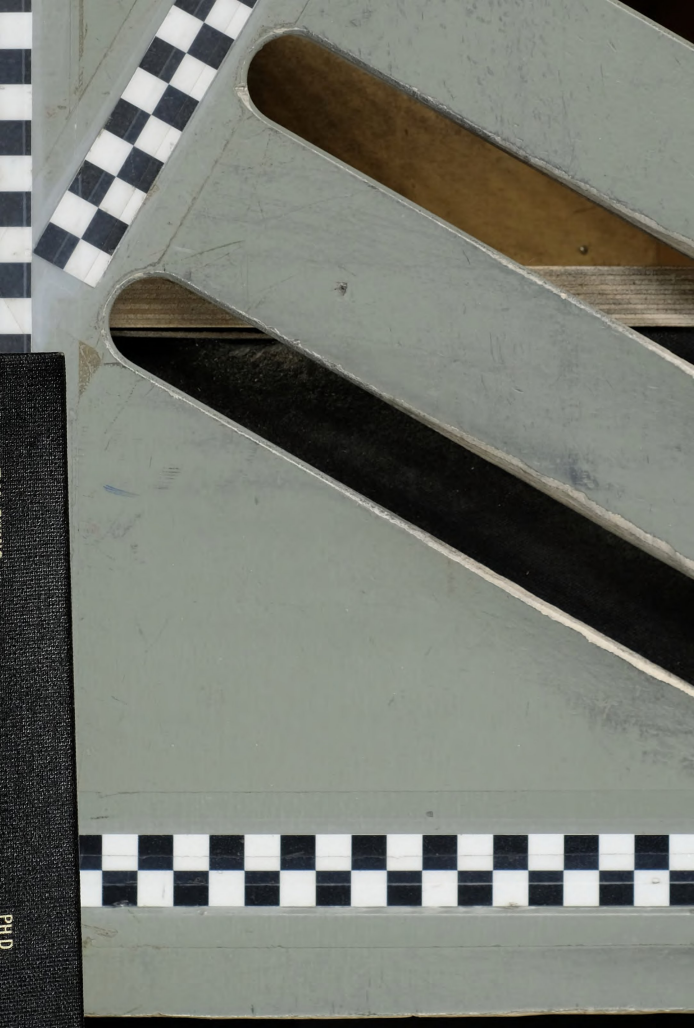




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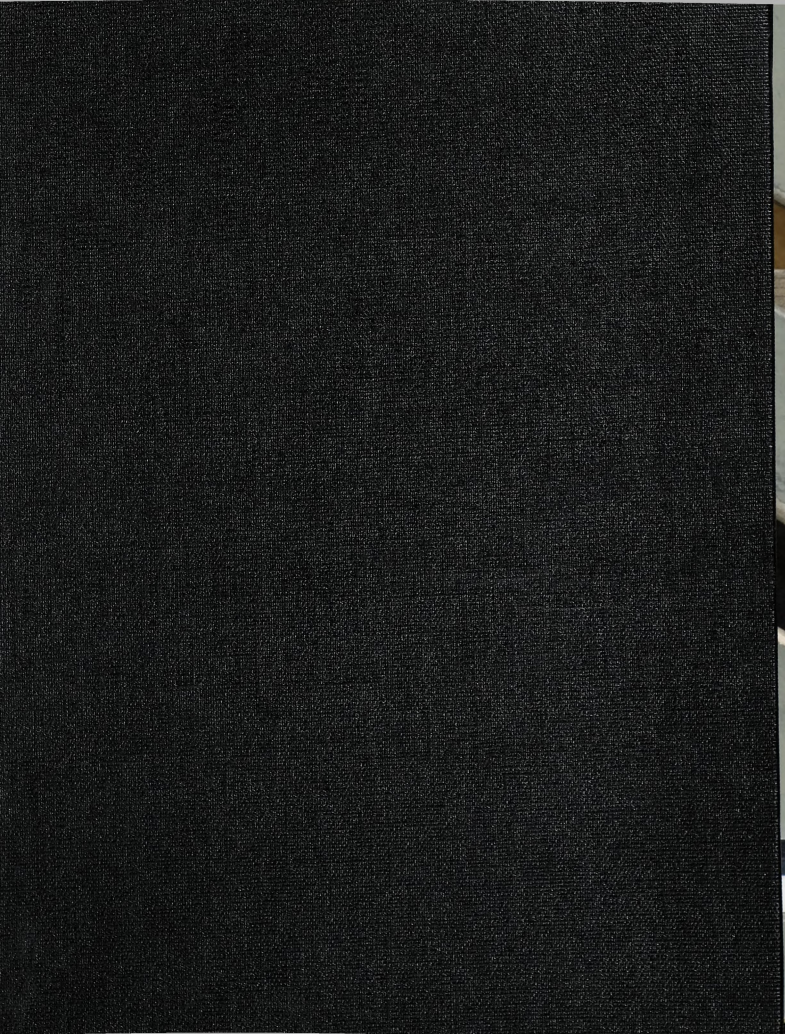
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Final State Scattering in Z Production via the
Quark - Gluon Interaction

By

THOMAS MITCHEL EWING

A DISSERTATION

Presented to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS

1989



ABSTRACT

analytical calculation of photon scattering between final state quark and a final lepton in the process $q g \Rightarrow q \bar{q} q \Rightarrow Z^0 q \Rightarrow l^+ l^- q$. Result is found as ratio to the same process without final state scattering. Motivation for the calculation is as a first step towards calculating photon scattering between final leptons and spectator quarks or debris particles, possibly predicting an observable asymmetry in the Z^0 decay curve.



Dedication

thesis is dedicated to Jon Pumplin for the right suggestions.

ts to Lansing Community College, especially Claude Watson and Melanie
s, for providing their computer for printing this manuscript.

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e main text is written to be read quickly without detail and is thus not lengthy. The proofs and calculations are contained in a bulky set of Appendices which are referenced in the text.

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Chapter One

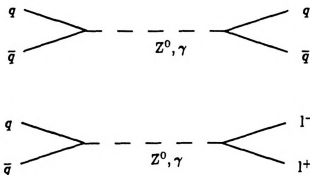
Statement of the Problem

Section 1.1 ... General Introduction

The 1984 status of particle physics is summarised in the comprehensive review article of Eichten, Hinchliffe, Lane, and Quigg (EHLQ)¹. Of the traditional four forces, the gravitational force is largely unincorporated (though since that date more promise has evolved from the further development of string theory). The theory of strong interactions rests comfortably in quantum chromodynamics (QCD), though the experiments are difficult and development is slow. The most remarkable success is the union of the weak and electromagnetic theories via the application of non-Abelian transformation invariance and spontaneous symmetry breaking, introducing the Higgs sector. As described in that article, the investigation of the Higgs sector will await the next generation of accelerators.

With present machines the 1983 verification of the Z^0 and W predicted masses² constituted the first hallmark in the experimental resolution of the electro-weak theory. Following this, the second level of refinement will be experiments extending the precision of the theory, the most recent of these being the $e^+ e^-$ determination of the Z^0 mass and width at SLAC³, and further resolution at FNAL¹¹.

Regarding the production of Z^0 , the zeroth order (Drell-Yan) processes are:



where its interference with the photon process is noted.

The analytic calculation of these amplitudes presents no difficulty and is given in EHLQ¹. Experimentally the measured invariant mass of the final state particles peaks at the Z^0 mass, (and for the case of a quark anti-quark pair would appear as a two jet phenomenon).

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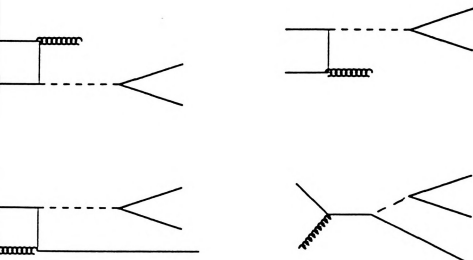
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With the improvement in the experimental precision it is important to do the calculations to first and higher order in perturbation theory.

First order diagrams are:



These calculations are summarized in the workshop of Berger, et al.⁴. The processes, known as 2 particles in and 3 out, represent a degree upwards in the complexity of their analytic solutions. Further, to forward the amplitudes to cross sections, the kinematics of phase space of 2 in and 3 out processes including soft gluon correction of the first two diagrams above is non-trivial. Even so, and in the precision of knowing the Z^0 mass and width, the calculations have been carried out and occupy a standard place in the literature.^{4,5,6}

Particular current interest is great precision in the width of the Z^0 , as this reflects the number of quark and lepton flavors that exist.

In the EHLQ paper:

$$\Gamma_Z = \frac{G_F M_Z^3 \sqrt{2}}{3\pi} \left[1 - 2x_W + \frac{8x_W^2}{3} \right] D$$

where D is the number of kinematically accessible quark and lepton doublets...."

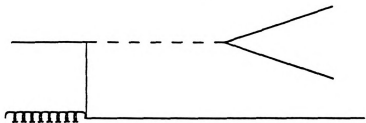
To have faith in first order experimental agreement, it must be held that the processes measured are free from contamination by other processes that would be undetectable in the measurement. For example, the first order contribution of

l^+l^-q :

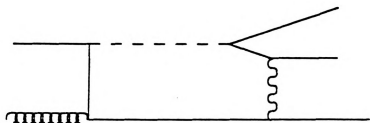
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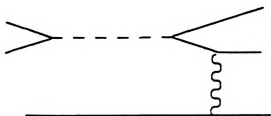
be contaminated by the rescattering of one of the decay leptons with a final state quark.



would cause a change in the invariant mass plot from that inherently caused by the Z^0 decay for measurements made at FNAL, though not those at CERN.

The calculational difficulty of diagrams containing a loop increases dramatically with the number of vertices or 'points'. Three and four point diagrams are standard. The above is a five point diagram. There must be developed a systematic method making these diagrams amenable. 't Hooft and Veltman have devoted a technique paper⁷ to procedures for these integrals. The complexity and large number of terms involved necessitate the aid of algebraic manipulation programs, such as the SCHOONSCHIP program written by van der Meer⁸.

Worse than the rescattering that occurs above, (which is first order), rescattering could appear as a second order contamination via a spectator or debris



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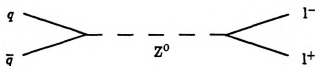
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g the invariant mass plot to an even higher degree. This zeroth order estimation is very hard to estimate as it involves interaction with particles involved in the Z^0 production. An assumption can be made that the ratio of zeroth order rescattering to the zeroth order (Drell-Yan) diagram alone is approximately the same as the above loop diagram to the first order term. So the effect of all rescattering can be estimated by calculating the latter

thesis is the estimation of this effect via the calculation of the latter ratio. This is restricted to final state leptons, as the case for final state quarks is done elsewhere.⁹

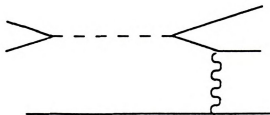
1.2 ... Introduction to this Thesis

At first order, first order Z^0 production can take place according to the usual Drell-Yan process, a quark-antiquark pair annihilating into a lepton pair:



A more accurate assessment of both the Z^0 mass and the width of the decay curve involves a number of questions. To this end it is important to examine possible contributions to shifts of the peak and asymmetric distortions of this width.

The determination of one possible source of contribution begins with the question: WHAT IS THE PROBABILITY OF A LEPTON SCATTERING WITH A SPECTATOR QUARK ON THE WAY OUT ?



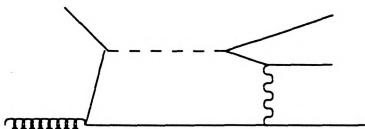
In the next step, one can solve the reduced problem in which the spectator and

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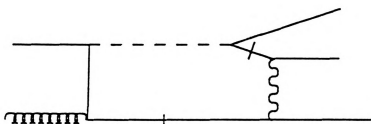
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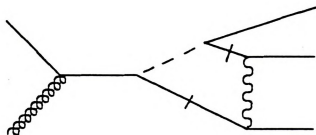
production quarks emerge from a common gluon source:



is the calculation of the above five-point and equivalent diagrams under restriction of two on-shell lines (The introduction of the on-shell restriction is to make the LOOP integrations manageable, as they reduce the dimensionality to two dimensions. It is expected that if the diagrams here examine a small effect under these restrictions, no great enhancement would be seen upon their relaxation. This point is further mentioned in Chapter 5 ... 'Con-



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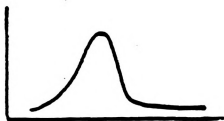
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e obtained, its interference RATIO to the process without scattering

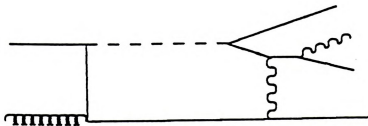
$$\frac{\text{[Feynman diagrams]}^2}{\left| \text{[Feynman diagrams]} + \text{[Feynman diagrams]} \right|^2}$$

produce an observable asymmetry in the Z^0 production curve.



asymmetry would experimentally appear in data gathered at FNAL
in data from SLAC, a comparison of the two would reveal an experi-
limit of resolution of this effect.

original motivation for the project was as a step in explaining some ex-
tial results which later died away - excess photon production in the Z^0
process not explainable by bremsstrahlung. The next step in that now
motivation would have been:



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Diagram diagrams are hard to evaluate and the value of this work includes development of the skills and procedures for working them, independent of particular project. A follow up calculation would replace the scattering with a gluon and final state leptons with quarks, greatly enhancing the ratio in the ratio of the strong coupling constant to $\frac{e^2}{4\pi}$. This is essentially the same calculation except the interference term is zero due to the colorlessness of the BORN and LOOP diagrams, the BORN term being a color singlet and the LOOP a color octet. For this reason, the lepton anti-lepton annihilation calculated here might still dominate, and is also easier to compare with experiments.

The mentioned gluon scattering calculation has already been finished⁹ and the results will be compared with this photon scattering calculation in the 'Conclusions'.

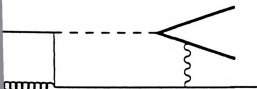
The RATIO is immediately expected to be of order α ($1/137$) because of the two added electromagnetic vertices, this could be considerably improved by the relatively short (10^{-24} sec.) lifetime of the Z^0 keeps the decay products in the locality of the exiting quark and available to the re-scattering.

This is of the general type described by 't Hooft and Veltman, though it uses the use of on-shell approximations, and uses the SCHOONSCHIP program to calculate the TRACES and perform algebraic substitutions.

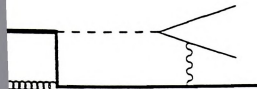
Section 1.3 ... Division of the Problem into Allowable Helicity Combinations

Massless particles only plus or minus projections of the spin onto the momenta exist.

Photon line :



Quark line:



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well known¹² and demonstrated in Appendix A that the final state lepton anti-lepton must have opposing helicities, while the incoming and out-going quarks must have the same helicity.

means for each helicity of the GLUON there are four allowable quark combinations:

	Gluon (*1)	Initial Quark (*1/2)	Final Quark (*1/2)	Lepton (*1/2)	Anti-lepton (*1/2)
	+	+	+	+	-
	+	+	+	-	+
	+	-	-	+	-
	+	-	-	-	+
	-	+	+	+	-
	-	+	+	-	+
	-	-	-	+	-
	-	-	-	-	+

the various helicity states are orthogonal, as are all quantum mechanical set states, interference with a Born helicity state will only be non-zero for OP state of the same helicity.

Therefore the answer is formed separately for each of the eight permitted helicities, labelled A-H in the above table.

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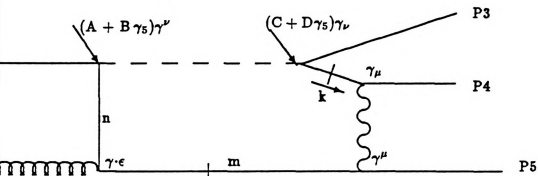
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Section 1.4 Notation

Sub-Section 1.4.1Diagram Notation

The five point lepton scattering diagram will serve to illustrate the notation used for all diagrams in the problem.



The linear momenta are labelled clockwise $P1....P5$ beginning with the GLUON. m is defined as the variable LOOP momentum and for convenience:

$$m = k - (P4 + P5)$$

$$n = k - (P4 + P5) + P1 = m + P1$$

For vector dot products are indicated by the contiguous writing of two four vectors, for example, $P4P5$ indicates the dot product of $P4$ with $P5$.

The space part of a four vector is indicated by a bar; e.g. $\bar{P4}$ is the space part of $P4$.

Sub-Section 1.4.2 The Metric

The metric used is

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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and the energy-momentum four vector-

$$P^\mu = E_0, P_x, P_y, P_z$$

$$P_\mu = -E_0, P_x, P_y, P_z$$

The dot products in this metric are negative:

$$P^\mu P_\mu = -m^2$$

This metric is consistent with the Minkowski metric $P^\mu = P_\mu = (P_x, P_y, P_z, iE_0)$ which is used by the SCHOONSCHIP program.

Sub-Section 1.4.3 Gamma Matrices

The original Dirac equation,

$$(\alpha \cdot p - E + \beta \cdot m)\psi = 0$$

to be consistent with the Klein-Gordan equation required that

$$\alpha_i \alpha_j + \alpha_j \alpha_i = 2\delta_{ij}$$

$$\alpha^2 = \beta^2 = 1$$

$$\beta \alpha_i + \alpha_i \beta = 0$$

and α and β must be Hermetian.

The Dirac equation may be written in terms of Gamma matrices for any metric provided they and their dot products with p_μ reproduce the original Dirac equation, leading to

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$$

and further defining

$$\gamma_5 = i\alpha_1\alpha_2\alpha_3$$

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the metric used here, the Gamma matrices, defined in terms of the Dirac matrices, become:

$$\gamma^i = i\beta\alpha_i$$

$$\gamma^0 = i\beta$$

and change the Dirac equation to be:

$$(i\gamma \cdot p - m)\psi = 0$$

becomes:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

the Minkowski metric used by SCHOONSCHIP:

$$\gamma^i = i\beta\alpha_i$$

$$\gamma^4 = -\beta$$

with the same Dirac equation as above:

$$(i\gamma \cdot p - m)\psi = 0$$

becomes:

$$\gamma_5 = \gamma^1\gamma^2\gamma^3\gamma^4$$

for further detail and comparisons are in Appendix B.

Section 1.4.4 The *Epf* Function

In particular the Gamma matrix set used by SCHOONSCHIP introduces a function *Epf* defined as

$$TRACE(\gamma_5 \not{A} \not{B} \not{C} \not{D}) = i4\epsilon_{\alpha\beta\gamma\delta} A^\alpha B^\beta C^\gamma D^\delta = 4 \text{ Epf}(A, B, C, D)$$

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where $\epsilon_{\alpha\beta\gamma\delta}$ is the usual Levi-Cevita totally anti-symmetric four tensor.

the Minkowski metric is used by SCHOONSCHIP, then

$$P^\mu = P_\mu = (P_x, P_y, P_z, iE_0)$$

and the Epf function is always a pure imaginary number.

For this problem, as there are only four independent linear momenta, all Epf functions, following the usual properties of Levi-Cevita tensors, must reduce to only one Epf :

$$Epf(P_1, P_3, P_4, P_5)$$

This is a Lorentz invariant and may be calculated directly in any convenient reference frame and co-ordinate system, or by the usual evaluation of Levi-Cevita tensor products via determinants, which of course gives the same result.

These are both done in Appendix C, and the result is

$$Epf(P_1, P_3, P_4, P_5) = [4(P_1P_4)(P_3P_5)(P_1P_5)(P_3P_4) - [(P_1P_4)(P_3P_5) + (P_1P_5)(P_3P_4) - (P_1P_3)(P_4P_5)]^2]^{1/2}$$



Section 1.5 Preparing the Integrals

Sub-Section 1.5.1 The Integral

The Cutowsky rule for an on-shell condition gives:

$$\frac{1}{(k^2) (k - (P_4 + P_5))^2} \Rightarrow \frac{1}{2} (2\pi i)^2 \delta(k^2) \delta([k - (P_4 + P_5)]^2)$$

With this, the amplitude of the sample diagram of the previous section is formed by the Feynman rules of Appendix D:

$$M = S * I$$

where S collects all the multiplicative scale factors:

$$S = \frac{1}{2} (2\pi i)^2 e^2 \frac{1}{(2\pi)^4}$$

and I is the LOOP integral of the above sample diagram:

$$\int \frac{d^4 k \delta(k^2) \delta([k - P_4 - P_5]^2) [\bar{u}(P_4) \gamma_\mu (\gamma \cdot k) (C + D \gamma_5) \gamma_\nu v(P_3)] [\bar{u}(P_5) \gamma^\mu (\gamma \cdot m) (\gamma \cdot \epsilon) (\gamma \cdot n) (A + B \gamma_5) \gamma^\nu u(P_2)]}{[(k - P_4)^2] [(k - P_4 - P_5 + P_1)^2] [(k + P_3)^2 + M_s^2 - i\Gamma M_s]}$$

Sub-Section 1.5.2 The Frame of Reference

The first delta function will dictate that $|\vec{k}|^2 = k_0^2$.

The second, $\delta([k - (P_4 + P_5)]^2)$, gives

$$k^2 - 2k \cdot (P_4 + P_5) + (P_4 + P_5)^2 = 0$$

with

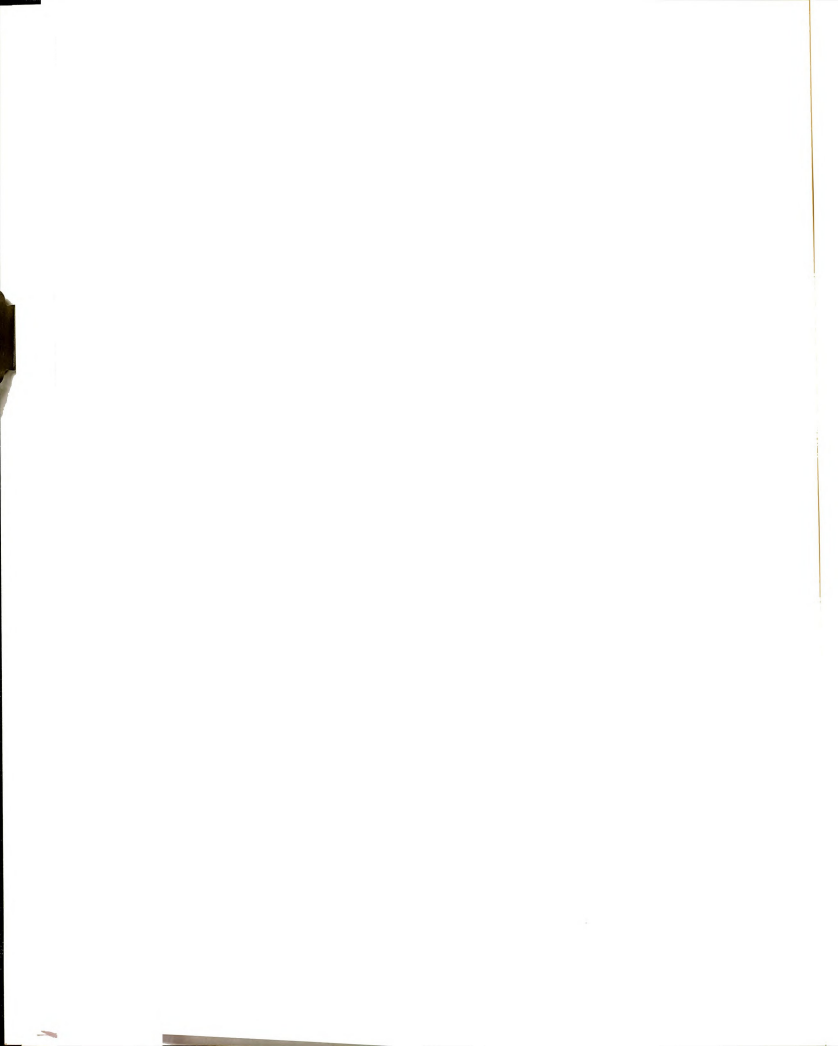
$$k^2 = P_4^2 = P_5^2 = 0$$

ults in

$$k \cdot (P_4 + P_5) = P_4 P_5$$

which assumes a particularly simple form only in the rest frame of $\overline{P_4}$ and $\overline{P_5}$:

$$k_0(E_4 + E_5) = -P_4 P_5$$



d, since also in this frame:

$$(P4 + P5)^2 = -(E4 + E5)^2 = 2(P4P5)$$

en

$$k0 = E4 = E5 = (-P4P5/2)^{1/2}$$

ablishing that in this frame $k0$ is CONSTANT.

the enormous simplification gained when the magnitude of $\vec{k}$ remains constant states this rest frame of $\vec{P4} + \vec{P5}$ to be the frame of reference for this diagram. The diagram for anti-lepton scattering is evaluated by interchanging the labels $P3$ and $P4$ (except for the lepton line trace), the same frame of reference holds throughout the problem.

te that in this frame the energy, E_x , of any other particle, PX , of the problem is also simply given:

$$PX \cdot (P4 + P5) = -2(k0)E_x = PXP4 + PXP5$$

$$E_x = \frac{PXP4 + PXP5}{-2k0}$$

b-Section 1.5.3 Removal of the Delta functions

delta functions may be simply removed by integrating over $d|\vec{k}|$ and $dk0$ provided the JACOBIAN between the set $(|\vec{k}|, k0)$ and the two arguments of the delta functions is included. This Jacobian is calculated in the chosen frame of reference in Appendix F and evaluates to:

$$J = \frac{-1}{4(P4P5)}$$

Jacobian is now absorbed into the scale factor S :

$$S = \frac{1}{2} (2\pi i)^2 e^2 \left[\frac{1}{(2\pi)^4} \right] \frac{-1}{4(P4P5)}$$

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the remaining integration over $|k|^2 d\Omega_k$,

$$|k|^2 = \frac{-P_4 P_5}{2} = \text{constant}$$

can come outside the integral sign and be absorbed into the scale factor:

$$\begin{aligned} S &= \frac{1}{2} (2\pi i)^2 e^2 \left[\frac{1}{(2\pi)^4} \right] \frac{-1}{4(P_4 P_5)} \left[\frac{-P_4 P_5}{2} \right] \\ &= - \left(\frac{1}{137} \right) \left(\frac{1}{16\pi} \right) \end{aligned}$$

thus the sample LOOP integral reduces to being over the directions of $\vec{k}$:

$$= \int \frac{d\Omega_k}{[(k - P_4)^2] [(k - P_4 - P_5 + P_1)^2] [(k + P_3)^2 + M_z^2 - i\Gamma M_z]} \frac{[\bar{u}(P_4)\gamma_\mu(\gamma \cdot k)(C + D\gamma_5)\gamma_{\nu\rho}(P_3)] [\bar{u}(P_5)\gamma^\mu(\gamma \cdot m)(\gamma \cdot \epsilon)(\gamma \cdot n)(A + B\gamma_5)\gamma^\nu u(P_2)]}{}$$

with the understanding that

the frame of reference is $\overline{P_4} + \overline{P_5} = 0$

$$) |\vec{k}|^2 = k_0^2$$

$$) k_0 = (-P_4 P_5 / 2)^{1/2}$$

$$) E_x = \frac{P_X P_4 + P_X P_5}{-2k_0}$$

is applies to all the various LOOP integrals in the problem.

b-Section 1.5.3 The Divergence Regulation

Examining the above LOOP integral, the first denominator factor, arising from the photon line propagator, produces a singularity when the direction of k becomes co-linear with P_4 , resulting in a logarithmically infinite value for the integral. This infinity does not occur in Nature - the process is finite - and that is reflected in the calculation by the coefficient of the logarithmically infinite

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term becoming zero when the five point and the four point LOOP diagrams are summed. To allow the doing of the diagrams separately, a small photon mass is added to regularize the infinity. When the diagrams are added it is observed that the coefficient of the log of this fictitious photon mass becomes identically zero, producing no ill effects when the photon mass is then allowed to go to zero and its log to infinity. The appearance of the zero coefficient does in fact serve as a check that the various parts of the calculation are fitting together correctly.

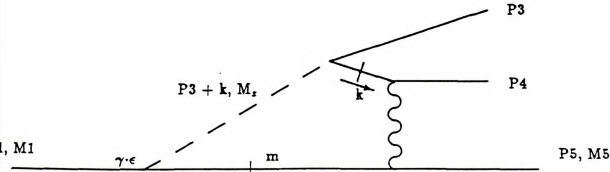
$$I = \int \frac{d\Omega_k [\bar{u}(P4)\gamma_\mu(\gamma \cdot k)(C + D\gamma_5)\gamma_\nu v(P3)] [\bar{u}(P5)\gamma^\mu(\gamma \cdot m)(\gamma \cdot \epsilon)(\gamma \cdot n)(A + B\gamma_5)\gamma^\nu u(P2)]}{[(k - P4)^2 + M_z^2] [(k - P4 - P5 + P1)^2] [(k + P3)^2 + M_z^2 - i\Gamma M_z]}$$

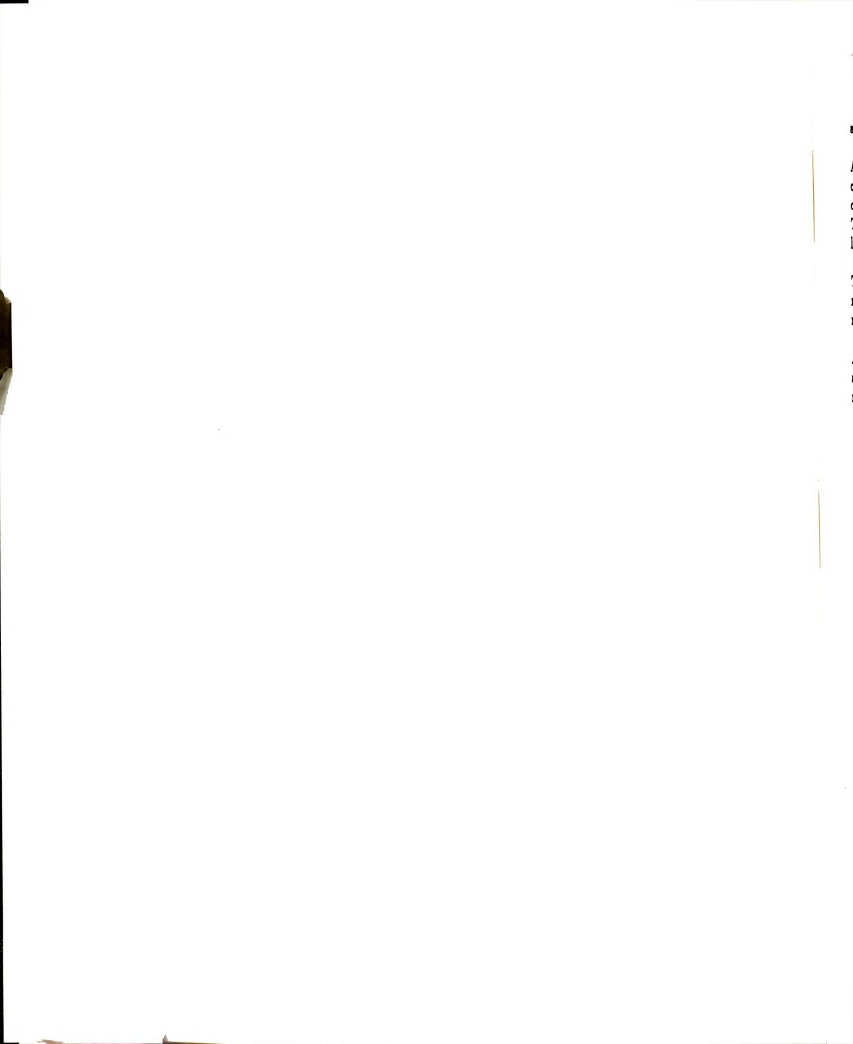
Passing to the second denominator factor $(k - (P4 + P5) - P1)^2$, it appears to possess a similar infinity. It is shown in Appendix G that at the very direction of $\vec{k}$ that this zero is produced, the value of the numerator also becomes zero, rendering the singularity benign. The non-appearance of this infinity also serves as a check that the integral calculation is proceeding correctly. A small quark mass is carried to allow this check.

$$I = \int \frac{d\Omega_k [\bar{u}(P4)\gamma_\mu(\gamma \cdot k)(C + D\gamma_5)\gamma_\nu v(P3)] [\bar{u}(P5)\gamma^\mu(\gamma \cdot m)(\gamma \cdot \epsilon)(\gamma \cdot n)(A + B\gamma_5)\gamma^\nu u(P2)]}{[(k - P4)^2 + M_z^2] [(k - P4 - P5 + P1)^2 + M_{quark}^2] [(k + P3)^2 + M_z^2 - i\Gamma M_z]}$$

Section 1.6 Evaluating a Preliminary Diagram

Prior to beginning the actual calculation it is expedient to calculate a preliminary diagram which contains all the essential features of the larger calculation, yet is simplified sufficiently that it may be evaluated analytically (without the aid of SCHOONSCHIP or projection techniques)





and the companion anti-lepton scattering diagram.

All the external fermion particles carry Spin zero, which makes them unphysical, yet experience has shown that Spin does not greatly affect the magnitude of the calculation.

The result of these diagrams should serve as an estimation for the larger calculation, and should that calculation differ significantly, the causes ascertained.

These diagrams are detailed and completely evaluated in Appendix W. The results will be stated in Chapter 5 ... 'Conclusions', and compared with the results of the larger calculation.

An advantage occurs from the simplicity of this diagram: it is possible to assign a non-zero mass to the out-going quark, and to see the effect of varying this mass on the result. The effect is also described in Chapter 5.

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Chapter Two

The Calculation

Section 2.1 Introduction

As the result will be the ratio of the LOOP plus BORN cross-section to the BORN cross-section, two calculations are needed. The first, the Born amplitude, is a tree structure and does not involve an integral. The calculation will highlight a trick for projecting the spinor product of different linear momenta, i.e. $\bar{u}(P3)v(P4)$, onto a basis set more amenable to calculation, and will introduce a projection of the gluon polarization vector onto a chosen set of linear momenta; both these projections will also be used in the more complicated LOOP calculation. Other than these projections, the Born term calculation is straight-forward.

The LOOP calculation, which of course involves an integration, will use the same spinor replacement technique, which then allows the TRACE of the numerator expressions to be taken. Then the same replacement of the polarization vector is made, resulting in an expression of several hundred terms, each of which must be integrated.

Some of these terms contain a factor of $Ep f(k, A, B, C)$ in the numerator which would make the integration over directions of $\vec{k}$ quite difficult. Improvement in these terms is obtained by replacing k_μ in all $Ep f$ factors with a linear expansion onto four momenta of the problem, after which all $Ep f$'s become $Ep f(P1, P3, P4, P5)$ which of course is the only independent $Ep f$ in the problem, and since it no longer depends on k_μ , can come outside the integral.

The approximately 50 remaining terms in the LOOP integral can be classified into integrals based on the number - from 0 to 3 - of $PX \cdot k$ factors in the numerator and the number of similar denominator factors... three for the five point diagrams, two for the four point. Each integral may then be reduced to a simpler integral through a process of adding and subtracting appropriate terms to allow some of these $PX \cdot k$ factors to cancel with denominator factors.

This reduction results in a set of 19 integrals which can not then be further reduced and must be solved analytically.

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In this manner a single complex number is produced for the LOOP amplitude for the chosen helicity state and set of physical parameters. This is added to the Born amplitude, an absolute square taken, and the RATIO to the absolute square of the Born term determined.

This is repeated for all physically allowable sets of input parameters and helicity states.

Section 2.2 ... Calculating the BORN Term

Sub-Section 2.2.1Introduction

The Born term is the sum of two diagrams, called the five point and the four point, by analogy with their counterparts in the LOOP calculation.



Sub-Section ... 2.2.2 The BORN Amplitude

$$BORN = BORN_{five} + BORN_{four}$$

The five point amplitude is:

$$BORN_{five} = \frac{[\bar{u}(P4)(C + D\gamma_5)\gamma_\nu v(P3)] [\bar{u}(P5)(\gamma \cdot \epsilon)(\gamma \cdot P5 - \gamma \cdot P1)(A + B\gamma_5)\gamma^\nu u(P2)]}{-2(P1P5)MZP}$$

$$BORN_{four} = \frac{TRACE[v(P3)\bar{u}(P4)(C + D\gamma_5)\gamma_\nu] TRACE[u(P2)\bar{u}(P5)(\gamma \cdot \epsilon)(\gamma \cdot P5 - \gamma \cdot P1)(A + B\gamma_5)\gamma^\nu]}{-2(P1P5)MZP}$$

where MZP is the Z^0 particle propagator:

$$MZP = 2(P3P4) + M_z^2 - i\Gamma M_z$$

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and the four point amplitude is:

$$BORN_{four} = \frac{[\bar{u}(P4)(C + D\gamma_5)\gamma_\nu v(P3)] [\bar{u}(P5)(A + B\gamma_5)\gamma_\nu(\gamma \cdot P1 + \gamma \cdot P2)(\gamma \cdot \epsilon)u(P2)]}{2(P1P2)M Z P}$$

$$BORN_{four} = \frac{TRACE[v(P3)\bar{u}(P4)(C + D\gamma_5)\gamma^\nu] TRACE[u(P2)\bar{u}(P5)(A + B\gamma_5)\gamma^\nu(\gamma \cdot P1 + \gamma \cdot P2)(\gamma \cdot \epsilon)]}{2(P1P2)M Z P}$$

Sub-Section 2.2.3Projecting the SPINOR product

In Appendix H it is demonstrated that a spinor product of the form $v(p)\bar{u}(q)$ may be replaced by $(1 \pm \gamma_5)\gamma \cdot A$ where A is a four vector given by

$$A_\mu = \frac{(p \cdot s)q_\mu + (q \cdot s)p_\mu - (p \cdot q)s_\mu \pm E p f(\mu, q, p, s)}{\sqrt{16(p \cdot s)(q \cdot s)}}$$

where s_μ is an arbitrary four vector of the problem, and the plus-minus signs are determined by the combination of helicities.

This permits the substitutions:

$$v(P3)\bar{u}(P4) = (1 \pm \gamma_5)\gamma \cdot B$$

and

$$u(P2)\bar{u}(P5) = (1 \pm \gamma_5)\gamma \cdot CU$$

where

$$B_\mu = \frac{(P3P5)P4_\mu + (P4P5)P3_\mu - (P3P4)P5_\mu \pm E p f(\mu, P3, P4, P5)}{\sqrt{16(P4P5)(P3P5)}}$$

and

$$CU_\mu = \frac{(P1P2)P5_\mu + (P1P5)P2_\mu - (P2P5)P1_\mu \pm E p f(\mu, P1, P2, P5)}{\sqrt{16(P1P2)(P1P5)}}$$

which enable the SCHOONSCHIP program of Appendix I to easily calculate the TRACES.

Sub-Section 2.2.4Removal of Z^0 vertex $(A + B\gamma_5)$ and strong coupling constant

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The above introduction of the $(1 \pm \gamma_5)$ factors into the TRACES nullifies the presence of the Z^0 vertex coefficient $(A + B\gamma_5)$ as demonstrated in Appendix E. The algebra of Gamma matrices causes the γ_5 in the coefficient to become redundant and the remaining factor $(A - B)(A + B)(C + D)(C - D)$ can come outside the TRACE process where it simply cancels with the same factor that will later come outside the LOOP integration before the final ratio is taken. For this reason the Z^0 vertex coefficient does not affect the ratio and will not further be carried in the calculation of either the Born or the LOOP terms. This cancellation is also true of the GLUON vertex coefficient, which will thus also not be carried. This is quite desirable as it removes any presence of the strong coupling constant with all its attendant uncertainties from the result.

Sub-Section 2.2.5 ... Projection of the Gluon Polarisation vector

Next the polarization vector of the gluon is replaced by

$$\epsilon_\mu = \frac{(P_1 P_2) P_{5\mu} + (P_1 P_5) P_{2\mu} \pm (Epf(\mu, P_1, P_2, P_5))}{\sqrt{(-P_1 P_2)(P_1 P_5)(P_2 P_5)}}$$

where $\pm$ is the GLUON-HELICITY. This replacement is derived in Appendix J.

Sub-Section 2.2.6 The BORN Term result

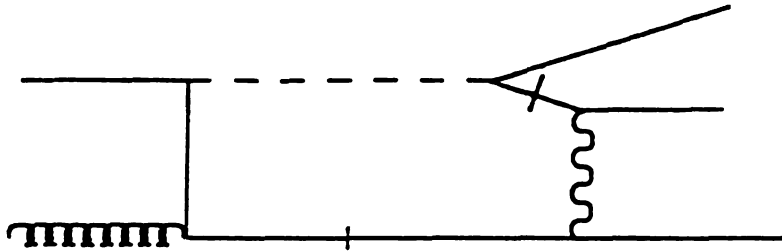
The Born term amplitude is now entirely evaluated in terms of the set of algebraic dot products that form the set of physical parameters for this problem. The SCHOONSCHIP program of Appendix I outputs this expression in a FORTRAN compatible form. It is then incorporated into a FORTRAN program which will use it to determine a single complex number- the BORN amplitude- for each physically permissible set of parameters for the chosen helicity state.

Section 2.3 ... Calculating the LOOP diagrams

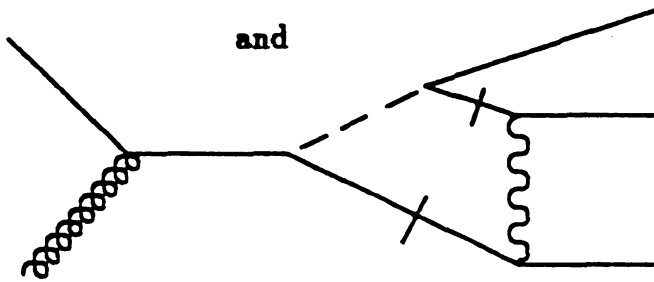
Sub-Section 2.3.1 ...Introduction

The LOOP term is the sum of the four diagrams:

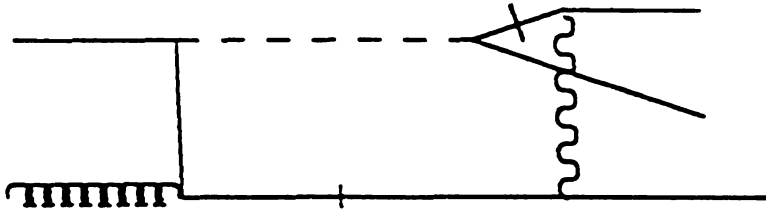
those for lepton scattering:



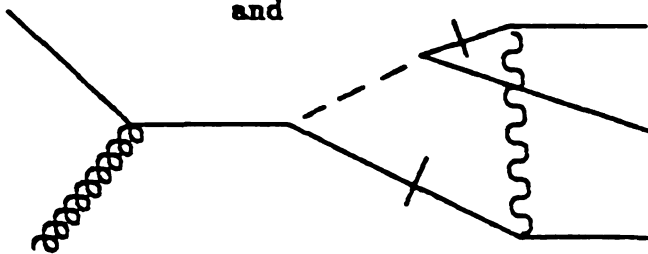
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and those for anti-lepton scattering:



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Once the lepton scattering diagrams are solved, the anti-lepton scattering ones are obtainable from that solution largely by re-labelling the linear momenta, (except in the lepton TRACE itself). Thus it is sufficient to demonstrate the lepton solutions only:

$$I_{loop} = I_{five} + I_{four}$$

$$I_{five} = \int \frac{d\Omega_k \text{ TRACE} [v(P3)\bar{u}(P4)\gamma_\mu(\gamma \cdot k)(C + D\gamma_5)\gamma_\nu] \text{ TRACE} [u(P2)\bar{u}(P5)\gamma^\mu(\gamma \cdot m)(\gamma \cdot \epsilon)(\gamma \cdot n)(A + B\gamma_5)\gamma^\nu]}{[(k - P4)^2 + M_s^2] [(k - P4 - P5 + P1)^2 + M_{quark}^2] [(k + P3)^2 + M_s^2 - i\Gamma M_s]}$$

$$I_{four} = \frac{1}{2P1P2} \int \frac{d\Omega_k \text{ TRACE} [v(P3)\bar{u}(P4)\gamma_\mu(\gamma \cdot k)(C + D\gamma_5)\gamma_\nu] \text{ TRACE} [u(P2)\bar{u}(P5)\gamma^\mu(\gamma \cdot m)(A + B\gamma_5)\gamma^\nu(\gamma \cdot \epsilon)]}{[(k - P4)^2 + M_s^2] [(k + P3)^2 + M_s^2 - i\Gamma M_s]}$$

Sub-Section 2.3.2 ... Spinor and Polarisation Replacement, Dropping of the Z^0 vertex coefficient $(A + B\gamma_5)$ and strong coupling constant

In each integral the same spinor four vector replacements as used for the BORN term are made, with the concurrent dropping of the Z^0 vertex coefficient $(A + B\gamma_5)$, and the GLUON vertex strong coupling constant.

$$I_{five} = \int \frac{d\Omega_k \text{ TRACE} [(1 \pm \gamma_5)(\gamma \cdot B)\gamma_\mu(\gamma \cdot k)\gamma_\nu] \text{ TRACE} [(1 \pm \gamma_5)(\gamma \cdot CU)\gamma^\mu(\gamma \cdot m)(\gamma \cdot \epsilon)(\gamma \cdot n)\gamma^\nu]}{[(k - P4)^2 + M_s^2] [(k - P4 - P5 + P1)^2 + M_{quark}^2] [(k + P3)^2 + M_s^2 - i\Gamma M_s]}$$

$$I_{four} = \frac{1}{2P1P2} \int \frac{d\Omega_k \text{ TRACE} [(1 \pm \gamma_5)(\gamma \cdot B)\gamma_\mu(\gamma \cdot k)\gamma_\nu] \text{ TRACE} [(1 \pm \gamma_5)(\gamma \cdot CU)\gamma^\mu(\gamma \cdot m)\gamma^\nu(\gamma \cdot \epsilon)]}{[(k - P4)^2 + M_s^2] [(k + P3)^2 + M_s^2 - i\Gamma M_s]}$$

TRACES are taken by the SCHOONSCHIP program listed in Appendix K, followed by substitution of the spinor and polarization four vectors, again as in the Born terms.

Sub-Section 2.3.3 ... Linear Replacement of k_μ

The result contains factors not present in the simpler Born calculation, due to the presence here of the LOOP momentum k_μ . These factors are of the form $\mathcal{P}f(k, A, B, C)$, and an attempt to integrate with them as they stand would be very unwieldy. Improvement is obtained by making a linear projection of k_μ in such factors onto a set of four chosen linear momenta of the problem - $S1_\mu, S2_\mu, S3_\mu, S4_\mu$.

$$k_\mu = (C1)S1_\mu + (C2)S2_\mu + (C3)S3_\mu + (C4)S4_\mu$$

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The $C1, C2, C3, C4$ can be found by forming the set of four dot products with μ :

$$S1 \cdot k = (C1)S1S1 + (C2)S2S1 + (C3)S3S1 + (C4)S4S1$$

$$S2 \cdot k = (C1)S1S2 + (C2)S2S2 + (C3)S3S2 + (C4)S4S2$$

$$S3 \cdot k = (C1)S1S3 + (C2)S2S3 + (C3)S3S3 + (C4)S4S3$$

$$S4 \cdot k = (C1)S1S4 + (C2)S2S4 + (C3)S3S4 + (C4)S4S4$$

This set of four equations can be inverted to yield $C1, C2, C3, C4$ in terms of the various dot products. This is done in Appendix L and yields:

$$C1 = CC11(S1 \cdot k) + CC12(S2 \cdot k) + CC13(S3 \cdot k) + CC14(S4 \cdot k)$$

$$C2 = CC21(S1 \cdot k) + CC22(S2 \cdot k) + CC23(S3 \cdot k) + CC24(S4 \cdot k)$$

$$C3 = CC31(S1 \cdot k) + CC32(S2 \cdot k) + CC33(S3 \cdot k) + CC34(S4 \cdot k)$$

$$C4 = CC41(S1 \cdot k) + CC42(S2 \cdot k) + CC43(S3 \cdot k) + CC44(S4 \cdot k)$$

with $CC11, CC12, CC13, \dots, CC43, CC44$ given in terms of the dot products of external momenta, again in Appendix L

An auxiliary arrangement of the same result is:

$$k_\mu = (S1 \cdot k)\mathcal{F}1_\mu + (S2 \cdot k)\mathcal{F}2_\mu + (S3 \cdot k)\mathcal{F}3_\mu + (S4 \cdot k)\mathcal{F}4_\mu$$

where

$$\mathcal{F}1_\mu = (CC11)S1_\mu + (CC12)S2_\mu + (CC13)S3_\mu + (CC14)S4_\mu$$

$$\mathcal{F}2_\mu = (CC21)S1_\mu + (CC22)S2_\mu + (CC23)S3_\mu + (CC24)S4_\mu$$

$$\mathcal{F}3_\mu = (CC31)S1_\mu + (CC32)S2_\mu + (CC33)S3_\mu + (CC34)S4_\mu$$

$$\mathcal{F}4_\mu = (CC41)S1_\mu + (CC42)S2_\mu + (CC43)S3_\mu + (CC44)S4_\mu$$

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This, as well as being useful later, permits self consistency checks to be made on the calculation of CC11, CC12,....CC44 as:

$$S(\alpha)^\mu \mathcal{F}(\beta)_\mu = \delta_\alpha^\beta$$

which is obtained by dotting the above expression for k_μ with $S1, S2, S3, S4$.

$$S1^\mu k_\mu = (S1^\mu \mathcal{F}1_\mu)(S1 \cdot k) + (S1^\mu \mathcal{F}2_\mu)(S2 \cdot k) + (S1^\mu \mathcal{F}3_\mu)(S3 \cdot k) + (S1^\mu \mathcal{F}4_\mu)(S4 \cdot k)$$

and noting that $S1 \cdot k, S2 \cdot k, S3 \cdot k, S4 \cdot k$ are all independent.

For this problem:

$$S1_\mu = P1_\mu$$

$$S2_\mu = P3_\mu$$

$$S3_\mu = P4_\mu$$

$$S4_\mu = P5_\mu$$

It is to be noted that in the Appendix L inversion, the condition for a valid inversion - that the Wronskian determinant be non zero - is the same as will later be required in the RESULTS Section for a set of parameters to be physically acceptable and thus does not introduce a new restriction on the problem.

Sub-Section 2.3.4 ... Calculating the Remaining Set of Integrals

With the replacement in the Epf functions of k_μ by this linear projection onto $P1, P3, P4, P5$, all Epf 's reduce to $Epf(P1, P3, P4, P5)$ times some number of $PX \cdot k$ factors. The $Epf(P1, P3, P4, P5)$ previously evaluated in Appendix C comes through the integral sign and the terms remaining in the integration number about 30. They can be classified according to the number of numerator $PX \cdot k$ factors they contain.... from 0 to 3 for the five point integral and from 0 to 2 for the four point, reflecting the number of factors of k_μ in the original numerator.

Each class, which is an integral, can be reduced to a simpler integral by the process of adding and subtracting pieces which produce cancellation of a denominator factor. This reduction is performed in Appendix M, and results in a set of 19 integrals which can not then be further reduced and each must be solved analytically.

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The grouping and labelling of these 19 integrals is the last thing done by the SCHOONSCHIP program, which then produces FORTRAN compatible output of the calculation as it now stands, with each of the 19 integrals algebraically labelled by a symbol.

This output is incorporated into the FORTRAN program already containing the BORN Term result.

Sub-Section 2.3.5 Solving the 19 integrals

The 19 integrals are solved analytically in Appendix N and the solutions are inserted into the above mentioned FORTRAN program as a called sub-routine.

Sub-Section 2.3.6The LOOP term result

At this point expressions solely in terms of dot products of external momenta and the plus/minus sign of the $Epf(P1, P3, P4, P5)$ parameter have been inserted into the FORTRAN program for the LOOP term. For each physically allowable set of such parameters it will reduce to a single complex number.

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Section 2.4 ... Calculating the RATIO

Sub-Section 2.4.1 Introduction

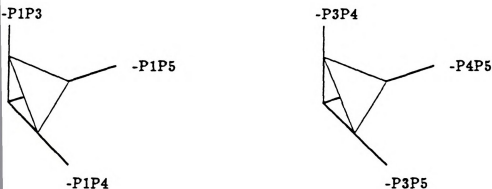
The FORTRAN program to generate the result is run separately for each of the eight helicity states of the problem, identified by the letters A-H as per the table in Section 1.2. The identifying letter is entered into the FORTRAN program as a parameter before compilation time.

Due to their size (each runs over 1600 lines of FORTRAN code), the two outputs of the SCHOONSCHIP LOOP code, one for LEPTON scattering, the other for ANTI-LEPTON, (each combining internally a five and a four point diagram), are compiled separately as FORTRAN functions and joined to the above FORTRAN program at LINK time.

The FORTRAN program contains permanently, (as functions), the eight output amplitudes of the SCHOONSCHIP BORN TERM code, one for each helicity state, and as a called sub-routine, the analytic solutions of the nineteen integrals.

Sub-Section 2.4.2 Physically Valid Set of Input Parameters

The program operates by examining in grid fashion all of phase space. It is demonstrated in Appendix P that all allowable values of the six dot products may be visualised as lying on triangular surfaces: (remembering that dot products in this metric are negative)



Each corner of the triangles intersects the axis at $\frac{E_{cm}^2}{2}$.

These points are covered in grid fashion in the FORTRAN program by allowing:

$$0 < -P1P4 < \frac{E_{cm}^2}{2}$$

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$$0 < -P_1P_5 < \frac{E_{cm}^2}{2} - (-P_1P_4)$$

$$-P_1P_3 = \frac{E_{cm}^2}{2} - (-P_1P_4) - (-P_1P_5)$$

and

$$0 < -P_3P_4 < \frac{E_{cm}^2}{2}$$

$$0 < -P_3P_5 < \frac{E_{cm}^2}{2} - (-P_3P_4)$$

$$-P_4P_5 = \frac{E_{cm}^2}{2} - (-P_3P_4) - (-P_3P_5)$$

The actual E_{cm} value of 120 GeV is, for ease of calculation, scaled down to 1.2 GeV, with the corresponding scaling of M_z to .937 GeV and Γ_z to .027 GeV. As the result is a dimensionless RATIO, it is unchanged by scaling.

The further restriction, that only certain combinations of input-output four vectors conserve separately all four components of energy-momenta (that is - only certain directional cones are allowed), is represented here by the requirement that P_1 , be real, which is that-

$$4(P_1P_4)(P_3P_5)(P_1P_5)(P_3P_4) > [(P_1P_4)(P_3P_5) + (P_1P_5)(P_3P_4) - (P_1P_3)(P_4P_5)]^2$$

again derived in Appendix P.

If the selected set of six dot products passes this inequality test, the FORTRAN program forwards it for processing. It processes both signs of $Epf(P_1, P_3, P_4, P_5)$, representing reflection of the output particles across the plane defined by the two input particles. (In the FORTRAN program $Epf(P_1, P_3, P_4, P_5)$ is labelled by the symbol EVL.)

Sub-Section 2.4.3 ... Processing the Point

The parameter labelling the helicity state is examined and the proper BORN TERM function called to produce that amplitude. It contains both four and five point diagram contributions, each multiplied by the appropriate propagator of the other ready to be combined to a single number through the combining denominator $4(P_1P_5)P_1P_2(MZP)$, (labelled PROPIV in the program). The other multiplicative factors, those arising from the two Spinor replacements and the gluon polarisation vector replacement, (labelled B0IV, CU0IV, and E0IV in the program), will also appear in the LOOP number and hence divide out when the ratio is taken. For this reason they are set equal to unity, for convenience.

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To evaluate the LOOP amplitude, the nineteen integrals for this set of dot products are evaluated, along with WIV, which is the determinant used in the expansion of k_μ , as per Appendix L. These results are fed into the linked LEPTON LOOP function which returns a single complex number.

As the ANTI-LEPTON diagram is the same as the LEPTON if one interchanges P3 with P4 (except in the LEPTON line TRACE), the dot products involving P3 or P4 are so interchanged, the sign of $Epf(P1, P3, P4, P5)$ switched to account for this interchange, the nineteen integrals re-evaluated along with WIV, and the results fed into the ANTI-LEPTON LOOP attached function, (which has the LEPTON line TRACE in minus reverse form in preparation for the switch), and the calculated number returned. (The minus sign is due to traversing the variable LOOP momentum k_μ in the opposite sense when writing the line TRACE for this diagram.)

Each of these numbers has already combined its four and five point contributions with appropriate multipliers in preparation for combining into a single number. The denominator for that combining is now folded in and the results joined as a single number for the LOOP TERM.

That number is then scaled by the scale factors $-(1/137)$ and $1/(16\pi)$. As in the BORN TERM, the factors B0IV, CU0IV, and E0IV arising from the Spinor and polarisation replacements are set to unity.

The LOOP number is added to the BORN number, squared, and divided by the square of the BORN number to form a RATIO, which is written to an output file along with the values of the six dot products that formed it.

The next point in the grid of phase space is examined and, if valid, processed.

This continues until all grid points have been examined.

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Chapter Three

The Results

Section 3.1 ... Introduction

This chapter concerns itself with the presentation of the results of the FORTRAN program which calculates the RATIO of the BORN + LOOP cross section to the BORN cross section.

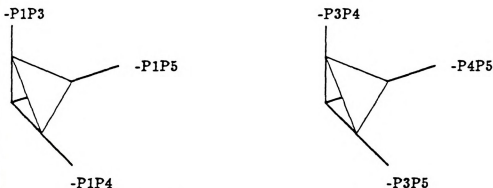
The results are presented separately for each of the eight allowed helicity states. The display of the data is as generated, that is in grid fashion, one dimensionless RATIO number for each physically allowable set of external dot products.

Rather than display the output from the small grid step size which actually generates the results (this is too much data to comfortably display), the program is rerun with a larger grid step convenient for displaying, the results of the two runs carefully compared to ascertain nothing interesting was lost in going to the larger grid size, and the larger size displayed.

The smaller stepped data is enclosed in Appendix U in case it is desired to examine it.

Section 3.2Choosing the Display Mode

As indicated at the close of the previous chapter, those points of phase space which are energetically allowable lie on two plane triangular surfaces:



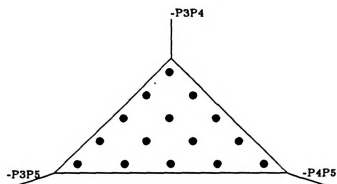
Each corner of the triangles intersects the axis at $\frac{E_{cm}^2}{2}$. The triangle on the right involves only outgoing dot products. For each point on this triangle, all points of the first (input) triangle are energetically allowable. The data is presented by selecting representative points on the output triangle and for each of those points displaying the RATIOS for a representative set of input points on the input triangle.

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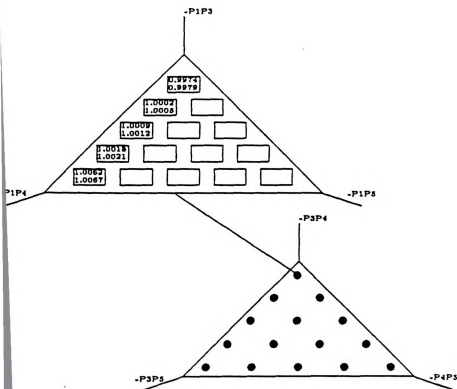
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View the output triangle straight on, and select 15 symmetrically spaced representative points: (the algorithm for selecting these is given in Appendix V)



For each output point generate an input triangle in which the RATIO has been calculated for each of 15 representative points in conjunction with the fixed output point:



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The missing RATIOS are due to the fact that not all the energetically allowable points also separately conserve each of four components of energy-momenta, that is not all survive the inequality test:

$$4(P1P4)(P3P5)(P1P5)(P3P4) > [(P1P4)(P3P5) + (P1P5)(P3P4) - (P1P3)(P4P5)]^2$$

derived in Appendix P.

Again, the display data has been carefully compared with that generated by the smaller grid size to assure it is indeed representative.

The dual values given for each RATIO represent the reflection of the output particles across the plane of the input particles. This reflection does not change the set of six dot products, but does alter the RATIO. It is accomplished by changing the sign of $Epf(P1, P3, P4, P5)$.

Section 3.3 ... Results for the Eight Helicity States

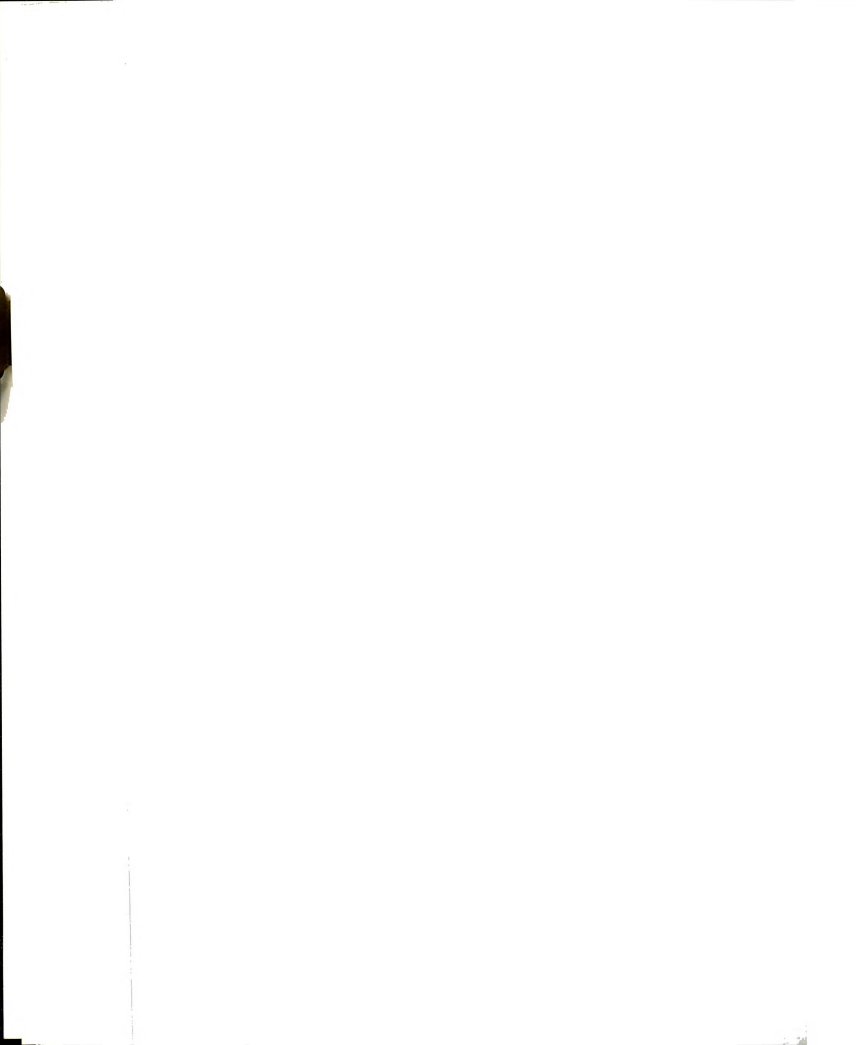
The following eight pages contain the display of the eight helicity states.





























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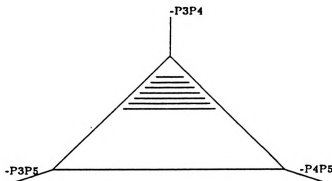
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Section 3.4 ... Focus on M_z Peak

Of particular interest is that area of phase space where the 'mass' carried off from the M_z decay is equal to M_z itself, that is: $-2(P3P4) = M_z^2$, and the Z^0 propagator becomes minimal:

$$\begin{aligned} MZP &= 2(P3P4) + M_z^2 - i\Gamma M_z \\ &= -i\Gamma M_z \end{aligned}$$

Therefore the values of P3P4 are slowly stepped through the M_z peak in a separate running of the FORTRAN program. This is done for the helicity state A, assuming it to be typical in this respect.



The results (enclosed in Appendix U), show NO particular distinction from any other area of phase space in the range of RATIOS produced.

As one of the goals of this project is to look for asymmetries in the Z^0 decay curve, the above data is plotted- RATIO vs. P3P4, as P3P4 is stepped through the M_z peak. In this graph, one point stands for the average value of the RATIO on one of the lines in the above diagram, averaged over 1000 points.

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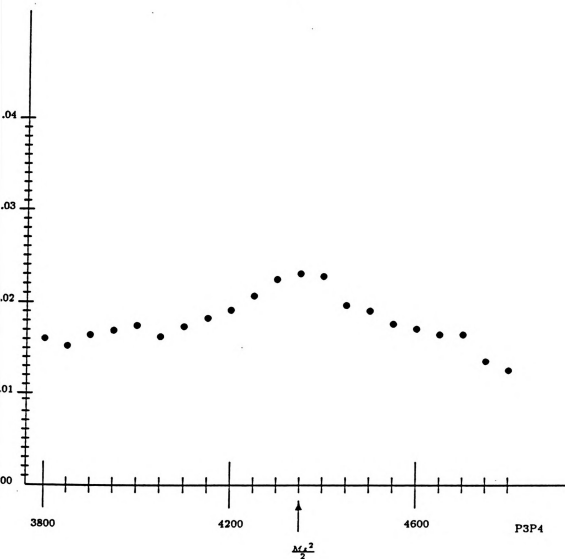
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This plot shows NO appreciable asymmetry, NOR shifting of the Z^0 peak.

Section 3.5 ... Edge Activity

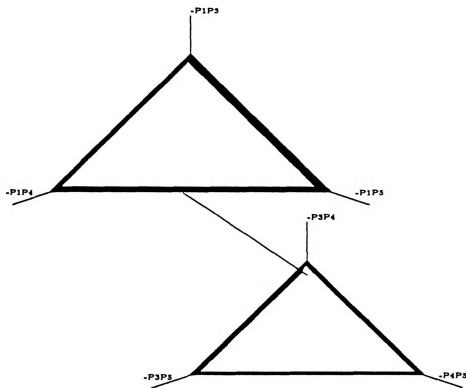
All the above results work on the interior of the triangles, staying away from the edges, that is away by 5 per cent.

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This is because as the edges are approached, the RATIO can become arbitrarily large due to the presence of dot products in denominator positions.

Section 3.6 ... Connecting with the Center of Mass System

It may be of interest to relate a set of dot products to outgoing scattering angles in the laboratory (center of mass) system. To this end, the polar angles of the three exiting particles in the center of mass system are derived in terms of dot products in Appendix P.

They are:

$$\cos(\theta_3)_{cm} = 1 - 2 \left[\frac{P1P3}{P3P4 + P3P5} \right]$$

$$\cos(\theta_4)_{cm} = 1 - 2 \left[\frac{P1P4}{P3P4 + P4P5} \right]$$

$$\cos(\theta_5)_{cm} = 1 - 2 \left[\frac{P1P5}{P3P5 + P4P5} \right]$$

Note that the new restrictions that seem to apply:

$$-(P1P3) < -(P3P4 + P3P5)$$

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$$-(P1P4) < -(P3P4 + P4P5)$$

$$-(P1P5) < -(P3P5 + P4P5)$$

are already contained in the restriction that $P1_y^2$ be greater than zero.

Section 3.7 ... Analysis of the Results

The pattern of results is approximately the same for the eight helicity states, in that almost all the RATIO's are near unity except for one 'corner' of phase space, where it might range from a value of 2 to near 50..

The largest ratio in the representative sets is 45.669, the smallest 0.0526.

The plot of RATIO vs. $P3P4$ showed NO noticeable asymmetry as the M_z peak was traversed, NOR shifting of the Z^0 peak.

For the small grid size, the values grew arbitrarily large as the edges of the triangles were approached, as expected due to the presence of these dot products in denominator positions. This increase is unimportant as the amount of phase space near the edge goes to zero faster than the amplitude grows and does not contribute to the integrated cross section. This is demonstrated in the solution to the preliminary diagram in Appendix W.

The largest values of the RATIO appeared in the phase space 'corner' of large $P4P5$ and small and equal $P3P4$ and $P3P5$. The 'input' triangle indicates small $P1P3$ and $P1P5$ and large $P1P4$. Comparison with the center of mass expressions of the preceeding section give particle 5 (the quark) staying in the forward direction, particle 4 (the lepton) reversing direction to emerge at 180° , particle 3 (the anti-lepton) emerging at 90° . Thus the largest effect of this process produces a high transverse momentum product!

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Chapter Four

Verification and Validation

Section 4.1 ... Introduction

There are six points at which verification of the calculation is enacted.

Section 4.2 ... Verifying $Epf(P1, P3, P4, P5)$

The value of $Epf(P1, P3, P4, P5)$ in terms of dot products of the problem is calculated two different ways in Appendix C - first by direct evaluation in a chosen reference frame and co-ordinate system, and second via the usual method of evaluating a determinant. Both methods give the same result.

Section 4.3 ... Verifying the Spinor replacement

This replacement, derived in Appendix H, for dis-similar momenta is non-standard and is thus checked carefully.

In Appendix Q it is applied to two simplistic diagrams in which the amplitudes and the spin averaged cross sections can be hand calculated. In each case, the square of the amplitude, (using the replacement), is equal to the cross section, (not using the replacement), verifying the correctness of it.

The second calculation is then slightly altered, to use the exact spinor replacements for this problem, and a SCHOONSCHIP routine written to evaluate and compare the amplitude squared and cross section. The successful comparison validates the correctness of the exact replacements and the correct use of SCHOONSCHIP syntax in evaluating the amplitude.

Section 4.4 ... Verifying the Born term

The eight helicity state amplitudes of the Born term are separately squared and added. This sum is then subtracted from the spin averaged cross section. If the amplitude calculations are correct, this will produce zero. It is checked as the last calculation of the SCHOONSCHIP program in Appendix I which generates the amplitudes. The zero is produced. This verifies that the least standard piece of the calculation, the Spinor replacement for dis-similar momenta, is working correctly. Less importantly, though of course also essential, it checks that the other factors of the Born amplitude are free of error.

As it operates on the amplitudes simultaneously as they are outputted into FORTRAN compatible format, there is no further possibility of transcription errors in

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Section 4.5 ... Verifying the LOOP numerator

It is noted in Appendix G that the quark singularity is benign because for precisely the value of k_μ at which it occurs, the numerator also approaches zero to the same order.

This zero is used as a check that the calculation is proceeding correctly.

After SCHOONSCHIP has taken the LOOP traces, substituted for the spinor and polarisation vectors, and replaced k_μ in all $Epf's$ by its linear expansion (as per Appendix L), then the replacement in the resulting expression of k_μ by precisely this value should still produce the zero, if all the substitutions were enacted correctly.

This check is performed by the SCHHONSHIP program of Appendix R, a truncated copy of the main SCHOONSCHIP loop program (Appendix K) that produces the FORTRAN compatible amplitudes. It is truncated after all the above substitutions are made and before the base integral groups are recognised and the nineteen integrals symbolically labelled.

The insertion of this expression for k_μ does produce the zero, indicating that at this point all the above substitutions were made correctly.

This zero took a half hour of SUN time to produce, reflecting the large number of terms present in the calculation at this point, all of which had to be correct to produce the zero.

Section 4.6 ... Validation of Divergent parts of Nineteen Integrals

Although the calculation itself is finite, each separate diagram is infinite, and most of the nineteen integrals possesses an infinite piece.

The infinite part of the diagram (and of the nineteen integrals) is regularised by the use of small photon and quark masses which are allowed to go to zero at the end of the calculation where the masses appear in log form, and their coefficients must sum to zero over both diagrams to avoid divergence as the mass goes to zero.

The appearance of these zero co-efficients verifys once again the correctness of the substitutions already once checked by the replacement of k_μ as per the above section.

It goes further and verifies the base integral groupings of Appendix M and the divergent parts of the analytic solutions of the nineteen integrals executed in Appendix N.

This check is enacted by using the SCHOONSCHIP loop program of Appendix K and instead of producing algebraic output, altering it to produce a numeric result for a specific set of dot products.

At the beginning of the program the six symbolic dot products are replaced

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with a set of six numbers that form a physically valid set. As SCHOONSCHIP does not evaluate logs, any logs of the above six dot products are evaluated separately and fed into the SCHOONSCHIP program. The same code that evaluates the nineteen integrals in the FORTRAN program is then inserted in the SCHOONSCHIP program, where it produces nineteen numbers from the set of six dot products. (Alternatively, these could have been evaluated in FORTRAN and the nineteen numbers fed in.)

The calculation is then performed numerically instead of algebraically, with the one exception of the logs of the fictitious photon and quark masses, which are carried as algebraic symbols.

The output of the SCHOONSCHIP program now becomes a complex number representing the convergent part of the amplitude being calculated and two algebraic symbols with coefficients for the logs of the photon and quark masses. These coefficients must sum to zero over both diagrams.

The altered SCHOONSCHIP program to perform this check is listed in Appendix S. The valid set of six dot products were arbitrarily chosen, and the production of the zero coefficients verified.

It is to be noted that the convergent parts of the nineteen integrals are not checked by this result.

Section 4.7 ... Numeric semi-check of nineteen integrals

Finally the convergent and divergent parts of the nineteen integrals are semi-checked by comparing (for a specific numeric set of dot products), the analytic result with the result of an approximate numerical grid integration.

The result is only approximate as the numerical integration necessarily diverges. This divergence is examined by introducing small numeric photon and quark masses and comparing runs with increasingly smaller masses, looking for an extrapolation to zero mass.

The approach to zero is quite tame, as expected for a logarithmic divergence, and even the presence of a minute mass, one-thousandth the dot product number, keeps the numerical integration from turning upwards. As the one-thousandth value is approached from the higher value of one-tenth, the integral does not vary, indicating insensitivity to the presence of the mass.

The numeric integrations are thus performed with these one-thousandth masses in place and the results compared with the analytic solutions.

The FORTRAN code for the numeric integrations and the results of the comparisons with the analytic solutions are in Appendix T. In all cases the results are within 30 per cent for a large grid size and become better as the grid size is reduced. In almost all cases the numeric results are above the analytic and by about the same percentage, and in almost all cases change the same as the grid size is reduced.

While not conclusive, this is indicative the analytic solutions are consistent as

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This method was used to identify errors in the nineteen analytic solutions by looking for those numeric comparisons that did not obey the systematic patterns. It was successful in finding three errors which when corrected led in each case to the integral falling into the pattern.

The search was instigated by the failure of the zero coefficients of the log terms of the above section to emerge. The three errors found were all in the divergent parts of integrals and when corrected led to the emergence of the zero coefficients, indicating no more errors in the divergent parts.

As the convergent parts of the integrals are functionally similar to the divergent, it is to be expected that any errors in them would produce similar pattern anomalies in the analytic-numeric comparison as did the divergent.

No remaining anomalies are present in the comparisons.

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Chapter Five

Conclusion

Section 5.1 ... Introduction

At this point the calculation has been made, and the data summarized and analyzed. It is time to draw the conclusions of the project against the questions raised in the Introduction, to see how it compares with estimations as well as other actual calculations that have been performed and to see what next projects might be undertaken.

Section 5.2 ... Conclusions

It has been ascertained that the influence of the re-scattering process here calculated is, except on the very edges of phase space and one 'hot' corner, minimal. It thus offers little promise of producing an observable consequence.

The specter raised in the Introduction, that the measured width of the Z^0 particle could be contaminated by the effects of rescattering is not at this point realized. This must be qualified as there is definitely a 'hot' corner in the amplitude that might contribute on a detailed phase space evaluation.

Several key techniques for working with diagrams of this complexity were gathered together: (a) the projection of a dis-similar momenta spinor product onto a momentum basis set of the problem. (b) a similar projection of a polarization vector (c) the projection of the LOOP momentum k_μ onto a momentum basis set of the problem, (d) the technique of reducing a loop integral containing three denominator factors to a sum of integrals containing at most two (e) the various steps for solving this still non-trivial set of integrals (f) the set of checks enacted at various points in the calculation to insure its correctness.

In addition it was determined once more that the Spin factors do not largely affect the value of an amplitude, as the simplified preliminary diagram yielded similar results to the full calculation.

Section 5.3 ... Relation to Other Work

In the 'Evaluation of a Preliminary Diagram' in Appendix W, the restriction of zero mass on the outgoing quark was lifted. It was found this did NOT produce an enhancement, indeed that the diagram was maximum at the zero mass.

This can crudely be understood in that the region of interaction for re-scattering of the outgoing particles is greatest if they all have like speeds; so if some have the speed of light (zero mass), all should, for maximum affect.

Also from that diagram an estimation of about 1 per cent was made for the

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result of the larger calculation. That estimation was fulfilled.

As to the restriction of the two internal lines being ON-SHELL, the preliminary diagram has been elsewhere¹⁰ calculated with this restriction removed, and the expectation (based on similar past calculations¹⁰) that the effect is not enhanced with the lifting of the restriction, fulfilled.

The larger calculation done with GLUON scattering and the M_z decaying to quarks has also been done elsewhere⁹ and results of the same low order found. (In GLUON scattering, the interference term is of course missing due to COLOR separateness between the BORN and LOOP digrams. This diminishment is offset somewhat by the increased strength of the strong interaction coupling constant over the electromagnetic one of the photon scattering.)

Section 5.4 ... Future Work

The most obvious next things to do would be to futher explore those corners of the extended Dalitz plot that generated the most promising effect. In particular, as those corners were in conjunction with the production of a high transverse momentum final state lepton, most acessible to experiment, see if these corners lead to changes in the expected angular distribution of products. As the RATIO in this corner reached a sizable value (near 50), a conclusion as to the negligible effect of this re-scattering process can not be drawn until the weighting factors of the phase space intergration are folded in.

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Footnotes

- .. Eichen, et al. 1984 'Reviews of Modern Physics' Vol 56, No 4, 579
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MSU, (1983-1984)

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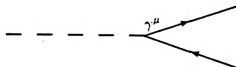
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Appendix A

Allowable Helicity Combinations

Consider a lepton line through one vertex:



The propagator numerator is:

$$\bar{u}^\pm \gamma_\mu v^?$$

and since

$$\bar{u}^\pm = \bar{u}^\pm \left(\frac{1 \mp \gamma_5}{2} \right)$$

then

$$\bar{u}^\pm \gamma_\mu v^? = \bar{u}^\pm \left(\frac{1 \mp \gamma_5}{2} \right) \gamma_\mu v^?$$

Then using

$$\gamma_5 \gamma_\mu = -\gamma_\mu \gamma_5$$

gives

$$\bar{u}^\pm \gamma_\mu v^? = \bar{u}^\pm \gamma_\mu \left(\frac{1 \pm \gamma_5}{2} \right) v^?$$

And since

$$\left(\frac{1 \pm \gamma_5}{2} \right) v^\pm = 0$$

$$\left(\frac{1 \pm \gamma_5}{2} \right) v^\mp = v^\mp$$

the propagator is zero unless $\bar{u}$ and v have opposite helicities.

Adding additional vertices to the electron line introduces additional gamma

matrix

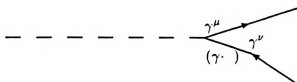
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matrices in pairs, not changing the result.



The same is true for any lepton line with internal line propagators and vertices that each contribute one gamma matrix.

However, if one particle is incoming, and the other outgoing, as in the quark line for this problem:



The propagator numerator is:

$$\bar{u}^{\pm} \gamma_{\mu} u^?$$

and by similar steps as above the helicities must be the same.

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Appendix B

Gamma Matrices, Metrics, and the Dirac Equation

The original Dirac equation,

$$(\alpha \cdot p - E + m)\psi = 0$$

to be consistent with the Klein-Gordan equation required that

$$\alpha_i \alpha_j + \alpha_j \alpha_i = 2\delta_{ij}$$

$$\alpha^2 = \beta^2 = 1$$

$$\beta \alpha_i + \alpha_i \beta = 0$$

and α and β must be Hermetian.

The Dirac equation may be written in terms of Gamma matrices.

For any metric:

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$$

In terms of the original Dirac matrices, γ_5 is:

$$\gamma_5 = i\alpha_1\alpha_2\alpha_3$$

This evaluation for γ_5 into the original Dirac matrices will be the same for all the metrics here considered.

In one such metric, for example the one used in the text by Bjorken-Drell:

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

and the energy-momentum four vector-

$$P^\mu = E_0 + P_x + P_y + P_z$$

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$$P_\mu = E_0 - P_x - P_y - P_z$$

dot products in this metric are positive:

$$P^\mu P_\mu = +m^2$$

the Gamma matrices, defined in terms of the Dirac matrices:

$$\gamma^0 = \beta$$

$$\gamma^i = \beta \alpha_i$$

ge the Dirac equation to:

$$(\gamma \cdot p - m)\psi = 0$$

ecomes:

$$\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$$

ther metric, the one used by 't Hooft - Veltman is:

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

the energy-momentum four vector-

$$P^\mu = E_0 + P_x + P_y + P_z$$

$$P_\mu = -E_0 + P_x + P_y + P_z$$

dot products in this metric are negative:

$$P^\mu P_\mu = -m^2$$

the Gamma matrices, defined in terms of the Dirac matrices:

$$\gamma^0 = i\beta$$

$$\gamma^i = i\beta\alpha_i$$

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nge the Dirac equation to:

$$(i\gamma \cdot p - m)\psi = 0$$

becomes:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

ariant of the above metric is the Minkowski metric:

$$g^{\mu\nu} = g_{\mu\nu} = \delta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

l the energy-momentum four vector-

$$P^\mu = P_\mu = P_x + P_y + P_z + iE_0$$

e dot products in this metric are also negative:

$$P^\mu P_\mu = -m^2$$

l the Gamma matrices, defined in terms of the Dirac matrices:

$$\gamma^i = i\beta\alpha_i$$

$$\gamma^4 = -\beta$$

nge the Dirac equation to:

$$(i\gamma \cdot p - m)\psi = 0$$

becomes:

$$\gamma_5 = \gamma^1\gamma^2\gamma^3\gamma^4$$

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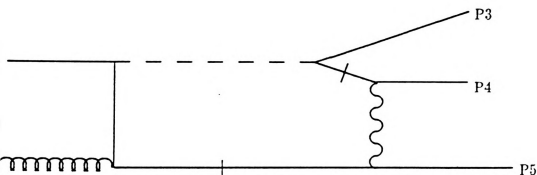
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Appendix C

Expanding $Ep f(P1, P3, P4, P5)$

As $Ep f(P1, P3, P4, P5)$ is a Lorentz invariant, it may be evaluated in any convenient reference frame and co-ordinate orientation. The reference system chosen here is the usual one for this problem- the rest frame of $\overline{P4}$ and $\overline{P5}$. The co-ordinate system is with $\overline{P4}$ taken as the plus z direction and $\overline{P3}$ in the x-z plane.



The four vectors- $P1, P3, P4, P5$ - have their components evaluated for this co-ordinate system in the solutions to Integrals I5, I8, and I18, in Appendix N. The results are- using as usual

$$ko = \sqrt{-P4P5/2}$$

and working in the Minkowski metric-

$$P4 = (0, 0, +ko, iko)$$

$$P5 = (0, 0, -ko, iko)$$

$$P3 = \left(\frac{\sqrt{(P3P4)(P3P5)}}{ko}, 0, \frac{(P3P4 - P3P5)}{2ko}, -i \frac{(P3P4 + P3P5)}{2ko} \right)$$

$$P1 = (P1_x, P1_y, \frac{(P1P4 - P1P5)}{2ko}, -i \frac{(P1P4 + P1P5)}{2ko})$$

where

$$P1_x = \pm \left(\frac{(P1P4)(P3P5) + (P1P5)(P3P4) - (P1P3)(P4P5)}{2ko\sqrt{(P3P4)(P3P5)}} \right)$$

$$P_{ij}^2 = \frac{4}{\sqrt{2}}$$

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$$\frac{4(P1P4)(P3P5)(P1P5)(P3P4) - [(P1P4)(P3P5) + (P1P5)(P3P4 - (P1P3)(P4P5))]^2}{4k_0^2(P3P4)(P3P5)}$$

appears in the above results. It is obtained in terms of the other dot products by squaring $P2$ and noting the particles have zero mass-

$$P2^2 = 0 = P3P4 + P3P5 + P4P5 - P1P3 - P1P4 - P1P5$$

$Ep f$ function is defined in terms of the Levi-Cevita totally anti-symmetric

$$Ep f(A, B, C, D) = \epsilon_{\alpha\beta\gamma\delta} A_\alpha B_\beta C_\gamma D_\delta$$

is immediately:

$$Ep f(P1, P3, P4, P5) = i(2k_0^2) P3_x P1_y$$

$$4(P1P4)(P3P5)(P1P5)(P3P4) - [(P1P4)(P3P5) + (P1P5)(P3P4 - (P1P3)(P4P5))]^2)^{1/2}$$

must be greater than 0 to make $P1_y$ real.

can be given a more symmetric form by re-arranging the numerator of

$$4ab - (a + b - c)^2 =$$

$$b - [a^2 + b^2 + c^2 + 2ab - 2ac - 2bc] = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$$

$$Ep f(P1, P3, P4, P5) = \pm i \{ [(P1P5)(P3P4)]^2 + [(P1P4)(P3P5)]^2 + [(P4P5)(P1P3)]^2 - 2(P1P5)(P3P4)(P1P4)(P3P5) - 2(P1P5)(P3P4)(P4P5)(P1P3) \}^{1/2}$$

is transparently symmetric under the interchange of $P3$ with $P4$, indicating that $Ep f(P1, P3, P4, P5)$ will have the same absolute value for both the muon and electron Loop diagrams for this problem, and in order to differ by minus sign required by the interchange of two vectors in the $Ep f$ function, pick up the oppositely signed square root of $\sqrt{P1_y^2}$.

any other possibly useful symmetric form could be generated by:

$$(a - b)^2 + (a - c)^2 + (b - c)^2 - a^2 - b^2 - c^2$$

The result is
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for $Ep f(P1, P3, P4, P5)$ agrees with that obtained by the determination of the four dimensional Levi-Cevita tensor:

$$[P3, P4, P5] * [Ep f(P1, P3, P4, P5)] = \begin{vmatrix} P1^2 & P1P3 & P1P4 & P1P5 \\ P1P3 & P2^2 & P3P4 & P3P5 \\ P1P4 & P3P4 & P3^2 & P4P5 \\ P1P5 & P3P5 & P4P5 & P5^2 \end{vmatrix}$$

same result as above:

$$\begin{aligned} f(P1, P3, P4, P5) = & \pm i([(P1P5)(P3P4)]^2 + [(P1P4)(P3P5)]^2 \\ & + [(P4P5)(P1P3)]^2 - 2(P1P5)(P3P4)(P1P4)(P3P5) \\ & - 2(P1P4)(P3P5)(P4P5)(P1P3) - 2(P1P5)(P3P5)(P4P5)(P1P3))^{1/2} \end{aligned}$$

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Appendix D

Feynman Rules

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Appendix B

Rules for Feynman Graphs

Expressions for cross sections are divided into two parts: first the invariant amplitude $\mathcal{M}$, which is a Lorentz scalar and in which physics lies, and second, the phase space and kinematical factors. In terms of $\mathcal{M}$, the expression for a differential cross section $d\sigma$ is, for spinless particles and for photons only,

$$d\sigma = \frac{1}{|\mathbf{v}_1 - \mathbf{v}_2|} \left(\frac{1}{2\omega_1} \right) \left(\frac{1}{2\omega_2} \right) |\mathcal{M}|^2 \frac{d^3k_1}{2\omega_1(2\pi)^3} \cdots \frac{d^3k_n}{2\omega_n(2\pi)^3} \times (2\pi)^4 \delta^4 \left(p_1 + p_2 - \sum_{i=1}^n k_i \right) S \quad (\text{B.1})$$

where $\omega_p = \sqrt{|\mathbf{p}|^2 + m^2}$ as usual and $\mathbf{v}_1$ and $\mathbf{v}_2$ are velocities of the incident collinear particles. This expression is then integrated over all undetected momenta $\cdots k_n$ of the final particles. The statistical factor S is obtained by including factor $1/m!$ if there are m identical particles in the final state:

$$S = \prod_i \frac{1}{m_i!}$$

For Dirac particles,¹ the factor $1/2\omega_p$ is replaced by m/E_p , and the statistical factor is again included; all other factors remain the same.

A differential decay rate of a particle of mass M is given in its rest frame by

$$d\omega = d\left(\frac{1}{\tau}\right) = \frac{1}{2M} |\mathcal{M}|^2 \frac{d^3k_1}{2\omega_1(2\pi)^3} \cdots \frac{d^3k_n}{2\omega_n(2\pi)^3} (2\pi)^4 \delta^4 \left(p - \sum_{i=1}^n k_i \right) S$$

¹ If one adopts the convention that Dirac spinors be normalized to $2m$ instead to unity as in Eq. (A.2), Eq. (B.1) applies as well to fermions. The energy projection operators are then simply $(m \pm \not{p})$ in place of (A.3).

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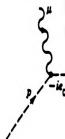


Fig. B.

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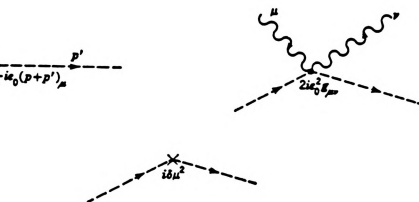


5.

Relativistic quantum mechanics

Electrodynamics of a Spin-zero Boson

Here there are three vertices, shown in Fig. B.2, corresponding to the inter-



B.2

in Lagrangian density

$$\mathcal{L}_I = -ie_0 : \varphi^\dagger \left(\overrightarrow{\frac{\partial}{\partial x_\mu}} - \overleftarrow{\frac{\partial}{\partial x_\mu}} \right) \varphi : A_\mu + e_0^2 : A^2 : : \varphi^\dagger \varphi : + \delta \mu^2 : \varphi^\dagger \varphi :$$

rules for these vertices are:

1. A factor $-ie_0(p + p')_\mu$, where p and p' are the momenta in the charged line.
2. A factor $+2ie_0^2 g_{\mu\nu}$ for each "seagull" graph.
3. A factor $i\delta\mu^2$ for each mass counterterm.
4. A factor $1/2$ for each closed loop containing only two photon lines, as shown in Fig. B.3.

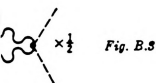


Fig. B.3

5. Renormalize the charge as in spinor electrodynamics.

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Meson-Nucleon Scattering

There are four interaction terms in the charge-independent theory:

$$\mathcal{H}_I = -\mathcal{L}_I = :ig_0\bar{\Psi}\gamma_4\tau_3\phi\Psi: - \delta m:\bar{\Psi}\Psi: - \frac{1}{2}\delta\mu^2:\phi\cdot\phi: + \frac{1}{4}\delta\lambda:(\phi\cdot\phi)^2:$$

illustrated in Figs. B.4 and B.5. The dotted line signifies that $I = 0$ only is

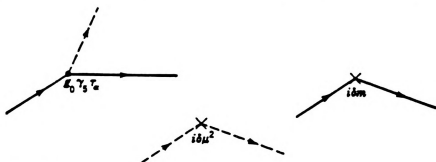
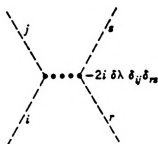


Fig. B.4

omitted from the meson pair ij to the pair rs , as shown by rule 2 below. The counterterms are treated as before and there is:

Fig. B.5



1. A factor $g_0\gamma_4\tau_3$ at each meson-nucleon vertex giving relative coupling strengths of $\sqrt{2}g_0$ for charged mesons and ± 1 for neutral ones to protons and neutrons, respectively.

2. A factor $-2i\delta\lambda\delta_{ij}\delta_{rs}$ at each four-meson vertex in Fig. B.5.

3. A factor $\frac{1}{2}$ for each closed loop containing two meson lines as in Fig. B.6.

Fig. B.6



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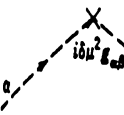
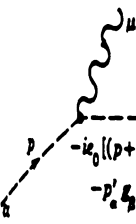


Fig. B.7

interaction 1

$\mathcal{L}' = -ie_0:$

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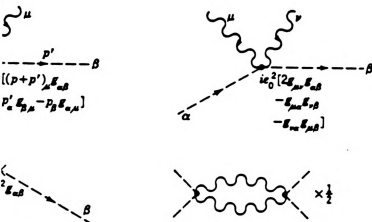
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Relativistic quantum mechanics

Dynamics of Spin-one Boson

vector boson propagator is $[-g_{\mu\nu} + k_\mu k_\nu / m^2](k^2 - m^2)^{-1}$ in place of the for massless photons, and the external line has a polarization factor ϵ_μ .

There are electrodynamics vertices shown in Fig. B.7 corresponding to an



on Lagrangian density

$$\mathcal{L} = \left[\left(\frac{\partial \varphi_\mu^*}{\partial x_\nu} \right) (A^\nu \varphi_\mu - A_\mu \varphi^\nu) - \left(\frac{\partial \varphi_\nu}{\partial x_\mu} \right) (A^\mu \varphi_\nu^* - A_\nu \varphi^{\mu*}) \right] + e_0^2 [A_\mu A^\mu \varphi_\nu^* \varphi^\nu - A_\mu \varphi^\mu A^\nu \varphi_\nu^*] + \delta \mu^2 \varphi_\mu^* \varphi^\mu$$

for these vertices as illustrated are:

factor $-ie_0(p' + p)_\mu g_{\alpha\beta} + ie_0 g_{\beta\mu} p'_\alpha + ie_0 p_\beta g_{\alpha\mu}$.

factor $+ie_0^2 [2g_{\mu\nu} g_{\alpha\beta} - g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\alpha\nu}]$.

factor $i\delta \mu^2 g_{\alpha\beta}$ for each mass counterterm.

factor $1/2$ for each closed loop containing only two photon lines.

For the derivation of these rules from canonical theory, effects of an anomalous magnetic moment term, and a regularization scheme see T. D. Lee and C. N. Yang, *Phys. Rev.*, **128**, 885 (1962).

In the above examples matrices are arranged in "natural order." For closed loops this means taking a trace. Isotopic indices are contracted with their mate at the other end of a boson line. In taking polarization sums for photons

$$\sum_\lambda \epsilon_\mu(k, \lambda) \epsilon_\nu(k, \lambda) \Rightarrow -g_{\mu\nu}$$

for mesons

$$\sum_\lambda \epsilon_\mu(k, \lambda) \epsilon_\nu(k, \lambda) \Rightarrow -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$$

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Appendix E

Ignoring the γ_5 in $A + B\gamma_5$

an expression of the form

$$(1 \pm \gamma_5) \gamma \cdot M \gamma \cdot N (A + B\gamma_5) \gamma \cdot P$$

$B\gamma_5$ can be commuted to the left:

$$(1 \pm \gamma_5) (A \pm B\gamma_5) \gamma \cdot M \gamma \cdot N \gamma \cdot P$$

The second $\pm$ depends on an even or odd number of commutations, though
 case it will always be even.

Using $\gamma_5\gamma_5 = 1$

$$A \pm A\gamma_5 \pm B\gamma_5 \pm B\gamma_5^2 = (A \pm B)(1 \pm \gamma_5)$$

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Appendix F

Evaluating the JACOBIAN

desired to evaluate a Jacobian such that

$$d(ko) d(|k|) = J \cdot d(|k|^2 - ko^2) \cdot d(k - (P4 + P5))^2$$

al, the Jacobian between (x, y) and (f, g) will be

$$J = \begin{vmatrix} \frac{\partial x}{\partial f} & \frac{\partial x}{\partial g} \\ \frac{\partial y}{\partial f} & \frac{\partial y}{\partial g} \end{vmatrix}$$

his case

$$f = x^2 - y^2$$

$$g = x^2 - y^2 + 2y(E4 + E5) + 2P4P5$$

$$df = (2x)dx - (2y)dy$$

$$dg = df + 2(E4 + E5)dy$$

$x = |k|$ and $y = ko$ and use has been made of the rest frame property $\vec{5} = 0$, and of:

$$g = (k - (P4 + P5))^2 = (P4 + P5)^2 - 2(P4 + P5) \cdot k + k^2$$

$$= 2P4P5 + 2ko(E4 + E5) + k^2$$

$$= 2P4P5 + 2y(E4 + E5) + f$$

$$dg = df + 2(E4 + E5)dy$$

$$dy = \frac{-1}{2(E4 + E5)} df + \frac{1}{2(E4 + E5)} dg$$

$$dx = \frac{1}{2x} df + \frac{y}{x} dy = \frac{1}{2x} df + \frac{y}{x} \left[\frac{-df}{2(E4 + E5)} + \frac{dg}{2(E4 + E5)} \right]$$

and using y

where use

and

$$J = \begin{vmatrix} \frac{1}{2x} - \frac{y}{2x(E4+E5)} & \frac{1}{2(E4+E5)} \\ \frac{-1}{2(E4+E5)} & \frac{1}{2(E4+E5)} \end{vmatrix}$$

ing $y = x = ko$ gives

$$J = \frac{1}{4ko(E4 + E5)}$$

$$J = \frac{-1}{4(P4P5)}$$

use has been made of

$$\delta(k^2) \Rightarrow |k| = ko$$

$$\delta(k - (P4 + P5))^2 \Rightarrow k^2 + 2ko(E4 + E5) + 2P4P5 = 0$$

$$\Rightarrow ko(E4 + E5) = -P4P5$$

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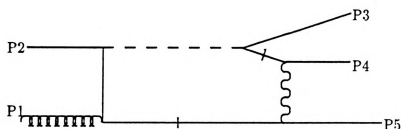
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Appendix G

Solving the Quark Singularity

in the diagram:



integral:

$$\Omega_k \frac{[\bar{u}(P4)\gamma_\mu(\gamma \cdot k)(C + D\gamma_5)\gamma_\nu v(P3)] [\bar{u}(P5)\gamma^\mu(\gamma \cdot m)(\gamma \cdot \epsilon)(\gamma \cdot n)(A + B\gamma_5)\gamma^\nu u(P2)]}{[(k - P4)^2 + M_z^2] [(k - P4 - P5 + P1)^2 + M_{quark}^2] [(k + P3)^2 + M_z^2 - i\Gamma M_z]}$$

ing with $m^2 = P1^2 = 0$:

$$n = m + P1 \Rightarrow n^2 = 2m \cdot P1$$

equals 0 whenever $\overline{m}$ and $\overline{P1}$ are co-linear:

$$\begin{aligned} 2m \cdot P1 &= 2(\overline{m} \cdot \overline{P1} - m_0 E1) \\ &= 2m_0 E1 [\cos\theta - 1] \end{aligned}$$

ever

$$\overline{m} = (\alpha - 1)\overline{P1}$$

means, because of zero masses, the four vectors are also parallel. $m = P1$

$$n = m + P1 = (\alpha - 1)P1 + P1 = \alpha P1$$

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 $k^2 = 0$

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$$n^2 = \alpha^2 P1^2 = 0$$

not free to assume any value. It is restricted by the other on-shell condition:
0:

$$m = (\alpha - 1)P1$$

$$m = k - (P4 + P5) = (\alpha - 1)P1$$

$$k = (\alpha - 1)P1 + P4 + P5$$

$$k^2 = 0 = (\alpha - 1)(P1P4 + P1P5) + P4P5$$

zero mass particles.

$$(\alpha - 1) = \frac{-P4P5}{P1P4 + P1P5}$$

$$\alpha = 1 + \frac{P4P5}{P1P4 + P1P5}$$

means the value of k at the singularity is

$$k_\mu = P4_\mu + P5_\mu - \left(\frac{P4P5}{P1P4 + P1P5} \right) P1_\mu$$

singularity uncovered is benign because the numerator of the above integral becomes zero when $m = (\alpha - 1)P1$ and $n = \alpha P1$.

$$\begin{aligned} \dots(\gamma \cdot m)(\gamma \cdot \epsilon)(\gamma \cdot n) \dots &= \dots(\gamma \cdot m)(\gamma \cdot \epsilon)(\gamma \cdot \alpha P1) \dots \\ &= \dots(\alpha - 1)(\gamma \cdot P1)(\gamma \cdot \epsilon)(\gamma \cdot \alpha P1) \dots \end{aligned}$$

using $(\gamma \cdot A)(\gamma \cdot B) = -(\gamma \cdot B)(\gamma \cdot A) + A \cdot B$ gives:

$$= \dots(\alpha - 1)[-(\gamma \cdot \epsilon)(\gamma \cdot P1) + \epsilon \cdot P1](\gamma \cdot \alpha P1) \dots$$

h, because $\epsilon \cdot P1 = 0$ becomes

$$\begin{aligned} &= \dots(\alpha - 1)\alpha(\gamma \cdot \epsilon)(\gamma \cdot P1)(\gamma \cdot P1) \dots \\ &= \dots(\alpha - 1)\alpha(\gamma \cdot \epsilon)(P1^2) \dots \end{aligned}$$

h goes to zero as $P1^2$, the same power as the denominator goes to zero.
is as $n^2 = \alpha^2 P1^2$

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this value

the fact that the numerator goes to zero when

$$k_{\mu} = P_{4\mu} + P_{5\mu} - \left(\frac{P_4 P_5}{P_1 P_4 + P_1 P_5} \right) P_{1\mu}$$

can be used to check the correctness of the SCHOONSCHIP program that calculates the LOOP term. At various points in that program the substitution of this value of k_{μ} should produce zero. This check is made in Appendix R.

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Appendix H

Spinor Replacement

It is desired to show that a Spinor product of dis-similar momenta may be replaced according to:

$$v^{(-)}(p3)\bar{u}^{(+)}(p4) = -(1 + \gamma_5)\gamma \cdot W$$

where W_μ is a four vector to be determined.

That this is a valid replacement may be seen by:

$$\chi^{+}(p3)\chi^{+}(p4) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = - \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} (\gamma \cdot \bar{W} - \gamma_0 W_0)$$

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \chi^{+}(p3)\chi^{+}(p4) = - \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -W_0 & \sigma \cdot \bar{W} \\ -\sigma \cdot \bar{W} & W_0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \chi^{+}(p3)\chi^{+}(p4) = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} (W_0 + \sigma \cdot \bar{W})$$

$$\chi^{+}(p3)\chi^{+}(p4) = (W_0 + \sigma \cdot \bar{W})$$

Since $(\chi^{+}(p3)\chi^{+}(p4))$ and $(W_0 + \sigma \cdot \bar{W})$ are 2 X 2 matrices, the four components of W_μ may be found by comparison. Q. E. D.

Similarly,

$$v^{(+)}(p3)\bar{u}^{(-)}(p4) = -(1 - \gamma_5)\gamma \cdot W$$

$$u^{(+)}(p3)\bar{u}^{(+)}(p4) = -(1 + \gamma_5)\gamma \cdot W$$

$$u^{(-)}(p3)\bar{u}^{(-)}(p4) = -(1 - \gamma_5)\gamma \cdot W$$

Now that it is established that this replacement is valid, W_μ may more simply be found by projecting onto a set of four linearly independent four vectors-

$$W_\mu = a_3(P3_\mu) + a_4(P4_\mu) + a_5(P5_\mu) + a_{\epsilon_{\mu\nu\lambda}} \epsilon_{\nu\lambda} Pf(\mu, P3, P4, P5)$$

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and the co-efficients (a_3, a_4, a_5, a_{eff}) determined as follows:

Starting with the defining statement for W^μ

$$(v^- \bar{u}^+)_{\alpha\beta} = -((1 + \gamma_5)\gamma \cdot W)_{\alpha\beta}$$

Multiply both sides by $(\gamma_\mu)_{\beta\alpha}$

$$(\gamma_\mu)_{\beta\alpha}(v^- \bar{u}^+)_{\alpha\beta} = \text{TRACE}(-\gamma_\mu \gamma_\nu W^\nu + \gamma_5 \gamma_\mu \gamma_\nu W^\nu)$$

$$\bar{u}^{(+)} \gamma_\mu v^{(-)} = -4W_\mu$$

The conjugate of this yields

$$\bar{v}^{(-)} \gamma_\mu u^{(+)} = 4W_\mu^*$$

These two results may be dotted with different arbitrary four vectors $R1_\mu$ and $R2_\mu$ and multiplied together to yield

$$\bar{v}^{(-)}(\gamma \cdot R1)u^{(+)}\bar{u}^{(+)}(\gamma \cdot R2)v^{(-)} = -16(W^* \cdot R1)(W \cdot R2)$$

$$\text{TRACE}\{v\bar{v}(\gamma \cdot R1)u\bar{u}(\gamma \cdot R2)\} = -16(W^* \cdot R1)(W \cdot R2)$$

and using projection operators:

$$v^-(p3)\bar{v}(p3)^- = \left(\frac{1 + \gamma_5}{2}\right)(\gamma \cdot p3)$$

$$u^+(p4)\bar{u}(p4)^+ = \left(\frac{1 + \gamma_5}{2}\right)(\gamma \cdot p4)$$

gives

$$\begin{aligned} \text{TRACE} \left[\frac{(1 + \gamma_5)}{2}(\gamma \cdot P3)(\gamma \cdot R1) \frac{(1 + \gamma_5)}{2}(\gamma \cdot P4)(\gamma \cdot R2) \right] &= -16(W^* \cdot R1)(W \cdot R2) \\ (1/2)\{\text{TRACE}[(\gamma \cdot P3)(\gamma \cdot R1)(\gamma \cdot P4)(\gamma \cdot R2)] + \text{TRACE}[\gamma_5(\gamma \cdot P3)(\gamma \cdot R1)(\gamma \cdot P4)(\gamma \cdot R2)]\} \\ &= -16(W^* \cdot R1)(W \cdot R2) \end{aligned}$$

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R1 = P5. $W \cdot P5$ can be found by temporarily letting R2 also equal P5:
 ing

$$\text{TRACE}[(\gamma \cdot A)(\gamma \cdot B)(\gamma \cdot C)(\gamma \cdot D)] = 4 [(A \cdot B)(C \cdot D) + (A \cdot D)(B \cdot C) - (A \cdot C)(B \cdot D)]$$

d

$$\text{TRACE}[\gamma_5(\gamma \cdot A)(\gamma \cdot B)(\gamma \cdot C)(\gamma \cdot D)] = 4 \text{Epf}(A, B, C, D)$$

e above equation yields

$$\left[\frac{(P4P5)(P3P5)}{4} \right]^{1/2} = (W \cdot P5)e^{i\theta}$$

$$\text{here } W = |W|e^{i\theta}, \quad W^* = |W|e^{-i\theta}.$$

ow let R2 once again be arbitrary. The equation now assumes the form:

$$\begin{aligned} & /2) \{ \text{TRACE}[(\gamma \cdot P3)(\gamma \cdot P5)(\gamma \cdot P4)(\gamma \cdot R2)] + \text{TRACE}[\gamma_5(\gamma \cdot P3)(\gamma \cdot P5)(\gamma \cdot P4)(\gamma \cdot R2)] \} \\ & = 16(W \cdot R2) \sqrt{\frac{(P4P5)(P3P5)}{4}} e^{-i\theta} \end{aligned}$$

nd taking the traces yields

$$\begin{aligned} & [(P3P5)P4 \cdot R2 + (P4P5)P3 \cdot R2 - (P3P4)P5 \cdot R2 - \text{Epf}(P3, P5, P4, R2)] \\ & = 16e^{-i\theta} \sqrt{\frac{(P4P5)(P3P5)}{4}} (W \cdot R2) \end{aligned}$$

$$\begin{aligned} & [(P4P5)P3_\mu + (P4P5)P4_\mu - (P3P4)P5_\mu - \text{Epf}(\mu, P3, P4, P5)] R2^\mu \\ & = e^{-i\theta} \sqrt{16(P4P5)(P3P5)} W_\mu R2^\mu \end{aligned}$$

nd since $R2^\mu$ is totally arbitrary,

$$W_\mu = \frac{e^{i\theta}}{\sqrt{16(P4P5)(P3P5)}} [(P4P5)P3_\mu + (P4P5)P4_\mu - (P3P4)P5_\mu - \text{Epf}(\mu, P3, P4, P5)]$$

SCH

Appendix I

SCHOONSHIP Program to Calculate
and Verify the BORN Term

C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
V
F
I
X
C
C
C
C
X
X
C
F
F
C

START SCHOONSHIP PROGRAM

BORN TERM CALCULATION

this program produces the BORN terms for the various HELECITY
 stares. As a check that these are taken correctly it adds
 all eight of them together, takes absolute square, then
 calculates the spin summed cross section, which does not
 use the SPINOR replacement, and takes the difference in
 the the two cross sections, which is zero if all is done
 correctly.

it produces the output in FORTRAN compatible format

V B,K,CU,M,E,N4,N5,P1,P2,P3,P4,P5,PD
 F C,F0,F1,F2,F3,F4,F5,F6,F7,F84,F85,F94,F95,F104,F105,F11
 I J1,J2,M1,M2,K3,K2
 X P13 = P34 + P35 + P45 - P14 - P15

SET SWITCHES

X FOUR = 1
 X FIVE = 1

P nstat
 P nlist
 C SET DOT PRODUCTS

X P1P3 = P13
 X P1P4 = P14
 X P1P5 = P15
 X P3P4 = P34
 X P3P5 = P35
 X P4P5 = P45

*fix

IN THIS SECTION, TAKE THE Trace, REPLACE EACH K BY IT'S
 LINEAR EXPANSION,
 AND REDUCE TO A FINAL ANSWER.

X LHEL = +1
 X QHEL = +1
 X GHEL = +1

SCALE FACTOR FOR THE AMPLITUDE IS ...

TAKE THE Trace

Z HA5=FIVE*F0*F1*F2*F11*F5*F85*F95*F105

Z HA
C
C
Id,F
Id,F
Id,F
Id,F
Id,F
*ye
Id,I
Id,
Id,
Id,
Id,
Id,
*ye
Id,
C
C
C
Id
*y
C
C
C
C
Id
Al
A
A
A
C
*
C
C
C
C

```

Z HA4=FOUR*F0*F1*F2*F11*F5*F84*F94*F104
C
C   SUBSTITUE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*(1+LHEL)*G6(J1) + 0.5*(1-LHEL)*G7(J1)
Id,F1=(G(J1,B))
Id,F2=(G(J1,M1))
Id,F11=0.5*(1+QHEL)*G6(J2) + 0.5*(1-QHEL)*G7(J2)
Id,F5=(G(J2,CU))
*yes
Id,F85=G(J2,E)
Id,F84=G(J2,M1)
Id,F95=G(J2,N5)
Id,F94=G(J2,N4)
Id,F105=G(J2,M1)
Id,F104=G(J2,E)
*yes
Id,Trick,Trace,J1
C
C   Trick AND Trace
C
Id,Trick,Trace,J2
*yes
C
C   START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N5(J-) = FIVE*(P1(J)-P5(J))
Al,Funct,N5(K2-) = FIVE*(P1(K2)-P5(K2))
Id,2,Dotpr,N4(J-) = - FOUR*(P3(J)+P4(J)+P5(J))
Al,Funct,N4(K2-) = - FOUR*(P3(K2)+P4(K2)+P5(K2))
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P   output
*yes
C
C   PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEpf*Epf(J,P3,P4,P5)
Al,Funct,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEpf*Epf(K3,P3,P4,P5)
C P   output
*yes
Id,B3 = BOIV*P4P5
Al,B4 = BOIV*P3P5
Al,B5 = -BOIV*P3P4
Al,BEpf = + LHEL*BOIV
Id,Trick
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P   output

```



```

*yes
Id,2,Dotpr,CU(J^-)=
  (CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
  +CUEpf*EpF(J,P1,P3,P5)+CUEpf*EpF(J,P1,P4,P5)
Al,Funcnt,CU(K^-)=
  (CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
  +CUEpf*EpF(K,P1,P3,P5)+CUEpf*EpF(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEpf = + QHEL*CU0IV
*yes
Id,Trick
Id,EDP1=0
Id,2,Dotpr,PD(J^-)=P4(J)+P5(J)
Al,Funcnt,PD(K2^-)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
C NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J^-)= E1*P1(J) + E3*P3(J) +
  E4*P4(J)+E5*P5(J)+EEpf*(EpF(J,P1,P3,P5)+EpF(J,P1,P4,P5))
Al,Funcnt,E(K^-)= E1*P1(K) + E3*P3(K) +
  E4*P4(K)+E5*P5(K)+EEpf*(EpF(K,P1,P3,P5)+EpF(K,P1,P4,P5))
Id,E1 = E0IV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = E0IV*GHEL*P1P5
Al,E4 = E0IV*GHEL*P1P5
Al,E5 = -E0IV*GHEL*(P1P3+P1P4)
Al,EEpf= -E0IV
*yes
Id,Trick
Id,EpF(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
Keep HA4,HA5
*next

```

```

Z Q
B I
Punch
Id,E'
B I
Keep
*next
Z HA
B
Keep
*next
C **
C
C
C
C
C
C
C
C
X L
X Q
X G
C
C
C
C
C
Z K
Z K
C
C
C
Id
Id
Id
Id
Id
*y
Id
Id
Id
Id
Id
*
I
C
C
C
I
*
C
C
C
:
```

```

Z QA = -2*P1P5*HA4 +2*(P3P4+P3P5+P4P5)*HA5
B EOIV,BOIV,CUOIV,EVL
Punch QA
Id,EVL=i*A
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep QA
*next
Z HA = QA*Conjg(QA)
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HA
*next
C *****
C
C
C
C IN THIS SECTION, TAKE THE Trace, REPLACE EACH K BY IT'S
C LINEAR EXPANSION,
C AND REDUCE TO A FINAL ANSWER.
C
C
C
C X LHEL = +1
C X QHEL = +1
C X GHEL = -1
C
C SCALE FACTOR FOR THE AMPLITUDE IS ...
C
C TAKE THE Trace
C
Z HB5=FIVE*F0*F1*F2*F11*F5*F85*F95*F105
Z HB4=FOUR*F0*F1*F2*F11*F5*F84*F94*F104
C
C SUBSTITUTE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*(1+LHEL)*G6(J1) + 0.5*(1-LHEL)*G7(J1)
Id,F1=(G(J1,B))
Id,F2=(G(J1,M1))
Id,F11=0.5*(1+QHEL)*G6(J2) + 0.5*(1-QHEL)*G7(J2)
Id,F5=(G(J2,CU))
*yes
Id,F85=G(J2,E)
Id,F84=G(J2,M1)
Id,F95=G(J2,N5)
Id,F94=G(J2,N4)
Id,F105=G(J2,M1)
Id,F104=G(J2,E)
*yes
Id,Trick,Trace,J1
C
C Trick AND Trace
C
Id,Trick,Trace,J2
*yes
C
C START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N5(J~) = FIVE*(P1(J)-P5(J))
Al,Funct,N5(K2~) = FIVE*(P1(K2)-P5(K2))

```



```

Id,2, Dotpr, N4(K2)
Al, Funct, N4(K2)
Id, P4DP4=0
Al, P5DP5=0
Al, EP1=0
C P output
*yes
C
C PUT IN THE
C
Id,2, Dotpr, B(K3)
Al, Funct, B(K3)
C P output
*yes
Id, B3 = BOIV*
Al, B4 = BOIV*
Al, B5 = -BOIV*
Al, BEpf = + L
Id, Trick
Id, P1DP1=0
Al, P3DP3=0
Al, P4DP4=0
Al, P5DP5=0
Id, P1DP3=P1P
Al, P1DP4=P1P
Al, P1DP5=P1P
Al, P3DP4=P3P
Al, P3DP5=P3P
Al, P4DP5=P4P
C P output
*yes
Id,2, Dotpr,
(CU1-CU2
+CUEpf+Eg+Eg
Al, Funct, Ck
(CU1-CU2
+CUEpf+Eg+Eg
Id, CU1 =
Al, CU2 = C
Al, CUS = C
Al, CUEpf =
*yes
Id, Trick
Id, EDP1=0
Id,2, Dotpr,
Al, Funct,
Id, P1DP1=
Al, P3DP3=
Al, P4DP4=
Al, P5DP5=
Id, P1DP3
Al, P1DP4
Al, P1DP5
Al, P3DP4
Al, P3DP5
Al, P4DP4
C P ou
*yes

```

```

pr,N4(J-) = - FOUR*(P3(J)+P4(J)+P5(J))
,N4(K2-) = - FOUR*(P3(K2)+P4(K2)+P5(K2))
=0
=0
0
put

```

IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.

```

pr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEpf*Epf(J,P3,P4,P5)
,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEpf*Epf(K3,P3,P4,P5)
put

```

```

BOIV*P4P5
BOIV*P3P5
-BOIV*P3P4
= + LHEL*BOIV

```

```

<
i=0
3=0
4=0
5=0
3=P1P3
4=P1P4
5=P1P5
4=P3P4
5=P3P5
5=P4P5
tput

```

```

tpr,CU(J-)=
-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
f*Epf(J,P1,P3,P5)+CUEpf*Epf(J,P1,P4,P5)
t,CU(K-)=
-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
f*Epf(K,P1,P3,P5)+CUEpf*Epf(K,P1,P4,P5)
= - CU0IV*(P3P5+P4P5-P1P5)
= CU0IV*P1P5
= CU0IV*(P3P4+P3P5+P4P5)
f = + QHEL*CU0IV

```

```

ck
i=0
otpr,PD(J-)=P4(J)+P5(J)
ct,PD(K2-)=P4(K2)+P5(K2)
P1=0
P3=0
P4=0
P5=0
P3=P1P3
P4=P1P4
P5=P1P5
P4=P3P4
P5=P3P5
P5=P4P5
tput

```

C
C
C
Id,
E4
Al,
E4
Id,
Al
Al
Al
Al
*y
Id
Id
Id
Al
Al
Al
Id
A
A
A
A
A
C
*
C
M
*
Z
E
T
I

```

C
C   NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J~)= E1*P1(J) + E3*P3(J) +
E4*P4(J)+E5*P5(J)+EEpf*(Epf(J,P1,P3,P5)+Epf(J,P1,P4,P5))
Al,Funct,E(K~)= E1*P1(K) + E3*P3(K) +
E4*P4(K)+E5*P5(K)+EEpf*(Epf(K,P1,P3,P5)+Epf(K,P1,P4,P5))
Id,E1 = EOIV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = EOIV*GHEL*P1P5
Al,E4 = EOIV*GHEL*P1P5
Al,E5 = -EOIV*GHEL*(P1P3+P1P4)
Al,EEpf= -EOIV
*yes
Id,Trick
Id,Epf(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
Keep HA,HB4,HB5
*next
Z QB = -2*P1P5*HB4 +2*(P3P4+P3P5+P4P5)*HB5
B EOIV,BOIV,CUOIV,EVL
Punch QB
Id,EVL=i*A
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HA,QB
*next
Z HB = QB*Conjg(QB)
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HB,HA
*next
C *****
C
C
C
C   IN THIS SECTION, TAKE THE Trace, REPLACE EACH K BY IT'S
C   LINEAR EXPANSION,
C   AND REDUCE TO A FINAL ANSWER.
C
C
C
C
X LHEL = +1
X QHEL = -1
X GHEL = +1
C
C   SCALE FACTOR FOR THE AMPLITUDE IS ...
C
C   TAKE THE Trace

```

C
Z HCSE
Z HC4
C
C
C
Id,F
Id,F
Id,F
Id,F
Id,F
*yep
Id,F
Id,F
Id,F
Id,F
Id,F
*ye
Id
C
C
C
Id,
*ye
C
C
Id
Al
Id
Al
Id
Al
Al
C
*y
C
C
Id
A
C
*
I
A
A
I
I
A
A
.

```

C
Z HC5=FIVE*F0*F1*F2*F11*F5*F85*F95*F105
Z HC4=FOUR*F0*F1*F2*F11*F5*F84*F94*F104
C
C   SUBSTITUE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*(1+LHEL)*G6(J1) + 0.5*(1-LHEL)*G7(J1)
Id,F1=(G(J1,B))
Id,F2=(G(J1,M1))
Id,F11=0.5*(1+QHEL)*G6(J2) + 0.5*(1-QHEL)*G7(J2)
Id,F5=(G(J2,CU))
*yes-
Id,F85=G(J2,E)
Id,F84=G(J2,M1)
Id,F95=G(J2,N5)
Id,F94=G(J2,N4)
Id,F105=G(J2,M1)
Id,F104=G(J2,E)
*yes
Id,Trick,Trace,J1
C
C   Trick AND Trace
C
Id,Trick,Trace,J2
*yes
C
C   START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N5(J-) = FIVE*(P1(J)-P5(J))
Al,Funct,N5(K2-) = FIVE*(P1(K2)-P5(K2))
Id,2,Dotpr,N4(J-) = - FOUR*(P3(J)+P4(J)+P5(J))
Al,Funct,N4(K2-) = - FOUR*(P3(K2)+P4(K2)+P5(K2))
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P   output
*yes
C
C   PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEpf*Epf(J,P3,P4,P5)
Al,Funct,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEpf*Epf(K3,P3,P4,P5)
C P   output
*yes
Id,B3 = B0IV*P4P5
Al,B4 = B0IV*P3P5
Al,B5 = -B0IV*P3P4
Al,BEpf = + LHEL*B0IV
Id,Trick
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5

```

Al,
C P
eye
Id,

Al,

Id
Al
Al
Al

yy

Id

Id

Id

Al

Id

A

A

A

I

A

A

A

A

A

C

eye

C

C

C

```

Al,P4DP5=P4P5
C P output
*yes
Id,2,Dotpr,CU(J~)=
  (CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
  +CUEpf*Epf(J,P1,P3,P5)+CUEpf*Epf(J,P1,P4,P5)
Al,Funct,CU(K~)=
  (CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
  +CUEpf*Epf(K,P1,P3,P5)+CUEpf*Epf(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEpf = + QHEL*CU0IV
*yes
Id,Trick
Id,EDP1=0
Id,2,Dotpr,PD(J~)=P4(J)+P5(J)
Al,Funct,PD(K2~)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
C NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J~)= E1*P1(J) + E3*P3(J) +
  E4*P4(J)+E5*P5(J)+EEpf*(Epf(J,P1,P3,P5)+Epf(J,P1,P4,P5))
Al,Funct,E(K~)= E1*P1(K) + E3*P3(K) +
  E4*P4(K)+E5*P5(K)+EEpf*(Epf(K,P1,P3,P5)+Epf(K,P1,P4,P5))
Id,E1 = E0IV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = E0IV*GHEL*P1P5
Al,E4 = E0IV*GHEL*P1P5
Al,E5 = -E0IV*GHEL*(P1P3+P1P4)
Al,EEpf= -E0IV
*yes
Id,Trick
Id,Epf(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C

```


[illegible]

```

Keep HA,HB,HC4,HC5
*next
Z QC = -2*P1P5*HC4 +2*(P3P4+P3P5+P4P5)*HC5
B EOIV,BOIV,CUOIV,EVL
Punch QC
Id,EVL=i*A
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HA,HB,QC
*next
Z HC = QC*Conjg(QC)
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HC,HB,HA
*next
C *****
C
C
C
C
C IN THIS SECTION, TAKE THE Trace, REPLACE EACH K BY IT'S
C LINEAR EXPANSION,
C AND REDUCE TO A FINAL ANSWER.
C
C
C
C
X LHEL = +1
X QHEL = -1
X GHEL = -1
C
C SCALE FACTOR FOR THE AMPLITUDE IS ...
C
C TAKE THE Trace
C
Z HD5=FIVE*F0*F1*F2*F11*F5*F85*F95*F105
Z HD4=FOUR*F0*F1*F2*F11*F5*F84*F94*F104
C
C SUBSTITUTE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*(1+LHEL)*G6(J1) + 0.5*(1-LHEL)*G7(J1)
Id,F1=(G(J1,B))
Id,F2=(G(J1,M1))
Id,F11=0.5*(1+QHEL)*G6(J2) + 0.5*(1-QHEL)*G7(J2)
Id,F5=(G(J2,CU))
*yes
Id,F85=G(J2,E)
Id,F84=G(J2,M1)
Id,F95=G(J2,N5)
Id,F94=G(J2,N4)
Id,F105=G(J2,M1)
Id,F104=G(J2,E)
*yes
Id,Trick,Trace,J1
C
C Trick AND Trace
C
Id,Trick,Trace,J2
*yes
C
C START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C

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Id,2,Dotpr,N5(J-) = FIVE*(P1(J)-P5(J))
Al,Funct,N5(K2-) = FIVE*(P1(K2)-P5(K2))
Id,2,Dotpr,N4(J-) = - FOUR*(P3(J)+P4(J)+P5(J))
Al,Funct,N4(K2-) = - FOUR*(P3(K2)+P4(K2)+P5(K2))
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P output
*yes
C
C PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEpf*EpF(J,P3,P4,P5)
Al,Funct,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEpf*EpF(K3,P3,P4,P5)
C P output
*yes
Id,B3 = B0IV*P4P5
Al,B4 = B0IV*P3P5
Al,B5 = -B0IV*P3P4
Al,BEpf = + LHEL*B0IV
Id,Trick
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
Id,2,Dotpr,CU(J-)=
(CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
+ CUEpf*EpF(J,P1,P3,P5)+CUEpf*EpF(J,P1,P4,P5)
Al,Funct,CU(K-)=
(CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
+ CUEpf*EpF(K,P1,P3,P5)+CUEpf*EpF(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEpf = + QHEL*CU0IV
*yes
Id,Trick
Id,EDP1=0
Id,2,Dotpr,PD(J-)=P4(J)+P5(J)
Al,Funct,PD(K2-)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5

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C P output

*yep

C

NOW PUT IN THE POLARIZATION VECTOR EXPANSION.

C

Id,2,Dotpr,E(J⁻)= E1*P1(J) + E3*P3(J) +
E4*P4(J)+E5*P5(J)+EEpf*(Ep f(J,P1,P3,P5)+Ep f(J,P1,P4,P5))

Al,Funct,E(K⁻)= E1*P1(K) + E3*P3(K) +
E4*P4(K)+E5*P5(K)+EEpf*(Ep f(K,P1,P3,P5)+Ep f(K,P1,P4,P5))

Id,E1 = EOIV*GHEL*(P3P5+P4P5-2*P1P5)

Al,E3 = EOIV*GHEL*P1P5

Al,E4 = EOIV*GHEL*P1P5

Al,E5 = -EOIV*GHEL*(P1P3+P1P4)

Al,EEpf= -EOIV

*yep

Id,Trick

Id,Ep f(P1,P3,P4,P5) = EVL

Id,P1DP1=0

Al,P3DP3=0

Al,P4DP4=0

Al,P5DP5=0

Id,P1DP3=P1P3

Al,P1DP4=P1P4

Al,P1DP5=P1P5

Al,P3DP4=P3P4

Al,P3DP5=P3P5

Al,P4DP5=P4P5

C P output

*yep

C

Keep HA,HB,HC,HD4,HD5

*next

Z QD = -2*P1P5*HD4 +2*(P3P4+P3P5+P4P5)*HD5

B EOIV,BoIV,CUOIV,EVL

Punch QD

Id,EVL=i*A

B PROIV,EOIV,BoIV,CUOIV,A,i

Keep HA,HB,HC,QD

*next

Z HD = QD*Conjg(QD)

B PROIV,EOIV,BoIV,CUOIV,A,i

Keep HD,HC,HB,HA

*next

C *****

C

C

C

C

C

C

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C

C

X LHEL = -1

X QHEL = +1

X GHEL = +1

C

C

SCALE FACTOR FOR THE AMPLITUDE IS ...

C
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C
C   TAKE THE Trace
C
Z HE5=FIVE*F0*F1*F2*F11*F5*F85*F95*F105
Z HE4=FOUR*F0*F1*F2*F11*F5*F84*F94*F104
C
C   SUBSTITUE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*(1+LHEL)*G6(J1) + 0.5*(1-LHEL)*G7(J1)
Id,F1=(G(J1,B))
Id,F2=(G(J1,M1))
Id,F11=0.5*(1+QHEL)*G6(J2) + 0.5*(1-QHEL)*G7(J2)
Id,F5=(G(J2,CU))
*yes
Id,F85=G(J2,E)
Id,F84=G(J2,M1)
Id,F95=G(J2,N5)
Id,F94=G(J2,N4)
Id,F105=G(J2,M1)
Id,F104=G(J2,E)
*yes
Id,Trick,Trace,J1
C
C   Trick AND Trace
C
Id,Trick,Trace,J2
*yes
C
C   START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N5(J~) = FIVE*(P1(J)-P5(J))
Al,Funct,N5(K2~) = FIVE*(P1(K2)-P5(K2))
Id,2,Dotpr,N4(J~) = - FOUR*(P3(J)+P4(J)+P5(J))
Al,Funct,N4(K2~) = - FOUR*(P3(K2)+P4(K2)+P5(K2))
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P output
*yes
C
C   PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J~)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEpf*EpF(J,P3,P4,P5)
Al,Funct,B(K3~)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEpf*EpF(K3,P3,P4,P5)
C P output
*yes
Id,B3 = B0IV*P4P5
Al,B4 = B0IV*P3P5
Al,B5 = -B0IV*P3P4
Al,BEpf = + LHEL*B0IV
Id,Trick
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5

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Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
Id,2,Dotpr,CU(J-)=
  (CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
  +CUEpf*Epf(J,P1,P3,P5)+CUEpf*Epf(J,P1,P4,P5)
Al,Funct,CU(K-)=
  (CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
  +CUEpf*Epf(K,P1,P3,P5)+CUEpf*Epf(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEpf = + QHEL*CU0IV
*yes
Id,Trick
Id,EDP1=0
Id,2,Dotpr,PD(J-)=P4(J)+P5(J)
Al,Funct,PD(K-)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
  NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J-)= E1*P1(J) + E3*P3(J) +
  E4*P4(J)+E5*P5(J)+EEpf*(Epf(J,P1,P3,P5)+Epf(J,P1,P4,P5))
Al,Funct,E(K-)= E1*P1(K) + E3*P3(K) +
  E4*P4(K)+E5*P5(K)+EEpf*(Epf(K,P1,P3,P5)+Epf(K,P1,P4,P5))
Id,E1 = E0IV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = E0IV*GHEL*P1P5
Al,E4 = E0IV*GHEL*P1P5
Al,E5 = -E0IV*GHEL*(P1P3+P1P4)
Al,EEpf= -E0IV
*yes
Id,Trick
Id,Epf(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output

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*yes
C
Keep HA,HB,HC,HD,HE4,HE5
*next
Z QE = -2*P1P5*HE4 +2*(P3P4+P3P5+P4P5)*HE5
B EOIV,BOIV,CUOIV,EVL
Punch QE
Id,EVL=i*A
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HA,HB,HC,HD,QE
*next
Z HE = QE*Conjg(QE)
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HE,HD,HC,HB,HA
*next
C *****
C
C
C
C
C IN THIS SECTION, TAKE THE Trace, REPLACE EACH K BY IT'S
C LINEAR EXPANSION,
C AND REDUCE TO A FINAL ANSWER.
C
C
C
C
X LHEL = -1
X QHEL = +1
X GHEL = -1
C
C SCALE FACTOR FOR THE AMPLITUDE IS ...
C
C TAKE THE Trace
C
Z HF5=FIVE*F0*F1*F2*F11*F5*F85*F95*F105
Z HF4=FOUR*F0*F1*F2*F11*F5*F84*F94*F104
C
C SUBSTITUE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*(1+LHEL)*G6(J1) + 0.5*(1-LHEL)*G7(J1)
Id,F1=(G(J1,B))
Id,F2=(G(J1,M1))
Id,F11=0.5*(1+QHEL)*G6(J2) + 0.5*(1-QHEL)*G7(J2)
Id,F5=(G(J2,CU))
*yes
Id,F85=G(J2,E)
Id,F84=G(J2,M1)
Id,F95=G(J2,N5)
Id,F94=G(J2,N4)
Id,F105=G(J2,M1)
Id,F104=G(J2,E)
*yes
Id,Trick,Trace,J1
C
C Trick AND Trace
C
Id,Trick,Trace,J2
*yes
C

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C   START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N5(J-) = FIVE*(P1(J)-P5(J))
Al,Funct,N5(K2-) = FIVE*(P1(K2)-P5(K2))
Id,2,Dotpr,N4(J-) = - FOUR*(P3(J)+P4(J)+P5(J))
Al,Funct,N4(K2-) = - FOUR*(P3(K2)+P4(K2)+P5(K2))
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P   output
*yes
C
C   PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEpf*Epf(J,P3,P4,P5)
Al,Funct,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEpf*Epf(K3,P3,P4,P5)
C P   output
*yes
Id,B3 = B0IV*P4P5
Al,B4 = B0IV*P3P5
Al,B5 = -B0IV*P3P4
Al,BEpf = + LHEL*B0IV
Id,Trick
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P   output
*yes
Id,2,Dotpr,CU(J-)=
  (CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
  +CUEpf*Epf(J,P1,P3,P5)+CUEpf*Epf(J,P1,P4,P5)
Al,Funct,CU(K-)=
  (CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
  +CUEpf*Epf(K,P1,P3,P5)+CUEpf*Epf(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEpf = + QHEL*CU0IV
*yes
Id,Trick
Id,EDP1=0
Id,2,Dotpr,PD(J-)=P4(J)+P5(J)
Al,Funct,PD(K2-)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4

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Al
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A
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A
A
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C
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I

```

Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
C NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J-)= E1*P1(J) + E3*P3(J) +
E4*P4(J)+E5*P5(J)+EEpf*(Epf(J,P1,P3,P5)+Epf(J,P1,P4,P5))
Al,Funct,E(K-)= E1*P1(K) + E3*P3(K) +
E4*P4(K)+E5*P5(K)+EEpf*(Epf(K,P1,P3,P5)+Epf(K,P1,P4,P5))
Id,E1 = EOIV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = EOIV*GHEL*P1P5
Al,E4 = EOIV*GHEL*P1P5
Al,E5 = -EOIV*GHEL*(P1P3+P1P4)
Al,EEpf= -EOIV
*yes
Id,Trick
Id,Epf(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
Keep HA,HB,HC,HD,HE,HF4,HF5
*next
Z QF= -2*P1P5*HF4 +2*(P3P4+P3P5+P4P5)*HF5
B EOIV,BOIV,CUOIV,EVL
Punch QF
Id,EVL=i*A
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HA,HB,HC,HD,HE,QF
*next
Z HF = QF*Conjg(QF)
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HF,HE,HD,HC,HB,HA
*next
C *****
C
C
C
C IN THIS SECTION, TAKE THE Trace, REPLACE EACH K BY IT'S
C LINEAR EXPANSION,
C AND REDUCE TO A FINAL ANSWER.
C
C
C
C
X LHEL = -1
X QHEL = -1
X GHEL = +1

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C
C   SCALE FACTOR FOR THE AMPLITUDE IS ...
C
C   TAKE THE Trace
C
Z HG5=FIVE*F0*F1*F2*F11*F5*F85*F95*F105
Z HG4=FOUR*F0*F1*F2*F11*F5*F84*F94*F104
C
C   SUBSTITUTE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*(1+LHEL)*G6(J1) + 0.5*(1-LHEL)*G7(J1)
Id,F1=(G(J1,B))
Id,F2=(G(J1,M1))
Id,F11=0.5*(1+QHEL)*G6(J2) + 0.5*(1-QHEL)*G7(J2)
Id,F5=(G(J2,CU))
*yes
Id,F85=G(J2,E)
Id,F84=G(J2,M1)
Id,F95=G(J2,N5)
Id,F94=G(J2,N4)
Id,F105=G(J2,M1)
Id,F104=G(J2,E)
*yes
Id,Trick,Trace,J1
C
C   Trick AND Trace
C
Id,Trick,Trace,J2
*yes
C
C   START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N5(J-) = FIVE*(P1(J)-P5(J))
Al,Funct,N5(K2-) = FIVE*(P1(K2)-P5(K2))
Id,2,Dotpr,N4(J-) = - FOUR*(P3(J)+P4(J)+P5(J))
Al,Funct,N4(K2-) = - FOUR*(P3(K2)+P4(K2)+P5(K2))
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P output
*yes
C
C   PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEpf*EpF(J,P3,P4,P5)
Al,Funct,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEpf*EpF(K3,P3,P4,P5)
C P output
*yes
Id,B3 = BOIV*P4P5
Al,B4 = BOIV*P3P5
Al,B5 = -BOIV*P3P4
Al,BEpf = + LHEL*BOIV
Id,Trick
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3

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Al
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ry
Id

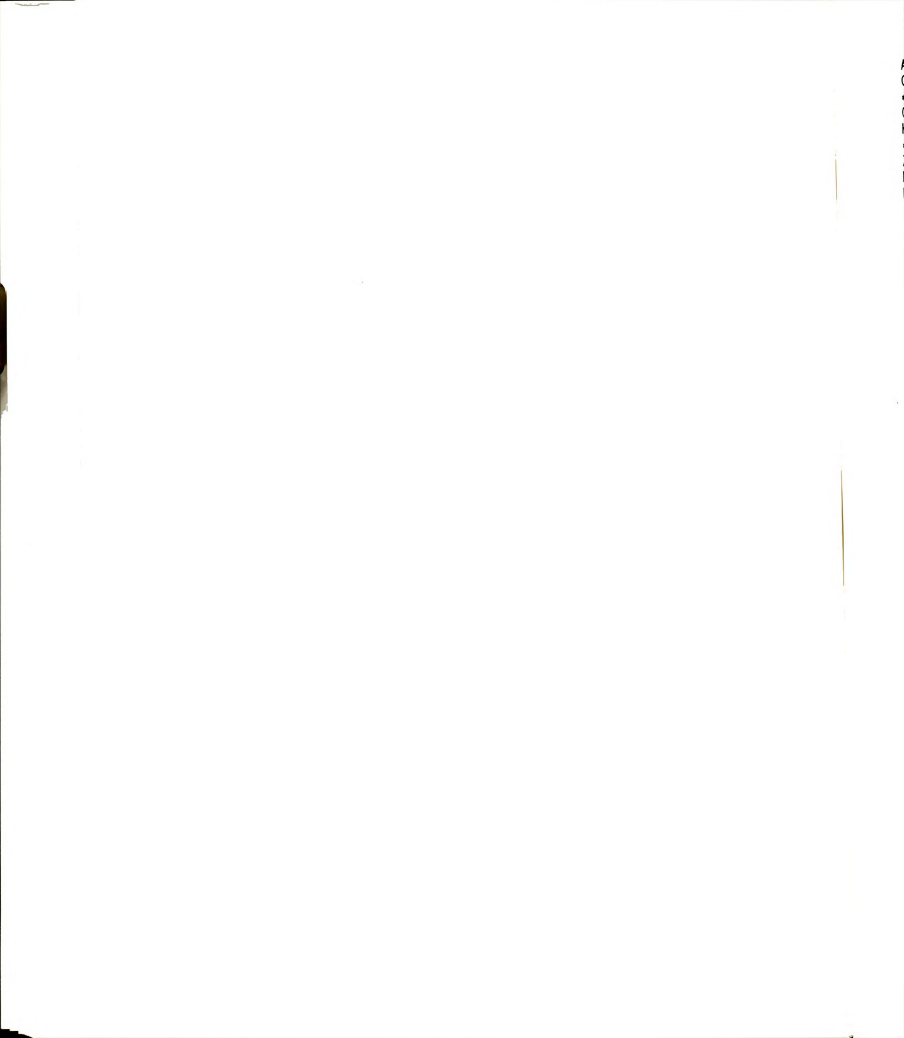
Al

Ic
Al
Al
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*
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C
C

```

Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
Id,2,Dotpr,CU(J^-)=
  (CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
  +CUEpf*EpF(J,P1,P3,P5)+CUEpf*EpF(J,P1,P4,P5)
Al,Funct,CU(K^-)=
  (CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
  +CUEpf*EpF(K,P1,P3,P5)+CUEpf*EpF(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEpf = + QHEL*CU0IV
*yes
Id,Trick
Id,EDP1=0
Id,2,Dotpr,PD(J^-)=P4(J)+P5(J)
Al,Funct,PD(K2^-)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
C NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J^-)= E1*P1(J) + E3*P3(J) +
  E4*P4(J)+E5*P5(J)+EEpf*(EpF(J,P1,P3,P5)+EpF(J,P1,P4,P5))
Al,Funct,E(K^-)= E1*P1(K) + E3*P3(K) +
  E4*P4(K)+E5*P5(K)+EEpf*(EpF(K,P1,P3,P5)+EpF(K,P1,P4,P5))
Id,E1 = E0IV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = E0IV*GHEL*P1P5
Al,E4 = E0IV*GHEL*P1P5
Al,E5 = -E0IV*GHEL*(P1P3+P1P4)
Al,EEpf= -E0IV
*yes
Id,Trick
Id,EpF(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5

```



```

A1,P4DP5=P4P5
C P output
*yes
C
Keep HA,HB,HC,HD,HE,HF,HG4,HG5
*next
Z QG = -2*P1P5*HG4 + 2*(P3P4+P3P5+P4P5)*HG5
B EOIV,BOIV,CUOIV,EVL
Punch QG
Id,EVL=i*A
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HA,HB,HC,HD,HE,HF,QG
*next
Z HG = QG*Conjg(QG)
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HG,HF,HE,HD,HC,HB,HA
*next
C *****
C
C
C
C IN THIS SECTION, TAKE THE Trace, REPLACE EACH K BY IT'S
C LINEAR EXPANSION,
C AND REDUCE TO A FINAL ANSWER.
C
C
C
C
X LHEL = -1
X QHEL = -1
X GHEL = -1
C
C SCALE FACTOR FOR THE AMPLITUDE IS ...
C
C TAKE THE Trace
C
Z HH5=FIVE*F0*F1*F2*F11*F5*F85*F95*F105
Z HH4=FOUR*F0*F1*F2*F11*F5*F84*F94*F104
C
C SUBSTITUTE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*(1+LHEL)*G6(J1) + 0.5*(1-LHEL)*G7(J1)
Id,F1=(G(J1,B))
Id,F2=(G(J1,M1))
Id,F11=0.5*(1+QHEL)*G6(J2) + 0.5*(1-QHEL)*G7(J2)
Id,F5=(G(J2,CU))
*yes
Id,F85=G(J2,E)
Id,F84=G(J2,M1)
Id,F95=G(J2,N5)
Id,F94=G(J2,N4)
Id,F105=G(J2,M1)
Id,F104=G(J2,E)
*yes
Id,Trick,Trace,J1
C
C Trick AND Trace
C
Id,Trick,Trace,J2

```

*
C
C
C
Ic
Al
Ic
A
Ic
A
A
C
*
C
C
C
I
A
C
*
I
A
A
I
I
A
A
I
A
A
A
C
S
S
C

```

*yes
C
C   START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N5(J-) = FIVE*(P1(J)-P5(J))
Al,Funct,N5(K2-) = FIVE*(P1(K2)-P5(K2))
Id,2,Dotpr,N4(J-) = - FOUR*(P3(J)+P4(J)+P5(J))
Al,Funct,N4(K2-) = - FOUR*(P3(K2)+P4(K2)+P5(K2))
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P   output
*yes
C
C   PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEpf*EpF(J,P3,P4,P5)
Al,Funct,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEpf*EpF(K3,P3,P4,P5)
C P   output
*yes
Id,B3 = B0IV*P4P5
Al,B4 = B0IV*P3P5
Al,B5 = -B0IV*P3P4
Al,BEpf = + LHEL*B0IV
Id,Trick
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P   output
*yes
Id,2,Dotpr,CU(J-)=
(CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
+CUepf*EpF(J,P1,P3,P5)+CUepf*EpF(J,P1,P4,P5)
Al,Funct,CU(K-)=
(CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
+CUepf*EpF(K,P1,P3,P5)+CUepf*EpF(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUepf = + QHEL*CU0IV
*yes
Id,Trick
Id,EDP1=0
Id,2,Dotpr,PD(J-)=P4(J)+P5(J)
Al,Funct,PD(K2-)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4

```


Al,P1
Al,P3
Al,P3
Al,P4
C P
*yep
C
C
Id,2
E4+
Al,F
E4+
Id,E
Al,E
Al,E
Al,E
Al,E
*yep
Id,
Id,
Id,
Al,
Al,
Id,
Al,
Al,
Al,
Al,
Al,
C
*y
K
er
Z
B
Pr
I
B
K
*
Z
B
K
*
Z
B
K

```

Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
C NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J^-)= E1*P1(J) + E3*P3(J) +
E4*P4(J)+E5*P5(J)+EEpf*(Ep f(J,P1,P3,P5)+Ep f(J,P1,P4,P5))
Al,Funct,E(K^-)= E1*P1(K) + E3*P3(K) +
E4*P4(K)+E5*P5(K)+EEpf*(Ep f(K,P1,P3,P5)+Ep f(K,P1,P4,P5))
Id,E1 = EOIV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = EOIV*GHEL*P1P5
Al,E4 = EOIV*GHEL*P1P5
Al,E5 = -EOIV*GHEL*(P1P3+P1P4)
Al,EEpf= -EOIV
*yes
Id,Trick
Id,Ep f(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P output
*yes
C
Keep HA,HB,HC,HD,HE,HF,HG,HH4,HH5
*next
Z QH = -2*P1P5*HH4 +2*(P3P4+P3P5+P4P5)*HH5
B EOIV,BOIV,CUOIV,EVL
Punch QH
Id,EVL=i*A
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HA,HB,HC,HD,HE,HF,HG,QH
*next
Z HH = QH*Conjg(QH)
B PROIV,EOIV,BOIV,CUOIV,A,i
Keep HH,HG,HF,HE,HD,HC,HB,HA
*next
Z HEL = PROIV**2*(HA+HB+HC+HD+HE+HF+HG+HH)
B PROIV,EOIV,BOIV,CUOIV,A
Id,A**2= - (-4*P14*P15*P34*P35+ (P15*P34+P14*P35-P13*P45)**2)
Keep HEL
*next
C *****
C
C THE EIGHT HELICITY STATE AMPLITUDES HAVE NOW BEEN
C CALCULATED.
C
C NOW FORM THE SPIN AVERAGED SUM CROSS SECTION

```

[illegible]

AND COMPARE IT WITH THE SUM OF THE EIGHT
ABOVE SQUARED, AS A CHECK THAT THEY WERE
ONE CORRECTLY.

IN THIS SECTION, TAKE THE Trace, REPLACE EACH K BY IT'S
LINEAR EXPANSION,
AND REDUCE TO A FINAL ANSWER.

,K,CU,M,E,E2,N4,N5,P1,P2,P3,P4,P5,PD
F0,F1,F2,F3,F4,F5,F65,F74,F75,F85,F9,F105,F11,A1,A2,A3,AA,
24,F125,F135,F64,F84,F104,F134
1,J2,M1,M2,M3,K3,K2

SCALE FACTOR FOR THE AMPLITUDE IS ...

TAKE THE Trace

XP55=FIVE*FO*F1*F2*F3*F4*F11*F5*F65*F75*F85*F9*F105*F125*F135
XP54=FIVE*FOUR*FO*F1*F2*F3*F4*F11*F5*F65*F75*F85*F9*F104*F124*F134
XP45=FOUR*FIVE*FO*F1*F2*F3*F4*F11*F5*F64*F74*F84*F9*F105*F125*F135
XP44=FOUR*FO*F1*F2*F3*F4*F11*F5*F64*F74*F84*F9*F104*F124*F134

SUBSTITUTE ACCORDING TO HELICITIES AND DIAGRAM

FO = 1
d,F0=0.5*0.5*(1+LHEL)*G6(J1) + 0.5*0.5*(1-LHEL)*G7(J1)
F1=(G(J1,P4))
F2=(G(J1,M1))
F3=(G(J1,P3))
F4=(G(J1,M2))
F11 = 1
d,F11=0.5*0.5*(1+QHEL)*G6(J2) + 0.5*0.5*(1-QHEL)*G7(J2)
F5=(G(J2,P5))
p
F65= (G(J2,M3))
F75=(G(J2,N5))
F85= (G(J2,M1))
F9=(G(J2,P2))
F105= (G(J2,M2))
F125=(G(J2,N5))
F135= (G(J2,M3))
F64= (G(J2,M1))
F74=(G(J2,N4))
F84= (G(J2,M3))
F104= (G(J2,M3))
F124=(G(J2,N4))
F134= (G(J2,M2))
p

Trick,Trace,J1

Trick AND Trace

Trick,Trace,J2

output

p
2,Dotpr,P2(K⁻) = P3(K4) + P4(K4) + P5(K4) - P1(K4)

Al,Func
C P ot
*yep
C
C S
C
Id,2,D
Id,2,D
Al, Fur
Id,2,L
Id,2,L
Al, Fur
C P
Id,P4
Al,P5
Al,P1
Al,P3
Al,ED
Al,E2
P ou
*yep
C
Id,P1
Al,P1
Al,P1
Al,P1
Al,P1
C P
*yep
C
Keep
*ne
Z E
Id,
Al,
Al,
Id,
Id,
B
Ke
*ne
Z S
Id
Id
B
Ke
*n
C
C
C
C
C
C
Z
B
*

,Funct,P2(K4~) = P3(K4) + P4(K4) + P5(K4) - P1(K4)

P output

rep

START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.

,2,2,Dotpr,N5(J~) = FIVE*(P1(J)-P5(J))

,2,2,Dotpr,N5(J~) = FIVE*(P1(J)-P5(J))

,Funct,N5(K2~) = FIVE*(P1(K2)-P5(K2))

,2,2,Dotpr,N4(J~) = - FOUR*(P3(J)+P4(J)+P5(J))

,2,2,Dotpr,N4(J~) = - FOUR*(P3(J)+P4(J)+P5(J))

,Funct,N4(K2~) = - FOUR*(P3(K2)+P4(K2)+P5(K2))

P output

,P4DP4=0

,P5DP5=0

,P1DP1 = 0

,P3DP3 = 0

,EDP1=0

,E2DP1=0

output

rep

,P1DP4=P1P4

,P1DP3=P1P3

,P1DP5=P1P5

,P3DP4=P3P4

,P3DP5=P3P5

,P4DP5=P4P5

P output

rep

exp EXP55,EXP54,EXP45,EXP44,HEL

next

EXP4 = Y*(Y1*EXP55 + Y23*EXP54 + Y23*EXP45 + Y4*EXP44)

,Y1 = 4*(P3P4+P3P5+P4P5)*(P3P4+P3P5+P4P5)

,Y23 = -2*P1P5*2*(P3P4+P3P5+P4P5)

,Y4 = 4*P1P5*P1P5

,Y=PROIV**2*EOIV**2*(-4)*P15*(P13+P14+P15)*(P35+P45-P15)

EOIV,PROIV

exp EXP4,HEL

next

SSEX = EXP4*BOIV**2*CUOIV**2*BO*CUO

,BO=16*P35*P45

,CUO=16*(P34+P35+P45)*P15

PROIV,EOIV,BOIV,CUOIV

exp HEL,SSEX

next

NOW SUTRACT THE SPIN AVERAGED CROSS SECTION
FROM THE SQUARED SUM OF THE HELECITY
STATES. THE RESULT SHOULD BE ZERO.

DIFF = HEL - SSEXP

PROIV,EOIV,BOIV,CUOIV

end

H

T
invar

$\epsilon_\mu =$

The

This
tick

wh

As

P
p
P

E
E
F
P
k

Appendix J

Projecting the Gluon Polarization Vector

The polarization vector of the GLUON particle may be expressed in Lorentz invariant form by projecting it onto a set of four vectors:

$$\epsilon_\mu = a_1 P_{1\mu} + a_4 P_{4\mu} + a_5 P_{5\mu} + a_{epf} E_{pf}(\mu, P_1, P_4, P_5)$$

The task is to find the four coefficients, a_1, a_4, a_5, a_{epf}

This can be done most simply in a co-ordinate system where the GLUON particle moves in the $+z$ direction. There the polarization vector has the form:

$$\epsilon_\mu = (1, \pm i, 0, 0)$$

where $\pm$ is the GLUON helicity state.

Assume the usual frame of reference, the rest frame of $\overline{P_4}$ and $\overline{P_5}$, then

$$\begin{aligned} P_1 &= (0, 0, E_1, iE_1) \\ P_4 &= (P_{4x}, 0, P_{4z}, iE_4) \\ P_5 &= (-P_{4x}, 0, -P_{4z}, iE_4) \end{aligned}$$

$$\begin{aligned} E_4 &= k_0 \\ E_1 &= \frac{P_1 P_4 + P_1 P_5}{-2k_0} \\ P_{4z} &= \frac{k_0(P_1 P_5 - P_1 P_4)}{P_1 P_4 + P_1 P_5} \\ P_{4x} &= \frac{k_0 \sqrt{4(P_1 P_4)(P_1 P_5)}}{P_1 P_4 + P_1 P_5} \\ k_0 &= \sqrt{-P_4 P_5 / 2} \end{aligned}$$

With the
are, using

where c_0
course w

$= 62314$

and as

then

As c

whi

With these values of P_1, P_3, P_4, P_5 , the non-zero terms of $Ep f(\mu, P_1, P_4, P_5)$ are, using

$$Ep f(A, B, C, D) = \epsilon_{\alpha\beta\gamma\delta} A_\alpha B_\beta C_\gamma D_\delta$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the usual Levi-Cevita totally anti-symmetric four tensor and of course we are in the Minkowski metric.

$$\begin{aligned} Ep f(\mu, P_1, P_4, P_5) &= \epsilon_{2314}(2, P_1, P_4, P_5) \\ &+ \epsilon_{2413}(2, P_1, P_4, P_5) + \epsilon_{2341}(2, P_1, P_4, P_5) \\ &+ \epsilon_{2314}(2, E_1, P_{4x}, iE_4) + \epsilon_{2431}(2, P_1, P_4, P_5) \\ &+ \epsilon_{2413}(2, iE_1, P_{4x}, -P_{4z}) + \epsilon_{2341}(2, E_1, iE_4, -P_{4x}) \\ &+ \epsilon_{2431}(2, iE_1, P_{4z}, -P_{4x}) \\ &= \epsilon_{2314}[(E_1)(P_{4x})(iE_4)] + \epsilon_{2413}[(iE_4)(P_{4x})(-P_{4z})] + \epsilon_{2341}[(E_1)(iE_4)(-P_{4x})] + \\ &\quad \epsilon_{2431}[(iE_1)(P_{4z})(-P_{4x})] \\ &= -2i(E_1)(E_4)(P_{4x}) \delta^2_\mu \end{aligned}$$

and as P_1, P_4, P_5 have no y component and $\epsilon_y = \pm i = (GH\bar{E}L)i$

then a_{epf} is immediately determined:

$$\begin{aligned} a_{epf} &= \frac{-GH\bar{E}L}{2(E_1)(P_{4x})(E_4)} \\ &= \frac{GH\bar{E}L}{\sqrt{-2(P_1P_4)(P_1P_5)(P_4P_5)}} \end{aligned}$$

As ϵ has but three deminsions and the space is four deminsional:

$$\epsilon_\mu P_1^\mu = 0 = a_4 P_1 P_4 + a_5 P_1 P_5$$

which gives:

$$a_4 = -a_5 \frac{P_1 P_5}{P_1 P_4}$$

and

give

By
sel

ϵ_p

and using

$$\epsilon_\mu P4^\mu = P4_x = a_1 P1P4 + a_5 P4P5$$

$$\epsilon_\mu P5^\mu = P5_x = -P4_x = a_1 P1P5 + a_4 P4P5$$

gives,

$$a_1 = \frac{(P1P4 - P1P5)}{(P1P4 + P1P5)} \frac{P4P5}{\sqrt{-2(P1P4)(P1P5)(P4P5)}}$$

$$a_4 = \frac{P1P5}{\sqrt{-2(P1P4)(P1P5)(P4P5)}}$$

$$a_5 = \frac{P1P4}{\sqrt{-2(P1P4)(P1P5)(P4P5)}}$$

By gauge invariance any amount of $P1_\mu$ can be added to ϵ_μ , so a_1 can be set to zero and the result becomes

$$\epsilon_\mu = \frac{1}{\sqrt{-2(P1P4)(P1P5)(P4P5)}} [(P1P5)P4_\mu + (P1P4)P5_\mu - (GHEL) \text{Epf}(\mu, P1, P4, P5)]$$



Appendix K

SCHOONSHIP Program for the LOOP Term

```

C
C
C   START SCHOONSHIP PROGRAM
C
C Calculate the LOOP term, algebraically
C with the 19 integrals labelled by a
C symbol. Output is by set of helicity
C states, and for either the two
C lepton or the two anti-lepton
C diagrams.
C
S LNMP,LNMQ,PI,FPIIV
V P1,P2,P3,P4,P5,S1,S2,S3,S4,U1,U2,U3,U4
C
C   FEED IN VALUES FOR THIS RUN
N 30,R0
C
C SET SWITCHES
C
X FOUR = 1
X FIVE = 1
X ELECT = 1
X POSIT = 0
X LHEL = +1
X QHEL = +1
X GHEL = +1
X MPARIT = +1
C
Outlimit,24000000
P nstat
P ninput
P nlist
C   SET DOT PRODUCTS
X P1P3 = ELECT*P13 + POSIT*P14
X P1P4 = ELECT*P14 + POSIT*P13
X P1P5 = P15
X P3P4 = P34
X P3P5 = ELECT*P35 + POSIT*P45
X P4P5 = ELECT*P45 + POSIT*P35
C
C
C   FOR CONVEINENCE , DEFINE SOME EXPRESSIONS AND THEIR RECIPROCALLS.
C
X D145 = P1P4 + P1P5
X D1 = D145
X D345 = P3P4 + P3P5
X MZP = 2*P3P4 + D3
C
C
C   NOW K CAN ALWAYS BE EXPANDED ON AN ARBRITARY BASIS SET S1,S2,S3,S4
C
K = C(1)*S1 + C(2)*S2 + C(3)*S3 + C(4)*S4
C
C   AND FOR THIS PROBLEM WE USE THE SET P1,P3,P4,P5
C
C S0....
C
X S1S2 = P1P3

```


X S1S3 = P1P4
 X S1S4 = P1P5
 X S2S3 = P3P4
 X S2S4 = P3P5
 X S3S4 = P4P5

C
 C TO THIS END, THE ORIGINAL EXPANSION OF K MAY BE DOTTED WITH EACH
 C OF THE BASIS TO FORM A 4 X 4 MATRIX, WHICH WHEN INVERTED
 C YIELDS.....

C C(1) = CC(1,1)*S1DK + CC(1,2)*S2DK + CC(1,3)*S3DK + CC(1,4)*S4DK

C W IS THE DETERMINENT FOR THIS INVERSION AND WIV IT'S RECIPROCAL

C
 X W = S1S2*S3S4*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)
 + S1S3*S2S4*(S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)
 + S1S4*S2S3*(S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)

C AND NOW, ALL THE CC'S USING THE NOTATION CC(4,3) = CC43

C
 X CC44 = WIV*(-2)*(S1S2*S1S3*S2S3)
 X CC43 = WIV*S1S2*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)
 X CC42 = WIV*S1S3*(S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)
 X CC41 = WIV*S2S3*(S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)
 X CC33 = WIV*(-2)*(S1S2*S1S4*S2S4)
 X CC32 = WIV*S1S4*(S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)
 X CC31 = WIV*S2S4*(S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)
 X CC22 = WIV*(-2)*(S1S3*S1S4*S3S4)
 X CC21 = WIV*S3S4*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)
 X CC11 = WIV*(-2)*(S2S3*S2S4*S3S4)

C
 *fix
 C
 C *****

C
 C IN THIS SECTION, TAKE THE TRACE, REPLACE EACH K BY IT'S
 C LINEAR EXPANSION,
 C AND REDUCE TO A FINAL ANSWER.

C
 C V B,K,CU,M,E,N,P1,P2,P3,P4,P5,PD
 F C,F0,F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,A1,A2,A3,AA
 I J1,J2,M1,M2,K3,K2

C
 C SCALE FACTOR FOR THE AMPLITUDE IS ...

C
 X FSCAL = 2*(P34+P35+P45)

C
 C TAKE THE TRACE

C
 Z EXP5=FSCAL*F0*F1*F2*F3*F4*F11*F5*F6*F7*F8*F9*F10

C
 C SUBSTITUE ACCORDING TO HELICITIES AND DIAGRAM

```

Id,F0=0.5*((1+LHEL)*(G6(J1))+(1-LHEL)*(G7(J1)))
Id,F1=(G(J1,B))
Id,F2=ELECT*(G(J1,M2)) + POSIT*(G(J1,M1))
Id,F3=(G(J1,K))
Id,F4=ELECT*(G(J1,M1)) + POSIT*(G(J1,M2))
Id,F11=0.5*((1+QHEL)*(G6(J2))+(1-QHEL)*(G7(J2)))
Id,F5=(G(J2,CU))
*yes
Id,F6=(G(J2,M2))
Id,F7=(G(J2,M))
Id,F8=G(J2,E)
Id,F9=(G(J2,N))
Id,F10=G(J2,M1)
*yes
Id,Trick,Trace,J1
C
C      TRICK AND TRACE
C
Id,Trick,Trace,J2
*yes
C
C      START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N(J^-) = M(J)+P1(J)
Al,Funct,N(K2^-) = M(K2)+P1(K2)
Id,MDM=0
Id,Dotpr,M(J^-)=K(J)-PD(J)
Al,Funct,M(J^-)=K(J)-PD(J)
Id,KDPD=P4P5
Id,KDK=0
Id,2,Dotpr,PD(J^-)=P4(J)+P5(J)
Al,Funct,PD(J^-)=P4(J)+P5(J)
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P OUTPUT
*yes
C
C      PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J^-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEPF*Epf(J,P3,P4,P5)
Al,Funct,B(K3^-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEPF*Epf(K3,P3,P4,P5)
C P OUTPUT
*yes
Id,B3 = B0IV*P4P5
Al,B4 = B0IV*P3P5
Al,B5 = -B0IV*P3P4
Al,BEPF = + LHEL*B0IV*(ELECT - POSIT)
Id,Trick
Id,KDK=0
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4

```



```

Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
Id,2,Dotpr,CU(J^-)=
  (CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
  +CUEPF*Ep(J,P1,P3,P5)+CUEPF*Ep(J,P1,P4,P5)
Al,Funct,CU(K^-)=
  (CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
  +CUEPF*Ep(K,P1,P3,P5)+CUEPF*Ep(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEPF = + QHEL*CU0IV
*yes
Id,Trick
Id,KDK=0
Al,KDPD=P4P5
Al,EDP1=0
Id,2,Dotpr,PD(J^-)=P4(J)+P5(J)
Al,Funct,PD(K2^-)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
C
C   NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J^-)= E1*P1(J) + E3*P3(J) +
  E4*P4(J)+E5*P5(J)+EEPF*(Ep(J,P1,P3,P5)+Ep(J,P1,P4,P5))
Al,Funct,E(K^-)= E1*P1(K) + E3*P3(K) +
  E4*P4(K)+E5*P5(K)+EEPF*(Ep(K,P1,P3,P5)+Ep(K,P1,P4,P5))
Id,E1 = E0IV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = E0IV*GHEL*P1P5
Al,E4 = E0IV*GHEL*P1P5
Al,E5 = -E0IV*GHEL*(P1P3+P1P4)
Al,EEPF = -E0IV
*yes
Id,Trick
Id,KDK=0
Id,Ep(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5

```

Al
C
*y
C
C
C
C
To

*
I
I
C
C
*
I
A
A
C
:
C

AI,P4DP5=P4P5

C P OUTPUT

*yep

C

C REPLACE K BY IT'S LINEAR EXPANSION.

C

Id,2,Funct,K(J⁻)=C(1)*P1(J)+C(2)*P3(J)+C(3)*P4(J)
+P5(J)*(1-C(3)-C(1)*D1*P45IV-C(2)*D345*P45IV)

*yep

Id,Trick

Id,Epf(P1,P3,P4,P5) = EVL

C

C P OUTPUT

*yep

Id,C(1) = CC11*P1DK+CC21*P3DK+(CC31-CC41)*P4DK+CC41*P4P5

AI,C(2) = CC21*P1DK+CC22*P3DK+(CC32-CC42)*P4DK+CC42*P4P5

AI,C(3) = CC31*P1DK+CC32*P3DK+(CC33-CC43)*P4DK+CC43*P4P5

AI,C(4) = CC41*P1DK+CC42*P3DK+(CC43-CC44)*P4DK+CC44*P4P5

C P OUTPUT

*yep

C

C THE AMPLITUDE IS NOW COMPLETELY

C IN THE FORM OF DOT PRODUCTS

C OF K WITH EXTERNAL MOMENTA.

C

C THE DOT PRODUCTS OF K NEED TO
C BE EVALUATED, FOR EXAMPLE...

C (P3DK)*(P4DK)*(P5DK)

C

C AND REMEMBER THIS FACTOR IS TO BE INTERGRATED OVER ALL K SPACE.

C

C THIS DEFINES THE A3 MATRIX....

C

C A3(4,3,1) = (S4DK)*(S3DK)*(S1DK) INTEGRAL OF, OVER

C

C ALL K SPACE.

C

C AND SIMILARLY A2(J,K) AND A1(J) MATRICES.

C

C THE A3, A2, AND A1 MATRICES ARE EVALUATED USING 19 SIMPLE
C INTEGRALS, EVALUATED EARLIER I1 THRU I19....

C

C REPLACE PDK 'S BY A MATRICES
C FOR THE FIVE POINT DIAGRAM, OR

C SIMILAR B MATRICES FOR THE FOUR POINT.

C

Id,P1DK = AA(1)

AI,P3DK = AA(2)

AI,P4DK = AA(3)

AI,P5DK = -AA(3) + P4P5

Id,AA(J⁻)*AA(K⁻)*AA(L⁻) = A3(J,K,L)

Id,AA(J⁻)*AA(K⁻) = A2(J,K)

Id,AA(J⁻) = A1(J)

Id,Symme,A3,1,2,3,A2,1,2

Keep
*new
S A
Z E
Id,
Al,
Al,
Al,
Al,
Al,
Al,
Al,
Al
Al
*y
Id
Al
Al
Al
Al
Al
Id
A
A
C
*
C
C
C
C
C
C
C
C

Keep EXP5

*next

S AO

Z EXP51 = AO*EXP5

Id,AO*A3(3,3,3) = A3333

Al,AO*A3(2,3,3) = A3332

Al,AO*A3(1,3,3) = A3331

Al,AO*A3(2,2,3) = A3322

Al,AO*A3(1,2,3) = A3321

Al,AO*A3(1,1,3) = A3311

Al,AO*A3(2,2,2) = A3222

Al,AO*A3(1,2,2) = A3221

Al,AO*A3(1,1,2) = A3211

Al,AO*A3(1,1,1) = A3111

*yep

Id,AO*A2(3,3) = A233

Al,AO*A2(2,3) = A232

Al,AO*A2(1,3) = A231

Al,AO*A2(2,2) = A222

Al,AO*A2(1,2) = A221

Al,AO*A2(1,1) = A211

Id,AO*A1(3) = A13

Al,AO*A1(2) = A12

Al,AO*A1(1) = A11

C P OUTPUT

*yep

C

C

C AND IN TERMS OF THE 19 INTEGRALS,

C THE ABOVE DEFINED D1 AND D3, THE

C A3,A2, AND A1 MATRICES,

C USING THE NOTATION....

C A3(1,3,3) = A3331

C

C

Id,A3333 = -0.5*(I19)

Al,A3332 = -0.25*(I13 - D3*I9)

Al,A3331 = -0.25*(I15 + 2*D1*I9)

Al,A3322 = -0.25*(I14 - 0.5*D3*I2 + 0.5*D3*D3*I6)

Al,A3321 = -0.125*(4*PI + 2*D1*I2 - D3*I3 - 2*D1*D3*I6)

Al,A3311 = -0.25*(I16 + D1*I3 + 2*D1*D1*I6)

Al,A3222 = 0.5*(I17 - D3*A222)

Al,A3221 = (0.25*I12 + 0.5*D1*I7 - 0.5*D3*A221)

Al,A3211 = (0.25*(I11 - D3*I8) + D1*A221)

Al,A3111 = 0.5*(I18 + 2*D1*A211)

Id,A233 = -0.5*(I9)

Al,A232 = -0.25*(I2 - D3*I6)

Al,A231 = -0.25*(I3 + 2*D1*I6)

Al,A222 = 0.5*(I7 - D3*A12)

Al,A221 = (0.25*(I1 + 2*D1*I4) - 0.5*D3*A11)

Al,A211 = 0.5*(I8 + 2*D1*A11)

*yep

Id,A13 = -0.5*(I6)

Al,A12 = 0.5*(I4 - D3*I10)

Al,A11 = 0.5*(I5 + 2*D1*I10)

C

C

C


```

yep PUT
Id,P13 C
C C
C Id,A0 C
C C
C C
C B WTV C
Keep *next C
C C
C C **
C C
C C
C C
C C
C V B
C F C
C I J
C C
C X R
C C
C Z
C C
C C
C Id
C Id
C Id
C Id
C Id
C *
```

```

*yes
Id,P13 = P34 + P35 + P45 - P14 - P15
C
C PUT IN A0
C
Id,A0 = I10
C
C
C
C
C THE FIVE POINT ANSWER IS NOW FORMED.
C
B WIV,D3,PI,FPIIV,CUOIV,BOIV,EoIV,EVL
Keep EXP51
*next
C
C NOW CALCULATE THE FOUR POINT AMPLITUDE
C
C *****
C
C
C
C IN THIS SECTION, TAKE THE TRACE, REPLACE EACH K BY IT'S
C LINEAR EXPANSION,
C AND REDUCE TO A FINAL ANSWER.
C
C
C
V B,K,CU,M,E,N,P1,P2,P3,P4,P5,PD
F C,F0,F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,A1,A2,A3,AA
I J1,J2,M1,M2,K3,K2
C
C
C SCALE FACTOR FOR THE AMPLITUDE IS ...
C
X FSCAL = 1.0
C
C
C TAKE THE TRACE
C
Z EXP4=FSCAL*F0*F1*F2*F3*F4*F11*F5*F6*F7*F8*F9*F10
C
C SUBSTITUE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*((1+LHEL)*(G6(J1))+(1-LHEL)*(G7(J1)))
Id,F1=(G(J1,B))
Id,F2=ELECT*(G(J1,M2)) + POSIT*(G(J1,M1))
Id,F3=(G(J1,K))
Id,F4=ELECT*(G(J1,M1)) + POSIT*(G(J1,M2))
Id,F11=0.5*((1+QHEL)*(G6(J2))+(1-QHEL)*(G7(J2)))
Id,F5=(G(J2,CU))
*yes
Id,F6=(G(J2,M2))
Id,F7=(G(J2,M))
Id,F8=G(J2,M1)
Id,F9=(G(J2,N))
Id,F10=G(J2,E)
*yes
Id,Trick,Trace,J1

```

C
C
C
Id,
*ye
C
C
C
Id
Al
Id
Id
Al
Id
Id
Id
Al
Id
A
A
C
*
C
C
C
I
A
C
:
:

```

C
C      TRICK AND TRACE
C
Id,Trick,Trace,J2
*yes
C
C      START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N(J-) = P3(J)+P4(J)+P5(J)
Al,Funct,N(K2-) = P3(K2)+P4(K2)+P5(K2)
Id,MDM=0
Id,Dotpr,M(J-)=K(J)-PD(J)
Al,Funct,M(J-)=K(J)-PD(J)
Id,KDPD=P4P5
Id,KDK=0
Id,2,Dotpr,PD(J-)=P4(J)+P5(J)
Al,Funct,PD(J-)=P4(J)+P5(J)
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P OUTPUT
*yes
C
C      PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEPF*EpF(J,P3,P4,P5)
Al,Funct,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEPF*EpF(K3,P3,P4,P5)
C P OUTPUT
*yes
Id,B3 = B0IV*P4P5
Al,B4 = B0IV*P3P5
Al,B5 = -B0IV*P3P4
Al,BEPF = + LHEL*B0IV*(ELECT - POSIT)
Id,Trick
Id,KDK=0
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
Id,2,Dotpr,CU(J-)=
(CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
+ CUEPF*EpF(J,P1,P3,P5)+CUEPF*EpF(J,P1,P4,P5)
Al,Funct,CU(K-)=
(CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
+ CUEPF*EpF(K,P1,P3,P5)+CUEPF*EpF(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEPF = + QHEL*CU0IV
*yes

```

```

Id,Trick
Id,KDK=0
Al,KDPD=P4P5
Al,EDP1=0
Id,2,Dotpr,PD(J^-)=P4(J)+P5(J)
Al,Funct,PD(K2^-)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
C
C   NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J^-)= E1*P1(J) + E3*P3(J) +
E4*P4(J)+E5*P5(J)+EEPF*(Ep f(J,P1,P3,P5)+Ep f(J,P1,P4,P5))
Al,Funct,E(K^-)= E1*P1(K) + E3*P3(K) +
E4*P4(K)+E5*P5(K)+EEPF*(Ep f(K,P1,P3,P5)+Ep f(K,P1,P4,P5))
Id,E1 = E0IV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = E0IV*GHEL*P1P5
Al,E4 = E0IV*GHEL*P1P5
Al,E5 = -E0IV*GHEL*(P1P3+P1P4)
Al,EEPF = -E0IV
*yes
Id,Trick
Id,KDK=0
Id,Ep f(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
C
C   REPLACE K BY IT'S LINEAR EXPANSION.
C
Id,2,Funct,K(J^-)=C(1)*P1(J)+C(2)*P3(J)+C(3)*P4(J)
+P5(J)*(1-C(3)-C(1)*D1*P45IV-C(2)*D345*P45IV)
*yes
Id,Trick
Id,Ep f(P1,P3,P4,P5) = EVL
C
C P OUTPUT
*yes
Id,C(1) = CC11*P1DK+CC21*P3DK+(CC31-CC41)*P4DK+CC41*P4P5

```


$AI, C(2) = CC21 * P1DK + CC22 * P3DK + (CC32 - CC42) * P4DK + CC42 * P4P5$
 $AI, C(3) = CC31 * P1DK + CC32 * P3DK + (CC33 - CC43) * P4DK + CC43 * P4P5$
 $AI, C(4) = CC41 * P1DK + CC42 * P3DK + (CC43 - CC44) * P4DK + CC44 * P4P5$
 C P OUTPUT

*yep

C
 C THE AMPLITUDE IS NOW COMPLETELY
 C IN THE FORM OF DOT PRODUCTS
 C OF K WITH EXTERNAL MOMENTA.

C THE DOT PRODUCTS OF K NEED TO
 C BE EVALUATED, FOR EXAMPLE...
 C $(P3DK) * (P4DK) * (P5DK)$

C AND REMEMBER THIS FACTOR IS TO BE INTERGRATED OVER ALL K SPACE.

C THIS DEFINES THE A3 MATRIX....

$A3(4,3,1) = (S4DK) * (S3DK) * (S1DK)$ INTEGRAL OF, OVER
 ALL K SPACE.

C AND SIMILARLY $A2(J,K)$ AND $A1(J)$ MATRICES.

C THE A3, A2, AND A1 MATRICES ARE EVALUATED USING 19 SIMPLE
 C INTEGRALS, EVALUATED EARLIER I1 THRU I19....

C REPLACE PDK 'S BY A MATRICES
 C FOR THE FIVE POINT DIAGRAM, OR
 C SIMILAR B MATRICES FOR THE FOUR POINT.

$Id, P1DK = AA(1)$
 $AI, P3DK = AA(2)$
 $AI, P4DK = AA(3)$
 $AI, P5DK = -AA(3) + P4P5$
 $Id, AA(J^-) * AA(K^-) * AA(L^-) = A3(J,K,L)$
 $Id, AA(J^-) * AA(K^-) = A2(J,K)$
 $Id, AA(J^-) = A1(J)$
 $Id, Symme, A3, 1, 2, 3, A2, 1, 2$
 Keep EXP51, EXP4

*next

S A0

Z EXP41 = A0 * EXP4

$Id, A0 * A3(3,3,3) = B3ERR$
 $AI, A0 * A3(2,3,3) = B3ERR$
 $AI, A0 * A3(1,3,3) = B3ERR$
 $AI, A0 * A3(2,2,3) = B3ERR$
 $AI, A0 * A3(1,2,3) = B3ERR$
 $AI, A0 * A3(1,1,3) = B3ERR$
 $AI, A0 * A3(2,2,2) = B3ERR$
 $AI, A0 * A3(1,2,2) = B3ERR$
 $AI, A0 * A3(1,1,2) = B3ERR$
 $AI, A0 * A3(1,1,1) = B3ERR$

*yep

Id,A0*A2(3,3) = B233
 Al,A0*A2(2,3) = B232
 Al,A0*A2(1,3) = B231
 Al,A0*A2(2,2) = B222
 Al,A0*A2(1,2) = B221
 Al,A0*A2(1,1) = B211
 Id,A0*A1(3) = B13
 Al,A0*A1(2) = B12
 Al,A0*A1(1) = B11
 C P OUTPUT

*yep

C
 C
 C AND IN TERMS OF THE 19 INTEGRALS,
 C THE ABOVE DEFINED D1 AND D3, THE
 C A3,A2, AND A1 MATRICES,
 C USING THE NOTATION....
 C A3(1,3,3) = A3331

C
 C
 C
 C
 C
 C PUT IN "B" SERIES

Id,B233 = -0.5*I15
 Al,B232 = -0.25*(4*PI - D3*I3)
 Al,B231 = -0.5*I16
 Al,B222 = 0.5*(I12 - D3*B12)
 Al,B221 = 0.5*(I11 - D3*B11)
 Al,B211 = I18
 Id,B13 = -0.5*I3
 Al,B12 = 0.5*(I1 - D3*I5)
 Al,B11 = I8

C PUT IN A0

*yep
 Id,P13 = P34 + P35 + P45 - P14 - P15
 *yep
 Id,A0 = I5

C THE ANSWER IS NOW FORMED.

C
 B WIV,D3,PI,FPIIV,CUOIV,BOIV,E0IV,EVL
 Keep EXP51,EXP41

*next

C
 C
 C NOW ADD THE FIVE AND FOUR POINT
 C DIAGRAMS TOGETHER

C
 C AND PUNCH THE FORTRAN
 C COMPATIBLE OUTPUT

C

Z EXP = EXP51 + EXP41

B WIV,D3,PI,FPIIV,CUOIV,BOIV,E0IV,EVL,I1,I2,I3,I4,I5,I6,
I7,I8,I9,I10,I11,I12,I13,I14,I15,I16,I17,I18,I1L

Punch EXP

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Appendix L

The Linear Expansion of k_μ

An arbitrary, though specific four-vector, k_μ , being composed of it's four components (k_x, k_y, k_z, k_0), may always be projected upon four other linearly independent four-vectors- S_1, S_2, S_3, S_4 , in that what we don't know about the four components of k_μ is traded for what we don't know about the four co-efficients of the expansion:

$$k_\mu = C1(S1_\mu) + C2(S2_\mu) + C3(S3_\mu) + C4(S4_\mu)$$

and in general the co-efficients are functions of k

$$k_\mu = [C1(k)](S1_\mu) + [C2(k)](S2_\mu) + [C3(k)](S3_\mu) + [C4(k)](S4_\mu)$$

As the equation is linear, the co-efficients for any k_μ may be determined in terms of the dot products:

$$S1 \cdot k, S2 \cdot k, S3 \cdot k, S4 \cdot k$$

or, to state it differently, what we don't know about the four co-efficients may be further traded for what we don't know about the four dot products.

This may be seen by forming the dot products:

$$S1 \cdot k = C1(S1 \cdot S1) + C2(S1 \cdot S2) + C3(S1 \cdot S3) + C4(S1 \cdot S4)$$

$$S2 \cdot k = C1(S2 \cdot S1) + C2(S2 \cdot S2) + C3(S2 \cdot S3) + C4(S2 \cdot S4)$$

$$S3 \cdot k = C1(S3 \cdot S1) + C2(S3 \cdot S2) + C3(S3 \cdot S3) + C4(S3 \cdot S4)$$

$$S4 \cdot k = C1(S4 \cdot S1) + C2(S4 \cdot S2) + C3(S4 \cdot S3) + C4(S4 \cdot S4)$$

which may be cast in matrix form

$$\begin{pmatrix} S1 \cdot k \\ S2 \cdot k \\ S3 \cdot k \\ S4 \cdot k \end{pmatrix} = \begin{pmatrix} S1 \cdot S1 & S1 \cdot S2 & S1 \cdot S3 & S1 \cdot S4 \\ S2 \cdot S1 & S2 \cdot S2 & S2 \cdot S3 & S2 \cdot S4 \\ S3 \cdot S1 & S3 \cdot S2 & S3 \cdot S3 & S3 \cdot S4 \\ S4 \cdot S1 & S4 \cdot S2 & S4 \cdot S3 & S4 \cdot S4 \end{pmatrix} \begin{pmatrix} C1 \\ C2 \\ C3 \\ C4 \end{pmatrix}$$

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The co-efficients $C1, C2, C3, C4$ are solved for in the usual Cramer's rule method. Let W be the determiniant of the above matrix. It is solved to be:

$$W = -(S1S2)(S3S4)\{-(S1S2)(S3S4) + (S2S3)(S1S4) + (S2S4)(S1S3)\} \\ - (S1S3)(S2S4)\{(S1S2)(S3S4) + (S2S3)(S1S4) + -(S2S4)(S1S3)\} \\ - (S1S4)(S2S3)\{(S1S2)(S3S4) - (S2S3)(S1S4) + (S2S4)(S1S3)\}$$

and the $C1$ numerator is the same with $S1$ replaced by k in the first column:

$$C1_{\text{numerator}} = 2(S1 \cdot k)(S2S3)(S2S4)(S3S4)$$

$$-(S1S2)(S3S4)\{-(S2 \cdot k)(S3S4) + (S2S3)(S4 \cdot k) + (S2S4)(S3 \cdot k)\} \\ -(S1S3)(S2S4)\{(S2 \cdot k)(S3S4) + (S2S3)(S4 \cdot k) + -(S2S4)(S3 \cdot k)\} \\ -(S1S4)(S2S3)\{(S2 \cdot k)(S3S4) - (S2S3)(S4 \cdot k) + (S2S4)(S3 \cdot k)\}$$

$$C1 = \frac{C1_{\text{numerator}}}{W}$$

Similarly for the $C2, C3, C4$ numerators.

Define the elements of the inverse matrix to be:

$$\begin{pmatrix} S1 \cdot S1 & S1 \cdot S2 & S1 \cdot S3 & S1 \cdot S4 \\ S2 \cdot S1 & S2 \cdot S2 & S2 \cdot S3 & S2 \cdot S4 \\ S3 \cdot S1 & S3 \cdot S2 & S3 \cdot S3 & S3 \cdot S4 \\ S4 \cdot S1 & S4 \cdot S2 & S4 \cdot S3 & S4 \cdot S4 \end{pmatrix}^{-1} = \begin{pmatrix} CC11 & CC12 & CC13 & CC14 \\ CC21 & CC22 & CC23 & CC24 \\ CC31 & CC32 & CC33 & CC34 \\ CC41 & CC42 & CC43 & CC44 \end{pmatrix}$$

Then

$$\begin{pmatrix} CC11 & CC12 & CC13 & CC14 \\ CC21 & CC22 & CC23 & CC24 \\ CC31 & CC32 & CC33 & CC34 \\ CC41 & CC42 & CC43 & CC44 \end{pmatrix} \begin{pmatrix} S1 \cdot k \\ S2 \cdot k \\ S3 \cdot k \\ S4 \cdot k \end{pmatrix} = \begin{pmatrix} C1 \\ C2 \\ C3 \\ C4 \end{pmatrix}$$

which gives four equations of the form:

$$C1 = CC11(S1 \cdot k) + CC12(S2 \cdot k) + CC13(S3 \cdot k) + CC14(S4 \cdot k)$$

Comparison with the Cramer's rule generated expressions for $C1, C2, C3, C4$

yield

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yields the $CC11$ $CC44$

$$CC11 = W^{-1}2(S2S3)(S2S4)(S3S4)$$

$$CC12 = -W^{-1}(S3S4)\{-(S1S2)(S3S4) + (S2S3)(S1S4) + (S2S4)(S1S3)\}$$

$$CC13 = -W^{-1}(S2S4)\{(S1S2)(S3S4) + (S2S3)(S1S4) + -(S2S4)(S1S3)\}$$

$$CC14 = -W^{-1}(S2S3)\{(S1S2)(S3S4) - (S2S3)(S1S4) + (S2S4)(S1S3)\}$$

$$CC21 = CC12$$

$$CC22 = W^{-1}2(S1S3)(S1S4)(S3S4)$$

$$CC23 = -W^{-1}(S1S4)\{(S1S2)(S3S4) + (S2S3)(S1S4) + -(S2S4)(S1S3)\}$$

$$CC24 = -W^{-1}(S1S3)\{(S1S2)(S3S4) - (S2S3)(S1S4) + (S2S4)(S1S3)\}$$

$$CC31 = CC13$$

$$CC32 = CC23$$

$$CC33 = W^{-1}2(S1S2)(S1S4)(S2S4)$$

$$CC34 = -W^{-1}(S1S2)\{-(S1S2)(S3S4) + (S2S3)(S1S4) + (S2S4)(S1S3)\}$$

$$CC41 = CC14$$

$$CC42 = CC24$$

$$CC43 = CC34$$

$$CC44 = W^{-1}2(S1S2)(S1S3)(S2S3)$$

In the above the fact that the particles have zero mass, that is that $S_\mu S^\mu = 0$ has been used.

Alternatively the terms may be rearranged to make explicit the dependence on the dot products:

$$k_\mu = (S1 \cdot k) \mathcal{F}1_\mu + (S2 \cdot k) \mathcal{F}2_\mu + (S3 \cdot k) \mathcal{F}3_\mu + (S4 \cdot k) \mathcal{F}4_\mu$$

where

$$\mathcal{F}1_\mu = (CC11)S1_\mu + (CC12)S2_\mu + (CC13)S3_\mu + (CC14)S4_\mu$$

$$\mathcal{F}2_\mu = (CC21)S1_\mu + (CC22)S2_\mu + (CC23)S3_\mu + (CC24)S4_\mu$$

$$\mathcal{F}3_\mu = (CC31)S1_\mu + (CC32)S2_\mu + (CC33)S3_\mu + (CC34)S4_\mu$$

$$\mathcal{F}4_\mu = (CC41)S1_\mu + (CC42)S2_\mu + (CC43)S3_\mu + (CC44)S4_\mu$$

As a check that the matrix elements $CC11.....CC44$ are correctly calculated, they are compared with the results of the relation:

$$S(\alpha)^\mu \mathcal{F}(\beta)_\mu = \delta_\alpha^\beta$$

which is obtained by dotting the above expression for k_μ with $S1, S2, S3, S4$.

$$S1^\mu k_\mu = (S1^\mu \mathcal{F}1_\mu)(S1 \cdot k) + (S1^\mu \mathcal{F}2_\mu)(S2 \cdot k) + (S1^\mu \mathcal{F}3_\mu)(S3 \cdot k) + (S1^\mu \mathcal{F}4_\mu)(S4 \cdot k)$$

and noting that $S1 \cdot k, S2 \cdot k, S3 \cdot k, S4 \cdot k$ are all independent.

For this problem:

$$S1_\mu = P1_\mu$$

$$S2_\mu = P3_\mu$$

$$S3_\mu = P4_\mu$$

$$S4_\mu = P5_\mu$$

Appendix M

Base Integral Reductions

After the TRACE is taken and the four linear substitutions invoked, the LOOP calculation has a numerator containing terms of three, two, one, or none of dot products with k_μ .

Each term, here called a 'base integral', can be integrated separately, and is potentially reducible to a simpler integral via cancellation of the dot products k_μ with similar dot products in the denominator.

For example, the 'base integral'

$$3(2, 1, 1) = \int \frac{d\Omega_k (P3 \cdot k) (P1 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

(with some co-efficient), from the five point diagram;

and

$$B1(3) = \int \frac{d\Omega_k (P4 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

(with some co-efficient), from the four point diagram, which only has two denominator factors to start with.

To establish some notation:

Those 'base integrals' of the five point diagram, (with three denominator factors), will be called 'A' type integrals and will be sub-typed A1, A2, or A3 to indicate the number of dot products of k_μ present. The first integral above is type A3.

Similarly, the four point diagram integrals (with two denominator factors) will be typed 'B', and sub-typed B1 or B2 to indicate the number of dot products k_μ present in the numerator. The second integral above is of type B1.

Further the designation will be indexed to show specifically, which dot products of k_μ are present. This index corresponds to the vectors position in the expansion of k_μ effected in Appendix L:

$$k_\mu = (C1)P1_\mu + (C2)P3_\mu + (C3)P4_\mu + (C4)P5_\mu$$

ning the index 1 to P1, the index 2 to P3, the index 3 to P4, and the index P5. The index 4 is removable by the on-shell condition:

$$P5 \cdot k = -P4 \cdot k + P4P5$$

the first example integral above is **A3(2, 1, 1)** and the second is **B1(3)**

reduction of all base integrals by the cancellation of factors is now enacted. Each case reduction is carried out until no further cancellation is possible. The integral is then expressed in terms of nineteen irreducible integrals which can be solved analytically. These nineteen integrals are listed and solved in appendix N.

reduction, whenever a term proportional to M_γ or M_{quark} is generated it is dropped, in recognition that both these masses will ultimately go to zero.

$$\begin{aligned} \mathbf{3, 3, 3}) &= \int \frac{d\Omega_k (P4 \cdot k) (P4 \cdot k) (P4 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_\xi^2 - i\Gamma M_\xi)} \\ &= \int \frac{d\Omega_k (P4 \cdot k) (P4 \cdot k) (P4 \cdot k)}{(\dots)(\dots)(\dots)} \\ &= -1/2 \left[\int \frac{d\Omega_k (-2(P4) \cdot k) (P4 \cdot k) (P4 \cdot k)}{(\dots)(\dots)(\dots)} \right] \\ &-1/2 \left[\int \frac{d\Omega_k (-2(P4) \cdot k + M_\gamma^2) (P4 \cdot k) (P4 \cdot k)}{(\dots)(\dots)(\dots)} - M_\gamma^2 \int \frac{d\Omega_k}{(\dots)(\dots)(\dots)} \right] \\ &-1/2 \left[\int \frac{d\Omega_k (-2(P4) \cdot k + M_\gamma^2) (P4 \cdot k) (P4 \cdot k)}{(\dots)(\dots)(\dots)} - 0 \int \frac{d\Omega_k}{(\dots)(\dots)(\dots)} \right] \\ &= -1/2 \left[\int \frac{d\Omega_k (P4 \cdot k) (P4 \cdot k)}{\bullet(\dots)(\dots)} \right] \\ &= -1/2 (\mathbf{I19}) \end{aligned}$$

$$\mathbf{3, 3, 2}) = \int \frac{d\Omega_k (P4 \cdot k) (P4 \cdot k) (P3 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_\xi^2 - i\Gamma M_\xi)}$$

$$\begin{aligned}
&= -1/2 \left[\int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(P4 \cdot k)(P3 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&+ \left[1/2 \left(\int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(P4 \cdot k)(2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{\bullet(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(P4 \cdot k)}{\bullet(\dots)(\dots)} \right) \right] \\
&= -1/2 [1/2 (\text{I13} - (M_z^2 - i\Gamma M_z)\text{I9})]
\end{aligned}$$

$$\begin{aligned}
3, 1) &= \int \frac{d\Omega_k}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \frac{(P4 \cdot k)(P4 \cdot k)(P1 \cdot k)}{\bullet(\dots)(\dots)} \\
&= -1/2 \left[\int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(P4 \cdot k)(P1 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&+ 4 \left[\int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(P4 \cdot k)(2P1 \cdot k - 2(P1P4 + P1P5))}{\bullet(\dots)(\dots)} + 2(P1P4 + P1P5) \int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(P4 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&= -1/4 [\text{I15} + 2(P1P4 + P1P5)\text{I9}]
\end{aligned}$$

$$\begin{aligned}
2, 2) &= \int \frac{d\Omega_k}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \frac{(P4 \cdot k)(P3 \cdot k)(P3 \cdot k)}{\bullet(\dots)(\dots)} \\
&= -1/2 \left[\int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(P3 \cdot k)(P3 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&+ 4 \left[\int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(P3 \cdot k)(2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{\bullet(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(P3 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&+ 4 \left[\text{I14} - 1/2(M_z^2 - i\Gamma M_z) \left(\int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{\bullet(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \right) \right] \\
&- 1/4 [\text{I14} - 1/2(M_z^2 - i\Gamma M_z)\text{I2} + 1/2(M_z^2 - i\Gamma M_z)(M_z^2 - i\Gamma M_z)\text{I6}]
\end{aligned}$$

$$\begin{aligned}
(2, 1) &= \int \frac{d\Omega_k \ (\dot{P}4 \cdot k) \ (P3 \cdot k) \ (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= -1/2 \left[\int \frac{d\Omega_k \ (P3 \cdot k) \ (P1 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&+ 1/4 \left[\int \frac{d\Omega_k \ (P1 \cdot k) \ (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{\bullet(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k \ (P1 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&+ \left[\int \frac{d\Omega_k \ (P1 \cdot k)}{\bullet(\dots)\bullet} - (M_z^2 - i\Gamma M_z)1/2 \left(\int \frac{d\Omega_k \ (2P1 \cdot k - 2(P1P4 + P1P5))}{\bullet(\dots)(\dots)} + 2(P1P4 + P1P5) \int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \right) \right] \\
&+ \left[1/2 \left(\int \frac{d\Omega_k \ (2P1 \cdot k - 2(P1P4 + P1P5))}{\bullet(\dots)\bullet} + 2(P1P4 + P1P5) \int \frac{d\Omega_k}{\bullet(\dots)\bullet} \right) - 1/2(M_z^2 - i\Gamma M_z) (\mathbf{I3} + 2(P1P4 + P1P5)\mathbf{I6}) \right] \\
&+ 1/4 [1/2(4\pi + 2(P1P4 + P1P5)\mathbf{I2}) - 1/2(M_z^2 - i\Gamma M_z) (\mathbf{I3} + 2(P1P4 + P1P5)\mathbf{I6})] \\
&+ 1/8 [4\pi + 2(P1P4 + P1P5)\mathbf{I2} - (M_z^2 - i\Gamma M_z)\mathbf{I3} - 2(P1P4 + P1P5)(M_z^2 - i\Gamma M_z)\mathbf{I6}]
\end{aligned}$$

$$\begin{aligned}
(3, 1, 1) &= \int \frac{d\Omega_k \ (P4 \cdot k) \ (P1 \cdot k) \ (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= -1/2 \left[\int \frac{d\Omega_k \ (P1 \cdot k) \ (P1 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&+ 1/4 \left[\int \frac{d\Omega_k \ (P1 \cdot k) \ (2P1 \cdot k - 2(P1P4 + P1P5))}{\bullet(\dots)(\dots)} + 2(P1P4 + P1P5) \int \frac{d\Omega_k \ (P1 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&+ \left[\mathbf{I16} + (P1P4 + P1P5) \left(\int \frac{d\Omega_k \ (2P1 \cdot k - 2(P1P4 + P1P5))}{\bullet(\dots)(\dots)} + 2(P1P4 + P1P5) \int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \right) \right] \\
&+ 1/4 [\mathbf{I16} + (P1P4 + P1P5)\mathbf{I3} + (1/2)2(P1P4 + P1P5)2(P1P4 + P1P5)\mathbf{I6}]
\end{aligned}$$

$$\begin{aligned}
\mathbf{2}, \mathbf{2}) &= \int \frac{d\Omega_k (P3 \cdot k) (P3 \cdot k) (P3 \cdot k)}{(-2P4 \cdot k + M_z^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&\left[\int \frac{d\Omega_k (P \cdot k) (P3 \cdot k) (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots)(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k (P3 \cdot k)^2}{(\dots)(\dots)\bullet} \right] \\
&= 1/2 [\mathbf{I17} - (M_z^2 - i\Gamma M_z) \mathbf{A2(2, 2)}]
\end{aligned}$$

$$\begin{aligned}
\mathbf{2}, \mathbf{1}) &= \int \frac{d\Omega_k (P3 \cdot k) (P3 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_z^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&\left[\int \frac{d\Omega_k (P3 \cdot k) (P1 \cdot k) (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots)(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k (P3 \cdot k)(P1 \cdot k)}{(\dots)(\dots)(\dots)} \right] \\
&\left[\int \frac{d\Omega_k (P3 \cdot k) (2P1 \cdot k - 2(P1P4 + P1P5))}{(\dots)(\dots)\bullet} + 2(P1P4 + P1P5) \int \frac{d\Omega_k (P3 \cdot k)}{(\dots)(\dots)\bullet} \right] - (M_z^2 - i\Gamma M_z) \mathbf{A2(2, 1)} \\
&= 1/2 [1/2 (\mathbf{I12} + 2(P1P4 + P1P5) \mathbf{I7}) - (M_z^2 - i\Gamma M_z) \mathbf{A2(2, 1)}]
\end{aligned}$$

$$\begin{aligned}
\mathbf{1}, \mathbf{1}) &= \int \frac{d\Omega_k (P3 \cdot k) (P1 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_z^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&\left[\int \frac{d\Omega_k (P1 \cdot k) (P1 \cdot k) (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots)(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \mathbf{A2(1, 1)} \right] \\
&\left[\int \frac{d\Omega_k (P1 \cdot k) (2P1 \cdot k - 2(P1P4 + P1P5))}{(\dots)(\dots)\bullet} + 2(P1P4 + P1P5) \int \frac{d\Omega_k (P1 \cdot k)}{(\dots)(\dots)\bullet} \right] - (M_z^2 - i\Gamma M_z) \mathbf{A2(1, 1)} \\
&\left[\mathbf{I11} + (P1P4 + P1P5) \left[\int \frac{d\Omega_k (2P1 \cdot k - 2(P1P4 + P1P5))}{(\dots)(\dots)\bullet} + 2(P1P4 + P1P5) \int \frac{d\Omega_k}{(\dots)(\dots)\bullet} \right] \right] - (M_z^2 - i\Gamma M_z) \mathbf{A2(1, 1)} \\
&= 1/2 [\mathbf{I11} + (P1P4 + P1P5) [\mathbf{I1} + 2(P1P4 + P1P5) \mathbf{I4}] - (M_z^2 - i\Gamma M_z) \mathbf{A2(2, 1)}]
\end{aligned}$$

The previous may be done another way.

$$\begin{aligned}
 \mathbf{A3}(2, 1, 1) &= \int \frac{d\Omega_k (P3 \cdot k) (P1 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
 &= 1/2 \left[\int \frac{d\Omega_k (P3 \cdot k) (P1 \cdot k) (2P1 \cdot k - 2(P1P4 + P1P5))}{(\dots)(\dots)(\dots)} + 2(P1P4 + P1P5)\mathbf{A2}(2, 1) \right] \\
 &= 1/2 \left[1/2 \left(\int \frac{d\Omega_k (P1 \cdot k) (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots) \bullet (\dots)} - (M_z^2 - i\Gamma M_z)\mathbf{I8} \right) + 2(P1P4 + P1P5)\mathbf{A2}(2, 1) \right] \\
 &= 1/2 [1/2 (\mathbf{I11} - (M_z^2 - i\Gamma M_z)\mathbf{I8}) + 2(P1P4 + P1P5)\mathbf{A2}(2, 1)]
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{A3}(1, 1, 1) &= \int \frac{d\Omega_k (P1 \cdot k) (P1 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
 &= 1/2 \left[\int \frac{d\Omega_k (P1 \cdot k) (P1 \cdot k) (2P1 \cdot k - 2(P1P4 + P1P5))}{(\dots)(\dots)(\dots)} + 2(P1P4 + P1P5)\mathbf{A2}(1, 1) \right] \\
 &= 1/2 [\mathbf{I18} + 2(P1P4 + P1P5)\mathbf{A2}(1, 1)]
 \end{aligned}$$

$$\begin{aligned}
\mathbf{A2(3,3)} &= \int \frac{d\Omega_k (P4 \cdot k) (P4 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= \int \frac{d\Omega_k (P4 \cdot k) (P4 \cdot k)}{(\dots)(\dots)(\dots)} \\
&= -1/2 \left[\int \frac{d\Omega_k (-2(P4 \cdot k) (P4 \cdot k))}{(\dots)(\dots)(\dots)} \right] \\
&= -1/2 \left[\int \frac{d\Omega_k (-2(P4 \cdot k + M_\gamma^2) (P4 \cdot k))}{(\dots)(\dots)(\dots)} - M_\gamma^2 \int \frac{d\Omega_k}{(\dots)(\dots)(\dots)} \right] \\
&= -1/2 \left[\int \frac{d\Omega_k (-2(P4 \cdot k + M_\gamma^2) (P4 \cdot k))}{(\dots)(\dots)(\dots)} - 0 \int \frac{d\Omega_k}{(\dots)(\dots)(\dots)} \right] \\
&= -1/2 \left[\int \frac{d\Omega_k (P4 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&= -1/2 \mathbf{I9}
\end{aligned}$$

$$\begin{aligned}
\mathbf{A2(3,2)} &= \int \frac{d\Omega_k (P4 \cdot k) (P3 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= -1/2 \left[\int \frac{d\Omega_k (P3 \cdot k)}{\bullet(\dots)(\dots)} \right] \\
&= -1/2 \left[1/2 \left(\int \frac{d\Omega_k (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{\bullet(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \right) \right] \\
&= -1/2 [1/2 (\mathbf{I2} - (M_z^2 - i\Gamma M_z)\mathbf{I6})]
\end{aligned}$$

$$\begin{aligned}
\mathbf{A2(3,1)} &= \int \frac{d\Omega_k (P4 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= -1/2 \left[\int \frac{d\Omega_k (P1 \cdot k)}{\bullet(\dots)(\dots)} \right]
\end{aligned}$$

$$\begin{aligned}
&= -1/4 \left[\int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \frac{(2P1 \cdot k - 2(P1P4 + P1P5))}{\bullet(\dots)(\dots)} + 2(P1P4 + P1P5) \int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \right] \\
&= -1/4 [\mathbf{I3} + 2(P1P4 + P1P5)\mathbf{I6}]
\end{aligned}$$

$$\begin{aligned}
\mathbf{A2}(2,2) &= \int \frac{d\Omega_k (P3 \cdot k) (P3 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= 1/2 \left[\int \frac{d\Omega_k (P3 \cdot k) (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots)(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k (P3 \cdot k)}{(\dots)(\dots)(\dots)} \right] \\
&= 1/2 [\mathbf{I7} - (M_z^2 - i\Gamma M_z)\mathbf{A1}(2)]
\end{aligned}$$

$$\begin{aligned}
\mathbf{A2}(2,1) &= \int \frac{d\Omega_k (P3 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= 1/2 \left[\int \frac{d\Omega_k (P1 \cdot k) (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots)(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k (P1 \cdot k)}{(\dots)(\dots)(\dots)} \right] \\
&= 1/2 \left[1/2 \left(\int \frac{d\Omega_k (2P1 \cdot k - 2(P1P4 + P1P5))}{(\dots)(\dots)\bullet} + 2(P1P4 + P1P5) \int \frac{d\Omega_k}{(\dots)(\dots)\bullet} \right) - (M_z^2 - i\Gamma M_z)\mathbf{A1}(1) \right] \\
&= 1/2 [1/2(\mathbf{I1} + 2(P1P4 + P1P5)\mathbf{I4}) - (M_z^2 - i\Gamma M_z)\mathbf{A1}(1)]
\end{aligned}$$

$$\begin{aligned}
\mathbf{A2}(1,1) &= \int \frac{d\Omega_k (P1 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= 1/2 \left[\int \frac{d\Omega_k (P1 \cdot k) (2P1 \cdot k - 2(P1P4 + P1P5))}{(\dots)(\dots)(\dots)} + 2(P1P4 + P1P5)\mathbf{A1}(1) \right] \\
&= 1/2 [\mathbf{I8} + 2(P1P4 + P1P5)\mathbf{A1}(1)]
\end{aligned}$$

$$\begin{aligned}
A1(3) &= \int \frac{d\Omega_k \quad (P4 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= \int \frac{d\Omega_k \quad (P4 \cdot k)}{(\dots)(\dots)(\dots)} \\
&= -1/2 \left[\int \frac{d\Omega_k \quad (-2(P4) \cdot k)}{(\dots)(\dots)(\dots)} \right] \\
&= -1/2 \left[\int \frac{d\Omega_k \quad (-2(P4) \cdot k + M_\gamma^2)}{(\dots)(\dots)(\dots)} - M_\gamma^2 \int \frac{d\Omega_k}{(\dots)(\dots)(\dots)} \right] \\
&= -1/2 \left[\int \frac{d\Omega_k \quad (-2(P4) \cdot k + M_\gamma^2)}{(\dots)(\dots)(\dots)} - 0 \int \frac{d\Omega_k}{(\dots)(\dots)(\dots)} \right] \\
&= -1/2 \left[\int \frac{d\Omega_k}{\bullet(\dots)(\dots)} \right] \\
&= -1/2(I6)
\end{aligned}$$

$$\begin{aligned}
A1(2) &= \int \frac{d\Omega_k \quad (P3 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= 1/2 \left[\int \frac{d\Omega_k \quad (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots)(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k}{(\dots)(\dots)(\dots)} \right] \\
&= 1/2 [I4 - (M_z^2 - i\Gamma M_z)I10]
\end{aligned}$$

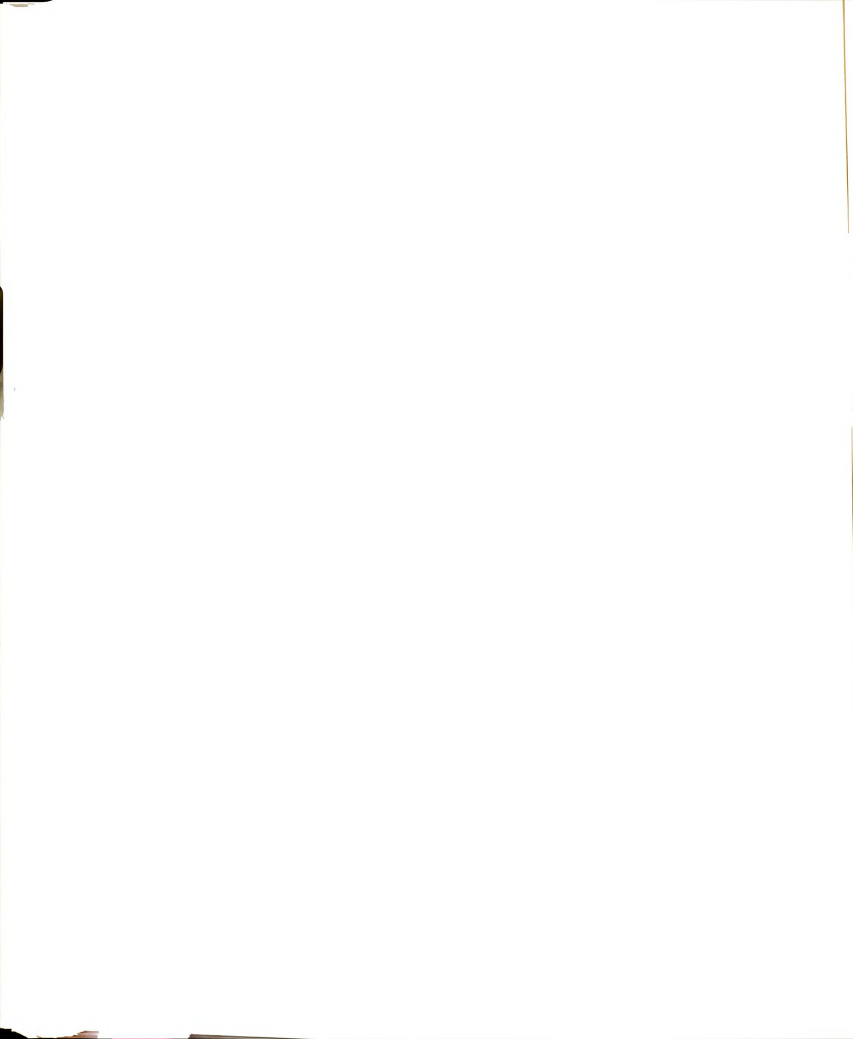
$$\begin{aligned}
A1(1) &= \int \frac{d\Omega_k \quad (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= 1/2 \left[\int \frac{d\Omega_k \quad (2P1 \cdot k - 2(P1P4 + P1P5))}{(\dots)(\dots)(\dots)} + 2(P1P4 + P1P5)I10 \right] \\
&= 1/2 [I5 + 2(P1P4 + P1P5)I10]
\end{aligned}$$

$$\begin{aligned}
\mathbf{B1(3)} &= \int \frac{d\Omega_k (P4 \cdot k)}{(-2(P4 \cdot k) + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= \int \frac{d\Omega_k (P4 \cdot k)}{(\dots)(\dots)} \\
&= -1/2 \left[\int \frac{d\Omega_k (-2(P4) \cdot k)}{(\dots)(\dots)} \right] \\
&= -1/2 \left[\int \frac{d\Omega_k (-2(P4) \cdot k + M_\gamma^2)}{(\dots)(\dots)} - M_\gamma^2 \int \frac{d\Omega_k}{(\dots)(\dots)} \right] \\
&= -1/2 \left[\int \frac{d\Omega_k (-2(P4) \cdot k + M_\gamma^2)}{(\dots)(\dots)} - 0 \int \frac{d\Omega_k}{(\dots)(\dots)} \right] \\
&= -1/2 \left[\int \frac{d\Omega_k}{\bullet(\dots)} \right] \\
&= -1/2(\mathbf{I3})
\end{aligned}$$

$$\begin{aligned}
\mathbf{B1(2)} &= \int \frac{d\Omega_k (P3 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= 1/2 \left[\int \frac{d\Omega_k (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k}{(\dots)(\dots)} \right] \\
&= 1/2 [\mathbf{I1} - (M_z^2 - i\Gamma M_z)\mathbf{I5}]
\end{aligned}$$

$$\begin{aligned}
\mathbf{B1(1)} &= \int \frac{d\Omega_k (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\
&= \mathbf{I8}
\end{aligned}$$

$$\mathbf{B2(3,3)} = \int \frac{d\Omega_k (P4 \cdot k) (P4 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$



$$= \int \frac{d\Omega_k (P4 \cdot k) (P4 \cdot k)}{(\dots)(\dots)}$$

$$= -1/2 \left[\int \frac{d\Omega_k (P4 \cdot k)}{\bullet(\dots)} \right]$$

$$= -1/2 \text{ I15}$$

$$\mathbf{B2(3,2)} = \int \frac{d\Omega_k (P4 \cdot k) (P3 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= -1/2 \left[\int \frac{d\Omega_k (P3 \cdot k)}{\bullet(\dots)} \right]$$

$$= -1/2 \left[1/2 \left(\int \frac{d\Omega_k (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{\bullet(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k}{\bullet(\dots)} \right) \right]$$

$$= -1/2 [1/2 (4\pi - (M_z^2 - i\Gamma M_z)\mathbf{I3})]$$

$$\mathbf{B2(3,1)} = \int \frac{d\Omega_k (P4 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= -1/2 \left[\int \frac{d\Omega_k (P1 \cdot k)}{\bullet(\dots)} \right]$$

$$= -1/2 (\mathbf{I16})$$

$$\mathbf{B2(2,2)} = \int \frac{d\Omega_k (P3 \cdot k) (P3 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= 1/2 \left[\int \frac{d\Omega_k (P3 \cdot k) (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k (P3 \cdot k)}{(\dots)(\dots)} \right]$$

$$= 1/2 [\mathbf{I12} - (M_z^2 - i\Gamma M_z) \mathbf{B1(2)}]$$

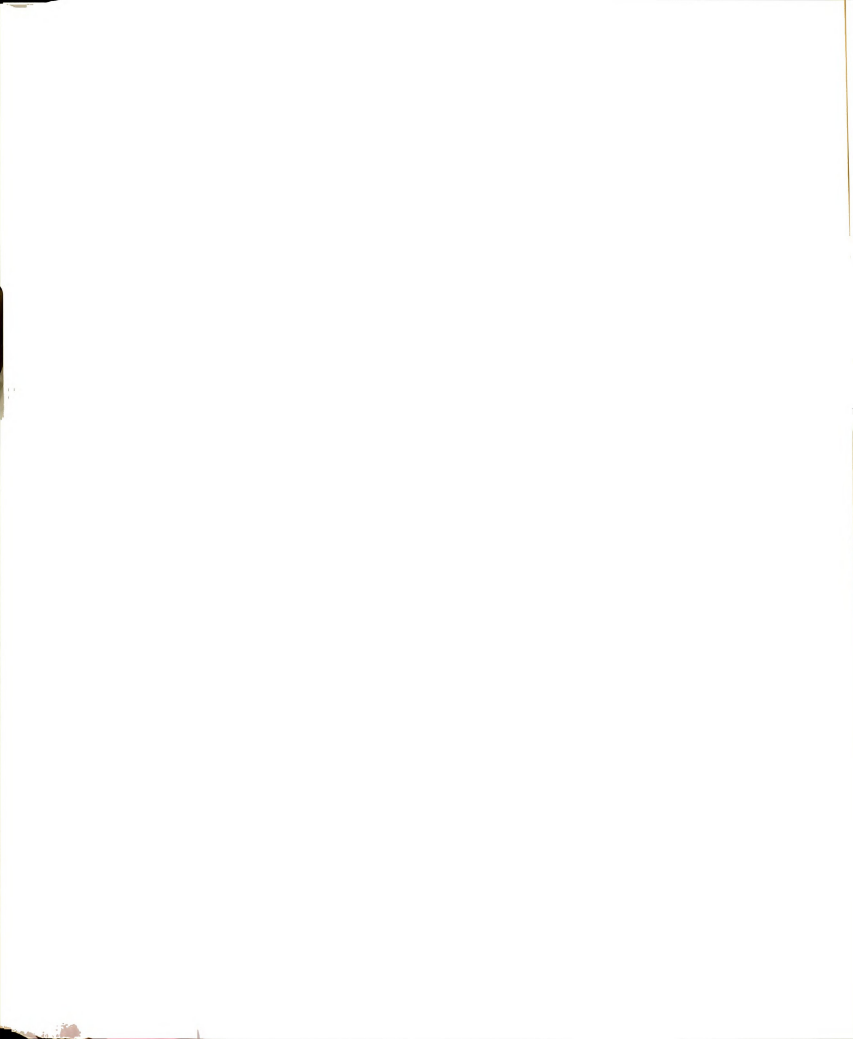
$$\mathbf{B2(2,1)} = \int \frac{d\Omega_k (P3 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_z^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= 1/2 \left[\int \frac{d\Omega_k (P1 \cdot k) (2P3 \cdot k + (M_z^2 - i\Gamma M_z))}{(\dots)(\dots)} - (M_z^2 - i\Gamma M_z) \int \frac{d\Omega_k (P1 \cdot k)}{(\dots)(\dots)} \right]$$

$$= 1/2 [\mathbf{I11} - (M_z^2 - i\Gamma M_z) \mathbf{I8}]$$

$$\mathbf{B2(1,1)} = \int \frac{d\Omega_k (P1 \cdot k) (P1 \cdot k)}{(-2P4 \cdot k + M_z^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= \mathbf{I18}$$



Appendix N

Analytic Solutions of Nineteen Irreducible Integrals

The calculation involves nineteen integrals which may not be further reduced and must be done analytically.

They are here listed, and in the remainder of this Appendix, analytically evaluated.

The notation to delineate each of the separate nineteen integrals is based on the following:

$$I = \int \frac{d\Omega_k \quad (P4 \cdot k) \quad (P1 \cdot k) \quad (P3 \cdot k)}{(-2P4 \cdot k + M_z^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

The numerator contains three factors, as does the denominator. As illustrated in Appendix M, some of these factors may be cancelled by the judicious adding and subtracting of balancing terms. Those which remain form the nineteen irreducible integrals. They are listed below in a notation where $\bullet$'s replace those factors of the notation integral above that have been removed by cancellation.

For example, in this notation the integral

$$\begin{aligned} & \int \frac{d\Omega_k \quad (P3 \cdot k)}{(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)} \\ &= \int \frac{d\Omega_k \quad \bullet \quad \bullet \quad (\dots)}{\bullet \quad (\dots) \quad \bullet} \end{aligned}$$

The nineteen irreducible integrals are:

$$I1 = \int \frac{d\Omega_k \quad \bullet \quad \bullet \quad \bullet}{(\dots) \quad \bullet \quad \bullet}$$

$$I2 = \int \frac{d\Omega_k \quad \bullet \quad \bullet \quad \bullet}{\bullet \quad (\dots) \quad \bullet}$$

$$\text{I3} = \int \frac{d\Omega_k}{\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & (\dots) \end{array}}$$

$$\text{I4} = \int \frac{d\Omega_k}{\begin{array}{ccc} \cdot & \cdot & \cdot \\ (\dots) & (\dots) & \cdot \end{array}}$$

$$\text{I5} = \int \frac{d\Omega_k}{\begin{array}{ccc} \cdot & \cdot & \cdot \\ (\dots) & \cdot & (\dots) \end{array}}$$

$$\text{I6} = \int \frac{d\Omega_k}{\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & (\dots) & (\dots) \end{array}}$$

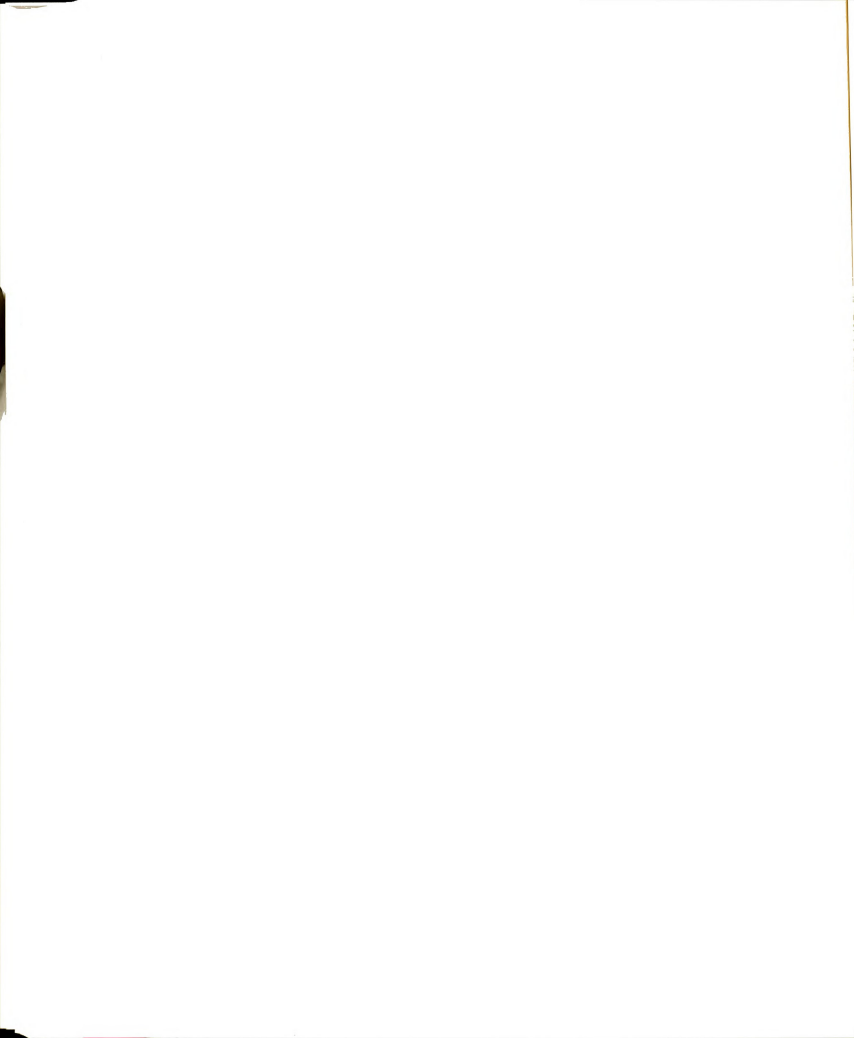
$$\text{I7} = \int \frac{d\Omega_k}{\begin{array}{ccc} \cdot & \cdot & (\dots) \\ (\dots) & (\dots) & \cdot \end{array}}$$

$$\text{I8} = \int \frac{d\Omega_k}{\begin{array}{ccc} \cdot & (\dots) & \cdot \\ (\dots) & \cdot & (\dots) \end{array}}$$

$$\text{I9} = \int \frac{d\Omega_k}{\begin{array}{ccc} (\dots) & \cdot & \cdot \\ \cdot & (\dots)(\dots) & \cdot \end{array}}$$

$$\text{I10} = \int \frac{d\Omega_k}{\begin{array}{ccc} \cdot & \cdot & \cdot \\ (\dots) & (\dots) & (\dots) \end{array}}$$

$$\text{I11} = \int \frac{d\Omega_k}{\begin{array}{ccc} \cdot & (\dots) & \cdot \\ (\dots) & \cdot & \cdot \end{array}}$$



$$I12 = \int \frac{d\Omega_k}{\frac{\bullet}{(\dots)} \frac{\bullet}{\bullet} \frac{(\dots)}{\bullet}}$$

$$I13 = \int \frac{d\Omega_k}{\frac{(\dots)}{\bullet} \frac{\bullet}{(\dots)} \frac{\bullet}{\bullet}}$$

$$I14 = \int \frac{d\Omega_k}{\frac{\bullet}{\bullet} \frac{\bullet}{(\dots)} \frac{(\dots)}{\bullet}}$$

$$I15 = \int \frac{d\Omega_k}{\frac{(\dots)}{\bullet} \frac{\bullet}{\bullet} \frac{\bullet}{(\dots)}}$$

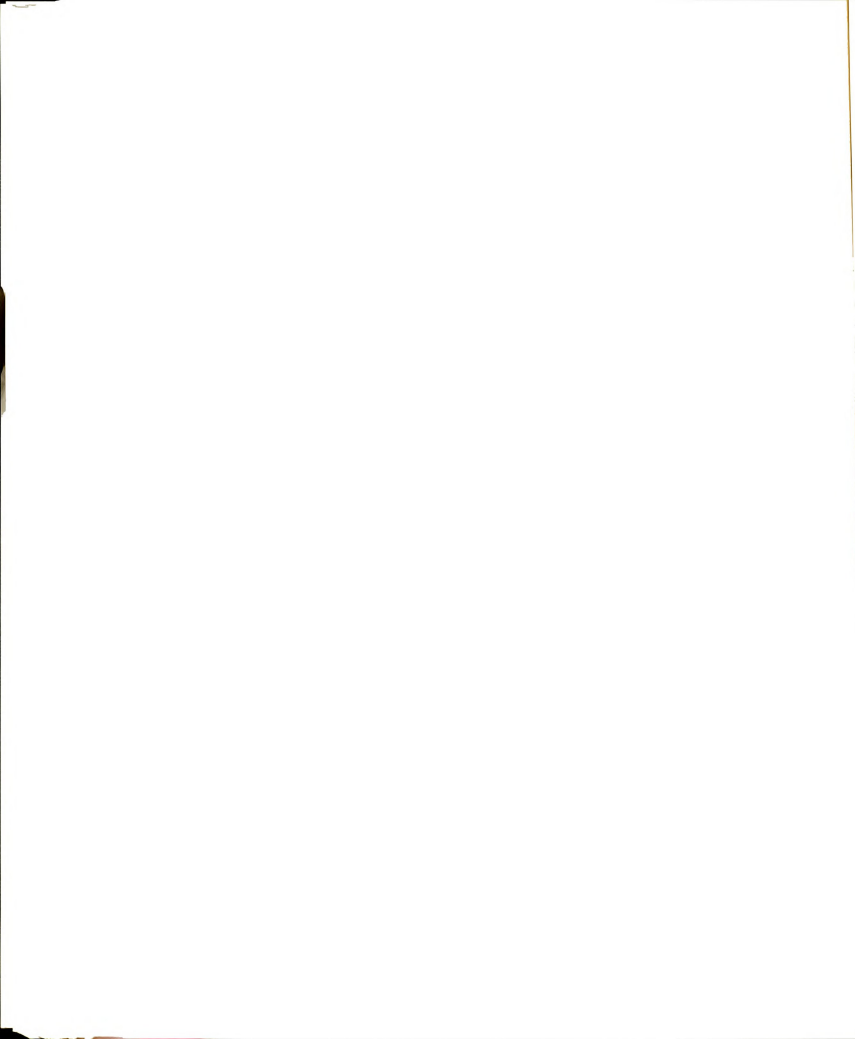
$$I16 = \int \frac{d\Omega_k}{\frac{\bullet}{\bullet} \frac{(\dots)}{(\dots)} \frac{\bullet}{\bullet}}$$

$$I17 = \int \frac{d\Omega_k}{\frac{\bullet}{(\dots)} \frac{\bullet}{(\dots)} \frac{(\dots)^2}{\bullet}}$$

$$I18 = \int \frac{d\Omega_k}{\frac{\bullet}{(\dots)} \frac{(\dots)^2}{\bullet} \frac{\bullet}{(\dots)}}$$

$$I19 = \int \frac{d\Omega_k}{\frac{(\dots)^2}{\bullet} \frac{\bullet}{(\dots)} \frac{\bullet}{(\dots)}}$$

The integrals may be grouped into 5 classes for solution- I1 - I3, I4 - I6, I7 - I9, I11 - I16, and I17 - I19, which are now solved for. The integral I10 is ; through on-shell conditions, expressable in terms of I1 - I9.



The Class of Integrals I1 - I3

These integrals are simple and may be done directly.

They are of the form:

$$I = \int \frac{d\Omega_k}{(\dots)}$$

In the co-ordinate system where the only three momentum in the integral is taken to be the z direction they assume the form:

$$\int_{4\pi} \frac{d(\cos\theta)d\phi}{(a \cos\theta + b)} = 2\pi \int_{-1}^1 \frac{dx}{ax + b} = \frac{2\pi}{a} \log \left(\frac{a+b}{-a+b} \right)$$

$$I1 = \int \frac{d\Omega_k}{-2(P4 \cdot k) + M_\gamma^2} = \int \frac{d\Omega_k}{-2ko^2 \cos\theta + 2ko^2 + M_\gamma^2}$$

$$= \frac{2\pi}{-2ko^2} \log \frac{M_\gamma^2}{4ko^2}$$

$$I1 = \frac{-2\pi}{P4P5} \log \left(\frac{-2(P4P5)}{M_\gamma^2} \right)$$

$$I2 = \int \frac{d\Omega_k}{2(P1 \cdot k) - 2(P1P4 + P1P5) + M_{quark}^2}$$

and using $2(E1)ko = -(P1P4 + P1P5)$

$$= \int \frac{d\Omega_k}{2(E1)ko \cos\theta + 2(E1)ko + M_{quark}^2}$$

$$= \frac{2\pi}{2(E1)ko} \log \frac{4(E1)ko}{M_{quark}^2}$$

$$I2 = \frac{-2\pi}{P1P4 + P1P5} \log \left(\frac{-2(P1P4 + P1P5)}{M_{quark}^2} \right)$$



$$I_3 = \int \frac{d\Omega_k}{2(P_3 \cdot k) + M_z^2 - i\Gamma M_z}$$

and using $2(E_3)k_0 = -(P_3P_4 + P_3P_5)$

$$\begin{aligned} &= \int \frac{d\Omega_k}{2(E_3)k_0 \cos\theta - 2(E_3)k_0 + M_z^2 - i\Gamma M_z} \\ &= \frac{2\pi}{2(E_3)k_0} \log \frac{M_z^2 - i\Gamma M_z}{-4(E_3)k_0 + M_z^2 - i\Gamma M_z} \\ I_3 &= \frac{-2\pi}{P_3P_4 + P_3P_5} \log \left(\frac{M_z^2 - i\Gamma M_z}{2(P_3P_4) + 2(P_3P_5) + M_z^2 - i\Gamma M_z} \right) \end{aligned}$$

The I4 - I6 Class of Integrals

These integrals are of the form:

$$I = \int \frac{d\Omega_k}{(\dots)(\dots)}$$

In a co-ordinate system where the left denominator defines the z direction and the right denominator is in the x-z plane, they assume the general form:

$$I = \int \frac{d\Omega_k}{(c \cos\theta + d) (e \cos\theta + f \sin\theta \cos\phi + g)}$$

then using

$$\int_0^{2\pi} \frac{d\phi}{a + b \cos\phi} = \frac{2\pi}{(a^2 - b^2)^{1/2}} \quad a > b > c$$

$$I = 2\pi \int_{-1}^1 \frac{dx}{(cx + d) ((e^2 + f^2)x^2 + 2gecx + g^2 - f^2)^{1/2}}$$

Then transforming $x = cx + d$

$$I = 2\pi \int_{-c+d}^{c+d} \frac{dx}{x (mx^2 + nx + p)^{1/2}}$$

where

$$m = c^2 + f^2$$

$$n = 2(gec - dm)$$

$$p = d^2m - 2gedc + c^2(g^2 - f^2)$$

which is solved using

$$\int \frac{dx}{xX^{1/2}} = \frac{-1}{\sqrt{p}} \log \frac{2\sqrt{pX} + nx + 2p}{x}$$

$$X = mx^2 + nx + p \quad p > 0$$

In general these solutions are intractable, however in the special cases involved in these integrals- $d = \pm c + M^2$, the solutions become:

first for $d = -c + M^2$

$$I = \frac{2\pi}{c(g+c)} \left[\log \left(\frac{(g + \sqrt{e^2 + f^2})^2}{(g+e)^2} \right) + \log \left(\frac{g - \sqrt{e^2 + f^2}}{g + \sqrt{e^2 + f^2}} \right) - \log \left(\frac{-2c}{M^2} \right) \right]$$

and for the other case $d = +c + M^2$

$$I = \frac{2\pi}{c(g-e)} \left[\log \left(\frac{(g-e)^2}{(g+\sqrt{e^2+f^2})^2} \right) + \log \left(\frac{g+\sqrt{e^2+f^2}}{g-\sqrt{e^2+f^2}} \right) - \log \left(\frac{M^2}{2c} \right) \right]$$

Now to use this general solution to find I4, I5 and I6.

$$\begin{aligned} \mathbf{I4} &= \int \frac{d\Omega_k}{(-2P4 \cdot k + M_\gamma^2) (2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)} \\ &= \int \frac{d\Omega_k}{(-2ko^2 \cos\theta + 2ko^2 + M_\gamma^2)(2P1_z(ko)\cos\theta + 2P1_x(ko)\sin\theta \cos\phi + 2(ko)E1 + M_{quark}^2)} \end{aligned}$$

$$\text{then using } P1_z = \frac{\overline{P1} \cdot \overline{P4}}{E4} = \frac{P1P4}{E4} + E1 = \frac{P1P4 - P1P5}{2ko} \text{ from } 2(ko)E1 = -(P1P4 + P1P5)$$

$$\text{and } P1_x = (E1 - P1_z^2)^{1/2} = \sqrt{(P1P4)(P1P5)/ko}$$

and noting that

$$\begin{aligned} c &= -2ko^2 \\ d &= 2ko^2 + M_\gamma^2 = -c + M_\gamma^2 \\ e &= P1P4 - P1P5 \\ f &= 2\sqrt{(P1P4)(P1P5)} \\ g &= -(P1P4 + P1P5) + M_{quark}^2 \\ e^2 + f^2 &= (2(ko)E1)^2 = (P1P4 + P1P5)^2 \\ g + e &= -2(P1P5) + M_{quark}^2 \end{aligned}$$

gives

$$\begin{aligned} \mathbf{I4} &= \frac{-2\pi}{2(P4P5)P1P5} \left[\log \left(\frac{(P1P4 + P1P5)^2}{P1P5^2} \right) + \log \left(\frac{M_{quark}^2}{-2(P1P4 + P1P5)} \right) - \left(\log \frac{-2(P4P5)}{M_\gamma^2} \right) \right] \\ &= \frac{-1}{(P4P5)P1P5} \left[\pi \log \left(\frac{P1P4 + P1P5}{P1P5} \right) + 0.5(P1P4 + P1P5)\mathbf{I2} + 0.5(P4P5)\mathbf{I1} \right] \end{aligned}$$

$$I5 = \int \frac{d\Omega_k}{(-2P4 \cdot k + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= \int \frac{d\Omega_k}{(-2ko^2 \cos\theta + 2ko^2 + M_\gamma^2)(2P3_z(ko)\cos\theta + 2P3_x(ko)\sin\theta \cos\phi - 2(ko)E3 + M_z^2 - i\Gamma M_z)}$$

then using $P3_z = \frac{\overline{P3} \cdot \overline{P4}}{E4} = \frac{P3P4}{E4} + E3 = \frac{(P3P4 - P3P5)}{2ko}$ from $2(ko)E3 = -(P3P4 + P3P5)$

and $P3_x = (E3^2 - P3_z^2)^{1/2} = \sqrt{(P3P4)(P3P5)}/ko$

and noting that

$$c = -2ko^2$$

$$d = 2ko^2 + M_\gamma^2 = -c + M_\gamma^2$$

$$e = P3P4 - P3P5$$

$$f = 2\sqrt{(P3P4)(P3P5)}$$

$$g = (P3P4 + P3P5) + M_z^2 - i\Gamma M_z$$

$$e^2 + f^2 = (2(ko)E3)^2 = (P3P4 + P3P5)^2$$

$$g + c = 2(P3P4) + M_z^2 - i\Gamma M_z$$

gives, using the symbol MZP for the Z particle propagator

$$MZP = 2(P3P4) + M_z^2 - i\Gamma M_z$$

$$I5 = \frac{2\pi}{(P4P5)MZP} \left[\log \left(\frac{(2(P3P5) + MZP)^2}{MZP^2} \right) + \log \left(\frac{M_z^2 - i\Gamma M_z}{2(P3P5) + MZP} \right) - \left(\log \frac{-2(P4P5)}{M_\gamma^2} \right) \right]$$

$$= \frac{1}{(P4P5)MZP} \left[2\pi \log \left(\frac{2(P3P5) + MZP}{MZP} \right) - (P3P4 + P3P5)I3 + (P4P5)I1 \right]$$

$$I6 = \int \frac{d\Omega_k}{(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= \int \frac{d\Omega_k}{(2(ko)E1 \cos\theta + 2(ko)E1 + M_{quark}^2)(2P3_z(ko)\cos\theta + 2P3_x(ko)\sin\theta \cos\phi - 2(ko)E3 + M_z^2 - i\Gamma M_z)}$$

then using $P3_z = \frac{\overline{P3} \cdot \overline{P1}}{E1} = \frac{P1P3}{E1} + E3$
with

$$E3 = \frac{P3P4 + P3P5}{2k_0} \quad E1 = \frac{P1P4 + P1P5}{2k_0}$$

to give

$$P3_z = \frac{-2k_0(P1P3 + \frac{(P1P4 + P1P5)(P3P4 + P3P5)}{4k_0^2})}{P1P4 + P1P5}$$

with $P3_x = (E3^2 - P3_z^2)^{1/2}$

and noting that

$$c = -(P1P4 + P1P5)$$

$$d = c + M_{quark}^2$$

$$e = \frac{2(P1P3)(P4P5) - (P1P4 + P1P5)(P3P4 + P3P5)}{(P1P4 + P1P5)}$$

$$f = 2(k_0)P3_x$$

$$g = -(P3P4 + P3P5) + M_z^2 - i\Gamma M_z$$

$$e^2 + f^2 = (2(k_0)E3)^2 = (P3P4 + P3P5)^2$$

$$g - e = 2(P3P5) + MZP - \frac{2(P1P3)(P4P5)}{(P1P4 + P1P5)} = \frac{R}{(P1P4 + P1P5)}$$

where

$$R = (2(P3P5) + MZP)(P1P4 + P1P5) - 2(P1P3)(P4P5)$$

using as usual, the symbol MZP for the Z particle propagator

$$MZP = 2(P3P4) + M_z^2 - i\Gamma M_z$$

$$\begin{aligned} \text{I6} &= \frac{-2\pi}{R} \left[\log \left(\frac{R^2}{(P1P4 + P1P5)^2 (2(P3P5) + MZP)^2} \right) - \log \left(\frac{M_z^2 - i\Gamma M_z}{2(P3P5) + MZP} \right) - \log \left(\frac{M_{quark}^2}{2c} \right) \right] \\ &= \frac{-1}{R} \left[4\pi \log \left(\frac{R}{(P1P4 + P1P5)(2(P3P5) + MZP)} \right) + (P3P4 + P3P5)\text{I3} - (P1P4 + P1P5)\text{I2} \right] \end{aligned}$$

The I7 - I9 Class of Integrals

These integrals are of the form:

$$I7 - I9 = \int \frac{d\Omega_k(A \cdot k)}{(\dots)(\dots)}$$

and in each case the solution will involve already solved integrals of the denominator only:

$$I_{left} = I_L = \int \frac{d\Omega_k}{(\dots) \bullet}$$

$$I_{right} = I_R = \int \frac{d\Omega_k}{\bullet (\dots)}$$

$$I_{both} = I_B = \int \frac{d\Omega_k}{(\dots)(\dots)}$$

The integrals of this class assume the general form:

$$I = \int \frac{d\Omega_k (A_x(ko)\sin\theta \cos\phi + A_y(ko)\sin\theta \sin\phi + A_z(ko)\cos\theta - A_0(ko))}{(c \cos\theta + d)(\epsilon \cos\theta + f \sin\theta \cos\phi + g)}$$

using the previously defined co-ordinate system of class I4 -I6.

Each term of the numerator may be solved separately. In an obvious notation:

$$I = I_x + I_y + I_z + I_0$$

I_y equals zero by ϕ symmetry, and by inspection:

$$I_0 = -A_0(ko)I_B$$

I_x is easily reduced:

$$\begin{aligned} I_x &= \frac{A_x ko}{f} \int \frac{f \sin\theta \cos\phi + \epsilon \cos\theta + g - \epsilon \cos\theta - g}{(c \cos\theta + d)(\epsilon \cos\theta + f \sin\theta \cos\phi + g)} \\ I_x &= \frac{A_x ko}{f} \left[I_L - g I_B - \frac{\epsilon}{c} \int \frac{(c \cos\theta + d - d)}{(c \cos\theta + d)(\epsilon \cos\theta + f \sin\theta \cos\phi + g)} \right] \\ &= \frac{A_x ko}{f} \left[I_L - (g - \frac{\epsilon d}{c}) I_B - \frac{\epsilon}{c} I_R \right] \end{aligned}$$

and the final term

$$I_z = \frac{A_z k o}{f} \int \frac{c \cos \theta + d - d}{(c \cos \theta + d)(c \cos \theta + f \sin \theta \cos \phi + g)} \\ = \frac{A_z k o}{f} [I_R - d I_B]$$

Combining the four terms:

$$I = -A_0(ko)I_B + \frac{A_z k o}{c}(I_R - d I_B) + \frac{A_r k o}{f} \left[I_L - (g - \frac{\epsilon d}{c})I_B - \frac{\epsilon}{c}I_R \right]$$

Now to use this general solution to find I7, I8 and I9.

$$I7 = \int \frac{d\Omega_k P3 \cdot k}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)}$$

$$I7 = \int \frac{d\Omega_k (P3_x(ko)\sin\theta \cos\phi + P3_y(ko)\sin\theta \sin\phi + P3_z(ko)\cos\theta - E3(ko))}{(-2ko^2 \cos\theta + 2ko^2 + M_\gamma^2)(2P1_z(ko)\cos\theta + 2P1_x(ko)\sin\theta \cos\phi + 2(ko)E1 + M_{quark}^2)}$$

$$\text{then using } P1_z = \frac{\overline{P1} \cdot \overline{P4}}{E4} = \frac{P1P4}{E4} + E1 = \frac{P1P4 - P1P5}{2ko}$$

$$\text{from } 2(ko)E1 = -(P1P4 + P1P5)$$

$$\text{and } P1_x = (E1^2 - P1_z^2)^{1/2} = \sqrt{(P1P4)(P1P5)}/ko$$

and noting that

$$c = -2ko^2$$

$$d = 2ko^2 + M_\gamma^2 = -c + M_\gamma^2$$

$$e = P1P4 - P1P5$$

$$f = 2\sqrt{(P1P4)(P1P5)}$$

$$g = -(P1P4 + P1P5) + M_{quark}^2$$

$$e^2 + f^2 = (2(ko)E1)^2 = (P1P4 + P1P5)^2$$

$$g + e = -2(P1P5) + M_{quark}^2$$

and adding

$$P3_z = \frac{\overline{P3} \cdot \overline{P4}}{E4} = \frac{P3P4}{E4} + E3 = \frac{P3P4 - P3P5}{2ko}$$

and to get $P3_x$

$$\overline{P1} \cdot \overline{P3} = P1P3 + E1E3 = P1_x P3_x + P1_y P3_y + P1_z P3_z$$

with $P1_y = 0$

$$\begin{aligned} P3_x &= 1/P1_x [P1P3 + E1E3 - P1_z P3_z] \\ &= ko/\sqrt{(P1P4)(P1P5)} \left[P1P3 + \frac{(P1P4 + P1P5)(P3P4 + P3P5)}{4ko^2} - \frac{(P1P4 - P1P5)(P3P4 - P3P5)}{4ko^2} \right] \\ &= \frac{S}{2(ko)\sqrt{(P1P4)(P1P5)}} \end{aligned}$$

where

$$S = P1P4(P3P5) + P1P5(P3P4) - P1P3(P4P5)$$

and using the general form gives:

$$\begin{aligned} I7 &= -ko(E3)I_B + \frac{P3P4 - P3P5}{2ko} \frac{ko}{c} (I_R - dI_B) \\ &+ \frac{(S)(ko)}{2ko\sqrt{(P1P4)(P1P5)}} \frac{1}{2\sqrt{(P1P4)(P1P5)}} \left[I_L + 2(P1P5)I_B - \frac{(P1P4 - P1P5)}{-2ko^2} I_R \right] \end{aligned}$$

$$\begin{aligned} I7 &= (P3P4)I4 + \frac{(P3P4 - P3P5)}{2(P4P5)} I2 \\ &+ \frac{S}{4(P1P4)P1P5} [I1 + 2(P1P5)I4 - \frac{(P1P4 - P1P5)}{P4P5} I2] \end{aligned}$$

$$\begin{aligned} I8 &= \int \frac{d\Omega_k P1 \cdot k}{(-2P4 \cdot k + M_z^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)} \\ &= \int \frac{d\Omega_k (P1_x(ko)\sin\theta \cos\phi + P1_y(ko)\sin\theta \sin\phi + P1_z(ko)\cos\theta - E1(ko))}{(-2ko^2 \cos\theta + 2ko^2 + M_z^2)(2P3_z(ko)\cos\theta + 2P3_x(ko)\sin\theta \cos\phi - 2(ko)E3 + M_z^2 - i\Gamma M_z)} \end{aligned}$$

$$\text{then using } P3_z = \frac{\overline{P3} \cdot \overline{P4}}{E4} = \frac{P3P4}{E4} + E3 = \frac{P3P4 - P3P5}{2ko}$$

$$\text{from } 2(ko)E3 = -(P3P4 + P3P5)$$

$$\text{and } P3_{\pm} = (E3^2 - P3_z^2)^{1/2} = \pm \sqrt{(P3P4)(P3P5)}/ko$$

$$P1_z = (P1P4 - P1P5)/2ko$$

$$P1_{\pm} = \frac{1}{P3_{\pm}} [P1P3 + E1E3 - P1_z P3_z] = \pm \frac{ko}{\sqrt{(P3P4)(P3P5)}} \frac{S}{2ko^2}$$

and noting that

$$c = -2ko^2$$

$$d = 2ko^2 + M_z^2 = -c + M_z^2$$

$$e = P3P4 - P3P5$$

$$f = \pm 2\sqrt{(P3P4)(P3P5)}$$

$$g = (P3P4 + P3P5) + M_z^2 - i\Gamma M_z$$

$$e^2 + f^2 = (2(ko)E3)^2 = (P3P4 + P3P5)^2$$

$$g + e = 2(P3P4) + M_z^2 - i\Gamma M_z$$

gives, using the symbol MZP for the Z particle propagator

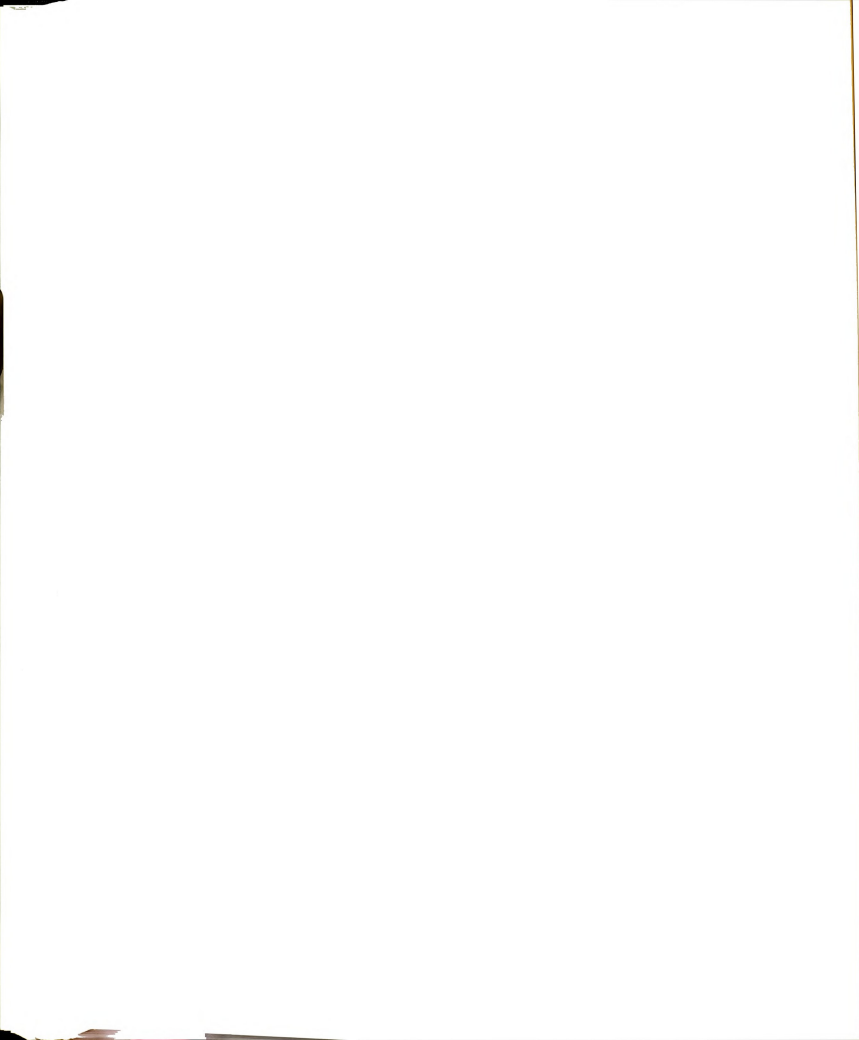
$$MZP = 2(P3P4) + M_z^2 - i\Gamma M_z$$

$$\begin{aligned} \mathbf{I8} &= -ko(E1)I_B + \frac{(P1P4 - P1P5)}{2ko} \frac{ko}{-2ko^2} (I_R - 2ko^2 I_B) \\ &+ \frac{S}{2ko\sqrt{(P3P4)(P3P5)}} \frac{ko}{2\sqrt{(P3P4)(P3P5)}} (I_L - (MZP)I_B - \frac{(P3P4 - P3P5)}{-2ko^2} I_R) \end{aligned}$$

(Note the $\pm$ arbitrariness became self-cancelling due to $\frac{d\Omega_k}{f}$ form.)

$$\mathbf{I8} = (P1P4)\mathbf{I5} + \frac{(P1P4 - P1P5)}{2(P4P5)}\mathbf{I3} + \frac{S}{4(P3P4)P3P5} \left[\mathbf{I1} - (MZP)\mathbf{I5} - \frac{(P3P4 - P3P5)}{P4P5}\mathbf{I3} \right]$$

$$\mathbf{I9} = \int \frac{d\Omega_k P4 \cdot k}{(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$



$$= \int \frac{d\Omega_k(P4_x(ko)\sin\theta \cos\phi + P4_y(ko)\sin\theta \sin\phi + P4_z(ko)\cos\theta - E4(ko))}{(2(ko)E1\cos\theta + 2(ko)E1 + M_{quark}^2)(2P3_z(ko)\cos\theta + 2P3_x(ko)\sin\theta \cos\phi - 2(ko)E3 + M_z^2 - i\Gamma M_z)}$$

then using $P3_z = \frac{\overline{P3} \cdot \overline{P1}}{E1} = \frac{P1P3}{E1} + E3$

with

$$E3 = \frac{P3P4 + P3P5}{2ko} \quad E1 = \frac{P1P4 + P1P5}{2ko}$$

to give

$$P3_z = \frac{-2ko(P1P3 + \frac{(P1P4 + P1P5)(P3P4 + P3P5)}{4ko^2})}{P1P4 + P1P5}$$

and

$$\begin{aligned} P3_x &= (E3^2 - P3_z^2)^{1/2} \\ &= [-\left(\frac{P1P3}{E1}\right)^2 - 2(P1P3)(E1/E3)]^{1/2} \\ &= \left(\frac{T}{A^2}\right)^{1/2} \end{aligned}$$

where

$$A = P1P4 + P1P5$$

and

$$T = 2(P1P3)[P1P3(P4P5) - P1P4(P3P4) - P1P4(P3P5) - P1P5(P3P4) - P1P5(P3P5)]$$

$$\begin{aligned} P4_z &= \frac{\overline{P4} \cdot \overline{P1}}{E1} = \frac{P1P4}{E1} + ko = -ko \frac{(P1P4 - P1P5)}{P1P4 + P1P5} \\ P4_x &= 1/P3_x[P4P3 + E4E3 - P4_zP3_z] \\ &= 1/P3_x[P3P4 + ko(E3) - \left(\frac{P1P3}{E1} + E3\right)\left(\frac{P1P4}{E1} + ko\right)] \\ &= \left(\frac{A^2}{T}\right)^2 \left(\frac{U}{A^2}\right) \end{aligned}$$

where

$$\begin{aligned} U &= P3P4(P1P5)^2 + P3P4(P1P4)P1P5 + P4P5(P1P3)P1P4 \\ &\quad - (P1P4)^2(P3P5) - P1P4(P1P5)P3P5 - P1P5(P1P3)P4P5 \end{aligned}$$

and noting that

$$c = -(P1P4 + P1P5) = -A$$

$$d = c + M_{quark}^2$$

$$\epsilon = \frac{W}{A}$$

where

$$W = 2(P1P3)(P4P5) - (P1P4 + P1P5)(P3P4 + P3P5)$$

$$f = 2(ko)P3_z = 2ko \left(\frac{T}{A^2} \right)^{1/2}$$

$$g = (P3P4 + P3P5) + M_z^2 - i\Gamma M_z$$

$$\epsilon^2 + f^2 = (2(ko)E3)^2 = (P3P4 + P3P5)^2$$

$$g - \epsilon = 2(P3P5) + MZP - \frac{2(P1P3)(P4P5)}{(P1P4 + P1P5)} = \frac{R}{(P1P4 + P1P5)}$$

where

$$R = (2(P3P5) + MZP)(P1P4 + P1P5) - 2(P1P3)(P4P5)$$

using as usual, the symbol MZP for the Z particle propagator

$$MZP = 2(P3P4) + M_z^2 - i\Gamma M_z$$

$$I9 = -ko^2 I_B + \frac{-ko(P1P4 - P1P5)}{A} \frac{-ko}{A} (I_R - AI_B)$$

$$+ \left(\frac{A^2}{T} \right)^{1/2} \left(\frac{ko}{2ko} \right) \left(\frac{A^2}{T} \right)^{1/2} \left(\frac{U}{A^2} \right) (I_L - (R/A)I_B + (W/A^2)I_R)$$

$$I9 = (P4P5/2)I6 - \frac{P4P5(P1P4 - P1P5)}{2A^2} I3$$

$$- \frac{P4P5(P1P4 - P1P5)}{2A} I6 + \frac{U}{2T} (I2 - (R/A)I6 + (W/A^2)I3)$$

The I10 Class Of Integral

It is desired to solve I10.

$$I10 = \int \frac{d\Omega_k}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_\xi^2 - i\Gamma M_\xi)}$$

This is effected by noting from Appendix L:

$$k_\mu = (S1 \cdot k)\mathcal{F}1_\mu + (S2 \cdot k)\mathcal{F}2_\mu + (S3 \cdot k)\mathcal{F}3_\mu + (S4 \cdot k)\mathcal{F}4_\mu$$

where

$$\mathcal{F}1_\mu = (CC11)S1_\mu + (CC12)S2_\mu + (CC13)S3_\mu + (CC14)S4_\mu$$

$$\mathcal{F}2_\mu = (CC21)S1_\mu + (CC22)S2_\mu + (CC23)S3_\mu + (CC24)S4_\mu$$

$$\mathcal{F}3_\mu = (CC31)S1_\mu + (CC32)S2_\mu + (CC33)S3_\mu + (CC34)S4_\mu$$

$$\mathcal{F}4_\mu = (CC41)S1_\mu + (CC42)S2_\mu + (CC43)S3_\mu + (CC44)S4_\mu$$

where CC11.....CC44 are given in that Appendix as assemblages of dot products.

In this problem-

$$S1 = P1$$

$$S2 = P3$$

$$S3 = P4$$

$$S4 = P5$$

so...

$$k_\mu = (P1 \cdot k)\mathcal{F}1_\mu + (P3 \cdot k)\mathcal{F}2_\mu + (P4 \cdot k)\mathcal{F}3_\mu + (P5 \cdot k)\mathcal{F}4_\mu$$

where

$$\mathcal{F}1_\mu = (CC11)P1_\mu + (CC12)P3_\mu + (CC13)P4_\mu + (CC14)P5_\mu$$

$$\mathcal{F}2_\mu = (CC21)P1_\mu + (CC22)P3_\mu + (CC23)P4_\mu + (CC24)P5_\mu$$

$$\mathcal{F}3_\mu = (CC31)P1_\mu + (CC32)P3_\mu + (CC33)P4_\mu + (CC34)P5_\mu$$

$$\mathcal{F}4_\mu = (CC41)P1_\mu + (CC42)P3_\mu + (CC43)P4_\mu + (CC44)P5_\mu$$

Further, in this problem, the on-shell condition

$$\delta[(P4 + P5 - k)]^2 = 0$$

implies

$$P5 \cdot k = P4P5 - P4 \cdot k$$

giving

$$k_\mu = (P1 \cdot k)\mathcal{F}1_\mu + (P3 \cdot k)\mathcal{F}2_\mu + (P4 \cdot k)(\mathcal{F}3_\mu - \mathcal{F}4_\mu) + (P4P5)\mathcal{F}4_\mu$$

Use of this is made by noting the other on-shell condition of this problem-
 $\delta(k^2) = 0$

$$0 = k^\mu k_\mu =$$

$$\begin{aligned} & \mathcal{F}1_\mu \mathcal{F}1^\mu (P1 \cdot k)(P1 \cdot k) + \mathcal{F}2_\mu \mathcal{F}2^\mu (P3 \cdot k)(P3 \cdot k) \\ & + (\mathcal{F}3_\mu - \mathcal{F}4_\mu)(\mathcal{F}3^\mu - \mathcal{F}4^\mu)(P4 \cdot k)(P4 \cdot k) + \mathcal{F}4_\mu \mathcal{F}4^\mu (P4P5)^2 \\ & + 2\mathcal{F}1_\mu \mathcal{F}2^\mu (P1 \cdot k)(P3 \cdot k) + 2\mathcal{F}1_\mu (\mathcal{F}3^\mu - \mathcal{F}4^\mu)(P1 \cdot k)(P4 \cdot k) \\ & + 2\mathcal{F}1_\mu \mathcal{F}4^\mu (P1 \cdot k)(P4P5) + 2\mathcal{F}2_\mu (\mathcal{F}3^\mu - \mathcal{F}4^\mu)(P3 \cdot k)(P4 \cdot k) \\ & + 2\mathcal{F}2_\mu \mathcal{F}4^\mu (P3 \cdot k)(P4P5) + 2(\mathcal{F}3_\mu - \mathcal{F}4_\mu)\mathcal{F}4^\mu (P4P5)(P4 \cdot k) \end{aligned}$$

As $k^2 = 0$, any integral of k^2 will also equal 0. In particular the integral of k^2 over the above I10 will be zero.

$$0 = \int \frac{d\Omega_k \quad k^\mu k_\mu}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

When the above expression for $k^\mu k_\mu$ is inserted into this integral, it produces a sum of integrals which still equals 0.

For example, one such term would be:

$$\int \frac{d\Omega_k \quad \mathcal{F}1_\mu \mathcal{F}1^\mu (P1 \cdot k)(P1 \cdot k)}{(-2P4 \cdot k + M_\gamma^2)(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

This integral, except for $\mathcal{F}1_\mu \mathcal{F}1^\mu$, is defined in Appendix M to have the notation **A2(1,1)**. The other integrals appearing in this sum have a similar notation defined in that Appendix. Using this notation, the expression for 0 becomes:

$$0 = \mathcal{F}1_\mu \mathcal{F}1^\mu \mathbf{A2}(1,1) + \mathcal{F}2_\mu \mathcal{F}2^\mu \mathbf{A2}(2,2) + (\mathcal{F}3_\mu - \mathcal{F}4_\mu)(\mathcal{F}3^\mu - \mathcal{F}4^\mu) \mathbf{A2}(3,3)$$

$$\begin{aligned}
& +\mathcal{F}4_\mu\mathcal{F}4^\mu(P4P5)^2\mathbf{I10} + 2\mathcal{F}1_\mu\mathcal{F}2^\mu\mathbf{A2}(2,1) + 2\mathcal{F}1_\mu(\mathcal{F}3^\mu - \mathcal{F}4^\mu)\mathbf{A2}(3,1) \\
& + 2\mathcal{F}1_\mu\mathcal{F}4^\mu\mathbf{A1}(1)(P4P5) + 2\mathcal{F}2_\mu(\mathcal{F}3^\mu - \mathcal{F}4^\mu)\mathbf{A2}(3,2) \\
& + 2\mathcal{F}2_\mu\mathcal{F}4^\mu\mathbf{A1}(2)(P4P5) + 2(\mathcal{F}3_\mu - \mathcal{F}4_\mu)\mathcal{F}4^\mu(P4P5)\mathbf{A1}(3)
\end{aligned}$$

In Appendix M, it is found that

$$\begin{aligned}
\mathbf{A2}(3,3) &= -1/2 \mathbf{I9} \\
\mathbf{A2}(3,2) &= -1/2 [\mathbf{I2} - (M_z^2 - i\Gamma M_z)\mathbf{I6}] \\
\mathbf{A2}(3,1) &= -1/4 [\mathbf{I3} + 2(P1P4 + P1P5)\mathbf{I6}] \\
\mathbf{A2}(2,2) &= 1/2 [\mathbf{I7} - (M_z^2 - i\Gamma M_z)\mathbf{A1}(2)] \\
\mathbf{A2}(2,1) &= 1/2 [\mathbf{I1} + 2(P1P4 + P1P5)\mathbf{I4} - (M_z^2 - i\Gamma M_z)\mathbf{A1}(1)] \\
\mathbf{A2}(1,1) &= [\mathbf{I8} + 2(P1P4 + P1P5)\mathbf{A1}(1)] \\
\mathbf{A1}(3) &= -1/2\mathbf{I6} \\
\mathbf{A1}(2) &= 1/2 [\mathbf{I4} - (M_z^2 - i\Gamma M_z)\mathbf{I10}] \\
\mathbf{A1}(1) &= 1/2 [\mathbf{I5} + 2(P1P4 + P1P5)\mathbf{I10}]
\end{aligned}$$

After these are substituted in the expression for 0, that expression becomes an algebraic expression for $\mathbf{I10}$, which may be solved to yield:

$$\mathbf{I10} = -\frac{\mathbf{I10}_{upper}}{\mathbf{I10}_{lower}}$$

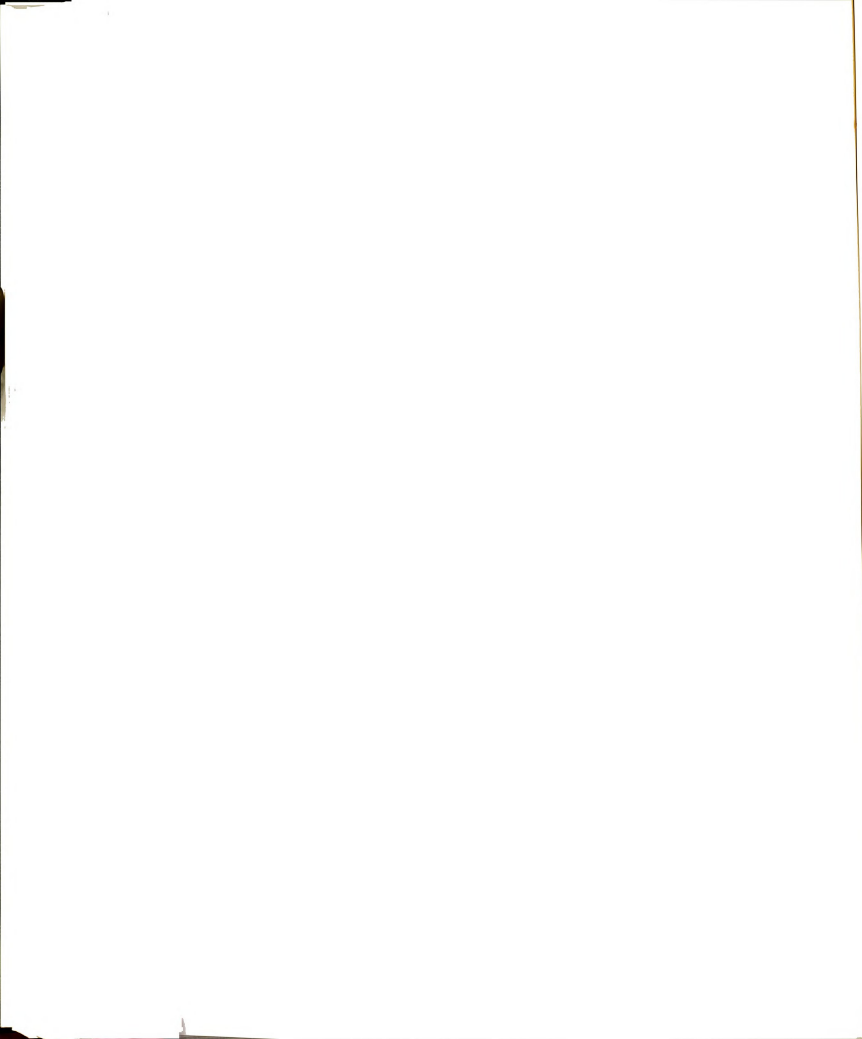
where

$$\begin{aligned}
\mathbf{I10}_{upper} &= \mathcal{F}1_\mu\mathcal{F}1^\mu(1/2\mathbf{I8}) + 1/2(P1P4 + P1P5)\mathbf{I5} \\
& + \mathcal{F}2_\mu\mathcal{F}2^\mu(1/2\mathbf{I7}) - 1/4(M_z^2 - i\Gamma M_z)\mathbf{I4} \\
& - (\mathcal{F}3_\mu - \mathcal{F}4_\mu)(\mathcal{F}3^\mu - \mathcal{F}4^\mu)(1/2)\mathbf{I9} \\
& + \mathcal{F}1_\mu\mathcal{F}2^\mu(1/2\mathbf{I1}) + (P1P4 + P1P5)\mathbf{I4} - 1/8(M_z^2 - i\Gamma M_z)\mathbf{I5} \\
& - \mathcal{F}1_\mu(\mathcal{F}3^\mu - \mathcal{F}4^\mu)(1/2\mathbf{I3}) + (P1P4 + P1P5)\mathbf{I6} + \mathcal{F}1_\mu\mathcal{F}4^\mu(P4P5)\mathbf{I5} \\
& - \mathcal{F}2_\mu(\mathcal{F}3^\mu - \mathcal{F}4^\mu)(1/2\mathbf{I2}) - (M_z^2 - i\Gamma M_z)\mathbf{I6} \\
& + \mathcal{F}2_\mu\mathcal{F}4^\mu(P4P5)\mathbf{I4} - (\mathcal{F}3_\mu - \mathcal{F}4_\mu)\mathcal{F}4^\mu(P4P5)\mathbf{I6}
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{I10}_{lower} &= \mathcal{F}1_\mu\mathcal{F}1^\mu((P1P4 + P1P5)^2) + \mathcal{F}2_\mu\mathcal{F}2^\mu(1/4)(M_z^2 - i\Gamma M_z)^2 \\
& + \mathcal{F}4_\mu\mathcal{F}4^\mu(P4P5)^2 + \mathcal{F}1_\mu\mathcal{F}2^\mu(-1/4(P1P4 + P1P5)(M_z^2 - i\Gamma M_z)) \\
& + \mathcal{F}1_\mu\mathcal{F}4^\mu(P4P5)(2(P1P4 + P1P5)) + \mathcal{F}2_\mu\mathcal{F}4^\mu(-(M_z^2 - i\Gamma M_z))
\end{aligned}$$

As $\mathbf{I1}.....\mathbf{I9}$ are now known, $\mathbf{I10}$ is now also known.



The I11 - I16 Class of Integrals

These integrals are of the form:

$$\text{I11} - \text{I16} = \int \frac{d\Omega_k(A \cdot k)}{(\dots)}$$

and in each case the solution is reducable to integrals already solved of the denominator only:

$$I_D = \int \frac{d\Omega_k}{(\dots)}$$

The integrals of this class assume the general form:

$$I = \int \frac{d\Omega_k (A_x(ko)\sin\theta \cos\phi + A_y(ko)\sin\theta \sin\phi + A_z(ko)\cos\theta - A_0(ko))}{(c \cos\theta + d)}$$

using the previously defined co-ordinate system of class I4 -I6.

Each term of the numerator may be solved separately. In an obvious notation:

$$I = I_x + I_y + I_z + I_0$$

I_x and I_y equal zero by ϕ symmetry, and by inspection:

$$I_0 = -A_0(ko)I_D$$

I_z is easily reduced:

$$I_z = \frac{A_z ko}{c} \int \frac{c \cos\theta + d - d}{(c \cos\theta + d)}$$

so,

$$I = -A_0(ko)I_D + \frac{A_z ko}{c} [4\pi - dI_D]$$

Now to use this general solution to find I11 - I16.

$$\text{I11} = \int \frac{d\Omega_k P1 \cdot k}{(-2(P4 \cdot k) + M_1^2)}$$

$$\text{I11} = \int \frac{d\Omega_k (P1_x(ko)\sin\theta \cos\phi + P1_y(ko)\sin\theta \sin\phi + P1_z(ko)\cos\theta - E1(ko))}{(-2ko^2 \cos\theta + 2ko^2 + M_1^2)}$$

$$\text{then using } P1_z = \frac{\overline{P1} \cdot \overline{P4}}{E4} = \frac{P1P4}{E4} + E1 = \frac{P1P4 - P1P5}{2k_0}$$

$$\text{from } 2(k_0)E1 = -(P1P4 + P1P5)$$

and noting that

$$c = -2k_0^2$$

$$d = 2k_0^2 + M_\gamma^2 = -c + M_\gamma^2$$

$$I11 = \left(\frac{P1P4 + P1P5}{2} \right) I1 + \left(\frac{P1P4 - P1P5}{2(P4P5)} \right) 4\pi + \left(\frac{P1P4 - P1P5}{2} \right) I1$$

$$I11 = 2\pi \left(\frac{P1P4 - P1P5}{P4P5} \right) + (P1P4)I1$$

and $I12$ is the identical with $P1$ replaced by $P3$

$$I12 = \int \frac{d\Omega_k P3 \cdot k}{(-2(P4 \cdot k) + M_\gamma^2)}$$

$$I12 = 2\pi \left(\frac{P3P4 - P3P5}{P4P5} \right) + (P3P4)I1$$

$$I13 = \int \frac{d\Omega_k P4 \cdot k}{(2(P1 \cdot k) - 2(P1P4 + P1P5) + M_{quark}^2)}$$

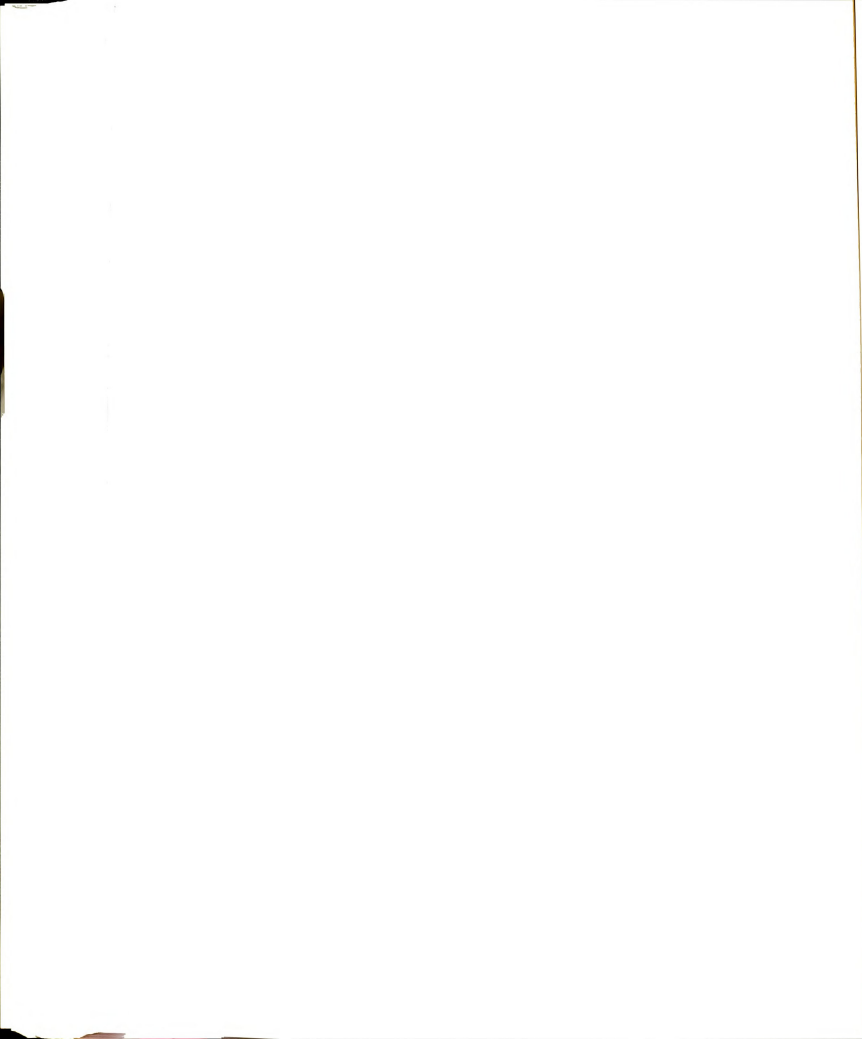
$$I13 = \int \frac{d\Omega_k P4 \cdot k}{(2(k_0)E1\cos\theta + 2(k_0)E1 + M_{quark}^2)}$$

$$c = -(P1P4 + P1P5)$$

$$d = c + M_{quark}^2 = c + 0 = c$$

$$P4_z = \frac{\overline{P4} \cdot \overline{P1}}{E1} = \frac{P1P4}{E1} + k_0 = -k_0 \frac{(P1P4 - P1P5)}{(P1P4 + P1P5)}$$

$$I13 = -k_0^2 I2 + \left(-k_0 \frac{(P1P4 - P1P5)}{(P1P4 + P1P5)} \right) k_0 \left(\frac{-4\pi}{(P1P4 + P1P5)} - I2 \right)$$



$$= -2\pi(P_4P_5) \frac{(P_1P_4 - P_1P_5)}{(P_1P_4 + P_1P_5)^2} + \left(\frac{(P_4P_5)(P_1P_5)}{(P_1P_4 + P_1P_5)} \right) \mathbf{I}_2$$

$$\mathbf{I}_{14} = \int \frac{d\Omega_k P_3 \cdot k}{(2(P_1 \cdot k) - 2(P_1P_4 + P_1P_5) + M_{quark}^2)}$$

$$\mathbf{I}_{14} = \int \frac{d\Omega_k P_3 \cdot k}{(2(ko)E_1 \cos\theta + 2(ko)E_1 + M_{quark}^2)}$$

$$c = -(P_1P_4 + P_1P_5)$$

$$d = c + M_{quark}^2 = c$$

$$P_{3z} = \frac{-2ko(P_1P_3 + \frac{(P_1P_4 + P_1P_5)(P_3P_4 + P_3P_5)}{4kc^2})}{P_1P_4 + P_1P_5}$$

$$= \frac{-[-2(P_1P_3)(P_4P_5) + (P_1P_4 + P_1P_5)(P_3P_4 + P_3P_5)]}{2ko(P_1P_4 + P_1P_5)}$$

$$\mathbf{I}_{14} = \left(\frac{P_3P_4 + P_3P_5}{2} \right) \mathbf{I}_2$$

$$+ \left(\frac{-2(P_1P_3)(P_4P_5) + (P_1P_4 + P_1P_5)(P_3P_4 + P_3P_5)}{2(P_1P_4 + P_1P_5)} \right) \left(\frac{4\pi}{P_1P_4 + P_1P_5} + \mathbf{I}_2 \right)$$

$$\mathbf{I}_{15} = \int \frac{d\Omega_k P_4 \cdot k}{(2(P_3 \cdot k) + M_i^2 - i\Gamma M_i)}$$

$$\mathbf{I}_{15} = \int \frac{d\Omega_k P_4 \cdot k}{(2E_3(ko)\cos\theta - 2(ko)E_3 + M_i^2 - i\Gamma M_i)}$$

and noting that

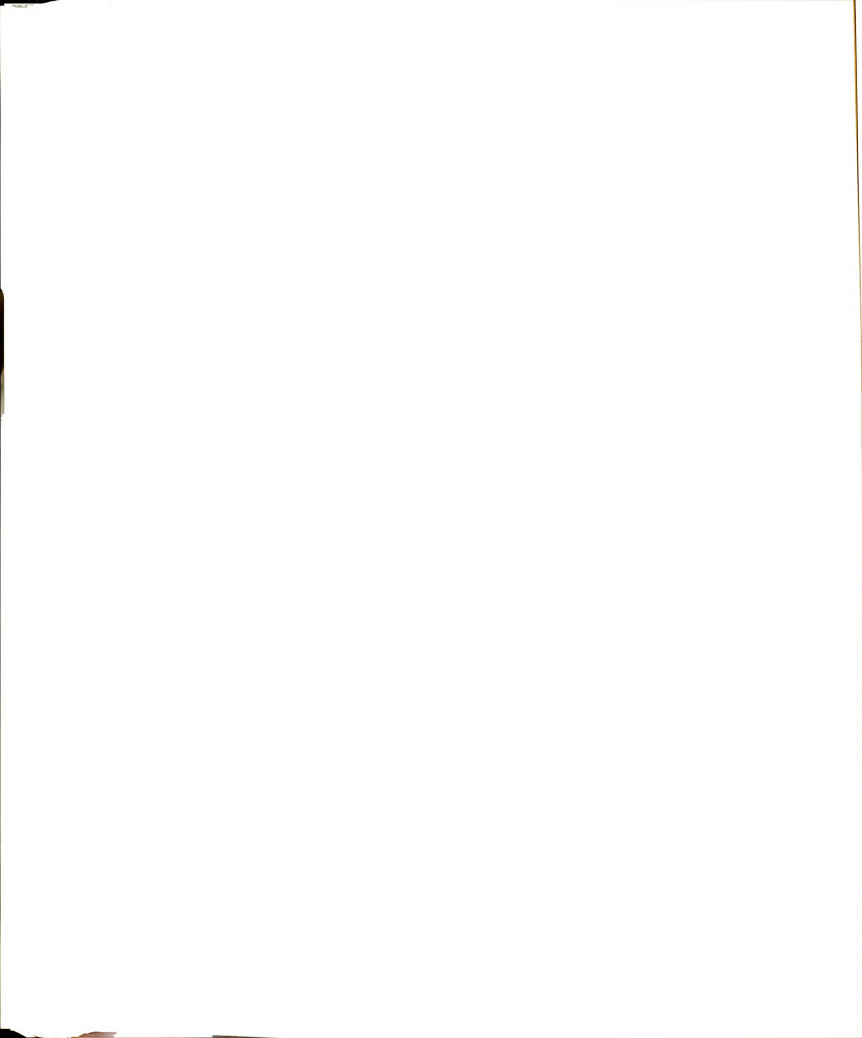
$$c = -(P_3P_4 + P_3P_5)$$

$$d = (P_3P_4 + P_3P_5) + M_i^2 - i\Gamma M_i$$

$$E_4 = ko$$

$$P_{4z} = \frac{\overline{P_4} \cdot \overline{P_1}}{E_1} = \frac{P_1P_4}{E_1} + ko = -ko \frac{(P_1P_4 - P_1P_5)}{P_1P_4 + P_1P_5}$$

$$\mathbf{I}_{15} = -ko^2 \mathbf{I}_3 + \left(-ko \frac{(P_3P_4 - P_3P_5)}{(P_3P_4 + P_3P_5)^2} \right) (-ko) (4\pi - [P_3P_4 + P_3P_5 + M_i^2 - i\Gamma M_i] \mathbf{I}_3)$$



$$= \left(\frac{(P_4 P_5)}{2(P_1 P_4 + P_1 P_5)^2} \right) (-4\pi(P_3 P_4 - P_3 P_5) + \mathbf{I}_3 [4(P_3 P_4)(P_3 P_5) + P_3 P_4 - P_3 P_5]) M Z P$$

$$\mathbf{I16} = \int \frac{d\Omega_k P_1 \cdot k}{(2(P_3 \cdot k) + M_z^2 - i\Gamma M_z)}$$

$$\mathbf{I16} = \int \frac{d\Omega_k P_1 \cdot k}{(2E_3(ko)\cos\theta - 2(ko)E_3 + M_z^2 - i\Gamma M_z)}$$

and noting that

$$c = -(P_3 P_4 + P_3 P_5)$$

$$d = (P_3 P_4 + P_3 P_5) + M_z^2 - i\Gamma M_z$$

$$E_1 = \frac{-(P_1 P_4 + P_1 P_5)}{2ko}$$

$$P_{1z} = \frac{\overline{P_3} \cdot \overline{P_1}}{E_3} + E_1$$

$$\begin{aligned} &= \frac{-2ko(P_1 P_3 + \frac{(P_1 P_4 + P_1 P_5)(P_3 P_4 + P_3 P_5)}{4ko^2})}{P_3 P_4 + P_3 P_5} \\ &= \frac{-[-2(P_1 P_3)(P_4 P_5) + (P_1 P_4 + P_1 P_5)(P_3 P_4 + P_3 P_5)]}{2ko(P_3 P_4 + P_3 P_5)} \end{aligned}$$

gives

$$\begin{aligned} \mathbf{I16} &= \left(\frac{(P_1 P_4 + P_1 P_5)}{2} \right) \mathbf{I3} \\ &- \left(\frac{[2(P_1 P_3)(P_4 P_5) - (P_1 P_4 + P_1 P_5)(P_3 P_4 + P_3 P_5)]}{2(P_1 P_4 + P_1 P_5)} \right) (4\pi - [P_3 P_4 + P_3 P_5 + M_z^2 - i\Gamma M_z] \mathbf{I3}) \end{aligned}$$

The I17 - I19 Class of Integrals

These integrals are of the form:

$$\text{I17 - I19} = \int \frac{d\Omega_k (A \cdot k)^2}{(\dots)(\dots)}$$

Most of the solution is expressable in terms of previously used I_L , I_R , and I_B and integrals also already solved involving just one of the numerator factors and one or both of the denominators:

$$I_{TopLeft} = I_{TL} = \int \frac{d\Omega_k (A \cdot k)}{(\dots) \bullet}$$

$$I_{TopRight} = I_{TR} = \int \frac{d\Omega_k (A \cdot k)}{\bullet (\dots)}$$

$$I_{TopBoth} = I_{TB} = \int \frac{d\Omega_k (A \cdot k)}{(\dots)(\dots)}$$

The solution is effected by expanding just one of the numerator factors into four terms:

$$I = \int \frac{d\Omega_k (A_x(ko)\sin\theta \cos\phi + A_y(ko)\sin\theta \sin\phi + A_z(ko)\cos\theta - A_0(ko)) (A \cdot k)}{(c \cos\theta + d)(e \cos\theta + f \sin\theta \cos\phi + g)}$$

using the previously defined co-ordinate system of class I4 - I6, producing the four terms, in an obvious notation:

$$I = I_x + I_y + I_z + I_0$$

First solve I_0 , by inspection:

$$I_0 = -A_0(ko)I_{TB}$$

and then I_z

$$I_z = \frac{A_z ko}{c} [I_{TR} - dI_{TB}]$$

and I_x

$$I_x = \frac{A_x ko}{f} \left[I_{TL} - \left(g - \frac{ed}{c} \right) I_{TB} - \frac{e}{c} I_{TR} \right]$$

These were done in the usual way of adding and subtracting the terms required

to produce cancellation of denominator factors.

I_y is harder. It is approached by also expanding the other numerator factor:

$$I_y = \int \frac{d\Omega_k (A_y(ko)\sin\theta \sin\phi) (A_x(ko)\sin\theta \cos\phi + A_y(ko)\sin\theta \sin\phi + A_z(ko)\cos\theta - A_0(ko))}{(c \cos\theta + d)(e \cos\theta + f \sin\theta \cos\phi + g)}$$

Three of the terms are zero by symmetry. The remaining term:

$$I_y = A_y^2 k o^2 \int \frac{d\Omega_k (\sin^2\theta \sin^2\phi)}{(c \cos\theta + d)(e \cos\theta + f \sin\theta \cos\phi + g)}$$

$$I_y = \frac{A_y^2 k o^2}{f^2} \int \frac{d\Omega_k (f^2 - f^2 \cos^2\theta - f^2 \sin^2\theta \sin^2\phi)}{(c \cos\theta + d)(e \cos\theta + f \sin\theta \cos\phi + g)}$$

which three terms may be called:

$$I_y = I_{y1} + I_{y2} + I_{y3}$$

By inspection:

$$I_{y1} = A_y^2 k o^2 I_B$$

and

$$I_{y2} = \frac{-A_y^2 k o^2}{c^2} \int \frac{d\Omega_k (c^2 \cos^2\theta + 2cd \cos\theta + d^2 - 2cd \cos\theta - d^2)}{(c \cos\theta + d)(e \cos\theta + f \sin\theta \cos\phi + g)}$$

$$I_{y2} = \frac{-A_y^2 k o^2}{c^2} [-2dI_R + 2d^2I_B - d^2I_B + dI_R + c(IC)]$$

$$I_{y2} = \frac{-A_y^2 k o^2}{c^2} [dI_R - d^2I_B - c(IC)]$$

where

$$IC = \int \frac{d\Omega_k \cos\theta}{(e \cos\theta + f \sin\theta \cos\phi + g)}$$

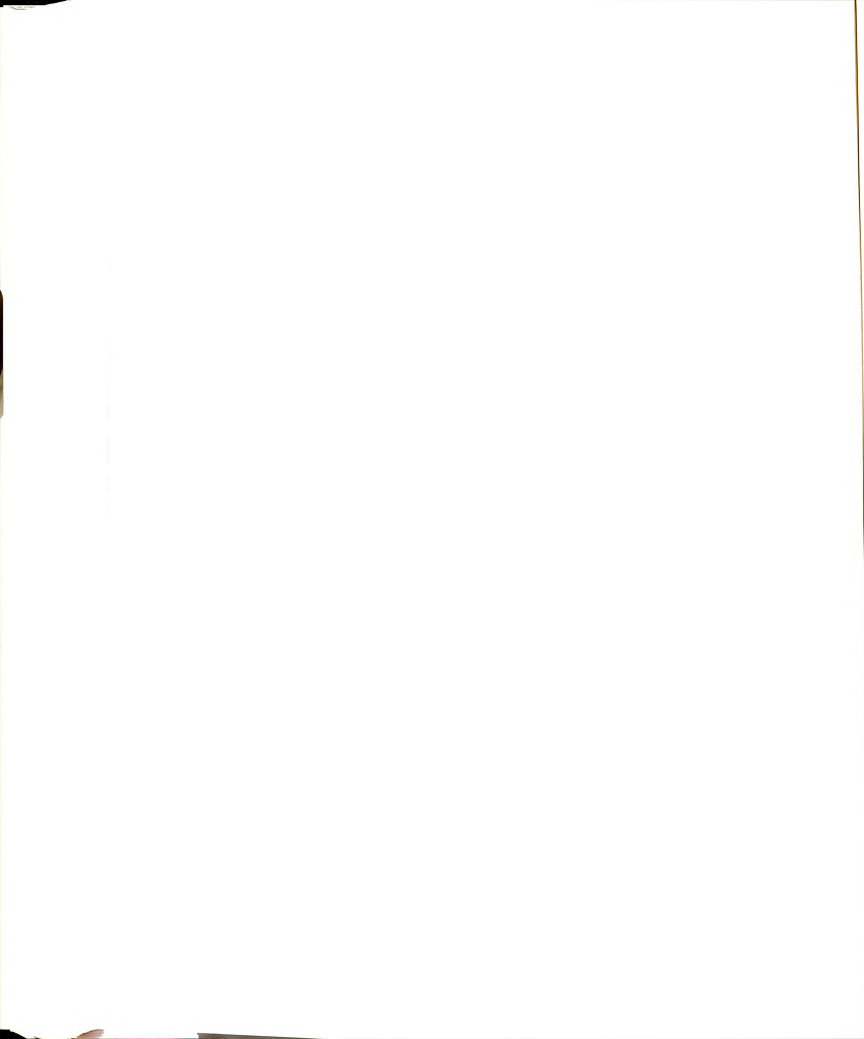
and remains to be evaluated.

Lastly,

$$I_{y3} = \frac{A_y^2 k o^2}{f^2} \int \frac{d\Omega_k (-f^2 \sin^2\theta \sin^2\phi)}{(c \cos\theta + d)(e \cos\theta + f \sin\theta \cos\phi + g)}$$

Then, using

$$f^2 \sin^2\theta \sin^2\phi = [(e \cos\theta + g) + (f \sin\theta \cos\phi)]^2 - 2(e \cos\theta + g)(f \sin\theta \cos\phi) - (e \cos\theta + g)^2$$



$$\begin{aligned}
&= [(e \cos \theta + g) + (f \sin \theta \cos \phi)]^2 - 2(e \cos \theta + g)[(e \cos \theta + g) + (f \sin \theta \cos \phi)] + (e \cos \theta + g)^2 \\
&= [(e \cos \theta + g) + (f \sin \theta \cos \phi)][-(e \cos \theta + g) + (f \sin \theta \cos \phi)] + (e \cos \theta + g)^2 \\
I_{y3} &= \frac{A_y^2 k o^2}{f^2} \left[\int \frac{d\Omega_k (e \cos \theta + g)}{(c \cos \theta + d)} - \int \frac{d\Omega_k (f \sin \theta \cos \phi)}{(c \cos \theta + d)} - \int \frac{d\Omega_k (\epsilon^2 \cos^2 \theta + 2ge \cos \theta + g^2)}{(c \cos \theta + d)(e \cos \theta + f \sin \theta \cos \phi + g)} \right]
\end{aligned}$$

The second term is zero.

The remaining become:

$$I_{y3} = \frac{-A_y^2 k o^2}{f^2} \left[\frac{-e}{c} 4\pi + \frac{ed}{c} I_L - g I_L + \frac{2ge}{c} (I_R - d I_B) + g^2 I_B + \epsilon^2 \int \frac{d\Omega_k (\cos^2 \theta)}{(c \cos \theta + d)(e \cos \theta + f \sin \theta \cos \phi + g)} \right]$$

Then, using the results of I_{y2} on the $\cos^2$ integral:

$$I_{y3} = \frac{-A_y^2 k o^2}{f^2} \left[\frac{-e}{c} 4\pi - (g - \frac{ed}{c}) I_L + \frac{2ge}{c} (I_R - d I_B) + g^2 I_B - \frac{\epsilon^2}{c^2} (d I_R - d^2 I_B - c(IC')) \right]$$

and the complete integral for this class of integrals:

$$\begin{aligned}
I &= I_0 + I_z + I_x + I_{y1} + I_{y2} + I_{y3} \\
\text{II7 - II9} &= -A_0(ko) I_{TB} + \frac{A_z ko}{c} [I_{TR} - d I_{TB}] + \frac{A_x ko}{f} \left[I_{TL} - (g - \frac{ed}{c}) I_{TB} - \frac{\epsilon}{c} I_{TR} \right] \\
&\quad + A_y^2 k o^2 I_B + \frac{A_y^2 k o^2}{c^2} [d I_R - d^2 I_B - c(IC')] \\
&\quad - \frac{A_y^2 k o^2}{f^2} \left[\frac{-e}{c} 4\pi - (g - \frac{ed}{c}) I_L + \frac{2ge}{c} (I_R - d I_B) + g^2 I_B - \frac{\epsilon^2}{c^2} (d I_R - d^2 I_B - c(IC')) \right]
\end{aligned}$$

It remains to evaluate IC.

$$IC = \int \frac{d\Omega_k \cos \theta}{(e \cos \theta + f \sin \theta \cos \phi + g)}$$

Using

$$\int_0^{2\pi} \frac{d\phi}{a + b \cos \phi} = \frac{2\pi}{(a^2 - b^2)^{1/2}} \quad a > b > c$$

$$IC = 2\pi \int_{-1}^1 \frac{x dx}{((\epsilon^2 + f^2)x^2 + 2g\epsilon x + g^2 - f^2)^{1/2}}$$

$$IC = 2\pi \int_{-1}^1 \frac{x dx}{(mx^2 + nx + p)^{1/2}}$$

where

$$m = \epsilon^2 + f^2$$

$$n = 2g\epsilon$$

$$p = g^2 - f^2$$

which is solved using

$$\int_{-1}^1 \frac{x dx}{X^{1/2}} = \frac{\sqrt{X}}{m} - \left(\frac{n}{2m}\right) \left(\frac{1}{\sqrt{in}}\right) \log(2\sqrt{mX} + 2mx + n)$$

Noting

$$X(+1) = (g + \epsilon)^2$$

$$X(-1) = (g - \epsilon)^2$$

and

$$\frac{(g + \epsilon)\sqrt{\epsilon^2 + f^2} + 2(\epsilon^2 + f^2) + 2(g\epsilon)}{(g - \epsilon)\sqrt{\epsilon^2 + f^2} - 2(\epsilon^2 + f^2) + 2(g\epsilon)} = \frac{(g + \sqrt{\epsilon^2 + f^2})(\epsilon + \sqrt{\epsilon^2 + f^2})}{(g - \sqrt{\epsilon^2 + f^2})(\epsilon + \sqrt{\epsilon^2 + f^2})}$$

gives

$$IC = \frac{2\pi}{(\epsilon^2 + f^2)} \left[2\epsilon - \frac{g\epsilon}{\sqrt{\epsilon^2 + f^2}} \log \left(\frac{g + \sqrt{\epsilon^2 + f^2}}{g - \sqrt{\epsilon^2 + f^2}} \right) \right]$$

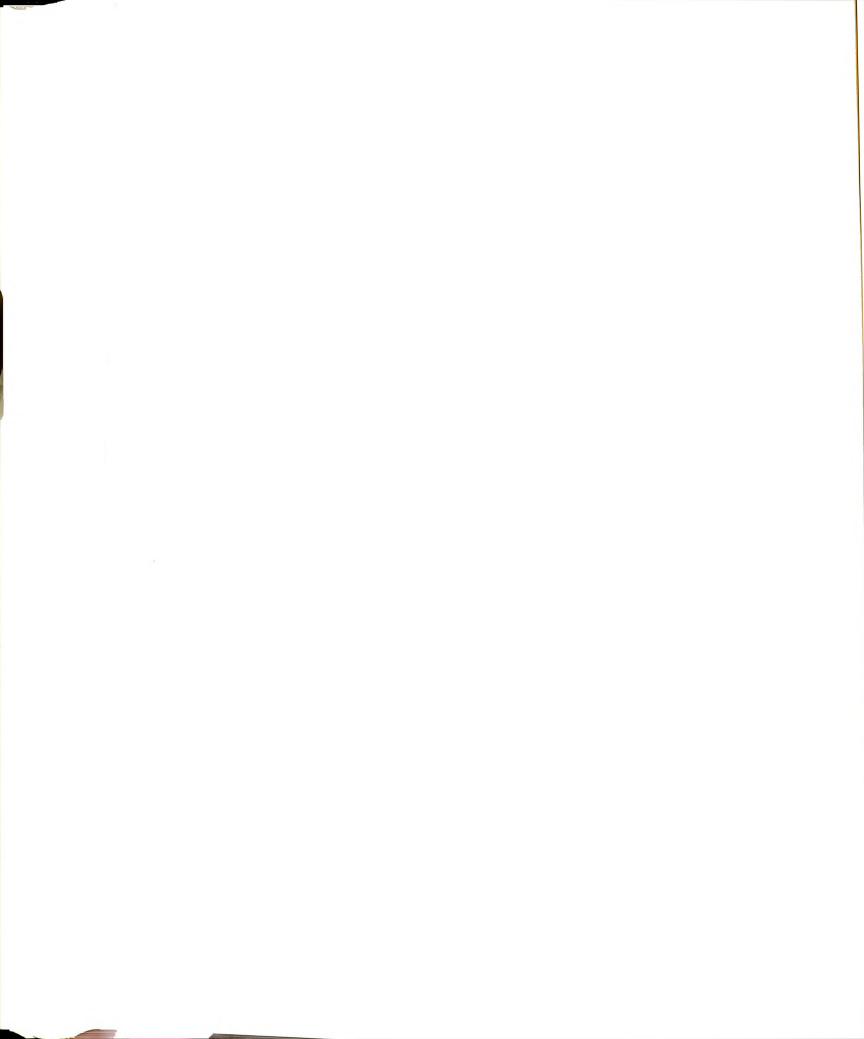
The log is the same as appears in the I1 - I3 class integrals and will be expressible in terms of them.

This class of integrals thus has solution:

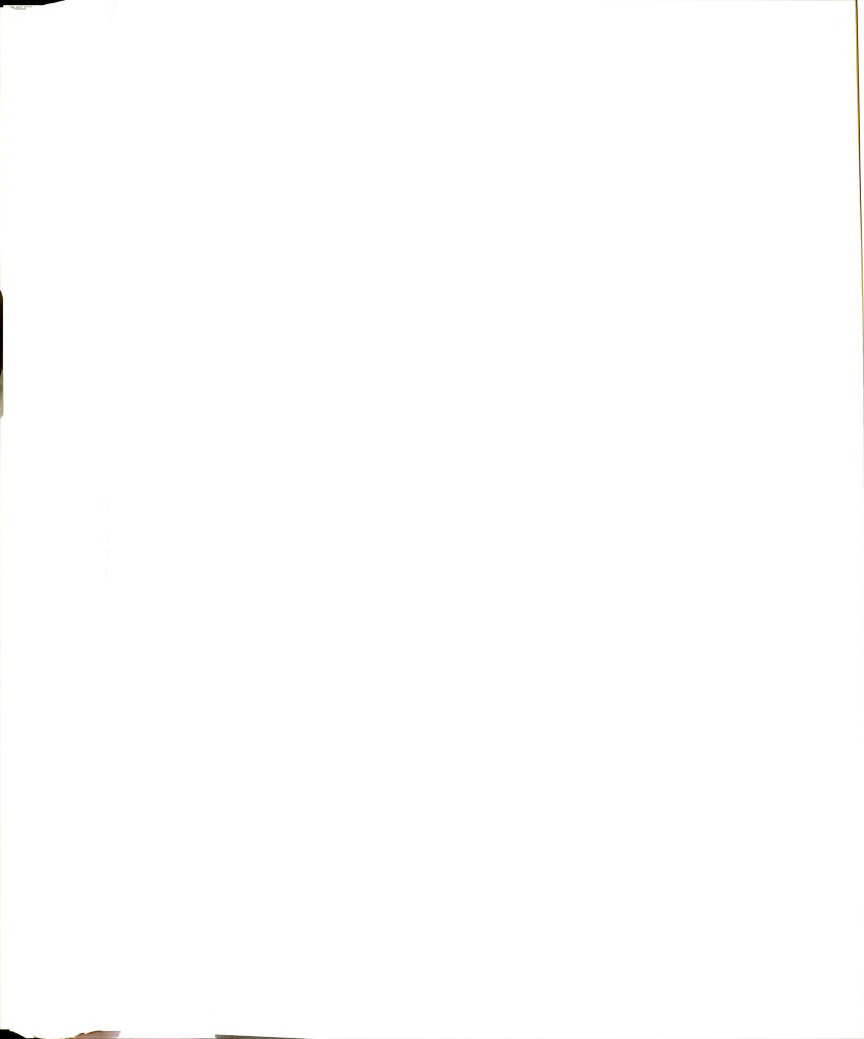
$$\begin{aligned} \text{I17} - \text{I19} = & -A_0(k\epsilon)I_{TB} + \frac{A_z k\epsilon}{c} [I_{TR} - dI_{TB}] + \frac{A_x k\epsilon}{f} \left[I_{TL} - (g - \frac{\epsilon d}{c})I_{TB} - \frac{\epsilon}{c}I_{TR} \right] \\ & + \frac{A_y^2 k\epsilon^2}{f^2} \left[\left(\frac{-\epsilon(f^2 + \epsilon^2)}{c(f^2 + \epsilon^2)} + \epsilon/c \right) 4\pi + (g - \frac{\epsilon d}{c})I_L + \left(f^2(1 - \frac{d^2}{c^2}) - (g - \frac{\epsilon d}{c})^2 \right) I_B + N \right] \end{aligned}$$

where

$$N = \left(\frac{d}{c^2}(f^2 + \epsilon^2) - \frac{2g\epsilon}{c} \right) I_R + \frac{(f^2 + \epsilon^2)}{c} \frac{2\pi}{(f^2 + \epsilon^2)} \frac{g\epsilon}{\sqrt{\epsilon^2 + f^2}} \log \left(\frac{g + \sqrt{\epsilon^2 + f^2}}{g - \sqrt{\epsilon^2 + f^2}} \right)$$



$$\begin{aligned}
\text{I17} - \text{I19} = & -A_0(k_0)I_{TB} + \frac{A_s k_0}{c} [I_{TR} - dI_{TB}] + \frac{A_r k_0}{f} \left[I_{TL} - \left(g - \frac{\epsilon d}{c}\right) I_{TB} - \frac{\epsilon}{c} I_{TR} \right] \\
& + \frac{A_v^2 k_0^2}{f^2} \\
& \left[\left(g - \frac{\epsilon d}{c}\right) I_L - \left(g - \frac{\epsilon d}{c}\right)^2 I_B + 1/c \left(\left[\frac{d}{c} (f^2 + \epsilon^2) - 2g\epsilon \right] I_R + \frac{2\pi g\epsilon}{\sqrt{\epsilon^2 + f^2}} \log \left(\frac{g + \sqrt{\epsilon^2 + f^2}}{g - \sqrt{\epsilon^2 + f^2}} \right) \right) \right]
\end{aligned}$$



Now to use this general solution to find I17, I18 and I19.

$$\begin{aligned} \text{I17} &= \int \frac{d\Omega_k (P3 \cdot k)^2}{(-2P4 \cdot k + M_7^2)(2P3 \cdot k + M_7^2 - i\Gamma M_z)} \\ \text{I17} &= \int \frac{d\Omega_k (P3_x(ko)\sin\theta \cos\phi + P3_y(ko)\sin\theta \sin\phi + P3_z(ko)\cos\theta - E3(ko))(P3 \cdot k)}{(-2ko^2\cos\theta + 2ko^2 + M_7^2)(2P1_z(ko)\cos\theta + 2P1_x(ko)\sin\theta \cos\phi + 2(ko)E1 + M_{quark}^2)} \\ I_L &= \text{I1} \quad I_R = \text{I2} \quad I_B = \text{I4} \quad I_{TL} = \text{I12} \quad I_{TR} = \text{I14} \quad I_{TB} = \text{I7} \end{aligned}$$

The parameters are the same as for the I7 solution.

$$\text{using } P1_z = \frac{\overline{P1} \cdot \overline{P4}}{E4} = \frac{P1P4}{E4} + E1 = \frac{P1P4 - P1P5}{2ko}$$

$$\text{from } 2(ko)E1 = -(P1P4 + P1P5)$$

$$\text{and } P1_x = (E1 - P1_z^2)^{1/2} = \pm \sqrt{(P1P4)(P1P5)}/ko$$

and noting that

$$\begin{aligned} c &= -2ko^2 \\ d &= 2ko^2 + M_7^2 = -c + M_7^2 \\ e &= P1P4 - P1P5 \\ f &= \pm 2\sqrt{(P1P4)(P1P5)} \\ g &= -(P1P4 + P1P5) + M_{quark}^2 \\ e^2 + f^2 &= (2(ko)E1)^2 = (P1P4 + P1P5)^2 \\ g + e &= -2(P1P5) + M_{quark}^2 \\ ge &= -[P1P4^2 - P1P5^2] \end{aligned}$$

and adding

$$\frac{2\pi}{\sqrt{e^2 + f^2}} \log \left(\frac{g + \sqrt{e^2 + f^2}}{g - \sqrt{e^2 + f^2}} \right) = \frac{2\pi}{(P1P4 + P1P5)} \log \left(\frac{M_{quark}^2}{-2(P1P4 + P1P5)} \right) = \text{I2}$$

$$P3_z = \frac{\overline{P3} \cdot \overline{P4}}{E4} = \frac{P3P4}{E4} + E3 = \frac{P3P4 - P3P5}{2ko}$$

and to get P3_x

$$\overline{P1} \cdot \overline{P3} = P1P3 + E1E3 = P1_x P3_x + P1_y P3_y + P1_z P3_z$$

which, since $P1_y = 0$

$$\begin{aligned}
 P3_x &= 1/P1_x [P1P3 + E1E3 - P1_z P3_z] \\
 &= \pm ko / \sqrt{(P1P4)(P1P5)} \left[P1P3 + \frac{(P1P4 + P1P5)(P3P4 + P3P5)}{4ko^2} - \frac{(P1P4 - P1P5)(P3P4 - P3P5)}{4ko^2} \right] \\
 &= \frac{\pm S}{2(ko)\sqrt{(P1P4)(P1P5)}}
 \end{aligned}$$

where

$$S = P1P4(P3P5) + P1P5(P3P4) - P1P3(P4P5)$$

Now in addition, is needed $P3_y^2$:

$$\begin{aligned}
 P3_y^2 &= E3^2 - P3_x^2 - P3_z^2 \\
 &= \frac{4(P3P4)P3P5(P1P4)P1P5 - S^2}{4ko^2(P1P4)P1P5}
 \end{aligned}$$

and using the general form gives:

$$\begin{aligned}
 I17 &= -ko(E3)I7 + \frac{P3P4 - P3P5}{2(-2ko^2)}(I14) + \frac{P3P4 - P3P5}{2}(I7) \\
 &+ \frac{(S)(ko)}{2ko\sqrt{(P1P4)(P1P5)}} \frac{1}{2\sqrt{(P1P4)(P1P5)}} \left[I12 - \frac{P1P4 - P1P5}{(-2ko^2)}(I14) + 2(P1P5)I7 \right] \\
 &+ \left[\frac{4(P3P4)P3P5(P1P4)P1P5 - S^2}{4(P1P4)(P1P5)4(P1P4)P1P5} \right] \\
 &\left[-2(P1P5)I1 - 4(P1P5)^2I4 + \frac{1}{-2ko^2} (-(P1P4 + P1P5)^2 + P1P4^2 - P1P5^2) I2 \right]
 \end{aligned}$$

where the $\pm$ arbitrariness has cancelled out.

$$\begin{aligned}
 I17 &= (P3P4)I7 + \frac{P3P4 - P3P5}{2(P4P5)}(I14) + \frac{(S)}{4(P1P4)(P1P5)} \left[I12 - \frac{P1P4 - P1P5}{(P4P5)}I14 + 2(P1P5)I7 \right] \\
 &+ \left[\frac{4(P3P4)P3P5(P1P4)P1P5 - S^2}{16(P1P4)^2(P1P5)^2} \right] \left[-2(P1P5)I1 - 4(P1P5)^2I4 - \left(\frac{(P1P4 + P1P5)}{P4P5} \right) (2P1P5)I2 \right]
 \end{aligned}$$

$$\mathbf{I18} = \int \frac{d\Omega_k (P1 \cdot k)^2}{(-2P4 \cdot k + M_\gamma^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= \int \frac{d\Omega_k (P1_x(ko)\sin\theta \cos\phi + P1_y(ko)\sin\theta \sin\phi + P1_z(ko)\cos\theta - E1(ko)) (P1 \cdot k)}{(-2ko^2\cos\theta + 2ko^2 + M_\gamma^2)(2P3_z(ko)\cos\theta + 2P3_x(ko)\sin\theta \cos\phi - 2(ko)E3 + M_z^2 - i\Gamma M_z)}$$

$$I_L = \mathbf{I1} \quad I_R = \mathbf{I3} \quad I_B = \mathbf{I5} \quad I_{TL} = \mathbf{I11} \quad I_{TR} = \mathbf{I16} \quad I_{TB} = \mathbf{I8}$$

The parameters are the same as for the **I8** solution.

$$\text{Using } P3_z = \frac{P3 \cdot P4}{E4} = \frac{P3P4}{E4} + E3 = \frac{P3P4 - P3P5}{2ko}$$

$$\text{from } 2(ko)E3 = -(P3P4 + P3P5)$$

$$\text{and } P3_x = (E3^2 - P3_z^2)^{1/2} = \pm \sqrt{(P3P4)(P3P5)}/ko$$

$$P1_z = (P1P4 - P1P5)/2ko$$

$$P1_x = 1/P3_x [P1P3 + E1E3 - P1_z P3_z] = \frac{\pm ko}{\sqrt{(P3P4)(P3P5)}} \frac{S}{2ko^2}$$

and noting that

$$c = -2ko^2$$

$$d = 2ko^2 + M_\gamma^2 = -c + M_\gamma^2$$

$$e = P3P4 - P3P5$$

$$f = \pm 2\sqrt{(P3P4)(P3P5)}$$

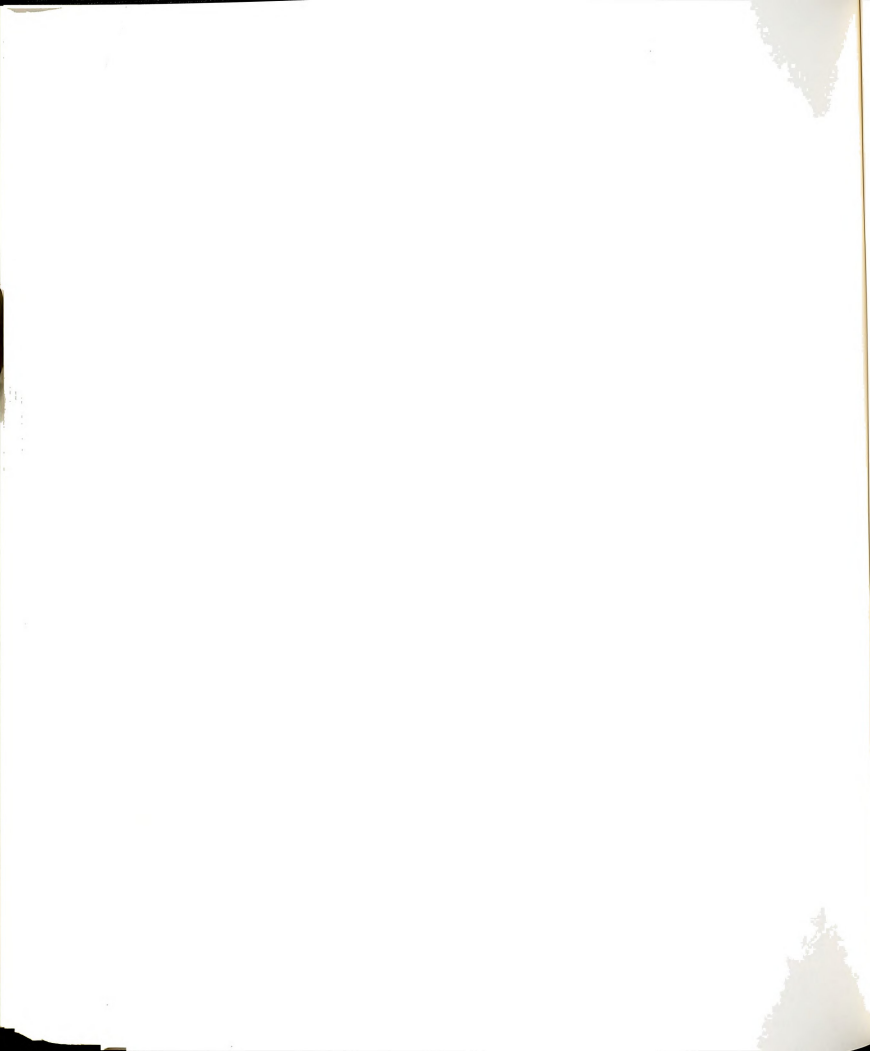
$$g = (P3P4 + P3P5) + M_z^2 - i\Gamma M_z$$

$$= -(P3P4 - P3P5) + MZP$$

$$e^2 + f^2 = (2(ko)E3)^2 = (P3P4 + P3P5)^2$$

$$g + e = 2(P3P4) + M_z^2 - i\Gamma M_z$$

$$ge = (P3P4 - P3P5) [-(P3P4 + P3P5) + MZP]$$



using, as usual, the symbol MZP for the Z particle propagator

$$MZP = 2(P3P4) + M_z^2 - i\Gamma M_z$$

and now adding

$$\frac{2\pi}{\sqrt{e^2 + f^2}} \log \left(\frac{g + \sqrt{e^2 + f^2}}{g - \sqrt{e^2 + f^2}} \right) = \frac{2\pi}{(P3P4 + P3P5)} \log \left(\frac{2(P3P4) + MZP}{M_z^2 - i\Gamma M_z} \right) = I3$$

$$P1_z = \frac{\overline{P1} \cdot \overline{P4}}{E4} = \frac{P1P4}{E4} + E1 = \frac{P1P4 - P1P5}{2k0}$$

Now in addition, is needed $P1_y^2$:

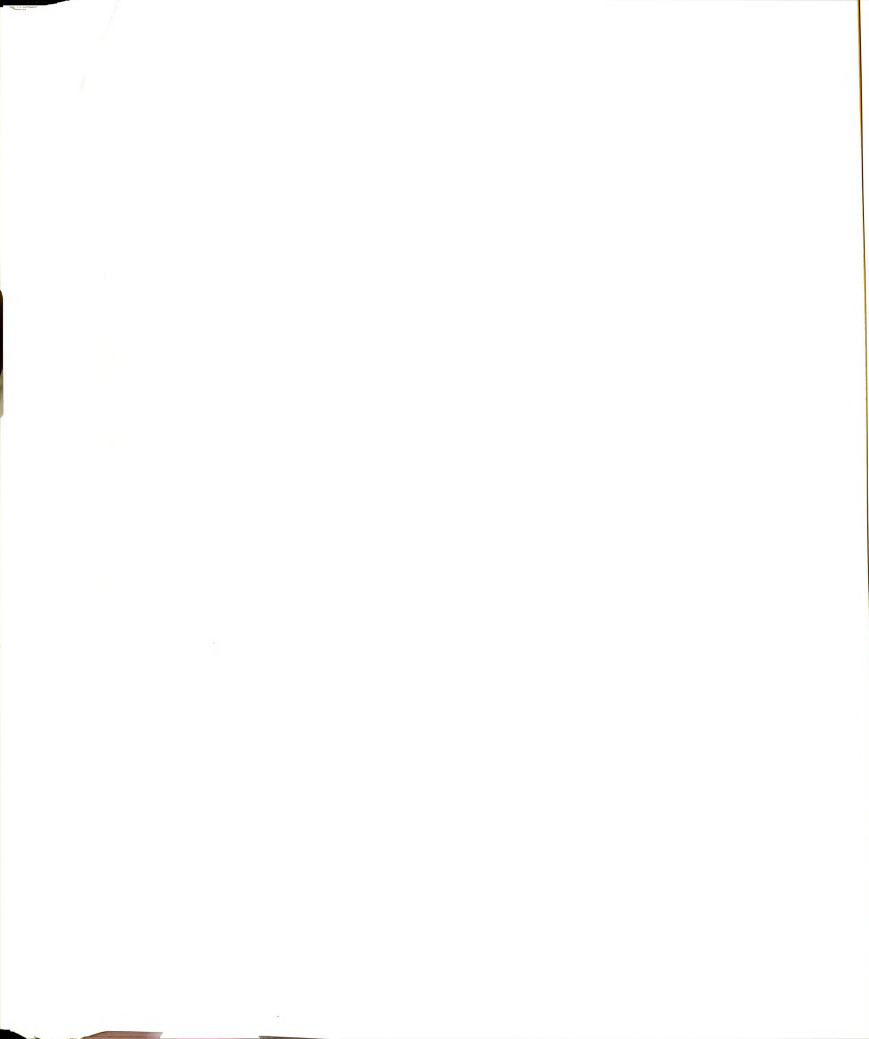
$$\begin{aligned} P1_y^2 &= E1^2 - P1_z^2 - P1_z^2 \\ &= \frac{4(P3P4)P3P5(P1P4)P1P5 - S^2}{4k0^2(P3P4)P3P5} \end{aligned}$$

and using the general form gives:

$$\begin{aligned} I18 &= -k0(E1)I8 + \frac{P1P4 - P1P5}{2(-2k0^2)}(I16) + \frac{P1P4 - P1P5}{2}(I8) \\ &+ \frac{(S)(k0)}{2k0\sqrt{(P3P4)(P3P5)}} \frac{1}{2\sqrt{(P3P4)(P3P5)}} \left[I11 - \frac{P3P4 - P3P5}{(-2k0^2)}(I16) - (P3P4 - P3P5)I8 \right] \\ &+ \left[\frac{4(P3P4)P3P5(P1P4)P1P5 - S^2}{16(P3P4)^2(P3P5)^2} \right] \\ &\left[(MZP)I1 - (MZP)^2I5 - \frac{I3}{2k0^2} (- (P3P4 + P3P5)^2 - (P3P4 - P3P5) [- (P3P4 - P3P5) + MZP]) \right] \end{aligned}$$

where the $\pm$ arbitrariness has cancelled out.

$$\begin{aligned} I18 &= (P1P4)I8 + \frac{P1P4 - P1P5}{2(P4P5)}I16 + \frac{(S)}{4(P3P4)(P3P5)} \left[I11 - \frac{P3P4 - P3P5}{(P4P5)}I16 - (MZP)I8 \right] \\ &+ \left[\frac{4(P3P4)P3P5(P1P4)P1P5 - S^2}{16(P3P4)^2(P3P5)^2} \right] \end{aligned}$$



$$\left[(MZP)I1 - (MZP)^2I5 - \frac{1}{P4P5} \{ (P3P4 + P3P5)2(P3P5) + (P3P4 - P3P5)MZP \} I3 \right]$$

$$I19 = \int \frac{d\Omega_k (P4 \cdot k)^2}{(2P1 \cdot k - 2(P1P4 + P1P5) + M_{quark}^2)(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= \int \frac{d\Omega_k (P4_x(ko)\sin\theta \cos\phi + P4_y(ko)\sin\theta \sin\phi + P4_z(ko)\cos\theta - E4(ko)) (P4 \cdot k)}{(2(ko)E1\cos\theta + 2(ko)E1 + M_{quark}^2)(2P3_z(ko)\cos\theta + 2P3_x(ko)\sin\theta \cos\phi - 2(ko)E3 + M_z^2 - i\Gamma M_z)}$$

$$I_L = I2 \quad I_R = I3 \quad I_B = I6 \quad I_{TL} = I13 \quad I_{TR} = I15 \quad I_{TB} = I9$$

The parameters are the same as for the I9 solution.

$$\text{Using } P3_z = \frac{\overline{P3 \cdot P1}}{E1} = \frac{P1P3}{E1} + E3$$

with

$$E3 = \frac{P3P4 + P3P5}{2ko} \quad E1 = \frac{P1P4 + P1P5}{2ko}$$

to give

$$P3_z = \frac{-2ko(P1P3 + \frac{(P1P4 + P1P5)(P3P4 + P3P5)}{4ko^2})}{P1P4 + P1P5}$$

and

$$P3_x = (E3^2 - P3_z^2)^{1/2}$$

$$= \pm \left[- \left(\frac{P1P3}{E1} \right)^2 - 2(P1P3)(E1/E3) \right]^{1/2}$$

$$= \pm \left(\frac{T}{A^2} \right)^{1/2}$$

where

$$A = P1P4 + P1P5$$

and

$$T = 2(P1P3)[P1P3(P4P5) - P1P4(P3P4) - P1P4(P3P5) - P1P5(P3P4) - P1P5(P3P5)]$$

$$\begin{aligned}
P4_z &= \frac{\overline{P4} \cdot \overline{P1}}{E1} = \frac{P1P4}{E1} + ko = -ko \frac{(P1P4 - P1P5)}{P1P4 + P1P5} \\
P4_x &= 1/P3_x [P4P3 + E4E3 - P4_z P3_z] \\
&= 1/P3_x [P3P4 + ko(E3) - (\frac{P1P3}{E1} + E3)(\frac{P1P4}{E1} + ko)] \\
&= \pm \left(\frac{A^2}{T} \right)^{1/2} \left(\frac{U}{A^2} \right)
\end{aligned}$$

where

$$\begin{aligned}
U &= P3P4(P1P5)^2 + P3P4(P1P4)P1P5 + P4P5(P1P3)P1P4 \\
&\quad - (P1P4)^2(P3P5) - P1P4(P1P5)P3P5 - P1P5(P1P3)P4P5
\end{aligned}$$

and noting that

$$\begin{aligned}
c &= -(P1P4 + P1P5) = A \\
d &= c + M_{quark}^2 = c + 0 = c \\
\epsilon &= \frac{W}{A}
\end{aligned}$$

where

$$\begin{aligned}
W &= 2(P1P3)(P4P5) - (P1P4 + P1P5)(P3P4 + P3P5)(P1P4 + P1P5) \\
f &= 2(ko)P3_x = \pm 2ko \left(\frac{T}{A^2} \right)^{1/2} \\
g &= (P3P4 + P3P5) + M_z^2 - i\Gamma M_z \\
e^2 + f^2 &= (2(ko)E3)^2 = (P3P4 + P3P5)^2 \\
ge &= W/A [- (P3P4 + P3P5) + M_z^2 - i\Gamma M_z] \\
g - e &= 2(P3P5) + MZP - \frac{2(P1P3)(P4P5)}{(P1P4 + P1P5)} = \frac{R}{(P1P4 + P1P5)}
\end{aligned}$$

where

$$R = (2(P3P5) + MZP)(P1P4 + P1P5) - 2(P1P3)(P4P5)$$

using, as usual, the symbol MZP for the Z particle propagator

$$MZP = 2(P3P4) + M_z^2 - i\Gamma M_z$$

and adding

$$\frac{2\pi}{\sqrt{\epsilon^2 + f^2}} \log \left(\frac{g + \sqrt{\epsilon^2 + f^2}}{g - \sqrt{\epsilon^2 + f^2}} \right) = \frac{2\pi}{(P3P4 + P3P5)} \log \left(\frac{2(P3P4) + MZP}{M_i^2 - i\Gamma M_i} \right) = I3$$

and now in addition, is needed $P1_y$:

$$\begin{aligned} P4_y^2 &= E4^2 - P4_x^2 - P4_z^2 \\ &= k\sigma^2 - \frac{k\sigma^2(P1P4 - P1P5)^2}{A^2} - \frac{U^2}{A^2 T} \\ &= \frac{2(k\sigma^2)(P1P4)(P1P5)}{A^2} - \frac{U^2}{A^2 T} \end{aligned}$$

and using the general form gives:

$$\begin{aligned} I19 &= (-k\sigma^2)I9 + \left(\frac{k\sigma^2(P1P4 - P1P5)}{A^2} \right) I15 + \left(\frac{k\sigma^2(P1P4 - P1P5)}{A} \right) I9 \\ &+ \left(\frac{A^2}{T} \right)^{1/2} \left(\frac{U}{A^2} \right) \left(\frac{1}{2} \right) \left(\frac{A^2}{T} \right)^{1/2} \left(I13 + \left(\frac{W}{A^2} \right) I15 + \left(\frac{W}{A} \right) I9 - (P3P5 - P3P4 + MZP)I9 \right) \\ &\quad + (P4_y^2) \left(\frac{1}{4} \right) \left(\frac{A^2}{T} \right) \\ &\left[\left(\frac{R}{A} \right) I2 - \left(\frac{R}{A} \right)^2 I6 - \frac{1}{A} \left([P3P4 + P3P5]^2 - \frac{W}{A} [-(P3P4 - P3P5) + MZP] \right) I3 \right] \end{aligned}$$

where the $\pm$ arbitrariness has cancelled out.

$$\begin{aligned} I19 &= \left(\frac{(P4P5)(P1P5)}{A} \right) I9 - \left(\frac{(P4P5)(P1P4 - P1P5)}{2A^2} \right) I15 \\ &+ \frac{U}{2T} \left(I13 + \left(\frac{W}{A^2} \right) I15 + \left(\frac{W}{A} \right) I9 - (P3P5 - P3P4 + MZP)I9 \right) \\ &\quad + \left[\frac{-P4P5(P1P4)P1P5}{2T} - \frac{U^2}{4T^2} \right] \\ &\left[\left(\frac{R}{A} \right) I2 - \left(\frac{R}{A} \right)^2 I6 - \frac{1}{A} \left([P3P4 + P3P5]^2 - \frac{W}{A} [-(P3P4 - P3P5) + MZP] \right) I3 \right] \end{aligned}$$

Appendix O

FORTRAN Program to Calculate the Result

```

PROGRAM thesis
IMPLICIT NONE
INTEGER LHEL,QHEL,GHEL
REAL ECM,PI,ALPHA
COMPLEX MZ
PARAMETER (LHEL = +1, QHEL = +1, GHEL = +1, ECM = 5.80)
PARAMETER (ALPHA = 1.0/137,PI = 3.14157,MZ = (93.5,2.7))
INTEGER EPF_PARITY
REAL P1P3,P1P4,P1P5,P3P4,P3P5,P4P5,LIMIT,P1Y_SQD,P1Y
REAL PP1P3,PP1P4,PP3P5,PP4P5
REAL WIV,FPIIV
REAL PROPIV,E0,B0,CUO,E0IV,B0IV,CUOIV,RATIO
COMPLEX I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15
COMPLEX I16,I17,I18,I19
COMPLEX EVL,BORN_TERM,BORN_LOOP_TERM,MZP,MZPIV
COMPLEX NUMERATOR,ELECTRON_LOOP_TERM,POSITRON_LOOP_TERM
COMPLEX E_LOOP,P_LOOP
CHARACTER SET_HEL_CODE*1,HEL_CODE*1
OPEN ( UNIT = 10, FILE = 'TOUTPUT', STATUS = 'NEW' )
LIMIT = -2*ECM**2
HEL_CODE = SET_HEL_CODE(LHEL,QHEL,GHEL)
PRINT*,'HEL CODE = ',HEL_CODE
DO 1000 P3P4 = 0,LIMIT,-10
  DO 1000 P3P5 = 0,LIMIT-P3P4,-10
    P4P5 = LIMIT - P3P5 - P3P4
    DO 1000 P1P4 = 0,LIMIT,-10
      DO 1000 P1P5 = 0,LIMIT-P1P4,-10
        P1P3 = LIMIT - P1P5 - P1P4
        P1Y_SQD = ( 4*P1P4*P1P5*P3P4*P3P5)
        + (P1P5*P3P4*P1P4*P3P5-P4P5*P1P3)**2
        PRINT*,'P1Y SQD = ',P1Y_SQD
        IF ( P1Y_SQD .GE. 0 ) THEN
          PRINT*,'P1P3 = ',P1P3
          PRINT*,'P1P4 = ',P1P4
          PRINT*,'P1P5 = ',P1P5
          PRINT*,'P3P4 = ',P3P4
          PRINT*,'P3P5 = ',P3P5
          PRINT*,'P4P5 = ',P4P5
          DO EPF_PARITY = -1, +1, 2
            P1Y = EPF_PARITY*P1Y_SQD**(0.5)
            PRINT*,'PIY = ',P1Y
            EVL = CMPLX(0.0,P1Y)
            PRINT*,'EVL = ',EVL
            BORN_TERM = BORN(HEL_CODE,P1P3,P1P4,P1P5,
              + P3P4,P3P5,P4P5,EVL)
            MZP = 2*P3P4 + MZ
            MZPIV = 1.0/MZP
            PROPIV = 1.0/(-2*P1P5*2*(P3P4+P3P5+P4P5))
            BORN_TERM = PROPIV*MZPIV*BORN_TERM
            C
            E0 = -4*P1P5*(P3P5+P4P5-P1P5)*(P3P4+P3P5+P4P5)
            E0 = 1
            E0IV = 1.0/E0**(0.5)
            B0 = 1
            C
            B0 = 0.125*(P3P4*P3P5)**(0.5)
            B0IV = 1.0/B0**(0.5)
            C
            CUO = 0.125*(P3P4+P3P5+P4P5)*P1P5
            CUO = 1
            CUOIV = 1.0/CUO**(0.5)

```

```

FPIIV = 0.25*(1/PI)
BORN_TERM = EOIV*BOIV*CUOIV*BORN_TERM
PRINT*, 'BORN_TERM = ', BORN_TERM
I1 = (0,0)
I2 = (0,0)
I3 = (0,0)
I4 = (0,0)
I5 = (0,0)
I6 = (0,0)
I7 = (0,0)
I7 = (0,0)
I8 = (0,0)
I9 = (0,0)
I10 = (0,0)
I11 = (0,0)
I12 = (0,0)
I13 = (0,0)
I14 = (0,0)
I15 = (0,0)
I16 = (0,0)
I17 = (0,0)
I18 = (0,0)
I19 = (0,0)
WIV = 0
CALL INTEGRALS (P1P3,P1P4,P1P5,P3P4,P3P5,P4P5,
  I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
  I11,I12,I13,I14,I15,I16,I17,I18,I19,
  WIV,FPIIV,
  MZ)
ELECTRON_LOOP_TERM = E_LOOP(HEL_CODE,P1P3,P1P4,P1P5,
  P3P4,P3P5,P4P5,
  I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
  I11,I12,I13,I14,I15,I16,I17,I18,I19,
  WIV,FPIIV,BOIV,CUOIV,EOIV,EVL,
  MZ)
ELECTRON_LOOP_TERM = ELECTRON_LOOP_TERM/(2*P4P5)
PP1P3 = P1P4
PP1P4 = P1P3
PP3P5 = P4P5
PP4P5 = P3P5
EVL = -EVL
C EO = -4*P1P5*(PP3P5+PP4P5-P1P5)*(P3P4+PP3P5+PP4P5)
EO = 1
EOIV = 1.0/EO**(0.5)
BO = 1
C BO = 0.125*(P3P4+PP3P5)**(0.5)
BOIV = 1.0/BO**(0.5)
CUO = 1
C CUO = 0.125*(P3P4+PP3P5+PP4P5)*P1P5
CUOIV = 1.0/CUO**(0.5)
CALL INTEGRALS (PP1P3,PP1P4,P1P5,P3P4,PP3P5,PP4P5,
  I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
  I11,I12,I13,I14,I15,I16,I17,I18,I19,
  WIV,FPIIV,
  MZ)
POSITRON_LOOP_TERM = P_LOOP(HEL_CODE,PP1P3,PP1P4,P1P5,
  P3P4,PP3P5,PP4P5,
  I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,

```

```

+                               I11,I12,I13,I14,I15,I16,I17,I18,I19,
+                               WIV,FPIIV,BOIV,CUOIV,EQIV,EVL,
+                               MZ)
+                               POSITRON LOOP TERM = POSITRON LOOP TERM/(2*PP4P5)
+                               LOOP_TERM = ELECTRON LOOP TERM - POSITRON LOOP_TERM
+                               LOOP_TERM = LOOP_TERM*BOIV*CUOIV*EQIV
+                               LOOP_TERM = LOOP_TERM/(2*(P3P4+P3P5+P4P5))
+                               PRINT*, 'LOOP TERM = ', LOOP_TERM
+                               LOOP_TERM = LOOP_TERM*ALPHA/(8*PI*P4P5)
+                               NUMERATOR = LOOP_TERM + BORN_TERM
+                               RATIO = NUMERATOR*CONJG(NUMERATOR)
+                               RATIO = RATIO/(BORN_TERM*CONJG(BORN_TERM))
+                               PRINT*, 'RATIO = ', RATIO
+                               WRITE ( 10,200 ) HEL_CODE,P1P3,P1P4,P1P5,P3P4,P3P5,
+                               P4P5,RATIO
200      FORMAT (A1,6F7.2,F12.7)
+                               ENDDO
+                               ENDIF
1000 CONTINUE
END

CHARACTER FUNCTION SET_HEL_CODE(LHEL,QHEL,GHEL)
INTEGER LHEL,QHEL,GHEL
  IF ((LHEL.EQ.+1).AND.(QHEL.EQ.+1)
+    .AND.(GHEL.EQ.+1)) THEN
+    PRINT*, 'A'
+    SET_HEL_CODE = 'A'
+    RETURN
  ELSEIF ((LHEL.EQ.+1).AND.(QHEL.EQ.+1)
+    .AND.(GHEL.EQ.-1)) THEN
+    PRINT*, 'B'
+    SET_HEL_CODE = 'B'
+    RETURN
  ELSEIF ((LHEL.EQ.+1).AND.(QHEL.EQ.-1)
+    .AND.(GHEL.EQ.+1)) THEN
+    PRINT*, 'C'
+    SET_HEL_CODE = 'C'
+    RETURN
  ELSEIF ((LHEL.EQ.+1).AND.(QHEL.EQ.-1)
+    .AND.(GHEL.EQ.-1)) THEN
+    PRINT*, 'D'
+    SET_HEL_CODE = 'D'
+    RETURN
  ELSEIF ((LHEL.EQ.-1).AND.(QHEL.EQ.+1)
+    .AND.(GHEL.EQ.+1)) THEN
+    PRINT*, 'E'
+    PRINT*, 'EVL = ',EVL
+    SET_HEL_CODE = 'E'
+    RETURN
  ELSEIF ((LHEL.EQ.-1).AND.(QHEL.EQ.+1)
+    .AND.(GHEL.EQ.-1)) THEN
+    PRINT*, 'F'
+    SET_HEL_CODE = 'F'
+    RETURN
  ELSEIF ((LHEL.EQ.-1).AND.(QHEL.EQ.-1)
+    .AND.(GHEL.EQ.+1)) THEN
+    PRINT*, 'G'

```



```

        SET_HEL_CODE = 'G'
        RETURN
    ELSEIF ((LHEL .EQ. -1) .AND. (QHXL .EQ. -1)
    + .AND. (GHXL .EQ. -1)) THEN
        PRINT*, 'H'
        SET_HEL_CODE = 'H'
        RETURN
    ENDIF
RETURN
END

COMPLEX FUNCTION BORN(HEL_CODE,P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL,QA,QB,QC,QD,QE,QF,QG,QH
CHARACTER HEL_CODE*1

IF (HEL_CODE .EQ. 'A') THEN
    BORN = QA(P13,P14,P15,P34,P35,P45,EVL)
    RETURN
ENDIF

IF (HEL_CODE .EQ. 'B') THEN
    BORN = QB(P13,P14,P15,P34,P35,P45,EVL)
    RETURN
ENDIF

IF (HEL_CODE .EQ. 'C') THEN
    BORN = QC(P13,P14,P15,P34,P35,P45,EVL)
    RETURN
ENDIF

IF (HEL_CODE .EQ. 'D') THEN
    BORN = QD(P13,P14,P15,P34,P35,P45,EVL)
    RETURN
ENDIF

IF (HEL_CODE .EQ. 'E') THEN
    BORN = QE(P13,P14,P15,P34,P35,P45,EVL)
    RETURN
ENDIF

IF (HEL_CODE .EQ. 'F') THEN
    BORN = QF(P13,P14,P15,P34,P35,P45,EVL)
    RETURN
ENDIF

IF (HEL_CODE .EQ. 'G') THEN
    BORN = QG(P13,P14,P15,P34,P35,P45,EVL)
    RETURN
ENDIF

IF (HEL_CODE .EQ. 'H') THEN
    BORN = QH(P13,P14,P15,P34,P35,P45,EVL)
    RETURN
ENDIF

```


RETURN
END

```

SUBROUTINE INTEGRALS (P1P3,P1P4,P1P5,P3P4,P3P5,P4P5,
C      I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
C      I11,I12,I13,I14,I15,I16,I17,I18,I19,
C      WIV,FPIIV,
C      D3)

COMPLEX D3,MZP,R,LN3,LN5,LN6,MZP1,ZI10D,RUPPER,      110
C      MZPUP,MZPIV,RIV,ZI10U,UPPER,                  120
C      I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I96,IR,        130
C      I11,I12,I13,I14,I15,I16,I17,I18,I19          140
REAL    LN1,LN2,LN4,RLOWER,MZPLW,I9U,I9L,I93,IW,IS,  150
C      LOWER,WIV
REAL    P1P3,P1P4,P1P5,P3P4,P3P5,P4P5,API,FPIIV,K02,K02IV
DOUBLE PRECISION CC44,CC43,CC42,CC41,CC33,CC32,CC31,
C      CC22,CC21,CC11

API = 3.142857143      200
P13IV = 1/P1P3         570
P14IV = 1/P1P4         580
P15IV = 1/P1P5         590
P34IV = 1/P3P4         600
P35IV = 1/P3P5         610
P45IV = 1/P4P5         620
MZP = 2*P3P4 + D3     630
S1S2 = P1P3           640
S1S3 = P1P4           650
S1S4 = P1P5           660
S2S3 = P3P4           670
S2S4 = P3P5           680
S3S4 = P4P5           690
W = S1S2*S3S4*(-S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)  700
C + S1S3*S2S4*( S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)  710
C + S1S4*S2S3*( S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)  720
WIV = 1.0/W           730
CC44 = WIV*(-2)*(S1S2*S1S3*S2S3)                 740
CC43 = WIV*S1S2*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)  750
CC42 = WIV*S1S3*( S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)  760
CC41 = WIV*S2S3*( S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)  770
CC33 = WIV*(-2)*(S1S2*S1S4*S2S4)                 780
CC32 = WIV*S1S4*( S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)  790
CC31 = WIV*S2S4*( S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)  800
CC22 = WIV*(-2)*(S1S3*S1S4*S3S4)                 810
CC21 = WIV*S3S4*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)  820
CC11 = WIV*(-2)*(S2S3*S2S4*S3S4)                 830
R = -2*P4P5*P1P3*2*P3P5*(P1P4+P1P5) + MZP*(P1P4+P1P5)  840
LN1 = LOG (-2*P4P5 )                               850
LN2 = LOG(-2*(P1P4+P1P5))                           860
LN3 = LOG((2*P3P5+MZP)/D3)                           870
LN4 = LOG((P1P4+P1P5)/P1P5)                         880
LN5 = LOG(MZP/(2*P3P5+MZP))                         890
LN6 = LOG((2*P3P5+MZP)*(P1P4+P1P5)/R)               900
K02 = 0.5*(-P4P5)                                     1060
K02IV = 1/K02                                         1070
D145 = P1P4 + P1P5                                  1080

```


D1 = D145	1090
D145IV = 1/D145	1100
D345 = P3P4 + P4P5	1110
D345IV = 1/D345	1120
MZP = 2*P3P4 + D3	1130
I1 = -2*API*P45IV*(LN1)	1140
I2 = -2*API*D145IV*(LN2)	1150
I3 = +2*API*D345IV*LN3	1160
R = -2*P4P5*P1P3+2*P3P5*D1+MZP*D1	1200
MZP1 = MZP	1210
RUPPER = CONJG(R)	1220
RLOWER = R*CONJG(R)	1230
MZPUP = CONJG(MZP1)	1240
MZPLW = MZP1*CONJG(MZP1)	1250
RIV = RUPPER/(RLOWER)	1260
MZPIV = MZPUP/(MZPLW)	1270
I4 = - api*P45IV*P15IV*LN4 -	1290
C 0.5*D1*P45IV*P15IV*I2 - 0.5*P15IV*I1	1300
I5 = P45IV*MZPIV*(-4*api*LN5 - D345*I3 + P4P5*I1)	
I6 = RIV*(+4*api*LN6 - D345*I3 + D1*I2)	
I7 = 0.25*(P3P5*P15IV*P3P4*P14IV-P1P3*P4P5*P14IV*P15IV)*	1360
C (I1 - (P1P4-P1P5)*P45IV*I2 + 2*P1P5*I4)	1370
C -0.5*(P3P5-P3P4)*P45IV*I2 + P3P4*I4	1380
I8 = 0.25*(P1P4*P34IV*P1P5*P35IV-P1P3*P4P5*P34IV*P35IV)*	1390
C (I1 - (P3P4-P3P5)*P45IV*I3 - MZP*I5)	1400
C -0.5*(P1P5-P1P4)*P45IV*I3 + P1P4*I5	1410
I9U = P3P4*P1P4*P1P5*P3P4*P1P5*P1P5*P4P5*P1P3*P1P4	1420
C - P4P5*P1P3*P1P5-P1P4*P1P4*P3P5-P1P5*P1P4*P3P5	1430
I9L = 2*P1P3*	1440
C (P1P3*P4P5-P1P4*P3P4-P1P4*P3P5-P1P5*P3P4-P1P5*P3P5)	1450
I93 = 2*P4P5*P1P3*D145IV*D145IV-(P3P4*P3P5)*D145IV	1460
I96 = -2*P4P5*P1P3*D145IV + 2*P3P5 + MZP	1470
I9 = 0.5*I9U*(1/I9L)*(I2*I93-I3-I96*I6)	1480
C + 0.5*P4P5*(P1P5-P1P4)*D145IV*(D145IV*I3+I6)	1490
C + 0.5*P4P5*I6	1500
I11 = 2*API*P45IV*(P1P4-P1P5) + P1P4*I1	
I12 = 2*API*P45IV*(P3P4-P3P5) + P3P4*I1	
I13 = -2*API*P4P5*D145IV*D145IV*(P1P4 - P1P5) +	1560
C P4P5*P1P5*D145IV*I2	
I14 = 0.5*(P3P4 + P3P5)*I2 +	1580
C 0.5*(D145*D345-2*P1P3*P4P5)*D145IV*(4*API*D145IV + I2)	1590
I15 = -2*API*P4P5*D345IV*D345IV*(P3P4 - P3P5) +	1600
C 0.5*P4P5*(1+D345IV*D345IV*(P3P4-P3P5)*(D345+D3))*I3	1700
I16 = 0.5*D145*I3 +	1620
C 0.5*(D145*D345 - 2*P1P3*P4P5)*	1630
C D345IV*D345IV*(4*API - I3*(D345 + D3))	1640
I17 = P3P4*I7 + 0.5*P45IV*(P3P4-P3P5)*I14 +	1720
C 0.25*I5*P14IV*P15IV*(I12 - (P1P4-P1P5)*P45IV*I14 + 2*P1P5*I7) +	1730
C 0.0625*P14IV*P14IV*P15IV*P15IV*(4*P3P4*P3P5*P1P4*P1P5-IS*IS)*	1740
C (-2*P1P5*I1 - 4*P1P5*P1P5*I4 - P45IV*D145*2*P1P5*I2)	1750
I18 = P1P4*I8 + 0.5*P45IV*(P1P4-P1P5)*I16 +	1760
C 0.25*I5*P34IV*P35IV*(I11-(P3P4-P3P5)*P45IV*I16-(2*P3P4*D3)*I8)+	1770
C 0.0625*P34IV*P34IV*P35IV*P35IV*(4*P3P4*P3P5*P1P4*P1P5-IS*IS)*	1780
C (	1790
C (MZP)*I1-MZP*MZP*I5	1800
C -P45IV*(D345+2*P3P5+(P3P4-P3P5)*MZP)*I3	1810
C)	1820
IW = 2*P1P3*P4P5 - D145*D345	1830

```

172
IR = -2*P4P5*P1P3 + D1*(2*P3P5 + MZP) 1840
I19 = P4P5*P1P5*D145IV*I9 -
C 0.5*D145IV*D145IV*P4P5*(P1P4-P1P5)*I15 + 1850
C 0.5*I9U*(1/I9L)* 1860
C (I13 + D145IV*D145IV*IW*I15*(D145IV*IW-P3P4-P3P5-D3)*I9) + 1870
C 1880
C (-0.5*P4P5*P1P4*P1P5*(1/I9L) 1890
C -0.25*I9U*I9U*(1/I9L)*(1/I9L) 1900
C )* 1910
C( 1920
C (D145IV*IR)*I2 1930
C -D145IV*D145IV*IR*IR*I6 1940
C -D145IV*(D345*D345+D145IV*IW*(P3P4-P3P5-MZP))*I3 1950
C ) 1960
U1DU1=+2.*CC41*CC31*P4P5+2.*CC41*CC21*P3P5+2.*CC41*CC11*P1P5+2.* 2000
CCC31*CC21*P3P4+2.*CC31*CC11*P1P4+2.*CC21*CC11*P1P3 2010
U1DU2=+CC42*CC31*P4P5+CC42*CC21*P3P5+CC42*CC11*P1P5+CC41*CC32*P4 2020
CP5+CC41*CC22*P3P5+CC41*CC21*P1P5+CC32*CC21*P3P4+CC32*CC11*P1P4+C 2030
CC31*CC22*P3P4+CC31*CC21*P1P4+CC22*CC11*P1P3+CC21*P1P3+CC21*P1P3 2040
U1DU3=+CC43*CC31*P4P5+CC43*CC21*P3P5+CC43*CC11*P1P5+CC41*CC33*P4 2050
CP5+CC41*CC32*P3P5+CC41*CC31*P1P5+CC33*CC21*P3P4+CC33*CC11*P1P4+C 2060
CC32*CC31*P3P4+CC32*CC11*P1P3+CC31*CC21*P1P3+CC31*P1P3+CC31*P1P4 2070
U1DU4=+CC44*CC31*P4P5+CC44*CC21*P3P5+CC44*CC11*P1P5+CC43*CC41*P4 2080
CP5+CC43*CC21*P3P4+CC43*CC11*P1P4+CC42*CC41*P3P5+CC42*CC31*P3P4+C 2090
CC42*CC11*P1P3+CC41*CC31*P1P4+CC41*CC21*P1P3+CC41*P1P5 2100
U2DU2=+2.*CC42*CC32*P4P5+2.*CC42*CC22*P3P5+2.*CC42*CC21*P1P5+2.* 2110
CCC32*CC22*P3P4+2.*CC32*CC21*P1P4+2.*CC22*CC21*P1P3 2120
U2DU3=+CC43*CC32*P4P5+CC43*CC22*P3P5+CC43*CC21*P1P5+CC42*CC33*P4 2130
CP5+CC42*CC32*P3P5+CC42*CC31*P1P5+CC33*CC22*P3P4+CC33*CC21*P1P4+C 2140
CC32*CC31*P1P4+CC32*CC21*P1P3+CC32*P3P4+CC31*CC22*P1P3 2150
U2DU4=+CC44*CC32*P4P5+CC44*CC22*P3P5+CC44*CC21*P1P5+CC43*CC42*P4 2160
CP5+CC43*CC22*P3P4+CC43*CC21*P1P4+CC42*CC41*P1P5+CC42*CC32*P3P4+C 2170
CC42*CC21*P1P3+CC42*P3P5+CC41*CC32*P1P4+CC41*CC22*P1P3 2180
U3DU3=+2.*CC43*CC33*P4P5+2.*CC43*CC32*P3P5+2.*CC43*CC31*P1P5+2.* 2190
CCC33*CC32*P3P4+2.*CC33*CC31*P1P4+2.*CC32*CC31*P1P3 2200
U3DU4=+CC44*CC33*P4P5+CC44*CC32*P3P5+CC44*CC31*P1P5+CC43*CC42*P3 2210
CP5+CC43*CC41*P1P5+CC43*CC32*P3P4+CC43*CC31*P1P4+CC43*P4P5+CC4 2220
C2*CC33*P3P4+CC42*CC31*P1P3+CC41*CC33*P1P4+CC41*CC32*P1P3 2230
U4DU4=+2.*CC44*CC43*P4P5+2.*CC44*CC42*P3P5+2.*CC44*CC41*P1P5+2.* 2240
CCC43*CC42*P3P4+2.*CC43*CC41*P1P4+2.*CC42*CC41*P1P3+0. 2250
Z110U = U1DU1*(0.5*I8+0.5*D1*I5)+U2DU2*(0.5*I7-0.25*D3*I4) 2260
C - (U3DU3-2*U3DU4+U4DU4)*0.5*I9 2270
C - U1DU2*(0.5*I1-D1*I4-0.5*D3*I5) 2280
C - (U1DU3-U1DU4)*(0.5*I3-D1*I6) 2290
C + U1DU4*P4P5*I5-(U2DU3-U2DU4)*0.5*(I2-D3*I6)+U2DU4*P4P5*I4 2300
C - (U3DU4-U4DU4)*P4P5*I6 2310
Z110D = U1DU1*D1*D1+U2DU2*0.25*D3*D3+U4DU4*P4P5*P4P5 2320
C -1.0*U1DU2*D1*D3+2*U1DU4*P4P5*D1-U2DU4*P4P5*D3 2330
UPPER = -Z110U*CONJG(Z110D) 2340
LOWER = Z110D*CONJG(Z110D) 2350
I10 = UPPER/(LOWER) 2360
RETURN
END 5010

```

```

COMPLEX FUNCTION E_LOOP(HEL_CODE,P13,P14,P15,P34,P35,P45,
+ I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,

```

```

+      I11,I12,I13,I14,I15,I16,I17,I18,I19
+      WIV,FPIIV,BOIV,CUOIV,EoIV,EVL,
+      D3)

```

```

REAL P13,P14,P15,P34,P35,P45
REAL WIV,FPIIV,BOIV,CUOIV,EoIV
COMPLEX EVL,ELA,ELB,ELC,ELD,ELE,ELF,ELG,ELH
COMPLEX I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15,
+      I16,I17,I18,I19
COMPLEX D3
CHARACTER HEL_CODE*1

```

```

IF (HEL_CODE .EQ. 'A') THEN
  E_LOOP = ELA(P13,P14,P15,P34,P35,P45,
+      I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
+      I11,I12,I13,I14,I15,I16,I17,I18,I19,
+      WIV,FPIIV,BOIV,CUOIV,EoIV,EVL
+      D3)

```

```

  RETURN
ENDIF

```

```

IF (HEL_CODE .EQ. 'B') THEN
  E_LOOP = ELB(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

```

```

IF (HEL_CODE .EQ. 'C') THEN
  E_LOOP = ELC(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

```

```

IF (HEL_CODE .EQ. 'D') THEN
  E_LOOP = ELD(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

```

```

IF (HEL_CODE .EQ. 'E') THEN
  E_LOOP = ELE(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

```

```

IF (HEL_CODE .EQ. 'F') THEN
  E_LOOP = ELF(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

```

```

IF (HEL_CODE .EQ. 'G') THEN
  E_LOOP = ELG(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

```

```

IF (HEL_CODE .EQ. 'H') THEN
  E_LOOP = ELH(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

```

```

RETURN

```

END

```

COMPLEX FUNCTION P_LOOP(HEL_CODE,P13,P14,P15,P34,P35,P45,
+      I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
+      I11,I12,I13,I14,I15,I16,I17,I18,I19
+      WIV,FPIIV,BOIV,CUOIV,EoIV,EVL,
+      D3)

REAL P13,P14,P15,P34,P35,P45
REAL WIV,FPIIV,BOIV,CUOIV,EoIV
COMPLEX EVL,PLA,PLB,PLC,PLD,PLE,PLF,PLG,PLH
COMPLEX I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15,
+      I16,I17,I18,I19
COMPLEX D3
CHARACTER HEL_CODE*1

IF (HEL_CODE .EQ. 'A') THEN
  P_LOOP = PLA(P13,P14,P15,P34,P35,P45,
+      I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
+      I11,I12,I13,I14,I15,I16,I17,I18,I19,
+      WIV,FPIIV,BOIV,CUOIV,EoIV,EVL
+      D3)
  RETURN
ENDIF

IF (HEL_CODE .EQ. 'B') THEN
  P_LOOP = PLB(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

IF (HEL_CODE .EQ. 'C') THEN
  P_LOOP = PLC(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

IF (HEL_CODE .EQ. 'D') THEN
  P_LOOP = PLD(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

IF (HEL_CODE .EQ. 'E') THEN
  P_LOOP = PLE(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

IF (HEL_CODE .EQ. 'F') THEN
  P_LOOP = PLF(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

IF (HEL_CODE .EQ. 'G') THEN
  P_LOOP = PLG(P13,P14,P15,P34,P35,P45,EVL)
  RETURN
ENDIF

IF (HEL_CODE .EQ. 'H') THEN

```



```

P_LOOP = PLH(P13,P14,P15,P34,P35,P45,EVL)
RETURN
ENDIF

RETURN
END

```

C Expression QA

```

COMPLEX FUNCTION QA(P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL
COMPLEX TLPART,HDPART,XTOTAL

```

```

XTOTAL = 0
HDPART = 1
TLPART=256*P34*P35*P45*P14*P15**2-256*P34*P35*P45*P15**3+512*P3
1 4*P35*P45**2*P14*P15+768*P34*P35*P45**2*P15**2-512*P34*P35*P45*
1 *3*P15+256*P34*P35**2*P45*P14*P15+256*P34*P35**2*P45*P15**2-256
1 *P34*P35**2*P45**2*P15+512*P34*P35**2*P14*P15**2-256*P34*P35**2
1 *P15**3-256*P34*P45**2*P14*P15**2-256*P34*P45**2*P15**3+256*P34
1 *P45**3*P14*P15+512*P34*P45**3*P15**2-256*P34*P45**4*P15+256*P3
1 4**2*P35*P45*P14*P15+1024*P34**2*P35*P45*P15**2-512*P34**2*P35*
1 P45**2*P15+256*P34**2*P35*P14*P15**2-512*P34**2*P35*P15**3-256*
1 P34**2*P45*P14*P15**2
TLPART=TLPART-512*P34**2*P45*P15**3+256*P34**2*P45**2*P14*P15+1
1 024*P34**2*P45**2*P15**2-512*P34**2*P45**3*P15+512*P34**3*P45*P
1 15**2-256*P34**3*P45**2*P15-256*P34**3*P15**3+256*P35*P45**2*P1
1 4*P15**2+256*P35*P45**2*P15**3-256*P35*P45**3*P15**2+512*P35**2
1 *P45*P14*P15**2+256*P35**2*P45*P15**3-512*P35**2*P45**2*P15**2-
1 256*P35**3*P45*P15**2+256*P35**3*P14*P15**2
XTOTAL=XTOTAL+HDPART*TLPART
HDPART=EVL
TLPART=-256*P34*P35*P45*P15+512*P34*P35*P15**2+256*P34*P45*P15*
1 *2-256*P34*P45**2*P15-256*P34**2*P45*P15+256*P34**2*P15**2+256*
1 P35*P45*P15**2+256*P35**2*P15**2
XTOTAL=XTOTAL+HDPART*TLPART
QA=XTOTAL
RETURN
END

```

*end

C Expression QB

```

COMPLEX FUNCTION QB(P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL
COMPLEX TLPART,HDPART,XTOTAL

```

```

XTOTAL=0.
HDPART=1
TLPART=512*P34*P35*P45**2*P14*P15+256*P34*P35*P45**2*P15**2-102
1 4*P34*P35*P45**3*P15+256*P34*P35**2*P45*P14*P15-256*P34*P35**2*
1 P45*P15**2-512*P34*P35**2*P45**2*P15+256*P34*P45**3*P14*P15+512
1 *P34*P45**3*P15**2-512*P34*P45**4*P15-256*P34**2*P35*P45*P15**2
1 -256*P34**2*P35*P45**2*P15+256*P34**2*P45**2*P15**2-256*P34**2*
1 P45**3*P15+768*P35*P45**3*P14*P15+512*P35*P45**3*P15**2-768*P35
1 *P45**4*P15+768*P35**2*P45**2*P14*P15+256*P35**2*P45**2*P15**2-
1 768*P35**2*P45**3*P15+256*P35**3*P45*P14*P15-256*P35**3*P45**2*

```

```

1 P15*256*P45**4*P14*P15
  TLPART=TLPART+256*P45**4*P15**2-256*P45**5*P15
  XTOTAL=XTOTAL+HDPART*TLPART
  HDPART=EVL
  TLPART=-256*P34*P35*P45*P15-256*P34*P45**2*P15-512*P35*P45**2*P
1 15-256*P35**2*P45*P15-256*P45**3*P15
  XTOTAL=XTOTAL+HDPART*TLPART
  QB=XTOTAL
  RETURN
END

```

*end

C Expression QC

```

COMPLEX FUNCTION QC(P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL
COMPLEX TLPART,HDPART,XTOTAL

```

```

  XTOTAL=0.
  HDPART=1
  TLPART=256*P34*P35*P45**2*P14*P15+512*P34*P35*P45**2*P15**2-512
1 *P34*P35*P45**3*P15+512*P34*P35**2*P45*P14*P15+256*P34*P35**2*P
1 45*P15**2-1024*P34*P35**2*P45**2*P15-512*P34*P35**3*P45*P15+256
1 *P34*P35**3*P14*P15-256*P34*P35**3*P15**2+256*P34**2*P35*P45*P1
1 5**2-256*P34**2*P35*P45**2*P15-256*P34**2*P35**2*P45*P15-256*P3
1 4**2*P35**2*P15**2+256*P35*P45**3*P14*P15+256*P35*P45**3*P15**2
1 -256*P35*P45**4*P15+768*P35**2*P45**2*P14*P15+512*P35**2*P45**2
1 *P15**2-768*P35**2*P45**3*P15+768*P35**3*P45*P14*P15+256*P35**3
1 *P45*P15**2
  TLPART=TLPART-768*P35**3*P45**2*P15-256*P35**4*P45*P15+256*P35*
1 **4*P14*P15
  XTOTAL=XTOTAL+HDPART*TLPART
  HDPART=EVL
  TLPART=256*P34*P35*P45*P15+256*P34*P35**2*P15+256*P35*P45**2*P1
1 5+512*P35**2*P45*P15+256*P35**3*P15
  XTOTAL=XTOTAL+HDPART*TLPART
  QC=XTOTAL
  RETURN
END

```

*end

C Expression QD

```

COMPLEX FUNCTION QD(P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL
COMPLEX TLPART,HDPART,XTOTAL

```

```

  XTOTAL=0.
  HDPART=1
  TLPART=256*P34*P35*P45*P14*P15**2+512*P34*P35*P45*P15**3+256*P3
1 4*P35*P45**2*P14*P15-1280*P34*P35*P45**2*P15**2-256*P34*P35*P45
1 **3*P15+512*P34*P35**2*P45*P14*P15-512*P34*P35**2*P45*P15**2-51
1 2*P34*P35**2*P45**2*P15-256*P34*P35**2*P14*P15**2-256*P34*P35**
1 3*P45*P15+256*P34*P35**3*P14*P15+512*P34*P45**2*P14*P15**2+768*
1 P34*P45**2*P15**3-768*P34*P45**3*P15**2+256*P34**2*P35*P45*P14*
1 P15-1024*P34**2*P35*P45*P15**2-512*P34**2*P35*P45**2*P15-256*P3
1 4**2*P35*P14*P15**2+256*P34**2*P35*P15**3-512*P34**2*P35**2*P45
1 *P15

```

```

TLPART=TLPART+256*P34**2*P35**2*P14*P15-256*P34**2*P35**2*P15**
1 2-256*P34**2*P45*P14*P15**2+768*P34**2*P45*P15**3-768*P34**2*P4
1 5**2*P15**2-256*P34**3*P35*P45*P15-256*P34**3*P35*P15**2-256*P3
1 4**3*P45*P15**2+256*P34**3*P15**3+512*P35*P45**2*P14*P15**2+256
1 *P35*P45**2*P15**3-512*P35*P45**3*P15**2+256*P35**2*P45*P14*P15
1 **2-256*P35**2*P45**2*P15**2+256*P45**3*P14*P15**2+256*P45**3*P
1 15**3-256*P45**4*P15**2

```

```

XTOTAL=XTOTAL+HDPART*TLPART

```

```

HDPART=EVL

```

```

TLPART=256*P34*P35*P45*P15-256*P34*P35*P15**2+256*P34*P35**2*P1
1 5-512*P34*P45*P15**2+256*P34**2*P35*P15-256*P34**2*P15**2-256*P
1 35*P45*P15**2-256*P45**2*P15**2

```

```

XTOTAL=XTOTAL+HDPART*TLPART

```

```

QD=XTOTAL

```

```

RETURN

```

```

END

```

```

*end

```

C Expression QE

```

COMPLEX FUNCTION QE(P13,P14,P15,P34,P35,P45,EVL)

```

```

REAL P13,P14,P15,P34,P35,P45

```

```

COMPLEX EVL

```

```

COMPLEX TLPART,HDPART,XTOTAL

```

```

XTOTAL=0.

```

```

HDPART=1

```

```

TLPART=-256*P34*P35*P45*P14*P15**2-512*P34*P35*P45*P15**3-256*P
1 34*P35*P45**2*P14*P15+1280*P34*P35*P45**2*P15**2+256*P34*P35*P4
1 5**3*P15-512*P34*P35**2*P45*P14*P15-512*P34*P35**2*P45*P15**2+5
1 12*P34*P35**2*P45**2*P15+256*P34*P35**2*P14*P15**2+256*P34*P35*
1 *3*P45*P15-256*P34*P35**3*P14*P15-512*P34*P45**2*P14*P15**2-768
1 *P34*P45**2*P15**3+768*P34*P45**3*P15**2-256*P34**2*P35*P45*P14
1 *P15+1024*P34**2*P35*P45*P15**2+512*P34**2*P35*P45**2*P15+256*P
1 34**2*P35*P14*P15**2-256*P34**2*P35*P15**3+512*P34**2*P35**2*P4
1 5*P15

```

```

TLPART=TLPART-256*P34**2*P35**2*P14*P15+256*P34**2*P35**2*P15**
1 2-256*P34**2*P45*P14*P15**2-768*P34**2*P45*P15**3+768*P34**2*P4
1 5**2*P15**2+256*P34**3*P35*P45*P15+256*P34**3*P35*P15**2+256*P3
1 4**3*P45*P15**2-256*P34**3*P15**3-512*P35*P45**2*P14*P15**2-256
1 *P35*P45**2*P15**3+512*P35*P45**3*P15**2-256*P35**2*P45*P14*P15
1 **2+256*P35**2*P45**2*P15**2-256*P45**3*P14*P15**2-256*P45**3*P
1 15**3+256*P45**4*P15**2

```

```

XTOTAL=XTOTAL+HDPART*TLPART

```

```

HDPART=EVL

```

```

TLPART=256*P34*P35*P45*P15-256*P34*P35*P15**2+256*P34*P35**2*P1
1 5-512*P34*P45*P15**2+256*P34**2*P35*P15-256*P34**2*P15**2-256*P
1 35*P45*P15**2-256*P45**2*P15**2

```

```

XTOTAL=XTOTAL+HDPART*TLPART

```

```

QE=XTOTAL

```

```

RETURN

```

```

END

```

```

*end

```

C Expression QF

```

COMPLEX FUNCTION QF(P13,P14,P15,P34,P35,P45,EVL)

```

```

REAL P13,P14,P15,P34,P35,P45

```

```

COMPLEX EVL

```

```

COMPLEX TLPART,HDPART,XTOTAL

```

```

XTOTAL=0.
HDPART=1
TLPART=-256*P34*P35*P45**2*P14*P15-512*P34*P35*P45**2*P15**2+51
1 2*P34*P35*P45**3*P15-512*P34*P35**2*P45*P14*P15-256*P34*P35**2*
1 P45*P15**2+1024*P34*P35**2*P45**2*P15-512*P34*P35**3*P45*P15-25
1 6*P34*P35**3*P14*P15-256*P34*P35**3*P15**2-256*P34**2*P35*P45*P
1 15**2+256*P34**2*P35*P45**2*P15-256*P34**2*P35**2*P45*P15-256*P
1 34**2*P35**2*P15**2-256*P35*P45**3*P14*P15-256*P35*P45**3*P15**
1 2-256*P35*P45**4*P15-768*P35**2*P45**2*P14*P15-512*P35**2*P45**
1 2*P15**2+768*P35**2*P45**3*P15-768*P35**3*P45*P14*P15-256*P35**
1 3*P45*P15**2
TLPART=TLPART+768*P35**3*P45**2*P15+256*P35**4*P45*P15-256*P35*
1 *4*P14*P15
XTOTAL=XTOTAL+HDPART*TLPART
HDPART=EVL
TLPART=256*P34*P35*P45*P15+256*P34*P35**2*P15+256*P35*P45**2*P1
1 5-512*P35**2*P45*P15+256*P35**3*P15
XTOTAL=XTOTAL+HDPART*TLPART
QF=XTOTAL
RETURN
END
*end

```

C Expression QG

```

COMPLEX FUNCTION QG(P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL
COMPLEX TLPART,HDPART,XTOTAL

```

```

XTOTAL=0.
HDPART=1
TLPART=-512*P34*P35*P45**2*P14*P15-256*P34*P35*P45**2*P15**2+10
1 24*P34*P35*P45**3*P15-256*P34*P35**2*P45*P14*P15+256*P34*P35**2
1 *P45*P15**2+512*P34*P35**2*P45**2*P15-256*P34*P45**3*P14*P15-51
1 2*P34*P45**3*P15**2+512*P34*P45**4*P15-256*P34**2*P35*P45*P15**
1 2+256*P34**2*P35*P45**2*P15-256*P34**2*P45**2*P15**2+256*P34**2
1 *P45**3*P15-768*P35*P45**3*P14*P15-512*P35*P45**3*P15**2+768*P3
1 5*P45**4*P15-768*P35**2*P45**2*P14*P15-256*P35**2*P45**2*P15**2
1 *768*P35**2*P45**3*P15-256*P35**3*P45*P14*P15-256*P35**3*P45**2
1 *P15-256*P45**4*P14*P15
TLPART=TLPART-256*P45**4*P15**2+256*P45**5*P15
XTOTAL=XTOTAL+HDPART*TLPART
HDPART=EVL
TLPART=-256*P34*P35*P45*P15-256*P34*P45**2*P15-512*P35*P45**2*P
1 15-256*P35**2*P45*P15-256*P45**3*P15
XTOTAL=XTOTAL+HDPART*TLPART
QG=XTOTAL
RETURN
END
*end

```

C Expression QH

```

COMPLEX FUNCTION QH(P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL
COMPLEX TLPART,HDPART,XTOTAL

```



```

      XTOTAL=0.
      HDPART=1
      TLPART=-256*P34*P35*P45*P14*P15**2+256*P34*P35*P45*P15**3-512*P
1 34*P35*P45**2*P14*P15-768*P34*P35*P45**2*P15**2-512*P34*P35*P45
1 **3*P15-256*P34*P35**2*P45*P14*P15-256*P34*P35**2*P45*P15**2+25
1 6*P34*P35**2*P45**2*P15-512*P34*P35**2*P14*P15**2+256*P34*P35**
1 2*P15**3+256*P34*P45**2*P14*P15**2+256*P34*P45**2*P15**3-256*P3
1 4*P45**3*P14*P15-512*P34*P45**3*P15**2+256*P34*P45**4*P15-256*P
1 34**2*P35*P45*P14*P15-1024*P34**2*P35*P45*P15**2+512*P34**2*P35
1 *P45**2*P15-256*P34**2*P35*P14*P15**2+512*P34**2*P35*P15**3+256
1 *P34**2*P45*P14*P15**2
      TLPART=TLPART+512*P34**2*P45*P15**3-256*P34**2*P45**2*P14*P15-1
1 024*P34**2*P45**2*P15**2+512*P34**2*P45**3*P15-512*P34**3*P45*P
1 15**2+256*P34**3*P45**2*P15+256*P34**3*P15**3-256*P35*P45**2*P1
1 4*P15**2-256*P35*P45**2*P15**3+256*P35*P45**3*P15**2-512*P35**2
1 *P45*P14*P15**2-256*P35**2*P45*P15**3+512*P35**2*P45**2*P15**2+
1 256*P35**3*P45*P15**2-256*P35**3*P14*P15**2
      XTOTAL=XTOTAL+HDPART*TLPART
      HDPART=EVL
      TLPART=-256*P34*P35*P45*P15+512*P34*P35*P15**2+256*P34*P45*P15*
1 *2-256*P34*P45**2*P15-256*P34**2*P45*P15+256*P34**2*P15**2+256*
1 P35*P45*P15**2+256*P35**2*P15**2
      XTOTAL=XTOTAL+HDPART*TLPART
      QH=XTOTAL
      RETURN
      END
C Expression ELC
      COMPLEX FUNCTION ELC(P13,P14,P15,P34,P35,P45,EVL)
      REAL P13,P14,P15,P34,P35,P45
      COMPLEX EVL
      COMPLEX TLPART,HDPART,XTOTAL

      XTOTAL=0.
      HDPART=1
      TLPART=256*P34*P35*P45**2*P14*P15+512*P34*P35*P45**2*P15**2-512
1 *P34*P35*P45**3*P15+512*P34*P35**2*P45*P14*P15+256*P34*P35**2*P
1 45*P15**2-1024*P34*P35**2*P45**2*P15-512*P34*P35**3*P45*P15+256
1 *P34*P35**3*P14*P15-256*P34*P35**3*P15**2+256*P34**2*P35*P45*P1
1 5**2-256*P34**2*P35*P45**2*P15-256*P34**2*P35**2*P45*P15-256*P3
1 4**2*P35**2*P15**2+256*P35*P45**3*P14*P15+256*P35*P45**3*P15**2
1 -256*P35*P45**4*P15+768*P35**2*P45**2*P14*P15+512*P35**2*P45**2
1 *P15**2-768*P35**2*P45**3*P15+768*P35**3*P45*P14*P15+256*P35**3
1 *P45*P15**2
      TLPART=TLPART-768*P35**3*P45**2*P15-256*P35**4*P45*P15+256*P35*
1 **4*P14*P15
      XTOTAL=XTOTAL+HDPART*TLPART
      HDPART=EVL
      TLPART=256*P34*P35*P45*P15+256*P34*P35**2*P15+256*P35*P45**2*P1
1 5+512*P35**2*P45*P15+256*P35**3*P15
      XTOTAL=XTOTAL+HDPART*TLPART
      ELC=XTOTAL
      RETURN
      END
*end

C Expression ELD
      COMPLEX FUNCTION ELD(P13,P14,P15,P34,P35,P45,EVL)
      REAL P13,P14,P15,P34,P35,P45

```

COMPLEX EVL
COMPLEX TLPART, HDPART, XTOTAL

XTOTAL=0.

HDPART=1

TLPART=256*P34*P35*P45*P14*P15**2*512*P34*P35*P45*P15**3+256*P3
1 4*P35*P45**2*P14*P15-1280*P34*P35*P45**2*P15**2-256*P34*P35*P45
1 **3*P15+512*P34*P35**2*P45*P14*P15-512*P34*P35**2*P45*P15**2-51
1 2*P34*P35**2*P45**2*P15-256*P34*P35**2*P14*P15**2-256*P34*P35**
1 3*P45*P15+256*P34*P35**3*P14*P15-512*P34*P45**2*P14*P15**2+768*
1 P34*P45**2*P15**3-768*P34*P45**3*P15**2+256*P34**2*P35*P45*P14*
1 P15-1024*P34**2*P35*P45*P15**2-512*P34**2*P35*P45**2*P15-256*P3
1 **2*P35*P14*P15**2+256*P34**2*P35*P15**3-512*P34**2*P35**2*P45
1 *P15

TLPART=TLPART+256*P34**2*P35**2*P14*P15-256*P34**2*P35**2*P15**
1 2+256*P34**2*P45*P14*P15**2+768*P34**2*P45*P15**3-768*P34**2*P4
1 5**2*P15**2-256*P34**3*P35*P45*P15-256*P34**3*P35*P15**2-256*P3
1 4**3*P45*P15**2+256*P34**3*P15**3+512*P35*P45**2*P14*P15**2+256
1 *P35*P45**2*P15**3-512*P35*P45**3*P15**2+256*P35**2*P45*P14*P15
1 **2-256*P35**2*P45**2*P15**2+256*P45**3*P14*P15**2+256*P45**3*P
1 15**3-256*P45**4*P15**2

XTOTAL=XTOTAL+HDPART*TLPART

HDPART=EVL

TLPART=256*P34*P35*P45*P15-256*P34*P35*P15**2+256*P34*P35**2*P1
1 5-512*P34*P45*P15**2+256*P34**2*P35*P15-256*P34**2*P15**2-256*P
1 35*P45*P15**2-256*P45**2*P15**2

XTOTAL=XTOTAL+HDPART*TLPART

ELD=XTOTAL

RETURN

END

*end

C Expression ELE

COMPLEX FUNCTION ELE(P13,P14,P15,P34,P35,P45,EVL)

REAL P13,P14,P15,P34,P35,P45

COMPLEX EVL

COMPLEX TLPART, HDPART, XTOTAL

XTOTAL=0.

HDPART=1

TLPART=-256*P34*P35*P45*P14*P15**2-512*P34*P35*P45*P15**3-256*P
1 34*P35*P45**2*P14*P15+1280*P34*P35*P45**2*P15**2+256*P34*P35*P4
1 5**3*P15-512*P34*P35**2*P45*P14*P15-512*P34*P35**2*P45*P15**2+5
1 12*P34*P35**2*P45**2*P15+256*P34*P35**2*P14*P15**2+256*P34*P35*
1 *3*P45*P15-256*P34*P35**3*P14*P15-512*P34*P45**2*P14*P15**2-768
1 *P34*P45**2*P15**3+768*P34*P45**3*P15**2-256*P34**2*P35*P45*P14
1 *P15+1024*P34**2*P35*P45*P15**2+512*P34**2*P35*P45**2*P15+256*P
1 34**2*P35*P14*P15**2-256*P34**2*P35*P15**3+512*P34**2*P35**2*P4
1 5*P15

TLPART=TLPART-256*P34**2*P35**2*P14*P15+256*P34**2*P35**2*P15**
1 2-256*P34**2*P45*P14*P15**2-768*P34**2*P45*P15**3+768*P34**2*P4
1 5**2*P15**2+256*P34**3*P35*P45*P15+256*P34**3*P35*P15**2+256*P3
1 4**3*P45*P15**2-256*P34**3*P15**3-512*P35*P45**2*P14*P15**2-256
1 *P35*P45**2*P15**3+512*P35*P45**3*P15**2-256*P35**2*P45*P14*P15
1 **2+256*P35**2*P45**2*P15**2-256*P45**3*P14*P15**2-256*P45**3*P
1 15**3+256*P45**4*P15**2

XTOTAL=XTOTAL+HDPART*TLPART

HDPART=EVL

```

TLPART=256*P34*P35*P45*P15-256*P34*P35*P15**2+256*P34*P35**2*P1
1 5-512*P34*P45*P15**2+256*P34**2*P35*P15-256*P34**2*P15**2-256*P
1 35*P45*P15**2-256*P45**2*P15**2
XTOTAL=XTOTAL+HDPART*TLPART
ELE=XTOTAL
RETURN
END

```

•end

C Expression ELF

```

COMPLEX FUNCTION ELF(P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL
COMPLEX TLPART,HDPART,XTOTAL

```

```

XTOTAL=0.
HDPART=1
TLPART=-256*P34*P35*P45**2*P14*P15-512*P34*P35*P45**2*P15**2+51
1 2*P34*P35*P45**3*P15-512*P34*P35**2*P45*P14*P15-256*P34*P35**2*
1 P45*P15**2+1024*P34*P35**2*P45**2*P15-512*P34*P35**3*P45*P15-25
1 6*P34*P35**3*P14*P15-256*P34*P35**3*P15**2-256*P34**2*P35*P45*P
1 15**2+256*P34**2*P35*P45**2*P15+256*P34**2*P35**2*P45*P15+256*P
1 34**2*P35**2*P15**2-256*P35*P45**3*P14*P15-256*P35*P45**3*P15**
1 2+256*P35*P45**4*P15-768*P35**2*P45**2*P14*P15-512*P35**2*P45**
1 2*P15**2+768*P35**2*P45**3*P15-768*P35**3*P45*P14*P15-256*P35**
1 3*P45*P15**2
TLPART=TLPART+768*P35**3*P45**2*P15+256*P35**4*P45*P15-256*P35*
1 **4*P14*P15
XTOTAL=XTOTAL+HDPART*TLPART
HDPART=EVL
TLPART=256*P34*P35*P45*P15+256*P34*P35**2*P15+256*P35*P45**2*P1
1 5+512*P35**2*P45*P15+256*P35**3*P15
XTOTAL=XTOTAL+HDPART*TLPART
ELF=XTOTAL
RETURN
END

```

•end

C Expression ELG

```

COMPLEX FUNCTION ELG(P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL
COMPLEX TLPART,HDPART,XTOTAL

```

```

XTOTAL=0.
HDPART=1
TLPART=-512*P34*P35*P45**2*P14*P15-256*P34*P35*P45**2*P15**2+10
1 24*P34*P35*P45**3*P15-256*P34*P35**2*P45*P14*P15+256*P34*P35**2
1 *P45*P15**2+512*P34*P35**2*P45**2*P15-256*P34*P45**3*P14*P15-51
1 2*P34*P45**3*P15**2+512*P34*P45**4*P15+256*P34**2*P35*P45*P15**
1 2+256*P34**2*P35*P45**2*P15-256*P34**2*P45**2*P15**2+256*P34**2
1 *P45**3*P15-768*P35*P45**3*P14*P15-512*P35*P45**3*P15**2+768*P3
1 5*P45**4*P15-768*P35**2*P45**2*P14*P15-256*P35**2*P45**2*P15**2
1 +768*P35**2*P45**3*P15-256*P35**3*P45*P14*P15+256*P35**3*P45**2
1 *P15-256*P45**4*P14*P15
TLPART=TLPART-256*P45**4*P15**2+256*P45**5*P15
XTOTAL=XTOTAL+HDPART*TLPART
HDPART=EVL

```



```

TLPART=-256*P34*P35*P45*P15-256*P34*P45**2*P15-512*P35*P45**2*P
1 15-256*P35**2*P45*P15-256*P45**3*P15
XTOTAL=XTOTAL+HDPART*TLPART
ELC=XTOTAL
RETURN
END

```

```

*end

```

C Expression ELH

```

COMPLEX FUNCTION ELH(P13,P14,P15,P34,P35,P45,EVL)
REAL P13,P14,P15,P34,P35,P45
COMPLEX EVL
COMPLEX TLPART,HDPART,XTOTAL

```

```

XTOTAL=0.
HDPART=1
TLPART=-256*P34*P35*P45*P14*P15**2+256*P34*P35*P45*P15**3-512*P
1 34*P35*P45**2*P14*P15-768*P34*P35*P45**2*P15**2+512*P34*P35*P45
1 **3*P15-256*P34*P35**2*P45*P14*P15-256*P34*P35**2*P45*P15**2+25
1 6*P34*P35**2*P45**2*P15-512*P34*P35**2*P14*P15**2+256*P34*P35**
1 2*P15**3+256*P34*P45**2*P14*P15**2+256*P34*P45**2*P15**3-256*P3
1 4*P45**3*P14*P15-512*P34*P45**3*P15**2+256*P34*P45**4*P15-256*P
1 34**2*P35*P45*P14*P15-1024*P34**2*P35*P45*P15**2+512*P34**2*P35
1 *P45**2*P15-256*P34**2*P35*P14*P15**2+512*P34**2*P35*P15**3+256
1 *P34**2*P45*P14*P15**2
TLPART=TLPART+512*P34**2*P45*P15**3-256*P34**2*P45**2*P14*P15-1
1 024*P34**2*P45**2*P15**2+512*P34**2*P45**3*P15-512*P34**3*P45*P
1 15**2+256*P34**3*P45**2*P15+256*P34**3*P15**3-256*P35*P45**2*P1
1 4*P15**2-256*P35*P45**2*P15**3+256*P35*P45**3*P15**2-512*P35**2
1 *P45*P14*P15**2-256*P35**2*P45*P15**3+512*P35**2*P45**2*P15**2+
1 256*P35**3*P45*P15**2-256*P35**3*P14*P15**2
XTOTAL=XTOTAL+HDPART*TLPART
HDPART=EVL
TLPART=-256*P34*P35*P45*P15+512*P34*P35*P15**2+256*P34*P45*P15*
1 *2-256*P34*P45**2*P15-256*P34**2*P45*P15+256*P34**2*P15**2+256*
1 P35*P45*P15**2+256*P35**2*P15**2
XTOTAL=XTOTAL+HDPART*TLPART
ELH=XTOTAL
RETURN
END

```

C Expression ELB

```

COMPLEX FUNCTION ELB(P13,P14,P15,P34,P35,P45,
+ I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
+ I11,I12,I13,I14,I15,I16,I17,I18,I19,
+ WIV,FPIIV,BOIV,CUOIV,E0IV,EVL
+ D3)

REAL P13,P14,P15,P34,P35,P45
REAL WIV,FPIIV,BOIV,CUOIV,E0IV
COMPLEX EVL
COMPLEX I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15,
+ I16,I17,I18,I19
COMPLEX D3
COMPLEX TLPART,HDPART,XTOTAL

```

```

XTOTAL = 0
HDPART = 1

```

```

      TLPART=256*P34*P35*P45*P14*P15**2-256*P34*P35*P45*P15**3+512*P3
1  4*P35*P45**2*P14*P15-768*P34*P35*P45**2*P15**2-512*P34*P35*P45*
1  *3*P15+256*P34*P35**2*P45*P14*P15+256*P34*P35**2*P45*P15**2-256
1  *P34*P35**2*P45**2*P15+512*P34*P35**2*P14*P15**2-256*P34*P35**2
1  *P15**3-256*P34*P45**2*P14*P15**2-256*P34*P45**2*P15**3+256*P34
1  *P45**3*P14*P15+512*P34*P45**3*P15**2-256*P34*P45**4*P15+256*P3
1  4**2*P35*P45*P14*P15+1024*P34**2*P35*P45*P15**2-512*P34**2*P35*
1  P45**2*P15+256*P34**2*P35*P14*P15**2-512*P34**2*P35*P15**3-256*
1  P34**2*P45*P14*P15**2
      TLPART=TLPART-512*P34**2*P45*P15**3+256*P34**2*P45**2*P14*P15+1
1  024*P34**2*P45**2*P15**2-512*P34**2*P45**3*P15+512*P34**3*P45*P
1  15**2-256*P34**3*P45**2*P15-256*P34**3*P15**3+256*P35*P45**2*P1
1  4*P15**2+256*P35*P45**2*P15**3-256*P35*P45**3*P15**2+512*P35**2
1  *P45*P14*P15**2+256*P35**2*P45*P15**3-512*P35**2*P45**2*P15**2-
1  256*P35**3*P45*P15**2+256*P35**3*P14*P15**2
      XTOTAL=XTOTAL+HDPART*TLPART
      HDPART=EVL
      TLPART=-256*P34*P35*P45*P15+512*P34*P35*P15**2+256*P34*P45*P15*
1  *2-256*P34*P45**2*P15-256*P34**2*P45*P15+256*P34**2*P15**2+256*
1  P35*P45*P15**2+256*P35**2*P15**2
      XTOTAL=XTOTAL+HDPART*TLPART
      PLA=XTOTAL
      RETURN
      END
*end

C Expression PLB
      COMPLEX FUNCTION PLB(P13,P14,P15,P34,P35,P45,EVL)
      REAL P13,P14,P15,P34,P35,P45
      COMPLEX EVL
      COMPLEX TLPART,HDPART,XTOTAL

      XTOTAL=0.
      HDPART=1
      TLPART=512*P34*P35*P45**2*P14*P15+256*P34*P35*P45**2*P15**2-102
1  4*P34*P35*P45**3*P15+256*P34*P35**2*P45*P14*P15-256*P34*P35**2*
1  P45*P15**2-512*P34*P35**2*P45**2*P15+256*P34*P45**3*P14*P15+512
1  *P34*P45**3*P15**2-512*P34*P45**4*P15-256*P34**2*P35*P45*P15**2
1  -256*P34**2*P35*P45**2*P15+256*P34**2*P45**2*P15**2-256*P34**2*
1  P45**3*P15+768*P35*P45**3*P14*P15+512*P35*P45**3*P15**2-768*P35
1  *P45**4*P15+768*P35**2*P45**2*P14*P15+256*P35**2*P45**2*P15**2-
1  768*P35**2*P45**3*P15+256*P35**3*P45*P14*P15-256*P35**3*P45**2*
1  P15+256*P45**4*P14*P15
      TLPART=TLPART+256*P45**4*P15**2-256*P45**5*P15
      XTOTAL=XTOTAL+HDPART*TLPART
      HDPART=EVL
      TLPART=-256*P34*P35*P45*P15-256*P34*P45**2*P15-512*P35*P45**2*P
1  15-256*P35**2*P45*P15-256*P45**3*P15
      XTOTAL=XTOTAL+HDPART*TLPART
      PLB=XTOTAL
      RETURN
      END
*end

C Expression PLC
      COMPLEX FUNCTION PLC(P13,P14,P15,P34,P35,P45,EVL)
      REAL P13,P14,P15,P34,P35,P45
      COMPLEX EVL

```

COMPLEX TLPART,HDPART,XTOTAL

```

XTOTAL=0.
HDPART=1
TLPART=256*P34*P35*P45**2*P14*P15+512*P34*P35*P45**2*P15**2-512
1 *P34*P35*P45**3*P15+512*P34*P35**2*P45*P14*P15+256*P34*P35**2*P
1 45*P15**2-1024*P34*P35**2*P45**2*P15-512*P34*P35**3*P45*P15+256
1 *P34*P35**3*P14*P15-256*P34*P35**3*P15**2+256*P34**2*P35*P45*P1
1 5**2-256*P34**2*P35*P45**2*P15-256*P34**2*P35**2*P45*P15-256*P3
1 4**2*P35**2*P15**2+256*P35*P45**3*P14*P15+256*P35*P45**3*P15**2
1 -256*P35*P45**4*P15+768*P35**2*P45**2*P14*P15+512*P35**2*P45**2
1 *P15**2-768*P35**2*P45**3*P15+768*P35**3*P45*P14*P15+256*P35**3
1 *P45*P15**2
TLPART=TLPART-768*P35**3*P45**2*P15-256*P35**4*P45*P15+256*P35
1 *4*P14*P15
XTOTAL=XTOTAL+HDPART*TLPART
HDPART=EVL
TLPART=256*P34*P35*P45*P15+256*P34*P35**2*P15+256*P35*P45**2*P1
1 5+512*P35**2*P45*P15+256*P35**3*P15
XTOTAL=XTOTAL+HDPART*TLPART
PLC=XTOTAL
RETURN
END

```

*end

C Expression PLD

COMPLEX FUNCTION PLD(P13,P14,P15,P34,P35,P45,EVL)

REAL P13,P14,P15,P34,P35,P45

COMPLEX EVL

COMPLEX TLPART,HDPART,XTOTAL

```

XTOTAL=0.
HDPART=1
TLPART=256*P34*P35*P45*P14*P15**2+512*P34*P35*P45*P15**3+256*P3
1 4*P35*P45**2*P14*P15-1280*P34*P35*P45**2*P15**2-256*P34*P35*P45
1 **3*P15+512*P34*P35**2*P45*P14*P15-512*P34*P35**2*P45*P15**2-51
1 2*P34*P35**2*P45**2*P15-256*P34*P35**2*P14*P15**2-256*P34*P35**
1 3*P45*P15+256*P34*P35**3*P14*P15+512*P34*P45**2*P14*P15**2+768*
1 P34*P45**2*P15**3-768*P34*P45**3*P15**2+256*P34**2*P35*P45*P14*
1 P15-1024*P34**2*P35*P45*P15**2-512*P34**2*P35*P45**2*P15-256*P3
1 4**2*P35*P14*P15**2+256*P34**2*P35*P15**3-512*P34**2*P35**2*P45
1 *P15
TLPART=TLPART+256*P34**2*P35**2*P14*P15-256*P34**2*P35**2*P15**
1 2+256*P34**2*P45*P14*P15**2+768*P34**2*P45*P15**3-768*P34**2*P4
1 5**2*P15**2-256*P34**3*P35*P45*P15-256*P34**3*P35*P15**2-256*P3
1 4**3*P45*P15**2+256*P34**3*P15**3+512*P35*P45**2*P14*P15**2+256
1 *P35*P45**2*P15**3-512*P35*P45**3*P15**2+256*P35**2*P45*P14*P15
1 **2-256*P35**2*P45**2*P15**2+256*P45**3*P14*P15**2+256*P45**3*P
1 15**3-256*P45**4*P15**2
XTOTAL=XTOTAL+HDPART*TLPART
HDPART=EVL
TLPART=256*P34*P35*P45*P15-256*P34*P35*P15**2+256*P34*P35**2*P1
1 5-512*P34*P45*P15**2+256*P34**2*P35*P15-256*P34**2*P15**2-256*P
1 35*P45*P15**2-256*P45**2*P15**2
XTOTAL=XTOTAL+HDPART*TLPART
PLD=XTOTAL
RETURN
END

```


C Expression PLE

COMPLEX FUNCTION PLE(P13,P14,P15,P34,P35,P45,EVL)

REAL P13,P14,P15,P34,P35,P45

COMPLEX EVL

COMPLEX TLPART,HDPART,XTOTAL

XTOTAL=0.

HDPART=1

TLPART=-256*P34*P35*P45*P14*P15**2-512*P34*P35*P45*P15**3-256*P

1 34*P35*P45**2*P14*P15-1280*P34*P35*P45**2*P15**2-256*P34*P35*P4

1 5**3*P15-512*P34*P35**2*P45*P14*P15-512*P34*P35**2*P45*P15**2+5

1 12*P34*P35**2*P45**2*P15+256*P34*P35**2*P14*P15**2+256*P34*P35*

1 1*3*P45*P15-256*P34*P35**3*P14*P15-512*P34*P45**2*P14*P15**2-768

1 *P34*P45**2*P15**3*768*P34*P45**3*P15**2-256*P34**2*P35*P45*P14

1 *P15+1024*P34**2*P35*P45*P15**2+512*P34**2*P35*P45**2*P15+256*P

1 34**2*P35*P14*P15**2-256*P34**2*P35*P15**3+512*P34**2*P35**2*P4

1 5*P15

TLPART=TLPART-256*P34**2*P35**2*P14*P15+256*P34**2*P35**2*P15**

1 2-256*P34**2*P45*P14*P15**2-768*P34**2*P45*P15**3*768*P34**2*P4

1 5**2*P15**2+256*P34**3*P35*P45*P15+256*P34**3*P35*P15**2+256*P3

1 4**3*P45*P15**2-256*P34**3*P15**3-512*P35*P45**2*P14*P15**2-256

1 *P35*P45**2*P15**3+512*P35*P45**3*P15**2-256*P35**2*P45*P14*P15

1 **2+256*P35**2*P45**2*P15**2-256*P45**3*P14*P15**2-256*P45**3*P

1 15**3+256*P45**4*P15**2

XTOTAL=XTOTAL+HDPART*TLPART

HDPART=EVL

TLPART=256*P34*P35*P45*P15-256*P34*P35*P15**2+256*P34*P35**2*P1

1 5-512*P34*P45*P15**2+256*P34**2*P35*P15-256*P34**2*P15**2-256*P

1 35*P45*P15**2-256*P45**2*P15**2

XTOTAL=XTOTAL+HDPART*TLPART

PLE=XTOTAL

RETURN

END

*end

C Expression PLF

COMPLEX FUNCTION PLF(P13,P14,P15,P34,P35,P45,EVL)

REAL P13,P14,P15,P34,P35,P45

COMPLEX EVL

COMPLEX TLPART,HDPART,XTOTAL

XTOTAL=0.

HDPART=1

TLPART=-256*P34*P35*P45**2*P14*P15-512*P34*P35*P45**2*P15**2+51

1 2*P34*P35*P45**3*P15-512*P34*P35**2*P45*P14*P15-256*P34*P35**2*

1 P45*P15**2+1024*P34*P35**2*P45**2*P15+512*P34*P35**3*P45*P15-25

1 6*P34*P35**3*P14*P15+256*P34*P35**3*P15**2-256*P34**2*P35*P45*P

1 15**2+256*P34**2*P35*P45**2*P15+256*P34**2*P35**2*P45*P15+256*P

1 34**2*P35**2*P15**2-256*P35*P45**3*P14*P15-256*P35*P45**3*P15**

1 2-256*P35*P45**4*P15-768*P35**2*P45**2*P14*P15-512*P35**2*P45**

1 2*P15**2+768*P35**2*P45**3*P15-768*P35**3*P45*P14*P15-256*P35**

1 3*P45*P15**2

TLPART=TLPART+768*P35**3*P45**2*P15+256*P35**4*P45*P15-256*P35*

1 4*P14*P15

XTOTAL=XTOTAL+HDPART*TLPART

HDPART=EVL

```

      TLPART=256*P34*P35*P45*P15+256*P34*P35**2*P15+256*P35*P45**2*P1
1 5+512*P35**2*P45*P15+256*P35**3*P15
      XTOTAL=XTOTAL+HDPART*TLPART
      PLF=XTOTAL
      RETURN
      END

```

*end

C Expression PLG

```

      COMPLEX FUNCTION PLG(P13,P14,P15,P34,P35,P45,EVL)
      REAL P13,P14,P15,P34,P35,P45
      COMPLEX EVL
      COMPLEX TLPART,HDPART,XTOTAL

```

XTOTAL=0.

HDPART=1

```

      TLPART=-512*P34*P35*P45**2*P14*P15-256*P34*P35*P45**2*P15**2+10
1 24*P34*P35*P45**3*P15-256*P34*P35**2*P45*P14*P15+256*P34*P35**2
1 *P45*P15**2+512*P34*P35**2*P45**2*P15-256*P34*P45**3*P14*P15-51
1 2*P34*P45**3*P15**2+512*P34*P45**4*P15+256*P34**2*P35*P45*P15**
1 2+256*P34**2*P35*P45**2*P15-256*P34**2*P45**2*P15**2+256*P34**2
1 *P45**3*P15-768*P35*P45**3*P14*P15-512*P35*P45**3*P15**2+768*P3
1 5*P45**4*P15-768*P35**2*P45**2*P14*P15-256*P35**2*P45**2*P15**2
1 +768*P35**2*P45**3*P15-256*P35**3*P45*P14*P15+256*P35**3*P45**2
1 *P15-256*P45**4*P14*P15

```

TLPART=TLPART-256*P45**4*P15**2+256*P45**5*P15

XTOTAL=XTOTAL+HDPART*TLPART

HDPART=EVL

TLPART=-256*P34*P35*P45*P15-256*P34*P45**2*P15-512*P35*P45**2*P

1 15-256*P35**2*P45*P15-256*P45**3*P15

XTOTAL=XTOTAL+HDPART*TLPART

PLG=XTOTAL

RETURN

END

*end

C Expression PLH

```

      COMPLEX FUNCTION PLH(P13,P14,P15,P34,P35,P45,EVL)
      REAL P13,P14,P15,P34,P35,P45
      COMPLEX EVL
      COMPLEX TLPART,HDPART,XTOTAL

```

XTOTAL=0.

HDPART=1

```

      TLPART=-256*P34*P35*P45*P14*P15**2+256*P34*P35*P45*P15**3-512*P
1 34*P35*P45**2*P14*P15-768*P34*P35*P45**2*P15**2+512*P34*P35*P45
1 **3*P15-256*P34*P35**2*P45*P14*P15-256*P34*P35**2*P45*P15**2+25
1 6*P34*P35**2*P45**2*P15-512*P34*P35**2*P14*P15**2+256*P34*P35**
1 2*P15**3+256*P34*P45**2*P14*P15**2+256*P34*P45**2*P15**3-256*P3
1 4*P45**3*P14*P15-512*P34*P45**3*P15**2+256*P34*P45**4*P15-256*P
1 34**2*P35*P45*P14*P15-1024*P34**2*P35*P45*P15**2+512*P34**2*P35
1 *P45**2*P15-256*P34**2*P35*P14*P15**2+512*P34**2*P35*P15**3+256
1 *P34**2*P45*P14*P15**2

```

TLPART=TLPART+512*P34**2*P45*P15**3-256*P34**2*P45**2*P14*P15-1

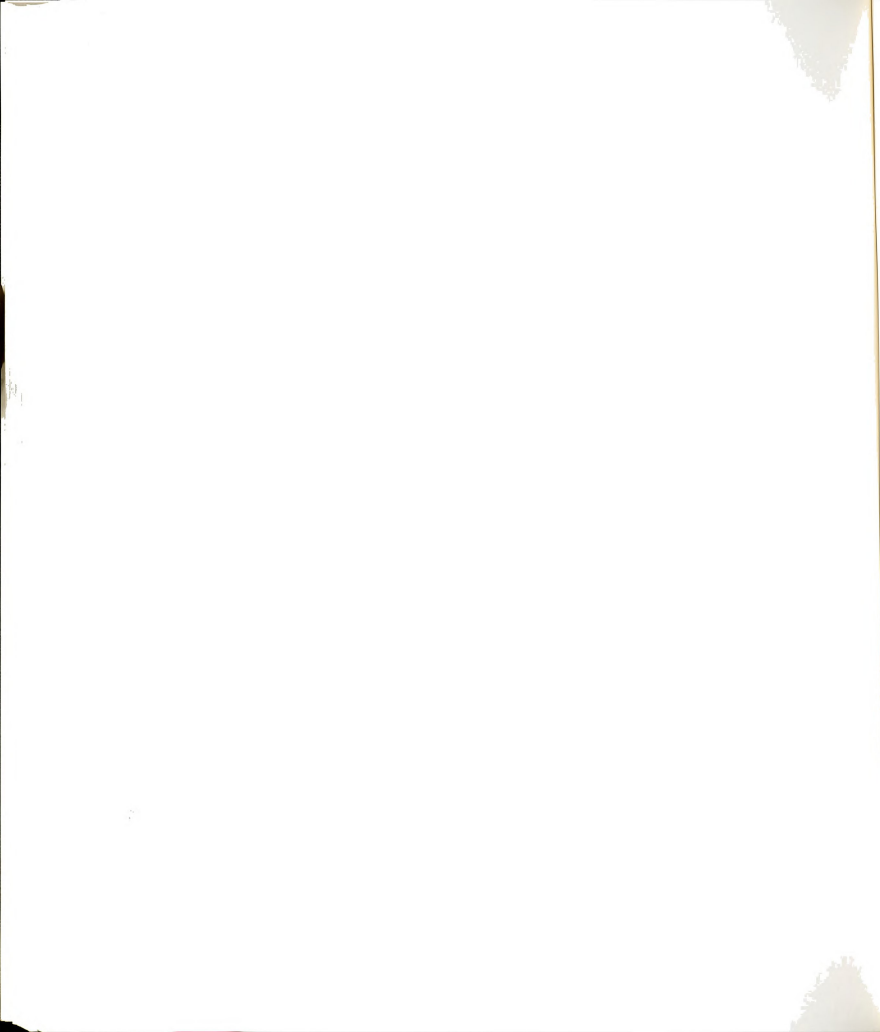
1 024*P34**2*P45**2*P15**2+512*P34**2*P45**3*P15-512*P34**3*P45*P

1 15**2+256*P34**3*P45**2*P15+256*P34**3*P15**3-256*P35*P45**2*P1

1 4*P15**2-256*P35*P45**2*P15**3+256*P35*P45**3*P15**2-512*P35**2

1 *P45*P14*P15**2-256*P35**2*P45*P15**3+512*P35**2*P45**2*P15**2+

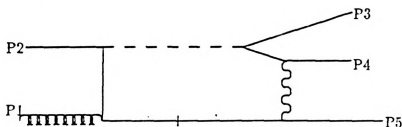
```
1 256*P35**3*P45*P15**2-256*P35**3*P14*P15**2
  XTOTAL=XTOTAL+HDPART*TLPART
  HDPART=EVL
  TLPART=-256*P34*P35*P45*P15+512*P34*P35*P15**2+256*P34*P45*P15*
1 *2-256*P34*P45**2*P15-256*P34**2*P45*P15+256*P34**2*P15**2+256*
1 P35*P45*P15**2+256*P35**2*P15**2
  XTOTAL=XTOTAL+HDPART*TLPART
  PLH=XTOTAL
  RETURN
  END
```



Appendix P

Ascertaining the Physical Range of Parameters

All interacting systems of two particles in and three particles out possess six degrees of freedom- five continuous ones and a plus-minus degree which represents the reflection of the three output particles across the plane defined by the two input particles.



It is common to work in the center of mass system and represent the degrees of freedom by the energies and directions of the particles. This representation is described here though not used. For this calculation it is more appropriate to work in the rest frame of $\bar{P}4$ and $\bar{P}5$, with the degrees of freedom represented by ranges of dot products.

Section 1 ... Center of Mass System

Sub-Section 1.1 ...Fixing the Parameters

For the input particles, one parameter E_{cm} fixes their values:

$$P1^\mu = (0, 0, \frac{E_{cm}}{2}, \frac{E_{cm}}{2})$$

$$P2^\mu = (0, 0, -\frac{E_{cm}}{2}, \frac{E_{cm}}{2})$$

(The assignment of the minus sign to one of the input particle's momentum is the fixing of the plus-minus degree of freedom.)

For the output particles, three parameters fix the directions:

Pick particle 4 to be in the x-z plane:

$$P4^\mu = [E4 \sin(\theta_4), 0, E4 \cos(\theta_4), E4]$$

and θ_4 becomes the second parameter.

Then θ_5 and ϕ_5 become the third and fourth parameters, fixing the direction of $P5^\mu$.

$$P5^\mu = [E5 \sin(\theta_5) \cos(\phi_5), E5 \sin(\theta_5) \sin(\phi_5), E5 \cos(\theta_5), E5]$$

and $P3$ becomes fixed by energy-momentum conservation:

$$P3^\mu = [-(P4_x + P5_x), -(P4_y + P5_y), -(P4_z + P5_z), (E_{cm} - E4 - E5)]$$

Now select $E3$ as the fifth and last parameter.

Then $E4$ and $E5$ are determined by the simultaneous solution of:

$$E4 + E5 = E_{cm} - E3$$

and the zero mass of $P3$

$$P3^2 = 0 \Rightarrow$$

$$E3^2 = [E4 \sin(\theta_4) + E5 \sin(\theta_5) \cos(\phi_5)]^2 + [E5 \sin(\theta_5) \sin(\phi_5)]^2 + [E4 \cos(\theta_4) + E5 \cos(\theta_5)]^2$$

Sub-Section 1.2 ... The Center of Mass Outgoing Polar Angles

To find the polar angles made by the three outgoing particles in the center of mass frame of reference system, first note, taking particle 3 as an example, that:

$$(P1 + P2) \cdot P3 = -(E1 + E2)E3 = -2(E1)E3$$

but also:

$$(P1 + P2) \cdot P3 = (P3 + P4 + P5) \cdot P3 = P3P4 + P3P5$$

giving:

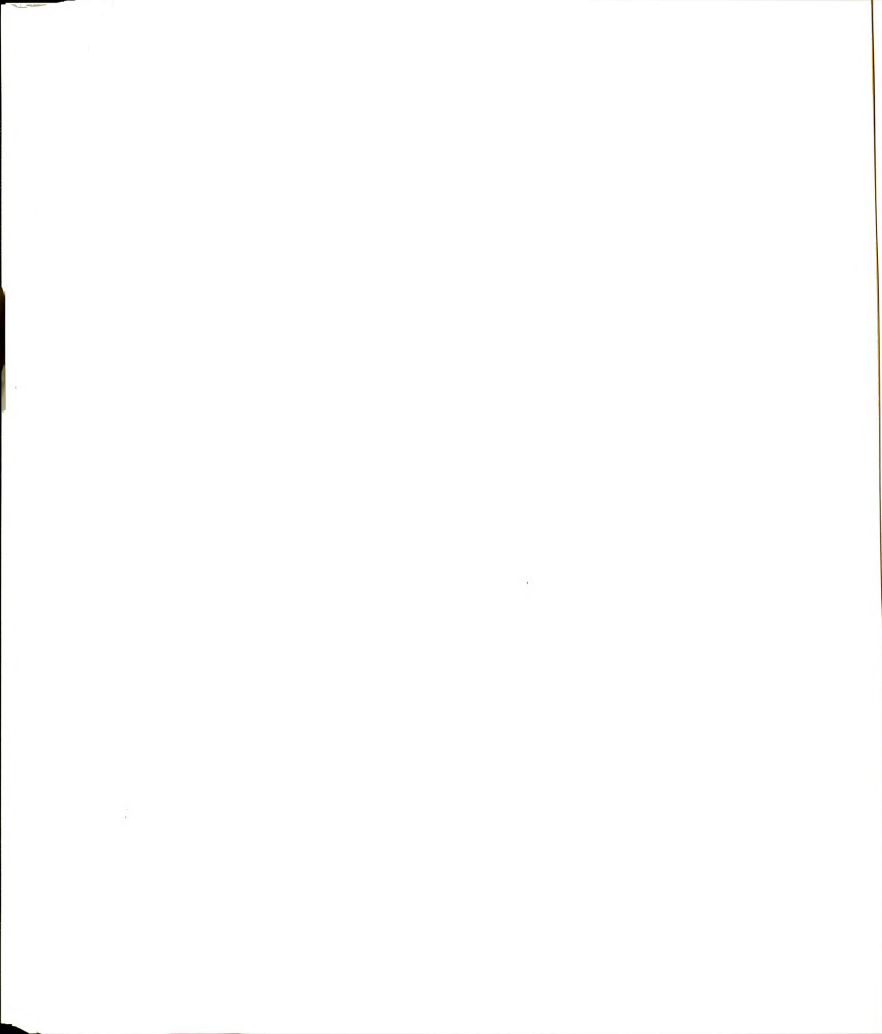
$$(E1)E3 = \frac{P3P4 + P3P5}{-2}$$

Then to find $\cos \theta_3$:

$$\overline{P1} \cdot \overline{P3} = |P1| |P3| \cos \theta_3 = (E1) E3 \cos \theta_3$$

but also:

$$\overline{P1} \cdot \overline{P3} = P1P3 + (E1)E3$$



giving:

$$\cos \theta_3 = 1 + \frac{P_1 P_3}{(E_1) E_3}$$

then using the above for $(E_1) E_3$:

$$\cos \theta_3 = 1 - \frac{2(P_1 P_3)}{P_3 P_4 + P_3 P_5}$$

and similarly for $\cos \theta_4$ and $\cos \theta_5$:

$$\cos \theta_4 = 1 - \frac{2(P_1 P_4)}{P_3 P_4 + P_4 P_5}$$

$$\cos \theta_5 = 1 - \frac{2(P_1 P_5)}{P_3 P_5 + P_4 P_5}$$

Section 2 ... Another System

For this problem the chosen set is:

the five continuous-

$$\begin{aligned} P_1 P_4 \\ P_1 P_5 \\ P_3 P_4 \\ P_3 P_5 \\ P_4 P_5 \end{aligned}$$

and the plus-minus degree-

$$\text{the sign of } E p f(P_1, P_3, P_4, P_5)$$

As these are all Lorentz invariants, the combinations of values that are physically allowable may be ascertained in any convenient reference frame and co-ordinate system. The reference frame chosen here is the usual one for this problem- the rest frame of P_4 and P_5 . The co-ordinate system is with P_4 taken as the plus z direction and P_3 in the x-z plane.

The four vector P_2 is removable by energy-momentum conservation

$$P_2 = P_3 + P_4 + P_5 - P_1$$

The remaining four vectors- P_1, P_3, P_4, P_5 - have their components evaluated

for this co-ordinate system in the solutions to Integrals I5, I8, and I18. in Appendix N.

The results are- using as usual $ko = \sqrt{-P4P5/2}$:

$$P4 = (0, 0, +ko, ko)$$

$$P5 = (0, 0, -ko, ko)$$

$$P3 = \left[\frac{\sqrt{(P3P4)(P3P5)}}{ko}, 0, \frac{(P3P4 - P3P5)}{2ko}, -\frac{(P3P4 + P3P5)}{2ko} \right]$$

$$P1 = \left[P1_x, P1_y, \frac{(P1P4 - P1P5)}{2ko}, -\frac{(P1P4 + P1P5)}{2ko} \right]$$

where

$$P1_x = \pm \left(\frac{(P1P4)(P3P5) + (P1P5)(P3P4) - (P1P3)(P4P5)}{2ko\sqrt{(P3P4)(P3P5)}} \right)$$

$$P1_y^2 = \frac{4(P1P4)(P3P5)(P1P5)(P3P4) - [(P1P4)(P3P5) + (P1P5)(P3P4) - (P1P3)(P4P5)]^2}{4ko^2(P3P4)(P3P5)}$$

$P1P3$ appears in the above results. It is obtained from the above expression for $P2$ by squaring and noting the particles have zero mass-

$$P2^2 = 0 = P3P4 + P3P5 + P4P5 - P1P3 - P1P4 - P1P5$$

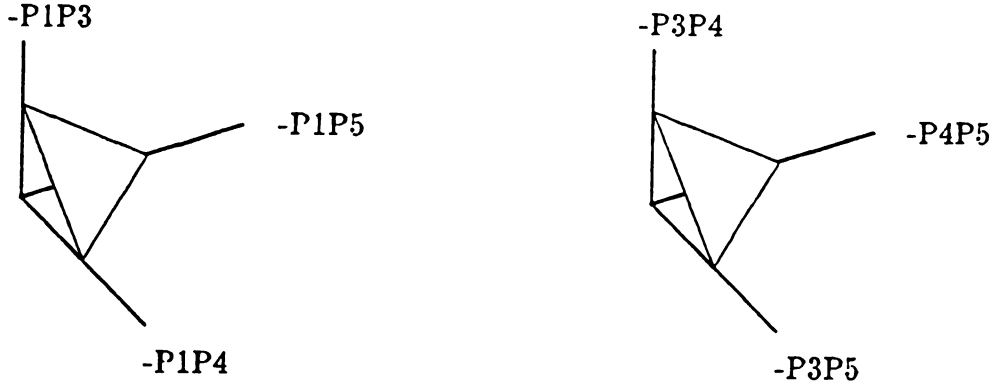
Selecting a value for the Center of Mass energy imposes a restriction on the set of dot products

$$(P1 + P2)^2 = (P3 + P4 + P5)^2 = -E_{cm}^2$$

and removing $P2$ as usual-

$$P1P3 + P1P4 + P1P5 = P3P4 + P3P5 + P4P5 = -E_{cm}^2/2$$

The allowable values may be visualized as lying on triangular surfaces



where each corner of the triangles intersects the axis at $\frac{E_{cm}^2}{2}$.

These points are generateable in the output FORTRAN program by allowing

$$0 < -P1P4 < \frac{E_{cm}^2}{2}$$

$$0 < -P1P5 < \frac{E_{cm}^2}{2} - (-P1P4)$$

$$-P1P3 = \frac{E_{cm}^2}{2} - (-P1P4) - (-P1P5)$$

and

$$0 < -P3P4 < \frac{E_{cm}^2}{2}$$

$$0 < -P3P5 < \frac{E_{cm}^2}{2} - (-P3P4)$$

$$-P4P5 = \frac{E_{cm}^2}{2} - (-P3P4) - (-P3P5)$$

These possibilities are then restricted by the requirement that $P1_y$ be real, which is that-

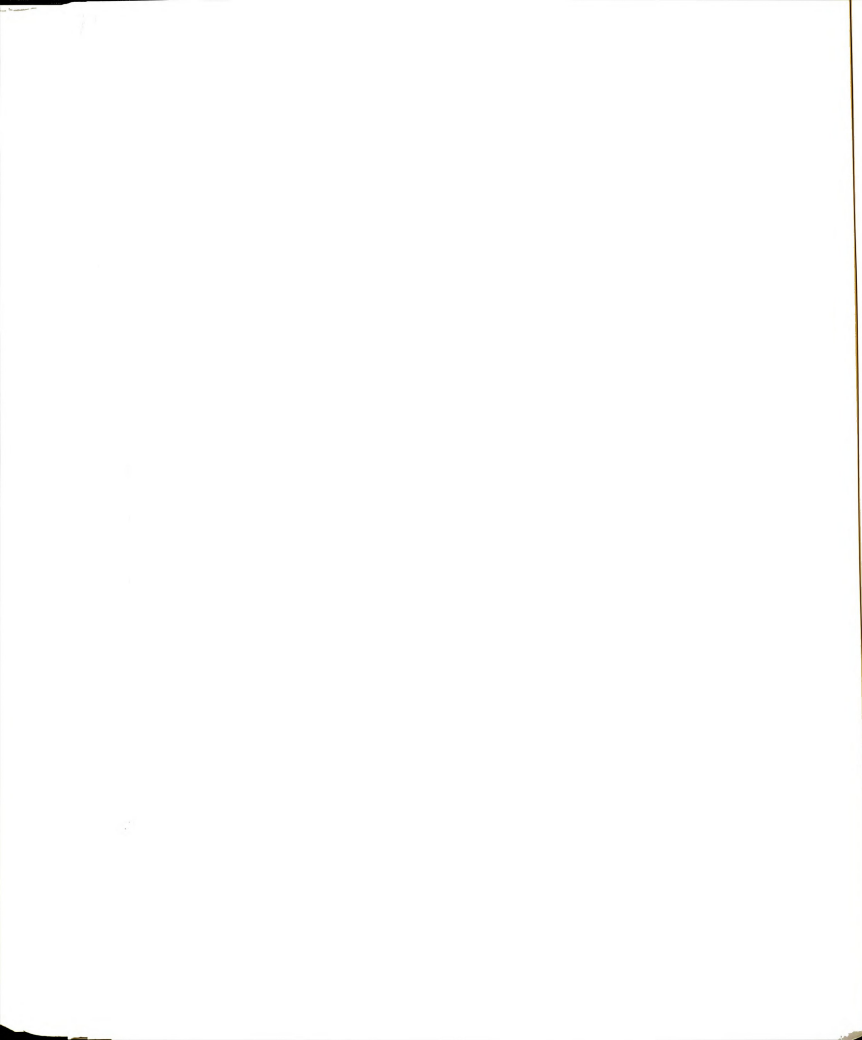
$$4(P1P4)(P3P5)(P1P5)(P3P4) > [(P1P4)(P3P5) + (P1P5)(P3P4) - (P1P3)(P4P5)]^2$$

A set of six dot products which is on the two triangles and also obeys this restriction may be regarded as a physically realizable point and processed.

The remaining degree of freedom, the plus-minus sign appears as that sign



which is selected for $\sqrt{P1_y^2}$, (with $P2_y$ picking up the opposing sign). As $Epf(P1, P3, P4, P5)$, evaluated in Appendix B in this same reference frame and co-ordinate system is equal to $i(2ko^2)(P3_r)(P1_y)$, the sign of $P1_y$ enters the calculation as the sign of $Epf(P1, P3, P4, P5)$.



Appendix Q

Verification of Spinor Replacement

Section 1 Introduction

In Appendix H it is demonstrated that a spinor product of the form $v(p)\bar{u}(q)$ may be replaced by $(1 \pm \gamma_5)\gamma \cdot A$ where A is a four vector, and is given by

$$A_\mu = \frac{(p \cdot s)q_\mu + (q \cdot s)p_\mu - (p \cdot q)s_\mu \pm \text{Epf}(\mu, q, p, s)}{\sqrt{16(p \cdot s)(q \cdot s)}}$$

where s_μ is an arbitrary four vector of the problem, and the sign of γ_5 is determined by the combination of helicities.

This permits the substitutions:

$$v(P3)\bar{u}(P4) = (1 \pm \gamma_5)\gamma \cdot B$$

and

$$v(P2)\bar{u}(P5) = (1 \pm \gamma_5)\gamma \cdot CU$$

where

$$B_\mu = \frac{(P3P5)P4_\mu + (P4P5)P3_\mu - (P3P4)P5_\mu \pm \text{Epf}(\mu, P3, P4, P5)}{\sqrt{16(P4P5)(P3P5)}}$$

and

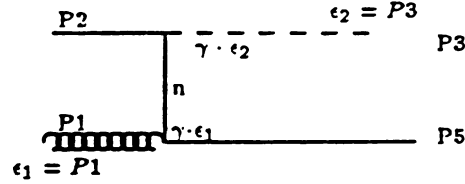
$$CU_\mu = \frac{(P1P2)P5_\mu + (P1P5)P2_\mu - (P2P5)P1_\mu \pm \text{Epf}(\mu, P1, P2, P5)}{\sqrt{16(P1P2)(P1P5)}}$$

which enable the SCHOONSCHIP program of Appendix I to easily calculate the TRACES.

Here this result is checked in two examples by squaring the amplitude it leads to and comparing it to the cross section obtained by the standard method spin sum replacement of: $u(p)\bar{u}(p) = \gamma \cdot p$

Section 2 Example 1

Consider the following simplistic diagram in which the polarization of the incident gluon and the exiting Z^0 are simply set equal to their linear momenta:



Overall energy momenta conservation dictates:

$$P1 + P2 = P3 + P5$$

which gives immediately, using zero mass particles:

$$P1P2 = P3P5$$

$$P2P3 = P1P5$$

$$P1P3 = P2P5$$

$$P2P5 = P3P3 - P1P5$$

and the amplitude is proportional to:

$$\bar{u}(P5) \gamma \cdot \epsilon_1 \gamma \cdot n \gamma \cdot \epsilon_2 v(P2)$$

Then using

$$(\gamma \cdot \epsilon_1)(\gamma \cdot n) = (\gamma \cdot P1) \gamma \cdot (P1 - P5) = -[(\gamma \cdot P1)(\gamma \cdot P5)] = [(\gamma \cdot P5)(\gamma \cdot P1) - P1P5]$$

gives:

$$\bar{u}(P5) [(\gamma \cdot P5)(\gamma \cdot P1) - P1P5] (\gamma \cdot \epsilon_2) v(P2)$$

and because of the Dirac equation $\bar{u}(P5) \gamma \cdot P5 = 0$ becomes:

$$-(P1P5) \text{ TRAC'E } [v(P2) \bar{u}(P5) \gamma \cdot \epsilon_2]$$

and using the above definition of CU :

$$-(P1P5) \text{ TRAC'E } [(1 + \gamma_5)(\gamma \cdot CU) (\gamma \cdot \epsilon_2)]$$

$$= -4(P1P5)[CU \cdot P3]$$

Using the expansion of CU :

$$= \frac{-4(P1P5)[(P1P2)P3P5 + (P1P5)P2P3 - (P2P5)P1P3 \pm Epf(P3, P1, P2, P5)]}{\sqrt{16(P1P2)(P1P5)}}$$

noting $P3 = P1 + P2 - P5$ makes the Epf zero

$$\begin{aligned} &= \frac{-4(P1P5)[(P3P5)^2 + (P1P5)^2 - (P2P5)^2]}{\sqrt{16(P1P2)(P1P5)}} \\ &= \frac{-4(P1P5)[-2(P3P5)(P1P5)]}{\sqrt{16(P1P2)(P1P5)}} \end{aligned}$$

and squaring to produce the cross section

$$Amplitude\epsilon^2 = 4(P1P5)^2(P3P5)P2P3$$

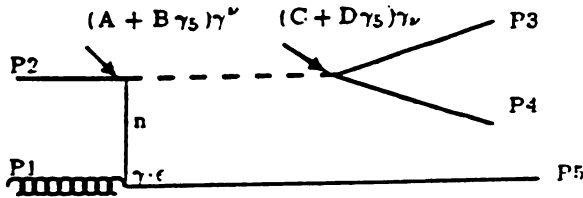
This is to be checked with the spin summed cross section:

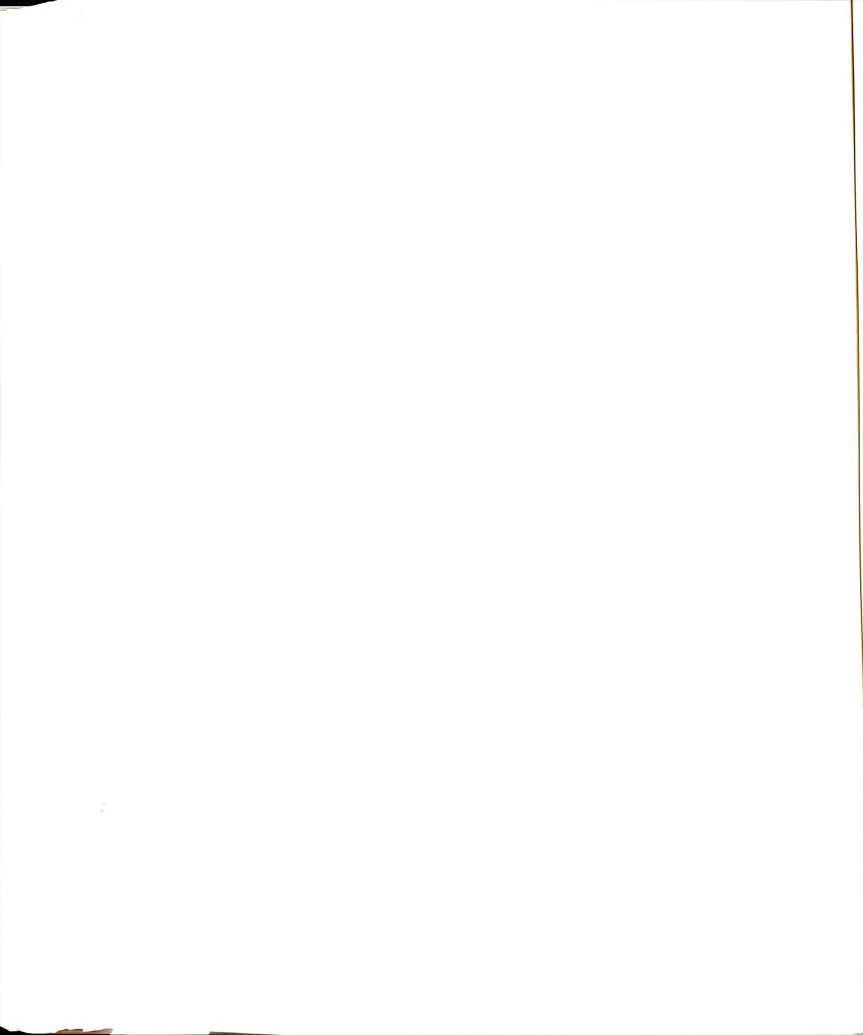
$$\begin{aligned} &\bar{u}(P5) \gamma \cdot \epsilon_1 \gamma \cdot n \gamma \cdot \epsilon_2 v(P2) \bar{v}(P2) \gamma \cdot \epsilon_2 \gamma \cdot n \gamma \cdot \epsilon_1 u(P5) \\ &= -(P1P5)^2 \text{TRACE} \left[\frac{(1 + \gamma_5)}{2} \gamma \cdot P5 \gamma \cdot P3 \gamma \cdot P2 \gamma \cdot P3 \right] \\ &= 4(P1P5)^2(P3P5)P2P3 \end{aligned}$$

giving agreement.

Section 3 Example 2

Consider a second example :





First evaluate the spin summed cross section.

The quark line yields:

$$\begin{aligned} & \bar{u}(P5) \gamma \cdot \epsilon \gamma \cdot n \gamma^\nu v(P2) \bar{v}(P2) \gamma^\mu \gamma \cdot n \gamma \cdot \epsilon^* u(P5) \\ &= \text{TRACE}[u(P5) \bar{u}(P5) \gamma \cdot \epsilon \gamma \cdot n \gamma^\nu \frac{(1+\gamma_5)}{2} (\gamma \cdot P2) \gamma^\mu \gamma \cdot n \gamma \cdot \epsilon^*] \end{aligned}$$

then using the Example 1 relationship between ϵ , n , and the Dirac equation:

$$\begin{aligned} &= \frac{4(P1P5)^2}{2} \text{TRACE}[(1+\gamma_5) \gamma \cdot P5 \gamma^\nu \gamma \cdot P2 \gamma^\mu] \\ &= \frac{4(P1P5)^2}{2} [4(P5_\nu P2_\mu + P5_\mu P2_\nu - (P2P5)g_{\mu\nu}) - 4 \text{Epf}(P5, \mu, P2, \nu)] \end{aligned}$$

and from the lepton line:

$$\begin{aligned} & \bar{u}(P4) \gamma_\nu v(P3) \bar{v}(P3) \gamma_\mu u(P4) \\ &= \frac{1}{2} \text{TRACE}[(1+\gamma_5) \gamma \cdot P4 \gamma_\nu \gamma \cdot P3 \gamma] \\ &= \frac{1}{2} (4) [4(P4^\nu P3^\mu + P4^\mu P3^\nu - (P3P4)g^{\mu\nu}) - 4 \text{Epf}(P4, \mu, P3, \nu)] \end{aligned}$$

and the product:

$$\begin{aligned} &= 16(P1P5)^2 \{ (P4P5)P2P3 + P2P4)P3P5 - P3P4(P2P5) - \text{Epf}(P4, P2, P3, P5) \\ & \quad (P3P5)P2P4 + (P2P5)P4P5 - P3P4(P2P5) - \text{Epf}(P4, P4, P3, P2) \\ & \quad - (P3P4)P2P5 - (P3P4)P2P5 + 4(P3P4)P2P5 + 0 \\ & \quad - \text{Epf}(P5, P3, P2, P4) - \text{Epf}(P5, P4, P2, P3) + 0 \\ & \quad + [\text{Epf}(P5, \mu, P2, \nu)] * [\text{Epf}(P4, \mu, P3, \nu)] \} \\ &= 16(P1P5)^2 [2(P4P5)P2P3 + 2(P2P4)P3P5 + 2 \left| \begin{matrix} P4P5 & P3P5 \\ P2P4 & P2P3 \end{matrix} \right|] \\ &= 16(P1P5)^2 4(P4P5)P2P3 \end{aligned}$$

Now calculate and compare the amplitude squared.

$$\begin{aligned} & \bar{u}(P4) \gamma \cdot \epsilon \gamma^\nu v(P3) \bar{v}(P5) \gamma \cdot \epsilon^* \gamma \cdot n \gamma_\mu v(P2) \\ &= \text{TRACE}[v(P3) \bar{v}(P4) \gamma \cdot \epsilon \gamma^\nu] \text{TRACE}[v(P2) \bar{u}(P5) \gamma \cdot \epsilon^* \gamma \cdot n \gamma_\mu] \end{aligned}$$

$$\begin{aligned}
&= \text{TRACE} [(1 + \gamma_5) \gamma \cdot B \gamma^\nu] 2(P1P5) \text{TRACE} [(1 + \gamma_5) \gamma \cdot CU \gamma_\mu] \\
&= 4(B^\nu)(P1P5) 4(CU_\nu)
\end{aligned}$$

Then using

$$B_\mu = \frac{(P3P5)P4_\mu + (P4P5)P3_\mu - (P3P4)P5_\mu \pm \text{Epf}(\mu, P3, P4, P5)}{\sqrt{16(P4P5)(P3P5)}}$$

and a slightly different CU

$$CU_\mu = \frac{(P2P3)P5_\mu + (P3P5)P2_\mu - (P2P5)P3_\mu - \text{Epf}(\mu, P3, P2, P5)}{\sqrt{16(P1P3)(P3P5)}}$$

with

$$\begin{aligned}
B \cdot CU &= 0 + P4P5(P3P5)P2P3 + P4P5(P2P3)P3P5 + 0 \\
&\quad - P3P5(P2P5)P3P4 + P3P5^2(P2P4) + P2P3(P3P5)P4P5 \\
&\quad - P3P5 [\text{Epf}(P4, P3, P2, P5)] - P3P5 [\text{Epf}(P2, P3, P4, P5)] \\
&\quad - [\text{Epf}(\mu, P3, P4, P5)] * [\text{Epf}(\mu, P2, P3, P5)]
\end{aligned}$$

$$\begin{aligned}
&= 3(P4P5)P2P3(P3P5) - P3P5(P2P5)P3P4 \\
&\quad + P3P5^2(P2P4) - \begin{vmatrix} P2P3 & P2P4 & P2P5 \\ P3^2 & P3P4 & P3P5 \\ P3P5 & P4P5 & P5^2 \end{vmatrix}
\end{aligned}$$

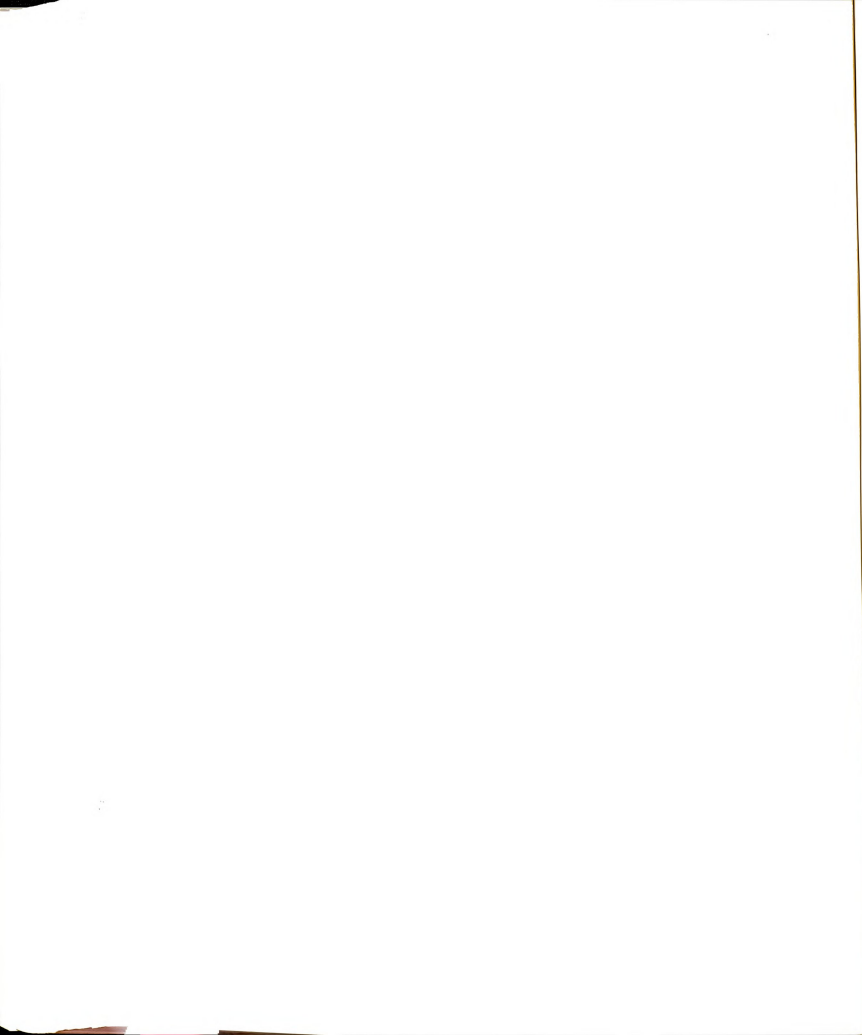
all over appropriate denominator,

$$\begin{aligned}
&= \frac{4(P4P5)P2P3(P3P5)}{\sqrt{16(P3P4)(P3P5)}\sqrt{16(P1P3)(P3P5)}} \\
&= 4(16)(P1P5)^2(P4P5)(P2P3)
\end{aligned}$$

giving agreement.

Now do again using the regular projection of CU:

$$CU_\mu = \frac{(P1P2)P5_\mu + (P1P5)P2_\mu - (P2P5)P1_\mu \pm \text{Epf}(\mu, P1, P2, P5)}{\sqrt{16(P1P2)(P1P5)}}$$



This will give a result that SCHOONSHIP will be asked to calculate the final step of.

$$\begin{aligned}
B \cdot CU &= \\
&-P_4P_5(P_2P_5)P_1P_3 + P_4P_5(P_1P_5)P_2P_3 \\
&+P_4P_5(P_1P_2)P_3P_5 - (P_4P_5) \text{Epf}(P_3, P_1, P_2, P_5) \\
&-P_3P_5(P_2P_5)P_1P_4 + P_3P_5(P_1P_5)P_2P_4 \\
&+P_3P_5(P_1P_2)P_4P_5 - (P_3P_5)\text{Epf}(P_4, P_1, P_2, P_5) \\
&+P_3P_4(P_2P_5)P_1P_5 - P_3P_4(P_1P_5)P_2P_5 + (P_2P_5)\text{Epf}(P_1, P_3, P_4, P_5) \\
&-(P_1P_5)\text{Epf}(P_3, P_3, P_4, P_5) + [\text{Epf}(\mu, P_3, P_4, P_5)] * [\text{Epf}(\mu, P_1, P_2, P_5)] \\
\\
&= -P_4P_5(P_2P_5)P_1P_3 + P_4P_5(P_1P_5)P_2P_3 - P_3P_5(P_1P_5)P_2P_3 \\
&-P_3P_5(P_2P_5)P_1P_4 + P_3P_5(P_1P_5)P_2P_4 + 2(P_4P_5)P_1P_2(P_3P_5) \\
&+(P_2P_5 + P_1P_5 - P_3P_5 + P_4P_5) \text{Epf}(P_1, P_3, P_4, P_5) \\
&\quad + \begin{vmatrix} P_1P_3 & P_1P_4 & P_1P_5 \\ P_2P_3 & P_2P_4 & P_2P_5 \\ P_3P_5 & P_4P_5 & P_5^2 \end{vmatrix} \\
\\
&= -2(P_4P_5)P_2P_5(P_1P_3 + 2(P_4P_5)P_1P_5(P_2P_3) \\
&+2(P_4P_5)P_1P_2(P_3P_5) + 2(P_4P_5) \text{Epf}(P_1, P_3, P_4, P_5) \\
\\
&= \frac{2(P_4P_5) [P_1P_2(P_3P_5) + P_1P_5(P_2P_3) - P_2P_5(P_1P_3) + \text{Epf}(P_1, P_3, P_4, P_5)]}{\sqrt{16(P_1P_2)P_1P_5(16)P_3P_5(P_4P_5)}}
\end{aligned}$$

This is evaluated by the attached SCHOONSHIP program to be:

$$\text{Amplitude}^2 = 64(P_1P_5)^2(P_4P_5)P_2P_3$$

giving agreement.

Appendix R

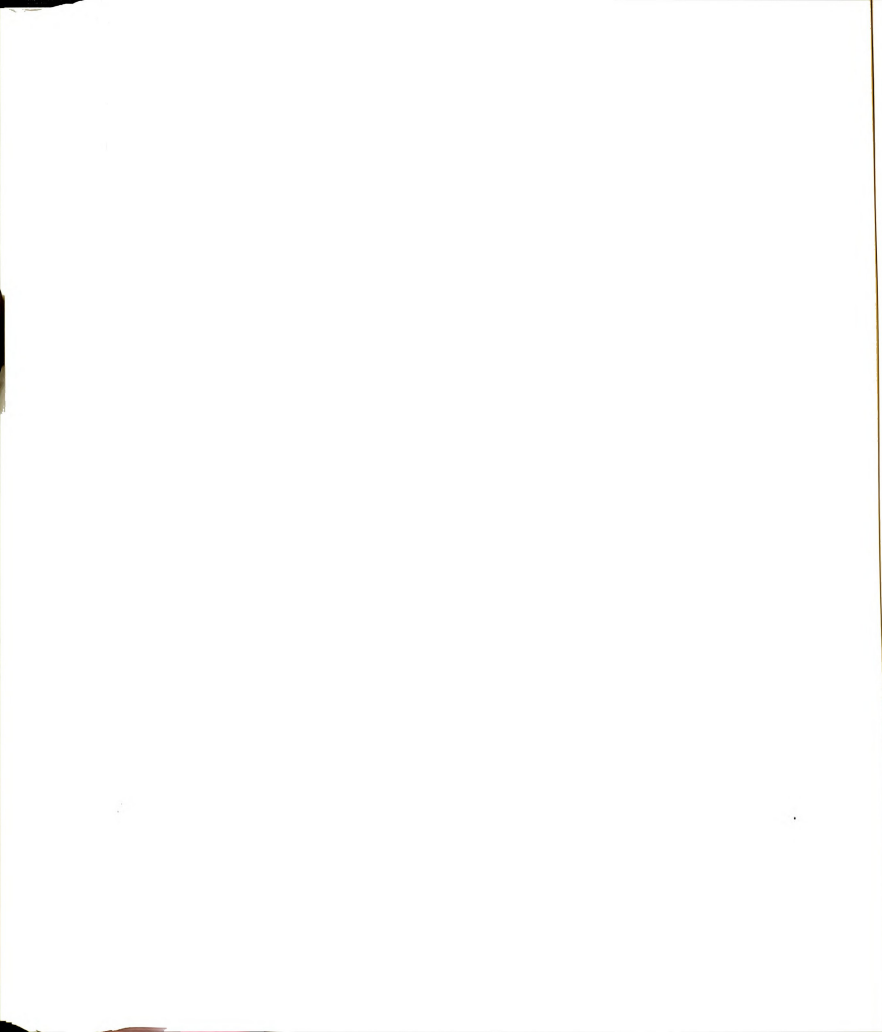
Verification of SCHOONSHIP LOOP Program, TRACE Calculation



```

C
C
C   START  SCHOONSHIP  PROGRAM
C
C   ALGEBRAIC TEST OF LOOP - TRACE PROGRAM
C
C   this program tests the spinor, polarization replacements
C   and the expansion of Ku , by after these replacements
C   have all been made, replacing Ku by an expression that
C   should force a zero result , as per this appendix,
C   if all these replacements have been made correctly.
C
C
S LNMP, LNMQ, PI, FPIIV
A P14, P13, P15, P34, P35, P45
V P1, P2, P3, P4, P5, S1, S2, S3, S4, U1, U2, U3, U4
C
C   FEED IN VALUES FOR THIS RUN
N 20
C
C   SET  SWITCHES
C
X FOUR = 0
X FIVE = 1
X ELECT = 1
X POSIT = 0
X LHEL = +1
X QHEL = +1
X GHEL = +1
X MPARIT = +1
C
P nstat
P ninput
Outlimit, 12000000
P nlist
C   SET DOT PRODUCTS
X P1P3 = ELECT*P13 + POSIT*P14
X P1P4 = ELECT*P14 + POSIT*P13
X P1P5 = P15
X P3P4 = P34
X P3P5 = ELECT*P35 + POSIT*P45
X P4P5 = ELECT*P45 + POSIT*P35
C
C
C   FOR CONVEINENCE , DEFINE SOME EXPRESSIONS AND THEIR RECIPROCALLS.
C
X D145 = P1P4 + P1P5
X D1 = D145
X D345 = P3P4 + P4P5
X MZP = 2*P3P4 + D3
C
C   NOW K CAN ALWAYS BE EXPANDED ON AN ARBRITARY BASIS SET S1, S2, S3, S4
C
C    $K = C(1)*S1 + C(2)*S2 + C(3)*S3 + C(4)*S4$ 
C
C   AND FOR THIS PROBLEM WE USE THE SET P1, P3, P4, P5

```



```

C
C S0....
C
X S1S2 = P1P3
X S1S3 = P1P4
X S1S4 = P1P5
X S2S3 = P3P4
X S2S4 = P3P5
X S3S4 = P4P5
C
C   TO THIS END, THE ORIGINAL EXPANSION OF K MAY BE DOTTED WITH EACH
C   OF THE BASIS TO FORM A 4 X 4 MATRIX, WHICH WHEN INVERTED
C   YIELDS.....
C
C    $C(1) = CC(1,1)*S1DK + CC(1,2)*S2DK + CC(1,3)*S3DK + CC(1,4)*S4DK$ 
C
C   W IS THE DETERMINENT FOR THIS INVERSION AND WIV IT'S RECIPROCAL
C
X W = S1S2*S3S4*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)
    + S1S3*S2S4*( S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)
    + S1S4*S2S3*( S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)
C
C   AND NOW, ALL THE CC'S USING THE NOTATION CC(4,3) = CC43
C
X CC44 = WIV*(-2)*(S1S2*S1S3*S2S3)
X CC43 = WIV*S1S2*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)
X CC42 = WIV*S1S3*( S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)
X CC41 = WIV*S2S3*( S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)
X CC33 = WIV*(-2)*(S1S2*S1S4*S2S4)
X CC32 = WIV*S1S4*( S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)
X CC31 = WIV*S2S4*( S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)
X CC22 = WIV*(-2)*(S1S3*S1S4*S3S4)
X CC21 = WIV*S3S4*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)
X CC11 = WIV*(-2)*(S2S3*S2S4*S3S4)
C
*fix
C
C *****
C
C
C   IN THIS SECTION, TAKE THE TRACE, REPLACE EACH K BY IT'S
C   LINEAR EXPANSION,
C   AND REDUCE TO A FINAL ANSWER.
C
C
V B,K,CU,M,E,N,P1,P2,P3,P4,P5,PD
F C,F0,F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,A1,A2,A3,AA
I J1,J2,M1,M2,K3,K2
C
C
C   SCALE FACTOR FOR THE AMPLITUDE IS ...
C
X FSCAL = 0.5*FPIIV*(1/137)*(FIVE*(P34+P35+P45) + FOUR)
C
C
C   TAKE THE TRACE
C

```

```

Z EXP=FSCAL*F0*F1*F2*F3*F4*F11*F5*F6*F7*F8*F9*F10
C
C   SUBSTITUE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*((1+LHEL)*(G6(J1))+(1-LHEL)*(G7(J1)))
Id,F1=(G(J1,B))
Id,F2=ELECT*(G(J1,M2)) + POSIT*(G(J1,M1))
Id,F3=(G(J1,K))
Id,F4=ELECT*(G(J1,M1)) + POSIT*(G(J1,M2))
Id,F11=0.5*((1+QHEL)*(G6(J2))+(1-QHEL)*(G7(J2)))
Id,F5=(G(J2,CU))
*yes
Id,F6=(G(J2,M2))
Id,F7=(G(J2,M))
Id,F8=FIVE*(G(J2,E)) + FOUR*(G(J2,M1))
Id,F9=(G(J2,N))
Id,F10=FIVE*(G(J2,M1)) + FOUR*(G(J2,E))
*yes
Id,Trick,Trace,J1
C
C   TRICK AND TRACE
C
Id,Trick,Trace,J2
*yes
C
C   START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N(J-) = FIVE*(M(J)+P1(J)) - FOUR*(P3(J)+P4(J)+P5(J))
Al,Func,N(K2-) = FIVE*(M(K2)+P1(K2)) - FOUR*(P3(K2)+P4(K2)+P5(K2))
Id,MDM=0
Id,Dotpr,M(J-)=K(J)-PD(J)
Al,Func,M(J-)=K(J)-PD(J)
Id,KDPD=P4P5
Id,KDK=0
Id,2,Dotpr,PD(J-)=P4(J)+P5(J)
Al,Func,PD(J-)=P4(J)+P5(J)
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P OUTPUT
C Id,2,Dotpr,K(J-) = P1(J)
C Id,2,Func,K(J-) = P1(J)
*yes
C
C   PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEPF*Epf(J,P3,P4,P5)
Al,Func,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEPF*Epf(K3,P3,P4,P5)
C P OUTPUT
*yes
Id,B3 = BOIV*P4P5
Al,B4 = BOIV*P3P5
Al,B5 = -BOIV*P3P4
Al,BEPF = + LHEL*BOIV*(ELECT - POSIT)
Id,Trick
Id,KDK=0
Id,P1DP1=0
Al,P3DP3=0

```

```

Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
Id,2,Dotpr,CU(J~)=
  (CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
  +CUEPF*EpF(J,P1,P3,P5)+CUEPF*EpF(J,P1,P4,P5)
Al,Funct,CU(K~)=
  (CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
  +CUEPF*EpF(K,P1,P3,P5)+CUEPF*EpF(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEPF = + QHEL*CU0IV
*yes
Id,Trick
Id,KDK=0
Al,KDPD=P4P5
Al,EDP1=0
Id,2,Dotpr,PD(J~)=P4(J)+P5(J)
Al,Funct,PD(K2~)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
C
C   NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J~)= E1*P1(J) + E3*P3(J) +
  E4*P4(J)+E5*P5(J)+EEPF*(EpF(J,P1,P3,P5)+EpF(J,P1,P4,P5))
Al,Funct,E(K~)= E1*P1(K) + E3*P3(K) +
  E4*P4(K)+E5*P5(K)+EEPF*(EpF(K,P1,P3,P5)+EpF(K,P1,P4,P5))
Id,E1 = EOIV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = EOIV*GHEL*P1P5
Al,E4 = EOIV*GHEL*P1P5
Al,E5 = -EOIV*GHEL*(P1P3+P1P4)
Al,EEPF = -EOIV
*yes
Id,Trick
Id,KDK=0
Id,EpF(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0

```



```

Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
C
C   REPLACE K BY IT'S LINEAR EXPANSION.
C
Id,P5DK = -P4DK + P45
C
Id,2,Funct,K(J-)=C(1)*P1(J)+C(2)*P3(J)+C(3)*P4(J)
      +C(4)*P5(J)
*yes
Id,Trick
Id,Epf(P1,P3,P4,P5) = EVL
C
C P OUTPUT
*yes
Id,C(1) = CC11*P1DK+CC21*P3DK+(CC31-CC41)*P4DK+CC41*P4P5
Al,C(2) = CC21*P1DK+CC22*P3DK+(CC32-CC42)*P4DK+CC42*P4P5
Al,C(3) = CC31*P1DK+CC32*P3DK+(CC33-CC43)*P4DK+CC43*P4P5
Al,C(4) = CC41*P1DK+CC42*P3DK+(CC43-CC44)*P4DK+CC44*P4P5
C P OUTPUT
*yes
Id,Epf(P1,P3,P4,P5) = EVL
B WIV,CU0IV,PI,FPIIV,BOIV,E0IV,EVL
Keep EXP
*next
C
C   now begin the replacement of Ku by that which should force a zero.
C
Z exp0 = D0*EXP
Id,Ainbe,D0*WIV = 1
Id,D0 = W
B D145IV,CU0IV,PI,FPIIV,BOIV,E0IV,EVL
Keep exp0
*next
C
Z exp00 = exp0
Id,2,Dotpr,K(J-) = (P4(J) + P5(J)) - D145IV*P4P5*P1(J)
C Id,2,Funct,K(J-) = (P4(J) + P5(J)) - D145IV*P4P5*P1(J)
Id,Epf(P1,P3,P4,P5) = EVL
Id,P1DP1 = 0
Al,P1DP3 = P1P3
Al,P1DP4 = P1P4
Al,P1DP5 = P1P5
Al,P3DP4 = P3P4
Al,P3DP5 = P3P5
Al,P4DP5 = P4P5
Al,P1DP1 = 0
Al,P3DP3 = 0
Al,P4DP4 = 0
Al,P5DP5 = 0
Id,P13 = P34 + P35 + P45 -P14 - P15

```

```

Id,Epf(P1,P3,P4,P5) = EVL
B D145IV,CUOIV,PI,FPIIV,BoIV,EoIV,EVL
Keep exp00
*next
Z exp1 = D0*exp00
Id,Ainbe,D0*D145IV^2 = 1
Id,D0 = P14*P14 + 2*P14*P15 +P15*P15
B D145IV,CUOIV,PI,FPIIV,BoIV,EoIV,EVL
Keep exp1
*next
Z exp2 = D0*exp1
Id,Ainbe,D0*D145IV = 1
Id,D0 = P14 +P15
B D145IV,CUOIV,PI,FPIIV,BoIV,EoIV,EVL
*end
C
C the test is now finished.
C the remainder of the program is not
C subject to this test as all Ku's will
C now be replaced by A and B expressions.
C
C

```

Appendix S

Verification of Divergence Regulation Co-efficient Cancellation



```

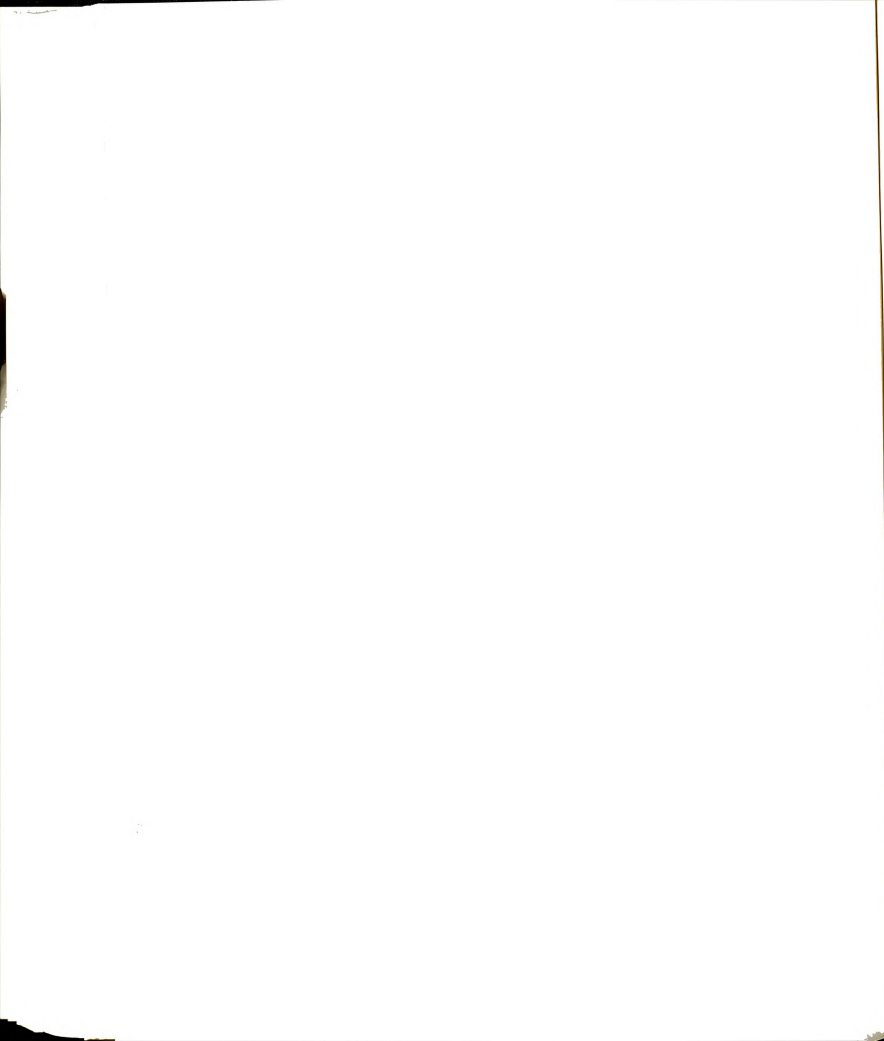
C
C
C   START SCHOONSHIP PROGRAM
C
C   CHECK DIVERGENCE CO-EFFICIENT
C   CANCELLATION
C
C
C   THIS PROGRAM CHECKS THE SCHOONSHIP LOOP
C   PROGRAM BY COMPUTING A NUMERICAL RESULT
C   FOR AN ARBITRARILY CHOSEN SET OF PARAMETERS.
C
C   IT CALCULATES AND USES THE NUMERICAL
C   VALUE OF THE 19 INTEGRALS FOR THIS
C   SET OF NUMERICAL INPUT.
C
C   IT LOOKS FOR THE COEFFICIENT OF THE
C   LOG PHOTON MASS AND THE COEFFICIENT
C   OF THE LOG QUARK MASS TO COME THROUGH
C   THE COMPLICATED CALCULATION AS ZERO,
C   INDICATING THAT THE TRACE CALCULATION
C   AND VARIOUS SUBSTITUTIONS, AND AT
C   LEAST THE DIVERGENT PART OF THE 19
C   INTEGRALS HAVE BEEN CODED CORRECTLY.
C
C
S LNMP,LNMQ,PI,FPIIV
V P1,P2,P3,P4,P5,S1,S2,S3,S4,U1,U2,U3,U4
X D3 = 93.7*93.7 - 2.7*93.7*i
C
C   FEED IN VALUES FOR THIS RUN
N 30,R0
X P13 = -0.5*66.0605782596549785908571076
      *66.0605782596549785908571076
X P14 = -0.5*70*70
X P15 = -0.5*15*15
X P34 = -0.5*92*92
X P35 = -0.5*20*20
X P45 = -0.5*25*25
C X P13 = P34 + P35 + P45 - P14 - P15
C
C   SET SWITCHES
C
X FOUR = 0
X FIVE = 1
X ELECT = 1
X POSIT = 0
X LHEL = +1
X QHEL = +1
X GHEL = +1
X MPARIT = +1
C
P nstat
P ninput
P nlist
C   SET DOT PRODUCTS
X P1P3 = ELECT*P13 + POSIT*P14
X P1P4 = ELECT*P14 + POSIT*P13

```

```

X P1P5 = P15
X P3P4 = P34
X P3P5 = ELECT*P35 + POSIT*P45
X P4P5 = ELECT*P45 + POSIT*P35
C
C
C   RECIPROCAL
C
X P13IV = ((P1P3)**-1)
X P14IV = ((P1P4)**-1)
X P15IV = ((P1P5)**-1)
X P34IV = ((P3P4)**-1)
X P35IV = ((P3P5)**-1)
X P45IV = ((P4P5)**-1)
C
C   FOR CONVEINENCE , DEFINE SOME EXPRESSIONS AND THEIR RECIPROCAL.
C
X K02 = 0.5*(-P4P5)
X K02IV = ((K02)**-1)
X D145 = P1P4 + P1P5
X D1 = D145
X D145IV = ((D145)**-1)
X D345 = P3P4 + P4P5
X D345IV = ((D345)**-1)
X MZP = 2*P3P4 + D3
C
C
C   NOW K CAN ALWAYS BE EXPANDED ON AN ARBRITARY BASIS SET S1,S2,S3,S4
C
C   K = C(1)*S1 + C(2)*S2 + C(3)*S3 + C(4)*S4
C
C   AND FOR THIS PROBLEM WE USE THE SET P1,P3,P4,P5
C
C   S0....
C
X S1S2 = P1P3
X S1S3 = P1P4
X S1S4 = P1P5
X S2S3 = P3P4
X S2S4 = P3P5
X S3S4 = P4P5
C
C   TO THIS END, THE ORIGINAL EXPANSION OF K MAY BE DOTTED WITH EACH
C   OF THE BASIS TO FORM A 4 X 4 MATRIX, WHICH WHEN INVERTED
C   YIELDS.....
C
C   C(1) = CC(1,1)*S1DK + CC(1,2)*S2DK + CC(1,3)*S3DK + CC(1,4)*S4DK
C
C   W IS THE DETERMINENT FOR THIS INVERSION AND WIV IT'S RECIPROCAL
C
X W = S1S2*S3S4*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)
  + S1S3*S2S4*( S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)
  + S1S4*S2S3*( S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)
X WIV = ((W)**-1)
C
C   AND NOW, ALL THE CC'S USING THE NOTATION CC(4,3) = CC43
C
X CC44 = WIV*(-2)*(S1S2*S1S3*S2S3)

```



```

X CC43 = WIV*S1S2*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)
X CC42 = WIV*S1S3*( S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)
X CC41 = WIV*S2S3*( S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)
X CC33 = WIV*(-2)*(S1S2*S1S4*S2S4)
X CC32 = WIV*S1S4*( S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)
X CC31 = WIV*S2S4*( S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)
X CC22 = WIV*(-2)*(S1S3*S1S4*S3S4)
X CC21 = WIV*S3S4*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)
X CC11 = WIV*(-2)*(S2S3*S2S4*S3S4)
C
*fix
C
C THE NINETEEN INTEGRALS WILL NOW BE EVALUATED.
C THEY WILL MAKE USE OF SIX LOGS WHICH ARE ...
C
C LN1 = LN(-2*P4P5)
C LN2 = LN(-2*(P1P4+P1P5))
C LN3 = LN((2*P3P5+MZP)/D3)
C LN4 = LN((P1P4+P1P5)/P1P5))
C LN5 = LN(MZP/(2*P3P5+MZP))
C LN6 = LN((2*P3P5+MZP)*(P1P4+P1P5)/R)
C WHERE R = -2*P4P5*P1P3+2*P3P5*(P1P4+P1P5)+MZP*(P1P4+P1P5)
C
C
C
C SET LOG VALUES
C
Z LN1 = ELECT*(6.43775164973640149840303733)
+ POSIT*(5.99146454710798198687044715)
Z LN2 = ELECT*(8.54188580400660892766904137)
+ POSIT*(8.43141741439483350344920756)
Z LN3 = ELECT*(-3.494604862965 - 1.863668333657*i)
+ POSIT*(-3.090158418852 - 2.427214215569*i)
Z LN4 = ELECT*(3.12578540180218879567703223)
+ POSIT*(3.01531701219041337145719842)
Z LN5 = ELECT*(.4167792613628 + 1.216889362411*i)
+ POSIT*(.01233281724963 + 1.780435244322*i)
Z LN6 = ELECT*(-.6569902501244 - 1.378299363866*i)
+ POSIT*(-.3589882796597 - 1.320996180611*i)
C
Keep LN1,LN2,LN3,LN4,LN5,LN6
*next
C
Z I1 = -2*PI*P4SIV*(LN1 - LNMP)
Z I2 = -2*PI*D14SIV*(LN2 - LNMQ)
Z I3 = +2*PI*D34SIV*LN3
C
Keep LN4,LN5,LN6,I1,I2,I3
*next
C
Z R = -2*P4P5*P1P3+2*P3P5*D1+MZP*D1
Z MZP1 = MZP
Keep LN4,LN5,LN6,I1,I2,I3,R,MZP1,PPD
*next
Z RUPPER = Conjg(R)
Z RLOWER = R*Conjg(R)
Z MZPUP = Conjg(MZP1)
Z MZPLW = MZP1*Conjg(MZP1)

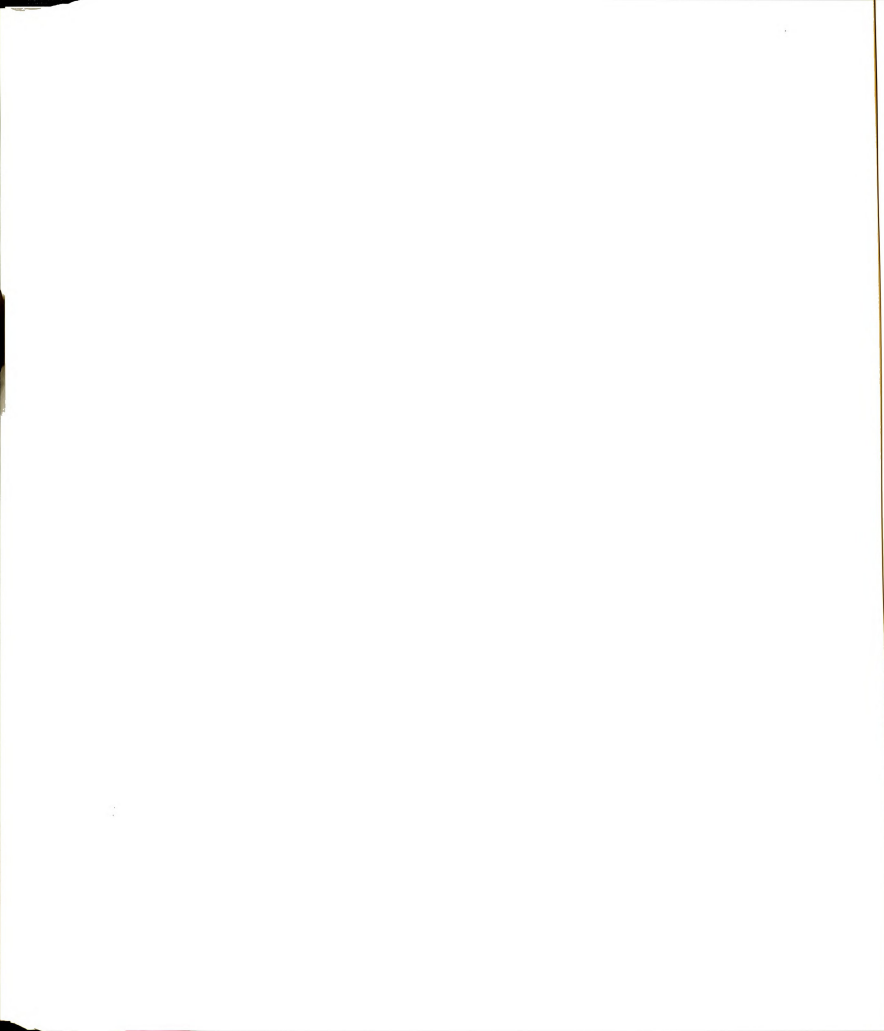
```



```

Keep LN4, LN5, LN6, I1, I2, I3, RUPPER, RLOWER, MZPUP, MZPLW
*next
Z RIV = RUPPER/(RLOWER)
Z MZPIV = MZPUP/(MZPLW)
Keep LN4, LN5, LN6, I1, I2, I3, RIV, MZPIV
*next
Z I4 = - PI*P45IV*P15IV*LN4 -
      0.5*D1*P45IV*P15IV*I2 - 0.5*P15IV*I1
Z I5 = -P45IV*MZPIV*(4*PI*LN5 + D345*I3 - P4P5*I1)
Z I6 = RIV*(4*PI*LN6 - D345*I3 + D1*I2)
C
Keep I1, I2, I3, I4, I5, I6, PPDIV, RIV
*next
C
Z I7 = 0.25*(P3P5*P15IV*P3P4*P14IV-P1P3*P4P5*P14IV*P15IV)*
      (I1 - (P1P4-P1P5)*P45IV*I2 + 2*P1P5*I4)
      - 0.5*(P3P5-P3P4)*P45IV*I2 + P3P4*I4
Z I8 = 0.25*(P1P4*P34IV*P1P5*P35IV-P1P3*P4P5*P34IV*P35IV)*
      (I1 - (P3P4-P3P5)*P45IV*I3 - MZP*I5)
      - 0.5*(P1P5-P1P4)*P45IV*I3 + P1P4*I5
Z I9U = P3P4*P1P4*P1P5*P3P4*P1P5*P1P5*P4P5*P1P3*P1P4
      - P4P5*P1P3*P1P5-P1P4*P1P4*P3P5-P1P5*P1P4*P3P5
Z I9L = 2*P1P3*
      (P1P3*P4P5-P1P4*P3P4-P1P4*P3P5-P1P5*P3P4-P1P5*P3P5)
C
Keep I1, I2, I3, I4, I5, I6, I7, I8, I9U, I9L, RIV
*next
C
X I93 = 2*P4P5*P1P3*D145IV*D145IV-(P3P4+P3P5)*D145IV
X I96 = -2*P4P5*P1P3*D145IV + 2*P3P5 + MZP
X I9F = 0.5*I9U*((I9L)**-1)*(I2+I93*I3-I96*I6)
Z I9 = I9F + 0.5*P4P5*(P1P5-P1P4)*D145IV*(D145IV*I3+I6)
      + 0.5*P4P5*I6
Keep I1, I2, I3, I4, I5, I6, I7, I8, I9, I9U, I9L, RIV
*next
Z I11 = 2*PI*P45IV*(P1P4-P1P5) + P1P4*I1
Z I12 = 2*PI*P45IV*(P3P4-P3P5) + P3P4*I1
Z I13 = -2*PI*P4P5*D145IV*D145IV*(P1P4 - P1P5) +
      P4P5*P1P5*D145IV*I2
Z I14 = 0.5*(P3P4 + P3P5)*I2 +
      0.5*(D145*D345-2*P1P3*P4P5)*D145IV*(4*PI*D145IV + I2)
Keep I1, I2, I3, I4, I5, I6, I7, I8, I9, I11, I12, I13, I14, I9U, I9L, RIV
*next
Z I15 = -2*PI*P4P5*D345IV*D345IV*(P3P4 - P3P5) +
      0.5*P4P5*(1+D345IV*D345IV*(P3P4-P3P5)*(D345+D3))*I3
Z I16 = 0.5*D145*I3 +
      0.5*(D145*D345 - 2*P1P3*P4P5)*
      D345IV*D345IV*(4*PI - I3*(D345 + D3))
C
Keep I1, I2, I3, I4, I5, I6, I7, I8, I9, I11, I12, I13, I14, I15, I16, I9U, I9L,
RIV
*next
Z I17 = P3P4*I7 + 0.5*P45IV*(P3P4-P3P5)*I14 +
      0.25*IS*P14IV*P15IV*(I12 - (P1P4-P1P5)*P45IV*I14 + 2*P1P5*I7) +
      0.0625*P14IV*P14IV*P15IV*P15IV*(4*P3P4*P3P5*P1P4*P1P5-IS*IS)*
      (-2*P1P5*I1 - 4*P1P5*P1P5*I4 - P45IV*D145*2*P1P5*I2)
Z I18 = P1P4*I8 + 0.5*P45IV*(P1P4-P1P5)*I16 +
      0.25*IS*P34IV*P35IV*(I11 - (P3P4-P3P5)*P45IV*I16-(2*P3P4+D3)*I8)+

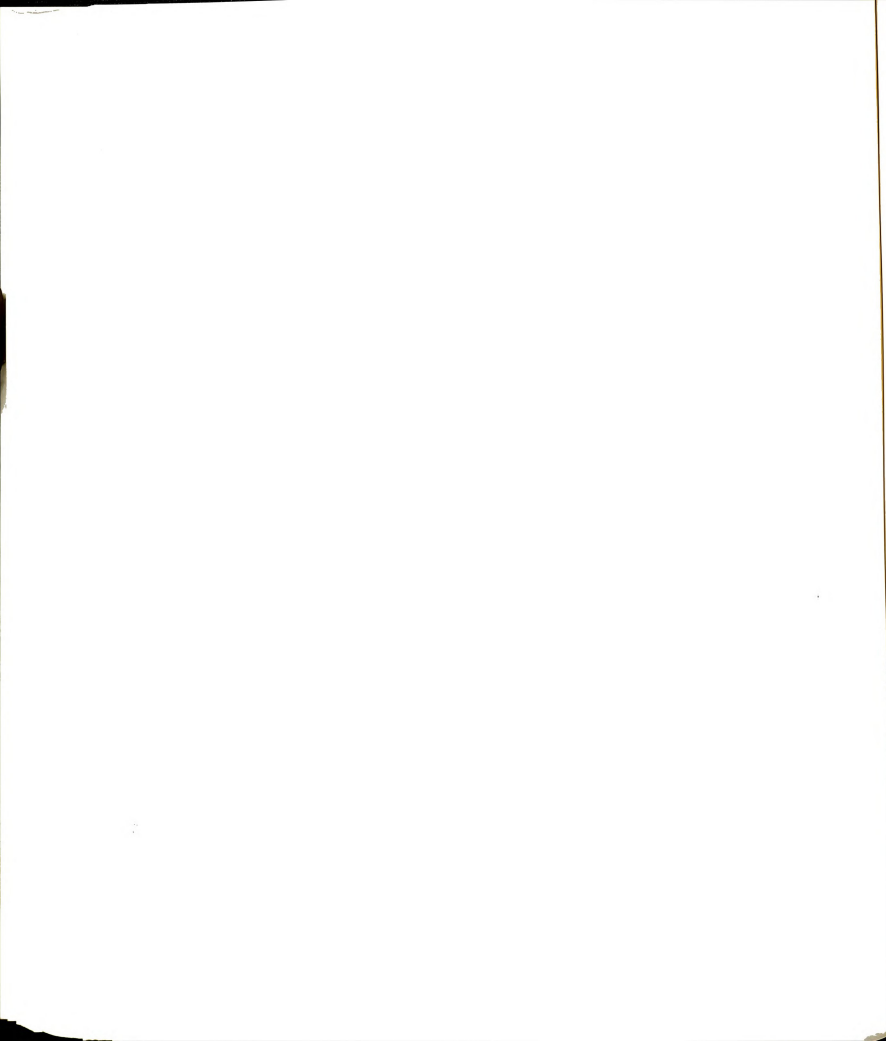
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      0.0625*P34IV*P34IV*P35IV*P35IV*(4*P3P4*P3P5*P1P4*P1P5-IS*IS)*
      ((MZP)*I1-MZP*MZP*I5-P45IV*(D345*2*P3P5+(P3P4-P3P5)*MZP)*I3)
Id,IS = P1P4*P3P5 + P1P5*P3P4 - P1P3*P4P5
C
Keep I1,I2,I3,I4,I5,I6,I7,I8,I9,I11,I12,I13,I14,I15,I16,I9U,I9L,
      I17,I18
*next
C
Z I19=P4P5*P1P5*D145IV*I9-0.5*D145IV*D145IV*P4P5*(P1P4-P1P5)*I15 +
      0.5*I9U*((I9L)**-1)*
      (I13 + D145IV*D145IV*IW*I15 - D145IV*IR)*I9) +
      (-0.5*P4P5*P1P4*P1P5*((I9L)**-1)-0.25*I9U*I9U*((I9L)**-1)*((I9L)**-1))*
      (
        (D145IV*IR)*I2
        -D145IV*D145IV*IR*IR*I6
        -D145IV*(D345*D345+D145IV*IW*(P3P4-P3P5-MZP))*I3
      )
C
C
Id,IW = 2*P1P3*P4P5 - D145*D345
Id,IR = -2*P4P5*P1P3 + D1*(2*P3P5 + MZP)
C
C
Keep I1,I2,I3,I4,I5,I6,I7,I8,I9,I11,I12,I13,I14,I15,I16,
      I17,I18,I19
*next
C
Z ZI10U = U1DU1*(0.5*I8+0.5*D1*I5)+U2DU2*(0.5*I7-0.25*D3*I4)
      - (U3DU3-2*U3DU4+U4DU4)*0.5*I9
      + U1DU2*(0.5*I1+D1*I4-0.5*D3*I5)
      - (U1DU3-U1DU4)*(0.5*I3+D1*I6)
      + U1DU4*P4P5*I5-(U2DU3-U2DU4)*0.5*(I2-D3*I6)+U2DU4*P4P5*I4
      - (U3DU4-U4DU4)*P4P5*I6
Z ZI10D = U1DU1*D1*D1+U2DU2*0.25*D3*D3+U4DU4*P4P5*P4P5
      -1.0*U1DU2*D1*D3+2*U1DU4*P4P5*D1-U2DU4*P4P5*D3
Id,2,Dotpr,U1(J-)=CC11*P1(J)+CC21*P3(J)+CC31*P4(J)+CC41*P5(J)
Al,2,Dotpr,U2(J-)=CC21*P1(J)+CC22*P3(J)+CC32*P4(J)+CC42*P5(J)
Al,2,Dotpr,U3(J-)=CC31*P1(J)+CC32*P3(J)+CC33*P4(J)+CC43*P5(J)
Al,2,Dotpr,U4(J-)=CC41*P1(J)+CC42*P3(J)+CC43*P4(J)+CC44*P5(J)
Id,Numer,P1DP1,0,P3DP3,0,P4DP4,0,P5DP5,0
Al,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
Keep I1,I2,I3,I4,I5,I6,I7,I8,I9,I11,I12,I13,I14,I15,I16,
      I17,I18,I19,ZI10U,ZI10D
*next
Z UPPER = -ZI10U*Conjg(ZI10D)
Z LOWER = ZI10D*Conjg(ZI10D)
Keep I1,I2,I3,I4,I5,I6,I7,I8,I9,I11,I12,I13,I14,I15,I16,
      I17,I18,I19,UPPER,LOWER
*next
Z I10 = UPPER/(LOWER)
Keep I1,I2,I3,I4,I5,I6,I7,I8,I9,I11,I12,I13,I14,I15,I16,
      I17,I18,I19,I10
*next

```



```

C
C   ALL THE NINETEEN INTEGRALS ARE NOW CALCULATED.
C
C *****
C
C
C   IN THIS SECTION, TAKE THE TRACE, REPLACE EACH K BY IT'S
C   LINEAR EXPANSION,
C   AND REDUCE TO A FINAL ANSWER.
C
C
C   V B,K,CU,M,E,N,P1,P2,P3,P4,P5,PD
C   F C,F0,F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,A1,A2,A3,AA
C   I J1,J2,M1,M2,K3,K2
C
C   SCALE FACTOR FOR THE AMPLITUDE IS ...
C
C   X FSCAL = 0.5*FPIIV*(1/137)*(P34+P35+P45)
C
C   TAKE THE TRACE
C
C   Z EXP5=FSCAL*F0*F1*F2*F3*F4*F11*F5*F6*F7*F8*F9*F10
C
C   SUBSTITUTE ACCORDING TO HELICITIES AND DIAGRAM
C
C   Id,F0=0.5*((1+LHEL)*(G6(J1))+(1-LHEL)*(G7(J1)))
C   Id,F1=(G(J1,B))
C   Id,F2=ELECT*(G(J1,M2)) + POSIT*(G(J1,M1))
C   Id,F3=(G(J1,K))
C   Id,F4=ELECT*(G(J1,M1)) + POSIT*(G(J1,M2))
C   Id,F11=0.5*((1+QHEL)*(G6(J2))+(1-QHEL)*(G7(J2)))
C   Id,F5=(G(J2,CU))
C   *yep
C   Id,F6=(G(J2,M2))
C   Id,F7=(G(J2,M))
C   Id,F8=G(J2,E)
C   Id,F9=(G(J2,N))
C   Id,F10=G(J2,M1)
C   *yep
C   Id,Trick,Trace,J1
C
C   TRICK AND TRACE
C
C   Id,Trick,Trace,J2
C   *yep
C
C   START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
C   Id,2,Dotpr,N(J-) = M(J)+P1(J)
C   Al,Funct,N(K2-) = M(K2)+P1(K2)
C   Id,MDM=0
C   Id,Dotpr,M(J-)=K(J)-PD(J)
C   Al,Funct,M(J-)=K(J)-PD(J)
C   Id,KDPD=P4P5
C   Id,KDK=0

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Id,2, Dotpr, PD(J-)=P4(J)+P5(J)
Al, Funct, PD(J-)=P4(J)+P5(J)
Id, P4DP4=0
Al, P5DP5=0
Al, EDP1=0
C P OUTPUT
*yes
C
C PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2, Dotpr, B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEPF*Epf(J,P3,P4,P5)
Al, Funct, B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEPF*Epf(K3,P3,P4,P5)
C P OUTPUT
*yes
Id, B3 = B0IV*P4P5
Al, B4 = B0IV*P3P5
Al, B5 = -B0IV*P3P4
Al, BEPF = + LHEL*B0IV*(ELECT - POSIT)
Id, Trick
Id, KDK=0
Id, P1DP1=0
Al, P3DP3=0
Al, P4DP4=0
Al, P5DP5=0
Id, P1DP3=P1P3
Al, P1DP4=P1P4
Al, P1DP5=P1P5
Al, P3DP4=P3P4
Al, P3DP5=P3P5
Al, P4DP5=P4P5
C P OUTPUT
*yes
Id,2, Dotpr, CU(J-)=
(CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
+ CUEPF*Epf(J,P1,P3,P5)+CUEPF*Epf(J,P1,P4,P5)
Al, Funct, CU(K-)=
(CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
+ CUEPF*Epf(K,P1,P3,P5)+CUEPF*Epf(K,P1,P4,P5)
Id, CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al, CU2 = CU0IV*P1P5
Al, CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al, CUEPF = + QHEL*CU0IV
*yes
Id, Trick
Id, KDK=0
Al, KDPD=P4P5
Al, EDP1=0
Id,2, Dotpr, PD(J-)=P4(J)+P5(J)
Al, Funct, PD(K2-)=P4(K2)+P5(K2)
Id, P1DP1=0
Al, P3DP3=0
Al, P4DP4=0
Al, P5DP5=0
Id, P1DP3=P1P3
Al, P1DP4=P1P4
Al, P1DP5=P1P5
Al, P3DP4=P3P4
Al, P3DP5=P3P5

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Al,P4DP5=P4P5
C P OUTPUT
*yes
C
C   NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J~)= E1*P1(J) + E3*P3(J) +
E4*P4(J)+E5*P5(J)+EEPF*(Ep(J,P1,P3,P5)+Ep(J,P1,P4,P5))
Al,Funct,E(K~)= E1*P1(K) + E3*P3(K) +
E4*P4(K)+E5*P5(K)+EEPF*(Ep(K,P1,P3,P5)+Ep(K,P1,P4,P5))
Id,E1 = EOIV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = EOIV*GHEL*P1P5
Al,E4 = EOIV*GHEL*P1P5
Al,E5 = -EOIV*GHEL*(P1P3+P1P4)
Al,EEPF = -EOIV
*yes
Id,Trick
Id,KDK=0
Id,Ep(P1,P3,P4,P5) = EVL
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
C
C   REPLACE K BY IT'S LINEAR EXPANSION.
C
Id,2,Funct,K(J~)=C(1)*P1(J)+C(2)*P3(J)+C(3)*P4(J)
+P5(J)*(1-C(3)-C(1)*D1*P45IV-C(2)*D345*P45IV)
*yes
Id,Trick
Id,Ep(P1,P3,P4,P5) = EVL
C
C P OUTPUT
*yes
Id,C(1) = CC11*P1DK+CC21*P3DK+(CC31-CC41)*P4DK+CC41*P4P5
Al,C(2) = CC21*P1DK+CC22*P3DK+(CC32-CC42)*P4DK+CC42*P4P5
Al,C(3) = CC31*P1DK+CC32*P3DK+(CC33-CC43)*P4DK+CC43*P4P5
Al,C(4) = CC41*P1DK+CC42*P3DK+(CC43-CC44)*P4DK+CC44*P4P5
C P OUTPUT
*yes
C
C   THE AMPLITUDE IS NOW COMPLETELY
C   IN THE FORM OF DOT PRODUCTS
C   OF K WITH EXTERNAL MOMENTA.
C
C
C
C   THE DOT PRODUCTS OF K NEED TO
C   BE EVALUATED, FOR EXAMPLE...
C   (P3DK)*P4DK)*P5DK)
C

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C   AND REMEMBER THIS FACTOR IS TO BE INTERGRATED OVER ALL K SPACE.
C
C   THIS DEFINES THE A3 MATRIX....
C
C       A3(4,3,1) = (S4DK)*(S3DK)*(S1DK)   INTEGRAL OF, OVER
C
C       ALL K SPACE.
C
C   AND SIMILARLY A2(J,K)   AND A1(J) MATRICES.
C
C   THE A3, A2, AND A1 MATRICES ARE EVALUATED USING 19 SIMPLE
C   INTEGRALS, EVALUATED EARLIER .... I1 THRU I19....
C
C   REPLACE PDK 'S BY A MATRICES
C   FOR THE FIVE POINT DIAGRAM, OR
C   SIMILAR B MATRICES FOR THE FOUR POINT.
C
C   Id,P1DK = AA(1)
C   A1,P3DK = AA(2)
C   A1,P4DK = AA(3)
C   A1,P5DK = -AA(3) + P4P5
C   Id,AA(J~)*AA(K~)*AA(L~) = A3(J,K,L)
C   Id,AA(J~)*AA(K~) = A2(J,K)
C   Id,AA(J~) = A1(J)
C   Id,Symme,A3,1,2,3,A2,1,2
C   Keep I1,I2,I3,I4,I5,I6,I7,I8,I9,I11,I12,I13,I14,I15,I16,
C       I17,I18,I19,I10,EXP5
C
C *next
C S A0
C Z EXP51 = A0*EXP5
C Id,A0*A3(3,3,3) = A3333
C A1,A0*A3(2,3,3) = A3332
C A1,A0*A3(1,3,3) = A3331
C A1,A0*A3(2,2,3) = A3322
C A1,A0*A3(1,2,3) = A3321
C A1,A0*A3(1,1,3) = A3311
C A1,A0*A3(2,2,2) = A3222
C A1,A0*A3(1,2,2) = A3221
C A1,A0*A3(1,1,2) = A3211
C A1,A0*A3(1,1,1) = A3111
C *yep
C Id,A0*A2(3,3) = A233
C A1,A0*A2(2,3) = A232
C A1,A0*A2(1,3) = A231
C A1,A0*A2(2,2) = A222
C A1,A0*A2(1,2) = A221
C A1,A0*A2(1,1) = A211
C Id,A0*A1(3) = A13
C A1,A0*A1(2) = A12
C A1,A0*A1(1) = A11
C P OUTPUT
C *yep
C
C   AND IN TERMS OF THE 19 INTEGRALS,

```

```

C      THE ABOVE DEFINED D1 AND D3, THE217
C      A3, A2, AND A1 MATRICES,
C      USING THE NOTATION....
C      A3(1,3,3) = A3331
C
C      Id, A3333 = -0.5*(I19)
C      Al, A3332 = -0.25*(I13 - D3*I9)
C      Al, A3331 = -0.25*(I15 + 2*D1*I9)
C      Al, A3322 = -0.25*(I14 - 0.5*D3*I2 + 0.5*D3*D3*I6)
C      Al, A3321 = -0.125*(4*PI + 2*D1*I2 - D3*I3 - 2*D1*D3*I6)
C      Al, A3311 = -0.25*(I16 + D1*I3 + 2*D1*D1*I6)
C      Al, A3222 = 0.5*(I17 - D3*A222)
C      Al, A3221 = (0.25*I12 + 0.5*D1*I7 - 0.5*D3*A221)
C      Al, A3211 = (0.25*(I11 - D3*I8) + D1*A221)
C      Al, A3111 = 0.5*(I18 + 2*D1*A211)
C      Id, A233 = -0.5*(I9)
C      Al, A232 = -0.25*(I2 - D3*I6)
C      Al, A231 = -0.25*(I3 + 2*D1*I6)
C      Al, A222 = 0.5*(I7 - D3*A12)
C      Al, A221 = (0.25*(I1 + 2*D1*I4) - 0.5*D3*A11)
C      Al, A211 = 0.5*(I8 + 2*D1*A11)
C      *yep
C      Id, A13 = -0.5*(I6)
C      Al, A12 = 0.5*(I4 - D3*I10)
C      Al, A11 = 0.5*(I5 + 2*D1*I10)
C
C
C
C      *yep
C
C      PUT IN A0
C
C      Id, A0 = I10
C
C
C
C
C      THE FIVE POINT ANSWER IS NOW FORMED.
C
C      B LNMP, LNMQ, PI, FPIIV, CUOIV, BOIV, EOIV, EVL
C      Keep I1, I2, I3, I4, I5, I6, I7, I8, I9, I11, I12, I13, I14, I15, I16,
C      I17, I18, I19, I10, EXP51
C      *next
C
C
C      *****
C
C      NOW CALCULATE THE FOUR POINT AMPLITUDE
C
C
C      IN THIS SECTION, TAKE THE TRACE, REPLACE EACH K BY IT'S
C      LINEAR EXPANSION,
C      AND REDUCE TO A FINAL ANSWER.
C
C
C
C      V B, K, CU, M, E, N, P1, P2, P3, P4, P5, PD

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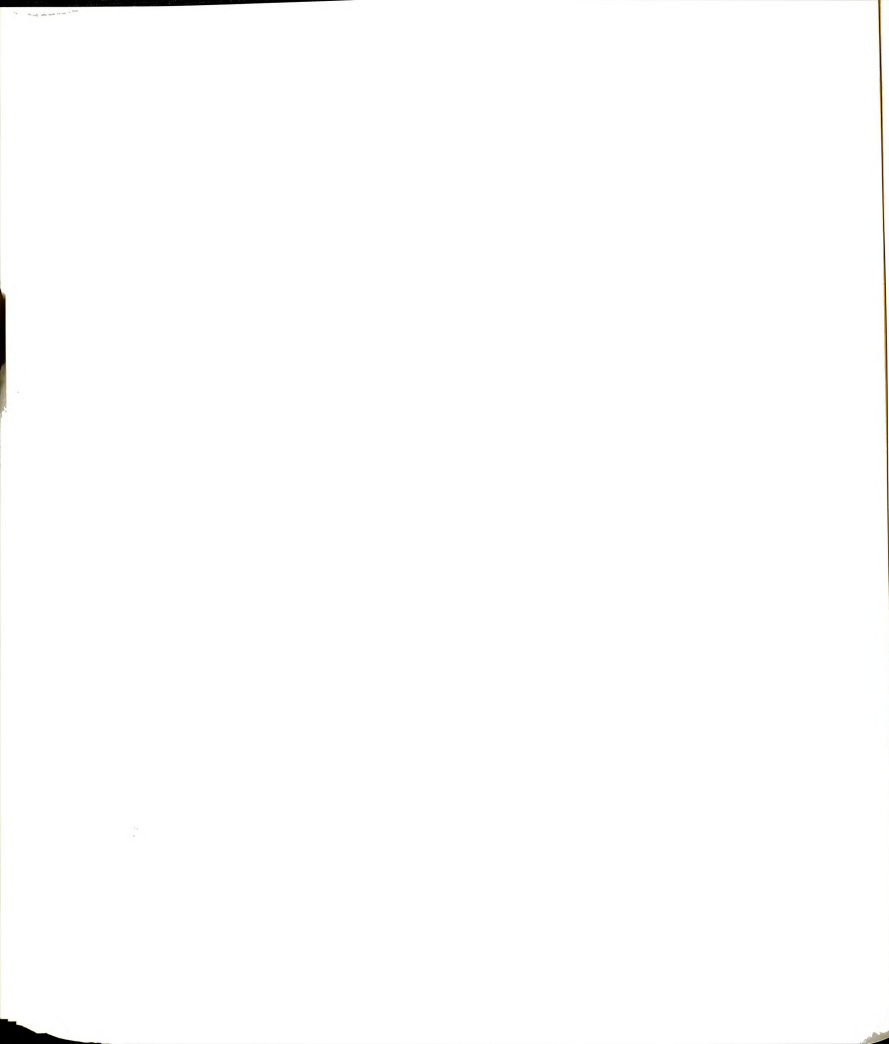
F C,F0,F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,A1,A2,A3,AA
I J1,J2,M1,M2,K3,K2
C
C
C   SCALE FACTOR FOR THE AMPLITUDE IS ...
C
X FSCAL = 0.5*FPIIV*(1/137)
C
C
C   TAKE THE TRACE
C
Z EXP4=FSCAL*F0*F1*F2*F3*F4*F11*F5*F6*F7*F8*F9*F10
C
C   SUBSTITUE ACCORDING TO HELICITIES AND DIAGRAM
C
Id,F0=0.5*((1+LHEL)*(G6(J1))+(1-LHEL)*(G7(J1)))
Id,F1=(G(J1,B))
Id,F2=ELECT*(G(J1,M2)) + POSIT*(G(J1,M1))
Id,F3=(G(J1,K))
Id,F4=ELECT*(G(J1,M1)) + POSIT*(G(J1,M2))
Id,F11=0.5*((1+QHEL)*(G6(J2))+(1-QHEL)*(G7(J2)))
Id,F5=(G(J2,CU))
*yes
Id,F6=(G(J2,M2))
Id,F7=(G(J2,M))
Id,F8=G(J2,M1)
Id,F9=(G(J2,N))
Id,F10=G(J2,E)
*yes
Id,Trick,Trace,J1
C
C   TRICK AND TRACE
C
Id,Trick,Trace,J2
*yes
C
C   START CONDENSATION SUBSTITUTIONS, SOME DIAGRAM DEPENDENT.
C
Id,2,Dotpr,N(J-) = P3(J)+P4(J)+P5(J)
Al,Funct,N(K2-) = P3(K2)+P4(K2)+P5(K2)
Id,MDM=0
Id,Dotpr,M(J-)=K(J)-PD(J)
Al,Funct,M(J-)=K(J)-PD(J)
Id,KDPD=P4P5
Id,KDK=0
Id,2,Dotpr,PD(J-)=P4(J)+P5(J)
Al,Funct,PD(J-)=P4(J)+P5(J)
Id,P4DP4=0
Al,P5DP5=0
Al,EDP1=0
C P OUTPUT
*yes
C
C   PUT IN THE ELSEWHERE EVALUATED SPINOR EXPRESSIONS.
C
Id,2,Dotpr,B(J-)=B3*P3(J)+B4*P4(J)+B5*P5(J)+BEPF*Ep f(J,P3,P4,P5)
Al,Funct,B(K3-)=B3*P3(K3)+B4*P4(K3)+B5*P5(K3)+BEPF*Ep f(K3,P3,P4,P5)
C P OUTPUT

```

```

*yes
Id,B3 = BOIV*P4P5
Al,B4 = BOIV*P3P5
Al,B5 = -BOIV*P3P4
Al,BEPF = + LHEL*BOIV*(ELECT - POSIT)
Id,Trick
Id,KDK=0
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
Id,2,Dotpr,CU(J^-)=
  (CU1-CU2)*P1(J)+CU2*P3(J)+CU2*PD(J)+CU5*P5(J)
  +CUEPF*EpF(J,P1,P3,P5)+CUEPF*EpF(J,P1,P4,P5)
Al,Funct,CU(K^-)=
  (CU1-CU2)*P1(K)+CU2*P3(K)+CU2*PD(K)+CU5*P5(K)
  +CUEPF*EpF(K,P1,P3,P5)+CUEPF*EpF(K,P1,P4,P5)
Id,CU1 = - CU0IV*(P3P5+P4P5-P1P5)
Al,CU2 = CU0IV*P1P5
Al,CU5 = CU0IV*(P3P4+P3P5+P4P5)
Al,CUEPF = + QHEL*CU0IV
*yes
Id,Trick
Id,KDK=0
Al,KDPD=P4P5
Al,EDP1=0
Id,2,Dotpr,PD(J^-)=P4(J)+P5(J)
Al,Funct,PD(K2^-)=P4(K2)+P5(K2)
Id,P1DP1=0
Al,P3DP3=0
Al,P4DP4=0
Al,P5DP5=0
Id,P1DP3=P1P3
Al,P1DP4=P1P4
Al,P1DP5=P1P5
Al,P3DP4=P3P4
Al,P3DP5=P3P5
Al,P4DP5=P4P5
C P OUTPUT
*yes
C
C   NOW PUT IN THE POLARIZATION VECTOR EXPANSION.
C
Id,2,Dotpr,E(J^-)= E1*P1(J) + E3*P3(J) +
  E4*P4(J)+E5*P5(J)+EFPF*(EpF(J,P1,P3,P5)+EpF(J,P1,P4,P5))
Al,Funct,E(K^-)= E1*P1(K) + E3*P3(K) +
  E4*P4(K)+E5*P5(K)+EFPF*(EpF(K,P1,P3,P5)+EpF(K,P1,P4,P5))
Id,E1 = EOIV*GHEL*(P3P5+P4P5-2*P1P5)
Al,E3 = EOIV*GHEL*P1P5
Al,E4 = EOIV*GHEL*P1P5

```



```

                                220
A1,E5 = -E0IV*GHSL*(P1P3+P1P4)
A1,EEPF = -E0IV
*yes
Id,Trick
Id,KDK=0
Id,Epf(P1,P3,P4,P5) = EVL
Id,P1DP1=0
A1,P3DP3=0
A1,P4DP4=0
A1,P5DP5=0
Id,P1DP3=P1P3
A1,P1DP4=P1P4
A1,P1DP5=P1P5
A1,P3DP4=P3P4
A1,P3DP5=P3P5
A1,P4DP5=P4P5
C P OUTPUT
*yes
C
C   REPLACE K BY IT'S LINEAR EXPANSION.
C
Id,2,Funct,K(J~)=C(1)*P1(J)+C(2)*P3(J)+C(3)*P4(J)
                +P5(J)*(1-C(3)-C(1)*D1*P45IV-C(2)*D345*P45IV)
*yes
Id,Trick
Id,Epf(P1,P3,P4,P5) = EVL
C
C P OUTPUT
*yes
Id,C(1) = CC11*P1DK+CC21*P3DK+(CC31-CC41)*P4DK+CC41*P4P5
A1,C(2) = CC21*P1DK+CC22*P3DK+(CC32-CC42)*P4DK+CC42*P4P5
A1,C(3) = CC31*P1DK+CC32*P3DK+(CC33-CC43)*P4DK+CC43*P4P5
A1,C(4) = CC41*P1DK+CC42*P3DK+(CC43-CC44)*P4DK+CC44*P4P5
C P OUTPUT
*yes
C
C   THE AMPLITUDE IS NOW COMPLETELY
C   IN THE FORM OF DOT PRODUCTS
C   OF K WITH EXTERNAL MOMENTA.
C
C
C   THE DOT PRODUCTS OF K   NEED TO
C   BE EVALUATED, FOR EXAMPLE...
C           (P3DK)*(P4DK)*(P5DK)
C
C   AND REMEMBER THIS FACTOR IS TO BE INTERGRATED OVER ALL K SPACE.
C
C
C   THIS DEFINES THE A3 MATRIX....
C
C           A3(4,3,1) = (S4DK)*(S3DK)*(S1DK)   INTEGRAL OF, OVER
C
C   ALL K SPACE.
C
C   AND SIMILARLY A2(J,K)   AND   A1(J)   MATRICES.
C
C
C   THE A3, A2, AND A1 MATRICES   ARE EVALUATED USING 19 SIMPLE

```

```

C  INTEGRALS, EVALUATED EARLIER .... I1 THRU I19....
C
C
C  REPLACE PDK 'S BY A MATRICES
C  FOR THE FIVE POINT DIAGRAM, OR
C  SIMILAR B MATRICES FOR THE FOUR POINT.
C
Id,P1DK = AA(1)
Al,P3DK = AA(2)
Al,P4DK = AA(3)
Al,P5DK = -AA(3) + P4P5
Id,AA(J-)*AA(K-)*AA(L-) = A3(J,K,L)
Id,AA(J-)*AA(K-) = A2(J,K)
Id,AA(J-) = A1(J)
Id,Symme,A3,1,2,3,A2,1,2
Keep I1,I2,I3,I4,I5,I6,I7,I8,I9,I11,I12,I13,I14,I15,I16,
      I17,I18,I19,I10,EXP51,EXP4

*next
S A0
Z EXP41 = A0*EXP4
Id,A0*A3(3,3,3) = B3ERR
Al,A0*A3(2,3,3) = B3ERR
Al,A0*A3(1,3,3) = B3ERR
Al,A0*A3(2,2,3) = B3ERR
Al,A0*A3(1,2,3) = B3ERR
Al,A0*A3(1,1,3) = B3ERR
Al,A0*A3(2,2,2) = B3ERR
Al,A0*A3(1,2,2) = B3ERR
Al,A0*A3(1,1,2) = B3ERR
Al,A0*A3(1,1,1) = B3ERR

*yep
Id,A0*A2(3,3) = B233
Al,A0*A2(2,3) = B232
Al,A0*A2(1,3) = B231
Al,A0*A2(2,2) = B222
Al,A0*A2(1,2) = B221
Al,A0*A2(1,1) = B211
Id,A0*A1(3) = B13
Al,A0*A1(2) = B12
Al,A0*A1(1) = B11
C P OUTPUT
*yep
C
C
C  AND IN TERMS OF THE 19 INTEGRALS,
C  THE ABOVE DEFINED D1 AND D3, THE
C  A3,A2, AND A1 MATRICES,
C  USING THE NOTATION....
C      A3(1,3,3) = A3331
C
C
C
C
C  PUT IN "B" SERIES
C
Id,B233 = -0.5*I15
Al,B232 = -0.25*(4*PI - D3*I3)
Al,B231 = -0.5*I16

```

```

Al,B222 = 0.5*(I12 - D3*B12)
Al,B221 = 0.5*(I11 - D3*B11)
Al,B211 = I18
Id,B13 = -0.5*I3
Al,B12 = 0.5*(I1 -D3*I5)
Al,B11 = I8
C
C  PUT IN A0
C
Id,A0 = I5
C
C
C
C  THE FOUR POINT ANSWER IS NOW FORMED.
C
B LNMP,LNMQ,PI,FPIIV,CU0IV,B0IV,E0IV,EVL
Keep EXP51,EXP41
*next
C
C
C  NOW ADD THE FIVE POINT AND FOUR POINT RESULTS
C
C
Z EXP = EXP51 + EXP41
B LNMP,LNMQ,PI,FPIIV,CU0IV,B0IV,E0IV,EVL
*end

```


Appendix T

Verification of 19 Irreducible Integrals



```

PROGRAM check 19 integrals                                090
COMPLEX D3,MZP,R,LN3,LN5,LN6,MZP1,ZI10D,RUPPER,          110
C MZPUP,MZPIV,RIV,ZI10U,UPPER,I,                        120
C I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I96,IR,                130
C I11,I12,I13,I14,I15,I16,I17,I18,I19,
C g,p,xx,i5u,i5l,z,II1,n
REAL LN1,LN2,LN4,RLOWER,MZPLW,I9U,I9L,I93,IW,IS,        150
C LOWER,S,MP,MQ,x,c,d,e,f,m,za,z0,z1,NN,NX,NZ,NO,AX,NY,NY1
REAL P1P3,P1P4,P1P5,P3P4,P3P5,P4P5,API,FPIIV,K02,K02IV,NX1,NZ1
DOUBLE PRECISION WIV,CC44,CC43,CC42,CC41,CC33,CC32,CC31,
C CC22,CC21,CC11
GAM = 2.7                                                170
MP = 1
MQ = 1
MZ = 93.7                                                180
D3 = ( 8779.69,-252.99)                                190
API = 3.142857143                                       200
FPIIV = 0.25*(1/API)                                    210
PRINT*,API                                              220
PRINT*, 'D3 = ',D3                                     230
PRINT*, 'P3P4 = 92'                                     240
P3P4 = 92                                                250
PRINT*, 'P3P5 = 20'                                     260
P3P5 = 20                                                270
PRINT*, 'P4P5 = 5'                                     280
P4P5 = 5                                                 290
P3P4 = -0.5*P3P4**2                                     300
P3P5 = -0.5*P3P5**2                                     310
P4P5 = -0.5*P4P5**2                                     320
P14MAX = (P3P4+P4P5)*P4P5*(P3P4+P3P5+P4P5)            330
P14MAX = P14MAX/((P3P4+P4P5)*(P3P5+P4P5) - P3P4*P3P5) 340
PRINT*, 'P14MAX = ',-((2*(-P14MAX))**(0.5))            350
PRINT*, 'P1P4 = 70'                                     360
P1P4 = 70                                                370
P1P4 = -0.5*P1P4**2                                     380
R = P3P4 + P4P5                                          390
S = P1P4*(P3P5+P4P5) - P4P5*(P3P4+P3P5+P4P5)          400
T = P1P4*P3P4*P3P5                                      410
DISC = (16*T*(T-R*S))**(0.5)                            420
P15UP = (- (2*R*S - T) + DISC)/(2*R*R)                  430
IF (P15UP) 370,367,367                                  440
367 P15UP = 0                                             450
370 P15DN = (- (2*R*S - T) - DISC)/(2*R*R)              460
IF (P15DN) 380,377,377                                  470
377 P15DN = 0                                             480
380 PRINT*, 'P15UP = ',-((2*(-P15UP))**(0.5))          490
PRINT*, 'P15DN = ',-((2*(-P15DN))**(0.5))              500
PRINT*, 'P1P5 = 15'                                     510
P1P5 = 15                                                520
P1P5 = -0.5*P1P5**2                                     530
P1P3 = P3P4+P3P5+P4P5-P1P4-P1P5                        540
PRINT*, 'P1P3 = ',P1P3                                  550
PRINT*, 'P1P3 = ',(-2*P1P3)**(0.5)                    560
P13IV = 1/P1P3                                           570
P14IV = 1/P1P4                                           580
P15IV = 1/P1P5                                           590
P34IV = 1/P3P4                                           600
P35IV = 1/P3P5                                           610

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P45IV = 1/P4P5	620
MZP = 2*P3P4 + D3	630
S1S2 = P1P3	640
S1S3 = P1P4	650
S1S4 = P1P5	660
S2S3 = P3P4	670
S2S4 = P3P5	680
S3S4 = P4P5	690
W = S1S2*S3S4*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)	700
C + S1S3*S2S4*(S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)	710
C + S1S4*S2S3*(S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)	720
WIV = 1.0/W	7.
CC44 = WIV*(-2)*(S1S2*S1S3*S2S3)	740
CC43 = WIV*S1S2*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)	750
CC42 = WIV*S1S3*(S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)	760
CC41 = WIV*S2S3*(S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)	770
CC33 = WIV*(-2)*(S1S2*S1S4*S2S4)	780
CC32 = WIV*S1S4*(S1S2*S3S4-S1S4*S2S3+S1S3*S2S4)	790
CC31 = WIV*S2S4*(S1S2*S3S4+S1S4*S2S3-S1S3*S2S4)	800
CC22 = WIV*(-2)*(S1S3*S1S4*S3S4)	810
CC21 = WIV*S3S4*(-S1S2*S3S4+S1S4*S2S3+S1S3*S2S4)	820
CC11 = WIV*(-2)*(S2S3*S2S4*S3S4)	830
PRINT*, 'WIV = ', WIV	
PRINT*, 'CC44 = ', CC44	
PRINT*, 'CC43 = ', CC43	
PRINT*, 'CC42 = ', CC42	
PRINT*, 'CC41 = ', CC41	
PRINT*, 'CC33 = ', CC33	
PRINT*, 'CC32 = ', CC32	
PRINT*, 'CC31 = ', CC31	
PRINT*, 'CC22 = ', CC22	
PRINT*, 'CC21 = ', CC21	
PRINT*, 'CC11 = ', CC11	
R = -2*P4P5*P1P3+2*P3P5*(P1P4+P1P5) + MZP*(P1P4+P1P5)	840
LN1 = LOG (-2*P4P5 + MP**2)	
LN2 = LOG(-2*(P1P4+P1P5) + MQ**2)	
LN3 = LOG((2*P3P5+MZP)/D3)	870
LN4 = LOG((P1P4+P1P5)/P1P5)	880
LN5 = LOG(MZP/(2*P3P5+MZP))	890
LN6 = LOG((2*P3P5+MZP)*(P1P4+P1P5)/R)	900
B0 = (8*P3P5*P4P5)**(-0.5)	910
CU0 = (8*P1P5*(P3P4+P3P5+P4P5))**(-0.5)	920
E0 = (-4*P1P5*(P3P5+P4P5-P1P5)*(P3P4+P3P5+P4P5))**(-0.5)	930
EVL1=(P1P5*P3P4+P1P4*P3P5-P4P5*P1P3)**2	940
EVL = (4*P1P4*P1P5*P3P4*P3P5 - EVL1)**(0.5)	950
PRINT*, 'B0 = ', B0	960
PRINT*, 'CU0 = ', CU0	970
PRINT*, 'E0 = ', E0	980
PRINT*, 'EVL = ', EVL	990
PRINT*, 'LN1 = ', LN1	1000
PRINT*, 'LN2 = ', LN2	1010
PRINT*, 'LN3 = ', LN3	1020
PRINT*, 'LN4 = ', LN4	1030
PRINT*, 'LN5 = ', LN5	1040
PRINT*, 'LN6 = ', LN6	1050
K02 = 0.5*(-P4P5)	1060
K02IV = 1/K02	1070
D145 = P1P4 + P1P5	1080

```

D1 = D145                                1090
D145IV = 1/D145                          1100
D345 = P3P4 + P4P5                      1110
D345IV = 1/D345                         1120
MZP = 2*P3P4 + D3                       1130
I1 = -2*API*P45IV*(LN1)                 1140
I = 0
S = .00001
Print*, 's= ', s
X = -1
1135 I = I + S/(P4P5*(X-1) + MP**2)
X = X + S
IF (X-1) 1135, 1137, 1137
1137 PRINT*, 'CHECK I1 = ', I*2*API
I2 = -2*API*D145IV*(LN2)                1150
I = 0
S = .00001
print*, 's= ', s
X = -1
1145 I = I + S/(-(P1P4+P1P5)*(X+1) + MQ**2)
X = X + S
IF (X-1) 1145, 1147, 1147
1147 PRINT*, 'CHECK I2 = ', I*2*API
I3 = +2*API*D345IV*LN3                  1160
I = 0
S = .00001
print*, 's= ', s
X = -1
1155 I = I + S/(-(P3P4+P3P5)*(X-1) + D3)
X = X + S
IF (X-1) 1155, 1157, 1157
1157 PRINT*, 'CHECK I3 = ', I*2*API
PRINT*, 'I1 = ', I1                    1170
PRINT*, 'I2 = ', I2                    1180
PRINT*, 'I3 = ', I3                    1190
R = -2*P4P5*P1P3+2*P3P5*D1+MZP*D1      1200
MZP1 = MZP                             1210
RUPPER = CONJG(R)                       1220
RLOWER = R*CONJG(R)                    1230
MZPUP = CONJG(MZP1)                    1240
MZPLW = MZP1*CONJG(MZP1)               1250
RIV = RUPPER/(RLOWER)                  1260
MZPIV = MZPUP/(MZPLW)                  1270
PRINT*, 'MZPIV = ', MZPIV              1280
Z0 = 2*((P1P4*P1P5)**(0.5))
I = 0
S = .01
print*, 's= ', s
X = -1
1163 Y = 0
Z1 = Z0* sin(acos(X))
1165 ZA = Z1*cos(Y)
II1 = (P1P4-P1P5)*X+ZA-P1P4-P1P5+MQ**2
I = I + S*S/((P4P5*(X-1) + MP**2)*II1)
Y = Y + S
IF (Y - 2*API) 1165, 1167, 1167
1167 X = X + S
IF (X-1) 1163, 1169, 1169

```

```

1169 PRINT*, 'CHECK I4 = ', I
      I4 = - api*P45IV*P15IV*LN4 -
C      0.5*D1*P45IV*P15IV*I2 - 0.5*P15IV*I1
      print*, 'I4 = ', I4
      c = p4p5
      d = -c + mp**2
      e = p1p4-p1p5
      f = 2*((p1p4*p1p5)**(0.5))
      g = -(p1p4+p1p5) + mq**2
      m = (p1p4+p1p5)**2
      n = 2*(g*e*c - d*m)
      p = d*d*m - 2*g*e*c*d + c*c*(g*g - f*f)
      x = c + d
      xx = m*x*x + n*x + p
      z = (p*xx)**(0.5)
      i4u = log((2*z + n*x + 2*p)/x)
      x = -c + d
      xx = m*x*x + n*x + p
      z = (p*xx)**(0.5)
      i4l = log((2*z + n*x + 2*p)/x)
      i4 = +2*api*(p**(-0.5)*(i4u-i4l))
      print*, 'i4detl = ', i4
      Z0 = 2*((P3P4*P3P5)**(0.5))
      I = 0
      S = .01
      print*, 's = ', s
      X = -1
1183 Y = 0
      Z1 = Z0* sin(acos(X))
1185 ZA = Z1*cos(Y)
      II1 = (P3P4-P3P5)*X+ZA+P3P4+P3P5+d3
      I = I + S*S/((P4P5*(X-1) + MP**2)*II1)
      Y = Y + S
      IF (Y - 2*API) 1185,1187,1187
1187 X = X + S
      IF (X-1) 1183,1189,1189
1189 PRINT*, 'CHECK I5 = ', I
      I5 = P45IV*MZPIV*(-4*api*LN5 - D345*I3 +P4P5*I1)
      print*, 'I5 = ', I5
      c = p4p5
      d = -c + mp**2
      e = p3p4-p3p5
      f = 2*((p3p4*p3p5)**(0.5))
      g = +(p3p4+p3p5) + d3
      m = (p3p4+p3p5)**2
      n = 2*(g*e*c - d*m)
      p = d*d*m - 2*g*e*c*d + c*c*(g*g - f*f)
      x = c + d
      xx = m*x*x + n*x + p
      z = (p*xx)**(0.5)
      i5u = log((2*z + n*x + 2*p)/x)
      x = -c + d
      xx = m*x*x + n*x + p
      z = (p*xx)**(0.5)
      i5l = log((2*z + n*x + 2*p)/x)
      i5 = 2*api*(p**(-0.5)*(i5u-i5l))
      print*, 'i5detl = ', i5
      e = (2*p1p3*p4p5-(p1p4+p1p5)*(p3p4+p3p5))/(p1p4+p1p5)

```

```

Z0 = ((p3p4+p3p5)**2 - e*e)**(0.5)
I = 0
S = .01
PRINT*, 'S = ', S
X = -1
1193 Y = 0
Z1 = Z0* sin(acos(X))
1195 ZA = Z1*cos(Y)
II1 = e*X+ZA+P3P4+P3P5+d3
I = I + S*S/((- (P1P4+P1P5)*(X+1) + MQ**2)*II1)
Y = Y + S
IF (Y - 2*API) 1195,1197,1197
1197 X = X + S
IF (X-1) 1193,1199,1199
1199 PRINT*, 'CHECK I6 = ', I
I6 = RIV*(+4*api*LN6 - D345*I3 + D1*I2)
PRINT*, 'I6 = ', I6
c = -p1p4-p1p5
d = +c + mq**2
e = (2*p1p3*p4p5-(p1p4+p1p5)*(p3p4+p3p5))/(p1p4+p1p5)
g = +(p3p4+p3p5) + d3
f = Z0
m = (p3p4+p3p5)**2
n = 2*(g*e*c - d*m)
p = d*d*m - 2*g*e*c*d + c*c*(g*g - f*f)
x = c + d
xx = m*x*x + n*x + p
z = (p*xx)**(0.5)
i6u = log((2*z + n*x + 2*p)/x)
x = -c + d
xx = m*x*x + n*x + p
z = (p*xx)**(0.5)
i6l = log((2*z + n*x + 2*p)/x)
i6 = 2*api*(p**(-0.5)*(i6u-i6l))
print*, 'i6detl = ', i6
Z0 = 2*((P1P4*P1P5)**(0.5))
NO = 0.5*(P3P4+P3P5)
AX = 0.5*((P1P4*P1P5)**(-0.5))*(P1P4*P3P5+P1P5*P3P4-P1P3*P4P5)
I = 0
s = .01
print*, 's= ', s
X = -1
2163 Y = 0
NZ = 0.5*(P3P4-P3P5)*X
NX = AX*sin(acos(X))
Z1 = Z0*sin(acos(X))
2165 ZA = Z1*cos(Y)
NN = NX*cos(Y) + NZ + NO
II1 = (P1P4-P1P5)*X+ZA-P1P4-P1P5+MQ**2
I = I + S*S*NN/((P4P5*(X-1) + MP**2)*II1)
Y = Y + S
IF (Y - 2*API) 2165,2167,2167
2167 X = X + S
IF (X-1) 2163,2169,2169
2169 PRINT*, 'CHECK I7 = ', I
Z0 = 2*((P3P4*P3P5)**(0.5))
NO = 0.5*(P1P4+P1P5)
AX = 0.5*((P3P4*P3P5)**(-0.5))*(P1P4*P3P5+P1P5*P3P4-P1P3*P4P5)

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I = 0
S = .01
print*, 's= ', s
X = -1
2263 Y = 0
      NZ = 0.5*(P1P4-P1P5)*X
      NX = AX*sin(acos(X))
      Z1 = Z0*sin(acos(X))
2265 ZA = Z1*cos(Y)
      NN = NX*cos(Y) + NZ + NO
      II1 = (P3P4-P3P5)*X+ZA+P3P4+P3P5+D3
      I = I + S*S*NN/((P4P5*(X-1) + MP**2)*II1)
      Y = Y + S
      IF (Y - 2*API) 2265,2267,2267
2267 X = X + S
      IF (X-1) 2263,2269,2269
2269 PRINT*, 'CHECK I8 = ', I
      e = (2*p1p3*p4p5-(p1p4+p1p5)*(p3p4+p3p5))/(p1p4+p1p5)
      Z0 = ((P3P4+P3P5)**2 - e*e)**(0.5)
      NO = 0.5*(P4P5)
      u = p3p4*p1p5*p1p5 + p3p4*p1p4*p1p5 + p4p5*p1p3*p1p4
      C      - p1p4*p1p4*p3p5 - p1p4*p1p5*p3p5 - p1p5*p1p3*p4p5
      t = 2*p1p3*(p1p3*p4p5-p1p4*p3p4-p1p4*p3p5-p1p5*p3p4-p1p5*p3p5)
      AX = ((-p4p5/2)**(0.5))*u/((p1p4+p1p5)*t**(0.5))
      I = 0
      S = .01
      print*, 's= ', s
      X = -1
2363 Y = 0
      NZ = 0.5*P4P5*(P1P4-P1P5)*X/(P1P4+P1P5)
      NX = AX*sin(acos(X))
      Z1 = Z0*sin(acos(X))
2365 ZA = Z1*cos(Y)
      NN = NX*cos(Y) + NZ + NO
      II1 = e*X+ZA+P3P4+P3P5+D3
      I = I + S*S*NN/((-P1P4+P1P5)*(X+1) + MQ**2)*II1)
      Y = Y + S
      IF (Y - 2*API) 2365,2367,2367
2367 X = X + S
      IF (X-1) 2363,2369,2369
2369 PRINT*, 'CHECK I9 = ', I
      I7 = 0.25*(P3P5*P15IV+P3P4*P14IV-P1P3*P4P5*P14IV+P15IV)*
      C      (I1 - (P1P4-P1P5)*P45IV*I2 + 2*P1P5*I4)
      C- 0.5*(P3P5-P3P4)*P45IV*I2 + P3P4*I4
      I8 = 0.25*(P1P4*P34IV+P1P5*P35IV-P1P3*P4P5*P34IV+P35IV)*
      C      (I1 - (P3P4-P3P5)*P45IV*I3 - MZP*I5)
      C - 0.5*(P1P5-P1P4)*P45IV*I3 + P1P4*I5
      I9U = P3P4*P1P4*P1P5+P3P4*P1P5*P1P5+P4P5*P1P3*P1P4
      C - P4P5*P1P3*P1P5-P1P4*P1P4*P3P5-P1P5*P1P4*P3P5
      I9L = 2*P1P3*
      C(P1P3*P4P5-P1P4*P3P4-P1P4*P3P5-P1P5*P3P4-P1P5*P3P5)
      I93 = 2*P4P5*P1P3*D145IV*D145IV-(P3P4+P3P5)*D145IV
      I96 = -2*P4P5*P1P3*D145IV + 2*P3P5 + MZP
      I9 = 0.5*I9U*(1/I9L)*(I2+I93*I3-I96*I6)
      C      + 0.5*P4P5*(P1P5-P1P4)*D145IV*(D145IV*I3+I6)
      C      + 0.5*P4P5*I6
      PRINT*, 'I7 = ', I7
      PRINT*, 'I8 = ', I8

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PRINT*, 'I9 = ', I9
PRINT*, 'I9U = ', I9U
PRINT*, 'I9L = ', I9L
PRINT*, 'I93 = ', I93
PRINT*, 'I96 = ', I96
I11 = 2*API*P45IV*(P1P4-P1P5) + P1P4*I1
PRINT*, 'I11 = ', I11
NX = 2*((P1P4*P1P5)**(0.5))
I = 0
S = .01
print*, 's= ', s
X = -1
2763 Y = 0
NX1 = NX * sin(acos(X))
2765 NX2 = NX1*cos(Y)
NN = (P1P4-P1P5)*X+NX2+P1P4+P1P5
I = I + S*S*0.5*NN/(P4P5*(X-1) + MP**2)
Y = Y + S
IF (Y - 2*API) 2765,2767,2767
2767 X = X + S
IF (X-1) 2763,2769,2769
2769 PRINT*, 'CHECK 11 = ', I
I12 = 2*API*P45IV*(P3P4-P3P5) + P3P4*I1
PRINT*, 'I12 = ', I12
NX = 2*((P3P4*P3P5)**(0.5))
I = 0
S = .005
print*, 's= ', s
X = -1
2963 Y = 0
NX1 = NX * sin(acos(X))
2965 NX2 = NX1*cos(Y)
NN = (P3P4-P3P5)*X+NX2+P3P4+P3P5
I = I + S*S*0.5*NN/(P4P5*(X-1) + MP**2)
Y = Y + S
IF (Y - 2*API) 2965,2967,2967
2967 X = X + S
IF (X-1) 2963,2969,2969
2969 PRINT*, 'CHECK 12 = ', I
I13 = -2*API*P4P5*D145IV*D145IV*(P1P4 - P1P5) +
C P4P5*P1P5*D145IV*I2
PRINT*, 'I13 = ', I13
NO = 0.5*(P4P5)
NZ = 0.5*P4P5*(P1P4-P1P5)/(P1P4+P1P5)
NX = (NO**2 - NZ**2)**(0.5)
I = 0
S = .01
print*, 's= ', s
X = -1
3063 Y = 0
NZ1 = NZ*X
NX1 = NX*sin(acos(X))
3065 NN = NX1*cos(Y) + NZ1 + NO
I = I + S*S*NN/(-(P1P4+P1P5)*(X+1) + MQ**2)
Y = Y + S
IF (Y - 2*API) 3065,3067,3067
3067 X = X + S
IF (X-1) 3063,3069,3069

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3069 PRINT*, 'CHECK I13 = ', I
      I14 = 0.5*(P3P4 + P3P5)*I2 +
C      0.5*(D145*D345-2*P1P3*P4P5)*D145IV*( 4*API*D145IV + I2)
      PRINT*, 'I14 = ', I14
      NZ = (2*p1p3*p4p5-(p1p4+p1p5)*(p3p4+p3p5))/(p1p4+p1p5)
      NX = ( (p3p4+p3p5)**2 - NZ**2)**(0.5)
      I = 0
      S = .01
      PRINT*, 'S = ', S
      X = -1
3193 Y = 0
      NZ1 = NZ*X
      NX1 = NX* sin(acos(X))
3195 NX2 = NX1*cos(Y)
      NN = NZ1+NX2+P3P4+P3P5
      I = I + S*S*0.5*NN/(-(P1P4+P1P5)*(X+1) + MQ**2)
      Y = Y + S
      IF (Y - 2*API) 3195,3197,3197
3197 X = X + S
      IF (X-1) 3193,3199,3199
3199 PRINT*, 'CHECK I14 = ', I
      I15 = -2*API*P4P5*D345IV*D345IV*(P3P4 - P3P5) +
C      0.5*P4P5*(1+D345IV*D345IV*(P3P4-P3P5)*(D345+D3))*I3
      PRINT*, 'I15 = ', I15
      NO = 0.5*(P4P5)
      NZ = 0.5*P4P5*(P3P4-P3P5)/(P3P4+P3P5)
      NX = ( NO**2 - NZ**2)**(0.5)
      I = 0
      S = .01
      print*, 's= ', s
      X = -1
3363 Y = 0
      NZ1 = NZ*X
      NX1 = NX*sin(acos(X))
3365 NN = NX1*cos(Y) + NZ1 + NO
      I = I + S*S*NN/(-(P3P4+P3P5)*(X-1) + D3)
      Y = Y + S
      IF (Y - 2*API) 3365,3367,3367
3367 X = X + S
      IF (X-1) 3363,3369,3369
3369 PRINT*, 'CHECK I15 = ', I
      I16 = 0.5*D145*I3 +
C      0.5*(D145*D345 - 2*P1P3*P4P5)*
C      D345IV*D345IV*( 4*API - I3*(D345 + D3))
      PRINT*, 'I16 = ', I16
      NZ = (2*p1p3*p4p5-(p1p4+p1p5)*(p3p4+p3p5))/(p1p4+p1p5)
      NX = ((p3p4+p3p5)**2 - NZ**2)**(0.5)
      I = 0
      S = .01
      PRINT*, 'S = ', S
      X = -1
3593 Y = 0
      NX1 = NX * sin(acos(X))
3595 NX2 = NX1*cos(Y)
      NN = NZ*X+NX2+P3P4+P3P5
      I = I + S*S*0.5*NN/(-(P3P4+P3P5)*(X-1) + D3)
      Y = Y + S
      IF (Y - 2*API) 3595,3597,3597

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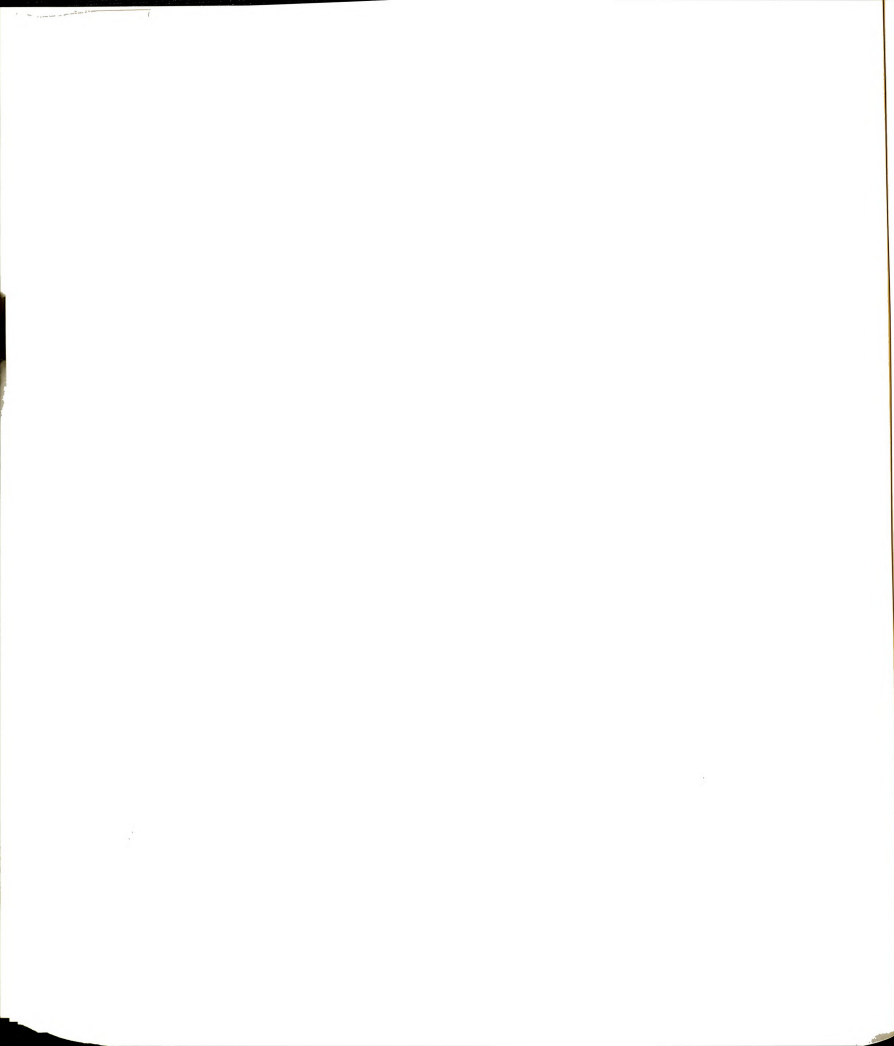
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3597 X = X + S
      IF ( X-1) 3593,3599,3599
3599 PRINT*, 'CHECK I16 = ', I
      Z0 = 2*((P1P4*P1P5)**(0.5))
      N0 = 0.5*(P3P4+P3P5)
      NX = 0.5*((P1P4*P1P5)**(-0.5))*(P1P4*P3P5+P1P5*P3P4-P1P3*P4P5)
      NZ = 0.5*(P3P4-P3P5)
      NY = (N0**2 - NZ**2 - NX**2)**(0.5)
      I = 0
      s = .01
      print*, 's= ', s
      X = -1
4163 Y = 0
      NZ1 = NZ*X
      NX1 = NX*sin(acos(X))
      NY1 = NY*sin(acos(X))
      Z1 = Z0*sin(acos(X))
4165 ZA = Z1*cos(Y)
      NN = NX1*cos(Y) + NY1*sin(Y) + NZ1 + N0
      II1 = (P1P4-P1P5)*X+ZA-P1P4-P1P5+MQ**2
      I = I + S*S*NN*NN/((P4P5*(X-1) + MP**2)*II1)
      Y = Y + S
      IF (Y - 2*API) 4165,4167,4167
4167 X = X + S
      IF ( X-1) 4163,4169,4169
4169 PRINT*, 'CHECK I17 = ', I
      Z0 = 2*((P3P4*P3P5)**(0.5))
      N0 = 0.5*(P1P4+P1P5)
      NX = 0.5*((P3P4*P3P5)**(-0.5))*(P1P4*P3P5+P1P5*P3P4-P1P3*P4P5)
      NZ = 0.5*(P1P4-P1P5)
      NY = (N0**2 - NZ**2 - NX**2)**(0.5)
      I = 0
      S = .01
      print*, 's= ', s
      X = -1
4263 Y = 0
      NZ1 = NZ*X
      NX1 = NX*sin(acos(X))
      NY1 = NY*sin(acos(X))
      Z1 = Z0*sin(acos(X))
4265 ZA = Z1*cos(Y)
      NN = NX1*cos(Y) + NY1*sin(Y) + NZ1 + N0
      II1 = (P3P4-P3P5)*X+ZA+P3P4+P3P5+D3
      I = I + S*S*NN*NN/((P4P5*(X-1) + MP**2)*II1)
      Y = Y + S
      IF (Y - 2*API) 4265,4267,4267
4267 X = X + S
      IF ( X-1) 4263,4269,4269
4269 PRINT*, 'CHECK I18 = ', I
      e = (2*p1p3*p4p5-(p1p4+p1p5)*(p3p4+p3p5))/(p1p4+p1p5)
      Z0 = ((P3P4+P3P5)**2 - e*e)**(0.5)
      N0 = 0.5*(P4P5)
      u = p3p4*p1p5*p1p5 + p3p4*p1p4*p1p5 + p4p5*p1p3*p1p4
      - p1p4*p1p4*p3p5 - p1p4*p1p5*p3p5 - p1p5*p1p3*p4p5
      t = 2*p1p3*(p1p3*p4p5-p1p4*p3p4-p1p4*p3p5-p1p5*p3p4-p1p5*p3p5)
      NZ = 0.5*P4P5*(P1P4-P1P5)/(P1P4+P1P5)
      NX = ((-p4p5/2)**(0.5))*u/((-p1p4+p1p5))*t**(0.5)
      NY = (N0**2 - NZ**2 - NX**2)**(0.5)

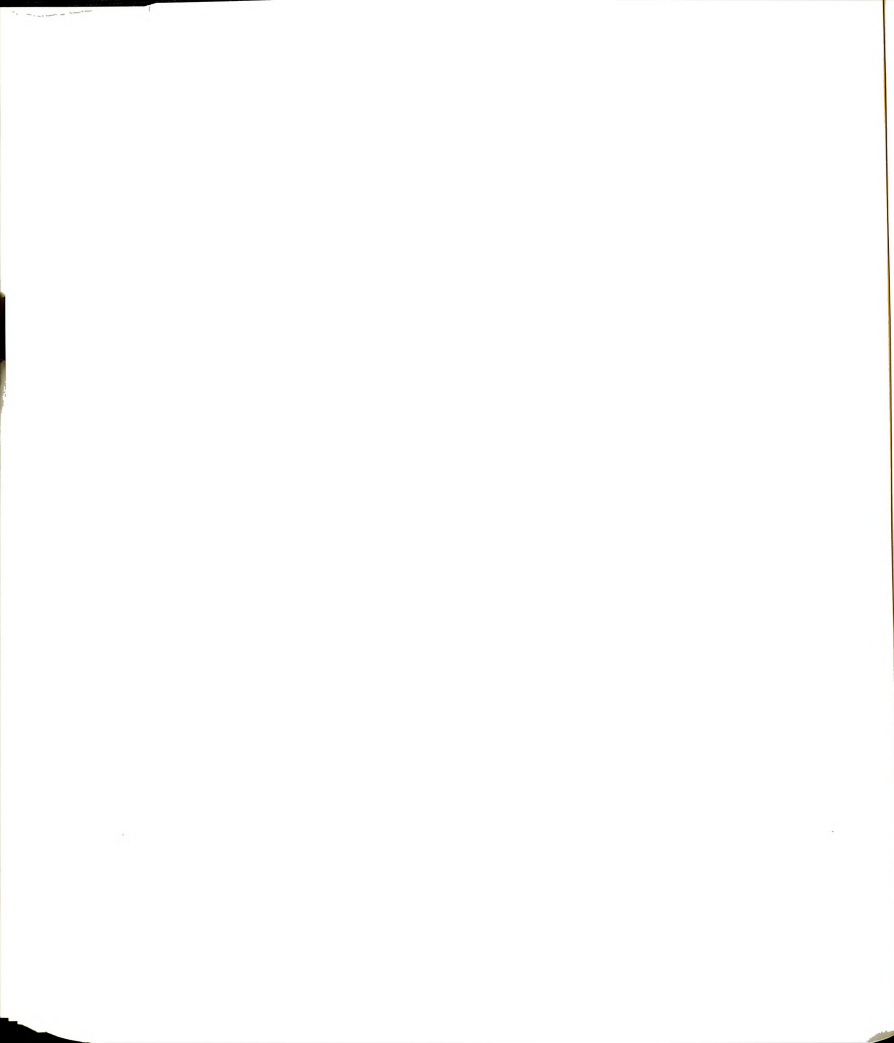
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```

I = 0
S = .01
print*, 's= ', s
X = -1
4363 Y = 0
      NZ1 = NZ*X
      NX1 = NX*sin(acos(X))
      NY1 = NY*sin(acos(X))
      Z1 = Z0*sin(acos(X))
4365 ZA = Z1*cos(Y)
      NN = NX1*cos(Y) + NY1*sin(Y) + NZ1 + NO
      II1 = e*X+ZA+P3P4+P3P5+D3
      I = I + S*S*NN*NN/((- (P1P4+P1P5)*(X+1) + MQ**2)*II1)
      Y = Y + S
      IF (Y - 2*API) 4365, 4367, 4367
4367 X = X + S
      IF (X-1) 4363, 4369, 4369
4369 PRINT*, 'CHECK I19 = ', I
      IS = P1P4*P3P5 + P1P5*P3P4 - P1P3*P4P5
      I17 = P3P4*I7 + 0.5*P45IV*(P3P4-P3P5)*I14 +
C 0.25*IS*P14IV*P15IV*(I12 - (P1P4-P1P5)*P45IV*I14 + 2*P1P5*I7) +
C 0.0625*P14IV*P14IV*P15IV*P15IV*(4*P3P4*P3P5*P1P4*P1P5-IS*IS)*
C (-2*P1P5*I1 - 4*P1P5*P1P5*I4 - P45IV*D145*2*P1P5*I2)
      I18 = P1P4*I8 + 0.5*P45IV*(P1P4-P1P5)*I16 +
C 0.25*IS*P34IV*P35IV*(I11 - (P3P4-P3P5)*P45IV*I16 - (2*P3P4+D3)*I8) +
C 0.0625*P34IV*P34IV*P35IV*P35IV*(4*P3P4*P3P5*P1P4*P1P5-IS*IS)*
C
C      (
C      (MZP)*I1-MZP*MZP*I5
C      -P45IV*(D345*2*P3P5+(P3P4-P3P5)*MZP)*I3
C      )
      IW = 2*P1P3*P4P5 - D145*D345
      IR = -2*P4P5*P1P3 + D1*(2*P3P5 + MZP)
      I19 = P4P5*P1P5*D145IV*I9 -
C      0.5*D145IV*D145IV*P4P5*(P1P4-P1P5)*I15 +
C      0.5*I9U*(1/I9L)*
C      (I13 + D145IV*D145IV*IW*I15+(D145IV*IW-P3P4-P3P5-D3)*I9) +
C      (
C      -0.5*P4P5*P1P4*P1P5*(1/I9L)
C      -0.25*I9U*I9U*(1/I9L)*(1/I9L)
C      )*
C(
C (D145IV*IR)*I2
C -D145IV*D145IV*IR*IR*I6
C -D145IV*(D345*D345+D145IV*IW*(P3P4-P3P5-MZP))*I3
C )
      PRINT*, 'I17 = ', I17
      PRINT*, 'I18 = ', I18
      PRINT*, 'I19 = ', I19
      Z0 = 2*((P1P4*P1P5)**(0.5))
      NO = 0.5*(P3P4+P3P5)
      NX = 0.5*((P1P4*P1P5)**(-0.5))*(P1P4*P3P5+P1P5*P3P4-P1P3*P4P5)
      NZ = 0.5*(P3P4-P3P5)
      NY = (NO**2 - NX**2 - NZ**2)**(0.5)
      I = 0
      s = .01
      print*, 's = ', s
      X = -1
5163 Y = 0

```



```

NZ1 = NZ*X
NX1 = NX*sin(acos(X))
NY1 = NY*sin(acos(X))
Z1 = Z0*sin(acos(X))
5165 ZA = Z1*cos(Y)
NN = NX1*cos(Y) + NY1*sin(Y) + NZ1 + NO
II1 = (P1P4-P1P5)*X+ZA-P1P4-P1P5+MQ**2
I = I + S*S/((2*NN+D3)*(P4P5*(X-1) + MP**2)*II1)
Y = Y + S
IF (Y - 2*API) 5165,5167,5167
5167 X = X + S
IF (X-1) 5163,5169,5169
5169 PRINT*, 'CHECK I10 = ', I
U1DU1=+2.*CC41*CC31*P4P5+2.*CC41*CC21*P3P5+2.*CC41*CC11*P1P5+2.* 2000
CCC31*CC21*P3P4+2.*CC31*CC11*P1P4+2.*CC21*CC11*P1P3 2010
U1DU2=+CC42*CC31*P4P5+CC42*CC21*P3P5+CC42*CC11*P1P5+CC41*CC32*P4 2020
CP5+CC41*CC22*P3P5+CC41*CC21*P1P5+CC32*CC21*P3P4+CC32*CC11*P1P4+C 2030
CC31*CC22*P3P4+CC31*CC21*P1P4+CC22*CC11*P1P3+CC21**2*P1P3 2040
U1DU3=+CC43*CC31*P4P5+CC43*CC21*P3P5+CC43*CC11*P1P5+CC41*CC33*P4 2050
CP5+CC41*CC32*P3P5+CC41*CC31*P1P5+CC33*CC21*P3P4+CC33*CC11*P1P4+C 2060
CC32*CC31*P3P4+CC32*CC11*P1P3+CC31*CC21*P1P3+CC31**2*P1P4 2070
U1DU4=+CC44*CC31*P4P5+CC44*CC21*P3P5+CC44*CC11*P1P5+CC43*CC41*P4 2080
CP5+CC43*CC21*P3P4+CC43*CC11*P1P4+CC42*CC41*P3P5+CC42*CC31*P3P4+C 2090
CC42*CC11*P1P3+CC41*CC31*P1P4+CC41*CC21*P1P3+CC41**2*P1P5 2100
U2DU2=+2.*CC42*CC32*P4P5+2.*CC42*CC22*P3P5+2.*CC42*CC21*P1P5+2.* 2110
CCC32*CC22*P3P4+2.*CC32*CC21*P1P4+2.*CC22*CC21*P1P3 2120
U2DU3=+CC43*CC32*P4P5+CC43*CC22*P3P5+CC43*CC21*P1P5+CC42*CC33*P4 2130
CP5+CC42*CC32*P3P5+CC42*CC31*P1P5+CC33*CC22*P3P4+CC33*CC21*P1P4+C 2140
CC32*CC31*P1P4+CC32*CC21*P1P3+CC32**2*P3P4+CC31*CC22*P1P3 2150
U2DU4=+CC44*CC32*P4P5+CC44*CC22*P3P5+CC44*CC21*P1P5+CC43*CC42*P4 2160
CP5+CC43*CC22*P3P4+CC43*CC21*P1P4+CC42*CC41*P1P5+CC42*CC32*P3P4+C 2170
CC42*CC21*P1P3+CC42**2*P3P5+CC41*CC32*P1P4+CC41*CC22*P1P3 2180
U3DU3=+2.*CC43*CC33*P4P5+2.*CC43*CC32*P3P5+2.*CC43*CC31*P1P5+2.* 2190
CCC33*CC32*P3P4+2.*CC33*CC31*P1P4+2.*CC32*CC31*P1P3 2200
U3DU4=+CC44*CC33*P4P5+CC44*CC32*P3P5+CC44*CC31*P1P5+CC43*CC42*P3 2210
CP5+CC43*CC41*P1P5+CC43*CC32*P3P4+CC43*CC31*P1P4+CC43**2*P4P5+CC4 2220
C2*CC33*P3P4+CC42*CC31*P1P3+CC41*CC33*P1P4+CC41*CC32*P1P3 2230
U4DU4=+2.*CC44*CC43*P4P5+2.*CC44*CC42*P3P5+2.*CC44*CC41*P1P5+2.* 2240
CCC43*CC42*P3P4+2.*CC43*CC41*P1P4+2.*CC42*CC41*P1P3+0. 2250
ZI10U = U1DU1*(0.5*I8+0.5*D1*I5)+U2DU2*(0.5*I7-0.25*D3*I4) 2260
C - (U3DU3-2*U3DU4+U4DU4)*0.5*I9 2270
C + U1DU2*(0.5*I1+D1*I4-0.5*D3*I5) 2280
C - (U1DU3-U1DU4)*(0.5*I3+D1*I6) 2290
C + U1DU4*P4P5*I5-(U2DU3-U2DU4)*0.5*(I2-D3*I6)+U2DU4*P4P5*I4 2300
C - (U3DU4-U4DU4)*P4P5*I6 2310
ZI10D = U1DU1*D1*D1+U2DU2*0.25*D3*D3+U4DU4*P4P5*P4P5 2320
C -1.0*U1DU2*D1*D3+2*U1DU4*P4P5*D1-U2DU4*P4P5*D3 2330
UPPER = -ZI10U*CONJG(ZI10D) 2340
LOWER = ZI10D*CONJG(ZI10D) 2350
I10 = UPPER/(LOWER) 2360
PRINT*, 'I10 = ', I10 2370
END 5010

```


_HSC0000\$DUA3: [EWING]CK. LOG: 8

```

$ IF F$MODE() .EQS. "BATCH" THEN $ EXIT
$ RUN CHECK
  3.142857
D3 = (8779.690, -252.9900)
P3P4 =
P3P5 =
P4P5 =
P14MAX = -94.14882
P1P4 =
P15UP = 0.0000000E+00
P15DN = -23.00508
P1P5 =
P1P3 = -2182.000
P1P3 = 66.06058
BO = 1.4142136E-03
CUO = 4.8393107E-04
EO = 3.4219094E-05
EVL = 899726.6
LN1 = 5.993961
LN2 = 8.542081
LN3 = (-3.090159, -2.427214)
LN4 = 3.125785
LN5 = (1.2334378E-02, 1.780435)
LN6 = (0.4495091, -1.008301)
CHECK I1 = (0.1881661, 0.0000000E+00)
CHECK I2 = (2.0959381E-02, 0.0000000E+00)
CHECK I3 = (4.2695566E-03, 3.3561315E-03)
I1 = (0.1883816, 0.0000000E+00)
I2 = (2.0953396E-02, 0.0000000E+00)
I3 = (4.3826397E-03, 3.4424122E-03)
MZPIV = (1.9288890E-03, 1.5457852E-03)
CHECK I4 = (1.2800620E-03, 0.0000000E+00)
I4 = (7.5656187E-04, 0.0000000E+00)
i4det1 = (1.0532890E-03, 0.0000000E+00)
CHECK I5 = (2.2208611E-04, 2.6291382E-04)
I5 = (1.2245527E-04, 2.1099522E-04)
i5det1 = (1.4313067E-04, 2.0318378E-04)
S = 5.0000002E-04
CHECK I6 = (8.5144438E-06, 4.8571172E-05)
I6 = (9.2996797E-06, 4.2994034E-05)
i6det1 = (5.9150848E-06, 4.8653204E-05)
CHECK I7 = (-5.436626, 0.0000000E+00)
CHECK I8 = (-0.5183365, -0.6392409)
CHECK I9 = (-4.7151849E-04, -2.4306704E-03)
I7 = (-4.466591, 0.0000000E+00)
I8 = (-0.3241602, -0.4922062)
I9 = (-2.1155991E-04, -5.3360919E-04)
I11 = (-388.0707, 0.0000000E+00)
I12 = (-674.0468, 0.0000000E+00)
I13 = (-0.6314969, 0.0000000E+00)
I14 = (-80.43964, 0.0000000E+00)
I15 = (-0.2915253, -6.7722343E-02)
I16 = (-7.577801, -8.108868)
I17 = (10072.33, 0.0000000E+00)
I18 = (469.3852, 892.2405)
I19 = (-103.7464, -261.4523)
CHECK I10 = (1.0828019E-06, 3.0718329E-06)

```

```

I1 = (0.1883816, 0.0000000E+00)
I2 = (2.0953396E-02, 0.0000000E+00)
I3 = (4.3826397E-03, 3.4424122E-03)
MZPIV = (1.9288890E-03, 1.5457852E-03)
CHECK I4 = (1.2800620E-03, 0.0000000E+00)
I4 = (7.5656187E-04, 0.0000000E+00)
i4det1 = (1.0532890E-03, 0.0000000E+00)
CHECK I5 = (2.2208611E-04, 2.6291382E-04)
I5 = (1.2245527E-04, 2.1099522E-04)
i5det1 = (1.4313067E-04, 2.0318378E-04)
S = 9.9999998E-03
CHECK I6 = (3.6448710E-05, 2.6391298E-04)
I6 = (9.2996797E-06, 4.2994034E-05)
i6det1 = (5.9150848E-06, 4.8653204E-05)
CHECK I7 = (-5.426626, 0.0000000E+00)
CHECK I8 = (-0.5183365, -0.6392409)
CHECK I9 = (-4.7151849E-04, -2.4306704E-03)
I7 = (-4.466591, 0.0000000E+00)
I8 = (-0.3241602, -0.4922062)
I9 = (-2.1155991E-04, -5.3360919E-04)

```

_HSC000\$DUA3: [EWING]CK. LOG: 1

```

I11 = (-388.0707, 0.0000000E+00)
I12 = (-674.0468, 0.0000000E+00)
I13 = (-0.6314969, 0.0000000E+00)
I14 = (-80.43964, 0.0000000E+00)
I15 = (-0.2915253, -6.7722343E-02)
I16 = (-7.577801, -8.108868)
I17 = (10072.33, 0.0000000E+00)
I18 = (469.3852, 892.2405)
I19 = (-103.7464, -261.4523)
S = 1.0000000E-03
CHECK I10 = (7.6385157E-07, 2.8371851E-06)
I10 = (5.7549897E-07, 4.0432174E-06)
%FOR-F-ENDDURREA, end-of-file during read
unit -4 file SYS$INPUT:
user PC 000037E2
-RMS-E-EDF, end of file detected
%TRACE-F-TRACEBACK, symbolic stack dump follows
module name      routine name      line      rel PC

```

```

0000C0D6
0000BFE3
0000BC60
0000B716
00002FE2

```

```

CHECK19INTEGRAL CHECK19INTEGRALS      477
EWING      job terminated at 1-JUL-1988 16:56:00.85

```

Accounting information:

Buffered I/O count:	68	Peak working set size:	529
Direct I/O count	58	Peak page file size:	787
Page faults:	539	Mounted volumes:	0
Charged CPU time:	0 00:10:31.50	Elapsed time:	0 00:45:41..

_HSC0000DUA3 LEWINGJCK. LOG: 1

CHECK I7 = (-5.436626, 0.0000000E+00)
 S = 9.9999998E-03
 CHECK I8 = (-0.5183365, -0.6392409)
 S = 9.9999998E-03
 CHECK I9 = (-4.7151849E-04, -2.4306704E-03)
 I7 = (-4.466591, 0.0000000E+00)
 I8 = (-0.3241602, -0.4922062)
 I9 = (-2.1155991E-04, -5.3360919E-04)
 S = 5.0000002E-04
 CHECK I11 = (-387.2682, 0.0000000E+00)
 S = 5.0000002E-04
 CHECK I12 = (-674.6837, 0.0000000E+00)
 S = 5.0000002E-04
 CHECK I13 = (-0.3193493, 0.0000000E+00)
 S = 5.0000002E-04
 CHECK I14 = (-70.59887, 0.0000000E+00)
 S = 5.0000002E-04
 CHECK I15 = (-0.5000000, -0.5654656)
 S = 5.0000002E-04
 CHECK I16 = (-8.238111, -12.45711)
 I11 = (-388.0707, 0.0000000E+00)
 I12 = (-674.0468, 0.0000000E+00)
 I13 = (-0.6314969, 0.0000000E+00)
 I14 = (-80.43964, 0.0000000E+00)
 I15 = (-0.2915253, -6.7722343E-02)
 I16 = (-7.577801, -8.108868)
 S = 9.9999998E-03
 CHECK I17 = (23191.27, 0.0000000E+00)
 S = 9.9999998E-03
 CHECK I18 = (1225.885, 1556.307)
 S = 9.9999998E-03
 CHECK I19 = (1.5155446E-02, 2.4578951E-02)
 I17 = (10072.33, 0.0000000E+00)
 I18 = (469.3852, 892.2405)
 I19 = (-103.7464, -261.4523)
 S = 9.9999998E-03
 CHECK I10 = (1.0828019E-06, 3.0718329E-06)
 I10 = (5.7549897E-07, 4.0432174E-06)

%FOR-F-ENDDURREA, end-of-file during read

unit -4 file SYS\$INPUT.;

user PC 00004474

-RMS-E-EOF, end of file detected

%TRACE-F-TRACEBACK, symbolic stack dump follows

module name	routine name	line	rel PC
-------------	--------------	------	--------

0000CCD6
 0000CBE3
 00009860
 00009316
 00003A74

CHECK19INTEGRAL CHECK19INTEGRALS

EWING job terminated at 4-JUL-1988 11:08:05.74

Accounting information:

Buffered I/O count: 72

Direct I/O count: 69

Page faults: 539

Charged CPU time: 0 01:28:07.84

Peak working set size: 535

Peak page file size: 787

Mounted volumes: 0

Elapsed time: 0 01:39:23.

```

CHECK I7 = (-5.436626, 0.0000000E+00)
S = 9.9999998E-03
CHECK I8 = (-0.5183365, -0.6392409)
S = 9.9999998E-03
CHECK I9 = (-4.6512054E-04, -2.4429935E-03)
I7 = (-4.466591, 0.0000000E+00)
I8 = (-0.3241602, -0.4922062)
I9 = (-2.1155991E-04, -5.3360919E-04)
I11 = (-388.0707, 0.0000000E+00)
S = 9.9999998E-03
CHECK I11 = (-487.2787, 0.0000000E+00)
I12 = (-674.0468, 0.0000000E+00)
S = 4.9999999E-03
CHECK I12 = (-753.0253, 0.0000000E+00)
I13 = (-0.6314969, 0.0000000E+00)
S = 9.9999998E-03
CHECK I13 = (-1.124600, 0.0000000E+00)
I14 = (-80.43964, 0.0000000E+00)
S = 9.9999998E-03
CHECK I14 = (-327.2178, 0.0000000E+00)
I15 = (-0.2915253, -6.7722343E-02)
S = 9.9999998E-03
CHECK I15 = (-0.2977193, -6.8340585E-02)
I16 = (-7.577801, -8.108868)
S = 9.9999998E-03
CHECK I16 = (-12.40340, -13.92181)
S = 4.9999999E-03
CHECK I17 = (21520.46, 0.0000000E+00)
S = 4.9999999E-03
CHECK I18 = (967.8455, 1364.273)
S = 4.9999999E-03
CHECK I19 = (1.6154986E-02, 1.6571850E-02)
I17 = (19075.81, 0.0000000E+00)
I18 = (749.0870, 1194.346)
I19 = (1.3806876E-02, 8.2438448E-03)
S = 9.9999998E-03
CHECK I10 = (1.0650439E-06, 3.0762460E-06)
I10 = (5.7549897E-07, 4.0432174E-06)
%FOR-F-ENDDURREA, end-of-file during read
unit -4 file SYS$INPUT.:
user PC 0000454B
-RMS-E-EOF, end of file detected
%TRACE-F-TRACEBACK, symbolic stack dump follows
module name      routine name      line      rel PC

```

```

0000CED6
0000CDE3
00009A60
00009516
00003B4B

```

```

CHECK19INTEGRAL CHECK19INTEGRALS
EWING job terminated at 5-NOV-1988 16:14:22.00

```

```

Accounting information:
Buffered I/O count:      67
Direct I/O count:       54
Page faults:            539
Charged CPU time:      0 00:01:38.80

```

```

Peak working set size: 536
Peak page file size: 837
Mounted volumes: 0
Elapsed time: 0 00:01:49.

```

```

CHECK I7 = (-5.436626, 0.000000E+00)
s = 9.9999998E-03
CHECK I8 = (-0.5183365, -0.6392409)
s = 9.9999998E-03
CHECK I9 = (-4.6512054E-04, -2.4429935E-03)
I7 = (-4.466591, 0.000000E+00)
I8 = (-0.3241602, -0.4922062)
I9 = (-2.1155991E-04, -5.3360919E-04)
I11 = (-388.0707, 0.000000E+00)
s = 9.9999998E-03
CHECK I11 = (-487.2787, 0.000000E+00)
I12 = (-674.0468, 0.000000E+00)
s = 4.9999999E-03
CHECK I12 = (-753.0253, 0.000000E+00)
I13 = (-0.6314969, 0.000000E+00)
s = 9.9999998E-03
CHECK I13 = (-1.124600, 0.000000E+00)
I14 = (-80.43964, 0.000000E+00)
s = 9.9999998E-03
CHECK I14 = (-327.2178, 0.000000E+00)
I15 = (-0.2915253, -6.7722343E-02)
s = 9.9999998E-03
CHECK I15 = (-0.2977193, -6.8340585E-02)
I16 = (-7.577801, -8.108868)
s = 9.9999998E-03
CHECK I16 = (-12.40340, -13.92181)
s = 4.9999999E-03
CHECK I17 = (21520.46, 0.000000E+00)
s = 4.9999999E-03
CHECK I18 = (967.8455, 1364.273)
s = 4.9999999E-03
CHECK I19 = (1.6154986E-02, 1.6571850E-02)
I17 = (19075.81, 0.000000E+00)
I18 = (749.0870, 1194.346)
I19 = (1.3806876E-02, 8.2438448E-03)
s = 9.9999998E-03
CHECK I10 = (1.0650439E-06, 3.0762460E-06)
I10 = (5.7549897E-07, 4.0432174E-06)
%FOR-F-ENDDURREA, end-of-file during read
unit -4 file SYS$INPUT.;
user PC 0000454B
-RMS-E-EOF, end of file detected
%TRACE-F-TRACEBACK, symbolic stack dump follows
module name routine name

```

line

rel PC

```

0000CED6
0000CDE3
00009A60
00009516
00003B48

```

```

CHECK19INTEGRAL CHECK19INTEGRALS 657
EWING job terminated at 5-NOV-1988 16:14:22.00

```

```

Accounting information:
Buffered I/O count: 67
Direct I/O count: 54
Page faults: 539
Charged CPU time: 0 00:01:38.80

```

_SC00C\$DUA3 (EWING)CK LG: 2

```

CHECK I7 = (-5 436626, 0. 0000000E+00)
S = 9 9999998E-03
CHECK I8 = (-0. 5183365, -0. 6392409)
S = 9 9999998E-03
CHECK I9 = (-4 7151849E-04, -2. 4306704E-03)
I7 = (-4. 466591, 0. 0000000E+00)
I8 = (-0. 3241602, -0. 4922062)
I9 = (-2. 1155991E-04, -5. 3360919E-04)
I11 = (-388. 0707, 0. 0000000E+00)
S = 9 9999998E-03
CHECK I11 = (-487. 2792, 0. 0000000E+00)
I12 = (-674 0468, 0. 0000000E+00)
S = 4. 9999999E-03
CHECK I12 = (-753. 6212, 0. 0000000E+00)
I13 = (-0. 6314969, 0. 0000000E+00)
S = 9. 9999998E-03
CHECK I13 = (-1. 143458, 0. 0000000E+00)
I14 = (-80 43964, 0. 0000000E+00)
S = 9. 9999998E-03
CHECK I14 = (-327. 1871, 0. 0000000E+00)
I15 = (-0. 2915253, -6. 7722343E-02)
S = 9 9999998E-03
CHECK I15 = (-0. 2986189, -7. 0020974E-02)
I16 = (-7 577801, -8. 108868)
S = 9 9999998E-03
CHECK I16 = (-12. 40342, -13. 92181)
S = 5. 0000002E-04
CHECK I17 = (19989. 04, 0. 0000000E+00)
S = 9. 9999998E-03
CHECK I18 = (1225. 885, 1556. 307)
S = 9. 9999998E-03
CHECK I19 = (1. 5155446E-02, 2. 4578951E-02)
I17 = (18432. 83, 0. 0000000E+00)
I18 = (697. 3430, 1197. 104)
I19 = (8 3091952E-02, 1. 4633661E-02)
S = 9. 9999998E-03
CHECK I10 = (1. 0828019E-06, 3. 0718329E-06)
I10 = (5. 7549897E-07, 4. 0432174E-06)
%FOR-F-ENDDURREA, end-of-file during read
unit -4 file SYS$INPUT.:
user PC 000044E1
-RMS-E-EOF, end of file detected
%TRACE-F-TRACEBACK, symbolic stack dump follows
module name - routine name - line rel PC
0000CCD6
0000CBE3
00009860
00009316
00003AE1
CHECK19INTEGRAL CHECK19INTEGRALS ..... 647 .....
EWING job terminated at 8-JUL-1988 20:08:20.36

Accounting information:
Buffered I/O count: 68 Peak working set size: 535
Direct I/O count: 59 Peak page file size: 787
Page faults: 539 Mounted volumes: 0
Charged CPU time: 0 00:22:52.56 Elapsed time: 0 00:32:51.

```

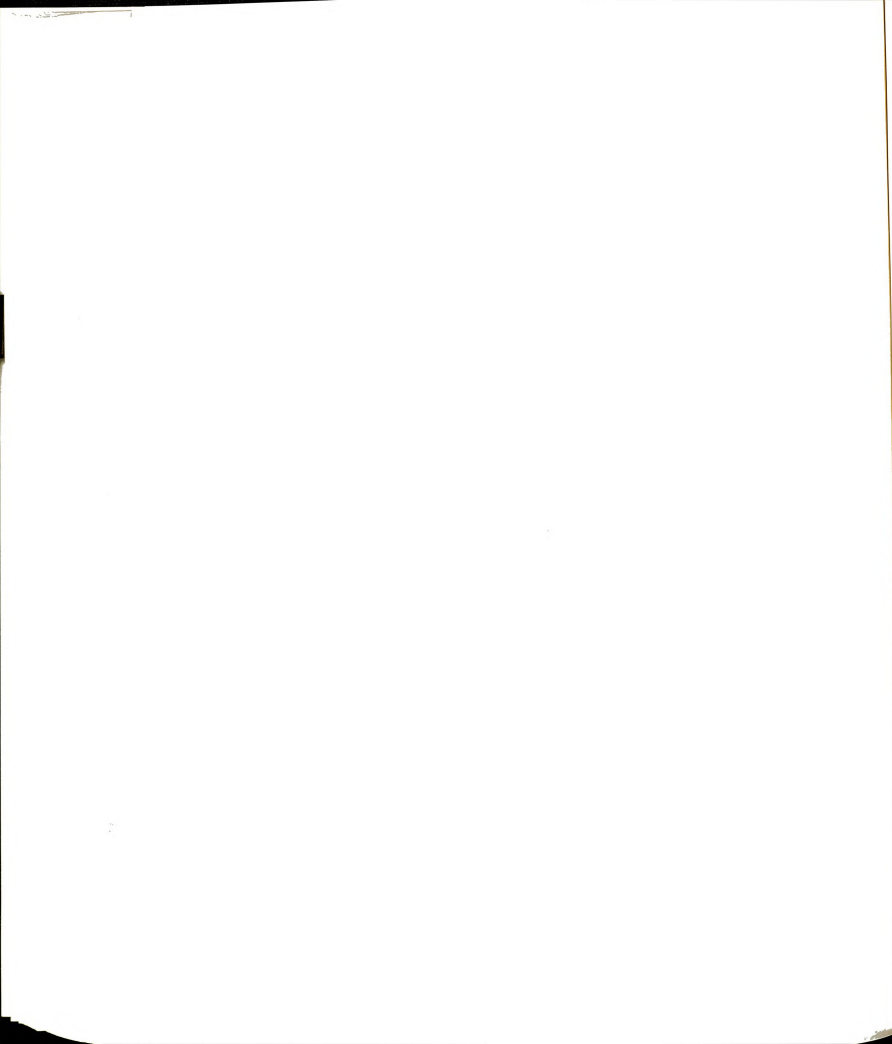
Appendix U

FORTRAN Output -Sample Results

-17.28	-10.00	-40.00	-10.00	-10.00	-47.28	1.0001823
-17.28	-10.00	-40.00	-10.00	-10.00	-47.28	0.9995154
-7.28	-10.00	-50.00	-10.00	-10.00	-47.28	1.0004421
-7.28	-10.00	-50.00	-10.00	-10.00	-47.28	0.9997410
-17.28	-20.00	-30.00	-10.00	-10.00	-47.28	1.0012978
-17.28	-20.00	-30.00	-10.00	-10.00	-47.28	1.0009155
-7.28	-20.00	-40.00	-10.00	-10.00	-47.28	1.0004318
-7.28	-20.00	-40.00	-10.00	-10.00	-47.28	1.0001115
-17.28	-30.00	-20.00	-10.00	-10.00	-47.28	1.0014023
-17.28	-30.00	-20.00	-10.00	-10.00	-47.28	1.0016459
-7.28	-30.00	-30.00	-10.00	-10.00	-47.28	1.0006382
-7.28	-30.00	-30.00	-10.00	-10.00	-47.28	1.0005274
-17.28	-40.00	-10.00	-10.00	-10.00	-47.28	0.9998592
-17.28	-40.00	-10.00	-10.00	-10.00	-47.28	1.0003670
-7.28	-40.00	-20.00	-10.00	-10.00	-47.28	1.0004268
-7.28	-40.00	-20.00	-10.00	-10.00	-47.28	1.0004545
-7.28	-50.00	-10.00	-10.00	-10.00	-47.28	1.0004847
-7.28	-50.00	-10.00	-10.00	-10.00	-47.28	1.0004878
-17.28	-10.00	-40.00	-10.00	-20.00	-37.28	1.0000499
-17.28	-10.00	-40.00	-10.00	-20.00	-37.28	0.9999858
-7.28	-10.00	-50.00	-10.00	-20.00	-37.28	1.0001203
-7.28	-10.00	-50.00	-10.00	-20.00	-37.28	1.0000110
-27.28	-20.00	-20.00	-10.00	-20.00	-37.28	1.0001280
-27.28	-20.00	-20.00	-10.00	-20.00	-37.28	1.0001181
-17.28	-20.00	-30.00	-10.00	-20.00	-37.28	1.0000650
-17.28	-20.00	-30.00	-10.00	-20.00	-37.28	1.0000538
-7.28	-20.00	-40.00	-10.00	-20.00	-37.28	1.0000736
-7.28	-20.00	-40.00	-10.00	-20.00	-37.28	1.0000571
-27.28	-30.00	-10.00	-10.00	-20.00	-37.28	1.0002115
-27.28	-30.00	-10.00	-10.00	-20.00	-37.28	1.0002140
-17.28	-30.00	-20.00	-10.00	-20.00	-37.28	1.0000709
-17.28	-30.00	-20.00	-10.00	-20.00	-37.28	1.0000695
-7.28	-30.00	-30.00	-10.00	-20.00	-37.28	1.0000588
-7.28	-30.00	-30.00	-10.00	-20.00	-37.28	1.0000566
-17.28	-40.00	-10.00	-10.00	-20.00	-37.28	1.0000805
-17.28	-40.00	-10.00	-10.00	-20.00	-37.28	1.0000807
-7.28	-40.00	-20.00	-10.00	-20.00	-37.28	1.0000527
-7.28	-40.00	-20.00	-10.00	-20.00	-37.28	1.0000530
-27.28	-10.00	-30.00	-10.00	-30.00	-27.28	0.9999997
-27.28	-10.00	-30.00	-10.00	-30.00	-27.28	1.0000006
-17.28	-10.00	-40.00	-10.00	-30.00	-27.28	1.0000060
-17.28	-10.00	-40.00	-10.00	-30.00	-27.28	1.0000060
-7.28	-10.00	-50.00	-10.00	-30.00	-27.28	1.0000273
-7.28	-10.00	-50.00	-10.00	-30.00	-27.28	1.0000241
-37.28	-20.00	-10.00	-10.00	-30.00	-27.28	1.0000234
-37.28	-20.00	-10.00	-10.00	-30.00	-27.28	1.0000242
-27.28	-20.00	-20.00	-10.00	-30.00	-27.28	1.0000116
-27.28	-20.00	-20.00	-10.00	-30.00	-27.28	1.0000124
-17.28	-20.00	-30.00	-10.00	-30.00	-27.28	1.0000155
-17.28	-20.00	-30.00	-10.00	-30.00	-27.28	1.0000159
-7.28	-20.00	-40.00	-10.00	-30.00	-27.28	1.0000300
-7.28	-20.00	-40.00	-10.00	-30.00	-27.28	1.0000296
-27.28	-30.00	-10.00	-10.00	-30.00	-27.28	1.0000151
-27.28	-30.00	-10.00	-10.00	-30.00	-27.28	1.0000153
-17.28	-30.00	-20.00	-10.00	-30.00	-27.28	1.0000153
-17.28	-30.00	-20.00	-10.00	-30.00	-27.28	1.0000157
-7.28	-30.00	-30.00	-10.00	-30.00	-27.28	1.0000250
-7.28	-30.00	-30.00	-10.00	-30.00	-27.28	1.0000250

-47.28	-10.00	-10.00	-10.00	-40.00	-17.28	0.9999987
-47.28	-10.00	-10.00	-10.00	-40.00	-17.28	1.0000116
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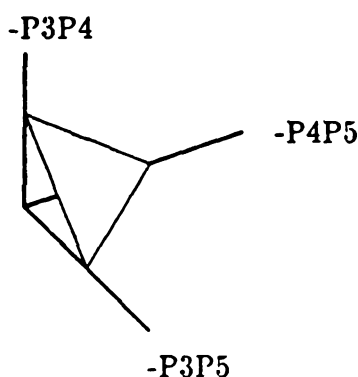
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Appendix V

Choosing the Output Data Display Points .

Given that the allowable output data points lie on a triangular surface:



where each corner of the triangles intersects the axis at $\frac{E_{cm}^2}{2}$.

Each edge of the triangle is obviously of length $(\sqrt{2}) (\frac{E_{cm}^2}{2})$

To find the angle between the plane of the triangle and any of the three co-ordinate planes, for example the x-y plane:

The normal to the triangle is $N1 = i + j + k$

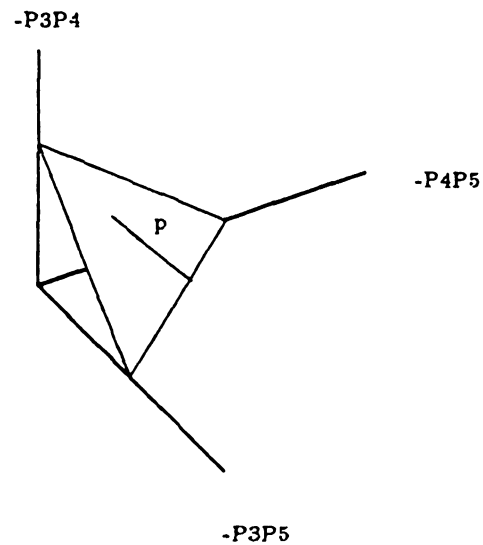
The normal to the x-y plane is $N2 = k$

$$N1 \cdot N2 = 1 = |N1| |N2| \cos \theta = (1)(\sqrt{3}) \cos \theta$$

$$\cos \theta = \frac{1}{\sqrt{3}} \quad \sin \theta = \frac{\sqrt{2}}{\sqrt{3}}$$

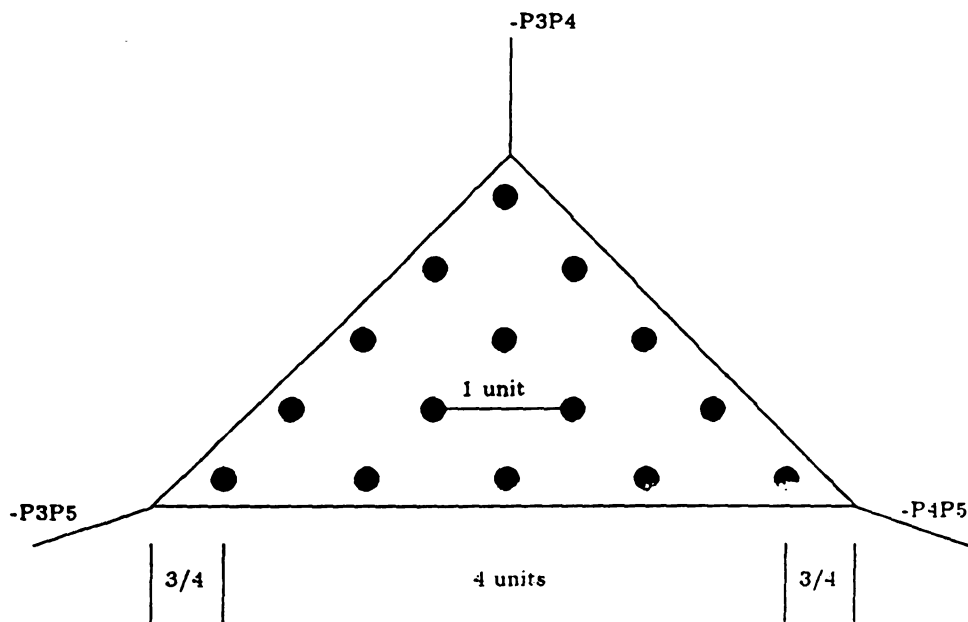
So if p measures the distance 'up' the face of the triangle from the base line in the x-y plane, p is related to the 'z' co-ordinate, that is the value of $-P3P4$, by:

$$-P3P4 = p \frac{\sqrt{2}}{\sqrt{3}}$$



Viewing the triangle straight on and selecting 15 symmetrically spaced representative points.

Allow the internal distance between any two points to be one unit, with an additional $\frac{3}{4}$ unit to the corner, as shown.



Then $5 \frac{1}{2}$ units equals the edge length of $(\sqrt{2}) \left(\frac{E_{cm}^2}{2}\right)$ or

$$1 \text{ unit} = \left(\frac{2}{11}\right) (\sqrt{2}) \left(\frac{E_{cm}^2}{2}\right)$$

The width of each border is $\frac{1}{4} \sqrt{3}$ units, and the vertical distance between rows is $\frac{1}{2} \sqrt{3}$ units, so if $N = 0, 1, 2, 3, 4$ indicates the horizontal row number, then the distance p from the lower edge to that row is given by:

$$\begin{aligned} p &= \left[\left(\frac{1}{4} \sqrt{3}\right) + N \left(\frac{1}{2} \sqrt{3}\right) \right] [\text{length of one unit}] \\ &= \left[\left(\frac{1}{4} \sqrt{3}\right) + N \left(\frac{1}{2} \sqrt{3}\right) \right] \left[\frac{2}{11} \sqrt{2} \frac{E_{cm}^2}{2} \right] \\ &= \left(\frac{\sqrt{3}\sqrt{2}}{11} \right) \left(\frac{E_{cm}^2}{2} \right) \left(\frac{1}{2} + N \right) \end{aligned}$$

and since

$$-P_3P_4 = p \frac{\sqrt{2}}{\sqrt{3}}$$

then

$$-P_3P_4 = \left(\frac{2}{11}\right) \left(\frac{E_{cm}^2}{2}\right) \left(\frac{1}{2} + N\right)$$

and the values of $-P_3P_4$ to run the FORTRAN program at to generate these display points are given by allowing N to range through its values 0,1,2,3,4. In a similar fashion the values of the other axis, $-P_3P_5$ and $-P_4P_5$ are generated,

$$-P_3P_5 = \left(\frac{2}{11}\right) \left(\frac{E_{cm}^2}{2}\right) \left(\frac{1}{2} + L\right)$$

$$-P_4P_5 = \left(\frac{2}{11}\right) \left(\frac{E_{cm}^2}{2}\right) \left(\frac{1}{2} + M\right)$$

and the set N, L, M locates the display point.

The set of three values for the other triangle, $-P_1P_3$, $-P_1P_4$, $-P_1P_5$ are generated in an identical fashion.

Appendix W

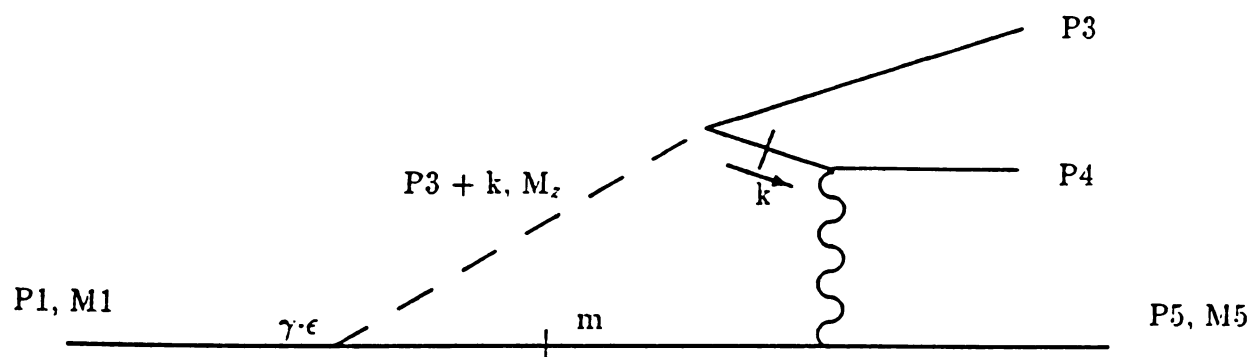
Evaluation of a Preliminary Diagram

Section 1 Introduction

Consider the diagram of a single, massive, incoming particle of mass $M_1 > M_z$, which decays to M_z and an outgoing 'quark'. Assign Spin zero to all external particles. Allow the outgoing 'quark' to have mass, M_5 .

Section 2 The Diagrams

This four point lepton scattering diagram will serve to illustrate the notation used for both diagrams in the problem.



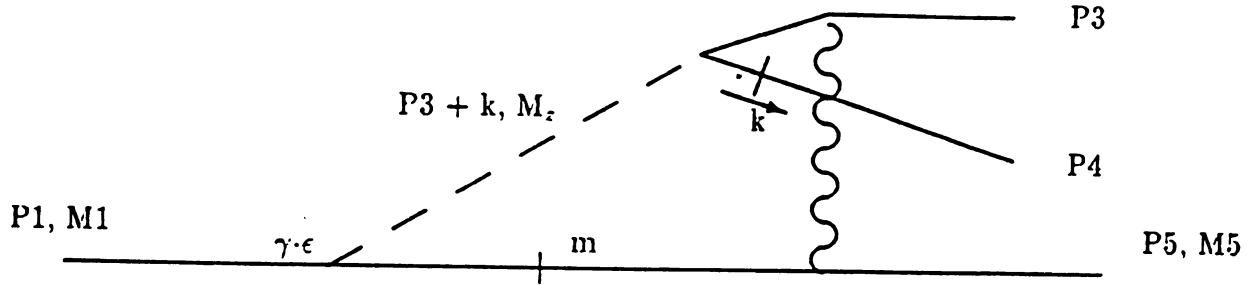
The linear momenta are labelled clockwise $P_1 \dots P_5$ beginning with the GLUON. k_μ is defined as the variable LOOP momenta and for convenience:

$$m = k - (P_4 + P_5)$$

Four vector dot products are indicated by the contiguous writing of two four vectors, for example, $P_4 P_5$ indicates the dot product of P_4 with P_5 .

The three part of a four vector is indicated by a bar; e.g. $\overline{P_4}$ is the space part of P_4 .

The anti-lepton scattering diagram is:



Section 3 Preparing the Calculation

The Cutowsky² rule for an on-shell condition gives:

$$\frac{1}{(k^2) [(k - (P4 + P5))^2 + M_5^2]} \Rightarrow \frac{1}{2} (2\pi i)^2 \delta(k^2) \delta([k - (P4 + P5)]^2 + M_5^2)$$

With this, the amplitude of the sample diagram of the previous section is formed by the Feynman rules of Appendix D:

Let

$$M = S * I$$

where S collects all the multiplicative scale factors:

$$S = \frac{1}{2} (2\pi i)^2 e^2 \frac{1}{(2\pi)^4}$$

and I is the LOOP integral of the above diagram:

$$I = \int \frac{d^4 k \delta(k^2) \delta[(k - P4 - P5)^2 + M_5^2] (k + P4)_\mu g^{\mu\nu} (P4 + P5 - k + P5)_\nu}{[(k - P4)^2] [(k + P3)^2 + M_z^2 - i\Gamma M_z]}$$

Sub-Section 3.1 The Frame of Reference

The first delta function will dictate that $|\vec{k}|^2 = k o^2$.

The second, $\delta([k - (P4 + P5)]^2 + M_5^2)$, gives

$$k^2 - 2k \cdot (P4 + P5) + (P4 + P5)^2 + M_5^2 = 0$$

and with

$$k^2 = P_4^2 = 0$$

and

$$P_5^2 = -M_5^2$$

results in

$$k \cdot (P_4 + P_5) = P_4 P_5$$

which assumes a particularly simple form only in the rest frame of $\overline{P}_4$ and $\overline{P}_5$:

$$k_0(E_4 + E_5) = -P_4 P_5$$

And, since also in this frame:

$$(P_4 + P_5)^2 = -(E_4 + E_5)^2 = 2(P_4 P_5) - M_5^2$$

from the above result

$$-\frac{(P_4 P_5)^2}{k_0^2} = 2(P_4 P_5) - M_5^2$$

then

$$k_0^2 = \frac{(P_4 P_5)^2}{M_5^2 - 2(P_4 P_5)}$$

establishing that in this frame k_0 is CONSTANT.

The enormous simplification gained when the magnitude of $\vec{k}$ remains constant dictates this rest frame of $\overline{P}_4 + \overline{P}_5$ to be the frame of reference for the lepton diagram (and similarly $\overline{P}_3 + \overline{P}_5$ for the anti-lepton).

Note that in this frame the energy, E_x , of any other particle, P_X , of the problem is also simply given:

$$P_X \cdot (P_4 + P_5) = -E_x(E_4 + E_5) = E_x \frac{P_4 P_5}{k_0} = P_X P_4 + P_X P_5$$

or

$$k_0(E_x) = \frac{k_0^2}{P_4 P_5} (P_X P_4 + P_X P_5)$$

Sub-Section 3.2 The SPIN Factor

The SPIN factor in the above integral can be evaluated to:

$$\begin{aligned}
 (k + P4)_\mu g^{\mu\nu} (P4 + P5 - k + P5)_\nu \\
 &= 2(P5 \cdot k) + 2(P4P5) \\
 &= 2\overline{P5} \cdot \bar{k} - 2(ko)E5 + 2(P4P5)
 \end{aligned}$$

then using the above equivalent for $(ko)E5$

$$= 2\overline{P5} \cdot \bar{k} - 2 \frac{ko^2}{P4P5} (P4P5 - M_5^2) + 2(P4P5)$$

Sub-Section 3.3 Removal of the Delta functions

Using

$$d^4k = d|k| dko ko^2 d\Omega_k$$

The delta functions may be simply removed by integrating over $d|k|$ and dko provided the JACOBIAN between the set $(|k|, ko)$ and the two arguments of the delta functions is included. This Jacobian is the same as for the larger problem. It is calculated in the chosen frame of reference in Appendix F and evaluates to:

$$J = \frac{-1}{4(P4P5)}$$

The Jacobian is now absorbed into the scale factor S:

$$\begin{aligned}
 S &= \frac{1}{2} (2\pi i)^2 \epsilon^2 \left[\frac{1}{(2\pi)^4} \right] \frac{-1}{4(P4P5)} \\
 S &= \frac{1}{137} \frac{1}{(2\pi)} \frac{1}{4(P4P5)}
 \end{aligned}$$

Thus the sample LOOP integral reduces to being over $ko^2 d\Omega_k$. ko^2 may be absorbed into the scale factor:

$$S = \frac{1}{137} \frac{1}{(2\pi)} \frac{ko^2}{4(P4P5)}$$

leaving the integral over the directions of $\bar{k}$:

$$I = \int \frac{d\Omega_k (k + P4)_\mu g^{\mu\nu} (P4 + P5 - k + P5)_\nu}{[(k - P4)^2] [(k + P3)^2 + M_z^2 - i\Gamma M_z]}$$

with the understanding that

(i) the frame of reference is $\overline{P4} + \overline{P5} = 0$

(ii) $|\bar{k}|^2 = k o^2$

(iii) $k o^2 = \frac{(P4P5)^2}{M_z^2 - 2(P4P5)}$

(iv) $k o(E_x) = \frac{k o^2}{P4P5} (PXP4 + PXP5)$

This applies to both LOOP integrals in the problem.

Sub-Section 3.4 The Divergence Regulation

In examining the above LOOP integral, the first denominator factor, arising from the photon line propagator, produces a singularity when the direction of $\bar{k}$ becomes co-linear with $\overline{P4}$, resulting in a logarithmically infinite value for the integral. This infinity does not occur in Nature - the process is finite - and that is reflected in the calculation by the coefficient of the logarithmically infinite term becoming zero when the LEPTON scattering and the ANTI-LEPTON scattering LOOP diagrams are summed. To allow the doing of the diagrams separately, a small photon mass is added to regularize the infinity. When the diagrams are added it is observed that the coefficient of the log of this fictitious photon mass becomes identically zero, producing no ill effects when the photon mass is then allowed to go to zero and its log to infinity. The appearance of the zero coefficient does in fact serve as a check that the various parts of the calculation are fitting together correctly.

$$I = \int \frac{d\Omega_k (k + P4)_\mu g^{\mu\nu} (P4 + P5 - k + P5)_\nu}{[(k - P4)^2 + M_\gamma^2] [(k + P3)^2 + M_z^2 - i\Gamma M_z]}$$

Section 4 ... The Calculation

$$I = \int \frac{d\Omega_k [2(P5 \cdot k) + 2(P4P5)]}{(-2P4 \cdot k + M_\gamma^2) (2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

In a co-ordinate system with $\overline{P4}$ in the $+z$ direction and using the above expansion of the SPIN factor with $\overline{P5} = -\overline{P4}$:

$$= \int \frac{d\Omega_k [-2|\overline{P5}| k o \cos \theta - 2 \frac{k o^2}{|\overline{P4P5}|} (P4P5 - M_\gamma^2) + 2(P4P5)]}{(-2k o^2 \cos \theta + 2k o^2 + M_\gamma^2) (2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$= \int \frac{d\Omega_k (a \cos \theta + b)}{(c \cos \theta + d) (2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

where

$$a = -2|\overline{P5}| k o$$

and since $|\overline{P5}| = |\overline{P4}| = E4 = k o$

$$a = -2k o^2$$

$$b = -2 \frac{k o^2}{(P4P5)} (P4P5 - M_\gamma^2) + 2(P4P5)$$

$$c = -2k o^2 = a$$

$$d = 2k o^2 + M_\gamma^2 = -c + M_\gamma^2$$

$$I = \int \frac{d\Omega_k \left(\frac{a}{c} [c \cos \theta] + \frac{ad}{c} \right)}{(c \cos \theta + d) (2P3 \cdot k + M_z^2 - i\Gamma M_z)} + \int \frac{d\Omega_k \left(b - \frac{ad}{c} \right)}{(c \cos \theta + d) (2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

$$I = I_a + I_b$$

First solve I_a :

$$I_a = \frac{a}{c} \int \frac{d\Omega_k (c \cos \theta + d)}{(c \cos \theta + d) (2P3 \cdot k + M_z^2 - i\Gamma M_z)} = \int \frac{d\Omega_k}{(2P3 \cdot k + M_z^2 - i\Gamma M_z)}$$

noting $\frac{a}{c} = 1$.

As this is integrated over all angles, the co-ordinate system may be switched to

$\overline{P3}$ in the $+z$ direction.

$$\begin{aligned}
 I_a &= \int_{-1}^{+1} \frac{d\phi d(\cos\theta)}{2(E3)ko \cos\theta - 2(E3)ko + M_z^2 - i\Gamma M_z} \\
 &= \frac{-2\pi}{2(E3)ko} \log \left(\frac{-4(E3)ko + M_z^2 - i\Gamma M_z}{M_z^2 - i\Gamma M_z} \right) \\
 I_a &= -\frac{2\pi(P4P5)}{ko^2(2P3P4 + 2P3P5)} \log \left(\frac{-4\frac{ko^2}{P4P5}(P3P4 + (P3P5) + M_z^2 - i\Gamma M_z)}{M_z^2 - i\Gamma M_z} \right)
 \end{aligned}$$

Now to solve I_b . It remains in the original co-ordinate system of $\overline{P4}$ in the $+z$ direction and taking $\overline{P3}$ to be in the x - z plane:

$$I_b = (b - \frac{ad}{c}) \int \frac{d\Omega_k}{(-2ko^2 \cos\theta + 2ko^2 + M_z^2) (2P3_z(ko) \cos\theta + 2P3_x(ko) \sin\theta \cos\phi - 2(ko)E3 + M_z^2 - i\Gamma M_z)}$$

Using $\frac{a}{c} = 1$

$$I_b = (b - d) \int \frac{d\Omega_k}{(c \cos\theta + d) (e \cos\theta + f \sin\theta \cos\phi + g)}$$

$$\text{noting } P3_z = \frac{\overline{P3} \cdot \overline{P4}}{E4} = \frac{P3P4}{E4} + E3 = \frac{P3P4}{ko} + E3$$

then

$$\begin{aligned}
 c &= -2ko^2 \\
 d &= 2ko^2 + M_z^2 = -c + M_z^2 \\
 e &= 2(ko)P3_z = 2(P3P4) + 2(ko)E3 = 2(P3P4) + \frac{2ko^2}{P4P5}(P3P4 + P3P5)
 \end{aligned}$$

$$g = \frac{-2ko^2}{P4P5}(P3P4 + P3P5) + M_z^2 - i\Gamma M_z$$

$$e^2 + f^2 = (2(ko)E3)^2 = \left[\frac{2ko^2}{P4P5}(P3P4 + P3P5) \right]^2$$

then using

$$\int_0^{2\pi} \frac{d\phi}{a + b \cos\phi} = \frac{2\pi}{(a^2 - b^2)^{1/2}} \quad a > b > c$$

$$I_b = (b-d)2\pi \int_{-1}^1 \frac{dx}{(cx+d) ((e^2+f^2)x^2 + 2ge x + g^2 - f^2)^{1/2}}$$

Then transforming $x = cx + d$

$$I = (b-d)2\pi \int_{-c+d}^{c+d} \frac{dx}{x (mx^2 + nx + p)^{1/2}}$$

where

$$m = e^2 + f^2$$

$$n = 2(gec - dm)$$

$$p = d^2m - 2gedc + c^2(g^2 - f^2)$$

which is solved using

$$\int \frac{dx}{xX^{1/2}} = \frac{-1}{\sqrt{p}} \log \frac{2\sqrt{pX} + nx + 2p}{x}$$

$$X = mx^2 + nx + p \quad p > 0$$

$$I_b = \frac{(b-d)2\pi}{c(g+e)} \left[\log \left(\frac{(g + \sqrt{e^2 + f^2})^2}{(g+e)^2} \right) + \log \left(\frac{g - \sqrt{e^2 + f^2}}{g + \sqrt{e^2 + f^2}} \right) - \log \left(\frac{-2c}{M_z^2} \right) \right]$$

Now noting

$$\begin{aligned} g + e &= 2(P3P4) + M_z^2 - i\Gamma M_z \\ &= MZP \end{aligned}$$

using the symbol MZP for the Z particle propagator

$$MZP = 2(P3P4) + M_z^2 - i\Gamma M_z$$

$$g + \sqrt{e^2 + f^2} = M_z^2 - i\Gamma M_z$$

$$g - \sqrt{e^2 + f^2} = -\frac{4ko^2}{P4P5}(P3P4 + P3P5) + M_z^2 - i\Gamma M_z$$

$$\begin{aligned} \frac{b-d}{c} &= \frac{-2ko^2(P4P5 - M_z^2) + 2(P4P5)^2}{-2ko^2(P4P5)} + 1 \\ &= \frac{-2(P4P5)}{ko^2} \end{aligned}$$

gives:

$$I_b = \frac{-4\pi(P4P5)}{(ko^2)MZP} \left[\log \left(\frac{(M_z^2 - i\Gamma M_z)^2}{MZP^2} \right) + \log \left(\frac{-\frac{4ko^2}{P4P5}(P3P4 + P3P5) + M_z^2 - i\Gamma M_z}{M_z^2 - i\Gamma M_z} \right) - \left(\log \frac{4ko^2}{M_z^2} \right) \right]$$

Combining I_a and I_b , folding in the scale factor $S = \frac{ko^2}{137(2\pi)4(P4P5)}$, and replacing $ko^2 = \frac{P4P5^2}{[M5^2 - 2(P4P5)]}$:

$$\begin{aligned} M &= S * I = S * (I_a + I_b)M = \frac{1}{4(137)} \text{ times} \\ &\frac{-2}{MZP} \left[\log \left(\frac{(M_z^2 - i\Gamma M_z)^2}{MZP^2} \right) \right] \\ &- \left(\frac{2}{MZP} + \frac{1}{2(P3P4) + 2(P3P5)} \right) \left[\log \left(\frac{-\frac{4(P4P5)}{[M5^2 - 2(P4P5)]}(P3P4 + P3P5) + M_z^2 - i\Gamma M_z}{M_z^2 - i\Gamma M_z} \right) \right] \\ &- \left(\frac{2}{MZP} \right) \left[-\log \left(\frac{(P4P5)^2}{[M5^2 - 2(P4P5)][M_z^2]} \right) \right] \end{aligned}$$

The anti-lepton scattering diagram produces the identical result with P4 and P3 interchanged and an overall minus sign to account for traversing the variable LOOP momentum k_μ in the opposite direction when writing the lepton line TRACE.

When the two diagrams are combined, the leading term, which is symmetrical in P3 and P4, vanishes, and the co-efficient of the $\log(M_z^2)$ term becomes zero, leaving:

$$\begin{aligned} M_{both} &= \frac{1}{4(137)} \text{ times} \\ &- \left(\frac{2}{MZP} + \frac{1}{2(P3P4) + 2(P3P5)} \right) \left[\log \left(\frac{-\frac{4(P4P5)^2}{[M5^2 - 2(P4P5)]}(P3P4 + P3P5) + M_z^2 - i\Gamma M_z}{M_z^2 - i\Gamma M_z} \right) \right] \\ &+ \left(\frac{2}{MZP} + \frac{1}{2(P3P4) + 2(P4P5)} \right) \left[\log \left(\frac{-\frac{4(P3P5)^2}{[M5^2 - 2(P3P5)]}(P3P4 + P4P5) + M_z^2 - i\Gamma M_z}{M_z^2 - i\Gamma M_z} \right) \right] \\ &- \left(\frac{2}{MZP} \right) \left[-\log \left(\frac{(P4P5)^2[M5^2 - 2(P3P5)]}{(P3P5)^2[M5^2 - 2(P4P5)]} \right) \right] \end{aligned}$$

First to examine the effects of $M5^2$, the mass of the outgoing 'quark'. Remembering that dot products in this metric are negative, it is clear from its

position in the factors that the matrix element obtains its greatest value when M5 is zero ! With that replacement we have:

$$M_{both} = \frac{1}{4(137)} \text{ times}$$

$$- \left(\frac{2}{MZP} + \frac{1}{2(P3P4 + P3P5)} \right) \left[\log \left(1 + \frac{2(P3P4 + P3P5)}{M_i^2 - i\Gamma M_i} \right) \right]$$

$$+ \left(\frac{2}{MZP} + \frac{1}{2(P3P4 + P4P5)} \right) \left[\log \left(1 + \frac{2(P3P4 + P4P5)}{M_i^2 - i\Gamma M_i} \right) \right]$$

$$- \left(\frac{2}{MZP} \right) \left[-\log \left(\frac{P4P5}{P3P5} \right) \right]$$

To examine the effect of setting either $(P3P4 + P3P5)$ or $(P3P4 + P4P5)$ to zero and thereby gaining a large co-efficient, noting that $\log(1+x) \Rightarrow x$ as $x \rightarrow 0$ shows that these terms will at most become unity.

It is the final term that dominates:

$$M_{both} = \frac{1}{4(137)} \left(\frac{2}{MZP} \right) \left[\log \left(\frac{P4P5}{P3P5} \right) \right]$$

Notice this term, (as well as the terms above) goes to zero when $P3P5 = P4P5$, as it should, for in this case there is no distinguishing the lepton scattering diagram from the anti-lepton one, and the two are subtracted.

It becomes a maximum when either $P3P5$ or $P4P5$ is zero, indicating one of the leptons is co-linear (assuming $M5 = 0$) with the outgoing 'quark'.

Conservation of energy-momentum dictates that:

$$M1^2 = (P3 + P4 + P5)^2$$

$$= 2(P3P4 + P3P5 + P4P5)$$

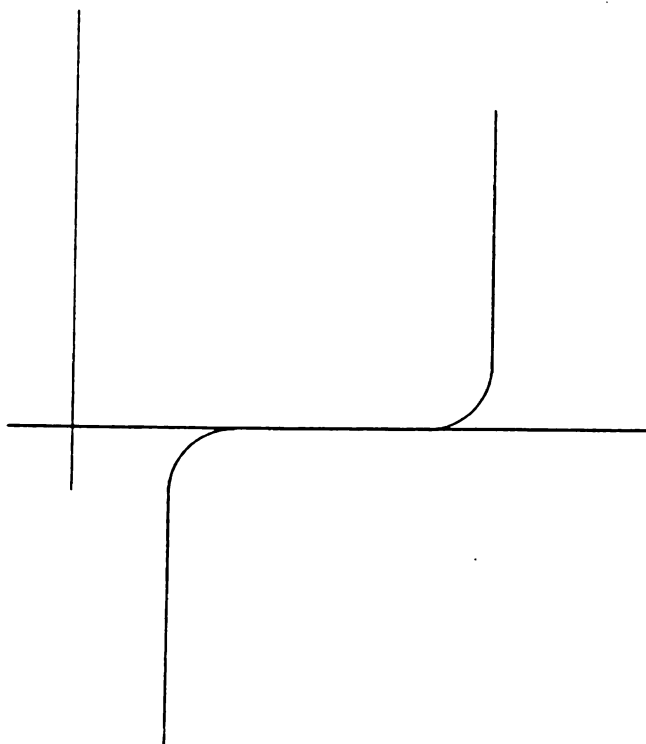
so that $P3P5$ and $P4P5$ must add to a constant:

$$P3P5 + P4P5 = \frac{M1^2}{2} - P3P4$$

$$P3P5 = \left[\frac{M1^2}{2} - P3P4 \right] - P4P5$$

$$\log \left(\frac{P4P5}{P3P5} \right) = \log \left[\frac{\frac{2(P4P5)}{M1^2 - 2(P3P4)}}{1 - \frac{2(P4P5)}{M1^2 - 2(P3P4)}} \right]$$

Thus $\log \frac{P_4 P_5}{P_3 P_5}$ graphs as $\log(\frac{x}{1-x})$ vs. x with $x = \frac{2(P_4 P_5)}{M^2 - 2(P_3 P_4)}$:



To realize the divergence does not contribute (due to the disappearance of phase space where it occurs), find the average between 0 and 1/2:

$$\text{Average} = \frac{1}{L} \int_0^L f(x) dx$$

$$\begin{aligned} \text{Average} &= \frac{1}{1/2} \left(\int_0^{1/2} \log\left(\frac{x}{1-x}\right) dx \right) \\ &= 2 \left(\int_0^{1/2} \log(x) dx - \int_0^{1/2} \log(1-x) dx \right) \end{aligned}$$

and using:

$$\int \log(x) dx = x \log(x) - x$$

and l'Hopital's rule:

$$\text{limit as } x \rightarrow 0 \text{ of } x \log(x) = \frac{\log(x)}{1/x} = \frac{1/x}{-1/x^2} = 0$$

gives

$$Average = 2 \left(\log \left(\frac{1}{2} \right) - 1 \right)$$

so approximately,

$$Average = -4$$

so an 'average' value of the matrix element is

$$M_{both} = (4) \frac{1}{4(137)} \frac{1}{MZP}$$

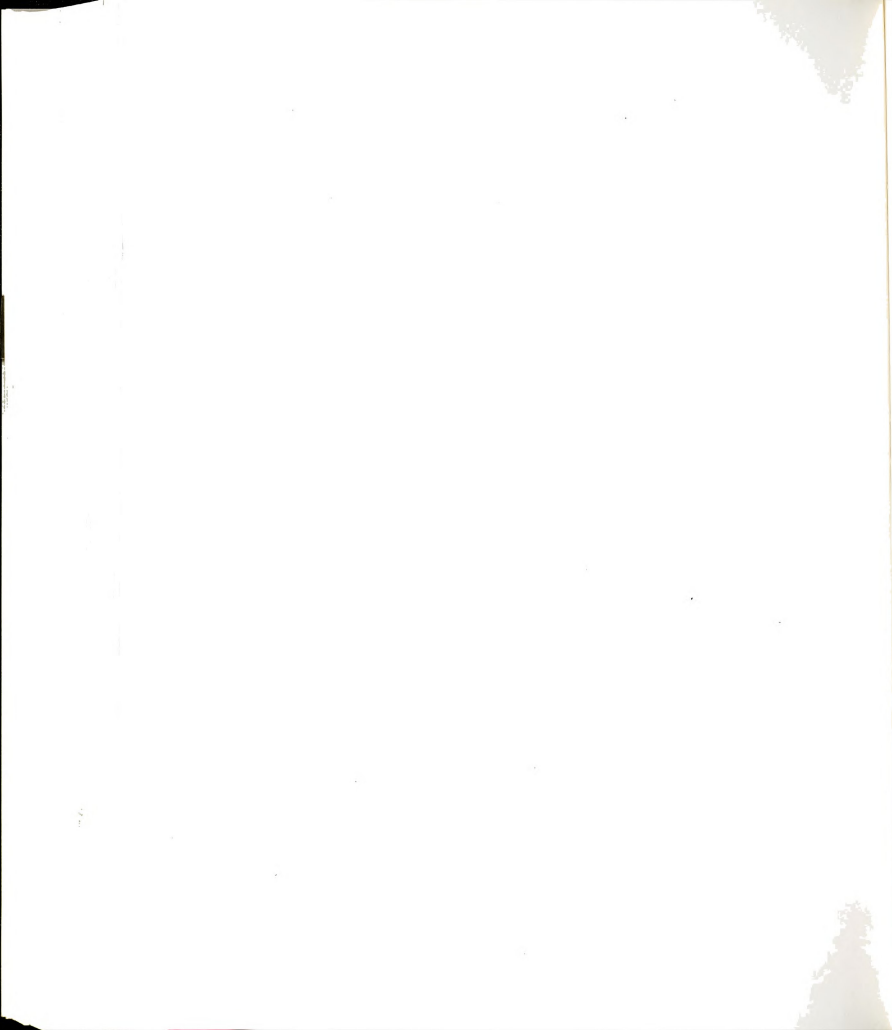
and since the matrix element for the same diagram without scattering (the BORN term) is just:

$$M_{Born} = \frac{1}{MZP}$$

then the RATIO of $\frac{|Born+Loop|^2}{|Born|^2}$ becomes:

$$RATIO = 1 + 2 \left(\frac{4}{4(137)} \right) + \left(\frac{4}{4(137)} \right)^2$$

so the scattering diagrams can expect to contribute on the order of 1.0 per cent.





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