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THEORY OF DRYING

Thesis for the Degree of Ph. D.
MICHIGAN STATE UNIVERSITY
Jaw-kai Wang
1958

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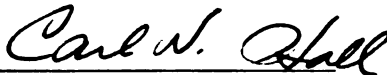
THE THEORY OF DRYING

presented by

Jaw-Kai Wang

has been accepted towards fulfillment
of the requirements for

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Major professor

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THEORY OF DRYING

By

Jaw-kai Wang

AN ABSTRACT

Submitted to Michigan State University of Agriculture and
Applied Science in partial fulfillment of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Engineering

Year 1958

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Carl W. Hall, Nov. 26, 1958

ABSTRACT

Based on the information gathered by previous workers, a theory, concerning the moisture movement inside a kernel of grain during drying process, has been proposed. Mathematical equations have been used to express the proposed drying theory and were solved under various drying conditions.

Calculated and experimental drying data for corn were compared. Over a time span of seventy hours and for six different drying conditions, the maximum deviation was found to be less than ten percent of the difference between the initial and final moisture of the corn.

It was concluded, contrary to previous beliefs, that the vapor diffusivity of corn could be regarded as a constant for the temperature range of 60 to 100 deg F and moisture content from 30 to 15 percent db.

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REVIEW OF LITERATURE

Drying in agricultural work refers to the removal of moisture until the moisture content of the product is in equilibrium with the surrounding air, usually twelve to fourteen percent, wet basis (wb). (Hall, 1957)

Since the introduction of mechanized harvesting, considerable emphasis has been placed on the drying of agricultural products which has led to the study of the movement of water through hygroscopic materials. However, much of the research work in drying of agricultural products reported from 1940 to 1945 has not delved into the theory of drying but has been concerned mainly with field results. (Hall, 1957) Among the few researchers Sherwood(1936), Newman(1931), Ceaglske(1937), Hougén(1940), Henry(1939), Babbitt(1940), Simmond(1953), Van Arsdell(1947), Fenton(1941), Hukill(1947), Hall and Rodríguez-Arias(1958), who have concerned themselves with the theoretical aspect of the study of water movement in hygroscopic materials, various hypothesis have been advanced to explain the mechanism by which the moisture moves. It is now generally agreed by researchers, that the drying of a hygroscopic material can be divided into two or more distinct phases. The earlier of these, characterized by phenomena similar to evaporation from a free liquid surface, is known as the constant-rate period. One or more later periods, characterized by a steadily decreasing drying rate, is known as the

falling-rate period. (Van Arsdell, 1947) (Hall, 1957) Drying of agricultural products usually falls in the falling-rate period. Babbitt(1940) has proved beyond doubt that the vapor pressure is the main driving force for the moisture movement during a drying process. In his experiment, the sample had the vapor pressure gradient in one direction and the moisture concentration gradient in the opposite direction. He found the moisture moved against the moisture concentration gradient and that the total moisture migration is proportional to the vapor pressure difference. In its differential form, the relationship expressed by him is,

$$\frac{\partial M'}{\partial t} = D \frac{\partial^2 P'}{\partial x^2} \quad \text{equation 1}$$

where M' = moisture content

p' = vapor pressure

D = diffusion constant

t = time

x = distance

Edward and Wray(1936) showed in their experiment, in which vapor pressure difference accross the sample was kept constant while the temperature was varied over a wide range, that the rate of moisture migration was virtually constant irrespective of temperature.

It is important to bear in mind that equation 1 is true only when the vapor pressure is the dominant factor controlling the internal movement of moisture. At higher or lower moisture content other than that indicated by Babbitt(1940), other factors, like, for instance, capillary force might

become the governing mechanism and equation 1 will no longer hold.

Vapor pressure of agricultural products, almost without exception, is a function of both the moisture content and temperature. (Dexter, 1955) (Hall, 1958) Consequently, in general, equation 1 is an equation with two unknowns and to solve it an additional equation would be needed.

Henry(1939) has suggested a general approach for solving certain type of simultaneous partial differential equations.

Up to now, most of the researchers have avoided the task of solving simultaneous partial differential equations by making certain assumptions, some of them have since been known to be wrong and others may be shown to be incorrect. Van Arsdell(1947) has cited the works by various researchers who took the difference in moisture concentration as the driving force and concluded that the diffusivity D_c in the following equation is highly dependent upon the moisture content.

$$\frac{\partial M}{\partial t} = D_c \frac{\partial^2 M}{\partial x^2} \quad \text{equation 2}$$

where D_c is the diffusivity when moisture concentration is used for the driving force.

Van Arsdell realized that the vapor pressure difference should be taken as the main governing mechanism for moisture diffusion through a hygroscopic material, but in his work he assumed that temperature throughout the medium was uniform during the drying process, or in other words, that the diffusion process is an isothermal one.

The temperature variation inside a spherical medium

where surface evaporation is taking place has been solved by Wang(Appendix). In general, except for extremely low drying rate, it can be shown that the temperature inside a kernel of grain is not uniform.

For materials that have smaller individual dimensions than a grain, King and Cassie (1940, Part I)have shown that during the uptake of moisture by wool fibres the temperature of wool fibres were anything but in equilibrium with its surroundings. They have stated in their paper "The result of this paper showed that slow rate(of the uptake of moisture by wool fibres) is entirely due to external factors and have no relation to the surface structure of the colloids".

Equation 3 has been used by many research workers to correlate drying data (Simmond, 1953) (Hall and Rodriguez-Arias, 1958).

$$\frac{M - M_e}{M_o - M_e} = e^{-k\theta} \quad \text{equation 3}$$

When applied to the heating or cooling of a substance, equation 3 has been known as the Newton's equation and it applies only when the temperature can be regarded as uniform throughout the substance and the heat exchange between the substance and its surroundings is proportional to their temperature difference. In using this equation to drying, the effect of internal resistance to moisture movement has been entirely neglected.

Simmond(1953) has indicated that in forced air shallow bed drying, the air velocity usually has only negligible effect on the drying rate. Since air velocity is of great importance

in determining the film coefficient between the grains and its surroundings, it is therefore apparent that the internal resistance must be the main factor in determining drying rate.

A modified form of equation 3 has been suggested by Hall (1958b) and it can be used to approximate the experimental drying curves to great accuracy.

$$\frac{M - M_e}{M_o - M_e} = \sum_1^N A_n e^{-k_n \theta} \quad \text{equation 4}$$

N 1, 2, 3,, N -1, N.

Since both equation 3 and 4 are not derived from basic considerations, it is therefore little surprise that the values of drying constant, k , have to be determined experimentally and no effort has been successful in predicting, under various drying conditions, the values of drying constant.

It is worthwhile to note, however, that the theoretically derived drying equation, under certain conditions, has a form very close to that of equation 4. (See, for instance, equation 24)

INTRODUCTION

A rigorous treatment of the diffusion of water vapor through hygroscopic materials would require the usage of statistical mechanics or kinetic theory. However, as in many engineering problems, the primary interest of this thesis is not in the motions of individual water molecules, but rather in the phenomenological result. Thus, to avoid the formidable task of cumbersome calculation which would result if the statistical mechanics or kinetic theory approach were used, the concept of continuum is introduced. Using this concept, vapor pressure and moisture concentration or moisture content can be defined at a point. This makes the description of the diffusion process in mathematical terms a much easier job. It would be well to bear in mind, however, that the use of a continuum concept to construct a working theory unifying a large body of observational knowledge and permitting the deduction of useful conclusions is only justified by its outcomes. But this is nothing new in modern physics. Indeed, even the most advanced atomic and nuclear theories are regarded as useful conceptual schemes rather than as reality.

The moisture content or moisture concentration in dry basis (db) is expressed by the ratio $\frac{\delta m}{\rho \delta v}$, where δm is the weight of water in δv , δv is the volume and ρ is the specific weight of the dry material. It is obvious when δv becomes so small as to contain relatively few molecules, the moisture

content fluctuates substantially with time as molecules pass into and out of the volume, so it is impossible to speak about a definite value of $\frac{\delta m}{\rho \delta v}$. If the smallest volume which can be regarded as continuous is $\delta v'$, then moisture content (db) at a point is defined as

$$M' = \lim_{\delta v \rightarrow \delta v'} \frac{\delta m}{\rho \delta v}$$

It is known that the vapor pressure exerted by most agricultural products is dependent upon its moisture content and temperature. (Hall, 1957) Considering any δv , all the water contained inside the volume δv may either be absorbed by the hygroscopic material or exist in vapor form in the pore space of the medium. If δv is small enough, the vapor pressure exerted by the medium should be always in equilibrium with the vapor pressure existing in the surrounding pore space. Furthermore, if δv is small enough, the moisture content and temperature can be regarded as uniform throughout δv at all time and therefore, vapor pressure at a point can be defined in terms of M' and T' in the following way,

$$P' = \lim_{\delta v \rightarrow \delta v'} P' \left(\frac{\delta m'}{\rho \delta v}, T' \right)$$

As cited in the Review of Literature, previous work shows that for the moisture range involved in the drying of agricultural products, that the moisture migration between two points in a hygroscopic material is proportional to the vapor pressure difference between the two points. (Babbitt, 1940) (Edward and Wray, 1936)

Thus, the drying process can be regarded as the diffusion

of water vapor through a hygroscopic material which may set free and release or absorb and immobilize some of the diffusing vapor, depend on whether the vapor pressure in its surrounding is greater or smaller than that exerted by itself. When absorption takes place, heat will be evolved; when evaporation takes place, heat will be needed as latent heat. Heat produced or taken up in this fashion will diffuse through the same medium and thus affect the extent to which the vapor will diffuse. The vapor is thought of reaching from one point of the medium to another by diffuse through the pore space. The pores are envisaged as a continuous network of space included in the solid, the dimension of the pore is small as compared with δV and its distribution is uniform, so that the whole medium can be regarded as a continuum.

THE GENERAL SOLUTION OF THE DIFFERENTIAL EQUATIONS GOVERNING
THE MOISTURE MOVEMENT THROUGH HYGROSCOPIC MATERIAL

NOMENCLATURE:

P'	Vapor pressure, psia
M'	Moisture content, lb of water/ lb of dry matter
$\bar{M}$	Average moisture content of the kernel
M_0	Initial moisture content
M_e	Equivalent moisture content of drying air
T'	Temperature, degree Fahrenheit (deg F)
r	Radius, feet (ft)
K	Heat conductivity, BTU/ deg F-ft ² -hour/ ft
D'	Vapor permeability, lb/ psia-ft ² -hour/ ft
k	Heat diffusivity, $K/\rho c$, ft ² / hour
D	Vapor diffusivity, D'/ρ , ft ² / hour-psia
t	Time, hour
q	Latent of moisture, BTU/ lb of moisture
ρ	Specific weight of medium, lb/ ft ³
c	Specific heat of medium, BTU/ deg F-lb
c_n	Constants, $n = 1, 2, 3, \dots$
h	Film coefficient
a_n	Eigen values
λ_k	Eigen values

ASSUMPTIONS UNDERLYING THE MATHEMATICAL ANALYSIS

In order to derive and solve the partial differential equations governing the diffusion of moisture through hygroscopic materials, the following assumptions have been made. The validity and applicability of those assumptions in drying process, within its range of temperature and moisture variation, will be presented in the Discussion;

- (1) The medium is isotropic.
- (2) The rate of moisture migration between two points in the medium is proportional to the vapor pressure difference. Other factors, like capillary force, is assumed to be negligible.
- (3) The relationship between vapor pressure, temperature and moisture content is linear, or,

$$P' = c_1 + c_2 M' + c_3 T'$$
- (4) The quantities K , D , q , ρ , c are constants.
- (5) The Structure and volume of the medium remain constant during the drying.

If the above are true, then, for a spherical specimen,

THE VAPOR DIFFUSION EQUATION

$$\rho \frac{\partial M'}{\partial t} = \frac{D'}{r^2} \frac{\partial^2}{\partial r^2} (r^2 \frac{\partial P'}{\partial r}) \quad \text{equation 5}$$

THE HEAT DIFFUSION EQUATION

$$c \rho \frac{\partial T'}{\partial t} = \frac{K}{r^2} \frac{\partial^2}{\partial r^2} (r^2 \frac{\partial T'}{\partial r}) - q \rho \frac{\partial M'}{\partial t} \quad \text{equation 6}$$

Let $T = rT'$, $P = rP'$, $M = rM'$ and using the relationship

$P = c_1 r + c_2 M + c_3 T$ to eliminate M in equations 5 and 6.

$$k \frac{\partial^2 T}{\partial r^2} = \frac{q}{c c_2} \frac{\partial P}{\partial t} + \left(1 - \frac{c_3 q}{c c_2}\right) \frac{\partial T}{\partial t} \quad \text{equation 7}$$

$$D \frac{\partial^2 P}{\partial r^2} = \frac{1}{c_2} \frac{\partial P}{\partial t} + \frac{c_3}{c_2} \frac{\partial T}{\partial t} \quad \text{equation 8}$$

Multiply equation 7 by s_x/k and divide through equation 8 by D, then add,

$$\frac{\partial^2}{\partial r^2} (P + S_x T) = \frac{\partial}{\partial t} \left\{ \left(\frac{1}{D c_2} + \frac{s_x q}{k c c_2} \right) P + \left(\frac{s_x}{k} - \frac{c_3 q s_x}{c c_2 k} - \frac{c_3}{D c_2} \right) T \right\} \quad \text{equation 9}$$

choose s_x such that

$$\frac{1}{D c_2} + \frac{s_x q}{k c c_2} = \frac{1}{k} - \frac{c_3 q}{k c c_2} - \frac{c_3}{D c_2 s_x} = \mu_x \quad \text{equation 10}$$

eliminate s_x in equation 10 and solve for

$$-\frac{c_3 q}{D k c c_2^2} = (\mu_x - \frac{1}{D c_2}) \left(\mu_x + \frac{c_3 q}{k c c_2} - \frac{1}{k} \right) \quad \text{equation 10a}$$

or,

$$\mu_x = \frac{1}{2 D k} \left[D \left(1 - \frac{c_3 q}{c c_2} \right) + \frac{k}{c_2} \pm \sqrt{\left(D \left(1 - \frac{c_3 q}{c c_2} \right) + \frac{k}{c_2} \right)^2 - \frac{4 D k}{c_2}} \right] \quad \text{equation 10b}$$

equation 10b gives two roots of μ_x , say, μ_1 and μ_2 .

Using μ_1 , μ_2 and equation 10, corresponding values of s_1 and s_2 can be found.

Equation 9 can now be simplified to

$$\frac{\partial^2}{\partial r^2} (P + S_x T) = \mu_x \frac{\partial}{\partial t} (P + S_x T) \quad \text{equation 11}$$

Assume P and T have the following solutions,

$$P_x = \sum_{n=0}^{\infty} A_n \sin \alpha_n r e^{-\alpha_n^2 t / \mu_x} \quad \text{equation 12}$$

$$T_x = \sum_{k=0}^{\infty} B_k \sin \lambda_k r e^{-\lambda_k^2 t / \mu_x} \quad \text{equation 13}$$

when $x=1$ $P=P_1$, $T=T_1$ and $x=2$, $P=P_2$, $T=T_2$.

Since equation 11 is only a rearrangement of equations 7 and 8, it is clear that any solution of equation 11 must also be a solution to equations 7 and 8. The solution of equation 11 are $P+s_1T$ and $P+s_2T$. But when $x=1$, $P=P_1$ and $T=T_1$; when $x=2$, $P=P_2$ and $T=T_2$.

$$\text{So, } P+s_1T = P_1+s_1T_1 \quad \text{equation 14}$$

$$P+s_2T = P_2+s_2T_2 \quad \text{equation 15}$$

or,

$$P = \frac{S_2 P_1 + S_1 P_2 + S_2 S_1 (T_1 - T_2)}{S_2 - S_1} \quad \text{equation 16}$$

$$T = \frac{P_1 - P_2 + S_1 T_1 - S_2 T_2}{S_1 - S_2} \quad \text{equation 17}$$

Equations 16 and 17 are general solution to the simultaneous partial differential equations, provided that equations 12 and 13 are true.

BOUNDARY CONDITIONS FOR SHALLOW BED DRYING

To show that equations 12 and 13 are formal solutions of P_x , T_x , it is necessary to show that the eigen values α_n , λ_k and coefficients A_n and B_k can be found so that equations 16 and 17 satisfy certain initial and boundary conditions.

Suppose a spherical shaped kernel of radius b is in equilibrium with its surroundings and that at time $t=0$ the vapor pressure and the temperature of its surroundings are suddenly altered from P_0 to P_a and T_0 to T_a .

Expressing P and T as functions of r and t , or, $P=P(r,t)$ and $T=T(r, t)$, the initial and boundary conditions are,

$$P(0, t) = \lim_{r \rightarrow 0} r P'(r, t) = 0 \quad \text{equation 18}$$

$$T(0, t) = \lim_{r \rightarrow 0} r T'(r, t) = 0 \quad \text{equation 19}$$

$$P(b, t) = b P'(b, t) = 0 \quad t \geq 0 \quad \text{equation 20}$$

$$\left(h + \frac{\partial}{\partial r}\right) T'(b, t) = 0 \quad t \geq 0 \quad \text{equation 21}$$

$$P(r, 0) = r P_0 \quad \text{equation 22}$$

$$T(r, 0) = r T_0 \quad \text{equation 23}$$

$$\text{where } P_0 = P_0' - P_a, \quad T_0 = T_0' - T_a.$$

SOLUTION

Applying the above conditions to equations 12 and 13 the eigen values and coefficients in those equations were found to be

$$A_n = (-1)^{n+1} \frac{2b}{n\pi} P_0$$

$$\alpha_n = \frac{n\pi r}{b} \quad n = 1, 2, 3, \dots$$

$$B_k = \frac{4 T_0 (\sin \lambda_k b - b \lambda_k \cos \lambda_k b)}{\lambda_k (2 b \lambda_k - \sin 2 b \lambda_k)} \quad k = 1, 2, 3, \dots$$

λ_k are the roots of

$$\tan \lambda_k b = \frac{b \lambda_k}{1 - hb}$$

CALCULATION

The average moisture content of a kernel at any time is,

$$\bar{M} = \frac{1}{c_2 V} \iiint_V \left(\frac{P}{r} - C_1 - C_2 \frac{T}{r} + P_0 \right) dV$$

If the external vapor pressure only were changed, as Rodriguez-Arias has conducted his experiment, then $B_k = 0$ and

$$P = \frac{S_2 P_1 - S_1 P_2}{S_2 - S_1}, \quad T = \frac{P_2 - P_1}{S_2 - S_1}$$

Hence,

$$\begin{aligned} \bar{M} &= \frac{1}{c_2 V} \iiint_V \left(\frac{P - c_3 T}{r} \right) dV - \frac{C_1}{C_2} \\ &= \frac{1}{c_2 V (S_2 - S_1)} \iiint_V \frac{1}{r} (S_2 P_1 + c_3 P_1 - S_1 P_2 - c_3 P_2) dV - \frac{C_1}{C_2} + \frac{P_0}{C_2} \\ &= \frac{1}{c_2 V (S_2 - S_1)} \iiint_V \frac{1}{r} [P_1 (S_2 + c_3) - P_2 (S_1 + c_3)] dV - \frac{C_1}{C_2} + \frac{P_0}{C_2} \\ &= \frac{S_2 + c_3}{c_2 V (S_2 - S_1)} \iiint_V \sum_1^{\infty} A_n \sin \frac{n \pi r}{b} e^{-n^2 \pi^2 t / b^2 \mu_1} dV \\ &\quad - \frac{S_1 + c_3}{c_2 V (S_2 - S_1)} \iiint_V \sum_1^{\infty} A_n \sin \frac{n \pi r}{b} e^{-n^2 \pi^2 t / b^2 \mu_2} dV \\ &\quad - \frac{C_1}{C_2} + \frac{P_0}{C_2} \end{aligned}$$

Assume that the order of integration and summation are interchangeable,

$$\begin{aligned} &= \frac{4 \pi (S_2 + c_3)}{c_2 V (S_2 - S_1)} \sum_1^{\infty} A_n e^{-n^2 \pi^2 t / b^2 \mu_1} \int_0^b r \sin \frac{n \pi r}{b} dr \\ &\quad - \frac{4 \pi (S_1 + c_3)}{c_2 V (S_2 - S_1)} \sum_1^{\infty} A_n e^{-n^2 \pi^2 t / b^2 \mu_2} \int_0^b r \sin \frac{n \pi r}{b} dr - \frac{C_1}{C_2} + \frac{P_0}{C_2} \end{aligned}$$

$$= \frac{6 P_0}{C_2(s_2-s_1)} \left\{ \sum_1^{\infty} \frac{1}{n^2 \pi^2} \left[(S_2 + C_3) e^{-n^2 \pi^2 t / b^2 \mu_1} - (S_1 + C_3) e^{-n^2 \pi^2 t / b^2 \mu_2} \right] \right\} - \frac{C_1}{C_2} + \frac{P_0}{C_2}$$

equation 24

To check the applicability of the above development, experimental data by Rodriguez-Arias(1956) were used.

A pseudo radius of shelled corn of $r = 0.014$ ft, which is the average of the width, length and thickness of yellow dent hybrid corns picked at random, was used in the calculation. The specific weight and heat of yellow dent hybrid corn were taken to be $\rho = 72.4$ lb/ft³, $c = 0.5$ BTU/lb-deg F. From page 87 figure 9 of Rodriguez-Arias' thesis the following relationships between P' , T' and M' of shelled corn were calculated. The value of c_3 was taken from the figure as $c_3 = \frac{\partial P'}{\partial T'}$, the slope of an isostere whose moisture content $M' = M_0 + \frac{1}{2}(M_0 - M_g)$. Using the data of figure 9 (p. 87, Rodriguez-Arias, 1956) curves of vapor pressure vs. moisture content were constructed. Values of c_2 were taken from those P-M curves. $C_2 = \frac{\partial P'}{\partial M'}$, the slope of an isotherm at the air temperature.

For drying at 60 deg F, $c_1 = -0.43$, $c_2 = 0.011$, $c_3 = 0.007$

86 deg F, $c_1 = -1.12$, $c_2 = 0.021$, $c_3 = 0.0145$

100 deg F, $c_1 = -2.25$, $c_2 = 0.022$, $c_3 = 0.0265$

where c_1 , c_2 and c_3 are constants in the following equation. $P' = c_1 + c_2 M' + c_3 T'$

equation 25

Equation 25 is only approximations to the actual experimental data it should be used within the range of moisture content from 25-30 to 13-15 percent db and ± 6 deg F.

Assuming a set of value for D and K. The values of μ_1 and μ_2 can then be calculated from equation 10b.

Adjust the values of K and D so that $\bar{M}$, calculated from equation 24, agrees with the experimental data at more than two different time intervals.

The values of vapor diffusivity, heat conductivity and heat diffusivity of yellow dent hybrid corn were calculated to be $D = 3.37 \times 10^{-5}$ ft²/ hour-psia, $K = 0.117$ BTU/ hour-deg F-ft and $k = 326 \times 10^{-5}$ ft²/ hour.

Using the above values, drying rates of shelled corn were predicted. The theoretically calculated curves were, in figures 1, 2 and 3, compared with experimental data obtained by Rodriguez-Arias.

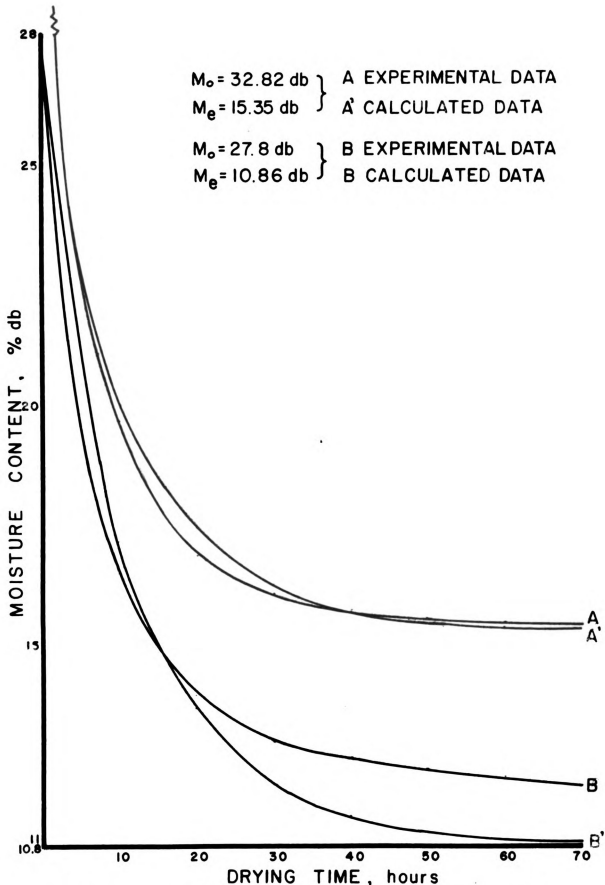


FIGURE 1 COMPARISON OF EXPERIMENTAL AND
 CALCULATED DATA ON DRYING SHELLD
 CORN AT 100 DEGREE F.

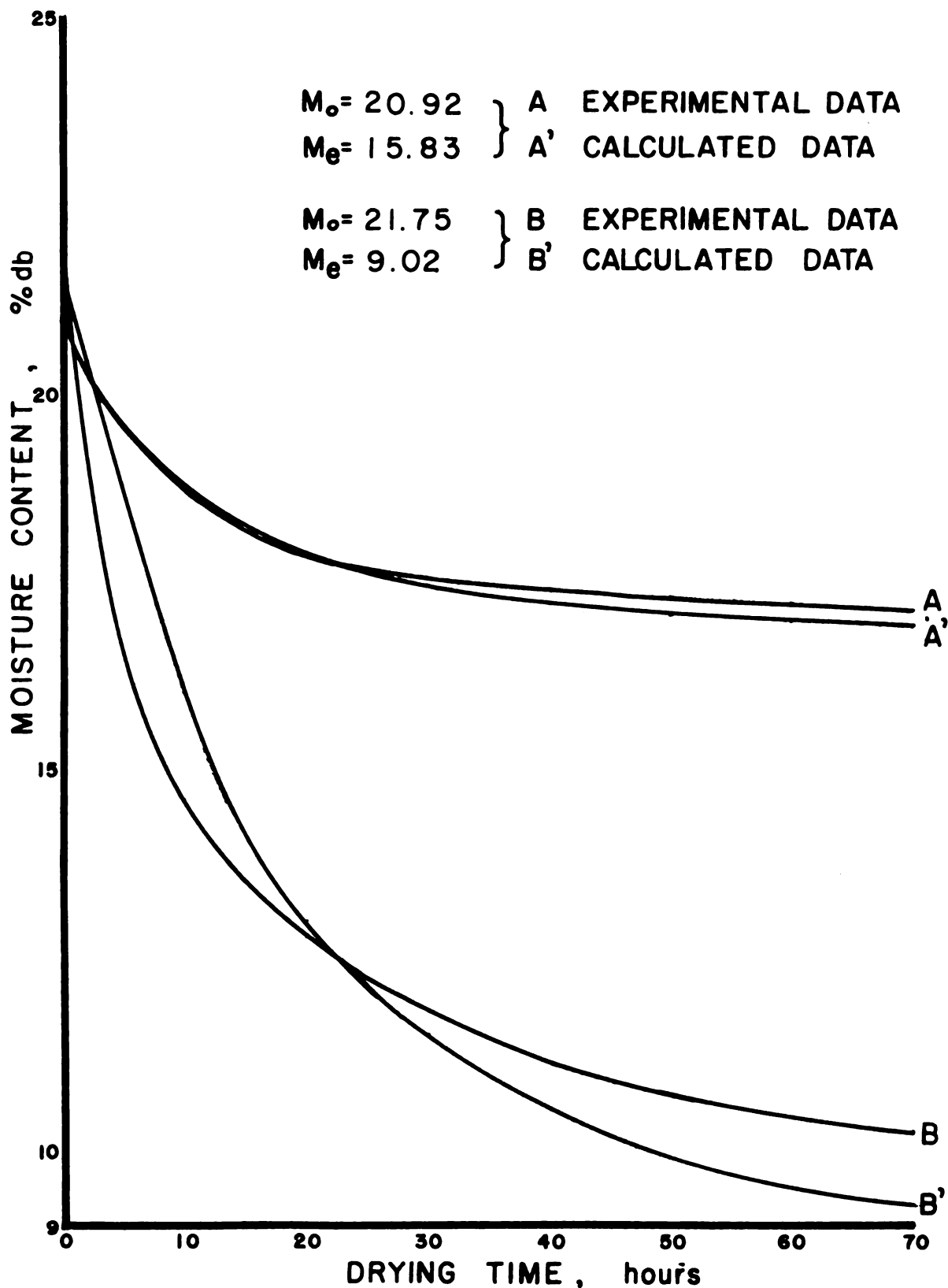


FIGURE 2 COMPARISON OF EXPERIMENTAL AND
 CALCULATED DATA ON DRYING SHELLLED
 CORN AT 86 DEGREE F.

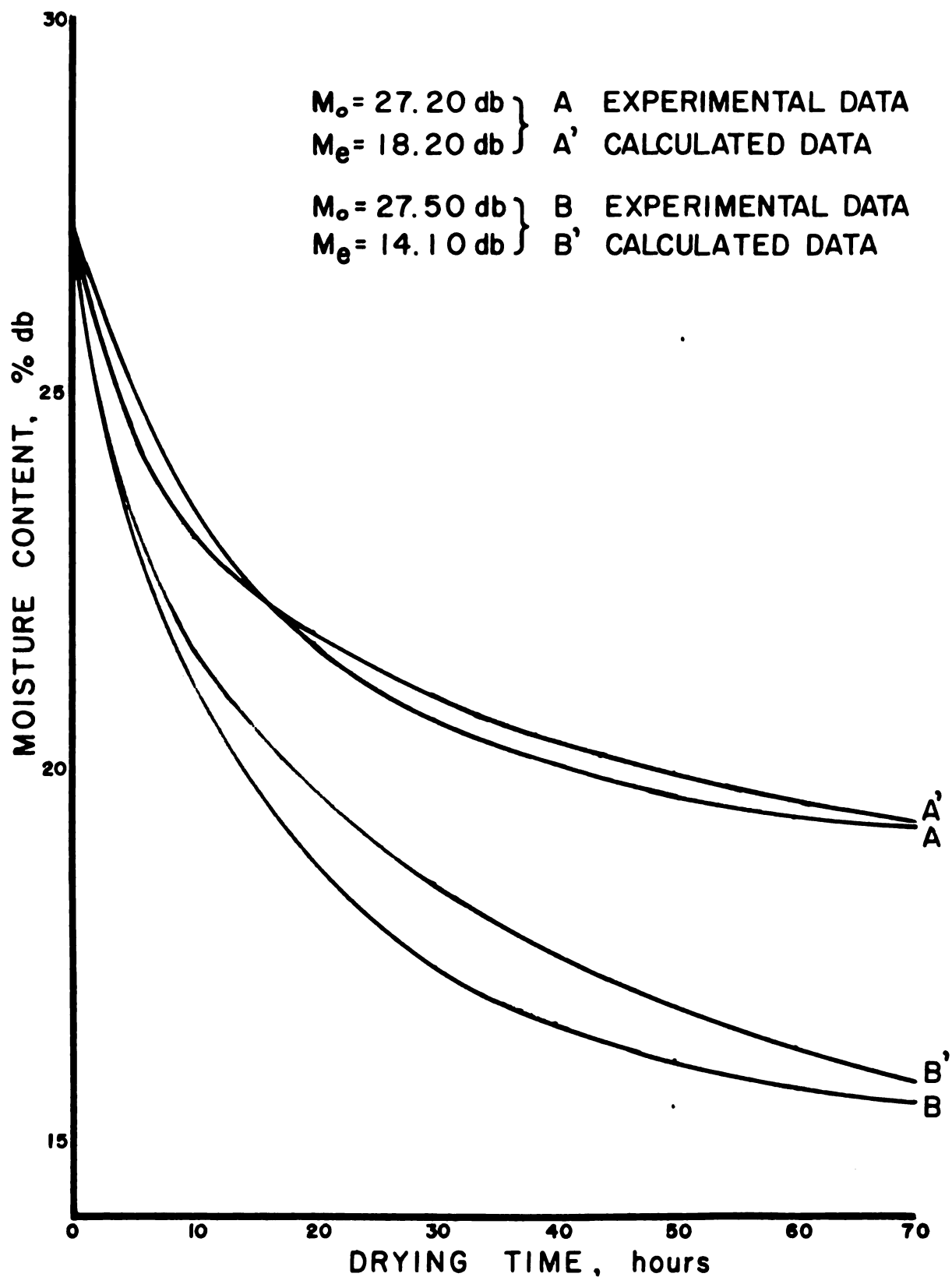


FIGURE 3 COMPARISON OF EXPERIMENTAL AND
 CALCULATED DATA ON DRYING SHELLLED
 CORN AT 60 DEGREE F.

DISCUSSION

In the above treatment, certain assumptions have been made to keep the resultant equations simple enough so that they can be solved. Nearly all the assumptions are good approximations when applied to a limited range of temperature and moisture content variation which are encountered in usual drying process. Outside their limitations, the equations may give correctly the general nature of the phenomena but close numerical agreement should not be expected.

Probably the worst assumption made was that the vapor pressure of a hygroscopic material depends linearly upon the local moisture content and temperature. It was not noted without gratitude that this assumption gave good numerical approximations to experimental data even when used to a greater moisture content variation than it was intended. However, it can not escape the attention, that the agreement was much better when applied to smaller range of moisture variation. In general, for grains, excluding the extreme high or low moisture content, the linearity can be taken as a good approximation between 25 percent to 15 percent db, and may be used between 30 percent to 13 percent moisture content db.

The boundary conditions concerning the temperature variation is worthy of consideration. It is clear, for more rigorous treatment of the problem, that the effect of sur-

face evaporation should be taken into consideration. But these more complicated boundary conditions are beyond the realm of Sturm-Liouville system and although the problem of temperature distribution inside a sphere with surface evaporation has been solved by Wang(Appendix), the calculation is formidable.

When cooling effect of surface evaporation is neglected, the calculated temperature inside a kernel may be several degrees higher. Consequently, the calculated vapor pressure will be higher. But, since the numerical value of c_3 is generally in the order of 10^{-2} and is small compared to the numerical value of c_1 , a few degree deviation in the calculated temperature distribution will not effect the numerical value of vapor pressure to any great extent. When boundary conditions in page 13 were used, the effect on the drying rate produced by the small deviation of the calculated vapor pressure distribution from the true vapor pressure may be compensated by a small alternation on the numerical value of vapor diffusivity D . Thus, the calculated value of D as shown was expected to be a little lower than the true value, however, the deviation should be small.

When the drying rate only is wanted, the most important boundary condition is that the vapor pressure at the surface is always in equilibrium with its surroundings. (King and Cassie, 1940) Furthermore, there are no data available on either the vapor diffusivity or the heat conductivity of small grains and method of successive approximation has to be emp-

loyed to calculate them. To simplify the calculation, the less influential boundary condition of the temperature distribution on the boundary was altered and equation 21 was used instead of the more complicated boundary. The use of equation 21 can be expected to give a reasonably good result of the total moisture migration or the drying the drying rate. The good agreement was shown in figure 1,2, and 3. However, it is not capable of giving correct values on internal distribution of either temperature or vapor pressure during the drying process. When internal temperature or vapor pressure data are desired, the correct boundary condition must be used.

Nevertheless, contrary to previous belief, the good agreement between the calculated and the experimental data has shown that for all engineering purposes, the vapor diffusivity D can be regarded as a constant in drying process for kernels of corn. Since the properties, like density or heat conductivity, of other grains do not differ greatly from corn, it seems there is no reason to assume that the contrary is true for other grains.

For other hygroscopic materials, the assumption that the vapor diffusivity is not dependent upon the local moisture content was supported by experiments cited in previous chapter in which the moisture migration was shown to be constant when same vapor pressure difference were maintained between two planes but moisture contents were varies.

In conclusion, the effect of the temperature change due

to the loss of moisture at points inside a kernel of grain during a drying process is important and has a profound influence on the moisture diffusion rate. When this effect is being taken into consideration, the theoretically predicted drying rate of shelled corn agreed reasonably well with experimental data.

SUMMARY

It was proposed that the drying of agricultural products can be regarded as the diffusion of water vapor through hygroscopic materials, which may set free and release, or absorb and immobilize the diffusing vapor, depending on whether the vapor pressure of its surrounding is greater or less than the vapor pressure exerted by the material itself. When absorption takes place, heat will be needed as latent heat. Heat produced or taken in this fashion will diffuse through the same medium and thus affect the extent to which the vapor will diffuse. The vapor is thought of reaching from one point of the medium to another by diffuse through the pore space.

Based on the above theory, two equations were formulated. These are the vapor diffusion equation, equation 5, and the heat diffusion equation, equation 6.

$$\rho \frac{\partial M'}{\partial t} = \frac{D'}{r^2} \frac{\partial^2}{\partial r^2} (r^2 \frac{\partial P'}{\partial r}) \quad ; \quad c\rho \frac{\partial T'}{\partial t} = \frac{K}{r^2} \frac{\partial^2}{\partial r^2} (r^2 \frac{\partial T'}{\partial r}) - q\rho \frac{\partial M'}{\partial t}$$

When applied to drying of agricultural products by forced air at a certain temperature, and when the difference of initial and final moisture content of the product is about fifteen percent db, it was found that the relationship between the vapor pressure, the moisture content and the temperature at any point inside the product can be approximated by the following equation,

$$P' = C_1 + C_2 M' + C_3 T'$$

using this relationship, the solution of the governing

differential equations were found to be,

$$P = \frac{S_2 P_1 + S_1 P_2 + S_2 S_1 (T_1 - T_2)}{S_1 - S_2} \quad ; \quad T = \frac{P_1 - P_2 + S_1 T_1 - S_2 T_2}{S_1 - S_2}$$

where P_1 , P_2 and T_1 , T_2 were given by the following expressions with $x = 1, 2$.

$$P_x = \sum_{n=0}^{\infty} A_n \sin \alpha_n r e^{-\alpha_n^2 t / \mu_x} \quad T_x = \sum_{k=0}^{\infty} B_k \sin \lambda_k r e^{-\lambda_k^2 t / \mu_x}$$

μ_x was defined by equation 10b in page 11.

A set of boundary and initial conditions were derived by considering, with simplifying assumptions, the true physical conditions under which Rodriguez-Arias (1956) has conducted his experiments.

The coefficients and eigen values in equations 12 and 13 were obtained by applying the boundary and initial conditions to equations 12 and 13.

The average moisture content $\bar{M}$ was then derived and was given by equation 24 in page 15.

Using equation 24 and by successive approximation the numerical values of the heat conductivity, K , and vapor diffusivity, D , were calculated.

$$D = 3.37 \times 10^{-5} \text{ ft}^2 / \text{hour-psia}$$

$$K = 0.117 \text{ BTU} / \text{hour-deg F-ft}^2 / \text{ft}$$

Drying rates of shelled corn at six different drying conditions were predicted, the theoretically calculated drying curves were compared with experimental data obtained by Rodriguez-Arias (1956) in figure 1, 2, 3.

RECOMMENDATIONS FOR FURTHER STUDY

1. Calculate the diffusivity and heat conductivity of other small grains and examine the applicability of the proposed drying theory to other small grains.
2. Determine by experiments the value of vapor diffusivity and heat conductivity of small grains. Check the validity of the proposed drying theory.
3. The relationship between the vapor pressure, the moisture content and temperature of small grains at high moisture content (above 25 percent db) is needed.
4. Investigate the possibility of applying the same mathematical technique to solve the deep layer drying problem.

APPENDIX

THE TEMPERATURE DISTRIBUTION IN A COMPOSITE SPHERE WITH THIN SKIN WHEN MOISTURE EVAPORATION AND HEAT EXCHANGE TAKE PLACE AT THE SURFACE

(1) ASSUMPTIONS:

- a. The composite sphere is assumed to be isotopic to heat transfer.
- b. Due to the fact that the thickness of skin $b - a$ is small compared with the radius a of the inner sphere, the temperature of the skin is assumed to be a function of time only, $v = v(t)$, $a \leq r \leq b$.

(2) GOVERNING DIFFERENTIAL EQUATION:

$$\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial w(r,t)}{\partial r} \right] = \frac{\partial w(r,t)}{\partial t} \quad \text{equation A-1}$$

(3) INITIAL AND BOUNDARY CONDITIONS:

$$w(r, 0) = V \quad \text{equation A-2}$$

$$w(a, t) = v(t) \quad \text{equation A-3}$$

$$c M \frac{\partial v(t)}{\partial t} = -K \frac{\partial w(a,t)}{\partial r} + h(V - v(t)) - \phi(t) \quad \text{equation A-4}$$

where c specific heat of medium

M Total mass of skin

K heat conductivity of the medium

V initial temperature of sphere, a constant

w temperature of inner sphere

$v(t)$ temperature of skin

$\phi(t)$ heat loss due to evaporation

α heat diffusivity, $K/\rho c$

(4) SOLUTION:

$$w(r, t) = \frac{1}{r} \sum_{n=0}^{\infty} \left[F_n e^{-\lambda_n^2 a t} - E_n \lambda_n^2 a \int_0^t \Phi(\tau) e^{-\lambda_n^2 a(t-\tau)} d\tau \right] \text{SIN} \lambda_n r$$

where,

$$F_n = \frac{v \left[1 - a \text{COS} \lambda_n a + \left(\frac{1}{a} + \frac{\lambda_n c M a a}{k - a h} \right) \text{SIN} \lambda_n a \right]}{\frac{\lambda_n a}{2} \left[1 + \frac{c M a a}{k - a h} \right] - \frac{\text{SIN} 2 \lambda_n a}{4} \left[1 - \frac{c M a a}{k - a h} \right]}$$

$$E_n = \frac{\text{COS} \lambda_n a - 1}{\frac{\lambda_n a}{2} \left[1 + \frac{c M a a}{k - a h} \right] - \frac{\text{SIN} 2 \lambda_n a}{4} \left[1 - \frac{c M a a}{k - a h} \right]}$$

$$\Phi(t) = \frac{h a v - a \phi(t)}{h}$$

 λ_n are the roots of the following equation

$$\text{TAN} \lambda_n a = \frac{k \lambda_n}{\frac{k}{a} - h + c M \lambda_n^2 a}$$

$$v(t) = w(a, t)$$

THE APPLICATION OF DRYING THEORY TO DEEP LAYER DRYING

(1) INTRODUCTION:

In shallow bed drying, the vapor pressure and temperature of the drying air can be taken as constants. This is not true in deep layer drying. To apply the theory of drying to deep layer drying, the drying condition, which is a function of the time and depth, would have to be given.

Based on the following assumptions, a set of partial differential equations, governing the vapor concentration and the temperature of the drying air, were formulated and solved.

(2) NOMENCLATURE:

s_b	bulk(grain and air) specific heat, constant
s_a	specific heat of air, constant
V	velocity of drying air, constant
q	latent heat of moisture, constant
ρ	bulk density, constant
T	air temperature
M	grain moisture content
C	vapor concentration of drying air
ρ_0	density of air, constant
c_1, c_2, c_3	constants

(3) ASSUMPTIONS:

1. The temperature distribution inside a small volume $\delta v'$ of the bulk is uniform.
2. $M = c_1 + c_2 C + c_3 T$

(4) GOVERNING PARTIAL DIFFERENTIAL EQUATIONS:

$$\frac{\partial C}{\partial t} = -V \frac{\partial C}{\partial x} + \rho \frac{\partial M}{\partial t} \quad \text{equation A-5}$$

$$S_b \frac{\partial T}{\partial t} = -\rho_a S_a V \frac{\partial T}{\partial x} - q \rho \frac{\partial M}{\partial t} \quad \text{equation A-6}$$

(5) GENERAL SOLUTIONS:

$$T = \frac{\alpha_1 \alpha_2 (C_1 - C_2) + \alpha_2 \beta_1 T_1 - \alpha_1 \beta_2 T_2}{\alpha_2 \beta_1 - \alpha_1 \beta_2}$$

$$C = \frac{\beta_1 \beta_2 (T_1 - T_2) + \alpha_1 \beta_2 C_1 - \alpha_2 \beta_1 C_2}{\alpha_1 \beta_2 - \alpha_2 \beta_1}$$

where,

$$C_x = A e^{\eta (x - t/\mu_x)}$$

$$T_x = B e^{v (x - t/\mu_x)} \quad x = 1, 2$$

$$\mu_x V = (\rho C_2 - 1) - \frac{\beta_x}{\alpha_x} \left(\frac{q \rho C_2}{\rho_a S_a} \right) = \frac{\alpha_x}{\beta_x} \rho C_3 - \frac{S_b + q \rho C_3}{\rho_a S_a}$$

$$x = 1, 2$$

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