Social Organization of the Class to Facilitate Mathematics Learning and Instruction

    Artzt, A. "Student teams in mathematics class."

    Blanchard, K., & Zigarmi, P. "Models for change in schools."

    Brophy, J. "Successful teaching strategies for the inner-city child."

    Brophy, J., & Putnam, J. "Classroom management in the elementary grades."

    Cusick, P. Inside high school.

Davis, J. "Teachers, kids, and conflict: Ethnography of a junior high school."

Doyle, W. "Making managerial decisions in classrooms."

Emmer, E., & Evertson, C. "Effective management at the beginning of the school year in junior high classes."

Everhart, R. "The fabric of meaning in a junior high school."

Rounin, J. Discipline and group management in classrooms.

Slavin, R. Using student team learning.

Thelen, H. "Authenticity, legitimacy and productivity: A study of the tensions underlying emotional activity."

Turner, L. M. "GMC+TC+PR = A formula for survival in a non-academic mathematics class."

*General Mathematics Project's Reading List (Appendix C) for complete reference.

APPENDIX C

GENERAL MATHEMATICS PROJECT READING LIST

Appendix C

General Mathematics Project Reading List

Anderson, L. (1981). Short-term student responses to classroom instruction. The Elementary School Journal, 82(2), 100-103.

Anderson, R. D. (1983). Arithmetic in the Computer/Calculator Age, National Academy of Science, Washington, DC.

Artzt, A. (1979). Student teams in mathematics class. Mathematics Teacher, 72(7), 505-508.

Berman, B., & Friederwitzer, F. J. (1983). Teaching fractions without numbers. School Science and Mathematics, 83(1), 77-82.

Bishop, A. J. (1975). Opportunities for attitude development within lessons. A paper presented at the International Conference on Mathematics Education, Nyiregjaza, Hungary.

Blanchard, K., & Zigarmi, P. (1982). Models for change in schools. In J. Price, & J. D. Gawronski (Eds.), Changing school mathematic (pp.36-41). Reston, VA: National Council of Teachers of Mathematics.

Brophy, J. E. (1983). Successful teaching strategies for the inner-city child. Phi Delta Kappa, 63(8), 527-529.

Brophy, J. E. (1982). Classroom organization and management. A paper presented at the National Institute of Education conference on "Implications of Research on Teaching for Practice," Airline House, Warrenton, VA.

Brophy, J. E., & Evertson, C. M. (1982). Chapter 2: Teacher expectations. Student characteristics and teaching, 8-24. New York: Longman.

Brophy, J. E., & Good, T. L. (1982). Chapter 2: Teacher expectations. Teacher-student relationships: Causes and consequences, 30-41. New York: Holt, Rinehart and Winston.

Brophy, J. E., & Putnam, J. (1979). Classroom management in the elementary grades (Research Series No. 32). East Lansing: Michigan State University, Institute for Research on Teaching.

Butts, T. (1980). Posing problems properly. In S. Krulick & R. Reys (Eds.), Problem solving in school mathematics (pp. 23-33). Reston, VA: Association for Supervision and Curriculum Development.

Carpenter, T. (1980). NAEP Note: Problem Solving. Mathematics Teacher, 73(6), 427-432.

Copeland, W. D. (1980). Teaching-learning behaviors and the demands of the classroom environment. The Elementary School Journal, 80(4), 163-170.

Cusick, P. (1972). Inside high school. New York: Holt, Rinehart and Winston.

Davis, J. (1972). Teachers, kids and conflict: Ethnography of a junior high school. In J. P. Spradley & D. W. McCurdy (Eds.), The cultural experience: Ethnography in a complex society (pp. 103-119) Chicago: Science Research.

Dillon, J. T. (1983). Teaching and the art of questioning. Fastback 194. Bloomington, IN: Phi Delta Kappa.

Doyle, W. (1979). Making managerial decisions in classrooms. In D. Duke (Ed.), Classroom management. 78th yearbook of the national society for the study of education (pp.42-75). Chicago: University of Chicago.

Driscoll, M. (1983). Communicating mathematics. Research within reach: Secondary school mathematics (pp. 31-39). Washington, DC: National Institute of Education.

Driscoll, M. (1983). Estimation: A prerequisite for success in secondary school mathematics. Research within reach: Secondary school mathematics (pp. 107-115). Washington, DC: National Institute of Education, Department of Education.

Emmer, E., & Evertson, C. (1982) Effective management at the beginning of the school year in junior high classes. Journal of Educational Psychology, 74(4), 485-498.

Emmer, E. T., & Evertson, C. M. (1981). Adjusting instruction for special groups. Organizing and managing the junior high classrom. Austin: The University of Texas, The Research and Dvelopment Center for Teacher Education, 151-173.

Emmer, E. T., & Evertson, C. (1980). Effective classroom management at the beginning of the school year in junior high classes (R & D Report No. 6107). Austin: The University of Texas at Austin, The Research and Development Center for Teacher Education.

Emmer, E. T., & Evertson, C. M. (1981). Maintaining your management system. Organizing and managing the junior high classroom. Austin: The University of Texas, The Research and Development Center for Teacher Education, 101-121.

Everhart, R. (1979). The fabric of meaning in a junior high school. Theory into practice, 18(3), 152-157.

Evertson, C. (1982). Differences in instructional activities in higher- and lower-achieving junior high english and math classes. The Elementary School Journal, 82(4), 329-350.

Finkel, D. L., & Monk, G. S. (1983). Teachers and learning groups: Dissolution of the Atlas complex. In C. Bouton & R. Y. Garth (Eds.), Learning in groups: New directions for teaching and learning, 14 (pp. 83-97). San Francisco: Jossey-Bass.

Fisher, C. W., & Berliner, D. C. (1981). Teaching behaviors, academic learning time and student achievement. Journal of Classroom Interaction, 71(1), 2-15.

Gibson, M. A. (1982). Reputation and respectability: How competing cultural systems affect students' performance in school. Anthropology and Education Quarterly, 13(1), 3-26.

Good, T., & Grouws, D. (1979). The Missouri mathematics effectiveness project: An experimental study in fourth-grade classrooms. Journal of Educational Psychology, 71(3), 355-362.

Hart, K. M. (1982) I know what I believe: Do I believe what I know? Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Athens, GA.

Jencks, S. M. (1980). Why blame the kids? We teach mistakes? The Arithmetic Teacher, 28(2), 38-42.

Jurascheck, W. (1983). Piaget and middle school mathematics. School Science and Mathematics, 83(1), 4-13.

Kounin, J. (1977). Discipline and group management in classrooms. New York: Holt, Rinehart and Winston.

Levin, T., & Long, R. (1981). Feedback and corrective procedures. In R. Brandt & N. Modrak (Eds.), Effective instruction. Alexandria, VA: Association for Supervision and Curriculum Development.

Manning, B. (1984). A self-communication structure for learning mathematics. School Science and Mathematics, 84(1), 43-51.

McCaleb, J. L., & White, J. A. (1980). Critical dimensions in evaluating teacher clarity. Journal of Classroom Interaction, 15(2), 27-30.

McLaughlin, M. W., & Marsh, D. D. (1978). Staff development and school change. Teachers College Record, 80(1), 1-14.

Miltz, R. J. (1982). Development and evaluation of a manual for improving teachers' expectations (Report No. 26). Palo Alto: Stanford University, Stanford Center for Research Development in Teaching.

Moos, R. H. (1979). Chapter I: Framework for evaluating environments. Evaluating Educational Environments (pp. 1-21). San Francisco: Jossey-Bass.

Moos, R. H. (1979). Chapter 7: Social environments of secondary school classes. Evaluating educational environments (pp. 136-158). San Francisco: Jossey-Bass.

Moos, R. H. (1979). Appendix B: Classroom environment scale scoring key. Evaluating educational environments (pp. 287-296). San Francisco: Jossey-Bass.

Olson, J. (1981). Teacher influence in the classroom: A context for understanding curriculum translation. Instructional Science, 10, 259-275.

Pereir-Mendoza, L., & May, S. (1983). The environment: A teaching aid. School Science and Mathematics, 83(1), 54-60.

Resnick, L. B. (1975). Task analysis in instructional design: Some cases from mathematics. In D. Klahr (Ed.), Cognition and instruction (pp.149-198). Hillsdale, NJ: Lawrence Erlbaum.

Rowe, M. B. (1978). Wait, wait, wait. School Science and Mathematics. 78(3), 207-216.

Rudnitsky, A. (1981). Talking mathematics with children. The Arithmetic Teacher, 28(8) 14-17.

Slavin, R. (1978). Using student team learning. Baltimore: Johns Hopkins University, Center for Social Organizastion of Schools.

Thelen, H. (1982). Authenticity, legitimacy and productivity: A study of the tensions underlying emotional activity. Journal of Curriculum Studies, 14(1), 29-41.

Turner, L. M. (1979). GMC + TC + PR = A formula for survival in a nonacademic mathemtics class. Mathematics Teacher, 72(8), 580-583.

Wearne-Hiebert, D., & Hiebert, J. (1983). Junior high school students' understanding of fractions. School Science and Mathematics, 83(2), 96-106.

APPENDIX D

AUTHORS AND TITLES OF LITERATURE FOR THE GENERAL MATHEMATICS PROJECT'S INITIAL INTERVENTION PERIOD (PHASE III)

Appendix D

Authors and Titles of Literature for the General Mathematics Project's Initial Intervention Period (Phase III)*

1. Modifying the Mathematics Content and Task Selection

   Berman, B., & Frienderwitzer, F. "Teaching fractions without numbers."

   Driscoll, M. "Estimation: A prerequisite for success in secondary school mathematics."

   Driscoll, M. "Understanding fractions: A prerequisite for success in secondary school mathematics."

   Hiebert, J., & Wearne-Hiebert, D. C. "Junior high school students' understanding of fractions"

   Juraschek, W. "Piaget and middle school mathematics."

   Resnick, L. B. "Task analysis in instructional design: Some cases from mathematics."

2. Increasing the Quality and Quantity of Communication About the Mathematical Content

   Bishop, A. J. "Opportunities for attitude development within lessons."

   Driscoll, M. "Communicating mathematics."

   Hart, K. M. "I know what I believe; Do I believe what I know."

   McCaleb, J. L., & White, J. A. "Critical dimensions in evaluating teacher clarity."

3. Using the Social Organization of the Class to Facilitate Mathematics Learning and Instruction

   Brophy, J. E. "Classroom organization and management."

   Emmer, E. T., & Evertson, C. M. "Maintaining your management system."

   Emmer, E. T., & Evertson, C. M. "Adjusting instruction for special groups."

   Gibson, M. A. "Reputation and respectabilty: How competing cultural systems affect students' performance in school."

*See the GMP's reading list (Appendix C) for complete reference.

APPENDIX E

PAIRED OBSERVATIONS: OBSERVER RELIABILITY

Appendix E

Paired Observations: Observer Reliability

The following is a set of observational vignettes from researcher Lanier and myself. These observations were made of one of Pamela Kaye's Pre-Intervention lessons on the addition of integers. The purpose for paired observations was to ascertain the reliability of the primary observer's (in this case, mine) field notes. The focus was on trying to capture as much of the lesson, the teacher's talk and interactions as possible.

Field notes from GMP Researcher: Lanier

8:08 a.m.
Ms. Kaye, "You're so patient this morning. Well, I want to alter a little bit--what I planned for you to do. It will be the same topic as yesterday--addition of integers--but the rules will vary (be a little different) from yesterday's."

Example	Explanation Model: The Number Line
$^{+}2 + {}^{+}3 = {}^{+}5$	$^{+}5$
$^{-}2 + {}^{-}3 = {}^{-}5$	$+{}^{-}2$
$^{+}5 + {}^{-}2 = {}^{+}3$	
$^{-}2 + {}^{-}7 = {}^{-}9$	-3 -2 -1 0 1 2 3 4 5
	$^{+}5 + {}^{-}2 = {}^{+}3$

After illustrating an example or two--as a review--on the number line Ms. Kaye points out that the difference in today's work in contrast to yesterday's work will be in the way the problems are set up. Today there will be three addends which will be put in horizontal equation form or vertical form.

Example

(1) $({}^{+}6 + {}^{+}4) + {}^{-}2$
$^{+}10 \quad + {}^{-}2$
$^{+}8$

(2) $^{+}2$
$^{-}8$
$^{+}4$

(3) $^{-}4$
$^{-}10$
$^{+}3$

She indicates that the parentheses tell you to operate (clear) there first and then add the other term. She asked for a response to the second problem and a student immediately responds, "Negative two."
Ms. Kaye, "How did you get that so quickly?"

The student tells her, "I just figured it out," then says, "I put all the plusses together, found the difference and took the sign of the larger."
She agrees with that strategy and shows a variation of it on the third example and then assures the students the number line will work every time.

GMP Researcher: Madsen-Nason

8:08 a.m.
Ms. Kaye, "You people are so patient this morning. Please turn to page 183. I am going to alter this a little bit. Yesterday it seemed to me that no one had a problem with adding integers."
(Ms. Kaye puts the following on the chalkboard to check the class):

$$^{+}2 + {}^{+}3 = {}^{+}5$$

Ms. Kaye, "If you were going to add these, what would you do?"
A student answers, "Plus five."
Ms. Kaye, (to the class), "The only problem you had yesterday was when the signs were mixed up."
She writes on the board:

$$^{+}5 + {}^{-}2 =$$

Ms. Kaye, to a student, "What would you get?"
A student says, "Plus three."
Ms. Kaye, "O.K. (to another student), what would you have if you had this."
(on the board)

$$^{+}2 + {}^{-}7 =$$

A student, "Minus five."
Ms. Kaye, "Today I want you to do the same kinds of problems except there are parentheses in them. Remember when you have parentheses you look at just what's inside the parentheses first."
She writes on the board:

$$({}^{+}6 + {}^{+}4) + {}^{-}2 =$$

Ms. Kaye, (to the class), "What do you have inside the parentheses?"
A student, "Plus eight."
Ms. Kaye finishes the problem, then writes:

$$13d. \quad \begin{array}{r} {}^{+}2 \\ {}^{-}8 \\ +4 \\ \hline \end{array}$$

She says, to a student, "What do you get?"
The student responds, "Minus two."
Another student asks Ms. Kaye, "Could you go, 4 + 2 = 6 and then $^{-}8$ to get minus two?"

Ms. Kaye says instead of using the number line it would be all right to just combine the like numbers first.
She says, "Say that you do this (on the board)."

$$\begin{array}{r} {}^{-}4 \\ {}^{+}7 \\ {}^{-}10 \\ {}^{+}3 \\ \hline \end{array}$$

A student says, "Negative four."
This student tells her how he combined the like signs to get the answer.
Ms. Kaye shows the class how to do the problem using the number line.

APPENDIX F

INTERVIEW QUESTION FOR CLASSROOM CONSULTATION UNIT DURING INTERVENTION YEARS

Appendix F

Interview Questions for Classroom Consultation Unit During Intervention Years

1. As you reflect on this unit, list and describe any and all important changes from the way you have previously taught this content.

2. What has remained the same?

3. Briefly list each change again in one or two words. What are the effects, if any, of each change you have mentioned on the students? On your role as a teacher? (Include both positive and negative effects)

4. What, if anything, about this unit do you feel you will repeat the next time you teach this content?

5. What, if anything, about the way you taught this unit do you think will transfer to your teaching of other units of content?

6. As you reflect on the following experiences you have had in the Project since the start of the year, what has been of value to you from each category?
   - Readings
   - Discussions
   - Planning Sessions with the Classroom Consultant
   - Feedback from the Classroom Consultant

7. What has not been of value?
   - Readings
   - Discussions
   - Planning Sessions with the Classroom Consultant
   - Feedback from the Classroom Consultant

8. On the basis of the views you expressed in Questions 6 & 7, what changes do you think would be helpful in planning future experiences?

APPENDIX G

STUDENT DATA FROM PAMELA KAYE'S GENERAL MATHEMATICS CLASS

Appendix G

Student Data From Pamela Kaye's General Mathematics Class

1. The General Mathematics Project: Phases I and II (1981-1982)

   Student psychometric data was not gathered during this period. However, copies were made of Pamela Kaye's Grade/Record book to enable researchers to examine the students' levels of achievement in the class.

2. The General Mathematics Project: Phase III (1982-1983)

   IOWA Test of Basic Skills (8th grade results)
   Shaw-Hiehle Computational Test (September, January, June)
   MGMP Probability Unit (Pretest and Posttest)

3. The General Mathematics Project: Phase IV (1983-1984)

   IOWA Test of Basic Skills (8th Grade Results)
   Stanford Diagnostic Mathematics Test (Blue Level: September, January, June)
   Learning Environment Inventory (September and June)
   Shaw-Hiehle Computational Test (September, January, June)
   MGMP Probability & Similarity Units (Pretest and Posttest)
   Intensive Intervention Fraction Unit (Pretest and Posttest)
   Student Responses to End of the Semester Questionnaire (January, June)
   Copy of Pamela Kaye's Grade/Record Book of General Math

4. The Post Project Years 1 and 2 (1984-1985 and 1985-1986)
   The Shaw-Hiehle Computational Test (September, January, June)

APPENDIX H

SHAW-HIEHLE INDIVIDUALIZED COMPUTATIONAL SKLLS PROGRAM

Appendix H

Shaw-Hiehle: Individualized Computational Skills Program

Whole Numbers

1. $17 + 21$
2. $48 + 7$
3. $87 + 62$
4. $869 + 653$
5. $707 + 8 + 64 + 1491$

6. $29 - 16$
7. $43 - 25$
8. $146 - 98$
9. $460 - 373$
10. $3067 - 948$

11. 61×7
12. 84×16
13. 104×75
14. 439×160
15. 1001×4008

16. $4\overline{)3284}$
17. $9\overline{)146}$
18. $68\overline{)849}$

19. $17\overline{)1803}$
20. $782\overline{)15652}$

Answers

1.__________
2.__________
3.__________
4.__________
5.__________
6.__________
7.__________
8.__________
9.__________
10.__________
11.__________
12.__________
13.__________
14.__________
15.__________
16.__________
17.__________
18.__________
19.__________
20.__________

Common Fractions

Answers

1.
$$\frac{1}{4} + \frac{3}{8} =$$

2.
$$\frac{2}{3} + \frac{5}{6} + \frac{5}{12} =$$

3.
$$3\frac{2}{3} + 2\frac{3}{5}$$

4.
$$\frac{4}{5} - \frac{1}{5} =$$

5.
$$\frac{3}{4} - \frac{1}{5} =$$

6.
$$3\frac{2}{3} - 1\frac{3}{4}$$

7.
$$\frac{1}{5} \times \frac{2}{7} =$$

8.
$$4\frac{2}{5} \times 15 =$$

9.
$$5 \div \frac{1}{4} =$$

10.
$$2\frac{2}{5} \div 6 =$$

1.__________

2.__________

3.__________

4.__________

5.__________

6.__________

7.__________

8.__________

9.__________

10.__________

Decimal Fractions

Answers

31.. $\begin{array}{r} 2.006 \\ 13.08 \\ +\ 121.745 \\ \hline \end{array}$

32. $2.1 + 8.09 + 16.004 =$

33. $\begin{array}{r} 18.66 \\ -\ 7.45 \\ \hline \end{array}$

34. $\begin{array}{r} 16.4 \\ -\ 3.78 \\ \hline \end{array}$

35. $19.004 - 16.007 =$

36. $\begin{array}{r} .31 \\ \times\ .50 \\ \hline \end{array}$

37. $\begin{array}{r} 14 \\ \times\ .0002 \\ \hline \end{array}$

38. $\begin{array}{r} 12.07 \\ \times\ 2.01 \\ \hline \end{array}$

39. $.05\overline{)25.055}$

40. $.04\overline{)800}$

31.________

32.________

33.________

34.________

35.________

36.________

37.________

38.________

39.________

40.________

Percents

Answers

41.

$$\frac{4}{100} = \_\_\_\_\_\_ \%$$

42.

$$\frac{2}{5} = \_\_\_\_\_\_ \%$$

43.

$$.76 = \_\_\_\_\_\_ \%$$

44.

$$50\% = \_\_\_\_\_\_$$

45.

$$\_\_\_\_\_\_ \% \text{ of } 64 = 16$$

46.

$$75\% \text{ of } \_\_\_\_\_\_ = 12$$

47.

$$66\frac{2}{3}\% \text{ of } 27 = \_\_\_\_\_\_$$

48.

$$150 \text{ is } \_\_\_\_\_\_ \% \text{ of } 100$$

49.

$$200\% \text{ of } 7 = \_\_\_\_\_\_$$

50.

$$1.2\% \text{ of } \$4000 = \_\_\_\_\_\_$$

41.________

42.________

43.________

44.________

45.________

46.________

47.________

48.________

49.________

50.________

Practical Arithmetic

	Answers
1. What is the cost of 6 pencils at 60¢ a dozen?	1.________
2. At 4¢ each, how many pencils can be bought for 24¢?	2.________
3. A team won $\frac{1}{4}$ of its games. If they played 20 games, how many did they win?	3.________
4. A man drove his car 216 miles on 12 gallons of gas. How many miles did he get to a gallon?	4.________
5. Carl spent 25 % of his money for some presents. What percent did he have left?	5.________
6. If $\frac{1}{4}$ inch on a map represents 3 miles, how many miles would one inch represent?	6.________
7. If a garden is 20 ft . by 35 ft., how many feet of fence are needed to enclose it?	7.________
8. How many square feet of carpet are needed to cover the floor of a room 10 ft. by 12 ft.?	8.________
9. What do you pay for goods marked $13.50 with a discount of 20% ?	9.________
10. A man receives a rate of $3.00 per hour for a 40-hour week. If he receive $1\frac{1}{2}$ times the regular rate for overtime, how much will he earn working a 50-hour week?	10.________

Conversion Table--Raw Score to Grade Equivalence
Computation Test 7-9, Forms A and B

Raw Score	Grade Equivalent	Raw Score	Grade Equivalent
1	3.1	31	6.9
2	3.1	32	7.1
3	3.1	33	7.3
4	3.1	34	7.5
5	3.1	35	7.6
6	3.1	36	7.7
7	3.2	37	7.9
8	3.2	38	8.1
9	3.3	39	8.3
10	3.4	40	8.5
11	3.6	41	8.6
12	3.7	42	8.7
13	3.9	43	8.9
14	4.2	44	9.1
15	4.4	45	9.2
16	4.6	46	9.4
17	4.7	47	9.5
18	4.8	48	9.6
19	5.1	49	9.8
20	5.3	50	9.9
21	5.4	51	10.1
22	5.6	52	10.2
23	5.7	53	10.4
24	5.8	54	10.5
25	6.1	55	10.8
26	6.3	56	11.1
27	6.4	57	11.1
28	6.5	58	11.1
29	6.6	59	11.1
30	6.8	60	11.1

APPENDIX I

LEVELS OF TOPICS FOR OBSERVATIONS AND INTERVIEWS

Appendix I

Levels of Topics for Observations and Interviews*

	Instructional Improvement Categories											
	Mathematical Content				Communication Patterns				Lesson Structure			
	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Pre-Intervention Year												
3/15/82	1	2	1	1	1	1	0	1	1	1	1	1
3/16/82	1	2	1	1	1	1	0	1	1	1	1	1
3/17/82	1	1	1	1	1	1	0	1	1	1	1	1
3/18/82	1	1	1	1	1	1	0	1	1	1	1	1
3/19/82	1	1	1	1	1	1	0	1	1	1	1	1
Interview	1	1	1	1	1	0	0	1	1	1	1	1
Intervention Year 1												
Regular Instruction Period-1												
9/2/82	1	0	2	2	1	1	1	0	1	0	2	2
9/7/82	1	2	1	1	2	1	2	1	1	2	1	1
9/8/82	2	1	1	2	2	1	2	1	2	2	2	2
9/20/82	1	1	2	2	1	1	1	1	1	0	2	1
10/5/82	1	2	1	2	1	1	1	1	1	1	1	1

*Classroom Consultant was present during the observation.

	Instructional Improvement Categories											
	Mathematical Content				Communication Patterns				Lesson Structure			
	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Classroom Consultation Period-1												
10/12/82*	2	2	2	2	1	2	1	1	1	2	2	1
10/15/82	2	2	2	2	1	1	1	1	1	1	2	1
10/21/82*	2	2	2	2	1	1	1	1	1	1	2	1
10/22/82	1	2	1	1	1	1	1	0	1	1	1	1
10/29/82*	1	1	1	1	1	1	0	0	1	1	0	1
Classroom Consultation-1 Interview												
1/82*	2	2	2	2	2	0	0	3	3	3	2	0
Regular Instruction Period-2												
1/5/83	2	2	1	1	2	2	2	0	2	2	2	2
1/10/83	2	2	2	2	1	1	1	0	1	2	2	2
2/8/83	2	2	3	2	2	1	2	2	2	2	2	2
2/17/83	2	1	3	2	2	1	2	2	2	2	2	1
2/23/83	2	2	2	2	2	1	2	0	1	2	2	2

	Instructional Improvement Categories											
	Mathematical Content				Communication Patterns				Lesson Structure			
	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Classroom Consultation Period-2												
4/18/83*	1	1	3	2	1	2	1	0	1	1	2	2
4/20/83*	1	2	3	2	1	2	2	0	2	2	2	2
4/21/83*	2	2	3	3	3	2	3	0	3	3	3	2
4/26/83*	2	2	3	3	3	3	3	3	1	3	3	2
4/27/83*	2	3	3	3	3	3	3	3	3	3	3	3
Classroom Consultation-2 Interview												
6/83*	2	0	3	3	2	0	3	3	3	0	3	0
Regular Instruction Period-3												
5/4/83	2	3	3	2	2	3	3	0	3	3	0	2
5/17/83	2	3	3	2	3	3	3	0	3	3	3	3
Teacher Interview: Intervention Year-1												
6/83	3	0	3	3	2	0	0	3	3	0	2	0

	Instructional Improvement Categories											
	Mathematical Content				Communication Patterns				Lesson Structure			
	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
<u>Intervention Year 2</u>												
Regular Instruction Period-1												
9/1/83	3	3	3	2	3	3	0	0	2	3	2	2
9/6/83	3	3	3	3	3	3	3	0	2	3	2	2
9/7/83	3	3	3	3	3	3	3	0	2	3	2	3
9/12/83	3	3	2	3	3	3	3	0	2	3	3	3
9/15/83	3	3	2	3	3	3	3	0	3	3	3	3
Classroom Consultation Period-1												
9/27/83*	3	3	2	2	3	3	3	0	2	3	3	2
10/5/83*	3	3	2	3	3	3	3	0	2	3	3	2
10/10/83*	3	3	2	3	3	3	3	0	3	3	3	2
10/11/83	3	3	2	3	3	3	3	0	2	3	2	2
10/12/83*	3	3	2	3	3	3	3	0	2	3	3	2

Instructional Improvement Categories												
	Mathematical Content				Communication Patterns				Lesson Structure			
	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Regular Instruction Period-2												
11/7/83	1	1	2	2	2	2	2	0	1	3	2	1
11/17/83	3	2	2	1	2	2	2	0	3	3	1	2
12/12/83	3	2	2	3	3	2	2	3	3	2	3	2
1/30/84	1	1	3	3	2	3	2	3	3	2	2	2
2/6/84	3	3	3	3	3	3	3	0	1	3	3	2
Classroom Consultation Period-2												
3/12/84*	2	2	3	3	3	3	3	0	2	2	3	2
3/13/84*	2	2	3	3	2	3	2	0	2	3	3	2
3/14/84*	3	3	3	3	3	3	3	0	1	3	3	2
3/15/84*	3	3	3	3	3	3	2	0	2	3	3	2
3/16/84*	3	3	3	3	3	3	3	0	1	3	3	2

	Instructional Improvement Categories											
	Mathematical Content				Communication Patterns				Lesson Structure			
	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Regular Instruction Period-3												
5/11/84	1	1	1	2	2	3	2	0	1	3	2	2
5/29/84	3	3	2	3	3	3	3	0	3	3	3	2
Intervention Year 2: Interview												
6/84	3	3	3	3	3	0	3	3	3	3	2	2
Post Project Year 1												
10/09/84	3	3	2	3	3	3	3	0	2	3	3	2
11/15/84	2	3	2	3	3	3	3	0	3	3	3	2
11/29/84	3	3	2	3	3	3	3	0	3	3	3	2
02/22/85	3	3	3	3	3	3	3	0	3	3	3	2
05/22/85	2	3	3	3	2	3	3	0	3	2	2	2

Instructional Improvement Categories												
	Mathematical Content				Communication Patterns				Lesson Structure			
	or	pr	to	ta	in	dr	di	f/e	sc	ld	la	ec
Post Project Year 2												
09/25/85	2	3	3	3	3	3	3	0	3	3	3	3
11/08/85	3	3	2	3	3	3	3	0	3	3	3	3
12/11/85	3	3	2	3	3	3	3	0	2	3	3	2
12/12/85	2	3	2	3	3	3	3	0	3	3	3	2
12/16/85	2	3	2	2	2	3	2	0	3	2	2	3
Post Project Interview												
7/86	3	3	2	3	3	0	3	3	3	3	3	0
MGMP Teacher Presentation												
11/86	3	3	2	3	3	0	3	0	3	0	0	0

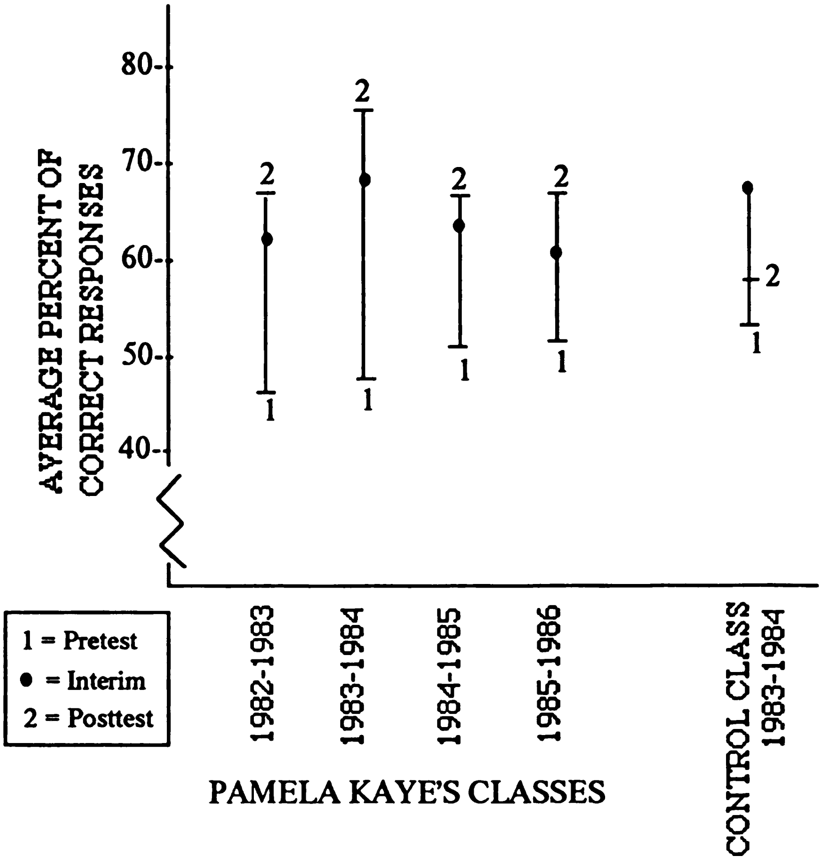
APPENDIX J

SHAW-HIEHLE COMPUTATIONAL RESULTS

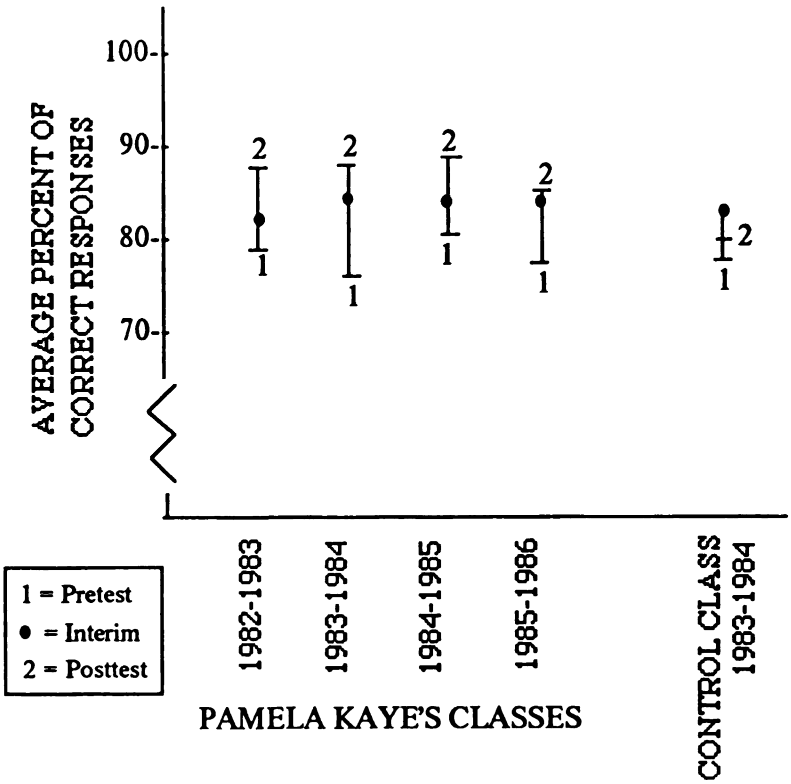
Appendix J

Shaw-Hiehle Computational Results

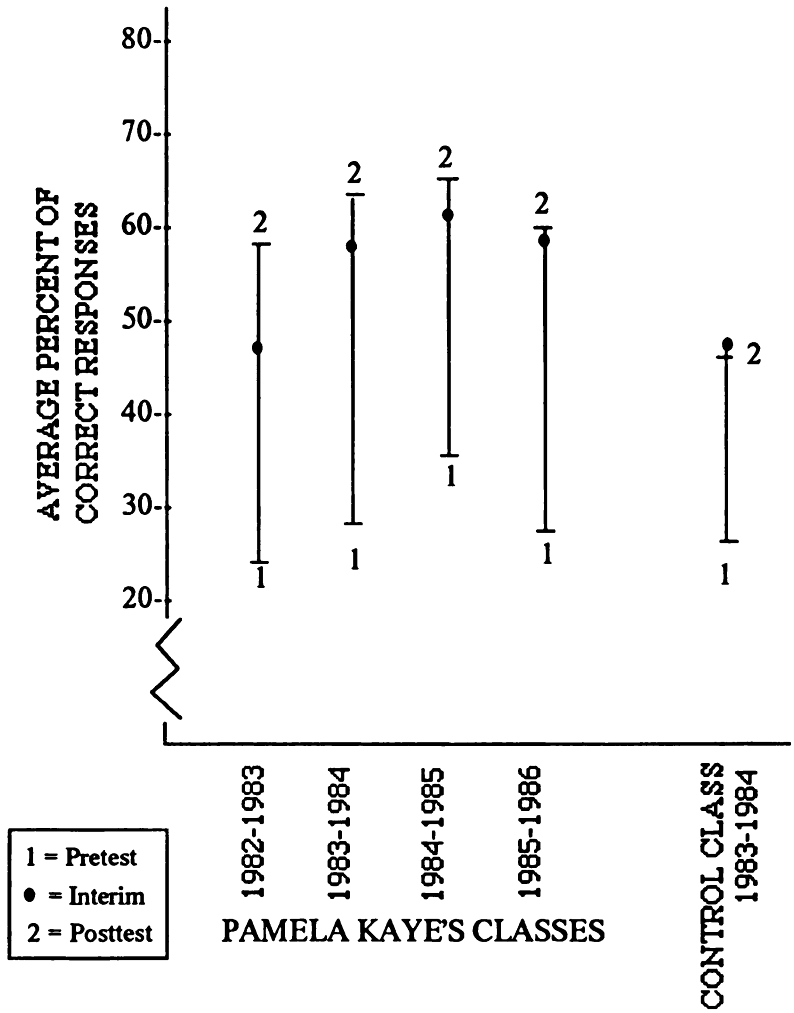
TOTAL TEST



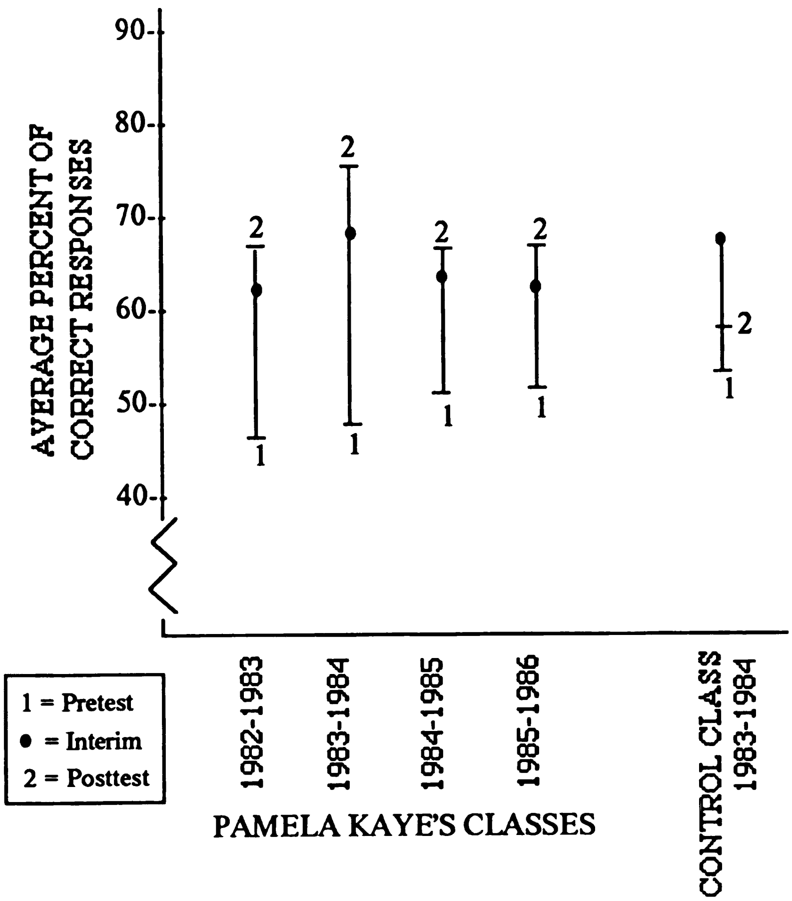
Subtest: Whole Numbers



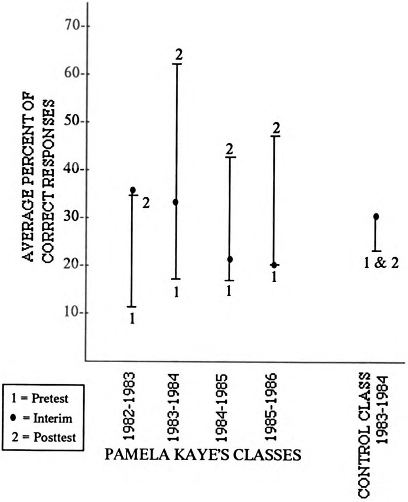
Subtest: Fractions



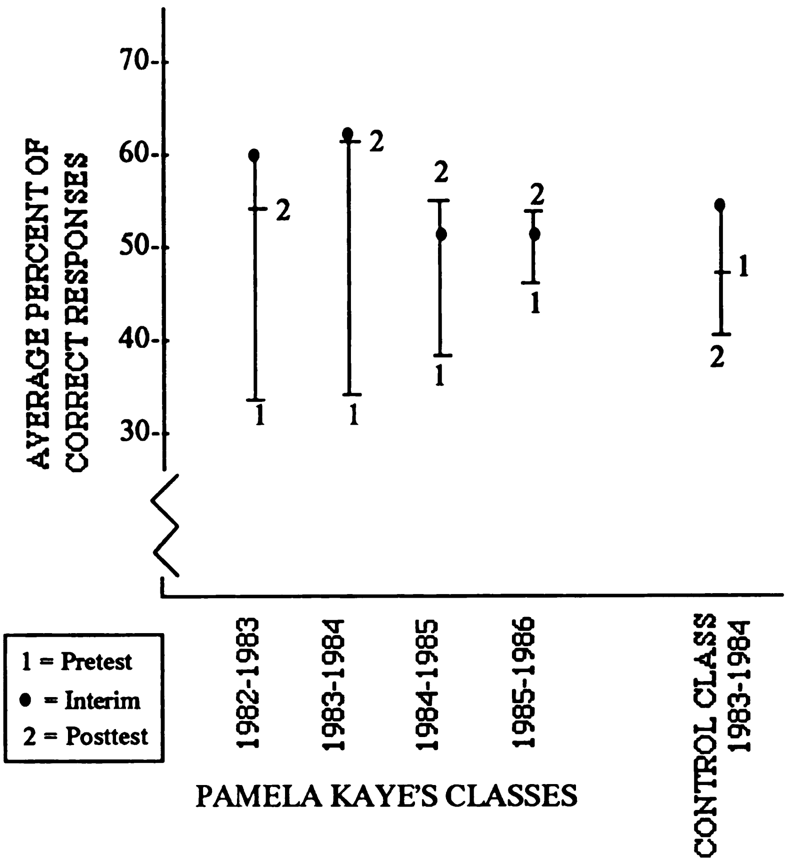
Subtest: Decimals



Subtest: Percents



Subtest: Applications (Practical Problems)



APPENDIX K

SELECTED OBSERVATIONAL VIGNETTES AND INTERVIEW SEGMENTS FOR THE TOPIC LEVELS IN THE INSTRUCTIONAL IMPROVEMENT CATEGORIES

Appendix K

Selected Observational Vignettes and Interview Segments for the Topic Levels in the Instructional Improvement Categories

MATHEMATICS CONTENT/TASKS

Orientation: Observations

Level 1----

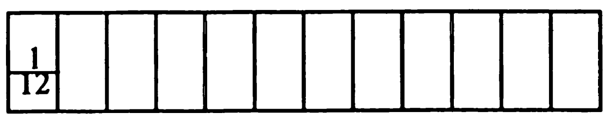
8:15
Ms. Kaye writes the following problems on the chalkboard:

$$-1 + +6 = \qquad\qquad -4 + +2 =$$

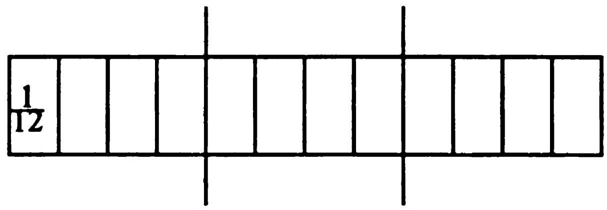
The students copy the problems on their papers. Ms. Kaye explains all they have to do to get the answer is to subtract the two numbers and use the sign of the larger number.

She tells the students, "All right, I want you to do a whole bunch of these problems on pages 179 and 180 in your books." Ms. Kaye reads the following directions in the book to the students, "In each of the above problems do you see that the absolute value of the sum is the sum of the absolute value of the two addends and that the sum of two positive integers is a positive integer?" She then tells the students, "Well, just ignore what they tell you and do the problems like we did them on the chalkboard." (3/15/82 p.2)

Level 2----
10:40
Ms. Kaye has just finished reviewing the answers to a test on percents with the students. She goes to the chalkboard and draws a rectangle and divides it into twelfths.



She tells the students, "I want you to do something about fractions today. We need to go back to it because we are having trouble with percents for this part. I want you to make a rectangle and I want you to divide it into twelve little squares. Make twelve squares in a line." James tells her, "I can't fit twelve squares on my paper!" (He was trying to draw twelve large 100-square grids like the ones they have been using in the decimal and percent units.) Tom says to him, "Just do little squares--not the big ones!" Ms. Kaye responds, "Draw one row with twelve squares." She then tells the students, "I want you to take that box and divide it up into fourths. Four equal parts." Ms. Kaye divides her rectangle into four equal parts:



She says to the students, "All right, ladies and gentlemen, I just counted over four squares and put a line, and counted four more squares and and put a line. Now do I have fourths?" The students respond, "No, you have thirds."

Susan says, "You would only count over three squares for fourths." Ms. Kaye asks, "Then tell me why some of you would have counted four squares?" Jeff responds, "Because of the four in fourths." Ms. Kaye says, "Tom, what did I do?" Tom replys, "Well, I took four groups of three squares." Ms. Kaye tells the students, "When I am trying to see fourths I try to look at the whole box and divide that up into four parts." (1/10/83 p.4)

Level 3----

10:20
Ms. Kaye says to the students, "We talked briefly about absences yesterday. There are twenty-five people in this class today and one is absent. Steve, how would you figure out the percent absent?" Steve, "You would multiply the twenty-five by four and get one hundred and multiply the four by one and that would give you the the four percent."

Ms. Kaye continues, "If we had four people gone then what percent would be absent?" Steve replies, "Eight percent." Jim corrects him by saying, "Sixteen percent." Ms. Kaye says, "Right. Suppose I want to check the percent of students in the class that are absent at 11:20? If no one is here what is the percent absent?" The students tell her it would be one hundred percent.

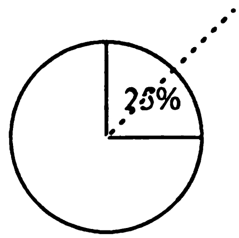
Ms. Kaye tells the students, "I run into a problem with my fifth and sixth hours because I don't have twenty-five students in there--I only have twenty-four people in each class." Steve says, "Well, you just divide twenty-four into three." Ms. Kaye asks, "Why?" He replies, "Because it's just the way you're supposed to do it."

10:25
Ms. Kaye tells the students, "There are three ways to write a percent. You can write it as a decimal, as a fraction, and as a percent. You would write it like this."

$$24\overline{)\,3.00}$$

Steve tells Ms. Kaye, "You would have twelve and a half percent."Ms. Kaye says, "That's right, what fraction does that equal?" He tells her, "One-eighth, because in a circle you take one-quarter which is twenty-five percent and a half of a quarter is one-eighth which is one-half of twenty-five percent which is twelve and a half percent."

10:26
Ms. Kaye draws a diagram of Steve's exlanation on the chalkboard:



She tells the students, "I want you to see that there are a lot of different ways to represent percents." (1/5/83 p.3)

Orientation: Interviews

Level 1----

Anne: Pamela, when you hear the term General Math, what is the first thing that comes to mind?

Pamela: I suppose it's computation: add, subtract, multiply, and divide--whole numbers, decimals, and fractions. (Pre-Project)

Level 2----

Anne: Now, when you hear the term General Math, what's the first thing that comes to mind?

Pamela: At this point in time it's the project (GMP). General math is different than it was in the past. You are still working with add, subtract, multiply, and divide of whole numbers, decimals, and percents, but it's with a conceptual understanding rather than with a computational focus. Everything we've done in the project has led me to believe that the major problem the students are having is that they really don't have any understanding if what is going on. (Interim)

Level 2----

Pamela spoke of her plans for next year and the kinds of things she wanted to do with the mathematics content.

Pamela: I would like to spend more time with the decimals unit and I think that would probably not alter the time I use with the percent unit--it would just make it more workable. I don't think I need to spend anymore time with the percent unit, but if I would spend more time with decimals the percent unit would be more efficient. I would definitely do the fractions unit again, and continue working with actual hands-on activities. I will also take a strong look at putting fractions, decimals, and percents together. Probably the biggest thing that needs to be done is to do an overall plan and then try and figure out where things slide in from there--whether fractions, decimals, and percents really do belong all together in one big unit. (Interim)

Level 3----

Anne: How important do you think it would be for students to develop skills in fractions?

Pamela: I think fraction concepts are very important, I use them in the Probability and Simlarity Units and so forth. I still think computation is important, but not as important as before. The concepts are real important because they will take you into decimals, percents, and so forth. There is no way they can do percents adequately until they have a grasp of both fractions and decimals. (Interim)

Presentation: Observations

Level 1----

8:08 Ms. Kaye tells the students, "I have a lengthy assignment for you and it will probably take two days. We are moving into something that will be kind of hard and that is why I want you to do all of these problems. Please copy down these problems I have written on the chalkboard".

1. $^{+}.6$	2. $^{+}.7$	3. $^{+}.05$	4. $^{+}.05$	5. $^{+}.45$	6. $^{+}.25$
$^{-}.09$	$^{-}.04$	$^{-}.2$	$^{-}.6$	$^{-}.2$	$^{-}.6$

The students copy the problems on their papers.

8:10
Ms. Kaye explains to the students that the problems they were having on the assignment were mistakes made because they were unable to tell which number was bigger. She tells them, "I want you to put a check mark on the number in each pair that you think is the biggest." As the students do this Ms. Kaye walks around the room checking the students work.

8:13
Ms. Kaye says, "Are you all finished? Now, I want you to go and fill in all the empty parts with zeros." She puts zeros in the problems on the chalkboard.

1. $^{+}.60$	2. $^{+}.70$	3. $^{+}.05$	4. $^{+}.05$	5. $^{+}.45$	6. $^{+}.25$
$^{-}.09$	$^{-}.04$	$^{-}.20$	$^{-}.60$	$^{-}.20$	$^{-}.60$

8:15
Ms. Kaye tells the students, "The easiest way to tell which number is bigger is to fill in the same number of zeros and take the decimal out and then compare the numbers. I think it is easier to compare them by pulling the decimals off and putting on the zeros. Then the only problem will be in telling what sign you will have. We have done positives and negatives and we have subtracted decimals so what we're just doing is combining them." (3/18/82 p.1)

Level 2----

8:05 Ms. Kaye writes the following problem on the chalkboard:

+1 1/2 + + 7/8 =

She shows the students what the problems look like on the numberline.

One student tells her the answer is, "Plus one and one-fourth." A second student tells her the answer is, "Plus one and seven-eighths." Ms. Kay tells the students that the easiest thing to do is to find out how many eighths you have and then add the numbers together. She shows them that they have one and four-eighths and seven-eighths and then asks, "What do you have?" One student replies, "One and eleven-eighths." Ms. Kaye asks the students what is wrong with the problem and they tell her, "It's top heavy!" She changes the answer to two and seven-eighths and tells the students this answer is kind of hard to show on a number line.

(3/16/82 p.1)

Level 2----

10:33
Ms. Kaye writes the word PROBABILITY on the chalkboard. One student tells her, "I'm no good at this." She responds, "I wasn't either until I found out what this thing called Probability was all about. What do you think Probability is all about?" James says, "Guessing." Ms. Kaye asks, "Guessing about what?" Another student tells her, "Probability is like tossing up heads and tails on coins." James says, "Like you can't absolutely know. Like it is an estimate, a lucky guess, like heads and tails."

Ms. Kaye tells the students, "So, it deals with the number of times something occurs or happens. Your heads and tails are a good example."

10:35 Ms. Kaye continues, "Are there any other instances where probability is used in real life?" Steve tells her, "Dice, cards, horseraces." Ms. Kaye says, "I'm going to put an ' A' here and it is a shortcut way of writing the 'probability of A'." She writes on the chalkboard:

$$P(A) = \frac{\text{Number of times something happens}}{\text{Number of total things possible}}$$

Ms. Kaye tells the students, "If we took a look at James' heads and tails thing, there are only two chances for something to happen and only one way for it to happen. So the probability to get heads is one-half and that would be written like this."

$$P(H) = \frac{1}{2}$$

Ms. Kaye continues, "I have two blocks here." She puts a yellow and a blue cubic-inch block in a can. She continues, "One yellow and one blue. I want you to pick one out." She hands the can to Susan. Susan picks out a blue block and Ms. Kaye records the choice on the chalkboard. She continues this with five more students and records the results on the chalkboard as follows:

Blue 1 1 1

Yellow 1 1 1

As the students are picking blocks from the can Ms. Kaye asks, "What is the next one going to be?" The students start engaging in guessing and most are certain that if a blue block was picked the next one must be yellow. Ms. Kaye walks around the room and gives most students a chance to choose a block from the can. As this is done a tally is kept on the chalkboard.

The students are attentive and all are participating. They seem to enjoy guessing and seeing if their guesses are correct.

10:41
Ms. Kaye asks the students, "Do you think these blocks have a memory? Do you think they know when they are to be picked up? If you draw one hundred times from this can what do you think I would get?" One student tells her, "You might come out with the same number of each." Another student says, "You have a fifty-fifty chance." Ms. Kaye responds, "You are right. In fact the chances are the same. What I want you to put on your paper is the number of times something will happen out of the number of possibilities for drawing blue." She writes on the chalkboard:

$$P(B) = \frac{1}{2}$$

Ms. Kaye continues, "So the probability of drawing a blue block would be one-half. What is the probability of getting a yellow?" She writes on the chalkboard:

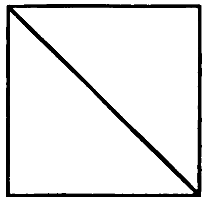
$$P(Y) = \frac{1}{2}$$

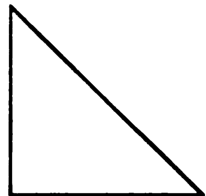
Ms. Kaye tells the students, "So what happens when you add them together? You would get one-half and one-half which would equal one whole." James responds, "That may be why so many people gamble." (2/8/83 p.3)

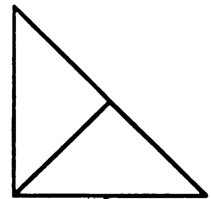
Level 3----

Ms. Kaye is teaching the students how to make a set of Tangrams by folding and tearing a square piece of construction paper. The students are following her directions as she demonstrates what to do. This is a part of the lesson development when Ms. Kaye relates the Tangram pieces to fractional parts of a whole.

8:12
Ms. Kaye tells the students, "I want you to cut the square (#1) you made on it's diagonal--that fold you just did. Now, if you compare your triangle (#2) to the original square how do they compare?" The students tell her that the triangle is one half of the original square. She continues, "I want you to fold your triangle so you have the fold along the height (#3). Now, cut it in half. How many pieces do you have?" A student tells her, "Three." Ms. Kaye asks, "Then how do each of those pieces compare with the whole square?" Some of the students tell her, "One-third."


#1


#2


#3

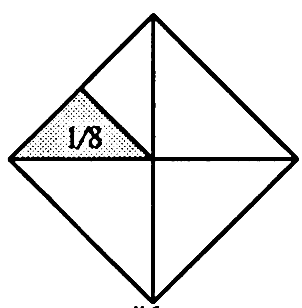
Other students tell Ms. Kaye that the pieces are one-fourth. Ms. Kaye tells the students, "There are three pieces here, but they aren't the same size. So, even though three pieces make one whole square each piece is not one-third of the whole." She goes to the chalkboard as she is talking and draws the following:

8:16
Ms. Kaye says to the students, "What you have then is this. You have here one-fourth, because we took one-half of a half and that gave us one-fourth. Now, take the large triangle and find the middle of the base and bring the tip down to the middle of the base. Once you fold (#4) it I want you to cut it (#5)."


#4


#5

Ms. Kaye continues, "Now, how does the small triangle compare to the one-fourth size piece?" Most students tell her that it is one-half of the one-fourth sized piece. Ms. Kaye says, "Now, can I figure that out--what would that be?" She draws the following on the chalkboard.

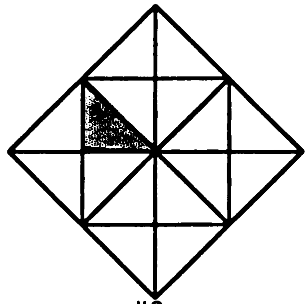


#6

She continues, "Because it is half of a fourth what would it be?" The students tell her, "One-eighth." She asks, "How could I show that it is one-eighth? If I marked off eight parts on the square then does it seem like this is one-eighth to you?" The students tell her, "Yes."

The students are then asked to fold and cut a smaller right triangle from another of their Tangram pieces. Ms. Kaye asks the students,"Now, after you cut off the small triangle from the trapezoid (#7) how does that small right triangle compare to the large square (#8)?" Ms. Kaye draws a square and writes the fraction relationships next to it.

#7



#8

1/2 of 1/2 = 1/4
1/2 of 1/4 = 1/8
1/2 of 1/8 = 1/16
1/2 of 1/16 = ?

She asks the students, "Now, don't do this but tell me what you would get if you cut those pieces in half? How could you compare them?" The students respond, "Each piece would be one-thirty-second." Ms. Kaye tells them, "You have just done multiplication of fractions." (9/1/83 p. 2-5)

Presentation: Interviews

Level 1----

Pamela was asked to discuss the difference between "showing" and "teaching."

Pamela: The difference between "showing" and "teaching" is that I can tell you how to multiply fractions by multiplying the top times the top and the bottom times the bottom and you will get your answer, and you then reduce it. I could also tell them they could cancel out and show them how to do that. That's what I mean by "showing". If I am trying to teach them how to do it I try to give them some kind of understanding of why it works the way it does.

In multiplication of fractions, for instance, I start out with reducing and the students do a week of reducing and then we go into addition and subtraction then I go directly to multiplication of fractions.We talk about doing three-fourths times one-third and the fact that it doesn't make any difference whether you do three times one or one times three--and we back it up and say, "There's three-thirds, what would that reduce to?" I don't use the term cancelling--I tell the students they are reducing inside the problem. We spend a long time reducing until they understand reducing inside the problem. Before, in my first couple years of teaching I "showed" students how to do that. I really think I am teaching them something now. I am teaching them why the problem works the way it does, and why it is all right to do that in that kind of problem. (Pre-Project)

Level 1--

Anne: Describe what went on in your class today.

Pamela: I went through and made various attempts trying to teach a percent lesson today in a different manner than I have ever taught it before. It was a complete bomb out today for a lot of reasons, part of it being that I was assuming some things they did not have. One of them was a general understanding of fractions. One of the examples that I tried to do to explain what I was going after was to use two erasers--I never used that before in fractions. I have drawn pictures before--I don't know why I didn't draw a picture, but I didn't. I was getting nowhere quick. I finally gave up and I asked the students to do some problems I had written on the board. I discovered that many of them already did know what they were doing...at least they knew the procedure I wanted them to use. So, I said, "We will call a halt to this." I put part of the assignment up and then I went to work one-on-one with the students who were still having difficulty. (Pre-Project)

Level 2----

Pamela was asked to describe a "typical day" in her General Mathematics class in this interview segment she talks about the lesson presentation.

Pamela: Then we will go through the days lesson together. Much of the rest of the class period we work problems together. They will copy the problems down as we go over them. We talk a lot about math.The students interact with each other and with me. (Final)

Level 3----

Anne: How has your thinking changed about teaching general mathematics?

Pamela: I think probably the biggest change is I have gotten a lot more flexible in my thinking in terms of teaching. I have always tried to find different ways of approaching the same thing, because I know students don't learn in just one way--but I have always felt like I didn't have a lot of resources. I feel much more comfortable with that now. I think the percent unit was when it really hit me--not only were my students getting more flexible in their thinking but I was getting more flexible in my methods of presentation and that was enabling me to be a much better teacher. (Final)

<u>Topic: Observations</u>

Level 1----

8:02
Ms. Kaye tells the students, "We will start the class today with another 100% quiz. Remember to reduce your answers."

8:04
The students copy on their papers the following problems that Ms. Kaye writes on the chalkboard:

1. $\frac{2}{3} + \frac{2}{3}$
2. $3/5 + 1/5 =$
3. $\frac{4}{5} + \frac{1}{2}$
4. $\frac{2}{3} + \frac{1}{9} =$
5. $1\frac{2}{3} + 2\frac{1}{3}$
6. $5\ 5/8 + 2\ 7/8 =$
7. $6\ 5/6 + 1\ 3/8$
8. $10\ 5/12 + 13\ 7/15$

8:05
A few students talk to Ms. Kaye as she puts the problems on the chalkboard.Ms. Kaye walks around the room checking on how the students are progressing. Most students are still copying the problems from the chalkboard.

8:09
Two students finish the quiz and put their papers on Ms. Kaye's desk. They take their seats and chat with each other. Ms. Kaye continues helping individual students.

8:12
There is a lot of activity around the room as students finish their quiz and put them on Ms. Kaye's desk. Ms. Kaye stops working with one student at her desk in order to tell two other students to be quiet. There are five students at Ms. Kaye's desk waiting for help. They take their seats after their questions are answered.

8:14
Ms. Kaye stands at the front of the room and tells the students, "All right ladies and gentlemen, I would like to work one-on-one with you today so I want you to work on the puzzle that is printed on the back of the papers I just returned to you." The puzzle contains fraction problems the students need to solve in order to complete a path for a trip to Boston.

From 8:15 until 8:50 the students work together and socialize as they solve the puzzle. (10/30/81 p. 2-3)

Level 2----

Ms. Kaye is teaching a Similarity Unit (Middle Grades Mathematics Project) The students just completed an activity where they measured the lengths and widths of seven rectangles and determined (by comparing the corresponding sides) which ones are similar. Ms. Kay has finished asking the students questions about the areas and perimeters of the rectangles and now moves to having the students consider the ratios of the sides.

8:31
Ms. Kaye says, "Let's look at the ratios of the sides for a moment. We talked about the ratio of the short side to the long side. Are there any of those ratios that are the same?" Most of the students seem to think that they're not. Ms. Kaye asks, "Are they similar?" A student replies, "Yes, some of them reduce to three-fourths." (3/15/84 p.3)

Level 2----

Ms. Kaye gives the students a beginning of class quiz on percents. The following problems are written on the chalkboard:

Review 5/21

On your 100-Square Grids show these:

1. 50%	4. 250%	7. 100%
2. 5%	5. 25%	8. 10%
3. .5%	6. 2.5%	9. 1%
		10. .1%

Level 3----

9:07
Ms. Kaye tells the students, "We are going to continue today with the 'Make a Table' problems. We are going to do this problem: How many different coin arrangements can be made that would total 13 cents?" The students have a piece of lined paper for note taking and controlled practice activities. Some of the students say to Ms. Kaye they think this problem will be hard. Ms. Kaye tells them, "Give mean example of how you would make 13 cents." She draws the following chart on the chalkboard:

Dimes	Nickles	Cents
0	2	3

9:08 All of the students are working on this problem. One student asks Ms. Kaye, "Can you have zeros in two spots?" Ms. Kaye says to the student, "Yes, if that's all you can get. Does anyone have more than four different ways?" One student tells her, "There are a lot of them!" As the students continue working on the problem Ms. Kaye walks around monitoring their progress. (9/13/85 p.1)

Topic: Interviews

Level 1----

Pamela Kaye talked previously about the orientation and the mathematical content in the general mathematics class as computations of whole numbers, fractions, and decimals. In this selection she talks about her abilities and weaknesses in teaching general mathematics with respect to the content.

Pamela: I feel pretty good about my ability to teach general math--the basic computation things. The thing I would like to know more about and which I do not feel I have a strong background in is in specific applications, beyond checkbooks. I mean like what you are going to buy at the grocery store --that kind of thing. I would like to give them more specifc applications that they could see right now in their lives and that they could use.

My problem is that sometimes I can't come up with things that I think are really interesting to the students. I don't know how to make multiplication of fractions something that they can really get involved in. (Pre-Project)

Level 2----

Anne: Did you change the content that you taught the first semester?

Pamela: I still taught fractions, decimals and percents. I still taught area, volume, and the kinds of things that I have done before. In terms of the content, I did not change the content, but what was being taught within those areas of content did change. We spent a lot more time on them. We spent a lot more time talking about, "What does one third really mean?" In the past I have only talked about adding the third and the half and then changing fractions to common denominators and that kind of stuff. (Fraction Intensive Intervention Year 1)

Level 2----

Pamela Kaye is talking about using new units of content to reinforce her students' computational skills.

Pamela: The probability unit is not something that I would have said would have been important to teach by itself. My students would survive quite nicely if they never had any probability, thank you. But what I see the probability unit being worthwhile is that it is a method to get at those computational skills. What I am saying is that in the end computational ability is what we are heading for and in the end, if I had a student who knew everything else in my class I wouldn't feel real terrible abut it if he didn't get probability. I do think it is a nice vehicle for teaching the other things. Problem solving I think is the final goal we are trying to reach and we spend an awful lot of time with computational things, because if you can't compute you are still going to have problems with problem solving. (Interim)

Level 3----

Pamela: Looking at the probability and similarity units and how they tie into fractions and decimals and percents--that makes a lot more sense now. When I look at the integer and algebra units, and the success the students had with that, well it just seems logical that we do it this way. I never thought of an algebra unit with these students before--all I thought about was add, subtract, multiply, and divide the whole numbers, fractions, and decimals. Then you had a smattering of everything--I never linked it back to those same things. Maybe it does have to do with the concepts, but I think it was just not really having an idea of what was valuable and the fact that you were couching those same skills within something else. (Post Project)

Level 3----

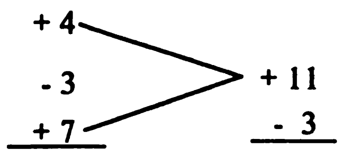
Anne: What motivates your students to learn the mathematics content?

Pamela: It's not the cutesie, neatsy things that we do. It gives us a different way of approaching the content, doing the problem solving unit, doing the probabilty unit, doing the similarity unit. To me, these units provide a different approach that will help them to look at mathematics in a way they hadn't thought of before. When they find out they can do it that's when they will learn it. In the problem solving situation sometimes they do get interested and say, "Gee I want to really stick with this until I get it." They do tend to perservere and you know that perserverence is not one of the general math students' main traits. (Post Project)

<u>Task: Observations</u>

Level 1----

8:05
Ms. Kaye tells the students, "We will continue with pages 189 and 190 in your books. I would caution you that when you get to three fractions it would be easier to work with two fractions first and then work with the third." Ms. Kaye shows the students an example on the chalkboard using whole numbers:



Ms. Kaye continues, "I don't want to do anymore here with you. I will be around to you and work with you individually."

The students form their usual working groups in order to finish the assignment (which contains 128 problems). Some groups contain four students, some contain two or three students. The groups were not selected by Ms. Kaye, but were students who wanted to work together.

Ms. Kaye talks to a student who was absent on the previous day when the students started working on their assignment. She says to him, "Are you all set?" He replies, "No, I wasn't here." Ms. Kaye tells him, "Well, why don't you put the first problem down on your paper." The student writes the first problem down and solves it. He says, "Oh, I get it now!"

8:10
Two students are checking their work together. One student tells the other, "Reduce it down...reduce eleven-twelfths down." The second student says, "Oh, yeah." The student who was absent joins the two students and tells them, "This is so simple, anyone can do it." (3/19/82 p.1)

Level 2----

10:25
The students are completing a beginning of class review activity. Ms. Kaye tells them, "Ladies and gentlemen, would you look up here for a moment? When you finish the review I want you to take your pieces out and take one of the halves and see if you can make a half by combining two other colors. Use two different colors and see if you can make a half. See how many different combinations that you can get to make one-half." Ms. Kaye writes on the chalkboard:

combine: 1 1 + 1 1 1 = 5
combine: 1/3 + 1/6 = 1/2

Some students who have completed their review assignment start the task by taking out their fraction pieces and start fitting them together to make one-half.

10:34
Ms. Kaye tells the class they may make other fractions than one-half (such as fractional equivalents for one-fourth or one-third). Most of the students have their pieces out and are working together on the task.

10:43
Ms. Kaye directs several students to go to the chalkboard and record their results. The students write their answers and sign their names:

1/6 + 1/8 + 1/8 + 1/12 = 1/2 Bob

1/12 + 1/6 + 1/8 + 1/6 = 1/2 Sue

1/3 + 1/8 + 1/12 = 1/2 Nancy

1/12 + 1/12 + 1/12 + 1/12 + 1/12 + 1/12 = 1/2 Fred

1/3 + 1/6 = 1/2 Steve

1/12 + 1/12 + 1/12 + 1/12 + 1/6 = 1/2 Randy

1/4 + 1/6 + 1/12 = 1/2 Ken

1/3 + 1/8 + 1/12 = 1/2 Dawn (10/15/82 p.2)

Level 3----

8:00
The students were given the following beginning of class activity:

Write an algebraic expression for these statements

1. The quotient of x divided by a
2. The sum of r and s
3. The product of m, n, and p
4. The sum of A, B, C, and D
5. The square of x
6. The cube of a
7. The square root of b
8. The difference between a squared and b squared

8:06
The students are directed by Ms. Kaye to exchange their papers and check the answers.

8:10
Ms. Kaye asks the students to open their textbooks to page 238 as they are going to continue their work with algebraic expressions. She discusses the algebraic expressions with the students. An example from this discussion of the problems follows:

8:15
Ms. Kaye asks April, "April, what do you think of this next expression, 'The electromagnetic force (E) divided by the resistance (R).'" April tells her, "I would divide E by R."

8:17
Ms. Kaye continues, "What would you do with the next expression? The product of the weight of the body (W) and the square of the velocity (V) divided by the product of the acceleration of gravity (G) and the radius of the circle (R)." Karen says, "I would write W dot V^2 then I would divide that by G dot R on the bottom."

From 8:17 to 8:32 Ms. Kaye and the students discuss the problems in the book. Ms. Kaye asks the students how they think they would write each expression. All the students are paying attention.

8:32
Ms. Kaye tells the students, "Let's go to the next part where they (the authors) turn it around. Here you will write down the following algebraic exressions in words." One student asks, "All by ourselves?" Ms. Kaye tells him, "No, we will work on these together." Another student asks, "Well can some of us do them by ourselves if we want to?" Ms. Kaye replies, "Of course you can--by all means."

Many students choose to work on this assignment by themselves while Ms. Kaye continues working with the rest of the students. The sample includes:

a. $x + 10$

b. $8n$

c. $a/15$

d. b^2

e. $z - 5$

i. $R_1 + R_2$

n. $h(b + b^2)$ (5/22/85 p. 1-8)

<u>Tasks: Interviews</u>

Level 1----

Anne: What motivates your students to complete tasks?

Pamela: Probably the biggest motivator is grades. If students just complete their assignment and they do the majority of it correctly they will get five points for the day. If they don't do it and they don't do it often enough then they will get an interim report sent to their parents. This is really a lot more threatening than motivating--it is much more negative than positive.

Anne: What motivates your students to learn and understand mathematics?

Pamela: Well, I don't think there is much interest on their part for the mathematics. For those students I don't think they really care about understanding what happens as much as they care about having me tell them how to work the problems so they can get the assignment done. Most of the students just want to get the assignment over with. (Pre-Project)

Level 2----

Anne: What motivates your students to complete their assignments?

Pamela: There is a certain amount of motivation in just getting it done. That usually happens when they first come in. As time goes on the motivation is in their intrinsic interest in the task. In the problem solving unit there was some intrinsic interest in the problems themselves that served as the motivator. The students would pursue a problem just for the sake of the problem. They were interested in finding out what they were going to come up with. (Interim)

Level 2----

Anne: You have changed the strategies you have used in teaching the content of general mathematics.

Pamela: Yes, we have done cut-outs before, but not to the degree that I did it this time. Once they were cut out I used them with the students. I have not done a lot of that in the past. We have drawn and cut out things so we could look at thirds or sixths or whatever, but I have never utilized the cut-outs very much. My drawing of pictures was something I assumed the students knew. I thought they would have no problem understanding the ideas I was representing. I found out that wasn't true. I now have them actually create their own pictures as we go along. (Interim)

Level 3----

Anne: What do you think about the role of drill and practice in general math?

Pamela: Oh, drill and practice. I think it is real important but I think it needs to be done differently than we have done it in the past. If we are talking about the kinds of drill and practice where students sit down by themselves and do drill and practice alone, that is a waste of time. What we need to do is more drill and practice with the class as a whole where students are not doing more seatwork but are doing more controlled practice during direct instruction. Drill and practice where we are doing problems together as a group is important. We now work on problems where we try one problem individually and then as a group we look at it and disect it and decide what was right or wrong about it. I don't know if that is really drill and practice.

I know that if I don't do something over and over it just doesn't stick with me. So, I really believe drill and practice is important--it just doesn't need to be done in the way that we have traditionally done it. I believe that drill and practice can be done within the context of problem solving. (Post Project)

Level 3----

Anne: What changes would you make that would improve general mathematics?

Pamela: Obviously changing the content--no, not the content, the content is a lot the same as it was. I mean changing the task selection. I say the content is still the same because when I teach probability it is not really a new content area, but I now use it to reinforce the fractions, decimals, and the percents. In the end, while the content of probability is nice, it is not one of my priorities. I am talking about changing the tasks that we used to do to get at the content we wanted to teach. (Post Project)

COMMUNICATION

Interaction: Observations

Level 1----

8:27
Ms. Kaye and the students are checking the answers to review problems that were written on the chalkboard. Ms. Kaye asks the students, "For number 9, what is the answer?"

$$+\frac{1}{6} = +\frac{1}{6}$$
$$-\frac{2}{3} = -\frac{4}{6}$$

Ms. Kaye asks Tim, "Tim, which one is the larger number?" Tim replies, "The four-sixths." Ms. Kaye tells him, "Yes, it is easier if you change it then look at the size of the numbers." She solves the problem (-3/6).

Tammy asks, "Isn't the answer one-half?" Ms. Kaye tells her, "Yes, if you reduce it down--but what is the sign?" Tammy says, "Negative." Ms. Kaye solves the next problem for the students:

$$+\frac{2}{3} = +\frac{8}{12}$$
$$-\frac{3}{4} = -\frac{9}{12}$$

Ms. Kaye tells the students their answer should by negative one-twelfth. Mary frowns. Ms. Kaye says, "Mary, you are frowning, what is the trouble?" Mary says, "I don't get it." Ms. Kaye repeats the rule for her, "When you have unlike signs you subtract. When you have like signs you add." (3/18/82 p.3)

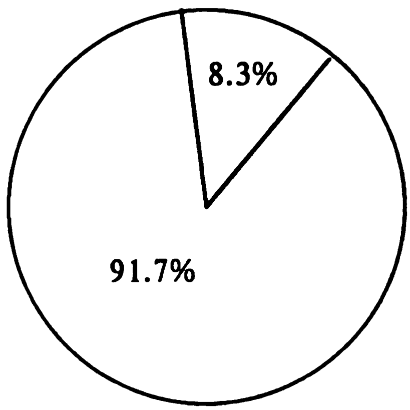
8:55
The students get their papers out to check the answers. Ms. Kaye did not have to remind them it was time to check their work. They seemed to know that five minutes before the end of the class period all card playing and socializing had to cease (at least until the papers were checked). Before Ms. Kaye starts reading the answers Steve asks, "Can we go around and have each one read one answer?" A student responds to his request by saying, "That's kindergarten!" Ms. Kaye reads the answers to the assignment. (1/16/82 p.4)

Level 2 (controlled practice)----

10:26
Ms. Kaye tells the students, "Let's say you had a test at the end of this hour and I gave you twelve problems. Let's say you missed one of those problems. What percent would you have missed? I want you to figure out the decimal and the percent for one-twelfth." The class starts working on the problem. Ms. Kaye walks around checking the students as they work.

Ms. Kaye stands in front of the class and asks Ray, "What do you have to do to solve this problem?" Ray doesn't answer and Steve raises his hand. Ms. Kaye says, "All right, Steve." Steve responds, "You would divide the twelve into the one." Ms. Kaye says, "The fraction one-twelfth means one divided by twelve. Now Steve, give the rest of the class a chance to figure the answer out." The students keep working on the problem. Steve says, "It is going to be eight point three percent." Ms. Kaye says to the students, "Steve says this is the number that I missed on the test, eight point three percent and this is the number I got right--three point seven percent. He says that when you add them up you get ten percent."

Steve tells Ms. Kaye, "No, it would have to be ninety-two point seven percent because you have to get to one-hundred." Ms. Kaye asks, "So then how many percent did you get right?" Mary, "The rest of the pie." Ms. Kaye draws Mary's description on the chalkboard:



Ms. Kaye continues, "Mary, am I going to put ninety-one point seven in my grade book?"
Mary tells her, "No, you will put ninety-two percent in your book."
Ms. Kaye says, "Right. (1/5/83 p.3)

Level 3----

9:47
Ms. Kaye is showing the students how to multiply fractions.
She tells them, "So, the final answer would be one-fourth."
She has written the following on the chalkboard:

$$\frac{2}{3} \times \frac{3}{8} = \frac{6}{24} \div \frac{6}{6} = \frac{1}{4}$$

Jeff asks, "Why did you do it that way? How come? I thought you were just supposed to multiply the tops. Why didn't you get a common denominator?"

Ms. Kaye seeing Jeff's error in thinking uses another example, she says, "Let's try this one."

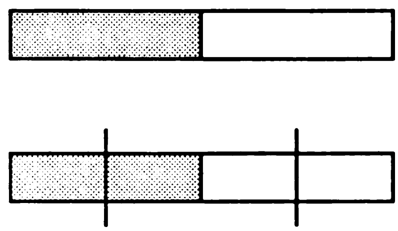
She writes the following on the chalkboard:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

This doesn't help Jeff understand the problem.

Ms. Kaye says, "All right, Jeff, the reason for how come we have to do that is that 3 times 4 means 3 sets of 4--so, 1/2 times 1/2 means the <u>same</u> thing. It means I want to take 1/2 and cut it in half."

She draws the following picture:

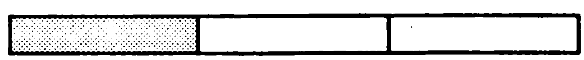


Ms. Kaye tells the students, "Draw a picture of 1/2 and cut it in half."

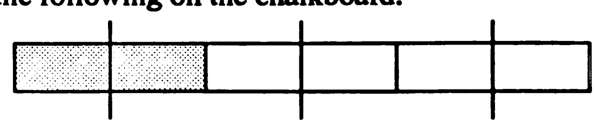
The students draw the illustration in the same way as Pamela has done on the chalkboard.

A student tells her, "They turn into fourths!"
Ms. Kaye asks, "How much is shaded?"
A student tells her, "1/4."
Ms. Kaye continues, "Do this, draw a 1/3 fraction bar."

The students follow Ms. Kaye's directions and draw the same illustration as she does on the chalkboard.



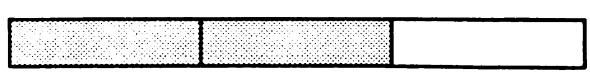
Ms. Kaye continues, "Now, I want you to take 1/2 of 1/3."
The students cut their one-third fraction bar in half.
Ms. Kaye draws the following on the chalkboard:



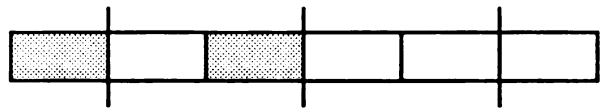
1/3 X 1/2 =

A student tells her, "So, what you get is 1/6."
Ms. Kaye asks the students, "Will you draw a 2/3 fraction bar?"

The students draw a 2/3 fraction bar as follows:



Ms. Kaye draws a fraction bar on the chalkboard and then asks the students to take 1/2 of the 2/3. She draws on the chalkboard as the students draw on their papers:



A student says, "If I take 1/2 of the 2/3 I get 2/6 and that reduces to 1/3."

Jeff says, "I see it now." (12/11/85 p.7-9)

Interaction: Interviews

Level 1----

Pamela Kaye is describing the differences between her first and fourth hour general math classes. She reported that her first hour was much more quiet and much less responsive than her fourth hour class.

Pamela: The first hour class runs well. I don't have to spend a lot of time corraling people. I am not sure they are any more attentive than my fourth hour group which is usually bouncing off the walls. Now that I think about it I am not sure that they are any more attentive--I think that was a misconception on my part earlier when I thought the class was really going well. Now, I am not so sure. I think I was misreading their apparent attention because I wasn't getting a lot of expression. They were being quiet and they appeared to be listening but I wasn't getting the other signs I look for like facial expressions. (Pre-Project)

Level 2----

Pamela: One big thing I have done this year has been questioning my students as to how they arrived at a particular answer. I question both right and wrong answers. Questioning right answers is something I have not done a lot of in the past. I have questioned students about the processes when I knew they were doing them wrong. This time I focused on questioning students on concepts as well as on their answers. That has been distinctly different from what I have done in the past. (Intervention 1/83)

Level 3----

Anne: What different teaching techniques have you tried?

Pamela: I think the questioning if students responses in large group situations. That is probably on of the biggest changes that I have made. I don't just focus on asking questions where students give me a single answer response.I try to find out where the students got their answers from and if the question is, for example, "What is two and three?" and their response is, "Five", well, I then ask, "How did you get five?" I have tried to work on questioning students responses throughout the year. I think it has been most helpful to both me and the students for me to know where their misconceptions are even when they give me the right answer. It is also helpful for students because they see other classmates, whom they have always perceived as having the right answers, didn't necessarily know what they were talking about either. (Interim)

Level 3----

Pamela: I use a half and a whole for the students to constantly refer to. When we get to something like five-fourths my students don't tell me anymore, "Five-fourths is a top-heavy fraction so it will likely fall over and we have to do something to it and so you divide four into five." I don't give that rule anymore. I ask the students, "Is five-fourths as simple as you can get?" They tell me, "No."I then ask, "What is the matter with it?" The students say, "It is more than a whole." Then I ask, "If it is more than a whole how much more is it?" We go on through a way to find it. They see that a way to find it would be to find out how many fours there are in five--they see there is one whole and another one part--which is one-fourth. I talk about it as an improper fraction. I discovered one of the things I was doing was avoiding language such as denominator, numerator, and improper fraction because I felt it was getting in the way of the general math students' learning. Actually, it was the fact that they did not know these words that was getting in the way of their learning. They couldn't communicate with me about the things they didn't understand. I now try to stick with some more proper terms so the students and I can communicate. If they want me to draw a parallelogram instead of saying, "Draw one of those smushed-down boxes that got tipped over on one of its corners." They must use the definitions. I would have never believed that with general math students. I never believed that this was important.That may not be a big deal to you, but to me I really get excited when general math students can talk to me and to each other about what was happening. That just didn't happen much before. (MGMP Presentation)

Level 3----

Pamela: I do a lot of whole class diagnosis of where the students are by the questions I ask of them. An interesting thing happened during the last year of the General Math Project. It was at the end of the first semester when many students changed classes. I had the new students sit in class for a week before I asked them questions. I could see the new students looking around the room quite perplexed when one of my old students would respond to a question I asked. I would say something like, "What is the answer to number two?" And my old students would give me their answer and then follow that with a description of the process they used. When I finally called on the new students they just gave me the answer I would have to ask, "How did you get that?" What I didn't realize was that my first semester students had gotten to the point where they didn't wait for me to ask, "How did you get that answer?"They just gave me their reasoning along with the answer. That took the new students about two weeks to figure out what was going on in this class. (MGMP Presentation)

<u>Directions: Observations</u>

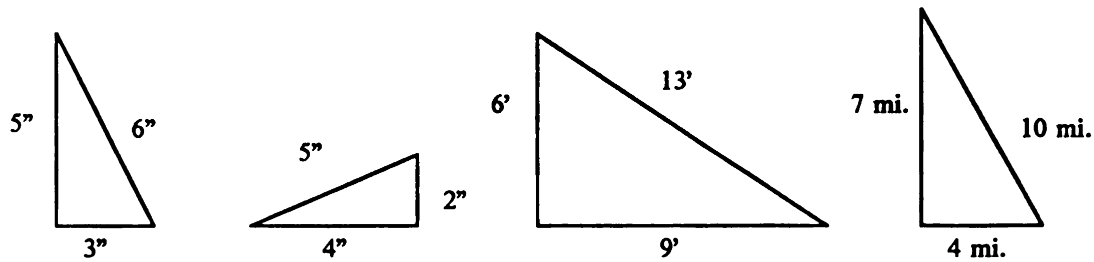
Level 1----

10:05
Ms. Kaye writes the directions on the chalkboard for a set of review problems on the area and perimeter of triangles:

Review Sheets-Hand In When Done

The bell to begin the class rings and the students are in the room but many are not in their seats. One student enters the room and notices the desks are arranged in groupings of four. He asks Ms. Kaye, "Are we working in groups today?" She tells him that they are.

Find the area and perimeter of the following.
Label Answer



A=___ A=___ A=___ A=___

P=___ P=___ P=___ P=___

10:08
The students are in their groups and are working on the review sheet. Ms. Kaye is taking attendance and dealing with other things at her desk. There seems to be some confusion in the groups as to what the students are to do on the review sheet.

10:11
Ms. Kaye looks over her copy of the review sheet and announces to the students, "Folks, I forgot to put Area and Perimeter on the triangles in the middle of the page. You will need to put them in yourselves." She returns to her work. One student from a group calls me over to the group and asks me to tell them what they were supposed to do because they don't understand both the directions on the review sheet and the ones that Ms. Kaye has just given them. (10/5/82 p.1)

Level 2----

10:41
Ms. Kaye writes the problem on the chalkboard:

1 2 3 4 5
----
4 5 6 7 8

Ms. Kaye says to the students, "James, what is the rule for this one?"
James tells her, "Take the term number and add one." Ms. Kaye says, "We take the term number and multiply it by the constant difference and then add one." Steve repeats what Ms. Kaye said.

10:43
Ms. Kaye tells the students, "What we are trying to figure out is a way to say that. The term number multiplied by one and add three." Steve suggests, "We could go t x 1 + 3. Is that a rule?" Ms. Kaye asks Mary "Steve said t x 1 + 3. If you take the fourth term and apply his rule do you get the answer?" Mary says, "Yes. Because four times one is four and three added to that makes it seven." Ms. Kaye, "Well, let's do number three together." Jeff says, "I dont want to do that one, it's too hard!" Ms. Kaye writes o the chalkboard:

1	2	3	4
4	9	14	19

Ms. Kaye continues, "Take the first term number and notice that we're trying to get an answer of four. You have to take one away so you would take the term number times five minus one and that would equal the result. Now, I would like to see what you can do with the rest of these numbers." (4/18/83 p.6)

Level 3----

9:08
Ms. Kaye hands out a commercially prepared worksheet from the problem solving unit the class is working on. She reads the directions as the students follow along. The directions are:

WORKSHEET 3--Make a Sequence

Compute the first seven terms in each sequence.

EXAMPLE: Use the term number, multiply it by 5.

1st Term	2nd Term	3rd Term	4th Term	5th Term
1 x 5	2 x 5	3 x 5	4 x 5	___ x ___
5	10	15	______	______

Ms. Kaye asks the students, "It says to COMPUTE. What does that mean?"The students tell her, "It means to figure it out." Ms. Kaye continues, It says to use the term number, multiply it by five. Would you please write the fifth term?" The students complete the example as Ms. Kaye walks around the room checking their answers.

One student tells her the fifth term would be twenty-five. Another student asks her if he can use a calculator. Ms. Kaye tells the students,"So, you would multiply the term number by the number that is showing on your page." Ms. Kaye quickly checks to see if all the students are correctly working the problem. She tells them, "You can continue from there." (9/20/85 p.1)

Directions: Interviews
There were no interview statements to indicate different levels of the teacher's thinking about giving directions to students.

<u>Discussion: Observations</u>

Level 1----

10:53
At the end of the period Ms. Kaye summarizes a lesson on fractions. She asks Sue, "Sue, will you tell us what the difference is between an improper fraction and a proper fraction?" Sue tells her, "An improper fraction is bigger on the top. It is top-heavy." Ms. Kaye asks, "Bigger than what?" Sue replies, "Bigger than the bottom."

Ms. Kaye asks, "James, tell us what is a proper fraction?" James says, "It is a fraction where the top is smaller." Steve says, "A proper fraction is smaller than a whole." Ms. Kaye asks, "Jeff, how do you identify fractions equal to one?" Jeff tells her, "They are even." Ms. Kaye asks,"What do you mean?" Jeff, "I mean they are the same."

Ms. Kaye writes on the chalkboard:

Proper	$\frac{\text{SMALLER}}{\text{BIGGER}}$	smaller than a whole
Improper	$\frac{\text{BIGGER}}{\text{SMALLER}}$	larger than a whole
One	$\frac{\text{SAME}}{\text{SAME}}$	equal to one whole

(10/29/82 p.4)

Level 2----

10:53
Ms. Kaye tells the students, "Now, I want you to take your 100-square grids and shade in one-half of the large squares." The students follow her directions. She asks, "How many squares did youend up shading?" The students tell her,"Fifty." She continues, "Can you tell me what percent that is?" Donald tells her, "Fifty, no, one-half, no, fifty percent." Ms. Kaye says, "Fifty percent." Donald says, "Right, that's what I said." Ms. Kaye says, "You had trouble going backwards. From fifty percent you had trouble going to a fraction. Fifty percent is fifty out of one hundred squares. It is the same as one out of two squares." (1/10/83 p.5)

Level 3----

8:26
Let's look over the next problem. How many halves is that altogether?" The picture is written on the chalkboard:

Frank tells her, "Six." Joan tells her, "Five." Ms. Kaye asks Joan, "How?" Joan says, "Count them." Ms. Kaye, "Alright." She draws the following on the board:

2 1/2 = 5/2

Jack says, "Ms. Kaye, you made a mistake. You were supposed to have five-sixths instead because you have six halves." Ms. Kaye asks him, "All right, how many should we have?" Jack replies, "Six." Frank tells him,"Only two. Look, there are six here, right? But the bottom number is the number of pieces in only one of them." Richard tells Ms. Kaye, "I was all right until Jack confused me."

Ms. Kaye tells the students, "If I have two and one-half and if I shade in the other empty half would I have six?"

Some of the students start arguing whether or not the result is three or six. Ms. Kaye says, "Frank, these people are confused. Can you help them out? Can you explain it to them? Frank says, "It is because one thing is divided into six parts." Ms. Kaye says, "You have one whole and it is divided into how many parts?" Frank tells her, "Two." She continues, "So you have two-halves, two-halves, and one-half more. So that would be what?" The students tell her, "Five-halves."

Ms. Kaye tells the students, "Jack brought up a very important concept in fractions, Dave, can you tell me why he said six?" Dave, "Because you had six halves." (11/15/84 p.4)

<u>Discussion:Interviews</u>

Level 1----

Pamela: One day recently we were working on interest problems and I was putting up problems like buying motorcycles and cross country skis and Calvin Klein jeans and we did sales tax. For that lesson there was a lot of interest. I'm sure part of it was the topics and it was the kinds of things they were interested in. That is the opposite from the usual. I usually have to pull responses from them. They were giving responses without having to be specifically called on by me. There were more than the regular two students who usually answer.

Anne: So, when you have more than two or three students who respond to you then, for you, that is an indication the class period is going well?

Pamela: Yes, in that class it is. (Pre-Project)

Level 2----

Anne: Is there any difference this year in the opportunities the students had to respond to or discuss the content?

Pamela: I think the fact that they were talking more in groups gave them a different way of responding than they would have had if I would have been doing the little I have done in the past. Although in the past they would have talked to one another, there was a difference in the quality of the conversation. The previous conversations would have been more just "How do you do this?"-type.

Anne: Do you think they discussed more as a whole group this year?

Pamela: I am not sure in terms of the total group discussion exactly how much more. It's hard to compare because we tried to do so much during the course of the year. Certainly when the Consultant was in the room there was more discussion going on. But I think the discussion that went on was of much better quality. I would like to try and draw more from them. I was not making real serious efforts at trying to get them to discuss--I was just trying to get through this year's materials. That was something I had never done before.

Anne: If you can recall the time you taught area and perimeter, what kind of discussion took place between you and your students?

Pamela: There was more discussion going on there. Again, in what I would have formally done in the past would have been to show them how to do one type of problem and they would only work on that kind. They really don't have to think what the content of the problem was. In the perimeter and area unit they did discuss more because their problems were not fitting into one particular content area. (Problem Solving Intensive Intervention-Year 1)

Level 3---

Anne: Would you comment on your first Yearly Improvement Plan strategy, "Engaging the students in more talking about mathematics."

Pamela: It is still one thing that I need to keep uppermost in my mind. I think it is the most important thing we can do--to get students talking about mathematics. I found it is real easy to fall back into telling them or asking right/wrong questions. It got to the point that in my trying to do more discussion my students were anticipating my "How did you get that?" questions after a while. I noticed that particularly at the start of the second semester when the group of new students came in to the class. As I began to ask the new students those questions they just looked at me in an expression that said, "What do you mean, how did I get that?" Then I found my old students responding almost snobbily with, "I got it this way..." and then going through their routine. So I do think it is really important but it is something that you just have to keep working at all the time. (Final)

Level 3----

Pamela: My class is still very teacher-oriented. There is a lot more conversation that goes on back and forth now than has ever been done before. That is much improved. (Post Project)

Feedback/Expectations: Observations

Level 1----

8:59
Ms. Kaye reads the answers to the daily assignment. Todd says,"Wow! I got all of those wrong!"

Ms. Kaye responds, "I am not surprised about you guys missing those problems because they had fractions in them and we haven't had fraction problems like those as yet. We will be doing those tomorrow." (3/15/82 p.6)

Level 2----

Ms. Kaye has just finished reviewing the answers to a test that had been given the previous day.

10:32
Ms. Kaye tells the students, "Overall I was really pleased with your test scores. I think you did a super job." (1/10/83 p.3)

Level 3---- There were no selections from the classroom observations at Level 3.

Feedback/Expectations : (Interviews)

Level 1----

Anne: Describe the general mathematics students.

Pamela: Well, skill-wise, they are students who are still having trouble with basic operations. As a group they don't have enough computational skills to survive in algebra class. I have a number of girls and boysthat could make it in a regular class if they just had a little more confidence in their math ability. Probably the biggest characteristic would be that they are totally convinced they can't do math and they generally aren't real excited about school, but the number one factor is that they think there is no way they can ever do math. They think since nobody has been able to teach them up to this point, that there is no point in even attempting to try go learn it now because they probably couldn't do it now either. (Pre-Project)

Level 2----

Anne: Describe the general math students.

Pamela: I think they're students who have missed out in math somewhere and they need help getting over the hump. I think many of them believe they can't learn math when they first come in here. If I were to describe them as a total group I would say they are average, fun-loving students. Probably the biggest thing that stuck in my mind was that now there is a chance for the students to kind of come up front. When they see they can succeed they get more ambitious and become a lot more fun to work with. (Final)

Level 3----

(Pamela is talking about her students' reactions to feedback on a computational test.)

Anne: Are you more inclined to use more pre- and post-testing of a unit of instruction based on the feedback you got from your students?

Pamela: Oh, yes. It is so simple, but I would not have expected that kind of result. I don't show them the grade levels they are at--I just show them by how many grade levels they have increased since they took the test at the beginning of the school year. I just show them, for example, that they had increased by one and a half grade levels. When I start next year that will be part of my opening statement. I will tell them , "Here is what you will be able to do if you want to." I'll show them what this year's students did and how many grade levels they went up. I want them to know that if they want to do that I can teach them. I don't know if that will have a lot of significance to them at that point until they see their own grade level increase, but it may. Generally, when they come in here they hate math and are totally frightened or disgusted with it. They don't want to have anything to do with it. If they can se there is some light at the end of the tunnel, I think that will be significant to them. (Fraction Intensive Intervention Year 1)

Level 3----

Anne: Has your perception changed with regard to your students learning the content of the problem solving unit?

Pamela: Well, since I believed when I started this unit that they didn't have any understanding of it at all--yes, I guess my perceptions have changed. I guess it's not that I thought the students were incapable of doing them, it is just that I didn't know how to teach them. I felt like it was something that I couldn't do with a general math class. It wasn't their inadequacies as much as it was mine. I believe that they can now do problem solving and I know now that I can do something to facilitate their learning in how to do problem solving. I have changed radically in what I think they and I were capable of doing together.

(Problem Solving Intervention Year 1)

Level 3----

Pamela: Most of my students start out in the beginning of the school year with about a sixth grade ability level on the Shaw-Hiehle Computation Test. By the end of the year they are at the eighth grade level. That's still not where I want them to be, but they have gained a lot. There are extreme cases where I have had one student that gained five grade levels in one year. He didn't have any concept of what was going on, but as soon as you showed him a few basic things he went crazy! The attitude of most of my students' when they first come into my class is, "People have been telling me about this stuff for nine years and you aren't going to make this any different for me."

I give my students the Shaw-Hiehle Computation Test in the fall and again in January and I don't tell them what their scores are or what their exact grade levels are, but I do tell them how much they have improved. I will say, "You have improved a grade level and a half in one semester. Normally I would expect to have only half a grade level gain in one semester." Suddenly I see a difference in their opinion of what they can do in mathematics. Occasionally this has been dramatic. (MGMP Presentation)

LESSON STRUCTURE

Start of Class: Observations

Level 1----

8:00
Ms. Kaye is in the classroom by her desk. She is handing out lined paper and selling pencils to students. As she does this she talks with the students. The students are entering the room and socializing with one another and are taking their seats slowly.

8:03
Ms. Kaye is at her desk talking to a woman who came in the room. As she takes attendance she chats with the woman.

8:07
The students are in their seats and are chatting quietly with one another. Most of the students seem ready to begin the class. (3/15/82 p.1)

Level 2----

10:05
As the students enter the room they obtain a packet of worksheets from Ms. Kaye's desk. Ms. Kaye has the following written on the chalkboard:

General Math
-3 worksheets

Make sure you read the Wizard's hints. We'll go over the problems at the end.

The students take their seats and wait for Ms. Kaye to begin the lesson for the day. (4/27/83 p.1)

Level 3----

8:00
Ms. Kaye reminds the students as they come into the room that they are to pick up a beginning of class review sheet from a shelf behind her desk. This is a review of geometry terms that the students have been studying for the past few days.

As the students work on their review sheet Ms. Kaye takes attendance.

Randy asks, "Ms. Kaye, is this a test?" Ms. Kaye tells him, "No, if you want to you can work with somebody on it." Some students choose to work together and some work alone. Those students who are working together are discussing the meaning of the terms they are trying to find. The left hand side of the review sheet contains a listing of the geometry terms and the right side contains the definitions.
(9/15/83 p.1)

<u>Start of Class: Interviews</u>

Level 1----

Anne: Describe the start of a typical day in general math class.

Pamela: A typical day in general math class usually starts out by my handing out pencils to anywhere from two to five students. Also, trying to make sure that everybody has paper because they usually don't remember to bring that. Frequently, I have to give out a loaner book because someone usually forgets to bring their book. Then I take attendance. (Pre-Project)

Level 2----

Anne: Have you perceived any changes in your classroom this semester?

Pamela: I think the thing that stands out most to me is the beginning of class. It always seemed to me like it took a lot of time to get started. I think we ought to be able to get started in a couple of minutes and it has been five, six, or seven minutes to get everybody started.That's always been a frustration to me. I shifted part way through the semester to doing a review at the beginning of the class period. My students pick up their materials and now know that if it says "Review" at the top they're supposed to sit down and work on that. If I am consistent with having a review page at least three times a week then that helps a great deal.

One other thing, I used to be the passer-out of materials, but this semester I have tried to stay very consistent with having the students pick up the materials they would need from my desk before they sit down. They don't wait for me to pass them out. Every semester it takes a little longer to get everybody started, so this has been a management change for me. This has functioned better for me than trying to get everybody organized and get their attention--because now they are fairly well settled. (Fraction Intensive Intervention Year 1)

Level 3---

Anne: You said that you have changed the organization of your classroom instruction this year. What effects did that have on the students?

Pamela: I would say for probably at least a third of them it had an effect on them, even for the remaining part of the year. There were students that I could depend on for always coming in, picking up their materials, sitting down and taking a look at what was on the board and deciding whether it was something they were supposed to be doing. That only happened sporatically in the past, even though I had the materials out it was still a matter of my having to urge the students to get going. This year there were more students involved in math. They still took time to socialize along the way, but they were getting ready for the class to start instead of waiting for me to get them ready.

Anne: What was the effect of this on your role as a teacher?

Pamela: It made it a lot easier, I could deal with the little management things we have to do like the tardies, and handing out books, and get my attendance out. I don't have to stop in the middle of the class period to do that. I could even get some extra math in--like when some student would raise a question that was not part of the unit. I could talk to him before the lesson got underway.
(Problem Solving Intervention Year 1)

Level 3----

Anne: Can you think of anymore strategies you used this year to teach the content?

Pamela: Not necessarily in terms of the story problem unit. There are things I have worked on all year long--one of them was the idea of doing a preview at the beginning of the class and talking about what we were going to do that day and what we were going to do for the seatwork. That kind of thing I worked on all year long and I did continue it after the problem solving unit. I had their weekly assignments written on the chalkboard, just like I had done for the Algebra Twos. There were a number of students who became accustomed to coming in and looking at it. That was helpful to the general math students and it was somehing I plan to continue next year. (Problem Solving Intervention Year 1)

Level 3----

Anne: Have you changed any of your teaching techniques during the project?

Pamela: I added a review at the start of the period. I thought that went really well. It gave me some sanity while I got things rollng, and I also think that's probably the key to some of the high scores that the students had on the computation test. The over and over continuous review helped them recall what they were doing. (Final)

Level 3----

8:00
Ms. Kaye reminds the students as they come into the room that they are to pick up a beginning of class review sheet from a shelf behind her desk. This is a review of geometry terms that the students have been studying for the past few days.

As the students work on their review sheet Ms. Kaye takes attendance.

Randy asks, "Ms. Kaye, is this a test?" Ms. Kaye tells him, "No, if you want to you can work with somebody on it." Some students choose to work together and some work alone. Those students who are working together are discussing the meaning of the terms they are trying to find. The left hand side of the review sheet contains a listing of the geometry terms and the right side contains the definitions.
(9/15/83 p.1)

Lesson Development: Observations

Level 1----

8:08
Ms. Kaye has finished with taking attendance and the students are waiting for her to begin the lesson. She tells them, "You people are so patient with me this morning. Please turn to page 183. I am going to alter this a little bit. Yesterday it seemed to me that no one had a problem with adding integers." She writes the following on the chalkboard:

$$+2 \quad + \quad +3 \quad =$$

Ms. Kaye tells the students, "If you were going to add these, what would you do?" Fred tells her, "Plus five." Ms. Kaye says to the class, "The only problem you had yesterday was when the signs were mixed up." She writes the following on the chalkboard:

$$+5 \quad + \quad -2 \quad =$$

She asks Gene, "What would you get?" He tells her, "Plus three." Ms. Kaye continues, "All right, Debra, what would you have if you had this." She writes on the chalkboard:

$$+2 \quad + \quad -7 \quad =$$

Debra answers, "Minus five."

Ms. Kaye tells the students, "Today I want you to so the same kinds of problems except there are parentheses in them. Remember when you have parentheses you look at just what's inside the parentheses first."
She writes:

$$(+6 \quad + \quad +4) \quad + \quad -2$$

Ms. Kaye tells the students, "What do you have inside the parentheses?"
A student tells her, "Plus eight." Ms. Kaye writes on the chalkboard:

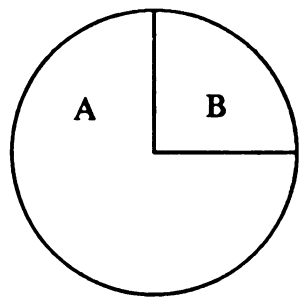
$$+2 \quad + \quad -8 \quad + \quad +4 \quad =$$

Ms. Kaye asks, "What do you get?" A student answers, "Minus two."

8:15
Ms. Kaye tells the students, "On page 189, I want you to work two rows of problems, they are rows two and four. There are some decimals in them. I want you to try them. I am not going to give you any more help on these now. Just try them." (3/18/82 p.1)

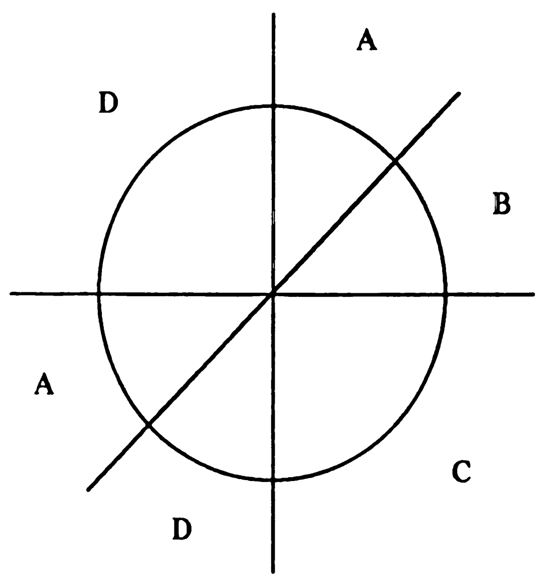
Level 2---

10:28
Ms. Kaye tells the student, "Look at your spinners. Some of you could tell me right away that the probability of 'A' would be more. What do you think the probability of 'A' would be on this spinner?" She draws the following spinner on the chalkboard:

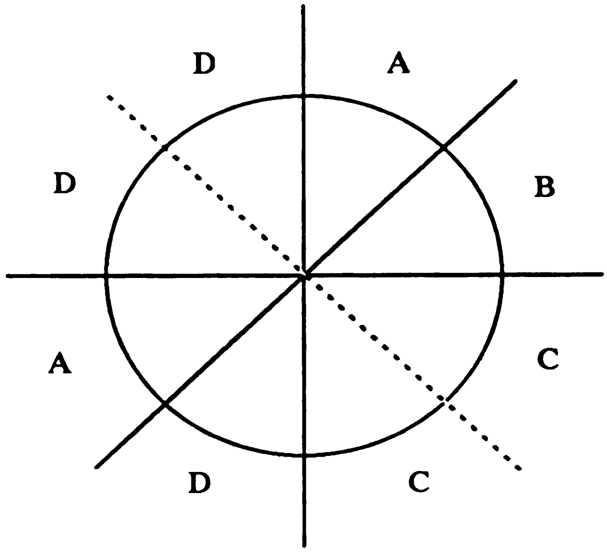


Ms. Kaye tells the students, "You were able to tell me that the probability of 'A' would be three-fourths without first thinking that it would be seventy-five hundredths. You could also tell me that the probability of 'B' would be one-fourth without first thinking it would be twenty-five hundredths. How did you know that?" One student tells her, "Because it looked like one-fourth of a circle."

Ms. Kaye asks the students, "Look at spinner number two. How many parts are there that are not equal to the rest?" She draws the spinner on the chalkboard:



A student tells her, "Two." Ms. Kaye tells the students, "If I cut the D and C into parts, then how many D's do I have?" Ms. Kaye draws the following:



Ms. Kaye asks, "What would be the probability of the pieces?" The students are able to tell her the probablities for the different pieces. She writes them on the chalkboard:

P(A) = 2/8
P(B) = 1/8
P(C) = 2/8
P(D) = 3/8

Ms. Kaye says, "James, If we were spinning for free lunches what would you want to pick for your letter?" James tells her, "I would pick D." Ms. Kaye tells the class she wants them to do number three by themselves. (10/22/82 p.2)

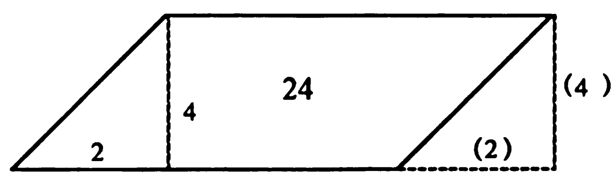
Level 3---

8:39
Ms. Kaye asks the students to take out the envelopes from their math folders which contain blue paper geometric pieces. She says, "Now, I want you to pull out all those little blue pieces that we were working on last Friday. Also all those little blue square papers that you have." She is referring to blue grid paper the students used. She continues, "What I want you to do is to have one person with the squared sheet draw the outline of one of the geometric pieces on it and I want both of you to figure out the area of that piece. Do the same thing for all your geometric pieces."

Ms. Kaye asks the students, "What is the area of piece fourteen?" A student says, "Twenty-one." Ms. Kaye asks, "How did you get that?" The student replies, "I just counted up the squares after I traced it." Ms. Kaye asks the students, "Is there any other way I could have done that?" Another student tells her, "You could figure it out by multiplying seven times three." Ms. Kaye says, "All right, take out piece number seven and trace that. Then figure out the area.. Put one of the sides of the piece on one of the lines on your grid paper. It is easier to do it that way. Just line up the bottom side with one of the lines." The students start working.

Jeff says, "Can give my estimate now?" Ms. Kaye tells him, "Yes." He says, "I got thirty-two and a half." Other students give their estimates for the area of this parallelogram. Their answers range from 28 to 32 squares. Ms. Kaye says to Mary, "Tell me how you figured out your answer?" Mary explains, "You just add up all the halves first then you add up the whole squares." Ms. Kaye asks, "How did you figure the number of squares, John?" John, "I remembered it from the last time." Ms. Kaye, "Oh?" John, "Well, I really multiplied."

Ms. Kaye asks, "Is there anything you could have done with the triangles?" Ms. Kaye has a drawing of the parallelogram on the board as she talks to the students:



A student tells her, "Well, you could put them together and you would get a square." Ms. Kaye says, "All right, if you did that you would have a four by two rectangle. So then you would have twenty-four squares in the rectangle and eight more which would give you thirty-two for the total. All your estimates were really good." (9/12/83 p.4)

Level 3---

Anne: Have you changed any of your teaching techniques during the project?

Pamela: I added a review at the start of the period. I thought that went really well. It gave me some sanity while I got things rolling, and I also think that's probably the key to some of the high scores that the students had on the computation test. The over and over continuous review helped them recall what they were doing. (Final)

Level 3----

8:00
Ms. Kaye reminds the students as they come into the room that they are to pick up a beginning of class review sheet from a shelf behind her desk. This is a review of geometry terms that the students have been studying for the past few days.

As the students work on their review sheet Ms. Kaye takes attendance.

Randy asks, "Ms. Kaye, is this a test?" Ms. Kaye tells him, "No, if you want to you can work with somebody on it." Some students choose to work together and some work alone. Those students who are working together are discussing the meaning of the terms they are trying to find. The left hand side of the review sheet contains a listing of the geometry terms and the right side contains the definitions. (9/15/83 p.1)

Level 3----

Pamela: Much of the class time, probably 20 minutes, is now spent in controlled practice activities. It is probably even more than that, because we are all working on problems together. (Final)

<u>Lesson Development: Interviews</u>

Level 1----

Anne: Is there a time when you felt the direct instruction has gone well and when you gave them the seatwork activity you find out it is not going well?

Pamela: Yes. I have guessed wrong on various occasions. At that point I may try to do some individual instruction. Sometimes I will have several people asking questions and so I will go around and after I find the fifth student asking me the same question, it is obvious that I have made an error. I have not explained something well enough. I just go back up to the front of the classroom and call the students attention back again. If there are students who already know what they're doing they can kind of ignore me. I don't like doing that, and I try to avoid it because it is really hard to draw them back again. Sometimes there is no other way of doing it at that point.

Anne: Is it due to an omission of something on your part or do you think there is some kind of miscommunication between you and the students?

Pamela: It could be either. Sometimes it has been that although what I was explaining was perfectly adequate, the students missed something. Like they could have missed the third step in a three-step process, and I had made the assumption that they knew the first two steps when they really didn't. Sometimes it has been that they just didn't put it together. (Pre-Project)

Level 1----

Anne: Could you give me an example of some teaching techniques you use?

Pamela: Most of what I do in my classroom is what works for me. I usually do a group lecture to start with and then I try to work one to one with the students. (Pre Project)

Level 2----

Not in interviews

Level 3----

Pamela: Much of the class time, probably 20 minutes, is now spent in controlled practice activities. It is probably even more than that, because we are all working on problems together. (Final)

Lesson Assigment: Observations

Level 1----

8:42
The students are working on their assignment and are not interacting with one another. Ms. Kaye is at her desk checking papers and writing the answers to the assignment on a piece of paper. Randy asks her, "How do you do this one?" Ms. Kaye walks over to Randy's desk to help him. Randy tells her, "See, I added this and this, is that right?" Ms. Kaye tells him, "Yes, that's correct." He tells her, "Is that all? That's all I have to do?" Ms. Kaye assures him that this was all he was supposed to do. Steve looks at Randy and says, "If everybody was done we could play cards." Steve is referring to the four students who form one of the daily card playing groups after they hand in their assignments. (3/17/82 p.3)

Level 2----

10:15
Ms.Kaye hands out a set of worksheets that have been stapled together to each student. The students have been working on the word problems on these worksheets for the past few days. Ms. Kaye tells the students, "This is what we are working on today. I would like to come around and see how you are doing with them. Part of the way through the hour we will check them." The students start working on their assignment. Two students are working on the asignment together. One student is explaining to the other how to solve the problem. The students are working together in groups discussing and solving the problems. (4/20/83 p.3)

Level 3----

10:20
The students are working on their assignment in pairs and in small groups. Their task is to solve a collection of word problems. Ms. Kaye is circulating around the room checking their progress. Two students are discussing a problem concerning misplaced box tops on containers of colored marbles. All of the students are working on the assignment and all are working together on the problems.Between 10:34 and 10:43 all of the students are working on their problems.

(Observer comment at the end of the period)
One of the most interesting outcomes of today's lesson was that the students remained engaged and on task for most of the class period. Most of the students worked in groups of two or more. It was unusual for me to see how engaged the students were in the word problems. Prior to Ms. Kaye's unit the students indicated little interest in problem solving. (4/27/83 p.4-6)

Lesson Assignment: Interviews

Level 1----

Anne: What is a typical lesson assignment period like?

Pamela: I spend anywhere from twenty to thirty minutes working with the kids on what they're supposed to be doing for the day. Then I give them a homework assignment that they try to complete in class. A lot of my time is spent trying to get students on task. Hopefully, the majority of my time is spent being able to work with the students one-on-one.

Anne: What would an a typical assignment period be like?

Pamela: If the students would immediately grasp whatever it was that I was trying to explain to them. Where I wouldn't have to explain it to them four or five times and in four or five different ways. Where the students would sit quietly and do their work with no problems and without bothering anybody else. (Pre Project)

Level 2---

Anne: What have you changed in your teaching style since you started this year?

Pamela: Well, I did utilize groups a little more. Not in the way that I had intended to at the beginning of the year, but I did allow more use of group work to do lesson assignments. In the process I found the students were discussing among themselves more. (Fraction Intensive Intervention-Year 1)

Level 3----

Anne: What was different in the way you taught your general math classes this year?

Pamela: Much of the work the students did was in a team situation. Instead of giving them an assignment of certain problems and having each student work on them alone, they worked on them together. That's probably the biggest single difference, students working cooperatively on problems. The students were talking back and forth to one another. In the problem solving unit they had something concrete to work with and I found it improved the quality of their questioning one another. Had they not been in groups I don't think I would have seen the same kinds of test results because I think they would have given up quickly. If I had used the same exact material and said, "All right, you've got to sit in your rows and work on this stuff," I think I would have gotten about five minutes work out of most of them--they would have just quit.

(Problem Solving Intensive Intervention Year 1)

End of Class: Observations

Level 1----

8:50
The students have finished their assignment and are chatting with one another.Six students have received "pink slips" from a student messenger from the main office indicating that they must see the assistant principal or the counselor for various reasons. Ms. Kaye sends these students to the office.

Ms. Kaye uses this time to talk to several students about non- math related things. She looks at the clock and announces, "Needless to say folks, with the number of people gone, we will correct papers later." She starts gathering up the student's papers as the students continue chatting or playing cards. Some students prepare to leave.

9:00
The bell rings and the students are dismissed. (3/16/82 p.4)

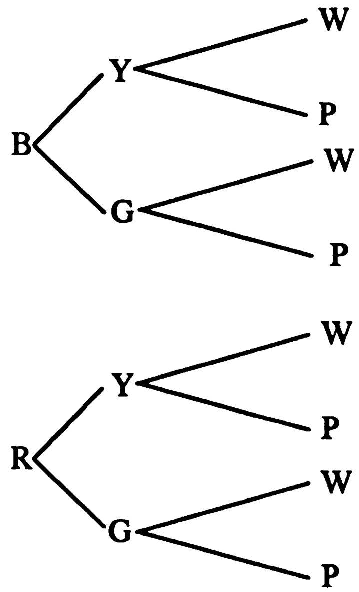
Level 2----

10:53
Ms. Kaye says to the students, "Would you turn your worksheets over, please? I would like you to do this instead of what I had planned for the rest of the hour. I would like you to do another probability tree, only this time we are going to be using different colored coins or chips. The first chip will be blue on one side and red on the other. The second chip will be yellow on one side and green on the other. The third chip will be white on one side and pink on the other side o her, "Pink! That's cute!" Ms. Kaye says, "Cute, I like that! Now, Freda is going to flip one chip. What do you think she could get? I want you to tell me the outcomes of those colors." Ms. Kaye writes in the chalkboard:

Blue/Red	Yellow/Green	White/Pink	OUTCOMES
CHIP 1	CHIP 2	CHIP 3	

10:57
Ms. Kaye asks the students, "If Freda flips the first chip, what could she get?" The students tell her, "Blue or red." Ms. Kaye says, "All right, if Tom flips the second chip what could he get?" The students tell her, "Yellow or green." Ms. Kaye continues, "If Sue flips the third chip, what could she get?" The students tell her, "Pink or white." Ms. Kaye says, "Then I want you to make a probability tree for the outcomes of the tosses of the three chips."

10:59
The students are working and Ms. Kaye is checking their answers. She goes to the chalkboard and shows them what the tree would look like:

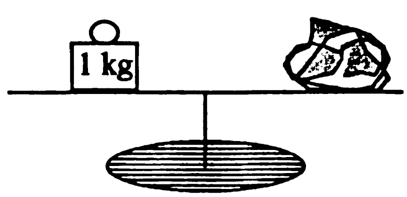


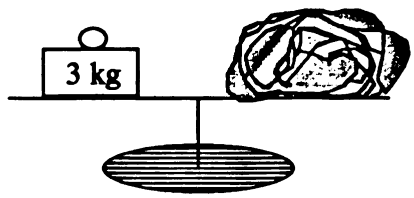
11:00
The bell rings and Ms. Kaye dismisses the students. (2/23/83 p.5)

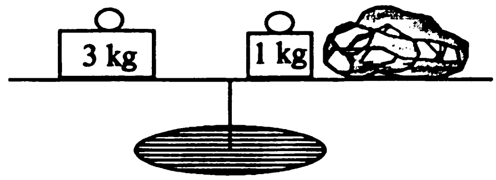
Level 3----

9:50
Ms. Kaye tells the students, "Your last question for the day--for those of you who get the answer there will be extra points. You have a balance scale and you are trying to weight this meat. You have only four weights to work with....a 1 kilogram, a 3 kilogram, a 9 kilogram, and a 27 kilogram weight. You have bundles of meat ranging from 1 to 40 kilograms and you have to weigh them. You can weigh them with these just four weights."

Ms. Kaye demonstrates for the students how to weigh a 1kg., a 3kg., and a 2kg. bundle of meat:







9:54
The students start working on the problem. Ms. Kaye tells the students they can take their math folders and their review sheet home to study for their test. The students continue working as Ms. Kaye monitors their progress on the problem.

9:59
Ms. Kaye asks, "Does everybody have their notes from the board on this problem so it can be turned it in tomorrow?" The students tell her they do. The bell rings and Ms. Kaye dismisses them. (9/26/85 p. 9-10)

End of Class: Interiew

Level 1----

Anne: What motivates your general math students to complete their assignments?

Pamela: I have some alternatives in my room for them like playing checkers, backgammon, cards, that sort of thing. That is definitely intentional on my part and that is interwoven with the social part of general math that I think is important. In fact, I often find my students are trying to strike deals with me, they'll say, "If we take this home and promise to bring it back tomorrow can we play cards?" So I think that is a motivator and one of the purposes of having these items available.

Anne: Did you instigate that yourself or did your students?

Pamela: I did. I have received substantial criticism from the administration for that, but I fully believe this is important for these students. They also know I won't hassle them about talking with one another if they are done with their assignments. I have made the statement any number of times, that I don't care if they're talking as long as they're done with their work and that the person they are talking with is also done. (Pre Project)

Level 2---

No evidence of Level 2 in the Interviews.

Level 3---

Anne: What changes could be made that would improve general mathematics classes?

Pamela: We need to improve the time we spend on mathematics. We have got to do that in all of the classes and specifically in general mathematics. That includes getting the students on task when they first come in the room with a little review and working them down right to the very end of the hour. Oh, yes, they do complain that they don't have any time to socialize anymore but it is important that they work until the end. (Post Project)

APPENDIX L

CLASSROOM CONSULTATION: MATHEMATICAL CONTENT

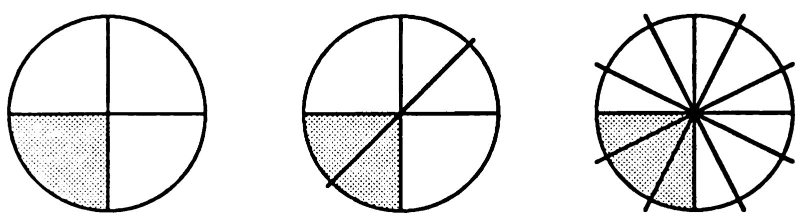
Appendix L

Classroom Consultation: Mathematical Content

Pamela and Perry discussed fraction equivalents and related tasks as they planned for the next day's lesson. Prior to their planning session he gave Pamela the following written suggestions for the lesson.

Perry:
For tomorrow, we might work more with equivalency by giving them symbols as stimulus and asking them to construct appropriate pictures. An added notion could be dealing with less than/more than.

1. Draw a circle picture for 1/4.
   Draw a picture that shows 1/4 to be equal to 2/8 and to 3/12.



2. Draw a picture that shows:
   1/4 is not equivalent to 1/3.
   1/4 is not equivalent to 1/6.
   1/4 is not equivalent to 3/8

3. Which of the following is true?

   1/4 = 2/8 = 3/12

   1/4 < 1/3

   1/4 > 1/6

   1/4 < 3/8

2. Have another task that might be the same/similar to one above using 2/3

Perry suggested the students draw pictures of the fraction symbols and compare them. During the planning session, he and Pamela Kay designed a task that would involve students in comparing different fractions, ordering fractions and deciding if fractions were equivalent or non-equivalent. The following is their discussion:

Perry:...moving on to the equivalency stuff...

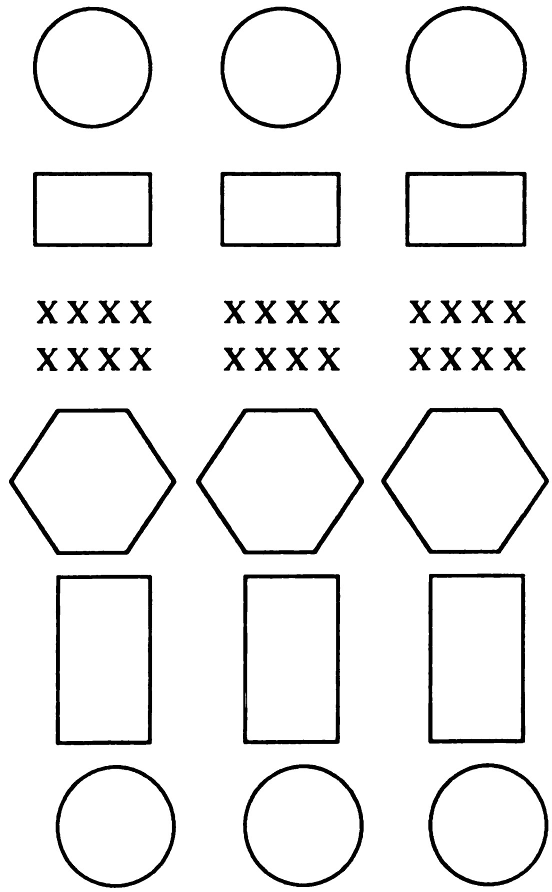
> Pamela: I intended to go ahead and use the less than and the greater than signs and the unequal and equal sign with them tomorrow. This wouldn't be too confusing to them would it?
>
> Perry: No. The thing I thought about was to put three fractions on the top of the paper and then to have the students place the symbols on with the fractions. I was trying to get at the notion of ordering and putting the fraction with the symbol.
>
> Pamela: Can I use the word "middle" instead of "next" on the sheet?
>
> Perry: Yes.
>
> Pamela: All right, I'll think about it.
>
> Perry: I tried to think about the things that would be close. Are you giving them the picture?
>
> Pamela: I could give them a circle first and then use a rectangle for the next set, etc.
>
> Perry: Yes.
>
> Pamela: I can do a number of these.
>
> Perry: I think this gets us into the equivalency stuff--this will bring more non-examples for them and I think that is important. You might put some fractions that are equivalent and that would be interesting.

In this session Perry talked with Pamela about selecting a task that encouraged the development of the concept of equivalency and one that would be interesting enough to engage the students. He suggested students do the following: (a) work with fractional equalities and inequalities, (b) order sets of fractions, and (c) link fractional symbols with their pictorial representations. As a result of this planning session, Pamela designed the following worksheet as one math task she would give her students the next day:

GENERAL MATHEMATICS

Place each group of fractions in order from largest to smallest.
Use your fraction pieces to check the sizes.
Draw a picture in the figures provided, showing each fraction.

FRACTIONS	IN ORDER	LARGEST	NEXT	SMALLEST
$\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{2}$				
$\frac{1}{3}$ $\frac{2}{5}$ $\frac{2}{4}$				
$\frac{1}{8}$ $\frac{1}{4}$ 1				
$\frac{3}{5}$ $\frac{3}{8}$ $\frac{3}{12}$				
$\frac{3}{10}$ $\frac{5}{10}$ $\frac{1}{10}$				
$\frac{3}{6}$ $\frac{4}{8}$ $\frac{2}{4}$				



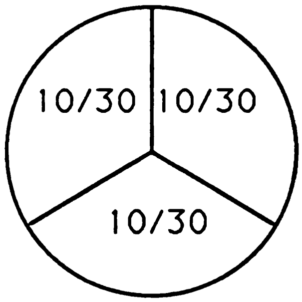
The following describes the class period in which Pamela implemented Perry's suggestions. The focus of this description is the mathematical content and tasks. The direct instruction began when Pamela Kaye reviewed the previous day's work at the board. During this time she involved the students in a discussion about patterns for equivalent fractions.

> Ms. Kaye says to the class, "We found equivalent fractions for one-third and one-half and one-fourth, but we didn't for two-thirds yet. What do you suppose you would do to get fractions equal to two-thirds?"

On the board she has written:

2/3 = 4/6 = 6/9 = __/12 = . . . = __/30

She continues, "All right, Miss Dyer, what would you do for two-thirds?" Mary, "You get eight by timesing, I'm not too good at explaining it though."
Ms. Kaye to Mary, "I think your doing a fine job. What would you do for 30? Melanie what did you get?"
Melanie, "I'm not quite sure."
Ms. Kaye, "Well, how many of those would be in one third?"
Melanie, "Ten."
Ms. Kaye writes on the board:

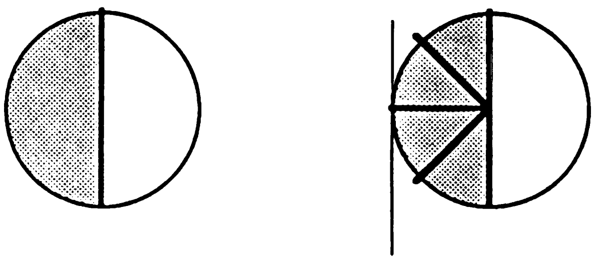


She tells the students, "You had the ten-thirtieths for one-third and then another ten-thirtieths for one more third and then another ten-thirtieths for one-third."

Randy, "Yeah, but that last one doesn't count!"
Randy means that since they were only interested in two-thirds and not three-thirds they only needed to be concerned with twenty of the thirty pieces.

Pamela Kaye then reviewed addition of fractions equal to one whole, addition of fractional equivalents (1/2 + 1/2, 1/2 + 2/4, 1/2 + 4/8) and addition of non-equivalents (1/2 + 1/3 + 1/6). The students used the pieces from their fraction kits to determine which combinations gave sums equal to one whole. Pamela drew a picture of 1/2 + 4/8 = ? to help students understand the answer:

Ms. Kaye, "What is one-half plus four-eighths?"
William, "Five-eighths, cause two goes into eight and so you use the eight."
Ms. Kaye, "If I wanted to draw a picture ..."



She continues, "So one-half and four-eighths is equal to one-half. So what is one-half and one-half equal to?"
The students, "One whole."
William, "I don't get it. I didn't reduce."
Ms. Kaye, "Well, you did another error in there but we'll get to it eventually."

With the review completed Pamela Kaye started the lesson for the day. She discussed the relationship of three-fifths to one whole, the inequality symbols, and ordering sets of fractions. As she talked, the students used their fraction pieces and followed her instructions:

Ms. Kaye, "I want you to think about three-fifths. I would like you to get your pieces out. Is three-fifths greater or less than or equal to one whole?"

The students take their fractional pieces from their envelopes and
lay them on the whole circles. They observe three-fifths is less than one whole.

Pamela writes on the board three-fifths less than one whole and below that five-fifths equals one whole.

Pamela asks, "How many fifths would you need to make it bigger than one whole?"
Randy, "Six."
On the board she has:

3/5 = less than

5/5 = equal to

6/5 = greater than

She continues, "Does anyone remember the sign for less than or greater than?"
Karla responds, "An alligator."
Kenneth says, "Mrs. Jones showed me."
Randy, "No, it's Pac Man." (meaning Pac Man's mouth)
Ms. Kaye, "My algebra students forget this all the time."
Karla, "That's surprising, albegra students are supposed to know everything! I know the big part goes to the biggest fraction, because you want to eat the biggest part of the pie."

Ms. Kaye continues. "Which one of these signs, the A sign or the B sign would you use with three-fifths?"

B A

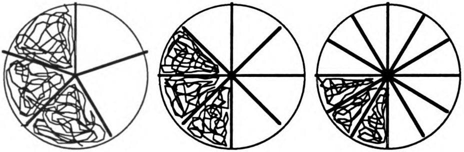
She tells the class, "The three-fifths is less than one and you would write it like this."
Karla, "Those were on the assessment test, you know, those problems like which is bigger?"
Ms. Kaye, "Sometimes those are hard."
Karla, "Yeah, sometimes."
Randy, "Yeah, but I understand it now."

Ms. Kaye, "Suppose I gave you three fractions?"
Randy, "You can't do it with three fractions."
Ms. Kaye writes on the board:

1/2 1/3 1/5

The students look at the fractions one-half, one-third, and one-fifth on the board. One of the students say, "One-half is bigger, one-third is next, and one-fifth is smallest."

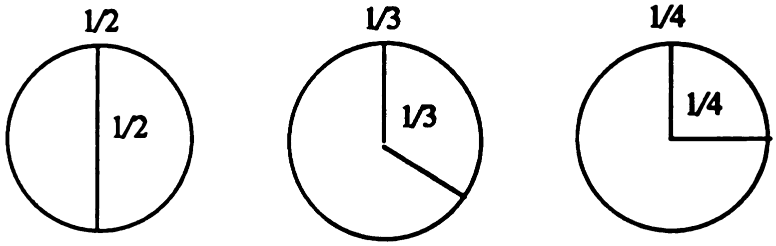
Ms. Kaye, "What about three-fifths, three-eighths, and three-twelvths?"
Karla lays her fraction parts over the whole and compares three-fifths, three-eights, three-twelfths, then looks at Ms. Kaye, and says, "The three-fifths is bigger than eighths and eighths are bigger than twelfths."
Ms. Kaye draws pictures on the three fractions on the board to show the class.



In this presentation Pamela combined the use of manipulatives, pictorial representations and the fractional symbols to help the students understand fraction inequalities and order.

The students were given the lesson assignment and were told if they didn't finish the worksheet during the period it would be considered homework. Pamela reminded them to use their fraction kit pieces to check their answers.

Ms. Kaye tells the class, "All right, I have one thing for you to do today ..."
Kenneth, "Can we work in groups?"
Ms. Kaye says to the class, "Notice that the directions tell you to use your pieces to check if you need to."
Dr. Lanier goes to the board and writes the first problem on the board for the students.



William says to Leonard, "I'll give you some answers if you'll let me use your eraser."

Ms. Kaye, passing near William, hears it and says, "You turkey!"
Dr. Lanier, says to the class, "Here is the first one I've done for you."

Ms. Kaye tells the class that if they don't get done with this it will be their homework ... As the bell rings the students are putting their papers in the folders and are putting them in books to take home.

The bell rings and the students leave.

After the students leave Randy tells Ms. Kaye, "This is the first time I ever understood fractions! It's the first time I could ever do them!"

At the end of the lesson Perry wrote his feedback regarding the lesson and gave it to Pamela. The feedback that focused on the Mathematical Content included the following comments:

<u>Reviewing the Mathematical Tasks</u>:

1. The concept of equivalence was reinforced with good use of terminology including <u>equivalence</u>, <u>multiples</u>, <u>denominators</u>, <u>wholes</u>.
2. Also, good elicitation of student thoughts.
3. <u>Suggestion</u>: Push the linkage between pictorial, circle parts, and symbol.
4. Your use of <u>non-examples</u> with equivalence and into the ordering was also effective.
5. Compared to yesterday, the consideration for the verbal input of others was astounding, especially given the extensive period of development.
6. <u>Observation</u>: Generally, that the length of time without a task other than listening is perilous with a general math class.
7. Worksheets look good.

After Pamela Kaye read Perry's feedback she responded to his comment regarding the amount of time she spent talking with the students (number 6 above). She told him she knew she could have stopped the discussion/presentation at anytime but chose not to because the students were asking good questions and seemed attentive to her and each other. She said she believed they seemed to grasp the ideas in the lesson.

Pamela decided to continue the lesson another day because she wanted to review this assignment and give them another worksheet which reviewed the content covered the past three days. The worksheet for the next day included activities similar to those in the worksheet she and Perry had worked on previously and included work that: (a) linked the pictorial with the symbolic representations of fractions;(b) used manipulatives; (c) used number patterns and relationships in fractional equivalents; (d) used correct and incorrect answers to understand fractional operations; and (e) showed fraction equivalency through part/whole pictorial representations.

This description of a planning session, the selected observations and the Consultant's feedback focused on the Category of Mathematical Content. The Topics included in this Category are Orientation, Presentation, Topic and Task. The Orientation of both the lesson and Consultation session focused on students' understanding fraction concepts. The Presentation of the lesson included the use of manipulatives and pictures to talk about and describe the fraction concepts. The students were actively engaged in working along with Pamela as they explored the ideas. The mathematical Topic, although familiar to the students, was presented in a different way-through the use of manipulatives and illustrations. The Tasks the students were asked to do for the lesson assignment and during the lesson development were interesting to them and encouraged the development of their understandings of the fraction concepts.

APPENDIX M

CHANGES IN PAMELA KAYE'S GENERAL MATHEMATICS CURRICULUM

Appendix M

Changes in Pamela Kaye's General Mathematics Curriculum

Pre-Intervention Year

Review of Whole Numbers:
Add, Subtract, Multiply, Divide

Review of Decimals:
Add, Subtract, Multiply, Divide

Review of Fractions:
Add, Subtract, Multiply, Divide

Percents:
Computation

Geometry:
Measurement, Shapes
Area Formulas

First Intervention Year

First Semester

1 wk. Orientation
  Consumer Math
1 wk. Measurement
3 wks. Using Formulas
2 wks. Decimals
3 wks. Calculators
5 wks. Fractions
1 wk. Percents
  Concepts
  Computation
  Problem Solving
3 wks. Coordinate Graphing
  Semester Exams

Second Semester

1 wk. Orientation
2 wks. Coordinate Graphing
  Fractions
  Decimals
3 wks. Probability
4 wks. Introduction to Algebra
3 wks. Statistics, Graphs, Charts
5 wks. Consumer Use of
  Mathematics
2 wks. Review and Testing

Second Intervention Year

First Semester

1 wk. Pretesting
  Tangrams
  Pentominoes
1 wk. Areas-- Concept of Formulas
1 wk. Review Fractions
  Concepts and Relationships
3 wks. Problem Solving Strategies
2 wks. Factors & Multiples
1 wk. Review Fractions
1 wk. Fractions
  Addition and Multiplication
4 wks. Decimals
  Concepts
  Relation to Fractions
1 wk. Percents
  Introduction to Concept
1 wk. Review and Testing

Second Semester

1 wk. Review
  Fractions
  Problem Solving Strategy
  Making Tables
3 wks. Fractions
5 wks. Probability Unit
4 wks. Similarity Unit
2 wks. Graphing
2 wks. Integers
2 wks. Algebra
2 wks. Reviews
  Problem Solving
  Estimation

Second Post Project Year

First Semester

1 wk. Surface Area and Volume
  Making Manipulatives
  Tangrams
3 wks. Problem Solving Strategies
  Guess and Check
  Making Tables
  Finding Patterns
3 wks. Factors and Multiples
1 wk. Fraction Bars
  Manipulatives
  Fraction Concepts
5 wks. Fraction Concepts
  Part-to-Whole
3 wks. Decimal Concepts
  Part-to-Whole
1 wk. Review

Second Semester

4 wks. Percents
  Part-to-Whole
4 wks. Probability unit
4 wks. Similarity unit
4 wks. Algebra
  Integers
  Operations
  Symbols
  Formulas
1 wk. Review
  Examinations

APPENDIX N

CLASSROOM CONSULTATION: COMMUNICATION PATTERNS

Appendix N

Classroom Consultation: Communication Patterns

The Instructional Improvement Category, Communication Patterns, and its Topics are the focus of the planning session and Perry's feedback. The planning session for the first day involved a discussion in which Perry stressed the importance of Pamela's establishing patterns of communication with her students about fractions and fraction concepts. The first Consultation segment describes their discussion regarding the start of class activity.

> Perry: Let's take a look at the task that you thought about.
>
> Pamela: Well, the first day of this unit is going to be somewhat problematic if we just have a discussion and try to get the answers from the students. I could ask them, "What does a fraction mean?" I would like to give them something to work from and go from there. I could have them draw what a fraction looks like and then start a discussion from there. We can talk about where fractions are used.
>
> Perry: We can also talk about what the terms numerator and denominator mean and maybe how they could remember them.
>
> Pamela: I did like Helen Taylor suggested in our Teacher-Researcher Meeting and told the students that numerator went up and denominator went down. When we talked about it it just made more sense to do it Helen's way.

Pamela Kaye talked with Perry about wanting to engage the students in some kind of controlled practice activity that would enable them to begin to talk about the meaning of fractions. As the planning session continued Perry suggested to Pamela that she link fraction symbols (i.e., 3/4) with written words (i.e., three-fourths) and pictorial representations.

> Perry: I would like you to do an embellishment of the fraction discussion. You might put the fraction numeral up or the word up on the board first. You might say it was a number and then attach the symbol to the number. You could write out three-fourths and then attach the numeral 3/4 to it. You could begin with, "What does it mean?"
>
> In years past I have had students who drew erroneous pictures for 3/4. I had one student who, for 3/4, wrote 3 large sticks and one small stick. We had a long

discussion about what that meant to him. That's a real example of a mistake that a lot of people make. It's helpful to look at wrong examples. If students have a wrong idea then another wrong idea like this creates dissonance for them.

Pamela: All right.

Pamela was not convinced of the value in using wrong examples with her students during the first day. In spite of her hesitation she indicated to Perry she would try this strategy. He and Pamela discussed the importance of talking about fraction ideas with the students and provided her with the following suggestions.

Perry: What you want to do is to get them to talk about the fractions and the ideas of fractions.

Pamela: All right, we could work with wrong and right ideas. Would you from there go into the notation of numerators and denominator?

Perry: From what we talked about we want to focus on the students' notions of fractions rather than going into the names.

Pamela: We used sets with that when we talked about the meaning of a fraction. If we do that we can use the number line at the same time.

Perry: We should talk about what we mean by the shaded and unshaded part in relation to the whole.

Pamela: What I do is that I usually tell the students when we talk about fractional parts like we mean the shaded part.

Perry: With the region idea you can talk about congruence and equivalent parts.

Pamela: "Right, we can talk about equal and unequal ..."

Perry: I think that I would conclude this introduction with the notion given to the students that in this fraction unit we're basically going to be dealing with these fractions: one-half, one-third, two-thirds, one-fourth, three-fourths, one-fifth, one-eighth, and then tenths and twelfths. It's really important that you re-emphasize the notion of part-to-whole throughout your presentation.

The suggestions Perry made during this session included an emphasis on using correct/incorrect ideas, sets, numbers lines, shaded/unshaded regions and congruent/equivalent parts. He mentioned the importance of encouraging students to talk about these ideas. The planning session concluded with Pamela writing her lesson plan for the class period. This plan is as follows.

1. Fractions - as they relate to number symbol and math language.

2. Include some wrong ideas, counter examples. Look at shaded and unshaded parts. Use sets four objects and eight objects. Use the numberline.

3. Talk about the fractions one-half, one-third, one-fourth, two-thirds, three-fourths, one-fifth, and eighths.

4. Students do their own representations using shaded and unshaded regions.

5. Brainstorm - what does a fraction mean? Where do we use fractions?

6. Talk about fractions as comparisons of parts to wholes. Numerator means up, denominator means down.

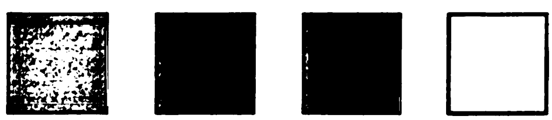
7. Have students make their fraction kits.

Pamela's lesson plan contained the elements of the Consultation session and reflected her interpretation of Perry's suggestions. The following is a description of the lesson which took place after the above planning session. The focus of the selected vignette is on the communication between the students and Pamela.

Ms. Kaye, "Let's find out what you know. Mr. Biggs, what does that say?"
Ms. Kaye has written the following on the chalkboard:

Three-fourths

Richard, "Three-fourths."
Ms. Kaye, "Have you seen seen it written any other way?"
Richard, "You could have four squares and three of them colored. That would equal three-fourths."
Ms. Kaye draws the following:

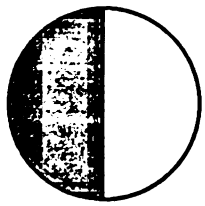


Ms. Kaye, "Does anyone see another way?"
Kenneth, "Draw a three, put a line under it and then put a four under that."
Ms. Kaye writes the following on the chalkboard:

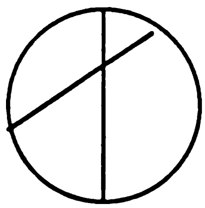
$$\frac{3}{4}$$

Ms. Kaye asks, "Is there another way?"
Kenneth says, "Draw a circle with three-fourths of it colored in."
Ms. Kaye, "Like this?"
She draws the following:

Kenneth, "No, it has part of it shaded in."
Ms. Kaye, "Oh, like this?"
She draws the following:

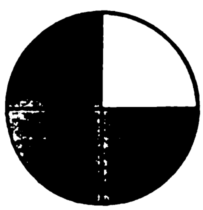


Kenneth, "No, do a line cutting across it then draw another line cutting it down."
Ms. Kaye draws the following:

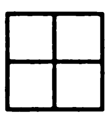


Kenneth, "No, like this." (He uses his finger and draws a vertical line in the air.) make it straight. "Make the line straight up. Then make the other line straight across." (He uses his finger to draw a horizontal line in the air.)

Pamela follows Kenneth's directions and draws the following:



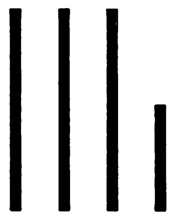
Ms. Kaye, "Is there another way?"
Richard, "Probably."
Kevin, "You could make a square and put a line in vertically, and then another one across."
Ms. Kaye draws this on the chalkboard:



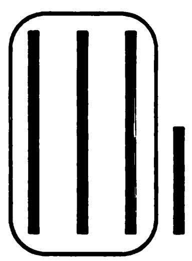
Ms. Kaye, "Which line is vertical?"
Kenneth, "It's the up and down line."
Ms. Kaye, "Then what is horizontal?"
A student, "The line going across."

Ms. Kaye takes the square and writes the word vertical on the vertical line and horizontal on the horizontal line.

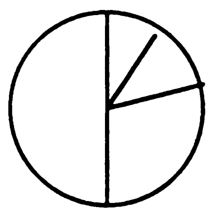
Ms. Kaye, "Do all the parts have to be equal?"
The students, "No."
Kenneth, "The parts don't have to be equal--it just makes it neater."
Ms. Kaye, "If I do this, is this three fourths?"
She draws the following:



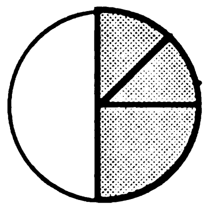
Kenneth, "No, that's three and a half."
Ms. Kaye, "What could I do to get three-fourths?"
Mary, "Put a circle around three of them."
Ms. Kaye follows Mary's directions and draws the following:



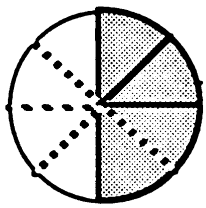
Ms. Kaye, "Well, what if I had done my circle like this?"
She draws the following:



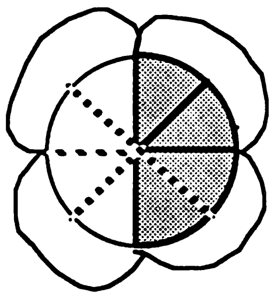
Ms. Kaye continues, "What if I had shaded it in like this."



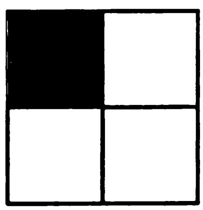
Shara, "They're not equal pieces."
Kenneth, "Well, you could make them equal, you could put in extra lines like this."
Ms. Kaye divides the circle she made with dotted lines:



One student says, "Now you could shade in every two of them."
Ms. Kaye does the following:



One student says, "It is just like a flower."
Kenneth, "You could make one of them squares just shade in one of them squares and just shade in one of them squares and leave the others blank."



Ms. Kaye, "If I ask what fraction is represented by this what would you then say? Would three-fourths be right or would it be wrong?"
The students, "No."
Ms. Kaye, "What I usually mean is that the shaded part is what we are talking about. Let's look at what we've done so far."

Pamela continued questioning the students for other ways to represent three-fourths, one-half, and four-eighths. She then started a discussion of the terms numerator and denominator.

Ms. Kaye writes the following on the chalkboard:

1 numerator
2 denominator

Ms. Kaye continues, "Have you seen these words before?"
A student responds, "Yes, last year our teacher told us once."
Ms. Kaye, "Could I say there are two pieces in the whole circle and the word whole is the one I want to emphasize. If you have a circle that means you have one piece in the whole. What would it look like if you had two over one?"
A student, "You would need two circles."

Ms. Kaye writes the following on the board:

1 part shaded
2 pieces in the whole

Ms. Kaye, "What I would like you to put on your paper now is this."

Ms. Kaye writes the following on the chalkboard:

numerator	1	1 part shaded
denominator	2	2 pieces in the whole

Kenneth, "Is there a quiz or something on this?"
Ms. Kaye, "Does this look like a quiz? I want you to underline the word 'part' and the word 'whole'."

Pamela asked the students to write the symbol, the terms, and the words to describe the meaning of the term one-half. This was followed by a discussion of the meaning of the terms for the fraction three-sevenths. She then returned to the fraction three-fourths and reviewed the meaning of numerator and denominator.

Ms. Kaye asks the students to draw four Xs on their papers.
Ms. Kaye, "How could you show three-fourths? You could circle three of them, right?"
A student says, "You could but it is more sophisticated to put a square around them."
William, "I made stars out of mine."

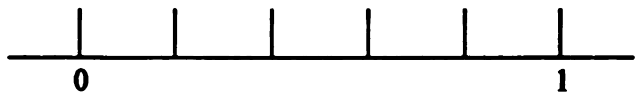
Ms. Kaye, "If I asked what fractional part are not circled then what would you say?"

The students, "One-fourth."
Ms. Kaye, "If I had four parts shaded out of four parts then what would I have?"

Ms. Kaye writes on the board

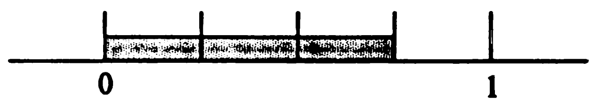
$$\frac{4}{4} = 1 \qquad \frac{0}{4}$$

Ms. Kaye, "We are going to look at part of the line and we are going to call this a number line with these segments. How are you going to show 3/4's?"
Kenneth tells Ms. Kaye, "Put in four lines between the zero and one."
Ms. Kaye draws the following:

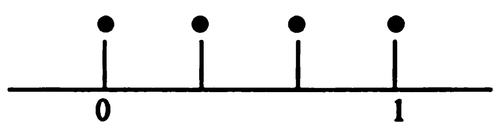


A student responds, "No, you just need three lines!"
Ms. Kaye changes the drawing to three lines.

The students say, "No, not like that!"
Ms. Kaye, "Melanie, what would you do?"
Melanie, "Make four spaces with three lines and shade them in like this."
Ms. Kaye draws the illustration:



She continues, "Someone has just shown us where a lot of mistakes occur. What Kenneth did was to count all the points on the line like this."



Ms. Kaye, "There are four points on that line but you need to have four segments, four line segments."
Kenneth, "Give us an illustration of that Ms. Kaye."

Ms. Kaye, "Class, what is a fraction?"
The students, "Parts to a whole."
Ms. Kaye, "What are the names for the parts?"
The students, "Numerators and denominators."
Ms. Kaye, "Can anyone think of a way to help us remember numerator and denominator?"
A student, "D for down and the other one is on top."

Ms. Kaye, "We are going to stop here for now. I am going to pass out some papers and magic markers and I want you to color in the circles according to the colors I have written on the papers."

Ms. Kaye collects the notes the students have taken during the lesson and hands out three sheets with circles on them. Each circle is divided into different equal parts. For the rest of the period the students make their fraction kit pieces.

This observation describes how Pamela introduced the unit on fraction concepts to her general mathematics students. The presentation just described took 40 minutes and all the students attended and participated in the discussion. Perry observed the lesson and gave Pamela his feedback regarding the lesson. The following is part of the feedbak he gave to Pamela.

1. Discussion and interaction about the TOPIC was very effective. It clarified the need for <u>parts</u> to be equal and congruent. I think some students are still very shakey about it. However, I believe continuing the emphasis and modeling will crystalize the notion for them.

   One suggestion on discussion: Consciously draw into the conversation more students. The participation of most of the students was good, but there were some who took over. Their dominance might interfere with optimal participation by all.

2. My hunch is that the linkage between the <u>whole</u> and your use of pictures, symbols, numerators, denominators, parts, in discussing fractions went relatively well for most students. Perhaps this linkage could be reinforced by beginning each illustration/explanation with a whole region or set of pictures and natural language descriptions by you.

   The linkage could also be reinforced by observing that the shaded and unshaded parts together make a whole. The zero-fourths was super!

3. Language--The use of geometric terms would have aided students considerably in their instruction to you on where to draw line segments to divide the figures, for example, diameters, diagonals, vertex, bisector, etc. Don't dwell on them but a brief explanation would, I think, facilitate their communication about the content.

4. The number line model is one that more use can be made of. They were pretty shakey with that.

5. For tomorrow I think you should begin with a brief discussion that emphasizes the whole in the context of the set or region and then the numberline would be productive. Then finish the kits following some variation activity.

Perry's feedback to Pamela focused on the various ways to represent the concepts through communication during the lesson presentation and the discussions with the students. In addition, he pointed out to her his observation that she ignored to bring some students into the conversation. His observations and suggestions provided Pamela with information on what she could think about as she planned for the next lesson. After Pamela read Perry's written feedback she talked with him. A portion of their conversation follows.

> Pamela: I know there was a problem with the students responding. It was hard to get the people pulled into the conversation today. I know I should be getting more. I'll work on that.
>
> You talked about the parts being equal and congruent when we talked about parts of the fraction if we agree that they needed to be the same shape and size I think the students will understand what congruent is. I'll probably just be using "equal parts". You said that it's important to have the linkage between the whole fraction and the pictures and symbols because you felt the students' linkage was weak.
>
> Perry: Yes, I thought that ...
>
> Pamela:
> Well, if we went ahead and did the addition thing like you suggested then ...
>
> Perry: I didn't want to get into that right away, but I thought if we just got a little bit into it that the students would be able to do it. Did you like the zero-fourths discussion you did in class?
>
> Pamela: Yeah, I thought it was something worth reviewing.
>
> Perry: I thought it just reinforced the notion of the whole. If you just reinforce the whole and don't make big deal about it I think you can just keep going back to it.
>
> Pamela: I thought I would review the part-to-whole and the congruent idea with the numerator and denominator terms. I thought I would draw some figures like that and have them do three-fourths of a rectangle and three-fourths of a circle and so on and then we can talk about equivalencies. You mentioned using a number line.
>
> Perry: Today I was kind of shakey in my part of the explanation.
>
> Pamela: Yes, me too. I started using the word distance and then went into the word segment.
>
> Perry: I was trying to reinforce the part-to-whole notion.

> Pamela: With the line segments I thought I would start with just using zero to one and then we can talk about thirds and quarters, three-fourths, and then go into parts of line segments and different areas of the line for example we can talk about the line segment between zero and one.
>
> Perry: I would stay with zero as the point of origin.
>
> Pamela: All right, I'll just do a couple of fractions and stay with the distance between zero and one. We skipped over the uses of fraction entirely. I was thinking about doing it after we talked about it. I like what you suggested for the worksheet.

While Pamela was pleased with Perry's positive reaction to her instruction, she responded more to his criticism of the places in the lesson where improvements were needed. The planning session, classroom instruction, and the feedback session focused on the importance of providing a variety of opportunities for students to communicate about the meanings of fraction concepts. Classroom Consultation during this unit helped Pamela Kaye improve her instruction in the Topics of Instructional Interactions and Discussions. Consultation also helped her find ways to integrate improvements in the Categories of Mathematical Content and Communication Patterns.

APPENDIX O

CLASSROOM CONSULTATION: LESSON STRUCTURE

Appendix O

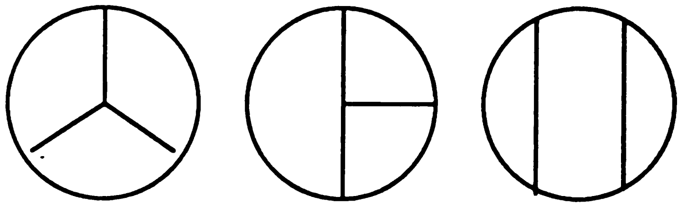
Classroom Consultation: Lesson Structure

The third Category, the Lesson Structure (social organization), was one Pamela did not consider until Phase IV. Throughout the Classroom Consultation unit Perry talked with Pamela frequently about ways she could improve her practice in this Category. The following planning session, description of the lesson, and the feedback session after the lesson focus on strategies Pamela started to implement to improve the Lesson Structure in the classroom. During a planning session Perry and Pamela discussed the lesson plan for the next day over the telephone. He suggested Pamela start with a review at the beginning of the period, then have the students complete their fraction kits, and wrap-up the period with a prepared worksheet activity where the students would use their fraction kits. The following classroom observation describes the organization strategies Pamela used to improve student participation and task engagement during direct instruction.

> The students walked into the room, picked up colored markers and scissors Ms. Kaye had on her desk, and started to complete their fraction kits. Ms. Kaye was in the hall until the bell rang and didn't notice the students started working on their own.
>
> When she entered the room she noticed them on task and decided to let them go ahead and finish their fraction kits as she took attendance. The students finished coloring and cutting out the pieces for their fraction kits for the next 20 minutes.
>
> Ms. Kaye writes the following on the chalkboard:

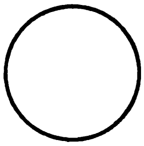
WORDS ► NUMBER/SYMBOL ► PICTURES ► OBJECTS ► REAL WORLD

A FRACTION IS: $\frac{\text{TOP}}{\text{BOTTOM}}$



A B C

1.



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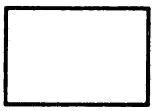
2.



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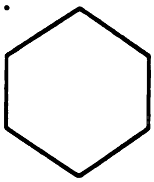
3.



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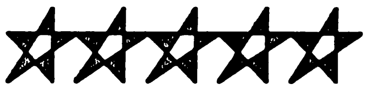
4.



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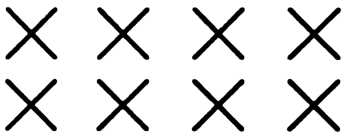
5.



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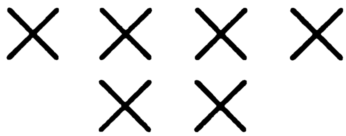
6.



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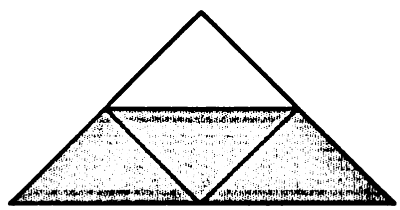
7.

3/4

ONE WHOLE SEGMENT

Pamela started the class with a review of the content which was covered the previous day. This beginning of class activity focused the students' attention on the fraction concepts. Pamela began the review with the students and asked several students to go to the chalkboard and work the problems.

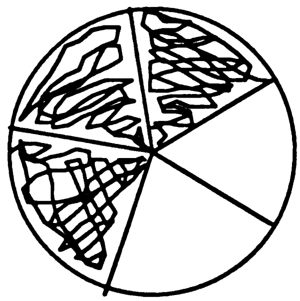
> Ms. Kaye is talking to the class. "Yesterday we talked about what a fraction was and I want to review that with you today. Gary, what does 3/4's mean?"
> Gary, (pauses a while) "There are four parts in an object and you want only three of them."
> Ms. Kaye, "Gary had the right idea here. You have three parts in the whole and you want to take three parts of them. Fractions compare parts to the whole thing. When we looked at the parts what did we call them?"
>
> Kenneth, "Numerators and denominators."
> Ms. Kaye, "Randy, is there a way to remember those?"
> Randy, "U means up, D means down. Also, N means north."
>
> Ms. Kaye, "If I asked you to write on the board the symbol for 3/4's what would you put?"
> Kenneth goes to the board, takes the chalk and writes the numeral 3/4's on the board.
>
> Ms. Kaye, "Can someone show me a picture of three-fourths?" Richard goes to the board and draws a triangle with an upside down triangle inside it and shades in three of the four pieces.



Pamela Kaye continued to discuss the part-to-whole concept of fraction as the students were asked to explain which of the three circles on the board (A, B, or C) divided into thirds. Ms. Kaye moved to the seven review problems she wrote on the chalkboard and told the students to write them out on lined paper.

> Ms. Kaye, "All right, on the circle, I want you to show me the fraction three-fifths."
>
> The students write the problem on their papers.
>
> One student said, "How can we do that?"
> Ms. Kaye, "Well, you do have a circle that is divided into five parts. You could

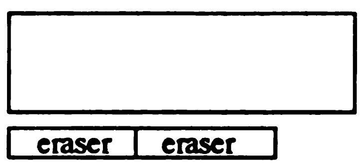
use that. That would help you with it."
Ms. Kaye tells the class, "O.K., I need a volunteer, Gary come up here and do the first on, three-fifths, for me. One thing that I've noticed is that they get a lot harder when you have to do them up here."
Gary draws the following:



The students comment on Gary's drawing of the circle by saying, "They aren't equal!"

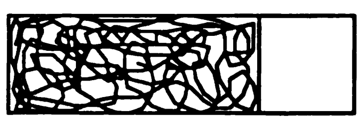
Ms. Kaye, "What's wrong?"
The students say, "They need to be even!"
Gary straightens up his drawing.

Ms. Kaye says, "O.K., Holly."
Holly goes to the board and draws or attempts to draw a solution to problem number two. She doesn't know how to divide the rectangle into three equal parts. She uses the length of the chalkboard eraser laying end to end and marking off the distance, to help her.



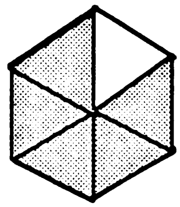
Mary says, "Careful Holly, this is a real toughy!"

Holly realizes that the eraser is too long and will not divide the rectangle into three equal parts and she says, "I can't do it!" She still attempts to divide it into thirds and she finally shades in about two-thirds of it.

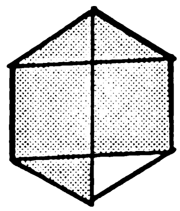


Dr. Lanier says, "I think that might be one-half. <u>One part</u> that is shaded and two parts that are unshaded."
The students respond, "No, he's just trying to confuse us!"
Dr. Lanier, "So, we have to compare the parts to the whole thing?" The students agree.

Ms. Kaye, "Let's go to five-sixths. Melanie."
Ms. Kaye calls Melanie up to the board to answer problem number three.
Melanie draws a nexagon and divides it into six equal parts and shades five parts.



Richard responds, "I didn't do mine that way -- I did mine the easy way!"
Ms. Kaye, "What did you do Richard? Show us."
Richard goes to the chalkboard, draws a hexagon and shades it this way:



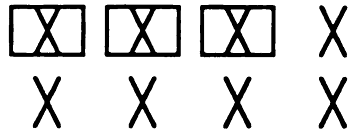
Ms. Kaye, "Richard did five-sixths this way, is he right?"
The students respond, "No."
Ms. Kaye, "Why is that not five-sixths?"
Karla, "Because the corner pieces are not the same size as the middle pieces are!"...
Kenneth interjects, "I didn't know they had to be congruent."

Ms. Kaye says, "William would you please do two-fifths of the stars?"
She asks William go to the board and show two-fifths of the five stars. William goes to the board and puts a line through two of the five stars.
Ms. Kaye, "You could circle two parts."

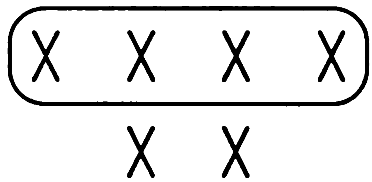
2/5

She continues, "O.K., how about number five Randy?"
Randy goes to the board and puts squares around three of the eight X's.
Ms. Kaye says, "How many parts are there in whole set?"
The students respond, "Eight."
She says, "How many parts are represented?"
The students respond, "Three."

3/8



Ms. Kaye, "Kenneth will you please do one half for number six?" Kenneth goes to the board and puts squares around three of the six X's.
Ms. Kaye, "You had to split it into two groups of equal number and then circle one of those groups. Can I do this?"

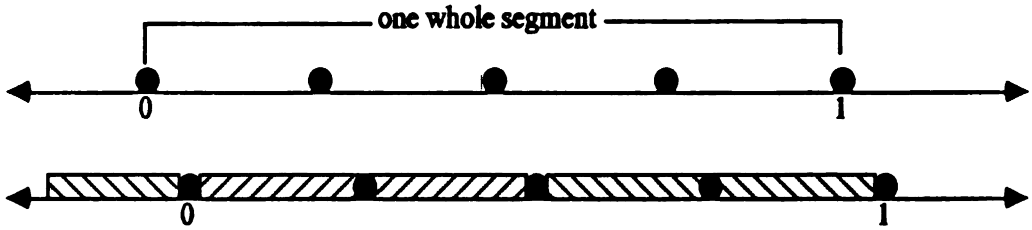


Carol, "No!"
Ms. Kaye, "Why?"
Carol, "Cause if you fold those over they don't match!"
Ms. Kaye, "Well, if you talk about sets of things instead of regions then you aren't talking about them being congruent. You still have one out of two equal groups."

(Observer's Inference)
Carol is confusing fractional parts of a set of things with the notion of congruent regions. Although Ms. Kaye doesn't pursue this line of questioning with Carol she does talk about it in terms of how to divide the class of people in the different rows in her room.

Ms. Kaye continues, "O.K., Mary I'd like you to do number seven. You have to divide the line segment into three-fourths."

Mary draws the following on the chalkboard:



Ms. Kaye looks at Mary's drawing and says, "Can you give her a hand, Melanie?"
Melanie goes to the chalkboard and draws the following:

> Ms. Kaye, "In one whole segment if you want three-fourths and if you have to color them in, what are you coloring in? If I look at the whole segment between zero and one, how many parts did Mary divide it into?"
> The students respond, "Two parts,"
> Dr. Lanier, "Well, what was Mary thinking about?"
> Mary, "I was thinking of the whole thing, the whole line from arrow to arrow!"

After the class period Pamela met with Perry to review the lesson. This lesson was selected to described how Pamela used strategies to improve the Lesson Structure through a beginning of class activity, controlled practice and students working at the chalkboard. Perry's feedback to Pamela also considered communication patterns and the mathematical task. Regarding the mathematical task, he wrote:

> Tasks: Linkage of concept "whole" to other concepts, e.g., part numerator, denominator, etc., well reinforced. Such a pattern should continue. Congruence of parts considerations seemed to have impact on several students ... Linkage between words, symbols, pictures, etc., was grasped by most of the students. You need to continue to be mindful of the need to make these linkages.

His feedback on communication included:

> Communication: Discussion, questions, interactions were good. Students were attentive -- even if noisy. Further, there were many interested and smiley faces. I think increasing dissonance via non-examples and seeking the way kids are thinking might enrich the interaction. The way they talked about congruence suggests they can handle mathematical terminology pretty well.

He felt interaction and participation of students had improved greatly, and that many more students were commenting and interacting on the problems.

Perry's suggestion to Pamela regarding the lesson structure focused on pairing the students to complete the task of coloring and cutting the circles for their fraction kits more efficiently.

> Social Organization: I think circle production might have gone faster with as much learned, by having students in pairs. However, you now have a set for each student which provides many degrees of freedom.

Although Perry did not give Pamela Kaye written feedback regarding the way she used student volunteers to work at the chalkboard, and the beginning of class activity, he mentioned to her during the discussion later he thought the lesson went very well. Part of this interaction follows:

Perry: I felt that the distribution of your questions and the participation from the students was a lot better today. I was, overall, quite happy with today's work. They could have dealt with a little more substance but I felt overall it was pretty good.

Pamela: My feeling was that by having things out before class that when they came in they could get started right away. It would have been much better if I had done a review first, as you suggested, and then had them coloring later. They could have done something more of substance. I felt it was taking quite a while to get them settled down and focused on working.

Perry's feedback on the improved patterns of communication in the lesson today was the result of Pamela Kaye's strategies to improve the lesson structure, such as student boardwork, controlled practice and the start of class activity.

The strategies Pamela Kaye used to improve the lesson structure of the classroom were becoming integrated with her strategies to improve the communication patterns and mathematical content. As a result the overall quality of both instruction and learning improved as more students became actively engaged in the lesson.

Summary of the Classroom Consultation Period

Pamela Kaye was interviewed at the end of the Fraction Unit Intervention on her perceptions of changes that may have taken place during this period with respect to her instruction and the students' learning. In responding to the questions, Pamela Kaye mentioned the implementation of strategies in all three Instructional Improvement Categories. Selected questions from this interview and Pamela's responses are organized according to the Instructional Improvement Category and related Topics.

Anne: As you reflect on this unit, list and describe any and all important changes from the way you taught this content last year.

Pamela: Mathematical Content: Orientation

I think that there was a lot more emphasis placed this time on part-to-whole relationship. The meaning of the part to whole, thanks to Perry. He reminded me daily that I needed to emphasize the part-to-whole relationship.

We used the greater than, less than, equal to and not equal to symbols which we had not done before ... And did a lot of comparisons with one whole and one-half. That is something that I have not done before.

LIST OF REFERENCES

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A system of pay, autonomy, career opportunities. (1986, May 21). Education Week.

Adler, M. J. (1982). The paideia proposal: An educational manifesto. New York: Macmillan.

An agenda for action: Recommendations for school mathematics of the 1980s. (1980). Reston, VA: National Council of Teachers of Mathematics.

Andreae, J. (1972). Developing open classrooms in New Rochelle. In E. G. Nyquist & G. R. Howes (Eds.), Open education: A source-book for parents and teachers. New York: Bantam Books.

Apelman, M. (1981). The role of the advisor in the inservice education of elementary school teachers: A case study. Unpublished doctoral dissertation. University of Colorado, Denver.

Armentrout, W. W. (1971). What should the purpose(s) of American education be? Dubuque, IA: Kendall/Hunt.

Begle, E. G., & Gibb, E. G. (1980). Why do research? In R. J. Shumway (Ed.), Research in mathematics education. Reston, VA: National Council of Teachers of Mathematics.

Berliner, D., Fisher, C., Filby, N., & Marliave, R. (1978). Executive summary of beginning teacher evaluation study. San Francisco: Far West Regional Laboratory for Educational Research and Development.

Berman, P., & McLaughlin, M. W. (1978). Federal programs supporting educational change, Vol. VIII: Implementing and sustaining innovations. Santa Monica, CA: The Rand Corp.

Bestor, A. E. (1952). Aimlessness in education. Scientific Monthly, 75 (8).

Bidwell, J. K. (1968). A new look at old committee reports. The Mathematics Teacher, 61.

Blanchard, K., & Zigarma, P. (1982). Models for change in schools. In J. J. Price & J. D. Gawronski (Eds.), Changing school mathematics: A responsive process. Reston, VA: National Council of Teachers of Mathematics.

Boyer, E. L. (1983). High school. New York: Harper & Row.

Brophy, J. (1979). Teacher behavior and its effects. Journal of Educational Psychology, 71.

Brubaker, N., Lanier, P., Buschman, J., Patriarca, L., Madsen-Nason, A., & Schmidt, D. (1983). A ninth grade general mathematics teacher's perception of her changing beliefs about students, instruction, and teacher role. Unpublished manuscript. Michigan State University, Institute for Research on Teaching, College of Education.

Cain, R. W. (1985). Mathematics in secondary schools: Four points of view. In C. R. Hirsch (Ed.), The secondary school mathematics curriculum. Reston, VA: The National Council of Teachers of Mathematics.

Carnegie Forum on Education and the Economy (1986). A nation prepared: Teachers for the 21st century. New York: The Carnegie Corporation.

Carpenter, T. P., Corbitt, M. K., Kepner, H. K., Lindquist, M. M., & Reyes, R. E. (1981). Results from the second mathematics assessment of the national assessment of educational progress. Reston, VA: National Council of Teachers of Mathematics.

Carson, P. (1986, June). Curriculum Guide for Integrated Mathematics--9. (Available from Ann Arbor Public Schools, Ann Arbor, MI).

Chambers, D. L. (Ed.). (1986). A guide to curriculum planning in mathematics. Madison, WI: Wisconsin Department of Public Instruction.

Chance, W. (1986). "...the best of educations": Reforming America's public schools in the 1980s. Olympia, WA: John D. and Catherine T. MacArthur Foundation.

Cohen, D. K. (1987). Teaching practice: Plus que ca change... . Unpublished manuscript. Michigan State University, East Lansing.

Committee on the Definitions of College Entrance Requirements. (1904). Committee of 1903. (Bulletin 10). New York: American Mathematical Society.

Committee on the Function of Mathematics in General Education of the Commission on the Secondary School Curriculum of the Progressive Education Association. (1940). Mathematics in General Education. New York: D. Appleton-Century.

Conant, J .B. (1959). The American high school today. New York: McGraw-Hill.

Coxford, A. F. (Ed.). (1970). A history of mathematics education in the United States and Canada. Washington, DC: National Council of Teachers of Mathematics.

Cuban, L. (1986). Persistent instruction: Another look at constancy in the classroom. Phi Delta Kappan, 7-11.

Cuban, L. (1984). Transforming the frog into a prince: Effective schools research, policy and practice at the district level. Harvard Educational Review, 54 (2), 129-151.

Cuban, L. (1984). How teachers taught: Constancy and change in American classrooms, 1890-1980. New York: Longman.

Cuban, L. (1979). Determinants of curriculum change and stability, 1870-1970. In Jon Schaffarzick & Gary Sykes (Eds.), Value Conflicts and Curriculum Issues. Berkeley, CA: McCutchan.

Cuban, L. (1970). To make a difference: Teaching in the inner city. New York: The Free Press, Macmillan.

Cummings, C. (1980). Teaching makes a difference. Edmonds, WA: Teaching.

Dolan, D. T., & Williamson, J. (1983). Teaching problem-solving strategies. Menlo Park, CA: Addison-Wesley.

Driscoll, M. (1983). Breaking vicious cycles: Remediation in secondary school mathematics. In M. Driscoll (Ed.), Research within reach: Secondary school mathematics. Washington, DC: National Institute of Education.

Driscoll, M. (1983). Research within research: Secondary school mathematics Washington DC: National Institute of Education.

Dunkin, M. J., & Biddle, B. J. (1974). The study of teaching. New York: Holt, Rinehart, & Winston.

Education Commission of the States. (1983). A Summary of Major Reports on Education. Denver: Author.

Endinger, L. V., Hauts, P. L., & Meyer, D. V. (Eds.). (1981). Education in the 80s: Curriculum challenges. Washington, DC: National Education Association.

Educational Testing Service. (1988). The mathematics report card: Are we measuring up? (Rep. No. 17-M-01). Princeton, NJ: Author.

Edwards, S. (1981). Changing teacher practice: A synthesis of relevant research. (Report No. 9008). Austin: The University of Texas

Erlwanger, S. H. (1973). Benny's conception of rules and answers in IMP mathematics. Journal of Children's Mathematical Behavior. 1 (2).

Fullan, M., & Pomfret, A. (1977). Research on curriculum and instruction implementation. Review of Educational Research, 47, 335-397.

Gage, N. (1979). The generality of dimensions of teaching. In P. Peterson & H. Walberg (Eds.), Research on teaching: Concepts, findings, and implications. Berkeley, CA: McCutchan.

Good, T., & Brophy, J. (1974). An empirical investigation: Changing teacher and student behavior. Journal of Education Psychology, 66.

Good, T. L., & Biddle, B. J. (1987). Teacher thought and teacher behavior in mathematics instruction: The need for observational resources. Unpublished manuscript. Columbia: The University of Missouri.

Goodlad, J. I. (1972). Staff development: The league model. Theory into practice. 11(4).

Goodlad, J. (1980). A place called school. In B. Gross & R. Gross (Eds.), The great school debate: Which way for American education? New York: Simon & Schuster.

Goodlad, J. I. (1984). A place called school., New York: McGraw-Hill.

Gordon, E. W. (1972). Defining equal opportunity. In F. Mosteller & D. P. Moynihan (Eds.), On equality of educational opportunity. New York: Vintage Books.

Hall, G. E., & Loucks, S. (1979). Implementing innovations in schools: A concerns-based approach. Austin: The University of Texas.

Hall, G. E., Wallace, R. D., & Dossett, W. A. (1973). A developmental conceptualization of the adoption process within educational institutions. Austin: The University of Texas.

Harman, W. (1972). Changing United States society: Implications for schools. In Alternative educational future in the United States and in Europe methods, issues and policy relevance. France: Center for Educational Research and Innovation (CERI), Organization for Economic Cooperation and Development.

Holmes Group. (1986). Tomorrow's teachers: A report of the Holmes Group. East Lansing: Michigan State University.

Howson, A. (1979). A critical analysis of curriculum development in mathematical education. New Trends in Mathematics Teaching. New York: UNESCO.

International Commission on the Teaching of Mathematics. (1911). The American report: Mathematics in the public and private secondary schools of the United States. (USBE Bulletin 1911, no. 16). Washington, DC: U. S. Government Printing Office.

James, T., & Tyack, D. (1983). Learning from past efforts to reform the high school. Phi Delta Kappan, 64.

Joyce, B., & Showers, B. (1980). Improving inservice training: The messages of research. Educational Leadership, 37.

Joyce, B., & Weil, M. (1986). Models of teaching. Englewood Cliffs, NJ: Prentice-Hall.

Kline, M. (October, 1961). Math teaching assailed as peril to U.S. scientific progress. New York University Alumni News.

Knowles, M. (1978). The adult learner: A neglected species. Houston: Gulf.

Lanier, P. E. (1981). Ninth-grade general mathematics adds up to trouble. IRT Communication Quarterly. 4 (4).

Lanier, P. E. (1983). The general math project: Consultation. Unpublished manuscript, Michigan State University, Institute for Research on Teaching, East Lansing.

Lappan, G., Fitzgerald, W., Winter, M. J., & Phillips, E. (1986). Middle grades mathematics project: Similarity and equivalent fractions. Menlo Park, CA: Addison-Wesley.

Lewin, K. (1951). Field theory in social science. New York: Harper & Bros.

Lewin, K. (1947). Frontiers in group dynamics: Concept, methods, and reality in social science, social equilibria and social change. Human Relations, 1, 5-41.

Mann, D. (1978). Making change happen? New York: Teachers College, Columbia University.

McCall, G., & Simmons, J. (1969). Issues in participant observation. Reading, MA: Addison-Wesley Publishing.

McDonald, F., & Elias, P. (1976). Executive summary report: Beginning Teacher Evaluation Study, Phase II. Princeton, NJ: Educational Testing Service.

McLaughlin, M. W. (1978). Implementation as mutual adaptation: Change in classroom organization. In D. Mann (Ed.). Making Change Happen? (pp. 19-31). New York: Teachers College Press.

McLaughlin, M. W., & Marsh, D. D. (1978). Staff development and school change. Teachers College Record, 80 (1).

McKnight, C. C., Crosswhite, F. J., Dossey, J. A., Kifer, E., Swafford, J. O., Travers, K. J., & Cooney, T. J. (1987). The underachieving curriculum: Assessing U.S. school mathematics from an international perspective. Champaign, IL: Stripes.

Madsen-Nason, A., & Lanier, P. E. (1986). Pamela Kaye's general math class: From a computational to a conceptual orientation. (Research Series No. 172). East Lansing: Michigan State University, Institute for Research on Teaching.

Mitchell, E. M., Ortiz, F. I., & Mitchell, T. K. (1982). Controlling the impact of rewards and incentives on teacher task performance: Executive summary (NIE-G-80-0154). Washington, DC: National Institute of Education.

Murray, F. B. (1986). Goals for the reform of teacher education: An executive summary of the Holmes Group report. Phi Delta Kappan, 68 (1).

National Advisory Committee on Mathematical Education. (1975). Overview and analysis of school mathematics grades K-12. Washington, DC: Conference Board of the Mathematical Science.

National Commission on Excellence in Education. (1983). A nation at risk: The imperative for educational reform. Washington, DC: U.S. Government Printing Office.

National Committee on Mathematical Requirements of the MAA. (1922). The reorganization of mathematics in secondary education (USBE Bulletin 1921, no. 32). Washington, DC: U. S. Government Printing Office.

National Committee on Mathematical Requirements of the MAA. (1923). The reorganization of mathematics in secondary education. Washington, DC: Author.

National Council of Teachers of Mathematics. (1980). The revolution in school mathematics: A challenge for administrators and teachers. Washington, DC: Author.

National Council of Teachers of Mathematics. (1961). The revolution in school mathematics: A challenge for administrators and teachers. Washington, DC: Author.

N.E.A. (1893). Report of the committee on secondary school studies. (Document 205). Washington, DC: Author.

N.E.A. (1894). Report of the committee of ten on secondary school studies with the reports of the conferences arranged by the committees. New York: American Book.

Oja, S. N. (1980). Adult development is implicit in staff development. Journal of Staff Development. 1 (2).

Patriarca, L. A. (1983). Looking at and looking for instruction: Patterns of Teaching in ninth grade general mathematics classrooms. Unpublished manuscript, Michigan State University, Institute for Research on Teaching, East Lansing.

Peterson, P. L. (1983). Research on teaching. In T. A. Romberg & D. M. Stewart (Eds.), School Mathematics: Options for the 1990s. Proceedings of the Conference (pp. 99-108). Madison: University of Wisconsin-Madison.

Potter, M. A. (1950). Mathematics to Use. Boston: Ginn.

Powell, A. G., Farrar, E., & Cohen, D. K. (1985). The shopping mall high school: Winners and losers in the educational market place. Boston: Houghton Mifflin.

Reeve, W. D. (1924). The general trend of mathematics education in secondary schools. Mathematics Teacher, 15, 449-458.

Reeve, W. D. (1920). General mathematics for the high school: Its purpose and content. Educational Administration and Supervision, 6, 258-273.

Romberg, T. A. (1988). Introduction to the standards. In T. A. Romberg (Chair), Curriculum and evaluation standards for school mathematics (pp. 1-13). Reston, VA: National Council of Teachers of Mathematics.

Romberg, T. A. (1983). Change and options in school mathematics. In T. A. Romberg & D. M. Stewart (Eds.), School Mathematics: Options for the 1990s. Proceedings of the Conference (pp. 31-35). Madison: University of Wisconsin-Madison.

Romberg, T. A., & Carpenter, T. P. (1986). Research on teaching and learning mathematics: Two disciplines on scientific inquiry. In M. L. Wittrock (Ed.), The third handbook of research on teaching. New York: MacMillan.

Rosenshire, B. (1971). Teaching behaviors and student achievement. London: National Foundation for Educational Research.

Schloring, R. (Chair). (1945). The second report of The Commission on Post-War Plans. The Mathematics Teacher, 38, 195-221.

Schloring, R. (1926). Suggestions for the solution of an important problem that has arisen in the last quarter of a century. In R. Schloring (Ed.), A general survey of progress in the last twenty-five years (pp. 58-105). New York: Teachers College Press.

Sarason, S. B. (1971). The culture of school and the problem of change. Boston: Allyn & Bacon.

Schein, E. H. (1972). Professional education. New York: McGraw-Hill.

Seymour, D. (1987). Dale Seymour Publications. Palo Alto, CA: Seymour.

Shaw-Hiehle Computation Test. (1972). Boston: Houghton-Mifflin.

Showers, B. (1983, April). Coaching: A training component for facilitating transfer of training. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Quebec, Canada.

Stannic, G. M. A. (1986). The growing crisis in mathematics education in the early twentieth century. Journal for Research in Mathematics Education, 17, 190-205.

Stein, E. I. (1974). Teacher's Guide: Refresher mathematics. Boston: Allyn & Bacon.

Stenhouse, L. (1978). Case study and case records: Towards a contemporary history of education. British Educational Research Journal. 4 (2).

Staff. (1984, July 20). Most educational reforms just change the problem. Notes & News, 11 (20).

Tucker, M., & Mandel, D. (1986). The Carnegie report: A call for redesigning the schools. Phi Delta Kappan, 68 (1).

U.S. Bureau of Education. (1918). Cardinal principles of secondary education (Bulletin No. 35). Washington, DC: U.S. Government Printing Office.

Usiskin, Z. (1985). We need another revolution in secondary school mathematics. In C.R. Hirsch (Ed.), The secondary school mathematics curriculum. Reston, VA: The National Council of Teachers of Mathematics.

Watzlawick, P., Weakland, J. H., & Fisch, R. (1974). Change: Principles of problem formation and problem resolution. New York: W. W. Norton.

Wilson, S. M., & Shulman, L. S. (1987). 150 different ways of knowing: Representations of knowledge in teaching. In J. Calderhead, (Ed.), Exploring teacher thinking. Sussex: Holt, Rinehart, and Winston.