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ABSTRACT

THE EFFECT OF INFORMATION ON THE BEHAVIORS OF SECURITY PRICE AND TRADING VOLUME

By

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The price-volume relationship and the effect of the information arrival process on price and volume have been extensively examined by many authors. However, as noted by Hal R. Varian[1], little analytical work has been done on the effect of diverse beliefs arising from heterogeneous information on the behavior of price and trading volume. In particular, little theoretical work has been done on the relationship between the nature of information and the behavior of security price and trading volume. The objective of the three essays in this dissertation is to investigate theoretically and empirically the impact of information characteristics and heterogeneous beliefs on security price and trading volume.

The first essay develops a theoretical model in a noisy rational expectations equilibrium framework incorporating heterogeneous information and diverse beliefs. The quality of information is characterized by individual investor's confidence and the variability of opinion across investors. The effects of these two characteristics of information on security price and volume are examined. It is found that when the market is confidence driven, large trading volume

normally accompanies large price variability. When the market is consensus driven, price variability is accompanied by low trading volume. Also, caution needs to be exercised when attempting to use price and volume to measure information content.

The second essay similarly develops a theoretical model relating security price and volume reaction to earnings announcements. A potentially asymmetric price-volume relationship emerges from the theoretical model depending on investor optimism or pessimism just prior to the announcement and the effect of the announcement on investor uncertainty. Empirical tests using daily CRSP returns, Media General's Trading Volume Tapes, Compustat, and Lynch, Jones and Ryan's Institutional Brokers Estimate System database are developed to examine the model. Empirical evidence is consistent with the asymmetric response of price and volume to good news and bad news announcements according to the theory.

The third essay develops a statistical test for estimating the onset and duration of security price and trading volume responses to new information. It extends the analysis of Hillmer and Yu (1979) by allowing a dependent relationship between price and volume. The dependent relationship between price and volume is addressed by orthogonalizing one market attribute with respect to the other. However, the resulting statistical test may provide biased estimates of the onset and duration of market responses to new information (see Giliberto (1985)). A practical procedure for implementing the statistical test is then prescribed. The statistical test allowing dependence is compared to the Hillmer and Yu (1979) and Pincus (1983) tests in simulations of real world responses to information.

1. Varian, H. R. "Differences of Opinion in Financial Markets." Working Paper, University of Michigan (March 1988).

DEDICATION

This dissertation is dedicated to my wife, Mun Chee Chan, my son, Kyle, my daughter, Kellie, and my mother, Lam Heung.

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Obviously, this dissertation could not have been completed without substantial assistance. Research resources and computer facilities provided by both Michigan State University and Indiana University proved quite valuable. Committee members Ann Kremer and Dale Domain will never know the true extent of my thanks for their help and comments. Most of all, I thank my committee chairman, Kirt Butler, for all his help and advice. I will never be able to repay him for his dedication to my work and career.

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CHAPTER ONE: INTRODUCTION

The relationship between stock prices and trading volume has interested practitioners and financial economists for many years. Price and volume are widely used by financial analysts as market sentiment indicators to gauge rallies and declines, to forecast bull and bear markets, and to predict market turning points. For example, a lot of technical analysts agree on these principles:[1]

1. A price rise accompanied by expanding volume is a normal market characteristic and has no implications so far as a potential trend reversal is concerned.
2. A rally which reaches a new (price) high on expanding volume but whose overall level of activity is lower than the previous rally is suspect and warns of a potential trend reversal.
3. A rally which develops on contracting volume is suspect and warns of a potential trend reversal.

Technical analysts generally also believe that security return and volume are associated and that volume changes may presage price changes. Bernstein [1983] states that an investor can predict the movement of the stock market if he can predict volume.[2]

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[1]Martin J. Pring, "Technical Analysis Explained" (New York: McGraw-Hill, 1985, p.149.

[2]Peter L. Bernstein, "The Volume Indicator, Refurbished, and Retained," Peter L. Bernstein, Inc., April 1, 1983.

To the financial economist, price-volume relationship has important implications for understanding the microstructure of financial markets. Several important works have been developed to explain the impact of the rate of information flow and the way information is disclosed on price-volume relation.[3] Some researchers also believe that if price and volume are jointly determined, incorporating volume information into event studies will improve the power of test statistics.[4] Price-volume relationship is also important for identifying the empirical price distributions of speculative assets including options and futures. It is a common belief that speculative prices follow either the stable Paretian distribution with infinite variances or a mixture of distributions with different conditional variances.[5] On the other hand, volume information could

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[3]See for example Copeland, T. E. "A Model of Asset Trading under the Assumption of Sequential Information Arrival." The Journal of Finance 31 (September 1976), 1149-1168.

Morse, D. "Asymmetrical Information in Securities Markets and Trading Volume." Journal of Financial and Quantitative Analysis 15 (March 1980), 1129-1148.

Jennings, R. H., L. T. Starks, and J. C. Fellingham. "An Equilibrium Model of Asset Trading with Sequential Information Arrival." Journal of Finance, 36 (March 1981), 143-161.

[4]Richardson, G, S. E. Sefcik, and R. Thompson. "A Test of Dividend Irrelevance Using Volume Reaction to a Change in Dividend Policy." Journal of Financial Economics, 17 (Dec. 1986), 313-333.

[5]See, for example, Epps, T.W., and M. L. Epps. "The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications or the Mixture-of-Distributions Hypothesis." Econometrica 44 (March 1976), 305-321.

provide a good proxy for the changing variance,[6] providing useful insight concerning the behavior of price around the event day in event studies.

Empirical studies in the accounting literature have used security prices and trading volume to measure the effects of new public information on financial markets.[7] Traditionally, price changes are used to measure the effect of informativeness.[8] Trading volume, on the other hand, is employed by researchers as a measure of consensus among investors[9] or of the information content of an event.[10]

Tauchen, G. E., and M. Pitts. "The Price Variability-Volume Relationship on Speculative Markets." Econometrica 51 (March 1983), 485-505.

[6]Rogalski, R. J. "The Dependence of Prices and Volume." The Review of Economics and Statistics 36 (may 1978), 268-274.

[7]Imhoff, E. A. Jr., and G. J. Lobo. "Information Content of Analysts' Composite Forecast Revisions." Journal of Accounting Research 22, No. 26 Autumn 1984, 541-554.

[8]Atiase, R. K. "Predisclosure Information, Firm Capitalization and Security Price Behavior Around Earnings Announcements." Journal of Accounting Research 23 (1985), 21-35.

Beaver, W. H. "The Information Content of Annual Earnings Announcements." Empirical Research in Accounting: Selected Studies. Supplement to Journal of Accounting Research 6 (1968), 67-92.

[9]Bamber, L. S. "The Information Content of Annual Earnings Releases: A Trading Volume Approach." Journal of Accounting Research 24 (Spring 1986), 40-56.

[10]Lakonishok, J., and T. Vermaelen. "Tax-Induced Trading around Ex-Dividend Days." Journal of Financial Economics 16(July 1986), 287-319.

Pincus, M. "Information Characteristics of Earnings Announcements and Stock Market Behavior." Journal of Accounting Research 21 (Spring 1983), 155-183.

The price-volume relationship and the effect of the information arrival process on price and volume have been extensively examined by many authors. However, as noted by Hal R. Varian,[11] little analytical work has been done on the effect of diverse beliefs arising from heterogeneous information on the behavior of price and trading volume. In particular, little theoretical work has been done on the relationship between the nature of information and the behavior of security price and trading volume. The objective of this study is to investigate theoretically and empirically the impact of information characterisitics and heterogeneous beliefs on security price and trading volume. The remainder of this dissertation is organized in this manner. Chapter 2 develops a theoretical model in a noisy rational expectations equilibrium framework incorporating heterogeneous information and diverse beliefs. The quality of information is characterized by individual investor's confidence and the variability of opinion across investors. The effects of these two characteristics of information on security price and volume are examined. The conclusion is that when the market is confidence driven, large trading volume normally accompanies large price variability. When the market is consensus driven, price variability is

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[11]Varian, H. R. "Differences of Opinion in Financial Markets." Working Paper, University of Michigan (March 1988).

accompanied by low trading volume. Also, caution needs to be taken when we try to use price and volume to measure information content. Chapter 3 employs earnings announcement as a source of information to study how security price and volume react to good news and bad news. In a framework similar to Chapter 2, a theoretical model relating earnings announcements to security price and volume reaction is first developed. Empirical tests using daily CRSP returns, Media General's Trading Volume Tapes, Compustat, and Lynch, Jones and Ryan's Institutional Brokers Estimate System database are then developed to examine the model. Empirical evidence is consistent with the asymmetric response of price and volume to good news and bad news announcement according to the theory. Chapter 4 attempts to develop a multivariate statistical technique using security price and trading volume to measure the adjustment speed of financial market to new information disclosure. The technique serves to detect the point in time when the market attributes begin to react to the news and the point in time when the reaction is over. Simulation studies are conducted to confirm its properties.

CHAPTER 2: THE EFFECT OF DIVERGENT OPINIONS ON SECURITY PRICES AND TRADING VOLUME

Abstract

This essay develops a single-period rational expectation model with noise and diverse beliefs to investigate the effect of investors' divergent opinions on the behavior of prices and trading volume. Information characteristics in terms of investor's confidence in his forecast and diversity of opinion are employed to analyze the effects of heterogeneous information on equilibrium price and trading volume.

I. Introduction

Security prices and trading volume are the two most widely reported financial variables by the news media. What kind of insight about investors' opinions in securities can we gain from prices and trading volume? The relationship between security prices and trading volume has interested practitioners and financial economists for many years. Price and volume are popularly used by financial analysts as market sentiment indicators to gauge rallies and declines, to forecast bull and bear markets, and to predict market turning points. To the financial economist, the price and volume relationship has important implications for understanding the microstructure of financial markets, for event studies, and for identifying the empirical price distributions of speculative assets including options and futures. Empirical studies in the accounting literature have used security prices and trading volume to measure the effects of new public information on financial markets. In general, a public disclosure can cause a precision effect and/or a consensus effect. Precision measures the gain of knowledge and consensus measures the extent of agreement among agents caused by the new information. Traditionally, price changes are used to measure the effect of informativeness (Beaver [1968] and Atiase [1985]). It has also been used as a measure of information content (Beaver, Lambert, and Ryan [1987]). Trading volume, on the other hand, is employed by researchers as a measure of consensus among investors (Beaver [1968], Morse [1981], and Bamber [1987]) or of the information content of an event (Beaver [1968], Lakonishok and Vermaelen [1986], Morse [1981], Ro [1981], and Pincus [1983]).

The objective of this essay is to investigate the effect of investors' divergent opinions induced by information disclosure on the behavior of prices and trading volume. A single-period rational expectation model with noise and diverse beliefs of the sort examined by Admati [1985], Varian [1987], and Diamond and Verrecchia [1981] is developed. Information characteristics in terms of precision and consensus will be formally defined and incorporated into the model in order to analyze the effects of heterogeneous information on equilibrium price and trading volume. The rest of the chapter is divided into five sections. Section II describes the market structure, the demand for the risky asset, and its price at equilibrium. Noise trading is allowed to exist in the economy. Section III defines the precision and the consensus effects induced among investors by information disclosure. The effect of precision and consensus on price variability and trading volume are analyzed in Sections IV and V. Section VI discusses the circumstances in which the separate effects of precision and consensus may be observed based on price variability and trading volume. In most cases it is impossible to use either price or volume alone to measure information content.

II. Market Structure

A two-asset single period model is developed in which investors have different endowments of wealth and identical initial beliefs. Investors may have diverse preferences, but all investors have a negative exponential utility function for wealth and all their preferences exhibit constant absolute risk tolerance. During the period, a different signal is observed by each investor, causing him to have different expectations regarding the final price of each asset. Another ingredient of this model is the existence of noise in the form of random supply and demand of the risky asset.

IIA. Assumptions

(A1) Population

There are two groups of traders in the market. The first group is the diversely informed investors who trade to maximize their utilities subject to their budget constraints. This group is composed of I investors, indexed by $i = 1,2,\ldots,I$. The second group is the liquidity traders (noise traders) who submit their demand orders according to their liquidity needs. In general, noise trading can exist in different forms. It might be caused by some trade of a nonspeculative nature such as for life-cycle or liquidity reasons. It can be caused by some traders lacking perfect knowledge of the market structure, or by agents who do not know the realized aggregate endowment, as in Diamond and Verrecchia (1981). In a recent article, Trueman (1988) argues that noise trading can ensue from the incentive of the investment funds manager which is related to investors' perceptions of his ability. In this analysis, noise trading comes only from liquidity trading. Therefore, the supply per capita of the risky asset is assumed to be the realization of a random variable $\tilde{Z}$. Trades from this group are assumed to arrive at the market in a random fashion and constitute the exogenous noise in the economy.

(A2) **Assets**

There are only two assets in the economy: a riskless bond with known payoff and a risky asset with uncertain payoff $\tilde{U}$. The realizations of $\tilde{U}$ are given by U. Both the risky and the riskless asset pay off in a single consumption good. The riskless bond serves as numeraire and each unit yields one unit of the consumption good. That is, the return to one unit of the riskless asset is unity. No consideration is given to time preference since it would only obscure the analysis.

(A3) Endowments

Each informed investor i (i = 1,2...I) is endowed with risky asset D_{0i} and riskless bond B_{0i}. Assume that the total endowment of the risky asset is held by the informed investors and the net holding of the risky asset by the liquidity traders is zero. Let Z_0 be the total per-capita supply of the risky asset. Then

$$\Sigma_i D_{0i} = Z_0. \tag{1}$$

(A4) Preferences

Every investor i has a negative exponential utility function for wealth w of the consumption good given by:

$$U_i(w) = -\exp(-w/r_i) \tag{2}$$

where investor i exhibits constant absolute risk tolerance r_i.

(A5) Information

At the beginning of the period, every investor has the same prior beliefs about the risky asset's uncertain end-of-period payoff $\tilde{U}$ which is believed to be normally distributed with mean M and variance V. During the period, each investor i receives information Y_i concerning the liquidating value $\tilde{U}$ of the risky asset,

$$\tilde{Y}_i = \tilde{U} + \tilde{n} + \tilde{\epsilon}_i \tag{3}$$

where $\tilde{n}$ is the common noise normally distributed with mean 0 and variance N, and $\tilde{\epsilon}_i$ is the idosyncratic noise term normally distributed with mean zero and variance S_i. Note that $\bar{n} > 0$ implies that investors as a whole are optimistic about the liquidating value of the risky asset, and $\bar{n} < 0$ implies that they are pessimistic. It is assumed that $\tilde{U}$, $\tilde{n}$ and $\tilde{\epsilon}$ are independent of each other. Also the $\tilde{\epsilon}_i$ are independent across all investors, $E[\tilde{\epsilon}_i, \tilde{\epsilon}_j] = 0$ for $i \neq j$. Following Admati [1985], we assume that the variance of $\tilde{\epsilon}_i$ is uniformly bounded, and that,

$$(\Sigma_i \tilde{Y}_i)/I = \tilde{U} + \tilde{n} \quad \text{almost surely.} \tag{4}$$

As the idiosyncratic noise terms $\tilde{\epsilon}_i$ are aggregated across investors, the law of large numbers causes them to converge almost surely to their mean of zero.

Investors submit their buy and sell orders to the auctioneer based on the information they receive during the period. The liquidity traders submit orders randomly. The total per-capital supply of the risky asset net of liquidity trading is a random variable $\tilde{Z}$ with mean Z_0 and variance approaching ∞. The assumption of liquidity trading implies that $\tilde{Z}$ is independent of $\tilde{U}$, $\tilde{n}$, and $\tilde{\epsilon}_i$. Through this exchange of assets, a new equilibrium price P for the risky asset is established. At the end of the period, every agent liquidates his holdings of the two assets and consumes them. Let D_i and B_i be investor i's holdings of the risky and the riskless asset at the end of the transaction. The objective of each investor is to maximize his expected utility of terminal consumption at the end of the period,

$$E_i[-\exp(-r_i^{-1}(D_i\tilde{U} + B_i))] \tag{5}$$

$E_i[.]$ is the expectation operator of investor i based on his own information. The terminal wealth $D_i\tilde{U} + B_i$ is to be consumed at the end of the period. The budget constraint of investor i is given by,

$$D_iP + B_i = D_{0i}P + B_{0i}. \tag{6}$$

IIB. Definition of Rational Expectations Equilibrium

Following Admati [1985], and Diamond and Verrecchia [1981], the rational expectations equilibrium for the finite economy is defined as the price $\tilde{P}$ and allocation functions $D_i(\tilde{Y}_i, \tilde{P}_i)$ and $B_i(\tilde{Y}_i, \tilde{P}_i)$ for all i = 1,2...I such that

a) $\tilde{P}$ is $(\tilde{U} + \tilde{n}, \tilde{Z})$ measurable;

b) $[D_i(\tilde{Y}_i, \tilde{P}), B_i(\tilde{Y}_i, \tilde{P})] \in \arg\max_{D_i} E[-\exp(r_i^{-1}(D_i\tilde{U}+B_i))|\tilde{Y}_i]$

subject to $D_iP + B_i = D_{0i}P + B_{0i}$;

c) $\Sigma_i D_i(\tilde{Y}_i, \tilde{P}) = \tilde{Z}$ almost surely.

Note that conditional on $\tilde{Y}_i = Y_i$, $\tilde{U}$ is normally distributed with mean and variance as follows:

$$E[\tilde{U}|\tilde{Y}_i = Y_i] = M + \beta_i(Y_i - M)$$
$$Var(\tilde{U}|\tilde{Y}_i = Y_i) = Var(\tilde{U}) - \beta_i Var(\tilde{U}),$$

where

$$\beta_i = Var(\tilde{U})/Var(\tilde{Y}_i) = V/(V + N + S_i).$$

The distribution of $\exp\{-r_i^{-1}(D_i\tilde{U} + B_i)\}$ conditional on $\tilde{Y}_i = Y_i$ is lognormal. Direct computation yields

$$E[-\exp\{-r_i^{-1}(D_i\tilde{U} + B_i)\}|\tilde{Y}_i = Y_i]$$
$$= -\exp\{-r_i^{-1}(D_iE[\tilde{U}|\tilde{Y}_i = Y_i] + B_i) + 1/2(r_i^{2})D_i^{2}Var(\tilde{U}|\tilde{Y}_i = Y_i)\}.$$

Since an exponential function is strictly increasing in its exponent, the maximization of (5) becomes

$$\mathrm{Max}_{D_i}\{r_i^{-1}(D_iE[\tilde{U}|\tilde{Y}_i = Y_i] + B_i) - 1/2(r_i^{-2})D_i^{2}Var(\tilde{U}|\tilde{Y}_i = Y_i)\},$$

subject to $B_i = D_{0i}\tilde{P} + B_{0i} - D_i\tilde{P}$

and its unique solution is provided by the first order condition. After determining individual i's optimal demand for the risky asset, we can solve for the risky asset's equilibrium price by using the market clearing condition

(1). The results are summarized below:

Lemma 1:

(a) Investor i's demand for the risky asset conditional on $\tilde{Y}_i = Y_i$ is given by:

$$D_i = \frac{r_i[M + V(V + N + S_i)^{-1}(Y_i - M) - P]}{V - V^2(V + N + S_i)^{-1}} \tag{7}$$

(b) The equilibrium price of the risky asset is a linear function of the form:

$$\tilde{P} = (1 - A)M + A(\tilde{U} + \tilde{n}) - B\tilde{Z} \tag{8}$$

$$\text{where } A = \{\Sigma_i[(r_i/\sigma_i^2)V(V + N + S_i)^{-1}]\}B , \tag{9}$$

$$\sigma_i^2 = \mathrm{Var}[\tilde{U}|\tilde{Y}_i = Y_i] = V - V^2(V + N + S_i)^{-1} \tag{10}$$

$$\text{and } B = [1/\Sigma_i(r_i/\sigma_i^2)]. \tag{11}$$

Proof: See Appendix

III. Definition of Information Characteristics

The precision effect is measured by the variability of each agent's observed signal about the unknown value of the risky asset. Consensus on the other hand is a measure of the degree of agreement among different agents. It is measured by the degree of dispersion of opinions among the agents. Suppose we have two financial analysts A and B forecasting the value of a stock at the end of the period. At the start of the period, before new information about the company is released, analyst A's forecast is $50 with high and low being $45 and $55, while analyst B's figure is $50 with high being $60 and low $40. After the information is announced, A revises his forecast to $55 per share and B to $65 per share. However, A's high and low are $57 and $53, while B's are $70 and $60. The smaller post-information high-low spreads for both A and B's forecast reflect the precision effect induced by the information. Both A's and B's uncertainty about the unknown value of the stock become less. However, the information has induced a weaker consensus between A and B about the expected value of the stock. In this study, the precision effect of information follows the standard definitions. The consensus effect between two agents i and j is usually defined as the correlation coefficient between their diverse opinions, $\tilde{Y}_i$ and $\tilde{Y}_j$. This definition, however, is not very descriptive of the overall consensus in the economy. Therefore, this study develops a different and yet intuitive definition of consensus.

(1) Precision of information for investor i, Θ_i, is defined by

$$\ln[1/\mathrm{Var}(\tilde{Y}_i)] = \ln[1/(V + N + S_i)] \tag{12}$$

The precision effect for each agent is simply defined as the inverse of the variability of his signal since less variability means more precision.

(2) Consensus among the investors on the liquidating value $\tilde{U}$ of the risky asset induced by the observation of information, ϕ_L, is defined by:

$$\ln[1/(\Sigma_i S_i)] \tag{13}$$

Consensus is defined as the inverse of the dispersion of each agent's opinion from the mean consensus opinion since less dispersion of opinion means more consensus. From equations (3) and the independence of idiosyncratic noise across investors, the dispersion of agent i's opinion from the average opinion is

$$E(\tilde{Y}_i - \Sigma_i \tilde{Y}_i/I)^2 = E(\tilde{\epsilon}_i - \Sigma_i \tilde{\epsilon}_i/I)^2 = S_i.$$

The second equality is true because equation (3) implies that $\Sigma_i \tilde{\epsilon}_i$ converges to zero. Adding across all agents and taking the inverse yields the definition for consensus in equation (13).

IV. Mean-Variance Analysis of the Change in Price

In this section, we analyze the effects of precision θ_i and consensus ϕ_L on the mean and variance of the price change induced by the new information. The change in price is the new equilibrium price minus the beginning price P_0. Based on the equilibrium price given in Lemma 1b, the mean and the variance of the change in price conditional on the beginning price and the supply $\tilde{Z}$ are respectively given by

$$E(\Delta\tilde{P}|P_0,\ \tilde{Z} = Z) = M - BZ - P_0 \tag{14}$$

and $$Var(\Delta\tilde{P}|P_0,\ \tilde{Z} = Z) = A^2(V + N). \tag{15}$$

To examine the comparative statics of the expected change in price and the variance of the $\Delta\tilde{P}$ in terms of the precision and consensus effects, we take the derivative of the mean and the variance with respect to each effect while holding the other constant. Some preliminary results prove useful.

<u>Lemma 2</u>: By allowing N and S_i to vary with precision θ_i and consensus ϕ_L, we have:

(a) If ϕ_L is kept constant, then

(i) $dN/d\theta_i + dS_i/d\theta_i = -(V + N + S_i)$ for all i;

(ii) $\Sigma_{j=1}^{I} dS_j/d\theta_i = 0$ for all i; and

(iii) $dA/d\theta_i > 0$, and $dB/d\theta_i < 0$ for all i.

(b) If θ_i is kept constant, then

(i) $dN/d\phi_L + dS_i/d\phi_L = 0$ for all i;

(ii) $\Sigma_{i=1}^{I} dS_i/d\phi_L = -(\Sigma S_i)$; and

(iii) $dA/d\phi_L = dB/d\phi_L = 0$.

Proof: See Appendix.

Theorem 1: If consensus ϕ_L about the risky asset's liquidating value is kept constant, then

(a) An increase (decrease) in information precision about the value of the risky asset for all agents will increase (decrease) the expected change in asset price.

(b) The effect of information precision on the variance of the price change is indeterminate. However, if $S_i < (V + N)$, then an increase in information precision will lead to an increase in price variability. If the agents are symmetrically informed ($S_i = S$ for all i), then an increase in information precision will also lead to an increase in price variability provided $S < (V + N)$. The price variability will decrease if $S > (V + N)$.

Proof of Theorem 1: See Appendix.

<u>Theorem 2</u>: If every agent i's information precision Θ_i is kept constant, then

(a) A change in consensus has no effect on the expected change in price.

(b) An increase (decrease) in consensus will increase (decrease) the variance of the change in price.

<u>Proof of Theorem 2</u>:

(a) By taking the first derivative of the expected change in price given P_0 and Z with respect to ϕ_L, the result follows directly from lemma 2(b)(iii).

(b) The first derivative of $Var(\Delta P | P_0 , Z = Z)$ with respect to ϕ_L is $A^2(dN/d\phi_L) + 2A(V + N)(dA/d\phi_L)$. By lemma 2(b), $dN/d\phi_L$ is positive and $dA/d\phi_L$ is zero, hence the result.

(q.e.d.)

V. Volume of Trade

In this section, the trading volume consequences of the effects of precision and consensus about the value of the risky asset are analyzed. The results developed are based on the assumption that all agents are symmetrically informed about the value of the risky asset, that is, $S_i = S_j$ for all i and j. When the agents are asymmetrically informed, the effects of precision and consensus on the volume of trade become uncertain.

The overall trading volume after the information disclosure is given by:

$$\tilde{T} = 1/2\ \Sigma_i |\tilde{D}_i - D_{0i}| \tag{16}$$

Since $\tilde{Y}_i$ is normally distributed, so is $\tilde{D}_i$ which is a linear function of $\tilde{Y}_i$. Let $\tilde{X}_i$ be the net demand $(\tilde{D}_i - D_{0i})$ of agent i. From the expression for $\tilde{D}_i$, it can be shown that the expected value (μ_i) and the variance (σ'^2_i) of $\tilde{X}_i$ given D_{0i} are respectively given by:

$$r_i BZ/[V - V^2(V + N + S_i)^{-1}] - D_{0i}, \text{ and} \tag{17}$$

$$\frac{r_i^2\{V^2(V + N + S_i)^{-2}S_i + [V(V + N + S_i)^{-1} - A](V + N)\}}{[V - V^2(V + N + S_i)^{-1}]^2}. \tag{18}$$

Since $\tilde{X}_i$ is normally distributed with mean μ_i and variance σ'^2_i, the expected value of the absolute value of $\tilde{X}_i$ is given by

$$2[\sigma_i^2\phi_i(0) - \mu_i\Phi_i(0)] + \mu_i \tag{19}$$

where $\phi_i(0)$ is the normal density function with mean μ_i and variance σ'^2_i evaluated at zero, and $\Phi_i(0)$ is the corresponding cumulative normal distribution function. Therefore, aggregating the expected absolute net demand over all agents yields the expression for the expected trading volume given the initial demands D_{0i}:

$$E[\tilde{T}|Z, D_{0i}] = \Sigma_i[\sigma'^2_i\phi(0) - \mu_i\Phi(0)] \tag{20}$$

Note that the aggregate expected net demand $\Sigma_i\mu_i$ is zero. Before we can state the effects of precision and consensus on trading volume, we need the following preliminary results:

Lemma 3: Assume that the agents are symmetrically informed. Then

(a) If consensus is kept constant, $d\mu_i/d\theta_i = 0$, and $d\sigma'^2_i/d\theta_i > 0$;

(b) If precision is kept constant, $d\mu_i/d\phi_L = 0$, and $d\sigma'^2_i/d\phi_L < 0$.

Proof:

If $S_i = S$ for all i, then μ_i becomes r_iZ/Σ_ir_i which is a constant, hence the result is obtained. Similarly, it is trivial that $d\mu_i/d\phi_L = 0$. In the symmetrical case, it can be easily shown that $V(V + N + S)_{-1} - A = 0$, and therefore σ'^2_i becomes

$$\frac{r_i^2\{V^2(V + N + S)^{-2}S + [V(V + N + S)^{-1} - A](V + N)\}}{[V - V^2(V + N + S)^{-1}]^2}$$

$$= r_i^2[V^2(V + N + S)^{-2}S]/[V - V^2(V + N + S)^{-1}]^2 \tag{21}$$

By taking the first derivative of equation (5.6) with respect to Θ and applying lemma 2a, it can be easily shown that $d\sigma'^2_i/d\Theta_i > 0$. Similarly, by applying lemma 2b to the first derivative, $d\sigma'^2_i/d\phi_i < 0$ is obtained.

(q.e.d.)

We can now state the effects of information on the behavior of trading volume.

<u>Theorem 3</u>: If the agents are symmetrically informed, an increase (decrease) in the precision of information about the value of the underlying risky asset will increase (decrease) the expected volume of trade.

<u>Proof</u>: The first derivative of the expected volume of trade E(T) with respect to Θ is given by:

$$\Sigma_i[(dE(T)/d\sigma'_i)(d\sigma'_i/d\Theta) + (dE(T)/d\mu_i)(d\mu_i/d\Theta)] \qquad (22)$$

Note that $dE(T)/d\sigma'_i = \Sigma_i[(\mu_i^2/\sigma'_i + \sigma'_i)\phi(0) - (\mu_i^2/\sigma'_i)\phi(0)]$, which is in turn equal to $\Sigma_i[\sigma'_i\phi(0)] > 0$. By lemma 3a, $d\mu_i/d\Theta$ is zero and $d\sigma'_i/d\Theta$ is positive; hence $dE(T)/d\Theta > 0$.

[q.e.d.]

Theorem 4: If the agents are symmetrically informed, an increase (decrease) in the consensus of information about the value of the underlying risky asset will decrease (increase) the expected volume of trade.

Proof: The steps are exactly the same as in the proof of theorem 3, except that the results of lemma 3b are used instead.

[q.e.d.]

VI. The Effect of Information on Price Variability and Trading Volume

The interaction of confidence and consensus and their effects on the risky asset's price variability and trading volume are summarized in this section. Let $\uparrow\theta$ denote an increase in confidence for all agents and $\uparrow\phi$ an increase in consensus among investors. Similarly let $\downarrow\theta$ and $\downarrow\phi$ denote a decrease in confidence and consensus respectively. Let "$\uparrow\theta \gg \downarrow\phi$" denote the event that the effect on price variability or trading volume of an increase in confidence dominates that of a decrease in consensus. In the following discussion, we assume that (1) $V + N > S_i$, and (2) all agents are symmetrically informed. Now, consider the effect of "$\uparrow\theta \gg \downarrow\phi$" on the behavior of the change in price and trading volume. By theorem 1, $\uparrow\theta$ will increase the price variability, but by theorem 2, $\downarrow\phi$ will lead to a decrease in price variability, ceteris paribus. Since the effect of $\uparrow\theta$ dominates that of $\downarrow\phi$, we expect to observe a "moderate" increase in price variability. Assuming everything else constant, theorem 3 implies that $\uparrow\theta$ leads to an increase in trading volume, and theorem 4 indicates that $\downarrow\phi$ will increase the trading volume as well. Since both $\uparrow\theta$ and $\downarrow\phi$ exert an upward pressure on trading volume, we should observe an extraordinarily large volume. Similarly, if the information disclosure induces $\uparrow\theta$, but does not influence the consensus among the agents, then theorem 1 and theorem 3 imply that we should observe a 'large' price variability and a 'large' increase in trading volume. The effects on price variability and trading volume of various combinations of change in confidence and consensus are presented in Table 3.1.

Table 3.1

The Effect of Confidence and Consensus on Price Variability and Trading Volume

	Price Variablity		Trading Volume	
	Degree of Increase	Degree of Decrease	Degree of Increase	Degree of Decrease
1. $\uparrow\theta \gg \downarrow\phi$	+		+ + +	
2. $\uparrow\theta$ only	+ +		+ +	
3. $\uparrow\theta \gg \uparrow\phi$	+ + +		+	
4. $\uparrow\phi \gg \downarrow\theta$	+			- - -
5. $\uparrow\phi$ only	+ +			- -
6. $\uparrow\phi \gg \uparrow\theta$	+ + +			-
7. $\downarrow\phi \gg \uparrow\theta$		-	+ + +	
8. $\downarrow\phi$ only		- -	+ +	
9. $\downarrow\phi \gg \downarrow\theta$		- - -	+	
10. $\downarrow\theta \gg \uparrow\phi$		-		- - -
11. $\downarrow\theta$ only		- -		- -
12. $\downarrow\theta \gg \downarrow\phi$		- - -		-

+ + +: extraordinarily large increase;
+ + : large increase;
+ : moderate increase;
- - -: extraordinarily large decrease;
- - : large decrease; and
- : moderate decrease.

The results in Table 3.1 enable us to draw inferences about the characteristics of new information based on observed changes in price variability and trading volume. Suppose we observe a negligible change in price variability and trading volume after a news announcement is released. In this case, either the news announcement did not contain new information or the effect of consensus is greater than the effect of confidence (cases 4 through 9 in Table 3.1). If we observe instead a decrease in price variability and an increase in trading volume, we can conjecture from cases 7 through 9 in the table that there is a decrease in consensus and an increase in confidence among the investors, and the consensus effect dominates the confidence effect. From the price variability and trading volume that we observe, we can still infer about the possible combinations of the confidence and the consensus effect induced by the information. Cases 2, 5, 8, and 11 represent those situations in which isolated effects of confidence and consensus are observed. In all other cases, it is extremely difficult to use either the trading volume or price variablity as a measure or proxy for the information content or consensus effect.

VII. Conclusion

This essay develops a theoretical model in a noisy rational expectations equilibrium framework incorporating heterogeneous information and diverse beliefs. The quality of information is characterized by individual investor's confidence and the variability of opinion across investors. The effects of these two characteristics of information on security price and volume are examined. The conclusion is that when the market is confidence-driven, large trading volume normally accompanies large price variability. When the market is consensus-driven, price variability is accompanied by low trading volume. Also, caution needs to be taken when we try to use price and volume to measure information content. As stated in Theorem 1, the effect of investors' confidence on price variability is not clear. However, if the variance of the idiosyncratic noise is small relative to the variance of the unknown risky payoff and the common noise, then increase in confidence will lead to increase in price variability.

Appendix

Proof of Lemma 1(a):

The first order condition of

$$\mathrm{Max}_{D_i}\{r_i^{-1}(D_i E[\tilde{U}|\tilde{Y}_i = Y_i] + B_i) - 1/2(r_i^{-2})D_i^2 \mathrm{Var}(\tilde{U}|\tilde{Y}_i = Y_i)\},$$

subject to $B_i = D_{0i}\tilde{P} + B_{0i} - D_i\tilde{P}$

is given by

$$r_i^{-1}\left[E(\tilde{U}|\tilde{Y}_i) - \tilde{P}\right] - r_i^{-2} D_i \mathrm{Var}(\tilde{U}|\tilde{Y}_i) = 0$$

Solving for D_i gives the individual demand equation.

Proof of Lemma 1(b):

The market clearing condition is that $\Sigma_i D_i = Z$ almost surely. Therefore, aggregating across all individual demands yields

$$\Sigma_i \frac{r_i[E(\tilde{U}|\tilde{Y}_i) - P]}{\mathrm{Var}(\tilde{U}|\tilde{Y}_i)} = Z$$

Solving for P gives

$$\tilde{P} = \left[\Sigma_i \frac{r_i E(\tilde{U}|\tilde{Y}_i)}{\mathrm{Var}(\tilde{U}|\tilde{Y}_i)} - Z\right] \frac{1}{\left[\Sigma_i \frac{r_i}{\mathrm{Var}(\tilde{U}|\tilde{Y}_i)}\right]}$$

Denote the rightmost term on the right hand side as B. By substituting the expression for $E[\tilde{U}|\tilde{Y}_i = Y]$ into the price function above, we obtain

$$\tilde{P} = \left[1 - B\,\Sigma_i \frac{r_i \mathrm{Var}(\tilde{U})/\mathrm{Var}(\tilde{Y})}{\mathrm{Var}(\tilde{U}|\tilde{Y}_i)}\right]M + B\left[\Sigma_i \frac{r_i \mathrm{Var}(\tilde{U})/\mathrm{Var}(\tilde{Y})}{\mathrm{Var}(\tilde{U}|\tilde{Y}_i)}\;\tilde{Y}_i\right] - BZ$$

Substitute $\tilde{Y}_i = \tilde{U} + \tilde{n} + \tilde{\epsilon}_i$ in to the price function. Rewrite the first term on the right hand side of the equation as $(1 - A)M$. If it is justified to write

$$B\left[\Sigma_i \frac{r_i Var(\tilde{U})/Var(\tilde{Y})}{Var(\tilde{U}|\tilde{Y}_i)} \quad \tilde{\epsilon}_i\right] = 0$$

then we have $\tilde{P} = (1 - A)M + A(\tilde{U} + \tilde{n}) - BZ$. See Admati [1985].

<u>Proof of Lemma 2(a)</u>:

(i) By the definition of Θ_i, we have

$$d\Theta_i/d\Theta_i = 1 = d[\ln(V + N + S_i)^{-1}]/d\Theta_i$$

$$= -(dN/d\Theta_i + dS_i/d\Theta_i)/(V + N + S_i)$$

This implies that $dN/d\Theta_i + dS_i/d\Theta_i = (V + N + S_i)$.

(ii) By keeping ϕ_L constant while changing Θ_i, we have

$$d\phi_L/d\Theta_i = 0 = d\ln(1/\Sigma_j S_j)/d\Theta_i = -(\Sigma S_j)^{-1}\Sigma_j(dS_j/d\Theta_i)$$

which implies that $\Sigma_j(dS_j/d\Theta_i) = 0$.

(iii) First, show that $dB/d\Theta_i < 0$. From the expression of B in lemma 1b, we have $dB/d\Theta_i$ given by:

$$-B^2\Sigma_i\Big[[-r_iV^2(dN/d\Theta_i + dS_i/d\Theta_i)/(V + N + S_i)^2]$$
$$/[V - V^2(V + N + S_i)^{-1}]^2\Big]$$

By the result of 2(a)(i), $dB/d\Theta_i < 0$. Next show that $dA/d\Theta_i > 0$. From the expression of A in lemma 1b, rewrite A as CB where C is

$$\Sigma_i[(r_i/\sigma_i^2)V(V + N + S_i)^{-1}].$$

By the chain rule of differentiation and rearrangement, it can be shown that the first derivative of A with respect to Θ_i is given by:

$$-CB^2\Big[\Sigma_i r_i V^2(V+N+S_i)^{-1}]/(\sigma_i^2)^2\Big] + B\Big[\Sigma_i r_i V^2(V+N+S_i)^{-1}]/(\sigma_i^2)^2\Big]$$
$$= B^2\Big[-[\Sigma_i r_i V(V+N+S_i)^{-1}/\sigma_i^2][\Sigma_i r_i V^2(V+N+S_i)^{-1}/(\sigma_i^2)^2]$$
$$+ (1/B)[\Sigma_i r_i V^2(V+N+S_i)^{-1}/(\sigma_i^2)^2]\Big]$$

By adding the term, A, to the expression above, and substracting A from the first term, the derivative can be rewritten as:

$$A + B^2\{(\Sigma_i(r_i/\sigma_i^2))[\Sigma_i r_i V^2(V+N+S_i)^{-1}/(\sigma_i^2)^2]$$

$$-(\Sigma_i(r_i/(\sigma_i^2)^2)[\Sigma_i r_i V^2(V+N+S_i)^{-1}/(\sigma_i^2)]\}$$

From the expression for σ_i^2 in lemma 1(b), it is easy to show that $V^2(V + N + S_i)^{-1} = V - \sigma_i^2$. Then, we replace $V^2(V + N + S_i)^{-1}$ with $(V-\sigma_i^2)$ in the expression above, and by cancelling terms, we obtain the following expression:

$$A + B^2[(\Sigma_i(r_i/(\sigma_i^2)^2)(\Sigma_i r_i) - (\Sigma_i(r_i/(\sigma_i^2))^2]$$

$$= A + B^2(\Sigma_i r_i)^2[\Sigma_i(1/\sigma_i^2)^2 f_i - (\Sigma_i(1/\sigma_i^2)f_i)^2]$$

where $f_i = r_i/(\Sigma_i r_i)$ and $\Sigma_i f_i = 1$. Since f_i can be regarded as a density function, the second term can be considered as the variance of $(1/\sigma_i)$ which is positive. Also, since A is positive, we conclude that $dA/d\Theta_i$ is positive.

<u>Proof of Lemma 2(b)</u>:

(i) By the definition of ϕ_L, we have

$$d\Theta_i/d\phi_L = 0 = -(dN/d\phi_L + dS_i/d\phi_L)/(V + N + S_i)$$

which implies the stated result.

(ii) By differentiating ϕ_L with respect to ϕ_L, we have

$$d\phi_L/d\phi_L = 1 = -(\Sigma S_i)^2 \Sigma_i dS_i/d\phi_L$$

which implies 4(b)(ii).

(iii) The first derivative of B with respect to ϕ_L is given by:

$$-B^2\Sigma_i\Big[[-r_iV^2(dN/d\phi_L+ dS_i/d\phi_L)/(V + N + S_i)^2]$$
$$/[V - V^2(V + N + S_i)^{-1}]^2\Big]$$

By 4(b)(i), we can conclude that $dB/d\phi_L = 0$. To show that the derivative of A with respect to ϕ_L is zero, rewrite A as CB as done in 4(a)(iii). Then $dA/d\phi_L$ is given by:

$$B(dC/d\phi_L) + C(dB/d\phi_L)$$
$$= B\ \Sigma_i\Big[[V-V^2(V+N+S_i)^{-1}][-r_iV(dN/d\phi_L+ dS_i/d\phi_L)/(V+N+S_i)^2]$$
$$- [r_iV(V+N+S_i)^{-1}][V^2(dN/d\phi_L+ dS_i/d\phi_L)/(V+N+S_i)^2]\Big]$$
$$/[V-V^2(V+N+S_i)^{-1}]^2 + C(dB/d\phi_L)$$

Again, by the result of 4(b)(i), we have $dA/d\phi_L = 0$.

(q.e.d.)

<u>Proof of Theorem 1</u>:

(a) The expected change in price given P_0 and Z is given by:

$$E(\Delta\tilde{P}|P_0, \tilde{Z} = Z) = M - BZ - P_0$$

Taking the first derivative with respect to Θ_i while keeping ϕ_L constant yields:

$$dE(.)/d\Theta_i = -Z(dB/d\Theta_i)$$

By the result of lemma 2(a)(iii), we have $dE(.)/d\Theta_i > 0$.

(b) The variance of the change in price given P_0 and Z is

$$Var(\Delta\tilde{P}|P_0, \tilde{Z} = Z) = A^2(V + N).$$

The derivative of the variance with respect to Θ_i is $A^2(dN/d\Theta_i) + 2A(V + N)(dA/d\Theta_i)$. By lemma 2a,

$$(dN/d\Theta_i) = -dS_i/d\Theta_i - (V + N + S\ )$$

In the proof for lemma 2a(iii), we have demonstrated that $dA/d\Theta_i>0$ is given by:

$$A + B^2(\Sigma_i r_i)^2[\Sigma_i(1/\sigma_i^2)^2 f_i - (\Sigma_i(1/\sigma_i^2)f_i)^2]$$

which can be rewritten as A + A' where A' > 0. Therefore, $dVar(\Delta P|.)/d\Theta_i$ can be simplified to:

$$A^2(-dS_i/d\Theta_i) - A^2S_i + A^2(V + N) + 2AA'(V + N)$$

which can be either negative or positive, and the sign is therefore indeterminate. However, if $S_i < (V + N)$, then

$dVar(\Delta P|.)/d\theta_i > 0$. If the agents are symmetrically informed, that is, $S_i = S$ for all i, then $A' = 0$, $dS_i/d\theta_i = 0$ by lemma 2a(ii), and $dVar(\Delta P|.)/d\theta_i > 0$ given $S_i < (V + N)$. If $S_i > (V + N)$, then $dVar(\Delta P|.)/d\theta_i < 0$.

(q.e.d.)

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CHAPTER THREE: THE EFFECT OF FORECAST BIAS AND INVESTOR DISAGREEMENT ON SECURITY PRICES AND TRADING VOLUME

Abstract

In a two-asset competitive equilibrium model incorporating subjective prior beliefs, the response of share price to an earnings surprise is shown to be asymmetric if 1) investors are optimistic or pessimistic about the unknown future value of the risky asset, and 2) if positive and negative surprises have differential effects on uncertainty. Trading volume also reacts asymmetrically to positive and negative surprises when the effect of uncertainty is taken into consideration. These theoretical results are consistent with empirically observed relationships between share prices and trading volume.

THE EFFECT OF FORECAST BIAS AND INVESTOR DISAGREEMENT ON SECURITY PRICES AND TRADING VOLUME

I. Introduction

This essay develops a two-asset competitive equilibrium model with subjective prior beliefs to incorporate the effect of an earnings announcement on share price and trading volume. In this model, price response to the size of a positive or negative earnings surprise is symmetrical if investors' expected liquidating value of the risky asset is unbiased. Price response is asymmetric if 1) investors are optimistic or pessimistic about the unknown future value of the risky asset, and 2) if positive and negative surprises have differential effects on uncertainty. Trading volume also reacts asymmetrically to positive and negative surprises when the effect of uncertainty is taken into consideration. When the behavior of price and volume are examined jointly, an asymmetric price-volume relationship can exist even in a perfect market.

The relationship between stock prices and trading volume has interested finance practitioners and financial economists for many years. Price and volume are widely used by financial analysts as market sentiment indicators to gauge rallies and declines, to forecast bull and bear markets, and to predict market turning points. To the financial economist, the price and volume relationship has important implications for understanding the microstructure of financial markets, for event studies, and for identifying the empirical price distributions of speculative assets including options and futures.

Beginning with Osborne [1959], the price-volume relationship has been studied from a variety of empirical perspectives. Price-volume studies have examined both equity and futures markets and have included price change intervals ranging from the individual transaction level (Wood, McInish and Ord [1985]) to two months (Morgan [1976] and Rogalski [1978]). In an early article, Granger and Morgenstern [1963] studied the relationship between price indices and aggregate exchange volume using spectral analysis of weekly data and found no association. More recent articles have focused on individual securities. Karpoff [1987] surveys the empirical literature and categorizes empirically documented relationships between contemporaneous changes in the price and trading volume of individual stocks as follows. First, there is an association between absolute changes in price and volume. Crouch [1970], Westerfield [1977], Rogalski [1978], and Tauchen and Pitts [1983] find a positive association between absolute price changes and volume. Epps and Epps [1976] find a positive association between the variance of price change and volume. Clark [1973] and Harris [1983] find a positive association between squared price change and volume. Second, these and other authors (Smirlock and Starks [1985] and Harris [1986]) find a positive relationship between price change and volume. Third, trading volume is higher when prices increase than when prices decrease (Ying [1966], Morgan [1976], Harris [1986], and Richardson, Sefcik and Thompson [1986]). Karpoff [1987] argues that these results could all be true if the price-volume relationship is asymmetric. In particular, the correlation between volume and positive price changes could be positive while the correlation could be negative and smaller in magnitude for negative price changes. This asymmetry could exist in markets in which short positions are more costly than long positions. In a dissenting paper, Wood, McInish and Ord [1985] find evidence of an asymmetry in the opposite direction using trade-to-trade data.

The price-volume relationship has been examined from different theoretical perspectives as well. Copeland [1976], Morse [1980], and Jennings, Starks and Fellingham [1981] model the price-volume relationship with a sequential information arrival process. Clark [1973], Epps and Epps [1976], Tauchen and Pitts [1983], and Harris [1983] develop equilibrium models for the stochastic dependence between transaction volume and changes in security price and employ the relationship in modeling the distribution of stock price changes. However, no theoretical model has addressed the observed asymmetry in the relationship between stock price and trading volume.

The next section describes the model of trade and the equilibrium conditions attained before and after an earnings announcement. The responses of price and volume to earningssurprises are developed as well as the relationship between price and volume responses. Section III develops empirical tests of the hypotheses in Section II and reports results for a sample of quarterly earnings announcements. Section IV summarizes and concludes the paper.

II. A Two-Asset Competitive Equilibrium Model

Consider a simple two-period framework as depicted by the following time line.

```
Expectation           Earnings              Ỹ
 Formation          Announcement         Realized

  |___________________|___________________|
  0     Pre-          1    Post-          2
        Announcement       Announcement
  |<--- Period ------>|<-- Period ------->|
```

We assume that there are n investors in the economy. Noise trading by liquidity traders is allowed to exist in the economy but the liquidity traders as a group have no net holdings of either the riskless or the risky asset. There are two assets, one with an unknown payoff and one with a certain payoff with zero rate of return. The unknown or risky security value at the end of the second period is a random variable ($\tilde{Y}$) which is initially (at t=0) believed to be normally distributed with mean M and variance V. Each investor i has a constant absolute risk tolerance utility function for wealth with coefficient of risk tolerance r_i

$$U_i(w) = -\exp(-w/r_i).$$

Each investor maximizes his expected utility of end-of-period wealth

$$E_i[-\exp(-r_i^{-1}(D_i\tilde{Y} + B_i))]$$

subject to $D_i\tilde{P} + B_i = D_{0i}\tilde{P} + B_{0i}$

where r_i ≡ constant absolute risk tolerance

B_{0i} ≡ initial holdings of the riskless asset

D_{0i} ≡ initial holdings of the risky asset

B_{2i} ≡ end-of-period holdings of the riskless asset

D_{2i} ≡ end-of-period holdings of the risky asset

$\tilde{P}_0$ ≡ initial equilibrium price of the risky asset.

Note that the right hand side of the budget constraint, $D_{0i}P + B_{0i}$, represents individual i's initial wealth conditional on the market value of his holdings of the risky asset. The equilibrium price of the risky asset has yet to be determined by the market auctioneer who functions to match aggregate demand with aggregate supply. Once a marketprice is announced by the auctioneer, each investor will re-allocate his initial wealth between the riskless asset and the risky asset to the extent that his utility of wealth is maximized. Therefore B_{0i} and B_{2i} may not be the same. To focus on the effect of earnings on $\tilde{Y}$, decompose investor i's signal concerning $\tilde{Y}$ into

$$\tilde{I}_{0i} = \tilde{Y} + \tilde{e}_{0i} = d_{0i}\tilde{X}_{0i} + G_{0i}. \qquad (1)$$

The idiosyncratic error term $\tilde{e}_i$ is assumed to be independent of $\tilde{Y}$ and is normally distributed with mean zero and variance S_{0i}. Before an earnings announcement, each investor has an expectation X_{0i} of future earnings $\tilde{X}$ to be announced at time $t = t_a$ and interprets the impact of this expectation on security price according to his earnings interpretation coefficient d_{0i}. Each investor also has an expectation G_{0i} of a random variable $\tilde{G}$ independent of $\tilde{X}$ which represents the impact of all other factors on the value of the risky asset. Investor i's signal or expectation of the value of the risky asset is then $I_i = d_{0i}X_{0i} + G_{0i}$.

With subjective prior beliefs, each investor forms an expectation about $\tilde{Y}$

$$E_{0i}[\tilde{Y}] = E[\tilde{Y}|\tilde{I}_{0i} = I_{0i}] = M + \frac{V}{(V + S_{0i})}(d_{0i}X_{0i} + G_{0i} - M)$$

with conditional variance (uncertainty)

$$V_{0i} = \mathrm{Var}[\tilde{Y}|\tilde{I}_{0i} = I_{0i}] = V - \frac{V^2}{(V + S_{0i})},$$

where $E_{0i}[.]$ represents the expectation of investor i based on his information set at time t = 0. Investors then trade on their diverse beliefs during the first period. First, the market auctioneer announces a price P for the risky asset. Given P, each investor determines his/her optimal demand for the risky asset by maximizing

$$E[-\exp(-r_i^{-1}(D_i\tilde{Y} + B_i))|\tilde{I}_i = I_i]$$

subject to $\quad D_iP + B_i = D_{0i}P + B_{0i} = W_0$

where W_0 is the initial wealth. We already know that conditional on $\tilde{I}_{0i} = I_{0i}$, $\tilde{Y}$ is normally distributed with mean and variance as follows:

$$E[\tilde{Y}|\tilde{I}_{0i} = I_{0i}] = M + \beta_i(I_{0i} - M)$$

$$\mathrm{Var}(\tilde{Y}|\tilde{I}_{0i} = I_{0i}) = \mathrm{Var}(\tilde{Y}) - \beta_i\mathrm{Var}(\tilde{Y}),$$

where

$$\beta_i = \mathrm{Var}(\tilde{Y})/\mathrm{Var}(\tilde{I}_{0i}) = V/(V + S_{0i}).$$

Since $\tilde{Y}$ is normal, the distribution of $\exp\{-r_i^{-1}(D_i\tilde{Y}+ B_i)\}$ conditional on $\tilde{I}_i = I_i$ is lognormal. Direct computation with the substitution of B_i with the budget constraint yields

$$E[-\exp\{-r_i^{-1}(D_i\tilde{Y} + B_i)\}|\tilde{I}_{0i} = I_{0i}]$$

$$= E[-\exp\{-r_i^{-1}(D_i\tilde{Y} + W_0 - D_iP)\}|\tilde{I}_{0i} = I_{0i}]$$

$$= -\exp\{-r_i^{-1}(D_iE[\tilde{Y}|\tilde{I}_{0i} = I_{0i}] + W_0 - D_iP) +$$

$$1/(2r_i^{-2})D_i^2\mathrm{Var}(\tilde{Y}|\tilde{I}_{0i} = I_{0i})\}.$$ [12]

Since an exponential function is strictly increasing in its exponent, the maximization of the objective function becomes

$$\underset{D_i}{\mathrm{Max}}\Big[r_i^{-1}(D_iE[\tilde{Y}|\tilde{I}_{0i}=I_{0i}] + W_0 - D_iP)$$

$$- 1/(2r_i^{-2})D_i^2\mathrm{Var}(\tilde{Y}|\tilde{I}_{0i} = I_{0i})\Big]$$

The first order condition with respect to D_i yields

$$(1/r_i)(E[\tilde{Y}|\tilde{I}_{0i}=I_{0i}] - P) - 1/(r_i^2)D_i\mathrm{Var}(\tilde{Y}|\tilde{I}_{0i}=I_{0i}) = 0.$$

---

[12] Let X be normally distributed with mean μ and with variance σ. Then exp[X] is lognormal with mean $\exp[\mu + 1/2\sigma^2]$.

Individual i's optimal demand for the risky asset based on the price P is therefore equal to $r_i(E[\tilde{Y}|\tilde{I}_i = I_i]-P)/Var(\tilde{Y}|\tilde{I}_i = I_i)$. Each individual submits his demand/supply order to the market auctioneer who will then match the aggregate demand with total supply and adjust the market price. This kind of iteration process continues until the market price announced by the auctioneer equates total demand to total supply. If the clearing price is P_0, the equilibrium demand of investor i for

the risky asset is given by

$$D_{0i} = \frac{r_i[E_{0i}[\tilde{Y}] - P_0]}{V_{0i}} \tag{4}$$

After determining individual i's equilibrium demand for the risky asset, we can solve for the risky asset's equilibrium price P_0 by using the market clearing condition

$$\Sigma_i D_i = Z_0$$

where Z_0 is the supply of the risky asset being traded initially. Substituting equation (4) into the market clearing condition, we obtain

$$Z_0 = \Sigma_i\{r_i[E_{0i}(\tilde{Y}) - P_0]/V_{0i}\}$$

$$= \Sigma_i[r_i E_{0i}(\tilde{Y})/V_{0i}] - P_0(\Sigma_i r_i/V_{0i}).$$

Rewriting this equation for P_0 gives

$$P_0 = \frac{\Sigma_i\ [r_i E_{0i}[\tilde{Y}]/V_{0i}]}{\Sigma_i (r_i/V_{0i})} - \frac{Z_0}{\Sigma_i (r_i/V_{0i})} \tag{5}$$

At the end of the first period (t = 1), the firm discloses actual earnings X^*. The information content of the disclosed earnings may change investors' beliefs. We assume that the signal observed by investor i after the earnings announcement is represented by

$$\tilde{I}_{1i} = \tilde{Y} + \tilde{e}_{1i} = d_{1i} X^* + G_{1i} \tag{6}$$

Investor i's revised expectations are then $I_{1i} = d_{1i}X^* + G_{1i}$. Each investor's interpretation of earnings after the announcement, d_{1i}, may differ from the pre-announcement earnings interpretation coefficient d_{0i}. The new error term e_{1i} is assumed to be independent of $\tilde{Y}$ and is normally distributed with mean zero and variance S_{1i}. Based on the information content of X^*, investors revise their expectations and their subjective posterior beliefs about $\tilde{Y}$ such that

$$E_{1i}[\tilde{Y}] = E[\tilde{Y}|\tilde{I}_{1i} = I_{1i}] = M + V(V + S_{1i})^{-1}(d_{1i} X^* + G_{1i} - M), \tag{7}$$

and $$V_{1i} = Var[\tilde{Y}|\tilde{I}_{1i} = I_{1i}] = V - V^2(V + S_{1i})^{-1} \tag{8}$$

As a result, the equilibrium demand of investor i and the equilibrium price of the risky asset are respectively given by

$$D_{1i} = \frac{r_i[E_{1i}[\tilde{Y}] - P_1]}{V_{1i}} \qquad (9)$$

and

$$P_1 = \frac{\Sigma_i\ [r_i E_{1i}[\tilde{Y}]/V_{1i}]}{\Sigma_i(r_i/V_{1i})} - \frac{Z_1}{\Sigma_i(r_i/V_{1i})} \qquad (10)$$

where Z_1 is the supply of the risky asset being traded during the post-announcement period.

A. The Effect of an Earnings Surprise on Security Price

To simplify our analysis, assume that the variance of the idiosyncratic error is the same for all investors at a particular point in time (i.e. $S_{0i} = S_0$ and $S_{1i} = S_1$ for all i). Equations (3) and (8) then imply that at a particular point in time investors' conditional uncertainty regarding the unknown liquidating value $\tilde{Y}$ is constant across investors (i.e. $V_{0i} = V_0$ and $V_{1i} = V_1$ for all i). Then P_0 and P_1 can be written as:

$$P_0 = \Sigma_i w_i\ E_{0i}[\tilde{Y}] - (V_0 Z_0)/\Sigma_i r_i \qquad (11)$$

$$P_1 = \Sigma_i w_i\ E_{1i}[\tilde{Y}] - (V_1 Z_1)/\Sigma_i r_i\ , \quad \text{where } w_i = r_i/(\Sigma_i r_i) \qquad (12)$$

Note that $\Sigma_i w_i = 1$. The change in price $(P_1 - P_0)$ due to the earnings announcement is given by:

$$P_1 - P_0 = \Sigma_i w_i (E_{1i}[\tilde{U}] - E_{0i}[\tilde{U}]) - (V_1 Z_1 - V_0 Z_0)/(\Sigma_i r_i) \tag{13}$$

This equation states that the change in price is affected by the weighted average of the change in expectation formations about the underlying value and the change in supply of the risky asset caused by the earnings announcement. The weight w_i is the percentage of investor i's constant risk tolerance in the total risk tolerance of the economy. To decompose the price change equation further, insert the expressions for $E_{1i}\tilde{U}$ and $E_{0i}\tilde{U}$ into equation (13). Since $S_i = S$ and $s_i = s$ for all i, replace $V(V + S)^{-1}$ and $V(V + s)^{-1}$ by A and a respectively. Equation (13) can be rewritten as

$$\begin{aligned} P_1 - P_0 = {} & \Sigma_i w_i (A_1 d_{1i} X^* - A_0 d_{0i} X_{0i}) + \Sigma_i w_i (A_1 G_{1i} - A_0 G_{0i}) \\ & + \Sigma_i w_i (A_0 - A_1) M - (V_1 Z_1 - V_0 Z_0)/(\Sigma_i r_i) \ . \end{aligned} \tag{14}$$

Traditionally, an earnings surprise is defined as the deviation of actual earnings from the mean consensus forecast, $X_* - \Sigma_i w_i X_{0i}$. By adding and subtracting $\Sigma_i w_i (A_i d_{1i} X_{0i})$ to the first term in equation (14), the change in price due to an earnings announcement can be seen to be linearly related to the size of the earnings surprise.

In order to isolate the effect of an earnings surprise, we further assume that all investors have the same risk tolerance ($r_i = r$). The weight w_i for each investor in equation (14) becomes 1/n. Let X_0 be the simple average earnings forecast ($\Sigma_i X_{0i}/n$) before the announcement. By adding and substracting $A_1 d_{1i} X_0$ in the first term, equation (14) becomes

$$P_1 - P_0 = A_1 d_1 (X^* - X_0) + (A_1 d_1 - A_0 d_1) X_0 + (A_1 G_1 - A_0 G_0) + (A_0 - A_1)M - (V_1 Z_1 - V_0 Z_0)/(nr) \quad (15)$$

where $d_0 = \Sigma_i d_{0i}/n$, $d_1 = \Sigma_i d_{1i}/n$, $G_1 = (\Sigma_i G_{1i})/n$, and $G_0 = (\Sigma_i G_{0i})/n$. Equation (15) states that the change in price caused by an earnings announcement is determined by (i) the size of the earnings surprise ($X^* - X_0$), (ii) the change in interpretation and uncertainty about the unknown stock value, (iii) the revision of the growth forecast ($G_1 - G_0$), and (iv) the change in supply of the risky asset being traded due to the earnings announcement.

Equation (15) can be used to examine the symmetry of a price response to an earnings surprise. Assume that (i) the average forecast of growth does not change ($G_0 = G_1$), and (ii) investors' pre- and post-announcement average interpretations of the impact of earnings on value does not change ($d_0 = d_1 = d$), and (iii) the number of investors is large enough so that the last term in equation (15) is negligible. After rearrangement, the price change equation becomes

$$P_1 - P_0 = A_1 d(X^* - X_0) + (A_1 - A_0)(dX_0 + G_0 - M) \ . \qquad (16)$$

It is obvious from equation (16) that the magnitude of the price change is directly proportional to the size of the earnings surprise. The second term represents the pre-announcement consensus forecast revision of the underlying expected value of the risky asset. The term $(dX_0 + G_0 - M)$ is a measure of market sentiment about the unknown value of the risky asset. If the term is zero, the market does not revise its expectation before announcement. If the term is positive (negative), the market is optimistic (pessimistic).

Three testable hypotheses about the reaction of price to the earnings announcement can be established at this point.

Proposition 1

If 1) the market consensus about the value of the risky asset does not change before the earnings announcement, or 2) the earnings announcement does not affect the degree of uncertainty among the investors about the asset's value, then the reaction of price to both negative and positive earnings surprises is symmetrical.

Under either condition, the second term in equation (16) is zero and the magnitude of share price response to the earnings announcement is linearly related to the magnitude of the earnings surprise according to $P_1 - P_0 = A_1 d(X^* - X_0)$.

Proposition 2

Suppose the market consensus about the value of the risky asset is revised upward before the earnings announcement such that $(dX_0 + G_0 - M) > 0$. Then,

1) If the earnings announcement reduces the uncertainty about the value of the risky asset among investors ($A_0 < A_1$), then for the same size earnings surprise the absolute change in price is larger for a favorable surprise than for an unfavorable surprise.
2) If the earnings announcement induces more uncertainty among investors ($A_0 > A_1$), then the absolute change in price is smaller for a favorable surprise than for an unfavorable surprise.

Proposition 3

Suppose the market consensus about the value of the risky asset is revised downward before the earnings announcement such that $(dX_0 + G_0 - M) < 0$.

1) If the earnings announcement reduces the uncertainty about the value of the risky asset among investors ($A_0 < A_1$), then for the same size earnings surprise the

absolute change in price is smaller for a favorable surprise than for an unfavorable surprise.

2) If the earnings announcement induces more uncertainty among investors ($A_0 > A_1$), the absolute change in price is larger for a favorable surprise than for an unfavorable surprise.

Intuitively, if investors are optimistic about the value of the risky asset before the earnings announcement, a large negative earnings surprise is likely to cause more uncertainty among investors ($A_1 < A_0$). A large positive earnings surprise is more likely to confirm their beliefs and hence resolve their uncertainty to some extent ($A_1 > A_0$). Equation (16) implies that the price change is more responsive to a positive surprise than to a negative surprise. Similarly, if the investors are pessimistic, a large positive surprise is more likely to introduce more uncertainty into their beliefs. A large negative surprise will confirm their expectations. In this case, equation (16) implies that the price change is more responsive to negative earnings surprise than to positive surprise.

III. Earnings Surprise and Trading Volume

This section develops the effect of an earnings announcement on the behavior of the volume of trade. Conditional on the original equilibrium demand D_{0i}, investor i's volume of trade induced by the earnings announcement is $(D_{1i} - D_{0i})$. Equations (4) and (9) of Section II yield the net trade of investor i in the risky asset induced by the earnings announcement:

$$T_i = D_{1i} - D_{0i} = \frac{r_i[E_{1i}[\tilde{Y}] - P_1]}{V_{1i}} - \frac{r_i[E_{0i}[\tilde{Y}] - P_0]}{V_{0i}} \tag{17}$$

Assuming the variance of the idiosyncratic error is constant across investors (i.e. $S_{0i} = S_0$ and $S_{1i} = S_1$) yields

$$T_i = \left[r_i(M + A_1(d_{1i}X^* + G_{1i} - M) - [\Sigma_i w_i(M + A_1(d_{1i}X^* + G_{1i} - M)])\right]/V_1$$

$$- Z_1/\Sigma_i r_i - D_{0i}$$

$$= \left[\, r_i A_1[(d_{1i}X^* + G_{1i}) - \Sigma_i w_i(d_{1i}X^* + G_i)] \,\right]/V_1 - Z_1/\Sigma_i r_i - D_{0i} \tag{18}$$

If all investors agreed on the value of the risky asset after an earnings announcement, no trades would be necessary to drive price to its new equilibrium level. Trade occurs after the earnings announcement solely because of the information content that causes diverse opinions (i.e. prior probabilities) among investors about the future activity of the firm. That is, trading volume changes because investors interpret information differently and not because of the new information itself. This result is consistent with Varian (1985).

Next, the effect of an earnings surprise on the behavior of trading volume is examined. By substituting equations (5) and (10) into equation (17), and assuming $r_i = r$ for all investors, T_i can be written as:

$$T_i = \Big[r[(E_{1i}[\tilde{Y}]-\overline{E_1[Y]}) - V_1/(nr)]\Big]/V_1 -$$

$$- \Big[r[(E_{0i}[\tilde{Y}]-\overline{E_0[Y]}) - V_0/(nr)]\Big]/V_0$$

$$= \Big[(V_0E_{1i}[\tilde{Y}] - V_1E_{0i}[\tilde{Y}]) - (V_0\overline{E_1[Y]} - V_1\overline{E_0[Y]})\Big]/(V_0V_1/r) \tag{19}$$

where $\overline{E_1[Y]} = \Sigma_i E_{1i}[\tilde{Y}]/n$ and $\overline{E_0[Y]} = \Sigma_i E_{0i}[\tilde{Y}]/n$. The overall volume of trade is given by:

$$T = \Sigma_i |T_i|/2$$

$$= \Big[\Sigma_i |(V_0E_{1i}[\tilde{Y}] - V_1E_{0i}[\tilde{Y}]) - (V_0\overline{E_1[Y]} - V_1\overline{E_0[Y]})|\Big]/2(V_0V_1/r) \tag{20}$$

The functional form of T makes further analysis difficult.

To alleviate this problem, note that the numerator is the sample mean absolute deviation (MAD) of $(V_0E_1[\tilde{Y}] - V_1E_0[\tilde{Y}])$.[13] $E_1[\tilde{Y}]$ and $E_0[\tilde{Y}]$ are normally distributed because they are expectations conditional on the information I_0 and I_1 which are normally distributed. Therefore, there exists a one-to-one correspondence between the MAD and the variance. Since $E_1[\tilde{Y}]$ and $E_0[\tilde{Y}]$ are normally normally distributed, the numerator of equation (19) can be approximated by the variance of $(V_0E_1[\tilde{Y}] - V_1E_0[\tilde{Y}])$. Then T can be approximated by a function τ of the form:

$$\tau = \frac{Var(V_0E_1[\tilde{Y}] - V_1E_0[\tilde{Y}])}{2(V_0V_1/r)} \tag{21}$$

By substituting equations (2) and (7) into the numerator of (21) and dropping the subscript i, we have:

$$Var(V_0E_1[\tilde{Y}] - V_1E_0[\tilde{Y}])$$

$$= Var[(V_0A_1d_1X^* - V_1A_0d_0X_0) + (V_0A_1G_1 - V_1A_0G_0)$$

$$+ (V_0M - V_1M - V_0A_1M - V_1A_0M)]$$

---

[13]. $E_0(\tilde{Y})$ and $E_1(\tilde{Y})$ represent the expectation of the unknown payoff Y conditional on the information set at t=0 and t=1 respectively. The expressions for $E_0(\tilde{Y})$ and $E_1(\tilde{Y})$ are the same as equations (2) and (7) without the subscript i. In fact the expressions given by (2) and (7) are analogous to the outcomes of the ith experiment with $E_0(\tilde{Y})$ and $E_1(\tilde{Y})$. Therefore, the interpretation (d), earnings forecast (X_0), and the growth factor (G0) are random variables.

$$= \mathrm{Var}[(V_0A_1d_1X^* - V_1A_0d_0X_0) + (V_0A_1G_1 - V_1A_0G_0)] \tag{22}$$

As in the development of equation (16), assume that investors' pre- and post-announcement average interpretations of the impact of earnings on value do not change ($d_0 = d_1 = d$). Furthermore, to concentrate on the impact of an earnings announcement on trading volume, assume that $G_0 = G_1 = G$. Adding and substracting $V_0A_1dX_0$ in the first term of (22) yields:

$$\mathrm{Var}[V_0A_1d(X^* - X_0) + d(V_0A_1 - V_1A_0)X_0 + (V_0A_1 - V_1A_0)G] \tag{23}$$

Now let $$X_0 = \bar{X} + \epsilon_0 \tag{24}$$

where $\bar{X}$ is the mean consensus earnings forecast across all investors and the error term ϵ_0 has zero mean and finite variance $\mathrm{Var}(\epsilon_0)$ and is independent of $\bar{X}$, d, and G. Substituting X_0 into equation (23) and rearranging yields

$$\mathrm{Var}(V_0E_1[\tilde{Y}] - V_1E_0[\tilde{Y}])$$

$$= \mathrm{Var}[V_0A_1d(X^* - \bar{X}) - V_0A_1d(\epsilon_0) + d(V_0A_1 - V_1A_0)\bar{X} + d(V_0A_1 - V_1A_0)\epsilon_0] + \mathrm{Var}[(V_0A_1 - V_1A_0)G]$$

$$= (V_0A_1X^* - V_1A_0\bar{X})\mathrm{Var}(d) + (V_1A_0)^2E(d^2)\mathrm{Var}(\epsilon_0) + (V_0A_1 - V_1A_0)^2\mathrm{Var}(\bar{G})$$

$$= [V_0A_1(X^* - \bar{X}) + (V_0A_1 - V_1A_0)\bar{X}]^2\mathrm{Var}(d) + (V_1A_0)^2E(d^2)\mathrm{Var}(\epsilon_0) + (V_0A_1 - V_1A_0)^2\mathrm{Var}(G) \qquad (25)$$

Therefore, from equation (21),

$$\tau = (r/(2V_0V_1)\left[[V_0A_1(X^* - \bar{X}) + (V_0A_1 - V_1A_0)\bar{X}]^2\mathrm{Var}(d) + (V_1A_0)^2\mathrm{Var}(\epsilon_0)E(d^2) + (V_0A_1 - V_1A_0)^2\mathrm{Var}(G)\right] \qquad (26)$$

Observe from equation (26) that even when the earnings surprise is zero trade still occurs if the earnings announcement has an impact on investors' beliefs about the value of the risky asset. Based on equation (26), we can establish two hypotheses about the response of trading volume to an earnings announcement.

Proposition 4

If the earnings announcement resolves uncertainty about the future value of the risky asset among the investors (i.e., $V_0 > V_1$ and $A_0 < A_1$), then the volume of trade associated with a favorable earnings surprise $[(X^* - \bar{X}) > 0]$ is larger than that associated with an unfavorable earnings surprise $[(X^* - \bar{X}) < 0]$ of the same magnitude.

This proposition can be demonstrated by inspecting the first term in equation (26). If $V_0 > V_1$ and $A_0 < A_1$, then every component in the first term is positive. If $V_0 < V_1$ and $A_0 > A_1$, then $(V_0A_1 - V_1A_0)$ is negative and so is the cross-product component in the first term. Therefore, the extent of a volume increase to a negative earnings surprise is less than that of a positive surprise. Similarly, the impact of an increase in uncertainty on trading volume is presented below:

Proposition 5:

If the earnings announcement induces more uncertainty about the future value of the risky asset among the investors, that is, $V_0 < V_1$ and $A_0 > A_1$, then the size of the volume of trade associated with favorable earnings surprise is less than that associated with the unfavorable earnings surprise of the same magnitude.

A summary of the five propositions is provided in the Table 3.1.

IV. Empirical Tests

Price and volume responses to earnings announcements are categorized in Table 1 according to pre-announcement forecast revisions and changes in uncertainty after the announcement. According to equation (16), price response to an earnings announcement depends on the size and direction of forecast revision before the announcement and change in the uncertainty of investors after the announcement. Four testable hupotheses about price responses to earnings announcements arising from equation (16) are summarized as follow:

H1: If the market consensus about the value of the risky asset is revised upward before an earnings announcement and the announcement reduces uncertainty about the value of the risky asset among investors, then the absolute change in price is more responsive to a favorable earnings surprise than to an unfavorable surprise.

H2: If the market consensus about the value of the risky asset is revised upward before an earnings announcement and the announcement induces more uncertainty about the value of the risky asset among investors, then the absolute change in price is less responsive to a favorable earnings surprise than to an unfavorable surprise.

H3: If the market consensus about the value of the risky asset is revised downward before an earnings announcement and the announcement reduces uncertainty about the value of the risky asset among investors, then the absolute change in price is less responsive to a favorable earnings surprise than to an unfavorable surprise.

H4: If the market consensus about the value of the risky asset is revised downward before an earnings announcement and the announcement induces more uncertainty about the value of the risky asset among investors, then the absolute change in price is more responsive to a favorable earnings surprise than to an unfavorable surprise.

According to equation (27), trading volume response depends on the change in uncertainty. Two testable hypotheses about trading volume arising from equation (27) are summarized below:

H5: If an earnings announcement resolves uncertainty about the value of the risky asset among investors, then the volume of trade is more responsive to a favorable earnings surprise than to an unfavorable surprise.

H6: If an earnings announcement increases uncertainty about the value of the risky asset, then the volume of trade is less responsive to a favorable earnings surprise than to an unfavorable surprise.

A. Variables and Data

This section discusses the measures and proxies adopted in empirically testing the hypotheses derived from equations (16) and (26). Tests of these hypotheses require: i) an estimate of the market consensus about the future value of the risky asset, ii) a measure of uncertainty about the future value of the risky asset, iii) a measure of earnings surprise, and iv) security price and volume response to the earnings announcement.

Lynch, Jones and Ryan's Institutional Brokers Estimate System (I/B/E/S) is our source of earnings forecast data. The mean and standard deviation of financial analysts reporting quarterly estimates of earnings per share to I/B/E/S are adopted as proxies for investors' consensus expectation and the uncertainty of investors about the future value of the risky asset. Earnings announcement dates are retrieved from I/B/E/S and from the Wall Street Journal when not reported by I/B/E/S.

Earnings surprise is defined as actual earnings per share minus the adjusted mean consensus forecast standardized by price on the announcement day. The I/B/E/S adjustment factor (ADJFAC) is used to adjust consensus forecasts for stock splits, stock dividends, and new issues. Quarterly earnings per share are taken from Standard and Poor's Compustat database. The standard deviation of analyst forecasts is often used to standardize the level of earnings across companies. This is inappropriate here since the standard deviation of analyst forecasts is one of the independent variables under study.

The sample period includes the third quarter of 1984 through the fourth quarter of 1987. The sampling criteria include: i) each firm must have at least three analysts during the sample period, ii) every firm must have monthly forecasts reports at least three months before the earnings announcement date and one month after, and iii) firms must survive the sample period. Daily returns to the sample companies and to the value weighted market index are taken from CRSP. Trading volume data is supplied by Media General.

B. Empirical Design

Consider the following diagram depicting the timing of financial analyst forecasts and quarterly earnings announcements.

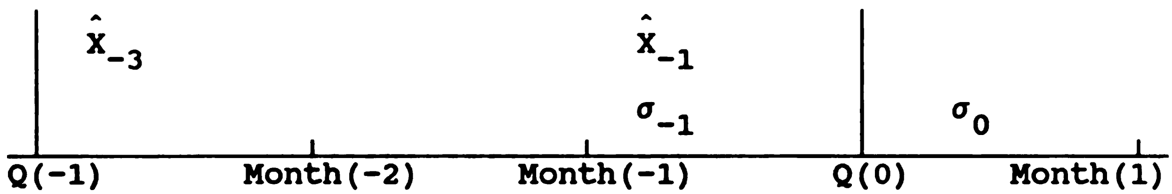


Table 3.2 categorizes the six hypotheses according to i) change in the uncertainty of investor expectations (for price and volume response), and ii) change in mean consensus expectations (for price response). Change in uncertainty induced by the earnings announcement is measured by the difference between the standard deviation of analyst forecasts in the months before and after the announcement date ($(\sigma_0) - \sigma_{-1})$). Revision in the mean consensus opinion about the future value of the risky asset is measured by change in the mean analyst forecast in the month prior to the quarterly earnings announcement date from the mean consensus forecast three months prior to the announcement ($(\hat{X}_{-1} - \hat{X}_{-3})$). This design eliminates some earnings announcements from the sample because there are not enough analysts following that firm.

A two-day window is employed to capture the reaction of price and volume to an announcement as well as any reaction to information leakage about actual earnings immediately prior to the announcement. Price reaction to an earnings announcement is examined in hypotheses 1 through 4. Two proxies for price reaction are employed to see if results are sensitive to expected stock return assumptions. One is the two-day geometric mean return

$$((1+Ri_{-1})*(1+Ri_0)]-1)^{\frac{1}{2}}$$

and the other is the two-day mean excess return over a value weighted market index

$$((ER_{-1} + ER_0)/2) = ((R_{-1}-Rm_{-1})/2 + (R_0-Rm_0)/2.$$

The negative one subscript refers to the day before an earnings announcement and the zero subscript refers to the day of an announcement. The effect of an earnings announcement on trading volume is examined in Hypotheses 5 and 6. The two-day average trading volume

$$(V_{-1} + V_0)/2$$

is used to capture the volume response to an announcement as well as any information leakage immediately prior to an announcement.

Earnings announcements from the fourth quarter of 1984 through the second quarter of 1987 are categorized along these two dimensions. Observations in each group in Table 3.2 are then aggregated across all quarters. The econometric model designed to investigate asymmetric price and volume reaction to favorable and unfavorable earnings surprises is

$$\left| \text{RESPONSE}_i \right| = \beta_0 + \beta_1 * \text{DUMMY}_i + \beta_2 * \left| \text{SURPRISE}_i \right| + \beta_3 * \text{DUMMY}_i * \left| \text{SURPRISE}_i \right| + e_i \; . \tag{27}$$

Earnings surprise (SURPRISE) for the ith observation is defined as (Actual EPS - Mean Forecast * ADJFAC)/P_0. The dummy variable has a value of one if SURPRISE is positive and a value of zero if SURPRISE is negative. The response variable RESPONSE_i is the two-day average return for Hypotheses 1 through 4 and the two-day average volume for Hypotheses 5 and 6.

Equation (27) tests whether change in the dependent variable (price or trading volume) in response to a unit increase of favorable earnings surprise is different in absolute magnitude from change in response to a unit increase of unfavorable surprise. The model is equivalent to two separate regressions:

SURPRISE > 0 => DUMMY = 1

$$\left| \mathrm{RESPONSE}_i \right| = \beta_0 + \beta_2 + (\beta_1 + \beta_3) \left| \mathrm{SURPRISE}_i \right| + e_i$$

SURPRISE < 0 => DUMMY = 0

$$\left| \mathrm{RESPONSE}_i \right| = \beta_0 + \beta_1 * \left| \mathrm{SURPRISE}_i \right| + e_i.$$

The coefficient β_3 serves to capture any asymmetric effect. Significance tests on the coefficient β_3 are one-tailed t-tests. The null hypothesis is listed in each cell of Table 2.

VII. Empirical Results

Tables 3.3 to 3.10 display the regression results for H1 through H4 based on the two-day geometric mean returns and the two-day average excess returns respectively. Tables 3.11 and 3.12 report the results for testing H5 and H6. As can be seen from the tables, the directions for the price and volume response to favorable and unfavorable earnings surprise are consistent with the implications of the theory. However, only H4 is significantly supported by the empirical evidence.

VIII. Conclusions

This paper develops a two-asset competitive equilibrium model with heterogeneous expectations and constant absolute risk aversion to investigate the effect of an earnings announcement on the change in price and trading volume. Theoretically, price response to a good or bad news earnings announcement is symmetrical only if investors' expectation of the liquidating value of the risky asset is unbiased. The response is asymmetric if (1) investors have biased opinions about the unknown value of the asset, or (2) the positive and negative earnings surprises have differential effects on the uncertainty of investors' expectations. Trading volume also reacts differently to positive and negative earnings surprises when the effect of uncertainty induced by the earnings announcement is taken into consideration even if the investors' expectations are unbiased. An asymmetric relationship between the change in price and trading volume exists even if there is no differential transaction cost between short and long positions. The asymmetry can exist in either direction, depending on whether the earnings announcement causes more or less uncertainty about the value of the risky asset among the investors. The empirical tests weakly support the theory developed.

TABLE 3.1 SUMMARY OF PROPOSITIONS 1 THROUGH 6

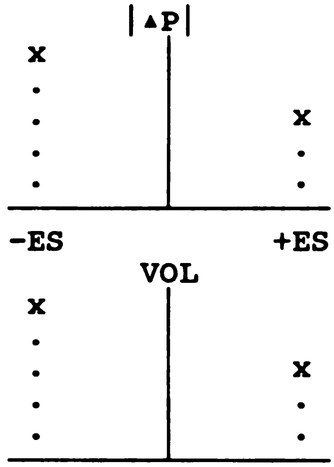
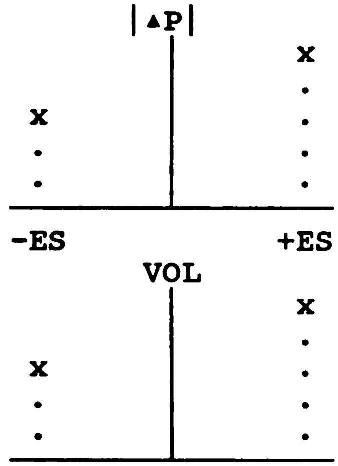
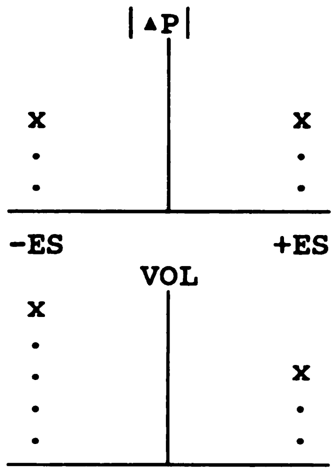
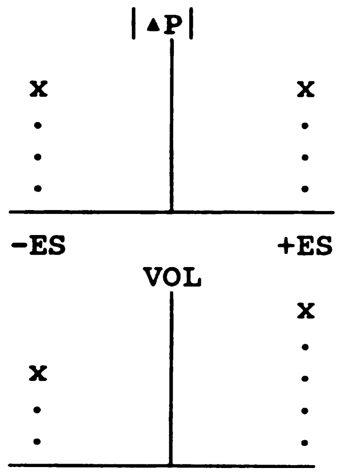
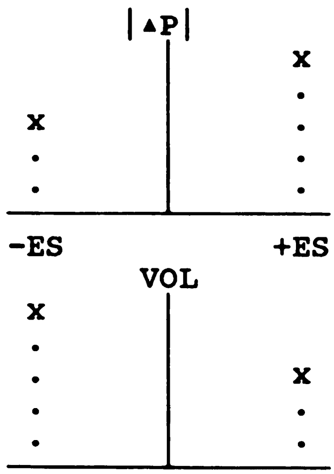
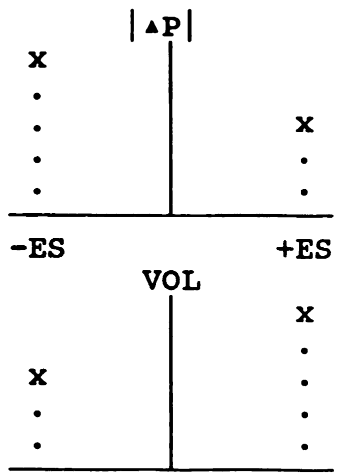
PRE-ANNOUNCEMENT FORECAST REVISION	UNCERTAINTY AFTER EARNINGS ANNOUNCEMENT	
	INCREASE	DECREASE
1. UPWARD		
2. NO CHANGE		
3. DOWNWARD		

TABLE 3.2 SUMMARY OF EMPIRICAL TESTS OF ASYMMETRY

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">CHANGE IN UNCERTAINTY AROUND THE TiME OF AN EARNINGS ANNOUNCEMENT</td></tr>
<tr><td>Reduced
$\sigma_0 - \sigma_{-1} < 0$</td><td>Increased
$\sigma_0 - \sigma_{-1} > 0$</td></tr>
<tr><td colspan="2">PRICE RESPONSE</td><td></td><td></td></tr>
<tr><td rowspan="2">MEAN CONSENSUS FORECAST REVISION BEFORE AN EARNINGS ANNOUNCEMENT</td><td>Upward
$\hat{X}_{-1} - \hat{X}_{-3} > 0$</td><td>H1: $\beta_3 \leq 0$</td><td>H2: $\beta_3 \geq 0$</td></tr>
<tr><td>Downward
$\hat{X}_{-1} - \hat{X}_{-3} < 0$</td><td>H3: $\beta_3 \geq 0$</td><td>H4: $\beta_3 \leq 0$</td></tr>
<tr><td colspan="2">VOLUME RESPONSE</td><td>H5: $\beta_3 \leq 0$</td><td>H6: $\beta_3 \geq 0$</td></tr>
</table>

Table 3.3

Test of Hypothesis 1 Using Two-Day Geometric Mean Return as Dependent Variable.

Dependent Variable: RESPONSE

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	0.00127	0.00042	3.508	0.0158
Error	286	0.03455	0.00012		
C Total	289	0.03582			

Root MSE	0.01099	R-square	0.0355
Dep Mean	0.01392	Adj R-sq	0.0254
C.V.	78.97483		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >|T|
INTERCEP	1	0.011785	0.00105458	11.175	0.0001
DUMMY	1	0.002449	0.00143717	1.704	0.0895
SURPRISE	1	0.058612	0.02566401	2.284	0.0231
INTER	1	0.033827	0.07290147	0.464	0.6430

Table 3.4

Test of Hypothesis 2 Using Two-Day Geometric Mean Return as Dependent Variable.

Dependent Variable: RESPONSE

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	0.00006	0.00002	0.198	0.8975
Error	272	0.02679	0.00010		
C Total	275	0.02685			

Root MSE	0.00994	R-square	0.0022
Dep Mean	0.01238	Adj R-sq	-0.0089
C.V.	80.28807		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.012664	0.00111440	11.364	0.0001
DUMMY	1	-0.000128	0.00141221	-0.091	0.9277
SURPRISE	1	-0.002075	0.06193654	-0.033	0.9733
INTER	1	-0.037370	0.08396224	-0.445	0.6566

Table 3.5

Test of Hypothesis 3 Using Two-Day Geometric Mean Return as Dependent Variable.

Dependent Variable: RESPONSE

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	0.00019	0.00006	0.463	0.7084
Error	600	0.08266	0.00014		
C Total	603	0.08285			

Root MSE	0.01174	R-square	0.0023
Dep Mean	0.01238	Adj R-sq	-0.0027
C.V.	94.83586		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.012366	0.00067593	18.295	0.0001
DUMMY	1	-0.000151	0.00103482	-0.146	0.8843
SURPRISE	1	0.009891	0.01190614	0.831	0.4064
INTER	1	-0.024784	0.02614147	-0.948	0.3435

Table 3.6

Test of Hypothesis 4 Using Two-Day Geometric Mean Returns as Dependent Variable.

Dependent Variable: RESPONSE

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	0.00289	0.00096	6.669	0.0002
Error	664	0.09591	0.00014		
C Total	667	0.09880			

Root MSE	0.01202	R-square	0.0293
Dep Mean	0.01243	Adj R-sq	0.0249
C.V.	96.67977		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.012749	0.00061397	20.764	0.0001
DUMMY	1	-0.001970	0.00098353	-2.003	0.0455
SURPRISE	1	0.011784	0.00374932	3.143	0.0017
INTER	1	0.042791	0.02174958	1.967	0.0495

Table 3.7

Test of Hypothesis 1 Using Two-Day Average Excess Returns as Dependent Variable.

Dependent Variable: RESPONSE

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	0.00102	0.00034	3.373	0.0189
Error	286	0.02874	0.00010		
C Total	289	0.02975			

Root MSE	0.01002	R-square	0.0342
Dep Mean	0.01241	Adj R-sq	0.0240
C.V.	80.77937		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.011019	0.00096181	11.456	0.0001
DUMMY	1	0.001188	0.00131075	0.907	0.3654
SURPRISE	1	0.062726	0.02340645	2.680	0.0078
INTER	1	0.026740	0.06648863	0.402	0.6879

Table 3.8

Test of Hypothesis 2 Using Two-Day Average Excess Returns as Dependent Variable.

Dependent Variable: RESPONSE

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	0.00023	0.00008	0.978	0.4033
Error	272	0.02151	0.00008		
C Total	275	0.02174			

Root MSE	0.00889	R-square	0.0107
Dep Mean	0.01078	Adj R-sq	-0.0002
C.V.	82.51684		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.010883	0.00099663	10.920	0.0001
DUMMY	1	-0.000738	0.00126273	-0.584	0.5595
SURPRISE	1	0.070576	0.05539110	1.274	0.2037
INTER	1	-0.039284	0.07445819	-0.528	0.5982

Table 3.9

Test of Hypothesis 3 Using Two-Day Average Excess Returns as Dependent Variable.

Dependent Variable: RESPONSE

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	0.00026	0.00009	0.697	0.5540
Error	600	0.07409	0.00012		
C Total	603	0.07435			

Root MSE	0.01112	R-square	0.0035
Dep Mean	0.01161	Adj R-sq	-0.0015
C.V.	95.76569		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.011578	0.00064648	17.909	0.0001
DUMMY	1	-0.000386	0.00098446	-0.392	0.6951
SURPRISE	1	0.015298	0.01252128	1.222	0.2223
INTER	1	-0.015362	0.02535855	-0.606	0.5449

Table 3.10

Test of Hypothesis 4 Using Two-Day Average Excess Returns as Dependent Variable.

Dependent Variable: RESPONSE

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	0.00196	0.00065	4.661	0.0031
Error	664	0.09295	0.00014		
C Total	667	0.09491			

Root MSE	0.01183	R-square	0.0206
Dep Mean	0.01177	Adj R-sq	0.0162
C.V.	100.51809		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.012439	0.00060441	20.580	0.0001
DUMMY	1	-0.002510	0.00096822	-2.592	0.0098
SURPRISE	1	0.006305	0.00369095	1.708	0.0881
INTER	1	0.042868	0.02141096	2.002	0.0457

Table 3.11

Test of Hypothesis 5 Using Two-Day Average Trading Volume as Dependent Variable.

Dependent Variable: VOLUME

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	522053928	174017976	5.685	0.0007
Error	891	27273812090	30610339		
C Total	894	27795866018			

Root MSE	5532.66112	R-square	0.0188
Dep Mean	4331.24972	Adj R-sq	0.0155
C.V.	127.73822		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	3558.336157	270.42977348	13.158	0.0001
DUMMY	1	1222.684040	417.58652131	2.928	0.0035
SURPRISE	1	8897.814088	4485.0699235	1.984	0.0476
INTER	1	19064	21937.461554	0.869	0.3851

Table 3.12

Test of Hypothesis 6 Using Two-Day Average Trading Volume as Dependent Variable.

Dependent Variable: VOLUME

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	360909650	120303216	3.671	0.0120
Error	926	30346326034	32771410		
C Total	929	30707235685			

Root MSE	5724.63190	R-square	0.0118
Dep Mean	4460.85645	Adj R-sq	0.0086
C.V.	128.33033		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	4095.785966	284.33120143	14.405	0.0001
DUMMY	1	265.128447	403.90846281	0.656	0.5117
SURPRISE	1	29098	9299.0543781	3.129	0.0018
INTER	1	-18615	13282.507100	-1.401	0.1614

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CHAPTER FOUR: THE ESTIMATION OF MARKET SPEED OF ADJUSTMENT USING SECURITIES PRICES AND TRADING VOLUME

ABSTRACT

This essay develops a statistical test for estimating the onset and duration of security price and trading volume responses to new information. It extends the analysis of Hillmer and Yu (1979) by allowing a dependent relationship between security price and trading volume. The dependent relationship between price and volume is addressed by orthogonalizing one market attribute with respect to the other. The resulting statistical test provides biased estimates of the onset and duration of market responses to new information (see Giliberto (1985)). A practical procedure for implementing the statistical test is then prescribed. The statistical test allowing dependence is compared to the Hillmer and Yu (1979) and Pincus (1983) tests in simulations of real world responses to information.

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I. Introduction

This paper develops a statistical procedure for estimating the onset and duration of security price and trading volume responses to new information. Study of the adjustment period is important for testing market efficiency and for understanding the way in which markets respond to new information such as earnings announcements. For example, Pincus (1983) examines the relationship between the duration of market (price and volume) adjustment and earnings predictability. He concludes that firms with harder to predict earnings streams have longer adjustment periods. Defeo (1986) finds that price adjustment duration depends upon firm size, reporting lag, and whether the announcement is of annual or quarterly earnings.

Statistical techniques for identifying the time of a change in the mean (Hinkley (1970) and Lee and Heghinian (1977)) and variance (Wichern, Miller and Hsu (1976)) of a time series have been proposed in the literature. The drawback of these methods, as Hillmer and Yu (1979) point out, is that they are very complicated and difficult to implement. Hillmer and Yu (1979) introduce a statistical technique that signals the point of time when the market begins to react to new information and the time when the reaction stops. Change in the mean or variance of a market attribute signals the onset and duration of the adjustment period. Hillmer and Yu's technique allows only one market attribute to be analyzed at a time. However, it is very general in that it allows the market attribute to be price, volume of trade, frequency of trade, number of block trades, or any other attribute which responds to information.

Pincus (1983) extends Hillmer and Yu's conceptualization of the adjustment period with a maximum likelihood procedure which incorporates price and trading volume. In the development of his MLE procedure, Pincus assumes that returns and volume are independent. Pincus goes on to observe that the variance of trading volume is constant over his time series and hence omits the variance of abnormal volume from his estimation procedure. These assumptions are inconsistent with empirical findings regarding changes in price and trading volume (e.g. Tauchen and Pitts (1983) and Harris (1986)).

A shortcoming common to these estimation methods is that they examine a single market attribute at a time. Beaver (1968), Copeland (1976), and Tauchen and Pitts (1983) demonstrate that volume and price are jointly determined by the arrival of information. Morse (1981) similarly argues that price and volume be used together to measure the information content of an event. Pincus (1983) comes the closest to a general method for identifying the adjustment period by including both price and volume in his maximum likelihood procedure. But by assuming price and volume are independent, he does not retain the full power of his test.

This essay develops a statistical test for estimating the onset and duration of security price and trading volume responses to new information. It extends the analysis of Hillmer and Yu (1979) by allowing a dependent relationship between security price and trading volume. The dependent relationship between price and volume is addressed by orthogonalizing one market attribute with respect to the other. The resulting statistical test provides biased estimates of the onset and duration of market responses to new information (see Giliberto (1985)). A practical procedure for implementing the statistical test is then prescribed. The statistical test allowing dependence is compared to the Hillmer and Yu (1979) and Pincus (1983) tests in simulations of real world responses to information.

In this essay, the technique of Hillmer and Yu is extended to incorporate both return and volume into estimation of the adjustment period. The technique explicitly allows for jointly dependent return and volume response to new information. This conceptualization of the adjustment period is discussed in Section II. Section III reformulates Hillmer and Yu's (1979) statistical test assuming independent price and volume changes. Section IV addresses the dependent relationship between price and volume by orthogonalizing one market attribute with respect to the other. Note that the resulting statistical test provides biased estimates of the onset and duration of market responses to new information (see Giliberto (1985)). Section V discusses implementation of the statistical test in practice. Section VI demonstrates use of the test statistic with 4 illustrative simulations.

II. Characterization of the Adjustment Process

In most previous empirical studies, the adjustment period is defined as the length of time around a public disclosure date during which the distribution of return is different from when there is no new information. This adjustment period is represented by the interval $t_{R2} - t_{R1}$ in Figure 1. However, volume may change before price as investors anticipate information release (see Figure 1). For instance, Morse (1981) contends that trading before the public disclosure of information may ensue from an increase in the differences in beliefs about the probability of different signals being released by the public announcement. These differences may be caused by asymmetric information among investors before the event date. Estimation of the adjustment period using return alone tends to underestimate the length of the adjustment period and cause an upward bias in estimation of the speed of adjustment. We define the adjustment period as the total length of time that both price and volume take to fully reflect new information. For example, in Figure 1 this period is given by (t_{R2} - t_{V1}). Although the process of information emission and interpretation is complex and unobservable, using both price and volume together may provide further insight into how the market reacts to information.

III. Estimation of the Adjustment Period

Empirical findings on the correlation between price changes and trading volume around the time of an earnings announcement are mixed. The conflicting evidence may arise from sample differences such as earnings predictability (Pincus (1983)), the magnitude of analysts' earnings forecast revisions (Jennings and Starks (1985)), firm size, reporting lags, or quarterly versus annual announcements (Defeo (1986)). Hillmer and Yu (1979) develop a statistical test for identifying the onset and duration of market response to information by assuming independence between market attributes including price and trading volume. This section reformulates Hillmer and Yu (1979) according to the conceptualization of adjustment period in Section II.

Consider the situation where return and volume are independent. Let V and R denote the level of trading volume and the rate of change of price, respectively. Suppose V and R are generated by the following stochastic processes:

$$R_t = \mu_R + \sigma_R S(R_t) \tag{1}$$

$$V_t = \mu_V + \sigma_V S(V_t) \tag{2}$$

where μ and σ are the respective means and standard deviations. Let $S(R_t)$ and $S(V_t)$ be independent and identically distributed normal random variables with mean zero and variance one. The covariance between $S(R_t)$ and $S(V_t)$ is zero by assumption.

Figure 2 depicts the market reaction of either return or volume around a public disclosure date. Suppose information is released at date t_0. Either market attribute may have begun to respond at t_1 prior to the announcement date. After the announcement, the market attributes (price and volume) continue to adjust until t_2 in order to fully reflect the effect of the new information. During the adjustment period $[t_1, t_2]$, the mean level of volume usually increases. However, return could either increase or decrease, depending on whether the news is good news or bad news. We develop one-tailed test statistics for a bad news scenario and a good news scenario in the remainder of this section.

A. Bad News Scenario

When the information released is bad news, we would expect the price level to decrease and the trading volume to increase. The partial sums of the deviation from the mean for R_t and V_t over the interval $[t_s, k]$ can be expressed as

$$SR_k = \sum_{t = t_s}^{k} (R_t - \mu_R) \tag{3}$$

$$SV_k = \sum_{t = t_s}^{k} (V_t - \mu_V). \tag{4}$$

Since SR_k and SV_k both have independent increments over time, their behaviors follow a Wiener process. The expected values of SR_k and SV_k are zero under the hypothesis that the mean levels do not change. Their respective standard deviations are $\sigma_R\sqrt{t}$ and $\sigma_V\sqrt{t}$ which are functions of time. As k increases beyond t_1 in Figure 1, we would expect SR_k to decrease and SV_k to increase.

In order to signal the beginning of the market reaction, we need to determine the crossing boundary ($B(R_j)=0$ and $B(V_j)=0$) for SR_k and SV_k such that, under the null hypothesis of no change in μ_R and μ_V, the probability of <u>either</u> SR_k or SV_k drifting beyond the boundary is less than some preset value α. That is,

$$\Pr\left[(SR_j \leq B(R_j) \cup SV_j \geq B(V_j)) \text{ for some } j \leq k \,\middle|\, \text{constant means}\right] = \alpha$$

Under the independence assumption, this probability can be simplified to

$$\Pr\left[(SR_j \le B(R_j);\ j \le k\right] + \Pr\left[SV_j \ge B(V_j);\ j \le k\right]$$

$$- \Pr\left[(SR_j \le B(R_j);\ j \le k\right] * \Pr\left[(SV_j \ge B(V_j);\ j \le k\right]$$

$$= \Phi\left[B(R_j)/(\sigma_R\sqrt{j})\right] + \left[1 - \Phi[B(V_j)/(\sigma_V\sqrt{j})]\right]$$

$$- \Phi\left[B(R_j)/(\sigma_R\sqrt{j})\right] * \left[1 - \Phi[B(V_j)/(\sigma_V\sqrt{j})]\right] = \alpha \qquad (5)$$

where $\Phi(.)$ is the standard cumulative normal distribution. We can normalize $B(R_j)$ and $B(V_j)$ with σ_R and σ_V such that

$$B(R_j)/\sigma_R = -B(V_j)/\sigma_V.$$

Then

$$\Phi[B(R_j)/(\sigma_R\sqrt{j})] = 1 - \Phi[B(V_j)/(\sigma_V\sqrt{j})]. \qquad (6)$$

Substituting equation (6) into (5) and simplifying,

$$B(R_j) = (\sigma_R\sqrt{j})\Phi^{-1}[1-\sqrt{(1-\alpha)}]$$

$$B(V_j) = (\sigma_V\sqrt{j})\Phi^{-1}[\sqrt{(1-\alpha)}]. \qquad (7)$$

The table below presents values of $B(R_j)$ and $B(V_j)$ at different levels of significance α's assuming no change in the processes R_j and V_j according to equation (7).

	$\alpha = 1\%$	$\alpha = 5\%$	$\alpha = 10\%$
$B(R_j)$	$-2.57\sigma_R\surd j$	$-1.96\sigma_R\surd j$	$-1.64\sigma_R\surd j$
$B(V_j)$	$2.57\sigma_V\surd j$	$1.96\sigma_V\surd j$	$1.64\sigma_V\surd j$

$B(R_j)$ and $B(V_j)$ are crossing boundaries that can be used to detect response of the market attributes to new information at the $\alpha\%$ significance level.

Figure 3 describes the behavior of SR_t and SV_t over time when μ_R and μ_V are allowed to change. Initially, SR_t and SV_t are fluctuating around zero. According to the definition of adjustment period in Section II, we want to detect the point at which either price or volume change. Of course, both means could shift at the same time. At t_1, the mean of V_t increases and/or the mean of R_t decreases. As R_t and V_t drift away from zero, the test statistic should not signal a change until SR_t and SV_t go beyond $B(R_t)$ and $B(V_t)$.

Let T_R and T_V be the amount of time from t_s that SR_t and SV_t take to reach $B(R_t)$ and $B(V_t)$, respectively. In order to estimate t_1, we need to determine the mean time that SR_t and SV_t take to deviate from the zero level and cross the boundaries. At the point of time when the mean of R_t

changes to μ_R', SR_t becomes a new Wiener process with mean $(\mu_R' - \mu_R)$. Similarly, SV_t will fluctuate with a new mean $(\mu_V' - \mu_V)$ if the mean level of trading activity changes to reflect the new information. The probability distribution of the first passage time for SR_t to drift from zero to $B(R_t)$ is given by:

$$\Pr\{T_R<t\} = \Phi\left[[-B(R_t)t + (\mu_R' - \mu_R)t]/(\sigma_R t^{1/2})\right]$$
$$+ \exp\left[2(\mu_R'-\mu_R)B(R_t)/\sigma_R^{2}\right] * \Phi\left[[-B(R_t)-(\mu_R'-\mu_R)t]/(\sigma_R t^{1/2})\right]. \tag{8}$$

From (8), the expected first passage time for SR_t determined by using the moment generating method is $B(R_t)/(\mu_R'-\mu_R)$. Similarly, the expected first passage time for SV_t is $B(V_t)/(\mu_V'-\mu_V)$. An unbiased estimate for t_1 is

$$\hat{t}_1 = \mathrm{Min}\left[\, T_R - \frac{B(R_{T_R})}{\mu_R' - \mu_R},\ T_V - \frac{B(R_{\ T_V})}{\mu_V' - \mu_V} \,\right]. \tag{9}$$

Detecting the ending point, t_2, of the adjustment period of the market attributes under the bad news scenario is similar to the detection of t_1. Beyond t_2, we would expect SR_k to increase and SV_k to decrease. Again, to signal the end of the market reaction, we need to determine the set of crossing boundaries ($B(R_j) > 0$, $B(V_j) < 0$) for SR_k and SV_k such that, under the null hypothesis of no change in μ_p and μ_v, the probability of either SR_k or SV_k drifting beyond the boundary is less than some preset value α. Similar to the derivation of Equation (5) under the independence assumption,

$$\Big[1- \Phi[B(R_j)/(\sigma_R\surd j)]\Big] + \Phi\Big[B(V_j)/(\sigma_V\surd j)\Big] - \Big[1 - \Phi[B(R_j)/(\sigma_R\surd j)] * \Phi[B(V_j)/(\sigma_V\surd j)]\Big] = \alpha. \tag{10}$$

Following (6), we can solve for β_j and β_j':

$$B(R_j) = (\sigma_R\surd j)\Phi^{-1}(\surd(1-\alpha))$$

$$B(V_j) = (\sigma_V\surd j)\Phi^{-1}[1-\surd(1-\alpha)]. \tag{11}$$

The following table presents the values of $B(R_j)$ and $B(V_j)$ at different levels of significance assuming no change in the processes R_t and V_t.

	α = 1%	α = 5%	α = 10%
$B(R_j)$	$2.57\sigma_R\surd j$	$1.96\sigma_R\surd j$	$1.64\sigma_R\surd j$
$B(V_j)$	$-2.57\sigma_V\surd j$	$-1.96\sigma_V\surd j$	$-1.64\sigma_V\surd j$

As in the derivation of equation (9), an unbiased estimate of t_2 is given by

$$\hat{t}_2 = \text{Max}\left[T_R - \frac{B(R_{T_R})}{\mu_R' - \mu_R},\ T_V - \frac{B(V_{T_V})}{\mu_V' - \mu_V} \right]. \qquad (12)$$

B. Good News Scenario

Empirical evidence indicates that both the price level and trading volume should increase in response to unexpected good news. The partial sums, SR_k and SV_k, of the deviation from the mean for R_t and V_t over the interval $[t_s, k]$ are given by Equations (3) and (4). SR_k and SV_k have independent increments and their behaviors follow a Wiener process. Under the null hypothesis of no change in the mean levels, the expected values of SR_k and SV_k are zero and their standard deviations are $\sigma_R\sqrt{t}$ and $\sigma_V\sqrt{t}$, respectively. As k increases beyond t_1, we would expect SR_k and SV_k to increase as R_t and V_t reflect the new information. To detect the beginning of the market reaction at a $(1-\alpha)$ significance level, we need to determine the set of crossing boundaries ($B(R_j) > 0$, $B(V_j) > 0$) for SR_k and SV_k such, that under the null hypothesis, the probability of <u>either</u> SR_k or SV_k exiting the boundary is less than α. That is,

$$\Pr\left[(SR_j \geq B(R_j)) \cup (SV_j \geq B(V_j)) \text{ for some } j \leq k \,\middle|\, \text{constantmeans}\right] = \alpha.$$

Under the independence assumption, this probability can be decomposed into

$$Pr[(SR_j \geq B(R_j)); j \leq k] + Pr[(SV_j \geq B(V_j)); j \leq k]$$

$$- Pr[(SR_j \geq B(R_j)); j \leq k] * Pr[(SV_j \geq B(V_j)); j \leq k]$$

$$= (1 - \Phi[B(R_j)/(\sigma_R/j)]) + (1 - \Phi[B(V_j)/(\sigma_V/j)])$$

$$- (1 - \Phi[B(R_j)/(\sigma_R/j)]) * (1 - \Phi[B(V_j)/(\sigma_V/j)]) = \alpha. \quad (13)$$

By normalizing $B(R_j)$ and $B(V_j)$ with σ_R and σ_V such that $B(R_j)/\sigma_R = - B(V_j)/\sigma_V$, then

$$\Phi[B(R_j) /(\sigma_R/j)] = \Phi[B(V_j)/(\sigma_V/j)] \ . \quad (14)$$

Substituting equation (14) into (13), we can solve for

$$B(R_j) = (\sigma_R/j)\Phi^{-1}[/(1-\alpha)] \quad \text{and}$$

$$B(V_j) = (\sigma_V/j)\Phi^{-1}[/(1-\alpha)]. \quad (15)$$

The values of $B(R_t)$ and $B(V_t)$ assuming different values of α are:

	α = 1%	α = 5%	α = 10%
$B(R_j)$	$2.57\sigma_R/j$	$1.96\sigma_R/j$	$1.64\sigma_R/j$
$B(V_j)$	$2.57\sigma_V/j$	$1.96\sigma_V/j$	$1.64\sigma_V/j$

IV. Dependent Return and Trading Volume Changes

Assume that R_t and V_t are generated by the same processes as specified by (1) and (2) except that $B(R_t)$ and $B(V_t)$ are bivariate normal with correlation coefficient equal to r. Although the relaxed assumption is simple, determination of the joint distribution of the first passage time of two stochastically dependent variables is difficult. Also, there may not be an explicit solution for the expected first passage time of either SR_t or SV_t exiting the boundaries. While lower or upper bounds could be found, it does not help in estimating the reaction time t_1 or t_2.

As an alternative route toward a solution, consider the following model based on the relaxed assumption

$$R_t = b_0 + b_1 * V_t + \epsilon_t \tag{16}$$

where ϵ_t has zero mean. Since R_t and V_t are correlated, the variable V_t explains part of the variation of R_t. The constant and the residual term ϵ_t capture that part of the variation in R_t which is not explained by V_t. Suppose there is a shift in the mean levels of R_t and V_t due to the arrival of new information. If the mean level of V_t shifts before those components of R_t which are independent of V_t, then R_t will also change according to the sign and the size of b_1. However, the term $(b_0 + \epsilon_t)$ will not reflect the change of V_t. If the mean level of R_t shifts first, then b_0

must change and hence the sum $(b_0 + \epsilon_t)$ will reflect the shift. If the mean levels of R_t and V_t independently shift at the same time, then $(b_0 + \epsilon_t)$ will not reflect the total change if $r > 0$.

Now, consider shifts in the variances of R_t and V_t. If the change in variance of V_t comes first, $(b_0 + \epsilon_t)$ will not capture the effect on R_t. If the variance of R_t shifts first, then it must come from a change in the variance of ϵ_t. If both variances change independently at the same moment, then $(b_0 + \epsilon_t)$ will not reflect the total change if $r>0$.

Under this formulation, $(b_0 + \epsilon_t)$ is an instrumental variable for R_t that can capture the response time of R_t with respect to that of V_t while remaining uncorrelated with V_t. We transform R_t and V_t into two new random variables, R_t^* and V_t^*, that are stochastically independent. In vector notation,

$$\begin{bmatrix} R_t^* \\ V_t^* \end{bmatrix} = \begin{bmatrix} R_t - bV_t \\ V_t \end{bmatrix} = \begin{bmatrix} \mu_R - b\mu_V + \sigma_R S(R_t) - b\sigma_V S(V_t) \\ \mu_V + \sigma_V S(V_t) \end{bmatrix}$$

where b is such that

$$E[(R_t-bV_t)-(\mu_R-b\mu_V)][(V_t-\mu_V)] = 0. \tag{17}$$

A solution for b exists if the variance-covariance matrix between R_t and V_t is non-singular. From (14), $b = r\sigma_R/\sigma_V$, which is simply the regression coefficient of R_t^* on V_t^*.

R_t* is the residual plus the constant term obtained by regressing R_t against V_t. It is easily shown that the transformed variables R_t* and V_t* are independent with means $(\mu_R - b\mu_v)$ and μ_v and variances $(1-r^2)\sigma_R^2$ and σ_v^2 respectively. The same analysis as in Section IIIA can then be repeated on R_t* and V_t*. Using the first passage time approach with T_R and T_V, the estimates for t_1 and t_2[14] are, respectively,

$$\hat{t}_1 = \text{Min}\left[T_R - \frac{\surd[T_R(1-r^2)]\ \sigma_R\Phi^{-1}[1-\surd(1-\alpha)]}{(\mu_R'-\mu_R) + r(\sigma_R/\sigma_V)(\mu_V'-\mu_V)},\ T_V - \frac{(\surd T\ )\sigma_V\Phi^{-1}\surd(1-\alpha)}{(\mu_V'-\mu_V)}\right]$$

and

$$\hat{t}_2 = \text{Max}\left[T_V - \frac{\surd[T_V(1-r^2)]\ \sigma_V\Phi^{-1}[1-\surd(1-\alpha)]}{(\mu_V'-\mu_V) + r(\sigma_V/\sigma_R)(\mu_R'-\mu_R)},\ T_R - \frac{(\surd T)\ \sigma_R\Phi^{-1}\surd(1-\alpha)}{(\mu_R'-\mu_R)}\right]. \tag{18}$$

If the correlation coefficient r is zero, the estimates for t_1 and t_2 are the same as those in Section IIIA.

V. Empirical Estimation Procedures

Procedures for detecting a shift in the mean, variance, and correlation coefficient of the market attributes are described in this section. Since the estimation of the beginning and the end point of the adjustment period are

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[14] The estimate for t_1 based on the orthogonalization of R_t and V_t may be biased. For reference, see Giliberto (1985).

similar, the following discussion focuses on estimation of the beginning point t_1.

A. Shift in Mean

Suppose the variance of R_t and V_t and the correlation between R_t and V_t remain constant. An empirical procedure to estimate the time of a shift in the mean of either R_t or V_t is described below.

STEP A1.

Refer to Figure 2. Arbitrarily estimate t_1 by plotting the behavior of the market attributes versus time by using moving averages, exponential smoothing, or visually. Then, pick a preliminary test interval $[t_s, t_m]$ where t_s is well before t_1 and t_m lies between t_0 and t_2.

STEP A2.

Calculate the sample correlation coefficient between V_t and R_t for $t \in [t_s, t_1]$,

$$\hat{r} = \frac{\sum_{t=t_s}^{t_m} (V_t - \bar{V})(R_t - \bar{R})}{\left[\sum_{t = t_s}^{t_m} (V_t - \bar{V})^2\right]^{1/2} \left[\sum_{t = t_s}^{t_m} (R_t - \bar{R})^2\right]^{1/2}}, \quad (19)$$

where $\bar{V} = \sum_{t=t_s}^{t_m} V_t/(t_m - t_s + 1)$ and $\bar{R} = \sum_{t=t_s}^{t_m} R_t/(t_m - t_s + 1)$.

Anderson (1984, p.109) shows that

$$(t_m - t_s + 1)^{1/2} \frac{\hat{r}}{(1 - \hat{r}^2)^{1/2}}$$

has a t-distribution with (t_m-t_s-1) degrees of freedom. Therefore, for a test of the hypothesis H0: r = 0 against the alternative hypothesis $r \neq 0$ at significance level α, we would reject the null hypothesis if the test statistic is greater than $t_{N-2}(\alpha)$.[15]

STEP A3.

If $\hat{r}$ is found to be significant, then go to Step A6. Otherwise, follow Steps A4 and A5 below.

STEP A4.

Calculate the following six statistics over the preliminary test interval $[t_s, t_m]$:

$$\hat{\mu}_R = \sum_{t=t_s}^{t_1} R_t/(t_1 - t_s + 1),$$

$$\hat{\mu}_R' = \sum_{t=t_1+1}^{t_m} R_t/(t_m - t_1),$$

[15] An equivalent test is to regress R_t against V_t and test if the estimated coefficient is significant or not. If it is, then R_t and V_t are correlated. However equation (16) is needed for the estimation of t_1 later.

$$\hat{\mu}_V = \sum_{t=t_s}^{t_1} V_t/(t_1 - t_s + 1),$$

$$\hat{\mu}_V' = \sum_{t=t_1+1}^{t_m} V_t/(t_m - t_1),$$

$$\hat{\sigma}_R{}^2 = \sum_{t=t_s}^{t_1} (R_t - \hat{\mu}_R)^2/(t_m - t_s + 1),$$

and
$$\hat{\sigma}_V^2 = \sum_{t=t_s}^{t_1} (V_t - \hat{\mu}_V)^2/(t_m - t_s + 1). \tag{20}$$

STEP A5.

Calculate SR_t, SV_t, $B(R_t)$, and $B(V_t)$ according to Equations (3), (4), and (7) for all $t \geq t_s$ until the crossing signals are sent. Record T_R and T_V. Then estimate t_1 by using equation (9). With the newly estimated t_1, repeat from Step A4 until the estimated t_1 converges to the desired confidence limit.

STEP A6.

Replace R_t and V_t in (3) and (4) with R_t' and V_t'. Replace σ_R in (7) with the sample estimate $\hat{\sigma}_R\sqrt{}(1-r^2)$. Determine SR_t, SV_t, $B(R_t)$, and $B(V_t)$ for all $t \geq t_s$ until the crossing signals are sent. Then record T_R and T_V. Estimate t_1 by substituting all the statistics in Step 4 and the sample correlation coefficient in (12) into (11). Repeat this step until the estimated t_1 converges to the desired confidence limit.

B. Shift in Variance

Suppose the mean of R_t and V_t and the correlation between R_t and V_t remain constant. An empirical procedure to estimate the time of a shift in the variance of either R_t or V_t is described below.

<u>STEP B1.</u>

Follow Steps A1 and A2 in Part IVA. Then calculate the sample correlation coefficient over the interval $[t_s, t_m]$ using the corresponding sample means:

$$r = \frac{\sum_{t=t_s}^{t_m} [(V_t - \bar{V})^2 - \mu_V]\ [(R_t - \bar{R})^2 - \mu_R]}{(t_m - t_s + 1)(\hat{\sigma}_R)(\hat{\sigma}_V)} . \tag{21}$$

If the sample correlation coefficient is significantly different from zero, then go to Step B4. Otherwise, follow Steps B2 and B3 below.

<u>STEP B2.</u>

Calculate the following statistics:

$$\hat{\mu}_R = \sum_{t=t_s}^{t_1} (R_t - \bar{R})^2/(t_1 - t_s + 1),$$

$$\hat{\mu}_R' = \sum_{t=t_1+1}^{t_m} (R_t - \bar{R})^2/(t_m - t_1),$$

$$\hat{\mu}_V = \sum_{t=t_s}^{t_1} (V_t - \bar{V})^2/(t_1 - t_s + 1),$$

$$\hat{\mu}_V' = \sum_{t=t_1+1}^{t_m} (V_t - \bar{V})^2/(t_m - t_1),$$

$$\hat{\sigma}_R^2 = \sum_{t=t_s}^{t_1} [(R_t - \bar{R})^2 - \mu_R]^2/(t_m - t_s + 1),$$

and $$\hat{\sigma}_V^2 = \sum_{t=t_s}^{t_1} [(V_t - \bar{V})^2 - \mu_V]^2/(t_m - t_s + 1). \tag{22}$$

where $\bar{R}$ and $\bar{V}$ are sample means over the period $[t_s, t_1]$.

STEP B3.

Replace R_t and V_t in (3) and (4) with $(R_t-\bar{R})^2$ and $(V_t-\bar{V})^2$, respectively. Use the estimates from (19) to estimate t_1 with (9). Repeat this step until t_1 converges as in Step A5.

Step B4.

Replace R_t and V_t in (3) and (4) with $(R_t*-\bar{R}*)^2$ and $(V_t*-\bar{V}*)^2$ and σ_R in $B(R_t)$ with $\sigma_R\surd(1-r^2)$. Determine SR_t, SV_t, $B(R_t)$, and $B(V_t)$ for all $t \geq t_s$ until the crossing signals are sent. Record T_R and T_V. Then substitute $\hat{r}$ and the estimates in Step B2 into (11) to estimate t_1. Repeat until convergence.

The mean and variance shift identification procedures prescribed above focus only on the estimation of the beginning point t_1. Estimation for the end point t_2 is basically the same. Roughly estimate t_2 and then arbitrarily select a preliminary test interval $[t_s, t_m]$ where t_s is after t_0 but before t_2 and t_m is far enough beyond t_2 to capture the end of the adjustment period.

C. Shift in Correlation Coefficient

Suppose we want to detect an increase in the correlation coefficient between R_t and V_t. Once the preliminary test interval $[t_s, t_m]$ is identified as above, calculate

$$\hat{\mu} = \sum_{t=t_s}^{t_1} (R_t - \bar{R})(V_t - \bar{V}) /[(t_1 - t_s + 1)\hat{\sigma}_R\hat{\sigma}_V]$$

$$\sigma^2 = \sum_{t=t_s}^{t_m} [(R_t - \bar{R})(V_t - \bar{V})/(\hat{\sigma}_R\hat{\sigma}_V) - \mu]^2/(t_m - t_s + 1),$$

where $\bar{V}$, $\bar{R}$, $\hat{\sigma}_R$, and $\hat{\sigma}_V$ are estimated over the period $[t_s, t_m]$. Next, calculate for all $k \geq t_s$

$$Sr_k = \sum_{t=t_s}^{k} [(R_t - \bar{R})(V_t - \bar{V})/(\hat{\sigma}_R\hat{\sigma}_V) - \mu]$$

until Sr_k crosses some boundary $B(R_j)$ at the desired confidence level. From Equation (5), we want $B(R_j)$ such that, conditional on no change in the correlation coefficient,

$$\Pr[(Sr_j \geq B(R_j));\ j \leq k] = (1 - \Phi[B(R_j)/(\hat{\sigma}\surd j)]) = \alpha.$$

Solving for $B(R_j)$, we have

$$B(R_j) = (\hat{\sigma}\surd j)\Phi^{-1}(1-\alpha).$$

From here, the procedures for estimating t_1 and t_2 are the same as those discussed previously. Similarly, for the detection of a decrease in the correlation coefficient, we want $B(R_j)$ such that

$$\Pr[(Sr_j \leq B(R_j));\ j \leq k] = \Phi[B(R_j)/(\hat{\sigma}\surd j)] = \alpha,$$

and therefore the crossing boundary is

$$B(R_j) = (\hat{\sigma}\surd j)\Phi^{-1}(\alpha).$$

VI. Simulation Results

This section discusses the results of 4 illustrative simulations. The first and second examples assume that R_t

and V_t are independent and that a change in the mean and then the variance of R_t and V_t are generated with equations (1) and (2). The third example assumes that the correlation coefficient between R_t and V_t changes at some point in time but that the mean and variance of R_t and V_t do not change. The fourth example simulates a change in the mean levels of R_t and V_t when return and volume are dependent. The beginning and ending points and hence the length of the adjustment period are estimated.

<u>EXAMPLE 1:</u> Shifts in Mean Levels (R_t independent of V_t)

Return and volume time series are generated 5000 times with the following models to simulate a shift in mean levels:

$$
\begin{aligned}
R_t &= 0.15 + 0.05\ S(R_t) && \text{for } t \in [1,\ 35] \text{ and } [77,\ 120] \\
&= 0.05 + 0.05\ S(R_t) && \text{for } t \in [36,\ 76] \\
V_t &= 5000 + 500\ S(V_t) && \text{for } t \in [1,\ 30] \text{ and } [73,\ 120] \\
&= 7000 + 500\ S(V_t) && \text{for } t \in [31,\ 72]\ ,
\end{aligned}
$$

where $S(R_t)$ and $S(V_t)$ are independent and normally distributed with mean zero and variance one. Each time, the beginning point (t_1) and the end point (t_2) of R_t and V_t are estimated using the steps described in Section VA with five iterations toward convergence. Table 1 compares the resulting mean estimates of t_1, t_2 and the adjustment period duration to those obtained with Pincus' maximum likelihood estimator. Numbers in parentheses represent standard deviations. Both procedures correctly identified the onset and duration of the separate adjustment periods for return and trading volume when R_t and V_t are independent.

<u>EXAMPLE 2</u> Shifts in Variance Levels (R_t independent of V_t)

Assume that the means do not shift but that the return and volume variances shift at some point in time. The following return and volume time series are generated 5000 times:

$$
\begin{aligned}
R_t &= 0.15 + 0.05\, S(R_t) && \text{for } t \in [1, 36] \text{ and } [77, 120] \\
&= 0.15 + 0.15\, S(R_t) && \text{for } t \in [37, 76] \\
V_t &= 5000 + 500\, S(V_t) && \text{for } t \in [1, 30] \text{ and } [77, 120] \\
&= 5000 + 1000\, S(V_t) && \text{for } t \in [31, 76]\ ,
\end{aligned}
$$

B_t and B_t' are again independent and normal with mean zero and variance one. The number of iterations toward convergence is again limited to 5. The summary in Table 2 indicates that the procedure of Section VB provided very close estimates of the onset and duration of the adjustment period. Pincus does not provide a maximum likelihood estimator for a shift in variance.

EXAMPLE 3: Shift in Correlation Coefficient

Assume that the means and the variances of R_t and V_t are constant. $S(R_t)$ and $S(V_t)$ are bivariate normal with zero means and variances one. Time series are generated 1000 times by:

$$R_t = 0.15 + 0.05\, S(R_t) \quad \text{for } t \in [1, 120]$$
$$V_t = 5000 + 500\, S(V_t) \quad \text{for } t \in [1, 120] \;.$$

The correlation coefficient between $S(R_t)$ and $S(V_t)$ is zero over the intervals $t \in [1,32]$ and $t \in [74,120]$. The correlation coefficient shifts to 0.7188 over the interval $t \in [33,73]$. Using the steps in Section VC, t_1 and t_2 are estimated with five iterations toward convergence in each simulation. Table 3 summarizes the results. Again, the onset and duration of the adjustment period are correctly identified.

EXAMPLE 4: Shifts in Means Levels (R_t Related to V_t))

Return and volume time series are generated 500 times with 5 iterative steps toward convergence according to the following:

$$\begin{aligned} R_t &= 0.15 + 0.05\, S(R_t) && \text{for } t \in [1, 99] \\ &= 0.05 + 0.05\, S(R_t) && \text{for } t \in [100, 200] \\ V_t &= 5000 + 500\, S(V_t) && \text{for } t \in [1, 99] \\ &= 8000 + 500\, S(V_t) && \text{for } t \in [100, 200] \end{aligned}$$

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$S(R_t)$ and $S(V_t)$ are bivariate normal with zero means, variances one, and correlation coefficient 0.6. In order to highlight the effect of non-zero correlation between R_t and V_t on the estimates of t_1 and t_2, assume that the means of R_t and V_t shift at the same time. Three groups of simulations are performed. The first group employs the estimation procedures with orthogonalization as described in Section V. The second group does not use orthogonalization but simply (and inappropriately) applies the procedure of Section IIIA assuming independence between R_t and V_t. The third group adopts Pincus' MLE procedure assuming independence between R_t and V_t.

The summary results presented in Table 4 indicate that the estimators which assume R_t and V_t are dependent compare favorably to the estimators assuming independence. However, a cautionary note is necessary. Giliberto (1985), among others, have shown that orthogonalization leads to biased estimators. If done appropriately, orthogonalization can extract no more information from the data than can OLS regression performed separately on each market attribute. In this case, appropriate use of the regression analysis reduces to the case of independence set out in equations (9) and (12). These equations reduce to Hillmer and Yu's (1979) if a single market attribute is examined at a time.

FIGURE 1

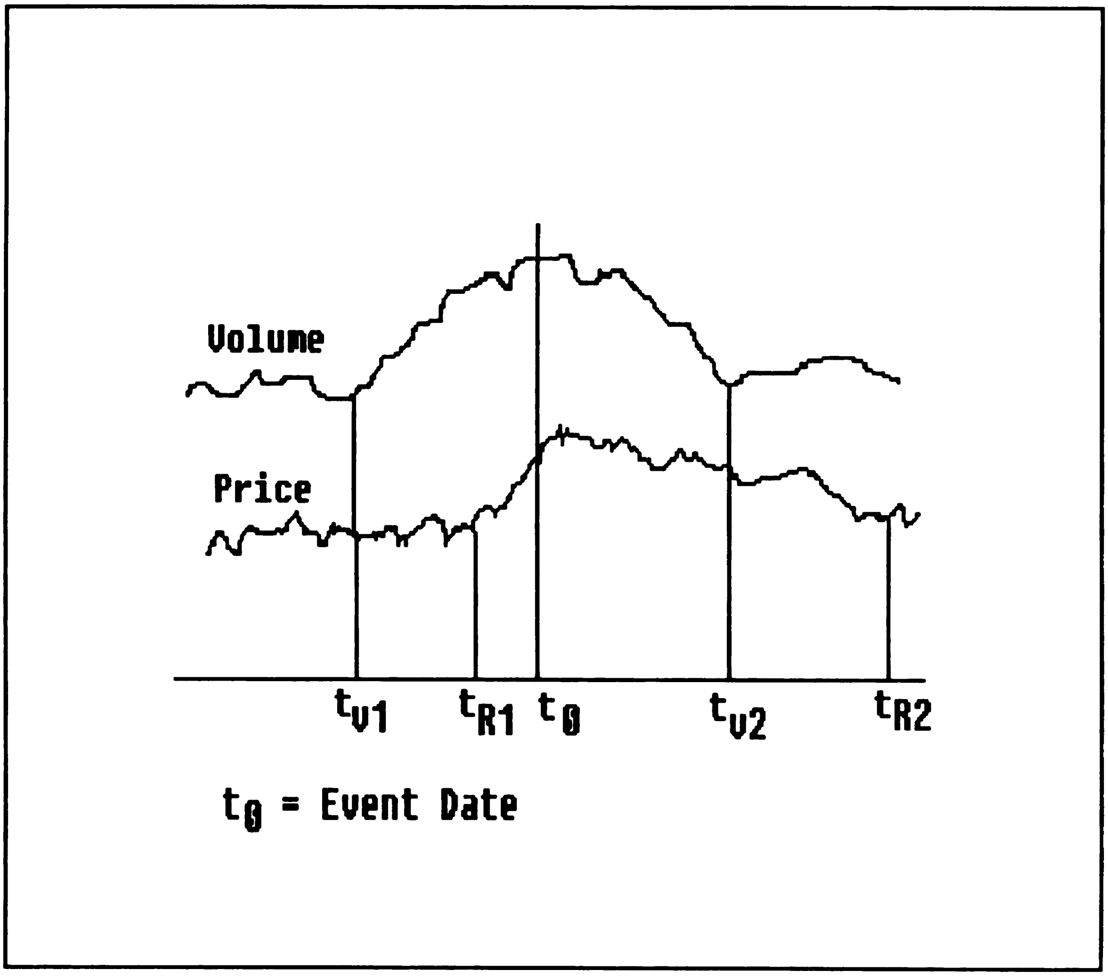

FIGURE 2

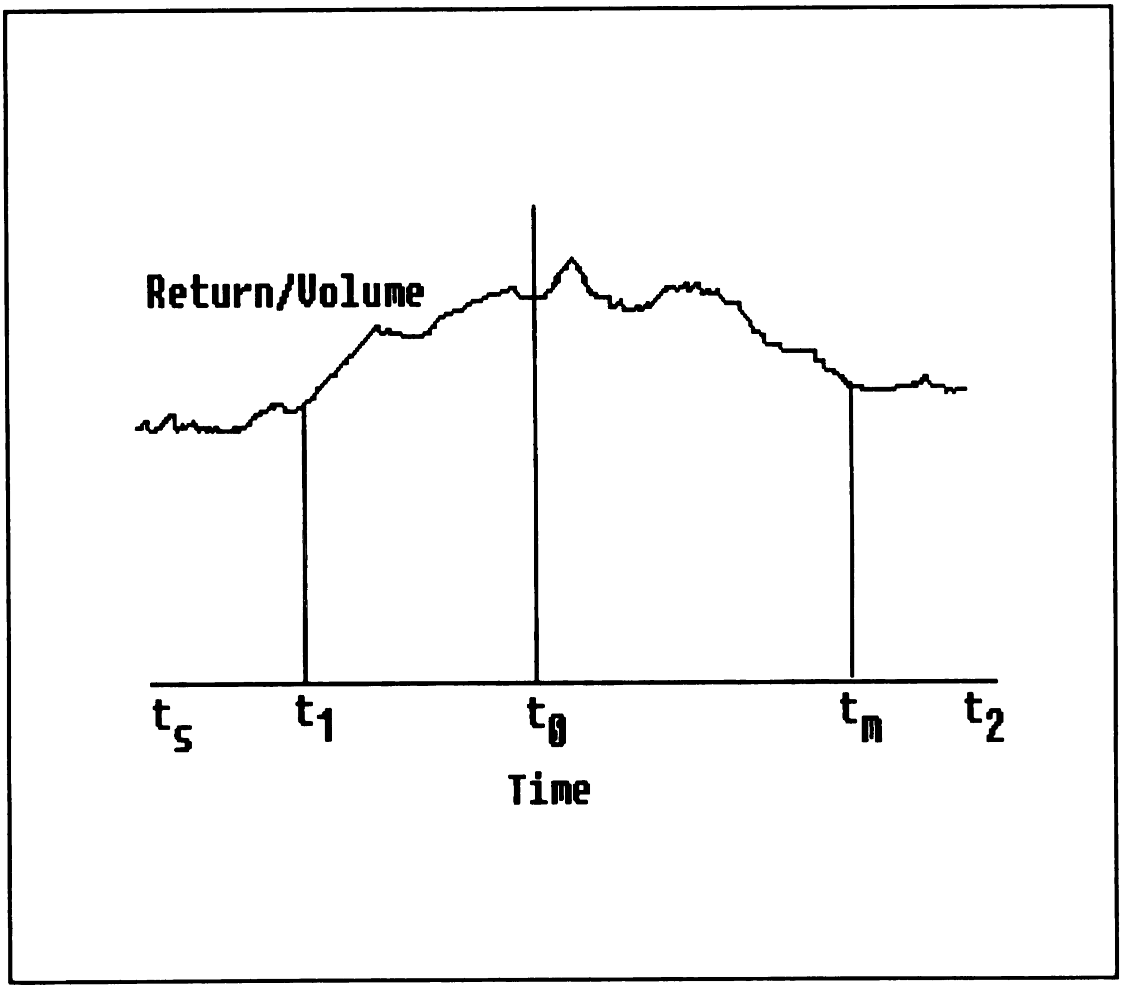

FIGURE 3

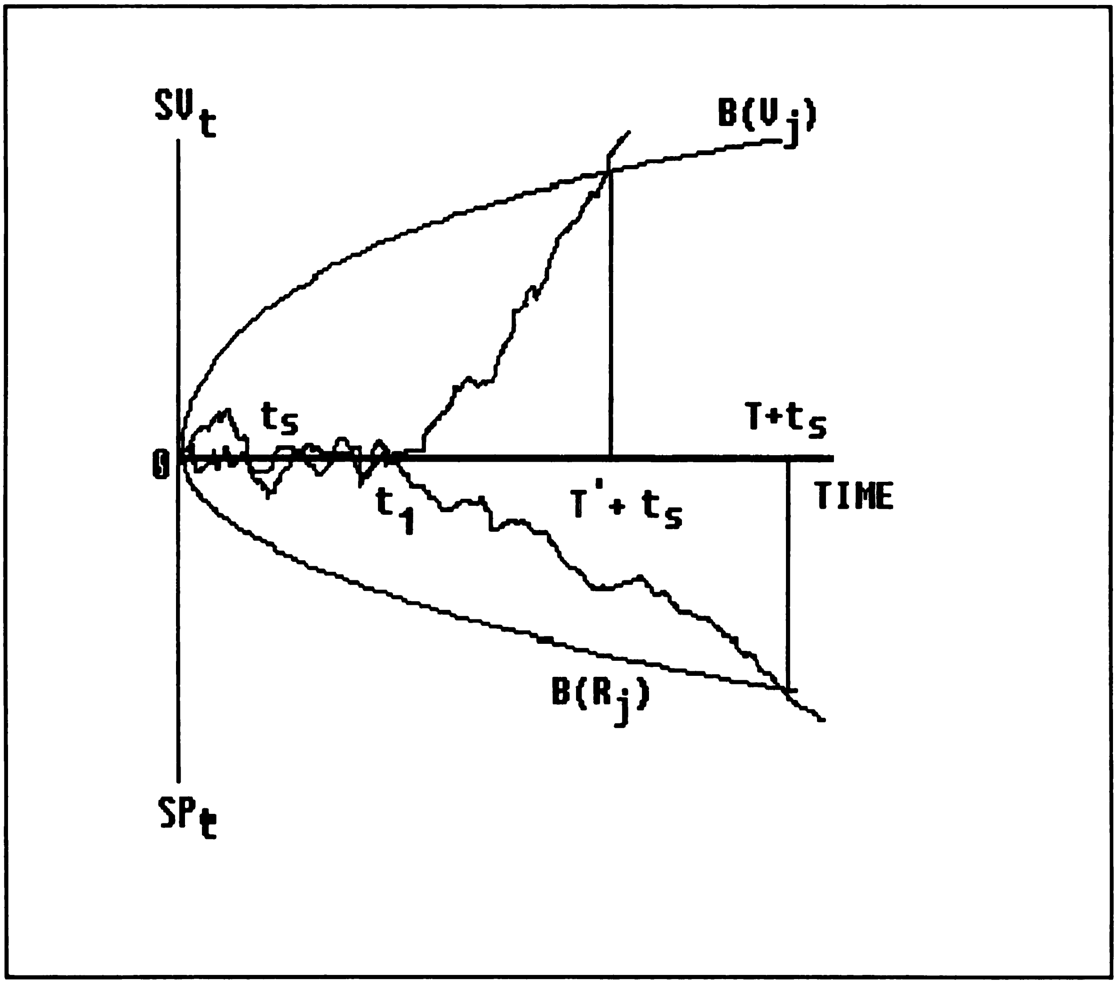

TABLE 4.1 SHIFT IN MEAN LEVELS - R_t AND V_t INDEPENDENT

		t1	t2	Adjustment Period
True Value	Return	36	77	41
	Volume	31	72	41
Section IVA	Return	35.94 (1.97)	77.05 (2.96)	45.69 (3.06)
	Volume	31.36 (0.53)	72.92 (0.74)	45.69 (3.06)
Pincus	Return	36.42 (2.39)	77.75 (3.97)	45.44 (4.88)
	Volume	32.31 (2.27)	70.97 (6.46)	45.44 (4.88)

TABLE 4.2 SHIFT IN VARIANCE LEVELS - R_t AND V_t INDEPENDENT

		t1	t2	Adjustment Period
True Value	Return	36	77	41
	Volume	31	72	41
Section IVB	Return	36.24 (2.54)	77.31 (5.41)	46.38 (8.20)
	Volume	30.93 (5.87)	71.89 (6.82)	46.38 (8.20)

TABLE 4.3 SHIFT IN CORRELATION COEFFICIENT

		t1	t2	Adjustment Period
True Value	r	33	74	41
Section IVC	r	33.15 (5.71)	73.96 (4.98)	40.81 (7.13)

TABLE 4.4 SHIFT IN MEAN LEVELS - R_t RELATED TO V_t

		t1
True Value	Return Volume	100 100
Section IVA	Return	99.50 (0.61)
	Volume	99.45 (0.56)
Section III	Return	98.03 (4.59)
	Volume	98.47 (5.08)
Pincus	Return	97.24 (3.89)
	Volume	96.43 (4.02)

CHAPTER FIVE: CONCLUSIONS

The objective of this study is to investigate theoretically and empirically the impact of information characterisitics and heterogeneous beliefs on security price and trading volume. Chapter 2 develops a theoretical model in a noisy rational expectations equilibrium framework incorporating heterogeneous information and diverse beliefs. The quality of information is characterized by individual investor's confidence and the variability of opinion across investors. The effects of these two characteristics of information on security price and volume are examined. The conclusion is that when the market is confidence-driven, large trading volume normally accompanies large price variability. When the market is consensus-driven, price variability is accompanied by low trading volume. Also, caution needs to be taken when we try to use price and volume to measure information content. Chapter 3 employs earnings announcement as a source of information to study how security price and volume react to good news and bad news. In a framework similar to Chapter 2, a theoretical model relating earnings announcements to security price and volume reaction is first developed. Empirical tests using daily CRSP returns, Media General's Trading Volume Tapes, Compustat, and Lynch, Jones and Ryan's Institutional Brokers Estimate System database are then developed to examine the model. Empirical evidence is consistent with the theory regarding the asymmetric response of price and volume to good news and bad news announcement.

A. Comments

Despite all the interesting results in chapter 2, a major weakness of the model employed in the analysis is that the competitive equilibrium constructed is arguably not stable. The price of the risky asset in the competitive equilibrium is a linear function of individuals' observations. The demand functions of the investors are characterized by the equilibrium price. Therefore, in order to trade, the investors must know the equilibrium price. If the economy as constructed was allowed to repeat itself, the investors would learn about the relationship between the equilibrium price and the individuals' signals. As they learned from history, they would get to suspect that the price of the risky asset contained valuable information about the uncertain payoff. Therefore, after knowing the most current equilibrium price, each investor would have the incentive to acquire more information to learn about the risky payoff. They would form new demands incorporating their new posterior beliefs. In this case, the competitive price may fail to clear the market and the competitive equilibrium collapses. As for chapter 3, the basic theoretical model suffers from the same weakness as in chapter 2.

B. Future Research Direction

In recent years, there have been several important theoretical works showing that historical prices as used in technical analysis is useful to investors.[16] Therefore, it would be interesting to extend the analysis in chapter 2 into a two-period model in order to investigate how price and volume behave over time in the confidence-driven and the consensus-driven economy. The additional assumption to be adopted is that prices are not fully revealing and traders have rational conjectures about the relationship between prices and the observed signals. The results would shed some light on whether using historical price and volume information in technical analysis is useful or not.

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[16] For example, see Brown D.P., and R.H. Jennings, "On Technical Analysis," Forthcoming in the Review of Financial Studies.

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