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A CONDITIONAL VARIANCE APPROACH TO THE TIME-SERIES BEHAVIOR OF INTEREST RATES

by

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ABSTRACT

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By

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The purpose of this dissertation is to examine the time-series behavior of interest rates. Traditional macroeconomics has concentrated on the level of interest rates with little attention being paid to higher order moments. But given the erratic behavior of interest rates over the last decade, such an approach seems dubious. In this study the recently developed autoregressive conditional heteroskedasticity (ARCH) and generalized (GARCH) processes are employed to examine interest rates. The examination begins with an application of the Phillips-Perron tests to determine whether interest rates possess a unit root. The finding is that interest rates possess one unit root and are best described as an ARIMA(0,1,q). Having examined the level of interest rates, the conditional variance is then modeled. Here application of the GARCH processes shows interest rates to possess time dependent heteroskedasticity as well as excess kurtosis. Given these results, the GARCH model with t-distributed errors is employed to explain the kurtosis. While the use of the conditional t is successful in explaining some of the kurtosis, the conditional t can not fully account for its presence in interest rates.

Having modeled the conditional variance of interest rates, the third

and fourth chapters of this dissertation examine what factors influence the conditional variance, and how the response of the conditional variance to these factors changes over alternate monetary operating procedures. This is accomplished by introducing unanticipated money and the date of FOMC meetings into the conditional variance equation. The finding is that unanticipated money has a significant impact upon the conditional variance only during the period in which the Federal Reserve was targeting a monetary aggregate. Changes in the range of the federal funds rate were also found to influence the conditional variance of interest rates. Here however the impact of funds rate changes was found to die out as the as the length to maturity increased.

The final chapter concludes the study and raises areas for further research.

To Karl S. Willson, Ph.D.
June 11, 1910 - February 4, 1986

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CHAPTER ONE

1.1 Introduction

In recent years the topic of interest rate volatility and its role in economic activity has taken an increasingly important place in discussions of macroeconomics. This discussion has been particularly acute since the implementation of the reserve aggregate targeting scheme in October 1979.[1] For example, Slovin and Shushka(1983) and Garner(1986), among others,[2] have examined the impact of interest rate volatility on the money demand function while Evans(1984) and Tatom(1984, 1985) have studied its impact on output. The topic of interest rate volatility has also appeared in the literature on the term structure of interest rates. A number of studies have proposed a time-varying term premium which exhibits a positive correlation with the level of interest rate volatility. As such studies by Jones and Roley(1983) and Mankiw(1986), among others,[3] have estimated a time-varying term premium using various measures of interest rate volatility.

In these and other studies, an important question is how to measure interest rate volatility. To date the literature seems to offer no decisive answer; thus empirically, researchers tend to employ a variety of techniques. Some studies have measure volatility using the sample variance. But while such an estimator may be unbiased, such a technique implicitly constrains the level of interest rate volatility to be constant for the chosen time interval. But many macroeconomic time series, such as output, inflation, exchange rates, and interest rates, have been found to exhibit both volatile and tranquil periods. Thus constant volatility appears to be an overly restrictive assumption for many macroeconomic time-series.

Another popular method for measuring volatility involves the use of a moving standard deviation or variance. Such a method is popular because it overcomes the homoskedasticity constraint implicit in the use of the sample variance. Instead by employing a moving standard deviation or variance, interest rate volatility can fluctuate on a period-by-period basis. Unfortunately such a technique has many difficulties. First, as noted by Engle(1982, 1983), moving standard deviations or variances involve misspecified equations for the mean, thus yielding biased estimates of the variance. To further complicate the matter, the moving standard deviation or variance process results in estimators of volatility which are sensitive to the ad hoc specification of the process. An example of this can be found in Tatom(1984). Here specification of interest rate volatility as a four quarter moving standard deviation yields substantially different results than when interest rate volatility is estimated by a twenty quarter moving standard deviation.[4] Diebold and Nerlove(1986) note that a further weakness of this method is its failure to efficiently use information. For example, the moving standard deviation approach is predicated on the assumption that volatility changes over time, yet such a process ignores available information. Diebold and Nerlove argue that volatility is better measured using the conditional second moment as rational decision-making agents will employ all relevant and available information so as to eliminate any uncertainty which could be explained by already existing information.

An alternative method for measuring interest rate volatility involves the Autoregressive Conditional Heteroskedasticity (ARCH) model developed by Engle(1982, 1983) and later generalized (GARCH) by Bollerslev(1986). The ARCH process explicitly differentiates between the conditional and unconditional variances; the process attempts to measure the conditional variance using innovations from the conditional mean equation. Specifically, given the equation for the conditional mean, the simplest form of the ARCH(q) error process is:

$$\epsilon_t | \Omega_{t-1} \sim N\ (o,\ h_t)$$

$$h_t = \alpha_o + \sum_{j=1}^{q} \alpha_j\ \epsilon^2_{t-j} \tag{1.1}$$

where h_t is the conditional variance, Ω_{t-1} is the set of all relevant information at time t-1, $\alpha_j > 0$ for all j, and ϵ_t is a serially uncorrelated disturbance of the mean equation and is assumed to be conditionally normally distributed. Thus while the errors may be uncorrelated, they are not independent in that they may be related through their conditional second moments. Rather the conditional variance is a linear function of past squared innovations, and as such the model may exhibit serial dependence in it's squared innovations. As a result of this feature, large values of the conditional variance tend to be followed by other large values, and small values tend to be followed by small values, thus yielding periods of volatile behavior as well as periods of tranquility. Another appealing feature of the ARCH process is that it is consistent with the leptokurtic, or fat-tailed, unconditional distributions found in many financial time series.[5]

A subsequent generalization of the ARCH process, the GARCH model, was introduced by Bollerslev(1986). In equation (1.1),

$\alpha_j > 0$ and $\sum_{j=1}^{q} \alpha_j < 1$ so as to insure that the conditional variance is both positive and stationary. The GARCH parameterization permits a less stringent lag structure than the ARCH(q) process and in doing so eliminates the difficulties associated with the non-negativity constraint imposed in (1.1). Given an appropriate specification of the mean, the GARCH(p,q) model can be written as :

$$\text{(1.2)} \qquad \begin{aligned} &\epsilon_t | \Omega_{t-1} \sim N\ (0, h_t) \\ &h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j\ \epsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_j h_{t-j}. \end{aligned}$$

Thus in the GARCH model the conditional variance is a nonlinear function of the past squared errors. If p = 0 in equation (1.1) then the GARCH(p,q) model simply reduces to the ARCH(q) model in (1.1).

Over the last five years ARCH and GARCH models have been successfully utilized to explain the time-series behavior of a wide variety of macroeconomic data. For example, Engle(1982, 1983) employed the ARCH process to model the volatility of inflation in both the United States and the United Kingdom. ARCH and GARCH models have also been applied to the time-series behavior of exchange rates in studies by Diebold and Nerlove (1989), Bollerslev (1986), Milhoj (1987), Baillie and Bollerslev (1989), and Engle and Bollerslev (1986), among others. Engle, Lilien, and Robins (1987) and Engle, Bollerslev, and Wooldridge (1988) have used the ARCH-in-the-mean or ARCH-M and GARCH-M processes as they are known respectively, to examine the question of time-varying risk premia in excess holding period yields on financial assets. The volatility of stock prices returns has been examined with a GARCH model by Baillie and

DeGennaro(1988a, 1988b). Finally Weiss(1984) employed ARMA models with ARCH errors to explain the behavior of sixteen different macroeconomic time series.

This study proceeds as follows. The next Chapter develops a univariate GARCH model of government interest rates across the maturity spectrum using daily data. First, interest rates are examined to determine whether or not they are stationary. If interest rates are weakly stationary then they possess a time-invariant mean and variance. However if interest rates are nonstationary then such time-series properties will not exist and standard statistical testing will be biased and invalid. The traditional remedy for such a problem is to "difference" the variable an appropriate number of times.[6] Thus recent tests developed by Perron(1986), Phillips(1987), and Phillips and Perron(1988) are utilized to determine whether daily interest rates are "difference stationary."[7] Given the results of the unit root tests, ARIMA models of the interest rates are developed. Furthermore, diagnostic tests are applied to determine whether or not daily interest rates exhibit conditional heteroskedasticity. The finding is that the daily rates do exhibit conditional heteroskedasticity, as well as serial correlation and excess kurtosis. The chapter concludes with an application of the GARCH (p,q) model to the time-series behavior of interest rates.

Chapter Three explores the idea of how money supply announcements influence financial markets. Over the last decade a considerable amount of research has been devoted to the question of how news, particularly in the form of unanticipated money, influences the level of interest rates. Few studies however have addressed how money supply announcements affect the volatility of interest rates; those that do usually infer the impact on volatility from the change in the level of interest rates. But changes in volatility can occur from a movement in the mean or from movement in the error term. In this Chapter money supply announcements are introduced into the GARCH(p,q) model of interest rates developed in the previous chapter. Using GARCH estimates of the conditional variance as a measure of volatility, it is possible to systematically assess how money supply announcements impact not only the level of interest rates, but also their volatility. The introduction of unanticipated money into the GARCH process is done over three periods according to the procedure for monetary policy being employed by the Federal Reserve. The finding is that unanticipated money had it's most significant impact on both the level and the conditional variance of interest rates during the period when the Federal Reserve was thought to be targeting a monetary aggregate.

Chapter Four begins with an examination of interest rate volatility over the period 1974 to 1988. In October 1979 the Federal Reserve, in attempting to gain greater control over the growth of monetary aggregates, switched from an operating procedure which smoothed interest rates to one which targeted the level of nonborrowed reserves. Critics argued that such a policy would result in increasing levels of interest rate volatility and reduced levels of economic welfare.[8] It is now generally accepted that interest rates did indeed become more volatile after the October 1979 change in operating procedures. What remains a question is to what degree did interest rate volatility increase? And was the behavior of volatility homogeneous over the nonborrowed reserve procedure or did interest rate volatility vary during this period? The answers to these questions are important if we are to assess the effectiveness of the nonborrowed reserve experiment. In this Chapter the GARCH(p,q) models, both with and without unanticipated money, are further examined to see what evidence they provide regarding the choice of a monetary policy operating regime and the historical behavior of interest rate volatility. The results reveal that interest rate volatility did increase with the switch to a reserve-oriented procedure in October 1979, but that it was not homogeneous over the period October 1979 to October 1982. Rather following an initial surge, interest rate volatility declined until its abandonment in October 1982. Such a result is consistent with the hypothesis that learning on the part of economic agents about the nonborrowed reserve procedure was an important factor in the behavior of interest rates.[9]

Finally Chapter Five concludes with a summary of our results as well as some suggestions for future research.

ENDNOTES

1. The Federal Reserve can not independently control both interest rates and a monetary aggregate. If the Federal Reserve decides to target a monetary aggregate, traditional macroeconomics implies a greater volatility of interest rates than would exist if the Federal Reserve were targeting interest rates. For a recent discussion of the volatility of interest rates under alternative operating procedures see Tinsley, von zur Muehlen, and Fries(1982). It should be further noted that some debate has existed as to whether or not the Federal Reserve actually began targeting a monetary aggregate in October 1979. For opposing views on this topic see Poole(1982) and Spindt and Tarhan(1987).

2. Other papers examining this question include Brunner and Meltzer(1964), Baba, Starr, and Hendry(1985), Rasche(1986), and McGibany and Nourzad(1986).

3. For other papers see Shiller, Campbell, and Schoenholtz(1983), Modigliani and Shiller(1973), Fama(1976), Mishkin(1982), and Engle, Lilien, and Robins(1987).

4. This is particularly true for the literature on how the money demand function is affected by interest rate volatility. Here the results appear quite sensitive to how the moving variance is specified. This result also appears to true in other studies of macroeconomic variables and interest rate volatility.

5. For example Baillie and Bollerslev(1989) find a GARCH process with a conditional t density explains the leptokurtosis present in exchange rates. Bollerslev(1987) finds a similar result for stock prices.

6. For a discussion of the ramifications of overdifferencing a variable versus underdifferencing it see Plosser and Schwert(1978).

7. This is a phrase used by Nelson and Plosser(1982) to imply variables which must be differenced so as to acheive stationarity.

8. For example, B. Friedman(1982) and Brimmer(1983) argue that volatile interest rates destabilize capital markets, increase uncertainty, and raise required rates of return on long-term investment. Such an impairment of the market retards investment thus reducing economic welfare.

9. Papers that argue for learning behavior with regard to the new procedure include Loeys(1985), Rasche(1986), and Baxter(1989). Collectively these papers contend that the adoption of the new procedure in October 1979 increased uncertainty and thus the volatility of interest rates. As economic agents learned of the Federal Reserve's new procedure, uncertainty decreased as did the volatility of interest rates.

CHAPTER TWO

UNIT ROOTS AND GARCH EFFECTS

2.1 Introduction

An abundance of empirical evidence suggests that interest rates follow a random walk process. Previous studies by Phillips and Pippenger(1976), Mishkin(1978), Pesando(1978, 1979, 1981), and Mankiw and Miron(1986), among others, suggest that interest rates, particularly long-term rates, follow a random walk process, or at least can be approximated as a martingale sequence. Such a specification suggests that over a short interval of time, the predictable change in interest rates should be minimal. However, while equations for the conditional mean have been thoroughly examined, empirical work has either ignored the conditional variance or treated it as a constant. Given the erratic and volatile behavior of interest rates during the late 1970's and early 1980's, the assumption of a constant variance seems dubious at best. In this Chapter, the statistical distribution of government interest rates is considered. Daily data was obtained for seven government interest rates: the 3-month, 6-month, and 12-month T-bill rates, and the 3-year, 5-year, 10-year, and 20-year rates on government bonds. The data, obtained from the Federal Reserve, are quotes of bid rates collected from a survey of dealers between 3:00 and 3:30 on each day financial markets were open.[1] The various interest rates are examined over four periods: January 1, 1974 through October 4, 1979; October 10, 1979 through October 6, 1982; October 7, 1982 to January 31, 1984; and February 1, 1984 through March 16, 1988. The data was divided into various periods so as to avoid problems or biases due to the way monetary policy was implemented at the time. For example, on October 6, 1979 the Federal Reserve switched from targeting the federal funds rate to targeting the level of nonborrowed reserves; thus our first period, January 1974 to October 6, 1979, is consistent in that the thrust of monetary policy during this time was the attainment of interest rate targets. In October 1982 the Federal Reserve officially abandoned it's nonborrowed reserves target while in February 1984 a switch was made from a lagged reserve accounting system to a contemporaneous one. Each of these changes has strong implications for the conduct of monetary policy in general and the applicability of ARCH and GARCH processes to the modeling of interest rates in particular. Thus by examining daily interest rates over periods where the operating procedures are approximately consistent, the inclusion of any interest rate volatility attributed to procedural changes is minimized.

In the remainder of this chapter, specific issues relevant to the time-series behavior of daily interest rates are examined. First, new tests developed by Perron(1988), Phillips(1987), and Phillips and Perron(1988) are employed to examine the stationarity of daily interest rates. Next the results of the Phillips-Perron tests are combined with a series of diagnostic tests to allow for specification of a conditional mean equation. The results of the diagnostic tests are also of use in examining other properties of interest rates; specifically whether or not daily interest rates exhibit conditional heteroskedasticity and whether or not their distribution is leptokurtic, or fat-tailed. Given the results of these tests, GARCH models with Gaussian errors are fitted to the time-series process. The finding here is that while the GARCH model explains the presence of conditional heteroskedasticity, it fails to fully account for the degree of excess kurtosis present in daily interest rate data. Finally this Chapter concludes with an examination of a GARCH model with an alternative distribution, the conditional t, as a way of explaining the time-series behavior of interest rates.

2.2 Unit Root Testing

In the time series representation of macroeconomic variables, tests of the unit root hypothesis are important for a variety of reasons. First the presence of a unit root implies that the variable is stochastic nonstationary, or in the terms of Nelson and Plosser(1982), is "difference stationary" as opposed to "trend stationary". A trend stationary process implies that the variable exhibits stationary fluctuations around a deterministic trend while a difference stationary process implies that the variable is inherently nonstationary with no tendency to follow a deterministic trend. As such the work of Plosser and Schwert(1978), as well as Dickey, Bell, and Miller(1986), note that classical inference procedures may be invalid when a nonstationary variable is regressed on a group of explanatory variables. In recent years research by Nelson and Plosser(1982), as well as Perron(1988), has revealed that a wide variety of macroeconomic data, including interest rates[2], can be characterized as such. This result has very strong implications for economic theory. The traditional assumption in economic theory is that variables have stationary time series properties. However if a time series is nonstationary, random shocks occuring in the distant past will continue to have a significant influence upon the variable in the present period. In fact in a nonstationary series a random shock affects all future values of the variable with the same influence as in the present period because the process has an infinite memory. Thus at any point in time the value of a variable reflects the summation of all past errors as well as the accumulated effects of the initial conditions.[3] And unlike a stationary series, past and present shocks are of equal importance in the time-series behavior of a nonstationary variable.

The existence of a unit root also has strong implications for forecasting. If a series is weakly stationary then, by definition, the series will exhibit both a constant mean and variance. For a nonstationary series the mean will not be a constant and, as Dickey, Bell, and Miller (1986) note, forecasts of the series mean into the future "will either explode or behave like a polynomial..."[4]. Concurrently the variance of a nonstationary series will approach infinity as the forecast horizon increases. Thus the nonstationarity of a variable has important implications for the use of a time series model for forecasting purposes.

The traditional approach to testing for the presence of a unit root involves use of the class of test statistics developed by Fuller (1976) and Dickey and Fuller (1981). The Dickey-Fuller statistics however are predicated on the assumption that the underlying data-generating mechanism is a random walk model with no drift. Thus the critical values are valid only in the case where the variable is driftless and the error term is independent and identically distributed with mean zero and variance σ^2. However in the event that errors are autocorrelated or that they display conditional heteroskedasticity, the Dickey-Fuller statistics are biased and the augmented Dickey-Fuller test statistics are more appropriate. The problem of conditional heteroskedasticity may be of particular importance here as studies by Weiss (1984) and Engle, Lilien, and Robins (1987) have noted the existence of conditional heteroskedasticity in monthly and quarterly interest rates, respectively. One remedy to this problem is to utilize tests recently developed by Perron (1988), Phillips (1987), and Phillips and Perron (1988). Implementation of the Phillips-Perron test involves ordinary least squares(OLS) computation of the following regressions:

$$(2.2.1) \qquad r_t = \tilde{\mu} + \tilde{\beta}\ (t-n/2) + \tilde{\alpha}\, r_{t-1} + \tilde{u}$$

$$(2.2.2) \qquad r_t = \mu^* + \alpha^* r_{t-1} + u^*_t$$

$$(2.2.3) \qquad r_t = \hat{\alpha}\, r_{t-1} + \hat{u}_t$$

where r_t is the level of the interest rate in period t, $\tilde{\mu}$ and μ^* are drift parameters, n is the sample size, (t - n/2) is a determinsitic trend, and $\tilde{u}_t$, u^*_t, and $\hat{u}_t$ represent error terms which allow for the possibility of conditional heteroskedasticity. Calculation of the Phillips-Perron statistics requires consistent estimation of the sum of the error terms. To this end, error-covariance corrections of the Newey and West (1987) type are employed. The Newey and West corrections guarantee a positive semi-definite estimate of the variance necessary for calculation of the Phillips-Perron statistics.

In equation (2.2.1) the null hypotheses H_0^1: $\tilde{\mu} = 0$, $\tilde{\beta} = 0, \tilde{\alpha} = 1$; $H_0^2: \tilde{\beta} = 0, \tilde{\alpha} = 1$; and $H_0^3: \tilde{\alpha} = 1$ are tested against stationary alternatives by the test statistics $Z(\Phi_2)$, $Z(\Phi_3)$, and $Z(t_{\tilde{\alpha}})$, respectively. Under the null hypothesis the 1-percent and 5-percent critical values for $Z(\Phi_2)$, $Z(\Phi_3)$, and $Z(t_{\tilde{\alpha}})$ are 6.09 and 4.68, 8.27 and 6.25, and -3.96 and -3.41, respectively. For a discussion of the algebraic nature of the Phillips-Perron test statistics see Perron (1988).

In equation (2.2.2) the test statistics $Z(\Phi_1)$ and $Z(t_{\alpha*})$ are employed to test the null hypothesis of a unit root $H_0^4{:}\mu^* = 0$, $\alpha^* = 1$ and $H_0^5{:}\alpha^* = 1$. Here the 1-percent and 5-percent critical values are 4.59 and 6.43 for H_0^4 and -3.43 and -2.86 for H_0^5, respectively.

Finally in equation (2.2.3) the null hypothesis $H_0^6{:}\hat{\alpha} = 1$ is tested against a stationary alternative by the test statistic $Z(t_{\hat{\alpha}})$. The corresponding critical values at the 1-percent and 5-percent level in this case -2.58 and -1.95.

A complete set of Phillips-Perron tests statistics for the daily government interest rates over the four different periods is contained in Tables 2.1 through 2.4. In the most general case, equation (2.2.1) allows for the possibility of both drift and a deterministic trend, as well as testing for a unit root. Of the twenty-eight applications of equation (2.2.1), in twenty-seven of those cases the test statistic $Z(\Phi_2)$ does not allow the null hypothesis of a unit root to be rejected at either the 1-percent or the 5-percent level. As noted by Perron(1986), if the null hypothesis H_0^1 can not be rejected then a more powerful test involves the test statistic $Z(t_{\alpha*})$ from equation (2.2.2). In none of these twenty-seven cases can the unit root hypothesis be rejected. The one case where the null hypothesis of equation (2.2.1) is rejected by the $Z(\Phi_2)$ test statistic is the 20-year bond rate from February 1984 through March 1988. Here the $Z(\Phi_2)$ statistic was 4.813 where the 1-percent and 5-percent critical values are 6.09 and 4.68, respectively. So while the unit root hypothesis is marginally rejected at the 5-percent level, it can not be rejected at the 1-percent level. Given the maginal rejection of the

TABLE 2.1

Phillips-Perron Tests for Unit Roots
January 1, 1974 - October 4, 1979

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
$Z(\Phi_2)$	1.515	1.467	1.209	1.105	1.179	1.574	1.400
$Z(\Phi_3)$	2.105	1.996	1.532	1.215	1.219	1.431	1.372
$Z(t_{\tilde{\alpha}})$	-0.961	-0.362	-0.223	-0.387	-0.588	-0.736	-0.545
$Z(\Phi_1)$	0.341	0.324	0.453	0.894	1.184	1.426	1.451
$Z(t_{\alpha^*})$	-0.579	-0.475	-0.581	-0.952	-1.129	-1.015	-1.209
$Z(t_{\hat{\alpha}})$	0.427	0.522	0.614	0.794	0.907	1.245	1.104

The 1-percent and 5-percent critical values for $Z(\Phi_2)$ are 6.09 and 4.68; for $Z(\Phi_3)$ are 8.27 and 6.25; for $Z(t_{\tilde{\alpha}})$ are -3.96 and -3.41; for $Z(\Phi_1)$ are 6.43 and 4.59; for $Z(t_{\alpha^*})$ are -3.43 and -2.86; and for $Z(t_{\hat{\alpha}})$ are -2.58 and -1.95, respectively.

The truncation lag used in these tests equals 22.

Table 2.2

Phillips-perron Tests for Unit Roots

October 10, 1979 - October 6, 1982

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
$Z(\Phi_2)$	1.584	1.638	1.648	1.618	1.637	1.616	1.649
$Z(\Phi_3)$	2.353	2.447	2.470	2.399	2.403	2.342	2.363
$Z(t_{\tilde{\alpha}})$	-1.862	-1.955	-1.955	-1.815	-1.704	-1.436	-1.316
$Z(\Phi_1)$	1.826	2.181	2.397	2.416	2.440	2.421	2.473
$Z(t_{\alpha*})$	-1.898	-2.083	-2.188	-2.185	-2.185	-2.162	-2.172
$Z(t_{\hat{\alpha}})$	-0.566	-0.476	-0.384	-0.093	0.0095	0.119	0.190

The 1-percent and 5-percent critical values for $Z(\Phi_2)$ are 6.09 and 4.68; for $Z(\Phi_3)$ are 8.27 and 6.25; for $Z(t_{\tilde{\alpha}})$ are -3.96 and -3.41; for $(Z\Phi_1)$ are 6.43 and 4.59; for $Z(t_{\alpha*})$ are -3.43 and -2.86; and for $Z(t_{\hat{\alpha}})$ are -2.58 and -1.95, respectively.

The truncation lag used in these tests equals 22.

Table 2.3

Phillips - Perron Tests for Unit Roots

October 7, 1982 - January 31, 1984

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
$Z(\Phi_2)$	1.779	1.498	1.322	1.850	1.635	1.772	1.798
$Z(\Phi_3$	2.258	2.088	1.847	2.620	2.167	2.183	2.018
$Z(t_{\tilde{\alpha}})$	-2.027	-2.023	-1.906	-2.253	-2.049	-2.072	-1.975
$Z(\Phi_1)$	1.842	1.207	1.059	0.684	0.687	0.898	1.196
$Z(t_{\alpha*})$	-1.714	-1.457	-1.367	-1.051	-0.938	-1.000	-1.133
$Z(t_{\hat{\alpha}})$	0.705	0.405	0.377	0.409	0.613	0.793	0.943

The 1-percent and 5-percent critical values for $Z(\Phi_2)$ are 6.09 and 4.68; for $Z(\Phi_3)$ are 8.27 and 6.25; for $Z(t_{\tilde{\alpha}})$ are -3.96 and -3.41; for $(Z\Phi_1)$ are 6.43 and 4.59; for $Z(t_{\alpha*})$ are -3.43 and -2.86; and for $Z(t_{\hat{\alpha}})$ are -2.58 and -1.95, respectively.

For critical values see the note in Table 2.1.

The truncation lag used in these tests equals 22.

Table 2.4

Phillips - Perron Tests for Unit Roots

February 1, 1984 - March 16, 1988

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
$Z(\Phi_2)$	1.490	1.309	0.989	0.962	0.939	0.877	4.813
$Z(\Phi_3)$	1.511	1.342	1.019	0.965	0.895	0.831	5.805
$Z(t_{\tilde{\alpha}})$	-1.721	-1.631	-1.408	-1.371	-1.308	-1.241	-3.220
$Z(\Phi_1)$	-1.180	1.050	0.926	0.934	0.971	0.972	1.310
$Z(t_{\alpha*})$	-0.996	-0.995	-0.977	-0.971	-0.973	-1.001	0.019
$Z(t_{\hat{\alpha}})$	-1.353	-1.266	-1.130	-1.146	-1.179	-1.147	-1.594

The 1-percent and 5-percent critical values for $Z(\Phi_2)$ are 6.09 and 4.68; for $Z(\Phi_3)$ are 8.27 and 6.25; for $Z(t_{\tilde{\alpha}})$ are -3.96 and -3.41; for $(Z\Phi_1)$ are 6.43 and 4.59; for $Z(t_{\alpha*})$ are -3.43 and -2.86; and for $Z(t_{\hat{\alpha}})$ are -2.58 and -1.95, respectively.

For critical values see note in Table 2.1.

hypothesis H_0^1, the test statistic $Z(t_{\hat{\alpha}})$ for equation (2.2.3) was examined to further investigate the possibility of a unit root. In this case the null hypothesis of $\hat{\alpha} = 1$ can not be rejected at either the 1-percent or the 5-percent level for the 20-year bond rate. In conclusion there is strong evidence to suggest that daily interest rates over a variety of monetary policy operating regimes possess a unit root and can thus be classified as nonstationary. This also points to the need for first-differencing the series if the model is to exhibit the desirable statistical characteristics.[5] These results tend to confirm the earlier findings of Nelson and Plosser(1982) and Perron(1986), as well as broaden their scope by considering a more inclusive data set over a variety of time periods.

2.3 Diagnostic Testing

Given the results of the Phillips-Perron tests, daily interest rates were first differenced to achieve stationarity. With the error term, u_t, which is initially assumed to be normally distributed, the model is of the form:

$$\text{(2.3.1)} \qquad \Delta r_t = u_t$$
$$u_t | \Omega_{t-1} \sim N(0, \omega_0)$$

where Δ is the first-difference operator, Ω_{t-1} is the set of all pertinent information available at time t-1, and ω_0 is the conditional variance which is assumed to be normally distributed. Tables 2.5, 2.6, 2.7, and 2.8 present the results for each of the four periods, along with statistics for a variety of diagnostic tests. First, the Ljung and

Box(1978) portmanteau test statistic, Q(k), tests for serial correlation up to the k^{th} order in u_t and is asymptotically equivalent to an Lagrange Multiplier (LM) test. Here the null hypothesis is that u_t is white noise; the alternate hypothesis being that u_t follows an AR(p) or MA(p) process. The Ljung and Box test is a test of the joint hypothesis that all autocorrelation coefficients are zero and as such is chi-square distributed with k-p-q degrees of freedom where p and q correspond to the ARMA(p,q) specification of the conditional mean. In this case Q(10) yields critical values of 18.307 and 15.987 at the 5-percent and 10-percent levels, respectively. In our study the null hypothesis of no serial correlation is rejected in the majority of cases. This can be seen in Tables 2.9 through 2.12 where moving average parameters for all seven interest rates over the four different periods are estimated. The results reveal that with the exception of the 20-year rate during the 1979 to 1982 period, and the 6-month, 3-year, 5-year, 10-year, and 20-year rates during the 1982 to 1984 period, the moving average parameters are significantly different from zero and the interest rates follow a martingale sequence. For the six interest rates were the moving average terms are insignificant, the random walk process is more appropriate. Because our data involves a survey of dealers taken over a half-hour interval, the possibility of survey error exists. This may at least partially explain the autocorrelation present in the data and the significance of the moving average parameters. It is also consistent with the work of Perron(1986) and Schwert(1987) who argue that many economic time series, including interest rates, may contain moving average components.

Tables 2.5 through 2.8 also present the Ljung and Box test

Table 2.5

Diagnostic Testing

January 1, 1974 - October 4, 1979

$$\Delta r_t = \epsilon_t$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, \omega_0)$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
ω_0	.0122 (.0002)	.0071 (.0001)	.0059 (.0001)	.0041 (.0001)	.0029 (.0001)	.0016 (.00003)	.0011 (.00002)
Log L	1117.573	1513.134	1640.031	1907.953	2143.409	2559.701	2845.943
Q(10)	44.827	41.796	58.239	42.409	40.638	23.501	37.248
$Q^2(10)$	520.614	316.650	107.190	78.841	61.256	83.616	83.475
M_3	-0.422	0.294	0.352	0.270	-0.311	-0.208	0.275
M_4	12.834	7.982	7.938	6.763	7.953	7.986	8.083

Standard errors in parentheses.

Table 2.6

Diagnostic Testing for Arch Effects

October 10, 1979 - October 6, 1982

$$\Delta r_t = \epsilon_t$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, \omega_0)$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
ω_0	0.836 (.0029)	0.647 (.0023)	0.463 (.0017)	0.398 (.0015)	.0331 (.0012)	.0255 (.0010)	0.229 (.0010)
LogL	-134.157	-37.477	87.563	145.205	213.173	313.566	353.619
Q(10)	25.053	21.683	24.102	21.368	21.791	17.343	9.535
$Q^2(10)$	30.044	11.760	7.376	24.628	21.281	39.614	46.016
M_3	0.237	0.271	0.005	-0.038	-0.257	-0.191	-0.115
M_4	5.466	5.359	4.754	4.950	4.794	4.206	4.085

Standard errors in parentheses.

Table 2.7

Diagnostic Testing

October 7, 1982 - January 31, 1984

$$\Delta r_t = \epsilon_t$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, \omega_0)$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
ω_0	.0074 (.0004)	.0091 (.0004)	.0086 (.0003)	.0100 (.0003)	.0088 (.0003)	.0079 (.0003)	.0072 (.0003)
LogL	338.126	304.683	313.659	289.750	310.585	328.501	342.451
Q(10)	21.888	9.565	11.816	10.499	8.954	8.225	9.058
$Q^2(10)$	21.846	46.542	44.004	36.733	16.971	23.005	41.300
M_3	0.399	-0.797	-1.570	-1.184	-1.196	-0.722	-0.421
M_4	5.664	9.116	13.714	11.906	13.115	9.333	6.756

Standard errors in parentheses.

Table 2.8

Diagnostic Testing for ARCH Effects

February 1, 1984 - March 16, 1988

$$\Delta r_t = \epsilon_t$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, \omega_0)$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
ω_0	.0071 (.0002)	.0067 (.0001)	.0054 (.0001)	.0077 (.0002)	.0078 (.0002)	.0080 (.0002)	.0070 (.0003)
LogL	1083.560	1116.805	1221.444	1038.260	1027.797	1019.147	772.036
Q(10)	16.894	29.202	31.007	24.547	28.028	17.387	17.168
$Q^2(10)$	283.042	121.303	54.741	32.515	42.136	45.275	34.020
M_3	-0.390	-0.995	-1.075	-0.737	-0.617	-0.565	-0.121
M_4	9.276	13.124	14.195	10.114	9.211	8.329	3.942

Standard errors in parentheses.

Table 2.9

Estimating MA(q) Parameters

January 1, 1974 - October 4, 1979

$$\Delta r_t = u_t$$
$$u_t = \epsilon_t + \sum_{i=1}^{k} \theta_i \epsilon_{t-i}$$
$$\epsilon_t | \Omega_{t-1} \sim N(0, \omega_0)$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
θ_1	.1295	.1338	.1554	.1116	.1194	.0728	.1065
	(.0179)	(.0186)	(.0209)	(.0195)	(.0211)	(.0189)	(.0166)
θ_2	-0.136	.0588	.0663	.0546	.0264	---	---
	(.0146)	(.0176)	(.0223)	(.0207)	(.0220)		
θ_3	.0010	---	---	.0692	.0686	---	---
	(.0143)			(.0223)	(.0218)		
θ_4	.0066	---	---	---	---	---	---
	(.0177)						
θ_5	.0971	---	---	---	---	---	---
	(.0168)						
ω_0	.0120	.0069	.0058	.0040	.0029	.0016	.0011
	(.0002)	(.0001)	(.0001)	(.0001)	(.0001)	(.0000)	(.0000)
LogL	1134.704	1528.013	1659.614	1922.816	2157.968	2563.499	2855.017
Q(10)	13.694	8.770	11.961	8.741	10.424	14.937	16.327
$Q^2(10)$	549.449	317.155	108.677	81.906	47.161	91.124	116.349
M_3	-0.392	0.271	0.299	0.183	-0.418	-0.227	0.254
M_4	11.863	7.884	7.249	6.715	8.552	8.223	8.203

Standard errors in parentheses.

Table 2.10

Estimating MA(q) Parameters

October 10, 1979 - October 6, 1982

$$\Delta r_t = u_t$$
$$u_t = \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$$
$$\epsilon_t | \Omega_{t-1} \sim N(0, \omega_0)$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
ω_0	.0821 (.0028)	.0642 (.0023)	.0460 (.0017)	.0391 (.0015)	.0323 (.0012)	.0253 (.0011)	.0227 (.0010)
θ_1	.1273 (.0351)	.0829 (.0354)	.0835 (.0372)	.1031 (.0336)	.1008 (.0325)	.0667 (.0327)	.0625 (.0377)
LogL	-127.787	-34.666	90.340	149.622	223.439	315.344	352.179
Q(10)	9.606	12.550	14.624	9.790	12.138	13.416	6.462
$Q^2(10)$	29.115	12.025	8.019	24.722	21.839	38.820	44.169
M_3	0.229	0.274	0.015	-0.006	-0.152	-0.174	-0.098
M_4	5.472	5.344	4.732	4.760	4.619	4.147	4.026

Standard errors in parentheses.

Table 2.11

Estimating MA(q) Parameters

October 7, 1982 - January 31, 1984

$$\Delta r_t = u_t$$
$$u_t = \epsilon_t + \sum_{i=1}^{k} \theta_i \epsilon_{t-i}$$
$$\epsilon_t | \Omega_{t-1} \sim N(0, \omega_0)$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
θ_1	.1588	.0804	.1070	.0558	.0608	.0698	.0786
	(.0662)	(.0578)	(.0468)	(.0522)	(.0656)	(.0597)	(.0666)
ω_0	.0073	.0092	.0086	.0100	.0089	.0079	.0073
	(.0004)	(.0004)	(.0003)	(.0003)	(.0003)	(.0003)	(.0003)
LogL	338.794	303.071	312.982	287.610	308.502	326.510	340.564
Q(10)	13.314	6.830	6.767	9.124	7.082	5.868	6.219
$Q^2(10)$	18.816	45.296	36.170	35.602	17.082	22.956	41.057
M_3	0.406	-0.774	-1.517	-1.192	-1.213	-0.729	-0.409
M_4	5.521	8.855	13.443	11.825	13.169	9.392	6.756

Standard errors in parentheses.

Table 2.12

Estimating MA(q) Parameters

February 1, 1984 - March 16, 1988

$$\Delta r_t = u_t$$
$$u_t \;\; \epsilon_t + \sum_{i=1}^{k} \theta_i \; \epsilon_{t-i}$$
$$\epsilon_t | \Omega_{t-1} \sim N\,(0,\; \omega_0)$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
θ_1	.1018	.1250	.1293	.1117	.1201	.0833	.1318
	(.0174)	(.0206)	(.0235)	(.0300)	(.0306)	(.0321)	(.0369)
ω_0	.0070	.0065	.0053	.0076	.0078	.0079	.0068
	(.0002)	(.0002)	(.0001)	(.0002)	(.0002)	(.0002)	(.0003)
LogL	1088.855	1125.734	1230.255	1045.082	1035.730	1022.818	777.883
Q(10)	7.276	11.652	12.340	9.447	9.397	8.925	6.990
$Q^2(10)$	258.657	124.111	57.657	36.581	45.972	47.493	34.141
M_3	-0.294	-0.854	-0.940	-0.696	-0.581	-0.545	-0.100
M_4	8.886	11.408	12.379	9.572	8.884	8.198	3.885

Standard errors in parentheses.

statistic, $Q^2(k)$, which can be used to test for serial dependence in the time-dependent conditional variance. The $Q^2(k)$ statistic is asymptotically chi-square distributed with k degrees of freedom and, under a null hypothesis of no time-dependent conditional heteroskedasticity, is equivalent to a Lagrange Multiplier (LM) test for an ARCH(k) process. Examination of the $Q^2(10)$ statistics for daily interest rates over the various periods overwhelmingly rejects the null hypothesis of no ARCH effects. It is also interesting to note that in most cases conditional heteroskedasticity appears to be strongest at the short end of the maturity spectrum. This should not be surprising however in that under the expectations hypothesis, short-term rates would be expected to exhibit greater volatility than long-term rates.[6]

Finally Tables 2.5 through 2.8 present statistics M_3 and M_4 which are measures of the sample skewness and kurtosis of the unconditional distribution based on the residuals of the model. Under the assumption of conditionally normal errors, the asymptotic distribution of $M_3 \sim N(0, 6/n)$ and $M_4 \sim N(0, 24/n)$. Examination of the M_4 statistic for every interest rate over each of the four periods reveals that in every case the sample kurtosis exceeds three standard deviations and the unconditional distributions are leptokurtic, or fat-tailed. In general, this fact is particularly pronounced in the relatively short-term rates.

2.4 GARCH Models with Conditionally Normal Errors

Given the results of the previous section an ARIMA (0,1,q) model with ARCH effects was fitted to daily data for the various interest rates over all four periods. A number of other studies have applied GARCH(p,q) models to a variety of macroeconomic data. For example Baillie and

Bollerslev(1989), Bollerslev(1987), and Bollerslev(1986) find the GARCH (1,1) process accurately reflects the behavior of the conditional variance of exchange rates, stock prices, and inflation, respectively. In this study daily interest rate data is also modeled as a GARCH (1,1) process so that the conditional variance equation can be written as:

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}. \tag{2.4.1}$$

Combining equation (2.4.1) with the ARIMA (0,1,q) model of the conditional mean yields the ARIMA-GARCH process:

$$\begin{aligned} \Delta r_t &= u_t \\ u_t &= \epsilon_t + \sum_{i=1}^{k} \Theta_i \epsilon_{t-i} \\ \epsilon_t | \Omega_{t-1} &\sim N(0, h_t) \\ h_t &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}. \end{aligned} \tag{2.4.2}$$

Simultaneous maximum likelihood estimates of the parameters in the GARCH (1,1) model, using the Berndt, Hall, Hall, and Hausman(1972) algorithm, are presented in Tables 2.13 through 2.16. From the results it can be seen that the conditional heteroskedasticity present in the daily data is well approximated by a GARCH (1,1) process. In all cases the estimated parameters α_1 and β_1 are highly significant and the Ljung and Box $Q^2(k)$ statistic reveals no additional heteroskedasticity present in the data. Another interesting point revealed by the tables is the fact that while the diagnostic tests point to much stronger conditional heteroskedasticity in the short-term rates, examination of the parameters α_1 and β_1 shows the estimates to be rather homogeneous across the maturity spectrum. It should be noted that in almost all cases $\alpha_1 + \beta_1$ is close to unity. When $\alpha_1 + \beta_1 = 1$ the process is known as integrated in GARCH (IGARCH) and the unconditional variance is infinite.[7] The finding that $\alpha_1 + \beta_1$ approaches unity is not unique to this study, rather it appears to be commonplace in GARCH (1,1) models of financial time series[8].

Finally the question of the kurtosis of daily interest rates is examined. While a GARCH (1,1) model with normal errors explains some of the kurtosis, such a model can not completely explain the leptokurtic unconditional distribution of interest rates. This can be seen by the fact that despite the introduction of the GARCH model, the sample kurtosis statistics, M in Tables 2.13 through 2.16, still exceeds the theoretical kurtosis level by at least three standard deviations. In conclusion while an ARIMA (0,1,q) model with GARCH effects, assuming errors are normally distributed, can account for the level of serial correlation and the presence of conditional heteroskedasticity, it can only partially explain the severe excess kurtosis present in daily interest rate data.

2.5 Models with a Conditional t Distribution

While the use of the conditional normal distribution in a GARCH (1,1) model accounts for the degree of conditional heteroskedasticity in daily interest rates, it reduces but does not eliminate the level of excess kurtosis. Such a result is not unique; in fact Baillie and Bollerslev(1989), Bollerslev(1987), and Milhoj(1987) find a similar result in examining daily exchange rate data while Baillie and DeGennaro(1988a) get the same result for stock prices. A common remedy for this shortcoming has been to employ a standardized t-distribution rather than a normal distribution in explaining conditional residuals. Previous

Table 2.13

Daily GARCH Models

January 1, 1974 - October 4, 1979

$$\Delta r_t = u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \theta_3 \epsilon_{t-3}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1}$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
θ_1	.1523	.1208	.1349	.1424	.1464	.1542	.1726
	(.0261)	(.0289)	(.0296)	(.0289)	(.0287)	(.0265)	(.0299)
θ_2	.0053	.0594	.0864	.0635	.0243	---	---
	(.0306)	(.0310)	(.0313)	(.0296)	(.0306)		
θ_3	-.0531	---	---	.0657	.0800	---	---
	(.0292)			(.0282)	(.0273)		
θ_4	.0009	---	---	---	---	---	---
	(.0266)						
θ_5	.0794	---	---	---	---	---	---
	(.0261)						
α_0	.0002	.0001	.0001	.0002	.0001	.0001	.0001
	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)
α_1	.2060	.0731	.0682	.1225	.1330	.1352	.1536
	(.0183)	(.0096)	(.0090)	(.0122)	(.0133)	(.0128)	(.0152)
β_1	.8060	.9085	.9123	.8393	.8309	.8346	.7719
	(.0140)	(.0109)	(.0107)	(.0123)	(.0146)	(.0141)	(.0242)
LogL	1562.568	1745.709	1790.468	2011.694	2248.369	2661.106	2936.105
Q(10)	7.888	11.263	10.503	10.685	11.554	11.192	15.110
$Q^2(10)$	3.739	2.232	4.961	5.628	8.966	13.757	5.792
M_3	0.588	0.736	0.504	.0310	0.031	-0.142	0.140
M_4	7.437	7.817	6.482	8.316	7.427	6.298	5.879

Standard errors in parentheses.

Table 2.14

Daily Garch Models

October 10, 1979 - October 6, 1982

$$\Delta r_t = u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t \; \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1}$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
θ_1	.1404	.0903	.0873	.1122	.0937	.0685	---
	(.0404)	(.0400)	(.0402)	(.0412)	(.0401)	(.0397)	
α_0	.0023	.0016	.0018	.0015	.0010	.0008	.0005
	(.0009)	(.0008)	(.0014)	(.0005)	(.0004)	(.0003)	(.0002)
α_1	.0602	.0263	.0269	.0471	.0524	.0599	.0543
	(.0150)	(.0096)	(.0139)	(.0111)	(.0121)	(.0148)	(.0139)
β_1	.9121	.9481	.9328	.9155	.9155	.9095	.9242
	(.0212)	(.0193)	(.0397)	(.0215)	(.0208)	(.0229)	(.0207)
LogL	-106.975	-28.784	93.680	162.397	238.132	332.939	372.739
Q(10)	9.270	13.782	13.57	10.546	12.663	12.987	11.717
$Q^2(10)$	10.657	9.875	6.171	7.118	3.308	3.412	6.180
M_3	0.378	0.338	0.087	0.227	0.108	0.054	0.103
M_4	5.110	5.324	4.813	5.200	4.684	3.921	3.782

Standard errors in parentheses.

Table 2.15

Daily GARCH Models

October 7, 1982 - January 31, 1984

$$\Delta r_t = u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}$$

	3 Month	6 Month	1 Year	3 Year	5 Year	10 Year	20 Year
θ_1	.1241 (.0617)	---	.0708 (.0649)	---	---	---	---
α_0	.0000 (.0001)	.0002 (.0001)	.0002 (.0001)	.0003 (.0001)	.0003 (.0001)	.0002 (.0001)	.0002 (.0001)
α_1	.0107 (.0053)	.0277 (.0071)	.0302 (.0105)	.0306 (.0112)	.0295 (.0150)	.0361 (.0158)	.0512 (.0176)
β_1	.9902 (.0083)	.9465 (.0143)	.9260 (.0225)	.9284 (.0237)	.9224 (.0344)	.9306 (.0253)	.9175 (.0242)
LogL	349.399	322.212	331.246	310.906	327.641	346.255	360.600
Q(10)	14.068	5.778	3.520	7.852	7.642	6.890	5.713
$Q^2(10)$	17.220	13.536	4.400	3.980	1.691	2.075	5.333
M_3	0.387	-0.435	-0.999	-0.630	-0.845	-0.476	-0.080
M_4	5.100	6.997	10.295	9.605	10.876	8.070	6.057

Standard errors in parentheses.

Table 2.16

Daily GARCH Models

February 1, 1984 - March 16, 1988

$$\Delta\ r_t = u_t$$

$$u_t = \epsilon_t + \Theta_1\ \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N\ (0, h_t)$$

$$h_t = \alpha_0 + \alpha_1\ \epsilon^2_{t-1} + \beta_1\ h_{t-1}$$

	3 Month	6 Month	1 Year	3 Year	5 Year	10 Year	20 Year
Θ_1	.0682 (.0360)	.1278 (.0360)	.1154 (.0370)	.1114 (.0362)	.1213 (.0367)	.0905 (.0380)	.1152 (.0413)
α_0	.0002 (.0000)	.0003 (.0001)	.0003 (.0001)	.0003 (.0001)	.0004 (.0001)	.0003 (.0001)	.0001 (.0001)
α_1	.1100 (.0121)	.1081 (.0127)	.1077 (.0109)	.1318 (.0131)	.1230 (.0126)	.0884 (.0129)	.0485 (.0145)
β_1	.8618 (.0125)	.8455 (.0183)	.8409 (.0196)	.8357 (.0202)	.8359 (.0205)	.8811 (.0195)	.9358 (.0216)
LogL	1201.424	1215.343	1299.326	1093.039	1078.256	1053.822	790.253
Q(10)	6.246	7.828	12.975	9.180	7.374	6.532	4.672
$Q^2(10)$	14.596	11.189	13.280	12.022	16.183	10.723	14.991
M_3	0.135	-0.052	-0.082	-0.198	-0.241	-0.323	-0.032
M_4	5.787	4.946	5.131	5.319	6.022	6.869	3.588

Standard errors in parentheses.

studies have found the conditional t-distribution in a GARCH model to adequately account for excess kurtosis in financial time series.

The standardized t-distribution has a log likelihood function which can be expressed as:

$$\text{LogL} = n[\log \Gamma(\frac{v+1}{2}) - \log \Gamma(\frac{v}{2}) - 1/2 \log (v-2)]$$

$$- \frac{1}{2} \sum_{t=1}^{n} [\log h_t + (v+1) \log (1 + \epsilon_t^2 h^{-1}_t (v-2)^{-1}]$$

where v is the degrees of freedom and Γ is the gamma function. One of the appealing features of the standardized t-distribution is that while it approaches the normal distribution as v approaches zero, if $v^{-1} > 0$ then the standardized t exhibits more leptokurtosis than the normal distribution. As such the standardized t-distribution was integrated into the GARCH process with the model being:

$$\begin{aligned}
&\Delta r_t = u_t \\
&u_t = \epsilon_t + \sum_{i=1}^{k} \Theta_i \epsilon_{t-i} \\
(2.5.1) \qquad &\epsilon_t | \Omega_{t-1} \sim t\ (0, h_t, v) \\
&h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1}.
\end{aligned}$$

Unfortunately, an application of the conditional t distribution appears to be of little value in solving the problem of excess kurtosis present in daily interest rate data. For example, maximum likelihood estimates of equation (2.5.1) are presented for the period February 1984 through March 1988 in Table 2.17.[9] Also presented is the theoretical kurtosis under a t-distribution, $3(\hat{v} - 2)/(\hat{v} - 4)$, where $\hat{v}$ is the estimated degrees of freedom. While the conditional t-distribution has been successful in explaining excess kurtosis in other studies, an examination of Table 2.17 reveals that it is of limited value in explaining the kurtosis found in daily interest rates. In the case of the 10- and 20-year bond rates, the estimates of v^{-1} are .1791 and .1142, respectively. The Likelihood Ratio (LR) test statistic for $v^{-1} = 0$, under the null hypothesis that errors are conditionally normal, is decisively rejected. For these two interest rates the theoretical kurtosis closely reflects the actual level of kurtosis in the unconditional distribution. For the remaining interest rates however the standardized t-distribution is less successful. For these interest rates v^{-1} exceeds .20, thus yielding unreasonably large estimates of the degree of kurtosis. In fact for the t distribution the fourth moment only exists for estimates of v greater than 4; in the case of the 3-month and 6-month Treasury bill rates $v^{-1} > .25$ thereby making the theoretical kurtosis undefined. While the $LR_{v^{-1}=0}$ statistic overwhelmingly rejects the hypothesis of conditionally normal errors, for these five daily interest rates the use of the standardized t-distribution seems questionable.

To further examine the applicability of the conditional t to the problem of excess kurtosis, v^{-1} was estimated without allowing the estimate to iterate. The results are shown in Table 2.18. Estimates of v^{-1} range from .0713 to .1831; these estimates being considerably lower than the estimates in Table 2.17. In all cases these estimates closely coincide with the excess kurtosis found in daily interest rates.

2.6 Conclusion

The purpose of this chapter has been to investigate the time-series behavior of daily interest rates. This task is accomplished by

Table 2.17

Daily GARCH Models

February 1, 1984 - March 16, 1988

$$\Delta r_t = u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim t\ (0, h_t, v)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
θ_1	.0685	.1022	.0828	.0837	.0988	.0677	.1048
	(.0308)	(.0309)	(.0316)	(.0316)	(.0317)	(.0327)	(.0391)
α_0	.0003	.0004	.0003	.0004	.0005	.0002	.0001
	(.0001)	(.0001)	(.0001)	(.0002)	(.0002)	(.0001)	(.0001)
α_1	.1173	.1084	.0817	.1048	.1131	.0770	.0469
	(.0318)	(.0313)	(.0231)	(.0287)	(.0307)	(.0209)	(.0188)
β_1	.8630	.8553	.8756	.8533	.8420	.9006	.9339
	(.0308)	(.0361)	(.0338)	(.0372)	(.0388)	(.0259)	(.0304)
v^{-1}	.2751	.2726	.2316	.2187	.2234	.1791	.1142
	(.0004)	(.0004)	(.0014)	(.0022)	(.0441)	(.0291)	(.0354)
LogL	1254.209	1261.942	1341.797	1128.155	1118.430	1088.460	795.689
Q(10)	6.239	8.658	14.973	10.880	8.583	7.731	4.936
$Q^2(10)$	16.387	13.549	21.136	11.581	15.646	10.385	15.226
M_3	0.116	-0.073	-0.158	-0.272	-0.273	-0.334	-0.036
M_4	5.818	4.985	5.475	5.811	6.268	7.081	3.599
$Lr_{1/v}=0$	105.570	93.198	84.942	70.232	80.348	69.276	10.872
$\frac{3(\hat{v}-2)}{(\hat{v}-4)}$	undef	undef	21.868	13.490	15.605	6.790	4.261

Table 2.18

Daily GARCH Models

February 1, 1984 - March 16, 1988

$$\Delta r_t = u_t$$

$$u_t = \epsilon_t + \Theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim t\ (0,\ H_t,\ v)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 H_{t-1}$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
Θ_1	.0759 (.0329)	.1119 (.0336)	.0876 (.0331)	.0850 (.0326)	.0987 (.0325)	.0684 (.0325)	.1077 (.0398)
α_0	.0002 (.0001)	.0003 (.0001)	.0002 (.0001)	.0004 (.0001)	.0004 (.0002)	.0002 (.0001)	.0001 (.0001)
α_1	.0999 (.0205)	.0926 (.0192)	.0753 (.0180)	.0968 (.0240)	.1025 (.0255)	.0769 (.0208)	.0460 (.0167)
β_1	.8534 (.0277)	.8461 (.0308)	.8713 (.0303)	.8542 (.0342)	.8406 (.0373)	.9011 (.0258)	.9347 (.0273)
v^{-1}	.1632 ---	.1424 ---	.1557 ---	.1630 ---	.1710 ---	.1831 ---	.0713 ---
Log L	1248.964	1254.168	1338.931	1126.935	1117.233	1008.456	794.974
Q(10)	6.008	8.315	14,512	10.752	8.557	7.698	4.854
$Q^2(10)$	15.525	12.883	20.365	11.616	15.588	10.380	15.171
M_3	0.106	-0.066	-0.153	-0.272	-0.281	-0.334	-0.035
M_4	5.854	4.981	5.431	5.814	6.309	7.082	3.597

Standard errors in parentheses.

integrating an ARIMA model into the ARCH process developed by Engle(1982) and later generalized (GARCH) by Bollerslev(1986). The results point to a number of interesting conclusions. First, daily government interest rates, regardless of the monetary operating regime in existence at the time, possess one unit root and are well approximated by an ARIMA (0,1,q) process. Second, daily interest rates exhibit conditional heteroskedasticity and severe excess kurtosis. As such a GARCH (1,1) model with normal errors accounts well for the level of conditional heteroskedasticity, but can not fully account for the severe excess kurtosis. Finally while the GARCH(1,1) model with a conditional t-distribution has been successful in other studies, in this study it proves to be of limited value in explaining the excess kurtosis unless the inverted degrees of freedom parameter is not allowed to iterate.

ENDNOTES

1. While the collection of data was done using the survey method outlined herin, how long such a survey method has been employed to gather daily interest rate data is unknown.

2. Nelson and Plosser(1982) and Perron(1986) examine the unit root question using the quarterly bond yield.

3. To see this assume the a variable y can be described by the following nonstationary process:

   $$y_t = c + y_{t-1} + e_t$$

   where c is an initial condition and e is and error term which is white noise. Using successive substitutions this nonstationary process can also be written:

   $$y_t = ct + \sum_{i=0}^{n} e_{t-i}.$$

4. See Dickey, Bell, and Miller(1986) page 13.

5. In all twenty-eight cases the Phillips-Perron test statistics for the first-differenced data were examined to see if the data required additional differencing. In no case was it necessary to difference interest rates a second time to acheive stationary.

6. If long-term interest rates are averages of present and expected future short-term rates then long-term rates should exhibit less volatility than short-term rates. For the seminal work in this area see Shiller(1979).

7. For a discussion of the IGARCH process see Engle and bollerslev (1986).

8. For example, Baillie and Bollerslev(1987) find this to be true for exchange rates.

9. Unfortunately applications of the conditional t distribution to all interest rates in other periods were not available. In those cases where results were available, degrees of freedom estimates were very similar to those found in Table 2.13. In these cases the conditional t distribution again was found to be of little value in explaining the level of excess kurtosis.

CHAPTER THREE

GARCH MODELS AND MONEY SUPPLY ANNOUNCEMENTS

3.1 Introduction

Over the last decade considerable research has been devoted to studying the impact of money supply announcements on interest rates as well as other financial variables. The primary emphasis of these studies has been to assess the impact of money supply announcements on the level of interest rates. The research has shown that when an unexpected increase in money supply occurs, both short-term and long-term interest rates rise. Similarly unexpected decreases in the money supply subsequently lead to reductions in both short-term and long-term rates. This result is true for both pre-October 1979 and post-October 1979 periods. For anticipated money, the results are mixed with some studies finding it to be statistically significant while others find it has no effect[1]. While the emphasis of these studies is on the response of the level of interest rates to money supply announcements, few studies examine how these announcements influence interest rate volatility. Those that do typically infer interest rate volatility from changes in the level of interest rates as a result of the announcement[2]. The purpose of this chapter is to examine how money supply announcements influence interest rate volatility across a variety of Federal Reserve operating procedures. This is accomplished by introducing money supply announcements into the daily GARCH (1,1) models derived in the previous chapter.

3.2 Previous Literature

To date a considerable amount of research exists which examines how money supply announcements impact asset prices. Following Cornell(1983), the basic equation can be written:

$$(3.2.1) \quad \Delta A_t = \gamma_0 + \gamma_1 \; EM_t + \gamma_2 \; UM_t + \epsilon_t$$

where ΔA_t is the change in the asset price in period t, EM_t and UM_t represent the expected and unexpected change in the money supply, respectively, and ϵ_t is an error term which is assumed to be white noise. Under the efficient markets hypothesis, current levels of asset prices should reflect all currently known information; thus changes in asset prices should only occur when new information is received by the market. To the degree that money supply announcements provide new information in the form of unanticipated changes in the money supply, asset prices should change. Thus a priori we would expect $\gamma_2 \neq 0$. Since anticipated money provides no new information to the market, a priori we would expect $\gamma_1 = 0$.

Equation (3.2.1) has been examined for a variety of assets. Cornell (1983), and Pearce and Roley (1983, 1985) have all examined the impact of money supply announcements on stock prices. While stock prices appear to respond negatively to monetary shocks in the post October 1979 period, no consensus appears to exist on the effect of money surprises in the pre-October 1979 period[3]. For exchange rates, equation (3.2.1) has been employed by Cornell(1983), Engel and Frankel(1984), Hardouvelis(1984), Hakkio and Pearce(1985), Edwards(1983,1984), and Ito and Roley(1987) to see how the exchange rate responds to money supply announcements. The conclusion here is that positive monetary surprises lead to an appreciation of the dollar against foreign currencies for the post-October 1979 period only[4].

In a similar manner equation (3.2.1) has also been applied to the behavior of interest rates across the maturity spectrum. Under the assumption that interest rates follow a random walk[5], equation (3.2.1) is generally expressed as:

$$(3.2.2) \qquad \Delta r_t = \gamma_0 + \gamma_1 \, EM_t + \gamma_2 \, UM_t + \epsilon_t$$

where Δr_t is the first-difference of interest rates and all remaining variables are as defined earlier. Over the last decade there has been extensive research on equation (3.2.2), or some variation of it. Studies by Urich and Wachtel(1981), Roley(1982,1983), Cornell(1983), Roley and Troll(1983), Hardouvelis(1984), and Loeys(1985), among others, all note that interest rates, both short- and long- term, rise when unanticipated increases in the money supply are announced. Furthermore, they also note that interest rates exhibit their greatest response to unanticipated money during the period October 1979 to October 1982 when the Federal Reserve was targeting the level of nonborrowed reserves. While such a result is now readily accepted, debate on a variety of issues continues.

Possibly the most discussed issue is the theory of how money supply announcements influence interest rates. To this end a variety of hypotheses have been proposed; the most popular of all theories is the expected liquidity effect hypothesis[6]. According to this hypothesis it is anticipated that any unexpected change in the money supply will be corrected in future periods. Assuming inflationary expectations are fixed, an unexpectedly large increase in the announced money supply causes both nominal and real interest rates to rise due to expectations that the Federal Reserve will counteract the increase shortly. In a similar fashion substantial decreases in the announced money supply, if unexpected, lead to falling interest rates.

As such the expected liquidity effect hypothesis is predicated on a number of crucial assumptions[7]. First, agents must believe that the Federal Reserve is credible, and as such, adheres at least in part, to it's monetary growth rate targets. Second, any corrective action taken by the Federal Reserve to achieve the target growth rate must be perceived to take place in the near future. Finally it must be assumed that the money supply shock is of a permanent nature; if the shock is temporary then the money supply will simply return to it's long run growth rate and no corrective action need be taken. Given these assumptions, the relevance of the expected liquidity hypothesis may depend upon the Federal Reserve operating regime. During the period October 1979 through October 1982 when the Federal Reserve targeted a monetary aggregate, the reaction of interest rates to unanticipated money should be quite strong. This is because under the given assumptions, economic agents believe the Federal Reserve will quickly counteract any deviation of the money stock from it's long-run growth path. However, when the Federal Reserve attempts to control the money supply using the federal funds rate as a target, as existed prior to October 1979, the reaction of interest rates to unanticipated money should be weaker since the operating regime explicitly focuses on the stabilization of interest rates rather than the control of the money stock.

A second popular theory to explain the response of interest rates to money supply announcements is the inflation premium hypothesis. According to this theory, unexpected changes in the published money supply will lead economic agents to revise their expectations of future money growth and inflation. Specifically, if unanticipated increases in the money supply are not expected to be offset, this leads to rising inflationary expectations and thus increasing nominal interest rates. The opposite result holds true for unanticipated decreases in the money supply. Like the expected liquidity effect hypothesis, assumptions are a crucial element in the inflation premium hypothesis. First, despite the fact that the Federal Reserve has previously announced it's monetary growth rates, it either lacks credibility or unbeknown to agents, has altered it's policy. Second, money shocks are assumed to be permanent rather than temporary. As a result of these two assumptions, announcements of money supply increases which are unanticipated act as a signal to financial markets that the Federal Reserve has adopted a more lenient monetary policy. In a similar manner, unexpected decreases in the announced money supply are interpreted as a sign of a more restrictive monetary policy. Under the inflation premium hypothesis the response of interest rates to announcements of unanticipated money also depends on the monetary regime. If the Federal Reserve targets a monetary aggregate rather than interest rates, then deviations of the money stock from it's growth path cause a relatively larger increase in interest rates since economic agents interpret this as a sign of a more expansionary monetary policy.

It is noteworthy to compare the underlying nature of the two theories delineated so far. Both view unanticipated increases in the money supply as a signal; the difference lies in that the inflation premium hypothesis views such an announcement as a signal of a more expansionary monetary policy, where the expected liquidity theory views the same increase as an indication of future tightening. Furthermore, both theories argue that when employing a monetary aggregate targeting regime, announcements of unanticipated money provide a stronger signal to agents of future policy than would occur under an interest rate smoothing scheme. However, because both theories agree on the response of interest rates to unanticipated money, it is impossible to differentiate between them on the basis of interest rates alone[8].

A third theory advanced for explaining the response of interest rates to unanticipated money shocks employs the argument that while announcements yield information about the money supply, they simultaneously provide information about money demand. Thus the theory is referred to as the money demand hypothesis[9]. Given a positive relationship between money demand and expected future output[10], an unanticipated increase in the money supply provides information about the expected future level of economic activity. With an increase in expected economic activity in the future will come a corresponding increase in the expected future demand for real money balances. If the expected future money demand increase exceeds expectations of the future growth of the money supply, then anticipated nominal interest rates in the future will also rise. But rational economic agents, expecting nominal interest rates to rise in future periods, will sell bonds in the present period so as to avoid capital losses. As a result, assuming inflationary expectations are fixed, present nominal and real interest rates will rise. Like the previously outlined theories, assumptions play a vital role in the credibility of the money demand effect. Here money demand shocks are seen to have more permanence, or at least are slower to return to their original state, than the corresponding money supply shock. While such an assumption may be acceptable if the Federal Reserve targets a monetary aggregate, it seems questionable if the Federal Reserve utilizes a policy of stabilizing interest rates.

While the major issue surrounding money supply announcements has been the development of a theory to explain the response of asset prices, other issues exist which may be relevant for the present study. One of the primary areas of concern has been the measurement of expected money. Since both expected and unexpected money are not directly observable, some method of deriving these variables is necessary. As noted by Urich and Wachtel(1981), "the most problematic aspect of any study of unanticipated change in economic activity is the procedure used to develop a proxy for expectations."[11] One method has been to use ARIMA models to develop estimates of expected and unexpected money. However the inferiority of the ARIMA-based estimates, relative to the survey method, has been documented by Urich and Wachtel(1981) and Belongia and Sheehan(1987). Recently, survey data from Money Market Services, Inc. has been accepted as the standard measure of expectations. The tendency has been to use the median of the Money Market Services' weekly survey as an estimate of the expected weekly change in the money stock; unexpected money is then the difference between the published change and the anticipated change. A key question with respect to the use of survey data involves the potential bias of the survey median. If the median is biased then agents are not rational in that available information is not employed efficiently. Specifically the standard test for unbiasedness can be written as:

$$(3.2.3) \quad \Delta M_t = \Phi_0 + \Phi_1 EM_t + \epsilon_t$$

where the null hypothesis $\Phi_0 = 0$ and $\Phi_1 = 1$ utilizes an F-test. The rationality of the Money Market Services' survey has been examined in a number of studies including those of Grossman(1981), Roley(1983), Urich and Wachtel(1984), and Engel and Frankel(1984) who have found the survey data to be unbiased. Contrary results however, have been found by Hafer(1983) and Deaves, Melino, and Pesando(1987) who found the survey data to be biased after 1979. The general conclusion to date appears to be that while some evidence exists that the survey data is biased, it is a superior estimator relative to ARIMA models.

A final issue to be addressed here involves the length of the interval over which interest rates are measured. Ideally interest rate changes are measured over the smallest possible interval so as to avoid contamination due to the arrival of new information which may be relevant to the behavior of interest rates. Belongia and Sheehan recognize this point when they state, "measuring the change in interest rates across a period of one day or more necessarily confuses the announcement effect with the reaction to other new information."[12] The preferred interval in the research today is from 3:30 p.m. to 5:00 p.m. on the afternoon of the announcement. Because of the use of daily data in this study, the measurement interval is the twenty four hour period between survey observations on consecutive days.

3.3 The Model

Given the results derived in Chapter Two, the GARCH(1,1) model can be easily extended to allow for the inclusion of money supply announcements. In general, introducing exogenous variables into a GARCH(p,q) process for interest rates allows the equation for the mean to be rewritten as:

$$(3.3.1) \qquad \Delta r_t = \gamma_i X_{it} + u_t$$

where X_{it} is a vector of relevant exogenous variables to be included and u_t is an error term which may be serially correlated. The unanticipated weekly change in the money supply may be interpreted as an exogenous variable in that on the day of the publication, the statistical release does not change the money supply, but rather provides information about the estimated level of the money supply for the period ending on Wednesday of the previous week. Introducing unanticipated changes in the money supply into equation (3.3.1) yields:

$$(3.3.2) \qquad \Delta r_t = \gamma_1 UM_t + u_t$$

where UM_t is the unexpected change in the money supply[13]. Money is measured using M1. UM_t is the actual change in M1 minus expected money, where expected money is taken as the median value of the Money Market Services survey[14]. Equation (3.3.2) can be interpreted as saying that unanticipated changes in the money supply may alter the level of interest rates. Equation (3.3.2) is similar to the standard money supply announcement equation, like equation (3.2.2), with a few exceptions.

First, most money supply announcement equations include a constant term although its only relevance appears to be in testing for market efficiency. Depending upon the particular study being examined, the constant may or may not be significant. In this study the results of Chapter Two reveal that the first difference of daily interest rates is free of drift or deterministic trend; that being the case a constant term was excluded from equation (3.3.2)[15].

A second and more dramatic difference of this model from the standard form involves the use of daily data. Typically empirical estimation of an equation such as (3.3.2) employs weekly data. However in this study daily interest rate data is employed. By using a model with daily data the problems associated with a change in the day on which the money supply is announced are avoided. There is also a gain in efficiency as this model uses information on non-announcement days as well as money supply announcements which occur on irregular days.[16] As such the variables UM takes on its respective survey value on the day of the money supply announcement, and is set equal to zero on all non-announcement days. In Chapter Two the level of interest rates was found to follow a martingale sequence. Thus given the ARIMA(0,1,q) specifications derived in Chapter Two the GARCH(1,1) model with money supply announcement effects can be written:

$$\begin{aligned} &\Delta r_t = \gamma_1 UM_t + u_t \\ &u_t = \epsilon_t + \Theta_1 \epsilon_{t-1} \\ (3.3.3) \qquad &\epsilon_t | \Omega_{t-1} \sim N(0, h_t) \\ &h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} \end{aligned}$$

where u_t is a MA(1) error and ϵ_t is a serially uncorrelated error process. Model (3.3.3) can now be utilized to examine the the impact of unanticipated money on interest rates. Given the previously delineated theories and empirical studies, a priori we would also expect unanticipated changes in the money supply to result in increasing interest rates. Given that the strength of the market response depends upon the markets perception of Federal Reserve policy, we would expect γ_1 to be greater than zero and to be larger during the period October 1979 through October 1982 when the Federal Reserve targeted nonborrowed reserves. This result is consistent with any of the three hypotheses outlined in section 3.2 and with the majority of empirical research to date.

A further extension of model (3.3.3) is possible however. Implicit in models like (3.3.2) or (3.3.3) is the assumption that unanticipated changes in the money supply only influence the change in the level of interest rates, but have no impact upon their volatility. This assumption however is easily testable within the framework of a GARCH(p,q) model. To date, empirical research has tended to ignore the impact of unanticipated changes in the money supply on interest rate volatility, or has inferred it's impact from changes in the mean. But the volatility of interest rates may change because of either a change in unanticipated money, a change in the response of interest rates to unanticipated money, or a change in the volatility of the error term. Those papers which do address the issue, such as Roley(1982,1983) and Evans(1981), typically find that unanticipated money significantly alters the volatility of interest rates, particularly after the adoption of the new operating procedure in October 1979. In fact Roley(1982) argues that about 34 percent of the increased volatility of 3-month Treasury bills after

October 1979 can be attributed to the increased response of economic agents to unanticipated money while an additional 6 to 9 percent can be attributed to an increasingly volatile money supply during this period. But studies such as these typically utilize a variance decomposition approach, and in doing so employ the sample variance as a measure of volatility. Given the weaknesses of using this estimator of volatility, as discussed in Chapter One, an alternative approach seems appropriate. Nevertheless the assumption that unanticipated money changes only have a direct impact on the mean of interest rates seems overly restrictive. Rather if interest rate volatility is measured using the conditional variance then the GARCH(p,q) models derived earlier can be employed to systematically assess the impact of unanticipated money on not only the level of interest rates, but also their volatility. As such the GARCH process can be written:

$$\begin{aligned} &\Delta r_t = \gamma_1 UM_t + u_t \\ &u_t = \epsilon_t + \theta_1 \epsilon_{t-1} \\ (3.3.4) \qquad &\epsilon_t | \Omega_{t-1} \sim N(0, h_t) \\ &h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + D_1 \mid UM_t \mid . \end{aligned}$$

Since any change in unanticipated money would be expected to increase the volatility of interest rates, the absolute value of UM_t is included as an exogenous variable in the conditional variance equation where the sign of the coefficient of the unanticipated money variable is expected to be positive[17]. Furthermore, to the extent that market agents are indeed more responsive to unanticipated changes in M1 when the Federal Reserve targets a monetary aggregate, we would also expected unanticipated money to have a greater impact upon interest rate volatility during the period October 1979 to October 1982.

3.4 Results

Models (3.3.3) and (3.3.4) were estimated for the periods January 1, 1978 to October 4, 1979; October 10, 1979 through October 6, 1982; and October 7, 1982 to January 26, 1984; using all seven government interest rates in this study.[18] These are the periods during which the Federal Reserve targeted the federal funds rate, the level of nonborrowed reserves, and the level of borrowed reserves, respectively. As noted in some of the previously outlined theories and empirical studies, during the period October 1979 through October 1982, money supply announcements should have their strongest effects. As in the previous chapter, MLE of models (3.3.3) and (3.3.4) were obtained using the Berndt, Hall, Hall, and Hausman(1972) algorithim. The results are reported in Tables 3.1 through 3.6. In all cases the GARCH(1,1) model continues to perform well. The parameters α_1 and β_1 are all significant at the 5-percent level and examination of the Q^2 (10) statistic reveals no evidence of remaining conditional heteroskedasticity. Unlike the earlier models however, the introduction of unanticipated money has rendered the moving average error terms insignificant in some cases.

The results in Table 3.1 through 3.3 all confirm that unanticipated changes in the money supply exert a positive influence on interest rates. Again Q(10) and $Q^2(10)$ are measures of serial dependence in the conditional first and second moments and M_3 and M_4 are measures of skewness and kurtosis. Here all parameter estimates are significant at the 5-percent

Table 3.1

Daily GARCH Models With UM

January 1, 1978 - October 4, 1979

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1}$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0095	.0138	.0151	.0094	.0083	.0080	.0048
	(.0038)	(.0031)	(.0032)	(.0021)	(.0022)	(.0022)	(.0020)
θ_1	.0671	.0919	---	.2210	---	---	---
	(.0526)	(.0537)		(.0619)			
α_0	.0002	.0002	.0003	.0008	.0001	.0001	.0001
	(.0001)	(.0001)	(.0001)	(.0001)	(.0000)	(.0000)	(.0000)
α_1	.2318	.1328	.1277	.3124	.2086	.1989	.1755
	(.0435)	(.0300)	(.0207)	(.0732)	(.0305)	(.0259)	(.0329)
β_1	.7773	.8263	.8130	.3993	.7635	.7308	.7141
	(.0357)	(.0329)	(.0335)	(.0881)	(.0195)	(.0406)	(.0552)
LogL	456.221	581.440	617.301	732.031	784.552	839.102	932.517
Q(10)	16.292	6.201	8.047	6.455	12.467	9.660	15.029
$Q^2(10)$	6.228	2.327	4.630	1.631	2.182	6.710	6.775
M_3	0.462	0.584	1.000	1.590	0.220	-0.193	-0.002
M_4	5.227	6.606	6.715	13.293	8.311	6.396	6.900

Standard errors in parentheses.

Table 3.2

Daily GARCH Models

October 10, 1979 - October 6, 1982

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1}$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0626 (.0071)	.0647 (.0062)	.0537 (.0054)	.0512 (.0048)	.0442 (.0044)	.0322 (.0049)	.0285 (.0050)
θ_1	.1018 (.0398)	.0549 (.404)	.0440 (.0398)	.0735 (.0409)	.0638 (.0391)	.0428 (.0395)	---
α_0	.0018 (.0008)	.0013 (.0006)	.0015 (.0008)	.0013 (.0005)	.0010 (.0004)	.0008 (.0004)	.0005 (.0002)
α_1	.0804 (.0193)	.0438 (.0146)	.0449 (.0183)	.0626 (.0164)	.0728 (.0179)	.0646 (.0179)	.0575 (.0154)
β_1	.8993 (.0232)	.9353 (.0205)	.9209 (.0328)	.9030 (.0245)	.8978 (.0246)	.9037 (.0267)	.9226 (.0221)
LogL	-73.820	7.038	128.559	193.860	267.781	348.929	385.841
Q(10)	4.088	8.119	7.571	5.402	7.753	10.367	7.437
$Q^2(10)$	10.349	11.501	7.695	7.625	3.819	4.462	5.988
M_3	0.256	0.215	-0.095	0.076	-0.049	-0.008	0.054
M_4	4.604	4.937	4.367	4.771	4.283	3.764	3.712

Standard errors are in parentheses.

Table 3.3

Daily GARCH Models With UM

October 10, 1982 - January 26, 1984

$$\Delta r_t = \gamma_1 UM_t + \mu_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0323	.0377	.0319	.0315	.0313	.0270	.0234
	(.0044)	(.0045)	(.0043)	(.0043)	(.0041)	(.0036)	(.0032)
θ_1	.0886	---	.0537	---	---	---	---
	(.0603)		(.0603)				
α_0	.0000	.0001	.0001	.0002	.0003	.0002	.0002
	(.0000)	(.0000)	(.0001)	(.0001)	(.0001)	(.0001)	(.0001)
α_1	.0162	.0201	.0204	.0248	.0288	.0348	.0484
	(.0077)	(.0061)	(.0076)	(.0101)	(.0163)	(.0170)	(.0175)
β_1	.9768	.9636	.9494	.9392	.9221	.9298	.9174
	(.0136)	(.0118)	(.0169)	(.0209)	(.0344)	(.0265)	(.0265)
LogL	364.872	342.502	348.548	322.650	341.570	357.285	369.241
Q(10)	13.240	5.844	2.815	5.584	5.331	5.306	4.542
$Q^2(10)$	8.899	13.388	5.041	3.275	1.067	1.512	4.186
M_3	0.401	-0.614	-1.196	-0.842	-1.050	-0.674	-0.266
M_4	5.283	7.377	11.305	10.039	11.853	8.472	6.015

Standard errors in parentheses.

level; all estimates of γ_1 for the period October 1979 to October 1982 lie between 0.0285 and 0.0647, while the same estimates have a range of 0.0048 to 0.0151 for the pre-October 1979 data and 0.0234 to 0.0377 for the post-October 1982 data. These estimates appear similar to those reported in other studies.[19] What is of note in Tables 3.1, 3.2, and 3.3 is that the response of interest rate changes to unanticipated money appears sensitive to the choice of monetary regime. For example, during the nonborrowed reserve targeting regime, an unanticipated change in the money supply of $1 billion would increase the 3-month Treasury bill by 6.26 basis points while the 20-year bond rate would increase by 2.85 basis points. During the federal funds rate targeting regime, a similar increase in unanticipated money would increase the 3-month Treasury bill rate by 0.95 basis points and the 20-year rate by 0.48 basis points. In fact in every case the response of the level of interest rates is at least 3.5 times greater during the nonborrowed reserve targeting regime than when the federal funds rate was employed as the target. Finally for the post-1982 data, a $1 billion increase in unanticipated money would raise 3-month T-bill rates by 3.23 basis points and 20-year rates by 2.34 basis points. What is interesting to note is that during the post-1982 period, interest rates are more responsive to unanticipated money than during the pre-1979 period, but less responsive than during the October 1979 to October 1982 period. A similar result is reported by Huizinga and Leiderman(1987), Roley(1986), and Loeys(1985). Roley(1987) argues that such a result depends upon the persistence of money demand shocks, the response of the Federal Reserve to deviations of the money supply from its target range, and the choice of reserve accounting system. Another possible explanation for this result is that economic agents had some difficulty distinguishing a discernable policy change in October 1982. As such while the Federal Reserve de-emphasized M1 after October 1982, economic agents still perceived M1 growth as being an important policy objective and continued to use money supply announcements as a signal of future policy.[20]

Tables 3.4 through 3.6 present the GARCH model where unanticipated money is introduced into the equation for the conditional variance. The new feature of these models is that we have allowed unanticipated money to influence the conditional second moment of interest rates. Here the parameter D_1 represents the response of the conditional variance to an unanticipated change in the money supply. Examination of D_1 in Tables 3.4 through 3.6 shows that the impact of unanticipated money upon the conditional variance depends upon the monetary policy operating regime. For the period October 1979 to October 1982, the parameter D_1 is significantly greater than zero at the 5-percent level for all interest rates. Furthermore a Likelihood Ratio(LR) test of the null hypothesis $D_1 = 0$ is rejected for all rates at the 5-percent level. Estimates of D_1 range from 0.0013 to 0.0060 and, given that the mean of the conditional variance during this period ranged from 0.0225 to 0.0800, it can be seen that large innovations in the money supply would significantly increase the conditional variance of interest rates.

The results for the other periods are much different. For the pre-1979 data D is significantly different from zero at the 5-percent level in three cases; those cases being the 3-month, 6-month, and 20-year rates. Again the null hypothesis $D_1 = 0$ is tested using a LR test with the finding that the $D_1 = 0$ is rejected at the 5-percent level for only the 3-month and 6-month rates. What is interesting to note is that in each

Table 3.4

Daily GARCH Models with UM

January 1, 1978 - October 4, 1979

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} + D_1 |UM_t|$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0100 (.0050)	.0135 (.0039)	.0147 (.0034)	.0093 (.0035)	.0087 (.0020)	.0081 (.0022)	.0050 (.0014)
θ_1	.0988 (.0569)	.0868 (.0579)	---	.2192 (.0621)	---	---	---
α_0	.0000 (.0001)	.0001 (.0001)	.0002 (.0001)	.0009 (.0002)	.0002 (.0000)	.0001 (.0000)	.0002 (.0001)
α_1	.2834 (.0472)	.2065 (.0374)	.1425 (.0227)	.3650 (.0851)	.1962 (.0283)	.1994 (.0262)	.1736 (.0375)
β_1	.7180 (.0382)	.7420 (.0383)	.7999 (.0347)	.3067 (.1016)	.7635' (.0207)	.7299 (.0431)	.6663 (.0739)
D_1	.0015 (.0004)	.0008 (.0002)	.0002 (.0002)	.0001 (.0002)	-.0001 (.0001)	.0000 (.0001)	-.0002 (.0001)
LogL	461.715	584.014	617.582	734.425	785.347	839.108	934.019
Q(10)	14.313	5.896	8.238	6.739	11.924	9.627	14.877
$Q^2(10)$	7.563	2.405	4.127	1.514	2.374	6.618	7.046
M_3	0.234	0.412	0.988	1.673	0.255	-0.188	0.041
M_4	4.625	6.759	6.655	13.830	8.066	6.381	6.577
$LR_{D_1}=0$	10.988	6.625	0.079	4.788	0.632	0.000	3.004

Standar errors are in parentheses.

Table 3.5

Daily GARCH Models With UM

October 10, 1979 - October 6, 1982

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + D_1 |UM_t|$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0631	.0649	.0540	.0509	.0452	.0325	.0289
	(.0092)	(.0084)	(.0072)	(.0077)	(.0063)	(.0061)	(.0058)
θ_1	.0978	.0533	.0447	.0757	.0632	.0444	---
	(.0391)	(.0382)	(.0398)	(.0399)	(.0395)	(.0386)	
α_0	.0002	-.0006	.0001	-.0005	-.0002	.0003	.0001
	(.0009)	(.0005)	(.0006)	(.0004)	(.0003)	(.0004)	(.0002)
α_1	.0749	.0464	.0470	.0736	.0804	.0657	.0560
	(.0187)	(.0145)	(.0160)	(.0152)	(.0171)	(.0184)	(.0173)
β_1	.9025	.9302	.9181	.8842	.8811	.8946	.9182
	(.0237)	(.0226)	(.0273)	(.0191)	(.0215)	(.0270)	(.0234)
D_1	.0048	.0056	.0038	.0060	.0040	.0020	.0013
	(.0019)	(.0015)	(.0012)	(.0010)	(.0009)	(.0007)	(.0006)
LogL	-71.008	13.657	133.254	205.809	275.812	352.473	387.839
Q(10)	3.924	8.267	8.176	6.037	8.460	10.908	7.772
$Q^2(10)$	9.419	11.935	8.553	4.429	1.588	4.174	4.553
M_3	0.206	0.093	-0.184	-0.090	-0.170	-0.093	-0.004
M_4	4.545	4.756	4.374	4.370	4.046	3.697	3.632
$LR_{D_1=0}$	5.624	13.238	9.390	23.898	16.062	7.088	3.992

Standard errors in parentheses.

Table 3.6

Daily GARCH Models With UM

October 7, 1982 - January 26, 1984

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + D_1 |UM_t|$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0316 (.0046)	.0379 (.0046)	.0320 (.0043)	.0313 (.0042)	.0311 (.0038)	.0272 (.0038)	.0241 (.0037)
θ_1	.0821 (.0618)	---	.0547 (.0602)	---	---	---	---
α_0	.0000 (.0001)	.0001 (.0001)	.0001 (.0001)	.0003 (.0002)	.0003 (.0002)	.0001 (.0001)	.0001 (.0001)
α_1	.0170 (.0086)	.0199 (.0061)	.0205 (.0078)	.0254 (.0106)	.0290 (.0176)	.0330 (.0168)	.0431 (.0158)
β_1	.9739 (.0164)	.9641 (.0117)	.9506 (.0165)	.9375 (.0221)	.9201 (.0370)	.9335 (.0261)	.9261 (.0233)
D_1	.0002 (.0002)	.0000 (.0002)	.0000 (.0002)	-.0001 (.0003)	-.0002 (.0003)	.0001 (.0002)	.0003 (.0002)
LogL	365.294	342.506	348.587	322.708	341.712	357.389	369.864
Q(10)	11.968	5.870	2.799	5.548	5.285	5.477	5.018
$Q^2(10)$	8.440	13.465	4.999	3.217	1.076	1.562	4.399
M_3	0.342	-0.614	-1.186	-0.832	-1.034	-0.682	-0.296
M_4	5.294	7.379	11.249	10.014	11.720	8.489	6.064
$LR_{D_{1=0}}$	0.844	0.008	0.078	0.116	0.284	0.208	1.246

Standard errors in parentheses.

of these two cases, the corresponding parameter estimate D_1 is at least 3.2 times larger during the October 1979 to October 1982 period than during the pre-October 1979 period. From Table 3.6 estimates of D_1 reveal that while unanticipated money has an impact upon the level of interest rates, its impact upon the conditional variance is never statistically significant at the 5-percent level. Thus during those periods when the Federal Reserve was not targeting nonborrowed reserves, unanticipated money had little influence upon the conditional variance of interest rates. These results are not surprising given that during the pre-1979 and post-1982 periods the Federal Reserve was to some degree smoothing interest rates.

An alternative approach to measuring the impact of unanticipated money on the conditional variance of interest rates is to examine the total impact of a unit change in the unanticipated money on the conditional variance. Since the conditional variance, h_t, depends upon the amount of unanticipated money and the lagged value of the conditional variance, h_{t-1}, the total impact of a unit change in UM_t on the conditional variance can be measured by $D_1 \ (1-\beta_1)^{-1}$. Results, shown in Table 3.7, reveal a striking difference in the total impact of unanticipated money under alternative monetary policy regimes. Because D_1 is insignificant in most cases prior to October 1979 and after October 1982, the total impact on the conditional variance of unanticipated money in most cases is zero. The two exceptions are for the 3-month and 6-month rates in the pre-October 1979 period. Here the total impacts are significant but small, 0.0053 and 0.0031, respectively. In contrast, during the period of the nonborrowed reserve procedure the total impact is always significantly different from zero and large in relative magnitude. For

Table 3.7

Total Effect of a Unit Change In UM

On the Conditional Variance of Interest Rates

	1/1/78 - 10/4/79	10/10/79 - 10/6/82	10/7/82 - 1/26/84
3 Month	0.0053*	0.0492*	.0077
6 Month	0.0031*	0.0802*	.0011
12 Month	0.0010	.0464*	.0008
3 Year	0.0001	.0518*	-.0016
5 Year	-.0004	.0336*	-.0025
10 Year	.0001	.0190*	.0015
20 Year	-.0006*	.0159*	.0041

*Significantly different from 0 at the 5-percent level. In all remaining cases the variable was not significantly different from 0 at either the 5-percent or 10-percent level.

example as compared to the pre-October 1979 period, the total impact is 9 times geater during the nonborrowed reserve period for the 3-month rate and 25 times greater for the 6-month rate. The significance of the total impact is that unanticipated money not only increases the conditional variance on the day of the announcement, but has a persistent effect in that the conditional variance on forthcoming days is also increased. One possible explanation is that the changes in the level and volatility of interest rates change the expectations of economic agents. Given changes in expectations, economic agents will then take actions which result in further fluctuations in interest rates.

3.5 Symmetry of Responses

One extension of Tables 3.4 through 3.6 is to examine whether the response of the mean and conditional variance to unanticipated money is symmetrical. Roley (1982) examined a similar question with the finding that for the period prior to October 1979, only unusually large negative money shocks, occurring when money growth was above the target range, had an impact upon the three-month Treasury bill rate. For the period February 1980 to November 1982, positive money shocks caused an increase in the three-month rate while negative shocks were only significant when the money growth rate was below the range set by the Federal Reserve. Furthermore Roley found that positive money shocks caused a significant increase in interest rate volatility after February 1980. In this study Tables 3.8, 3.9, and 3.10 introduce both positive money shocks, UM_t^+, and negative money shocks, UM_t^-. The results show that prior to October 1979 only UM_t^+ had an impact on interest rates. Between October 1979 and October 1982 both UM_t^+ and UM_t^- led to increases in the mean of interest

Table 3.8

Daily GARCH Models with UM

January 1, 1978 - October 4, 1979

$$\Delta r_t = \gamma_1 UM_t^+ + \gamma_2 UM_t^- + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + D_1 |UM_t^+| + D_2 |UM_t^-|$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0450	.0373	.0324	.0138	.0141	.0151	.0087
	(.0091)	(.0051)	(.0051)	(.0029)	(.0029)	(.0035)	(.0025)
γ_2	-.0062	-.0004	.0030	.0031	.0036	.0028	.0020
	(.0047)	(.0045)	(.0036)	(.0029)	(.0023)	(.0027)	(.0019)
θ_1	.1331	.0897	---	.1977	---	---	---
	(.0564)	(.0556)		(.0504)			
α_0	.0001	.0001	.0002	.0001	.0001	.0001	.0001
	(.0001)	(.0001)	(.0001)	(.0000)	(.0000)	(.0000)	(.0000)
α_1	.2422	.2123	.1584	.1325	.2078	.2021	.1553
	(.0473)	(.0352)	(.0292)	(.0178)	(.0299)	(.0261)	(.0310)
β_1	.7383	.7480	.8017	.8451	.7692	.7558	.7326
	(.0389)	(.0348)	(.0351)	(.0108)	(.0207)	(.0353)	(.0549)
D_1	.0022	.0007	.0003	.0001	-.0001	.0001	-.0001
	(.0008)	(.0004)	(.0003)	(.0001)	(.0001)	(.0001)	(.0001)
D_2	.0005	.0005	.0001	-.0001	-.0001	.0000	-.0001
	(.0003)	(.0002)	(.0002)	(.0001)	(.0001)	(.0001)	(.0001)
LogL	476.853	595.741	626.638	739.534	789.794	845.275	936.486
Q(10)	16.028	6.374	7.968	10.448	13.255	10.848	14.724
$Q^2(10)$	8.151	2.916	2.783	3.686	2.175	6.953	6.461
M_3	0.145	0.383	0.944	1.144	0.207	-0.254	0.031
M_4	4.389	6.732	6.756	10.639	8.021	6.172	6.699

Table 3.9

Daily GARCH Models with UM

October 10, 1979 - October 6, 1982

$$\Delta r_t = \gamma_1 UM_t^+ + \gamma_2 UM_t^- + u_t$$

$$U_T = \epsilon_T + \theta_1 \epsilon_{T-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + D_1 |UM_t^+| + D_2 |UM_t^-|$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0593 (.0115)	.0621 (.0106)	.0477 (.0087)	.0467 (.0091)	.0422 (.0082)	.0299 (.0082)	.0276 (.0077)
γ_2	.0710 (.0172)	.0693 (.0150)	.0666 (.0139)	.0589 (.0121)	.0486 (.0104)	.0359 (.0093)	.0290 (.0087)
θ_1	.0983 (.0393)	.0537 (.0390)	.0418 (.0404)	.0752 (.0399)	.0643 (.0401)	.0455 (.0392)	---
α_0	.0003 (.0009)	-.0005 (.0006)	.0002 (.0006)	-.0005 (.0004)	-.0001 (.0004)	.0003 (.0004)	.0002 (.0002)
α_1	.0739 (.0191)	.0464 (.0144)	.0455 (.0161)	.0718 (.0149)	.0770 (.0171)	.0629 (.0181)	.0523 (.0160)
β_1	.9042 (.0240)	.9314 (.0221)	.9189 (.0281)	.8881 (.0186)	.8867 (.0216)	.9004 (.0260)	.9264 (.0221)
D_1	.0050 (.0020)	.0058 (.0016)	.0037 (.0012)	.0058 (.0011)	.0043 (.0010)	.0023 (.0008)	.0016 (.0006)
D_2	.0035 (.0031)	.0043 (.0022)	.0039 (.0019)	.0058 (.0015)	.0026 (.0012)	.0006 (.0010)	-.0003 (.0008)
LogL	-70.742	13.979	133.923	206.117	276.516	353.907	390.508
Q(10)	4.039	8.619	8.225	6.296	8.680	11.240	8.467
$Q^2(10)$	9.197	11.525	8.248	4.320	1.487	4.575	4.154
M_3	0.192	0.068	-0.171	-0.080	-0.193	-0.128	-0.036
M_4	4.536	4.727	4.416	4.389	4.083	3.686	3.602

Table 3.10

Daily GARCH Models with UM

October 7, 1982 - January 26, 1984

$$\Delta r_t = \gamma_1 UM_t^+ + \gamma_2 UM_t^- + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + D_1|UM_t^+| + D_2|UM_t^-|$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0346 (.0065)	.0386 (.0563)	.0298 (.0056)	.0331 (.0051)	.0319 (.0046)	.0284 (.0044)	.0259 (.0043)
γ_2	.0275 (.0060)	.0361 (.0063)	.0360 (.0063)	.0285 (.0071)	.0299 (.0065)	.0247 (.0072)	.0214 (.0070)
θ_1	.0818 (.0637)	---	---	---	---	---	---
α_0	.0000 (.0001)	.0001 (.0001)	.0002 (.0001)	.0003 (.0001)	.0003 (.0002)	.0001 (.0001)	.0001 (.0001)
α_1	.0179 (.0096)	.0202 (.0063)	.0176 (.0069)	.0253 (.0104)	.0290 (.0172)	.0324 (.0165)	.0444 (.0169)
β_1	.9591 (.0217)	.9578 (.0132)	.9540 (.0160)	.9405 (.0198)	.9228 (.0341)	.9344 (.0260)	.9212 (.0261)
D_1	.0004 (.0003)	.0001 (.0002)	.0001 (.0002)	.0000 (.0003)	-.0001 (.0003)	.0002 (.0003)	.0004 (.0003)
D_2	-.0001 (.0002)	-.0003 (.0002)	-.0003 (.0003)	-.0004 (.0004)	-.0004 (.0004)	.0000 (.0004)	.0000 (.0004)
LogL	366.385	343.667	349.543	323.153	341.882	357.540	370.328
Q(10)	12.412	6.180	2.834	5.356	4.918	5.314	4.860
$Q^2(10)$	9.444	14.093	5.755	3.348	1.105	1.624	4.666
M_3	0.325	-0.613	-1.191	-0.827	-1.030	-0.690	-0.300
M_4	5.427	7.209	11.244	9.881	11.624	8.416	5.955

rates. During this period UM_t^+ led to increases in the conditional variance for all rates while UM_t^- increased the conditional variance for the six-month, twelve-month, three-year, and five-year rates. For the conditional variance, UM_t^+ had a stronger impact than UM_t^-, thus suggesting that positive shocks had a greater impact on interest rate volatility. Finally, after October 1982, both UM_t^+ and UM_t^-, caused the mean of interest rates to rise, but had no impact upon the conditional variance.

3.6 Explaining Kurtosis

Tables 3.4 through 3.6 provide significant evidence of kurtosis which is not eliminated by the introduction of unanticipated money. One possibility is that there exist variables other than unanticipated money which can explain the kurtosis. One such variable may be the range of the federal funds rate. Prior to October 1979 the Federal Reserve smoothed interest rates by manipulating the federal funds rate; as such changes in the range over which the federal funds rate could vary would be expected to alter the conditional variance of interest rates and hence the degree of kurtosis. Following October 1979, if movement of the federal funds rate is to some degree still being used by the Federal Reserve, then a similar argument can be made for the period October 1979 to October 1982. Tables 3.11 and 3.12 introduce a change in the range of the federal funds rate into previously developed GARCH models. Here ΔFFR is defined as the change in the absolute value of the midpoint of the federal funds rate range. On those days during which the FOMC changed the range, ΔFFR takes on the value of the absolute change in the midpoint of the range; on all other days it equals zero. The results reveal that prior to October 1979 changes in the federal funds rate had a significant impact upon the conditional variance across the maturity spectrum. Furthermore the introduction of ΔFFR reduced, but didnot eliminate the kurtosis problem. For October 1979 to October 1982 a similar result occurs. Here ΔFFR is significant for all Treasury bill rates as well as for the 3-year bond rate. In this case however, the estimates of D_2 are lower than in the pre-October 1979 period. This suggests that while a change in the range of the federal funds rate had an impact on interest rate volatility after October 1979, its effect was more pronounced prior to October 1979. Finally like the pre-1979 period,

the inclusion of ΔFFR reduced the level of kurtosis.

Finally, in Tables 3.13 and 3.14 the GARCH models with UM_t and ΔFFR were estimated using the conditional t distribution. Given the difficulties with estimating V^{-1} in Chapter two, V^{-1} was estimated in Tables 3.13 and 3.14 without letting it iterate. The results support the use of the conditional t; all estimates lie between 0.0728 and 0.2062. Furthermore, likelihood ratio (LR) tests overwhelmingly reject the hypothesis $V^{-1} = 0$ in every case; here the relevant critical value at the five percent level equals 3.84. Thus, the use of the conditional t without iteration appears useful in modelling the behavior of interest rates.

3.7 Conclusion

Over the last decade a number of studies have addressed the issue of how money supply announcements influence the level of interest rates. Few studies have however addressed how these announcements influence the volatility of interest rates. In this chapter, using the conditional variance as a measure of interest rate volatility, the findings are that during the period when the Federal Reserve targeted nonborrowed reserves,

Table 3.11

Daily GARCH Models with UM and ΔFFR

January 1, 1978 - October 4, 1979

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + D_1 |UM_t| + D_2 \Delta FFR$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0097	.0126	.0133	.0068	.0087	.0082	.0042
	(.0048)	(.0039)	(.0036)	(.0032)	(.0020)	(.0018)	(.0015)
θ_1	.0961	.1010	---	.2224	---	---	---
	(.0558)	(.0560)		(.0563)			
α_0	.0001	.0002	.0002	.0008	.0002	.0001	.0001
	(.0001)	(.0001)	(.0001)	(.0001)	(.0000)	(.0000)	(.0000)
α_1	.2566	.1565	.1372	.4205	.2641	.1759	.2031
	(.0432)	(.0333)	(.0312)	(.0811)	(.0384)	(.0227)	(.0415)
β_1	.7123	.7190	.7504	.1429	.6719	.7381	.6568
	(.0337)	(.0402)	(.0416)	(.0489)	(.0294)	(.0406)	(.0660)
D_1	.0006	.0003	.0002	.0002	.0000	.0000	-.0001
	(.0001)	(.0001)	(.0001)	(.0001)	(.0001)	(.0001)	(.0000)
D_2	.0312	.0267	.0208	.0734	.0086	.0020	.0025
	(.0103)	(.0057)	(.0044)	(.0245)	(.0016)	(.0009)	(.0007)
LogL	466.255	595.694	629.113	767.640	791.053	841.970	938.297
Q(10)	17.385	9.573	11.909	11.507	13.290	9.974	15.875
$Q^2(10)$	9.556	4.970	4.069	1.673	2.448	5.837	9.769
M_3	0.092	-0.035	0.569	0.762	-0.201	-0.325	-0.181
M_4	4.332	6.211	5.704	10.069	6.879	6.287	6.147

Table 3.12

Daily GARCH Models With UM and ΔFFR

October 10, 1979 - October 6, 1982

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} + D_1 |UM_t| + D_2 \Delta FFR$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0613 (.0092)	.0621 (.0086)	.0520 (.0074)	.0498 (.0075)	.0444 (.0064)	.0326 (.0060)	.0289 (.0058)
θ_1	.0926 (.0387)	.0547 (.0378)	.0506 (.0390)	.0793 (.0392)	.0642 (.0393)	.0442 (.0387)	---
α_0	.0005 (.0008)	-.0002 (.0006)	.0003 (.0006)	-.0005 (.0004)	-.0001 (.0003)	.0002 (.0004)	.0001 (.0002)
α_1	.0602 (.0162)	.0368 (.0134)	.0325 (.0135)	.0643 (.0145)	.0752 (.0175)	.0657 (.0184)	.0574 (.0172)
β_1	.8972 (.0216)	.9186 (.0234)	.9117 (.0245)	.8830 (.0182)	.8807 (.0213)	.8977 (.0268)	.9220 (.0229)
D_1	.0046 (.0020)	.0057 (.0016)	.0042 (.0012)	.0063 (.0010)	.0040 (.0009)	.0019 (.0007)	.0011 (.0006)
D_2	.0283 (.0084)	.0173 (.0049)	.0123 (.0039)	.0069 (.0030)	.0030 (.0026)	-.0005 (.0021)	-.0012 (.0016)
LogL	-65.119	18.830	138.569	208.577	276.398	352.510	388.072
Q(10)	3.554	8.138	8.361	7.015	8.969	10.933	7.798
$Q^2(10)$	7.378	9.692	5.346	4.147	1.758	4.718	4.818
M_3	0.165	0.103	-0.171	-0.097	-0.174	-0.088	0.008
M_4	4.385	4.450	4.095	4.217	3.999	3.692	3.616

Table 3.13

Daily GARCH Models With UM and ΔFFR

January 1, 1978 - October 4, 1979

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t, V)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} + D_1 |UM_t| + D_2 \Delta FFR$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0063 (.0045)	.0123 (.0036)	.0143 (.0035)	.0066 (.0029)	.0078 (.0019)	.0075 (.0017)	.0051 (.0014)
θ_1	.0492 (.0551)	.0877 (.0506)	---	.1831 (.0531)	---	---	---
α_0	.0001 (.0001)	.0001 (.0001)	.0003 (.0002)	.0004 (.0001)	.0001 (.0000)	.0001 (.0000)	.0001 (.0001)
α_1	.2352 (.0512)	.1589 (.0460)	.1230 (.0429)	.4822 (.1108)	.1716 (.0420)	.1358 (.0396)	.0651 (.0332)
β_1	.7256 (.0421)	.7375 (.0527)	.7082 (.0710)	.1620 (.0681)	.7340 (.0485)	.7913 (.0540)	.7346 (.1123)
D_1	.0003 (.0001)	.0002 (.0001)	.0002 (.0001)	.0004 (.0001)	.0000 (.0001)	.0000 (.0001)	.0000 (.0001)
D_2	.0310 (.0140)	.0217 (.0081)	.0148 (.0059)	.0256 (.0094)	.0014 (.0016)	.0013 (.0010)	.0017 (.0010)
V^{-1}	.1176 (---)	.1704 (---)	.1608 (---)	.2062 (---)	.1803 (---)	.1717 (---)	.1693 (---)
LogL	477.270	620.719	657.090	832.093	829.733	871.527	971.450
Q(10)	17.595	9.457	11.619	12.644	13.241	10.616	15.547
$Q^2(10)$	9.251	3.992	4.427	2.080	2.272	6.150	11.968
M_3	0.190	0.001	0.621	1.057	0.041	-0.354	-0.381
M_4	4.670	6.236	5.918	11.465	7.512	6.612	7.196

Table 3.14

Daily GARCH Models With UM and ΔFFR

October 10, 1979 - October 6, 1982

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t, V)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + D_1 |UM_t| + D_2 \Delta FFR$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0565	.0587	.0513	.0458	.0418	.0305	.0273
	(.0085)	(.0078)	(.0068)	(.0065)	(.0058)	(.0058)	(.0055)
θ_1	.0946	.0489	.0443	.0664	.0684	.0492	---
	(.0373)	(.0359)	(.0377)	(.0376)	(.0384)	(.0378)	
α_0	.0009	-.0001	.0003	-.0006	-.0001	.0003	.0001
	(.0012)	(.0008)	(.0008)	(.0004)	(.0004)	(.0004)	(.0003)
α_1	.0485	.0297	.0289	.0660	.0721	.0643	.0545
	(.0182)	(.0150)	(.0153)	(.0187)	(.0209)	(.0215)	(.0193)
β_1	.9025	.9242	.9140	.8914	.8857	.8973	.9276
	(.0285)	(.0293)	(.0309)	(.0241)	(.0275)	(.0338)	(.0257)
D_1	.0043	.0059	.0044	.0052	.0036	.0019	.0010
	(.0025)	(.0022)	(.0016)	(.0013)	(.0012)	(.0009)	(.0006)
D_2	.0248	.0126	.0096	.0049	.0017	-.0004	-.0010
	(.0109)	(.0063)	(.0046)	(.0034)	(.0030)	(.0026)	(.0019)
V^{-1}	.1252	.1271	.1055	.1120	.0999	.0789	.0728
	(---)	(---)	(---)	(---)	(---)	(---)	(---)
LogL	-51.358	34.244	149.861	222.766	286.649	357.592	392.418
Q(10)	3.946	7.205	8.788	7.862	9.027	11.124	7.992
$Q^2(10)$	7.224	9.340	5.389	4.118	1.662	4.705	4.909
M_3	0.184	0.105	-0.180	-0.054	-0.159	-0.088	0.017
M_4	4.505	4.654	4.144	4.328	4.039	3.705	3.642

unanticipated money had a significant impact not only on the level of interest rates but also on their volatility. This impact was persistent in that unanticipated money increased volatility not only on the day of the announcement, but also on subsequent days. Furthermore while unanticipated money had a significant but weaker effect on the level of interest rates both before October 1979 and after October 1982, it had little impact upon interest rate volatility during these periods.

ENDNOTES

1. A summary of the literature on money supply announcements and asset prices can be found in Sheehan(1985).

2. Roley(1982) is one of the few papers which directly addresses the issue of how money supply announcements influence interest rate volatility. Here volatility is measured using the root-mean-square error of the 3-month Treasury bill yield.

3. This is the conclusion of Sheehan(1985).

4. Again this is the conclusion of Sheehan(1985).

5. Belongia and Sheehan(1987) note that the dependent variable, the change in the level of interest rates, may be misspecified. Rather they note that it may be more theoretically correct to specify the dependent variable as:

   $r_t - {}_{t-1}E(r_t)$

   where ${}_{t-1}E(r_t)$ is the expected level of the treasury bill at time t.

6. Some authors refer to this as the policy anticipations effect.

7. The assumptions are outlined in the expected liquidity effect and the two forthcoming theories are outlined in Sheehan(1985).

8. All three of the theories outlined in this section argue that an unexpected increase in the money supply will cause interest rates to rise. As such the response of interest rates can not be used to differentiate between the theories. To do this other assets such as stocks or foreign currencies must also be analyzed. Only by using multiple assets is it possible to see which theory is "correct." Cornell(1983) attempts to do this but is unable to come to a conclusive answer.

9. For a detailed analysis of this theory see Nichols, Small, and Webster(1983).

10. In this case the present level of money demand depends on expected future output. See Fama(1982).

11. See page 1065.

12. See pages 351 - 352.

13. The unexpected change in the money supply($ billion) is simply defined as the actual change in the money supply minus the change which was expected. Here the expected change is simply the median

of the Money Market Services, Inc. survey. The actual change is the first announced value minus the first revised estimate of the money supply from the previous week.

14. During the period of this study the definition of money, M1, changed. Again I follow the work of Hafer(1986). Prior to February 1980 money is defined as old M1. For the period February 1980 through November 1981, M1B is employed as money. finally for the post-November 1981 data the current definition of M1 is employed.

15. The equations in this section were also run with a constant in the mean. In no case was the constant significant at the 5-percent or 10-percent level.

16. Prior to February 8, 1980 the money supply data was announced on Thursday. After that the announcements were made on Friday until February 1984 at which point announcements were again made on Thursdays. Some studies, such as Roley(1982), differentiate between the Thursday and Friday announcements. By using daily data there is divide the sample according to the day on which the announcement is made.

17. It is also necessary to assume this so as to insure that the conditional variance is positive. If this assumption is not made, then a sufficiently large unanticipated decrease in the money supply may result in the conditional variance to be negative.

18. Money Market Services data was available beginning January 1, 1978. As such this date is used as the starting point for the pre-October 1979 analysis. Phillips-Perron test statistics and diagnostic tests suggest that prior to the introduction of unanticipated money, interest rates over the period January 1, 1978 to October 4, 1979 are best approximated by either a random walk process or an ARIMA(0,1,1) model.

19. Roley(1987) finds that with regard to the 3-month Treasury bill rate, $\gamma_1 = 0.0078$ for the pre-October 1979 period and 0.0587 for the early portion of the nonborrowed reserve period.

20. See Roley(1986).

CHAPTER FOUR

MONETARY CONTROL PROCEDURES AND INTEREST RATE VOLATILITY

4.1 Introduction

The choice of an operating procedure by the Federal Reserve for monetary policy has important implications for the volatility of interest rates, particularly in the short-run. Over the last two decades, the Federal Reserve has adopted two or at most three different operating procedures. While different procedures have been employed, since the mid-1970's the long-run focus of Federal Reserve policy has been upon control of the money supply. Prior to October 1979, monetary policy involved maintaining the federal funds rate within a narrow range over short-run intervals. By moving interest rates along what was believed to be a stable and predictable short-run money demand function, the Federal Reserve could control the money supply. However such a procedure proved faulty in that rigid adherence to a federal funds rate target resulted in the Federal Reserve consistently missing the target for the money stock. Thus the procedure was deemed inadequate in that it's ability to control the money supply seemed lacking.

Beginning in October 1979, monetary policy was switched from short-run targeting of the federal funds rate to a procedure which concentrated on attainment of a target level of nonborrowed reserves. This procedure was in place until October 1982 at which time Federal Reserve policy moved to indirectly control the federal funds rate by targeting the level of borrowed reserves. Given that the Federal Reserve can not independently determine both interest rates and a monetary aggregate, the adoption of a reserve-oriented operating procedure, such as the nonborrowed reserve procedure adopted in October 1979, would be expected to lead to greater control over the money stock but with increased volatility of interest rates, particularly in the short-run. Critics contend however that the additional interest rate volatility induced by the move to a such a procedure would impose substantial costs on the economy. For example Friedman(1982) and Brimmer(1983) argue that interest rate volatility reduces economic efficiency by interfering with the efficient functioning of capital markets. Evans(1984,1985), Tatom(1984), and Dutkowsky(1987) further argue that interest rate volatility reduces real output by reducing either aggregate demand or aggregate supply.

The purpose of this chapter is, given recent historical experience, to examine the volatility of interest rates under alternative monetary control procedures. This is accomplished by using the GARCH(p,q) process to examine interest rate volatility over the various monetary policy regimes, with particular emphasis on the period October 1979 through October 1982. This chapter proceeds as follows. The next section reviews the literature on how the level of interest rate volatility was effected by the Federal Reserve's choice of an operating procedure. Here special emphasis is given to empirical estimates of interest rate volatility and the techniques used to derive these estimates. In the third section a GARCH(p,q) model is utilized to derive estimates of the conditional variance of interest rates. Using these estimates as a measure of volatility, the time-series behavior of interest rates is compared both across and within operating regimes. Finally the fourth section of this chapter examines the impact of unanticipated money on interest rate volatility over the nonborrowed reserve operating period. Some literature suggests that because of the uncertainty caused by the introduction of a new policy regime, the response of interest rates to unanticipated money showed considerable temporal variation over the period October 1979 to October 1982. Using the GARCH(1,1) model with unanticipated money developed in the previous chapter, the question of the magnitude of the interest rate response, and its impact on interest rate volatility, over the nonborrowed reserve targeting period is examined.

4.2 Previous Literature

The choice of a monetary policy operating procedure has important implications for the volatility of interest rates, particularly short-term rates. Relative to alternative operating procedures, a reserve-oriented procedure implies greater levels of interest rate volatility, at least in the short run. In particular, subsequent to the adoption of the nonborrowed reserves operating procedure on October 6, 1979, interest rates, particularly short-term rates, became much more volatile. One popular technique for measuring this volatility has been the use of the sample standard deviation or variance. As such a number of studies have estimated the level of interest rate volatility existing under alternative operating procedures. Roley(1983), measuring the change in the three-month Treasury bill rate from 3:30 p.m. to 5:00 p.m. on the day the Federal Reserve announces the money supply, finds that the sample variance for this 1 1/2 hour interval each week is over thirty times greater for the period October 1979 to October 1982 than for the period prior to October 1979. Huizinga and Leiderman(1987), estimating interest rate volatility by the use of the sample standard deviation, find that on money supply announcement days, interest rates are almost four times more volatile during the October 1979 to October 1982 period than in the subsequent sixteen months. Walsh(1982), employing the standard deviation of weekly interest rates, finds that after the October 1979 regime change, three- and six-month Treasury bills, and twenty-year bond rates became 4.3, 5.9, and 5.0 times more volatile, respectively. Finally Johnson(1981), employing a similar technique concludes, "In the year since October 6, 1979, the standard deviation of the weekly change in rates on Treasury securities of various maturities has been three to four times greater than in the previous 11 years."[1] As noted in the preceding chapters however, the techniques utilized in these studies implicitly constrain volatility to be constant over the chosen time interval, and as such are of limited value as a measure of volatility.

Another popular technique to study the change in interest rate volatility as a result of changing operating procedures has been the use of a moving standard deviation or variance. For example, Spindt and Tarhan(1987), using standard deviations from a five-week centered moving average of weekly interest rate data find that interest rates across the maturity spectrum were between 2.8 and 3.6 times more volatile from October 1979 to October 1982 than for the nine years preceding the policy change. In a similar fashion they find that with the October 1982 change in operating procedure, interest rate volatility across the maturity spectrum was reduced by at least forty percent. Johnson(1981) estimated interest rate volatility using the standard deviation from a centered moving average of weekly interest rates. From a three-week, five-week, and seven-week centered moving average, she found that three-month Treasury bills were 2.4 to 2.8 times more volatile for the period October 1979 to September 1980 than for the period January 1968 to September 1979. Long-term interest rates also exhibited greater volatility for the same period as five-year Treasury notes were 2.8 to 3.1 times more volatile and twenty-year bond rates were 3.0 to 3.7 times more volatile. As noted in Chapter One however, while the use of such a technique in estimating volatility negates the homoskedasticity constraint implied by using the sample variance, such a technique possesses few desireable econometric properties. Furthermore interest rate volatility is extremely sensitive to the specification of the moving average process. An example of this can be found in Tatom(1984) who, in studying the relationship between output, money, and interest rate volatility, employs both a twenty-quarter and four quarter moving standard deviation of the Aaa bond yield from 1926 to 1984. These two estimators of volatility yield very different results as the four-quarter standard deviation shows volatility increased in late 1979 and peaked in the first half of 1980. The twenty-quarter moving standard deviation reveals instead that interest rate volatility began increasing in late 1978 and increased every quarter until the first half of 1982 at which time it peaked. Thus the use of a moving standard deviation or variance also seems inappropriate.

As noted in a number of studies, unanticipated money also played a key role in the volatility of interest rates over the various policy regimes. Roley(1982), employing a variance decompostion approach, found that a change in how market agents responded to unanticipated money accounted for 34 percent of the increased volatility of the three-month Treasury bill yield after the October 1979 change in operating procedure. From this result he concludes, "Moreover, in comparison to the period before the introduction of the reserve-aggregate monetary control procedure, interest rates would nevertheless have recorded a substantial increase in volatility even if money growth happened to fall within its long-run range."[2] The same conclusion is also reached by Evans(1983) who, also employing the variance decomposition approach, concludes that 28.2 percent of the increased volatility of the three-month Treasury bill rate after October 1979 can be accounted for by an increased responsiveness by agents to unanticipated changes in the money supply. Evans also extends the analysis to long-term rates with the finding that increased responsiveness accounts for 25.7 percent of the increased volatility in five-year rates, 27.8 percent in ten-year rates, and 27.4 percent in twenty-year rates.

Inherent in much of the research outlined in the preceding paragraphs is the implicit assumption that interest rate volatility changes across monetary policy operating procedures, but is homogeneous under a given operating procedure. But this need not be so. Rather economic agents may learn of the policy change over time; this in turn having important implications for not only the volatility of interest rates but also for the long-run effects of a monetary aggregate targeting regime. To date there is some evidence to suggest that learning may play an important role. For example, Lewis(1988) has shown that uncertainty regarding U.S. monetary policy in the late 1970's and early 1980's resulted in increased conditional variances of the forecast errors of exchange rates. Using a Bayesian learning model, Lewis shows that as agents gathered more information about U.S. monetary policy during this period, the conditional variance of forecast errors slowly and systematically decreased. Rasche(1986) notes the possible impact of learning on interest rates during the October 1979 to September 1982 period when he states, "the 1979 switch to the nonborrowed reserves procedure was one without precedent in the history of the Federal Reserve System, and it may have prompted a considerable period of learning for market participants."[3] Instead of employing the level of interest rates, Rasche uses the change in the natural log of interest rates so as to measure interest rate changes on a percentage basis, the argument here being that the previously outlined measures of volatility using levels overstates the increase in volatility subsequent to the October 1979 procedure change. Measuring interest rate volatility as the standard deviation of percentage changes in weekly interest rate data, he finds that in the year following the adoption of the nonborrowed reserve procedure, interest rate volatility was 1.8 to 2.2 times larger than during the previous ten years. In the final two years of the nonborrowed reserve procedure, October 1980 to September 1982, interest rate volatility decreased substantially. In fact in the final year of the nonborrowed reserve procedure, interest rate volatility across the maturity spectrum was 8 to 31 percent lower than in the first year of the procedure.

Some studies of money supply announcements also suggest that learning may have had important implications for the volatility of interest rates during the nonborrowed reserves procedure. Cornell(1983) and Roley(1982,1983) argue that uncertainty regarding Federal Reserve policy after October 6, 1979 played an important role in the behavior of interest rates. Loeys(1985), utilizing a moving regression approach on three-month Treasury bill rates finds that the magnitude of the interest rate response to unanticipated money increased significantly after October

1979. Loeys further notes that the response of the three-month rate varies substantially over the October 1979 to October 1982 period. In 1979 the response of interest rates to unanticipated money rose dramatically, then in early 1981 the response began a systematic decline which would continue even after the abandonment of the nonborrowed reserve procedure in October 1982. Loeys thus concludes that a policy change, such as that which occurred on October 6, 1979, causes an initial period of increased uncertainty for economic agents, thereby raising the responsiveness of interest rates to money supply announcements. Over time, as uncertainty is reduced, the responsiveness of interest rates to unanticipated money declines. Belongia, Hafer, and Sheehan(1988) utilize a time-varying parameter approach to the question of interest rate responsiveness to unanticipated money over the period February 1978 to November 1983. Like Loeys(1985) they find considerable variation in the response during the October 1979 to September 1982 period. However their estimates show that the response does not substantially increase following the October 1979 policy shift; rather the response of interest rates to unanticipated money peaks in mid-1981 and then subsequently declines throughout the remainder of the nonborrowed reserve period. They therefore argue that a host of other factors, such as the 1980 credit controls or the 1981 introduction of NOW accounts, rather than uncertainty regarding the October 1979 regime change, may have been responsible for the temporal instability of interest rate responsiveness. Finally Baxter(1988), in examining short-term interest rates, concludes that while the interest rate response to unanticipated money is not consistent with a simple Bayesian learning model, learning may nonetheless play a key role in understanding the temporal behavior of interest rate volatility.

4.3 Conditional Variance Estimates

To investigate the homogeneity of interest rate volatility a GARCH(p,q) model of interest rates across the maturity spectrum was developed. In order to examine interest rate volatility over a long period of time, weekly data was utilized over the entire period of our sample, January 1974 to March 1988.[4] Following Nelson and Plosser(1982), Perron(1988), and Schwert(1987), a linear rather than log-linear specification for the conditional mean equation was used.[5] Extending the model employed in previous chapters, the resulting GARCH model took the form:

$$
\begin{aligned}
&\Delta r_t = u_t \\
(4.3.1)\qquad &u_t = \epsilon_t + \sum_{i=1}^{q} \theta_i \, \epsilon_{t-i} \\
&\epsilon_t | \Omega_{t-1} \sim N(0, h_t) \\
&h_t = \alpha_0 + \alpha_1 \, \epsilon_{t-1}^2 + \beta_1 \, h_{t-1} + \rho_1 \, REG_t
\end{aligned}
$$

where REG_t is a regime dummy. Both theory and empirical evidence suggest that interest rate volatility rose during the nonborrowed reserve regime.[6] Thus REG_t takes on a value of one during the October 1979 to October 1982 period and zero at all other times. From Table 4.1 the GARCH(1,1) model is again found to be appropriate. In all cases the parameters α_1 and β_1 are significant; all $Q^2(10)$ statistics are well below the critical value thus suggesting that the conditional heteroskedasticity in weekly interest rates has been accounted for. Furthermore the parameter estimates for the regime dummy, ρ_1, are all significant at the five-percent level, thus confirming the impact of the October 1979 nonborrowed reserve experiment on the volatility of interest rates. Using the conditional variance as an estimator of volatility, the estimated conditional variance values, $\hat{h}_t$, derived from the GARCH(1,1) model were examined. Specifically the behavior of $\hat{h}_t$ was examined according to the various monetary policy regimes; Table 4.2 shows the mean values of $\hat{h}_t$ over a variety of periods. Examination of Table 4.2 reveals that for the period January 1974 through September 1979, the mean estimates of the conditional variance ranged from 0.009 to 0.079. As expected the largest values of the conditional variance occur at the short end of the maturity spectrum with estimates being smallest on long-term rates. For the October 1979 through September 1982 period, the average value of the conditional variance rose substantially, ranging from 0.156 on the twenty-year rate to 0.702 on the three-month Treasury bill. For this period the average conditional variance was 8.9 times greater for the three-month Treasury bill, 10.6 times greater for the six-month rate, and 8.4 times larger for the one-year bill than in the preceding period. A similar analysis for long-term rates reveals that $\hat{h}_t$ for the three-year, five-year, ten-year, and twenty-year are 8.7, 8.8, 13.4, and 17.3 times greater, respectively, during the October 1979 to September 1982 period. Beginning in October 1982 the Federal Reserve de-emphasized the role of the money stock in monetary policy. Thus for the period October 1982 through January 1984 the mean value of the estimated conditional variance revealed a substantial decline. For the three-month, six-month, and one-year Treasury bills the average $\hat{h}_t$ fell to 0.059, 0.051, and 0.048, respectively. These results show short-term rates to be considerably less volatile during the borrowed reserve procedure than during the nonborrowed reserve procedure. The same result is true of long-term rates; $\hat{h}_t$ equals 0.048, 0.048, 0.051, and

Table 4.1

Weekly GARCH Model

January 1974 - March 1988

$$\Delta r_t = u_t$$

$$u_t = \epsilon_t + \Sigma \theta_i \epsilon_t$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} + \rho_1 REG_t$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
θ_1	.0568 (.0430)	.0509 (.0431)	.0888 (.0413)	.1030 (.0436)	.0883 (.0435)	.0721 (.0406)	.1194 (.0406)
θ_2	-.0468 (.0419)	.0439 (.0455)	.0429 (.0403)	---	---	---	---
θ_3	-.0138 (.0408)	.0132 (.0377)	.0561 (.0356)	---	---	---	---
θ_4	.1047 (.0415)	.0706 (.0354)	---	---	---	---	---
α_0	.0033 (.0006)	.0055 (.0012)	.0055 (.0011)	.0069 (.0018)	.0038 (.0009)	.0012 (.0003)	.0006 (.0002)
α_1	.3020 (.0303)	.2209 (.0304)	.1386 (.0329)	.2019 (.0345)	.1871 (.0291)	.2049 (.0281)	.1711 (.0285)
β_1	.6624 (.0254)	.6428 (.0469)	.7046 (.0494)	.6030 (.0670)	.6976 (.0437)	.7576 (.0311)	.7934 (.0286)
ρ_1	.0783 (.0191)	.0729 (.0188)	.0530 (.0097)	.0545 (.0128)	.0294 (.0061)	.0136 (.0035)	.0106 (.0025)
LogL	-48.212	-2.010	33.047	57.356	110.577	228.857	294.596
Q(10)	9.356	5.290	10.221	11.409	11.777	12.805	12.681
$Q^2(10)$	2.773	5.140	3.625	6.754	9.258	10.596	4.857
M_3	-0.394	-0.234	-0.215	-0.276	-0.393	-0.330	-0.021
M_4	7.124	5.886	7.823	5.501	5.758	5.528	4.444

Table 4.2

Conditional Variance Averages
Weekly Interest Rates

January 1974 - March 1988

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
Pre Oct 79:							
Jan 74-Sept 79	0.079	0.043	0.038	0.031	0.025	0.013	0.009
NBR Regime:							
Oct 79-Sept 82	0.702	0.454	0.321	0.270	0.221	0.174	0.156
Oct 79-Sept 80	0.662	0.430	0.322	0.302	0.232	0.176	0.160
Oct 80-Sept 81	0.860	0.520	0.327	0.258	0.221	0.170	0.151
Oct 81-Sept 82	0.583	0.411	0.314	0.251	0.212	0.175	0. 158
Oct 79-Mar 80	0.576	0.363	0.288	0.296	0.216	0.164	0.151
Apr 80-Sept 80	0.748	0.497	0.355	0.307	0.247	0.188	0.169
Oct 80-Mar 81	0.909	0.546	0.353	0.298	0.264	0.212	0.177
Apr 81-Sept 81	0.811	0.493	0.300	0.218	0.177	0.129	0.124
Oct 81-Mar 82	0.592	0.415	0.329	0.281	0.248	0.212	0.196
Apr 82-Sept 82	0.573	0.407	0.299	0.222	0.176	0.138	0.121
Post Oct 82:							
Oct 82-Jan 84	0.059	0.051	0.048	0.048	0.048	0.051	0.043
Feb 84-Mar 88	0.044	0.041	0.033	0.041	0.042	0.043	0.034

0.043 for the three-year, five-year, ten-year, and twenty-year rates, respectively.

It may be erroneous to assume that interest rate volatility is homogeneous under a given monetary operating procedure. In particular if learning occurs as postulated by some of the previously outlined studies, then following the introduction of a new procedure, such as that which occurred on October 6, 1979, interest rate volatility would be expected to intially increase due to uncertainty and then decrease from its higher level as economic agents learn of the new procedure. To see whether the pattern of interest rate volatility under the nonborrowed reserve procedure is consistent with that of learning, the estimated values of the conditional variance, derived from (4.4.1), are examined over the period October 1979 to September 1982. Figures 4.1 through 4.9 reveal the weekly estimates of the conditional variance; Table 4.2 summarizes these results. Examination of these results reveal a number of interesting points. First those studies which assume interest rate volatility to be constant under the nonborrowed reserves procedure are clearly incorrect. Rather between October 1979 and September 1982 interest rates exhibited periods of extreme volatility as well as periods of relative tranquility. Second, the results reveal that following the introduction of the nonborrowed reserve regime, interest rate volatility increased dramatically. However while interest rate volatility during this period increased, examination of the conditional variance estimates across the maturity spectrum reveals that volatility was not continuously increasing and that it did not peak until 1980 or early 1981. By this time other factors, such as the implementation of credit controls by the Carter administration in March

1980 or the introduction of NOW accounts also had an impact upon the time-series behavior of interest rates.

Another possible explanation for the sharp, periodic increases in the conditional variance is systematic changes in the federal funds rate by the FOMC. In Figures 4.1 through 4.3 vertical lines are drawn during the weeks when the FOMC altered the range of the federal funds rate. If changes in the range at the federal funds rate by the Federal Reserve represent a change in monetary policy, then these changes would be expected to influence not only the level, but also the volatility of interest rates. As evidenced by Figures 4.1 through 4.3, despite the fact that the Federal Reserve was targeting a monetary aggregate, fundamental shifts in the range of the federal funds rate had a strong impact on the conditional variance of interest rates. Tables 4.3 and 4.4 examine this point where ρ_2 measures the impact of changes in the midpoint of the federal funds rate range on the conditional variance. Here estimates range from 0.0452 on the three-month rate to 0.0015 on the ten-year rate. It is interesting to note that estimates of ρ_2 are significant for short rates but not long rates. This suggests that alterations in the federal funds rate cause an increase in the volatility of short-term rates, with little or no impact on the volatility of long-term rates. In conclusion while the introduction of the nonborrowed reserve regime undoubtedly altered the level of interest rate volatility, it is impossible to state that the increase in volatility is solely the result of the change in operating procedure. Rather other exogenous factors exerted a significant influence on interest rate volatility during this period.

Finally the conditional variance estimates are examined to see whether or not they are consistent with the pattern of interest rate volatility which would be expected if learning were occurring. Examination of Figures 4.1 through 4.3 reveals that after the peak in 1980 or early 1981, conditional variance estimates show a general downward trend with the exception of the late-1981 or early-1982 period during which time the conditional variance again increases substantially. The increases in late-1981 or early-1982 may again be due to changes in the range of the federal funds rate. Table 4.2 provides further evidence that not only did the conditional variance increase dramatically during the October 1979 to September 1982 period, but over that period the conditional variance exhibited a decling pattern. To see this note that during the first year of the nonborrowed reserve regime, the mean of the conditional variance estimates ranges from 0.662 on the three-month Treasury bill to 0.160 on the twenty-year bond rate. During the second year of the new regime, the average level of $\hat{h}_t$ decreases for all long-term rates. In the final year of the nonborrowed reserve regime, the mean value of $\hat{h}_t$ is again lower than existed in the first year. For all but the ten-year interest rate, the mean value of the conditional variance is also lower than that which existed in the second year of the new regime. Finally, the final six months of the nonborrowed reserve policy, April 1982 to September 1982, reveal that the conditional variance estimates in these cases are substantially lower than those which existed for the final year as a whole, and are at least five percent lowerin every case except the three- and six-month rates than existed in the first year of the policy. For the later rates the mean of the final six months is only 1.7 and 1.0 percent lower, respectively.

The results of these tests are generally consistent with the findings of previous studies such as Rasche(1986). Like previous studies

Figure 4.1
Weekly Conditional Variance Estimates

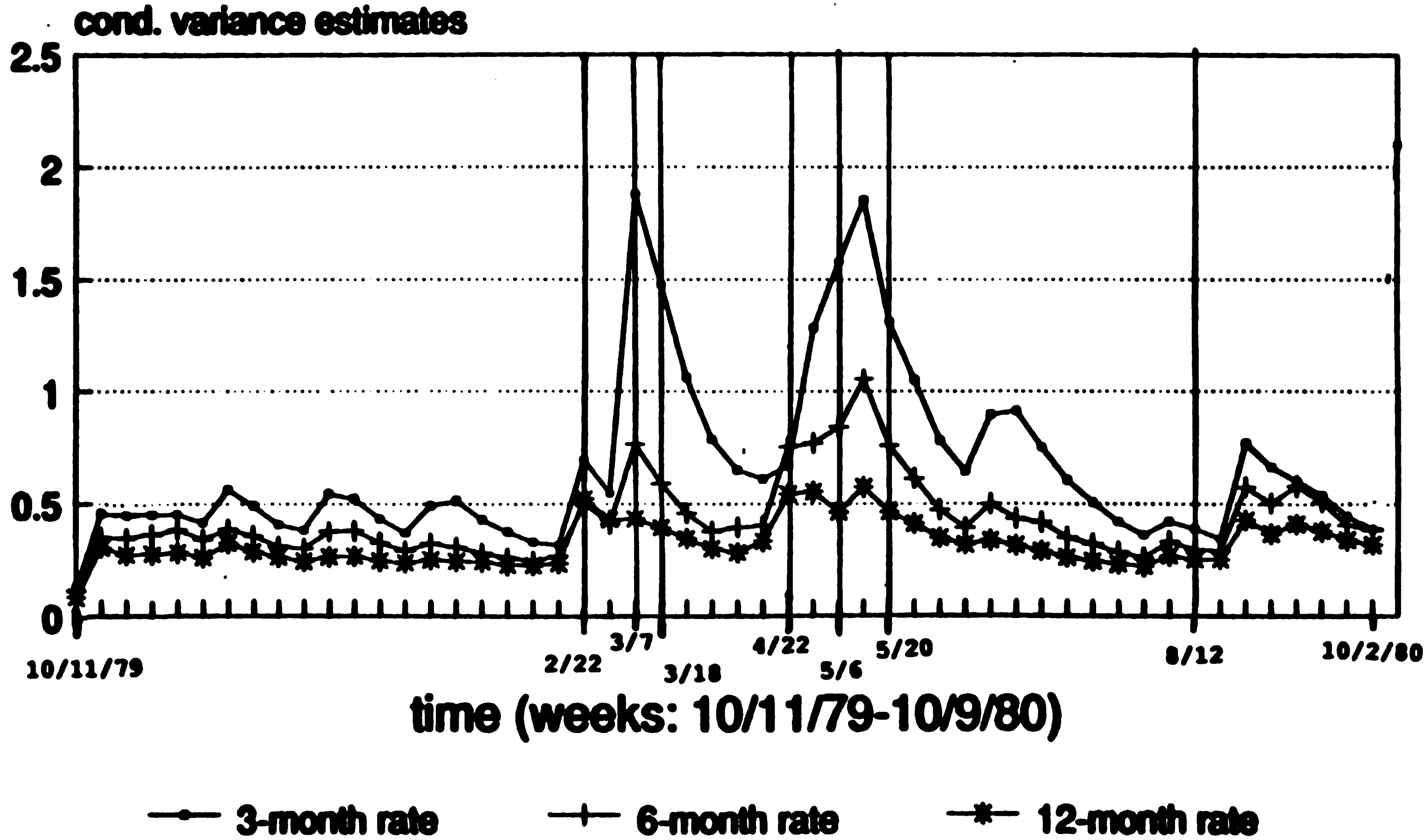

Figure 4.1 (cont'd)

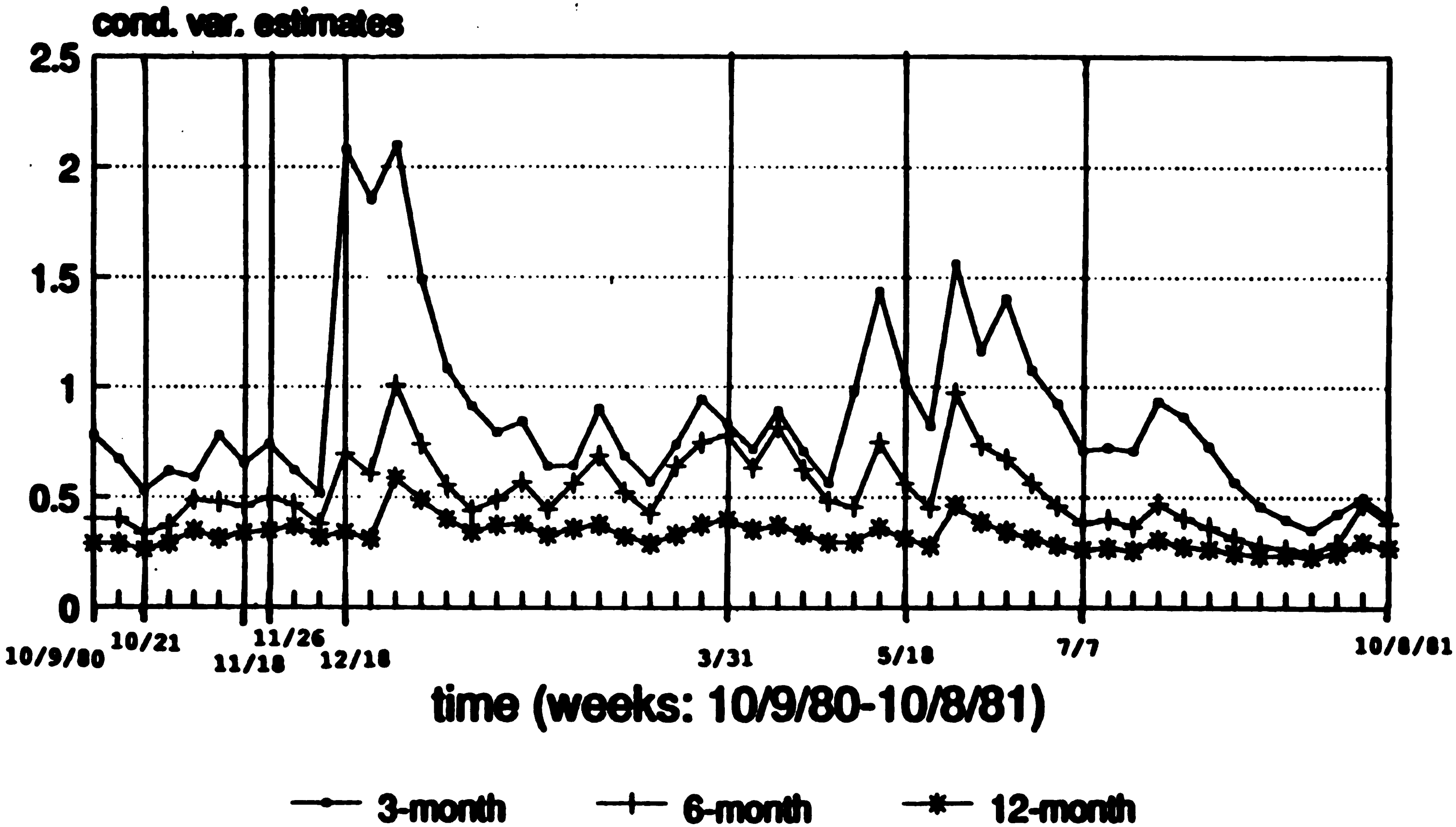

Figure 4.1 (cont'd)

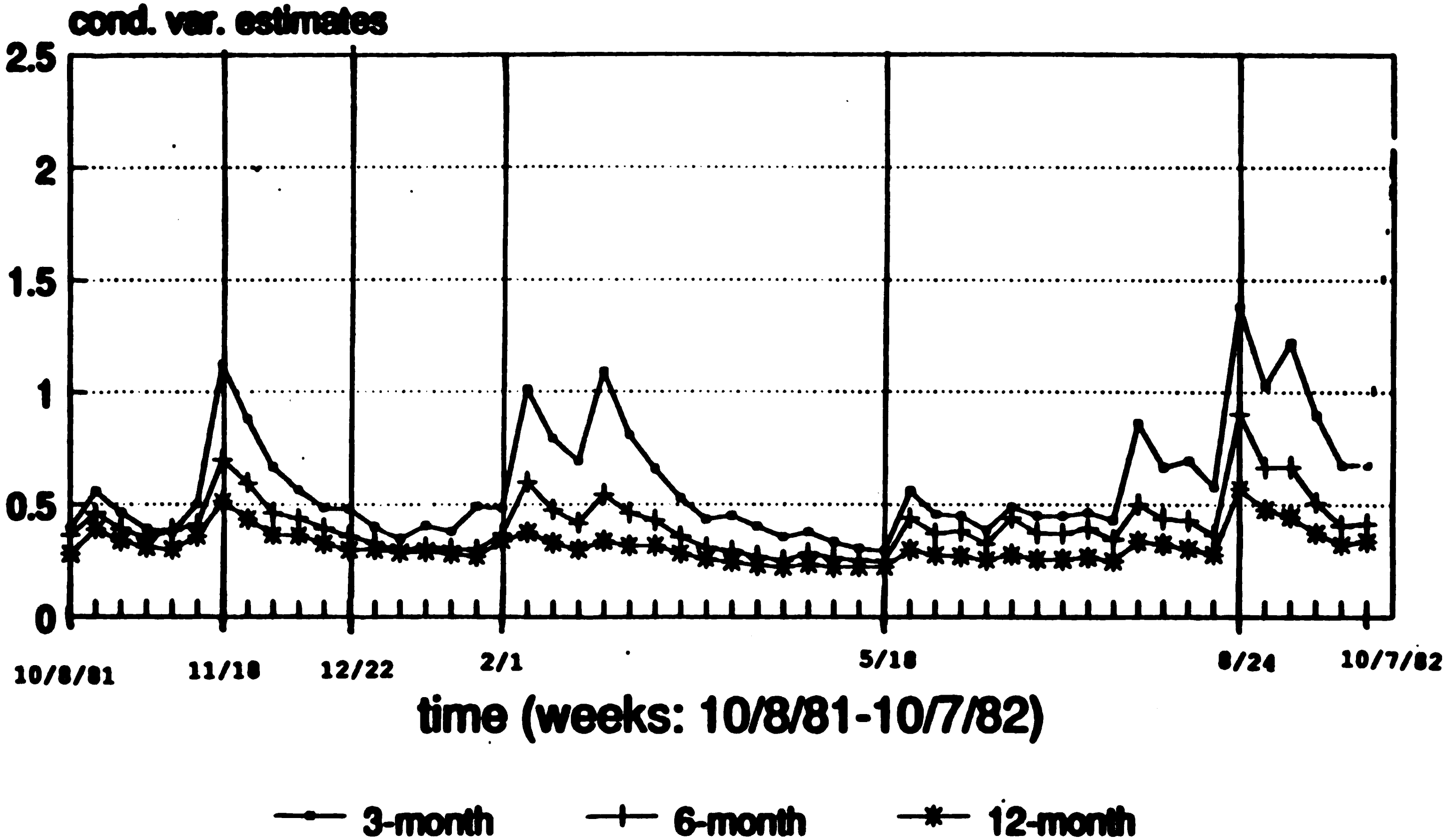

Figure 4.2
Weekly Conditional Variance Estimates

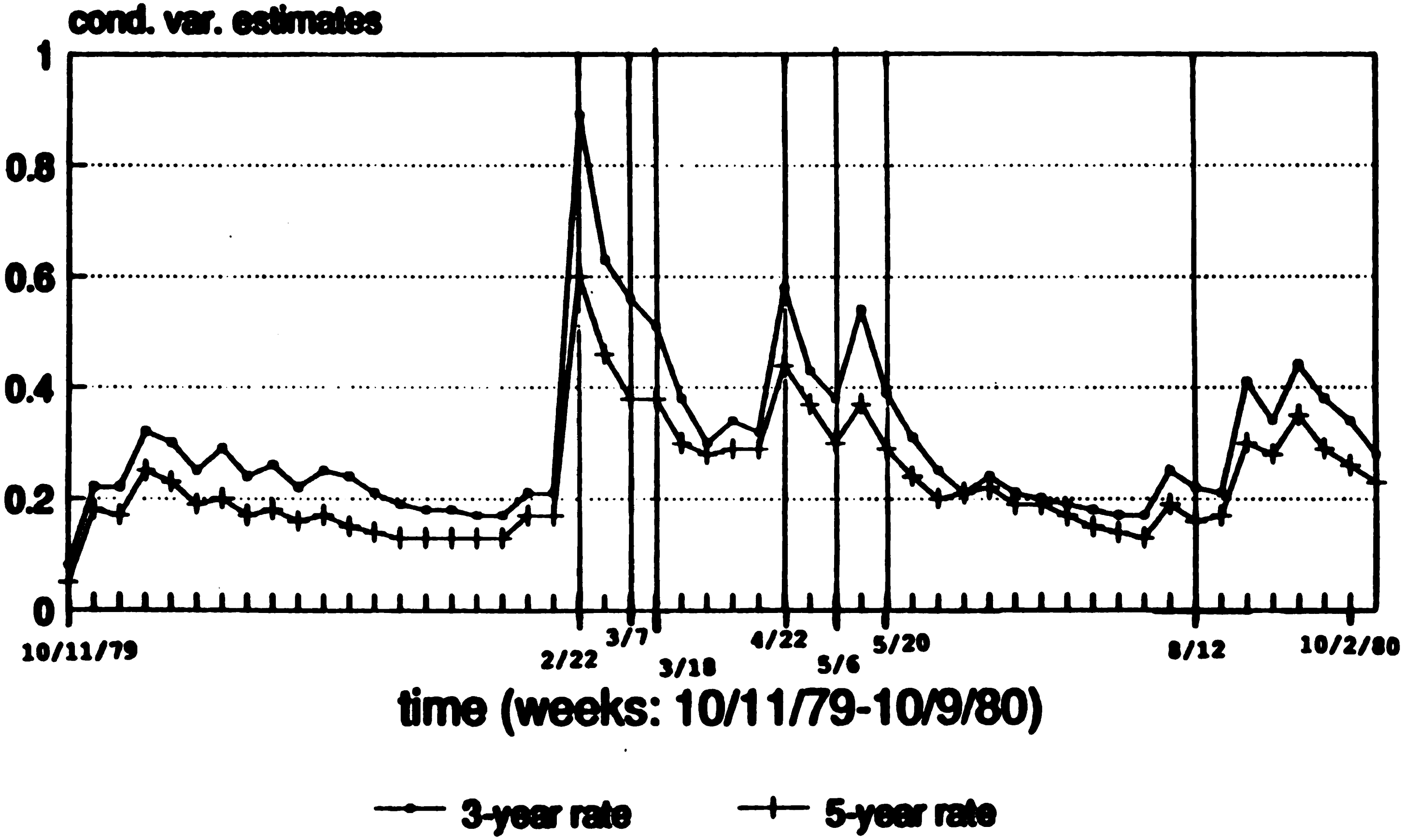

Figure 4.2 (cont'd)

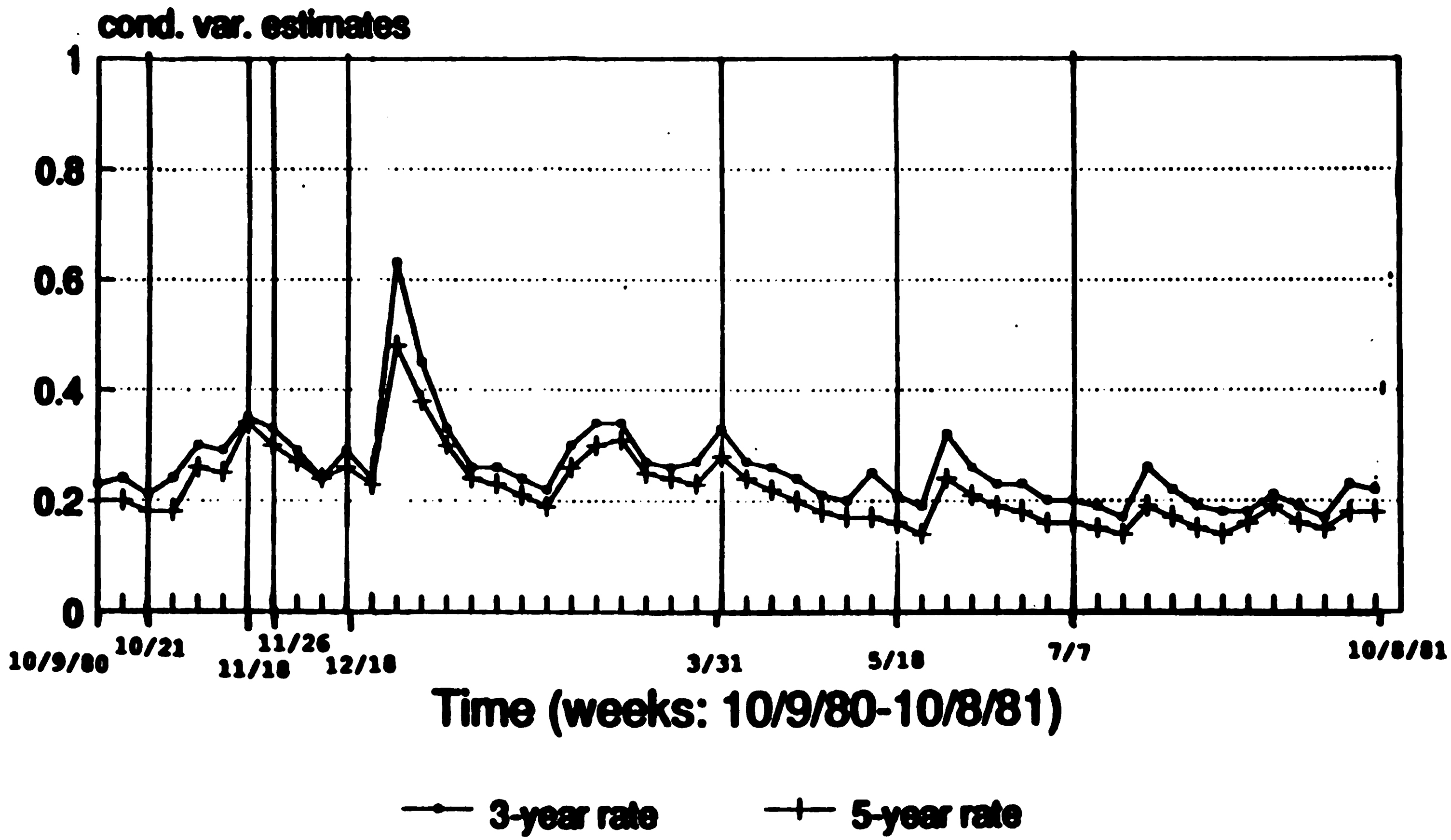

Figure 4.2 (cont'd)

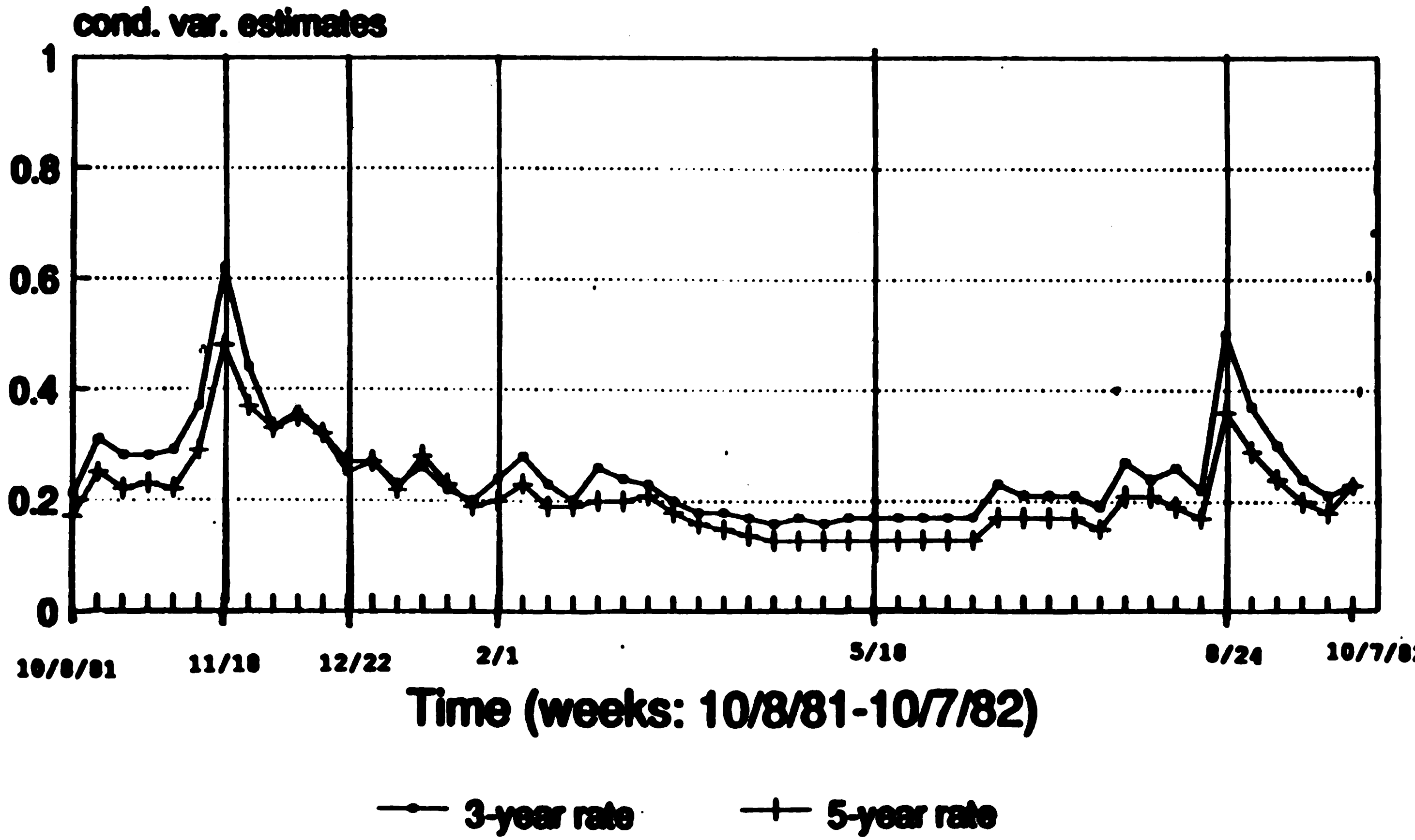

Figure 4.3
Weekly Conditional Variance Estimates

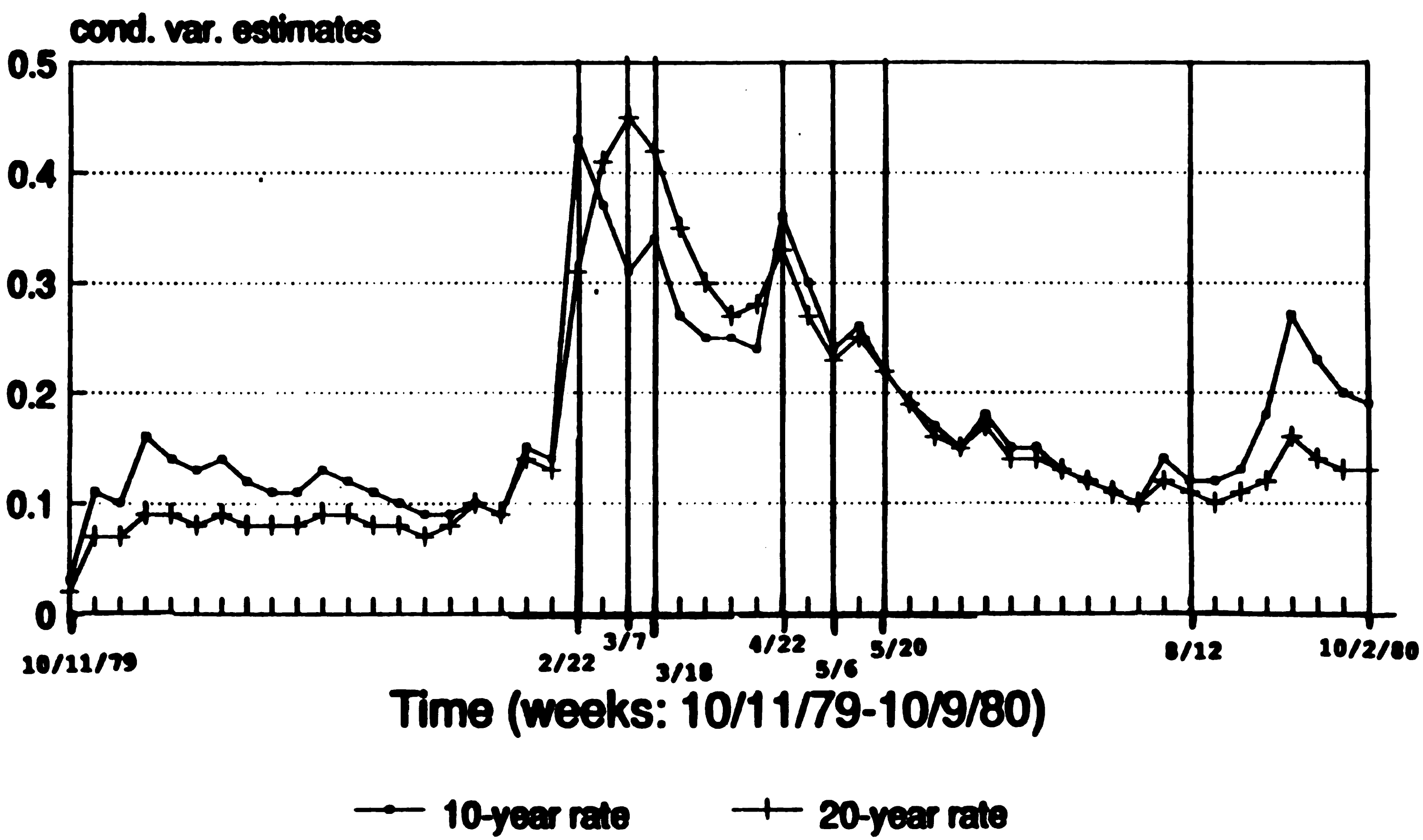

Figure 4.3 (cont'd)

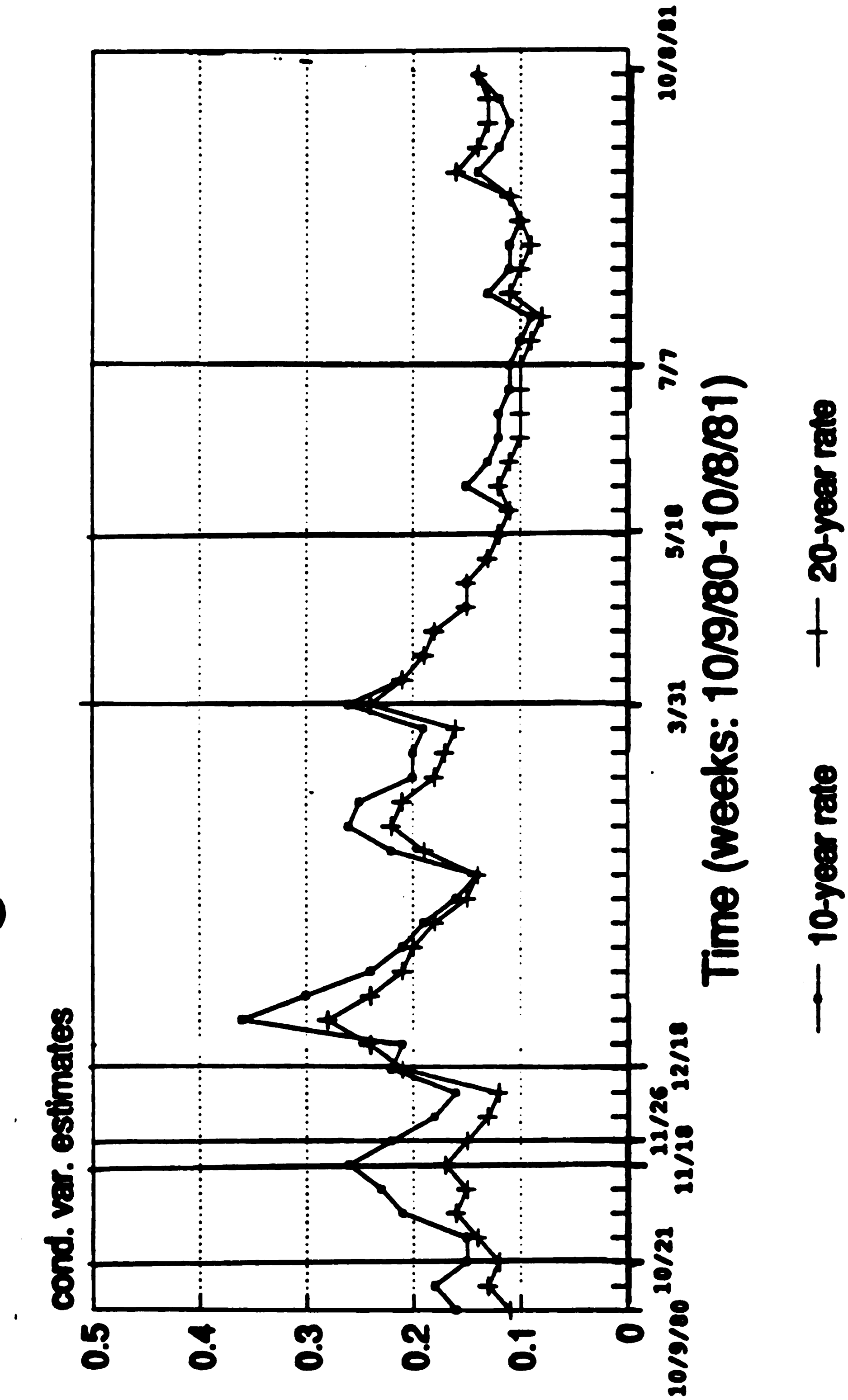

Figure 4.3 (cont'd)

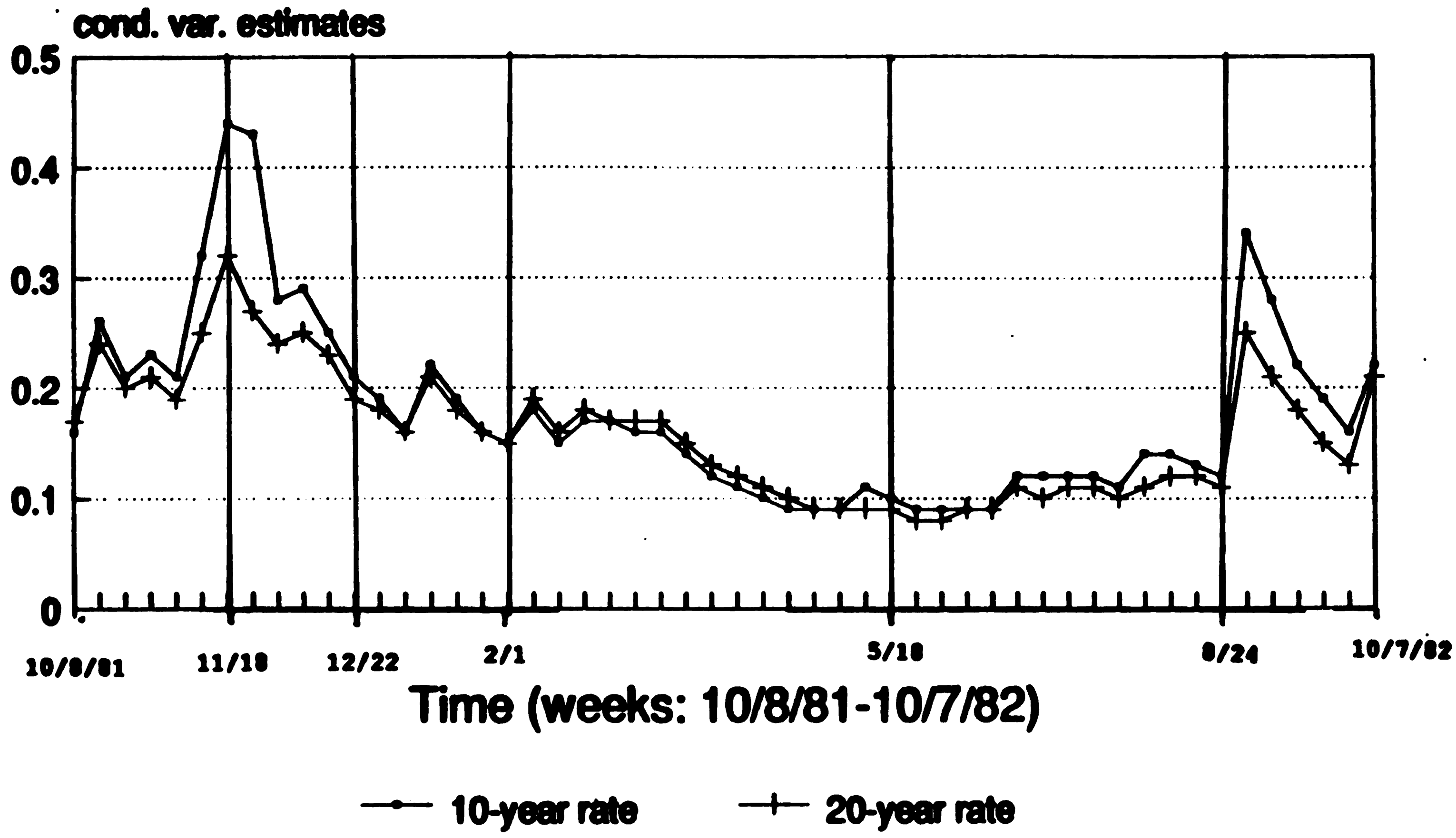

Table 4.3

Weekly GARCH Model

January 1, 1974 - march 16, 1988

$$\Delta\ r_t = u_t$$

$$u_t = \epsilon_t + \Sigma\ \theta_i\ \epsilon_{t-i}$$

$$\epsilon_t | \Omega_{t-1} \sim N\ (0,\ h_t)$$

$$h_t = \alpha_0 + \alpha_1\ \epsilon^2_{t-1} + \beta_1\ h_{t-1} + \rho_1\ \text{REG} + \rho_2\ \Delta\text{FFR}$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
θ_1	.0914 (.0679)	.1276 (.0610)	.0789 (.0656)	.1018 (.0420)	.1572 (.0619)	.0726 (.0407)	.1120 (.0440)
θ_2	-.0624 (.0746)	.0778 (.0589)	.0604 (.0572)	---	---	---	---
θ_3	-.0052 (.0629)	.0908 (.0562)	.1181 (.0516)	---	---	---	---
θ_4	.0759 (.0631)	.0403 (.0615)	---	---	---	---	---
α_0	.0018 (.0006)	.0000 (.0003)	-.0003 (.0002)	.0043 (.0014)	.0003 (.0002)	.0011 (.0003)	.0003 (.0001)
α_1	.2100 (.0584)	.0441 (.0183)	.0162 (.0148)	.1777 (.0348)	.0207 (.0122)	.1990 (.0284)	.1584 (.0257)
β_1	.7097 (.0519)	.8963 (.0245)	.9452 (.0168)	.6770 (.0627)	.9372 (.0241)	.7646 (.0313)	.8101 (.0262)
ρ_1	.0892 (.0392)	.0258 (.0122)	.0192 (.0061)	.0375 (.0099)	.0236 (.0059)	.0126 (.0036)	.0082 (.0027)
ρ_2	.0452 (.0148)	.0301 (.0054)	.0238 (.0038)	.0189 (.0076)	.0062 (.0024)	.0015 (.0018)	.0035 (.0013)
LogL	21.847	69.361	73.645	60.561	165.876	229.056	296.933
Q(10)	6.656	10.815	5.347	11.187	2.552	12.721	13.095
$Q^2(10)$	1.982	1.900	4.829	7.486	11.014	10.922	4.824
M_3	-0.083	0.246	0.480	-0.311	-0.234	-0.340	-0.118
M_4	4.891	4.253	4.206	5.624	6.307	5.560	4.371

Table 4.4

Weekly GARCH Model

January 1, 1974 - March 16, 1988

$$\Delta r_t = u_t$$

$$u_t = \epsilon_t + \Sigma \theta_i \epsilon_{t-i}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t, V)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} + \rho_1 \text{REG} + \rho_2 \Delta\text{FFR}$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
θ_1	.0400 (.0602)	.1218 (.0566)	.0696 (.0572)	.1307 (.0392)	.1228 (.0517)	.1025 (.0396)	.1155 (.0427)
θ_2	-.0121 (.0631)	.0680 (.0550)	.0593 (.0519)	---	---	---	---
θ_3	-.0084 (.0569)	.0947 (.0557)	.0842 (.0530)	---	---	---	---
θ_4	.0843 (.0592)	.0332 (.0554)	---	---	---	---	---
α_0	.0016 (.0009)	.0002 (.0004)	-.0002 (.0002)	.0023 (.0011)	.0000 (.0001)	.0006 (.0003)	.0002 (.0001)
α_1	.1652 (.0544)	.0448 (.0235)	.0126 (.0171)	.1368 (.0408)	.0063 (.0109)	.1430 (.0368)	.1182 (.0319)
β_1	.7344 (.0584)	.8973 (.0315)	.9533 (.0187)	.7777 (.0544)	.9645 (.0170)	.8377 (.0354)	.8545 (.0304)
ρ_1	.0769 (.0431)	.0281 (.0152)	.0146 (.0064)	.0218 (.0098)	.0177 (.0069)	.0088 (.0042)	.0072 (.0030)
ρ_2	.0419 (.0179)	.0235 (.0076)	.0189 (.0052)	.0161 (.0082)	.0063 (.0029)	.0008 (.0023)	.0026 (.0016)
V^{-1}	.1394	.1138	.1114	.1591	.1720	.1576	.1194
LogL	33.461	76.820	80.253	79.448	182.558	248.599	308.844
Q(10)	8.641	9.917	7.838	10.229	3.044	11.356	12.795
$Q^2(10)$	1.729	2.229	6.336	5.824	13.829	8.400	4.121
M_3	-0.140	0.300	0.580	-0.309	-0.199	-0.393	-0.107
M_4	5.303	4.417	4.548	6.222	6.415	6.323	4.616

our results show that interest rate volatility over the nonborrowed reserve policy regime is not homogeneous, but rather, declines over the course of the period. This later result is consistent with the expected behavior of interest rate volatility if learning plays an important role.

4.4 Unanticipated Money and the Conditional Variance

In Chapter Three the impact of unanticipated money on the level and volatility of interest rates during the nonborrowed reserves operating procedure was explored. There it was discovered that following the October 6, 1979 change in operating procedures, the response of interest rates across the maturity spectrum to a change in unanticipated money increased substantially. Furthermore it was found that unanticipated money supply changes increased interest rate volatility as they were found to exert a significant influence upon the conditional variance of interest rates. In the previous sections of this chapter however, it was noted that the response of interest rates to unanticipated money may change over the course of a given operating procedure. In fact a number of previously outlined studies show this to be true. One possible explanation for this phenomena would be that following a change in operating regime, economic agents experience high degrees of uncertainty as they attempt to discern the Federal Reserve's policy; as such they respond relatively strongly to "news" which may provide information regarding the new policy. But over time as agents experience the new policy they adjust their behavior accordingly and the response of interest rates to unanticipated money decreases. The October 6, 1979 change from a federal funds rate targeting regime to a nonborrowed reserve regime may be such a change. Baxter(1988) notes the potential role of learning on the time-series behavior of interest rates in the post-October 1979 period when she states, "The key implication of the learning explanation is that the response of financial markets to 'news' about the money supply process should decrease over time in a specific way."[7] Thus a priori there exists the possibility that the response of interest rates to unexpected money may not be homogeneous over the course of the nonborrowed reserve procedure.

How interest rates and interest rate volatility respond to unanticipated money has important implications for the choice of a monetary policy operating regime. Analysis from earlier sections of this chapter reveal that following the introduction of the nonborrowed reserve procedure, interest rate volatility increased dramatically. Much of this increase in volatility has been attributed to the heightened response of interest rates to unanticipated money. But if learning does influence the behavior of interest rates, then analyses which treat the response of interest rates to unanticipated money as constant are flawed, and lead us to some inappropriate conclusions regarding the effectiveness of a nonborrowed reserve regime. In particular conclusions drawn about the appropriateness of a nonborrowed reserve procedure, if agents are engaged in the learning process, would be expected to overestimate the impact of unanticipated money on both the level and volatility of interest rates.

To explore whether or not the response of interest rates to unexpected money is constant over the nonborrowed reserve regime, the GARCH(1,1) model with unanticipated money derived in Chapter Three is employed over various subsets of the October 1979 to October 1982 period. Recalling equation (3.3.4):

$$\begin{aligned}
&\Delta r_t = \gamma_1 UM_t + u_t \\
(4.4.1) \qquad & u_t = \epsilon_t + \theta_1 \epsilon_{t-1} \\
&\epsilon_t | \Omega_{t-1} \sim N(0, h_t) \\
&h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} + D_1 |UM_t|
\end{aligned}$$

where UM_t equals the level of unanticipated money on the day of the money supply announcement and zero on all nonannouncement days.[8] Daily data for rates across the maturity spectrum is again employed. To examine the response of interest rates to unexpected money over the course of the new regime, model (4.4.1) was estimated for the periods October 6, 1979 to October 6, 1982; for the first year of the new regime, October 6, 1979 to September 30, 1980; and for the last two years of the new regime, October 1, 1980 to October 6, 1982. The results are presented in Tables 4.5 through 4.7 where Table 4.5 repeats the results shown in Table 3.5 of the previous chapter.

The results from estimating model (4.4.1) over various periods during the nonborrowed reserve regime yield some striking results. From Table 4.5, over the October 10, 1979 to October 6, 1982 period, unanticipated money exerts a positive influence on all interest rates across the maturity spectrum. Here the response of the level of interest rates to unanticipated money ranges from 0.0540 to 0.0649 on Treasury bills and from 0.0289 to 0.0509 on longer-term interest rates where all estimates of are significantly different from zero at the five-percent level. Estimates of the response of the conditional variance to unanticipated money, D_1, range from 0.0038 to 0.0056 on short-term bill rates and from 0.0013 to 0.0060 on long-term rates. Again all estimates are significantly different from zero at the five-percent level. Examination of Table 4.6 reveals estimates for model (4.4.1) for the period October 6, 1979 to September 30, 1980, the first year during which the nonborrowed reserve operating procedure was in place. Here estimates of γ_1 range from 0.0670 to 0.0770 on short-term rates and 0.0378 to 0.0632 on long-term rates. All estimates are again significantly different from zero at the five-percent level with γ_1 being at least 1.17 times larger for short-term rates during the first year than for the entire period. For long-term rates estimates of γ_1 are at least 1.24 times larger during the first year than for the period as a whole. With respect to the impact of unexpected money on the conditional variance of interest rates, estimates of D_1 for the first year of the new procedure range from 0.0052 to 0.0065 on Treasury bill rates and from 0.0016 to 0.0066 on long-term rates. Again all estimates differ significantly from zero at the five-percent level with the exception of D_1 on the twenty-year rate which has a t-statistic of 1.6. With the exception of the six-month Treasury bill, estimates of D_1 for the initial year of the new procedure are at least ten percent, and in four of the seven cases at least twenty percent, higher than estimates of D_1 for the entire period. Thus the first year of the new period appears markedly different from the behavior of interest rates during the period as a whole; both the level and conditional variance of interest rates during the first year appear far more responsive to unanticipated changes in the money stock.

Examination of Table 4.7 reveals further evidence of the nonhomogeneity of interest rates to unexpected money supply changes during the nonborrowed reserve regime. Model (4.4.1) was estimated for the remainder of the nonborrowed reserve procedure, October 1, 1980 through October 6, 1982. Following the learning theory discussed earlier, a priori the impact of unanticipated money on the level and volatility of interest rates should be less than that which exists for the first year results. The results in Table 4.7 point this out. Estimates of γ_1, the response of the level of interest rates to unexpected money, range from 0.0230 to 0.0682. In every case estimates of γ_1 are significant at the five-percent level with estimates during the first year of the new

Table 4.5

Daily GARCH Models with UM

October 10, 1979 - October 6, 1982

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + D_1 |UM_t|$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0631 (.0092)	.0649 (.0084)	.0540 (.0072)	.0509 (.0077)	.0452 (.0063)	.0325 (.0061)	.0289 (.0058)
θ_1	.0978 (.0391)	.0533 (.0382)	.0447 (.0398)	.0757 (.0399)	.0632 (.0395)	.0444 (.0386)	---
α_0	.0002 (.0009)	-.0006 (.0005)	.0001 (.0006)	-.0005 (.0004)	-.0002 (.0003)	.0003 (.0004)	.0001 (.0002)
α_1	.0749 (.0187)	.0464 (.0145)	.0470 (.0160)	.0736 (.0152)	.0804 (.0171)	.0657 (.0184)	.0560 (.0173)
β_1	.9025 (.0237)	.9302 (.0226)	.9181 (.0273)	.8842 (.0191)	.8811 (.0215)	.8946 (.0270)	.9182 (.0234)
D_1	.0048 (.0019)	.0056 (.0015)	.0038 (.0012)	.0060 (.0010)	.0040 (.0009)	.0020 (.0007)	.0013 (.0006)
LogL	-71.008	13.657	133.254	205.809	275.812	352.473	387.839
Q(10)	3.924	8.267	8.176	6.037	8.460	10.908	7.772
$Q^2(10)$	9.419	11.935	8.553	4.429	1.588	4.174	4.553
M_3	0.206	0.093	-0.184	-0.090	-0.170	-0.093	-0.004
M_4	4.545	4.756	4.374	4.370	4.046	3.697	3.632
$LR_{D_1=0}$	5.624	13.238	9.390	23.898	16.062	7.088	3.992

Standard errors in parentheses.

Table 4.6

Daily GARCH Models with UM

October 10, 1979 - September 30, 1980

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} + D_1 |UM_t|$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
γ_1	.0770	.0764	.0670	.0632	.0559	.0440	.0378
	(.0138)	(.0121)	(.0116)	(.0112)	(.0101)	(.0102)	(.0092)
θ_1	.1020	.1042	.0451	.1533	.1278	.0479	.1282
	(.0699)	(.0683)	(.0791)	(.0734)	(.0822)	(.0742)	(.0759)
α_0	-.0004	-.0005	-.0005	-.0009	-.0005	-.0001	.0000
	(.0009)	(.0008)	(.0005)	(.0003)	(.0003)	(.0004)	(.0003)
α_1	.0498	.0325	.0310	.0624	.0615	.0683	.0877
	(.0302)	(.0292)	(.0241)	(.0217)	(.0239)	(.0382)	(.0392)
β_1	.9137	.9327	.9313	.8973	.8984	.8943	.8824
	(.0454)	(.0448)	(.0353)	(.0219)	(.0250)	(.0420)	(.0412)
D_1	.0065	.0054	.0052	.0066	.0045	.0027	.0016
	(.0029)	(.0019)	(.0020)	(.0012)	(.0012)	(.0012)	(.0010)
LogL	11.300	43.970	64.164	78.580	113.377	126.642	156.399
Q(10)	3.823	10.512	13.180	7.179	9.829	11.043	6.762
$Q^2(10)$	8.937	5.367	4.769	12.835	11.105	7.780	8.113
M_3	0.220	0.055	-0.215	-0.087	-0.281	-0.266	-0.023
M_4	3.230	3.277	3.878	4.229	4.106	4.009	3.364

Standard errors in parentheses.

Table 4.7

October 1, 1980 - October 6, 1982

$$\Delta r_t = \gamma_1 UM_t + u_t$$

$$u_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} + D_1 |UM_t|$$

	3 Month	6 Month	12 Month	3 Year	5 Year	10 Year	20 Year
α_1	.0665	.0682	.0552	.0498	.0407	.0277	.0230
	(.0109)	(.0096)	(.0079)	(.0069)	(.0066)	(.0057)	(.0056)
θ_1	.1032	.0481	.0560	.0553	.0405	.0477	.0341
	(.0486)	(.0473)	(.0472)	(.0460)	(.0454)	(.0454)	(.0506)
α_0	.0020	.0009	.0016	.0005	.0009	.0005	.0004
	(.0018)	(.0017)	(.0017)	(.0006)	(.0009)	(.0005)	(.0005)
α_1	.0695	.0361	.0392	.0509	.0672	.0468	.0291
	(.0277)	(.0211)	(.0252)	(.0201)	(.0233)	(.0193)	(.0200)
β_1	.8956	.9223	.9125	.9146	.8799	.9277	.9491
	(.0303)	(.0481)	(.0611)	(.0338)	(.0442)	(.0341)	(.0418)
D_1	.0030	.0049	.0013	.0019	.0021	.0004	.0002
	(.0029)	(.0024)	(.0015)	(.0011)	(.0012)	(.0008)	(.0008)
LogL	-72.442	-19.141	.80.309	138.993	171.339	231.032	240.662
Q(10)	2.915	2.446	2.978	3.688	5.607	11.691	11.460
$Q^2(10)$	9.330	9.531	8.926	7.823	5.829	7.530	7.078
M_3	0.135	0.049	-0.189	-0.144	-0.152	-0.062	0.024
M_4	4.477	4.726	4.088	3.964	3.831	3.336	3.266

Standard Errors in parentheses.

procedure being at least 1.12 times greater than the estimates for the last two years. For five of the seven interest rates, γ_1 is at least twenty-percent smaller during the last two years of the nonborrowed reserve procedure. Thus our results are in agreement with those of Loeys(1985) who finds that after the first year of the new policy, the response of the level of interest rates to unanticipated money decreased significantly.

Given that over the course of the nonborrowed reserve regime the response of the level of interest rates to unanticipated money declined significantly, it is also worth investigating how interest rate volatility was influenced by money supply announcements over the same period. A priori if agents become less responsive to unexpected money over the course of the new regime, unanticipated money would be expected to have a smaller impact on interest rate volatility. From Table 4.7, estimates of D_1 again show striking differences over the course of the nonborrowed reserve regime. For the last two years of the new regime, estimates of D_1 range from 0.0013 to 0.0030 on Treasury bills and from 0.0002 to 0.0021 on longer-term rates. These estimates are significantly lower than similar estimates for the first year of the new procedure. Furthermore for the period October 1980 to October 1982, D_1 is statistically significant at the five percent level for only the six-month rate and at the ten percent level for only the three-month and six-month rates. In the remaining four cases D_1 is not significantly different from zero. Thus unlike the first year of the nonborrowed reserves procedure, unanticipated money had relatively little effect upon the conditional variance of interest rates. This result is further reinforced by Table 4.8 which presents estimates of the total impact, $D_1\ (1-\beta_1)^{-1}$, of unanticipated money on the conditional variance of interest rates. These results show the dramatically more pronounced effect of unanticipated money on the conditional variance during the first year than the last two years of the new monetary regime. For those cases where the total impact is significantly different from zero during the October 1980 to October 1982 period, in two of these cases the total impact is at least 2.5 times smaller than during the first year of the new procedure and in the third case, the six-month Treasury bill, it is 1.27 times smaller. The results further reveal that unanticipated money's impact on the conditional variance is not homogeneous over the course of the nonborrowed reserve regime, but rather show a pattern consistent with the learning theory outlined earlier.

4.5 Conclusion

The conclusion of many studies over the last decade has been that following the adoption of the nonborrowed reserve procedure by the Federal Reserve on October 6, 1979, interest rate volatility was found to increase substantially. At the crux of this increased volatility was an increased responsiveness by market agents to unanticipated changes in the money supply. The purpose of this chapter has been to examine these issues in the context of a GARCH(p,q) model. Specifically, a limited number of previous studies suggest that interest rate volatility was not constant over the nonborrowed reserve regime. Rather, while interest rate volatility was initially high following the adoption of the new procedure, over the course of the regime interest rate volatility declined as economic agents learned of the Federal Reserve's new policy. Concurrent with this learning process was a reduction in the responsiveness of agents to news of the money supply process. The results of this chapter suggest

Table 4.8

Total Effect of a Unit Change In UM_t on the Conditional Variance of Interest Rates

TIME

	10/10/79-10/6/82	10/6/79-9/30/80	10/1/80-10/6/82
3 Month	0.0492*	00753*	0.0287
6 Month	0.0802*	0.0802*	0.0631*
12 Month	0.0464*	0.0757*	0.0149
3 Year	0.0518*	0.0643*	0.0222**
5 Year	0.0336*	0.0443*	0.0175**
10 Year	0.0190*	0.0255*	0.0055
20 Year	0.0159*	0.0136**	0.0039

* - significant at the five-percent level.

** - significant at the ten-percent level.

the following conclusions. Like other studies, interest rate volatility was found to increase dramatically after the adoption of the new procedure. Interest rate volatility was not constant over the course of the new procedure however. Rather it increased throughout late 1979 and early 1980, but then began a declining pattern which is generally consistent with the theory that economic agents learn of new policies over time. There are however sharp increases in volatility in either late 1981 or early 1982 suggesting that while the Federal Reserve claimed to be targeting a monetary aggregate, changes in the range of the federal funds rate had a pronounced effect upon interest rate volatility.

Possibly the most interesting finding of this chapter involves the response of interest rates to unexpected money. Unlike other studies the results here suggest that during the first year of the new regime, unanticipated money exerted a strong and significant impact upon both the level and volatility of interest rates. But after the first year, unanticipated money was shown to have a reduced impact on the level of interest rates with little or no impact upon interest rate volatility.

Finally the results of this chapter have a number of important implications for the choice of a monetary policy operating regime. Analysis of the effectiveness of the 1979 to 1982 experiment with a nonborrowed reserve procedure based on either the first year of the new regime or the procedure as a period of homogeneous interest rate behavior are seriously flawed. Rather sharp differences in the time-series behavior of interest rates and interest rate volatility exist over the period, differences which appear to be consistent with the idea that economic agents learned of the new policy over time. Thus the results suggest that more work needs to be done on how the choice of a policy regime effects interest rate volatility, particularly allowing for the possibility of learning on the part of economic agents.

ENDNOTES

1. See Johnson(1981) p. 2.

2. See Roley(1982) p. 15.

3. See Rasche(1986) p. 47.

4. Here interest rates are taken on the Thursday of each week.

5. Despite the comments of Rasche(1986) noted earlier, most time-series studies of interest rates use a levels rather than logarithmic specification. Nelson and Plosser(1982), for example, justify this by saying, "The tendency of economic time series to exhibit variation that increases in mean and dispersion in proportion to absolute level, motivates the transformation to natural logs..."(p. 141).

6. Based on the argument in Rasche(1986), an alternative specification of the conditional mean equation would be:

   $\Delta \ln r_t = u_t$

   where ln refers to the change in the natural log of interest rates. Such a specification was tried with the finding that the logarithmic transformation of interest rates did not change the GARCH model; the parameters α_1 and β_1 were unchanged by the change in specification of the conditional mean equation. Furthermore while the logarithmic transformation did decrease the estimated value of the conditional variance, h_t, it had little impact upon the relationship between values of h_t over various operating regimes and did not alter the conclusions derived from Table 4.2 and Figures 4.1 through 4.7

7. See Baxter(1988) p. 5.

8. Here unanticipated money is defined as in Chapter Three.

CHAPTER FIVE

SUMMARY AND CONCLUSIONS

5.1 Introduction

Over the last few decades a wide variety of studies have examined the time-series behavior of interest rates. Some of these studies have examined how new information alters the level of interest rates; others focus on the behavior of interest rates over a variety of monetary policy regimes. As a whole the emphasis of these studies has been upon the representation of the conditional mean, with the finding in most studies that interest rates follow a random walk. While equations for the conditional mean have been thoroughly examined, few studies have examined the time-series behavior of the conditional variance. Rather most studies have either implicitly or explicitly treated it as a constant. But given the increasingly volatile nature of interest rates during the late 1970's and early 1980's, such an assumption seems questionable and merits investigation.

This dissertation has been concerned with a conditional variance approach to the time-series behavior of interest rates. Specifically using ARIMA models with a GARCH error process, the behavior of the conditional variance of daily government interest rates across the maturity spectrum has been examined. The purpose of this chapter is twofold. First, this chapter reviews the findings of the preceding chapters including the application of the GARCH model to interest rates and the further introduction of news, in the form of money supply announcements, into the GARCH model. The second part of this chapter examines a host of issues and areas in which this study can be further extended. Given the recent advancements in time-series econometrics, a variety of further research possibilities exist.

5.2 Unit Roots and ARCH Effects

In this study both the conditional mean and variance of interest rates are examined. This is accomplished using the autoregressive conditional heteroskedasticity (ARCH) process developed by Engle(1982, 1983) and later generalized (GARCH) by Bollerslev(1986). The first step in modelling any economic variable using the GARCH process involves correct specification of the mean equation. If the equation for the mean is correctly specified, then the GARCH process yields efficient estimates of the conditional variance. To date an abundance of evidence suggests that interest rates follow a random walk process, or at least can be represented as a martingale sequence. In this study daily data for a variety of government interest rates, over different monetary policy regimes, are used to examine the statistical distribution of interest rates. First, using new tests developed by Phillips(1987), Perron(1986), and Phillips and Perron(1986), daily government interest rates are examined to determine whether or not they are stochastic nonstationary. The finding is that interest rates possess one unit root, thus confirming as well as extending the results of Perron(1986) and Nelson and Plosser(1982). Using first differences, maximum likelihood estimation reveals the existence of serial correlation, conditional heteroskedasticity, and excess kurtosis. While an ARIMA(0,1,q) model with Gaussian errors appears to account for the serial correlation, such a model is inadequate to explain either the excess kurtosis or the conditional heteroskedasticity. To model the behavior of the conditional variance the GARCH(p,q) process is applied to the various interest rates with the finding that a GARCH(1,1) model with conditionally normal errors well approximates the conditional heteroskedasticity found in daily interest rate data. As is common in financial time series however, the GARCH model with conditionally normal errors is inadequate to explain the excess kurtosis found in interest rates. A common remedy is to employ the GARCH process with conditionally t-distributed errors. Using the conditional t distribution, as well as exogenous variables in the conditional variance, the degree of kurtosis is reduced but not eliminated.

5.3. Unanticipated Money in the GARCH Model

In addressing the time-series behavior of interest rates, a number of studies have examined how interest rates are influenced by the arrival of new information about pertinent economic variables. In particular a variety of studies have explored how interest rates respond to weekly announcements of changes in the money supply. The consensus of this literature is that unanticipated changes in the money supply cause interest rates to rise, although the theory as to why this occurs is still a topic of considerable debate. A further result from this literature is that the responsiveness of interest rates to unanticipated money varies according to the monetary operating regime in existence at the time. Here interest rates showed the greatest response to unanticipated money when the Federal Reserve targeted nonborrowed reserves; the magnitude of the interest rate response was smallest when the federal funds rate was employed as the operating target.

The difficulty with these studies is that while they measure the response of the level of interest rates to unanticipated money, they implicitly assume that unanticipated money has no impact upon the volatility of interest rates. The few studies which do address this issue conclude that unanticipated money exerted a significant and positive impact on interest rate volatility, particularly after the adoption of the nonborrowed reserve procedure on October 6, 1979.

In this study unanticipated changes in the money supply were introduced into the GARCH(1,1) model of daily government interest rates developed in Chapter Two. Using the conditional variance estimates derived from the GARCH(1,1) model as a proxy for interest rate volatility, a number of interesting conclusions can be drawn about the impact of unanticipated money on the time-series behavior of interest rates. First, like other studies, the results reveal that the magnitude of the interest rate response to unanticipated money depends upon the interest rate chosen and the monetary operating regime. Here the response to unexpected money is found to be greatest for short-term rates such as the three-month and six-month Treasury bills and smallest for long-term rates such as the ten-year and twenty-year bond rates. Furthermore the response to unanticipated money was found to be greatest during the period when the Federal Reserve targeted nonborrowed reserves and smallest during the federal funds rate targeting regime with the magnitude of the response being between the two when the operating target was borrowed reserves.

A second conclusion to result from the introduction of unanticipated money into the GARCH model involves the behavior of the conditional variance. Here it was found that unexpected changes in the money supply significantly increased the conditional variance of interest rates only during the period October 1979 to October 1982 when the Federal Reserve was targeting the level of nonborrowed reserves. For this period the results reveal that unanticipated changes in the money supply had a persistent effect upon the conditional variance; that is the conditional variance increased not only on the day of the monetary surprise but also on subsequent days. Furthermore the magnitude of the response of the conditional variance was found to decline as the length to maturity increased.

Finally the impact of unanticipated money on interest rates during the nonborrowed reserve regime was further decomposed so as to examine whether or not the behavior of interest rates during this period was homogeneous. A number of empirical studies suggest that because the adoption of the nonborrowed reserve procedure on October 6, 1979 was one without precedent, a considerable period of learning on the part of market participants was involved. As a result of this learning period it is argued that interest rate volatility over the October 1979 to October 1982 period was not homogeneous, but rather exhibited a declining pattern consistent with learning. It was found in this study that interest rate volatility over the length of the nonborrowed reserve regime was not homogeneous, but rather was consistent with the theory that learning may have played an important role. It was further discovered that while unanticipated money had a significant impact upon the conditional variance of interest rates during the first year of the nonborrowed reserve regime, after the first year unanticipated money had a minimal impact upon the conditional variance of interest rates.

5.4. Questions for Future Research

Conditional variance models such as the GARCH model utilized here provide an interesting approach to studying the time-series behavior of interest rates. Nonetheless this study points to a number of avenues for further research. One possible extension would be to employ a multivariate, rather than univariate, GARCH model as a way of examining the temporal persistence of interest rates. In interest rates along the maturity spectrum are simultaneously determined, then a multivariate GARCH model provides an enriched framework for analyzing the time series behavior of interest rates.[1] The benefit of the multivariate framework is that it allows not only the conditional variances, but also the conditional covariances to vary over time. That is to say, one may examine whether or not the conditional covariances also exhibit serial dependence.

One interesting application of the multivariate process would be to employ a multivariate GARCH-in-the-mean (GARCH-M) model as a way of estimating risk premia in the term structure of interest rates. Engle, Lilien, and Robins(1987), employing a univariate ARCH-M model on quarterly interest rate data, estimate a time varying risk premia. An attempt to incorporate the GARCH-M model into this study using biweekly interest rate data yielded extremely poor results, thus leaving some question as to the applicability of the univariate GARCH model in estimating risk premia. Bollerslev, Engle, and Wooldridge(1986) recongnize this point as well when they state, "There is also some evidence that the risk premia are better represented by the covariances with the implied market than by own variances".[2] Thus the multivariate GARCH-M model may provide a superior means of estimating time varying risk premia. As such the introduction of the multivariate GARCH-M process into this study would provide some interesting alternatives. Incorporated into the results of Chapter Three, which shows the relevance of unanticipated money for the conditional variance, the multivariate GARCH-M model would allow the possibility of examining how money supply announcements affect not only the conditional variances, but also the conditional covariances of interest rates across the maturity spectrum. In such a model it would thus be possible to examine if and by how much money supply announcements alter risk premia.

While the multivariate GARCH-M model may provide a rich framework for estimating time varying risk premia, a number of other issues exist which provide further possibilities for future research. From the univariate specification of the GARCH model, the finding is that interest rates possess one unit root and thus need to be first-differenced in order to acheive stationarity. The extension of such a result to the multivariate GARCH framework would seem to imply that all interest rates in a multivariate model should be first-differenced. However the simultaneous modeling of interest rates raises the possibility that all interest rates across the maturity spectrum may exhibit a common driving force or factor. This raises the question of how many independent unit roots to impose on a group of interest rates. Such a question can now be examined due to a recent test developed by Johansen(1988). The Johansen method explicitly tests for the number of common unit roots or trends in a multivariate model. The application of this test to other types of financial data, such as exchange rates by Baillie and Bollerslev(19889, reveals that the imposition of one unit root for each exchange rate imposes too many unit roots on the system. Application of the Johansen test to a subset of the data in this study yielded a similar conclusion.[3]

Interest rates, as a group, appear to possess at least one unit root; intuitively this is not surprising if one notices the common behavior of the conditional variance estimates in Figures 4.1 through 4.7 of Chapter Four. Given that first-differencing is a standard remedy for nonstationarity, the alternative is to model the system in levels. But since such a system imposes no unit roots, it too is clearly inappropriate. Thus considerable research needs to be done, and many questions need to be answered, before representing interest rates in a multivariate GARCH framework.

ENDNOTES

1. Ceteris paribus, if long rates are a function of the current short rate, among other variables, then innovations in the current short rate imply corresponding changes in current long rates.

2. See Bollerslev, Engle, and Wooldridge(1986) p. 11.

3. In a separate paper related to the topic of interest rates and unit roots, I performed the Johansen test on a series of three short term and three long term interest rates. In both cases the Johansen test revealed the presence of one common unit root in the system of three interest rates. Thus a multivariate model which used the first-difference of all three rates would clearly be imposing too many unit roots on the model.

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