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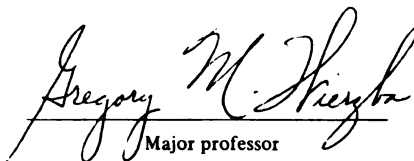
A Systematic Means for Active Filter Synthesis

presented by

Kenneth Vincent Noren

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A SYSTEMATIC MEANS FOR ACTIVE FILTER SYNTHESIS

By

Kenneth Vincent Noren

A THESIS

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ABSTRACT

A SYSTEMATIC MEANS FOR ACTIVE FILTER SYNTHESIS

By

Kenneth Vincent Noren

A systematic method of designing new active filters with op-amps is presented. Active filter synthesis with operational amplifiers is an area of knowledge which has never been fully completed. Many designs have been presented, but there is no way to generate circuits with a set of design rules. Few new designs are being published today. The purpose of this thesis is to present a systematic way of generating active filters which can then be constructed for practical use. It uses a combination of computer programs which have been developed over the last decade. Enroute to designing two new circuits, it shows where some of the major problems with active filter design with op-amps can arise.

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CHAPTER 1

Introduction

1.1 Introduction

In the 1960's, the advancement of solid state technology introduced a compact and inexpensive version of the operational amplifier to the electronics world. It is difficult to imagine a device having a bigger impact on how electronics engineers and technicians would design and build circuits, than this miniaturized op-amp. It has found its way into almost every area of applications, from simple amplification to switched capacitor circuits. Another area, which this thesis will focus on, is active filter synthesis.

In the 1970's, active filter synthesis using op-amps reached a peak, but it had many unanswered questions. With the explosion of the digital electronics area, these questions remained unanswered and somewhat forgotten. This thesis attempts to look at some of those questions and also provide a framework for the design of active filters.

1.2 Current designs

In the first phase of active RC filter design using op-amps, the initial step is to realize a circuit with a biquadratic characteristic equation with a suitable filter function at the output of one or more of the op-amps in the circuit. The op-amps used in this stage are typically modeled as ideal op-amps. These ideal op-amps have the properties of having an infinite gain-bandwidth product (gbp), zero output impedance and infinite input impedance, among other characteristics. Modeling an op-amp with these traits is a fairly

simple task. One way is to impose upon the op-amp the restrictions that the input current into either node of the op-amp is zero and also the voltage across the input terminals is zero. These restrictions seem to model an op-amp well, for if one has ever measured these two values in a circuit containing op-amps, in comparison to other node voltages and branch currents, these values are minuscule. Many, if not most circuits fall into the above category. There are very few ways of approaching the design problem at this stage. It appears to be somewhat of a trial and error session. Some basic one op-amp building blocks can be found in many active filter circuits, one of these being the integrator. Another would be the inverting amplifier. These building blocks are then typically cascaded together with one or more feedback paths interconnecting the different blocks in order to realize some filter transfer function. It is noticeable that most active filters containing two or more op-amps have this topological trait about them. This seems to limit the number of available topologies. It is perhaps just the easiest way to do the designing in addition to being more pleasing to the eye. Whatever the strategies used in designing active filters, there presently does not seem to exist any systematic way of creating the circuits.

1.3 Op-Amp Relocation

One systematic way of performing active filter design has been developed. It is called Op-Amp Relocation (OAR) [1]. This method of design will be described in more detail later in Chapter 2. A key thing to note about OAR is that a number of unique topologies can be generated from a existing design. OAR provides a systematic way to generate many circuits in the initial design stage. In fact, the number of circuits generated from a single topology may be greater than the total number of existing circuits in a particular category.

1.4 Non-Ideal Effects of Op-Amps

The second stage of active filter design takes into account the fact that very few op-amps in use today have ideal properties. Slight variations in these properties can lead to big changes in the circuits performance. The biggest culprit is the frequency dependence of the gain of an op-amp, or more commonly termed having a gbp of less than infinity. This occurs because of a pole in the gain versus frequency response of the op-amp. Furthermore, the op-amp may have higher order poles close to the point at which its gain approaches unity. The problems which can arise from these poles are at least three.

First, a filter with a characteristic equation having a specific selectivity, Q , and center frequency, ω_0 , can have these driven far from the values for which they were designed for. In fact, Q can become so high as to place the poles of the characteristic equation into the right half plane in the frequency domain. This is typically caused by the first pole in the op-amp's transfer function. Additionally this first pole generates extra terms in the circuits characteristic equation driving it from a biquadratic to an n th order equation, where n is two plus the number of op-amps. The new poles generated from these terms may also be in the right half plane.

The second pole in the response of an op-amp also gives rise to an increase in the number of poles in the circuit. The added poles here tend to be very large. These will also be termed parasitic poles due to a two pole model for the op-amp, and they tend to be more subtle than the first two problems. These poles may also be in the right hand plane, causing instability.

Taking these problems into account makes the job of active RC filter design in which a realistic model for the op-amps is used much more complicated. A variety of ways have been introduced, to reduce these effects on the behavior of the circuit, ranging from sensitivity analysis to active and passive compensation [8-13]. This is yet, for the

most part, another unanswered question, for it isn't really clear which may be the best way.

1.5 Preview

In Chapter 2, a means for the ideal synthesis stage of design will be given.

In Chapter 3, problems in the second stage of design will be outlined, and they are the non-ideal factors which come into play, both inband errors and a stability problem.

In Chapter 4, a method for classifying and solving the inband problem will be given.

Chapter 5 deals with a solution to the high frequency stability problem.

Chapter 6 gives a example of both a newly designed notch filter and a bandpass filter. Some of the principles in Chapters 4 and 5 are given.

Chapter 7 gives conclusions and new paths for future research.

1.6 Computer Aided Design Tools

The use of computer tools is a must in active filter design. Without them, the design of several filters could be a tremendously long process. Some of the tools which have been developed in universities have proven to be greatly beneficial in the research for this thesis.

SLAP [2], Symbolic Linear Analysis Program, written by Vikek Joshi, allows one to look at the transfer function of the design in terms of the symbolic elements it is composed of.

ZNAP [6], Zenith Nodal Analysis Program written by James Svoboda, Clarkson University, allows the user to very quickly look at the poles and zeros of a transfer function in a circuit.

OAR [3], Op-Amp Relocation also written by James Svoboda, lets the computer generate the many available circuits from a given topology. Additionally, it does a stability check and then shows the poles and zeros of a given transfer function.

PSPICE, the pc version of SPICE, Simulated Program with Integrated Circuit Emphasis, written by A. Vladimirescu, A.R.Newton, and D.O.Pederson from University of California, Berkley. PSPICE is put out by MicroSim Corporation. The original SPICE was written by L. Nagel, also from University of California, Berkley.

CHAPTER 2

Ideal Filter Synthesis

2.1 Introduction

The ideal filter synthesis stage begins with designing a structure which will give the circuit designer one or more suitable filter functions. At this point in time, this is a very difficult, if not an impossible chore. There appears to be a limit on the number of possible topologies and it seems we are close to this limit, as there are few worthwhile new circuits being published. Noticeably, the existing circuits tend to be quite simple in appearance. They may have been done by combining basic building blocks together or using a signal flow charts, among other means. Yet as mentioned before, the designs seem to have been done more by trial and error rather than systematically. One scheme allows the designer a methodical way of creating different topologies from an existing structure. The method is Op-Amp Relocation.

2.2 Why Op-Amp Relocation?

Op-Amp Relocation (OAR) allows one to create new structures. The structures are created using a very simple set of design rules. It elegantly uses a combination of matrix theory and graph theory to generate its circuits. The generated circuits can look random and haphazard, but they have a mathematical order about them. Take the circuit in Figure 2.1, for example. It does not seem likely that one just sits down and creates a circuit like that. Yet OAR does the job beautifully. In fact, from the circuit, one can generate

another 383 circuits. There are probably less than 10 four op-amp circuits existing as of today. This represents more than a 3700% increase. As with any new design, some of these may be good or bad. OAR gives a larger pool of potentially good circuits to choose from. This fact alone is alone is justification for the use of OAR.

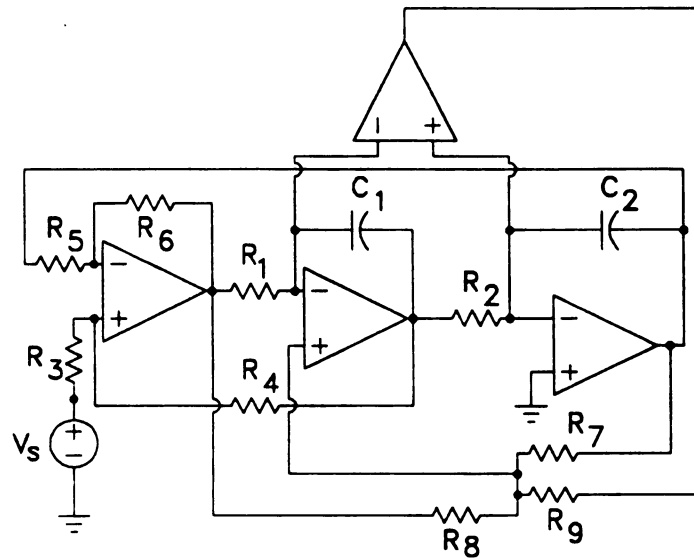


Figure 2.1 active filter with op-amps

2.3 Op-Amp Relocation

To begin to give the reader an ideal of just what the concept of Op-Amp Relocation is, a start with some basic fundamental concepts is necessary. The first concept involves the introduction of two fictitious devices. The devices are shown in Figure 2.2 [4].

The nullator is a two terminal device which is characterized by the property of having zero current through and zero voltage across its terminals, at all times. The norator is also a two terminal device, but with the property of having any current through it for any given voltage across it, or any voltage across it for any given current through it. In other words, no constraints. With a moments thought, one can probably conclude that

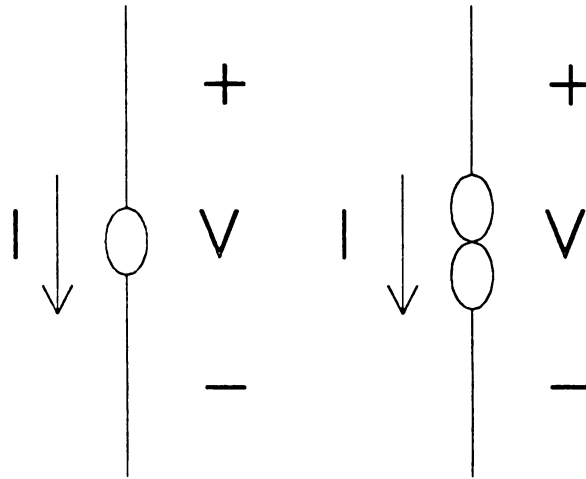


Figure 2.2 nullator and norator

no other device, not even an open circuit or a short circuit has these properties in their entirety.

At first, using these devices can be a little unnerving for the practical designer who needs to work with real devices! However, as time progresses, one becomes more and more comfortable with the thought using the nullator and norator as their mathematical properties become very important. Additionally, many real world devices can be modeled using nullators and norators, so getting comfortable with them can prove to be beneficial to the circuit designer.

It was stated earlier that an ideal op-amp has the property of having zero current through its input terminals and zero voltage across them. The voltage is driven to zero by means of feedback. Obviously the input of an ideal op-amp may be modeled with a nullator. Now for the output of the op-amp, one with experience in evaluating op-amp circuits, notes that the current through the output of the op-amp is never taken into consideration. It is not used in the ideal op-amp circuit evaluation because it has no constraints, causing problems if Kirchoff's current law at the node or Kirchoff's voltage law around a loop containing the output of the op-amp is used. Additionally, the output

of an ideal op-amp can take on any voltage. Because of these lack of constraints on the output of the op-amp it can be modeled as a norator. The pair together is termed a nullor and can be used to model an ideal op-amp. In fact, it has been shown that any infinite-gain controlled current or voltage source is precisely equal to a nullor [4]. Since the ideal op-amp is a voltage controlled voltage source with a gain of infinity, this is reassuring.

Using nullators and norators in circuit design introduces some unique mathematical properties in the analysis of the structure. In this thesis, the chosen method of analysis will be to sum currents at each node and form an admittance matrix. One could also use a loop analysis technique. The former was chosen because SLAP uses it to do its analysis. It is also easier.

The general matrix equation of a circuit done using the nodal analysis technique has the form of

$$I = Y \times V,$$

where I is the current column vector, usually consisting of known currents, V is the voltage column vector, usually consisting of unknown node voltages, and Y is the admittance matrix, consisting of the admittances of the elements used in the circuit. In other words, the constraints between the unknown node voltages and the known currents. Now suppose the circuit contains k nullators, l norators, and m nullors. A look at the two nodes associated with a nullator, call them e and f for convenience, are shown in Figure 2.3.

There is no current through the nullator, thus the current vector will remain the same. The admittance matrix contains only elements which can have current flowing through them via multiplication by a voltage. The nullator does not belong there. The nullator does add one constraint in the circuit, however. $V(e)$ must be equal to $V(f)$. Thus, since nodes e and f have been treated as separate nodes, there exists the following

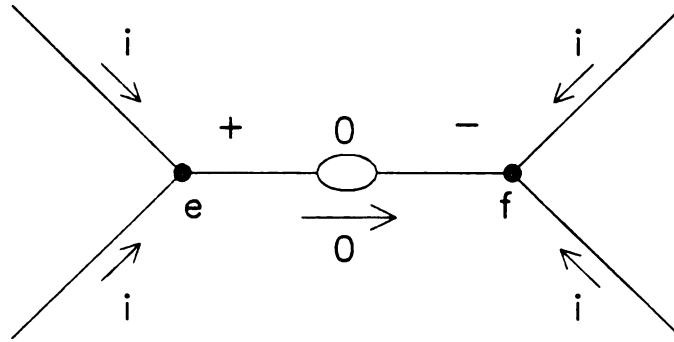


Figure 2.3 nullator in a circuit

system of equations:

$$\begin{bmatrix} \dot{\Sigma i_e} \\ \vdots \\ \dot{\Sigma i_r} \\ \vdots \end{bmatrix}_{n \times 1} = \begin{bmatrix} \begin{matrix} e & f \end{matrix} \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}_{n \times n} \times \begin{bmatrix} \dot{V_e} \\ \vdots \\ V_f \\ \vdots \end{bmatrix}_{n \times 1}.$$

But since $V(e) = V(f)$, $V(f)$ can be replaced with $V(e)$ leaving

$$\begin{bmatrix} \dot{\Sigma i_e} \\ \vdots \\ \dot{\Sigma i_r} \\ \vdots \end{bmatrix}_{n \times 1} = \begin{bmatrix} \begin{matrix} e & f \end{matrix} \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}_{n \times n} \times \begin{bmatrix} \dot{V_e} \\ \vdots \\ V_e \\ \vdots \end{bmatrix}_{n \times 1}.$$

Equivalently for this system of equations,

$$\begin{bmatrix} \dot{\Sigma i}_e \\ \vdots \\ \dot{\Sigma i}_f \\ \vdots \end{bmatrix}_{n \times 1} = \begin{bmatrix} e + f \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} \times \begin{bmatrix} \dot{V}_e \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}_{(n-1) \times 1}$$

$\begin{matrix} (n \times (n-1)) \end{matrix}$

$V(f)$ can be dropped and column f added to column e immediately after writing the node equations. If node e or f happens to be the ground node, it is even easier, $V(e) = V(f) = 0$ and thus columns e or f may be deleted, as it will be multiplied by zero anyway. The other will not be considered as it is the ground node and no currents are summed there.

Now suppose there is a norator between nodes g and h , as shown in Figure 2.4.

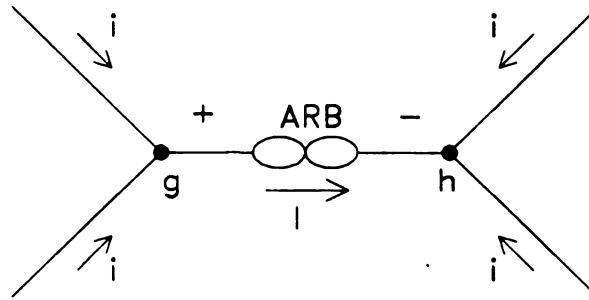


Figure 2.4 norator in a circuit

There are no constraints on the norator, so it can't possibly enter into the admittance matrix, so an alternative approach is chosen. Labeling the current through the norator as I , the following current vector exists:

$$\begin{bmatrix} \vdots \\ \Sigma i_g - I \\ \vdots \\ \Sigma i_h + I \\ \vdots \end{bmatrix}_{n \times 1} = \begin{bmatrix} g \\ \vdots \\ \vdots \\ h \\ \vdots \end{bmatrix}_{n \times 1} \times \begin{bmatrix} \dot{V}_e \\ \vdots \\ \vdots \\ \dot{V}_h \\ \vdots \end{bmatrix}_{n \times 1}$$

$\begin{matrix} (n \times n) \end{matrix}$

This will work because the current through it is arbitrary, as is a current source, for instance. The changes in the current vector occur because current I is leaving node g and entering into node h . To remove the current I from the system, add the two equations together, or effectively add row g to row h . This gives

$$\begin{bmatrix} \vdots \\ \Sigma i_g + \Sigma i_h \\ \vdots \\ \vdots \end{bmatrix}_{((n-1) \times 1)} \quad g + h = \begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}_{((n-1) \times n)} \times \begin{bmatrix} \vdots \\ V_g \\ \vdots \\ V_h \\ \vdots \end{bmatrix}_{n \times 1}.$$

If node g or h is the ground node, simply throw the equation out, or delete the h th or g th row. This will be the case if the output of an op-amp is being modeled as a norator. The rationale for doing this is as follows.

The effect of k nullators in a circuit is to reduce the system of equations to n equations and $n-k$ unknowns, no non-trivial solution to the circuit. The effect of l norators is to leave $n-l$ equations and n unknowns, that is no unique solution. In an RC circuit containing op-amps, k and l must obviously be equal (barring the introduction of any other nullators or norators) to m , since the op-amps are modeled as nullors. If this is the case, there are $n-m$ equations and $n-m$ unknowns, a solvable system of equations.

From the discussion, an easy algorithm can be developed for evaluating rc circuits containing op-amps [4]. It is to first treat the circuit as if no nullors were present. After the admittance matrix is completed, insert the nullors. Secondly, perform the row and column operations due to the nullors. From here, carry on in a normal fashion in figuring out the prospective node voltages.

It should be noted from above that in an RC op-amp circuit each norator is assigned to a specific nullator. In practical terms, the input of an op-amp has an output.

The nodal analysis routine does not take pairings into account. Consider a two op-amp circuit. There are two norators and two nullators in the circuit. It makes no difference whether norator one is paired to nullator one or nullator two in writing the nodal equations and carrying out the algorithm. So when two op-amps are used to implement the circuit, choose to connect the inputs of the op-amps to the nodes associated with the nullators and there are a choice of two nodes of where to connect the output. (One of the norator nodes must always be grounded, for a practical circuit) The second op-amp's output is connected to the remaining node. Nullator-norator pairing has been achieved. The end result is always the same for any pairing, as nothing has changed in the RC-nullor topology. In general, for an n nullor circuit, there are n choices for the first pairing a norator with a given nullator. Choosing the first pair there are $n-1$ nullators left. Choosing another nullator, there are $n-1$ choices in which to pair it with a norator. Choosing yields $(n-2)$ nullors left. By the Multiplication Principal of Combinational Analysis (MPCA), there are $n \times (n-1)$ ways of picking two pairs, $n \times (n-1) \times (n-2)$ ways of picking three pairs, or $n \times (n-1) \times (n-2) \times \dots \times 2 \times 1 = n!$ ways of choosing n pairs. This leads to property one of OAR [4].

PROPERTY 1:

Given a circuit containing n nullators and n grounded norators, there exists at least $n!$ equivalent op-amp circuits.

At this point, a definition is needed. An m -nullator tree will be defined to be a connected nullator network which has m nullators, $m+1$ nodes and no closed paths.

A nullator tree is shown in Figure 2.5. For an $n+1$ node, n nullator tree, it is noted that there are many ways to connect the nodes via n nullators.

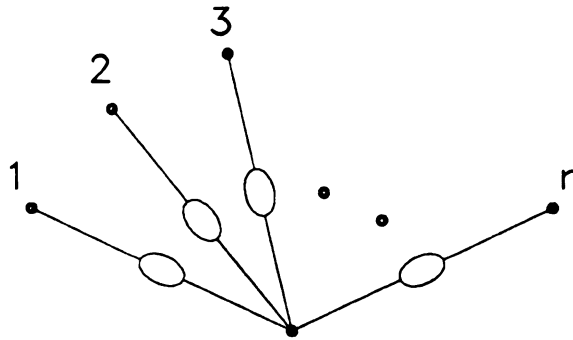


Figure 2.5 nullator tree

In fact, from [5], this leads to property 2.

PROPERTY 2:

Given a connected nullator network, where there are m nullators, there exists $(m+1)^{(m-1)}$ m -nullator trees.

The effect of a nullator tree is very simple to understand. Take, for example, the tree shown in Figure 2.6.

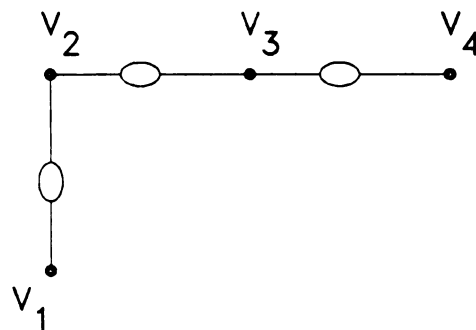


Figure 2.6 nullator tree in a circuit

The effect this has on the admittance matrix is to add the columns of three of the nodes to the fourth. It matters not which row chosen. The voltages at each of these nodes are all equal, since each nullator has a voltage drop of zero across it. Furthermore, a little work will show that it matters not how the nodes are connected together with nullators, as long as they are connected by any tree structure. The final outcome is always the same, the rows all get summed together and the node voltages are all equal. This leads to Theorem 1 [1].

THEOREM 1:

Given a circuit which contains n nullators and n grounded norators where there are i disjoint m_j -nullator trees, then there exists at least $n! \prod_{j=1}^i (m_j + 1)^{(m_j - 1)}$ equivalent op-amp circuits.

Proof: Suppose that an op-amp circuit N_1 which contains $k+1$ nodes is given. Insert nullors for the op-amps. Label the nullator trees 1, 2, . . . , i . Label the nodes of the designated first nullator tree 1, 2, . . . , m_1+1 . Label the nodes of the second designated nullator tree m_1+2 , m_1+3 , . . . , m_1+m_2+2 . Label the nodes of the i th nullator tree $(m_1 + m_2 + \dots + m_{i-1} + i)$, . . . , $(m_1 + m_2 + \dots + (m_i+i))$. Define a ground node. Form the admittance matrix of N_1 , $Y_{1 \ (k+1) \times (k+1)}$. Now add the columns of the connected nullator trees. Delete the rows corresponding to the norators. There now exists an admittance matrix $Y_{1 \ ((k-n) \times (k-n))}$. The determinant of this matrix is the characteristic polynomial.

Now consider $N_2, N_3, \dots, N_x$ created by forming a different nullator tree where $x = (m_1+1)^{(m_1-1)}$. In forming $Y_{x \ ((k-n) \times (k-n))}$ add columns 1, 2, . . . , m_1+1 together into one of the corresponding columns, call it column j . Since all the nodes corresponding to the columns are all connected via nullators, column j remains unchanged. If one of the nodes had been the ground node, all the columns are associated with those nodes are deleted as they would have been previously. Again no change. Thus each admittance matrix

corresponding to a different tree structure formed by rearranging the nullators in the first tree are the same. Thus there are $(m_1+1)^{(m_1-1)}$ equivalent circuits.

The same holds true for each of the remaining $i-1$ nullator trees. Thus, by using MPCA the total number of equivalent circuits the product of the number of trees for each i nullator structure, or equivalently $\prod_{j=1}^i (m_j + 1)^{(m_j - 1)}$. By property 1, there exists $n!$ equivalent op-amp circuits for each different nullator-norator network. Again using MPCA there are $n! \prod_{j=1}^i (m_j + 1)^{(m_j - 1)}$ equivalent circuits.

2.4 OAR and ideal filter synthesis

The means to generate circuits from OAR were intended to be used for circuits having any given transfer function between two nodes, one grounded. This thesis sticks strictly with a transfer function that takes on a one of four suitable filter functions. In addition to this, more emphasis will be placed on the band pass filter, although it is not necessary. These four filter functions are as follows:

1.) high pass function

$$T(s) = \frac{H_{hp} s^2}{s^2 + s(\omega_o/Q_o) + \omega_o^2}$$

2.) low pass function

$$T(s) = \frac{H_{lp} \omega_o^2}{s^2 + s(\omega_o/Q_o) + \omega_o^2}$$

3.) bandpass function

$$T(s) = \frac{H_p(\omega_o/Q_o)s}{s^2 + s(\omega_o/Q_o) + \omega_o^2}$$

4.) band reject or notch function

$$T(s) = \frac{H_n(s^2 + \omega_n^2)}{s^2 + s(\omega_o/Q_o) + \omega_o^2}$$

2.5 An example of OAR in the generation of a circuit

This section will give an example of Theorem 1 as presented in section 2.3. In addition it will also introduce a means to sort through the numerous generated circuits via the OAR program. It is a short example, but very representative of what will happen in a larger circuit. Consider the circuit Kerwin-Heulsmann-Newcomb (KHN) filter [7] and its nullor equivalent circuit shown in Figure 2.7.

The circuit exhibits a band-pass filter function at node 6. Let m_1 be the nullator tree connected between nodes 0, 3, and 4 and let m_2 be the nullator between nodes 1 and 2. From Property 2, there are $(2 + 1)^{(2-1)} = 3$ trees associated with m_1 . Then from Theorem 1 there are $3! \times 3 \times 1 = 18$ different circuits which can be generated from the seed circuit above. In addition, there are $2^3 = 8$ different sign configurations which must be checked out for each circuit. This is $18 \times 8 = 144$ circuits which can be tested.

From the above example, a circuit designer, to do a complete job, would invest time in checking out each circuit. A program has been written which will cut this time down substantially. It is called Op-Amp Relocation (OAR). OAR calculates the transfer function of a circuit and then does a stability check. If the circuit is unstable, it is thrown out. If stable, the transfer function is saved, along with the configuration of the op-amps. As a finale, the total number of stable circuits is given to us at the end of the program. An example input to OAR is given in Table 2.1.

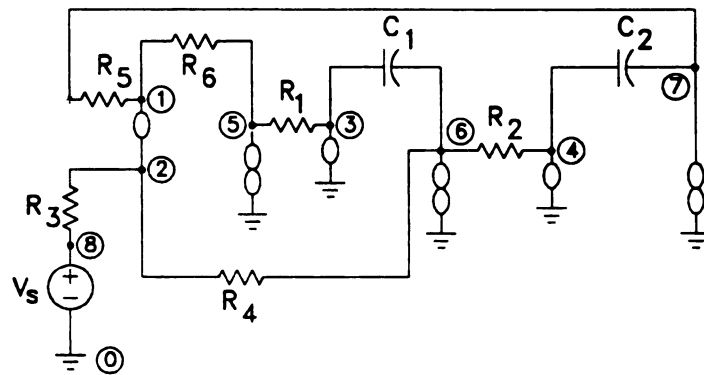
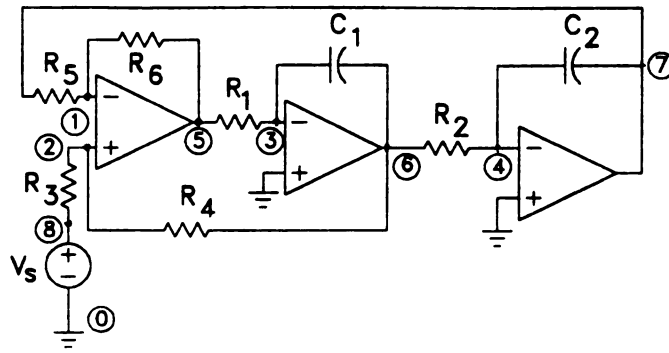


Figure 2.7 KHN active filter and equivalent circuit

Table 2.1 example OAR input

13							
3	5	0	0	res	.3845E+04	.0000E+00	.0000E+00
4	6	0	0	res	.1000E+04	.0000E+00	.0000E+00
1	8	0	0	res	.1000E+04	.0000E+00	.0000E+00
1	6	0	0	res	.5000E+05	.0000E+00	.0000E+00
2	7	0	0	res	.1000E+04	.0000E+00	.0000E+00
2	5	0	0	res	.1000E+04	.0000E+00	.0000E+00
4	7	0	0	cap	.3247E-07	.0000E+00	.0000E+00
3	6	0	0	cap	.3247E-07	.0000E+00	.0000E+00
1	2	5	0	noal	.0000E+00	.2000E+06	.5100E+07
0	3	6	0	noal	.0000E+00	.2000E+06	.5100E+07
0	4	7	0	noal	.0000E+00	.2000E+06	.5100E+07
8	0	0	0	vs	.1000E+01	.0000E+00	.0000E+00
6	0	0	0	ocr	.0000E+00	.0000E+00	.0000E+00

The circuit is first configured in the OAR editor. When commanded to save the circuit, Table 2.1 is the resulting output, saved in a file. The number thirteen at the top is the number of components in the circuit. The components are then listed below. The first four columns are the nodes of the devices; then the abbreviation for the device name; finally component values. Op-Amps are slightly different than the passive two terminal devices. The chosen op-amp model is the dominant pole model, stated by noa1. The user also has the option of ioa, noa2, or noa3 which depict different models of op-amps. The first, ioa depicts an ideal op-amp. The second and third, noa2 and noa3, depict two pole models for the op-amps, with noa2 having the second pole at the gbp and noa3 having the second pole placed at the choice of the user. The d.c. open loop gain for each op-amp is 200000, indicted by the second column. The gain-bandwidth product is given by .5100E+07 radians. These numbers are also determined by the user. The above component values correspond to a center frequency of 2500Hz. and a selectivity, Q, of 50. The output for the above program is shown in Table 2.2.

By theorem one there were 144 circuits as shown above. OAR then checked out each circuit for stability and determined that 14 out of these were stable (for the particular chosen op-amp model). The number of good possibilities has been considerably narrowed down. The sets of three numbers at the top of each group refers to the op-amps pairings used in each circuit. The first two numbers show the connection of the nullator, or equivalently the input terminals to the op-amp. The positive terminal hook-up is shown first. The last number refers to the connection of the ungrounded norator terminal, or equivalently the output of the op-amp. There were three op-amps, and hence three sets of numbers.

Next, it may be necessary to know the poles and zeros of the transfer functions above. OAR, through another module called poles and zeros (PZ), does this. In addition to putting out the output shown in Table 2.2, OAR outputs the input to PZ. The output from PZ, and hence the poles and zeros of the transfer function are shown in Table 2.3.

Table 2.2 example of an OAR output

GENERAL FORM OF Nth ORDER TRANSFER FUNCTION:

$$N(0) * S^{**} N + N(1) * S^{**} N-1 + N(2) * S^{**} N-2 \dots + N(N)$$

$$D(0) * S^{**} N + D(1) * S^{**} N-1 + D(2) * S^{**} N-2 \dots + D(N)$$

SYSTEM ORDER = 5

The scaling factors of the transfer functions are:

Impedance scaling factor: .1414E-03

Frequency scaling factor: .2296E-03

THE STABLE CIRCUITS

(+ input, - input, output)

Ground Node : 0

(1, 2, 5); (0, 3, 6); (0, 4, 7)

N(0) = .000000D+00

N(1) = .000000D+00

N(2) = .000000D+00

N(3) = -.247208D+07

N(4) = -.291218D+10

N(5) = -.102342D+06

D(0) = .100000D+01

D(1) = .293630D+04

D(2) = .275793D+07

D(3) = .808935D+09

D(4) = .582794D+08

D(5) = .104389D+11

(0, 3, 5); (2, 1, 6); (0, 4, 7)

N(0) = .000000D+00

N(1) = -.114799D+04

N(2) = -.135448D+07

N(3) = -.496709D+07

N(4) = -.291219D+10

N(5) = -.102343D+06

D(0) = .100000D+01

D(1) = .120283D+04

D(2) = .716982D+06

D(3) = .815089D+09

D(4) = .672305D+08

D(5) = .104390D+11

(4, 0, 5); (0, 3, 6); (1, 2, 7)

N(0) = .000000D+00

N(1) = .000000D+00

N(2) = .000000D+00

N(3) = .000000D+00

N(4) = -.289468D+10

N(5) = .000000D+00

D(0) = .100000D+01

D(1) = .176535D+04

D(2) = .138465D+07

D(3) = .808878D+09

D(4) = .757594D+08

D(5) = .104390D+11

(1, 2, 5); (0, 4, 6); (3, 0, 7)

N(0) = .000000D+00
 N(1) = .000000D+00
 N(2) = .000000D+00
 N(3) = .000000D+00
 N(4) = -.289468D+10
 N(5) = .000000D+00

D(0) = .100000D+01
 D(1) = .594401D+03
 D(2) = .138463D+07
 D(3) = .808893D+09
 D(4) = .757595D+08
 D(5) = .104390D+11

(1, 2, 5); (4, 0, 6); (0, 3, 7)

N(0) = .000000D+00
 N(1) = .000000D+00
 N(2) = .000000D+00
 N(3) = .000000D+00
 N(4) = -.289468D+10
 N(5) = .000000D+00

D(0) = .100000D+01
 D(1) = .594401D+03
 D(2) = .136807D+07
 D(3) = .796646D+09
 D(4) = .400279D+08
 D(5) = .104388D+11

(1, 2, 5); (4, 3, 6); (0, 4, 7)

N(0) = .000000D+00
 N(1) = .000000D+00
 N(2) = .000000D+00
 N(3) = -.247208D+07
 N(4) = -.291218D+10
 N(5) = -.102342D+06

D(0) = .100000D+01
 D(1) = .293630D+04
 D(2) = .274965D+07
 D(3) = .804072D+09
 D(4) = .493359D+08
 D(5) = .104389D+11

(4, 3, 5); (2, 1, 6); (0, 4, 7)

N(0) = .000000D+00
 N(1) = -.114799D+04
 N(2) = -.135448D+07
 N(3) = -.496709D+07
 N(4) = -.291219D+10
 N(5) = -.102343D+06

D(0) = .100000D+01
 D(1) = .120283D+04
 D(2) = .716982D+06
 D(3) = .810241D+09
 D(4) = .582871D+08
 D(5) = .104389D+11

(4, 0, 5); (4, 3, 6); (1, 2, 7)

N(0) = .000000D+00
 N(1) = .000000D+00
 N(2) = .134423D+07
 N(3) = .247995D+07
 N(4) = -.289467D+10
 N(5) = .000000D+00

D(0) = .100000D+01
 D(1) = .176535D+04
 D(2) = .134948D+07
 D(3) = .803965D+09
 D(4) = .668156D+08
 D(5) = .104390D+11

(1, 2, 5); (0, 4, 6); (3, 4, 7)

N(0) = .000000D+00
 N(1) = .000000D+00
 N(2) = .000000D+00
 N(3) = .000000D+00
 N(4) = -.289468D+10
 N(5) = .000000D+00

D(0) = .100000D+01
 D(1) = .176535D+04
 D(2) = .207236D+07
 D(3) = .810158D+09
 D(4) = .757669D+08
 D(5) = .104390D+11

(1, 2, 5); (4, 3, 6); (0, 3, 7)

N(0) = .000000D+00
 N(1) = .000000D+00
 N(2) = .000000D+00
 N(3) = -.247208D+07
 N(4) = -.291218D+10
 N(5) = -.102342D+06

D(0) = .100000D+01
 D(1) = .176535D+04
 D(2) = .206193D+07
 D(3) = .801547D+09
 D(4) = .404062D+08
 D(5) = .104388D+11

(4, 3, 5); (0, 3, 6); (1, 2, 7)

N(0) = .000000D+00
 N(1) = .000000D+00
 N(2) = .000000D+00
 N(3) = .000000D+00
 N(4) = -.289468D+10
 N(5) = .000000D+00

D(0) = .100000D+01
 D(1) = .176535D+04
 D(2) = .138680D+07
 D(3) = .810154D+09
 D(4) = .846819D+08
 D(5) = .104391D+11

(3, 0, 5); (4, 3, 6); (1, 2, 7)

N(0) = .000000D+00
 N(1) = .000000D+00
 N(2) = .134423D+07
 N(3) = .247995D+07
 N(4) = -.289467D+10
 N(5) = .000000D+00

D(0) = .100000D+01
 D(1) = .176535D+04
 D(2) = .661771D+06
 D(3) = .801424D+09
 D(4) = .578857D+08
 D(5) = .104389D+11

(3, 0, 5); (1, 2, 6); (3, 4, 7)

N(0) = .000000D+00
 N(1) = .114799D+04
 N(2) = .135448D+07
 N(3) = .229183D+05
 N(4) = -.291216D+10
 N(5) = -.102342D+06

D(0) = .100000D+01
 D(1) = .115692D+04
 D(2) = .134406D+07
 D(3) = .805072D+09
 D(4) = .493417D+08
 D(5) = .104389D+11

(1, 2, 5); (0, 3, 6); (3, 4, 7)

N(0) = .000000D+00
 N(1) = .000000D+00
 N(2) = .000000D+00
 N(3) = -.247208D+07
 N(4) = -.291218D+10
 N(5) = -.102342D+06

D(0) = .100000D+01
 D(1) = .293630D+04
 D(2) = .275793D+07
 D(3) = .810196D+09
 D(4) = .672017D+08
 D(5) = .104390D+11

Stable circuits: 14

Total circuits: 144

Table 2.3 example of a PZ output

(+input, -input, output) (+input, -input, output) (+input, -input, output)

Unscaled Poles and Zeros

Ground Node : 0

(1, 2, 5); (0, 3, 6); (0, 4, 7)

-.610809D+02 .156470D+05
 -.254981D+07 -.495806D-02
 -.514499D+07 -.416336D+02
 -.509395D+07 .416338D+02
 -.610809D+02 -.156470D+05

-.153062D+00 -.143912D-18
 -.513083D+07 .127758D-06

(0, 3, 5); (2, 1, 6); (0, 4, 7)

-.155104D+03 .155867D+05
 -.487265D+05 .361881D+07
 -.514109D+07 .307989D-02
 -.155104D+03 -.155867D+05
 -.487269D+05 -.361881D+07

-.401764D+04 .202074D+06
 -.153063D+00 -.560860D-12
 -.513082D+07 -.193436D+00
 -.401764D+04 -.202074D+06

(4, 0, 5); (0, 3, 6); (1, 2, 7)

-.155876D+03 .156470D+05
 -.129036D+07 .337891D+07
 -.510784D+07 -.223961D+01
 -.155876D+03 -.156470D+05
 -.129036D+07 -.337891D+07

(1, 2, 5); (0, 4, 6); (3, 0, 7)

-.155870D+03 .156467D+05
 -.177768D+05 .511606D+07
 -.255301D+07 .461288D-01
 -.155870D+03 -.156467D+05
 -.177760D+05 -.511606D+07

(1, 2, 5); (4, 0, 6); (0, 3, 7)

-.604185D+02 .157664D+05
 -.210475D+05 .508369D+07
 -.254666D+07 .425383D-01
 -.604185D+02 -.157664D+05
 -.210467D+05 -.508369D+07

(1, 2, 5); (4, 3, 6); (0, 4, 7)

-.369338D+02 .156940D+05
 -.254981D+07 .387132D-01
 -.551631D+07 -.185549D+01
 -.472268D+07 .173713D+01
 -.369338D+02 -.156940D+05

-.153062D+00 -.143912D-18
 -.513083D+07 .127758D-06

(4, 3, 5); (2, 1, 6); (0, 4, 7)

-.131844D+03 .156334D+05
 -.538873D+05 .361158D+07
 -.513081D+07 .320775D-02
 -.131844D+03 -.156334D+05
 -.538877D+05 -.361158D+07

-.401764D+04 .202074D+06
 -.153063D+00 -.560860D-12
 -.513082D+07 -.193436D+00
 -.401764D+04 -.202074D+06

(4, 0, 5); (4, 3, 6); (1, 2, 7)

-.133538D+03 .156948D+05
 -.123310D+07 .334620D+07
 -.522240D+07 -.183995D+00
 -.133538D+03 -.156948D+05
 -.123310D+07 -.334620D+07

.198135D+06 -.100672D-07
 -.206171D+06 .100672D-07
 .000000D+00 .000000D+00

(1, 2, 5); (0, 4, 6); (3, 4, 7)

-.131901D+03 .156351D+05
 -.256681D+07 .442778D+07
 -.255499D+07 -.178074D-02
 -.256682D+07 -.442778D+07
 -.131901D+03 -.156351D+05

(1, 2, 5); (4, 3, 6); (0, 3, 7)

-.368211D+02 .157184D+05
 -.257208D+07 .440546D+07
 -.254463D+07 -.152308D-02
 -.257209D+07 -.440546D+07
 -.368211D+02 -.157184D+05

-.153062D+00 -.143912D-18
 -.513083D+07 .127758D-06

(4, 3, 5); (0, 3, 6); (1, 2, 7)
 -.179622D+03 .156346D+05
 -.129233D+07 .338268D+07
 -.510386D+07 .237847D-03
 -.179622D+03 -.156346D+05
 -.129233D+07 -.338268D+07

(3, 0, 5); (4, 3, 6); (1, 2, 7)
 -.133884D+03 .157192D+05
 -.234159D+06 .301913D+07
 -.722029D+07 -.764917D-01
 -.133884D+03 -.157192D+05
 -.234158D+06 -.301913D+07

.198135D+06 -.100672D-07
 -.206171D+06 .100672D-07
 .000000D+00 .000000D+00

(3, 0, 5); (1, 2, 6); (3, 4, 7)
 -.863329D+02 .156839D+05
 -.843320D+06 .437385D+07
 -.335208D+07 -.115147D+01
 -.863329D+02 -.156839D+05
 -.843322D+06 -.437385D+07

.198135D+06 -.736373D-07
 -.206170D+06 .328059D-01
 -.513082D+07 -.345856D+00
 -.153063D+00 .387565D-13

(1, 2, 5); (0, 3, 6); (3, 4, 7)
 -.851202D+02 .156350D+05
 -.256560D+07 .107252D+02
 -.531469D+07 -.958762D+00
 -.490842D+07 .950893D+00
 -.851202D+02 -.156350D+05

-.153062D+00 -.143912D-18
 -.513083D+07 .127758D-06

The roots of the denominator determine the actual Q's and center frequency of the bandpass filter function. A look at the above output show that they are obviously not the same. This is because in choosing the design components ideal op-amps were used. In OAR, non-ideal op-amps are used. This gives way for a wide range of different frequency responses for the generated circuits. Some of the responses will be closer to their ideal performance than others.

Consider the original circuit, the very first listed circuit. From pole defining equations it is seen that it has a center frequency of 2490.32 Hz and a Q of 128.08. Far from the ideal values of 2500 and 50. This is a -0.38% error in the center frequency and a 128.08% error in Q. Next consider the third circuit shown. It has a center frequency of 2490.42Hz., a -0.38% error. The Q of this circuit is 50.19, an error of only 0.38%! Not perfection, but a vast improvement over the original circuit.

It is up to the circuit designer to choose which circuit best represents the ideal circuit. This is no easy job. A set of rules in the upcoming chapters will aid the circuit designer in this task.

CHAPTER 3

Effects of non-ideal op-amps

3.1 Introduction

In the initial stage, the active filter design process consists of recognizing a transfer function through resistors, capacitors, and op-amps. Typically, the op-amps are ideal. They are much easier to work with. The non-ideal op-amp has a gain which is quite frequency dependent. The ideal op-amp has a infinite gain-bandwidth product. The non-ideal op-amp has a gain of less than infinity, and also several poles along the frequency spectrum. This is this biggest cause of causing the circuit to deviate from it's ideal response. The gain vs. frequency response of a typical op-amp is shown in Figure 3.1.

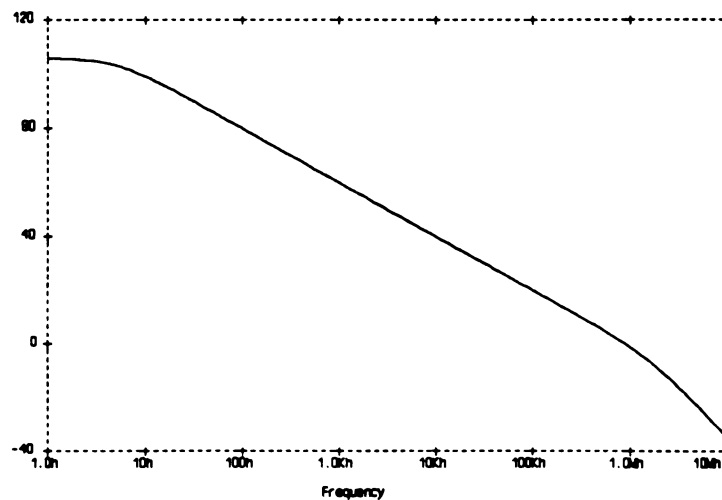


Figure 3.1 non-ideal op-amp response

Frequently, it is sufficient to model the op-amp as a voltage controlled voltage source with one pole in the frequency response. The controlling voltage is the input to the op-amp with typically the input impedance taken to be infinite. This is termed the dominant pole model. The transfer function for the dominant pole model is $V_{in}/V_{out} = A_o\omega_o/(s + \omega_o)$ where A_o is the dc open loop gain. Typically $A_o = 100,000\text{--}200,000$ and $\omega_o = 5\times 2\pi\text{--}25\times 2\pi$ rad, for common op-amps. Other characteristics assumed in the model are infinite input impedance, infinite common-mode rejection ratio, and zero output impedance. The gain vs. frequency response is a substantial departure from that of an ideal op-amp. One might have to ask oneself why not just stick to this model in the initial ideal design process? The answer is that if each and every op-amp from a manufacturer had the same frequency response, it may be reasonable to do this. However, currently, solid state technology is not to the point where every op-amp of even the same model can be modeled identically. Moreover, there are many different models of op-amps with many different qualities on the market. This makes this approach nearly impossible. It is better to be able to design a circuit with ideal op-amps and hope that when using non-ideal op-amps the circuit's performance will not vary significantly when non-ideal op-amps are used. There has been extensive work done along these lines. The results of the work are commonly referred to as making the circuit insensitive to the finite gain-bandwidth product of the op-amp. This chapter will look at the problem of the varying circuit performance due to finite gain-bandwidth product of the op-amp.

3.2 The KHN active filter

Consider the ideal KHN circuit shown in Figure 2.7.

When ideal op-amps are used, the circuit has the solution shown in Table 3.1, where $G_i = 1/R_i$.

Table 3.1 SLAP output

DENOMINATOR IS :

$$\begin{aligned} & -sC2*sC1*G6*G4 \\ & -sC2*sC1*G6*G3-sC2*G6*G4*G1-sC2*G5*G4*G1-G5*G4*G2*G1 \\ & -G5*G3*G2*G1 \end{aligned}$$

numerator for V5 is :

$$\begin{aligned} & -sC2*sC1*G6*G3 \\ & -sC2*sC1*G5*G3 \end{aligned}$$

numerator for V6 is :

$$\begin{aligned} & sC2*G6*G3*G1 \\ & +sC2*G5*G3*G1 \end{aligned}$$

numerator for V7 is :

$$\begin{aligned} & -G6*G3*G2*G1 \\ & -G5*G3*G2*G1. \end{aligned}$$

The above is courtesy of SLAP. It should be noted from above that the following filter functions exist in the circuit. At node 5 there exists a high pass filter. At node 6 there exists a bandpass filter. At node 7, a low pass filter exists. This is a nice package to have in one circuit.

Suppose that a bandpass filter is desired from the above. Then the transfer function at node 6 can easily be shown to be equation (3.1). Equations (3.2), (3.3), and (3.4) give the equations for the center frequency gain H_{bp} , the selectivity Q_o , and the center frequency f_o :

$$\frac{V_6}{V_m} = \frac{s[R_6(1/R_5 + 1/R_6)/C_1R_1R_3(1/R_3 + 1/R_4)]}{s^2 + s[R_6(1/R_5 + 1/R_6)/C_1R_1R_4(1/R + 1/R_4)] + [R_6/C_2C_1R_2R_3]} \quad (3.1)$$

$$H_{bp} = R_4/R_3 \quad (3.2)$$

$$f_o = [R_4/C_2C_1R_3R_2R_1]^{1/2}(1/2\pi) \quad (3.3)$$

$$Q_o = \frac{C_1R_1R_4^2(1/R_3 + 1/R_4)^2}{C_2R_4R_3R_2(1/R_3 + 1/R_4)^2} \quad (3.4)$$

Then if the following is selected:

$$\begin{aligned} C_1 &= C_2 = 32.467\text{nF} \\ R_2 &= R_3 = R_5 = R_6 = 1\text{k}\Omega \\ R_1 &= 3.844\text{k}\Omega \\ R_4 &= 50\text{k}\Omega, \end{aligned}$$

this yields an $H_{op} = 50$, a $Q_o = 50$, and an $f_o = 2500\text{Hz}$. The frequency response is given in Figure 3.2, which is a SPICE output.

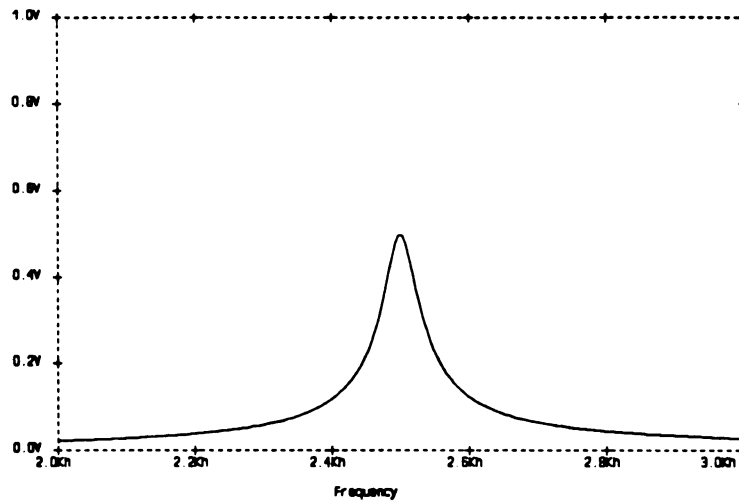


Figure 3.2 ideal filter frequency response

The plot in Figure 3.2 has the characteristics that the filter has been designed for. If, however, a dominant pole model is used the results change drastically. Consider the comparison of the ideal vs. non-ideal frequency response shown in Figure 3.3.

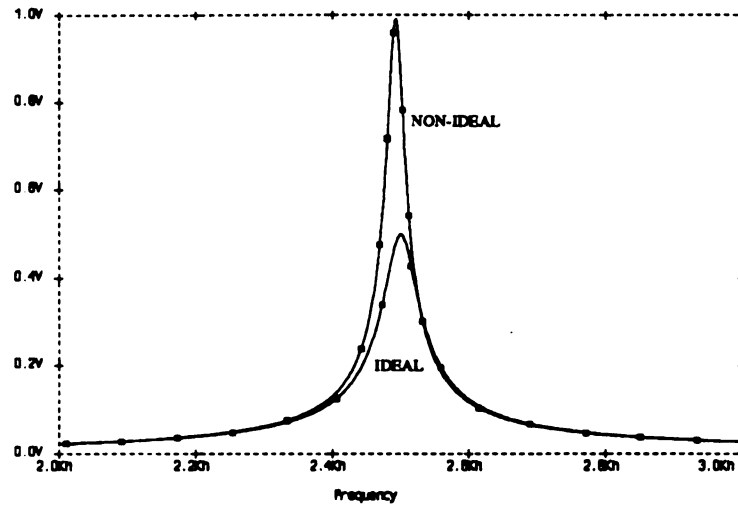


Figure 3.3 ideal vs. non-ideal filter response

There is a substantial change in the characteristics at the bandpass output of the KHN filter. Surprisingly, this is not bad. Many circuits designed with ideal op-amps will not perform at all, when non-ideal op-amps are used. The large spike in the non-ideal characteristic represents a great enhancement of the Q_o . In many circuits, it is easy to drive Q_o much larger than this. The result can be instability. The complete solution to the circuit with the one pole model, where $A(s) \equiv 1/B_1$, from SLAP is shown in Table 3.2.

This is a complicated solution. Now, if given this as a design equation in order to construct a bandpass filter with the same parameters as before, one could prepare oneself for a nearly impossible task. Additionally it is evident that because of the different B_1 terms there will be many different responses corresponding to the many different op-amps available on the market, with frequency responses not alike. One solution to the problem is the Wilson-Bedri-Bowron [8] approximation, as will be shown. The approximation allows one to get a handle on the immense task at hand, albeit using some constraints.

It is noted in the above solution that the frequency response of the circuit changes substantially. A quick look through the solution will show that there are many terms multiplied by one or more $1/A(s)$ terms and some terms remain untouched by these terms.

Table 3.2 SLAP output with non-ideal op-amps

DENOMINATOR IS :

$$\begin{aligned}
 & sC2*sC1*G6*G4*B3*B2*B1 \\
 & +sC2*sC1*G6*G4*B3*B2+sC2*sC1*G6*G4*B3*B1 \\
 & +sC2*sC1*G6*G4*B3+sC2*sC1*G6*G4*B2*B1+sC2*sC1*G6*G4*B2+sC2*sC1*G6*G4*B1 \\
 & +sC2*sC1*G6*G4+sC2*sC1*G6*G3*B3*B2*B1+sC2*sC1*G6*G3*B3*B2 \\
 & +sC2*sC1*G6*G3*B3*B1+sC2*sC1*G6*G3*B3+sC2*sC1*G6*G3*B2*B1 \\
 & +sC2*sC1*G6*G3*B2+sC2*sC1*G6*G3*B1+sC2*sC1*G6*G3+sC2*sC1*G5*G4*B3*B2*B1 \\
 & +sC2*sC1*G5*G4*B3*B1+sC2*sC1*G5*G4*B2*B1+sC2*sC1*G5*G4*B1 \\
 & +sC2*sC1*G5*G3*B3*B2*B1+sC2*sC1*G5*G3*B3*B1+sC2*sC1*G5*G3*B2*B1 \\
 & +sC2*sC1*G5*G3*B1+sC2*G6*G4*G1*B3*B2*B1+sC2*G6*G4*G1*B3*B2 \\
 & +sC2*G6*G4*G1*B3+sC2*G6*G4*G1*B2*B1+sC2*G6*G4*G1*B2+sC2*G6*G4*G1 \\
 & +sC2*G6*G3*G1*B3*B2*B1+sC2*G6*G3*G1*B3*B2+sC2*G6*G3*G1*B2*B1 \\
 & +sC2*G6*G3*G1*B2+sC2*G5*G4*G1*B3*B2*B1+sC2*G5*G4*G1*B3 \\
 & +sC2*G5*G4*G1*B2*B1+sC2*G5*G4*G1+sC2*G5*G3*G1*B3*B2*B1 \\
 & +sC2*G5*G3*G1*B2*B1+sC1*G6*G4*G2*B3*B2*B1+sC1*G6*G4*G2*B3*B2 \\
 & +sC1*G6*G4*G2*B3*B1+sC1*G6*G4*G2*B3+sC1*G6*G3*G2*B3*B2*B1 \\
 & +sC1*G6*G3*G2*B3*B2+sC1*G6*G3*G2*B3*B1+sC1*G6*G3*G2*B3 \\
 & +sC1*G5*G4*G2*B3*B2*B1+sC1*G5*G4*G2*B3*B1+sC1*G5*G3*G2*B3*B2*B1 \\
 & +sC1*G5*G3*G2*B3*B1+G6*G4*G2*G1*B3*B2*B1+G6*G4*G2*G1*B3*B2 \\
 & +G6*G4*G2*G1*B3+G6*G3*G2*G1*B3*B2*B1+G6*G3*G2*G1*B3*B2 \\
 & +G5*G4*G2*G1*B3*B2*B1+G5*G4*G2*G1*B3+G5*G4*G2*G1+G5*G3*G2*G1*B3*B2*B1 \\
 & +G5*G3*G2*G1
 \end{aligned}$$

numerator for V1 is :

$$\begin{aligned}
 & sC2*sC1*G6*G3*B3*B2*B1 \\
 & +sC2*sC1*G6*G3*B3*B2+sC2*sC1*G6*G3*B3*B1 \\
 & +sC2*sC1*G6*G3*B3+sC2*sC1*G6*G3*B2*B1+sC2*sC1*G6*G3*B2+sC2*sC1*G6*G3*B1 \\
 & +sC2*sC1*G6*G3+sC2*sC1*G5*G3*B3*B2*B1+sC2*sC1*G5*G3*B3*B1 \\
 & +sC2*sC1*G5*G3*B2*B1+sC2*sC1*G5*G3*B1+sC2*G6*G3*G1*B3*B2*B1 \\
 & +sC2*G6*G3*G1*B3*B2+sC2*G6*G3*G1*B2*B1+sC2*G6*G3*G1*B2 \\
 & +sC2*G5*G3*G1*B3*B2*B1+sC2*G5*G3*G1*B2*B1+sC1*G6*G3*G2*B3*B2*B1 \\
 & +sC1*G6*G3*G2*B3*B2+sC1*G6*G3*G2*B3*B1+sC1*G6*G3*G2*B3 \\
 & +sC1*G5*G3*G2*B3*B2*B1+sC1*G5*G3*G2*B3*B1+G6*G3*G2*G1*B3*B2*B1 \\
 & +G6*G3*G2*G1*B3*B2+G5*G3*G2*G1*B3*B2*B1+G5*G3*G2*G1
 \end{aligned}$$

numerator for V2 is :

$$\begin{aligned}
 & sC2*sC1*G6*G3*B3*B2 \\
 & +sC2*sC1*G6*G3*B3+sC2*sC1*G6*G3*B2+sC2*sC1*G6*G3 \\
 & +sC2*G6*G3*G1*B3*B2+sC2*G6*G3*G1*B2+sC1*G6*G3*G2*B3*B2+sC1*G6*G3*G2*B3 \\
 & +G6*G3*G2*G1*B3*B2+G5*G3*G2*G1
 \end{aligned}$$

numerator for V3 is :

$$\begin{aligned}
 & sC2*G6*G3*G1*B3*B2 \\
 & +sC2*G6*G3*G1*B2+sC2*G5*G3*G1*B3*B2+sC2*G5*G3*G1*B2
 \end{aligned}$$

$$+G6*G3*G2*G1*B3*B2+G5*G3*G2*G1*B3*B2$$

numerator for V4 is :

$$\begin{aligned} &-G6*G3*G2*G1*B3 \\ &-G5*G3*G2*G1*B3 \end{aligned}$$

numerator for V5 is :

$$\begin{aligned} &sC2*sC1*G6*G3*B3*B2 \\ &+sC2*sC1*G6*G3*B3+sC2*sC1*G6*G3*B2+sC2*sC1*G6*G3 \\ &+sC2*sC1*G5*G3*B3*B2+sC2*sC1*G5*G3*B3+sC2*sC1*G5*G3*B2+sC2*sC1*G5*G3 \\ &+sC2*G6*G3*G1*B3*B2+sC2*G6*G3*G1*B2+sC2*G5*G3*G1*B3*B2+sC2*G5*G3*G1* \\ &B2 \\ &+sC1*G6*G3*G2*B3*B2+sC1*G6*G3*G2*B3+sC1*G5*G3*G2*B3*B2+sC1*G5*G3*G2* \\ &B3 \\ &+G6*G3*G2*G1*B3*B2+G5*G3*G2*G1*B3*B2 \end{aligned}$$

numerator for V6 is :

$$\begin{aligned} &-sC2*G6*G3*G1*B3 \\ &-sC2*G6*G3*G1-sC2*G5*G3*G1*B3-sC2*G5*G3*G1 \\ &-G6*G3*G2*G1*B3-G5*G3*G2*G1*B3 \end{aligned}$$

numerator for V7 is :

$$\begin{aligned} &G6*G3*G2*G1 \\ &+G5*G3*G2*G1 \end{aligned}$$

If, as in the ideal case, A goes to infinity, all of the B terms drop out and there are a few remaining terms. These remaining terms are precisely the solution to the circuit when ideal op-amps are used as shown in the first example of an output from SLAP.

If a one pole rolloff model is used, then $A(s) = A_o\omega_o/(s + \omega_o) \approx A_o\omega_o/s$ at frequencies higher than the cutoff frequency, ω_o . Then the $s/(A_o\omega_o)$ terms will add three extra poles to the circuit, one for each op-amp. These will be termed parasitic poles. The poles located about the ideal design poles will be termed in-band poles. Now if the ideal equation has the form of $a_2s^2 + a_1s + a_0$, the solution now has the form of $[f_3(A_o\omega_o)]s^5 + [f_4(A_o\omega_o)]s^4 + [f_3(A_o\omega_o)]s^3 + [f_2(A_o\omega_o) + a_2]s^2 + [f_1(A_o\omega_o) + a_1]s + a_0$, where each $f_j(A_o\omega_o)$ term is a summation of terms, each with $(A_o\omega_o)^j$, $j = 1, 2$, or 3 , term in the denominator. Thus, each $f_j(A_o\omega_o)$ tends to zero as the gain-bandwidth product, $A_o\omega_o$, tends to infinity. Now since each term in front of the s^5 , s^4 , and s^3 does contain one or more $A_o\omega_o$ terms in the denominator, the coefficients in front of these terms will tend to be very small in comparison to the coefficients in front of the s^2 , s^1 , and s^0 terms. Thus, to a reasonable approximation, at frequencies about the design frequencies, only the in-band poles need to be considered. It may be noted that this is a valid approximation provided that the gain-bandwidth product, and hence $A_o\omega_o$, is fairly large. These poles still tend to be different than the design poles, since the $f_j(A_o\omega_o)$ terms must be used when calculating the center frequency and selectivity. At higher frequencies, three extra poles are introduced into the denominator. These are the parasitic poles for the circuit, using a one-pole model. Most generally, the denominator can be written in the following fashion,

$$D(s) = \left(\sum_{j=0}^k a_j s^j \right) \left(\sum_{l=0}^n b_l s^l \right) \quad (3.5)$$

where the coefficients b_l are to be determined, the coefficients a_j are the in-band coefficients, k is the order of the ideal polynomial, and n is the number of op-amps. In order for a circuit to be stable, both sets of these poles must lie in the LHP on the

frequency domain.

It can be seen that the stability of the parasitic poles depend heavily on the sign of the op-amp itself, more so than the in-band poles. If, for instance, the polarity of one of the op-amps in the above equation is changed, this corresponds to the B_i term for that particular op-amp, or in terms of the model, simply changing signs in the voltage controlled voltage source being used as a model. This has a big impact on the $f_i(A_{oi}\omega_{oi})$ terms, which in turn has a big impact in front of the coefficients in for the s^5 , s^4 , and s^3 terms. It has a lesser impact on the other coefficients as the change is a smaller percentage of the overall coefficient, as it is dominated by the ideal a_i term. These parasitic poles are thus very dependent on the sign configuration of the op-amp, and result in probably the biggest cause for instability in a breadboarded circuit. When one tries to build this seemingly good circuit, noise will push the circuit into instability and the circuit will lock up at either positive or negative 15 volts. This very subtle problem is one that still needs to be addressed.

The second source of instability are the in-band poles themselves. It is possible to drive Q_o very high. At the other extreme, it may be possible to drive it negative. The example of a Q_o being driven high was shown in the example of the KHN circuit. A very high Q can give rise to a very large gain near the center frequency, and the overall voltage can go beyond the upper limit of the op-amp's output. It can be so high that it is possible for noise to actually excite the signal past this limit.

Thirdly, although the effects of a one pole model have been described in this chapter, a two pole model may more accurately reflect a real op-amp [9]. It will be shown that if a circuit used to implement a biquadratic equation has n op-amps, then this model will give rise to a $2n+2$ order equation in s , as opposed to an $n+2$ order equation. Although the one pole model may reflect accurately the in-band errors, the two pole model more accurately reflects the stability of the parasitic poles which may arise in the circuit. Unfortunately, stability of the parasitic poles in a circuit which utilizes a one pole

model no way insures stability when a two pole model is employed. An even more complicated situation now arises.

This chapter introduced the problems due to the finite gbp of an op-amp. The finite gbp then complicates circuit design and gives rise to both inband errors and an inherent stability problem which arises due to the parasitic poles. The inband problem will be addressed in the upcoming chapter.

CHAPTER 4

Reduction of in-band errors

4.1 Introduction

It was shown that when a circuit is designed with ideal op-amps, the in-band poles have a tendency to shift when the circuit is employed using non-ideal op-amps. These errors are mainly caused by the gain-bandwidth product of the op-amp having a finite value. Furthermore it was noted that the coefficients in front of the now non-ideal biquadratic equation describing the in-band poles of the circuit go from just a few terms, to an equation containing many terms. This new equation is the cause of serious problems for the circuit designer who is trying to design the circuit with specific parameters. There have been several approaches to the solution of this problem [8-13]. This chapter introduces one of those solutions, the Wilson-Bedri-Bowron (WBB) [8] approximation. Additionally, it is shown where the main causes of deviation in other filter function parameters come from. The approximation allows one to get a grip on the design process, although a couple of constraints must be invoked, one of the most important is that matched op-amps are employed, or equivalently each op-amp has the same gbp, for the approximation to be used in its simplest form. This constraint is impractical if one is trying to match individual chips, of the same model, each containing one op-amp. It is very practical, however, if all of the op-amps being used are on the same chip, as it has been shown that this can indeed give rise to 'equivalent' op-amps.

4.2 The Wilson-Bedri-Bowron approximation [8]

For an arbitrary N op-amp circuit realizing some random second order transfer function, the denominator polynomial may be written as

$$D(s) = s^2\alpha f_2(A) + s\beta f_1(A) + \gamma f_0(A), \quad (4.1)$$

where the coefficients α , β , and γ are the ideal design coefficients of the circuit, and the $f_i(A)$ are functions of the open loop gain and tend to one as A tends to infinity. Then the center frequency ω_o and the selectivity, Q_o , can be written as:

$$\omega_o^2 = \frac{\gamma f_0(\infty)}{\alpha f_2(\infty)} \quad (4.2)$$

$$Q_o = \frac{\alpha \gamma f_0(\infty) f_2(\infty)}{\beta f_1(\infty)}. \quad (4.3)$$

In general:

$$f_j(A) = 1 + \sum_{i=0}^j \frac{d_{ij}}{A_j} + \sum_{i=0}^j \sum_{k=0}^j \frac{d_{ijk}}{A_j A_k} + \dots + \frac{d_{1,2,\dots,N}}{A_1 A_2 \dots A_N} \quad (4.4)$$

where A_j is the response of the op-amp and all repeated-suffix coefficients are zero due to the bilinear nature of the circuit. Adopting the dominant pole model for an op-amp, where $A_i = A_{oi}\omega_{oi}/(s + \omega_{oi})$ is a reasonable approximation, and at reasonable frequencies the higher order terms in $f_j(A)$ can be dropped, since A_o is typically on the order of 100,000 ~ 200,000. Then let

$$\frac{d_i}{A} = \sum_{j=1}^n \frac{d_{ij}}{A_j}, \quad (4.5)$$

using the fact that matched op-amps will be employed. Thus $f_i(A)$ becomes

$$f_i(A) = 1 + \frac{d_i}{A}. \quad (4.6)$$

Dividing by γ , equation 4.1 becomes

$$D(s) = s^2 T_o^2 (1 + d_2/A) + s(T_o/Q_o)(1 + d_1/A) + (1 + d_o/A), \quad (4.7)$$

where $T_o = 1/\omega_o$ = the period of the denominator. Also from (4.5),

$$\begin{aligned} \frac{d_i}{A} &= \frac{d_i(s + \omega_o)}{A_o \omega_o} \\ &= \frac{d_i s}{A_o \omega_o} + \frac{d_i}{A_o}. \end{aligned} \quad (4.8)$$

Then from (4.7),

$$\begin{aligned} D(s) &= s^2 T_o^2 (1 + s d_2/gb + d_2/A_o) + s T_o/Q_o (1 + s d_1/gb + d_1/A_o) \\ &\quad + (1 + s d_o/gb + d_o/A_o), \end{aligned} \quad (4.9)$$

where $gb = A_o \omega_o$. Collecting the s terms gives

$$\begin{aligned}
D(s) &= s^3(T_o^2 d_2 / gb) + s^2(T_o^2 + T_o^2 d_2 / A_o + T_o d_1 / Q_o gb) \\
&\quad + s(T_o d_1 / Q_o A_o + T_o / Q_o + d_o / gb) + (1 + d_o / A_o) \\
&= s^3(T_o^2 d_2 / gb) + s^2 T_o^2 (1 + d_2 / A_o + d_1 / T_o Q_o gb) \\
&\quad + s(T_o / Q_o)(1 + d_1 / A_o + Q_o d_o / T_o gb) + (1 + d_o / A_o). \tag{4.10}
\end{aligned}$$

Factoring out $(1 + d_o / A_o)$ and noting that

$$\frac{1}{(1 + d_o / A_o)} \equiv (1 - d_o / A_o), \tag{4.11}$$

gives

$$\begin{aligned}
D(s) &= [1 + d_o / A_o] [s^3(T_o^2 d_2 / gb)(1 - d_o / A_o) \\
&\quad + s^2 T_o^2 (1 + d_2 / A_o + d_1 / T_o Q_o gb)(1 - d_o / A_o) \\
&\quad + s(T_o / Q_o)(1 + d_1 / A_o + Q_o d_o / T_o gb)(1 - d_o / A_o) + 1]. \tag{4.12}
\end{aligned}$$

Since $1/A_o \gg 1/A_o^2$, for large A_o , equation (4.12) further factors to

$$\begin{aligned}
D(s) &= [1 + d_o / A_o] [s^3(T_o^2 d_2 / gb) + s^2 T_o^2 (1 + d_2 / A_o - d_o / A_o + d_1 / T_o Q_o gb) \\
&\quad + s(T_o / Q_o)(1 + d_1 / A_o + Q_o d_o / T_o gb - d_o / A_o) + 1] \tag{4.13}
\end{aligned}$$

$$= (1 + d_o / A_o) \bar{D}(s). \tag{4.14}$$

Now the poles of $D(s)$ are exactly the poles of $\bar{D}(s)$. Thus the poles of $\bar{D}(s)$ will be solved for to give the center frequency ω and selectivity Q . At frequencies about the design poles, the coefficient for the s^3 term is much smaller than the coefficients in front of the other s

terms since it is multiplied by the d_i/gb term and is negligible inband. Thus $\bar{D}(s)$ can assume to have one high frequency pole and two inband poles. $\bar{D}(s)$ can be rewritten as

$$\begin{aligned}\bar{D}(s) &= (s\tau_d + 1)(s^2T^{*2} + s(T^*/Q^*) + 1) \\ &= s^3(\tau_d T^{*2}) + s^2(T^* + \tau_d T^*/Q^*) + s(\tau_d + T^*/Q^*) + 1,\end{aligned}\quad (4.15)$$

where

T^* is the realized period,

Q^* is the realized selectivity,

τ_d is defined to be an auxiliary time constant, the reciprocal of which is the higher frequency pole, and the above quantities need to be determined. Then define, in terms of the ideal design parameters, the following

$$\Delta t \equiv \frac{\Delta T_o}{T_o}, \quad (4.16)$$

which is the fractional change in T_o , and

$$\Delta q \equiv \frac{\Delta(1/Q_o)}{(1/Q_o)}, \quad (4.17)$$

which is the fractional change in the inverse of Q_o . Then it is easy to write the realized period and inverse selectivity in terms of changes in the design parameters as

$$T^* \equiv T_o(1 + \Delta t) \text{ and} \quad (4.18)$$

$$1/Q^* \equiv 1/Q_o(1 + \Delta q), \quad (4.19)$$

or equivalently,

$$f^* = f_o/(1 + \Delta t) \approx f_o(1 - \Delta t) \quad (4.20)$$

$$Q^* = Q_o/(1 + \Delta q) \approx Q_o(1 - \Delta q) \quad (4.21)$$

for small Δt and Δq . Now

$$\Delta t = - \frac{\Delta f_o}{f_o} \quad (4.22)$$

$$\Delta q = - \frac{\Delta Q_o}{Q_o} . \quad (4.23)$$

Substituting in the values for T^* and Q^* into (4.15) gives

$$\bar{D} = (s\tau_d + 1)\{s^2T_o^2(1 + \Delta t)^2 + s(T_o/Q_o)(1 + \Delta t)(1 + \Delta q) + 1\}. \quad (4.24)$$

Since the goal at this point is to minimize the fractional errors, for small deviations the assumption $\Delta t, \Delta q \gg (\Delta t)^2, (\Delta q)^2$, and $(\Delta t)(\Delta q)$ can be made, so (4.24) becomes

$$\begin{aligned} \bar{D}(s) &= (s\tau_d + 1)\{s^2T_o^2(1 + 2\Delta t) + s(T_o/Q_o)(1 + \Delta t + \Delta q) + 1\} \\ &= s^3\tau_d T_o^2(1 + 2\Delta t) + s^2(T_o^2(1 + 2\Delta t) + \tau_d(T_o/Q_o)(1 + \Delta t + \Delta q) \\ &\quad + s(T_o/Q_o(1 + \Delta t + \Delta q) + \tau_d) + 1. \end{aligned} \quad (4.25)$$

There are now two equations for $\bar{D}(s)$ in equation (4.13) and equation (4.25). Thus the two can be equated and Δt , Δq , and τ_d can be solved for. Equating the s^3 coefficients gives

$$\begin{aligned}\tau_d T_o^2(1 + 2\Delta t) &= T_o^2(d_2/gb) \\ \tau_d + \tau_d 2\Delta t &= d_2/gb.\end{aligned}\tag{4.26}$$

For small τ_d and Δt , $\tau_d \gg \tau_d 2\Delta t$ and

$$\tau_d \approx d_2/gb.\tag{4.27}$$

Equating the s^2 coefficients gives:

$$\tau_d T_o/Q_o(1 + \Delta t + \Delta q) + T_o^2(1 + 2\Delta t) = T_o^2(1 + d_2/A_o - d_o/A_o + d_1/T_o Q_o gb).\tag{4.28}$$

Since $1 \gg \Delta t, \Delta q$,

$$\tau_d/T_o Q_o + 1 + 2\Delta t = 1 + d_2/A_o - d_o/A_o + d_1/T_o Q_o gb.\tag{4.29}$$

Subtracting 1 from both sides and substituting d_2/gb for τ_d gives

$$\Delta t = (1/2)[d_2/A_o - d_o/A_o] + (1/2)(1/Q_o T_o)[d_1/gb - d_2/gb]\tag{4.30}$$

$$\approx (1/2)(\omega_o/Q_o)[(d_1 - d_2)/gb]\tag{4.31}$$

$$= (1/2)(f_o/Q_o)[(d_1 - d_2)/gbp],\tag{4.32}$$

showing a fractional error in frequency proportional to the frequency, but inversely proportional to the design selectivity Q_o .

Lastly equating the s coefficients gives

$$T_o/Q_o(1 + \Delta t + \Delta q) + \tau_d = T_o/Q_o(1 + d_1/A_o + Q_o/T_o(d_2/gb) - d_o/A_o)$$

$$(Q_o/T_o)\tau_d + 1 + \Delta t + \Delta q = 1 + d_1/A_o + Q_o/T_o(d_o/gb) - d_o/A_o. \quad (433)$$

This means

$$\begin{aligned} \Delta q &= (d_1 - d_o)/A_o + Q_o/T_o(d_o/gb - d_2/gb) - \Delta t \\ &= 1/A_o(d_1 - d_o) + \omega_o Q_o((d_o - d_2)/gb) - \Delta t \end{aligned} \quad (434)$$

$$\approx \omega_o Q_o((d_o - d_2)/gb) - \Delta t. \quad (435)$$

Since $gb = 2\pi gbp$

$$\Delta q \approx f_o Q_o[(d_o - d_2)/gbp] - \Delta t. \quad (436)$$

So from (4.32), if $d_1 = d_2$ can be selected, the deviation from the design frequency can be reduced substantially. If $d_o = d_2$ can be selected then the deviation from Q_o is substantially reduced. Ideally $d_2 = d_1 = d_o$ would give optimal results in terms of the design parameters, but this is not always possible.

In summary, it should be noted that in the WBB approximation, three important constraints were used:

- i) A_o very large
- ii) τ_d is very small, (associated with the above)

iii) Δt and Δq are very small.

The goal is to minimize the errors in the design poles. The closer to minimizing the errors, the better the approximation will be.

The WBB approximation thus gives a handle on tackling an op-amp problem. By the proper selection of the resistors and capacitors composing d_2 , d_1 , and d_0 it is possible to design an active filter with ideal op-amps and not have the performance stray from the ideal design.

It is also possible that some of the A^2 terms may come into effect. In this situation, a similar approach may be taken in order to minimize the errors. This could be classified as yet another problem and will not be touched on here.

4.3 Error approximation for H_{lp} , H_{bp} , H_{hp} [17]

In the last section a means to approximate the shifting of the poles of the denominator was presented. Since transfer functions of five different types of filters are being looked at, it is necessary to develop a means to evaluate the error in other parameters found in a given filter function. The parameters which will be addressed here are the deviation from the ideal low frequency gain for a lowpass filter, the ideal high frequency gain for a highpass filter, and the ideal center frequency gain for a bandpass filter. The results are reached by applying a WBB approach.

The ideal lowpass filter has the form of

$$T(s) = \frac{H_{lp}}{s^2(1/\omega_o^2) + s(1/\omega_o Q_o) + 1} \quad (437)$$

The most general non-ideal transfer function is

$$T(s) = \frac{s^2 g_2(A) + s g_1(A) + \lambda h_0(A)}{s^2 \alpha f_2(A) + s \beta f_1(A) + \gamma f_0(A)} \quad (438)$$

where $f_j(A)$ is the same as (4.4) and

$$h_j(A) = 1 + \sum_{i=0}^n \frac{n_{ij}}{A_j} + \sum_{i=0}^n \sum_{k=0}^n \frac{n_{ijk}}{A_j A_k} + \dots + \frac{n_{1,2,\dots,N}}{A_1 A_2, \dots, A_N} . \quad (4.39)$$

The $g_i(A)$ term is a little more special. It tends to zero as A tends to infinity as no coefficients of s show up in the ideal equation. Then

$$g_j(A) = \sum_{i=0}^n \frac{n_{ij}}{A_j} + \sum_{i=0}^n \sum_{k=0}^n \frac{n_{ijk}}{A_j A_k} + \dots + \frac{n_{1,2,\dots,N}}{A_1 A_2, \dots, A_N} \quad (4.40)$$

and

$$\frac{n_i}{A} \equiv \sum_{j=1}^N \frac{n_{ij}}{A_j} , \quad (4.41)$$

so that

$$h_i(A) = 1 + \frac{n_i}{A} \quad (4.42)$$

and

$$g_i = \frac{m_i}{A} . \quad (4.43)$$

The n_i terms are typically ratios of the passive components and where the m_i terms tend to be the summation of several products of passive admittances, so the distinction between the two is made. Putting the denominator in the form of (4.15) the characteristic equation becomes

$$T(s) = \frac{s^2(1/\gamma)(m_2/A) + s(1/\gamma)(m_1/A) + H_{ip}(1 + n_o/A)}{(1 + d_o/A_o)(s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^*Q^* + 1)}, \quad (4.44)$$

where the same constraints apply as before. Then with

$$\frac{n_i}{A} = \frac{n_i s}{A_o \omega_o} + \frac{n_i}{A_o} \quad (4.45)$$

and

$$\frac{m_i}{A} = \frac{m_i s}{A_o \omega_o} + \frac{m_i}{A_o}, \quad (4.46)$$

this gives

$$T(s) = \frac{s^2(1/\gamma)(m_2 s/gb + m_2/A_o) + s(1/\gamma)(m_1 s/gb + m_1/A_o) + H_{ip}(1 + n_o s/gb + n_o/A_o)}{(1 + d_o/A_o)(s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^*Q^* + 1)}. \quad (4.47)$$

In order to find the low frequency gain, it is sufficient to find the magnitude as $s \rightarrow 0$. Then it becomes necessary to define

$$\Delta h_{ip} \equiv - \frac{\Delta H_{ip}}{H_{ip}}, \quad (4.48)$$

so that

$$H_{ip}^* \equiv H_{ip}(1 - \Delta h_{ip}) \quad (4.49)$$

This leaves

$$|T(s \rightarrow 0)| = H_{ip}^* = \frac{H_{ip}(1 + n_o/A)}{(1 + d_o/A)} \quad (4.50)$$

$$H_{ip}^* \approx H_{ip} \quad (4.51)$$

and

$$\Delta h_{ip} = 0. \quad (4.52)$$

as the n_o and d_o terms are much too small to consider. Surprisingly, there is no change in the low frequency gain when non-ideal op-amps are employed.

The ideal highpass filter has the form of

$$T(s) = \frac{s^2(H_{hp}/\omega_o^2)}{s^2(1/\omega_o^2) + s(1/\omega_o Q_o) + 1} \quad (4.53)$$

Then when non-ideal op-amps are employed,

$$T(s) = \frac{s^2 \lambda h_2(A) + s g_1(A) + g_0(A)}{s^2 \alpha f_2(A) + s \beta f_1(A) + \gamma f_0(A)} \quad (4.54)$$

where $h_i(A)$ takes the form of (4.42), and $g_i(A)$ takes the form of (4.43). Then the transfer function, $T(s)$ takes the form of (4.44) and

$$\begin{aligned} T(s) &= \frac{s^2(H_{hp}/\omega_o^2)(1 + n_2/A) + s(1/\gamma)(m_1/A) + (1/\gamma)(m_0/A)}{(1 + d_o/A_o)(s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^*Q^* + 1)} \\ &= \frac{s^2(H_{hp}/\omega_o^2)(1 + sn_2/gb + n_2/A_o) + s(1/\gamma)(sm_1/gb + m_1/A_o) + (1/\gamma)(sm_0/gb + m_0/A_o)}{(1 + d_o/A_o)(s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^*Q^* + 1)} \end{aligned}$$

$$= \frac{s^3(H_{np}n_2/\omega_o^2gb) + s^2[H_{np}/\omega_o^2 + (H_{np}/\omega_o^2)(n_2/A_o) + (1/\gamma)(m_1/gb)] + s[(1/\gamma)(m_1/A) + (1/\gamma)(m_o/gb)] + [(1/\gamma)(m_o/A)]}{(1 + d_o/A_o)(s\tau_d + 1)(s^2/\omega^2 + s/\omega^*Q^* + 1)} \quad (455)$$

At $s = \omega^*$, $s\tau_d \ll 1$, and $(1 + n_o/A_o) \cong 1$, and (4.55) becomes

$$\begin{aligned} T(s) &= \frac{-j\omega^3(H_{np}n_2/\omega_o^2gb) - \omega^{*2}[H_{np}/\omega_o^2 + (H_{np}/\omega_o^2)(n_2/A_o) + (1/\gamma)(m_1/gb)] + j\omega^*[(1/\gamma)(m_1/A) + (1/\gamma)(m_o/gb)] + [(1/\gamma)(m_o/A)]}{1/Q^*} \\ &= \{-j\omega^3(H_{np}n_2/\omega_o^2gb) - \omega^{*2}[H_{np}/\omega_o^2 + (H_{np}/\omega_o^2)(n_2/A_o) + (1/\gamma)(m_1/gb)] + j\omega^*[(1/\gamma)(m_1/A) + (1/\gamma)(m_o/gb)] + [(1/\gamma)(m_o/A)]\}Q^*. \end{aligned} \quad (456)$$

Then

$$H_{np}^* = \{-j\omega^3(H_{np}n_2/\omega_o^2gb) - \omega^{*2}[H_{np}/\omega_o^2 + (H_{np}/\omega_o^2)(n_2/A_o) + (1/\gamma)(m_1/gb)] + j\omega^*[(1/\gamma)(m_1/A) + (1/\gamma)(m_o/gb)] + [(1/\gamma)(m_o/A)]\}, \quad (457)$$

where the following will be defined

$$H_{np}^* = H(1 - \Delta h_{np}). \quad (458)$$

The magnitude of (4.57) needs to be found in order to determine H_{np}^* . At reasonable frequencies the ω^2 term is the dominating factor and so

$$H_{np}^* \approx \omega^{*2}[H_{np}/\omega_o^2 + (H_{np}/\omega_o^2)(n_2/A_o) + m_1/\gamma gb]. \quad (459)$$

The dominating terms in this equation are H_{np}/ω_o^2 and $m_1/\gamma gb$, so (4.59) further reduces to

$$\begin{aligned}
H_{hp}^* &\approx \omega^2 [H_{hp}/\omega_o^2 + m_1/\gamma gb] \\
&\approx \omega^2 H_{hp}/\omega_o^2 + \omega^2 (m_1/\gamma gb) \\
&\approx H_{hp}/(1 + \Delta t)^2 + (m_1/\gamma gb)(\omega_o^2/(1 + \Delta t)^2) \\
&\approx H_{hp}[1 - 2\Delta t + \omega_o^2(m_1/\gamma gb H_{hp})(1 - 2\Delta t)] \\
&\approx H_{hp}[1 + \omega_o^2(m_1/\gamma gb H_{hp}) - 2\Delta t(1 + \omega_o^2(m_1/\gamma gb H_{hp}))]. \tag{4.60}
\end{aligned}$$

So

$$\Delta h_{hp} = 2\Delta t(1 + \omega_o^2(m_1/\gamma gb H_{hp})) - \omega_o^2(m_1/\gamma gb H_{hp}). \tag{4.61}$$

The $\omega_o^2(m_1/\gamma gb H_{hp})$ term must be observed, but the important factor is to note the errors are proportional to $2\Delta t$.

Lastly, in this section, there is the change in H_{hp} due for the bandpass circuit. The lowpass and highpass show nothing interesting, but the bandpass should prove to be different.

As before, the ideal bandpass filter has the form of

$$T(s) = \frac{H_{bp}(1/\omega_o Q_o)s}{s^2(1/\omega_o^2) + s(1/\omega_o Q_o) + 1}. \tag{4.62}$$

The most general non-ideal transfer function is

$$T(s) = \frac{s^2 g_2(A) + s \gamma h_1(A) + g_o(A)}{s^2 \alpha f_2(A) + s \beta f_1(A) + \gamma f_o(A)}. \tag{4.63}$$

Then

$$\begin{aligned}
T(s) &= \frac{s^2(1/\gamma)g_z(A) + s(H_{bp}/\omega_o Q_o)h_i(A) + 1/\gamma g_o(A)}{(1 + d_o/A_o)(s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^*Q^* + 1)} \\
&= \frac{s^2(1/\gamma)(m_z/A) + s(H_{bp}/\omega_o Q_o)(1 + n_i/A) + 1/\gamma(m_o/A)}{(1 + d_o/A_o)(s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^*Q^* + 1)} \\
&= \frac{s^2(1/\gamma)(m_z s/gb + m_z/A_o) + s(H_{bp}/\omega_o Q_o)(1 + n_i s/gb + n_i/A_o) + (1/\gamma)(m_o s/gb + m_o/A_o)}{(1 + d_o/A_o)(s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^*Q^* + 1)}, \tag{4.64}
\end{aligned}$$

in the usual fashion. Further factoring, and using the fact that $1 \gg d_o/A_o$, the numerator and the denominator become

$$\begin{aligned}
N(s) &= s^3 m_z/\gamma gb + s^2[m_z/\gamma A_o + (H_{bp}/\omega_o Q_o)n_i/gb] + s[H_{bp}/\omega_o Q_o(1 + n_i/A_o) + m_o/\gamma gb] + m_o/\gamma A_o \tag{4.65}
\end{aligned}$$

$$D(s) = (s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^*Q^* + 1), \tag{4.66}$$

and the magnitudes of $D(s)$ and $N(s)$ need to be known to find the gain at the center frequency ω^* . For $D(j\omega^*)$, seeing that $1 \gg \omega^* \tau_d$, in band and substituting in the actual ω and Q for the denominator as defined before,

$$D(j\omega^*) \approx j\omega^*/\omega^*Q^*, \tag{4.67}$$

so that

$$|D(j\omega^*)| \approx 1/Q^*$$

$$= \frac{(1 + \Delta q)}{Q_o} \quad (4.68)$$

For the numerator,

$$\begin{aligned} N(s) = & -j(\omega^*)^3 m_2 / \gamma g b - (\omega^*)^2 [m_2 / \gamma A_o + (H_{bp} / \omega_o Q_o) n_1 / g b] + j\omega^* [H_{bp} / \omega_o Q_o (1 + n_1 / A_o) + \\ & + m_o / \gamma g b] + m_o / \gamma A_o. \end{aligned} \quad (4.69)$$

In looking for the magnitude of the denominator, all the n_i and m_i terms are sufficiently small so that they do not come into play at reasonable frequencies. For the bandpass circuits in this thesis, where $f_o \leq 10,000$ Hz, all of the terms dropped out. The exception can come from the $-j(\omega^*)^3 m_2 / \gamma g b$ term because of the cubing of the frequency. For bandpass filters of $\sim 30,000$ Hz or greater, this can have a significant effect and should be considered. In many good designs, this term does not even exist.

Excluding $-j(\omega^*)^3 m_2 / \gamma g b$, the magnitude of the numerator becomes

$$\begin{aligned} |N(j\omega^*)| & \approx \omega^* H_{bp} / \omega_o Q_o \\ & = \frac{H_{bp}}{(1 + \Delta t) Q_o} \end{aligned} \quad (4.70)$$

Defining the following

$$\Delta h_{bp} \equiv - \frac{\Delta H_{bp}}{H_{bp}} \quad (4.71)$$

so that

$$H_{bp}^* \equiv H_{bp}(1 - \Delta h_{bp}) \quad (4.72)$$

With (4.68) and (4.70), the magnitude of $T(j\omega^*)$ becomes

$$|T(j\omega^*)| = H_{bp}^* = \frac{H_{bp}}{(1 + \Delta t)(1 + \Delta q)}, \quad (4.73)$$

and so

$$\Delta h_{bp} \approx \Delta t + \Delta q. \quad (4.74)$$

In practice, the Δq term tends to have the biggest effect on gain at this point. It is noticeable to many circuit designers that the gain at the center frequency tends to have a deviation from the desired gain which follows very closely the deviation from the desired Q to the realized Q . The above gives the reason why.

4.4 Error approximation for notch filters [17]

This section will deal solely with the notch filter. The analysis will deal with the main causes of deviation in the notch frequency, and the gain of the response at the notch frequency, which is ideally zero.

The ideal notch filter transfer function is

$$T(s) = \frac{H_n/\omega_o^2(s + \omega_n^2)}{s^2(1/\omega_o^2) + s(1/\omega_o Q_o) + 1} \quad (4.75)$$

At $s = j\omega_n$, the numerator becomes $H_n/\omega_o^2(-\omega_n^2 + \omega_n^2)$, and at the notch frequency the signal has been successfully blocked out. The most general transfer function for a notch filter realized with non-ideal op-amps is

$$T(s) = \frac{s^2 \lambda h_2(A) + s g_1(A) + \phi h_0(A)}{s^2 \alpha f_2(A) + s \beta f_1(A) + \gamma f_0(A)}. \quad (4.76)$$

In the usual fashion

$$T(s) = \frac{(H_n/\omega_o^2)s^2(1 + n_2/A) + s(m_1/\gamma A) + H_n(\omega_n^2/\omega_o^2)(1 + n_0/A)}{(1 + d_0/A_o)(s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^*Q^* + 1)}, \quad (4.77)$$

and since the point where the numerator hits a minimum is being sought, only the numerator will be considered. Thus let $T(s) = N(s)/D(s)$ be defined. Also define

$$\eta \equiv H_n \omega_n^2 / \omega_o^2, \quad (4.78)$$

and now

$$\begin{aligned} N(s) = & s^3(H_n/\omega_o^2)(n_2/gb) + s^2(H_n/\omega_o^2 + (H_n/\omega_o^2)(n_2/A_o) + m_1/\gamma gb) \\ & + s(m_1/\gamma A_o + \eta n_0/gb) + \eta(1 + n_0/A_o). \end{aligned} \quad (4.79)$$

Factoring out $\eta(1 + n_0/A_o)$, gives $N(s) = \eta(1 + n_0/A_o)\tilde{N}(s)$ where

$$\begin{aligned} \tilde{N}(s) = & s^3(1/\omega_n^2)(n_2/gb)(1 - n_0/A_o) + s^2[(1/\omega_n^2 + (1/\omega_n^2)(n_2/A_o) + m_1/\gamma \eta gb)(1 - n_0/A_o)] \\ & + s[(m_1/\gamma \eta A_o + n_0/gb)(1 - n_0/A_o)] + 1 \\ \approx & s^3(n_2/\omega_n^2 gb) + s^2(1/\omega_n^2 + n_2/A_o \omega_n^2 + m_1/\eta \gamma gb - n_0/\omega_n^2 A_o) \end{aligned}$$

$$+ s(m_1/A_o\gamma\eta + n_o/gb) + 1. \quad (4.80)$$

To find the minimum of $N(s)$, the minimum of $\tilde{N}(s)$ is found. Similar to the approach of the denominator poles the numerator may be written in the following form,

$$\begin{aligned} \tilde{N}(s) &= (s\tau_n + 1)(s^2/\omega_n^{*2} + s/\omega_n^*Q_n^* + 1) \\ &= (s\tau_n + 1)(s^2T_n^{*2} + sT_n^*D^* + 1), \end{aligned} \quad (4.81)$$

where the following parameters will be defined

$$T_n^* = 1/\omega_n^* \equiv T_n(1 + \Delta n) \text{ and} \quad (4.82)$$

$$D^* = 1/Q_n^* = 1/Q_n + \Delta d. \quad (4.83)$$

Since the ideal parameter for Q_n is ∞ , then

$$D^* = \Delta d \quad (4.84)$$

Also

$$f_n^* = f_n(1 - \Delta n) \quad (4.85)$$

$$Q_n^* = 1/\Delta d. \quad (4.86)$$

Then

$$N(s) = s^3(\tau_n T_n^{*2}) + s^2(\tau_n T_n^* D^* + T_n^{*2}) + s(\tau_n + T_n^* D^*) + 1. \quad (4.87)$$

Substituting (4.82) and (4.83) into (4.87) gives,

$$\bar{N}(s) = s^3(\tau_n T_n^2(1 + 2\Delta n) + s^2(\tau_n \Delta d T_n + T_n(1 + 2\Delta n)) + s(\tau_n + T_n \Delta d) + 1, \quad (4.88)$$

with the same constraints as before, i.e. $\Delta n, \Delta d \gg \Delta n \Delta d, (\Delta n)^2, (\Delta d)^2$, again this constraint is possible because the goal is to minimize these terms. Now comparing equations (4.80) and

(4.88) the s^3 terms can be compared and

$$\tau_n T_n^2(1 + 2\Delta n) = n_2 T_n^2 / gb. \quad (4.89)$$

Since $\tau_n \gg \tau_n 2\Delta n$,

$$\tau_n = n_2 / gb. \quad (4.90)$$

For the s terms,

$$\tau_n + T_n \Delta d = m_1 / \gamma \eta A_o + n_o / gb$$

$$\Delta d = \omega_n [(m_1 / \gamma \eta A_o) + (n_o - n_2) / gb] \quad (4.91)$$

$$\approx \omega_n (n_o - n_2) / gb. \quad (4.92)$$

For the s^2 terms,

$$T_n^2 + T_n^2 n_2 / A_o + m_1 / \gamma \eta gb = \tau_n \Delta d T_n + T_n^2(1 + 2\Delta n)$$

$$= T_n^2 + 2\Delta n T_n^2 + \tau_n \Delta d T_n. \quad (4.93)$$

So

$$T_n^2 n_2 / A_o + m_1 / \gamma \eta g b = T_n^2 2\Delta n + \tau_n \Delta d T_n$$

$$2T_n^2 \Delta n = T_n^2 n_2 / A_o + m_1 / \gamma \eta g b - \tau_n \Delta d T_n$$

$$\Delta n = 1/2 [n_2 / A_o + (m_1 / \gamma \eta g b) \omega_n^2 - \tau_n \Delta d \omega_n] \quad (4.94)$$

$$\approx 1/2 \omega_n^2 (m_1 / \eta \gamma g b). \quad (4.95)$$

Frequently $\omega_n = \omega_o$, leaving

$$\Delta n = 1/2 \omega_n^2 (m_1 / \gamma H_n g b). \quad (4.96)$$

Thus, for the numerator of the notch:

$$Q_n^* = 1/\Delta d$$

$$f_n^* = f_n (1 - \Delta n)$$

and $\Delta n = \pi f_n^2 (m_1 / \gamma \eta g b p)$

$$\Delta d = f_n ((n_o - n_2) / g b p).$$

The gain at the notch frequency tends to be a simpler derivation. Given some non-ideal center frequency ω^* , and some non-ideal notch frequency ω_n^* and that the numerator and the denominator are factored in the form of (4.15) and (4.81), then

$$T(s) = \frac{\eta(1 + n_o/A)(s\tau_n + 1)(s^2/\omega_n^{*2} + s/\omega_n^* Q_n^* + 1)}{(1 + d_o/A_o)(s\tau_d + 1)(s^2/\omega^{*2} + s/\omega^* Q^* + 1)}. \quad (4.97)$$

Since $(s\tau_i + 1) \equiv 1$ at the notch frequency, and $(1 + n_o/A_o)$, $(1 + d_o/A_o) \equiv 1$. Then at $s =$

$j\omega_n^*$ the transfer function becomes

$$\begin{aligned} T(s) &= \frac{\eta(1/Q_n^*)}{-(\omega_n^*/\omega^*)^2 + 1 + j(\omega_n^*/\omega^*Q^*)} \\ &= \eta(\Delta d)[1 - (\omega_n^*/\omega^*)^2 + j(\omega_n^*/\omega^*Q_n^*)]^{-1}. \end{aligned} \quad (4.98)$$

Defining

NG = the gain at notch frequency, ideally zero.

Then

$$\begin{aligned} NG &= |T(s)| \\ &= \eta(\Delta d)[1 - 2(\omega_n^*/\omega^*)^2 + (\omega_n^*/\omega^*)^4 + (\omega_n^*/\omega^*Q^*)^2]^{-1/2} \\ &= \eta(\Delta d)\{1 - 2[\omega_n(1 + \Delta t)/\omega_o(1 + \Delta n)]^2 + [\omega_n(1 + \Delta t)/\omega_o(1 + \Delta n)]^4 \\ &\quad + [\omega_n(1 + \Delta t)(1 + \Delta q)/(1 + \Delta n)(\omega_o Q_o)]^2\}^{-1/2}, \end{aligned} \quad (4.99)$$

and it stops here. However, in many notch filters $\omega_o = \omega_n$, and this can be used to simplify the expression. Using this, and that Δn , Δt , and Δq , are very small quantities, equation (4.99) becomes

$$\begin{aligned} M(\omega_n) &\approx H_n(\Delta d)\{1 - 2(1 + 2\Delta t - 2\Delta n) + (1 + 4\Delta t - 4\Delta n) \\ &\quad + 1/Q_o^2(1 + 2\Delta t + 2\Delta q - 2\Delta n)\}^{-1/2}, \end{aligned} \quad (4.100)$$

at this point, this is sufficient to describe the magnitude of the notch at the center frequency. It is most important to note that if $\Delta d = 0$, meaning infinite Q_n^* , that the magnitude reduces down to the ideal value of 0. Perhaps simplifying things too much, however used in the research for this thesis effectively is the further simplification of

$$\Delta n \sim \Delta t, \quad (4101)$$

in which case the magnitude reduces further to

$$NG = H_n(\Delta d)[(1 + 2\Delta q)/Q_o^2]^{-1/2} \quad (4102)$$

$$\approx H_n(\Delta d)Q_o(1 - \Delta q), \quad (4103)$$

which can be used to predict the magnitude of the errors, but not to reduce them, as the key factor is still $\Delta d = 0$.

The gain at the notch frequency can be reduced substantially if $\Delta d = 0$. This is somewhat surprising since Δd consists of the n_0 and n_2 terms.

4.5 Summary

A final summary of the error terms will be as follows. For the bandpass, highpass and lowpass filters, there is:

$$f^* = f_o(1 - \Delta t)$$

$$Q^* = Q_o(1 - \Delta q)$$

$$H_{lp}^* = H_{lp}(1 - \Delta h_{lp})$$

$$H_{hp}^* = H_{hp}(1 - \Delta h_{hp})$$

$$H_{bp}^* = H_{bp}(1 - \Delta h_{bp}),$$

where

$$\Delta t \approx (1/2)(f_o/Q_o)[(d_1 - d_2)/gbp]$$

$$\Delta q \approx f_o Q_o [(d_o - d_2)/g_{bp}] - \Delta t$$

$$\Delta h_{hp} \approx 0$$

$$\Delta h_{bp} \approx 2\Delta t(1 + 2\pi f_o^2(m_1/\gamma g_{bp} H_{hp})) - 2\pi f_o^2(m_1/\gamma g_{bp} H_{hp})$$

$$\Delta h_{bp} \approx \Delta t + \Delta q.$$

Then for the notch filter,

$$Q_n^* = 1/\Delta d$$

$$f_n^* = f_n(1 - \Delta n)$$

$$NG \approx H_n(\Delta d)Q_o(1 - \Delta q),$$

where

$$\Delta d \approx f_n((n_o - n_2)/g_{bp})$$

$$\Delta n \approx \pi f_n^2(m_1/\gamma \eta g_{bp})$$

4.6 Minimization of the errors

The past sections showed that a circuit has a chance of attaining its ideal performance if the possibility of selecting d_i and n_i properly exists. The circuit designer who may have tried this knows that at this point there is a dilemma to be faced. Most typically, there are a number of constraints due to the ideal design parameters. Combining these together with the constraints imposed by the d_i and n_i terms makes for still a confusing problem. Although it is possible that the terms may cancel out naturally, this is not probable. A better method has been proposed [16]. If resistors could be inserted into the circuit so that they did not appear in the ideal design equations, but did however appear in the n_i and d_i terms, the circuit could then be designed with ideal op-amps using the given original R's and C's, and the

added resistors could then be chosen to satisfy the n_i and d_i constraints. This is indeed the case.

The most reasonable place to place the resistors would be in a location where the resistor drew no current, thus not showing up in the ideal transfer function. The best spot in a circuit consisting of ideal op-amps, would be between any two nodes on a nullator tree, as all the nodes have the same voltage. Both nodes must however belong to the same nullator tree. Obviously then there are a number of places where these resistors can be inserted. The exact number is $\sum_{i=1}^m \sum_{j=1}^{m_i} j$ where m_i is the number of nullators on a given tree and m is the number of distinct trees. There are other locations, but these gave the best results. There is, up to this point, no systematic way of knowing exactly where to place these resistors. It must be done by trial and error. Even so, there is no guarantee that a circuit can be made to approach its ideal behavior. Some examples will be given shortly.

CHAPTER 5

Second Pole Effects

5.1 Introduction

It has been shown that using a dominant pole model for an op-amp results in two critical deviations from an ideal transfer function to a non-ideal transfer function. The first, is the shifting of the in-band poles which may or may not be driven to instability. The second is the addition of, in most cases, n poles at higher frequencies, where n is the number of op-amps. These poles must lie in the left half plane of the frequency spectrum in order for a circuit to function if the dominant pole model sufficiently models the op-amp. In practice, it can occur that a circuit which uses the one pole model has in band errors which are nearly zero and high frequency poles which are stable. Yet, when constructed in lab, the circuit may not be functional, typically oscillating at some locked in frequency, for example, for no apparent reason. It is because a one pole model may not suitably model an op-amp [14]. A real op-amp is more realistically a transfer function consisting of many poles and zeros. It turns out that other sets of parasitic poles, other than the ones that arise from a dominant pole, may be important if a designed circuit is to exist physically. This is verified from experimental results. The poles which will be addressed in this chapter are parasitic poles due to an op-amp model which depicts the gain vs. frequency response of an op-amp as a two pole transfer function.

5.2 Rationale for using a two pole model in filter design

A normal op-amp is actually a many pole and zero device. There are circuits which perform well in lab, so it seems reasonable that either one of two things can be happening. Either not all the poles are affecting circuit stability, or the effect of each of the poles is to induce poles into the circuit all of which are stable. Either way, it is impractical to do any type of analysis using this model. The two-pole model was chosen solely on the basis that it agrees with experimental results, in terms of stability, to a very large degree, in *all* cases in the research done in this thesis. As was done to solve the preceding in-band error problems, a suitable model must be chosen to look at the effects of parasitic poles, and presently it is the two-pole model.

The method of designing the circuits used in this thesis was to first choose a seed circuit and design it with ideal parameters. Secondly, OAR was used to generate more circuits out of this seed circuit. A two pole model was used in the OAR run. OAR then produced a handful of circuits which were stable when this model of the op-amp was employed. The best circuits were chosen and then constructed in lab. In all cases, the circuits which were stable when a two pole model was used for the op-amps, performed as well as well as the in-band errors would allow. At this point, the only justification for using the two-pole model for in active network synthesis to predict circuit stability, is that it has been successful in the work done for this thesis.

5.3 Solution to the two pole model [14]

As mentioned, the method used in this thesis to design active filters was to use the two-pole model in OAR which then produced a list of stable circuits. That is provided any existed at all. A question may arise as to whether or not there is any control over these new sets of parasitic roots. The answer is yes, to a degree. These new sets of poles are

dependent on the topology of the circuit. It's difficult to get a handle on the non-ideal transfer function which will arise when the two pole model is employed in a circuit. If this approach is used the symbolic coefficients of a now 2ⁿth order equation have to be known in order to ensure stability. The passive elements in the circuit then need to be picked so that the coefficients in the equation satisfy stability requirements. The values of the passive components also determine the filter parameters. Together this gives rise to a very complicated problem. The following paragraphs will show an alternate method.

Suppose, as before, an *i*th order transfer function is to be realized by *n* op-amps. Then, if a two-pole model is the chosen model for the op-amps, a 2*n*+*i* order characteristic polynomial is exhibited. For a 2nd order equation to be implemented with 3 op-amps an eighth order transfer function will exist. A one pole model, however, gives rise to a much less, but still possibly, complex fifth, or more generally an (*n*+*i*)th order equations. The proposed method, first proposed by Randall Geiger in [14], consists of solving this *n*+*i* order equation to predict stability for the 2**n*+*i* order equation. The idea can applied to any *n* op-amp circuit. The used restrictions of matched op-amps with large gbps must be invoked.

The most general transfer characteristic for a transfer function using *n* identical op-amps can be expressed as

$$T(s) = \frac{N_0(s) + N_1(s)/A + N_2(s)/A^2 + \dots + N_n(s)/A^n}{D_0(s) + D_1(s)/A + D_2(s)/A^2 + \dots + D_n(s)/A^n}, \quad (5.1)$$

where $N_0(s)$ and $D_0(s)$ are the ideal polynomials and the $N_i(s)$ and $D_i(s)$ terms are introduced because of the non ideal op-amps. If a one pole roll off model approximation is used, where $A = \text{gb/s}$, there is the addition of parasitic poles. For large gb, as before, at low frequencies about the design poles, the $1/A$ terms may be neglected. At high frequencies, D_0 and N_0 terms may be neglected, and under this circumstance, the non ideal polynomial can be factored in the following fashion

$$T(s) = \sum_{j=0}^n b_j(\tau s)^j \sum_{i=1}^m a_i s^i \quad (5.2)$$

where n is the number of op-amps, $\tau = 1/gb$, and m is the order of the ideal polynomial. The roots of (5.2) need to be known in order to determine the parasitic poles. Furthermore, let $A(s) = 1/(\tau s)$ for the one pole model. Then for a two pole model,

$$A(s) = \frac{1}{\tau s(1 + \theta \tau s)}, \quad (5.3)$$

where θ is defined such that it identifies the location of the second pole of the op-amp where $\omega_2 = 1/\theta\tau$. It is noted that if $\theta = 0$, the second pole is located at infinity and the op-amp defining equation, $A(s)$ reduces to that of the one pole model.

The poles of the equation

$$\sum_{j=0}^n b_j(\tau s)^j = 0 \quad (5.4)$$

must be determined where it will be chosen to solve for τs instead of s , or find the solution to

$$\prod_{j=0}^n (\tau s - p_j) = 0. \quad (5.5)$$

Then the parasitic poles are located at p_j/τ .

It is not the intent here to solve for these poles, the only intent is to produce some means of evaluating stability when a two pole model is using the fact that the above poles have already been solved for. Thus, at this point, it is assumed that the parasitic poles when the one pole op-amp model is employed are known, and this information will be used to determine the parasitic poles when a two pole model is employed.

To estimate the parasitic poles when going from a one pole model to a two pole

model involves the simple substitution of $\tau s \rightarrow \tau s(1 + \theta \tau s)$. Making this substitution everywhere in the equation defining the parasitic poles, it is reasonable to assume that $(\tau s(1 + \theta \tau s) - p_i)$ still yields the solution to the now $2n$ order equation describing the parasitic poles, except that the above equation may be used to locate the roots due to a two pole model being used. Again, assuming that p_i is known, then

$$\tau s(1 + \theta \tau s) - p_i = 0 \quad (5.5)$$

and using the known solution to the biquadratic equation,

$$\begin{aligned} s &= 1/(2\tau^2)\{-\tau \pm [\tau^2 + 4\theta\tau^2 p_i]^{1/2}\} \\ &= 1/(2\theta\tau^2)\{-\tau \pm \tau[1 + 4\theta p_i]^{1/2}\} \\ &= 1/(2\theta\tau)\{-1 \pm [1 + 4\theta p_i]^{1/2}\}. \end{aligned} \quad (5.6)$$

The most general form for p_i is that it is complex, so let $p_i = -\alpha_i + j\beta_i$ and note that if β exists, then p_i^* also exists. After defining p_i , the above now has a predetermined solution [15] and that solution for s is

$$\begin{aligned} s &= -(1/2\theta\tau)\{1 \pm [(1 - 4\theta\alpha)/2 + (1 - 8\theta\alpha + 16\theta^2\alpha^2 + 16\theta^2\beta^2)^{1/2}/2]^{1/2} \\ &\quad \pm j[(4\theta\alpha - 1)/2 + (1 - 8\theta\alpha + 16\theta^2\alpha^2 + 16\theta^2\beta^2)^{1/2}/2]^{1/2}\}. \end{aligned} \quad (5.7)$$

Thus, by solving for the parasitic roots when a one pole-model is utilized, the means to evaluate the poles when a two-pole model is substituted, and then stability of the circuit of the circuit can be determined.

Suppose the parasitic poles when a one pole op-amp is used have been successfully solved for. Furthermore, suppose the situation occurs where, for each p_i the following condition, $\alpha \gg \beta$, or equivalently 'nearly' real roots. The pole defining equation can be written as

$$s = -(1/2\tau\theta)\{1 \pm [(1 - 4\theta\alpha)/2 + (1 - 8\theta\alpha + 16\theta^2(\alpha^2 + \beta^2))^{1/2}/2]^{1/2} \pm j[(4\theta\alpha - 1)/2 + (1 - 8\theta\alpha + 16\theta^2(\alpha^2 + \beta^2))^{1/2}/2]^{1/2}\}. \quad (5.8)$$

Because $\alpha \gg \beta$, then surely $\alpha^2 \gg \beta^2$, and the above becomes

$$\begin{aligned} s &= -1/(2\tau\theta)\{1 \pm [(1 - 4\theta\alpha)/2 + ((1 - 4\theta\alpha)^2)^{1/2}/2] \\ &\quad \pm j[-(1 - 4\theta\alpha)/2 + ((1 - 4\theta\alpha)^2)^{1/2}/2] \\ &= -1/(2\tau\theta)\{1 \pm [(1 - 4\theta\alpha)/2]\}^{1/2} \end{aligned} \quad (5.9)$$

and the term $4\theta\alpha$ is always greater than or equal to zero, since θ cannot be less than zero, and if α is less than zero, this analysis would not have begun.

At this point, two regions in which $4\theta\alpha$ may fall into need to be considered. The first is

$0 < 4\theta\alpha < 1$, the second is

$4\theta\alpha > 1$,

with the idea that α is constant and θ can be varied. Now if $4\theta\alpha = 0$, then clearly roots of s are always real and negative, so the parasitic poles are always stable. If $4\theta\alpha = 1$, then there is a double negative real root at $1/(2\tau\theta)$. These two end points define the two extremes

in which the real part of s will lie if $0 < 4\theta\alpha < 1$. If $4\theta\alpha > 1$, then $[(1 - 4\theta\alpha)/2]^{1/2}$ is always imaginary and the real part of s is always $-1/(2\tau\theta)$. Thus at this point if given an n op-amp circuit where the one-pole model for the op-amps has been used, and one in which the parasitic poles are real, or 'nearly' real, then the parasitic poles will be stable when a two pole model for the op-amp is substituted in for the one-pole model.

It should be stressed that this does not mean that the circuit will be stable when a two-pole model is utilized, but only that the parasitic poles have negative real parts. If the second pole of the op-amp is not out far enough on the frequency axis, it has the potential to drive the in-band poles into instability. This fact may or may not have to be taken into consideration when designing active filters with op-amps. Most op amps on the market achieve this. If both of the constraints above are met, the circuit can be constructed with a high degree of certainty that it will be stable.

The above analysis is important for several reasons. First, if given a symbolic transfer function in which a two pole model is utilized, the coefficients may be determined for this by proper choice of passive components. The resulting equation may be tested for stability. This is very impractical and messy. If the transfer function has the typical biquadratic denominator, and n op-amps are used to construct the circuit, a $2n$ order equation has to be solved.

If a one pole model is utilized, solving the parasitic equation is reduced to an n th order equation. This is much simpler. The coefficients can then be chosen so that this equation may have real roots. Still difficult, but simpler. Most of the active filters looked at in this thesis contained three or four op-amps. It is then possible to determine the constraints on the passive components which will make this equation have real roots. When then two pole model is utilized, stability can be insured and as pointed out earlier, the circuit can be constructed in lab with some degree of certainty that it will operate.

5.4 Example

Going back to the example given in chapter 3, it is noted that circuits 1, 6, and 14 have the parasitic roots shown in Table 5.1.

Table 5.1 parasitic roots when a one pole op-amp model is used

Unscaled Poles

Ground Node : 0

(1, 2, 5); (0, 3, 6); (0, 4, 7) (1)

-.254981D+07 -.495806D-02
-.514499D+07 -.416336D+02
-.509395D+07 .416338D+02

(1, 2, 5); (4, 3, 6); (0, 4, 7) (6)

-.254981D+07 .387132D-01
-.551631D+07 -.185549D+01
-.472268D+07 .173713D+01

(1, 2, 5); (0, 3, 6); (3, 4, 7) (14)

-.256560D+07 .107252D+02
-.531469D+07 -.958762D+00
-.490842D+07 .950893D+00

These sets of roots clearly fit the mold of having a real part much greater than the imaginary parts. A input to OAR is given in Table 5.2.

Table 5.2 input to OAR

13							
3	5	0	0	res	.3845E+04	.0000E+00	.0000E+00
4	6	0	0	res	.1000E+04	.0000E+00	.0000E+00
1	8	0	0	res	.1000E+04	.0000E+00	.0000E+00
1	6	0	0	res	.5000E+05	.0000E+00	.0000E+00
2	7	0	0	res	.1000E+04	.0000E+00	.0000E+00
2	5	0	0	res	.1000E+04	.0000E+00	.0000E+00
4	7	0	0	cap	.3247E-07	.0000E+00	.0000E+00
3	6	0	0	cap	.3247E-07	.0000E+00	.0000E+00
1	2	5	0	noa3	.1570E+07	.2000E+06	.5100E+07
0	3	6	0	noa3	.1570E+07	.2000E+06	.5100E+07
0	4	7	0	noa3	.1570E+07	.2000E+06	.5100E+07
8	0	0	0	vs	.1000E+01	.0000E+00	.0000E+00
6	0	0	0	ocr	.0000E+00	.0000E+00	.0000E+00

The difference between this input file and that of chapter 2, is that the two pole model is utilized for the op-amps. This is distinguished by the first column which defines the location of the second pole, at 1.57×10^6 radians. The open loop d.c. gain and the gain-bandwidth product are identical to that of chapter 2. As before, the input is run through OAR which determines the stable circuits, then PZ which gives the poles and zeros of the transfer function. The output of PZ is shown in Table 5.3.

It is evident that these are the circuits that were predicted to be two pole stable from above. Thus the Geiger approximation is valid.

For a circuit to be designed properly, these poles must be taken into consideration. The proposed method is to get a handle on the much simpler third order equation and insure that the roots are real. This may be done for an n op-amp circuit, with the complexity of the problem increasing with n. It will be done in this example for a third order equation, as it is assumed that a second and first order equation are much too trivial.

It was assumed several times that the parasitic poles were placed far from the inband poles. It is necessary to solve an equation like equation (5.2). At high frequencies above the inband poles, the capacitors can be approximated as short circuits. By shorting out the capacitors, the remaining characteristic equation poles are due simply to the pole of the op-amps, and these are the parasitic poles.

Consider the KHN circuit with the capacitors shorted out shown in Figure 5.1. SLAP is again used to determine the characteristic equation, and the output of SLAP is shown in Table 5.4.

Then with $B_1 = 1/A(s) \approx s/gb = s\tau$, and sorting out the terms gives

$$\begin{aligned} (s\tau)^3[(G_6 + G_5)(G_4 + G_3)] + (s\tau)^2[(3G_6 + 2G_5)(G_4 + G_3)] + (s\tau)[(3G_6 + G_5)(G_4 + G_3)] \\ + [G_6(G_4 + G_3)] = 0, \end{aligned} \quad (5.10)$$

Table 5.3 output of PZ

(+input, -input, output) (+input, -input, output) (+input, -input, output)

Unscaled Poles and Zeros

Ground Node : 0

(1, 2, 5); (0, 3, 6); (0, 4, 7)

-.604940D+02 .156479D+05
 -.785939D+06 .271653D+07
 -.803559D+06 .272638D+07
 -.784914D+06 .184015D+07
 -.604941D+02 -.156479D+05
 -.803902D+06 -.272075D+07
 -.785597D+06 -.272217D+07
 -.784873D+06 -.184014D+07

-.153063D+00 -.150728D-12
 -.800408D+06 .272299D+07
 -.800408D+06 -.272299D+07

(1, 2, 5); (4, 3, 6); (0, 4, 7)

-.368085D+02 .156952D+05
 -.702330D+06 .259777D+07
 -.887203D+06 .284274D+07
 -.784864D+06 .184015D+07
 -.368085D+02 -.156952D+05
 -.887202D+06 -.284273D+07
 -.702333D+06 -.259777D+07
 -.784858D+06 -.184015D+07

-.153063D+00 -.150728D-12
 -.800408D+06 .272299D+07
 -.800408D+06 -.272299D+07

(1, 2, 5); (0, 3, 6); (3, 4, 7)

-.844211D+02 .156357D+05
 -.743488D+06 .265761D+07
 -.838061D+06 .278226D+07
 -.792816D+06 .184366D+07
 -.844211D+02 -.156357D+05
 -.838003D+06 -.278226D+07
 -.743578D+06 -.265762D+07
 -.792767D+06 -.184365D+07

-.153063D+00 -.150728D-12
 -.800408D+06 .272299D+07
 -.800408D+06 -.272299D+07

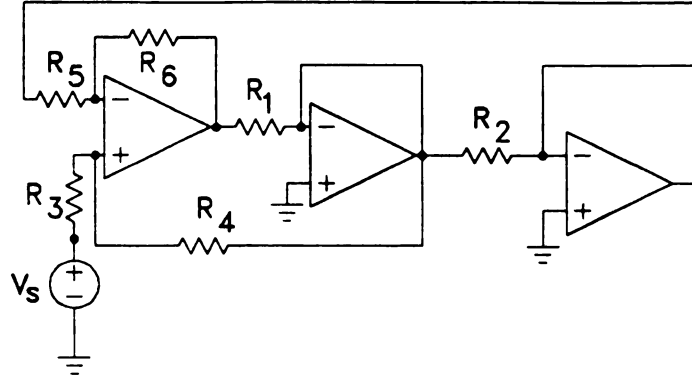


Figure 5.1 circuit to determine parasitic roots

Table 5.4 denominator used to determine the parasitic roots

DENOMINATOR IS :

$$\begin{aligned}
 &G6 \cdot G4 \cdot B3 \cdot B2 \cdot B1 \\
 &+ G6 \cdot G4 \cdot B3 \cdot B2 + G6 \cdot G4 \cdot B3 \cdot B1 + G6 \cdot G4 \cdot B3 + G6 \cdot G4 \cdot B2 \cdot B1 + G6 \cdot G4 \cdot B2 \\
 &+ G6 \cdot G4 \cdot B1 + G6 \cdot G4 + G6 \cdot G3 \cdot B3 \cdot B2 \cdot B1 + G6 \cdot G3 \cdot B3 \cdot B2 + G6 \cdot G3 \cdot B3 \cdot B1 + G6 \cdot G3 \cdot B3 \\
 &+ G6 \cdot G3 \cdot B2 \cdot B1 + G6 \cdot G3 \cdot B2 + G6 \cdot G3 \cdot B1 + G6 \cdot G3 + G5 \cdot G4 \cdot B3 \cdot B2 \cdot B1 + G5 \cdot G4 \cdot B3 \cdot B1 \\
 &+ G5 \cdot G4 \cdot B2 \cdot B1 + G5 \cdot G4 \cdot B1 + G5 \cdot G3 \cdot B3 \cdot B2 \cdot B1 + G5 \cdot G3 \cdot B3 \cdot B1 + G5 \cdot G3 \cdot B2 \cdot B1 + G5 \cdot G3 \cdot B1
 \end{aligned}$$

as the root defining equation. It is much easier to find the roots of $s\tau$. When the values for the passive components are substituted in, and multiplied by 10^6 , equation (5.10) becomes

$$4(s\tau)^3 + 10(s\tau)^2 + 8(s\tau) + 2 = 0, \text{ dividing by 2 gives}$$

$$2(s\tau)^3 + 5(s\tau)^2 + 4(s\tau) + 1 = 0 \quad (5.11)$$

and $s\tau$ has the roots of

$$s\tau = -0.5, -1.0 \text{ and } -1.0. \text{ The roots of } s \text{ are}$$

$$s = -2.55 \times 10^6, -5.1 \times 10^6, -5.1 \times 10^6,$$

and these are close to the parasitic poles found in Table 5.1.

A practical application of how this may be used in the design process is as follows. It is known that the polynomial $2(s\tau)^3 + 5(s\tau)^2 + 4(s\tau) + 1 = 0$ has real roots. So taking equation (5.10) and dividing by $(G_4 + G_5)$ gives

$$(s\tau)^3(G_6 + G_5) + (s\tau)^2(3G_6 + 2G_5) + (s\tau)(3G_6 + G_5) + G_6 = 0, \quad (5.12)$$

further dividing by G_6 gives

$$(s\tau)^3(1 + G_5/G_6) + (s\tau)^2(3 + 2G_5/G_6) + (s\tau)(3 + G_5/G_6) + 1 = 0, \quad (5.13)$$

and it is seen that if $G_5/G_6 = 1$, then all of the equations meet the conditions of equation (5.11), and will have real roots. This means that in a design process that it would be good procedure to have an additional constraint of $R_5 = R_6$. It may not be the only solution, but it is indeed a solution to the problem of stability in this particular circuit. This example worked out very nicely. Many good circuits looked at in this thesis had this property. Each circuit however must be treated differently, and may not work out as nicely.

Obviously a four op-amp circuit needs more work. This chapter is only intended to give a start on the remaining work that needs to be done.

CHAPTER 6

Examples of new circuits

6.1 Introduction

In Chapter 2, a means to develop new op-amp circuit structures out of old circuit structures was introduced. Circuits were generated from an existing seed circuit using OAR. An example was given in which 18 circuits were generated from one circuit.

In Chapter 3, it was shown that circuit performance can deviate from ideal performance by a substantial amount when a more realistic one-pole model for the op-amp is used. These errors were essentially two, inband errors and the existence of parasitic poles. These poles had little effect on the circuit near the design frequencies, but have to be stable in order for the circuit to function properly when built in lab.

Chapter 4 dealt with the reduction of the in-band errors by first explaining where they came from, and secondly adding resistors to null out the effects of the gbp of the op-amp.

Chapter 5 dealt with the idea of using a two pole model in order to predict whether or not the circuit would operate in lab when constructed.

In this chapter, two new circuits are constructed which put together the ideas of the last few chapters. The circuits are designed, simulated on computer, and then constructed in lab. One of the circuits has an improved bandpass filter and the other an improved notch filter.

6.2 The KHN filter revisited

Consider again the KHN circuit in Figure 2.7.

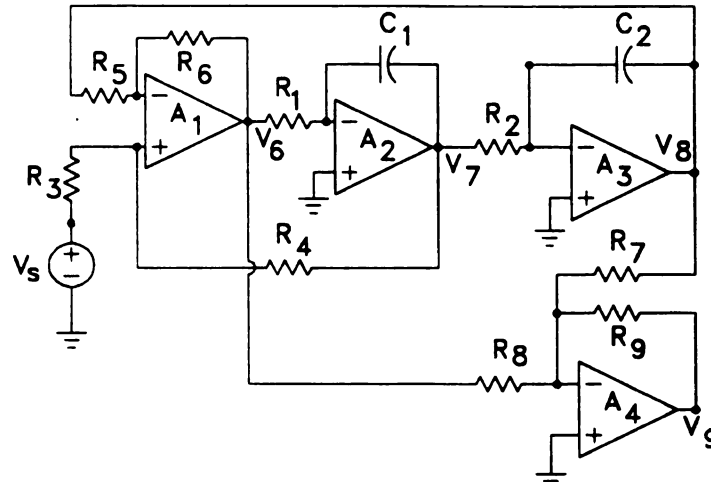


Figure 6.1 KHN filter with summer

It was noted that the circuit exhibits highpass, lowpass, and bandpass filter functions. Adding a fourth op-amp which performs the operation of summing, gives rise to the circuit in Figure 6.1.

The outputs which are summed are the highpass and lowpass functions. This gives rise to a notch function at the output of the summer. This circuit is actually manufactured by National Semiconductor and is part number AF100.

6.3 Choosing circuit parameters

The first step in the design process is to look at the symbolic transfer function of the output, so that resistors may be chosen in order to satisfy circuit parameters. The circuit is typed into a file and run on SLAP. The output is shown in Table 6.1.

Table 6.1 transfer functions for KHN circuit with summer

DENOMINATOR IS :

$$sC2*sC1*G8*G6*G4 \\ +sC2*sC1*G8*G6*G3+sC2*G8*G6*G4*G1+sC2*G8*G5*G4*G1 \\ +G8*G5*G4*G2*G1+G8*G5*G3*G2*G1$$

numerator for V1 is :

$$sC2*sC1*G8*G6*G3 \\ +G8*G5*G3*G2*G1$$

numerator for V6 is :

$$sC2*sC1*G8*G6*G3 \\ +sC2*sC1*G8*G5*G3$$

numerator for V7 is :

$$-sC2*G8*G6*G3*G1 \\ -sC2*G8*G5*G3*G1$$

numerator for V8 is :

$$G8*G6*G3*G2*G1 \\ +G8*G5*G3*G2*G1$$

numerator for V9 is :

$$-sC2*sC1*G9*G6*G3 \\ -sC2*sC1*G9*G5*G3-G7*G6*G3*G2*G1-G7*G5*G3*G2*G1.$$

This gives the following transfer functions for the circuit.

$$\frac{V_6}{V_{in}} = \frac{s^2[(R_5 + R_6)/(R_3 + R_4)](R_4/R_5)}{s^2 + (1/C_1)[(R_5 + R_6)/(R_3 + R_4)](R_3/R_1R_5) + (1/C_1C_2)(R_4/R_1R_2R_5)} \quad (6.1)$$

$$\frac{V_7}{V_{in}} = \frac{-s(1/C_1)[(R_5 + R_6)/(R_3 + R_4)](R_4/R_1R_5)}{s^2 + s(1/C_1)[(R_5 + R_6)/(R_3 + R_4)](R_3/R_1R_5) + (1/C_1C_2)(R_4/R_1R_2R_5)} \quad (6.2)$$

$$\frac{V_8}{V_{in}} = \frac{(1/C_2C_1)[(R_5 + R_6)/(R_3 + R_4)](R_4/R_1R_2R_5)}{s^2 + s(1/C_1)[(R_5 + R_6)/(R_3 + R_4)](R_3/R_1R_5) + (1/C_1C_2)(R_4/R_1R_2R_5)} \quad (6.3)$$

$$\frac{V_9}{V_{in}} = \frac{-[(R_3 + R_4)/(R_5 + R_6)](R_4R_9/R_5R_9)[s^2 + R_9/R_7R_2R_1]}{s^2 + s(1/C_1)[(R_5 + R_6)/(R_3 + R_4)](R_3/R_1R_5) + (1/C_1C_2)(R_4/R_1R_2R_5)} \quad (6.4)$$

For the highpass, bandpass, and lowpass outputs, V_6 , V_7 , and V_8 are the same transfer functions as in Chapter 3, as they should be. The ideal parameters which describe the filter function are chosen as in Chapter 3. The values selected for the components are

$$C_1 = C_2 = 32.467\text{nF}$$

$$R_1 = 3.844\text{k}\Omega$$

$$R_2 = R_3 = R_5 = R_6 = R_7 = R_8 = R_9 = 1\text{k}\Omega$$

$$R_4 = 50\text{k}\Omega,$$

which yields

$$H_{bp} = 50$$

$$Q_0 = 50$$

and $f_0 = 2500.$

For the notch filter, there exists the following constraints on the ideal parameters

$$H_n = (R_3 + R_4)/(R_5 + R_6)(R_4R_8/R_5R_9) \quad (6.5)$$

$$f_n = [1/2\pi][R_9/C_2C_1R_7R_2R_1]^{-1/2}. \quad (6.6)$$

With the choice of component values, this gives

$$H_n = 1.96$$

$$f_n = 2500\text{Hz}.$$

It should be noted that different values of resistors were tried, which yielded different results for the ideal parameters, but the above gave the best results, in terms of circuit performance (computer simulation). Also, only the bandpass and the notch filter will be observed, since the highpass and lowpass characteristics are only as good as the bandpass.

At this point the circuit is ready to run through OAR.

6.4 OAR, the generation of a new topology

A two pole model is selected for the OAR run for the reasons discussed in Chapter 5. OAR allows for a one pole model, and certainly many circuits will appear good, but the circuits designed in this thesis were chosen with the idea of creating a circuit which could actually be constructed in lab - a nice feature. Furthermore, the second pole was placed far enough in so that it was worse than any op-amp on the market. These features help insure the circuit's usefulness.

From the topology of the circuit, there is one nullator tree with only one nullator and one nullator tree with three nullators. From the theory presented in Chapter 2, this gives rise to

$$4 \sum_{j=1}^2 (m_j + 1)^{(m_j - 1)}$$

$$= 24(4^2 \times 1), \text{ since } m_1 = 1 \text{ and } m_2 = 3,$$

$$= 384$$

different circuits and a total of

$$(2^4)(384)$$

$$= 6144$$

circuits which must be checked for stability by OAR, taking the different sign configurations of the input terminals into account. The output of OAR is then run through PZ as before. The output of PZ is shown in Table 6.2.

From the output of OAR and PZ, there are 13 stable circuits. The first set of roots are the denominator poles, used in finding the best bandpass filter. The second set are the numerator zeros of the output at node 9. These are used to determine the best notch filter. In order to determine f^* and Q^* the following formulas were used:

$$f^* = [1/2\pi][(\text{re}(\text{ccp}))^2 + (\text{im}(\text{ccp}))^2]^{1/2} \quad (6.7)$$

$$Q^* = (2\pi f^* / \text{re}(\text{ccp})), \quad (6.8)$$

where ccp is the complex conjugate pair. A look at circuit 1 show the original circuit and shows it having a $f^* = 2490.4\text{Hz}$ a -0.38% change, and a Q^* of 129.34, a 158.7% increase, very substantial. The measure of the notch filter will be the quality factor Q_n of the notch, ideally at infinity. The original circuit shows a Q_n^* of 161.89, a far cry from infinity. The notch frequency is $f_n^* = 2490.33$, off -0.39% from the original design parameter.

At this point, it is necessary to pick the best notch filter and bandpass filter out of the set. For the bandpass filter, circuit 11 shows promise. From the poles and zeros it is determined that it has a $f^* = 2499.9$ or a -0.002% error, and a $Q^* = 127.95$ or a 155.9% error. The Q errors are not significantly better than that of the original circuit. The frequency errors are much improved. The circuit also has an added feature which will become clearer, that makes it much more attractive and generous than the original circuit.

Table 6.2 PZ output for KHN with summer

(+input, -input, output) (+input, -input, output) (+input, -input, output)

Unscaled Poles and Zeros

Ground Node : 0

(1, 2, 6); (0, 3, 7); (0, 4, 8); (0, 5, 9) (1)

-.604904D+02 .156479D+05
 -.779520D+06 .183483D+07
 -.775883D+06 .271432D+07
 -.803616D+06 .271284D+07
 -.780068D+06 .142962D+07
 -.779877D+06 -.142962D+07
 -.779864D+06 -.183498D+07
 -.799923D+06 -.271906D+07
 -.779531D+06 -.270807D+07
 -.604903D+02 -.156479D+05

.483242D+02 .156467D+05
 -.787307D+06 .271852D+07
 -.792165D+06 .270825D+07
 .483243D+02 -.156467D+05
 -.791821D+06 -.270800D+07
 -.787651D+06 -.271877D+07

(1, 2, 6); (0, 3, 7); (0, 4, 8); (4, 5, 9) (2)

-.604904D+02 .156479D+05
 -.779520D+06 .183483D+07
 -.775883D+06 .271432D+07
 -.803616D+06 .271284D+07
 -.780068D+06 .142962D+07
 -.779877D+06 -.142962D+07
 -.779864D+06 -.183498D+07
 -.799923D+06 -.271906D+07
 -.779531D+06 -.270807D+07
 -.604903D+02 -.156479D+05

-.236886D+02 .156466D+05
 -.777504D+06 .271012D+07
 -.801896D+06 .271669D+07
 -.236886D+02 -.156466D+05
 -.801658D+06 -.271673D+07
 -.777742D+06 -.271008D+07

(1, 2, 6); (5, 3, 7); (0, 4, 8); (0, 5, 9) (3)

-.611413D+02 .156483D+05
 -.746508D+06 .147251D+07
 -.680187D+06 .260557D+07
 -.864177D+06 .283862D+07

-.848507D+06 .177361D+07
 -.746510D+06 -.147251D+07
 -.848357D+06 -.177315D+07
 -.864278D+06 -.283876D+07
 -.680050D+06 -.260542D+07
 -.611414D+02 -.156483D+05

.483242D+02 .156467D+05
 -.787307D+06 .271852D+07
 -.792165D+06 .270825D+07
 .483243D+02 -.156467D+05
 -.791821D+06 -.270800D+07
 -.787651D+06 -.271877D+07

(1, 2, 6); (0, 3, 7); (0, 4, 8); (3, 5, 9) (4)

-.604904D+02 .156479D+05
 -.779520D+06 .183483D+07
 -.775883D+06 .271432D+07
 -.803616D+06 .271284D+07
 -.780068D+06 .142962D+07
 -.779877D+06 -.142962D+07
 -.779864D+06 -.183498D+07
 -.799923D+06 -.271906D+07
 -.779531D+06 -.270807D+07
 -.604903D+02 -.156479D+05

.483557D+02 .156837D+05
 -.775942D+06 .270898D+07
 -.791517D+06 .271431D+07
 .483558D+02 -.156837D+05
 -.791363D+06 -.271382D+07
 -.776096D+06 -.270948D+07

(1, 2, 6); (5, 3, 7); (0, 4, 8); (0, 3, 9) (5)

-.852417D+02 .156603D+05
 -.206863D+06 .191643D+07
 -.825162D+06 .272688D+07
 -.713209D+06 .188750D+07
 -.139442D+07 .187788D+07
 -.852417D+02 -.156603D+05
 -.139441D+07 -.187793D+07
 -.825074D+06 -.272707D+07
 -.206864D+06 -.191643D+07
 -.713209D+06 -.188703D+07

.244850D+02 .156222D+05
 -.795435D+06 .271499D+07
 -.332654D+09 -.591019D+00
 .244847D+02 -.156222D+05
 -.795435D+06 -.271499D+07

(1, 2, 6); (5, 3, 7); (0, 4, 8); (4, 5, 9) (6)

-370261D+02 .156953D+05
 -.735380D+06 .146182D+07
 -.713756D+06 .264144D+07
 -.841333D+06 .280567D+07
 -.848939D+06 .177962D+07
 -.735283D+06 -.146196D+07
 -.849371D+06 -.177959D+07
 -.841460D+06 -.280607D+07
 -.713656D+06 -.264113D+07
 -.370272D+02 -.156953D+05

.211673D+00 .156938D+05
 -.697092D+06 .259001D+07
 -.882332D+06 .283441D+07
 .211691D+00 -.156938D+05
 -.882323D+06 -.283441D+07
 -.697101D+06 -.259001D+07

(1, 2, 6); (4, 3, 7); (0, 4, 8); (0, 5, 9) (7)

-368071D+02 .156952D+05
 -.779719D+06 .183552D+07
 -.696822D+06 .258951D+07
 -.882856D+06 .283488D+07
 -.779933D+06 .142949D+07
 -.779936D+06 -.142949D+07
 -.780211D+06 -.183543D+07
 -.882075D+06 -.283505D+07
 -.697180D+06 -.258927D+07
 -.368075D+02 -.156952D+05

.726606D+02 .156938D+05
 -.697718D+06 .258978D+07
 -.881779D+06 .283461D+07
 .726606D+02 -.156938D+05
 -.881765D+06 -.283461D+07
 -.697731D+06 -.258978D+07

(1, 2, 6); (0, 3, 7); (3, 4, 8); (0, 5, 9) (8)

-.844168D+02 .156357D+05
 -.787427D+06 .183821D+07
 -.737878D+06 .265003D+07
 -.833686D+06 .277413D+07
 -.780075D+06 .142964D+07
 -.779893D+06 -.142968D+07
 -.787730D+06 -.183845D+07
 -.833903D+06 -.277554D+07
 -.737715D+06 -.264951D+07
 -.844167D+02 -.156357D+05

.243082D+02 .156345D+05
 -.727278D+06 .276059D+07
 -.852266D+06 .266678D+07

.243084D+02 -.156345D+05
 -.852162D+06 -.266681D+07
 -.727286D+06 -.276059D+07

(1, 2, 6); (4, 3, 7); (0, 4, 8); (4, 5, 9) (9)

-.368071D+02 .156952D+05
 -.779719D+06 .183552D+07
 -.696822D+06 .258951D+07
 -.882856D+06 .283488D+07
 -.779933D+06 .142949D+07
 -.779936D+06 -.142949D+07
 -.780211D+06 -.183543D+07
 -.882075D+06 -.283505D+07
 -.697180D+06 -.258927D+07
 -.368075D+02 -.156952D+05

.211673D+00 .156938D+05
 -.697092D+06 .259001D+07
 -.882332D+06 .283441D+07
 .211691D+00 -.156938D+05
 -.882323D+06 -.283441D+07
 -.697101D+06 -.259001D+07

(1, 2, 6); (0, 3, 7); (3, 4, 8); (4, 5, 9) (10)

-.844168D+02 .156357D+05
 -.787427D+06 .183821D+07
 -.737878D+06 .265003D+07
 -.833686D+06 .277413D+07
 -.780075D+06 .142964D+07
 -.779893D+06 -.142968D+07
 -.787730D+06 -.183845D+07
 -.833903D+06 -.277554D+07
 -.737715D+06 -.264951D+07
 -.844167D+02 -.156357D+05

-.478953D+02 .156710D+05
 -.722097D+06 .262502D+07
 -.857279D+06 .280057D+07
 -.478953D+02 -.156710D+05
 -.857280D+06 -.280057D+07
 -.722096D+06 -.262502D+07

(1, 2, 6); (5, 3, 7); (0, 4, 8); (4, 3, 9) (11)

-.613811D+02 .157075D+05
 -.221012D+06 .190194D+07
 -.810474D+06 .272086D+07
 -.714157D+06 .189350D+07
 -.139383D+07 .189083D+07
 -.613811D+02 -.157075D+05
 -.139384D+07 -.189106D+07
 -.810698D+06 -.272075D+07
 -.221016D+06 -.190198D+07

-.713995D+06 -.189315D+07

-.237590D+02 .156689D+05
 -.780060D+06 .271056D+07
 -.332685D+09 -.591072D+00
 -.237592D+02 -.156689D+05
 -.780061D+06 -.271056D+07

(1, 2, 6); (4, 3, 7); (0, 4, 8); (3, 5, 9) (12)

-.368071D+02 .156952D+05
 -.779719D+06 .183552D+07
 -.696822D+06 .258951D+07
 -.882856D+06 .283488D+07
 -.779933D+06 .142949D+07
 -.779936D+06 -.142949D+07
 -.780211D+06 -.183543D+07
 -.882075D+06 -.283505D+07
 -.697180D+06 -.258927D+07
 -.368075D+02 -.156952D+05

.112058D-01 .157311D+05
 -.690801D+06 .258823D+07
 -.876610D+06 .283271D+07
 .112491D-01 -.157311D+05
 -.876603D+06 -.283271D+07
 -.690808D+06 -.258822D+07

(1, 2, 6); (0, 3, 7); (3, 4, 8); (3, 5, 9) (13)

-.844168D+02 .156357D+05
 -.787427D+06 .183821D+07
 -.737878D+06 .265003D+07
 -.833686D+06 .277413D+07
 -.780075D+06 .142964D+07
 -.779893D+06 -.142968D+07
 -.787730D+06 -.183845D+07
 -.833903D+06 -.277554D+07
 -.737715D+06 -.264951D+07
 -.844167D+02 -.156357D+05

.242268D+02 .156715D+05
 -.721007D+06 .275819D+07
 -.846428D+06 .266575D+07
 .242269D+02 -.156715D+05
 -.846441D+06 -.266575D+07
 -.720994D+06 -.275819D+07.

6.5 The bandpass circuit

The new bandpass circuit is shown Figure 6.2.

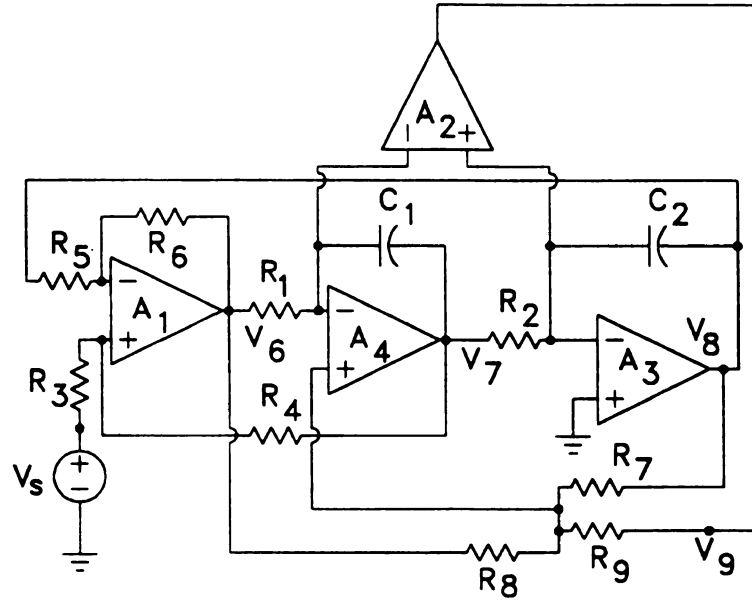


Figure 6.2 new circuit with a good bandpass function

Once the choice of a good bandpass filter has been made, the next step is to add compensation resistors to 'tune' out the errors. For this, SLAP is used to symbolically determine the values of the compensation resistors. With no compensation resistors, the output of SLAP is shown below. Only the denominator is used for the analysis, since it describes the bandpass filter parameters, so this is the only portion of the output of SLAP which is shown. It is shown in Table 6.3.

The terms have been sorted out according to the power of the s term in front of them. From this, it is seen that

$$d_2 = \frac{[G_9 G_6 G_3 - G_9 G_5 G_4 + 3G_8 G_6 G_4 + 3G_8 G_6 G_3 + G_8 G_5 (G_4 + G_3) + G_7 G_6 (G_4 + G_3)]}{G_8 G_6 G_4 + G_8 G_6 G_3} \quad (6.9)$$

Table 6.3 denominator for the new bandpass circuit

Denominator:

36

$$\begin{aligned}
&+sC2*sC1*G9*G6*G4*B4 \\
&+sC2*sC1*G9*G6*G4*B3 \\
&+sC2*sC1*G9*G6*G4*B1 \\
&+sC2*sC1*G9*G6*G4 \\
&+sC2*sC1*G9*G6*G3*B4 \\
&+sC2*sC1*G9*G6*G3*B3 \\
&+sC2*sC1*G9*G6*G3*B1 \\
&+sC2*sC1*G9*G6*G3 \\
&+sC2*sC1*G9*G5*G4*B1 \\
&+sC2*sC1*G9*G5*G3*B1 \\
&+sC2*sC1*G8*G6*G3*B4 \\
&-sC2*sC1*G8*G5*G4*B4 \\
&+sC2*sC1*G7*G6*G4*B4 \\
&+sC2*sC1*G7*G6*G3*B4
\end{aligned}$$

$$\begin{aligned}
&+sC2*G9*G6*G4*G1*B4 \\
&+sC2*G9*G6*G4*G1*B3 \\
&+sC2*G9*G6*G4*G1 \\
&+sC2*G9*G5*G4*G1*B4 \\
&+sC2*G9*G5*G4*G1*B3 \\
&+sC2*G9*G5*G4*G1 \\
&+sC2*G7*G6*G4*G1*B4 \\
&+sC2*G7*G5*G4*G1*B4 \\
&-sC1*G8*G5*G4*G2*B4 \\
&-sC1*G8*G5*G3*G2*B4 \\
&+sC1*G7*G6*G4*G2*B4 \\
&+sC1*G7*G6*G3*G2*B4
\end{aligned}$$

$$\begin{aligned}
&-G9*G6*G3*G2*G1*B3 \\
&+G9*G5*G4*G2*G1*B4 \\
&+G9*G5*G4*G2*G1*B3 \\
&+G9*G5*G4*G2*G1 \\
&+G9*G5*G3*G2*G1*B4 \\
&+G9*G5*G3*G2*G1 \\
&+G7*G6*G4*G2*G1*B4 \\
&+G7*G6*G3*G2*G1*B4 \\
&+G7*G5*G4*G2*G1*B4 \\
&+G7*G5*G3*G2*G1*B4.
\end{aligned}$$

$$d_1 = \frac{[2C_2G_8G_6G_4G_1 + 2C_2G_8G_5G_4G_1 + C_2G_7(G_6 + G_5)G_4G_1 - C_1G_9G_5(G_4 + G_3)G_2 + C_1(G_7G_6(G_4 + G_3)G_2)]}{[C_2G_8G_6G_4G_1 + C_2G_8G_5G_4G_1]} \quad (6.10)$$

$$d_0 = \frac{[-G_8G_6G_3G_2G_1 + 2G_8G_5G_4G_2G_1 + G_8G_5G_3G_2G_1 + G_7G_6(G_4 + G_3)G_2G_1 + G_7G_5(G_4 + G_3)G_2G_1]}{[(G_5G_4G_2G_1 + G_5G_3G_2G_1)G_8]} \quad (6.11)$$

At this point, a mathematical tool such as MATHCAD is useful in evaluating the above equations. One could do the above by hand, but it is a very tedious process. A computer program can do it quickly. Substituting the values for the resistances and the capacitances yields the values

$$d_2 = 5.961$$

$$d_1 = 3$$

$$d_0 = 2.039,$$

and these terms are the determining factors in approximating the frequency and selectivity errors. From these and equations (4.32) and (4.35),

$$\Delta t = 2\pi \times 2500 \times (3 - 5.961) \times 2 \times 50 \times 5.1 \times 10^6$$

$$= -9.1191 \times 10^{-5}$$

$$\Delta q = (2\pi \times 50 / 5.1 \times 10^6) (2.039 - 5.961)$$

$$= -0.60382.$$

From this, the center frequency and selectivity are

$$f^* = f_0 / (1 + \Delta t)$$

$$f^* = 2500.2$$

$$Q^* = Q_0 / (1 + \Delta q)$$

$$= 126.21.$$

These are close to the measured values for f^* and Q^* . Thus the WBB approximation is been valid for the circuit.

The next step is to add in compensation resistors to effectively tune out the errors.

From Chapter 4, it is seen that there are

$$\sum_{i=0}^2 \sum_{j=1}^{m_i} = 7$$

different resistors which can be placed in the circuit which will not affect the ideal equation, where $m_2 = 3$ and $m_1 = 1$. As stated earlier, there is no systematic way of knowing where to place the resistors to ensure a positive effect. The method used in this thesis was to individually place each resistor in the location and vary it to see what effect it had on f^* and Q^* . The program used in this thesis was ZNAP. SLAP could also be effectively used. Some of the resistors had a minimal effect, some of them a detrimental effect, and a few a positive effect. The ones with the positive effect were kept and placed in the circuit. They are labeled R_x and R_y and shown in Figure 6.3.

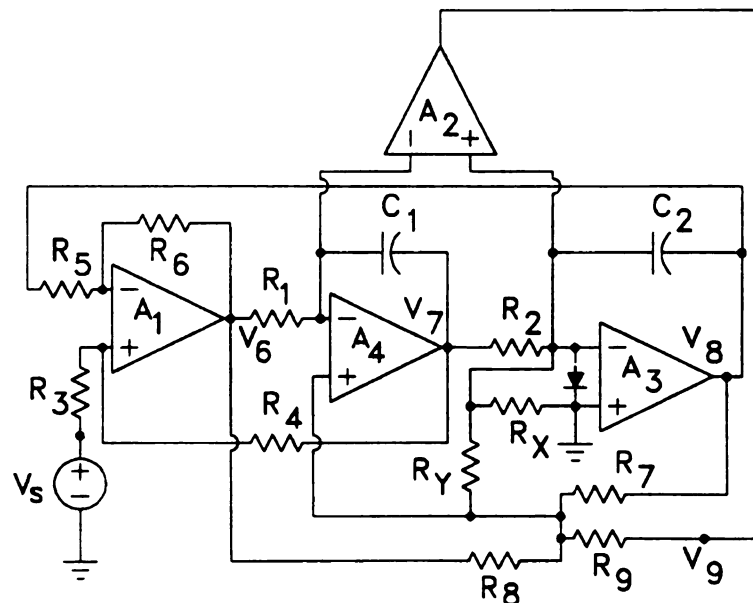


Figure 6.3 new bandpass circuit with compensation resistors

There is a diode placed at the third op-amp. It was found necessary to place the diode there to avoid "latch up", or oscillation at some seemingly arbitrary frequency. This is a known problem, but is yet unsolved and is considered at future research topic.

The biggest problem in the circuit was to null out the Q errors. When R_x was added, it did just that. However, it had a slightly negative effect on f' . Thus, it was necessary to introduce R_y to 'recompensate' the frequency errors, introduced by R_x .

At this point it is necessary to know the approximate values for the resistors that will have the effect on nulling out the errors. Again, SLAP is used to determine the symbolic solution to the problem. The output of the SLAP run is shown in Table 6.4.

This denominator is much larger than even the first run. From the above, it can be shown that the new error defining terms are

$$d_2 = \frac{[G_8 G_6 G_3 - G_8 G_5 G_4 + 3G_9 G_6 G_4 + 3G_9 G_6 G_3 + G_9 G_5 (G_4 + G_3) G_7 G_6 (G_4 + G_3) + G_Y G_6 (G_4 + G_3)]}{(G_9 G_6 G_4 + G_9 G_6 G_3)} \quad (6.12)$$

$$d_1 = \frac{[2C_2 G_9 G_6 G_4 G_1 + 2C_2 G_9 G_5 G_4 G_1 + C_2 G_7 (G_6 + G_5) G_4 G_1 - C_1 G_8 G_5 (G_4 + G_3) G_2 + C_2 G_Y (G_6 + G_5) G_4 G_1 - C_1 G_Y G_8 G_5 (G_4 + G_3) + C_1 G_X G_9 G_6 (G_4 + G_3)]}{[C_2 G_9 G_6 G_4 G_1 + C_2 G_9 G_5 G_4 G_1]} \quad (6.13)$$

$$d_0 = \frac{[-G_9 G_6 G_3 G_2 G_1 + 2G_9 G_5 G_4 G_2 G_1 + G_9 G_5 G_3 G_2 G_1 + G_7 G_6 (G_4 + G_3) G_2 G_1 + G_7 G_5 (G_4 + G_3) G_2 G_1 + G_Y G_9 G_5 (G_4 + G_3) G_1 + G_Y G_7 (G_6 + G_5) G_4 G_1 + G_Y G_5 (G_4 + G_3) G_2 G_1 + G_X G_9 (G_6 + G_5) G_4 G_1]}{[(G_5 G_4 G_2 G_1 + G_5 G_3 G_2 G_1) G_8]} \quad (6.14)$$

There is no easy way to solve for the proper values for R_x and R_y , but there is a unique solution to the problem. The way it was solved for in this thesis was to use MATHCAD and a number of iterations. There are probably other methods.

Table 6.4 output of SLAP

Denominator:

56

+sC2*sC1*GY*G6*G4*B4
 +sC2*sC1*GY*G6*G3*B4
 +sC2*sC1*GX*G6*G4*B4
 +sC2*sC1*GX*G6*G3*B4
 +sC2*sC1*G9*G6*G4*B4
 +sC2*sC1*G9*G6*G4*B3
 +sC2*sC1*G9*G6*G4*B1
 +sC2*sC1*G9*G6*G4
 +sC2*sC1*G9*G6*G3*B4
 +sC2*sC1*G9*G6*G3*B3
 +sC2*sC1*G9*G6*G3*B1
 +sC2*sC1*G9*G6*G3
 +sC2*sC1*G9*G5*G4*B1
 +sC2*sC1*G9*G5*G3*B1
 +sC2*sC1*G8*G6*G3*B4
 -sC2*sC1*G8*G5*G4*B4
 +sC2*sC1*G7*G6*G4*B4
 +sC2*sC1*G7*G6*G3*B4
 +sC2*GY*G6*G4*G1*B4
 +sC2*GY*G5*G4*G1*B4
 +sC2*GX*G6*G4*G1*B4
 +sC2*GX*G5*G4*G1*B4
 +sC2*G9*G6*G4*G1*B4
 +sC2*G9*G6*G4*G1*B3
 +sC2*G9*G6*G4*G1
 +sC2*G9*G5*G4*G1*B4
 +sC2*G9*G5*G4*G1*B3
 +sC2*G9*G5*G4*G1
 +sC2*G7*G6*G4*G1*B4
 +sC2*G7*G5*G4*G1*B4
 -sC1*GY*G8*G5*G4*B4
 -sC1*GY*G8*G5*G3*B4
 +sC1*GY*G7*G6*G4*B4
 +sC1*GY*G7*G6*G3*B4
 -sC1*G8*G5*G4*G2*B4
 -sC1*G8*G5*G3*G2*B4
 +sC1*G7*G6*G4*G2*B4
 +sC1*G7*G6*G3*G2*B4
 +GY*G9*G5*G4*G1*B2
 +GY*G9*G5*G3*G1*B2
 +GY*G7*G6*G4*G1*B4
 +GY*G7*G5*G4*G1*B4
 +GY*G5*G4*G2*G1*B4
 +GY*G5*G3*G2*G1*B4
 +GX*G5*G4*G2*G1*B4
 +GX*G5*G3*G2*G1*B4
 -G9*G6*G3*G2*G1*B3
 +G9*G5*G4*G2*G1*B4
 +G9*G5*G4*G2*G1*B3
 +G9*G5*G4*G2*G1
 +G9*G5*G3*G2*G1*B4

+G9*G5*G3*G2*G1
+G7*G6*G4*G2*G1*B4
+G7*G6*G3*G2*G1*B4
+G7*G5*G4*G2*G1*B4
+G7*G5*G3*G2*G1*B4.

It was found that if

$$R_x = 33.1129\text{k}\Omega \text{ and}$$

$$R_y = 265.08\text{k}\Omega,$$

then the goal of $d_2 = d_1 = d_0$ has been achieved. This yields

$$d_2 = d_1 = d_0 = 9.7333 \text{ and}$$

$$\Delta t = \Delta q = 0,$$

and the theoretical error for the bandpass parameters are 0. A run from PSPICE, shown in Figure 6.4 yields the following information.

$$f^* = 2500.428, \text{ a deviation of } 0.017\%$$

$$H_{bp}^* = 52.8537, \text{ a deviation of } 5.7075\%$$

$$Q^* = 51.049, \text{ a deviation of } 2.098\%.$$

The above bandpass filter is much improved from the original circuit's bandpass filter. The original circuit has no chance of being tuned. Although the circuit does not display zero errors as predicted by the WBB analysis, they are small. It is believed that the errors are due to the A^2 terms coming into play. Also, R_x and R_y can be "overtuned" to further reduce the errors. The purpose of this example was to show the initial stage and ideas introduced, so this will not be done. A final comparison of the responses of the ideal circuit, original circuit, and the improved circuit are shown in Figure 6.4. A new circuit has been successfully developed.

6.6 The notch filter

As shown circuit 12 appeared to be a much better notch filter than the original circuit was. The circuit is shown in Figure 6.5.

To get a symbolic handle on the circuit, again SLAP is used. The output, showing only the necessary denominator terms and the output at node voltage 9, is shown in Table 6.5.

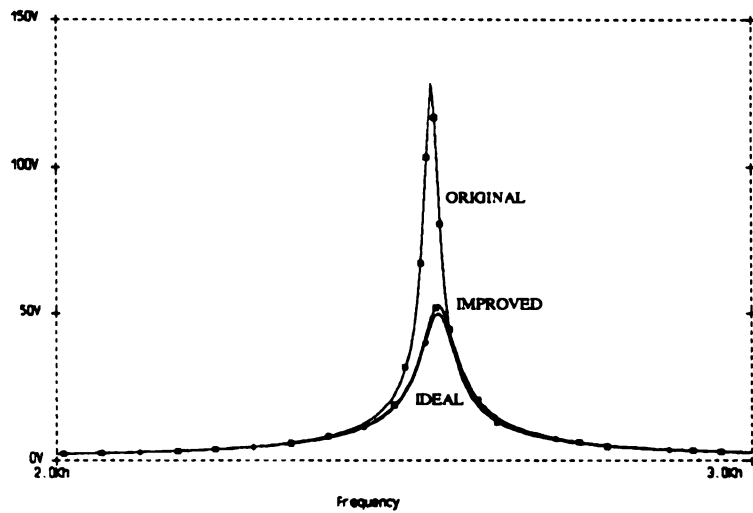


Figure 6.4 ideal, original, and improved bandpass responses

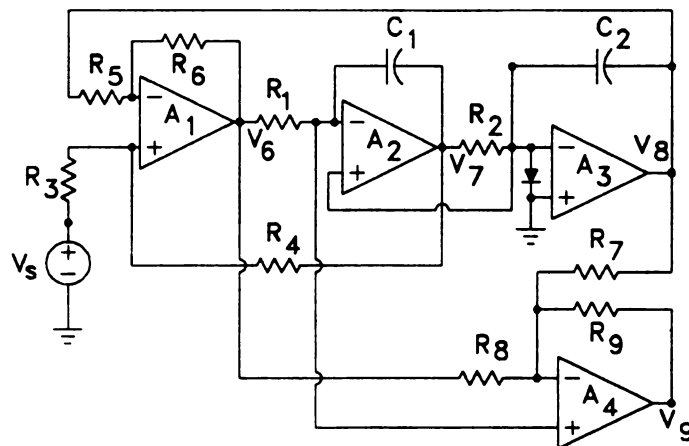


Figure 6.5 the circuit with the best notch filter

Table 6.5 notch circuit transfer function

Denominator:

38

$+sC2*sC1*G9*G6*G4*B4$
 $+sC2*sC1*G9*G6*G4*B3$
 $+sC2*sC1*G9*G6*G4*B2$
 $+sC2*sC1*G9*G6*G4*B1$
 $+sC2*sC1*G9*G6*G4$
 $+sC2*sC1*G9*G6*G3*B4$
 $+sC2*sC1*G9*G6*G3*B3$
 $+sC2*sC1*G9*G6*G3*B2$
 $+sC2*sC1*G9*G6*G3*B1$
 $+sC2*sC1*G9*G6*G3$
 $+sC2*sC1*G9*G5*G4*B1$
 $+sC2*sC1*G9*G5*G3*B1$
 $+sC2*sC1*G8*G6*G4*B4$
 $+sC2*sC1*G8*G6*G3*B4$
 $+sC2*sC1*G7*G6*G4*B4$
 $+sC2*sC1*G7*G6*G3*B4$

$+sC2*G9*G6*G4*G1*B4$
 $+sC2*G9*G6*G4*G1*B3$
 $+sC2*G9*G6*G4*G1*B2$
 $+sC2*G9*G6*G4*G1$
 $+sC2*G9*G6*G3*G1*B2$
 $+sC2*G9*G5*G4*G1*B4$
 $+sC2*G9*G5*G4*G1*B3$
 $+sC2*G9*G5*G4*G1$
 $+sC2*G8*G6*G4*G1*B4$
 $+sC2*G8*G5*G4*G1*B4$
 $+sC2*G7*G6*G4*G1*B4$
 $+sC2*G7*G5*G4*G1*B4$

$-G9*G6*G3*G2*G1*B3$
 $+G9*G5*G4*G2*G1*B4$
 $+G9*G5*G4*G2*G1*B3$
 $+G9*G5*G4*G2*G1$
 $+G9*G5*G3*G2*G1*B4$
 $+G9*G5*G3*G2*G1$
 $+G8*G5*G4*G2*G1*B4$
 $+G8*G5*G3*G2*G1*B4$
 $+G7*G5*G4*G2*G1*B4$
 $+G7*G5*G3*G2*G1*B4$

V9:

16

$-sC2*sC1*G8*G6*G3*B3$
 $-sC2*sC1*G8*G6*G3*B2$
 $-sC2*sC1*G8*G6*G3$
 $-sC2*sC1*G8*G5*G3*B3$
 $-sC2*sC1*G8*G5*G3*B2$

-sC2*sC1*G8*G5*G3

+sC2*G9*G6*G3*G1*B2

+sC2*G9*G5*G3*G1*B2

+sC2*G7*G6*G3*G1*B2

+sC2*G7*G5*G3*G1*B2

-G9*G6*G3*G2*G1*B3

-G9*G5*G3*G2*G1*B3

-G7*G6*G3*G2*G1*B3

-G7*G6*G3*G2*G1

-G7*G5*G3*G2*G1*B3

-G7*G5*G3*G2*G1

.END

From the numerator

$$n_2 = \frac{2(C_2 C_1 G_8 (G_6 + G_5) G_3)}{C_2 C_1 G_8 (G_6 + G_5) G_3} \quad (6.15)$$

$$m_1 = -C_2 G_9 (G_6 + G_5) G_3 G_1 - C_2 G_7 (G_6 + G_5) G_3 G_1 \quad (6.16)$$

$$n_0 = \frac{G_9 (G_6 + G_5) G_3 G_2 G_1 + G_7 (G_6 + G_5) G_3 G_2 G_1}{G_7 (G_6 + G_5) G_3 G_2 G_1} \quad (6.17)$$

From the denominator

$$\gamma = G_9 G_5 (G_4 + G_3) G_2 G_1 \quad (6.18)$$

and substituting in the values of the passive components gives

$$n_2 = 2$$

$$m_1 = -3.367 \times 10^{-20}$$

$$n_1 = 2$$

$$\gamma = 2.653 \times 10^{-16}$$

From equations (4.92) and (4.96) this gives

$$\begin{aligned} \Delta n &= (1/2)(2\pi \times 2500)^2 (-3.367 \times 10^{-20} / 2.653 \times 10^{-16} \times 1.96 \times 5.1 \times 10^6) \\ &= -0.001566 \end{aligned}$$

$$\begin{aligned} \Delta d &= 2\pi \times 2500 \times (2 - 2) / 5.1 \times 10^6 \\ &= 0. \end{aligned}$$

From equations (4.85), (4.86), and (4.100), the frequency, quality factor, and magnitude at the notch frequency are

$$\begin{aligned} f_n^* &= f_n / (1 + \Delta n) \\ &= 2500 / (1 - 0.001566) \\ &= 2503.922 \end{aligned}$$

$$Q^* = \infty$$

$$NG = 0,$$

and these values are close to those predicted by SPICE. The exact figures from SPICE shown in Figure 6.6 are

$$f_n^* = 2504.0$$

$$NG = 0.0099.$$

At this point, the next step is to try to minimize the m_1 term as the n_2 and n_0 terms are already equal. The same scheme as with the bandpass circuit was tried, but to no avail. The circuit must be accepted as it sits. It has a slightly better notch frequency than the original circuit, but a much improved quality factor and notch depth. This is further illustrated by Figure 6.6.

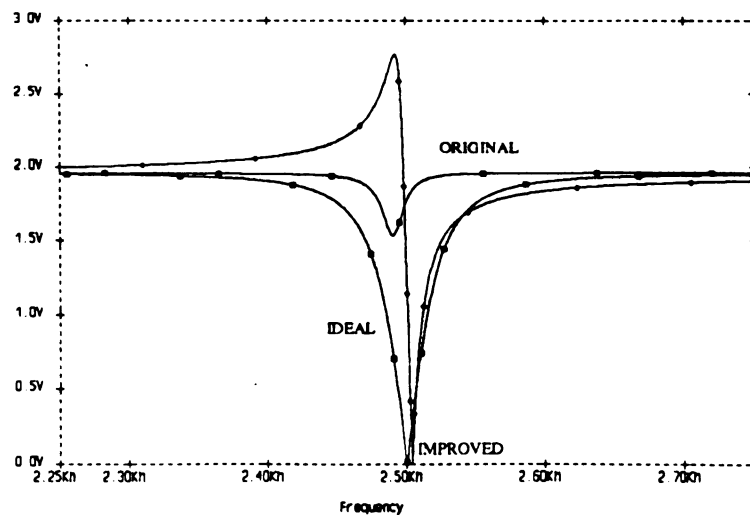


Figure 6.6 notch filter comparison

Figure 6.6 shows a peak near the notch frequency for the improved notch filter. This is not good. In order to better illustrate the notch filter, the capacitors were changed to give a new notch frequency of 240Hz.

Figure 2.7 gives the ideal, improved, and original responses in the range of 235 to 245Hz. It is evident that the new circuit outperforms the original circuit, in that it is both nearer to the desired frequency, but also has a notch depth which approaches zero, the original does not. The original cannot completely filter out a signal.

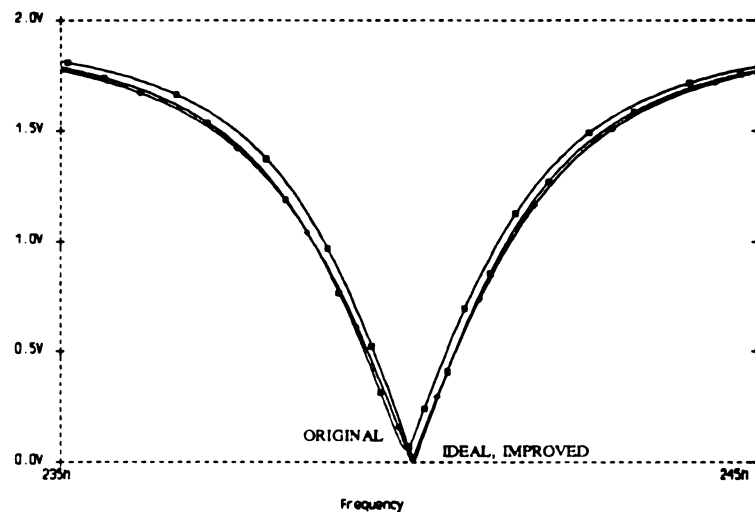


Figure 6.7 notch filter comparison around 240Hz.

To better exploit this, Figure 6.8 shows the response in db between 239.9 and 240.1Hz. In this range, it is much clearer that there is an improvement.

A notch filter has been successfully developed.

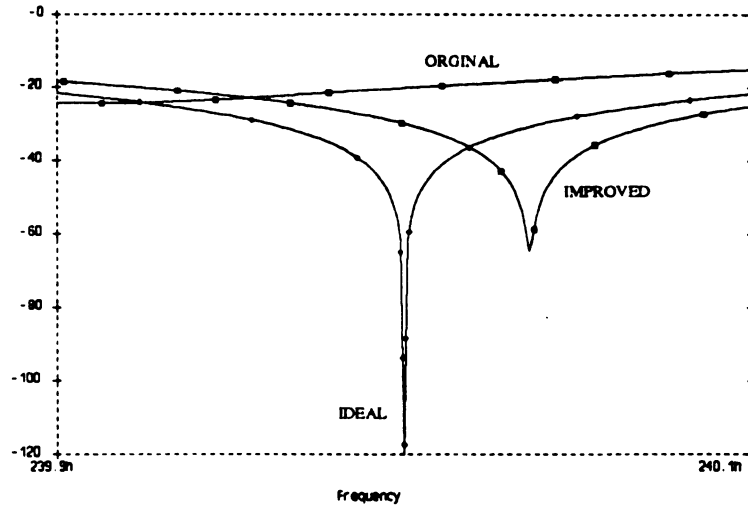


Figure 6.8 detailed notch filter depth

6.9 Experimental results

As stated earlier, one may design filters which perform better on paper. The purpose of this chapter was to design circuits that could not only perform by pencil, but that also can be built in the lab and used in practical applications. This section of Chapter 6 will present those results.

The KHN bandpass circuit was designed for a center frequency of 4.249kHz. The circuit was designed and tested for sensitivity to the gain-bandwidth product of the op-amps composing it. The TL084, a quad amplifier unit, was selected [17] because it has exceptional [16] on chip matching of the gbp. 25 different chips were used in the circuit, with measurements taken on the center frequency and gain at center frequency of the circuit. The results are in Table 6.6.

Table 6.6 bandpass results

f_o max	f_o min	H_{bp} max	H_{bp} min
4249.37	4247.44	55.6983	53.14

These are good results as the circuit both operated and showed minimal sensitivity to the gbp of the TL084s.

The notch filter was designed for $f_n = 235\text{Hz}$. It also operated nicely in lab. This was tested with 10 different TL084s. The results are in Table 6.7.

Table 6.7 notch results

$f_{n\text{max}}$	$f_{n\text{min}}$	NG
235.24	235.26	-40db

This again shows nice results.

CHAPTER 7

Conclusions and Future Research

7.1 Conclusions

It is shown that there is a systematic way to approach active filter synthesis using op-amps. Some of the problems have been outlined, and guidelines for handling these problems given. Two new high performing circuits have been designed to verify the solution process, at a time when very few new circuits are being published. The approach is simple and straight forward, which will aid in the design process.

7.2 Future research

Some areas of potential research using some of the topics in this thesis are as follows:

- 1.) Many existing active filters can be taken, new structures generated and analyzed. This may lead to a circuit which will be considered the "best" circuit. This can lead to a greater understanding of active filter synthesis using op-amps.
- 2.) The parasitic pole problem is not completely solved. Although a two pole model has shown to be experimentally true in predicting parasitic pole stability, there is no mathematical proof. It may be that in certain structures that a one pole model may suffice. It may also lead to a more intuitive way to predict whether or not a circuit will be stable, from a topological point of view. Here perhaps there are reasons why only certain configurations of op-amp polarities work. This work may also give rise to a solution in choosing component values for a op-amp network employing more

than three op-amps, as was done in chapter 5.

- 3.) Although the approach given was for active filters, the same approach can be used on any circuit employing more than one op-amp realizing some transfer function. The same approach to stability can be used, and a WBB approach to the errors resulting from non ideal op-amps can be done so that errors may be nulled out.
- 4.) Analysis may be done with output resistances of the op-amps taken into consideration. There perhaps may be a simple way of dealing with the deviations which may occur from a finite output resistance of the op-amp.
- 5.) The circuit in Figure 6.3 showed a diode at the input of op-amp three. This diode was placed there by trial and error. It is put there for instability which occurs when that particular op-amp saturates due to overdriving it. It is still unknown where it is exactly needed, in addition to the necessary polarity of the diode. This problem also needs to be solved.

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BIBLIOGRAPHY

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