ABSTRACT

MANUFACTURING PROGRESS FUNCTIONS

By

Dayr Ramos Americo dos Reis

The Manufacturing Progress Function (MPF) may be generally defined as the relationship in which the labor input per unit used in the manufacture of a product tends to decline by a constant percentage as the cumulative quantity produced is doubled. Where this relationship is present, it may be represented by a straight line in a double logarithmic scale.

The prime objective of this study is to contribute a general symbolic-analytic model of the manufacturing progress phenomenon.

Once the general model is established, an equally important objective is to respond to the need for a coherent systematic approach to be used in predicting the developments of the adaptation process in industrial concerns of almost any kind.

The work is broadly divided into four major parts. Chapter II contains a comprehensive review of the historical development of the Manufacturing Progress Function and a summary of the more important contributions to the progress

curve literature that are relevant to this dissertation.

The field of progress functions lacks notation uniformity, precise definition of the variables and functional relationships involved, and formal mathematical proofs of several assumed results. A coherent mathematical exposition can be the basis for the derivation of new important results. Towards this end an original theoretical systematization is offered in Chapter III. Chapter IV represents a continuation of the mathematical exposition initiated in Chapter III. Two related topics of practical relevance are approached: the integration of progress functions and the debatable problem of their aggregation. Original approximations are proposed for both problems.

A general symbolic-analytic model of the manufacturing progress phenomenon is offered in Chapter V. In Chapter VI the manufacturing progress model presented in Chapter V is tested with real data from nine manufacturers representing five different industries. A hundred and fifty-nine separate cases of product and process startups that occurred in four different countries and nine distinct plants are analyzed. In addition, aggregate data was obtained for whole industries in one country, yielding nine more startups.

The descriptive efficiency of the proposed model is generally supported by the results of regression analysis of the startups and startup parameters obtained from the participating industrial firms.

The findings of this research and previous findings by two other authors constitute adequate evidence to suggest that the model can be developed into an effective means of predicting the mathematical slope (parameter b) of a new startup. The author believes that the parameter model approach presented in Chapter V has proved to be superior to other existing methods for estimating the parameters of a new startup.

The dissertation is concluded (Chapter VII) with a discussion of the industrial implications of the findings reported in Chapter VI. The importance of recognizing and predicting the manufacturing progress phenomenon is related to several decision-making functions that are encountered in an industrial setting as well as in economic planning at the national level. An overall design of a computerized Manufacturing Progress Function (MPF) System is suggested.

MANUFACTURING PROGRESS FUNCTIONS

By

Dayr Ramos Americo dos Reis

A DISSERTATION

Submitted to

Michigan State University

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

Department of Management

1977

TO MY FATHER

JULIO AMERICO DOS REIS, Aer.Eng.

A PIONEER OF THE BRAZILIAN

AIRFRAME INDUSTRY

(1910-1977)

IN MEMORIAM

ACKNOWLEDGMENTS

The author wishes to express appreciation for the guidance and encouragement provided by Dr. Richard F. Gonzalez, as advisor throughout the thesis. The time and assistance of Dr. Stanley E. Bryan, Dr. Dole A. Anderson and Dr. Phillip L. Carter have also been of great value.

The author is also indebted to a large number of individuals in industry for their helpful cooperation. These individuals, who must unfortunately remain anonymous, provided the essential data for the dissertation.

I am grateful to Mrs. Doris Singer who so patiently assisted in revising the manuscript.

Without my wife, Maria de Fatima, who assisted me with editing and typing and was a source of constant inspiration, this work might never have been completed.

DAYR R.A. DOS REIS

February, 1977

TABLE OF CONTENTS

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (Cont'd)

TABLE OF CONTENTS (cont'd)

LIST OF TABLES

ASSEM

(FIR

ASSE

STOR

NPF

FIR

PAR

(FI

PA

(F

FI

FI

PA

P

P

LIST OF TABLES (cont'd)

LIST OF TABLES (cont'd)

LIST OF TABLES (cont'd)

LIST OF TABLES (cont'd)

LIST OF TABLES (cont'd)

LIST OF FIGURES

LIST OF FIGURES (cont'd)

INTRODUCTI

Pres

the level

In

not

It

ever

G

have

CHAPTER I

INTRODUCTION: THE PERCEPTION OF THE PROBLEMATIC SITUATION

Presumably, the process of economic progress involves three levels of activity: invention, innovation and transformation. Invention is a new idea, the discovery of relationships not before perceived. Innovation is the pioneering application of an invention. Transformation constitutes the substitution of existing processes and outputs by those which innovation has already shown to be superior or preferable. The end result of transformation is economic progress.

It is perhaps a platitude to say that the industrial environment in the United States and abroad - including some modern less developed countries - has been characterized by an ever increasing rate of technological innovation and transformation. Some degree of innovation and transformation is shared by virtually all manufacturing industries and their component enterprises.

Given that rapid innovation and transformation processes have become the environment as well as the internal life of a large number of industries - here and abroad - it is appropriate and opportune to investigate the possible influence of those processes upon the manufacturing activities of the affected firms and their consequences in terms of decision-making.

Adaptation to Innovation and Transformation

Innovation or transformation imply change, and change generates the need for adaptation. In the implementation of a new product, the following represent general decision-making areas in which the adaptation phenomenon might have to be taken into account:

1. Long-range forecasting and planning for capacities and locations.
2. Selection of equipment and processes
3. Production design
4. Job design and work measurement
5. Location of the system
6. Facility layout
7. Short-range forecasting
8. Inventory control
9. Aggregate planning and scheduling
10. Scheduling and production control
11. Maintenance
12. Quality control
13. Labor and cost control

Such a list indicates that the adaptation phenomenon pervades the whole activity of the production system. It also suggests the wide variety of talents that might be involved in resolving the problems generated by system adaptation to change.

Producti

he developmen

oduction pro

ture. In re

uction can

of the produc

anufacturing

or the hour

mulative pr

different ma

his asserti

The oc

cturing is

he manual

erated ad

on-labor

etitive r

upervisor

ntrol pe

labor empl

creasing

Thi

significa

em. Pra

labor tim

lling p

Productivity, System Adaptation and Decision-Making. The development and implementation of new products or new production processes can alter the means and methods of manufacture. In responding to these changes the production organization can experience a period of adaptation. The efforts of the production system give rise to notable increases in manufacturing productivity. In fact, it is generally known that the hours required to produce a new product decrease as cumulative production increases. Several studies in many different manufacturing areas and other situations support this assertion.[1]

The occurrence of this adaptation phenomenon in manufacturing is rarely a consequence of direct-labor learning of a manual task. The phenomenon usually results from an integrated adaptation effort on the part of direct-labor, indirect-labor and technical personnel. It relies primarily on cognitive rather than manual learning. Managers, engineers, supervisors, machine operators, maintenance men, quality control personnel, purchasing personnel and other indirect-labor employees can all make important contributions toward increasing the efficiency of a manufacturing process.

This type of manufacturing progress phenomenon may have a significant effect on many decision-making activities in a firm. Practically every manufacturing concern has to forecast labor time and cost per unit of new products in order to set selling prices, plan delivery schedules, estimate capital , ar and spa

ns in prod

d control f

rial list

changes i

"1. Pri

the

th

2. De

be

ca

3. P

"

(

4.

5.

6.

labor and space needs, and the like. Large, sustained increments in productivity can influence a variety of planning and control functions. N.Baloff[2] suggests the following partial list of decision-making areas that can be influenced by changes in productivity:

"1. Price-setting. The level of productivity affects the direct manufacturing costs and, in many cases, the allocation of overhead costs.

2. Delivery commitments. Delivery commitments cannot be made reliably unless the rate of product output can be correctly anticipated.

3. Production scheduling. The rate of output of a "bottleneck" can materially affect the scheduling of other processes in a sequential manufacturing operation.

4. Purchasing and raw material inventory. These activities must be synchronized with the productivity of the production processes.

5. Manpower requirements. Akin to the scheduling problem, the total labor requirements in sequential manufacturing operations may depend on the rate of output of a new bottleneck process.

6. Work standards. The determination of work standards and wage incentives is both hazardous and

diff

7. Cos

The

pro

ser

Howev

ition of

or chang

s. one

ing th

nomenon

predict

eciabl

Man

ted fo

d somet

aptatic

ufact

Th

the

lation

ons re

ducts

e gener

difficult under conditions of changing productivity.

7. Cost accounting, budgeting, and cost control. These control procedures are sensitive to changing productivity, as well as being motivationally sensitive - a volatile combination."

However, before considering the above list, the initial decision of whether or not to implement the new product, the major change or the new process must be made. In order to do this, one must count upon a valid and reliable tool for predicting the pattern and the magnitude of the adaptation phenomenon. Moreover, unless such productivity changes can be predicted at the beginning of a manufacturing startup, an appreciable degree of costly uncertainty can result.

Manufacturing progress functions (MPFs) have been documented for years in the literature as a means of describing and sometimes predicting the pattern and the magnitude of the adaptation phenomenon.

Manufacturing Progress Function Defined

The manufacturing progress function (MPF) has been used in the literature to describe several different proposed relationships between the labor input involved in the operations required to manufacture a product and the volume of production. For the purpose of this dissertation the MPF can be generally defined as the relationship in which the labor nt per un

line by a

duced is

be repr

ula. Chap

is subjec

The

have

servati

Uni

re mod

nufactu

late,

the a

el to

stems.

Fo

ecturi

r yea

und f

st po

rve.

P

ers of

input per unit used in the manufacture of a product tends to decline by a constant percentage as the cumulative quantity produced is doubled. Where this relationship is present, it may be represented by a straight line in a double logarithmic scale. Chapter III is aimed at a complete clarification of this subject.

Scope

The following observations made by the author since 1959 have served the purpose of setting the scope for this dissertation.

Universalization. The need for making the progress curve model more generally applicable in labor-intensive manufacture has been stressed for years in the literature.[3] To date, however, its importance has been mainly recognized by the aerospace industry apart some few extensions of the model to other labor and machine-intensive production systems.[4]

Form of the Model. The adequate form of the manufacturing progress function model has also been discussed for years. In the following work empirical support has been found for some of the forms proposed in the literature.[5] The most popular are the cumulative-average curve and the unit curve.

Parameters Estimation. The estimation of the parameters of the progress function for different products and

processes has been of significant concern, although few authors have tried the empirical approach.[6] It must be understood that without a valid and reliable method for estimating the parameters of the manufacturing progress function before the inception of the production process, its usefulness as a predicting tool can be virtually dismissed.

When Does Learning End? The occurrence and predictability of steady-state plateaux in the curve have also been the subject of much argument and little empirical research effort.[7]

Comparative Studies. Comparative studies of productivity values across facilities located in different countries are missing in the literature of the progress curve. Such an approach might provide thoughtful insights in theorizing about the adaptation phenomenon.

Better Theoretical Systematization. The field of progress functions lacks notation uniformity, precise definition of the variables and relationships involved, and proofs of several results assumed. A coherent mathematical exposition can be the basis for the derivation of new theoretical results.

GEN

Note on

Mode
ensible s
not ne
ionship
the same t
of the ref
of interes

A di
planatio
ly repr
of theory
are,
ly ex
ssess
ain
theor

of sc
of the
and se

Th
del's a

GENERAL AND SPECIFIC OBJECTIVES OF THE STUDY

A Note on Models

Models are taken to be analogues of existing or conceivable systems, resembling their referent systems in form but not necessarily in content.[8] They exhibit structural relationships among elements found in the referent system. At the same time they are abstractions, omitting some aspects of the referent systems and duplicating only those that are of interest for the purposes at hand.

A distinction can be drawn between representation and explanation and hence between models and theory. Models need only represent the referent system; explanation is the role of theory. While many theories may, in fact, be models, not all are, for as A. Kaplan suggests,[9] theories need not actually exhibit the structure they assert the referent system to possess. On the other hand, models that do not purport to explain are not theories. Models can also be used to predict phenomena without necessarily explaining them.

Models have long been considered the central necessity of scientific procedure. Models can increase understanding of the referent systems, aid in the development of theory, and serve as a framework for experimentation.

The disadvantages of modeling in general, stem from the model's artificiality, simplification, and idealization, and consequ

ings from

ral syst

s a crucia

Symb

inistra

ial, an

In additio

ing inc

General St

The

subject m

The

eneral s

phenomen

On

that obj

matic

of the a

any kind

Sy

purpose

ication

tives:

the consequent difficulties and dangers in making inferential leaps from a model to the real world. As a representation of a real system, the model's representativeness of that system is a crucial issue.

Symbolic models are of special interest in social and administrative science and may take various forms, such as verbal, analytic (in the mathematical sense), or numerical. In addition, flow charts and similar pictorial models are becoming increasingly popular.

General Statement of Objectives

The overall end purpose is to advance knowledge on the subject matter of Manufacturing Progress Functions.

The prime objective of the study is to contribute a general symbolic-analytic model of the manufacturing progress phenomenon.

Once the general model is established, an equally important objective is to respond to the need for a coherent systematic approach to be used in predicting the developments of the adaptation process in industrial concerns of almost any kind.

Subobjectives. The foregoing general statement of purpose can be broken down into a number of layers of investigation leading to the following more specific subobjectives:

1. A review of the literature that is of relevance to the objectives of the dissertation.

2. An investigation of the theory of the manufacturing progress function aiming at a systematization of the existing body of knowledge, and at the derivation of new theoretical results that will settle the question of the parameters estimation of the general model.

3. The conception of a general symbolic-analytic model of the system adaptation phenomenon, including the development of a method for using the model to predict the course of future startups.

4. The testing of the model in a number of real world situations by using data from diverse industrial operations.

5. The possibilities of implementing the model into practical use at firm level and national level.This signifies that the model can be practically integrated into the strategical planning of an industrial concern as well as used for macroeconomic decision-making.

CHAPTER II

THE EXISTING BODY OF KNOWLEDGE

The present chapter contains a review of the historical development of the manufacturing progress function, as well as a summary of the more important contributions to the progress curve literature which are of interest to the purpose of this dissertation.

THE WRIGHT MODEL

Historically this model is the first published attempt to relate labor hours and cumulative production.[1] After fifteen years of empirical studies in the airframe industry Dr. T.P. Wright discovered the following hyperbolic functional relation:

$$\bar{y} = ax^{-b} \tag{2.1}$$

where,

$\bar{y}$ = the cumulative average number of direct labor hours (or related cost) required per unit of output;

x = the cumulative units of output;

a = a parameter of the model, i.e., the labor hours required to produce the initial unit of production; (note that for $x = 1$,

$\bar{y} = a\ (1)^{-b} = a)$

b = a parameter of the model, i.e., a constant dependent upon the rate of progress.It is an index of the rate of decrease in labor hours during the start up (usually , $0 < b < 1$).

The so-called learning curve equation (2.1) is such that the logarithms of $\bar{y}$ and x are in a linear relationship, given by:

$$\log \bar{y} = \log a - b \log x \qquad (2.2)$$

Such relationship exhibits the convenient property that every time x doubles, successive values of $\bar{y}$ are a constant multiple (some fraction between 0 and 1) of the preceding value. This fraction of $\bar{y}$, expressed as a percentage, is commonly called the learning factor, or the learning curve "slope", according to Wright's unusual terminology.[2] Thus:

$$\text{Wright's "slope"} \triangleq \frac{(2x)^{-b}}{x^{-b}} = 2^{-b} \qquad (2.3)$$

Learning curves are often identified by this percentage. The most popular learning curve from 1930-1950 was the 80 per cent curve.

Factors in Labor Reduction. Wright attributed the reduction of labor time per unit as production quantity is

increased to four factors: (i) The improvement in proficiency of a workman. (ii) The greater spread of machine and fixture set up time in large quantity production. (iii) The economy in labor which greater tooling can give as the quantity increases. And finally, (iv) as more and more tooling and standardization of procedure is introduced, it is possible to use less skilled labor.[3] Although he did not offer a theoretical explanation of the relationship, Wright suggested that the improvements in worker proficiency that occur with increased experience are the main causative factors.[4] He argued that this effect was particularly noticeable in assembly operations.

Material, Labor and Overhead Versus Quantity. Wright holds that the three major costs of production vary with the cumulative quantity of production. It is worthy to note that although it has been a widely acknowledged fact that labor and overhead vary as quantity produced is increased, there has been a surprising lack of development on the direct material curve which Wright originally suggested.[5] He hypothesized that in addition to the reduction of waste, greater cutting efficiency and more economical purchasing partially explain the reduction in material cost. Other possible factors are reductions in price of materials because of quantity purchases and greater vendor efficiency.

The Value of Wright's Model. Many magazine articles were written about the learning curve during Worl War II .

By that time, acceptance of its theory had increased as rapidly as the aviation industry itself.

By the end of World War II, the major aircraft companies had generally recognized the value of the curve by its application to their specific production data. The smaller firms, however, comprising most of the subcontractors, had little knowledge of its use.

As to the learning-curve equation, it should be noted that $\bar{y}$ approaches zero as $\underline{x}$ increases. This feature as well as the linear logarithmic relationship were sometimes criticized. A number of models have been proposed since Wright's initial work. However, the Wright formulation continues in almost universal use in the airframe industry.[6] H. Asher notes that most airframe producers, and a large number of Air Force personnel, accept and use the Wright model.[7]

THE FORTIES

After Wright's pioneering contribution a number of modifications of the original model have been proposed. Some are similar to Wright's hyperbolic function formulation. Others include additional parameters so as to fit certain World War II production data that were not well explained by the early models. Empirical verification of the modified models has been fragmentary, and they have found little acceptance in the airframe industry. A discussion of some of the approaches to the progress curve that appeared in the forties

is given below.

The Crawford Model

J. R. Crawford has contributed much to the literature of progress functions. He was associated with the Stanford Research Institute and the Air Materiel Command immediately following World War II. His chief contribution was a study of two hundred jobs in the airframe-manufacturing process, which resulted in a new formulation of the progress curve. Crawford's equation for the learning curve can be expressed as follows:[8]

$$\bar{y}_L = ax^{-b} \tag{2.4}$$

where:

$\bar{y}_L$ = lot-average direct-labor hours per unit of output

x = cumulative units of output

$\underline{a}$ and $\underline{b}$ = parameters with the same meaning as in Wright's model.[9]

Clearly, the difference from the Wright formulation resides in the definition of the dependent variable. In the Wright model $\bar{y}$ is calculated as a <u>cumulative</u>-average manhour expenditure, whereas in the Crawford model $\bar{y}$ is calculated as the average manhour expenditure for the "lot" produced in a single month.

In reality, Crawford's model cannot be considered a truly <u>unit</u> curve, since $\bar{y}_L$ does not refer to the labor hours consumed in the xth. unit but simply has to do with the average number of direct-labor hours per unit, expended for the "lot" produced in the kth. month. In Chapter III, this form is further discussed as well as its relationship with other forms proposed in the literature.

According to H. Asher, a number of Crawford's attempts to develop a progress curve of simpler equations did not yield forms acceptable to the airframe industry or to the Air Force.[10]

Crawford makes a relevant observation that will be of interest at a later point, specifically, that the shape of an individual operator's progress curve is a function of the mental effort required of the operator to perform the task. For Crawford, mental effort expended is also a function of the complexity of the job and of the lack of experience of the worker. Thus, when the work requires more (less) mental effort, the ratio of improvement increases (decreases). This is true when the job is complex (simple) requiring great (little) mental effort, and also when the operator is less (more) experienced and requires more (less) mental effort to learn the technique involved.[11]

It seems that Crawford was the first author to suspect a link between the rate of improvement in a job and its degree of complexity and novelty. This relationship would come to be investigated by Hirsch, Conway, Schultz, and others, some years later.

In addition to proposing his modified learning curve, Crawford also noticed that different slopes might exist for different airframes - a result that was largely ignored by the rest of the industry.[12]

Bureau of Labor Statistics Studies

The Productivity and Technological Development Division of the Bureau of Labor Statistics, Department of Commerce,has prepared a number of studies that deal with progress-curve data, and their content will be summarized below.

The Liberty Vessel Study. The first progress curve data on ship construction was reported by Montgomery in 1943.[13] The study deals with the EC2 Liberty ship of 10,800 dead weight tons (TDW). The man-hour requirements data represented both direct and indirect labor hours. The direct man-hours had to be estimated from actual man-hours required for a group of ships; the total was then averaged for the number of units within the group. Between December 1941, when the first two ships were completed, and April 1943, when nine hundred ships where delivered, the average man-hours required per vessel decreased by more than one-half.

Wartime Shipbuilding Study. Another relevant study was done at the Bureau of Labor Statistics by A.D. Searle.[14]

The study is similar to that made by F.J. Montgomery except that it covered Liberty and Victory ships, tankers and standard cargo vessels. The labor data used to determine labor requirements are <u>total</u> man-hours defined in the same way as in the Montgomery study, already described.The data covers the period between December 1941 and December 1944 with emphasis on the interval between April 1943 and December 1944. A.D. Searle notes that each individual shipyard exhibited a different learning curve as to the Wright "slope". However, the "slopes" were almost identical for the different types of vessels considered, i.e., about 80 per cent decline in man-hour requirements between doubled quantities.[15] The study concludes that differences in the types of vessel are <u>less</u> significant in determining the rate of man-hour requirement reduction, than the production function differences between individual yards. The above two studies demonstrate the existence of the progress phenomenon in ship construction programs. Moreover, they constitute a pioneering effort towards studying learning curves outside the airframe industry.

Wartime Productivity Changes in the Airframe Industry. This is the third in a series of studies which were carried out by the Bureau of Labor Statistics. Conducted by Kenneth A. Middleton it deals with productivity changes.[16] He noticed that there was approximately a tripling of production per man-hour during World War II and he attempts to identify the factors responsible for the observed productivity increment.

Middleton holds that the remarkable reduction of labor cost per unit is the result of technological changes such as increasing standardization of models, introduction of more sophisticated hand tools, and gauges. The rate of adoption of the new methods and new production techniques depends on management willingness to break with traditional methods.

Kenneth A. Middleton also attempted to construct an industry wide production index by relating man-hours _per pound_ of airframe and the cumulative production _pounds_ in the particular facility.[17]. He found that the decrease in man-hours required per pound is _similar_ for _all_ types of air planes and that this index is useful for comparative purposes, although somewhat more man-hours per pound were required for a one engine fighter than for standard four engine bombers. Thus, using the above indicator of productivity, Middleton concludes that the 70 per cent progress curve is more representative of the aircraft industry during World War II than the normally assumed 80 per-cent curve.[18] It is worthy to note that Middleton also has found considerable variation in the "slope" of the curve from one plane to another.

Airframe Companies Publications

Besides J.R. Crawford's contribution while working at the Lockheed Aircraft Corporation several progress-curve studies had also been prepared by other airframes companies in the forties, like Chance Vought Aircraft, Inc.,[19] and the

Boeing Airplane Company.[20]

The Chance Vought study was designed to instruct company personnel in the application of the cumulative average curve, being similar to the Wright presentation, and hence will not be discussed herein. The study prepared at Boeing has some features relevant to the scope of this dissertation and will be briefly reviewed.

The Experience Curve. This short book - one of the earliest company publications available - presents a concise statement of the unit curve, the cumulative average curve and the total curve as they were used by Boeing Cost Accounting Department. The pamphlet provides a mathematical presentation of the progress curve. From the equation for the unit curve, i.e.,

$$y = ax^{-b} \tag{2.5}$$

the exact expression for the total labor hours expended in the manufacturing of units one up to and including the nth. unit is given by:

$$y_{T_n} = a \sum_{i=1}^{n} x_i^{-b} \tag{2.6}$$

Since equation (2.6) requires calculating every unit value from 1 to n, an approximation was suggested by the authors, in which the limits of integration were formed by extending the range of the summation index so as to reduce the error inherent in approximating a discrete function by a continuous function. Thus:

$$y_{T_n} = a \sum_{i=1}^{n} x_i^{-b} \cong a \int_{0.5}^{x_i+0.5} x^{-b}\, dx =$$

$$= \frac{a}{1-b} \left[(x_i + 0.5)^{1-b} - 0.5^{1-b} \right] \qquad (2.7)$$

Studies Financed by the Air Force

The Source Book of World War II Basic Data. This Air Force study of the progress curve published in 1947 has been the principal source of data for several empirical progress-curve studies.[21] It contains data from every facility engaged in the production of military models from 1940 through mid-1945, presenting unit progress curves for each type of aircraft and one progress curve for all aircraft produced during the war period. The curves obtained by using the least-squares method represent direct labor hours per pound of airframe weight versus the cumulative number of airframes produced. The following table presents the parameters of the progress curve obtained in the Source Book of World War II Basic Data:

TABLE 2.1

PARAMETERS OF THE PROGRESS CURVE

Type of Aircraft	Man-hours per Pound at Unit Number One (a)	Wright's "Slope" (%)
Fighters	18.5	79
Bombers	16.0	77
Transports	16.0	77

The Crawford-Strauss Study. An analysis of World War II production experience was made by Crawford & Strauss in 1947.[22] The major purpose of the study has to do with the acceleration of airframe production; but since progress curves are important tools in any airframe production program, they are given careful attention.

The data was derived from the "Source Book"[23], and was based on the production data of 118 Worl War II models. Weighted average direct man-hours per pound at specific plane numbers were determined both for the industry as a whole and for each type of airplane (fighters, bombers and transports). The resulting unit-progress-curve equation for all models was

$$y = 14.3\ x^{-0.32668} \qquad (2.8)$$

From (2.8) and (2.3) it is easy to see that:

$$a = 14.3 \quad \text{and} \quad \text{Wright's "slope"}=2^{-0.32668}=0.797.$$

It was probably this study that established the "80 per-cent rule" as standard in the airframe industry.

Crawford & Strauss found that at least three major factors were responsible for the dispersion of the individual progress curves around equation (2.8): (i) type of aircraft (fighters, bombers or transports); (ii) newness of the model and of the facility, and (iii) particular circumstances and problems that surround production of each individual model.

As to the relative position of the fighter, bomber and transport curves, they make the point that the bomber learning curve is the lowest because (i) bomber programs during World War II were given priority over most other programs, and (ii) the size of bomber aircraft permitted greater access in the assembly operation. The fighter curve is the highest because: (i) fighters are complex aircraft and (ii) their design changes often.

In connection with the newness of models and facilities, the 118 models were classified as: (1) Proven models produced in experienced facilities; (2) proven models produced in new facilities, and (3) new models produced in new or experienced facilities. Weighted average curves like the previous

asis of

Krauss o

roduced

per poun

is not m

been pro

nued in

aircraft

which th

curve f

is cons

raft.

T

been f

the sp

tion o

the a

thath

tion,

When

was s

desig

When

ity,

ty,

previous ones were obtained for each class above. On the basis of visual inspection of such curves Crawford and Strauss concluded that: The early units of a proven model produced in an experienced facility require fewer man-hours per pound than the average for all aircrafts. This advantage is not maintained, however, after a few hundred units have been produced. The progress curve for proven models produced in new facilities follows the average curve for all aircraft until a few hundred units have been delivered,after which the former curve falls slightly below the latter. The curve for new models produced in either new or old facilities is consistently higher than the average curve for all aircraft. These results are summarized in Table 2.2 .

The third factor - or cluster of factors - which has been found to affect the level of progress is described as the special circumstances and problems that surround production of each individual model. Some of these are listed by the authors as: (i) The length of the production run, (ii) whether or not the model has been engineered for mass production, (iii) whether or not proven engineering was available when production started, (iv) whether or not high production was started from low production tools, (v) introduction of design changes, (vi) whether or not old tools were available when production started, (vii) availability of materials or component parts, (viii) availability of experienced manpower, (ix) relative priority attached to a given model, (x) efficiency of operating controls,(xi) frequency of scheduled

INFLUEN

ON TH

...		
...	C C u	
...	C t f	

changes

economi

degree

ized

to the

partic

produc

measur

affect

TABLE 2.2

INFLUENCE OF THE NEWNESS OF MODELS AND FACILITIES ON THE RELATIVE POSITION OF PROGRESS CURVES

	facility	
model	new	old
old	curve follows the avg. curve until a few hundred units; then it falls below	curve is lower than the avg. curve only up to a few hundred units
new	curve is higher than the avg. curve for all planes	curve is higher than the avg. curve for all planes.

changes and degree of pressure attached to a program, (xii) economical and uneconomical use of outside production,(xiii) degree to which feeder plants and outlying areas were utilized in order to tap a wider labor market, (xiv) whether or not the plant layout was favorable to the production of a particular model and (xv) availability of specialized high production machinery.[24]

The authors conclude that it would be difficult to measure the effect of special circumstances or to weigh direct labor progress curves and industry averages for these

factors. However, they believe that the industry average established in the study presents a reliable picture of the relationship of direct man-hours per pound to cumulative production during World War II, since they include the cumulative effect of all particular circumstances and problems which surrounded the production program, for each model.

The Stanford-B Model. After World War II a number of economists and econometricians became interested in learning curve research. The Air Force had also come to recognize the importance of the progress curve, and sponsored several research projects with private organizations to develop further the application of the theory. Probably the best known of these were the studies carried out by the Stanford Research Institute and by the Rand Corporation. Since the latter were mainly published after 1950 they will be considered in the next section.

Under contract with the United States Air Force, Air Materiel Command, the Stanford Research Institute made a study on the Relationships for Determining the Optimum Expansibility of the Elements of a Peace-Time Aircraft Procurement Program in order to determine the means of measuring the maximum rates in aircraft production programs. Since the maximum expansibility rates depend to a large extent on the rate of manufacturing time reduction, a decision was made to analyze the direct man-hour progress curve. According to the final report the project has resulted in an improved relationship involving direct-labor hours, airframe unit weight and cumulative production.[25]

The Stanford-B formula or the learning formula with a B-factor can be expressed as follows:

$$y = a\ (x + B)^n \qquad (2.9)$$

where:

y = direct man-hours required for cumulative unit number x.

B = equivalent-units of experience available at the start of a manufacturing program; $1 \leq B \leq 10$, 4 being a typical value.

n has a similar meaning as the b-exponent in the Wright model. Usually $-1 < n < 0$, -0.5 being a typical value.

For the case $B=0$, $\underline{a}$ represents man-hours for the first unit. The curve is asymptotic to a straight line with slope $\underline{n}$, and approaches such a line over its entire length as $\underline{b}$ approaches zero. The effect of the B-factor is to round the initial portion of the curve thus providing a better approximation in determining the number of labor hours (and cost) for the first units produced.

Such a curve - or its variants - have satisfactorily described the general features of many large scale military aircraft production programs, including the Boeing B-17,B-47, B-52, and B-707 programs.[26]

Its is worthy to mention that the reasons for preferring equation (2.9) rather than equation (2.5) or (2.1) is not that the former fits the empirical data more closely. In fact, the Stanford study does not intend to establish this point. Rather, equation (2.9) is preferred merely because its parameters are more suitable to predicting airframe production performance than the parameters of the conventional progress curves.

Progress Curves in Great Britain and France

Among British and French manufacturers the progress-curve concept was also popular as evidenced by the contributions of E. Mensforth and P. Guibert.

In Great Britain. The first published evidence of progress curves existing in Great Britain was indicated by Eric Mensforth, who was associated with the ministry of aircraft production in World War II.[27] Mensforth found that British experience with progress curves was rather similar to that of American aircraft production. British figures exhibit the same general trend within limits of 75-85 percent.[28]

Mensforth considers the peak volume of production to be a major factor in the declining of direct-labor hours per unit. He suggests that when the scale of production is greater, it permits more specialization and earlier attainment of full dexterity. To illustrate this point Mensforth cites a case of two factories which were producing the same aircraft with similar equipment and tooling, at rates of 15 and 55 planes per week. The actual man-hour of the latter were half that of the former. Since both of the factories were on a piecework basis and the same rate was paid, the earnings of the one producing 55 units per week were 200 per cent higher.

In France. The most comprehensive study of the progress curve prior to 1945 was published in France by P. Guibert.[29] The book was translated into English by the U.S. Air Force but this translation has not received widespread circulation.[30] Guibert considers the rate of production as an important variable influencing unit labor cost. He also views the progress curve as approaching a plateau after a large number of units have been produced. The beginning of the plateau depends on the rate of production. His rationale for considering unit man-hour cost as a function of the rate of production is that to achieve a given production rate, it is necessary to design and build adequate tools, a dominating time factor.[31] Guibert points out that although the number of certain tools must increase with the rate of production, there are many which can only be considered for

larger series (e.g.,the tools of the authomatic lathes,high cost dies for presses, jigs for simultaneous assembling and drilling), and these take a long time to build.

Through empirical studies, Guibert obtained one progress-curve equation that can be used for any rate of production:

$$y = m + \frac{(\alpha - 1)(\alpha - m)(1 - m)(A - 1)}{x(\alpha - 1) + A(1 - m) - (\alpha - m)} \qquad (2.10)$$

where:

y = unit man-hour cost expressed as a ratio of the man-hour cost of the Ath airplane produced

x = number of units (planes) produced

A = the number of units in process when peak production is attained.

α = the value of y when x = 1

m = the value of the horizontal asymptote (plateau).

Guibert found that m,α,A may be approximated by empirical equations where the only independent variable is the rate of production a. Therefore, for a given rate of production a, it is possible to determine m,α,A and to establish y as a function of x through equation (2.10).[32]

Other Contributions

The contributions to progress-curve literature that are summarized in this chapter were selected for review because they appeared to have added substance to the development of the concept. It would be unfair not to include some other contributions by individuals who seem to have been deeply interested in augmenting the knowledge about learning curves.

A.B. Berghell. A mathematical treatment of the progress curve may be found in Berghell's book entitled Production Engineering in the Aircraft Industry,[33] which contains a chapter on "Learning Curves". Aside from Wright's article, this is perhaps the most popular discussion of progress curves in the forties. Berghell shows the mathematical relationships existing among the cumulative average curve, the unit curve, and the cumulative total curve. He also offers an approximation for the equation of the unit curve, derived by the use of empirical equations.[34]

A different problem to those who work with learning curves is that of estimating curves for aircraft not yet built. Berghell suggests that if the cumulative average man-hour data are plotted against cumulative output for, say, four different aircraft, the resulting slopes of these average curves will not be significantly different from one another[35] (i.e., the b values will be similar), but the level of the curves (i.e., the a values) will be different. The spread between the curves can be reduced by dividing direct labor hours by airframe weight. Since Berghell assumes that airframe weight remains the same for each plane, the slopes do not change. Berghell argues that a heavier airframe will generally require fewer labor-hours per pound than a lighter airframe at the same cumulative unit number. Therefore,it is expected that, in terms of direct labor-hours per unit the curves for the heaviest airframes appear at a higher level, but in terms of direct labor-hours per pound, the curves for the heaviest airframes are on a lower level. Berghell then reads off the direct labor-hours per pound at units 50 and 100 for the four aircraft, plots these eight values against airframe weight on logarithmic grids, and obtains two linear curves showing the relationship between direct labor-hours per pound and airframe weight, one curve for unit number 50 and the other curve for unit number 100. The man-hours per pound of airframe values for any new airplane are then obtained at units 50 and 100 by interpolation for the given airframe weight. These two cumulative average man-hour per pound points are all that is needed to draw the cumulative average progress curve for the new airplane.

I.M. Laddon. Laddon was Executive Vice-President of Consolidated Vultee Aircraft Corporation. The article he presents contains a description of change in the production function as the total quantity to be produced by a company is increased. The author explains the difference in production methods when sixty units are to be produced as opposed to several thousand. The company performed much better than the customary 80 per-cent curve.[36]

G.W. Carr. One writer who believes the progress curve should take on the "S" shape is G.W. Carr.[37] He found that, in several cases, the rate of labor decrease in a given airframe varied, yielding different values of b instead of one constant value. Carr did not offer a formal relationship embodying these conclusions, but another modified model, the "Boeing hump-curve", is a recognition of his findings.[38] The concavity early in the series was also recognized by the Stanford Research Institute.[39] Carr argues that such concavity results from hiring inexperienced crews at different points in time during the production of the first several lots of airframes. The empirical data available to P.Guibert from the French airframe industry apparently exhibited the same early concavity.[40]

THE FIFTIES

A number of decisive contributions to the theory and development of progress curves were published in the 1950s. There was an effort by some researchers to extend the concept to other labor-intensive industries outside the airframe industry. Also, studies continued to be financed by the Air Force in several research institutes of the country.

Extensions of the Concept Outside the Airframe Industry

Werner Hirsch's Studies and Research. Hirsch's studies were mainly published in <u>Econometrica</u> and the <u>Review of Economics and Statistics</u>.[41] In one of his works,[42] Hirsch computed a total of 22 empirical progress functions, based upon historical data from one of the country's largest machine tool manufacturers. The results of the study may be summarized as follows:

(i) The hypothesis of linearity (when in logarithmic coordinates) between the labor per unit and cumulative output was confirmed for all cases. In 17 out of 22 cases, the correlation coefficient was found greater than 0.85. In only one instance this coefficient was as small as 0.59.

(ii) The 22 empirical progress functions revealed a negative slope, i.e., labor hours per unit decrease as cumulative production increases.

(iii) The average progress ratio[43] was 19.3 per cent and the range (16.5 - 24.8).[44]

(iv) Another important conclusion was that different kinds of operations exhibit different "slopes". Thus, the 80 per cent learning curve cannot be universally applied. In fact, Hirsch has shown that assembly operations are characterized by significantly higher progress ratios than are machining operations. An explanation offered was that in the former the labor content is <u>higher</u> than in the latter.

(v) Compounded experience. In a paper [45], Armen Alchian suggested that older, more experienced manufacturing facilities exhibit a greater rate of decline than do new facilities. To test this hypothesis Hirsch used the data of a manufacturer who had produced more than 15 lots of a certain semi-automatic turret lathe, in the same plant, when he initiated production of a greatly improved model. In more than one respect the new model resembled the previous one, and work on the first appeared to have constituted valuable experience. In a sense, then, the improved model was built with the help of much previously accumulated experience. Thus, the question was whether the progress ratio of the new lathe would be significantly greater than that of the old one. Progress functions for both lathes were calculated, and progress ratios (both for assembly and machining work) of the model built with the compounded experience were found significantly larger than those of the first model. This conclusion is consistent with Alchian's "experience" hypothesis.

Hirsch was probably one of the first researchers to recognize the value of the manufacturing progress function outside the airframe industry.His contribution represents one of few published empirical studies in non-airframe industries.

Stanley E. Bryan's Study. In an insightful article[46] Professor Bryan related the manufacturing progress phenomenon to value concepts and collected data on its existence in a large company's footwear plant. He points out that in certain types of procurement like the contracting for special and non-standard equipment, competition ceases to be a decisive influence in price determination. In such cases , pricing becomes a matter of negotiation. In negotiation, price is related to cost to a greater extent than to the utility of the product. Consequently, the method of estimating and compiling cost of producing an item becomes crucial both to the buyer and to the seller. If the manufacturer does not take into account the progress phenomenon in his estimate he would quote unrealistically high cost on the labor portion of the contract. If the buyer is not aware of that phenomenon he would accept the seller's estimate as being fair.[47] According to Professor Bryan's data, this particular manufacturer of footwear experienced an approximately 90 per cent "slope" progress curve.

The Schulz and Conway Study.[48] The existence of the progress phenomenon had been known for some time at IBM-Endicott, but until 1955 no systematic study had ever been made to determine whether the decline in direct-labor hours per unit of output followed any predictable pattern. With this in mind, a group of graduate students from the Industrial Engineering School at Cornell University, under the guidance of Dr. Andrew Schulz, Jr., defined the following objectives for a study of labor hours reduction trends: (i) To furnish a better basis for product pricing, product replacement, and decision to manufacture. (ii) To provide a better basis for estimating and planning the amounts of space and manpower required by a proposed manufacturing program, and (iii) To help in the planning and budgeting of engineering or other staff effort for cost reduction activities.

The study carried out to develop the Manufacturing Progress Function consisted in: (i) The collection of a number of series of labor hours with the corresponding production quantities, together with all the supporting information that could be obtained, such as: engineering changes, manufacturing methods and tooling changes, personnel turnover rates, and the like. (ii) The analysis of the information so as to associate cause and effect, and to isolate the relevant variables, and (iii) The generalization of the results obtained from the analysis. The conclusions reached were based on a detailed study of an accounting machine and verified by sample checks on other IBM machines.

The following factors were found to influence the decline in labor hours per unit with increased cumulative output:

(1) The degree of similarity of a machine to a predecessor or to other machines produced had a significant effect on the rate of progress: the less similarity, the greater the rate of progress.[49]

(2) Single-product departments exhibit greater rates of progress than do multi-product departments.

(3) Product redesign, and tool and methods improvements result in a greater rate of progress.

(4) Increasing rates of production as a machine program accelerates result in economies.

(5) Management progress in scheduling and supervision results in a greater rate of progress.

(6) Increased planning prior production will result in a lower initial cost and a reduced rate of progress.

(7) Worker learning through repetition, changes in method, and reduction in scrap and rework result in a greater rate of progress.

According to the authors of the study, the last item has either been overemphasized or erroneously indicated as the main causal factor in the literature prior to 1959. They indicate that operator learning in the true sense of performance of a fixed task is of negligible importance in most manufacturing progress. Changes in tooling, methods and product design that are usually the result of management and engineering effort rather than operator learning in any sense, have been found much more significant.[50]

Terminology. An understanding of the following terminology will facilitate further discussion:

(i) Manufacturing Progress Function Hours - Those hours over and above the estimated hours which are caused by the introduction of a new unit into a manufacturing system.

(ii) Manufacturing Progress Function Cost - The cost associated with the MPF hours.

(iii) Ultimate Unit of Production - That point in cumulative production at which the reduction in manufacturing hours per unit from the first unit in the month to the last unit for the month is between 2% and 3% and thus can be considered nominal.

(iv) Ultimate Hours - The estimated hours required to produce the unit at the ultimate unit of production.

(v) Standard Parts - Component parts used in previous machines.

(vi) Labor Value Index (LVI) - A measure of a component part value determined as follows:

$$\text{LVI} = \frac{\text{Estimated unit}}{\text{hours at ultimate}} \times \frac{\text{Parts usage}}{\text{per machine}} \times \frac{\text{No. of machs. produced}}{\text{per month at ultimate}}$$

Problem of Aggregation. In order to make the study, it was necessary to accumulate a great deal of historical data on manufacturing and assembly hours for the machines and units under consideration. An analysis of the data showed that different types of operations exhibited varying progress trends. As a result, the operations were broken down as follows:

(1) Final Assembly Operations: (a) Mechanical Assembly; (b) Wiring; (c) Inspection, Test and Clear trouble.

(2) Sub-Assembly Operations: (a) Single-product departments; (b) Multi-product departments.

(3) Manufactured Parts: (a) Standard Parts; (b) New Parts, further subdivided into: New Parts with LVI $>$ 140 and New Parts with LVI $\leq$ 140.

These breakdowns are referred to as categories. In analysing the data further for determining the MPF curves, the data was handled by categories for a machine. For example, when studying the trends in cost reduction for parts having LVI $\leq$ 140, all parts for a given machine having LVI $\leq$ 140 were handled as one figure, rather than as individual components. It was felt that the study of separate components would tend to give misleading results, whereas a group of parts would be more representative of the category.

The intent was to break the labor content down into categories that have reasonably uniform behavior with regard

<u>to the rate of progress</u>. Such classifications, however, are subject to the criticism that for application of the function, much subjective judgement is required until more knowledge is gained.[51]

Form of the Model and Parameter Determination. From the investigation of historical data Schultz and Conway adopted the Crawford model for the manufacturing progress curve:

$$\bar{y}_L = ax^{-b} \tag{2.11}$$

where:

a = the direct-labor hours required to produce the initial unit of production.

b = a constant dependent upon the rate of progress.

x = cumulative unit of production

$\bar{y}_L$ = lot-average direct-labor hours per unit of output.

This same curve when plotted on full logarithmic paper becomes a straight line. Such line will be designated by the term progress line. The method of least squares was used so as to obtain the optimum fit of the progress line to the raw data. To this end, a code was developed in order to compute the following information for <u>each</u> of the aforementioned categories: (1) The <u>a</u> parameter; (2) The <u>b</u> parameter; (3) The value of y for the thousandth unit; (this permits the plotting of the least squares trend line between <u>a</u> and y_{1000}); (4) The

Wright "slope" percentage; (5) The confidence limits; (6) The coefficients of correlation of the data.

Plotting the Progress Curve for a New Machine. The application of the "slope" percentages thus derived requires judgement for assembly operations. Since such values were obtained mainly from the study of a particular machine (call it the basis machine), all new machines to which a manufacturing progress function study is to be applied must be compared to the basis machine as to complexity and novelty, as well as to any similar predecessors to the machine under consideration. Empirical tables were developed that permit such comparisons and the final selection of the Wright "slope" for a given category of operations. In such tables, the greater the complexity of the focused machine as compared to the basis machine, the greater the rate of progress to be experienced in future manufacturing.Also, the greater the similarity with respect to a predecessor or to other machines produced, the smaller will be the rate of progress. Two other factors, namely the amount of tooling completed prior to initial production, and the rate of production at the ultimate month influence the "slope" determination for some categories. In the referred tables, the greater the percentage of tooling completed prior the inception of production, the smaller will be the future rate of progress for the category involved. In addition, the greater the rate of production scheduled for the ultimate month, the greater the rate of progress.

The next problem in plotting the progress curve for a new machine is the determination of the ultimate unit of production. As a result of the study carried out by the Cornell Group, the following locations of ultimate units were recommended: (1) Manufactured Parts - 12 months; (2) Final Assembly - 18-24 months. Increased complexity, rate of production at ultimate and novelty of a machine will dictate the ultimate units at or near the 24 month end of the range. These relationships are presented in tables similar to those employed in selecting the Wright "slope" for the category.

In a paper,[52] Schreiner describes the technique used at IBM-Endicott to apply the Manufacturing Progress Function Procedure developed by the Cornell Group to their manufacturing activities. In order to plot the progress curve for a new machine it would be sufficient to know the values for the parameters a and b. However, this procedure requires the knowledge of the direct-labor hours consumed by the production of the initial unit (i.e., the a-parameter). Since knowledge of the manufacturing progress function cost is required prior to the inception of production, this information would not be available. A different approach is used instead.

When a machine has been designed and tested, Cost Engineering prepares an estimate on the machine (using, for example, predetermined times). These estimates are carried out for each component, sub-assembly, and final assembly by operation assuming optimum conditions, i.e., the Methods Engineer considers all tooling complete and operator learning also complete. The estimates are then separated into the foremen-tioned seven categories. Next, considering one category at a time, the direct-labor hours for the category are totalled. This value, y_u, is represented in Figure 1 by the horizontal line at which the progress curve should level off.[53]

The next step is to determine, in units of production, <u>when</u> the progress curve is expected to level off, that is, to

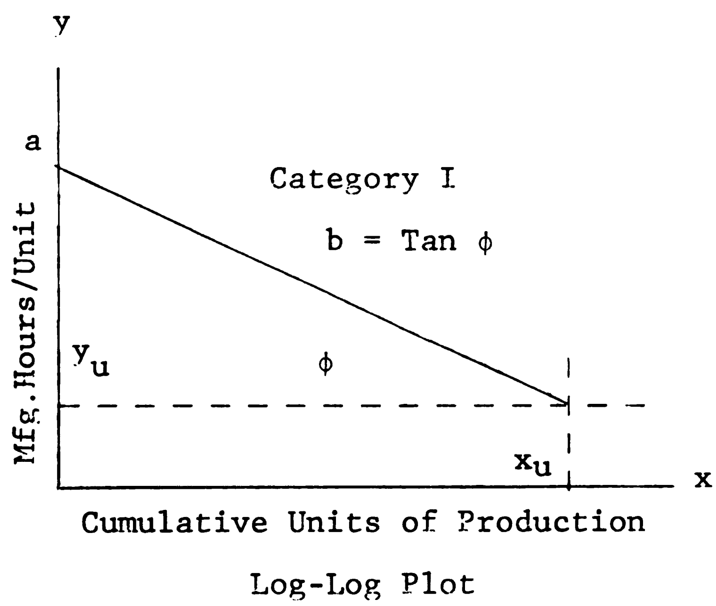


FIGURE 1

Plotting the Progress Line For a Given Category select the <u>ultimate</u> unit of production. This is done by comparing the machine being studied to predecessor machines for which the progress rates are known . As mentioned before, three characteristics are employed in the selection of ultimates : (1) The relative complexity or difficulty of the operations ; (2) The novelty or newness of the machine as compared to earlier operations ; (3) The number of units to be produced per month at ultimate . These characteristics permit to obtain the entries to the proper tables of ultimates , already referred . Having selected the ultimate unit of production x_u , it is now possible to locate point (x_u , y_u) of the progress line (see Figure 1).

After the mathematical slope of the progress line through point (x_u, y_u) is determined, it will be straightforward to draw the progress curve. Again the focused machine is compared to previous machines as to complexity of operations and novelty. The comparisons together with the expected rate of production at ultimate yield entries to the proper tables of "slopes" already mentioned. Since the Wright "slope" is given by equation (2.3), it is easy to see that the angle ϕ between the progress line and the horizontal axis can be calculated by:

$$\phi = tg^{-1}\left[\frac{\log(\text{Wright's"slope"})}{\log 2}\right] \qquad (2.12)$$

The knowledge of angle ϕ makes it possible to draw the progress line through point (x_u, y_u) and to determine the a value for the category (Figure 1). Knowing the a and b parameters for each of the categories it is possible to draw the progress curve on arithmetic paper (Figure 2).

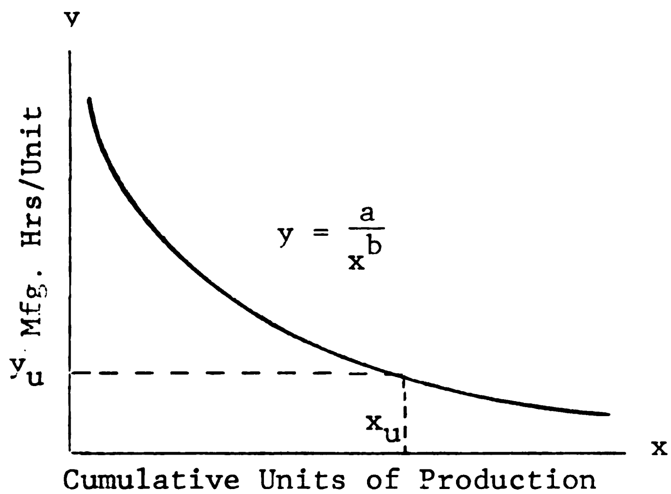


FIGURE 2

Progress Curve in Arithmetic Graph

Airframe Companies Publications

Cost-estimating manuals and the results of progress-curve studies have been prepared by the Glenn L. Martin Company,[54] by Northrop Aircraft, Inc.,[55] North American Aviation, Inc.,[56] and the Boeing Airplane Company.[57]

The Martin and Northrop studies were designed to train company personnel in the use of the cumulative average curve.

The North American publication was prepared for the instruction of the purchasing personnel in the use of the learning curve in procurement of subcontracted parts. It is a concise statement of how a purchasing agent may apply the learning curve to the many procurement problems which arise in the course of his duty. The latter three studies are similar to the Wright presentation, and hence, will not be discussed further.

The Boeing publication which was authored by W.F.Brown presents the learning curve as it is used by Boeing, in relatively easy to understand form. In speaking about the production of automobile bodies, the author holds that whereas time reduction continues to take place even after a large cumulative output has been reached, this reduction will be insignificant and may take a long time to realize.[58] This is contrary to the opinion held by Boeing personnel in the past, which was:after a certain number of units have been produced, the progress curve reaches a plateau.[59]

It should be remembered that most of the publications, do not contain empirical data to support statements made , but presumably, the statements are based on company "experience".

Studies Financed by the Air Force

The RAND Corporation of Stanta Monica, California,under contract with the Air Force, has prepared several studies of the pro

the Cr

of int

and As

A.

charac

during

within

are (

we fo

fundam

(3) D

altern

found

ted

on w

both

direc

Eich

Tact

that

be a

leas

ther

the progress curve based on the same data that were used in the Crawford-Strauss and Stanford studies already reviewed. Of interest to this dissertation are the studies by Alchian and Asher, to be summarized in the following paragraphs.

A. Alchian's Study . This study[60] investigates several characteristics of the new airframe startups that ocurred during World War II. In regards to the progress curve, Alchian poses several questions, the most pertinent of which are (1) How long does the decline in unit labor hours continue for a given model? (2) Does the progress curve correspond fundamentally to a linear function on log-log scale? and (3) Does one progress curve with given parameters _a_ and _b_ adequately describe the consumption of direct-labor hours per pound of airframe for _all_ models?

As to the first question, no cessation in the manhour decline was evident to the author. This conclusion was based on visual examination of the graphs presented in the _Source Book of World War II Basic Data: Airframe Industry, Volume I_, already cited.

As to the second question, no attempt is made in Alchian's study to establish whether or not a suitable alternative exists to the linear progress curve because of the inadequate amount of data available for the study. It should be added that fitting the Crawford model to the data via least squares regression analysis yielded correlation coefficients exceeding 0.80 in the twenty-two startups analyzed.

In answer to the third question, Alchian's study reveals that the slope and height of a single progress curve do not represent the unit man-hour expenditures required for all aircraft models. Different values of parameters a and b were found for different airframes. Moreover, classifying the airframes by type (fighter, bomber, and so on) still resulted in diverse a and b values within each class. These findings appear to erase any doubt about the validity of the "universal 80% - learning curve".

H.Asher's Study . In the second RAND study, Asher attempts to demonstrate that the progress curve in linear terms does not describe accurately the relationship between unit manhours and the cumulative output.[61] To show that the progress curve departs from linearity after a certain cumulative production number has been reached, Asher examines hourly data for a number of individual producing departments. It is concluded that the linear approximation is reasonable for all departments for an initial quantity of airframes. However, the different departments exhibit non-similar slopes for these linear segments . If an analysis of actual data reveals that departmental progress curves do, in fact, have significantly different slopes from each other, then the unit curve (the sum of the departmental curves) cannot be linear.Instead, with linear but nonparallel departmental curves, the composite unit curve must be convex on logarithmic grids and must approach as a limit the flattest of the departmental curves. This mathematical consideration alone is sufficient to demonstrate Asher's point of view. He then, proceeded to sum the department curves. The aggregate unit curve shows that it begins to level off at approximately unit 125, and the author claims that if a linear extrapolation was made between units 100 to 1,000, the estimating error would be around 25 per-cent. With some reservation about the limited samples examined in the study, Asher concludes that beyond certain values of cumulative output, the progress curves examined develop convexity and thus the conventional linear progress cannot be considered an accurate description of the relation-ship between unit labor hours and cumulative output.[62]

In examining the continuous-declining characteristics of the progress curve, Asher found definite discontinuities in the individual shop data at high cumulative outputs. In addition, the discontinuities persisted even when the data from different departments were aggregated into total labor hour expenditure for an airframe.

Asher also investigated the "slopes" that occur in different producing departments. Using the Crawford model,he found that an aggregate of sheet metal work, machine shop work, and materials processing are characterized by "slope" percentages varying from 76% to 87%. Major and final assembly work exhibited a faster rate of manhour decrease, yielding "slopes" between 69% and 75%.

The studies by Alchian and Asher, together with Hirsch's study in the machine-tool industry,may represent the most

Other c

connect

Raythe

as the

he is

manufa

mance,

take-o

standa

and op

one o

the m

inclu

are o

there

there

Such

Progr

pages

objective and rigorous empirical investigation of the progress curve concept produced in the fifties.

Other contributions

Two other contributions are worth mentioning in this connection, namely the articles by Koen[63] and Andress.[64]

Francis T. Koen, of the Missile Systems Division, Raytheon Company, uses the manufacturing progress function as the basis for what he calls "Dynamic Evaluation". With it, he is able to (1) predict production trouble, (2) estimate manufacturing costs, (3) check estimates and budget performance, (4) predict personnel requirements, (5) help determine make-or-buy decisions, (6) determine relation between standard cost and quantity values, and (7) establish budgets and optimum production schedules.

Frank J. Andress believes that "product innovation" is one of the primary criterions upon which the usefulness of the manufacturing progress function should be based. This includes situations where both major and minor design changes are often incorporated while the product is in production, where new products are frequently introduced, and/or where there are frequent production runs at well-spaced intervals. Such companies would be at the upper end of the manufacturing progress function and could, thus, realize the maximum advantages of the improvement rate.

THE SIXTIES AND SEVENTIES

In the last fifteen years there is evidence of a renewed interest in the subject of progress functions, particularly outside the airframe industry. The contributions in this period are of a somewhat diversified nature. There are pioneering extensions of the concept to machine-intensive industries as well as to other labor-intensive industries not studied before. At least one leading company in the field of electronic data processing systems continues to develop and apply the theory of progress functions in its manufacturing operations. The progress curve is also demonstrated to be valid in service activities like overhaul and maintenance. Furthermore, their users seem more careful in identifying its caveats and possible deviations.

Extension to Machine-Intensive Manufacture

Baloff is best known for extending the use of the learning curve model to machine-intensive production systems[65] as well as to other labor intensive manufactures different from the airframe industry.[66]

He suggests the use of a modified version of the model in highly mechanized manufacture (steel, glass-manufacturing, paper products and electrical-products).[67] Some years later the startup model is extended to three examples of labor intensive manufacture - automobile assembly, apparel manufacture and the production of large musical instruments.[68]

Baloff is highly skeptical with respect to past approaches used in parameter estimation. As indicated earlier, only a few solutions to the parameter prediction have been proposed in the literature.

A well known approach to the estimation of the b parameter is to assume simply that its value will remain constant for all startups in an industry, regardless of changes in product type, processing facility, or company origin. This assumption, which was apparently quite popular in the airframe industry at one time, continues to find support in the literature.[69] The violence to reality that results has been stressed by Baloff.[70] The assumption of a constant b parameter is inconsistent with the results of empirical examinations of a large number of airframe startups that took place during World War II and in postwar years.[71]

A more refined approach is based on the assumption that the startups of similar products or processes will experience identical startup curves. This approach essentially uses the empirically derived parameters of past startups as best estimates of one or both of the parameters in a future startup of a physically similar product or process. Unfortunately, experience suggests that variations in parameter values are not to be explained this simply. It has been shown by Baloff that the startups of steel processes that are physically very similar exhibit significantly different a and b parameters, even if the comparison is restricted to a single company in the industry or to a single plant.[72] Interplant comparisons of the startups of certain classes of airframes have yielded equivalent findings.[73]

A third approach that has been mentioned in the literature represents a refinement of the similarity concept. Here the focus is on recognizing and evaluating variables or factors that influence the values of one or both of the parameters for a given startup.[74] An evaluation of these factors in relation to experience with past startups could then presumably serve as a basis for generating estimates of the model parameters for a future startup. According to Baloff, there has been no published account of the reliability of this type of factor approach. Lacking such information, its utility in practice must remain an open question.[75]

Baloff argues that a different approach to the parameter estimation problem is suggested by the existence of a strong relationship between the a and b parameters among different startups in the steel and airframe industries and in the results of a laboratory research on group problem-solving. Though still tentative, this relationship appears to hold promise of being developed into an objective means of estimating the b parameter of the model, given a measurement of the initial productivity of a startup.

The existence of the relationship was initially reported by Asher in the airframe industry nearly two decades ago.[76]

Asher found a strong direct correlation between the a and b parameters of the startup model among 12 postwar startups of fighter-class airframes. The strength of the relationship observed by Asher was notable, allowing him to fit the following model to the 12 pairs of parameter values:

$$\log b = \log m + n \log a \qquad (2.13)$$

Following the lead of Asher, Baloff extended this parameter model approach to the steel industry and to the glass manufacturing industry.[77] The results of a laboratory experiment on group problem-solving also provided Baloff evidence of an inverse correlation between the parameters of the startup model in a learning situation that is similar to an industrial startup.[78] Later he would come to find support for the parameter model approach in laboratory experiments with group adaptation to a business game[79] and in automobile startups.[80]

Baloff's empirical contributions may well represent a turning point in the theory and application of the manufacturing progress function.

Contributions by IBM Personnel

After the Schultz and Conway study already reviewed in an earlier section, IBM personnel became very active in the subject of progress functions. Articles by P.B. Metz, J.G. Kneip and J.H. Russel will be briefly commented on in the following paragraphs.

a no

of t

cons

tan

a sp

tana

pren

poin

tion

boun

thr

con

ca

th

The Metz Nomograph. In 1962, Philip Metz presented a nomograph designed to simplify and expedite the application of the unit progress curve.[81] In essence, the nomograph is a consistent means of estimating the direct-labor hours requirement per cumulative unit relative to the known requirement of a specific cumulative unit. The values of three parameters, namely, (x_u, y_u) and the Wright's "slope" percentage are prerequisite to the application of the progress curve. The point (x_u, y_u) represents a known point in cumulative production. A typical problem is to determine the direct-labor hours for specific units of pre-defined lots and the average direct-labor hours for given lots. The analytical solution consists in calculating segments of areas under the progress curve through integration between known limits. The nomograph expedites the computational effort. Moreover, the probability of a computational error is lessened by the reduced number of required calculations. According to Metz, the accuracy of the obtained solutions is at least commensurate with the procedures normally employed in establishing the required parameters.

The Kneip Maintenance Progress Function. In order to effect a transition to a maintenance progress function, J.G.Kneip makes the following adjustments in concepts: (1) Whereas the manufacturing progress function describes the characteristics of a system through which all of the products must flow, the maintenance progress function describes the system through which each unit of product flows according to some probability

distribution. That is, a unit may not require service or it may require a number of service calls; (2) Whereas in the manufacturing progress function the dependent variable (cost or labor time) is a production efficiency characteristic, in the maintenance progress function the dependent variable is a quality, reliability and serviceability characteristic.[82]

Having in mind these modifications Kneip adopted the following model:

$$y = ax^b \tag{2.14}$$

where:

y = the average of the maintenance required on a sample of machines from each month's production of a given production period, expressed in <u>number of calls or in service hours</u> per unit of production.

a = the value of y for the first unit

x = the cumulative production

b = parameter dependent on the rate of progress.

In order to evaluate the applicability of the modified model in describing the improvement in maintenance requirement, two products, an electro-mechanical machine and an electronic machine were selected. The data on the first product based on the first twenty-seven months of the product's manufacture were taken from the field service histories for the ninety-day warranty period. The second product data were obtained from two sources: (i) Development models which were mechanically

operated, simulating an office installation of an equivalent ninety-day warranty period and (ii) production models in actual customer installations. These data were derived from the warranty service reported during the first eight months of the product's manufacture.

The data were fitted to the logarithmic function by the method of least squares. On the strength of the results of the regression analysis Kneip concludes that the maintenance progress function exists, that it relates warranty period maintenance to cumulative production, and that the relationship is represented by equation (2.14).

Any manufacturing organization which assumes some responsibility for its products through a warranty period should find the maintenance progress function of value in predicting warranty maintenance requirements on new products and in the analysis of the maintenance experience on development models when a maintenance criterion must be met by future production.

The Russell Study. The purpose of J.H.Russell's article[83] is to explain the deviations from the log-linear model that result from the addition or subtraction of parallel production lines. A system of parallel lines operating at a fixed rate of learning is used in a computer program[84] to observe the results of varying the number of lines employed. The result is the generation of a specific progress function model for each product profile thus simulated. Therefore, the progress function can become more useful when the model for the

specific product is predictable.

From the simulation of various product profiles Russell concludes that the addition and subtraction of parallel lines of production do create significant deviations in the progress function. The author believes that these changes form the major trend lines of the progress function as applied to a specific product. In addition, Russell points out that the effects of major product improvements, operational problems, inventory policy, lead-time changes, and modifications in accounting method will also cause deviations to the progress function. Each should be evaluated separately and its effect overlaid on the trend lines of the product profile. The study by Russell represents a pioneering effort towards using simulation methodology in the investigation of adaptation phenomena.

Other Contributions

For their intrinsic interest and relevance to the field of manufacturing progress functions, four more contributions will be concisely reviewed in the subsequent paragraphs.

Setting Management Goals. James M. White[85] of Stevens Institute of Technology, believes that the manufacturing progress function is highly useful in almost any situation in which there is some criterion by which to measure the improvement phenomenon, and which is initially in what would be considered an "uncontrolled state". Possible cases that would fall within this category are: (1) Reduction of losses due to waste, (2) reduction of scrap and rejects, (3) decreasing accident rates, (4) increasing capacity because of poor planning and control of resources, (5) reducing clerical errors, and other situations. In his article White demonstrates how to use the progress curve in setting goals for improvement in waste control. The other cases above mentioned are potential situations where progress curve theory could be used to predict the expected rate of improvement thus permitting the achievement of worthwhile results in management planning.

Multi-Product Industries. Paul F. Williams[86], an industrial engineer for United Control Corporation, holds that the manufacturing progress function can be a very beneficial tool for multi-product industries. He claims there are two general classes of application of the function. The first is for use on "initial quantities" of production. This would be used for scheduling a new product, or one on which insufficient records were kept by which to make a manufacturing progress function. The second classification would be for "follow-on-quantities". This would be used for a product presently being produced, or one on which sufficient records were kept by which to make a progress function. In addition, the function is valuable for the following reasons: (1) It provides a systematic, consistent, and objective method of forecasting production information; (2) It can be used to estimate average production costs of both "initial" and "follow-on-quantities"; (3) It is a graphic technique and,

thus, easy to use; (4) It can be used by management as a yard stick to measure manufacturing performance, and (5) When it is properly applied, it can minimize the error of estimation.

Petroleum Refining. W.B.Hirschmann believes that the progress curve is an underlying natural characteristic of organized activity, just as the normal curve is an accurate depiction of normal, random distribution of anything, from human I.Q.'s to the size of tomatoes.[87] He plotted the performance of individual catalytic cracking units at a point in time against their age at that time. The dependent variable was current capacity expressed as a percentage of design capacity. The first unit in 1958 was one and one-half years old and by that time had achieved approximately 116% of design capacity. The second unit was four years old and had achieved about 125% of design capacity. The older units show that the performance rapidly improved in the first few years, and continued at a slower rate in later years.Another plotting of successive annual points for an individual cracking unit indicates growth occurring in a step-wise fashion. However, the pattern of improvement resembles the inverse of a progress function on arithmetic paper.

If the parameters are changed so that the number of days to process 100,000 barrel is plotted against cumulated throughput on a logarithmic paper, a declining straight line can be drawn through the points as indicated by Figure 3.

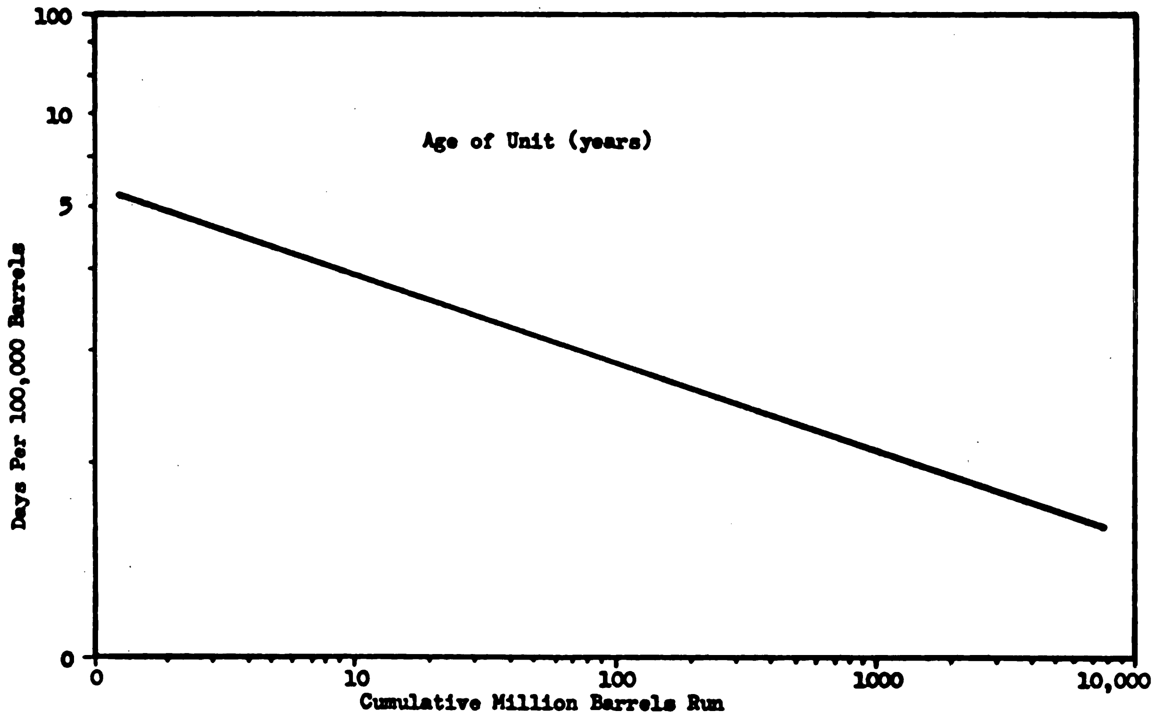


FIGURE 3

Progress Line For a Catalytic Cracking Unit

This Line has a slope of about 90%, as might be expected from a machine-paced operation which involves comparatively little direct labor.

Hirschmann further states that a manufacturing progress function can be determined for any industry. A logarithmic plot of man-hours per barrel versus cumulated barrels of crude oil refined in the United States since 1860 was made. He found similar declines for the United States basic steel industry and for the United States electric power industry. Hirschmann's study indicates that the manufacturing progress function is not only applicable to the aircraft industry,but also to non-aircraft industries.Together with the studies by Baloff, it represents a pioneering effort towards extending the progress curve model to highly automated industries where the adaptation phenomenon might be thought to be either non-existent or too small to be of value.

Limits of the Progress Curve. An article by W. J. Abernathy and K. Wayne looks at the progress curve in a new way.[88] It shows the unforeseen consequences of following the strategy of reducing costs in a product through steady increases in volume: rising fixed costs,a narrowly specialized work group, and a withered capacity for innovation, to name just three. To illustrate the changes that accompany a cost-minimizing strategy, the authors use the case of Ford Motor Company and its model T. The kinds of changes that took place can be grouped into six categories - product; capital equipment and process technology; task characteristics and process structure; scale; material inputs; and labor. Each category is briefly described as follows.

(1) Product: Standardization increases, models change less frequently and the product line offers less diversity. As the implementation of the strategy continues, the total contribution improves with acceptance of lower margins accompanying larger volume.

(2) Capital Equipment and Process Technology: Vertical integration expands and specialization in process equipment, machine tools, and facilities increase. The rate of capital investment rises while the flexibility of these investments declines.

(3) Task Characteristics and Process Structure: The throughput time improves and the division of labor is extended as the production process is rationalized and oriented more toward a line-flow operation. The amount of direct supervision decreases as the labor input falls.

(4) Scale: The process is segmented to take advantage of economies of scale.

(5) Material Inputs: Through either vertical integration or capture of sources of supply, material inputs come under control. Costs are reduced by forcing suppliers to develop materials that meet process needs.

(6) Labor: the heightening rationalization of the process leads to greater specialization in labor skills and may ultimately lessen workers' pride in their jobs and concern for product quality.

The authors point out that the same pattern of change in the six categories that characterizes Ford history also describes periods of major reduction in other industries.

Implications for Management. According to Abernathy and Wayne, management needs to recognize that conditions stimulating innovation are different from those favoring efficient, high-volume, established operations. The unfortunate implication is that product innovation is the enemy of cost efficiency and vice-versa. The authors point two courses of action that some major companies have followed. One is to maintain efforts to continue development of the existing high--volume product lines. This requires setting the industry pace in periodically inaugurating major product changes while stressing cost reduction via the learning curve between model changes. This course of action, exemplified by IBM, amounts to maintaining comparatively less efficient overall operations. The second course of action is to take a decentralized approach in which separate organizations or plants in the corporate framework adopt different strategies within the same line of business. One organization in the company will pursue profits with a traditional product to the limit of the learning curve while others will develop new products and processes.

SUMMARY

The present chapter contains a review of the historical development of the manufacturing progress function and a summary of the more important contributions to the progress curve literature that are relevant to this dissertation.

Although the progress curve was discovered in 1922, it was largely unknown until World War II.

T.P. Wright is given credit for originating the formulation of the progress curve theory in 1936. His statement, that cumulative average man-hours per unit decline by a constant percentage every time the output is doubled, remains the most popular formulation in existence.

In the forties, a number of modifications of the origi nal model were proposed. However, empirical verification of the modified models has been fragmentary and they have found little acceptance. In spite of this, Crawford contributed the "unit" learning curve and noticed that different "slopes" might exist for different airframes. He is probably the first author to perceive the link between rate of progress in a job and its degree of complexity and novelty.

Several relevant contributions to the development of progress curves were published in the 1950s. There is an effort by some researchers like Hirsch, Bryan, Schultz and Conway to extend the concept to labor-intensive industries other than the airframe industry. In addition, studies continued to be financed by the United States Air Force in some research institutes of the country. The studies by Alchian and Asher, in the airframe industry,together with Hirsch's study in the machine-tool industry,and Schultz and Conway's research in manufacturing of electronic and electro-mechanical products may well represent the most objective and rigorous empirical investigation of the progress curve concept produced in the fifties.

The last fifteen years have seen some pioneering extensions of the progress curve concept to machine-intensive industries as well as to diverse labor-intensive industries. The studies carried out by Baloff constitute the most comprehensive investigation of the progress function in the mentioned period.

Review of the Main Hypotheses

It is worthwhile to recapitulate the principal hypotheses raised by the forementioned authors and generally accepted:

(1) Linearity , when in logarithmic coordinates , between the labor per unit and cumulative output was generally confirmed except in Asher's study.

(2) There is no one single progress curve that can be universally applied to all types of operations involved in the manufacturing of a given product. Also, there is not a curve that can represent the adaptation phenomenon for all products in a given firm or industry.

(3) Assembly operations experiment significantly higher rates of progress than machining operations.

(4) As to the newness or novelty hypothesis the findings are contradictory. According to Crawford the rate of progress is an increasing function of the complexity of the job and of the lack of experience of the worker. Schulz and Conway agree with Crawford when they state that the degree of similarity of a machine to a predecessor has a significant effect on the rate of progress; the less similarity, the greater the rate of progress. They also observed that the greater complexity, the less the rate of progress. Nevertheless, Alchian and Hirsch found in diverse settings that the greater the similarity of a product to a predecessor, the greater the rate of progress experienced.

(5) There exists a strong correlation between the parameters of the startup model among startups that occurred in a given production facility (Baloff) and among startups that occurred in different facilities (Asher).

(6) Plateaux predictability continues to be a controversial issue.

CHAPTER III

MANUFACTURING PROGRESS FUNCTIONS: A MATHEMATICAL EXPOSITION

The interested reader of the literature on manufacturing progress functions will have certainly noticed that the field lacks uniformity as to mathematical notation and more precise definition of the variables involved. A number of theoretical results are stated and used without formal mathematical demonstration. Assumptions are often implicit in the derivation of important results.

In spite of being discrete, the manufacturing progress function may be advantageously treated as a continuous function under certain conditions. The consequent simplification achieved in the final formulas and calculations using the progress function is worthwhile. Nevertheless,these features have been largely neglected in the literature. Such deficiencies and others that will be pointed out in the course of this chapter have led to a state of confusion in the design, interpretation and evaluation of related empirical research.

In the present chapter the mathematical treatment of progress functions will be rewritten. The systematic exposition here proposed is probably novel in the literature.

FOUR TYPES OF PROGRESS FUNCTIONS

There are at least four types of progress functions, although they are mathematically related to one another. All four can be expressed as power functions. They are similar in form but have different meaning. The following notations, terminology and definitions will be used in this dissertation.

Type 1: The Unit Progress Function

The functional relationship specified for the unit progress function is:

$$y = ax^{-b} \qquad (3.1)$$

where

x = the cumulative unit of production

y = the direct-labor hours required to produce that xth unit in cumulative production

a and b = parameters; $a > 0$, $0 \leq b \leq 1$

Interpretation of Parameters a and b. Parameter a represents the direct labor hours required to produce the initial unit of production: for $x = 1$, $y = a(1)^{-b} = a$. Parameter b is dependent upon the rate of progress. Geometrically, it is the slope of the progress line, as already mentioned in Chapter 2 (p.12). Another interpretation is suggested by the concept of elasticity of a function. The elasticity of Y at the point x is given by:[1]

$$\frac{Ey}{Ex} \triangleq - \frac{dy/y}{dx/x} = - \frac{dy}{dx} \cdot \frac{x}{y} \qquad (3.2)$$

But, according to (3.1):

$$\frac{dy}{dx} = \frac{d}{dx}(ax^{-b}) = - abx^{-b-1}$$

and

$$\frac{x}{y} = \frac{x}{ax^{-b}} = \frac{1}{a}x^{1+b}$$

Replacing $\frac{dy}{dx}$ and $\frac{x}{y}$ in (3.2) by their equivalents already obtained, it follows that

$$\frac{Ey}{Ex} = b \qquad (3.3)$$

Thus, parameter b may be called the progress elasticity,i.e., the ratio of an infinitely small relative change in the direct labor requirement associated with a correspondingly small relative change in cumulative output. For example, in the "eighty per cent curve" parameter b is 0.32, i.e., a 1 per cent increase in cumulative output is associated with a 0.32 per cent decline in direct labor requirement.Recalling that the rate of progress has been defined as the complement to one of the "slope", in the case of the"eighty per cent curve" the rate of progress is

$$(1 - 0.80) = 0.20$$

Theref

elasti

a posi

sent a

defin

funct

domai

The f

y = y

expr

but

whe

Al

an

th

Therefore, the rate of progress corresponding to a progress elasticity or parameter b of 0.32 is about 0.20.

Domain and Range of y. Usually (but not always) x is a positive integer. In some instances, however, x may represent a continuous variable.[2] For x = 0, the function is not defined. Thus, in general, the domain X of the unit progress function y is a subset of the positive reals. Usually, the domain X of y is a subset of the set of positive integers.

The range set Y of y is a set of positive real numbers.[3] The function y is the set of ordered pairs (x,y) where $y = y(x) = ax^{-b}$. These specifications may be more compactly expressed by using set theory notation. Thus, generally

$$X = \text{Dom } y = \{x : x \in R_+\}$$

but usually

$$X = \text{Dom } y = \{x : x \in N\}$$

where

R_+ = the set of positive reals

N = the set of positive integers: 1,2,3,...

Also

$$Y(X) = \text{Ran } y = \{y : y = ax^{-b},\ x \in R_+ (\text{or } \in N),\ a \in R_+,\ b \in I\} \subset R_+$$

and

$$y = \{(x, y(x)) : x \in X\}$$

where I = the unit interval of real numbers.

Type 2: The Cumulative Average Progress Function

The functional relationship specified for the cumulative average progress function is the following:

$$\bar{y} = ax^{-b} \qquad (3.4)$$

where

$\bar{y}$ = the cumulative average direct labor hours per unit

x = the cumulative number of units produced

$\underline{a}$ and $\underline{b}$ = parameters; $a > 0$, $0 \leq b \leq 1$

Meaning of a, b and $\bar{y}$. Parameters $\underline{a}$ and $\underline{b}$ have the same meaning as in the case of the unit progress function.As to the meaning of $\bar{y}$, assume a table is available containing a series of values of $\underline{x}$ and the corresponding direct labor hours input per unit, y, for each $\underline{x}$:

x	x_1	$x_2 \ldots x_i \ldots x_n$
y	y_1	$y_2 \ldots y_i \ldots y_n$

The cumulative average direct labor hours per unit up to and including unit x_i is defined as follows:

$$\bar{y}_i \triangleq \frac{1}{x_i} \sum_{j=1}^{i} y_j \; , \quad i = 1,2,\ldots,n \qquad (3.5)$$

Equation (3.4) means that the hyperbolic rule of correspondence is now between the set of x_is and the set of $\bar{y}_i$s where the $\bar{y}_i$s are calculated according to (3.5).

Domain and Range of $\bar{y}$ Using set theory notation:

$$X = \text{Dom } \bar{y} = \{x : x \varepsilon R_+\} \quad \text{(in general)}$$

or

$$X = \text{Dom } \bar{y} = \{x : x \varepsilon N\} \quad \text{(more usually)}$$

Also

$$\bar{Y}(X) = \text{Ran } \bar{y} = \{\bar{y} : \bar{y} = ax^{-b}, x \varepsilon R_+ (\text{or } \varepsilon N),\ a \varepsilon R_+, b \varepsilon I\} \subset R_+$$

$$\bar{y} = \{(x, \bar{y}(x)) : x \varepsilon X\}$$

where the $\bar{y}$s have the meaning given by (3.5)

Type 3: The Cumulative Total Progress Function

The cumulative total progress function is expressed by:

$$y_T = ax^B \qquad (3.6)$$

where

y_T = cumulative total direct labor hours

x =the cumulative number of units produced

a and B = parameters; $a > 0$, $0 \leq B \leq 1$

Meaning of a, B, and y_T. Again, parameter a represents the direct labor hours required to produce the initial unit of production. Parameter B may be seen as the elasticity of y_T at the point x. As to the meaning of y_T, assuming as before that a table of values of x and y is available, the cumulative total direct labor hours expended in producing the first x_i units is given by:

ati

wea

cr

so

Y,

d

the

ty

whe

$$y_{T_i} \triangleq \sum_{j=1}^{i} y_j \quad , \quad i = 1,2,\ldots,n \qquad (3.7)$$

Equation (3.6) signifies that the rule of association is now between the set of x_is and the set of y_{T_i}s calculated according to (3.7).

Domain and Range of y_T. Using set theory notation:

$$X = \text{Dom } y_T = \{x : x \varepsilon \mathcal{R}_+\} \qquad \text{(in general)}$$

or

$$X = \text{Dom } y_T = \{x : x \varepsilon \mathcal{N}\} \qquad \text{(more often)}$$

Also

$$Y_T(X) = \text{Ran } y_T = \{y_T : y_T = ax^B, x\varepsilon \mathcal{R}_+ (\text{or } \varepsilon \mathcal{N}), a \varepsilon \mathcal{R}_+, B \varepsilon \mathcal{I}\} \subset \mathcal{R}_+$$

and

$$y_T = \{(x, y_T(x)) : x \varepsilon X\}$$

where the y_Ts have the meaning given by (3.7).

Type 4: The Lot Average Progress Function

The lot average progress function is expressed by:

$$\bar{y}_L = ax^{-b} \qquad (3.8)$$

where

$\bar{y}_L$ = lot average direct labor hours per unit

x =the cumulative number of units produced

a and b = parameters; $a > 0$, $0 \leq b \leq 1$

Meaning of a, b, and $\bar{y}_L$. Parameter a represents the direct labor hours required to produce the initial unit of production. Parameter b is the elasticity of $\bar{y}_L$ at the point x. The meaning of $\bar{y}_L$ may be approached as follows. Let

x_k = the counting from the first unit produced up to and including the last unit of lot k, $k = 1,2,\ldots,m$.

Similarly,

x_{k-1} = the number of units from the first unit produced up to and including the last unit of lot (k-1).

Therefore

$x_k - x_{k-1}$ = number of units in lot k.

The lot average direct labor hours per unit considering lot k is defined as follows:

$$\bar{y}_{L_k} = \frac{y_{T_k} - y_{T_{k-1}}}{x_k - x_{k-1}}, \quad k = 1,2,\ldots,m \qquad (3.9)$$

In equation (3.8), the rule of correspondence is now between the set of x_ks and the set of $\bar{y}_{L_k}$s calculated according to (3.9).

Domain and Range of $\bar{y}_L$. Again, employing set theory notation:

$$X = \text{Dom } \bar{y}_L = \{x : x \varepsilon \mathcal{R}_+\} \qquad \text{(in general)}$$

or

$$X = \text{Dom } \bar{y}_L = \{x : x \varepsilon \mathcal{N}\} \qquad \text{(more frequently)}$$

Also

$$\bar{Y}_L(X) = \text{Ran } \bar{y}_L = \{\bar{y}_L : \bar{y}_L = ax^{-b}, x \varepsilon \mathcal{R}_+ (\text{or } \varepsilon \mathcal{N}), a \varepsilon \mathcal{R}_+, b \varepsilon \mathcal{I}\} \subset \mathcal{R}_+$$

$$\bar{y}_L = \{(x, \bar{y}_L(x)) : x \varepsilon X\}$$

where the $\bar{y}_L$'s have the meaning given by (3.9).

Fitting the Progress Function to Empirical Data. The crucial problem has been the conformity of the forementioned functional relationships to empirical reality. It is worth mentioning that several other forms have been considered in the literature of progress functions.[4] However, the power function continues to be universally employed by the researchers in the field. The preference for the power function may be justified on the following grounds: (1) Adequate fitting to the available data as produced by the firms involved, and (2) simplicity of utilization, particularly when in the form of double-logarithmic graph. In the following section it will be shown how the four types of progress functions previously described are mathematically related to one another.

FUNDAMENTAL PROBLEMS

Statement of Fundamental Problems

Almost always progress function users have faced the following four basic problems:

Problem I. Given $\bar{y} = \bar{y}\ (x)$, determine:

$$y_T = y_T(x)\ ,\ y = y(x),\ \text{and}\ \bar{y}_L = \bar{y}_L(x)$$

Problem II. Given $y = y(x)$, determine:

$$y_T = y_T(x),\ \bar{y} = \bar{y}(x),\ \text{and}\ \bar{y}_L = \bar{y}_L(x)$$

Problem III. Given $y_T = y_T(x)$, determine:

$$\bar{y} = \bar{y}(x),\ y = y(x),\ \text{and}\ \bar{y}_L = \bar{y}_L(x)$$

Problem IV. Given $\bar{y}_L = \bar{y}_L(x)$, determine:

$$y_T = y_T(x),\ \bar{y} = \bar{y}(x),\ \text{and}\ y = y(x).$$

Whenever the expression "<u>given</u> the function..." is used in this chapter, it signifies that the focused function can be statistically fitted to the empirical data and moreover, that the chosen function is the one that yields the best fitting of the four types of progress functions already

defined. From such point on the other three progress functions may be determined through mathematical analysis, as follows.

Solution of Problem I

(a) Given the cumulative average progress function $\bar{y} = \bar{y}(x)$, the cumulative total progress function may be determined by using definition equation (3.5):

$$y_T(x) = \bar{y}(x) \cdot x \qquad (3.10)$$

In case the discrete function $\bar{y} = \bar{y}(x)$ is approximated by $\bar{y} = ax^{-b}$, it follows from (3.10) that:

$$y_T(x) = ax^{1-b}, \quad x = 1,2,... \qquad (3.11)$$

(b) The unit progress function will be determined by subtracting the cumulative total hours consumed in the production of the first (x-1) units from the cumulative total hours consumed in the production of the first <u>x</u> units. The difference is exactly the the hours consumed in producing unit <u>x</u> alone:

$$y(x) = y_T(x) - y_T(x-1) \qquad (3.12)$$

or according to (3.11) and (3.12):

$$y(x) = a\left[x^{1-b} - (x-1)^{1-b}\right], x=1,2,... \qquad (3.13)$$

(c) The lot average progress function is easily determined by adopting the same notation of the previous section. Therefore, according to equations (3.9) and (3.11):

$$\bar{y}_{L_k} \triangleq \frac{y_{T_k} - y_{T_{k-1}}}{x_k - x_{k-1}} = \frac{a x_k^{1-b} - a x_{k-1}^{1-b}}{x_k - x_{k-1}}$$

or

$$\bar{y}_{L_k} = \frac{a(x_k^{1-b} - x_{k-1}^{1-b})}{x_k - x_{k-1}} \quad , \; k=1,2,\ldots,m, x_0=0 \qquad (3.14)$$

Using equation (3.14) and given the number of units in each lot $\underline{k}$, $k=1,2,\ldots,m$, it is possible to calculate a table of values of $\bar{y}_{L_k}$ as a function of x_k and x_{k-1} and thus to determine $\bar{y}_L = \bar{y}_L(x)$.

Using the Continuity Assumption to Approximate Solutions . As it was previously noted, progress functions are often discrete. Nevertheless it is viable to approximate solutions by assuming continuity of the progress function in the interval of interest. Take, for example equation (3.13). For values of $\underline{x}$ not very close to the first units of production:

$$\bar{y}(x) \cong \frac{1}{x} \int_0^x y(x)\, dx \qquad (3.15)$$

Equation (3.15) follows from the definition of the average or mean value of function $y = y(x)$ over the interval from zero to x. It is assumed that y(x) is continuous over the interval of interest.[5] Since $\bar{y} = \bar{y}(x)$ is being approximated by $\bar{y} = ax^{-b}$, it follows from (3.15) that

$$\int_0^x y(x)\ dx \cong \bar{y}(x)\cdot x = ax^{1-b} \qquad (3.16)$$

Derivation of (3.16) with respect to x yields:

$$y(x) \cong a\ (1-b)x^{-b} \qquad (3.17)$$

Equation (3.16) is the same as (3.11). Recall that equation (3.11) was derived by assuming that the discrete function $\bar{y} = \bar{y}(x)$ could be approximated by $\bar{y} = ax^{-b}$.

Calculation of the unit progress function through (3.17) is evidently faster and simpler than through the exact equation (3.13) even if machine computation is considered. Since $\bar{y} = \bar{y}(x)$ is given, it suffices to multiply the $\bar{y}$s by (1-b) in order to obtain the ys.

It is worth mentioning that the function:

$$f(\log x) = \log a + \log(1-b) - b \log x$$

is asymptote of

$$F\ (\log x) = \log \{a \left[x^{1-b} - (x-1)^{1-b}\right]\}$$

This result is used very often in the literature but no formal proof is offered except the recourse to geometrical intuition. In Appendix B this author states it as a theorem and contributes a formal mathematical demonstration. The rationale for approximating equation (3.13) by equation (3.17) (for values of x not close to the first units) stems from this theorem.

Solution of Problem II

(a) Given the unit progress function y = y(x), the cumulative total direct labor hours up to and including unit x is given by definition (3.7):

$$y_T(x) \triangleq \sum_x y(x) \qquad x = 1,2,... \tag{3.18}$$

In case the discrete function y = y(x) is approximated by $y = ax^{-b}$, then:

$$y_T(x) = \sum_x ax^{-b} = a \sum_x x^{-b}, \quad x = 1,2,... \tag{3.19}$$

(b) The cumulative average progress function is determined through definition (3.5) and equation (3.18):

$$\bar{y}(x) \triangleq \frac{y_T(x)}{x} = \frac{\sum_x y(x)}{x} \tag{3.20}$$

Therefore, in case y=y(x) is approximated by $y=ax^{-b}$, the cumulative average function is given by:

$$\bar{y}(x) = \frac{a}{x} \sum_{x} x^{-b}, \quad x = 1,2,... \tag{3.21}$$

(c) In order to determine the lot average progress function, given the unit progress function, let:

x_k = the counting from the first unit produced up to and including the last unit of lot k, $k = 1,2,...,m$, $x_0 = 0$.

x_{k-1} = the counting from the first unit produced up to and including the last unit of lot (k-1).

In case the discrete function $y=y(x)$ is approximated by $y=ax^{-b}$, equation (3.19) yields:

$$y_T(x_k) = a \sum_{1}^{x_k} x^{-b}$$

and

$$y_T(x_{k-1}) = a \sum_{1}^{x_{k-1}} x^{-b}$$

The total direct labor hours expended in lot k is obtained by subtracting the last equation from the first one:

$$y_{T_k} = a\left(\sum_{1}^{x_k} x^{-b} - \sum_{1}^{x_{k-1}} x^{-b}\right) \tag{3.22}$$

or

$$y_{T_k} = a \sum_{j=1}^{x_k - x_{k-1}} (x_{k-1}+j)^{-b}, \quad k=1,2,\ldots,m, x_0=0 \tag{3.23}$$

The lot average direct labor hours for lot k will be given by:

$$\bar{y}_{L_k} = \frac{a}{x_k - x_{k-1}} \sum_{j=1}^{x_k - x_{k-1}} (x_{k-1}+j)^{-b} \tag{3.24}$$

Given the number of units in each lot k, $k=1,2,\ldots,m$, it is possible to calculate a table of values of $\bar{y}_{L_k}$ as a function of x_k and x_{k-1} through equation (3.24) and thus to determine $\bar{y}_L = \bar{y}_L(x)$.

Approximating the Summation Formulas by Integrals. The formulas involving summations may be approximated by improper integrals. Take, for example, formula (3.19):

$$y_T(x) \cong a \int_0^x x^{-b} = a \lim_{x_1 \to 0} \int_{x_1}^x x^{-b} = \frac{a}{1-b} x^{1-b} \tag{3.25}$$

Using definition (3.5) and the approximation in (3.25), the cumulative average progress function given by equation (3.21) may be expressed as:

$$\bar{y}(x) \triangleq \frac{y_T(x)}{x} \cong \frac{a}{1-b} x^{-b} \tag{3.26}$$

Also, it follows from equation (3.25) that:

$$y_T\ (x_k) \cong \frac{a}{1-b}\ x_k^{1-b}$$

and

$$y_T(x_{k-1}) \cong \frac{a}{1-b}\ x_{k-1}^{1-b}$$

Thus,

$$\bar{y}_{L_k} = \frac{y_T(x_k) - y_T(x_{k-1})}{x_k - x_{k-1}} \cong \frac{a(x_k^{1-b} - x_{k-1}^{1-b})}{(1-b)(x_k - x_{k-1})} \qquad (3.27)$$

Solution of Problem III

This problem is trivial once the solution to problem I is presented.

(a) Given the cumulative total progress function, $y_T = y_T(x)$, the cumulative average progress function is merely:

$$\bar{y}\ (x) \triangleq \frac{y_T(x)}{x} \qquad (3.28)$$

In case $y_T = y_T(x)$ is approximated by $y_T = ax^B$, the cumulative average progress function will be of the form:

$$\bar{y}(x) = ax^{B-1}\ , \quad x=1,2,... \qquad (3.29)$$

(b) The unit progress function may be obtained as in Problem I:

$$y(x) = y_T(x) - y_T(x-1)$$

Given that $y_T = y_T(x)$ is of the form $y_T = ax^B$,

$$y(x) = a\left[x^B - (x-1)^B\right] \quad , \; x=1,2,... \qquad (3.30)$$

(c) The lot average progress function is determined by the same method already explained in the case of Problem I. The resulting expression is:

$$\bar{y}_{L_k} = \frac{a(x_k^B - x_{k-1}^B)}{x_k \quad x_{k-1}} \quad , \; k=1,2,...,m, \quad x_0 = 0 \qquad (3.31)$$

Solution of Problem IV

(a) Given the lot average progress function $\bar{y}_L = \bar{y}_L(x)$, the total direct labor hours consumed by the manufacturing of lot k is given by:

$$y_{T_k} = \bar{y}_{L_k}(x_k - x_{k-1}) \quad , \; k=1,2,...,m, \; x_0=0 \qquad (3.32)$$

The cumulative total direct labor hours since the inception of production up to and including unit x_k (the last of lot k) is, therefore:

$$y_T(x_k) = \sum_{j=1}^{k} \bar{y}_{L_j}(x_j - x_{j-1}) \quad , \; k=1,2,...,m, \; x_0=0 \qquad (3.33)$$

In case $\bar{y}_L = \bar{y}_L(x)$ is being approximated by $\bar{y}_L = ax^{-b}$, the cumulative total direct labor hours from unit number one up to and including unit x_k is, according to (3.33):

$$y_T(x_k) = a \sum_{j=1}^{k} x_j^{-b}(x_j - x_{j-1}),\ k=1,2,\ldots,m,\ x_0=0 \qquad (3.34)$$

(b) The cumulative average direct labor hours per unit up to x_k will be, by definition:

$$\bar{y}\,(x_k) \triangleq \frac{y_T(x_k)}{x_k} \qquad (3.35)$$

If $\bar{y}_L = \bar{y}_L(x)$ is being approximated by $\bar{y}_L = ax^{-b}$, then, according to (3.34) and (3.35):

$$\bar{y}(x_k) = \frac{a}{x_k} \sum_{j=1}^{k} x_j^{-b}\,(x_j - x_{j-1}),\ k=1,2,\ldots,m,\ x_0=0 \qquad (3.36)$$

(c) Given the lot average progress function $\bar{y}_L = \bar{y}_L(x)$, the unit progress function cannot be exactly determined. However, an assumption can be made, i.e., that the lot average direct labor hours considering lot k is approximately equal to the lot average direct labor hours considering lot k deleted by its last unit x_k. Symbolically,

$$\bar{y}_{L_k} \triangleq \frac{y_{T_k}}{x_k - x_{k-1}} \cong \frac{y_{T_k} - y(x_k)}{x_k - 1 - x_{k-1}} \qquad (3.37)$$

If (3.37) holds, then:

$$y(x_k) \cong \bar{y}_{L_k} \qquad (3.38)$$

i.e., the unit progress function can be approximated by the lot average progress function. An alternative way of checking the validity of (3.38), given the same assumption (3.37) is as follows.

The cumulative total direct labor hours up to and including unit x_k is, according to (3.33):

$$y_T\ (x_k) = \sum_{j=1}^{k} \bar{y}_{L_j}\ (x_j - x_{j-1}) =$$

$$= \sum_{j=1}^{k-1} \bar{y}_{L_j} (x_j - x_{j-1}) + \bar{y}_{L_k} (x_k - x_{k-1}) \qquad (3.39)$$

In view of (3.37), the cumulative total direct labor hours up to and including unit $(x_k - 1)$ may be expressed as follows:

$$y_T(x_k - 1) \cong \sum_{j=1}^{k-1} \bar{y}_{L_j} (x_j - x_{j-1}) + \bar{y}_{L_k} (x_k - 1 - x_{k-1}) \qquad (3.40)$$

Subtracting (3.40) from (3.39) yields the same result as in (3.38):

$$y\ (x_k) \cong \bar{y}_{L_k}$$

In case $\bar{y}_L = \bar{y}_L(x)$ is approximated by $\bar{y}_L = ax^{-b}$, then:

$$y\ (x_k) \cong ax_k^{-b} \tag{3.41}$$

Note. Problem IV as well as item (c) of Problems I, II, and III are not formally treated in the literature. The unit progress function and the lot average progress function are taken as the same model. The approximation and the assumption behind it are not explained. The confusion may have its origin in the fact that the unit progress function is rarely encountered in a real world situation. Labor expenditure is typically recorded on a monthly, as opposed to a per-unit basis. As a result, direct labor hours data are normally calculated and reported in relation to standard accounting time periods, yielding average direct labor hours per unit figures for the "lot" of product produced during the accounting period. Also, cumulative output statistics (x) indicate the total output of the product (from inception of manufacture) that is achieved <u>at the end</u> of the accounting period. Thus, what exists in practice is the lot average function $\bar{y}_L = \bar{y}_L(x)$ as previously defined, not the unit function y = y (x).

Nevertheless, in the literature the lot average function is designated by the term "unit function"...

PARAMETER CALCULATION

Application of the progress function to practical problems involves two requirements:

(1) It must be known which type of progress function best fits the empirical data. In practice the cumulative average progress function and the lot average progress function are very popular . In some special situations where data is available in a per unit basis the unit progress function may also be considered.

(2) Parameters a and b must be known or somehow calculated so that the progress function can be applied.

In this section it is assumed that the cumulative average progress function (type 2) is the best fitted to the available empirical data. Once given the type of progress function there exist two general classes of problems involving parameter calculation:

(A) Given a point in the progress curve and the slope of the progress line, determine the progress function.

(B) Given two points in the progress curve, determine the progress function.

Class (A) involves solely parameter a determination since parameter b (the slope of the progress line) is given. In class (B) both parameters must be calculated.

Each class contains four problems. Their statements and respective solutions will be the subject of this section.

Statement of Problem A1

Given a point in the cumulative average progress curve, $(x_i, \bar{y}(x_i))$ and the slope of the progress line (parameter b), determine the cumulative average progress function $\bar{y} = \bar{y}(x)$.

Solution of Problem A1

Since, by assumption, $\bar{y} = ax^{-b}$, one must have:

$$\bar{y}\ (x_i) = ax_i^{-b}$$

$$\therefore \qquad a = \bar{y}\ (x_i)\ x_i^{b} \qquad (3.42)$$

Parameter b is given and parameter a can be calculated by (3.42). Thus, problem A1 is solved, i.e.,

$$\bar{y} = ax^{-b} = \bar{y}(x_i)\ x_i^{b}\ x^{-b} \qquad (3.43)$$

Statement of Problem A2

Given a point in the cumulative total progress curve, $(x_i, y_T(x_i))$, and parameter $\underline{b}$, determine the cumulative average progress function $\bar{y} = \bar{y}(x)$.

Solution of Problem A2

From equation (3.11):

$$y_T(x_i) = a x_i^{1-b}$$

$$\therefore \quad a = \frac{y_T(x_i)}{x_i^{1-b}} \tag{3.44}$$

Since $\underline{b}$ is known and $\underline{a}$ can be calculated by (3.44), problem A2 is solved. The cumulative average progress function will be given by:

$$\bar{y} = a x^{-b} = \frac{y_T(x_i)}{x_i^{1-b}} \; x^{-b} \tag{3.45}$$

Statement of Problem A3

Given a point in the unit progress curve, $(x_i, y(x_i))$ and parameter $\underline{b}$, determine the cumulative average progress function $\bar{y} = \bar{y}(x)$.

Solution of Problem A3

According to equation (3.13):

$$y(x_i) = a \left[x_i^{1-b} - (x_i - 1)^{1-b} \right]$$

$$\therefore \qquad a = \frac{y(x_i)}{x_i^{1-b} - (x_i - 1)^{1-b}} \qquad (3.46)$$

Again, since b is known and a can be calculated through (3.46), problem A3 is solved. The cumulative average progress function will be given by:

$$\bar{y} = ax^{-b} = \frac{y(x_i)}{x_i^{1-b} - (x_i - 1)^{1-b}} x^{-b} \qquad (3.47)$$

Statement of Problem A4

Given a point in the lot average progress curve, $(x_k, \bar{y}_L(x_k))$ and parameter b, determine the cumulative average progress function $\bar{y} = \bar{y}(x)$.

Solution of Problem A4

One possible solution is to use approximation (3.36), i.e.,

$$\bar{y}_L(x_k) \cong y(x_k)$$

In this case, problem A4 is reduced to problem A3 already solved.

If the number of units in lot $\underline{k}$ is known, say, n_k, then equation (3.14) may be used to determine parameter $\underline{a}$. Since $x_{k-1} = x_k - n_k$, it follows from (3.14):

$$a = \frac{\bar{y}_{L_k} n_k}{x_k^{1-b} - (x_k - n_k)^{1-b}} \tag{3.48}$$

Thus, the cumulative average progress function will be expressed by:

$$\bar{y} = ax^{-b} = \frac{\bar{y}_{L_k} n_k}{x_k^{1-b} - (x_k - n_k)^{1-b}} x^{-b} \tag{3.49}$$

Statement of Problem B1

Given two points in the cumulative average progress curve, $(x_i, \bar{y}(x_i))$ and $(x_j, \bar{y}(x_j))$, determine the cumulative average progress function $\bar{y} = \bar{y}(x)$.

Solution of Problem B1

Since, by assumption, $\bar{y} = ax^{-b}$, one must have:

$$\bar{y}(x_i) = ax_i^{-b} \tag{3.50}$$

and

$$\bar{y}\ (x_j) = ax_j^{-b} \qquad (3.51)$$

Dividing (3.50) by (3.51), taking logarithms and solving for b, yields:

$$b = \frac{\log\left[\bar{y}(x_i)/\bar{y}(x_j)\right]}{\log\ (x_j/x_i)} \qquad (3.52)$$

After calculating b, parameter a may be calculated from (3.50) or from (3.51). Therefore, using (3.50):

$$a = \bar{y}(x_i)\ x_i^b \qquad (3.53)$$

Once parameters b and a are calculated, problem B1 is solved. The cumulative average progress function will be given by:

$$\bar{y} = ax^{-b}$$

where b and a are calculated through equations (3.52) and (3.53), respectively.

Statement of Problem B2

Given two points in the cumulative total progress curve, $(x_i,\ y_T(x_i))$ and $(x_j,\ y_T(x_j))$, determine the cumulative average progress function $\bar{y} = \bar{y}(x)$.

Solution of Problem B2

According to equation (3.11):

$$y_T\ (x_i) = a x_i^{1-b} \tag{3.54}$$

and

$$y_T\ (x_j) = a x_j^{1-b} \tag{3.55}$$

Dividing (3.54) by (3.55), applying logarithms and solving for b, yields:

$$b = 1 - \frac{\log\ \left[y_T(x_i)/y_T(x_j)\right]}{\log\ (x_i/x_j)} \tag{3.56}$$

Parameter a may be calculated through equation (3.54) or (3.55). Thus, from (3.54):

$$a = \frac{y_T(x_i)}{x_i^{1-b}} \tag{3.57}$$

The cumulative average progress function will be expressed by

$$\bar{y} = a x^{-b}$$

where b and a are given by equations (3.56) and (3.57), respectively.

Statement of Problem B3

Given two points in the unit progress curve, $(x_i, y(x_i))$ and $(x_j, y(x_j))$, determine the cumulative average progress function $\bar{y} = \bar{y}(x)$.

Solution of Problem B3

(a) An approximate solution may be developed by using equation (3.17). Since $(x_i, y(x_i))$ and $(x_j, y(x_j))$ are points in the unit progress curve, one must have:

$$y(x_i) \simeq a\ (1 - b)\ x_i^{-b} \tag{3.58}$$

and

$$y(x_j) \simeq a\ (1-b)\ x_j^{-b} \tag{3.59}$$

for x_i and x_j not very close to the first units of the series. Dividing (3.58) by (3.59), taking logarithms and solving for b, yields:

$$b \cong \frac{\log\ \left[y(x_i)/y(x_j)\right]}{\log\ (x_j/x_i)} \tag{3.60}$$

Also, from (3.58):

$$a \cong \frac{y(x_i)\ x_i^b}{(1 - b)} \tag{3.61}$$

The cumulative average progress function will be given by

$$\bar{y} = ax^{-b}$$

where b and a are calculated through (3.60) and (3.61), respectively.

(b) A more exact solution stems from the application of formula (3.13). Thus,

$$y(x_i) = a\left[x_i^{1-b} - (x_i - 1)^{1-b}\right] \tag{3.62}$$

and

$$y(x_j) = a\left[x_j^{1-b} - (x_j - 1)^{1-b}\right] \tag{3.63}$$

Dividing (3.62) by (3.63) yields:

$$\frac{y(x_i)}{y(x_j)} = \frac{x_i^{1-b} - (x_i - 1)^{1-b}}{x_j^{1-b} - (x_j - 1)^{1-b}} \tag{3.64}$$

Now, let:

$$x_i = A,\ (x_i - 1) = C,\ \ x_j = D,\ \ (x_j - 1) = E,$$

$$1 - b = B, \text{ and } \frac{y(x_i)}{y(x_j)} = F$$

Then equation (3.64) reduces to

$$F = \frac{A^B - C^B}{D^B - E^B} \tag{3.65}$$

where B is the only unknown. Equation (3.65) may be solved by Newton-Raphson method as follows.

In order to apply the method, equation (3.65) is written in the form $f(B) = 0$, i.e.,

$$A^{B} - C^{B} - F(D^{B} - E^{B}) = 0 \tag{3.66}$$

An assumption is made that $f(B)$ is analytic for values of the variable B near the root sought.[6] Also, suppose that B_n is a known approximate value of the root.[7] Then, the Taylor's expansion about B_n may be formed as follows:

$$f(B_n + h) = f(B_n) + hf'(B_n) + \frac{h^2}{2!} f''(B_n) + \ldots = 0 \tag{3.67}$$

If all terms of second and higher degree in h are neglected, then:

$$h = -f(B_n)/f'(B_n) \tag{3.68}$$

and

$$B_{n+1} = B_n - f(B_n)/f'(B_n) \tag{3.69}$$

Applying iteration formula (3.69), yields:

$$f(B_n) = A^{B_n} - C^{B_n} - F(D^{B_n} - E^{B_n}) \tag{3.70}$$

$$f'(B_n) = A^{B_n} \ln A - C^{B_n} \ln C - F(D^{B_n} \ln D - E^{B_n} \ln E) \tag{3.71}$$

and

$$B_{n+1} = B_n - \frac{A^{B_n} - C^{B_n} - F(D^{B_n} - E^{B_n})}{A^{B_n}\ln A - C^{B_n}\ln C - F(D^{B_n}\ln D - E^{B_n}\ln E)} \tag{3.72}$$

After some iterations it is possible to obtain an adequate approximation B^* of B.[8] Thus, parameter b may be calculated with the required approximation from:

$$b \cong 1 - B^* \tag{3.73}$$

Parameter a may be calculated from (3.62) or (3.63). Thus:

$$a = \frac{y(x_i)}{x_i^{B^*} - (x_i - 1)^{B^*}} \tag{3.74}$$

The cumulative average progress function will be given by:

$$\bar{y} = ax^{-b} = \frac{y(x_i)}{\left[x_i^{B^*} - (x_i - 1)^{B^*}\right]} x^{B^*-1} \tag{3.75}$$

where B^* is calculated through iteration formula (3.72).

Statement of Problem B4

Given two points in the lot average progress curve, $(x_k, \bar{y}_L(x_k))$ and $(x_1, \bar{y}_L(x_1))$, determine the cumulative average progress function $\bar{y} = \bar{y}(x)$.

One possible solution is to use approximation (3.36). Then:

$$\bar{y}_{L_k} \cong y\,(x_k)$$

and

$$\bar{y}_{L_1} \cong y(x_1)$$

In this case, problem B4 is reduced to problem B3 already solved.

If the number of units in lot k and lot 1 are known, say, n_k and n_1, a more exact solution may be developed by using equation (3.14), as follows:

$$\bar{y}_{L_k} = \frac{a\left[x_k^{1-b} - (x_k - n_k)^{1-b}\right]}{n_k} \qquad (3.76)$$

and

$$\bar{y}_{L_1} = \frac{a\left[x_1^{1-b} - (x_1 - n_1)^{1-b}\right]}{n_1} \qquad (3.77)$$

Dividing (3.76) by (3.77) yields:

$$\frac{\bar{y}_{L_k}}{\bar{y}_{L_1}} = \frac{\left[x_k^{1-b} - (x_k - n_k)^{1-b}\right]}{\left[x_1^{1-b} - (x_1 - n_1)^{1-b}\right]} \cdot \frac{n_1}{n_k} \qquad (3.78)$$

Let:

$$x_k = A\ ,\ (x_k - n_k) = C,\ x_1 = D,\ (x_1 - n_1) = E,$$

$$(1 - b) = B \ , \ \text{and} \ \frac{\bar{y}_{L_k} / \bar{y}_{L_1}}{n_1 / n_k} = F \tag{3.79}$$

Then, equation (3.78) becomes:

$$F = \frac{A^B - C^B}{D^B - E^B} \tag{3.80}$$

where B is the only unknown. Equation (3.80) may be solved for B by Newton-Raphson method, as in Problem B3. The iteration formula is again (3.72). The coefficients A, C, D, E, and F are now given by (3.79). After a number of iterations an adequate approximation B^* of B is reached. Thus, parameter b may be calculated with the required approximation from:

$$b \cong 1 - B^*$$

Parameter a may be calculated from (3.76) or from (3.77). Therefore:

$$a = \frac{\bar{y}_{L_k} n_k}{x_k^{B^*} - (x_k - n_k)^{B^*}} \tag{3.81}$$

The cumulative average progress function will be given by:

$$\bar{y} = ax^{-b} = \frac{\bar{y}_{L_k} n_k}{x_k^{B^*} - (x_k - n_k)^{B^*}} x^{B^* - 1} \tag{3.82}$$

where B^* is calculated through iteration formula (3.72) with A, C, D, E, and F given by (3.79).

SUMMARY

The field of manufacturing progress functions lacks notation uniformity, precise definition of the variables and functional relationships involved, and formal proofs of important results.

The fact that the progress function may be advantageously treated as a continuous function so as to simplify final formulas and speed up solutions has been largely neglected in the literature. Instead, formidable formulas that are given to computers to digest are preferred in the name of exactness...

In this chapter an original mathematical exposition of the progress function is offered. Initially, four types of progress function are identified. Functional relationships for the four types are clearly and compactly defined with recourse to set theory notation. Parameters _a_ and _b_ are carefully explained and suggestively interpreted.

In a second section, four fundamental problems which users might have faced consciously or unconsciously, are formally stated and solved, at times by more than one method.

Finally, parameter calculation problems are classified and solved by exact or approximate formulas.

CHAPTER IV

QUADRATURE AND SUMMATION OF PROGRESS FUNCTIONS

This chapter represents a continuation of the mathematical exposition on progress functions initiated with Chapter III. Two related topics of practical relevance are now approached: the integration of progress functions and the debatable problem of their aggregation. Original approximations are proposed for both problems.

QUADRATURE OF PROGRESS FUNCTIONS

Cumulative Total Hours Calculation

Let the manufacturing progress function be expressed as a unit progress function:

$$y = ax^{-b} \tag{4.1}$$

Also, let (x_u, y_u) be the point in the progress curve where a plateau begins (Figure 4). This point has already been defined in Chapter II (page 39). For the moment it is assumed that it can be practically determined.

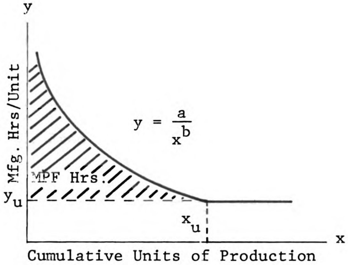


FIGURE 4

Progress Curve Ultimate Point (x_u, y_u)
and Manufacturing Progress Function Hours.

Recall that with equation (4.1) holding for the unit curve, the exact expression for the cumulative total hours is given by:

$$y_T = a \sum_x x^{-b} \qquad (4.2)$$

Approximation Methods . (a) An approximation method that eliminates calculation of every unit value in equation (4.2) was suggested in a study prepared by Boeing Airplane Company, already mentioned in Chapter II (page 20 , equation 2.7). Essentially, the integral is taken of equation (4.1) with respect to x between the limits 0.5 and $(x_u + 0.5)$ so as to improve the approximation. Thus:

$$y_T \cong \int_{0.5}^{x_u+0.5} ax^{-b}\,dx = \frac{a}{1-b}\left[(x_u+0.5)^{1-b} - 0.5^{1-b}\right] \qquad (4.3)$$

(b) However, a simpler approximation was also suggested in Chapter III. From equation (3.25) it follows that

$$y_T(x_u) \cong \frac{a}{1-b}\, x_u^{1-b} \qquad (4.4)$$

for x_u not very close to the first units in the series.

But, according to equation (4.1):

$$a = y_u x_u^{b} \qquad (4.5)$$

Therefore, from (4.4) and (4.5):

$$y_T\ (x_u) \cong \frac{x_u y_u}{1-b} \qquad (4.6)$$

where $y_T(x_u)$ does not depend on parameter a.

The same reasoning might be applied to equation (4.3). Hence, from (4.3) and (4.5), it follows that:

$$y_T(x_u) \cong \frac{x_u^{b}\ y_u}{1-b}\left[(x_u+0.5)^{1-b} - 0.5^{1-b}\right] \qquad (4.7)$$

(c) A third approach to be proposed herein is suggested by Euler-MacLaurin summation formula:[1]

$$\sum_{i=0}^{n} f(x_i) = \frac{1}{h}\int_{x_0}^{x_n} f(x)\,dx + \frac{1}{2}\left[f(x_0) + f(x_n)\right] + \frac{B_2\ h}{2!}\left[f'(x_n) - f'(x_0)\right] +$$

$$+ \frac{B_4\ h^3}{4!}\left[f'''(x_n) - f'''(x_0)\right] + \ldots +$$

$$+ \frac{B_{2p-2}\ h^{2p-3}}{(2p-2)!}\left[f^{(2p-3)}(b) - f^{(2p-3)}(a)\right]$$

$$+ n\,\mu\,\frac{B_{2p}\ h^{2p}}{(2p)!} \qquad (4.8)$$

Formula (4.8) is useful for finding the approximate sum of any number of consecutive values of a function when these values are given for equidistant values of x, provided the integral $\int_{x_0}^{x_n}(x)dx$ can be easily evaluated. In this formula h is the distance between the equidistant values of x, so that $n h = x_n - x_0$. The last term is a remainder where μ designates an average value of $f^{(2p)}(x)$ between x_0 and x_n . The B's are Bernoulli numbers. Recall that these are the numbers $B_1, B_2, \ldots, B_n$, defined by the expansion of the following generating function:[2]

$$\frac{x}{e^x - 1} = 1 + \frac{B_1 x}{1!} + \frac{B_2\ x_2}{2!} + \ldots + \frac{B_n x^n}{n!} + \ldots \qquad (4.9)$$

convergent provided that $|x| < 2$. Also, recall from the related theory that:

$$B_{2n+1} = 0 \quad , \qquad n = 1,2,3,...$$

and $$B_1 = -1/2$$

The B's with even indices have alternate signs and the following values:

$$B_2 = \frac{1}{6},\ B_4 = -\frac{1}{30},\ B_6 = \frac{1}{42},\ B_8 = -\frac{1}{30},$$

$$B_{10} = \frac{5}{66},\ B_{12} = -\frac{691}{2730},\ B_{14} = \frac{7}{6},\ B_{16} = \frac{3617}{510},$$

$$B_{18} = \frac{43867}{798},\ B_{20} = -\frac{174611}{330},\ B_{22} = \frac{854513}{138},\ \ldots$$

Replacing the B's in (4.8) for their values, yields:

$$\sum_{i=0}^{n} f(x_i) = \frac{1}{h}\int_{x_0}^{x_n} f(x)dx + \frac{1}{2}\left[f(x_0) + f(x_n)\right] + \frac{h}{12}\left[f'(x_n) - f'(x_0)\right]$$

$$- \frac{h^3}{720}\left[f'''(x_n) - f'''(x_0)\right] + \frac{h^5}{30240}\left[f^{v}(x_n) - f^{v}(x_0)\right]$$

$$- \frac{h^7}{1209600}\left[f^{vii}(x_n) - f^{vii}(x_0)\right] - R \qquad (4.10)$$

The Inherent Error in Euler-MacLaurin Formula . In (4.10) the terms on the right side, beginning with $(h/12)[f'(x_n) - f'(x_0)]$, form an asymptotic series. In computing with such a series it is important to know what term to stop with in order to get the most accurate result. One get the most accurate result by stopping with the term just before the smallest, since according to C.V.L. Charlier, in stopping with any term in Euler-MacLaurin' formula the error committed is less than twice the first neglected term. It can also be shown that the first two terms of Euler-MacLaurin formula will give a more accurate result than Simpson's Rule.[4]

By taking <u>the first three terms</u> in formula (4.10), the third formula for approximating equation (4.2) can be derived as follows:

$$y_T = a \sum_{1}^{x_u} x^{-b} \cong a \{ \int_1^{x_u} f(x)\,dx + \frac{1}{2}[f(1) + f(x_u)] + \frac{1}{12}[f'(x_n) - f'(1)] \} \qquad (4.11)$$

Since, $f(x) = x^{-b}$, $\int_1^{x_u} x^{-b}\,dx = \frac{1}{1-b}(x_u^{1-b} - 1)$, $f(1) = 1$, $f(x_u) = x_u^{-b}$, $f'(x) = -bx^{-(b+1)}$, $f'(x_u) = -b\,x_u^{-(b+1)}$, and $f'(1) = -b$, it follows from (4.11) that:

$$y_T(x_u) \cong a \left[\frac{1}{1-b}(x_u^{1-b} - 1) + \frac{1}{2}(1 + x_u^{-b}) + \frac{b}{12}(1 - x_u^{-(b+1)}) \right] \qquad (4.13)$$

or from (4.5):

$$y_T(x_u) \cong y_u x_u^b \left[\frac{1}{1-b}(x_u^{1-b}-1) + \frac{1}{2}(1+x_u^{-b}) + \frac{b}{12}(1-x_u^{-(b+1)}) \right] \qquad (4.14)$$

Manufacturing Progress Function Hours Calculation

In Chapter II the term "MPF hours" was defined as those hours over and above the estimated hours which are caused by the introduction of a new unit into a manufacturing system. The MPF hours or the direct labor hours associated with the manufacturing progress function can be measured by the shaded area in Fig.4. Knowledge of the MPF hours may be of extreme importance to management in deciding about the implementation onf a new product or a major change in existing products.The practical development of such matters will be delayed until a later chapter. In the following paragraphs four methods for calculating the MPF hours are proposed and assessed.

Exact Method. An obvious approach is suggested by equation (4.2). Noting that the shaded area in Figure 4 is the difference between the total area under the progress curve and the rectangular area $x_u \cdot y_u$, and also using equation (4.1), the MPF hours consumed by units number 1 up to and including unit x_u can be exactly determined as follows:

$$\text{MPF Hours} = a \sum_{1}^{x_u} x^{-b} - x_u \cdot y_u$$

$$= a \sum_{1}^{x_u} x^{-b} - x_u \cdot a x_u^{-b}$$

$$= a \left(\sum_{1}^{x_u} x^{-b} - x_u^{1-b} \right) \qquad (4.15)$$

Also, according to (4.5):

$$\text{MPF Hours} = y_u x_u^b \sum_{1}^{x_u} x^{-b} - x_u y_u$$

$$= x_u y_u \left(\frac{1}{x_u^{1-b}} \sum_{1}^{x_u} x^{-b} - 1 \right) \qquad (4.16)$$

Note that formula (4.16) does not require the knowledge of parameter <u>a</u>. Formulas (4.15) and (4.16) are better suited for machine computation. Tables can be programmed and developed on a digital computer.

Approximate Methods. Equations (4.3), (4.4) and (4.13) suggest three possible approximations to the problem of calculating the MPF hours. Again, subtraction of the rectangular area $x_u y_u$ from the total area under the progress curve, and equation (4.1) yield the following results:

(a) <u>Using equation (4.3)</u>

$$\text{MPF Hours} \cong \frac{a}{1-b} \left[(x_u+0.5)^{1-b} - 0.5^{1-b} \right] - x_u y_u$$

$$= a \left\{ \frac{1}{1-b} \left[(x_u+0.5)^{1-b} - 0.5^{1-b} \right] - x_u^{1-b} \right\} \qquad (4.17)$$

Also, according to (4.5):

$$\text{MPF Hours} \cong x_u y_u \left[\frac{(x_u+0.5)^{1-b} - 0.5^{1-b}}{(1-b)\, x_u^{1-b}} - 1 \right] \qquad (4.18)$$

(b) Using equation (4.4)

$$\text{MPF Hours} \cong \frac{a}{1-b} x_u^{1-b} - x_u y_u = \frac{ab}{1-b} x_u^{1-b} \qquad (4.19)$$

Also, from (4.5):

$$\text{MPF Hours} \cong \frac{x_u^b y_u}{1-b} x_u^{1-b} - x_u y_u$$

$$= \frac{b}{1-b} x_u y_u \qquad (4.20)$$

Equations (4.19) and (4.20) are much simpler than (4.17) and (4.18) and are well suited for manual calculation.

(c) Using equation (4.13)

$$\text{MPF Hours} \cong a \left[\frac{1}{1-b} (x_u^{1-b} - 1) + \frac{1}{2} (1 + x_u^{-b}) + \frac{b}{12} (1 - x_u^{-(b+1)}) \right] - x_u y_u$$

$$= a \left[\frac{1}{1-b} (x_u^{1-b} - 1) + \frac{1}{2} (1 + x_u^{-b}) + \frac{b}{12} (1 - x_u^{-(b+1)}) - x_u^{1-b} \right] \qquad (4.21)$$

or from (4.5):

$$\text{MPF Hours} \cong x_u y_u \left\{ \frac{1}{x_u^{1-b}} \left[\frac{1}{1-b} (x_u^{1-b} - 1) + \frac{1}{2} (1 + \frac{1}{x_u^b}) + \frac{b}{12} (1 - \frac{1}{x_u^{b+1}}) \right] - 1 \right\} \qquad (4.22)$$

Equations (4.21) and (4.22) derived from Euler-MacLaurin summation formula are the most accurate of the three proposed approximations. They can be easily handled by a pocket calculator.

Accuracy of the Proposed Formulas

In order to demonstrate the accuracy of formulas (4.3), (4.4), and (4.13) the cumulative total hours values were machine computed for x_u = 1, 2, 3, 4, 5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, and 999 using the exact formula (4.2) and also calculated with a HP-25 Scientific Programmable Pocket Calculator through the approximation formulas. In all cases parameter a was taken as 100 hours. Table 4.1 was developed for b = 0.152003 (90% progress curve). The percent deviations from the exact values are also shown in the referred table. Such results were plotted on semi-log graph (Figure 5).

THE AGGREGATION PROBLEM

Consider the sum of two progress functions given by:

$$y_1 = a_1 x^{-b_1}$$

and

$$y_2 = a_2 x^{-b_2}$$

The sum is

$$y_1 + y_2 = a_1 x^{-b_1} + a_2 x^{-b_2} \tag{4.23}$$

TABLE 4.1

EXACT AND APPROXIMATE CUMULATIVE TOTAL HOURS; a = 100, b = 0.152003 (90% curve)

x_u	Formula (4.2)	Formula (4.3)	% dev.	Formula (4.4)	% dev.	Formula (4.13)	% dev.
1	100.000000	100.800810	0.801	117.924946	17.9	100.000000	0
2	190.000000	190.968574	0.510	212.264916	11.7	190.036653	0.0193
3	274.620600	275.658438	0.378	299.366415	9.01	274.661171	0.0148
4	355.620600	356.695583	0.302	382.076873	7.44	355.662119	0.0117
5	433.919300	435.017196	0.253	461.668402	6.13	433.961129	0.0096
10	799.447900	800.592343	0.143	831.003177	3.95	799.490071	0.0053
20	1460.776000	1461.943251	0.080	1495.805815	2.40	1460.818365	0.0029
30	2072.689300	2073.863996	0.057	2109.599805	1.78	2072.731946	0.0021
40	2654.271300	2655.449680	0.044	2692.450641	1.44	2654.314187	0.0016
50	3214.195500	3215.376092	0.037	3253.322754	1.22	3214.238600	0.0013
100	5814.101800	5815.287344	0.020	5855.981338	0.720	5814.146067	0.0008
200	10496.406400	10497.59535	0.011	10540.76709	0.423	10496.45236	0.0004
300	14820.402800	14821.59509	0.008	14866.10092	0.308	14820.45157	0.0003
400	18926.783400	18927.97795	0.006	18973.38198	0.246	18926.83417	0.0003
500	22878.510500	22879.70727	0.005	22925.78159	0.207	22878.56334	0.0002
999	41182.189600	41183.39816	0.003	41231.41311	0.120	41182.25397	0.0002

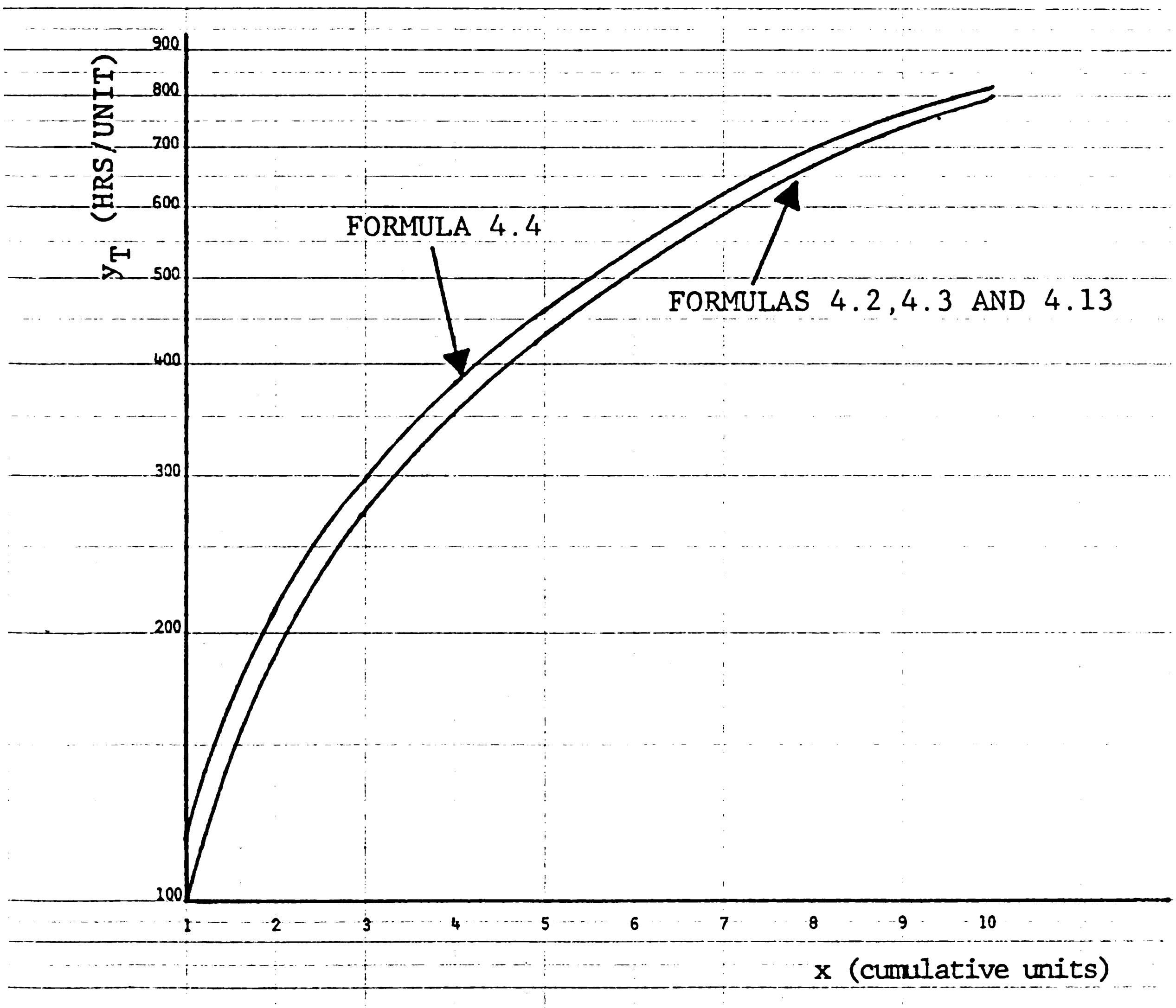


FIGURE 5 - EXACT AND APPROXIMATE CUMULATIVE TOTAL HOURS vs. CUMULATIVE PRODUCTION
$a = 100$, $b = 0.152003$ (90% CURVE)

A log-log plot of $(y_1 + y_2)$ versus $\underline{x}$ is a convex curve whose shape depends upon parameters (a_1, a_2, b_1, b_2). The plot will be linear if and only if $b_1 = b_2 = b$:

$$y_1 + y_2 = (a_1 + a_2)\ x^{-b} \tag{4.24}$$

Strictly speaking, if the model is assumed to hold for two separate portions of a task it cannot also be assumed to hold for their sum unless the separate curves have equal slopes - which will not in general be the case. Several authors contend that this fact precludes the use of the linear model for operations, departments, sections and total of the same project.[5]

Notwithstanding, it is the purpose of this section to show that it is theoretically correct and even desirable in practical work to approximate the sum of $\underline{m}$ progress functions of different parameters by a progress function of the same functional form as the addend functions.

Statement of the Problem

Given $\underline{m}$ progress functions

$$y_1 = a_1 x^{-b_1},\ y_2 = a_2 x^{-b_2}, \ldots, y_j = a_j x^{-b_j}, \ldots, y_m = a_m x^{-b_m},$$

determine a function of the form

$$y = Ax^{-B} \quad , \ A > 0 \quad , \quad 0 \leq B \leq 1$$

such that

$$y \cong \sum_{j=1}^{m} y_j$$

Solution of the Aggregation Problem

Expressing a Progress Function as a Power Series. The result known as Taylor's series may be posed as follows:

$$y(x)=y(c)+\frac{y'(c)}{1!}(x-c)+\frac{y''(c)}{2!}(x-c)^2+\ldots+\frac{y^{(n)}(c)}{n!}(x-c)^n+\ldots \tag{4.25}$$

Taylor's series may be used for the expansion of any given function y(x) in a power series in (x-c), provided the expansion exists. For a function to admit of a Taylor's series expansion in powers of (x-c), it is necessary that the function be finite and possess finite derivatives of all orders at x = c. The series in the right member of (4.25) will have a sum equal to y(x) whenever the series converges.[6]

Let $y(x) = ax^{-b}$. Therefore, expanding y(x) into Taylor's series yields:

$$y(x) = ax^{-b}$$

$$y'(x) = -abx^{-(b+1)}$$

$$y''(x) = ab(b+1)\ x^{-(b+2)}$$

$$y'''(x) = -ab(b+1)(b+2)\ x^{-(b+3)}$$

$$\vdots$$

$$y^{(n)}(x) = (-1)^n\ ab(b+1)(b+2)\ldots(b+n-1)x^{-(b+n)},\quad n=1,2,3,\ldots$$

or equivalently

$$y^{(n)}(x) = (-1)^n\ \frac{a(b-1)!b(b+1)(b+2)\ldots(b+n-1)}{(b-1)!}\ x^{-(b+n)}$$

$$= (-1)^n \frac{a(b+n-1)!}{(b-1)!} x^{-(b+n)} , \quad n = 0, 1, 2,...$$

Also

$$y(c) = ac^{-b}$$
$$y'(c) = -abc^{-(b+1)}$$
$$y''(c) = ab(b+1)\ c^{-(b+2)}$$
$$y'''(c) = -ab(b+1)(b+2)\ c^{-(b+3)}$$
$$\vdots$$
$$y^{(n)}(c) = (-1)^n \frac{a(b+n-1)!}{(b-1)!} c^{-(b+n)} , \quad n = 0,1,2,...$$

By substituting these values in equation (4.25), it follows that:

$$y=ax^{-b}=ac^{-b} - \frac{abc^{-(b+1)}}{1!} (x-c)+ \frac{ab(b+1)c^{-(b+2)}}{2!} (x-c)^2 - ...$$

$$+ (-1)^n \frac{a(b+n-1)!}{n!(b-1)!} c^{-(b+n)} (x-c)^n + ...$$

$$= \sum_{i=0}^{\infty} (-1)^n \frac{a(b+n-1)!}{n!(b-1)!} c^{-(b+n)} (x-c)^n \qquad (4.26)$$

Existence of the Expansion. In order to prove that the expansion in (4.26) exists, it suffices to prove that

$$\lim_{n\to\infty} R_n = 0$$

where R_n is the remainder term of the expansion. It is given by:

$$R_n = \frac{y^{(n)} [c + \theta\ (x-c)]}{n!} (x-c)^n, \ 0<\theta<1 \qquad (4.27)$$

Since $y^{(n)}(x) = (-1)^n ab(b+1)\dots(b+n-1)x^{-(b+n)}, n=1,2,\dots$ it follows that:

$$y^{(n)}[c+\theta(x-c)] = (-1)^n ab(b+1)\dots(b+n-1)[c+\theta(x-c)]^{-(b+n)}$$

Hence, from (4.27):

$$R_n = (-1)^n \frac{ab(b+1)(b+2)\dots(b+n-1)}{[c+\theta(x-c)]^{b+n}} \cdot \frac{(x-c)^n}{n!} \tag{4.28}$$

But $|y^{(n)}(x)|$ decreases with x, and $0 < \theta < 1$. Thus:

$$|y^{(n)}[c+\theta(x-c)]| < y^{(n)}(|x-c|) \tag{4.29}$$

for all $c - \frac{c}{1+\theta} < x < c + \frac{c}{1-\theta}$

Also, let N be a fixed positive integer greater than $2|x-c|$. And for $n>N$, put $n = N + k$. Then,

$$\frac{|x-c|}{N+1} < \frac{1}{2}, \frac{|x-2|}{N+2} < \frac{1}{2}, \dots, \frac{|x-c|}{N+k} < \frac{1}{2} \tag{4.30}$$

Since

$$\frac{(x-c)^n}{n!} = \frac{(x-c)^N}{N!}\left(\frac{x-c}{N+1}\right)\left(\frac{x-c}{N+2}\right)\cdots\left(\frac{x-c}{N+k}\right),$$

it follows that

$$\left|\frac{(x-c)^n}{n!}\right| < \frac{(x-c)^N}{N!} \cdot \frac{1}{2^k}$$

And for $n > N$,

$$|R_n| < y^{(n)}(|x-c|)\ \frac{|x-c|^N\ 2^N}{N!} \cdot \frac{1}{2^n}$$

When $n \to \infty$, x, c and N remain fixed but $\frac{1}{2^n} \to 0$. Hence

$$R_n \to 0$$

for all $c - \frac{c}{1+\Theta} < x < c + \frac{c}{1-\Theta}$

Therefore , Taylor's expansion given by (4.26) is valid for all x in the interval above.

Interval of Convergence. The interval of convergence for (4.26) may be practically determined by d'Alembert criterion (ratio test) applied to the corresponding series of absolute values. Let

$$u_n = (-1)^n\ \frac{a(b+n-1)!}{n!(b-1)!}\ c^{-(b+n)}(x-c)^n$$

Then

$$u_{n+1} = (-1)^{n+1}\ \frac{a\ (b+n)!}{(n+1)!(b-1)!}\ c^{-(b+n+1)}(x-c)^{n+1}$$

And after the necessary simplifications

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{b+n}{n+1} \cdot \frac{x-c}{c}\right| = \left|\frac{n+b}{n+1}\right| \left|\frac{x-c}{c}\right|$$

Hence

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = 1 \cdot \left|\frac{x-c}{c}\right| < 1 \quad \text{for convergence} \tag{4.31}$$

It follows from (4.31) that the interval of convergence is

$$0 < x < 2c \tag{4.32}$$

At the end points of the interval of convergence, the simple ratio test can never be effective. Convergence or divergence will have to be tested by other criteria.

At the end point x = 2c the series has:

$$u_n = (-1)^n \frac{a(b+n-1)!}{n!(b-1)!} c^{-(b+n)} c^n$$

$$= (-1)^n \frac{a(n+1)(n+2)\ldots(n+b-1)}{c^b (b-1)!} \tag{4.33}$$

Since the series has alternate signs and the nth term in (4.33) numerically decreases to zero as n becomes infinite the series is convergent at x = 2c. To see this:

$$\lim_{n\to\infty} \frac{a(n+1)(n+2)\ldots(n+b-1)}{c^b (b-1)!} = \frac{a}{c^b (b-1)!} \lim_{n\to\infty} n^{b-1} = 0$$

(recall that $0 \leq b \leq 1$ in a progress function)

At the end point x = 0 the progress function is not defined. Thus, the Taylor's expansion given by (4.26) is valid for all x such that

$$0 < x \leq 2c \tag{4.34}$$

Deriving the Aggregation Formula. Using (4.26) to expand the progress functions

$$y_j = a_j x^{-b_j} \qquad , \quad j=1,2,\ldots,m$$

about x=c:

$$y_j = a_j x^{-b_j} = a_j c^{-b_j} - \frac{a_j b_j c^{-(b_j+1)}}{1!}(x-c) +$$

$$+ \frac{a_j b_j (b_j+1) c^{-(b_j+2)}}{2!} (x-c)^2 - \ldots$$

$$+ (-1)^n \frac{a_j (b_j+n-1)!\, c^{-(b_j+n)}}{n!\,(b_j-1)!} (x-c)^n + \ldots \qquad (4.35)$$

valid for $0 < x \leq 2c$, $j=1,2,\ldots,m$, $n = 0,1,2,\ldots$

From the theory on power series, assuming that the series are convergent in some interval, then they can be added or subtracted term by term for each value of x common to their intervals of convergence. Since the y_j's are convergent in the same interval, their sum is also a convergent series in the same interval. Thus:

$$\sum_{j=1}^{m} y_j = \sum_{j=1}^{m} a_j c^{-b_j} - \left[\sum_{j=1}^{m} \frac{a_j b_j c^{-(b_j+1)}}{1!}\right](x-c) +$$

$$\left[\sum_{j=1}^{m} \frac{a_j b_j (b_j+1) c^{-(b_j+2)}}{2!}\right](x-c)^2 - \ldots$$

$$+ \left[\sum_{j=1}^{m} (-1)^n \frac{a_j (b_j+n-1)!\, c^{-(b_j+n)}}{n!\,(b_j-1)}\right](x-c)^n + \ldots \qquad (4.36)$$

valid for $0 < x \leq 2c$

Recall that the initial statement of the problem is to find a progress function of the form

$$y = Ax^{-B} \tag{4.37}$$

such that

$$y \cong \sum_{j=1}^{m} y_j \tag{4.38}$$

Expansion of (4.37) into a Taylor's series about x=c yields:

$$y=Ax^{-B}=Ac^{-B} - \frac{ABc^{-(B+1)}}{1!}(x-c)+\frac{AB(B+1)c^{-(B+2)}}{2!}(x-c)^2 - \ldots$$

$$+ (-1)^n \frac{A(B+n-1)!c^{-(B+n)}}{n!(B-1)!} (x-c)^n + \ldots \tag{4.39}$$

valid for $0 < x \leq 2c$, $n = 0,1,2,\ldots$

From (4.38) one must have:

$$Ac^{-B} - \frac{ABc^{-(B+1)}}{1!}(x-c) + \frac{AB(B+1)c^{-(B+2)}}{2!}(x-c)^2 - \dots$$

$$+(-1)^n \frac{A(B+n-1)!c^{-(B+n)}}{n!(B-1)!}(x-c)^n + \dots = \sum_{j=1}^{m} a_j c^{-b_j} -$$

$$\left[\sum_{j=1}^{m} \frac{a_j b_j}{1!} c^{-(b_j+1)}\right](x-c) + \left[\sum_{j=1}^{m} \frac{a_j b_j (b_j+1) c^{-(b_j+2)}}{2!}\right](x-c)^2 - \dots$$

$$+ \left[\sum_{j=1}^{m} (-1)^n \frac{a_j (b_j+n-1)! c^{-(b_j+n)}}{n!(b_j-1)!}\right](x-c)^n + \dots \qquad (4.40)$$

If the expansions in (4.40) are truncated after the second term and the method of undetermined coefficients is employed, it follows that:

$$Ac^{-B} = \sum_{j=1}^{m} a_j c^{-b_j} \qquad (4.41)$$

and

$$ABc^{-(B+1)} = \sum_{j=1}^{m} a_j b_j c^{-(b_j+1)} \qquad (4.42)$$

Solving (4.41) and (4.42) for A and B yields:

$$B = \frac{\sum_{j=1}^{m} a_j b_j c^{-b_j}}{\sum_{j=1}^{m} a_j c^{-b_j}} \qquad (4.43)$$

and

$$A = c^B D \qquad (4.44)$$

where D is the denominator in (4.43)

Thus, given $y_j = a_j x^{-b_j}$, $a_j > 0$, $0 \leq b_j \leq 1$, it is possible to find $y = Ax^{-B}$, ($A > 0$, $0 < B < 1$) such that

$$y \cong \sum_{j=1}^{m} y_j$$

the parameters of y being given by equations (4.43) and (4.44).

Proposition. Let $b_* = \min b_j$ and $b^* = \max b_j$, $j = 1,2,\ldots,m$. Then

$$b_* < B < b^*$$

<u>Proof</u>. Since

$$\sum_{j=1}^{m} a_j b_j c^{-b_j} > b_* \sum_{j=1}^{m} a_j c^{-b_j} \qquad (4.45)$$

and

$$\sum_{j=1}^{m} a_j b_j c^{-b_j} < b^* \sum_{j=1}^{m} a_j c^{-b_j} \qquad (4.46)$$

then according to (4.43), (4.45) and (4.46)

$$b_* < B < b^* \qquad (4.47)$$

Consequently, since $0 \leq b_j \leq 1$ it follows from (4.47) that

$$0 < B < 1$$

Accuracy of the Proposed Aggregation Method

The following sample calculation and graph are aimed at demonstrating the accuracy of the proposed aggregation method which is based upon Taylor's series expansion. Let

$$y_1 = 150\ x^{-0.322}, \quad y_2 = 350\ x^{-0.152} \text{ and } y_3 = 500\ x^{-0.515}$$

be three hypothetical progress functions. The calculations and graph plotting are carried out according to the succeeding steps:

(1) Each individual function $y_1(x)$, $y_2(x)$, and $y_3(x)$ is computed for x = 1,2,3,4,5,10,20,30,40,50,100,200,300, 400,500 and 1000 (Table 4.2, columns (2),(3), and (4)).

(2) The aggregate function is then calculated for each given $\underline{x}$ by summing up the corresponding values $y_1(x)$, $y_2(x)$, and $y_3(x)$ for the three addend functions. Call it the <u>exact</u> aggregate function (Table 4.2, column (5)).

(3) Parameters B and A of the <u>approximate</u> aggregate function are computed through formulas (4.43) and (4.44). From (4.33) one must have

$$0 < x \leq 2c \quad \therefore \quad c \geq \frac{x}{2}$$

Also, from the given data

$$x \in [1, 1000]$$

TABLE 4.2

EXACT AND APPROXIMATE AGGREGATE

PROGRESS FUNCTION-INTERVAL [1, 1000]

(1)	(2)	(3)	(4)	(5)	(6)	(7)
x	$y_1 = 150/x^{.322}$	$y_2 = 350/x^{.152}$	$y_3 = 500/x^{.515}$	$\sum_1^3 y_j$	$y \cong 664/x^{.213}$	% dev.
1	150.000	350.000	500.000	1000.000	664.000	33.6
2	119.994	315.001	349.896	784.891	572.860	27.0
3	105.307	296.173	283.957	685.437	525.462	23.3
4	95.9904	283.501	244.855	624.346	494.230	20.8
5	89.3353	274.047	218.273	581.655	471.289	19.0
10	71.4647	246.643	152.746	470.854	406.601	13.6
20	57.1689	221.979	106.891	386.039	350.791	9.13
30	50.1716	208.711	86.7466	345.629	321.767	6.90
40	45.7328	199.781	74.8013	320.315	302.642	5.52
50	42.5621	193.119	66.6808	302.362	288.594	4.55
100	34.0480	173.807	46.6627	254.518	248.982	2.18
200	27.2370	156.427	32.6542	216.318	214.807	0.70
300	23.9033	147.077	26.5004	197.481	197.034	0.23
400	21.7885	140.784	22.8512	185.424	185.323	0.054
500	20.2779	136.090	20.3704	176.738	176.721	0.010
1000	16.2215	122.481	14.2551	152.958	152.464	0.32

Therefore, by taking the mid-point of the interval:

$$c = \frac{x}{2} = \frac{1000}{2} = 500$$

$$B = \frac{\frac{150 \times 0.322}{500^{0.322}} + \frac{350 \times 0.152}{500^{0.152}} + \frac{500 \times 0.515}{500^{0.515}}}{\frac{150}{500^{0.322}} + \frac{350}{500^{0.152}} + \frac{500}{500^{0.515}}} = \frac{37.705864}{176.737834}$$

$$= 0.213343$$

and

$$A = c^{B} \cdot D = 500^{0.213343} \times 176.737834 = 664.064309$$

Thus, the aggregate function may be approximated by:

$$y \cong 664\ x^{-0.213} \qquad (4.48)$$

(4) The approximate aggregate function in (4.48) is calculated for the same values of $\underline{x}$ as already mentioned in step 1 (Table 4.2 , column 6).

(5) Per cent deviations of the approximate values obtained in step (4) with respect to the corresponding exact values from step (2) are evaluated (Table 4.2, column (7)).

(6) The exact and the approximate aggregate functions are plotted on arithmetic graph paper (Figure 6).

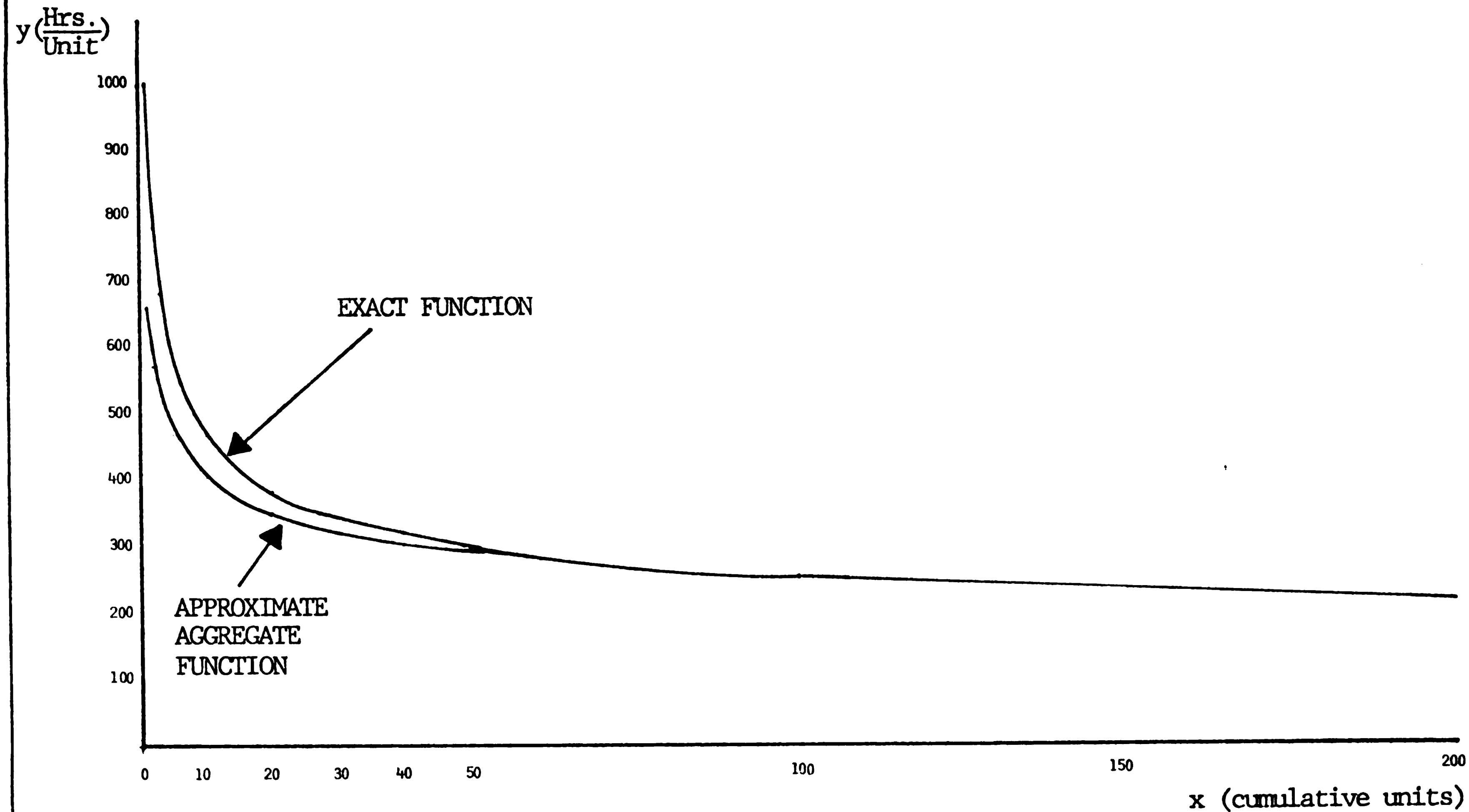


FIGURE 6 EXACT AND APPROXIMATE AGGREGATE PROGRESS FUNCTIONS - INTERVAL [1, 1000]

The results in Table 4.2 and Figure 6 show that the approximate aggregate function in (4.48) is quite effective in representing the exact aggregate function after the 40th or 50th unit up to the 1000th unit in the series.

If the focus is on the early units of the series it suffices to change the value of c and recalculate the parameters A and B for a new approximate aggregate function in that interval. For example, assuming that the interval of interest is now

$$x \in [1, 20]$$

then

$$c = \frac{x}{2} = \frac{20}{2} = 10$$

$$B = \frac{\frac{150 \times 0.322}{10^{0.322}} + \frac{350 \times 0.152}{10^{0.152}} + \frac{500 \times 0.515}{10^{0.515}}}{\frac{150}{10^{0.322}} + \frac{350}{10^{0.152}} + \frac{500}{10^{0.515}}} = \frac{139.165507}{470.853278}$$

$$= 0.295560$$

and

$$A = c^B \cdot D = 10^{0.295560} \times 470.853278 = 929.920021$$

The aggregate function may be represented by

$$y \cong 930\ x^{-0.296} \tag{4.49}$$

A second table is calculated by following steps 1-5 previously mentioned (Table 4.3). The results are also plotted on arithmetic graph paper (Figure 7). It is manifest that the approximate aggregate function in (4.49) fairly well represents the exact aggregate function in the interval [1, 20] and even beyond the 20th unit.

TABLE 4.3

EXACT AND APPROXIMATE AGGREGATE PROGRESS FUNCTION-INTERVAL [1, 20]

x	$\sum_{1}^{3} y_j$	$y \cong 930\ x^{-0.296}$	% dev.
1	1000.000	930.000	7.00
2	784.891	757.492	3.49
3	685.437	671.823	1.99
4	624.346	616.983	1.18
5	581.655	577.548	0.71
10	470.854	470.417	0.093
20	386.039	383.158	0.75
30	345.629	339.825	1.68
40	320.315	312.085	2.57
50	302.362	292.138	3.38
100	254.518	237.948	6.51

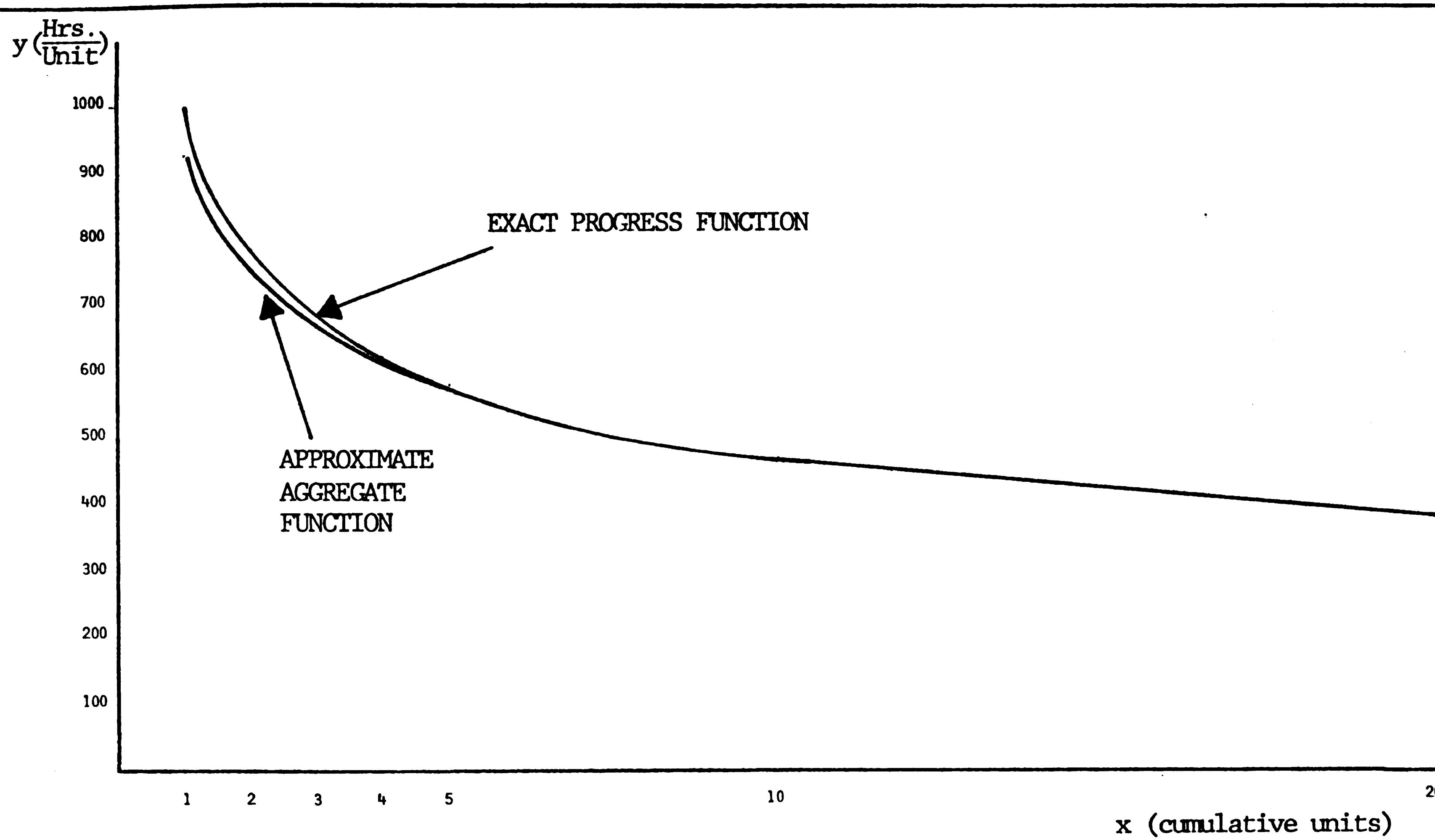


FIGURE 7 EXACT AND APPROXIMATE AGGREGATE PROGRESS FUNCTIONS - INTERVAL [1,20]

Accuracy in the Quadrature of
Approximate Aggregate Functions

The main interest of approximate aggregation of progress functions may reside in estimating the total direct labor hours and the MPF hours to be consumed by a prospective manufacturing program where one or several new products are involved. Since this is one of the potential applications of the manufacturing progress function to be treated in chapter VII no further comments are necessary at this point. However, to show how accurate the integration of the approximate aggregate function is the following sample checking is carried out. The same data of the previous subsection is assumed.

(1) For each given progress function $y_1(x)$, $y_2(x)$ and $y_3(x)$ the <u>exact</u> total hours expended in the manufacturing of unit 1 up to and including unit 999 are machine computed through formula (4.2). Thus:

$$y_{T_1} = 150 \sum_{1}^{999} x^{-0.322} = 150 \times 158.562733 = 23784.40995$$

$$y_{T_2} = 350 \sum_{1}^{999} x^{-0.152} = 350 \times 411.821896 = 144137.6636$$

$$y_{T_3} = 500 \sum_{1}^{999} x^{-0.515} = 500 \times 57.3720489 = 28686.02445$$

(2) The total hours expended in the three products (or activ-ities) represented by progress functions $y_1(x)$, $y_2(x)$ and $y_3(x)$ are computed

$$\sum_{1}^{3} y_{T_i} = 196608.0981$$

(3) Now the approximate aggregate function in (4.48) is used in order to calculate the same total hours consumed by $y_1(x)$, $y_2(x)$ and $y_3(x)$. Since the focused interval is sufficiently large, formula (4.4) may be used:

$$y_T \cong \frac{A}{1-B} x^{1-B} = \frac{664}{0.787} \times 999^{0.787} = 193575.9442$$

(4) The per cent deviation of the approximate result obtained in step 3 with respect to the exact result from step 2 is computed:

$$\% \text{ dev.} = \frac{193576 - 196608}{196608} \times 100\% = -1.54\ \%$$

It may be concluded from step (4) that even using the less accurate formula (4.4) the value of the integral is approximated with reasonable accuracy.

Again, considering the interval [1, 20] and going through the same previously mentioned steps, the following results are obtained:

$$\text{exact total hours} = \sum_{1}^{3} y_{T_i} = 150 \sum_{1}^{20} x^{-0.322} + 350 \sum_{1}^{20} x^{-0.152} + 500 \sum_{1}^{20} x^{-0.515} =$$

$$= 10388.7271$$

From formula (4.4) and equation (4.49):

$$\text{approximate total hours} = y_T \cong \frac{930}{0.704} \times 20^{0.704} = 10885.17477$$

$$\%\ \text{dev.} = \frac{10885-10389}{10389} \times 100\% = 4.78\%$$

Or from formula (4.3) and equation (4.49)

$$y_T \cong \frac{930}{0.704}\left(20.5^{0.704} - 0.5^{0.704}\right) = 10265.1192$$

$$\%\ \text{dev.} = \frac{10265-10389}{10389} \times 100\% = -\ 1.19\%$$

CHAPTER V

A MODEL OF THE PROGRESS PHENOMENON IN MANUFACTURING INDUSTRIES

The objectives of the study included the derivation of a symbolic-analytic model of the manufacturing progress phenomenon. Such a model will be presented in this chapter.

This purpose cannot be accomplished without a means of understanding how system adaptation can exist in a manufacturing concern. It is worthwhile to expend some effort in questioning the causes of, or reasons for, the systematic gains in productivity embodied in the progress curve,however tentative the resulting explanation might be.

Although system adaptation can exist in a wide variety of production systems, methodological problems can impose serious restrictions on a study of the phenomenon. These problems, and their influence on the study, are also discussed in this chapter.

A CONCEPT OF MANUFACTURING PROGRESS

The manufacturing progress function is essentially an empirical concept. There is no rational or deductive proof that can be advanced to support it or the assumptions upon which it is based. While these assumptions may be intuitively appealing, they must be empirically validated. The three assumptions upon which the MPF is based are: (1) The amount of time required to complete a given task or unit of production will decrease each successive time the task is undertaken; (2) The rate at which this reduction in unit time will occur will be a decreasing one, and (3) This reduction in unit time will follow a specific predictable pattern.

It has been demonstrated in the literature that these assumptions are valid for many classes of products and manufacturing processes. Moreover, they may serve the purpose of providing a preliminary characterization of the manufacturing progress phenomenon.

Factors in Manufacturing Progress

There are several factors which may influence or contribute to manufacturing progress: (1) The production worker, (2) Management, and (3) Staff or supporting personnel. Their contribution will vary over time and in relative importance from one industrial environment to another.

The Production Worker. It seems reasonable to expect that the production worker will improve his performance as he becomes more acquainted with a new task. This should be evident in a reduction in the time to complete a unit of production as well as a reduction in the number of rejects. It is also suggested that the learning of the worker will have a rather immediate and significant effect in terms of manufacturing progress but the change in rate of production he contributes will not extend over a long period of time, particularly in routine and repetitive tasks.

Management and/or Supervision. An efficient supervision should be able to motivate the worker towards a reduction in the time necessary to complete a unit of production. Similarly, management may be able to impress upon the staff personnel the need for further savings through their efforts. In a situation where more than one product is being manufactured simultaneously in a plant the attention dispensed by management to one or more of the products will influence the magnitude of manufacturing progress for that product or products. On the other hand, the relative attention or lack thereof given one product over time may be reflected in a progress curve exhibiting plateaux of no improvement followed by sharp drops reflecting sudden management emphasis.

Staff and/or Supporting Personnel. It is suggested that a substantial portion of the manufacturing progress will be the result of the efforts of the supporting personnel who in many organizations are listed under the headings of the various staff functions. It is also suggested that the improvements contributed by the staff functions will be more of a continuing, long-ranging and long-lasting nature and that they will be the significant factor after the initial learning has been accomplished by the production workers. Some of the typical contributions from the various staff offices are discussed in the following paragraphs.

Tooling. The type of tooling used, the degree of completion or development prior to production and the changes made during production will influence the magnitude of manufacturing progress.

Methods Engineering. The extent to which work methods in detail are designed prior to production and then the emphasis on improving these methods during production through work simplification and similar programs will affect total manufacturing improvements.

Production Planning and Control. This function may facilitate progress through improved planning, routing, scheduling, dispatching, and follow-up. This should result in an increased utilization of machines, tools, and labor , and ultimately in a reduction in the direct labor hours necessary to manufacture a unit of production.

Materials Management. When a materials management concept of organization is used this function may include the production planning and control, the purchasing of raw materials and component parts, the control and the storage of preproduction, in-process and finished inventory and the materials handling function. If it is performed effectively, material shortages and the disruption of production should be reduced or eliminated.

Product Engineering. This function is responsible for product design, specifications, testing, and the like. The extent to which manufacturability was considered prior to production and the degree of change required subsequently during production will influence total manufacturing improvement.

Quality Control. The extent to which a quality assurance program can reduce rework and repair operations, and scrap losses will affect the total manufacturing progress.

The degree to which each and all of the above listed factors may influence manufacturing progress will depend upon the amount of available or possible improvement remaining immediately after the first unit of production has been built. It seems reasonable to accept that a company that has spent much time and money getting the entire organization ready to start production of the first unit has a smaller potential for manufacturing improvement after production has started than a company that has started unprepared and plans to remove the inadequacies during manufacture. There is simply more room for progress for the second company than for the first. Let indices 1 and 2 designate the prepared and the unprepared company, respectively. Thus, in terms of the power function model, their progress functions may be written as follows:

$$y_1 = a_1 x^{-b_1} \quad , \quad \text{and} \quad y_2 = a_2 x^{-b_2}$$

It seems logical to expect that

$$a_1 < a_2 \quad , \quad \text{and} \quad b_1 < b_2$$

provided that the greater potential for improvement of the second company be fully realized. The fact that there exists more room for improvement does not necessarily imply that the available improvement will materialize. By a similar reasoning, company 1 will probably produce the first unit in less time than company 2 because it is better prepared to do so. However, it may also happen that the inequality $a_1 < a_2$ will not come true in spite of the greater preproduction effort of company 1. Management action may be the crucial determinant of actual progress in the manufacturing organization, once there exists a potential for improvement. The degree to which such a potential is exploited is related to the drive and resourcefulness of management and its skill in stimulating supervisors and technical people to be creative and workers to be productive.

Hypotheses About the Progress Phenomenon - A Reexamination

It is believed that in theoretical work a minimum number of hypotheses is desirable in explaining any kind of phenomenon. It is our contention that a reduction can be achieved in the number of available hypotheses about the manufacturing progress phenomenon and that the explanatory power of the remaining hypotheses can be greatly enhanced.

A Rationale for the Novelty Hypothesis. According to Crawford the rate of progress is a function of the experience of the worker. The greater the lack of experience of the worker, the greater the rate of improvement. Schultz and Conway agree with Crawford when they state that the degree of similarity of a machine to a predecessor has a significant effect on the rate of progress; the less similarity, the greater the rate of progress (see Chapter II, pp 16, pp 38, and pp 68). An organization which has had experience doing a similar type of work probably will not exhibit much improvement when the "new" job begins because there is not much room left for improvement in the "new" operation. By the same token, an organization which is unfamiliar with this type of product and with this type of work will have a greater potential for manufacturing progress. This is a possible rationale for the novelty hypothesis already mentioned in Chapter II.

Preproduction Preparation, a Kind of Experience. The novelty hypothesis can be strengthened if preparation prior to production can be considered a kind of experience -- particularly if the fabrication of prototypes is part of the preproduction effort.

The Assembly and Machining Progress Hypothesis. Some researchers have found that assembly operations exhibit significantly higher rates of progress than machining operations (see Chapter II, pp 34 , and pp 50). The usual explanation is that in assembly work there is a relatively large scope for learning; in machine work the ability to reduce labor hours is greatly restricted by the fact that the machines cannot "learn" to run any faster.[1] However, this hypothesis can be explained by the novelty hypothesis.Consider, for example Hirsch's study of the progress phenomenon in machine-tool manufacturing.[2] This study represents one of the few instances where the Assembly and Machining Progress hypothesis was explicitly tested. According to Hirsch, the possible reasons why the machining improvement rate is considerably less than the assembly improvement rate are: (1) Many times the same part is used in both old and new models, or even in different machines that, after many lots have been produced, the actual improvement rate for each additional lot becomes almost negligible; (2) The accomplished machine operator can do little to improve his efficiency, since most of his basic motions are the same, regardless of the part being produced, and (3) The third reason is due to the great inconsistency in the ratio of new to old parts used in the assembly of the different machines. Thus, machines comprised of a large quantity of new parts will have a high rate of improvement, while machines with few new parts will have a low improvement rate. Hirsch's explanation is consistent with the findings of Schultz and Conway.[3]

The Parameter Correlation Hypothesis. As mentioned in Chapter II a strong correlation between the parameters of the progress curve was found by H. Asher and N. Baloff. In Asher's study the correlation was found among startups that occurred in different facilities. In Baloff's study the correlation was among startups that occurred in the same facility. However, the findings of both studies are consistent, i.e., low values of parameter a are associated with small b-parameter values, and high values of a with large, b values. A tentative explanation is that management reacts to the reported level of initial productivity in ways that could tend to accelerate or decelerate the progress phenomenon. Startups that show a relatively "poor" beginning frequently have more technical resources allocated to them, and are generally "nurtured" to a greater extent, than startups that begin relatively well. Furthermore, judging from the comments of many production personnel, it appears that the motivation for progress within the production organization will be inversely correlated with the initial productivity. The motivation for productivity improvement is apparently stronger for "poor" startups than for those that show a relatively large initial productivity. Both the "internal" motivation of the production organization and "external" management pressure seem to be influenced in this way.[4]

MANUFACTURING PROGRESS - - A MODEL

The conceptual framework presented in the previous section offers a basis for a model of the progress phenomenon in manufacturing industries.

It seems desirable that such a model preserve the simplicity of Wright's formulation while reflecting the generally accepted linearity hypothesis. Recall that this hypothesis states that linearity exists - - when in logarithmic coordinates - - between the labor per unit input and cumulative output.

The model should also provide a means of predicting the exact course of a startup, not just its functional form. The usefulness of the model could be enhanced appreciably if some method of predicting the a and b parameters of the model for a given startup could be developed. The parameter correlation hypothesis and the novelty hypothesis provide some basis to develop such a method.

In explaining these hypotheses a common element emerged: the potential for improvement. Implicit in the explanation given in the last section was the assumption that the potential for improvement can be somehow estimated a priori and that management will strive to reach the estimated productivity target. A model of the progress phenomenon should recognize the potential for improvement and include a method for determining its magnitude.

The Traditional Model: Some Qualifications.

The power function $y = ax^{-b}$ offers a very efficient description of the way in which the progress phenomenon occurs in a manufacturing concern. The model has successfully explained a large number of startups in diverse industrial settings. This is well documented in the literature (Chapter II). Nevertheless, the use of the device as a predictor requires some clear definition of the estimating values of the parameters of the function, namely, the a and b parameters.

Much of its use in the airframe industry implies the use of the manhour expenditure on the first airframe produced as a basis for extrapolation and use of a uniform 80% characteristic to define the Wright "slope". However, it seems doubtful, that the labor expenditure on the first unit produced can be determined with any accuracy. Even in the airframe industry parameter a represents a theoretical manhour expenditure on the first airframe produced. This is so because the actual manhours figure is rarely known. Labor expenditure is typically recorded on a monthly as opposed to a per-unit basis. As a result, the general practice in empirical investigations has been to determine the $\bar{y}$ value (the average value) and the corresponding value of x, for a "lot" of several airframe units. The size of a lot will correspond to the number of units produced during the month-long accounting period. Therefore, for the first empirical observation, x_1 will represent the number of units produced during the first mo

na

b

t

t

d

e

f

month of production, and $\bar{y}_1$ will be the average number of manhours per unit expended on these x_1 units.

Use of this lot convention, whose adoption is dictated by industrial accounting procedures, has several consequences: the value of the a parameter is a theoretical one, except in the rare instance that only one airframe unit is produced during the first month of production. It can be argued that, even if the first accounting period yielded a cost for the first unit of an airframe, this cost figure would be suspect. It is a rare cost system that could accurately determine the cost of a single unit - - let alone the first unit - - of an airframe, given the nature of the production process and the industry.[5]

Moreover, it seems unlikely that first piece labor hours can be determined soon enough to enable their use as a predictor. To be useful for production decisions related to engineering effort, production planning, manpower planning, or design changes, estimates far in advance of production are necessary.

Estimating the Ultimate Point (x_u, y_u)

Given the difficulties in estimating the manhours consumed by the first unit produced, the progress function may still be determined if some other point in future production is estimated. The so-called ultimate point (x_u, y_u) was already defined in Chapter II (p.39). Consider now some aspects of its determination in practical work.

Conventional estimating procedures, say the Methods-Time Measurement system (M.T.M.), are usually intended to give an estimate of the time that will be required after the operation has "settled down".[6] This estimate corresponds to the ordinate y_u of the ultimate point. However, it is necessary to associate the conventional estimate y_u with a specific point in cumulative production, x_u. The most satisfactory way of doing this is to try not to disturb the existing estimating procedure but to examine its past performance to determine how long it took after production began for the actual time to decrease to the vicinity of the estimate.

Presumably, (x_u, y_u) should be the point where the plateau begins. In practice it may not happen exactly this way. This fact does not impede the determination of the progress function. It suffices that the estimated labor hours per unit at ultimate (y_u) be reached at point x_u in cumulative production. Therefore (x_u, y_u) continues to be a point on the progress curve and can be used instead of the first point. In firms that have experience with production standards the plateau can be anticipated with reasonable accuracy.

Once determination of point (x_u, y_u) is made, the difficulties of obtaining first item labor hour are avoided. In addition, the estimate y_u is a practical means of defining the potential for improvement. It represents a productivity target that management will try to reach. The potential for improvement is given by $(a - y_u)$, i.e., the difference between the first unit labor hours and the ultimate unit labor hours.

Estimating Parameter b

The parameter correlation hypothesis provides a means of determining the b-parameter. As previously mentioned, this hypothesis was confirmed in at least two empirical studies. The findings of our research - - to be presented in Chapter 6 - - also disclose the possibility of formalizing this relationship in terms of a parameter model in which the two parameters could be functionally related in the case of startups that occurred in the same plant. Such results indicate the usefulness of the following empirical equation:

$$b = m + n \ln \alpha \qquad (5.1)$$

where $$\alpha = \frac{a}{y_u}$$

and m, n are constants in the model.

A Symbolic-Analytic Model of the Manufacturing Progress Phenomenon

It is suggested by this author that the manufacturing progress phenomenon can be described and its course predicted by the following empirical equations:

$$y = \alpha x^{-b} \tag{5.2}$$

$$b = m + n \ln \alpha \tag{5.3}$$

where

$$y = \frac{y}{y_u}$$

$$\alpha = \frac{a}{y_u}$$

and

(x_u, y_u) is the estimated ultimate point.

If equations (5.2) and (5.3) prove to be empirically valid, prediction of parameters a and b for a new startup can be developed as follows.

Equation (5.2) yields for $x = x_u$:

$$1 = \alpha x_u^{-b}$$

$$\therefore \alpha = x_u^{b}$$

By substitution in equation (5.3):

$$b = m + n \ln \left(x_u^{b}\right)$$

or solving for b

$$b = \frac{m}{1 - n \ln x_u} \tag{5.4}$$

Finally,

$$a = x_u^{b} = x_u^{\left(\frac{m}{1-n \ln x_u}\right)} \qquad (5.5)$$

and

$$a = y_u x_u^{\left(\frac{m}{1-n \ln x_u}\right)} \qquad (5.6)$$

Thus, parameters a and b can be predicted by using formulas (5.4) and (5.6).[7]

GENERAL METHODOLOGY OF THE EMPIRICAL RESEARCH

An examination of the progress phenomenon in several manufacturing concerns was a central feature of the study objectives. The intention is to test the model offered in the last section by using real data. A first methodological decision was the selection of an adequate sample. As will be apparent the empirical nature of the study did place very definite practical limitations on the sample. The selection of the study sample was largely shaped by two conditions - - industrial cooperation and availability of reliable and usable data.

Criteria in Sample Selection

Since empirical studies in non-airframe manufacturing firms are rare, one of the criteria used in selecting the sample was the desire to extend the model to a large number of concerns outside the airframe industry.

A rapid rate of implementation of new products or major changes in existing products was another criterion. The reasons for this requirement are: the availability of sufficient historical data for meaningful analysis and the future usefulness of the research findings in predicting manufacturing startups.

Another aim was to gather data from facilities located in diverse countries for the purpose of comparison.

It appeared desirable to analyse manhour data experienced in production situations which have not used a progress curve as a control device.

Finally it was intended to obtain aggregate data for whole industries, for the purpose of possible macroeconomic applications.

The Sample

Nine manufacturing firms representing five different industries were ultimately selected for inclusion in the study sample. These firms will be designated by letters A,B, C,D,E,F,G,H, and I.

Firms A,B,C, and D are members of the electronic data processing systems industry.

Firms A,B, and C are respectively the Brazilian, Canadian, and Japanese representatives of the same multinational enterprise in the field of business machines manufacturing. In firm A twelve different examples of manufacturing startups were analyzed. One startup was studied in each of the firms B and C. The processes considered cover the manufacturing of both conventional and advanced products in the field. Startups in these three firms occurred in a period ranging from 1960 until mid-1973.

Firm D is a leading manufacturer of business machines in the United States.[8] The net sales for 1958 amount to over half a billion dollars. Data was obtained for two different programs. The first set of data cover man-hours expended in the manufacture of an electronic data processing system. The second set represents small computer components and one control unit. A total of fourteen startups were available for study. The data obtained was that for assembly operations only and covers a period ranging from 1953 until 1960.

Firm E is an American firm in the electronics manufacturing field, which started operations in 1946.[9] Sales ran approximately ten million dollars annually in 1960. Small electronic motors is the speciality of the company. The firm's output for which data was obtained consists entirely of specialized components used in electronic data processing systems and automated control systems. In firm E fifteen startups were analyzed. The manufacture of the focused computer components started in 1949 and ended in 1951. Only assembly time data was obtained from firm E.

Firm F is one of the oldest printing press and bindery machine manufacturers in the United States.[10] It is considered a leader in its field. Sales in 1959 were over twenty-five million dollars.At firm F data was obtained for all products manufactured from 1950 until 1960 yielding a total of twenty-five startups. Data obtained from this manufacturer represents both assembly and machining time.

Firm G is one of the oldest Brazilian shipyards. Its shipbuilding capacity was 10,000 tons of steel in 1973. The sales in the same year were over ten million dollars.At this firm data was obtained for only one type of ship. However , the nineteen startups made available cover all manufacturing operations involved in its production. Startups occurred in a period ranging from August 1971 until March 1973.

Firm H is the leading Brazilian manufacturer of oil drilling tools and equipment. At this firm data was made available for only three of the products manufactured. Data represents assembly time. The three startups occurred from July 1971 until May 1973.

Firm I is a Brazilian manufacturer of light airplanes. Data was obtained for parts, major assemblies and complete airframes, yielding a total of sixty-nine startups that occurred in two different plants during 1972.

Whole Industries. Aggregate data was obtained for the Brazilian Mechanical and Electrical Industry in a period that is considered the "take-off" period of Brazilian industrialization in these fields (1960 - 1964). This industry will be represented by letter J. Data is subdivided into nine groups as follows: (J1) Castings, (J2) Forgings, (J3) Mechanical Machinery, (J4) Electrical Machinery, (J5) Industrial Equipment, (J6) Locomotives, Wagons and Railway Equipment, (J7) Shipbuilding, (J8) Roadbuilding Tractors and Equipment, and (J9) Farm Machinery. A more detailed description of the products included in each aggregate is found in the following Chapter.[11]

Summarizing the above, nine manufacturers - - representing five different industries - - are included in the sample. A hundred and fifty-nine separate cases of product and process startups that occurred in four different countries and nine distinct plants have been analyzed . In addition aggregate data was obtained for whole industries in one country, yielding nine more startups.

Measuring the Progress Phenomenon

After selecting the sample, a second methodological question had to be considered, that is, the way of measuring the startups.

Each of the startups was analyzed in relation to two variables, an index of productivity (dependent variable) and a measure of cumulative output (independent variable), as indicated in equation (5.2). The progress of the different startups was examined periodically in relation to the normal production statistics developed by the participating firms. As to the dependent variable, direct-labor hours data are usually calculated and reported in relation to standard accounting time periods, yielding average direct-labor hours per unit figures for the lot of product produced during the accounting period. As to the independent variable, cumulative output statistics indicate the total output of the product (from inception of manufacture) that is achieved at the end of the accounting period.

Direct comparisons of productivity between different products or processes are not possible with a direct-labor per unit measure. For example, process 1 may have a theoretical ultimate of y_{u1}, whereas process 2's theoretical ultimate is y_{u2}. It is meaningless to compare the productivity of the two processes in absolute terms. In addition, the direct-labor hours per unit measure creates a disclosure problem; the participating firms do not wish to have the actual labor hours per unit of their operations and products revealed in absolute terms.

These inconveniences are largely avoidable. As already mentioned, it is standard practice to develop predetermined engineered standards of performance (y_u). Thus it is possible to define an index of productivity ($\frac{y}{y_u}$) that relates the actual direct-labor hours per unit consumed to the production standard and take it as the independent variable. This is the meaning of y in equation (5.2).

A final general methodological question refers to the measurement period used in the study. Production records are summarized on a monthly basis in all individual firms. The physical output of the various products and operations, as well as the productivity indices, are calculated at the end of each calendar month. In most cases, monthly observations give a good description of the course of a startup. In a few examples, bi-weekly or weekly observations might have provided a better representation of the progress phenomenon. The main effect was a reduction in sample size, which has some statistical implications.

In the next chapter the findings in the firms and industries will be described and analyzed. There, specific methodology will be introduced whenever needed.

CHAPTER VI

EMPIRICAL FINDINGS

In this chapter the manufacturing progress model presented in chapter V will be tested with real data from several industries. The analysis of the startups encountered in each of the industries studied has been divided into six separate sections, in this order: (1) Manufacturing of Electronic Data Processing Systems and Components (Firms A,B,C,D and E); (2) Manufacturing of Bindery Machines and Printing Presses (Firm F); (3) Shipbuilding (Firm G); (4) Manufacturing of Oil Drilling Tools and Equipment (Firm H); (5) Production of Airframes (Firm I) and (6) Mechanical and Electrical Industry (designated by letter J).

STARTUP ANALYSIS IN THE MANUFACTURING OF ELECTRONIC DATA PROCESSING SYSTEMS AND COMPONENTS

Characteristics of the Products and Manufacturing Processes Studied

Wherever possible, some of the characteristics of products and processes studied will be revealed. For obvious reasons, however, a complete disclosure is not possible.

Firms A,B and C. The Products studied at firms A, B and C comprise card punch machines of two types and the central processing unit (CPU) of a third generation computer. The card punches will be designated by letters X (a modern model) and Y (an older model). Technically, the focused products and components are electromechanical or electronic. The card punch is one of the units in the so-called peripheral equipment of almost all electronic data processing systems. Before any data can be read and properly processed, it must be recorded in proper form. Data recording is performed by the card punch. Bearing a striking resemblance to an electric typewriter, the card punch's keyboard contains alphabetic , numeric, and special characters. Recording is accomplished by stroking the keys one key at a time, as the card moves from right to left. Modern card punches print or interpret directly over the columns as they punch holes in the card.

The central processing unit studied is a two-level memory system, made up of large-size processor (main) storage which functions as a backing storage for smaller, monolithic buffer storage. The processor is capable of requesting eight bytes from the buffer every 80 nanoseconds. Memory capacity is up to 3,100,000 characters.

The manufacturing of these products is broadly divided into the following processes: (1) parts machining, (2) subassembly, (3) final assembly and (4) testing. Because of the various features and combinations of features available to customers, the major portion of the machines is built to customer order. It is not a mass production organization in comparison with an appliance or an automobile manufacturer. The machine shop delivers parts on a short production run basis. The subassembly departments produce units in lots, and the final assembly departments operate on a continuous production basis. Engineering changes resulting from customer requests for new features, cost reductions, or standardization of components or units are introduced throughout production of a machine.

Table 6.1 summarizes the startups analyzed in firms A, B and C of the same multinational company.

Firms D and E. The products of firm D comprise several second generation computer components. Although these parts would at a later date be assembled by the buyer into a unified data processing system, each one of the computer components was especially designed and different from the other units. The company made these to order for one of the larger electronics manufacturers in the United States. It should be stressed that the data obtained was for assembly operations only. Close to 70 per cent of the parts are procured from outside vendors, and these are charged to material cost.

The products of firm E for which data was obtained consist entirely of second generation computer components and data storage units. As with firm D, only assembly data was obtained from firm E, because 1) Almost 80 per cent of the parts going into the assembly are procured from outside

TABLE 6.1

STARTUPS ANALYSED IN FIRMS A, B AND C

Firm	Country	Startup Code	General Description
A	Brazil	A1,A2,A3, A4 and A5	Startups A1 through A4 comprise the final assembly operations of the CPU of a third generation computer. Startup A5 corresponds to the total final assembly of that unit.
		A6	Total manufacturing,card punch X.
		A7	Parts machining, card punch X.
		A8	Subassembly, card punch X.
		A9	Final Assembly, card punch X.
		A10	Testing, card punch X.
		A11	Final Assembly and Testing, card punch X.
		A12	Assembly of major units of card punch Y.
B	Canada	B1	Final Assembly and Testing, card punch X.
C	Japan	C1	Final Assembly, card punch Y.

vendors, and 2) manhour data from parts produced by the company are misleading - - company E has a policy of manufacturing certain parts in lot sizes greater than assembly lots. The parts in excess of assembly needs are stored for future assembly needs or issued as spare parts orders are received. Table 6.2 summarizes the startups studied in firms D and E.

TABLE 6.2

STARTUPS ANALYZED IN FIRMS D AND E

Firm	Country	Startup Code	General Description
D	United States	D1	Control Unit no. 2
		D2	High Speed Printer no. 1
		D3	High Speed Printer no. 2
		D4	Card Reader Unit no. 1
		D5	Tape Perforation Unit no.1
		D6	Magnetic Tape Unit no. 1
		D7	Control Unit no. 1A
		D8	Computer no. 1A
		D9	Computer no. 2A
		D10	Computer no. 3A
		D11	Computer no. 4A
		D12	Computer no. 5A
		D13	Computer no. 6A
		D14	Computer no. 7A

TABLE 6.2 (cont'd)

Firm	Country	Startup Code	General Description
E	United States	E1	Computer no. 2B
		E2	Computer no. 3B
		E3	Computer no. 4B
		E4	Computer no. 5B
		E5	Computer no. 6B
		E6	Data Storage Unit no. 1B
		E7	Data Storage Unit no. 2B
		E8	Data Storage Unit no. 3B
		E9	Computer no. 7B
		E10	Computer no. 8B
		E11	Control Unit no. 1B
		E12	Computer no. 1B

Startup Measurement and Data

In firms A, B and C data is usually reported in relation to standard accounting time periods (month), yielding average direct labor hours per unit figures for the lot of product delivered during the accounting period. However, in the case of startups A1 through A5, the direct labor hours were available on a per unit basis. To obtain a common basis of analysis cumulative average man-hours per unit ($\bar{y}$) were computed for all startups analyzed in this dissertation.

The dependent variable in equation (5.2) is a productivity index given by:

$$\mathcal{y} = \frac{\bar{y}}{y_u}$$

This index relates the actual performance of the operation (y) to predetermined, engineered standards of performance (y_u). Therefore, for each startup, the cumulative average man-hours per unit figures were divided by the respective engineered estimate y_u.

Cumulative output statistics indicate the total output of the product (from inception of manufacture) that is achieved at the end of the accounting period. In the case of startups A1 through A5, the completion of each unit is not necessarily achieved at the end of the accounting period.

Startup data in this modified form pertaining to firms A,B and C is shown in Tables 6.3 through 6.7 (Appendix F).

In firms D and E data is usually reported in lot-average form. In the case of startups D1 through D6 data was available on a per unit basis. Cumulative average man-hours per unit figures ($\bar{y}$) were computed for all startups. Since engineered estimates (y_u) were not available, the dependent variable $\mathcal{y}$ was calculated by dividing the cumulative average figures ($\bar{y}$) by the y value of the last observation in each startup.

The job order cost system used by Company D is similar to job order cost systems found in other situations.Briefly, when an order is placed on a computer unit, a ledger is set up for the particular order. The particular computer will have a unit number assigned to it and the various operations on the unit are assigned job numbers. The worker's time tickets are then posted to the particular job number and then to the ledger. The company charges the time of the inspection force to direct labor.

Firm E has a modified job order cost system of accumulating time data for fixed interval units in the total quantity produced. Thus, the first two lots consist of five units each. Thereafter time data is accumulated for lots of ten , and occasionally for a lot of twenty.

Data in this modified form for firms D and E is shown in Tables 6.8 through 6.10 (Appendix F).

Startup Analysis

The descriptive efficiency of the manufacturing progress model, given by equation (5.2) is supported by the results of regression analysis of the startups obtained from firms A,B,C,D and E. The detailed results are displayed in Table 6.11, startups A1 through E12 (Appendix F). In each case, the results were obtained by regressing the data relating to the startup phase against the log-transformed form of the proposed model using the least-squares criterion.

Clearly, by taking logarithms of both sides of equation (5.2), it follows that

$$\log y = \log a - \text{b} \log \text{x} \qquad (6.1)$$

All regressions were machine-computed (CDC 6500) using a FORTRAN program written by this author (Appendix G).

Table 6.11 shows the coefficients of correlation (r) and determination (r^2), the t-ratios, the a parameter values and the b parameter values obtained in each regression, as well as the number of observations (N) upon which the regressions are based, the Wright "slope" and the startup codes previously assigned.

The closeness of fit can be judged by the correlation coefficients (r) or the coefficients of determination (r^2). The coefficient of determination, being the square of the correlation coefficient, is more sensitive to any deviations from the regression line. Thus, it is a more revealing measure of the closeness of fit. Also, it indicates the proportion of the total variance of the dependent variable that is explained by the regression line.

The t-ratios given in the table measure the relative sizes of the regression coefficients (b) and the standard deviation of the regression coefficient. The t-ratio is obtained by dividing the calculated regression coefficient b by its own standard error.

Examination of the t-ratios and coefficients of determination (Table 6.11) pertaining to startups A1 through E12 will demonstrate the general efficiency of the fit provided by the regression model. The r^2 values vary from 0.590 to 1.000, with a median value of 0.960. Hence, in one-half of the cases,the startup model explains 96% or more of the total variance in the dependent variable. In only three out of forty cases does the regression fail to explain at least 80% of the variance. Also, in only six out of forty cases,the coefficients of determination were less than 0.910.

The t-ratios are also generally impressive, ranging from 2.08 to 61.2. (In the case of startup D7 the t-ratio was extremely large.) The median value is 10.1.

If one is willing to make the necessary assumptions of normality and common variance, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.99 level of significance in all cases except the following: D5, where the level is 0.975; A7, where the level is 0.95; A8 and D11, where the level is 0.90, and D8 where the level is 0.75.

Parameter Analysis

The parameter model given by equation (5.3) is supported by the results of regression analysis of the startup parameters obtained from firms A,D and E. Data from firms B and C is insufficient for a parameter analysis.

For firm A, the degree of correlation in the available data can be judged from Table 6.12, where the *a* and b values of the different process startups have been presented in order of decreasing *a* value for each product analyzed. Startups A5, A6 and A11 are not included because they represent aggregation of individual processes already taken into account in the table. Startup A12 is not included because it refers to an older model of card punch. This startup occurred some ten years before the startups that are considered in the table.

TABLE 6.12

FIRM A PARAMETERS a AND b

Startup Code	General Description	a	b
A3	Final assembly operations	2.707	0.311
A4	of the CPU of a	2.492	0.231
A1	third generation	1.898	0.173
A2	computer	1.722	0.157
A10	Testing, card punch X	4.876	0.205
A9	Final Assembly, cd punch X	3.253	0.132
A7	Parts Machining, cd punch X	1.721	0.0875
A8	Subassembly, cd punch X	1.516	0.0509

The strength of the relationships can be assessed from the regression results shown in Tables 6.13 and 6.14.

TABLE 6.13

PARAMETER MODEL REGRESSION RESULTS

(FIRM A, CPU OF A 3rd. GENERATION COMPUTER)

Number of Observations	m	n	r^2	t-ratio
4	-0.0177	0.305	0.889	4.01

TABLE 6.14

PARAMETER MODEL REGRESSION RESULTS

(FIRM A, CARD PUNCH X)

Number of Observations	m	n	r^2	t-ratio
4	0.0090	0.118	0.954	6.46

Both the coefficients of determination (r^2) and the t-ratios are sufficiently large to conclude that the parameter model given by equation (5.3) provides a good description of the data available from firm A.

If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.975 level of significance in the case of card punch X (Table 6.14) and near the 0.95 level in the case of the central processing unit (Table 6.13).

It can be concluded that the parameter model expressed by equation (5.3) was found empirically valid among processes of the same product, for card punch X and for the central processing unit of a third generation computer. The assembling of the central processing unit is a final "product" for firm A.

The available data was insufficient to check the empirical validity of the model among products of firm A.

As to firms D and E, their data may be used as an indirect means of supporting the credibility of the correlation between the *a* and b parameters. Since the engineered estimates (y_u) for each startup were not available, a difficulty arose as to the way of calculating the indices of productivity (y) as defined in (5.2). As previously mentioned, the indices of productivity in the case of firms D and E startups (Tables 6.8, 6.9 and 6.10) were calculated by dividing the cumulative average figures ($\overline{y}$) by the y value of the last observation in each startup. Intuitively, the last observation in each startup is closer to a possible steady-state phase (estimated by y_u) than any other previous observation in that startup, provided that the progress function is a strictly decreasing one. Thus, the a and b parameters pertaining to startups D1 through E12 (Table 6.11, Appendix) were obtained by regressing the data in this modified form against the log-transformed form of the proposed model (equation 6.1), using the least-squares criterion.

The degree of correlation in the available data can now be judged from Table 6.15 (Firm D) and Table 6.16 (Firm E), where the a and b values of the different product startups have been presented in order of decreasing a value.

The strength of the relationships can be assessed from the regression results shown in Tables 6.17 and 6.18.

Although the regression results are not quite as favorable as those obtained in the case of firm A, they still substantiate the possibility that the parameter model would have been empirically supported if the true estimates y_u were used in order to calculate the productivity indices y in the case of firms D and E.This possibility is strengthned if the data is adjusted according to the following rationale. Intuitively, if a generic startup X exhibits a smaller percentual decline in the labor hours per unit from the (x - 1)th observation to the xth observation (the xth observation being the last one available) than another startup Y, then startup X is probably closer to a steady-state phase than startup Y. From a practical point of view , the beginning of a plateau might be defined as that point in cumulative production at which the reduction in labor hours per unit from the (x-1)th

TABLE 6.15

FIRM D PARAMETERS *a* AND b

Startup Code	General Description	*a*	b
* D12	Computer no. 5A	7.002	0.406
D10	Computer no. 3A	6.945	0.409
D13	Computer no. 6A	6.683	0.409
* D7	Control Unit no. 1A	5.252	0.333
D11	Computer no. 4A	3.521	0.220
* D2	High Speed Printer no. 1	3.230	0.297
* D6	Magnetic Tape Unit no. 1	2.882	0.205
* D1	Control Unit no. 2	2.649	0.368
* D4	Card Reader Unit no. 1	2.483	0.275
D9	Computer no. 2A	1.625	0.0929
* D3	High Speed Printer no. 2	1.341	0.0936
D14	Computer no. 7A	1.333	0.0533
D5	Tape Perforation Unit no. 1	1.230	0.0543
D8	Computer no. 1A	1.156	0.0226

TABLE 6.16

FIRM E PARAMETERS *a* AND b

Startup Code	General Description	*a*	b
E9	Computer no. 7B	9.602	0.365
E1	Computer no. 2B	8.889	0.377
* E11	Control Unit no. 1B	5.948	0.211
* E6	Data Storage Unit no.1B	4.497	0.146
E7	Data Storage Unit no.2B	4.359	0.276
* E12	Computer no. 1B	4.349	0.238
E4	Computer no. 5B	4.034	0.235
* E5	Computer no. 6B	3.911	0.175
* E3	Computer no. 4B	3.548	0.174
* E10	Computer no. 8B	3.076	0.179
E8	Data Storage Unit no.3B	3.017	0.214
* E2	Computer no. 3B	2.471	0.142

TABLE 6.17

PARAMETER MODEL REGRESSION RESULTS

(FIRM D)

Number of Observations	m	n	r^2	t-ratio
14	0.0242	0.204	0.870	8.97

TABLE 6.18

PARAMETER MODEL REGRESSION RESULTS

(FIRM E)

Number of Observations	m	n	r^2	t-ratio
12	0.0312	0.161	0.687	4.69

observation to the xth observation (the xth observation being the last one) is some negligible figure. Recall that this procedure was adopted in a previously mentioned research by Schultz and Conway (Chapter II, p.39).

The startups with an asterisk in Tables 6.15 and 6.16 have the largest percentual decline in the labor hours per unit from the (x - 1)th to the xth observation (the xth observation being the last one). From the available data , startups D12 , D1 , D4 and D3 have a percentual decline greater than 3% in the labor hours per unit from the (x-1)th observation to the last observation x. If they are excluded from the regression computation, then the results obtained from the remaining startups in Table 6.15 are as shown in Table 6.19.

TABLE 6.19

PARAMETER MODEL REGRESSION RESULTS

(FIRM D, STARTUPS D12, D1 , D4 and D3 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
10	-0.0039	0.211	0.973	16.98

Now, if all startups with more than 1% of percentual decline are excluded from the computation (i.e., all startups with an asterisk in Table 6.15), then the regression results obtained from the remaining startups in Table 6.15 are as shown in Table 6.20.

TABLE 6.20

PARAMETER MODEL REGRESSION RESULTS

(FIRM D, STARRED STARTUPS IN TABLE 6.15 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
7	-0.0070	0.211	0.988	20.29

The increasingly improved regression results from Tables 6.17, 6.19 and 6.20 seem to support the conjecture that the parameter model would have been empirically validated among products of firm D if the true estimates y_u were available for calculating the productivity indices y in the case of firm D startups. The results from Table 6.20 show that if one is willing to make the assumptions of normality and common variance, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.995 level of significance.

Similarly, in the case of firm E, Tables 6.21 and 6.22 exhibit the regression results when the startups with more than 2% and 1% of percentual decline, respectively, are excluded from the regression computation. Again, the increasingly improved regression results from Tables 6.18, 6.21 and 6.22 seem to support the conjecture that the parameter model would have been empirically validated among products of firm E if the true estimates y_u were available for calculating the productivity indices y in the case of firm E startups. The results from Table 6.22 show that if one is willing to make the assumptions

TABLE 6.21

PARAMETER MODEL REGRESSION RESULTS
(FIRM E, STARTUPS E11 AND E10 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
10	-0.0204	0.171	0.760	5.03

TABLE 6.22

PARAMETER MODEL REGRESSION RESULTS
(FIRM E, STARRED STARTUPS IN TABLE 6.15 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
5	0.0530	0.143	0.967	9.35

of normality and common variance, the null hypotheses $\rho=0$ and $\beta=0$ can be rejected at the 0.995 level of significance.

STARTUP ANALYSIS IN THE MANUFACTURING OF BINDERY MACHINES AND PRINTING PRESSES

Characteristics of the Products and Manufacturing Processes Studied

The products studied at firm F comprise bindery machines and printing presses of several types. Technically , the focused products are electromechanical. The means and methods of manufacture are labor-intensive and feature a large component of assembly work. The manufacture of these products is broadly divided into machining and assembly work. The company initiates the production process only after orders are received. Normally, action on the first few purchase orders is delayed until enough additional orders are received to make up what is considered a sufficient number of units for the lot. Over 95% of the parts used in assembly are manufactured by the company. It may be added that a large percentage of the parts are bought in the form of unfinished castings and then machined to specification in the plant.

Table 6.23 summarizes the startups analyzed in firm F.

Startup Measurement and Data

In firm F,data is reported in lot-average form. Cumulative average man-hours per unit figures ($\overline{y}$) were computed for all startups. Since engineered estimates (y_u) were not

TABLE 6.23

STARTUPS ANALYZED IN FIRM F (U.S.)

Startup Code	Designation
F1 through F16	Binding machines (no.1 through no.16)
F17 through F28	Printing presses (no.1 through no.12)

available, the dependent variable *y* was calculated by dividing the cumulative average figures ($\overline{y}$) by the y value of the last observation in each startup. Cumulative output statistics (x) indicate the total output of the product (from inception of manufacture) that is achieved at the last unit of each lot.

The job order cost system used by Company F is similar to the cost system found in Company E or in Company D. For this reason no further description is necessary.Data obtained from this manufacturer represents both assembly and machining time.

Data in this modified form for firm F may be found in Table 6.24 (Appendix F).

Startup Analysis

The descriptive efficiency of the manufacturing progress model given by equation (5.2) is also supported by the results of regression analysis of the startups obtained from F. Examination of the t-ratios and coefficients of determination (Table 6.11) pertaining to startups F1 through F28 reveals that the r^2 values vary from 0.657 to 0.999, with a median value of 0.985. Hence, in one-half of the cases, the startup model explains 98.5% or more of the total variance in the dependent variable. In only two out of twenty-eight cases does the regression fail to explain at least 88.9% of the variance.

The t-ratios are also generally large, ranging from 3.05 to 81.05, with a median value of 16.3.

If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.99 level of significance in all cases except the following: F3 and F13, where the level is 0.975; F2, where the level is near 0.95 and F15, where the level is 0.90.

Parameter Analysis

As in the case of firms D and E, for which the engineered estimates (y_u) were not available, the data from firm F may still be used as an indirect means of supporting the credibility of the correlation between the $\underline{a}$ and $\underline{b}$ parameters.

The procedure adopted for firms D and E is played again . The initial degree of correlation in the available data from firm F can be judged from Table 6.25, where the *a* and b values of the different product startups have been arranged in order of decreasing *a* value.

TABLE 6.25

FIRM F PARAMETERS *a* AND b

Startup Code	Designation	*a*	b
F16	Binding machine no.16	4.94	0.246
F24	Printing press no. 8	4.74	0.228
F20	Printing press no. 4	4.47	0.235
F13	Binding machine no.13	3.85	0.271
F1	Binding machine no. 1	3.57	0.238
F17	Printing press no. 1	3.42	0.237
F15	Binding machine no.15	3.22	0.219
F22	Printing press no. 6	3.17	0.201
F28	Printing press no.12	3.16	0.227
F23	Printing press no. 7	2.80	0.133
F19	Printing press no. 3	2.71	0.186
F18	Printing press no. 2	2.70	0.176
F14	Binding machine no.14	2.59	0.172
F25	Printing press no. 9	2.41	0.120
F21	Printing press no. 5	2.02	0.123

TABLE 6.25 (cont'd)

Startup Code	Designation	a	b
F5	Binding machine no. 5	2.01	0.147
F6	Binding machine no. 6	1.99	0.150
F4	Binding machine no. 4	1.97	0.184
F27	Printing press no.11	1.95	0.131
F9	Binding machine no. 9	1.93	0.136
F3	Binding machine no. 3	1.83	0.110
F12	Binding machine no.12	1.80	0.148
F10	Binding machine no.10	1.80	0.128
F7	Binding machine no. 7	1.77	0.123
F26	Printing press no.10	1.57	0.0731
F2	Binding machine no. 2	1.49	0.0604
F11	Binding machine no.11	1.48	0.0758
F8	Binding machine no. 8	1.36	0.0551

The strength of the relationship can be assessed from the regression results shown in Table 6.26.

Tables 6.27 and 6.28 exhibit the regression results when the startups with more than 5% and 2% of ultimate percentual decline, respectively, are excluded from the regression computation.

TABLE 6.26

PARAMETER MODEL REGRESSION RESULTS

(FIRM F)

Number of Observations	m	n	r^2	t-ratio
28	0.0292	0.150	0.835	11.5

TABLE 6.27

PARAMETER MODEL REGRESSION RESULTS

(FIRM F, STARTUP F4 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
27	0.0244	0.153	0.865	12.6

TABLE 6.28

PARAMETER MODEL REGRESSION RESULTS

(FIRM F,STARTUPS F4 AND F5 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
26	0.0229	0.154	0.867	12.5

The improved regression results from Tables 6.27 and 6.28 seem to indicate that the parameter model would have been better supported among products of firm F if the true estimates y_u were available for calculating the productivity indices y in the case of firm F startups. Although the coefficient of determination (r^2) is not quite as favorable as those obtained in the case of firms D and E, it demonstrates that little of the variance in parameter b was left unexplained by the regression (See Table 6.28). The t-ratio also remains reliably large. The results from Table 6.28 show that if normality and common variance are assumed, then the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.995 level of significance.

Since the products studied at Firm F comprise bindery machines and printing presses it seemed worthwhile to check if the parameter model would be supported among products of the same kind. The initial degree of correlation can be assessed from Tables 6.29 (binding machines) and 6.30 (printing presses) where the a and b parameter values have been arranged in order of decreasing a value (Appendix F).

The strength of the relationships can be judged from the regression results exhibited in Tables 6.31 (binding machines) and 6.32 (printing presses).

Tables 6.33 and 6.34 exhibit the regression results for binding machines when the startups with more than 2% and 1% of ultimate percentual decline, respectively, are excluded

TABLE 6.31

PARAMETER MODEL REGRESSION RESULTS

(FIRM F, BINDING MACHINES)

Number of Observations	m	n	r^2	t-ratio
16	0.0270	0.162	0.884	10.3

TABLE 6.32

PARAMETER MODEL REGRESSION RESULTS

(FIRM F, PRINTING PRESSES)

Number of Observations	m	n	r^2	t-ratio
12	0.0160	0.153	0.809	6.50

TABLE 6.33

PARAMETER MODEL REGRESSION RESULTS

(FIRM F, BINDING MACHINES, F4 AND F5 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
14	0.0209	0.165	0.922	11.9

TABLE 6.34

PARAMETER MODEL REGRESSION RESULTS

(FIRM F, BINDING MACHINES; F4,F5,F3 AND F2 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
12	0.0304	0.158	0.924	11.0

from the regression computation.

Similarly, Tables 6.35 and 6.36 show the regression results for printing presses when startups with ultimate percentual decline greater than 0.5% and 0.3%, respectively, are excluded from the regression computation. Again, the results seem to indicate that the parameter model would have been even better supported among products of the <u>same</u> kind at firm F, had the true estimates y_u been available for calculating the productivity indices y in the case of the focused startups.

The regression results from Tables 6.34 (binding machines) and 6.36 (printing presses) show that if normality and common variance are assumed, then the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.995 level of significance in both cases.

TABLE 6.35

PARAMETER MODEL REGRESSION RESULTS

(FIRM F, PRINTING PRESSES , F28 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
11	0.0170	0.148	0.834	6.73

TABLE 6.36

PARAMETER MODEL REGRESSION RESULTS

(FIRM F, PRINTING PRESSES; F28,F21,F17 AND F22 EXCLUDED)

Number of Observations	m	n	r^2	t-ratio
8	0.0204	0.139	0.862	6.13

STARTUP ANALYSIS IN SHIPBUILDING

Characteristics of The Product and Manufacturing Processes Studied

The product studied at firm G is a ferry-boat of 1,250 dead-weight tons (DWT) of which five units were built. The means and methods of manufacture are labor-intensive and feature a large component of assembly work.

The production of these ships is divided into three main groups of operations called: (1) Services, (2) Hull and (3) Equipment.

The Services group includes the following operations: machining, transportation and carpentry , and accounts for approximately 11 per cent of the total manhours expended in the production of the ship.

The Hull group comprises the following operations: lofting, laying out, manual cutting, automatic cutting, bending, subassembly preparation, hull blocks subassembly, manual welding, automatic welding, chamfering, carbon chamfering, planing, and scaffolding. This group uses approximately 55 per cent of the total manhours expended in the production of the ship. The three underlined operations use 75 per cent of the labor hours expended in the group, or 41 per cent of the total manhours expended in the construction of the ship.

The Equipment group comprehends the following operations: piping (fabrication), piping (assembly), parts fabrication, deck equipment subassembly, machinery subassembly, electrical assembly, machinery assembly, carpentry, painting, and rigging. This group uses approximately 34 per cent of the total manhours expended in the production of the ship.

Table 6.37 summarizes the startups analyzed in Firm G.

Startup Measurement and Data

In firm G the data was available on a per unit (ship) basis. The reports issued by the Production Planning and Control Department of that firm exhibit the direct labor hours per unit expended in each startup from G1 through G19. Cumulative average manhours per unit ($\overline{y}$) were computed for all startups analyzed. Then, for each startup the cumulative average manhours per unit figues were divided by the respective engineered estimate y_u yielding the productivity index figures (y) of equation (5.2).

Cumulative output statistics indicate the total output of the product (ships) from inception of manufacture. The completion of each ship is not necessarily achieved at the end of the accounting period.

Startup data in this modified form is shown in Table 6.38 (Appendix F).

TABLE 6.37

STARTUPS ANALYZED IN

FIRM G (BRAZIL)

Startup Code	General Description
G1	Production of Complete Ferry-Boats
G2	Services Group of Operations
G3	Hull Group of Operations
G4	Equipment Group of Operations
G5	Carpentry (fabrication)
G6	Manual Cutting (Hull)
G7	Hull Blocks Subassembly
G8	Manual Welding
G9	Piping (fabrication)
G10	Piping (assembly)
G11	Parts Fabrication
G12	Deck Equipment Subassembly
G13	Machinery Subassembly
G14	Electrical Assembly
G15	Machinery Assembly
G16	Carpentry (assembly)
G17	Painting
G18	Rigging
G19	Manual Cutting (Equipment)

Startup Analysis

The descriptive efficiency of the manufacturing progress model given by equation (5.2) is generally supported by the results of the regression analysis of the startups obtained from firm G.

Examination of the coefficients of determination (Table 6.11) pertaining to startups G1 through G19 reveals that the r^2 values vary from 0.054 to 0.999, with a median value of 0.924. Hence, in one-half of the cases, the startup model explains 92.4% or more of the total variance in the dependent variable. In six out of nineteen cases the regression fails to explain at least 81.7% of the variance. However, in three of these six cases the available data cannot be considered reliable. According to the Chief-Engineer of the Production Planning and Control Department, the data pertaining to startups G13, G15 and G8 should not be considered reliable since the timekeepers were new in the job and were being trained for that purpose.

Unfortunately it was not possible to have the data corrected within the period of time that the author allocated for his research in the focused shipyard. Therefore, it seems advisable to exclude startups G13, G15 and G8 from the present analysis. In doing so, the new median value for r^2 will be 0.929. Now, in only three out of sixteen cases does the regression fail to explain at least 81.7% of the total variance in the dependent variable. The t-ratios are generally large, ranging from 2.39 to 53.08, with a median value of 6.27.

If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.99 level of significance in all cases except the following: G1 and G4, where the level is 0.975; G12 and G2, where the levels are 0.95 or near 0.95, respectively, and G7 and G10, where the level is 0.90.

Parameter Analysis

The parameter model given by equation (5.3) is supported by the results of regression analysis of the startup parameters obtained from firm G. The degree of correlation in the available data can be judged from Table 6.39, where the a and $\underline{b}$ values of the different process startups have been presented in order of decreasing a value for the only product analyzed. Startups G1, G2, G3 and G4 are not included because they represent aggregation of individual processes already taken into account in the table. Startups G13, G15 and G8 were excluded for reasons previously explained.

The strength of the relationship can be judged from the regression results shown in Table 6.40.

Both the coefficient of determination r^2 and the t-ratio are sufficiently large to conclude that the parameter model given by equation (5.3) provides a good description

TABLE 6.39

FIRM G PARAMETERS a AND b

Startup Code	General Description	a	b
G16	Carpentry (assembly)	2.109	0.2663
G14	Electrical Assembly	1.964	0.2517
G10	Piping (assembly)	1.955	0.1397
G9	Piping (fabrication)	1.448	0.1131
G11	Parts Fabrication	1.361	0.1007
G17	Painting	1.356	0.1058
G18	Rigging	1.293	0.0900
G6	Manual Cutting (Hull)	1.275	0.0878
G5	Carpentry (fabrication)	1.273	0.0727
G19	Manual Cutting (Equipment)	1.150	0.0509
G7	Hull Blocks Assembly	1.101	0.0303
G12	Deck Equipment Subassembly	1.022	0.0363

TABLE 6.40

PARAMETER MODEL REGRESSION RESULTS

(FIRM A, FERRY-BOAT OF 1250 DWT)

Number of Observations	m	n	r^2	t-ratio
12	0.0110	0.298	0.867	8.07

of the data from firm G. If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.995 level of significance.

It can be concluded that the parameter model expressed by equation (5.3) is empirically supported among processes of the same product, in the case of the particular type of ferry-boat analysed in firm G. Since this was the only product studied in that firm the empirical validity of the parameter model among its products cannot be established.

STARTUP ANALYSIS IN THE MANUFACTURING OF OIL DRILLING AND PRODUCTION EQUIPMENT

Characteristics of The Products and Manufacturing Processes Studied

The products studied at firm H may be generally described as tools and equipment for oil drilling and production.

The manufacturing of these products is broadly divided into: (1) Machining operations and (2) Assembly operations. The bulk of the machining work corresponds to the cutting of internal threads. The assembly operations are mainly performed by welding. Table 6.41 summarizes the startups analyzed in firm H.

TABLE 6.41

STARTUPS ANALYZED IN

FIRM H (BRAZIL)

Startup Code	Designation
H1	Packers
H2	Cement Retainers
H3	Shoes

Startup Measurement and Data

In firm H the data was available in lot-average form. Cumulative average manhours per unit figures ($\overline{y}$) were computed for all startups. Then, for each startup the cumulative average manhours per unit figures were divided by the respective engineered estimate y_u yielding the productivity index figures (y) of equation (5.2).

Cumulative output statistics indicate the total output of the product (from inception of manufacture) that is achieved at the end of the accounting period (month).

Startup data in this modified form is shown in Table 6.42 (Appendix F).

Startup Analysis

The descriptive efficiency of the manufacturing progress model given by equation (5.2) is generally supported by the results of the regression analysis of the startups available from firm H (Table 6.11). In all cases the startup model explains 84.5% or more of the total variance in the independent variable. If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.99 level of significance in two cases (H3 and H2), and at the 0.95 level in one case (H1).

Parameter Analysis

The small number of observations available is not sufficient for a reliable regression analysis of the startup parameters obtained from firm H.Notwithstanding,the existing data favors the parameter model given by equation (5.3).

The degree of correlation in the available data can be judged from Table 6.43, where the a and b parameter values of the different product startups have been arranged in order of decreasing *a* value.

TABLE 6.43

FIRM H PARAMETERS *a* AND b

Startup Code	General Description	*a*	b
H3	Shoes	3.132	0.2564
H2	Cement Retainers	2.520	0.1524
H1	Packers	1.450	0.0702

The strength of the relationship can be assessed from the regression results shown in Table 6.44.

TABLE 6.44

PARAMETER MODEL REGRESSION RESULTS

(FIRM H, OIL DRILLING AND PRODUCTION EQUIPMENT)

Number of Observations	m	n	r^2	t-ratio
3	-0.0220	0.2236	0.905	3.08

If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.80 level of significance.

STARTUP ANALYSIS IN THE AIRFRAME INDUSTRY

Characteristics of the Products and Manufacturing Processes Studied

The products studied at firm I comprise light airplanes of four types, to be designated by the letters W,X,Y and Z.

Airplane W is a two-seat plane fabricated with steel tubes, wood and fabric, and propelled by a 90 HP engine. It is intended for primary training purposes.

Airplane X is an executive four-seat plane, entirely fabricated with aluminum alloys and propelled by a 180 HP engine.

Airplane Y is the military version of airplane X. Finally, airplane Z is a two-or-three-seat 290 HP-engine plane, intended for intermediate training and acrobatics, and equipped for navigation instruction.

The means and methods of manufacture in the airframe industry are labor-intensive and feature a large component of assembly work. Airframe production can be roughly classified into eleven classes of operations, as follows: (1) Machining Operations, (2) Sheet metal work, (3) Processing, including painting, heat treating, etc., (4) Primary Assembly,

(5) Wing subassembly, (6) Fuselage subassembly, (7) Miscellaneous subassembly (e.g., control surfaces and electrical tubing), (8) Major wing assembly, (9) Major fuselage assembly, (10) Final Assembly, and (11) Ramp work and miscellaneous operations.

This classification of operations gives a good indication of the sequential nature of the production flow. The producing departments are interrelated; the output of one serves as the input of others. The assembly operations are almost totally labor-paced. The machining and fabrication operations are partially machine-paced, but owing to the discontinuous job-lot nature of much of this work, labor-pacing is still very important.

Table 6.45 summarizes the startups analyzed in firm I.

Startup Measurement and Data

In firm I the data was available both in lot-average and in cumulative-average form ($\overline{y}$). For each startup, the cumulative average manhours per unit figures were divided by the respective engineered estimate y_u, yielding the productivity index figures (y) of equation (5.2).

Cumulative output statistics indicate the total output of the product (airplanes) from inception of manufacture . Startup data in this modified form is shown in Table 6.46 (Startup I1 through I4).

TABLE 6.45

STARTUPS ANALYZED
IN FIRM I (BRAZIL)

Startup Code	General Description
I1	Total Manufacturing, airplane W
I2	Total Manufacturing, airplane X
I3	Total Manufacturing, airplane Y
I4	Major Assemblies , airplane Z
I5 through I69	Spare parts, airplanes W,X,Y and Z

TABLE 6.46

AIRFRAME INDUSTRY (BRAZIL)
FIRM I

Startup Code — x = cumulative production — y = productivity index

Startup Code											
I1	x...	26	52	83	116	146	164				
	y...	1.30	1.29	1.18	1.11	1.08	1.12				
I2	x...	5	15	25	35	45	55	60	65	75	80
	y...	1.45	1.43	1.40	1.35	1.29	1.21	1.18	1.16	1.13	1.11
I3	x...	5	10	15	20	25	30	34	39	40	
	y...	1.54	1.46	1.39	1.33	1.28	1.20	1.20	1.17	1.17	
I4	x...	2	5	9	14	20	28	36	44	52	
	y...	2.64	2.27	2.05	1.87	1.69	1.53	1.42	1.35	1.29	

In the case of startups I5 through I69, parameters a and b as well as the engineered estimates (y_u) for each spare part were already available in firm I. However, the startup data was not made available by that firm. The modified parameter α was calculated by dividing parameter a by the corresponding engineered estimate y_u. Thus, Table 6.47 (Appendix F) exhibits the parameters α and b for each spare part. The data is arranged in order of decreasing α value.

Startup Analysis

The descriptive efficiency of the manufacturing progress model given by equation (5.2) is supported by the results of the regression analysis of the startups available from firm I (Table 6.11). In all cases the startup model explains 78.5% or more of the total variance in the dependent variable. If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.995 level of significance in all cases.

Parameter Analysis

The parameter model given by equation (5.3) is supported by the results of regression analysis of the startup parameters obtained from firm I. The degree of correlation in the available sata can be judged from Tables 6.48 (Products) and 6.47 (Spare parts, Appendix F).

TABLE 6.48

FIRM I PARAMETERS *a* AND b

(PRODUCTS)

Startup Code	General Description	*a*	b
I4	Major Assemblies, airplane Z	3.240	0.226
I3	Manufacturing, airplane Y	1.994	0.142
I1	Manufacturing, airplane W	1.877	0.106
I2	Manufacturing, airplane X	1.833	0.103

The strength of the relationships can be judged from the regression results exhibited in Tables 6.49 (products) and 6.50 (spare parts).

TABLE 6.49

PARAMETER MODEL REGRESSION RESULTS

(FIRM I PRODUCTS, STARTUPS I1 THROUGH I4)

Number of Observations	m	n	r^2	t-ratio
4	-0.0179	0.209	0.965	7.43

TABLE 6.50

PARAMETER MODEL REGRESSION RESULTS)

(FIRM I, SPARE PARTS, STARTUPS I5 THROUGH I69)

Number of Observations	m	n	r^2	t-ratio
65	1.424×10^{-4}	0.217	1.000	5.22×10^{3}

Both the coefficients of determination r^2 and the t-ratios are sufficiently large to conclude that the parameter model given by equation (5.3) provides a good description of the data from firm I. If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.975 level of significance,in the case of products (Table 6.49) and at the 0.9995 level , in the case of spare parts (Table 6.50).

It can be concluded that the parameter model given by equation (5.3) is empirically supported among products of firm I and also among spare parts of products of firm I.

STARTUP ANALYSIS IN THE MECHANICAL AND ELECTRICAL INDUSTRY

An industry is an aggregate of components. It can be argued that if progress in components is widespread, it should be reflected in aggregate performance. W.B. Hirschmann has shown that a logarithmic plot of manhours per barrel versus cumulative barrels of crude oil refined in the United States since 1860 results in a fairly regular type of decline as such a reasoning would lead us to expect. Hirschmann has shown that other industries (e.g. electric power and basic steel industries) also exhibit similar declines.[1] It seems reasonable to infer that improvement curve patterns can exist in other areas as well.

In this section the data gathered from the Brazilian mechanical and electrical industry during its "take-off" period (1960-1964) is analyzed and discussed. The data is found to support the manufacturing progress model given by equations (5.2) and (5.3).

The Product Groups Studied

As mentioned in Chapter V, the product groups analyzed in the Brazilian mechanical and electrical industry are the following: (J1) Castings, (J2) Forgings, (J3) Mechanical Machinery, (J4) Electrical Machinery, (J5) Industrial Equipment, (J6) Locomotives, Wagons and Railway Equipment, (J7) Shipbuilding, (J8) Roadbuilding Tractors and Equipment, and

(J9) Farm Machinery.

According to the source document[2] the above classification was used as a basis for the determination of the input-output matrix of the mechanical and electrical industrial sector of the Brazilian economy. The products considered in each group have similar technological processing. A list of the most representative products in each group is given below.

(J1) Castings. Cast iron parts processed by manual molding and machine molding. Cast steel parts produced by manual molding and machine molding.

(J2) Forgings. Forged carbon and alloy steel parts produced by drop forging or press forging processes.

(J3) Mechanical Machinery. Machine-tools, cranes, centrifugal pumps and stonebreakers.

(J4) Electrical Machinery. Generators and alternators, transformers, DC and AC motors.

(J5) Industrial Equipment. Liquid gas tanks, coolers, and steam boilers.

(J6) Locomotives, Wagons and Railway Equipment. Vans, ore wagons, passenger-cars, electrical locomotives, diesel-hydraulic and diesel-electric locomotives.

(J7) Shipbuilding (powerplants excluded). Passenger-liners, cargo ships, freezers, tank-ships, tugs, motor-boats, barges, flatboats and ferry-boats.

(J8) Roadbuilding Tractors and Equipment. Bulldozers, caterpillar tractors, levellers, dump trucks, scrapers, asphalters, crushers and loaders.

(J9) Farm Machinery. Farm tractors (micro, light, medium and heavy), ploughs, sowing machines and harvesters.

Startup Measurement and Data

For each startup in industry J, the data was available in manhours per metric ton, for each year of the period (1960-1964). Also available was the number of metric tons produced per year for each group of products considered. Cumulative average manhours per metric ton figures ($\bar{y}$) were computed for all startups. Then, for each startup, the cumulative average manhours per unit figures were divided by the y value of the last observation in each startup (1964 figure) yielding the productivity index figures (y) of equation (5.2).

Cumulative output statistics indicate the aggregate output (in metric tons) of all products in each focused group from inception of manufacture (1960). Startup data in this modified form is shown in Table 6.51 (Appendix F).

Startup Analysis

The descriptive efficiency of the manufacturing progress model given by equation (5.2) is supported by the results of the regression analysis of the startups from industry J. Examination of the coefficients of determination (Table 6.11) pertaining to startups J1 through J9 reveals that the r^2 values vary from 0.882 to 0.992, with a median value of 0.979. Hence, in one-half of the cases, the startup model explains 97.9% or more of the total variance in the dependent variable. In only two out of nine cases does the regression fail to explain at least 90.4% of the variance.

The t-ratios are also generally large, ranging from 4.74 to 19.38, with a median value of 11.82.

If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.995 level of significance in all cases except the following : J7, J6 and J9, where the level is 0.975.

Parameter Analysis

Since the last observation in each startup corresponds to the beginning of a plateau (1964), the data from industry J can be used as a direct means of supporting the credibility of the correlation between a and $\underline{b}$ parameters. The degree of correlation in the available data can be judged from Table 6.52, where the a and $\underline{b}$ values of the different product group

startups have been arranged in order of decreasing *a* value.

TABLE 6.52

INDUSTRY J PARAMETERS *a* AND b

Startup Code	General Description	*a*	b
J8	Roadbuilding Tractors and Equip.	5.960	0.1598
J4	Electrical Machinery	3.708	0.0969
J7	Shipbuilding	3.293	0.0883
J9	Farm Machinery	3.219	0.0891
J5	Industrial Equipment	2.672	0.0732
J1	Castings	2.423	0.0655
J6	Locomotives,Wagons and Rwy.Equip.	2.058	0.0594
J3	Mechanical Machinery	1.968	0.0498
J2	Forgings	1.831	0.0457

The strength of the relationship can be judged from the regression results exhibited in Table 6.53.

Both the coefficient of determination r^2 and the t-ratio are sufficiently large to conclude that the parameter model given by equation (5.3) provides a good description of the data from industry J. If the assumptions of normality and common variance are made, the null hypotheses $\rho=0$ and $\beta=0$

TABLE 6.53

PARAMETER MODEL REGRESSION RESULTS

(INDUSTRY J, STARTUPS J1 THROUGH J9)

Number of Observations	m	n	r^2	t-ratio
9	-0.0133	0.0909	0.965	9.07

can be rejected at the 0.995 level of significance.

It can be concluded that the parameter model given by equation (5.3) is empirically supported among groups of products of similar technology within the mechanical and electrical industrial sector of the Brazilian economy in the period 1960-1964.

SUMMARY

In the foregoing chapter the manufacturing progress model presented in chapter V was tested with real data from nine manufacturers representing five different industries. A hundred and fifty-nine separate cases of product and process startups that occurred in four different countries and nine distinct plants have been analyzed. In addition , aggregate data was obtained for whole industries in one country, yielding nine more startups.

The major points that have emerged from the discussion can be recapitulated as follows.

The descriptive efficiency of the manufacturing progress model given by equation (5.2) is generally supported by the results of regression analysis of the startups obtained from firms A,B,C,D,E,F,G,H,I, and industry J. Considering all startups, the coefficients of determination r^2 vary from 0.590 to 1.000, with a median value of 0.9685. Hence, in one-half of the cases, the startup model explains 96.85% or more of the total variance in the dependent variable. In only ten per cent of the cases does the regression fail to explain at least 81.7% of the total variance in the dependent variable.

The t-ratios are also generally impressive, ranging from 2.08 to 99.99, with a median value of 10.89. If the assumptions of normality and common variance are made, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.95 level of significance in 95% of the startups analyzed.

The descriptive efficiency of the parameter model given by equation (5.3) is generally supported by the results of regression analysis of the startup parameters from firms A,D,E,F,G,H,I and industry J. Data available from firms B and C was insufficient for a parameter regression analysis. The parameter model is empirically supported among processes of the same product (firms A and G), among products within the same plant (firms D,E,F,H and I), and among groups of products of similar technology within the same industrial sector of a foreign economy (industry J).

The findings of this research together with similar results obtained by Asher in the airframe industry and by Baloff in the steel, glass manufacturing, and automobile industries[3] constitute adequate evidence to suggest that the parameter model can be developed into an effective means of predicting the parameter <u>b</u> of a new startup. However, additional investigation and validation is a mandatory requirement for successful industrial application of the model.

One aspect of the model that requires attention is the stability of the coefficients <u>m</u> and <u>n</u>. In Appendix D this subject is explored by using the available data.

CHAPTER VII

IMPLICATIONS OF THE FINDINGS

The present study is concluded with a discussion of the industrial implications of the findings reported in Chapter VI. The importance of recognizing and predicting the manufacturing progress phenomenon is now related to several decision-making functions that are encountered in an industrial concern as well as in economic planning at the national level. An overall design of a computerized Manufacturing Progress Function (MPF) System is also suggested.

IMPLICATIONS AT FIRM LEVEL

In Chapter II the term "Manufacturing Progress Function Hours" (MPF Hrs) was defined as those direct-labor hours over and above the estimated hours which are caused by the introduction of a new unit into a manufacturing system. The term "Manufacturing Progress Function Cost" (MPF Cost) was defined as the cost associated with the MPF hours. In Chapter IV some methods for calculating the MPF hours were proposed and assessed.

Knowledge of the MPF hours and cost may be of extreme importance to management, first, in deciding whether or not to put a proposed product or a major change in an existing product into production, and secondly, in pricing a product to recoup these introductory costs.

The Manufacturing Progress Function is applicable not only to complete machines, but to smaller units or features as well. Even manufacturers who produce new models each year seldom produce units which are completely new. The MPF can then be applied to major changes in existing products to determine their effect on the cost. Many of the proposed changes are of a cost reduction nature. But not every proposed change can result in lower costs. It may be advisable to submit all major changes to an MPF study. If the MPF Cost exceeds a given percentage of the anticipated savings for a predetermined period (say, one or two years), the change might well be returned to Product Engineering for reanalysis. Also, if more than one choice exists for Engineering Changes, the Manufacturing Progress Function provides additional cost information for the proper selection.

Once it has been decided to produce a new machine other questions must be answered. How many assembly personnel will be required each month while production is building up? How much floor space will be required and when? The Manufacturing Progress Function provides answers to these questions. In addition, the knowledge of the MPF Cost - - prior to and during production - - makes possible the channeling of special engineering and staff efforts to those programs or activities of a program which show high MPF Costs. By doing this, it is possible to effect a more rapid rate of progress and reduce the MPF Cost through engineering changes or methods changes.

In the following subsections the practical use of the Manufacturing Progress Function at firm level is illustrated with a sample problem.

Problem Statement

The management of firm X wishes to know the MPF Hours and Cost associated with the implementation of a certain product P so as to decide whether or not to put product P into production and to price it in order to recoup the intro ductory MPF Cost.

The manufacturing of product P comprehends the following operations: (1) Machining, (2) Subassembly, (3) Mechanical Assembly, (4) Electrical Assembly and (5) Testing.

The following data is also known:

TABLE 7.1

MANUFACTURING PROGRESS FUNCTION AND COST DATA

Operation	y_u (hrs/unit)	T_u (months)	r (monetary units per hour)
Machining	30	12	10
Subassembly	10	13	9
Mechanical Assy.	6	20	6
Electrical Assy.	20	18	10
Testing	4	24	12

TABLE 7.2

PRODUCTION SCHEDULE

(PRODUCT P)

Month	Year 1	Cum.	Year 2	Cum.
JAN	10	10	40	280
FEB	10	20	40	320
MAR	10	30	40	360
APR	10	40	40	400
MAY	10	50	40	440
JUN	10	60	70	510
JUL	20	80	80	590
AUG	20	100	80	670
SEP	20	120	80	750
OCT	40	160	80	830
NOV	40	200	80	910
DEC	40	240	80	990

In Table 7.1, T_u is the estimated period of time (in months) elapsed from the inception of production until the ultimate unit of production x_u is reached. Recall that x_u can be practically defined as that point in cumulative production at which the reduction in manufacturing hours per unit from the first unit in the month to the last unit for the month is between 2% and 3% (or some other negligible figure) and thus can be considered nominal. The y_u values correspond to the estimated direct-labor hours required to produce a unit at the ultimate unit of production. The r values are the labor and burden hourly rates for each type of manufacturing operation. The production schedule of product P appears in Table 7.2 .

From past research it is anticipated that parameters b and a of the operation startups for product P will be related as follows:

$$b = 0.0410 + 0.132 \ln a \qquad (7.1)$$

It is also known that the best fit to the empirical data was achieved with the unit progress function.

Management also wishes to know how many assembly personnel and how much floor space will be required each month while production is building up, considering a time horizon of 12 months.

Solution

(a) Determination of The Ultimate Unit of Production For Each Operation.

Once T_u is estimated for each category of operation from historical data, the value of x_u is obtained from the production schedule by searching for the unit in cumulative production that corresponds to the estimated T_u. For example, for the machining operation, T_u = 12 months (see Table 7.1).

From the production schedule (Table 7.2) the unit in cumulative production that corresponds to T_u = 12 months is the 240th unit. Thus, x_u = 240. The other x_u values for each operation can be determined in a similar way. The results are exhibited in Table 7.3, together with the corresponding y_u values, for better visualization.

TABLE 7.3

ULTIMATE HOURS AND UNITS

(PRODUCT P)

Operation	x_u	y_u
Machining	240	30
Subassembly	280	10
Mechanical Assy.	670	6
Electrical Assy.	510	20
Testing	990	4

(b) Calculation of Parameter b Values.

From empirical equation (7.1) it follows that m=0.0410 and n=0.132. Since the best fit to the empirical data was achieved with the unit progress function, parameter b values can be calculated through formula (5.4), as follows:

Machining $$b = \frac{m}{1 - n \ln x_u} = \frac{0.0410}{1 - 0.132 \ln 240} = 0.148$$

Subassembly $b = \dfrac{0.0410}{1 - 0.132 \ln 280} = 0.160$

Mech. Assy. $b = \dfrac{0.0410}{1 - 0.132 \ln 670} = 0.291$

Elect.Assy. $b = \dfrac{0.0410}{1 - 0.132 \ln 510} = 0.232$

Testing $b = \dfrac{0.0410}{1 - 0.132 \ln 990} = 0.458$

(c) Calculation of Parameter a Values

The parameter a value of each manufacturing operation can be calculated through formula (5.6) as exhibited in Table 7.4 .

TABLE 7.4

PARAMETER a VALUES CALCULATED
ACCORDING TO FORMULA (5.6)

Operation	x_u	y_u	b	a
Machining	240	30	0.148	67.51
Subassembly	280	10	0.160	24.63
Mech.Assy.	670	6	0.291	39.86
Elect. Assy.	510	20	0.232	84.95
Testing	990	4	0.458	94.20

For example, in the case of the machining operation, formula (5.6) yields:

$$a = y_u \, x_u^b = 30 \times 240^{0.148} = 67.51$$

(d) MPF Hours and Cost Calculation

Exact Method

The MPF Hours and Cost associated with the implementation of product P may be calculated as exhibited in Table 7.5 . In column (1) and for each operation, the exact total hours expended in the manufacturing of unit 1 up to and including unit x_u were machine-computed[1] through formula (4.2), repeated below for easy reference:

$$y_T = a \sum_{1}^{x_u} x^{-b} \qquad (7.2)$$

For example, in the case of the machining operation, the total hours expended in the manufacturing of unit 1 up to and including unit 240 is given by:

$$y_T\ (240) = 67.51 \sum_{x=1}^{240} x^{-0.148} = 8{,}420.5498$$

In column (2), the rectangular areas $x_u \, y_u$ are also calculated (see Figure 4, Chapter IV). In column (3), the MPF Hours are computed by subtracting the values in column (2) from the corresponding values in column (1). Finally, the

MPF Cost figures in column (5) are calculated by multiplying the MPF Hours in column (3) by the corresponding labor and burden rates given in column (4). The totals for the product are presented at the bottom of the table.

TABLE 7.5

MPF HOURS AND COST CALCULATION
(EXACT METHOD)

	(1)	(2)	(3)	(4)	(5)
Operation	y_T (x_u)	$x_u y_u$	MPF Hrs.	r	MPF Cost
Machining	8,420.5498	7,200	1,220.5498	10	12,205.50
Subassembly	3,321.0222	2,800	521.0222	9	4,689.20
Mech. Assy.	5,637.5737	4,020	1,617.5737	6	9,705.44
Elect.Assy.	13,223.9939	10,200	3,023.9939	10	30,239.94
Testing	7,184.8316	3,960	3,224.8316	12	38,697.98
Totals	37,787.9712	28,180	9,607.9712		95,538.06

Approximate Method

The MPF Hours and Cost associated with the implementation of product P may be calculated as exhibited in Table 7.6. For each operation, the MPF Hours in column (4) are computed according to formula (4.18), repeated below:

$$\text{MPF Hours} \cong x_u\ y_u \left[\frac{(x_u + 0.5)^{1-b} - 0.5^{1-b}}{(1 - b)\ x_u^{1-b}} - 1 \right] \qquad (7.3)$$

TABLE 7.6

MPF HOURS AND COST CALCULATION

(APPROXIMATE METHOD)

	(1)	(2)	(3)	(4)	(5)	(6)
Operation	x_u	y_u	b	MPF Hrs.	r	MPF Cost
Machining	240	30	0.148	1,221.80	10	12,218.00
Subassembly	280	10	0.160	521.95	9	4,697.55
Mech. Assy.	670	6	0.291	1,618.56	6	9,711.36
Elect. Assy.	510	20	0.232	3,026.29	10	30,262.90
Testing	990	4	0.458	3,228.90	12	38,746.80
	Totals............			9,617.50		95,636.61

For example, in the case of the Subassembly operation, it follows that

$$\text{MPF Hours} = 280 \times 10 \left(\frac{280.5^{0.840} - 0.5^{0.840}}{0.840 \times 280^{0.840}} - 1\right)$$

$$= 521.95$$

The MPF Cost figures (column 6) are calculated by multiplying the MPF Hours (column 4) by the corresponding labor and burden rates given in column (5). The totals for the product are presented at the bottom of the table. Note that formulas (4.20) or (4.22) might have been used instead for calculating the MPF Hours per operation.[2] In any case the approximate results exhibited in Table 7.6 compare well with those from Table 7.5.

(e) Total Labor Requirements

<u>Exact Method</u>

The total labor hours associated with the implementation of product P, considering a planning horizon of 24 months, may be calculated as shown in Table 7.7. For each operation, the total labor hours were machine-computed through exact formula (7.2). Total labor hours for product P are presented at the bottom of the table.

<u>Approximate Method</u>

The total labor hours associated with the introduction of product P may be advantageously calculated by employing aggregation formulas (4.43) and (4.44) derived in chapter IV. These formulas are particularly useful when the number of products and/or the number of operations in the manufacturing of a given product are large. The calculations are carried out as follows.

TABLE 7.7

TOTAL LABOR HOURS FOR PRODUCT P

(EXACT METHOD)

Operation	Calculation
Machining	$y_{T_1} = 67.51 \sum_{1}^{990} x^{-0.148} = 28,229.92690$
Subassy	$y_{T_2} = 24.63 \sum_{1}^{990} x^{-0.160} = 9,615.009714$
Mech.Assy	$y_{T_3} = 39.86 \sum_{1}^{990} x^{-0.291} = 7,445.505004$
Elect.Assy	$y_{T_4} = 84.95 \sum_{1}^{990} x^{-0.232} = 22,044.95553$
Testing	$y_{T_5} = 94.20 \sum_{1}^{990} x^{-0.458} = 7,184.831639$
	Total ...74,520.22878

(1) Aggregate Parameter Calculation

From (4.33), one must have

$$0 < x \leq 2c$$

Also, from the data

$$x \in [1,\ 990]$$

Therefore, by taking the mid-point of the interval:

$$c = \frac{x}{2} = \frac{990}{2} = 495$$

By using parameter a and b values from Table 7.4 and applying formulas (4.43) and (4.44) with c = 495, the following results are obtained:

$$B = \left(\frac{67.51 \times 0.148}{495^{0.148}} + \frac{24.63 \times 0.160}{495^{0.160}} + \frac{39.86 \times 0.291}{495^{0.291}} + \frac{84.95 \times 0.232}{495^{0.232}} + \frac{94.20 \times 0.458}{495^{0.458}}\right) \div \left(\frac{67.51}{495^{0.148}} + \frac{24.63}{495^{0.160}} + \frac{39.86}{495^{0.291}} + \frac{84.95}{495^{0.232}} + \frac{94.20}{495^{0.458}}\right) = \frac{14.5442}{68.2623} = 0.2131$$

and

$$A = c^{B}\ D = 495^{0.2131} \times 68.2623 = 256.0951\ ,$$ where

D is the denominator of B.

(2) Aggregate Progress Function for Product P

In view of the above results the aggregate progress function for P in the given planning period (24 months) can be written as follows:

$$y = 256.1\ x^{-0.2131} \qquad (7.4)$$

(3) Total Labor Hours Calculation

The total labor requirement associated with the introduction of product P into the manufacturing system can now be determined by using formula (4.4) as follows:

$$y_T = \frac{A}{1-B}\ x_u^{1-B} = \frac{256.1}{1-0.2131} \times 990^{1-0.2131} = 74,089.28$$

The per cent deviation with respect to the exact result given in Table 7.7 is -0.58%.

(f) Labor and Space Requirements On
A Periodic Basis

Exact Method

As an example, the labor hours and space required by the Final Assembly (Mechanical and Electrical) and Testing operations can be determined for year 1, on a monthly basis, as exhibited in Tables 7.8, 7.9, and 7.10. In each table (column 3), the total labor hours y_T from inception of production up to and including the last unit of the ith monthly lot of production are machine-calculated according to formula (7.2). For example, the total hours expended in the Mechanical Assembly of product P from the beginning of production up to and

TABLE 7.8

LABOR AND SPACE REQUIREMENTS

(MECH. ASSY, EXACT METHOD)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Month	Cum. Prod.	y_T	y_L	n	(n')	S (m^2)	(S') (m^2)
Jan	10	262.4548	262.4548	1	1.5	6	9.0
Feb	20	443.1955	180.7407	1	1.0	6	6.0
Mar	30	598.8934	155.6979	1	0.89	6	5.3
Apr	40	740.1337	141.2403	1	0.80	6	4.8
May	50	871.4692	131.3355	1	0.74	6	4.4
Jun	60	995.3966	123.9274	1	0.70	6	4.2
Jul	80	1,226.7595	231.3629	1	1.3	6	7.8
Aug	100	1,441.7975	215.0380	1	1.2	6	7.2
Sep	120	1,644.6459	202.8484	1	1.2	6	7.2
Oct	160	2,023.2437	378.5978	2	2.2	12	13.2
Nov	200	2,375.0450	351.8013	2	2.0	12	12.0
Dec	240	2,706.8551	331.8101	2	1.9	12	11.4
		Totals....	2,706.8551	15	(15.4)	90	(92.5)

including the last unit of the "lot" produced in March (lot 3) are given by:

$$y_{T_3} = 39.86 \sum_{x=1}^{30} x^{-0.291} = 598.8934$$

TABLE 7.9

LABOR AND SPACE REQUIREMENTS

(ELECT.ASSY, EXACT METHOD)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Month	Cum. Prod.	y_T	y_T	n	(n')	S (m^2)	(S') (m^2)
JAN	10	606.5843	606.5843	3	3.4	15	17
FEB	20	1,058.6636	452.0793	3	2.6	15	13
MAR	30	1,460.1249	401.4613	2	2.3	10	11.5
APR	40	1,831.5912	371.4663	2	2.1	10	10.5
MAY	50	2,182.1436	350.5524	2	2.0	10	10.0
JUN	60	2,516.8424	334.6988	2	1.9	10	9.5
JUL	80	3,150.4663	633.6239	4	3.6	20	18.0
AUG	100	3,748.1947	597.7284	3	3.4	15	17.0
SEP	120	4,318.7567	570.5620	3	3.2	15	16.0
OCT	160	5,398.6586	1,079.9019	6	6.1	30	30.5
NOV	200	6,417.1926	1,018.5340	6	5.8	30	29.0
DEC	240	7,389.3200	972.1274	6	5.5	30	27.5
		Totals...	7,389.3200	42	(41.9)	210	(209.5)

TABLE 7.10

LABOR AND SPACE REQUIREMENTS

(TESTING , EXACT METHOD)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Month	Cum. Prod.	y_T	y_L	n	(n')	S (m^2)	(S') (m^2)
JAN	10	498.4215	498.4215	3	2.8	18	16.8
FEB	20	770.1003	271.6788	2	1.6	12	9.6
MAR	30	984.7455	214.6452	1	1.2	6	7.2
APR	40	1,168.8237	184.0782	1	1.1	6	6.6
MAY	50	1,332.9817	164.1580	1	0.93	6	5.6
JUN	60	1,482.7963	149.8146	1	0.85	6	5.1
JUL	80	1,751.6943	268.8980	2	1.5	12	9.0
AUG	100	1,991.3166	239.6223	1	1.4	6	8.4
SEP	120	2,209.8999	218.5833	1	1.2	6	7.2
OCT	160	2,602.0582	392.1583	2	2.2	12	13.2
NOV	200	2,951.3915	349.3333	2	2.0	12	12.0
DEC	240	3,269.9788	318.5873	2	1.8	12	10.8
		Totals...	3,269.9788	19	(18.6)	114	(111.5)

In column (4) of each focused table the labor hours y_{L_i} expended in the production of the ith lot (i.e., the lot produced in the ith month) are computed as follows:

$$y_{L_i} = y_{T_i} - y_{T_{(i-1)}}$$

For example, the labor hours expended in Mechanical Assembly of the lot produced in March are calculated as below:

$$y_{L_3} = 598.8934 - 443.1955 = 155.6979$$

In column (5) of each table considered, the number of personnel required on a monthly basis is computed assuming that a person works 176 hours per month (one shift). Thus, column (5) is calculated by dividing column (4) figures by 176 and rounding the results to the nearest integer. For example, the Mechanical Assembly operation of product P during March would require:

$$n_3 = \frac{156}{176} = 0.89 \cong 1 \quad \text{operator}$$

As another example, the Electrical Assembly operation in March would require

$$n_3 = \frac{401}{176} = 2.3 \cong 2 \quad \text{operators}$$

Column (6) in each table contains non-rounded figures for <u>n</u>.

In column (7) of each focused table, the number of square meters required on a monthly basis is computed assuming the following coefficients of working area per person: (i) Mechanical Assembly: 6 m^2 per person; (ii) Electrical Assembly: 5 m^2 per person and (iii) Testing: 6 m^2 per person. For example, the area required for the mechanical assembling of product P during March is given by:

$$S_3 = 1 \times 6 = 6 \ m^2$$

The same coefficients were applied to the non-rounded figures of column (6) so as to obtain column (8) results in each table considered.

The labor hours and space required by the Final Assembly and Testing operations are summarized in Table 7.11. The lot figures in column (3) were obtained by totalling the corresponding lot figures in Tables 7.8, 7.9 and 7.10. For example, the total hours expended in the Final Assembly and Testing operations of product P during March are computed as follows:

$$y_{L_3} = 155.6979 + 401.4613 + 214.6452 = 771.8044$$

The total number of personnel required on a monthly basis (column 4) is obtained by summing up the corresponding figures in Tables 7.8, 7.9 and 7.10. Thus, the total number of personnel required for assembling and testing product P

TABLE 7.11

LABOR AND SPACE REQUIREMENTS

(FINAL ASSY.AND TESTING, EXACT METHOD)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Month	Cum. Prod.	y_L	n	(n')	S (m^2)	(S') (m^2)
JAN	10	1,367.4606	7	7.7	39	42.8
FEB	20	904.4988	6	5.2	33	28.6
MAR	30	771.8044	4	4.4	22	24.0
APR	40	696.7848	4	4.0	22	21.9
MAY	50	646.0459	4	3.7	22	20.0
JUN	60	608.4408	4	3.5	22	18.8
JUL	80	1,133.8848	7	6.4	38	34.8
AUG	100	1,052.3887	5	6.0	27	32.6
SEP	120	991.9937	5	5.6	27	30.4
OCT	160	1,850.6580	10	10.5	54	56.9
NOV	200	1,719.6686	10	9.8	54	53.0
DEC	240	1,622.5248	10	9.2	54	49.7
	Totals...	13,366.1539	76	(76.0)	414	(413.5)

during March is

$$n_3 = 1 + 2 + 1 = 4$$

Similarly, column (6) figures were obtained by summing up the corresponding figures in Tables 7.8, 7.9 and 7.10. For example, the total area required for assembling and testing product P during March is given by:

$$S_3 = 6 + 10 + 6 = 22 \text{ m}^2$$

Columns (5) and (7) of Table 7.11 reflect the results obtained when the calculations are carried out using the non-rounded columns of Tables 7.8, 7.9 and 7.10.

Approximate Method

Column (3) of Tables 7.8, 7.9 and 7.10 might have been calculated through approximate formulas (4.3), (4.4) or (4.13). If formula (4.3) is used, the labor and space requirements on a monthly basis can be quickly determined. For example , Table 7.12 was computed for the case of the Testing operation. The results compare well with those from Table 7.10.

(g) Aggregate Labor and Space Requirements on a Periodic Basis

If management is just interested in knowing the total labor hour and space requirements of the Final Assembly and

TABLE 7.12

LABOR AND SPACE REQUIREMENTS

(TESTING , APPROXIMATE METHOD)

(1) Month	(2) Cum. Prod.	(3) y_T	(4) y_L	(5) n	(6) S (m^2)
JAN	10	605.4	605.4	3	18
FEB	20	881.5	276.1	2	12
MAR	30	1,098.1	216.6	1	6
APR	40	1,283.4	185.3	1	6
MAY	50	1,448.4	165.0	1	6
JUN	60	1,598.9	150.5	1	6
JUL	80	1,868.6	269.7	2	12
AUG	100	2,108.9	240.3	1	6
SEP	120	2,327.9	219.0	1	6
OCT	160	2,720.7	392.8	2	12
NOV	200	3,070.5	349.8	2	12
DEC	240	3,389.4	318.9	2	12
		Totals	3,389.4	19	114

Testing operations for aggregate planning purposes, the procedure developed in (f) or (g) can be shortened by establishing an aggregate progress function of the three focused operations in the time interval chosen . The following steps are carried out considering year 1 data:

(1) Aggregate Parameter Calculation

From (4.33), one must have

$$0 < x \leq 2c$$

Also, from the data

$$x \varepsilon [1,\ 240]$$

Thus, by taking the mid-point of the interval:

$$c = \frac{x}{2} = \frac{240}{2} = 120$$

By using parameter a and b values from Table 7.4 and applying formulas (4.43) and (4.44) with c = 120, the following results are obtained:

$$B = \left(\frac{39.86 \times 0.291}{120^{0.291}} + \frac{84.95 \times 0.232}{120^{0.232}} + \frac{94.20 \times 0.458}{120^{0.458}}\right) \div$$

$$\left(\frac{39.86}{120^{0.291}} + \frac{84.95}{120^{0.232}} + \frac{94.20}{120^{0.458}}\right) = \frac{14.1861}{48.3877} = 0.2932$$

and

$$A = c^{B}\ D = 120^{0.2932} \times 48.3877 = 196.9477,$$ where

D is the denominator of B.

(2) Aggregate Progress Function for Final Assembly and Testing Operations

The aggregate progress function for the Mechanical and Electrical Assembly and Testing of product P can be written as:

$$y = 196.9\ x^{-0.2932} \qquad (7.5)$$

(3) Aggregate Labor and Space Requirements

The aggregate labor and space requirements for the Final Assembly and Testing operations can now be calculated on a monthly basis as exhibited in Table 7.13.

The total labor hours y_T in column (3) are calculated through approximate formula (4.4). For example, the total hours expended in the production of unit 1 up to and including unit 40 are given by:

$$y_T = \frac{A}{1-B}\ x^{1-B} = \frac{196.9}{1-0.2932} \times 40^{1-0.2932} = 3{,}778.2$$

The total labor hours y_{L_i} for each monthly lot <u>i</u> (column 4) are calculated as follows:

$$y_{L_i} = y_{T_i} - y_{T_{(i-1)}}$$

For example, the total labor hours for the lot produced during April is given by:

$$y_{L_4} = 3{,}778.2 - 3{,}083.0 = 695.2$$

TABLE 7.13

AGGREGATE LABOR AND SPACE REQUIREMENTS

(FINAL ASSY AND TESTING, AGGREGATE PROGRESS FUNCTION METHOD)

(1) Months	(2) Cum. Prod.	(3) y_T	(4) y_L	(5) %	(6) n'	(7) S' (m^2)
JAN	10	1,418.2	1,418.2	+3.7	8.0	46
FEB	20	2,314.8	896.6	-0.87	5.1	29
MAR	30	3,083.0	768.2	-0.47	4.4	25
APR	40	3,778.2	695.2	-0.23	4.0	23
MAY	50	4,423.7	645.5	-0.08	3.7	21
JUN	60	5,032.1	608.4	-0.00	3.5	20
JUL	80	6,166.7	1,134.6	0.06	6.5	37
AUG	100	7,220.2	1,053.5	0.10	6.0	34
SEP	120	8,213.2	993.0	0.10	5.6	32
OCT	160	10,065	1,851.9	0.06	10.5	60
NOV	200	11,785	1,719.5	-0.01	9.8	56
DEC	240	13,405	1,620.8	-0.10	9.2	52
		Totals ...	13,405.4		76.3	435

The per cent deviations of the results in column (4) with respect to the corresponding results in column (3) of Table 7.11 are calculated for comparison purposes (see column 5). Finally, the number of personnel required (column 6) and the necessary space (column 7) are calculated on a monthly basis as was done for Tables 7.8, 7.9 and 7.10. It is assumed an average coefficient of 5.7 m^2 per person. The results were not rounded to the nearest integer so that they can be compared with those exhibited in Table 7.11. It can be seen that the aggregate progress function method is quite effective and expeditious in determining the aggregate labor and space requirements for a given manufacturing program.

MULTINATIONALS AND THE COST OF LEARNING

Knowledge of the progress functions of similar products in plants of the same multinational enterprise which are located in different countries can be relevant for decision-making purposes at parent company level.

As an example, suppose that the startups of the same product P in plant B (country B) and plant C (country C) are characterized by the following estimates of ultimate points, respectively:

$$(x_u, y_u)_B = (910, 8.0)$$

and

$$(x_u, y_u)_C = (1910, 16.0)$$

Parameter b values are also estimated from past research:

$$b_B = 0.174 \quad \text{and} \quad b_C = 0.161$$

The corresponding MPF Hours and Total Hours up to unit 1910 are calculated as follows:

<u>Product P in Plant B</u>

$$(\text{MPF Hours})_B = x_u\, y_u \left[\frac{(x_u + 0.5)^{1-b} - 0.5^{1-b}}{(1 - b)\, x_u^{1-b}} - 1 \right]$$

$$= 910 \times 8.0 \left(\frac{910.5^{0.826} - 0.5^{0.826}}{0.826 \times 910^{0.826}} - 1 \right) \cong 1520$$

$$y_{T_B} \simeq (\text{MPF Hours})_B + x_u y_u = 1520 + 1910 \times 8 = 16,800 \text{ (hours)}$$

<u>Product P in Plant C</u>

$$(\text{MPF Hours})_C = 1910 \times 16.0 \left(\frac{1910.5^{0.839} - 0.5^{0.839}}{0.839 \times 1910^{0.839}} - 1 \right) \cong 5,836$$

$$y_{T_C} \cong (\text{MPF Hours})_C + x_u y_u = 5,836 + 1910 \times 16.0$$

$$= 36,396 \quad \text{(hours)}$$

Ceteris paribus, it would cost less to launch product P in plant B than in plant C of the same multinational company.

A similar rationale is valid for operations within the manufacturing cycle of the same product. For example, due to plant advantage in implementation costs, plant C might have possibly the machining and subassembly operations of product P, while plant B would be in charge of the final assembly and testing operations of the same product.

IMPLICATIONS AT NATIONAL LEVEL

Empirical estimates of progress functions can be used on a firm as well as a national level for numerous purposes. Progress function estimates can be useful for national planning in such areas as industry or nationwide forecasts of labor requirements and cost, allocation of labor and materials, and similar determinations.

If aggregate progress functions are available (e.g., industry J, chapter VI), it is possible to use them as a means of estimating future labor and space requirements for sectors of an economy. If similar functions are available for materials - - as Wright suggested in his pioneering article[3] - - it is possible to allocate materials according to planned requirements.

These potentialities may find even greater application in developing countries that have macroeconomic planning mechanisms of a more or less centralized nature. The necessary calculations would be carried out in the same way as explained in the last section.

Determining the Magnitude of Contracts

A problem of some interest is to determine the size of a government contract which will furnish the proper production base by day X in the future. The base of production for product P is the cumulative output of P by day X. The problem has some importance because of the interest in estimating production bases for various products under rearmament programs. It has also a general interest for programs of varied nature that are to be implemented by government funding.

The problem can be posed as follows, for the case of airframes: Having a requirement of R units of airplane P to be produced after day X with a fixed input of y_T direct manhours, what must be the size C of the contract which will insure post day X unit costs low enough to make possible the production of R units after day X?

Solution

If x_1 is the serial number of the last plane produced to date, we solve

$$y_T = a \int_{x_{1+C}}^{x_1+C+R} x^{-b}\, dx$$

$$= \frac{a}{1-b} \left[(x_1 + C + R)^{1-b} - (x_1 + C)^{1-b} \right]$$

for the value of C.

The preceding equation may be written as follows:

$$f(C) = (x_1+C+R)^{1-b} - (x_1+C)^{1-b} - \frac{y_T(1-b)}{a} = 0 \qquad (7.6)$$

Solving by Newton-Raphson's method, yields:

$$f(C_1) = (x_1+C_1+R)^{1-b} - (x_1+C_1)^{1-b} - \frac{y_T(1-b)}{a}$$

$$f'(C_1)=(1-b)(x_1+C_1+R)^{-b} - (1-b)(x_1+C_1)^{-b}$$

$$C_2=C_1 - \frac{(x_1+C_1+R)^{1-b} - (x_1+C_1)^{1-b} - \frac{y_T(1-b)}{a}}{(1-b)\left[(x_1+C_1+R)^{-b} - (x_1+C_1)^{-b}\right]} \qquad (7.7)$$

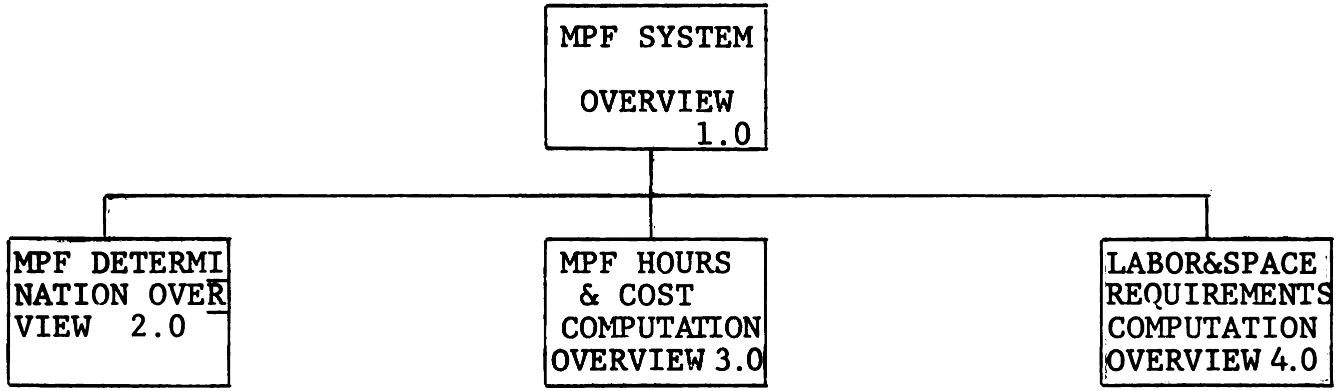
After some iterations of formula (7.7) it is possible to find an approximation C^* of C such that (7.6) is satisfied.

A COMPUTERIZED MPF SYSTEM

In this section an overall design of a computerized Manufacturing Progress Function System is suggested by using a system design and documentation technique known as HIPO (Hierarchy plus Input-Process-Output).[4] Because the functions of a system are described and not its organization and logic, a HIPO description provides information on "what a system does", and is thereby useful at most stages of planning, development, and implementation.

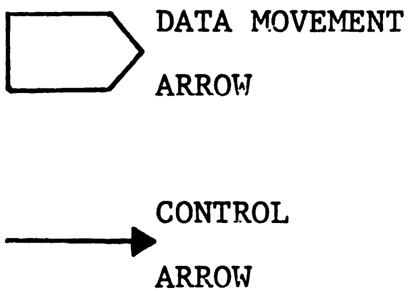
The fundamental version of a HIPO package is developed during the initial design phase and is called the initial design package. This package is prepared by the design group and gives the overall design of the proposed (or modified) system. At the initial design level, the HIPO package is lacking in details necessary for implementation but adequately gives the scope of the project and can be used by management for scheduling and cost estimation.

The following diagrams illustrate our initial conception of a computerized MPF System. The detail design phase can be developed by translating the calculation procedures exemplified in the previous sections into FORTRAN programs.



HIERARCHY DIAGRAM

LEGEND



DESCRIPTION SECTION

1.0 OVERVIEW DIAGRAM OF THE MPF SYSTEM THAT ESTABLISHES THE MPFs FOR PRODUCTS/OPERATIONS, CALCULATES THE MPF HOURS AND COST OF PRODUCTS/MAJOR CHANGES AND THEIR LABOR & SPACE REQUIREMENTS

2.0 OVERVIEW DIAGRAM OF MPF DETERMINATION FOR PRODUCTS/OPERATIONS AND PARAMETER DETERMINATION FOR PRODUCTS/OPERATIONS

3.0 OVERVIEW DIAGRAM OF MPF HOURS AND COST COMPUTATION

4.0 OVERVIEW DIAGRAM OF LABOR AND SPACE REQUIREMENTS COMPUTATION

FIGURE 8 VISUAL TABLE OF CONTENTS OF THE PACKAGE DESCRIBING THE MPF SYSTEM.

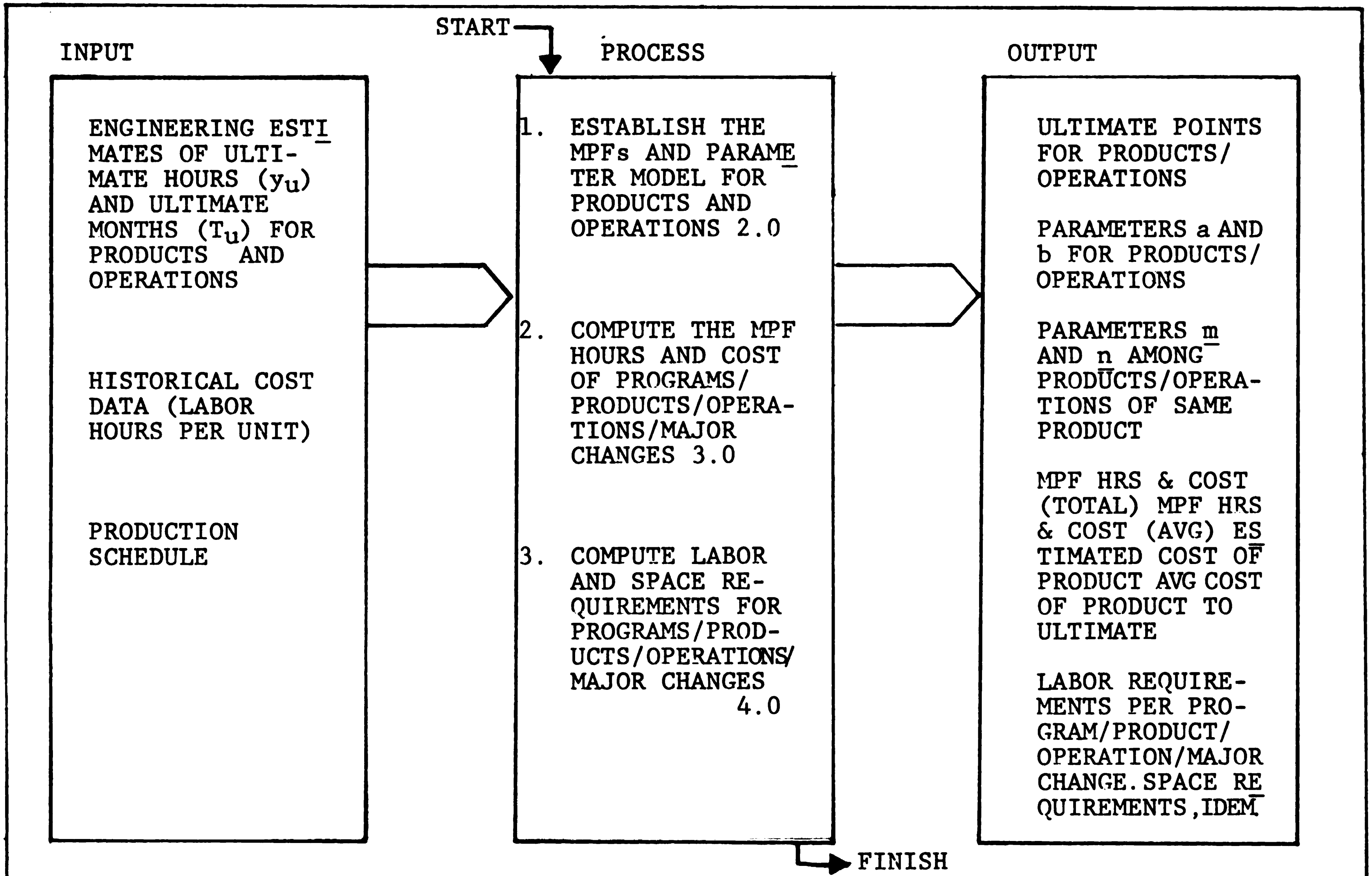


FIGURE 9. OVERVIEW DIAGRAM NUMBERED 1.0 OF THE MPF SYSTEM (THE HIGHEST-LEVEL DIAGRAM)

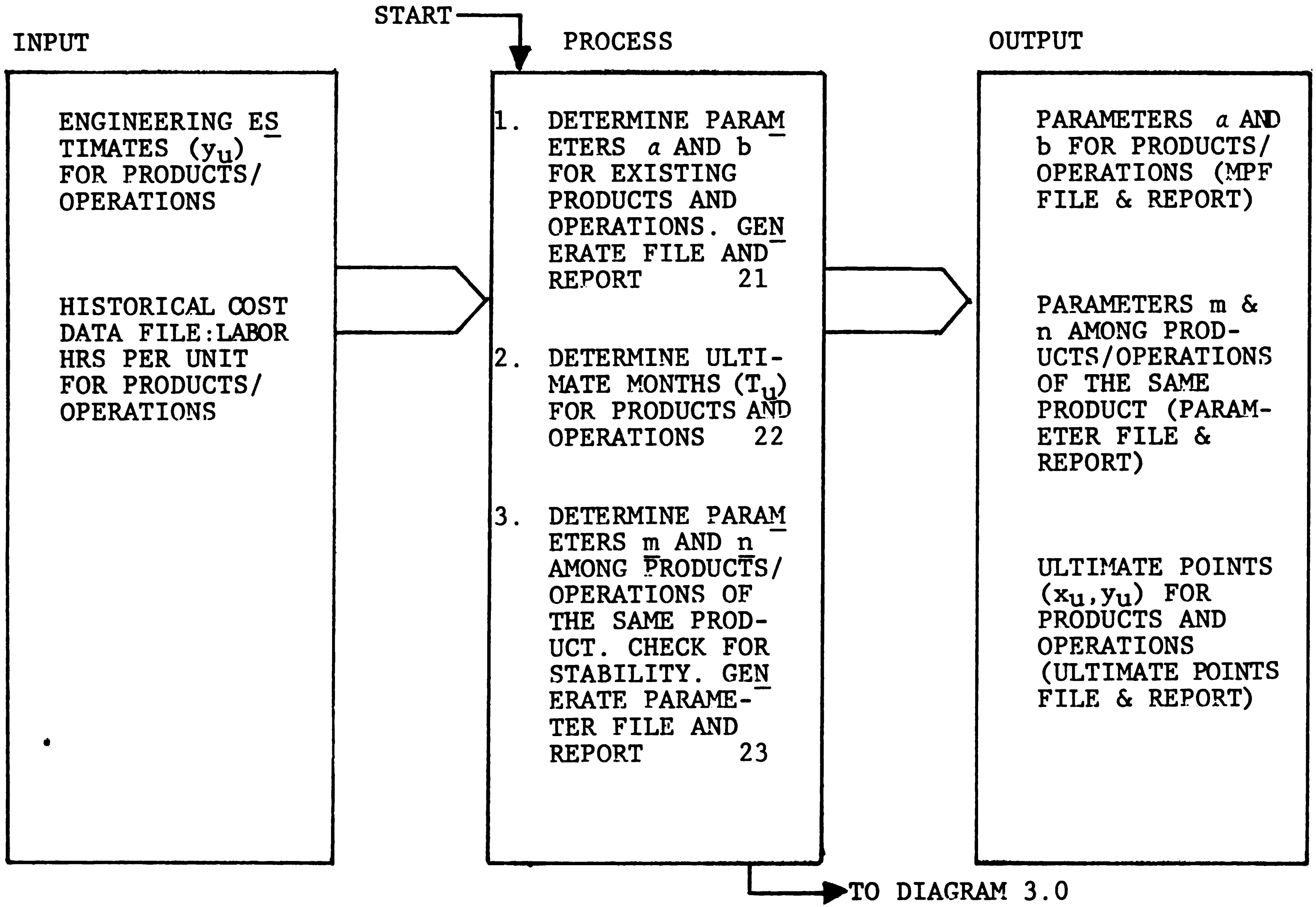


FIGURE 10 - OVERVIEW DIAGRAM NUMBERED 2.0 OF THE MPF SYSTEM

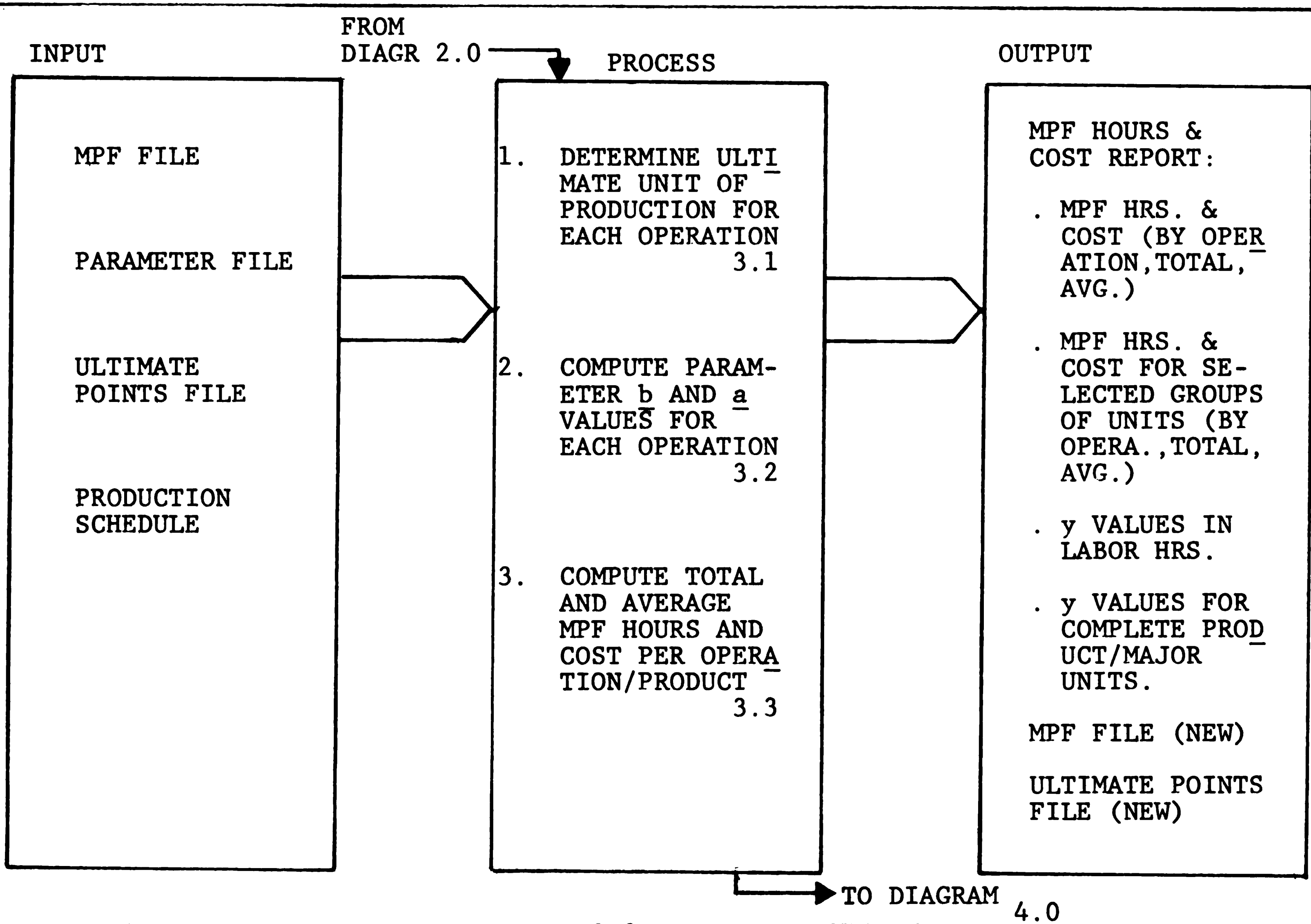


FIGURE 11. OVERVIEW DIAGRAM NUMBERED 3.0 OF THE MPF SYSTEM.

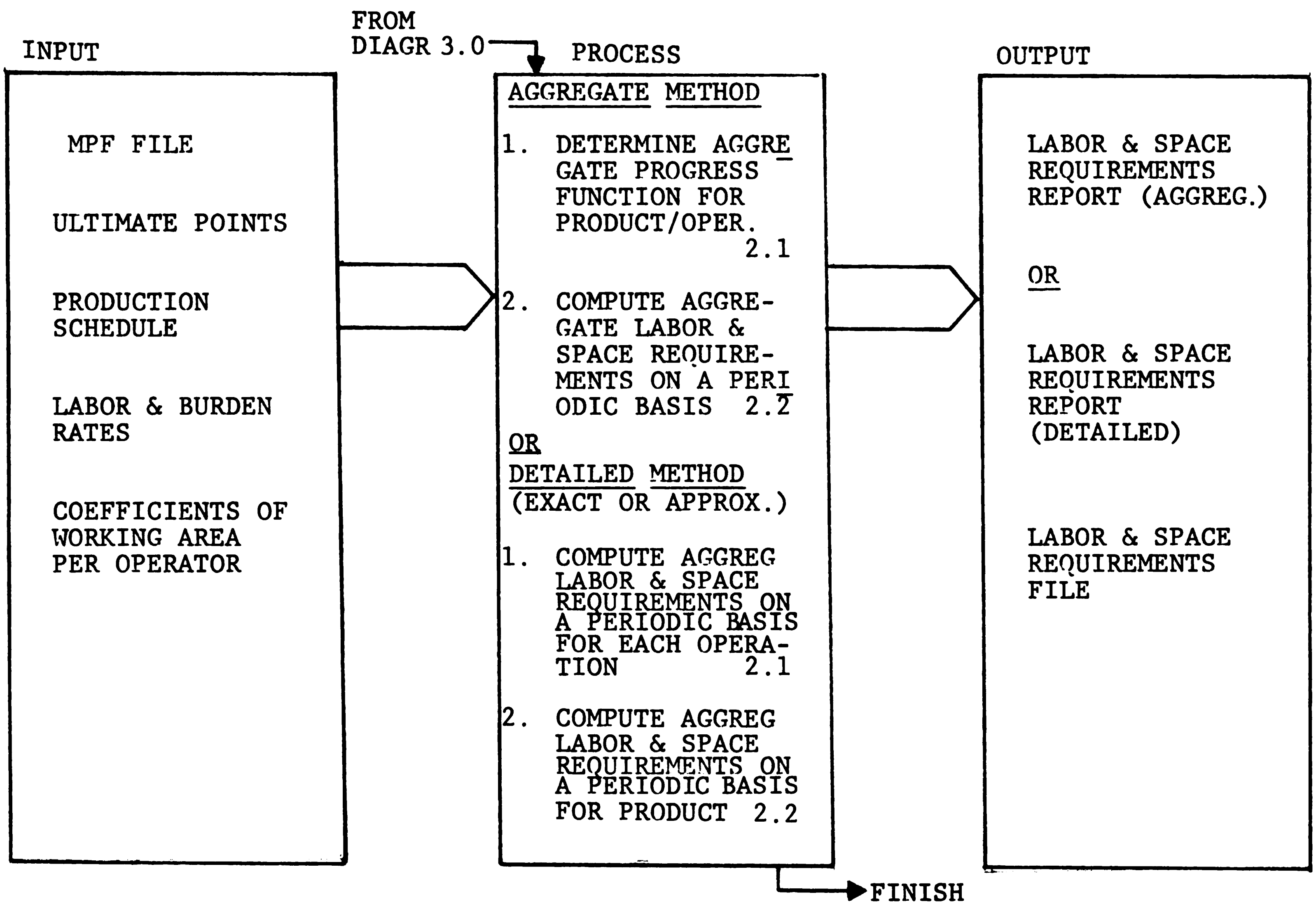


FIGURE 12 - OVERVIEW DIAGRAM NUMBERED 4.0 OF THE MPF SYSTEM

CHAPTER VIII

SUMMARY AND CONCLUSIONS

The overall purpose of this study was to advance knowledge on the subject matter of Manufacturing Progress Functions. For the purpose of this dissertation the Manufacturing Progress Function was generally defined as the relationship in which the labor input per unit used in the manufacture of a product tends to decline by a constant percentage as the cumulative quantity produced is doubled. Where this relationship is present, it may be represented by a straight line in a double logarithmic scale.

The prime objective of the study was to contribute a general symbolic-analytic model of the manufacturing progress phenomenon.

Once the general model was established, an equally important objective was to respond to the need for a coherent systematic approach to be used in predicting the developments of the adaptation process in industrial concerns of almost any kind.

The foregoing general statement of purpose was broken down into a number of layers of investigation leading to the following more specific subobjectives:

1. A review of the literature that is of relevance to the objectives of the dissertation.
2. An investigation of the theory of the Manufacturing Progress Function aiming at a systematization of the existing body of knowledge, and at the derivation of new theoretical results that will settle the question of the parameters estimation of the general model.
3. The conception of a general symbolic-analytic model of the system adaptation phenomenon, including the development of a method for using the model to predict the course of future startups.
4. The testing of the model in a number of real world situations by using data from diverse industrial operations.
5. The possibilities of implementing the model at firm level and national level.

A brief recapitulation of the major points that have emerged from each chapter of the dissertation will serve to demonstrate that the aforementioned objectives were attained.

Chapter II contains a comprehensive review of the historical development of the Manufacturing Progress Function and a summary of the more important contributions to the progress curve literature that are relevant to this dissertation.

Although the progress curve was discovered in 1922, it was largely unknown until World War II. T.P. Wright is given credit for originating the formulation of the progress curve theory in 1936. His statement, that cumulative average manhours per unit decline by a constant percentage every time the output is doubled, remains the most popular formulation in existence.

From 1940-1949, a number of modifications of the original model were proposed. J.R. Crawford contributed the unit learning curve and noticed that different rates of progress might exist for different airframes.

Several relevant contributions were published in the 1950s.There was a pioneering effort by some researchers like W.Z. Hirsch, S.E. Bryan, R.W. Conway and A. Schultz, Jr. to extend the concept of the progress curve to labor-intensive industries other than the airframe industry.In addition,the studies by A. Alchian and H. Asher, in the airframe industry, together with Hirsch's study in the machine-tool industry, and Schultz and Conway's research in manufacturing of electronic and electro-mechanical products represent objective and rigorous empirical investigations of the progress curve concept.

The last fifteen years have seen some pioneering extensions of the Manufacturing Progress Function to machine-intensive industries as well as to diverse labor-intensive industries. The studies carried out by N. Baloff constitute the most comprehensive investigation of the progress function during this time.

It is worthwhile to review the main hypotheses raised by the aforementioned authors: (i) Linearity, when in logarithmic coordinates, between the labor per unit input and cumulative output was generally confirmed except in Asher's study; (ii) There is no one single progress curve that can be universally applied to all types of operations involved in the manufacturing of a given product or to all products in a given firm or industry; (iii) Assembly operations experience higher rates of progress than machining operations; (iv) The findings are contradictory with respect to the novelty hypotheses. Crawford, Schultz and Conway agree that the less similar a new product is to a predecessor, the greater the rate of progress experienced by the new product. Nevertheless, Alchian and Hirsch found in diverse settings that the greater the similarity of a product to a predecessor, the greater the rate of progress experienced by the new product; (v) Baloff found a strong correlation between the parameters of the startup model among startups that occurred in the same production facility whereas Asher noticed a strong correlation among startups that occurred in different facilities; (vi) Plateaux predictability continues to be a controversial issue.

The field of progress functions lacks notation uniformity, precise definition of the variables and functional relationships involved, and formal mathematical proofs of several assumed results. A coherent mathematical exposition can be the basis for the derivation of new important results. Such theoretical systematization is offered in Chapter III .

Initially, four types of progress functions are identified. Functional relationships for the four types are clearly and compactly defined with recourse to set theory notation. Parameters $\underline{a}$ and $\underline{b}$ are explained and interpreted. In a second section, four fundamental problems which users might have faced consciously or unconsciously, are formally stated and solved, at times by more than one method. Finally, parameter calculation problems are classified and solved by exact or approximate formulas.

Chapter IV represents a continuation of the mathematical exposition initiated in Chapter III. Two related topics of practical relevance are approached: the integration of progress functions and the debatable problem of their aggregation. Original approximations are proposed for both problems.

A general symbolic-analytic model of the manufacturing progress phenomenon is offered in Chapter V. Some effort is also expended in questioning the causes of the systematic gains in productivity embodied in the progress curve. Thus, a tentative concept of manufacturing progress is proposed where the number of available hypotheses is reduced to a minimum and the explanatory power of the remaining hypotheses is greatly enhanced.

It is suggested that the manufacturing progress phenomenon can be described and its course predicted by the following empirical equations:

$$\mathit{y} = \mathit{a} x^{-b} \tag{8.1}$$

and

$$b = m + n \ln \mathit{a} \tag{8.2}$$

where

$$\mathit{y} = y/y_u$$

$$\mathit{a} = a/y_u$$

and

(x_u, y_u) is the estimated ultimate point.

In Chapter VI the manufacturing progress model presented in Chapter V is tested with real data from nine manufacturers representing five different industries. A hundred and fifty-nine separate cases of product and process startups that occurred in four different countries and nine distinct plants have been analyzed. In addition, aggregate data was obtained for whole industries in one country, yielding nine more startups.

The major conclusions that have emerged from the empirical research are the following:

(1) The descriptive efficiency of the manufacturing progress model given by equation (8.1) is generally supported by the results of regression analysis of the startups from firms A,B,C,D,E,F,G,H,I, and industry J. The coefficients of determination r^2

vary from 0.59 to 1.000, with a median value of 0.969. In only ten per cent of the cases does the regression fail to explain at least 81.7% of the total variance in the dependent variable. The t-ratios are generally impressive, ranging from 2.08 to 99.99, with a median value of 10.9. If normality and common variance are assumed, the null hypotheses $\rho = 0$ and $\beta = 0$ can be rejected at the 0.95 level of significance in 95% of the startups analyzed.

(2) The descriptive efficiency of the parameter model given by equation (8.2) is generally supported by the results of regression analysis of the startup parameters from firms A,D,E,F,G,H,I and industry J. The parameter model is supported among operations of the same product (firms A and G), among products within the same plant (firms D,E,F,H and I), and among groups of products of similar technology within the same industrial sector of a foreign economy (industry J).

(3) The findings of this research and previous findings by Asher and Baloff constitute adequate evidence to suggest that the parameter model can be developed into an effective means of predicting the parameter $\underline{b}$ of a new startup. However, additional investigation and validation is necessary for

successful industrial application of the model.

The dissertation is concluded (Chapter VII) with a discussion of the industrial implications of the findings reported in Chapter VI. The importance of recognizing and predicting the manufacturing progress phenomenon is related to several decision-making functions that are encountered in an industrial setting as well as in economic planning at the national level. Finally, an overall design of a computerized Manufacturing Progress Function (MPF) System is suggested.

APPENDICES

APPENDIX A

APPENDIX A

MATHEMATICAL PROOFS

(1) A FORMAL PROOF THAT $\lim_{x \to 0} ax^{-b} = +\infty$

The above statement means that for each positive number M it is possible to find a positive number δ (depending on M in general) such that

$$\frac{a}{x^b} > M \quad \text{when} \quad 0 < |x| < \delta$$

To prove this, note that

$$\frac{a}{x^b} > M \quad \text{when} \quad 0 < x^b < \frac{a}{M}$$

or

$$0 < |x| < \left(\frac{a}{M}\right)^{1/b}$$

Choosing $\delta = \left(\frac{a}{M}\right)^{1/b}$, the required result follows.

(2) A FORMAL PROOF THAT $\lim_{x \to \infty} ax^{-b} = 0$

The above statement means that for any positive number ε it is possible to find a positive number $N = N(\varepsilon)$ such that

$$|ax^{-b}| < \varepsilon \quad \text{whenever} \quad x > N$$

To prove this, note that for

$$|ax^{-b}| < \varepsilon$$

one must have

$$x > \left(\frac{a}{\varepsilon}\right)^{1/b}$$

Choosing $N = \left(\frac{a}{\varepsilon}\right)^{1/b}$, the required result follows.

APPENDIX B

APPENDIX B

EQUATION OF THE ASYMPTOTE TO THE CURVE REPRESENTING THE LOGARITHM OF THE UNIT PROGRESS FUNCTION

The following theorem of Mathematical Analysis is used:

"If an infinite branch of a curve has an asymptote $y = cx + d$, the coefficients c and d will be given by

$$c = \lim_{x\to\infty} \frac{y}{x} \quad \text{and} \quad d = \lim_{x\to\infty} (y - cx) \qquad (B.1)$$

where the point M(x,y) remains on the branch of the curve. And, conversely, if the limits in (B.1) exist when M(x,y) moves to infinity along a branch of the curve, the straight line $y = cx + d$ will be an asymptote of that branch.[1] "

$$\text{Let} \quad F\left[\log(x)\right] = \log\left\{ a\left[x^{1-b} - (x-1)^{1-b}\right]\right\} \qquad (B.2)$$

One must show that

$f(\log x) = \log\left[a(1-b)\right] - b \log x$ is asymptote to $F\left[\log(x)\right]$ given by (B.2)

Coefficient c of the Asymptote

According to the forementioned theorem:

$$c = \lim_{(\log x)\to\infty} \frac{\log a \left[x^{1-b} - (x-1)^{1-b}\right]}{\log x}$$

$$= \lim_{x\to\infty} \frac{\log a + \log \left[x^{1-b} - (x-1)^{1-b}\right]}{\log x}$$

Applying L'Hospital's rule, it follows that:

$$\lim_{x\to\infty} \frac{\log a + \log \left[x^{1-b} - (x-1)^{1-b}\right]}{\log x}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{\ln 10} \cdot \frac{1}{\left[x^{1-b} - (x-1)^{1-b}\right]} (1-b) \left[x^{-b} - (x-1)^{-b}\right]}{\frac{1}{\ln 10} \cdot \frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{(1-b)\ x \left[-bx^{-b-1} - \frac{b(b+1)}{2!} x^{-b-2} - \ldots\right]}{\left[(1-b)\ x^{-b} + \frac{(1-b)b}{2!} x^{-b-1} + \ldots\right]}$$

$$= \lim_{x\to\infty} \frac{-bx^{-b}}{x^{-b}} = -b$$

Coefficient d of the Asymptote

According to the previously stated theorem and using the result $c = -b$, it follows that:

$$d = \lim_{(\log x)\to\infty} \{\log a \left[x^{1-b} - (x-1)^{1-b}\right] + b \log x\}$$

$$= \lim_{x\to\infty} \log \{a \left[x^{1-b} - (x-1)^{1-b}\right] x^{b}\}$$

$$= \log a + \lim_{x\to\infty} \log \{x^{b} \left[x^{1-b} - (x-1)^{1-b}\right]\}$$

$$= \log a + \lim_{x\to\infty} \log \{x^{b} \left[(1-b)x^{-b} + \frac{(1-b)}{2!} bx^{-b-1} + \ldots\right]\}$$

$$= \log a + \lim_{x\to\infty} \log \left[(1-b) + \frac{(1-b)}{2!} b \frac{1}{x} + \ldots\right]$$

$$= \log a + \log \lim_{x\to\infty} \left[(1-b) + \frac{(1-b)}{2!} b \frac{1}{x} + \ldots\right]$$

$$= \log a + \log (1-b) = \log \left[a(1-b)\right]$$

<u>Equation of the Asymptote to F $\left[\log(x)\right]$</u>

Since $c = -b$ and $d = \log \left[a\ (1-b)\right]$, the equation of the asymptote f(log x) to

$$F\left[\log(x)\right] = \log \{a \left[x^{1-b} - (x-1)^{1-b}\right]\}$$

is, according to the forementioned theorem:

$$f(\log x) = - b \log x + \log \left[a(1-b)\right] \qquad \text{(q.e.d.)}$$

APPENDIX C

APPENDIX C

PARAMETER FORMULAS DERIVATION

In chapter V it was suggested that the manufacturing progress phenomenon can be described and its course predicted by the following equations

$$y = \alpha x^{-b} \qquad \text{(C1)}$$

and $$b = m + n \ln \alpha \qquad \text{(C2)}$$

where $$y = \frac{y(x)}{y_u}$$

$$\alpha = \frac{a}{y_u}$$

and (x_u, y_u) is the estimated ultimate point.

If the best fit to the available data is achieved with the cumulative-average progress function, the productivity index y will have to be defined as

$$y = \frac{\bar{y}(x)}{y_u}$$

In this case, prediction of parameters a and b for a new startup can be developed as follows.

Equation (C1) yields for $x = x_u$:

$$\frac{\bar{y}(x_u)}{y_u} = a\, x_u^{-b} \tag{C3}$$

If the best fit is given by the cumulative-average progress function $\bar{y} = ax^{-b}$, then:

$$\bar{y}(x_u) = ax_u^{-b} \tag{C4}$$

From Chapter II, equation (3.17) :

$$y\ (x_u) \cong a(1-b)\ x_u^{-b} \tag{C5}$$

Dividing (C4) by (C5), yields:

$$\frac{\bar{y}(x_u)}{y(x_u)} = \frac{1}{1-b} \tag{C6}$$

Substituting in (C3) the result obtained in (C6) and solving for a, yields:

$$a = \frac{1}{1-b}\, x_u^{b} \tag{C7}$$

Substituting in (C2) the result obtained in (C7) it follows that:

$$b = m + n\ \ln\ (\frac{1}{1-b}\, x_u^{b}) \qquad \text{or}$$

$$n \ln \left(\frac{x_u^b}{1-b}\right) - b + m = 0 \tag{C8}$$

Equation (C8) can be solved for b by the Newton-Raphson Method as follows:

$$n \left[b \ln x_u - \ln (1-b)\right] - b + m = 0$$

$$f(b_1) = n \left[b_1 \ln x_u - \ln (1-b_1)\right] - b_1 + m = 0$$

$$f'(b_1) = n \left(\ln x_u + \frac{1}{1-b_1}\right) - 1 = 0$$

$$b_2 = b_1 - \frac{n\left[b_1 \ln x_u - \ln(1 - b_1)\right] - b_1 + m}{n\left[\ln x_u + \frac{1}{1-b_1}\right] - 1} \tag{C9}$$

After some iterations of formula (9) it is possible to obtain an adequate approximation b^* of b. By substituting b^* in (C7), it follows that:

$$\alpha = \frac{1}{1-b} x_u^{b^*}$$

and

$$a = y_u \, \alpha = \frac{y_u \, x_u^{b^*}}{1-b} \tag{C10}$$

APPENDIX D

APPENDIX D

STABILITY OF COEFFICIENTS *m* AND *n* OF THE PARAMETER MODEL

The following is a check on the stability of coefficients m and n of the parameter model:

$$b = m + n \ln \underline{a}$$

In the case of firm D (14 products), sample 1 was taken randomly from products D1 through D14. Sample 2 is formed by the remaining products. Data tables are in page 268. In the case of firm F (28 products), sample 1 was taken randomly from products F1 through F28. Sample 2 is formed by the remaining products. Data tables are in pages 278-79 .

The following calculations were then carried out: (1) Estimation of the regression of b on ln a for samples 1 and 2 of each firm; (2) Variances about regressions 1 and 2; (3) Difference between the two regression coefficients n_1 and n_2: variance, standard error and confidence limits; (4) Difference between the two intercepts m_1 and m_2: variance, standard error and confidence limits; (5) Test of the hypothesis that the variances about the two regressions are equal.

It is concluded that the difference between the two regression coefficients is not statistically significant.The same goes for the difference between the two intercepts.

In the final section of Appendix D a comparison is made between two approaches for predicting parameter b and a values of new startups, namely, by using average b and a values taken from past data and by employing the parameter model approach proposed in Chapter V. Sample 1 (Table D.5) was drawn randomly from firm F binding machines and simulates past data. Sample 2 (Table D.6) is formed by the remaining types of binding machines produced by firm F. Predictions were then carried out for parameters b and a of the products in Sample 2 by using data available from Sample 1. The results of the comparison are exhibited in Tables D.7 and D.8. The actual values of b and a appear in column (2) of each table, respectively. The average b's and a's - - calculated from Sample 1 values - - are in column (3). Per cent deviations of the predictions made by using average values with respect to the actual values are in column (4). The predictions for b and a made by using the parameter model are in column (5). Detailed calculations precede Tables D.7 and D.8. Per cent deviations of the parameter model predictions b_p and a_p with respect to the actual values of b and a are in column (6) of each table.

The mean absolute deviation is defined as

$$\text{M.A.D.} = \frac{\Sigma \ |\text{Prediction Errors}|}{\text{No. of Predictions}}$$

From the results in Tables D.7 and D.8, the following M.A.D. values were calculated through the above formula:

(1) Predictions using averages of past data

b parameter : M.A.D. = 0.0815

a parameter : M.A.D. = 1.24

(2) Predictions using the parameter model

b parameter : M.A.D. = 0.0497

a parameter : M.A.D. = 0.502

Similar results were obtained with the data pertaining to printing presses produced by firm F and with the data available from other firms (e.g., firms G and I). The author believes that the parameter model contributed in Chapter V has proved to be superior to other existing methods for estimating the parameters of new startups.

FIRM D

(14 products)

TABLE D.1

SAMPLE 1, FIRM D

PRODUCTS	a	ln a	b
D3	1.341	0.2934	0.0936
D11	3.521	1.259	0.220
D4	2.483	0.9095	0.275
D14	1.333	0.2874	0.0533
D5	1.230	0.2070	0.0543
D7	5.252	1.658	0.333
D12	7.002	1.946	0.406

TABLE D.2

SAMPLE 2, FIRM D

PRODUCTS	a	ln a	b
D10	6.945	1.938	0.409
D1	2.649	0.9742	0.368
D2	3.230	1.173	0.297
D6	2.882	1.059	0.205
D9	1.625	0.4855	0.0929
D8	1.156	0.1450	0.0226
D13	6.683	1.900	0.409

1- ESTIMATION OF THE REGRESSION OF b ON ln a

(LET b = y and ln a = x)

REGRESSION 1 (sample 1)

n' = no. of observations = 7

$\Sigma x = 6.5603; \; \bar{x} = \frac{\Sigma x}{n'} = 0.9372$

$\Sigma(x - \bar{x})^2 = x^2 - (\Sigma x)^2/n' = 9.1597 - 6.5603^2/7 = 3.0115$

$\Sigma y = 1.4352; \; \bar{y} = \frac{\Sigma y}{n'} = 0.2050$

$\Sigma(y - \bar{y})^2 = \Sigma y^2 - (\Sigma y)^2/n' = 0.4143 - 1.4352^2/7 = 0.1200$

$\Sigma(x-\bar{x})(y-\bar{y}) = \Sigma xy - \Sigma x \Sigma y/n' = 1.9233 - \frac{6.5603 x 1.4352}{7} = 0.5783$

The regression coefficient n_1 is given by

$$n_1 = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(x - \bar{x})^2} = \frac{0.5783}{3.0115} = 0.1920$$

The regression equation of y on x is then:

$Y = \bar{y} + n_1 (x - \bar{x}) = 0.2050 + 0.1920 (x - 0.9372)$

or $Y = 0.0251 + 0.1920 x$

or $B = 0.0251 + 0.1920 \ln \underline{a}$

where: $m_1 = 0.0251$ (intercept) and $n_1 = 0.1920$ (slope)

REGRESSION 2 (sample 2)

$n'' = 7$

$\Sigma x = 7.6747; \; \bar{x} = \Sigma x/n'' = 1.0964$

$\Sigma(x - \bar{x})^2 = 11.0691 - 7.6747^2/7 = 2.6547$

$\Sigma y = 1.8035; \; \bar{y} = \Sigma y/n'' = 0.2567$

$\Sigma(y - \bar{y})^2 = 0.6094 - 1.8035^2/7 = 0.1447$

$$(x - \bar{x})(y - \bar{y}) = 2.5421 - \frac{7.6747 \times 1.8035}{7} = 0.5648$$

$$n_2 = \frac{0.5648}{2.6547} = 0.2128 \quad \text{(regression coefficient)}$$

The regression equation of y on x is then:

$$Y = \bar{y} + n_2(x - \bar{x}) = 0.2576 + 0.2128\,(x - 1.0964)$$

or $Y = 0.0244 + 0.2128\,x$

or $B = 0.0244 + 0.2128 \ln \underline{a}$

where: $m_2 = 0.0244$(intercept) and $n_2 = 0.2128$ (slope)

2 - VARIANCE ABOUT THE REGRESSION

VARIANCE ABOUT REGRESSION 1

The sum of squares about the regression is given by

$$\Sigma(y - Y)^2 = \Sigma(y - \bar{y})^2 - \Sigma(Y - \bar{y})^2 \tag{D.1}$$

Sum of squares of observations y about their mean

$$\Sigma(y - \bar{y})^2 = 0.1200 \qquad \text{(already calculated)}$$

Sum of squares due to the regression

$$\Sigma(Y - \bar{y})^2 = b^2\,\Sigma(x-\bar{x})^2 = [\Sigma(x-\bar{x})(y-\bar{y})]^2/\Sigma(x-\bar{x})^2$$

$$= 0.5783^2/3.0115$$

$$= 0.1111$$

Substituting in (D.1) yields:

$$\Sigma(y - Y)^2 = 0.1200 - 0.1111 = 0.0089$$

Hence, the variance about the regression is estimated by:

$$S_1^2 = \frac{0.0089}{5} = 0.00178$$

(Since the sum of squares about the regression is based on $n' - 2 = 7 - 2 = 5$ degrees of freedom.)

VARIANCE ABOUT REGRESSION 2

Similarly, we have:

$$\Sigma(y - \bar{y})^2 = 0.1447$$

$$\Sigma(Y - \bar{y})^2 = 0.5648^2/2.6547 = 0.1202$$

$$\therefore \Sigma(y - Y)^2 = 0.1447 - 0.1202 = 0.0245$$

Hence, the variance about the regression is estimated by:

$$S_2^2 = 0.0245/5 = 0.00490$$

3 - THE DIFFERENCE BETWEEN THE TWO REGRESSION COEFFICIENTS n_1 AND n_2.

Let σ^2 be the error variance, which is usually estimated by combining the mean squares about the two regressions, i.e.,

if S_1^2 = variance about regression 1 (slope n_1) with ϕ_1 degrees of freedom and S_2^2 = variance about regression 2 (slope n_2) with ϕ_2 degrees of freedom

then σ^2 is estimated by

$$S^2 = (\phi_1 S_1^2 + \phi_2 S_2^2)/(\phi_1 + \phi_2)$$

The variance of the regression coefficient is given by:

$$V(n) = \frac{\sigma^2}{\Sigma(x - \bar{x})^2} \qquad \text{(D.2)}$$

Substituting S for σ in (D.2) yields:

$$V(n_1) = S^2 / \Sigma_1(x - \bar{x})^2$$

and

$$V(n_2) = S^2 / \Sigma_2(x - \bar{x})^2$$

where $\Sigma_1(x - \bar{x})^2$ is based on the observations from which n_1 was calculated and similarly for $\Sigma_2(x - \bar{x})^2$. Therefore, since n_1 and n_2 are independent estimates:

$$V(n_1 - n_2) = S^2 \left[\frac{1}{\Sigma_1(x-\bar{x})^2} + \frac{1}{\Sigma_2(x-\bar{x})^2} \right] \qquad \text{(D.3)}$$

and $$\text{S.E.}(b_1 - b_2) = S \left[\frac{1}{\Sigma_1(x-\bar{x})^2} + \frac{1}{\Sigma_2(x-\bar{x})^2} \right]^{\frac{1}{2}} \qquad \text{(D.4)}$$

These enable confidence limits for the difference to be calculated, using the value of t with $(\phi_1 + \phi_2)$ degrees of freedom. If the variances about the two regressions cannot be assumed equal, then the confidence limits must be calculated by another method. We assume that the variances are equal (this assumption will be tested in a later section).

Thus, σ^2 is estimated by

$$s^2 = 0.5\ (0.00178 + 0.00490) = 0.00334$$

The variance and the standard error of the difference between n_1 and n_2 are calculated as follows:

$$V(n_1 - n_2) = s^2 \left[\frac{1}{\Sigma_1 (x-x)^2} + \frac{1}{\Sigma_2 (x-x)^2} \right] = 0.00334 \left[\frac{1}{3.0115} + \frac{1}{2.6547} \right]$$

$$= 0.002367$$

$$\text{S.E. } (n_1 - n_2) = (0.002367)^{\frac{1}{2}} = 0.0487$$

<u>Confidence Limits</u>

Using t with $(\phi_1 + \phi_2) = 10$ degrees of freedom, the 95% confidence limits are:

$$(n_1 - n_2) \pm 2.23 \text{ S.E. } (n_1 - n_2) = (0.1920 - 0.2128) \pm 2.23 \times 0.0487$$

$$= -0.1294 \text{ to } 0.0878$$

Since the confidence limits include zero, the difference between the regression coefficients n_1 and n_2 is not statistically significant.

4 - THE DIFFERENCE BETWEEN THE TWO INTERCEPTS m_1 AND m_2

The variance of the intercept m is given by:

$$V(m) = \sigma^2 \left[\frac{1}{n} + \frac{\bar{x}^2}{\Sigma(x-\bar{x})^2} \right] \qquad (D.5)$$

Substituting S for σ in (D.5) yields

$$V(m_1) = S^2 \left[\frac{1}{n'} + \frac{\bar{x}_1^2}{\Sigma_1(x-\bar{x})^2} \right]$$

$$V(m_2) = S^2 \left[\frac{1}{n''} + \frac{\bar{x}_2^2}{\Sigma_2(x-\bar{x})^2} \right]$$

Since m_1 and m_2 are independent estimates and $n' = n''$

$$V(m_1-m_2) = S^2 \left[\frac{2}{n'} + \frac{\bar{x}_1^2}{\Sigma_1(x-\bar{x})^2} + \frac{\bar{x}_2^2}{\Sigma_2(x-\bar{x})^2} \right]$$

$$= 0.00334 \left(\frac{2}{7} + \frac{0.9372^2}{3.0115} + \frac{1.0964^2}{2.6547}\right)$$

$$= 0.003441$$

$$S.E.\ (m_1 - m_2) = (0.003441)^{\frac{1}{2}} = 0.0587$$

Confidence Limits

Using t with $(\phi_1 + \phi_2) = 10$ degrees of freedom, the 95% limits are

$$(m_1 - m_2) \pm 2.23 \times S.E.(m_1 - m_2) = (0.0251-0.0244) \pm 2.23 \times 0.0587$$
$$= -0.1302 \text{ to } 0.1316$$

Since the confidence limits include zero, the difference between the intercepts m_1 and m_2 is not statistically significant.

Note. If the variances about the two regressions cannot be assumed equal, then the confidence limits must be calculated by another method. In the following section the hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ is tested with $H_\phi : \sigma_1^2 \neq \sigma_2^2$.

5 - TEST OF THE HYPOTHESIS THAT THE VARIANCES ABOUT THE TWO REGRESSIONS ARE EQUAL.

When the populations are normally distributed and the samples are independent, the ratio of two samples variances is distributed according to the F distribution and has the test statistic

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$$

where S_1^2 and S_2^2 are the sample variances and σ_1^2 and σ_1^2 are the population variances from which the samples were taken. The hypothesis is that $\sigma_1^2 = \sigma_2^2$, and the estimates of these two parameters are represented by their unbiased estimates S_1^2 and S_2^2. Consequently, the test statistics reduces to

$$F = S_1^2/S_2^2$$

Since $S_1^2 = 0.00490$ and $S_2^2 = 0.00178$,

$$F = 0.00490/0.00178 = 2.76$$

We have also:

$$\phi_1 = \phi_2 = 5 \quad \text{degrees of freedom}$$

From the table of the variance ratio (F - Distribution), the 5% and 1% points are 5.05 and 11.0, respectively.

Thus, the hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ is accepted.

FIRM F

(28 products)

TABLE D.3

SAMPLE 1, FIRM F

PRODUCTS	a	ln a	b
F24	4.74	1.556	0.228
F18	2.70	0.9933	0.176
F11	1.48	0.3920	0.0758
F12	1.80	0.5878	0.148
F21	2.02	0.7031	0.123
F17	3.42	1.230	0.237
F7	1.77	0.5710	0.123
F20	4.47	1.497	0.235
F28	3.16	1.151	0.227
F19	2.71	0.9969	0.186
F13	3.85	1.348	0.271
F26	1.57	0.4511	0.0731
F25	2.41	0.8796	0.120
F1	3.57	1.273	0.238

TABLE D.4

SAMPLE 2, FIRM F

PRODUCTS	a	ln a	b
F5	2.01	0.6981	0.147
F6	1.99	0.6881	0.150
F4	1.97	0.6780	0.184
F27	1.95	0.6678	0.131
F9	1.93	0.6575	0.136
F3	1.83	0.6043	0.110
F10	1.80	0.5878	0.128
F2	1.49	0.3988	0.0604
F8	1.36	0.3075	0.0551
F16	4.94	1.597	0.246
F15	3.22	1.169	0.219
F22	3.17	1.154	0.201
F23	2.80	1.030	0.133
F14	2.59	0.9517	0.172

1 - ESTIMATION OF THE REGRESSION OF b ON ln a

(LET b = y AND ln a = x)

REGRESSION 1

n' = no. of observations = 14

$\Sigma x = 13.6298; \; \bar{x} = \Sigma x/n' = 0.9736$

$\Sigma(x-\bar{x})^2 = \Sigma x^2 - (\Sigma x)^2/n' = 15.2147 - 13.6298^2/14 = 1.9453$

$\Sigma y = 2.4609; \; \bar{y} = \Sigma y/n' = 0.1758$

$\Sigma(y-\bar{y})^2 = \Sigma y^2 - (\Sigma y)^2/n' = 0.4882 - 2.4609^2/14 = 0.05563$

$$\Sigma(x-\bar{x})(y-\bar{y}) = \Sigma xy - \Sigma x \Sigma y/n' = 2.6998 - \frac{13.6298 \times 2.4609}{14}$$

$$= 0.3040$$

The regression coefficient n_1 is given by

$$n_1 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \frac{0.3040}{1.9453} = 0.1563$$

The regression equation of y on x is then:

$$Y = \bar{y} + n_1 (x - \bar{x}) = 0.1758 + 0.1563 (x - 0.9736)$$

or $Y = 0.0236 + 0.1563 x$

or $B = 0.0236 + 0.1563 \ln a$

where: m_1 = 0.0236 (intercept) and n_1 = 0.1563 (slope)

REGRESSION 2

$n'' = 14$

$\Sigma x = 11.1896;\ \bar{x} = \Sigma x/n'' = 0.7993$

$\Sigma(x-\bar{x})^2 = 10.4784 - 11.1896^2/14 = 1.5350$

$\Sigma y = 2.0725;\ \bar{y} = \Sigma y/n'' = 0.1480$

$\Sigma(y-\bar{y})^2 = 0.3449 - 2.0725^2/14 = 0.03810$

$$\Sigma(x-\bar{x})(y-\bar{y}) = 1.8717 - \frac{11.1896 \times 2.0725}{14} = 0.2152$$

$$n_2 = \frac{0.2152}{1.5350} = 0.1402$$

The regression equation of y on x is then:

$Y = \bar{y} + n_2\ (x-\bar{x}) = 0.1480 + 0.1402\ (x - 0.7993)$

or $Y = 0.0359 + 0.1402\ x$

or $B = 0.0359 + 0.1402 \ln \underline{a}$

where m_2 = 0.0359 (intercept) and n_2 = 0.1402 (slope)

2 - VARIANCE ABOUT THE REGRESSION

VARIANCE ABOUT REGRESSION 1

The sum of squares about the regression is given by

$$\Sigma(y-Y)^2 = \Sigma(y-\bar{y})^2 - \Sigma(Y-\bar{y})^2 \qquad \text{(D.6)}$$

Sum of squares of observations y about their mean

$$\Sigma(y - \bar{y})^2 = 0.05563$$

Sum of squares due to the regression

$$\Sigma(Y-\bar{y})^2 = b^2\ \Sigma(x-\bar{x})^2 = [\Sigma(x-\bar{x})(y-\bar{y})]^2/\Sigma(x-\bar{x})^2$$

$$= 0.3040^2/1.9453$$

$$= 0.04751$$

Substituting in (D.6) yields:

$$\Sigma(y-Y)^2 = 0.05563 - 0.04751 = 0.00812$$

Hence, the variance about the regression is estimated by

$$S_1^2 = \frac{0.00812}{12} = 0.000677$$

(Since the sum of squares about the regression is based on $n' - 2 = 14 - 2 = 12$ degrees of freedom.)

VARIANCE ABOUT REGRESSION 2

Similarly, we have:

$$\Sigma(y - \bar{y})^2 = 0.03810$$

$$\Sigma(Y - \bar{y})^2 = 0.2152^2/1.5350 = 0.03017$$

$$\therefore \Sigma(y - Y)^2 = 0.03810 - 0.03017 = 0.00793$$

Hence, the variance about the regression is estimated by:

$$S_2^2 = 0.00793/12 = 0.000661$$

3 - THE DIFFERENCE BETWEEN THE TWO REGRESSION COEFFICIENTS

σ^2, the error variance is estimated by combining S_1^2 and S_2^2 as follows:

$$S^2 = 0.5\ (0.000677 + 0.000661)$$

$$= 0.000669$$

The variance and the standard error of the difference between n_1 and n_2 are calculated as follows

$$V(n_1 - n_2) = S^2 \left[\frac{1}{\Sigma_1 (x-\bar{x})^2} + \frac{1}{\Sigma_2 (x-\bar{x})^2} \right] = 0.000669 \left[\frac{1}{1.9453} + \frac{1}{1.5350} \right]$$

$$= 0.001463$$

$$S.E.\ (n_1 - n_2) = (0.001463)^{\frac{1}{2}} = 0.0382$$

Confidence Limits

Using t with $(\phi_1 + \phi_2) = 24$ degrees of freedom, the 95% confidence limits are

$$(n_1-n_2) \pm 2.06\ S.E.(n_1-n_2) = (0.1563-0.1402) \pm 2.06 \times 0.0382$$

$$= -0.0626 \text{ to } 0.0948$$

Since the confidence limits include zero, the difference between the regression coefficients n_1 and n_2 is not statistically significant.

4 - THE DIFFERENCE BETWEEN THE TWO INTERCEPTS m_1 AND m_2.

The variance and the standard error of the difference $(m_1 - m_2)$ are calculated as follows:

$$V(m_1-m_2) = s^2 \left[\frac{2}{n'} + \frac{\bar{x}_1^2}{\Sigma_1 (x-\bar{x})^2} + \frac{\bar{x}_2^2}{\Sigma_2 (x-\bar{x})^2} \right]$$

$$= 0.000669 \left[\frac{2}{14} + \frac{0.9736^2}{1.9453} + \frac{0.7993^2}{1.5350} \right]$$

$$= 0.0007000$$

$$S.E.(m_1 - m_2) = (0.0007000)^{\frac{1}{2}} = 0.0265$$

Confidence Limits

Using t with $(\phi_1 + \phi_2) = 24$ degrees of freedom, the 95% limits are

$$(m_1-m_2) \pm 2.06\ S.E.(m_1-m_2)=(0.0236-0.0359) \pm 2.06 \times 0.0265$$

$$= -0.0669 \text{ to } 0.0423$$

Since the confidence limits include zero, the difference between the intercepts m_1 and m_2 is not statistically significant.

5 - TEST OF THE HYPOTHESIS $H_0:\sigma_1^2 = \sigma_2^2$

Since $S_1^2 = 0.000677$ and $S_2^2 = 0.000661$,

$$F = \frac{0.000677}{0.000661} = 1.02$$

We have also:

$$\phi_1 = \phi_2 = 12 \text{ degrees of freedom.}$$

From the table of the variance ratio (F-Distribution), the 5% and 1% points are 2.69 and 4.16, respectivelly.

Thus, the hypothesis H_0: $\sigma_1^2 = \sigma_2^2$ is accepted.

PREDICTING PARAMETER b AND a VALUES

OF NEW STARTUPS (FIRM F, BINDING MACHINES)

TABLE D.5

SAMPLE 1 (SIMULATES PAST DATA)

PRODUCT	a	ln a	b
F15	3.22	1.169	0.219
F12	1.80	0.5878	0.148
F14	2.59	0.9517	0.172
F9	1.93	0.6575	0.136
F1	3.57	1.273	0.238
F6	1.99	0.6881	0.150

TABLE D.6

SAMPLE 2 (SIMULATES NEW STARTUPS)

PRODUCT	a	ln a	b
F7	1.77	0.5710	0.123
F8	1.36	0.3075	0.0551
F10	1.80	0.5878	0.128
F11	1.48	0.3920	0.0758
F13	3.85	1.348	0.271
F16	4.94	1.597	0.246

REGRESSION EQUATION (FROM SAMPLE 1)

$$b = 0.0518 + 0.141 \ln \underline{a}$$

"ULTIMATES"

PRODUCT	x_u	y_u
F7	37	236
F8	32	241
F10	33	168
F11	39	215
F13	88	295
F16	110	169

PREDICTIONS

<u>Product F7</u>

Parameter b

Parameter b is calculated through equation (C.9), as follows:

$$b_2 = b_1 - \frac{n \left[b_1 \ln x_u - \ln (1-b_1) \right] - b_1 + m}{n \left(\ln x_u + \frac{1}{1-b_1} \right) - 1}$$

Set $b_1 = 0.100$

$$b_2 = 0.100 - \frac{0.141\ [0.100\ \ln 37 - \ln(1-0.100)] - 0.100+0.0518}{0.141\ (\ln 37 + \frac{1}{1-0.100}) - 1}$$

$$= 0.100 - \frac{0.0176}{-0.334} = 0.153$$

Set $b_2 = 0.153$

$$b_3 = 0.153 - \frac{0.141\ [0.153\ \ln 37 - \ln\ (1-0.153)] - 0.153+0.0518}{0.141\ (\ln 37 + \frac{1}{1-0.153}) - 1}$$

$$= 0.153 - \frac{0.000112}{-0.324} \cong 0.153.$$

Parameter a

Parameter a is calculated through equation (C.10) as follows:

$$a = \frac{x_u^b}{1-b} = \frac{37^{0.153}}{1-0.153} = 2.05$$

Product F8

Parameter b

Set $b_1 = 0.100$

$$b_2 = 0.100 - \frac{0.141\ [0.100\ \ln 32 - \ln\ (1-0.100)] - 0.100+0.0518}{0.141\ (\ln 32 + \frac{1}{1-0.100}) - 1}$$

$$= 0.364$$

Set $b_2 = 0.364$

$$b_3 = 0.364 - \frac{0.141 \left[0.364 \ln 32 - \ln(1-0.364)\right] - 0.364+0.0518}{0.141 \left(\ln 32 + \frac{1}{1-0.364}\right) - 1}$$

$$= 0.121$$

Set $b_3 = 0.121$

$$b_4 = 0.121 - \frac{0.141 \left[0.121 \ln 32 - \ln(1-0.121)\right] - 0.121+0.0518}{0.141 \left(\ln 32 + \frac{1}{1-0.121}\right) - 1}$$

$$= 0.144$$

Set $b_4 = 0.144$

$\therefore b_5 \cong 0.144$

Parameter a

$$\underline{a} = \frac{x_u^b}{1-b} = \frac{32^{0.144}}{1-0.144} = 1.92$$

Product F10

Similarly we have:

Parameter b

$b = 0.146$

Parameter a

$$\underline{a} = \frac{x_u^b}{1-b} = \frac{33^{0.146}}{1-0.146} = 1.95$$

Product F11

Parameter b

b = 0.157

Parameter a

$$\underline{a} = \frac{x_u^b}{1-b} = \frac{39^{0.157}}{1-0.157} = 2.11$$

Product F13

Parameter b

b = 0.251

Parameter a

$$\underline{a} = \frac{x_u^b}{1-b} = \frac{88^{0.251}}{1-0.251} = 4.11$$

Product F16

Parameter b

b = 0.306

Parameter a

$$\underline{a} = \frac{x_u^b}{1-b} = \frac{110^{0.306}}{1-0.306} = 6.07$$

RESULTS

TABLE D.7

PREDICTED PARAMETER b VALUES

(1)	(2)	(3)	(4)	(5)	(6)
PRODUCT	b (actual)	$\bar{b}$	Δ%	b_p	Δ%
F7	0.123	0.177	44	0.153	24
F8	0.0551	0.177	221	0.144	161
F10	0.128	0.177	38	0.146	14
F11	0.0758	0.177	134	0.157	107
F13	0.271	0.177	-35	0.251	-7.4
F16	0.246	0.177	-28	0.306	24

TABLE D.8

PREDICTED PARAMETER a VALUES

(1)	(2)	(3)	(4)	(5)	(6)
PRODUCT	a (actual)	$\bar{a}$	Δ%	a_p	Δ%
F7	1.77	2.52	42	2.05	16
F8	1.36	2.52	85	1.92	41
F10	1.80	2.52	40	1.95	8.3
F11	1.48	2.52	70	2.11	42
F13	3.85	2.52	-35	4.11	6.7
F16	4.94	2.52	-49	6.07	23

APPENDIX E

APPENDIX E

A CODE FOR CALCULATING $a\sum_{1}^{x_u}$ WITH AN

HP-25 SCIENTIFIC PROGRAMMABLE POCKET CALCULATOR

Switch to PRGM mode and press f PRGM to clear program memory and display step 00. Then key in the list of keys below:

Keys	Comments
1	
STO +1	
RCL 1	
ENTER	
b	Enter parameter b value
f y^x	
g 1/x	
a	Enter parameter a value
x	
STO +2	
f SST 6	
RCL 1	
x_u	Enter x_u value
−	
g x=0	
R/S	
GTO 01	

To run the program switch to automatic RUN mode and press f PRGM so that the calculator will begin execution from step 00. Press R/S to start execution. When execution stops press RCL 2 to retrieve the result stored in register R2

It takes approximately 20 seconds to compute a $\sum_{1}^{10} x^{-b}$

APPENDIX F

DATA TABLES (CHAPTER VI)

TABLE 6.3

FINAL ASSEMBLY OF THE CPU OF A
THIRD GENERATION COMPUTER (FIRM A)

Startup Code	Productivity Index (y) at Cum. Unit 1	2	3	4	5
A1	1.87	1.68	1.65	1.49	1.39
A2	1.68	1.59	1.49	1.38	1.30
A3	2.70	2.18	1.94	1.76	1.63
A4	2.40	2.26	1.95	1.80	1.67
A5	2.12	1.92	1.72	1.58	1.47

TABLE 6.4

MANUFACTURING OF CARD PUNCH X (FIRM A)

Startup Code	Productivity Index (y) at Cumulative Unit 94	183	277	421	584	757
A6	1.53	1.39	1.34	1.29	1.21	1.16
A7	1.22	1.05	1.00	1.00	1.00	1.00
A8	1.24	1.13	1.09	1.15	1.11	1.08
A9	1.71	1.65	1.63	1.53	1.38	1.29
A10	1.89	1.68	1.60	1.42	1.31	1.24
A11	1.81	1.67	1.61	1.47	1.34	1.26

TABLE 6.5

ASSEMBLY OF MAJOR UNITS

OF CARD PUNCH Y (FIRM A)

Startup	Productivity Index (y) at Cumulative Unit					
Code	111	155	214	287	394	575
A12	1.34	1.31	1.29	1.27	1.21	1.16

TABLE 6.6

FINAL ASSEMBLY & TESTING OF

CARD PUNCH X (FIRM B)

Startup	Productivity Index (y) at Cumulative Unit					
Code	15	40	52	358	1007	1912
B1	2.44	2.52	2.37	1.61	1.38	1.22

TABLE 6.7

FINAL ASSEMBLY OF CARD PUNCH Y (FIRM C)

Startup	Productivity Index (y) at Cumulative Unit										
Code	40	84	134	195	264	340	410	490	580	680	800
C1	3.07	2.73	2.52	2.33	2.18	2.06	1.99	1.91	1.84	1.76	1.70
	920	1050	1180	1320	1470	1610	1780	1940	2140		
	1.64	1.59	1.54	1.50	1.46	1.43	1.39	1.37	1.33		

TABLE 6.8

ASSEMBLY OF 2nd GENERATION COMPUTER UNITS (FIRM D, PROGRAM # 1)

Startup	Productivity Index (y) at Cumulative Unit										
Code	1	2	3	4	5	6	7	8	9	10	11
D1	2.68	2.02	1.75	1.60	1.48						
D2	3.12	2.59	2.38	2.26	2.06	1.92	1.81	1.73	1.66	1.59	1.54
D3	1.31	1.28	1.23	1.19	1.16	1.13	1.11	1.09	1.09	1.08	
D4	2.47	2.05	1.89	1.66							
D5	1.23	1.16	1.19	1.16	1.12	1.10					
D6	2.86	2.57	2.23	2.23	2.05	1.99	1.89	1.83	1.84	1.82	1.79

(cont'd)

	12	13	14	15	16	17	18	19	20	21	22
D6	1.74	1.69	1.69	1.68	1.66	1.64	1.60	1.58	1.54	1.52	1.49

TABLE 6.9

ASSEMBLY OF SMALL COMPUTER

COMPONENTS (FIRM D, PROGRAM # 2)

Startup	Productivity Index (*y*) at Cumulative Unit									
Code	5	10	15	20	23	30	37	38	39	40
D7	3.07	2.44		1.95		1.70	1.57			
D8	1.13	1.07		1.09		1.08			1.06	
D9	1.38	1.33		1.25		1.18				1.14
D10	3.51	2.82		2.05		1.70				
D11	2.41	2.12		2.03		1.53				
D12	3.56	2.83	2.39		1.90					
D13	3.37	2.73		1.96		1.64				
D14	1.23	1.16	1.16	1.14		1.12		1.09		

TABLE 6.10

ASSEMBLY OF COMPUTER COMPONENTS

AND DATA STORAGE UNITS (FIRM E)

Startup	Productivity Index (y) at Cumulative Unit											
Code	5	10	20	30	40	50	60	70	80	90	100	110
E1	4.35	4.58	2.86	2.31	2.03		1.94	1.82	1.73	1.65		
E2	1.89	1.80	1.71	1.55	1.47		1.31					
E3	2.59	2.32	2.25	2.09	1.92	1.80	1.70	1.59				
E4	2.73	2.28	2.06	1.91	1.73	1.61	1.51	1.43				
E5	2.89	2.57	2.32	2.33	2.15	2.00	1.86	1.74	2.43	2.30	2.18	2.07
E6	3.34	3.24	2.96	2.85	2.70	2.59	2.55	2.50				
E7	2.94	2.19	1.79	1.77	1.60	1.49						
E8	2.03	2.00	1.56	1.48	1.35	1.29						
E9	5.14	4.08	3.47	2.87	2.53	2.31	2.10	1.94				
E10	2.23	2.14	1.75	1.72	1.59	1.53	1.44					
E11	3.79	4.03	3.28	3.07	2.76	2.59	2.52	2.38	2.21			
E12	2.98	2.53	2.04	1.99	1.85		1.65	1.56				

TABLE 6.11

PROGRESS FUNCTION REGRESSION RESULTS

STARTUP CODE	PARAMETER A	PARAMETER B	WEIGHT SLOPE	NO. OF OBS.	CORREL. COEFF.	DETERM. COEFF.	T-RATIOS
A1	1.893	-.1734	.8867	5	-.963	.927	-6.172
A2	1.722	-.1572	.8968	5	-.964	.929	-6.266
A3	2.707	-.3111	.8060	5	-1.000	.999	-61.153
A4	2.492	-.2310	.8520	5	-.966	.933	-6.449
A5	2.173	-.2278	.8539	5	-.985	.969	-9.763
A6	2.728	-.1273	.9156	6	-.993	.986	-16.791
A7	1.721	-.0875	.9412	6	-.852	.726	-3.252
A8	1.515	-.0509	.9653	6	-.787	.620	-2.555
A9	3.253	-.1322	.9125	6	-.922	.850	-4.770
A10	4.876	-.2047	.8677	6	-.992	.985	-16.094
A11	4.101	-.1736	.8866	6	-.978	.956	-9.352
A12	2.034	-.0865	.9418	6	-.976	.953	-8.958
B1	4.197	-.1606	.8947	6	-.977	.955	-9.226
C1	7.230	-.2180	.8598	20	-.997	.994	-56.122
D1	2.649	-.3675	.7751	5	-.999	.997	-32.111
D2	3.233	-.2974	.8137	11	-.993	.985	-24.394
D3	1.341	-.0936	.9372	10	-.983	.966	-14.974
D4	2.483	-.2746	.8267	4	-.992	.984	-11.041
D5	1.230	-.0543	.9631	6	-.891	.795	-3.933
D6	2.892	-.2052	.8674	22	-.995	.989	-43.148
D7	5.252	-.3326	.7941	5	-1.000	1.000	*******
D8	1.155	-.0226	.9845	5	-.768	.590	-2.077
D9	1.625	-.0929	.9376	5	-.984	.969	-9.690
D10	6.945	-.4085	.7534	4	-.996	.992	-15.766
D11	3.521	-.2202	.8584	4	-.907	.822	-3.037
D12	7.002	-.4061	.7547	4	-.993	.986	-11.886

TABLE 6.11

PROGRESS FUNCTION REGRESSION RESULTS

STARTUP CODE	PARAMETER A	PARAMETER B	WEIGHT SLOPE	NO. OF OBS.	CORREL. COEFF.	DETERM. COEFF.	T-RATIOS
D13	6.683	-.4085	.7534	4	-.995	.990	-14.307
D14	1.333	-.0533	.9637	6	-.973	.947	-8.484
E1	8.889	-.3768	.7701	9	-.972	.945	-10.933
E2	2.471	-.1422	.9061	6	-.956	.914	-6.529
E3	3.548	-.1736	.8867	8	-.960	.922	-8.410
E4	4.034	-.2348	.8498	8	-.990	.979	-16.818
E5	3.911	-.1746	.8860	8	-.963	.928	-8.787
E6	4.497	-.1463	.9035	12	-.956	.915	-10.353
E7	4.359	-.2763	.8257	6	-.982	.964	-10.383
E8	3.017	-.2140	.8621	6	-.972	.945	-8.311
E9	9.602	-.3651	.7764	8	-.993	.986	-20.276
E10	3.075	-.1794	.8831	7	-.982	.965	-11.747
E11	5.948	-.2109	.8640	9	-.954	.910	-8.431
E12	4.349	-.2376	.8482	7	-.994	.989	-21.107
F1	3.565	-.2376	.8482	8	-.996	.993	-29.102
F2	1.488	-.0604	.9590	5	-.870	.757	-3.053
F3	1.825	-.1099	.9266	5	-.943	.889	-4.897
F4	1.971	-.1836	.8805	5	-.967	.936	-6.599
F5	2.014	-.1472	.9030	7	-.982	.964	-11.634
F6	1.991	-.1500	.9013	5	-.999	.998	-43.161
F7	1.773	-.1232	.9182	5	-.999	.998	-44.283
F8	1.357	-.0551	.9625	8	-.965	.931	-8.996
F9	1.932	-.1362	.9099	5	-.990	.981	-12.350
F10	1.796	-.1275	.9154	5	-.984	.969	-9.718
F11	1.477	-.0759	.9488	5	-1.000	.999	-64.737
F12	1.832	-.1482	.9024	5	-.999	.999	-49.665

TABLE 6.11

PROGRESS FUNCTION REGRESSION RESULTS

STARTUP CODE	PARAMETER A	PARAMETER B	WRIGHT SLOPE	NO. OF OBS.	CORREL. COEFF.	DETERM. COEFF.	T-RATIOS
F13	3.849	-.2712	.8286	4	-.987	.974	-8.625
F14	2.590	-.1716	.8879	6	-.991	.981	-14.430
F15	3.213	-.2187	.8593	3	-.995	.990	-9.856
F16	4.937	-.2464	.8430	6	-.984	.968	-10.927
F17	3.423	-.2371	.8484	5	-.999	.999	-48.625
F18	2.700	-.1761	.8851	6	-.999	.998	-48.471
F19	2.705	-.1860	.8790	6	-.994	.989	-18.933
F20	4.463	-.2352	.8496	6	-.994	.988	-18.224
F21	2.015	-.1230	.9183	6	-.975	.952	-8.923
F22	3.171	-.2007	.8701	6	-.999	.999	-53.901
F23	2.804	-.1330	.9119	24	-.998	.996	-70.719
F24	4.735	-.2283	.8536	9	-.994	.988	-24.348
F25	2.413	-.1199	.9202	18	-.986	.971	-23.308
F26	1.570	-.0731	.9506	10	-.811	.657	-3.916
F27	1.948	-.1313	.9130	12	-.976	.952	-14.133
F28	3.157	-.2268	.8546	12	-.999	.998	-81.052
G1	1.362	-.0877	.9410	5	-.953	.909	-5.480
G2	1.793	-.1055	.9295	5	-.870	.757	-3.058
G3	1.237	-.0681	.9539	5	-.988	.976	-11.066
G4	1.445	-.1066	.9287	5	-.933	.871	-4.493
G5	1.273	-.0727	.9508	5	-.965	.931	-6.352
G6	1.275	-.0878	.9410	5	-.986	.972	-10.224
G7	1.101	-.0303	.9792	5	-.842	.708	-2.699
G8	1.093	-.0050	.9965	5	-.233	.054	-.415
G9	1.448	-.1131	.9246	5	-.963	.927	-6.194
G10	1.955	-.1397	.9077	5	-.809	.655	-2.387

TABLE 6.11

PROGRESS FUNCTION REGRESSION RESULTS

STARTUP CODE	PARAMETER A	PARAMETER B	WEIGHT SLOPE	NO. OF OBS.	CORREL. COEFF.	DETERM. COEFF.	T-RATIOS
G11	1.361	-.1007	.9326	5	-.961	.924	-6.054
G12	1.022	.0363	1.0255	5	.904	.817	3.656
G13	1.072	.0661	1.0469	5	.630	.397	1.405
G14	1.964	-.2517	.8399	5	-.993	.986	-14.584
G15	1.642	-.0630	.9573	5	-.417	.174	-.794
G16	2.109	-.2663	.8315	5	-.999	.999	-53.080
G17	1.356	-.1058	.9293	5	-.997	.995	-23.676
G18	1.293	-.0900	.9395	5	-.987	.975	-10.842
G19	1.150	-.0509	.9653	5	-.999	.998	-40.179
H1	1.450	-.0702	.9525	5	-.919	.845	-4.042
H2	2.520	-.1524	.8998	7	-.991	.981	-16.126
H3	3.132	-.2564	.8372	5	-.969	.938	-6.746
I1	1.877	-.1058	.9293	6	-.934	.871	-5.208
I2	1.833	-.1033	.9309	10	-.886	.785	-5.401
I3	1.994	-.1424	.9060	9	-.980	.961	-13.165
I4	3.240	-.2255	.8553	9	-.992	.983	-20.297
J1	2.423	-.0655	.9556	5	-.990	.981	-12.467
J2	1.831	-.0457	.9688	5	-.984	.968	-9.558
J3	1.969	-.0498	.9661	5	-.989	.979	-11.824
J4	3.708	-.0969	.9350	5	-.994	.988	-15.996
J5	2.672	-.0732	.9505	5	-.996	.992	-19.377
J6	2.053	-.0594	.9597	5	-.939	.882	-4.745
J7	3.293	-.0833	.9406	5	-.951	.904	-5.326
J8	.447	.0575	1.0407	5	.991	.981	12.513
J9	5.960	-.1598	.8952	5	-.991	.982	-12.763
J10	3.219	-.0691	.9401	5	-.939	.882	-4.737

TABLE 6.24

MANUFACTURING OF BINDING MACHINES AND PRINTING PRESSES

FIRM F (U.S.)

Startup Code: x = cumulative production; y = productivity index

Startup Code									
F1	x...	10	13	19	23	30	36	51	56
	y...	2.03	1.94	1.78	1.71	1.62	1.53	1.39	1.35
F2	x...	1	4	12	17	25			
	y...	1.43	1.45	1.32	1.25	1.17			
F3	x...	7	11	16	23	27			
	y...	1.44	1.44	1.37	1.29	1.25			
F4	x...	2	3	4	5	8			
	y...	1.69	1.63	1.57	1.49	1.31			
F5	x...	5	8	10	13	16	18	21	
	y...	1.57	1.48	1.45	1.41	1.35	1.31	1.26	
F6	x...	6	10	18	28	37			
	y...	1.53	1.40	1.29	1.21	1.16			
F7	x...	6	16	24	32	41			
	y...	1.42	1.26	1.19	1.16	1.12			

TABLE 6.24 (cont'd)

Startup Code	x = cumulative production	y = productivity index						
F8	x... 9	17	22	30	39	48	57	66
	y...1.19	1.16	1.15	1.15	1.11	1.09	1.08	1.07
F9	x... 16	29	40	48	57			
	y...1.33	1.22	1.16	1.13	1.13			
F10	x... 12	18	25	39	50			
	y...1.32	1.24	1.18	1.11	1.11			
F11	x... 2	13	25	33	43			
	y...1.40	1.22	1.16	1.13	1.11			
F12	x... 3	4	8	15	22			
	y...1.54	1.46	1.32	1.21	1.14			
F13	x... 12	42	72	102				
	y...2.01	1.32	1.20	1.14				
F14	x... 10	25	40	55	70	85		
	y...1.71	1.54	1.39	1.29	1.24	1.20		
F15	x... 20	40	70					
	y...1.66	1.46	1.26					

TABLE 6.24 (cont'd)

Startup Code		x = cumulative production			y = productivity index		
F16	x...	20	40	60	80	100	120
	y...	2.28	2.06	1.85	1.70	1.57	1.47
F17	x...	5	15	30	45	60	
	y...	2.32	1.32	1.54	1.38	1.29	
F18	x...	10	30	50	70	90	110
	y...	1.79	1.50	1.36	1.27	1.22	1.18
F19	x...	10	20	35	50	65	80
	y...	1.73	1.59	1.41	1.30	1.24	1.19
F20	x...	10	35	60	85	110	135
	y...	2.54	1.99	1.75	1.59	1.46	1.37
F21	x...	4	14	34	54	74	94
	y...	1.64	1.55	1.31	1.22	1.18	1.14
F22	x...	5	15	30	50	70	90
	y...	2.28	1.85	1.61	1.46	1.35	1.27

TABLE 6.24 (cont'd)

Startup Code	x = cumulative production	y = productivity index							
F23	x... 10	30	55	85	111	140	167	195	
	y...2.03	1.80	1.67	1.57	1.50	1.44	1.40	1.38	
(cont'd)	x...225	253	283	313	343	373	403	433	
	y...1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.26	
(cont'd)	x...463	493	523	553	581	611	641	671	
	y...1.24	1.23	1.22	1.20	1.20	1.19	1.17	1.17	
F24	x... 6	16	36	66	96	136	186	236	286
	y... 2.99	2.55	2.17	1.89	1.72	1.55	1.41	1.32	1.27
F25	x... 10	20	40	70	90	115	145	175	205
	y... 1.76	1.70	1.51	1.52	1.45	1.39	1.34	1.31	1.29
(cont'd)	x... 235	255	270	285	295	305	315	327	339
	y... 1.26	1.24	1.23	1.21	1.20	1.20	1.19	1.18	1.18
F26	x... 10	20	35	50	60	80			
	y...1.31	1.24	1.18	1.35	1.13	1.12			
(cont'd)	x...100	120	150	168					
	y...1.11	1.10	1.08	1.07					

TABLE 6.24 (cont'd)

Startup Code — x = cumulative production — y = productivity index

Startup Code							
F27	x...	8	12	30	43	57	72
	y...	1.50	1.44	1.26	1.17	1.11	1.07
(cont'd)	x...	86	103	122	140	156	171
	y...	1.05	1.03	1.03	1.04	1.04	1.04
F28	x...	5	7	18	20	27	33
	y...	2.17	2.03	1.66	1.62	1.50	1.43
(cont'd)	x...	38	39	42	43	45	49
	y...	1.38	1.37	1.35	1.34	1.33	1.30

TABLE 6.29

PARAMETERS a AND b

(FIRM F, BINDING MACHINES)

Startup Code	a	b
F16	4.94	0.246
F13	3.85	0.271
F1	3.57	0.238
F15	3.22	0.219
F14	2.59	0.172
F5	2.01	0.147
F6	1.99	0.150
F4	1.97	0.184
F9	1.93	0.136
F3	1.83	0.110
F12	1.80	0.148
F10	1.80	0.128
F7	1.77	0.123
F2	1.49	0.0604
F11	1.48	0.0758
F8	1.36	0.0551

TABLE 6.30

PARAMETERS *a* AND b

(FIRM F, PRINTING PRESSES)

Startup Code	*a*	b
F24	4.74	0.228
F20	4.47	0.235
F17	3.42	0.237
F22	3.17	0.201
F28	3.16	0.227
F23	2.80	0.133
F19	2.71	0.186
F18	2.70	0.176
F25	2.41	0.120
F21	2.02	0.123
F27	1.95	0.131
F26	1.57	0.0731

TABLE 6.38

SHIPBUILDING

FIRM G (BRAZIL)

Startup Code	Productivity Index (y) at Cumulative Unit 1	2	3	4	5
G1	1.34	1.31	1.25	1.21	1.16
G2	1.74	1.74	1.65	1.57	1.45
G3	1.23	1.19	1.15	1.13	1.10
G4	1.41	1.39	1.31	1.24	1.19
G5	1.27	1.20	1.20	1.15	1.12
G6	1.28	1.20	1.14	1.14	1.11
G7	1.09	1.10	1.06	1.06	1.04
G8	1.08	1.11	1.09	1.09	1.07
G9	1.42	1.38	1.29	1.23	1.19
G10	1.84	1.92	1.77	1.59	1.47
G11	1.34	1.31	1.21	1.18	1.15
G12	1.02	1.04	1.08	1.08	1.07
G13	1.02	1.21	1.18	1.17	1.14
G14	1.93	1.70	1.50	1.37	1.30
G15	1.51	1.78	1.61	1.48	1.38
G16	2.10	1.77	1.57	1.46	1.37
G17	1.35	1.27	1.21	1.17	1.14
G18	1.29	1.21	1.19	1.14	1.11
G19	1.15	1.11	1.09	1.07	1.06

TABLE 6.42

OIL DRILLING TOOLS & EQUIPMENT

FIRM H (BRAZIL)

Startup Code	x = cumulative production y = productivity index						
H1	x...	24	44	45	63	65	
	y...	1.15	1.12	1.13	1.08	1.07	
H2	x...	1	21	61	71	101	131	132
	y...	2.56	1.51	1.30	1.37	1.31	1.18	1.19
H3	x...	3	5	13	20	33	
	y...	2.42	2.12	1.45	1.47	1.35	

TABLE 6.47

PARAMETERS a AND b

(FIRM I , SPARE PARTS)

Startup Code	a	b
I10	843.79	1.461
I9	247.29	1.196
I7	96.75	0.993
I37	46.02	0.8315
I5	12.80	0.554
I25	5.03	0.351
I17	4.87	0.344
I28	4.84	0.342
I27	4.385	0.321
I62	4.31	0.317
I68	4.31	0.317
I61	3.89	0.295
I65	3.78	0.289
I46	3.74	0.2865
I57	3.70	0.284
I54	3.645	0.281
I56	3.64	0.281
I6	3.438	0.268
I29	3.10	0.246

TABLE 6.47 (cont'd)

Startup Code	a	b
I41	3.01	0.240
I38	2.825	0.226
I66	2.81	0.2245
I8	2.705	0.216
I48	2.61	0.208
I15	2.47	0.196
I30	2.17	0.168
I22	2.11	0.162
I59	2.06	0.157
I69	1.98	0.148
I58	1.91	0.141
I67	1.65	0.109
I60	1.62	0.104
I64	1.62	0.104
I63	1.53	0.0928
I21	1.44	0.0799
I19	1.44	0.0790
I16	1.44	0.0786
I53	1.28	0.0542
I50	1.25	0.0485
I18	1.23	0.0442
I49	1.20	0.0396

TABLE 6.47 (cont'd)

Startup Code	a	b
I45	1.19	0.0379
I33	1.16	0.0318
I23	1.14	0.0290
I12	1.13	0.0264
I34	1.06	0.0131
I43	1.05	0.0116
I44	1.05	0.0103
I24	1.03	0.0071
I20	1.02	0.0053
I36	1.00	0.00
I26	1.00	0.00
I35	1.00	0.00
I11	1.00	0.00
I31	1.00	0.00
I13	1.00	0.00
I32	1.00	0.00
I14	1.00	0.00
I39	1.00	0.00
I40	1.00	0.00
I42	1.00	0.00
I47	1.00	0.00

TABLE 6.47 (cont'd)

Startup Code	a	b
I51	1.00	0.00
I52	1.00	0.00
I55	1.00	0.00

TABLE 6.51

MECHANICAL AND ELECTRICAL INDUSTRY

(BRAZIL : 1960-1964)

Startup Code	x = cumulative output (m.t.)			y= productivity index	
J1	x...49,500	101,100	155,100	216,600	287,100
	y... 1.19	1.14	1.12	1.08	1.06
J2	x...43,055	83,985	136,605	187,425	240,290
	y... 1.12	1.10	1.06	1.05	1.04
J3	x...41,800	86,600	133,100	179,600	231,800
	y... 1.16	1.11	1.10	1.08	1.06
J4	x...67,400	151,800	249,600	350,800	437,600
	y... 1.27	1.16	1.10	1.08	1.06
J5	x...60,500	125,000	172,600	273,600	388,200
	y... 1.19	1.14	1.10	1.07	1.04
J6	x...46,300	52,000	74,500	108,900	158,600
	y... 1.10	1.08	1.04	1.03	1.02
J7	x...14,100	39,400	73,350	113,050	153,750
	y... 1.38	1.34	1.25	1.17	1.12
J8	x... 2,240	4,190	11,190	34,390	52,540
	y... 1.70	1.64	1.32	1.10	1.07
J9	x... 2,800	9,700	36,700	72,300	116,900
	y... 1.51	1.50	1.33	1.16	1.10

APPENDIX G

REGRESSION ANALYSIS OF LOG-TRANSFORMED DATA -- A FORTRAN IV PROGRAM

```
      PROGRAM REGAN(INPUT,OUTPUT)
C*****PROGRAM***REGRESSION ANALYSIS OF LOG-TRANSFORMED DATA***            REGAN 10
C*****PROJECT**PHD, DISSERTATION #MANUFACTURING PROGRESS FUNCTIONS#**     REGAN 20
C*****BY**DAYR AMERICO DOS REIS**      **DATE AUGUST 21, 1976**           REGAN 30
C
C
C*****SET VALUES FOR CARD READER AND PRINTER                              REGAN 40
C
      NCRDR=5LINPUT                                                       REGAN 50
      NPRNT=6LOUTPUT                                                      REGAN 55
C
C*****INITIALIZE PAGE NUMBER                                              REGAN 56
      NPAGE=1                                                             REGAN 57
C
C*****PRINT HEADING, INCLUDING PAGE NUMBER                                REGAN 58
C
   80 WRITE(NPRNT,70)NPAGE                                                REGAN 59
   70 FORMAT(1H1,45X,*TABLE*,2X,*6.11*//32X,*PROGRESS*,1X,*FUNCTION*,1X,REGAN 60
     1*REGRESSION*,1X,*RESULTS*//12X,*STARTUP*,8X,*PARAMETER*,2X,*PARAMEREGAN 61
     2TER*,4X,*WRIGHT*,4X,*NO.*,1X,*OF*,4X,*CORREL.*,4X,*DETERM.*,4X,*T-REGAN 62
     3RATIOS*,11X,*PAGE *,I3/13X,*CODE*,14X,*A*,10X,*B*,9X,*SLOPE*,5X,*OREGAN 63
     4BS.*,5X,*COEFF.*,5X,*COEFF.*////)                                   REGAN631
C
C*****INCREMENT PAGE NUMBER                                               REGAN 64
      NPAGE=NPAGE+1                                                       REGAN 65
C*****INITIALIZE LINE CONTER                                              REGAN 66
      LINES=0                                                             REGAN 67
C
C*****READ ACTIVITY CODE,NUMBER OF OBSERVATIONS AND LAST RUN CODE         REGAN 70
C
   50 READ(NCRDR,10) CODE,N,N1                                            REGAN 80
   10 FORMAT(A3,I3,72X,I2)                                                REGAN 90
C
C*****CONVERT N TO FLOATING-POINT FORM                                    REGAN100
C
      AN=N                                                                REGAN110
C
C*****INITIALIZE ALL SUMS TO ZERO                                         REGAN120
C
      SUMX=0.                                                             REGAN130
      SUMY=0.                                                             REGAN140
      SUMXY=0.                                                            REGAN150
      SUMX2=0.                                                            REGAN160
      SUMY2=0.                                                            REGAN170
C
C*****READ INPUT VALUES, COMPUTE LOGARITHMS AND SUMS                      REGAN180
C
      DO 20 I=1,N                                                         REGAN190
      READ(NCRDR,30)AX,AY                                                 REGAN200
   30 FORMAT(2F10.2)                                                      REGAN210
      X=ALOG10(AX)                                                        REGAN220
      Y=ALOG10(AY)                                                        REGAN230
      SUMX=SUMX+X                                                         REGAN240
      SUMY=SUMY+Y                                                         REGAN250
      SUMXY=SUMXY+X*Y                                                     REGAN260
      SUMX2=SUMX2+X*X                                                     REGAN270
   20 SUMY2=SUMY2+Y*Y                                                     REGAN280
C
C*****CALCULATE AVERAGES                                                  REGAN290
C
      XAV=SUMX/AN                                                         REGAN300
      YAV=SUMY/AN                                                         REGAN310
C
C*****CALCULATE MODIFIED SUM OF SQUARES AND CROSS-PRODUCTS                REGAN320
C
      CSCP=SUMXY-SUMX*SUMY/AN                                             REGAN330
      CSSX=SUMX2-SUMX*SUMX/AN                                             REGAN340
      CSSY=SUMY2-SUMY*SUMY/AN                                             REGAN350
C
C*****CALCULATE REGRESSION PARAMETERS(SLOPE B AND INTERCEPT C)            REGAN360
C*****CALCULATE ANTILOG OF C AND WRIGHT#S SLOPE W                         REGAN370
C
      B=CSCP/CSSX                                                         REGAN380
      C=YAV-B*XAV                                                         REGAN390
      A=10.**C                                                            REGAN400
      W=2.**B                                                             REGAN405
```

```
C*****CALCULATE COEFFICIENTS OF CORRELATION R AND DETERMINATION R2     REGAN410
C                                                                       REGAN420
      R=CSCP/SQRT(CSSY*CSSX)                                            REGAN430
      R2=R*R
C                                                                       REGAN440
C*****CALCULATE T-STATISTICS                                            REGAN450
C                                                                       REGAN460
      TR=R*SQRT((AN-2.)/(1.-R2))
      TB=B*SQRT((AN-2.)*CSSX/(CSSY-B*CSCP))
C                                                                       REGAN470
C*****WRITE RESULTS                                                     REGAN480
C                                                                       REGAN490
      WRITE(NPRNT,40)CODE,A,B,N,M,R,R2,TB
   40 FORMAT(1H0,13X,A3,7X,F11.3,5X,F7.4,5X,F6.4,6X,I2,5X,F6.3,5X,F6.3, REGAN500
     16X,F7.3)
C                                                                       REGAN510
C*****CHECK FOR LAST RUN                                                REGAN520
C
      IF(N1)60,90,90                                                    REGAN530
C
C*****INCREMENT LINE COUNTER                                            REGAN540
C
   90 LINES=LINES+2                                                     REGAN550
C*****CHECK IF PAGE IS FULL                                             REGAN560
      IF(LINES .EQ. 52)GO TO 80                                         REGAN570
      GO TO 50                                                          REGAN580
   60 CALL EXIT                                                         REGAN590
      END
```

BIBLIOGRAPHICAL NOTES

BIBLIOGRAPHICAL NOTES

Chapter I

[1] Of special interest for the purpose of this study were: T.P. Wright, "Factors Affecting the Cost of Airplanes", Journal of Aeronautical Sciences 3, no.4 [February 1936] : 122-8; A. Alchian, "An Airframe Production Function", The RAND Corporation, P-108, October 10, 1949, and "Reliability of Progress Curves in Airframe Production", The RAND Corporation, RM-206-1, February 3, 1950; F.J. Andress,"The Learning Curve as a Production Tool", Harvard Business Review 32,no.1, [Jan-Feb 1954] : 87-97; K.J. Arrow and S.S. Arrow, "Methodological Problems in Airframe Cost-Performance Studies", The RAND Corporation, RM-456, September 20, 1950; K.J. Arrow and H. Bradley, "Cost Quality Relations in Bomber Airframes",The RAND Corporation, RM-536, February 6,1951; H. Asher, "Cost-Quantity Relationships in the Airframe Industry", The RAND Corporation, R-291, July 1, 1956; N. Baloff, "Manufacturing Startup: A Model", Unpublished Ph.D. dissertation, Stanford University, 1963; "Startups in Machine-intensive Production Systems", The Journal of Industrial Engineering 17, no.1 [Jan-Feb 1966]: 25-32; "The Learning Curve - Some Controversial Issues", Journal of Industrial Economics, [July 1966] : 275-82; "Estimating the Parameters of the Startup Model - An Empirical Approach", The Journal of Industrial Engineering 18, no.4 [April 1967] : 248-53, and "Extension of the Learning Curve - Some Empirical Results", Operations Research

Quarterly 22, no.4 [December 1971] : 329-40; N. Baloff and S. Becker, "A Model of Group Adaptation to Problem Solving Tasks", Human Performance and Organizational Behavior 3, [August 1968] : 217-38; S.E. Bryan, "A Melhoria da Produção no Brasil", Revista de Administração de Empresas 1, no.2 [Sep.-Dec. 1961] : 27-55, and "Fair Value and the Learning Curve", Purchasing, no. , [September 1954] : ; R. W. Conway and A. Schultz, Jr., "The Manufacturing Progress Function", The Journal of Industrial Engineering 10, no. 1 [Jan-Feb 1959] : 39-53; W. Z. Hirsch, "Manufacturing Progress Functions", The Review of Economics and Statistics 34, no. 2 [May 1952] : 145-55, and "Firm Progress Ratios",Econometrica 24, no.2 [April 1956] : 136-43; W.B. Hirschman, "Profit from the Learning Curve", Harvard Business Review 42, no. 1 [January 1964] : 125-39; M.D. Kilbridge, "Predetermined Learning Curves for Clerical Operations", The Journal of Industrial Engineering 10, no.3 [May-June 1959] : 206- ; F.K. Levy, "Adaptation in the Production Process", Management Science 11, no.4 [April 1965] : 136-54; Stanford Research Institute, An Improved Rational and Mathematical Explanation of the Progress Curve in Airframe Production. Stanford: SRI, August 10, 1949, and Relationships for Determining the Optimum Expansibility of the Elements of a Peacetime Aircraft Procurement Program. Stanford: SRI, December 31, 1949; P.F. Williams, "The Application of Manufacturing Improvement Curves in Multi-Product Industries", The Journal of Industrial Engineering 12, no.2 [Mar-Apr 1961] : 108-112.

2 Baloff, "Manufacturing Startup: A Model", pp.5-6.

3 Asher, "Cost-Quantity Relationships in the Airframe Industry"; Baloff, "Startups in Machine-intensive Production Systems"; Conway and Schultz, Jr., "The Manufacturing Progress Function", and R.P. Zieke, "Progress Curve Analysis in the Aerospace Industry", The Boeing Co., 1962.

4 Asher, "Cost-Quantity Relationships in the Airframe Industry" ; Baloff, "The Learning Curve - Some Controversial Issues"; Conway and Schultz, Jr., "The Manufacturing Progress Function"; Hirsch, "Firm Progress Ratios", and "Manufacturing Progress Functions", and Levy, "Adaptation in the Production Process".

5 Baloff, "Startups in Machine-intensive Production Systems", and "Extension of the Learning Curve - Some Empirical Results"; Bryan, "Fair Value and the Learning Curve"; Conway and Schultz, "The Manufacturing Progress Function" ; Hirsch, "Firm Progress Ratios", and "Manufacturing Progress Functions", and Wright, "Factors Affecting the Cost of Airplanes".

6 Alchian, "An Airframe Production Function", and "Reliability of Progress Curves in Airframe Production"; Asher , "Cost-QUantity Relationships in the Airframe Industry" ; Baloff, "Manufacturing Startup: A Model", "Estimating the Parameters of the Startup Model - An Empirical Approach", and "Extension of the Learning Curve - Some Empirical Results";

Conway and Schultz, Jr., "The Manufacturing Progress Function"; Hirsch, "Firm Progress Ratios", and "Manufacturing Progress Functions".

[7] Andress, "The Learning Curve as a Production Tool"; Asher, Cost-Quantity Relationships in the Airframe Industry"; Baloff, "The Learning Curve - Some Controversial Issues", and "Extension of The Learning Curve - Some Empirical Results" , and Conway and Scultz, Jr.,"The Manufacturing Progress Function".

[8] Kaplan, A., The Conduct of Inquiry: Methodology for Behavioral Science . San Francisco, Calif. : Chandler Publishing Co., 1965.

[9] Ibid.

Chapter II

[1] Wright, "Factors Affecting the Cost of Airplanes".The original formulation of the progress curve theory is attributed to T.P. Wright. Nevertheless, at least two authors believe that Leslie McDill should get the credit for its earliest idealization. According to Reguero (see M. A. Reguero, "An Economic Study of the Airframe Industry", Unpublished Ph.D. dissertation, Department of Economics, New York University, October, 1957, p.213), the credit for the first investigation which led to the formulation of the progress curve theory was given to Leslie McDill, who was commanding

officer at McCook Field (the predecessor of Wright-Patterson Air Force Base, near Dayton, Ohio) in 1925. Also, Max Stupar (see Max Stupar, "Forecasting of Airplane Man-Hours", Unpublished manuscript, Headquarters Air Materiel Command, 1942, p.7) says that about 1932 Leslie McDill studied costs of airplanes in various quantities and arrived at the conclusion that doubling cumulative airframe output was accompanied by an average reduction in man-hour consumption of about 20 per cent. This meant that the average labor requirement after doubling quantities of output was about 80 per cent of what it had been before. Neither of the above mentioned authors give the source of their information. Whether or not Wright originated the learning curve concept cannot be determined with certainty. Since McDill did not publish his findings, one will never know precisely what was the character and extension of his contribution. Nonetheless, it is granted that Wright was the first to make the theory known.

[2] This latter term is misleading since it is mathematically illogical - it is neither the slope of $\bar{y}$ nor of log $\bar{y}$.

[3] Wright, "Factors Affecting the Cost of Airplanes" , p. 124.

[4] Ibid., p. 124

[5] Ibid., p. 125.

[6] See J.M. White, "The Use of Learning Curve Theory in Setting Management Goals", <u>The Journal of Industrial</u>

Engineering 12, no.6 [November-December 1961] : 409-411, p. 410; see also Zieke, "Progress Curve Analysis in the Aero space Industry", p.2.

[7] Asher, "Reliability of Progress Curves in Airframe Production", p.16.

[8] Crawford's notation is slightly different. However, the present dissertation requires standardization of all symbols employed in order to make the discourse coherent to the reader, particularly in the chapters ahead.

[9] J.R. Crawford, Learning Curve, Ship Curve, Ratios, Related Data, Lockheed Aircraft Corporation, Burbank, California (n.d.), p.52.

[10] ——————, Estimating, Budgeting, and Scheduling, Lockheed Aircraft Corporation, Burbank, California, 1945, p.51.

[11] Ibid., p. 26

[12] Ibid.

[13] F.J. Montgomery, "Increased Productivity in the Construction of Liberty Vessels", Monthly Labor Review, no. [November 1943] : 861-64.

[14] A.D. Searle, "Productivity Changes in Selected Wartime Ship Building Programs", Monthly Labor Review 61, no.6 [December 1945] : 1132-47.

[15] Ibid., p. 1144.

[16] K.A. Middleton, "Wartime Productivity Changes in the Airframe Industry", Monthly Labor Review 61, no.2 [August 1945] : 215-25.

[17] The idea of working with this new function, i.e., man--hours per pound of airframe versus cumulative production in pounds is due to A.B. Berghell. See Berghell. Production Engineering in the Aircraft Industry. New York: McGraw-Hill Book Company, Inc., 1944, Chap.12.

[18] K.A. Middleton, "Wartime Productivity Changes in the Airframe Industry", p.221.

[19] E.A. Rutan, Theory of Learning Curves, Chance Vought Aircraft Incorporated, Dallas, Texas, October, 1948.

[20] Gordon W. Link and Don A. Ellis, The Experience Curve, Boeing Aircraft Company, Wichita, Kansas, December, 1945.

[21] "Source Book of World War II Basic Data", Airframe Industry, Vol.1, Air Materiel Command, Dayton, Ohio, 1947.

[22] J.R. Crawford, and E. Strauss, World War II Acceleration of Airframe Production, Air Materiel Command, Dayton , Ohio, 1947.

[23] "Source Book of World War II Basic Data", Air Materiel Command.

[24] J.R. Crawford, and E. Strauss, World War II Acceleration of Airframe Production, p. 13.

[25] *Relationships for Determining the Optimum Expansibility of the Elements of a Peace-time Aircraft Procurement Program*, Stanfors Research Institute, prepared for Air Materiel Command, USAF, December 31, 1949.

[26] A.Garg and P. Milliman, "The Aircraft Progress Curve - Modified for Design Changes", *The Journal of Industrial Engineering* 12, no.1 [Jan-Feb 1961] : 23-28.

[27] Eric Mensforth, "Airframe Production", Parts I and II, *Aircraft Production* 9, [September 1947] :343-50, and [October 1947] : 388-95.

[28] Ibid., p.392

[29] P. Guibert. *Le Plan de Fabrication Aéronautique*.Paris: Dunod, 1945.

[30] ————. *Mathematical Studies of Aircraft Construction*, Central Air Documents Office, Wright-Patterson Air Force Base, Dayton, Ohio.

[31] Ibid.,p.64.

[32] For example, Guibert found that *m* may be approximated by

$$m = 1 - \frac{20}{a + 81} \quad \text{where} \quad a = \text{rate of production}$$

Other empirical equations were derived for A and α, as well. See Guibert. *Le Plan de Fabrication Aéronautique*.

[33] A.B. Berghell. _Production Engineering in the Aircraft Industry_. New York: McGraw-Hill Book Company, Inc., 1944, Chap. 12.

[34] Ibid., p.177.

[35] Berghell does not state whether this common slope is 80 per cent or some other value.

[36] I.M. Laddon, "Reduction of Man-Hours in Aircraft production", _Aviation_ [May 1943] : 170-73.

[37] G.W. Carr, "Peacetime Cost Estimating Requires New Learning Curves", _Aviation_ 45, no.4 [April 1946] : 76-77.

[38] Boeing Transport Division, _Improvement Curve Handbook_, Renton, Washington: Boeing Airplane Company, 1956.

[39] _Relationships for Determining the Optimum Expansibility of the Elements of a Peace-time Aircraft Procurement Program, Stanford Research Institute_.

[40] Guibert, _Le Plan de Fabrication Aéronautique_.

[41] Werner Z. Hirsch, "Manufacturing Progress Functions", _The Review of Economics and Statistics_ 34, no.2 [May 1952] : 145-55, and "Firm Progress Ratios", _Econometrica_ 24, no.2 [April 1956] : 136-43.

[42] Hirsch, "Firm Progress Ratios".

43 Hirsch's terminology is suggestive: The exponent (-b) of the progress function $y = ax^{-b}$ is called "progress elasticity". The complement to one of Wright's "slope" is defined as the "progress ratio". It is easy to see that Hirsch's "progress ratio" and the exponent b vary in the same direction.

44 Computed for eight products only.

45 A. Alchian, "An Airframe Production Function", The RAND Corporation, P-108, October 10, 1949, p.12.

46 Stanley E. Bryan, "Fair Value and the Learning Curve", Purchasing [September 1954] :

47 Ibid., p. 97.

48 This account of the Schultz and Conway research is based on three main sources: (1) R.W. Conway and A. Schultz, Jr.,"The Manufacturing Progress Function", The Journal of Industrial Engineering 10, no.1 [Jan-Feb 1959] : 39-53; (2) Schreiner, D.A., "The Manufacturing Progress Function, Its Application to Operations at IBM, Endicott", Paper presented at 12th Annual ASQC Convention, Boston, Mass., May, 1958,and (3) information independently gathered by this author.

49 Rate of progress here has the same meaning as Hirsch's progress ratio. See note # 43.

50 R. W. Conway and A. Schultz, Jr., "The Manufacturing Progress Function", p.42.

51 Ibid., p.43.

52 Schreiner, D.A., "The Manufacturing Progress Function, Its Application to Operation at IBM, Endicott".

53 The model $y = ax^{-b}$ describes the progress phenomenon only up to point (x_u, y_u). After that a plateau is assumed. Recall that y_u is pratically defined as the point in cumulative production at which the reduction in manufacturing hours per unit from the first unit produced in the month to the last unit of the month is approximately 2-3%. See p.

54 A.W. Morgan, Experience Curves Applicable to the Aircraft Industry, The Glenn L. Martin Company, Baltimore, Maryland, 1952.

55 W.A. Rayborg, Mechanics of the Learning Curve,Northrop Aircraft, Inc., Hawthorne, California, March, 1952.

56 E.J. Blume and D. Peitzke, Purchasing with the Learning Curve, North American Aviation, Inc., Inglewood, California, August, 1953.

57 William F. Brown, The Improvement Curve, Boeing Airplane Company, Wichita, Kansas, March, 1955, and Roy W.Smith, William C. Lansing, and Henry G. Horton, Improvement Curve Handbook, Boeing Airplane Company, Seattle, Washington, 1956.

58 William F. Brown, The Improvement Curve, p.6.

59 The Experience Curve and Improvement Curve Study, Boeing Airplane Company, Wichita, Kansas (n.d.), p.9 .

60 A. Alchian,"Reliability of Progress Curves in Airframe Production".

61 H. Asher, "Cost-Quantity Relationships in the Airframe Industry".

62 Ibid., p.129.

63 F.T. Koen, "Dynamic Evaluation", Factory 117, no.9 [September 1959] : 98-103.

64 F.J. Andress, " The Learning Curve as a Production Tool", Harvard Business Review 32, no.1 [Jan-Feb 1954] :87-97.

65 N. Baloff, "Manufacturing Startup: A Model",Unpublished Ph.D. dissertation, Stanford University, 1963; "Startups in Machine-Intensive Production System", The Journal of Industrial Engineering 17, no.1 [Jan-Feb 1966] : 25-32, and "Estimating the Parameters of the Startup Model - An Empirical Approach", The Journal of Industrial Engineering 18,no.4 [April 1967] : 248-53.

66 N. Baloff, "Extension of the Learning Curve - Some Empirical Results", Operations Research Quarterly 22, no. 4 [December 1971] : 329-40.

67 Baloff, "Manufacturing Startup: A Model", and "Startups in Machine-Intensive Production Systems". Baloff's model is

the power function $y = ax^b$ where y is per cent performance efficiency (PPE), x is cumulative output (in appropriate units) and a and b are parameters of the model. The relationship is very similar to those previously used in labor-intensive manufacture. The per cent performance efficiency (PPE) is an index that relates the actual performance of the process on each product to predetermined, engineered standards of performance. For example, for a given product, a 70% PPE indicates that the process has performed at 70% of the standard output level for that product.

68 Baloff, "Extension of the Learning Curve - Some Empirical Results".

69 Hirschmann, "Profit From the Learning Curve".

70 Baloff, "Manufacturing Startup: A Model", and "Startups in Machine-Intensive Production Systems.

71 Alchian, "Reliability of Progress Curves in Airframe Production", and Asher, "Cost-Quantity Relationships in the Airframe Industry".

72 Baloff, "Manufacturing Startup: A Model", and "Startups in Machine-Intensive Production Systems".

73 Alchian, "Reliability of Progress Curves in Airframe Production", and Asher, "Cost-Quantity Relationships in the Airframe Industry".

74 Conway and Schultz, "The Manufacturing Progress Function".

75 Baloff,"Estimating the Parameters of the Startup Model - An Empirical Approach", p.250.

76 Asher, "Cost-Quantity Relationship in the Airframe Industry", pp 74-79.

77 Baloff, "Manufacturing Startup: A Model".

78 Baloff and Becker, "Group Learning Curves: A Power Function?", Working Paper, Graduate School of Business, Univeristy of Chicago.

79 S. Becker and N. Baloff, "Organization Structure and Problem Solving Behavior", Administrative Science Quarterly 14, no.6 [June 1969] : 260-71.

80 Baloff, "Extension of the Learning Curve - Some Empirical Results".

81 Philip B. Metz, "A Manufacturing Progress Function Nomograph", The Journal of Industrial Engineering 13, no.4 [July-Aug 1962] : 253-56.

82 J.G. Kneip, "The Maintenance Progress Function", The Journal of Industrial Engineering 16, no.6 [Nov-Dec 1965] : 398-400

83 J.H. Russell, "Progress Function Models and their Deviations", The Journal of Industrial Engineering 19, no.1 [January 1968] : 5-10.

84 J.H. Russell,"Predicting Progress Function Deviations", IBM TR 22.4 , 1967.

85 J.M.White, "The use of Learning Curve in Setting Management Goals",The Journal of Industrial Engineering 12,no.6 [Nov-Dec 1961] : 409-411.

86 P.F. Williams, "Application of Manufacturing Improvement Curves to Multi-Product Industries", The Journal of Industrial Engineering 12, no.2 [March-April 1961] :108-112.

87 W.B. Hirschmann, "Profit From the Learning Curve", Harvard Business Review 42, no.1 [Jan-Feb 1964] : 125-139, p. 125.

88 W.J. Abernathy and K. Wayne,"Limits of the Learning Curve", Harvard Business Review no.5 [Sept-Oct 1974]:109-119.

Chapter III

1 Allen, R.G.D. Mathematical Analysis for Economists . London: MacMillan & Co. Ltd., 1962, pp 251-55.

2 For example, in the airframe industry the independent variable is sometimes the weight of the airplane.In petroleum refining Hirschmann plotted a progress curve of performance of cracking units (expressed in days per 100,000 barrels) *versus* cumulative throughput (expressed in million barrels). In steel-making (Baloff),outputs are expressed in thousands of

tons, millions of lineal feet, and so on. See Chapter 2.

3 A formal proof that $\lim_{x \to \infty} (ax^{-b}) = 0$ and that $\lim_{x \to 0} (ax^{-b}) = \infty$ is given in Appendix A.

4 See, for example, Marlin U. Thomas, "Developing More Accurate Cubic Learning Curves", Manufacturing Engineering and Management, [July 1972] : 29-30.

5 Franklin, P. Differential and Integral Calculus . N. York: McGraw-Hill Book Company, Inc., 1953, p.366.

6 ——————. Methods of Advanced Calculus. N.York : McGraw-Hill Book Company, Inc., 1944, pp 26-28.

7 An initial value B_1 may be found by some means, e.g. by a graph or by tabulation.

8 If B_1 is a first approximation to the root of the equation $f(B)=0$, the Newton-Raphson formula for a better approximation B_2 is $B_2 = B_1 - f(B_1)/f'(B_1)$. If ε is the error in B_1, i.e., $B_1 = B + \varepsilon$, one can write:

$$B_2 = B + \varepsilon - f(B + \varepsilon)/f'(B + \varepsilon)$$

By suitably expanding $f(B + \varepsilon)$ and $f'(B + \varepsilon)$ one can show that, if ε is small, the error in B_2 is approximately $\frac{1}{2} \varepsilon^2 f''(B)/f'(B)$. This means that if the error in the first approximation is ε, the error in the next approximation is roughly proportional to ε^2, a result which is used when considering the rate of convergence of the Newton-Raphson formula.

Notes pages 101 - 143

Chapter IV

[1] For the derivation of Euler-MacLaurin' formula see de La Vallée-Poussin, Ch-J. Cours d'Analyse Infinitésimale, Paris: Gauthier-Villars, 1957, volume 2, Ch 11,pp 374-378.

[2] Ibid. Volume 2, Chapter 3, pp 78-80 and Ch. 11, pp 368-69.

[3] C.V.L. Charlier. Mechanik des Himmels, II,pp.13-16. See also Carl-Erik Fröberg, "Introduction to Numerical Analysis", Addison-Wesley Publishing Company, Inc., 1966, p.211 (This book is an English translation of Lärobok i numerisk analys by C E Fröberg, Lund, Sweden: Svenska Bokförlaget/Bonniers, 1962.

[4] Scarborough, J.B., Numerical Mathematical Analysis. Baltimore: The John Hopkins Press, 1966, p.193.

[5] Conway and Schultz, "The Manufacturing Progress Function", p.41 and pp 43-44.

[6] Franklin, P. Differential and Integral Calculus. New York: McGraw-Hill Book Company, Inc., 1953, p.443.

Chapter V

[1] See, for example Andress, "The Learning Curve as a Production Tool",p.89. See also Hirschmann,"Profit From the Learning Curve", p.126.

[2] Hirsch, "Manufacturing Progress Functions". See also Hirsch, "Firm Progress Ratios".

[3] Schultz and Conway, "The Manufacturing Progress Function", p.49.

[4] Baloff, "Manufacturing Startup: A Model", p.196. See also Schultz and Conway, "The Manufacturing Progress Function", p. 46.

[5] Baloff, "Manufacturing Startup: A Model", p. 16.

[6] For a brief account of work measurement systems that make use of predetermined times, see J.L. Riggs. Production Systems: Planning, Analysis, and Control. New York: John Wiley and Sons, Inc., 1970, Ch.9. The original M.T.M. is described in Maynard, H.B., G.J. Stegemerten,and J.L.Schwab. Methods-Time Measurement. New York: McGraw-Hill Book Company, Inc., 1948.

[7] Implicit in the derivation of formulas (5.4) and (5.6) is the assumption that the best fit to the available data is given by the unit progress function or by the lot-average progress function. In Appendix C parameter formulas are derived for the case where the best fit to the data is achieved with the cumulative-average progress function.

[8] The data for firm D was obtained from S.A. Billon, "Industrial Time Reduction Curves As Tools for Forecasting", Unpublished Ph.D. dissertation, Michigan State University, 1960.

9
Ibid. as to firm E.

10
Ibid. as to firm F.

11
Aggregate data for whole Brazilian industries was obtained from: Ministério do Planejamento e Coordenação Econômica, Escritório de Pesquisa Econômica Aplicada - EPEA, Plano Decenal de Desenvolvimento Econômico e Social-Diagnóstico Preliminar da Industria Mecânica e Elétrica, May 1966.

Chapter VI

1
Hirschmann, "Profit From the Learning Curve",pp 133-34.

2
Ministério do Planejamento e Coordenação Econômica, EPEA, Plano Decenal de Desenvolvimento Econômico e Social - Diagnóstico Preliminar da Indústria Mecânica e Elétrica, pp 59-60.

3
The findings by Asher and Baloff were already discussed in Chapter II.

Chapter VII

1
The calculations were carried out with an HP-25 Scientific Pocket Calculator. The program used is in Appendix E.

2
The relative accuracy of each approximate formula was previously discussed in chapter IV.

[3] Wright, "Factors Affecting the Cost of Airplanes".

[4] For a comprehensive introduction to the HIPO method, see Katzan, H. Jr., Systems Design and Documentation - - An Introduction to the HIPO Method. New York: Van Nostrand Reinhold Company, 1976. See also HIPO - A Design Aid and Documentation Technique, White Plains, New York, IBM Corporation, 1974, Form GC 20-1851.

Appendix B

[1] De La Vallée Poussin, Ch. J. Cours d'Analyse Infinitésimale, Paris: Gauthier-Villars, 1957, Volume 2, pp. 396-97.

BIBLIOGRAPHY

BIBLIOGRAPHY

ARTICLES

Abernathy, W.J., and Wayne, K."Limits of the Learning Curve". Harvard Business Review, no.5 (September-October 1974) : 109-19.

Andress, Frank J. "The Learning Curve as a Production Tool". Harvard Business Review 32, no.1 (January-February 1954) : 87-97.

Baloff, N. "Startups in Machine-intensive Production Systems". The Journal of Industrial Engineering 17, no.1 (January-February 1966) : 25:32.

———. "The Learning Curve - Some Controversial Issues". Journal of Industrial Economics (July 1966) : 275-82.

———. "Estimating the Parameters of the Startup Model - An Empirical Approach". The Journal of Industrial Engineering 18, no.4 (April 1967) : 248-53.

———. "Extension of the Learning Curve - Some Empirical Results". Operations Research Quarterly 22, no.4 (December 1971) : 329-40.

———, and Becker, S. "A Model of Group Adaptation to Problem Solving Tasks". Human Performance and Organization Behavior 3, (August 1968) : 217-38.

Becker, S., and Baloff, N. "Organization Structure and Problem Solving Behavior". Administrative Science Quarterly 14, no.6 (June 1969) : 260-71.

Bryan, Stanley E. "Fair Value and the Learning Curve". Purchasing (September 1954) .

———. "A Melhoria da Produção no Brasil". Revista de Administração de Empresas 1, no.2 (setembro-dezembro 1961) : 27-55.

Carr, G.W. "Peacetime Cost Estimating Requires New Learning Curves". Aviation 45, no.4 (April 1946) : 76-77.

Co

Ga

H

Conway, R.W., and Schultz, A. "The Manufacturing Progress Function". The Journal of Industrial Engineering 10, no.1 (January-February 1959) : 39-53.

Garg, A., and Milliman, P. "The Aircraft Progress Curve - Modified for Design Changes". The Journal of Industrial Engineering 12, no.1 (January-February 1961) : 23-28.

Hirsch, Werner Z. "Manufacturing Progress Functions". The Review of Economics and Statistics 34, no.2 (May 1952) : 145-55.

―――――――――. "Firm Progress Ratios". Econometrica 24, no.2 (April 1956) : 136-143.

Hirschmann, W.B. "Profit from the Learning Curve". Harvard Business Review 42, no.1 (January 1964) : 125-39.

Kilbridge, M.D. "Predetermined Learning Curves for Clerical Operations". The Journal of Industrial Engineering 10,no.3 (May-June 1959) .

Kneip, J.G. "The Maintenance Progress Function". The Journal of Industrial Engineering 16, no.6 (November-December 1965) : 398-400.

Koen, F.T. "Dynamic Evaluation". Factory 117, no.9, (September 1959) : 98-103.

Laddon, I.M. "Reduction of Man-Hours in Aircraft Production". Aviation (May 1943) : 170-73.

Levy, F.K. "Adaptation in the Production Process". Management Science 11, no.4 (April 1965) : 136-54.

Mensforth, E. "Airframe Production", Parts I and II, Aircraft Production 9, (September 1947) : 343-50 and (October 1947) : 388-95.

Metz, P.B. "A Manufacturing Progress Function Nomograph". The Journal of Industrial Engineering 13, no.4 (July-August 1962) : 253-56.

Middleton, K.A. "Wartime Productivity Changes in the Airframe Industry". Monthly Labor Review 61, no.2 (August 1945) : 215-25.

Montgomery, F.J. "Increased Productivity in the Construction of Liberty Vessels". Monthly Labor Review, (November 1943): 861-64.

Russel, J.H. "Progress Function Models and their Deviations". The Journal of Industrial Engineering 19, no.1 (January 1968) : 5-10.

Searle, A.D., and Gody, G.S. "Productivity Changes in Selected Wartime Ship Building Programs". Monthly Labor Review 61, no.6 (December 1945) : 1132-47.

Thomas, M.U. "Developing More Accurate Cubic Learning Curves". Manufacturing Engineering and Management, (July 1972) : 29-30.

White, J.M. "The Use of Learning Curve Theory in Setting Management Goals". The Journal of Industrial Engineering 12, no.6 (November-December 1961) : 409-11.

Williams, P.F. "The Application of Manufacturing Improvement Curves in Multi-Product Industries". The Journal of Industrial Engineering 12, no.2 (March-April 1961) : 108-12.

Wright, T.P. "Factors Affecting the Cost of Airplanes". Journal of Aeronautical Sciences 3, no.4 (February 1936): 122-28.

BOOKS

Allen, R.G.D. Mathematical Analysis for Economists. London: MacMillan & CO LTD, 1962.

Berghell, A.B. Production Engineering in the Aircraft Industry. New York: McGraw-Hill Book Company, Inc., 1944.

de La Vallée-Poussin, C.J. Cours d'Analyse Infinitésimale. Paris: Gauthier-Villars, 1959, Volume I.

—————————————. Cours d'Analyse Infinitésimale. Paris: Gauthier-Villars, 1957, Volume II.

Franklin, P. Differential and Integral Calculus. New York: McGraw-Hill Book Company, Inc., 1953.

——————————. Methods of Advanced Calculus. New York: McGraw-Hill Book Company, Inc., 1944.

Fröberg, C.E. Introduction to Numerical Analysis. New York: Addison-Wesley Publishing Company, Inc., 1966.

——————————. Lärobok i numerisk analys. Lund, Sweden: Svenska Bokförlaget/Bonniers, 1962.

Kaplan, A. The Conduct of Inquiry: Methodology for Behavioral Science. San Francisco, Calif.: Chandler Publishing Co., 1965.

Maynard, H.B., Stegemerten, G.J., and Schwab, J.L. Methods-Time Measurement. New York: McGraw-Hill Book Company, Inc., 1948.

Riggs, J.L. Production Systems :Planning, Analysis, and Control. New York: John Wiley and Sons, Inc., 1970.

Scarborough, J.B. Numerical Mathematical Analysis. Baltimore: The John Hopkins Press, 1966.

REPORTS, PAPERS, AND OTHER PUBLICATIONS

Alchian, A. "An Airframe Production Function". The RAND Corporation, P-108, October 10,1949.

―――――――. "Reliability of Progress Curves in Airframe Production". The RAND Corporation, RM-206-1, February 3, 1950.

Arrow, K.J., and Arrow, S.S. "Methodological Problems in Airframe Cost-Performance Studies". The RAND Corporation, RM-456, September 20,1950.

―――――――, and Bradley, H. "Cost Quality Relations in Bomber Airframes". The RAND Corporation, RM-536, February 6, 1951.

Asher, H. Cost-Quantity Relationships in the Airframe Industry. The RAND Corporation, R-291, July 1, 1956.

Boeing Airplane Company. The Experience Curve and Improvement Curve Study. Wichita, Kansas (n.d.).

Boeing Transport Division. Improvement Curve Handbook. Renton, Washington: Boeing Airplane Company, 1956.

Brazil, Ministério do Planejamento e Coordenação Econômica, EPEA, Plano Decenal de Desenvolvimento Econômico e Social - Diagnóstico Preliminar da Indústria Mecânica e Elétrica, Brasilia, May 1966.

Brown, W.F. The Improvement Curve. Boeing Airplane Company, Wichita, Kansas, March 1955.

Blume, E.J. and Peitzke, D. Purchasing with the Learning Curve. North American Aviation, Inc., Inglewood, California, August, 1953.

Crawford, J.R. Learning Curve, Ship Curve, Ratios, Related Data. Lockheed Aircraft Corporation, Burbank, California (n.d.).

Crawford, J.R. Estimating, Budgeting, and Scheduling. Lockheed Aircraft Corporation, Burbank, California, 1945.

————————, and Strauss, E. World War II Acceleration of Airframe Production. Air Materiel Command, Dayton, Ohio, 1947.

Guibert, P. Mathematical Studies of Aircraft Construction. Central Air Documents Office, Wright-Patterson Air Force Base, Dayton, Ohio.

Link, G.W., and Ellis, D.A. The Experience Curve. Boeing Aircraft Company, Wichita, Kansas, December, 1945.

Morgan, A.W. Experience Curves Applicable to the Aircraft Industry. The Glenn L. Martin Company, Baltimore, Maryland, 1952.

Rayborg, W.A. Mechanics of the Learning Curve. Northrop Aircraft, Inc., Hawthorne, California, March, 1952.

Russell, J.H. "Predicting Progress Function Deviations", IBM TR 22.4, 1967.

Rutan, E.A. Theory of Learning Curves. Chance Vought Aircraft, Inc., Dallas, Texas, October, 1948.

Schreiner, D.A. "The Manufacturing Progress Function, Its Application to Operations at IBM, Endicott", Paper presented at 12th Annual ASQC Convention, Boston, Mass., May, 1958.

Smith, R.W., Lansing, W.C., and Horton, H.G. Improvement Curve Handbook. Boeing Airplane Company, Seattle, Washington, 1956.

Source Book of World War II Basic Data, Airframe Industry, Vol. I, Air Materiel Command, Dayton, Ohio, 1947.

Stanford Research Institute. An Improved Rational and Mathematical Explanation of the Progress Curve in Airframe Production. Stanford: SRI, August 10, 1949.

————————————————. Relationships for Determining the Optimum Expansibility of the Elements of a Peacetime Aircraft Procurement Program. Stanford: SRI, December 31, 1949 (prepared for Air Materiel Command, USAF).

Zieke, R.P. "Progress Curve Analysis in the Aerospace Industry". An Engineer's thesis presented to the faculty of the Department of Industrial Engineering, Stanford University, June, 1962. (Published by the Boeing Company, Seattle,Washington, June, 1962.)

UNPUBLISHED MATERIALS

Baloff, N. "Manufacturing Startup: A Model". Unpublished Ph.D. dissertation, Stanford University, 1963.

————, and Becker, S. "Group Learning Curves: A Power Function?" Working Paper, Graduate School of Business, University of Chicago.

Billon, S.A. "Industrial Time Reduction Curves as Tools for Forecasting". Unpublished Ph.D. dissertation, Michigan State University, 1960.

Conway, R.W. and Schultz, A., Jr. "The Manufacturing Progress Function". Department of Industrial and Engineering Administration, Cornell University, (n.d.).

Reguero, M.A. "An Economic Study of the Airframe Industry". Unpublished Ph.D. dissertation, New York University, 1957.

Stupar, M. "Forecasting of Airplane Man-Hours". Unpublished manuscript, Headquarters Air Materiel Command, 1942.

GENERAL REFERENCES

Ezekiel, M. and Fox, K.A. <u>Methods of Correlation and Regression Analysis</u>. New York: John Wiley and Sons, 1959.

Hoel, P.G. <u>Introduction to Mathematical Statistics</u>. New York: John Wiley and Sons, 1965.

Reis, D.R.A. "Extensão da Função de Progresso da Produção a Algumas Indústrias Brasileiras". Dissertação de Mestrado em Administração de Empresas, EAESP-FGV, S.Paulo, 1973.

Tintner, G. <u>Econometrics</u>. New York: John Wiley and Sons, Inc., 1952.

————. <u>Mathematiques et Statistiques pour les Économistes</u>. Paris: Dunod, 1962.