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By

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ABSTRACT

A GAME-THEORETIC ANALYSIS OF TRANSFER PRICE NEGOTIATION UNDER INCOMPLETE INFORMATION CONDITIONS

By

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Negotiations between division managers have been widely used for transfer pricing decisions in many decentralized companies. The objectives of the study are (1) to characterize strategic behaviors of division managers when negotiating transfer prices, and (2) to investigate potential bargaining inefficiency problems resulted from negotiations under uncertainty conditions.

A bargaining model is constructed so that an uninformed selling division manager offers both prices and quantity of intermediate products to a buying division manager with private information. Three different solution concepts are computed depending on the level of central headquarters' intervention in pricing decisions: bargaining equilibrium prices (fully autonomous pricing decisions), constrained Pareto optimal prices (partially autonomous/centralized pricing decisions), and Pareto optimal prices (fully centralized pricing decisions). They are compared with respect to the individual divisions' and firm's overall profits.

The bargaining model is analyzed both for one-period and for two-period settings. The present study shows a significant loss of efficiency at the bargaining equilibrium price in the single-period model. The results become worse in the two-period model, in which uncertainty and time-preference (discounting factors) of the bargainers counterbalance in determination of bargaining equilibrium prices and consequent inefficiency problems. It is also found that a bargainer with incomplete information suffers more than his opponent from heavy discounting factors.

To my wife, Whang-Hee, and my parents

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INTRODUCTION

> The trend towards large decentralized business enterprises has increased the importance of the internal exchange of goods and services among various operations organized as responsibility centers (departments, divisions, subsidiaries). Internal transactions, usually called transfers, may represent a sizable activity for some responsibility centers even where the total volume of transfers is not substantial for the business entity as a whole (Benke and Edwards [1980], p. iii).

As business organizations become larger and more diversified, decentralization becomes an acceptable means of exercising management control for such complex organizations[1]. In particular, the existence and growth of large integrated and conglomerate business organizations has commonly resulted in the phenomenon of interdependency among divisions and consequent inter-divisional product transfer (McNally [1973], p. 13).

One of the rationales for decentralization is that

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[1] An earlier survey conducted by Mauriel and Anthony [1966] showed that of the 2,658 companies (with sales in excess of $20 million) in the sample, 81 percent had adopted the (decentralized) profit center concept to control and evaluate divisions. More recently, an empirical study by Caves [1980] in the U.S.A., U.K., France, and Italy over the period 1950-1970 reports a trend toward the use of decentralized structures in diversified firms.

corporate management personnel of large organizations are likely to experience bounded rationality (due to either limited abilities or limited information) when faced with complex decision problems and, therefore, prefer to take advantage of the division managers' special knowledge of local operational information[2]. Given a decentralized organizational structure, division managers are given autonomy to make pricing, production, and other operating decisions according to their individual interests.

The transfer pricing issue arises in decentralized firms when goods and services are transferred between the firm's various divisions, and divisional performances are evaluated in accordance with the profit center concept. Umapathy [1978] reports that 249 firms (85 percent) of the 291 largest decentralized manufacturing firms transfer products among profit centers (p. 169).

I.A Motivation

Over the past three decades there has been some research on transfer pricing issues. Much of this research has been devoted to finding optimal transfer pricing methods from a theoretical perspective. In general, various transfer pricing models developed by accounting and economic researchers fall into one of the following three categories:

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[2] Central management's resource decisions may be sub-optimal under uncertainty conditions, due to incomplete information (Waterhouse and Tiessen [1978] p. 72).

classical economic models, mathematical programming approaches, and behavioral models. The first two categories encounter strong objections because they call for centralized transfer pricing decisions and, thus, fail to achieve a main objective of decentralization: autonomous decision making. An attempt to make divisional managers act in accordance with the firm's overall profit maximization plan (i.e., goal congruence) often fails to allow autonomous decisions (i.e., divisional autonomy) (Thomas [1980]; Grabski [1985]). Schlachter [1986] recognizes the complexities involved in transfer pricing decisions in decentralized firms:

> Transfer pricing becomes a problem within the divisional structure because of certain managerial goals which are difficult to reconcile. On the one hand managers seek to decentralize routine production and exchange decisions to the level of people who would make them in the most informed way. On the other hand the decisions are expected to be in the best interest of the firm as a whole. Whenever managers are evaluated on the contribution their division has made to the company's overall profit, their interests may occasionally conflict with those of the company (p. 104).

Other limitations of the two models are due to the fact that the deterministic, cooperative, complete-information environment assumed in Hirshleifer's classical economic model rarely exists. For instance, we cannot easily find market prices for transferred goods, especially for highly customized intermediate goods. Many real business transactions are made under uncertainty[3]. In particular, when divisional cost/revenue information is not available to top management, divisional managers may not be willing to provide the truthful information necessary to optimize both the economic model and the mathematical programming model. Consequently, the existence of both private information and the possibility of strategic management behavior requires that transfer pricing problems be examined in an environment that clearly permits non-cooperative bargaining in an uncertain environment (Kaplan [1984]).

Empirical research shows that about twenty percent of U.S. firms use some form of negotiated transfer pricing although data on the extent or frequency of use is not available (Tang [1979]; Umapathy [1979] in Vancil [1979]). Wu and Sharp [1979] found that negotiated transfer pricing is the second most often used pricing method. Umapathy revealed a stronger statistic than the other studies. Twenty-three percent of the sample firms responded that the negotiation method was used most often for transfer pricing decisions. Umapathy also showed that large firms tend to use negotiated transfer pricing methods more than small firms (p. 183)[4].

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[3] Consider the examples of various price bidding, stock market transactions, and numerous collective bargaining agreements between companies and labor unions.

[4] An early survey conducted by Mautz [1968] showed similar findings. Of the 678 firms responding, 160 firms used negotiated transfer pricing methods. Of these 160 firms, 131 firms (or 81.8 percent) had annual sales in

Results from the above empirical studies demonstrate patterns in industry practices. First, adoption of negotiated transfer pricing schemes seems more widespread than commonly believed. Second, as companies become larger, they are more likely to adopt negotiated transfer pricing methods. However, empirical studies failed to provide information on the magnitude of total assets exchanged by means of negotiated transfer pricing methods.

Some researchers advocate transfer prices which are freely determined by the managers of the seller division and the buyer division. Watson and Baumler [1975] argue that transfer prices should be negotiated in order to help resolve conflicts of organizational subunits and as an aid to integration when profit centers are strongly differentiated and interdependent. Kaplan [1982] argues that negotiated transfer price systems seem to offer a desirable mechanism for permitting local managers to exploit special information about local opportunities[5]. Similar arguments are made by Grabski [1985] who states "work is needed in the area of negotiated transfer pricing" (p. 61).

Despite the popularity and importance of negotiated

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excess of $100 million. On the other hand, of the 518 firms using methods other than direct negotiation method, 360 firms (or 69 percent) had annual sales in excess of $100 million.

[5] Larson [1974] says that in a decentralized business organization negotiated transfer prices can achieve the two major objectives of decentralization: goal congruence and a high degree of autonomy.

transfer pricing methods, research in this area is minimal. Little has been done to explain the nature of the bargaining phenomena that occurs in negotiated transfer pricing. Two (or more) divisions with conflicting interests involved in transfer price bargaining can be analyzed by a game-theoretic model. Unlike existing accounting and economics research on transfer pricing, a game-theoretic model allows strategic interactions among divisions. The objective of this study is to analyze optimal strategies that could be used by division managers to arrive at an equilibrium point for the transfer price bargaining game. The study employs economic bargaining theory to develop optimal strategies for the bargaining processes in incomplete information environments. It will also investigate the properties of negotiated outcomes. A strategic non-cooperative bargaining model (Samuelson [1984]; Fudenberg and Tirole [1983]; Cramton [1984, 1985]), rather than the axiomatic cooperative bargaining model (Nash [1950]; Roth [1979]), is employed.

I.B. Research objective

In this study, I intend to formulate a game-theoretic environment in which two profit divisions in a decentralized firm negotiate transfer prices for intermediate products. This analysis of optimal bargaining strategies will provide a better understanding of how an independent division can most effectively advance its own interest (profit maximization) against other independent divisions in the transfer price negotiation process. The optimal strategies can be studied when optimal bargaining strategies are analyzed through a non-cooperative (strategic) approach in which the individual players' bargaining processes is explicitly modeled.

This paper explores two interesting questions about transfer price bargaining problems: First, what are the characteristics of the agreements made between division managers when they are allowed to negotiate the prices of the transferred goods with each other? For instance, is the agreement (i.e., the solution of the bargaining game) unique? The uniqueness of a bargaining agreement can allow predictions of potential bargaining outcomes of transfer pricing decisions. Is the agreement efficient? If not, such a bargaining agreement's inefficiency could discourage firms from adopting the negotiated transfer pricing method. Second, what strategic bargaining procedures should be adopted by each division manager in order to arrive at the equilibrium points?

This paper is organized as follows. The second chapter reviews previous literature on transfer pricing. The third chapter provides a description of recent developments in economic game theory and bargaining literature. The fourth chapter presents a game theoretic framework using a single period bargaining of negotiated transfer prices under incomplete information. The analysis of a single period setting is used when expanding the time-frame to a multi-period setting in the following section. The fifth chapter extends the analysis to a two-period bargaining setting with incomplete information and examines optimal strategies for possible solutions. The final section presents the conclusions, the implications, and the limitations of the study.

CHAPTER II

LITERATURE REVIEW

II.A. Introduction

Since the mid 1950s (Dean [1955]; Hirshleifer [1956] and [1957]), there has been some research on transfer pricing in both accounting and economics. Such research has focused on the development of a transfer pricing mechanism which satisfies three objectives: (1) performance evaluation of profit centers, (2) motivation of self-interested division mangers to maximize a firm's overall profits, and (3) stimulating division managers to increase their efficiency without sacrificing divisional autonomy[6] (Ronen and McKinney [1970] and Dejong et al. [1986]). Therefore, identifying an optimal transfer pricing mechanism which promotes two conflicting objectives, those of both decentralization and centralization[7] (i.e., integration), has proven difficult.

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[6] The third objective is to make division managers reconcile the first two conflicting objectives (i.e., division managers' self-interest maximization and firm's overall profit maximization) successfully.

[7] For a solution to this dilemma, see Watson and Baumler [1975] in the behavioral research review section.

Prior studies on transfer pricing are categorized into three different approaches according to which model type is employed - the classical economic model, the mathematical programming approach, and the behavioral model. The first model was primarily developed in economics literature, while the latter two were primarily developed in accounting literature[8].

II.B. Classical economic model

The traditional economic view on the transfer pricing problem began in the early 1950s (Cook [1955] and Hirshleifer [1956]). Hirshleifer was concerned with the problem of pricing products transferred between divisions, and with the maximization of a firm's overall profits. In his model, Hirshleifer assumed two profit centers, a manufacturing unit (the seller division) and a distribution unit (the buyer division); technological independence and demand independence; a competitive market for the final products; and no market for the intermediate products. In this setting, he claimed that output from both divisions should satisfy the rule that total marginal cost from both divisions equals the final product's external market price in order to maximize overall firm profits. Later he relaxed the assumption of no intermediate market and concluded that

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[8] Grabski [1985] provides a recent extensive literature review on transfer pricing.

marginal cost should be the basis for the transfer price when the intermediate products are traded in an imperfectly competitive market and market price should be the basis for the transfer price when there is a perfectly competitive market for intermediate products.

According to Abdel-khalik and Lusk [1974], a serious problem in Hirshleifer's model is that "marginal cost pricing might induce dysfunctional behavior of divisional managers." That is, when each manager reports cost information to headquarters for the purpose of transfer pricing calculations, the manager has an incentive to misrepresent cost and production structures in his divisions for manipulating divisional profitability[9].

Thomas [1980] also points out that centrally administered transfer prices are likely to deprive division managers of the sense of autonomy over the quantities of their inputs and outputs. As a result, Hirshleifer's approach "would (a) generate substantial behavioral perversities and (b) destroy division managers' senses of autonomy. ... Hirshleifer's transfer prices do nothing to resolve this internal conflict and the associated tensions that can be major sources of dysfunctional behavior" (pp.

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[9] Dejong et al. [1986] were also concerned with the possibility of cost misrepresentation. They found that when such a possibility exists the Hirshleifer economic model to maximize a firm's total profits cannot be obtained, because his model requires both completeness and truthful representation of cost information.

152-153).

Recognizing the deficiencies in the Hirshleifer model, Ronen and McKinney [1970] allow communication of divisional private information to headquarters and use a tax or subsidy for inducing truthful-telling of divisional private information. Under such a scheme, both seller and buyer enjoy monopolistic positions. They suggest that the transfer price of intermediate products is determined by the average revenue curve, while the quantity of output is determined by the intersection of the selling division's marginal cost and marginal revenue curves[10].

Ronen and McKinney's model is criticized for the following reasons: (1) The model is a static one which excludes time. Cost relationships in a single-period setting can be difficult to implement in a more dynamic setting. (2) The assumption of a linear production function may not exist in the real world. (3) There is a possibility of cost information manipulation by divisional managers, which may result in suboptimization. (4) The model considers only two-division and a single product case, making it difficult to implement the multi-division and multiple-product situation (see Abdel-khalik and Lusk [1974] for details).

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[10] Dejong et al. examined Ronen and McKinney's transfer pricing mechanism, and found Ronen and McKinney's is superior to Hirshleifer's mechanism.

II.C. Mathematical programming approach

Since the economic model is based on assumptions of a single intermediate product and one seller division and one buyer division, it cannot be applied to the case of multiple products and multiple divisions. In response to this failure, Dopuch and Drake [1964] developed a linear programming model in which opportunity costs are the correct transfer prices. Unlike the economic model, the mathematical programming model explicitly deals with external economies and diseconomies between divisions. Following the Dopuch and Drake model, Onsi [1970] developed a linear programming model based on the decomposition principle[11]. He concludes that the shadow price should be used as a measure of opportunity cost (i.e., lost contribution margin by not taking the next best alternative), which could be used to set the optimal transfer price[12].

Onsi [1970] discusses a crucial operational problem when one product has a high contribution margin, therefore, causing high shadow price of another product. In such a

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[11] For details, see Dantzig and Wolfe [1960] and also Baumol and Fabian [1964].

[12] Benke and Edwards [1980], consistent with the opportunity cost rule, suggest the general rule for selecting the most useful transfer price technique is that the transfer price (TP) should be equal to the standard variable cost (SVC) plus the contribution margin per unit on the outside sale given up by the company when a segment sells internally (LCM).

case the division manager's optimal plan was to buy a product with high shadow price externally, which leads to a suboptimal solution for the firm as a whole. To remedy this, Onsi suggests use of the motivation costs[13] (i.e., the difference between the division's maximization figure and that resulting from the corporate optimal solution) and they should be credited to the selling division's profitability plan in order to make the division follow the corporate optimal solution.

However, mathematical programming models are not free of criticism. There is no need for decentralization in the form of a profit center if the specification of inputs, outputs, and shadow prices are centrally determined by a linear program (see Abdel-khalik and Lusk [1974]). With a loss of divisional autonomy, division managers may lose motivation and consequently, productivity may fall. In addition, the models are single-period, complete information models, making them difficult to implement in real-life settings.

II.D. <u>Behavioral models</u>

There have been research approaches in which the price and market mechanisms are substituted for the companies' internal economies among divisions. Intracompany trades

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[13] In a more accurate term, it should be called "motivational revenue".

among divisions are substituted for trades among the macroeconomy firms in the market. Dean [1955] is an example of this research approach in which he suggests that the transfer pricing problems can be solved by three principles: (1) transfer prices should be negotiated, (2) negotiators should have perfect access to information, and (3) buyers and sellers should be completely free to deal outside the company.

From a positive theory of allocation perspective, Cyert and March [1963] argue that "performance is determined ... partly by the transfer payment rules they can arrange by bargaining with the other parts of the organization ... (such) rules result primarily from a long-run bargaining process rather than a problem solving solution" (p. 276). Lawrence and Lorsch [1967] claim that "successful firms facing uncertain environment are able to resolve effectively interdepartmental conflict, and the most important means of resolving this conflict is confrontation, i.e., negotiation."

Based on Lawrence and Lorsch's claim, Watson and Baumler [1975] assert that both decentralization and integration goals can be achieved by the proper use of transfer pricing. They conclude that while decentralization is sacrificed to achieve the integration goal (i.e., optimize a firm's joint profit) in the mathematical programming model, negotiated transfer pricing can be used "to enhance organizational differentiation and to facilitate organizational integration" (p. 473). Caplan [1971] also argues that it will be necessary to examine a long-run (i.e., multiperiod) negotiation to see whether the conflict among divisions can be eliminated while facilitating both differentiation and integration. Spicer [1988] insists that negotiation provides a cost-effective way to deal with information impactedness[14] due to incomplete information within the firm.

Many empirical studies show that negotiated transfer pricing methods are adopted more widely than commonly believed (Mautz [1968]; Tang [1979]; Umapathy [1979]; Wu and Sharp [1979]). Wu and Sharp point out that "Negotiated prices ... may be optimal in the practical sense in view of the degree of complexity of the transfer pricing issue and if the added dimension of international, political and economical environment. ... the most favorable way to settle the (transfer price) dispute is through negotiation at the local level (p. 85)." Results of the above surveys seem to call for more research on the characteristics of negotiated transfer pricing methods.

Kaplan [1982] says "Lacking a perfectly competitive

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[14] Spicer and Ballew [1983] defined Information impactedness as " a central, derivative condition resulting from the combination of opportunism and uncertainty conditions. It arises whenever there is an asymmetric distribution of incomplete information between parties that cannot be overcome at low cost (p. 80)."

market for the intermediate product and being aware of the limitations of cost-based pricing rules, perhaps the most practical method for establishing a transfer price is through negotiation between the managers of the two divisions" (p. 492). Cats-Baril et al. [1988] suggest a transfer pricing policy that encourages price negotiation between the selling division and buying division. They say that in a decentralized organization composed of divisions organized as profit centers, freedom to negotiate should not be discouraged. They also argue that transfer prices should be determined by timely negotiations between the individual divisions without interference from corporate headquarters. If not, firms will be unable to properly adapt themselves in a market with rapid product changes.

Using 54 pairs of subjects doing transfer pricing negotiations, Schlachter [1986] investigates the economic and organizational forces which impact the transfer of products between domestic divisions. The negotiation process is analyzed into two stages: prenegotiation expectations and negotiations. He finds that prenegotiation conditions are not significantly associated with current negotiation behaviors. However, Schlachter gives little attention to information constraints faced by division managers in transfer pricing negotiations.

II.E. Summary

In essence, decentralization means freedom for lower level managers to make decisions and subsequent actions. Accordingly decentralization is said to be directly related to division managers' decisions on transfer pricing mechanisms and the transfer prices may, thus, be determined optimally from a bargaining process between the seller division and the buyer division (see Wu and Sharp [1979] and Kaplan [1982])[15]. In a decentralized firm, central authority should intervene only when an impasse develops in the negotiation process to protect the firm's overall profit maximization[16].

Larson [1971] defines an appropriate transfer price as the one which results in the division manager making the same decision that corporate management would make in viewing the overall benefit to the firm (p. 20). As to appropriate transfer prices, Horngren and Foster [1987] share a similar viewpoint about the appropriate transfer price (i.e., an optimal transfer price should lead each subunit manager to make optimal decisions so as to maximize the profits of the organization as a whole). They suggest three criteria in choosing a transfer pricing method: promotion of goal congruence, promotion of managerial

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15 See Dean [1955] for a similar argument (p. 68).

16 Divisional conflict developed during negotiation process can be reduced by introducing an arbitration provision into the negotiation process.

effort, and promotion of subunit autonomy. Therefore, division managers ideally make the same transfer pricing decisions (including output decisions) as the central corporate headquarter would make, while preserving divisional autonomy. In this study I investigate whether the firm's overall profit maximization objective (as a measure of efficiency of transfer price negotiation) could be fulfilled by allowing the negotiation of transfer prices between division managers.

CHAPTER III

GAME THEORETIC MODEL (BARGAINING MODEL[17])

III.A. Introduction

Rubinstein [1982] defines the "bargaining problem" as follows:

> Two individuals have before them several possible contractual agreements. Both have interests in reaching agreement but their interests are not entirely identical. What "will be" the agreed contract, assuming that both parties behave rationally? (p. 97)

In a transfer pricing decision process, in which two divisions of a firm try to reach agreeable prices for intermediate products while maximizing each division's profits (by behaving rationally), the situation can be matched with Rubinstein's description. Two profit-maximizing division managers (of two profit centers) are trying to reach agreements on transfer prices of intermediate products, while maximizing their own payoffs from the bargaining outcomes.

Since corporate management wants to maximize a firm's overall profit, a principal-agent formulation of the

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17 The terms 'bargaining' and 'negotiation' are used interchangeably in this paper.

transfer pricing mechanism may show the effect of corporate management's intervention in a limited decentralized environment. The principal-agent model, as a special form of the game-theoretic model, features accounting information as a contracting device between a risk-neutral principal (corporate management in this study) and a risk- and effort-averse agents (both selling and buying divisions). In the principal-agent formulation, the focal point is the principal's problem; to design an incentive scheme for inducing a desired level of effort by the agent for maximizing the principal's payoffs. In such a framework, corporate headquarters should first provide a reward scheme to divisional managers; then, the selling division manager and the buying division manager can engage in a game to negotiate quantity and prices of transferred products.

Unlike the principal-agent formulation, the focus of this study is on the subgame between the two division managers for negotiating over transferred products. Corporate management offers the reward scheme and then is excluded from the subgame between the division managers. This study will develop a better understanding of possible negotiation behavior of managers. Complete analysis of the transfer pricing problem in a principal-agent formulation is left for future research.

The purpose of this study is to analyze optimal strategies for division managers to arrive at an equilibrium point for the transfer price bargaining game. The next subsections will review previous literature on game theory in the bargaining process. Chapters Four and Five will formulate game-theoretic models for two bargaining situations under incomplete information conditions: a) a single period uncertainty case, followed by b) a two-period uncertainty case.

III.B. Previous literature on bargaining theory

Since the pioneering work of von Neumann and Morgenstein [1944] and Luce and Raiffa [1957], game theory has been researched extensively. Bargaining between rational and utility maximizing individuals was investigated by Nash in his seminal paper "The Bargaining Problem" in 1950. Following Nash's work, many researchers have explored various facets of bargaining problems, and a variety of solutions have been suggested[18].

Over recent decades, game-theoretic analyses of bargaining problems have taken one of two alternative approaches. The first alternative is the axiomatic cooperative approach[19]. A formal theory is developed to

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[18] For example, Harsanyi [1956], [1967], [1968a], [1968b], and [1977]; Kalai [1977]; Roth [1979]; Samuelson [1980] and [1984]; Rubinstein [1982]; Chatterjee and Samuelson [1983]; Sobel and Takahashi [1983]; Cramton [1984] and [1985]; Fudenberg et al. [1985].

[19] See Luce and Raiffa [1957], Bishop [1963], Roth [1979], and Shubik [1982] for a detailed description of various solution concepts for cooperative bargaining games. Especially, Roth [1979] provides a comprehensive survey of

predict the outcome of games in which certain axioms are satisfied without explicitly modeling the bargaining process that arises from a particular strategic situation. Instead, it attempts to map the bargainers' preferences and beliefs over many possible outcomes into a unique outcome[20]. If the mapping function predicts a unique outcome over a wide range of bargaining problems, it is possible to use the cooperative approach to predict and suggest bargaining outcomes without studying the complex process of bargaining. A potential weakness of this approach is the difficulty of finding a good predictor of bargaining outcomes that is independent of the strategic possibilities of the bargainers.

The second alternative approach is the non-cooperative strategic approach, in which a direct model of the bargaining process is constructed and equilibrium bargaining behavior is studied. In other words, a particular extensive form of the bargaining game is specified, given by a set of preferences and beliefs, and then the equilibria of the game is investigated. Much of the usefulness of non-cooperative bargaining models rests on their specification of a unique equilibrium, with explicit consideration of strategic

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the literature on the axiomatic cooperative approach for bargaining games.

[20] Even though multiple equilibria are more common in a bargaining game, Nash constructed a set of axioms to derive the uniqueness of equilibrium points.

bargaining behavior and the preferences and information sets of the bargainers.

This paper uses the non-cooperative approach in extensive form[21] for the following reason. The extensive form of the game explains the strategic options available to the bargainers, and then the resulting unique equilibrium strategy of each bargainer determines the optimal bargaining behavior. According to Spicer [1988], one key factor is the possibility of opportunistic behavior where there is uncertainty and only a small number of buyers and sellers involved in transactions. This opportunistic behavior can involve strategic distortions and misrepresentation of intentions and outcomes (p. 306). Therefore, one of the significant benefits from using the non-cooperative approach in extensive form is that it enables us to model the strategic use of private information in bargaining with incomplete information (uncertainty). The non-cooperative approach will prescribe optimal bargaining strategies to division managers who negotiate over the prices of transferred goods, when faced with uncertainty[22].

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[21] A normal form is also possible for the non-cooperative approach, but it suppresses bargainers' strategic behavior which is one of the major interests in this study.

[22] Cramton [1985] argues that the cooperative approach is likely to be more valuable in unstructured and general game settings while the non-cooperative approach is more effective in highly structured settings in which the strategic options for the bargainers are restricted by many constraints.

In Nash's first paper [1950], his axiomatic analysis is limited to a nonzero-sum two-person bargaining game which involves two individuals who have the opportunity to collaborate for several mutual benefits[23]. In order to avoid the existence of multiple Nash equilibria, Nash adopts as axioms several properties that seem natural for the solution and then shows that the axioms determine the solution of the game uniquely. In this "axiomatic approach" to the bargaining game, Nash constructed the following four "fair" and "reasonable" properties (i.e., axioms) that should be satisfied at the unique equilibrium point, when the bargaining parties[24] are sufficiently "knowledgeable" and "rational": (1) Independence of equivalent utility representation, (2) A symmetric bargaining game, (3) Independence of irrelevant alternatives, and (4) Pareto optimality. The Nash Theorem states that there is a unique solution for a bargaining game, in which the axioms are satisfied[25]. In fact, the Nash solution is a function which

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[23] Five key assumptions with regard to the bargaining players are (1) the players have von Neumann-Morgenstein style utility functions; (2) the players are highly rational; (3) each player can accurately compare his desires for various things; (4) each has equal bargaining skill; and (5) each player has full knowledge of the tastes and the preferences of the others.

[24] Throughout this paper I will use the terms 'bargainer', 'bargaining individual', 'bargaining party', and 'player' interchangeably.

[25] A discussion of the Nash Theorem is provided in Appendix A.

selects the unique outcome which maximizes the geometric average of the gains which the players realize by reaching a unanimous agreement, instead of payoffs at disagreement points[26]. Even though Nash contributed significantly to the development of the economic bargaining theory, his axiomatic approach is difficult to adopt for a study in which strategic bargaining behavior of division managers are investigated.

Long before Nash's work, a negotiation model was proposed by Zeuthen [1930]. Zeuthen's model was originally based on direct analysis of a collective bargaining process on the labor market. The key element of Zeuthen's model is that instead of making just one demand, each player can adjust their demands over time by making concessions when their demands are incompatible. The principle object of Zeuthen's analysis is to determine, at any given period in which an agreement has not yet been reached, which player should make the next concession. He argues that the player

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[26] In Nash's next paper [1953], he extended his earlier paper [1950] by presenting a model of bargaining as a non-cooperative game which could derive a Nash solution that was regarded as a complementary to the axiomatic derivation in his 1950 paper. A cooperative game is reduced to a non-cooperative game that makes the players' steps of negotiation in the cooperative game become moves in the noncooperative game in which the demands are declared simultaneously. Then, an equilibrium pair is a pair (x_1, x_2) such that x_1 is the best reply which player 1 could make to x_2, and x_2 is the best reply which player 2 could make to x_1. An equilibrium demand vector is any pair $x = (x_1, x_2)$ which gives neither player an incentive to raise or lower his demand, given the fixed demand of the other player.

less willing to face the risk of conflict will be forced to concede.

Later, Harsanyi ([1956] and [1958]) compared Zeuthen's model with Nash's and found that both analyses are mathematically equivalent[27]. That is, both solutions arrive at the same agreement point where the profits are equally divided between bargainers in a symmetric bargaining game. He also claims that Zeuthen's approach actually supplements Nash's somewhat abstract axiomatic treatment of a bargaining model by providing a plausible psychological model for the actual bargaining process.

Myerson [1986] comments on the contribution of Nash's work to the development of game theory as follows:

> The fundamental principle of game theory is that any definite theory or social plan that predicts and prescribes behavior for all players in a game must designate a Nash equilibrium ... if this theory or plan is to be understood by all the players and is not to impute irrational behavior to any player at any point in time (p. 3).

Despite the clear contribution of Nash's work (for static complete information bargaining situations) to the initial development of bargaining theory, his study has some inherent limitations in dealing with more realistic bargaining situations. Two issues of great importance which are inherent in most bargaining games are the existences of

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[27] See Harsanyi [1956] on pp. 147-149 and Bishop [1963] for a detail.

"private information"[28] and "time horizon". These issues are not only important in deciding the strategic behavior of the bargainers, but are also vital in determining the outcome of a bargaining game. In reality, there exists information asymmetry (incomplete information) in most bargaining situations, including negotiated transfer pricing (Spicer [1988]). For instance, each bargainer is not certain about the preference and information set of the opposing bargainer, even if the bargainer is confident of his own information. This uncertainty implies that bargaining is a learning process (about revealed private information) that takes place over time. The time preferences of the bargainers (the issue of "time horizon") is also crucial in determining the final bargaining agreement. For example, the bargainers should be impatient to come to an agreement, for if it does not matter when they arrive at an agreement, it does not matter if they agree at all (Cramton [1985], p. 3).

Samuelson [1980, 1984] investigates a similar uncertain bargaining situation, in which one party has strictly better information about the potential value of the transaction

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[28] Zeuthen assumes that each party has perfect knowledge, by which each party can estimate correctly the probability that the other party will definitely reject a certain offer. It is the lack of knowledge (thus, different beliefs) that makes two individuals negotiate. Without the differing opinions, there would be no need to bargain. Thus, his model also suffers from this problem which would interfere with its operation as a positive model.

than the other. He argues that, in all circumstances, the presence of information asymmetry precludes a mutually beneficial sale of goods, even though the transferred good is more valuable to the buyer than to the seller. Total gains (for the two bargainers) from the exchange is not obtained in the bargaining game with asymmetric information. In other words, the presence of uncertainty results in inefficient bargaining agreements.

Since the static (one-period) model fails to capture the pattern of reciprocal concessions in the everyday practice of bargaining, Rubinstein [1982] examines the effect of a "time" element on the game's outcomes. He proposes that an equilibrium pair of bargaining strategies should satisfy an additional requirement - subgame perfect equilibrium[29]. The equilibria are formed not only by the strategies chosen at the beginning of the game, but also by the strategies planned after all possible histories in every subgame (p. 99). Thus, each player's strategy must be a best response to the other's strategy at every subgame. In his dynamic (multi-period) version of the bargaining game, Rubinstein [1982] analyzes bargaining as a sequence of alternating offers in a setting of complete information. In turn, each player has to make an offer; after one player has made an offer, the other must either accept it or reject

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[29] The concept of subgame perfectness was originally developed by Selten [1975].

it (and make a counter-offer).

In a game to divide a good of worth equal to 1 unit of value between two players (a seller and a buyer); if they agree, each receives his agreed share. If they fail to agree, both receive none. The game is equivalent to one in which two players decide to split the total gains from a trade agreement (equal to 1) in a negotiated transfer pricing situation. Rubinstein claims that if the players discount the future payoffs (by discounting factors d_1 and d_2), then there exists a unique perfect equilibrium where an agreement is made immediately in the first period and gives $(1 - d_2)/(1 - d_1 d_2)$ to the initial offeror, and $[d_2(1 - d_1)]/(1 - d_1 d_2)$ to the opponent[30]. Due to perfect information, the bargainers are fully informed and are able to perfectly predict the future. Therefore, they are willing to agree with a reasonable initial offer in order to avoid any cost of delay. In the limit, as time between offers goes to zero (thus, d_1 and d_2 go to one), the initial offeror's advantage of making the first offer disappears and then the gains are divided equally at the equilibrium. This example is of interest since each bargainer has an incentive to respond to the other's offer as quickly as possibly.

Several papers incorporated the "information" element into the bargaining game and analyzed both sequential and

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[30] A simple proof of uniqueness of a perfect equilibrium is provided by Sutton [1986], pp. 710-711.

incomplete information aspects of the game[31]. Sobel and Takahashi [1983] analyzed a model of one-sided offers and one-sided uncertainty: the seller makes all the offers, given his uncertainty about the buyer's valuation of the good. They analyzed a two-period model and found a unique sequential equilibrium. Optimal strategy at the equilibrium was a pricing plan that was optimal from each period forward conditional on the seller's learning about the buyer's true reservation price by the previous refusal to accept the seller's proposal (p. 412). This bargaining process can be cast in the framework of the Cob-Web Theorem in economics, in which two bargainers approach the equilibrium point by learning about the opponent's valuation. Fudenberg et al. [1985], working with the same model, prove a uniqueness of the equilibrium when the seller's reservation price is strictly lower than the lower bound of the buyer's reservation price which is the private information for the buyer. Rubinstein [1985] analyzes one-sided uncertainty, alternating offer models. Unlike other studies, the uncertainty is not on one bargainer's reservation price, but it is on the discount rate (i.e., time preference) of one of the bargainers. He partitions the bargainer with private

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[31] Kreps and Wilson [1982] provide a concept of sequential equilibrium as a natural generalization of subgame perfection to dynamic games with incomplete information. In a sequential equilibrium, each player's gaming strategy must be the best response to the other's strategy in every information set.

information into either 'weak' or 'strong' type depending on higher/lower discounting factors. Similar to Fudenberg et al., Rubinstein proves a unique bargaining sequential equilibrium.

Perry's [1986] model differs from previous papers in that he considers fixed bargaining costs per period without discounting future payoffs. In Perry's alternating offer model, in which both players consider a permanent disagreement by leaving the game as the worst outcome, the players can arrive at the equilibrium in which the transaction takes place in the first period. The total waiting costs are a measure of impatience of the players and determine who will make the first offer. Perry's conclusion is similar to Rubinstein's [1982] model, in which a perfect equilibrium occurs at the first period under a complete information condition.

CHAPTER IV

THE ONE-PERIOD WITH INCOMPLETE INFORMATION MODEL

IV.A. Introduction

The Nash solution to the two-person bargaining problem can be applied only to a situation in which there is no uncertainty about the payoff (i.e., complete information on the bargainers' preferences) each bargainer will receive from a particular feasible agreement. However, in most real bargaining cases, the payoff to each bargainer will depend on a certain feasible agreement as well as on some external factors such as uncertainty about the other player's preference or payoffs. Uncertainty in transfer pricing decisions allows division managers' opportunism to be a part of strategic bargaining behavior. An investigation of how strategic behavior affects the dimensions of intrafirm transfer between buying and selling divisions is necessary (Spicer [1988]). To illustrate, look at a bilateral monopoly bargaining game under uncertainty conditions to investigate how transfer pricing decisions by negotiation could be made.

In a bilateral monopoly bargaining situation, each party has their own valuation of the contract agreement.

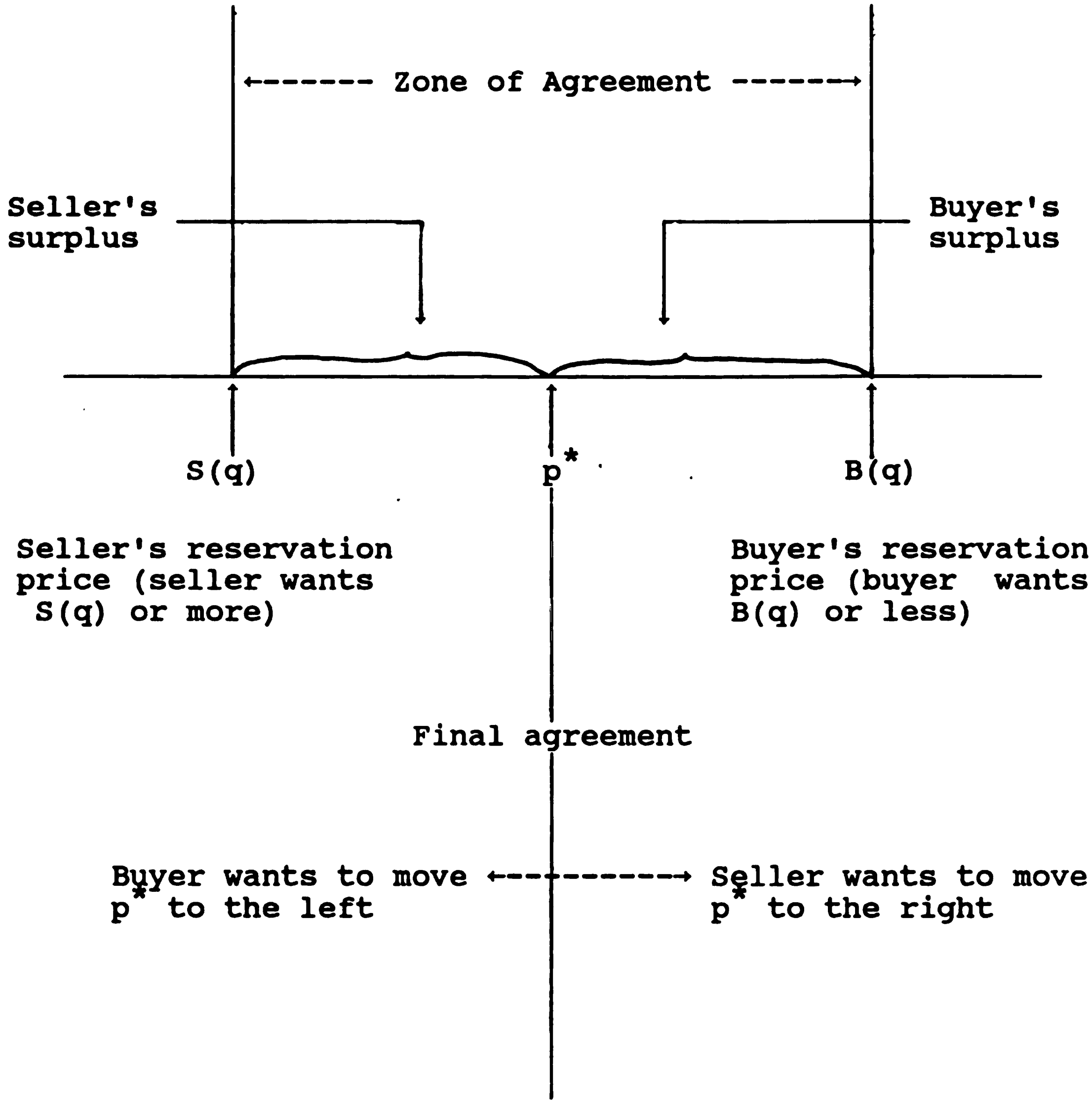
The seller has a reservation price, S(q), that represents the minimum valuation. The buyer also has a reservation price, B(q), that represents the maximum payout (see Figure 1).

As shown in Figure 1, when $S(q) < B(q)$, both the seller and the buyer try to maximize their own surplus (i.e., gains from a bargaining agreement). In other words, the seller wants to maximize the surplus, $(p^* - S(q)) \cdot q$, where p^* is the final mutually agreed value for the bargaining. Similarly, the buyer wants to maximize the surplus, $(B(q) - p^*) \cdot q$. On the other hand, if $B(q) < S(q)$, that is, the seller's minimum value is greater than the buyer's maximum value, then there is no zone of agreement: there is no mutually agreeable bargaining settlement (i.e., permanent disagreement).

Figure 2 shows that the bargaining game can be constructed for several different cases, depending on attributes such as who is informed, who moves first (including counteroffers), number of bargaining periods, and bargainers' discount rates. One-sided uncertainty is assumed with the buyer informed and the seller uninformed. It is also assumed that the uninformed seller moves first. If the informed bargainer (i.e., the buyer) were to move first, he would immediately offer a transfer price at the opponent's true valuation of the product given his knowledge on the opponent's valuation. Therefore, it is more

Figure 1

The Geometry of Bargaining



Source: Raiffa [1982] p. 46

Figure 2

Attributes of Bargaining Game

A. Information structure:

		Informed Buyer: Yes	Informed Buyer: No
Informed Seller	Yes	Certainty	One-sided Uncertainty
	No	One-sided Uncertainty	Two-sided Uncertainty

B. Other attributes:

1. Who moves first?

   a. Informed player
   b. Uninformed player
   c. Counteroffer*

2. Number of bargaining periods

   a. Single-period
   b. Multi-period (finite- or infinite-horizon)

3. Time preference (Discounting factors)*

   a. Same discount rates ($\delta_s = \delta_b$)
   b. Different discount rates (δ_s < or > δ_b)

* Not applicable to the single-period model

interesting to study the case in which the uninformed bargainer (the seller) makes the first offer (i.e., the price and quantity of the product) to the buyer. Regardless of who is uninformed, a case with informed buyer/uninformed seller is symmetrical to a case with informed seller/uninformed buyer. However, two-sided uncertainty case is not investigated in this study because of its extreme analytical complexity. Also, counteroffers are not an issue in a single-period model.

This chapter examines the negotiated interdivisional transfer pricing problems in a single-period setting with incomplete information, in which the manager of the selling division plays the role of an uninformed bargaining party. Consequently, I will explore some possible changes in the solution to the problem when there is uncertainty about the other division manager's true valuation of the transfer product.

The single-period incomplete information model is investigated as a stepping stone for the analysis of a two-period one-sided incomplete information model in Chapter Five[32]. In addition, the price of transferred goods is a function of the quantity of goods. Therefore, we need to set the price as a function of quantity instead of as an independent parameter.

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32 A bargaining solution of the one-period, complete information setting is discussed in Appendix B.

IV.B. Model

When transfer prices of intermediate products are determined in a decentralized firm, there can be three different ways for price determination. First, the firm can give price decision authority to division managers and let them negotiate freely over prices and quantity of the products. The outcome of the negotiation is called a pure bargaining price denoted as p^*. Second, central headquarters can decide the firm's optimal price based on divisional cost information while giving division managers veto power to reject this price. This centrally-determined price will be called a constrained Pareto optimal price denoted as p^{**}. Finally, the firm can determine the firm's overall optimal price without giving veto power to division managers. This price without veto power will be called a pure Pareto optimal price and denoted as p^{***}. This study compares these three different transfer pricing procedures from the standpoint of the divisions' profits, and firm's total profits; and also investigates potential bargaining inefficiency problems which exist in pure bargaining situations.

After the product is produced by the selling division, it is transferred to the buying division. Consider the case of transfer price determination through bargaining between two division managers: a seller and a buyer. Information asymmetry is characterized as one-sided incomplete information in which the buyer knows the seller's reservation price, while the latter only knows a subjective probability distribution of the former's reservation price. Each manager has perfect information on his own reservation price.

Let S(q) denote the seller's reservation price (production cost per unit as a function of quantity), which is the least monetary value acceptable as a transfer payment of goods (i.e., production costs); B(q) denotes the buyer's reservation price (selling price per unit as a function of quantity), which is the maximum monetary value paid for transferred goods[33]. The following assumptions are needed for the model:

(A-1) There are two division managers - a buyer and a seller. Each manager knows their own reservation price, B(q) and S(q), respectively.

(A-2) The seller's valuation S(q) is common knowledge to both managers, while the buyer's valuation B(q) is not known to the seller. The seller initially regards a parametric value b of B(q) as a uniformly distributed variable over [0, 1][34]. The cumulative probability distribution function,

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[33] S(q) and B(q) are derived from the seller's product supply function and the buyer's demand function for the transferred product, respectively. See more details in the numerical example following the model.

[34] A uniform distribution over [0, 1] is assumed for analytic simplicity. An analysis using a general uniform distribution over $[b_l, b_u]$ is discussed in Appendix C. If

F(b), from which the parameter b is drawn independently is common knowledge.

(A-3) Both managers are risk-neutral and, therefore, want to maximize the expected payoffs (bargaining gains) from the bargaining process. Risk-neutrality is assumed, so managers are trying to maximize expected monetary gains from bargaining.

(A-4) The negotiated transfer price is a function of the quantity of the transferred goods, $p = p(q)$. This assumption is necessary since division managers tend to negotiate over prices as well as quantity of the products[35].

(A-5) The quantity of transferred products is non-negative since division managers cannot trade products in a negative quantity. That is, $q \geq 0$[36].

(A-6) There is no outside market available to either division manager to sell or buy the products. Otherwise, the transfer prices are negotiated among three bargainers, which makes the present study of strategic bargaining processes between division managers more complex and, therefore, is left for future study.

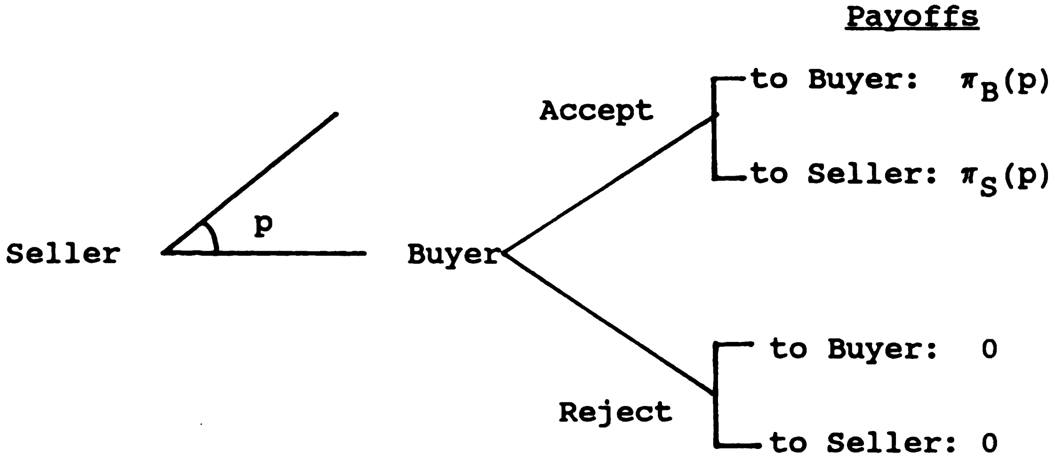
$b_l = b_u$, the problem becomes a certainty case which is discussed in Appendix B.

[35] The assumption (A-4) is different from that of the bargaining theory in economics in which only a single prize is the objective of bargaining, whereas this study assumes price as a function of quantity of the transferred product.

[36] A more complex version for this assumption would be to let buyer offer quantity q, then let seller offer a price based on the negotiated quantity.

Figure 3

Extensive form of a single-period one-sided incomplete information game*



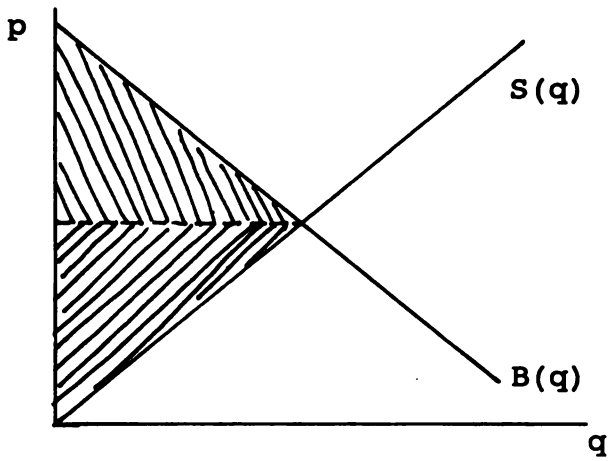
$$\pi_S = p \cdot q - \int_0^q S(q)\ dq = p \cdot q - \frac{q^2}{2}$$

$$\pi_B = \int_0^q (b - q)\ dq - p \cdot q = b \cdot q - \frac{q^2}{2} - p \cdot q$$

* The seller has incomplete information about the buyer's valuation.

Figure 4

Payoffs for seller and buyer



$$S(q): \quad p = q$$

$$\pi_S = pq - \int_0^q S(q)\, dq$$

$$= pq - (q^2/2)$$

$$B(q): \quad q = b - p$$

$$\pi_B = \int_0^q B(q)\, dq - pq$$

$$= bq - (q^2/2) - pq$$

(A-7) The seller setsprices and quantities for the transfer products and make an offer to the buyer. The buyer either accepts the prices and quantity or refuses them, and receives payoffs as in Figure 3. Additionally, I will consider a certain bargaining rule: The seller moves first and makes a non-negative price offer p per unit for quantity q. Then, the buyer can purchase the goods at this price or reject the offer in a single-period setting. A simple drawing of an extensive form of the present model is given in Figure 3. If the buyer accepts the transfer price offered by the seller, the payoffs are $(p(q) - S(q)) \cdot q$ and $(B(q) - p(q)) \cdot q$ for the seller and the buyer, respectively. If the buyer rejects the prices, both managers will have zero payoffs (i.e., disagreement payoffs). As shown in Figure 4, S(q) and B(q) are defined as follows:

$$S(q): \quad q = p \quad \text{and, therefore, } p = q$$

$$B(q): \quad q = b - p. \quad \text{Thus, } p = b - q \tag{4-1}$$

Thus, payoffs for the seller and the buyers (π_S and π_B) are

$$\pi_S = p \cdot q - \int_0^q S(q)\ dq = p \cdot q - \frac{q^2}{2}$$

$$\pi_B = \int_0^q (b - q)\ dq - p \cdot q = b \cdot q - \frac{q^2}{2} - p \cdot q \tag{4-2}$$

The first example of the transfer price model is a pure bargaining game solution. Here, two independent divisions negotiate over the intermediate product which was manufactured by one division regarding its price and quantity without any intervention from the corporate headquarter. This bargaining game can be solved by backward induction:

1. Buyer's decision: Buyer will accept the seller's offer p only when the gain is $\pi_B \geq 0$.

$$\pi_B = b \cdot q - \frac{q^2}{2} - p \cdot q \geq 0 \tag{4-3}$$

Since $q > 0$, $2b - q - 2p = 2b -(b - p) - 2p$

$$= b - p \geq 0$$

So, the buyer accepts the offer p if $b \geq p$. The probability of such an incident will be

$$\Pr\{p \text{ is accepted}\} = \Pr\{b \geq p\} = \frac{1 - p}{1 - 0} \tag{4-4}$$

2. Seller's offer: Seller will choose offer price p to maximize the expected payoff, π_S, given the distribution of the buyer's reservation price. Therefore, the seller's problem can be written as:

$$\underset{p}{\mathrm{Max}} \int_0^1 [\pi_S \cdot \Pr\{p \text{ is accepted}\} + 0 \cdot \Pr\{p \text{ is rejected}\}]\, db$$

$$\text{subject to } q = b - p \tag{4-5}$$

Equation (4-5) can be rewritten as follows:

$$\underset{p}{\text{Max}} \int_0^1 \left[(p \cdot q - \frac{q^2}{2})(1 - p)\right] db \qquad (4\text{-}6)$$

$$\underset{p}{\text{Max}} \int_0^1 \left[(p \cdot (b - p) - \frac{(b - p)^2}{2})(1 - p)\right] db$$

After a few steps of algebraic rearrangement[37],

$$\underset{p}{\text{Max}} \left[\frac{1 - p}{6}\right]\left[-9p^2 + 6p - 1\right] \qquad (4\text{-}7)$$

The first-order-condition (hereafter, F.O.C.) for (4-7) is[38]

$$\text{F.O.C.:} \quad 27p^2 - 30p + 7 = 0 \qquad (4\text{-}8)$$

From the F.O.C., two solutions for the bargaining equilibrium price p can be derived.

$$p = \frac{5 \pm 2}{9} = \frac{1}{3} \text{ or } \frac{7}{9} \qquad (4\text{-}9)$$

However, the second-order-condition (hereafter, S.O.C.) for the maximization problem should be negative in order to obtain the maximum point.

$$\text{S.O.C.:} \quad 54p - 30 = \begin{cases} -12 & \text{for } p = 1/3 \text{ or} \\ 12 & \text{for } p = 7/9 \end{cases} \qquad (4\text{-}10)$$

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37 For details, see Appendix C.

38 A full mathematical procedure is discussed in Appendix C.

Therefore, the equilibrium price for the seller's profit maximization problem in this single- period bargaining game is

$$p^* = \frac{1}{3} \approx .333 \tag{4-11}$$

At the equilibrium price p^* the payoff for the seller is

$$\pi_S{}^* = (p^* \cdot q^* - \frac{q^{*2}}{2}) \cdot (1 - p^*)$$

$$= \left[(\frac{1}{3}) \cdot (\frac{1}{3}) - \frac{(\frac{1}{3})^2}{2}\right] \cdot (\frac{2}{3}) = \frac{1}{27} \approx .0370 \tag{4-12}$$

The expected payoff for the buyer is

$$\pi_B{}^* = \int_{1/3}^{1} \left[b \cdot q^* - \frac{q^{*2}}{2} - p^* \cdot q^* \right] db$$

$$= \frac{1}{2} (b^2 q^* - bq^{*2} - 2bp^* \cdot q^*) \Big|_{1/3}^{1}$$

$$= (\frac{1}{2}) \left[(\frac{8}{9}) q^* - (\frac{2}{3}) q^{*2} - 2(\frac{2}{3}) p^* \cdot q^* \right]$$

$$= (\frac{1}{2}) \left[(\frac{8}{9})(\frac{1}{3}) - (\frac{2}{3})(\frac{1}{3})^2 - 2(\frac{2}{3})(\frac{1}{3}) \cdot (\frac{1}{3})\right]$$

$$= \frac{1}{27} \approx .0370 \tag{4-13}$$

The firm's total profit is

$$\pi_F^* = (\pi_S^* + \pi_B^*) = \frac{2}{27} \approx .0741 \qquad (4\text{-}14)$$

The seller can obtain the equal amount of payoffs like the buyer by taking advantage of offering a product price to the buyer, despite the disadvantage in the information structure. The seller's exploitation of the buyer is possible because both bargainers know that the game will not continue in the second period and the buyer will not reject the seller's offer unless the buyer wants zero payoff. However, the equilibrium solution p^* fails to maximize the firm's total profit (i.e., Pareto optimal solution). By relaxing the condition of no intervention from corporate headquarters in the pricing decision, the constrained Pareto optimal transfer price, p^{**}, can be determined. It describes the case in which central headquarters decides the optimal transfer price maximizing the firm's total expected profit, while each division manager has the right to refuse to adopt the centrally-determined product price. The constrained Pareto optimal solution can be computed in the following way:

$$\underset{p}{\text{Max}} \int_0^1 [\pi_S + \pi_B] \cdot \Pr\{p \text{ is accepted}\}\, db$$

$$\text{subject to } q = b - p \qquad (4\text{-}15)$$

$$\underset{p}{\text{Max}} \int_0^1 \left[(b \cdot q - q^2) \cdot \left(\frac{1 - p}{1 - 0}\right)\right] db$$

subject to $q = b - p$

$$\max_{p} \int_0^1 \left[[b(b - p) - (b - p)^2]\cdot(1 - p)\right] db$$

$$\max_{p} \int_0^1 \left[(b\cdot p - p^2)\cdot(1 - p)\right] db$$

$$\max_{p} \ (1 - p)\cdot\left[\frac{p}{2} - p^2\right] \tag{4-16}$$

The F.O.C. of (4-16) is

F.O.C.: $6p^2 - 6p + 1 = 0$

Thus, the constrained Pareto optimal solution price p_*^* is

$$p^{**} = \frac{3 - \sqrt{3}}{6} \approx .2113 \tag{4-17}$$

At the price of p^{**}, the payoffs π_S^{**} and π_B^{**} and then the total firm's profit π_F^{**} are

$$\pi_S^{**} = (p^{**}\cdot q^{**} - \frac{q^{**2}}{2})\cdot(1 - p^{**})$$

$$= \left[\frac{3 - \sqrt{3}}{6}\cdot\frac{3 - \sqrt{3}}{6} - (\frac{1}{2})(\frac{3 - \sqrt{3}}{6})^2\right](1 - \frac{3 - \sqrt{3}}{6})$$

$$= \frac{3 - \sqrt{3}}{72} \approx .0176 \tag{4-18}$$

$$\pi_B^{**} = \int_{(3-\sqrt{3})/6}^{1} \left[b\cdot q^{**} - \frac{q^{**2}}{2} - p^{**}\cdot q^{**} \right] db$$

$$= \frac{1}{2} \left(b^2 q^{**} - bq^{**2} - 2bp^{**} \cdot q^{**} \right) \Big|_{(3-\sqrt{3})/6}^{1}$$

$$= \frac{\sqrt{3}}{36} \approx .0481 \tag{4-19}$$

The firm's total profit is

$$\pi_F^{**} = (\pi_S^{**} + \pi_B^{**}) = \frac{3 + \sqrt{3}}{72} \approx .0657 \tag{4-20}$$

The final case is a (pure) Pareto optimal transfer price, p^{***}, which is determined when the full authority for price determination is given to central headquarters and neither division manager has the right to refuse the price[39]. The pure Pareto optimal price is determined as follows:

$$\underset{p}{\text{Max}} \int_0^1 [\pi_S + \pi_B] \, db \quad \text{subject to } q = b - p \tag{4-21}$$

$$\underset{p}{\text{Max}} \int_0^1 [b \cdot q - q^2] \, db \quad \text{subject to } q = b - p$$

$$\underset{p}{\text{Max}} \int_0^1 [b \cdot (b - p) - (b - p)^2] \, db$$

39 The pure Pareto optimal transfer price is a variation of Hirshleifer's [1956] price when incomplete information exists regarding transfer product costs.

$$\underset{p}{\text{Max}} \left[\frac{p}{2} - p^2 \right] \tag{4-22}$$

The F.O.C. of (4-22) is

$$\text{F.O.C. :} \quad \frac{1}{2} - 2p = 0$$

Thus, the Pareto optimal solution price p^{***} which maximizes firm's total profit from this product's transfer transaction is

$$p^{***} = \frac{1}{4} \approx .25 \tag{4-23}$$

At the Pareto optimal price p^{***} the payoff for the seller is

$$\pi_S^{***} = p^{***} \cdot q^{***} - \frac{q^{***2}}{2}$$

$$= (\frac{1}{4}) \cdot (\frac{1}{4}) - \frac{(\frac{1}{4})^2}{2} = \frac{1}{32} \approx .03125 \tag{4-24}$$

The expected payoff for the buyer is

$$\pi_B^{***} = \int_{1/4}^{1} \left[b \cdot q^{***} - \frac{q^{***2}}{2} - p^{***} \cdot q^{***} \right] db$$

$$= \frac{1}{2} \left(b^2 q^{***} - b q^{***2} - 2bp^{***} \cdot q^{***} \right) \Big|_{1/4}^{1}$$

$$= \frac{3}{16} = .1875 \tag{4-25}$$

Then, the firm's total profit is

$$\pi_F^{***} = (\pi_S^{***} + \pi_B^{***}) = \frac{7}{32} \approx .2188 \tag{4-26}$$

By comparing the bargaining equilibrium price p^* (4-11) with both the constrained Pareto optimal price p^{**} (4-17) and the pure Pareto optimal price p^{***} (4-23), it turns out that p^* is an inefficient profit allocation, since it results in a sub-optimal allocation of intrafirm resources under incomplete information conditions. Such an inefficient allocation is a consequence of the existence of the uncertainty condition, under which the seller enjoys the privilege of sole price offeror and exploits the opponent (buyer) during the negotiation process. Table 1 also shows that $\pi_S^* \geq \pi_S^{***}$ and $\pi_B^* \leq \pi_B^{***}$. This illustrates the fact that under the incomplete information condition the price offeror (the seller) takes the advantage of offering price and exploits the bargaining opponent (the buyer). It also shows $\pi_F^* \leq \pi_F^{***}$. In other words, the two divisions fail to share total possible gains exhaustively, due to the existence of incomplete information. In general, these results could hold when there exists incomplete information for any of the bargainers and the uninformed bargainer offers transfer prices (including the quantity of the

TABLE 1

COMPARISON OF PAYOFFS TO MANAGERS AND FIRM AT DIFFERENT SOLUTION PRICES

	Payoffs (Profits) of		
	Seller(π_S)	Buyer(π_B)	Firm(π_F)
$p^* = .333$	.0370#	.0370	.0741
$p^{**} = .211$	.0176	.0481	.0657
$p^{***} = .25$	.03125	.1875##	.21875###

p^* is the bargaining equilibrium price.
p^{**} is the constrained Pareto optimal price.
p^{***} is the pure Pareto optimal price.

#: the seller's largest payoff
##: the buyer's largest payoff
###: the firm's largest profits

products). So incomplete information plays a key role for bargaining inefficiency. Central headquarters can increase the total profits by not allowing division managers veto power over a centrally-administered transfer price for the intermediate product.

With veto power given to managers, the buying division manager can obtain a higher payoff than that from the individual bargaining solution. One interesting result is that constrained Pareto optimal price p^{**} is lower than the pure Pareto optimal price p^{***} obtained without veto power given to division managers. Consequently Table 1 shows $\pi_F^{*} \geq \pi_F^{**}$, which illustrates that the central headquarters fails to increase the firm's total profits due to the uncertainty with regard to the buyer's valuation from the standpoint of the central headquarters. In case of constrained Pareto optimal price, the central office tends to shade the product price (i.e., intentionally lower the price) in fear of the price being rejected by any of the division managers. However, the buyer's best interest is the pure Pareto optimal solution in the given model setting, while the seller's best performance is obtained when the transfer price is negotiated independently between division managers.

IV.C. Summary

In this chapter I examine transfer price determination in a simple single-period bilateral monopolistic bargain setting. The results show that under an incomplete information condition, the problem of a Pareto inefficient profit allocation exists even in a single period setting due to the fact that only the uninformed bargainer (i.e., the seller) offers the transfer price to the informed bargainer (i.e., the buyer). The seller exploits the buyer to obtain a higher payoff from the trade negotiation but fails to exhaust all the bargaining gains,resulting lower firm profits. If the negotiation starts with the informed buyer's price offer to the uninformed seller, the buyer can obtain all the bargaining gains, so there will be no bargaining inefficiency problem.

However, the firm can impose pure Pareto optimal price (p^{***}) in order to improve allocation efficiency at a sacrifice of divisional autonomy in transfer pricing decisions. The purpose of the single-period model is to provide a convenient stepping stone for an analysis of a multi-period model in the following chapter. Thus, we need to shift from a static model to a dynamic process in order to gain more insights into a realistic bargaining framework.

In a single period setting, a manager has no opportunity to signal his reservation price by communicating private information to the other manager during the course of bargaining. However, in the multi-period bargaining setting, one can exploit the other's private information by examining the responses of the opponent. Despite such an information updating procedure, an inefficiency problem still exists in the multi-period setting with the incomplete information condition. The next chapter will extend analysis to a multi-period (two-period) setting in which the buyer (informed division manager) reveals private information to the seller (uninformed division manager) by responding to (either accept or reject) the sequence of the seller's offer.

CHAPTER V

THE TWO-PERIOD WITH INCOMPLETE INFORMATION MODEL

V.A. Introduction

In real bargaining situations, it is common to observe bargaining taking place over time, especially when there exists incomplete information at least for one bargainer. A seller and a buyer continue bargaining until they reach a final agreement or until the negotiations fail. The fundamental question in every negotiation is how the total available bargaining surplus is distributed between participating players. In general, bargaining agreements depend on (i) the characteristics of the information that the players have and (ii) the relative impatience of players to reach final agreements. Consideration of the first factor involves a dichotomous classification of each player's information structure: complete vs. incomplete information. The second factor, called the relative impatience of bargainers, can be represented by the bargainers' costs of delayed agreement (i.e., discount factors) which give them incentives to have early bargaining agreements since their payoffs are discounted over time.

Unlike the one-period bargaining model, in two-period

(in general, multi-period) bargaining situations with incomplete information each manager can learn about the other manager's private information from each round of the bargaining process. For instance, the buying division manager's private information about the reservation price can be communicated or signaled to the selling division manager through decisions (by either accepting or rejecting the seller's offers) over bargaining sessions[40]. Subsequently the seller revises the probabilistic expectation of the buyer's private information using Bayes' rule[41]. Then, in the following period the seller will offer a new price given updated expectations. According to Harsanyi [1967, 1968a, 1968b], optimal bargaining strategies must constitute a Bayesian Nash equilibrium, in which a player's strategy must be the best reply based on that player's probabilistic expectation of other player's private information.

For general analyses of transfer price bargains taking place in multiple stages, it is necessary to consider two important factors before determining appropriate equilibrium concepts in a multi-period, incomplete-information

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40 Rubinstein [1985] assumes that asymmetric information concerns one of the player's discount factors. Thus in his model, the private information on such discount factors will be transmitted.

41 Harsanyi ([1967], [1968a], and [1968b]) describes a revision of this subjective probability function by Bayes' rule in multi-period setting. But, there is no such probability revision in a single-period setting.

bargaining setting. One is the discount rate of each player which converts future payoffs (expected utilities) of the player into equivalent present payoffs (expected utilities). The other is the question of how a player will revise beliefs (according to Bayes' rule) about the opponent's private information. Consequently, we need to examine the effect of the belief revision process on the final transfer price agreed upon at the bargaining equilibrium point. The sequential equilibrium concept is used here in which each manager is required to behave in a sequentially rational manner (Kreps and Wilson [1982]). At any point in time during the bargaining process, a manager's revised probability and the strategy selection must be part of an optimal strategy for the remainder of the game given these revised beliefs. Information signalling and consequent belief revision processes are commonly observed in many real bargaining situations including transfer price bargaining (Cramton [1984]).

Like many studies, the present study investigates uncertainty over the bargainer's true valuation[42]. Furthermore, I investigate the effect of uncertainty on the efficiency of bargaining outcomes. Unlike other studies, I construct a more realistic setting in which division managers negotiate over both prices and quantity of the

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[42] In Rubinstein [1985], the uncertainty is over the discount rate of one of the bargainers.

transferred products.

V.B. Model

This chapter develops a model of a two-period transfer price negotiation under a one-sided incomplete information condition. In the model, two division managers (a selling division and a buying division) are bargaining over transfer prices for some intermediate product made by the selling division - a production division. Unless they reach an agreement in the first period, their respective payoffs in the second period are discounted, so that they have incentives to come to an early agreement. However, an early bargaining agreement may not be easy to reach because one manager (the seller in the present model) has incomplete information about the preferences of the other. In particular, the seller does not know how much the buyer values the product. In an incomplete information framework, the managers have incentives to hide their private information which consequently delays an agreement. At the same time, discount factors provide incentives to come to an early agreement. These two confounding factors make the multi-period bargaining with incomplete information more difficult to analyze.

The seller offers a transfer price for the product, so that the buyer has an opportunity to purchase it. The buyer either accepts the offer price or rejects and waits for a new offer made in the next period. If the buyer accepts the seller's offer, the bargaining process is completed. On the other hand, if the buyer rejects the initial offer and the seller realizes that there are opportunities for positive gains from future agreement, it is reasonable to assume that in the second period the seller will continue to offer new prices based on the updated beliefs about the buyer's preferences.

Three important issues are examined. The first issue is the bargaining equilibrium concept of the transfer pricing decision. In addition, we investigate strategies used by division managers to come to a final transfer pricing agreement. Answers to these questions can provide an optimal bargaining process for division managers in determining transfer prices for intermediate products when division managers are given autonomy to determine the prices and the quantity of the intermediate products by negotiations. The second issue is whether potential bargaining impasses caused by incomplete information would result in bargaining inefficiency (in an ex-post sense) at the time of transfer price determination. If a bargaining inefficiency exists, two division managers may fail to make a Pareto optimal transaction and, therefore, potential bargaining benefits may not have been fully exhausted. In order to examine the effect of uncertainty on bargaining solutions, the present study excludes counteroffer case.

When the informed player makes counteroffer, it could make the final agreement immediately and diffuse the effect of uncertainty on bargaining inefficiency. Finally, the discount rates represent some form of penalty for not arriving at an early agreement. It is natural that the heavier the penalties are, the earlier an agreement would be made. I will examine how discount rates affect the timing and characteristics of the final bargaining agreement.

In addition to the assumptions for a single-period model [(A-1) through (A-6)] including a change in assumption (A-7), it is necessary to make a few additional assumptions:

(A-7') The seller offers a price for the transfer product to the buyer. The buyer either accepts the price or refuses and waits for the seller to offer a new price in the second period, given the buyer's own valuation of the product; the buyer chooses a binary action function $A(p, b, \beta) \in \{accept, reject\}$.

(A-8) There exist fixed discount factors, δ_s and δ_b, for the seller and the buyer, respectively[43]. These rates are common knowledge to both managers, and $0 < \delta_s, \delta_b \leq 1$.

(A-9) The bargaining can continue for two periods until a

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[43] Discount rates could be called either penalties (costs) for prolonged agreement or costs of bargaining. However they are different from fixed per-period bargaining costs used by Perry [1982] in which only trivial results of equilibria are found. In a special case of trade negotiation like arm's-length negotiations (including negotiations by telephone conversation), both discount rates approach one as the time period goes to zero.

final agreement is reached or until no agreement is made in the second period[44]. This assumption prevents the bargainers from walking away without a bargaining agreement in the first period even when substantial gains from trade exists.

(A-10) The supply and demand functions for both division managers, S(q) and B(q), are assumed to remain the same over the entire bargaining process.

Two division managers, a seller and a buyer, are negotiating the price of the transfer product which costs the seller S(q) to produce and are worth B(q) to the buyer. Like the one-period model in Chapter Four, S(q) and B(q) are defined as follows:

$$S(q):\quad q = p \quad \text{and, therefore, } p = q$$

$$B(q):\quad q = b - p. \quad \text{Thus, } p = b - q \qquad (5\text{-}1)$$

Without loss of generality, we assume that $S(q) \leq B(b_1, q)$ since the buyer with $S(q) > B(b_1, q)$ does not enter the trade negotiations because the seller would not offer prices below product valuation. At every period of the game, the seller makes an offer p, which is either accepted or rejected by the buyer. As the bargaining process continues over time, both managers face costs of delaying an agreement. When the buyer accepts the price offer p_t at the period of t (for t = 1, 2), the expected payoffs that both

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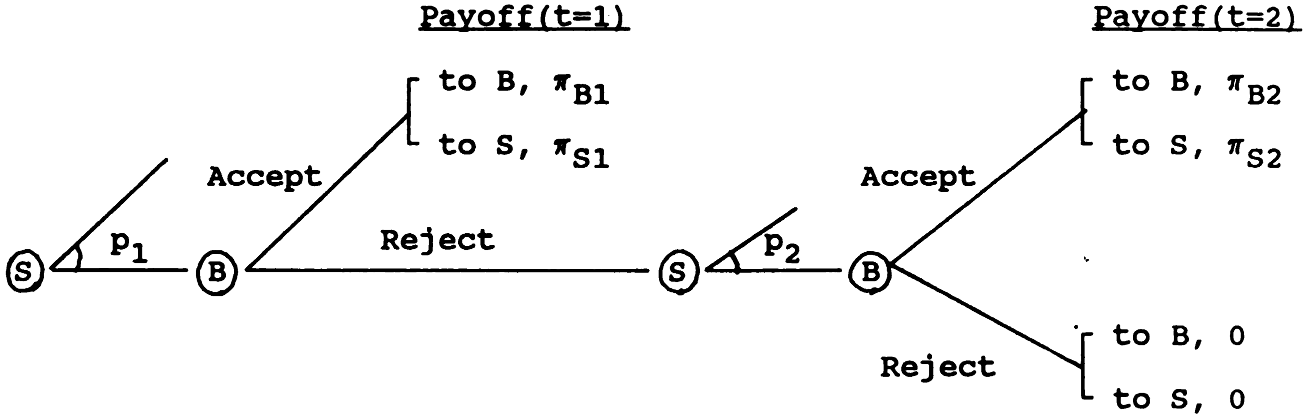
[44] See Figure 5 for the two-period game shown in an extensive form.

Figure 5

Extensive form of a two-period one-sided incomplete information game



S indicates the seller.
B indicates the buyer.
Payoff(t=1) indicates payoffs in the first period.
Payoff(t=2) indicates payoffs in the second period.

Also,

$$\pi_{B1} = \left[b \cdot q_1 - \frac{q_1^{\,2}}{2} - p_1 \cdot q_1 \right]$$

$$\pi_{S1} = \left[p_1 \cdot q_1 - \frac{q_1^{\,2}}{2} \right]$$

$$\pi_{B2} = \delta_b \cdot \left[b \cdot q_2 - \frac{q_2^{\,2}}{2} - p_2 \cdot q_2 \right]$$

$$\pi_{S2} = \delta_s \cdot \left[p_2 \cdot q_2 - \frac{q_2^{\,2}}{2} \right]$$

division managers will receive are

$$\pi_{Bt} = \delta_b^{\,t-1} \cdot \left[\int_0^q B(q)\ dq - p_t \cdot q \right] \quad \text{and}$$

$$= \delta_b^{\,t-1} \cdot \left[b \cdot q_t - \frac{q_t^{\,2}}{2} - p_t \cdot q_t \right]$$

$$\pi_{St} = \delta_s^{\,t-1} \cdot \left[p_t \cdot q - \int_0^q S(q)\ dq \right]$$

$$= \delta_s^{\,t-1} \cdot \left[p_t \cdot q_t - \frac{q_t^{\,2}}{2} \right]$$

$$\text{for } t = 1,\ 2 \tag{5-2}$$

Should they fail to reach an agreement, they will receive zero payoff.

In the case of the one-sided incomplete information, the buyer knows the seller's costs as well as his own valuation, but the seller does not know the buyer's valuation. The seller can only assess the parametric value b in the buyer's valuation to be given by the twice differentiable uniform distribution F(b) with a density function f(b) for $b \geq 0$. By the sequential equilibrium concept the buyer can gradually reveal private information, and the seller can update beliefs accordingly. The seller's offer price maximizes expected profit conditional on what is learned from the buyer's rejection of the seller's price in the previous period[1]. The seller considers the buyer's first refusal to accept an offer as an indication that the buyer's parametric value b is below a certain value (called cutoff valuation, $\beta(p_1)$, defined in the next paragraph) whereas the buyer uses the seller's first offer to predict the next price offer and decide whether to wait or to accept the present offer.

First, define $\beta(p_1)$ to be the cutoff valuation at which, the seller believes, a certain type of buyer is indifferent between accepting and rejecting the seller's offer p_1 at the first period. The next step is to assume that if the buyer's true value b is less than $\beta(p_1)$, the buyer is better off rejecting p_1 and waiting for a lower offer in the next period[2]. To the seller, therefore, a rejection of p_1 by the buyer means that b is less than $\beta(p_1)$. The seller now revises F(b) over a new range $[b_1, \beta(p_1)]$, and then offers a new price p_2 in the next period. Thus, the uninformed seller revises probabilistic belief about the buyer's valuation by truncating F(b) from the right.

Like the one-period model, this two-period model can be

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[1] This procedure is similar to the concept of no-commitment bargaining equilibrium in Sobel and Takahashi [1983].

[2] $\beta(p_1)$ is calculated by the seller based on revised belief after reviewing the buyer's response to the current price offer. Computation of $\beta(p_1)$ is discussed in Appendix D in a detail.

solved by backward induction[47].

1. Buyer's decision in the second period: The buyer will accept the seller's second price offer p_2 only when the expected payoff is $\pi_{B2} \geq 0$.

$$\pi_{B2} = \delta_b \cdot \left[b \cdot q_2 - \frac{{q_2}^2}{2} - p_2 \cdot q_2 \right] \geq 0 \tag{5-3}$$

$\delta_b > 0$ and $q > 0$, so $2b - q_2 - 2p_2$

$$= 2b - (b - p_2) - 2p_2 = b - p_2 \geq 0$$

Thus, the buyer accepts the second offer p_2 if $b \geq p_2$. The probability of his acceptance is to be[48]

$$\Pr\{p_2 \text{ is accepted} | \beta\} = \Pr\{b \geq p_2 | \beta\}$$

$$= \frac{\beta - p_2}{\beta - b_1} \quad \text{where } b_1 \leq p_2 \leq \beta \tag{5-4}$$

2. Seller's offer in the second period: The seller will offer the second price p_2 to maximize expected payoff, π_{S2}, given belief of the buyer's reservation price as uniformly distributed over $[b_1, \beta]$. The seller's problem is written as,

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47 The solution technique is similar to Cramton [1982].

48 For notational convenience, I will suppress functional dependence of β on p_1. That is, $\beta = \beta(p_1)$.

$$\underset{p_2}{\text{Max}} \int_{b_1}^{\beta} [\ \pi_S \cdot \Pr\{p_2 \text{ is accepted} | \beta\}\ ]\ db \qquad (5\text{-}5)$$

By (5-2), the seller's problem (5-5) can be rewritten as,

$$\underset{p_2}{\text{Max}} \int_{b_1}^{\beta} \delta_s \cdot \left[(p_2 \cdot q_2 - \frac{{q_2}^2}{2})(\frac{\beta - p_2}{\beta - b_1})\right]\ db \qquad (5\text{-}6)$$

$$\underset{p_2}{\text{Max}} \int_{b_1}^{\beta} \delta_s \cdot \left[(p_2 \cdot (b - p_2) - \frac{(b - p_2)^2}{2})(\frac{\beta - p_2}{\beta - b_1})\right]\ db$$

Equivalently,

$$\underset{p_2}{\text{Max}} \left[\frac{\beta - p_2}{6}\right]\left[-9{p_2}^2 + 6p_2(\beta + b_1) - (\beta^2 + \beta \cdot b_1 + {b_1}^2)\right] \qquad (5\text{-}7)$$

The F.O.C. for (5-7) is[49]

$$27{p_2}^2 - 6p_2(5\beta + 2b_1) + (7\beta^2 + 7\beta \cdot b_1 + {b_1}^2) = 0$$

From the F.O.C., two solutions for the bargaining equilibrium price p_2 can be derived.

$$p_2 = \frac{(5\beta + 2b_1) \pm \sqrt{(4\beta^2 - \beta \cdot b_1 + {b_1}^2)}}{9}$$

From the second order condition, only one can be the solution price for the seller's profit maximization problem.

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[49] The algebra needed for the computation is similar to the one used in the one-period model. See Appendix C for details.

$$p_2{}^* = \frac{(5\beta + 2b_1) - \sqrt{(4\beta^2 - \beta \cdot b_1 + b_1{}^2)}}{9} \tag{5-8}$$

Since p_2 is $[b_1, \beta]$ and $p_2 \leq b$ from (5-3) and (5-4),

$$p_2 = \min\{ \max(b_1, \frac{(5\beta + 2b_1) - \sqrt{(4\beta^2 - \beta \cdot b_1 + b_1{}^2)}}{9}), \beta\} \tag{5-9}$$

3. Buyer's decision in the first period: The buyer's indifference valuation $\beta(p_1)$ is chosen in order to be indifferent in profits between accepting and rejecting the seller's first offer p_1.

$$\pi_{B1}(\beta, p_1) = \pi_{B2}(\beta, p_1) \tag{5-10}$$

$$\text{So } \beta \cdot q_1 - \frac{q_1{}^2}{2} - p_1 q_1 = \delta_b \cdot \left[\beta \cdot q_2 - \frac{q_2{}^2}{2} - p_2 q_2\right]$$

Substituting $q_1 = \beta - p_1$ and $q_2 = \beta - p_2$ from (5-1) for the above condition yields

$$\beta(\beta-p_1) - \frac{(\beta-p_1)^2}{2} - p_1(\beta-p_1) = \delta_b\left[\beta(\beta-p_2) - \frac{(\beta-p_1)^2}{2} - p_2(\beta-p_2)\right]$$

After a few algebraic steps, it can be rearranged as

$$\frac{1}{2}(\beta - p_1)^2 = \frac{1}{2} \cdot \delta_b(\beta - p_2)^2$$

$\beta > p_1$ and $\beta > p_2$[50], so

$$(\beta - p_1) = \sqrt{\delta_b}\cdot(\beta - p_2)$$

and, therefore, $\beta = \dfrac{p_1 - \sqrt{\delta_b}\cdot p_2}{1 - \sqrt{\delta_b}}$ (5-11)

Substitution of (5-9) for p_2 in (5-11) yields[51]

$$\beta(p_1) = \max\{\ \min\left(\frac{p_1 - \sqrt{\delta_b}\cdot b_1}{1 - \sqrt{\delta_b}},\ \frac{Q + R}{2(4\delta_b - 24\sqrt{\delta_b} + 27)}\right),\ p_1\}$$

where[52] $Q = 6p_1(9 - 4\sqrt{\delta_b}) - \sqrt{\delta_b}\cdot b_1(12 - 5\sqrt{\delta_b})$

$$R = \sqrt{\delta_b[b_1^2(36-24\sqrt{\delta_b}+96\delta_b) - 12b_1\cdot p_1(4\sqrt{\delta_b}+3) + 144p_1]}$$ (5-12)

4. Seller's offer in the first period: Given the value p_2 and β in (5-9) and (5-12), the seller chooses p_1 to maximize expected bargaining gains. The seller's problem is written as

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50 Unless $\beta > p_1$ and $\beta > p_2$ the buyer will accept neither p_1 in the first period nor p_2 offered in the second period.

51 Appendix D shows in more detail the algebra needed in computation.

52 Neither Q nor R represents particular mathematical concepts. They are denoted simply for algebraic convenience.

$$\underset{p_1}{\text{Max}} \int_{b_l}^{b_u} [\pi_{S1} + \pi_{S2}]\, db \tag{5-13}$$

Since the parameter b in the buyer's cost function B(q) is uniformly distributed over $[b_l, b_u]$, the two probabilities in the problem (5-13) are

$$\Pr\{p_1 \text{ is accepted}\} = \Pr\{b - \beta \geq 0\} = \frac{b_u - \beta}{b_u - b_l}, \text{ and}$$

$$\Pr\{p_2 \text{ is accepted} | p_1 \text{ is rejected}\} = \frac{\beta - p_2}{\beta - b_l} \tag{5-14}$$

The seller's problem (5-13) can be rewritten as

$$\underset{p_1}{\text{Max}} \int_{b_l}^{b_u} \left[(p_1 q_1 - \frac{q_1^2}{2})(\frac{b_u - \beta}{b_u - b_l}) + \delta_s (p_2 q_2 - \frac{q_2^2}{2})(\frac{\beta - p_2}{\beta - b_l})\right] db$$

subject to

$$p_2 = \min\{\max(b_l, \frac{(5\beta + 2b_l) - \sqrt{(4\beta^2 - \beta \cdot b_l + b_l^2)}}{9}), \beta\}$$

$$\beta(p_1) = \max\{\min(\frac{p_1 - \sqrt{\delta_b} \cdot b_l}{1 - \sqrt{\delta_b}}, \frac{Q + R}{2(4\delta_b - 24\sqrt{\delta_b} + 27)}), p_1\}$$

and $\beta(p_1) \leq b_u$

where $Q = 6p_1(9 - 4\sqrt{\delta_b}) - \sqrt{\delta_b}\cdot b_1(12 - 5\sqrt{\delta_b})$

$$R = \sqrt{\delta_b[b_1^2(36-24\sqrt{\delta_b}+96\delta_b) - 12b_1\cdot p_1(4\sqrt{\delta_b}+3) + 144p_1} \qquad (5\text{-}15)$$

The objective function of (5-15) is rewritten as follows:

$$\max_{p_1}\Bigg[\left(\frac{b_u - \beta}{6}\right)\{-9p_1^2 + 6p_1(b_u+b_1) - (b_u^2+b_u\cdot b_1+b_1^2)\}$$

$$+ \delta_s\left[\frac{\beta - p_2}{6(\beta-b_1)}\right]\{-9p_2^2(b_u-b_1) + 6p_2(b_u^2-b_1^2) - (b_u^3-b_1^3)\}\Bigg] \qquad (5\text{-}16)$$

The F.O.C. of (5-16) is

$$\beta_p\{9p_1^2 - 6p_1(b_u + b_1) + (b_u^2 + b_u\cdot b_1 + b_1^2)\}$$

$$- (b_u - \beta)\{18p_1 - 6(b_u+b_1)\} - \delta_s\left[\frac{(\beta_p-p_{2p})(\beta-b_1) - (\beta-p_2)\beta_p}{(\beta-b_1)^2}\right]$$

$$\cdot\{9p_2^2(b_u-b_1) - 6p_2(b_u^2-b_1^2) + (b_u^3-b_1^3)\}$$

$$- \delta_s\left(\frac{\beta-p_2}{\beta-b_1}\right)\{18p_2\cdot p_{2p}(b_u-b_1) - 6p_{2p}(b_u^2-b_1^2)\} = 0 \qquad (5\text{-}17)$$

where β_p and p_{2p} are the partial derivatives of β and p_2 with respect to p_1.

To simplify the analysis without a loss in generality, it is assumed that $b_u = 1$ and $b_l = 0$. That is, the buyer's valuation b is uniformly distributed over $[0, 1]$[53]. This assumption simplifies p_2 and β so that

$$p_2 = \min \{ \max (0, \frac{\beta}{3}), \beta\}$$

$$\text{and } \beta = \max \{\min (\frac{p_1}{1 - \sqrt{\delta_b}}, \frac{3p_1}{3 - 2\sqrt{\delta_b}}), p_1\} \tag{5-18}$$

In addition, (5-17) can be simplified as follows:

$$\begin{aligned} &\beta_p(9p_1^2 - 6p_1 + 1) - (1 - \beta)(18p_1 - 6) \\ &- \delta_s\left[\frac{(\beta_p \cdot p_2 - \beta \cdot p_{2p})}{\beta^2}\right] \cdot (9p_2^2 - 6p_2 + 1) \\ &- \delta_s\left[\frac{(\beta - p_2)}{\beta}\right] \cdot (18p_2 \cdot p_{2p} - 6p_{2p}) = 0 \end{aligned} \tag{5-19}$$

In (5-18) $0 < \delta_b \leq 1$ and $p_1 > 0$, so

$$\frac{p_1}{1 - \sqrt{\delta_b}} > \frac{3p_1}{3 - 2\sqrt{\delta_b}} > p_1$$

and then,

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[53] The results of this study hold for other uniform distributions and other probability distribution functions.

$$\beta = \frac{3p_1}{3 - 2\sqrt{\delta_b}} \text{ and } p_2 = \frac{\beta}{3} = \frac{p_1}{3 - 2\sqrt{\delta_b}} \tag{5-20}$$

But $0 < \beta \leq 1$, so $0 < p_1 < (3 - 2\sqrt{\delta_b})/3$.

By letting $(3 - 2\sqrt{\delta_b})^{-1} = K$, the following variables can be simplified.

$\beta = 3p_1 \cdot K$ and $p_2 = p_1 \cdot K$,

$\beta_p = 3K$ and $p_{2p} = K$

The simplified F.O.C. (5-19) is equivalent to

$$81K \cdot p_1^2 - 6(2\delta_s K^2 + 6K + 3)p_1 + (4\delta_s K + 3K + 6) = 0 \tag{5-21}$$

The seller tries to maximize her expected profits, so the second order condition for the objective function should be negative.

$$162K \cdot p_1 - 6(2\delta_s K^2 + 6K + 3) < 0$$

Thus, by the S.O.C. the equilibrium prices and the buyer's indifference valuation are as follows:

$$p_1^* = \frac{(2\delta_s K^2 + 6K + 3) - \sqrt{4\delta_s^2 K^4 + 24\delta_s K^3 - 3(8\delta_s - 3)K^2 - 18K + 9}}{27K}$$

$$p_2^* = p_1^* \cdot K \text{ and } \beta = 3p_1^* \cdot K$$
$$\text{where } K = (3 - 2\sqrt{\delta_b})^{-1} \tag{5-22}$$

Following the equilibrium strategy (p_1^*, p_2^*, β), the seller's expected payoff is

$$E\pi_S^* = (p_1^* q_1^* - \frac{q_1^{*2}}{2})(1-\beta) + \delta_s(p_2^* q_2^* - \frac{q_2^{*2}}{2})(\beta - p_2^*)$$

(5-23)

Then, the buyer's expected payoff is

$$E\pi_B^* = \int_{\beta}^{1} (\pi_{B1}^*)\, db + \int_{p_2^*}^{\beta} (\pi_{B2}^*)\, db$$

$$E\pi_B^* = \frac{(1-\beta)}{2}\cdot[(1+\beta)q_1^* - q_1^{*2} - 2p_1^* q_1^*]$$

$$+ \frac{(\beta - p_2^*)}{2}\,\delta_b\cdot[(\beta + p_2^*)q_2^* - q_2^{*2} - 2p_2^* q_2^*]$$

(5-24)

V.C. <u>Numerical example</u>

There is a manufacturing firm with a production division (i.e., seller) and a marketing division (i.e., buyer). The production division's production cost function (i.e., S(q)) derived from its supply function is:

S(q): $q = p$

where q is the number of units produced

p is price per unit (5-25)

The buyer's (marketing division's) demand function is known to be

$$B(q): \quad q = b - p \tag{5-26}$$

The parameter b in the demand functions only known to the seller that it is uniformly distributed in [0, 1].

According to (5-23) and (5-24), payoffs for the seller and the buyer from the price negotiation depend on the values of discount rates used by both managers. Table 2 provides computed payoffs for two division managers as well as the firm for selected discount rates. It shows that the more patient the managers are as reflected by higher values of δ_s and δ_b, the lower the first and second period prices, with the second period price being lower than the first period price. When the discount rates are equal to one for both managers the selling division manager enjoys very large ex-ante expected profits while the counterpart manager has a very small expected payoff from the bargaining agreement. This can be explained by the fact that the seller values the present and future payoffs the same. The seller offers the same price (.3333) in both the first and the second period and discourage the buyer from accepting the second price offer. On the other hand when the burden of the discounting

Table 2

Two-period payoffs from bargaining equilibrium when $\delta_s = \delta_b$

δ_s	δ_b	p_1^*	p_2^*	$E\pi_S$	$E\pi_B$	$E\pi_F$
1.00	1.00	.3333	.3333	.0370	.0370	.0741
.99	.99	.3367	.3333	.0367	.0367	.0733
.95	.95	.3502	.3333	.0352	.0352	.0704
.90	.90	.3675	.3333	.0333	.0333	.0667
.50	.50	.4218	.2660	.0274	.0321	.0595
.00	.00	.3333	.1111	.0370	.0370	.0741

δ_s: discount rate for the seller

δ_b: discount rate for the buyer

p_1^*: bargaining equilibrium price in the first period

p_2^*: bargaining equilibrium price in the second period

$E\pi_S$: ex-ante expected bargaining payoff for the seller

$E\pi_B$: ex-ante expected bargaining payoff for the buyer

$E\pi_F$: ex-ante expected bargaining payoff for the firm

Table 3

Two-period payoffs from bargaining equilibrium when $\delta_s \neq \delta_b$ with $\delta_s = .95$

δ_s	δ_b	p_1^*	p_2^*	$E\pi_S$	$E\pi_B$	$E\pi_F$
.95	1.00	.3333	.3333	.0352	.0370	.0722
.95	.95	.3502	.3333	.0352	.0352	.0704
.95	.90	.3675	.3333	.0352	.0333	.0685
.95	.75	.4186	.3301	.0350	.0285	.0635
.95	.50	.4428	.2792	.0366	.0292	.0658
.95	.00	.4085	.1362	.0517	.0221	.0739

Table 4

Two-period payoffs from bargaining equilibrium when $\delta_s \neq \delta_b$ with $\delta_b = .95$

δ_s	δ_b	p_1^*	p_2^*	$E\pi_S$	$E\pi_B$	$E\pi_F$
1.00	.95	.3502	.3333	.0370	.0352	.0722
.95	.95	.3502	.3333	.0352	.0352	.0704
.90	.95	.3502	.3333	.0333	.0352	.0685
.75	.95	.3502	.3333	.0278	.0352	.0630
.50	.95	.3502	.3333	.0185	.0352	.0537
.00	.95	.3333	.3173	.0027	.0380	.0407

factor becomes heavier as δ_s and δ_b decline, the seller offers a higher price in order to complete the negotiation in the first period and to avoid bargaining in the second period.

At the limit when δ_s and δ_b equal to one, both bargainers value the future payoffs worthless and the bargaining games will lead to a single-period model with the same equilibrium solution. As shown in the previous chapter for the one-period model, the existence of incomplete information results in an inefficient bargaining outcome. Therefore, it is necessary to compare the bargaining outcome with the constrained Pareto optimal solution. Similar to (4-13) for the single-period model, the constrained Pareto optimal solution for the two-period model is computed in the following way:

$$\underset{p_1}{\text{Max}} \int_0^1 \Big[(bq_1 - q_1^2)(1 - \beta) + \delta_s(p_2q_2 - \frac{q_2^2}{2})(\beta - p_2)$$

$$+ \delta_b(bq_2 - \frac{q_2^2}{2} - p_2q_2)(\beta - p_2)\Big]$$

$$\text{subject to } q = b - p \qquad (5\text{-}27)$$

Equivalently,

$$\underset{p_1}{\text{Max}} \left[(1 - \beta)\left(\frac{p_1}{2} - p_1^2\right) + \delta_s(\beta - p_2)\left(\frac{p_2^2}{2}\right) + \delta_b \cdot \left(\frac{p_2 - 3p_2^2}{2}\right)(\beta - p_2)\right]$$

The F.O.C. is

$$6(\delta_s K^3 - 3\delta_b K^3 + 3K)p_1^2 + 2(2\delta_b K^2 - 3K - 2)p_1 + 1 = 0 \qquad (5\text{-}28)$$

By the S.O.C., the constrained Pareto optimal solution is

$$p_1^{**} = \frac{-(2\delta_b K^2 - 3K - 2) - \sqrt{4\delta_b^2 K^4 + 6(\delta_b - \delta_s)K^3 - (8\delta_b - 9)K^2 - 6K + 4}}{6(\delta_s K^3 - 3\delta_b K^3 + 3K)}$$

$$p_2^{**} = p_1^{**} \cdot K \text{ and } \beta = 3p_1^{**} \cdot K$$
$$\text{where } K = (3 - 2\sqrt{\delta_b})^{-1} \qquad (5\text{-}29)$$

Substituting p_1^{**}, p_2^{**} and β yields the payoffs at the constrained Pareto optimal prices. Table 3 shows decreased payoffs in general for the firm and both bargainers using the strategy (p_1^{**}, p_2^{**}, β). The pure Pareto optimal prices (p^{***}) in the two-period model is identical to the ones in the one-period model (i.e., Table 1) because division managers are not given veto power. Whatever transfer prices are imposed by central headquarters in the first period must be accepted by both managers.

Tables 2 through 5 also provide the evidence that when discount rates are equal to one (i.e., no discounting for the future value) the seller has exactly the same payoffs as those in the single-period bargaining case, while the buyer suffers a decrease in the payoffs. Tables 3 and 4 show that as heavier discount factors (i.e., lower δ_s and δ_b) are levied the managers have lower payoffs and the firm's profits decrease in a similar fashion. In Table 3 as δ_b is changed from 1.0 to 0.0 when δ_s is set to be .95, the expected payoffs for the buyer drops 40% from .0370 to .0221, while the seller's payoffs increase by 47% from .352 to .517. On the other hand, Table 4 shows as δ_s is changed from 1.0 to 0.0 when δ_b is fixed at .95, the seller's expected payoffs drop 93% from .0370 to .0027, while the buyer's profits increase only 8% from .352 to .380. These results illustrate that a burden of heavy discounting factors to the uninformed bargainer is more serious than the informed bargainer. The effect of discounting factor with a combination with incomplete information structure can hurt the seller who is uninformed in the present model. However, the present study could not separate the effect of discounting factors on the bargaining inefficiency from that of incomplete information.

In the two-period model, the information incompleteness among bargainers results in a significant decrease in division managers' payoffs and the firm's overall profits. The information structure in the bargaining game is surely acritical determinant of bargaining efficiency. The degree

Table 5

Two-period payoffs from constrained Pareto optimal solution

δ_s	δ_b	p_1^{**}	p_2^{**}	$E\pi_S$	$E\pi_B$	$E\pi_F$
1.00	1.00	.2113	.2113	.0176	.0481	.0657
.99	.99	.2115	.2094	.0174	.0482	.0656
.95	.95	.2118	.2016	.0166	.0484	.0650
.90	.90	.2112	.1916	.0158	.0486	.0644
.50	.50	.1988	.1253	.0133	.0493	.0626
.00	.00	.2113	.0704	.0176	.0481	.0657

δ_s: discount rate for the seller

δ_b: discount rate for the buyer

p_1^{**}: constrained Pareto optimal price in the first period

p_2^{**}: constrained Pareto optimal price in the second period

$E\pi_S$: ex-ante expected payoff for the seller

$E\pi_B$: ex-ante expected payoff for the buyer

$E\pi_F$: ex-ante expected payoff for the firm of bargaining efficiency can be improved as the transfer prices between divisions are determined by the central headquarter. The firm can have larger profits than under pure bargaining by implementing pure Pareto optimal solution prices (in Table 1)[10] in the two-period bargaining game.However, Table 5 shows that like the single-period model the constrained Pareto optimal price in the two-period model could not increase the firm's overall profits, possibly due to central headquarters' excessive shading in process of transfer price determination.

V.D. **Summary**

With the one-sided incomplete information, the bargaining efficiency is impaired significantly in the two-period model. Despite popular use of the negotiation method in the transfer pricing decision, information incompleteness generally leads to bargaining inefficiency. Unlike the single-period model, the buyer can benefit from his superior information when trade negotiation can be incurred in two periods (also multiple periods in general). During the course of the bargaining the buyer is able to reveal his private information. The seller updates his belief on the

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[10] One of the main objectives of this study is to investigate the impact of information incompleteness on bargaining inefficiency. Therefore, I have ignored the centralized cost of obtaining both the seller's and the buyer's information to the firm in implementing either the constrained Pareto optimal prices (p^{**}) or the pure Pareto optimal prices (p^{***}).

buyer's private information.

In the multiple-period bargaining situation, the buyer often delays the final agreement until the last period. It decreases ex-ante payoffs for the seller and increases ex-ante expected payoffs for the buyer. However, it prevents achievement of the goal congruence objective of decentralization, even though it gives a sense of divisional autonomy to each manager. Time pressure (i.e., expressed in discount rates for managers) tends to force an early bargaining agreement. In the given bargaining model, a heavy discount rate tends to decrease the seller's payoffs and the firm's overall profit while the buyer's payoffs are increased.

In fact, discount rates and incomplete information have counter-effects on bargaining efficiency. Certainly there should be some benefit to the firm by reducing information asymmetry between division managers negotiating transfer prices in multiple period games. It should be crucial to the firm's profitability to find some equilibrium prices where the level of the discount rates and the amount of uncertainty counterbalance to produce the firm's highest overall profit.

CHAPTER VI

SUMMARY AND CONCLUSIONS

This study adopts bargaining game theory arguments from the economics literature to investigate the nature of negotiated transfer prices at the equilibrium point. The negotiated transfer pricing process is considered under incomplete information conditions which could be used to explain observed transfer pricing policies and practices. The present study provides evidence about how two division managers can arrive at a unique equilibrium price in a multi-period bargain where uncertainty exists as to the managers' preferences. However, it shows that there are potential allocation inefficiency problems under incomplete information conditions, which results in a failure to achieve optimal intrafirm resource allocations.

Different from others, the present study constructs a model where two division managers negotiate both prices and quantity of the transferred products simultaneously. It proves that there are some bargaining inefficiency problems which occur when managers are given autonomy to negotiate over transfer products freely. Such bargaining inefficiency results from both incomplete information (uncertainty) and discount rates for division managers who negotiate in multiple periods. The bargaining inefficiency problem is more significant in the two-period model than the single-period model. Furthermore, as heavier discounting factors are imposed the bargainer with incomplete information suffers a larger decline in expected gains. Unfortunately, no clear conclusions about the interaction of factor on bargaining inefficiency can be drawn from the present research.

Regardless of certain criticisms of the negotiated transfer pricing method, negotiation is widely used. Therefore, we need to understand the strategic behavior of negotiating managers. The game-theoretic approach provides an explanation of how rational individuals (i.e., division managers) build their strategies when facing other rational individuals with conflicting interests. Such a description of strategic behavior with incomplete information and conflicting interests is central to an understanding of the process followed by divisional managers in negotiating transfer prices. Understanding the role of negotiation in transfer pricing may ultimately aid corporate management in structuring the negotiations to maximize the benefits to the company.

Systematic study of transfer pricing negotiations can be extended to the settings that are not considered in the present study. First, due to the restricted setting in this study, its conclusions may not be readily applicable to understanding the whole nature of negotiated transfer prices and actual bargaining behavior of division managers. In order to better comprehend the conditions under which a negotiated transfer pricing mechanism is used, we need more experimental research which investigates the price negotiation mechanism. Experimental research can serve as the next major breakthrough following theory development (Maher [1982]).

Second, this research studies one completion of price negotiation for a single object. However, I exclude from the present study many on-going interpersonal and psychological relationships between managers. Such on-going relationships can result in different strategic behaviors of managers in non-cooperative bargaining games.

Third, this study ignores some organizational characteristics which are relevant for explaining which transfer pricing mechanisms are used. Spicer [1988] suggests that we need to understand organizational and transactional conditions and organizational processes to explain observed transfer pricing policies and practices.

Fourth, assuming risk-neutrality and fixed discount factors for the managers' time preferences in a two-period model is a convenient way to simplify the model. A more general model would let division managers continue negotiation until all the potential bargaining gains are exhausted. Even though managers are assumed to be risk-neutral, they are willing to take risks during the course of negotiation by the nature of bargaining (Cramton [1982]). In addition, the assumption of the seller's role as the price offeror can be relaxed to allow both the seller and the buyer to make offers and counteroffers repetitively. By relaxing the above assumptions, we can make a more positive description of real transfer bargaining situations.

Finally, I focus on the subgame between managers in the process of determining transfer prices for understanding strategic bargaining behavior of managers, and exclude the corporate management from the game. To do this, we assume that the reward scheme for managers are exogenously determined. However, it would be necessary to consider a Principal-agent framework to obtain a comprehensive view of incentive mechanisms in decentralized companies.

APPENDICES

APPENDIX A

NASH THEOREM

A.1. Assumptions

The first set of crucial assumptions about two bargainers' utility functions is:

Assumption set 1 (Utility function assumption):

(1) Two players are assumed to be risk-neutral and have a von Neumann-Morgenstein utility function. Therefore, they are concerned with the maximization of expected utilities (or, profits) from bargaining results.

(2) The utility function $U(\cdot)$ is continuous, strictly increasing and concave.

Both bargainers have a utility for each possible outcome, and it should be assumed that the set of ordered pairs representing both bargainers' utilities for each outcome is convex.

Assumption 2 (Convexity assumption):

The set S in the utility plane is assumed to be compact, non-empty containing at minimum the disagreement point, and convex. That is, if two outcomes exist yielding utilities x_1 and x_2 to the buyer, and y_1 and y_2 to the seller, then in the set S there are potential outcomes yielding a vector $p(x_1y_1) + (1 - p)(x_2y_2)$, with $0 < p < 1$, to each bargainer.

Beyond the convexity assumption, according to Nash, the outcome of the negotiation possess following six properties:

Assumption 3 (Properties of the solution to the bargaining game):

(1) Individual rationality: The outcome of the negotiation should leave both bargainers no worse off than they would be if they did not achieve an agreement.

(2) Feasibility: The outcome of the bargaining should be a feasible outcome chosen from the set of all possible outcomes.

(3) Independence of utility function scale: The outcome of the bargaining should not depend on the scale used to measure the utility functions of the bargainers; that is, the solution of the bargaining game is not altered by an order-preserving, linear transformation of the utility functions of bargainers.

(4) Pareto optimality: The outcome of bargaining should be such that no other settlement exists that would make one bargainer better off and the other worse off.

(5) Independence of irrelevant alternatives: If x is

the mutually agreed outcome of a bargain consisting of a given set of alternative settlements, x should also be the outcome of any bargain consisting of some subset of these settlements, provided that x is present in the subset.

(6) Symmetry: Suppose the set of possible settlements is entirely symmetric. This means that for every settlement having utilities x to the buyer and y to the seller, a settlement exists that has utility y to the buyer and x to the seller. Second, assume that utility functions are scales so that the bargainers' utilities for no-agreement are equal; in this case, the outcome of the bargaining should have equal utility to both bargainers.

Assumption 4 (Individual player):

Individual players in a bargaining game are rational and perfectly knowledgeable not only about the occurred state but also about the opponent's utility function and strategy.

A bargaining game is described by a set $N = \{1, \ldots, n\}$ of players and a pair (S, d) where S is a non-empty compact (i.e., closed and bounded) convex subset of R^n representing the feasible utility payoffs to the players, and d is an element of S corresponding to the disagreement outcome. It was also assumed that there is at least one point s in S such that $d < s$. Nash described B as the set of all such bargaining games. He defined a solution to the bargaining problem to be a function $f\colon B \to R^n$ such that $f(S, d)$ is an element of S for any (S, d) in B. Then,

> <u>Nash's Theorem</u>: There is a unique solution possessing properties 1-4. It is the function $f = F$ defined by $F(S,d) = x$ such that $x \geq d$ and
>
> $$\prod_{i=1}^{n} (x_i - d_i) > \prod_{i=1}^{n} (y_i - d_i)$$ for all y in S such that $y \geq d$ and $y \neq x$.

APPENDIX B

THE ONE-PERIOD WITH COMPLETE INFORMATION MODEL

Consider two managers of two independent divisions (i.e., a seller and a buyer as profit centers) in a decentralized firm. The producer-seller manufactures intermediate products and sells them to the buyer for further processing before making final sales. During the negotiation process on transfer prices of the intermediate products, a seller and a buyer must negotiate a mutually agreeable outcome. If not, there will be no exchange of intermediate products between the two divisions without outside intervention.

In a simple bargaining case under the complete information condition, the two division managers begin the bargaining game knowing how much the product is worth to the opponent: the buyer knows how much it costs for the seller to produce the intermediate product and the seller knows the worth of the product to the buyer. Similar to the analysis of the incomplete information model, an equilibrium price can be found by a backward induction.

1. Buyer's decision: He will accept the seller's

offer p when his payoff $\pi_B \geq 0$.

$$\pi_B = b \cdot q - \frac{q^2}{2} - p \cdot q \geq 0 \tag{B-1}$$

Since $q > 0$, $2b - q - 2p = 2b - (b - p) - 2p$

$$= b - p \geq 0$$

So, the buyer accepts the seller's offer p if $b \geq p$.

2. Seller's offer: The seller will choose offer price p to maximize his profit π_S. Therefore, the seller's problem can be written as,

$$\underset{p}{\text{Max}} \left[pq - \frac{q^2}{2} \right] \quad \text{subject to } q = (b - p)$$

Equivalently,

$$\text{Max} \left[-\frac{1}{2}(3p^2 - 4bp + b^2) \right] \tag{B-2}$$

The F.O.C. of (B-2) is

$$- 6p + 4b = 0$$

Therefore, $p^c = (2b/3)$ is the seller's profit maximization price[1]. Substituting the optimal price into (B-1) and (B-2) yields

---

[1] The reason why the seller is not able to offer $p = b$ even though the buyer is willing to accept is because the seller's profit at $p = b$ is zero. It occurs because the seller's profit is defined as $\pi_S = (pq - q^2/2)$.

$$\pi_S{}^C = \frac{b^2}{6} \tag{B-3}$$

$$\pi_B{}^C = \frac{1}{2}(b - p)^2 = \frac{b^2}{18} \tag{B-4}$$

and $\pi_F{}^C = \pi_S{}^C + \pi_B{}^C = \dfrac{2b^2}{9}$ (B-5)

APPENDIX C

THE ONE-PERIOD WITH INCOMPLETE INFORMATION MODEL

This appendix explains the algebra used for computing the bargaining solution discussed in Chapter Four.

S(q) and B(q) are defined as follows:

S(q): $q = p$ and, therefore, $p = q$

B(q): $q = b - p$. Thus, $p = b - q$

Payoffs for the seller and the buyers (π_S and π_B) are

$$\pi_S = p \cdot q - \int_0^q S(q)\, dq = p \cdot q - \frac{q^2}{2}$$

$$\pi_B = \int_0^q (b - q)\, dq - p \cdot q = b \cdot q - \frac{q^2}{2} - p \cdot q$$

__Bargaining equilibrium price__

1. Buyer's decision: Buyer will accept the seller's offer p only when the gain is $\pi_B \geq 0$.

$$\pi_B = b \cdot q - \frac{q^2}{2} - p \cdot q \geq 0 \tag{C-1}$$

Since $q > 0$, $2b - q - 2p = 2b - (b - p) - 2p$

$$= b - p \geq 0$$

So, the buyer accepts the offer p if $b \geq p$. The probability of such an incident will be

$$\Pr\{p \text{ is accepted}\} = \Pr\{b \geq p\} = \frac{b_u - p}{b_u - b_l} \tag{C-2}$$

2. Seller's offer: Seller will choose offer price p to maximize the expected payoffs, π_S, given the distribution of the buyer's reservation price. Therefore, the seller's problem can be written as:

$$\max_p \int_{b_l}^{b_u} [\pi_S \cdot \Pr\{p \text{ is accepted}\} + 0 \cdot \Pr\{p \text{ is rejected}\}] \, db$$

$$\text{subject to } q = b - p \tag{C-3}$$

$$\max_p \int_{b_l}^{b_u} \left[(p \cdot q - \frac{q^2}{2})(\frac{b_u - p}{b_u - b_l})\right] db$$

$$\max_p \int_{b_l}^{b_u} \left[(p \cdot (b - p) - \frac{(b - p)^2}{2})(\frac{b_u - p}{b_u - b_l})\right] db$$

$$\max_p \left[\frac{pb^2}{2} - p^2 b - \frac{(b-p)^3}{6}\right](\frac{b_u - p}{b_u - b_l}) \Bigg|_{b_l}^{b_u}$$

$$\max_p \left[\frac{b_u - p}{b_u - b_l}\right]\left[\frac{p(b_u^2 - b_l^2)}{2} - p^2(b_u - b_l) - \frac{(b_u - p)^3 - (b_l - p)^3}{6}\right]$$

$$\underset{p}{\text{Max}} \quad \left[\frac{b_u - p}{6}\right]\left[-9p^2 + 6p(b_u + b_l) - (b_u^2 + b_u \cdot b_l + b_l^2)\right] \tag{C-4}$$

The F.O.C. of the problem (C-4) with respect to p is

$$9p^2 - 6p(b_u + b_l) + (b_u^2 + b_u \cdot b_l + b_l^2) - 18p(b_u - p)$$
$$+ 6(b_u + b_l)(b_u - p)$$
$$= 27p^2 - 6p(5b_u + 2b_l) + (7b_u^2 + 7b_u \cdot b_l + b_l^2) = 0 \tag{C-5}$$

From the quadratic equation in (C-5), two solutions for the bargaining equilibrium price p can be derived.

$$p = \frac{3(5b_u+2b_l) \pm \sqrt{9(5b_u+2b_l)^2 + 27(7b_u^2+7b_u \cdot b_l+b_l^2)}}{27}$$

$$p = \frac{(5b_u + 2b_l) \pm \sqrt{(4b_u^2 - b_u \cdot b_l + b_l^2)}}{9}$$

However, the second-order-condition (hereafter, S.O.C.) for the maximization problem should be negative in order to obtain the maximum optimal point.

S.O.C.: $54p - 6 \cdot (5b_u + 2b_l) < 0$

Therefore,

$$p^* = \frac{(5b_u + 2b_l) - \sqrt{(4b_u^2 - b_u \cdot b_l + b_l^2)}}{9} \tag{C-6}$$

Constrained Pareto optimal solution

$$\underset{p}{\mathrm{Max}} \int_{b_l}^{b_u} [\pi_S + \pi_B] \cdot \Pr\{p \text{ is accepted}\}\, db$$
$$\text{subject to } q = b - p \tag{C-7}$$

$$\underset{p}{\mathrm{Max}} \int_{b_l}^{b_u} \left[(b \cdot q - q^2) \cdot \left(\frac{b_u - p}{b_u - b_l}\right)\right] db$$
$$\text{subject to } q = b - p$$

$$\underset{p}{\mathrm{Max}} \int_{b_l}^{b_u} \left[[b(b - p) - (b - p)^2] \cdot \left(\frac{b_u - p}{b_u - b_l}\right)\right] db$$

$$\underset{p}{\mathrm{Max}} \int_{b_l}^{b_u} \left[(b \cdot p - p^2) \cdot \left(\frac{b_u - p}{b_u - b_l}\right)\right] db$$

$$\underset{p}{\mathrm{Max}} \quad (b_u - p) \cdot \left[\frac{p(b_u + b_l)}{2} - p^2\right] \tag{C-8}$$

The F.O.C. of (C-8) is

F.O.C.: $6p^2 - 2(3b_u + b_l)p + b_u(b_u + b_l) = 0$

Thus, the constrained Pareto optimal solution price p^{**} is

$$p^{**} = \frac{(3b_u + b_l) - \sqrt{(3b_u^{\,2} + b_l^{\,2})}}{6} \tag{C-9}$$

Pure Pareto Optimal Price

$$\underset{p}{\mathrm{Max}} \int_{b_l}^{b_u} [\pi_S + \pi_B]\, db \qquad \text{subject to } q = b - p \tag{C-10}$$

$$\underset{p}{\text{Max}} \int_{b_l}^{b_u} [b \cdot q - q^2] \, db \qquad \text{subject to } q = b - p$$

$$\underset{p}{\text{Max}} \int_{b_l}^{b_u} [b \cdot (b - p) - (b - p)^2] \, db$$

$$\underset{p}{\text{Max}} \quad (b_u - b_l) \cdot \left[\frac{p(b_u + b_l)}{2} - p^2\right] \tag{C-11}$$

The F.O.C. of (C-11) is

$$\text{F.O.C. :} \quad \frac{(b_u + b_l)}{2} - 2p = 0$$

Thus, the pure Pareto optimal solution price p^{***} is

$$p^{***} = \frac{(b_u + b_l)}{4} \tag{C-12}$$

APPENDIX D

THE TWO-PERIOD WITH INCOMPLETE INFORMATION MODEL

In this appendix I provide the algebra used for computing the buyer's indifference valuation $\beta(p_1)$.

<u>Computation of $\beta(p_1)$ in (5-12)</u>

In (5-9) and (5-11), we have

$$p_2 = \min\{ \max(b_1, \frac{(5\beta + 2b_1) - \sqrt{(4\beta^2 - \beta \cdot b_1 + b_1^2)}}{9}), \beta\}$$

$$\text{and } \beta = \frac{p_1 - \sqrt{\delta_b} \cdot p_2}{1 - \sqrt{\delta_b}} \tag{D-1}$$

First, when $p_2 = b_1$

$$\beta = \frac{p_1 - \sqrt{\delta_b} \cdot p_2}{1 - \sqrt{\delta_b}} \tag{D-2}$$

Second, substituting $p_2 = [(5\beta+2b_1) - \sqrt{(4\beta^2-\beta \cdot b_1+b_1^2)}]/9$ into p_2 in β in (D-1) yields

$$(1 - \sqrt{\delta_b})\beta = p_1 - \sqrt{\delta_b}p_2$$

$$= p_1 - \sqrt{\delta_b}\left[\frac{(5\beta+2b_1) - \sqrt{(4\beta^2-\beta\cdot b_1+b_1{}^2)}}{9}\right]$$

So, $(9-9\sqrt{\delta_b})\beta = 9p_1 - \sqrt{\delta_b}[(5\beta+2b_1) - \sqrt{(4\beta^2-\beta\cdot b_1+b_1{}^2)}]$

$$(9-4\sqrt{\delta_b})\beta + 2\sqrt{\delta_b}b_1 - 9p_1 = \sqrt{\delta_b}\cdot\sqrt{(4\beta^2-\beta\cdot b_1+b_1{}^2)}$$

Squaring both sides yields

$$(12\delta_b - 72\sqrt{\delta_b} + 81)\beta^2 + (36\sqrt{\delta_b}\cdot b_1 - 162p_1 - 15\delta_b\cdot b_1 + 72\sqrt{\delta_b}\cdot p_1)\beta + (3\delta_b\cdot b_1{}^2 - 36\sqrt{\delta_b}\cdot b_1\cdot p_1 + 81p_1{}^2) = 0 \tag{D-3}$$

Solving (D-3) for β,

$$\beta = \frac{Q + R}{2(4\delta_b - 24\sqrt{\delta_b} + 27)}$$

where $Q = 6p_1(9 - 4\sqrt{\delta_b}) - \sqrt{\delta_b}\cdot b_1(12 - 5\sqrt{\delta_b})$

$$R = \sqrt{\delta_b[b_1{}^2(36-24\sqrt{\delta_b}+96\delta_b) - 12b_1\cdot p_1(4\sqrt{\delta_b}+3) + 144p_1} \tag{D-4}$$

Finally, when $p_2 = \beta$,

$$(1 - \sqrt{\delta_b})\beta = p_1 - \sqrt{\delta_b}\cdot\beta$$

Therefore, $\beta = p_1$. (D-5)

Combining three values of β for different values of p_2, we get

$$\beta(p_1) = \max\{ \min(\frac{p_1 - \sqrt{\delta_b}\cdot b_1}{1 - \sqrt{\delta_b}}, \frac{Q + R}{2(4\delta_b - 24\sqrt{\delta_b} + 27)}), p_1\}$$

where $Q = 6p_1(9 - 4\sqrt{\delta_b}) - \sqrt{\delta_b}\cdot b_1(12 - 5\sqrt{\delta_b})$

$$R = \sqrt{\delta_b[b_1{}^2(36-24\sqrt{\delta_b}+96\delta_b) - 12b_1\cdot p_1(4\sqrt{\delta_b}+3) + 144p_1}$$

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