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PROPERTIES OF UNIT ROOT TESTS WITH HETEROGENEOUS AND DEPENDENT ERRORS

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ABSTRACT

PROPERTIES OF UNIT ROOT TESTS WITH HETEROGENEOUS AND DEPENDENT ERRORS

By

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The properties of unit root tests are considered when errors follow a generalized autoregressive conditional heteroskedasticity (GARCH) and a serial correlation in the variable's autoregressive representation. The standard Dickey-Fuller tests and various augmented and transformed versions of unit root tests are conducted. The primary concern is whether the proportion of rejections under the null hypothesis agrees with the nominal size of the test.

We also consider the use of different spectral windows with different numbers of truncation terms in the long run variance estimate in Phillips-Perron test statistics.

Finally Phillips-Perron tests based on the true values of innovation variance and long run variance are examined to help answer the cause of inaccuracies of Phillips-Perron tests.

Careful cautions in interpreting the results of the existing unit root tests are suggested and further theoretical work seems to be necessary.

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CHAPTER I

INTRODUCTION

It is commonly accepted that many economic time series are nonstationary. There are two very different models of nonstationarity, trend stationarity and difference stationarity, that have been considered extensively in both theoretical and empirical work.

In the trend stationary model, a variable is viewed as being composed as a deterministic (often linear) trend and a stationary deviation from the trend. In this case the variable can be detrended by regressing it on a suitable function of time, and the residuals of the regression are considered as a stationary series. A trend stationary variable has a tendency to revert to the trend, following a shock, and so it is meaningful to talk about the prediction of the series, even long into the future.

The difference stationary (or stochastic trend) model sees the variable as an accumulation of stationary differences. In this case, following Box and Jenkins (1970), differencing the series yields a stationary series. A random walk is a leading example of a difference stationary model, but a random walk has independently and identically distributed (iid) differences, and in the general difference stationary model we require only that the differences be stationary. In this model a shock to the variable will persist over time, and the uncertainty about the level of the variable grows larger as the horizon of the forecast increases. A difference stationary series contains a unit root in its autoregressive representation, and so a difference stationary series is also said to have a unit root.

Because the distinction between trend stationarity and difference stationarity is important both theoretically and empirically, there has been considerable interest in determining whether common economic time series are difference stationary or trend stationary. The recent interest in this empirical question was stimulated by the work of Nelson and Plosser (1982) and Nelson and Kang (1981, 1984), who questioned the prevailing view (at that time) that economic time series were trend stationary. In particular, Nelson and Plosser applied the tests of Dickey (1976), Fuller (1976) and Dickey and Fuller (1979) to test the hypothesis of difference stationarity in a large number of annual U.S. time series. They found strong evidence in favor of the unit root hypothesis, in that the unit root hypothesis could not be rejected for 13 out of 14 series considered.

The Dickey-Fuller tests will be described in detail in the next chapter. Briefly, they involve the regression of the variable on its own lagged value, and testing whether the coefficient of the lagged value equals unity. The regression may or may not include an intercept or time trend. The test does not follow the usual distribution theory based on the normal or t-distributions, even asymptotically. Dickey (1976) and Fuller (1976) provide the necessary distribution theory, but under the restrictive assumption that the errors in the regression are iid normal.

More recent work has extended the work of Dickey and Fuller in many different directions. The proper trearment of trend (or, more generally, other exogenous regressors) in the Dickey-Fuller regression has been discussed by Evans and Savin (1981, 1984), Nankervis and Savin (1985, 1987), Schmidt (1989) and Schmidt and Phillips (1989), among others. Tests of joint hypotheses (such as unit root <u>and</u> intercept equal to zero) have been proposed by Dickey and Fuller (1979) and Perron (1988). The robustness of the Dickey-Fuller tests to non-normality has been studied by Godfrey and Tremayne (1988) and Schmidt (1989). Extensive reviews of the unit root literature can be found in Dickey, Bell and Miller (1986) and Diebold and Nerlove (1989).

In this thesis we are concerned with the consequences of the failure of the assumption that the errors are iid. The asymptotic distribution of the Dickey-Fuller statistics has been derived by Phillips (1987) and Phillips and Perron (1988) under assumptions that allow fairly general forms of heterogeneity and autocorrelation. An immediate implication of these results is that heteroskedasticity and distributional heterogeneity do not affect the asymptotic validity of the tests. However, autocorrelation changes the asymptotic distribution of the test statistics and therefore must be accommodated in some way. This has been done in two ways in the literature.

The first way in which autocorrelated errors are accommodated in Dickey-Fuller tests is to use the so-called augmented tests of Said-Dickey (1984, 1985) and Solo (1984). These tests add lagged differences of the variable to the regression, and are valid asymptotically for series with a general autoregressive moving average (ARMA) representation.

The second way in which autocorrelated errors are handled is to find a transformed version of the test statistic which has asymptotically the standard Dickey-Fuller distribution. These tests based on transformed statistics are typically called Phillips-Perron tests, after Phillips (1987) and Phillips and Perron (1988), and are also valid asymptotically. They require consistent estimation of two nuisance parameters, as will be discussed in more detail in Chapter 4.

The asymptotic properties of the Dickey-Fuller tests in the presence of heteroskedasticity and autocorrelation are therefore well understood. Their finite sample properties are not well understood, however. In this thesis we use Monte Carlo methods to investigate the finite sample properties of Dickey-Fuller tests in the presence of heteroskedasticity and autocorrelation. We are primarily concerned with the accuracy of the tests, in the sense of whether the proportion of rejections under the null hypothesis agrees at least roughly with the nominal size of the test. We hope to provide some guidelines as to how well the various unit root tests can be expected to perform in practice. We also hope to identify circumstances under which they can be expected to be accurate and circumstances under which they cannot.

The plan of the thesis is as follows. Chapter 2 describes the tests to be considered in this thesis and introduces some necessary notation. Chapter 3 deals with the unit root tests when errors follow a generalized autoregressive conditional heteroskedasticity (GARCH) process. The main interest is to see how the tests perform in conditions under which the asymptotic theory almost breaks down; namely, when the GARCH process is close to being integrated and when the intercept in the GARCH process is close to zero.

Chapter 4 investigates the sensitivity of the Dickey-Fuller tests to autocorrelated errors. Schwert (1989), Godfrey and Tremayne (1988) and Phillips and Perron (1988) have shown that the Dickey-Fuller tests and their Phillips-Perron corrected versions are rather inaccurate, even for very large sample sizes, when the errors are highly autocorrelated. Their experiments used estimates of the "long run variance" parameter based on the method of Newey and West (1987), which basically amounts to the Bartlett (1950) window in the spectral estimation literature. We also consider the use of the Parzen (1961) and Bohman (1961) windows, with different numbers of terms in the sum defining the long run variance estimate, to see whether the use of a different spectral window can improve the Phillips-Perron tests.

In Chapter 5 we consider Phillips-Perron tests based on the true values of the innovation variance and the long run variance (which would be unknown in actual applications) to see whether the inaccuracies of the Phillips-Perron tests are due primarily to problems in estimation of these nuisance parameters, or whether they are intrinsic to the asymptotic theory for the tests. Finally, Chapter 6 contains our concluding remarks.

CHAPTER II

VARIANTS OF THE DICKEY-FULLER TESTS

In this chapter we present the Dickey-Fuller tests and their transformed versions which will be employed in the following chapters in investigating the reliability of unit root tests. The tests based on White's (1980) covariance matrix correction, the augmented Dickey-Fuller tests (Said-Dickey (1984)) and the Phillips-Perron tests (Phillips (1987) and Phillips-Perron (1988)) are introduced, as well as the standard Dickey-Fuller tests (Fuller (1976) and Dickey (1976)). The main interest in this thesis is the performances of the standard Dickey-Fuller tests and their various modifications under different error processes. We now describe these tests. All of the tests are based on either regression (1) or regression (2) below.

(1) $Y_t = \alpha + \beta Y_{t-1} + u_t \qquad t = 1, \ldots, T,$

(2) $Y_t = \alpha + \beta Y_{t-1} + \delta t + u_t \qquad t = 1, \ldots, T.$

Regression (1) is a regression of the variable , say "Y", on intercept and its lagged value, while regression (2) adds a time trend. In either case the unit root hypothesis corresponds to $\beta = 1$.

(a) The standard Dickey-Fuller $\hat{\rho}_\mu$ test is based on the statistic $T(\hat{\beta}-1)$, where $\hat{\beta}$ is the OLS estimate of β in regression (1), while the $\hat{\rho}_\tau$ test is based on the same statistic except that it uses the OLS estimate $\hat{\beta}$ from regression (2). Similarly, the Dickey-Fuller $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$ tests are based on the t-statistics for the hypothesis $\beta=1$ in regression (1) and (2) respectively, that is $\hat{\tau} = (\hat{\beta}-1)/sd(\hat{\beta})$ where $sd(\hat{\beta})$ is the standard error of the OLS estimate $\hat{\beta}$.

(b) White (1980) presented a covariance matrix estimator which is consistent in the presence of heteroskedasticity. Although the asymptotic theory for this covariance matrix correction does not extend to the case of a unit root in the variable or in its variance, we expect that the White tests will help in the conditional heteroskedasticity error model. We denote these tests with the suffix ".w"; that is, $\hat{\tau}_\mu$.w and $\hat{\tau}_\tau$.w. This correction is not available for the $\hat{\rho}_\mu$ and $\hat{\rho}_\tau$ tests, based on the estimated OLS coefficients, because these tests do not use an estimate of the variance of $\hat{\beta}$.

(c) We consider the augmented Dickey-Fuller tests (Said-Dickey(1984)), including either one or two lagged changes in Y in the regression. Said and Dickey(1984) proposed unit root tests using an approximation of an autoregressive-moving average model by an autoregression.

They argue that an unknown ARIMA(p,1,q) process can be approximated by an autoregressive model if the lag length (ℓ) in the autoregressive model increases with the sample size T, at a controlled rate less than $T^{1/3}$ [i.e. $\ell = O(T^{1/3})$]. However, this rule obviously does not specify the precise value of ℓ for a given T. Said and Dickey use the Dickey-Fuller regression t test for a unit root ($\beta = 1$) in the "augmented" model

$$(3) \qquad \Delta Y_t = \alpha + \beta Y_{t-1} + \sum_{i=1}^{\ell} r_i \, \Delta Y_{t-i} + u_t,$$

where $\Delta Y_t = Y_t - Y_{t-1}$.

The t-statistic for the hypothesis $\beta = 1$ in this model has the same limit distributions as those tabulated by Dickey (1976) and Fuller (1976) when $\ell \to \infty$ in (3) as $T \to \infty$, at the rate given above. We denote the tests based on the augmented models with the suffixes ".sd1" and ".sd2", respectively (e.g. $\hat{\tau}_\mu$.sd1 is the t-statistic when one lagged ΔY is added to the regression (1) above).

(d) Under weakly dependent and heterogeneously distributed errors, the limiting distributions of the regression coefficient and the t-statistic are represented in terms of standard Brownian motion in Theorem 3.1 in Phillips (1987). The limiting distribution of the test statistic depends on the variance ratio (σ_u^2/σ^2). Here σ_u^2 is the "innovation variance" and σ^2 is the "long run variance", defined as follows:

$$(4) \qquad \sigma_u^2 = \lim_{T\to\infty}(1/T) \sum_{t=1}^{T} var(u_t),$$

$$(5) \qquad \sigma^2 = \lim_{T\to\infty}(1/T) var(\sum_{t=1}^{T} u_t).$$

By using consistent variance estimators $\hat{S}_{t\ell}^2$, $\tilde{S}_{t\ell}^2$ (for σ^2) and $\hat{S}_u^2$, $\tilde{S}_u^2$ (for σ_u^2), Phillips (1987) and Phillips and Perron (1988) provide transformed versions of the test statistics which have the same asymptotic distribution as the standard Dickey-Fuller statistics. When the innovations are independently and identically distributed, $\sigma_u^2 = \sigma^2$, transformation of the test statistics would be unnecessary. Here $Z(\hat{\rho}_\mu)$, $Z(\hat{\rho}_\tau)$, $Z(\hat{\tau}_\mu)$ and $Z(\hat{\tau}_\tau)$ denote the transformed statistics for $\hat{\rho}_\mu$, $\hat{\rho}_\tau$, $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$, respectively. The new test statistics they propose are the following.

$$(6) \quad Z(\hat{\rho}_\mu) = T(\hat{\beta}-1) - (1/2)\ (\hat{S}_{t\ell}^2 - \hat{S}_u^2)\ /\ \{T^{-2} \sum_{t=1}^{T} (Y_t - \bar{y})^2\},$$

$$(7) \quad Z(\hat{\rho}_\tau) = (\hat{\beta}-1)\ \{\sum_{t=1}^{T} (Y_{t-1} - \bar{y}_{-1})^2\} - (1/2)(\hat{S}_{t\ell}^2 - \hat{S}_u^2)$$

$$[\hat{S}_{t\ell}\{T^{-2} \sum_{t=1}^{T} (Y_t - \bar{y})^2\}^{1/2}]^{-1},$$

where $\hat{S}_{t\ell}^2 = T^{-1} \sum_{t=1}^{T} \hat{u}_t^2 + 2T^{-1} \sum_{s=1}^{\ell} w_{s\ell} \sum_{t=s+1}^{T} \hat{u}_t\hat{u}_{t-s}$ ($\hat{u}_t$ is OLS

residuals from regression equation (1).),

$\bar{y}_{-1} = T^{-1} \Sigma Y_{t-1}$,

$\hat{S}_u^2$ = standard error of regression equation (1),

$\hat{\beta}$ = OLS estimate from equation (1).

(8) $Z(\hat{\tau}_\mu) = T(\tilde{\beta}-1) - (1/2)(\tilde{S}_{t\ell}^2 - \tilde{S}_u^2)M^{-1}$,

(9) $Z(\hat{\tau}_\tau) = (\hat{\beta}-1) / \{\tilde{S}_{t\ell}(c_1)^{1/2}\} - (1/2)(\tilde{S}_{t\ell}^2 - \tilde{S}_u^2)$
$(\tilde{S}_{t\ell} M^{1/2})^{-1}$,

where c_1 = first diagonal element of the matrix $(X'X)^{-1}$ (X denotes the T×3 matrix of explanatory variables in equation (2).),

$\tilde{\beta}$ = OLS estimate from regression equation (2),

$\tilde{S}_{t\ell}^2 = T^{-1} \sum_{t=1}^{T} \tilde{u}_t^2 + 2T^{-1} \sum_{s=1}^{\ell} w_{s\ell} \sum_{t=s+1}^{T} \tilde{u}_t \tilde{u}_{t-s}$ ($\tilde{u}_t$ is the OLS residuals from regression equation (2).),

$\tilde{S}_u^2$ = standard error of regression equation (2),

$$M = (1 - T^{-2})T^{-2}(\sum_{t=1}^{T} Y_t^2) - 12(T^{-5})(\sum_{t=1}^{T} tY_t)^2 + 12(T^{-4} + T^{-5})(\sum_{t=1}^{T} tY_t)(\sum_{t=1}^{T} Y_t) + (4T^{-3} + 6T^{-4} + 2T^{-5})(\sum_{t=1}^{T} Y_t)^2.$$

We consider the so-called Phillips-Perron tests (equation (6)-(9)), using either one or two terms in the estimation of the long run variance σ^2. We denote these with the suffixes ".pp1" and ".pp2", respectively. Like the augmented Dickey-Fuller tests, the Phillips-Perron tests are designed to handle residual autocorrelation.

The regression equations considered in this paper are equations (1) and (2); that is, one with an intercept and the other with an intercept and a time trend. The intercept is sometimes referred to as representing "drift", because when $\beta = 1$ a nonzero intercept implies a deterministic linear trend in Y. In discussing the effects of drift, it is important to make a distinction between the data generating process and the regression equation that generates the test statistics. The asymptotic and finite sample distributions of Dickey (1976) and Fuller (1976) were calculated under the assumption that the data generating process is a random walk without drift. But as Schmidt (1989) pointed out, using the Dickey-Fuller critical values is invalid when the time series process is a random walk with drift. Schmidt extensively tabulated the critical values for different values of sample size and drift. Nankervis and Savin (1985) also provided the distribution of the t-statistic for the case of unit root with drift. However, when the regression equation includes a time trend, the distributions of test statistics based on that equation do not depend on the value of the intercept. Therefore, for time series that contain drift, tests based on regression (2) would be a safer choice than tests based on equation (1).

All simulations in this paper are carried on IBM PC compatible using double precision on Lahey FORTRAN complier. The simulations are based on 10,000 replications for each choice of parameters. Normal random deviates were obtained from the routine GASDEV (using subroutine RAN3) of Press, Flannery, Teukolsky and Vetterling (1986). The required time to compute for a given choice of parameters on a 16 MHz 80386 PC ranged from 1.8 hours for T = 100 to 18 hours for T = 1000.

CHAPTER III

TESTS WITH CONDITIONAL HETEROSKEDASTICITY

In this chapter we consider the unit root tests of Dickey (1976), Fuller (1976), and Dickey and Fuller (1979, 1981). The tabulated distributions for these tests assume that the errors in the variable's autoregressive representation are normal white noise, but we consider the case that they are conditionally heteroskedastic.

Phillips (1987) has derived the asymptotic distribution of the Dickey-Fuller statistics under assumptions which allow for quite general weakly dependent and heterogeneously distributed errors. The asymptotic distribution of the Dickey-Fuller statistics depend on two nuisance parameters (σ_u^2 and σ^2), the innovation variance and the long run variance, that were defined in the last chapter. In the presence of autocorrelation, the Dickey-Fuller statistics do not have the same asymptotic distribution as they would without autocorrelation because σ^2 is not equal to σ_u^2, and adjustments to the tests are necessary. In the case of heteroskedasticity, however, such adjustments are unnecessary. This is so because σ^2 and σ_u^2 are equal (as

long as they exist), and the standard Dickey-Fuller distribution is asymptotically correct. Thus the sensitivity of the Dickey-Fuller tests to heteroskedasticity can be only a small sample problem.

This problem has been examined by several authors, including Godfrey and Tremayne (1988), Peters and Veloce (1988) and Phillips and Perron (1988). Godfrey and Tremayne investigated the performance of Dickey-Fuller tests and Phillips-Perron tests in the case of heteroskedastic errors. Their heteroskedastic innovations were generated by first obtaining independent drawings from a chi-squared distribution with 25 degrees of freedom, and then by scaling an independently drawn standardized Gaussian variate with the square root of drawings from the chi-squared distribution. The Dickey-Fuller tests are quite accurate even for small sample sizes. The Phillips-Perron tests performed well for T = 250 but poorly for T = 25. Peters and Veloce (1988) also provide Monte Carlo evidence for the case in which there is a conditional heteroskedasticity in the form of a generalized ARCH(GARCH) model of the type proposed by Bollerslev (1986). In their experiments, the Dickey-Fuller tests, the augmented Dickey-Fuller tests and the Phillips-Perron tests all reject too often when the null hypothesis is true, but not by a large amount.

We consider Dickey-Fuller tests in the presence of conditional heteroskedasticity of the GARCH form. The

GARCH model was developed by Bollerslev (1986) as a generalization of the ARCH model of Engle (1982). The GARCH model explicitly recognizes the difference between the conditional and the unconditional variance. In the ARCH process the conditional variance is specified as a linear function of past sample innovations, whereas the GARCH process allows the lagged conditional variances to enter as well. Specifically, if we let u_t denote the error in the Dickey-Fuller regression, and if we let h_t denote its variance conditional on information available at time t-1 (I_{t-1}), the error model is specified as follows.

(10) $u_t = \Gamma_t(\Phi_0 + \Phi_1 u_{t-1}^2 + \Phi_2 h_{t-1})^{1/2}$,

(11) $\Gamma_t \sim IN(0,1)$,

(12) $u_t | I_{t-1} \sim N(0,h_t)$,

(13) $h_t = \Phi_0 + \Phi_1 u_{t-1}^2 + \Phi_2 h_{t-1}$.

For $\Phi_2 = 0$ the process reduces to the ARCH(1) process, and for $\Phi_1 = \Phi_2 = 0$, u_t is simple white noise. If $\Phi_0 > 0$ and $\Phi_1 + \Phi_2 < 1$, the unconditional variance of the u_t exists and equals $\Phi_0 / (1 - \Phi_1 - \Phi_2)$.

The first aim of this chapter is to examine the robustness of the Dickey-Fuller tests to an integrated variance. The necessary condition for the variance process to be integrated, in the GARCH(1,1) model, is that Φ_1 and Φ_2 sum to one. Because the integrated GARCH (IGARCH) model has an infinite unconditional variance, the assumptions of Phillips (1987) are not satisfied, the limits defining the parameters σ^2 and σ_u^2 do not exist, and we should expect the distribution of the Dickey-Fuller statistics to be affected, even asymptotically. (Although we will not consider it in this paper, Hansen (1988) provides another model called the heteroskedastic cointegrated model (HCI). The HCI model is quite flexible and does not need to be specified explicitly, as do the ARCH models.) We conjecture that the accuracy of the Dickey-Fuller tests in the presence of heteroskedasticity will depend on how close the variance process is to being integrated.

The GARCH model has proved particularly useful in describing asset price fluctuations and in modelling time-varying risk premia, and many researchers have found a near-integrated variance process in this model. For example, Engle and Bollerslev (1986) apply the GARCH model to a time series of weekly data on the exchange rate between the U.S. dollar and the Swiss franc from July, 1973 through August, 1985, for a total of 632 observations. Other examples can be found in Baillie and Bollerslev (1989), Domowitz and Hakkio (1985), Diebold and Nerlove (1989), Engle, Lillien and Robins (1987), Bollerslev, Engle and Wooldridge (1988), and Diebold and Pauly (1988).

The second important case to be considered here is the case which the intercept (Φ_0) in the GARCH process has the value of zero. Nelson (1988) has shown that in the IGARCH (1,1) model with no drift ($\Phi_0 = 0$), h_t and u_t converge to zero almost surely. We will therefore call a GARCH model with $\Phi_0 = 0$ "degenerate". This is not a proper model for modelling changing volatility and the persistence of shocks to volatility unless one's economic model calls for variance to die out asymptotically. Here we conjecture that the performance of the Dickey-Fuller tests under GARCH errors will depend on the closeness of the intercept (Φ_0) to zero.

We investigate these two conjectures with a Monte Carlo experiment. Our errors will follow the GARCH(1,1) process, so we can control the closeness of the process to being integrated or degenerate by varying the parameters in the GARCH process.

For the Dickey-Fuller tests based on t-statistics, we also consider the test statistics which employ the heteroskedasticity-consistent covariance matrix estimator proposed by White (1980). The asymptotic theory for the White correction does not apply to the case in which the variable or its variance process is integrated. Furthermore, the asymptotic theory for the White correction requires the existence of the fourth moment of the error, and this condition is violated for some of the values that we consider for the GARCH parameters. (The condition for the existence of the unconditional fourth moment is

$$(14) \qquad 3\Phi_1^2 + 2\Phi_1\Phi_2 + \Phi_2^2 < 1 \quad ;$$

see Bollerslev (1986)).

The augmented Dickey-Fuller tests and the Phillips-Perron tests are also considered in this experiment.

A. Design of the Experiment

In this experiment we will consider the Dickey-Fuller $\hat{\rho}_\mu$, $\hat{\rho}_\tau$, $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$ tests. We will also consider their Phillips-Perron and Said-Dickey extensions, with ℓ = 1, 2 in each case. Finally, for the tests based on t-statistics ($\hat{\tau}_\mu$ and $\hat{\tau}_\tau$) we will consider the White covariance matrix correction.

Our data generating process will be an integrated process with GARCH errors. Thus the data generating process will be of the form of equation (1), with $\beta = 1$. Without loss of generality we set $Y_0 = 0$. We assume no drift; that is, $\alpha = 0$. Non-zero drift is potentially relevant, as argued by Schmidt (1989), West (1988) and others, but will not be considered in this chapter.

As specified in equation (12), the error u_t is distributed as $N(0,h_t)$ with the conditional variance h_t generated by equation (13). We set the initial variance $h_0 = 1$. This specific choice of h_0 does not affect the performance of the tests, because the test results are invariant to different values of h_0 as long as Φ_0 is changed proportionally so that the ratio of (Φ_0 / h_0) is held constant. We therefore need to vary only four parameters in the course of our experiments; Φ_0, Φ_1, Φ_2 and T.

Three types of experiments are considered. The first set of experiments (Tables 1-6) varies the extent to which the process is close to being integrated by varying the sum of $(\Phi_1+\Phi_2)$. This is done both by varying Φ_2 for fixed Φ_1 and vice-versa. In these experiments we set the initial variance h_0 equal to the long-run variance (when it exists) by setting $\Phi_0 = h_0(1 - \Phi_1 - \Phi_2)$ with $h_0 = 1$. Since only (Φ_0 / h_0) matters, this is equivalent to fixing Φ_0 and varying h_0 as $(\Phi_1 + \Phi_2)$ varies, except in the integrated case. In this setup, as $(\Phi_1 + \Phi_2)$ approaches one (as the process become nearly integrated), Φ_0 approaches zero (the process becomes nearly degenerate) too. This mixes two different types of parameter changes, but it automatically happens if we set the initial variance equal to the unconditional variance.

Our second set of experiments (Tables 7-14) fixes Φ_1 and Φ_2 and varies Φ_0. This fixes the degree of near integration and varies the degree of near degeneracy, and it therefore implicitly varies the relationship between the initial and the unconditional variance.

Our third set of experiments (Tables 15-16) follows the first set in varying the sum $(\Phi_1 + \Phi_2)$, but these experiments hold Φ_0 constant. This fixes the intercept and varies the degree of near-integration, and it also implicitly varies the relationship between the initial and the unconditional variance.

The results of the experiments are the percentages of rejections for a number of tests, though we also accumulated frequency distributions and moments for the various test statistics. Our main interest is in the performance of the Dickey-Fuller tests ($\hat{\rho}_\mu$, $\hat{\rho}_\tau$, $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$). However, the tests using White's (1980) heteroskedasticity-consistent covariance matrix, the augmented Dickey-Fuller tests (Said-Dickey (1984)) and the Phillips-Perron tests (Phillips (1987) and Phillips-Perron (1988)) are also considered.

B. Results of the Experiment

In the first set of experiments we vary ($\Phi_1 + \Phi_2$) while setting $\Phi_0 = h_0(1 - \Phi_1 - \Phi_2)$. In these experiments we set the initial variance equal to the unconditional variance (when it exists), and this implies that the progress becomes nearly integrated at the same time that the value of Φ_0 approaches zero. As the degree of near-integrated changes, the corresponding change in the value of Φ_0 happens automatically.

Table 1 reports the proportion of rejections under the null hypothesis for a 5% lower tail test, for T = 100, Φ_1 = .3 and Φ_2 = .30, .60, .65, .69 and .70. The results generally confirm our expectation that the accuracy of the tests should depend upon how close the variance process is to being integrated. For the smaller value of Φ_2 the tests reject too often, and the rejection rates are significantly different from 5%, but the differences from 5% are not really very large. For example, when $\Phi_2 = .30$, the proportions of rejections are .059, .069, .059 and .064 for $\hat{\tau}_\mu$, $\hat{\tau}_\tau$, $\hat{\rho}_\mu$ and $\hat{\rho}_\tau$, respectively. However, when the variance process is close to being integrated, the overrejection problem becomes more amd more serious for all tests. For example, the proportions of rejections for the $\hat{\tau}_\mu$ tests are .059, .074, .081, .114 and .412 for $\Phi_2 = .30, .60, .65, .69$ and .70, respectively. Also proportions of rejections for $\Phi_1 = .3$ and $\Phi_2 = .70$ are .412, .353, .294 and .243 for $\hat{\tau}_\mu$, $\hat{\tau}_\tau$, $\hat{\rho}_\mu$ and $\hat{\rho}_\tau$, respectively. The Phillips-Perron tests and the Said-Dickey tests behave in a similar manner, although the problem is a little less severe for the $\hat{\rho}$ tests than for the $\hat{\tau}$ tests. For instance, for $\Phi_2 = .70$ the proportions of rejections for $\hat{\tau}_\mu$.pp1 and $\hat{\tau}_\mu$.sd1 are .417 and .404, respectively, while, the proportions of rejections for $\hat{\rho}_\mu$.pp1 and $\hat{\rho}_\mu$.sd1 are .298 and .300, respectively. There is not much difference in the performance between the pp1 and pp2 tests, or between sd1 and sd2 tests. The White covariance matrix correction helps the performance of the tests but does not correct the overrejection problem completely. For instance, the proportion of rejections for the $\hat{\tau}_\mu$.w test when $\Phi_2 = .70$ is .121 which is much lower than the proportion of .412 for the unmodified standard $\hat{\tau}_\mu$ test, although it is still well above the nominal size of the tests, 5%. The same pattern occurs for the $\hat{\tau}_\tau$ when $\Phi_2 = .7$. The proportion of rejections for $\hat{\tau}_\tau$.w is .101 which is much lower than .353 for $\hat{\tau}_\tau$. Similarly, for the near-integrated case of $\Phi_2 = .69$, the proportions of rejections are .114 and .068 for $\hat{\tau}_\mu$ and $\hat{\tau}_\mu$.w respectively. The fourth moment existence condition (14) requires $\Phi_2 <$ .606 when $\Phi_1 = .3$, but the performance of $\hat{\tau}_\mu$.w and $\hat{\tau}_\tau$.w for $\Phi_2 = .65$, .69 and .70, which violate the fourth moment condition, does not show any discontinuity in the performance of the tests. The tests using the White covariance matrix correction are less severely affected in the integrated case compared to the other tests. The proportions of rejections for $\hat{\tau}_\mu$.w and $\hat{\tau}_\tau$.w is still better than the others for ($\Phi_1 = .3$, $\Phi_2 = .3$), (.3, .60), (.3, .65) and (.3, .69), but not by a large amount. In the integrated case the performances of the standard Dickey-Fuller tests, the Phillips-Perron tests and the Said-Dickey tests deteriorate rapidly compared to White's tests. For example, the proportion of rejections for $\hat{\tau}_\mu$ deteriorates from .114 when $\Phi_2 = .69$ to .413 when $\Phi_2 = .7$, but only from .068 to .121 for $\hat{\tau}_\mu$.w.

Although we have calculated results for upper tail and two tail tests as well as lower tail tests, we will discuss only the lower tail tests in detail, since the upper tail and the two tail tests are typically of little interest.

Table 2 and 3 report the proportions of rejection for the upper tail and the two tail tests, respectively, when Φ_1 = .3 and Φ_2 = .30, .60, .65, .69 and .70. Table 2 shows a slight overrejection for (.3,.3), (.3,.6), (.3,.65) and (.3,.69) for all tests and underrejection for the integrated variance error process. There is a serious problem only in the integrated case. For example, the proportion of rejections of $\hat{\tau}_\mu$ for (.3,.69) is .056, but .007 for (.3,.70). The White covariance matrix correction, the Phillips-Perron tests and the augmented Dickey-Fuller tests do not help at all. They do not improve the performance of the tests compared to the standard Dickey-Fuller tests.

Table 3 generally shows the same pattern as Table 1. For large values of Φ_2 all tests reject too often and when the variance process is close to being integrated, the overrejection problem becomes serious. The White correction tests work much better than the Phillips-Perron tests and the Said-Dickey tests, especially for the integrated variance process.

Table 4 reports the proportions of rejections for the lower tail test with T = 100, Φ_1 = .1, and Φ_2 = 0, .50, .80, .85, .89 and .90. We are again manipulating the nearness to integration of the variance process, but compared to the case of Table 1 the process has smaller value of Φ_1. All the results reported in Table 1 apply here, with only minor modifications, but the problem is much less severe. For example, even in the integrated case (Φ_2 = .90) the proportions of rejections are only in the range of .09 for $\hat{\tau}_\tau$ tests and .07 for the $\hat{\rho}$ tests. The White covariance matrix correction is not very useful for the parameter values in Table 4. It performed well only in the integrated case and for the $\hat{\tau}_\mu$ test, and for most Φ_2 values it actually made the performance worse. The fourth moment existence condition holds here for all except the two largest values of Φ_2. Again we can not find any discontinuity in the performance of the White tests due to the failure of the fourth moment existence condition. The $\hat{\tau}_\tau$.sd tests improve the performance by a small amount, although all other Phillips-Perron tests and other Said-Dickey tests do not help at all. It can be pointed out that $\hat{\tau}_\tau$.sd2 performs better than $\hat{\tau}_\tau$.sd1 for all parameter values in Table 4.

Table 5 is once again for T = 100 and for the lower tail test. Now we set Φ_2 = .30 and let Φ_1 = .30, .60, .65, .69 and .70. As Φ_1 is increased in this experiment the variance process is closer to being integrated. The same patterns that were evident in Table 1 are still evident in Table 5. For large values of Φ_1 the tests reject too often. The proportions of rejections are increasing as the variance process is closer to being integrated. In this integrated case the results in Table 5 are substantially worse than the corresponding results in Table 1. For example, the proportion of rejection for $\hat{\tau}_\mu$ with $\Phi_1 = .70$ and $\Phi_2 = .3$ is .888, whereas the same test with $\Phi_1 = .3$ and $\Phi_2 = .70$ is .412. Again $\hat{\tau}_\mu$.w and $\hat{\tau}_\tau$.w perform relatively well and the proportions of rejection stay between .074 and .053 except in the integrated variance case. The proportions of rejections of $\hat{\tau}_\mu$.w and $\hat{\tau}_\tau$.w in the integrated process case are well above their nominal size; however, their performances are still much better than the Phillips-Perron tests and the augmented Dickey-Fuller tests. The Phillips-Perron tests and the augmented Dickey-Fuller tests do not help at all, except that $\hat{\tau}_\mu$.sd2 and $\hat{\tau}_\tau$.sd2 do somewhat better than the other tests in the non-integrated case.

Table 6 reports results of experiments which we increase the sample size. For the three (Φ_1,Φ_2) pairs (.30,.60), (.30,.65) and (.30,.70), we set T = 100, 500 and 1000. Since the tests are asymptotically valid except in the integrated case, the performance of the tests would improve as the sample size increases, at least for the first two (Φ_1,Φ_2) pairs. This turns out to be so, but the rate of improvement is extremely slow. For example, when $(\Phi_1,\Phi_2) = (.30,.60)$ the proportions of rejections for the $\hat{\tau}_\mu$ test are .074, .070 and .061 for T = 100, 500 and 1000, respectively. In the integrated case the tests become worse rather than better as the sample size increases, suggesting the need for nonstandard asymptotics in the case of an integrated variance. In any case, it is clear that in determining the accuracy of the tests the sample size is not an extremely important factor, at least over an empirically relevant range of sample sizes.

Tables 7-14 report the second set of experiments. We fix Φ_1 and Φ_2 and vary Φ_0. Thus we are fixing the degree to which the process is nearly integrated, and varying the value of Φ_0. We are also implicitly varying the relationship between the initial and the unconditional variance. In Table 7, we fix $\Phi_1 = .3$, $\Phi_2 = .65$ and $h_0 = 1.0$ and vary Φ_0 from 0 to 100. The proportions of rejections for the various tests generally decrease with the increase in Φ_0. As the values of Φ_0 increase, the proportions of rejections are decreasing toward the nominal size. However, the proportions of rejections remain larger than the nominal size. The percentages of rejections remain near .08 for the most of tests even with $\Phi_0 = 100$. The tests perform reasonably well for large Φ_0 but the overrejection problem becomes extremely severe as Φ_0 approaches zero. We can see that the proportions of rejections of $\hat{\tau}_\mu$.w and $\hat{\tau}_\tau$.w approach the nominal size most closely when $\Phi_0 = 100$, compared to the standard Dickey-Fuller tests, the Phillips-Perron tests and the augmented Dickey-Fuller tests. For instance, the proportion of rejections of $\hat{\tau}_\mu$.w when $\Phi_0 = 100$ is .058.

Table 8 shows the percentages of rejections for the integrated case; i.e., $\Phi_1 = .3$ and $\Phi_2 = .7$. The results are very much similar to the results discussed for the near-integrated case in Table 7. There is a serious overrejection problem as the value of Φ_0 approaches zero, but the degree of seriousness is actually less here than in the near-integrated case in Table 7.

Tables 9 and 10 provide similar results when $(\Phi_1, \Phi_2) = (.1, .85)$ and $(\Phi_1, \Phi_2) = (.1, .9)$ repectively. The same pattern occurs here as in Tables 7-8 (when $\Phi_1 = .3$). The tests overreject when Φ_0 is small, and by an amount that increases as the value of Φ_0 approaches zero. The interesting point here is that the degree of overrejection depends upon the magnitude of Φ_1 and the overrejection problem is <u>less</u> severe in the integrated case rather than the near-integrated case. For example, Table 9 ($\Phi_1 = .1$, $\Phi_2 = .85$) and Table 10 ($\Phi_1 = .1$, $\Phi_2 = .9$) have less severe problems than Table 7 ($\Phi_1 = .3$, $\Phi_2 = .65$) and Table 8 ($\Phi_1 = .3$, $\Phi_2 = .7$) respectively, sometimes even an underrejection problem shows up for a large value of Φ_0 in Table 10.

Tables 11-14 increase the sample size from T = 100 to T = 1000 for the same values of Φ_0 (0, .01, 1.0 and 100) and for the same sets of other parameters ($\Phi_1 = .3$, $\Phi_2 = .65$), ($\Phi_1 = .3$, $\Phi_2 = .70$), ($\Phi_1 = .1$, $\Phi_2 = .85$) and ($\Phi_1 = .1$, $\Phi_2 = .9$). In the case when $\Phi_0 = 0$, increasing the sample size causes the performance of the tests to deteriorate, with the exception of the tests using the White covariance matrix correction. Actually $\hat{\tau}_{\mu}$.w and $\hat{\tau}_{\tau}$.w are better off with the increase of sample size when Φ_0 is equal to zero. When Φ_0 is not equal to zero, the increase in the sample size generally improves the accuracy of the tests, although this does not always hold true. In cases of improvement, the rate is often very slow. Increasing the sample size also tends to make the White correction tests reject too seldom for large values of Φ_0.

The interesting finding in the second set of experiments is that the important factor in determining the performance of the unit root tests in the presence of GARCH(1,1) errors is the intercept parameter Φ_0 in the variance process. A serious overrejection problem happens when Φ_0 is very small. This problem is even worse the larger the value of Φ_1 is and the larger the sample size is. The performance of the tests is not much influenced by how close the variance process is to being integrated.

The final set of experiments (Tables 15 and 16) confirms the findings of the second set of experiments. In our first set of experiments we set $\Phi_0 = h_0(1 - \Phi_1 - \Phi_2)$. In this setup as $(\Phi_1 + \Phi_2)$ approaches one, Φ_0 simultaneously approaches zero. Therefore the poor performance of the tests can be caused by either of these factors. However, from the results of the second set of experiments it is clear that the important factor in these experiments is the change in Φ_0, not the change in $(\Phi_1 + \Phi_2)$. This evidence is also supported by the results of the third set of experiments.

In the third set of experiments we fix Φ_0 and Φ_1, and we vary $(\Phi_1 + \Phi_2)$ by varying Φ_2. In Table 15 and 16 we pick T = 100, h_0 = 1.0 and Φ_0 = .01. According to the results of Tables 7-10, Φ_0 = .01 should correspond to a moderate degree of overrejection for the Dickey-Fuller tests. It turns out to be so. In Table 15 we set Φ_1 = .3 and vary Φ_2 from .3 to .7. Unlike the first set of experiments, the proportions of rejections do not change very much as $(\Phi_1 + \Phi_2)$ approaches unity. For example, when Φ_1 = .3 and with Φ_0 fixed at .01, changing Φ_2 from .65 to .70 hardly changes the percentages of rejections for the various tests. In Table 16 we set Φ_1 = .1 and vary Φ_2 from 0 to .9. We can also find a similar patterns of small changes in the performances of the various tests in Table 16. For example, when Φ_1 = .1, changing Φ_2 from .89 to .9 results in only small changes in the proportions of rejections for the various tests. It appears that the extent to which the GARCH error process is nearly integrated is not as important as the extent to which the magnitude of Φ_0 is close to zero (that is the extent to which the variance process is nearly degenerate).

C. Conclusion

In this chapter we have conducted a Monte Carlo experiment to investigate the reliability of the various Dickey-Fuller tests when the errors in the variable's autoregressive representation follow a GARCH(1,1) process. Specifically we are interested in whether the proportions of rejections under the null hypothesis (having a unit root) agrees with the nominal size of the test.

The standard Dickey-Fuller tests tend to reject too often in the presence of conditional heteroskedasticity. The problem seems to be less severe for tests based on the value of ß ($\hat{\rho}_\mu$ and $\hat{\rho}_\tau$) than for tests based on the t-statistic ($\hat{\tau}_\mu$ and $\hat{\tau}_\tau$). White's (1980) covariance matrix correction helps the performance of the $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$ tests most of time, but does not solve the problem entirely. Unsurprisingly the Phillips-Perron tests performed poorly throughout the tests. This is as expected because the Phillips-Perron corrections are designed to handle autocorrelation, not heteroskedasticity.

The same pattern of the results also occur for the augmented Dickey-Fuller tests (Said-Dickey (1984)). Again we might expect these poor results because this test is also designed to handle with autocorrelation, not heteroskedasticity.

In choosing the parameter values for our experiment, we emphasized how close the GARCH variance process equation

(13) is to being integrated ($\Phi_1 + \Phi_1 = 1$) and how close to zero the value of Φ_0 is. We found that the various Dickey-Fuller tests were seriously unreliable when the variance process has the value of Φ_0 close to zero or very nearly so. This was so more or less regardless of how close the variance process was to being integrated.

CHAPTER IV

TESTS WITH AUTOCORRELATED ERRORS

This chapter consists of two sections. The first section investigates the reliability of the various unit root tests in the presence of serially correlated errors. The second section uses three different lag windows (that is, Bartlett's, Parzen's and Bohman's lag windows) in the estimation of the long run variance to see how the Phillips-Perron tests perform for each different lag window. Also ten different lag truncation values (from one to ten) are tried in the estimate of the long run variance to see if there is a predictable pattern in the performance of the tests, and hopefully to be able to make some practical recommendation on the choice of the lag truncation value in applied work.

A. MA(1) and AR(1) error processes

Fuller (1976), Dickey (1976), Dickey and Fuller (1979, 1981) and Evans and Savin (1981,1984) proposed unit roots tests based on a first order autoregressive (AR(1)) model, and Said and Dickey (1984) allowed a more general ARIMA (p,1,q) progress. All of these authors assumed that the innovations are iid normal. Independence and homoskedasticity in the error process are quite restrictive assumptions. It is more realistic to ease these strong assumptions to allow various forms of serial dependence in the error structure. The most common autocorrelated error specification has been the firsr order autoregressive (AR(1)),

(15) $u_t = \rho u_{t-1} + \epsilon_t$, $t = 1, \ldots, T$,

where for stationarity $|\rho| < 1$, and the ϵ_t are random variables with $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma_\epsilon^2$ and $E(\epsilon_t \epsilon_s) = 0$ for $t \neq s$. Although the AR(1) error model has been popular, there are many instances when moving average errors are justified by economic theory (for example, Nicholls, Pagan and Terrell (1975)), and, in addition, models with MA or ARMA errors may often be a better representation of the data generating process.

For MA(1) errors, we have

(16) $u_t = \epsilon_t + \theta \epsilon_{t-1}$, $t = 1, \ldots, T$,

where again $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma_\epsilon^2$, $E(\epsilon_t \epsilon_s) = 0$ for $t \neq s$, and $|\theta| < 1$ for invertibility. This weakened error process was assumed by several authors, including Schwert (1989), Phillips and Perron (1988), Peters and Veloce (1989) and Godfrey and Tremayne (1988). All of them assumed MA(1) errors in the data generating process.

Schwert (1989) showed that tests for unit roots developed by Dickey and Fuller are sensitive to the assumption that the data are generated by a pure AR process. Specifically, the distributions tabulated by Dickey (1976) and Fuller (1976) can be very different when the error in the data generating process has an MA component. Furthermore, the tests of Said-Dickey (1984) and Phillips (1987) do not seem to work well when the MA parameter is large, even though they should be correct asymptotically. In particular, the Phillips-Perron tests do not come close to their asymptotic distributions even for rather large sample sizes. Godfrey and Tremayne (1988) also addressed the question of whether or not unit root tests are robust to the failure of the assumption of iid innovations, and whether or not the statistics of Phillips (1987) and Phillips and Perron (1988) designed for such situations perform satisfactorily in finite samples. They allowed MA(1) errors with the moving average parameter equal to .5. They found that the Phillips-Perron tests may need a rather large sample size to be reliable.

The aim of the first section of this chapter is to confirm the foundings of Schwert (1989), Godfrey and Tremayne (1988), and Phillips and Perron (1988) and to complement their results in the sense that we allow AR(1) errors in addition to MA(1) errors.

1. Design of the Experiments

In the experiments we will consider tests based on the same two regressions as before; that is, equation (1) and (2). Tests considered here are the standard Dickey-Fuller tests, the White covariance matrix correction tests, the Phillips-Perron tests, and the augmented Dickey-Fuller tests. These tests were explained in Chapter 2.

Our data generating processes are MA(1) errors and AR(1) errors. Otherwise they follow the regression equation (1) with $\beta = 1$. Without loss of generality we set $Y_0 = 0$. We assume no drift; that is, $\alpha = 0$. Most of our experiments are done with a sample size of T = 100. Sample sizes of T = 200 and T = 500 are considered for selected parameters. The MA parameter is set equal to -.8, -.5, -.2, .2, .5, and .8. The same values of ρ are considered for the case of AR(1) errors. The "base case" in the tables means that there is no AR or MA component in the data generating process, so that in the base case the errors are iid. Each experiment is replicated 10,000 times to create the sampling distribution for the test statistics. Although we have results for frequency distributions and moments, we only report the proportions of rejections for the lower tail tests. The lower tail tests are our main concern because the usual alternative hypothesis is that the process is stationary ($\beta < 1$).

2. Results of the Experiment

The main concern of the experiments is how close the proportions of rejections for the various tests are to the nominal size, 5%. Tables 17-20 report the results for MA(1) and AR(1) error process.

Table 17 gives the proportions of rejections under the null hypothesis for a 5% lower tail test for T = 100. It shows that MA errors affect the performance of the tests very much. The rejection rates are negatively related to the MA parameter. All the tests considered reject too often for negative values of θ and reject too seldom for positive value of θ. The degree of departure from the nominal size is much higher for the negative values of θ than for the positive values. The proportions of rejections of all the tests for θ = -.8 are close to 1.0. Thus MA errors definitely affect the performances of the tests. Although we do not report them here, the 5% quantiles for the test statistics reflect an extreme skewedness of the distributions toward negative values. For example, the 5% quantile for $\hat{\rho}_{\tau}$.pp2 reaches to -105.87 for θ = -.8. The Phillips-Perron tests show much the same pattern as the uncorrected tests; that is, the percentages of rejections decrease with an increase in the value of the MA parameters. The Phillips-Perron corrections seem to improve the performance of the tests, but the degree of improvement is very small. For example, the proportion of rejections for

$\hat{\tau}_\mu$.pp1 when $\theta = -.2$ is .105 compared to .132 for $\hat{\tau}_\mu$ and the proportion of rejections for $\hat{\tau}_\mu$.pp1 when $\theta = -.8$ is .985 compared to .997 for $\hat{\tau}_\mu$. Because of the serial dependence in the error structure, $\sigma^2 \neq \sigma_u^2$, we do not expect that the Phillips-Perron tests will have the same distribution as the standard Dickey-Fuller tests. However, the minimal improvement in the performance of the Phillips-Perron tests compared to the uncorrected tests is an unfortunate result. The augmented Dickey-Fuller tests perform well compared to the other tests. For example, the $\hat{\tau}_\mu$.sd2 and $\hat{\tau}_\tau$.sd2 tests perform somewhat better. However, the percentages of rejections are still very different from nominal size for large negative MA parameters. Schwert (1989) pointed out that the error caused by the approximation of an ARIMA process by the AR(1) process is large for MA parameters greater in absolute value than .8. The proportions of rejections for the Said-Dickey test range from .643 for $\hat{\tau}_\mu$.sd2 to .999 for $\hat{\rho}_\mu$.sd1.

The tests based on the White covariance matrix correction perform worse than the uncorrected tests for negative MA parameters, and better for positive MA parameters. It is understandable that the Phillips-Perron tests and the Said-Dickey tests, which were designed for residual autocorrelation, outperform the White tests designed for heteroskedasticity. It is interesting that the Said-Dickey tests outperform the Phillips-Perron tests for all values of the MA parameters, although $\hat{\tau}_{\mu}$.sd1 and $\hat{\tau}_{\tau}$.sd1 still clearly show the overrejection problem for θ = -.8. In particular, the performances of $\hat{\tau}_{\mu}$.sd2 and $\hat{\tau}_{\tau}$.sd2 are reasonably good. For example, the proportions of rejections for $\hat{\tau}_{\mu}$.sd2 are .049, .047 and .038 when θ = -.2, .2 and .5, respectively. The proportions of rejections for the base case are close to the nominal size as expected. The rejection rates for the base case should be close to those of the Dickey-Fuller tests because there is no serial correlation in the error structure, and it turns out to be so.

In Table 18 we increase the sample size to T = 200 and T = 500 to investigate the effects of the increase in the sample size on the performance of the tests when θ = -.5 or .5. The results for all tests for the base case converge to nominal size. When θ = -.5 the increase in the sample size actually deteriorates the reliability of all the tests with the exception of the Phillips-Perron tests. For example, proportion of rejections for $\hat{\tau}_{\mu}$ with T = 100 is .571 compared to .622 with T = 500. Although the increase in the sample size improves the performance of the Phillips-Perron tests, the proportions of rejections are still well above the nominal size. For instance, the proportion of rejections for $\hat{\tau}_{\mu}$.pp1 when θ = -.8 with T = 100 is .457 compared to .415 with T = 500. When θ = .5 many of the tests' performances deteriorate as the sample size grows, but by a small amount. It can be pointed out from Tables 17 and 18 that the most important factor determining the reliability of the tests is the degree of the dependence in the error structure, not sample size.

In the following we discuss the effects of AR(1) errors on the performance of tests. We consider same parameter values with T = 100; that is, ρ = -.8, -.5, -.2, .2, .5 and .8. As mentioned earlier, several authors have done Monte Carlo experiments of the unit root tests for the case of MA(1) erros, but no one has done these experiments for the case of AR(1) errors (as far as we know). Remembering that the AR(1) is a popular error specification, especially for annual data sets, the results of these experiments are interesting.

Table 19 reports the rejection rates under the null hypothesis for a 5% lower tail test for T = 100. The test results show a similar pattern as in the MA(1) error case. The tests reject too often for negative values of ρ and reject too seldom for ρ = .2 or .5. The degree of overrejection or underrejection is usually less severe than in the MA(1) error case. However, for the AR(1) case the underejection problem for the $\hat{\rho}_\mu$ and $\hat{\rho}_\tau$ tests when ρ = .5 or .8 is more severe than in the case of MA(1) error with θ = .5 or .8. Again the Phillips-Perron tests do not seem to work well. They do better than the standard Dickey-Fuller tests, but the degree of improvement is minimal. However, the Said-Dickey $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$ tests perform well. The proportions of rejections of $\hat{\tau}_\mu$.sd and $\hat{\tau}_\tau$.sd stay around 5% for all AR parameters. The rejection rates range from .046 to .059. $\hat{\rho}_\mu$.sd and $\hat{\rho}_\tau$.sd are better than the other $\hat{\rho}$ tests, but still reject too often for negative AR parameters and reject too seldom for positive AR parameters. Comparing the Said-Dickey tests with one and two lagged Δy's added to the regression equation, the tests using two lagged Δy's have a tendency to reject less often than those using one for $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$, and vice versa for $\hat{\rho}_\mu$ and $\hat{\rho}_\tau$. However, the Phillips-Perron tests with $\ell = 2$ always reject more often than the tests with $\ell = 1$ regardless of the choice of tests and the value of the parameters. The White correction worsens the performance of the various tests for negative values of the AR parameter and improves their performance slightly for positive values of the AR parameter. It is an unexpected result that the White tests work better than the Phillips-Perron tests when $\rho = .2$ or .5. For example, the proportion of rejections of $\hat{\tau}_\mu$.w when $\rho = .5$ is .044 compared to .028 for $\hat{\tau}_\mu$.pp1.

In Table 20 the sample size is increased to T = 200 and T = 500 with $\rho = -.5$ or .5. All tests generally improve as the sample size grows, but the degree of convergence to the nominal size is slow and the proportions of rejections are still well above 5% when $\rho = -.5$ and below 5% when $\rho = .5$.

The augmented Dickey-Fuller $\hat{\tau}$ tests are fairly reliable when $\rho = -.5$ or $.5$.

3. Conclusion

The simulations in this section confirm Schwert's (1989) results. Including MA(1) or AR(1) errors in the data generating process can make the critical values tabulated by Dickey (1976) and Fuller (1976) very misleading. Generally the proportions of rejections for the various tests are well above the nominal size for negative MA or AR parameters and are below the nominal size for positive MA or AR parameters. The White tests (designed for heteroskedasticity) do not seem to work well, which is understandable. However, it is an unexpected result that the Phillips-Perron corrections do not improve the performance of tests significantly, because they are designed for serial dependence in the error structure. The Said-Dickey augmentations certainly help the performance of the tests, especially in the AR case. In particular, the proportions of rejections of the Said-Dickey augmented $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$ tests are close to the nominal size. The Phillips-Perron tests seem to be more sensitive to model misspecification than the Said-Dickey tests. The performances of all of the tests depend more critically upon the degree of dependence in the error process than upon the sample size.

These simulation results give rise to a warning against the careless use of the Dickey-Fuller tests in economic time series when the errors are dependent. There is an obvious need for tests that are more reliable in the presence of dependent errors.

B. Lag Windows

Phillips (1987) and Phillips and Perron (1988) provided transformed statistics which have the same asymptotic distribution as the usual Dickey-Fuller statistics, even given autocorrelated or heteroskedastic errors. The transformed statistics require consistent estimates of two parameters, the innovation variance σ_u^2 and the long run variance σ^2, defined (as in Chapter 2) by

$$(17)\quad \sigma_u^2 = \lim_{T\to\infty} (1/T) \sum_{t=1}^{T} \mathrm{var}(u_t), \quad \sigma^2 = \lim_{T\to\infty} (1/T)\ \mathrm{var}\left(\sum_{t=1}^{T} u_t\right).$$

Consistent estimation of σ_u^2 is trivial. Phillips suggests as a consistent estimate of σ^2 the estimator $S_{t\ell}^2$ defined by

$$(18)\qquad S_{t\ell}^2 = T^{-1} \sum_{t=1}^{T} \hat{u}_t^2 + 2T^{-1} \sum_{\tau=1}^{\ell} \sum_{t=\tau+1}^{T} \hat{u}_t\ \hat{u}_{t-\tau}$$

Here "ℓ" controls how many autocorrelations are used in $S_{t\ell}^2$ and will be called the lag truncation value. Phillips (1987, Theorem 4.2) shows that consistency of $S_{t\ell}^2$ requires

$\ell \to \infty$ as $T \to \infty$ but at a controlled rate (e.g. $\ell = O(T^{1/4})$). This procedure assumes that there may be strong dependence among adjacent events, but events separated by a long distance are almost independent. Therefore, terms such as $E(u_t u_{t-\tau})$ with long lags $\tau > \ell$ can be ignored in the consistent estimator, $S_{t\ell}^2$. Although choosing a specific value of ℓ is an empirical matter, we can obtain a rough idea of the appropriate value of ℓ by examining the sample autocovariances of $u_t = y_t - y_{t-1}$. A natural procedure would be to choose the value of ℓ for which the autocovariance function is insignificantly different from zero for all lags greater than ℓ. Although rather crude, it is certainly preferable to choosing a completely arbitrary value of ℓ. However, there is an inherent danger in this approach. Because the sample autocovariances are themselves autocorrelated, the sample autocovariance function will decay more slowly than the theoretical autocovariance function, as pointed out in Priestley (1981). A problem with the long run variance estimate $S_{t\ell}^2$ is that there is a possibility that large negative serial covariances can make $S_{t\ell}^2$ negative. To see how likely this is we calculated the number of negative values of $S_{t\ell}^2$ in our experiments of the last section, based on 10,000 replications. The experiments are conducted in the same manner for both AR, MA errors; that is, we use the parameter values -.8, -.5, -.2, 0,.2, .5, and .8. We also changed the value of the lag truncation term ℓ in $S_{t\ell}^2$ from 1 through 10 to see how the value of ℓ affects the number of negative $S_{t\ell}^2$. For example, in Table 21 the notation pp10 indicates the number of negative values of $S_{t\ell}^2$ for the case of $\ell = 10$. The number of negative values of $S_{t\ell}^2$ increases with the value of ℓ for all MA parameters except $\theta = -.2$ and for all positive AR parameters. The base case (iid errors) shows a similar pattern. When $\rho = -.8$ or $-.5$, the number of negatives is large for odd values of ℓ and small for the even values of ℓ; however, the number of negatives gradually decreases with the value of ℓ. We certainly have evidence in this experiment that in some cases, for example, $\ell = 1$ and $\rho = -.8$ or $-.5$, there are too many negative numbers for the estimator $S_{t\ell}^2$ to be useful.

An appropriate weighting scheme can be used to guarantee a positive value of $S_{t\ell}^2$. Thus we write

$$\tilde{S}_{t\ell}^2 = (1/T) \sum_{t=1}^{T} \hat{u}_t^2 + (2/T) \sum_{i=1}^{\ell} \sum_{t=i+1}^{T} w(i) \; \hat{u}_t \hat{u}_{t-i}. \tag{19}$$

For example, Newey and West (1987) suggest $w(i) = i/(\ell+i)$, $i=1, \ldots, \ell$, and this guarantees a positive estimate of σ^2 (for any ℓ). Now it is true that the long run variance $\sigma^2 = 2\pi f_u(0)$ where $f_u(\lambda)$ is the spectral density of the error u. Thus there is a close connection between the problem of estimating σ^2 and the problem of estimating the spectral density function. The choice of a weighting function w(i) is equivalent to the choice of a "window" for spectral density estimation. The Newey-West weighting corresponds to the well known Bartlett window. But the literature on the estimation of the spectrum suggests other windows, and they too guarantee the positive value of the long run variance estimate. In this section two more lag windows in addition to Bartlett's window are considered; that is , Parzen's and Bohman's lag windows.

The first point of this section is to investigate the performance of the Phillips-Perron tests using Parzen's and Bohman's lag windows in the estimation of the long run variance. The second point is to investigate the choice of the lag truncation value (ℓ). For each lag window we tried values of ℓ from 1 through 10 to see how the value of ℓ makes a difference in the performance of the tests.

1. Lag Windows

We know that the true spectral density function is non-negative. Therefore, it is desirable that the estimate should share this property. Three lag windows considered here all guarantee a positive estimate of the spectral density function. The widely adopted form of the spectral density estimate is

$$(20) \qquad \hat{f}_u(\lambda) = (1/2\pi) \sum_{s=-(\ell+1)}^{s=\ell+1} \lambda(s)\ \hat{R}(s)\ \cos(s\lambda),$$

where ℓ is the truncation point. $\{\lambda(s)\}$ is a sequence of decreasing weights which is called the 'lag window' and $\hat{R}(s)$ is the sample autocovariance. The sequence $\{\lambda(s)\}$ involves a truncation point ℓ which controls its rate of decay. The weights given to $\hat{R}(s)$ decrease with increasing s. In particular, suppose that $\lambda(s)$ is of scale parameter form (that is, $\lambda(s) = k(s/\ell)$, and that $\lambda(s)$ is a bounded, even function. Then

(a) $\lambda(0) = 1$,

(b) $\lambda(w) = \lambda(-w)$,

(c) $\lambda(w) = 0,\ w > 1$.

All three lag window generators considered here have the lag window of scale parameter form. Variation of the window parameter ℓ simply 'stretches' or 'contracts' the function, so that ℓ acts as a scale parameter.

Bartlett(1950) proposed the lag window having the form

$$(21) \qquad \lambda(s) = \begin{cases} 1 - (|s| / \ell), & |s| \leq \ell, \\ 0, & |s| > \ell. \end{cases}$$

This is the window used in equation (20) in Phillips (1987). This lag window applies linearly decreasing weights to the autocovariance up to lag ℓ and zero weights thereafter.

Parzen (1961) suggested the lag window of the following form

$$(22) \qquad \lambda(s) = \begin{cases} 1 - 6(s / \ell)^2 + 6(|s| / \ell)^3, & |s| \le \ell/2, \\ 2\{(1 - (|s| / \ell)\}^3, & (\ell/2) \le |s| \le \ell, \\ 0, & |s| > \ell. \end{cases}$$

Bohman (1961) window is

$$(23) \quad \lambda(s) = \{1 - (|s| / \ell)\} \cos\{\pi(|s| / \ell)\} + [\sin\{\pi(|s| / \ell)\} / \pi].$$

The properties of the estimate of the spectral density function depend on the choice of ℓ and on the form of the window. It is not surprising that, even allowing for a suitable choice of ℓ, different windows can lead to estimates with different properties. Priestley (1981) illustrates various criteria for judging the different windows from a statistical point of view. However, our main criterion here is that the lag window should guarantee the positive estimate of the spectral density function. All three lag windows mentioned earlier satisfy this requirement. In addition they satisfy the relationship $\lambda(1) = 0$. When $\lambda(1) \neq 0$, there is a discontinuity in $\lambda(s)$ at 1. The Parzen window's weighting function has much sharper peaks which are nearer to $w = 0$ than Bartlett's lag window. As Neave (1972) pointed out, the total weight in the range $w > 1/2$ is only 18.75 percent in Parzen's window and 23.9 percent in Bohman's window, compared to about 50 percent in Bartlett's window. Parzen's window is satisfactory on the basis of the arguments above. Bohman's window has not received as much attention, but it is clear that Bohman's window has properties similar to Parzen's.

More discussions about the spectral analysis and lag window generators may be found in Priestley (1981) and Dhrymes (1974).

2. Design of the Experiment

Once again the two types of regression equations considered are one with a intercept and lagged dependent variable and the other with a time trend added; that is, equations (1) and (2), respectively. The data generating process is equation (1) with $\beta = 1$, having first order moving average or first order autoregressive error processes with the same set of parameters considered previously (-.8, -.5, -.2, .2, .5 and .8). The base case is Gaussian white noise; i.e., $\rho = \theta = 0$ as before. Only the Phillips-Perron tests with different values of ℓ are examined here, but the standard uncorrected Dickey-Fuller tests are also reported for purposes of comparison. Here the value of ℓ is varied from 1 through 10. As before, the suffix 'pp1' indicates the Phillips-Perron test with $\ell = 1$, and similarly for other values of ℓ. All simulations are replicated 10,000 times for sample size T = 100 with each different choice of parameters. We are mainly interested in comparing the three different lag windows in the performance of the Phillips-Perron type tests. We are also interested in how the lag truncation value used in $S_{t\ell}^2$ affects the performance of the tests. Although the investigation of the sample autocorrelations of u_t will help in selecting an appropriate choice of ℓ, the choice of ℓ is an empirical matter. As Phillips (1987) pointed out, we expect that small values of should be sufficient because the sample autocorrelations of first differenced economic time series usually decay quickly.

3. Results of the Experiments

Tables 22-27 report the proportions of rejections under the null hypothesis for 5% lower tail test, for each value of the MA parameters (-.8, -.5, -.2, .2, .5, .8) and for the base case. Similarly, Tables 28-33 report the same results of the experiments for AR parameters (-.8, -.5, -.2, .2, .5, .8). As pointed out earlier, the Phillips-Perron tests perform better than the standard Dickey-Fuller tests when the error process is MA(1) or AR(1), although the degree of improvement is small. However, as the last columns of the Tables 22-27 show, for the base case, the performance of the Phillips-Perron tests deteriorate somewhat as the value of ℓ grows.

In general, however, the value of ℓ in $S_{t\ell}^2$ makes little difference in the performance of the tests. The performances of the tests depend mainly on the parameters of the error process. The direction of the effect of ℓ on the performance of the tests varies. For some parameters larger ℓ improves the test, but for other parameters vice versa. Usually the best value of ℓ is around five, depending somewhat upon the lag window used. Parzen's and Bohman's lag windows generally seem to work somewhat better than Bartlett's window does, but the difference between them is minimal. There is a tendency for Parzen's and Bohman's lag windows to need a larger value of ℓ than Bartlett's lag window to get the best performance in each test. Usually Parzen's and Bohman's lag windows need 4-6 terms compared to 3-4 terms for Bartlett's lag window. The performance of the tests depends little on the kind of lag windows employed in calculating $\tilde{S}_{t\ell}^2$. This is understandable because the difference between the lag windows is the weight applied to each of the sample autocovariances. For example, the weights given to each sample autocovariances for $\ell = 10$ are shown in the following table.

ℓ	1	2	3	4	5	6	7	8	9	10
Bartlett	1.82	1.64	1.45	1.27	1.09	.91	.73	.55	.36	.18
Parzen	1.91	1.68	1.35	.99	.65	.38	.19	.08	.02	.00
Bohman	1.92	1.72	1.43	1.11	.79	.50	.28	.12	.04	.00

When comparing lag windows, there is a small variation among the first three weights of each lag window. After that, Parzen's weights decrease more rapidly than the other lag windows. The Bartlett window's weights appear to decrease at the slowest rate. Parzen's and Bohman's window add more weights on the adjacent events and add very little weights on the events which are separated by the long intervals, for example 1.91, 1.92 for $\ell = 1$ and .00 and .00 for $\ell = 10$, repectively. However, the weight of Bartlett's window for $\ell = 10$ is .18 which is large when compared to Parzen's and Bartlett's window. Parzen's and Bohman's weighting scheme fit better to the assumption 2.1 (d) in Phillips (1987), which controls the extent of the dependence in the process u_t. This explains why Parzen's and Bohman's lag windows perform somewhat better than Bartlett's window. However, it has to be remembered that the difference in the performance between three lag windows is small and the overall performances of all three windows are poor.

4. Conclusion

None of the lag windows employed in the experiments of this chapter improves the reliability of the tests significantly. Parzen's and Bohman's lag windows generally perform slightly better than Bartlett's window. Although choosing the value of ℓ is an empirical matter, it was shown in the experiment that a value of ℓ around five would be a reasonable choice for empirically relevant sample sizes.

CHAPTER V

TESTS USING THE TRUE VALUES OF σ^2 AND σ_u^2

The limiting distributions of $T(\hat{\beta}-1)$ and t-statistic given in Theorem 3.1 in Phillips (1987) depend on the ratio of the unknown nuisance parameters σ_u^2 and σ^2; that is

$$(24)\quad T(\hat{\beta}-1) \rightarrow (1/2)\ \{W(1)^2 - (\sigma_u^2 / \sigma^2)\} / \left(\int_0^1 W(r)^2\, dr\right),$$

$$(25)\quad \hat{\tau}_\mu \rightarrow (\sigma / 2\sigma_u)\ \{W(1)^2 - (\sigma_u^2 / \sigma^2)\} / \left\{\int_0^1 W(r)^2 dr\right\}^{1/2}.$$

where $W(r)$ is a standard Wiener process. Phillips (1987) and Phillips and Perron (1988) provide transformed versions of these statistics whose asymptotic distributions are the same as those tabulated by Dickey (1976) and Fuller (1976). The transformed statistics also depend on the ratio of σ_u^2 and σ^2. These statistics are not directly useful because of the unknown values of the nuisance parameters. As discussed in Chapters 2 and 4, a solution to this problem is to replace σ_u^2 and σ^2 by consistent estimates. However, σ^2 is not an easy parameter to estimate well. In this chapter we actually calculate the true value of σ_u^2 and σ^2, for AR(1) and MA(1) errors, and we use the true values of σ_u^2 and σ^2 instead of using consistent estimates. We therefore eliminate the possible inaccuracy of the tests arising from errors in estimation of (σ_u^2, σ^2). By taking this approach we hope to determine whether the inaccuracy of the Phillips-Perron tests is due to problems in estimating these nuisance parameters, or whether it is intrinsic to the use of the asymptotic distribution theory in finite samples. We first consider MA(1) error case; that is, $u_t = \epsilon_t + \theta\epsilon_{t-1}$ with $\epsilon_t \sim iid(0,\sigma_\epsilon^2)$. We can calculate the true values of σ_u^2 and σ^2 as follows:

$$\sigma_u^2 = \lim_{T\to\infty} (1/T) \sum_{t=1}^{T} E(u_t^2) = (1 + \theta^2)\, \sigma_\epsilon^2, \tag{26}$$

$$\sigma^2 = \lim_{T\to\infty} (1/T)\ E\left(\sum_{t=1}^{T} u_t\right)^2 = (1 + \theta)^2\, \sigma_\epsilon^2. \tag{27}$$

We next consider the AR(1) error case; that is, $u_t = \rho u_{t-1} + \epsilon_t$ with $\epsilon_t \sim iid(0,\sigma_\epsilon^2)$. Here we obtain

$$\sigma_u^2 = \lim_{T\to\infty} (1/T) \sum_{t=1}^{T} E(u_t^2) = \sigma_\epsilon^2 / (1-\rho^2), \tag{28}$$

$$\sigma^2 = \lim_{T\to\infty} (1/T)\ E\left(\sum_{t=1}^{T} u_t\right)^2 = \sigma_\epsilon^2 / (1-\rho)^2. \tag{29}$$

These formulas can be found in Phillips (1987). We then use (26) and (27) for the case of MA(1) errors and (28) and (29) for the case of AR(1) errors, instead of S_u^2 and $S_{t\ell}^2$ in equation (21) and (22) in Phillips(1987) and equations (6), (7), (8) and (9) in Chapter 2 (page 341 in Phillips and Perron (1988)).

A. Design of the Experiment

The framework of the experiment is basically the same as in Chapter 4. Two regression equations are considered here as usual.

(a) $Y_t = \alpha + \beta Y_{t-1} + u_t,$

(b) $Y_t = \alpha + \beta Y_{t-1} + \delta t + u_t.$

Obviously each corresponds to a regression of Y on lagged Y, but they differ as to whether they include a time trend. The data generating process is (1) with $\alpha = 0$ and $\beta = 1$. An MA(1) error is generated by equation (16) and an AR(1) error is generated by equation (15).

The parameter values $\theta = -.8, -.5, -.2, .2, .5, .8$ and $\rho = -.8, -.5, -.2, .2, .5, .8$ are used in the data generating process, with T = 100. The same experiments have been done with larger sample sizes, T = 200, 500 and 1000, for the parameter values of -.8, -.5, .5 and .8. Here again only the lower tail test results are reported. All simulations are based upon 10,000 replications.

B. Results of the Experiments

The standard Dickey-Fuller, the Phillips-Perron tests and the Phillips-Perron true value tests are compared in Tables 34-39. We denote the Phillips-Perron true value tests with the suffix 'ppt'. For example, τ.ppt denotes the Phillips-Perron t-statistic (equation (8) and (9) in Chapter 2) using the true value of (σ^2, σ_u^2). Table 34 reports the percentages of rejections for the lower tail test when T = 100. As expected the Phillips-Perron true value tests outperform the standard Dickey-Fuller tests and the Phillips-Perron tests. However, the closeness of the rejection rates to the nominal size still depends strongly upon the parameter values. For negative values of θ (-.8 or -.5), the proportions of rejections for the $\hat{\tau}_\mu$.ppt and $\hat{\tau}_\tau$.ppt are well above 5%, ranging from .160 to .919. This overrejection problem is much less severe for $\hat{\rho}_\mu$.ppt and $\hat{\rho}_\tau$.ppt for the same parameter values. For θ (-.2, .2, .5, .8), the proportions of rejections for $\hat{\tau}_\mu$.ppt, $\hat{\tau}_\tau$.ppt and $\hat{\rho}_\mu$.ppt are not very different from the nominal size, but the rejection rates of $\hat{\rho}_\tau$.ppt ranges from .033 to .075. Thus $\hat{\rho}_\tau$.ppt performed poorly compared to the other Phillips-Perron true value tests for positive θ or small negative θ. However, the performances of $\hat{\tau}_\mu$.ppt and $\hat{\tau}_\tau$.ppt are much more sensitive to large negative MA parameters than $\hat{\rho}_\mu$.ppt and $\hat{\rho}_\tau$.ppt. For example, when θ = -.8, the proportions of rejections for $\hat{\tau}_\mu$.ppt and $\hat{\tau}_\tau$.ppt are .811 and .919

respectively, compared to .113 and .069 for $\hat{\rho}_{\mu}$.ppt and $\hat{\rho}_{\tau}$.ppt respectively.

Table 35 reports the proportions of rejection for the lower tail tests for the AR(1) case, with value of ρ equal to (-.8, -.5, -.2, .2, .5, .8). A similar pattern of the results occurs as in Table 34. The performance of the Phillips-Perron true value tests when ρ = -.8, -.5 and .8 are poor overall. The proportion of rejections decreases with the increasing value of the AR parameter, but it then increases again for ρ=.8. For example, when ρ =.8 the proportions of rejections for $\hat{\tau}_{\mu}$.ppt, $\hat{\tau}_{\tau}$.ppt, $\hat{\rho}_{\mu}$.ppt and $\hat{\rho}_{\tau}$.ppt are .117, .231, .140 and .276, respectively. These overrejection problems are worse when ρ = .8 than when θ = .8 in the MA(1) model for all ppt tests. But the overrejection problem of $\hat{\tau}_{\mu}$.ppt and $\hat{\tau}_{\tau}$.ppt when ρ = -.5 or -.8 is less severe than it was in the MA model when θ = -.5 or -.8. For example, the proportion of rejections for $\hat{\tau}_{\mu}$.ppt with ρ = -.8 is .376, whereas the proportion of rejections for the same test with θ = -.8 is .811.

Tables 36 and 37 provide the percentages of rejections when θ = -.8, -.5, .5 and .8 with the increased sample sizes, T = 200, 500 and 1000. The increase in the sample size improves the performances of the tests most of time, but there are some exceptions. For example, $\hat{\rho}_{\tau}$.ppt does worse as T increases. The proportion of rejections when θ

= -.8 are still well above 5% even for T = 1000. For instance, the proportions of rejections at T = 1000 for $\hat{\tau}_\mu$.ppt, $\hat{\tau}_\tau$.ppt, $\hat{\rho}_\mu$.ppt and $\hat{\rho}_\tau$.ppt are .586, .838, .142 and .160 respectively. Generally the increase in the sample size helps the performance when θ = -.5, .5 and .8, for all Phillips-Perron true value tests. Their proportions of rejections are close to 5% with the exception of $\hat{\tau}_\mu$.ppt and $\hat{\tau}_\tau$.ppt when θ = -.5. The proportions of rejections of $\hat{\tau}_\mu$.ppt and $\hat{\tau}_\tau$.ppt when θ = -.5 are .073 and .093 respectively.

Tables 38 and 39 provide the test results for the AR(1) case with sample sizes of T = 200, 500 and 1000. All the proportions of rejections approach the nominal size reasonably close, except that for all of the .ppt tests the rejection rates are still well above 5% even with T = 1000 when ρ = -.8. The proportions of rejections in this case for $\hat{\tau}_\mu$.ppt, $\hat{\tau}_\tau$.ppt, $\hat{\rho}_\mu$.ppt and $\hat{\rho}_\tau$.ppt are .155, .244, .094 and .115 respectively. The improvement in performance is especially noticeable for $\hat{\rho}_\mu$.ppt and $\hat{\rho}_\tau$.ppt when ρ = -.8. The proportions of rejections of $\hat{\rho}_\mu$ and $\hat{\rho}_\tau$ are 0.000, 0.000 compared to .064 and .067 for $\hat{\rho}_\mu$.ppt and $\hat{\rho}_\tau$.ppt, repectively, with T = 1000.

C. Conclusion

The Phillips-Perron tests are considerably more accurate when they are based on the true values of σ_u^2 and σ^2

than when they are based on consistent estimates of these nuisance parameters. This is not surprising, and it suggests that it may be possible to improve the Phillips-Perron tests considerably by finding better ways to estimate the nuisance parameters. However, the tests still perform rather poorly when the errors are highly autocorrelated. The tests are based on asymptotics, and apparently their convergence to the asymptotic distribution is very slow when the errors are highly autocorrelated. To find tests that are accurate in this case will apparently require the use of more sophisticated distribution theory, or new tests.

CHAPTER VI

CONCLUDING REMARKS

This thesis has considered the properties of unit root tests when the errors in the variable's autoregressive representation are heteroskedastic or autocorrelated. We considered the standard Dickey-Fuller tests and various augmented and transformed versions of these tests that have been suggested in the literature. Chapter 2 provided a detailed description of these tests.

In Chapter 3 we considered heteroskedasticity, in the form of a generalized autoregressive conditional heteroskedasticity (GARCH) process. The Dickey-Fuller tests are known to be robust to heteroskedasticity asymptotically, but their finite sample robustness to heteroskedasticity is questionable. The reliability of the tests was found to depend heavily on how close the intercept in the GARCH process is to zero. When the intercept equals zero, the GARCH process asymptotically degenerate (the error process converges to zero almost surely), and near degeneracy causes severe problems for the unit root tests. Specifically, the tests overreject severely in this case. Worse yet, the sample size is enlarged. The use of the White (1980) heteroskedasticity-consistent covariance matrix estimator helps somewhat but does not really solve the problem. Further research is necessary to provide a test that will work better in the presence of heteroskedasticity of this form.

In Chapter 4 we considered the Dickey-Fuller tests and their Phillips-Perron (1988) extensions when the errors are AR(1) or MA(1). From Schwert (1987, 1989) and Phillips-Perron (1988) it is known that the tests overreject severely when there is strong negative autocorrelation, and they underreject when there is strong positive autocorrelation. We confirm these findings. We also considered the use of different spectral windows in the estimation of the long run variance parameter that is needed for the Phillips-Perron corrections. Previous work has used the Bartlett window, and we also considered the Parzen and Bohman windows. We found that the Parzen and Bohman windows work better than the Bartlett window, though the degree of improvement is discouragingly small. We also experimented with different values of the truncation term in the long run variance estimates. Some values worked better than others, but no value worked satisfactorily. A value of about five seemed best for sample size 100 and moderate to severe autocorrelation.

An interesting question is whether the Phillips-Perron

tests perform poorly because the long run variance estimates are poor, or whether the problem is intrinsic to the rate of convergence of the statistics to their asymptotic distributions. One way to answer this question is to see how well the tests work if we use the true value of the long run variance. This was done in Chapter 5. The results are again discouraging. Using the true value of the long run variance helps the performance of the tests considerably, but they are still very unreliable.

Overall, the results of this thesis suggest the need for considerable caution in interpreting the results of existing unit root tests. Further theoretical work seems to be called for to find tests that are more reliable. A reasonable conjecture is that the problem with the existing unit root tests is that they are based on ordinary least squares estimation, and, given heteroskedasticity or autocorrelation, some form of generalized least squares estimation would probably be superior. Perhaps tests based on generalized least squares estimation will be more accurate in finite samples than the current tests. This seems to be a direction of research well worth pursuing.

APPENDIX

TABLE 1

Proportions of Rejections, 5% Lower Tail Test

$T = 100,\ \Phi_0 = h_0\ (1 - \Phi_1 - \Phi_2)$

	$(\Phi_1,\ \Phi_2)$				
	(.3,.30)	(.3,.60)	(.3,.65)	(.3,.69)	(.3,.70)
$\hat{\tau}_\mu$	.0591	.0735	.0814	.1138	.4125
$\hat{\tau}_\mu$.w	.0650	.0631	.0617	.0679	.1207
$\hat{\tau}_\mu$.pp1	.0591	.0746	.0838	.1134	.4174
$\hat{\tau}_\mu$.pp2	.0601	.0745	.0844	.1157	.4214
$\hat{\tau}_\mu$.sd1	.0564	.0728	.0791	.1084	.4043
$\hat{\tau}_\mu$.sd2	.0518	.0661	.0726	.0995	.3820
$\hat{\tau}_\tau$	.0688	.0829	.0889	.1108	.3531
$\hat{\tau}_\tau$.w	.0739	.0698	.0687	.0704	.1011
$\hat{\tau}_\tau$.pp1	.0710	.0849	.0932	.1122	.3602
$\hat{\tau}_\tau$.pp2	.0718	.0872	.0930	.1146	.3632
$\hat{\tau}_\tau$.sd1	.0642	.0795	.0845	.1074	.3457
$\hat{\tau}_\tau$.sd2	.0532	.0642	.0693	.0909	.3171
$\hat{\rho}_\tau$	.0589	.0732	.0780	.0954	.2944
$\hat{\rho}_\mu$.pp1	.0600	.0729	.0796	.0972	.2981
$\hat{\rho}_\mu$.pp2	.0611	.0740	.0807	.0988	.2982
$\hat{\rho}_\mu$.sd1	.0612	.0752	.0826	.1022	.3009
$\hat{\rho}_\mu$.sd2	.0642	.0782	.0839	.1005	.2941
$\hat{\rho}_\tau$	.0637	.0742	.0790	.0914	.2429
$\hat{\rho}_\tau$.pp1	.0675	.0782	.0813	.0964	.2495
$\hat{\rho}_\tau$.pp2	.0680	.0808	.0852	.0997	.2553
$\hat{\rho}_\tau$.sd1	.0697	.0843	.0871	.1014	.2502
$\hat{\rho}_\tau$.sd2	.0754	.0858	.0910	.1029	.2475

TABLE 2

Proportions of Rejections, 5% Upper Tail Test

$T = 100,\ \Phi_0 = h_0(1 - \Phi_1 - \Phi_2)$

	(Φ_1, Φ_2)				
	(.3,.30)	(.3,.60)	(.3,.65)	(.3,.69)	(.3,.70)
$\hat{\tau}_\mu$	.0496	.0547	.0569	.0557	.0070
$\hat{\tau}_\mu$.w	.0495	.0548	.0575	.0558	.0071
$\hat{\tau}_\mu$.pp1	.0497	.0540	.0572	.0564	.0070
$\hat{\tau}_\mu$.pp2	.0502	.0530	.0558	.0564	.0073
$\hat{\tau}_\mu$.sd1	.0489	.0533	.0564	.0564	.0073
$\hat{\tau}_\mu$.sd2	.0532	.0563	.0604	.0597	.0089
$\hat{\tau}_\tau$	.0509	.0525	.0557	.0532	.0119
$\hat{\tau}_\tau$.w	.0495	.0592	.0648	.0667	.0226
$\hat{\tau}_\tau$.pp1	.0484	.0512	.0540	.0526	.0110
$\hat{\tau}_\tau$.pp2	.0481	.0500	.0530	.0509	.0100
$\hat{\tau}_\tau$.sd1	.0518	.0541	.0558	.0539	.0119
$\hat{\tau}_\tau$.sd2	.0547	.0571	.0595	.0597	.0136
$\hat{\rho}_\mu$	.0493	.0552	.0562	.0551	.0070
$\hat{\rho}_\mu$.pp1	.0498	.0541	.0571	.0562	.0072
$\hat{\rho}_\mu$.pp2	.0492	.0529	.0560	.0565	.0072
$\hat{\rho}_\mu$.sd1	.0484	.0534	.0562	.0557	.0074
$\hat{\rho}_\mu$.sd2	.0521	.0557	.0594	.0593	.0087
$\hat{\rho}_\tau$	.0542	.0556	.0555	.0563	.0169
$\hat{\rho}_\tau$.pp1	.0524	.0548	.0562	.0535	.0156
$\hat{\rho}_\tau$.pp2	.0496	.0523	.0537	.0516	.0147
$\hat{\rho}_\tau$.sd1	.0518	.0534	.0563	.0535	.0157
$\hat{\rho}_\tau$.sd2	.0484	.0557	.0537	.0521	.0162

TABLE 3

Proportions of Rejections, 5% Two Tail Test

$T = 100, \ \Phi_0 = h_0(1 - \Phi_1 - \Phi_2)$

	(Φ_1, Φ_2)				
	(.3,.30)	(.3,.60)	(.3,.65)	(.3,.69)	(.3,.70)
$\hat{\tau}_\mu$	.0601	.0736	.0815	.1083	.0365
$\hat{\tau}_\mu$.w	.0628	.0636	.0659	.0745	.0864
$\hat{\tau}_\mu$.pp1	.0606	.0745	.0827	.1087	.3488
$\hat{\tau}_\mu$.pp2	.0613	.0759	.0836	.1101	.3513
$\hat{\tau}_\mu$.sd1	.0577	.0737	.0822	.1065	.3393
$\hat{\tau}_\mu$.sd2	.0553	.0705	.0782	.1025	.3233
$\hat{\tau}_\tau$	.0660	.0794	.0856	.1050	.2901
$\hat{\tau}_\tau$.w	.0691	.0729	.0736	.0779	.0764
$\hat{\tau}_\tau$.pp1	.0640	.0781	.0851	.1028	.2978
$\hat{\tau}_\tau$.pp2	.0643	.0773	.0867	.1056	.3022
$\hat{\tau}_\tau$.sd1	.0618	.0747	.0829	.0999	.2874
$\hat{\tau}_\tau$.sd2	.0562	.0687	.0753	.0922	.2646
$\hat{\rho}_\mu$	.0559	.0695	.0777	.0935	.2206
$\hat{\rho}_\mu$.pp1	.0568	.0729	.0794	.0936	.2242
$\hat{\rho}_\mu$.pp2	.0603	.0724	.0814	.0962	.2266
$\hat{\rho}_\mu$.sd1	.0591	.0753	.0833	.0953	.2251
$\hat{\rho}_\mu$.sd2	.0639	.0756	.0848	.0996	.2169
$\hat{\rho}_\tau$	.0644	.0780	.0810	.0917	.1856
$\hat{\rho}_\tau$.pp1	.0639	.0769	.0827	.0913	.1934
$\hat{\rho}_\tau$.pp2	.0634	.0760	.0846	.0933	.1989
$\hat{\rho}_\tau$.sd1	.0650	.0781	.0860	.0918	.1937
$\hat{\rho}_\tau$.sd2	.0682	.0826	.0904	.0976	.1943

TABLE 4

Proportions of Rejections, 5% Lower Tail Test

$T = 100,\ \Phi_0 = h_0\ (1 - \Phi_1 - \Phi_2)$

	(Φ_1, Φ_2)					
	(.1,0)	(.1,.5)	(.1,.8)	(.1,.85)	(.1,.89)	(.1,.9)
$\hat{\tau}_\mu$	.0472	.0493	.0558	.0568	.0651	.0913
$\hat{\tau}_\mu$.w	.0664	.0673	.0658	.0649	.0696	.0768
$\hat{\tau}_\mu$.pp1	.0490	.0528	.0578	.0592	.0676	.0925
$\hat{\tau}_\mu$.pp2	.0507	.0535	.0591	.0616	.0693	.0952
$\hat{\tau}_\mu$.sd1	.0499	.0516	.0568	.0581	.0665	.0918
$\hat{\tau}_\mu$.sd2	.0462	.0479	.0526	.0547	.0637	.0891
$\hat{\tau}_\tau$	.0573	.0591	.0617	.0639	.0680	.0874
$\hat{\tau}_\tau$.w	.0808	.0805	.0804	.0803	.0802	.0847
$\hat{\tau}_\tau$.pp1	.0588	.0601	.0653	.0652	.0718	.0911
$\hat{\tau}_\tau$.pp2	.0634	.0644	.0678	.0686	.0749	.0942
$\hat{\tau}_\tau$.sd1	.0531	.0554	.0593	.0595	.0640	.0828
$\hat{\tau}_\tau$.sd2	.0477	.0498	.0509	.0548	.0601	.0784
$\hat{\rho}_\mu$	.0495	.0533	.0566	.0575	.0599	.0681
$\hat{\rho}_\mu$.pp1	.0515	.0543	.0579	.0591	.0640	.0707
$\hat{\rho}_\mu$.pp2	.0553	.0572	.0602	.0612	.0648	.0742
$\hat{\rho}_\mu$.sd1	.0568	.0576	.0611	.0635	.0667	.0737
$\hat{\rho}_\mu$.sd2	.0595	.0602	.0631	.0657	.0691	.0782
$\hat{\rho}_\tau$	.0536	.0541	.0561	.0568	.0575	.0658
$\hat{\rho}_\tau$.pp1	.0560	.0573	.0605	.0616	.0648	.0712
$\hat{\rho}_\tau$.pp2	.0610	.0611	.0641	.0647	.0674	.0772
$\hat{\rho}_\tau$.sd1	.0608	.0617	.0640	.0645	.0693	.0782
$\hat{\rho}_\tau$.sd2	.0679	.0692	.0727	.0744	.0745	.0835

TABLE 5

Proportions of Rejections, 5% Lower Tail Test

$T = 100,\ \Phi_0 = h_0(1 - \Phi_1 - \Phi_2)$

	(Φ_1, Φ_2)				
	(.30,.3)	(.60,.3)	(.65,.3)	(.69,.3)	(.70,.3)
$\hat{\tau}_\mu$	.0591	.0962	.1032	.1142	.8880
$\hat{\tau}_\mu$.w	.0650	.0562	.0535	.0528	.3457
$\hat{\tau}_\mu$.pp1	.0591	.0930	.1008	.1109	.8902
$\hat{\tau}_\mu$.pp2	.0601	.0914	.0988	.1091	.8917
$\hat{\tau}_\mu$.sd1	.0564	.0860	.0941	.1042	.8830
$\hat{\tau}_\mu$.sd2	.0518	.0725	.0803	.0879	.8570
$\hat{\tau}_\tau$	.0688	.1064	.1155	.1285	.8327
$\hat{\tau}_\tau$.w	.0739	.0578	.0543	.0541	.2999
$\hat{\tau}_\tau$.pp1	.0710	.1065	.1158	.1269	.8335
$\hat{\tau}_\tau$.pp2	.0718	.1070	.1167	.1271	.8341
$\hat{\tau}_\tau$.sd1	.0642	.0961	.1049	.1183	.8207
$\hat{\tau}_\tau$.sd2	.0532	.0713	.0782	.0861	.7895
$\hat{\rho}_\mu$	.0589	.0933	.1017	.1100	.8203
$\hat{\rho}_\mu$.pp1	.0600	.0904	.0999	.1081	.8202
$\hat{\rho}_\mu$.pp2	.0611	.0897	.0987	.1079	.8159
$\hat{\rho}_\mu$.sd1	.0612	.0934	.0999	.1094	.8207
$\hat{\rho}_\mu$.sd2	.0642	.0899	.0950	.1042	.8072
$\hat{\rho}_\tau$	.0637	.0976	.1079	.1185	.7249
$\hat{\rho}_\tau$.pp1	.0675	.1002	.1097	.1209	.7220
$\hat{\rho}_\tau$.pp2	.0680	.1003	.1090	.1197	.7138
$\hat{\rho}_\tau$.sd1	.0697	.1039	.1118	.1217	.7255
$\hat{\rho}_\tau$.sd2	.0754	.0989	.1061	.1167	.7083

TABLE 6

Proportions of Rejections, 5% Lower Tail Test

$\Phi_0 = h_0(1 - \Phi_1 - \Phi_2)$

(Φ_1, Φ_2):	(.3, .6)			(.3, .65)			(.3, .7)		
T :	100	500	1000	100	500	1000	100	500	1000
$\hat{\tau}_\mu$	.0735	.0698	.0607	.0814	.0817	.0726	.4125	.8765	.9676
$\hat{\tau}_\mu$.w	.0631	.0463	.0454	.0617	.0454	.0426	.1207	.1040	.1004
$\hat{\tau}_\mu$.pp1	.0746	.0703	.0609	.0838	.0809	.0714	.4174	.8783	.9677
$\hat{\tau}_\mu$.pp2	.0745	.0692	.0601	.0844	.0796	.0699	.4214	.8776	.9687
$\hat{\tau}_\mu$.sd1	.0728	.0694	.0597	.0791	.0782	.0725	.4043	.8757	.9689
$\hat{\tau}_\mu$.sd2	.0661	.0639	.0578	.0726	.0731	.0693	.3820	.8677	.9667
$\hat{\tau}_\tau$	.0829	.0755	.0683	.0889	.0912	.0833	.3531	.8138	.9374
$\hat{\tau}_\tau$.w	.0698	.0422	.0427	.0687	.0371	.0355	.1011	.0535	.0488
$\hat{\tau}_\tau$.pp1	.0849	.0747	.0682	.0932	.0907	.0826	.3602	.8149	.9386
$\hat{\tau}_\tau$.pp2	.0872	.0756	.0669	.0930	.0894	.0814	.3632	.8162	.9391
$\hat{\tau}_\tau$.sd1	.0795	.0732	.0676	.0845	.0876	.0815	.3457	.8113	.9402
$\hat{\tau}_\tau$.sd2	.0642	.0671	.0614	.0693	.0806	.0771	.3171	.8000	.9352
$\hat{\rho}_\mu$	.0732	.0684	.0641	.0780	.0799	.0756	.2944	.8190	.9469
$\hat{\rho}_\mu$.pp1	.0729	.0678	.0633	.0796	.0779	.0749	.2981	.8161	.9465
$\hat{\rho}_\mu$.pp2	.0740	.0678	.0628	.0807	.0785	.0738	.2982	.8138	.9461
$\hat{\rho}_\mu$.sd1	.0752	.0683	.0632	.0826	.0783	.0750	.3009	.8165	.9457
$\hat{\rho}_\mu$.sd2	.0782	.0659	.0628	.0839	.0790	.0731	.2941	.8130	.9470
$\hat{\rho}_\tau$	.0742	.0733	.0655	.0790	.0885	.0794	.2429	.7354	.8934
$\hat{\rho}_\tau$.pp1	.0782	.0729	.0653	.0813	.0878	.0782	.2495	.7329	.8945
$\hat{\rho}_\tau$.pp2	.0808	.0743	.0652	.0852	.0850	.0774	.2553	.7310	.8948
$\hat{\rho}_\tau$.sd1	.0843	.0744	.0651	.0871	.0881	.0781	.2502	.7339	.8958
$\hat{\rho}_\tau$.sd2	.0858	.0714	.0644	.0910	.0851	.0779	.2475	.7273	.8932

TABLE 7

Proportions of Rejections, 5% Lower Tail Test

$T = 100,\ \Phi_1 = .3,\ \Phi_2 = .65,\ h_0 = 1$

Φ_0 :	0	.00001	.0001	.001	.01	.1	1	10	100
$\hat{\tau}_\mu$	.6540	.6010	.4537	.2499	.1185	.0795	.0749	.0743	.0743
$\hat{\tau}_\mu$.w	.1749	.1586	.1297	.0960	.0722	.0610	.0588	.0584	.0584
$\hat{\tau}_\mu$.pp1	.6581	.6032	.4561	.2508	.1202	.0811	.0758	.0747	.0746
$\hat{\tau}_\mu$.pp2	.6612	.6075	.4580	.2532	.1199	.0817	.0769	.0762	.0762
$\hat{\tau}_\mu$.sd1	.6486	.5920	.4421	.2373	.1136	.0773	.0726	.0722	.0720
$\hat{\tau}_\mu$.sd2	.6182	.5500	.4074	.2183	.1015	.0707	.0669	.0663	.0663
$\hat{\tau}_\tau$	.5767	.5393	.4243	.2410	.1183	.0878	.0817	.0810	.0810
$\hat{\tau}_\tau$.w	.1437	.1378	.1232	.1006	.0788	.0681	.0653	.0649	.0650
$\hat{\tau}_\tau$.pp1	.5819	.5448	.4283	.2468	.1213	.0906	.0862	.0854	.0854
$\hat{\tau}_\tau$.pp2	.5867	.5486	.4309	.2502	.1234	.0908	.0865	.0861	.0861
$\hat{\tau}_\tau$.sd1	.5718	.5298	.4092	.2328	.1127	.0828	.0784	.0775	.0775
$\hat{\tau}_\tau$.sd2	.5356	.4891	.3670	.2018	.0944	.0687	.0642	.0639	.0639
$\hat{\rho}_\mu$	.5190	.4639	.3342	.1757	.0956	.0775	.0749	.0747	.0747
$\hat{\rho}_\mu$.pp1	.5183	.4659	.3373	.1812	.0985	.0787	.0765	.0761	.0761
$\hat{\rho}_\mu$.pp2	.5148	.4643	.3403	.1803	.0996	.0799	.0776	.0775	.0775
$\hat{\rho}_\mu$.sd1	.5217	.4610	.3319	.1802	.1010	.0818	.0795	.0792	.0792
$\hat{\rho}_\mu$.sd2	.5079	.4426	.3148	.1740	.0990	.0829	.0813	.0808	.0808
$\hat{\rho}_\tau$	.4214	.3868	.2884	.1650	.0954	.0785	.0760	.0758	.0758
$\hat{\rho}_\tau$.pp1	.4290	.3924	.2972	.1702	.1004	.0809	.0785	.0782	.0782
$\hat{\rho}_\tau$.pp2	.4305	.3933	.3022	.1746	.1025	.0838	.0823	.0820	.0820
$\hat{\rho}_\tau$.sd1	.4250	.3870	.2929	.1673	.1023	.0869	.0847	.0846	.0846
$\hat{\rho}_\tau$.sd2	.4163	.3705	.2794	.1646	.1033	.0897	.0876	.0874	.0875

TABLE 8

Proportions of Rejections, 5% Lower Tail Test

$T = 100,\ \Phi_1 = .3,\ \Phi_2 = .7,\ h_0 = 1$

Φ_0 :	0	.00001	.0001	.001	.01	.1	1	10	100
$\hat{\tau}_\mu$	.4125	.3990	.3484	.2326	.1208	.0774	.0711	.0700	.0699
$\hat{\tau}_\mu$.w	.1207	.1173	.1072	.0889	.0684	.0539	.0528	.0521	.0520
$\hat{\tau}_\mu$.pp1	.4174	.4044	.3516	.2362	.1213	.0787	.0710	.0700	.0698
$\hat{\tau}_\mu$.pp2	.4214	.4095	.3542	.2396	.1234	.0804	.0733	.0721	.0721
$\hat{\tau}_\mu$.sd1	.4043	.3919	.3409	.2269	.1153	.0736	.0678	.0669	.0668
$\hat{\tau}_\mu$.sd2	.3820	.3698	.3173	.2092	.1077	.0700	.0637	.0631	.0631
$\hat{\tau}_\tau$	.3531	.3441	.3064	.2082	.1165	.0828	.0775	.0766	.0766
$\hat{\tau}_\tau$.w	.1011	.1004	.0980	.0882	.0701	.0593	.0558	.0550	.0550
$\hat{\tau}_\tau$.pp1	.3602	.3510	.3109	.2159	.1185	.0843	.0787	.0782	.0782
$\hat{\tau}_\tau$.pp2	.3622	.3531	.3155	.2181	.1218	.0865	.0806	.0800	.0800
$\hat{\tau}_\tau$.sd1	.3457	.3374	.2969	.2013	.1138	.0792	.0747	.0740	.0740
$\hat{\tau}_\tau$.sd2	.3171	.3083	.2678	.1815	.0966	.0676	.0624	.0619	.0618
$\hat{\rho}_\mu$	.2944	.2850	.2439	.1615	.0992	.0808	.0773	.0772	.0773
$\hat{\rho}_\mu$.pp1	.2981	.2890	.2471	.1665	.1033	.0823	.0794	.0794	.0792
$\hat{\rho}_\mu$.pp2	.2982	.2885	.2483	.1652	.1043	.0855	.0813	.0807	.0806
$\hat{\rho}_\mu$.sd1	.3009	.2884	.2461	.1681	.1061	.0850	.0819	.0817	.0817
$\hat{\rho}_\mu$.sd2	.2941	.2812	.2362	.1654	.1066	.0873	.0841	.0837	.0838
$\hat{\rho}_\tau$	.2429	.2358	.2054	.1416	.0946	.0779	.0755	.0753	.0753
$\hat{\rho}_\tau$.pp1	.2495	.2434	.2129	.1488	.0992	.0814	.0796	.0792	.0791
$\hat{\rho}_\tau$.pp2	.2553	.2484	.2197	.1560	.1019	.0847	.0819	.0816	.0816
$\hat{\rho}_\tau$.sd1	.2502	.2433	.2124	.1499	.1029	.0868	.0841	.0836	.0836
$\hat{\rho}_\tau$.sd2	.2475	.2397	.2103	.1521	.1047	.0907	.0883	.0878	.0877

TABLE 9

Proportions of Rejections, 5% Lower Tail Test

$T = 100,\ \Phi_1 = .1,\ \Phi_2 = .85,\ h_0 = 1$

Φ_0 :	0	.00001	.0001	.001	.01	.1	1	10	100
$\hat{\tau}_\mu$	.4220	.4193	.3969	.2671	.1008	.0524	.0464	.0457	.0457
$\hat{\tau}_\mu$.w	.1194	.1187	.1157	.1042	.0823	.0626	.0593	.0590	.0590
$\hat{\tau}_\mu$.pp1	.4266	.4235	.4022	.2720	.1042	.0538	.0486	.0479	.0479
$\hat{\tau}_\mu$.pp2	.4322	.4291	.4039	.2762	.1064	.0556	.0497	.0493	.0492
$\hat{\tau}_\mu$.sd1	.4196	.4167	.3885	.2636	.0986	.0530	.0483	.0476	.0476
$\hat{\tau}_\mu$.sd2	.3961	.3926	.3661	.2437	.0924	.0505	.0457	.0453	.0452
$\hat{\tau}_\tau$	.3521	.3503	.3319	.2350	.0974	.0591	.0556	.0552	.0551
$\hat{\tau}_\tau$.w	.1008	.1014	.1027	.1062	.0968	.0773	.0735	.0728	.0727
$\hat{\tau}_\tau$.pp1	.3612	.3591	.3397	.2397	.1027	.0618	.0581	.0577	.0576
$\hat{\tau}_\tau$.pp2	.3657	.3624	.3438	.2438	.1054	.0655	.0605	.0597	.0596
$\hat{\tau}_\tau$.sd1	.3519	.3500	.3303	.2297	.0918	.0561	.0516	.0510	.0510
$\hat{\tau}_\tau$.sd2	.3238	.3216	.3007	.2071	.0840	.0509	.0468	.0467	.0465
$\hat{\rho}_\mu$	.2690	.2662	.2456	.1565	.0738	.0546	.0534	.0533	.0531
$\hat{\rho}_\mu$.pp1	.2695	.2663	.2468	.1621	.0761	.0574	.0552	.0548	.0548
$\hat{\rho}_\mu$.pp2	.2718	.2700	.2510	.1662	.0771	.0593	.1575	.0573	.0573
$\hat{\rho}_\mu$.sd1	.2731	.2699	.2486	.1604	.0785	.0614	.0596	.0592	.0592
$\hat{\rho}_\mu$.sd2	.2676	.2643	.2415	.1563	.0812	.0646	.0622	.0621	.0619
$\hat{\rho}_\tau$	.2176	.2156	.2033	.1385	.0701	.0555	.0539	.0537	.0536
$\hat{\rho}_\tau$.pp1	.2292	.2264	.2129	.1465	.0764	.0598	.0589	.0587	.0587
$\hat{\rho}_\tau$.pp2	.2328	.2316	.2190	.1534	.0818	.0634	.0619	.0618	.0617
$\hat{\rho}_\tau$.sd1	.2275	.2257	.2134	.1455	.0786	.0639	.0623	.0621	.0621
$\hat{\rho}_\tau$.sd2	.2296	.2281	.2153	.1504	.0867	.0727	.0708	.0702	.0702

Table 10

Proportions of Rejections, 5% Lower tail Test

$T = 100,\ \Phi_1 = .1,\ \Phi_2 = .9,\ h_0 = 1$

Φ_0	0	.00001	.0001	.001	.01	.1	1	10	100
$\hat{\tau}_\mu$	.0913	.0913	.0902	.0826	.0553	.0385	.0365	.0362	.0363
$\hat{\tau}_\mu$.w	.0768	.0769	.0766	.0737	.0612	.0445	.0420	.0419	.0417
$\hat{\tau}_\mu$.pp1	.0925	.0925	.0917	.0854	.0580	.0391	.0367	.0369	.0368
$\hat{\tau}_\mu$.pp2	.0952	.0950	.0946	.0879	.0599	.0415	.0385	.0384	.0384
$\hat{\tau}_\mu$.sd1	.0918	.0918	.0908	.0828	.0563	.0399	.0364	.0361	.0360
$\hat{\tau}_\mu$.sd2	.0891	.0891	.0877	.0792	.0541	.0371	.0337	.0333	.0333
$\hat{\tau}_\tau$	.0874	.0873	.0863	.0796	.0605	.0454	.0426	.0424	.0425
$\hat{\tau}_\tau$.w	.0847	.0847	.0849	.0843	.0739	.0603	.0556	.0552	.0550
$\hat{\tau}_\tau$.pp1	.0911	.0911	.0907	.0833	.0627	.0471	.0447	.0442	.0442
$\hat{\tau}_\tau$.pp2	.0942	.0940	.0937	.0877	.0670	.0504	.0475	.0474	.0474
$\hat{\tau}_\tau$.sd1	.0828	.0826	.0814	.0757	.0572	.0443	.0418	.0415	.0415
$\hat{\tau}_\tau$.sd2	.0784	.0783	.0779	.0718	.0519	.0397	.0373	.0371	.0371
$\hat{\rho}_\mu$	.0681	.0681	.0679	.0653	.0567	.0527	.0510	.0510	.0511
$\hat{\rho}_\mu$.pp1	.0707	.0707	.0706	.0671	.0590	.0558	.0552	.0550	.0550
$\hat{\rho}_\mu$.pp2	.0742	.0741	.0733	.0702	.0613	.0581	.0570	.0567	.0567
$\hat{\rho}_\mu$.sd1	.0737	.0736	.0732	.0703	.0631	.0584	.0569	.0566	.0566
$\hat{\rho}_\mu$.sd2	.0782	.0780	.0771	.0746	.0647	.0601	.0593	.0593	.0593
$\hat{\rho}_\tau$	.0658	.0657	.0653	.0624	.0536	.0504	.0504	.0505	.0506
$\hat{\rho}_\tau$.pp1	.0712	.0712	.0710	.0690	.0605	.0555	.0561	.0561	.0561
$\hat{\rho}_\tau$.pp2	.0772	.0770	.0766	.0736	.0644	.0592	.0607	.0609	.0608
$\hat{\rho}_\tau$.sd1	.0782	.0780	.0776	.0725	.0654	.0612	.0614	.0617	.0617
$\hat{\rho}_\tau$.sd2	.0835	.0835	.0830	.0793	.0720	.0691	.0699	.0699	.0699

Table 11

Proportions of Rejections, 5% Lower Tail Test

$\Phi_1 = .3, \quad \Phi_2 = .65, \quad h_0 = 1$

Φ_0 :	0		.01		1.0		100	
T :	100	1000	100	1000	100	1000	100	1000
$\hat{\tau}_\mu$	.6540	.9975	.1185	.0847	.0749	.0701	.0743	.0698
$\hat{\tau}_\mu$.w	.1749	.1616	.0722	.0429	.0588	.0420	.0584	.0421
$\hat{\tau}_\mu$.pp1	.6581	.9976	.1202	.0829	.0758	.0686	.0746	.0684
$\hat{\tau}_\mu$.pp2	.6612	.9975	.1199	.0828	.0769	.0672	.0762	.0670
$\hat{\tau}_\mu$.sd1	.6486	.9973	.1136	.0838	.0726	.0702	.0720	.0700
$\hat{\tau}_\mu$.sd2	.6182	.9965	.1015	.0792	.0669	.0670	.0663	.0670
$\hat{\tau}_\tau$	.5767	.9922	.1183	.0977	.0817	.0782	.0810	.0781
$\hat{\tau}_\tau$.w	.1437	.0957	.0788	.0370	.0653	.0348	.0650	.0346
$\hat{\tau}_\tau$.pp1	.5819	.9923	.1213	.0965	.0862	.0781	.0854	.0778
$\hat{\tau}_\tau$.pp2	.5867	.9929	.1234	.0961	.0865	.0764	.0861	.0761
$\hat{\tau}_\tau$.sd1	.5718	.9922	.1127	.0949	.0784	.0782	.0775	.0780
$\hat{\tau}_\tau$.sd2	.5356	.9902	.0944	.0912	.0642	.0729	.0639	.0728
$\hat{\rho}_\mu$	.5190	.9960	.0956	.0839	.0749	.0735	.0747	.0735
$\hat{\rho}_\mu$.pp1	.5183	.9966	.0985	.0838	.0765	.0726	.0761	.0725
$\hat{\rho}_\mu$.pp2	.5148	.9966	.0996	.0830	.0776	.0723	.0775	.0721
$\hat{\rho}_\mu$.sd1	.5217	.9962	.1010	.0825	.0795	.0729	.0792	.0729
$\hat{\rho}_\mu$.sd2	.5079	.9957	.0990	.0794	.0813	.0715	.0808	.0713
$\hat{\rho}_\tau$	.4214	.9841	.0954	.0892	.0760	.0774	.0758	.0772
$\hat{\rho}_\tau$.pp1	.4290	.9841	.1004	.0885	.0785	.0760	.0782	.0759
$\hat{\rho}_\tau$.pp2	.4305	.9852	.1025	.0884	.0823	.0750	.0820	.0750
$\hat{\rho}_\tau$.sd1	.4250	.9841	.1023	.0880	.0847	.0755	.0846	.0753
$\hat{\rho}_\tau$.sd2	.4163	.9830	.1033	.0866	.0876	.0758	.0875	.0757

Table 12

Proportions of Rejections, 5% Lower Tail Test

$\Phi_1 = .3, \quad \Phi_2 = .7, \; h_0 = 1$

Φ_0 :	0		.01		1.0		100	
T :	100	1000	100	1000	100	1000	100	1000
$\hat{\tau}_\mu$	.4125	.9676	.1208	.1165	.0711	.0889	.0699	.0886
$\hat{\tau}_\mu$.w	.1207	.1004	.0684	.0360	.0528	.0338	.0520	.0337
$\hat{\tau}_\mu$.pp1	.4174	.9677	.1213	.1149	.0710	.0881	.0698	.0874
$\hat{\tau}_\mu$.pp2	.4214	.9687	.1234	.1135	.0733	.0880	.0721	.0875
$\hat{\tau}_\mu$.sd1	.4043	.9689	.1153	.1139	.0678	.0893	.0668	.0889
$\hat{\tau}_\mu$.sd2	.3820	.9667	.1077	.1095	.0637	.0839	.0631	.0835
$\hat{\tau}_\tau$	.3531	.9374	.1165	.1387	.0775	.1073	.0766	.1072
$\hat{\tau}_\tau$.w	.1011	.0488	.0701	.0235	.0558	.0223	.0550	.0222
$\hat{\tau}_\tau$.pp1	.3602	.9386	.1185	.1380	.0787	.1054	.0782	.1051
$\hat{\tau}_\tau$.pp2	.3622	.9391	.1218	.1365	.0806	.1057	.0800	.1053
$\hat{\tau}_\tau$.sd1	.3457	.9402	.1138	.1364	.0747	.1067	.0740	.1060
$\hat{\tau}_\tau$.sd2	.3171	.9352	.0966	.1297	.0624	.1002	.0618	.0996
$\hat{\rho}_\mu$	.2944	.9469	.0992	.1175	.0773	.0989	.0773	.0989
$\hat{\rho}_\mu$.pp1	.2981	.9465	.1033	.1176	.0794	.0988	.0792	.0983
$\hat{\rho}_\mu$.pp2	.2982	.9461	.1043	.1157	.0813	.0979	.0806	.0973
$\hat{\rho}_\mu$.sd1	.3009	.9457	.1061	.1185	.0819	.0990	.0817	.0985
$\hat{\rho}_\mu$.sd2	.2941	.9470	.1066	.1147	.0841	.0963	.0838	.0962
$\hat{\rho}_\tau$	.2429	.8934	.0946	.1298	.0755	.1098	.0753	.1095
$\hat{\rho}_\tau$.pp1	.2495	.8945	.0992	.1306	.0796	.1108	.0791	.1104
$\hat{\rho}_\tau$.pp2	.2553	.8948	.1019	.1289	.0819	.1075	.0816	.1073
$\hat{\rho}_\tau$.sd1	.2502	.8958	.1029	.1313	.0841	.1109	.0836	.1106
$\hat{\rho}_\tau$.sd2	.2475	.8932	.1047	.1267	.0883	.1063	.0877	.1061

Table 13

Proportions of Rejections, 5% Lower Tail Test

Φ_1 =.1, Φ_2 =.85, h_0 = 1

Φ_0:	0		.01		1.0		100	
T:	100	1000	100	1000	100	1000	100	1000
$\hat{\tau}_\mu$	.4220	.9922	.1008	.0705	.0464	.0500	.0457	.0497
$\hat{\tau}_\mu$.w	.1194	.0967	.0823	.0518	.0593	.0477	.0590	.0475
$\hat{\tau}_\mu$.pp1	.4266	.9926	.1042	.0702	.0486	.0498	.0479	.0497
$\hat{\tau}_\mu$.pp2	.4322	.9933	.1064	.0702	.0497	.0496	.0492	.0495
$\hat{\tau}_\mu$.sd1	.4196	.9911	.0986	.0688	.0483	.0495	.0476	.0492
$\hat{\tau}_\mu$.sd2	.3961	.9917	.0924	.0674	.0457	.0483	.0452	.0483
$\hat{\tau}_\tau$	.3521	.9758	.0974	.0714	.0556	.0492	.0551	.0490
$\hat{\tau}_\tau$.w	.1008	.0467	.0968	.0489	.0735	.0426	.0727	.0425
$\hat{\tau}_\tau$.pp1	.3612	.9773	.1027	.0731	.0581	.0509	.0576	.0506
$\hat{\tau}_\tau$.pp2	.3657	.9786	.1054	.0724	.0605	.0510	.0596	.0508
$\hat{\tau}_\tau$.sd1	.3519	.9775	.0918	.0727	.0516	.0501	.0510	.0500
$\hat{\tau}_\tau$.sd2	.3238	.9760	.0840	.0710	.0468	.0501	.0465	.0499
$\hat{\rho}_\mu$	.2690	.9890	.0738	.0640	.0534	.0536	.0531	.0534
$\hat{\rho}_\mu$.pp1	.2695	.9892	.0761	.0647	.0552	.0533	.0548	.0529
$\hat{\rho}_\mu$.pp2	.2718	.9894	.0771	.0644	.0575	.0534	.0573	.0534
$\hat{\rho}_\mu$.sd1	.2731	.9893	.0785	.0633	.0596	.0541	.0592	.0539
$\hat{\rho}_\mu$.sd2	.2676	.9894	.0812	.0622	.0622	.0537	.0619	.0538
$\hat{\rho}_\tau$	.2176	.9564	.0701	.0571	.0539	.0466	.0536	.0465
$\hat{\rho}_\tau$.pp1	.2292	.9582	.0764	.0581	.0589	.0482	.0587	.0481
$\hat{\rho}_\tau$.pp2	.2328	.9583	.0818	.0597	.0619	.0494	.0617	.0494
$\hat{\rho}_\tau$.sd1	.2275	.9599	.0786	.0581	.0623	.0480	.0621	.0477
$\hat{\rho}_\tau$.sd2	.2296	.9591	.0867	.0576	.0708	.0490	.0702	.0489

Table 14

Proportions of Rejections, 5% Lower Tail Test

$\Phi_1 = .1,\ \Phi_2 = .9,\ h_0 = 1$

Φ_0 :	0		.01		1.0		100	
T :	100	1000	100	1000	100	1000	100	1000
$\hat{\tau}_\mu$	.0913	.4991	.0553	.0569	.0365	.0409	.0363	.0409
$\hat{\tau}_\mu$.w	.0768	.0533	.0612	.0349	.0420	.0276	.0417	.0276
$\hat{\tau}_\mu$.pp1	.0925	.5007	.0580	.0561	.0367	.0410	.0368	.0406
$\hat{\tau}_\mu$.pp2	.0952	.4992	.0599	.0572	.0385	.0421	.0384	.0415
$\hat{\tau}_\mu$.sd1	.0918	.4973	.0563	.0575	.0364	.0416	.0360	.0412
$\hat{\tau}_\mu$.sd2	.0891	.4915	.0541	.0579	.0337	.0407	.0333	.0406
$\hat{\tau}_\tau$	.0874	.4297	.0605	.0648	.0426	.0530	.0425	.0531
$\hat{\tau}_\tau$.w	.0847	.0247	.0739	.0246	.0556	.0216	.0550	.0217
$\hat{\tau}_\tau$.pp1	.0911	.4313	.0627	.0663	.0447	.0537	.0442	.0537
$\hat{\tau}_\tau$.pp2	.0942	.4315	.0670	.0658	.0475	.0543	.0474	.0544
$\hat{\tau}_\tau$.sd1	.0828	.4315	.0572	.0685	.0418	.0545	.0415	.0544
$\hat{\tau}_\tau$.sd2	.0784	.4254	.0519	.0639	.0373	.0521	.0371	.0519
$\hat{\rho}_\mu$	.0681	.3892	.0567	.0697	.0510	.0640	.0511	.0639
$\hat{\rho}_\mu$.pp1	.0707	.3905	.0590	.0700	.0552	.0632	.0550	.0632
$\hat{\rho}_\mu$.pp2	.0742	.3882	.0613	.0699	.0570	.0637	.0567	.0637
$\hat{\rho}_\mu$.sd1	.0737	.3896	.0631	.0704	.0569	.0635	.0566	.0635
$\hat{\rho}_\mu$.sd2	.0782	.3882	.0647	.0704	.0593	.0636	.0593	.0637
$\hat{\rho}_\tau$	.0658	.3207	.0536	.0667	.0504	.0630	.0506	.0624
$\hat{\rho}_\tau$.pp1	.0712	.3209	.0605	.0684	.0561	.0636	.0561	.0636
$\hat{\rho}_\tau$.pp2	.0772	.3214	.0644	.0693	.0607	.0639	.0608	.0639
$\hat{\rho}_\tau$.sd1	.0782	.3234	.0654	.0686	.0614	.0639	.0617	.0638
$\hat{\rho}_\tau$.sd2	.0835	.3199	.0720	.0684	.0699	.0636	.0699	.0636

TABLE 15

Proportions of Rejections, 5% Lower Tail Test

$T = 100,\ h_0 = 1,\ \Phi_0 = .01$

	(Φ_1, Φ_2)				
	(.3,.30)	(.3,.60)	(.3,.65)	(.3,.69)	(.3,.70)
$\hat{\tau}_\mu$	.0591	.0964	.1185	.1229	.1208
$\hat{\tau}_\mu$.w	.0649	.0710	.0722	.0706	.0684
$\hat{\tau}_\mu$.pp1	.0592	.0967	.1202	.1243	.1213
$\hat{\tau}_\mu$.pp	.0601	.0967	.1199	.1261	.1234
$\hat{\tau}_\mu$.sd1	.0564	.0906	.1136	.1182	.1153
$\hat{\tau}_\mu$.sd2	.0518	.0817	.1015	.1091	.1077
$\hat{\tau}_\tau$	.0688	.1035	.1183	.1196	.1165
$\hat{\tau}_\tau$.w	.0740	.0776	.0788	.0723	.0701
$\hat{\tau}_\tau$.pp1	.0710	.1057	.1213	.1216	.1185
$\hat{\tau}_\tau$.pp2	.0718	.1084	.1234	.1254	.1218
$\hat{\tau}_\tau$.sd1	.0642	.1004	.1127	.1158	.1138
$\hat{\tau}_\tau$.sd2	.0532	.0813	.0944	.0994	.0966
$\hat{\rho}_\mu$	.0590	.0835	.0956	.1010	.0992
$\hat{\rho}_\mu$.pp1	.0600	.0847	.0985	.1037	.1033
$\hat{\rho}_\mu$.pp2	.0611	.0857	.0996	.1040	.1043
$\hat{\rho}_\mu$.sd1	.0613	.0854	.1010	.1069	.1061
$\hat{\rho}_\mu$.sd2	.0642	.0861	.0990	.1055	.1066
$\hat{\rho}_\tau$	.0637	.0848	.0954	.0961	.0946
$\hat{\rho}_\tau$.pp1	.0675	.0903	.1004	.1020	.0992
$\hat{\rho}_\tau$.pp2	.0681	.0931	.1025	.1044	.1019
$\hat{\rho}_\tau$.sd1	.0697	.0931	.1023	.1049	.1029
$\hat{\rho}_\tau$.sd2	.0754	.0945	.1033	.1068	.1047

TABLE 16

Proportions of Rejections, 5% Lower Tail Test

$T = 100,\ h_0 = 1,\ \Phi_0 = .01$

	(Φ_1, Φ_2)					
	(.1, 0)	(.1,.5)	(.1,.8)	(.1,.85)	(.1,.89)	(.1,.9)
$\hat{\tau}_\mu$	.0472	.0493	.0885	.1008	.0671	.0553
$\hat{\tau}_\mu$.w	.0664	.0673	.0809	.0823	.0711	.0612
$\hat{\tau}_\mu$.pp1	.0490	.0528	.0907	.1042	.0692	.0580
$\hat{\tau}_\mu$.pp2	.0507	.0536	.0937	.1064	.0714	.0599
$\hat{\tau}_\mu$.sd1	.0499	.0516	.0856	.0986	.0693	.0563
$\hat{\tau}_\mu$.sd2	.0462	.0479	.0772	.0924	.0652	.0541
$\hat{\tau}_\tau$	.0573	.0591	.0911	.0974	.0700	.0605
$\hat{\tau}_\tau$.w	.0808	.0808	.0951	.0968	.0810	.0739
$\hat{\tau}_\tau$.pp1	.0588	.0601	.0938	.1027	.0733	.0627
$\hat{\tau}_\tau$.pp2	.0634	.0644	.0974	.1054	.0771	.0670
$\hat{\tau}_\tau$.sd1	.0531	.0554	.0856	.0918	.0653	.0572
$\hat{\tau}_\tau$.sd2	.0477	.0498	.0752	.0840	.0614	.0519
$\hat{\rho}_\mu$	.0495	.0533	.0693	.0738	.0607	.0567
$\hat{\rho}_\mu$.pp1	.0515	.0544	.0711	.0761	.0638	.0590
$\hat{\rho}_\mu$.pp2	.0553	.0572	.0729	.0771	.0658	.0613
$\hat{\rho}_\mu$.sd1	.0568	.0576	.0735	.0785	.0668	.0631
$\hat{\rho}_\mu$.sd2	.0595	.0602	.0751	.0812	.0698	.0647
$\hat{\rho}_\tau$	.0536	.0541	.0673	.0701	.0581	.0536
$\hat{\rho}_\tau$.pp1	.0560	.0573	.0725	.0764	.0658	.0605
$\hat{\rho}_\tau$.pp2	.0610	.0612	.0781	.0818	.0684	.0644
$\hat{\rho}_\tau$.sd1	.0608	.0617	.0749	.0786	.0699	.0654
$\hat{\rho}_\tau$.sd2	.0679	.0692	.0817	.0867	.0748	.0720

Table 17

Proportions of Rejections, 5% Lower Tail Test

T = 100

MA:	BASE	-.8	-.5	-.2	.2	.5	.8
$\hat{\tau}_\mu$	.0497	.9965	.5714	.1324	.0299	.0265	.0270
$\hat{\tau}_\mu$.w	.0683	.9971	.6076	.1630	.0402	.0345	.0343
$\hat{\tau}_\mu$.pp1	.0533	.9848	.4571	.1050	.0361	.0284	.0273
$\hat{\tau}_\mu$.pp2	.0531	.9825	.4413	.1031	.0391	.0313	.0290
$\hat{\tau}_\mu$.sd1	.0492	.8802	.2025	.0595	.0559	.0861	.1138
$\hat{\tau}_\mu$.sd2	.0472	.6425	.0923	.0489	.0465	.0377	.0311
$\hat{\tau}_\tau$	.0525	1.0000	.7987	.2009	.0157	.0090	.0081
$\hat{\tau}_\tau$.w	.0805	1.0000	.8344	.2579	.0337	.0179	.0161
$\hat{\tau}_\tau$.pp1	.0567	1.0000	.7047	.1650	.0252	.0146	.0128
$\hat{\tau}_\tau$.pp2	.0613	1.0000	.6957	.1643	.0288	.0169	.0148
$\hat{\tau}_\tau$.sd1	.0517	.9799	.3046	.0703	.0637	.1177	.1675
$\hat{\tau}_\tau$.sd2	.0488	.8094	.1273	.0510	.0472	.0315	.0200
$\hat{\rho}_\mu$	.0506	.9985	.6408	.1604	.0163	.0046	.0026
$\hat{\rho}_\mu$.pp1	.0524	.9899	.4938	.1193	.0303	.0194	.0169
$\hat{\rho}_\mu$.pp2	.0554	.9862	.4725	.1149	.0342	.0251	.0227
$\hat{\rho}_\mu$.sd1	.0561	.9778	.4331	.1126	.0325	.0204	.0172
$\hat{\rho}_\mu$.sd2	.0600	.9370	.3510	.1086	.0316	.0107	.0034
$\hat{\rho}_\tau$	.0500	1.0000	.8290	.2127	.0092	.0022	.0013
$\hat{\rho}_\tau$.pp1	.0547	1.0000	.6988	.1617	.0238	.0114	.0088
$\hat{\rho}_\tau$.pp2	.0611	1.0000	.6933	.1626	.0287	.0151	.0121
$\hat{\rho}_\tau$.sd1	.0593	.9995	.6047	.1460	.0304	.0158	.0129
$\hat{\rho}_\tau$.sd2	.0717	.9936	.4979	.1461	.0322	.0071	.0025

Table 18

Proportions of Rejections, 5% Lower Tail Test

	BASE			MA -.5			MA .5		
T:	100	200	500	100	200	500	100	200	500
$\hat{\tau}_\mu$	.0497	.0499	.0496	.5714	.5976	.6216	.0265	.0240	.0251
$\hat{\tau}_\mu$.w	.0683	.0613	.0512	.6076	.6150	.6262	.0345	.0299	.0267
$\hat{\tau}_\mu$.pp1	.0533	.0505	.0505	.4571	.4385	.4153	.0284	.0296	.0305
$\hat{\tau}_\mu$.pp2	.0531	.0513	.0513	.4413	.4010	.3451	.0313	.0319	.0290
$\hat{\tau}_\mu$.sd1	.0492	.0502	.0507	.2025	.2237	.2331	.0861	.0890	.0930
$\hat{\tau}_\mu$.sd2	.0472	.0472	.0490	.0923	.1078	.1139	.0377	.0346	.0373
$\hat{\tau}_\tau$	.0525	.0581	.0521	.7987	.8356	.8492	.0090	.0089	.0086
$\hat{\tau}_\tau$.w	.0805	.0733	.0569	.8344	.8532	.8546	.0179	.0151	.0110
$\hat{\tau}_\tau$.pp1	.0567	.0600	.0528	.7047	.6893	.6405	.0146	.0192	.0199
$\hat{\tau}_\tau$.pp2	.0613	.0625	.0539	.6957	.6534	.5643	.0169	.0254	.0251
$\hat{\tau}_\tau$.sd1	.0517	.0533	.0504	.3046	.3557	.3624	.1177	.1236	.1147
$\hat{\tau}_\tau$.sd2	.0488	.0495	.0485	.1273	.1485	.1506	.0315	.0318	.0311
$\hat{\rho}_\mu$	.0506	.0516	.0476	.6408	.6646	.6834	.0046	.0044	.0041
$\hat{\rho}_\mu$.pp1	.0524	.0535	.0477	.4938	.4829	.4660	.0194	.0192	.0188
$\hat{\rho}_\mu$.pp2	.0554	.0557	.0494	.4725	.4325	.3834	.0251	.0269	.0267
$\hat{\rho}_\mu$.sd1	.0561	.0549	.0481	.4331	.4457	.4499	.0204	.0180	.0178
$\hat{\rho}_\mu$.sd2	.0600	.0569	.0500	.3510	.3566	.3539	.0107	.0080	.0073
$\hat{\rho}_\tau$	.0500	.0456	.0501	.8290	.8449	.8743	.0022	.0019	.0014
$\hat{\rho}_\tau$.pp1	.0547	.0495	.0512	.6988	.6736	.6618	.0114	.0111	.0148
$\hat{\rho}_\tau$.pp2	.0611	.0513	.0515	.6933	.6317	.5734	.0151	.0176	.0229
$\hat{\rho}_\tau$.sd1	.0593	.0515	.0520	.6047	.6005	.6245	.0158	.0127	.0147
$\hat{\rho}_\tau$.sd2	.0717	.0540	.0524	.4979	.4852	.5005	.0071	.0046	.0043

Table 19

Proportions of Rejections, 5% Lower Tail Test

T = 100

AR:	-.8	-.5	-.2	.2	.5	.8
$\hat{\tau}_\mu$	.7981	.3627	.1144	.0288	.0349	.0941
$\hat{\tau}_\mu$.w	.8276	.4038	.1421	.0372	.0440	.1126
$\hat{\tau}_\mu$.pp1	.6086	.2243	.0891	.0317	.0280	.0608
$\hat{\tau}_\mu$.pp2	.6838	.2500	.0908	.0347	.0253	.0468
$\hat{\tau}_\mu$.sd1	.0506	.0505	.0496	.0497	.0520	.0541
$\hat{\tau}_\mu$.sd2	.0460	.0475	.0475	.0483	.0487	.0522
$\hat{\tau}_\tau$	.9580	.5651	.1727	.0134	.0094	.0358
$\hat{\tau}_\tau$.w	.0603	.6273	.2249	.0285	.0162	.0490
$\hat{\tau}_\tau$.pp1	.8705	.4007	.1345	.0195	.0079	.0200
$\hat{\tau}_\tau$.pp2	.9137	.4464	.1415	.0230	.0085	.0142
$\hat{\tau}_\tau$.sd1	.0518	.0514	.0508	.0520	.0544	.0590
$\hat{\tau}_\tau$.sd2	.0486	.0484	.0473	.0489	.0484	.0565
$\hat{\rho}_\mu$	.8444	.4287	.1386	.0122	.0006	.0000
$\hat{\rho}_\mu$.pp1	.5872	.2309	.0985	.0233	.0031	.0000
$\hat{\rho}_\mu$.pp2	.6998	.2686	.0995	.0285	.0070	.0002
$\hat{\rho}_\mu$.sd1	.2336	.1577	.0929	.0254	.0029	.0001
$\hat{\rho}_\mu$.sd2	.2404	.1645	.0978	.0276	.0036	.0001
$\hat{\rho}_\tau$	.9669	.5972	.1828	.0062	.0001	.0001
$\hat{\rho}_\tau$.pp1	.8380	.3701	.1309	.0168	.0014	.0001
$\hat{\rho}_\tau$.pp2	.9118	.4391	.1397	.0216	.0029	.0002
$\hat{\rho}_\tau$.sd1	.3292	.2210	.1176	.0218	.0017	.0001
$\hat{\rho}_\tau$.sd2	.3448	.2349	.1281	.0267	.0020	.0001

Table 20

Proportions of Rejections, 5% Lower Tail Test

AR:	-.5			.5		
T:	100	200	500	100	200	500
$\hat{\tau}_\mu$	.3627	.3764	.3833	.0349	.0342	.0351
$\hat{\tau}_\mu$.w	.4038	.3946	.3907	.0440	.0393	.0374
$\hat{\tau}_\mu$.pp1	.2243	.1951	.1609	.0280	.0255	.0265
$\hat{\tau}_\mu$.pp2	.2500	.2155	.1723	.0253	.0235	.0258
$\hat{\tau}_\mu$.sd1	.0505	.0494	.0499	.0520	.0495	.0495
$\hat{\tau}_\mu$.sd2	.0475	.0475	.0490	.0487	.0470	.0491
$\hat{\tau}_\tau$	.5651	.5898	.5880	.0094	.0085	.0096
$\hat{\tau}_\tau$.w	.6273	.6186	.6018	.0162	.0131	.0111
$\hat{\tau}_\tau$.pp1	.4007	.3416	.2509	.0079	.0078	.0080
$\hat{\tau}_\tau$.pp2	.4464	.3827	.2760	.0085	.0099	.0096
$\hat{\tau}_\tau$.sd1	.0514	.0541	.0507	.0544	.0545	.0492
$\hat{\tau}_\tau$.sd2	.0484	.0518	.0490	.0484	.0512	.0486
$\hat{\rho}_\mu$	.4287	.4432	.4461	.0006	.0003	.0004
$\hat{\rho}_\mu$.pp1	.2309	.2058	.1784	.0031	.0027	.0028
$\hat{\rho}_\mu$.pp2	.2686	.2314	.1878	.0070	.0071	.0072
$\hat{\rho}_\mu$.sd1	.1577	.1637	.1612	.0029	.0025	.0022
$\hat{\rho}_\mu$.sd2	.1645	.1661	.1607	.0036	.0025	.0026
$\hat{\rho}_\tau$	.5972	.6011	.6255	.0001	.0001	.0000
$\hat{\rho}_\tau$.pp1	.3701	.3058	.2516	.0014	.0007	.0006
$\hat{\rho}_\tau$.pp2	.4391	.3550	.2831	.0029	.0027	.0027
$\hat{\rho}_\tau$.sd1	.2210	.1987	.1990	.0017	.0011	.0006
$\hat{\rho}_\tau$.sd2	.2349	.2065	.2004	.0020	.0009	.0008

Table 21

Number of Negative Long-run Variance Estimates
(out of 10,000 replications)

T = 100

MA:	-.8	-.5	-.2	.2	.5	.8	BASE
pp1	0	0	36	0	0	0	0
pp2	0	0	19	0	0	0	0
pp3	0	1	10	2	0	0	0
pp4	1	0	5	5	2	2	2
pp5	10	3	9	11	11	15	14
pp6	12	4	4	6	18	28	31
pp7	34	7	3	19	52	86	101
pp8	58	10	8	18	106	167	188
pp9	78	27	8	37	166	244	275
pp10	109	38	8	40	216	348	405

AR:	-.8	-.5	-.2	.2	.5	.8
pp1	6204	1328	0	0	0	0
pp2	0	1	0	0	0	0
pp3	479	27	2	0	0	0
pp4	1	5	2	0	0	0
pp5	54	14	10	8	1	0
pp6	1	1	6	18	8	0
pp7	10	4	17	51	38	2
pp8	3	11	19	104	114	11
pp9	4	9	40	168	195	34
pp10	5	10	44	231	345	68

Table 22

Proportions of Rejections, 5% Lower Tail Test

Bartlett's Lag Window, T = 100

MA:	-.8	-.5	-.2	.2	.5	.8	BASE
$\hat{\tau}_\mu$	.9965	.5714	.1324	.0299	.0265	.0270	.0497
$\hat{\tau}_\mu$.pp1	.9848	.4571	.1050	.0361	.0284	.0273	.0533
$\hat{\tau}_\mu$.pp2	.9825	.4413	.1031	.0391	.0313	.0290	.0531
$\hat{\tau}_\mu$.pp3	.9834	.4474	.1062	.0392	.0319	.0299	.0548
$\hat{\tau}_\mu$.pp4	.9850	.4626	.1110	.0388	.0320	.0301	.0552
$\hat{\tau}_\mu$.pp5	.9872	.4799	.1148	.0382	.0316	.0299	.0556
$\hat{\tau}_\mu$.pp6	.9894	.4960	.1181	.0381	.0305	.0289	.0556
$\hat{\tau}_\mu$.pp7	.9910	.5126	.1225	.0375	.0291	.0277	.0566
$\hat{\tau}_\mu$.pp8	.9928	.5270	.1259	.0367	.0290	.0271	.0556
$\hat{\tau}_\mu$.pp9	.9942	.5396	.1303	.0359	.0279	.0263	.0561
$\hat{\tau}_\mu$.pp10	.9949	.5507	.1328	.0346	.0271	.0255	.0557
$\hat{\tau}_\tau$	1.0000	.7987	.2009	.0157	.0090	.0081	.0525
$\hat{\tau}_\tau$.pp1	1.0000	.7047	.1650	.0252	.0146	.0128	.0567
$\hat{\tau}_\tau$.pp2	1.0000	.6957	.1643	.0288	.0169	.0148	.0613
$\hat{\tau}_\tau$.pp3	1.0000	.7117	.1725	.0288	.0168	.0148	.0632
$\hat{\tau}_\tau$.pp4	1.0000	.7317	.1817	.0274	.0155	.0137	.0640
$\hat{\tau}_\tau$.pp5	1.0000	.7510	.1912	.0259	.0138	.0117	.0634
$\hat{\tau}_\tau$.pp6	1.0000	.7696	.1988	.0237	.0119	.0104	.0609
$\hat{\tau}_\tau$.pp7	1.0000	.7864	.2029	.0216	.0109	.0099	.0587
$\hat{\tau}_\tau$.pp8	1.0000	.8011	.2068	.0190	.0104	.0096	.0572
$\hat{\tau}_\tau$.pp9	1.0000	.8138	.2101	.0176	.0102	.0093	.0547
$\hat{\tau}_\tau$.pp10	1.0000	.8240	.2131	.0158	.0096	.0090	.0531

Table 23

Proportions of Rejections, 5% Lower Tail Test

Bartlett's Lag Window, T = 100

MA:	-.8	-.5	-.2	.2	.5	.8	BASE
$\hat{\rho}_{\mu}$	.9985	.6408	.1604	.0163	.0046	.0026	.0506
$\hat{\rho}_{\mu}$.pp1	.9899	.4938	.1193	.0303	.0194	.0169	.0524
$\hat{\rho}_{\mu}$.pp2	.9862	.4725	.1149	.0342	.0251	.0227	.0554
$\hat{\rho}_{\mu}$.pp3	.9868	.4774	.1191	.0361	.0263	.0242	.0568
$\hat{\rho}_{\mu}$.pp4	.9885	.4934	.1253	.0356	.0265	.0243	.0581
$\hat{\rho}_{\mu}$.pp5	.9907	.5113	.1304	.0348	.0254	.0224	.0593
$\hat{\rho}_{\mu}$.pp6	.9923	.5299	.1376	.0336	.0234	.0206	.0597
$\hat{\rho}_{\mu}$.pp7	.9941	.5481	.1427	.0321	.0210	.0188	.0596
$\hat{\rho}_{\mu}$.pp8	.9951	.5637	.1485	.0303	.0194	.0164	.0596
$\hat{\rho}_{\mu}$.pp9	.9959	.5787	.1528	.0282	.0165	.0144	.0592
$\hat{\rho}_{\mu}$.pp10	.9966	.5924	.1573	.0254	.0138	.0117	.0575
$\hat{\rho}_{\tau}$	1.0000	.8290	.2127	.0092	.0022	.0013	.0500
$\hat{\rho}_{\tau}$.pp1	1.0000	.6988	.1617	.0238	.0114	.0088	.0547
$\hat{\rho}_{\tau}$.pp2	1.0000	.6933	.1626	.0287	.0151	.0121	.0611
$\hat{\rho}_{\tau}$.pp3	1.0000	.7122	.1749	.0294	.0141	.0115	.0642
$\hat{\rho}_{\tau}$.pp4	1.0000	.7376	.1879	.0274	.0123	.0100	.0662
$\hat{\rho}_{\tau}$.pp5	1.0000	.7629	.1981	.0251	.0104	.0083	.0657
$\hat{\rho}_{\tau}$.pp6	1.0000	.7826	.2079	.0206	.0081	.0058	.0636
$\hat{\rho}_{\tau}$.pp7	1.0000	.8024	.2165	.0170	.0048	.0033	.0609
$\hat{\rho}_{\tau}$.pp8	1.0000	.8194	.2223	.0133	.0035	.0023	.0571
$\hat{\rho}_{\tau}$.pp9	1.0000	.8336	.2259	.0108	.0025	.0015	.0524
$\hat{\rho}_{\tau}$.pp10	1.0000	.8440	.2285	.0086	.0020	.0011	.0499

Table 24

Proportions of Rejections, 5% Lower Tail Test

Parzen's Lag Window, T = 100

MA:	-.8	-.5	-.2	.2	.5	.8	BASE
$\hat{\tau}_\mu$	.9965	.5714	.1324	.0299	.0265	.0270	.0497
$\hat{\tau}_\mu$.pp1	.9921	.5155	.1183	.0315	.0258	.0249	.0515
$\hat{\tau}_\mu$.pp2	.9839	.4499	.1032	.0369	.0290	.0280	.0530
$\hat{\tau}_\mu$.pp3	.9805	.4274	.0997	.0398	.0326	.0309	.0541
$\hat{\tau}_\mu$.pp4	.9801	.4254	.1010	.0407	.0333	.0323	.0545
$\hat{\tau}_\mu$.pp5	.9815	.4337	.1041	.0404	.0340	.0326	.0551
$\hat{\tau}_\mu$.pp6	.9826	.4450	.1070	.0402	.0338	.0331	.0552
$\hat{\tau}_\mu$.pp7	.9844	.4590	.1104	.0403	.0336	.0325	.0560
$\hat{\tau}_\mu$.pp8	.9863	.4727	.1136	.0404	.0331	.0321	.0562
$\hat{\tau}_\mu$.pp9	.9882	.4863	.1172	.0393	.0319	.0310	.0565
$\hat{\tau}_\mu$.pp10	.9899	.4988	.1205	.0395	.0314	.0295	.0569
$\hat{\tau}_\tau$	1.0000	.7987	.2009	.0157	.0090	.0081	.0525
$\hat{\tau}_\tau$.pp1	1.0000	.7386	.1806	.0195	.0103	.0092	.0548
$\hat{\tau}_\tau$.pp2	1.0000	.6894	.1642	.0258	.0153	.0133	.0577
$\hat{\tau}_\tau$.pp3	1.0000	.6802	.1609	.0303	.0185	.0169	.0609
$\hat{\tau}_\tau$.pp4	1.0000	.6896	.1633	.0323	.0198	.0175	.0629
$\hat{\tau}_\tau$.pp5	1.0000	.7078	.1694	.0314	.0188	.0174	.0646
$\hat{\tau}_\tau$.pp6	1.0000	.7262	.1760	.0304	.0161	.0165	.0661
$\hat{\tau}_\tau$.pp7	1.0000	.7431	.1832	.0292	.0164	.0151	.0653
$\hat{\tau}_\tau$.pp8	1.0000	.7602	.1897	.0276	.0149	.0137	.0644
$\hat{\tau}_\tau$.pp9	1.0000	.7758	.1963	.0261	.0131	.0119	.0634
$\hat{\tau}_\tau$.pp10	1.0000	.7882	.2023	.0234	.0122	.0112	.0613

Table 25

Proportions of Rejections, 5% Lower Tail Test

Parzen's Lag Window, T = 100

MA:	-.8	-.5	-.2	.2	.5	.8	BASE
$\hat{\rho}_\mu$	.9985	.6408	.1604	.0163	.0046	.0026	.0506
$\hat{\rho}_\mu$.pp1	.9961	.5713	.1406	.0225	.0097	.0074	.0513
$\hat{\rho}_\mu$.pp2	.9882	.4826	.1161	.0317	.0211	.0186	.0531
$\hat{\rho}_\mu$.pp3	.9840	.4497	.1112	.0366	.0276	.0255	.0547
$\hat{\rho}_\mu$.pp4	.9832	.4473	.1111	.0391	.0298	.0286	.0564
$\hat{\rho}_\mu$.pp5	.9847	.4582	.1138	.0397	.0311	.0291	.0576
$\hat{\rho}_\mu$.pp6	.9866	.4717	.1200	.0402	.0307	.0292	.0590
$\hat{\rho}_\mu$.pp7	.9877	.4845	.1249	.0396	.0305	.0283	.0605
$\hat{\rho}_\mu$.pp8	.9893	.5000	.1288	.0390	.0291	.0268	.0614
$\hat{\rho}_\mu$.pp9	.9981	.5176	.1345	.0378	.0268	.0246	.0610
$\hat{\rho}_\mu$.pp10	.9926	.5316	.1398	.0353	.0249	.0230	.0610
$\hat{\rho}_\tau$	1.0000	.8290	.2127	.0092	.0022	.0013	.0500
$\hat{\rho}_\tau$.pp1	1.0000	.7704	.1866	.0154	.0041	.0032	.0517
$\hat{\rho}_\tau$.pp2	1.0000	.6919	.1596	.0255	.0126	.0094	.0563
$\hat{\rho}_\tau$.pp3	1.0000	.6727	.1562	.0319	.0181	.0157	.0615
$\hat{\rho}_\tau$.pp4	1.0000	.6787	.1608	.0339	.0209	.0174	.0639
$\hat{\rho}_\tau$.pp5	1.0000	.6975	.1696	.0336	.0200	.0172	.0666
$\hat{\rho}_\tau$.pp6	1.0000	.7181	.1807	.0326	.0176	.0146	.0677
$\hat{\rho}_\tau$.pp7	1.0000	.7394	.1892	.0302	.0150	.0130	.0681
$\hat{\rho}_\tau$.pp8	1.0000	.7591	.1969	.0271	.0126	.0107	.0678
$\hat{\rho}_\tau$.pp9	1.0000	.7768	.2053	.0236	.0099	.0077	.0665
$\hat{\rho}_\tau$.pp10	1.0000	.7909	.2136	.0199	.0075	.0057	.0642

Table 26

Proportions of Rejections, 5% Lower Tail Test

Bohman's Lag Window, T = 100

MA:	-.8	-.5	-.2	.2	.5	.8	BASE
$\hat{\tau}_\mu$	.9965	.5714	.1324	.0299	.0265	.0270	.0497
$\hat{\tau}_\mu$.pp1	.9905	.4992	.1141	.0329	.0263	.0249	.0516
$\hat{\tau}_\mu$.pp2	.9826	.4407	.1013	.0377	.0302	.0284	.0530
$\hat{\tau}_\mu$.pp3	.9797	.4249	.0999	.0403	.0329	.0315	.0540
$\hat{\tau}_\mu$.pp4	.9804	.4279	.1020	.0406	.0334	.0325	.0547
$\hat{\tau}_\mu$.pp5	.9822	.4397	.1050	.0405	.0340	.0325	.0554
$\hat{\tau}_\mu$.pp6	.9839	.4524	.1092	.0403	.0336	.0327	.0560
$\hat{\tau}_\mu$.pp7	.9856	.4683	.1127	.0404	.0332	.0323	.0561
$\hat{\tau}_\mu$.pp8	.9877	.4824	.1161	.0397	.0322	.0313	.0563
$\hat{\tau}_\mu$.pp9	.9897	.4960	.1195	.0396	.0314	.0299	.0572
$\hat{\tau}_\mu$.pp10	.9908	.5111	.1234	.0387	.0301	.0287	.0577
$\hat{\tau}_\tau$	1.0000	.7987	.2009	.0157	.0090	.0081	.0525
$\hat{\tau}_\tau$.pp1	1.0000	.7386	.1765	.0213	.0113	.0098	.0552
$\hat{\tau}_\tau$.pp2	1.0000	.6894	.1622	.0274	.0165	.0142	.0587
$\hat{\tau}_\tau$.pp3	1.0000	.6802	.1618	.0314	.0193	.0172	.0616
$\hat{\tau}_\tau$.pp4	1.0000	.6896	.1664	.0321	.0193	.0175	.0632
$\hat{\tau}_\tau$.pp5	1.0000	.7078	.1728	.0308	.0185	.0173	.0656
$\hat{\tau}_\tau$.pp6	1.0000	.7262	.1801	.0304	.0175	.0155	.0657
$\hat{\tau}_\tau$.pp7	1.0000	.7431	.1886	.0281	.0152	.0143	.0650
$\hat{\tau}_\tau$.pp8	1.0000	.7602	.1948	.0266	.0135	.0127	.0643
$\hat{\tau}_\tau$.pp9	1.0000	.7758	.2013	.0240	.0124	.0114	.0622
$\hat{\tau}_\tau$.pp10	1.0000	.7882	.2052	.0216	.0115	.0105	.0598

Table 27

Proportions of Rejections, 5% Lower Tail Test

Bohman's Lag Window, T = 100

MA:	-.8	-.5	-.2	.2	.5	.8	BASE
$\hat{\rho}_\mu$	.9985	.6408	.1604	.0163	.0046	.0026	.0506
$\hat{\rho}_\mu$.pp1	.9954	.5508	.1340	.0248	.0119	.0092	.0509
$\hat{\rho}_\mu$.pp2	.9867	.4686	.1134	.0336	.0240	.0207	.0537
$\hat{\rho}_\mu$.pp3	.9831	.4464	.1099	.0377	.0286	.0266	.0557
$\hat{\rho}_\mu$.pp4	.9836	.4512	.1115	.0392	.0308	.0292	.0571
$\hat{\rho}_\mu$.pp5	.9858	.4650	.1161	.0406	.0309	.0293	.0588
$\hat{\rho}_\mu$.pp6	.9874	.4785	.1230	.0402	.0306	.0290	.0603
$\hat{\rho}_\mu$.pp7	.9890	.4951	.1278	.0393	.0298	.0276	.0610
$\hat{\rho}_\mu$.pp8	.9909	.5131	.1328	.0385	.0275	.0251	.0613
$\hat{\rho}_\mu$.pp9	.9923	.5291	.1392	.0368	.0254	.0231	.0610
$\hat{\rho}_\mu$.pp10	.9939	.5448	.1439	.0346	.0237	.0203	.0609
$\hat{\rho}_\tau$	1.0000	.8290	.2127	.0092	.0022	.0013	.0500
$\hat{\rho}_\tau$.pp1	1.0000	.7494	.1806	.0174	.0050	.0038	.0527
$\hat{\rho}_\tau$.pp2	1.0000	.6830	.1573	.0271	.0150	.0122	.0573
$\hat{\rho}_\tau$.pp3	1.0000	.6714	.1581	.0335	.0202	.0166	.0625
$\hat{\rho}_\tau$.pp4	1.0000	.6857	.1642	.0346	.0212	.0183	.0654
$\hat{\rho}_\tau$.pp5	1.0000	.7069	.1743	.0339	.0194	.0157	.0675
$\hat{\rho}_\tau$.pp6	1.0000	.7313	.1856	.0315	.0164	.0141	.0685
$\hat{\rho}_\tau$.pp7	1.0000	.7519	.1960	.0286	.0137	.0113	.0682
$\hat{\rho}_\tau$.pp8	1.0000	.7721	.2032	.0243	.0111	.0088	.0673
$\hat{\rho}_\tau$.pp9	1.0000	.7888	.2126	.0206	.0078	.0062	.0642
$\hat{\rho}_\tau$.pp10	1.0000	.8051	.2193	.0176	.0058	.0045	.0621

Table 28

Proportions of Rejections, 5% Lower Tail Test

Bartlett's Lag Window, T = 100

AR:	-.8	-.5	-.2	.2	.5	.8
$\hat{\tau}_\mu$	.7981	.3627	.1144	.0288	.0349	.0941
$\hat{\tau}_\mu$.pp1	.6086	.2243	.0891	.0317	.0280	.0608
$\hat{\tau}_\mu$.pp2	.6838	.2500	.0908	.0347	.0253	.0468
$\hat{\tau}_\mu$.pp3	.6730	.2574	.0947	.0364	.0254	.0401
$\hat{\tau}_\mu$.pp4	.7081	.2749	.0985	.0361	.0248	.0366
$\hat{\tau}_\mu$.pp5	.7176	.2893	.1019	.0351	.0246	.0344
$\hat{\tau}_\mu$.pp6	.7393	.3038	.1053	.0351	.0253	.0330
$\hat{\tau}_\mu$.pp7	.7486	.3177	.1077	.0347	.0245	.0315
$\hat{\tau}_\mu$.pp8	.7639	.3300	.1107	.0344	.0242	.0302
$\hat{\tau}_\mu$.pp9	.7749	.3422	.1141	.0338	.0240	.0301
$\hat{\tau}_\mu$.pp10	.7859	.3524	.1154	.0323	.0235	.0302
$\hat{\tau}_\tau$	.9580	.5651	.1727	.0134	.0094	.0358
$\hat{\tau}_\tau$.pp1	.8705	.4007	.1345	.0195	.0079	.0200
$\hat{\tau}_\tau$.pp2	.9137	.4464	.1415	.0230	.0085	.0142
$\hat{\tau}_\tau$.pp3	.9135	.4643	.1510	.0234	.0093	.0113
$\hat{\tau}_\tau$.pp4	.9334	.4955	.1594	.0222	.0091	.0098
$\hat{\tau}_\tau$.pp5	.9394	.5186	.1666	.0215	.0086	.0099
$\hat{\tau}_\tau$.pp6	.9495	.5392	.1729	.0197	.0082	.0094
$\hat{\tau}_\tau$.pp7	.9547	.5612	.1781	.0174	.0077	.0093
$\hat{\tau}_\tau$.pp8	.9601	.5796	.1808	.0159	.0074	.0092
$\hat{\tau}_\tau$.pp9	.9636	.5955	.1842	.0142	.0073	.0095
$\hat{\tau}_\tau$.pp10	.9666	.6072	.1839	.0130	.0071	.0096

Table 29

Proportions of Rejections, 5% Lower Tail Test

Bartlett's Lag Window, T = 100

AR:	-.8	-.5	-.2	.2	.5	.8
$\hat{\rho}_{\mu}$	.8444	.4287	.1386	.0122	.0006	.0000
$\hat{\rho}_{\mu}$.pp1	.5872	.2309	.0985	.0233	.0031	.0000
$\hat{\rho}_{\mu}$.pp2	.6998	.2686	.0995	.0285	.0070	.0002
$\hat{\rho}_{\mu}$.pp3	.6819	.2757	.1047	.0311	.0102	.0009
$\hat{\rho}_{\mu}$.pp4	.7286	.2992	.1102	.0316	.0121	.0016
$\hat{\rho}_{\mu}$.pp5	.7348	.3157	.1170	.0313	.0123	.0020
$\hat{\rho}_{\mu}$.pp6	.7601	.3366	.1221	.0298	.0111	.0022
$\hat{\rho}_{\mu}$.pp7	.7712	.3542	.1269	.0277	.0107	.0024
$\hat{\rho}_{\mu}$.pp8	.7894	.3699	.1322	.0263	.0095	.0026
$\hat{\rho}_{\mu}$.pp9	.8001	.3839	.1368	.0237	.0084	.0023
$\hat{\rho}_{\mu}$.pp10	.8102	.3976	.1396	.0210	.0061	.0023
$\hat{\rho}_{\tau}$	.9669	.5972	.1828	.0062	.0001	.0001
$\hat{\rho}_{\tau}$.pp1	.8380	.3701	.1309	.0168	.0014	.0001
$\hat{\rho}_{\tau}$.pp2	.9118	.4391	.1397	.0216	.0029	.0002
$\hat{\rho}_{\tau}$.pp3	.9104	.4632	.1507	.0232	.0038	.0002
$\hat{\rho}_{\tau}$.pp4	.9345	.5015	.1624	.0220	.0037	.0003
$\hat{\rho}_{\tau}$.pp5	.9417	.5306	.1752	.0190	.0030	.0003
$\hat{\rho}_{\tau}$.pp6	.9529	.5591	.1836	.0160	.0021	.0003
$\hat{\rho}_{\tau}$.pp7	.9584	.5814	.1884	.0134	.0011	.0001
$\hat{\rho}_{\tau}$.pp8	.9630	.5991	.1934	.0105	.0007	.0001
$\hat{\rho}_{\tau}$.pp9	.9674	.6161	.1965	.0081	.0006	.0001
$\hat{\rho}_{\tau}$.pp10	.9717	.6317	.1986	.0061	.0005	.0001

Table 30

Proportions of Rejections, 5% Lower Tail Test

Parzen's Lag Window, T = 100

AR:	-.8	-.5	-.2	.2	.5	.8
$\hat{\tau}_\mu$	.7981	.3627	.1144	.0288	.0349	.0941
$\hat{\tau}_\mu$.pp1	.7089	.2910	.1005	.0292	.0302	.0755
$\hat{\tau}_\mu$.pp2	.6193	.2263	.0889	.0325	.0276	.0558
$\hat{\tau}_\mu$.pp3	.6266	.2232	.0872	.0354	.0251	.0468
$\hat{\tau}_\mu$.pp4	.6501	.2336	.0893	.0376	.0255	.0410
$\hat{\tau}_\mu$.pp5	.6724	.2484	.0924	.0381	.0254	.0383
$\hat{\tau}_\mu$.pp6	.6885	.2620	.0959	.0374	.0260	.0351
$\hat{\tau}_\mu$.pp7	.7042	.2755	.0984	.0375	.0267	.0341
$\hat{\tau}_\mu$.pp8	.7191	.2867	.1012	.0370	.0269	.0324
$\hat{\tau}_\mu$.pp9	.7343	.2995	.1044	.0366	.0265	.0310
$\hat{\tau}_\mu$.pp10	.7438	.3107	.1070	.0365	.0263	.0302
$\hat{\tau}_\tau$	.9580	.5651	.1727	.0134	.0094	.0358
$\hat{\tau}_\tau$.pp1	.9191	.4799	.1524	.0152	.0079	.0251
$\hat{\tau}_\tau$.pp2	.8783	.4062	.1345	.0204	.0082	.0185
$\hat{\tau}_\tau$.pp3	.8859	.4072	.1346	.0242	.0088	.0140
$\hat{\tau}_\tau$.pp4	.9009	.4289	.1394	.0255	.0096	.0116
$\hat{\tau}_\tau$.pp5	.9135	.4562	.1466	.0261	.0101	.0102
$\hat{\tau}_\tau$.pp6	.9259	.4793	.1538	.0251	.0103	.0099
$\hat{\tau}_\tau$.pp7	.9340	.5014	.1618	.0242	.0103	.0098
$\hat{\tau}_\tau$.pp8	.9410	.5190	.1667	.0232	.0100	.0099
$\hat{\tau}_\tau$.pp9	.9483	.5363	.1717	.0215	.0093	.0095
$\hat{\tau}_\tau$.pp10	.9530	.5539	.1773	.0197	.0087	.0096

Table 31

Proportions of Rejections, 5% Lower Tail Test

Parzen's Lag Window, T = 100

AR:	-.8	-.5	-.2	.2	.5	.8
$\hat{\rho}_\mu$	.8444	.4287	.1386	.0122	.0006	.0000
$\hat{\rho}_\mu$.pp1	.7368	.3317	.1185	.0177	.0015	.0000
$\hat{\rho}_\mu$.pp2	.6045	.2327	.0974	.0254	.0040	.0001
$\hat{\rho}_\mu$.pp3	.6150	.2247	.0931	.0293	.0072	.0002
$\hat{\rho}_\mu$.pp4	.6528	.2405	.0955	.0332	.0110	.0007
$\hat{\rho}_\mu$.pp5	.6803	.2600	.1005	.0343	.0135	.0013
$\hat{\rho}_\mu$.pp6	.7010	.2809	.1056	.0352	.0145	.0017
$\hat{\rho}_\mu$.pp7	.7214	.2964	.1097	.0355	.0153	.0022
$\hat{\rho}_\mu$.pp8	.7357	.3100	.1157	.0347	.0160	.0024
$\hat{\rho}_\mu$.pp9	.7520	.3265	.1208	.0333	.0158	.0030
$\hat{\rho}_\mu$.pp10	.7656	.3449	.1244	.0318	.0150	.0034
$\hat{\rho}_\tau$	.9669	.5972	.1828	.0062	.0001	.0001
$\hat{\rho}_\tau$.pp1	.9203	.4850	.1534	.0109	.0005	.0001
$\hat{\rho}_\tau$.pp2	.8526	.3785	.1314	.0183	.0017	.0002
$\hat{\rho}_\tau$.pp3	.8682	.3822	.1312	.0240	.0033	.0002
$\hat{\rho}_\tau$.pp4	.8910	.4167	.1379	.0276	.0044	.0002
$\hat{\rho}_\tau$.pp5	.9109	.4515	.1462	.0273	.0058	.0003
$\hat{\rho}_\tau$.pp6	.9256	.4800	.1564	.0257	.0061	.0003
$\hat{\rho}_\tau$.pp7	.9360	.5080	.1664	.0245	.0061	.0004
$\hat{\rho}_\tau$.pp8	.9439	.5321	.1751	.0227	.0051	.0004
$\hat{\rho}_\tau$.pp9	.9514	.5548	.1833	.0197	.0042	.0004
$\hat{\rho}_\tau$.pp10	.9573	.5741	.1882	.0160	.0030	.0004

Table 31

Proportions of Rejections, 5% Lower Tail Test

Parzen's Lag Window, T = 100

AR:	-.8	-.5	-.2	.2	.5	.8
$\hat{\rho}_\mu$	.8444	.4287	.1386	.0122	.0006	.0000
$\hat{\rho}_\mu$.pp1	.7368	.3317	.1185	.0177	.0015	.0000
$\hat{\rho}_\mu$.pp2	.6045	.2327	.0974	.0254	.0040	.0001
$\hat{\rho}_\mu$.pp3	.6150	.2247	.0931	.0293	.0072	.0002
$\hat{\rho}_\mu$.pp4	.6528	.2405	.0955	.0332	.0110	.0007
$\hat{\rho}_\mu$.pp5	.6803	.2600	.1005	.0343	.0135	.0013
$\hat{\rho}_\mu$.pp6	.7010	.2809	.1056	.0352	.0145	.0017
$\hat{\rho}_\mu$.pp7	.7214	.2964	.1097	.0355	.0153	.0022
$\hat{\rho}_\mu$.pp8	.7357	.3100	.1157	.0347	.0160	.0024
$\hat{\rho}_\mu$.pp9	.7520	.3265	.1208	.0333	.0158	.0030
$\hat{\rho}_\mu$.pp10	.7656	.3449	.1244	.0318	.0150	.0034
$\hat{\rho}_\tau$	.9669	.5972	.1828	.0062	.0001	.0001
$\hat{\rho}_\tau$.pp1	.9203	.4850	.1534	.0109	.0005	.0001
$\hat{\rho}_\tau$.pp2	.8526	.3785	.1314	.0183	.0017	.0002
$\hat{\rho}_\tau$.pp3	.8682	.3822	.1312	.0240	.0033	.0002
$\hat{\rho}_\tau$.pp4	.8910	.4167	.1379	.0276	.0044	.0002
$\hat{\rho}_\tau$.pp5	.9109	.4515	.1462	.0273	.0058	.0003
$\hat{\rho}_\tau$.pp6	.9256	.4800	.1564	.0257	.0061	.0003
$\hat{\rho}_\tau$.pp7	.9360	.5080	.1664	.0245	.0061	.0004
$\hat{\rho}_\tau$.pp8	.9439	.5321	.1751	.0227	.0051	.0004
$\hat{\rho}_\tau$.pp9	.9514	.5548	.1833	.0197	.0042	.0004
$\hat{\rho}_\tau$.pp10	.9573	.5741	.1882	.0160	.0030	.0004

Table 32

Proportions of Rejections, 5% Lower Tail Test

Bohman's Lag Window, T = 100

AR:	-.8	-.5	-.2	.2	.5	.8
$\hat{\tau}_\mu$	.7981	.3627	.1144	.0288	.0349	.0941
$\hat{\tau}_\mu$.pp1	.6798	.2713	.0968	.0298	.0293	.0711
$\hat{\tau}_\mu$.pp2	.6140	.2216	.0877	.0336	.0270	.0532
$\hat{\tau}_\mu$.pp3	.6376	.2267	.0878	.0365	.0251	.0449
$\hat{\tau}_\mu$.pp4	.6585	.2397	.0905	.0381	.0256	.0398
$\hat{\tau}_\mu$.pp5	.6795	.2547	.0939	.0381	.0259	.0362
$\hat{\tau}_\mu$.pp6	.6979	.2691	.0972	.0375	.0265	.0342
$\hat{\tau}_\mu$.pp7	.7151	.2836	.1003	.0376	.0268	.0330
$\hat{\tau}_\mu$.pp8	.7310	.2964	.1037	.0366	.0266	.0312
$\hat{\tau}_\mu$.pp9	.7424	.3089	.1066	.0368	.0264	.0304
$\hat{\tau}_\mu$.pp10	.7543	.3209	.1092	.0359	.0262	.0295
$\hat{\tau}_\tau$	.9580	.5651	.1727	.0134	.0094	.0358
$\hat{\tau}_\tau$.pp1	.9063	.4581	.1476	.0158	.0077	.0238
$\hat{\tau}_\tau$.pp2	.8768	.4009	.1328	.0218	.0081	.0171
$\hat{\tau}_\tau$.pp3	.8918	.4158	.1357	.0245	.0089	.0129
$\hat{\tau}_\tau$.pp4	.9061	.4412	.1429	.0260	.0099	.0112
$\hat{\tau}_\tau$.pp5	.9199	.4671	.1506	.0260	.0103	.0100
$\hat{\tau}_\tau$.pp6	.9309	.4917	.1592	.0249	.0108	.0100
$\hat{\tau}_\tau$.pp7	.9387	.5140	.1657	.0237	.0100	.0090
$\hat{\tau}_\tau$.pp8	.9471	.5323	.1708	.0224	.0094	.0096
$\hat{\tau}_\tau$.pp9	.9524	.5514	.1761	.0203	.0088	.0096
$\hat{\tau}_\tau$.pp10	.9577	.5714	.1818	.0179	.0083	.0096

Table 33

Proportions of Rejections, 5% Lower Tail Test

Bohman's Lag Window, T = 100

AR:	-.8	-.5	-.2	.2	.5	.8
$\hat{\rho}_\mu$	.8444	.4287	.1386	.0122	.0006	.0000
$\hat{\rho}_\mu$.pp1	.6998	.3034	.1127	.0188	.0017	.0000
$\hat{\rho}_\mu$.pp2	.5974	.2227	.0953	.0268	.0049	.0001
$\hat{\rho}_\mu$.pp3	.6330	.2296	.0938	.0310	.0082	.0004
$\hat{\rho}_\mu$.pp4	.6626	.2482	.0976	.0339	.0128	.0012
$\hat{\rho}_\mu$.pp5	.6887	.2703	.1026	.0346	.0140	.0017
$\hat{\rho}_\mu$.pp6	.7134	.2898	.1080	.0353	.0151	.0022
$\hat{\rho}_\mu$.pp7	.7320	.3067	.1143	.0348	.0160	.0024
$\hat{\rho}_\mu$.pp8	.7474	.3221	.1197	.0342	.0159	.0029
$\hat{\rho}_\mu$.pp9	.7629	.3415	.1240	.0321	.0158	.0034
$\hat{\rho}_\mu$.pp10	.7766	.3567	.1280	.0307	.0140	.0036
$\hat{\rho}_\tau$	.9669	.5972	.1828	.0062	.0001	.0001
$\hat{\rho}_\tau$.pp1	.8990	.4542	.1463	.0118	.0010	.0001
$\hat{\rho}_\tau$.pp2	.8497	.3701	.1297	.0200	.0020	.0002
$\hat{\rho}_\tau$.pp3	.8781	.3936	.1321	.0257	.0039	.0002
$\hat{\rho}_\tau$.pp4	.8986	.4312	.1413	.0280	.0051	.0002
$\hat{\rho}_\tau$.pp5	.9181	.4651	.1522	.0263	.0062	.0003
$\hat{\rho}_\tau$.pp6	.9322	.4961	.1615	.0252	.0064	.0003
$\hat{\rho}_\tau$.pp7	.9413	.5248	.1719	.0233	.0055	.0004
$\hat{\rho}_\tau$.pp8	.9496	.5487	.1815	.0200	.0044	.0005
$\hat{\rho}_\tau$.pp9	.9568	.5710	.1880	.0166	.0033	.0004
$\hat{\rho}_\tau$.pp10	.9603	.5893	.1927	.0141	.0025	.0003

Table 34

Proportions of Rejections, 5% Lower Tail Test

T = 100

MA:	-.8	-.5	-.2	.2	.5	.8
$\hat{\tau}_\mu$	.9965	.5714	.1324	.0299	.0265	.0270
$\hat{\tau}_\mu$.ppl	.9948	.4571	.1050	.0361	.0284	.0273
$\hat{\tau}_\mu$.ppt	.8109	.1598	.0568	.0498	.0511	.0515
$\hat{\tau}_\tau$	1.0000	.7987	.2009	.0157	.0090	.0081
$\hat{\tau}_\tau$.ppl	1.0000	.7047	.1650	.0252	.0146	.0128
$\hat{\tau}_\tau$.ppt	.9186	.2510	.0655	.0534	.0561	.0570
$\hat{\rho}_\mu$	.9985	.6408	.1604	.0163	.0046	.0026
$\hat{\rho}_\mu$.ppl	.9899	.4938	.1193	.0303	.0194	.0169
$\hat{\rho}_\mu$.ppt	.1129	.0413	.0426	.0558	.0593	.0602
$\hat{\rho}_\tau$	1.0000	.8290	.2127	.0092	.0022	.0013
$\hat{\rho}_\tau$.ppl	1.0000	.6988	.1617	.0238	.0114	.0088
$\hat{\rho}_\tau$.ppt	.0689	.0275	.0327	.0614	.0723	.0754

Table 35

Proportions of Rejections, 5% Lower Tail Test

T = 100

AR:	-.8	-.5	-.2	.2	.5	.8
$\hat{\tau}_\mu$	.7981	.3627	.1144	.0288	.0349	.0941
$\hat{\tau}_\mu$.ppl	.6086	.2243	.0891	.0317	.0280	.0608
$\hat{\tau}_\mu$.ppt	.3759	.1053	.0561	.0521	.0659	.1171
$\hat{\tau}_\tau$	.9580	.5651	.1727	.0134	.0094	.0358
$\hat{\tau}_\tau$.ppl	.8705	.4007	.1345	.0195	.0079	.0200
$\hat{\tau}_\tau$.ppt	.5532	.1670	.0660	.0580	.0936	.2306
$\hat{\rho}_\mu$	.8444	.4287	.1386	.0122	.0006	.0000
$\hat{\rho}_\mu$.ppl	.5872	.2309	.0985	.0233	.0031	.0000
$\hat{\rho}_\mu$.ppt	.1427	.0527	.0451	.0580	.0752	.1395
$\hat{\rho}_\tau$	.9669	.5972	.1828	.0062	.0001	.0001
$\hat{\rho}_\tau$.ppl	.8380	.3701	.1309	.0168	.0014	.0001
$\hat{\rho}_\tau$.ppt	.1608	.0477	.0400	.0697	.1162	.2755

Table 36

Proportions of Rejections, 5% Lower Tail Test

MA:	-.8				-.5			
T:	100	200	500	1000	100	200	500	1000
$\hat{\tau}_{\mu}$	.9965	.9976	.9992	.9993	.5714	.5976	.6216	.6209
$\hat{\tau}_{\mu}$.pp1	.9948	.9880	.9861	.9875	.4571	.4385	.4153	.3986
$\hat{\tau}_{\mu}$.ppt	.8109	.7830	.6982	.5863	.1598	.1267	.0887	.0726
$\hat{\tau}_{\tau}$	1.0000	1.0000	1.0000	1.0000	.7987	.8356	.8492	.8572
$\hat{\tau}_{\tau}$.pp1	1.0000	.9999	.9999	.9998	.7047	.6893	.6405	.6319
$\hat{\tau}_{\tau}$.ppt	.9186	.9341	.9048	.8379	.2510	.2010	.1264	.0932
$\hat{\rho}_{\mu}$	.9985	.9991	.9998	.9996	.6408	.6646	.6834	.6849
$\hat{\rho}_{\mu}$.pp1	.9899	.9922	.9912	.9920	.4938	.4829	.4660	.4559
$\hat{\rho}_{\mu}$.ppt	.1129	.1459	.1525	.1421	.0413	.0456	.0489	.0524
$\hat{\rho}_{\tau}$	1.0000	1.0000	1.0000	1.0000	.8290	.8449	.8713	.8760
$\hat{\rho}_{\tau}$.pp1	1.0000	.9999	1.0000	.9998	.6988	.6736	.6618	.6511
$\hat{\rho}_{\tau}$.ppt	.0689	.1074	.1551	.1604	.0275	.0345	.0433	.0468

Table 37

Proportions of Rejections, 5% Lower Tail Test

MA:	.5				.8			
T:	100	200	500	1000	100	200	500	1000
$\hat{\tau}_\mu$	.0265	.0240	.0251	.0265	.0270	.0248	.0265	.0263
$\hat{\tau}_\mu$.ppl	.0284	.0296	.0305	.0319	.0273	.0285	.0298	.0308
$\hat{\tau}_\mu$.ppt	.0511	.0503	.0495	.0515	.0515	.0507	.0492	.0514
$\hat{\tau}_\tau$	.0090	.0089	.0086	.0076	.0081	.0076	.0079	.0070
$\hat{\tau}_\tau$.ppl	.0146	.0192	.0199	.0197	.0128	.0161	.0178	.0177
$\hat{\tau}_\tau$.ppt	.0561	.0578	.0536	.0512	.0570	.0580	.0537	.0508
$\hat{\rho}_\mu$	.0046	.0044	.0041	.0043	.0026	.0025	.0025	.0026
$\hat{\rho}_\mu$.ppl	.0194	.0192	.0188	.0199	.0169	.0173	.0163	.0178
$\hat{\rho}_\mu$.ppt	.0593	.0570	.0492	.0557	.0602	.0581	.0495	.0558
$\hat{\rho}_\tau$	.0022	.0019	.0014	.0017	.0013	.0004	.0009	.0009
$\hat{\rho}_\tau$.ppl	.0114	.0111	.0148	.0143	.0088	.0091	.0118	.0117
$\hat{\rho}_\tau$.ppt	.0723	.0561	.0525	.0490	.0754	.0569	.0526	.0492

Table 38

Proportions of Rejections, 5% Lower Tail Test

AR:	-.8				-.5			
T:	100	200	500	1000	100	200	500	1000
$\hat{\tau}_\mu$	.7981	.8165	.8356	.8343	.3627	.3764	.3833	.3809
$\hat{\tau}_\mu$.ppl	.6086	.5285	.4045	.3234	.2243	.1951	.1609	.1454
$\hat{\tau}_\mu$.ppt	.3759	.3057	.2069	.1546	.1053	.0849	.0661	.0609
$\hat{\tau}_\tau$	.9580	.9728	.9786	.9808	.5651	.5898	.5880	.6000
$\hat{\tau}_\tau$.ppl	.8705	.8136	.6741	.5683	.4007	.3416	.2509	.2198
$\hat{\tau}_\tau$.ppt	.5532	.4725	.3381	.2437	.1670	.1247	.0811	.0672
$\hat{\rho}_\mu$	.8444	.8607	.8754	.8767	.4287	.4432	.4461	.4463
$\hat{\rho}_\mu$.ppl	.5872	.5106	.3992	.3286	.2309	.2058	.1784	.1654
$\hat{\rho}_\mu$.ppt	.1427	.1377	.1086	.0937	.0527	.0521	.0506	.0542
$\hat{\rho}_\tau$	.9669	.9773	.9853	.9854	.5972	.6011	.6255	.6338
$\hat{\rho}_\tau$.ppl	.8380	.7574	.6341	.5322	.3701	.3058	.2516	.2190
$\hat{\rho}_\tau$.ppt	.1608	.1562	.1409	.1152	.0477	.0472	.0497	.0483

Table 39

Proportions of Rejections, 5% Lower Tail Test

AR:	.5				.8			
T:	100	200	500	1000	100	200	500	1000
$\hat{\tau}_\mu$	.0349	.0342	.0351	.0355	.0941	.0898	.0908	.0919
$\hat{\tau}_\mu$.ppl	.0280	.0255	.0265	.0271	.0608	.0576	.0567	.0562
$\hat{\tau}_\mu$.ppt	.0659	.0575	.0519	.0533	.1171	.0913	.0640	.0595
$\hat{\tau}_\tau$	.0094	.0085	.0096	.0085	.0358	.0319	.0327	.0310
$\hat{\tau}_\tau$.ppl	.0079	.0078	.0080	.0065	.0200	.0185	.0172	.0151
$\hat{\tau}_\tau$.ppt	.0936	.0777	.0586	.0533	.2306	.1485	.0827	.0662
$\hat{\rho}_\mu$	.0006	.0003	.0004	.0002	.0000	.0000	.0000	.0000
$\hat{\rho}_\mu$.ppl	.0031	.0027	.0028	.0026	.0000	.0001	.0001	.0000
$\hat{\rho}_\mu$.ppt	.0752	.0687	.0532	.0568	.1395	.1012	.0702	.0643
$\hat{\rho}_\tau$	.0001	.0001	.0000	.0000	.0001	.0000	.0000	.0000
$\hat{\rho}_\tau$.ppl	.0014	.0007	.0006	.0011	.0001	.0000	.0000	.0000
$\hat{\rho}_\tau$.ppt	.1162	.0776	.0596	.0540	.2755	.1519	.0871	.0674

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