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By

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ABSTRACT

ESSAYS IN EMPIRICAL INTERNATIONAL FINANCE

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This dissertation consists of three essays:

1. "Impact of EMS Membership on its Nominal Exchange Rate Volatility" compares the exchange rate volatilities before and after the advent of the EMS (European Monetary System), comparing members' currency volatilities with the non-EMS currency volatilities. Multivariate as well as univariate GARCH models indicate that the existence of the EMS has coincided with a marked reduction in the volatilities of intra-EMS exchange rates. However, in the EMS versus non-EMS cases, or between-non-EMS currency cases, some countries show at least constant volatilities. Hence, we cannot say that this stability results from the system itself. Furthermore, member countries' exchange rate volatilities against the US dollar show different patterns under their exchange-rate mechanisms.

2. "Multivariate Cointegration Tests and Long-Run Purchasing Power Parity Theory" reexamines the relationship between prices and exchange rates by multivariate cointegration tests developed by Johansen (1988). This method uses vector autoregressive processes. Cointegrating vectors between prices and exchange rates are determined simultaneously by maximum likelihood estimation. This study also reexamines the stationarity of the levels of price series and relative price indexes and find that some are I(2). After analyzing purchasing power parity (PPP) doctrines and finding evidence that PPP holds even after 1973, results are compared with the conventional unit root tests for PPP which showed unfavorable, but low

power, results. Short-run dynamics are analyzed in the last section.

3. "Multivariate Cointegration Tests for a Set of Foreign Exchange Rates and a Comparative Study of the Forecasting Accuracy of the Random-Walk and the Error-Correction Models" begins with the work of Baillie and Bollerslev (1989), which showed the existence of one long-run equilibrium relation between a set of seven daily exchange rate series by multivariate tests for unit roots. After eliminating some currencies redundant to this relationship, this study finds that the EMS currencies, with the German mark as a driving currency, contribute to such a long-run relationship. Although each exchange rate series has a univariate representation as a random walk, it appears that the random-walk model does not outperform our error-correction model in out-of-sample forecasting accuracy.

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CHAPTER I

INTRODUCTION

In March 1973 the Bretton Woods system of fixed but adjustable exchange rates collapsed and the economies moved to a system of floating exchange rates. This change was a reflection of the failure of the Bretton Woods system to deal effectively with the fundamental current account imbalances. During the early 1970s, the prevailing academic view was that flexible exchange rates would solve the increasingly obvious problems of the Bretton Woods system and thereby create a far less difficult environment for the management of domestic monetary and fiscal policies. What, then, of the case made for flexible exchange rates by its proponents? There were essentially five claims made by the early advocates of flexible exchange rates, that is, such writers as: Friedman (1968), Sohman (1969), Johnson (1970) and Machlup (1970). First, flexible exchange rates move to offset a country's relative price level; under flexible exchange rates, if we let the exchange rate depreciate, it compensates for the price increase to maintain the country's competitive position. This is known as purchasing power parity (PPP). Second, such a regime would be a relatively stable one, in contrast with the supposed inherent instability of the Bretton Woods system. Third, both Friedman (1968) and Sohmen (1969) argued that floating exchange rates would isolate a country from shocks emanating from the rest of the world. The fourth argument for floating rates is the independence they give a country in pursuing monetary policy, and the final justification for the regime is that, in principle, central banks need not hold foreign exchange reserves, since official intervention will be zero.

However, the recent history of currency gyrations under the prevailing floating exchange rate regime suggests that most of the propositions advanced in the articles by the early advocates are doubtful since fllexible exchange rates have not performed as expected. First, many authors have found little evidence of the empirical validity of PPP after 1973 (e.g., Frenkel, 1981). Furthermore, large and frequent exchange rate changes have produced a range of unforseen and generally disruptive side effects throughout the economies of the industrialized countries. Two widely-cited papers by Meese and Rogoff (1983a, 1983b) were the first studies to provide extensive and fairly convincing evidence that existing models of systematic exchange rate behavior could not outperform a random-walk model, even when the forecasts of systematic behavior were based on the <u>ex post</u> realized value of the explanatory variables. Therefore, to reduce such an exchange rate volatility, even among member countries, the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS) was established in March 1979. The purpose of this dissertation is to review and evaluate empirically some parts of international finance which are related to the arguments mentioned above. It consists of three essays: the first essay examines the volatility of bilateral exchange rates of the EMS currencies to see whether they stabilized exchange rates among member countries compared with non-EMS currency volatilities. In the second essay, purchasing power parity after the establishment of the floating exchange rate system is tested to see whether it exists after 1973. In the last essay, the random-walk model of exchange rate determination is compared with the error-correction model.

In the first essay (Chapter II), the EMS currency volatility is tested for its stability. The EMS came into operation in March 1979 to create "a zone of monetary stability in Europe," comprising "greater stability at home and abroad." It is a system of fixed, though adjustable, exchange rates. However, the dynamics of the EMS represent a considerable challenge to economists. Among many arguments against the EMS, this essay examines exchange rate volatilities before and after the advent of the EMS, comparing members' currency volatilities with the non-EMS currency volatilities. GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models are introduced to stress the importance of the stylized leptokurtic characteristic in the exchange rate series with the student t-distribution, and also the possibility of a time-dependent conditional heteroskedasticity. After estimating the univariate GARCH models, multivariate GARCH approaches are given, since nonzero covariance among exchange rate innovations requires a joint estimate of sets of regressions and since exchange rates are bilateral rates, it should affect all rates if a new information comes to the foreign exchange market.

In the second essay (Chapter III), the PPP theory after the 1970s is examined. The general view is that a currency's equilibrium level is best associated with its international purchasing power parity. Such a relationship between prices and exchange rates is generally rejected after the 1970s [e.g., Frenkel, 1981; Dornbusch, 1980]. However, the conventional tests disregard the fact that levels of price indexes (and also some first-differenced CPIs) and exchange rates are nonstationary. This essay carefully tests for unit roots in the price indexes and relative price indexes to determine whether they have two unit roots by Dickey and Pantula (1987) tests. Then PPP hypothesis is tested in the multivariate context developed by Johansen (1988). This method gives more efficient estimates, since it not only allows for general dynamic properties of the structure of the underlying process, but it also gives maximum likelihood estimates.

The third essay (Chapter IV) starts with the work of Baillie and Bollerslev (1989), which showed, by means of multivariate tests for unit roots, the existence of one long-run equilibrium relation between a set of seven daily exchange rate series. This result indicates a perceptible deviation from weak-form efficiency for each of the exchange rates, because, in the first-order error-correction model, if two or more exchange rates are cointegrated, part of the changes will usually be predictable. First, after eliminating some currencies redundant to this relation, we find a driving currency for such a long-run relations, and then we analyze this long-run equilibrium with the remaining currencies. Second, as we confirm in our study, each exchange rate series has a univariate representation as a random walk, but since a vector of the first differenced exchange rates should have a lagged error-correction term applied to it, we compare the forecasting accuracy of an error-correction model with that of a random-walk model.

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I. IMPACT OF EMS MEMBERSHIP ON ITS NOMINAL EXCHANGE RATE VOLATILITY: AN APPROACH WITH UNIVARIATE AND MULTIVARIATE GARCH MODELS

1. Introduction

A number of studies considered the evidence that the EMS has reduced exchange rate volatility. Ungerer, et. al (1983) noted that 'the exchange rate variability of EMS currencies has diminished since the introduction of the system---'[1] and updated the conclusion with a later paper (1986). The European Commission (1982), Ungerer (1983), Dennis and Nellis (1984), Bank of England (1984), and Rogoff (1985) also studied the variability of EMS currencies.

In the notable study by Ungerer, et al (1983, 1986), variable approaches to this question were used with various choices of exchange rates (bilateral, effective, nominal, and real), data frequency (daily, weekly, and monthly), and the level and change in exchange rates.

However, all of these studies which have tested for a downward shift in exchange-rate volatility for members of the EMS post-March, 1979 have generally relied on the unconditional distribution, independently and identically drawn from a normal distribution. It is by now an accepted fact that exchange-rate distributions tend to be leptokurtic (fat-tailed, highly-peaked) and that the variance shifts through time with new information available at time t-1, as noted by Taylor and Artis (1988). They applied non-parametric tests for volatility shifts which do not require actual estimation of the distribution parameters as well as tests for a shift in the conditional variance with a random walk with Autoregressive Conditional Heteroskedasticity (ARCH) disturbances. They found a significant reduction in the conditional variance of exchange rate

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[1] Ungerer, et al, (1983), pp 8-9.

for the EMS currencies against the D-mark and signs of a significant rise in the conditional variance against the US dollar (see Taylor and Artis, 1988 p. 12) However, they didn't demonstrate how to derive the likelihood ratio test which played a key role in their tests for shift in volatility. To derive a likelihood ratio test is not easy, given different observations and/or different distributions in each period i.e., Pre-EMS and Post-EMS. Also, after discussing the leptokurtic distribution in the exchange-rate change in one section, they ignored this distribution in their ARCH model and estimated the parameters under the normality assumption.

This paper will stress the importance of this stylized leptokurtic characteristic with the student t-distribution and also the possibility of a time-dependent conditional heteroskedasticity with multivariate GARCH (generalized ARCH) models as well as univariate models. I will test intra-EMS volatility against the Italian lira instead of the Deutsche mark (D-mark) to eliminate any possible impact of the role of the D-mark as a reserve currency or leading currency in the EMS. The US dollar will be used as a base currency to test the volatility change for non-EMS currencies, and the Pound Sterling will also be used to see whether there will be any difference in measuring volatility with a choice of a base currency.

In Section 1 I use unit root tests to check the stationarity in the weekly exchange-rate series. In Section 2 the univariate GARCH models are used to explain how the time-dependent conditional heteroscedasticity is built after diagnostic tests with Ljung-Box Q (k) and $Q^2(k)$ statistics to check serial correlations. In Section 3 the test results of the EMS currency volatility after March 13, 1979 are analyzed, and in Section 4 the multivariate GARCH models are estimated. Some conclusions are given in Section 5.

2. Tests for a Unit Root in Weekly Exchange Rate Series

Autoregressive time series with a unit root have been the subject of much recent attention in the econometrics literature. In part, this is because the unit root hypothesis is of considerable interest, not only with data from financial and commodity markets where it has a long history, but also with macroeconomic time series. Initially, the research was confined to cases where the sequence of innovations driving the model is independent with common variance. Frequently, it was assumed that the innovations were iid$(0,\sigma^2)$ or, further, that they were iid $N(0,\sigma^2)$. However, independence and homoskedasticity are rather strong assumptions to make about the errors in most empirical econometric work. There is now a substantial body of research that exchange-rate series exhibit time-dependent heteroskedasticity (see Baillie and Bollerslev (1989), Bollerslev (1987), Milhøj (1987), McCurdy and Morgan (1983) and the references therein.).

I have used the unit root test methods of Phillips (1987) and Phillips and Perron (1986, 1988) which are robust to a wide variety of serial correlation and time-dependent heteroskedasticity. These tests involve computing the OLS regressions:

$$s_t = \tilde{\mu} + \tilde{\beta}(t-T/2) + \tilde{\alpha}s_{t-1} + \tilde{u}_t \tag{2-1}$$

$$s_t = \mu^* + \alpha^* s_{t-1} + u_t^* \tag{2-2}$$

and

$$s_t = \hat{\alpha} s_{t-1} + \hat{u}_t \tag{2-3}$$

where s_t is the log of spot exchange rates, T denotes the sample size, and the innovation sequences $\tilde{u}_t$, u_t^* and $\hat{u}_t$ are allowed to follow a wide variety of stochastic behavior including conditional heteroskedasticity. The testing strategy recommended by Phillips and Perron is to start Eq. (2-1) and to test the null hypothesis Ho^1: $\tilde{\mu}=0$, $\tilde{\beta}=0$, $\tilde{\alpha}=1$ and Ho^2: $\tilde{\beta}=0$, $\tilde{\alpha}=1$ by means of the statistics $Z(\Phi_2)$ and $Z(\Phi_3)$ respectively. If Ho^1 and Ho^2 can be rejected, then one should next test Ho^3 : $\tilde{\alpha} = 1$ by means of the $Z(t_{\tilde{\alpha}})$ statistic. If Ho^1 and Ho^2 can not be rejected (i.e. they show both random walk and random walk with drift), then the strategy is to proceed to exclude the time trend and to test Ho^4 : $\mu^* = 0$ and $\alpha^* = 1$ by the use of the $Z(\Phi_1)$ test statistic for testing a unit root without drift.

Individual unit root tests of the null hypothesis on (2-2) and (2-3) of the form Ho^5: $\alpha^* = 1$ and Ho^6: $\hat{\alpha}=1$ are tested by the statistics $Z(t_{\alpha}*)$ and $Z(t_{\hat{\alpha}})$; see Phillips and Perron (1988) for the precise formula for each test statistic.

In our analysis, I took weekly spot exchange rate data from the New York Foreign Exchange Market between January 3, 1973 and September 28, 1988. The series were constructed by taking observations every Wednesday, and in the event of the market being closed, an observation on the next business day (i.e. Thursday; if the market was closed on that Thursday also, then Friday, and so on) was used. The data provided by the Federal Reserve System, are bid prices taken at noon, constituting a total sample of 827 observations for the EMS currencies and 8 major countries[2] against the US dollar.

Six different unit root test statistics were estimated for all currencies. Calculating the test statistics requires that consistent estimates of the variances of the sum of the disturbances $\tilde{u}_t$, u_t^* and $\hat{u}_t$ in (2-1) to (2-3) and a truncation lag, ℓ, corresponding to the maximum order of non-zero autocorrelation in the disturbances be chosen; see Phillips and Perron (1986) and Newey and West (1987) for details. Hence, the statistics were computed for $\ell = 0,2,4,6$ and 10, but were found to be remarkably similar for different values of ℓ. The results with lag 10 are reported in Table 1.

Both simple unit root tests of the t-statistic type, $Z(t_{\alpha}*)$ and $Z(t_{\tilde{\alpha}})$, confirm the unit root with drift. At the same time, the $Z(\Phi_1)$ statistics accepts the random walk without drift, and the inclusion of a time trend and use of the $Z(\Phi_2)$ statistics show the same results. However, the $Z(t_{\hat{\alpha}})$ statistics reject the random walk without a drift at the usual 95% level for the Swiss franc.

The overall indication is that there is strong evidence for the presence of unit root with a drift for all currency series, and hence, all the series appear to be stationary in their first differences.

[2]The EMS currencies include West German D-mark, French franc, Italian lira, Belgian franc, Netherlands guilder, Irish pound, and Danish krone. The other major currencies include the US dollar with weighted value, Canadian dollar, Pound sterling, Austrian shilling, Swiss franc, Japanese yen, Swedish krona, and Norwegian krone.

3. Models with Time-Dependent Conditional Heteroskedasticity; GARCH (1,1)

For time series analysis, the autoregressive heteroskedastic process (ARCH) type of model has proven useful in several different economic applications. Among many others, see Engle (1982), Engle and Kraft (1982), Coulson and Robins (1985), Engle, Lilien and Robins (1987), and Weiss (1984). However in this paper, the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) is considered for empirical study of the EMS currency volatility, since it allows for a much flexible lag structure (see Baillie and Bollerslev (1987) and Bollerslev (1986, 1987) for its applications with conditional t-distributed errors.).

3.1 Implication of GARCH Model

The first set of data consists of weekly exchange rates of EMS currencies against the US dollar and EMS currencies against the Mark from January 3, 1973 until September 28, 1988 for a total of 827 observations. The log of spot rates, s_t, are converted to continuously compounded rates of return[3],

$$y_t = 1000 \times (s_t - s_{t-1}).$$

The dependent variable y_t denotes the change in the logarithm of the exchange rates between time t and t-1 and is shown to be stationary in its first difference from the results of Section 1. The full model is then:

$$y_t = b_o + u_t$$

$$u_t \mid \Omega_{t-1} \sim D(0,h_t)$$

$$h_t = \omega_o + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1}$$

[3] For convenience of calculation, I multiplied 1000 by Δs_t, which doesn't change the statistical results.

where Ω_{t-1} is the set of all relevant and available information at time t-1, and where $D(0,h_t)$ represents some distribution with mean 0 and variance h_t. The assumed process is a regression model with innovations that have either conditional normal or student t densities with time-dependent variance. The conditional-variance equation is assumed to follow a generalized ARCH (or GARCH) model.

Before estimating the coefficients of GARCH models, the serial correlations are checked for implications of the GARCH model. First, most currencies were found to have moving average terms with significant levels. For example, the D-mark against the US dollar shows the value of Q(10) = 22.5 in the Ljung-Box (1978) portmanteau test statistic[4] for up to tenth-order serial correlation in $(y_t - \hat{b}_0)$, which is very significant at any reasonable level in the corresponding asymptotic χ^2_{10} distribution. After adding those moving average disturbance terms, the value of Q(10) is reduced to 8.5, which is not significant at any reasonable level (see Table 3, Column 1). This Q(10) reduction is the same for other currencies, with some exceptions, for example, the D-mark against the

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[4]This is a test of the joint hypothesis that all autocorrelation coefficients are zero and as such as chi-square with M-p-q degrees of freedom.

$$Q(\hat{r}) = n(n+2) \sum_{k=1}^{M} \hat{r}_a^2(k) \;/\; (n-k) \sim \chi^2(M-p-q)$$

$$\text{where } \hat{r}_a(k) = \sum_{t=k+1}^{n} \hat{a}_t \hat{a}_{t-k} \;/\; \sum_{t=1}^{n} \hat{a}_t^2 \quad (k=1,2,\cdots)$$

and $(1-\phi_1 L- \ldots -\phi_p L^p)\omega_t = (1-\phi_1 L- \ldots -\phi_q L^q)\, a_t$, where $\{a_t\} \sim$ iid $N(0,\sigma^2)$ with a discrete time series $\omega_1,.,.,.,\omega_n$. In the case of Q(10), critical values of those yield 18.307 and 15.987 at 5% and 10% level, respectively.

Italian lira and the D-mark against the Netherlands guilder need no moving average disturbance terms at all. After considering these moving average disturbance terms, we have

$$y_t = b_o + \theta(L)\epsilon_t \tag{3-1}$$

$$\theta(L)\epsilon_t = \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} \tag{3-2}$$

$$\epsilon_t \mid \Omega_{t-1} \sim D(o, h_t) \tag{3-3}$$

$$E(\epsilon^2{}_t|\Omega_{t-1}) = h_{t|t-1} = \omega_o + \alpha_1\epsilon^2_{t-1} + \beta_1 h_{t-1} \tag{3-4}$$

On the other hand, $(y_t - \hat{b}_o)^2$ is clearly not uncorrelated over time to all currencies, as reflected by the significant Ljung-Box test statistic for absence of serial correlation in the square, $Q^2(10)$, which is distributed asymptotically as a χ^2_{10} distribution (see McLeod and Li, 1983). For example, when we don't use GARCH model, the D-mark against the US dollar shows $Q^2(10)=21.2$, a very significant indication of the presence of serial correlation (see Table 3). The null hypothesis of no ARCH effects can be decisively tested with the $Q^2(k)$ statistic. Some series could have the squared residuals which appear to be autocorrelated even though the residuals do not (for our example, the Swiss franc against the US dollar; see Table 2-1). This absence of serial dependence in the conditional first moments, along with the dependence in the conditional second moments, is one of the implications of the ARCH or GARCH (p,q) model given by Eqs. (3-1) to (3-4) (see Bollerslev, 1987).

With the GARCH model we estimated the parameters by the Berndt, Hall, Hall and Hausman (1974) algorithm. The maximum likelihood estimates of the parameters are presented in Tables 2-1 and 2-2 with asymptotic standard errors in parentheses. The summary of the relevant test statistics are shown in Table 3; for example, the Ljung-Box test statistic for the standardized residuals, $\hat{\epsilon}_t\hat{h}_t^{-\frac{1}{2}}$ and the standardized squared residuals, $\hat{\epsilon}_t^2\hat{h}_t^{-1}$, from the estimated GARCH (1,1) model takes the values Q(10)=6.13 and $Q^2(10)$=3.48, respectively, for the D-mark against the US dollar, which doesn't indicate any further serial dependence. On the other hand, the hypothesis of the constant conditional variance fails with $LR_{\alpha=\beta=0}$ test statistics (see Table 4), which is highly significant at any level in the corresponding asymptotic χ_2^2 distribution.[5]

As can be seen from Tables 2-1 and 2-2, the estimated values for $\alpha+\beta$ are close to 1[6] for some currencies, indicating the probable existence of an integrated GARCH, or IGARCH process; see Bollerslev (1989), Engle and Bollerslev (1986). The autoregressive term i.e. the coefficients of h_{t-1}^2 are highly significant, which tells us that changes in volatility of exchange rates have a high degree of persistence.

It is also interesting to note that the implied estimate of the conditional kurtosis[7], $3(\hat{\nu}-2)(\hat{\nu}-4)^{-1}$ is in close accordance with the sample analogue for $\hat{\epsilon}_t^4\hat{h}_t^{-2}$ (which is k in Table 3) for most currencies (see Tables

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[5] I didn't show all test results in the Table for other exchange rates, but obviously they have the same results; see each Table.

[6] The GARCH (1,1) process is wide-sense stationary iff $\alpha + \beta < 1$. See Bollerslev, T (1986) for the proof. The time series $\{X_t, t\in\mathbf{Z}\}$, with index set $\mathbf{Z}=\{0,\pm1,\pm2,\cdots\}$ is said to be wide-sense stationary or covariance stationary if

(i) $E|X_t|^2 < \infty$ for all $t\in\mathbf{Z}$,

(ii) $EX_t = m$ for all $t\in\mathbf{Z}$,

and

(iii) $\gamma_x(r,s) = \gamma_x(r+t, s+t)$ for all $r,s,t\in\mathbf{Z}$, where $\gamma_x(r,s) = Cov(X_r,X_s)$. If $\alpha + \beta \geq 1$, then it blows up and we have an explosive ARMA model (see Bollerslev (1989) for discussions about IGARCH.).

[7] From Kendall and Stuart (1969), the fourth moment is equal to $E(\epsilon_t^4 \mid \Omega_{t-1}) = 3(\nu-2)(\nu-4)^{-1}h_{t|t-1}^2, \nu>4$.

2-1 and 2-2). This means that even in the weekly data, the t-distributed GARCH (1,1) model works well[8]. This estimate of the conditional kurtosis differs significantly from the normal value of three, as seen by the $LR_{1/\nu=0}$ test for the GARCH (1,1) model with conditional normal errors with χ^2_1 distribution (see Table 4). The estimated value of each ν^{-1} is the inverse of the degree of freedom parameter (see Tables 2-1 and 2-2).

In conclusion, as expected, GARCH models worked very well for my purposes and this model is used to test the EMS currency volatility.

3.2 Tests for EMS Currency Volatility

3.2.1 Test Method

Because EMS implemented the Exchange Rate Mechanism (ERM)[9] in March 13, 1979, to test the volatility, I will designate the time period before March 13, 1979 as pre-ERM and after March 13, 1979 as post-ERM and see whether there is a difference in volatility in both periods.

To test volatility we simply could test the following null hypothesis:

H_o : Pre-ERM $\hat{\omega}_i, \hat{\alpha}_{1i}, \hat{\beta}_{1i}$ are same as those of Post-ERM in our Eq.

(3-4) $h_{ti} = \omega_i + \alpha_{1i}\epsilon^2_{t-1} + \beta_{1i}h_{t-1}$ $(i = 1,2)$.

However, if H_o is rejected, does this imply increasing volatility, decreasing volatility, or neither? We can find no distinction between

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[8]Baillie and Bollerslev (1987, 1989) have found that with weekly data the assumption of normality is generally appropriate.

[9]Presently, Belgium, Luxembourg, Denmark, France, the Federal Republic of Germany, Ireland, Italy, and the Netherlands participate in the Exchange Rate Mechanism. Great Britain, Spain, Portugal and Greece are not in the ERM, but in the EMS. Hereafter, the term ERM is used to indicate these countries or their currencies.

them. One possible way to structure our test would be to test $\hat{\omega}_i$, $\hat{\alpha}_{1i}$, and $\hat{\beta}_{1i}$, but, $\hat{\alpha}$'s and $\hat{\beta}$'s have really nothing to do with volatility levels. Therefore, $\hat{\omega}_i'$s will be used to test the change in volatility, i.e., the differences in $\hat{\omega}_i$'s.

$$y_t = b_o + \theta(L)\epsilon_t \tag{3-1}$$

$$\theta(L)\epsilon_t = \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} \tag{3-2}$$

$$\epsilon_t \mid \Omega_{t-1} \sim D(0,h_t) \tag{3-3}$$

$$h_t = \omega_o + \omega_1 D_t + \alpha_1\epsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{3-4'}$$

$$\text{where } D_t \begin{cases} = 1 \text{ if post-ERM} \\ = 0 \text{ if pre-ERM} \end{cases}$$

$$H_o : \omega_1 = 0$$

3.2.2 Test Results of ERM Currency Volatility

First, the nominal exchange rate volatilities were estimated against intra-ERM (D-mark against Lira, D-mark against French franc, and D-mark against the Netherlands guilder). The existence of the ERM since 1979 has coincided with a marked reduction in the volatility of exchange rates within the ERM. This was one major goal of the system, and to this end, the intervention arrangement and other elements of the exchange rate mechanism were established (see Table 2-1, Column 1 to 3). However, in terms of the nominal volatility against the US dollar, the ERM currency volatility increased during the ERM period. It is statistically significant at the 5% level for the cases which I have studied with the D-mark, Danish krone, the Netherlands guilder and Belgian franc against the US dollar (see Table 2-2). To compare the volatility level change between ERM currencies with that between non-ERM currencies, I have estimated the volatility of the Canadian dollar, Swiss franc, and Pound sterling against the US dollar, which showed an increase in nominal volatility during the ERM period in each case (see Table 2-1, last three columns and Table 5 for the summary of t statistics). Figures 1, 3 and 4 confirm these changes, showing the residual movements in our model. Figure 1 shows that after the ERM system there was a decrease in the volatility in the case of intra-ERM currencies, while Figure 3 shows an increase in ERM currency volatilities against the US dollar. Figure 4 reveals an increase in volatility between non-ERM currencies after March, 1979.

In the previous case we checked the volatility level changes against two key currencies, the US dollar and the D-mark, which are both major reserve currencies and transaction currencies. In addition, we used the D-mark, because West Germany is the leading country in the ERM. However, due to those factors, the measure of the exchange rate volatility might be distorted. To eliminate this problem we used the Pound sterling instead of the US dollar and the Italian lira instead of the D-mark as base currencies and tested the significance of the change in the level of the volatility again. The results are shown in Table 6.

As expected, in the case of intra-ERM currencies (the French franc and the Netherlands guilder against the Italian lira), there were significant decreases in the volatility after March, 1979 (see the first two columns in Table 6). But in the case of the ERM currencies, (Italian lira and Netherlands guilder) against the non-ERM currencies (Pound sterling), the Netherlands guilder, which showed an increase in volatility after March, 1979 when it is measured against the US dollar, showed at least constant volatility after March, 1979 (see Table 6, fourth column). Also, between non-ERM currency volatility, the Swiss franc, which showed an increase in volatility against the US dollar, accepts the null hypothesis that there was no change in the volatility even after March, 1979 (see Table 6 column 5). These results are confirmed in Figures 2, 5, and 6. Figures 5a and 6b imply the constant volatility movements.

As we suspected that United Kingdom might try to stabilize her currency volatility against the other ERM currencies, as they are her neighbors, we tested the non-ERM currency volatility against the Japanese yen and Canadian dollar as base currencies. Table 7 shows that the Yen/guilder and Yen/Sfr had at least constant volatilities again, and we can confirm our results.

The clear diminution of exchange-rate volatility in the case of intra-ERM is certainly consistent with the view that the system has been successful in contributing to exchange-rate stability among participating countries. However, as is shown in Tables 2-1 and 2-2, in the exchange rate volatilities against the US dollar the volatility of the ERM currencies showed different patterns under their exchange-rate mechanisms from those of the non-ERM currencies. Hence, we can say that decreasing volatility of the intra-ERM does closely follow the increasing volatility against the US dollar. This was already noted by Cohen (1981), who said that "--- effort to maintain the joint float could increase the volatility of fluctuations between participating and non-participating currencies---" (see p. 14). It appears that such effort may do so at the cost of increased instability of exchange rates against the US dollar.[10]

Even though there was a significant reduction in volatility after joining the ERM and although the study as a whole suggest fairly distinct patterns to the results, no strong conclusions as to cause or effect can be drawn. For example, it is impossible to say how far the reduced volatility among ERM currencies is due to the operations of the ERM itself. In addition, even if the coincident fall in volatility among ERM versus non-ERM currencies and the constant volatility among the non-ERM currencies are not a reliable indication of the way in which the ERM rates would have behaved in the absence of the system, it does nevertheless somewhat weaken the claim that the reduction in the volatility of inter-ERM rates is due to the creation of the system alone.

4. The Multivariate Generalized ARCH Approach

In previous sections we estimated the univariate GARCH models, and they offered good statistical descriptions of exchange rate movements. However, they are not satisfying compared to a multivariate model because the multivariate approach gives some advantages for the following reasons:

1) Nonzero covariances among exchange rate innovations require joint estimation of sets of regression if efficient estimation in parameters is to be achieved.

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[10] Also Marston (1980) says that the volatility of the dollar exchange rate of that member country disturbs economic relationships between the two members of the union by changing cross-exchange rates between member currencies.

2) Exchange rates are bilateral rates, and if new information comes to the foreign exchange market (e.g., the US money supply, the US budget deficit, the German trade surplus, etc.), it should affect all rates as market dealers change their demands of specific currency and it affects their portfolios.

The multivariate ARCH (q) model was originally introduced in Engle and Kraft (1982), and then used by Diebold and Nerlove (1986). Later it was generalized by Bollerslev, Engle and Wooldridge (1988). Baillie and Bollerslev (1987) modelled risk premia in forward exchange-rate markets with a multivariate GARCH approach.

4.1 Estimation of Multivariate GARCH Models

In order to deal with the multivariate GARCH (1,1) model, the following SUR system is estimated for the set of N currencies:

$$y_t = b_o + \epsilon_t \tag{4-1}$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, H_t) \tag{4-2}$$

$$Vech(H_t) = C_o + C_1 D_{1t} + A_1 Vech(\epsilon_{t-1}\epsilon'_{t-1}) + B_1 Vech(H_{t-1}), \tag{4-3}$$

where y_t is vector of first-differenced N currencies and b_o and ϵ_t are Nx1 vector of constants and innovation vector. The Vech (.) denotes the column-stacking operator of the lower portion of a symmetric matrix. C_o and C_1 are N(N+1)/2 x 1 vector, and A_1 and B_1 are N(N+1)/2 x N(N+1)/2 matrices.

The conditional log likelihood function for (4-1)-(4-3) for the single time period t can be expressed as

$$L_t(\theta) = -N/2 \log 2\pi - 1/2 \log|H_t(\theta)| - 1/2\ \epsilon_t(\theta)' H_t^{-1}(\theta) \epsilon_t(\theta),$$

where all the parameters have been combined into $\theta' = (b'_o, C'_o, C'_1, Vec(A_1)', Vec(B_1)')$. Thus, conditional on the initial values, the log likelihood function for the sample 1,2,------,T is given by

$$L(\theta) = \sum_{t=1}^{T} L_t(\theta).$$

As is obvious from univariate case, the log likelihood function $L(\theta)$ depends on the parameter θ in a nonlinear form, and the maximization of $L(\theta)$ requires iterative methods.

While the multivariate GARCH (1,1) of manageable size is considered here, a natural simplification is to assume that each covariance depends only on its own past values. We restrict our attention to two currencies, the Italian lira and the Netherlands guilder, first against the Deutsche mark and then against other cross currencies. Weekly data from the FRB tape are used, as in the univariate GARCH models.

The model considered here becomes the bivariate GARCH model

$$y_{it} = b_i + \epsilon_{it} \qquad (4\text{-}1)'$$

$$\epsilon_t | \Omega_{t-1} \sim N(0, H_t) \qquad (4\text{-}2)'$$

$$h_{ijt} = C_{oij} + C_{1ij}D_{1t} + \alpha_{ij}\epsilon_{i\,t-1}\epsilon_{jt-1} + \beta_{ij}h_{ijt-1} \quad i,j = 1,2 \qquad (4\text{-}3)'$$

where subscript i refers to the ith elements of the corresponding vector and ij to the ijth element of the corresponding matrix.

4.2 Model Estimates

The maximum likelihood estimates of the model obtained by the BHHH (1974) algorithm for the case of the Netherlands guilder (y_{1t}) and the Italian lira (y_{2t}) against the Deutsche mark are:[11]

$$\begin{bmatrix} y_{1t} \\ \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \underset{(0.06)}{-0.001} \\ \\ \underset{(0.22)}{0.61^{**}} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \\ \epsilon_{2t} \end{bmatrix} \tag{4-4a}$$

$$\begin{bmatrix} h_{11t} \\ h_{21t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} \underset{(0.08)}{0.31^{**}} \\ \underset{(1.17)}{3.15^{**}} \\ \underset{(2.03)}{16.49^{**}} \end{bmatrix} + \begin{bmatrix} \underset{(0.01)}{0.05^{**}} & \epsilon^2_{1\,t-1} \\ \underset{(0.02)}{0.104^{**}} & \epsilon_{1\,t-1}\epsilon_{2\,t-1} \\ \underset{(0.03)}{0.31^{**}} & \epsilon^2_{2\,t-1} \end{bmatrix} + \begin{bmatrix} \underset{(0.05)}{0.93^{**}} & h_{11\,t-1} \\ \underset{(0.05)}{0.73^{**}} & h_{12\,t-1} \\ \underset{(0.03)}{0.61^{**}} & h_{22\,t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} \underset{(0.08)}{-0.29^{**}} \\ \underset{(1.13)}{-2.91^{**}} \\ \underset{(1.79)}{-11.59^{**}} \end{bmatrix} D_{1t} \tag{4-4b}$$

log likelihood function = -3970.3668.

In the case of the Netherlands guilder against the D-mark, in the conditional covariance equation of the h_{11t}, 0.31 is the intercept, α = 0.05, and β = 0.93, with -0.29 as the intercept change after March 1979. The h_{21t} is for the conditional covariance of the Netherlands guilder against the Italian lira, and the significant value of the coefficient of D_{1t}(i.e., -2.91) shows the decreasing volatility after ERM as already

[11] Hereafter, * indicates significance at the 5% level and ** at the 1% level. Asymptotic standard errors are in parentheses under corresponding parameter estimates.

verified in the univariate case (see Table 6). The h_{22t} is for the conditional covariance of the Italian lira against the D-mark, and the significant value of the coefficient D_{1t}(i.e., -11.59) also shows the decreasing volatility after joining the ERM.

The estimates for the model are appealing, and the estimated value for each coefficient is reasonable and highly significant, lending some support for the arguments that time series for exchange rates works well under the GARCH model and that the intra-ERM currencies show decreasing volatilities after participating in the ERM system. However, the likelihood ratio between the univariate GARCH (-2758.883 for DM-LIRA and -2017.937 for DM-Netherlands guilder) and the multivariate GARCH (-3970.3668) implies that the multivariate GARCH is more efficient than the univariate model. Compared with the univariate model, significant coefficients of α and β are achieved in the case of Guilder/Lira.

I estimated the Lira and the Netherlands guilder against the Japanese yen as one test of the change in volatility of the ERM against the non-ERM. In the univariate GARCH model, the decreasing volatility was shown at the 5% significant level in the case of the Yen-Lira and at least no change in volatility in the case of the Yen-Netherlands guilder (see Table 7). Even in the case of the ERM currency against the non-ERM currency, there was at least constant volatility after March 13, 1979 and demonstrated that the reduction in the volatility of ERM currency could result even in ERM vs. non-ERM cases. The following multivariate GARCH estimates assured such a claim. The volatility of the Yen-Lira (y_{1t}) is decreased after March, 1979 with a 5% significant level and the volatility of the Yen-Guilder (y_{2t}) is shown to be at least constant. Here, again, the multivariate model becomes more efficient than univariate models when we consider their likelihood functions. Furthermore, the constant term in the case of Yen/Guilder [see Eq. (4-5a)] shows significance at the 5% level, which was not significant in the univariate case.

$$\begin{bmatrix} y_{1t} \\ \\ y_{2t} \end{bmatrix} = \begin{bmatrix} 1.389^{**} \\ (0.403) \\ 0.604^{*} \\ (0.367) \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \\ \epsilon_{2t} \end{bmatrix} \qquad (4\text{-}5a)$$

$$\begin{bmatrix} h_{11t} \\ \\ h_{12t} \\ \\ h_{22t} \end{bmatrix} = \begin{bmatrix} 30.63^{**} \\ (2.82) \\ 19.89^{**} \\ (2.65) \\ 21.41^{**} \\ (3.71) \end{bmatrix} + \begin{bmatrix} 0.205^{**} & \epsilon^2_{1\ t-1} \\ (0.02) & \\ 0.196 & \epsilon_{1\ t-1}\epsilon_{2\ t-1} \\ (0.02) & \\ 0.186^{**} & \epsilon^2_{2\ t-1} \\ (0.02) & \end{bmatrix} + \begin{bmatrix} 0.67^{**} & h_{11\ t-1} \\ (0.02) & \\ 0.67^{**} & h_{12\ t-1} \\ (0.02) & \\ 0.701^{**} & h_{22\ t-1} \\ (0.025) & \end{bmatrix}$$

$$+ \begin{bmatrix} -5.65^{*} \\ (2.89) \\ 2.72 \\ (3.14) \\ -0.45 \\ (3.43) \end{bmatrix} D_{1t} \qquad (4\text{-}5b)$$

log likelihood function = -5170.6051

Lastly, the volatilities of the Canadian dollar and the Swiss franc against the Japanese yen were estimated to see whether among non-ERM currencies they have increased volatilities after March, 1979. The following multivariate estimates show that, in the case of the Swiss franc against the Yen, there was a decreasing volatility after March, 1979 at the 5% significant level. In the case of univariate estimates, it showed at least constant volatility (see Table 7). In Eqs. (4-6a) and (4-6b),

y_{1t} denotes the first-differenced Canadian dollar/Yen, and y_{2t} denotes the first-differenced Yen/Swiss franc.

$$\begin{bmatrix} y_{1t} \\ \\ y_{2t} \end{bmatrix} = \begin{bmatrix} -0.91^{*} \\ (0.47) \\ \\ 0.61 \\ (0.41) \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \\ \epsilon_{2t} \end{bmatrix} \tag{4-6a}$$

$$\begin{bmatrix} h_{11t} \\ \\ h_{12t} \\ \\ h_{22t} \end{bmatrix} = \begin{bmatrix} 9.62^{**} \\ (1.89) \\ -8.05^{**} \\ (2.52) \\ 16.01^{**} \\ (3.97) \end{bmatrix} + \begin{bmatrix} 0.08^{**}\ \epsilon_{1\ t-1}^{2} \\ (0.01) \\ 0.09^{**}\ \epsilon_{1\ t-1}\ \epsilon_{2\ t-2} \\ (0.02) \\ 0.16^{**}\ \epsilon_{2\ t-1}^{2} \\ (0.03) \end{bmatrix} + \begin{bmatrix} 0.86^{**}\ h_{11\ t-1} \\ (0.01) \\ 0.80^{**}\ h_{12\ t-1} \\ (0.04) \\ 0.77^{**}\ h_{22\ t-1} \\ (0.03) \end{bmatrix}$$

$$+ \begin{bmatrix} 5.30^{**} \\ (1.43) \\ 1.79 \\ (1.88) \\ -5.24^{*} \\ (2.84) \end{bmatrix} D_{1t} \tag{4-6b}$$

log likelihood function = -5844.3448

As we have seen, the estimates of multivariate GARCH models are efficient relative to univariate GARCH estimates, and it is important to have simultaneous multivariate estimation for the reasons mentioned. However, the magnitude and sign of the coefficients which were used to test for the change in the volatility after joining in the ERM did not vary with the multivariate estimates.

5. Conclusion

The European Monetary System was established on March 13, 1979. After ten years, the time is ripe to evaluate this scheme and consider its possible future contribution to European and world-wide monetary relations as well as to European integration.

I have empirically studied the after-EMS currency volatilities and demonstrated them with multivariate GARCH models as well as with univariate GARCH models. Although the intra-EMS showed stable volatility after March, 1979, one can not say that these stable exchange volatilities result from the system itself, because we have found that even in some ERM vs. non-ERM cases, as well as among non-ERM currency cases, there existed at least constant volatilities. Furthermore, decreasing volatility of intra-ERM closely follows the increasing volatility against the US dollar, and an effort to maintain the joint float increases the volatility of fluctuations between participating currencies and the US dollar. Proposals for policy coordination among the major industrial economies have been discussed in recent years. But if such proposals utilize the successful EMS-member coordination for stable exchange rates, they should be considered carefully, because our experience indicates it is not used without cost.

Table 1

Phillips-Perron Unit Root Tests on Exchange Rate Series

$$y_t = \tilde{\mu} + \tilde{\beta}(t-n/2) + \tilde{\alpha} y_{t-1} + u_t$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u^*_t \ , \quad y_t = \hat{\alpha} y_{t-1} + \hat{u}_t$$

$Z(\Phi_2)$: $\tilde{\mu} = 0$ $\tilde{\beta} = 0$ $\tilde{\alpha} = 1$

$Z(\Phi_3)$: $\tilde{\beta} = 0$ and $\tilde{\alpha} = 1$, $Z(t_{\hat{\alpha}})$: $\hat{\alpha} = 1$

$Z(t_{\tilde{\alpha}})$: $\tilde{\alpha} = 1$, $Z(\Phi_1)$: $\mu^* = 0$ and $\alpha^* = 1$, $Z(t_{\alpha^*})$: $\alpha^* = 1$

Lag = 10 in Newey and West(1987)

Currencies (against US$)	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_1)$	$Z(t^*_{\alpha})$	$Z(t_{\hat{\alpha}})$
Canadian $	0.8985	1.1039	-0.4762+	1.6543	-1.3388	-1.2617
Pound Sterling	1.1994	0.9803	-1.3970	1.3142	-1.4455	-0.7816
Irish Pound	0.9044	1.0334	-1.0969	1.4514	-1.2731	-1.1898
Italian lira	0.9932	1.9399	-0.8229	2.8737	-1.3907	-1.7145
French franc	1.0638	0.7877	-1.4448	0.5014	-0.8800	-0.5495
Belgian franc	1.1551	0.7876	-1.4757	0.7641	-1.2138	-0.0503
Danish krone	1.0127	0.6799	-1.4258	0.5445	-1.0358	-0.2049
Deutsche mark	1.8170	1.5909	-1.8384	2.3862	-1.9050	0.9654
Dutch guilder	1.4877	1.2487	-1.6841	1.8510	-1.7106	0.7888
Swedish krona	1.0141	0.8579	-1.4159	0.5575	-0.7591	-0.7956
Aust.schilling	1.8367	1.6774	-1.8514	2.5031	-1.9090	0.9662
Swiss franc	2.6288	2.5395	-2.1850	3.5574	-2.1850	1.3835+
Japanese yen	1.3073	1.7751	-1.3598	1.3454	-0.1924	-1.6233
Norweg.krone	3.1458	2.1024	-2.3415	0.5861	-1.0746	-0.1906
weighted-US dollar	0.8523	0.5976	-1.2980	0.8362	-1.2577	-0.3342

Key: * indicates significance at the .01 percentile and ** at .05 percentile
+ indicates significance at the .95 percentile and ++ at .99 percentile

Note: Under the null hypothesis, the 95% and 99% critical values of $Z(t_{\tilde{\alpha}})$, $Z(\Phi_3)$ and $Z(\Phi_2)$ are -.94 and -.33, 4.68 and 6.09, and 6.25 and 8.27, respectively, and values for $Z(t_{\tilde{\alpha}})$ are -3.96 and -3.441 at 1% and 5%. Also, at the 95% and 99% level the critical values of $Z(t_{\hat{\alpha}})$, $Z(t^*_{\alpha})$ and $Z(\Phi_1)$ are 1.28 and 2.00, -0.07 and 0.6, and 4.59 and 6.43. For $Z(t_{\hat{\alpha}})$ and $Z(t^*_{\alpha})$, values are -2.58 and -1.95, -3.43 and -2.86 at 1% and 5%, respectively [see Phillips and Perron (1986)].

Table 2-1

Estimation of GARCH Models with D-mark and US dollar as Base Currency

$$y_t = b_o + u_t \ ; \ u_t = \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} \ ; \ \epsilon_t | \Omega_{t-1} \sim D(0, h_t, \nu);$$
$$h_t = \omega_o + \omega_1 D_t + \alpha_1\epsilon^2_{t-1} + \beta_1 h_{t-1}$$

Parameters & Diagnostic Statistics	INTRA-EMS			NON-EMS		
	DM-LIRA	DM-NGL	DM-FFR	US\$-CN\$	US\$-SFR	US\$-UK£
Log L	-2758.883	-2017.937	-2684.132	-2452.528	-3482.335	-3297.485
b_o	0.607**	0.047	0.816*	-0.209	0.641	-0.671
	(0.238)	(0.062)	(0.382)	(0.169)	(0.608)	(0.567)
θ_1	---	---	0.219**	0.068*	0.069*	0.009
			(0.048)	(0.037)	(0.035)	(0.037)
θ_2	---	---	0.147**	0.117**	0.106**	0.078*
			(0.035)	(0.036)	(0.026)	(0.037)
ω_o	19.397**	0.222**	66.990**	2.263**	156.463**	15.239**
	(2.182)	(0.005)	(2.799)	(0.902)	(21.779)	(3.481)
ω_1	-14.580**	-0.209**	-40.981**	3.385**	140.164**	14.626**
	(1.722)	(0.058)	(2.739)	(1.430)	(39.797)	(5.437)
α_1	0.266**	0.048**	0.274**	0.169**	0.311**	0.109**
	(0.033)	(0.005)	(0.047)	(0.045)	(0.080)	(0.023)
β_1	0.631**	0.947**	---	0.694**	---	0.768**
	(0.033)	(0.004)		(0.079)		(0.047)
ν^{-1}	normal	normal	normal	0.201**	0.212**	normal
				(0.007)	(0.056)	
m_3	1.996	0.597	4.754	-0.868	0.240	-0.292
m_4	13.109	8.941	51.352	8.788	5.583	6.561
Q(10)	12.404	10.296	17.116	12.631	6.209	12.666
$Q^2(10)$	18.750	4.571	0.382	8.582	141.254	3.714
$3(\hat{\nu}-2)/(\hat{\nu}-4)$	N.A.	N.A.	N.A.	9.0	11.36	N.A.

Note:
1. Asymptotic standard errors are in parentheses under corresponding parameter estimates.
2. * indicates significance at the 5 % level and ** at the 1 % level.
3. NGL stands for the Netherlands guilder, CN\$ stands for the Canadian dollar, SFR for the Swiss franc, and FFR for the French franc.
4. ν denotes the degree of freedom with student t density.

Table 2-2

Estimation of GARCH Models with EMS Currencies against the US Dollar

Parameters & Diagnostics Statistics	US$-DM	US$-DKR	US$-NGL	US$-BFR
Log L	-3316.977	-3309.170	-3287.396	-3287.438
b_o	0.543	-18.033**	0.422	0.216
	(0.515)	(0.513)	(0.452)	(0.536)
θ_1	0.080*	0.081*	0.078*	0.084**
	(0.036)	(0.038)	(0.037)	(0.034)
θ_2	0.152**	0.119**	0.150**	0.140**
	(0.035)	(0.035)	(0.035)	(0.035)
ω_o	7.066**	12.020**	6.124**	5.773*
	(0.168)	(4.883)	(2.819)	(2.551)
ω_1	12.572*	18.715*	10.754*	12.079*
	(5.742)	(8.348)	(4.996)	(5.546)
α_1	0.168**	0.185**	0.156**	0.153**
	(0.039)	(0.045)	(0.034)	(0.036)
β_1	0.782**	0.726**	0.793**	0.798**
	(0.044)	(0.064)	(0.042)	(0.040)
ν^{-1}	0.159**	0.169**	0.123**	0.152**
	(0.032)	(0.031)	(0.020)	(0.013)
m_3	0.314	-0.067	0.349	0.321
m_4	6.093	4.190	6.257	5.947
Q(10)	6.129	4.190	4.903	5.861
$Q^2(10)$	3.484	4.453	4.903	3.290
$3(\hat{\nu}-2)/(\hat{\nu}-4)$	5.650	6.115	4.464	5.319

Note: DKR stands for the Danish krone and BFR stands for the Belgian franc.

Table 3

Summary Statistics with the Implication of GARCH(1,1) Model

	$y_t - \hat{b}_o$			GARCH(1,1)-t		
	Q(10)	$Q^2(10)$	k	Q(10)	$Q^2(10)$	k
EMS CURRENCY						
US$-DM	8.52	21.21	11.72	6.13	3.48	6.09
US$-DKR	8.45	34.52	10.77	4.19	4.46	5.21
US$-NGL	9.93	33.43	13.01	6.26	4.90	5.12
US$-BFR	7.64	29.45	12.39	5.86	3.29	5.95
NON-EMS CURRENCY						
US$-CN$	18.84	27.98	17.28	12.63	8.58	8.79
US$-SFR	3.03	145.52	6.20	6.21	141.25	5.58
UK£-US$	10.94	38.43	6.07	12.67	3.71	6.56

Note: Q(10) and $Q^2(10)$ denote the Ljung-Box (1978) portmanteau tests for up to tenth-order serial correlation in the levels and the squares which are standardized, respectively. They have χ^2 distributions with a degree of freedom of 10, which has values of 15.987 and 18.307 with p= 0.10 and 0.05, respectively. k is the usual measure of kurtosis given by the fourth sample moment divided by the square of the 2nd moment.

Table 4

Likelihood Ratio Tests

	US$-DM	US$-NGL	US$-DKR	US$-BFR	US$-CN$	US$-SFR	US$-UK£
$LR_{1/\nu=0}$	46.334	39.112	28.578	41.994	77.694	56.966	---
$LR_{\alpha=\beta=0}$	88.410	68.994	93.428	97.472	81.936	105.728	104.758

Note : For our reference, $\chi^2_1 = 6.638$ (with P=0.01) and $\chi^2_2 = 9.210$ (with P=0.01).

Table 5

Summary of T-Statistics for Shift in Volatility after March.1979

$$y_t = b_o + u_t \; ; \; u_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} ; \; \epsilon_t | \Omega_{t-1} \sim D(0, h_t, \nu);$$

$$h_t = \omega_o + \omega_1 D_t + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1} \text{ ,where } D_t = 1 \text{ if Post-ERM}$$
$$= 0 \text{ if Pre-ERM}$$

$$H_o : \omega_1 = 0$$

	ω_1	S.E.	t-statistic
INTRA-ERM			
DM-LIRA	-14.580	1.722	-8.467
DM-NGL	-0.209	0.058	-3.603
DM-FFR	-40.981	2.739	-14.962
NON-ERM			
US$-CN$	3.385	1.430	2.367
US$-SFR	140.164	39.797	3.522
US$-UK	14.626	5.437	2.690
ERM VS US$			
US$-DM	12.572	5.742	2.190
US$-DKR	18.715	8.348	2.242
US$-NGL	10.754	4.996	2.153
US$-BFR	12.079	5.546	2.178

Note: $t_{0.05} = 1.645$, $t_{0.025} = 1.960$, and $t_{0.01} = 2.326$.

Table 6

Estimation of GARCH Models with the Italian Lira and the Pound Sterling as Base Currency

$$y_t = b_o + u_t \;;\; u_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} ;\; \epsilon_t | \Omega_{t-1} \sim D(0, h_t, \nu) \;;$$

$$h_t = \omega_o + \omega_1 D_t + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1}$$

Parameters & Diagnosics Statistics	ERM				NON-ERM	
	INTRA-ERM		ERM VS NON-ERM			
	FFR-LIRA	NGL-LIRA	UK£-LIRA	UK£-NGL	UK£-SFR	UK£-CN$
Log L	-2773.098	-2775.547	-3173.470	-3162.507	-3331.852	-3258.569
b_o	0.161	0.934**	0.716*	-0.785*	-1.214**	-0.134
	(0.220)	(0.178)	(0.389)	(0.449)	(0.509)	(0.395)
θ_1	---	---	0.002	0.092**	0.044	0.0
			(0.039)	(0.039)	(0.038)	(0.037)
θ_2	---	---	0.006	0.019	0.033	0.067*
			(0.037)	(0.036)	(0.039)	(0.036)
ω_o	86.453**	84.280**	9.160**	9.818**	33.748**	8.985**
	(3.368)	(3.945)	(1.423)	(2.642)	(6.909)	(3.819)
ω_1	-64.247**	-63.423**	4.513**	-1.009	-0.681	21.505**
	(3.426)	(3.704)	(1.472)	(1.042)	(3.845)	(9.042)
α_1	0.392**	0.532**	0.145**	0.081**	0.201**	0.152**
	(0.036)	(0.056)	(0.018)	(0.018)	(0.037)	(0.064)
β_1	---	---	0.781**	0.850**	0.646**	0.755**
			(0.024)	(0.034)	(0.058)	(0.064)
ν^{-1}	normal	normal	normal	normal	normal	0.185**
						(0.001)
m_3	-1.584	7.852	0.438	-0.592	-0.638	-0.936
m_4	26.800	85.781	7.414	6.129	4.903	11.186
Q(10)	11.792	8.696	13.275	9.754	7.028	28.704
$Q^2(10)$	3.554	8.499	15.476	2.312	2.462	1.348
$3(\hat{\nu}-2)/(\hat{\nu}-4)$	N.A.	N.A.	N.A.	N.A.	N.A.	7.271

Note: 1. Asymptotic standard errors are in parentheses under corresponding parameter estimates.
2. * indicates significance at 5 % level and ** at 1 % level.

Table 7

Estimation of GARCH Models with the Japanese Yen and the Canadian Dollar as Base Currency

$$y_t = b_o + u_t \ ; \ u_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} ; \ \epsilon_t | \Omega_{t-1} \sim D(0, h_t, \nu) \ ;$$

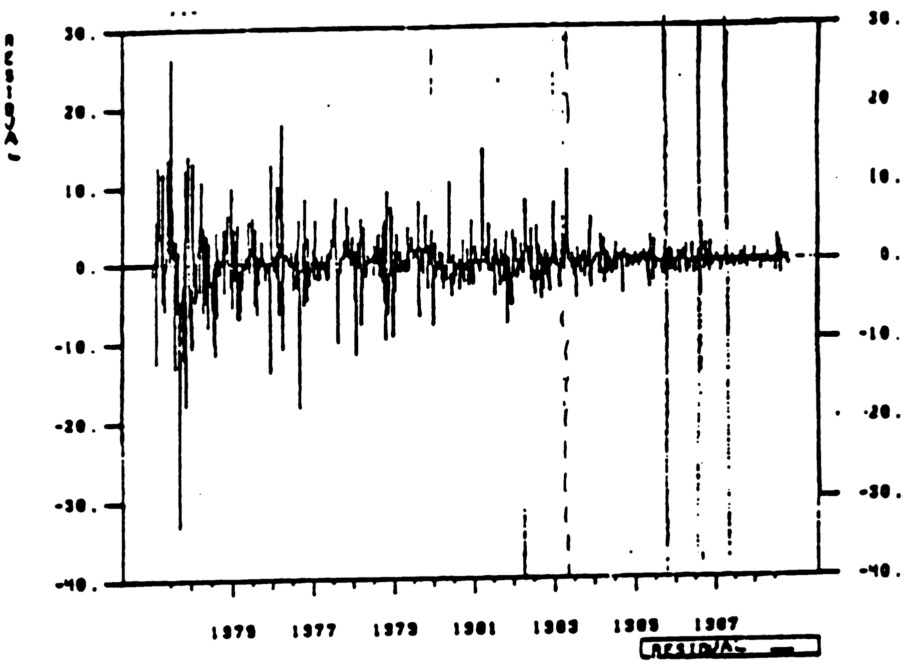
$$h_t = \omega_o + \omega_1 D_t + \alpha_1 \epsilon^2_{t-1} + \beta_1 h_{t-1}$$

Parameters & Diagnosic Statistics	ERM VS NON-ERM			NON-ERM VS NON-ERM	
	YEN-LIRA	YEN-NGL	CAN$-NGL	CAN$-YEN	YEN-SFR
Log L	-3286.487	-3205.630	-3297.810	-3383.972	-3256.098
b_o	1.929**	0.211	-0.625	-0.931*	0.355
	(0.420)	(0.477)	(0.517)	(0.540)	(0.393)
θ_1	---	0.077*	0.090**	0.070*	---
		(0.036)	(0.037)	(0.034)	
θ_2	---	0.097**	0.137**	0.069*	---
		(0.035)	(0.036)	(0.038)	
ω_o	15.829**	9.516*	12.176**	1.826**	12.911**
	(4.261)	(4.548)	(5.528)	(0.458)	(5.138)
ω_1	-3.669*	3.170	11.721*	2.523**	-0.743
	(2.099)	(3.010)	(6.171)	(0.564)	(3.915)
α_1	0.101**	0.109**	0.148**	0.041**	0.195**
	(0.016)	(0.034)	(0.034)	(0.004)	(0.044)
β_1	0.824**	0.821**	0.761**	0.944**	0.755**
	(0.031)	(0.049)	(0.059)	(0.004)	(0.047)
ν^{-1}	normal	0.158**	0.110**	normal	0.158**
		(0.031)	(0.022)		(0.035)
m_3	1.434	0.574	-0.156	-0.965	0.243
m_4	12.459	6.101	4.607	11.057	5.110
Q(10)	8.312	6.345	8.860	9.945	9.892
$Q^2(10)$	6.840	4.507	8.553	2.123	6.303
$3(\hat{\nu}-2)/(\hat{\nu}-4)$	N.A.	5.576	4.178	N.A.	5.576

Note: 1. Asymptotic standard errors are in parentheses under corresponding parameter estimates.
2. * indicates significance at the 5 % level and ** at the 1 % level.

Figure 1

A. Exchange Rate Volatility of Netherlands guilder against the Deutsche mark



B. Exchange Rate Volatility of Belgian franc against the Deutsche mark

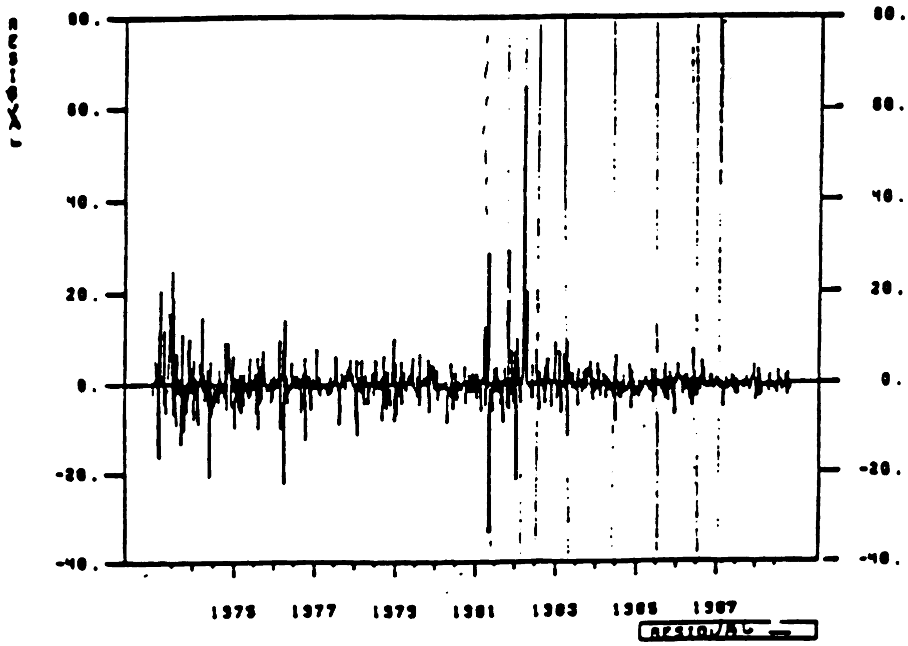
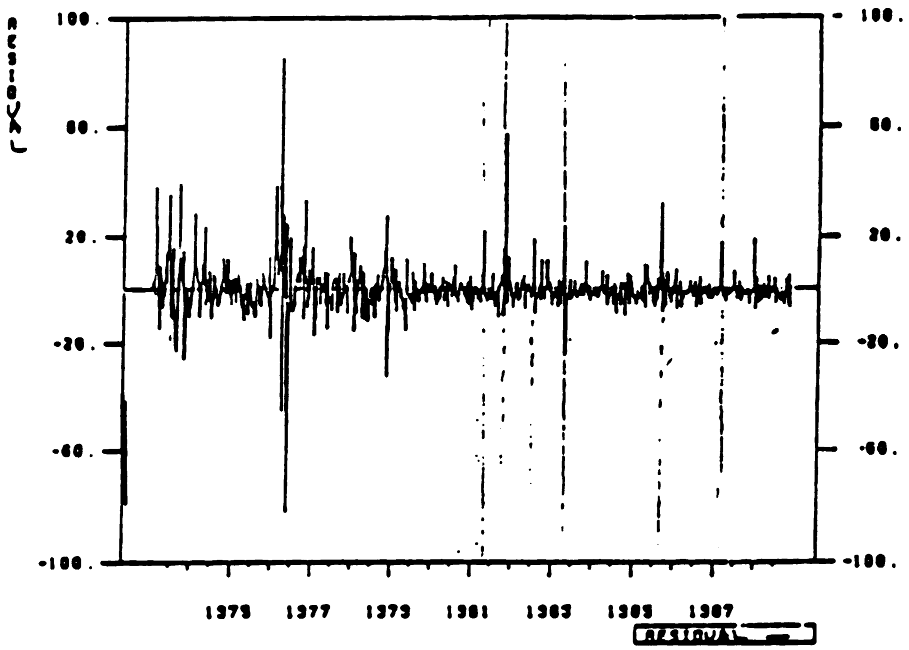


Figure 2

A. Exchange Rate Volatility of Netherlands guilder against the Italian lira



B. Exchange Rate Volatility of Belgian franc against the Italian lira

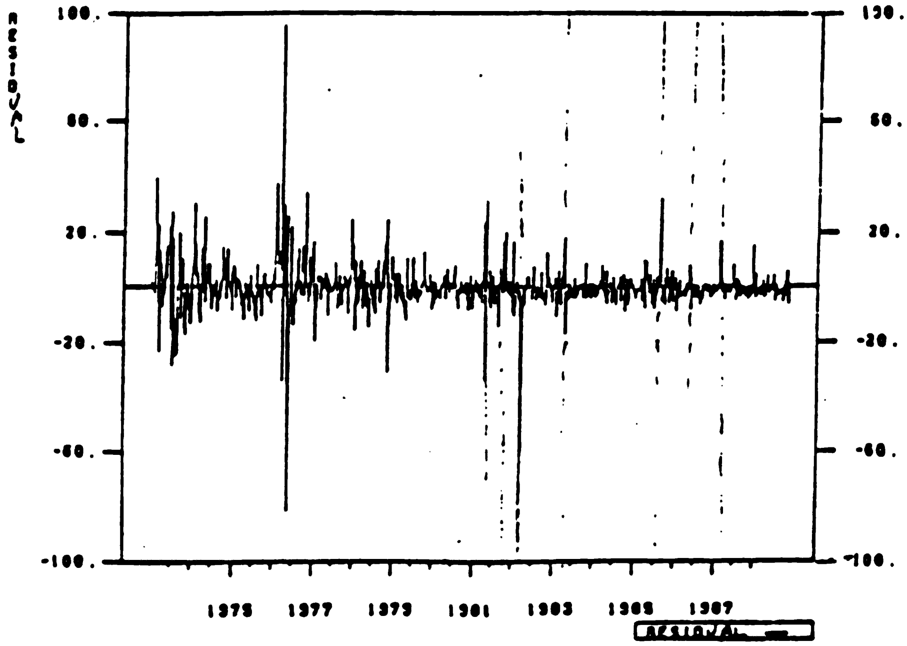
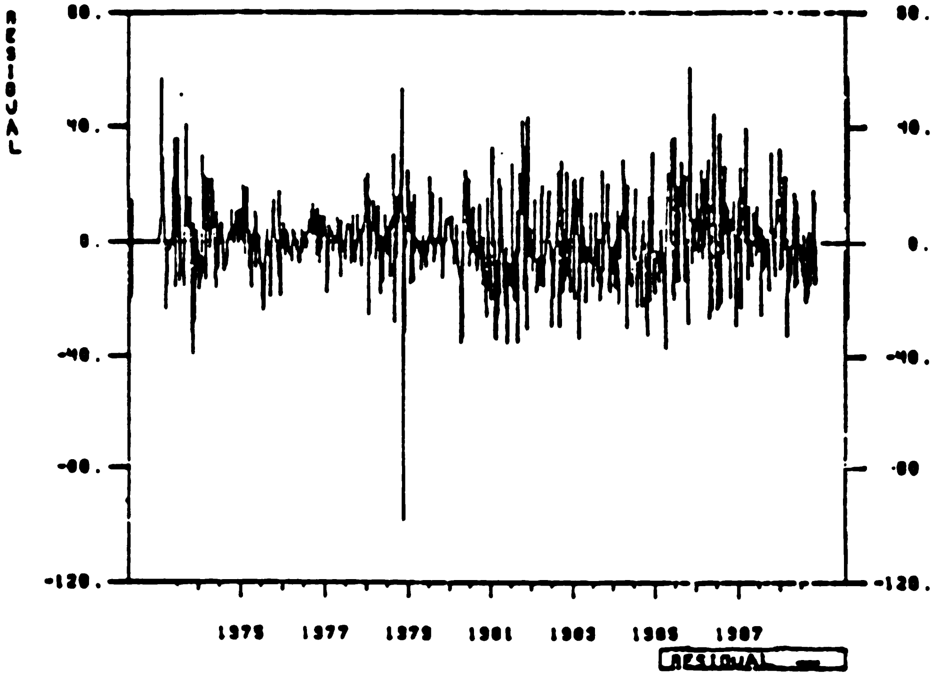


Figure 3

A. Exchange Rate Volatility of Netherlands guilder against the US dollar



B. Exchange Rate Volatility of Deutsche mark against the US dollar

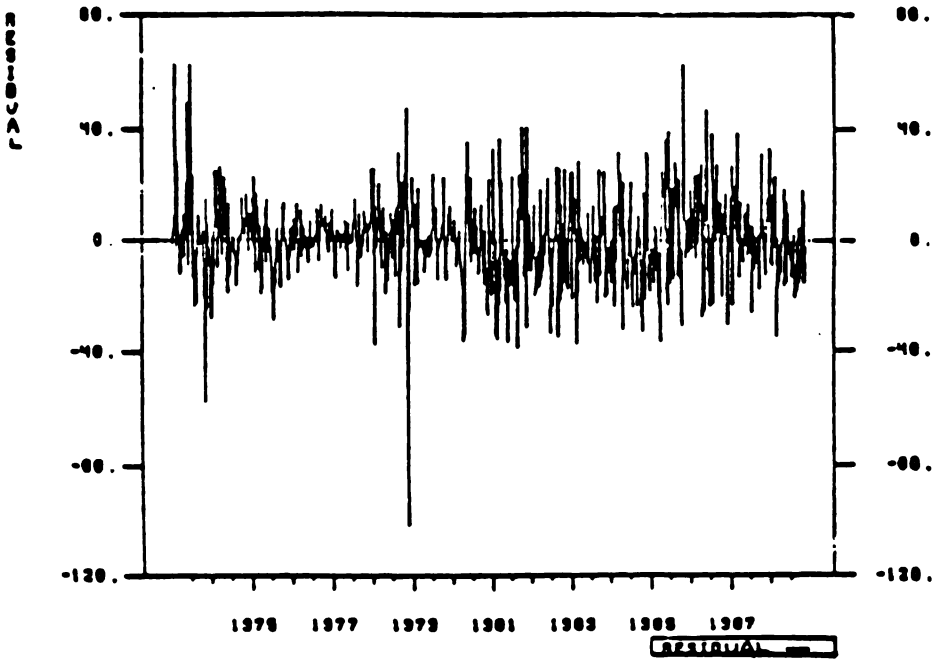
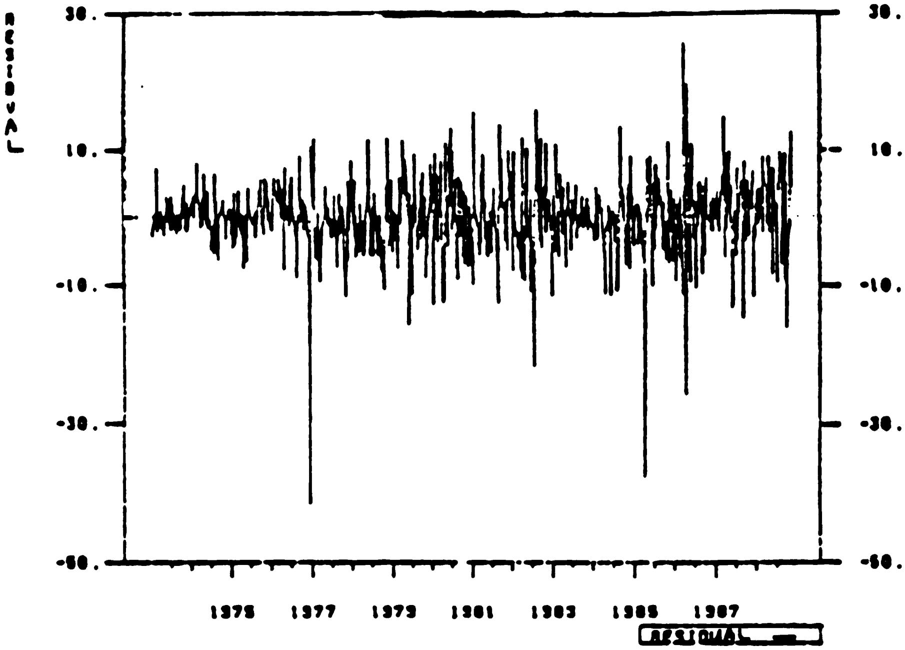


Figure 4

A. Exchange Rate Volatility of Canadian dollar against the US dollar



B. Exchange Rate Volatility of Swiss franc against the US dollar

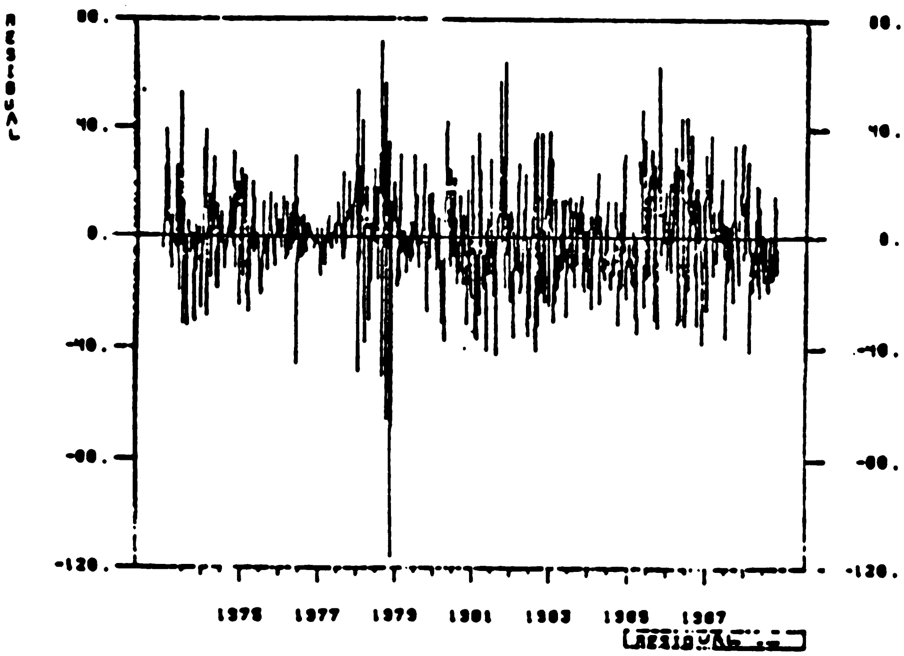
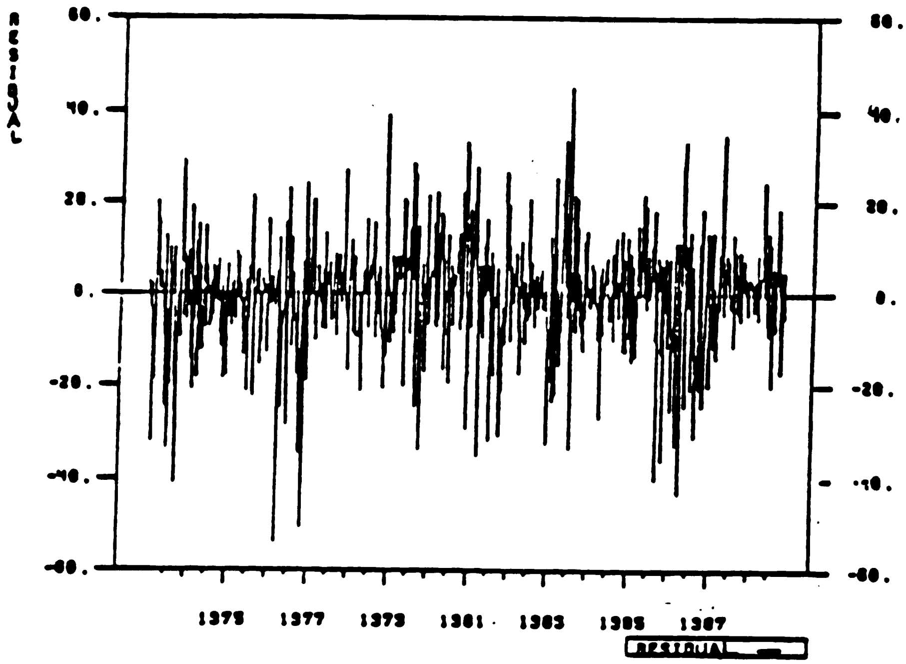


Figure 5

A. Exchange Rate Volatility of Netherlands guilder agianst the UK pound



B. Exchange Rate Volatility of Italian lira against the UK pound

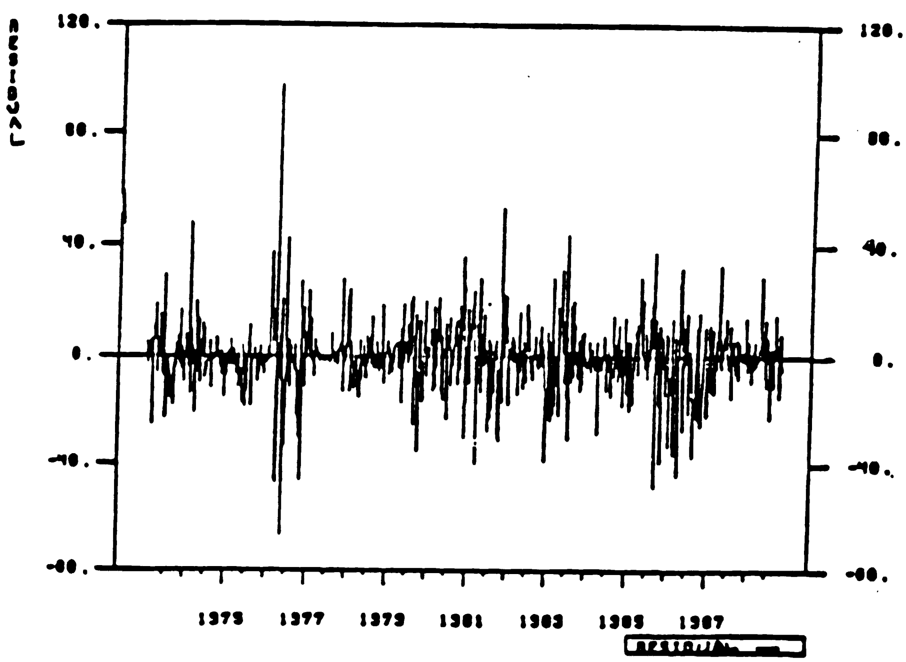
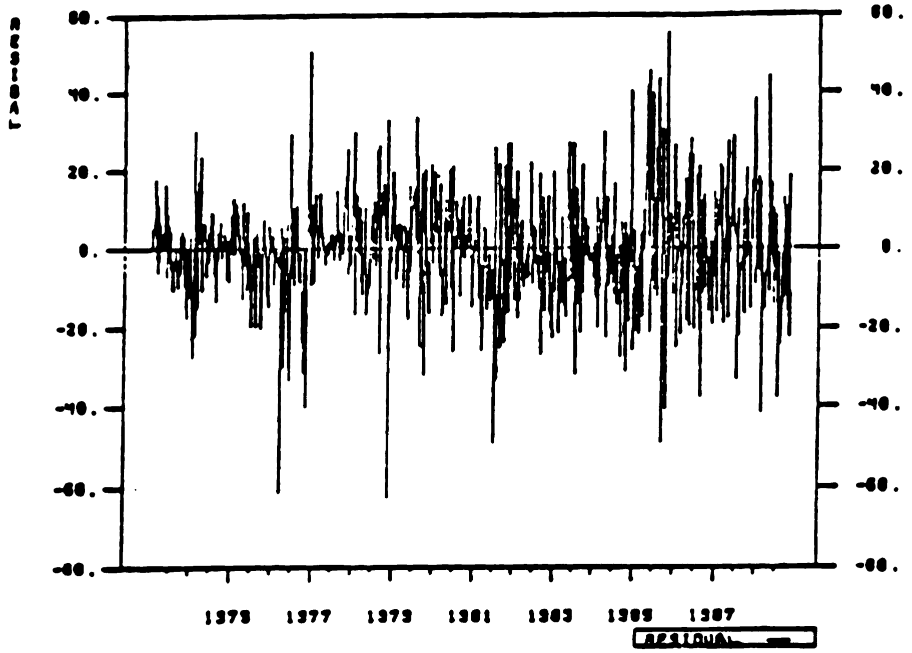
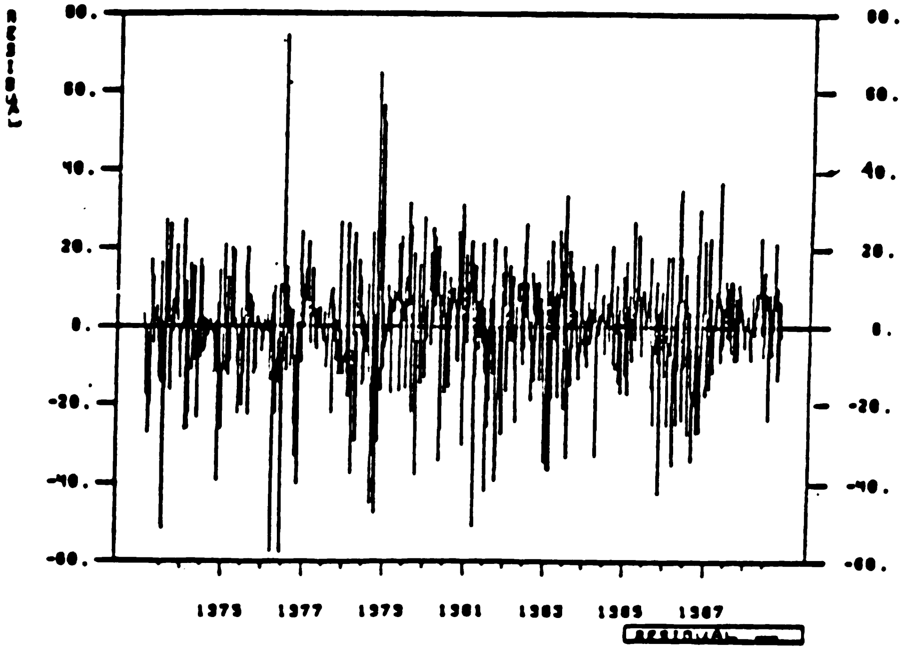


Figure 6

A. Exchange Rate Volatility of Canadian dollar against the UK pound



B. Exchange Rate Volatility of Swiss franc against the UK pound



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II. MULTIVARIATE COINTEGRATION TESTS AND LONG-RUN PURCHASING POWER PARITY THEORY

1. INTRODUCTION

Many theoretical and empirical models of purchasing power parity (PPP) theory have been built since the 1970s. This theory generally refers to the proposition that exchange-rate changes will be proportional to relative goods prices. Most authors have found little evidence in favor of the empirical validity of PPP after 1973 [see, for example, Frenkel (1981) and Dornbusch (1980). For a general review and discussion, see Officer (1984) and Dornbusch (1988)]. However, previous tests neglect the fact that the levels of price indexes and spot exchange rates are nonstationary [for example, Frenkel (1981)]. Since a cointegrated system allows individual time series to be integrated of order one but a linear combination of the series to be stationary, many recent papers are devoted to the estimation and testing of long-run relationships of PPP using the techniques of cointegration [Baillie and Selover (1987), Enders (1988), Edison and Fisher (1989), and Corbae and Ouliaris (1988)]. However, their unit root tests on the price indexes or the relative prices are limited to only one unit root in the price series, whereas we find that two unit roots are needed in some series. Furthermore, they ignore both the endogeneity of price series[1] and the error structures between variables[2]. In addition, most previous work use the Engle and Granger (1987) test

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[1] Krugman (1978) confirmed that tests which recognize the endogeneity of both prices and exchange rates give results considerably more favorable to PPP.

[2] Hakkio (1984) used SUR (seemingly unrelated regressions) to take into account error structures; however, in virtually all the equations of Hakkio's model where the nominal exchange rate is related to prices are estimated, AR(1) disturbances with a coefficient near unity, which suggests there may be a nonstationary residual and hence a lack of cointegration.

statistics in which the critical values are constructed for one sample size (100 observations) and only for the bivariate case in the regression estimates[3].

In contrast, this paper carefully tests for unit roots in the price indexes and relative price indexes to determine whether they have two unit roots. If some price indexes are really I(2), the conventional tests of PPP under the assumption of I(1) are wrong and subject to the spurious regression critique [e.g., Phillips (1986)]. After these univariate unit root tests, multivariate tests developed by Johansen (1988) are implemented in testing for cointegration vectors between exchange rates and relative price indexes. Those two variables are determined simultaneously by specifying a model with a vector autoregressive process. This approach gives more efficient estimates than the conventional regression estimates, since it not only allows for general dynamic properties of the structure of the underlying process, but it also gives maximum likelihood estimates. [We will discuss more properties of this test in Section 3.]

This paper is organized in five sections. The second section carries out univariate unit root tests on exchange rates and price series. It is critical to establish that the individual series are I(1), because the Johansen methodology is based on this assumption. We have used the CPI as the price series as in most previous studies[4]. Section 3 develops

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[3] Engle and Yoo (1987) have calculated critical values for more variables and sample sizes, but using the same general approach.

[4] The main choice of price indexes is among CPIs (e.g., Baillie and Selover (1987), Rogoff and Meese (1988), Corbae and Ouliaris (1988), and Edison and Fischer (1989)) and WPIs (e.g., Frenkel (1978, 1981) and Enders (1988)). On the other hand, Dornbusch (1988) ruled out WPIs on the argument that conceptually

the hypothesis tests of cointegration vectors between relative price indexes and exchange rates with Johansen's (1988) test method, before conventional unit root tests for PPP are done to compare with our results. Short-run dynamics are analyzed in Section 4, and in Section 5, the evidence is summarized.

We use monthly spot exchange rates obtained from the New York Foreign Exchange Market from January, 1974 through October, 1988. The sample is chosen to span the most recent floating exchange rate period. The data were provided by the FRB and comprised 179 observations for the currencies of Canada, Italy, France, Denmark, West Germany, the Netherlands, Switzerland, and Japan vis a vis the US dollar. The data are observations representing the value on the last business day of the month of these currencies' noon quotes(bid rates). As price levels, we used the consumer price indexes (CPI), not adjusted for seasonality, provided by the International Monetary Fund for the same period for each of the countries.

2. UNIVARIATE TESTS FOR UNIT ROOTS IN MONTHLY EXCHANGE RATES AND CONSUMER PRICE INDEXES

In this study of a unit root test for exchange rates and price series, the Augmented Dickey-Fuller (ADF) and Phillips and Perron (1986, 1988) tests are presented [see Said and Dickey (1984, 1985) for details about the ADF test]. The ADF tests are extended versions of Dickey and

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they are poorly defined, being neither producer nor consumer price indexes, and the preference is given to GDP deflators that, he thinks, have a clear methodological definition. Cost-of-living and materials price indexes are used also.

Fuller tests. Said and Dickey argue that the D-F procedure, which was originally developed for autoregressive representations of known order, remains valid asymptotically with

$$\Delta y_t = \alpha + \beta\, y_{t-1} + \gamma t + \sum_{j=1}^{p} \rho_j \Delta y_{t-1-j} + \epsilon_t \qquad (2\text{-}1)$$

where $\Delta y_t = y_t - y_{t-1}$ and t denotes a time trend. The null hypothesis of $\hat{\beta} = 0$ is tested with $\hat{\tau}$ tables of Fuller (1976, p. 373) compared to the standard t statistic on the $\hat{\beta}$, which is estimated when subjected to $\alpha = \gamma = 0$, $\hat{\tau}_\mu$ when $\gamma=0$, and $\hat{\tau}_\tau$ when Eq. (2-1) is estimated in unrestricted form. Phillips and Perron (1986, 1988) develop an alternative procedure for testing the presence of a unit root in a general time series, which includes ARIMA models with heterogeneously as well as identically distributed innovations. The tests involve computing three conventional least square regressions defined from:

$$y_t = \hat{\alpha}\, y_{t-1} + \hat{u}_t \qquad (2\text{-}2)$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u_t^* \qquad (2\text{-}3)$$

$$y_t = \tilde{\mu} + \tilde{\beta}\,(t - 1/2\, T) + \tilde{\alpha}\, y_{t-1} + \tilde{u}_t \qquad (2\text{-}4)$$

where T denotes the sample size and the innovation sequence $\{\hat{u}_t\}$, $\{u_t^*\}$, and $\{\tilde{u}_t\}$ allow for dependent and heterogeneously distributed time series.[5] In Eq. (2-2), the null hypothesis of a unit root, i.e., $\hat{\alpha} = 1$ against $\hat{\alpha} < 1$, is tested with adjusted t-statistics $Z(t_{\hat{\alpha}})$. In Eq. (2-3), the null hypothesis of $\alpha^* = 1$ is tested with $Z(t_{\alpha}*)$ and with $Z(\Phi_1)$ for $\mu^* = 0$ and $\alpha^* = 1$. Given Eq. (2-4), which allows a time trend and a fitted drift, we can test the hypothesis H_o: $\tilde{\alpha} = 1$, H_1: $\tilde{\beta} = 0$, $\tilde{\alpha} = 1$ and H_2: $\tilde{\mu} = 0$, $\tilde{\beta} = 0$ and $\tilde{\alpha} = 1$ by means of the statistics $Z(t_{\tilde{\alpha}})$, $Z(\Phi_3)$ and $Z(\Phi_2)$. The test

[5] See Phillips and Perron (1988) for precise conditions on $\{u_t\}$.

statistics require consistent estimates of the variances based on truncated (let the bound be ℓ) sample autocovariances [see Phillips and Perron (1988) for details]. Therefore the choice of ℓ will be very important, since a small ℓ would give biased estimate of the variance and too large a value for ℓ would include terms insignificantly different from zero. Inevitably, the choice of ℓ will be an empirical matter. The formulas for ADF and Phillips and Perron test statistics are not presented here. Both the ADF and Phillips and Perron (1986, 1988) test methods have low power [Schwert ,1987] when the series is generated by mixed ARIMA process with a root in the moving average polynomial close to unity. Schwert mentioned the US CPI series as one of those cases. We will examine the CPI series in detail later in this section.

Before we present unit root tests, a preliminary investigation of the sample autocorrelation functions will help us decide on an appropriate choice of differences in time series. In Table 1, the sample autocorrelations of exchange rate levels show positive and persistent movements. However, they decay quickly in the first differences. With this evidence, we presume that spot exchange rates are I(1), and the results of applying the Phillips and Perron test and ADF test statistics support the hypotheses of a unit root in the levels of the logarithms of spot exchange rates [Table 2]. Table 2 reports that the unit root hypothesis cannot be rejected for any currency, by using any truncation lags from 0 to 10 in the Phillips and Perron tests. For reasons of space, only the results for $\ell = 10$ are given in Table 2. The ADF tests also fail to reject the null hypothesis of a unit root in all cases, as reported in Table 2.

On the other hand, in the case of CPI series, the sample autocorrelations for the levels of logged CPI series are positive; they fail to damp and have very smooth and persistent movements [Table 3]. Apart from the CPIs of Netherlands, Denmark, Switzerland, Japan and France, the other 4 CPIs exhibit the slowly decaying autocorrelations again in their first differences [Table 4]. Based on these autocorrelation structures, it appears that the CPI series have at least one unit root, and some series may be I(2). First, to check whether the CPI series have at least one unit root, the Phillips and Perron tests will be applied. Then we will examine whether some series possess two unit roots by the Dickey and Pantula tests (1987). In testing a unit root with the Phillips and Perron tests, we try to choose the appropriate truncation lags. The test statistics are very sensitive to lag lengths; they change from significant to insignificant as ℓ increase [see Tables 5A through 5D]. However, in the case of $Z(\Phi_2)$'s, the test statistics reverse to be significant at the .05 percentile, at relatively high lags. Hence, we should probably give a selection of results with differing lag lengths. Table 6 reports the results of the Phillips-Perron tests. For CPI series it is not possible to reject the null hypothesis of at least one unit root. Using the $Z(t_{\tilde{\alpha}})$ statistics the null hypothesis of a unit root can not be rejected for the CPIs of the Netherlands, Switzerland, and Japan. Using the $Z(t_{\alpha}*)$ statistics it is also not possible to reject the null hypothesis for the other countries' CPIs. Because the examination of the sample autocorrelation functions of the first differences suggests that some of the CPI series may be I(2), we examine whether some series possess two unit roots. One simple method is to replace y_t in Eqs. (2-2) through

(2-4) with Δy_t and then to test for a second unit root with the Phillips and Perron test statistics. The results can be seen in Table 7(A); they suggest no second unit root with every test statistic, except some $Z(t_{\hat{\alpha}})$ statistics. According to the $Z(t_{\hat{\alpha}})$ statistics, for the USA, France, Italy, and Canada CPI series, the null hypotheses of a second unit root can not be rejected, a result expected from the previous examination of the autocorrelation functions. However, because Sen (1986) suggests that such a procedure lacks power, the Hasza and Fuller (1979) and Dickey and Pantula (1987) tests are preferred to test second unit roots in the CPI series. Furthermore, since the Hasza and Fuller (1979) F-statistics tend to be biased towards finding unit roots [Dickey and Pantula, 1987], the Dickey and Pantula (1987) testing strategy is used. Their proposed procedure starts with tests for the presence of three unit roots and for two, etc. The method for determining the order of differencing starts with the regression:

$$\Delta^3 y_t = \mu + \beta \Delta^2 y_{t-1} + \epsilon_t. \qquad (2\text{-}5)$$

The hypothesis of need for a third unit root is tested by the t-statistics on $\hat{\beta}$ compared to the τ_μ or τ tables of Fuller (1976), depending on whether or not a constant term is included in the regression. If the above hypothesis is rejected, the next step in the Dickey and Pantula procedure is to test two unit roots with the regression by H_o: $\beta_1 = 0$:

$$\Delta^3 y_t = \mu + \beta_1 \Delta y_{t-1} + \beta_2 \Delta^2 y_{t-2} + \epsilon_t. \qquad (2\text{-}6)$$

Eqs. (2-5) and (2-6) were augmented as in the OLS regression for the Augmented Dickey Fuller (ADF) tests [see Eq.(2-1)], since the disturbances were autocorrelated. Table 7(B) shows our Dickey and Pantula test results. Since three unit roots are rejected in all series, this table contains only the results of two unit root tests. The Dickey and Pantula tests suggest that most of the CPI series in our study are I(2). These results suggest that care is required in testing for cointegration between spot exchange rates, which are I(1), and the CPI series, which are I(2). This appears to be a problem in many previous studies [for example, Corbae and Ouliaris (1988), Edison and Fisher (1989), Taylor and Artis (1988) and Adler and Lehman (1983)].

If the level of the CPI series is typically I(2), our major concern is whether we can use relative price indexes as a variable to test PPP theory in the equation:

$$s_t = \alpha + \gamma(p_t - p_t^*) \qquad (2\text{-}7)$$

If relative price indexes are I(2), then we cannot perform conventional cointegration tests with Eq. (2-7), since the exchange rate series are all I(1). Table 8 reports that in all relative CPIs the null hypotheses of one unit root can not be rejected under the Phillips and Perron test. When we go further with two-unit-root tests with the Dickey and Pantula method, two unit roots are rejected in most series, except in the cases of France-Germany and France-USA [Table 9][6]. Thus, univariate unit roots tests suggest conclusively that, for most countries studied here, the levels of exchange rates and relative consumer price indexes must be at least first differenced to achieve stationarity. However, if we want to use the relative prices of France-Germany, for example, then we take second differences. This implies that, even for the relative price

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[6] For reference, I also tested for a second unit root with the Phillips and Perron tests. Second unit roots are rejected with most test statistics, and only the $Z(t_{\hat{\alpha}})$ accepts second unit roots for the case of Italy-Germany [see Table 9].

series, we must test whether those relative price series possess two unit roots, since most of the CPI series in this study are I(2), and it is possible for the difference between these CPI series to be I(2), I(1) or I(0). In the next section, tests of cointegration vectors will be performed to determine whether a linear combination of the series implied by PPP, i.e., $s_t = \alpha + \gamma(p-p^*)_t$, results in stationary trend.

3. HYPOTHESIS TESTING OF COINTEGRATION VECTORS BETWEEN EXCHANGE RATES AND RELATIVE PRICES AND LONG-RUN PPP THEORY

The equilibrium relationship in PPP says that the exchange rate reflects the relative price ratio between concerned countries. This relation can be expressed by $s_t = \alpha + \gamma(p-p^*)_t$, where s and $p-p^*$ denote the spot exchange rate and the difference between domestic and foreign price levels in logged terms, respectively. (This notation is used throughout the remainder of the text.)

Since we are looking for the long-run relation between exchange rates and relative prices, the concept of cointegration is important; if the variables are I(d), in our case I(1), then it will generally be true that a linear combination of those variables, i.e., $\beta' x_t = s_t + \gamma(p-p^*)_t$, will also be I(d). However, it may happen that $\beta' x_t \sim I(d-b)$ where $b > 0$; s_t and $(p-p^*)_t$ are then said to be cointegrated of order d,b. Here β is a cointegration vector.

Testing the PPP hypothesis on such a cointegrating vector is conventionally done with regressions of the spot exchange rates on relative prices, with prices given exogenously. In this section, multivariate tests developed by Johansen (1988) are implemented in testing for cointegrating vectors between exchange rates (s_t) and relative prices $(p-p^*)_t$ simultaneously by specifying a model with a vector autoregressive process (VAR). The VAR model is:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} \cdots + \Gamma_{k-1} \Delta x_{t-k+1} - \pi x_{t-k} + \epsilon_t \qquad (3\text{-}1)$$

where $x_t = (s_t,\ p_t - p_t^*)'$ and $\epsilon_1 \cdots \epsilon_T$ are iid $N(0,\Lambda)$. This is expressed as a traditional first-differenced VAR-model except for the term πx_{t-k}. If rank (π) is not full rank, then the coefficient matrix (π) may convey information about the long-run structure of our chosen data. Johansen's method of hypothesis testing of cointegrating vectors formulates the hypothesis of reduced rank ($=r$) in π, or one which implies that there are matrices α and β of order v($=$# of variables)x r, such that $\pi = \alpha\beta'$ where $\beta' x_t \sim I(0)$. The properties of the Johansen's test are as follows;

1. The maximum likelihood estimator of the space spanned by β is the space spanned by r canonical variates corresponding to the r largest squared canonical correlations between the residuals of x_{t-k} and Δx_t, corrected for the effect of the lagged differences of the x process.

2. The likelihood ratio test statistic for the hypothesis that there are, at most, r cointegration vectors is given by

$$-2\ln Q = -T \sum_{i=r+1}^{v} \ln(1-\hat{\lambda}_i)$$

where $\hat{\lambda}_{r+1}, \cdots, \hat{\lambda}_v$ are the v-r smallest squared canonical correlations.

3. Under the hypothesis that there are r cointegrating vectors, the estimate of the cointegration space as well as π and Λ are consistent, and the likelihood ratio test statistic of this hypothesis is asymptotically distributed as

$$tr\{ \int_0^1 BdB' [\int_0^1 B(u)B(u)'du]^{-1} \int_0^1 dBB' \}$$

where B is a v-r dimensional Brownian motion with covariance matrix I. The proofs of these results are presented in Johansen (1988).

Given these properties we test for cointegration using maximum likelihood estimates. First, to Eq.(3-1) we add constant terms and seasonal dummies, since they turn out to be very significant. Hence, our model will be:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} \cdot \cdot + \Gamma_{k-1} \Delta x_{t-k+1} - \pi x_{t-k} + \text{constant} + \sum_{i}^{11} k_i Q_{it} + \epsilon_t \qquad (3\text{-}2)$$

where the 6th term in RHS denotes seasonal dummies. Before testing the cointegrating vectors in $\pi = \alpha\beta'$ we must decide how many lags are needed to get uncorrelated residuals in Eq.(3-2). With the VAR model, the "k" is determined when the residuals in the data clearly passes the test for no autocorrelation. The estimates Γ, π, and Λ are given in Table 10 for the case of USA-Germany exchange rates and prices. The hypothesis of cointegrating vectors in the variables (s and $p\text{-}p^*$) according to Johansen's test method is H_o: $\pi = \alpha\beta'$, where α and β are 2 x r matrices with rank (π) $= r \leq v$ ($=2$). If $r = 0$, we can say that there is no cointegrating vector between s and ($p\text{-}p^*$) [Johansen (1988) and Johansen and Juselius (1988)].

First we tested for cointegration between U.S. variables and four other country variables: W. Germany, Canada, Switzerland, and Japan. Next, to eliminate the effect of the USA variables, we tested for cointegration among W. German variables and three other country variables: Denmark, Japan, and Switzerland. (For reference, Frenkel (1981) says that departures from PPP are a USA phenomenon.). Table 11 reports the results of calculating the likelihood ratio tests of cointegrating vectors in each case. In Table 11 we find that the null hypothesis of no cointegrating vector is rejected at the 95% or greater quantile in every case except that of Canada-USA. In cases where we reject the hypotheses of no cointegrating vector, the null hypotheses of at most one cointegrating vectors are accepted, and we can say that there is one cointegrating vector between the exchange rates and relative prices, except in the Canada-USA case. With these test results, we estimated the cointegrating relations, β, in the second column in $\hat{V}$ from Table 12. In this case it seemed natural to normalize β by the coefficient of s=-1. The normalized coefficients of $(p-p^*)$ are reported on Table 13 as $(-\phi/\alpha)$. This made it straightforward to interpret the cointegrating vector in terms of an error-correction mechanism measuring the excessive movement of exchange rates, where the equilibrium relation is given by $s = \gamma(p-p^*)$ + constant. Similarly, $\hat{\alpha}$'s can be found in the second column in the matrix of estimated Alphas in Table 12. α is discussed further in Section 4.

Wald tests for the significance of each coefficient in the cointegrating vector were done with the hypothesis

$k'\beta = (1,0)\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} = 0$ or $= (0,1)\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} = 0$, where β_1 is the coefficient of the exchange rate(s), and β_2 is that of relative price index $(p-p^*)$[7]. As can be seen in Table 13, the elements of cointegrating vectors are significant at any reasonable level. We also formulate a linear restriction on the cointegrating vectors and use Wald tests to see whether the traditional PPP theory that the coefficients of s and $(p-p^*)$ are equal

[7] The test statistic is $T^{1/2}K'\hat{\beta} / \{ (\hat{\lambda}_1^{-1} - 1) (K'\hat{\gamma}\hat{\gamma}'K) \}^{1/2}$ with χ_1^2. $\hat{\lambda}_1$ is the maximal eigenvalue, and β is the corresponding eigenvector, and the remaining eigenvector forms $\hat{\gamma}$, [see Johansen and Juselius (1988) Corollary 3.17.].

with opposite signs is acceptable. The restriction in a matrix formulation can be expressed as:

$$k'\beta = (1,\ -1)\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} = 0.$$

For example, in the case of Germany - USA, the Wald test statistic is 0.7346, which is less than $x_1^2(p=0.99) = 6.635$ and the null hypothesis of identical coefficients with opposite sign in s and $(p-p^*)$ is not rejected. The other cases can be seen in Table 13. In the case of Sfr/U\$, which has a large coefficient, the null hypothesis of identical coefficients with opposite signs is not rejected at $x^2_{0.95}$ (with 1 degree of freedom), but it is rejected at $x^2_{0.98}$. The Sfr/Mark also has a large coefficient on the relative price indexes, but the null hypothesis is not rejected. Only in the Yen/Mark case is the null hypothesis of identical coefficients with opposite signs rejected.

From these results, we are unable to reject the hypothesis of the simple version of PPP Theory; it generally holds under our multivariate approach. Therefore, we see that the alleged failure of PPP after 1973 is due to imprecise parameter estimates and improper specification of error structures. This result is opposite to Frenkel's (1981), where he has imprecise parameter estimates and concludes that departures from PPP are a USA phenomenon, because we have cases where the PPP theory holds when it involves the US dollar and the US price level. This result also is similar to that of Hakkio (1984), who is unable to reject PPP theory when it is viewed in a multivariate context. However, this approach is different from his; first, we recognize the cointegration problem which is not treated in his paper, and second, his paper uses instrumental variables for relative prices, which did not need to use when using the simultaneous approach due to Johansen.

To this point, we use cointegration tests in a multivariate context. Now we will see which results one can expect with conventional univariate cointegration tests. We applied the conventional univariate unit root tests to the deviations from PPP, defined as Z_t, with Z_t = constant + $\bar{s}_t$ - $(\bar{p} - \bar{p}^*)_t$, using Said and Dickey (1984, 1985), Phillips and Perron (1986, 1988) procedures and Dickey and Fuller's likelihood ratio tests (1979, 1981). First, in ADF tests [Table 14], only in the Denmark-Germany and Swiss-Germany cases are the null hypotheses of a unit root rejected at the 0.10 and 0.025 significance levels, respectively. In other cases which showed long-run equilibrium relations (see Table 13), the null hypotheses can not be rejected. Next, we used Phillips and Perron tests, and surprisingly, in the two cases cited, in which we reject the null hypotheses of a unit root by ADF tests, the null hypotheses of a unit root can not be rejected using $Z(t_{\tilde{\alpha}})$ statistics.[8] Lastly, the low power of univariate unit root tests is examined again with the Dickey and Fuller likelihood ratio tests (1981), by comparing them with the Phillips and Perron tests (1986, 1988). Dickey and Fuller (1981) assume that the time series is adequately represented by the model

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[8] The ADF test is H_o: $\beta_2 = 1$ with

$$y_t = \beta_o + \beta_1 t + \beta_2 y_{t-1} + \sum_{i=1}^{p} \phi(y_{t-i} - y_{t-1-i}) + \epsilon_t.$$

The $Z(t_{\hat{\alpha}})$ in the Phillips and Perron tests is H_o: $\beta_2 = 1$, with $y_y = \beta_o + \beta_1(t-T/2) + \beta_2 y_{t-1} + u_t$. See Section 2 for the conditions on $\{u_t\}$.

$$Z_t = \beta_o + \beta_1 t + \alpha Z_{t-1} + \sum_{j=1}^{p} \phi_j (Z_{t-j} - Z_{t-j-1}) + \epsilon_t \qquad (3\text{-}4)$$

where Z_t are deviations from PPP and ϵ_t are independent identically - distributed $(0,\sigma^2)$ random variables. The hypotheses are:

H_1: $\beta_o = \beta_1 = 0$, $\alpha = 1$

H_2: $\beta_1 = 0$, $\alpha = 1$ and

H_o: $\beta_o = 0$, $\alpha = 1$ with $Z_t = \beta_o + \alpha Z_{t-1} + \epsilon_t$.

The test statistics of Φ_2, Φ_3, and Φ_1 for each of above hypotheses are given in Dickey and Fuller (1981, P. 1063). The test of these hypotheses is basically the same as that of Phillips and Perron. However, Table 14 reports that even when we test the same hypothesis, most of the Phillips and Perron tests (1986, 1988) accept the hypotheses of H_o and H_1, which are rejected by the Dickey and Fuller tests(in the case of Φ_3, the results are mixed). For example, in the case of Denmark-Germany, to test the hypothesis that $\beta_o=\beta_1=0$ and $\alpha=1$ against the general alternative of Eq.(3-4) we first compute

$$\Phi_2 = \frac{(RSS_1 - RSS_2)/m}{RSS_2\ /\ (T-k)} = \frac{.01878 - .01461}{3(.01461/\ 171)} = 17.81,$$

where RSS_1 denotes the restricted residual sum of squares, and RSS_2 denotes unrestricted residual sum of squares, m denotes number of restrictions, T denotes total number of observations, and k denotes number of coefficients. As there were 179 observations in the regression, the 97.5% point of the distribution of Φ_2, as given in Dickey and Fuller (1981, P. 1063), was 5.40. Therefore, the hypothesis $\beta_o = \beta_1 = 0$ and $\alpha = 1$ is rejected at the 2.5 percent level. On the other hand, under the Phillips and Perron test statistics, the hypothesis of $\beta_o = \beta_1 = 0$ and $\alpha = 1$ in $Z_t = \beta_o + \beta_1(t-T/2) + \alpha Z_{t-1} + u_t$ is tested with the $Z(\Phi_2)$ statistic, which is basically identical to Φ_2 in the Dickey and Fuller tests (1981). However, the $Z(\Phi_2)$, 4.07 is less than 6.25 at the .95 percentile, and the null hypothesis of a unit root is accepted. That one test accepts the null hypothesis of a unit root, and the other test rejects the hypothesis can be seen in other cases in Table 14.

4. SHORT-RUN DYNAMICS

Based on the estimated coefficient matrix of $\hat{\pi}$, which conveys information about the long-run PPP as discussed above, we can explore short-run movements between exchange rates and relative prices in the context of the error-correction model.

Error-correction models are usually interpreted by the partial adjustment approach of Engle and Granger (1987), but another interpretation is the rational expectations approach of Campbell and Shiller (1988). The former says that, most of the time, the economy system is out of equilibrium, but there is a tendency for the system to return to equilibrium [see Engle and Granger (1987) and Johansen and Juselius (1988) for this interpretation.]. On the other hand, Campbell and Shiller have an alternative interpretation for cointegrated models. They say that the error-correction model may also arise because one variable forecasts another. Engle and Granger believe that the motivation for cointegration is that equilibrium error causes changes in the variables of the model. However, Campbell and Shiller emphasize the possibility that the equilibrium error results from agents' forecasts of these changes. According to Campbell and Shiller, cointegration can arise even in a well-organized market with no adjustment costs, where there is no true causal role for the equilibrium error. It can arise when agents forecast and have rational expectations.

In this study, the exchange rates and consumer price indexes have very different characteristics; it is generally accepted that the exchange rate, which is the relative price of two durable assets (monies), can be best treated by within an analysis of asset prices, which strongly depends on expectations concerning the future. On the other hand, aggregate price indexes reflect the prices of goods and services and are less sensitive to news. This distinction between aggregate price indexes and asset prices results in short-run deviations from PPP, and the stickiness exhibited by the aggregate price indexes may reflect the cost of price adjustment, which results in finite nominal contracts. Given such differences between exchange rates and consumer price indexes, we cannot avoid the partial adjustment approach in our analysis of short-run movements.

In our VAR model,

$$\Delta x_t = \hat{\Gamma}_1 \Delta x_{t-1} + \cdots + \hat{\Gamma}_{k-1} \Delta x_{t-k-1} - \hat{\pi} x_{t-k} + \widehat{\text{constant}} + \sum_{i=1}^{11} \hat{k}_i Q_{it} \qquad (4\text{-}1)$$

where $\Delta x_t = (\Delta s_t, \Delta(p-p^*)_t)'$, and $\hat{\Gamma}$ and $\hat{\pi}$ are matrices of order 2x2, and where seasonal dummy Q_{it} is 11x1 [see Table 10 for an example of USA-Germany]. Changes in exchange rates and relative prices at time t are expressed with lagged changes in the variables and error-correction terms as well as constants and seasonal dummies. If we replace the π with $\alpha\beta'$ to Eq.(4-1) which we derived in the previous section, $\beta' x_t$ can be used as an instrument or an error-correction term to estimate an error-correcting model, and α can be interpreted as the weight with which deviations from PPP enter the equations of our system. In this case α can be given an economic interpretation as the speed of adjustment towards the estimated equilibrium state. The MLEs of the error-correction model, in the case of USA-Germany, is as follows:

$$\begin{bmatrix} \Delta s_t \\ \\ \Delta rp_t \end{bmatrix} = \begin{bmatrix} .29^{**} & .80 \\ (.079) & (.75) \\ .008 & .22^{**} \\ (.008) & (.08) \end{bmatrix} \begin{bmatrix} \Delta s_{t-1} \\ \\ \Delta rp_{t-1} \end{bmatrix} + \begin{bmatrix} -.015 & .95 \\ (.079) & (.73) \\ .003 & .11^{**} \\ (.008) & (.08) \end{bmatrix} \begin{bmatrix} \Delta s_{t-2} \\ \\ \Delta rp_{t-2} \end{bmatrix}$$

$$- \begin{bmatrix} .023 \\ -.004 \end{bmatrix} (1, \ -.68) \begin{bmatrix} s_{t-3} \\ rp_{t-3} \end{bmatrix} + \begin{bmatrix} -.014 \\ (.013) \\ -.004^{**} \\ (.001) \end{bmatrix} + \sum_{i=1}^{11} k_i Q_{it} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}$$

In this equation, s denotes the logged exchange rates, rp denotes the logged relative prices, i.e., $\ln(p / p^*)$, and Δs and Δrp denote the first differenced term of each variable. The fifth term represents the coefficients of seasonal dummies, which are not reported for reasons of space. With two lagged first-differenced terms, the residuals clearly pass the test for being uncorrelated (the diagnostic tests are done with Box-Pierce statistics.). In the parentheses are standard errors, and ** denotes significance at the .05 level. The third term in the RHS includes the long-run equilibrium relationship discussed in the previous section; the first vector is α and the second one is β' in $\pi = \alpha\beta'$. The significance of π, which is shown as $\alpha\beta'$ in the above equation, is reported in Table 10. The vector, β, i.e., $(1, -.68)'$ has significant elements, and it was not possible to reject the null hypothesis of identical elements with opposite signs as $(1, -1)'$, which implies the long-run PPP [Table 13]. Under such restrictions on β, we construct a likelihood ratio test[9] for the hypothesis that the second element of α is zero, for this element is small when we compare it with first element. This comparison implies that the cointegration relation enters only the first equation. However, this hypothesis is rejected at the 99% level.

The first equation with Δs_t as a dependent variable is expressed as:

$$\Delta s_t = \underset{(.079)}{.29^{**}} \Delta s_{t-1} + \underset{(.75)}{.80} \Delta rp_{t-1} - \underset{(.079)}{.015} \Delta s_{t-2} + \underset{(.73)}{.95} \Delta rp_{t-2}$$

$$-.02(1.0^{**} s_{t-3} - .68^{**} rp_{t-3}) - \underset{(.013)}{.014} + \sum_{i=1}^{11} k_i Q_{it} + \epsilon_t .$$

This equation can be interpreted as demonstrating how the change in exchange rates at time t is related to lagged changes in exchange rates and prices, with deviations from long-run PPP. Only the coefficients of the changes in exchange rate at lag one and the departures from long-run PPP are significant at the .05 level, which explain the changes in exchange rate at time t. The deviations from the long-run PPP are entered into the parentheses in the 5th term in the RHS, and the coefficient of 0.02 indicates the speed of adjustment towards the estimated equilibrium states.

For Denmark, during the sample period, the estimated error-correction model against Germany is as follows:

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[9] The test statistic for this hypothesis of α is $-2\ln(Q) = T\{ \ln(1 - \lambda_1) - \ln(1 - \lambda_1^*)\}$ with χ_1^2, where λ_1 is eigenvalues under α and β restrictions, and λ_1^* is eigenvalues under β restrictions. The α restriction is $\alpha = \binom{1}{0} (\alpha_1, 0)$, and the β restriction is $\beta_1 = -\beta_2$ [see Johansen and Joselius (1988), Theorem 2.4 and its proof.].

$$\begin{bmatrix} \Delta s_t \\ \\ \Delta rp_t \end{bmatrix} = \begin{bmatrix} .19^{**} & .08 \\ (.08) & (.10) \\ .16^{*} & -.02 \\ (.06) & (.08) \end{bmatrix} \begin{bmatrix} \Delta s_{t-1} \\ \\ \Delta rp_{t-1} \end{bmatrix} + \begin{bmatrix} -.015 & .09 \\ (.08) & (.90) \\ .05 & -.05 \\ (.06) & (.08) \end{bmatrix} \begin{bmatrix} \Delta s_{t-2} \\ \\ \Delta rp_{t-2} \end{bmatrix}$$

$$+ \begin{bmatrix} -.02 & -.06 \\ (.08) & (.09) \\ .11 & -.05 \\ (.06) & (.08) \end{bmatrix} \begin{bmatrix} \Delta s_{t-3} \\ \\ \Delta rp_{t-3} \end{bmatrix} - \begin{bmatrix} .067 \\ \\ .038 \end{bmatrix} (1, \ -.90) \begin{bmatrix} s_{t-4} \\ \\ rp_{t-4} \end{bmatrix} + \begin{bmatrix} .079^{**} \\ (.027) \\ -.033 \\ (.02) \end{bmatrix}$$

$$+ \sum_{i=1}^{11} k_i Q_{it} + \begin{bmatrix} \epsilon_{1t} \\ \\ \epsilon_{2t} \end{bmatrix}$$

The fourth term in the RHS includes the long-run relationship shown in Table 13. Three lagged first-differenced terms in the RHS are chosen to eliminate correlated residuals. ** denotes significance at the .005 level, * denotes significance at the .025, and + denotes significance at the .05 level. The fourth term comes from the vector autoregressive estimates of $\pi = \alpha\beta'$ and the π has significance with the matrix of

$$\pi = \begin{bmatrix} -.06^{**} & .06^{**} \\ (.02) & (.02) \\ \\ .03 & -.03^{*} \\ (.02) & (.02) \end{bmatrix} .$$

In the vector β, i.e., $(1, \ -.90)'$, the null hypothesis of that both elements are zero is rejected and the null hypothesis of identical elements with opposite signs $(1, -1)'$ is accepted. This implies the long-run PPP [see the previous section.].

With Δs_t as a dependent variable, the equation becomes

$$\Delta s_t = \underset{(.08)}{.19^{**}} \Delta s_{t-1} + \underset{(.10)}{.08} \Delta rp_{t-1} - \underset{(.08)}{.015} \Delta s_{t-2} + \underset{(.90)}{.09} \Delta rp_{t-2} - \underset{(.08)}{.02} \Delta s_{t-3}$$

$$- \underset{(.09)}{.06} \Delta rp_{t-3} - .07\,(1.0^{**} s_{t-4} - .90^{**}\, rp_{t-4}) + \underset{(.03)}{.079^{**}} + \sum_{i=1}^{11} k_i Q_{it} + \epsilon_t .$$

The first, seventh, and eighth coefficients are significant at the .05 level. Therefore, like the USA-Germany case, the changes in exchange rates at time t are primarily explained by the changes in exchange rates at lag one and departures from long-run PPP as well as the constant terms. The speed of adjustment is approximately 0.07 and it is marginally higher than that in the USA-Germany case.

We have examined the short-run dynamics and speeds of adjustment with vector autoregressive regressions and derived the long-run relationship from $\pi = \alpha\beta'$ as proposed by Johansen (1988). We conclude that PPP theory holds in the long-run. However, we observe short-run deviations from PPP.

5. CONCLUSIONS

We have reviewed some of the evidence on prices and exchange rates, with the intention of testing the validity of the PPP. Using univariate unit root tests, we find that most CPI's and some of relative price indexes are I(2). Therefore, we cannot expect a stationary result with conventional cointegration tests if we use these price series. Cointegration between exchange rates and relative prices is tested in a multivariate context, using MLE estimates of vector autoregressive processes developed by Johansen (1988). In most cases tests of cointegrating vectors reject the null hypotheses of no cointegration in relative prices and exchange rates. The null hypotheses of identical coefficients with opposite signs between exchange rates and relative price indexes are usually not rejected. From these results, we are unable to reject the hypothesis of PPP theory in the post-1973 data. This result is contrary to that of Frenkel (1981), who has imprecise parameter estimates and rejects PPP in the USA data. The result resembles that of Hakkio (1984), who finds support for PPP in a multivariate context. It differs from his, because we recognize the cointegration problem that he did not address. The results suggest that second unit root tests must be done for price series, and that a multivariate approach to testing PPP theory is needed for more precise parameter estimates.

Table 1
Logged Monthly Spot Exchange Rates

A. Sample Autocorrelations

Lag	Level				First differenced			
	Yen/U$	DM/U$	Yen/DM	DK/DM	Yen/U$	DM/U$	Yen/DM	DK/DM
1	.97	.97	.98	.98	.35	.31	.37	.22
2	.94	.94	.95	.97	-.00	.10	.06	.08
3	.92	.91	.90	.96	.06	.08	.08	.04
4	.89	.88	.87	.94	.15	.05	.09	-.00
5	.79	.85	.84	.95	.08	.06	-.02	.14
6	.76	.81	.81	.94	-.04	.04	-.02	.03
7	.68	.77	.78	.92	-.00	.04	.05	-.02
8	.65	.73	.76	.89	.05	.07	.00	.02
9	.61	.69	.73	.88	-.02	.04	-.02	.01
10	.57	.64	.70	.86	-.03	.06	-.05	-.01
11	.54	.59	.67	.85	.03	.02	-.08	-.01
12	.51	.55	.65	.83	.02	-.01	-.09	-.02

Note: DK denotes Danish krone.

B. Diagrams of ACF (Autocorrelation Function) and PACF (Partial Autocorrelation Function) in the case of Yen / U$

LEVEL

FIRST DIFFERENCED

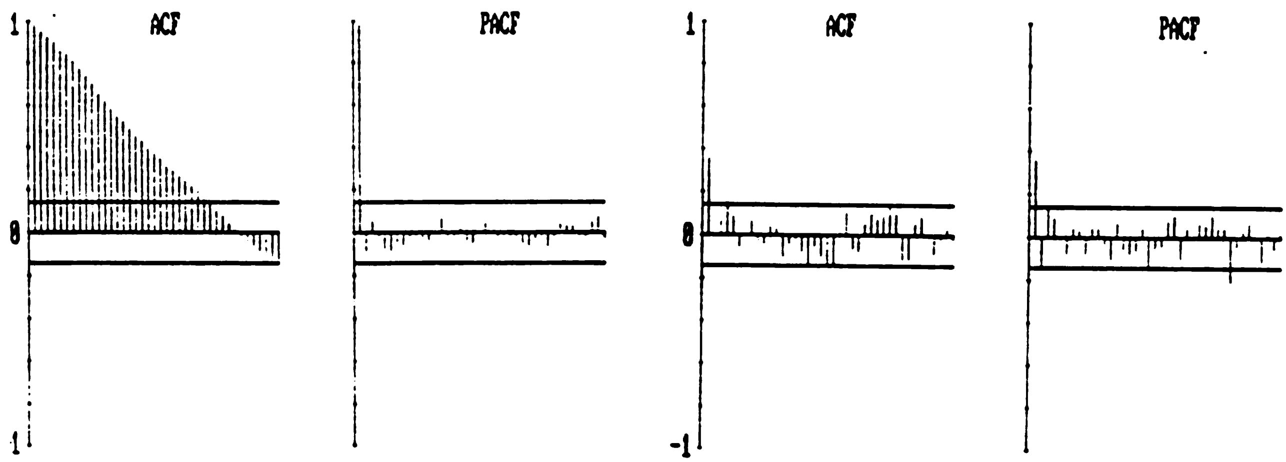

Table 2

Unit Root Tests on Logged Monthly Exchange Rate Series

Currencies	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_1)$	$Z(t_{\alpha}^{*})$	$Z(t_{\hat{\alpha}})$	ADF (Said & Dickey(1984))
	Phillips-Perron test						
Against U$							
Canada	1.372	1.219	-0.082	1.752	-1.574	-0.052	-1.787(2)
Italy	1.091	1.259	-0.844	1.823	-1.454	-1.141	-1.571(2)
France	0.926	0.652	-1.333	0.682	-1.122	0.134	-1.359(2)
Denmark	0.794	0.529	-1.215	0.658	-1.144	-0.131	-1.302(2)
Germany	1.124	1.051	-1.487	1.484	-1.435	-1.184	-1.587(2)
Holland	1.098	0.940	-1.464	1.389	-1.464	-1.013	-1.542(2)
Switzerland	2.135	2.161	-1.997	2.874	-1.901	-1.899	-2.114(2)
Japan	1.424	1.900	-1.254	1.280	0.008	-1.591	-2.005(3)
Against D-mark							
Canada	1.694	1.671	-1.786	2.119	-1.638	-1.656	-1.431(1)
Italy	4.458	6.982**	-1.732	8.234**	-2.665	-3.225++	-2.705(2)
France	2.653	4.603	-2.282	3.365	-0.756	-2.192+	-2.715(3)
Denmark	0.877	3.712	-1.143	4.265	-0.863	2.593**	-1.551(2)
Holland	1.985	2.065	-1.888	1.565	-1.155	0.812	-1.752(2)
Switzerland	4.412	3.775	-2.452	4.316	-2.555	-0.841	-2.805(3)
Japan	5.218*	3.841	-2.989	0.577	-0.544	-0.920	-3.207(3)

Note: 1. Phillips-Perron tests are with truncated lag = 10 in Newey and West (1987).
"*" indicates significance at the .01 percentile and "**" at .05.
"+" indicates significance at the .95 percentile and "++" at .99.
[See Table 6 for critical values]

$$y_t = \tilde{\mu} + \tilde{\beta}(t-n/2) + \tilde{\alpha} y_{t-1} + \tilde{u}_t$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u_t^* \quad , \quad y_t = \hat{\alpha} y_{t-1} + \hat{u}_t$$

$Z(\Phi_2)$: $\tilde{\mu} = 0$ $\tilde{\beta} = 0$ $\tilde{\alpha} = 1$,

$Z(\Phi_3)$: $\tilde{\beta} = 0$ and $\tilde{\alpha} = 1$, $Z(t_{\hat{\alpha}})$: $\hat{\alpha} = 1$

$Z(t_{\tilde{\alpha}})$: $\tilde{\alpha} = 1$, $Z(\Phi_1)$: $\mu^* = 0$ and $\alpha^* = 1$, $Z(t_{\alpha^*})$: $\alpha^* = 1$

2. For Said and Dickey test, test statistics are from Fuller(1976,p.373). "(1)" represents τ statistic, "(2)" represents τ_μ and "(3)", τ_τ .

Table 3
Logged Consumer Price Indexes

A. Sample Autocorrelations for Level

Lag	Italy	Germany	France	Nether- lands	Canada	Denmark	Swiss	Japan	U.S.
1	.98	.98	.98	.98	.98	.98	.98	.97	.98
2	.97	.97	.97	.96	.97	.97	.97	.95	.97
3	.95	.95	.95	.94	.95	.95	.95	.93	.95
4	.94	.94	.94	.93	.94	.94	.94	.91	.94
5	.92	.92	.93	.91	.92	.92	.92	.89	.92
6	.91	.91	.91	.89	.90	.91	.90	.87	.91
7	.89	.89	.90	.87	.89	.89	.89	.85	.89
8	.87	.88	.88	.86	.87	.87	.87	.83	.88
9	.86	.86	.87	.84	.86	.86	.85	.81	.86
10	.85	.85	.85	.82	.84	.84	.84	.79	.85
11	.83	.83	.84	.80	.83	.83	.83	.77	.83
12	.82	.82	.82	.79	.81	.81	.81	.75	.82

B. Sample Partial Autocorrelations

Lag	Italy	Germany	France	Nether- lands	Canada	Denmark	Swiss	Japan	U.S.
1	.98	.98	.98	.98	.98	.98	.98	.97	.98
2	-.01	.00	-.01	-.01	-.01	-.01	-.03	-.29	-.01
3	-.01	-.02	-.01	.00	-.01	-.01	-.01	.03	-.00
4	-.01	-.01	-.01	-.09	-.01	.00	-.04	-.05	-.01
5	-.01	-.01	-.01	.01	-.00	-.00	.03	-.01	-.00
6	-.01	-.00	-.01	-.02	-.00	-.00	-.02	-.02	-.01
7	-.01	-.00	-.01	-.02	-.01	-.01	-.01	-.04	-.01
8	-.00	-.01	-.01	-.01	-.01	-.01	.00	-.01	-.00
9	-.00	-.01	-.01	.01	-.01	-.00	.02	-.01	-.00
10	-.00	-.00	-.01	.01	-.01	-.00	.00	-.03	-.01
11	-.01	-.00	-.01	-.00	-.01	-.00	.02	-.01	-.01
12	-.01	-.01	-.01	-.02	-.01	-.01	-.01	.01	-.01

Table 4

A. Sample Autocorrelations for First Differenced Logged Consumer Price Indexes

Lag	Italy	Germany	Nether-lands	France	Canada	Denmark	Swiss	Japan	U.S.
1	.52	.38	-.10	.64	.20	.07	.26	-.12	.63
2	.37	.23	.12	.23	.27	.05	.25	.06	.49
3	.27	.26	-.05	.11	.29	-.05	.11	.10	.40
4	.25	.20	.11	-.00	.21	.03	.14	.04	.34
5	.23	.17	.04	.16	.24	-.21	-.03	.10	.32
6	.32	.11	.19	.15	.18	.13	-.11	.08	.31
7	.25	.13	.03	.14	.28	.03	-.10	-.05	.33
8	.21	.20	.09	-.05	.22	.08	.01	.07	.33
9	.14	.18	-.04	.05	.16	.03	.01	.11	.40
10	.13	.21	.01	.02	.24	-.04	.12	-.01	.38
11	.09	.25	.05	-.03	.16	-.10	.13	.19	.34
12	.03	.09	.26	-.02	.09	.06	.16	.03	.22

B. Sample Autocorrelations for Second Differenced CPI

Lag	Italy	Germany	Nether lands	France	Canada	Denmark	Swiss	Japan	U.S.
1	-.06	-.02	.01	-.14	-.05	-.01	-.03	.02	-.11
2	.07	.02	.11	.08	.19	.05	.15	.01	.08
3	.03	.15	-.02	.13	.22	-.06	.10	.18	.04
4	.07	.09	.11	-.04	.11	.05	.13	.06	.04
5	-.00	.09	.08	.09	.17	-.21	-.09	.04	.07
6	.22	.01	.20	.10	.09	.15	.19	.03	.02
7	.06	.04	.06	.19	.22	.02	-.08	-.02	.09
8	.08	.12	.09	.03	.15	.08	.04	.01	-.00
9	.00	.05	-.03	.11	.07	.03	.02	.01	.19
10	.05	.11	.01	.11	.19	-.03	.09	.07	.11
11	.04	.19	.08	.02	.10	-.11	.08	.04	.17
12	.01	-.08	.27	.04	.02	.06	.15	.04	-.08

Table 5A

Phillips-Perron Unit Root Tests on Logged Consumer Price Indexes of West Germany

$$y_t = \tilde{\mu} + \tilde{\beta}(t-n/2) + \tilde{\alpha} y_{t-1} + \tilde{u}_t$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u^*_t \quad , \quad y_t = \hat{\alpha} y_{t-1} + \hat{u}_t$$

$$Z(\Phi_2) : \tilde{\mu} = 0 \; \tilde{\beta} = 0 \; \tilde{\alpha} = 1$$

$$Z(\Phi_3) : \tilde{\beta} = 0 \text{ and } \tilde{\alpha} = 1 \; , \; Z(t_{\hat{\alpha}}) : \hat{\alpha} = 1$$

$$Z(t_{\tilde{\alpha}}) : \tilde{\alpha} = 1 \; , \; Z(\Phi_1) : \mu^* = 0 \text{ and } \alpha^* = 1, \quad Z(t_{\alpha*}) : \alpha* = 1$$

Lags in Newey and West(1987)

Lags	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_1)$	$Z(t_{\alpha}^{*})$	$Z(t_{\hat{\alpha}})$
lags = 0	19.651++	77.677++	1.148++	109.696++	-5.633**	12.261++
lags = 2	10.515++	32.768++	0.660++	46.760++	-4.239**	7.966++
lags = 4	7.851++	21.648++	0.475++	30.927++	-3.724**	6.449++
lags = 6	6.696++	16.689++	0.419++	23.705++	-3.473**	5.622++
lags = 8	5.973+	13.818++	0.401++	19.402++	-3.302*	5.064++
lags = 10	5.334+	11.900++	0.357++	16.421++	-3.140*	4.638++
lags = 12	4.716+	10.527++	0.259++	14.186++	-2.971*	4.289++
lags = 14	4.212	9.548++	0.145++	12.495++	-2.820	4.006++
lags = 20	3.470	8.033+	-0.032++	9.471++	-2.558	3.435++
lags = 25	3.173	7.513+	-0.116++	8.025++	-2.428	3.123++
lags = 30	2.988	7.308+	-0.195++	7.033++	-2.329	2.887++
lags = 35	2.989	7.285+	-0.214++	6.336+	-2.269	2.706++
lags = 40	2.840	7.373+	-0.231++	5.804+	-2.223	2.558++
lags = 45	2.812	7.528+	-0.210++	5.398+	-2.196	2.436++
lags = 50	2.794	7.731+	-0.178++	5.072+	-2.177	2.333++
lags = 55	2.784	7.965+	-0.129++	4.807+	-2.164	2.243++
lags = 60	2.779	8.221+	-0.060++	4.588	-2.157	2.165++

Key: * indicates significance at the .05 percentile and ** at .01
+ indicates significance at the .95 percentile and ++ at .99

Note: See Table 6 for critical values.

Table 5B
Phillips-Perron Unit Root Tests on Logged Consumer Price Indexes of Switzerland

$$y_t = \tilde{\mu} + \tilde{\beta}(t-n/2) + \tilde{\alpha} y_{t-1} + \tilde{u}_t$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u^*_t \ , \quad y_t = \hat{\alpha} y_{t-1} + \hat{u}_t$$

$$Z(\Phi_2) : \tilde{\mu} = 0 \ \tilde{\beta} = 0 \ \tilde{\alpha} = 1$$

$$Z(\Phi_3) : \tilde{\beta} = 0 \text{ and } \tilde{\alpha} = 1 \ , \ Z(t_{\hat{\alpha}}) : \hat{\alpha} = 1$$

$$Z(t_{\tilde{\alpha}}) : \tilde{\alpha} = 1 \ , \quad Z(\Phi_1) : \mu^* = 0 \text{ and } \alpha^* = 1, \quad Z(t_{\alpha^*}) : \alpha^* = 1$$

Lags in Newey and West(1987)

Lags	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_1)$	$Z(t_{\alpha}^*)$	$Z(t_{\hat{\alpha}})$
lags = 0	1.734	26.810++	-0.562+	40.165++	-1.833	8.604++
lags = 2	1.493	14.758++	-0.816+	21.626++	-1.619	6.284++
lags = 4	1.458	10.802++	-0.995	15.074++	-1.500	5.222++
lags = 6	1.479	9.099++	-1.089	11.931++	-1.444	4.624++
lags = 8	1.499	8.232+	-1.129	10.091++	-1.419	4.234++
lags = 10	1.522	7.725+	-1.168	8.796++	-1.398	3.936++
lags = 12	1.560	7.424+	-1.227	7.796++	-1.372	3.688++
lags = 14	1.605	7.273+	-1.288	7.022++	-1.347	3.483++
lags = 20	1.705	7.315+	-1.397	5.565+	-1.308	3.056++
lags = 25	1.758	7.619+	-1.447	4.834+	-1.291	2.813++
lags = 30	1.797	8.044+	-1.482	4.317	-1.281	2.625++
lags = 35	1.808	8.534++	-1.489	3.937	-1.278	2.475++
lags = 40	1.793	9.061++	-1.473	3.647	-1.282	2.351++
lags = 45	1.758	9.613++	-1.437	3.418	-1.291	2.246++
lags = 50	1.693	10.164++	-1.366	3.237	-1.312	2.157++
lags = 55	1.627	10.728++	-1.281	3.089	-1.337	2.079++

Key: * indicates significance at the .05 percentile and ** at .01
+ indicates significance at the .95 percentile and ++ at .99

Note: See Table 6 for critical values.

Table 5C
Phillips-Perron Unit Root Tests on Logged Consumer Price Indexes of the United States

$$y_t = \tilde{\mu} + \tilde{\beta}(t-n/2) + \tilde{\alpha} y_{t-1} + \tilde{u}_t$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u^*_t \ , \quad y_t = \hat{\alpha} y_{t-1} + \hat{u}_t$$

$Z(\Phi_2)$: $\tilde{\mu} = 0$ $\tilde{\beta} = 0$ $\tilde{\alpha} = 1$

$Z(\Phi_3)$: $\tilde{\beta} = 0$ and $\tilde{\alpha} = 1$, $Z(t_{\hat{\alpha}})$: $\hat{\alpha} = 1$

$Z(t_{\tilde{\alpha}})$: $\tilde{\alpha} = 1$, $Z(\Phi_1)$: $\mu^* = 0$ and $\alpha^* = 1$, $Z(t_{\alpha^*})$: $\alpha^* = 1$

Lags in Newey and West(1987)

Lags	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_1)$	$Z(t_{\alpha^*})$	$Z(t_{\hat{\alpha}})$
lags = 0	26.016++	178.503++	0.663++	258.906++	-6.768**	18.125++
lags = 2	11.336++	64.957++	0.179++	94.419++	-4.554**	10.916++
lags = 4	7.783++	40.790++	-0.020++	59.096++	-3.817**	8.613++
lags = 6	6.175++	30.304++	-0.140++	43.556++	-3.424*	7.374++
lags = 8	5.212+	24.442++	-0.232++	34.705++	-3.158*	6.565++
lags = 10	4.530	20.701++	-0.319++	28.921++	-2.949*	5.976++
lags = 12	4.040	18.144++	-0.398+	24.851++	-2.783	5.524++
lags = 14	3.692	16.323++	-0.465+	21.850++	-2.654	5.164++
lags = 20	3.116	13.205++	-0.603+	16.274++	-2.403	4.417++
lags = 25	2.890	11.941++	-0.669+	13.596++	-2.278	4.007++
lags = 30	2.768	11.255++	-0.700+	11.782++	-2.195	3.701++
lags = 35	2.701	10.898++	-0.703+	10.474++	-2.141	3.462++
lags = 40	2.662	10.746++	-0.687+	9.481++	-2.102	3.268++
lags = 45	2.641	10.736++	-0.649+	8.706++	-2.077	3.106++
lags = 50	2.628	10.806++	-0.596+	8.084++	-2.061	2.969++
lags = 55	2.620	10.949++	-0.533+	7.572++	-2.049	2.850++
lags = 60	2.615	11.139++	-0.459+	7.146++	-2.042	2.746++

Key: * indicates significance at the .05 percentile and ** at .01
+ indicates significance at the .95 percentile and ++ at .99

Note: See Table 6 for critical values.

Table 5D
Phillips-Perron Unit Root Tests on Logged Consumer Price Indexes of Italy

$$y_t = \tilde{\mu} + \tilde{\beta}(t-n/2) + \tilde{\alpha} y_{t-1} + \tilde{u}_t$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u^*_t \ , \quad y_t = \hat{\alpha} y_{t-1} + \hat{u}_t$$

$$Z(\Phi_2) \ : \ \tilde{\mu} = 0 \ \tilde{\beta} = 0 \ \tilde{\alpha} = 1$$

$$Z(\Phi_3) \ : \ \tilde{\beta} = 0 \text{ and } \tilde{\alpha} = 1 \ , \ Z(t_{\hat{\alpha}}) \ : \ \hat{\alpha} = 1$$

$$Z(t_{\tilde{\alpha}}) \ : \ \tilde{\alpha} = 1 \ , \quad Z(\Phi_1) : \mu^* = 0 \text{ and } \alpha^* = 1, \quad Z(t_{\alpha^*}) \ : \ \alpha^* = 1$$

Lags in Newey and West(1987)

Lags	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_1)$	$Z(t^*_{\alpha})$	$Z(t_{\hat{\alpha}})$
lags = 0	44.544++	285.750++	2.463++	389.278++	-8.247**	19.287++
lags = 2	20.149++	103.165++	1.593++	140.986++	-5.665**	11.575++
lags = 4	14.146++	64.535++	1.319++	87.946++	-4.812**	9.117++
lags = 6	11.055++	47.534++	1.136++	64.248++	-4.301**	7.772++
lags = 8	9.147++	38.091++	0.987++	50.807++	-3.946**	6.892++
lags = 10	7.938++	32.219++	0.885++	42.223++	-3.699**	6.265++
lags = 12	7.046++	28.234++	0.790++	36.204++	-3.502**	5.785++
lags = 14	6.383++	25.404++	0.712++	31.759++	-3.346**	5.403++
lags = 20	5.210+	20.584++	0.595++	23.443++	-3.034*	4.603++
lags = 25	4.636	18.655++	0.541++	19.368++	-2.856	4.152++
lags = 30	4.240	17.663++	0.482++	16.581++	-2.717	3.813++
lags = 35	3.981	17.223++	0.440++	14.573++	-2.613	3.547++
lags = 40	3.802	17.126++	0.395++	13.062++	-2.531	3.331++
lags = 45	3.688	17.253++	0.378++	11.893++	-2.471	3.153++
lags = 50	3.610	17.530++	0.376++	10.960++	-2.426	3.003++
lags = 55	3.559	17.910++	0.400++	10.202++	-2.393	2.874++
lags = 60	3.526	18.363++	0.452++	9.573++	-2.370	2.762++

Key: * indicates significance at the .05 percentile and ** at .01
+ indicates significance at the .95 percentile and ++ at .99

Note: See Table 6 for critical values.

Table 6

Phillips-Perron Unit Root Tests on Logged Consumer Price Indexes

$$y_t = \tilde{\mu} + \tilde{\beta}(t-n/2) + \tilde{\alpha} y_{t-1} + \tilde{u}_t$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u_t^* , \quad y_t = \hat{\alpha} y_{t-1} + \hat{u}_t$$

$Z(\Phi_2)$: $\tilde{\mu} = 0$ $\tilde{\beta} = 0$ $\tilde{\alpha} = 1$

$Z(\Phi_3)$: $\tilde{\beta} = 0$ and $\tilde{\alpha} = 1$, $Z(t_{\hat{\alpha}})$: $\hat{\alpha} = 1$

$Z(t_{\tilde{\alpha}})$: $\tilde{\alpha} = 1$, $Z(\Phi_1)$: $\mu^* = 0$ and $\alpha^* = 1$, $Z(t_{\alpha^*})$: $\alpha^* = 1$

Lags are in Newey and West(1987)

CPI	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_1)$	$Z(t_{\alpha}^*)$	Truncation lags
Germany	2.898	7.285+	-0.214++	6.336+	-2.269	35
United States	2.641	10.730++	-0.649+	8.706++	-2.077	45
Netherlands	5.162+	6.979+	-1.100	6.426+	-3.011*	40
Japan	7.585++	7.177+	-3.505*	6.598++	-3.449**	40
France	1.693	10.165++	1.366++	3.237	-1.312	50
Italy	3.802	17.126++	0.395++	13.062++	-2.531	40
Denmark	3.192	9.556++	0.163++	7.928++	-2.481	25
Swiss	1.705	7.315+	-1.397	5.565+	-1.308	20
Canada	2.997	14.744++	-0.294++	10.452++	-2.256	40

Key: * indicates significance at the .05 percentile and ** at .01
+ indicates significance at the .95 percentile and ++ at .99

Note: Under the null hypothesis the 95% and 99% critical values of $Z(t_{\tilde{\alpha}})$, $Z(\Phi_3)$ and $Z(\Phi_2)$ are -.94 and -.33,4.68 and 6.09,and 6.25 and 8.27 respectively and for $Z(t_{\tilde{\alpha}})$ are -3.96 and -3.441 at 1% and 5%.Also at the 95% and 99% level the critical values of $Z(t_{\hat{\alpha}})$,$Z(t_{\alpha}^*)$ and $Z(\Phi_1)$are 1.28 and 2.00,-0.07 and 0.6,and 4.59 and 6.43.For $Z(t_{\hat{\alpha}})$ and $Z(t_{\alpha}^*)$ -2.58 and -1.95,-3.43 and -2.86 at 1% and 5% respectively.
[See Phillips and Perron(1986)]

Table 7

Second Unit Root Tests on Logged Consumer Price Indexes

A. Phillips-Perron Test ($\Delta y_t = y_t - y_{y-1}$)

$$\Delta y_t = \tilde{\mu} + \tilde{\beta}(t-n/2) + \tilde{\alpha}\Delta y_{t-1} + \tilde{u}_t$$

$$\Delta y_t = \mu^* + \alpha\Delta y^*_{t-1} + u^*_t \;,\; \Delta y_t = \alpha\Delta \hat{y}_{t-1} + \hat{u}_t$$

$Z(\Phi_2)$: $\tilde{\mu} = 0$ $\tilde{\beta} = 0$ $\tilde{\alpha} = 1$, $Z(\Phi_3)$: $\tilde{\beta} = 0$ and $\tilde{\alpha} = 1$, $Z(t_{\hat{\alpha}})$: $\hat{\alpha} = 1$

$Z(t_{\tilde{\alpha}})$: $\tilde{\alpha} = 1$, $Z(\Phi_1)$: $\mu^* = 0$ and $\alpha^* = 1$, $Z(t_{\alpha^*})$: $\alpha^* = 1$

CPI	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_1)$	$Z(t^*_{\alpha})$	$Z(t_{\hat{\alpha}})$	Truncation lags
Germany	287.16++	195.41++	-9.39**	181.75++	-6.82**	-2.86**	35
United States	65.32++	43.73++	-5.28**	32.92++	-3.31**	-1.22	45
Netherlands	859.13++	579.63++	-17.44**	433.24++	-10.79**	-5.56**	40
Japan	1880.04++	1224.78++	-26.94**	1097.16++	-19.02**	-12.60**	40
France	131.61++	88.87++	-8.67**	37.98++	-3.58**	-0.96	50
Italy	104.59++	71.77++	-7.13**	43.11++	-4.05**	-1.05	40
Denmark	775.19++	520.61++	-17.41**	612.54++	-14.81**	-5.19**	25
Swiss	446.05++	298.91++	-12.75**	427.44++	-12.48**	-7.09**	20
Canada	621.80++	421.79++	-15.79**	345.24++	-10.17**	-1.91	40

Key: * indicates significance at the .05 percentile and ** at .01
+ indicates significance at the .95 percentile and ++ at .99

Note: See Table 6 for critical values.

B. Dickey-Pantula tests for two unit roots in logged CPI

Country	Germany	USA	Netherlands	Japan	France
Test Results	$\tau = -1.19$	$\tau_\mu = -2.11$	$\tau = -1.543$	$\tau = -2.09$	$\tau = -.77$
which imply	I(2)	I(2)	I(2)	I(1)	I(2)

Country	Italy	Denmark	Switzerland	Canada
Test Results	$\tau = -.12$	$\tau_\mu = -3.81$	$\tau_\mu = -3.88$	$\tau_\mu = -2.0$
which imply	I(2)	I(1)	I(1)	I(2)

Note: see Dickey and Pantula(1987) for test statistics.

Table 8

Phillips-Perron Tests for One Unit Root on Logged Relative Price Ratios

Lags in Newey and West(1987) are in parenthesis.

Relative Prices		$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_1)$	$Z(t_{\alpha}^{*})$	$Z(t_{\hat{\alpha}})$
1.Against USA CPI							
Germany	(10)	1.439	7.475+	-1.175	9.726++	-1.438	-1.924
	(40)	1.716	6.131	-1.495	3.853	-1.243	-1.677
Canada	(10)	1.721	2.239	-1.849	1.748	-0.840	0.696
	(40)	1.517	2.463	-1.735	1.735	-0.790	0.443
Netherlands	(10)	2.632	4.110	-1.928	3.187	0.468	-0.159
	(40)	3.217	4.323	-2.308	1.386	0.054	-0.652
Japan	(10)	7.103++	5.947	-3.242*	2.246	0.741	0.847
	(40)	6.499++	5.527	-3.279*	0.845	0.117	-0.040
Swiss	(10)	2.865	6.452+	-0.774	9.305++	-2.378	-3.042**
	(40)	2.075	4.027	-1.017	3.841	-1.857	-2.298*
Italy	(10)	3.002	10.995++	-0.128++	13.792++	-2.404	-0.489
	(40)	2.481	11.682++	-0.448+	5.030+	-1.971	-0.805
France	(10)	1.105	3.809	-0.664+	5.085+	-1.421	0.270
	(40)	1.643	4.102	-1.447	2.465	-1.294	-0.303
Denmark	(10)	2.348	3.792	-2.065	3.014	-1.018	0.160
	(40)	1.770	4.435	-1.682	2.228	-1.002	-0.163
2.Against German CPI							
Canada	(10)	2.214	12.369++	-1.110	12.939++	-1.827	-0.512
	(40)	2.149	17.054++	-1.028	4.482	-1.669	-0.835
Netherlands	(10)	9.579++	7.212+	-3.266	10.683++	-4.340**	-4.569**
	(40)	6.669++	5.044	-3.067	6.598++	-3.524**	-3.663**
Japan	(10)	12.508++	8.824++	-4.356**	12.659++	-4.858**	-4.298**
	(40)	9.091++	6.376+	-3.920*	9.461++	-4.277**	-3.579**
Swiss	(10)	3.224	2.247	-0.682+	1.779	-1.782	-1.643
	(40)	2.450	1.681	-0.418+	1.949	-1.903	-1.627
Italy	(10)	5.342+	20.389++	0.530++	26.584++	-3.184*	-0.992
	(40)	3.212	16.529++	0.707++	8.508++	-2.399	-1.094
France	(10)	4.912+	16.577++	1.260++	21.162++	-2.924*	-0.823
	(40)	2.385	15.545++	0.496++	6.689++	-2.068	-0.991
Denmark	(10)	2.163	9.246++	-1.024	9.698++	-1.836	-0.972
	(40)	2.152	12.697++	-0.550+	3.627	-1.819	-1.094

Key: * indicates significance at the .05 percentile and ** at .01
+ indicates significance at the .95 percentile and ++ at .99

Note : Truncation lags are 10 and 40. 10 was chosen since this is normal size in usual cases, and 40 was chosen since this lag was needed in our cases. The choice of lags did not affect our conclusion that these series are at least I(1).

Table 9
Second Unit Root Tests on Logged Relative Price Ratios

Relative Prices		Phillips and Perron Test $Z(t_{\tilde{\alpha}})$	$Z(t_{\alpha}^{*})$	$Z(t_{\hat{\alpha}})$	Dickey and Pantula Test
1.Against USA CPI					
Germany	(10)	-6.063**	-5.837**	-3.628**	τ = -2.92**
	(40)	-6.705**	-6.397**	-4.234**	
Canada	(10)	-13.144**	-13.258**	-12.718**	τ = -2.34**
	(40)	-21.891**	-22.131**	-21.139**	
Netherlands	(10)	-10.715**	-10.596**	-9.327**	τ = -1.83+
	(40)	-15.025**	-14.746**	-12.803**	
Japan	(10)	-16.429**	-15.644**	-14.602**	τ = -1.90+
	(40)	-26.707**	-25.491**	-23.457**	
Swiss	(10)	-10.424**	-9.171**	-6.043**	τ_{μ} = -3.30**
	(40)	-14.752**	-12.563**	-7.671**	
Italy	(10)	-5.141**	-4.624**	-2.605**	τ_{μ} = -2.98*
	(40)	-5.451**	-4.756**	-2.287*	
France	(10)	-7.474**	-7.219**	-5.800**	τ = -1.362
	(40)	-9.498**	-9.025**	-6.981**	
Denmark	(10)	-12.585**	-12.678**	-12.211**	τ_{μ} = -4.918**
	(40)	-20.331**	-20.556**	-19.712**	
2.Against German CPI					
Canada	(10)	-15.487**	-15.065**	-6.750**	τ_{μ} = -3.921**
	(40)	-24.789**	-23.886**	-9.531**	
Netherlands	(10)	-11.129**	-10.224**	-9.888**	τ = -2.46**
	(40)	-16.208**	-14.796**	-14.246**	
Japan	(10)	-14.494**	-12.995**	-12.755**	τ = -3.104**
	(40)	-23.028**	-20.779**	-20.176**	
Swiss	(10)	-15.520**	-14.718**	-14.918**	τ = -2.42**
	(40)	-24.334**	-22.902**	-23.228**	
Italy	(10)	-7.404**	-5.847**	-2.143*	τ_{μ} = -3.17**
	(40)	-9.373**	-6.793**	-1.802	
France	(10)	-7.474**	-7.219**	-5.800**	τ = -0.92
	(40)	-10.319**	-7.910**	-2.518*	
Denmark	(10)	-13.488**	-13.306**	-8.739**	τ_{μ} = -5.99**
	(40)	-22.098**	-21.721**	-13.359**	

Key:1. In Phillips and Perron test statistics, * indicates significance at the .05 percentile and ** at the .01 . + indicates significance at the .95 percentile and ++ at the .99

2. In Dickey and Pantula test statistics, * indicates significance at the 0.025, * at the 0.05 and + at the 0.10.

3. Lags in Newey and West(1987) are in parentheses for Phillips and Perron tests.

Table 10

Vector Autoregressive Estimates for the USA-Germany

$$\Delta x_t = -\Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} - \pi x_{t-3} + \text{const} + \sum_{i=1}^{11} \hat{k}_i Q_{it} + \epsilon_t$$

$$\Gamma_1 = \begin{bmatrix} \underset{(.079)}{.29^{**}} & \underset{(.75)}{.80} \\ \underset{(.008)}{.008} & \underset{(.08)}{.22^{**}} \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} \underset{(.079)}{-.015} & \underset{(.73)}{.95} \\ \underset{(.008)}{.003} & \underset{(.08)}{.11} \end{bmatrix} \quad \pi = \begin{bmatrix} \underset{(.02)}{-.02^{*}} & \underset{(.01)}{.01} \\ \underset{(.001)}{.004^{**}} & \underset{(.001)}{-.003^{*}} \end{bmatrix}$$

Note: x_t = (logged Mark/U\$, logged $CPI_{USA\text{-}Germany}$)'.
t-statistics are in the parentheses;
Key: ** at the .005%, and * at the .05 %.

Estimated Correlations and Variances of Regression Residuals

$$\Gamma_{\epsilon} = \begin{bmatrix} .58382E\text{-}03 & \\ -.69431E\text{-}01 & .30837E\text{-}02 \end{bmatrix}$$

Box-Pierce Q-statistics

	$\Delta p_{USA\text{-}Germany}$	Δ(U\$/Mark)
Q(39)	48.32	37.71

Note: $\chi^2_{30}(p = 0.01) = 50.892$.

Table 11

Test Statistics for the Hypothesis for Various Values of Cointegration Vectors between Log of Exchange Rates and Log of Relative Prices

o AGAINST THE U.S.A.(1974,3 ~ 1988,11)

# of cointe-gration vectors	-2 ln(Q) W.Germany	Switzerland	Japan	Canada
r = 0	13.003*	22.135**	13.062*	6.013
r ≤ 1	0.325	2.056	0.725	1.002

Key: ** denotes significance at the 97.5% quantile and * at the 95% quantile.

o AGAINST THE GERMANY(1974,3 ~ 1988,11)

# of cointe-gration vectors	-2 ln(Q) Switzerland	Japan	Denmark
r = 0	12.152*	25.706**	14.277**
r ≤ 1	1.489	1.315	3.088

Table 12

The Eigenvalues $\hat{\lambda}$ and Eigenvectors $\hat{V}$ and Estimated Alphas

	Germany-USA	Swiss-USA	Japan-USA
Eigenvalues $\hat{\lambda}$	(0.0018, 0.0695)	(0.0116, 0.108)	(0.0042, 0.0688)
Eigenvectors $\hat{V}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} -1.908 & -6.363 \\ -6.595 & 4.353 \end{bmatrix}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} 2.949 & -6.005 \\ 3.332 & 9.068 \end{bmatrix}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} 1.45 & -7.14 \\ -10.27 & 6.32 \end{bmatrix}$
Alpha x 10^3	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} -0.914 & -3.696 \\ -0.053 & 0.722 \end{bmatrix}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} 3.151 & -1.396 \\ 0.058 & 1.327 \end{bmatrix}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} 1.66 & -0.34 \\ -0.02 & 1.70 \end{bmatrix}$

	Swiss--Germany	Japan-Germany	Denmark-Germany
Eigenvalues $\hat{\lambda}$	(0.0084, 0.0591)	(0.0074, 0.129)	(0.017, 0.0619)
Eigenvectors $\hat{V}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} 3.004 & -13.107 \\ -44.960 & 24.662 \end{bmatrix}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} 5.236 & -2.182 \\ 10.755 & 21.557 \end{bmatrix}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} 6.856 & -41.937 \\ -0.994 & 37.831 \end{bmatrix}$
Alpha x 10^3	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} -0.132 & -3.105 \\ -0.326 & 0.128 \end{bmatrix}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} 1.841 & 0.960 \\ -0.033 & 2.462 \end{bmatrix}$	$\begin{array}{l} s \\ rp \end{array}\begin{bmatrix} 0.513 & -1.602 \\ 0.745 & 0.908 \end{bmatrix}$

Note: s denotes logged spot exchange rates and rp denotes logged relative price indexes.

Table 13 Tests of PPP

in $\alpha\bar{s}_t + \phi(\bar{p} - \bar{p}^*)_t = \epsilon_t$, where ϵ_t is stationary.

Exchange Rates	α	ϕ	k	$-\phi/\alpha$	Wald Tests for $\alpha = -\phi$
Mark/U$	-6.36**	4.35**	3	0.68	0.7436
	(146.15)	(5.72)			
Sfr /U$	-6.0**	9.06**	3	1.51	5.05
	(88.21)	(157.48)			
Yen/U$	-7.14**	6.32*	6	0.89	0.112
	(310.4)	(4.84)			
Sfr/Mark	-13.1**	24.86+	4	1.89	0.863
	(209.26)	(3.36)			
Yen/Mark	-2.2*	21.5**	2	9.7	38.39**
	(4.54)	(105.06)			
Kroner	-41.94**	37.83**	4	0.9	5.67
/Mark	(432.23)	(16735.36)			

Note: 1. In the parentheses we have the Wald test result on the significance of each coefficient. ** indicates significance at $\chi^2_1(p=0.01)$, * at $\chi^2_1(p=0.05)$, and + indicates significance at $\chi^2_1(p=0.1)$.

2. "k" denotes the lag term in the vector autoregressive regression [see Eq.(3-2)].

3. $-\alpha/\phi$ equals the coefficient of γ in the equation of $s_t = \alpha + \gamma(p - p^*) +$ constant.

Table 14
Univariate Unit Root Tests to Deviations from PPP

1. Dickey and Fuller(1981) F-Test :

$$Z_t = \beta_o + \beta_1 t + \alpha Z_{t-1} + \sum_{j=1}^{p} \phi_j (Z_{t-j} - Z_{t-j-1}) + \epsilon_t$$

2. Phillips and Perron(1986,1988) :

$$Z_t = \beta_o + \beta_1 (t-T/2) + \phi Z_{t-1} + \epsilon_t$$

	Φ_1	Φ_2	Φ_3	ADF	$Z(t_{\tilde{\alpha}})$
	$H_o:(\beta_1,\alpha)=(0,1)$	$H_1:(\beta_o,\beta_1,\alpha)=(0,0,1)$	$H_2:(\beta_o,\beta_1,\alpha)=(\beta_o,0,1)$		
Denmark (1)	17.81**	16.29**	5.48+	Z_τ=-3.20+	-3.07
-Germany(2)	4.07	4.22	4.92*		
Swiss (1)	7.91**	11.59**	9.49**	Z_τ=-3.98**	-3.17
-Germany (2)	3.55	4.58	5.68*		
Swiss (1)	5.09*	4.62+	5.2	Z_μ=-1.74	-1.75
-USA (2)	1.74	1.19	1.57		
Germany (1)	9.74**	7.51**	1.38	Z_μ=-1.50	-1.29
-USA (2)	0.83	0.60	0.90		
Japan (1)	8.41**	6.57**	1.33	Z_μ=-1.07	-1.27
-USA (2)	1.08	1.14	1.03		

Note: 1.The "(1)" indicates the likelihood ratio statistics under Dickey and Fuller(1981). The distributions are on page 1063 of that paper. ** indicates significance at the 99 % level, * at the 95 %, and + at the 90 % level.

2.The "(2)" indicates the Phillips and Perron test statistics of $Z(\Phi_1)$, $Z(\Phi_2)$ and $Z(\Phi_3)$ respectively.
* indicates significance at the .95 quantile.
The $Z(t_{\tilde{\alpha}})$ is under the null hypothesis of $\phi = 1$ and all test statistics accept the null of a unit root.

3.The distribution of ∴ is from Table 8.5.2 of Fuller(1976).
+ indicates significance at the .10 and ** at the .025.

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III. MULTIVARIATE COINTEGRATION TESTS FOR A SET OF FOREIGN EXCHANGE RATES AND A COMPARATIVE STUDY OF THE FORECASTING ACCURACY OF THE RANDOM-WALK AND THE ERROR-CORRECTION MODELS

1. INTRODUCTION

This study begins with the work of Baillie and Bollerslev (1989) on the common stochastic trends in a system of exchange rates. In their paper, multivariate tests for unit roots showed the existence of one long-run relationship between a set of seven daily exchange rate series during 1980:3:1 through 1985:1:28. This result indicates a perceptible deviation from weak-form efficiency for each of the exchange rates because in the first order error-correction model, if two or more prices of different currencies are cointegrated, part of the changes will usually be predictable. Several questions may be raised by this result, but the discussion centers around two related questions. First, one cointegrating factor between seven exchange rates arises because any two, any three, or any four, etc. are cointegrated. Are any rates redundant to this relationship, or is there one or more driving currency? Second, although each exchange rate series has a univariate representation as a martingale, which is similar to a random walk [see Section 3 for discussion], it is also true that a vector of the first differenced exchange rates should have a lagged error-correction term applied to it, since there is one cointegrating vector between the rates. This result implies that the daily exchange rate in each of the rates is partly determined by an I(0) equilibrium error and will in general be predictable. One interesting question is whether this representation can be used in forecasting, and if it outperforms the random walk. This question is related to the interpretation of the error-correction model;

the error-correction models for cointegrated economic variables are commonly interpreted by Engle and Granger (1987) as reflecting partial adjustment of one variable to another. The motivation for cointegration is that an equilibrium error causes changes in the variables of the model. However, Campbell and Shiller (1988) have an alternative approach, maintaining that the error-correction model may also arise because one variable forecasts another. Campbell and Shiller emphasize the possibility of these changes and that cointegration can arise even in a well-organized market with no adjustment costs. This study follows the general interpretation of error-correction models by Engle and Granger (1987) and analyzes the two questions.

This paper is divided into 5 sections. Section 2 tests for cointegrated vectors on a set of exchange rates, by pair, threesome, and so forth, to find the redundant rates and driving currencies. After examining the random-walk representation of daily exchange rate series in Section 3, out-of-sample forecasts are performed in Section 4 to determine the accuracy of forecasting in the error-correction model compared to the random-walk model. Conclusions follow.

I took the same daily spot exchange rate data which were used by Baillie and Bollerslev (1989), from the New York Foreign Exchange Market between March 1, 1980 and January 28, 1985, which constitutes a total of 1,245 observations. The data were originally provided by Data Resources Incorporated (DRI) and are opening bid prices for the UK pound, West German mark, French franc, Italian lira, Swiss franc, Japanese yen, and Canadian dollar vis-a-vis the US dollar.

2. MULTIVARIATE TESTS FOR UNIT ROOTS IN A SET OF EXCHANGE RATES

With Baillie and Bollerslev's finding of one cointegrating vector, we can estimate the parameters in this long-run relationship between the set of seven exchange rates, and secondly, find redundant currencies in the set by Johansen's technique, which was used by the authors. By redundant currencies we mean currencies that have zero coefficients in the cointegrating vector. Before proceeding, we should briefly explain the Johansen test: Hypothesis testing of a cointegrating vector is originally done with regression estimates; however, the multivariate tests developed by Johansen (1988) are implemented in testing for cointegrating vectors between exchange rate series simultaneously by specifying a model with a vector autoregressive process (VAR). This approach gives more efficient estimates than the conventional regression estimates, since it not only takes into account the error structure of the underlying process, which the conventional regression estimates do not, but it also gives maximum likelihood estimates. The VAR form of our model looks like

$$\Delta s_t = \Gamma \Delta s_{t-1} - \pi s_{t-2} + \epsilon_t \qquad (1)$$

where s_t is a vector of logged daily exchange rates of 7 currencies; UK pound, German mark, Japanese yen, Canadian dollar, French franc, Italian lira, and Swiss franc vis-a-vis the US dollar. ϵ_1 --- ϵ_T are iid $N(0,\Lambda)$. This model is the first differenced form of

$$s_t = \pi_1 s_{t-1} + \pi_2 s_{t-2} + \epsilon_t. \qquad (t=1,\text{---},T)$$

Using $\Delta = 1\text{-}L$, where L is the lag operator, we have the model (1), when

$$\Gamma = -1 + \pi_1 \text{ and } \pi = 1 - \pi_1 - \pi_2.$$

The model (1) is expressed as a traditional first differenced VAR model except for the term πs_{t-2}. If the rank (π) is not full, then the coefficient matrix (π) may convey information about the long-run structure of the chosen data. Johansen's hypothesis testing of cointegration vectors formulates the hypothesis of reduced rank ($=r$) in π, or the hypothesis which implies that there are matrices α and β of order v(= # of variables) x r such that $\pi = \alpha\beta'$, where $\beta' s_t \sim I(0)$. The properties of Johansen's test are as follows:

1. The maximum likelihood estimator of the space spanned by β is the space spanned by r canonical variates, corresponding to the r largest squared canonical correlations between the residuals of s_{t-2} and Δs_t, corrected for the effect of the lagged differences of the s process.

2. The likelihood ratio test statistic for the hypothesis that there are at most r cointegration vectors is

$$-2\ln Q = -T \sum_{i=r+1}^{v} \ln(1-\hat{\lambda}_i)$$

where $\hat{\lambda}_{r+1}$, ----, $\hat{\lambda}_v$ are the v-r smallest squared canonical correlations.

3. Under the hypothesis that there are r cointegrating vectors, the estimate of cointegration space as well as π and Λ are consistent, and the likelihood ratio test statistic of this hypothesis is asymptotically distributed as

$$\mathrm{tr}\left(\int_0^1 B\,dB' \left[\int_0^1 B(u)B(u)'du \right]^{-1} \int_0^1 dB\,B' \right)$$

where B is a v-r dimensional Brownian motion with covariance matrix I. The proofs and derivation of each parameter are presented in Johansen (1988).

Given such properties, in order to find currencies redundant to the existence of one long-run relationship between a set of seven daily exchange rates, I first performed Wald tests to examine the significance of each coefficient in the cointegrating vector among seven currencies with the hypothesis

$$k'\beta = (\ 0,\ 0,\ 1,\ 0,\ \cdots,\ 0) \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_7 \end{bmatrix} = 0$$

where β_i is the coefficient to be tested for significance. The test statistic is $T^{1/2}k'\hat{\beta}\ /\ \{\ (\ \hat{\lambda}_i^{-1} - 1)(\ k'\hat{\gamma}\hat{\gamma}'k\ )\}^{1/2}$ with χ^2_1. Here $\hat{\lambda}_1$ is the maximal eigenvalue, $\hat{\beta}$ the corresponding eigenvector, and the remaining eigenvectors form $\hat{\gamma}$ [see Johansen and Juselius (1988) Corollary 3.17].

As can be seen in Table 2, most elements of cointegrating vector are significant at the 99% level, except those of the UK pound and Japanese yen against the US dollar. With this result, we can expect that the Pound and the Yen may be redundant currencies to the existence of a long-run relationship. This turns out to be true, and without the U$/pound or U$/yen, a set of six currencies still have at least one long-run relationship [Table 1]. Also, without both the Pound and Yen, one long-run relationship still exists with the remaining five currencies (German mark, French franc, Italian lira, Canadian dollar, and Swiss franc against the US dollar) [see Table 1 column 4]. This result implies that, during the sample period, both the Japanese yen and the UK pound played no important role for the existence of a long-run relationship between industrial-country currencies.

Even if each of the five remaining exchange rates had a significant coefficient in the long-run relation, I continued to test cointegrating vectors by reducing specific currencies one by one, so as to examine whether there are other redundant currencies, or to find some driving currencies [see Table 1]. First, without the Swiss franc/U$ rate, I found one cointegrating vector in the remaining six exchange rates. Second, I tested whether the Canadian dollar is redundant by testing cointegrating vectors without the Pound, Yen, Swiss franc and Canadian dollar. The remaining currencies, Mark, Lira and French franc, showed one cointegrating vector at the 95% level. Further reduction of a specific exchange rate can not reject the null hypothesis of no cointrgrating vector [see Table 1, Part 2]. We can conclusively say that the redundant currencies in the long-run relations between the seven currencies are the UK pound, Japanese yen, Swiss franc, and Canadian dollar. The remaining currencies, Mark, Lira and French franc, are major European Monetary System (EMS) currencies [1] and contribute to a long-run relation during our sample period. This result is consistent with the general view that the EMS has been successful in contributing to exchange rate stability among participating countries. However, this relative stability of internal EMS currencies is coincident with a lack of external tension in the system due to a strong and rising US dollar during the sample period. Here we can raise a question whether there are any causality relations between EMS stability and the US dollar strength. During the sample period, the strong and rising dollar was influenced by the monetary policy in the United States that led to high nominal and real US interest rates, both

[1]The EMS currencies include the German mark, French franc, Italian lira, Belgian franc, Netherlands guilder, Irish pound, and Danish krone. Among them the Mark acts as a leading currency and the French franc and Lira are from major industrial countries.

in absolute terms and relative to other countries. The value of the European Currency Unit (ECU) in terms of dollars, which had been as high as U$ 1.44 at the end of 1979, had fallen to a little less than a dollar by the turn of 1982, and was less than 0.7 at the beginning of 1985 [see Charts]. Even if the US dollar gradually appreciated relative to European currencies during our sample period, giving Germany a favorable current account and making a continuous difference in the participating countries' external positions, our cointegration test shows that the EMS experienced relative internal stability. This may be explained as follows: first, the EMS-participating members made more cooperative efforts for exchange rate stability between them; the EMS had five realignments from March 23, 1981 to July 22, 1985. Secondly, a more reasonable explanation is that the Mark, the leading EMS currency, did not come under upward pressure within the EMS, largely because of strong capital flows to the United States due to the strong US dollar. With cointegrating vector tests without the Mark, we accept the null hypothesis of no cointegration vector with six remaining currencies. This may imply that the Mark played a key role as a driving currency, as did three other EMS currencies together, to have a long-run relationship during our sample period [see Table 1, Part 2]. Since we found redundant currencies in the long-run relationships, we explored short-run movements of exchange rates and the comparison the forecasting accuracy of the random-walk and the error-correction models from the set of remaining currencies, Mark, Lira and French franc, from that point on.

In the VAR model, the vector of changes in the three exchange rates at time t are expressed with lagged changes in the exchange rates and error-correction terms as well as a constant term.

$$\Delta s_t = \text{constant} + \Gamma\ \Delta s_{t-1} - \pi s_{t-2} + \epsilon_t$$

where Δs_t is a vector of changes of logged exchange rates of the Mark, French franc, and Lira against US dollar, and Γ and π are matrices of order 3x3, respectively. The estimated error-correction model is reported in Table 4; with one lagged first differenced term, the residuals for the exchange rate data clearly passed the test for no autocorrelation. Based on the estimated coefficient matrix of $\hat{\pi}$, which conveys information about the long-run relationship between a set of three exchange rates, we can explore short-run movements between these exchange rates with Johansen's method. Maximum likelihood estimates of α and β in $\pi = \alpha\beta'$ are derived, and Table 3 reports the estimates of cointegrating vector β as the third column in $\hat{V}$ and α as the third column in the matrix of Alphas. Tests for the significance of each element of the cointegrating vector are reported, and the results show that all elements are significant at the 99% level by Wald tests. Here we can raise a question: Each exchange rate series is said to have a univariate representation of being a martingale. But we found that a vector of the first differenced exchange rates should have a lagged error-correction term applied to it, since there is one cointegrating vector between them. Can this representation be used in forecasting? Or, does it outperform the random-walk forecasts? In Section 3, the univariate representation of a random walk is examined and we will compare the forecasting accuracy of an error-correction model with that of a random-walk model in Section 4.

3. ARIMA MODELS OF EXCHANGE RATE SERIES

Univariate autoregressive moving average models for the endogenous variables of a dynamic simultaneous equations system can be interpreted as a form of solution to the system [see Zellner and Palm (1974) and Wallis (1977)]. Under this methodology, if the log of bilateral exchange rate is generally approximated by a random-walk model, then the stochastic processes generating the exogenous variables should also be random-walk models. For example, consider the following monetary model (Baillie and Selover (1987)):

$$s_t = a_1(m_t - m^*_t) + a_2(y_t - y^*_t) + a_3(r_t - r^*_t) + a_4E_t(p_{t+1}-p^*_{t+1}) + \epsilon_t.$$

where s_t is the logarithm of the nominal exchange rates, m,y and r represent the logarithms of domestic money supply, real output and short term interest rates, E_tp_{t+1} is the expected domestic rate of inflation; asterisks denote foreign quantities and ϵ_t is a stochastic disturbance term. If the exchange rate is a random-walk model, then $m_t - m^*_t$, $y_t - y^*_t$, $r_t - r^*_t$, etc should also be random-walk models. In this section we will examine the ARIMA model of exchange rate series to see its implication of random-walk models.

Our empirical analysis begins with fitting univariate ARIMA models to the individual exchange rate series. When we plot the sample autocorrelation functions (ACF) for a sample of 1245 observations, it can be seen from Table 5 that all of the autocorrelations of the seven exchange rate series lie outside of the bounds $\pm$ 1.96 $n^{-½}$, which implies significance different from zero at the 5% level. The partial autocorrelation functions (PACF) strongly suggest that the appropriate models for this data are AR(1) processes. After first differencing, the autocorrelations of the five currencies excluding the Canadian dollar and Italian lira, lie between the bounds $\pm$ 1.96 $n^{-\frac{1}{2}}$, and we can not reject the hypothesis that the first differenced one is a white process i.e., (0,1,0). In the case of the Canadian dollar against the US dollar, however, inspection of the graph of ACF and PACF of the first differenced suggests that the appropriate model for this series may be an ARIMA (p,1,0) process [see Table 5-B]. I estimated AR(p) models for p = 1,2,. . .,12 and checked the significance of each AR coefficient; none of the coefficients have significant values at any reasonable levels. From the PACF graph, we may spot an autoregressive seasonal at lag nine; however, it is not significant at the 95% level. The implication of ARIMA (p,1,0) is applied to the Italian lira against the US dollar, also [see Table 5-C]. As can be seen from the PACF, the log of first differenced Lira may have significant autoregressive seasonals at lag 13 and 23. However, they turn out to be insignificant in each coefficient at the 95% level.

Overall, a random-walk model appears to describe the stochastic process of each daily spot exchange rate series adequately as $\Delta s_t = \epsilon_t$. This result imposes strong a priori restrictions in any exchange rate model. The error-correction model, where the vector of first differenced exchange rates have a lagged error-correction term is:

$\Delta s_t = \Delta s_{t-1} - \pi s_{t-1} + \epsilon_t$, in the VAR form.

The result for a specific rate i from the VAR showed as

$\Delta s_{it} = \sum_{j=1}^{N} \theta_j \Delta s_{jt-1} - \sum_{j=1}^{N} \pi_j s_{jt-2} + \epsilon_{it}$, with i = 1,2,. .,N.

If the matrix of π has significant elements [see Table 4], does this error-correction model outperform the random-walk model? We will examine the forecasting accuracy of the error-correction model relative to the random-walk model in the next section.

4. A COMPARATIVE STUDY OF THE FORECASTING ACCURACY OF THE ERROR-CORRECTION AND THE RANDOM-WALK MODELS

This section compares the out-of-sample forecasting accuracy of the error-correction model (hereafter, ECM) and the random-walk model for the French franc/U$ rates. The selected currency is an EMS currency for two reasons: first, after eliminating the redundant currencies from a set of seven currencies, the three currencies (Lira, Mark and French franc) vis a vis the US dollar show one long-run relationship in the cointegrating vector tests and three currencies make the computing work easier; and, secondly, among three currencies, the Franc/U$ rate has significant coefficients in the error-correction term compared to the other two currencies [see Table 4]. In our experiment, five models are set to compare their forecasting accuracies; in addition to the random-walk and the error-correction models, two modified versions of error-correction models and an unrestricted VAR model are included. The specific form of each model is given below. The parameters of each model are estimated on the basis of the most up-to-date information available at the time of a given forecast. This is accomplished by using rolling regressions to re-estimate the parameters of each model every forecast period. First, the random-walk model uses the current spot rate as a predictor of all future spot rates. I estimated a random-walk model with a drift term, which is very significant.

$$\Delta s_t = \text{constant} + \epsilon_t \qquad (1)$$

The second model, the ECM, is:

$$\Delta s_{it} = \alpha + \sum_{j=1}^{N}\phi_j \Delta s_{jt-1} - \psi(\beta' s_{t-2}) + \epsilon_t \qquad i = 1,2,..,N. \qquad (2)$$

where $\beta' s_{t-2}$, with β and s_{t-2} as vectors, is the deviation from the long-run relation which we obtained in Section 2. Since the coefficient of Δs_{t-1} for Lira rates is insignificant in our study [see Table 4], I modify the ECM to have the third model, ECM-1, where insignificant coefficients are excluded:

$$\Delta s_{it} = \alpha + \sum_{j}\phi_j \Delta s_{jt-1} - \psi(\beta' s_{t-2}) + \epsilon_t \qquad i = 1,2,..,N. \qquad (3)$$

Also, I estimate the ECM with an error-correction term in the RHS as the fourth model, ECM-2:

$$\Delta s_{it} = \alpha - \psi(\beta' s_{t-1}) + \epsilon_t \qquad (4)$$

Finally, I use the VAR without any restrictions as our fifth model;

$$\Delta s_t = \phi \Delta s_{t-1} - \pi s_{t-2} + \epsilon_t \qquad (5),$$

where s_t is a vector of logged exchange rates of the Franc/U$, Mark/U$ and Lira/U$.

These five models are estimated by OLS over a daily data series starting in March 1, 1980 and extending through April 30, 1984. Data ranging over May 1, 1984 to January 28, 1985 have been retained for ex-post out-of-sample forecasting exercises. Forecasts are generated at horizons of one through thirty days. Out-of-sample accuracy for each model is measured by three statistics: mean error (ME); mean absolute error (MAE); and the principle criterion, root mean square error (RMSE) [see Meese and Rogoff (1983,a) for their definitions]. Table 6 lists the forecast errors for the Franc/U$ rates at specific horizons. Each parenthesis contains a rank for each model. The striking feature of Table 6-A is that the random-walk model doesn't achieve lower RMSE than our ECM;

Although the differences in the RMSEs are small, the ECM still outperforms the random-walk model. The modified versions of ECM, i.e., Eqs. (3) and (4), and the unrestricted VAR do not outperform the random-walk model or our ECM. Overall, with RMSEs, the random--walk forecasts are not more accurate than our ECM forecasts. The MAE, which is less sensitive to outlier observations, shows a slightly different pattern [see Table 6-B]; our ECM outperforms the random-walk model up to horizon 12, and from horizon 18, the random-walk forecasts outperform our ECM forecasts. However, the difference in forecasting errors of MAEs is very small compared to that of the RMSEs. In this case, also, the modified versions of ECM and the unrestricted VAR do not outperform our model or the random-walk model. The mean errors of the various models are listed in Table 6-C. They are smaller relative to the corresponding MAE, indicating that the models do not systematically over- or under-predict.

Overall, with our examination of forecasting accuracy, we may conclude that the random-walk forecast is less accurate than our error-correction model, in which a vector of first differenced exchange rates has a lagged error-correction term. Although the forecasting errors (especially, RMSEs and MAEs) are not significantly different from each other (the difference between the error-correcton model and the random-walk model is 0.5 %, on average), an obvious conclusion is that the random-walk model can not outperform the error-correction model.

5. CONCLUSIONS

This study found that the EMS currencies contributed to the stability of a set of seven currencies, and that the stabilities of EMS currencies were coincident to the strong US dollar during our sample period, raising the question of whether we can find a causal relationship between them. We made a comparative study of the forecasting accuracy of the error-correction and the random-walk models. Our error-correction forecasts showed a little improvement in accuracy compared to the random-walk forecasts. This result reminds us of the Meese and Rogoff demonstration of the superiority of the random-walk model not only to asset market models but also to all economic time series models, generally; Meese and Rogoff (1983,a,b) found that the VAR models' forecasts did not improve on their structural models, both being no better than a random-walk model. However, their VAR model with lagged explantory variables did not consider cointegration, i.e., whether the exchange rate and a given set of explanatory variables are cointegrated. Cointegration was rejected by Baillie and Selover (1987), for example. As shown by Engle and Granger (1987), cointegration is a necessary and sufficient condition for a vector of variables to bear an equilibrium relationship. Considering the cointegration, I applied the VAR methodology to our model while introducing the error-correction term, which proves to be significantly different from zero, as an independant variable. The result was that the random-walk model did not outperform the forecastinng performance of the VAR model with an error-correction term applied to it. With root mean square error statistics, the random-walk model was marginally less than the error-correction model. This study did not compare the forecasting accuracy of our error-correction model with that of asset-market models. In Woo's paper (1985), a monetary model with lagged dependent variables outperforms the random-walk in forecasting the

Mark/U$ rates at one- to twelve-month horizons. However, before comparing the forecasting accuracies, it is useful to check whether the exchange rate and a given set of explanatory variables are cointegrated as mentioned above. Then it is necessary to compare the stochastic processes of structural and time-series exchange-rate models according to the methodology developed by Zellner and Palm (1974) [See Ahking and Miller (1987) for its application to the exchange rate series. They rejected the univariate representation of the asset-market models.].

Table 1
Multivariate Tests for Cointegration Vectors in the Logarithms of Daily Spot Exchange Rates[a]

Dates: 1980:3:1 through 1985:1:28
Exchange rates(U$ against domestic currency): UK pound, German mark(DM), Japanese yen, Canadian dollar, French franc(Ffr), Italian Lira and Swiss franc(Sfr)

r	7 rates	r	6 rates w/o Pound	r	6 rates w/o Yen	r	5 rates w/o Yen & Pound	r	6 rates w/o Sfr	Quantiles 95%	Quantiles 99%
6	0.97	5	1.57	5	1.57	4	0.08	5	1.52	4.2	5.2
5	5.41	4	5.76	4	6.32	3	4.17	4	5.87	12.0	15.6
4	11.31	3	12.57	3	14.23	2	11.64	3	11.92	23.8	28.5
3	23.14	2	27.71	2	35.29	1	30.81	2	30.45	38.6	44.5
2	46.21	1	47.51	1	60.91*	0	61.38*	1	53.92	57.2	63.9
1	77.36	0	79.61*	0	93.27**			0	83.37*	78.1	86.6
0	124.64**									103.1	112.7

r	4 rates w/o Yen, Pound & Sfr	r	6 rates w/o DM	r	3 rates w/ EMS[b]	r	2 rates w/ DM & Ffr	r	2 rates w/ DM & Lira	r	2 rates w/ lira & Ffr
3	1.39	5	0.07	2	1.25	1	0.04	1	0.12	1	0.10
2	6.55	4	4.66	1	8.32	0	3.91	0	7.01	0	9.75
1	17.41	3	11.25	0	27.23*						
0	39.87*	2	25.12								
		1	49.09								
		0	77.76								

a. Tests for r cointegration vectors in a VAR(1). This is a likelihood ratio test, $-2\ln(Q_r)$, for there being at most r cointegrating vectors with $r = 0, 1, 2, —, (p-1)$, where p denotes the number of variables [See Johansen (1988) for the details of the tests.].

b. EMS in our Table denotes the currencies of DM, Ffr and Lira.
* Denotes significance at the 95 % percentile.
** Denotes significance at the 99 % percentile.

Table 2

Eigenvalues, Eigenvectors and Estimated Alpha Coefficients with 7 Daily Exchange Rates[a]

EIGENVALUES:

(0.00078, 0.00357, 0.00473, 0.00947, 0.01839, 0.02474, 0.03733)

EIGENVECTORS(ν):

$$\begin{bmatrix} 14.9559 & 3.8858 & 3.18280 & -15.6548 & 11.2340 & 2.0151 & -2.2797 \\ -4.7153 & -12.136 & 13.5431 & -41.8736 & -3.0814 & 35.958 & -87.952 \\ -7.9889 & -3.4311 & -15.809 & -5.43268 & 2.96194 & 2.1741 & 2.3036 \\ 18.2713 & -21.944 & -21.109 & -11.5129 & -52.381 & -23.485 & -45.857 \\ -9.9867 & 13.769 & -0.2498 & 10.64385 & 11.1770 & -2.5308 & -70.601 \\ -5.2988 & -4.9388 & -3.2733 & 13.86174 & -19.861 & -16.256 & 111.360 \\ 11.6183 & 3.2015 & -6.6825 & 34.53311 & 12.8260 & -0.3785 & 51.6742 \end{bmatrix}$$

ESTIMATE OF ALPHA *1000:

$$\begin{bmatrix} 0.05409 & 0.01680 & -0.05443 & -0.46629 & 0.44189 & -0.23807 & -0.31666 \\ 0.01663 & 0.07516 & -0.20700 & -0.43194 & 0.41452 & 0.45515 & -0.14963 \\ -0.02860 & -0.01200 & -0.36097 & -0.12794 & 0.51288 & -0.08337 & -0.05450 \\ 0.03004 & -0.04313 & -0.92046 & -0.04543 & -0.02583 & 0.08178 & -0.28720 \\ 0.01949 & 0.20188 & -0.17814 & -0.29764 & 0.51495 & 0.39532 & -0.52211 \\ 0.04275 & 0.18192 & -0.23069 & -0.33169 & 0.24361 & 0.23297 & -0.07205 \\ 0.07593 & 0.02410 & -0.20922 & -0.30047 & 0.65882 & 0.47703 & 0.01148 \end{bmatrix}$$

TEST FOR SIGNIFICANCE OF ESTIMATED BETA[b]:

BETA' =

(-2.28,	-87.95**,	2.30,	-45.86**,	-70.62**,	111.35**,	51.675**)
(0.40)	(109.37)	(0.69)	(21.62)	(449.87)	(653.90)	(83.19)

---

a. The exchange rates (U$ against domestic currency) are as follows in order: UK pound; German mark; Japanese yen; Canadian dollar; French franc; Italian lira; and Swiss franc.

b. The parentheses have the results of Wald tests with χ^2_1(p= 99.0%) = 6.63.

** Denotes significance at the 99 % level.

Table 3

Eigenvalues, Eigenvectors and Estimated Alpha Coefficients
With 3 Daily Exchange Rates (D-mark, French franc and Italian lira)

EIGENVALUES:

(0.00100627, 0.00567086, 0.015118)

EIGENVECTORS($\hat{\nu}$):

$$\begin{bmatrix} -27.36069 & 66.85266 & -52.68962 \\ -6.71682 & -36.13997 & -115.37290 \\ 14.72755 & -5.64971 & 152.96214 \end{bmatrix}$$

ESTIMATE OF ALPHA *1000:

$$\begin{bmatrix} -0.089277 & 0.079985 & -0.055994 \\ -0.088014 & 0.018223 & -0.205984 \\ -0.086243 & -0.015824 & -0.027654 \end{bmatrix}$$

TEST FOR SIGNIFICANCE OF ESTIMATED BETA:

$$\text{BETA}' = \begin{pmatrix} -52.69^{**}, & -115.37^{**}, & 152.96^{**} \\ (10.15) & (187.96) & (1794.21) \end{pmatrix}$$

Note: The pharentheses have the results of Wald tests with $\chi^2_1(p = 99.0\ \%) = 6.63$.

** Denotes significance at the 99 % level.

Table 4

Vector Autoregressive Estimates with Lag 1 for 3 Daily Exchange Rates: D-mark, French franc and Italian lira

- $\Delta s_t = \text{constant} + \Gamma \Delta s_{t-1} - \pi s_{t-2} + \epsilon_t$
  where $s_t = (\text{ D-mark(DM), French franc(FFR), Italian lira(LIRA) })'$

$$\begin{pmatrix} \Delta DM_t \\ \Delta FFR_t \\ \Delta LIRA_t \end{pmatrix} = \begin{pmatrix} \underset{(0.073)}{0.056} \\ \underset{(0.077)}{0.179^{**}} \\ \underset{(0.06)}{0.03} \end{pmatrix} + \begin{pmatrix} \underset{(0.07)}{0.1} & \underset{(0.06)}{-0.09} & \underset{(0.07)}{-0.06} \\ \underset{(0.08)}{0.18^{**}} & \underset{(0.08)}{-0.20^{***}} & \underset{(0.08)}{-0.04} \\ \underset{(0.07)}{0.29^{***}} & \underset{(0.06)}{0.016} & \underset{(0.07)}{-0.37^{***}} \end{pmatrix} \begin{pmatrix} \Delta DM_{t-1} \\ \Delta FFR_{t-1} \\ \Delta LIRA_{t-1} \end{pmatrix}$$

$$- \begin{pmatrix} \underset{(0.007)}{-0.01} & \underset{(0.01)}{0.004} & \underset{(0.01)}{-0.01} \\ \underset{(0.008)}{-0.01^{*}} & \underset{(0.01)}{0.02^{**}} & \underset{(0.01)}{-0.03^{**}} \\ \underset{(0.007)}{-0.003} & \underset{(0.009)}{0.004} & \underset{(0.001)}{-0.005} \end{pmatrix} \begin{pmatrix} DM_{t-2} \\ FFR_{t-2} \\ LIRA_{t-2} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \end{pmatrix}$$

* Denotes significance at the 95 % level, ** at the 97.5 % level and *** at the 99.5 % level.

- VARIANCE AND CORRELATION MATRIX

	ΔDM	ΔFFR	ΔLIRA
ΔDM	.48983E-04	.89	.91
ΔFFR		.55737E-04	.89
ΔLIRA			.39647E-04

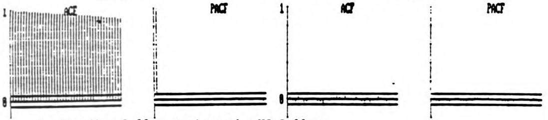
- BOX-PIERCE Q-STATISTICS

	Q(105)
ΔDM	105.245
ΔFFR	113.096
ΔLIRA	143.060

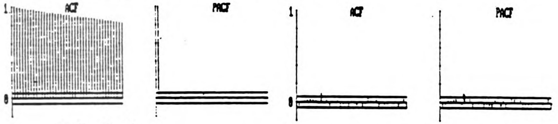
Table 5
Autocorrelation Functions (ACF) and Partial Autocorrelation Functions (PACF)

A. General Results of Five Exchange Rate Series Excluding Can. $ and Lira

Level | First Diferenced



B. Canadian Dollar Against the US Dollar



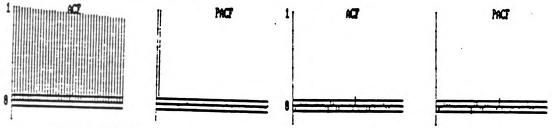
AR Coefficients

-.039	-.011	.016	.027	.034	.028	.011	-.014	.087	-.003	.036	-.031

Ratio of AR Coefficients to (1.96*standard error)

-.704	-.200	.298	.495	.620	.518	.197	-.262	1.568	-.588	.649	-.573

C. Italian Lira Against the US Dollar



AR Coefficients

-.053	-.000	.041	.018	.016	.008	-.004	.014	.038	.020	.013	-.032
-.070	.045	.040	-.034	-.052	.017	.008	-.001	.015	-.031	.079	-.050

Ratio of AR Coefficients to (1.96*standard error)

-.950	-.006	.736	.331	.297	.160	-.082	.264	.703	.367	.246	-.579
-1.27	.822	.734	-.622	-.949	.319	.141	-.024	.276	-.568	1.432	-.912

Table 6
Initial Estimate Period: 1980:3:3 ~ 1984:4:30
Forecasing Period: 1984:5:1 ~ 1984:12:7

A. Forecasting Percentage RMSE (Root Mean Square Error)

Horizon	Random Walk	ECM	ECM-1	ECM-2	VAR
3	.7519(2)	.7481(1)	.7625(5)	.7543(3)	.7588(4)
6	.7552(2)	.7510(1)	.7650(5)	.7572(3)	.7582(4)
9	.7521(2)	.7484(1)	.7623(5)	.7544(3)	.7593(4)
12	.7495(2)	.7458(1)	.7592(5)	.7520(3)	.7537(4)
15	.7478(2)	.7439(1)	.7577(5)	.7503(3)	.7531(4)
18	.7529(2)	.7496(1)	.7631(5)	.7556(3)	.7541(4)
21	.7593(2)	.7554(1)	.7695(5)	.7620(4)	.7607(3)
24	.7622(2)	.7583(1)	.7724(5)	.7648(4)	.7646(3)
27	.7703(2)	.7666(1)	.7807(5)	.7730(4)	.7704(3)
30	.7744(2)	.7703(1)	.7850(5)	.7770(4)	.7768(3)

B. Forecasting Percentage MAE (Mean Absolute Error)

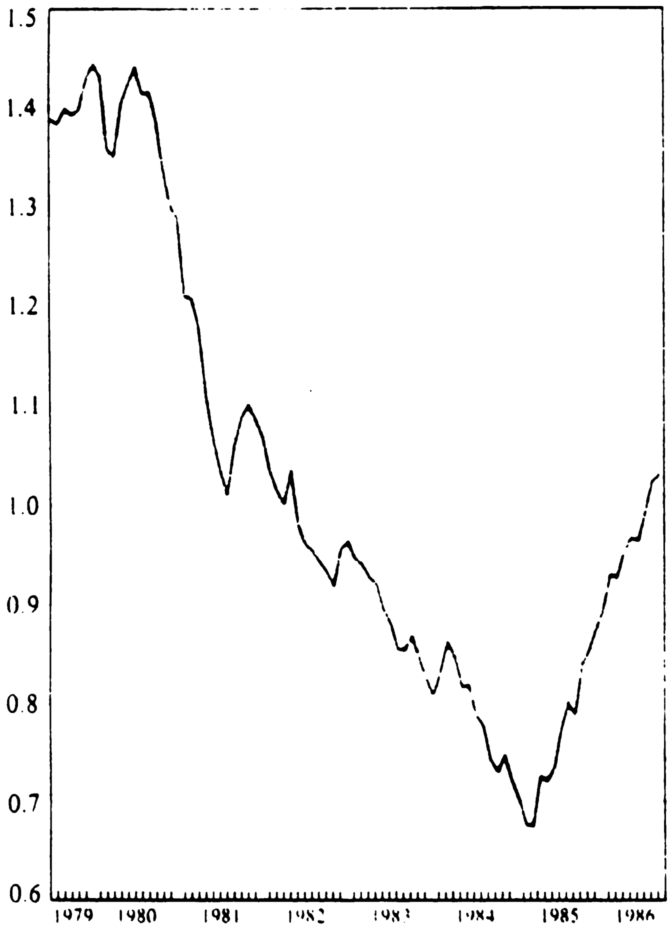
Horizon	Random Walk	ECM	ECM-1	ECM-2	VAR
3	.5361(2)	.5354(1)	.5454(4)	.5376(3)	.5517(5)
6	.5423(2)	.5409(1)	.5513(5)	.5431(3)	.5503(4)
9	.5381(2)	.5375(1)	.5478(5)	.5399(3)	.5521(4)
12	.5339(2)	.5338(1)	.5426(4)	.5361(3)	.5433(5)
15	.5302(1)	.5302(1)	.5392(4)	.5330(3)	.5412(5)
18	.5328(1)	.5337(2)	.5417(5)	.5353(3)	.5382(4)
21	.5373(1)	.5378(2)	.5464(5)	.5404(3)	.5433(4)
24	.5362(1)	.5367(2)	.5455(5)	.5392(3)	.5438(4)
27	.5457(1)	.5471(2)	.5556(5)	.5488(4)	.5477(3)
30	.5464(1)	.5473(2)	.5564(5)	.5494(3)	.5539(4)

C. Forecasting Percentage ME (Mean Error)

Horizon	Random Walk	ECM	ECM-1	ECM-2	VAR
3	-.0277(3)	-.0125(1)	-.0155(2)	-.0180(4)	-.1022(5)
6	-.0293(4)	-.0136(1)	-.0170(2)	-.0185(3)	-.0968(5)
9	-.0163(4)	-.0000(1)	-.0034(2)	-.0042(3)	-.0966(5)
12	-.0209(4)	-.0044(1)	-.0082(3)	-.0080(2)	-.0918(5)
15	-.0121(4)	-.0045(3)	.0008(1)	.0014(2)	-.0922(5)
18	-.0073(3)	-.0088(4)	.0049(1)	.0063(2)	-.0861(5)
21	-.0057(1)	-.0105(4)	.0062(2)	.0077(3)	-.0856(5)
24	-.0099(4)	-.0071(3)	.0030(1)	.0037(2)	-.0913(5)
27	-.0078(3)	-.0088(4)	.0045(1)	.0058(2)	-.0852(5)
30	-.0100(4)	-.0069(3)	.0024(1)	.0035(2)	-.0825(5)

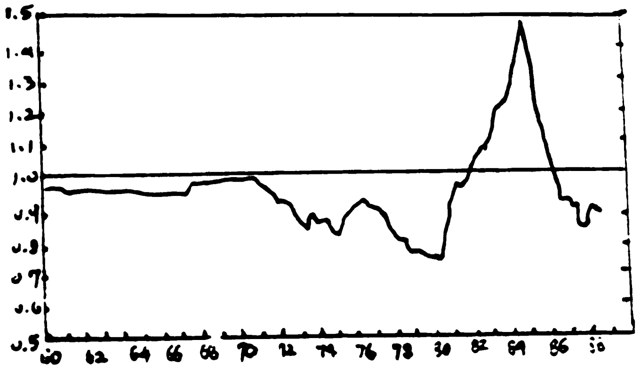
Charts
Movements of the European Currency Unit (ECU) Against the US Dollar

A. US Dollar per ECU, Momthly Average



Source: International Monetary Fund, *International Financial Statistics*, various issues.

B. ECU per US Dollar, Quarterly Average



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