This is to certify that the

thesis entitled

On the Application of the Boundary Element
Method to Plane Orthotropic Elasticity

presented by

Javad Katibai

has been accepted towards fulfillment
of the requirements for

Master's degree in Mechanics

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Date May 12, 1989

ON THE APPLICATION OF BOUNDARY ELEMENT METHOD TO PLANE ORTHOTROPIC ELASTICITY

By

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AN ABSTRACT OF A THESIS

Submitted to

Michigan State University

in partial fulfillment of the requirements

for the degree of

MASTER OF SCIENCE

Department of Metallurgy, Mechanics, and Material Science

1989

ABSTRACT

ON THE APPLICATION OF THE BOUNDARY ELEMENT METHOD

TO PLANE ORTHOTROPIC ELASTICITY

By

Javad Katibai

Application of the boundary element method to elastostatic, orthotropic, plane stress problems is presented. The direct boundary element method is utilized. The boundary is approximated as piecewise straight and, on each straight boundary element, constant traction and linear displacement variations are assumed. This model allows for traction discontinuities at the boundary element interfaces. Three forms of the influence functions are discussed for the orthotropic materials depending upon a relation among the material constants. However, only two practical cases are solved. Results are obtained for four example problems and are compared to finite element solutions(NASTRAN).

ACKNOWLEDGEMENTS

The author wishes to express his gratitude and deep appreciation to his advisor, Professor Nicholas Altiero, for his able guidance and patience during the course of this research, and for his spending many hours with the author in the preparation of this thesis.

The author also wishes to thank his parents for their continuous encouragement and moral support in the past several years.

The financial support of Garrett Turbine Engine Company, Phoenix, Arizona, is also gratefully acknowledged.

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CHAPTER I

INTRODUCTION

Boundary solution techniques are becoming increasingly popular with engineers and have been applied for the solution of a wide range of problems including two and three dimensional elasticity. The most frequently used method employs the fundamental solution of the governing equations as an influence function and constructs the solution to the problem of interest by superposition. This method is presented under different names such as "Boundary Integral Equation Method", "Boundary Integral Method",etc. In its most general form this technique consists of subdividing the boundary of the region under consideration into a collection of elements; hence the name "Boundary Element Method" [1,2].

Consider the classical mixed boundary value problem of linear elastostatics, shown in Figure I-1, consisting of an elastic body, R, loaded by specified tractions, t_i, on portion B_t of the boundary and specified displacements, u_i, on the remainder of the boundary B_u. Body forces are neglected here. The stress fields and displacements everywhere in R, subject to the given boundary conditions, are sought. Numerical solutions are sought for this problem which allow for orthotropic material properties and arbitrary body shape and loading conditions.

The Boundary Element Method derives from the statement of the problem in the form of an integral equation. The integrands consist of known influence functions and both known and unknown boundary conditions. The influence functions satisfy the differential equations exactly. Hence, the solution of displacements and stresses also satisfies the differential equations exactly. Thus, the resolution of high stress gradients by the Boundary Element Method is very good.

The integration path of the integral equations of the Boundary Element Method is around the boundary of the body. Thus, for numerical purposes, the discretization needs to be done only on the boundary . This is in contrast to the Finite Element Method in which discretization is done over the entire domain. The net result of this difference is that the Boundary Element Method requires less data preparation effort to solve a problem.

The Boundary Element Method, however, is not without its own share of problems. Though the Boundary Element Method has been used extensively for isotropic problems, literature on its application to anisotropic material problems is relatively sparse[6,11,12,13]. The sparsity of existing literature on anisotropic materials is not the only problem. Many inaccuracies present in these research papers and technical reports make application of the method difficult. It is one of the objectives of this dissertation to clearly define the influence functions and to offer a systematic solution that is easy to understand and implement.

The application of the Boundary Element Method to the isotropic case is discussed in chapter II of this dissertation. The general problem is formulated and all of the assumptions are outlined.

In chapter III, the application of the Boundary Element Method is extended to orthotropic materials. It is demonstrated that the mathematical formulation is the same except that the influence functions are different from the isotropic case. For plane problems involving orthotropic elastic materials, there are three forms of the influence functions depending upon the relationship among the four material constants [6].

The Boundary Element programs, ORTHO.CASE1 and ORTHO.CASE2, described in chapter III, are based on the direct approach. The boundary discretization consists of straight segments in which displacements are assumed to vary linearly and tractions are assumed to be constant.

In the final chapter, results of the program are presented and it is compared to the isotropic case. Some conclusions are drawn from these results and recommendations are made.

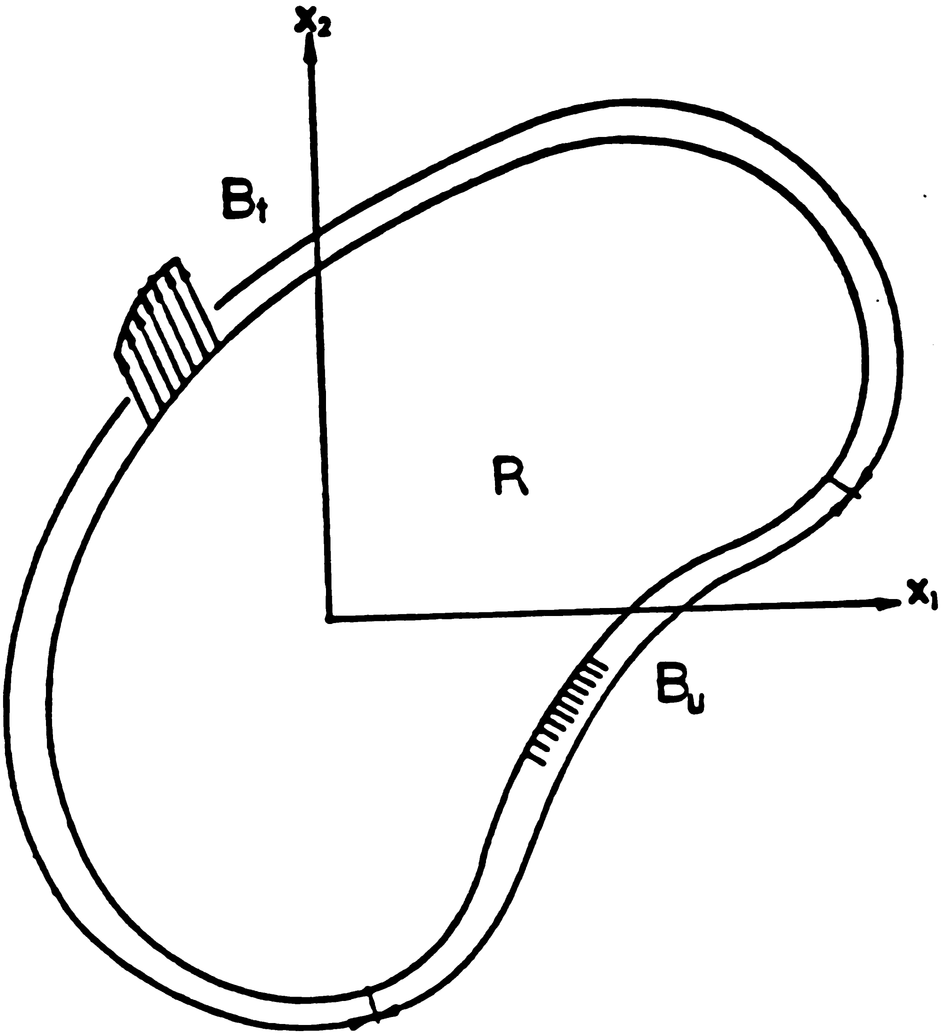


Figure I.1   Mixed boundary value problem

CHAPTER II

APPLICATION OF BOUNDARY ELEMENT METHOD TO ISOTROPIC MATERIALS

II.1 DESCRIPTION OF THE BOUNDARY ELEMENT METHOD

For the plane boundary-value problem of linear elasticity illustrated in Figure II.1, the displacement at a point x on B is related to the displacements and tractions at all other points on B by Somigliana's identity, [1] i.e.

$$\alpha_{ij}(x)u_j(x)+\int_B (uc)_{i.j}(x,\bar{x})u_j(\bar{x})d\bar{s}=\int_B (uR)_{i.j}(x,\bar{x})t_j(\bar{x})d\bar{s} \quad \text{(II.1)}$$

where the integral on the left hand side is interpreted in the Cauchy principal-value sense. The function $(uc)_{i.j}(x,\bar{x})$ is the displacement, $u_i(x)$, due to a unit displacement discontinuity, $c_j(\bar{x})$, applied in the infinite elastic plane and $(uR)_{i.j}(x,\bar{x})$ is the displacement, $u_i(x)$ due to a unit force, $R_j(\bar{x})$, applied in the infinite elastic plane.

The coefficients $\alpha_{ij}(x)$ are equal to $0.5\delta_{ij}$ if the boundary is smooth at x, where δ_{ij} is the Kronecker delta. Otherwise $\alpha_{ij}(x)$ depends on the corner angle at x.

At a point x in R, the displacements and stresses can be calculated from the equations

$$u_i(x)=\int_B (uR)_{i.j}(x,\bar{x})t_j(\bar{x})d\bar{s}-\int_B (uc)_{i.j}(x,\bar{x})u_j(\bar{x})d\bar{s} \quad \text{(II.2)}$$

$$\sigma_{ij}(x)=\int_B (\sigma R)_{ij.k}(x,\bar{x})t_k(\bar{x})d\bar{s}-\int_B (\sigma c)_{ij.k}(x,\bar{x})u_k(\bar{x})d\bar{s} \qquad \text{(II.3)}$$

where the influence function $(\sigma R)_{ij.k}(x,\bar{x})$ and $(\sigma c)_{ij.k}(x,\bar{x})$ are the stress component $\sigma_{ij}(x)$ due to a unit force, $R_k(\bar{x})$, and a unit displacement discontinuity, $c_k(\bar{x})$, respectively, applied in the infinite plane.

At each point x on B and in each direction, either $u_j(x)$ or $t_j(x)$ is known. Therefore, equation (II.1) can be used to solve for the unknown values of $u_j(x)$ and $t_j(x)$, thus giving complete boundary information. The displacements and stresses at any internal point can then be determined by simple integration using equations (II.2) and (II.3).

It can be shown that, for plane stress and material isotropy the influence functions or the Green's functions (in this dissertation the terms influence function and Green's function are used interchangably) of equations (II.1),(II.2) and (II.3) are given by [3]

$$(uR)_{i.k}=[-(3-\nu)\log\rho+(1+\nu)q_iq_k]/(8\pi G) \qquad \text{(II.4)}$$

$$(uc)_{1.1}=[2(1+\nu)(\bar{n}_1q_1^{\,3}-\bar{n}_2q_2^{\,3})+(1-\nu)\bar{n}_1q_1+(3+\nu)\bar{n}_2q_2]/(4\pi\rho)$$

$$(uc)_{1.2}=[2(1+\nu)(-\bar{n}_2q_1^{\,3}-\bar{n}_1q_2^{\,3})+(1+3\nu)\bar{n}_2q_1+(3+\nu)\bar{n}_1q_2]/(4\pi\rho)$$

$$(uc)_{2.1}=[2(1+\nu)(-\bar{n}_2q_1^{\,3}-\bar{n}_1q_2^{\,3})+(3+\nu)\bar{n}_2q_1+(1+3\nu)\bar{n}_1q_2]/(4\pi\rho)$$

$$(uc)_{2.2}=[2(1+\nu)(-\bar{n}_1 q_1{}^3+\bar{n}_2 q_2{}^3)+(1-\nu)\bar{n}_2 q_2+(3+\nu)\bar{n}_1 q_1]/(4\pi\rho) \tag{II.5}$$

$$(\sigma R)_{11.1}=[-2(1+\nu)q_1{}^3-(1-\nu)q_1]/(4\pi\rho)$$

$$(\sigma R)_{12.1}=[2(1+\nu)q_2{}^3-(3+\nu)q_2]/(4\pi\rho)$$

$$(\sigma R)_{22.1}=[2(1+\nu)q_1{}^3-(1+3\nu)q_1]/(4\pi\rho)$$

$$(\sigma R)_{11.2}=[2(1+\nu)q_2{}^3-(1+3\nu)q_2]/(4\pi\rho)$$

$$(\sigma R)_{12.2}=[2(1+\nu)q_1{}^3-(3+\nu)q_1]/(4\pi\rho)$$

$$(\sigma R)_{22.2}=[-2(1+\nu)q_2{}^3-(1-\nu)q_2]/(4\pi\rho) \tag{II.6}$$

$$(\sigma c)_{11.1}=G(1+\nu)[(1+4q_1{}^2-8q_1{}^4)\bar{n}_1+2q_1q_2(1-4q_1{}^2)\bar{n}_2]/(2\pi\rho^2)$$

$$(\sigma c)_{12.1}=G(1+\nu)[(1-8q_1{}^2q_2{}^2)\bar{n}_2+2q_1q_2(1-4q_1{}^2)\bar{n}_1]/(2\pi\rho^2)$$

$$(\sigma c)_{22.1}=G(1+\nu)[(1-8q_1{}^2q_2{}^2)\bar{n}_1+2q_1q_2(1-4q_2{}^2)\bar{n}_2]/(2\pi\rho^2)$$

$$(\sigma c)_{11.2}=(\sigma c)_{12.1}$$

$$(\sigma c)_{12.2}=(\sigma c)_{22.1}$$

$$(\sigma c)_{22.2}=G(1+\nu)[(1+4q_2{}^2-8q_2{}^4)\bar{n}_2+2q_1q_2(1-4q_2{}^2)\bar{n}_1]/(2\pi\rho^2) \tag{II.7}$$

where

$$\rho = [(x_1-\bar{x}_1)^2+(x_2-\bar{x}_2)^2]^{1/2}$$

$$q_1=(x_1-\bar{x}_1)/\rho$$

$$q_2=(x_2-\bar{x}_2)/\rho$$

$\bar{n}_1$ and $\bar{n}_2$ are the components of the outward-directed normal unit vector to the boundary at $\bar{x}$.

II.2 PIECEWISE LINEAR ELEMENTS

Equation (II.1) can be solved numerically if the boundary B is approximated by N straight segments, as shown in Figure II.2.

For this model, equation (II.1) can be written as

$$\alpha_{ij}(x^{(n)})u_j(x^{(n)})+\sum_{m=1}^{N}\int_m (uc)_{i.j}(x^{(n)},\bar{x})u_j(\bar{x})d\bar{s} = \sum_{m=1}^{N}\int_m (uR)_{i.j}(x^{(n)},\bar{x})t_j(\bar{x})d\bar{s} \quad \text{(II.8)}$$

where $x^{(n)}$ is the location of boundary node n.

The displacements and tractions in each segment m can be approximated using shape functions so that

$$u_j(\bar{x})=u_j^{(m-1)}N_1(\xi)+u_j^{(m)}N_2(\xi) \qquad \text{(II.9)}$$

$$t_j(\bar{x})=t_j^{m} \qquad \text{(II.10)}$$

where

$$N_1(\xi)=0.5(1-\xi)$$

$$N_2(\xi)=0.5(1+\xi)$$

$$\bar{x}=N_1(\xi)x^{(m-1)}+N_2(\xi)x^{(m)}$$

$$d\bar{s}=0.5(s_m-s_{m-1})d\xi \qquad \text{(II.11)}$$

$$d\bar{s}=0.5\Delta s_m d\xi \qquad \text{(II.12)}$$

where ξ is a local coordinate for the segment m with value -1 at node m-1, value 0 at the center of the segment, and value 1 at node m. Note that $u_j^{(m)}$ is the displacement at node m whereas t_j^{m}

is the traction on the element m.

Note that the order of differentiation of $t_i(x)$ in the interval is less than that of $u_j(x)$. This model allows for discontinuities of t_j on the boundary.

If equations (II.9-12) are substitued into equation (II.8) the following is obtanied

$$2\alpha_{ij}{}^{(n)}u_j{}^{(n)}+\sum_{m=1}^{N}\Delta s_m\Big[\int_m (uc)_{i.j}(x^{(n)},\xi)N_1(\xi)d\xi u_j{}^{(m-1)}$$
$$+\int_m (uc)_{i.j}(x^{(n)},\xi)N_2(\xi)d\xi u_j{}^{(m)}\Big]=\sum_{m=1}^{N}\int_m (uR)_{i.j}(x^{(n)},\xi)d\xi\Delta s_m t_j{}^m \quad \text{(II.13)}$$

or

$$2\alpha_{ij}{}^{(n)}\ u_j{}^{(n)}+\sum_{m=1}^{N}[A_{i.j}{}^{(m.n)}u_j{}^{(m-1)}+B_{i.j}{}^{(m.n)}\ u_j{}^{(m)}\ ]$$
$$=\sum_{m=1}^{N}C_{i.j}{}^{(m.n)}\ \hat{F}_j{}^m \qquad \text{(II.14)}$$

where

$$n=1,\ldots\ldots\ldots\ldots\ldots,N$$
$$\alpha_{ij}{}^{(n)}=\alpha_{ij}(x^{(n)})$$
$$u_j{}^{(n)}=u_j(x^{(n)})$$
$$A_{i.j}{}^{(m.n)}=\Delta s_m\int_m (uc)_{i.j}(x^{(n)},\xi)N_1(\xi)d\xi \quad \text{(II.15)}$$
$$B_{i.j}{}^{(m.n)}=\Delta s_m\int_m (uc)_{i.j}(x^{(n)},\xi)N_2(\xi)d\xi \quad \text{(II.16)}$$

$$C_{i.j}^{(m.n)} = \int_m (uR)_{i.j}(x^{(n)},\xi)d\xi \qquad (II.17)$$

$$\hat{F}^m = \Delta s_m t_j^m \qquad (II.18)$$

Note that the functions $(uc)_{i.j}$ and $(uR)_{i.j}$ are singular when the point of coordinate ξ approaches the node n. This is the case when the element m ends or begins with the node n, i.e. when m=n or m=n+1 (n=m or n=m-1). In this presentation, the terms involving logarithmic functions are evaluated analytically and the Cauchy principal-value integrals involving $1/\rho$ terms are evaluated numerically. One can also evaluate the logarithmic terms numerically provided that a proper table of integrals for numerical solution is used. Note that it is not necessary to mathematically calculate α_{ij} since it only contributes to the diagonal of $(uc)_{i.j}$. The value of this diagonal term can be determined quite easily using rigid-body considerations. This will be shown later.

All nonsingular integrals can be evaluated by numerical integration using Guass-Legendre quadrature. If it is defined that(see Figure II.3):

$$\hat{x}^{(m)} = 0.5\ [x^{(m)} + x^{(m-1)}]$$

$$a_i = x_i^{(n)} - \hat{x}_i^{(n)}$$

$$b_i = x_i^{(m)} - \hat{x}_i^{(m)}$$

$$R = a_i a_i - 2a_i b_i \xi + b_i b_i \xi^2$$

then substitution into equations (II.15-18) gives expressions that are easy to program.

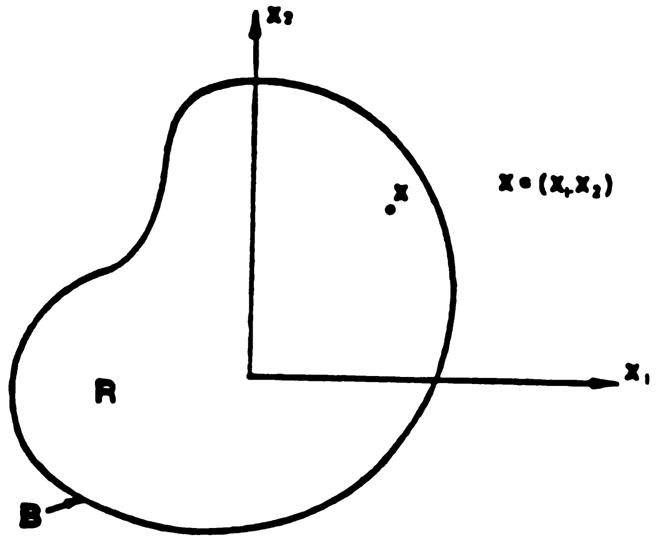


Figure II.1 Plane boundary value problem

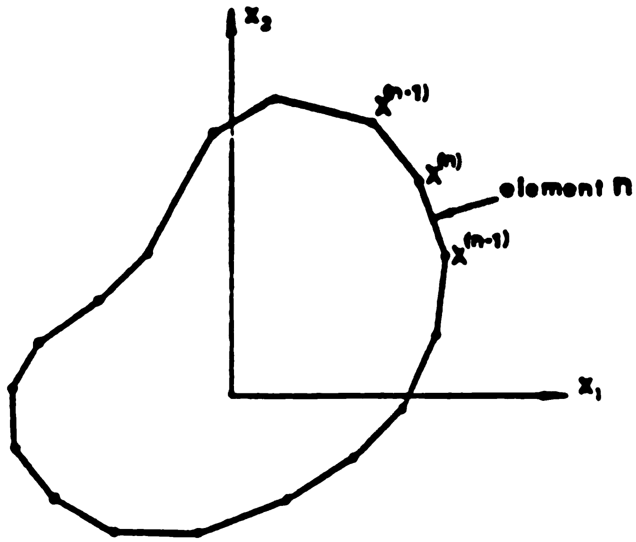


Figure II.2 Boundary discretization

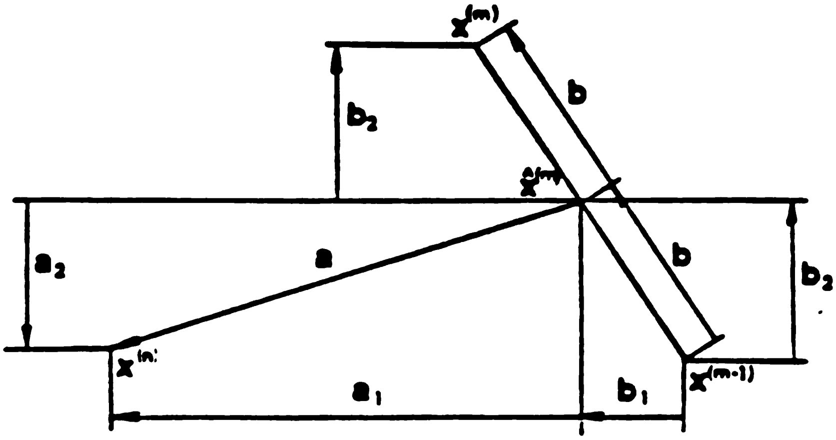


Figure II.3 Definition of a_i, b_i, $\hat{x}^{(m)}$

CHAPTER III

APPLICATION OF THE BOUNDARY ELEMENT METHOD TO ORTHOTROPIC MATERIALS

III.1 INTRODUCTION

For an orthotropic material for which the x_1-x_2 axes are alligned with the principal material directions the strain-stress laws for plane stress reduce to

$$
\begin{aligned}
\epsilon_{11} &= \frac{1}{E_x}\sigma_{11} - \frac{\nu_x}{E_x}\sigma_{22} \qquad E_y \\
\epsilon_{22} &= -\frac{\nu_x}{E_x}\sigma_{11} + \frac{1}{E_y}\sigma_{22} \\
\epsilon_{12} &= \frac{1}{2E_s}\sigma_{12} \qquad\qquad\qquad\qquad \text{(III.1)}
\end{aligned}
$$

where ν_x is Poisson's ratio and the constants E_x(Young's modulus in the x_1 direction), E_y(Young's modulus in the x_2 direction), and E_s are the longitudinal, transverse, and axial shear moduli of elasticity, respectively. We shall select the x_1 and x_2 axes such that $E_x > E_y$. In general, orthotropic materials should satisfy a fourth order polynomial equation in which the coefficients are in terms of the aforementioned elastic constants, [4]:

$$\frac{1}{E_x}\mu^4 + 2\left(\frac{1}{2E_s} - \frac{\nu_x}{E_x}\right)\mu^2 + \frac{1}{E_y} = 0 \tag{III.2}$$

The roots of this equation can be expressed as

$$\mu^2 = \frac{E_x}{2E_s} - \upsilon_x \pm \sqrt{\left(\frac{E_x}{2E_s} - \upsilon_x\right)^2 - \frac{E_x}{E_y}} \tag{III.3}$$

where the nature of the roots depends on the quantity under the radical sign. Note that μ^2 is real if

$$\left(\frac{E_x}{2E_s} - \upsilon_x\right)^2 \geq \frac{E_x}{E_y} \tag{III.4}$$

For isotropic materials, the equality of (III.4) is satisfied. Furthermore, it appears that the inequality (III.4) is satisfied for materials of practical interest. Table III.1 lists many materials that are widely used in industry, all of which satisfy inequality (III.4).

III.2 THE INFLUENCE FUNCTIONS FOR ORTHOTROPIC MATERIALS

To find the influence functions or the Green's functions, a two-dimensional infinite orthotropic plane is considered [6]. The equilibrium equation, the compatibility equation, and the boundary conditions at infinity are satisfied using the technique of Fourier Transforms. The boundary conditions at infinity are that the stresses and their first derivatives go to zero.

Three forms of the Green's function have been found [6]. These three forms of the Green's function correspond to the nature of the roots of equation (III.2). However, for practical purposes, the materials for which the material constants satisfy inequality (III.4) will be considered here.

Let us define the distance between the field and the source points in terms of Cartesian coordinate as:

$$r_x = x_1 - \bar{x}_1$$
$$r_y = x_2 - \bar{x}_2$$

where x_1 and x_2 are the coordinate of the field point and $\bar{x}_1$ and $\bar{x}_2$ are the coordinate of the source point. Then, the influence functions $(uR)_{i.j}(x,\bar{x})$ and $(uc)_{i.j}(x,\bar{x})$ have the form [6]:

$$(uR)_{1.1}(x,\bar{x}) = D_{11}\ T_7 + D_{12}\ T_8$$
$$(uR)_{1.2}(x,\bar{x}) = D_{22}\ T_6$$
$$(uR)_{2.1}(x,\bar{x}) = D_{32}\ T_6$$
$$(uR)_{2.2}(x,\bar{x}) = D_{41}\ T_7 + D_{42}\ T_8 \qquad \text{(III.5)}$$
$$(uc)_{1.1} = -E_{11}\ T_1\ \bar{n}_1 - E_{12}\ T_2\ \bar{n}_1 - E_{51}\ T_3\ \bar{n}_2 - E_{52}\ T_4\ \bar{n}_2$$
$$(uc)_{1.2} = -E_{51}\ T_3\ \bar{n}_1 - E_{52}\ T_4\ \bar{n}_1 - E_{31}\ T_1\ \bar{n}_2 - E_{32}\ T_2\ \bar{n}_2$$
$$(uc)_{2.1} = -E_{21}\ T_3\ \bar{n}_1 - E_{22}\ T_4\ \bar{n}_1 - E_{61}\ T_1\ \bar{n}_2 - E_{62}\ T_2\ \bar{n}_2$$
$$(uc)_{2.2} = -E_{61}\ T_1\ \bar{n}_1 - E_{62}\ T_2\ \bar{n}_1 - E_{41}\ T_3\ \bar{n}_2 - E_{42}\ T_4\ \bar{n}_2 \qquad \text{(III.6)}$$

and μ_1 and μ_2 are the roots of equation (III.3), $\mu_1 \geq \mu_2$. Also

$$
\begin{aligned}
E_{11} &= -(d_1 + d_3 - d_4) / (4 \pi d_5) \\
E_{12} &= -(d_1 - d_3 - d_4) / (4 \pi d_6) \\
E_{21} &= d_4 / (4 \pi) \\
E_{22} &= -(2d_2 - d_1 d_4) / (4 \pi d_5 d_6) \\
E_{31} &= (1 + (d_4 / d_3)) / (4 \pi d_5) \\
E_{32} &= (1 - (d_4 / d_3)) / (4 \pi d_6) \\
E_{41} &= -1 / (4 \pi) \\
E_{42} &= (d_1 - 2d_4) / (4 \pi d_5 d_6) \\
E_{51} &= E_{41} \\
E_{52} &= -E_{42} \\
E_{61} &= -d_3 E_{31} \\
E_{62} &= d_3 E_{32} \qquad\qquad \text{(III.7)} \\
D_{11} &= \frac{E_{11}}{E_x} - \frac{\upsilon_x E_{31}}{E_x} \\
D_{12} &= \frac{E_{12}}{E_x} - \frac{\upsilon_x E_{32}}{E_x} \\
D_{22} &= -\frac{E_{22}}{E_x} + \frac{\upsilon_x E_{42}}{E_x} \\
D_{32} &= D_{22} \\
D_{41} &= d_3 D_{11}
\end{aligned}
$$

$$D_{42} = -d_3\, D_{12} \tag{III.8}$$

where

$$d_1 = 2\left(\frac{E_x}{2E_s} - \upsilon_x\right)$$

$$d_2 = \frac{E_y}{E_x}$$

$$d_3 = \sqrt{\frac{E_y}{E_x}}$$

$$d_4 = -\upsilon_x$$

$$d_5 = \mu_1 + \mu_2$$

$$d_6 = \begin{cases} \mu_1 - \mu_2 & \text{case I} \\ 1 & \text{case II} \end{cases} \tag{III.9}$$

the functions T_1 through T_8 depend on the relationship among the elastic constants. Two cases are considered. In case 1:

$$\left(\frac{E_x}{2E_s} - \upsilon_x\right)^2 > \frac{E_x}{E_y}$$

and the roots of equation (III.2) can be expressed as

$$\mu_{1,2} = \sqrt{\frac{E_x}{2E_s} - \upsilon_x \pm \sqrt{\left[\frac{E_x}{2E_s} - \upsilon_x\right]^2 - \frac{E_x}{E_y}}}$$

If we let

$$r_1^2 = r_x^2 + \mu_1^2 r_y^2$$
$$r_2^2 = r_x^2 + \mu_2^2 r_y^2$$

then

$$T_1 = \frac{r_x}{r_1^2} + \frac{r_x}{r_2^2}$$

$$T_2 = \frac{r_x}{r_1^2} - \frac{r_x}{r_2^2}$$

$$T_3 = \frac{\mu_1 r_y}{r_1^2} + \frac{\mu_2 r_y}{r_2^2}$$

$$T_4 = \frac{\mu_1 r_y}{r_1^2} - \frac{\mu_2 r_y}{r_2^2}$$

$$T_6 = \tan^{-1}\left(\frac{\mu_1 r_y}{r_x}\right) - \tan^{-1}\left(\frac{\mu_2 r_y}{r_x}\right)$$

$$T_7 = \log (r_1) + \log (r_2)$$

$$T_8 = \log (r_1) - \log (r_2)$$

In case 2:

$$\left(\frac{E_x}{2E_s} - \nu_x\right)^2 = \frac{E_x}{E_y}$$

where

$$\mu_1 = \mu_2 = \sqrt{\frac{E_x}{2E_s} - \nu_x}$$

and

$$T_1 = \frac{2\ r_x}{r_1^{\ 2}}$$

$$T_2 = \frac{-2\mu_1 r_x r_y^{\ 2}}{r_1^{\ 4}}$$

$$T_3 = \frac{2\mu_1 r_y}{r_1^{\ 2}}$$

$$T_4 = \frac{r_x^{\ 2} r_y - \mu_1^{\ 2} r_y^{\ 3}}{r_1^{\ 4}}$$

$$T_6 = \frac{r_x\ r_y}{r_1^{\ 2}}$$

$$T_7 = 2 \log (\ r_1)$$

$$T_8 = \frac{r_y^{\ 2}}{r_1^{\ 2}}$$

III.3 PROBLEM FORMULATION

To determine the complete set of boundary information , i.e., the displacements and the tracions at all chosen boundary points, one must solve the numerical form of Somigliana's identity as formulated in Chapter II. The formulation of the problem for orthotropic materials is exactly the same as in the isotropic case except that the influence functions $(uc)_{i.j}(x,\bar{x})$ and $(uR)_{i.j}(x,\bar{x})$ are different.

III.4 COMPARISON OF THE ISOTROPIC AND ORTHOTROPIC INFLUENCE FUNCTIONS

The isotropic influence functions listed in chapter II were compared to the orthotropic influence functions presented in this chapter. This comparison was made by numerically calculating the values of those equations for different field and source points. The computer program TEST in Appendix B performs these calculations. See Figure III.1 for the flow chart of the the progrm TEST.

To better understand the differences and characteristics of these influence functions, the following examples were examined. Field points and source points were selected as shown in Figure III.2. The values of the influence functions in chapter I for the isotropic case were calculated based upon the Poisson's ratio and shear modulus of 0.25 and 1.25, respectively. Tables III.2-7 exhibit the results for each of the source points. These tables also list the results for the influence functions of cases 1 and 2 assuming near-isotropic and isotropic conditions, respectively.

The values calculated using the isotropic influence functions of chapter I are identical to the case 2 influence functions. This is expected since isotropic materials belong to the case 2 of the orthotropic influence functions. It appears that the case 1 is also suitable for isotropic materials provided that the material properties selected are slightly anisotropic to satisfy the inequality of equation III.4. Note that the $(uR)_{1.1}$ of orthotropic influence functions are off by a constant. This constant represents rigid body motion and has no effect on the boundary element formulation.

III.5 PIECEWISE LINEAR ELEMENTS

Somigliana's identity can be solved numerically for the case of orthotropic materials just as in the isotropic case by approximating the boundary B with N straight segments, as shown in Figure II.2. The displacemnts and tractions in each segment m are approximated using shape functions so that

$$u_j(\bar{x}) = u_j^{(m-1)} N_1(\xi) + u_j^{(m)} N_2(\xi)$$
$$t_j(\bar{x}) = t_j^{m}$$

where

$$N_1(\xi) = 0.5\ (1 - \xi)$$

$$N_2(\xi) = 0.5\ (1 + \xi)$$
$$\bar{x} = N_1(\xi)x^{(m-1)} + N_2(\xi)x^{(m)}$$
$$d\bar{s} = 0.5(s_m - s_{m-1})d\xi$$
$$d\bar{s} = 0.5\ \Delta s_m d\xi \qquad \text{(III.10)}$$

and ξ is a local coordinate for the segment m with value -1 at node m-1, value 0 at the center of the segment, and value 1 at node m. By substituting equations (III.10) into Somigliana's identity the following is obtained

$$2\alpha_{ij}{}^{(n)}u_j{}^{(n)} + \sum_{m=1}^{N} \Delta s_m \left[\int_m (uc)_{i.j}(x^{(n)},\xi)\ N_1(\xi)d\xi u_j{}^{(m-1)}\right.$$

$$\left.+\int_m (uc)_{i.j}(x^{(n)},\xi)N_2(\xi)d\xi u_j{}^{(m)}\right] = \sum_{m=1}^{N} \int_m (uR)_{i.j}(x^{(n)},\xi)d\xi\Delta s_m t_j{}^m$$

or

$$2\alpha_{ij}{}^{(n)}u_j{}^{(n)} + \sum_{m=1}^{N}[A_{i.j}{}^{(m.n)}u_j{}^{(m-1)} + B_{i.j}{}^{(m.n)}u_j{}^{(m)}]$$
$$= \sum_{m=1}^{N} C_{i.j}{}^{(m.n)}\hat{F}_j \qquad \text{(III.11)}$$

where

$$n=1,\ldots\ldots\ldots,N$$
$$\alpha_{ij}{}^{(n)} = \alpha_{ij}(x^{(n)})$$

$$u_j^{(n)}=u_j(x^{(n)})$$

$$A_{i.j}^{(m.n)}=\Delta s_m \int_m (uc)_{i.j}(x^{(n)},\xi)N_1(\xi)d\xi$$

$$B_{i.j}^{(m.n)}=\Delta s_m \int_m (uc)_{i.j}(x^{(n)},\xi)N_2(\xi)d\xi$$

$$C_{i.j}^{(m.n)}=\int_m (uR)_{i.j}(x^{(n)},\xi)d\xi$$

$$\hat{F}_j^m= \Delta s_m t_j^m$$

III.6 SINGULAR TERMS

As in the isotropic case, the functions $(uc)_{i.j}$ and $(uR)_{i.j}$ are singular as ξ approaches the node n. This is the case when the element m ends or begins with the node n, i.e. when m=n, m=n+1 (n=m,m-1). Again all the integrals, singular or non-singular, with the exception of the singular terms involving logarithmic functions, are evaluated numerically.

The singular terms involving logarithmic functions are evaluated analytically. However, one can evaluate those singular integrals involving logarithmic functions numerically provided that a proper table of integrals is used.

III.7 NUMERICAL INTEGRATION

The numerical integration is performed utilizing the Guass-Legendre quadrature formulae. The following is a prelude to the integrations. If it is defined that

$$\hat{x}^{(m)} = 0.5\ [x^{(m)} + x^{(m-1)}]$$
$$a_i = x_i^{(n)} - \hat{x}_i^{(n)}$$
$$b_i = x_i^{(m)} - \hat{x}_i^{(m)}$$

then

$$r_x = a_1 - b_1\xi$$
$$r_y = a_2 - b_2\xi$$
$$r_i^2 = (a_1 - b_1\xi)^2 + \mu_i^2(a_2 - b_2\xi)^2$$
$$i = 1,2$$

Also

$$\bar{n}_1 = \frac{b_2}{b} = \frac{2b_2}{s_m - s_{m-1}}$$

$$\bar{n}_2 = -\frac{b_1}{b} = -\frac{2b_1}{s_m - s_{m-1}}$$

For the case 1 influence functions

$$\left.\begin{matrix} A_{1.1}{}^{(m.n)} \\ B_{1.1}{}^{(m.n)} \end{matrix}\right\} = -E_{11}b_2 \int_{-1}^{1} \left(\frac{a_1-b_1\xi}{r_1^{\,2}} + \frac{a_1-b_1\xi}{r_2^{\,2}}\right)(1 \mp \xi)\, d\xi$$

$$-E_{12}b_2 \int_{-1}^{1} \left(\frac{a_1-b_1\xi}{r_1^{\,2}} - \frac{a_1-b_1\xi}{r_2^{\,2}}\right)(1 \mp \xi)\, d\xi$$

$$+E_{51}b_1 \int_{-1}^{1} \left[\frac{\mu_1(a_2-b_2\xi)}{r_1^{\,2}} + \frac{\mu_2(a_2-b_2\xi)}{r_2^{\,2}}\right](1 \mp \xi)\, d\xi$$

$$+E_{52}b_1 \int_{-1}^{1} \left[\frac{\mu_1(a_2-b_2\xi)}{r_1^{\,2}} - \frac{\mu_2(a_2-b_2\xi)}{r_2^{\,2}}\right](1 \mp \xi)\, d\xi \qquad \text{(III.11a)}$$

and $A_{2.2}{}^{(m.n)}$ and $B_{2.2}{}^{(m.n)}$ have the same expressions as $A_{1.1}{}^{(m.n)}$ and $B_{1.1}{}^{(m.n)}$ where E_{11}, E_{12}, E_{51} and E_{52} are replaced by E_{61}, E_{62}, E_{41} and E_{42} respectively. Also

$$\left.\begin{matrix} A_{1.2}{}^{(m.n)} \\ B_{1.2}{}^{(m.n)} \end{matrix}\right\} = -E_{51}b_2 \int_{-1}^{1} \left[\frac{\mu_1(a_2-b_2\xi)}{r_1^{\,2}} + \frac{\mu_2(a_2-b_2\xi)}{r_2^{\,2}}\right](1 \mp \xi)\, d\xi$$

$$-E_{52}b_2 \int_{-1}^{1} \left[\frac{\mu_1(a_2-b_2\xi)}{r_1^{\,2}} - \frac{\mu_2(a_2-b_2\xi)}{r_2^{\,2}}\right](1 \mp \xi)\, d\xi$$

$$+E_{31}b_1 \int_{-1}^{1} \left(\frac{a_1-b_1\xi}{r_1^{\,2}} + \frac{a_1-b_1\xi}{r_2^{\,2}}\right)(1 \mp \xi)\, d\xi$$

$$+E_{32}b_1 \int_{-1}^{1} \left(\frac{a_1-b_1\xi}{r_1^{\,2}} - \frac{a_1-b_1\xi}{r_2^{\,2}}\right)(1 \mp \xi)\, d\xi \qquad \text{(III.11b)}$$

and $A_{2.1}^{(m.n)}$ and $B_{2.1}^{(m.n)}$ have the same expressions as $A_{1.2}^{(m.n)}$ and $B_{1.2}^{(m.n)}$ where E_{51}, E_{52}, E_{31} and E_{32} are replaced by E_{21}, E_{22}, E_{61} and E_{62} respectively. Finally

$$C_{1.1}^{(m.n)} = \int_{-1}^{1} \{D_{11}[\log(r_1)+\log(r_2)]+D_{12}[\log(r_1)-\log(r_2)]\}d\xi$$

$$C_{1.2}^{(m.n)} = \int_{1}^{1} D_{22}\{\tan^{-1}\left(\frac{\mu_1(a_2-b_2\xi)}{a_1-b_1\xi}\right)-\tan^{-1}\left(\frac{\mu_2(a_2-b_2\xi)}{a_1-b_1\xi}\right)\}d\xi$$

$$C_{2.1}^{(m.n)} = \int_{-1}^{1} D_{32}\{\tan^{-1}\left(\frac{\mu_1(a_2-b_2\xi)}{a_1-b_1\xi}\right)-\tan^{-1}\left(\frac{\mu_2(a_2-b_2\xi)}{a_1-b_1\xi}\right)\}d\xi$$

$$C_{2.2}^{(m.n)} = \int_{-1}^{1} \{D_{41}[\log(r_1)+\log(r_2)]+D_{42}[\log(r_1)-\log(r_2)]\}d\xi$$

(III.11c)

For the case 2 influence functions

$$\begin{array}{l} A_{1.1}{}^{(m.n)} \\ B_{1.1}{}^{(m.n)} \end{array} = -E_{11}b_2 \int_{-1}^{1} \left(\frac{2(a_1-b_1\xi)}{r_1{}^2}\right)(1 \mp \xi)\, d\xi$$

$$+E_{12}b_2 \int_{-1}^{1} \left(\frac{2\mu_1(a_1-b_1\xi)(a_2-b_2\xi)^2}{r_1{}^4}\right)(1 \mp \xi)\, d\xi$$

$$+E_{51}b_1 \int_{-1}^{1} \left(\frac{2\mu_1(a_2-b_2\xi)}{r_1{}^2}\right)(1 \mp \xi)\, d\xi$$

$$-E_{52}b_1 \int_{-1}^{1} \left(\frac{\mu_1^2\,(a_2-b_2\xi)^3}{r_1{}^4}\right)(1 \mp \xi)\, d\xi$$

$$+E_{51}b_1 \int_{-1}^{1} \left(\frac{(a_1-b_1\xi)^2\,(a_2-b_2\xi)}{r_1{}^4}\right)(1 \mp \xi)\, d\xi \qquad \text{(III.11d)}$$

and $A_{2.2}{}^{(m.n)}$ and $B_{2.2}{}^{(m.n)}$ have the same expressions as $A_{1.1}{}^{(m.n)}$ and $B_{1.1}{}^{(m.n)}$ where E_{11}, E_{12}, E_{51} and E_{52} are replaced by E_{61}, E_{62}, E_{41} and E_{42} respectively. Also

$$\begin{array}{l} A_{1.2}{}^{(m.n)} \\ B_{1.2}{}^{(m.n)} \end{array} = -E_{51}b_2 \int_{-1}^{1} \left(\frac{2\mu_1(a_2-b_2\xi)}{r_1{}^2}\right)(1 \mp \xi)\, d\xi$$

$$-E_{52}b_2 \int_{-1}^{1} \left(\frac{(a_1-b_1\xi)^2(a_2-b_2\xi)}{r_1^{\,4}}\right)(1 \mp \xi)\, d\xi$$

$$+E_{52}b_2 \int_{-1}^{1} \left(\frac{\mu_1^2\,(a_2-b_2\xi)^3}{r_1^{\,4}}\right)(1 \mp \xi)\, d\xi$$

$$+E_{31}b_1 \int_{-1}^{1} \left(\frac{2(a_1-b_1\xi)}{r_1^{\,2}}\right)(1 \mp \xi)\, d\xi$$

$$-E_{32}b_1 \int_{-1}^{1} \left(\frac{2\mu_1(a_1-b_1\xi)(a_2-b_2\xi)^2}{r_1^{\,4}}\right)(1 \mp \xi)\, d\xi \qquad \text{(III.11e)}$$

and $A_{2.1}^{(m.n)}$ and $B_{2.1}(m.n)$ have the same expressions as $A_{1.2}^{(m.n)}$ and $B_{1.2}^{(m.n)}$ where E_{51}, E_{52}, E_{31} and E_{32} are replaced by E_{21}, E_{22}, E_{61} and E_{62} respectivly.

$$C_{1.1}^{(m.n)} = \int_{-1}^{1} \left(D_{11} 2\log(r_1) + D_{12}\left(\frac{(a_2-b_2\xi)^2}{r_1^{\,2}}\right)\right) d\xi$$

$$C_{1.2}^{(m.n)} = \int_{-1}^{1} D_{22}\left(\frac{(a_1-b_1\xi)(a_2-b_2\xi)}{r_1^{\,2}}\right) d\xi$$

$$C_{2.1}^{(m.n)} = \int_{-1}^{1} D_{32}\left(\frac{(a_1-b_1\xi)(a_2-b_2\xi)}{r_1^{\,2}}\right) d\xi$$

$$C_{2.2}^{(m.n)} = \int_{-1}^{1} \left(D_{41} 2\log(r_1) + D_{42}\left(\frac{(a_2-b_2\xi)}{r_1^{\,2}}\right)\right) d\xi \qquad \text{(III.11f)}$$

III.8) PRELUDE TO COMPUTER PROGRAMMING

Equations (III.11) can be written in matrix form as

$$[uc]\{u\}=[uR]\{\hat{F}\} \qquad \text{(III.12)}$$

This is a system of equations relating nodal displacements to resultant segment forces. In order to solve a well-posed elasticity problem, it is necessary to re-pose this system of equations in terms of nodal forces. Therefore, a transformation

$$\{F\}=[\Gamma]\{\hat{F}\}$$

relating the vector of nodal forces $\{F\}$ to the vector of segment forces $\{\hat{F}\}$, is introduced into the system (III.12). The simplest physical interpretation of the transformation is to replace the segment forces by nodal forces equal to the average of the segment forces adjacent to each node, or

$$F_i^{(n)} = 0.5\ [\hat{F}_i^{(n)} + \hat{F}_i^{(n+1)}]$$

The form of $[\Gamma]$ for this transformation is

$$[\Gamma] = 0.5 \begin{bmatrix} I & I & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & I & I & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & I & I & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & I & I & 0 & \cdots & 0 \\ \cdots & & & & & & & \\ I & 0 & 0 & 0 & 0 & 0 & \cdots & I \end{bmatrix}$$

where I is a 2x2 identity matrix.

For an odd number of nodes, the inverse of $[\Gamma]$ is

$$[\Gamma]^{-1} = \begin{bmatrix} I & -I & I & -I \ldots\ldots I \\ I & I & -I & I \ldots\ldots -I \\ -I & I & I & -I \ldots\ldots I \\ I & -I & I & I \ldots\ldots -I \\ \ldots & \ldots & \ldots & \ldots\ldots\ldots \\ -I & I & -I & I \ldots\ldots I \end{bmatrix}$$

and (III.12) becomes

$$[uc]\{u\}=[uR][\Gamma]^{-1}\{F\} \qquad \text{(III.13)}$$

It should be noted that for an even number of nodes, $[\Gamma]$ has no inverse.

A perturbation is introduced into the system of equations (III.13) in order to enforce the equilibrium conditions, i.e.

$$\sum_{i=1}^{N} F_1^{(i)}=0$$
$$\sum_{i=1}^{N} F_2^{(i)}=0$$
$$\sum_{i=1}^{N} (x_1^{(i)}F_2^{(i)}-x_2^{(i)}F_1^{(i)})=0 \qquad \text{(III.14)}$$

or in matrix form

$$[Q]\{F\}=0 \qquad \text{(III.15)}$$

where

$$[Q] = \begin{bmatrix} 1 & 0 & 1 & 0 & \ldots 1 & 0 \\ 0 & 1 & 0 & 1 & \ldots 0 & 1 \\ -x_2^{(1)} & x_1^{(1)} & -x_2^{(2)} & x_1^{(2)} & \ldots -x_2^{(N)} & x_1^{(N)} \end{bmatrix}$$

Coupling equations (III.14) and (III.15):

$$\begin{bmatrix} uc \\ 0 \end{bmatrix} \{u\} = \begin{bmatrix} uR^* & Q^T \\ Q & 0 \end{bmatrix} \begin{Bmatrix} \{F\} \\ \{\lambda\} \end{Bmatrix} \qquad \text{(III.16)}$$

where $[uR^*] = [uR]\,[\Gamma]^{-1}$.

III.9 PLANE STRESS, ORTHOTROPIC BOUNDARY ELEMENT PROGRAM

The boundary element program described in this section can be used to solve plane stress, orthotropic elastostatics problems. Linear interpolation is used for the displacements and constant segment tractions are assumed. The logic of the program for calculations for orthotropic materials belonging to case 1 and case 2 is the same. The only difference is that different equations are used to solve for material constants E_{ij} and D_{ij}.

The flow chart for the program is shown in Figure (III.3). In the first part, the input data is read. Then the weights and points for the numerical integration are assigned to the array W(L) and R(L). Next, E_{ij} and D_{ij} are calculated. In the fourth part of the program, the entries of the matrices [uc] and [uR] are calculated based upon the relationship among the material constants. In part five, the diagonal of the matrix [uc] is calculated. In the sixth part of the program, the operation [uR] $[\Gamma]^{-1}$ is performed, and in part seven, the system of equations are rearranged in order to have all the unknowns on the same side of the system of equations. Once the system of equations is solved by the Gauss elimination method, the unknowns are printed in part nine of the program. Finally, the stresses and displacements are calculated for the field points.

III.9.1 INPUTTING THE PROBLEM DATA

The order and format in which the variables and arrays must be submitted to the program are as follows:

TITLE	-The title of the problem. This must be written in column 1 through 80.
N	-The number of nodes on the boundary. It must be an odd number. Enter in free format.
EX, EY, ES, PRX	-Material properties. Enter in free format.
X(I), Y(I)	-Coordinates of nodes I through N, entered counter-clockwise in free format.
J, K	-Nodes, J, and corresponding directions, K, at which displacements are specified (zero and non-zero). Enter in free format and end by inputting 0, 0.

J,K,F(2*J+K+2) -Nodes, J, and corresponding directions, K, at which non-zero forces are specified, and specified values F. Enter in free format and end by inputting 0,0,0.

III.9.2 CALCULATION OF MATERIAL PROPERTIES E_{ij} and C_{ij}

The program calculates E_{ij} and C_{ij} based on the criterion of part three and equations (III.7-8).

III.9.3 CALCULATION OF COEFFICIENT MATRICES

A more detail flowchart for part four of the program is shown in Figure III.4. The order of the loops was chosen in order to avoid repeating operations.

For each value of loop J, associated with the element J, parts of the (2x2) singular submatrices, shown in Figure III.5, are calculated. Note that only singular, logarithmic terms are evaluted in this loop. This type of function appears only in the $(uR)_{i.j}$ influence functions.

In the other loop, I, all of the nonsingular integrals and non-logarithmic singular integrals are evaluated numerically. The positions in the matrices are illustrated in Figure III.6. The functions for the numerical integration are calculated using equations (III.11a-f).

The diagonal terms of the matrix $(uc)_{i.j}$ are calculated after completion of the loops J and I. Each diagonal term is calculated by algebraically summing row elements in the same row as the desired diagonal term. To show this, consider the equation (III.12)

$$[uc]\{u\}=[uR]\{\hat{F}\}$$

If rigid body displacement is applied to the body so that

$$u_1^{(1)}=u_1^{(2)}=u_1^{(3)}=u_1^{(4)}\ldots=u_1^{(n)}$$
$$u_2^{(1)}=u_2^{(2)}=u_2^{(3)}=u_2^{(4)}\ldots=u_2^{(n)}$$

then there are no stresses. Therefore, $\{\hat{F}\}$ is a null vector that

$$uc_{(2n-1)(2n-1)}= -\sum_{\substack{m=1\\m\neq n}}^{N} uc_{(2n-1)(2m-1)}$$

$$uc_{(2n-1)(2n)}= -\sum_{\substack{m=1\\m\neq n}}^{N} uc_{(2n-1)(2m)}$$

$$uc_{(2n)(2n-1)}= -\sum_{\substack{m=1\\m\neq n}}^{N} uc_{(2n)(2m-1)}$$

$$uc_{(2n)(2n)}= -\sum_{\substack{m=1\\m\neq n}}^{N} uc_{(2n)(2m)}.$$

III.9.4 TRANSFORMATION OF COEFFICIENT MATRIX uR

This part of the program performs the operation

$$[uR]^{*}=[uR][\Gamma]^{-1}$$

The most efficient way to obtain the matrix $[uR]^{*}$ is not to create $[\Gamma]^{-1}$ and postmultiply by $[uR]$, but to modify $[uR]$ itself.

The alogrithm for this modification is better understood by examining the example of a 6 x 6 matrix:

$$\begin{bmatrix} [uR_{11}]^{*} & [uR_{12}]^{*} & [uR_{13}]^{*} \\ [uR_{21}]^{*} & [uR_{22}]^{*} & [uR_{23}]^{*} \\ [uR_{31}]^{*} & [uR_{32}]^{*} & [uR_{33}]^{*} \end{bmatrix} = \begin{bmatrix} uR_{11} & uR_{12} & uR_{13} \\ uR_{12} & uR_{22} & uR_{23} \\ uR_{31} & uR_{32} & uR_{33} \end{bmatrix} \begin{bmatrix} I & -I & I \\ I & I & -I \\ -I & I & I \end{bmatrix}$$

in which the entries $[uR_{2i}]^{*}$ are

$$[uR_{21}]^{*}= [uR_{21}] + [uR_{22}] - [uR_{23}]$$

$$[uR_{22}]^{*}= -[uR_{21}] + [uR_{22}] + [uR_{23}]$$

$$[uR_{22}]^{*}= -([uR_{21}] + [uR_{22}] - [uR_{23}])+2[uR_{22}]$$

or $$[uR_{22}]^{*}= -[uR_{21}] +2[uR_{22}]^{*}$$

In general it can be shown that

$$[uR_{k1}]^{*}=[uR_{k1}]+[uR_{k2}]-[uR]_{k3}]+[uR]_{k4}-\ldots[uR_{kn}] \qquad \text{(III.17)}$$

and

$$[uR_{ki}]^{*}=-[uR_{k(i-1)}]^{*}+2[uR_{ki}] \quad , \; i=2,\ldots,N. \qquad \text{(III.18)}$$

The flowchart for this modification is illustrated in Figure III.7. In the first loop, the first two columns of $[uR]^*$ are obtained using equation (III.17). In the second loop, the other entries of $[uR]^*$ are generated using equation (III.18).

III.9.5 CONSOLIDATION OF KNOWN AND UNKNOWNS BOUNDARY VALUES

This part of the program consists of a reorganization of the system of equations (III.16) so that all unknowns go to the left hand side and all knowns go to the right hand side of the equations. This procedure is illustrated with the flow chart of Figure III.8. The array NTBC(K,I) which specifies the boundary condition at node K, direction I, is used to decide when to interchange columns, i.e. if NTBC(K,I)=0, there is a specified force, and no interchange is necessary.

The final vector of knowns is multiplied by the matrix of coefficients to produce the right-hand side of the final system of equations.

The final system of equations is then solved by Guass elimination procedure.

III.9.6 CALCULATIONS OF STRESSES AND DISPLACEMENTS AT THE ELEMENTS AND NODES

In this part of the program, the stresses at each element and displacements at each nodes are evaluated.

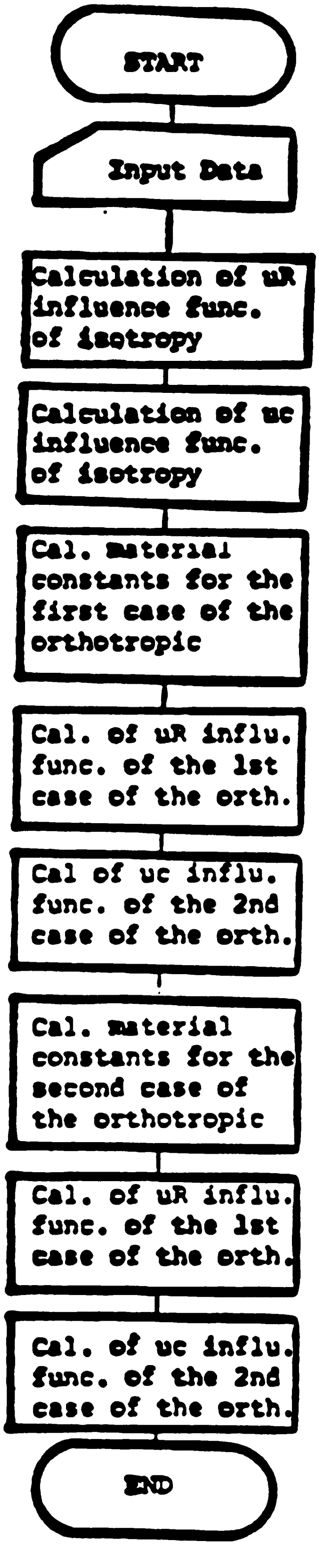


Figure III.1 Flowchart of the program TEST

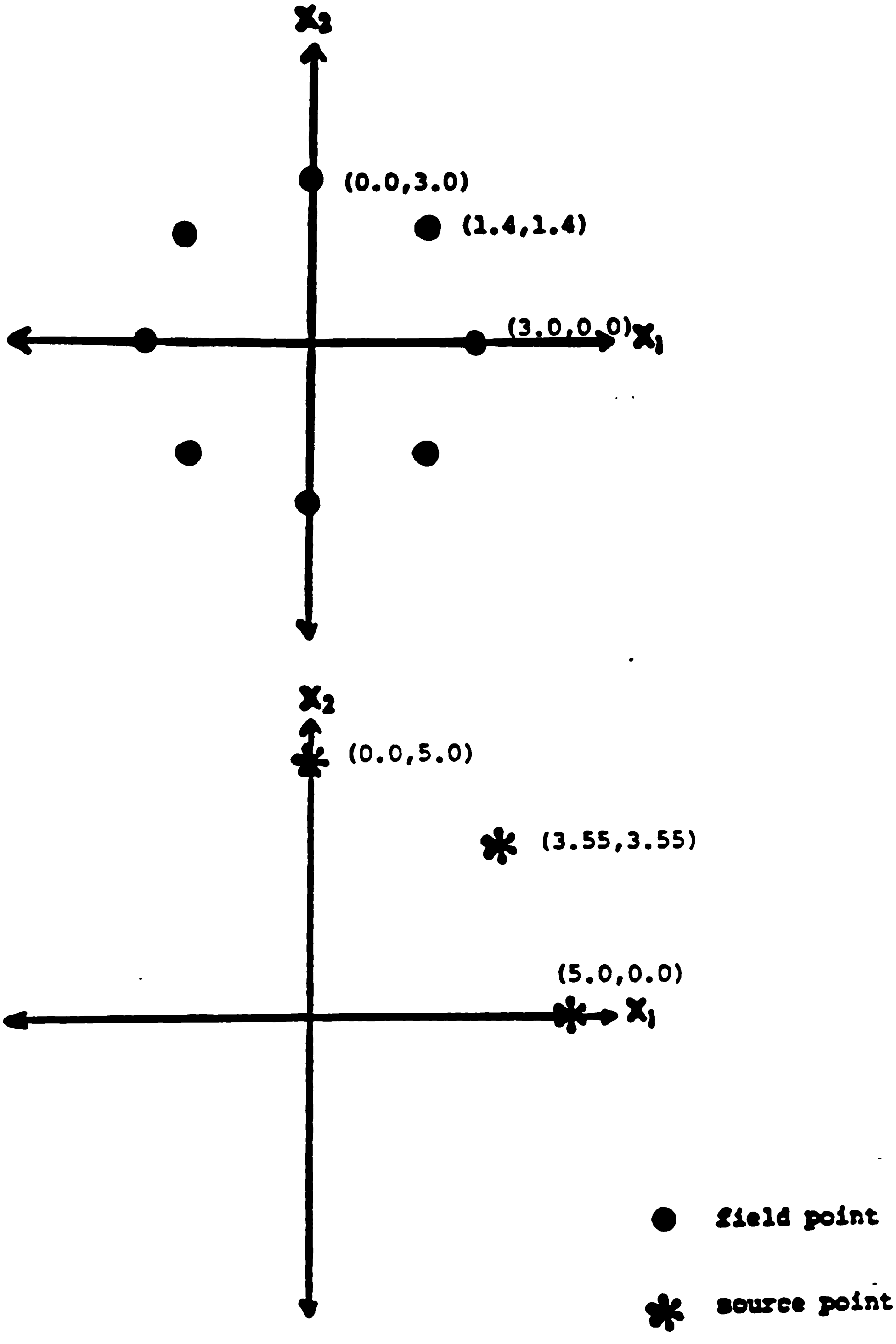


Figure III.2 Field and source points selected for the example problem

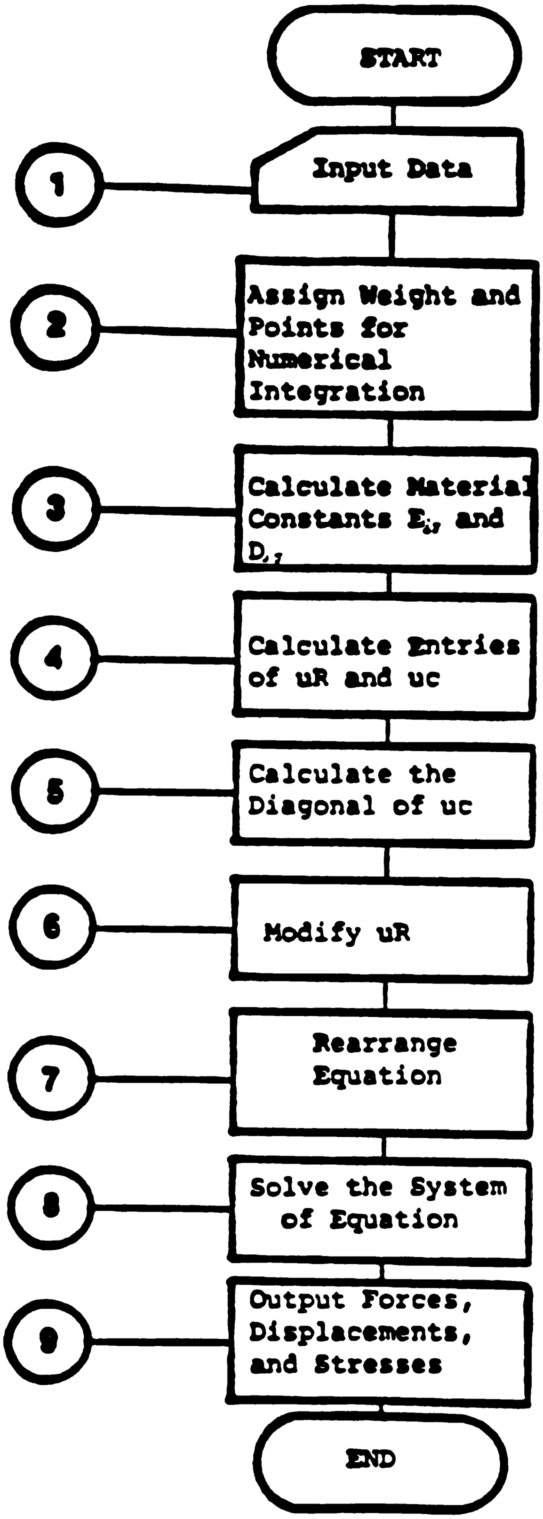


Figure III.3 Flowchart of the program ORTHO.CASE1 and ORTHO.CASE2

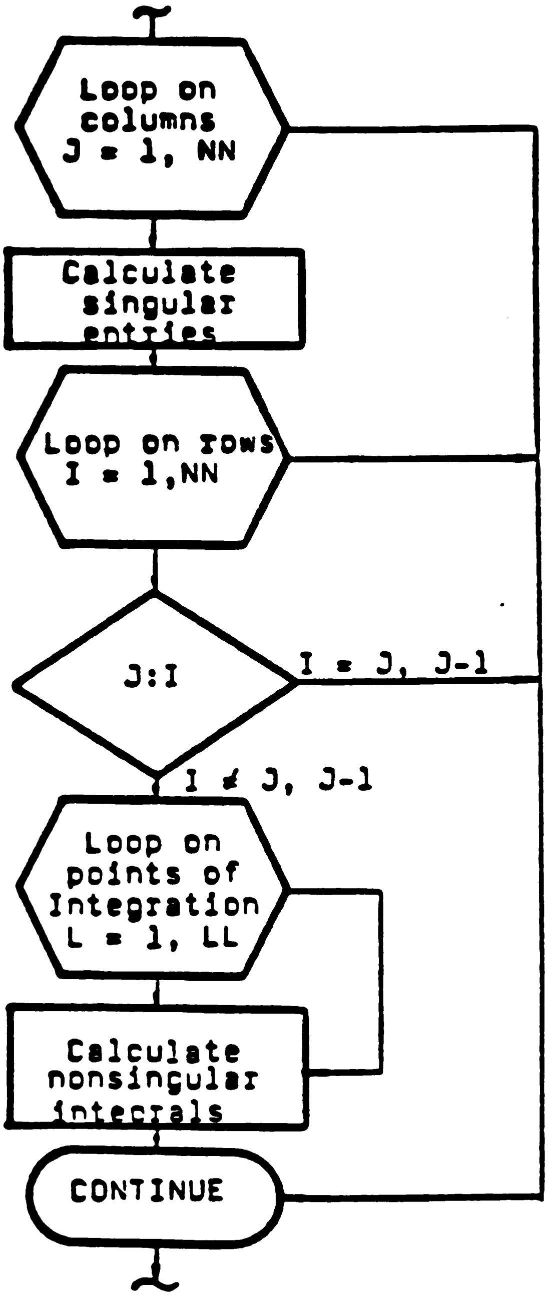


Figure III.4 Flowchart for the calculations of the coefficient matrix

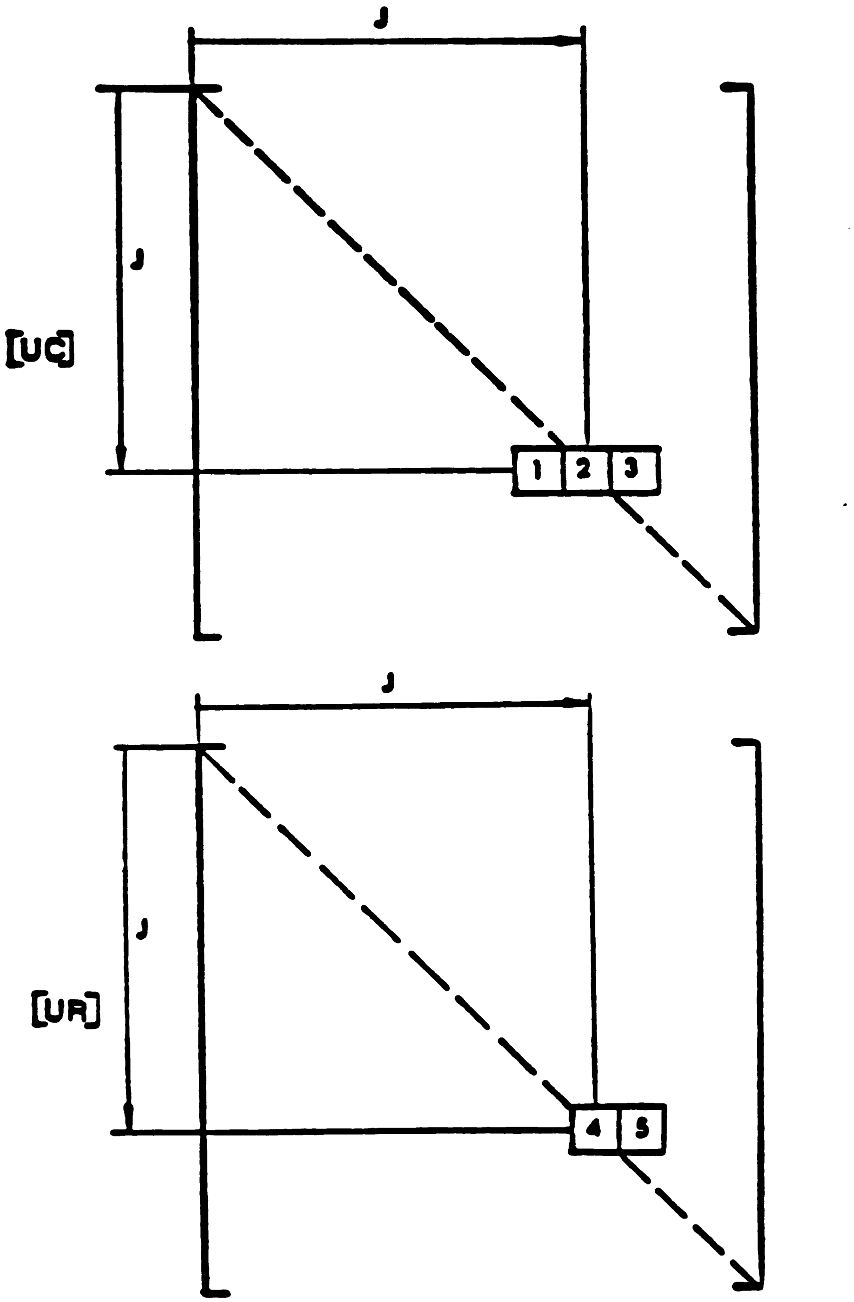
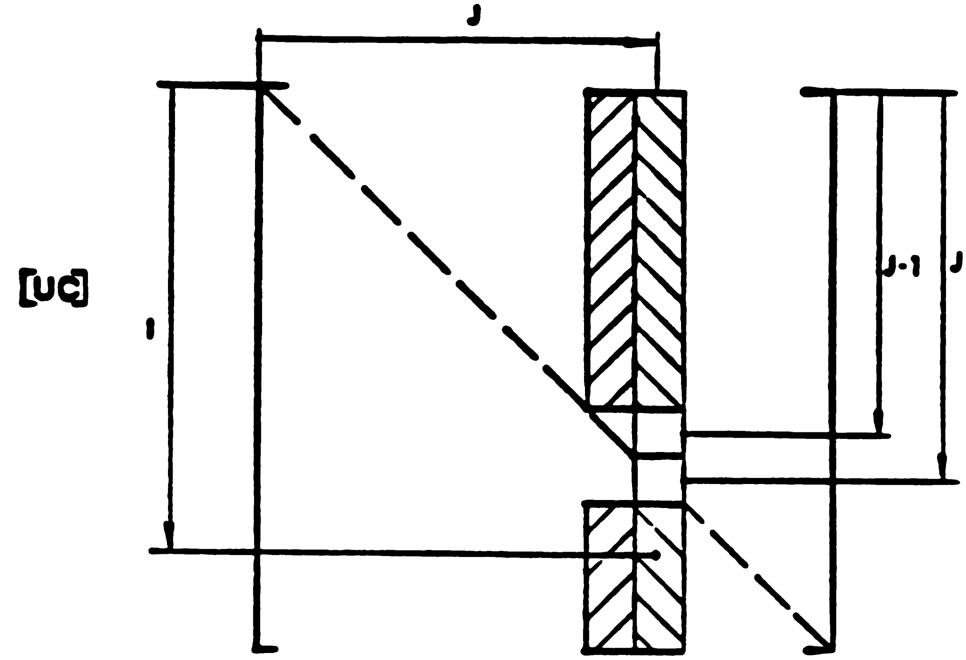


Figure III.5 Singular contribution in [uc], [uR]



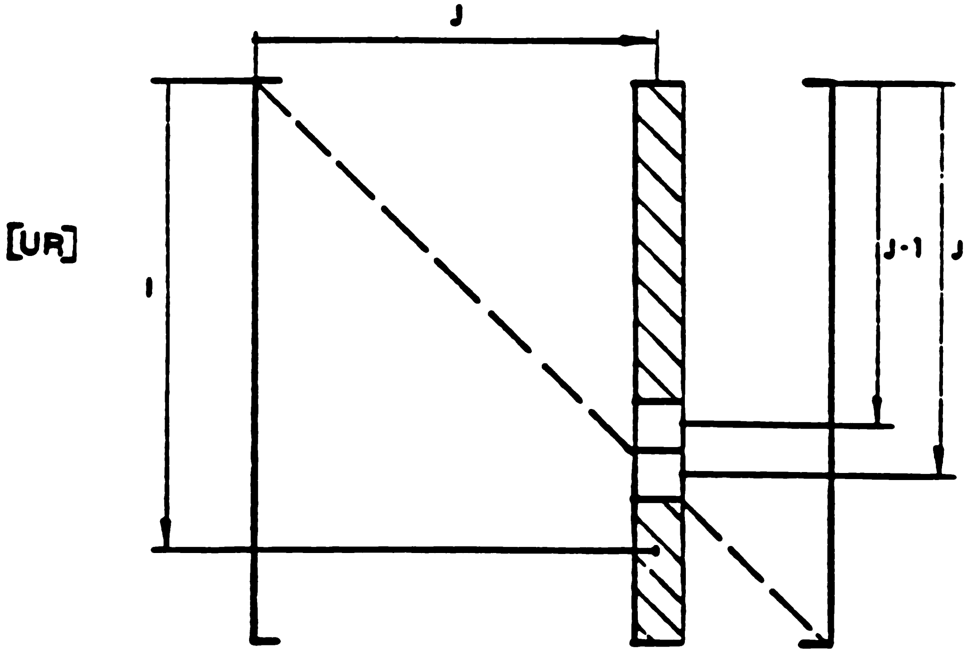


Figure III.6 Non-singular contribution in [uc], [uR]

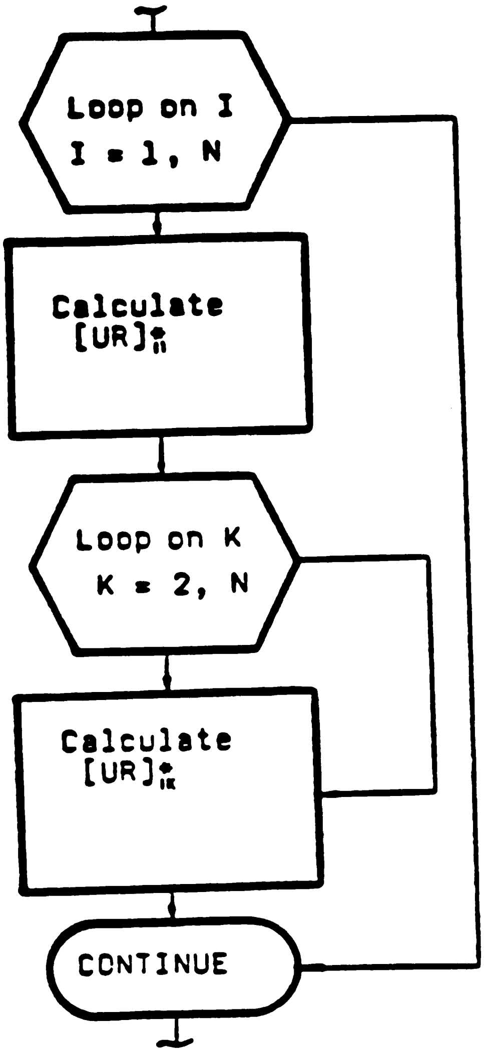


Figure III.7 Flowchart for modification of uR

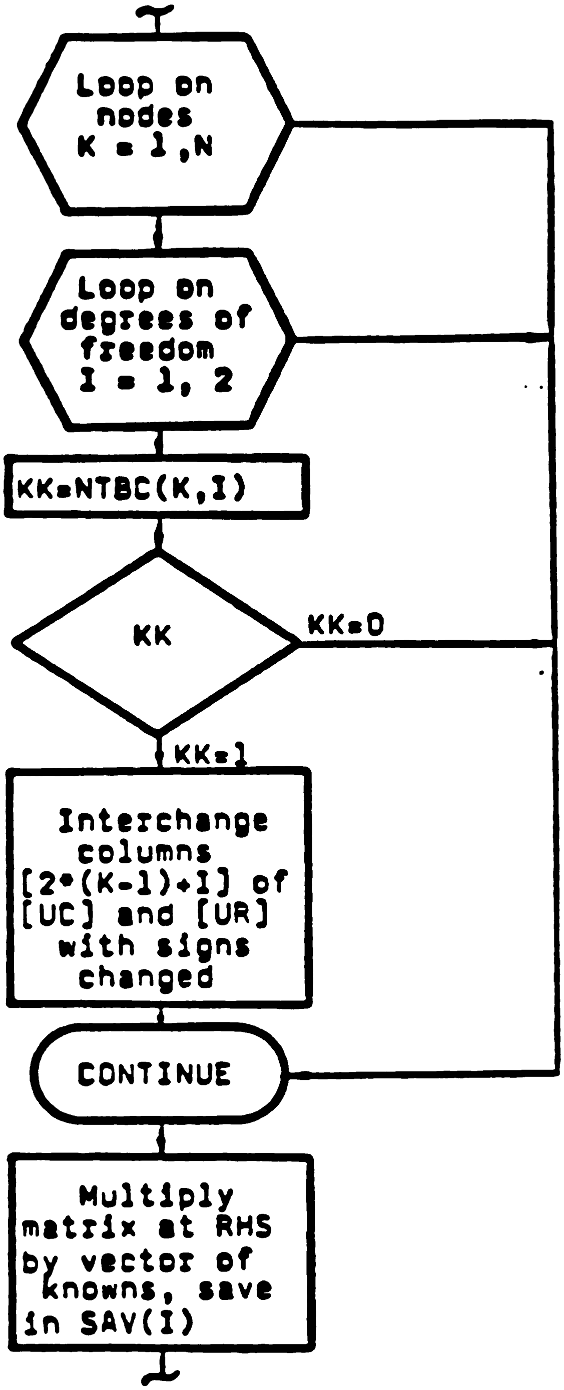


Figure III.8 Flowchart for rearranging equations into final form

TABLE III.1 SOME COMPOSITE MATERIALS WIDELY USED IN INDUSTRY

MATERIAL	EX (GPA)	EY (GPA)	ES (GPA)	POISSON'S RATIO IN X-DIRECTION
ALUMINA/EPOXY	230.0	21.0	7.0	0.28
BORON/EPOXY	210.0	19.0	4.8	0.25
BORON/EPOXY TYPE B(4)/5505	204.0	18.5	5.6	0.23
ULTRA-LIGHT MODULUS GRAPHITE/EPOXY	290.0	6.2	4.8	0.25
HIGH-MODULUS GRAPHITE/EPOXY	220.0	6.9	4.8	0.25
GRAPHITE/EPOXY TYPE T300/5208	181.0	10.3	7.17	0.28
GRAPHITE/EPOXY TYPE AS/3501	138.0	8.96	7.17	0.30
HIGH-STRENGTH GRAPHITE/EPOXY	145.0	10.0	4.8	0.25
KELVAR 49 ARAMID/EPOXY	76.0	5.5	2.1	0.34
S-GLASS/EPOXY	55.0	16.0	7.6	0.28
E-GLASS/EPOXY	45.0	12.0	5.5	0.28
GLASS/EPOXY TYPE SCOTCHPLY 1002	38.6	8.3	4.5	0.26
CONSTITUENT PLIES				
C65	39.8	10.3	4.5	0.30
R65	18.5	18.5	7.2	0.29
R75	33.5	17.7	9.7	0.39
C75	42.1	15.2	6.9	0.33

TABLE III.2 THE VALUES OF THE INFLUENCE FUNCTION UR
FOR THE SOURCE POINT X=0.00 Y=5.00

FIELD POINTS COORDINATE

UR ISOTROPIC INFLUENCE FUNCTION

X	Y				
3.000	0.000	UR11=-0.44940	UR12=-0.05486	UR21=-0.05486	UR22=-0.39089
1.400	1.400	UR11=-0.35332	UR12=-0.04200	UR21=-0.04200	UR22=-0.26165
0.400	3.000	UR11=-0.19019	UR12=-0.02391	UR21=-0.02391	UR22=-0.07542
0.200	3.000	UR11=-0.18974	UR12=-0.01231	UR21=-0.01231	UR22=-0.06786
0.000	3.000	UR11=-0.18961	UR12= 0.00000	UR21= 0.00000	UR22=-0.06527
-0.200	3.000	UR11=-0.18974	UR12= 0.01231	UR21= 0.01231	UR22=-0.06786
-0.400	3.000	UR11=-0.19019	UR12= 0.02391	UR21= 0.02391	UR22=-0.07542
-1.400	1.400	UR11=-0.35332	UR12= 0.04200	UR21= 0.04200	UR22=-0.26165
-3.000	0.000	UR11=-0.44940	UR12= 0.05486	UR21= 0.05486	UR22=-0.39089
-1.400	-1.400	UR11=-0.50850	UR12= 0.02596	UR21= 0.02596	UR22=-0.39552
-0.400	-3.000	UR11=-0.56886	UR12= 0.00620	UR21= 0.00620	UR22=-0.44514
-0.200	-3.000	UR11=-0.56883	UR12= 0.00311	UR21= 0.00311	UR22=-0.44465
0.000	-3.000	UR11=-0.56883	UR12= 0.00000	UR21= 0.00000	UR22=-0.44449
0.200	-3.000	UR11=-0.56883	UR12=-0.00311	UR21=-0.00311	UR22=-0.44465
0.400	-3.000	UR11=-0.56886	UR12=-0.00620	UR21=-0.00620	UR22=-0.44514
1.400	-1.400	UR11=-0.50850	UR12=-0.02596	UR21=-0.02596	UR22=-0.39552

UR CASE 1 INFLUENCE FUNCTION

3.000	0.000	UR11=-0.57668	UR12=-0.05482	UR21=-0.05482	UR22=-0.39170
1.400	1.400	UR11=-0.48032	UR12=-0.04200	UR21=-0.04200	UR22=-0.26178
0.400	3.000	UR11=-0.31666	UR12=-0.02392	UR21=-0.02392	UR22=-0.07469
0.200	3.000	UR11=-0.31621	UR12=-0.01232	UR21=-0.01232	UR22=-0.06703
0.000	3.000	UR11=-0.31608	UR12= 0.00000	UR21= 0.00000	UR22=-0.06440
-0.200	3.000	UR11=-0.31621	UR12= 0.01232	UR21= 0.01232	UR22=-0.06703
-0.400	3.000	UR11=-0.31666	UR12= 0.02392	UR21= 0.02392	UR22=-0.07469
-1.400	1.400	UR11=-0.48032	UR12= 0.04200	UR21= 0.04200	UR22=-0.26178
-3.000	0.000	UR11=-0.57668	UR12= 0.05482	UR21= 0.05482	UR22=-0.39170
-1.400	-1.400	UR11=-0.63603	UR12= 0.02597	UR21= 0.02597	UR22=-0.39588
-0.400	-3.000	UR11=-0.69660	UR12= 0.00621	UR21= 0.00621	UR22=-0.44555
-0.200	-3.000	UR11=-0.69657	UR12= 0.00311	UR21= 0.00311	UR22=-0.44505
0.000	-3.000	UR11=-0.69656	UR12= 0.00000	UR21= 0.00000	UR22=-0.44489
0.200	-3.000	UR11=-0.69657	UR12=-0.00311	UR21=-0.00311	UR22=-0.44505
0.400	-3.000	UR11=-0.69660	UR12=-0.00621	UR21=-0.00621	UR22=-0.44555
1.400	-1.400	UR11=-0.63603	UR12=-0.02597	UR21=-0.02597	UR22=-0.39588

UR CASE 2 INFLUENCE FUNCTION

3.000	0.000	UR11=-0.57374	UR12=-0.05486	UR21=-0.05486	UR22=-0.39089
1.400	1.400	UR11=-0.47766	UR12=-0.04200	UR21=-0.04200	UR22=-0.26165
0.400	3.000	UR11=-0.31453	UR12=-0.02391	UR21=-0.02391	UR22=-0.07542
0.200	3.000	UR11=-0.31408	UR12=-0.01231	UR21=-0.01231	UR22=-0.06786
0.000	3.000	UR11=-0.31395	UR12= 0.00000	UR21= 0.00000	UR22=-0.06527
-0.200	3.000	UR11=-0.31408	UR12= 0.01231	UR21= 0.01231	UR22=-0.06786
-0.400	3.000	UR11=-0.31453	UR12= 0.02391	UR21= 0.02391	UR22=-0.07542
-1.400	1.400	UR11=-0.47766	UR12= 0.04200	UR21= 0.04200	UR22=-0.26165
-3.000	0.000	UR11=-0.57374	UR12= 0.05486	UR21= 0.05486	UR22=-0.39089
-1.400	-1.400	UR11=-0.63284	UR12= 0.02596	UR21= 0.02596	UR22=-0.39552
-0.400	-3.000	UR11=-0.69320	UR12= 0.00620	UR21= 0.00620	UR22=-0.44514
-0.200	-3.000	UR11=-0.69317	UR12= 0.00311	UR21= 0.00311	UR22=-0.44465
0.000	-3.000	UR11=-0.69317	UR12= 0.00000	UR21= 0.00000	UR22=-0.44449
0.200	-3.000	UR11=-0.69317	UR12=-0.00311	UR21=-0.00311	UR22=-0.44465
0.400	-3.000	UR11=-0.69320	UR12=-0.00620	UR21=-0.00620	UR22=-0.44514
1.400	-1.400	UR11=-0.63284	UR12=-0.02596	UR21=-0.02596	UR22=-0.39552

TABLE III.3 THE VALUES OF THE INFLUENCE FUNCTION UR
FOR THE SOURCE POINT X=5.00 Y=0.00

UR ISOTROPIC INFLUENCE FUNCTION

FIELD POINTS COORDINATE X	Y				
3.000	0.000	UR11=-0.06527	UR12= 0.00000	UR21= 0.00000	UR22=-0.18961
1.400	1.400	UR11=-0.26165	UR12=-0.04200	UR21=-0.04200	UR22=-0.35332
0.400	3.000	UR11=-0.37869	UR12=-0.05689	UR21=-0.05689	UR22=-0.42882
0.200	3.000	UR11=-0.38478	UR12=-0.05588	UR21=-0.05588	UR22=-0.43927
0.000	3.000	UR11=-0.39089	UR12=-0.05486	UR21=-0.05486	UR22=-0.44940
-0.200	3.000	UR11=-0.39699	UR12=-0.05382	UR21=-0.05382	UR22=-0.45923
-0.400	3.000	UR11=-0.40309	UR12=-0.05279	UR21=-0.05279	UR22=-0.46878
-1.400	1.400	UR11=-0.39552	UR12=-0.02596	UR21=-0.02596	UR22=-0.50850
-3.000	0.000	UR11=-0.44449	UR12= 0.00000	UR21= 0.00000	UR22=-0.56883
-1.400	-1.400	UR11=-0.39552	UR12= 0.02596	UR21= 0.02596	UR22=-0.50850
-0.400	-3.000	UR11=-0.40309	UR12= 0.05279	UR21= 0.05279	UR22=-0.46878
-0.200	-3.000	UR11=-0.39699	UR12= 0.05382	UR21= 0.05382	UR22=-0.45923
0.000	-3.000	UR11=-0.39089	UR12= 0.05486	UR21= 0.05486	UR22=-0.44940
0.200	-3.000	UR11=-0.38478	UR12= 0.05588	UR21= 0.05588	UR22=-0.43927
0.400	-3.000	UR11=-0.37869	UR12= 0.05689	UR21= 0.05689	UR22=-0.42882
1.400	-1.400	UR11=-0.26165	UR12= 0.04200	UR21= 0.04200	UR22=-0.35332

UR CASE 1 INFLUENCE FUNCTION

3.000	0.000	UR11=-0.19024	UR12= 0.00000	UR21= 0.00000	UR22=-0.19024
1.400	1.400	UR11=-0.38762	UR12=-0.04200	UR21=-0.04200	UR22=-0.35448
0.400	3.000	UR11=-0.50535	UR12=-0.05685	UR21=-0.05685	UR22=-0.43018
0.200	3.000	UR11=-0.51144	UR12=-0.05585	UR21=-0.05585	UR22=-0.44067
0.000	3.000	UR11=-0.51754	UR12=-0.05482	UR21=-0.05482	UR22=-0.45084
-0.200	3.000	UR11=-0.52364	UR12=-0.05379	UR21=-0.05379	UR22=-0.46072
-0.400	3.000	UR11=-0.52973	UR12=-0.05276	UR21=-0.05276	UR22=-0.47029
-1.400	1.400	UR11=-0.52172	UR12=-0.02597	UR21=-0.02597	UR22=-0.51019
-3.000	0.000	UR11=-0.57073	UR12= 0.00000	UR21= 0.00000	UR22=-0.57073
-1.400	-1.400	UR11=-0.52172	UR12= 0.02597	UR21= 0.02597	UR22=-0.51019
-0.400	-3.000	UR11=-0.52973	UR12= 0.05276	UR21= 0.05276	UR22=-0.47029
-0.200	-3.000	UR11=-0.52364	UR12= 0.05379	UR21= 0.05379	UR22=-0.46072
0.000	-3.000	UR11=-0.51754	UR12= 0.05482	UR21= 0.05482	UR22=-0.45084
0.200	-3.000	UR11=-0.51144	UR12= 0.05585	UR21= 0.05585	UR22=-0.44067
0.400	-3.000	UR11=-0.50535	UR12= 0.05685	UR21= 0.05685	UR22=-0.43018
1.400	-1.400	UR11=-0.38762	UR12= 0.04200	UR21= 0.04200	UR22=-0.35448

UR CASE 2 INFLUENCE FUNCTION

3.000	0.000	UR11=-0.18961	UR12= 0.00000	UR21= 0.00000	UR22=-0.18961
1.400	1.400	UR11=-0.38599	UR12=-0.04200	UR21=-0.04200	UR22=-0.35332
0.400	3.000	UR11=-0.50303	UR12=-0.05689	UR21=-0.05689	UR22=-0.42882
0.200	3.000	UR11=-0.50912	UR12=-0.05588	UR21=-0.05588	UR22=-0.43927
0.000	3.000	UR11=-0.51523	UR12=-0.05486	UR21=-0.05486	UR22=-0.44940
-0.200	3.000	UR11=-0.52133	UR12=-0.05382	UR21=-0.05382	UR22=-0.45923
-0.400	3.000	UR11=-0.52743	UR12=-0.05279	UR21=-0.05279	UR22=-0.46878
-1.400	1.400	UR11=-0.51986	UR12=-0.02596	UR21=-0.02596	UR22=-0.50850
-3.000	0.000	UR11=-0.56883	UR12= 0.00000	UR21= 0.00000	UR22=-0.56883
-1.400	-1.400	UR11=-0.51986	UR12= 0.02596	UR21= 0.02596	UR22=-0.50850
-0.400	-3.000	UR11=-0.52743	UR12= 0.05279	UR21= 0.05279	UR22=-0.46878
-0.200	-3.000	UR11=-0.52133	UR12= 0.05382	UR21= 0.05382	UR22=-0.45923
0.000	-3.000	UR11=-0.51523	UR12= 0.05486	UR21= 0.05486	UR22=-0.44940
0.200	-3.000	UR11=-0.50912	UR12= 0.05588	UR21= 0.05588	UR22=-0.43927
0.400	-3.000	UR11=-0.50303	UR12= 0.05689	UR21= 0.05689	UR22=-0.42882
1.400	-1.400	UR11=-0.38599	UR12= 0.04200	UR21= 0.04200	UR22=-0.35332

TABLE III.4 THE VALUES OF THE INFLUENCE FUNCTION UR
FOR THE SOURCE POINT X=3.55 Y=3.55

UR ISOTROPIC INFLUENCE FUNCTION

FIELD POINTS COORDINATE X	Y				
3.000	0.000	UR11=-0.34690	UR12= 0.01881	UR21= 0.01881	UR22=-0.22839
1.400	1.400	UR11=-0.24203	UR12= 0.06217	UR21= 0.06217	UR22=-0.24203
0.400	3.000	UR11=-0.19732	UR12= 0.02107	UR21= 0.02107	UR22=-0.31430
0.200	3.000	UR11=-0.21327	UR12= 0.01988	UR21= 0.01988	UR22=-0.33108
0.000	3.000	UR11=-0.22839	UR12= 0.01881	UR21= 0.01881	UR22=-0.34690
-0.200	3.000	UR11=-0.24275	UR12= 0.01785	UR21= 0.01785	UR22=-0.36186
-0.400	3.000	UR11=-0.25643	UR12= 0.01698	UR21= 0.01698	UR22=-0.37604
-1.400	1.400	UR11=-0.35654	UR12= 0.04543	UR21= 0.04543	UR22=-0.44141
-3.000	0.000	UR11=-0.45324	UR12= 0.05209	UR21= 0.05209	UR22=-0.52112
-1.400	-1.400	UR11=-0.47014	UR12= 0.06217	UR21= 0.06217	UR22=-0.47014
-0.400	-3.000	UR11=-0.52339	UR12= 0.05499	UR21= 0.05499	UR22=-0.46537
-0.200	-3.000	UR11=-0.52220	UR12= 0.05361	UR21= 0.05361	UR22=-0.45925
0.000	-3.000	UR11=-0.52112	UR12= 0.05209	UR21= 0.05209	UR22=-0.45324
0.200	-3.000	UR11=-0.52012	UR12= 0.05041	UR21= 0.05041	UR22=-0.44735
0.400	-3.000	UR11=-0.51922	UR12= 0.04856	UR21= 0.04856	UR22=-0.44160
1.400	-1.400	UR11=-0.44141	UR12= 0.04543	UR21= 0.04543	UR22=-0.35654

UR CASE1 INFLUENCE FUNCTION

3.000	0.000	UR11=-0.47389	UR12= 0.01882	UR21= 0.01882	UR22=-0.22813
1.400	1.400	UR11=-0.36848	UR12= 0.06211	UR21= 0.06211	UR22=-0.24264
0.400	3.000	UR11=-0.32281	UR12= 0.02108	UR21= 0.02108	UR22=-0.31534
0.200	3.000	UR11=-0.33881	UR12= 0.01989	UR21= 0.01989	UR22=-0.33218
0.000	3.000	UR11=-0.35397	UR12= 0.01882	UR21= 0.01882	UR22=-0.34806
-0.200	3.000	UR11=-0.36838	UR12= 0.01786	UR21= 0.01786	UR22=-0.36306
-0.400	3.000	UR11=-0.38209	UR12= 0.01699	UR21= 0.01699	UR22=-0.37729
-1.400	1.400	UR11=-0.48288	UR12= 0.04543	UR21= 0.04543	UR22=-0.44286
-3.000	0.000	UR11=-0.58004	UR12= 0.05206	UR21= 0.05206	UR22=-0.52281
-1.400	-1.400	UR11=-0.59735	UR12= 0.06211	UR21= 0.06211	UR22=-0.47152
-0.400	-3.000	UR11=-0.65092	UR12= 0.05495	UR21= 0.05495	UR22=-0.46644
-0.200	-3.000	UR11=-0.64974	UR12= 0.05358	UR21= 0.05358	UR22=-0.46027
0.000	-3.000	UR11=-0.64865	UR12= 0.05206	UR21= 0.05206	UR22=-0.45420
0.200	-3.000	UR11=-0.64766	UR12= 0.05039	UR21= 0.05039	UR22=-0.44825
0.400	-3.000	UR11=-0.64676	UR12= 0.04855	UR21= 0.04855	UR22=-0.44244
1.400	-1.400	UR11=-0.56870	UR12= 0.04543	UR21= 0.04543	UR22=-0.35704

UR CASE 2 INFLUENCE FUNCTION

3.000	0.000	UR11=-0.47124	UR12= 0.01881	UR21= 0.01881	UR22=-0.22839
1.400	1.400	UR11=-0.36637	UR12= 0.06217	UR21= 0.06217	UR22=-0.24203
0.400	3.000	UR11=-0.32166	UR12= 0.02107	UR21= 0.02107	UR22=-0.31430
0.200	3.000	UR11=-0.33761	UR12= 0.01988	UR21= 0.01988	UR22=-0.33108
0.000	3.000	UR11=-0.35273	UR12= 0.01881	UR21= 0.01881	UR22=-0.34690
-0.200	3.000	UR11=-0.36709	UR12= 0.01785	UR21= 0.01785	UR22=-0.36186
-0.400	3.000	UR11=-0.38077	UR12= 0.01698	UR21= 0.01698	UR22=-0.37604
-1.400	1.400	UR11=-0.48088	UR12= 0.04543	UR21= 0.04543	UR22=-0.44141
-3.000	0.000	UR11=-0.57758	UR12= 0.05209	UR21= 0.05209	UR22=-0.52112
-1.400	-1.400	UR11=-0.59448	UR12= 0.06217	UR21= 0.06217	UR22=-0.47014
-0.400	-3.000	UR11=-0.64773	UR12= 0.05499	UR21= 0.05499	UR22=-0.46537
-0.200	-3.000	UR11=-0.64654	UR12= 0.05361	UR21= 0.05361	UR22=-0.45925
0.000	-3.000	UR11=-0.64546	UR12= 0.05209	UR21= 0.05209	UR22=-0.45324
0.200	-3.000	UR11=-0.64446	UR12= 0.05041	UR21= 0.05041	UR22=-0.44735
0.400	-3.000	UR11=-0.64356	UR12= 0.04856	UR21= 0.04856	UR22=-0.44160
1.400	-1.400	UR11=-0.56575	UR12= 0.04543	UR21= 0.04543	UR22=-0.35654

TABLE III.5 THE VALUES OF THE INFLUENCE FUNCTION UC
FOR THE SOURCE POINT X=0.00 Y=5.00

FIELD POINTS COORDINATE

UC ISOTROPIC INFLUENCE FUNCTION

X	Y	UC11	UC12	UC21	UC22
3.000	0.000	0.03792	-0.03326	-0.02599	0.06952
1.400	1.400	-0.02056	-0.01129	0.04350	-0.05570
0.400	3.000	-0.02333	-0.04149	0.06801	-0.08697
0.200	3.000	-0.06104	0.01359	0.02541	-0.25412
0.000	3.000	-0.05968	0.00000	0.00000	-0.25863
-0.200	3.000	-0.06104	-0.01359	-0.02541	-0.25412
-0.400	3.000	-0.06474	-0.02531	-0.04826	-0.24132
-1.400	1.400	-0.00829	0.02387	-0.03685	-0.02246
-3.000	0.000	-0.01181	0.01026	-0.02871	-0.02166
-1.400	-1.400	0.01839	0.01990	0.00231	0.06673
-0.400	-3.000	0.00859	0.01364	-0.01081	0.03685
-0.200	-3.000	0.01494	0.00087	0.00161	0.06459
0.000	-3.000	0.01492	0.00000	0.00000	0.06466
0.200	-3.000	0.01494	-0.00087	-0.00161	0.06459
0.400	-3.000	0.01501	-0.00173	-0.00322	0.06437
1.400	-1.400	0.01467	-0.02189	0.00417	0.05325

UC CASE 1 INFLUENCE FUNCTION

X	Y	UC11	UC12	UC21	UC22
3.000	0.000	0.03782	-0.03317	-0.02600	0.06947
1.400	1.400	-0.02051	-0.01126	0.04347	-0.05577
0.400	3.000	-0.02328	-0.04137	0.06793	-0.08722
0.200	3.000	-0.06089	0.01360	0.02550	-0.25500
0.000	3.000	-0.05953	0.00000	0.00000	-0.25958
-0.200	3.000	-0.06089	-0.01360	-0.02550	-0.25500
-0.400	3.000	-0.06459	-0.02530	-0.04840	-0.24202
-1.400	1.400	-0.00827	0.02380	-0.03679	-0.02249
-3.000	0.000	-0.01178	0.01023	-0.02866	-0.02164
-1.400	-1.400	0.01835	0.01986	0.00239	0.06691
-0.400	-3.000	0.00857	0.01361	-0.01077	0.03698
-0.200	-3.000	0.01491	0.00087	0.00162	0.06482
0.000	-3.000	0.01488	0.00000	0.00000	0.06489
0.200	-3.000	0.01491	-0.00087	-0.00162	0.06482
0.400	-3.000	0.01497	-0.00173	-0.00323	0.06460
1.400	-1.400	0.01464	-0.02185	0.00410	0.05339

UC CASE 2 INFLUENCE FUNCTION

X	Y	UC11	UC12	UC21	UC22
3.000	0.000	0.03792	-0.03326	-0.02599	0.06952
1.400	1.400	-0.02056	-0.01129	0.04350	-0.05570
0.400	3.000	-0.02333	-0.04149	0.06801	-0.08697
0.200	3.000	-0.06104	0.01359	0.02541	-0.25412
0.000	3.000	-0.05968	0.00000	0.00000	-0.25863
-0.200	3.000	-0.06104	-0.01359	-0.02541	-0.25412
-0.400	3.000	-0.06474	-0.02531	-0.04826	-0.24132
-1.400	1.400	-0.00829	0.02387	-0.03685	-0.02246
-3.000	0.000	-0.01181	0.01026	-0.02871	-0.02166
-1.400	-1.400	0.01839	0.01990	0.00231	0.06673
-0.400	-3.000	0.00859	0.01364	-0.01081	0.03685
-0.200	-3.000	0.01494	0.00087	0.00161	0.06459
0.000	-3.000	0.01492	0.00000	0.00000	0.06466
0.200	-3.000	0.01494	-0.00087	-0.00161	0.06459
0.400	-3.000	0.01501	-0.00173	-0.00322	0.06437
1.400	-1.400	0.01467	-0.02189	0.00417	0.05325

TABLE III.6 THE VALUES OF THE INFLUENCE FUNCTION UC
FOR THE SOURCE POINT X=5.00 Y=0.00

FIELD POINTS COORDINATE

UC ISOTROPIC INFLUENCE FUNCTION

X	Y	UC11	UC12	UC21	UC22
3.000	0.000	UC11=-0.17031	UC12=-0.04492	UC21= 0.04492	UC22=-0.03930
1.400	1.400	UC11=-0.04104	UC12= 0.04092	UC21=-0.01719	UC22=-0.01515
0.400	3.000	UC11=-0.03053	UC12= 0.03367	UC21=-0.00577	UC22=-0.01824
0.200	3.000	UC11= 0.03797	UC12= 0.00114	UC21=-0.03463	UC22= 0.02164
0.000	3.000	UC11= 0.03635	UC12= 0.00207	UC21=-0.03304	UC22= 0.01983
-0.200	3.000	UC11= 0.03479	UC12= 0.00289	UC21=-0.03156	UC22= 0.01821
-0.400	3.000	UC11= 0.03329	UC12= 0.00361	UC21=-0.03017	UC22= 0.01676
-1.400	1.400	UC11= 0.07174	UC12=-0.00581	UC21=-0.01807	UC22= 0.01977
-3.000	0.000	UC11= 0.04258	UC12= 0.01123	UC21=-0.01123	UC22= 0.00983
-1.400	-1.400	UC11= 0.06126	UC12=-0.00064	UC21= 0.02103	UC22= 0.01688
-0.400	-3.000	UC11= 0.06845	UC12= 0.02631	UC21= 0.02830	UC22= 0.03447
-0.200	-3.000	UC11= 0.03479	UC12=-0.00289	UC21= 0.03156	UC22= 0.01821
0.000	-3.000	UC11= 0.03635	UC12=-0.00207	UC21= 0.03304	UC22= 0.01983
0.200	-3.000	UC11= 0.03797	UC12=-0.00114	UC21= 0.03463	UC22= 0.02164
0.400	-3.000	UC11= 0.03964	UC12=-0.00010	UC21= 0.03631	UC22= 0.02368
1.400	-1.400	UC11=-0.07202	UC12=-0.04558	UC21= 0.00395	UC22=-0.02658

UC CASE 1 INFLUENCE FUNCTION

X	Y	UC11	UC12	UC21	UC22
3.000	0.000	UC11=-0.17093	UC12=-0.04480	UC21= 0.04480	UC22=-0.03920
1.400	1.400	UC11=-0.04109	UC12= 0.04087	UC21=-0.01714	UC22=-0.01511
0.400	3.000	UC11=-0.03050	UC12= 0.03361	UC21=-0.00576	UC22=-0.01819
0.200	3.000	UC11= 0.03793	UC12= 0.00110	UC21=-0.03453	UC22= 0.02158
0.000	3.000	UC11= 0.03632	UC12= 0.00202	UC21=-0.03295	UC22= 0.01977
-0.200	3.000	UC11= 0.03477	UC12= 0.00284	UC21=-0.03148	UC22= 0.01816
-0.400	3.000	UC11= 0.03327	UC12= 0.00356	UC21=-0.03009	UC22= 0.01672
-1.400	1.400	UC11= 0.07193	UC12=-0.00588	UC21=-0.01804	UC22= 0.01972
-3.000	0.000	UC11= 0.04273	UC12= 0.01120	UC21=-0.01120	UC22= 0.00980
-1.400	-1.400	UC11= 0.06143	UC12=-0.00056	UC21= 0.02099	UC22= 0.01684
-0.400	-3.000	UC11= 0.06843	UC12= 0.02633	UC21= 0.02823	UC22= 0.03438
-0.200	-3.000	UC11= 0.03477	UC12=-0.00284	UC21= 0.03148	UC22= 0.01816
0.000	-3.000	UC11= 0.03632	UC12=-0.00202	UC21= 0.03295	UC22= 0.01977
0.200	-3.000	UC11= 0.03793	UC12=-0.00110	UC21= 0.03453	UC22= 0.02158
0.400	-3.000	UC11= 0.03960	UC12=-0.00006	UC21= 0.03622	UC22= 0.02362
1.400	-1.400	UC11=-0.07210	UC12=-0.04557	UC21= 0.00393	UC22=-0.02652

UC CASE 2 INFLUENCE FUNCTION

X	Y	UC11	UC12	UC21	UC22
3.000	0.000	UC11=-0.17031	UC12=-0.04492	UC21= 0.04492	UC22=-0.03930
1.400	1.400	UC11=-0.04104	UC12= 0.04092	UC21=-0.01719	UC22=-0.01515
0.400	3.000	UC11=-0.03053	UC12= 0.03367	UC21=-0.00577	UC22=-0.01824
0.200	3.000	UC11= 0.03797	UC12= 0.00114	UC21=-0.03463	UC22= 0.02164
0.000	3.000	UC11= 0.03635	UC12= 0.00207	UC21=-0.03304	UC22= 0.01983
-0.200	3.000	UC11= 0.03479	UC12= 0.00289	UC21=-0.03156	UC22= 0.01821
-0.400	3.000	UC11= 0.03329	UC12= 0.00361	UC21=-0.03017	UC22= 0.01676
-1.400	1.400	UC11= 0.07174	UC12=-0.00581	UC21=-0.01807	UC22= 0.01977
-3.000	0.000	UC11= 0.04258	UC12= 0.01123	UC21=-0.01123	UC22= 0.00983
-1.400	-1.400	UC11= 0.06126	UC12=-0.00064	UC21= 0.02103	UC22= 0.01688
-0.400	-3.000	UC11= 0.06845	UC12= 0.02631	UC21= 0.02830	UC22= 0.03447
-0.200	-3.000	UC11= 0.03479	UC12=-0.00289	UC21= 0.03156	UC22= 0.01821
0.000	-3.000	UC11= 0.03635	UC12=-0.00207	UC21= 0.03304	UC22= 0.01983
0.200	-3.000	UC11= 0.03797	UC12=-0.00114	UC21= 0.03463	UC22= 0.02164
0.400	-3.000	UC11= 0.03964	UC12=-0.00010	UC21= 0.03631	UC22= 0.02368
1.400	-1.400	UC11=-0.07202	UC12=-0.04558	UC21= 0.00395	UC22=-0.02658

TABLE III.7 THE VALUES OF THE INFLUENCE FUNCTION UC
FOR THE SOURCE POINT X=3.55 Y=3.55

FIELD POINTS COORDINATE

UC ISOTROPIC INFLUENCE FUNCTION

X	Y	UC11	UC12	UC21	UC22
3.000	0.000	0.02303	-0.01468	0.03622	0.09090
1.400	1.400	-0.10446	-0.06267	-0.06790	-0.10446
0.400	3.000	-0.14646	-0.00549	-0.03358	-0.03800
0.200	3.000	-0.02419	0.03166	-0.03773	-0.00619
0.000	3.000	-0.02165	0.03027	-0.03540	-0.00548
-0.200	3.000	-0.01948	0.02897	-0.03335	-0.00489
-0.400	3.000	-0.01763	0.02777	-0.03152	-0.00439
-1.400	1.400	0.04768	0.03349	-0.00296	0.01916
-3.000	0.000	0.01263	0.02056	-0.01070	0.00620
-1.400	-1.400	0.04537	0.02722	0.02949	0.04537
-0.400	-3.000	0.02629	0.02758	0.01345	0.04794
-0.200	-3.000	0.02502	0.01187	0.02758	0.04818
0.000	-3.000	0.02475	0.01204	0.02730	0.05038
0.200	-3.000	0.02443	0.01213	0.02691	0.05261
0.400	-3.000	0.02407	0.01215	0.02639	0.05487
1.400	-1.400	0.00502	-0.01788	0.02587	0.01248

UC CASE 1 INFLUENCE FUNCTION

X	Y	UC11	UC12	UC21	UC22
3.000	0.000	0.02298	-0.01462	0.03621	0.09119
1.400	1.400	-0.10419	-0.06252	-0.06773	-0.10419
0.400	3.000	-0.14691	-0.00562	-0.03353	-0.03791
0.200	3.000	-0.02426	0.03156	-0.03764	-0.00618
0.000	3.000	-0.02172	0.03018	-0.03532	-0.00547
-0.200	3.000	-0.01955	0.02888	-0.03327	-0.00488
-0.400	3.000	-0.01768	0.02768	-0.03145	-0.00438
-1.400	1.400	0.04771	0.03347	-0.00295	0.01912
-3.000	0.000	0.01262	0.02052	-0.01067	0.00619
-1.400	-1.400	0.04526	0.02715	0.02942	0.04526
-0.400	-3.000	0.02622	0.02750	0.01347	0.04790
-0.200	-3.000	0.02495	0.01183	0.02757	0.04816
0.000	-3.000	0.02468	0.01200	0.02730	0.05037
0.200	-3.000	0.02437	0.01210	0.02691	0.05261
0.400	-3.000	0.02401	0.01212	0.02640	0.05489
1.400	-1.400	0.00500	-0.01783	0.02582	0.01249

UC CASE 2 INFLUENCE FUNCTION

X	Y	UC11	UC12	UC21	UC22
3.000	0.000	0.02303	-0.01468	0.03622	0.09090
1.400	1.400	-0.10446	-0.06267	-0.06790	-0.10446
0.400	3.000	-0.14646	-0.00549	-0.03358	-0.03800
0.200	3.000	-0.02419	0.03166	-0.03773	-0.00619
0.000	3.000	-0.02165	0.03027	-0.03540	-0.00548
-0.200	3.000	-0.01948	0.02897	-0.03335	-0.00489
-0.400	3.000	-0.01763	0.02777	-0.03152	-0.00439
-1.400	1.400	0.04768	0.03349	-0.00296	0.01916
-3.000	0.000	0.01263	0.02056	-0.01070	0.00620
-1.400	-1.400	0.04537	0.02722	0.02949	0.04537
-0.400	-3.000	0.02629	0.02758	0.01345	0.04794
-0.200	-3.000	0.02502	0.01187	0.02758	0.04818
0.000	-3.000	0.02475	0.01204	0.02730	0.05038
0.200	-3.000	0.02443	0.01213	0.02691	0.05261
0.400	-3.000	0.02407	0.01215	0.02639	0.05487
1.400	-1.400	0.00502	-0.01788	0.02587	0.01248

CHAPTER IV

EXAMPLES AND DISCUSSION

IV.1 Examples

Four example problems are solved utilizing the Boundary Integral Method and the results are compared to NASTRAN. In the first example problem, a triangular shaped geometry is analyzed. The FEM and BEM model of this problem are shown in Figure IV.1 and IV.2, respectively. The structure is subjected to the following non-constant boundary conditions, on the side where x=0

$$u_x = -4.76\ y^2/2$$

$$u_y = -1.43\ y^2/2$$

on the side where y=0

$$u_x = -1.43\ x^2/2$$

$$u_y = -4.76\ x^2/2 - 167\ x$$

and on the side where x + y = 1 the following tractions are applied

$$t_x = y \cos \alpha - \sin \alpha$$

$$t_y = -\cos \alpha + x \sin \alpha$$

The orthotropic material properties of Graphite/Epoxy, AS/3501 was used and those properties are as follows:

$$E_x = 0.210084$$

$$E_y = 0.014006$$

$$E_s = 0.002994$$

$$\nu_x = 0.300400$$

This material belongs to first case of orthotropic formulated in chapter III. The problem was solved utilizing BEM by discretizing the boundary into 13 elements as shown in Figure IV.2. The NASTRAN model shown in Figure IV.1 contains six rectangular elements and four triangular elements. Table IV.1 lists the displacements calculated by these two codes for three points on the side where x + y =1.

In the second example, the same problem is solved, however, the number of nodes used on the boundary was increased to 19 in the BEM model as shown in Figure IV.4. Figure IV.3 exhibits the model used to solved the problem using NASTRAN. Table IV.2 shows the displacements calculated by both codes on the side where x + y = 1. In example problem three, a quarter of a plate with a hole in the middle subjected to a uniform distributed load of magnitude unity is solved. The FEM and BEM model of this problem are shown in Figures IV.5 and IV.6, respectively. Table IV.3 lists the displacements calculated with both codes. The material properties selected for this problem are as follows:

$$E_x = 2.50000$$

$$E_y = 2.50000$$

$$E_s = 0.50000$$

$$\nu_x = 0.25000$$

In the example problem four, the same problem was solved with 47 nodes on the boundary. The coordinates of the points selected and displacements calculated by NASTRAN and BEM are listed in Table IV.4.

Note that this material belongs to the second case of the orthotropic formulated in chapter III.

IV.2 DISCUSSION

Some conclusions can be drawn from the results of the four example problems.

The BEM results are in agreement with FEM results generated from the NASTRAN code.

As expected, it is much easier to prepare the data for BEM than for FEM. This factor can be quantified by comparing the input decks of NASTRAN and BEM listed in appendix C. This advantage of the BEM is even more profound when solving three dimensional problems.

For the same number of the boundary nodes, fewer equations are solved in the BEM program. However, since the BEM matrix of coefficients is neither symmetric nor banded, a greater effort is necessary for solving the equations. Conversely, the high number of equations in the FEM method are solved quite efficiently due to the symmetric and banded form of the global stiffness matrix.

The numerical method used to solve the singular terms in BEM seems to be adequate to obtain accurate results.

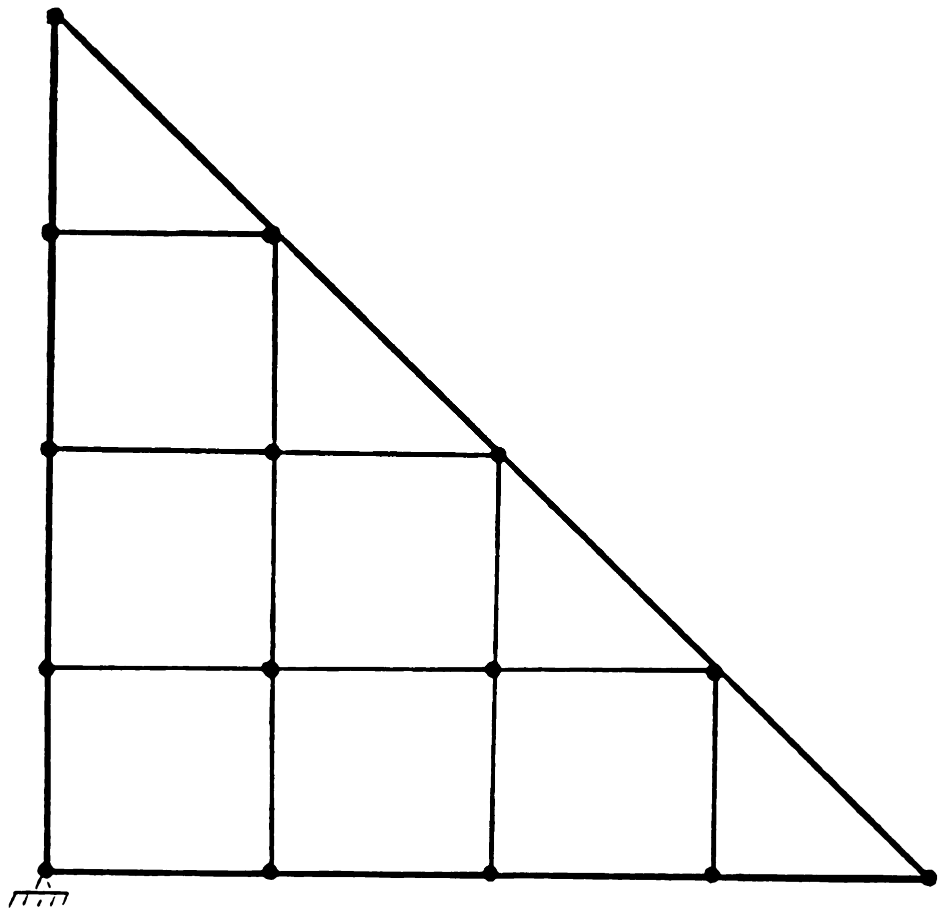


Figure IV.1 FEM model of example problem one

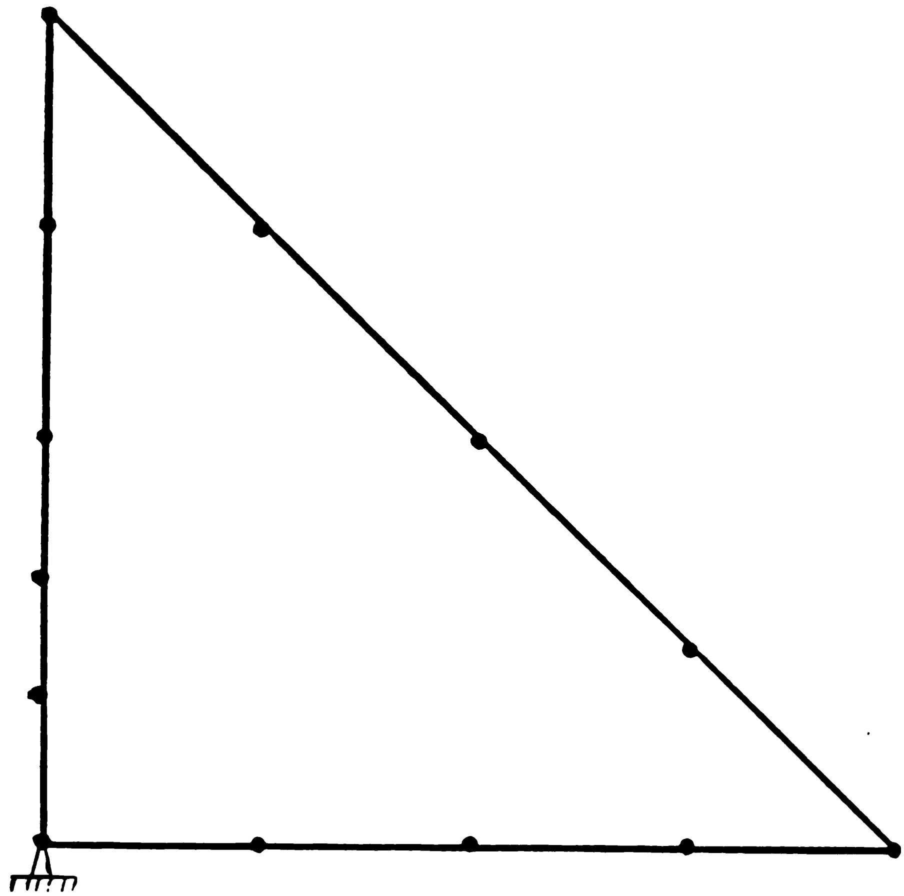


Figure IV.2   BEM model of example problem one

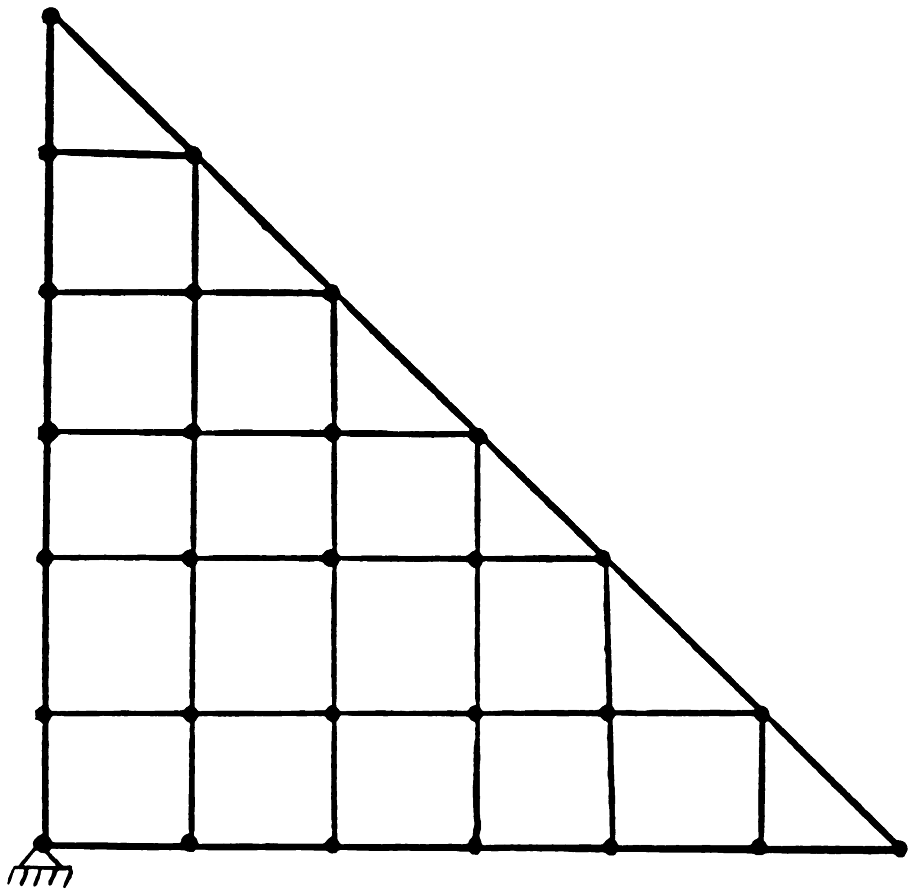


Figure IV.3   FEM model of example problem two

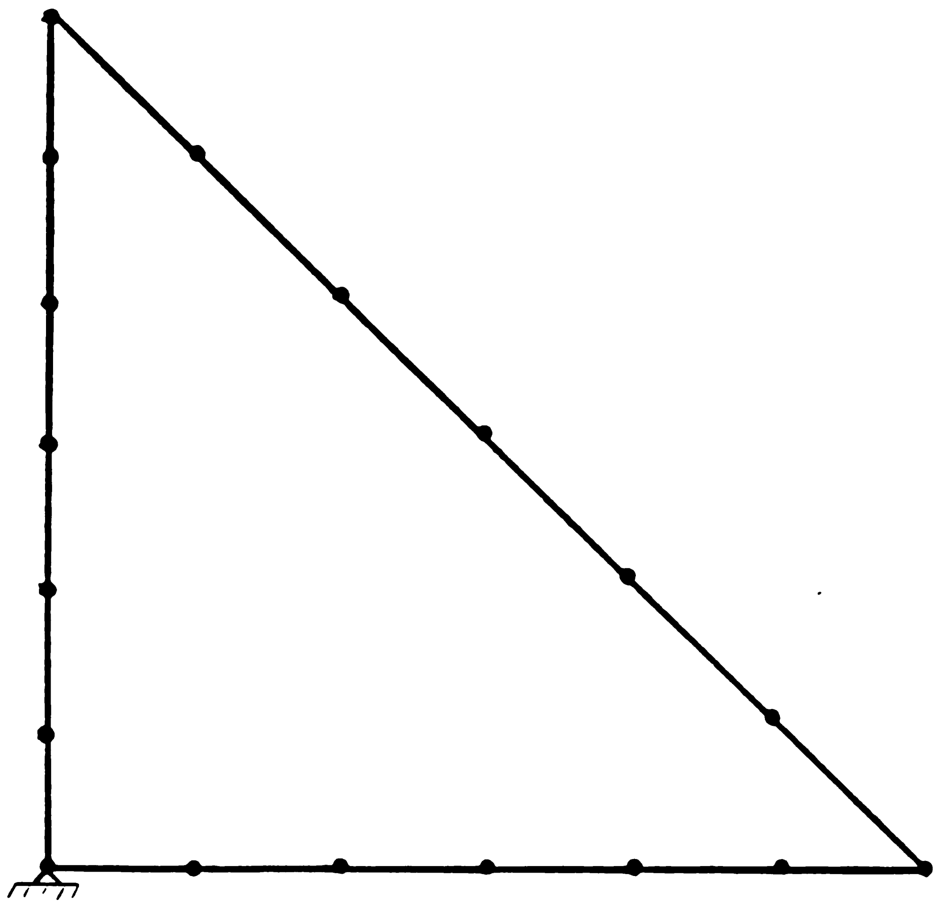


Figure IV.4   BEM model of example problem two

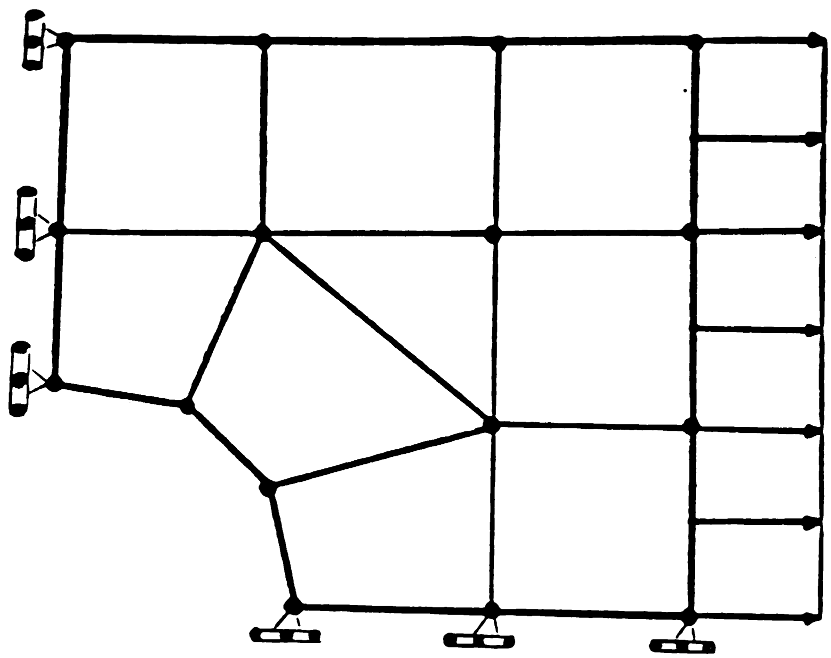


Figure IV.5 FEM model of example problem three

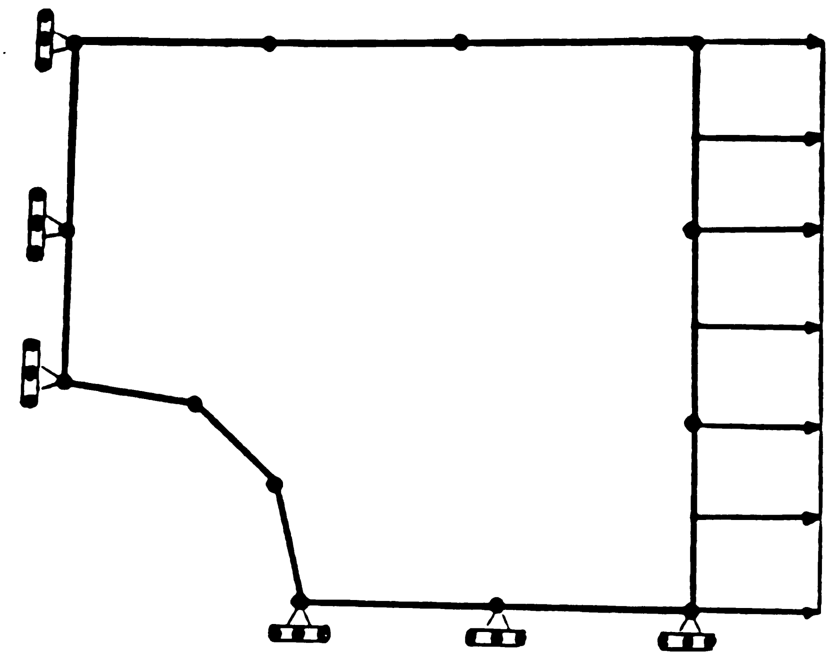


Figure IV.6 BEM model of example problem three

Table IV.1 Displacements calculated by NASTRAN and BEM for selected points in example problem one

		NASTRAN		BEM	
x-coord.	y-coord.	x-displ.	y-displ.	x-displ.	y-displ.
0.75000	0.25000	-3.41530	-120.95080	-3.85450	-121.34789
0.50000	0.50000	-8.81401	-81.34058	-8.80608	-81.37060
0.25000	0.75000	-19.53734	-44.36070	-19.62609	-43.95847

Table IV.2 Displacements calculated by NASTRAN and BEM for selected points in example problem two

		N A S T R A N		B E M	
x-coord.	y-coord.	x-displ.	y-displ.	x-displ.	y-displ.
0.66670	0.33333	-4.37904	-107.75980	-4.69846	-108.18671
0.83330	0.33333	-5.74310	-136.10740	-4.99990	-135.93440
0.50000	0.50000	-8.85747	-81.14751	-8.72976	-81.85050
0.33333	0.66670	-15.44165	-56.30618	-15.56597	-56.63600
0.16670	0.83330	-24.33414	-31.12919	-24.16266	-31.13294

Table IV.3 Displacements calculated by NASTRAN and BEM for selected points in example problem three

		N A S T R A N		B E M	
x-coord.	y-coord.	x-displ.	y-displ.	x-displ.	y-displ.
1.00000	1.73210	1.26791	-1.02519	1.38968	-1.06707
1.73210	1.00000	2.44562	-4.95746	2.55895	-5.58470
6.00000	2.00000	3.55484	0.25596	3.51277	0.17675
6.00000	4.00000	2.81648	0.28166	2.85083	0.14799
6.00000	6.00000	2.02305	0.10245	2.13055	-0.00495
4.00000	6.00000	1.24023	-0.65758	1.40719	-0.68455
2.00000	6.00000	0.55251	-1.29541	0.67339	-1.22614

Table IV.4 Displacements calculated by NASTRAN and BEM for selected points in example problem four

		NASTRAN		BEM	
x-coord.	y-coord.	x-displ.	y-displ.	x-displ.	y-displ.
0.00000	2.00000	0.00000	−1.88083	0.00000	−1.88414
0.45100	1.95000	0.73183	−1.82209	0.72876	−1.82348
1.00400	1.73000	1.60271	−1.31113	1.59667	−1.32311
1.20700	1.59500	1.92800	−1.19073	1.95500	−1.21781
1.57800	1.22900	2.56548	−0.90910	2.58499	−0.92374
1.73300	0.99800	2.85365	−0.74192	2.85780	−0.73754
1.95800	0.45800	3.25107	−0.33328	3.25357	−0.33842
2.00000	0.00000	3.33426	0.00000	3.36612	0.00000
2.50000	0.00000	3.39587	0.00000	3.38677	0.00000
3.00000	0.00000	3.42132	0.00000	3.42707	0.00000
3.50000	0.00000	3.49017	0.00000	3.50013	0.00000
4.00000	0.00000	3.59540	0.00000	3.60643	0.00000
4.50000	0.00000	3.72824	0.00000	3.73911	0.00000
5.00000	0.00000	3.88009	0.00000	3.89049	0.00000
5.50000	0.00000	4.04202	0.00000	4.05342	0.00000
6.00000	0.00000	4.20270	0.00000	4.20080	0.00000
6.00000	0.50000	4.16963	0.12081	4.17887	0.11215
6.00000	1.00000	4.07393	0.22900	4.08634	0.22791
6.00000	1.50000	3.92525	0.31446	3.93476	0.31618
6.00000	2.00000	3.73717	0.37126	3.74346	0.37306
6.00000	2.50000	3.52428	0.39822	3.52812	0.39875
6.00000	3.00000	3.29963	0.39818	3.30205	0.39685
6.00000	3.50000	3.07298	0.37662	3.07497	0.37333
6.00000	4.00000	2.85027	0.34002	2.85258	0.33488
6.00000	4.50000	2.63388	0.29447	2.63705	0.28757
6.00000	5.00000	2.42356	0.24476	2.42800	0.23575
6.00000	5.50000	2.21745	0.19406	2.22504	0.18013
6.00000	6.00000	2.01293	0.14377	2.04479	0.11650
5.50000	6.00000	1.81321	−0.06064	1.82731	−0.06623
5.00000	6.00000	1.61396	−0.26392	1.62430	−0.26772
4.50000	6.00000	1.41473	−0.46500	1.42356	−0.46756
4.00000	6.00000	1.21577	−0.66428	1.22324	−0.66589
3.50000	6.00000	1.01909	−0.86233	1.02511	−0.86346
3.00000	6.00000	0.82850	−1.05828	0.83295	−1.05956
2.50000	6.00000	0.64885	−1.24863	0.65178	−1.25084
2.00000	6.00000	0.48484	−1.42664	0.48666	−1.43064
1.50000	6.00000	0.33956	−1.58265	0.34137	−1.58909
1.00000	6.00000	0.21317	−1.70542	0.21744	−1.71397
0.50000	6.00000	0.10223	−1.78426	0.11337	−1.78898
0.00000	6.00000	0.00000	−1.81148	0.00000	−1.80821
0.00000	5.50000	0.00000	−1.78245	0.00000	−1.78932
0.00000	5.00000	0.00000	−1.74987	0.00000	−1.75586
0.00000	4.50000	0.00000	−1.72041	0.00000	−1.72691
0.00000	4.00000	0.00000	−1.69815	0.00000	−1.70474
0.00000	3.50000	0.00000	−1.68549	0.00000	−1.69092
0.00000	3.00000	0.00000	−1.68125	0.00000	−1.68303
0.00000	2.50000	0.00000	−1.67704	0.00000	−1.66485

Appendix A) Computer listing of the program ORTHO.CASE1 and ORTHO.CASE2

LISTING OF THE PROGRAM ORTHO.CASE1

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     PROGRAM ORTHO.CASE1
C
C     THIS PROGRAM EMPLOYS THE DIRECT BOUNDARY ELEMENT METHOD
C     USING STRAIGHT BOUNDARY ELEMENTS CHARACTERIZED BY LINEAR
C     DISPLACEMENTS AND CONSTANT TRACTIONS TO SOLVE PLANE STRESS
C     ORTHOTROPIC MATERIAL PROPERTIES PROBLEMS OF LINEAR ELASTICITY.
C     UNIT THICKNESS IS ASSUMED.
C
C     THE FOLLOWING IS THE USER'S DEFINED INPUT PARAMETERS,
C     VARIABLES AND ARRAYS:
C
C     N IS THE NUMBER OF THE NODES ON THE BOUNDARY. MUST BE ODD
C     NUMBER. ENTER IN FREE FORMAT.
C     EX IS THE LONGITUDINAL MODULUS OF ELASTICITY.
C     EY IS THE TRANSVERSE MODULUS OF ELASTICITY.
C     ES IS THE AXIAL SHEAR MODULUS OF ELASTICITY.
C     PRX IS THE POISSON'S RATIO IN THE X-DIRECTION.
C     X(I) AND Y(I) ARE THE COORDINATES OF NODES 1,N ENTERED IN
C     COUNTER-CLOCKWISE IN FREE FORMAT.
C     J,K ARE NODES, J, AND CORRESPONDING DIRECTION, K, AT WHICH
C     DISPLACEMENTS ARE SPECIFIED(ZERO AND NON-ZERO). ENTER IN
C     FREE FORMAT AND END BY INPUTTING 0, 0.
C     J,K,COND ARE NODES, J, AND CORRESPONDING DIRECTION, K, AT
C     WHICH NON-ZERO FORCES ARE SPECIFIED, AND SPECIFIED VALUE
C     COND.  ENTER IN FREE FORMAT AND END WITH BY INPUTTING 0, 0, 0.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      PARAMETER(MAXN=101)
      PARAMETER(MAXNN=2*MAXN,MAXNP3=MAXNN+3)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(MAXN),Y(MAXN)
      REAL*8  R(8),W0(8),W1(8),W2(8),MAT(2)
      REAL*8  RL(8),WL0(8),WL1(8),WL2(8)
      DIMENSION NTBC(MAXN,2),BC(MAXNN),SUMX(MAXN),SUMY(MAXN)
      REAL*8 UC(MAXNP3,MAXNP3),UR(MAXNP3,MAXNN),RHS(MAXNP3)
C
      PI=DACOS(-1.00D00)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     READ IN THE INPUT DATA FROM FILE 5
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      READ(5,*) N
      READ(5,*) EX,EY,ES,PRX
      READ(5,*) (X(I),Y(I),I=1,N)
C
      DO 9 I=1,N
      DO 9 K=1,2
9     NTBC(I,K)=0
C
11    READ(5,*)J,K
      IF(J.EQ.0)GOTO 12
      NTBC(J,K)=1
      GOTO 11
```

```
12    CONTINUE
C
      NN=2*N
      DO 15 I=1,NN
15    BC(I)=0.
C
16    READ(5,*) J,K,COND
      IF(J.EQ.0)GOTO 17
      I=2*(J-1)+K
      BC(I)=COND
      GOTO 16
17    CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CONVERT MATERIAL PROPERTIES TO MATERIAL COMPLIANCE CONSTANTS.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      C11=1/EX
      C22=1/EY
      C66=1/(2*ES)
      C12=-PRX/EX
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C    OUTPUT PRESCRIBED INFORMATION
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      WRITE(6,100)
      WRITE(6,150)
      DO 19 I=1,N
      II=2*I
      IIM1=II-1
      IF(NTBC(I,2).EQ.1) GOTO 20
      IF(NTBC(I,1).EQ.1)GOTO 21
      WRITE(6,200) I,X(I),Y(I),BC(IIM1),BC(II)
      GOTO 19
20    IF(NTBC(I,1).EQ.1) GOTO 22
      WRITE(6,201) I,X(I),Y(I),BC(IIM1),BC(II)
      GOTO 19
21    WRITE(6,202)I,X(I),Y(I),BC(IIM1),BC(II)
      GOTO 19
22    WRITE(6,203) I,X(I),Y(I),BC(IIM1),BC(II)
19    CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C     CALCULATE MATERIAL PROPERTIES IN THE  INFLUENCE FUNCTIONS
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      AUX=DSQRT(((C12+C66)/C11)*((C12+C66)/C11)-C22/C11)
      ROOT1=DSQRT((C12+C66)/C11+AUX)
      ROOT2=DSQRT((C12+C66)/C11-AUX)
```

```
C
      DUM1=(C12+C66)/C11
      DUM2=C22/C11
      DUM3=DSQRT(DUM2)
      RAD=DUM1*DUM1-DUM2
      DUM4=DSQRT(ABS(RAD))
      MAT(1)=DSQRT(DUM1+DUM4)
      MAT(2)=DSQRT(DUM1-DUM4)
      DUM4=C12/C11
      DUM5=MAT(1)+MAT(2)
      DUM6=MAT(1)-MAT(2)
      DUM1=2.*DUM1
C
      E11=-(DUM1+DUM3-DUM4)/(DUM5*4*PI)
      E12=-(DUM1-DUM3-DUM4)/(DUM6*4*PI)
      E21=DUM4/(4*PI)
      E22=-(2.*DUM2-DUM1*DUM4)/(DUM5*DUM6*4*PI)
      E31=(1.+DUM4/DUM3)/(DUM5*PI*4)
      E32=(1.-DUM4/DUM3)/(DUM6*PI*4)
      E41=-1./(4*PI)
      E42=(DUM1-2.*DUM4)/(DUM5*DUM6*4*PI)
      E51=-1./(4*PI)
      E52=-E42
      E61=-(DUM3+DUM4)/(DUM5*4*PI)
      E62=(DUM3-DUM4)/(DUM6*4*PI)
C
      D11=C11*E11+C12*E31
      D12=(C11*E12+C12*E32)
      D22=-C11*E22-C12*E42
      D32=D22
      DUM=DSQRT(C22/C11)
      D41=DUM*D11
      D42=-DUM*D12
C
      R(1)=-.96028985649753
      R(2)=-.79666647741362
      R(3)=-.52553240991632
      R(4)=-.18343464249565
      R(5)=-R(1)
      R(6)=-R(2)
      R(7)=-R(3)
      R(8)=-R(4)
C
      W0(1)=.10122853629038
      W0(2)=.22238103445337
      W0(3)=.31370664587789
      W0(4)=.36268378337836
      W0(5)=W0(1)
      W0(6)=W0(2)
      W0(7)=W0(3)
      W0(8)=W0(4)
C
      DO 10 L=1,8
      W1(L)=1.-R(L)
      W2(L)=1.+R(L)
   10 CONTINUE
C
```

```
C CCCCCCCCCCCCRCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C INITIALIZE MATRICES
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      NNP3=NN+3
      DO 1 I=1,NNP3
      DO 1 J=1,NNP3
      UC(J,I)=0.
      IF(I.GT.NN) GOTO 1
      UR(J,I)=0.
1     CONTINUE
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C LOOP ON COLUMN
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 2 J=1,N
      JJ=2*J
      IF(J.EQ.1) THEN
      JM1=N
      JJM2=NN
      ELSE
      JM1=J-1
      JJM2=JJ-2
      END IF
      IF(J.EQ.N) THEN
      JP1=1
      JJP1=1
      ELSE
      JP1=J+1
      JJP1=JJ+1
      END IF
C
      XM=(X(J)+X(JP1))/2.
      YM=(Y(J)+Y(JP1))/2.
      B1=X(JP1)-XM
      B2=Y(JP1)-YM
      BDB1=B1*B1+ROOT1*ROOT1*B2*B2
      BDB2=B1*B1+ROOT2*ROOT2*B2*B2
      TERM1=DLOG(BDB1)+DLOG(BDB2)+4.*LOG(2.)-4.
      TERM2=DLOG(BDB1)-DLOG(BDB2)
      UR(JJ-1,JJP1)=UR(JJ-1,JJP1)+D11*TERM1+D12*TERM2
      UR(JJ,JJP1+1)=UR(JJ,JJP1+1)+D41*TERM1+D42*TERM2
C
      XM=(X(J)+X(JM1))/2.
      YM=(Y(J)+Y(JM1))/2.
      B1=X(J)-XM
      B2=Y(J)-YM
      BDB1=B1*B1+ROOT1*ROOT1*B2*B2
      BDB2=B1*B1+ROOT2*ROOT2*B2*B2
      TERM1=DLOG(BDB1)+DLOG(BDB2)+4.*LOG(2.)-4.
      TERM2=DLOG(BDB1)-DLOG(BDB2)
      UR(JJ-1,JJ-1)=UR(JJ-1,JJ-1)+D11*TERM1+D12*TERM2
      UR(JJ,JJ)=UR(JJ,JJ)+D41*TERM1+D42*TERM2
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

```
C
C LOOP ON ROW
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 2 I=1,N
      II=2*I
C
      A1=X(I)-XM
      A2=Y(I)-YM
C
      ADA1=A1*A1+ROOT1*ROOT1*A2*A2
      ADA2=A1*A1+ROOT2*ROOT2*A2*A2
      ADB1=2.*(A1*B1+ROOT1*ROOT1*A2*B2)
      ADB2=2.*(A1*B1+ROOT2*ROOT2*A2*B2)
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C LOOP ON POINTS OF INTEGRATION
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 3 L=1,8
C
      R1=ADA1-ADB1*R(L)+BDB1*R(L)*R(L)
      R2=ADA2-ADB2*R(L)+BDB2*R(L)*R(L)
      A1B1=A1-B1*R(L)
      A2B2=A2-B2*R(L)
C
      T1=A1B1/R1+A1B1/R2
      T2=A1B1/R1-A1B1/R2
      T3=ROOT1*A2B2/R1+ROOT2*A2B2/R2
      T4=ROOT1*A2B2/R1-ROOT2*A2B2/R2
C
      TA11=-E11*B2*T1-E12*B2*T2+E51*B1*T3+E52*B1*T4
      TA22=-E61*B2*T1-E62*B2*T2+E41*B1*T3+E42*B1*T4
      TA12=-E51*B2*T3-E52*B2*T4+E31*B1*T1+E32*B1*T2
      TA21=-E21*B2*T3-E22*B2*T4+E61*B1*T1+E62*B1*T2
C
      UC(II-1,JJM2-1)=UC(II-1,JJM2-1)+W1(L)*W0(L)*TA11
      UC(II,JJM2)=UC(II,JJM2)+W1(L)*W0(L)*TA22
      UC(II-1,JJM2)=UC(II-1,JJM2)+W1(L)*W0(L)*TA12
      UC(II,JJM2-1)=UC(II,JJM2-1)+W1(L)*W0(L)*TA21
C
      UC(II-1,JJ-1)=UC(II-1,JJ-1)+W2(L)*W0(L)*TA11
      UC(II,JJ)=UC(II,JJ)+W2(L)*W0(L)*TA22
      UC(II-1,JJ)=UC(II-1,JJ)+W2(L)*W0(L)*TA12
      UC(II,JJ-1)=UC(II,JJ-1)+W2(L)*W0(L)*TA21
C
      ARGU1=ROOT1*A2B2
      ARGU2=ROOT2*A2B2
      IF(ABS(A1B1).LT.1.0E-3)THEN
      IF(ARGU1.GT.0.) TANT1=PI/2
      IF(ARGU1.LT.0.) TANT1=-PI/2
      IF(ARGU2.GT.0.) TANT2=PI/2
      IF(ARGU2.LT.0.) TANT2=-PI/2
      ELSE
      RATIO1=ROOT1*A2B2/A1B1
      TANT1=DATAN(RATIO1)
      RATIO2=ROOT2*A2B2/A1B1
```

```
      TANT2=DATAN(RATIO2)
      ENDIF
C
      F11=0.5*(D11*(DLOG(R1)+DLOG(R2))+D12*(DLOG(R1)-DLOG(R2)))
      F22=0.5*(D41*(DLOG(R1)+DLOG(R2))+D42*(DLOG(R1)-DLOG(R2)))
      F12=D22*(TANT1-TANT2)
      F21=D32*(TANT1-TANT2)
C
      IF(I.EQ.J.OR.I.EQ.JM1)THEN
      ELSE
      UR(II-1,JJ-1)=UR(II-1,JJ-1)+F11*W0(L)
      UR(II,JJ)=UR(II,JJ)+F22*W0(L)
      ENDIF
      UR(II-1,JJ)=UR(II-1,JJ)+F12*W0(L)
      UR(II,JJ-1)=UR(II,JJ-1)+F21*W0(L)
C
3     CONTINUE
2     CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C
C      CALCULATION OF THE DIAGONAL ENTRIES OF UC MATRIX.
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 59 I=1,NN
      SUMX(I)=0.
      SUMY(I)=0.
      DO 59 J=1,N
      JJ=2*J
      SUMX(I)=SUMX(I)+UC(I,JJ-1)
      SUMY(I)=SUMY(I)+UC(I,JJ)
 59   CONTINUE
      M=-1
      J=1
      DO 51 I=1,NN
      M=M+1
      UC(I,J)=-SUMX(I)
      UC(I,J+1)=-SUMY(I)
      IF(M.EQ.0)GOTO 51
      J=J+2
      M=-1
 51   CONTINUE
C     WRITE(6,455) ((UR(M,L),L=1,NN),M=1,NN)
C     WRITE(6,26)
C     WRITE(6,455) ((UC(M,L),L=1,NN),M=1,NN)
C455  FORMAT(10(1X,F5.3))
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C  POST-MULTIPLY BY GAMMA INVERSE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 6 I=1,N
      II=2*I
      IIM1=II-1
```

```
      A0=1.
      DO 4 K=2,N
C
      UR(IIM1,1)=UR(IIM1,1)+UR(IIM1,2*K-1)*A0
      UR(II,1)=UR(II,1)+UR(II,2*K-1)*A0
      UR(IIM1,2)=UR(IIM1,2)+UR(IIM1,2*K)*A0
      UR(II,2)=UR(II,2)+UR(II,2*K)*A0
C
4     A0=-A0
C
      DO 5 K=2,N
C
      UR(IIM1,2*K-1)=-UR(IIM1,2*K-3)+2.*UR(IIM1,2*K-1)
      UR(II,2*K-1)=-UR(II,2*K-3)+2.*UR(II,2*K-1)
      UR(IIM1,2*K)=-UR(IIM1,2*K-2)+2.*UR(IIM1,2*K)
5     UR(II,2*K)=-UR(II,2*K-2)+2.*UR(II,2*K)
C
6     CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C AUGMENT FOR EQUILIBRIUM
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      NNP1=NN+1
      NNP2=NN+2
      DO 7 I=1,N
      UC(2*I-1,NNP1)=-1.
      UC(2*I,NNP2)=-1.
      UR(NNP1,2*I-1)=1.
7     UR(NNP2,2*I)=1.
C
      DO 8 I=2,N
      UC(2*I-1,NNP3)=Y(I)-Y(1)
      UC(2*I,NNP3)=X(1)-X(I)
      UR(NNP3,2*I-1)=Y(1)-Y(I)
8     UR(NNP3,2*I)=X(I)-X(1)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C RE-ORCDER SYSTEM CBASED ON KNCOWN BOUNDARCY CONDITIONCS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 13 I=1,N
      DO 13 K=1,2
      II=2*(I-1)+K
      KK=NTBC(I,K)
      IF(KK.EQ.0) GOTO 13
      DO 14 L=1,NNP3
      TEM=UR(L,II)
      UR(L,II)=-UC(L,II)
      UC(L,II)=-TEM
14    CONTINUE
13    CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C DETERMINE KNOWN RIGHT-HAND SIDEM
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 18 I=1,NNP3
      RHS(I)=0.
      DO 18 L=1,NN
18    RHS(I)=RHS(I)+UR(I,L)*BC(L)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C SOLVE FCOR UNKNOWN BOUNDARY CONDITIONS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      NNP4=NN+4
      DO 23 I=1,NNP2
      PIVOT=0.
      DO 24 J=I,NNP3
      TEM=DABS(UC(J,I))
      IF(PIVOT.GE.TEM) GOTO 24
      PIVOT=TEM
      IPIVOT=J
24    CONTINUE
C
      IF(IPIVOT.EQ.I) GOTO 45
C
      DO 27 K=I,NNP3
      TEM=UC(I,K)
      UC(I,K)=UC(IPIVOT,K)
      UC(IPIVOT,K)=TEM
27    CONTINUE
C
      TEM=RHS(I)
      RHS(I)=RHS(IPIVOT)
      RHS(IPIVOT)=TEM
C
45    IP1=I+1
      DO 28 K=IP1,NNP3
      Q=-UC(K,I)/UC(I,I)
      UC(K,I)=0.
      RHS(K)=Q*RHS(I)+RHS(K)
      DO 29 J=IP1,NNP3
      UC(K,J)=Q*UC(I,J)+UC(K,J)
29    CONTINUE
28    CONTINUE
23    CONTINUE
C
      RHS(NNP3)=RHS(NNP3)/UC(NNP3,NNP3)
      DO 30 K=1,NNP2
      Q=0.
      DO 31 J=1,K
      Q=Q+UC(NNP3-K,NNP4-J)*RHS(NNP4-J)
31    CONTINUE
      RHS(NNP3-K)=(RHS(NNP3-K)-Q)/UC(NNP3-K,NNP3-K)
30    CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C PUT NODAL DISPLACEMENTS INTO KRHS VECTOR
C PUT NODAL FORCES INTO BC VECTOR
C OUTPUT VECTORS RHTS AND BC
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 32 I=1,N
      DO 32 K=1,2
      KK=NTBC(I,K)
      II=2*(I-1)+K
      IF(KK.EQ.1) THEN
      TEMP=BC(II)
      BC(II)=RHS(II)
      RHS(II)=TEMP
      ELSE
      END IF
32    CONTINUE
C
      WRITE(6,300)
      WRITE(6,325)
      DO 33 I=1,N
      II=2*I
      IIM1=II-1
33    WRITE(6,350) I,RHS(IIM1),RHS(II),BC(IIM1),BC(II)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C PRECMULTIPLY BCI BY GAMMA INVERSE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SAV1=BC(1)
      SAV2=BC(2)
      A0=-1.
      DO 34 I=2,N
      II=2*I
      IIM1=II-1
      BC(1)=BC(1)+A0*BC(IIM1)
      BC(2)=BC(2)+A0*BC(II)
34    A0=-A0
C
      DO 35 I=2,N
      II=2*I
      IIM1=II-1
      SAV3=BC(IIM1)
      SAV4=BC(II)
      BC(IIM1)=2.*SAV1-BC(IIM1-2)
      BC(II)=2.*SAV2-BC(II-2)
      SAV1=SAV3
      SAV2=SAV4
35    CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C COMPUTE BOUNDARY STRESSES AND OUTPUT
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6,400)
      WRITE(6,425)
      DO 36 I=1,N
      II=2*I
      IIM1=II-1
      IF(I.EQ.1) THEN
      IM1=N
      IIM2=2*N
```

```
      ELSE
      IM1=I-1
      IIM2=II-2
      END IF
      IIM3=IIM2-1
      XM=X(I)-X(IM1)
      YM=Y(I)-Y(IM1)
      SF=DSQRT(XM*XM+YM*YM)
      CTF=YM/SF
      STF=-XM/SF
      SIGNN=(CTF*BC(IIM1)+STF*BC(II))/SF
      SIGNT=(CTF*BC(II)-STF*BC(IIM1))/SF
C     SIGTT=PR*SIGNN+(1.+PR)*(CTF*(RHS(II)-RHS(IIM2))-STF*(RHS(IIM1)
C    +-RHS(IIM3)))*2./SF
      SIGTT=0.
      WRITE(6,450) I,SIGNN,SIGNT,SIGTT
36    CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     FORMATS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
100   FORMAT(//,' BOUNDARY NODES AND PRESCRIBED BOUNDARY CONDITIONS')
150   FORMAT(/,' NODE',6X,'X',9X,'Y',8X,'CONDITIONS')
200   FORMAT(/,I5,F10.5,F10.5,5X,'FX =',F10.5,3X,'FY =',F10.5)
201   FORMAT(/,I5,F10.5,F10.5,5X,'FX =',F10.5,3X,'UY =',F10.5)
202   FORMAT(/,I5,F10.5,F10.5,5X,'UX =',F10.5,3X,'FY =',F10.5)
203   FORMAT(/,I5,F10.5,F10.5,5X,'UX =',F10.5,3X,'UY =',F10.5)
C
300   FORMAT(//,' DISPLACEMENTS AND FORCES AT ALL BOUNDARY NODES')
325   FORMAT(/,' NODE',6X,'UX',11X,'UY',11X,'FX',11X,'FY')
450   FORMAT(/,I7,2X,F10.6,3X,F10.6,3X,F10.6)
350   FORMAT(/,I5,2X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
400   FORMAT(//,' STRESSES ON ALL BOUNDARY ELEMENTS')
425   FORMAT(/,' ELEMENT',3X,'SIGMA NN',5X,'SIGMA NT',5X,'SIGMA TT')
460   FORMAT(2X,'S11= ',E12.6,2X,'S12= ',E12.6,2X,'S16= '
     +,E12.6)
465   FORMAT(2X,'SE11= ',E12.6,2X,'SE12= ',E12.6,2X,'SE16= '
     +,E12.6)
470   FORMAT(24X,'S(2,2)= ',E12.6,2X,'S(2,6)= ',E12.6,2X)
475   FORMAT(24X,'SE22= ',E12.6,2X,'SE26= ',E12.6,2X)
480   FORMAT(46X,'S66= ',E12.6)
485   FORMAT(46X,'SE66= ',E12.6)
500   FORMAT(1X,' MATERIAL PROPERTY S(I,J) AFTER TRANSFORMATION ')
403   FORMAT(1X,' THROUGH ANGLE PHI= ',F5.2,1X,'DEGREE')
510   FORMAT(1X,///)
50    FORMAT(1X,'POISSON RATIO =',F6.3)
26    FORMAT(1X,///)
405   FORMAT(1X,/)
505   FORMAT(10(1X,F6.3))
      STOP
      END
```

LISTING OF THE PROGRAM ORTHO.CASE2

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     PROGRAM ORTHO.CASE2
C
C     THIS PROGRAM EMPLOYS THE DIRECT BOUNDARY ELEMENT METHOD
C     USING STRAIGHT BOUNDARY ELEMENTS CHARACTERIZED BY LINEAR
C     DISPLACEMENTS AND CONSTANT TRACTIONS TO SOLVE PLANE STRESS
C     ORTHOTROPIC MATERIAL PROPERTIES PROBLEMS OF LINEAR ELASTICITY.
C     UNIT THICKNESS IS ASSUMED.
C
C     THE FOLLOWING IS THE USER'S DEFINED INPUT PARAMETERS,
C     VARIABLES AND ARRAYS:
C
C     N IS THE NUMBER OF THE NODES ON THE BOUNDARY. MUST BE ODD
C     NUMBER. ENTER IN FREE FORMAT.
C     EX IS THE LONGITUDINAL MODULUS OF ELASTICITY.
C     EY IS THE TRANSVERSE MODULUS OF ELASTICITY.
C     ES IS THE AXIAL SHEAR MODULUS OF ELASTICITY.
C     PRX IS THE POISSON'S RATIO IN THE X-DIRECTION.
C     X(I) AND Y(I) ARE THE COORDINATES OF NODES 1,N ENTERED IN
C     COUNTER-CLOCKWISE IN FREE FORMAT.
C     J,K ARE NODES, J, AND CORRESPONDING DIRECTION, K, AT WHICH
C     DISPLACEMENTS ARE SPECIFIED(ZERO AND NON-ZERO). ENTER IN
C     FREE FORMAT AND END BY INPUTTING 0, 0.
C     J,K,COND ARE NODES, J, AND CORRESPONDING DIRECTION, K, AT
C     WHICH NON-ZERO FORCES ARE SPECIFIED, AND SPECIFIED VALUE
C     COND.  ENTER IN FREE FORMAT AND END WITH BY INPUTTING 0, 0, 0.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      PARAMETER(MAXN=101)
      PARAMETER(MAXNN=2*MAXN,MAXNP3=MAXNN+3)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(MAXN),Y(MAXN)
      REAL*8  R(8),W0(8),W1(8),W2(8),MAT(2)
      REAL*8  RL(8),WL0(8),WL1(8),WL2(8)
      DIMENSION NTBC(MAXN,2),BC(MAXNN),SUMX(MAXN),SUMY(MAXN)
      REAL*8 UC(MAXNP3,MAXNP3),UR(MAXNP3,MAXNN),RHS(MAXNP3)
C
      PI=DACOS(-1.00D00)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     READ IN THE INPUT DATA FROM FILE 5
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      READ(5,*) N
      READ(5,*) EX,EY,ES,PRX
      READ(5,*) (X(I),Y(I),I=1,N)
C
      DO 9 I=1,N
      DO 9 K=1,2
9     NTBC(I,K)=0
C
11    READ(5,*)J,K
      IF(J.EQ.0)GOTO 12
      NTBC(J,K)=1
      GOTO 11
```

```
12    CONTINUE
C
      NN=2*N
      DO 15 I=1,NN
15    BC(I)=0.
C
16    READ(5,*) J,K,COND
      IF(J.EQ.0)GOTO 17
      I=2*(J-1)+K
      BC(I)=COND
      GOTO 16
17    CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CONVERT MATERIAL PROPERTIES TO MATERIAL COMPLIANCE CONSTANTS.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      C11=1/EX
      C22=1/EY
      C66=1/(2*ES)
      C12=-PRX/EX
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C    OUTPUT PRESCRIBED INFORMATION
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      WRITE(6,100)
      WRITE(6,150)
      DO 19 I=1,N
      II=2*I
      IIM1=II-1
      IF(NTBC(I,2).EQ.1) GOTO 20
      IF(NTBC(I,1).EQ.1)GOTO 21
      WRITE(6,200) I,X(I),Y(I),BC(IIM1),BC(II)
      GOTO 19
20    IF(NTBC(I,1).EQ.1) GOTO 22
      WRITE(6,201) I,X(I),Y(I),BC(IIM1),BC(II)
      GOTO 19
21    WRITE(6,202)I,X(I),Y(I),BC(IIM1),BC(II)
      GOTO 19
22    WRITE(6,203) I,X(I),Y(I),BC(IIM1),BC(II)
19    CONTINUE
      PARAMETER(MAXN=101)
      PARAMETER(MAXNN=2*MAXN,MAXNP3=MAXNN+3)
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C    OUTPUT PRESCRIBED INFORMATION
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

```
C
      WRITE(6,50)PR
      WRITE(6,100)
      WRITE(6,150)
      DO 19 I=1,N
      II=2*I
      IIM1=II-1
      IF(NTBC(I,2).EQ.1) GOTO 20
      IF(NTBC(I,1).EQ.1)GOTO 21
      WRITE(6,200) I,X(I),Y(I),BC(IIM1),BC(II)
      GOTO 19
20    IF(NTBC(I,1).EQ.1) GOTO 22
      WRITE(6,201) I,X(I),Y(I),BC(IIM1),BC(II)
      GOTO 19
21    WRITE(6,202)I,X(I),Y(I),BC(IIM1),BC(II)
      GOTO 19
22    WRITE(6,203) I,X(I),Y(I),BC(IIM1),BC(II)
19    CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C     CALCULATE MATERIAL PROPERTIES IN THE  INFLUENCE FUNCTIONS
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      ARG1=(C12+C66)/C11
      ROOT=DSQRT(ARG1)
C
      DUM1=(C12+C66)/C11
      DUM2=C22/C11
      DUM3=DSQRT(DUM2)
      DUM4=C12/C11
      DUM5=2.*DSQRT(DUM1)
      DUM1=2.*DUM1
C
      E11=-(DUM1+DUM3-DUM4)/(DUM5*4*PI)
      E12=-(DUM1-DUM3-DUM4)/(4*PI)
      E21=DUM4/(4*PI)
      E22=-(2.*DUM2-DUM1*DUM4)/(DUM5*4*PI)
      E31=(1.+DUM4/DUM3)/(DUM5*PI*4)
      E32=(1.-DUM4/DUM3)/(PI*4)
      E41=-1./(4*PI)
      E42=(DUM1-2.*DUM4)/(DUM5*4*PI)
      E51=-1./(4*PI)
      E52=-E42
      E61=-(DUM3+DUM4)/(DUM5*4*PI)
      E62=(DUM3-DUM4)/(4*PI)
C
      D11=C11*E11+C12*E31
      D12=(C11*E12+C12*E32)
      D22=-C11*E22-C12*E42
      D32=D22
      D41=DUM3*D11
      D42=-DUM3*D12
C
      R(1)=-.96028985649753
```

```
      R(2)=-.79666647741362
      R(3)=-.52553240991632
      R(4)=-.18343464249565
      R(5)=-R(1)
      R(6)=-R(2)
      R(7)=-R(3)
      R(8)=-R(4)
C
      W0(1)=.10122853629038
      W0(2)=.22238103445337
      W0(3)=.31370664587789
      W0(4)=.36268378337836
      W0(5)=W0(1)
      W0(6)=W0(2)
      W0(7)=W0(3)
      W0(8)=W0(4)
C
      DO 10 L=1,8
      W1(L)=1.-R(L)
      W2(L)=1.+R(L)
10    CONTINUE
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C INITIALIZE MATRICES
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      NNP3=NN+3
      DO 1 I=1,NNP3
      DO 1 J=1,NNP3
      UC(J,I)=0.
      IF(I.GT.NN) GOTO 1
      UR(J,I)=0.
1     CONTINUE
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C LOOP ON COLUMN
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 2 J=1,N
      JJ=2*J
      IF(J.EQ.1) THEN
      JM1=N
      JJM2=NN
      ELSE
      JM1=J-1
      JJM2=JJ-2
      END IF
      IF(J.EQ.N) THEN
      JP1=1
      JJP1=1
      ELSE
      JP1=J+1
      JJP1=JJ+1
      END IF
C
```

```
      XM=(X(J)+X(JP1))/2.
      YM=(Y(J)+Y(JP1))/2.
      B1=X(JP1)-XM
      B2=Y(JP1)-YM
      BDB1=B1*B1+ROOT*ROOT*B2*B2
      TERM1=2.*DLOG(BDB1)+4.*LOG(2.)-4.
      TERM2=2.*B2*B2/BDB1
C
      UR(JJ-1,JJP1)=UR(JJ-1,JJP1)+D11*TERM1+D12*TERM2
      UR(JJ,JJP1+1)=UR(JJ,JJP1+1)+D41*TERM1+D42*TERM2
C
      XM=(X(J)+X(JM1))/2.
      YM=(Y(J)+Y(JM1))/2.
      B1=X(J)-XM
      B2=Y(J)-YM
C
      BDB1=B1*B1+ROOT*ROOT*B2*B2
      TERM1=2.*DLOG(BDB1)+4.*LOG(2.)-4.
      TERM2=2.*B2*B2/BDB1
C
      UR(JJ-1,JJ-1)=UR(JJ-1,JJ-1)+D11*TERM1+D12*TERM2
      UR(JJ,JJ)=UR(JJ,JJ)+D41*TERM1+D42*TERM2
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C LOOP ON ROW
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 2 I=1,N
      II=2*I
C
      A1=X(I)-XM
      A2=Y(I)-YM
C
      ADA1=A1*A1+ROOT*ROOT*A2*A2
      ADB1=2.*(A1*B1+ROOT*ROOT*A2*B2)
C
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C LOOP ON POINTS OF INTEGRATION
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 3 L=1,8
C
      R1=ADA1-ADB1*R(L)+BDB1*R(L)*R(L)
      A1B1=A1-B1*R(L)
      A2B2=A2-B2*R(L)
C
      T1=2.*A1B1/R1
      T2=-2.*ROOT*A1B1*A2B2*A2B2/(R1*R1)
      T3=2.*ROOT*A2B2/R1
      T4=(A1B1**2*A2B2-ROOT**2*A2B2**3)/(R1*R1)
C
      TA11=-E11*B2*T1-E12*B2*T2+E51*B1*T3+E52*B1*T4
      TA22=-E61*B2*T1-E62*B2*T2+E41*B1*T3+E42*B1*T4
      TA12=-E51*B2*T3-E52*B2*T4+E31*B1*T1+E32*B1*T2
      TA21=-E21*B2*T3-E22*B2*T4+E61*B1*T1+E62*B1*T2
```

```
C
      UC(II-1,JJM2-1)=UC(II-1,JJM2-1)+W1(L)*W0(L)*TA11
      UC(II,JJM2)=UC(II,JJM2)+W1(L)*W0(L)*TA22
      UC(II-1,JJM2)=UC(II-1,JJM2)+W1(L)*W0(L)*TA12
      UC(II,JJM2-1)=UC(II,JJM2-1)+W1(L)*W0(L)*TA21
C
      UC(II-1,JJ-1)=UC(II-1,JJ-1)+W2(L)*W0(L)*TA11
      UC(II,JJ)=UC(II,JJ)+W2(L)*W0(L)*TA22
      UC(II-1,JJ)=UC(II-1,JJ)+W2(L)*W0(L)*TA12
      UC(II,JJ-1)=UC(II,JJ-1)+W2(L)*W0(L)*TA21
C
      T6=A1B1*A2B2/R1
      T7=DLOG(R1)
      T8=A2B2*A2B2/R1
C
      F11=D11*T7+D12*T8
      F22=D41*T7+D42*T8
      F12=D22*T6
      F21=D32*T6
C
      IF(I.EQ.J.OR.I.EQ.JM1)THEN
      ELSE
      UR(II-1,JJ-1)=UR(II-1,JJ-1)+F11*W0(L)
      UR(II,JJ)=UR(II,JJ)+F22*W0(L)
      ENDIF
      UR(II-1,JJ)=UR(II-1,JJ)+F12*W0(L)
      UR(II,JJ-1)=UR(II,JJ-1)+F21*W0(L)
C
3     CONTINUE
2     CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C
C       CALCULATION OF THE DIAGONAL ENTRIES OF UC MATRIX.
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 59 I=1,NN
      SUMX(I)=0.
      SUMY(I)=0.
      DO 59 J=1,N
      JJ=2*J
      SUMX(I)=SUMX(I)+UC(I,JJ-1)
      SUMY(I)=SUMY(I)+UC(I,JJ)
 59   CONTINUE
      M=-1
      J=1
      DO 51 I=1,NN
      M=M+1
      UC(I,J)=-SUMX(I)
      UC(I,J+1)=-SUMY(I)
      IF(M.EQ.0)GOTO 51
      J=J+2
      M=-1
 51   CONTINUE
```

```
      WRITE(6,26)
      WRITE(6,505) ((UR(M,L),L=1,NN),M=1,NN)
      WRITE(6,26)
      WRITE(6,505) ((UC(M,J),J=1,NN),M=1,NN)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C  POST-MULTIPLY BY GAMMA INVERSE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 6 I=1,N
      II=2*I
      IIM1=II-1
      A0=1.
      DO 4 K=2,N
C
      UR(IIM1,1)=UR(IIM1,1)+UR(IIM1,2*K-1)*A0
      UR(II,1)=UR(II,1)+UR(II,2*K-1)*A0
      UR(IIM1,2)=UR(IIM1,2)+UR(IIM1,2*K)*A0
      UR(II,2)=UR(II,2)+UR(II,2*K)*A0
C
4     A0=-A0
C
      DO 5 K=2,N
C
      UR(IIM1,2*K-1)=-UR(IIM1,2*K-3)+2.*UR(IIM1,2*K-1)
      UR(II,2*K-1)=-UR(II,2*K-3)+2.*UR(II,2*K-1)
      UR(IIM1,2*K)=-UR(IIM1,2*K-2)+2.*UR(IIM1,2*K)
5     UR(II,2*K)=-UR(II,2*K-2)+2.*UR(II,2*K)
C
6     CONTINUE
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C AUGMENT FOR EQUILIBRIUM
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      NNP1=NN+1
      NNP2=NN+2
      DO 7 I=1,N
      UC(2*I-1,NNP1)=-1.
      UC(2*I,NNP2)=-1.
      UR(NNP1,2*I-1)=1.
7     UR(NNP2,2*I)=1.
C
      DO 8 I=2,N
      UC(2*I-1,NNP3)=Y(I)-Y(1)
      UC(2*I,NNP3)=X(1)-X(I)
      UR(NNP3,2*I-1)=Y(1)-Y(I)
8     UR(NNP3,2*I)=X(I)-X(1)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
```

```
C RE-ORDER SYSTEM BASED ON KNOWN BOUNDARY CONDITIONS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 13 I=1,N
      DO 13 K=1,2
      II=2*(I-1)+K
      KK=NTBC(I,K)
      IF(KK.EQ.0) GOTO 13
      DO 14 L=1,NNP3
      TEM=UR(L,II)
      UR(L,II)=-UC(L,II)
      UC(L,II)=-TEM
14    CONTINUE
13    CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C DETERMINE KNOWN RIGHT-HAND SIDE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 18 I=1,NNP3
      RHS(I)=0.
      DO 18 L=1,NN
18    RHS(I)=RHS(I)+UR(I,L)*BC(L)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C SOLVE FOR UNKNOWN BOUNDARY CONDITIONS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      NNP4=NN+4
      DO 23 I=1,NNP2
      PIVOT=0.
      DO 24 J=I,NNP3
      TEM=DABS(UC(J,I))
      IF(PIVOT.GE.TEM) GOTO 24
      PIVOT=TEM
      IPIVOT=J
24    CONTINUE
C
      IF(IPIVOT.EQ.I) GOTO 45
C
      DO 27 K=I,NNP3
      TEM=UC(I,K)
      UC(I,K)=UC(IPIVOT,K)
      UC(IPIVOT,K)=TEM
27    CONTINUE
C
      TEM=RHS(I)
      RHS(I)=RHS(IPIVOT)
      RHS(IPIVOT)=TEM
C
45    IP1=I+1
      DO 28 K=IP1,NNP3
```

```
      Q=-UC(K,I)/UC(I,I)
      UC(K,I)=0.
      RHS(K)=Q*RHS(I)+RHS(K)
      DO 29 J=IP1,NNP3
      UC(K,J)=Q*UC(I,J)+UC(K,J)
29    CONTINUE
28    CONTINUE
23    CONTINUE
C
      RHS(NNP3)=RHS(NNP3)/UC(NNP3,NNP3)
      DO 30 K=1,NNP2
      Q=0.
      DO 31 J=1,K
      Q=Q+UC(NNP3-K,NNP4-J)*RHS(NNP4-J)
31    CONTINUE
      RHS(NNP3-K)=(RHS(NNP3-K)-Q)/UC(NNP3-K,NNP3-K)
30    CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C PUT NODAL DISPLACEMENTS INTO KRHS VECTOR
C PUT NODAL FORCES INTO BC VECTOR
C OUTPUT VECTORS RHTS AND BC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 32 I=1,N
      DO 32 K=1,2
      KK=NTBC(I,K)
      II=2*(I-1)+K
      IF(KK.EQ.1) THEN
      TEMP=BC(II)
      BC(II)=RHS(II)
      RHS(II)=TEMP
      ELSE
      END IF
32    CONTINUE
C
      WRITE(6,300)
      WRITE(6,325)
      DO 33 I=1,N
      II=2*I
      IIM1=II-1
33    WRITE(6,350) I,RHS(IIM1),RHS(II),BC(IIM1),BC(II)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C PRE MULTIPLY BCI BY GAMMA INVERSE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SAV1=BC(1)
      SAV2=BC(2)
      A0=-1.
      DO 34 I=2,N
      II=2*I
      IIM1=II-1
```

```
      BC(1)=BC(1)+A0*BC(IIM1)
      BC(2)=BC(2)+A0*BC(II)
34    A0=-A0
C
      DO 35 I=2,N
      II=2*I
      IIM1=II-1
      SAV3=BC(IIM1)
      SAV4=BC(II)
      BC(IIM1)=2.*SAV1-BC(IIM1-2)
      BC(II)=2.*SAV2-BC(II-2)
      SAV1=SAV3
      SAV2=SAV4
35    CONTINUE
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C COMPUTE BOUNDARY STRESSES AND OUTPUT
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      WRITE(6,400)
      WRITE(6,425)
      DO 36 I=1,N
      II=2*I
      IIM1=II-1
      IF(I.EQ.1) THEN
      IM1=N
      IIM2=2*N
      ELSE
      IM1=I-1
      IIM2=II-2
      END IF
      IIM3=IIM2-1
      XM=X(I)-X(IM1)
      YM=Y(I)-Y(IM1)
      SF=DSQRT(XM*XM+YM*YM)
      CTF=YM/SF
      STF=-XM/SF
      SIGNN=(CTF*BC(IIM1)+STF*BC(II))/SF
      SIGNT=(CTF*BC(II)-STF*BC(IIM1))/SF
C     SIGTT=PR*SIGNN+(1.+PR)*(CTF*(RHS(II)-RHS(IIM2))-STF*(RHS(IIM1)
C    +-RHS(IIM3)))*2./SF
      SIGTT=0.
      WRITE(6,450) I,SIGNN,SIGNT,SIGTT
36    CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     FORMATS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
100   FORMAT(//,' BOUNDARY NODES AND PRESCRIBED BOUNDARY CONDITIONS')
150   FORMAT(/,' NODE',6X,'X',9X,'Y',8X,'CONDITIONS')
200   FORMAT(/,I5,F10.5,F10.5,5X,'FX =',F10.5,3X,'FY =',F10.5)
201   FORMAT(/,I5,F10.5,F10.5,5X,'FX =',F10.5,3X,'UY =',F10.5)
202   FORMAT(/,I5,F10.5,F10.5,5X,'UX =',F10.5,3X,'FY =',F10.5)
203   FORMAT(/,I5,F10.5,F10.5,5X,'UX =',F10.5,3X,'UY =',F10.5)
```

```
C
300   FORMAT(//,' DISPLACEMENTS AND FORCES AT ALL BOUNDARY NODES')
325   FORMAT(/,' NODE',6X,'UX',11X,'UY',11X,'FX',11X,'FY')
450   FORMAT(/,I7,2X,F10.6,3X,F10.6,3X,F10.6)
350   FORMAT(/,I5,2X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
400   FORMAT(//,' STRESSES ON ALL BOUNDARY ELEMENTS')
425   FORMAT(/,' ELEMENT',3X,'SIGMA NN',5X,'SIGMA NT',5X,'SIGMA TT')
460   FORMAT(2X,'S11= ',E12.6,2X,'S12= ',E12.6,2X,'S16= '
     +,E12.6)
465   FORMAT(2X,'SE11= ',E12.6,2X,'SE12= ',E12.6,2X,'SE16= '
     +,E12.6)
470   FORMAT(24X,'S(2,2)= ',E12.6,2X,'S(2,6)= ',E12.6,2X)
475   FORMAT(24X,'SE22= ',E12.6,2X,'SE26= ',E12.6,2X)
480   FORMAT(46X,'S66= ',E12.6)
485   FORMAT(46X,'SE66= ',E12.6)
500   FORMAT(1X,' MATERIAL PROPERTY S(I,J) AFTER TRANSFORMATION ')
403   FORMAT(1X,' THROUGH ANGLE PHI= ',F5.2,1X,'DEGREE')
510   FORMAT(1X,///)
50    FORMAT(1X,'POISSON RATIO =',F6.3)
26    FORMAT(1X,///)
405   FORMAT(1X,/)
505   FORMAT(10(1X,F5.2))
      STOP
      END
```

Appendix B) Computer listing of the program TEST

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     PROGRAM TEST
C
C     THIS PROGRAM CALCULATES THE VALUES OF THE ISOTROPIC AND BOTH
C     CASES OF  ORTHOTROPIC INFLUENCE FUNCTIONS.
C
C     INPUT DATA
C           N IS THE NUMBER OF THE FIELD POINTS.
C           PR IS THE POISSON'S RATIO. ISOTROPIC INFLUENCE FUNCTION.
C           G IS THE AXIAL SHEAR MODULUS OF ELASTICITY. ISOTROPIC
C           INFLUENCE FUNCTION.
C           EX IS THE LOGITUDINAL MODULUS OF ELASTICITY. ORTHOTROPIC
C           INFLUENCE FUNCTION.
C           EY IS THE TRANSVERSE MODULUS OF ELASTICITY. ORTHOTROPIC
C           INFLUENCE FUNCTION.
C           ES1 IS THE AXIAL SHEAR MODULUS. ORTHOTROPIC INFLUENCE
C           FUNCTION.
C           PRX1 IS  THE  POISSON'S  RATIO IN  THE  X-DIRECTION.
C           XS AND YS ARE THE X AND Y COORDINATE OF THE SOURCE
C           POINT.
C           X(I) AND Y(I) ARE THE COORDINATE OF THE FIELD POINTS
C           POINT.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 B(5),X(0:25),Y(0:25),LAMDA1,LAMDA2,MAT(5)
      COMPLEX*16 Z(2),ZE(2),ZD(2)
      PI=DACOS(-1.00D00)
      READ(5,*)N
      READ(5,*)PR,G
      READ(5,*) EX1,EY1,ES1,PRX1
      READ(5,*) EX2,EY2,ES2,PRX2
      READ(5,*) XS,YS
      READ(5,*) (X(I),Y(I),I=1,6)
      READ(5,*) (X(I),Y(I),I=7,12)
      READ(5,*) (X(I),Y(I),I=13,16)
      X(0)=1.4
      Y(0)=-1.4
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATIONS OF UR INFLUENCE FUNCTIONS FOR THE ISOTROPIC CASE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     WRITE(6,5501)
C     WRITE(6,20)
      DO 467 K=1,N
      RO=DSQRT((X(K)-XS)**2+(Y(K)-YS)**2)
      Q1=(X(K)-XS)/RO
      Q2=(Y(K)-YS)/RO
      UR11=((1.+PR)/(8*PI*G))*Q1*Q1-((3.-PR)/(8*PI*G))*DLOG(RO)
      UR12=((1.+PR)/(8*PI*G))*Q1*Q2
      UR21=((1.+PR)/(8*PI*G))*Q1*Q2
      UR22=((1.+PR)/(8*PI*G))*Q2*Q2-((3.-PR)/(8*PI*G))*DLOG(RO)
C     WRITE(6,2000) X(K),Y(K),UR11,UR12,UR21,UR22
467   CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

```
C
C     CALCULATIONS OF UC INFLUENCE FUNCTIONS FOR THE ISOTROPIC CASE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6,20)
      WRITE(6,5502)
      WRITE(6,20)
      DO 459 K=1,N
      RO=DSQRT((X(K)-XS)**2+(Y(K)-YS)**2)
      Q1=(X(K)-XS)/RO
      Q2=(Y(K)-YS)/RO
      VB=DSQRT((X(K)-X(K-1))**2+(Y(K)-Y(K-1))**2)
      VN1=2.*(Y(K)-Y(K-1))/VB
      VN2=-2.*(X(K)-X(K-1))/VB
      UC11=(1/(4*PI*RO))*(2*(1.+PR)*(VN1*Q1**3-VN2*Q2**3)+(1.-PR)*VN1
     +*Q1+(3.+PR)*VN2*Q2)
      UC12=(1/(4*PI*RO))*(2*(-1.-PR)*(VN2*Q1**3+VN1*Q2**3)+(1.+3.*PR)
     +*Q1*VN2+(3.+PR)*VN1*Q2)
      UC21=(-2*(1.+PR)*(VN2*Q1**3+VN1*Q2**3)+(3.+PR)*VN2*Q1
     ++(1.+3.*PR)*VN1*Q2)/(4*PI*RO)
      UC22=(2*(1.+PR)*(VN2*Q2**3-VN1*Q1**3)+(3.+PR)*VN1*Q1
     ++(1.-PR)*Q2*VN2)/(4*PI*RO)
      WRITE(6,2009) X(K),Y(K),UC11,UC12,UC21,UC22
459   CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     THE MATERIAL CONSTANTS FOR THE INFLUENCE FUNCTIONS OF THE
C     CASE 1.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      C11=1/EX1
      C22=1/EY1
      C66=1/(2*ES1)
      C12=-PRX1/EX1
      DUM1=(C12+C66)/C11
      DUM2=C22/C11
      DUM3=DSQRT(DUM2)
      RAD=DUM1*DUM1-DUM2
      MAT(1)=1.
      IF(ABS(RAD).LT.1.0E-03) GO TO 67
      IF(RAD.GT.0.) GO TO 68
      MAT(1)=-1
      MAT(2)=DSQRT(0.5*(DUM1+DUM3))
      MAT(3)=MAT(2)
      MAT(4)=DSQRT(0.5*(DUM3-DUM1))
      MAT(5)=-MAT(4)
      GO TO 69
67    MAT(1)=0.
68    DUM4=0.
      IF(MAT(1).GT.0.) DUM4=DSQRT(ABS(RAD))
      MAT(2)=DSQRT(DUM1+DUM4)
      MAT(3)=DSQRT(DUM1-DUM4)
      MAT(4)=0.
      MAT(5)=0.
69    CONTINUE
      DUM4=C12/C11
      DUM5=MAT(2)+MAT(3)
```

```
      DUM6=MAT(2)-MAT(3)
      IM=MAT(1)
      IF(IM.EQ.0) DUM6=DUM5
      IF(IM.EQ.-1) DUM6=2.*MAT(4)
      DUM1=2.*DUM1
      E11=-(DUM1+DUM3-DUM4)/(DUM5*4*PI)
      E12=-(DUM1-DUM3-DUM4)/(DUM6*4*PI)
      E21=DUM4/(4*PI)
      E22=-(2.*DUM2-DUM1*DUM4)/(DUM5*DUM6*4*PI)
      E31=(1.+DUM4/DUM3)/(DUM5*PI*4)
      E32=(1.-DUM4/DUM3)/(DUM6*PI*4)
      E41=-1./(4*PI)
      E42=(DUM1-2.*DUM4)/(DUM5*DUM6*4*PI)
      E51=-1./(4*PI)
      E52=-E42
      E61=-(DUM3+DUM4)/(DUM5*4*PI)
      E62=(DUM3-DUM4)/(DUM6*4*PI)
      D11=C11*E11+C12*E31
      D12=(C11*E12+C12*E32)
      D22=-C11*E22-C12*E42
      D32=D22
      DUM=DSQRT(C22/C11)
      D41=DUM*D11
      D42=-DUM*D12
C
      ARG=DSQRT(((C12+C66)/C11)**2-C22/C11)
      LAMDA1=DSQRT((C12+C66)/C11+ARG)
      LAMDA2=DSQRT((C12+C66)/C11-ARG)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATIONS OF UC INFLUENCE FUNCTIONS FOR THE ORTHOTRPOIC
C     MATERIALS OF CASE 1
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6,20)
      WRITE(6,6544)
      WRITE(6,20)
      DO 875 K=1,N
      R1=DSQRT((X(K)-XS)**2+LAMDA1**2*(Y(K)-YS)**2)
      R2=DSQRT((X(K)-XS)**2+LAMDA2**2*(Y(K)-YS)**2)
      T1=(X(K)-XS)/R1**2+(X(K-XS))/R2**2
      T2=(X(K)-XS)/R1**2-(X(K)-XS)/R2**2
      T3=LAMDA1*(Y(K)-YS)/R1**2+LAMDA2*(Y(K)-YS)/R2**2
      T4=LAMDA1*(Y(K)-YS)/R1**2-LAMDA2*(Y(K)-YS)/R2**2
      VB=DSQRT((X(K)-X(K-1))**2+(Y(K)-Y(K-1))**2)
      VN1=2.*(Y(K)-Y(K-1))/VB
      VN2=-2.*(X(K)-X(K-1))/VB
      UC11=(-E11*T1-E12*T2)*VN1+(-E51*T3-E52*T4)*VN2
      UC22=(-E61*T1-E62*T2)*VN1+(-E41*T3-E42*T4)*VN2
      UC12=(-E51*T3-E52*T4)*VN1+(-E31*T1-E32*T2)*VN2
      UC21=(-E21*T3-E22*T4)*VN1+(-E61*T1-E62*T2)*VN2
      WRITE(6,2009) X(K),Y(K),UC11,UC12,UC21,UC22
875   CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATIONS OF THE UR INFLUENCE FUNCTION FOR THE ORTHOTROPIC
C     MATERIALS OF CASE 1
```

```
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     WRITE(6,20)
C     WRITE(6,6545)
C     WRITE(6,20)
      DO 909 K=1,N
      R1=DSQRT((X(K)-XS)**2+LAMDA1**2*(Y(K)-YS)**2)
      R2=DSQRT((X(K)-XS)**2+LAMDA2**2*(Y(K)-YS)**2)
      ARGUM=X(K)-XS
      IF(ARGUM.NE.0.)THEN
      RATIO1=LAMDA1*(Y(K)-YS)/(X(K)-XS)
      RATIO2=LAMDA2*(Y(K)-YS)/(X(K)-XS)
      T6=DATAN(RATIO1)-DATAN(RATIO2)
      TANT1=DATAN(RATIO1)
      TANT2=DATAN(RATIO2)
      ELSE
      S1=LAMDA1*(Y(K)-YS)
      S2=LAMDA2*(Y(K)-YS)
      IF(S1.GT.0.)THEN
      TANT1=PI/2
      ELSE
      TANT1=-PI/2
      ENDIF
      IF(S2.GT.0.)THEN
      TANT2=PI/2
      ELSE
      TANT2=-PI/2
      ENDIF
      T6=TANT1-TANT2
      ENDIF
      T7=DLOG(R1)+DLOG(R2)
      T8=DLOG(R1)-DLOG(R2)
      UR11=D11*T7+D12*T8
      UR12=D22*T6
      UR21=D32*T6
      UR22=D41*T7+D42*T8
C     WRITE(6,2000) X(K),Y(K),UR11,UR12,UR21,UR22
909   CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     THE MATERIAL CONSTANTS FOR THE INFLUENCE FUNCTIONS OF THE
C     CASE 2.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      C11=1/EX2
      C22=1/EY2
      C66=1/(2*ES2)
      C12=-PRX2/EX2
      LAMDA=DSQRT((C12+C66)/C11)
C
      DUM1=(C12+C66)/C11
      DUM2=C22/C11
      DUM3=DSQRT(DUM2)
      DUM4=C12/C11
      DUM5=2.*DSQRT(DUM1)
      DUM1=2.*DUM1
C
```

```
      E11=-(DUM1+DUM3-DUM4)/(DUM5*4*PI)
      E12=-(DUM1-DUM3-DUM4)/(4*PI)
      E21=DUM4/(4*PI)
      E22=-(2.*DUM2-DUM1*DUM4)/(DUM5*4*PI)
      E31=(1.+DUM4/DUM3)/(DUM5*PI*4)
      E32=(1.-DUM4/DUM3)/(PI*4)
      E41=-1./(4*PI)
      E42=(DUM1-2.*DUM4)/(DUM5*4*PI)
      E51=-1./(4*PI)
      E52=-E42
      E61=-(DUM3+DUM4)/(DUM5*4*PI)
      E62=(DUM3-DUM4)/(4*PI)
C
      D11=C11*E11+C12*E31
      D12=(C11*E12+C12*E32)
      D22=-C11*E22-C12*E42
      D32=D22
      D41=DUM3*D11
      D42=-DUM3*D12
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATIONS OF UC INFLUENCE FUNCTIONS FOR THE ORTHOTRPOIC
C     MATERIALS OF CASE2
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6,20)
      WRITE(6,6546)
      WRITE(6,20)
      DO 885 K=1,N
      R1=DSQRT((X(K)-XS)**2+LAMDA**2*(Y(K)-YS)**2)
      T1=2*(X(K)-XS)/R1**2
      T2=-2*LAMDA*(X(K)-XS)*(Y(K)-YS)**2/R1**4
      T3=2*LAMDA*(Y(K)-YS)/R1**2
      T4=((X(K)-XS)**2*(Y(K)-YS)-LAMDA**2*(Y(K)-YS)**3)/R1**4
      VB=DSQRT((X(K)-X(K-1))**2+(Y(K)-Y(K-1))**2)
      VN1=2.*(Y(K)-Y(K-1))/VB
      VN2=-2.*(X(K)-X(K-1))/VB
      UC11=(-E11*T1-E12*T2)*VN1+(-E51*T3-E52*T4)*VN2
      UC22=(-E61*T1-E62*T2)*VN1+(-E41*T3-E42*T4)*VN2
      UC12=(-E51*T3-E52*T4)*VN1+(-E31*T1-E32*T2)*VN2
      UC21=(-E21*T3-E22*T4)*VN1+(-E61*T1-E62*T2)*VN2
      WRITE(6,2009) X(K),Y(K),UC11,UC12,UC21,UC22
885   CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATIONS OF THE UR INFLUENCE FUNCTION FOR THE ORTHOTROPIC
C     MATERIALS OF CASE2
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     WRITE(6,20)
C     WRITE(6,6547)
C     WRITE(6,20)
      DO 999 K=1,N
      R1=DSQRT((X(K)-XS)**2+LAMDA**2*(Y(K)-YS)**2)
      T6=(X(K)-XS)*(Y(K)-YS)/R1**2
      T7=2*DLOG(R1)
      T8=(Y(K)-YS)**2/R1**2
```

```
      UR11=D11*T7+D12*T8
      UR12=D22*T6
      UR21=D32*T6
      UR22=D41*T7+D42*T8
C     WRITE(6,2000) X(K),Y(K),UR11,UR12,UR21,UR22
999   CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     FORMATS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6544  FORMAT(18X,'UC INFLUNCE FUNCTION FOR CASE 1')
6546  FORMAT(18X,'UC INFLUNCE FUNCTION FOR CASE 2')
6545  FORMAT(18X,'UR INFLUNCE FUNCTION FOR CASE 1')
6547  FORMAT(18X,'UR INFLUNCE FUNCTION FOR CASE 2')
5501  FORMAT(2X,'X',7X,'Y',5X,'UR INFLUENCE FUNCTION OF ISOTROPY')
5502  FORMAT(2X,'X',7X,'Y',5X,'UC INFLUENCE FUNCTION OF ISOTROPY')
2000  FORMAT(F6.3,1X,F6.3,2X,'UR11=',F8.5,1X,'UR12=',F8.5,1X,'UR21='
     +,F8.5,1X,'UR22=',F8.5)
20    FORMAT(1X,/)
2009  FORMAT(F6.3,1X,F6.3,2X,'UC11=',F8.5,1X,'UC12=',F8.5,1X,'UC21='
     +,F8.5,1X,'UC22=',F8.5)
      STOP
      END
```

Appendix C) FEM and BEM input data for example problem two

NASTRAN INPUT DATA FOR EXAMPLE PROBLEM TWO

```
//KNIVDMA JOB (8282-16),'J. KATIBAI',MSGLEVEL=1,
// MSGCLASS=T,TIME=99,PRTY=7,NOTIFY=KNIVDM,
// USER=XXXXXX,PASSWORD=XXXXXXXX
//*MAIN SYSTEM=MVSA
//*FORMAT PR,DDNAME=,DEST=LOCAL
//*MAIN LINES=300,CARDS=500
// EXEC NASTRAN,WORKSP=3200,
/* ALTER=RF24D80,
// VSAPDSN='KNIVDM.VSAP.N19'
NASTRAN BUFFSIZE=5860
ID TRIANGULAR PROBLEM WITH 19 NODES
SOL 24
DIAG 16
TIME 200
$VSAP
$MERGE
$
$
CEND
TITLE= TRIANGULAR PROBLEM WITH 19 NODES
SUBTITLE= STATIC ANALYSIS TO CHECK DISPLACEMENTS
ECHO=SORT
$
SUBCASE 1
LABEL=STATIC ANALYSIS
SPC=100
LOAD=2
OLOAD=ALL
STRESS(VONMISES,PLOT)=ALL
DISP(PRINT)=ALL
$
BEGIN BULK
PARAM    AUTOSPC    YES
CQUAD4        12      1     246     268     273     247 0.
CQUAD4        17      1     263     264     269     268 0.
CQUAD4        18      1     245     263     268     246 0.
CQUAD4        22      1     259     260     265     264 0.
CQUAD4        23      1     258     259     264     263 0.
CQUAD4        24      1     244     258     263     245 0.
CQUAD4        27      1     255     256     261     260 0.
CQUAD4        28      1     254     255     260     259 0.
CQUAD4        29      1     253     254     259     258 0.
CQUAD4        30      1     243     253     258     244 0.
CQUAD4        32      1     239     238     257     256 0.
CQUAD4        33      1     240     239     256     255 0.
CQUAD4        34      1     241     240     255     254 0.
CQUAD4        35      1     242     241     254     253 0.
CQUAD4        36      1     232     242     253     243 0.
CTRIA3        37      1     256     257     261 0.
CTRIA3        38      1     260     261     265 0.
CTRIA3        39      1     264     265     269 0.
CTRIA3        40      1     268     269     273 0.
CTRIA3        41      1     247     273     231 0.
CTRIA3        42      1     238     229     257 0.
$
$   LCS.NAME = LOAD1    LOAD SET NUM IS             2
$
```

```
FORCE*                 2             257                                   1.0*FOE001
*FOE001 -1.389999986E-01-2.779999748E-02 0.000000000E+00                   LOAD1
FORCE*                 2             261                                   1.0*FOE002
*FOE002 -1.111999750E-01-5.559999868E-02 0.000000000E+00                   LOAD1
FORCE*                 2             265                                   1.0*FOE003
*FOE003 -8.339995146E-02-8.339995146E-02 0.000000000E+00                   LOAD1
FORCE*                 2             269                                   1.0*FOE004
*FOE004 -5.559999868E-02-1.111999750E-01 0.000000000E+00                   LOAD1
FORCE*                 2             273                                   1.0*FOE005
*FOE005 -2.779999748E-02-1.389999986E-01 0.000000000E+00                   LOAD1
GRID         229             1.000   0.000   0.000
GRID         231             0.000   1.000   0.000
GRID         232             0.000   0.000   0.000
GRID         238             0.833   0.000   0.000
GRID         239             0.667   0.000   0.000
GRID         240             0.500   0.000   0.000
GRID         241             0.333   0.000   0.000
GRID         242             0.167   0.000   0.000
GRID         243             0.000   0.167   0.000
GRID         244             0.000   0.333   0.000
GRID         245             0.000   0.500   0.000
GRID         246             0.000   0.667   0.000
GRID         247             0.000   0.833   0.000
GRID         253             0.167   0.167   0.000
GRID         254             0.333   0.167   0.000
GRID         255             0.500   0.167   0.000
GRID         256             0.667   0.167   0.000
GRID         257             0.833   0.167   0.000
GRID         258             0.167   0.333   0.000
GRID         259             0.333   0.333   0.000
GRID         260             0.500   0.333   0.000
GRID         261             0.667   0.333   0.000
GRID         263             0.167   0.500   0.000
GRID         264             0.333   0.500   0.000
GRID         265             0.500   0.500   0.000
GRID         268             0.167   0.667   0.000
GRID         269             0.333   0.667   0.000
GRID         273             0.167   0.833   0.000
MAT8*                  1 2.100800000E-01 1.400560000E-02 0.300400000E+00*MA2001
*MA2001  2.994010000E-03
$ GENERAL RIGID ELEMENTS
PSHELL*                1               1 1.000000000E+00
$
$       SPC2
$
SPC*                   2             229 3456            0.000           X1
SPC*                   2             231 3456            0.000           X1
SPC*                   2             232 3456            0.000           X1
SPC*                   2             238 3456            0.000           X1
SPC*                   2             239 3456            0.000           X1
SPC*                   2             240 3456            0.000           X1
SPC*                   2             241 3456            0.000           X1
SPC*                   2             242 3456            0.000           X1
SPC*                   2             243 3456            0.000           X1
SPC*                   2             244 3456            0.000           X1
SPC*                   2             245 3456            0.000           X1
SPC*                   2             246 3456            0.000           X1
```

```
SPC*            2               247 3456        0.000           X1
SPC*            2               253 3456        0.000           X1
SPC*            2               254 3456        0.000           X1
SPC*            2               255 3456        0.000           X1
SPC*            2               256 3456        0.000           X1
SPC*            2               257 3456        0.000           X1
SPC*            2               258 3456        0.000           X1
SPC*            2               259 3456        0.000           X1
SPC*            2               260 3456        0.000           X1
SPC*            2               261 3456        0.000           X1
SPC*            2               263 3456        0.000           X1
SPC*            2               264 3456        0.000           X1
SPC*            2               265 3456        0.000           X1
SPC*            2               268 3456        0.000           X1
SPC*            2               269 3456        0.000           X1
SPC*            2               273 3456        0.000           X1
$
$   LCS.NAME IS X2      SET ID NUM IS           1
$
SPC*            1               232 123         0.000           X2
SPC*            1               242            1-1.989999786E-02X2
SPC*            1               242            2-2.790499878E+01X2
SPC*            1               242 3           0.000           X2
SPC*            1               241            1-7.939994335E-02X2
SPC*            1               241            2-5.592498779E+01X2
SPC*            1               241 3           0.000           X2
SPC*            1               240            1-1.788999438E-01X2
SPC*            1               240            2-8.409498596E+01X2
SPC*            1               240 3           0.000           X2
SPC*            1               239            1-3.177999854E-01X2
SPC*            1               239            2-1.123959961E+02X2
SPC*            1               239 3           0.000           X2
SPC*            1               238            1-4.964999557E-01X2
SPC*            1               238            2-1.408139954E+02X2
SPC*            1               238 3           0.000           X2
SPC*            1               229            1-7.149999738E-01X2
SPC*            1               229            2-8.587998962E+01X2
SPC*            1               229 3           0.000           X2
SPC*            1               231            1-3.569999695E+01X2
SPC*            1               231            2-7.149999738E-01X2
SPC*            1               231 3           0.000           X2
SPC*            1               247            1-2.478999329E+01X2
SPC*            1               247            2-4.963999987E-01X2
SPC*            1               247 3           0.000           X2
SPC*            1               246            1-1.586799908E+01X2
SPC*            1               246            2-3.177999854E-01X2
SPC*            1               246 3           0.000           X2
SPC*            1               245            1-8.924999237E+00X2
SPC*            1               245            2-1.779999733E-01X2
SPC*            1               245 3           0.000           X2
SPC*            1               244            1-3.965869904E+00X2
SPC*            1               244            2 7.942855358E-02X2
SPC*            1               244 3           0.000           X2
SPC*            1               243            1-9.920599461E-01X2
SPC*            1               243            2-1.986899972E-02X2
SPC*            1               243 3           0.000           X2
SPCADD,100,1,2
```

BEM INPUT DATA FOR EXAMPLE PROBLEM TWO

```
19
0.21008403 0.014005602 0.00299401 0.3004
0 0 .1667 0 .3333 0 .5 0 .6667 0 .8333 0 1. 0. .8333 .1667 .6667 .3333
.5 .5 .3333 .6667 .1667 .8333 0 1 0 .8333 0 .6667 0 .5 0 .375 0 .25
0 .125
1 1
1 2
2 1
2 2
3 1
3 2
4 1
4 2
5 1
5 2
6 1
6 2
7 1
7 2
13 1
13 2
14 1
14 2
15 1
15 2
16 1
16 2
17 1
17 2
18 1
18 2
19 1
19 2
0 0
2 1 -0.0199
2 2 -27.905
3 1 -0.0794
3 2 -55.925
4 1 -0.1789
4 2 -84.095
5 1 -0.3178
5 2 -112.396
6 1 -0.4965
6 2 -140.814
7 1 -0.7150
7 2 -85.880
8 1 -0.139
8 2 -0.0278
9 1 -0.1112
9 2 -0.0556
10 1 -0.0834
10 2 -0.0834
11 1 -0.0556
11 2 -0.1112
12 1 -0.0278
12 2 -0.139
13 1 -35.7
```

13	2	-0.7150
14	1	-24.790
14	2	-0.4964
15	1	-15.868
15	2	-0.3178
16	1	-8.925
16	2	-0.1789
17	1	-5.02
17	2	-0.1005
18	1	-2.2313
18	2	-0.0447
19	1	-0.5578
19	2	-0.0118
0	0	0

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