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GROWTH AND PRODUCTION CONTROL
OF GOLDEN SHINER NOTEMIGONUS CRYSOLEUCAS (MITCHILL)
BY MANIPULATING TEMPERATURE AND DENSITY

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Sha Miao

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Niles R. Kevern
Major professor

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GROWTH AND PRODUCTION CONTROL

OF GOLDEN SHINER <u>NOTEMIGONUS</u> <u>CRYSOLEUCAS</u> (MITCHILL)

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ABSTRACT

GROWTH AND PRODUCTION CONTROL OF GOLDEN SHINER NOTEMIGONUS CRYSOLEUCAS (MITCHILL) BY MANIPULATING TEMPERATURE AND DENSITY

By

Sha Miao

Better control of the growth and production of the golden shiner to meet market demand is needed. An experimental study evaluated the simultaneous effects of two factors (temperature and fish density) with three treatment levels each. Three thermal regimes consisted of one constant temperature (24C) and two daily thermocycles. The temperatures of two thermocycles increased respectively from 22.7C to 26.8C and from 24.6C to 28.8C on a 12-hour-cycle, and then declined to their respective origins on the following 12-hour-cycle. Three fish densities were created by stocking five fish per 10-, 15-, and 30-gallon aquaria.

This investigation was accomplished using the systems analysis involving three modeling stages in sequence: a statistical, a deterministic, and a computer model. Data indicated that density was not a dominant factor during the 80-day experiment. However, the results suggested that two out of the three thermal treatments, 24C and 22.7C - 26.8C, should be alternatively applied to this conditioned system of four 20-day periods to control golden shiner production for market needs. The temperature of the dynamic system

should be constant at 24C for the first and last periods but cyclicly between 22.7C and 26.8C for the other periods.

TO

MY WIFE

Tu, Shunchi

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INTRODUCTION

The minnows (Cyprinidae) are one of the characteristic families of North American fishes and are of economic importance since they constitute the major portion of the food for game and commercial fishes (Markus, 1934). The advent of minnow farming was preceded by many years of harvesting of wild minnow stocks (Brown and Gratzek, 1979). A major problem of the bait industry is that it can not catch enough minnows during the summer when the sport fishery reaches its peak (Gordon, 1968). Hedges and Ball (1953) also pointed out that throughout Michigan there is a high summer demand for bait minnows, often considerably in excess of the supply. Bait dealers depend on natural supplies and are often forced to travel considerable distances in search of streams where bait fishes are available (Hedges and Ball, 1953). As wild stocks became more difficult to find and harvest, as a result of overharvesting, pollution, and other factors, interest grew in raising bait fish (Brown and Gratzek, 1979). In pond operations, however, the fish can be harvested as they are needed and a faster turnover between dealer and fisherman results (Hedges and Ball, 1953).

In February, 1983, Dr. Donald L. Garling surveyed the Michigan bait minnow supply. One hundred and fifty letters were sent to the wholesale bait dealers in Michigan. Three main questions were included in the survey (Appendix 1): (1) What is the status of the bait minnow industry in Michigan? (2) What are the needs of the Michigan bait minnow industry? (3) Can Michigan minnow farms be developed to meet those needs? Forty-nine responses were received including nine without answers. While many species were mentioned in the survey, my analysis concentrated on the five species having the highest production rates. The results are summarized in Tables 1 and 2. This survey showed that of the respondents annual production rates of minnows among these five species varied from a high of 1.8 million gallons to a low of 8.6 thousand gallons (1 gallon = 8 pounds) which were sold at $27 million and $199 thousand, respectively. Of the total production, the majority of bait minnows came from "purchasing" as compared to "netting and raising". Also, out-of-state producers played an important role in providing Michigan dealers with bait minnows. Seasonal shortages were evident throughout the year but were especially critical in July and August. The survey also showed that bait minnow dealers in Michigan will pay premium prices for a guaranteed constant supply of farmed raised minnows. The percentages of the dealers that will pay premium prices varied from a high of 89.0% for golden shiner to a low of 0.2% for emerald shiner. The

TABLE 1. Michigan bait minnow[a] supply in 1982-survey summarization

Species	Total[b] Production (Gallon)[c]	Revenue[d] (Dollar)	Sources of Bait Minnows(%)		Minnows Come From (%)	
			Net or Raise	Purchase	In-State producers	Out-of-State Producers
Fathead Minnow	14,990	330,385	6.0	94.0	22	78
White Sucker	12,429	274,674	42.0	58.0	45	55
Golden Shiner	8,653	199,716	66.0	34.0	28	72
Emerald Shiner	1,862,114	27,000,645	0.2	99.8	30	70
Grey Shiner	152,388	2,895,372	1.4	98.6	88	12

a. Fathead minnow, white sucker, golden shiner, emerald shiner and grey shiner were the bait minnows with the highest production in Michigan during 1982 survey.

b. Two dealers used "number of fish" as a production unit. Thus, the numbers of gallons for individuals were estimated by dividing their revenues by the average selling prices.For instance, the number of produced fathead minnows was 9500 and was sold at $680.00; the average selling price was estimated at $22.04 per gallon; therefore the production was roughly equal to 31 gallons. Another example for golden shiner, the number of production was 70000; was sold at $565.00; the average selling price was $23.08 per gallon; thus the production was roughly 24 gallons.

c. 1 gallon = 8 pounds.

d. Individual revenues were obtained by multiplying total production (gallons) by average selling price (dollars per gallon) without considering other units of selling price, including cents per dozen, cents per fish, and dollars per 100 fish.

Table 2. Michigan bait minnow demand in 1982-survey summarization

Species	Wholesalers (%) Having Difficult Months to Obtain Minnows by Netting												Wholesalers (%) Having Difficult Months to Obtain Minnows by Purchasing												Dealers(%)[a] Willingly Pay Premium Prices for Guaranteed Constant Supplies of Farmed Raised Minnows	
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	Yes	No
Fathead Minnow	22	22	22	-	-	-	22	22	11	-	-	22	10	10	10	10	-	10	-	-	-	10	10	10	21.0	79.0
White Sucker	22	22	22	11	11	33	33	33	22	-	-	11	21	21	21	7	7	14	21	36	7	7	7	21	2.0	98.0
Golden Shiner	30	30	30	20	20	30	40	40	40	10	10	20	40	40	30	30	20	30	40	50	-	10	10	30	89.0	11.0
Emerald Shiner	17	17	17	-	-	17	50	67	17	-	-	17	22	22	22	-	-	11	44	55	22	-	-	22	0.2	99.8
Grey Shiner	-	-	-	-	-	17	33	50	-	-	-	-	20	20	20	-	-	-	-	-	-	-	-	20	1.3	98.7

a. For individual species, the percentage of "Yes" was determined by the ratio of the partial production (produced by those who willingly pay a premium price) versus the total production. The complemental percentage denoted the "No".

results indicated an unmet demand for bait minnows that might be satisfied by an increase in minnow farming in Michigan.

In aquaculture, two variables that may influence recruitment to fish stocks are temperature and population density, both of which could influence growth and fecundity. Everhart and Youngs (1981) reported that the growth rate of fish depends to a large extent on temperature and most species of fish do not spawn unless the water temperature is within certain limits. A temperature fluctuating between 10 and 20C and averaging 15C does not necessarily have the same effect on organisms as a constant temperature of 15C (Odum, 1959). Biette and Geen (1980) demonstrated that the growth of young sockeye is greater under cyclic than constant temperatures. For brown trout, a fluctuating temperature could significantly increase their feeding, growth, and lipid deposition compared with constant temperatures at the mean of the fluctuations (Spigarelli et al., 1982). Odum (1959) stated that organisms normally subjected to variable temperatures in nature (as in most temperate regions) tend to be depressed, inhibited or slowed down by constant temperature. However, depending on the time of day the thermocycle was initiated, weight gain and testicular growth could be either stimulated, inhibited, or equal to that in fishes subjected to constant heat or constant cold (Spieler et al., 1977).

Le Cren (1965) stated that population density and growth rate are inversely related (cited by Smith et al., 1978). As population density increases, competition for nutrients, food, and living space usually intensifies, providing one of the most effective controls of both plant and animal populations (Odum, 1959). Meanwhile, metabolic wastes, which are directly proportional to population density, have been implicated as inhibitory to growth and toxic to fishes (Yu and Perlmutter, 1970). Smith et al. (1978) reported that high population density appeared to limit growth and gamete development regardless of food abundance. Water volume also appeared to limit numbers (tolerance density) of fish which can be supported in a specific volume of water (Smith et al., 1978). As a partial explanation for the reduction in reproduction and growth under crowded conditions, several investigators have suggested the presence of a water-borne, fish-produced represser that inhibits reproduction (Swingle, 1953; Rose and Rose, 1965, cited by Smith et al., 1978) and reduces growth rate (Yu and Perlmutter, 1970; Francis et al., 1974). However, Glaser and Kantor (1974) showed that spawning rate in medaka (Oryzias latipes) can be inhibited by social factors of crowding, irrespective of chemical conditions of the water (cited by Smith et al., 1978).

The golden shiner is an excellent bait minnow (Cooper, 1937; Prather, 1957; Scott and Crossman, 1973; Pflieger, 1975) and is well suited for pond culture (Forney, 1957;

Scott and Crossman, 1973; Pflieger, 1975). Table 2 shows that there is a high shortage of golden shiners throughout the year. It also indicates that 89% of the dealers were willing to pay a premium price for a guaranteed constant supply of farmed raised golden shiners. The objective of this research was to control the growth and production of the golden shiner Notemigonus crysoleucas (Mitchill) under two factors' stimuli. This demonstration has been illustrated by utilizing three related models; a linear stochastic model, a nonlinear deterministic model, and a computer simulation model. A linear stochastic model based on a split-plot design (Gill, 1978b; Petersen, 1985; Myers, 1979; Winer, 1971; Cochran and Cox, 1957) was designed to investigate the effect of interaction between temperature and stocking density on the growth of golden shiner. Each factor consists of three fixed levels. Two thermocycles and a constant temperature (24C) were introduced with respect to the temperature factor. Twenty-four C was determined by averaging the temperatures of spawning initiation (21C or 70F) and spawning cessation (27C or 80F) for golden shiner (Scott and Crossman, 1973; Pflieger, 1975; Brown and Gratzek, 1979). For density, 10-, 15- and 30-gallon sizes of aquarium were chosen. Five fish were stocked in each volume. It was reported by Hickman and Kilambi (1974), and Roseberg and Kilambi (1975) that a stocking density at 5 golden shiners per 15-gallon-water produces the best growth. There were a total of nine treatment combinations.

This study lasted eighty days. Individual fish weights were taken at the beginning of the study and at intervals of twenty days. As a result, the growth rate and carrying capacity were estimated at the end of each twenty-day interval. Carrying capacity here is defined as the growth potential or the maximum growth that is attained during the time period and under the conditions of this experiment. The growth rate and carrying capacity are a pair of parameters in the Logistic Equation (Ricker, 1975; Wilson and Bossert, 1971; Spain, 1982). Their magnitudes are affected with the intensity of the treatment combination. A better growth rate or carrying capacity reveals that a given treatment combination is more effective. There is no causal correlation between the paired parameters. This implies that a better growth rate is not necessarily accompanied by a better carrying capacity, or vice versa. Consequently, several sets of treatment combinations in a time sequence were statistically determined as optimum considering only growth rate. Likewise, other sets of treatment combinations might be accepted as optimum considering only carrying capacity. Computer modeling was used to clarify such a controversy.

In computer modeling, a mean and a standard deviation were computed from the initial body weights of the fish. With the mean and standard deviation, one million normal-random deviates were generated through Monte Carlo simulations (Rubinstein, 1981; Hewlett-Packard Company's

Manual, 1984; Sobol', 1974; Hammersley and Handscomb, 1964). The generated one million deviates represented equivalently one million fish body weights at the initial stage. Accordingly, the generated golden shiners were then simulatively stocked into a pretend "production system" described by the Logistic Model. A set of consecutive treatment combinations provided this production system with a series of dynamic inputs. In practice, however, the system's dynamic inputs were substituted with a sequence of the paired parameters; growth rate and carrying capacity. They were repeatedly estimated in four consecutive periods of twenty days. Secondly, the fourth-order Runge-Kutta method (Stiefel, 1963; Davis and Robinowitz, 1975; Todd, 1962; Weeg and Reed, 1966; Stark, 1970; Scraton, 1984) was selected to deal with the computer modeling because of its lower cost in terms of computer time while describing the Logistic Model more precisely. With the generated initial weights and the system's dynamic inputs (i.e. growth rate and carrying capacity), the computer modeling continuously predicted not only the total yield but the individual size of the one million generated golden shiners on a daily basis. The production cycle in number of days was determined under a desired yield or desired size. An optimum production cycle in terms of minimum time was thus identified according to desired yield, size or both. As a result, a particular set of the consecutive treatment combinations was finally screened as an optimum based on the best production cycle.

However, a less effective treatment combination does not always indicate a nuisance input. Such inputs, especially from the management point of view, may act as control valves to reduce an unwanted growth and production at different rates, and therefore to provide positive effects for the system. For example, sport fishermen may require various sizes of bait minnow in fishing. Our objectives are to provide neither a shortage nor an oversupply, but rather a supply that meets the demand of the market place. This research therefore demonstrates a feasible way by interchanging the system's inputs over time to periodically adjust the rates of growth and production. Eventually, the system's operation can be controlled at a desired pace to satisfy the market demand.

MATERIALS AND METHODS

Three stages in sequence were involved to develop and carry out this research. In conducting a stochastic modeling for the first stage, an experimental system was designed and its model was statistically examined. An indoor operational system was then built for the second stage and the system's performance in terms of exactness of temperatures control was statistically evaluated. In the final stage, a nonlinear deterministic model was studied using a computer simulation. The outputs of the computer modeling were statistically compared with one another. Consequently, a series of treatment inputs were determined as most suitable for serving market demands.

STOCHASTIC MODELING

The stochastic modeling formulates a linear statistical equation with split-plot design as follows:

$$\begin{aligned} Y_{tdap} = \mu &+ T_t + D_d + (TD)_{td} \\ &+ A_{(td)a} + P_p + (TP)_{tp} \\ &+ (DP)_{dp} + (TDP)_{tdp} \\ &+ (AP)_{(td)ap} + E_{(tdap)} \end{aligned}$$

where $t = 1, 2, 3$; $d = 1, 2, 3$; $a = 1, 2$; and $p = 1, 2, 3, 4$.

The right hand side of the above equation is composed of varieties of a system's inputs. These inputs represent average effects resulting from a particular factor or combination of factors at a certain quantitative level. The μ is described as the true mean of the distribution of Y for a population defined by the experimental conditions as a whole. The capital letters T and D represent temperature and density, respectively. The three treatment levels (t) in temperature were: (1) constant temperature of 24C, (2) low cyclic temperature from 22.7C to 26.8C, and (3) high cyclic temperature from 24.6C to 28.8C. Three density levels (d) were created by randomly distributing groups of five fish into aquaria of 10-, 15-, and 30-gallon. The $(TD)_{td}$ indicate the average effects of the interaction of temperature at level t and density at level d. The t levels of temperature and d levels of density are combined in all possible ways to make t x d = 3 x 3 = 9 treatment combinations. And for each treatment combination, there were two aquaria, or two replicates (a). The $A_{(td)a}$ denote random effects of aquaria nested within a treatment combination. Also known as "Error One", the $A_{(td)a}$ correspond to the $E_{(td)a}$ of the model for a completely randomized design without the repeated measurement.

As a result of the fish weight samplings repeatedly taken at intervals of twenty days, the P_p are effects of time at the various sampling points in the process of repeated measurement, where the sampling intervals are p =

1, 2, 3, 4. The $(TP)_{tp}$ and $(DP)_{dp}$ are the effects of the interactions of time with temperature and time with density, respectively. The $(TDP)_{tdp}$ are the effects of the interaction of three factors temperature, density, and time at the combined levels tdp. The interaction of an aquarium with time; $(AP)_{(td)ap}$ is not separable from the residual error within an aquarium; $E_{(tdap)}$. The $E_{(tdap)}$ is the residual error or net effect on Y of all unspecified factors of influence peculiar to aquarium a at interval p under the treatment combination td. The left hand side of the equation is the system's output symbolized by Y. Y is also a collective variable being repeatedly sampled every twenty days. By modeling the same equation, this collective variable Y may represent two different variables depending on what parameters are under study. If carrying capacity, a parameter in the Logistic Equation, is being estimated then variable Y are the total weights of 5 fish from each experimental unit. On the other hand, variable Y represents the weight ratio of the 5 fish as long as the growth rate, another parameter in the Logistic Equation, is being estimated. The weight ratio is the final weight divided by the initial weight over the study time periods (i.e., 20, 40, 60, 80 days). Therefore, the sampled variables Y_{tdap}, corresponding to a particular parameter, are the measurements being taken from aquarium a at interval p under the $(td)^{th}$ treatment combination. The total observations per estimated parameter, or sample size, are N = t x d x a x p = 3 x 3 x 2 x 4 = 72.

In order to prevent a possible influence on the experimental estimates, a few nuisance factors had to be eliminated or systematically controlled. The system's illumination included nine fluorescent light fixtures with four 34-watt bulbs each regulated to a light period from 0800 hours to 2000 hours each day. The aquaria were filled with cold tap water one week prior to the addition of fish to allow for aeration, iron precipitation, and chlorine removal. The water came from the Michigan State University water supply system originating in deep groundwater sources. The experimental fish were shipped by air from Arkansas (Finley Company Farms[a]). On arrival, five fish were randomly chosen and distributed into each aquarium and acclimatization begun. The system's conditions of acclimatization were identical to those of the experiment. The research began after twenty days of acclimatization. The fish received a daily diet of 3% of their body weights. This ration was divided into two equal parts given at 1000 hours and 1600 hours each. A commercial diet (Purina Trout Chow[b]) was the food source utilized for this feeding scheme and was kept refrigerated. The feeding activities were periodically terminated one day before to one day after the sampling date when the measurements of the fish

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[a] The address is P. O. Box 317, Lonoke, AR. 72086.

[b] Purina Trout Chow, Diet CR 6-30 #4 GRAN, manufactured by Glenooe Mills INC., Glencoe, Minnesota 55336.

weights and lengths were taken. Weights were determined to tenths of a gram using a top-loading Mettler balance. To avoid fish flapping and reduce the stress, the following method of handling each individual fish was used. This procedure showed no adverse effects on the fish: (1) prebalance the weight of a sandwich bag containing about a gram of water, (2) place an individual fish into the bag, and (3) weigh the entire bag. Meanwhile the aquaria's clean-ups were repeatedly performed during the weighing periods. This cleaning scheme consisted of (1) scrubbing walls of the heaters and aquaria, and (2) syphoning the metabolic wastes and feed residuals. After syphoning, each aquarium was refilled (about one-third of its volume) with conditioned water previously stored in a reservoir.

This study was followed by a statistical analysis performed in the following three consecutive steps: (1) tests of assumptions, (2) analysis of variance (ANOVA), and (3) Tukey's tests. Every test or analysis was repeated twice but alternatively with one of the two variables, weight and weight-ratio.

Tests of Assumptions

The assumptions underlying this statistical modeling may be summarized by $E_{tdap} \sim NID(0,\sigma^2)$. That is, the experimental errors are distributed "normally" and "independently", with mean zero and "homogeneous variance" σ^2 common to all treatment combinations.

The normality was examined by the Shapiro-Wilk test (Gill, 1978a) described as follows.

1. Determine individual random errors by

   $E_{(tdp)a} = Y_{tdpa} - \overline{Y}_{tdp.}$;

   where Y_{tdpa} denote the weight or weight-ratio observations of five fish; and $\overline{Y}_{tdp.}$ are the means of the observations Y_{tdpa} through a = 1, a = 2.

2. Order the N random errors by value ($E_1 \leq E_2 \leq \ldots \leq E_N$); where N = total sample size = 72.

3. Compute

   $$G = \sum_{i=1}^{M} B_{i,N} \, (E_{N-i+1} - E_i) \; ;$$

   where M = N / 2 = 36; and the values of $B_{i,N}$ are found in the extended tables of Appendix A.13.1 (Gill, 1978c).

4. If the test statistic,

   $$W = \frac{G^2}{SS_E} \; ,$$

   is less than the critical value $W_{\alpha,N}$ from the extended tables of A.13.2 (Gill, 1978c), one may reject the hypothesis of normality with probability of Type I error less than α; where SS_E = sum of squares of the random experimental error; $\alpha = 0.1$.

The test of independent errors can be omitted because the random experimental errors are no longer independently related due to repeated sampling. This problem resulting from the assumption's violation was internally solved by this particular modeling use of a split-plot design. The hypothesis of homogeneous variance was then examined by Bartlett's test (Gill, 1978a; Zar, 1984). In other words, we wish to test if the variance of weights or weight ratios is the same for all nine treatment combinations within each time interval. The test statistic within each period of time is

$$B = (\ln S_p^2)\left(\sum_{i=1}^{td} \nu_i\right) - \sum_{i=1}^{td} \nu_i \ln S_i^2 ,$$

where $\nu_i = A_i - 1$ and A_i is the number of duplicated aquaria per treatment combination i. The pooled variance, S_p^2, is calculated by

$$S_p^2 = \left(\sum_{i=1}^{td} SS_i\right) / \left(\sum_{i=1}^{td} \nu_i\right) ,$$

where td = the number of total treatment combinations = 9, and SS_i = the sum of the squares of the deviations from the sample's mean for a given treatment combination i. The S_i^2 is the sample variance corresponding with treatment combination i. The distribution of B is approximated by the Chi-Square distribution, with td-1 degrees of freedom, but a more accurate Chi-Square approximation is obtained by computing a correction factor,

$$C = 1 + \frac{1}{3(td-1)}\left(\sum_{i=1}^{td} \frac{1}{\nu_i} - \frac{1}{\sum_{i=1}^{td} \nu_i}\right) ,$$

with the corrected test statistic being

$$B_C = \frac{B}{C} ,$$

and the critical value is $\chi^2_{\alpha,td-1}$, with $\alpha = 0.2$.

Analysis of Variance (ANOVA)

ANOVA is a term of process for partitioning the variance of a random variable (Y) into orthogonal (independent) parts caused by treatments and experimental error. Namely its objectives are :(1) to obtain the precision (variance) of estimates of treatment means or differences, as well as (2) to test hypotheses about equality of treatment means and existence of interactions among factors. Table 3 details the ANOVA. Appendix 2 demonstrates the associated formulas to quantitatively determine the individual sums of squares.

The F ratios listed in Table 3 indicate that a series of corresponding hypotheses were being alternatively tested. Then Figure 1 shows the procedure of testing these hypotheses in a flow chart. Any hypothesis tested should be rejected if $F_i < F_{\alpha,df_1,df_2}$; where $F_i = F_1, F_2, \ldots F_7$ (Table 3); $\alpha = 0.05$; df_1 = associated numerator's degrees of freedom, and df_2 = associated denominator's degrees of freedom. A rejection or acceptance of any hypothesis test is then followed by making an appropriate decision.

Table 3. ANOVA of the weight/weight ratio.

Source of Variation	Degrees of Freedom (D.F.)	Sum of Squares	Mean Square	F Ratio
Total	(tdap)-1	SS_Y	MS_Y	
Temperature (T)	t-1	SS_T	MS_T	$MS_T/MS_{A/TD}=F_7$
Density (D)	d-1	SS_D	MS_D	$MS_D/MS_{A/TD}=F_6$
Interaction T-D	(t-1)(d-1)	SS_{TD}	MS_{TD}	$MS_{TD}/MS_{A/TD}=F_5$
Aquaria/TD[a]	td(a-1)	$SS_{A/TD}$	$MS_{A/TD}$	
Period (P)	p-1	SS_P	MS_P	$MS_P/MS_E=F_4$
Interaction T-P	(t-1)(p-1)	SS_{TP}	MS_{TP}	$MS_{TP}/MS_E=F_3$
Interaction D-P	(d-1)(p-1)	SS_{DP}	MS_{DP}	$MS_{DP}/MS_E=F_2$
Interaction T-D-P	(t-1)(d-1)(p-1)	SS_{TDP}	MS_{TDP}	$MS_{TDP}/MS_E=F_1$
Residuals[a]	td(a-1)(p-1)	SS_E	MS_E	

[a]. (Aquaria/TD) is $A_{(td)a}$. Recalling the model's equation, $A_{(td)a}$ is also known as Error I for a completely randomized design without repeated measurement. The residuals, or Error II, consist of $(AP)_{(td)ap}$ and $E_{(tdap)}$ and are inseparable due to the model's limitation. In fact we may separate $(AP)_{(td)ap}$ from $E_{(tdap)}$ by tagging all the fish. As a result, a new model containing two new components of $(FP)_{(tda)fp}$ and $E_{(tdafp)}$ (F denotes fish effect and f is the number of fish) would be used for the purpose of separating $(AP)_{(td)ap}$ from $E_{(tdap)}$. Again $(FP)_{(tda)fp}$ and $E_{(tdafp)}$ are inseparable due to the new model's limitation. Therefore, to what extent the partition of total variation should reach is a function of an acceptable precision in modeling from biological concerns. For example, without excluding the quantity $(AP)_{(td)ap}$ from Error II, which is the interaction effect of aquarium and time period, should not affect the model's precision significantly from biological considerations. Besides, tagging itself may possibly introduce unexpected physical effects into the system.

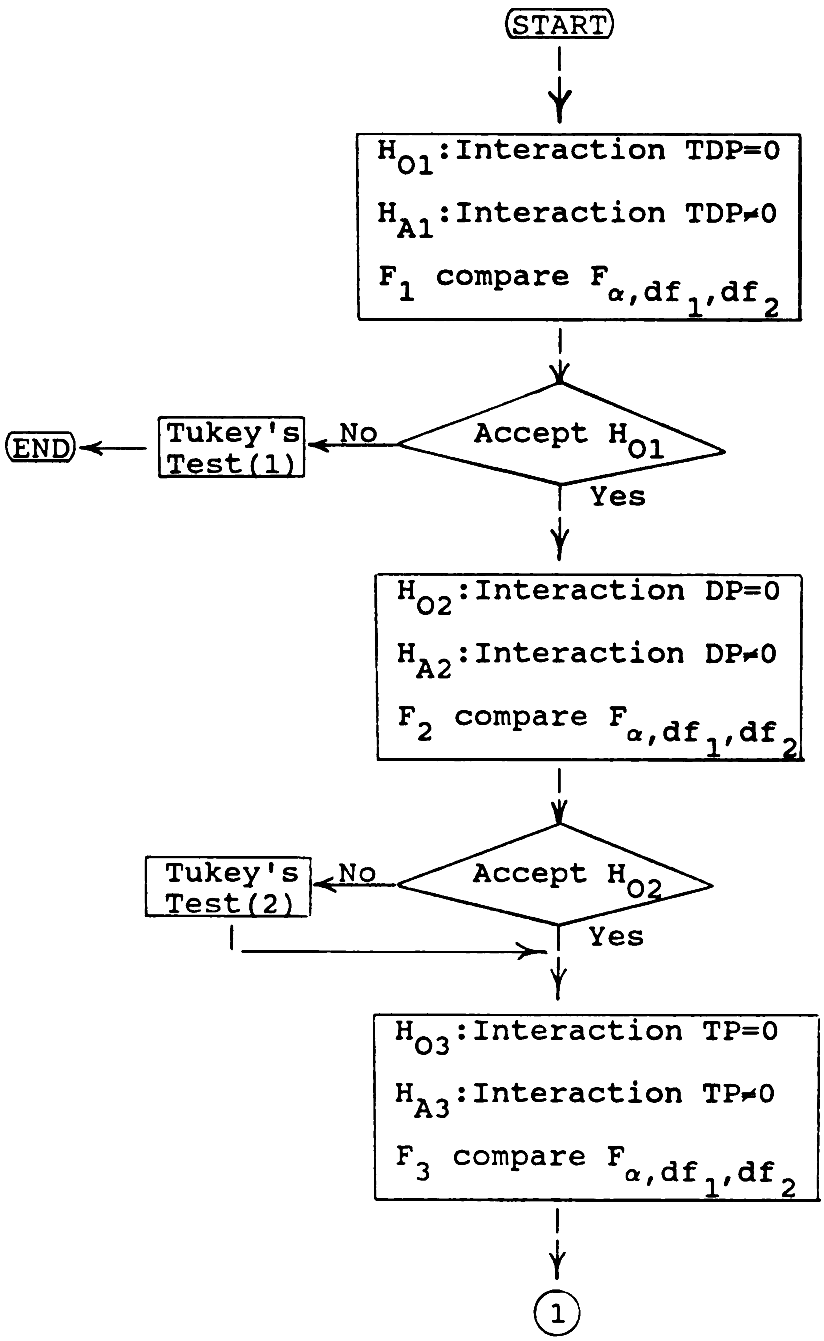


Figure 1. Flow chart of hypotheses tests on the weight/weight ratio followed by appropriate decisions.

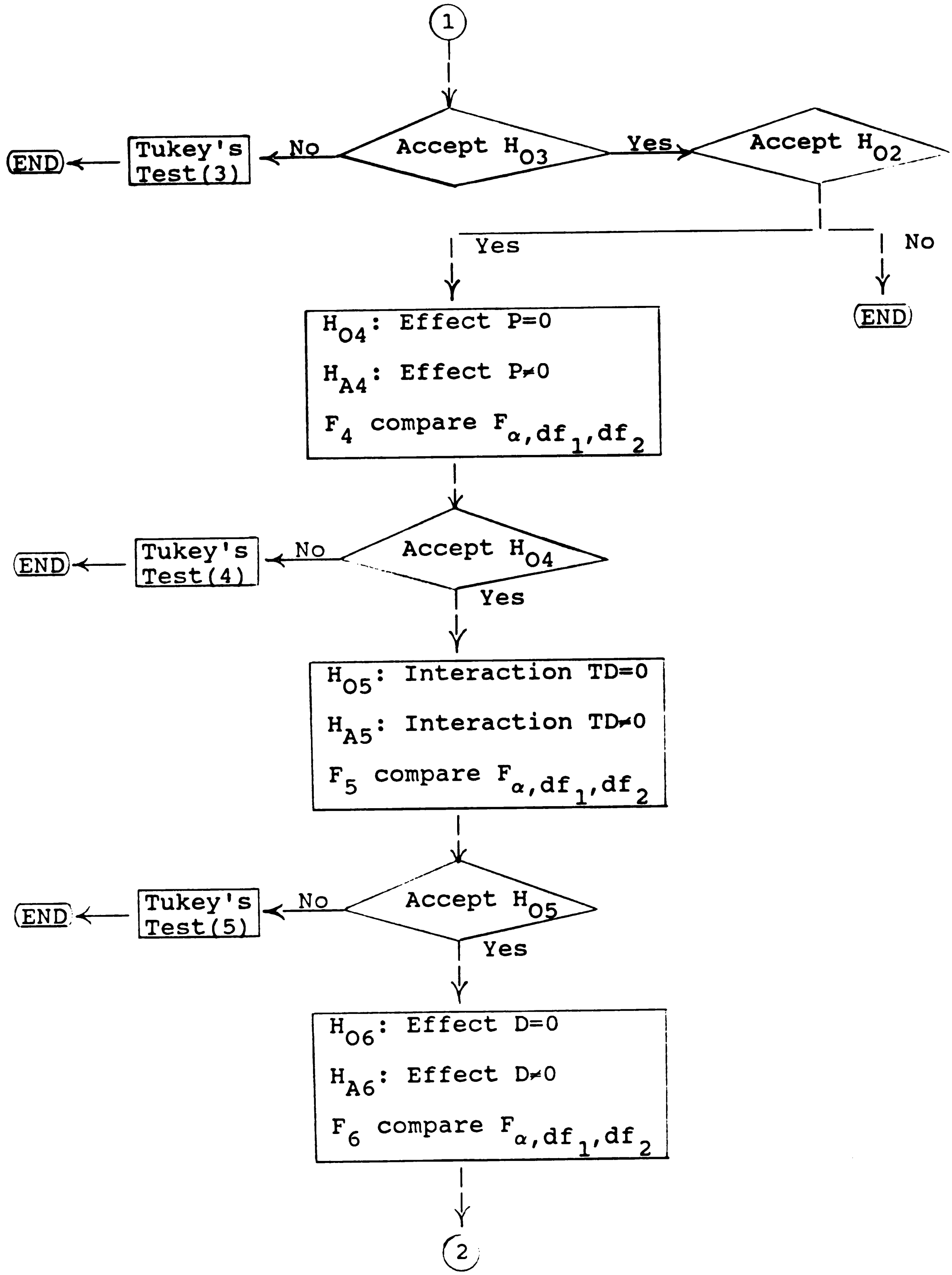
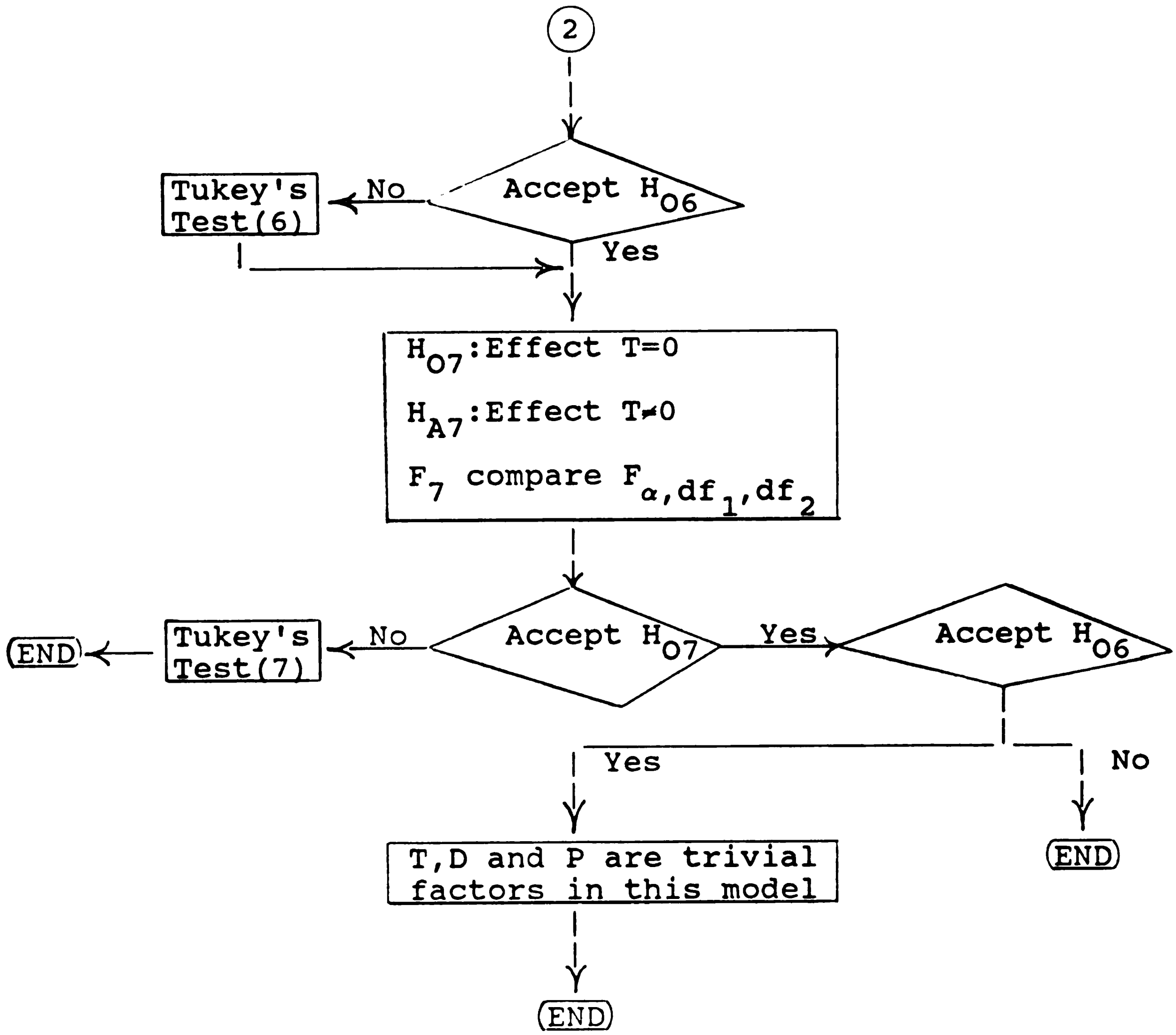


Figure 1 (cont'd.).



T = Temperature
D = Density
P = Period

Figure 1 (cont'd.).

Tukey's Test

Tukey developed an honestly significant difference (HSD) test that utilizes a t-like statistic based on the distribution of the Studentized range, that is, distribution of the ratio of the sample range to the sample standard deviation for K items from the normal distribution (Gill, 1978a). For the K(K-1)/2 possible comparisons of two means among a group of K means, the joint probability of Type I error may be set at α. Compare the difference between each pair of means from a balanced experiment (i.e. equal number of replicates for individual treatments) with a selected HSD. If the difference between two means, in absolute magnitude, exceeds a predetermined HSD, it may indicate that one correspondent treatment is significantly more effective than the other. However, my HSD may be computed in two different ways according to different decisions made through the flow chart shown in Figure 1.

I. HSD for split-plot design (Gill, 1986).

$$HSD = [(q_{\alpha,K,\nu})\sqrt{\frac{\sigma^2}{r}}\,] / \sqrt{2} \quad ,$$

$$\sigma^2 = [MS_{A/TD} + (P - 1)\ MS_E] / P \ ,$$

$$\nu = (\sigma^2)^2 / \{[MS_{A/TD}{}^2 + (P - 1)\ MS_E{}^2] / [KP^2(r - 1)]\} \ ,$$

K = number of compared means (i.e. treatments' means),

r = number of conditioned replicates,

P = 4 time periods, and

$\alpha = 0.05$

II. HSD for completely randomized design (Gill, 1978a).

$$HSD = (q_{\alpha, K, N-K}) \sqrt{\frac{MS_{A/TD}}{r}} ,$$

K = number of compared means (i.e. treatments' means),

r = number of conditioned replicates,

N = total sample size = 72, and

$\alpha = 0.05$

Table 4 demonstrates how the numbered Tukey's tests shown in Figure 1 are individually performed by using an appropriate HSD. In addition, the numerical numbers of both K and r are also given to each numbered Tukey's test (Table 4).

A DEVELOPMENT AND PERFORMANCE EVALUATION OF THE EXPERIMENTAL SYSTEM

Based on the previous model, an indoor operational system was built as follows. Three aquaria with respective sizes of 10-, 15-, and 30-gallon were assigned to a constant temperature at 24C, and to the other two cyclic thermal treatments. In order to maintain the desired temperatures, the aquaria were individually equipped with heaters at various wattages. All the heaters have an automatic electronic thermostat. Table 5 summarizes how the heaters were distributed into the aquaria according to the

Table 4. The demonstration of the numbered Tukey's tests in Figure 1 on the weight/weight ratio.

Numbered Tukey's Test from Figure 1	HSD	K	r	Remark
Tukey's Test (1)	I	9	2	Compare 9 means of treatment combinations within each time period.
Tukey's Test (2)	I	3	6	Compare 3 means of density treatments within each time period.
Tukey's Test (3)	I	3	6	Compare 3 means of thermal treatments within each time periods.
Tukey's Test (4)	I	4	18	Compare 4 means of time periods.
Time is no longer a factor below here				
Tukey's Test (5)	II	9	8	Compare 9 means of treatment combinations.
Tukey's Test (6)	II	3	24	Compare 3 means of density treatments.
Tukey's Test (7)	II	3	24	Compare 3 means of thermal treatments

Table 5. The distribution in number of heater(s) with correspondent wattages per aquarium for each thermal treatment.

Thermal Treatment	Aquarium Size					
	10-gallon		15-gallon		30-gallon	
24C	1 heater		1 heater		heater 1	heater 2
	50-watt		75-watt		100-watt	50-watt
22.7C to 26.8C	Heater #					
	1	2	1	2	1	2
	50-watt	50-watt	100-watt	50-watt	200-watt	100-watt
24.6C to 28.8C	Heater #					
	1	2	1	2	1	2
	100-watt	75-watt	200-watt	50-watt	300-watt	200-watt

desired temperatures and the aquaria sizes.

For each cyclic temperature, the three sizes of aquaria were placed into a tank. Inside each tank, the 10- and 15-gallon aquaria were seated on 4-leg steel frames, and as a result, were able to stand as high as the 30-gallon aquaria. After filling the tanks, the 3 aquaria emerged from the tanks' water level by 1-inch in height. With two 300-watt heaters installed diagonally in the tanks, the waterbath cyclicly controlled the temperatures of the aquaria at smoothly changing rates. Aeration was additionally applied inside of the tanks and their aquaria to facilitate the thermal circulation. For the 24C constant temperature aquaria, each was aerated with two air stones placed in its diagonal corners. The entire operational system was replicated twice. Table 6 details the items and the related quantities of the system's facilities.

The heaters used in aquaria receiving constant thermal treatment were adjusted to be continuously on to maintain the temperatures at 24C during this study. The heaters of the other tanks and their associated aquaria cycled on daily from 0800 hours to 2000 hours and then off during the next 12-hours. Such on-and-off activities therefore generated two thermocycles. Like feeding activities, the heating process was also periodically interrupted beause of cleanings and measurings.

The aquaria's temperatures were monitored by a

Table 6. Total required items and the related quantities of the system's facilities.

Item	Size	Quantity
Tank	length x width x height = 79" x 21.5" x 22"	4
10-gallon Aquarium	20" x 10.5" x 12.25"	6
15-gallon Aquarium	24.25" x 12.75" x 12.5"	6
30-gallon Aquarium	30" x 12.75" x 18.75"	6
Emergent Heater	50-watt	12
Emergent Heater	75-watt	4
Emergent Heater	100-watt	8
Submersible Heater	200-watt	6
Submersible Heater	300-watt	10
Air Compressor	0.5HP, 1725RPM	1
Tank-Air Stone	6"-stripe	24
Aquarium-Air Stone	marble shape	36
Air Tubing	regular	accordingly

thermistor usually four times daily: 8:00 AM, 8:00 PM, and before each feeding. Consequently, the accuracy of the temperature control was statistically evaluated through the recorded temperatures. The evaluation was performed in the following two ways: (1) one-sample Student's t test (Zar, 1984; Gill, 1978a), and (2) comparing simple linear regression (Zar, 1984).

The test statistic of one-sample t test is

$$t = \frac{\overline{T} - \mu}{S\ / \sqrt{N_T}} ,$$

where N_T denotes a number of the total temperature samplings from an individual aquarium receiving 24C-treatment throughout the experimental duration. Compute the mean temperature ($\overline{T}$) and standard deviation (S) of the N_T readings, followed by a hypothesis test of the aquarium temperature being controlled at μ = 24C. The hypotheses test for an individual aquarium receiving constant thermal treatment is

$H_O : \mu = 24C$ and $H_A : \mu \neq 24C$,

if $|t| \geq t_{\alpha(2), N_{T-1}}$, then reject H_O; where $\alpha(2)$ refers to the two-tailed probability of $\alpha = 0.01$.

Comparing two simple regression lines was used to examine the efficiency of the cyclic thermal treatments. The simple linear regression,

$\hat{Y} = a + bX$,

was determined (Appendix 3) for each individual tank at every interval of 20-days to reduce the possible variation in room temperature due to seasonal changes. Variables X and Y (Appendix 3) represent heating hours and averaged temperatures, respectively. Heating started daily at 8:00 AM and ended at 8:00 PM. Variable X ranges from 0 to 12 hours after coding. Practically speaking, there were no temperatures measured "exactly" at X = 0, or 8:00 AM. In other words, the true range of X, statistically speaking, should never include the point at X = 0. For each tank, the variable Y was computed by averaging the temperature readings taken from three sizes of aquaria at time X. Intercept "a" is the lowest point of a cyclic temperature estimated at X = 0, or 8:00 AM. Slope "b" denotes an increasing rate of temperature change per unit time. Variable $\hat{Y}$ differing from the observation variable Y, are the tank's temperatures predicted at hours X by the regression equation.

The thermal similarity of tanks is evaluated by comparing two treatment-related regression lines for the same time interval. The two compared regression lines thus correspond to two thermal-duplicated tanks. Therefore we may conclude that any two duplicated tanks are thermally alike at a given time interval when their corresponding regression lines possess statistically equal intercepts and slopes. Figure 2 demonstrates the testing procedure in comparing two regression lines. The related test statistics

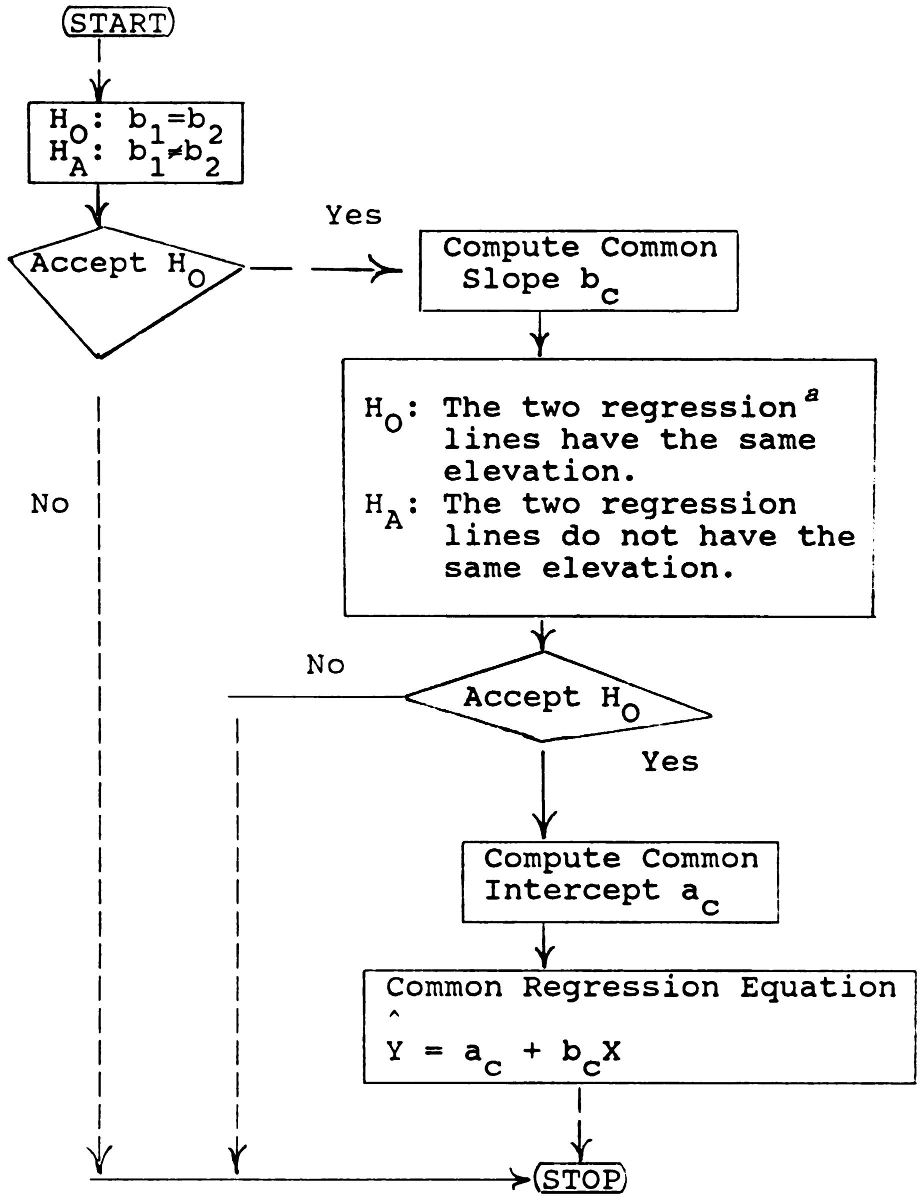


Figure 2. Flow Chart of the comparison of regression lines to evaluate the system's performance in cyclic thermal treatments.

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a. The test of difference between elevations may be considered the same as asking wether the two intercepts are different. However, it is not advisable to test H_O:$a_1 = a_2$ because no temperatures were able to be measured exactly at 8:00 AM (i.e. x=0). Therefore, the point of x=0 is statistically beyond the x's range.

as well as the computations of common slope (b_c) and intercept (a_c) are detailed in Appendix 4. Finally the common equation of linear regression

$$\hat{Y} = a_c + b_c X$$

is obtained if no hypotheses are rejected when quantitatively describing the qualities of the thermal treatments at a given time interval.

DETERMINISTIC MODELING BY COMPUTER SIMULATION

The nonlinear justification for Logistic Model was used to determine the optimum from the few selected sets of treatments according to the comparisons among their associated productions. The Logistic Equation is one well-accepted model proposed to characterize the growth of both individuals and populations. Some researchers sought to define the Logistic Model as a universal law of growth, others recognized it as a logical explanation for growth provided certain assumptions were met. This production model is mathematically expressed as

$$\frac{dW}{dt} = GW \left(\frac{K-W}{K}\right) ,$$

with two assumptions:

(1) the studied population is homogeneous; and

(2) the growth increases exponentially, when W is small.

The instantaneous output of the model, dW/dt, is the derivative of weight (W) with respect to time (t). The system's parameter G is the daily growth rate estimated by the natural logarithm of the ratio of final weight to initial weight for a time interval:

$$G = [\ln (\frac{W_t}{W_0})]/t \text{ , where } t = 20 \text{ days.}$$

The W_t/W_0, in fact, is the variable of weight ratio being repeatedly monitored at intervals of 20 days. K is the system's carrying capacity of a single fish, that is $K = W_t/5$. W_t is the weight of 5 fish in individual aquaria at time t, where t = days 20, 40, 60, and 80. Obviously W_t is the weight variable having been studied in stochastic modeling.

Apparently the system's output, dW/dt, is directly proportional to its own parameters, G and K. Meanwhile, a few sets of treatments in a time sequence may be all viewed as same optimum if they induce statistically equal G's and K's. Therefore, to compare the system's outputs (dW/dt's) associated with the sets of treatments in a statistical tie is a way to further determine the most optimum set. To do this, we must go to an approximate numerical (computer) technique for solution because of the nonlinearity in the Logistic Model. That is, changing the differential equation into a difference equation which we can solve recursively with a computer. The fourth order Runge-Kutta, a time efficient computer method with a high degree of precision, was selected to transform the Logistic Model into a difference equation,

$$W(t + \Delta t) = W(t) + 1/6\ (f_1 + 2f_2 + 2f_3 + f_4)$$

$$f_1 = (\Delta t)\ G_p\ W(t)\ \left[\frac{K_p - W(t)}{K_p}\right]$$

$$f_2 = (\Delta t)\ G_p\ \left[W(t) + \frac{f_1}{2}\right]\left\{\frac{K_p - [W(t) + f_1/2]}{K_p}\right\}$$

$$f_3 = (\Delta t)\ G_p\ \left[W(t) + \frac{f_2}{2}\right]\left\{\frac{K_p - [W(t) + f_2/2]}{K_p}\right\}$$

$$f_4 = (\Delta t)\ G_p\ [W(t) + f_3]\left\{\frac{K_p - [W(t) + f_3]}{K_p}\right\}$$

where $W(t)$ = individual weight of the fish at time t. t is initially set at 0, i.e. day zero; and $\Delta t = 1$ day. G_p and K_p are pairs of the system's parameters, growth rate and carrying capacity, respectively, constrained by a set of treatments associated with a sequence of time periods denoted by p = 1, 2, 3, 4. The system's initial condition is given by $W(t=0) = W(0)$, that is, the initial weight of a fish at day zero. One million values of $W(0)$ were generated from Monte Carlo Simulations using a given mean and standard deviation. The given mean and standard deviation were the mean and standard deviation of the initial weights of the studied fish.

A computer simulation based on the difference equation was therefore used with the one million initial weights of fish and a series of paired G_p and K_p. In other words, the total biomass production in grams from the system under a given set of sequential treatments was continuously generated on a daily basis. Appendix 5 presents a complete program of the computer simulation. By comparing the various outputs due to the various sets of treatments, we may approach a determination regarding the optimum set of sequential treatments. The optimum set may be defined by the following two ways with respect to the market demand:

(1) it is a set of sequential treatments capable of producing fish at a certain desired size in grams with the fewest operating days; or

(2) it is a set of sequential treatments capable of producing the greatest biomass in grams under a selected number of operating days, i.e. 80 days.

Two ways of analyzing the outputs were therefore designed.

Based on the shortest production cycle in days to reach a desired size, a statistical model was designed as follows,

$$Y_{ij} = \mu + SR_i + E_{ij} \ .$$

Y_{ij} is the j^{th} observed production cycle, or the number of days to reach a desired size using the i^{th} selection in the computer simulation. μ, a constant, is the true mean of the distribution of variable Y. SR_i is the average effect of the i^{th} selection of the simulation. E_{ij} is the random error associated with Y_{ij}. Perform Hartley's F-max test (Gill, 1978a) first, a test of homogeneous variance, involving the ratio of the largest to the smallest of the variances within the total simulations. As long as the assumption of equal variances is met, the ANOVA is conducted (Table 7). If the F ratio of the ANOVA is significantly large, then Tukey's test is applied to the multiple comparison. If not, the analysis ends. In Tukey's test, the HSD[a] (Honestly Significant Difference) is again used as an index to compare with any difference between two means, $\overline{Y}_{i.}$ and $\overline{Y}_{i'.}$, where $i \neq i'$. If the difference between $\overline{Y}_{i.}$ and $\overline{Y}_{i'.}$ exceeds the HSD, we may conclude that simulation run i is better than simulation run i', or vice versa. This test thus indicates which of the two compared sets of sequential treatments is the best for producing fish of a desired size in the shortest time.

To determine the optimum treatments to produce the largest biomass production in grams under a given number of operating days, the statistical model is

$$Y_{ij} = \mu + SR_i + E_{ij} .$$

Y_{ij} is the j^{th} observed mean weight in grams of the one

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a. $$HSD = (q_{\alpha,i,n-i})\sqrt{\frac{MS_E}{j}}$$, where $\alpha = 0.05$; $n = i \times j$ = sample size; i = a number of the simulation runs; j = a number of the observations (replicates) per simulation run.

Table 7. One way ANOVA with variable Y denoting the production cycle of reaching a desired size.

Source of Variation	Sum of Squares[a]	Degrees of Freedom	Mean Square	F Ratio[b]
Total	SS_Y	$i \times j - 1$	MS_Y	
Simulation Run	SS_{SR}	$i - 1$	MS_{SR}	$\frac{MS_{SR}}{MS_E}$
Residuals	SS_E	$i(j - 1)$	MS_E	

a.

$$SS_Y = \sum_{i=1}^{i} \sum_{j=1}^{j} Y_{ij}^2 - \left(\sum_{i=1}^{i} \sum_{j=1}^{j} Y_{ij}\right)^2 / (i \times j) ,$$

$$SS_{SR} = \left(\sum_{i=1}^{i} Y_{i.}^2 / j\right) - \left(\sum_{i=1}^{i} \sum_{j=1}^{j} Y_{ij}\right)^2 / (i \times j) ,$$

$$SS_E = SS_Y - SS_{SR} ,$$

where i = the number of computer simulations,
j = the number of observations in production cycle per simulation.

b. A critical value of F is chosen at $\alpha = 0.05$, i.e., $F_{\alpha,(i-1),i(j-1)} = F_{0.05,(i-1),i(j-1)}$.

million fish under the i^{th} simulation run. μ, a constant, is the true mean of the distribution of the variable Y. SR_i is the average effect of the i^{th} selection of the simulation run. E_{ij} is the random error associated with Y_{ij}. This modeling, design and analysis, is similar to the former, except that the variable Y is now defined differently.

RESULTS

The discussion of results is divided into three areas: (1) stochastic modeling, (2) performance of the system's operation, and (3) deterministic modeling by computer simulation.

STOCHASTIC MODELING

The experimental observations of weight and weight ratio are contained respectively in Tables 8 and 9. The results of this modeling are separately presented in the following sections: (1) tests of assumptions, (2) ANOVA, and (3) Tukey's tests.

Tests of Assumptions

In testing the hypothesis of normality, the 72 random errors with weight variable are shown in Tables 10. The test statistic is

$$W = \frac{G^2}{SS_E{}^a} = \frac{(9.20738)^2}{15.76} = 5.38 \quad ,$$

which is much greater than a critical value of $W_{0.1,72}$ =

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[a] SS_E = 15.76 is quoted from Table 14.

Table 8. The weight observations[a] per treatment combination at 4 periods (P) of 20-day.

Thermal Treatment	Density Treatment														
	10-gallon					15-gallon					30-gallon				
	Aquarium Number	P1	P2	P3	P4	Aquarium Number	P1	P2	P3	P4	Aquarium Number	P1	P2	P3	P4
24C	(1)	13.5	12.8	13.8	14.5	(7)	11.3	9.1	8.9	10.1	(13)	13.9	12.2	12.9	14.9
	(2)	10.2	10.4	10.8	12.2	(8)	12.1	11.8	12.4	12.8	(14)	11.3	10.2	10.5	12.2
	Sum	23.7	23.2	24.6	26.7	Sum	23.4	20.9	21.3	22.9	Sum	25.2	22.4	23.4	27.1
	Mean	11.85	11.60	12.30	13.35	Mean	11.70	10.45	10.65	11.45	Mean	12.60	11.20	11.70	13.55
	S^2	5.44	2.88	4.50	2.64	S^2	0.32	3.64	6.12	3.64	S^2	3.38	2.00	2.88	3.64
22.7C to 26.8C	(3)	8.7	9.9	10.8	11.4	(9)	9.9	9.1	9.3	9.5	(15)	11.6	13.0	14.0	16.5
	(4)	9.5	10.7	12.7	13.4	(10)	9.5	11.1	11.9	12.9	(16)	12.5	13.7	14.5	14.2
	Sum	18.2	20.6	23.5	24.8	Sum	19.4	20.2	21.2	22.4	Sum	24.1	26.7	28.5	30.7
	Mean	9.10	10.30	11.75	12.40	Mean	9.70	10.10	10.60	11.20	Mean	12.05	13.35	14.25	15.35
	S^2	0.32	0.32	1.81	2.00	S^2	0.08	2.00	3.38	5.78	S^2	0.41	0.24	0.12	2.64
24.6C to 28.8C	(5)	9.6	9.5	10.3	11.6	(11)	13.6	11.5	12.0	12.8	(17)	10.4	10.0	11.0	11.5
	(6)	10.9	11.1	11.3	12.7	(12)	12.0	13.0	13.3	15.3	(18)	12.8	13.5	14.4	15.0
	Sum	20.5	20.6	21.6	24.3	Sum	25.6	24.5	25.3	28.1	Sum	23.2	23.5	25.4	26.5
	Mean	10.25	10.30	10.80	12.15	Mean	12.80	12.25	12.65	14.05	Mean	11.60	11.75	12.70	13.25
	S^2	0.84	1.28	0.50	0.60	S^2	1.28	1.12	0.84	3.12	S^2	2.88	6.12	5.78	6.12

a. Each observation is the weight of 5 fish.

Table 9. The weight ratio observations[a] per treatment combination at 4 periods (P) of 20-day.

Thermal Treat-ment	Density Treatment														
	10-gallon					15-gallon					30-gallon				
	Aquarium Number	P1	P2	P3	P4	Aquarium Number	P1	P2	P3	P4	Aquarium Number	P1	P2	P3	P4
	(1)	0.894	0.948	1.078	1.051	(7)	1.507	0.805	0.978	1.135	(13)	1.299	0.878	1.057	1.155
	(2)	1.378	1.020	1.038	1.130	(8)	1.198	0.975	1.051	1.032	(14)	1.256	0.903	1.029	1.162
24C	Sum	2.272	1.968	2.116	2.181	Sum	2.705	1.730	2.029	2.167	Sum	2.555	1.781	2.086	2.317
	Mean	1.136	0.984	1.058	1.090	Mean	1.352	0.890	1.014	1.084	Mean	1.278	0.890	1.043	1.158
	S^2	0.117	0.002	0.0008	0.003	S^2	0.048	0.014	0.003	0.005	S^2	0.001	0.0003	0.0004	$2x10^{-5}$
	(3)	1.145	1.138	1.091	1.056	(9)	1.031	0.919	1.022	1.022	(15)	1.094	1.121	1.077	1.179
	(4)	0.950	1.126	1.187	1.055	(10)	0.922	1.168	1.072	1.084	(16)	1.096	1.096	1.058	0.979
22.7C to 26.8C	Sum	2.095	2.264	2.278	2.111	Sum	1.953	2.087	2.094	2.106	Sum	2.190	2.217	2.135	2.158
	Mean	1.048	1.132	1.139	1.056	Mean	0.976	1.044	1.047	1.053	Mean	1.095	1.108	1.068	1.079
	S^2	0.019	$7x10^{-5}$	0.005	$5x10^{-7}$	S^2	0.006	0.031	0.001	0.002	S^2	$2x10^{-6}$	0.0003	0.0002	0.02
	(5)	1.215	0.990	1.084	1.126	(11)	1.283	0.846	1.044	1.067	(17)	1.224	0.962	1.100	1.045
	(6)	0.973	1.018	1.018	1.124	(12)	1.188	1.083	1.023	1.150	(18)	1.153	1.055	1.067	1.042
24.6C to 28.8C	Sum	2.188	2.008	2.102	2.250	Sum	2.471	1.929	2.067	2.217	Sum	2.377	2.017	2.167	2.087
	Mean	1.094	1.004	1.051	1.125	Mean	1.236	0.964	1.034	1.108	Mean	1.188	1.008	1.084	1.044
	S^2	0.029	0.0004	0.002	$2x10^{-6}$	S^2	0.004	0.028	0.0002	0.003	S^2	0.002	0.004	0.0005	$4x10^{-6}$

a. Each observation is the weight ratio of 5 fish.

Table 10. Random errors, $E_{(tdp)a}$[a], of the weight.

Thermal Treatment	Density Treatment														
	10-gallon					15-gallon					30-gallon				
	Aquarium Number	P1	P2	P3	P4	Aquarium Number	P1	P2	P3	P4	Aquarium Number	P1	P2	P3	P4
24C	(1)	1.65	1.20	1.50	1.15	(7)	-0.40	-1.35	-1.75	-1.35	(13)	1.3	1.00	1.20	1.35
	(2)	-1.65	-1.20	-1.50	-1.15	(8)	0.40	1.35	1.75	1.35	(14)	-1.3	-1.00	-1.20	-1.35
22.7C to 26.8C	(3)	-0.40	-0.40	-0.95	-1.00	(9)	0.20	-1.00	-1.30	-1.70	(15)	-0.45	-0.35	-0.25	1.15
	(4)	0.40	0.40	0.95	1.00	(10)	-0.20	1.00	1.30	1.70	(16)	0.45	0.35	0.25	-1.15
24.6C to 28.8C	(5)	-0.65	-0.80	-0.50	-0.55	(11)	0.80	-0.75	-0.65	-1.25	(17)	-1.20	-1.75	-1.70	-1.75
	(6)	0.65	0.80	0.50	0.55	(12)	-0.80	0.75	0.65	1.25	(18)	1.20	1.75	1.70	1.75

a. $E_{(tdp)a} = Y_{tdpa} - \bar{Y}_{tdp.}$

0.9725. Therefore the random errors are normally distributed. Table 11 contains the other 72 random errors with the variable of weight ratio. The test statistic is

$$W = \frac{G^2}{SS_E{}^a} = \frac{(0.5605007)^2}{0.275311} = 1.14 \quad ,$$

which exceeds the critical value of $W_{0.1,72} = 0.9725$. The hypothesis of normality in weight ratios is still accepted.

The homogeneity of variances associated with the nine treatment combinations was examined within each time period. Tables 12 and 13 indicate that the assumption of equal variances is met.

Analysis of Variance (ANOVA)

Both of the ANOVA tables (Tables 14 and 15) reveal that the time factor (period) has a significant effect on the growth of fish. However, the multiple comparison among the three thermal treatments must be done for the same period of time because of the significant interaction effect between period and temperature (Tables 14 and 15).

Tukey's Tests

Since density is not an effective factor from the ANOVAs (Tables 14 and 15), the aquaria with various volumes at every time interval may be treated as a set of replicates as long as they receive the same thermal

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[a]. $SS_E = 0.275311$ is quoted from Table 15.

Table 11. Random errors, $E_{(tdp)a}$[a], of the weight ratio.

Thermal Treat-ment	Density Treatment														
	10-gallon					15-gallon					30-gallon				
	Aquarium Number	P1	P2	P3	P4	Aquarium Number	P1	P2	P3	P4	Aquarium Number	P1	P2	P3	P4
24C	(1)	-2.42	-0.36	0.20	-0.39	(7)	1.55	-0.85	-0.36	0.51	(13)	0.21	-0.12	0.14	-0.03
	(2)	2.42	0.36	-0.20	0.39	(8)	-1.55	0.85	0.36	-0.51	(14)	-0.21	0.12	-0.14	0.03
22.7C to 26.8C	(3)	0.97	0.06	-0.48	0	(9)	0.55	-1.25	-0.25	-0.31	(15)	-0.01	0.13	0.09	1.00
	(4)	-0.97	-0.06	0.48	0	(10)	-0.55	1.25	0.25	0.31	(16)	0.01	-0.13	-0.09	-1.00
24.6C to 28.8C	(5)	1.21	-0.14	0.33	0.01	(11)	0.47	-1.18	0.10	-0.41	(17)	0.36	-0.46	0.16	0.01
	(6)	-1.21	0.14	-0.33	-0.01	(12)	-0.47	1.18	-0.10	0.41	(18)	-0.36	0.46	-0.16	-0.01

a. $E_{(tdp)a}$, $Y_{tdpa} - \overline{Y}_{tdp.}$, are magnified by 10^{-1} in this table.

Table 12. Bartlett's test on homogeneity of variances regarding the weight.

Time Period	S_p^2	B	C	Test Statistic B_C	Critical Value $\chi^2_{\alpha, td-1}$	Is the Assumption of Equal Variances Respectively Associated with 9 Treatment Combinations Met?
Period I Day 1~Day 20	1.66	6.23	1.37	4.55	$\chi^2_{0.2,8}=11.03$ $\chi^2_{0.1,8}=13.36$	Yes
Period II Day 21~Day 40	2.18	3.64	1.37	2.66	$\chi^2_{0.2,8}=11.03$ $\chi^2_{0.1,8}=13.36$	Yes
Period III Day 41~Day 60	2.88	4.53	1.37	3.31	$\chi^2_{0.2,8}=11.03$ $\chi^2_{0.1,8}=13.36$	Yes
Period IV Day 61~Day 80	3.36	1.47	1.37	1.07	$\chi^2_{0.2,8}=11.03$ $\chi^2_{0.1,8}=13.36$	Yes

Table 13. Bartlett's test on homogeneity of variances regarding the weight ratio.

Time Period	S_p^2	B	C	Test Statistic B_C	Critical Value $\chi^2_{\alpha,td-1}$	Is the Assumption of Equal Variances Respectively Associated with 9 Treatment Combinations Met?
Period I	0.025	16.181	1.370	11.808	$\chi^2_{0.2,8}=11.03$	Fair
Day 1~Day 20					$\chi^2_{0.1,8}=13.36$	
Period II	0.009	13.874	1.370	10.124	$\chi^2_{0.2,8}=11.03$	Yes
Day 21~Day 40					$\chi^2_{0.1,8}=13.36$	
Period III	0.001	4.680	1.370	3.415	$\chi^2_{0.2,8}=11.03$	Yes
Day 41~Day 60					$\chi^2_{0.1,8}=13.36$	
Period IV	0.004	17.941	1.370	13.092	$\chi^2_{0.2,8}=11.03$	Fair
Day 61~Day 80					$\chi^2_{0.1,8}=13.36$	

Table 14. **ANOVA** of the weight.

Source of Variation	Degrees of Freedom (D.F.)	Sum of Squares	Mean Square	F Ratio	Probability
Total(Y)	71	222.107	3.128		
Temperature (T)	2	1.614	0.807	0.0969	
Density (D)	2	30.334	15.167	1.821	
Interaction (TD)	4	47.789	11.947	1.434	
Error I (A/TD)	9	74.970	8.330		
Period (P)	3	34.697	11.566	19.814[a]	$p<<0.0005$
Interaction (TP)	6	10.499	1.750	2.998[a]	$0.01<p<0.025$
Interaction (DP)	6	4.162	0.694	1.188	
Interaction (TDP)	12	2.282	0.190	0.326	
Residuals (E)	27	15.760	0.584		

a. The effect of treatment or treatment combination is significant.

Table 15. **ANOVA** of the weight ratio.

Source of Variation	Degrees of Freedom (D.F.)	Sum of Squares	Mean Square	F Ratio	Probability
Total(Y)	71	0.955525	0.013458		
Temperature (T)	2	0.001596	0.000798	0.0898	
Density (D)	2	0.004842	0.002421	0.272	
Interaction (TD)	4	0.018771	0.0046927	0.528	
Error I (A/TD)	9	0.079995	0.0088883		
Period (P)	3	0.218839	0.0729463	7.154[a]	$0.001<p<0.0025$
Interaction (TP)	6	0.245025	0.0408375	4.005[a]	$0.005<p<0.01$
Interaction (DP)	6	0.056241	0.0093735	0.919	
Interaction (TDP)	12	0.054905	0.0045754	0.449	
Residuals (E)	27	0.275311	0.0101967		

a. The effect of treatment or treatment combination is significant.

treatment. As a result, Tables 16 and 17 respectively display the rearranged data with related treatment means of the weight and weight ratio. By applying Tukey's method, the means of thermal treatments were compared at every time interval. Tables 18 and 19 present the optimum treatment(s) in terms of temperature on the basis of the 20-day-cycle.

PERFORMANCE OF THE SYSTEM'S OPERATION

Table 20 shows that the quality control of the constant thermal treatment (24C) was satisfactory. Additionally, each pair of duplicated tanks, receiving the same assigned cyclic thermal treatment, statistically possess equal slopes (b) and intercepts (a) for the same time interval (Tables 21 and 22). As a result of the equivalences, a common linear regression of temperature versus time for each cyclic thermal treatment is described on the basis of 20 days' cycle (Tables 23 and 24).

DETERMINISTIC MODELING BY COMPUTER SIMULATION

The stochastic modeling has identified some significant thermal treatments according to the variables of weight and weight ratio on the cycle basis of 20 days (Tables 18 and 19). By further processing the results of Tables 18 and 19,

Table 16. The means of thermal treatments of the weight at each time period.

Thermal Treatment	Aquaria Size (Gal)	P0	P20	P40	P60	P80
	10	15.1	13.5	12.8	13.8	14.5
	10	7.4	10.2	10.4	10.8	12.2
	15	7.5	11.3	9.1	8.9	10.1
24C	15	10.1	12.1	11.8	12.4	12.8
	30	10.7	13.9	12.2	12.9	14.9
	30	9.0	11.3	10.2	10.5	12.2
	mean =	10.0	12.050	11.083	11.550	12.783
	10	7.6	8.7	9.9	10.8	11.4
	10	10.0	9.5	10.7	12.7	13.4
22.7C	15	9.6	9.9	9.1	9.3	9.5
to	15	10.3	9.5	11.1	11.9	12.9
26.8C	30	10.6	11.6	13.0	14.0	16.5
	30	11.4	12.5	13.7	14.5	14.2
	mean =	9.9	10.283	11.250	12.200	12.983
	10	7.9	9.6	9.5	10.3	11.6
	10	11.2	10.9	11.1	11.3	12.7
24.6C	15	10.6	13.6	11.5	12.0	12.8
to	15	10.1	12.0	13.0	13.3	15.3
28.8C	30	8.5	10.4	10.0	11.0	11.5
	30	11.1	12.8	13.5	14.4	15.0
	mean =	9.9	11.550	11.433	12.050	13.150

Table 17. The means of thermal treatments of the weight ratio at each time period.

Thermal Treatment	Size of Aquarium (in Gallons)	P20	P40	P60	P80
	10	0.894	0.948	1.078	1.051
	10	1.378	1.020	1.038	1.130
	15	1.507	0.805	0.978	1.135
24C	15	1.198	0.975	1.051	1.032
	30	1.299	0.878	1.057	1.155
	30	1.256	0.903	1.029	1.162
	mean =	1.255	0.922	1.038	1.111
	10	1.145	1.138	1.091	1.056
	10	0.950	1.126	1.187	1.055
22.7C	15	1.031	0.919	1.022	1.022
to	15	0.922	1.168	1.072	1.084
26.8C	30	1.094	1.121	1.077	1.179
	30	1.096	1.096	1.058	0.979
	mean =	1.040	1.095	1.084	1.062
	10	1.215	0.990	1.084	1.126
	10	0.973	1.018	1.018	1.124
24.6C	15	1.283	0.846	1.044	1.067
to	15	1.188	1.083	1.023	1.150
28.8C	30	1.224	0.962	1.100	1.045
	30	1.153	1.055	1.067	1.042
	mean =	1.173	0.992	1.056	1.092

Table 18. Tukey's test on thermal treatments of the weight within each time period.

Time Period	Thermal Treatment	Treatment Mean	Difference Between Means	Determination According to HSD=1.62
Period I	24C	12.050		Treatments 24C and 24.6C to 28.8C are effective.
			0.500	
Day 1	24.6C~28.8C	11.550		
to			1.267	
Day 20	22.7C~26.8C	10.283		
Period II	24.6C~28.8C	11.433		There is no significant difference.
			0.183	
Day 21	22.7C~26.8C	11.250		
to			0.167	
Day 40	24C	11.083		
Period III	22.7C~26.8C	12.200		There is no significant difference.
			0.150	
Day 41	24.6C~28.8C	12.050		
to			0.500	
Day 60	24C	11.550		
Period IV	24.6C~28.8C	13.150		There is no significant difference.
			0.167	
Day 61	22.7C~26.8C	12.983		
to			0.200	
Day 80	24C	12.783		

Table 19. Tukey's test on thermal treatments of the weight ratio within each time period.

Time Period	Thermal Treatment	Treatment Mean	Difference Between Means	Determination According to HSD=0.0975
Period I	24C	1.255		Treatments
			0.082	24C and 24.6C
Day 1	24.6C~28.8C	1.173		to 28.8C are
to			0.133	effective.
Day 20	22.7C~26.8C	1.040		
Period II	22.7C~26.8C	1.095		Treatment
			0.103	22.7C to
Day 21	24.6C~28.8C	0.992		26.8C is
to			0.070	effective.
Day 40	24C	0.922		
Period III	22.7C~26.8C	1.084		
			0.028	There is no
Day 41	24.6C~28.8C	1.056		significant
to			0.018	difference.
Day 60	24C	1.038		
Period IV	24C	1.111		
			0.019	There is no
Day 61	24.6C~28.8C	1.092		significant
to			0.030	difference.
Day 80	22.7C~26.8C	1.062		

Table 20. Quality control of the constant thermal treatment (24C).

Aquarium Volume (in Gallons)	Samplings of Temperature (N_T)	Mean of Temperature (C)	Standard Deviation (S)	Test Statistic (t)	Critical $t_{0.01(2),125}$ [a]	Satisfaction
10	129	24.054	0.236	2.568	2.616	Yes
10	127	24.042	0.333	1.412	2.616	Yes
15	127	23.976	0.269	1.023	2.616	Yes
15	129	24.027	0.191	1.612	2.616	Yes
30	127	24.087	0.508	1.939	2.616	Yes
30	127	24.014	0.276	0.578	2.616	Yes

a. $t_{0.01(2),125}$ is a substitute because $t_{0.01(2),N_T-1}$ is not available.

Table 21. Quality control of the cyclic thermal treatment (22.7C to 26.8C).

		Slope b	Intercept a	Correlation Coefficient	Common Slope b_c	Common Intercept a_c
Period I	Tank 1	0.434	21.642	0.912	0.434	21.552
	Tank 2	0.435	21.461	0.921		
Day 1	Test Statistic t	0.0196	0.977			
to Day 20	Critical t	$t_{0.05(2),90}$ =1.987	$t_{0.05(2),91}$ =1.986			
Period II	Tank 1	0.333	23.099	0.914	0.336	22.989
	Tank 2	0.339	22.878	0.921		
Day 21	Test Statistic t	0.197	1.367			
to Day 40	Critical t	$t_{0.05(2),90}$ =1.987	$t_{0.05(2),91}$ =1.986			
Period III	Tank 1	0.306	23.129	0.901	0.327	22.776
	Tank 2	0.348	22.424	0.924		
Day 41	Test Statistic t	0.879	2.010			
to Day 60	Critical t	$t_{0.05(2),36}$ =2.028	$t_{0.05(2),37}$ =2.026			
Period IV	Tank 1	0.245	23.573	0.912	0.254	23.542
	Tank 2	0.263	23.511	0.926		
Day 61	Test Statistic t	0.588	0.341			
to Day 80	Critical t	$t_{0.05(2),50}$ =2.009	$t_{0.05(2),51}$ =2.009			

Table 22. Quality control of the cyclic thermal treatment (24.6C to 28.8C).

		Slope b	Intercept a	Correlation Coefficient	Common Slope b_c	Common Intercept a_c
Period I	Tank 1	0.412	23.680	0.850	0.403	23.656
	Tank 2	0.394	23.632	0.824		
Day 1 to Day 20	Test Statistic t	0.315	0.593			
	Critical t	$t_{0.05(2),90}$ =1.987	$t_{0.05(2),91}$ =1.986			
Period II	Tank 1	0.362	24.760	0.914	0.356	24.791
	Tank 2	0.349	24.822	0.898		
Day 21 to Day 40	Test Statistic t	0.366	0.055			
	Critical t	$t_{0.05(2),90}$ =1.987	$t_{0.05(2),91}$ =1.986			
Period III	Tank 1	0.355	24.543	0.878	0.341	24.705
	Tank 2	0.328	24.868	0.876		
Day 41 to Day 60	Test Statistic t	0.453	0.501			
	Critical t	$t_{0.05(2),38}$ =2.024	$t_{0.05(2),39}$ =2.023			
Period IV	Tank 1	0.314	25.010	0.918	0.307	25.149
	Tank 2	0.299	25.288	0.911		
Day 61 to Day 80	Test Statistic t	0.399	1.053			
	Critical t	$t_{0.05(2),50}$ =2.009	$t_{0.05(2),51}$ =2.009			

Table 23. Linear regression[a] of cyclic thermal treatment (22.7C to 26.8C) at various time interval.

Period I (Day 1 to Day 20)	Initial Temperature at X = 0	Final Temperature at X = 12	Range
$\hat{Y} = 21.552 + 0.434X$	21.6C	26.8C	5.2C
Period II (Day 21 to Day 40)			
$\hat{Y} = 22.989 + 0.336X$	23.0C	27.0C	4.0C
Period III (Day 41 to Day 60)			
$\hat{Y} = 22.776 + 0.327X$	22.8C	26.7C	3.9C
Period IV (Day 61 to Day 80)			
$\hat{Y} = 23.542 + 0.254X$	23.5C	26.6C	3.1C
Mean	22.7C	26.8C	4.1C
Standard Deviation	0.8057	0.1708	0.8660

a. $\hat{Y}$ are temperatures predicted at time X according to a regression line.

Table 24. Linear regression[a] of cyclic thermal treatment (24.6C to 28.8C) at various time interval.

Period I (Day 1 to Day 20)	Initial Temperature at X = 0	Final Temperature at X = 12	Range
$\hat{Y} = 23.656 + 0.403X$	23.7C	28.5C	4.8C
Period II (Day 21 to Day 40)			
$\hat{Y} = 24.791 + 0.356X$	24.8C	29.1C	4.3C
Period III (Day 41 to Day 60)			
$\hat{Y} = 24.705 + 0.341X$	24.7C	28.8C	4.1C
Period IV (Day 61 to Day 80)			
$\hat{Y} = 25.149 + 0.307X$	25.2C	28.8C	3.6C
Mean	24.6C	28.8C	4.2C
Standard Deviation	0.6377	0.2450	0.4967

a. $\hat{Y}$ are temperatures predicted at time X according to a regression line.

the carrying capacities and daily growth rates, influenced by various thermal treatments in a time sequence for a single fish, are summarized respectively in Tables 25 and 26. Tables 27 and 28 summarize four sets of dynamic computer inputs, determined by selecting the optimum thermal treatments through time periods from Tables 25 and 26, according to the optimums in daily growth rate and carrying capacity, respectively.

By providing the simulation system (Appendix 5) with the inputs (Tables 27 and 28) in order, the computer made printouts of mean and variance of the one million weights on a daily basis. These primary printouts are collected in Appendix 6. Tables 29 and 30 show the rearranged data extracted from the primary printouts to proceed ANOVA in the simulations.

The statistics in Hartley's F_{max} test are

$$F_{max} = \frac{S^2_{maximum}}{S^2_{minimum}} = \frac{9.1666667}{9.1666667} = 1.00 \quad \text{(Table 29)},$$

and

$$F_{max} = \frac{S^2_{maximum}}{S^2_{minimum}} = \frac{0.0000277}{0.0000236} = 1.17 \quad \text{(Table 30)}.$$

Table 25. Carrying capacity of a single fish associated with producing thermal treatment at 4 periods of 20-day.

Time Period	Thermal Treatment	Treatment Mean(W_t)[a]	Carrying Capacity(K)[b]	Remark
Period I	24C	12.050	2.410	Optimum
(Day 1 to	24.6C~28.8C	11.550	2.310	Optimum
Day 20)	22.7C~26.8C	10.283	2.057	
Period II	24.6C~28.8C	11.433	2.287	Viewed as optimum
(Day 21 to	22.7C~26.8C	11.250	2.250	
Day 40)	24C	11.083	2.217	
Period III	22.7C~26.8C	12.200	2.440	Viewed as optimum
(Day 41 to	24.6C~28.8C	12.050	2.410	
Day 60)	24C	11.550	2.310	
Period IV	24.6C~28.8C	13.150	2.630	Viewed as optimum
(Day 61 to	22.7C~26.8C	12.983	2.597	
Day 80)	24C	12.783	2.557	

a. W_t is a weight of 5 fish per aquarium at time t; where t = days 20, 40, 60 and 80.

b. K is carrying capacity of a single fish, i.e. $K = W_t/5$.

Table 26. Daily growth rate of a single fish associated with producing thermal treatment at 4 periods of 20-day.

Time Period	Thermal Treatment	Treatment Mean(W_t/W_0) [a]	Daily Growth Rate (G) [b]	Remark
Period I	24C	1.255	0.0113568	Optimum
(Day 1 to	24.6C~28.8C	1.173	0.0079782	Optimum
Day 20)	22.7C~26.8C	1.040	0.0019610	
Period II	22.7C~26.8C	1.095	0.0045377	optimum
(Day 21 to	24.6C~28.8C	0.992	-0.0004016	
Day 40)	24C	0.922	-0.0040605	
Period III	22.7C~26.8C	1.084	0.0040329	Viewed as optimum
(Day 41 to	24.6C~28.8C	1.056	0.0027244	
Day 60)	24C	1.038	0.0018648	
Period IV	24C	1.111	0.0052630	Viewed as optimum
(Day 61 to	24.6C~28.8C	1.092	0.0044005	
Day 80)	22.7C~26.8C	1.062	0.0030077	

[a] W_0 is the fish weight of an aquarium at time zero or the former time; W_t is the fish weight of an aquarium at time t or the latter time.

[b] G is the daily growth rate of fish, that is,

$$G = \ln\left(\frac{W_t}{W_0}\right)/t \quad \text{, where } t = 20 \text{ days.}$$

Table 27. The dynamic inputs of computer simulation based on the selection of optimum daily growth rate.

		Simulation 1	Simulation 2
Period I	Thermal Treatment	24C	24.6C~28.8C
(Day 1 to	Produced G	0.0113568	0.0079782
Day 20)	Produced K	2.410	2.310
Period II	Thermal Treatment	22.7C~26.8C	22.7C~26.8C
(Day 21 to	Produced G	0.0045377	0.0045377
Day 40)	Produced K	2.250	2.250
Period III	Thermal Treatment	22.7C~26.8C	22.7C~26.8C
(Day 41 to	Produced G	0.0040329	0.0040329
Day 60)	Produced K	2.440	2.440
Period IV	Thermal Treatment	24C	24C
(Day 61 to	Produced G	0.005263	0.005263
Day 80)	Produced K	2.557	2.557

G and K respectively denote daily growth rate and carrying capacity of a single fish.

Table 28. The dynamic inputs of computer simulation based on the selection of optimum carrying capacity.

		Simulation 3	Simulation 4
Period I (Day 1 to Day 20)	Thermal Treatment	24C	24.6C~28.8C
	Produced K	2.410	2.310
	Produced G	0.0113568	0.0079782
Period II (Day 21 to Day 40)	Thermal Treatment	24.6C~28.8C	24.6C~28.8C
	Produced K	2.287	2.287
	Produced G	-0.0004016	-0.0004016
Period III (Day 41 to Day 60)	Thermal Treatment	22.7C~26.8C	22.7C~26.8C
	Produced K	2.440	2.440
	Produced G	0.0040329	0.0040329
Period IV (Day 61 to Day 80)	Thermal Treatment	24.6C~28.8C	24.6C~28.8C
	Produced K	2.630	2.630
	Produced G	0.0044005	0.0044005

K and G respectively denote carrying capacity and daily growth rate of a single fish.

Table 29. Production cycles of reaching a desired size, 2.03[a] grams, of 4 simulations.

Selections of Simulation							
Simulation 1		Simulation 2		Simulation 3		Simulation 4	
Production Cycle (# of days)	Mean Weight of One Million Fish	Production Cycle (# of days)	Mean Weight of One Million Fish	Production Cycle (# of days)	Mean Weight of One Million Fish	Production Cycle (# of days)	Mean Weight of One Million Fish
33	2.0276947	57	2.0236032	45	2.0257604	65	2.0231091
34	2.0281563	58	2.0246502	46	2.0267513	66	2.0247735
35	2.0286193	59	2.0256959	47	2.0277416	67	2.0264357
36	2.0290835	60	2.0267406	48	2.0287311	68	2.0280955
37	2.0295490	61	2.0285272	49	2.0297199	69	2.0297530
38	2.0300157	62	2.0303107	50	2.0307080	70	2.0314083
39	2.0304836	63	2.0320910	51	2.0316954	71	2.0330612
40	2.0309527	64	2.0338680	52	2.0326820	72	2.0347118
41	2.0319749	65	2.0356418	53	2.0336678	73	2.0363601
42	2.0329960	66	2.0374124	54	2.0346529	74	2.0380060
Replicate = 10 Mean = 37.5 Variance = 9.1666667		Replicate = 10 Mean = 61.5 Variance = 9.1666667		Replicate = 10 Mean = 49.5 Variance = 9.1666667		Replicate = 10 Mean = 69.5 Variance = 9.1666667	

a. 2.03 is picked up for demonstration without any particular reason.

Table 30. Maximum yields[a] of a fixed production cycle (80-day) of 4 simulations.

Selections of Simulation							
Simulation 1		Simulation 2		Simulation 3		Simulation 4	
Maximum Yield (g)	Production Cycle (days)	Maximum Yield (g)	Production Cycle (days)	Maximum Yield (g)	Production Cycle (days)	Maximum Yield (g)	Production Cycle (days)
2.0701188	71	2.0462161	71	2.0584488	71	2.0330612	71
2.0718195	72	2.0479669	72	2.0600617	72	2.0347118	72
2.0735166	73	2.0497144	73	2.0616722	73	2.0363601	73
2.0752103	74	2.0514586	74	2.0632802	74	2.0380060	74
2.0769006	75	2.0531994	75	2.0648857	75	2.0396496	75
2.0785873	76	2.0549368	76	2.0664888	76	2.0412908	76
2.0802706	77	2.0566708	77	2.0680894	77	2.0429297	77
2.0819504	78	2.0584015	78	2.0696875	78	2.0445662	78
2.0836266	79	2.0601288	79	2.0712831	79	2.0462003	79
2.0852994	80	2.0618526	80	2.0728761	80	2.0478320	80
Replicate = 10 Mean = 2.07773 Variance = 0.0000261		Replicate = 10 Mean = 2.0540546 Variance = 0.0000277		Replicate = 10 Mean = 2.0656773 Variance = 0.0000236		Replicate = 10 Mean = 2.0404608 Variance = 0.0000247	

a. Maximum yield is still expressed by using the mean weight of one million fish for convenience.

Both of the F_{max}'s are much smaller than the critical $F_{max,\alpha,i,j-1} = 3.64$[a]. As a result, the assumption of homogeneous variance is met. Moreover the ANOVAs (Tables 31 and 32) indicate that the overall tests are significant.

Tukey's test is then used on the comparisons of means from Tables 29 and 30. The results of Tukey's test, summarized in Tables 33 and 34, conclude that the first selection of simulation enables the production of golden shiners to reach the management goals.

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a. $F_{max,\alpha,i,j-1} = 3.64$; where $\alpha = 0.25$; i = a number of the simulation runs = 4; j-1 = (a number of the replicates) - 1 = 10 - 1 = 9.

Table 31. ANOVA of computer modelings for reaching a desired size with shortest production cycle.

Source of Variation	Sum of Squares	D. F.	Mean Square	F Ratio
Total	6210	39		
Simulations	5880	3	1960	**213.818**[a] (p<<0.0005)
Residuals	330	36	9.1666666	

Table 32. ANOVA of computer modelings for obtaining a maximum harvest over 80 days.

Source of Variation	Sum of Squares	D. F.	Mean Square	F Ratio
Total	0.008454	39		
Simulations	0.007624	3	0.0025413	**110.49130**[a] (p<<0.0005)
Residuals	0.00083	36	0.000023	

[a] The effects due to the simulation runs are significantly different from one another.

Table 33. Tukey's test on computer modelings of reaching a desired size with shortest production cycle (days).

Selections of Simulation	Means of Simulation	Differences Between Means	Conclusion According to HSD = 3.681[a]	
1	37.5		First Priority	regarding this management goal
		12.0		
3	49.5		Second Priority	regarding this management goal
		12.0		
2	61.5		Third Priority	regarding this management goal
		8.0		
4	69.5		Fourth Priority	regarding this management goal

a. The true HSD = $(q_{\alpha,i,n-i})\sqrt{MS_E/j} = (q_{0.05,4,36})\sqrt{9.1666666/10}$. However $q_{0.05,4,36}$ is not available, therefore, $q_{0.05,4,30} = 3.845$ is chosen to replace it.

Table 34. Tukey's test on computer modelings of obtaining a maximum harvest over 80 days.

Selections of Simulation	Means of Simulation	Differences Between Means	Conclusion According to HSD = 0.0058312[a]
1	2.07773	0.0120527	First Priority regarding this management goal
3	2.0656773	0.0116227	Second Priority regarding this management goal
2	2.0540546	0.0135938	Third Priority regarding this management goal
4	2.0404603		Fourth Priority regarding this management goal

a. The true HSD = $(q_{\alpha,i,n-i})\sqrt{MS_E/j} = (q_{0.05,4,36})\sqrt{0.000023/10}$. However $q_{0.05,4,36}$ is not available, therefore, $q_{0.05,4,30} = 3.845$ is chosen to replace it.

DISCUSSION

Discussion of this research is focusd on two areas: (1) quality of the experimental system's operation; and (2) application and promotion of the research concept.

The intensities of the three thermal treatments were consistent over the duration of this research. Thus the system's performance in thermal control was highly satisfactory. However, the two studied cyclic temperatures as a secondary design differed from the primary desires. The primary desires were intended to be 24 ± 2C and 24 ± 4C with daily fluctuations of 4C and 8C, respectively. Budgetary constraints did not allow using the desired temperature cycles, forcing the use of a secondary design. In addition, the electrical supply to the laboratory only allowed each treatment combination to be replicated 2.5 times in terms of the needed heating and aeration. Constrained to two replicates per treatment combination, this experiment ran the risk of low statistical power. The replication used was the maximum attainable with the facilities available.

While the factor of density was shown to be insignificant, the thermal factor did have a significant effect on the growth and production of fish. The constant temperature, 24C, was the most effective treatment for the first and last periods of 20 days. The cyclic temperature, fluctuation between 22.7C and 26.8C, appeared to be the best treatment from day 21 through day 60. These results suggests that the two effective thermal treatments should be interchanged with each other over the four consecutive intervals to achieve the management goals.

The statistical information from this experiment enables scientists to conduct a series of additional studies efficiently. That is, with referred mean square error (MS_E), scientists can estimate the replication required per treatment (or treatment combination) in related studies to obtain expected statistical power ($1 - \beta$) of detecting treatment (or treatment combination) effects of specified magnitude at a specified level of significance ($1 - \alpha$).

Moreover, the results can be applied to at least two other aspects of aquaculture development: (1) utilizing the heated effluent waters of power plants as a substitute water source, and (2) introducing the modeling technique into the field of aquaculture. The aquaculture industry in the United States today is facing an increasing problem of water supply. More and more farming systems are getting less and less water out of deep wells because of lowered water tables. Such declines expose the serious fact of overpumping resulting from the high water demand by industrial plants and a vast increase in farming activities. In fact, overpumping is a common and inevitable means used to gain more profits in the management of aquaculture businesses for those farmers in developing countries. Consequently, water tables fall rapidly and usually contribute to environmental deterioration. The west coast of southern Taiwan, for example, has attracted thousands of aquaculture businesses and is now suffering from degeneration of water quality. The local scientists warn that overpumping is a major factor and must be prohibited immediately, otherwise the water table will continue to drop. As a result of the lowered water tables, the local stratum will be encroached upon by the sea and the ground water will be salinized. Another problem is heated effluent waters released from the cooling systems of power plants. These effluents cause concern among limnologists. The heated effluents may disturb the normal thermal structures of the aquatic environments and result in degradation of the system. Therefore, limnologists suggest that water used in the power-generating process should be returned to receiving waters after being cooled in pools, reservoirs, or cooling towers. But, as viewed by aquaculturists, such heated effluents are valuable resources providing tremendous amount of warm water for fish culture. By mixing waters properly, several desired temperatures (constant or cyclic) could be generated to produce fish to meet market demands. The application of "waste-heat aquaculture" may provide for efficient production and reduce ecological problems.

The three types of modeling introduced into this study contribute to a future trend in developing aquaculture using systems analysis. Statistical modeling is a commonly used routine for experimental design and analysis. This type of modeling is applied to investigate systems which can be only described by predictive models. Scientists from fields, such as biology, economics, and sociology, must accept the use of predictive models that are only approximately correct because of unknown factors. The unknown effects on the trait of interest are collectively referred to as random error, sometimes called experimental error. Studies of physical and chemical systems, on the other hand, can be described with related deterministic models. The input-output relationship of a given electronic system, for example, may be exactly expressed by a deterministic model without considering random error because of the certainty of the physical world. Computer modeling, in fact, is an extension from previous modelings; that is, scientists study a system by translating the original form of a mathematical model into a computer program. Future life scientists may be characterized more by their ability to use the computer for data analysis and simulation than by their ability to use the traditional microscope. This results from the fact that living systems involve a complex interaction of chemical and physical processes all of which are capable of being described in mathematical terms. These systems being more complex than any devised by the mind of man have for the most part resisted mathematical analysis by the classical methods so successfully employed by physicists and chemists. With the advent of computers, scientists have been able to use numerical methods to deal with these complex, multi-component systems.

The problems of managing aquaculture systems grow in magnitude and complexity as the mass production of fish involves environmental conditions which have physical, nutritional, physiological, and pathological manifestations. Each species of fish has specific environmental requirements in which it can best grow and reproduce. Because of this, anyone interested in raising fish for profit should make every effort to obtain all available information concerning the environmental requirements of the fish of interest and should attempt to maintain such an environment for the fish. In addition, a recognition of the intricacy of aquaculture development has grown with the expanding awareness that nothing in this world exists by itself. Everything is interrelated, and no longer is there any excuse for considering the farming of aquatic systems, the overproduction by agriculture accompanied by changing markets, the fluctuating of financial interest rates, the tumbling of agricultural land values, the increasing or decreasing of oil-prices, or a national deficit as a single, simple activity or phenomenon. By facing the complexity within and among systems, successful stewardship of aquaculture must rely deeply on systems analysis. Systems analysis through modeling, therefore, provides us with quantitative techniques and a methodological approach in dealing with planning, development, and management problems in real world systems. Aquaculture is now in a transitional stage evolving from reliance on crudely subsistent production techniques to more highly developed technologies. Unlike the past, modern aquaculture scientists are now making an endeavor at introducing systems and modeling concepts into the aquaculture area. Eventually they will use the systems at their disposal to manage aquaculture operations as effectively as a driver controls the course of an automobile to reach one's destination. The development of aquaculture will proceed more rapidly and efficiently if we apply the analytical techniques that have been developed for other fields to aquaculture. Appreciation of the whole may seem to make aquaculture management more difficult, but it also makes real and satisfactory solutions more certain.

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APPENDICES

Appendix 1. Michigan bait minnow supply survey.

1) What is your source of bait minnows? (please check one)

____ I net or raise minnows for sale (please go to question 2)
____ I purchase minnows for resale (please go to question 6)
____ A combination of both (please go to question 2)

2) My answers for the numbers of minnows netted or raised in the following section will be in (check one) dozens ____, pounds ____, or other ____ (please specify other as ____________________________).

3) Please list below how many bait minnows you netted last year:

Type	Small spr/sum/fall/wint	Medium spr/sum/fall/wint	Large spr/sum/fall/wint
Golden shiner			
Fathead minnow			
White sucker			
Other (list)			

4) When was it difficult to obtain minnows by netting during the year? ____ Never or fill in below:

Type	Small months in short supply	Medium mo. short supply	Large mo. short supply
Golden shiner			
Fathead minnow			
White sucker			
Other (list)			

5) How many bait minnows did you raise last year? ____ None or fill in below:

Type	Small spr/sum/fall/wint	Medium spr/sum/fall/wint	Large spr/sum/fall/wint
Golden shiner			
Fathead minnow			

Appendix 1 (cont'd.).

White sucker ____________________
Other (list)

____________ ____________________
____________ ____________________
____________ ____________________
____________ ____________________

*** IF YOU DID NOT PURCHASE ANY MINNOWS FOR RESALE, PLEASE GO TO QUESTION 17***

6) My answers for the numbers of bait minnows purchased for reslae in the following section will be in (check one) dozens ____, pounds ____, or other ____ (please specify other as ______________).

7) Please list the numbers of bait minnows you purchased for resale last year:

Type	Small spr/sum/fall/wint	Medium spr/sum/fall/wint	Large spr/sum/fall/wint
Golden shiner			
Fathead minnow			
White sucker			
Other (list)			

8) Have minnows been harder ____ or easier ____ to obtain (please check one and explain in the space below).

9) During what months are minnows difficult to obtain ? ____Never or fill in below:

Type	Small months in short supply	Medium mo. short supply	Large mo. short supply
Golden shiner			
Fathead minnow			
White sucker			
Other (list)			

Appendix 1 (cont'd.).

10) **Please list percentages** of your minnows which come **from in-state and out-of-state producers below:**

Type	In-state Producers netted (%)	minnow farms (%)	Out-of-state Producers (%)
Golden shiner			
Fathead minnow			
White sucker			
Other (list)			

11) **Have your sources** of bait minnows changed **significantly in the last few (3-5) years:** No ____ Yes ____ (please explain **yes answers below).**

12) **Please list your** in-state sources of farmed **bait minnows. Please provide addresses if available.** (use separate **sheet if necessary)**

13) **Do you know of any** other in-state sources of **farmed bait minnows?: Please provide names and addresses** if available. (use **separate sheet if necessary)**

14) **Are there any** specific reasons why you do not **buy minnows from the bait farms listed in** question 13? Please explain.

15) **Please** list **your average** purchase prices per dozen ____, **pound ____, or other** ____ (please **specify** other as ____________________) **bait minnows:**

Type	Small months in short supply	Medium mo. short supply	Large mo. short supply
Golden shiner			
Fathead minnow			
White sucker			
Other (list)			

16) **Do you expect** prices for bait minnows to increase ____, **remain about the same** ____, or decrease ____ in the coming year?

Appendix 1 (cont'd.).

17) Please list your average selling prices per dozen ____, pound ____, or other ____ (please specify other as ________________________) bait minnows:

Type	Small months in short supply	Medium mo. short supply	Large mo. short supply
Golden shiner			
Fathead minnow			
White sucker			
Other (list)			

18) Have your sales of bait minnows improved ____, remained about the same ____, or declined ____ over the last few (3-5) years. Please explain.

19) Do you expect bait sales will increase ____, remain about the same ____, or decline ____ over the next few (3-5) years. Please explain.

20) Would you be willing to pay premium prices for a guaranteed constant supply of farmed raised bait minnows ? ____Yes ____No (explain no).

Appendix 2. The formulas of respective sums of squares regarding the weight/weight ratio.

$$SS_Y = \sum_{t=1}^{3} \sum_{d=1}^{3} \sum_{a=1}^{2} \sum_{p=1}^{4} Y_{tdap}^{2} - Y_{....}^{2}/N^{a}$$

$$SS_T = \sum_{t=1}^{3} Y_{t...}^{2}/(dap) - Y_{....}^{2}/N$$

$$SS_D = \sum_{d=1}^{3} Y_{.d..}^{2}/(tap) - Y_{....}^{2}/N$$

$$SS_{TD} = \sum_{t=1}^{3} \sum_{d=1}^{3} Y_{td..}^{2}/(ap) - Y_{....}^{2}/N - SS_T - SS_D$$

$$SS_{A/TD} = \sum_{t=1}^{3} \sum_{d=1}^{3} \sum_{a=1}^{2} Y_{tda.}^{2}/p - \sum_{t=1}^{3} \sum_{d=1}^{3} Y_{td..}^{2}/(ap)$$

$$SS_P = \sum_{p=1}^{4} Y_{...p}^{2}/(tda) - Y_{....}^{2}/N$$

$$SS_{TP} = \sum_{t=1}^{3} \sum_{p=1}^{4} Y_{t..p}^{2}/(da) - Y_{....}^{2}/N - SS_T - SS_P$$

$$SS_{DP} = \sum_{d=1}^{3} \sum_{p=1}^{4} Y_{.d.p}^{2}/(ta) - Y_{....}^{2}/N - SS_D - SS_P$$

$$SS_{TDP} = \sum_{t=1}^{3} \sum_{d=1}^{3} \sum_{p=1}^{4} Y_{td.p}^{2}/a - Y_{....}^{2}/N - [SS_T + SS_D + SS_P + SS_{TD} + SS_{TP} + SS_{DP}]$$

$$SS_E = SS_Y - [SS_T + SS_D + SS_{TD} + SS_{A/TD} + SS_P + SS_{TP} + SS_{DP} + SS_{TDP}]$$

---

[a] N = total sample size = t x d x a x p = 3 x 3 x 2 x 4 = 72, where t = 3 thermal levels, d = 3 density levels, a = 2 aquarium replicates, and p = 4 time intervals (peiods).

Appendix 3. Obtaining the simple regression line for the four individual tanks during four individual 20-days intervals.

$$\text{Slope } b = \frac{N(\Sigma XY) - (\Sigma X)(\Sigma Y)}{N\Sigma X^2 - (\Sigma X)^2}$$

where X = heating hours;
Y = averaged temperature of tank; and
N = number of (X,Y).

$$\text{Intercept } a = \bar{Y} - b\bar{X}$$

$$\text{Total SS} = \Sigma Y^2 - (\Sigma Y)^2/N$$

$$\text{Regression SS} = \frac{(\Sigma XY - \Sigma X \Sigma Y/N)^2}{\Sigma X^2 - (\Sigma X)^2/N}$$

Residual SS = Total SS - Regression SS

Residual DF = Total DF - Regression DF = N - 2

where DF = degrees of freedom.

r = Correlation Coefficient = $\sqrt{r^2}$

$$r^2 = \frac{\text{Regression SS}}{\text{Total SS}}$$

Appendix 4. The formulas of testing hypotheses in Figure 2 and of computing common slope as well as intercept.

I. Comparing two slopes.

The test statistic is,

$$t = \frac{b_1 - b_2}{S_{b_1 - b_2}}$$

if $|t| \geq t_{\alpha(2),(N_1+N_2-4)}$, reject H_O: $b_1 = b_2$; otherwise accept H_O: $b_1 = b_2$; $\alpha = 0.05$.

$$S_{b_1 - b_2} = \sqrt{\frac{(S_{Y.X}^2)_P}{\Sigma X_1^2 - (\Sigma X_1)^2/N_1} + \frac{(S_{Y.X}^2)_P}{\Sigma X_2^2 - (\Sigma X_2)^2/N_2}}$$

$$(S_{Y.X}^2)_P = \frac{\text{Residual } SS_1 + \text{Residual } SS_2}{\text{Residual } DF_1 + \text{Residual } DF_2}$$

If H_O: $b_1 = b_2$, is accepted, then the common slop is

$$b_c = \frac{\Sigma X_1Y_1 - (\Sigma X_1)(\Sigma Y_1)/N_1 + \Sigma X_2Y_2 - (\Sigma X_2)(\Sigma Y_2)/N_2}{\Sigma X_1^2 - (\Sigma X_1)^2/N_1 + \Sigma X_2^2 - (\Sigma X_2)^2/N_2}$$

and then;

II. Comparing two elevations.

The test statistic is,

$$t = \frac{(\bar{Y}_1 - \bar{Y}_2) - b_c(\bar{X}_1 - \bar{X}_2)}{\sqrt{(S_{Y.X}^2)_C\ [1/N_1 + (1/N_2)\ (\bar{X}_1 - \bar{X}_2)^2/A_C]}}$$

if $|t| \geq t_{\alpha(2),(N_1+N_2-3)}$, reject H_O: the same elevation; otherwise accept H_O: the same elevation; $\alpha = 0.05$.

$$A_C = \Sigma X_1^2 - (\Sigma X_1)^2/N_1 + \Sigma X_2^2 - (\Sigma X_2)^2/N_2$$

$$B_C = \Sigma X_1Y_1 - (\Sigma X_1)(\Sigma Y_1)/N_1 + \Sigma X_2Y_2 - (\Sigma X_2)(\Sigma Y_2)/N_2$$

Appendix 4 (cont'd.).

$$C_C = \Sigma Y_1^{\,2} - (\Sigma Y_1)^2/N_1 + \Sigma Y_2^{\,2} - (\Sigma Y_2)^2/N_2$$

$$SS_C = C_C - B_C^{\,2}/A_C$$

$$(S_{Y.X}^{\;2})_C = \frac{SS_C}{N_1 + N_2 - 3}$$

If H_O: the same elevation, is accepted then the common intercept is

$$a_c = \bar{Y}_p - b_c\,\bar{X}_p,$$

$$\text{where } \bar{X}_p = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2},$$

$$\bar{Y}_p = \frac{N_1\bar{Y}_1 + N_2\bar{Y}_2}{N_1 + N_2}.$$

Thus, the common regression equation at a time interval of 20-day is $\hat{Y} = a_c + b_c\,X$.

This linear equation describes a thermal characteristic of the cyclic temperature treatment in a quantitative way;

where X = heating hours with a range of 0 to 12;

b_c = thermal increasing rate (C per hour);

a_c = the lowest point of the cyclic temperature, namely a temperature at $x = 0$; and

$\hat{Y}$ = a predicted temperature at a given X.

Appendix 5. Program of computer simulation.

```
*JOBCARD*,RG2,JC500,CM370000.
      FTN5.
      HAL,L*IMSL5.
      LGO.
*EOS
      PROGRAM SHA
      REAL K
      DIMENSION W(1000000, 0:80),R(1000000,0:1)
      DOUBLE PRECISION DSEED
      OPEN (2,FILE='OUTPUT')
      G=0.0
      K=0.0
      F1=0.0
      F2=0.0
      F3=0.0
      F4=0.0
      WMEANPD=0.0
      WVARPD=0.0
      DSEED=999999999.D0
      WBAR=1.9855556
      WSD=0.5661939
      NR=1000000
      WRITE (2,303)
 303  FORMAT ('0',10X,"MEAN AND VARIANCE OF 1000000 FISH
```

Appendix 5 (cont'd.).

```
     -FOR EACH DAY")
      WRITE (2,313)
 313  FORMAT(25X,"[FIRST      SELECTION]")
      WRITE (2,404)
 404  FORMAT('0',14X,"MEAN",25X,"VARIANCE")
      CALL GGNML[a] (DSEED,NR,R)
      DO 10 J=1,80
      S=0.0
      WTOTAL=0.0
      WSQRTOT=0.0
      DO 20 I=1,1000000
      IF (J .GE. 2) GO TO 100
      W(I,J-1)=R(I,J-1)*WSD+WBAR
 100  IF (J .LE. 20) THEN
                G=0.0113568[b]
                K=2.410[b]
      ENDIF
      IF ((J .GT. 20) .AND. (J .LE. 40)) THEN
                G=0.0045377[b]
                K=2.250[b]
      ENDIF
      IF ((J .GT. 40) .AND. (J .LE. 60)) THEN
                G=0.0040329[b]
                K=2.440[b]
```

Appendix 5 (cont'd.).

```
      ENDIF
      IF ((J .GT. 60) .AND. (J .LE. 80)) THEN
              G=0.005263^b
              K=2.557^b
      ENDIF
      F1=G*W(I,J-1)*((K-W(I,J-1))/K)
      F2=G*(W(I,J-1)+F1/2)*((K-(W(I,J-1)+F1/2))/K)
      F3=G*(W(I,J-1)+F2/2)*((K-(W(I,J-1)+F2/2))/K)
      F4=G*(W(I,J-1)+F3)*((K-(W(I,J-1)+F3))/K)
      W(I,J)=W(I,J-1)+(F1+2*F2+2*F3+F4)/6
      S=W(I,J)
      WTOTAL=WTOTAL+S
      WSQRTOT=WSQRTOT+S**2
  20  CONTINUE
  30  CONTINUE
      WMEANPD=WTOTAL/(I-1)
      WVARPD=(WSQRTOT-WTOTAL**2/(I-1))/(I-2)
      WRITE(2,505) WMEANPD,WVARPD
 505  FORMAT('0',11X,F10.7,21X,F9.7)
```

---

[a] Generate Standard Normal Deviates (GGNML) is a computer subroutine program from IMSL Library (1984).

[b] During the computer simulation, the numerical values of G and K may be changed periodically depending on the selection of simulations.

Appendix 5 (cont'd.).

```
  10    CONTINUE
         STOP
         END
```

Appendix 6. Original printouts of 4 selections of computer simulation.

	Selections of Computer Simulation							
Day	Selection 1		Selection 2		Selection 3		Selection 4	
	Mean Weight[a]	Variance	Mean Weight[a]	Variance	Mean Weight[a]	Variance	Mean Weight[a]	Variance
1	1.9730539	0.3198393	1.9716926	0.3208814	1.9730539	0.3198393	1.9716926	0.3208814
2	1.9756123	0.3151638	1.9728920	0.3172188	1.9756123	0.3151638	1.9728920	0.3172188
3	1.9781739	0.3105670	1.9740971	0.3136068	1.9781739	0.3105670	1.9740971	0.3136068
4	1.9807383	0.3060467	1.9753078	0.3100441	1.9807383	0.3060467	1.9753078	0.3100441
5	1.9833051	0.3016011	1.9765248	0.3065298	1.9833051	0.3016011	1.9765238	0.3065198
6	1.9858738	0.2972282	1.9777450	0.3030630	1.9858738	0.2972282	1.9777450	0.3030630
7	1.9884440	0.2929264	1.9789711	0.2996427	1.9884440	0.2929264	1.9789711	0.2996427
8	1.9910153	0.2886938	1.9802019	0.2962679	1.9910153	0.2886938	1.9802019	0.2962679
9	1.9935873	0.2845289	1.9814372	0.2929478	1.9935873	0.2845289	1.9814372	0.2929378
10	1.9961597	0.2804301	1.9826769	0.2896516	1.9961597	0.2804301	1.9826769	0.2896516

Appendix 6 (cont'd.).

11	1.9987320	0.2763959	1.9839208	0.2864084	1.9987320	0.2763959	1.9839208	0.2864084
12	2.0013039	0.2724247	1.9851687	0.2832073	2.0013039	0.2724247	1.9851687	0.2832073
13	2.0038751	0.2685152	1.9864203	0.2800476	2.0038751	0.2685152	1.9864203	0.2800476
14	2.0064452	0.2646659	1.9876756	0.2769286	2.0064452	0.2646659	1.9876756	0.2769286
15	2.0090139	0.2608757	4.9889344	0.2738494	2.0090139	0.2608757	1.9889344	0.2738494
16	2.0115809	0.2571431	1.9901935	0.2708094	2.0115809	0.2571431	1.9901965	0.2708094
17	2.0141457	0.2534670	1.9914617	0.2678078	2.0141457	0.2534670	1.9914617	0.2678078
18	2.0167082	0.2498462	1.9927299	0.2648440	2.0167082	0.2498462	1.9927299	0.2648440
19	2.0192680	0.2498462	1.9940008	0.2619173	2.0192680	0.2462796	1.9940008	0.2619173
20	2.0218249	0.2427660	1.9952745	0.2590270	2.0218249	0.2427660	1.9952745	0.2590270
21	2.0222669	0.2411231	1.9957784	0.2572938	2.0217734	0.2429064	1.9952177	0.2591751
22	2.0227107	0.2394929	1.9962840	0.2555740	2.0217218	0.2430470	1.9951610	0.2593234

Appendix 6 (cont'd.).

23	2.0231561	0.2378753	1.9967913	0.2568674	2.0216703	0.2431877	1.9951043	0.2594718
24	2.0236031	0.2362700	1.9973002	0.2521740	2.0216188	0.2433285	1.9950475	0.2596203
25	2.0240517	0.2346770	1.9978107	0.2504935	2.0215673	0.2434693	1.9949908	0.2597688
26	2.0245019	0.2330962	1.9983228	0.2488258	2.0215158	0.2436103	1.9949341	0.2599175
27	2.0249537	0.2315273	1.9988365	0.2471708	2.0214643	0.2437513	1.9948774	0.2600662
28	2.0254069	0.2297041	1.9935160	0.2455284	2.0214128	0.2438924	1.9948207	0.2602151
29	2.0258616	0.2284252	1.9998382	0.2438984	2.0213613	0.2440336	1.9947640	0.2603641
30	2.0263178	0.2268917	2.0003863	0.2422806	2.0213098	0.2441750	1.9947073	0.2605131
31	2.0267754	0.2253697	2.0009058	0.2406751	2.0212584	0.2443164	1.9946507	0.2606622
32	2.0272343	0.2238591	2.0014267	0.2390816	2.0212069	0.2444579	1.9945940	0.2608115
33	2.0276947	0.2223598	2.0019489	0.2375000	2.0211555	0.2445994	1.9945373	0.2609608
34	2.0281563	0.2208717	2.0024724	0.2359302	2.0211041	0.2447411	1.9944807	0.2611103

Appendix 6 (cont'd.).

35	2.0286193	0.2193948	2.0029973	0.2343721	2.0210526	0.2448829	1.9944241	0.2612598
36	2.0290835	0.2179287	2.0035234	0.2328255	2.0210012	0.2450248	1.9943675	0.2614094
37	2.0295490	0.2164736	2.0040508	0.2312904	2.0209498	0.2451667	1.9943108	0.2615592
38	2.0300157	0.2150292	2.0045793	0.2297666	2.0208984	0.2453088	1.9942542	0.2617090
39	2.0304836	0.2135954	2.0051091	0.2282541	2.0208470	0.2454509	1.9974976	0.2618589
40	2.0309527	0.2121723	2.0056400	0.2267526	2.0207957	0.2455931	1.9941411	0.2620090
41	2.0319749	0.2111458	2.0067049	0.2256716	2.0217899	0.2443812	1.9951776	0.2607321
42	2.0329960	0.2101242	2.0077688	0.2245955	2.0227835	0.2431753	1.9962136	0.2594617
43	2.0340161	0.2091072	2.0088318	0.2235244	2.0237765	0.2419755	1.9972491	0.2581977
44	2.0350351	0.2080950	2.0098937	0.2224582	2.0247688	0.2408916	1.9982839	0.2569401
45	2.0360530	0.2070874	2.0109546	0.2213969	2.0257604	0.2395937	1.9993182	0.2556887
46	2.0370698	0.2060845	2.0120145	0.2203405	2.0267513	0.2384117	2.0003519	0.2544435

Appendix 6 (cont'd.).

47	2.0380855	0.2050862	2.0130734	0.2192889	2.0277416	0.2372356	2.0013849	0.2532046
48	2.0391002	0.2040925	2.0141312	0.2182422	2.0287311	0.2360653	2.0024174	0.2519718
49	2.0401137	0.2031034	2.0151880	0.2172003	2.0297199	0.2349008	2.0034492	0.2507452
50	2.0411261	0.2021188	2.0162437	0.2161631	2.0307080	0.2337421	2.0044803	0.2495246
51	2.0421374	0.2011387	2.0172983	0.2151306	2.0316954	0.2325891	2.0055108	0.2483100
52	2.0431476	0.2001631	2.0183519	0.2141029	2.0326820	0.2314417	2.0065406	0.2471015
53	2.0441566	0.1991920	2.0194043	0.2130798	2.0336678	0.2303000	2.0075697	0.2458989
54	2.0451645	0.1982254	2.0204557	0.2120614	2.0346529	0.2291640	2.0085981	0.2447022
55	2.0461712	0.1972632	2.0215060	0.2110476	2.0356372	0.2280334	2.0096258	0.2435114
56	2.0471767	0.1963053	2.0225552	0.2100384	2.0366207	0.2269085	2.0106528	0.2423264
57	2.0481811	0.1953519	2.0236032	0.2090338	2.0376034	0.2257890	2.0116791	0.2411472
58	2.0491843	0.1944028	2.0246502	0.2080337	2.0385853	0.2246750	2.0127046	0.2399737

Appendix 6 (cont'd.).

59	2.0504863	0.1934580	2.0256959	0.2070682	2.0395664	0.2235664	2.0137294	0.2388060
60	2.0511872	0.1925175	2.0267406	0.2060472	2.0405466	0.2224632	2.0147534	0.2376440
61	2.0529251	0.1914442	2.0285272	0.2049176	2.0421860	0.2214922	2.0164290	0.2366233
62	2.0546596	0.1903756	2.0303107	0.2037930	2.0438231	0.2205244	2.0181024	0.2356060
63	2.0563908	0.1893118	2.0320910	0.2026733	2.0454578	0.2195599	2.0197736	0.2345920
64	2.0581187	0.1882528	2.0338680	0.2015586	2.0470901	0.2185986	2.0214425	0.2335814
65	2.0598432	0.1871985	2.0356418	0.2004488	2.0487200	0.2176404	2.0231091	0.2325742
66	2.0615643	0.1861489	2.0374124	0.1993439	2.0503475	0.2166855	2.0247735	0.2315703
67	2.0632820	0.1851040	2.0391797	0.1982439	2.0519726	0.2157337	2.0264326	0.2305698
68	2.0649964	0.1840638	2.0409438	0.1971488	2.0535953	0.2147851	2.0280955	0.2295725
69	2.0667073	0.1830283	2.0427045	0.1960585	2.0552155	0.2138397	2.0297530	0.2285786
70	2.0684148	0.1819974	2.0444620	0.1949731	2.0568334	0.2128974	2.0314083	0.2275879

Appendix 6 (cont'd.).

71	2.0701188	0.1809711	2.0462161	0.1938924	2.0584488	0.2119583	2.0330612	0.2266006
72	2.0718195	0.1799495	2.0479669	0.1928165	2.0600617	0.2110223	2.0347118	0.2256165
73	2.0735166	0.1789325	2.0497144	0.1917454	2.0616722	0.2100894	2.0363601	0.2246357
74	2.0752103	0.1779200	2.0514586	0.1906790	2.0632802	0.2091597	2.0380060	0.2236581
75	2.0769006	0.1769121	2.0531994	0.1896174	2.0648857	0.2082330	2.0396496	0.2226838
76	2.0785873	0.1759088	2.0549368	0.1885605	2.0664888	0.2073095	2.0412908	0.2217127
77	2.0802706	0.1749100	2.0566708	0.1875083	2.0680894	0.2063891	2.0429297	0.2207448
78	2.0819504	0.1739157	2.0584015	0.1864607	2.0696875	0.2054717	2.0445662	0.2497801
79	2.0836266	0.1729259	2.0601288	0.1854178	2.0712831	0.2045575	2.0462003	0.2188187
80	2.0852994	0.1719406	2.0618526	0.1843795	2.0728761	0.2036462	2.0478320	0.2178604

[a] It is the mean body weight of the one million fish.