This is to certify that the

thesis entitled

Structural Applications of Steel Fiber Reinforced Concrete: Analysis and Design

presented by

Abdeslam Reklaoui

has been accepted towards fulfillment
of the requirements for

M.S. degree in Civil Engineering

Parviz Soroushian
Major professor

Date 1/14/89

STRUCTURAL APPLICATIONS OF STEEL FIBER REINFORCED CONCRETE ANALYSIS AND DESIGN

By

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A THESIS

Submitted to
Michigan State University
In partial fulfillment of the requirements
of the degree of

MASTER OF SCIENCE

Department of Civil Engineering

1988

ABSTRACT

STRUCTURAL APPLICATIONS OF STEEL FIBER REINFORCED CONCRETE ANALYSIS AND DESIGN

By

Reklaoui Abdeslam

Structural analysis and design techniques were developed for reinforced concrete elements incorporating steel fibers as well as conventional reinforcement. The work was performed in three phases which were concerned with flexural, shear and torsional fibrous reinforced concrete elements. In each phase structural design guidelines were developed which accounted for the effects of steel fibers on strength and ductility of reinforced concrete elements. In the case of flexural elements emphasis was made on the improvement in ductility and the consequent relaxation of limits on the tension and compression steel ratio in the presence of steel fibers. As far as shear and torsional elements were concerned, consideration was given to the improvements in strength characteristics of elements resulting from steel fiber reinforcement.

The developed design guidelines were verified using relatively large numbers of flexural, shear and torsional test data reported in literature. The final guidelines were used to assess the effects of steel fiber reinforcement on the performance characteristics of reinforced concrete structural elements.

TO MY BELOVED PARENTS

ACKNOWLEDGMENTS

The writer wishes to express his sincere appreciation to his academic advisor, Dr. Parviz Soroushian, Assistant Professor of Civil and Environmental Engineering, for his encouragement, guidance, and aid throughout the writer's master of science studies. Thanks also to the other members of the thesis committee: Dr. R. Harichandran, Assistant Professor of Civil and Environmental Engineering, and Dr. A. Altiero Professor of Metallurgy, Mechanics And Materials Science.

Thanks are also extended to the Government of Morocco "Ministere de L'equipement, de la Formation Professionnelle et de la Formation des Cadres" for the financial assistance.

Last, but not least, special appreciation, admiration, and love to the writer's wife and his daughter for their continuous encouragement, and love.

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NOTATION

a	compressive stress block depth
A_l	cross section area of longitudinal reinforcement to resist torsion
A_s	area of tension steel
A_t	cross section area of transverse reinforcement resisting torsion
A_v	total area of shear reinforcement supplied by stirrup
A_{sp}	area of the compression steel
b	width of the rectangular section
c	neutral axis depth-compression zone
s	stirrups spacing
b_w	beam web width
C_c	compressive force of fibrous concrete
C_s	compressive force of steel bars
d_f	diameter of fiber
d_p	effective depth, distance from compression face to centroid of tension steel
d'	distance from compression face of the member to centroid of compression steel
E_c	modulus of elasticity of concrete
E_f	modulus of elasticity of steel fiber
f_i	curvature
f_r	modulus of rupture ($7.5(f'c)^{0.5}$ for plain concrete)
f_s	tension steel stress

f_y yield strength of longitudinal steel

f_{ys} yield strength of transverse steel

f_{ct} split cylinder tensile strength

f_{cu} the ultimate flexural strength of steel fiber

f_{tf} tensile strength of steel fiber reinforced concrete

f_{pf} post-cracking tensile resistance of steel fiber reinforced concrete

f'_c compression strength of Concrete, measured at 28 days after casting

f'_s compressive steel stress

f'_{cf} compressive strength of the steel fiber Reinforced Concrete

H the height of the rectangular section

l_f length of fiber

M_n nominal moment at section

p_b reinforcement ratio for the balanced strain condition

p reinforcement ratio, for tension steel

T_c tension force of fibrous concrete

T_s tension force of steel bars

T_u total torsional strength

V_c shear strength provided by concrete

V_f volume content of fiber

V_u shear force at section

T_{ct} torsional strength provided by concrete

T_{st} torsional strength provided by transverse steel

x width of the cross section

CHAPTER 1

INTRODUCTION

1.1 GENERAL

Steel Fiber Reinforced Concrete applications have grown rapidly in the 1970's and 80's. Steel Fiber Reinforced Concrete is now a standard construction material for some major secondary structures, including overlays on industrial floors, bridge decks and parking structures, mine tunnel lining, and dams. There are also major potentials for the application of steel fiber reinforced concrete to primary structural elements subjected to flexural, shear and torsional forces. Steel fibers can improve the ductility, strength and serviceability of structural elements. There is however, a strong demand for the development of structural analysis and design procedures for facilitating the commercial applications of steel fiber reinforced concrete to primary load-bearing structural elements.

Development of structural analysis and design guidelines has been the main thrust of this research. A brief background on the effects of steel fibers on the concrete material properties, and the flexural, shear and torsional performance characteristics of reinforced concrete elements is presented in this chapter. The available structural analysis and design techniques for fibrous reinforced concrete elements are introduced and critically evaluated using the reported experimental data for flexural, shear and torsional elements in chapters 2, 3 and 4, respectively. The structural analysis and design methodologies developed in this research for reinforced concrete elements subjected to flexural, shear and torsional forces are also described and verified in these chapters.

1.2 STEEL FIBER REINFORCED CONCRETE

Concrete is weak in tension and fails in a brittle manner under tensile and impact loads. These deficiencies generally result from the ease of initiation and propagation of microcracks, and also from the lack of post-cracking tensile resistance in conventional concrete materials [44]. Microcracks are generally initiated in concrete at the aggregate-cement paste interfaces as a result of the drying shrinkage, bleeding and settlement of cement paste in concrete. Under external load and environmental effects, microcracks tend to propagate and interconnect, causing the brittle failure of concrete. The propagation of microcracks in concrete can be effectively hindered through the use of closely-spaced short fibers, as shown in Figure 1.1a. The arrest of microcracks by steel fibers leads to improvements in the pre-peak tension behavior and tensile strength of concrete as shown in Figure 1.1b [7]*.

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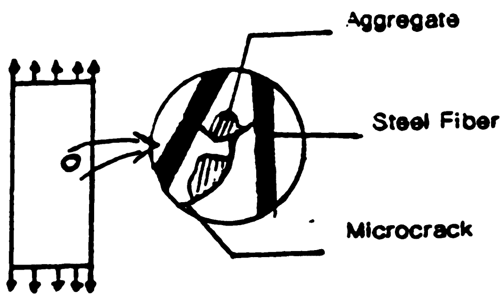
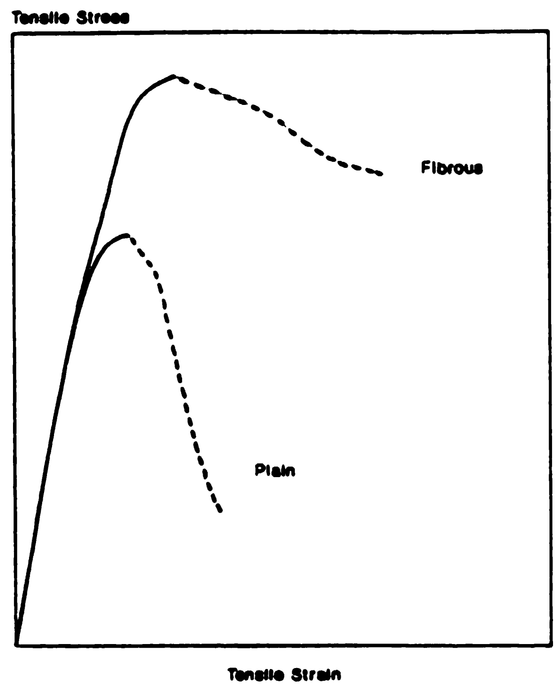
*Figures in brackets indicate reference number

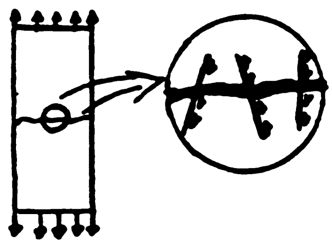


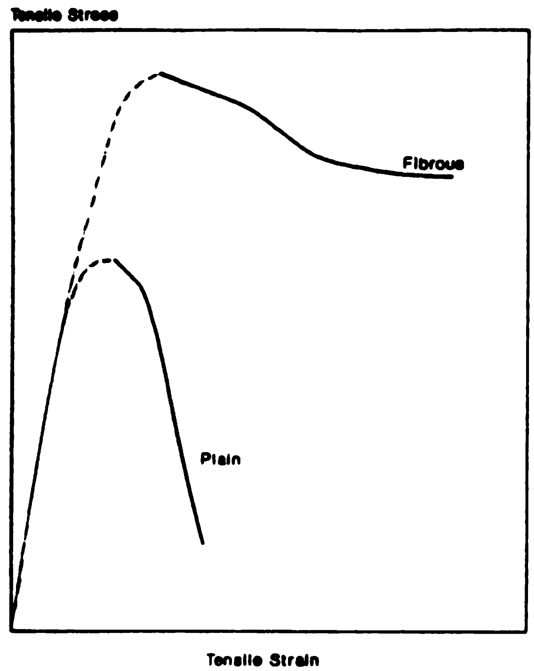


a) Microcrack Propagation   b) Pre-Peak Tensile Behavior

Figure 1.1 Effects of Steel Fibers on Microcrack Propagation and Pre-Peak Tensile Behavior of Concrete.

Upon the formation of a non stable microcrack system, which usually marks the achievement of the ultimate tensile strength in concrete, tensile deformations tend to concentrate in a limited number of macrocracks. Steel fibers bridge these cracks and restrain their widening through pull-out resistance as shown in Figure 1.2a . Steel fiber are thus highly effective in enhancing the post-peak ductility and energy absorption capacity of concrete under tensile stresses as shown in Figure 1.2b.

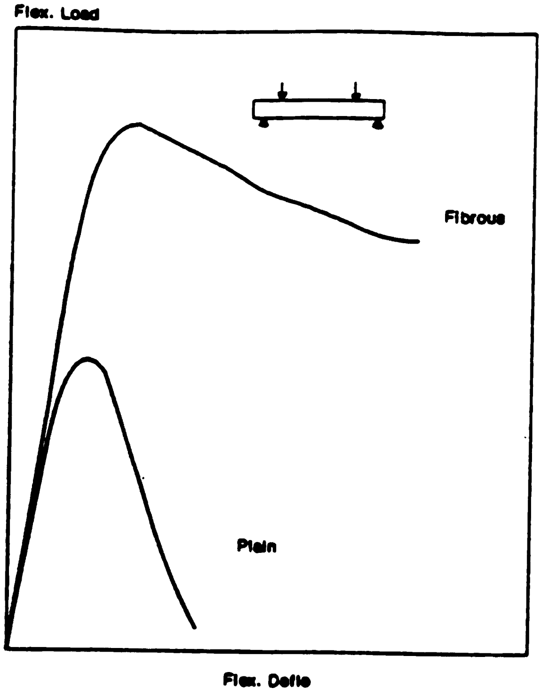




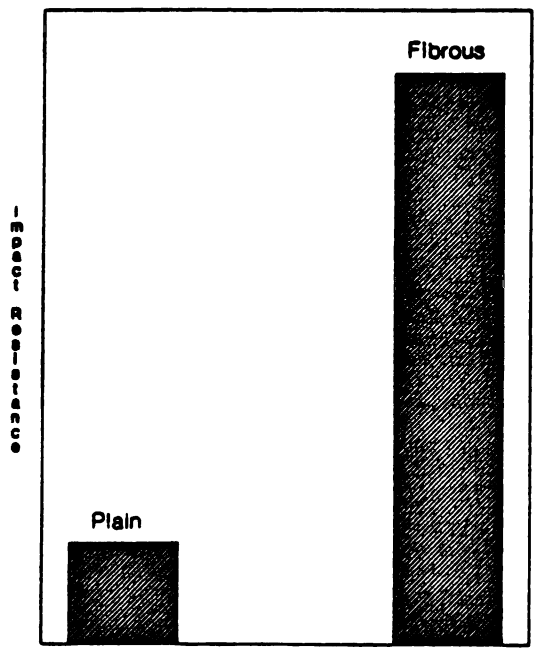
a) Pull-Out Action of fibers b) Post-Peak Tensile Behavior

Figure 1.2 Pull-Out Action of Steel Fibers and Post-Peak Tensile Behavior of Steel Fiber Concrete.

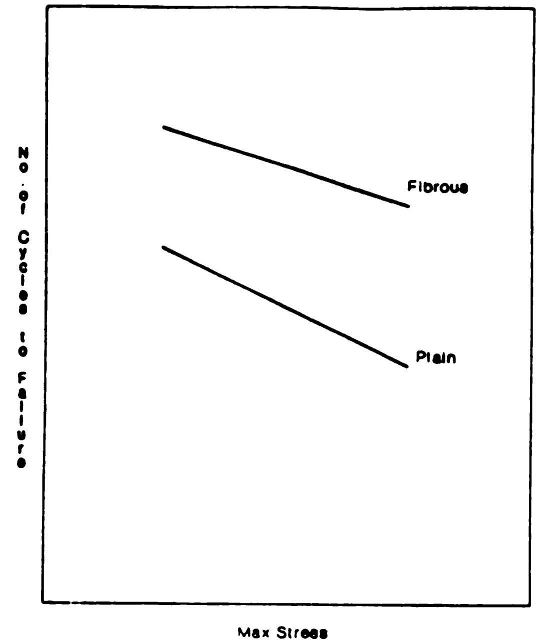
The crack-stabilization action of steel fibers in concrete also results in major improvements in the compressive and flexural performance characteristics, impact resistance, fatigue life and freeze-thaw durability of steel fiber reinforced concrete as shown in Figure 1.3a, 1.3b, 1.3c and Figure 1.3d respectively.



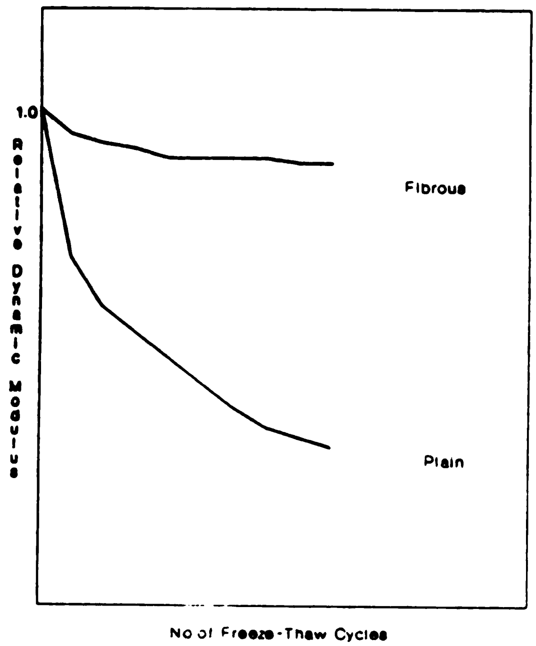
(a) Compressive and Flexural Ductility



(b) Impact Resistance



(c) Fatigue Life



(d) Freeze-Thaw Durability

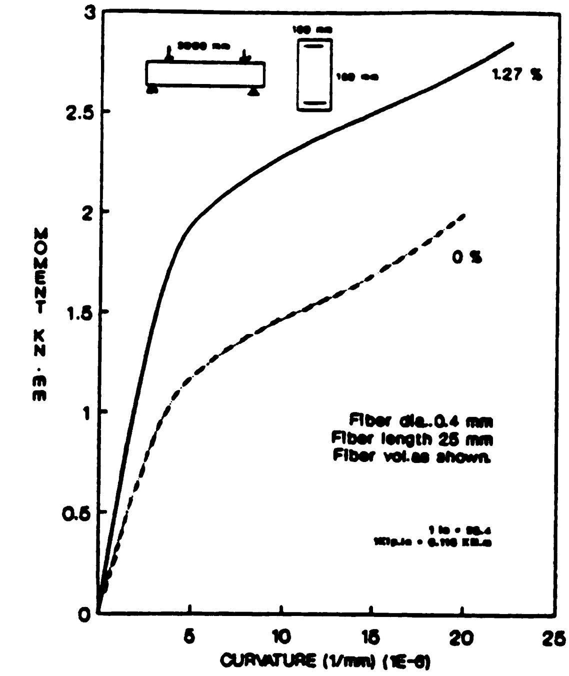
Figure 1.3 Improvements in Concrete Material Properties Resulting from Steel Fiber Reinforcement [45].

1.3 STEEL FIBERS IN FLEXURAL REINFORCED CONCRETE ELEMENTS

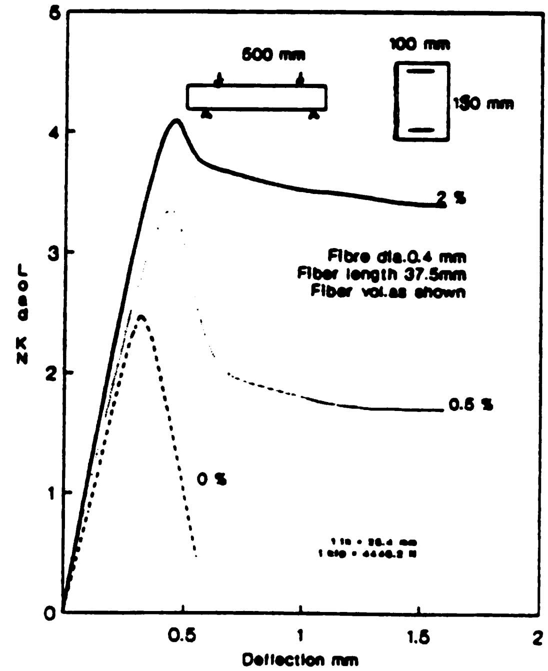
The crack-aresting action of steel fibers in concrete results in improved performance of fibrous reinforced concrete flexural elements under service conditions. Elements incorporating steel fibers have higher flexural stiffness (reduced deflections) and smaller crack widths when subjected to service loads [9].

Figure 1.4a presents test results which are indicative of the increase in flexural stiffness resulting from the application of steel fibers to conventional reinforced concrete elements. Steel fibers also improve the strength and especially post-peak ductility and toughness of reinforced concrete flexural elements as shown in Figure 1.4b. The flexural failure mode of fibrous reinforced concrete elements involves the formation of more closely-spaced cracks with reduced crack widths when compared with conventionally reinforced concrete elements as shown in Figure 1.4c.

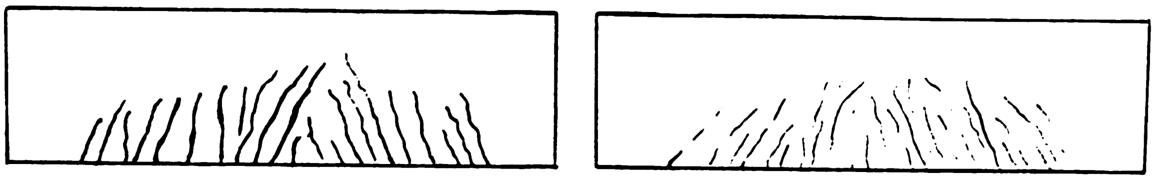
These advantages of the use of steel fibers in reinforced concrete flexural elements produce opportunities for reducing the required compressive steel ratios and increasing the maximum allowable tensile steel area (set to ensure the ductility of failure), and also for using high-strength materials (which generally tend to encourage brittle failure modes), and reducing the safety factors in flexural elements.



a) Stiffness



b) Strength and Ductility



Beam a: no fiber reinforcement   Beam b: with 1 % fiber reinforcement

c) Crack Characteristics

Figure 1.4 Steel Fiber Effects on The Flexural Performance of Reinforced Concrete Elements [4].

1.4 STEEL FIBERS IN REINFORCED CONCRETE ELEMENTS SUBJECTED TO SHEAR FORCES

Shear failure modes in reinforced concrete elements are generally brittle. Relatively high safety factors are used in shear design of reinforced concrete elements, which lead to the need for large ratios of transverse shear reinforcement. Congestion of shear reinforcement, and the cost and time involved in the placement of transverse steel are some undesirable aspects of reinforced concrete structures.

The improvements in tensile resistance, ductility and shear friction (across cracks) in concrete resulting from steel fiber reinforcement lend themselves particularly well to enhancing the performance characteristics of reinforced concrete elements under shear. Improvements up to 70 % or more in the shear resistance of reinforced concrete beams have been achieved through steel fiber reinforcement [27]. References [21, 22] have reported test results which indicate that fibers can change the failure mode of typical reinforced concrete beams from a brittle shear mode to a ductile flexural one. With the introduction of steel fibers the amount of costly and labor intensive transverse steel can be reduced, which also relaxes the congestion of reinforcing bars in structural elements. It has been suggested that the best results can be achieved through the combined use of steel fibers with a relatively small amount of transverse reinforcement [25].

1.5 STEEL FIBERS IN TORSIONAL REINFORCED CONCRETE ELEMENTS

Significant improvement in the strength and ductility of reinforced concrete elements subjected to torsional forces can be achieved through steel fiber reinforcement as shown in Figure 1.4 [31]. In the presence of steel fibers, the widening of cracks in torsional elements tends to be restrained a condition which leads to the formation of well-distributed (i.e., closely spaced) cracks with controlled widths [20, 31, 32].

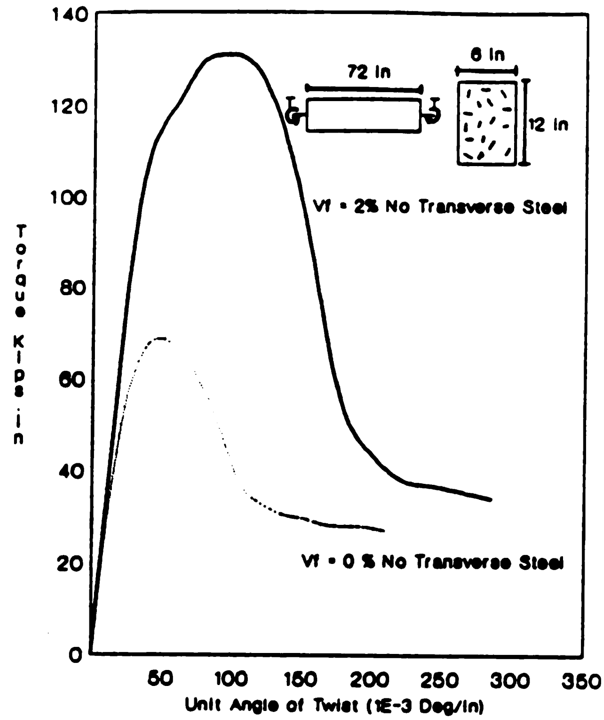


Figure 1.5 Typical Improvements in the Torsional Performance of Reinforced Concrete Elements Resulting From Steel Fiber Reinforcement [31].

The improvements in torsional performance of reinforced concrete elements resulting from fiber reinforcement tend to be more pronounced at higher volume fractions of fibers with larger aspect ratios [31, 34]. Optimum results seem to be achieved through the use of a combination of steel fibers and transverse reinforcement as shown in Figure 1.6.

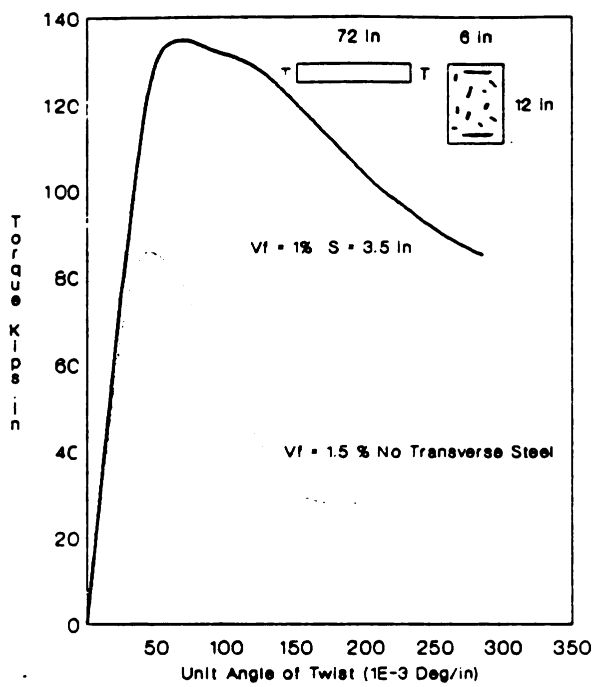


Figure 1.6 Joint Action of Steel Fibers and Transverse Reinforcement in Torsional Elements.

CHAPTER 2

EFFECTS OF STEEL FIBERS ON FLEXURAL BEHAVIOR OF REINFORCED CONCRETE BEAMS : REFINED ANALYSIS AND DESIGN

2.1 INTRODUCTION

The advantages of steel fiber reinforced concrete can be beneficial in structural elements subjected to flexural loads for improving the ductility of failure and thus releasing the strict limits on conventional reinforcement ratios (e.g maximum limit on tensile steel ratio) required to ensure a ductile failure mode. Steel fibers can facilitate the effective use of high strength materials in structural elements.

A review of the existing literature on the flexural behavior of reinforced concrete beams incorporating steel fibers reveals the following effects of fiber reinforcement on flexural behavior:

Fibers are effective in increasing the flexural stiffness and reducing deformations of cracked reinforced concrete sections. The increase in flexural rigidity results from the role of the fibers in inhibiting the growth and widening of cracks [1, 8, 13].

The load at which flexural cracks appear in reinforced concrete beams are twice the corresponding loads for the beam without steel fibers [1, 13].

The flexural cracks in steel fiber reinforced concrete members are more closely spaced, and have smaller widths when compared with those in conventional concrete elements subjected to flexural loads [2, 14].

The post-cracking behavior and ductility of the flexural reinforced concrete elements are significantly enhanced through the addition of steel fibers. This is a direct result of the desirable toughness and ductility of steel fiber reinforced concrete in compression and in tension [5, 6, 7, 8]. Steel fibers, through preventing a brittle crushing of concrete in compression, also provide the compression steel with lateral restraint against buckling at large strains, and thus further enhance the ductility flexural behavior in reinforced concrete elements.

The improvements in flexural strength resulting from steel fiber reinforcement are not large enough to give steel fibers the potential to fully substitute continuous reinforcing bars in reinforced concrete elements [15]. Optimum conditions in flexural elements may be achieved through the use of steel fibers together with conventional steel bars.

The purpose of this phase of research is to study the influence of steel fibers on the strength and ductility of reinforced concrete members subjected to flexural loads, and to develop design techniques for reinforced concrete flexural members incorporating steel fibers.

The flexural behavior of fibrous reinforced concrete elements is investigated using a refined modeling technique.

Based on the results of this investigation, practical design guidelines are produced which take advantage of the improvements in flexural ductility of reinforced concrete elements resulting from the presence of steel fibers.

2.2 REFINED FLEXURAL ANALYSIS

A refined analysis technique was developed for predicting the complete moment-curvature relationship of reinforced concrete sections in the presence of steel fibers. The proposed analysis procedure accounts for the post-cracking tensile resistance and the improved ductility of steel fiber reinforced concrete under tensile and compressive stresses [7].

The developed flexural analysis procedure has been based on the assumption that plane sections normal to the beam longitudinal axis remain plane after bending. This assumption indicates that the flexural strain distribution is linear across the section as shown in Figure 2.1. The tensile and compressive constitutive models of steel fiber reinforced concrete are assumed to be known (the constitutive models used in this study will be presented later).

In the developed flexural analysis procedure the complete moment-curvature relationship is constructed by finding the value of moment corresponding to different curvature input values. For each curvature, through a trial and adjustment technique, the neutral axis position at which the equilibrium of compressive and tensile forces at the cross section are satisfied is decided. With the neutral axis location and value of curvature known, the strains and thus the stresses at the cross section can be calculated as shown in Figure 2.1 and the moment of forces acting on the section corresponding to the input value of curvature can be computed. A computer program was developed to perform the above flexural analysis procedure; this program is described in Appendix "A" where a listing of the program is also given.

Typical moment-curvature diagrams generated using the developed program are given in Figures 2.2a and 2.2b as.

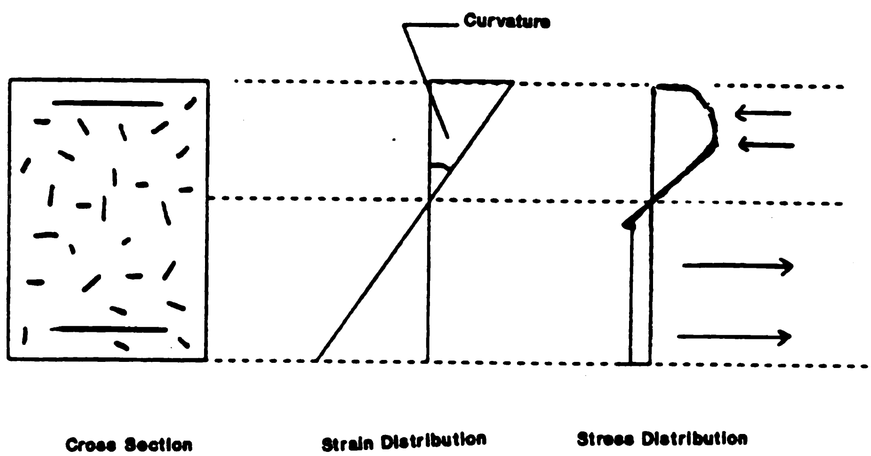
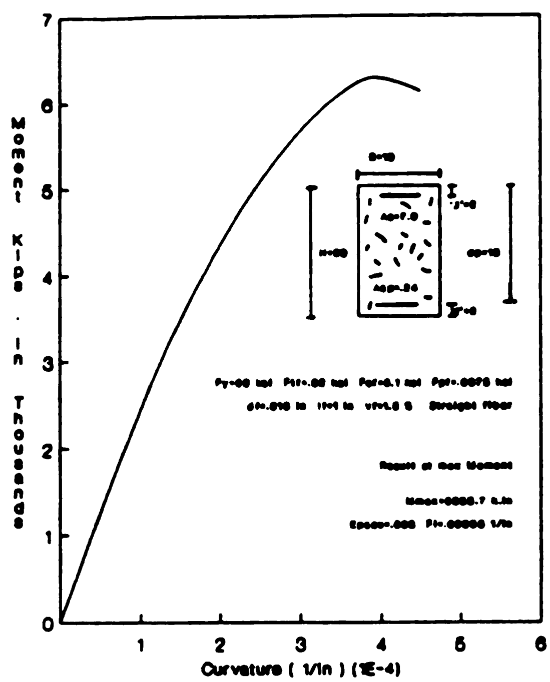
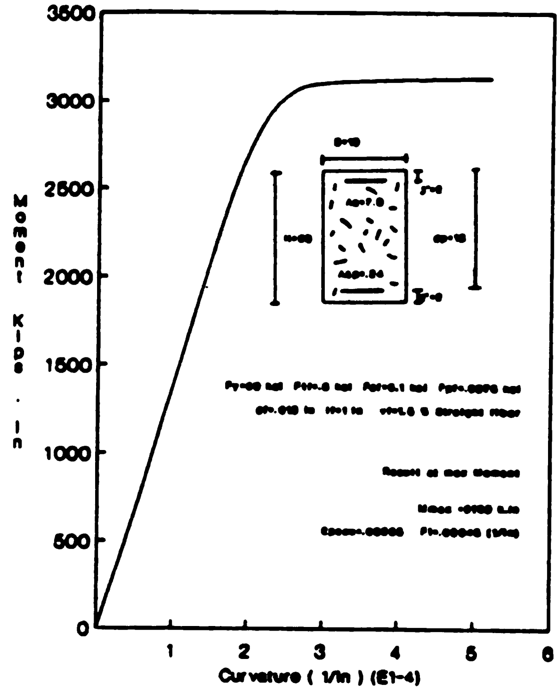


Figure 2.1 Refined Flexural Analysis Procedure.





a) V_f = 1.5 %; A_s = 7.9 in^2 b) V_f = 1.5 %; A_s = 3.8 in^2

Figure 2.2 Typical Moment-curvature Relationships.

2.3 CONSTITUTIVE MODELS

The refined flexural analysis procedure introduced in the previous section requires the compressive and tensile constitutive models of steel fiber reinforced concrete. The models used in this investigation are described below.

2.3.1 COMPRESSION MODEL

The compressive stress-strain relationship presented in References [18, 19] for steel fiber reinforced concrete was selected for use in this study. This constitutive model is a function of the following parameters.

The matrix compressive strength

Fiber reinforcement index $V_f l_f / d_f$

The model consists of 3 segments, a curvilinear ascending portion followed by a linear descending branch and a flat tail as shown in Figure 2.3. The equations for this model are given below:

$f = - f'_{cf} (\epsilon / \epsilon_p)^2 + 2f'_{cf} (\epsilon / \epsilon_p)$ for $\epsilon \leq$ p

$f = z (\epsilon - \epsilon_p) + f'_{cf} \geq f'_r$ for $\epsilon \geqslant$ p

Where :

f'_c = concrete compressive stress

ϵ = concrete compressive strain

ϵ_p = strain at peak stress

f'_{cf} = compressive strength of the steel fiber Reinforced Concrete

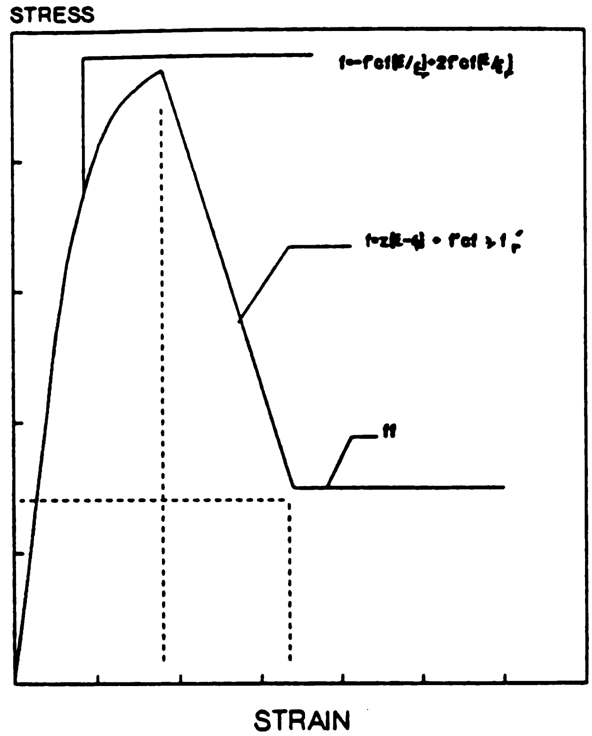


Figure 2.3 General Form of the Compressive Constitutive Model for Steel Fiber Reinforced Concrete.

z , f'_r = Coefficient to be derived, together with f'cf and ϵ_p, empirically in terms of the matrix compressive strength and the fiber reinforcement index.

The empirical expressions for coefficients of the above compressive constitutive model are given below

$$f'_{cf} = f'_c + 994\, V_f \frac{l_f}{d_f}$$

$$f'_r = 0.12\, f'_{cf} + 2000\, V_f \frac{l_f}{d_f}$$

$$z = -343\, f'_c \left(1 - 0.64 \left(V_f \frac{l_f}{d_f}\right)^{0.5} \right) \leq 0$$

$$p = \left(0.00079 + \frac{1.13}{f'_c} \right) V_f \frac{l_f}{d_f} + 0.0021$$

Note : f'_c , f'_{cf} and f'_r are in psi.

Where :

d_f = diameter of fiber

f'_c = concrete compressive strength at 28 days

V_f = volume content of fiber

l_f = length of fiber

2.3.2 TENSION MODEL

The tensile constitutive model of steel fiber reinforced concrete used in this study is one proposed in References [16, 17]. This model is shown in Figure 2.4 and involves four variables:

f_{tf} = tensile strength

E_f = modulus of elasticity

f_{pf} = post-cracking tensile resistance provided by the pull-out resistance of fibers.

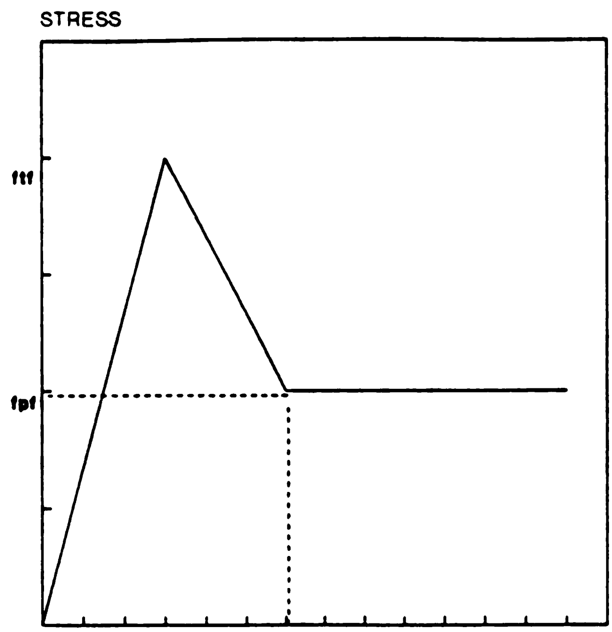


Figure 2.4 General Form of the Tensile Constitutive Model for Steel Fiber Reinforced Concrete.

The values of peak tensile stress (f_{tf}) and post-peak tensile resistance in the tensile constitutive model of Figure 2.4 can be obtained using the following expressions:

$$f_{tf} = f_{tm} \left(1 - \frac{V_f}{100} \right) + 0.5*0.41*\tau* \frac{V_f}{100} \frac{l_f}{d_f}$$

$$f_{pf} = 0.5*0.41*\tau* \frac{V_f}{100} \frac{l_f}{d_f}$$

The modulus of elasticity in tension was taken equal to that in compression, and the effects of steel fibers on elastic modulus were assumed to be negligible:

$$E_c = 57000 \, (f'_c)^{0.5} \quad \text{psi}$$

2.4 COMPARISON WITH TEST RESULTS

The analytical moment-curvature relationships of steel fiber Reinforced Concrete (obtained using the developed computer program and the constitutive models described earlier) are compared in Figures 2.5 - 2.7 with the experimental moment-curvature relationships reported in References [3, 1] in order to verify the accuracy of the analytical techniques. The comparison between experimental and analytical result is observed to be reasonable, as shown in Figures 2.5, 2.6 and 2.7.

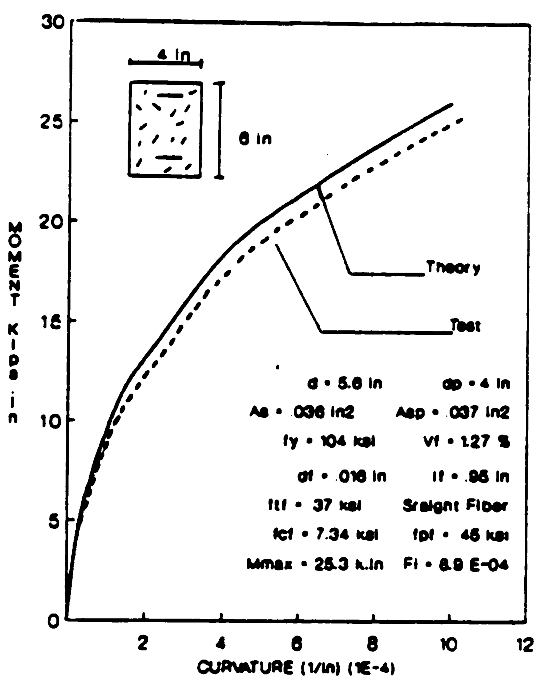


Figure 2.5 Comparison of Analytical and Experimental Moment-Curvature Relationships (Reference 3).

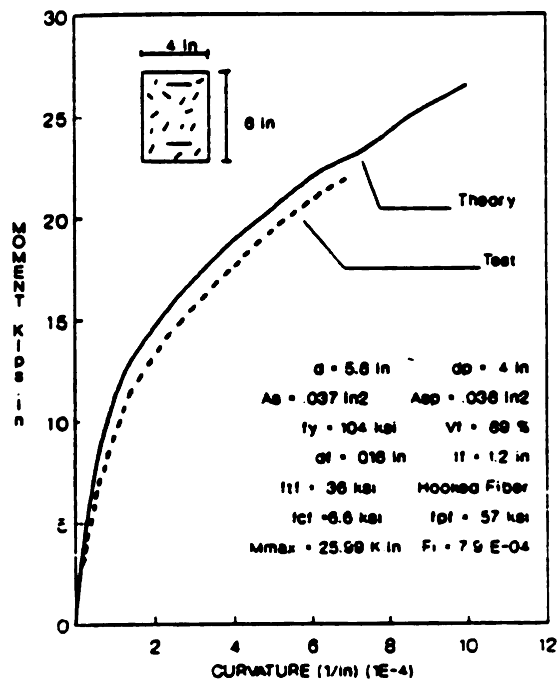


Figure 2.6 Comparison of Analytical and Experimental Moment-Curvature Relationships (Reference 3).

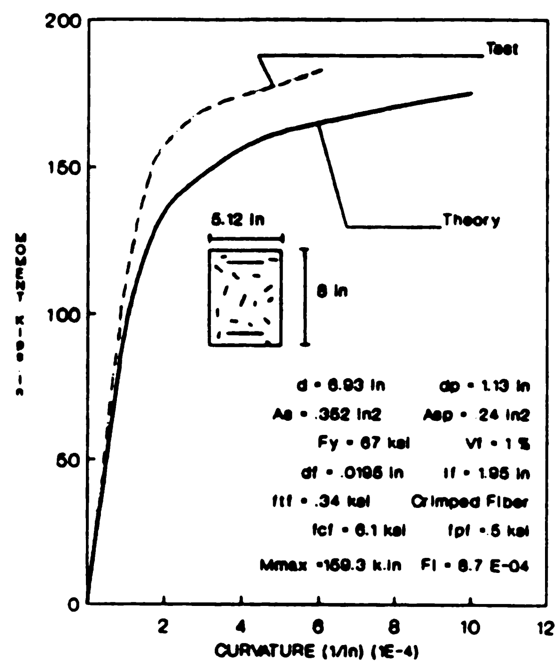


Figure 2.7 Comparison of Analytical and Experimental Moment-Curvature Relationships (Reference 1).

2.5 PARAMETRIC STUDIES

The flexural performance characteristics of reinforced concrete sections incorporating steel fibers are mainly dependent on the steel fiber and conventional reinforcement conditions, and also on the concrete strength. The developed analytical techniques were used in this part of the study to assess the effects of these factors on the flexural moment-curvature relationships of typical fibrous reinforced concrete sections.

The variable used in this parametric study are: a) fiber volume fraction (V_f); b) fiber length (l_f); c) fiber diameter (d_f); d) fiber type (Straight, Hooked and Crimped) e) tensile steel area (A_s); f) compression steel area (A'_s); g) rebars yield strength (f_y); and h) concrete compressive strength (f'_c). The ranges of these variables considered in this parametric study are given in Table 2.2.

The typical rectangular reinforced concrete beam section used as the basic section in the parametric study on flexural behavior is shown in Figure 2.8. The parametric study was performed by analytically deriving the moment-curvature relationship while one of the variables was changed with the others kept constant. The effects of each variable on moment-curvature relationship could thus be investigated separately.

Table 2.1 VARIABLES IN PARAMETRIC STUDIES

Sec. number (1)	Variable & value (2)		Fiber properties (3)				Longitudinal steel(4)			f'cf ksi (5)	ftf ksi (6)	Fig. number (7)
			Vf %	lf in	df in	tf*	As in2	A's in2	fy ksi			
10	tf*	-	1.5	1	.013	str	2.37	.24	60	5.1	.32	
11	tf*	-	1.5	1	.013	crp	2.37	.24	60	5.1	.32	2.9
12	tf*	-	1.5	1	.013	hok	2.37	.24	60	5.1	.32	
22	fy	40	.35	1	.013	str	2.37	.24	40	4.2	.27	
23	fy	80	.35	1	.013	str	2.37	.24	80	4.2	.27	2.10
24	fy	120	.35	1	.013	str	2.37	.24	120	4.2	.27	
25	fy	140	.35	1	.013	str	2.37	.24	140			
26	fy	40	1.5	1	.013	str	2.37	.24	40	5.1	.32	
27	fy	80	1.5	1	.013	str	2.37	.24	80	5.1	.32	2.11
28	fy	120	1.5	1	.013	str	2.37	.24	120	5.1	.32	
29	fy	140	1.5	1	.013	str	2.37	.24	140	5.1	.32	
16a	f'c	3	.4	1	.013	str	2.37	.24	60	3.3	.24	
17a	f'c	4	.4	1	.013	str	2.37	.24	60	4.3	.27	2.12
18a	f'c	6	.4	1	.013	str	2.37	.24	60	6.3	.33	
16b	f'C	3	1.5	1	.013	str	2.37	.24	60	3.3	.24	
17b	f'c	4	1.5	1	.013	str	2.37	.24	60	5.1	.32	2.13
18b	f'c	6	1.5	1	.013	str	2.37	.24	60	7.1	.38	
19a	A's	0	.5	1	.013	str	2.37	0	60	4.3	.27	
20a	A's	1.81	.5	1	.013	str	2.37	1.81	60	4.3	.27	2.14
21a	A's	2.74	.5	1	.013	str	2.37	2.74	60	4.3	.27	
19b	A's	0	1.5	1	.013	str	2.37	0	60	5.1	.32	
20b	A's	1.81	1.5	1	.013	str	2.37	1.81	60	5.1	.32	2.15
21b	A's	2.74	1.5	1	.013	str	2.37	2.74	60	5.1	.32	
1	As	.79	.35	1	.013	str	.79	.24	60	4.2	.27	
2	As	1.57	.35	1	.013	str	1.57	.24	60	4.2	.27	2.16
3	As	3.95	.35	1	.013	str	3.95	.24	60	4.2	.27	
4	As	6.32	.35	1	.013	str	6.32	.24	60	4.2	.27	
5	As	.79	1.5	1	.013	str	.79	.24	60	5.1	.32	
6	As	1.57	1.5	1	.013	str	1.57	.24	60	5.1	.32	2.17
7	As	3.95	1.5	1	.013	str	3.95	.24	60	5.1	.32	
8	As	6.32	1.5	1	.013	str	6.32	.24	60	5.1	.32	
13a	df	.02	0.5	1	.02	str	2.37	.24	60	4.2	.27	
14a	df	.013	0.5	1	.013	str	2.37	.24	60	4.4	.28	2.18
15a	df	.01	0.5	1	.01	str	2.37	.24	60	4.5	.28	
13b	df	.02	1.5	1	.02	str	2.37	.24	60	4.7	.3	
14b	df	.013	1.5	1	.013	str	2.37	.24	60	5.1	.32	2.19
15b	df	.01	1.5	1	.01	str	2.37	.24	60	5.4	.35	

Table 2.1 (cont'd)

Sec. number (1)	Variable & value (2)		Fiber properties (3) Vf %	lf in	df in	tf*	Longitudinal steel(4) As in2	A's in2	fy ksi	f'cf ksi (5)	ftf ksi (6)	Fig. number (7)
32a	lf	1	.5	1	.013	str	2.37	.24	60	4.3	.27	2.20
33a	lf	1.5	.5	1.5	.013	str	2.37	.24	60	4.6	.29	
34a	lf	2	.5	2	.013	str	2.37	.24	60	4.7	.3	
32b	lf	1	1.5	1	.013	str	2.37	.24	60	5.1	.32	2.21
33b	lf	1.5	1.5	1.5	.013	str	2.37	.24	60	5.7	.36	
34b	lf	2	1.5	2	.013	str	2.37	.24	60	6.2	.4	

Note :

tf* = Type of steel fiber.
crp = Crimped fiber
hok = Hooked fiber
str = Straight fiber.
1 in = 25.4 mm
1 ksi = 6.895 MPa
1 kip. in = 0.113 KN.m

Rectangular cross section (20 in height * 10 in width)

A_s = 2.37 in_2	l_f = 1 in	d' = 2 in
A_{sp} = 0.24 in^2	d_f = 0.013 in	d" = 2 in
fy = 60 ksi	Vf = 1.5 %	dp = 18 in
f'c = 4 ksi	Type of Fiber = Straigth	

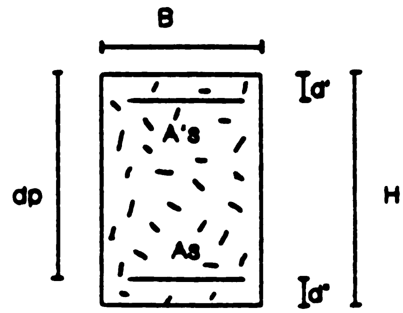


Figure 2.8 The Basic Rectangular Section in Parametric Study.

Figures 2.9 and 2.10 show the effects of the longitudinal steel yield strength on flexural performance for conditions with V_f = 0.5 % and V_f = 1.5 % respectively. The ultimate moment capacity is observed to increase with the increase in the yield strength of longitudinal steel. At the lower fiber volume fraction of 0.5 % in Figure 2.9, the increase in the reinforcing bar yield strength tends to damage the ductility of flexural behavior (by limiting post-peak deformations prior to failure). At high fiber volume fraction (1.5 % in Figure 2.10), however, the section remains ductile in spite of major increases in the reinforcing bar yield strength (up to the very high value of 120 ksi).

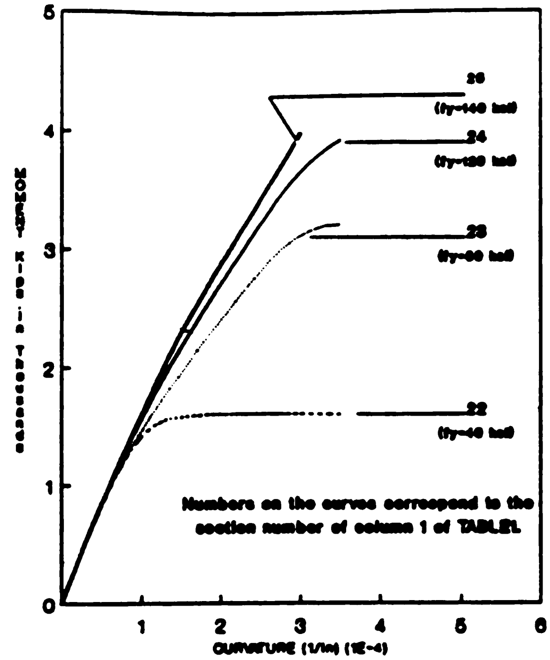


Figure 2.9 Calculated Moment-Curvature Relationship for V_f= 0.5 % (variable = f_y).

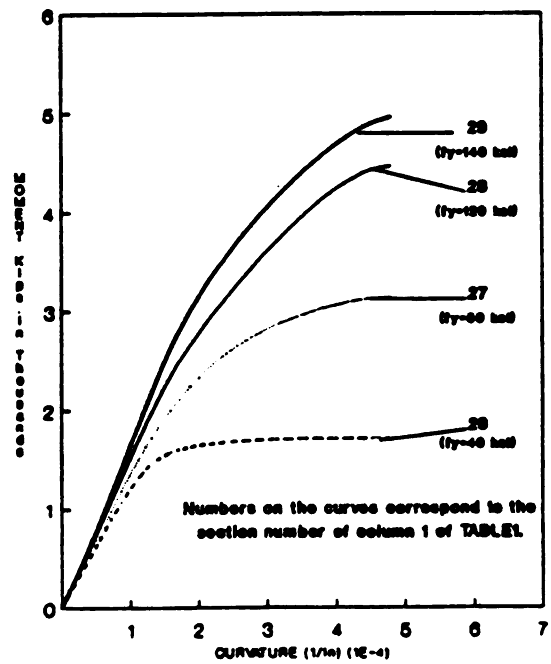


Figure 2.10 Calculated Moment-Curvature Relationship for V_f= 1.5 % (variable = f_y).

Figures 2.11 and 2.12 show the effects of concrete compressive strength on flexural behavior of reinforced concrete sections with 0.5 % and 1.5 % fiber volume fractions respectively. The increase in fiber volume fraction is observed to provide the section with higher flexural ductility even in conditions with relatively high concrete compressive strengths.

A comparison between the effects of compression steel area on flexural behavior of sections with 0.5 % and 1.5 % fiber volume fractions in Figures 2.13 and 2.14, respectively, indicates that at higher fiber volume fractions there is a lesser need of compression reinforcement for improving the flexural strength and ductility of reinforced concrete sections.

Figures 2.15 and 2.16 show the effects of tension reinforcement area on the flexural behavior of reinforced concrete sections at 0.5 % and 1.5 % fiber volume fractions, respectively. The increase in fiber volume fraction is observed to provide the section with desirable ductility characteristics, even with high tension steel areas which tend to encourage brittle failure modes prior to the yielding of tension steel in reinforced concrete sections with zero or low steel fiber contents.

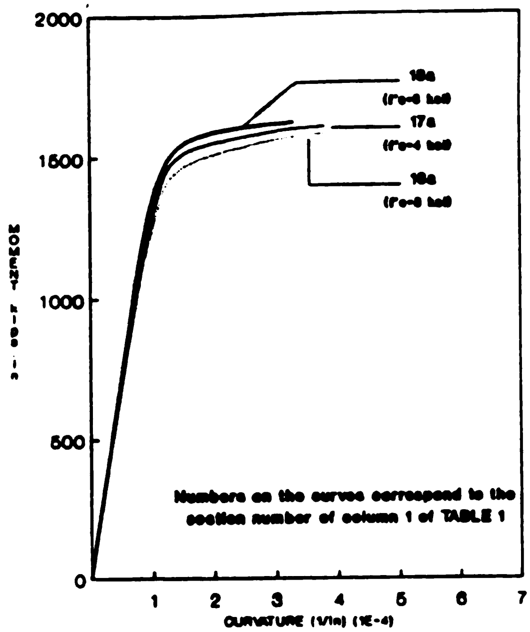


Figure 2.11 Calculated Moment-Curvature Relationship for V_f= 0.5 % (variable = f'_c).

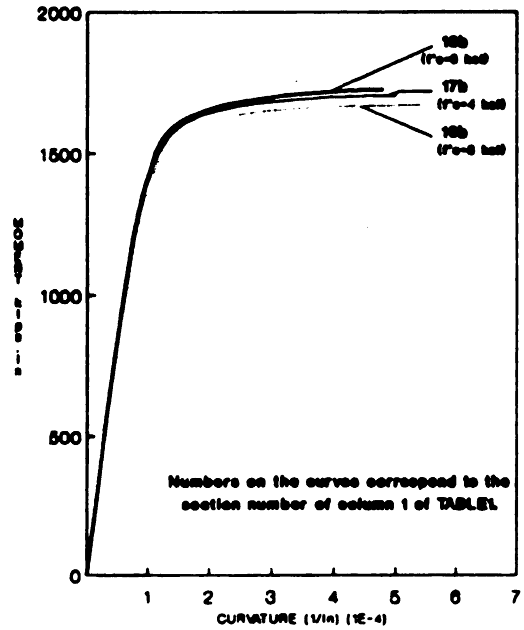


Figure 2.12 Calculated Moment-Curvature Relationship for V_f= 1.5 % (variable = f'_c).

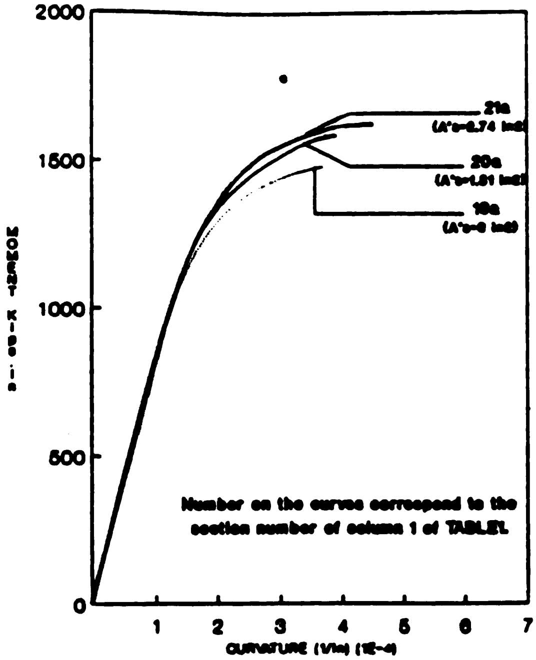


Figure 2.13 Calculated Moment-Curvature Relationship for V_f= 0.5 % (variable = A_{sp}).

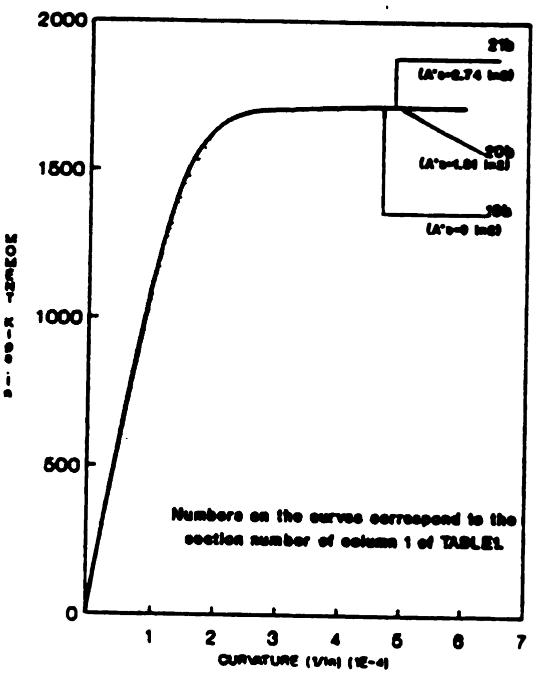


Figure 2.14 Calculated Moment-Curvature Relationship for V_f= 1.5 % (variable = A_{sp}).

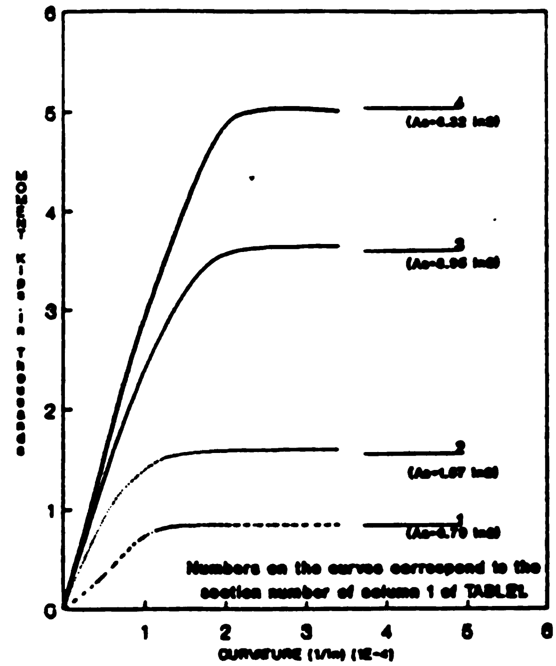


Figure 2.15 Calculated Moment-Curvature Relationship for V_f= 0.5 % (variable = A_s).

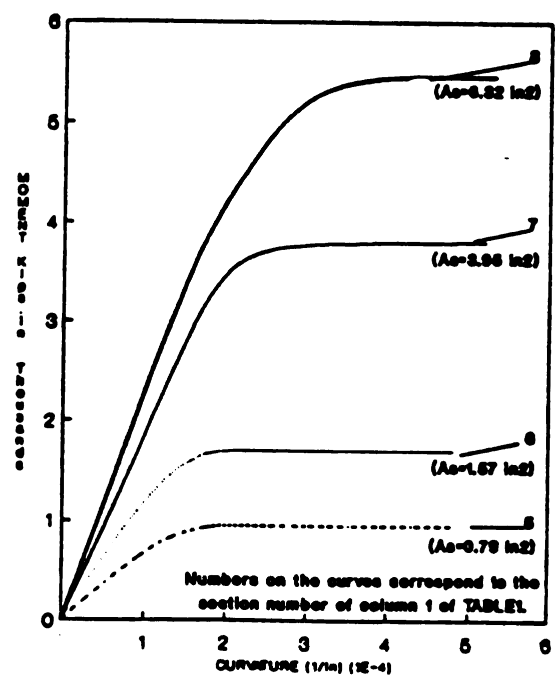


Figure 2.16 Calculated Moment-Curvature Relationship for V_f= 1.5 % (variable = A_s).

In general, one may conclude from the above discussion that the addition of steel fibers is effective in improving the ductility of reinforced concrete sections. Hence, the limits on maximum tension steel area can be relaxed (leading to the increase in the upper limits of the flexural strength which can be achieved using sections of specified dimensions) the need for compression steel can be reduced, and high - strength concrete and steel can be used without loss in ductility. Besides improving flexural ductility, steel fibers also provide the section with increased flexural strength.

Figures 2.17 and 2.18 clearly show that fibers with lower diameters tend to be more effective in improving the flexural strength and ductility of reinforced concrete sections.

Figures 2.19 and 2.20 show that the increase in fiber length results in enhanced flexural strength and ductility of reinforced concrete sections.

Figure 2.21 shows the influence of different fiber types (Straight, Crimped and Hooked) on the flexural load-deflection relationship of the base reinforced concrete section. Hooked fibers are observed to result in a slightly higher flexural strength than crimped and straight fibers.

Figures 2.22 and 2.23 show typical effects of fiber volume fraction for different fiber lengths and diameters respectively, on the flexural strength of reinforced concrete

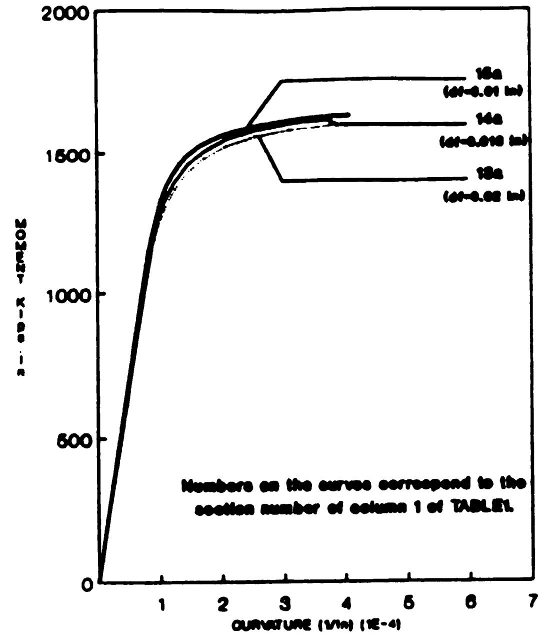


Figure 2.17 Calculated Moment-Curvature Relationship for V_f= 0.5 % (variable = fiber diameter).

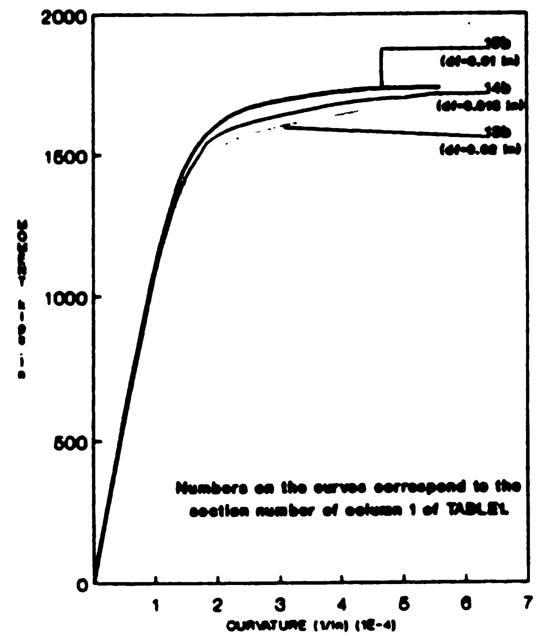


Figure 2.18 Calculated Moment-Curvature Relationship for V_f= 1.5 % (variable = fiber diameter).

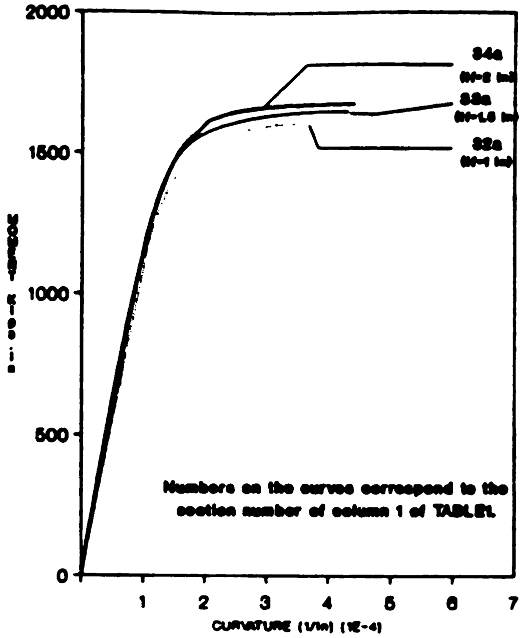


Figure 2.19 Calculated Moment-Curvature Relationship for V_f= 0.5 % (variable = fiber length).

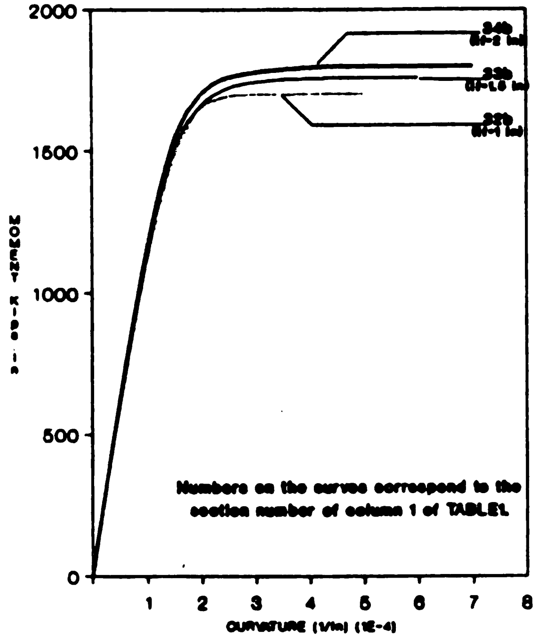


Figure 2.20 Calculated Moment-Curvature Relationship for V_f= 1.5 % (variable = fiber length).

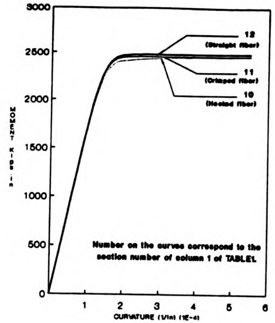


Figure 2.21 Calculated Moment-Curvature Relationship for V_f= 1.5 % (variable = fiber type).

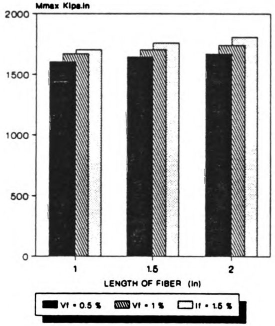


Figure 2.22 Calculated Ultimate Moment as Function of V_f for Different Fiber length.

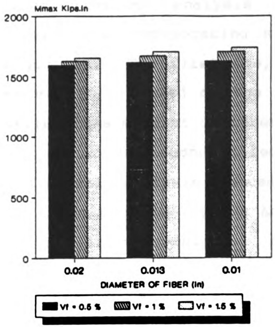


Figure 2.23 Calculated Ultimate Moment as Function of V_s With Different Fiber Diameter.

2.6 SIMPLIFIED FLEXURAL ANALYSIS AND DESIGN GUIDELINES

Two methods for flexural analysis and design of reinforced concrete sections incorporating steel fibers are introduced in this section. The first one, referred to as the "alternative method", is based on the observations of flexural stress distributions made at different stages of the refined flexural analysis. The second called the "ACI-based method", follows flexural analysis procedures which are modified versions of those proposed by the American Concrete Institute (ACI 318-83) for conventional concrete. The predictions of these two methods are compared with the results of experimental studies and refined flexural analysis. Conclusions are made regarding the practicality of these methods in application to flexural design of fibrous reinforced concrete sections.

2.6.1 "ALTERNATIVE" METHOD

The method introduced in this section has been based on the examination of flexural stress and strain distributions of peak flexural load obtained in refined analysis of fibrous reinforced concrete sections.

For the fibrous reinforced concrete section of Figures 2.24(a) and 2.24(b), Figure 2.24(c) shows the proposed linear flexural strain distribution at ultimate moment, which has been based on the fundamental assumption of this study which implies that plane section normal to the beam longitudinal axis remains plane after bending. A maximum compressive strain value of 0.002 is proposed as the criterion for the achievement of maximum bending moment. Figure 2.24(d) presents a theoretical stress obtained using the strain distribution of Figure 2.24(c) and the constitutive models of the material in tension and in compression. The simplified flexural stress distribution at peak bending moment, used in the "Alternative method" of flexural analysis, is presented in Figure 2.24(e). This stress distribution together with the proposed value of 0.002 for extreme compressive fiber strain at peak moment form the basis for the "Alternative" method of flexural analysis and design of fibrous reinforced concrete sections.

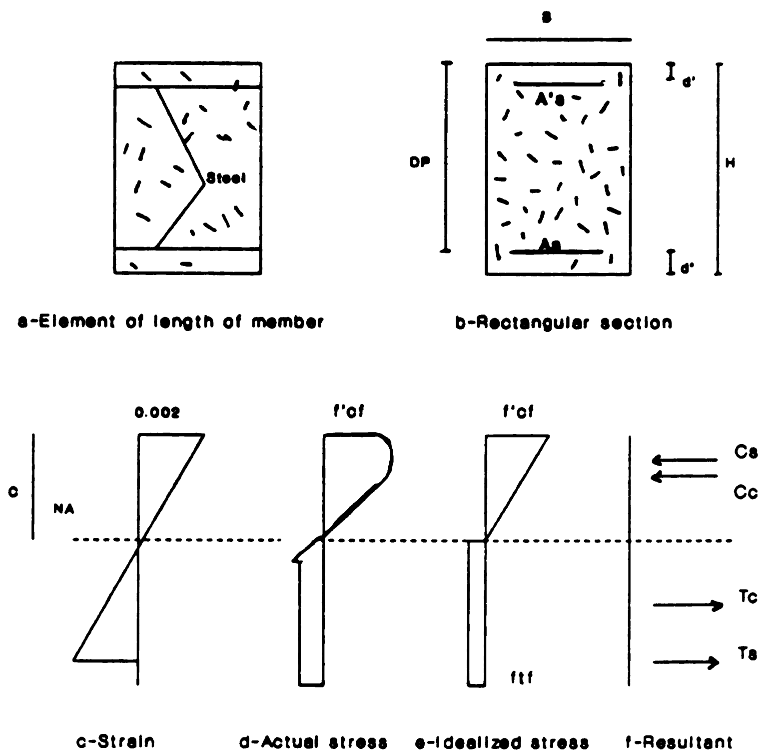


Figure 2.24 "ALTERNATIVE" Method of Flexural Analysis.

The flexural strength of fibrous Reinforced Concrete sections can be obtained in the "alternative" method through the performance of the following steps:

1 - Guess a value for c (the depth of the neutral axis in Figure 2.24(c))

2 - Find the strain and stress values; for compression and tension steel:

$$\epsilon_s = \frac{d-c}{c} * 0.002 \qquad (2\text{-}1a)$$

$$\epsilon'_s = \frac{c-d}{c} * 0.002 \qquad (2\text{-}2a)$$

$$f_s = E_s \epsilon_s = E_s \left(\frac{d-c}{c}\right) * 0.002 < f_y \qquad (2\text{-}3a)$$

$$f'_s = E_s \epsilon'_s = E_s \left(\frac{c-d'}{c}\right) * 0.002 < f_y \qquad (2\text{-}4a)$$

Compute the compression force C applied on the section:

$$C = C_c + C_s = f'_{cf} \left(\frac{c*b}{2}\right) + A'_s f'_s \qquad (2\text{-}5a)$$

compute the tension force T:

$$T = T_c + T_s = f_r (h-c)b + A_s f_s \qquad (2\text{-}6a)$$

Where :

C_c = compressive force of fibrous concrete

C_s = compressive force of steel bars

T_c = tension force of fibrous concrete

T_s = tension force of steel bars

$$f'_{cf} = f_c + 994 * \frac{V_f}{100} \frac{l_f}{d_f} \quad \text{(psi)} \qquad (2\text{-}7a)$$

$$f_r = 0.5 * 0.41 * \tau * \frac{V_f}{100} \frac{l_f}{d_f} \quad \text{(psi)} \qquad (2\text{-}8a)$$

Then compare T and C as components of the internal couple, they must be equal.

3 - If the difference between the compressive force (T) and the tension force (C) is close to zero (within acceptable limits) then the selected neutral axis depth is acceptable and the analysis can be completed by computing the ultimate moment corresponding to the flexural stresses, as described in the next step. Otherwise, the neutral axis depth should be adjusted and the procedure repeated from step 2 (increase c If $C_c + C_s < T_c + T_s$, and vice versa).

4 - Once the location of the neutral axis has been established, the bending moment capacity of section (M_n) can be obtained by taking the moment of all the forces applied on the section around the extreme compression fiber of the cross section see (Figure 2.24):

$$M_n = C_c \left(\frac{d-c}{3}\right) + C_s \, (d-d') - f_r \, (h-c) \, b \left(\frac{h-c}{2-d'}\right) \qquad (2\text{-}9a)$$

2.6.2 " ACI - BASED " METHOD

This method follows the conventional reinforced concrete flexural analysis procedure [20, 21], with some refinements which reflect the effects of steel fiber reinforcement. The maximum strain at fiber when the peak bending moment is assumed to be 0.003 as shown in Figure 2.25(c) for the cross section presented in Figure 2.25.

(a) and (b). The strain distribution is assumed to be linear. The simplified compressive stress distribution (which assumes a uniform compression stress) is shown in Figure 2.25(e). The steps followed in deciding the neutral axis depth and calculating the bending moment at peak flexural load, which are comparable with those in the previous method, are given below:

1 - Guess a value for the neutral axis depth, c.

2 - Compute the depth of the stress block: $a = \beta * c$, and compute ϵ_s and ϵ'_s using the strain distribution of Figure 2.25(c), and then find f_s and f'_s:

$$\epsilon_s = \frac{d-c}{c} * 0.003 \quad (2\text{-}1b)$$

$$\epsilon'_s = \frac{c-d}{c} * 0.003 \quad (2\text{-}2b)$$

$$f_s = E_s * \epsilon_s = E_s \frac{d-c}{c} * 0.003 \leq f_y \quad (2\text{-}3b)$$

$$f'_s = E_s * \epsilon'_s = E_s \frac{c-d'}{c} * 0.003 \leq f_y \quad (2\text{-}4b)$$

Compute the compressive force C:

$$C = C_c + C_s = 0.85\, f'_{cf}\, 0.85cb + A'_s\, f'_s \quad (2\text{-}5b)$$

compute the tension force T:

$$T = T_c + T_s = f_r\, (h-c)\, b + A_s\, f_s \quad (2\text{-}6b)$$

Where :

C_c = Compressive force of fibrous concrete

C_s = Compressive force of steel bars

T_c = Tension force of fibrous concrete

T_s = Tension force of steel bars

$$f'_{cf} = f_c + 994 * \frac{V_f}{100} \frac{l_f}{d_f} \qquad (2\text{-}7b)$$

$$f_r = 0.5 * 0.41 * \tau * \frac{V_f}{100} \frac{l_f}{d_f} \qquad (2\text{-}8b)$$

3 - If C and T balance each other (within the acceptable error) then go to the next step and find the flexural strength; otherwise adjust c and repeat from step 2.

4 - Once the location of the neutral axis has been established, the bending moment capacity of section can be obtained by taking the moment of all the forces applied on the section about the tension steel axis. (See Figure 2-25(c)):

$$M_n = C_c \left(\frac{d-a}{2}\right) + C_s \,(d-d') - f_r \,(h-c)\, b \left(\frac{h-c}{2-d'} \right) \qquad (2\text{-}9b)$$

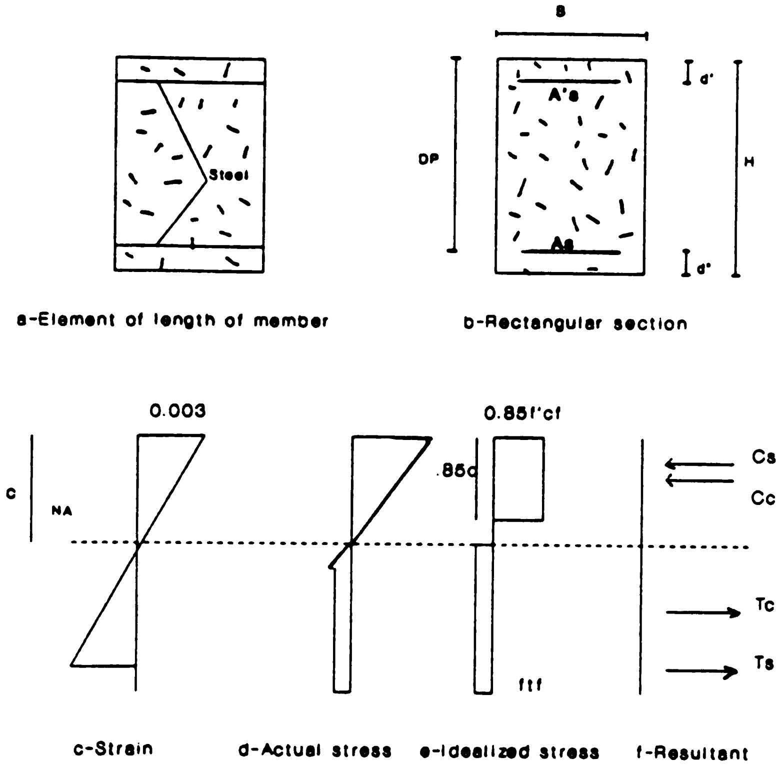


Figure 2.25 "ACI-Based Method" of Flexural Analysis

2.7 COMPARISON OF THEORETICAL PREDICTIONS WITH TEST RESULTS

The flexural test data presented in References [1, 3 and 20] were used to verify the "alternative" and "ACI -based" methods for predicting the flexural strength of reinforced concrete elements incorporating steel fibers. The cross-sectional properties of tested elements are given in Table 2.2 and Figure 2.26.

Table 2-2 also compares the theoretical flexural strengths as predicted by the refined computer analysis, and "alternative" and "ACI -based" methods with the flexural strength test results. While the refined flexural analysis shows the best comparison with test results (average error of only 2 %), both the "alternative" and "ACI-based" procedures also predict test results with a reasonable accuracy (average errors less than 10 %).

Considering the fact that the "ACI-based" method proposed for flexural analysis and design of reinforced concrete sections is similar to the conventional reinforced concrete flexural analysis procedure, this method is suggested for use in design of fibrous reinforced concrete flexural members.

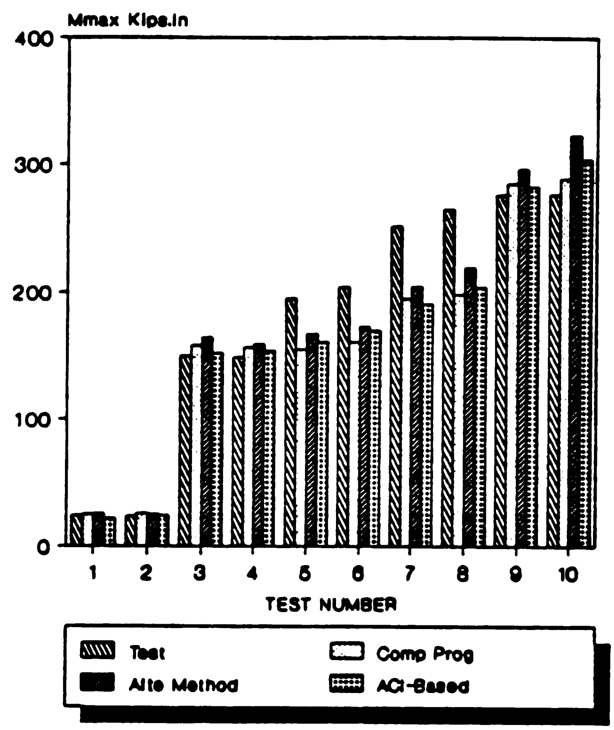


Figure 2.26 Comparison of Theoretical Predictions With Test Results.

Table 2.2 COMPARISON OF THEORETICAL WITH TEST RESULTS.

Reference		3	3	3	3	1	1	1	1	20	20
Section	B in	4	4	4	4	5.122	5.122	5.122	5.122	5.98	5.98
	H in	6	6	6	6	8	8	8	8	12.2	12.2
	d in	5.6	5.6	5.6	5.6	6.934	6.934	6.934	6.934	11.58	11.58
	dp in	.4	.4	.4	.4	1.131	1.131	1.131	1.131	.6	.6
Fiber Propreties	Vf %	1.27	.89	1.27	1.27	.5	1	.5	1	.5	1
	lf in	.95	1.2	.95	.95	1.95	1.95	1.95	1.95	1	1
	df in	.016	.016	.016	.016	.0195	.0195	.0195	.0195	.016	.016
	TF*	Strt	Hook	Strt	Hook	Crim	Crim	Crim	Crim	Strat	Strat
Rebar Steel Properties	As in2	.036	.036	.47	.47	.352	.352	.334	.334	.365	.365
	Asp in2	.036	.036	.036	.036	.24	.24	.24	.24	.24	.24
	fy ksi	104	104	66.7	66.7	67	67	90	90	67	67
Concrete Properties	ftf ksi	.37	.36	.4	.37	.31	.34	.31	.34	.3	.33
	fcf ksi	7.3	6.67	7.8	6.7	5.6	6.1	5.6	6.1	5.3	6.0
Flexural Strength Obtained	ACI (1)	26.2	25.1	164.4	159.5	165.7	173.4	204.8	219.6	296.5	323
	Alt (2)	21.6	24.5	152	154.5	161.3	170.1	190.7	204.3	282.8	303.9
	CP (3)	25.2	25.9	158.4	157	155.5	161	195	198.3	285.3	289.2
	Test(4)	24.4	24	150	149	195	204	252	265	276.3	276.7
<u>Theory</u> Test	(1)/(4)	1.07	1.04	1.09	1.07	.84	.84	.81	.82	1.07	1.16
	(2)/(4)	.88	1.02	1.01	.82	.82	.83	.76	.77	1.02	1.09
	(3)/(4)	1.03	1.08	1.05	.79	.79	.80	.77	.75	1.03	1.04

NOTE:

1 in = 25.4 mm
1 ksi = 6.895 MPa
1 kip.in = 0.113 KN.m
TF* = Fiber Type
CP = Computer Program
Alt =" Alternative Method"

2.8 MAXIMUM STEEL RATIO IN FIBROUS REINFORCED CONCRETE FLEXURAL MEMBER

An important contribution of fiber reinforcement to the flexural behavior of reinforced concrete elements is related to the improvement in ductility at failure. There are important potentials for taking advantage of this aspect of fiber reinforcement effects on flexural performance.

An important restriction in flexural design of conventional reinforced concrete beams is the maximum limit specified for the tension steel area to ensure yielding of steel prior to crushing of concrete for achieving a ductile failure. Fiber reinforced concrete has a desirable ductility in compression and it thus has relatively large strain capacities beyond the peak compressive stress. Hence, more tension steel can be used in reinforced concrete sections incorporating steel fibers without reducing the ductility below desirable levels.

The maximum limit on the tension steel ratio of conventional reinforced concrete sections without compressive steel is 0.75 times the balanced steel ratio. The balanced steel ratio (p_b) is the one for which yielding of steel takes place simultaneously with the crashing of concrete in compression:

$$p_b = \frac{0.85\beta f'_c}{f_y} \left(\frac{87000}{87000 + f_y} \right)$$

Where:

$$p = \frac{A_s}{b_w d}$$

β = ratio $\frac{a}{c}$ = ratio of the depth of rectangular stress block to the neutral axis depth.

$\beta = 0.85$ for $f'_c \leq 4000$ psi

$\beta = 0.85 - 0.05 (f'_c - 4000) / 1000 \geq 0.65$

for $f'_c > 4000$ psi

p_b = reinforcement ratio for the balanced strain condition

p = tension steel ratio

A_s = area of tension reinforcement

b_w = width of web

d = effective depth; distance from compression face to centroid of tension steel

f_y = yield strength of steel

f'_c = compressive strength of concrete, measured at 28 days after casting

The objective of this part of the study was to decide the maximum limit on tension steel ratio in fibrous sections, at which the flexural ductility corresponds to that of conventional reinforced concrete sections with $p = 0.75\ p_b$.

For this purpose a curvature ductility was defined as the maximum curvature at failure (Where the refined flexural analysis by computer shows a major drop in flexural resistance in the post-peak region) divided by the curvature corresponding to flexural yielding Figure 2.27 presents the flexural ductility of sections similar to the one shown in Figure 2.8, but with different tension steel ratios and fiber volume fractions. This figure which has been generated using the results of refined flexural analysis clearly shows the improvement in flexural ductility resulting from steel fiber reinforcement. The data presented in Figure 2.27 indicate that a ductility of about 1.2 is achieved at $p / p_b = 0.75$ in beams with a minimal fiber reinforcement volume fraction of 0.35 %. This level of ductility can be achieved in the presence of 1 % ($V_f l_f / d_f = 70$) fiber volume fraction at $p / p_b = 0.9$, and with the presence of 1.5 % ($V_f l_f / d_f = 105$) fiber volume fraction at $p / p_b = 1.05$.

Hence, in the presence of steel fibers the limits on maximum tension steel for ensuring flexural ductility can be relaxed, and smaller cross sections can be used in flexural elements to generate the required load capacity. The corresponding weight reductions and increase in useful height of the buildings and other structures can be an important economic advantages encouraging structural applications of steel fiber reinforced concrete.

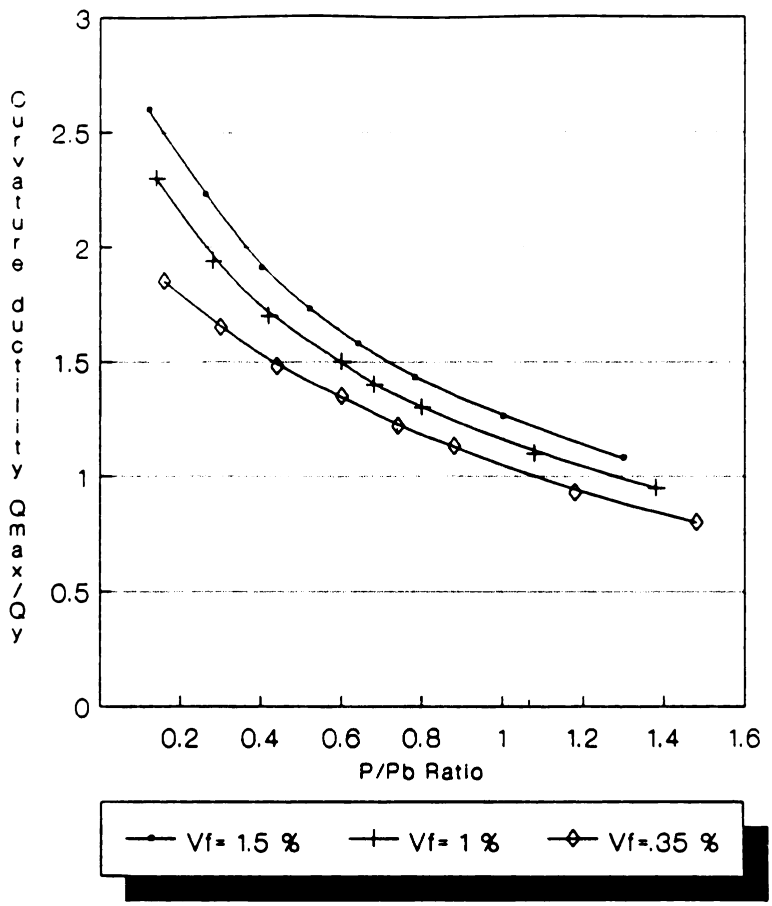


Figure 2.27 Relation Between Curvature Ductility (Q_u / Q_Y) And Tension Steel p / p_b Ratio.

2.9 EXAMPLE: FLEXURAL ANALYSIS OF A FIBROUS REINFORCED CONCRETE SECTION USING THE "ACI - BASED" METHOD

For the section given in Figure 1.32 find the flexural strength.

Given:

$B = 10$ in
$d = 18$ in
$A_s = 1.57$ in$_2$
$f_y = 60$ ksi
$d_f = 0.013$ in
$f_{tf} = .32$ ksi
$f_{cf} = 5.1$ ksi

$H = 20$ in
$d_p = 2$ in
$A_{sp} = 0.24$ in$_2$
$V_f = 1.5$ %
$l_f = 1$ in
Straight Fiber

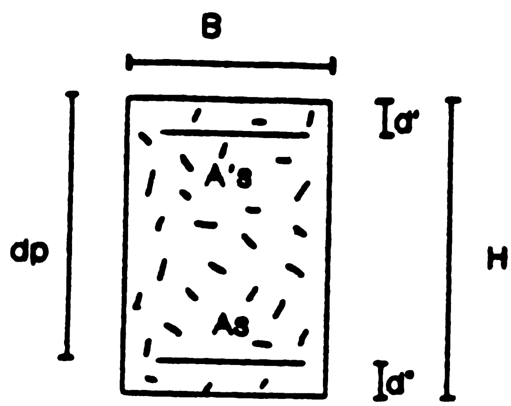


Figure 2.28 Example Problem

Solution:

Check tension steel ratio

$p / p_b = 0.24 < 1.05$ OK

Find flexural strength:

Assume f_s and $f'_s \leqslant f_y$.

Assume c (a trial value for the neutral axis depth);

guess a value of $c = 4$ in .

Using equations presented below (2-1b; 2-2b; 2-3b; 2-4b;

2-5b and 2-6b) compute respectively ϵ_s; ϵ'_s; f_s; f'_s; C and T, respectively.

$$\epsilon_s = \frac{d-c}{c} * 0.003 \qquad (2\text{-}1b)$$

$$\epsilon'_s = \frac{c-d}{c} * 0.003 \qquad (2\text{-}2b)$$

$$fs = E_s * \epsilon_s = E_s * \frac{d-c}{c} * 0.003 < f_y \qquad (2\text{-}3b)$$

$$f's = Es * \epsilon's = Es * \frac{c-d'}{c} * 0.003 < f_y \qquad (2\text{-}4b)$$

Compressive force:

$$C = C_c + C_s = 0.85\ f'_{cf}\ 0.85*c*b + A'_s * f'_s \qquad (2\text{-}5b)$$

Tension force:

$$T = T_c + T_s = f_r\ (h-c)*b + A_s*f_s \qquad (2\text{-}6b)$$

Compare C and T as explained earlier.

C = 217 kips and T = 15.6 kips

Since C exceeds T, c needs to be reduced

Assume a smaller value of c = 2.2 in

Repeat steps 1 to 5.

C = 83 kips and T = 107 kips

Since T exceeds C, c should be increased, Assume a larger value of c = 2.4, and repeat steps 1 to 5.

C = 104 kips and T = 107 kips

The two values are close enough (error less than 5 %). The analysis can thus be continued. Compute the ultimate bending moment by summing the moments of all forces acting on the cross section about the tension steel centroid:

$$M = C_c \left(\frac{d-a}{2}\right) + C_s \,(d-d') - f_r \,(h-c)\, b \left(\frac{h-c}{2-d'}\right) \qquad (2\text{-}9b)$$

The calculated value of flexural strength is:

M = 1653 kips . in

2.10 SUMMARY AND CONCLUSIONS

A refined flexural analysis procedure was developed for reinforced concrete sections incorporating steel fibers. The analysis procedure was verified using test results reported in the literature, and it was used for an analytical parametric study on the effects of different design variables on flexural strength and ductility of fibrous reinforced concrete sections. Based on the results of this parametric study simplified flexural analysis and design guidelines were developed for reinforced concrete elements incorporating steel fibers.

From the analytical studies conducted on the flexural behavior of fibrous reinforced concrete sections it was concluded that:

1 - Steel fiber reinforcement results in improvements in the flexural strength and especially the ductility of reinforced concrete sections.

2 - High strength steel and concrete can be efficiently used in flexural elements incorporating steel fibers with no significant adverse effects on the ductile behavior.

3 - Steel fiber reinforcement reduces the need for

compression steel to improve the ductile behavior in flexural elements. High tension steel ratios can be also used in flexural reinforced concrete elements incorporating steel fibers, resulting in higher flexural strengths, with the ductility being maintained at desirable levels.

The developed flexural analysis / design procedure is simple and conforms to the design guidelines of the American Concrete Institute (ACI 318 - 83). It accounts for the improvements in flexural strength resulting from steel fiber reinforcement (using a flexural stress distribution at failure which considers the improvements in tensile and compressive properties of the fibrous concrete).

The developed flexural design guideline also accounts for the positive effects of fiber reinforcement on the ductility of flexural behavior by increasing the maximum limit on tension steel area in the presence of fibers. Results of the proposed flexural analysis / design procedure compare reasonably well with test results.

CHAPTER 3

SHEAR ANALYSIS AND DESIGN OF REINFORCED CONCRETE ELEMENTS INCORPORATING STEEL FIBER

3.1 INTRODUCTION

Conventional reinforced concrete elements generally fail in a brittle manner under the action of shear forces. In order to safely resist shear, reinforced concrete elements should be provided with sufficient amounts of transverse shear reinforcement, a requirement that occasionally leads to undesirable congestion of steel and time consuming construction efforts. Steel fiber reinforcement is an alternative method of improving the strength and ductility of reinforced concrete elements subjected to shear forces. There are potentials for important gains in construction speed and material costs through partial substitution of conventional shear reinforcing bars with steel fibers.

This chapter summarizes the results of a study on the effects of steel fiber on the shear behavior of reinforced concrete elements. A new shear design equation is developed and it is used in an analytical parametric study on the shear strength of fibrous reinforced concrete elements as influenced by different design variables.

3.2 STEEL FIBER EFFECTS ON SHEAR STRENGTH OF REINFORCED CONCRETE ELEMENTS

A review of the published data is indicative of the effectiveness of steel fibers as shear reinforcement for improving the strength and ductility of reinforced concrete elements subjected to shear forces.

Steel fibers improve the shear behavior of elements mainly by increasing the cracking and tensile strength of concrete [21 and 22]. The fact that steel fibers provide tensile resistance in all directions is specially important in restraining shear cracks which develop with different orientations. Following the formation of cracks, steel fibers bridge the cracks and provide concrete with post-cracking tensile resistance.

Reference [23] suggests that conventional shear reinforcement (stirrups) can be substituted with steel fibers. The use of steel fibers as shear reinforcement in some reinforced concrete beams has been observed to double the shear strength and substantially improve the ductility under shear. Reference [27] suggests that shear strength of reinforced concrete elements can be increased almost linearly with the increase in fiber volume fraction.

Tests show that stirrups together with fiber reinforcement at volume fractions as low as 0.44 percent can be used effectively to provide sufficient shear capacity for beams to fail in a ductile flexural mode [24, 23]. Reference [25] reports that in reinforced concrete beams without stirrups a 45 percent increase in shear capacity can be achieved through reinforcement with 1.6 volume percentage of straight fibers (but the fiber reinforced concrete beam still failed in shear). The same Reference also reports that reinforced concrete beams without stirrups but with 1.1 volume percentage of steel fibers with deformed ends had an increase in shear capacity of 45 to 67 percent when compared with beams without fibers; and the beams failed in flexure.

The effectiveness of steel fibers in improving the shear performance of concrete elements seems to be dependant on the shear span-to -depth ratio (a / d) of the element. It has been shown in Reference [28] that the shear capacity of reinforced concrete beams without stirrups can be increased by 130 percent through reinforcement with 1.0 volume percentage of fibers in elements with an a / d ratio of 1.5. At an a / d ratio of 3.0, there is about 108 percent increase in shear strength of otherwise comparable reinforcement concrete beams resulting from reinforcement with 1.0 volume percentage of steel fibers [27].

The studies reviewed so far strongly indicate a fraction of the web reinforcement can be replaced by steel fibers, thus eliminating the need for large amounts of stirrups.

In spite of all the available evidence reported regarding the effectiveness of steel fibers as shear reinforcement, there are insufficient theories and equations available for predicting the shear capacity of reinforced concrete elements incorporating steel fibers.

3.3 SHEAR STRENGTH EQUATIONS

External shear forces are resisted in reinforced concrete elements by several internal mechanisms: shear strength of the uncracked concrete in compression, aggregate interlock, dowel action of flexural steel and transverse reinforcement. Steel fibers enhance the performance of these internal mechanisms. It is however, difficult to quantify the contribution of fibers to each mechanism based on the available test results [27].

A relatively simple semi-empirical shear design equation has been developed for fibrous reinforced concrete elements by simply modifying the design equations commonly used for conventionally reinforced concrete beams [31 and 32]. This simple approach considers that the factors influencing shear strength and formation of inclined cracks are so numerous and complex that a definitive conclusion regarding the correct mechanism of inclined cracking and shear resistance in the presence of fibers is difficult to establish [29]. The modification of ACI shear design equation in this approach basically involves substituting $(f'_c)^{0.5}$ in the ACI equation for the concrete contribution to shear strength with $.133f_r$ (f_r= Modulus of rupture of steel fibers reinforced concrete)

$V_c = 2(f'_c)^{0.5} b_w\, d$ for conventional reinforced concrete (3-1)

$V_c = .266 f_r\, b_w\, d$ for fibrous reinforced concrete (3-2)

Or in the alternative more complex but also more accurate equation:

$$V_c = (1.9 \ f'_c + 2500 \ p_w \frac{V_u d}{M_u}) \ b_w d \quad (3\text{-}3)$$

(for conventional reinforced concrete)

$$V_c = (0.253 \ f_r + 2500 \ p_w \frac{V_u d}{M_u}) \ b_w d \quad (3\text{-}4)$$

(for fibrous reinforced concrete)

Where :

- V_c = shear strength provided by concrete
- d = effective depth of the beam
- b_w = web width of a rectangular beam
- $p_w = A_s / b_w d$
- V_u = factored shear force at section
- M_u = factored moment at section
- f_r = modulus of rupture $(7.5 \ (f'_c)^{0.5}$ psi for plain concrete)
- f'_c = compressive strength of concrete

3.3.1 THE PROPOSED METHOD

The following method, which is a refined version of the one introduced in Reference [22], was developed in this study based on a simplified physical model of shear failure shown in Figure 3.1, noting that the effects of transverse reinforcement will be considered later.

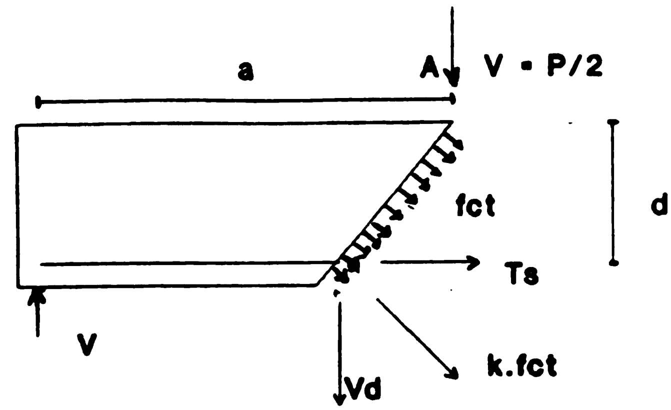


Figure 3.1 Simplified Model of Shear Failure in Fibrous Reinforced Concrete.

A shear strength equation can be derived based on the physical model of Figure 3.1, using the following assumptions:

- The shear diagonal crack extends through the beam depth.
- The main diagonal crack governing the shear failure of the element initiates at the load point A.
- Fibers provide the diagonal crack with post-cracking tensile resistance.
- The direction of tensile stresses in fibrous concrete is assumed to be perpendicular to the crack plane.

Using the above assumptions, an expression for ultimate shear strength can be derived as describe below:
Take the moment of applied forces in Figure 3.1 about point A

$$M_u = V_u\, a = f_{ys}\, A_s\, d + k_1\, f_{ct}\, b\, d^2 + V_d\, d \qquad (3\text{-}5)$$

Where :

f_{ys} = steel yield strength

k_1 = constant depending on the fraction of fibrous concrete tensile strength acting across the fracture plane.

$$V_d = p\, b^2 \;\; (\nu_1\, f_{cu} + \nu_2) \qquad (3\text{-}6)$$

which represents the contribution of dowel bars to shear strength in fiber reinforced concrete [30]

Where :

ν_1 and ν_2 = constant coefficients; and

f_{cu} = the ultimate flexural strength of steel fiber reinforced concrete

The ultimate shear force can thus be derived using equation (2-1):

$$V_u = f_{ys} A_s \frac{d}{a} + k_1 f_{ct} \frac{d}{a} bd + p(\nu_1 f_{cu} + \nu_2) b^2 \frac{d}{a} \qquad (3\text{-}7)$$

Regression analysis was carried out using the data reported in Reference [22] to find the unknown coefficients (ν_1 and ν_2) in the equation. using these empirical values of the coefficients, the following expression was obtained for predicting the shear strength of steel fiber reinforced concrete, accounting for the effects of transverse steel.

$$V_u = V_c + V_s \qquad (3\text{-}8)$$

Where:

$$V_c = (0.77 + (0.9 - 9.6p)p \frac{d}{a} \frac{f_y}{f_{ct}}) f_r bd \qquad (3\text{-}9)$$

$$V_s = \frac{A_v}{s} f_{ys} d \qquad (3\text{-}10)$$

Where:

A_v = total leg area of stirrups;

s = stirrups spacing;

f_r = modulus of rupture of steel fiber reinforced concrete; and

$f_{ct} = 67.6 \frac{V_f}{100} \frac{l_f}{d_f}$

3.4 COMPARISON WITH TEST RESULTS

Table 3.1 and Figures 3.2 through 3.4 present comparisons between test results and the predictions of the modified ACI (equation 3-2) and the proposed (equation 3-8) expressions. The test results were selected from a number of references. Obviously, the proposed equation compares favorably with test results. While the average ratio of ACI-based predictions to test results is 1.85, the average ratio of the proposed equation predictions to test results is 1.09.

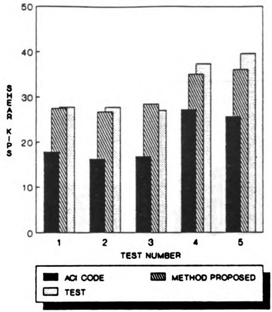


Figure 3.2 Experimental vs Analytical Shear Strength Values (Reference 2).

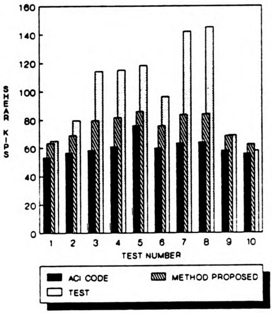


Figure 3.3 Experimental vs Analytical Shear Strength Values (Reference 27).

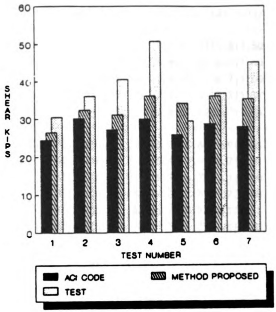


Figure 3.4 Experimental vs Analytical Shear Strength Values (Reference 43).

Table 3.1 EXPERIMENTAL VS. THEORETICAL SHEAR STRENGTHS OF FIBROUS CONCRETE AND FIBROUS REINFORCED CONCRETE ELEMENTS (TEST VS THEORY).

	Tes No	Ref	SECTION PROPERTIES B in	H in	FIBER DETAILS D in	Vf %	lf	df	HOOPS DETAILS Ast	S**	FY	F'C Ksi	TEST Kip (1)	ACI Kip (2)	TEST(A) ACI (1)/(2)	PROPOSED METHOD (3)	TEST(B) METOD (1)/(3)	AVERAGE (A)	(B)
F	1		8.25	12	11.60	0.2	1.18	.02	.51	14.50	63	8.5	27.8	16.8	1.60	26.54	1.01		
I	2		8.25	12	11.60	0.3	1.18	.02	.51	14.50	63	6.8	27.8	16.2	1.60	26.80	1.03		
B	3	2	8.25	12	11.60	0.5	1.18	.02	.51	14.50	63	7.3	27.1	16.7	1.60	28.54	0.94	1.52	1.02
R	4		8.25	12	11.60	0.2	1.18	.02	.51	14.50	63	7.3	37.4	27.3	1.37	35.09	1.06		
O	5		8.25	12	11.60	0.3	1.18	.02	.51	14.50	63	6.8	39.6	25.7	1.54	36.10	1.09		
U																			
S	1		7	10	8.40	0	2	.02	.02	6.40	82	6.8	14.6	11.9	1.21	14.30	1.01		
	2		7	10	8.40	0.4	2	.02	.02	6.40	82	6.4	17.9	12.7	1.21	15.50	1.13		
R	3		7	10	8.40	0.8	2	.02	.02	6.40	82	6.7	25.7	13.1	1.90	17.89	1.44		
E	4		7	10	8.40	1.2	2	.02	.02	6.40	82	7.2	25.8	13.7	1.80	18.30	1.35		
I	5	27	7	10	8.40	0.8	2	.02	.02	6.40	82	6.9	26.5	17.1	1.55	19.32	1.37	1.6	1.2
N	6		7	10	8.40	0.8	2	.02	.02	6.40	82	7.5	21.7	13.5	1.60	17.32	1.20		
F	7		7	10	8.40	0.8	2	.02	.02	6.40	82	6.5	31.9	14.3	2.20	18.80	1.70		
O	8		7	10	8.40	0.8	2	.02	.02	6.40	82	6.8	32.6	14.4	2.10	18.90	1.60		
R	9		7	10	8.40	0.8	2	.02	.02	6.40	82	6.8	15.5	13.1	1.17	15.40	1.01		
C	10		7	10	8.40	0.8	2	.02	.02	6.40	82	7.7	13.0	12.6	1.03	14.06	.89		
E																			
D	1		4	8	7.44	0.5	1.18	.02	.04	4.80	51	4.1	30.5	24.5	1.23	26.54	1.13		
	2		4	8	7.44	0.5	2	.02	.04	4.80	51	4.7	36.1	30.2	1.19	32.40	1.18		
C	3		4	8	7.44	1	1.18	.02	.04	4.80	51	4.2	40.5	27.2	1.50	31.10	1.30		
O	4	43	4	8	7.44	1	2	.02	.04	4.80	51	4.7	50.7	29.9	1.70	36.03	1.40	1.42	1.14
N	5		4	8	7.44	1	1.18	.02	.04	4.80	51	4.7	29.4	25.7	1.14	34.05	.86		
C	6		4	8	7.44	1	2	.02	.04	4.80	51	4.7	36.8	28.6	1.29	36.03	1.02		
R	7		4	8	7.44	1.5	1.18	.02	.04	4.80	51	4.8	45.0	27.8	1.62	35.20	1.27		
F	1		4	8	6.4	0.75	1.2	.02				7.6	19.5	4.9	3.90	16.90	1.16		
I	2		4	8	6.4	0.75	1.2	.02				7.6	12.8	4.8	2.50	13.70	.94		
B	3		4	8	6.4	0.75	1.2	.02				7.6	11.5	4.8	2.40	14.12	.96		
R	4		4	8	6.4	0.75	1.2	.02				7.6	9.45	4.8	1.80	10.70	.88		
O	5		4	8	6.4	1.50	1.2	.02				7.2	23.4	5.79	4.00	23.55	.99		
U	6		4	8	6.4	1.50	1.2	.02				7.2	20.0	5.93	3.30	19.30	1.03		
S	7		4	8	6.4	1.50	1.2	.02				7.2	19.8	5.93	3.20	18.00	1.09		
	8	22	4	8	6.4	1.50	1.2	.02				7.2	21.6	5.79	3.60	19.80	1.09	2.80	1.02
C	9		4	8	6.4	1.50	1.2	.02				7.2	15.2	5.43	2.70	16.50	.92		
O	10		4	8	6.4	0.75	1.2	.02				7.0	8.59	4.71	1.80	7.77	1.10		
N	11		4	8	6.4	0.75	1.2	.02				7.0	11.3	4.75	2.40	11.20	1.00		
C	12		4	8	6.4	0.75	1.2	.02				7.0	8.27	4.71	3.00	7.98	1.03		
R	13		4	8	6.4	0.75	1.2	.02				7.0	14.3	4.72	3.20	12.12	1.17		
E	14		4	8	6.4	0.75	1.2	.02				7.0	19.2	5.79	3.30	16.79	1.14		
T	15		4	8	6.4	0.75	1.2	.02				7.0	11.8	5.79	2.01	13.40	.88		

Note:

1 in = 25.4 mm

1 kip = 4448.2 N

1 ksi = 6.895 MPa

3.5 PARAMETRIC INVESTIGATION OF STEEL FIBER EFFECTS ON THE SHEAR STRENGTH OF REINFORCED CONCRETE ELEMENTS

The proposed shear strength equation was used to perform a parametric study on the effects of fiber reinforcement on shear strength of reinforced concrete elements with different material and geometric properties. The basic "standard" section used in the parametric study has the same cross-sectional properties as introduced in Figure 1.8, with No 3 stirrups placed at a spacing of 4 inches (102 mm).

Figure 3.5 presents the geometric and material properties of the "standard" section. Further discussions on the trends observed in the fiber reinforcement effects on shear strength of reinforced concrete elements with different characteristics are presented below.

Figures 3.5 though 3.11 present the results of the parametric study. Fiber reinforcement is observed in these figures to increase the shear strength more than it increases the flexural strength.

$A_s = 2.37\ in^2$

$A_{sp} = .24\ in^2$

$f_y = 60$ ksi

$f'_c = 4$ ksi

Type of Fiber = Straight

$l_f = 1$ in

$d_f = .013$ in

$V_f = 1.5$ %

$A_v = 0.04\ in^2$

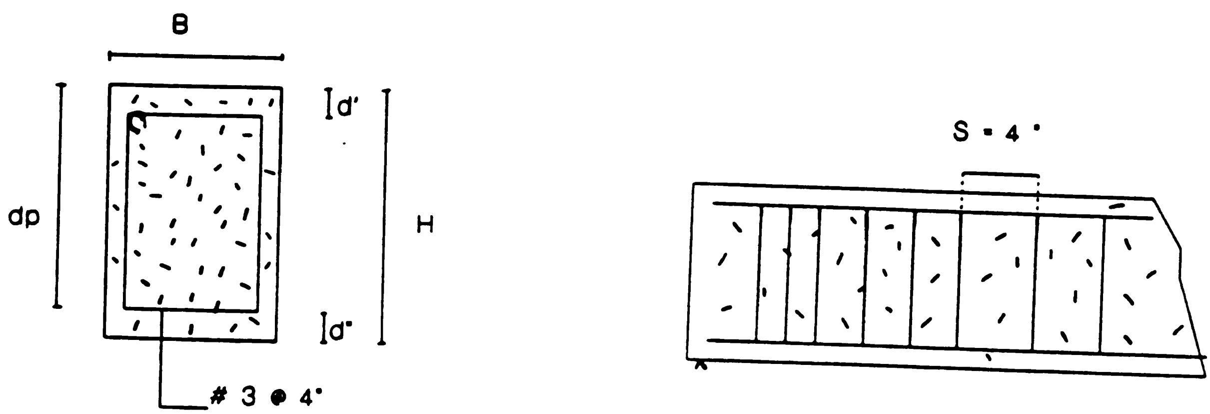


Figure 3.5 The Basic ("Standard") Reinforced Concrete Element.

3.5.1 EFFECTS OF THE LONGITUDINAL AND TRANSVERSE REINFORCEMENT

Variations in the longitudinal and transverse steel ratios and yield strengths, as shown in Figures 3.6, 3.7 and 3.8, respectively, have small effects on the improvements in the absolute values of shear strength resulting from steel fiber reinforcement.

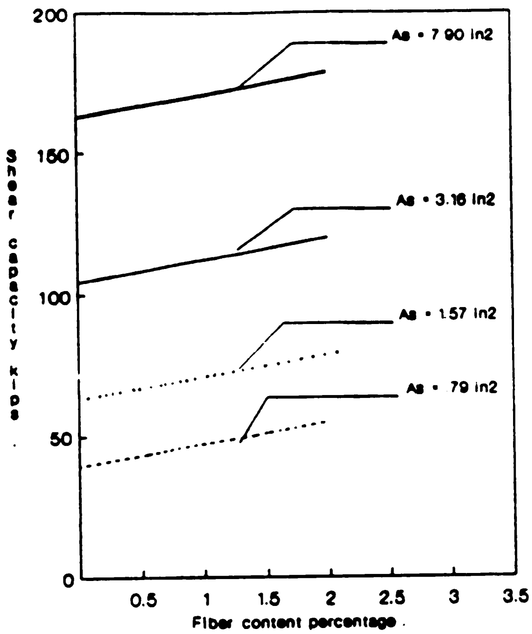


Figure 3.6 Effects of Fiber Volume Fraction on Shear Strength for Beams with Different Transverse Steel Areas.

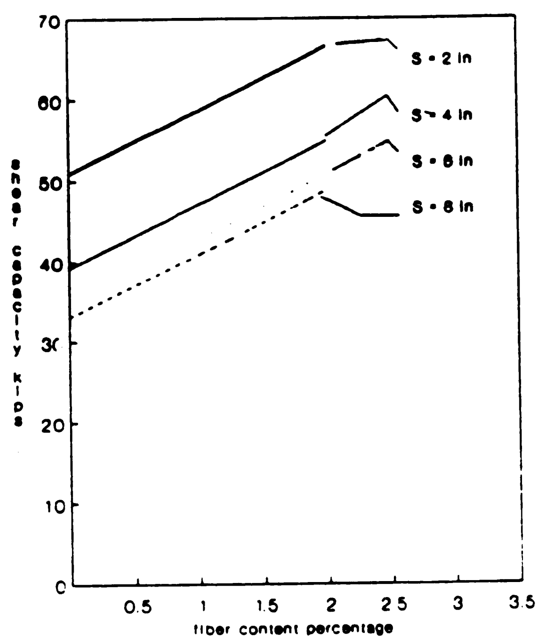


Figure 3.7 Effects of Fiber Volume Fraction on Shear Strength for Beams with Different Stirrup Spacings.

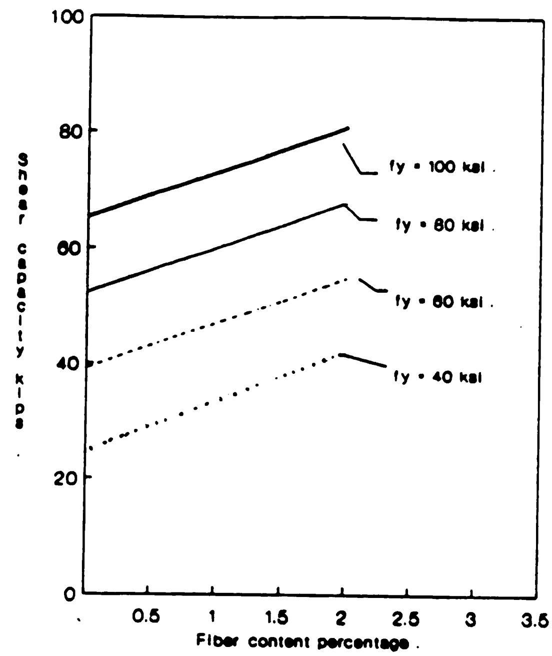


Figure 3.8 Effects of Fiber Volume Fraction on Shear Strength for Beams with Different Transverse Steel Yield Strengths.

3.5.2 EFFECT OF $\frac{a}{d}$ RATIO

Among the parameters studied the shear span-to-depth (a / d) ratio (which describes the ratio of bending moment to shear force at a section) seems to be the one with relatively significant effects on the shear strength of plain and fibrous elements. Figure 3.9 shows the effects of fiber reinforcement on shear strength of elements with different a / d ratios. The trends in fiber reinforcement effects on shear strength (as far as the absolute values of improvements in strength are concerned) seem to be largely independent on the shear span-to-depth (a / d) ratio.

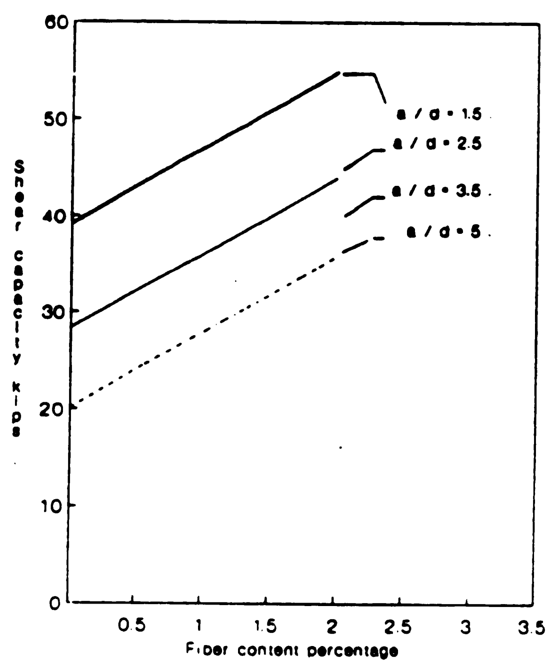


Figure 3.9 Effects of Fiber Volume Fraction on Shear Strength for Beams with Different Span-to-Depth (a / d) Ratios.

3.5.3 EFFECTS OF FIBER REINFORCEMENT PROPERTIES

Figures 3.10, 3.11 and 3.12 show the effects of volume fraction of fibers with different lengths, diameters and mechanical deformations on the shear strength of the "standard" reinforced concrete element. The increase in fiber volume fraction is observed to improve the shear strength rather significantly.

Longer fibers with smaller cross section dimensions, and those with mechanical deformations, are observed to give better theoretical results (if the practical problems with workability and fiber dispersability can be overcome). Typical improvements of the order of 50 % in shear strength of reinforced concrete elements are observed in Figure 3.10 - 3.12 to result from the addition of 2 % volume fraction of steel fibers.

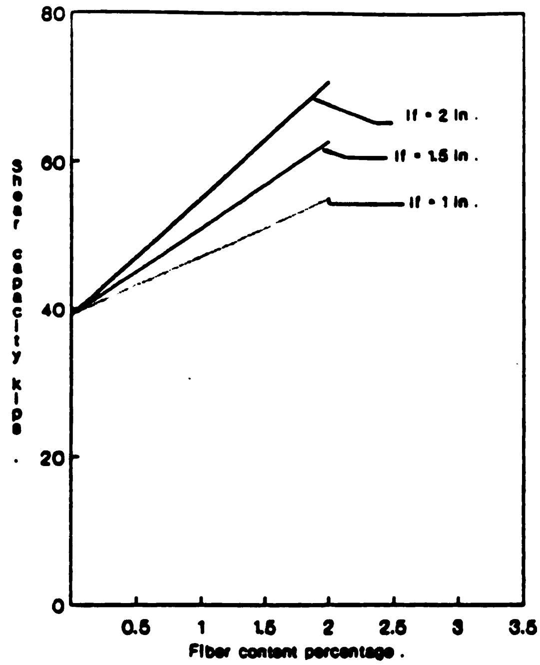


Figure 3.10 Effects of Fiber Volume Fraction on Shear Strength for Beams with Different Fiber Lengths.

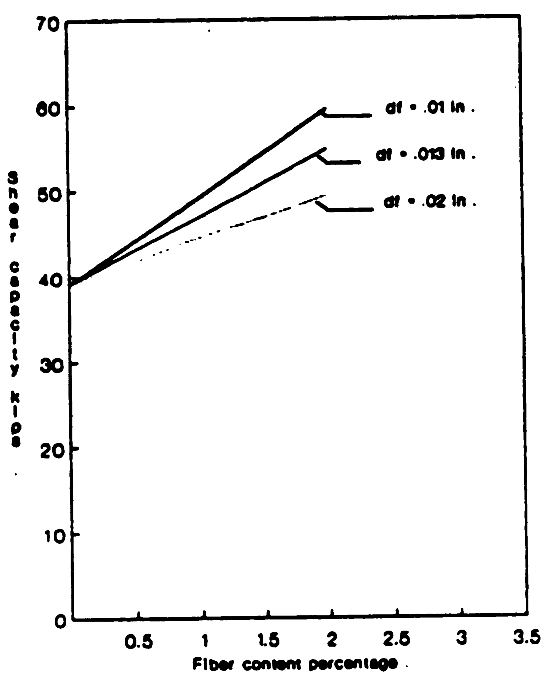


Figure 3.11 Effects of Fiber Volume Fraction on Shear Strength for Beams with Different Fiber Diameters

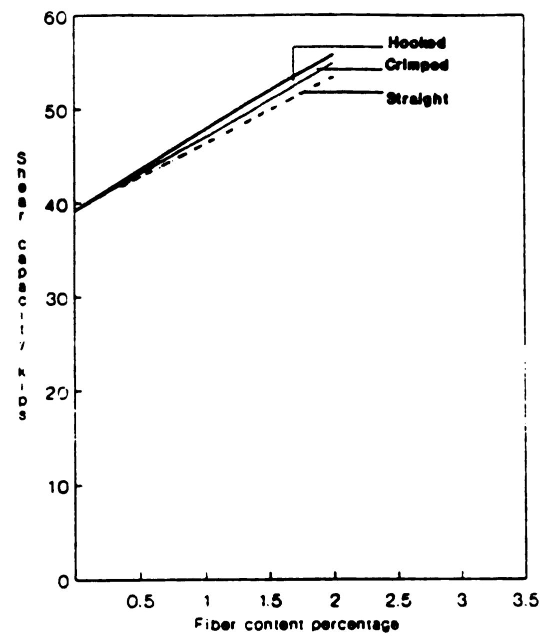


Figure 3.12 Effects of Fiber Volume Fraction on Shear Strength for Beams with Different Fiber Types.

3.6 SUMMARY AND CONCLUSION

Steel fiber reinforcement was found to be effective in improving the strength and ductility of reinforced concrete elements subjected to shear forces. Best results seem to be achieved when a combination of steel fibers and transverse reinforcement is used to provide shear resistance.

A shear strength equation was developed in this study for fibrous reinforced concrete elements based on the physics of shear failure in the presence of steel fibers. The proposed equation was shown to predict test results far better than the previous equations (which are simply modified versions of the conventional reinforced concrete equations).

The proposed shear strength equation was used in an analytical parametric study on the fiber effects of reinforcement on shear strength of reinforced concrete elements. The results indicated that improvements of the order of 50 % in shear strength can be expected in typical reinforced concrete elements as a result of reinforcement with 2 % volume fraction of steel fibers. Longer fibers with smaller cross-sectional dimensions and especially those with mechanical deformations were theoretically more effective in increasing the shear strength of reinforced concrete elements (ifthe practical problems with fresh mix workability and fiber dispersability can be overcome). The conventional reinforcement properties on The shear span-to-depth ratio of reinforced concrete elements did not significantly influence the absolute value of improvement in shear resistance resulting from fiber reinforcement.

CHAPTER 4

DESIGN OF FIBROUS REINFORCED CONCRETE ELEMENTS IN TORSION

4.1 INTRODUCTION

The effectiveness of steel fibers in enhancing the torsional strength of fibrous reinforced concrete elements is discussed in this chapter, and a practical method for predicting the torsional strength of fibrous reinforced concrete sections is introduced. The predictions of this method are compared with a variety of experimental results, and also with the predictions of other analytical methods. The developed analytical method is also used for a numerical parametric study on the torsional strength of fibrous reinforced concrete elements. The principle variables in this parametric study are the fiber reinforcement properties and the longitudinal and transverse reinforcement ratios.

4.2 BACKGROUND

From the experimental data reported on the torsional behavior of fibrous reinforced concrete elements it may be concluded that elements with both steel fibers and conventional torsional reinforcement perform better in torsion than elements containing only the conventional reinforcement [32, 33, 34]. The presence of fibers has been observed to increase the ductility, rotational capacity, stiffnesses, and damage tolerance of concrete elements under the action of torsional forces [31]. The type and aspect ratio of steel fibers are found to be important factors deciding their effectiveness in torsional elements [31]. The presence of both steel fibers and conventional transverse reinforcement has also been reported to be much more effective than steel fibers or transverse reinforcement alone [20, 31, 32].

It has been proposed in Reference [35] that the torsional strength of fibrous reinforced concrete elements can be adequately predicted using modified versions of the American Concrete Institute code (ACI 318-83) equations, which have been originally developed for conventional reinforced concrete elements.

The contribution of concrete to the ultimate torque in elements without fiber reinforcement is given by [38]:

$$T = 6\ (x^2+10)\ y\ (f'^{c})^{0.5} \tag{4-1}$$

Where:

x and y are the smaller and larger dimensions of the rectangular section respectively.

References [39, 40] suggest that the term $(f'c)^{0.5}$ in Equation (4-1) can be expressed in terms of the modulus of rupture (f_r) or the splitting tensile strength (f_{ct}) of concrete:

$$T = 1.56\ (\ x^2+10\ )\ y\ (f_r{}^2)^{0.5} \tag{4-2}$$

or

$$T = 1.6\ (\ x^2+10\ )\ y\ (f_{ct}{}^2)^{0.5} \tag{4-3}$$

These equations, according to References [39, 40] are directly applicable to steel fiber reinforced concrete as far as the values of f_r or f_{ct} for the fibrous concrete are available.

Reference [38] has suggested a modified version of the above equations for predicting the ultimate torsional strength of fibrous concrete:

$$T = x^2y/3 \ (0.85 \ f_r) \tag{4-4}$$

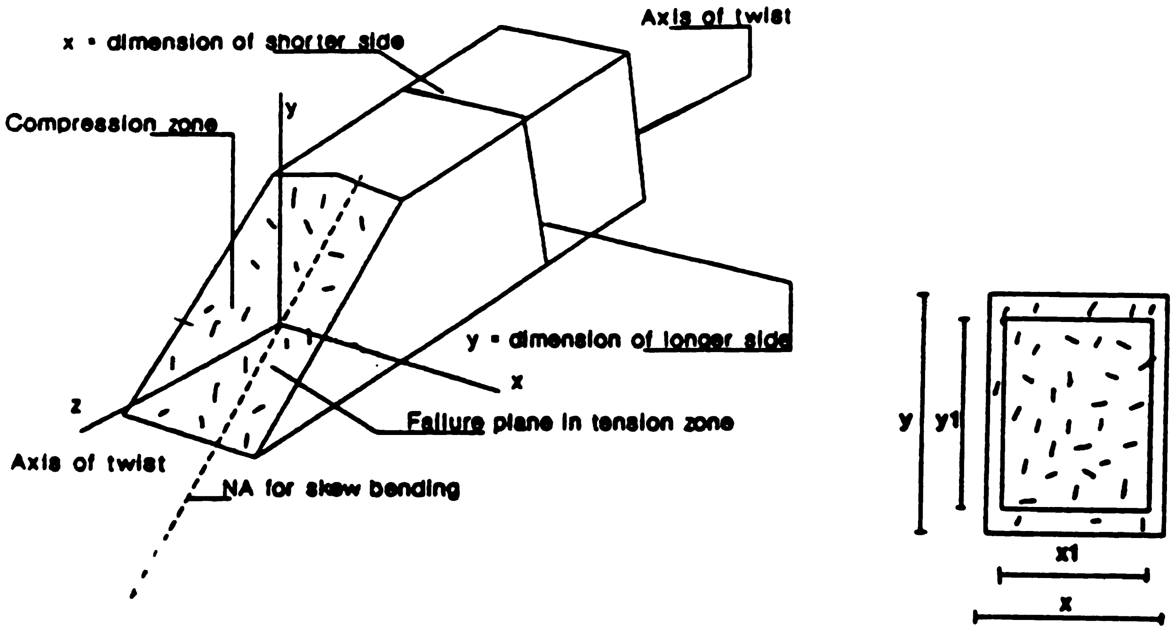
An alternative equation for The ultimate torque of fibrous concrete is given in Reference [34, 42]:

$$T = x^2y/3 \ (0.71 \ f_r) \tag{4-5}$$

A comprehensive examination of the accuracy of these alternative ultimate torsional strength equations is presented below following a brief discussion of the approach adopted in this study for torsional analysis and design of fibrous reinforced concrete elements.

4.2.1 PROPOSED APPROACH

Resistance against the action of torsional forces in fibrous reinforced concrete elements (T_u) is provided by both the fiber reinforced concrete (T_c) and the conventional reinforcement (T_s), as shown in Figure 4.1



a) Skew Bending of Rectangular Section b) Cross-Section

Figure 4.1 Mechanism of Resistance Against Torsion In Fibrous Reinforced Concrete Elements.

Based on a comprehensive verification of the torsional strength equations presented above using test results on torsional behavior of fibrous reinforced concrete elements reported in References [20, 31, 32, 33, 34], Equation (4-5) was selected as a simple and sufficiently accurate representation of fibrous reinforced concrete contributions to the torsional strength of reinforced concrete elements.

The contribution of conventional reinforcement to torsional strength of fibrous reinforced concrete elements is suggested to be similar to this contribution in non-fibrous concrete elements. The torsional strength of fibrous reinforced concrete elements (T_u) can therefore be calculated as follows:

$$T_u = T_{ct} + T_{st} \qquad (4-6)$$

Where:

$T_{ct} = \frac{x^2 y}{3} (0.71\ f_r)$

$T_{st} = \frac{x_1 y_1 A_t f_y}{s}$

T_u = total torsional strength

T_{ct} = torsional strength provided by concrete

T_{st} = torsional strength provided by hoop steel

= $(0.66 + 0.33\ y_1/x_1\ ) \leq 1.50$

x = smaller cross sectional dimension

y = larger cross sectional dimension

x_1 = smaller dimension of the confined core of the section subject to torsion

y_1 = smaller dimension of the confined core of the section subject to torsion

f_y = yield strength of steel

A_t = cross sectional area of transverse reinforcement resisting torsion

s = spacing of transverse reinforcement

f_r = modulus of ruptur of fiber reinforced concrete

= $490\ V_f l_f/d_f + 0.97 f_{rm}(1-V_f)$

Where:

V_f = fiber volume fraction

l_f = fiber length

d_f = fiber diameter

The minimum transverse reinforcement recommended is:

$$2A_t \geq \frac{50 b_w s}{f_y}$$

The minimum longitudinal reinforcement recommended is:

$$A_l \geq [400 \frac{xs}{f_y} - 2A_t](\ x_1 + y_1\ ) / s$$

Where:

b_w = width of web

A_l = area of longitudinal reinforcement to resist torsion

4.3 VERIFICATION OF TORSIONAL STRENGTH EQUATIONS

Torsional strength test results reported in References [20, 31, 32, 33 and 34] were used to verify the torsional strength expressions previously presented in Equations (4-1), (4-2), (4-3), (4-4), (4-5). The proposed equation (4-6) is also checked against test results on concrete specimens reinforced with both fibers and conventional rebars. The results of this comparison are shown in Tables 4.1 through 4.7, and in Figures 4.2 trough 4.8. A summary of the comparison is presented in Table 4.8. As can be seen Equation (4-6) (and also Equation (4-5), (which represents the concrete contribution in Equation 4-6)) show good comparisons with torsional strength test result for fibrous concrete elements with and without conventional reinforcement

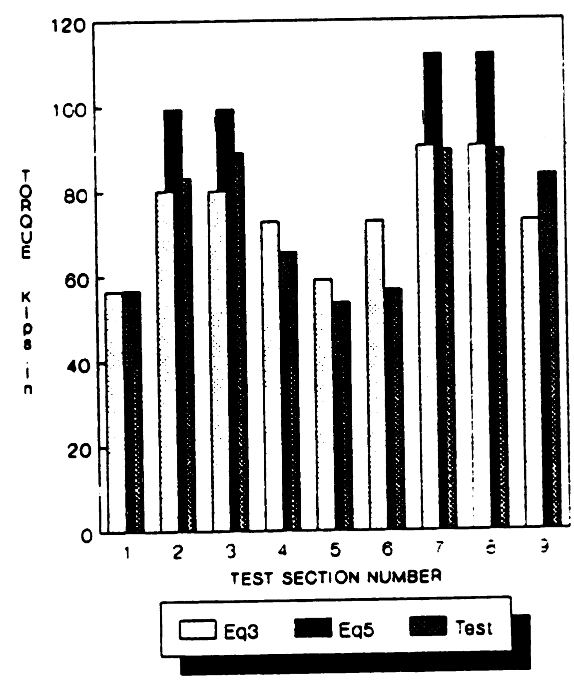
Table 4.1 COMPARISON OF TORSIONAL STRENGTH FOR FIBROUS CONCRETE SECTIONs (TEST VS THEORY).

Ref No	Test No	Fiber detail Vf%	df in	lf in	Conc Prop ksi f'c ksi	fr ksi	ft ksi	fct ksi	TORSIONAL STRENGTH Kips.in Eq.1 kip.in	Eq.2 kip.in	Eq.3 kip.in	Eq.4 kip.in	Eq.5* kip.in	Test kip.in	FIG No
	1	0	0	0	4.2	.38	.29	.48	53.40	45.60	56.60	46.90	39.20	56.70	
	2	1.5	.02	1.18	4.7	.98	.48	.81	55.50	84.70	80.10	119.00	99.40	83.20	
	3	1.5	.02	1.18	4.7	.98	.48	.81	55.30	84.70	80.10	119.00	99.40	89.10	
	4	1	.02	2	4.3	.55	.42	.71	53.40	57.80	72.90	67.00	55.90	65.70	
31	5	0	0	0	4.2	.4	.31	.51	53.40	46.70	59.10	48.80	40.70	53.80	4.3
	6	1	.02	2	4.3	.55	.42	.71	53.90	57.80	72.90	67.00	55.90	65.70	
	7	2	.02	1.18	4.9	1.1	.58	.97	56.30	91.90	90.40	134.00	111.90	89.60	
	8	2	.02	1.18	4.9	1.1	.58	.97	56.30	91.90	90.40	134.00	111.90	89.60	
	9	1	.02	1.18	4.3	.62	.42	.71	53.70	62.80	72.90	76.10	63.50	83.70	
AVERAGE THEORY / TEST									0.77	0.91	0.98	1.18	0.98		

Note:

Section width = 12 in — 1 in = 25.4 mm — 1 kip.in = 0.113 KN.m

Section height= 6 in — 1 ksi = 6.895 MPa — * This study



Eq5 (This study)

Figure 4.2 Torsional Strength Comparisons.

Table 4.2 COMPARISON OF TORSIONAL STRENGTH FOR FIBROUS CONCRETE SECTIONs (TEST VS THEORY).

Ref No	Test No	Fiber detail Vf%	df in	lf in	Conc Prop ksi f'c ksi	fr ksi	ft ksi	fct ksi	TORSIONAL STRENGTH Kips.in Eq.1 kip.in	Eq.2 kip.in	Eq.3 kip.in	Eq.4 kip.in	Eq.5* kip.in	Test kip.in	FIG No
	T1	0	016	1	5.5	.44	.29		58.50	50.10	41.06	54.31	45.36	86.80	
20	T2	0.5	016	1	5.2	.51	.38		57.30	55.11	48.70	62.70	52.40	120.50	4.4
	T3	1.5	016	1	5.1	.64	.50		58.40	63.90	58.40	78.20	65.32	125.30	
AVERAGE THEORY / TEST									0.52	0.51	0.45	0.59	0.49		

Note:

Section width = 12.2 in
Section height= 5.98 in
1 in = 25.4 mm
1 ksi = 6.895 MPa
1 kip.in = 0.113 KN.m
* This study

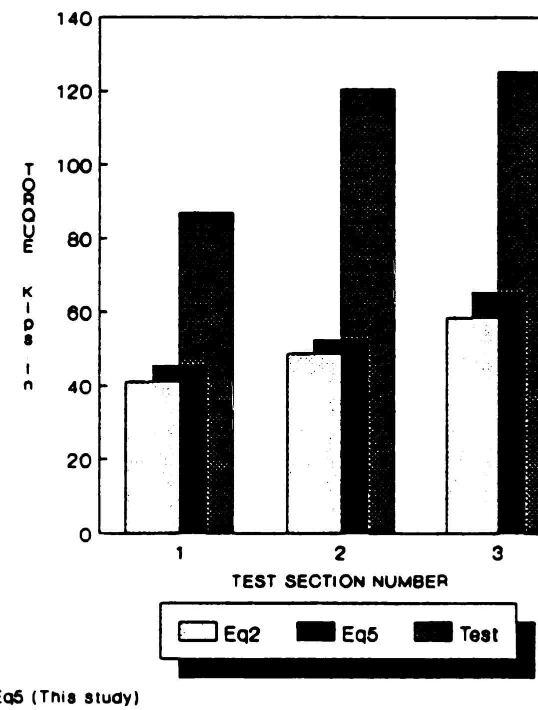


Figure 4.3 Torsional Strength Comparisons.

Table 4.3 COMPARISON OF TORSIONAL STRENGTH FOR FIBROUS CONCRETE SECTIONs (TEST VS THEORY).

Ref No	Test No	Vf%	df in	lf in	f'c ksi	fr ksi	ft ksi	fct ksi	Eq.1 kip.in	Eq.2 kip.in	Eq.3 kip.in	Eq.4 kip.in	Eq.5* kip.in	Test kip.in	FIG No
	1	.75	016	1.20	5.7	.57	.50		16.71	16.65	16.41	15.39	12.86	13.45	
33	2	.75	016	1.20	5.7	.57	.50		16.71	16.65	16.41	15.39	12.86	13.23	4.5
	3	.75	016	1.20	5.7	.57	.50		16.71	16.65	16.41	15.39	12.86	13.59	
AVERAGE THEORY / TEST									1.24	1.23	1.22	1.14	0.95		

Note:

Section width = 6 in

Section height= 4 in

1 in = 25.4 mm

1 ksi = 6.895 MPa

1 kip.in = 0.113 KN.m

* This study

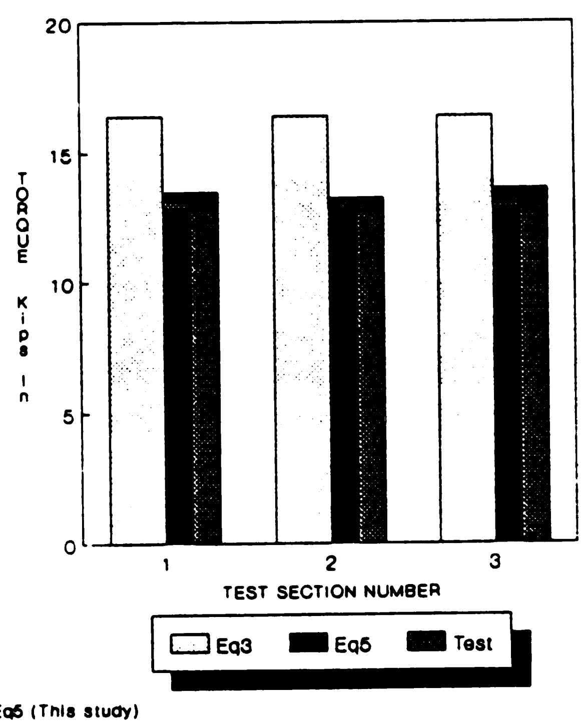


Figure 4.4 Torsional Strength Comparisons.

Table 4.4 COMPARISON OF TORSIONAL STRENGTH FOR FIBROUS CONCRETE SECTIONs (TEST VS THEORY).

Ref No	Test No	Fiber detail Vf%	Fiber detail df in	Fiber detail lf in	Conc Prop f'c ksi	Conc Prop fr ksi	Conc Prop ft ksi	Conc Prop fct ksi	TORSIONAL STRENGTH Eq.1 kip.in	Eq.2 kip.in	Eq.3 kip.in	Eq.4* kip.in	Eq.5 kip.in	Test kip.in	FIG No
	P1	0	0	0	5.9	.74	.30	.50	59.80	70.50	58.10	90.60	75.67	77.70	
	P2	.7	.02	1.18	5.8	.73	.40	.67	59.50	69.80	70.60	89.40	74.67	83.70	
	P3	0	.02	2	6.3	.72	.42	.71	61.20	69.20	73.40	88.10	73.58	77.70	
32	P4	1.5	.02	1.18	5.2	.93	.40	.67	57.40	81.90	70.60	113.0	94.38	76.20	4.6
	P5	2	.02	1.18	4.2	.75	.37	.61	53.40	70.90	66.30	91.60	76.50	65.70	
	P6	2	.02	2	6.9	1.2	.41	.63	63.10	97.20	71.10	147.00	122.80	89.60	
	P7	1.5	.02	1.18	5.8	.74	.37	.61	59.50	70.30	66.30	90.20	75.34	92.60	
	P8	2	.02	2	6.6	1.2	.42	.70	62.10	96.40	72.70	145.00	121.12	89.60	
AVERAGE THEORY / TEST									0.72	0.95	0.84	1.30	1.09		

Note:

Section width = 12 in   1 in = 25.4 mm   1 kip.in = 0.113 KN.m

Section height= 6 in   1 ksi = 6.895 MPa   * This study

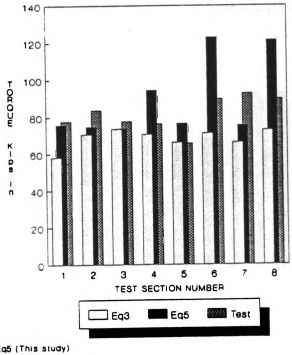


Figure 4.5 Torsional Strength Comparisons.

Table 4.5 COMPARISON OF TORSIONAL STRENGTH FOR FIBROUS CONCRETE SECTIONs (TEST VS THEORY).

Ref No	Test No	Fiber detail Vf%	df in	lf in	Conc Prop ksi f'c ksi	fr ksi	ft ksi	fct ksi	TORSIONAL STRENGTH Kips.in Eq.1 kip.in	Eq.2 kip.in	Eq.3 kip.in	Eq.4 kip.in	Eq.5* kip.in	Test kip.in	FIG No
	A1	.75	016	1.20	4.9	.64	.31		10.62	12.07	7.95	11.65	9.57	8.54	
	A2	.75	016	1.20	4.9	.64	.31		10.62	12.07	7.95	11.65	9.57	8.80	
	A3	.75	016	1.20	4.9	.64	.31		10.62	12.07	7.95	11.65	9.57	8.97	
	B1	.75	016	1.20	4.9	.64	.31		15.96	18.11	11.90	17.46	14.58	14.82	
34	B2	.75	016	1.20	4.9	.64	.31		15.96	18.11	11.90	17.46	14.58	14.63	4.7
	B3	.75	016	1.20	4.9	.64	.31		15.96	18.11	11.90	17.46	14.58	14.42	
	C1	.75	016	1.20	4.9	.64	.31		21.28	24.10	15.90	23.28	19.44	18.73	
	C2	.75	016	1.20	4.9	.64	.31		21.28	24.10	15.90	23.28	19.44	18.22	
	C3	.75	016	1.20	4.9	.64	.31		21.28	24.10	15.90	23.28	19.44	18.55	
	AVERAGE THEORY / TEST								1.14	1.29	0.85	1.25	1.04		

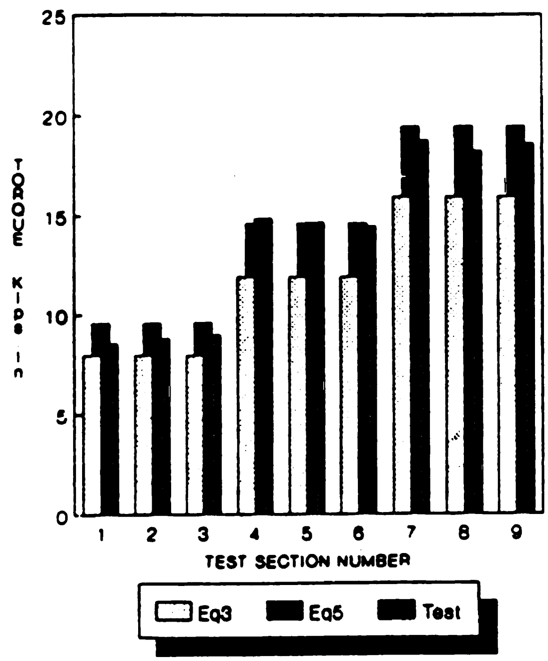
Note:

Section width = 4 in

Section height= 4 in for (A1, A2 and A3)
= 6 in for (B1, B2 and B3)
= 8 in for (C1, C2 and C3)

1 in = 25.4 mm
1 ksi = 6.895 MPa
1 kip.in = 0.113 KN.m

* This study



Eq5 (This study)

Figure 4.6 Torsional Strength Comparisons.

Table 4.6 COMPARISON OF TORSIONAL STRENGTH FOR FIBROUS CONCRETE SECTIONs (TEST VS THEORY).

Ref No	Test No	Fiber detail Vf%	Fiber detail df in	Fiber detail lf in	Conc Prop f'c ksi	Conc Prop fr ksi	Conc Prop ft ksi	Conc Prop fct ksi	Hoop detail Ast in2	Hoop detail fy ksi	Hoop detail s in	TORS STRENGTH Eq.6* kip.in	TORS STRENGTH Test kip.in	FIG No
	1	0	0	0	4.2	.38	.29	.48	.11	46	7	91.40	83.20	
	2	1.5	.02	1.18	4.7	.98	.48	.81	.11	46	7	151.60	149.00	
31	3	1	.02	2	4.3	.55	.42	.71	.11	46	7	129.00	179.00	4.8
	4	2	.02	1.18	4.9	1.1	.58	.97	.11	46	7	164.10	125.00	
	5	1	.02	1.18	4.3	.62	.42	.71	.11	46	3.5	168.00	146.00	
AVERAGE THEORY / TEST												1.14		

Note:

Section width = 12 in

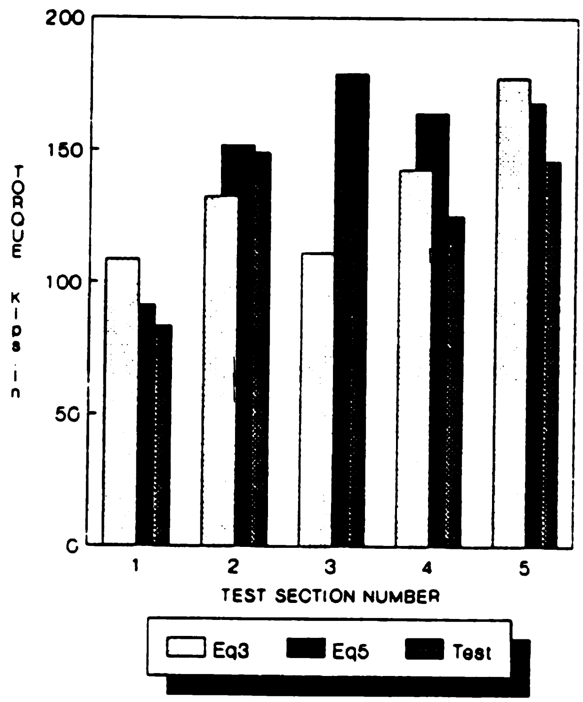
Section height= 6 in

1 in = 25.4 mm

1 ksi = 6.895 MPa

1 kip.in = 0.113 KN.m

* This study



Eq5 (This study)

Figure 4.7 Torsional Strength Comparisons.

Table 4.7 COMPARISON OF TORSIONAL STRENGTH FOR FIBROUS CONCRETE SECTIONs (TEST VS THEORY).

Ref No	Test No	Fiber detail Vf%	Fiber detail df in	Fiber detail lf in	Conc Prop ksi f'c ksi	Conc Prop fr ksi	Conc Prop ft ksi	Conc Prop fct ksi	Hoop detail Ast in2	Hoop detail fy ksi	Hoop detail S in	TORS STRENGTH Eq.6* kip.in	TORS STRENGTH Test kip.in	FIG No
	T1	0	016	1	5.5	.44	.29		.04	66	4	82.34	86.80	
20	T2	0.5	016	1	5.2	.51	.38		.04	66	4	89.38	120.50	3-7
	T3	1.5	016	1	5.1	.64	.50		.04	66	4	65.32	125.30	
AVERAGE THEORY / TEST												0.92		

Note:

Section width = 12.2 in
Section height= 5.98 in
1 in = 25.4 mm
1 ksi = 6.895 MPa
1 kip.in = 0.113 KN.m
* This study

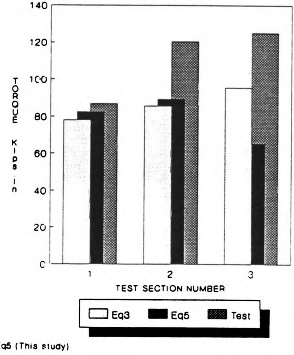


Figure 4.8 Torsional Strength Comparisons.

Table 4.8 AVERAGE OF THEORETICAL / EXPERIMENTAL TORSIONAL STRENGTH RATIO.

REF	31		20		33		32		30		Average	
EQ	FC	RFC	FC	RFC	FC	RFC	FC	RFC	FC	RFC	FC	RFC
Eq 1	0.72				1.24		1.14		0.77		0.97	
Eq 2	0.95				1.23		1.29		0.91		1.09	
Eq 3	0.84				1.22		0.85		0.98		0.97	
Eq 4	1.30				1.14		1.25		1.18		1.22	
Eq 5**	1.09				0.95		1.04		0.98		1.02	
Eq 6++				0.92						0.97		0.95

Note:

* FC = Fibrous Concrete

\+ FRC = Reinforced Fibrous Concrete

** Eq 5 used in this study for Fibrous Concrete

++ Eq 6 proposed in this study for Reinforced Fibrous Concrete

4.4 PARAMETRIC STUDIES

The developed equation for the prediction of torsional strength of fibrous reinforced concrete elements (Equation (4-6)) was used to investigate the effects of fiber reinforcement properties and conventional reinforcement on torsional strength of elements, as described below.

4.4.1 EFFECTS OF FIBER REINFORCEMENT PROPERTIES

Figures 4.10 and 4.11 show the effects of the volume fraction of straight-round steel fibers with different lengths and diameters on the torsional strength of a "Standard" reinforced fibrous element shown in Figure 4.9. Major improvements in torsional strength, of the order of 100 % , are observed to result from reinforcement by 2 % volume fraction of steel fibers. Longer fibers with smaller diameters (as far as dispersed satisfactorily in concrete matrix) seem to be more effective in increasing the torsional strength of fibrous reinforced concrete elements.

A_s = 2.37 in^2	l_f = 1 in	X = 10 in
A_{sp} = .24 in^2	d_f = .013 in	X1 = 8 in
f_y = 60 ksi	V_f = 1.5 %	Y = 20 in
f'_c = 4 ksi	A_t = 0.04 in^2	Y1 = 18 in

Type of Fiber = Straight

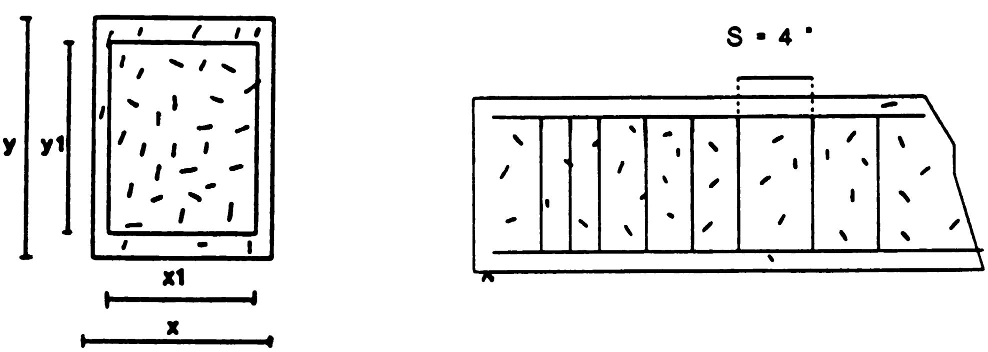


Figure 4.10 The Basic ("Standard") Reinforced Concrete Element.

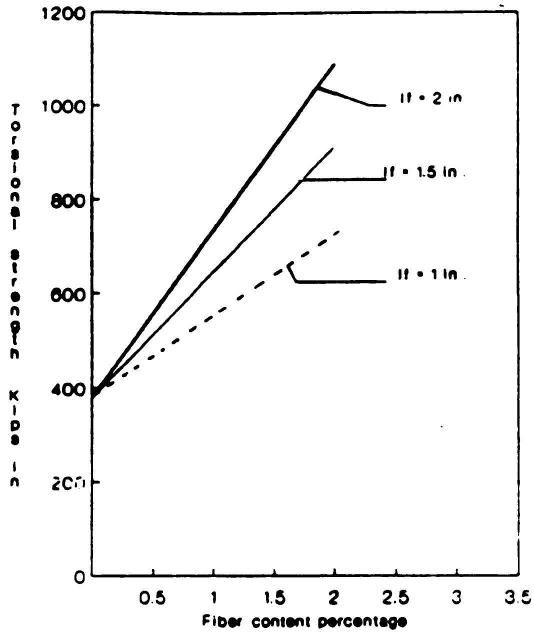


Figure 4.10 Effects of Fiber Content (V_f) on Torsional Strength at Different Fiber Lengths.

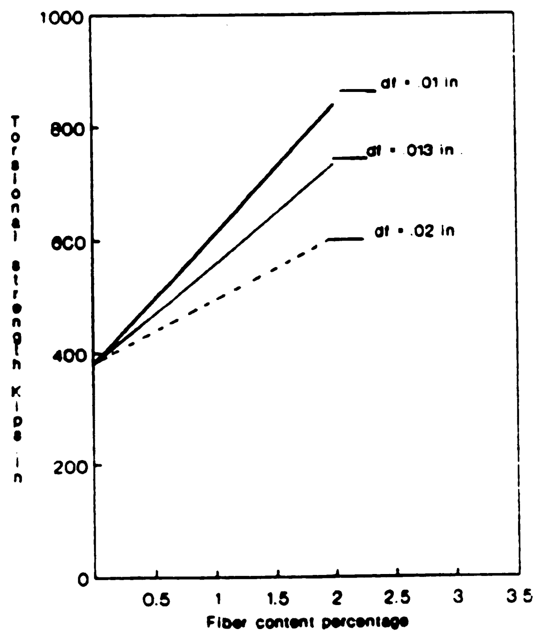


Figure 4.11 Effects of Fiber Content (V_f) on Torsional Strength at Different Fiber Diameters.

4.4.2 EFFECTS OF THE CONVENTIONAL REINFORCEMENT

Figure 4.12, 4.13, and 4.14 show the effects of transverse reinforcement leg area, yield strength and spacing on the relationship between torsional strength of the "Standard" element and fiber volume fraction. The torsional strength is observed to increase with the increase in leg area and yield strength, and decrease in spacing of the transverse steel. It should be noted that test results indicate that steel fibers are most effective in improving torsional performance when they are used together with a small amount of transverse reinforcement.

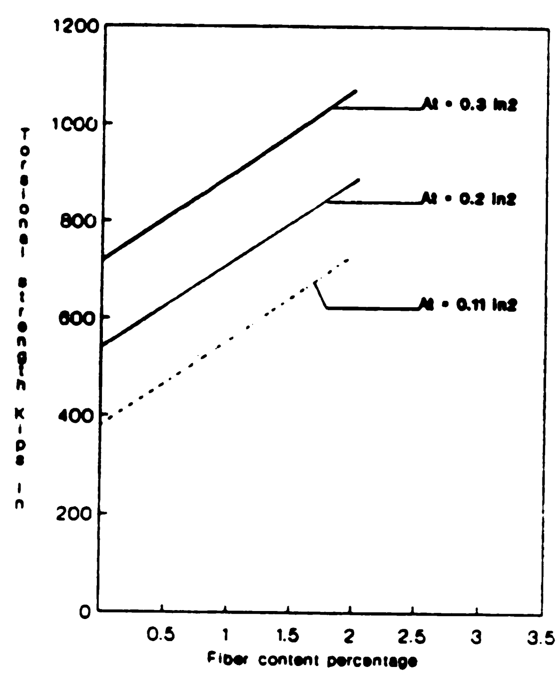


Figure 4.12 Effects of Fiber Content (V_f) on Torsional Strength at Different Transverse Steel Leg Areas

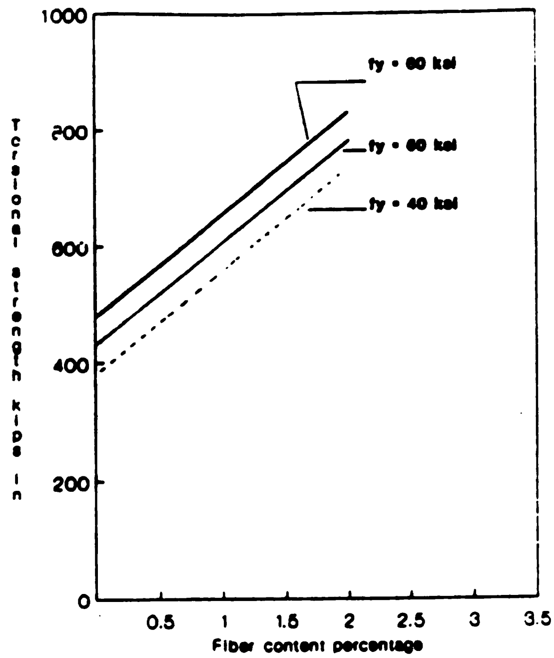


Figure 4.13 Effects of Fiber Content (V_f) on Torsional Strength at Different Transverse Steel Yield Strengths.

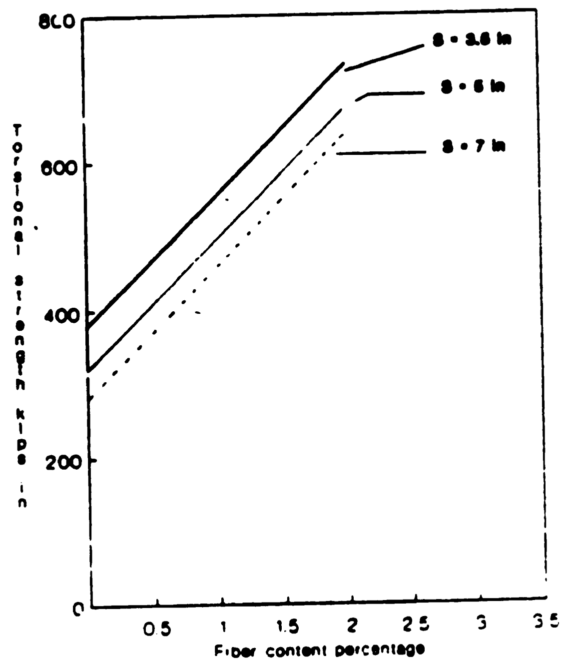


Figure 4.14 Effects of Fiber Content (V_f) on Torsional Strength at Different Transverse Steel Spacings.

4.5 SUMMARY AND CONCLUSION

A torsional strength equation was developed for predicting the torsional strength of fibrous reinforced concrete elements. The proposed equation was verified using torsional test results for fibrous concrete and fibrous reinforced concrete elements with wide ranges of material properties.

The proposed equation for predicting the torsional strength of fibrous elements, after being verified, was used in an analytical parametric study on the effects of fiber reinforcement and conventional steel properties on the torsional strength of reinforced concrete elements. The results showed that the torsional strength of reinforced concrete elements increase almost linearly with increasing fiber volume fraction. In the presence of a typical 2 % volume fraction of steel fibers with an aspect ratio of 70, the torsional strength of reinforced concrete elements increases by about 100 % over that of non-fibrous elements. The results also showed that the longer fibers with smaller diameters, as far as they are uniformly dispersed in the concrete matrix, are more effective in increasing torsional strength.

Conventional reinforcement properties seem to have relatively small effects on the effectiveness of fibers in increasing the torsional strength of reinforced concrete elements. Test results indicate that best results can be achieved when fibers are used in combination with relatively small amounts of conventional reinforcement.

CHAPTER 5
SUMMARY AND CONCLUSIONS

5.1 GENERAL

Steel fiber reinforced concrete has found growing applications in secondary structures. The material is currently under serious consideration for use in primary load-bearing reinforced concrete elements. Such applications of steel fiber reinforced concrete can be encouraged through the development of structural design guidelines. This investigation has been concerned with developing structural analysis and design techniques for fibrous reinforced concrete elements. Due consideration has been given to the improvements in the flexural, shear and torsional strength, ductility and cracking characteristics of reinforced concrete elements resulting from steel fiber reinforcement. The proposed equations and guidelines provide designers with practical tools for optimizing the structural design of reinforced concrete elements through the use of steel fibers.

It is also worth mentioning that much is left to be understood regarding structural performance characteristics of fibrous reinforced concrete elements. The equations and procedures presented in this study can be the basis for further developments as more experimental data become available in this area.

This investigation on the development of structural procedures for fibrous reinforced concrete elements has been performed in three phases, which were concerned with flexural, shear and torsional elements. A summary of the activities conducted in each phase of the study and the corresponding outcomes and conclusions are given below.

5.2 FLEXURAL ELEMENTS

A refined flexural analysis procedure was developed for reinforced concrete sections incorporating steel fibers. The analysis procedure was verified using test results reported in the literature, and it was used for an analytical parametric study on the effects of different design variables on flexural strength and ductility of fibrous reinforced concrete sections. Based on the results of this parametric study, a simplified flexural analysis and design guidelines were developed for reinforced concrete elements incorporating steel fibers.

From the analytical studies conducted on the flexural behavior of fibrous reinforced concrete sections it was concluded that:

1 - Steel fiber reinforcement results in improvements in the flexural strength and especially the ductility of reinforced concrete sections.

2 - High strength steel and concrete can be efficiently used in flexural elements incorporating steel fibers with no significant adverse effects on the ductile behavior.

3 - Steel fiber reinforcement reduces the need for

compression steel to improve the ductile behavior in flexural elements, High tension steel can be used in reinforced concrete flexural elements incorporating steel fibers, resulting in higher flexural strength, withthe ductility being maintained at a desirable level.

4 - The developed flexural analysis and design procedure is simple and conforms to the design guidelines of the American Concrete Institute (ACI 318 - 83). It account for the improvements in flexural strength resulting from steel fiber reinforcement using a flexural stress distribution at failure which neglects the improvements in tensile and compressive properties of the fibrous concrete. The developed flexural design guideline also considers the positive effects of fiber reinforcement on the ductility of flexural behavior by increasing the maximum limit on tension steel area. The result of the proposed simplified flexural analysis and design procedures compare reasonably well with test results.

5.3 SHEAR ELEMENTS

Steel fiber reinforcement was found to be an effective approach for improving the strength and ductility of failure in reinforced concrete elements subjected to shear forces. Best results seem to be achieved when a combination of steel fibers and transverse reinforcement is used to provide shear resistance.

A shear strength equation was developed in this study for fibrous reinforced concrete elements based on the physics of shear failure in the presence of steel fibers. The proposed equation was shown to predict test results far better than the previous equations (which are simply modified versions of the conventional reinforced concrete equations).

The proposed shear strength equation was used in an analytical parametric study on the fiber effects of reinforcement on shear strength of reinforced concrete elements. The results indicated that improvements of the order of 50 % in shear strength can be expected in typical reinforced concrete elements as a result of reinforcement with 2 % volume fraction of steel fibers. Longer fibers with smaller cross-sectional dimensions and especially those with mechanical deformations were theoretically more effective in increasing the shear strength of reinforced concrete elements (as far as the practical problems with fresh mix workability and fiber dispersability can be overcome). The conventional reinforcement properties on The shear span-to-depth ratio of reinforced concrete elements did not significantly influence the absolute value of improvements in shear resistance resulting from fiber reinforcement.

5.4 TORSIONAL ELEMENTS

A torsional strength equation was developed for predicting the torsional strength of fibrous reinforced concrete elements. The proposed equation was verified using torsional test results for fibrous concrete and fibrous reinforced concrete elements with wide ranges of material properties.

The proposed equation for predicting the torsional strength of fibrous elements, after being verified, was used in an analytical parametric study on the effects of fiber reinforcement and conventional steel properties on the torsional strength of reinforced concrete elements. The results showed that the torsional strength of reinforced concrete elements increase almost linearly with increasing fiber volume fraction. In the presence of a typical 2 % volume fraction of steel fibers with an aspect ratio of 70, the torsional strength of reinforced concrete elements increases by about 100 % over that of non-fibrous elements. The results also showed that the longer fibers with smaller diameters, as far as they are uniformly dispersed in the concrete matrix, are more effective in increasing torsional strength.

Conventional reinforcement properties seem to have relatively small effects on the effectiveness of fibers in increasing the torsional strength of reinforced concrete elements. Test results indicate that best results can be achieved when fibers are used in combination with relatively small amounts of conventional reinforcement.

REFERENCES

1. Swamy, R.N. and Sa'ad A - Al-Ta'an "Deformation and Ultimate Strength in Flexure of Reinforced Concrete Beams made with Steel Fiber Concrete" ACI Journal / September - October 1981. pp 335-405

2. Craig, R.J. "Light Weight Reinforced Fiber Concrete Behavior and uses" New Jersey Institute of Technology .USA RILEM 1986 VOL 1.

3. Kormeling, H.A., Reinhardt,H.W., and Shah, S.P. "Static and Fatigue Properties of Concrete Beams Reinforced with Continues Bars and with Fibers" ACI Journal / January - February 1980. pp 36-42

4. Swamy,R.N., Sa'ad A - Al-Ta'an and Sami A.R.Ali "Steel Fiber for Controlling Cracking and Deflection" Concrete International / August 1979. pp 41-45

5. Byasi, Z. "Mechanical Properties and Structural Application of Steel Fiber Reinforced concrete" Ph.D Thesis, Volume 2, Michigan State University 1987.

6. Hannant, D.J., "Fiber Cements and Fiber Concretes"; John Wiley and Sons Ltd.

7. Soroushian, P. and Byasi, Z. "Prediction of the Tensile Strength of Fiber Reinforced Concrete Acritique of the Composite Material Concept," ACI Convention on Fiber Reinforced Concrete, Baltimore, November, 1986.

8. Soroushian, P. and Byasi, Z. "Mechanical Properties of Fiber Reinforced Concrete" Seminar Procedings Composite Material and Structures Center Michigan State University February 1987.

9. Shah.S.P and Rangan.V. "Effect of Reinforcements on Ductility of Concrete" Journal of The Struct.Div ASCE VOL,96,N..St6, June 1970 pp 1167-1184.

10. Craig,R.J., "Design Procedures for Fibrous Concrete, Shear, Moment and Torsion, New Jersy Institute of Technology New York New, Jersy.

11. Park, R., and Panlay, T., "Reinforced Concrete Structures Department of Civil Engineering University of Canterbury, Christchurch, New Zealand pp 207.

12. Batson,G.R, Clarkson University, Potsdam, New York, USA C. Alguire, Sargent Lundy, Chicago Illinois, USA "Steel Fiber as Shear Reinforcement in Reinforced Concrete T - Beams".

13. Ot.Ibrahim "Control of Cracks In Reinforced Concrete Using Steel fibers" University of Elminior. Egypt. RILEM 1986 VOL 1.

14. SA.AL-Ta'an and JR Al-feel "Role of Steel Fiber Reinforcement in flexural Failure" University of Mosul, Mosul, Iraq RILEM 1986 Vol 1.

15. Swamy, R.N. and Al-Noori, K.A. "Flexural Behavior of Fiber Concrete With Conventional Steel Reinforcement", Proc. RILEM Symp. on Fiber Reinforced Cement And Concrete (Sept.1975). The Conctruction Press Ltd, Lancaster, 1975, pp 187-195.

16. Shah,S.P. Stroven, D.Dalhuison, and p.Steke Lenburg, in Testing and Test Method of Fiber Cement Composites, edited by R.N Swamy (The construction Press, lancaster 1978), pp 399-408.

17. D.Fanella and A.Naaman, ACI Journal., 82, pp 475 (1985).

18. Fanella, D.A and Naaman, A.E., "Stress-strain Properties of Fiber Reinforced Mortar In Compression" ACI Journal, July - August, 1985. pp 475-485.

19. Souroushian, P. and Cha - Don Lee."Steel Fiber Reinforced Concrete Under Compression: Performance characteristics and Constitutive Modeling" Michigan State University.

20. Abdul-wahad H.,M,S. and AL-Ausi and SH TH Tawfiq "Steel Fiber Reinforced Concrete Member Under Combined, Shear and Torsion " University of Technology, Baghdad, Iraq RILEM 1986 VOL 1.

21. Batson, G., "Use of Steel Fiber for Shear Reinforcement and Ductility" USA - Sweden joint seminar (NSF - STU), S. Shah and A.SKarendahil, editors, Swedish cement and Concrete Research Institute, Stockholm, 1985.

22. Takato Uomoto, Ranjan, Hitoshi Furukoshi, and Hideo Fujino "Shear Strength of Reinforced Concrete Beams with Steel Fiber reinforcement" RILEM 1986 VOL 1.

23. Batson , G; Jenkins, E.; and Spatney, R., "Steel Fibers as Shear Reinforcement in Beams" ACI Journal, Proceeding V.69 N 10, Oct, 1972 pp 640 - 644.

24. ACI commitee 544 State - of - Art Report on Design Consideration " Steel Fiber Reinforcement Concrete and Morter" being written.

25. William son, G.R., "Steel Fibers as Web Reinforcement in Reinforced Concrete", Proceedings, U.S.Army Science Conference, V.3, June 1978, West Point, New York.

26. Craig R.j, Unpublishid manuscript for text book of Fiber reinforced Concrete, New Jersy Institute of Technology, Newark, New Jersy, 1984.

27. Swamy R.N and Bahia H.M "The effectiveness of Steel Fibers as Shear Reinforcement", Concrete International, V.7,No.3, March, 1985, pp 35

28. Jindal R.L, "Shear and Moment Capacities of Steel Fiber Reinforced Concrete Beams", Fiber Reinforced Concrete International Symposium, Sp-81, American Concrete Institute, Detroit, Michigan, 1984.

29. Chu - Kia Wang Charees G.Salmon "Reinforced Concrete design" Fourth Edition "Shear Strenght and Shear Reinforcement" pp 121.

30. Swamy R.N and Bahia H.N "Influence of Fiber Reinforcement on the Dowel Resistance to Shear" ACI. Journal, Symposuim paper, Feb. 1979, pp 327 - 355.

31. Craig, R, John; Dunya, R.; Riaz, J.; and Shirazi, H., "Torsional Behavior of Reinforced Fibrous Concrete Beams", Fiber Reincorced Concrete, SP-81, American Concrete Institute, Detroit, 1984, pp 17-49.

32. Craig,R.J., James A. Parr, Eddy Germain, Victor. Mosquera and Stavros Kamilares "Fiber Reinforced in torsion" ACI Journal, November - December 1986 pp 934 - 942.

33. Mansur, M.A., and P. Paramasivan. "Fiber Reinforced Concrete Beams in Torsion, Bindin and Shear" ACI Jouran, January -February 1985 pp 33 - 39.

34. Mansur, M.A., BSC Eng, MSC Eng, PhD, MASCE "Bending - Torsion Interaction for Concrete Beams Reinforced with Steel Fibers " Magazine of Concrete Research: Vol 34, No, 121 : December 1982 pp 182 - 190.

35. Edgington, J., "Steel Fiber Reinforced Concret," Research Report Submitted to the Departement of Environment, Jan.1974. Also PhD Thesis, University of Surry , 1974.

36. Edgington, J., "Steel Fiber Reinforced Concrete," Intermediate Report, Departement of the Environment, Grston, Wat ford, March 1972.

37. Edgington, J., Hannant, D.J.; and Williams, R.I.T, "Steel Fiber Reinforced Concrete," Fiber Reinforced Materials, BRT Building Research Series, V.2 The Construction Press, New York, 1978, pp 112 - 129.

38. Hsu, Thomas, T.C, "Torsion of Structural Concrete-Plain Concret Rectangular Sections," Torsion of Structural Concret, Sp.18 American Concrete Institute, Detroit, 1986, pp 203 - 238.

39. Paul, S.L. and Sinnamon, G.k., "Concret Tunnel Liners: Structural Testing Of Sgmented Liners ,"Final Report, Departement of Transprtation, FRA and D-75-93 Augost, 1975.

40. Criswel, M.E, "Shear in Fiber Reinforced Concret," National Structural Engineering Conference, Augost 1976, Masison, Wisconsin American Society of Civil Engineers, New York, pp 1-20.

41. Turner, L. and V.C Davies,"Plain and Reinforced Concrete in Torsion, with Particular REferance to Reinforced Beams" Selected Engineering Papers No.165, London, Institution of civil Engineers, 1934 pp 31.

42. Swamy, R.N and Mangat p.s "Influence of Test Geometry on the Properties of Steel Fiber Reinforced Concrete," V1.4 pp 451 - 465, 1975.

22. Murty, D.S.R., T.Venkatach Aryulu "Fiber REinforced Concret Beams Subjected to SHEAR fORCE," Proceedings of the International Symposium on Fiber Reinforced Concret, December 16 -19, 1987, Madras, India.

44. Shah, S.P., "Application of Fracture Mechanics to Cementitions Composites," Martinus Publishers, 1985.

45 Soroushian, P , and Bayasi, Z. (editors) "Fiber reinforced Concrete Design and Application" Composite Materials and Structures Center, Michigan State University Feb 1987.

APPENDIX A

AN OUTLINE OF THE COMPUTER PROGRAM FOR REFINED FLEXURAL ANALYSIS OF FIBROUS REINFORCED CONCRETE SECTIONS

The developed computer program performs a step-by-step flexural analysis of reinforced concrete sections incorporating steel fibers, following the refined flexural analysis procedures described in chapter 2. Different parts of the program are outlined below.

Part 1 - Data input (only the matrix and fiber properties and not the composite properties are needed):

B = width of the cross section

H = height of the cross section

d = effective depth, distance from compression face to centroid of tension steel

d_p = distance from compression face of the member to centroid of compression steel

A_s = area of tension steel

A'_s = area of compression steel

f'_c = concrete compressive strength

f_y = yield strength of steel

V_f = volume content of fibers

l_f = length of fibers

d_f = diameter of fibers

Part 2 - Calculate the composite material properties:

Tensile and compressive strength and constitutive models of steel fiber reinforced concrete.

Part 3 - Input the new value of curvature (which is an increment higher than the previous value).

Part 4 - Assuming a linear strain distribution, through trial and adjustment, find the neutral axis position which satisfies (within acceptable errors) the equilibrium of tension and compression forces at the section.

Part 5 - Find the moment of compression and tension forces: Repeat from part 3 step 3 until flexural failure is marked by a major drop in force resistance and in stability of the section flexural behavior.

The completed flexural moment-curvature relationship of fibrous reinforced concrete sections can be produced following the above analysis procedure.

A complete print-out of the computer program which performs the above tasks for flexural analysis of fibrous reinforced concrete sections is given below.

```
10 DIM FI(300), M(300), EPSCU(300), CCC(300), TCC(300)
20 DIM Z%(2)
30 CLS
40 KEY OFF
50 PRINT TAB(20)    "FLEXURAL ANALYSIS OF FIBROUS R/C BEAMS"
60 PRINT TAB(20)    "***************************************"
70 PRINT:PRINT
80 INPUT "DO YOU WANT A HARD COPY OF GRAPH (Y/N)";YYNN$
90 PRINT
100 PRINT
110 'COLOR 31,5
120 LOCATE 23,63
130 BEEP :BEEP :BEEP
140 PRINT "PLEASE WAIT"
150 'COLOR 7,0
160 FOR I=1 TO 10000: NEXT I
170 REM-----------------------------------------------------
180 CLS :PRINT "ENTER SECTION GEOMETRY AND REINFORCEMENT PATTERN"
190      PRINT "-----------------------------------------------"
200 PRINT: PRINT: INPUT "ENTER THE RECTANGULAR BEAM WIDTH (in)";B
210 PRINT: INPUT "ENTER THE BEAM HEIGHT (in)";H
220 PRINT: INPUT "ENTER THE EFFECTIVE DEPTH OF BEAM (in)";D
230 PRINT: INPUT "ENTER THE DIST. FROM CENTER OF COMP. STEEL TO COMP. SURFACE (i
n)";DP
240 PRINT: INPUT "ENTER THE TENSION STEEL AREA (in2)";AS
250 PRINT: INPUT "ENTER THE COMP. STEEL AREA (in2)";ASP
260 PRINT: INPUT "ENTER REBAR YIELD STRENGTH (psi)";FY
270 REM-----------------------------------------------------
280 'COLOR 31,1
290 BEEP :BEEP
300 LOCATE 23,66
310 PRINT "CHECK DATA"
320 'COLOR 15,4
330 LOCATE 24,10
340 INPUT "INPUT O/K  (Y/N)  ";I$
350 LOCATE 20,66
360 'COLOR 7,0
370 IF I$="N" OR I$="n" OR I$="NO" OR I$="no" THEN CLS:GOTO 200
380 CLS:PRINT "ENTER STEEL FIBER PROPRETIES"
390     PRINT "--------------------------"
400 PRINT: INPUT "ENTER FIBER VOL. FRACTION (%)";VF
410 PRINT: INPUT "ENTER FIBER LENGTH (in)";LF
420 PRINT: INPUT "ENTER FIBER DIAMETER (in)";DF
430 PRINT: INPUT "ENTER FIBER TYPE: S(traight), H(ooked) or C(rimped)";FT$
440 IF FT$="S" THEN TU=320 ELSE IF FT$="H" THEN TU=450 ELSE IF FT$="C" THEN TU=3
00
450 'COLOR 31,1
460 BEEP :BEEP
470 LOCATE 15,66
480 PRINT "CHECK DATA"
490 'COLOR 15,4
500 LOCATE 24,10
510 INPUT "INPUT O/K  (Y/N)";L$
520 LOCATE 12,66
530 'COLOR 7,0
540 IF L$="N" OR L$="n" OR L$="NO" OR I$="no" THEN CLS:GOTO 400
550 REM---------------------------------------------------------------
560 CLS :PRINT "ENTER THE MATERIALS AND MECHANIQUE PROPERTIES"
570      PRINT "--------------------------------------------"
580 PRINT: INPUT "CHECKING THE POSSIBILITY OF EXCEEDING CRITICAL FIBLER LENGTH (
```

```
Y/N)";YN$
590 IF YN$="Y" OR YN$="y" THEN PRINT: INPUT "ENTER FIBER ULT. TENSILE STRENGTH (
psi)";FUF: LC=FUF*D/TU: IF L>=LC THEN FFF=FUF*.5*VF/100/(3.14*DF^2/4) ELSE FFF=.
5*.41*TU*VF/100*LF/DF
600 PRINT:PRINT: INPUT "DO YOU KNOW THE ULT. TENSILE STRENGTH OF FRC (Y/N)";NU$
610 IF NU$="N" OR NU$="n" THEN PRINT: INPUT "ENTER THE TENSILE STRENGTH OF CONC.
 MATRIX (psi), ABOUT 4*Fc^.5";FTM ELSE 640
620 IF YN$="Y" OR YN$="y" THEN FTF=FTM*(1-VF/100)+FFF ELSE FTF=FTM*(1-VF/100)+.5
*.41*TU*VF/100*LF/DF
630 IF NU$="N" OR NU$="n" THEN 650
640 PRINT: INPUT "ENTER ULT. TENSILE STRENGTH OF FRC (psi)";FTF
650 PRINT: INPUT "DO YOU KNOW THE POST-PEAK TENSILE STRENGTH OF FRC (Y/N)";NP$
660 IF NP$="Y" OR NP$="Y" THEN 690
670 IF YN$="Y" OR YN$="y" THEN FPF=FFF ELSE FPF=.5*.41*TU*VF/100*LF/DF
680 IF NP$="N" OR NU$="n" THEN 700
690 PRINT: INPUT "ENTER THE POST-PEAK TENSILE STRENGTH OF FRC (psi)";FPF
700 PRINT: INPUT "DO YOU KNOW THE COMP. STRENGTH OF FRC (Y/N)";NC$
710 IF NC$="N" OR NC$="n" THEN 740
720 PRINT: INPUT "ENTER THE COMP. STRENGTH OF FRC (psi)";FCF
730 IF NC$="Y" OR NC$="y" THEN 860
740 PRINT: INPUT "ENTER THE COMP. STRENGTH OF CONC. MATRIX (psi)";FC: FCF=FC+994
*VF/100*LF/DF
750 'COLOR 31,1
760 BEEP :BEEP
770 LOCATE 20,65
780 PRINT "CHECK DATA"
790 'COLOR 15,4
800 LOCATE 24,10
810 INPUT "INPUT O/K   (Y/N)";T$
820 LOCATE 12,66
830 'COLOR 7,0
840 REM---------------------------------------------------------------------

850 IF T$="N" OR T$="n" OR T$="NO" OR T$="no" THEN CLS :GOTO 580
860 CP=(AS*FY*(H-D)+ASP*FY*(H-DP)+FCF*B*H*H/2)/(AS*FY+ASP*FY+FCF*B*H)
870 FR=.12*FCF+2000*VF/100*LF/DF
880 EP=(.00079+1.13/(FCF-994*VF/100*LF/DF))*VF/100*LF/DF+.0021
890 Z=-343*(FCF-994*VF/100*LF/DF)*(1-.64*SQRT(VF/100*LF/DF)):IF Z>0 THEN Z=0
900 NFI=1: FI(NFI)=0: M(NFI)=0
910 SCREEN 2: KEY OFF
920 XMAX=.0010000001#: YMAX=20000000#*.5
930 WINDOW (-XMAX/10,-YMAX/10)-(XMAX,YMAX): CLS
940 LINE (0,0)-(0,YMAX): LINE (0,0)-(XMAX,0)
950 FOR NN=1 TO 10
960 LINE (0,YMAX*NN/10)-(XMAX/65,YMAX*NN/10)
970 LINE (XMAX*NN/10,0)-(XMAX*NN/10,YMAX/50)
980 IF NN=2 OR NN=4 OR NN=6 OR NN=8 THEN LOCATE 24,6+72*NN/10: PRINT USING ".###
#";XMAX*NN/10;
990 IF NN=2 OR NN=4 OR NN=6 OR NN=8 THEN LOCATE 22.5-22.5*NN/10,1: PRINT YMAX*NN
/10000;
1000 NEXT NN
1010 LOCATE 25,60: PRINT "CURVATURE (1/in)";
1020 LOCATE 1,10: PRINT "MOMENT (Kips.in)";
1030 LOCATE 3,12:PRINT "B (in)     =";B
1040 LOCATE 3,32:PRINT "H (in)     =";H
1050 LOCATE 4,12:PRINT "d (in)     =";D
1060 LOCATE 4,32:PRINT "dp (in)    =";DP
1070 LOCATE 5,12:PRINT "As (in2)   =";AS
1080 LOCATE 5,32:PRINT "Asp (in2)  =";ASP
1090 LOCATE 6,12:PRINT "Fy (psi)   =";FY
```

```
1100 LOCATE 6,32:PRINT "Vf (%)    =";VF
1110 LOCATE 7,12:PRINT "df (in)   =";DF
1120 LOCATE 8,33
1130 IF FT$="S" THEN PRINT "STRAIGHT FIBER"
1140 IF FT$="H" THEN PRINT "HOOKED FIBER"
1150 IF FT$="C" THEN PRINT "CRIMPED FIBER"
1160 IF YN$="N" OR YN$="n" THEN 1200
1170 LOCATE 9,32
1180 IF L>=LC THEN PRINT "Lf >= Lc"
1190 IF L<LC THEN PRINT "Lf < Lc"
1200 LOCATE 8,12:PRINT "Ftf (psi) =";FTF
1210 LOCATE 9,12:PRINT "Fcf (psi) =";FCF
1220 LOCATE 7,32:PRINT "Lf  (psi) =";LF
1230 LOCATE 10,12:PRINT "Fpf (in)  =";FPF
1240 REM ------------------------------------------------
1250 FIFIMAX=.001
1260 DFIFI=.00002
1270 FOR FIFI=.00002 TO FIFIMAX STEP DFIFI
1280 NFI=NFI+1
1290 FI(NFI)=FIFI
1300 EPSP1=-FIFI*CP
1310 EPSP2=FIFI*(H-CP)
1320 NA=CP+EPSP1/FIFI
1330 EPSSP=-(H-DP-NA)/(NA-CP)*EPSP1
1340 EPSS=EPSP1*(NA-H+D)/(NA-CP)
1350 FS=AS*EPSS*29000*1000
1360 IF ABS(FS)>FY*AS THEN FS=FY*AS*FS/ABS(FS)
1370 FSP=ASP*EPSSP*29000*1000
1380 EPSCX=(EPSP1)*(H-NA)/(CP-NA)
1390 GOSUB 2390
1400 EPSTX=-EPSP1*NA/(CP-NA)
1410 REM ------------------------------------------------------------
1420 GOSUB 2480
1430 '            PRINT "NA,EPSP1,TC,CC,FS,FSP,TOTAL=";NA;EPSP1;TC;CC;FS;FSP;TC+C
C+FS+FSP
1440 T1=TC+CC+FS+FSP
1450 NA=CP+EPSP2/FIFI
1460 EPSSP=-(H-DP-NA)/(NA-CP)*EPSP2
1470 EPSS=EPSP2*(NA-H+D)/(NA-CP)
1480 FS=AS*EPSS*29000*1000
1490 IF ABS(FS)>FY*AS THEN FS=FY*AS*FS/ABS(FS)
1500 FSP=ASP*EPSSP*29000*1000
1510 IF ABS(FSP)>FY*ASP THEN FSP=FY*ASP*FSP/ABS(FSP)
1520 EPSCX=EPSP2*(H-NA)/(CP-NA)
1530 GOSUB 2390
1540 EPSTX=-EPSP2*NA/(CP-NA)
1550 REM ---------------------------------------------------------
1560 GOSUB 2480
1570 '            PRINT "NA,EPSP2,TC,CC,FS,FSP,TOTAL=";NA;EPSP2;TC;CC;FS;FSP;TC+CC
+FS+FSP
1580 T2=TC+CC+FS+FSP
1590 EPSP3=(EPSP1+EPSP2)/2
1600 NA=CP+EPSP3/FIFI
1610 IF  NA > H THEN NA=H: EPSP3=FIFI*(NA-CP)
1620 IF NA < 0 THEN NA=0: EPSP3=FIFI*(NA-CP)
1630 IF ABS(CP-NA)/CP < .0001 THEN EPSP3=.55*EPSP1+.45*EPSP2: GOTO 1610
1640 EPSSP=-(H-DP-NA)/(NA-CP)*EPSP3: EPSS=EPSP3*(NA-H+D)/(NA-CP)
1650 FS=AS*EPSS*29000*1000: IF ABS(FS)>FY*AS THEN FS=FY*AS*FS/ABS(FS)
1660 FSP=AS*EPSSP*29000*1000: IF ABS(FSP)>FY*ASP THEN FSP=FY*ASP*FSP/ABS(FSP)
1670 EPSCX=EPSP3*(H-NA)/(CP-NA)
```

```
1680 GOSUB 2390
1690 EPSTX--EPSP3*NA/(CP-NA)
1700 REM -----------------------------------------------------------
1710 GOSUB 2480
1720 '              PRINT "NA,EPSP3,TC,CC,FS,FSP,TOTAL-";NA;EPSP3;TC;CC;FS;FSP;TC+C
C+FS+FSP
1730 T3-TC+CC+FS+FSP
1740 IF T1*T3 < 0 THEN T2-T3: EPSP2-EPSP3: GOTO 1780
1750 IF T2*T3 < 0 THEN T1-T3: EPSP1-EPSP3: GOTO 1780
1760 IF T3-T1 OR T2-T1 THEN EPSP3-EPSP1+.4*(EPSP2-EPSP1): GOTO 1610
1770       PRINT "SOMETHING IS WRONG, T3 IS NOT BETWEEN T1 AND T2": STOP
1780 IF ABS(T3) < .002*(B*H*FCF+FY*AS+FY*ASP) THEN 1790 ELSE 1590
1790 GG-H
1800 BB-EP/FIFI+NA
1810 DD-FCF*FIFI/EP
1820 EPSR-(FR-FCF+Z*EP)/Z
1830 EE-EPSR/FIFI+NA
1840 IF EPSCX <- EP THEN AR1--(GG^4/4-NA*GG^3*2/3+NA^2*GG^2/2)*FIFI/EP+2*(GG^3/3
-NA*GG^2/2)+NA^4/12*FIFI/EP+NA^3/3
1841 IF EPSCX <- EP THEN AR2--FIFI/EP*GG*(GG^2/3-NA*GG+NA^2)+2*GG*(GG/2-NA)+FIFI
/EP*NA^3/3+NA^2
1842 IF EPSCX <- EP THEN ARMCC-AR1/AR2
1850 REM -----------------------------------------------------------
1860 IF EPSCX > EP AND EPSCX <- EPSR THEN AR1-DD*(-FIFI/EP*BB^2*(BB^2/4-2*NA*BB/
3+NA^2/2)+2*BB^2*(BB/3-NA/2)+FIFI/EP*NA^4/12+NA^3/3)+GG^2*Z*(GG/3*FIFI-NA*FIFI/2
-EP/2)+GG^2*FCF/2-BB^2*Z*(BB/3*FIFI-NA/2*FIFI-EP/2)-BB^2*FCF/2
1870 IF EPSCX > EP AND EPSCX <- EPSR THEN AR2-DD*(-FIFI/EP*BB*(BB^2/3-NA*BB+NA^2
)+2*BB*(BB/2-NA)+FIFI/EP*NA^3/3+NA^2)+Z*GG*(GG/2*FIFI-NA*FIFI-EP)+FCF*GG-Z*BB*(B
B/2*FIFI-NA*FIFI-EP)-FCF*BB
1880 IF EPSCX > EP AND EPSCX <- EPSR THEN ARMCC-AR1/AR2
1890 IF EPSCX > EPSR THEN ARM1-DD*(-FIFI*BB^2/EP*(BB^2/4-2*NA*BB/3+NA^2/2)+2*BB^
2*(BB/3-NA/2)+FIFI*NA^4/EP/12+NA^3/3)+EE^2*Z*(EE*FIFI/3-NA*FIFI/2-EP/2)+EE^2*FCF
/2-BB^2*Z*(BB/3*FIFI-NA*FIFI/2-EP/2)-BB^2*FCF/2+FR/2*(GG^2-EE^2)
1900 IF EPSCX > EPSR THEN ARM2-DD*(-FIFI/EP*BB*(BB^2/3-NA*BB+NA^2)+2*BB*(BB/2-NA
)+FIFI/EP*NA^3/3+NA^2)+Z*EE*(EE/2*FIFI-NA*FIFI-EP)+FCF*EE-Z*BB*(BB/2*FIFI-NA*FIF
I-EP)-FCF*BB+FR*(GG-EE)
1910 IF EPSCX > EPSR THEN ARMCC-ARM1/ARM2
1920 IF EPSTX <- FTF/(57000!*FCF^.5) THEN ARMCT-NA/3
1930 IF EPSTX > FTF/(57000!*FCF^.5) THEN ARMCT1-57000!*FCF^.5*FIFI*(NA^3/6-NA/2*
(NA-FTF/(57000!*FCF^.5)/FIFI)^2+(NA-FTF/(57000!*FCF^.5)/FIFI)^3/3)+FPF/2*(NA-FTF
/(57000!*FCF^.5)/FIFI)^2
1940 IF EPSTX > FTF/(57000!*FCF^.5) THEN ARMCT2-57000!*FCF^.5*FIFI*(NA^2/2-NA*(N
A-FTF/(57000!*FCF^.5)/FIFI)+(NA-FTF/(57000!*FCF^.5)/FIFI)^2/2)+FPF*(NA-FTF/(5700
0!*FCF^.5)/FIFI)
1950 IF EPSTX > FTF/(57000!*FCF^.5) THEN ARMCT-ARMCT1/ARMCT2
1959 '      PRINT "NA,TC,CC,FS,FSP,ARMCC,ARMCT-";NA;TC;CC;FS;FSP;ARMCC;ARMCT
1960 M(NFI)--TC* ARMCT-CC*ARMCC-FS*(H-D)-FSP*(H-DP)
1970 REM -----------------------------------------------------------
1980 '                  PRINT "FI, M -";FI(NFI); M(NFI)
1990 LINE (FI(NFI-1),M(NFI-1))-(FI(NFI),M(NFI))
2000 IF INT(FIFI/DFIFI)><INT(FIFIMAX/DFIFI/4) AND INT(FIFI/DFIFI)><INT(2*FIFIMAX
/DFIFI/4) AND INT(FIFI/DFIFI)><INT(3*FIFIMAX/DFIFI/4) AND INT(FIFI/DFIFI)><INT(F
IFIMAX/DFIFI-2) THEN 2180
2010 VIEW ((.1*XMAX+FIFI)*640/1.1/XMAX-16,80)-((.1*XMAX+FIFI)*640/1.1/XMAX+16,10
0),,1
2020 WINDOW (-FCF,0)-(FTF,H): LINE (0,0)-(0,H)
2030 FOR X-NA TO H STEP (H-NA)/20
2040 EE-(X-NA)/(H-NA)*ABS(EPSCX)
2050 IF EE<-EP THEN SS--(-FCF*(EE/EP)^2+2*FCF*EE/EP) ELSE IF EE <- EPSR THEN SS-
-(FCF+Z*(EE-EP)) ELSE SS--FR
```

```
2060 IF X=NA THEN LINE (SS,X)-(SS,X) ELSE LINE -(SS,X)
2070 NEXT X
2080 FOR X=NA TO 0 STEP -NA/20
2090 EE=(NA-X)/NA*ABS(EPSTX)
2100 IF EE <= FTF/(57000!*FCF^.5) THEN SS=EE*57000!*FCF^.5 ELSE SS=FPF
2110 IF X=NA THEN LINE (SS,X)-(SS,X) ELSE LINE -(SS,X)
2120 NEXT X
2130 VIEW
2140 WINDOW (-XMAX/10,-YMAX/10)-(XMAX,YMAX)
2150 EPSCU(FIFI)=EPSCX
2160 CCC(FIFI)=CC
2170 TCC(FIFI)=TC
2180 NEXT FIFI
2190 FOR IQ=1 TO NFI
2200 IF IQ=1 THEN MMAX=M(IQ): GOTO 2220
2210 IF M(IQ) > MMAX THEN MMAX=M(IQ):EPSCU0=EPSCU(IQ):CCC0=CCC(IQ):TCC0=TCC(IQ):
FI0=FI(IQ)
2220 NEXT IQ
2230 LOCATE 6,53
2240 PRINT "EPSCU       =";EPSCX
2250 LOCATE 7,53
2260 PRINT "FI (1/in)  =";FI0
2270 LOCATE 3,53
2280 PRINT "AT max Moment";
2290 LOCATE 4,53
2300 PRINT "-------------";
2310 LOCATE 5,53
2320 PRINT "Mmax (K.IN)=";MMAX/1000
2330 LOCATE 8,53
2340 PRINT "CC (Kips)  =";CC/1000
2350 LOCATE 9,53
2360 PRINT "TC (Kips)  =";TC/1000
2370 IF YYNN$="Y" OR YYNN$="y" THEN GOSUB 2520
2380 END
2390 DD=FCF*FIFI/EP
2400 GG=H
2410 BB=EP/FIFI+NA
2420 EPSR=(FR-FCF+Z*EP)/Z
2430 EE=EPSR/FIFI+NA
2440 IF EPSCX<=EP THEN CC=-DD*(-FIFI/EP*GG*(GG^2/3-NA*GG+NA^2)+2*GG*(GG/2-NA)+FI
FI/EP*NA^3/3+NA^2)*B
2450 IF EPSCX>EP AND EPSCX<=EP+(FCF-FR)/Z THEN CC=-DD*(-FIFI/EP*BB*(BB^2/3-NA*BB
+NA^2)+2*BB*(BB/2-NA)+FIFI/EP*NA^3/3+NA^2)*B-(Z*((GG^2/2-NA*GG)*FIFI-EP*GG)+FCF*
GG-Z*((BB^2/2-NA*BB)*FIFI-EP*BB)-FCF*BB)*B
2460 IF EPSCX > EP+(FCF-FR)/Z THEN CC=-DD*(-FIFI/EP*BB*(BB^2/3-NA*BB+NA^2)+2*BB*
(BB/2-NA)+FIFI/EP*NA^3/3+NA^2)*B-(Z*((EE^2/2-NA*EE)*FIFI-EP*EE)+FCF*EE-Z*((BB^2/
2-NA*BB)*FIFI-EP*BB)-FCF*BB)*B-FR*(GG-EE)*B
2470 RETURN
2480 EC=57000!*FCF^.5
2481 FTFEC=FTF/EC
2483 '        PRINT "EPSTX, FTF/EC=";EPSTX;FTFEC
2490 IF EPSTX <= FTFEC THEN TC=NA^2/2*B*EC*FIFI
2500 IF EPSTX > FTFEC THEN TC=FTF*(FTF/EC/FIFI)/2*B+FPF*(NA-FTF/EC/FIFI)*B
2501 '            PRINT "TC,FTF,FPF,EC=";TC;FTF;FR;EC
2510 RETURN
2520 WINDOW (0,0)-(639,199)
2530 WIDTH "LPT1:",255
2540 LINESPACE9$=CHR$(27)+CHR$(65)+CHR$(8)
2550 LINESPACE6$=CHR$(27)+CHR$(65)+CHR$(12)
2560 GRAPH400$=CHR$(27)+CHR$(75)+CHR$(144)+CHR$(1)
```

```
2570 LPRINT LINESPACE9$;
2580 FOR J%=1 TO 9
2590 LPRINT
2600 NEXT J%
2610 FOR COL%=0 TO 79
2620 LPRINT SPACE$(9);
2630 LPRINT GRAPH400$;
2640 FOR ROW%=0 TO 199
2650 GET (8*COL%+7,ROW%)-(8*COL%,ROW%),Z%
2660 LPRINT CHR$(Z%(2))+CHR$(Z%(2));
2670 NEXT ROW%
2680 LPRINT
2690 NEXT COL%
2700 FOR J%=1 TO 10
2710 LPRINT
2720 NEXT J%
2730 LPRINT LINESPACE6$;
2740 RETURN
```