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FINITE ELEMENT ANALYSIS OF DRIP IRRIGATION HYDRAULICS USING QUADRATIC ELEMENTS AND A VIRTUAL EMITTER SYSTEM

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FINITE ELEMENT ANALYSIS OF DRIP IRRIGATION HYDRAULICS USING QUADRATIC ELEMENTS AND A VIRTUAL EMITTER SYSTEM

By

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ABSTRACT

FINITE ELEMENT ANALYSIS OF DRIP IRRIGATION HYDRAULICS USING QUADRATIC ELEMENTS AND A VIRTUAL EMITTER SYSTEM

By

Shaun Francis Kelly

As use of drip irrigation increases, the need to analyze large systems becomes more and more apparent. The purpose of this research was to conserve water and energy through the improved design of large drip irrigation systems by using the finite element method and a virtual emitter system. A differential equation describing flow in a drip irrigation system was developed and solved by using classical finite element methods. Several different methods were found to incorporate emitters in a virtual node system, where several emitters are combined into one element or node, as a point source. The virtual emitter methods were evaluated using linear and quadratic elements and found to be fast and efficient methods for analyzing the hydraulics of drip irrigation systems.

Approved Vincent F. Bralts
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Approved Larry Segerlind
Department Chairman

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LIST OF SYMBOLS

α	pipe constant
a	pipe constant, equation [7]
A	cross sectional area of pipe flow
C_e	linearized emitter constant
C_p	linearized pipe constant
D	diameter of pipe
D	Jacobian matrix, equation [21]
dx	length of pipe element
D_x	coefficients for differential equation
e	number of emitters per plant
$\{f^{(e)}\}$	element force vector
$\{f\}$	global force vector
f	dimensionless friction factor
F	reduction coefficient to account for emitters
g	acceleration of gravity
g	C_pZ
G	coefficients for differential equation
$\{H^{(e)}\}$	element nodal vector
$\{H\}$	global solution vector
h	pressure head
h'	average pressure head in the element
h_f	headloss due to friction

h_{fl}	headloss due to friction in a lateral line
h_i	pressure head for a given length ratio
ΔH	total friction loss
$\Delta H'$	total friction loss due to elevation
ΔH_i	friction loss in the lateral line
H_i	pressure head at node i
H_j	pressure head at node j
H_k	pressure head at node k
H_o	initial pressure estimate
H_o	pressure head at the origin, equation [13]
i	length ratio
$[K^{(e)}]$	element stiffness matrix
$[K]$	global stiffness matrix
k	emitter constant
k	friction loss coefficient, equation [23]
K	constant in Christianson equation [9]
k'	modified pipe constant
L	length of pipe
m	pipe flow exponent
n	iteration number
n	number of emitters
N	number of emitters
N_i	interpolation function for node i
N_j	interpolation function for node j
N_k	interpolation function for node k
q	emitter discharge

$\overline{q}$	mean emitter flow
Q	coefficients for differential equation
ΔQ	flow correction
Q	flow rate
q_e	emitter flow rate
q_i	emitter flow in the lateral line
q_{ij}	emitter flow in the submain unit
q_o	flow in the first emitter
Q_o	initial flow estimate
q_s	total flow in the submain unit
R_e	Reynolds number
R_i	energy drop ratio
$R_i'\Delta H'$	head loss due to elevation
$R_i'\Delta H_l'$	lateral head loss due to elevation
$R_i\Delta H$	head loss due to friction
$R_i\Delta H_l$	lateral head loss due to friction
$R_j'\Delta H_s'$	submain head loss due to elevation
$R_j\Delta H_s$	submain head loss due to friction
$\{R^{(e)}\}$	element residual vector
$\{R\}$	global residual vector
S_q	standard deviation of emitter flow
U_s	statistical uniformity coefficient as a percentage
V	average pipe flow velocity
V_k	coefficient of variation of the emitter constant
V_h	coefficient of variation of the hydraulic pressure
V_p	coefficient of variation of emitter plugging

V_q	coefficient of variation of emitter flow
v_x	average pipe fluid velocity
x	emitter discharge exponent
x^e	emitter discharge exponent
z	elevation head
Z	elevation
Z	solution vector, equation [20]

I. INTRODUCTION

For most of the world's population, agricultural production has been the major source of food for the last 4000 years. Historically the practice of irrigation has been an important part of food production for much of the world, especially in arid regions. With the world's population reaching an estimated 6.1 billion people by the year 2000, experts are predicting an increasing world food crisis. In recent years this crisis has become increasingly evident in some developing nations. In Africa, prolonged drought and severe lack of developed water supplies, coupled with high population density and the highest population growth rates in the world, have triggered a crippling famine. This is an area which even in the absence of drought cannot produce enough food for its own population. Irrigation systems will continue to play an important role in food production as we approach the year 2000, when the world will need fifty percent more food than today to secure at least the current state of nutrition (Holy, 1977).

Basically agriculture can be expanded in two directions - extensively by expanding cropland, and intensively through improved plant species and hybrids, irrigation, improved management schemes and proper use of chemicals and fertilizers. Irrigation is important for both expanding the available cropland and increasing production on current land. Worldwide, irrigated area has increased from 95 million hectares in 1950 to approximately 250 million hectares in 1985. This represents about 20% of the world's total cropland and accounts for over 40% of the total crop production (Power, 1986). It has been estimated that by the year 2000 the total area under irrigation will be 500 million hectares (Holy, 1977).

Expansion of irrigation will not continue unchecked, however, as freshwater resources are in short supply. In the United States we have become increasingly aware of diminishing water resources. Development of irrigation between 1960 and 1978 in the south central plains of the United States resulted in an explosion in feed grain production. This had a huge impact on the local economies, as feedlots and meat processing plants moved in. In 1958 there were 300,000 cattle on feed in Texas; by 1978 there were more than 4.9 million. Increased pumping demands on the vast Ogallala aquifer underlying this region led to dropping water tables, threatening the economy on the high plains. In five states where levels are dropping most pervasively - Arizona, Kansas, New Mexico, Oklahoma and Texas - net irrigated area has actually declined between 1978 and 1982 by 678,000 ha. or 14% (Postel, 1985). Of all the world's freshwater resources used today, 80% is used for irrigation, with an average efficiency of only 37% (Power, 1986). By increasing the water use efficiency of our irrigation systems, not only can we ease our ever-increasing demand on these water resources, but we can reduce the runoff of both water and soil. This can be achieved by improving more traditional irrigation techniques such as surface flooding and furrow irrigation. Efficiency can also be increased through improved management with irrigation scheduling techniques, and through use of new technologies in sprinkler design and drip irrigation.

Drip irrigation principles evolved in the early 1900s from experiments using drainage tiles for subsurface irrigation. The drip irrigation systems popular today were introduced for use in greenhouses in the early 1940's in England. Drip irrigation did not become widespread in practice until the advent of relatively inexpensive plastic pipe and emitters which became available in the 1960s. These allowed the development of economically feasible systems. Growth of drip irrigation usage has continued, with Australia, United States, and Israel leading the way. In 1980 the total area under drip irrigation in the United States was over 200,000 hectares. Drip irrigation techniques are being used in cotton and other traditional row crops, as well as in the landscape industry. Ninety-five percent of all irrigation in Michigan is drip or sprinkler. Most is used on fruits, vegetables and orchard crops, accounting for up to 250 million dollars (Michigan Ag. Statistics, 1987).

Despite this impressive expansion in the United States, drip irrigation represents less than 1% of the total irrigated area worldwide. Because of high capital costs it has primarily attracted growers of high-valued orchard crops, grapes and vegetables. These new technologies are to costly and complex to benefit many farmers in developing countries, but incorporating the principles behind these techniques and technologies could prove to be worthwhile.

Drip irrigation is the practice of applying water using low application rates and low pressures from emission points located near the plants' root areas. Water flows from the emission points and through the soil by capillarity and gravity. A drip irrigation system offers many unique advantages for efficient use of water and labor over conventional systems. An efficient drip irrigation system applies water close to the consumptive rate of the plants, or stores water temporarily in the soil depending on the growing conditions desired. Labor costs can be reduced because of the ease of automation and the use of chemical and fertilizer injection. Drip irrigation offers the farmer greater control of water delivery, and frequent irrigation maintains a stable soil moisture condition that enables the farmer to irrigate with water of a higher salinity.

A conventional drip irrigation system, (Figure 1), consists of a water supply and a pump, followed by a network of mainlines, submains, laterals, emitters, flow and pressure controls and filters. From the mainline, water is delivered to the various irrigation zones. Within each zone (20 - 50 ha) there are commonly a number of submain units (1 - 5 ha) with manifolds that feed water to the lateral. Emitters are placed along the lateral lines to discharge water near the bases of the plants.

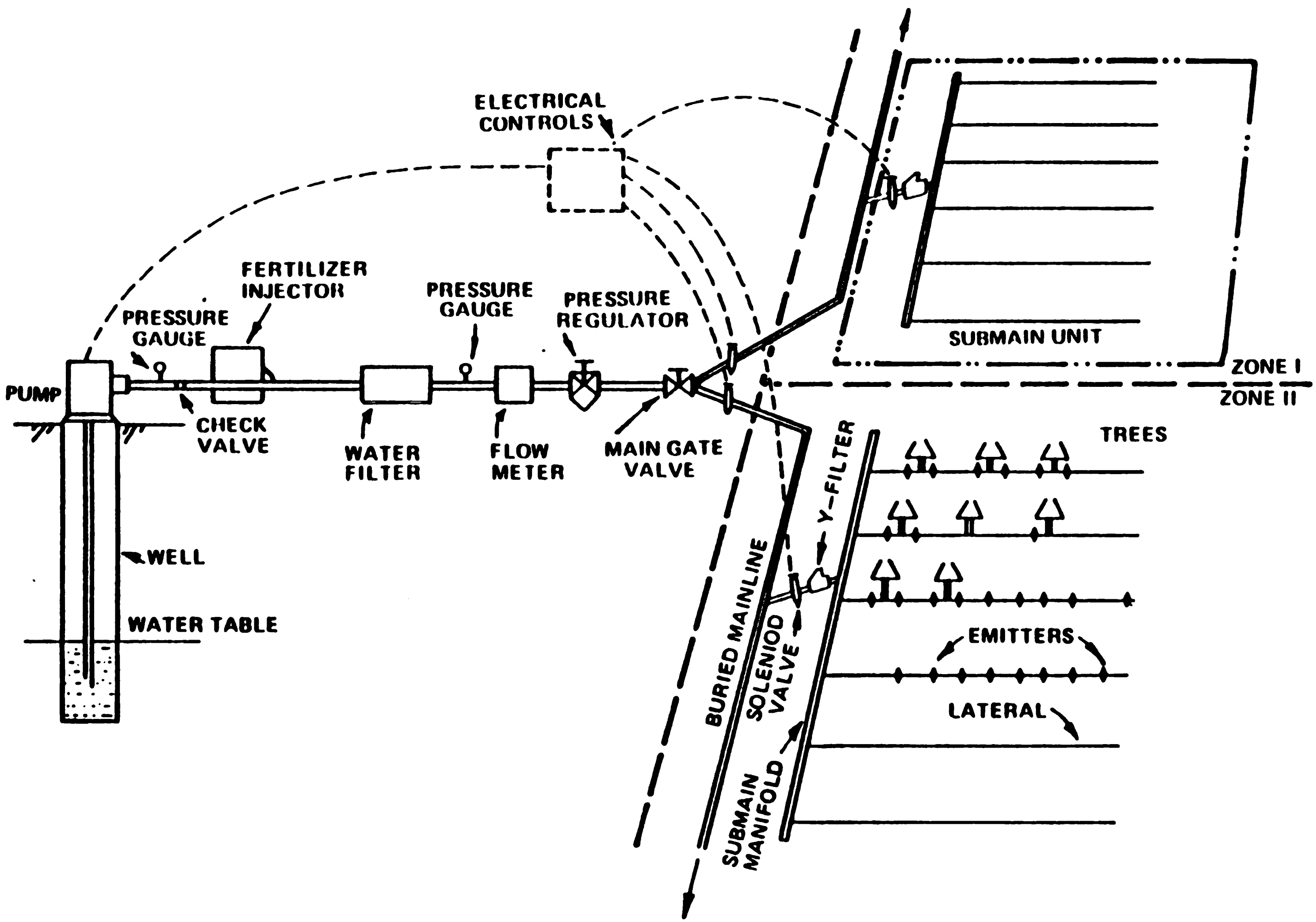

Figure 1. A typical drip irrigation system layout and components, (Bralts, 1983).

Design of a drip irrigation system generally proceeds from basic data on, climate, soil, water quality, topography and the crop to be irrigated. From this data, decisions about the amount of water required per area, and the basic intervals of irrigation are derived. Hydraulic design of an efficient drip irrigation system relies on proper choice of emitters, laterals and pipe sizing. Hydraulic design of the submain unit generally decides the success or failure of a drip system, because once components have been selected very little additional flow control is possible beyond varying irrigation times.

The measure of a good design of a drip irrigation system is defined by both economics and uniformity of water applied to meet the crop's needs. Much research has been conducted in the past on uniformity to assist in the improved design of drip irrigation systems. Keller and Karmeli (1974) developed the Emission Uniformity Concept. Wu and Gitlin (1974) developed the Emitter Flow Variation Concept, and Bralts (1987) developed the Statistical Uniformity Concept. These concepts were developed to analyze the performance of drip irrigation submain units.

1.1 Scope and Objectives

With the development of drip irrigation came an awareness of the need for accurate hydraulic design due to certain unique complexities in the system. Good hydraulic design requires uniformity of emitter discharge rates and economical pipe selection. Depending on the type selected, emitter discharge rate is more or less sensitive to changes in the lateral line pressure. These changes in pressure are unavoidable due to frictional losses and topographical effects. The frictional pressure loss changes constantly along a lateral because the flow of water in the pipe changes beyond each emitter. The frictional losses must, therefore, be recalculated after each emitter.

Generally, a large number of emitters must be considered even if only one lateral line is to be analyzed. If the submain unit is designed to include topographical effects, the task of calculating the pressure behind each emitter becomes enormous.

More traditional methods of hydraulic design for drip irrigation systems included graphical techniques, charts, slide rules, nomographs and simplified numerical solutions. One of the earliest was Polyplot (ICIANZ, 1970). Originally intended for use in sizing pipe diameters in multi-nozzle sprinkler systems, it was found to be a valuable aid for the design of drip irrigation networks. This technique doesn't actually calculate the pressure behind each emitter. The designer specifies a range of acceptable pressures assuring that the required uniformity and flow curves are selected to fit this range. Charts and nomographs specifically designed for drip irrigation lines were presented by Wu and Gitlin (1974). They incorporated both the dimensionless energy grade line concept and the uniformity coefficient for sloping lateral lines and submains. Keller and Rodrigo (1979) presented graphical methods and simple numerical solutions for the design of single sloping laterals and pairs of sloping laterals. Most numerical methods utilized the backstep method - an iterative technique to solve for flow rates and pressures in lateral lines based on assumed end line pressures. Howell and Hiler (1972) described a computerized drip irrigation design based on this technique.

The advent of economical and simple microcomputers has left no excuse for designers to avoid computer solutions to hydraulic network analysis of irrigation systems. The practicing engineer can discard tedious graphical and computational techniques and place more emphasis on the agricultural aspects of designing a drip irrigation system. Iterative techniques for flow analysis of pipe networks which solve the equations used to describe the hydraulic phenomena are well suited to computer implementation. Solutions obtained using Hardy-Cross, Newton-Raphson and the linear theory techniques have been available for a number of years.

The implementation of these techniques by irrigation engineers has been sparse at best. Edwards and Spencer (1972) presented design criteria for computer-based analysis of sprinkler irrigation systems using the Hardy Cross method. Solomon and Keller (1974), Wu and Fangmeir (1974) and Perold (1977) have used iterative techniques based on the backstep method. These techniques were expanded to drip irrigation submain units but were considered too cumbersome for practical use.

Bralts and Segerlind (1985) proposed using the finite element approach to analyze drip irrigation submain units. The method used linear theory techniques to obtain a set of flow equations. The advantages of the technique included minimal computer storage, simplicity of application to large irrigation networks and relative quickness of convergence. Bralts, Segerlind and Driscoll (1985) presented an interactive microcomputer program using this technique. In this paper the authors expressed their desire for additional improvement in execution speed, initial estimates, and convergence of solution. Bralts and Segerlind (1987) proposed a technique for incorporating a virtual node substructure. This technique would allow the concentration of the solution matrix into a more compact virtual node matrix, by combining multiple emitters and lateral lines into virtual nodes. The system was then solved using a stepwise expansion of the virtual node system until a final solution was obtained. This solution technique would allow the improved speed of convergence with large (10,000 or more) emitter systems. A complete validation of this technique has not been performed. However, it was found to be accurate for drip irrigation designs. The authors expressed a need for additional testing of the virtual node system to examine limits of application and efficiency of solution. This method is still the state-of-the-art technique for hydraulic design of drip irrigation submain units.

The goal of this research is to conserve water and energy through improved design of drip irrigation systems. Improved hydraulic design techniques will conserve water and energy by improving the uniformity of application. Hydraulic design of drip irrigation systems is well-suited for computer analysis using network analysis techniques. The focus of this research will be on hydraulic design of drip irrigation systems.

The specific objectives of this research are as follows.

1. Development of an equation to describe the flow phenomena in a drip irrigation system.

2. Investigate the use of virtual node substructures in hydraulic design of drip irrigation systems.

3. Define limits of application, efficiency of solution, improve speed of convergence, and improve initial estimates of the solution method using a virtual node concept.

II. REVIEW OF LITERATURE AND THEORY

In order to more fully understand the hydraulic design of drip irrigation systems using network analysis techniques, a literature review of a) hydraulic design, b) hydraulic network analysis, c) finite element methods, d) numerical solutions must be completed. A review of theory and literature of these topics is presented in this section.

2.1 Hydraulic Design of Drip Irrigation Systems

The classical equations of continuity and energy have been applied to drip irrigation to provide a theoretical basis for hydraulic design. This theory has been presented by various researchers. (Wu and Gitlin, 1974; Howell and Hiler, 1974; and Keller and Karmeli, 1974). The following development will closely follow the theory and nomenclature used by Bralts, Edwards and Wu (1987).

2.1.1 Hydraulics of Emitter Flow

An important element of a drip irrigation system is the emitter. To achieve uniformity of application each emitter must provide a constant rate of discharge at pressure heads of at least 10 m. Many emitter designs have been developed, from elaborate pressure compensating devices to long flow path and simple orifice type emitters. Emitter choice depends on the required flow rate (usually 2 - 10 liters/hr), water quality, and cost.

The flow characteristics of emitters have been shown by Karmeli (1977) and Wu, et al. (1979) to be defined by

$$q = kh^x \qquad [1]$$

where q = emitter discharge

k = constant characterizing discharge

h = working pressure head

x = emitter discharge exponent

Equation [1] can be derived from the energy and continuity equations of classic fluid hydraulics. The constants k and x are usually determined empirically for each emitter design. The discharges versus different operating heads are recorded and values of k and x in [1] are determined to best fit this data. The emitter discharge exponent characterizes the pressure-flow relationship and the flow regime. The value of x varies from 0, (fully pressure compensating), 0.5 (fully turbulent orifice type emitter), to 1, (laminar flow in long path emitters). With lower values of x, emitter discharge is affected less by pressure variations. The constant of proportionality, k, encompasses characteristics such as flow cross section, acceleration of gravity, viscosity, and characteristics of the nozzle.

2.1.2 Hydraulics of Lateral Line Flow

The pipe that supplies water to the emitters is referred to as the lateral line. Similarly, the manifold generally includes the pipe that supplies water to the laterals and includes flow from the laterals (Figure 2).

For design, the flow from the lateral of a manifold can be considered to be hydraulically steady, spatially varied, one-dimensional, incompressible pipe flow. The total flow through the pipe is changing, usually decreasing with respect to length. The

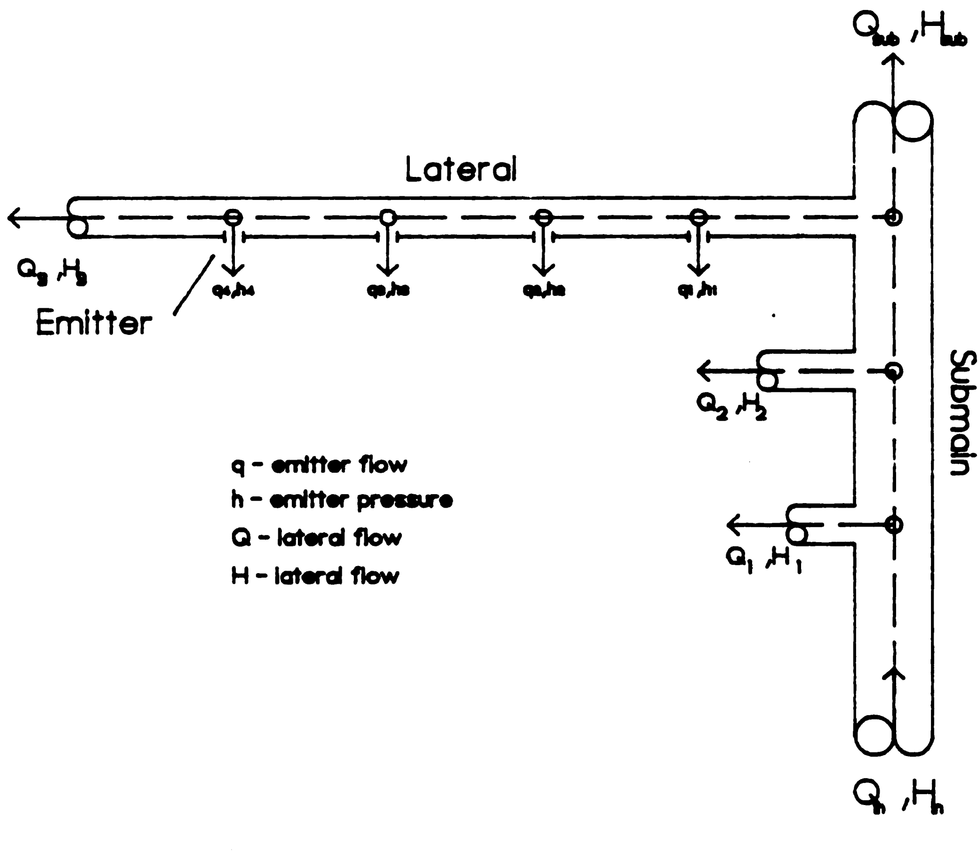

Figure 2. A submain unit with laterals and emitters.

pressure is also usually decreasing along the length of the pipe due to friction losses and elevation (Bralts, et al. 1987). The hydraulic design is simplified because a drip system is generally based on uniform layout of components. Thus a large system is composed of essentially a number of smaller identical components and the manifold can be considered a large lateral with large emitters (lateral lines) with a larger flow rate.

The design equations for laterals and manifolds are based on continuity and conservation of energy. Whereas the continuity equation remains unchanged the energy equation has an additional term to account for frictional losses. Drip irrigation pipes are normally considered as hydraulically smooth and their flow turbulent. Numerous equations to solve for head loss due to friction are available. Two equations, both empirical, are most often used in solving for head loss in drip irrigation. They are known as the Darcy-Weisbach and the Hazen-Williams equations and will be described as they are commonly used in drip irrigation.

A. Darcy-Weisbach Equation

The Darcy-Weisbach equation is written as follows,

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

where h_f = headloss due to friction (m)

f = dimensionless friction factor

L = length of pipe (m)

D = diameter of pipe (m)

g = acceleration of gravity (m/s^2)

V = average pipe flow velocity (m^2/s)

As lateral lines and manifold pipes can be considered hydraulically smooth with fully turbulent flow, the Blasius empirical formula is generally used and is substituted for f (Wu and Gitlin, 1974; Howell, et al. 1981). The Blasius formula is

$$f = \frac{0.3164}{R_e^{0.25}} \quad [3]$$

where f = friction factor

R_e = Reynolds number

($4000 \le R_e \le 100{,}000$)

Watters and Keller (1978) combined [2] and [3] at 20°C and found

$$h_f = 7.87 \times 10^5 \frac{Q^{1.75}}{D^{4.75}} L \quad [4]$$

where h_f = friction loss (m)

Q = flow rate (l/s)

D = diameter of pipe (mm)

L = length of pipe (m)

The friction factor, f, can also be taken from the Moody diagram (Figure 3).

Bezdek and Solomon (1978) presented an equation to approximate the Darcy-Weisbach friction factor as follows.

$$f = 0.0286 e^{e^{-(1.9875 \times 10^{-4}) R_e}} \quad [5]$$

B. Hazen-Williams Equation

The Hazen-Williams formula is commonly used in hydraulic design of drip irrigation (Keller and Karmeli, 1975). The Hazen-Williams formula is

$$h_f = 1.22 \times 10^{10} \left(\frac{Q^{1.852}}{C^{1.852} D^{4.875}} \right) L \quad [6]$$

where C is a pipe roughness coefficient, which for smooth pipe used in drip irrigation is usually given a value from 130 - 150. Q, D, and L are the same as in the Darcy-Weisbach equation [2].

C. Comparison of h_f

Both Hazen-Williams and Darcy-Weisbach were found to result in very similar solutions to drip irrigation hydraulic problems (Bralts, 1987). The Darcy-Weisbach which can be corrected for viscosity through Reynolds number is generally regarded as the best rational equation for solving hydraulic problems in drip irrigation. A comparison of the two equations using various friction factors for Darcy-Weisbach and values of C for Hazen-Williams are shown in Figure 4 (Howell, et al. 1979).

Both Hazen-Williams and Darcy-Weisbach were generalized into the form used by Wu and Gitlin (1975) and Wu, et al. (1979) as

$$h_f = -aQ^m L \qquad [7]$$

where h_f = headloss due to friction (m)

Q = total lateral line flow (m^3/s)

a = pipe constant (s/m^3)

L = lateral line length (m)

m = pipe flow exponent

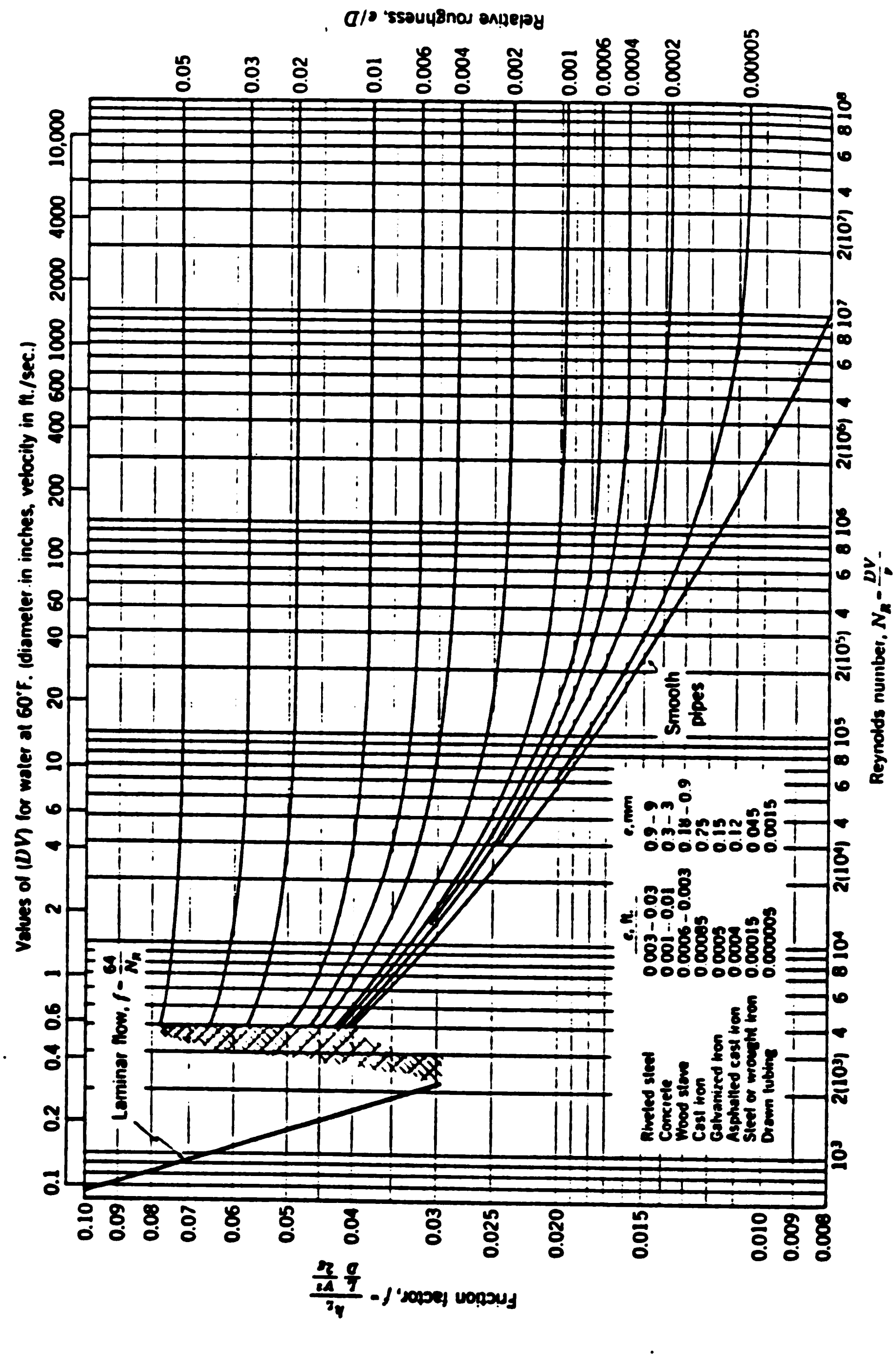

Figure 3. Moody Diagram (Benedict, 1980)

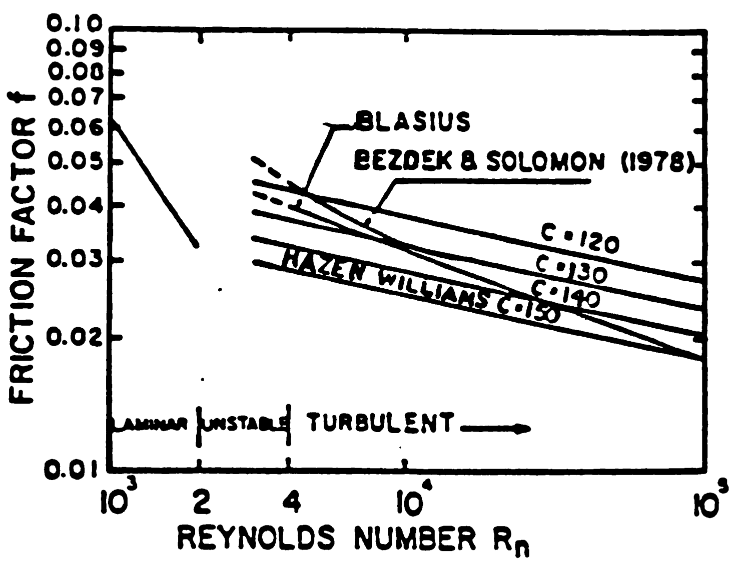

Figure 4. Comparison of Hazen-Williams vs. Darcy-Weisbach (Howell, et al.)

2.1.3 General Lateral Design

For lateral design, water flows through the entire lateral and gradually decreases as water is discharged along the length of the lateral. It is important to be able to calculate the head at any emitter on a lateral. Many methods have been used to calculate these pressures.

A close approximation of a pipe which loses water along its length (as in a drip lateral) in relation to the loss from an identical conduit which discharges water only at the end of the line is (Goldberg, et al. 1976).

$$h_{fl} = \frac{h_f}{3} \quad [8]$$

where h_{fl}, h_f are defined in Figure 5

A more precise procedure was suggested by Christianson who computed

$$h_f = \frac{FKLQ^m}{D^{2m}} \quad [9]$$

where

$$F = \frac{1}{m+1} + \frac{1}{2n} + \frac{(m-1)^{0.5}}{6N^2} \quad [10]$$

and m = velocity exponent use

1.85 for H-W

2.00 for D-W

N = number of emitters

n = 1.1

K = constant

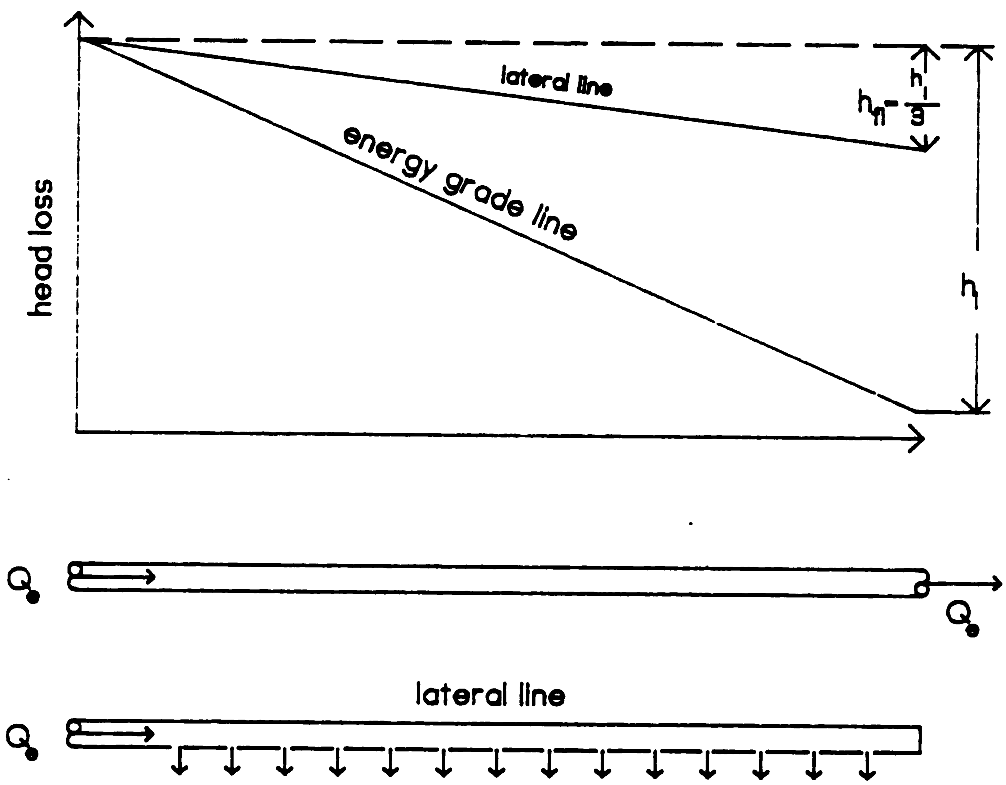

Figure 5. Approximation of friction loss in a lateral line using identical conduit which discharges water only at the end of the line.

On this basis it is possible to calculate the friction loss as though all of the water were flowing through the entire length of the pipe, then multiply by *F*, a reduction coefficient to compensate for discharge along the length of the pipe. As the number of emitters increases, the value of *F* approaches 0.33 which yields the same result as [8].

Wu and Gitlin (1975) presented the dimensionless energy grade line to approximate head loss at any point along any lateral line. Bralts, et al. (1987) expanded the concept to analyze manifolds. The following development follows closely that presented by Bralts, et al.(1987).

The dimensionless energy grade line procedure assumes that all emitters along a lateral line discharge the same flow. The shape of the energy grade line can be expressed dimensionlessly to be the energy drop ratio, R_i, as shown by Wu and Gitlin (1975).

$$R_i = \frac{\Delta H_i}{\Delta H} = 1 - (1 - i)^{m+1} \qquad [11]$$

where i = the length ratio l/L (Figure 6)

m = velocity exponent

1.85 for H-W

2.00 for D-W

and $L, \Delta H_i, \Delta H$ are defined in Figure 6

The shape of the dimensionless energy gradient line is shown in Figure 6 for various flow conditions. The total pressure along a lateral line can be expressed by summing the original pressure, the pressure head loss due to friction, and the pressure head loss or gain due to elevation. Wu, et al.(1979) developed the equation

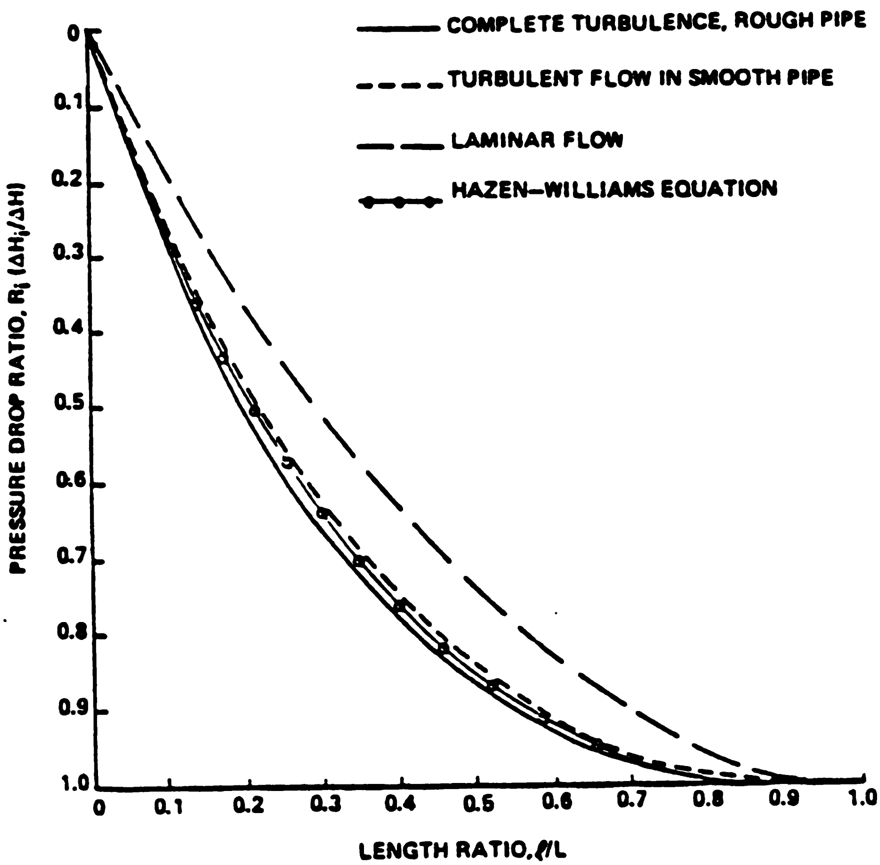

Figure 6. Shape of the dimensionless energy grade line (Wu, et al. 1974).

$$h_i = H_o - R_i \Delta H \pm R_i' \Delta H' \qquad [12]$$

where h_i = pressure head for a given length ratio

H_o = pressure head at the origin

$R_i \Delta H$ = pressure head loss due to friction

$R_i' \Delta H'$ = pressure head loss or gain due to elevation

Figure 7 shows the pressure variations due to various situations and the resulting energy grade line along a lateral.

Bralts, et al. (1987) combined the general emitter flow equation, [1], with [12] and divided it by the flow equations for the first emitter, q_o ($q_o = KH_o^x$). The resulting equation relates emitter flow anywhere along a lateral independent of the coefficient, *K*.

$$q_i = q_o \left[1 - R_i \left(\frac{\Delta H}{H_o} \right) \pm R_i' \left(\frac{\Delta H'}{H_o} \right) \right]^x \qquad [13]$$

where all the variables are as previously defined.

The dimensionless energy gradient line concept can also be used for drip irrigation submain manifolds. To approximate head loss along the manifold, lateral lines are considered similar to uniformly spaced emitters. Using the same concepts as for lateral lines, Bralts, et al. (1987) presented the following equation.

$$q_{ij} = q_s \left[1 - R_j \left(\frac{\Delta H_s}{H_s} \right) - R_i \left(\frac{\Delta H_l}{H_s} \right) \pm R_j' \left(\frac{\Delta H_s'}{H_s} \right) \pm R_i' \left(\frac{\Delta H_l'}{H_s} \right) \right]$$

where $R_j \Delta H_s$ = manifold head loss due to friction

$R_i \Delta H_l$ = lateral head loss due to friction

$R_j' \Delta H_s'$ = manifold head loss or gain due to elevation

$R_i' \Delta H_l'$ = lateral head loss or gain due to elevation

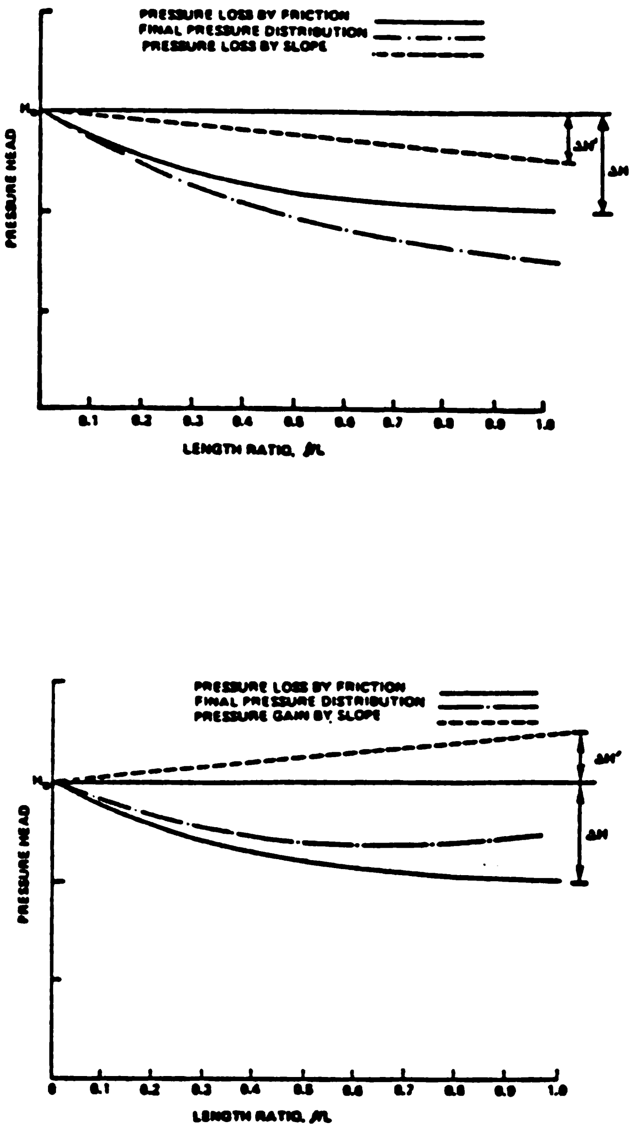

Figure 7. Pressure variations due to various situations (Wu, et al.).

q_s = total flow into the submain unit

x = flow equation exponent

q_{ij} = flow anywhere in the submain unit

Equation [14] can be used to approximate emitter flow at any point in the submain unit once the basic parameters of the submain unit hydraulic system are known. This is shown schematically in Figure 8.

The dimensionless energy grade line concepts presented are the best approximations to flow rates in a drip irrigation submain. It should be realized that these are only approximations because the procedure assumes all emitters flow at the same rate as well as assuming the relationship between flow and pressure in a lateral behave the same as an emitter. Equation [14] must also be used with caution because the exponent x is the same for lateral line and emitters alike. To obtain better estimates of flow, network analysis techniques need to be used.

2.1.4 The Statistical Uniformity Concept

The statistical uniformity concept consists of a statistical approach to the uniformity of emitter flow and irrigation application efficiency based on the coefficient of variation. The coefficient of variation gives a normalized measure of dispersion which can be used to compare the relative variation of different systems and is defined as the standard deviation divided by the mean of the quantity of interest. Bralts, et al., (1981) recommended the use of the statistical uniformity coefficient for drip irrigation lateral lines and can be defined by

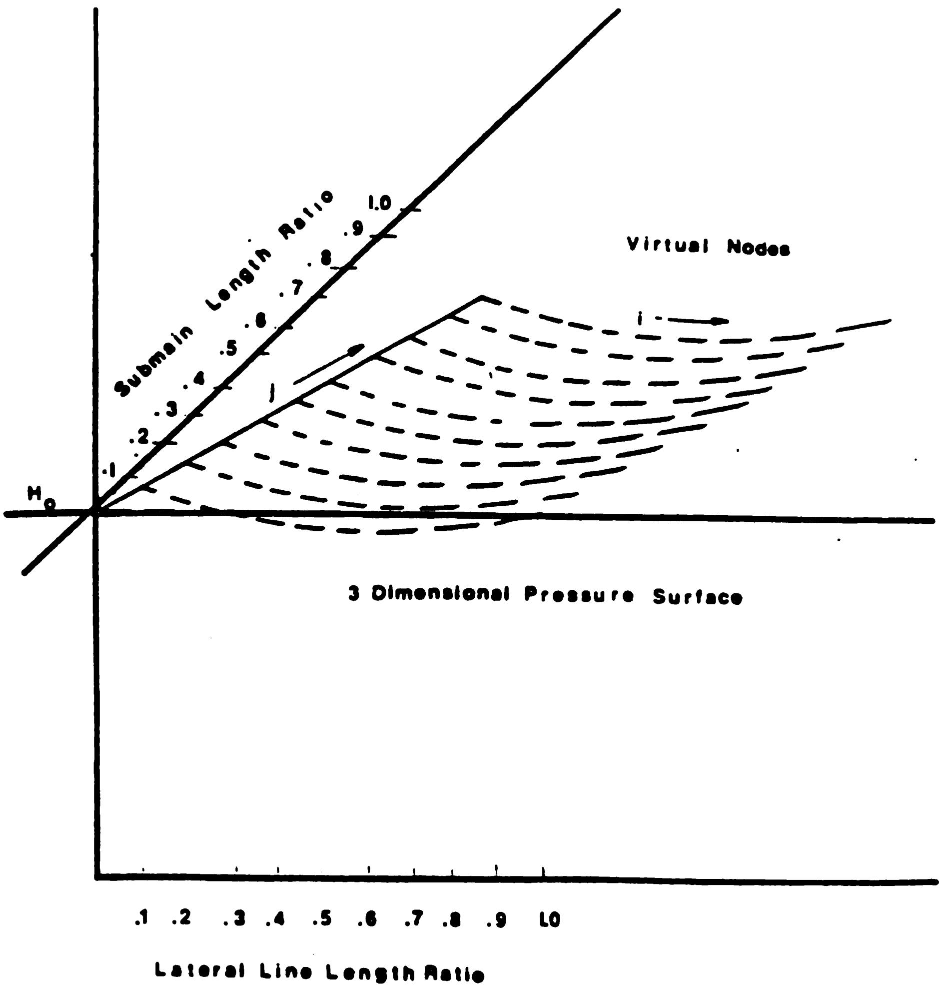

Figure 8. Dimensionless energy grade line extended to the submain unit.

$$U_s = 100(1 - V_q) = 100\left(1 - \frac{S_q}{\bar{q}}\right) \qquad [15]$$

where U_s = statistical uniformity coefficient as a percentage

V_q = the coefficient of variation of emitter flow

S_q = the standard deviation of emitter flow

$\bar{q}$ = the mean emitter flow rate

The variation of emitter flow in a drip irrigation submain unit is the result of a variety of factors. The primary factor is hydraulic design. Other important factors are emitter type, emitter manufacturing, emitter plugging and the number of emitters per plant. Bralts, et al., (1987) shows how the variation of emitter flow in the submain unit can be determined as a probabilistic combination of these relevant factors.

$$V_q = e^{-\frac{1}{2}}(V_p^2 + V_k^2 + x^2 V_h^2)^{\frac{1}{2}} \qquad [16]$$

where V_q = the coefficient of variation of emitter flow

V_p = the coefficient of variation of emitter plugging

V_k = the coefficient of variation of the emitter constant

V_h = the coefficient of variation of the hydraulic pressure

e = the number of emitters per plant

It is important that emitter flow variation or the uniformity of water distribution be known when a drip irrigation system is designed. These quantities can be calculated using the above equations to evaluate the accepability of any particular system. For drip irrigation systems the general criteria for an acceptable statistical uniformity, U_s, are 90% or greater, excellent; 80-90%, very good; 70-80%, fair; 60-70%, poor; and less than 60%, unacceptable.

2.2 Network Analysis Techniques

As stated earlier, the flow in a drip irrigation system can be considered to be hydraulically steady, spatially varied, one dimensional, incompressible pipe flow. Methods for approximating the flows and pressures were presented in the previous section. Network analysis techniques need to be used to more accurately describe the spatial variability of flow and pressure in such systems. The solution of any network problem must satisfy both continuity and energy principles. The continuity principle states that the net flow rate in any piping junction must be zero. Energy principles state that the net head loss around any loop of the network must be zero. Because of the non-linear relationship between flow and head it is not easy to calculate the flow and head at each point in the system. Network analysis techniques are generally implemented on a computer because of their iterative nature and the need for speed and accuracy.

The most popular computer-based network analysis techniques used today are based on Hardy-Cross, Newton-Raphson, and linear theory techniques. The use of these techniques by irrigation engineers for the design of drip irrigation systems has been limited. Solomon & Keller (1974), Wu & Fangmeir (1974) and Perold (1977) have used iterative techniques based on the backstep method. Bralts & Segerlind (1985) proposed a method based on a linearized set of flow equations using the finite element method.

All of the network analysis techniques described here can be derived from a linearization of the system of energy equations. The non-linear terms in the energy equation can be written in terms of flows or heads and expanded into a Taylor's series in the following manner (Wiggert, 1985). The function $F(Q)$ contains the non-linear terms of the energy equation.

$$F(Q) = F(Q_o) + F'(Q_o)(Q - Q_o) + F''(Q_o)\frac{(Q - Q_o)^2}{2} + \ldots \quad [17]$$

or in terms of the pressure heads $F(h)$.

$$F(H) = F(H_o) + F'(H_o)(H - H_o) + F''(H_o)\frac{(H - H_o)^2}{2} + \ldots \quad [18]$$

where H_o and Q_o are estimates of the pressure heads and flows at the point the solution is expanded about. It is usual to consider only the first two terms of the series expansions in the above equations to obtain a first order approximation of the non-linear terms. The Hardy-Cross, Newton-Raphson and linear theory methods will be presented in the following sections.

2.2.1 Hardy-Cross Method

The Hardy-Cross method was one of the first methods developed to analyze hydraulic pipe networks. This method was popular in the pre-computer days as a hand worked solution, but is still used in various forms as the basis of many computer algorithms for the solution of hydraulic network problems (Edwards and Spencer, 1972).

This loop method, first used by Hardy-Cross, is an iterative flow corrective technique using assumed pipe flow rates, based on continuity to solve the energy based loop equations of a hydraulic network. Flow adjustments are made according to the following equation which can be derived from the Taylor's series expansion [17].

$$Q - Q_o = \Delta Q = \frac{\Sigma h}{n \Sigma \frac{h}{Q}} \quad [19]$$

where Σh = net head loss around any closed loop (m)

n = the flow equation exponent

1.852 for H-W

2 for D-W

Q = the flow through an individual pipe (m^3/s)

Barlow and Markland (1969) used a second order approximation of the above equation for corrective flows and obtained a more rapid convergence.

The Hardy-Cross iterative solution is outlined in the following steps following Wiggert (1986).

1. Assume an initial estimate of the flow distribution in the network which satisfies continuity. An initial estimate closer to the correct solution results in faster convergence.
2. For each loop evaluate Q with [19]. The numerator should approach zero as the loops become balanced.
3. Update the flows in each pipe in all the loops using the following equation.

$$Q_{(n+1)} = Q_{(n)} + \Delta Q_{(n)} \qquad [20]$$

where n = the iteration number

Q = the corrected flow

ΔQ = the flow correction from [19]

4. Repeat steps 2 and 3 until a desired accuracy is attained.

The Hardy-Cross method is a simplified version of the method of successive approximations. A modified version of this method called the nodal method was also used by Barlow & Markland (1969). In this method the heads are solved for instead of the flows. The heads are successively approximated at each node using an equation similar to [19]. It can be derived from [18] and is given as

$$\Delta h = \frac{\Sigma Q}{n\Sigma\left(\frac{Q}{h}\right)} \qquad [21]$$

where Q is the sum of the flows into or out of a node and n, h and Q are as defined earlier in [19]. The solution follows the more traditional Hardy-Cross method only the heads are corrected instead of the flows. The Hardy-Cross method can easily be used to solve relatively small networks using either a hand held calculator or spreadsheet algorithm on a microcomputer. The continuity relations are in a sense "decoupled" from the solution of energy equations, they are satisfied initially with the assumed flows and remain satisfied throughout the solution (Wiggert, 1985). The fundamental drawback of the Hardy-Cross method is poor convergence due to the independent solution of loop and nodal equations.

2.2.2 Newton-Raphson Method

The Newton-Raphson method is a method of successive approximations similar to the Hardy-Cross method (Jeppson, 1982). The Hardy-Cross method solves each loop equation one at a time during each iteration. The Newton-Raphson method solves all of the loop equations simultaneously determining all of the corrective flows and head values in the network each iteration. Convergence in this case is quadratic.

The Newton-Raphson method commonly used in hydraulic network analysis solves for corrective flows using the following equation derived from the Taylor's series expansion of corrective flow equations.

$$\Delta Q_{(n+1)} = \Delta Q_{(n)} - Z_n \qquad [22]$$

where ΔQ_{n+1} = the vector of newly estimated corrective flows

ΔQ_n = the vector of previously estimated corrective flows

Z_n = the solution vector of the linear system of equations $D_n Z_n = F_n$

where F_n = matrix of corrective loop flow equations

and D = the Jacobian matrix

$$D = \begin{bmatrix} \frac{\partial F_1}{\partial \Delta Q_1} & \frac{\partial F_1}{\partial \Delta Q_2} & \cdots & \frac{\partial F_1}{\partial \Delta Q_n} \\ \frac{\partial F_2}{\partial \Delta Q_1} & \frac{\partial F_2}{\partial \Delta Q_2} & \cdots & \frac{\partial F_2}{\partial \Delta Q_n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \frac{\partial F_n}{\partial \Delta Q_1} & \frac{\partial F_n}{\partial \Delta Q_2} & \cdots & \frac{\partial F_n}{\partial \Delta Q_n} \end{bmatrix} \qquad [23]$$

The Newton Raphson technique can also be used to solve a set of nodal equations for corrective pressures, (ΔH). In this case ΔH is substituted in place of ΔQ in the above equations.

The convergence of the Newton-Raphson method is highly dependent upon a reasonable first approximation (Jeppson, 1977). Another disadvantage of the Newton-Raphson method is the need to evaluate first derivative terms in the Jacobian matrix D each iteration.

2.2.3 Linear Theory Method

The linear theory method of network analysis was first proposed by Wood & Charles (1972). This method has several advantages. The most significant is that convergence to the final result is rapid and does not require initial flow estimates or complicated differential equations for the solution.

Network analysis theory method solves both loop or nodal equations. The basic theory transforms the loop or nodal equations into a set of linear equations that can be easily be solved. An initial approximation of the flow rates is generally obtained by assuming laminar flow exists. The calculated flow rates are used to determine the coefficients in the equations each successive iteration. The transformation of the loop energy equation results in

$$\Sigma h_f = \Sigma K_{(n)} Q^{m-1} Q = \Sigma K_{(n)}' Q_{(n)}' = 0 \qquad [24]$$

where Q_n = approx. discharge

K' = modified pipe constant

The nodal equations can also be transformed in a similar manner that results in a highly accurate solution obtained with few iterations. However, in some cases convergence was never obtained. By far the most common application of the linear theory is with the loop equations.

2.2.4 Finite Element Method

The finite element method is a systematic numerical procedure used to solve many complex engineering problems. This method has been used with discrete elements such as in structural analysis and continuum element problems such as groundwater movement and heat transfer. The classical finite element method involves as integral formulation and a set of piece-wise smooth equations to approximate a quantity. The use of the finite element method for the solution of hydraulic network problems is a simple extension of the original development for structural assemblages (Bralts, 1987).

When laminar flow is present throughout the hydraulic network, the relationship between head loss and flow is linear and was analyzed using the nodal equations and the finite element method. This special case of the linear theory method was solved using the finite element procedure by Norrie & DeVries (1978). Bralts & Segerlind

(1985) used the finite element method for network analysis of a drip irrigation system. The following is presented from their development of what a single pipe element contributes to the continuity equation. Friction losses in tees and elbows were neglected. The energy equation is written for a straight pipe element, (Figure 9), and rearranged into the following equation.

$$Q = C_p(H_i - H_j) + C_p(Z_i - Z_j) \qquad [25]$$

where $$C_p = \frac{\left[|(Z_i + H_i) - (Z_j + H_j)|^{\frac{(1-m)}{m}} \right]}{k^{\frac{1}{m}}}$$

H_i & H_j = upstream and downstream static pressure heads

Q = flow through the pipe

k = friction loss coefficient

m = exponent in D-W or H-W

The finite element method utilizes the concept of an element stiffness matrix and element force vector to construct a system of equations. For element (e),(Figure 10), the system of nodal equations for s and t are

$$-Q_s^{(e-1)} + Q_s^{(e)} = 0 \qquad [26]$$

$$-Q_t^{(e)} + Q_t^{(e+1)} = 0 \qquad [27]$$

where flow into a node is negative and flow away from a node is positive. The contribution of element (e) to the above equations is

$$Q_s^{(e)} = C_p(H_s - H_t) + C_p(Z_s - Z_t) \qquad [28]$$

and

$$Q_t^{(e)} = -C_p(H_s - H_t) - C_p(Z_s - Z_t) \qquad [29]$$

The element matrices are the contribution of an element to the nodal equations that it touches,

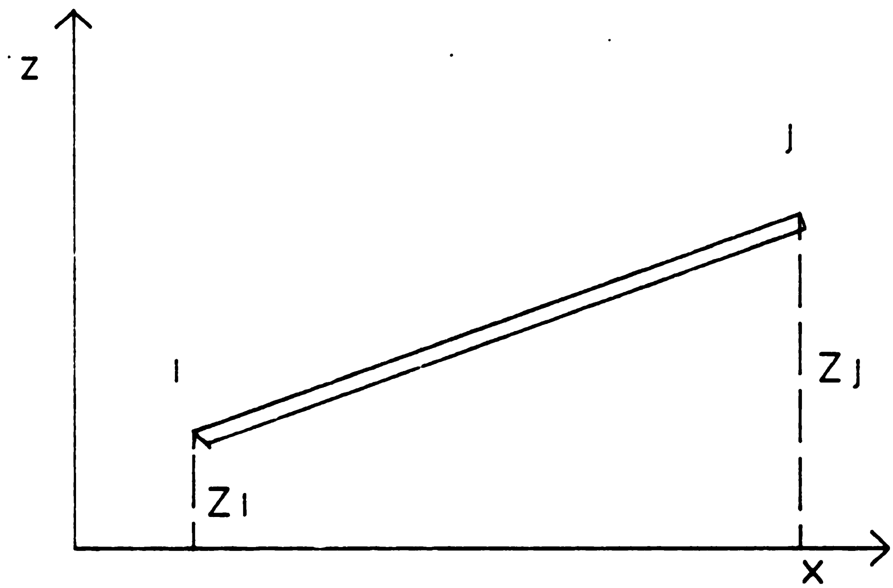

Figure 9. Straight Pipe Element

Figure 10. Three successive pipe elements

$$\begin{Bmatrix} Q_s^{(e)} \\ Q_s^{(e)} \end{Bmatrix} = \begin{bmatrix} C_p & -C_p \\ -C_p & C_p \end{bmatrix} \begin{Bmatrix} H_s \\ H_t \end{Bmatrix} - \begin{Bmatrix} C_p \Delta Z \\ -C_p \Delta Z \end{Bmatrix} \quad [30]$$

where $\Delta Z = Z_t - Z_s$.

Equation [30] has the standard finite element form

$$\{R^{(e)}\} = [K^{(e)}]\,\{H^{(e)}\} - \{f^{(e)}\} \quad [31]$$

where

$$[K^{(e)}] = \begin{bmatrix} C_p & -C_p \\ -C_p & C_p \end{bmatrix} \quad [32]$$

is the element stiffness matrix.

$$\{H^{(e)}\} = \begin{Bmatrix} H_s \\ H_t \end{Bmatrix} \quad [33]$$

is the vector containing the element nodal values and

$$\{f^{(e)}\} = \begin{Bmatrix} C_p \Delta Z \\ -C_p \Delta Z \end{Bmatrix} = \begin{Bmatrix} g \\ -g \end{Bmatrix} \quad [34]$$

is the element force vector. The product of $C_p Z$ is g and all other variables are previously defined.

The element matrices are assembled using a direct stiffness algorithm (Segerlind, 1984) and yield a system of equations which have a general matrix form as

$$[K]\,\{H\} - \{F\} = \{0\} \quad [35]$$

where $[K]$ is the global stiffness matrix

$\{F\}$ is the global force vector

$\{H\}$ is the vector containing the nodal pressure values for the network being solved.

Emitters in a drip irrigation system were implemented in the system by considering them to be a separate element (Figure 11).

$$Q = C_e(H_i - H_j) \qquad [36]$$

where C_e is the linearized coefficient for the generalized emitter flow equation as presented by Wu et al. (1979). The flow equation for an emitter junction (Figure 11) is

$$Q_s^{(e-1)} + Q_s^{(e+1)} = 0 \qquad [37]$$

An emitter head connected to node s is incorporated into the final system of equations as follows. First add the value of C_e directly to the diagonal value K_{ss} in $[K]$. Second add the value of $C_e H_t$ directly to rows of $\{F\}$. An example submain unit layout and the associated stiffness matrix was presented by Bralts (1987) and is shown in Figures 12 and 13.

Bralts and Segerlind (1985) developed a computer program to solve drip irrigation systems using the finite element method described above. They cited numerous advantages. The final system of equations is symmetric and banded. Proper numbering of the pipe junctions and emitters resulted in a relatively small bandwidth. A large drip irrigation design problem can be stored and solved in a relatively small computer (Bralts, et al. 1987). Another advantage of the finite element method is that once the equations are developed in the standard finite element form [31], then many of the existing finite element programs can be used to solve and assemble the equations.

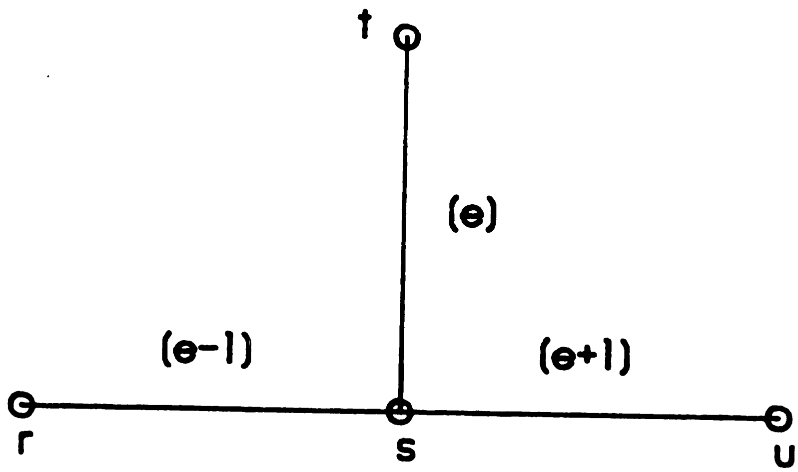

Figure 11. Two pipe elements and an emitter element

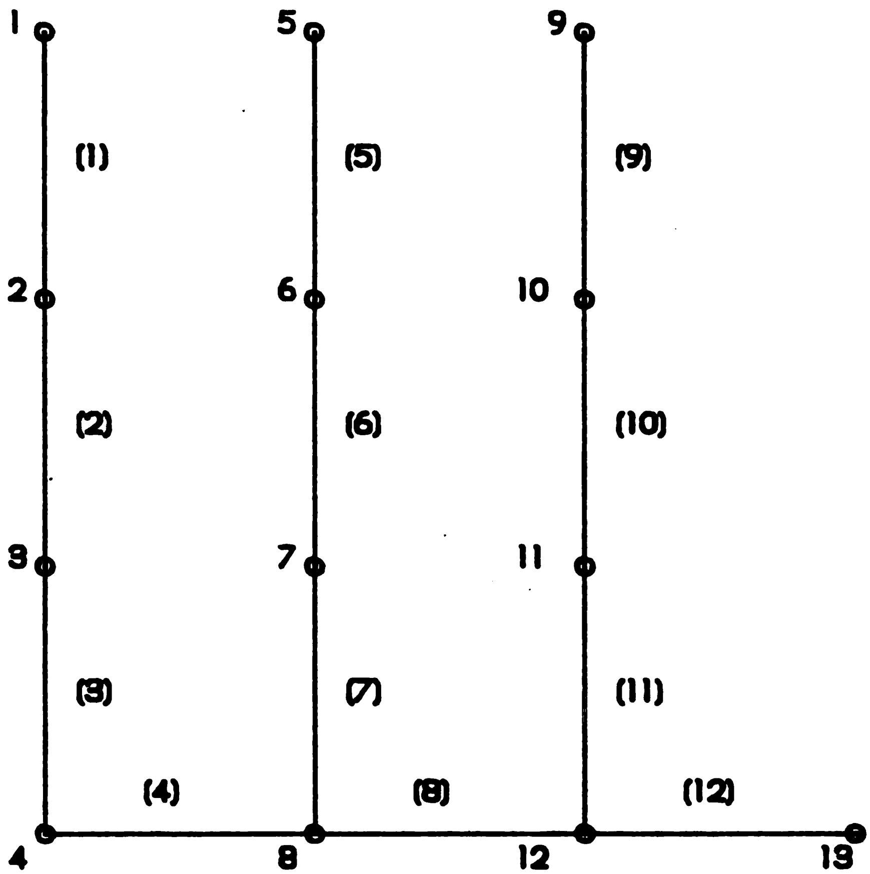

Figure 12. Example submain unit

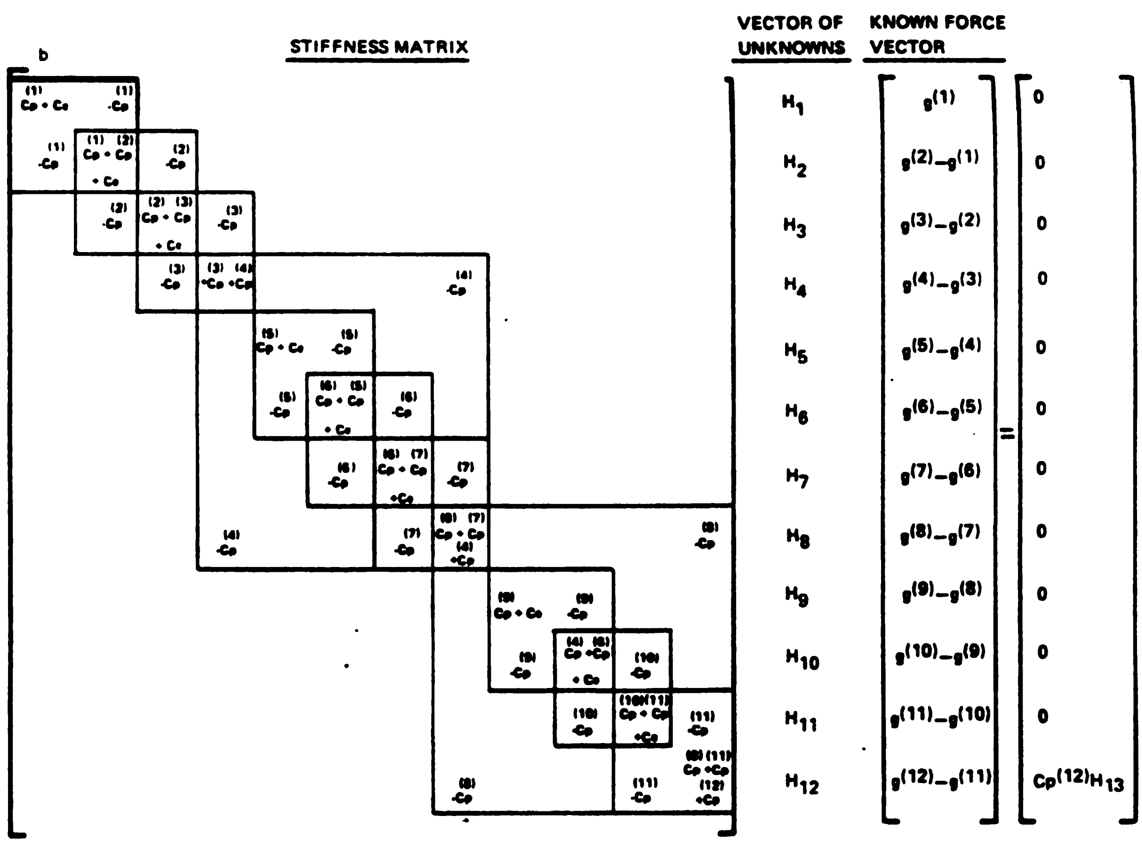

Figure 13. Example solution matrix (Bralts and Segerlind, 1985)

2.3 Numerical Solution Techniques

Use of substructures and various techniques for solving the resulting set of equations will be described in the following sections.

2.3.1 Virtual Nodes in the Finite Element Method

Bralts and Segerlind (1987) proposed a technique for incorporating a virtual node concept using the finite element approach to drip irrigation system hydraulic design. They proposed that this technique could allow the concentration of the solution matrix into a simple, more compact virtual node matrix. This solution technique could possibly improve the speed of convergence with large drip irrigation systems of 10,000 or more emitters. The motivation for this section is to find the scope of current use of such techniques and determine their applicability of such techniques to solving large drip irrigation network problems.

To analyze a large complex structure it is necessary, for economic, computer capacity or organizational reasons to divide it into a number of smaller substructures. Substructure coupling procedures have been developed for finite element analysis of large structures. Each substructure is analyzed as though it were a separate problem. The various substructures are then coupled together in such a way that equilibrium and continuity are satisfied at common boundaries.

Substructures have been used to analyze the separate components on an airplane where natural substructures occur, such as fuselage, wings and tail. The overall problem then involves only the variables on the boundaries of the substructure. The idea of the substructure may be nested. Each substructure may itself be substructured to any depth and identical substructures could greatly reduce calculations. Artificial substructures can also be introduced. A two dimensional problem consisting of a regular q x q grid of square elements could be solved by dividing the elements into four groups with a " + " shaped cut, then dividing each group into four parts again with a " + " shaped cut and so on. This was termed "nested dissection".

Utilizing a technique similar to substructuring, a large drip irrigation system could be analyzed using the finite element method. It could increase the speed of convergence as well as solve some of the problems that occur when storing such a large network with limited memory.

2.2.2 Solution of Linear Finite Element Equations

Once the energy and continuity equations for network analysis are put in standard finite element form,

$$[K]\{H\}-\{F\}=\{0\} \qquad [38]$$

there are numerous methods for solving these sets of linear equations. Bralts and Segerlind (1985) solved these resulting equations for a drip irrigation submain unit. They based their initial estimates of the nodal heads using the energy grade line concept (Wu, et al., 1975). Using these estimates they calculated the linearized flow coefficient, C_p, in $[K]$ and $\{F\}$. $[K]$ and $\{F\}$ were used to solve for new head values, $\{H\}$, using Gaussian elimination techniques. These new head values were used to calculate updated C_p values in $[K]$ and $\{F\}$. The process was repeated until changes on successive head values, $\{H\}$, were small and the solution was considered to have converged on the correct solution.

The principle direct methods for solving finite element sets of linear equations are all variants of Gaussian elimination. Two iterative methods that have come to prominence recently occur more frequently in finite element methods are the frontal technique and band matrix techniques.

It is thought that the use of these newer solution techniques could be useful when solving a large drip irrigation network on a small computer using the finite element method described by Segerlind and Bralts (1985). Some of these newer solution techniques could be applied along with a substructuring could be investigated using a large drip irrigation network.

2.4 Summary and Discussion

The review of literature and theory was written to provide the theoretical framework and motivation for the proposed research. It is hoped that this provides a basis with which to improve the techniques available to analyze the hydraulics of large drip irrigation systems using computer based network analysis techniques.

The review of hydraulics of drip irrigation systems was included, as it is the basis of hydraulic design of drip irrigation systems. The resulting non-linear pressure-flow relationships of emitter and pipe flow were presented as they occur in a drip system. Empirical relationships are generally used to calculate emitter flow and friction loss in pipes. Hazen-Williams and Darcy-Weisbach were used predominantly in drip irrigation design, with the Darcy-Weisbach considered as more accurate.

Various methods to get around the problems that the non-linearities of the pressure-flow relationship and the spatial variability in a drip lateral line were presented. Graphical and simplified computational approximations of head loss in pipes have prevailed for drip irrigation lateral design. The approximation technique that seemed best suited for computer use was the dimensionless energy grade line concept. This concept was expanded to include a whole submain unit and provides a good initial estimate for head loss for use in other network analysis techniques.

Network analysis techniques were presented and their use in drip irrigation design was found to be limited. All the methods use a linearized form of the energy and continuity equations that result in loop or nodal equations that are solved iteratively for flows or heads. The use of the finite element formulation of the nodal equations extends the methods of solution to the expansive techniques that are available in many finite element programs. The use of substructures for the solution of large drip irrigation networks was mentioned. The techniques could be applied to the finite element formulation of Bralts and Segerlind (1985). More efficient techniques are available for solving systems of linear equations such as the frontal method and sparse matrix techniques could be used to reduce storage requirements and speed convergence of large networks. Even with the speed and power of microcomputers seeming to increase every day, the utilization of efficient techniques opens the door for ever increasing analysis without the need for mainframe storage and speed.

III. METHODOLOGY

The review of literature has shown that network analysis techniques have seen limited use in the design off drip irrigation submain units. As use of drip irrigation grows, the need to analyze large irrigation networks with 10,000 or more emitters efficiently becomes more apparent. For row crops such as tomatoes, it is easy to imagine a submain unit of 1.5 ha with 100 m laterals spaced 1.5 m apart, with an emitter every 1 m. This system would contain 15,000 emitters. The goal of this research is to utilize the finite element method and network analysis techniques to reduce the number of nodal equations and obtain a virtual node system. Development of a procedure would lead to improved hydraulic design of drip irrigation systems to conserve water and energy.

3.1 Research Approach

Based upon the need for comprehensive hydraulic design techniques for drip irrigation networks, the following approaches are proposed for achieving the major research objectives.

Objective 1. Development of an equation to describe the flow phenomena in a drip irrigation system.

The approach to be followed under objective 1 will be to use conservation of energy and mass principles along with empirical equations describing the pipe friction losss and emitter flow to develop a general differential equation expressing the pressure head in a lateral line.

Objective 2. Investigate the use of virtual node concept in hydraulic design of drip irrigation systems.

The approach to be followed under objective 2 will be to develop the nodal equations for network analysis using a virtual node system. A virtual node system will enable large drip irrigation systems to be discretized into a manageable number of equations by combining emitters into virtual nodes Figure 14. After the nodal equations are developed, the finite element method will be used to solve for nodal pressures at each emitter.

Objective 3. Define limits of application, efficiency of solution, improve speed of convergence, and improve initial estimates of the solution method using a virtual node concept.

The approach to be followed under this objective is the development of a finite element based computer model using the concepts formulated in objectives one and two. The computer model will be designed such that various emitter types, lateral line sizes, and submain units can be analyzed. Using the computer model, speed of convergence, efficiency of solution, accuracy, and initial estimates of the different methods developed

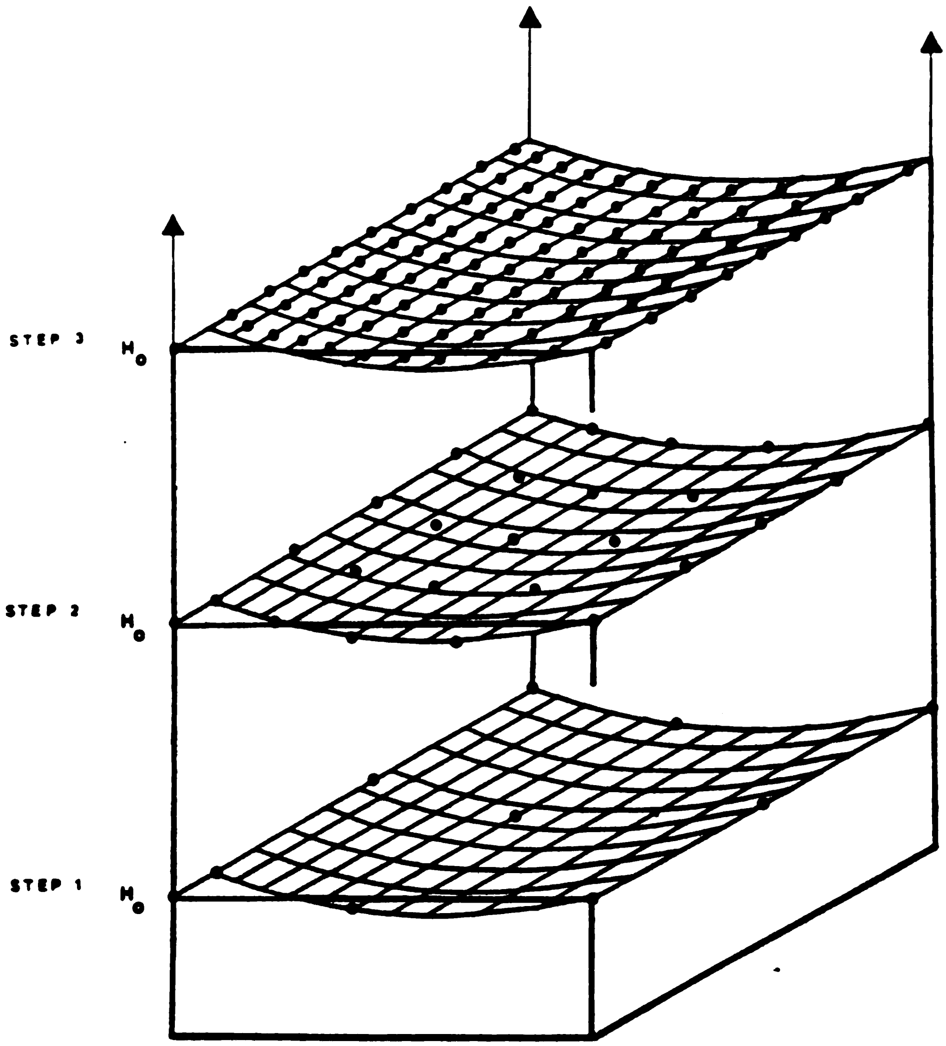

Figure 14. Virtual node substructure.

will be compared to an exact solution of a drip irrigation system. Once the advantages, disadvantages and limits of application are determined, its use in hydraulic design of drip irrigation systems will be evaluated.

3.2 Theoretical Development

The objective of the following development is to obtain a system of equations describing the flow phenomena in the submain, laterals and emitters of a trickle irrigation system. The resulting equations will allow the submain unit to be discretized into elements which can contain any number of laterals or emitters. By using conservation of energy and mass principles along with empirical equations describing pipe friction loss and emitter flow a differential equation can be developed to solve for streamline or potential flow in a submain unit. In streamline flow the quantity of interest is the flow rates or fluid velocities. For potential flow the quantity of interest is the pressure heads. In either case, once one of the quantities is solved for the other can be easily found using the empirical relations relating flow rates to pressure heads.

The following sections derive a differential equation in terms of potentials or pressure heads in a drip irrigation submain unit. The differential equation takes the following form

$$D_x \frac{\partial^2 h}{\partial x^2} + Gh + Q = 0 \qquad [39]$$

where h, is the pressure head or potential anywhere in the submain unit. D_x, G, and Q are coefficients in the differential equation and are functions of the pressure head and pipe section. Equation [39] is solved using the classical finite element method by applying Galerkin's formulation. The coefficients, D_x, G and Q are recalculated using the newest approximations for pressure head and an iterative solution is obtained.

First, it is shown how a grid of one dimensional linear elements is used to approximate pressure heads in the submain unit. Emitters are incorporated into [39] in three different ways with the G and Q terms. The method is then extended to use a grid of quadratic elements. Finally, it is shown how emitters can be incorporated into the solution of [39] by treating them as point sources.

3.2.1 Differential Equation for Pipe Flow

For the pipe element in Figure 15, the energy at point x, can be expressed using the energy equation for incompressible pipe flow as

$$E_x = h + z + \frac{v_x^2}{2g} \qquad [40]$$

Similiarly, the energy at point $x+dx$ is

$$E_{x+dx} = \left(h - \frac{\partial h}{\partial x}dx\right) + \left(z - \frac{\partial z}{\partial x}dx\right) + \frac{1}{2g}\left(v_x - \frac{\partial v_x}{\partial x}dx\right)^2 \qquad [41]$$

Using conservation of energy principles,the energy at point x is equal to the energy at point $x+dx$. The energy equation is written for the pipe element of length, dx in Figure 15 as

$$h + z + \frac{v_x^2}{2g} = \left(h - \frac{\partial h}{\partial x}dx\right) + \left(z - \frac{\partial z}{\partial x}dx\right) + \frac{1}{2g}\left(v_x - \frac{\partial v_x}{\partial x}dx\right)^2 + h_f \qquad [42]$$

where h_f is the head loss due to friction

h is the pressure head (m)

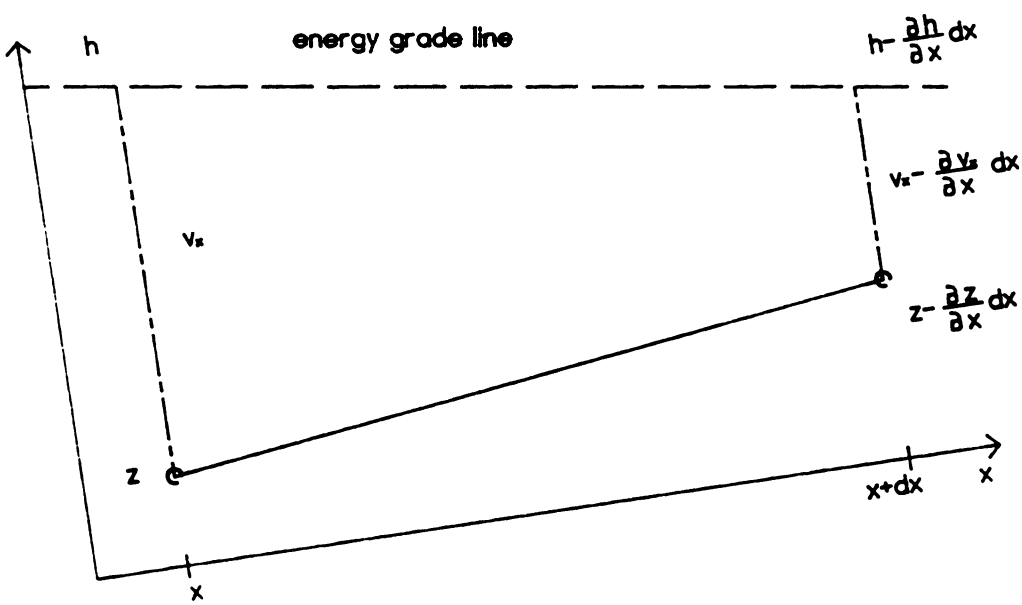

Figure 15. Pipe element

z is the elevation head (m)

v_x is the average fluid velocity (m/s)

dx is the length of the pipe element (m)

Subtracting h, z, and $v_x^2/2g$ from both sides of [42] gives

$$\frac{\partial h}{\partial x}dx+\frac{\partial z}{\partial x}dx+\frac{1}{2g}\left(v_x\frac{\partial v_x}{\partial x}dx\right)+\frac{1}{2g}\left(\frac{\partial v_x}{\partial x}dx\right)^2=h_f \qquad [43]$$

The headloss due to friction in the straight pipe element in Figure 15 is empirically calculated from the generalized equation, [7], discussed in the literature review and rearranged in terms of fluid velocities instead of flow rates as follows

$$h_f=av_x^m dx \qquad [44]$$

where h_f = head loss due to friction

v_x = average fluid velocity

dx = length of the pipe element

and if the Hazen-Williams formulation is used.

$$a=\frac{5.88}{C^{1.852}A^{0.5835}} \qquad [45]$$

where C = Hazen-Williams coefficient

A =cross sectional area through which flow occurs

m = 1.852

For the straight pipe element in Figure 15, the fluid velocity at point x and $x+dx$ are equal, ($v_x = v_{x+dx}$), and the first two terms of the energy equation, [43], are exactly the friction loss described by [44]. Equating them results in the following

$$av_x^m dx=\frac{\partial h}{\partial x}dx+\frac{\partial z}{\partial x}dx \qquad [46]$$

Equation [46] is then solved for the flow velocity, v_x, giving

$$v_x = \frac{1}{a^{\frac{1}{m}}}\left[\frac{\partial h}{\partial x}\right]^{\frac{1}{m}} + \frac{1}{a^{\frac{1}{m}}}\left[\frac{\partial z}{\partial x}\right]^{\frac{1}{m}} \quad [47]$$

The quantity $\partial z/\partial x$ in [47] is the slope of the pipe and is generally known as the change in elevation per unit length. The quantity of interest in [47] is the headloss per unit length, $\partial h/\partial x$. Equation [47] is linearized in terms of $\partial h/\partial x$ when an initial estimate of the headloss per unit length or $\Delta h/\Delta x$ is provided. This results in a linearized form of equation [47].

$$v_x = \left[\frac{1}{a^{\frac{1}{m}}}\left[\frac{\Delta h}{\Delta x}\right]^{\frac{1-m}{m}}\right]\frac{\partial h}{\partial x} + \frac{1}{a^{\frac{1}{m}}}\left[\frac{\partial z}{\partial x}\right]^{\frac{1}{m}} \quad [48]$$

The law of conservation of mass for steady state flow requires that the rate of fluid out of the pipe element be equal to the rate of flow into the pipe element. If flows into the pipe are considered positive and flows out of the pipe considered negative the mass balance for the pipe element of Figure 15 is

$$v_x A - \left(v_x + \frac{\partial v_x}{\partial x}dx\right)A = 0 \quad [49]$$

where A is the cross sectional area through which flow occurs. Equation [49] algebraically reduces to

$$-A\frac{\partial v_x}{\partial x} = 0 \quad [50]$$

The flow velocity given by [48] is substituted into [50] and the differentials are evaluated resulting in

$$D_x\frac{\partial^2 h}{\partial x^2} + Q_x = 0 \quad [51]$$

where

$$D_x = \frac{A}{a^{\frac{1}{m}}}\left[\frac{\Delta h}{\Delta x}\right]^{\frac{1-m}{m}} \qquad [52]$$

and

$$Q_x = \frac{A}{ma^{\frac{1}{m}}}\left[\frac{\Delta z}{\Delta x}\right]^{\frac{1-m}{m}} \qquad [53]$$

3.2.2 Differential equation for pipe flow including emitters

Using conservation of energy principles, the energy equation is written for the pipe element of length, dx which is losing fluid along its length at a rate q_e in Figure 16 the same as equation [42]. The generalized friction loss equation, [44], is related to the energy equation in exactly the same way as if no emitters were present. This results in the same equation for flow velocity,

$$v_x = \left[\frac{1}{a^{\frac{1}{m}}}\left[\frac{\Delta H}{\Delta x}\right]^{\frac{1-m}{m}}\right]\frac{\partial h}{\partial x} + \frac{1}{a^{\frac{1}{m}}}\left[\frac{\partial z}{\partial x}\right]^{\frac{1}{m}} \qquad [54]$$

If flows into the pipe are considered positive and flows out of the pipe considered negative the mass balance for the pipe element of Figure 16 is

$$A\frac{\partial v_x}{\partial x} - \frac{q_e}{dx} = 0 \qquad [55]$$

where A =cross sectional area through which flow occurs

q_e =flow out of the element due to emitter flow

The flow velocity given by [54] is substituted into [55] and the differentials are evaluated resulting in the following differential equation to describe the pressure head in the pipe element including emitters.

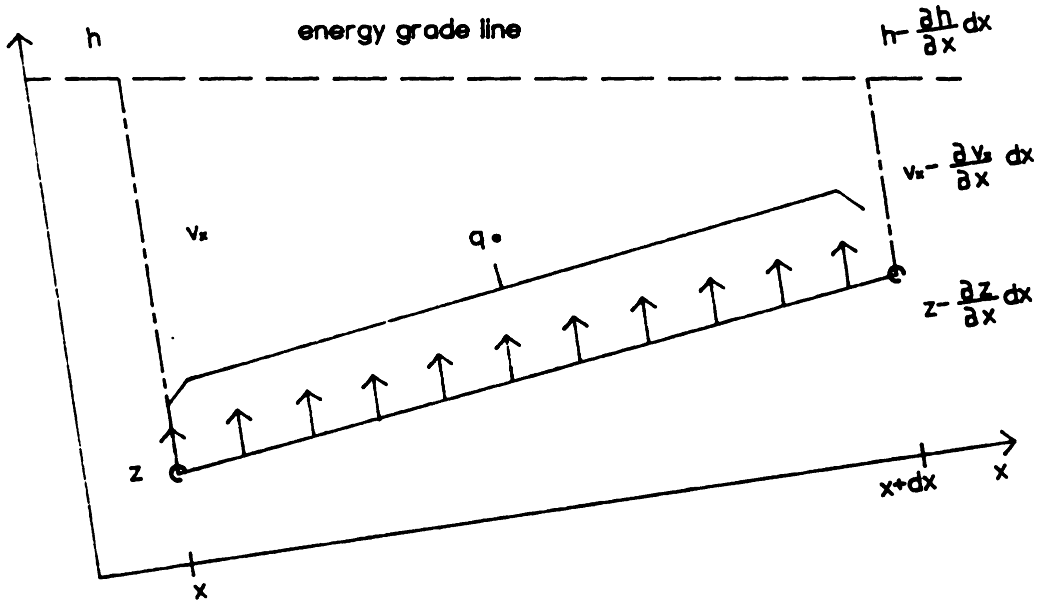

Figure 16. Pipe element with emitters.

$$D_x \frac{\partial^2 h}{\partial x^2} + Q_s - \frac{q_e}{dx} = 0 \quad [56]$$

The outflow from the pipe element, q_e, is related to pressure head in the element by the following equation for emitter flow.

$$q_e = nkh^{x_e} \quad [57]$$

where h = the pressure head at the emitter

x_e = emitter exponent

k = emitter constant

n = the number of emitters/element

The emitter outflow can be incorporated into the differential equation, [56], using one of the following three methods. The average pressure head in the element, h' can be calculated from previous estimates of pressure heads at both ends of the pipe element and emitters can be considered to flow at this average pressure head resulting in

$$Q_e = -\frac{nk(h')^{x_e}}{\Delta x} \quad [58]$$

hereafter referred to as method 1.

The emitter flow equation can be linearized in terms of pressure head, h, resulting in

$$G(h) = \left[\frac{nk(h')^{x_e - 1}}{\Delta x}\right] h \quad [59]$$

hereafter referred to as method 2.

The emitter flow equation, [57], can be expanded using the first two terms of a Taylor's series and estimated as

$$G(h) + Q_e = \left[\frac{x_e nk(h')^{x_e - 1}}{\Delta x}\right] h + (1 - x_e)\frac{nk}{\Delta x}(h')^{x_e} \quad [60]$$

hereafter referred to as method 3.

The resulting one-dimensional differential equation describing pressure heads in a pipe element can be summarized as follows.

$$D_x \frac{\partial^2 h}{\partial x^2} + G(h) + Q_e + Q_z = 0 \qquad [61]$$

The coefficients D_x, G, Q_e, and Q_z for three different methods is summarized in Table 1. The solution procedure is summarized in the following steps.

1. The submain unit or lateral is discretized into elements by specifying nodal points as well as specifying coordinate values.
2. Specify the approximation equation. In this case the one dimensional linear element is used.
3. Initial estimates for pressure heads, h, at nodes must be specified to calculate D_x, G, Q_e, and Q_z for each element. Subsequently, D_x, G, Q_e, and Q_z are recalculated using the most recent values of pressure heads.
4. Using Galerkin's method an equation is written for each unknown nodal value using D_x, G, Q_e, and Q_z.
5. The system of equations is modified to include values of constant pressure head.
6. Solve the system of equations.
7. Compare previous pressure heads with new values of pressure head. If the solution hasn't converged go to step 3.
8. When the solution has converged calculate other quantities of interest. (i.e. flow rates in average emitter flows, nodal flows.)

Table 1. Coefficients for [61].

Emitter Outflow Method	D_x	G	Q_e	Q_z
1	$\frac{A}{a^{\frac{1}{m}}}\left[\frac{\Delta h}{\Delta x}\right]^{\frac{1-m}{m}}$	0	$-\frac{nk(h')^{x_e}}{\Delta x}$	$\frac{A}{ma^{\frac{1}{m}}}\left[\frac{\Delta z}{\Delta x}\right]^{\frac{1-m}{m}}$
2	$\frac{A}{a^{\frac{1}{m}}}\left[\frac{\Delta h}{\Delta x}\right]^{\frac{1-m}{m}}$	$\left[\frac{nk(h')^{x_e-1}}{\Delta x}\right]h$	0	$\frac{A}{ma^{\frac{1}{m}}}\left[\frac{\Delta z}{\Delta x}\right]^{\frac{1-m}{m}}$
3	$\frac{A}{a^{\frac{1}{m}}}\left[\frac{\Delta h}{\Delta x}\right]^{\frac{1-m}{m}}$	$\left[\frac{x_e nk(h')^{x_e-1}}{\Delta x}\right]h$	$(1-x_e)\frac{nk}{\Delta x}(h')^{x_e}$	$\frac{A}{ma^{\frac{1}{m}}}\left[\frac{\Delta z}{\Delta x}\right]^{\frac{1-m}{m}}$

3.2.3 One-dimensional linear element using Galerkin's method

Equation [61] can be solved using the classical finite element method by applying Galerkin's formulation and one-dimensional linear elements. The residual equations for each element are as follows (Segerlind, 1984)

$$\{R^{(e)}\} = [k^{(e)}]\,\{h^{(e)}\} - \{f^{(e)}\} \qquad [62]$$

where

$$\{h^{(e)}\} = \begin{Bmatrix} h_i \\ h_j \end{Bmatrix} \quad [63]$$

$$[k^{(e)}] = \frac{D_x}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{G_e L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad [64]$$

and

$$\{f^{(e)}\} = \frac{Q_e L}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{Q_s L}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The element matrices are then combined using the direct stiffness procedure and the final system of equations,

$$[K]\,\{H\} - \{F\} = \{0\} \quad [66]$$

results.

3.2.4 One-dimensional quadratic element using Galerkin's method

Equation [61] can be solved using the classical finite element method by applying Galerkin's formulation and one-dimensional quadratic elements. The residual equations for an element are as follows.

$$\{R^{(e)}\} = [k^{(e)}]\,\{h^{(e)}\} - \{f^{(e)}\} \quad [67]$$

where

$$\{h^{(e)}\} = \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} \quad [68]$$

and

$$[k^{(e)}] = \frac{D_x}{6L}\begin{bmatrix} 14 & -16 & 2 \\ -16 & 32 & -16 \\ 2 & -16 & 14 \end{bmatrix} + G_e L \begin{bmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{bmatrix} \quad [69]$$

and

$$\{f^{(e)}\} = \frac{Q_e L}{6}\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} + \frac{Q_s L}{6}\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \quad [70]$$

3.2.5 Emitters as point sources

In this section it is shown how an emitter can be treated as a point source in a section of pipe. Consider the linear pipe element in Figure 17 with an emitter located at X_o. When emitters are considered this way emitter flow, q_e is no longer constant throughout the element as in the previous solution technique, but is a function of X.

As before, the flow in the pipe without emitters is

$$D_x \frac{\partial^2 h}{\partial x^2} + Q_s = 0 \quad [71]$$

The solution to [71] is solved using the finite element method and a residual equation is formed for each element. The residual equations are then modified as follows to incorporate emitters as point sources.

$$\{R^{(e)}\} = [k^{(e)}]\{h^{(e)}\} - \{f^{(e)}\} \quad [72]$$

For the linear element the modifications are as follows.

$$\{h^{(e)}\} = \begin{Bmatrix} h_i \\ h_j \end{Bmatrix} \quad [73]$$

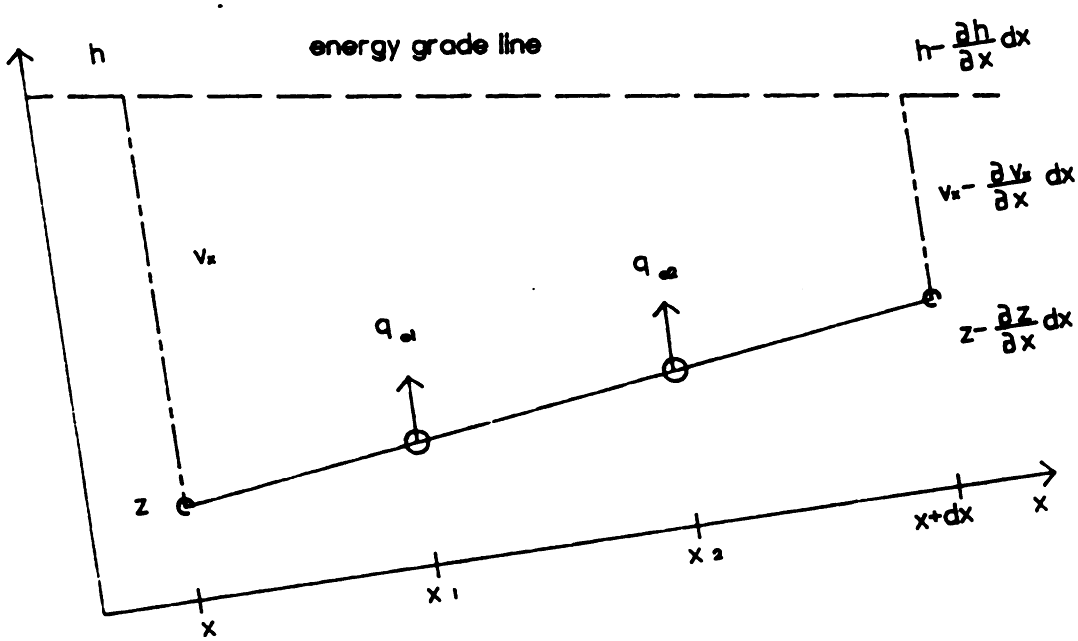

Figure 17. Pipe element with emitters as point sources.

$$[k^{(e)}] = \frac{D_x}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \alpha G_e \begin{bmatrix} N_i^2 & N_i N_j \\ N_j N_i & N_j^2 \end{bmatrix}_{X = X_0} \qquad [74]$$

$$\{f^{(e)}\} = \frac{Q_s L}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \alpha Q_e \begin{bmatrix} N_i \\ N_j \end{bmatrix}_{X = X_0} \qquad [75]$$

For the quadratic element the modifications are as follows.

$$\{h^{(e)}\} = \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} \qquad [76]$$

$$[k^{(e)}] = \frac{D_x}{6L}\begin{bmatrix} 14 & -16 & 2 \\ -16 & 32 & -16 \\ 2 & -16 & 14 \end{bmatrix} + \alpha G_e \begin{bmatrix} N_i^2 & N_i N_j & N_i N_k \\ N_j N_i & N_j^2 & N_j N_k \\ N_k N_i & N_k N_j & N_k^2 \end{bmatrix}_{X = X_0} \qquad [77]$$

$$\{f^{(e)}\} = \frac{Q_s L}{6}\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} + \alpha Q_e \begin{bmatrix} N_i \\ N_j \\ N_k \end{bmatrix}_{X = X_0} \qquad [78]$$

N_i, N_j and N_k are the interpolation functions for the element evaluated at the point X_o. If the emitter is located directly on a node that is shared by one or more elements the contribution of Q_e is divided according to the factor α.

IV. RESULTS AND ANALYSIS

In the results and discussion section, the methods derived in the theoretical development are analyzed using the computer model developed as research objectives. Utilizing the results of this analysis, the virtual node concept is then evaluated for use in hydraulic design of drip irrigation systems.

4.1 Computer Model (VNODE)

A computer program, (VNODE) was developed to incorporate the virtual node concepts with the finite element method to analyze the hydraulics of a drip irrigation lateral. (Appendix A). The computer program (VNODE) solves the general hydraulic network problem posed in a drip irrigation lateral, where flow is considered hydraulically steady, spatially varied, one-dimensional, incompressible pipe flow.

VNODE was developed in Turbo Pascal Ver. 4.0 on an AT class microcomputer. The problem is first formulated as a differential equation to describe flow in the drip irrigation lateral as described in equation [53]. The solution of this equation then follows the classical finite element method where the coefficients D_x, G and Q are all functions of the pressure head, h. The solution steps are described in the theoretical development. Figure 18 shows a general flow chart of the program.

The program can use one dimensional linear or quadratic elements and depending on the formulation used, the functions $D_x(h)$, $G(h)$ and $Q(h)$ can be changed. Friction loss is approximated using the Hazen-Williams formulation. The system of residual

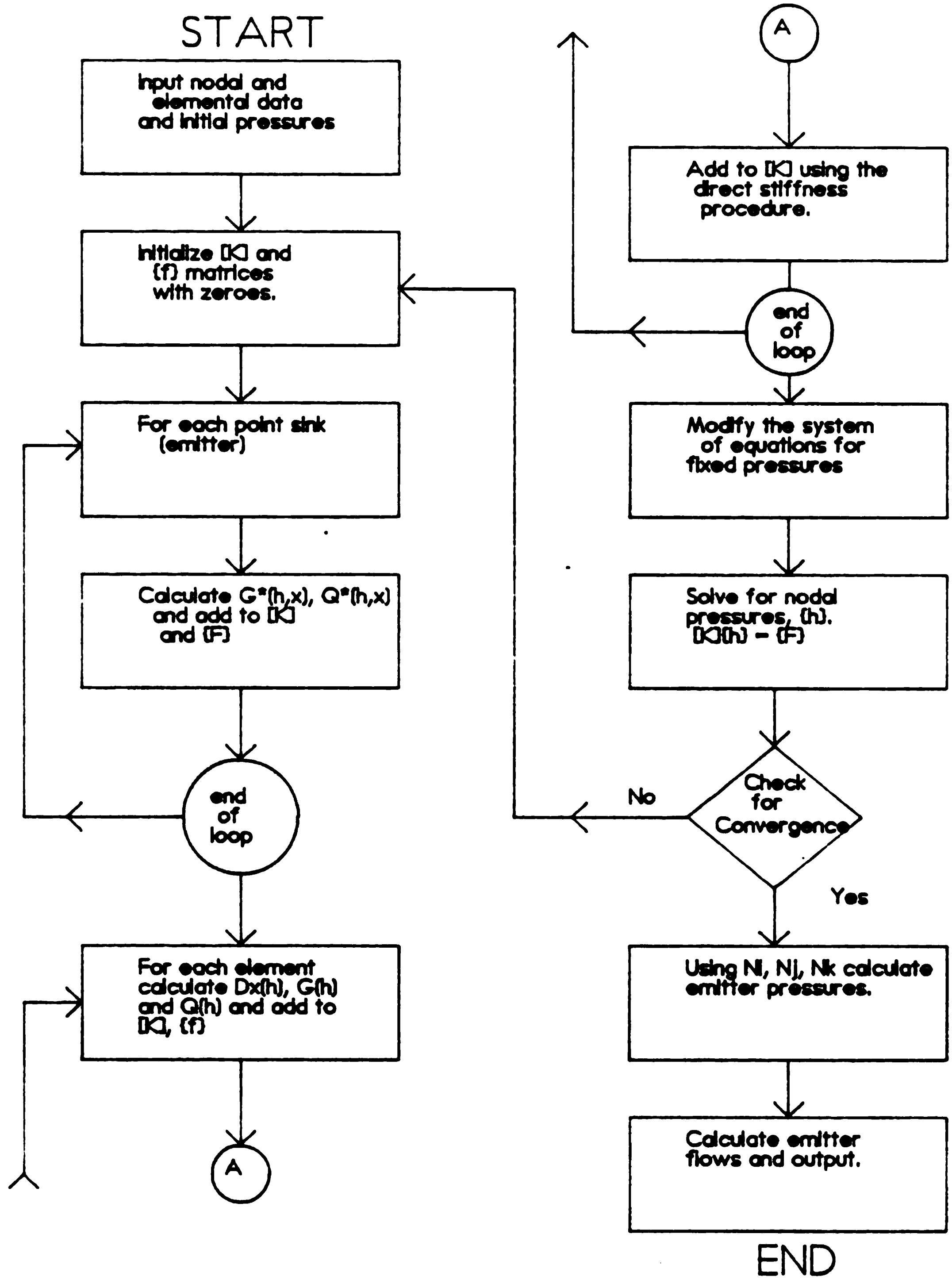

Figure 18. Flow chart for VNODE.

equations are solved for pressure heads at each node using Gaussian elimination. Pressure head in the lateral can then be calculated at specific points of interest (e.g. at emitters) using the interpolation functions for the element chosen.

4.2 Analysis

It was necessary to define a drip irrigation system to analyze with VNODE to facilitate comparisons between solution methods. A lateral line was chosen as opposed to a submain unit to keep the comparisons simple. The drip lateral in Figure 19 was selected for initial analysis. This is a lateral that might be commonly found in orchards in Michigan. The lateral line is polyethylene tubing (*C*=150) of 15.24 mm i.d. [0.6 in.] and total length of 250 m. [820 ft.]. Emitters are vortex type, k=9.14x10^{-7} [0.008], x_e=0.5 spaced every 5 m. [16 ft.]. Emitter flow at design pressures is about 17 lph [4.5 gph].

To determine how many elements would be needed to approximate the "exact" pressures in the lateral line, the system in Figure 19 was subdivided using an increasing number of nodes. It was observed from Figure 20 that 7 nodes would be sufficient to approximate the pressure gradient along the lateral. From this observation it was decided that further comparisons would be made using this 7 node configuration.

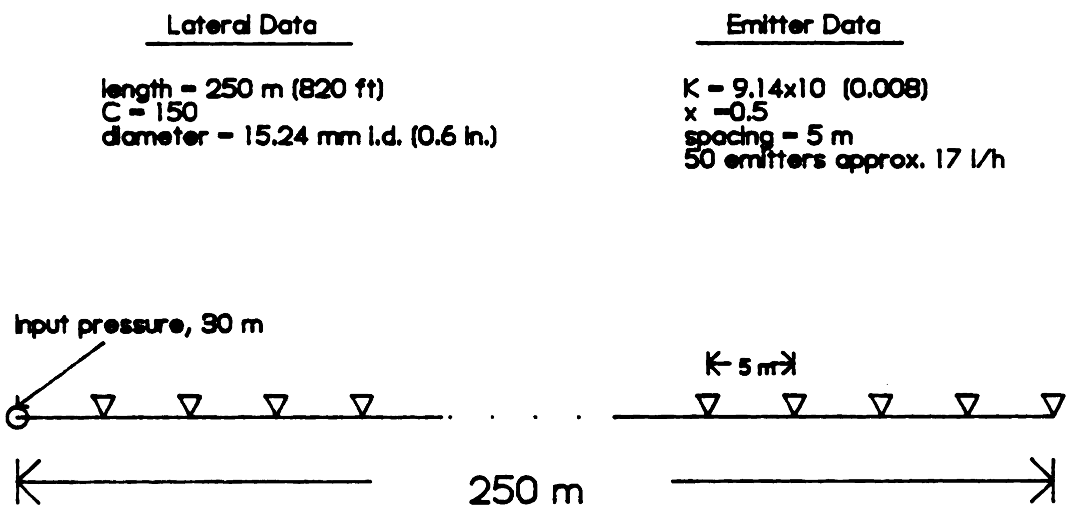

Figure 19. Lateral line.

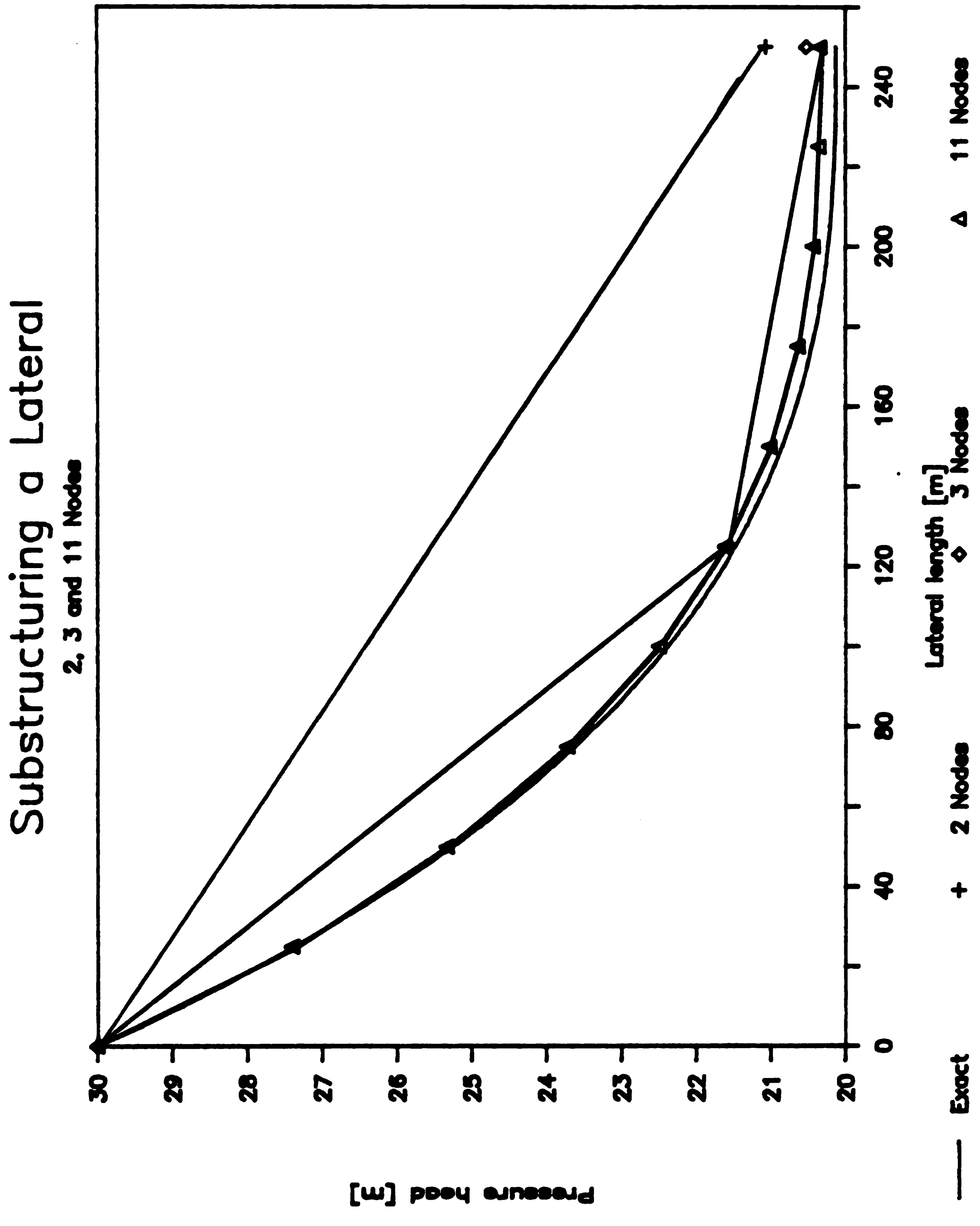

Figure 20. Pressures along the lateral line with various nodal subdivisions.

4.2.1 Accuracy and Speed of Convergence

The accuracy and speed of convergence were analyzed for the system in Figure 19 for each method developed in the theoretical development. The emitter flow equation was incorporated using the three methods as summarized in Table 1. In addition emitters were added to the nodes as point sources or distributed flow for both linear and quadratic elements. The results of this analysis is presented in Table 2. All analysis were performed using the same computer with calculation time for actual data computation, not including the time needed to input or output the data to and from the program. In general it was found that by including emitters as separate point sources increased computation time significantly. This is due to evaluating the interpolation functions N_i, N_j and N_k (for the quadratic element) at every point on the lateral line each iteration. By incorporating emitters a distributed flow along the lateral, the calculation time was reduced significantly without sacrificing accuracy.

Accuracy of each method was evaluated by comparing the total flows in the lateral of Figure 19 for each method. Since the most important design parameter in most drip irrigation systems is the uniformity of emitter flow it was calculated for each method. Table 2 shows the total submain flow calculated by each method and the exact solution. In general most methods overestimated the flow slightly. Coefficient of uniformity for emitter flow rate didn't change much regardless of the solution method.

The calculated emitter pressures were plotted against the exact solutions for each method in Figures 21-24. For most methods there was no major deviation in estimated pressures. With the linear method when emitters were added as point sources for method 2 and 3, major deviations occured. It is not sure exactly why these occured only in these cases. These errors could have been caused by the $G(h)$ term evaluated for point sources. An error in VNODE cannot possibly be ruled out although it was

Table 2. Accuracy and speed of convergence for methods of solution.

	Method	Element Type	Press. Unif.	Flow Unif.	Lateral flow [gpm]	Comp. time [sec]	Number of iter.
	Exact		88	94	3.43	70.13	15
No Point Sources	Method 1	Linear	88	94	3.45	2.19	8
		Quad.	88	94	3.45	1.42	8
	Method 2	Linear	88	94	3.45	2.63	9
		Quad.	88	94	3.46	1.92	9
	Method 3	Linear	88	94	3.45	2.19	8
		Quad.	88	94	3.46	1.92	8
Point Sources	Method 1	Linear	88	94	3.45	12.02	8
		Quad.	88	94	3.45	26.52	8
	Method 2	Linear	81	91	3.21	16.2	10
		Quad.	88	94	3.45	36.62	9
	Method 3	Linear	85	93	3.33	17.9	9
		Quad.	88	94	3.45	32.02	8

checked into. For most methods it is though that these deviations in flow rates are insignificant compared to the variation due to manufacturing most emmiters. One could probably calculate a "estimation uniformity" that would decrease the hydraulic uniformity according to this error estimation.

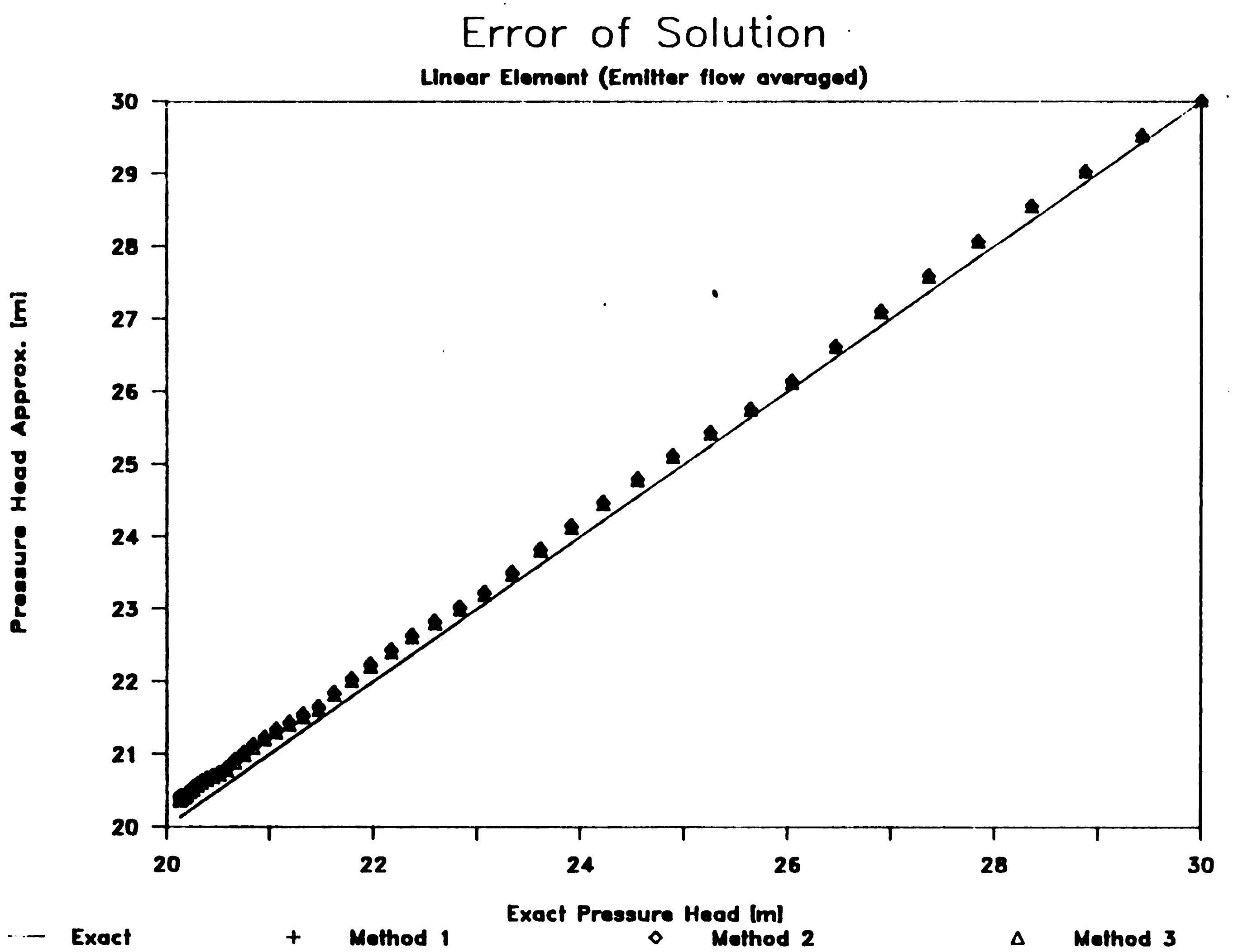

Figure 21. Exact emitter pressures vs. 7 node linear approx. (no point sources).

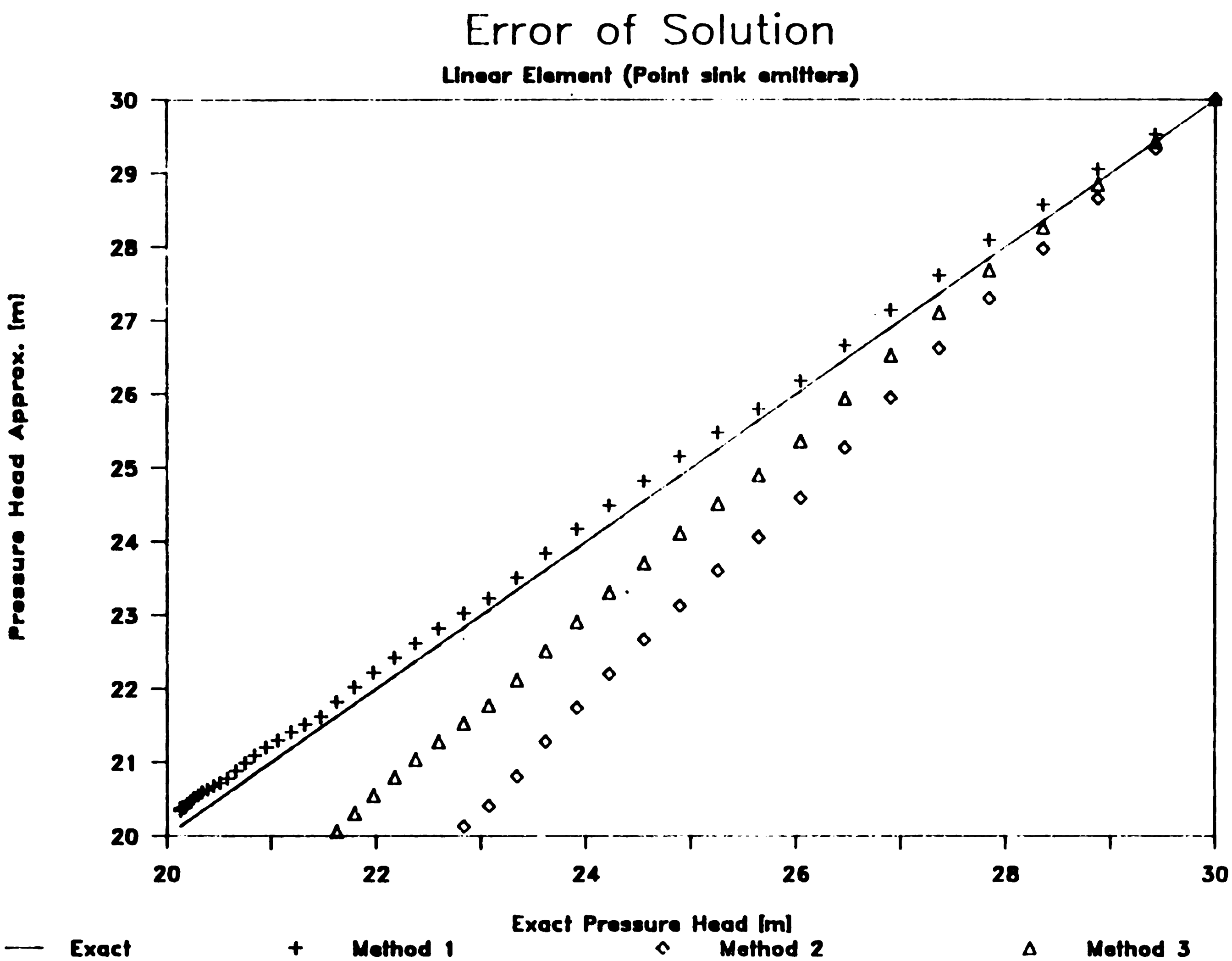

Figure 22. Exact emitter pressures vs. 7 node linear approx. (point sources).

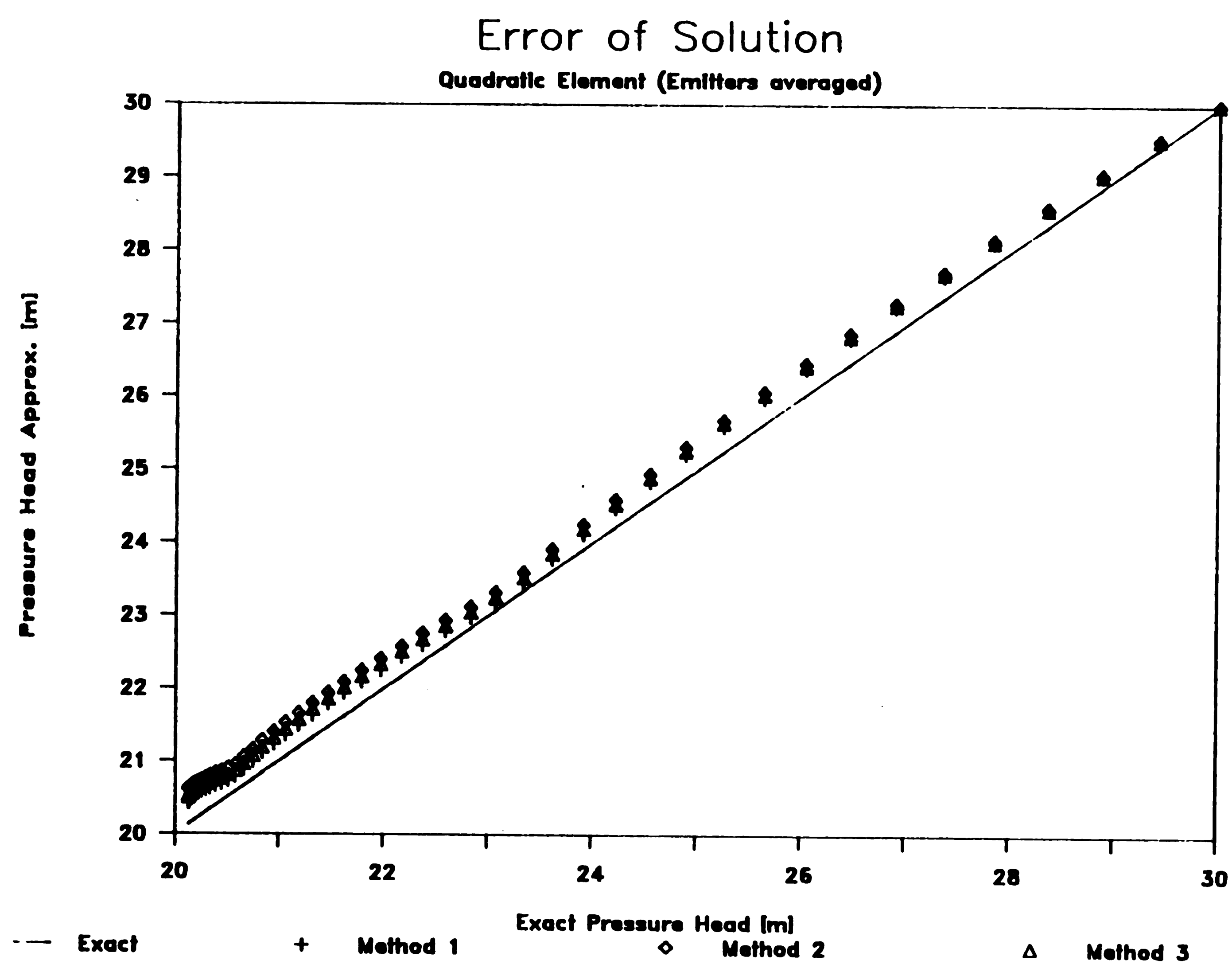

Figure 23. Exact emitter pressures vs. 7 node quadratic approx. (no point sources).

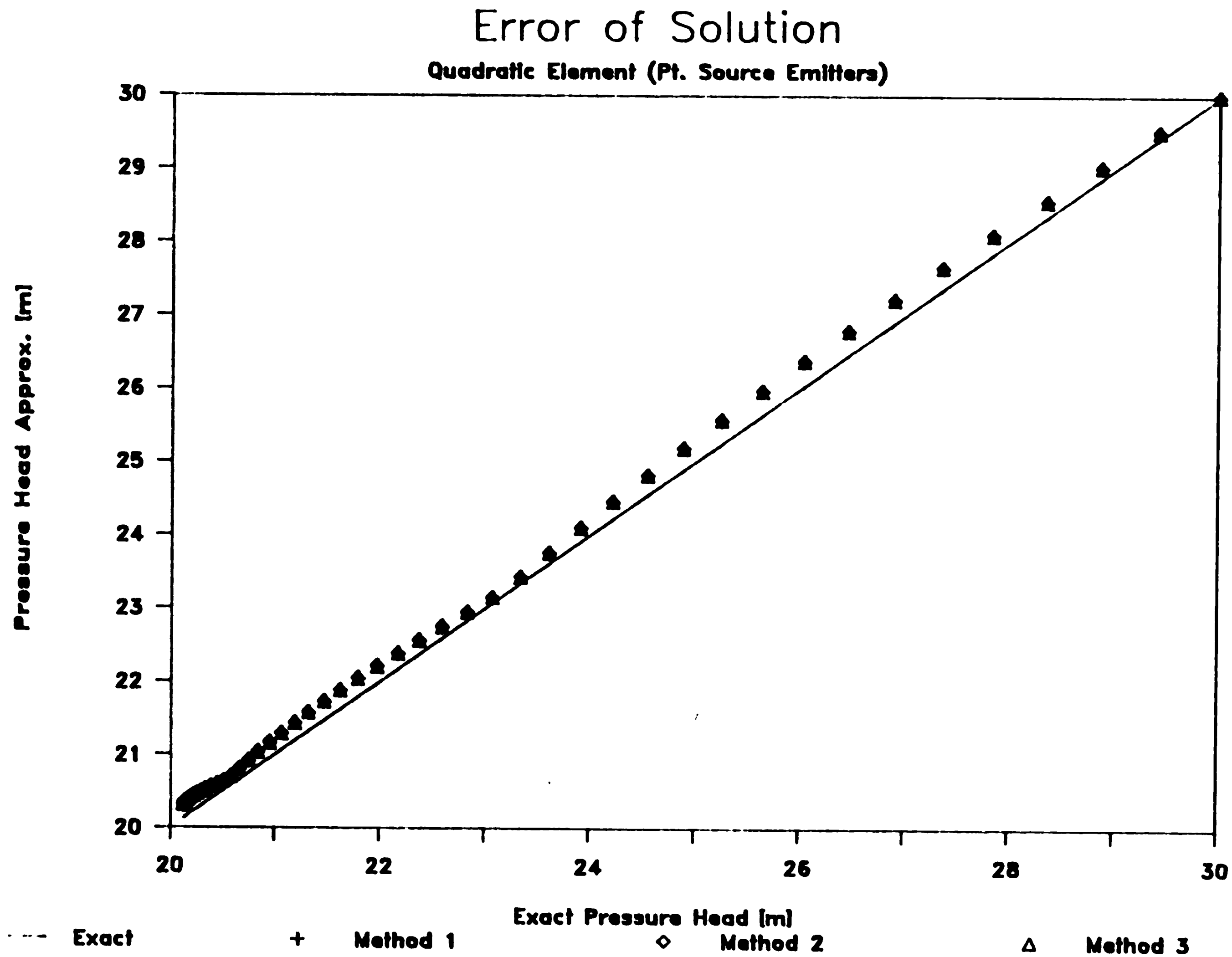

Figure 24. Exact emitter pressures vs. 7 node quadratic approx. (point sources).

4.2.2 Initial Estimates and Convergence

The sensitivity of initial estimates of pressure head at each node varied greatly depending on which solution method was used. Method 1, suffered from the fact that initial estimates at each node had to be fairly accurate in order for the solution to converge. For example, if a linear element is chosen and initial estimates at both nodes are the same then $\Delta h/\Delta x$ for the element is zero, and a zero results on the diagonal of the solution matrix causing the solution to stop. In addition, other initial estimates can be chosen to cause the solution matrix to have off diagonal terms causing the solution to diverge.

Methods 2 and 3 didn't suffer from this problem. This is due to the additional $G(h)$ term which adds directly to the stiffness matrix of setting any terms that ended up negative or zero when $[K_D]$ was added to the solution matrix.

The solution was considered arrived at when the maximum defect each iteration reached a predetermined value. For the system shown in Figure 19 a plot of the maximum defect from each iteration is shown in Figure 25. All methods converged to a relative accuracy after a about 8 iterations to a maximum defect of 0.01 for the system in Figure 19.

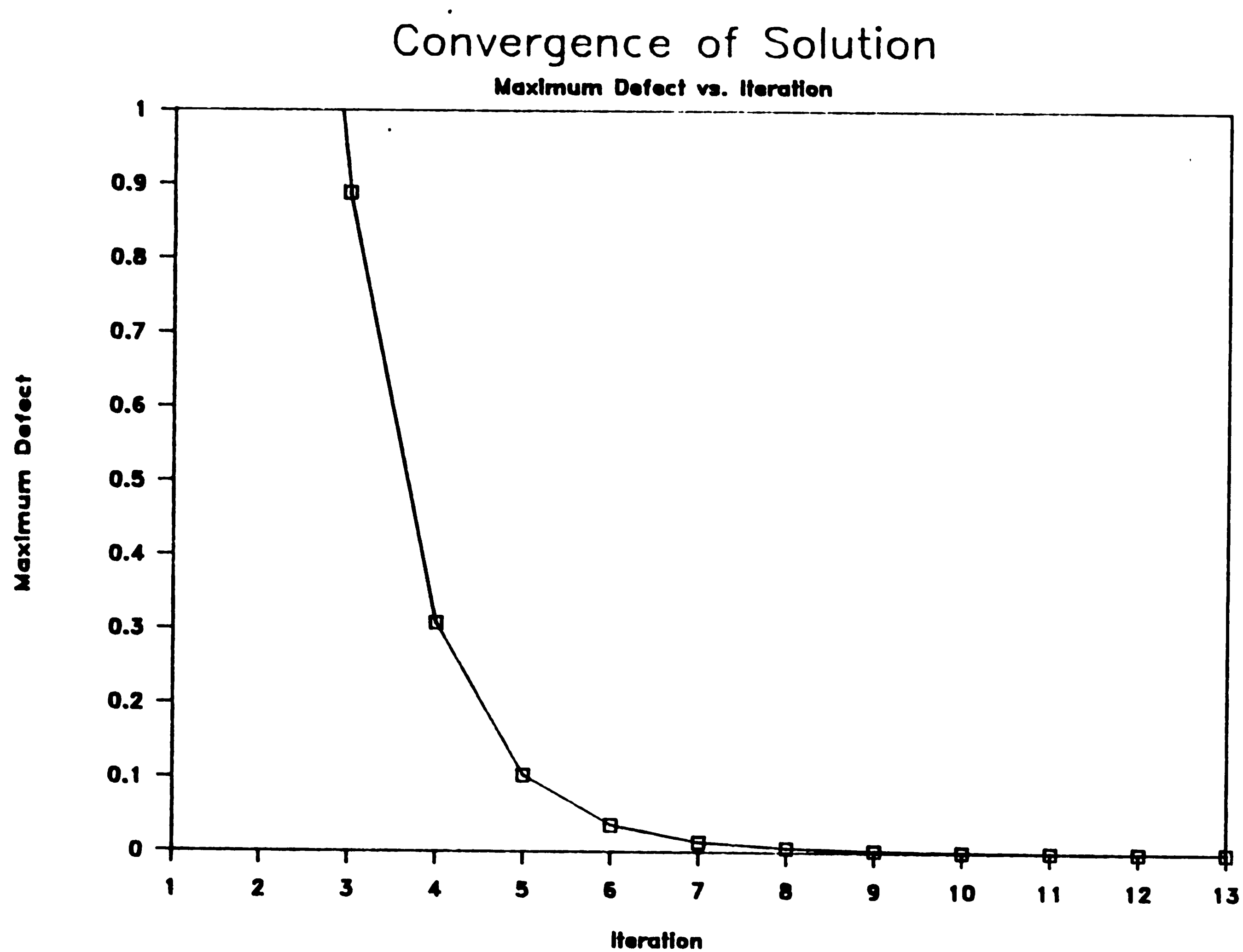

Figure 25. Iteration vs. maximum defect.

4.3 Summary Discussion

The results and discussion section have shown the advantages and disadvantages of using a virtual node concept in analyzing hydraulics of a drip irrigation system. One of the advantages of using the concepts demonstrated in this thesis are the speed with which an analysis can be carried out. By combining emitters into virtual nodes the computation speed with which an analysis can be carried out is increased tenfold. Large systems of 10,000 or more emitters can be analyzed more accurately by using a virtual node system. A very important limitation of the virtual emitter concept is the element should have the same pipe diameter and flow characteristics throughout for the method to be used sucessfully regardless of the number of emitters that are combined into virtual nodes. If the pipe diameter changes in the middle of the element the element should be divided into two elements at the point the change takes place and seperate flow characteristics of each element can be accounted for in the system of equations.

By incorporating emitter flow as described in the theoretical development, good initial estimates are not required to carry out the computer analysis and research shows that convergence is almost always guaranteed.

It was shown that the flow phenomena occuring in a drip irrigation lateral could be described by a differential equation [38]. The solution of the differential equation of this form is readily solved using classical finite element techniques. Using the concepts developed in this thesis, a large drip irrigation submain unit can be discretized into a convenient number of linear elements of any type and analyzed using a general finite element program.

V. CONCLUSIONS AND RECOMMENDATIONS

In this thesis, the use of virtual node concepts were investigated in the hydraulic design of drip irrigation systems. A differential equation was developed to describe the flow conditions in a drip lateral. This equation was solved using the finite element method and virtual node concepts in the theoretical development section. A computer program, VNODE, was written to evaluate these concepts. This resulted in a fast, efficient and accurate solution in most cases.

The specific conclusions of this research are:

1. Virtual node concepts and the finite element method can be used to analyze the hydraulics of a drip irrigation system.

2. Combining emitters into virtual nodes resulted in less computation time and approximated the exact solution closely.

3. Using a virtual node system, a good analysis can be performed without the need to directly solve for pressures at every emitter.

4. Using the differential equation developed, quadratic elements can be used to analyze

the flow in a drip irrigation lateral.

Recommendations for further research

1. In the theoretical development it was shown how slope or elevational effects could be incorporated into the solution, these effects could be included for further study.

2. To make this an effective design tool for analyzing hydraulics of drip systems VNODE could be incorporated into a more easily used program.

3. Large drip irrigation submain units could be analyzed using these methods using a general finite element program.

APPENDIX

APPENDIX A

Computer Model (VNODE)

Program Listing

```
program vnode;

uses Crt, Printer, Dos;

const
  MaxSize = 65;
  Xe = 0.5;
  Ke = 9.14E-7;
  C = 150;
  pi = 3.14159;

type
  MatrixType = array [1..MaxSize,1..MaxSize] of real;
  Vector = array [1..MaxSize] of real;


var
  k,Kg : MatrixType;
  oldh,h,z : Vector;
  m,iter,N,i,j,Ne,r,s,t,u : integer;
  Dx,G,Q,Mdbc,Sdbc,epl,lpl,Dia,NodeFlows : Vector;
  Itype : array[1..MaxSize] of integer;
  Psm,Psx,Psy,X,Y : Vector;
  Nel : array[1..MaxSize,1..4] of integer;
  out,data,plotfile : text;
  L,TotalFlow : real;
  filename : string[14];
  EFlow : real;
  Ps,Nps,Nfgn : integer;
  Pse,Fgn : array[1..Maxsize] of integer;
  defect,Gtemp,Qtemp : real;
  hour,min,sec,hndth : word;

function Power(Base,Pwr : Real) : Real;

{

   Function Power raises Base to the power Pwr.

}

begin { Power }
   if Base>0.0 then
      Power:=exp(Pwr*Ln(Base))
   else
      Power:=0.0;
end; { Power }
```

```
function Length(p : integer) : real;
var
  i,j : integer;

begin
  i:=Nel[p,1];
  if Itype[p] = 1 then j:=Nel[p,2] else j:=Nel[p,3];
  Length:=sqrt(sqr(X[i]-X[j])+sqr(Y[i]-Y[j]));
end;


function LinearNi(e : integer; Xo : real):real;
var
  j : integer;

begin
   j:=Nel[e,2];
   LinearNi:=(X[j]-Xo)/Length(e);
end;


function LinearNj(e : integer; Xo : real):real;
var
  i : integer;

begin
   i:=Nel[e,1];
   LinearNj:=(Xo-X[i])/Length(e);
 end;


function QuadNi(e : integer; Xo : real):real;
var
  i : integer;
begin
   i:=Nel[e,1];
   QuadNi:=(1-2*(Xo-X[i])/Length(e))*(1-(Xo-X[i])/Length(e));
end;

function QuadNj(e : integer; Xo : real):real;
var
  i : integer;
begin
   i:=Nel[e,1];
   QuadNj:=(4*(Xo-X[i])/Length(e))*(1-(Xo-X[i])/Length(e));
end;


function QuadNk(e : integer; Xo : real):real;
var
  i : integer;
begin
   i:=Nel[e,1];
   QuadNk:=(-1*(Xo-X[i])/Length(e))*(1-2*(Xo-X[i])/Length(e));
end;
```

```
function DxCalc(i,j,m:integer):real;

var
  base,a,XArea : real;

begin
   XArea:=pi*sqr(Dia[m])/4;
   Base:=abs((h[i]-h[j])/Length(m));
   a:= 5.88/(power(C,1.852)*power(XArea,0.5835));
   if (base=0.0) or (a=0.0) then DxCalc:=0 else
   DxCalc:=(XArea)/(power(base,0.46)*power(a,0.54));

end;



function Gcalc(m,Ps:integer):real;
var
  base : real;
  i,j,k : integer;

begin
   i := Nel[m,1];
   j := Nel[m,2];
   k := Nel[m,3];

   if Itype[m] = 1 then
     base := LinearNi(m,Psx[Ps])*h[i]+LinearNj(m,Psx[Ps])*h[j]
    else
     base := QuadNi(m,Psx[Ps])*h[i]+QuadNj(m,Psx[Ps])*h[j]+QuadNk(m,Psx[Ps])*h[k];
{   GCalc:=Xe*Ke*power(base,Xe-1);}
{   GCalc:=Ke*power(base,Xe-1); }
   GCalc:=0;

end;



function QCalc(m,Ps:integer):real;
var
  base : real;
  i,j,k : integer;

begin

   i := Nel[m,1];
   j := Nel[m,2];
   k := Nel[m,3];

   if Itype[m] = 1 then
     base := LinearNi(m,Psx[Ps])*h[i]+LinearNj(m,Psx[Ps])*h[j]
    else
     base := QuadNi(m,Psx[Ps])*h[i]+QuadNj(m,Psx[Ps])*h[j]+QuadNk(m,Psx[Ps])*h[k];
   Psy[Ps]:=base;
{   QCalc:=-1*(1-Xe)*Ke*power(base,Xe);}
   QCalc:=-1*Ke*power(base,Xe);
{   QCalc:=0;               }

end;
```

```
function  DxnCalc(h1,h2:real;m:integer):real;

var
  base,a : real;

begin
    writeln('****',Itype[m]:4,h1:10:3,h2:10:3,Length(m):10:3);
    Base:=abs((h1-h2)/Length(m));
    a:= 10.59/(power(C,1.852)*power(Dia[m],4.871));
    if (base=0.0) or (a=0.0) then DxnCalc:=0 else
    DxnCalc:=1/(power(base,0.46)*power(a,0.54));

end;



function  GnCalc(h1,h2:real;m:integer):real;
var
  base : real;

begin

    base := abs((h1+h2)/2);
{   GnCalc:=Xe*lpl[m]*epl[m]*Ke*power(base,Xe-1);}
{   GnCalc:=lpl[m]*epl[m]*Ke*power(base,Xe-1);}
    GnCalc:=0;

end;



function  QnCalc(h1,h2:real;m:integer):real;
var
  base : real;

begin
    base:=abs((h1+h2)/2);
{     QnCalc:=-1*(1-Xe)*lpl[m]*epl[m]*Ke*power(base,Xe);}
     QnCalc:=-1*lpl[m]*epl[m]*Ke*power(base,Xe);
{     QnCalc:=0;                              }

end;



procedure Solve(var a : MatrixType; var x : Vector; n : integer);

var
 i,j,k : integer;
 pivot,mult : real;

begin

{ forward procedure }

 for k:=1 to n do begin
    pivot := a[k,k];
    for j:=k to n+1 do a[k,j] := a[k,j]/pivot;
    for i:=k+1 to n do
      begin
       mult := a[i,k];
```

```
      for j:=k to n+1 do a[i,j] := a[i,j] - mult*a[k,j];
     end;
 end;

{ backward procedure }

 for i:=n downto 1 do begin
  for j:=1 to n do x[i] := x[i] - x[j]*a[i,j];
  x[i] := x[i] + a[i,n+1]
 end;

end;


procedure MatShow(var MatrixA: MatrixType; Rows, Columns: integer);

var
  I, J: integer;

begin
    for I:=1 to Rows do
       begin
          for J:=1 to Columns do
             write(MatrixA[I,J]:10);
          writeln;
       end;
    writeln;
end;


procedure DSPLinear1D(m : integer);

var
 i,j : integer;
 L : real;

begin
    i:=Nel[m,1];
    j:=Nel[m,2];

    L:=Length(m);

    K[i,i]:=K[i,i]+Dx[m]/L;
    K[j,j]:=K[j,j]+Dx[m]/L;
    K[i,j]:=K[i,j]-Dx[m]/L;
    K[j,i]:=K[j,i]-Dx[m]/L;

    K[i,i]:=K[i,i]+G[m]*L/3;
    K[j,j]:=K[j,j]+G[m]*L/3;
    K[i,j]:=K[i,j]+G[m]*L/6;
    K[j,i]:=K[j,i]+G[m]*L/6;

    K[i,N+1]:=K[i,N+1]+Q[m]*L/2;
    K[j,N+1]:=K[j,N+1]+Q[m]*L/2;

    Kg[i,i]:=Kg[i,i]+G[m]*L/3;
    Kg[j,j]:=Kg[j,j]+G[m]*L/3;  { The [Kg] matrix is saved so it is easier to  }
    Kg[i,j]:=Kg[i,j]+G[m]*L/6;  {  calculate the nodal flows later on      }
    Kg[j,i]:=Kg[j,i]+G[m]*L/6;
```

```
end;

procedure DSPQuad(m : integer);

type
  Coeff = array[1..3,1..3] of integer;

const
   CKg : Coeff = ((4,2,-1),(2,16,2),(-1,2,4));
   CKDx : Coeff = ((14,-16,2),(-16,32,-16),(2,-16,14));

var
 r,s,t,i,j : integer;
 L : real;

begin

   r:=Nel[m,1];
   s:=Nel[m,2];
   t:=Nel[m,3];

   L:=Length(m);

   K[r,N+1]:=K[r,N+1]+Q[m]*L/6;
   K[s,N+1]:=K[s,N+1]+4*Q[m]*L/6;
   K[t,N+1]:=K[t,N+1]+Q[m]*L/6;

   for i:=1 to 3 do
    for j:=1 to 3 do
     K[Nel[m,i],Nel[m,j]]:=K[Nel[m,i],Nel[m,j]]+G[m]*L*CKg[i,j]/30;

   for i:=1 to 3 do
    for j:=1 to 3 do
    K[Nel[m,i],Nel[m,j]]:=K[Nel[m,i],Nel[m,j]]+Dx[m]/6/L*CKDx[i,j];

   for i:=1 to 3 do
    for j:=1 to 3 do
     Kg[Nel[m,i],Nel[m,j]]:=Kg[Nel[m,i],Nel[m,j]]+G[m]*L*CKg[i,j]/30;

end;


procedure PsLinear(m : integer);
var
  Qstar,Gstar,Ni,Nj : real;
  i,j : integer;
begin
   i:=Nel[Pse[m],1];
   j:=Nel[Pse[m],2];
   Qstar:=Psm[m]*Qcalc(Pse[m],m);
   Gstar:=Psm[m]*Gcalc(Pse[m],m);
   Ni:=LinearNi(Pse[m],Psx[m]);
   Nj:=LinearNi(Pse[m],Psx[m]);

   K[i,N+1]:=K[i,N+1]+Qstar*Ni;
   K[j,N+1]:=K[j,N+1]+Qstar*Nj;

   K[i,i]:=K[i,i]+Gstar*sqr(Ni);
   K[j,j]:=K[j,j]+Gstar*sqr(Nj);
   K[i,j]:=K[i,j]+Gstar*Nj*Ni;
   K[j,i]:=K[j,i]+Gstar*Nj*Ni;
```

```
    Kg[i,i]:=Kg[i,i]+Gstar*sqr(Ni);
    Kg[j,j]:=Kg[j,j]+Gstar*sqr(Nj);
    Kg[i,j]:=Kg[i,j]+Gstar*Nj*Ni;
    Kg[j,i]:=Kg[j,i]+Gstar*Nj*Ni;
end;


procedure PsQuad(m : integer);
var
  Qstar,Gstar,Ni,Nj,Nk : real;
  i,j,r : integer;
begin
    i:=Nel[Pse[m],1];
    j:=Nel[Pse[m],2];
    r:=Nel[Pse[m],3];
    Qstar:=Psm[m]*Qcalc(Pse[m],m);
    Gstar:=Psm[m]*Gcalc(Pse[m],m);
    Ni:=QuadNi(Pse[m],Psx[m]);
    Nj:=QuadNj(Pse[m],Psx[m]);
    Nk:=QuadNk(Pse[m],Psx[m]);

    K[i,N+1]:=K[i,N+1]+Qstar*Ni;
    K[j,N+1]:=K[j,N+1]+Qstar*Nj;
    K[r,N+1]:=K[r,N+1]+Qstar*Nk;

    K[i,i]:=K[i,i]+Gstar*sqr(Ni);
    K[j,j]:=K[j,j]+Gstar*sqr(Nj);
    K[r,r]:=K[r,r]+Gstar*sqr(Nk);
    K[i,r]:=K[i,r]+Gstar*Ni*Nk;
    K[r,i]:=K[r,i]+Gstar*Nk*Ni;
    K[i,j]:=K[i,j]+Gstar*Nj*Ni;
    K[j,i]:=K[j,i]+Gstar*Nj*Ni;
    K[j,r]:=K[j,r]+Gstar*Nj*Nk;
    K[r,j]:=K[r,j]+Gstar*Nk*Nj;

    Kg[i,i]:=Kg[i,i]+Gstar*sqr(Ni);
    Kg[j,j]:=Kg[j,j]+Gstar*sqr(Nj);
    Kg[r,r]:=Kg[r,r]+Gstar*sqr(Nk);
    Kg[i,r]:=Kg[i,r]+Gstar*Ni*Nk;
    Kg[r,i]:=Kg[r,i]+Gstar*Nk*Ni;
    Kg[i,j]:=Kg[i,j]+Gstar*Nj*Ni;
    Kg[j,i]:=Kg[j,i]+Gstar*Nj*Ni;
    Kg[j,r]:=Kg[j,r]+Gstar*Nj*Nk;
    Kg[r,j]:=Kg[r,j]+Gstar*Nk*Nj;
end;


procedure Modify(node : integer);

var
  i : integer;

begin

  for i:=1 to N do
    begin
       K[node,i]:=0;
       K[i,N+1]:=K[i,N+1]-h[node]*K[i,node];
       K[i,node]:=0;
    end;
```

```
  K[Node,Node]:=1;
  K[Node,N+1]:=h[node];

end;

procedure Plotit;
begin
  write('Plotit Datafile name?  ');
  readln(filename);
  Assign(plotfile,filename);
  Rewrite(plotfile);
  for i:=1 to Nps do writeln(plotfile,Psx[i]:12:2,Psy[i]:12:2);
  Close(plotfile);
end;



begin
  ClrScr;
  write('Output File Name?   ');
  readln(filename);
  Assign(out,filename);
  Rewrite(out);

  write('Datafile name?  ');
  readln(filename);
  Assign(data,filename);
  Reset(data);

  read(data,N);
  writeln(out,N:4,' Nodes');
  writeln(out);
  writeln(out,'      X          Y        Head,h (m)    Elev,Z (m)');
  writeln(out);
  for i:=1 to N do
    begin
      read(data,X[i],Y[i],h[i],z[i]);
      writeln(out,X[i]:10:2,Y[i]:13:2,h[i]:13:2,z[i]:13:2);
    end;

  read(data,Ne);
  writeln(out);
  writeln(out,Ne:4,' Elements');
  writeln(out);
  writeln(out,'  i  j  k  m   Dx    G     Q    epl  lpl   Dia');
  writeln(out);
  for i:=1 to Ne do
   begin
    read(data,Itype[i]);
    if Itype[i]=1 then
     begin
      read(data,Nel[i,1],Nel[i,2],Dx[i],G[i],Q[i],epl[i],lpl[i],Dia[i]);
      writeln(out,Nel[i,1]:4,Nel[i,2]:4,Dx[i]:16:4,G[i]:8:4,Q[i]:8:4,epl[i]:6:3,lpl[i]:6:2,Dia[i]:9:5);
     end;
    if Itype[i]=2 then
     begin
       read(data,Nel[i,1],Nel[i,2],Nel[i,3],
          Dx[i],G[i],Q[i],epl[i],lpl[i],Dia[i]);
       writeln(out,Nel[i,1]:4,Nel[i,2]:4,Nel[i,3]:4,
```

```
          Dx[i]:12:4,G[i]:8:4,Q[i]:8:4,epl[i]:6:3,lpl[i]:6:2,Dia[i]:9:5);
      end;
    end;

  writeln(out);
  writeln(out);
  read(data,Nfgn);
  writeln(out,Nfgn:4,' Known Nodal Values');
  writeln(out);
  writeln(out,'    Element    Head, h (m)');
  writeln(out);
  for i:=1 to Nfgn do
    begin
      read(data,Fgn[i]);
      read(data,h[Fgn[i]]);
      writeln(out,Fgn[i]:10,h[Fgn[i]]:12:2);
    end;
  writeln(out);
  read(data,Nps,Ps);
  writeln(out,Nps:4,' Point Sinks (emitters)');
  writeln(out);
  writeln(out,'    Element      X        Y       Modifier');
  writeln(out);
  for i:=1 to Nps do
    begin
      read(data,Pse[i],Psx[i],Psy[i],Psm[i]);
      writeln(out,Pse[i]:10,Psx[i]:12:2,Psy[i]:12:2,Psm[i]:12:2);
    end;

  Close(data);

 SetTime(0,0,0,0);
 for i:=1 to N do oldh[i]:=h[i];
 defect:=0.11;
 iter:=0;

 while ((defect>=0.01) and (iter<=20)) do begin

  for i:=1 to Ne do begin
   if Itype[i]=1 then begin
     Dx[i]:=DxCalc(Nel[i,1],Nel[i,2],i);
   end;
   if Itype[i]=2 then begin
     Dx[i]:=DxCalc(Nel[i,1],Nel[i,3],i);
   end;
  end;

  for i:=1 to N do for j:=1 to N+1 do K[i,j]:=0;

  for i:=1 to N do for j:=1 to N+1 do Kg[i,j]:=0;

  if Ps=1 then for i:=1 to Nps do if Itype[Pse[i]]=1 then PsLinear(i) else PsQuad(i);

  if Ps=0 then begin
   for i:=1 to Ne do begin
   if Itype[i]=1 then begin
     G[i]:=GnCalc(h[Nel[i,1]],h[Nel[i,2]],i);
     Q[i]:=QnCalc(h[Nel[i,1]],h[Nel[i,2]],i);
    end;
   if Itype[i]=2 then begin
     G[i]:=GnCalc(h[Nel[i,1]],h[Nel[i,3]],i);
     Q[i]:=QnCalc(h[Nel[i,1]],h[Nel[i,3]],i);
```

```
    end;
   end;
  end;

 for i:=1 to Ne do if Itype[i]=1 then DSPLinear1D(i) else DSPQuad(i);

 for i:=1 to N do Nodeflows[i]:=K[i,N+1];

 for i:=1 to NFgn do Modify(Fgn[i]);

 {Matshow(k,n,n+1);for i:= 1 to Ne do writeln(Dx[i]:20:15);}

 Solve(K,h,n);

 defect:=0;

 for i:=1 to N do if (abs(h[i]-oldh[i]) >= defect)
            then defect:=abs(h[i]-oldh[i]);

 for i:=1 to N do oldh[i]:=h[i];

 writeln(iter:4,defect:8:4);

 iter:=iter+1;

end;

GetTime(hour,min,sec,hndth);

 writeln(out);
 writeln(out,' Calculation Time [sec] ',min*60+sec+hndth/100:7:3);
 writeln(out);
 writeln(out,' Number of Iterations',iter:4);
 writeln(out);
 writeln(out,' Maximum defect ',defect:7:5);
 writeln(out);
 writeln(out,'Final Coefficients Dx, G, and Q for each element');
 writeln(out);
 for m:=1 to Ne do begin
  i:=Nel[m,1];
  j:=Nel[m,2];
  writeln(out,Dx[m]:15:10,G[m]:15:10,Q[m]:15:10);
 end;
 for i:= 1 to N do for j:=1 to N do NodeFlows[i]:=NodeFlows[i]-Kg[i,j]*h[i];
 writeln(out);
 writeln(out,'      Final Nodal Pressures          NodalFlows {F}+[Kg]*{h} ');
 writeln(out);
 writeln(out,'     meters           feet        m3/sec            gpm');
 writeln(out,'Node');
 for i:= 1 to N do begin
    writeln(out,i:3,h[i]:10:2,(3.281*h[i]):20:2,'     ',
          NodeFlows[i]:15:8,(15835.31*NodeFlows[i]):15:2);
 end;
 writeln(out);
 TotalFlow:=0;
 for i:=1 to N do TotalFlow:=TotalFlow+NodeFlows[i];
 writeln(out,'           Total Submain Flow     ',TotalFlow:15:8,
       (TotalFlow*15835.31):15:2);
 writeln(out);
 writeln(out,'                ****** 1D Linear Elements ******');
 writeln(out);
 writeln(out,'        Element flows, Dx*Hf/L    Average Emitter Flows, Q[e]+G[e]*h ');
```

```
 writeln(out);
 writeln(out,'          m3/sec        gpm          m3/sec         gph');
 writeln(out,'Element');
 for m:=1 to Ne do begin
  if Itype[m]=1 then begin
  i:=Nel[m,1];
  j:=Nel[m,2];
  write(out,m:5,(Dx[m]*(h[i]-h[j])/Length(m)):15:8,
          (15835.31*Dx[m]*(h[i]-h[j])/Length(m)):12:2);
  if (epl[m]=0) or (lpl[m]=0) then writeln(out,epl[m]:17:8,epl[m]:17:2)
    else writeln(out,(Q[m]-G[m]*(h[i]+h[j])/2)/epl[m]/lpl[m]:17:8,
          (15835.31*60*(Q[m]-G[m]*(h[i]+h[j])/2)/epl[m]/lpl[m]):17:2);
  end;
 end;
 writeln(out);
  writeln(out,Nps:4,'  Point Sinks (emitters)');
  writeln(out);
  if Ps=1 then writeln(out,'Emitters considered as point sinks')
    else writeln(out,'Emitter flow averaged over the element');
  writeln(out);
  writeln(out,'    Element      X      Pres.[m]    Outflow [gph]');
  writeln(out);
  for i:=1 to Nps do
    begin
      Gtemp:=Gcalc(Pse[i],i);
      QTemp:=Qcalc(Pse[i],i);
      Eflow:=Psm[i]*(Gtemp*Psy[i]-Qtemp);
      writeln(out,Pse[i]:10,Psx[i]:12:2,Psy[i]:12:2,
        15835.31*60*Eflow:12:2,Psm[i]:12:2);
    end;
Close(out);
{Plotit;}

end.
```

APPENDIX B

Sample Output

7 Nodes

X	Y	Head,h (m)	Elev,Z (m)
0.00	0.00	30.00	0.00
41.67	0.00	29.00	0.00
83.33	0.00	28.00	0.00
125.00	0.00	27.00	0.00
166.67	0.00	26.00	0.00
208.33	0.00	25.00	0.00
250.00	0.00	24.00	0.00

6 Elements

i	j	k	m	Dx	G	Q	epl	lpl	Dia
1	2			0.0000	0.0000	0.0000	0.200	1.00	0.01524
2	3			0.0000	0.0000	0.0000	0.200	1.00	0.01524
3	4			0.0000	0.0000	0.0000	0.200	1.00	0.01524
4	5			0.0000	0.0000	0.0000	0.200	1.00	0.01524
5	6			0.0000	0.0000	0.0000	0.200	1.00	0.01524
6	7			0.0000	0.0000	0.0000	0.200	1.00	0.01524

1 Known Nodal Values

Element	Head, h (m)
1	30.00

51 Point Sinks (emitters)

Element	X	Y	Modifier
1	5.00	0.00	1.00
1	10.00	0.00	1.00
1	15.00	0.00	1.00
1	20.00	0.00	1.00
1	25.00	0.00	1.00
1	30.00	0.00	1.00
1	35.00	0.00	1.00
1	40.00	0.00	1.00
2	45.00	0.00	1.00
2	50.00	0.00	1.00
2	55.00	0.00	1.00
2	60.00	0.00	1.00
2	65.00	0.00	1.00
2	70.00	0.00	1.00
2	75.00	0.00	1.00
2	80.00	0.00	1.00
3	85.00	0.00	1.00
3	90.00	0.00	1.00
3	95.00	0.00	1.00

3	100.00	0.00	1.00
3	105.00	0.00	1.00
3	110.00	0.00	1.00
3	115.00	0.00	1.00
3	120.00	0.00	1.00
3	125.00	0.00	0.50
4	125.00	0.00	0.50
4	130.00	0.00	1.00
4	135.00	0.00	1.00
4	140.00	0.00	1.00
4	145.00	0.00	1.00
4	150.00	0.00	1.00
4	155.00	0.00	1.00
4	160.00	0.00	1.00
4	165.00	0.00	1.00
5	170.00	0.00	1.00
5	175.00	0.00	1.00
5	180.00	0.00	1.00
5	185.00	0.00	1.00
5	190.00	0.00	1.00
5	195.00	0.00	1.00
5	200.00	0.00	1.00
5	205.00	0.00	1.00
6	210.00	0.00	1.00
6	215.00	0.00	1.00
6	220.00	0.00	1.00
6	225.00	0.00	1.00
6	230.00	0.00	1.00
6	235.00	0.00	1.00
6	240.00	0.00	1.00
6	245.00	0.00	1.00
6	250.00	0.00	1.00

Calculation Time [sec] 2.190

Number of Iterations 8

Maximum defect 0.00453

Final Coefficients Dx, G, and Q for each element

0.0020426272	0.0000000000	-0.0000009670
0.0024604603	0.0000000000	-0.0000009070
0.0030791205	0.0000000000	-0.0000008661
0.0041248804	0.0000000000	-0.0000008413
0.0063850525	0.0000000000	-0.0000008292
0.0162336773	0.0000000000	-0.0000008253

	Final Nodal Pressures		NodalFlows {F}+[Kg]*{h}	
	meters	feet	m3/sec	gpm
Node				
1	30.00	98.43	-0.00002015	-0.32
2	25.96	85.18	-0.00003904	-0.62
3	23.27	76.34	-0.00003694	-0.58
4	21.62	70.93	-0.00003557	-0.56
5	20.74	68.06	-0.00003480	-0.55
6	20.41	66.95	-0.00003447	-0.55
7	20.36	66.81	-0.00001720	-0.27

Total Submain Flow -0.00021816 -3.45

****** 1D Linear Elements ******

Element flows, Dx*Hf/L   Average Emitter Flows, Q[e]+G[e]*h

Element	m3/sec	gpm	m3/sec	gph
1	0.00019801	3.14	-0.00000483	-4.59
2	0.00015897	2.52	-0.00000454	-4.31
3	0.00012204	1.93	-0.00000433	-4.11
4	0.00008646	1.37	-0.00000421	-4.00
5	0.00005166	0.82	-0.00000415	-3.94
6	0.00001720	0.27	-0.00000413	-3.92

51 Point Sinks (emitters)

Emitter flow averaged over the element

Element	X	Pres.[m]	Outflow [gph]	
1	5.00	29.52	4.72	1.00
1	10.00	29.03	4.68	1.00
1	15.00	28.55	4.64	1.00
1	20.00	28.06	4.60	1.00
1	25.00	27.58	4.56	1.00
1	30.00	27.09	4.52	1.00
1	35.00	26.61	4.48	1.00
1	40.00	26.12	4.44	1.00
2	45.00	25.75	4.41	1.00
2	50.00	25.42	4.38	1.00
2	55.00	25.10	4.35	1.00
2	60.00	24.78	4.32	1.00
2	65.00	24.45	4.29	1.00
2	70.00	24.13	4.27	1.00
2	75.00	23.81	4.24	1.00
2	80.00	23.48	4.21	1.00
3	85.00	23.20	4.18	1.00
3	90.00	23.00	4.17	1.00
3	95.00	22.81	4.15	1.00
3	100.00	22.61	4.13	1.00
3	105.00	22.41	4.11	1.00
3	110.00	22.21	4.09	1.00
3	115.00	22.01	4.07	1.00
3	120.00	21.82	4.06	1.00
3	125.00	21.62	2.02	0.50
4	125.00	21.62	2.02	0.50
4	130.00	21.51	4.03	1.00
4	135.00	21.41	4.02	1.00
4	140.00	21.30	4.01	1.00
4	145.00	21.20	4.00	1.00
4	150.00	21.09	3.99	1.00
4	155.00	20.99	3.98	1.00
4	160.00	20.88	3.97	1.00
4	165.00	20.78	3.96	1.00
5	170.00	20.72	3.95	1.00
5	175.00	20.68	3.95	1.00
5	180.00	20.64	3.94	1.00
5	185.00	20.60	3.94	1.00
5	190.00	20.56	3.94	1.00
5	195.00	20.51	3.93	1.00
5	200.00	20.47	3.93	1.00
5	205.00	20.43	3.93	1.00
6	210.00	20.40	3.92	1.00

6	215.00	20.40	3.92	1.00
6	220.00	20.39	3.92	1.00
6	225.00	20.39	3.92	1.00
6	230.00	20.38	3.92	1.00
6	235.00	20.38	3.92	1.00
6	240.00	20.37	3.92	1.00
6	245.00	20.37	3.92	1.00
6	250.00	20.36	3.92	1.00

APPENDIX C

Length [m]	Linear, 7 Node, Method 1: Pressure head [m]	Linear, 7 Node, Method 1: Emitter Flow [gph]	Linear, 7 Node P.S., Method 1: Pressure head [m]	Linear, 7 Node P.S., Method 1: Emitter Flow [gph]	Quadratic, 7 Node, Method 1: Pressure head [m]	Quadratic, 7 Node, Method 1: Emitter Flow [gph]
0	30		30		30	
5	29.52	4.72	29.52	4.72	29.51	4.72
10	29.03	4.68	29.05	4.68	29.03	4.68
15	28.55	4.64	28.57	4.64	28.56	4.64
20	28.06	4.6	28.09	4.6	28.11	4.6
25	27.58	4.56	27.61	4.56	27.66	4.57
30	27.09	4.52	27.14	4.52	27.23	4.53
35	26.61	4.48	26.66	4.48	26.8	4.5
40	26.12	4.44	26.18	4.44	26.39	4.46
45	25.75	4.41	25.8	4.41	25.98	4.43
50	25.42	4.38	25.48	4.38	25.59	4.39
55	25.1	4.35	25.15	4.35	25.21	4.36
60	24.78	4.32	24.82	4.33	24.83	4.33
65	24.45	4.29	24.49	4.3	24.47	4.3
70	24.13	4.27	24.17	4.27	24.12	4.27
75	23.81	4.24	23.84	4.24	23.78	4.23
80	23.48	4.21	23.51	4.21	23.45	4.21
85	23.2	4.18	23.23	4.19	23.17	4.18
90	23	4.17	23.03	4.17	22.98	4.16
95	22.81	4.15	22.82	4.15	22.79	4.15
100	22.61	4.13	22.62	4.13	22.6	4.13
105	22.41	4.11	22.42	4.11	22.43	4.11
110	22.21	4.09	22.22	4.09	22.25	4.1
115	22.01	4.07	22.02	4.08	22.09	4.08
120	21.82	4.06	21.82	4.06	21.93	4.07
125	21.62	4.04	21.62	4.04	21.78	4.06
130	21.51	4.03	21.51	4.03	21.63	4.04
135	21.41	4.02	21.41	4.02	21.49	4.03
140	21.3	4.01	21.3	4.01	21.36	4.01
145	21.2	4	21.2	4	21.23	4
150	21.09	3.99	21.09	3.99	21.11	3.99
155	20.99	3.98	20.99	3.98	20.99	3.98
160	20.88	3.97	20.88	3.97	20.88	3.97
165	20.78	3.96	20.78	3.96	20.78	3.96
170	20.72	3.95	20.72	3.95	20.72	3.95
175	20.68	3.95	20.68	3.95	20.68	3.95
180	20.64	3.94	20.63	3.94	20.65	3.95
185	20.6	3.94	20.59	3.94	20.62	3.94
190	20.56	3.94	20.55	3.94	20.59	3.94
195	20.51	3.93	20.51	3.93	20.56	3.94
200	20.47	3.93	20.47	3.93	20.54	3.94
205	20.43	3.93	20.43	3.92	20.52	3.93
210	20.4	3.92	20.4	3.92	20.5	3.93
215	20.4	3.92	20.39	3.92	20.48	3.93
220	20.39	3.92	20.39	3.92	20.46	3.93
225	20.39	3.92	20.38	3.92	20.45	3.93
230	20.38	3.92	20.38	3.92	20.44	3.93
235	20.38	3.92	20.37	3.92	20.43	3.93
240	20.37	3.92	20.36	3.92	20.43	3.92
245	20.37	3.92	20.36	3.92	20.42	3.92
250	20.36	3.92	20.35	3.92	20.42	3.92
Total Lateral Flow [gpm]		3.45		3.45		3.45
	Press.	Flow				
Uniformity	88%	94%	88%	94%	88%	94%
Execution Time [sec]	2.19		12.02		1.42	
# iter	8		8		8	
Defect	0.00453		0.00446		0.00509	

Quadratic 7 Node P.S. Method 1			Linear 7 Node Method 2			Linear 7 Node P.S. Method 2		
Length [m]	Pressure head [m]	Emitter Flow [gph]	Length [m]	Pressure head [m]	Emitter Flow [gph]	Length [m]	Pressure head [m]	Emitter Flow [gph]
0	30		0	30		0	30	
5	29.51	4.72	5	29.52	4.72	5	29.32	4.7
10	29.03	4.68	10	29.03	4.68	10	28.65	4.65
15	28.56	4.64	15	28.55	4.64	15	27.97	4.59
20	28.1	4.6	20	28.07	4.6	20	27.3	4.54
25	27.65	4.57	25	27.59	4.56	25	26.62	4.48
30	27.22	4.53	30	27.1	4.52	30	25.95	4.42
35	26.79	4.49	35	26.62	4.48	35	25.27	4.37
40	26.37	4.46	40	26.14	4.44	40	24.59	4.31
45	25.97	4.43	45	25.76	4.41	45	24.06	4.26
50	25.57	4.39	50	25.44	4.38	50	23.6	4.22
55	25.19	4.36	55	25.12	4.35	55	23.13	4.18
60	24.82	4.33	60	24.8	4.32	60	22.67	4.13
65	24.46	4.29	65	24.48	4.3	65	22.2	4.09
70	24.1	4.26	70	24.15	4.27	70	21.74	4.05
75	23.76	4.23	75	23.83	4.24	75	21.28	4.01
80	23.43	4.2	80	23.51	4.21	80	20.81	3.96
85	23.14	4.18	85	23.23	4.19	85	20.41	3.92
90	22.94	4.16	90	23.03	4.17	90	20.13	3.9
95	22.75	4.14	95	22.83	4.15	95	19.84	3.87
100	22.56	4.13	100	22.64	4.13	100	19.56	3.84
105	22.38	4.11	105	22.44	4.11	105	19.28	3.81
110	22.21	4.09	110	22.24	4.1	110	19	3.78
115	22.04	4.08	115	22.04	4.08	115	18.71	3.76
120	21.88	4.06	120	21.85	4.06	120	18.43	3.73
125	21.72	4.04	125	21.65	4.04	125	18.15	3.7
130	21.57	4.03	130	21.55	4.03	130	18	3.68
135	21.43	4.02	135	21.44	4.02	135	17.86	3.67
140	21.29	4.01	140	21.34	4.01	140	17.71	3.65
145	21.16	3.99	145	21.23	4	145	17.57	3.64
150	21.03	3.98	150	21.13	3.99	150	17.42	3.62
155	20.91	3.97	155	21.02	3.98	155	17.27	3.61
160	20.8	3.96	160	20.92	3.97	160	17.13	3.59
165	20.7	3.95	165	20.81	3.96	165	16.98	3.58
170	20.63	3.94	170	20.75	3.96	170	16.89	3.57
175	20.6	3.94	175	20.71	3.95	175	16.84	3.56
180	20.56	3.94	180	20.67	3.95	180	16.78	3.56
185	20.53	3.93	185	20.63	3.94	185	16.72	3.55
190	20.49	3.93	190	20.59	3.94	190	16.66	3.54
195	20.47	3.93	195	20.55	3.94	195	16.6	3.54
200	20.44	3.93	200	20.51	3.93	200	16.55	3.53
205	20.41	3.92	205	20.47	3.93	205	16.49	3.53
210	20.39	3.92	210	20.44	3.93	210	16.45	3.52
215	20.37	3.92	215	20.44	3.93	215	16.44	3.52
220	20.36	3.92	220	20.43	3.93	220	16.43	3.52
225	20.34	3.92	225	20.43	3.92	225	16.42	3.52
230	20.33	3.92	230	20.42	3.92	230	16.41	3.52
235	20.32	3.91	235	20.42	3.92	235	16.41	3.52
240	20.31	3.91	240	20.41	3.92	240	16.4	3.52
245	20.31	3.91	245	20.4	3.92	245	16.39	3.52
250	20.31	3.91	250	20.4	3.92	250	16.38	3.51
		3.45			3.45			3.21
	88%	94%		88%	94%		81%	91%
	26.52			2.63			16.2	
	8			9			10	
	0.00482			0.00969			0.00778	

Quadratic 7 Node Method 2			Quadratic 7 Node P.S. Method 2			Linear 7 Node Method 3		
Length [m]	Pressure head [m]	Emitter Flow [gph]	Length [m]	Pressure head [m]	Emitter Flow [gph]	Length [m]	Pressure head [m]	Emitter Flow [gph]
0	30		0	30		0	30	
5	29.52	4.72	5	29.51	4.72	5	29.52	4.72
10	29.05	4.68	10	29.03	4.68	10	29.03	4.68
15	28.59	4.64	15	28.56	4.64	15	28.55	4.64
20	28.15	4.61	20	28.1	4.6	20	28.06	4.6
25	27.71	4.57	25	27.66	4.57	25	27.58	4.56
30	27.28	4.54	30	27.22	4.53	30	27.09	4.52
35	26.87	4.5	35	26.79	4.5	35	26.61	4.48
40	26.46	4.47	40	26.38	4.46	40	26.12	4.44
45	26.07	4.43	45	25.97	4.43	45	25.75	4.41
50	25.68	4.4	50	25.58	4.39	50	25.42	4.38
55	25.31	4.37	55	25.19	4.36	55	25.1	4.35
60	24.94	4.34	60	24.82	4.33	60	24.78	4.32
65	24.59	4.31	65	24.46	4.29	65	24.45	4.29
70	24.25	4.28	70	24.1	4.26	70	24.13	4.27
75	23.91	4.25	75	23.76	4.23	75	23.81	4.24
80	23.59	4.22	80	23.43	4.2	80	23.48	4.21
85	23.32	4.19	85	23.15	4.18	85	23.2	4.18
90	23.12	4.18	90	22.95	4.16	90	23	4.17
95	22.94	4.16	95	22.76	4.14	95	22.81	4.15
100	22.76	4.14	100	22.57	4.13	100	22.61	4.13
105	22.58	4.13	105	22.39	4.11	105	22.41	4.11
110	22.41	4.11	110	22.22	4.09	110	22.21	4.09
115	22.25	4.1	115	22.05	4.08	115	22.01	4.07
120	22.09	4.08	120	21.88	4.06	120	21.82	4.06
125	21.94	4.06	125	21.73	4.04	125	21.62	4.04
130	21.8	4.05	130	21.58	4.03	130	21.51	4.03
135	21.66	4.04	135	21.44	4.02	135	21.41	4.02
140	21.53	4.03	140	21.3	4.01	140	21.3	4.01
145	21.4	4.02	145	21.17	4	145	21.2	4
150	21.28	4.01	150	21.04	3.98	150	21.09	3.99
155	21.16	4	155	20.92	3.97	155	20.99	3.98
160	21.06	3.98	160	20.81	3.96	160	20.88	3.97
165	20.95	3.98	165	20.7	3.95	165	20.78	3.96
170	20.9	3.97	170	20.64	3.95	170	20.72	3.95
175	20.86	3.97	175	20.6	3.94	175	20.68	3.95
180	20.83	3.96	180	20.57	3.94	180	20.64	3.94
185	20.8	3.96	185	20.53	3.94	185	20.6	3.94
190	20.77	3.96	190	20.5	3.93	190	20.56	3.94
195	20.74	3.95	195	20.47	3.93	195	20.51	3.93
200	20.72	3.95	200	20.45	3.93	200	20.47	3.93
205	20.69	3.95	205	20.42	3.92	205	20.43	3.93
210	20.68	3.95	210	20.4	3.92	210	20.4	3.92
215	20.66	3.95	215	20.38	3.92	215	20.4	3.92
220	20.64	3.95	220	20.37	3.92	220	20.39	3.92
225	20.63	3.94	225	20.35	3.92	225	20.39	3.92
230	20.62	3.94	230	20.34	3.92	230	20.38	3.92
235	20.61	3.94	235	20.33	3.92	235	20.38	3.92
240	20.61	3.94	240	20.32	3.91	240	20.37	3.92
245	20.6	3.94	245	20.32	3.91	245	20.37	3.92
250	20.6	3.94	250	20.32	3.91	250	20.36	3.92
		3.46			3.45			3.45
	88%	94%		88%	94%		88%	94%
	1.92			32.62			2.19	
	9			9			8	
	0.00865			0.00969			0.00453	

Linear
7 Node P.S.
Method 3

Length [m]	Pressure head [m]	Emitter Flow [gph]
0	30	
5	29.42	4.71
10	28.84	4.66
15	28.26	4.62
20	27.68	4.57
25	27.1	4.52
30	26.52	4.47
35	25.94	4.42
40	25.36	4.37
45	24.9	4.33
50	24.51	4.3
55	24.11	4.26
60	23.71	4.23
65	23.31	4.19
70	22.91	4.16
75	22.51	4.12
80	22.12	4.08
85	21.77	4.05
90	21.53	4.03
95	21.28	4.01
100	21.04	3.98
105	20.8	3.96
110	20.55	3.94
115	20.31	3.91
120	20.06	3.89
125	19.82	3.87
130	19.7	3.85
135	19.57	3.84
140	19.44	3.83
145	19.32	3.82
150	19.19	3.8
155	19.06	3.79
160	18.94	3.78
165	18.81	3.77
170	18.74	3.76
175	18.69	3.75
180	18.64	3.75
185	18.59	3.74
190	18.54	3.74
195	18.49	3.73
200	18.44	3.73
205	18.39	3.72
210	18.35	3.72
215	18.34	3.72
220	18.34	3.72
225	18.33	3.72
230	18.32	3.72
235	18.31	3.72
240	18.31	3.72
245	18.3	3.72
250	18.29	3.71
		3.33
	85%	93%
	17.9	
	9	
	0.00497	

Quadratic
7 Node
Method 3

Length [m]	Pressure head [m]	Emitter Flow [gph]
0	30	
5	29.52	4.72
10	29.04	4.68
15	28.58	4.64
20	28.13	4.61
25	27.69	4.57
30	27.26	4.53
35	26.83	4.5
40	26.42	4.46
45	26.03	4.43
50	25.64	4.4
55	25.26	4.36
60	24.89	4.33
65	24.53	4.3
70	24.19	4.27
75	23.85	4.24
80	23.52	4.21
85	23.25	4.19
90	23.05	4.17
95	22.86	4.15
100	22.68	4.14
105	22.51	4.12
110	22.34	4.1
115	22.17	4.09
120	22.01	4.07
125	21.86	4.06
130	21.72	4.05
135	21.58	4.03
140	21.44	4.02
145	21.32	4.01
150	21.19	4
155	21.08	3.99
160	20.97	3.98
165	20.87	3.97
170	20.81	3.96
175	20.77	3.96
180	20.74	3.95
185	20.71	3.95
190	20.68	3.95
195	20.65	3.95
200	20.63	3.94
205	20.61	3.94
210	20.59	3.94
215	20.57	3.94
220	20.55	3.94
225	20.54	3.94
230	20.53	3.93
235	20.52	3.93
240	20.52	3.93
245	20.51	3.93
250	20.51	3.93
		3.46
	88%	94%
	1.92	
	8	
	0.00947	

Quadratic
7 Node P.S.
Method 3

Length [m]	Pressure head [m]	Emitter Flow [gph]
0	30	
5	29.51	4.72
10	29.03	4.68
15	28.56	4.64
20	28.1	4.6
25	27.66	4.57
30	27.22	4.53
35	26.79	4.5
40	26.38	4.46
45	25.97	4.43
50	25.58	4.39
55	25.19	4.36
60	24.82	4.33
65	24.46	4.29
70	24.1	4.26
75	23.76	4.23
80	23.43	4.2
85	23.15	4.18
90	22.95	4.16
95	22.75	4.14
100	22.57	4.13
105	22.39	4.11
110	22.21	4.09
115	22.04	4.08
120	21.88	4.06
125	21.73	4.04
130	21.58	4.03
135	21.43	4.02
140	21.29	4.01
145	21.16	4
150	21.04	3.98
155	20.92	3.97
160	20.81	3.96
165	20.7	3.95
170	20.64	3.95
175	20.6	3.94
180	20.56	3.94
185	20.53	3.93
190	20.5	3.93
195	20.47	3.93
200	20.44	3.93
205	20.42	3.92
210	20.4	3.92
215	20.38	3.92
220	20.36	3.92
225	20.35	3.92
230	20.34	3.92
235	20.33	3.92
240	20.32	3.91
245	20.31	3.91
250	20.31	3.91
		3.45
	88%	94%
	32.02	
	8	
	0.00959	

Exact
50 Nodes

Length [m]	Pressure head [m]	Emitter Flow [gph]
0	30	
5	29.42	4.71
10	28.87	4.67
15	28.35	4.62
20	27.84	4.58
25	27.36	4.54
30	26.9	4.5
35	26.46	4.47
40	26.04	4.43
45	25.64	4.4
50	25.25	4.36
55	24.89	4.33
60	24.55	4.3
65	24.22	4.27
70	23.91	4.25
75	23.61	4.22
80	23.34	4.2
85	23.07	4.17
90	22.83	4.15
95	22.59	4.13
100	22.37	4.11
105	22.17	4.09
110	21.97	4.07
115	21.79	4.05
120	21.62	4.04
125	21.47	4.02
130	21.32	4.01
135	21.19	4
140	21.06	3.99
145	20.95	3.97
150	20.84	3.96
155	20.75	3.96
160	20.66	3.95
165	20.58	3.94
170	20.51	3.93
175	20.45	3.93
180	20.39	3.92
185	20.34	3.92
190	20.3	3.91
195	20.26	3.91
200	20.23	3.91
205	20.2	3.9
210	20.18	3.9
215	20.17	3.9
220	20.15	3.9
225	20.14	3.9
230	20.13	3.9
235	20.13	3.9
240	20.13	3.9
245	20.13	3.9
250	20.13	3.9
		3.43
	88%	94%
	70.13	
	13	
	0.00008	

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