This is to certify that the

dissertation entitled

Numerical Study Of An Underground
Heat Tube

presented by

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has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Mechanical Engineering

Merle C. Potter
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Date 10/16/89

NUMERICAL STUDY
OF
AN UNDERGROUND HEAT TUBE

By

Fauziah Sulaiman

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Mechanical Engineering

1989

604361 6

ABSTRACT

NUMERICAL STUDY OF AN UNDERGROUND HEAT TUBE

By

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The energy consumption of the air-to-air heat pump can be reduced, especially in winter, by using the soil as a heat source. A system of buried tube through which air is passed, has great potential in suppling higher temperature air than the ambient air, to the outside heat exchanger of the heat pump.

Heat transfer from the soil to the tube, including the possibility of formation of ice lenses around the tube, was investigated over a period of time in a cold season. Models of ice formation were developed in two types of tube, the circular tube and the square tube. Latent heat released due to the formation of ice were included in the models. Computer simulation utilizing finite difference equations were developed, using the explicit method, where forward differences were used in time and central differences were used in space.

The numerical results show the effects of increasing the moisture content of the soil, increasing the air flow rate in the tube, and the release of latent heat when soil freezes. The formation of ice around the tube

played a significant role in achieving a stabilized output air temperature at a short time.

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ACKNOWLEDGEMENTS

I would like to sincerely thank Dr. Merle C. Potter, my major advisor and Chairman of the doctoral committee, for the expert guidance, suggestions and knowledge he provided me throughout the research, as it made this research both a pleasing and rewarding experience.

Sincere thanks go to my doctoral committee, Dr. J. Beck, Dr. G. Ludden, and Dr. C. Somerton for their suggestions, time and support throughout this research.

Special thanks go to my husband and children, for their support, patience, and prayers throughout the entirety of my doctoral study. My family in Malaysia, especially my mother, deserves special recognition, for their support and prayers.

Finally, I would like to thank the Public Services Department of Malaysia and the University of Science Malaysia for providing me with a scholarship that made it possible for me to accomplish my doctoral degree.

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NOMENCLATURE

α	Thermal diffusivity
c	Heat capacity
h	Heat transfer coeeficient from soil to air in tube
h_{ice1}	Ice thickness above tube in 2-D
h_{ice2}	Ice thickness below tube in 2-D
h_{sur}	Heat transfer coefficient from soil to air at ground surface
k	Thermal conductivity
L	Latent heat of fusion of water
M	Moisture content of soil as a percentage by weight
m	mass
Nu	Nusselt number
ρ	Density
Pr	Prandtl number
r	Radial coordinate
r_{ice}	Ice thickness in 1-D
R	Radius of ice from center of tube
Ra	Rayleigh number
Re	Reynolds number
T	Temperature
t	Time
u	Velocity of air in the tube

x	Cartesian x coordinate
y	Cartesian y coordinate
z	Cartesian z coordinate
Δ	Change
∂	Partial differential

Subscripts

a	Air
atm	Atmosphere
D	Diameter
F	Farfield
L	Length
s	Soil
zsur	Location at ground surface

Array Indices

i	x direction
j	r direction in 1-D, y direction in 2-D
k	time in 1-D, z direction in 2-D
l	time in 2-D

CHAPTER 1

INTRODUCTION

1.1 Background

Due to the rising costs of gas and oil compared to the cost of electricity, many air-to-air heat pumps have been installed for residential use. These heat pumps are used for both cooling and heating. Since electricity is used to operate the air-to-air heat pumps, a leveling in the electric utilities throughout the year shows great promise if electricity can be used for heating as well as cooling. However, both the heat output of an air-to-air heat pump and its coefficient of performance are drastically reduced during operation in very cold weather. Thus, the use of heat pumps in regions where the heating load dominates the cooling load has been slow to develop.

In order to improve the performance of a heat pump, various modifications were made to the heat exchanger. According to a study by the Oak Ridge National Laboratory (1), the performance of a heat pump is significantly improved when the size of the heat exchangers is increased and the configuration of the heat exchangers is changed. These configuration changes include the use of larger evaporator flow rates, smaller compressor displacement, smaller motor size for the compressor and the fans and finally, an increase in the number of refrigerant circuits. Installation of more efficient fans and increasing the compressor efficiency also contributes to the improvement in the performance of the heat pump.

The performance of the air-to-air heat pump is primarily dependent upon outdoor air conditions. Many improvements have been made for operation in moderate climates. However, the problems caused by the operation of a heat pump in cold weather have never been solved economically, even with the improvements mentioned above. Since the air temperature is very low during the winter months, especially in northern climates, the possibility of providing higher temperature air to the outside heat exchanger of the air-to-air heat pump is considered. This would result in higher efficiency in the performance of the heat pump and thus, would significantly increase the heating load.

1.2 Water Systems

Many types of heat pumps have been tested to provide a higher temperature thermal reservoir. Surface water and well water systems have been used with water-to-air heat pumps to provide home heating and cooling (2). In winter, ground water is warmer than air and in summer, it is cooler. Therefore, ground water acts as a heat source in the winter and a heat sink in the summer.

However, a well must be drilled to tap the water so that it can be provided to the heat exchanger.

A system installed by a physics professor at Ohio State University for his home, has been in use for over twenty-two years without many problems (3). It was initially meant as an experiment but after achieving the system's coefficient of performance of 3.5, it became a money saver. Another ground water system installed at Bear Creek, Pennsylvania, has been able to supply hot water for a 2900-square-foot house and to heat a swimming pool (4).

There are some associated problems with the use of the systems, however, making them economically unattractive. The quality and the availability of groundwater makes the use of this system limited (5). Ground water causes scaling problems to the heat exchanger and the possibility of aquifer depletion is always threatening. Therefore, another well is probably needed to return the water to the aquifer in order to overcome the problem of aquifer depletion. This system is also known as the water reinjection system (see Figure 1). This system thus becomes uneconomical due to the high drilling costs of the wells and the restrictions due to the possibility of ground water contamination when reinjection is used.

1.3 Closed-Loop, Ground-Coupled Systems

Closed-loop ground-coupled systems are divided into two basic types, horizontal and vertical systems (see

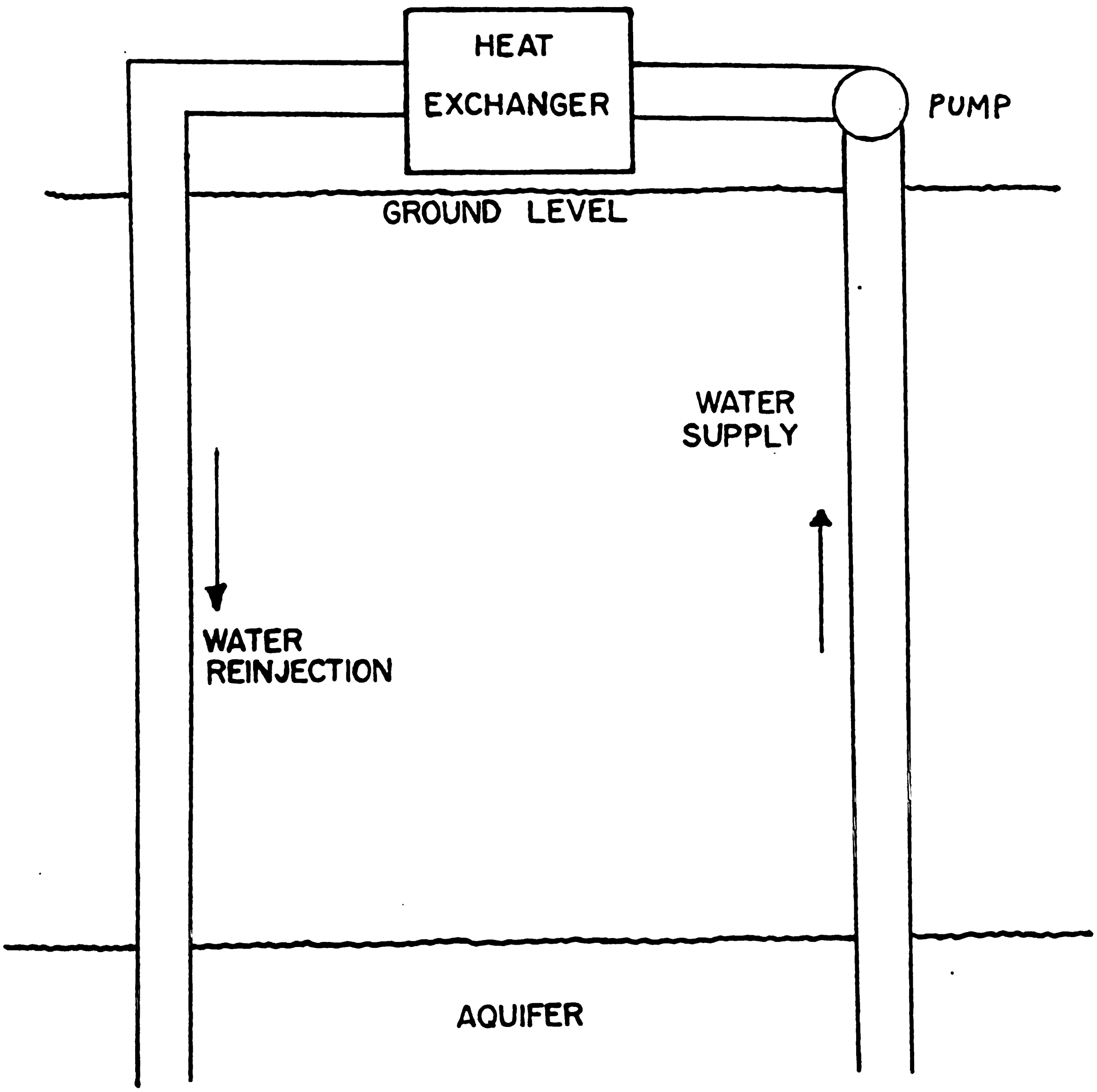

Figure 1. A Water Reinjection System.

Figures 2 and 3). A study (6) completed at Oklahoma State University has shown that a high coefficient-of-performance can be obtained by using the ground as a source or sink for a water-to-air heat pump. This is due to the relatively cool ground temperature in the summer and warm ground temperature in the winter.

A study (7) has indicated that the horizontal type which was buried at a depth of 1 meter below the ground surface, has been able to successfully supply heated water to a home in Britain. The system has been able to supply the 84 x 10^6 Btu annual demand of the residence. Another ground coupled horizontal system, buried 1.3 meters underground, was tested at Brookhaven National Laboratory (8). This system has been successful in supplying the heating demand of a residence at very low ambient temperatures without supplemental heating. The overall seasonal coefficient of performance of the system was 2.2. However, the performance of the horizontal type can be improved by utilizing several pipes and burying at greater depths.

The geometry of the vertical type is U-shaped, configured as one deep well or several shallow wells. A system with six shallow-well, U-tubes of unequal depth, which was installed at Knoxville, Tennessee, has been able to achieve an annual performance factor of 2.39, thus reducing the maximum heating season peak demand (9).

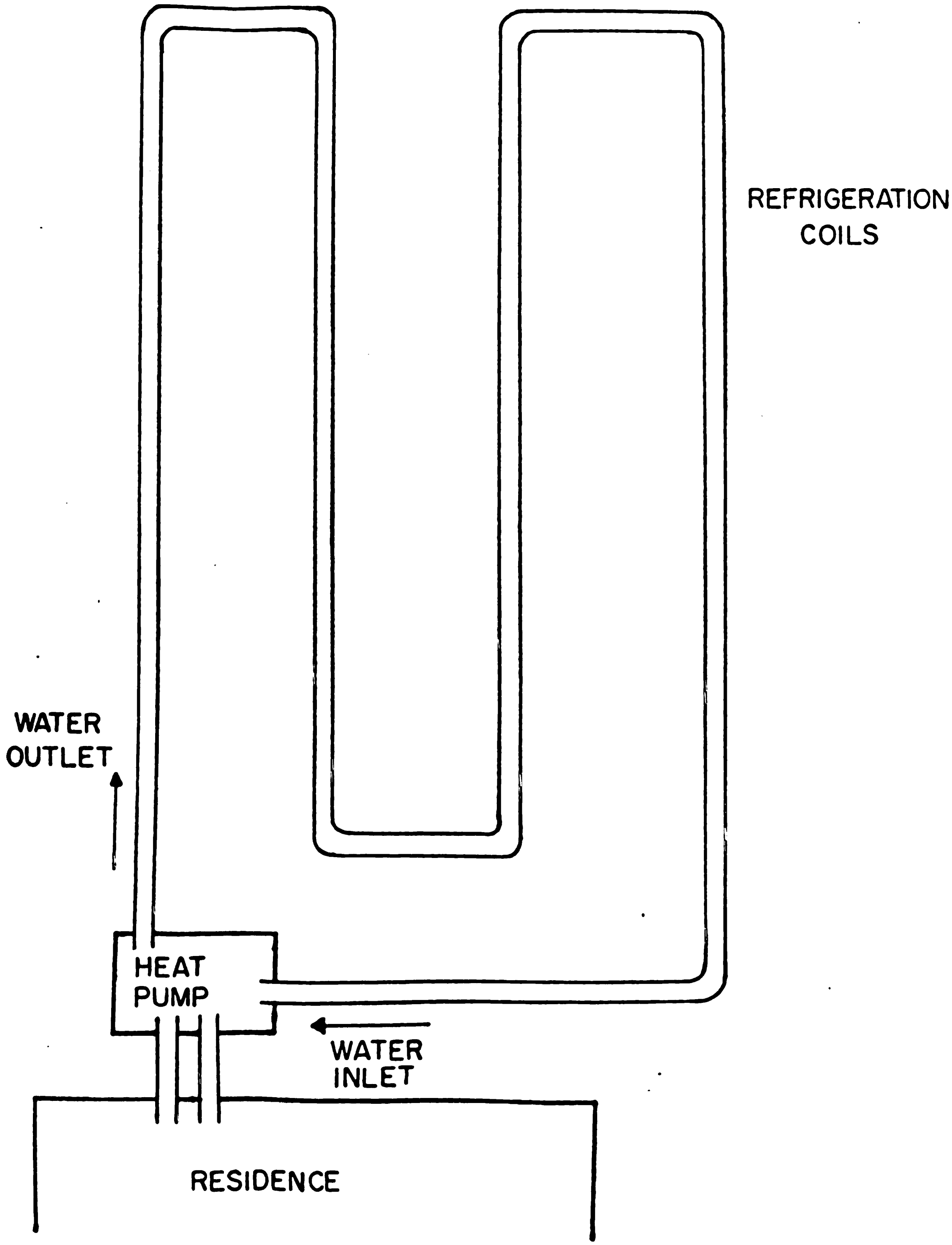

Figure 2. A Horizontal Closed-Loop, Ground-Coupled System.

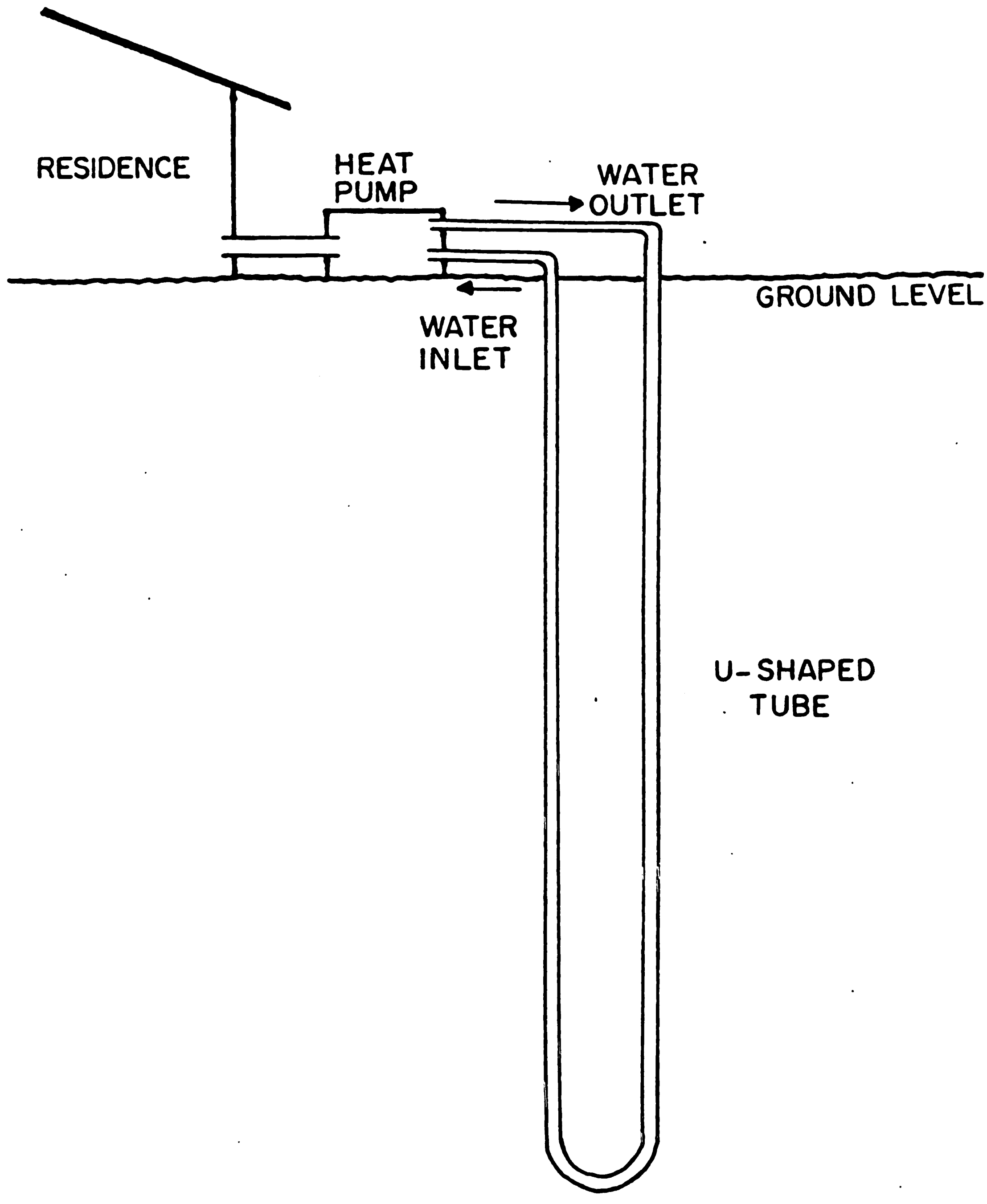

Figure 3. A Vertical Closed-Loop, Ground-Coupled System.

Another system utilizing thirty-seven shallow vertical couplings of 10 meters in length, was tested in Utby, Sweden, for determination of the ground's usefulness for seasonal heat storage (10). The resulting coefficient of performance of this system was approximately 3.0.

Oak Ridge National Laboratory has done many studies, as well as designs, on horizontal and vertical ground-coupled heat exchangers. One study on a horizontal ground-coil heat exchanger with soil freezing (11), indicated that the total energy absorbed by a fluid is higher when the fluid inlet temperature is lower than 25°F. The study showed that at the fluid inlet temperature of 26.3°F, the latent heat absorbed was 30.6% and when the fluid inlet temperature was lowered to 10°F, the latent heat absorbed increased to 63.9%. Therefore, the increase in the latent heat was the primary reason for the increase in the total energy absorbed.

Water is circulated between the heat pump and the horizontal or vertical tubes that make up the piping system that absorbs or rejects heat. In northern climates where freezing often occurs, a need of using a mixture of water and antifreeze is required for the heat pump operation. The possibility of refrigerant leaks has to be eliminated by using high quality refrigerant tubes which are quite costly. Also, small leaks must be repaired, and such work is difficult when the ground is frozen.

Because of the high initial cost of both the horizontal and vertical ground-coupled systems, and the relatively high cost of maintaining both systems, attention has been focussed in this study, on the air-to-air heat pump system.

1.4 Crawl Space Air Systems

Energy stored in the soil beneath a house can easily be utilized by using a crawl space assisted heat pump. In Alcoa, Tennessee, such a system (see Figure 4) has been tested and found to reduce the peak heating load and the peak cooling load (12). Air is first preconditioned by the crawl space soil in a recirculation mode before it is supplied to the outside air-to-air heat pump, therefore, resulting in a significant increase in the efficiency in the heat pump.

The study revealed that the crawl space maintained a temperature near 30°F even when the ambient temperature dropped below 0°F, thereby reducing peak winter utility demand whose highest electric demand coincided with the coldest ambient temperatures. It was also found that a reduction of 26.3% in the peak heating load purchased energy and 14.4% in the total heating season purchased energy when the crawl space assisted heat pump was in use. Unfortunately, despite the great potential of this system in producing higher temperature reservoir, the unavailability of crawl spaces in many homes is the major drawback.

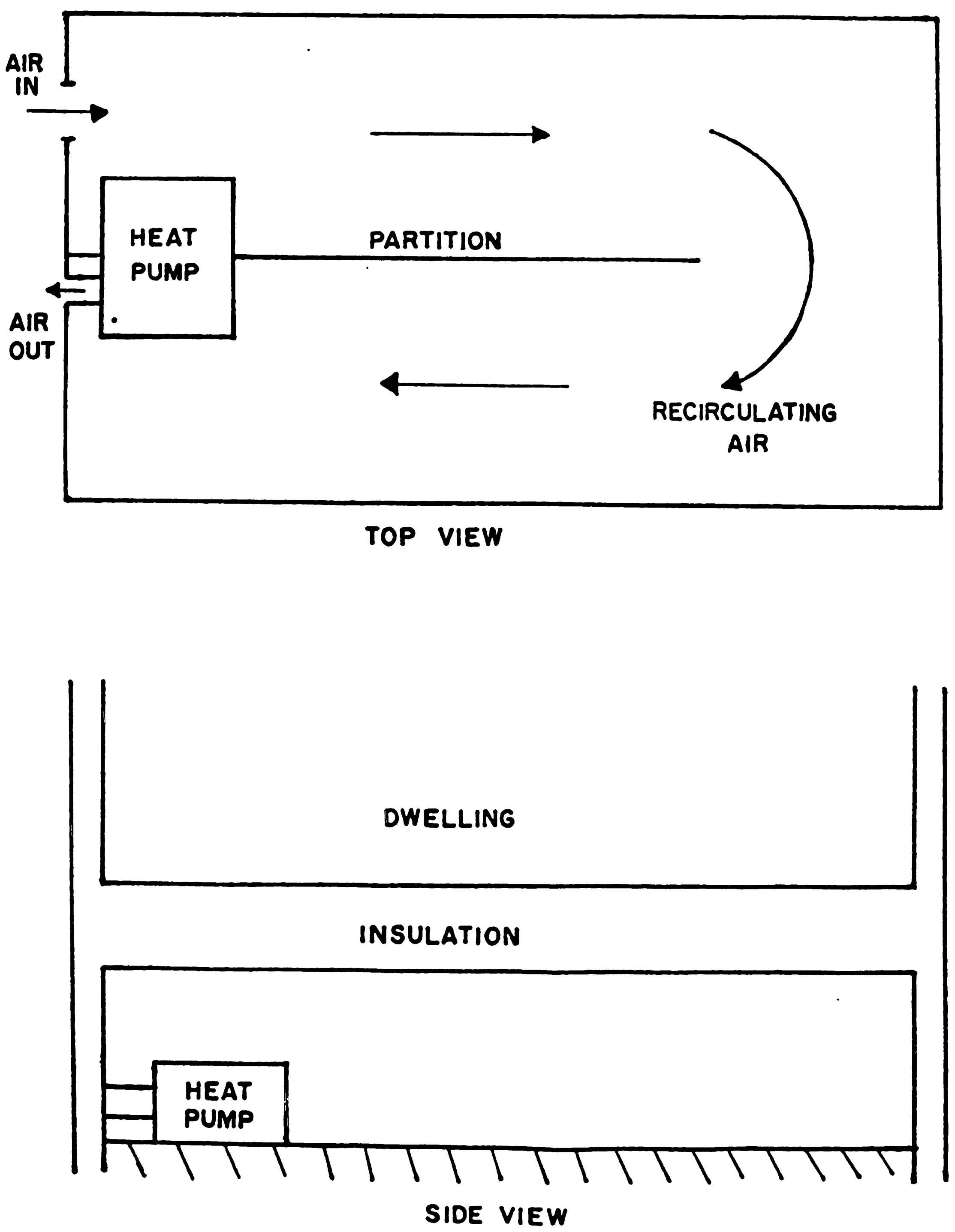

Figure 4. A Crawl Space Air System.

1.5 Buried Tube Air Systems

The energy consumption of the air-to-air heat pump can be significantly reduced, especially in the winter, by using the soil as a heat source. The potential of the soil as a moderator of the cold winter air temperatures has already been explored widely. Therefore, it is important to fully understand the heat transfer mechanisms in the soil in order to obtain the optimum benefits.

A system of buried tubes through which air is passed can easily be connected to an air-to-air heat pump, as shown in Figure 5. This system has great potential in supplying higher temperature air than the ambient air to the outside heat exchanger of the heat pump and overcoming many of the problems encountered in the other systems discussed above (13).

Heat transfer from the soil to the tube, including the formation of ice lenses around the tube, is investigated over a period of time during a cold period. Models of ice formation are developed in two types of tubes, the circular tube and the square tube; this will be described in detail in chapter 3. Computer simulation utilizing finite difference equations are developed, using the explicit method, where forward differences are used in time and central differences are used in space. Fortran Seventy-Seven programming language is used to translate the simulation. Factors that are taken into account include: the effects of the soil moisture content, the changes in

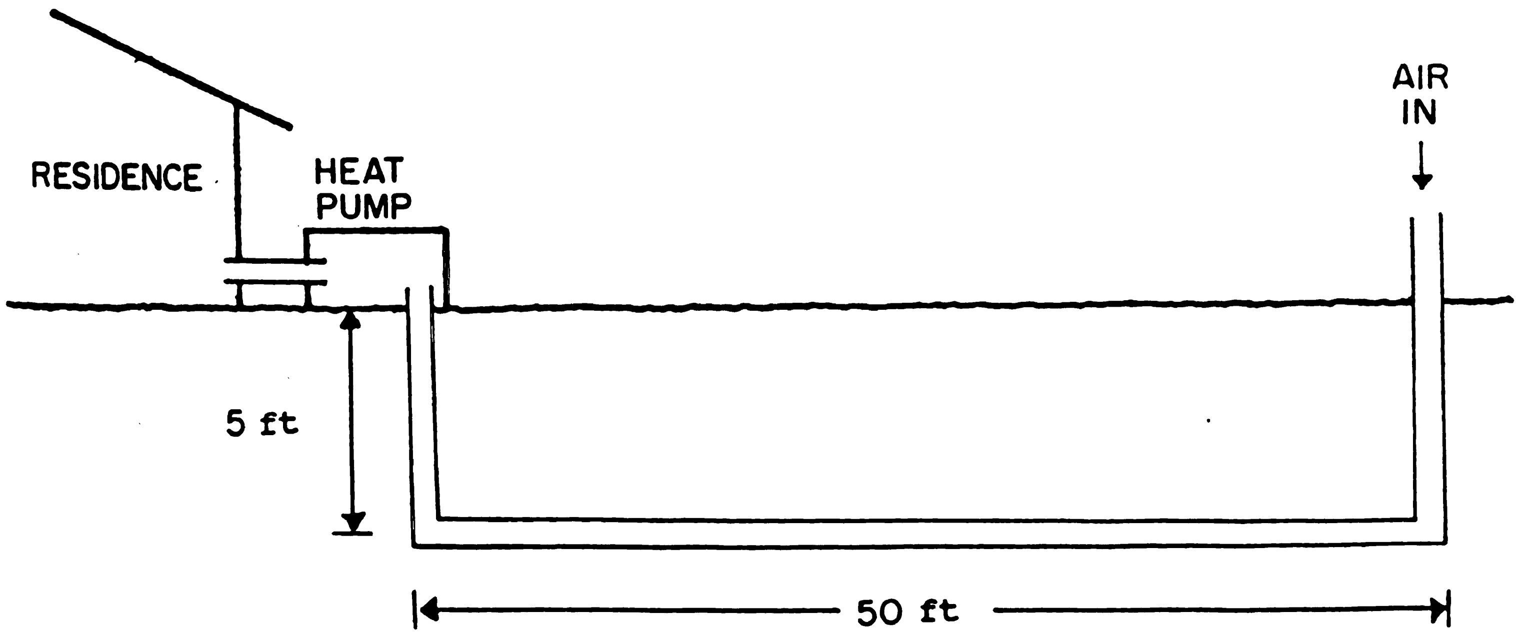

Figure 5. A Proposed Buried Tube Air System.

soil thermal properties according to the temperature and moisture content of the soil, and the release of latent heat when soil freezes. Parameters which may be varied are the moisture content of the soil, the velocity of air in the tube, the convective heat transfer coefficient, the length of the tube, and the maximum run time.

CHAPTER 2

THEORETICAL DEVELOPMENT

2.1 Introduction

Two models were developed in order to determine the heat transfer from the soil to the cold air in the buried tube. The one-dimensional model has a circular cross-sectional area and the two-dimensional model has a square cross-sectional area. The tube is buried 5 feet in the ground. It was found that during a heating season, heat transfer does not take place beyond 7.5 feet (14). Therefore, a volume of 12.5 feet depth, 15 feet in width, with 50 feet length of the pipe was chosen from which to extract the heat (see Figures 6 and 7). The initial temperature distribution in the soil was determined by using a second order polynomial derived from Neumann's Theory (15) for the two-dimensional model. In the one-dimensional model, the initial temperature distribution in the soil was assigned to be constant throughout.

2.2 Governing Heat Transfer Equations

The governing equation for the one-dimensional model is the one-dimensional transient diffusion equation

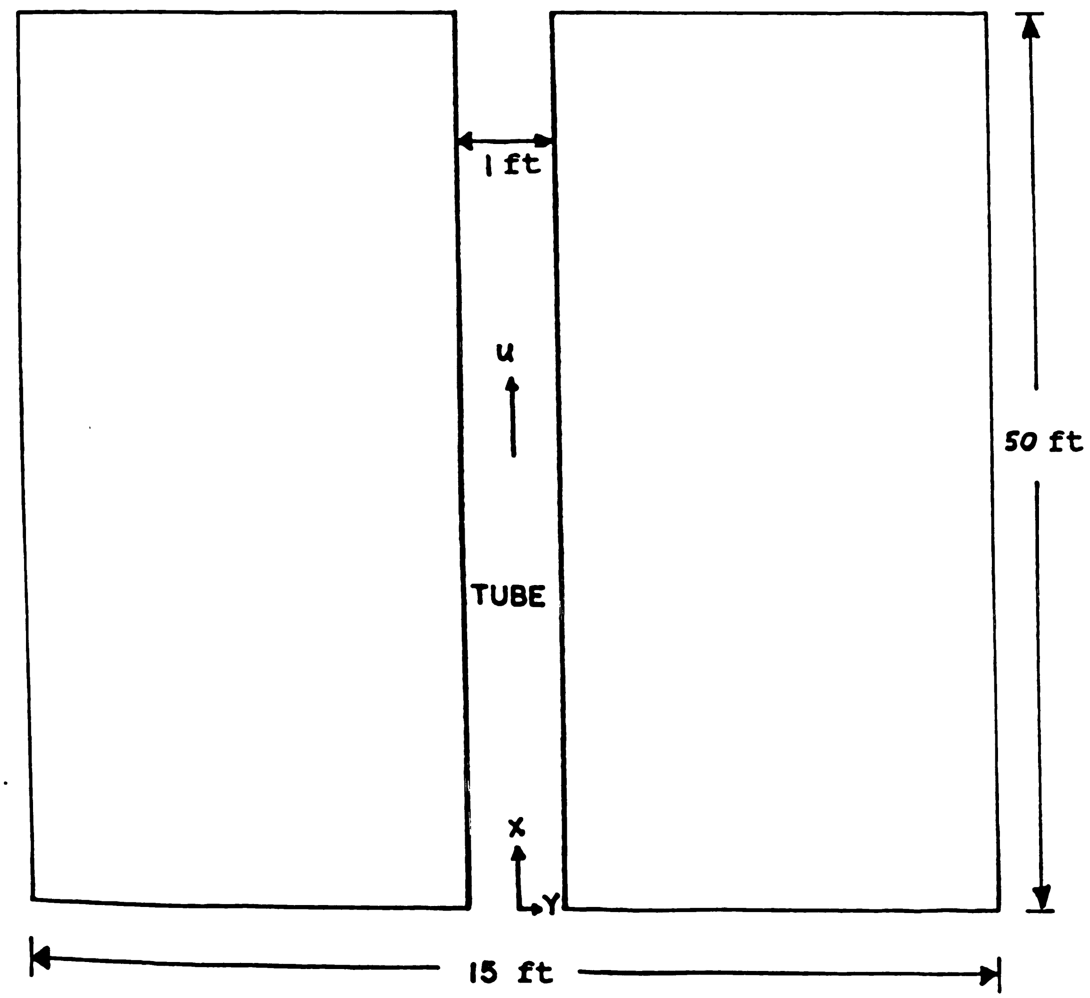

Figure 6. Top View of the Proposed Two-Dimensional Tube.

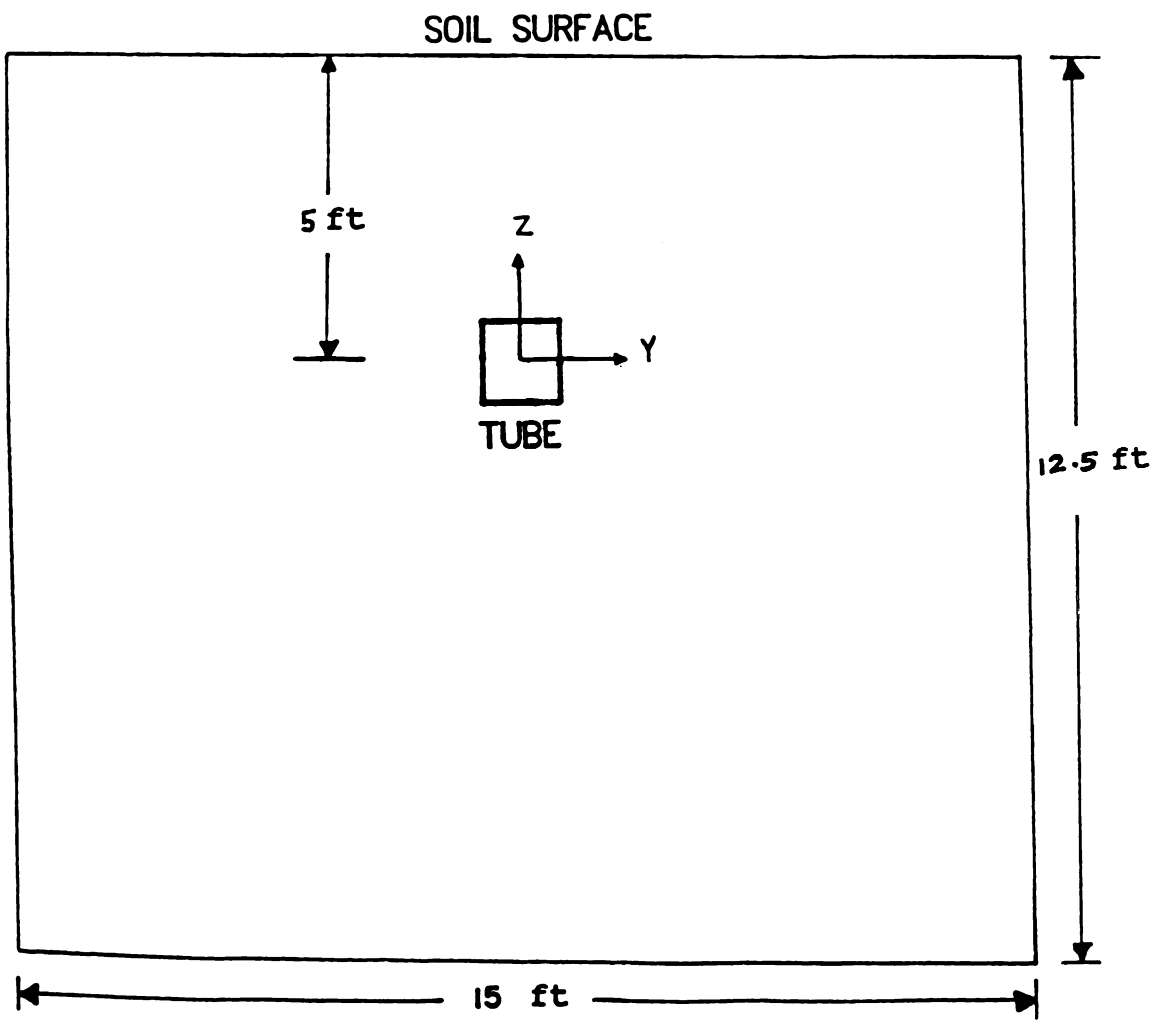

Figure 7. End View of the Proposed Two-Dimensional Tube.

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) \qquad (2.1)$$

where the thermal diffusivity is represented by

$$\alpha = \frac{k}{\rho c} \quad - \ ft^2/hr \qquad (2.2)$$

and the other parameters are as follows:

T = temperature - °F

k = conductivity - Btu /(ft-hr-°F)

ρ = density - lb_m/ft

c = specific heat - Btu/(lb_m-°F)

The governing equation for the two-dimensional model is the two-dimensional transient diffusion equation

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \qquad (2.3)$$

Heat transfer in the x-direction is quite small and has been neglected. Boundary conditions and the initial condition needed in the solution of the above diffusion equations will be presented in a later section.

A finite-difference method, which is often used in solving partial differential equations, was employed using forward differences in time and central differences in space; it is known as the explicit method. Solutions were obtained by placing a finite number of points in a selected grid pattern within the soil and air in the tube. At these points, or nodes, finite-difference equations for the temperatures were obtained by direct replacement of the partial differential equations or by an energy balance on an element. The error of the method is on the order of size of space step squared and the time step.

2.3 Finite Difference Methods

There are several finite difference methods that can be used to solve the heat diffusion equations. Descriptions of some of the methods follow:

1. The Implicit Method is similar to the explicit method outlined above, but instead, uses a backward difference approximation for the time step. It has the same order of error as the explicit method but is stable unconditionally. However, this method cannot be used to solve for a single unknown in terms of previously calculated values. It was also found that the method is not efficient for multi-dimensional problems (16).

2. The Crank Nicolson Method, which is known as the improved implicit method, is also unconditionally stable. This method averages the forward difference and backward

difference approximation for the time step. Unfortunately, this method produces a system which is no longer tridiagonal and thus, requires unnecessary computations (17).

3. Alternating Direction Implicit Method (ADI) of Peaceman and Rachford was proposed to overcome the problems encountered in the explicit and the implicit methods. This method uses an implicit method in one direction and uses an explicit method in another direction. It has, however, a bias on intermediate values and therefore is not to be greatly trusted (18).

According to a preliminary study, the purely explicit method was rated third from a comparison of nine methods of solving the heat diffusion equation (19). This was based on relative accuracy, ease of programming, computation time and computer storage requirement. However, the explicit method has a stability criterion which must not exceed 0.5. Due to this, substantial computing time will be needed since there is a need to employ small time steps. This is the method selected in this study.

2.4 Boundary and Initial Conditions

The boundary conditions in the one-dimensional model are as follows:

1. At $r = r_1$ (see Figure 14),

$$h\ (T_s - T_{air}) = k_s \frac{\partial T_s}{\partial r} \qquad (2.4)$$

where h is the convective heat transfer coefficient from the soil element to the air in the tube (the thin wall tube material was ignored) and k_s is the soil conductivity. T_s and T_{air} are the temperature of soil and air, respectively.

2. At $r = r_F$ (farfield),

$$T_s = T_F \qquad (2.5)$$

where T_F is the given constant farfield temperature.

The initial conditions at $t = 0$ are

$$\begin{aligned} T_{air} &= T_F \\ T_s &= T_F \end{aligned} \qquad (2.6)$$

where the initial temperature of air and soil are assigned to be the farfield temperature at all locations.

Boundary conditions for the two-dimensional model are:

1. At $y = h'$ and $-\frac{h'}{2} < z < \frac{h'}{2}$ (see Figure 29),

$$k_s \frac{\partial T}{\partial y} = h\ (T_s - T_{air}) \qquad (2.7)$$

where $h' = \Delta y = \Delta z$.

2. At $z = -h'$ and $-\frac{h'}{2} < y < \frac{h'}{2}$ (see Figure 18),

$$-k_s \frac{\partial T}{\partial z} = h\ (T_s - T_{air}) \qquad (2.8)$$

3. At $z = h'$ and $-\frac{h'}{2} < y < \frac{h'}{2}$ (see Figure 21),

$$k_s \frac{\partial T}{\partial z} = h\ (T_s - T_{air}) \qquad (2.9)$$

4. At $y = 0$ and $z > h'$ and $z < -h'$,

$$\frac{\partial T}{\partial y} = 0 \qquad (2.10)$$

The initial conditions are:

$$T_{air} = T_s(z = \text{tube height})$$

$$T_s = T_s(z) \qquad (2.11)$$

The air inlet condition is:

$$T_{air}(x = 0,t) = 0^\circ F \qquad (2.12)$$

As above, h is the convective heat transfer coefficient and k_s is the soil conductivity. The atmospheric air temperature is always maintained at 0°F and the soil temperature at 12.5 feet depth is always set at the deep soil temperature.

Due to symmetry, heat transfer need only be calculated on one side with a vertical plane bisecting the tube longitudinally (see Figure 8). An insulated condition is being imposed at each end of the tube since heat transfer there is assumed small and thus ignored (Figure 9). This condition is considered conservative as there is actually heat transfer occuring at each end of the tube.

2.5 Convective Heat Transfer Coefficient

The velocity of air flow in the tube is assumed to be sufficiently high to ensure a turbulent flow. The turbulent flow in the circular smooth tube is considered to be fully developed. The Nusselt number is thus calculated by using the Dittus-Boelter equation (20),

$$Nu_D = 0.023\ Re_D^{4/5}\ Pr^{0.4} \qquad (2.13)$$

where Re_D is the Reynolds number (based on diameter of the tube and average air velocity) and Pr is the Prandtl number. The convective heat transfer coefficient is thus obtained. For the square tube, the same equation is used but the

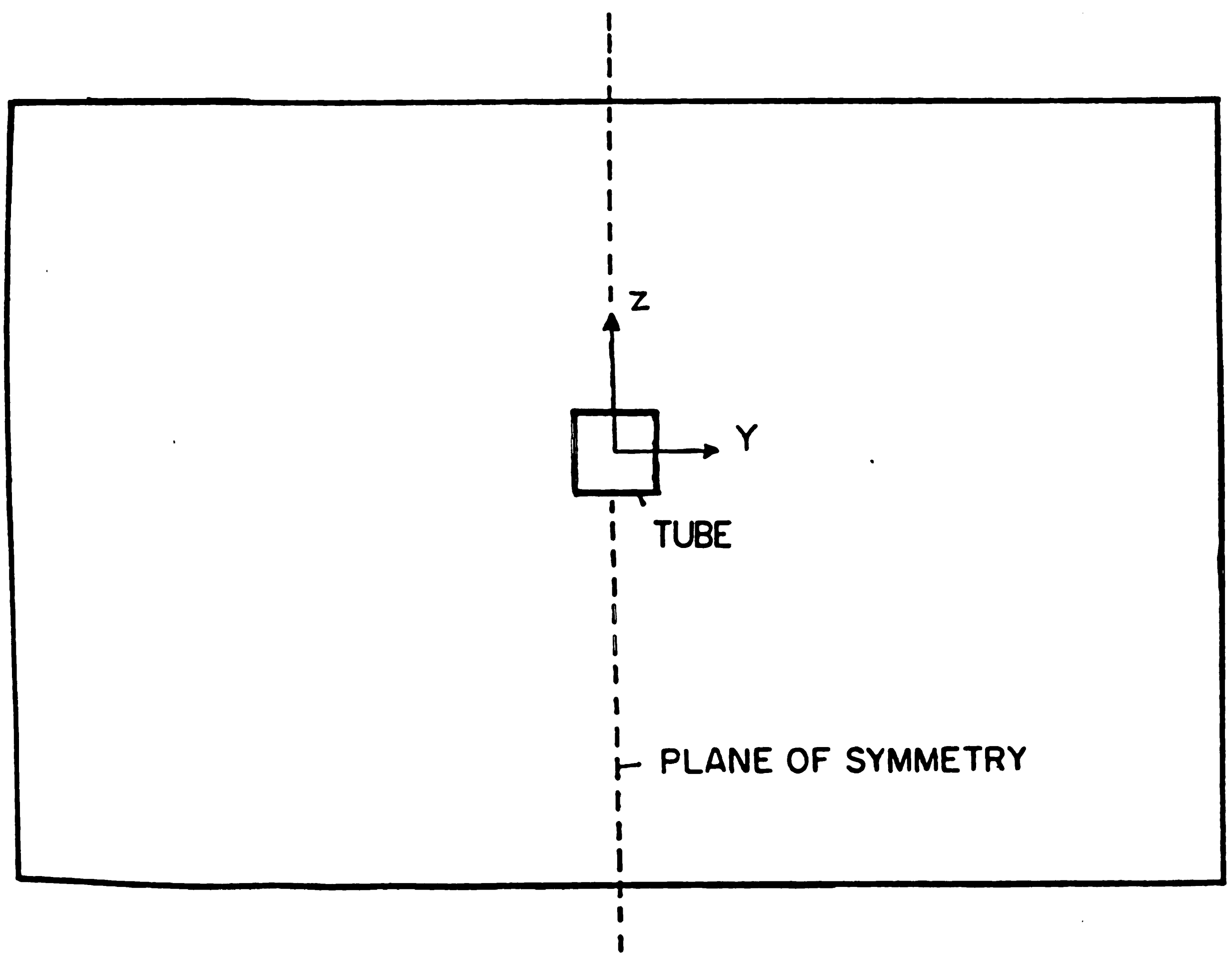

Figure 8. Plane of Symmetry for the Two-Dimensional Tube.

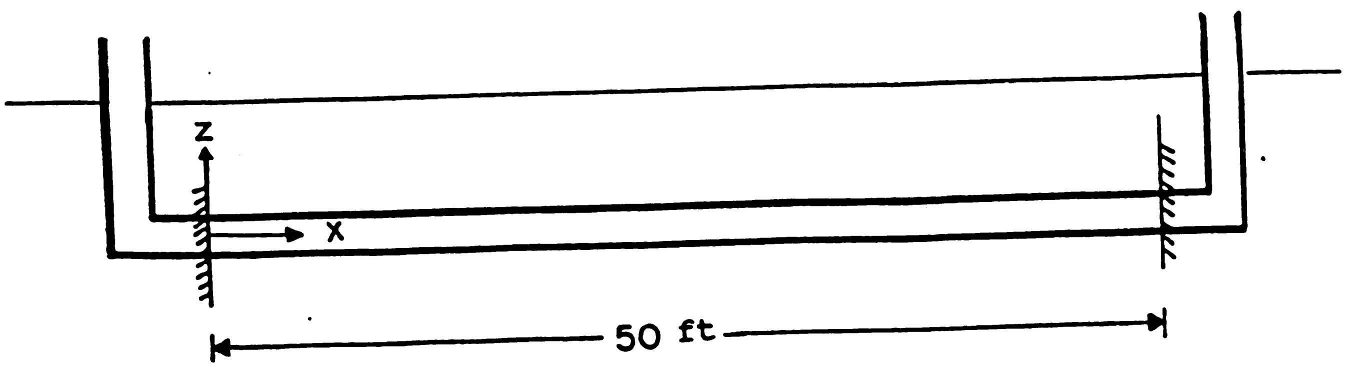

Figure 9. Insulated Condition at Both Ends of the Tube.

diameter of the circular tube is replaced by the hydraulic diameter of the square tube.

To be conservative, at the soil surface, free convection (worst case condition) is assumed. The correlation used for the calculation of Nusselt number is obtained by the improved correlations of McAdams (21),

$$Nu_L = 0.15\ Ra_L^{1/3} \qquad (2.14)$$

where Ra_L is the Rayleigh number.

2.6 Thermal Properties of the Soil

The soil conductivity and thermal diffusivity are relatively low in dry soil. However, moisture content in the soil increases the thermal conductivity and thus provides for increased heat transfer (22). In addition to moisture content, soil type and dry density must be known in order to estimate the thermal conductivity of the soil. Since there is a large variation even in similar soils, soil thermal properties were empirically determined.

During the process of freezing, heat capacity of the soil is significantly high. This is due to the release of latent heat of fusion in the moist soil. Frozen soil has the highest thermal conductivity which is 0.833 Btu/(hr-ft-°F) but a relatively low heat capacity of 0.2 Btu/(lb_m-°F). The high thermal conductivity results from the soil moisture completely changed to ice and that ice is more conductive than water. However, heat capacity of ice is lower than water, thus resulting in low heat capacity. Thawed soil has relatively low thermal conductivity and heat capacity which are 0.458 Btu/(hr-ft-°F) and 0.3 Btu/(lb_m-°F), respectively (23).

2.7 Initial Soil Temperature Distribution

Comparison with actual data of Michigan soil temperatures (24) and Neumann's theory (25), led to a linear distribution merged with a second-order polynomial for the two-dimensional model (Figure 10). It was noted that when the ground surface is frozen, the temperature variation in the frozen surface layer of the soil is linear, whereas in the thawed condition, the temperature variation in the surface layer is sinusoidal (26). However, this sinusoidal variation does not extend to a significant depth so it was not taken into account. In the one-dimensional model, the temperature in the soil was assigned a constant value equal to the constant farfield temperature.

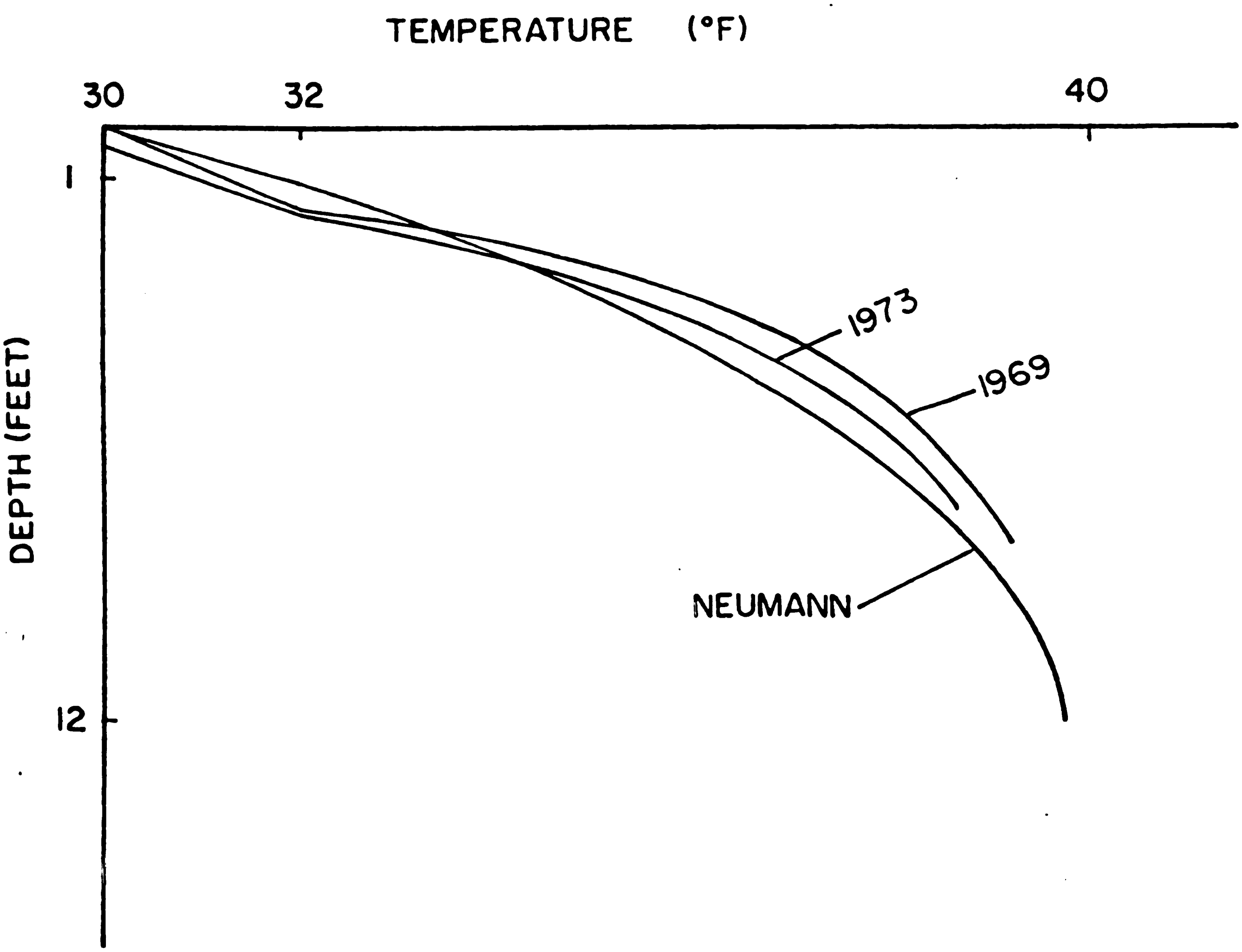

Figure 10. Soil Temperature vs Depth for Michigan Soil.

CHAPTER 3

NUMERICAL MODEL

3.1 Introduction

Two types of models were developed, one with cylindrical coordinates (the one-dimensional model) and the other with Cartesian coordinates (the two-dimensional model). The geometry for the numerical models are shown in Figures 11 and 12. The possibility of ice formation around the tube was taken into consideration and modelled appropriately. The two models provide for a comparison with the two-dimensional model being the more accurate. It is anticipated that the one-dimensional model may be sufficiently accurate.

3.2 Basic Equations for the One-Dimensional Model

The equations which describe the heat transfer in the one-dimensional model are as follows:

1. When the node is at the center of the tube (Figure 13), the following equation derived from a steady-state energy balance is used (see Appendix C):

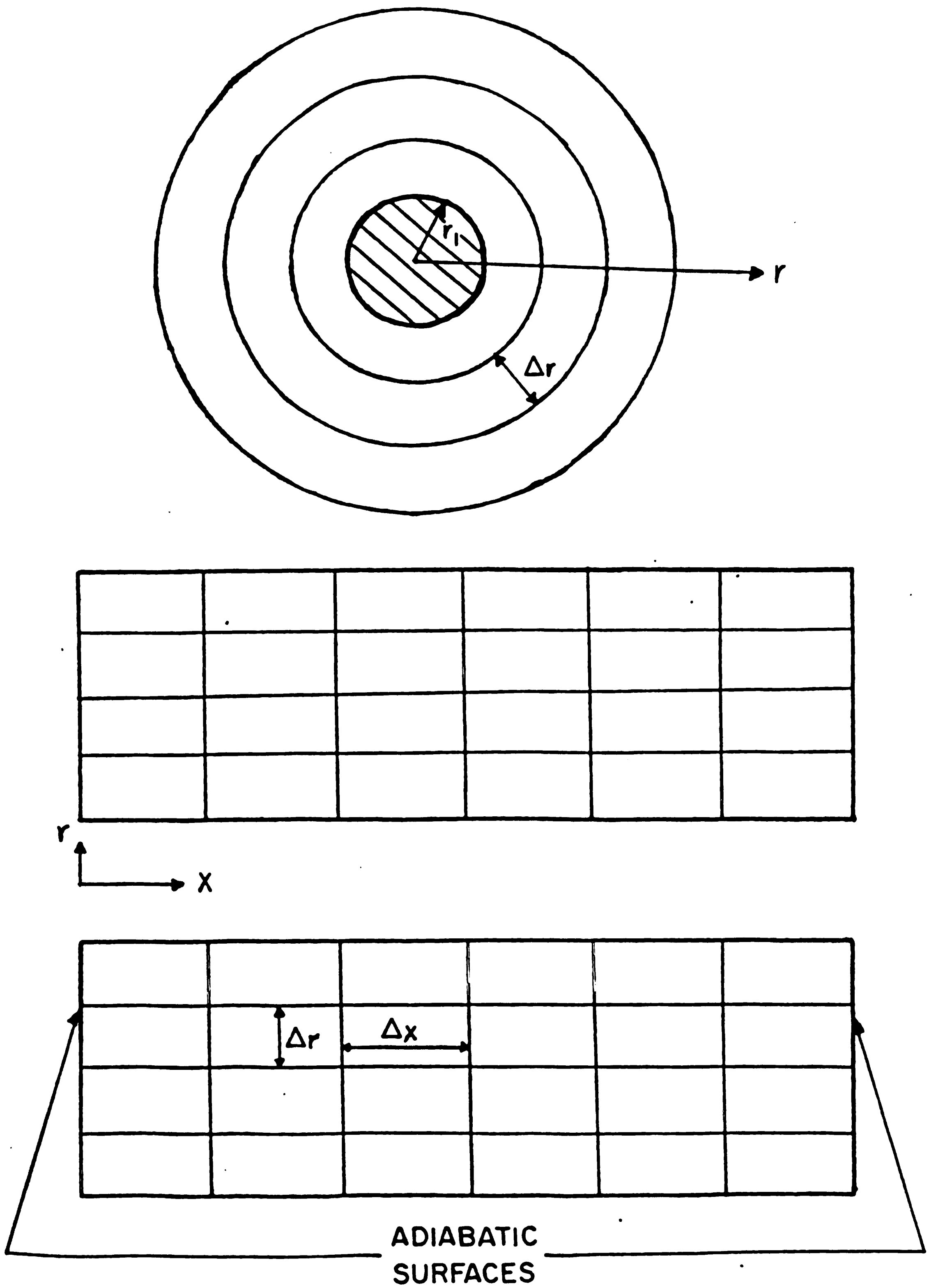

Figure 11. Geometry for the One-Dimensional Model.

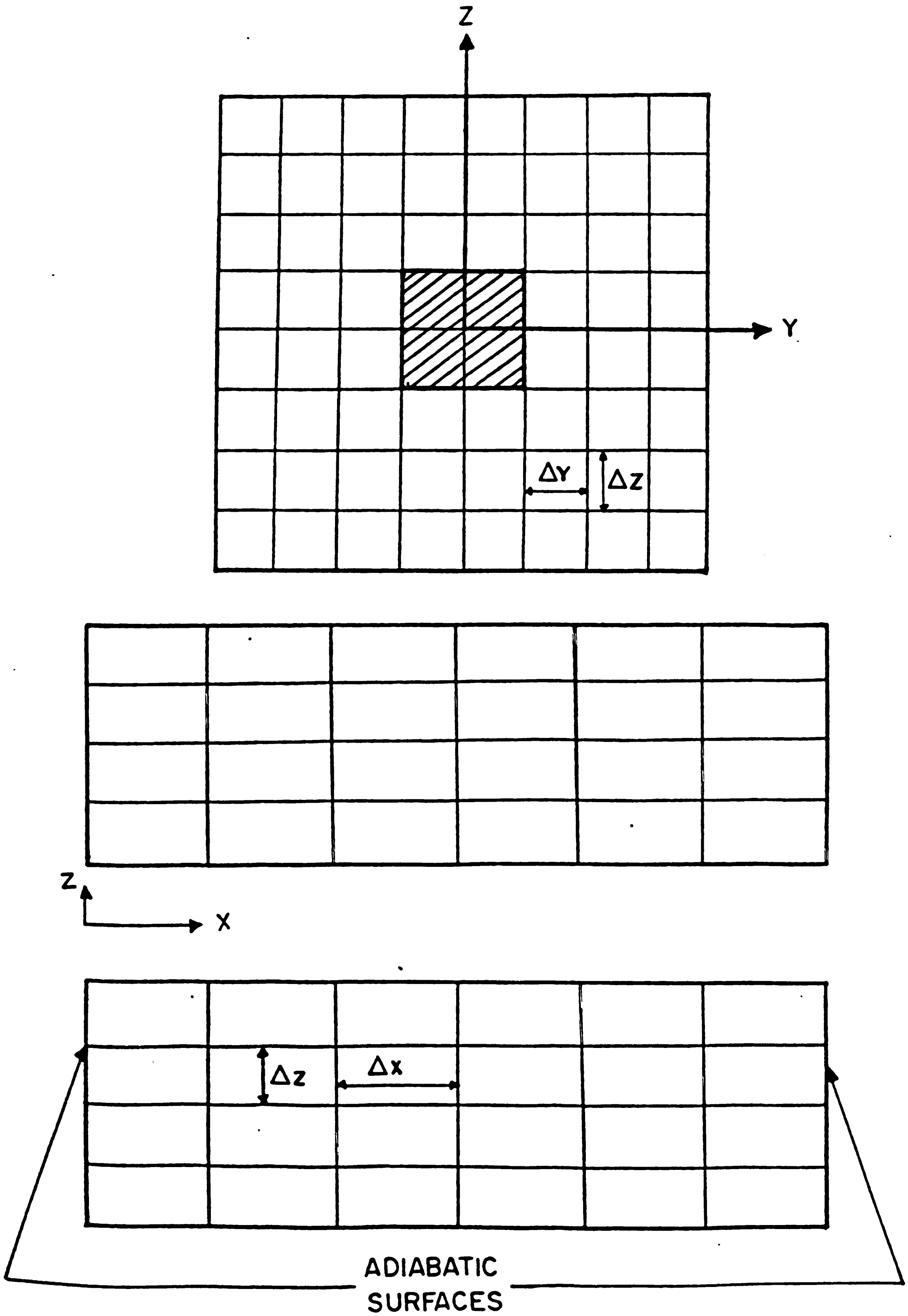

Figure 12. Geometry for the Two-Dimensional Model.

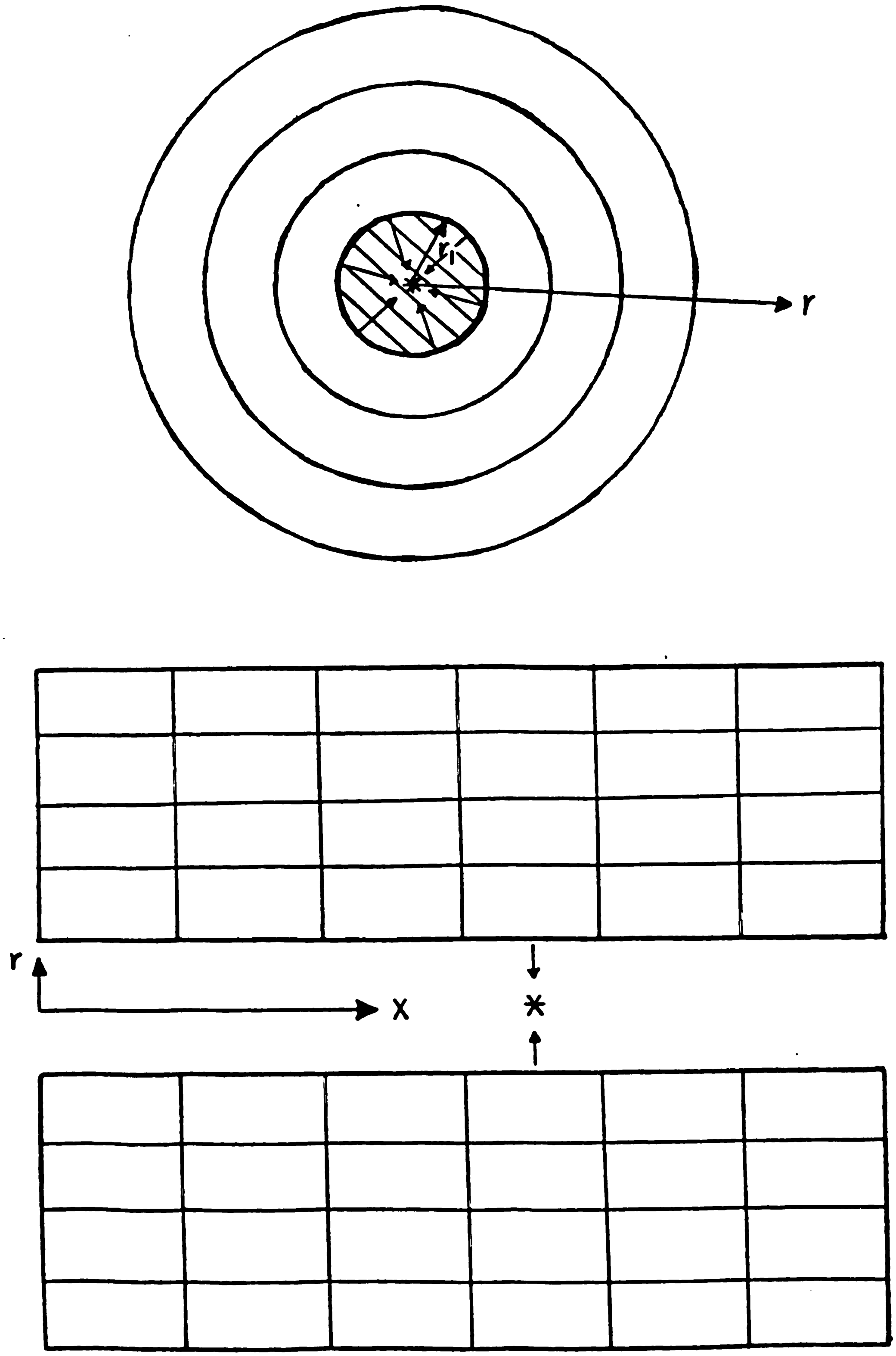

Figure 13. Node at the Center of the Circular Tube.

$$T^{k}_{i+1,0} = \frac{2h\Delta x}{\rho_a c_{pa} r_1 u} \left(T^{k}_{i,1} - T^{k}_{i,0} \right) + T^{k}_{i,0} \qquad (3.1)$$

where ρ_a is the density of air, c_{pa} is the heat capacity of air, u is the velocity of air and r_1 is the radius of the tube. The superscript "k" denotes the time and the subscripts denote x and r, respectively.

2. When the node borders on the tube (Figure 14), the following equation is obtained with application of the boundary condition given by equation 2.4 (see Appendix C):

$$T^{k+1}_{i,1} = \frac{\Delta t}{\rho_s \pi (r_1 + \frac{\Delta r}{4}) \Delta r \Delta x c_{ps}} \left[h\ 4\pi r_1 \Delta x \ (T^{k}_{i,0} - T^{k}_{i,1}) + k_{s2} \frac{4\pi (r_1 + \frac{\Delta r}{2}) \Delta x}{\Delta r} (T^{k}_{i,2} - T^{k}_{i,1}) \right] + T^{k}_{i,1} \qquad (3.2)$$

where ρ_s is the density of soil, c_{ps} is the heat capacity of soil and k_{s2} is the thermal conductivity of soil evaluated at $(T^{k}_{i,j+1} + T^{k}_{i,j})/2$. The calculated temperature will always be checked to determine if some of the soil is frozen. If $T^{k+1}_{i,1} < 32°F$ and $r_{ice} < \Delta r/2$, part of the soil

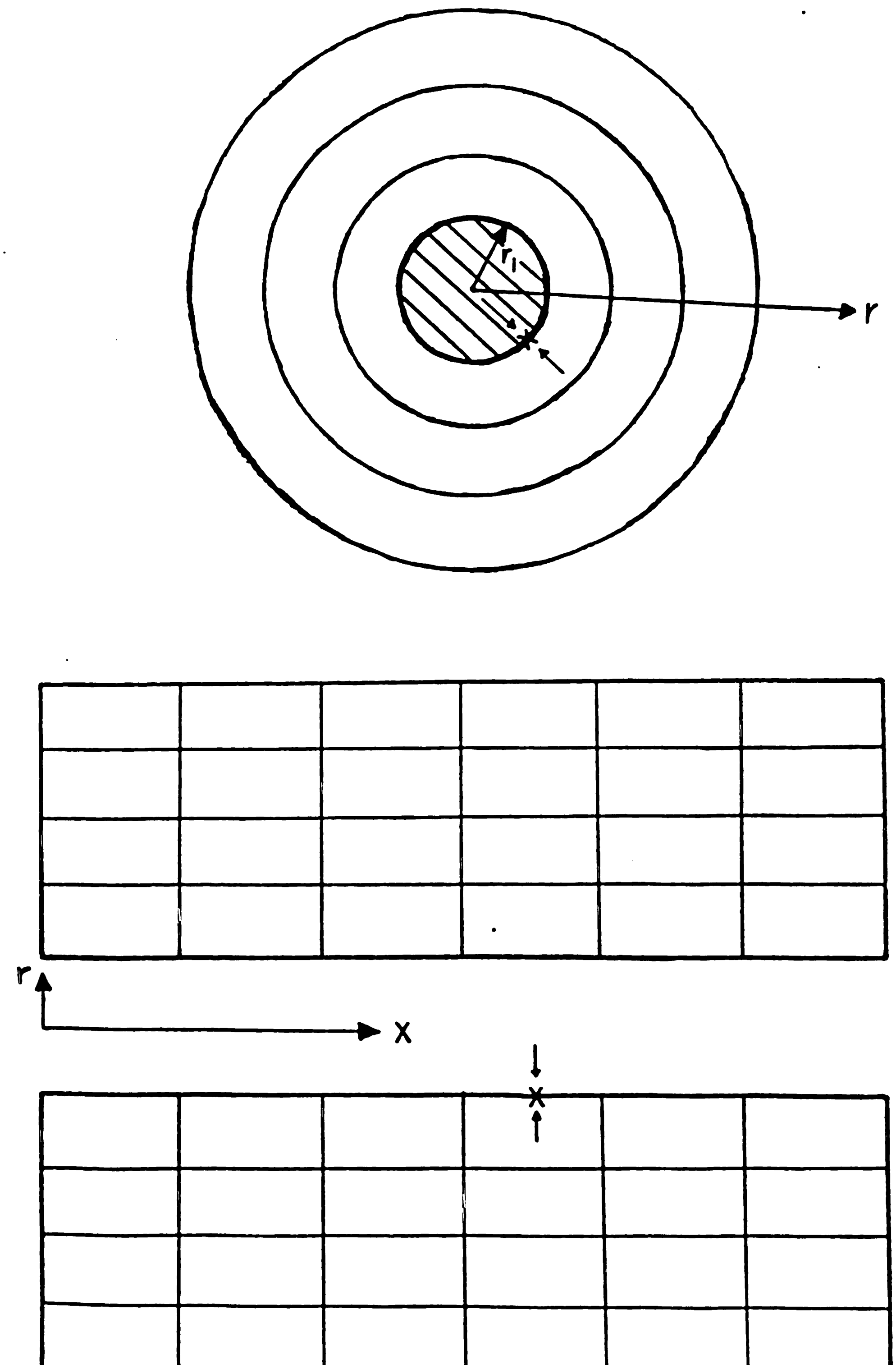

Figure 14. Node Bordering the Circular Tube.

element next to the tube will be frozen (Figure 15). The equation used for the ice formation is (see Appendix C),

$$\Delta r_{ice} = \frac{\Delta t}{\rho_s LM\ 2\pi R\ \Delta x}\left[\frac{\rho_s c_{ps}}{\Delta t}\pi(r_1 + \frac{\Delta r}{4})\Delta r\Delta x(T^{k+1}_{i,1} - T^{k}_{i,1})\right] \tag{3.3}$$

where L is the latent heat of fusion of water, M is the moisture content of the soil as a percentage by weight and R is the radius of ice formed from the center of the tube. If the element is completely frozen, the element directly next to it may freeze partially (Figure 16). The following equation is then used for the ice formation (see Appendix C):

$$\Delta r_{ice} = \frac{\Delta t}{\rho_s LM\ 2\pi R\ \Delta x}\left[\frac{\rho_s c_{ps}}{\Delta t}\ 2\pi r_2\Delta r\Delta x\ (T^{k+1}_{i,2} - T^{k}_{i,2})\right] \tag{3.4}$$

where k_{s1} is the thermal conductivity of soil evaluated at $(T^{k}_{i,j-1} + T^{k}_{i,j})/2$.

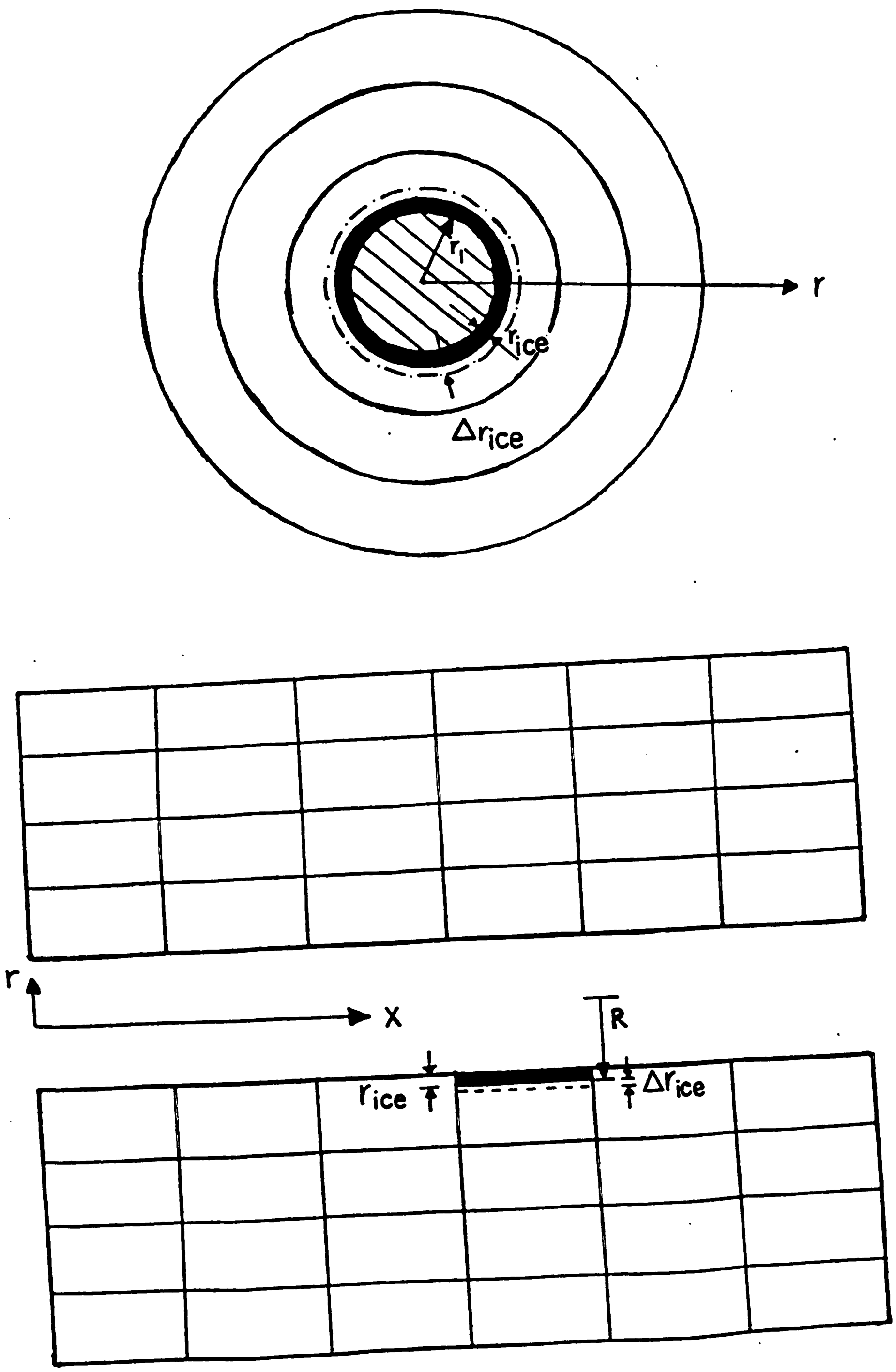

Figure 15. Ice Formed less than Δr/2 for the Circular Tube.

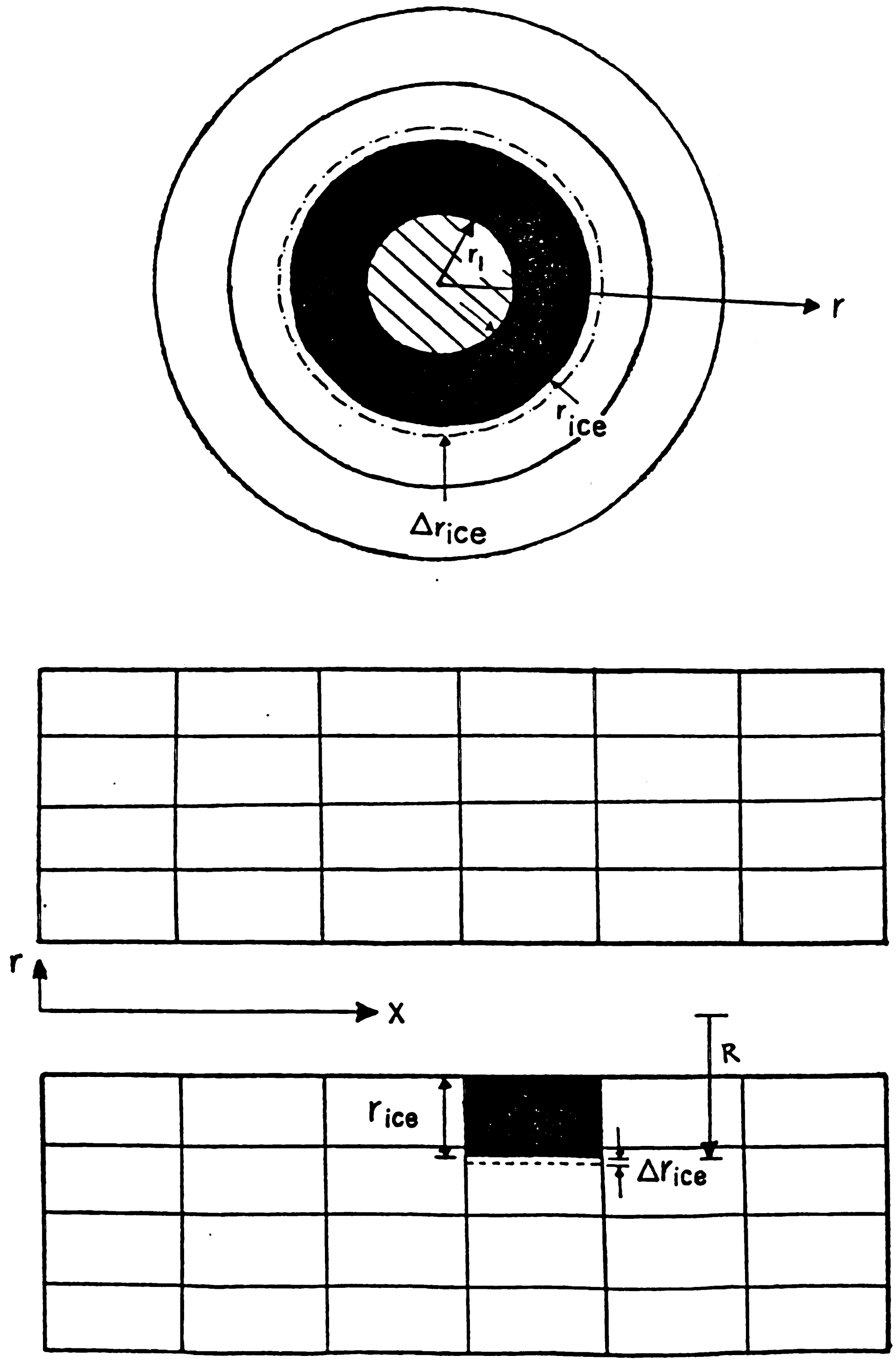

Figure 16. Ice Formed greater than Δr for the Circular Tube.

3. For the remaining soil elements, the following equation is used (see Appendix C):

$$T_{i,j}^{k+1} = \frac{\Delta t}{m_s c_{ps}} \left[k_{s1} \frac{2\pi(r_j - \frac{\Delta r}{2})\Delta x}{\Delta r} (T_{i,j-1}^{k} - T_{i,j}^{k}) + k_{s2} \frac{2\pi(r_j + \frac{\Delta r}{2})\Delta x}{\Delta r} (T_{i,j+1}^{k} - T_{i,j}^{k}) \right] + T_{i,j}^{k} \qquad (3.5)$$

3.3 <u>Basic Equations for the Two-Dimensional Model</u>

There are several equations that describe the heat transfer in the two-dimensional model. They are as follows:

1. As in the one-dimensional model, steady state is also assumed with an energy balance on the air element in the tube. The following equation is derived for the node at the center of the tube (Figure 17):

$$T_{i+1,0,0}^{l} = \frac{\Delta x}{m_a c_{pa} u} \left[2h\ \Delta x \Delta z\ (T_{i,-1,0}^{l} - T_{i,0,0}^{l}) + 2h\ \Delta x \Delta z\ (T_{i,1,0}^{l} - T_{i,0,0}^{l}) + 2h\ \Delta x \Delta y\ (T_{i,0.-1}^{l} - T_{i,0,0}^{l}) \right. \qquad (3.6)$$

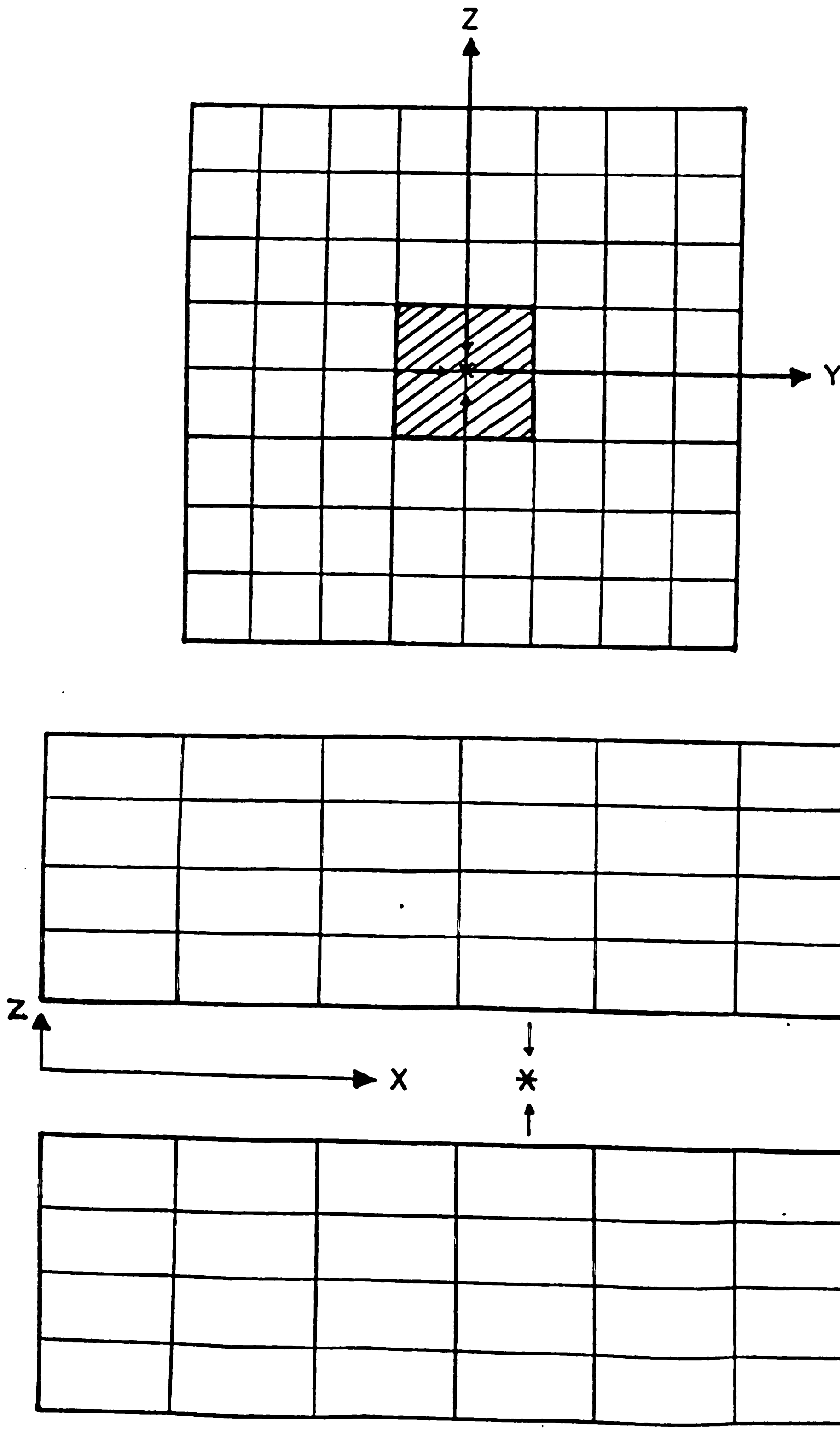

Figure 17. Node at the Center of the Square Tube.

$$+ 2h\,\Delta x \Delta y\,(T^{l}_{i,0,1} - T^{l}_{i,0,0}) \Big]$$
$$+ T^{l}_{i,0,0}$$

where m_a is the mass of air in an element; the superscript again refers to time and the three subscripts to x, y, z, respectively.

2. When the node borders on the center of the bottom face of the tube (Figure 18), the following equation is used (see Appendix C):

$$\begin{aligned} T^{l+1}_{i,0,-1} = \frac{\Delta t}{m_s c_{ps}} \Big[& k_{s1} \frac{\Delta x \Delta z}{\Delta y}\,(T^{l}_{i,-1,-1} - T^{l}_{i,0,-1}) \\ & + k_{s2} \frac{\Delta x \Delta z}{\Delta y}\,(T^{l}_{i,1,-1} - T^{l}_{i,0,-1}) \\ & + 2k_{s3} \frac{\Delta x \Delta y}{\Delta z}\,(T^{l}_{i,0,-2} - T^{l}_{i,0,-1}) \\ & + 2h\,\Delta x \Delta y\,(T^{l}_{i,0,0} - T^{l}_{i,0,-1}) \Big] \\ & + T^{l}_{i,0,-1} \end{aligned} \qquad (3.7)$$

If $T^{l+1}_{i,0,-1} < 32^{\circ}F$ and $h_{ice2} < \Delta z/2$, the soil element directly next to the node is partially frozen (Figure 19). The following equation is used for the ice formation (see Appendix C):

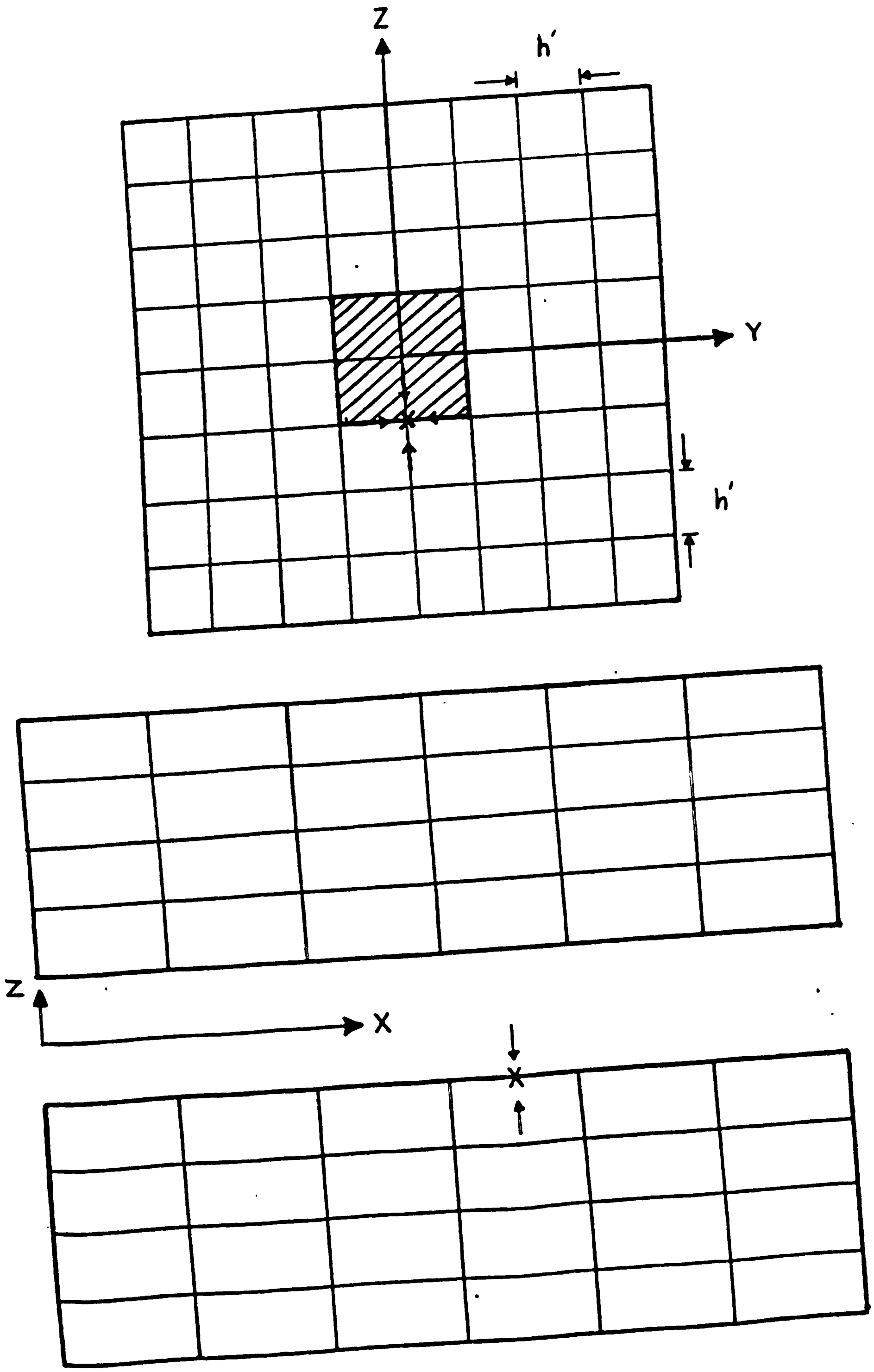

Figure 18. Node Bordering the Center of Bottom Face of the Square Tube.

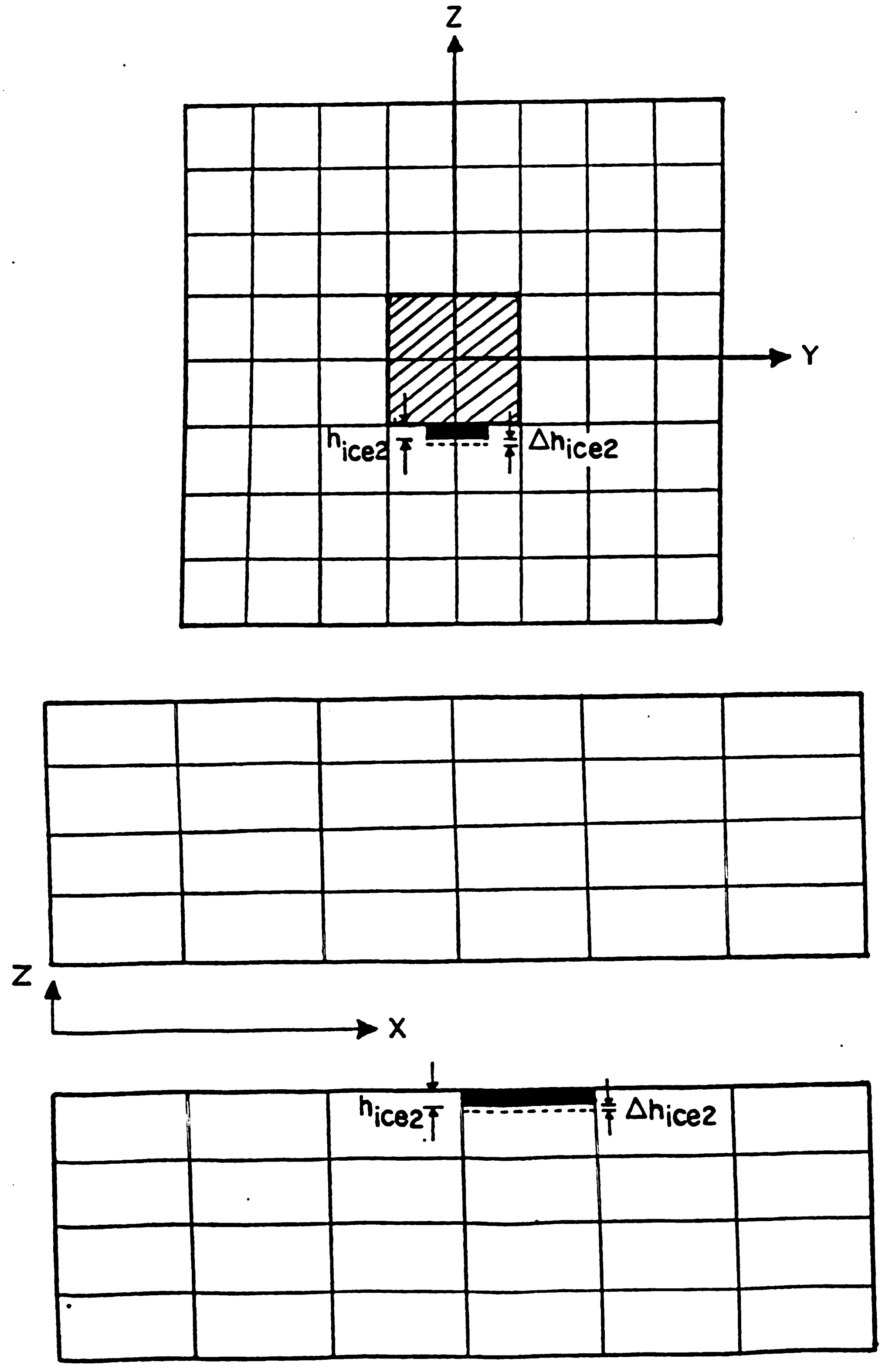

Figure 19. Ice Formed less than Δz/2 below the Square Tube.

$$\Delta h_{ice2} = \frac{\Delta t}{2\rho_s LM\ \Delta x \Delta y}\left[\frac{m_s c_{ps}}{\Delta t} (T^{l+1}_{i,0,-1} - T^{l}_{i,0,-1}) \right]$$

(3.8)

If the element is completely frozen, the element directly below the frozen element may be partially frozen (Figure 20). The equation for the ice formation is as follows:

$$\Delta h_{ice2} = \frac{\Delta t}{\rho_s LM\ \Delta x \Delta y}\left[\frac{m_s c_{ps}}{\Delta t} (T^{l+1}_{i,0,-2} - T^{l}_{i,0,-2}) \right]$$

(3.9)

3. When the node borders on the center of the top face of the tube (Figure 21), the equation used for heat transfer is similar to equation (3.7). The only difference is that a different boundary equation is applied whereby the soil-to-air convection term is positioned appropriately in the equation. The formation of ice is also similar but labelled differently (Figures 22 and 23).

4. When the node is at the bottom corner of the tube, special consideration is taken to account for the corner element with surface convection (Figure 24). For no ice formation the following equation is used (see Appendix C):

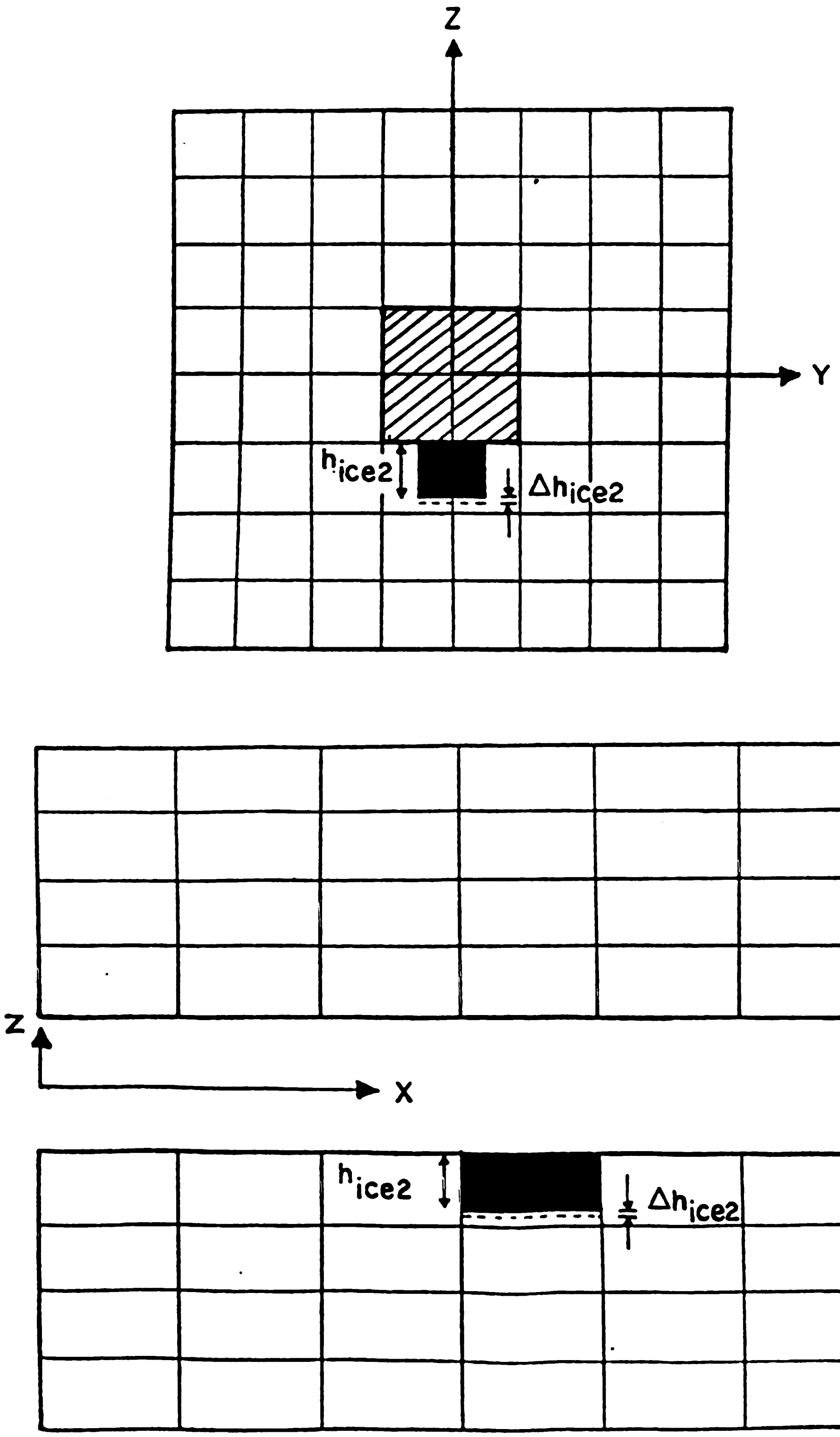

Figure 20. Ice Formed greater than Δz/2 below the Square Tube.

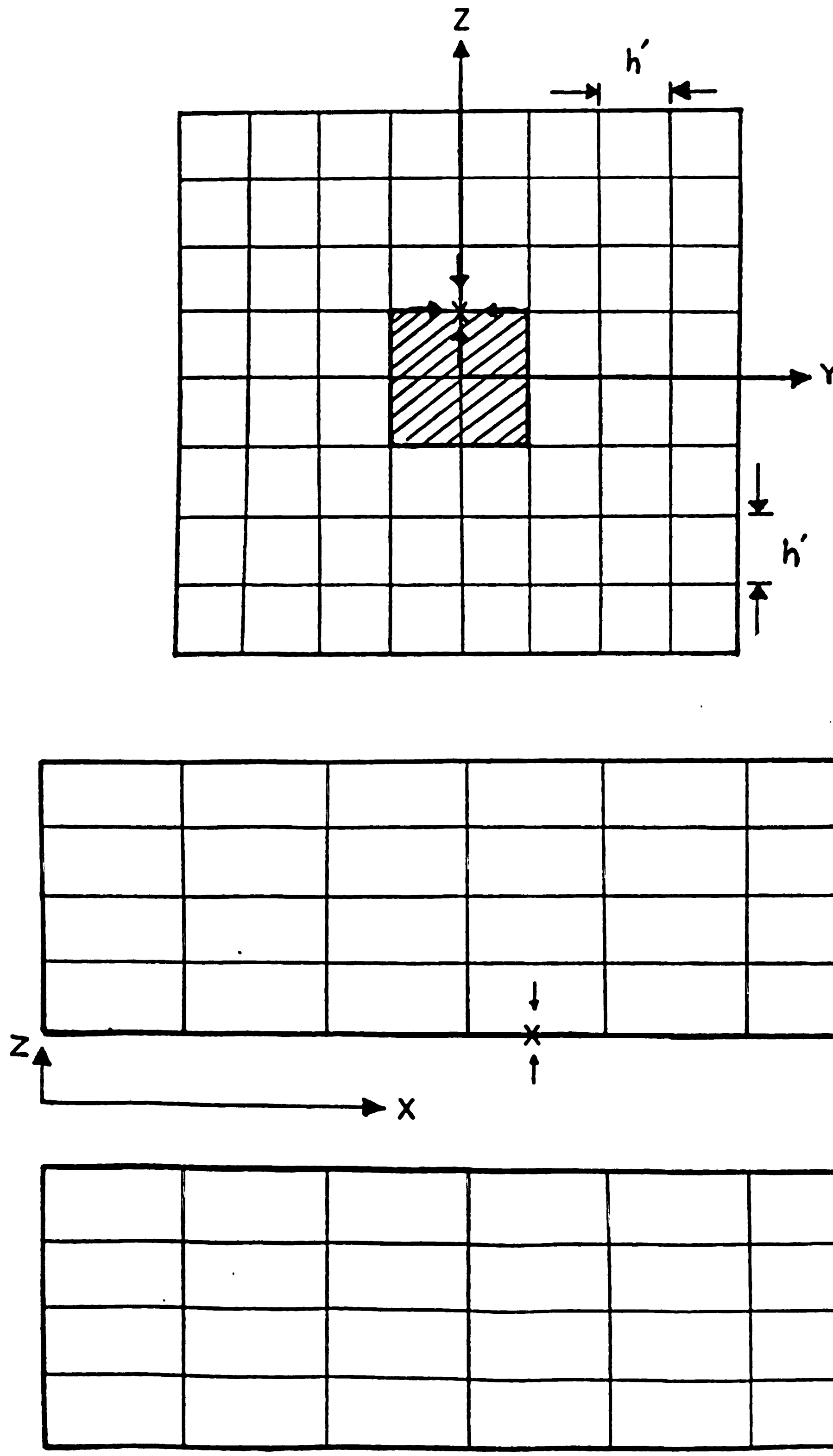

Figure 21. Node Bordering the Center of Top Face of the Square Tube.

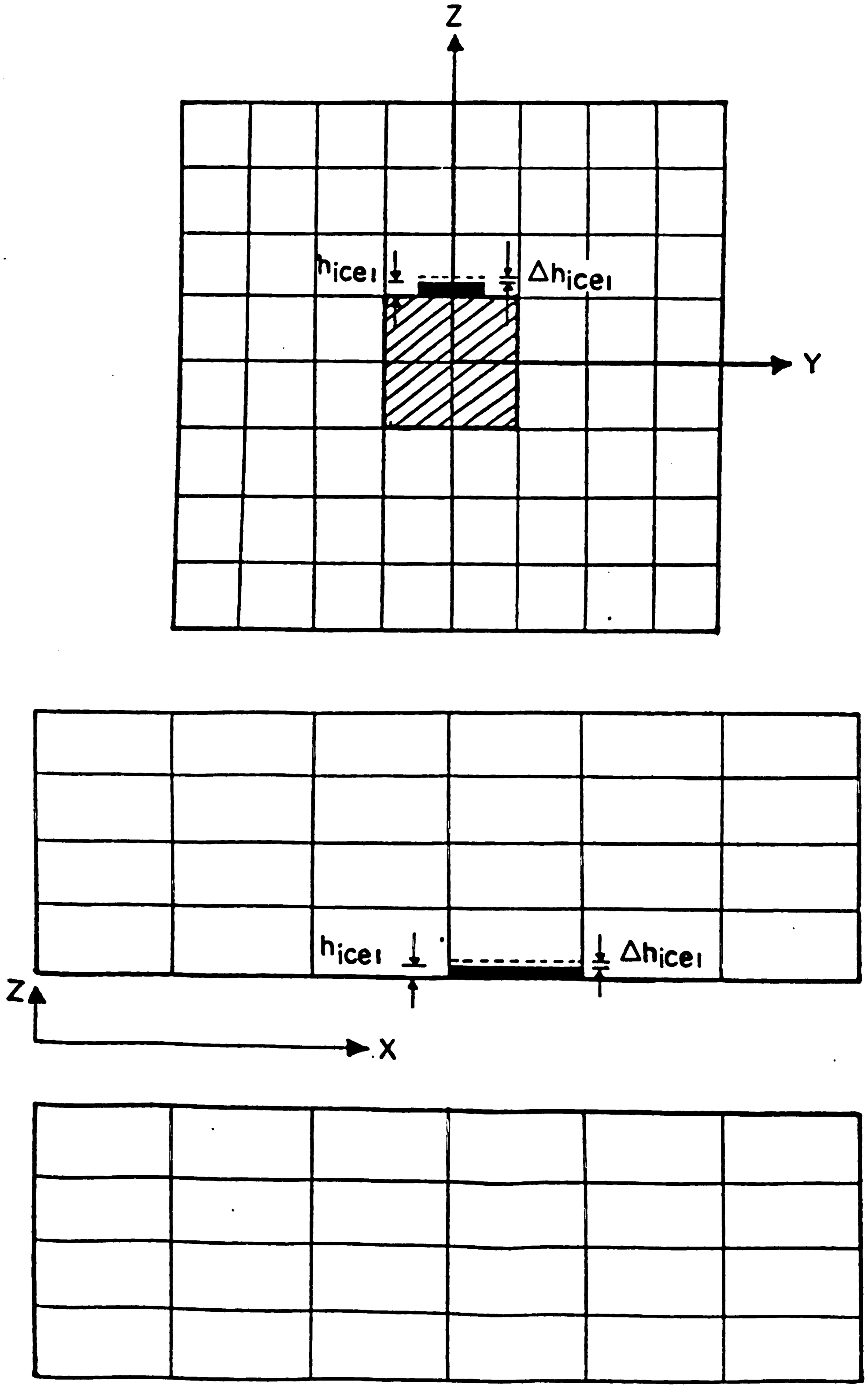

Figure 22. Ice Formed less than Δz/2 above the Square Tube.

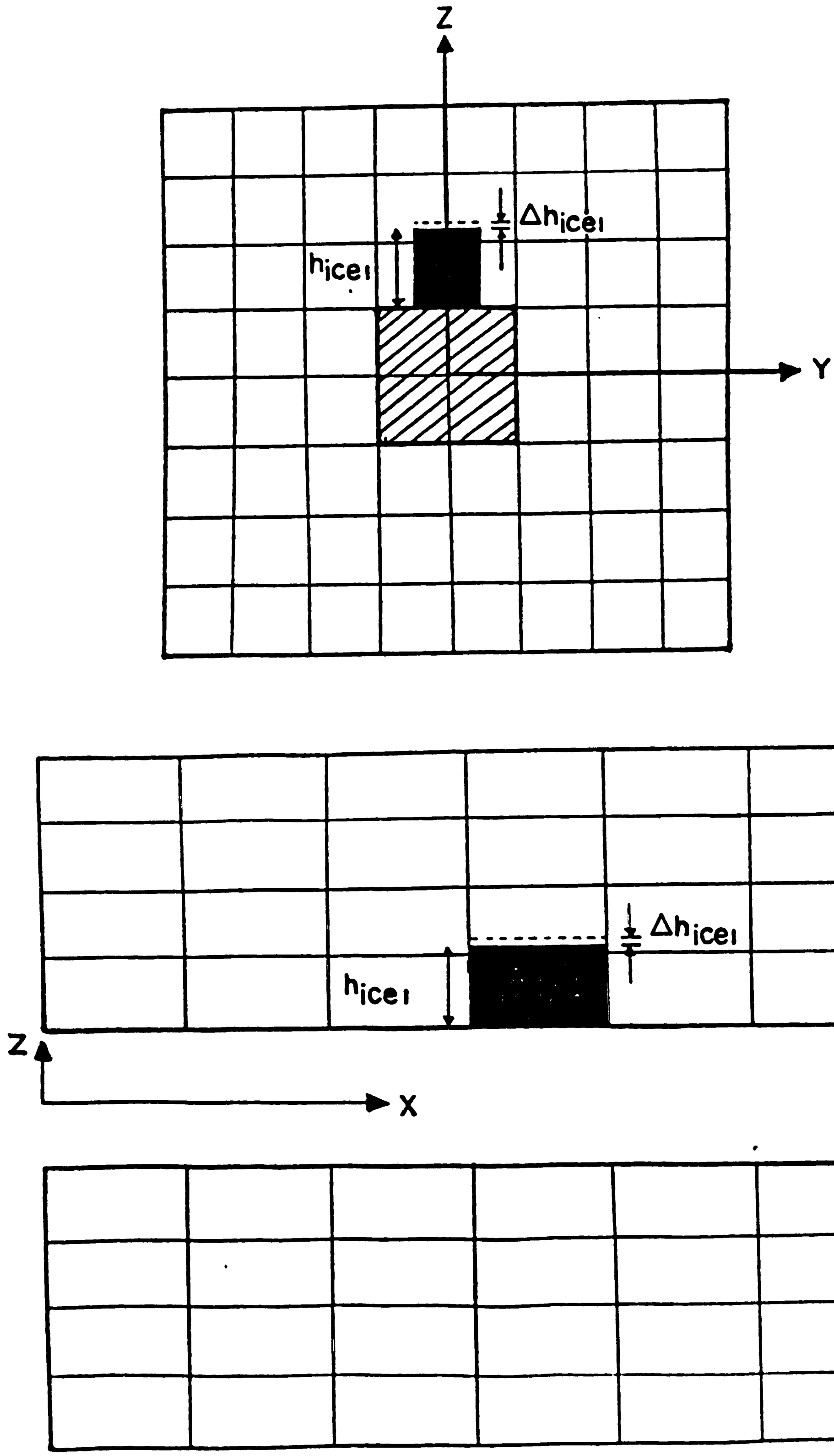

Figure 23. Ice Formed greater than $\Delta z/2$ above the Square Tube.

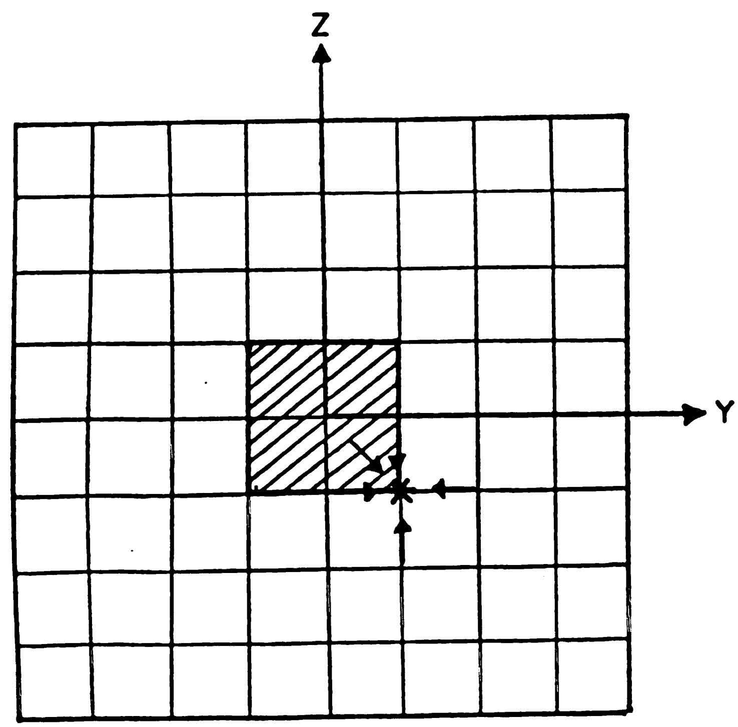

Figure 24. Node Bordering the Bottom Corner of the Square Tube.

$$T^{l+1}_{i,1,-1} = \frac{2\Delta t}{3m_s c_{ps}} \Big[k_{s1}\frac{\Delta x\Delta z}{\Delta y} (T^l_{i,0,-1} - T^l_{i,1,-1})$$

$$+ 2k_{s2}\frac{\Delta x\Delta z}{\Delta y} (T^l_{i,2,-1} - T^l_{i,1,-1})$$

$$+ 2k_{s3}\frac{\Delta x\Delta y}{\Delta z} (T^l_{i,1,-2} - T^l_{i,1,-1})$$

$$+ k_{s4}\frac{\Delta x\Delta y}{\Delta z} (T^l_{i,1,0} - T^l_{i,1,-1}) \qquad (3.10)$$

$$+ h\ \Delta x\Delta z\ (T^l_{i,0,0} - T^l_{i,1,-1})$$

$$+ h\ \Delta x\Delta y\ (T^l_{i,0,0} - T^l_{i,1,-1}) \Big]$$

$$+ T^l_{i,1,-1}$$

When there is some ice formation occuring in the element, $h_{ice2} < \Delta z/2$ (Figure 25), the following equation is then used to account for the release of latent heat of fusion (see Appendix C):

$$T^{l+1}_{i,1,-1} = \frac{2\Delta t}{3m_s c_{ps}} \Big[\rho_s LM\ \Delta x\ (\Delta y + \Delta z$$

$$+ 4h_{ice2} + 2\Delta h_{ice2})\ \frac{\Delta h_{ice2}}{\Delta t} \Big]$$

$$+ T^l_{i,1,-1} \qquad (3.11)$$

If $\Delta z/2 \le h_{ice2} \le 3\Delta z/2$, the element directly below the frozen corner element will be partially frozen (Figure 26). The following equation for that element is:

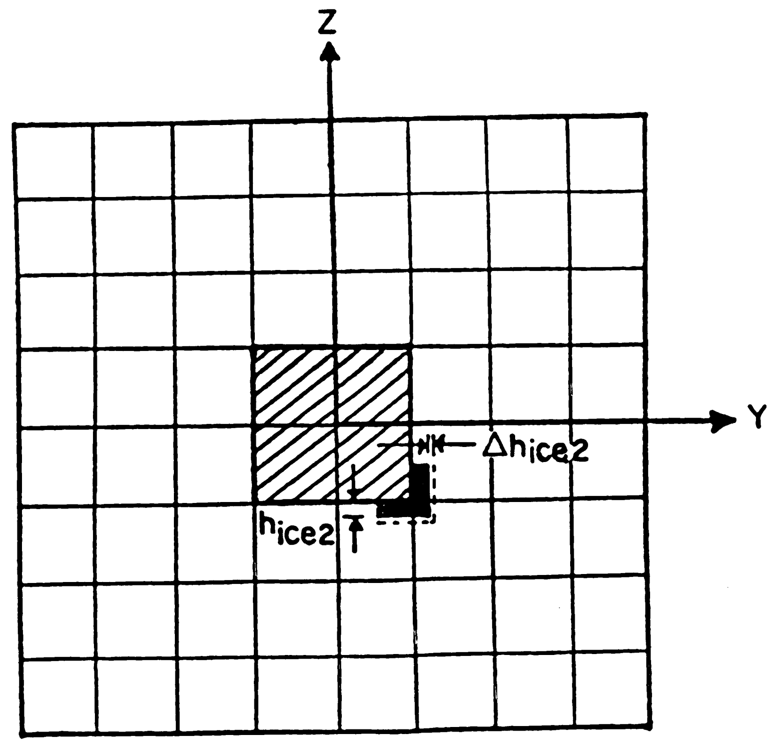

Figure 25. Ice Formed less than Δz/2 at the Bottom Corner of the Square Tube.

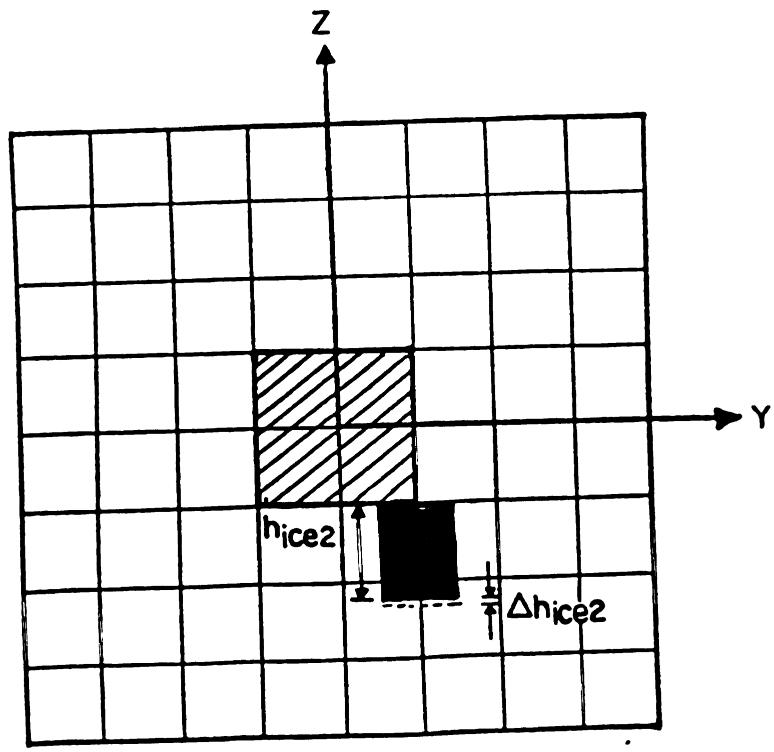

Figure 26. Ice Formed greater than Δz/2 below the Bottom Corner of the Square Tube.

$$T^{l+1}_{i,1,-2} = \frac{\Delta t}{m_s c_{ps}} \left[\rho_s LM \, \Delta x \Delta y \, \frac{\Delta h_{ice2}}{\Delta t} \right] + T^{l}_{i,1,-2} \tag{3.12}$$

The equation used for the element diagonal to the frozen corner element (Figure 27) is as follows:

$$T^{l+1}_{i,2,-2} = \frac{\Delta t}{m_s c_{ps}} \left[\rho_s LM \, \Delta x \, [\, 2(h_{ice2} - \tfrac{1}{2}\Delta z) + \Delta h_{ice2} \,] \, \frac{\Delta h_{ice2}}{\Delta t} \right] + T^{l}_{i,2,-2} \tag{3.13}$$

The equation used for the element directly to the right of the frozen corner element (Figure 28) is:

$$T^{l+1}_{i,2,-1} = \frac{\Delta t}{m_s c_{ps}} \left[\rho_s LM \, \Delta x \Delta z \, \frac{\Delta h_{ice2}}{\Delta t} \right] + T^{l}_{i,2,-1} \tag{3.14}$$

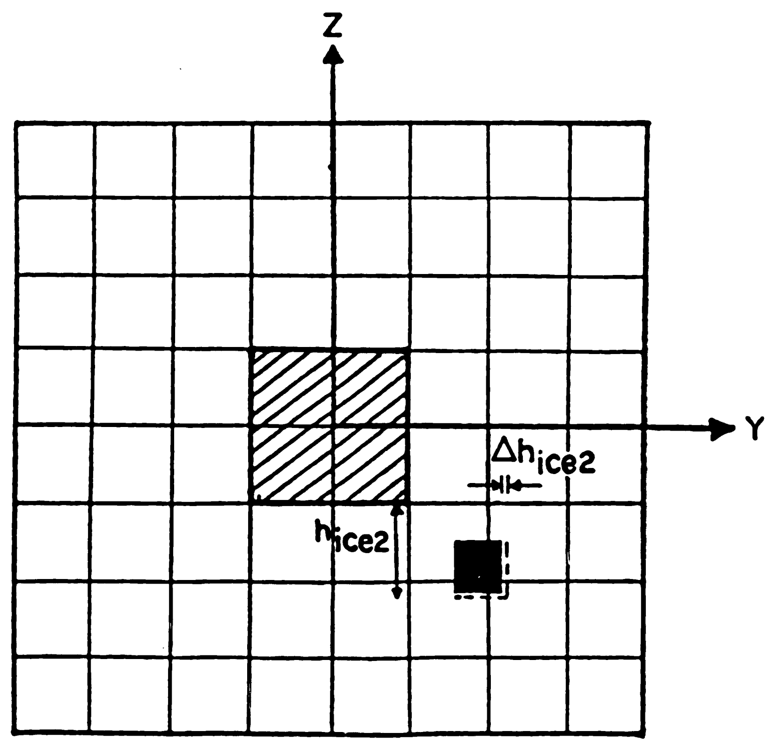

Figure 27. Ice Formed greater than $\Delta z/2$ diagonal to the Bottom Corner of the Square Tube.

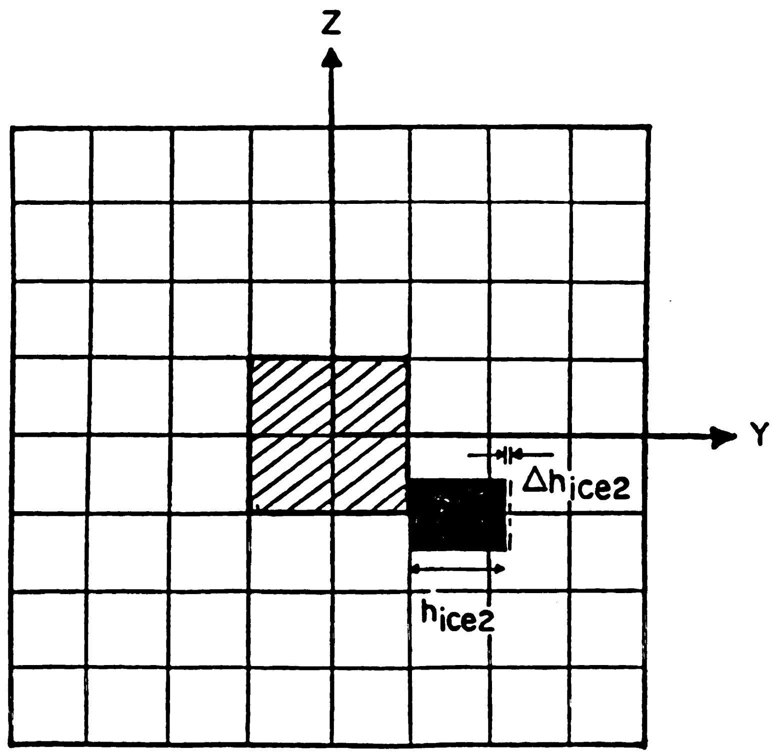

Figure 28. Ice Formed greater than $\Delta z/2$ adjacent to the Bottom Corner of the Square Tube.

5. When the node is at the top corner of the tube, similar equations are used for the frozen and unfrozen states. For the nodes surrounding the frozen top corner element, similar equations are used for the heat transfer in the element, taking into account the release of latent heat of fusion.

6. When the node borders the center of the side face of the tube (Figure 29), the following equation is used when no ice formation occurs:

$$
\begin{aligned}
T^{l+1}_{i,1,0} = \frac{\Delta t}{m_s c_{ps}} \Big[& 2h\ \Delta x \Delta z\ (T^{l}_{i,0,0} - T^{l}_{i,1,0}) \\
& + 2k_{s2} \frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,2,0} - T^{l}_{i,1,0}) \\
& + k_{s3} \frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,1,-1} - T^{l}_{i,1,0}) \\
& + k_{s4} \frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,1,1} - T^{l}_{i,1,0}) \Big] \\
& + T^{l}_{i,1,0}
\end{aligned}
\qquad (3.15)
$$

When $0 < h_{ice1} \leq \Delta z/2$ and $h_{ice2} = 0$ (see Figure 30), the following equation is used:

$$
T^{l+1}_{i,1,0} = \frac{\Delta t}{m_s c_{ps}} \left[\rho_s LM\ \Delta x \sqrt{(\Delta z)^2 + h^2_{ice1}}\ \frac{\Delta h_{ice1} + \Delta h_{ice2}}{\Delta t} \right] + T^{l}_{i,1,0} \qquad (3.16)
$$

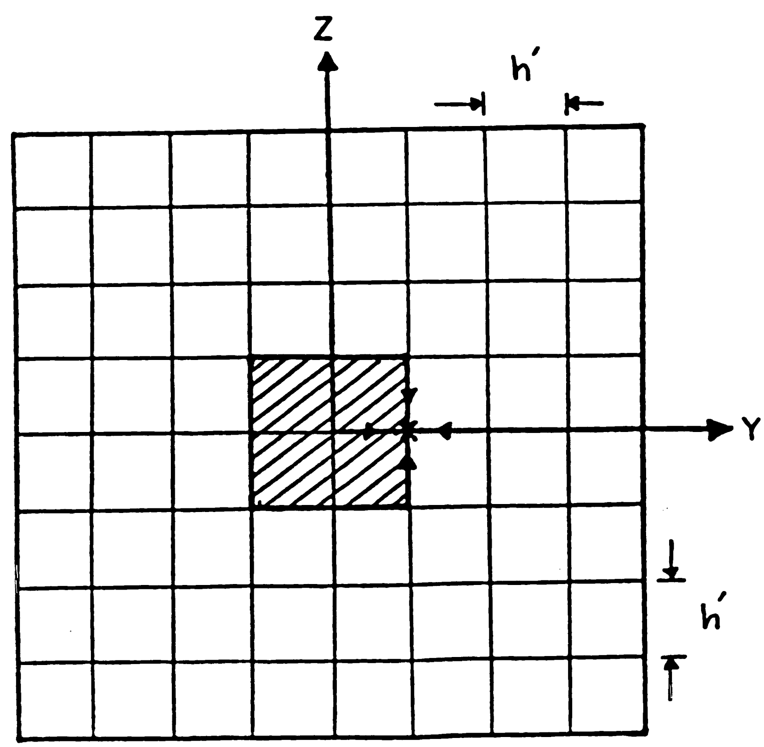

Figure 29. Node Bordering the Center of Side Face of the Square Tube.

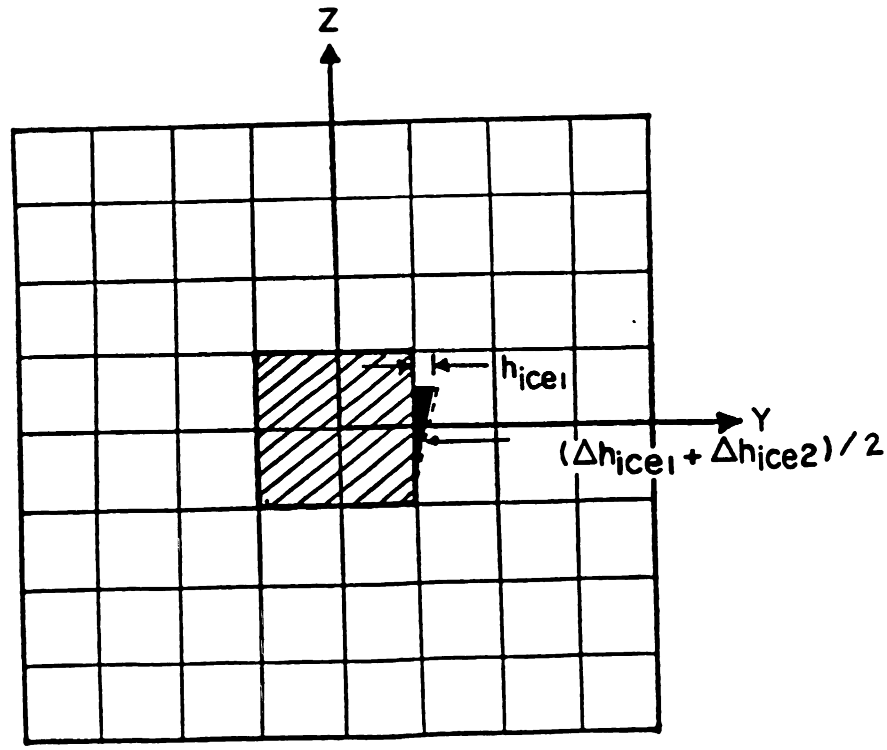

Figure 30. Ice Formed less than $\Delta z/2$ adjacent to the Square Tube with Condition 1.

When $0 < h_{ice1} \le \Delta z/2$ and $0 < h_{ice2} \le \Delta z/2$ (see Figure 31), the following equation is then used:

$$T_{i,1,0}^{l+1} = \frac{\Delta t}{m_s c_{ps}} \left[\rho_s LM \, \Delta x \sqrt{(\Delta z)^2 + (h_{ice1} - h_{ice2})^2} \; \frac{\Delta h_{ice1} + \Delta h_{ice2}}{\Delta t} \right] + T_{i,1,0}^{l} \tag{3.17}$$

When the element is completely frozen, the next element directly to the right of the frozen element may be partially frozen (Figure 32). The equation used for the heat transfer in that element is:

$$T_{i,2,0}^{l+1} = \frac{\Delta t}{m_s c_{ps}} \left[\frac{1}{2}\rho_s LM \, \Delta x \sqrt{(\Delta z)^2 + (h_{ice1} - h_{ice2})^2} \; \frac{\Delta h_{ice1} + \Delta h_{ice2}}{\Delta t} \right] + T_{i,2,0}^{l} \tag{3.18}$$

7. When the node borders the ground surface (Figure 33), the following heat transfer equation is used:

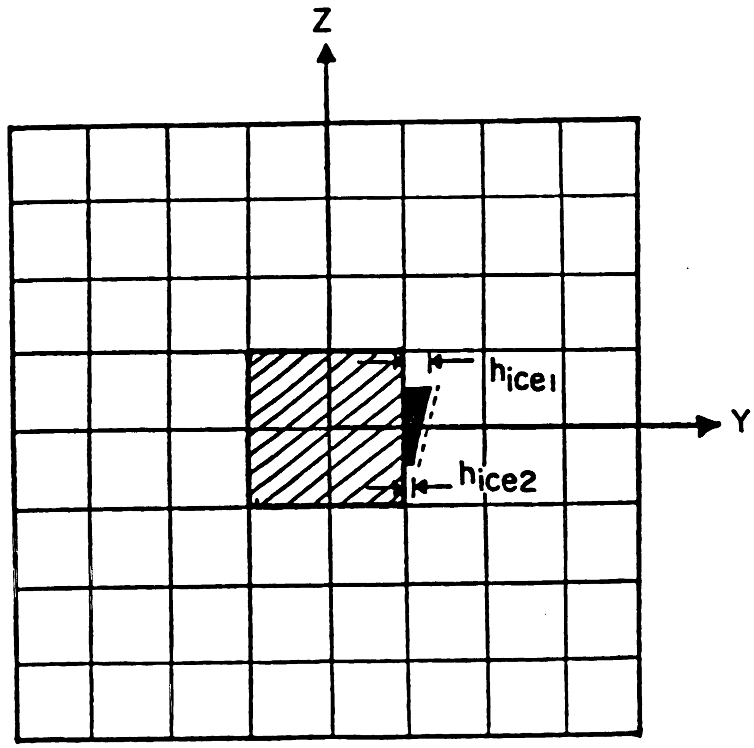

Figure 31. Ice Formed less than Δz/2 adjacent to the Square Tube with Condition 2.

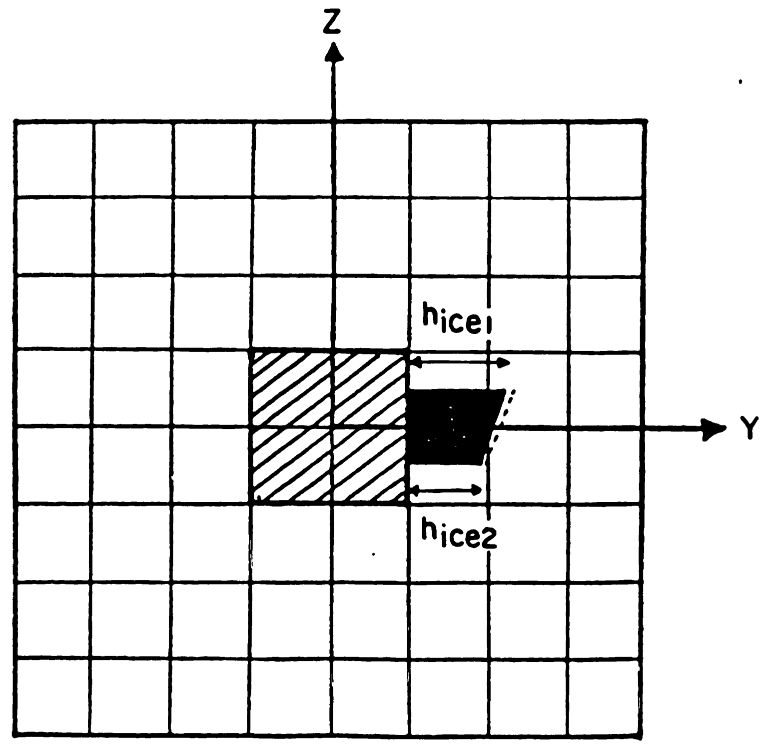

Figure 32. Ice Formed greater than $\Delta z/2$ adjacent to the Square Tube.

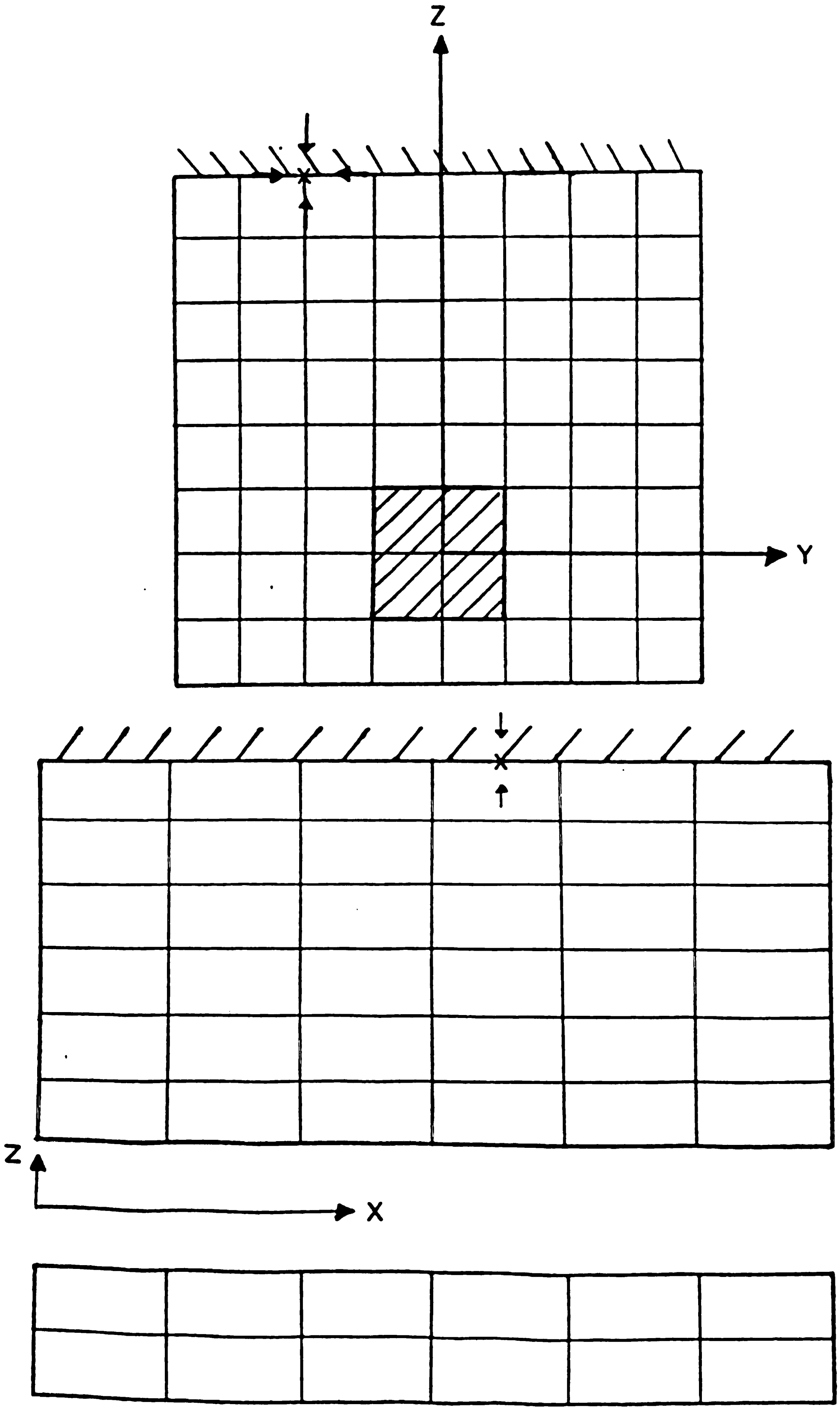

Figure 33. Node Bordering the Ground Surface.

$$T^{l+1}_{i,j,zsur} = \frac{\Delta t}{m_s c_{ps}} \Big[k_{s1} \frac{\Delta x \Delta z}{\Delta y} (T^l_{i,j-1,zsur} - T^l_{i,j,zsur}) + k_{s2} \frac{\Delta x \Delta z}{\Delta y} (T^l_{i,j+1,zsur} - T^l_{i,j,zsur}) + 2k_{s3} \frac{\Delta x \Delta y}{\Delta z} (T^l_{i,j,zsur-1} - T^l_{i,j,zsur}) + 2h_{sur} \Delta x \Delta y (T_{atm} - T^l_{i,j,zsur}) \Big] + T^l_{i,j,zsur} \quad (3.19)$$

where h_{sur} is the convective heat transfer coefficient of the soil to the outside air.

8. For the other soil elements, the following equation is used (see Appendix C):

$$T^{l+1}_{i,j,k} = \frac{\Delta t}{m_s c_{ps}} \Big[k_{s1} \frac{\Delta x \Delta z}{\Delta y} (T^l_{i,j-1,k} - T^l_{i,j,k}) + k_{s2} \frac{\Delta x \Delta z}{\Delta y} (T^l_{i,j+1,k} - T^l_{i,j,k}) + k_{s3} \frac{\Delta x \Delta y}{\Delta z} (T^l_{i,j,k-1} - T^l_{i,j,k}) + k_{s4} \frac{\Delta x \Delta y}{\Delta z} (T^l_{i,j,k+1} - T^l_{i,j,k}) \Big] + T^l_{i,j,k} \quad (3.20)$$

where k_{s1} is the conductivity of soil evaluated at $(T^l_{i,j-1,k} + T^l_{i,j,k})/2$, k_{s2} is the conductivity of soil evaluated at $(T^{1}_{i,j+1,k} + T^{1}_{i,j,k})/2$, k_{s3} is the conductivity of soil evaluated $(T^{1}_{i,j,k-1} + T^{1}_{i,j,k})/2$ and finally, k_{s4} is the conductivity of soil evaluated at $(T^{1}_{i,j,k+1} + T^{1}_{i,j,k})/2$.

3.4 Numerical Stability

The numerical model which was developed using the explicit method has a stability criterion. For a stable numerical solution, the formula used for the one-dimensional model is

$$\frac{\alpha \Delta t}{(\Delta r)^2} \leq \frac{1}{2} \tag{3.21}$$

and for the two-dimensional model, the formula used is

$$\frac{\alpha \Delta t}{(\Delta y)^2 + (\Delta z)^2} \leq \frac{1}{4} \tag{3.22}$$

where $\Delta y = \Delta z$. For reasonable accuracy, the time step and the space steps have to be made quite small (27).

Several tests were conducted to select the appropriate time step and space step for the one-dimensional model considering only the exit temperature of the tube. Tables 1, 2 and 3 indicate the tests that was conducted and selections that was made. It was found that a time step of 10 minutes and a space step of 5 ft in the x-direction would give reasonably accurate results. The space step in the r-direction is varied between 0.2 ft and 0.4 ft, allowing only 2 soil elements to freeze.

Table 1. Comparisons of the Exit Temperature of the Circular Tube with Different Time Steps.

Time Time (hr)	Δt = 1 min T_{out} (°F)	Δt = 10 mins T_{out} (°F)	Δt = 1 hr T_{out} (°F)
0.5	14.7540302	15.2320642	
1.0	13.5392723	13.7052917	16.5444374
1.5	13.4371080	13.4371080	
2.0	13.4371080	13.4371080	13.4371080
2.5	13.4371080	13.4371080	
3.0	13.4371080	13.4371080	13.4371080
3.5	13.4371080	13.4371080	
4.0	13.4371080	13.4371080	13.4371080
4.5	13.4371080	13.4371080	
5.0	13.4301677	13.2995367	12.4048958
5.5	12.9083624	12.8521147	
6.0	12.4164248	12.3375645	11.0715809

Here, Δr = 0.564 ft and Δx = 1.0 ft. The selected value for Δt is 10 mins.

Table 2. Comparisons of the Exit Temperature of the Circular Tube with Different Space Steps in the x-direction.

Time (hr)	Δx = 1 ft T_{out} (°F)	Δx = 5 ft T_{out} (°F)	Δx = 10 ft T_{out} (°F)
0.5	15.2320642	15.4745922	15.8068676
1.0	13.7052917	13.9133835	14.1865015
1.5	13.4371080	13.6633844	13.9620523
2.0	13.4371080	13.6633844	13.9620523
2.5	13.4371080	13.6633844	13.9620523
3.0	13.4371080	13.6633844	13.9620523
3.5	13.4371080	13.6633844	13.9620532
4.0	13.4371080	13.6633844	13.9620532
4.5	13.4371080	13.6633844	13.9620532
5.0	13.2995367	13.5441265	13.9620532
5.5	12.8521147	13.0398951	13.4373751
6.0	12.3375645	12.4092398	12.8577614

Here, Δt = 10 mins and Δr = 0.564 ft. The selected value for Δx is 5 ft.

Table 3. Comparisons of the Exit Temperature of the Circular Tube with Different Space Steps in the r-direction.

Time (hr)	Δr = 0.2 ft T_{out} (°F)	Δr = 0.4 ft T_{out} (°F)	Δr =0.564 ft T_{out} (°F)
0.5	13.6781063	14.8539438	15.4745922
1.0	13.6633844	13.6633844	13.9133835
1.5	13.6633844	13.6633844	13.6633844
2.0	12.9093580	13.6633844	13.6633844
2.5	12.0458002	13.6633844	13.6633844
3.0	11.3234854	13.6633844	13.6633844
3.5	10.8421450	13.4894409	13.6633844
4.0	10.5065155	12.7473459	13.6633844
4.5	10.3096199	12.0335646	13.6633844
5.0	10.2087717	11.3665028	13.5441265
5.5	10.1735430	10.6625042	13.0398951
6.0	10.1728191	10.0137520	12.4092398

Here, Δt = 10 mins and Δx = 5 ft. The selected value for Δr is between 0.2 and 0.4 ft, allowing only 2 soil elements to freeze.

3.5 Material Properties

Soil properties vary enormously and therefore empirical values must be determined (28). These values are tabulated in Table 4 :

Table 4. Values of the Soil Properties.

Normal Soil Conductivity	0.458	Btu/(hr-ft-°F)
Frozen Soil Conductivity	0.833	Btu/(hr-ft-°F)
Normal Soil Heat Capacity	0.3	Btu/(lb_m-°F)
Frozen Soil Heat Capacity	0.2	Btu/(lb_m-°F)
Soil Density	95.0	lb_m/ft^3

The air outside which is being fed into the tube is selected as a winter condition of 0°F at all times. Since the air in the tube may vary between 0°F to 40°F, the density of air at 20°F is chosen which is 0.08275 lb_m/ft^3(29). Atmospheric pressure is used since pressure varies only slightly from atmosphere. The corresponding heat capacity of air is 0.24 Btu/(lb_m-°F). The latent heat of fusion of water is 143.3 Btu/(lb_m-°F) and the heat capacity of water is 1.0 Btu/(lb_m-°F). The heat capacity of ice is 0.5 Btu/(lb_m-°F).

3.6 Program Layout

Initially, some parameters are input interactively so that several runs can be made without changing the contents of the program. The parameters are the element size, the moisture content in the soil, the velocity of air in the tube, the stability constraint and the convective heat transfer coefficient from the soil to the tube. The initial temperature distribution in the soil is then calculated and the temperature of air in the tube is assigned to be the temperature of soil at that depth.

The program next selects the heat capacity and the thermal conductivity of soil according to the temperature surrounding the node. Then it will select the appropriate heat transfer coefficient for calculation of the temperature at that node. Calculation of temperature of the air in the tube is accomplished only after the soil temperature is found at that time step.

3.7 Temperature Adjustments

Ice formation around the tube occurs if the temperature of the nodes bordering the tube is less than 32°F. Temperature is always reset to 32°F if the soil element is not completely frozen. When the element is completely frozen, the temperature will be calculated and will be less than 32°F. According to Jumikis (30), latent heat of fusion is released as long as there is water left in the soil element in the process of cooling (Figure 34). The release of latent heat of fusion will contribute significantly to the energy transfered to heat the air in the tube (31).

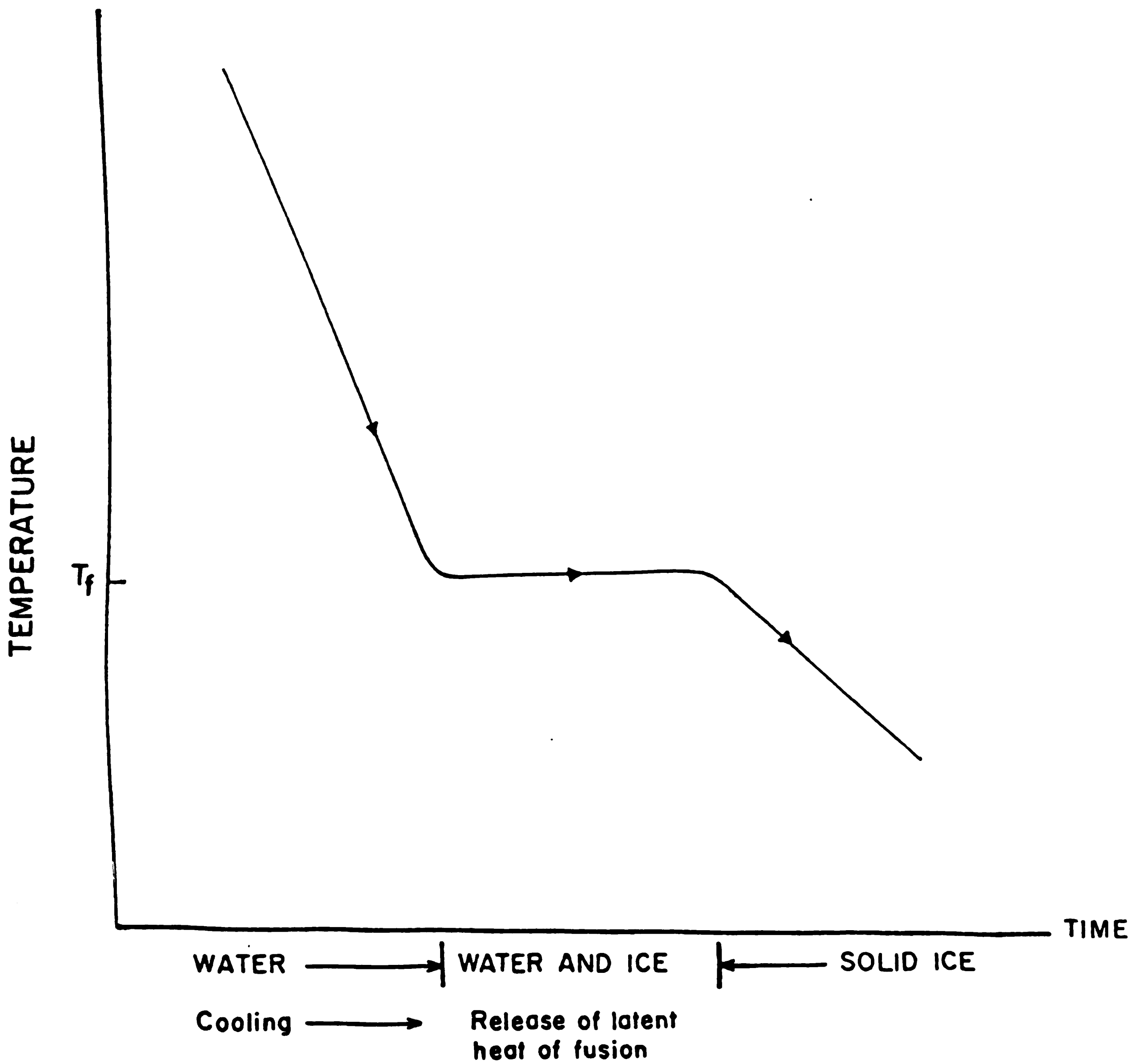

Figure 34. Temperature of Water vs Time in Cooling Process.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Background

Several parameters were varied in order to obtain data for comparisons. The parameters that were varied are the air flow rates, the convective heat transfer coefficient at the surface of the tube, and the moisture content in the soil. For the one-dimensional model, the computer simulation was in operation for 24 hours. However, considerable computing time is needed to run the two-dimensional model; and therefore, a run only between 1 to 2 hours was done to check the exit temperature with the one-dimensional model. Since the models were developed to simulate operation during a cold winter period, the air inlet temperature is set at 0°F.

Criterion for termination of the program is when the maximum time of continuous operation is done. For all the one-dimensional model runs, a maximum time of continuous operation of 24 hours was set.

4.2 Results for the One-Dimensional Model

The simulation predicted that a temperature of 12.4°F is obtained for 10 percent moisture content in the soil and 13.7°F for 50 percent moisture content in the soil at an air flow rate of 300 cubic feet per minute (u = 5 ft/s) after 24 hours of continous operation as indicated in Figure 35. This figure is a graph of the output air temperature of a 50 ft circular, 6.77-in-radius tube at an air velocity of 5 ft/s for different moisture contents in the soil as a function of time. Here, the output air temperature difference after 24 hours is 1.3°F.

Figure 36 shows the output air temperature of the 50 ft tube as a function of time for different moisture contents of the soil at an air flow rate of 600 cubic feet per minute (u = 10 ft/s). At 10 percent moisture content of the soil, the output air temperature of the tube is 8.5°F and at 50 percent moisture content of the soil, the output air temperature of the tube is 10.2°F after 24 hours of continuous operation. In this case, the difference in the output air temperature is 1.7°F. For both cases of 5 ft/s and 10 ft/s air velocity, the output air temperature of the tube for 50 percent moisture content in the soil is higher than the output air temperature for the 10 percent moisture. This is due to the higher heat capacity of the soil which depends on the soil moisture.

Figures 37 and 38 are the graphs of output air temperature of air velocities of 5 ft/s and 10 ft/s as a function of time at 10 and 50 percent moisture contents in the soil, respectively. At the beginning, the output temperature difference between the 5 ft/s air and 10 ft/s

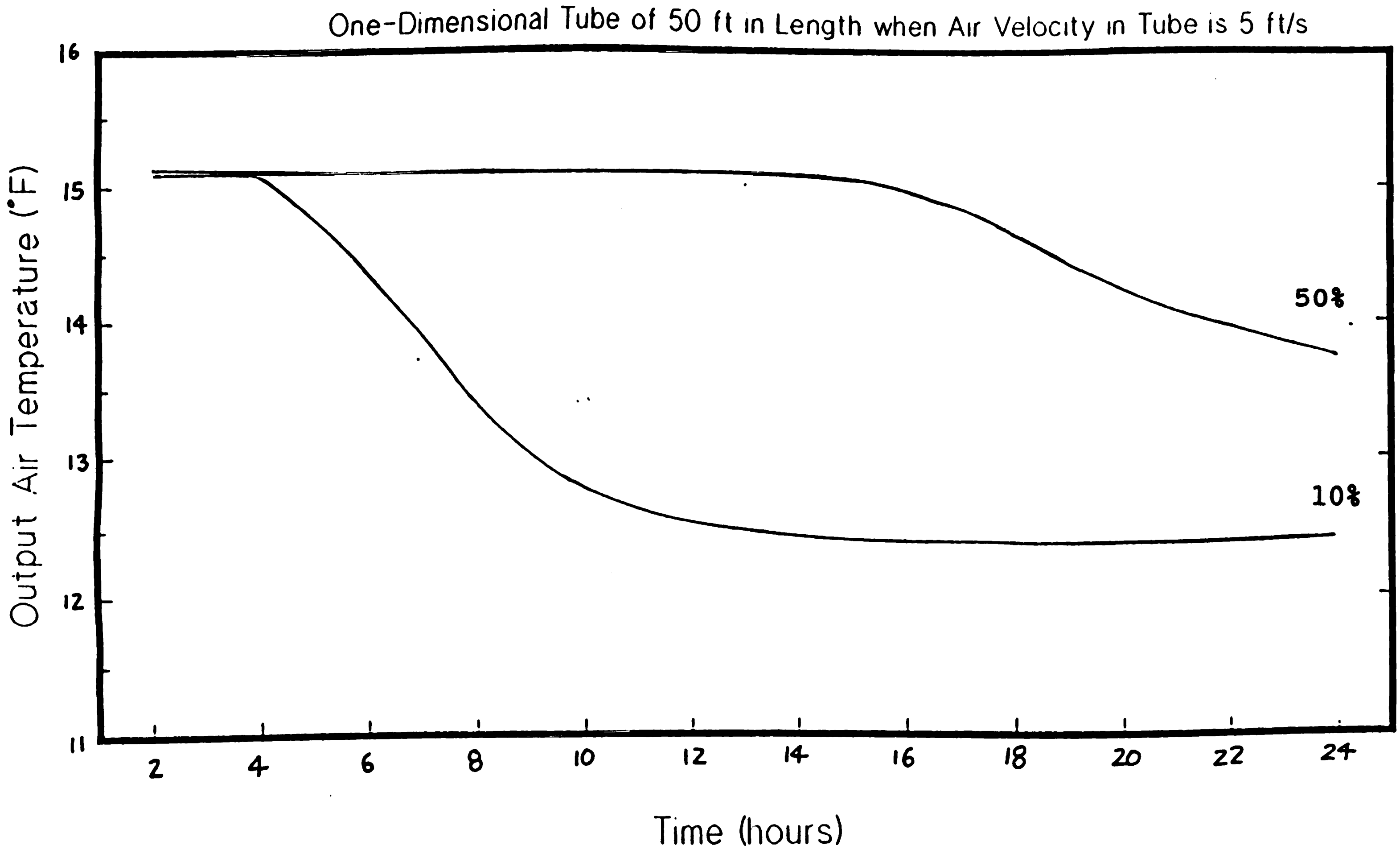

Figure 35. Output Air Temperature vs Time for 10% and 50% Moisture Contents.

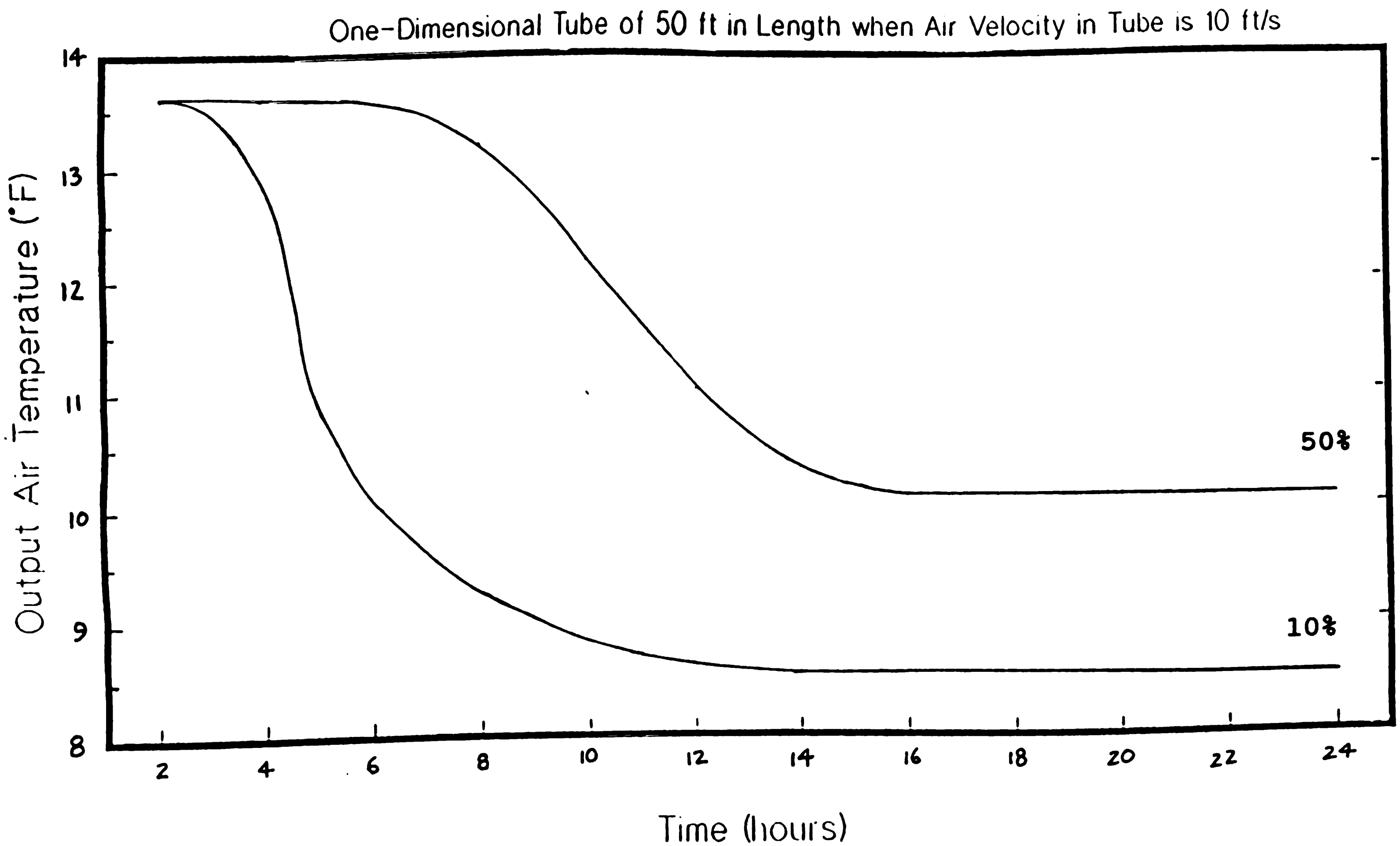

Figure 36. Output Air Temperature vs Time for 10% and 50% Moisture Contents.

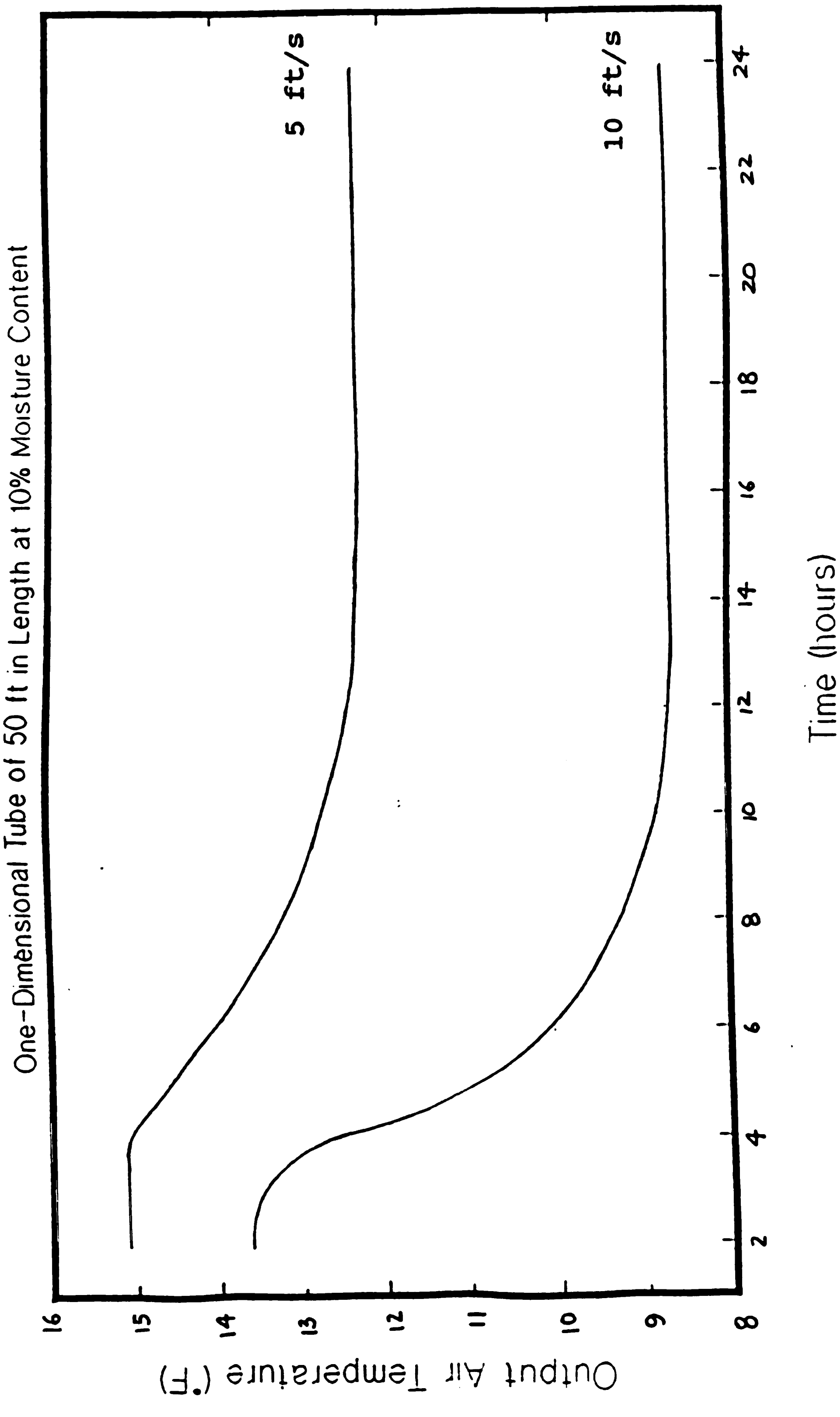

Figure 37. Output Air Temperature vs Time for Air Velocities of 5 and 10 ft/s.

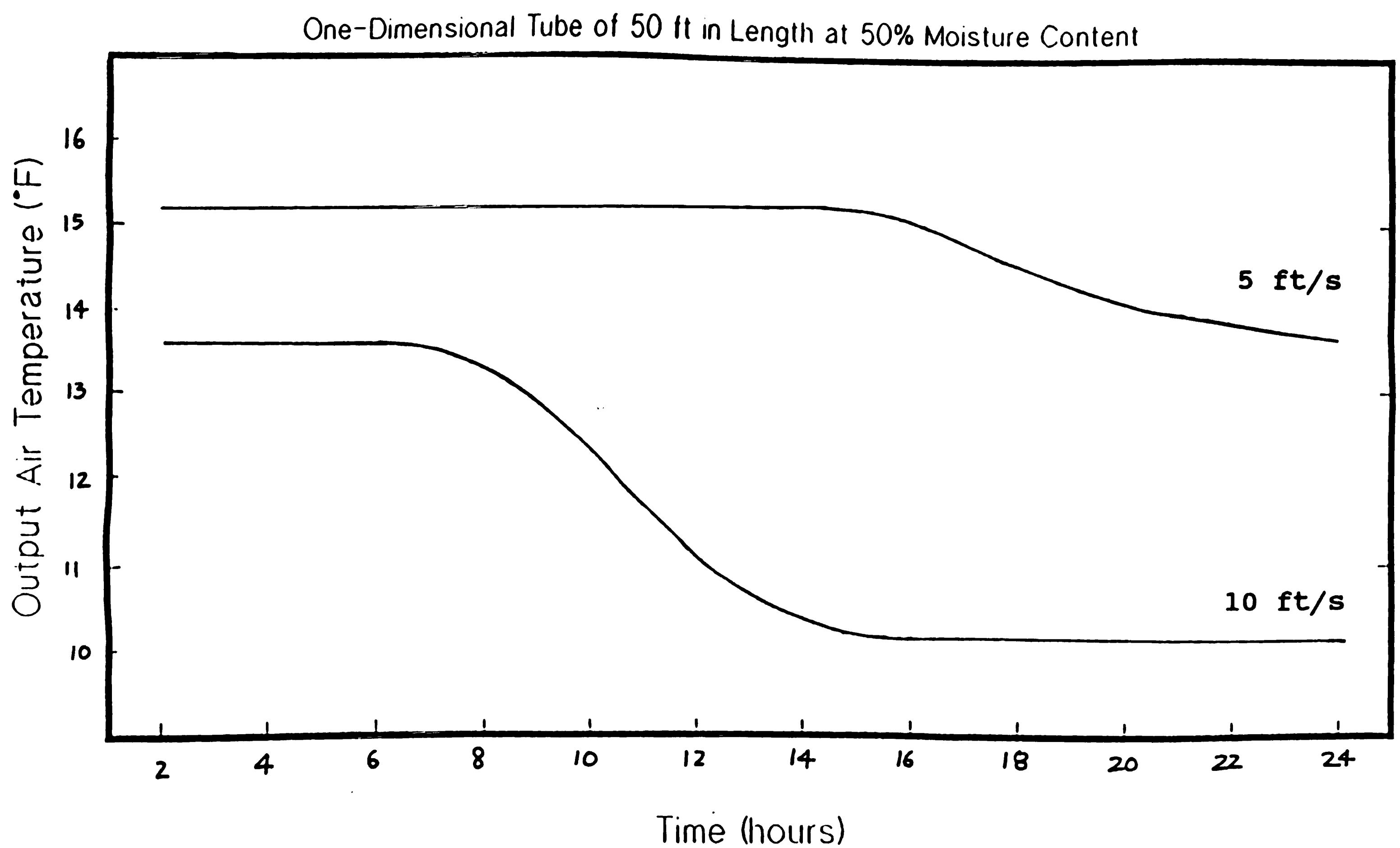

Figure 38. Output Air Temperature vs Time for Air Velocities of 5 and 10 ft/s.

air was approximately 1.5°F for both cases. After about 12 hours of continuous operation, the difference became close to 3.5°F for the 50 percent moisture content in the soil (see Figure 38), and 3.8°F for the 10 percent moisture content in the soil (see Figure 37).

For the one-dimensional tube of 50 ft in length with air velocities in the tube of 5 ft/s and 10 ft/s, Figures 39 and 40 show graphs of air temperature in the tube as a function of x-location along the tube at the 24th hour of continuous operation. It appears that for different moisture contents of the soil, the air temperature in the tube does not vary significantly at each x-location along the tube. For both cases of the air velocity in the tube of 5 ft/s (see Figure 39) and 10 ft/s (see Figure 40), the temperature difference between the 10 and 50 percent moisture content, does not exceed more than 1.7°F throughout the length of the tube.

Air temperature in the tube versus x-location along the tube at air velocities of 5 ft/s and 10 ft/s for 10 and 50 percent moisture contents in the soil is depicted in Figures 41 and 42, respectively. For the 10 percent moisture content in the soil, the temperature difference for the two air velocities along the tube varies between 0.54°F and 3.8°F and for the 50 percent moisture content in the soil, the temperature difference varies between 0.39°F and 3.51°F.

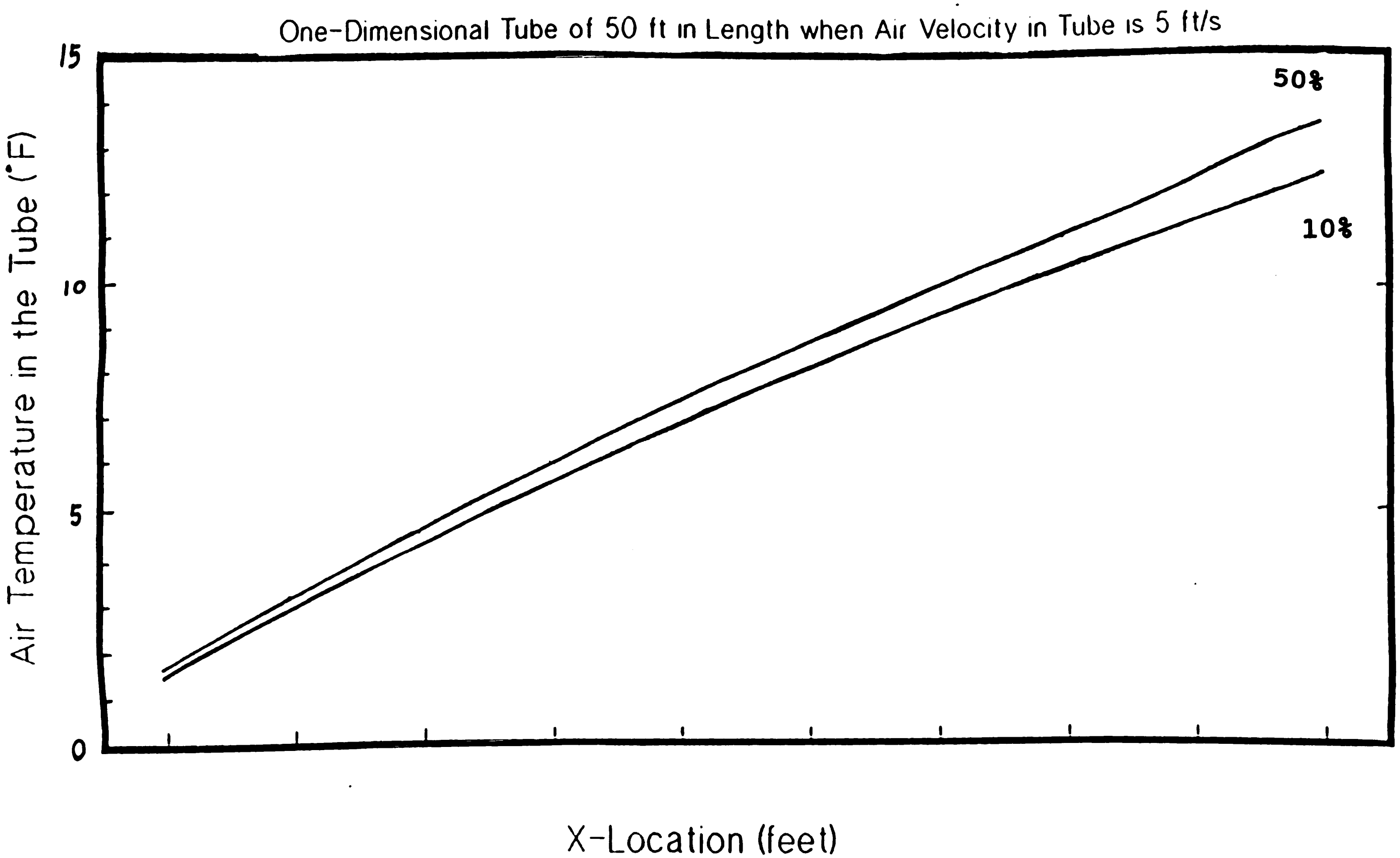

Figure 39. Air Temperature in the Tube vs X-Location at 24th Hour.

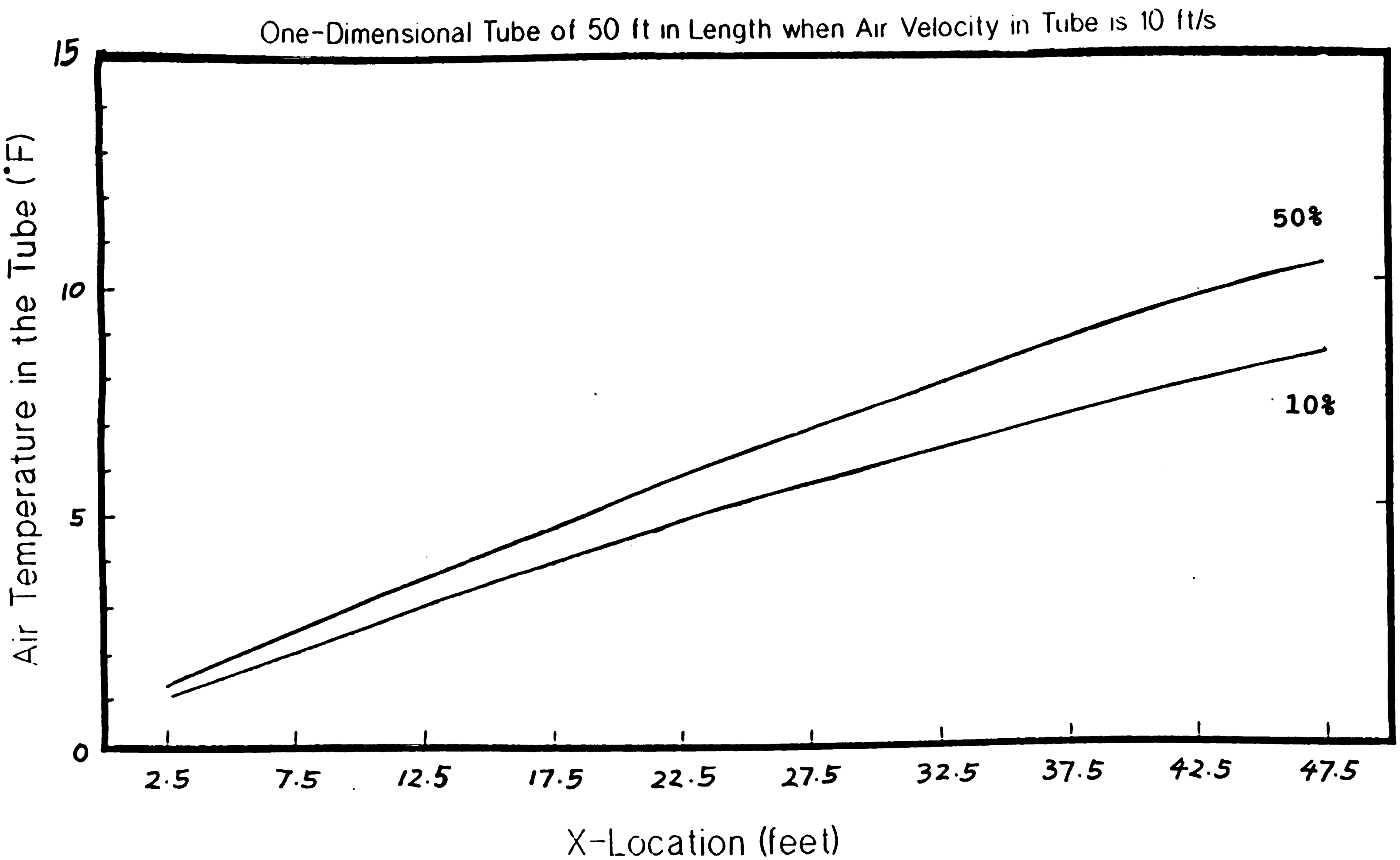

Figure 40. Air Temperature in the Tube vs X-Location at 24th Hour.

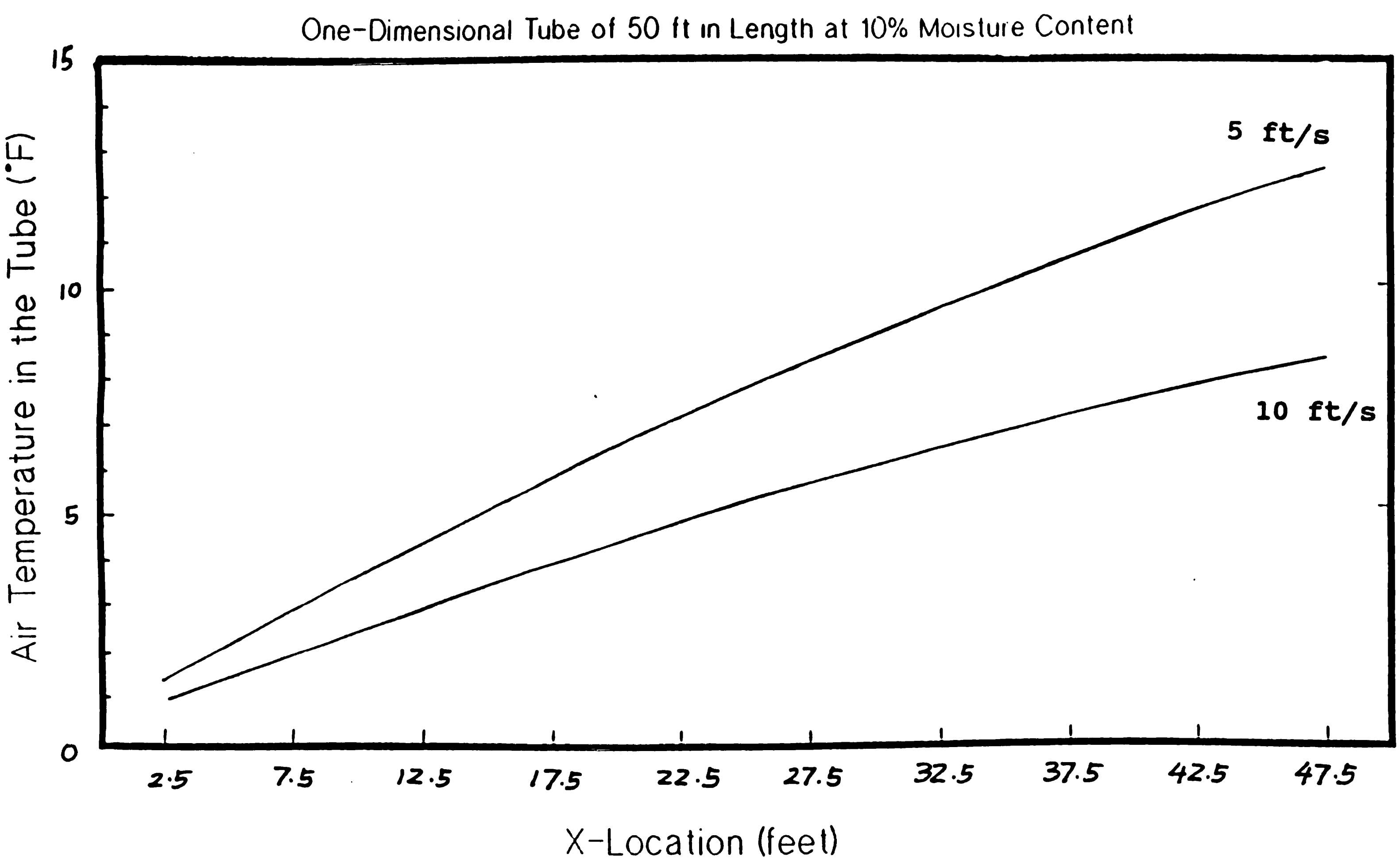

Figure 41. Air Temperature in the Tube vs X-Location at 24th Hour.

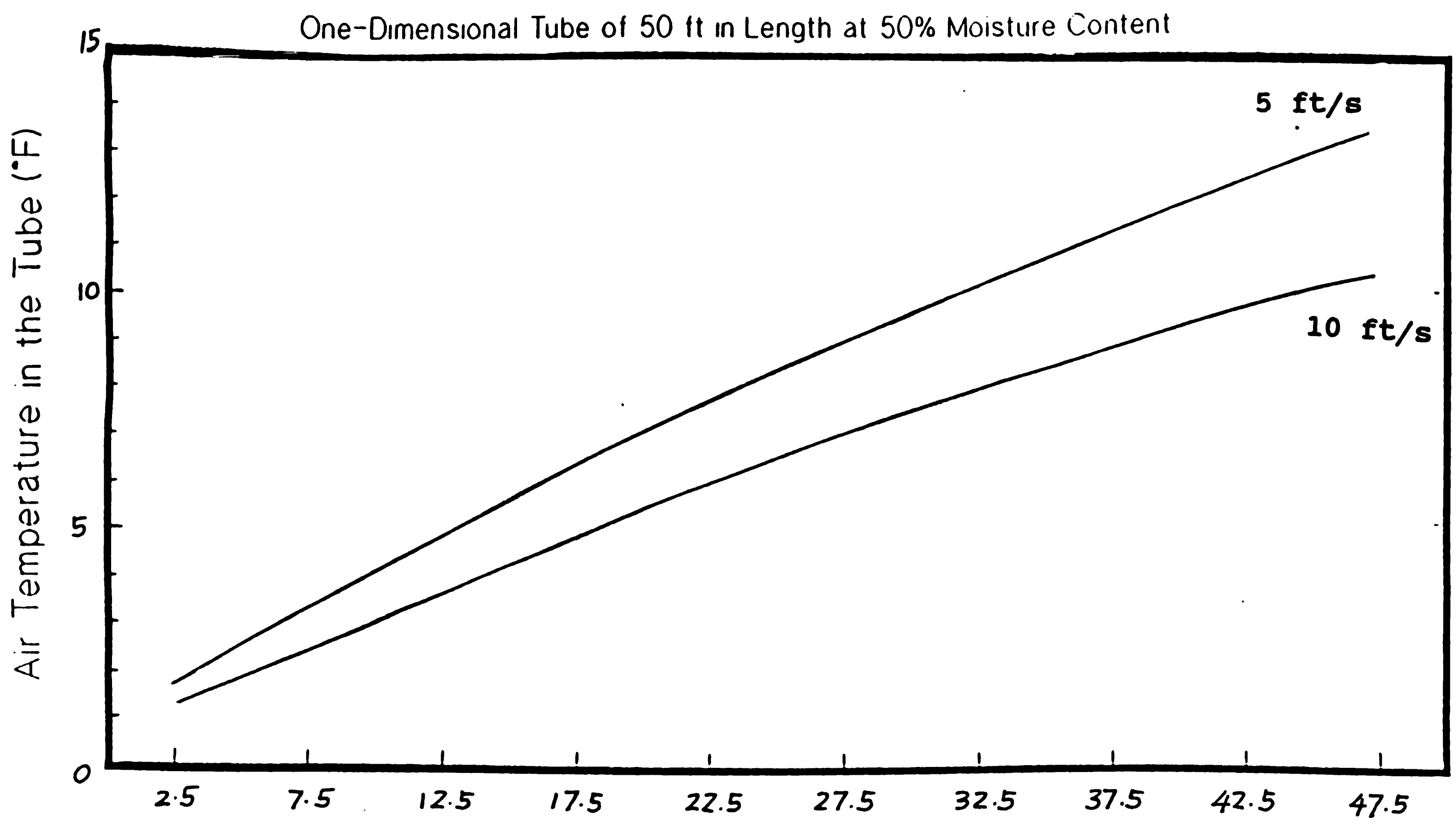

Figure 42. Air Temperature in the Tube vs X-Location at 24th Hour.

Figures 43, 44, 45 and 46 show the thickness of ice along the length of the tube at different air velocities and moisture contents of the soil. At an air velocity of 5 ft/s, Figures 43 and 44 show the ice formation at the 8th, 16th and 24th hour at 10 and 50 percent moisture content in the soil, respectively. Figures 45 and 46 show the thickness of ice formation at the 8th, 16th and 24th hour at an air velocity of 10 ft/s with 10 and 50 percent moisture content in the soil, respectively. It is observed that there was more ice formed when the moisture content of the soil is 10 percent for both cases of the air velocity. This is due to more time needed to freeze the moisture in the soil for higher moisture content.

4.3 Results for the Two-Dimensional Model

Criterion for termination of the two-dimensional model program is when the output air temperature achieved do not change from one time step to the next. This stability criterion is set at 1.0×10^{-10} °F.

Figure 47 shows the output air temperature as a function of time for 10 and 50 percent moisture content of the soil when the air flow rate is 600 cubic feet per minute. For the 10 percent moisture content in the soil, a stabilized output air temperature of 14.96°F was obtained after 1.09 hours of continuous operation whereas for the 50 percent moisture content in the soil, a stabilized output air temperature of 14.98°F was obtained after 1.98 hours of

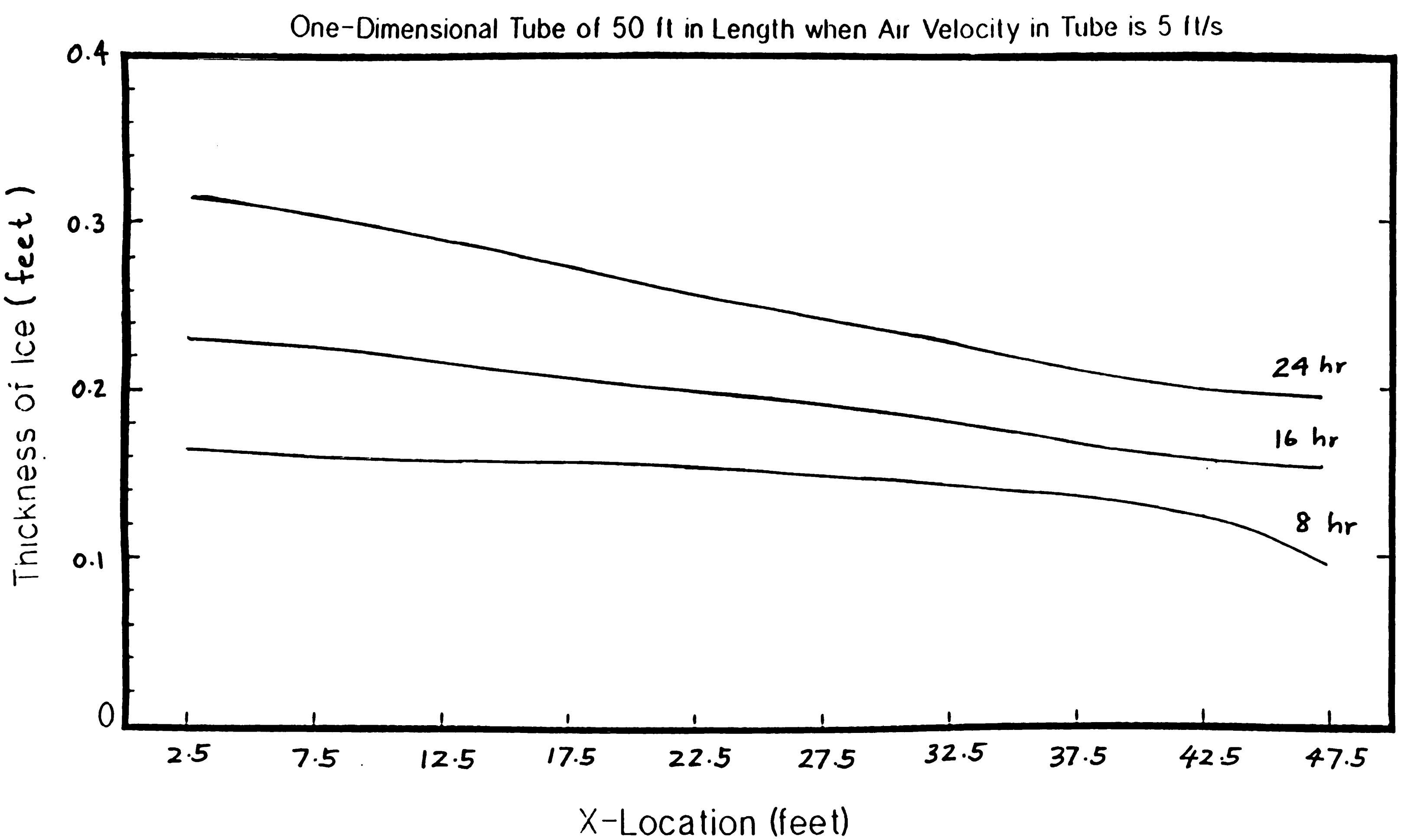

Figure 43. Thickness of Ice vs X-Location at 10% Moisture Content.

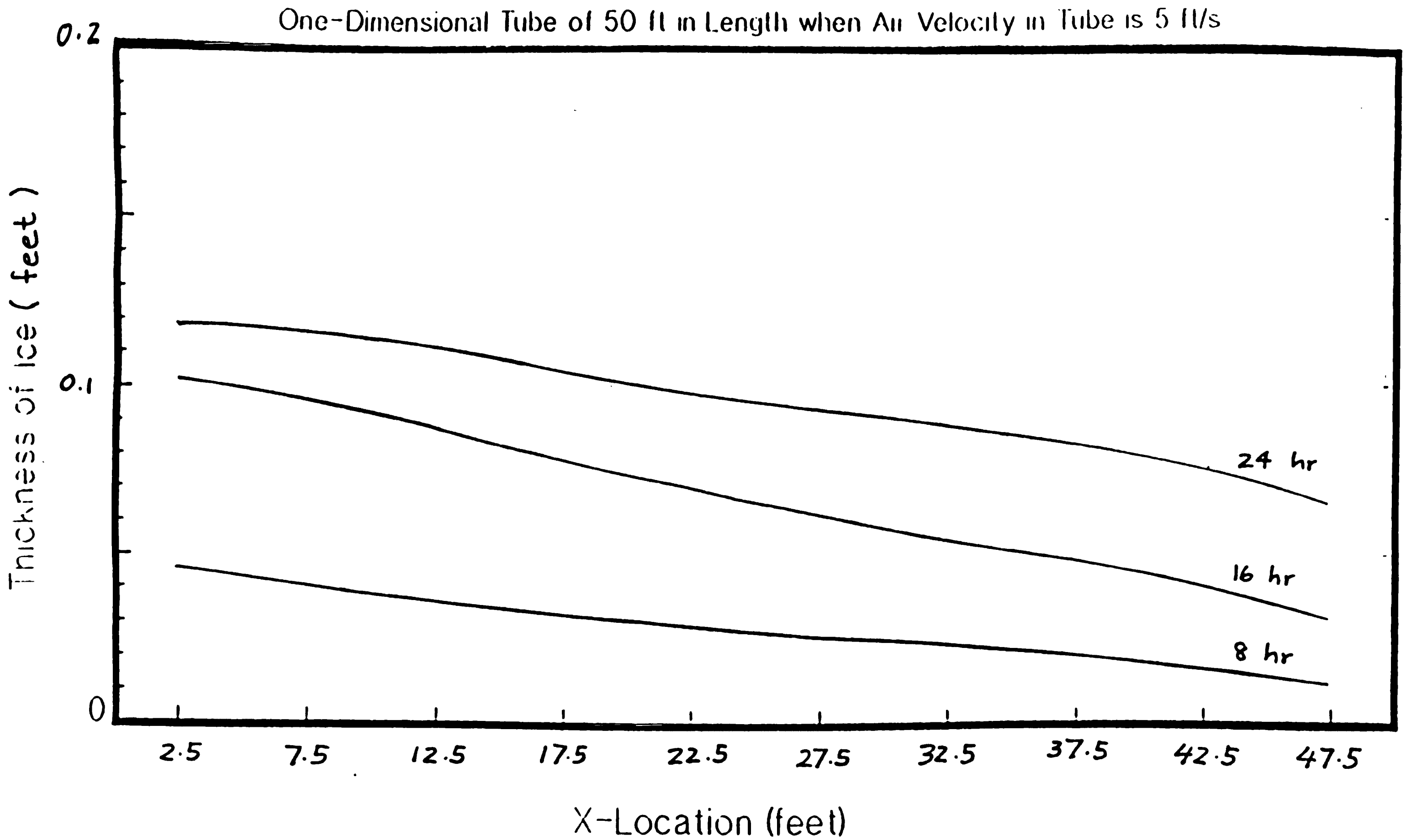

Figure 44. Thickness of Ice vs X-Location at 50% Moisture Content.

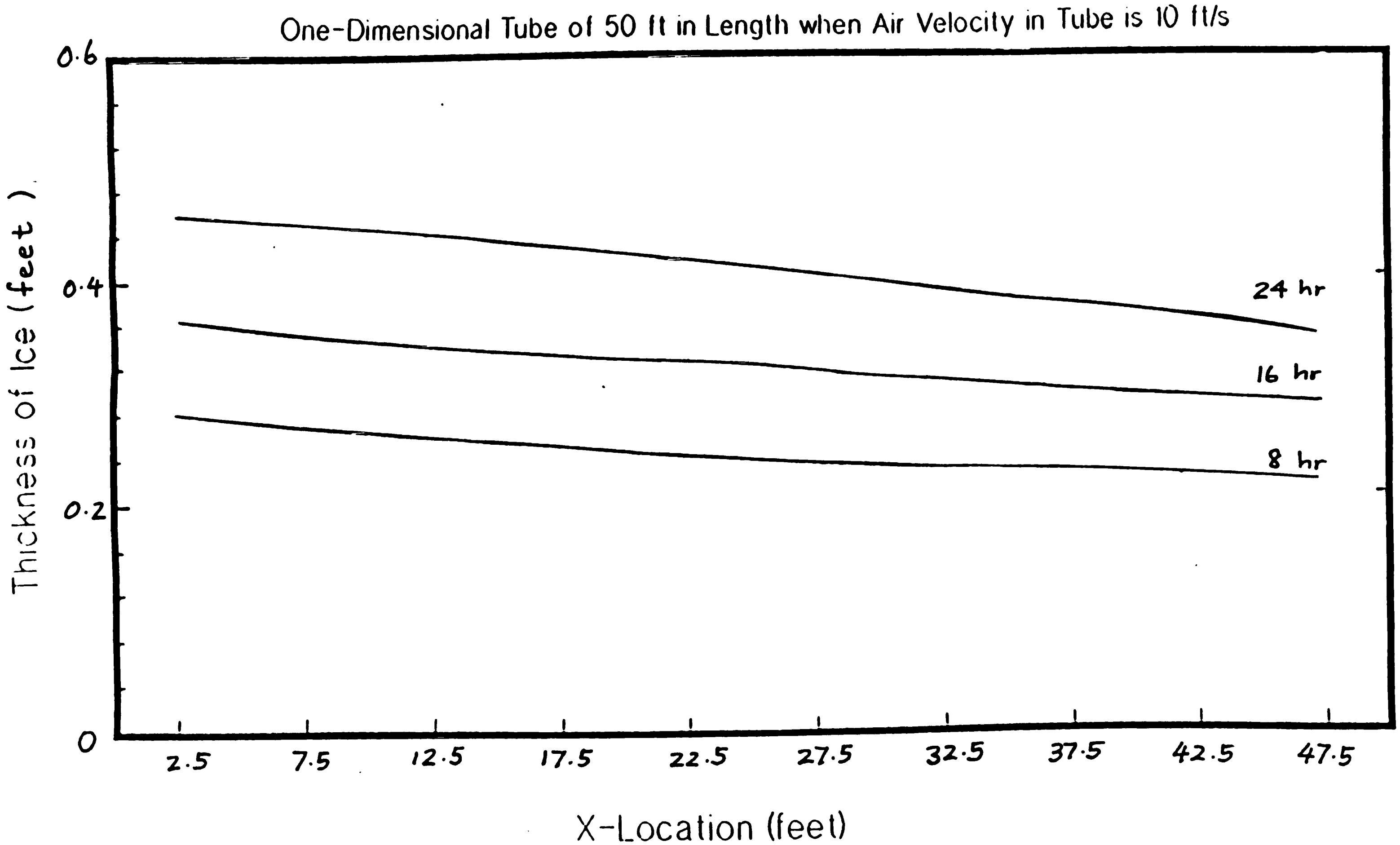

Figure **45.** Thickness of Ice vs X-Location at 10% Moisture Content.

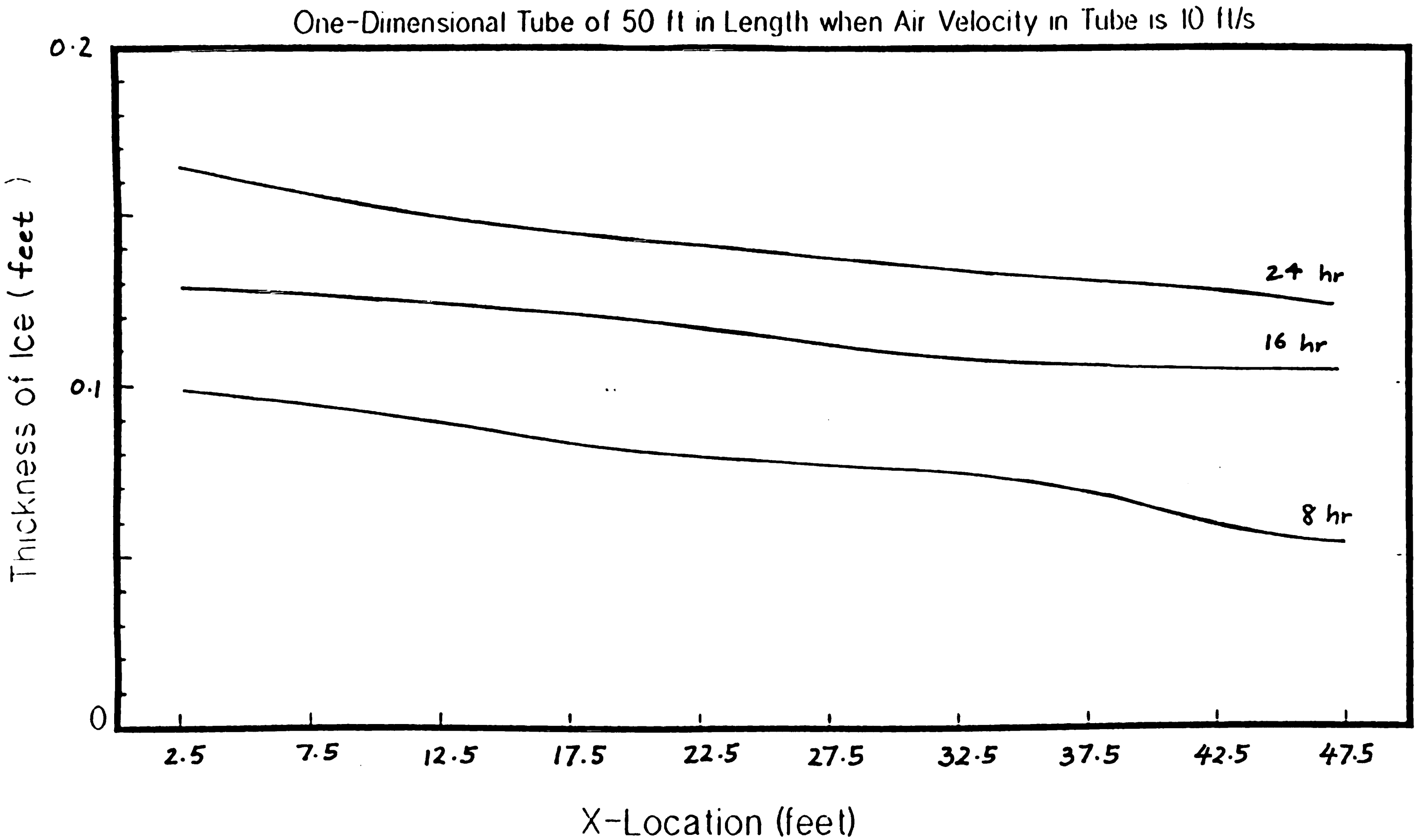

Figure **46.** Thickness of Ice vs X-Location at **50%** Moisture Content.

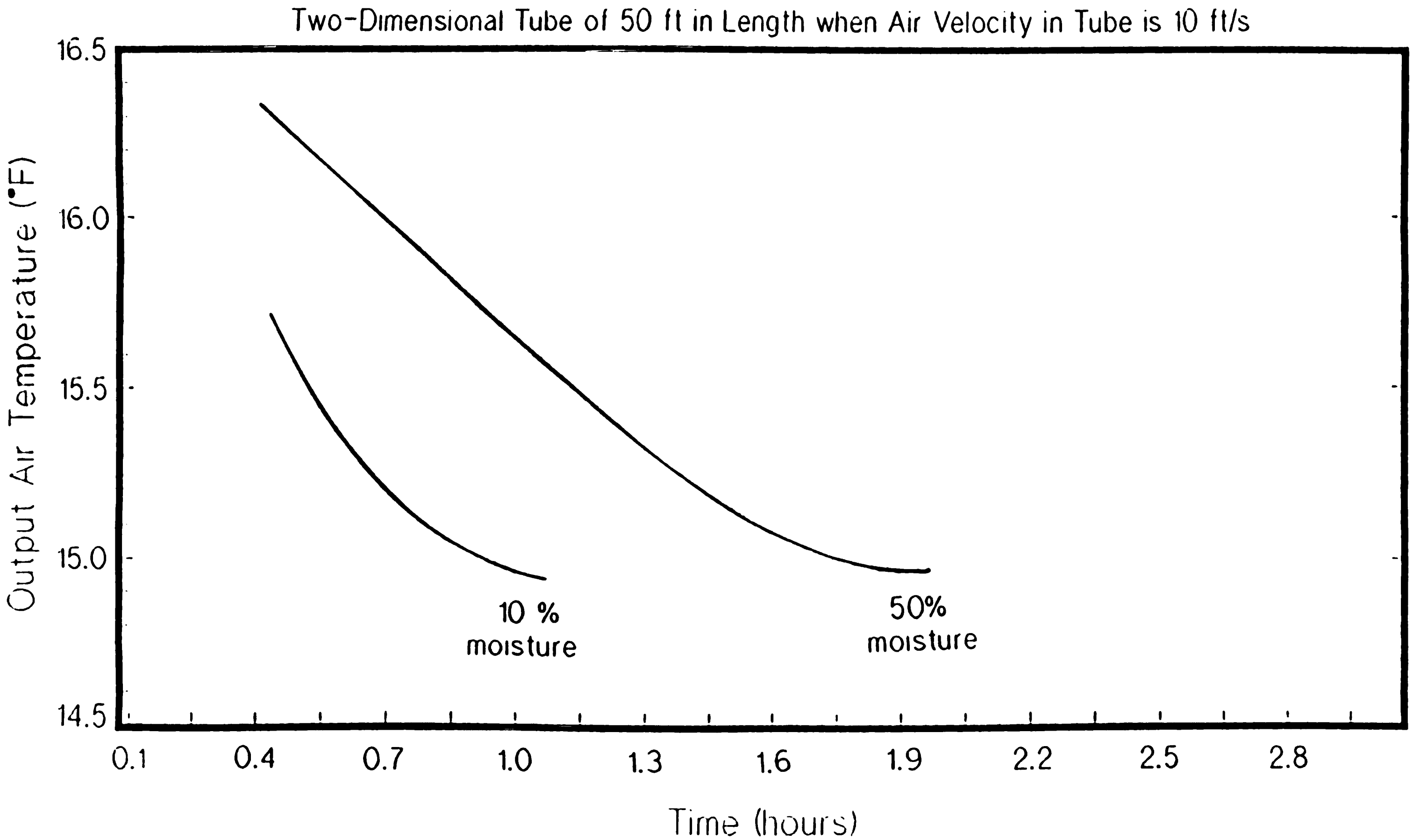

Figure **47.** Output Air Temperature vs Time for 10% and 50% Moisture Content.

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continuous operation. After the beginning of the simulation, at 0.5 hours of the tube's operation, the output temperature difference between the two cases was 4.1 percent. However, after stability, the temperature difference was only 0.2 percent.

Air temperature in the tube along the length of the tube is compared for 10 and 50 percent moisture content in the soil when the air velocity in the tube is 10 ft/s, as depicted in Figure 48. As in the one-dimensional tube, the difference in the air temperature in the tube at each x-location along the tube for the different moisture contents in the soil, does not vary significantly. The variation is only between 0.04°F and 0.2°F.

Since the output air temperature stabilized very quickly for both runs, the ice formations could not be observed due to program termination.

4.4 Comparisons for the One and Two-Dimensional Models

Output air temperature in the 50 feet long tube as a function of time for the one and two-dimensional models are graphed in Figures 49 and 50 for comparisons. For both cases, the air flow rate in the tube is 600 cubic feet per minute. Two moisture contents of the soil, 10 and 50 percent, were considered.

In Figure 49, where the soil's moisture content is 10 percent, the output air temperature for the two-dimensional tube was higher by 0.84°F, than the

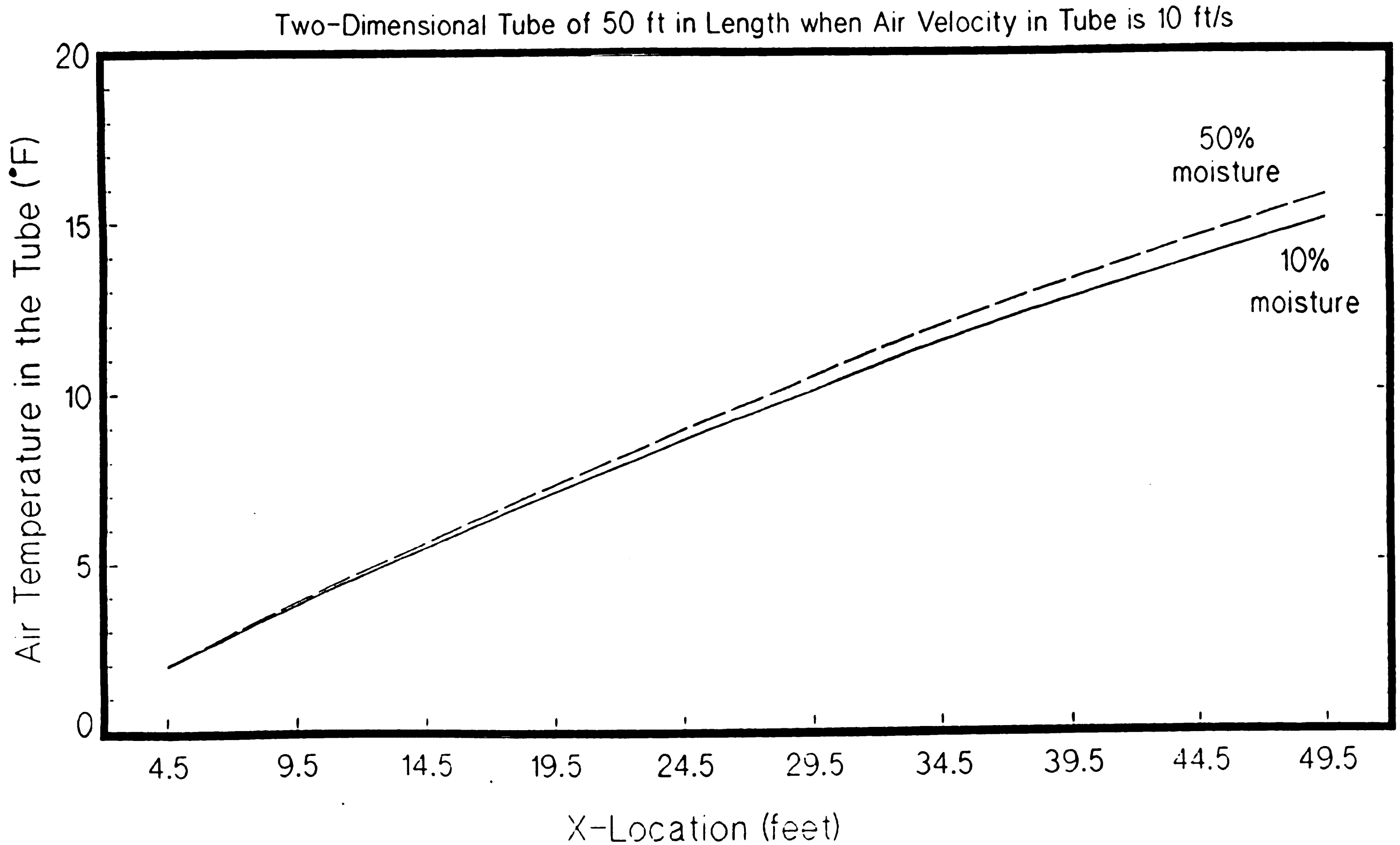

Figure **48.** Air Temperature in the Tube vs X-Location at the First Hour.

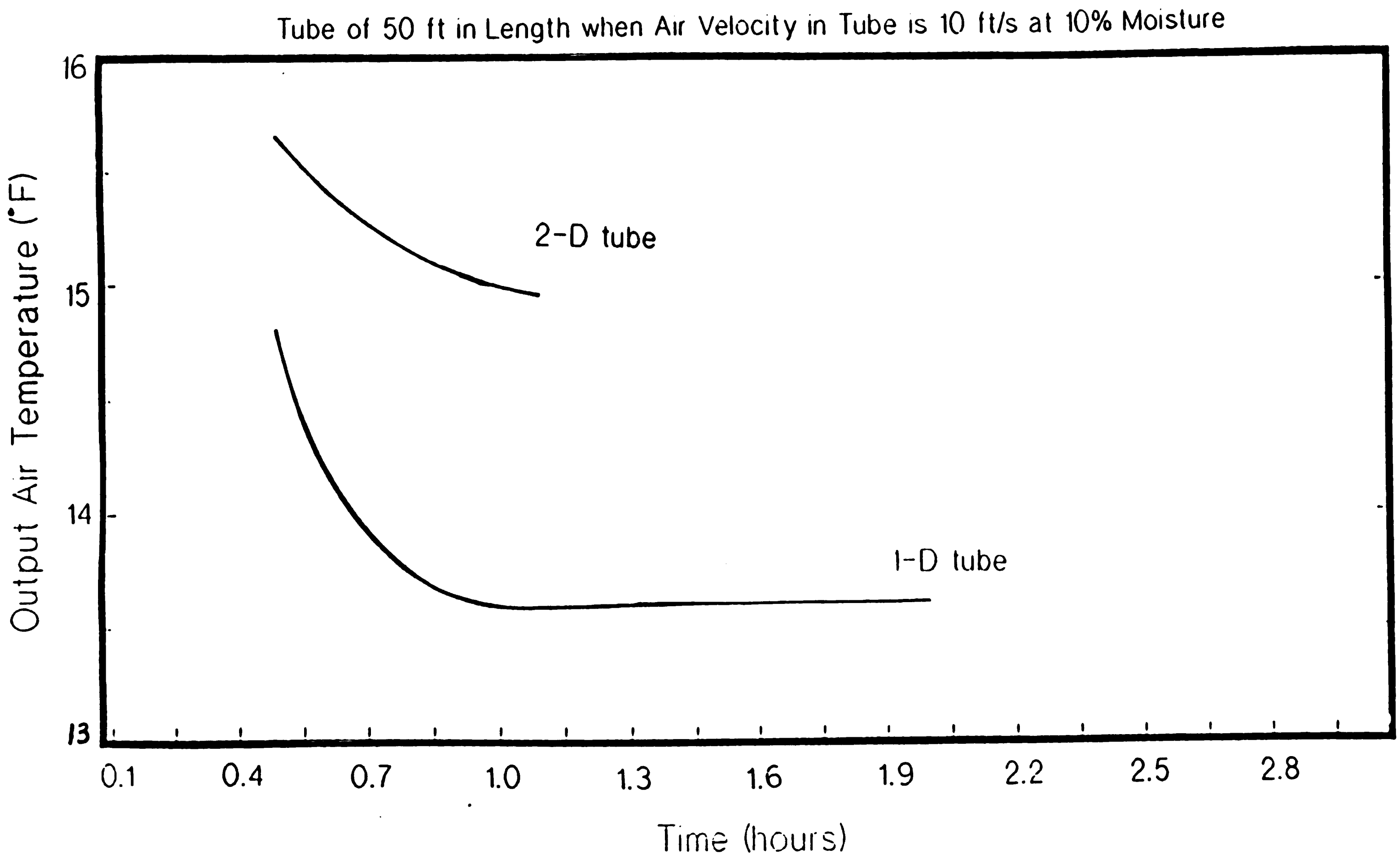

Figure **49.** Output Air Temperature vs Time for 1-D and 2-D Tubes.

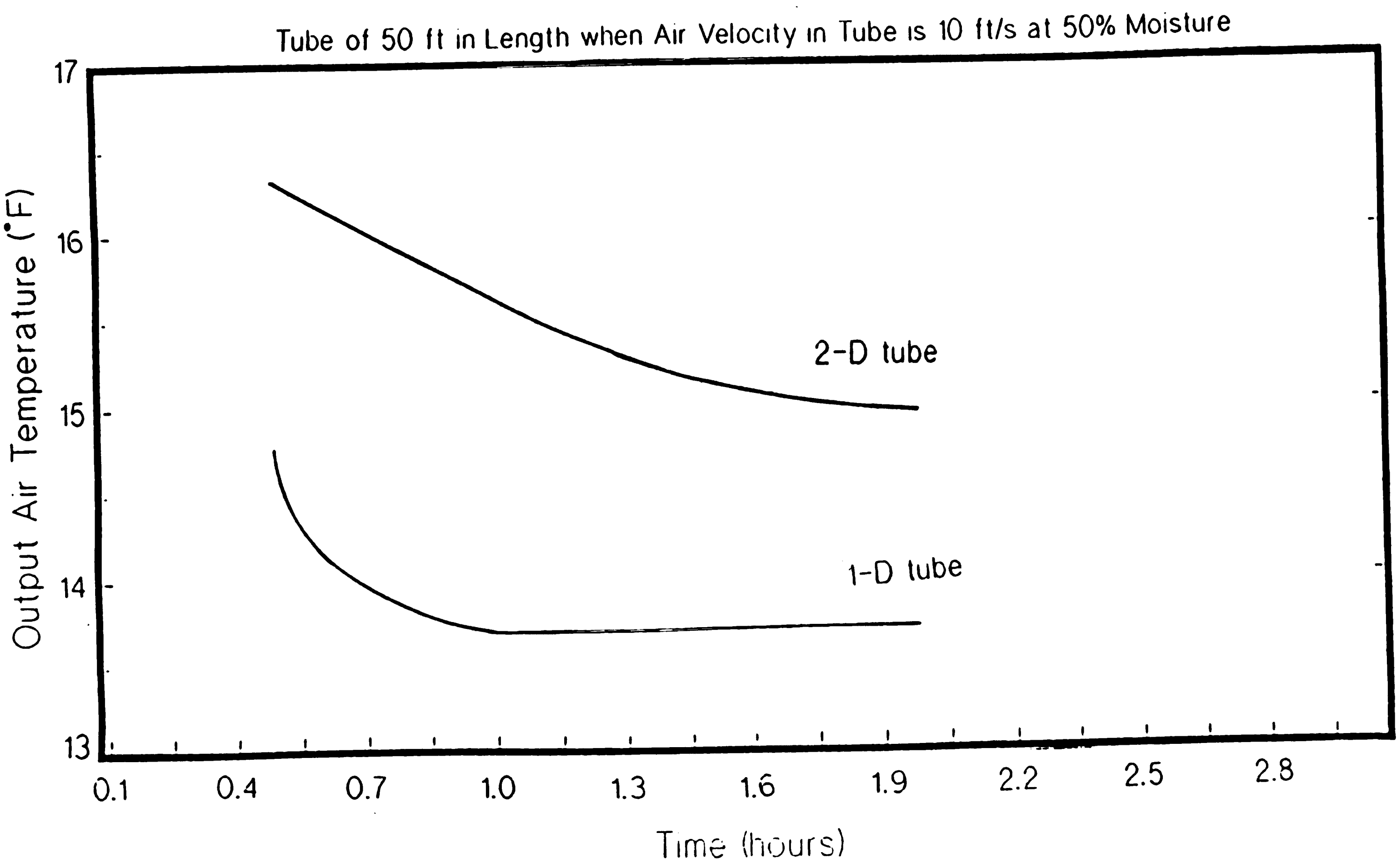

Figure **50**. Output Air Temperature vs Time for 1-D and 2-D Tubes.

one-dimensional tube at 0.5 hour of the tube's operation. However, after 1 hour, the output air temperature for the two-dimensional tube was higher than the one-dimensional tube by 1.31°F.

For the soil's moisture content of 50 percent (see Figure 50), the output air temperature at 0.5 hour of the tube's operation for two-dimensional tube, was higher than the one-dimensional tube by 1.59°F. However, after 2 hours, the output air temperature for the two-dimensional tube was higher by 1.32°F, when compared to the one-dimensional tube.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Two mathematical models were considered, one a circular tube with radial coordinates and the other a square tube with Cartesian coordinates, known here as the one- and two-dimensional models. From the study that has been accomplished, several conclusions can be stated.

Increasing the moisture content of the soil will increase the output air temperature of the tube. This is due to the increase in the heat capacity of the soil which depends on the moisture content of the soil. For the one-dimensional model with an air flow rate of 300 cubic feet per minute, when the moisture content in the soil is increased from 10 to 50 percent, the output air temperature of the tube is increased by 10.2 percent; with an air flow rate of 600 cubic feet per minute, the increment is 17.3 percent. As for the two-dimensional model, the increment is 0.2 percent when the air flow rate is 600 cubic feet per minute. It appears that with the increasing air flow rate, the increment gets larger.

The formation of ice significantly influences the increase in temperature of the air in the tube. In the

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one-dimensional tube with an air flow rate of 300 cubic feet per minute, when the moisture content of the soil is 10 percent, the output air temperature stabilized at about 12 hours of simulation time; and at 50 percent, the output air temperature continued to decrease gradually after 16 hours. With an air flow rate of 600 cubic feet per minute, the output air temperature stabilized at about 14 hours and 16 hours for 10 and 50 percent moisture content in the soil, respectively. This early stability phenomena in the output air temperature is due to the early occurance of the ice formation in the soil of 10 percent moisture content. For higher moisture content, additional time would be needed for larger ice formation and thus prolongs the occurance of the stabilized output air temperature. Therefore, the moisture content in the soil significantly affects the ice formation.

It appears that the output air temperature for the two-dimensional tube is higher than the one-dimensional tube. For the 10 percent moisture content in the soil, the output air temperature for the two-dimensional tube is higher by 9.1 percent than the one-dimensional tube after 1 hour of operation. For the 50 percent moisture content in the soil, the output air temperature for the two-dimensional tube was higher than the one-dimensional tube by 9.2 percent after 2 hours of operation.

5.2 Recommendations

The major problem encountered in this numerical study is that too much computer time is needed. This is due to the time step and space step being small. Therefore, some work should be done in modifying or refining the computer model. The program could be modified by having smaller space steps near the tube and larger ones away from the tube. This would reduce the amount of computer time without actually affecting the accuracy of the simulation. Another possibility to reduce the computer time is to use an approximate solution in calculating the ground temperature distribution.

One area where the computer model could be refined is to include the tube's material and thickness, and the effects of the soil's thermal contact with the tube. Also, the calculation of the heat transfer coefficient of the soil to the atmospheric air at the ground surface should be included with real outside conditions of variable temperature and wind speed.

Experimental research should be carried out for the validation of the numerical results to verify if the predicted results could actually be obtained.

Experimental research should also be carried out to determine the soil's thermal properties since different references give different values of thermal properties for the same soil. The soil's thermal conductivity and heat capacity actually depend on its composition, moisture content and temperature. Therefore, a suitable corelation should be obtained to include these effects.

According to a parametric study (32), the performance of the tube could be increased if the burial depth of the tube is increased. However, caution must be taken to avoid a permafrost region around the tube when in use in the northern climates.

This numerical study is limited to the tube's continuous operating condition. Study should be carried out in the "on/off" cyclic operating condition where air is allowed to enter the tube for 30 minutes and then shut off for the next 30 minutes for every hour. This is to allow the ground to recover between periods of the tube operation.

Heat pumps are also used for summer cooling. Further study is called for, in simulating summer operation. For summer simulation, soil moisture migration effects should be included in the computer model because the moisture content in the soil plays an important role in the soil thermal properties. However, research in soil moisture migration with soil temperature has been limited.

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BIBLIOGRAPHY

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32. Same as (11).

APPENDICES

APPENDIX A

COMPUTER PROGRAM FOR ONE-DIMENSIONAL MODEL

Appendix A

Computer Program for One-Dimensional Model

```
      PROGRAM ONEDIM
C ********************************************************************
C This program is to determine the thermal performance of an under-
C ground heat tube in one dimension. It performs a finite difference
C analysis on a circular buried tube through which air is flowing.
C Central differences are used to calculate the heat gain of each
C element and forward differences are used in the time step. This
C program also calculates the thickness of ice formation around the
C tube.
C ********************************************************************
C
C Variable List
C
C TFAR   Farfield Temperature of Soil
C TATM   Current Temperature of Atmosphere
C TEMP   Array Containing Temperatures of Soil and Air
C CPAIR  Heat Capacity of Air
C H      Convective Heat Transfer Coefficient
C DNAIR  Density of Air
C AKSOLN Thermal Conductivity of Normal Soil
C AKSOLF Thermal Conductivity of Frozen Soil
C CPSOLN Heat Capacity of Soil in a Thawed State
C CPSOLF Heat Capacity of Soil in a Frozen State
C CPSOIL Heat Capacity of Soil based  on its Current Temperature
C DNSOIL Density of Soil
C CPFUSE Heat of Fusion of Water
C CPWATR Heat Capacity of Water
C CPICE  Heat Capacity of Ice
C XINCRE Increment Size in the X-direction
C RINCRE Increment Size in the R-direction
C XNUM   Number of Increments in the X-direction
C RNUM   Number of Increments in the R-direction
C AMOIST Moisture Content of Soil as a Percentage by Weight
C VELCTY Velocity of Air
C TIMLOP Maximum Number of Timesteps to be Performed
C TIMEX                    ,,
```

```
C AMASOL Dry Mass of Soil
C AMASAR Mass of Air
C DELI   Array containing Change in Ice Formation
C RICE   Array containing Ice Thickness
C RI     Array containing Radius of Ice from Center of Tube
C AKMAJ  Coefficient of Heat Transfer Equation
C AKMAJ1              ,,
C AKMAJ2              ,,
C AK00                ,,
C AK1    Coefficient of the J-1 term Heat Transfer Coefficient
C AK2    Coefficient of the J+1 term Heat Transfer Coefficient
C I      Loop Counter Array Index
C J              ,,
C K              ,,
C P              ,,
C T              ,,
C A              ,,
C B              ,,
C
C ********************************************************************
C ********************************************************************
C
      INTEGER I,J,K,T,P,XNUM,RNUM,TIMEX,TIMLOP,CHOICE,A,B,ANS,UNIT
      DIMENSION TEMP(12,10,2),R(10),RICE(11,2),DELI(11,2),
     +RI(11,2)
C
C ********************************************************************
C Open Data File
C ********************************************************************
C
      OPEN (UNIT=8,NAME='RESULT.DAT',TYPE='UNKNOWN')
C
C ********************************************************************
C Initialize Test Variables
C ********************************************************************
C
```

```
      TFAR   = 39.4
      TATM   = 0.0
      CPAIR  = 0.24
      DNAIR  = 0.08275
      AKSOLN = 0.458
      AKSOLF = 0.833
      CPSOLN = 0.3
      CPSOLF = 0.2
      DNSOIL = 95.0
      CPFUSE = 143.3
      CPWATR = 1.0
      CPICE  = 0.5
      PI     = 3.141593
      K      = 1
      XINCRE = 5.0
      XNUM   = 10
      RNUM   = 8
      R(2)   = 0.564
      TIMLOP = 12
      TIMSTP = 0.16667
      TIMEX  = 12
C
      WRITE(*,*)' INPUT MOISTURE CONTENT OF THE SOIL'
      READ(*,*)AMOIST
      WRITE(8,*)' MOISTURE CONTENT OF SOIL =  ',AMOIST
      WRITE(*,*)' INPUT VELOCITY OF AIR IN THE TUBE'
      READ(*,*)VELCTY
      WRITE(8,*)' VELOCITY OF AIR IN THE TUBE =  ',VELCTY
      WRITE(*,*)' INPUT CONVECTIVE HEAT TRANSFER COEFFICIENT'
      READ(*,*)H
      WRITE(8,*)' CONVECTIVE HEAT TRANSFER COEFFICIENT =  ',H
      WRITE(*,*)' INPUT DELTA R'
      READ(*,*)RINCRE
      WRITE(8,*)' DELTA R =  ',RINCRE
C
C *********************************************************************
```

```
C Set Initial Temperature
C ***********************************************************************
C
      DO 100 I = 1,XNUM+2
         DO 110 J = 1,RNUM+2
            TEMP(I,J,K) = TFAR
110      CONTINUE
100   CONTINUE
C
C ***********************************************************************
C Calculate Radius of Nodes
C ***********************************************************************
C
      WRITE(8,*)'RADIUS OF NODES [R(3),R(4),.....]:'
      DO 120 J = 3,RNUM+2
         R(J) = R(J-1) + RINCRE
         WRITE(8,51)R(J)
51       FORMAT(1X,F8.5)
120   CONTINUE
C
C ***********************************************************************
C Calculate Time Step
C ***********************************************************************
C
      WRITE(8,54)TIMSTP
54    FORMAT(1X,'TIMSTP = ',F10.8)
C
C ***********************************************************************
C Write Initial Ice Thickness, Radius of Ice and Temperature of Air
C in the Tube
C ***********************************************************************
C
      WRITE(8,*)' INITIAL ICE THICKNESS, RADIUS OF ICE AND'
      WRITE(8,*)' TEMPERATURE OF AIR IN THE TUBE :'
      DO 130 I = 2,XNUM+1
         RICE(I,K) = 0.0
```

```
          RI(I,K) = R(2)
          DELI(I,K+1) = 0.0
          WRITE(8,55)RICE(I,K),RI(I,K),TEMP(I,1,K)
55        FORMAT(1X,F10.8,3X,F10.8,3X,F11.7)
130    CONTINUE
C
       WRITE(8,2)
2      FORMAT(1X,70('*'))
C




C **********************************************************************
C **********************************************************************
C ***                                                               ***
C ***     Loop Calculating Heat Transfer Throughout The Array       ***
C ***                                                               ***
C **********************************************************************
C **********************************************************************
C
       DO 200 T = 1,TIMEX
          WRITE(8,*)' T = ',T
          WRITE(8,3)
3         FORMAT(1X,70('@'))
          DO 210 P = 1,TIMLOP
             DO 230 I = 2,XNUM+1
                DO 240 J = 2,RNUM+1
C
C **********************************************************************
C Select Soil Heat Capacity
C **********************************************************************
C
       IF (TEMP(I,J,K) .GT. 32.0) THEN
          CPSOIL = CPSOLN + (AMOIST * CPWATR)
       ELSE
```

```
            IF (TEMP(I,J,K) .LT. 32.0) THEN
               CPSOIL = CPSOLF + (AMOIST * CPICE)
            ELSE
               CPSOIL = (CPSOLN+(AMOIST*CPWATR))*(RINCRE-RICE(I,K))
     +  /RINCRE + (CPSOLF+(AMOIST*CPICE))*RICE(I,K)/RINCRE
            END IF
         END IF
C
C ********************************************************************
C Select Soil Thermal Conductivity
C ********************************************************************
C
         TAVE1 = (TEMP(I,J-1,K) + TEMP(I,J,K)) / 2.0
         TAVE2 = (TEMP(I,J+1,K) + TEMP(I,J,K)) / 2.0
         IF (TAVE1 .GE. 32.0) THEN
            AKSOL1 = AKSOLN
         ELSE
            AKSOL1 = AKSOLF
         END IF
         IF (TAVE2 .GE. 32.0) THEN
            AKSOL2 = AKSOLN
         ELSE
            AKSOL2 = AKSOLF
         END IF
C
C ********************************************************************
C Select Appropriate Heat Transfer Coefficient
C ********************************************************************
C
         IF (J .EQ. 2) THEN
            AMASOL = DNSOIL * PI * (R(2) + (RINCRE/4.0)) * RINCRE *
     +               XINCRE
            AKMAJ = AMASOL * CPSOIL
            AK1 = H * 4.0 * PI * R(2) * XINCRE
            AK2 = AKSOL2 * 4.0 * PI * (R(2) + (RINCRE/2.0)) * XINCRE /
     +            RINCRE
```

```
            CHOICE = 1
C
         ELSE IF (J .EQ. 3) THEN
            AMASOL = DNSOIL * 2.0 * PI * R(3)  * RINCRE * XINCRE
            AKMAJ = AMASOL * CPSOIL
            AK1 = AKSOL1 * 2.0 * PI * (R(3) - (RINCRE / 2.0)) * XINCRE/
     +           RINCRE
            AK2 = AKSOL2 * 2.0 * PI * (R(3) + (RINCRE / 2.0)) * XINCRE/
     +           RINCRE
            CHOICE = 2
C
         ELSE
            AMASOL = DNSOIL * 2.0 * PI * R(J)  * RINCRE * XINCRE
            AKMAJ = AMASOL * CPSOIL
            AK1 = AKSOL1 * 2.0 * PI * (R(J) - (RINCRE / 2.0)) * XINCRE/
     +           RINCRE
            AK2 = AKSOL2 * 2.0 * PI * (R(J) + (RINCRE / 2.0)) * XINCRE/
     +           RINCRE
            CHOICE = 3
         END IF
C
C ************************************************************************
C Calculate The Temperature of The Element
C ************************************************************************
C
         TEMP(I,J,K+1) = TIMSTP * ( AK1 * (TEMP(I,J-1,K) - TEMP(I,J,K))
     +                           + AK2 * (TEMP(I,J+1,K) - TEMP(I,J,K)))
     +                           / AKMAJ + TEMP(I,J,K)
C
C ************************************************************************
C Calculating Ice Formation and Making Temperature Adjustments
C ************************************************************************
C
         IF (CHOICE .EQ. 1) THEN
            IF (TEMP(I,J,K+1) .LT. 32.0) THEN
               IF (RICE(I,K) .LE. (RINCRE/2.0)) THEN
```

```
               AKMAJ2 = DNSOIL * CPFUSE * AMOIST * 2.0 *
     +                     PI * RI(I,K) * XINCRE
               AK00 = AMASOL * CPSOIL / TIMSTP
               DELI(I,K+1) = TIMSTP *
     +                       (AK00 * (TEMP(I,J,K+1) - TEMP(I,J,K)))
     +                    / AKMAJ2
C
               IF (DELI(I,K+1) .LT. 0.0) THEN
                     DELI(I,K+1) = -DELI(I,K+1)
                     RICE(I,K+1) = RICE(I,K) + DELI(I,K+1)
                     RI(I,K+1) = R(2) + RICE(I,K+1)
               ELSE
                  RICE(I,K+1) = RICE(I,K) + DELI(I,K+1)
                  RI(I,K+1) = R(2) + RICE(I,K+1)
               END IF
C
               IF (RICE(I,K+1) .LT. (RINCRE/2.0)) THEN
                     TEMP(I,J,K+1) = 32.0
               END IF
            END IF
         ELSE
            RICE(I,K+1) = RICE(I,K)
            RI(I,K+1) = R(2) + RICE(I,K+1)
         END IF
C
C
      ELSE IF (CHOICE .EQ. 2) THEN
         IF (RICE(I,K) .GT. (RINCRE/2.0)) THEN
               AKMAJ2 = DNSOIL * CPFUSE * AMOIST * 2.0 *
     +                     PI * RI(I,K) * XINCRE
               AK00 = AMASOL * CPSOIL / TIMSTP
               DELI(I,K+1) = TIMSTP *
     +                       (AK00 * (TEMP(I,J,K+1) - TEMP(I,J,K)))
     +                    / AKMAJ2
C
               IF(DELI(I,K+1) .LT.0.0)THEN
```

```
                  DELI(I,K+1)=-DELI(I,K+1)
                  RICE(I,K+1) = RICE(I,K) + DELI(I,K+1)
                  RI(I,K+1) = R(2) + RICE(I,K+1)
                END IF
C
                  IF (RICE(I,K+1) .LT. (3.0*RINCRE/2.0)) THEN
                     IF (RICE(I,K+1) .GT. (RINCRE/2.0)) THEN
                        IF (TEMP(I,J,K+1) .LT. 32.0) THEN
                           TEMP(I,J,K+1) = 32.0
                        END IF
C
                     END IF
                  END IF
            END IF
C
C
         END IF
C
C
240               CONTINUE
C
230            CONTINUE
C
C **********************************************************************
C Reset Tube Entrance Temperature to Current Ambient Temperature
C **********************************************************************
C
         TEMP(1,1,K+1) = TATM
C
C **********************************************************************
C Match End Wall Temperature to Satisfy the Insulation Condition
C **********************************************************************
C
         TEMP(1,2,K+1) = TEMP(2,2,K+1)
C
C **********************************************************************
```

```
C Calculate The Temperature of Air in The Tube
C ********************************************************************
C
        AKMAJ1 = (2.0 * H * XINCRE) / (DNAIR * CPAIR * R(2) * VELCTY)
        DO 220 I = 2,XNUM+1
           TEMP(I,1,K+1)=AKMAJ1 * (TEMP(I-1,2,K+1) - TEMP(I-1,1,K+1))
     +                    + TEMP(I-1,1,K+1)
220     CONTINUE
C
C ********************************************************************
C Move K+1 Temperatures to K for Next Loop
C ********************************************************************
C
        DO 310 I = 2,XNUM+1
           DO 320 J = 1,RNUM+1
              TEMP(I,J,K) = TEMP(I,J,K+1)
320        CONTINUE
310     CONTINUE
C
C ********************************************************************
C Move K+1 Ice Formation to K for Next Loop
C ********************************************************************
C
        DO 330 I = 2,XNUM+1
           RICE(I,K) = RICE(I,K+1)
           RI(I,K) = RI(I,K+1)
330     CONTINUE
C
C
210        CONTINUE
C
C ********************************************************************
C Write Out The Temperature of Soil Elements
C ********************************************************************
C
        WRITE(8,*)' TEMPERATURE OF SOIL ELEMENTS FOR EACH X-INCREMENT:
```

```
     +'
        DO 500 J = 2,RNUM+2
           WRITE(8,13)TEMP(2,J,K),TEMP(3,J,K),TEMP(4,J,K),TEMP(5
     +,J,K),TEMP(6,J,K)
13         FORMAT(1X,5F11.7)
500     CONTINUE
        WRITE(8,*)
C
        DO 501 J = 2,RNUM+2
           WRITE(8,13)TEMP(7,J,K),TEMP(8,J,K),TEMP(9,J,K),
     +TEMP(10,J,K),TEMP(11,J,K)
501     CONTINUE
        WRITE(8,*)
C
C *********************************************************************
C Write Out The Temperature of Air in The Tube
C *********************************************************************
C
        WRITE(8,*)' TEMPERATURE OF AIR IN THE TUBE: '
        DO 225 I = 2,6
           WRITE(8,14)TEMP(I,1,K+1),TEMP(I+5,1,K+1)
14         FORMAT(1X,2F11.7)
225     CONTINUE
C
C *********************************************************************
C Calculate Time for Every TIMLOP
C *********************************************************************
C
        HOURS = TIMSTP * TIMLOP * T
        WRITE(*,16)TEMP(XNUM+1,1,K+1)
        WRITE(8,16)TEMP(XNUM+1,1,K+1)
16      FORMAT (1X,' OUTPUT TEMPERATURE OF TUBE IS ',F11.7)
        WRITE(*,17)HOURS
        WRITE(8,17)HOURS
17      FORMAT(1X,' TIME IS ',F12.8,' HOURS')
C
```

```
C *********************************************************************
C Write Out Ice Formation For Every TIMLOP
C *********************************************************************
        WRITE(8,*)
        WRITE(8,*)' THICKNESS OF ICE FORMED:'
        DO 600 I = 2,6
           WRITE(8,*)RICE(I,K),RICE(I+5,K)
600     CONTINUE
C
C
200     CONTINUE
C
C *********************************************************************
C If Maximum Time Step is Reached Write Out Message
C *********************************************************************
C
        WRITE(8,18)
18      FORMAT(1X,70('*'))
        WRITE(8,19)
19      FORMAT(1X,70('*'))
        WRITE(*,*)' MAXIMUM TIME EXCEEDED'
        WRITE(8,*)' MAXIMUM TIME EXCEEDED'
C
C *********************************************************************
C If Stable Temperature is Achieved, Write Out Message
C *********************************************************************
C
420     WRITE(*,*)' PROGRAM TERMINATED'
        WRITE(8,*)' PROGRAM TERMINATED'
        CLOSE(8)
        STOP
        END
```

APPENDIX B

COMPUTER PROGRAM FOR TWO-DIMENSIONAL MODEL

Appendix B

Computer Program for Two-Dimensional Model

```
      PROGRAM TWODIM
C **********************************************************************
C This program is to determine the thermal performance of an under
C ground heat tube in two dimension. It performs a finite difference
C analysis on a square buried tube through which air is flowing.
C Central differences are used to calculate the heat gain of each
C element and forward differences are used in the time step. The
C program also calculates the thickness of ice formation around the
C tube.
C **********************************************************************
C
C variable list
C
C TSUR     soil surface temperature
C TATM     current temperature of the atmosphere
C TSOILD   deep soil temperature
C TEMPT    temporary temperature of preprocessed increments
C TEMP     array containing temperatures of soil and air
C TDIFF    temperature difference
C TDIFFS   temperature difference required for stability
C CPAIR    heat capacity of air
C DNAIR    density of air
C AKSOLN   thermal conductivity of normal soil
C AKSOLF   thermal conductivity of frozen soil
C CPSOLN   heat capacity of soil in a thawed state
C CPSOLF   heat capacity of soil in a frozen state
C CPSOLC   heat capacity of soil in a freezing state
C DNSOIL   density of soil
C CPFUSE   heat of fusion of water
C CPWATR   heat capacity of water
C CPICE    heat capacity of ice
C XINCRE   the increment size in the x-direction
C YINCRE   the increment size in the y-direction
C ZINCRE   the increment size in the z-direction
C XNUM     number of increments in the x-direction
C YNUM     number of increments in the y-direction
```

```
C ZNUM     number of increments in the z-direction
C TNUM     the depth at which the tube is located vertically
C VELCTY   the velocity of air
C AMOIST   moisture content of soil as a percentage by weight
C HSUR     soil-air heat transfer coefficient at the surface
C H        soil-air heat transfer coefficient at the tube
C TIMEX    maximum number of time steps to be performed
C TIMLOP                       ,,
C AMASOL   mass of dry soil
C AMASAR   mass of air
C TUBEHT   location of tube vertically
C DELI1    array containing change in ice formation above the tube
C DELI2    array containing change in ice formation below the tube
C HICE1    array containing ice formation above the tube
C HICE2    array containing ice formation below the tube
C AKMAJ    coefficient of heat transfer equation
C AKMAJ1                   ,,
C AKMAJ2                   ,,
C AK00                     ,,
C AK1      coefficient of the J-1 term
C AK2      coefficient of the J+1 term
C AK3      coefficient of the K-1 term
C AK4      coefficient of the K+1 term
C AK5      coefficient of the air term
C AK6                  ,,
C AH1                  ,,
C AH2                  ,,
C DELTAE   energy released/absorbed due to formation/defrosting of ice
C I        loop counter array index
C J                  ,,
C K                  ,,
C L                  ,,
C P                  ,,
C T                  ,,
C A                  ,,
C B                  ,,
```

```
C C                        ,,
C
C ********************************************************************
C ********************************************************************
C
      INTEGER I,J,K,L,T,P,A,B,C,XNUM,YNUM,ZNUM,TNUM,TUBEHT,TIMEX,
     +TIMLOP,CHOICE,ANS,UNIT
      DIMENSION TEMP(52,17,26,2),HICE1(51,2),HICE2(51,2),DELI1(51,2)
     +,DELI2(51,2)
C
C ********************************************************************
C open data file
C ********************************************************************
C
      OPEN (UNIT=8,FILE='RESULT.DAT',STATUS='NEW')
C
C ********************************************************************
C initialize test variables
C ********************************************************************
C
      TSUR   = 30.0
      TATM   = 0.0
      TSOILD = 39.7
      CPAIR  = 0.24
      DNAIR  = 0.08275
      AKSOLN = 0.458
      AKSOLF = 0.833
      CPSOLN = 0.3
      CPSOLF = 0.2
      DNSOIL = 95.0
      CPFUSE = 143.3
      CPWATR = 1.0
      CPICE  = 0.5
      XINCRE = 1.0
      YINCRE = 0.5
      ZINCRE = 0.5
```

```
      XNUM   = 50
      YNUM   = 14
      ZNUM   = 24
      TNUM   = 10
      L      = 1
      HSUR   = 0.959
C
      WRITE(*,*)' INPUT MOISTURE CONTENT OF SOIL'
      READ(*,*)AMOIST
      WRITE(8,*)' MOISTURE CONTENT OF SOIL = ',AMOIST
      WRITE(*,*)' INPUT TEMPERATURE DIFFERENCE FOR STABILITY'
      READ(*,*)TDIFFS
      WRITE(8,*)' TEMPERATURE DIFFERENCE FOR STABILITY = ',TDIFFS
      WRITE(*,*)' INPUT VELOCITY OF AIR IN THE TUBE'
      READ(*,*)VELCTY
      WRITE(8,*)' VELOCITY OF AIR IN THE TUBE = ',VELCTY
      WRITE(*,*)' INPUT CONVECTIVE HEAT TRANSFER COEFFICIENT'
      READ(*,*)H
      WRITE(8,*)' CONVECTIVE HEAT TRANSFER COEFFICIENT = ',H
      WRITE(*,*)' INPUT TIMLOP'
      READ(*,*)TIMLOP
      WRITE(8,*)' TIMLOP = ',TIMLOP
C
C *********************************************************************
C set temperature of soil to its farfield temperature - the air
C temperature in the tube is set to soil temperature at that depth
C *********************************************************************
C
      DO 100 K = 1,ZNUM-2
         TEMPT = TSOILD - ((TSOILD - 32.0) * (0.001959 * FLOAT(K)
     +**2.0 - 0.001558 * FLOAT(K) - 0.000401))
         DO 110 I = 1,XNUM+2
            DO 120 J = 1,YNUM+3
               TEMP(I,J,K,L) = TEMPT
120         CONTINUE
110      CONTINUE
```

```
100     CONTINUE
C
        DO 130 K = ZNUM-1,ZNUM+1
           TEMPT = (TSUR - 32.0)/2.0 * FLOAT(K) - (23.0 / 2.0 * TSUR)
     ++400.0
           DO 140 I = 1,XNUM+2
              DO 150 J = 1,YNUM+3
                 TEMP(I,J,K,L) = TEMPT
150           CONTINUE
140        CONTINUE
130     CONTINUE
C
        K = ZNUM+2
        DO 160 I = 1,XNUM+2
           DO 170 J = 1,YNUM+3
              TEMP(I,J,K,L) = TATM
170        CONTINUE
160     CONTINUE
C
C
        WRITE(8,*)
        WRITE(8,*)' INITIAL TEMPERATURE OF THE NODES FOR ALL X :'
        WRITE(8,15)
        DO 180 I = 1,52
           WRITE(8,*)
           WRITE(8,70)
70         FORMAT(1X,'K',4X,'J = 1',5X,'J = 2',5X,'J = 3',5X,'J = 4',
     +5X,'J = 5',5X,'J = 6')
           DO 185 K = ZNUM+2,1,-1
              WRITE(8,80)K,TEMP(I,1,K,L),TEMP(I,2,K,L),TEMP(I,3,K,L),
     +TEMP(I,4,K,L),TEMP(I,5,K,L),TEMP(I,6,K,L)
80            FORMAT(1X,I2,6F11.7)
185        CONTINUE
C
           WRITE(8,*)
           WRITE(8,71)
```

```
71          FORMAT(1X,'K',4X,'J = 7',5X,'J = 8',5X,'J = 9',4X,'J = 10',
     +4X,'J = 11',4X,'J = 12')
            DO 190 K = ZNUM+2, 1,-1
               WRITE(8,80)K,TEMP(I,7,K,L),TEMP(I,8,K,L),TEMP(I,9,K,L),
     +TEMP(I,10,K,L),TEMP(I,11,K,L),TEMP(I,12,K,L)
190         CONTINUE
C
            WRITE(8,*)
            WRITE(8,72)
72          FORMAT(1X,'K',3X,'J = 13',4X,'J = 14',4X,'J = 15',4X,
     +'J = 16',4X,'J = 17')
            DO 195 K = ZNUM+2,1,-1
               WRITE(8,81)K,TEMP(I,13,K,L),TEMP(I,14,K,L),
     +TEMP(I,15,K,L),TEMP(I,16,K,L),TEMP(I,17,K,L)
81             FORMAT(1X,I2,5F11.7)
195         CONTINUE
C
180      CONTINUE
C
C ***********************************************************************
C calculate heat capacity of freezing soil
C ***********************************************************************
C
         CPSOLC = CPSOLN + (AMOIST * CPWATR) + (AMOIST * CPFUSE / 2.0)
         WRITE(8,3)CPSOLC
3        FORMAT(1X,' HEAT CAPACITY OF FREEZING SOIL = ',F7.3)
C
C ***********************************************************************
C calculate time step
C ***********************************************************************
C
         TIMSTP = XINCRE / VELCTY
         WRITE(8,4)TIMSTP
4        FORMAT(1X,' TIME STEP = ',F10.8)
C
C ***********************************************************************
```

```
C calculate tube location underground
C ********************************************************************
C
      TUBEHT = ZNUM - TNUM + 1
C
C ********************************************************************
C calculate mass of air in the tube
C ********************************************************************
C
      AMASAR = 4.0 * DNAIR * XINCRE * YINCRE * ZINCRE
C
C ********************************************************************
C calculate mass of soil in an element
C ********************************************************************
C
      AMASOL = DNSOIL * XINCRE * YINCRE * ZINCRE
C


C ********************************************************************
C write on data file initial temperature of air along the tube
C ********************************************************************
C
      WRITE(8,*)
      WRITE(8,*)' INITIAL TEMPERATURE OF AIR ALONG THE TUBE :'
      WRITE(8,15)
      WRITE(8,5)
5     FORMAT(1X,'I',1X,'TEMPERATURE')
      DO 200 I = 2,XNUM+1
         WRITE(8,6)I,TEMP(I,2,TUBEHT,L)
6        FORMAT(1X,I2,1X,F11.7)
200   CONTINUE
C
C ********************************************************************
C set ice formation around the tube to be zero initially
C ********************************************************************
C
```

```
          WRITE(8,*)
          WRITE(8,*)' INITIAL ICE THICKNESS :'
          WRITE(8,15)
          WRITE(8,7)
7         FORMAT(1X,'I',4X,'HICE1',5X,'HICE2')
          DO 210 I = 2,XNUM+1
             HICE1(I,L) = 0.0
             HICE2(I,L) = 0.0
             HICE1(I,L+1) = 0.0
             HICE2(I,L+1) = 0.0
             DELI1(I,L+1) = 0.0
             DELI2(I,L+1) = 0.0
             WRITE(8,8)I,HICE1(I,L),HICE2(I,L)
8            FORMAT(1X,I2,2F10.8)
210       CONTINUE
C
C
          WRITE(8,10)
          WRITE(8,10)
10        FORMAT(1X,70('*'))
C
C         ********************************************************
C     **************************************************************
C ******                                                      *******
C ******  LOOP CALCULATING HEAT TRANSFER THROUGHOUT THE ARRAY  *******
C ******                                                      *******
C     **************************************************************
C         ********************************************************
C
20        WRITE(*,*)' INPUT TIMEX'
          READ(*,*)TIMEX
          WRITE(8,*)' TIMEX = ',TIMEX
          DO 300 T = 1,TIMEX
             WRITE(8,*)' T = ',T
             WRITE(8,15)
15           FORMAT(1X,70('-'))
```

```
          DO 310 P = 1,TIMLOP
             DO 320 I = 2,XNUM+1
                DO 330 J = 2,YNUM+2
                   DO 340 K = 2,ZNUM+1
C
C
C **********************************************************************
C select soil heat capacity
C **********************************************************************
       IF (TEMP(I,J,K,L) .GT. 32.0) THEN
          CPSOIL = CPSOLN + (AMOIST * CPWATR)
       ELSE
          IF (TEMP(I,J,K,L) .LT. 32.0) THEN
             CPSOIL = CPSOLF + (AMOIST * CPICE)
          ELSE
             CPSOIL = CPSOLC
          END IF
       END IF
C
C **********************************************************************
C select soil thermal conductivity
C **********************************************************************
C
       TAVE1 = (TEMP(I,J-1,K,L) + TEMP(I,J,K,L)) / 2.0
       TAVE2 = (TEMP(I,J+1,K,L) + TEMP(I,J,K,L)) / 2.0
       TAVE3 = (TEMP(I,J,K-1,L) + TEMP(I,J,K,L)) / 2.0
       TAVE4 = (TEMP(I,J,K+1,L) + TEMP(I,J,K,L)) / 2.0
C
       IF (TAVE1 .GE. 32.0) THEN
          AKSOL1 = AKSOLN
       ELSE
          AKSOL1 = AKSOLF
       END IF
C
       IF (TAVE2 .GE. 32.0) THEN
          AKSOL2 = AKSOLN
```

```
         ELSE
            AKSOL2 = AKSOLF
         END IF
C
         IF (TAVE3 .GE. 32.0) THEN
            AKSOL3 = AKSOLN
         ELSE
            AKSOL3 = AKSOLF
         END IF
C
         IF (TAVE4 .GE. 32.0) THEN
            AKSOL4 = AKSOLN
         ELSE
            AKSOL4 = AKSOLF
         END IF
C
C        *************************************************
C ********************************************************************
C **     select the appropriate heat transfer coefficients        **
C ********************************************************************
C        *************************************************
C
C if node borders the center of bottom face of tube
C
         IF (J.EQ.2 .AND. K.EQ.TUBEHT-1) THEN
            AKMAJ = TIMSTP / (AMASOL * CPSOIL)
            AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
            AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
            AK3 = 2.0 * AKSOL3 * XINCRE * YINCRE / ZINCRE
            AK4 = 2.0 * H * XINCRE * YINCRE
            AK5 = 0.0
            AK6 = 0.0
            DELTAE = 0.0
            CHOICE = 1
C
C if node is directly below the node which borders the center of
```

```
C bottom face of tube
C
        ELSE IF (J.EQ.2 .AND. K.EQ.TUBEHT-2) THEN
           AKMAJ = TIMSTP / (AMASOL * CPSOIL)
           AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
           AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
           AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
           AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
           AK5 = 0.0
           AK6 = 0.0
           DELTAE = 0.0
           CHOICE = 2
C
C if node borders the center of top face of tube
C
        ELSE IF (J.EQ.2 .AND. K.EQ.TUBEHT+1) THEN
           AKMAJ = TIMSTP / (AMASOL * CPSOIL)
           AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
           AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
           AK3 = 2.0 * H * XINCRE * YINCRE
           AK4 = 2.0 * AKSOL4 * XINCRE * YINCRE / ZINCRE
           AK5 = 0.0
           AK6 = 0.0
           DELTAE = 0.0
           CHOICE = 3
C
C if node is directly above the node which borders the center of
C top face of tube
C
        ELSE IF (J.EQ.2 .AND. K.EQ.TUBEHT+2) THEN
           AKMAJ = TIMSTP / (AMASOL * CPSOIL)
           AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
           AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
           AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
           AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
           AK5 = 0.0
```

```
          AK6 = 0.0
          DELTAE = 0.0
          CHOICE = 4
C
C if node borders the bottom corner of tube
C
        ELSE IF (J.EQ.3 .AND. K.EQ.TUBEHT-1) THEN
          IF (HICE2(I,L) .LT. (0.5*ZINCRE)) THEN
C there could be some freezing, defrosting or no freezing at all
            AKMAJ = 2.0 * TIMSTP / (3.0 * AMASOL * CPSOIL)
            AK1 = 0.0
            AK2 = 0.0
            AK3 = 0.0
            AK4 = 0.0
            AK5 = 0.0
            AK6 = 0.0
            DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +DELI2(I,L+1) * (YINCRE + ZINCRE + 4.0 * HICE2(I,L) +
     +2.0 * DELI2(I,L+1)) / TIMSTP
            CHOICE = 5
C
          ELSE IF (HICE2(I,L) .EQ. (0.5*ZINCRE)) THEN
            IF (HICE2(I,L+1) .LT. (0.5*ZINCRE)) THEN
C some defrosting occurs
              AKMAJ = 2.0 * TIMSTP / (3.0 * AMASOL * CPSOIL)
              AK1 = 0.0
              AK2 = 0.0
              AK3 = 0.0
              AK4 = 0.0
              AK5 = 0.0
              AK6 = 0.0
              DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +DELI2(I,L+1) * (YINCRE + ZINCRE + 4.0 * HICE2(I,L) +
     +2.0 * DELI2(I,L+1)) / TIMSTP
              CHOICE = 6
C
```

```
                ELSE
C element is completely frozen
                   AKMAJ = 2.0 * TIMSTP / (3.0 * AMASOL * CPSOIL)
                   AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
                   AK2 = 2.0 * AKSOL2 * XINCRE * ZINCRE / YINCRE
                   AK3 = 2.0 * AKSOL3 * XINCRE * YINCRE / ZINCRE
                   AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
                   AK5 = H * XINCRE * ZINCRE
                   AK6 = H * XINCRE * YINCRE
                   DELTAE = 0.0
                   CHOICE = 7
                END IF
C
             ELSE
C element is completely frozen
                AKMAJ = 2.0 * TIMSTP / (3.0 * AMASOL * CPSOIL)
                AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
                AK2 = 2.0 * AKSOL2 * XINCRE * ZINCRE / YINCRE
                AK3 = 2.0 * AKSOL3 * XINCRE * YINCRE / ZINCRE
                AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
                AK5 = H * XINCRE * ZINCRE
                AK6 = H * XINCRE * YINCRE
                DELTAE = 0.0
                CHOICE = 8
             END IF
C
C if node is directly below the node which borders the bottom
C corner of tube
C
          ELSE IF (J.EQ.3 .AND. K.EQ.TUBEHT-2) THEN
             IF (HICE2(I,L) .LE. (0.5*ZINCRE)) THEN
                IF (HICE2(I,L+1) .GT. (0.5*ZINCRE)) THEN
C there is some freezing occuring
                   AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                   AK1 = 0.0
                   AK2 = 0.0
```

```
                  AK3 = 0.0
                  AK4 = 0.0
                  AK5 = 0.0
                  AK6 = 0.0
                  DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE * YINCRE *
     +DELI2(I,L+1) / TIMSTP
                  CHOICE = 9
C
               ELSE
C no freezing occurs
                  AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                  AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
                  AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
                  AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
                  AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
                  AK5 = 0.0
                  AK6 = 0.0
                  DELTAE = 0.0
                  CHOICE = 10
               END IF
C
            ELSE
C there could be some freezing or defrosting
               AKMAJ = TIMSTP / (AMASOL * CPSOIL)
               AK1 = 0.0
               AK2 = 0.0
               AK3 = 0.0
               AK4 = 0.0
               AK5 = 0.0
               AK6 = 0.0
               DELTAE = DNSOIL*CPFUSE*AMOIST*XINCRE*YINCRE*
     +DELI2(I,L+1)/TIMSTP
               CHOICE = 11
            END IF
C
C if node borders the center of side face of tube
```

```
C
       ELSE IF (J.EQ.3 .AND. K.EQ.TUBEHT) THEN
          IF (HICE1(I,L) .LT. (0.5*ZINCRE)) THEN
C there could be some freezing, defrosting or no freezing at all
             AKMAJ = TIMSTP / (AMASOL * CPSOIL)
             AK1 = 2.0 * H * XINCRE * ZINCRE
             AK2 = 2.0 * AKSOL2 * XINCRE * ZINCRE / YINCRE
             AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
             AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
             AK5 = 0.0
             AK6 = 0.0
             DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +SQRT(ZINCRE ** 2.0 + ABS(HICE1(I,L) - HICE2(I,L)) ** 2.0) *
     +(DELI1(I,L+1) + DELI2(I,L+1)) / TIMSTP
             CHOICE = 12
C
          ELSE IF (HICE1(I,L) .EQ. (0.5*ZINCRE)) THEN
             IF (HICE1(I,L+1) .LT. (0.5*ZINCRE)) THEN
C some defrosting occurs
                AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                AK1 = 0.0
                AK2 = 0.0
                AK3 = 0.0
                AK4 = 0.0
                AK5 = 0.0
                AK6 = 0.0
                DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +SQRT(ZINCRE ** 2.0 + ABS(HICE1(I,L) - HICE2(I,L)) ** 2.0) *
     +(DELI1(I,L+1) + DELI2(I,L+1)) / TIMSTP
                CHOICE = 13
C
             ELSE
C element is completely frozen
                AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                AK1 = 2.0 * H * XINCRE * ZINCRE
                AK2 = 2.0 * AKSOL2 * XINCRE * ZINCRE / YINCRE
```

```
                 AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
                 AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
                 AK5 = 0.0
                 AK6 = 0.0
                 DELTAE = 0.0
                 CHOICE = 14
              END IF
C
           ELSE
C element is completely frozen
              AKMAJ = TIMSTP / (AMASOL * CPSOIL)
              AK1 = 2.0 * H * XINCRE * ZINCRE
              AK2 = 2.0 * AKSOL2 * XINCRE * ZINCRE / YINCRE
              AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
              AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
              AK5 = 0.0
              AK6 = 0.0
              DELTAE = 0.0
              CHOICE = 15
           END IF
C
C if node borders the top corner of tube
C
        ELSE IF (J.EQ.3 .AND. K.EQ.TUBEHT+1) THEN
           IF (HICE1(I,L) .LT. (0.5*ZINCRE)) THEN
C there could be some freezing, defrosting or no freezing at all
              AKMAJ = 2.0 * TIMSTP / (3.0 * AMASOL * CPSOIL)
              AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
              AK2 = 2.0 * AKSOL2 * XINCRE * ZINCRE / YINCRE
              AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
              AK4 = 2.0 * AKSOL4 * XINCRE * YINCRE / ZINCRE
              AK5 = H * XINCRE * ZINCRE
              AK6 = H * XINCRE * YINCRE
              DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +DELI1(I,L+1) * (YINCRE + ZINCRE + 4.0 * HICE1(I,L) +
     +2.0 * DELI1(I,L+1)) / TIMSTP
```

```
                CHOICE = 16
C
             ELSE IF (HICE1(I,L) .EQ. (0.5*ZINCRE)) THEN
                IF (HICE1(I,L+1) .LT. (0.5*ZINCRE)) THEN
C some defrosting occurs
                   AKMAJ = 2.0 * TIMSTP / (3.0 * AMASOL * CPSOIL)
                   AK1 = 0.0
                   AK2 = 0.0
                   AK3 = 0.0
                   AK4 = 0.0
                   AK5 = 0.0
                   AK6 = 0.0
                   DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +DELI1(I,L+1) * (YINCRE + ZINCRE + 4.0 * HICE1(I,L) +
     +2.0 * DELI1(I,L+1)) / TIMSTP
                   CHOICE = 17
C
                ELSE
C element is completely frozen
                   AKMAJ = 2.0 * TIMSTP / (3.0 * AMASOL * CPSOIL)
                   AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
                   AK2 = 2.0 * AKSOL2 * XINCRE * ZINCRE / YINCRE
                   AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
                   AK4 = 2.0 * AKSOL4 * XINCRE * YINCRE / ZINCRE
                   AK5 = H * XINCRE * ZINCRE
                   AK6 = H * XINCRE * YINCRE
                   DELTAE = 0.0
                   CHOICE = 18
                END IF
C
             ELSE
C element is completely frozen
                AKMAJ = 2.0 * TIMSTP / (3.0 * AMASOL * CPSOIL)
                AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
                AK2 = 2.0 * AKSOL2 * XINCRE * ZINCRE / YINCRE
                AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
```

```
            AK4 = 2.0 * AKSOL4 * XINCRE * YINCRE / ZINCRE
            AK5 = H * XINCRE * ZINCRE
            AK6 = H * XINCRE * YINCRE
            DELTAE = 0.0
            CHOICE = 19
         END IF
C
C if node is directly above the node which borders the top
C corner of tube
C
      ELSE IF (J.EQ.3 .AND. K.EQ.TUBEHT+2) THEN
         IF (HICE1(I,L) .LE. (0.5*ZINCRE)) THEN
            IF (HICE1(I,L+1) .GT. (0.5*ZINCRE)) THEN
C there is some freezing occuring
               AKMAJ = TIMSTP / (AMASOL * CPSOIL)
               AK1 = 0.0
               AK2 = 0.0
               AK3 = 0.0
               AK4 = 0.0
               AK5 = 0.0
               AK6 = 0.0
               DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE * YINCRE *
     +DELI1(I,L+1) / TIMSTP
               CHOICE = 20
C
            ELSE
C no freezing occurs
               AKMAJ = TIMSTP / (AMASOL * CPSOIL)
               AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
               AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
               AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
               AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
               AK5 = 0.0
               AK6 = 0.0
               DELTAE = 0.0
               CHOICE = 21
```

```
                END IF
C
              ELSE
C there could be some freezing or defrosting
                AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                AK1 = 0.0
                AK2 = 0.0
                AK3 = 0.0
                AK4 = 0.0
                AK5 = 0.0
                AK6 = 0.0
                DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE * YINCRE *
     +DELI1(I,L+1) / TIMSTP
                CHOICE = 22
              END IF
C
C if node is diagonal to the node that borders the bottom
C corner of tube
C
            ELSE IF (J.EQ.4 .AND. K.EQ.TUBEHT-2) THEN
              IF (HICE2(I,L) .LE. (0.5*ZINCRE)) THEN
                IF (HICE2(I,L+1) .GT. (0.5*ZINCRE)) THEN
C there is some freezing occuring
                  AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                  AK1 = 0.0
                  AK2 = 0.0
                  AK3 = 0.0
                  AK4 = 0.0
                  AK5 = 0.0
                  AK6 = 0.0
                  DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +DELI2(I,L+1) * (2.0 * (HICE2(I,L) - 0.5 * ZINCRE) +
     +DELI2(I,L+1)) / TIMSTP
                  CHOICE = 23
C
                ELSE
```

```
C no freezing occurs
                  AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                  AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
                  AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
                  AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
                  AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
                  AK5 = 0.0
                  AK6 = 0.0
                  DELTAE = 0.0
                  CHOICE = 24
               END IF
C
            ELSE
C there could  be some freezing or defrosting
               AKMAJ = TIMSTP / (AMASOL * CPSOIL)
               AK1 = 0.0
               AK2 = 0.0
               AK3 = 0.0
               AK4 = 0.0
               AK5 = 0.0
               AK6 = 0.0
               DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +DELI2(I,L+1) * (2.0 * (HICE2(I,L) - 0.5 * ZINCRE) +
     +DELI2(I,L+1)) / TIMSTP
               CHOICE = 25
            END IF
C
C if node is directly to the right of the node that borders
C the bottom corner of tube
C
         ELSE IF (J.EQ.4 .AND. K.EQ.TUBEHT-1) THEN
            IF (HICE2(I,L) .LE. (0.5*ZINCRE)) THEN
               IF (HICE2(I,L+1) .GT. (0.5*ZINCRE)) THEN
C there is some freezing occuring
                  AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                  AK1 = 0.0
```

```
                    AK2 = 0.0
                    AK3 = 0.0
                    AK4 = 0.0
                    AK5 = 0.0
                    AK6 = 0.0
                    DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE * ZINCRE *
     +DELI2(I,L+1) / TIMSTP
                    CHOICE = 26
C
                 ELSE
C no freezing occurs
                    AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                    AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
                    AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
                    AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
                    AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
                    AK5 = 0.0
                    AK6 = 0.0
                    DELTAE = 0.0
                    CHOICE = 27
                 END IF
C
              ELSE
C there could be some freezing or defrosting
                 AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                 AK1 = 0.0
                 AK2 = 0.0
                 AK3 = 0.0
                 AK4 = 0.0
                 AK5 = 0.0
                 AK6 = 0.0
                 DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE * ZINCRE *
     +DELI2(I,L+1) / TIMSTP
                 CHOICE = 28
              END IF
C
```

```
C if node is directly to the right of the node that borders
C the center of side face of tube
C
      ELSE IF (J.EQ.4 .AND. K.EQ.TUBEHT) THEN
         IF (HICE1(I,L) .LE. (0.5*ZINCRE)) THEN
            IF (HICE1(I,L+1) .GT. (0.5*ZINCRE)) THEN
C there is some freezing occuring
               AKMAJ = TIMSTP / (AMASOL * CPSOIL)
               AK1 = 0.0
               AK2 = 0.0
               AK3 = 0.0
               AK4 = 0.0
               AK5 = 0.0
               AK6 = 0.0
               DELTAE = 0.5 * DNSOIL * CPFUSE * AMOIST * XINCRE *
     +SQRT(ZINCRE ** 2.0 + ABS(HICE1(I,L) - HICE2(I,L)) ** 2.0) *
     +(DELI1(I,L+1) + DELI2(I,L+1)) / TIMSTP
               CHOICE = 29
C
            ELSE
C no freezing occurs
               AKMAJ = TIMSTP / (AMASOL * CPSOIL)
               AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
               AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
               AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
               AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
               AK5 = 0.0
               AK6 = 0.0
               DELTAE = 0.0
               CHOICE = 30
            END IF
C
         ELSE
C there could be some freezing or defrosting
            AKMAJ = TIMSTP / (AMASOL * CPSOIL)
            AK1 = 0.0
```

```
              AK2 = 0.0
              AK3 = 0.0
              AK4 = 0.0
              AK5 = 0.0
              AK6 = 0.0
              DELTAE = 0.5 * DNSOIL * CPFUSE * AMOIST * XINCRE *
     +SQRT(ZINCRE ** 2.0 + ABS(HICE1(I,L) - HICE2(I,L)) ** 2.0) *
     +(DELI1(I,L+1) + DELI2(I,L+1)) / TIMSTP
              CHOICE = 31
            END IF
C
C if node is directly to the right of the node that borders
C the top corner of tube
C
          ELSE IF (J.EQ.4 .AND. K.EQ.TUBEHT+1) THEN
            IF (HICE1(I,L) .LE. (0.5*ZINCRE)) THEN
              IF (HICE1(I,L+1) .GT. (0.5*ZINCRE)) THEN
C there is some freezing occuring
                AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                AK1 = 0.0
                AK2 = 0.0
                AK3 = 0.0
                AK4 = 0.0
                AK5 = 0.0
                AK6 = 0.0
                DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE * ZINCRE *
     +DELI1(I,L+1) / TIMSTP
                CHOICE = 32
C
              ELSE
C no freezing occurs
                AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
                AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
                AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
                AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
```

```
                  AK5 = 0.0
                  AK6 = 0.0
                  DELTAE = 0.0
                  CHOICE = 33
               END IF
C
            ELSE
C there could be some freezing or defrosting
               AKMAJ = TIMSTP / (AMASOL * CPSOIL)
               AK1 = 0.0
               AK2 = 0.0
               AK3 = 0.0
               AK4 = 0.0
               AK5 = 0.0
               AK6 = 0.0
               DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE * ZINCRE *
     +DELI1(I,L+1) / TIMSTP
               CHOICE = 34
            END IF
C
C if node is diagonal to the node that borders the top
C corner of tube
C
         ELSE IF (J.EQ.4 .AND. K.EQ.TUBEHT+2) THEN
            IF (HICE1(I,L) .LE. (0.5*ZINCRE)) THEN
               IF (HICE1(I,L+1) .GT. (0.5*ZINCRE)) THEN
C there is some freezing occuring
                  AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                  AK1 = 0.0
                  AK2 = 0.0
                  AK3 = 0.0
                  AK4 = 0.0
                  AK5 = 0.0
                  AK6 = 0.0
                  DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +DELI1(I,L+1) * (2.0 * (HICE1(I,L) - 0.5 * ZINCRE) +
```

```
     +DELI1(I,L+1)) / TIMSTP
                 CHOICE = 35
C
             ELSE
C no freezing occurs
                 AKMAJ = TIMSTP / (AMASOL * CPSOIL)
                 AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
                 AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
                 AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
                 AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
                 AK5 = 0.0
                 AK6 = 0.0
                 DELTAE = 0.0
                 CHOICE = 36
             END IF
C
         ELSE
C there could be some freezing or defrosting
             AKMAJ = TIMSTP / (AMASOL * CPSOIL)
             AK1 = 0.0
             AK2 = 0.0
             AK3 = 0.0
             AK4 = 0.0
             AK5 = 0.0
             AK6 = 0.0
             DELTAE = DNSOIL * CPFUSE * AMOIST * XINCRE *
     +DELI1(I,L+1) * (2.0 * (HICE1(I,L) - 0.5 * ZINCRE) +
     +DELI1(I,L+1)) / TIMSTP
             CHOICE = 37
         END IF
C
C if node borders the ground surface
C
      ELSE IF (K .EQ. ZNUM+1) THEN
         AKMAJ = TIMSTP / (AMASOL * CPSOIL)
         AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
```

```
            AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
            AK3 = 2.0 * AKSOL3 * XINCRE * YINCRE / ZINCRE
            AK4 = 2.0 * HSUR * XINCRE * YINCRE
            AK5 = 0.0
            AK6 = 0.0
            DELTAE = 0.0
            CHOICE = 38
C
C if node is the center of the tube
C
         ELSE IF (J.EQ.2 .AND. K.EQ.TUBEHT) THEN
C soil temperature need not be calculated here - proceed to
C next element
            GO TO 340

C
         ELSE
C
C for other interior nodes
C
            AKMAJ = TIMSTP / (AMASOL * CPSOIL)
            AK1 = AKSOL1 * XINCRE * ZINCRE / YINCRE
            AK2 = AKSOL2 * XINCRE * ZINCRE / YINCRE
            AK3 = AKSOL3 * XINCRE * YINCRE / ZINCRE
            AK4 = AKSOL4 * XINCRE * YINCRE / ZINCRE
            AK5 = 0.0
            AK6 = 0.0
            DELTAE = 0.0
            CHOICE = 39
         END IF
C
C *******************************************************************
C calculate the temperature of the element
C *******************************************************************
C
         TEMP(I,J,K,L+1) = AKMAJ *
```

```
     +              (AK1 * (TEMP(I,J-1,K,L) - TEMP(I,J,K,L)) +
     +               AK2 * (TEMP(I,J+1,K,L) - TEMP(I,J,K,L)) +
     +               AK3 * (TEMP(I,J,K-1,L) - TEMP(I,J,K,L)) +
     +               AK4 * (TEMP(I,J,K+1,L) - TEMP(I,J,K,L)) +
     +               AK5 * (TEMP(I,J-1,TUBEHT,L) - TEMP(I,J,K,L)) +
     +               AK6 * (TEMP(I,J-1,TUBEHT,L) - TEMP(I,J,K,L)) +
     +               DELTAE) + TEMP(I,J,K,L)
C
C *******************************************************************
C calculate ice formations and make temperature adjustments
C *******************************************************************
C
        IF (CHOICE .EQ. 1) THEN
           IF (TEMP(I,J,K,L+1) .LT. 32.0) THEN
C
              IF (HICE2(I,L) .LE. (0.5*ZINCRE)) THEN
                 AKMAJ2 = TIMSTP / (2.0 * DNSOIL * CPFUSE * AMOIST *
     +XINCRE * YINCRE)
                 AKOO = AMASOL * CPSOLC / TIMSTP
                 DELI2(I,L+1) = AKMAJ2 *
     +              (AKOO * (TEMP(I,J,K,L+1) - TEMP(I,J,K,L)))
C
                 IF (DELI2(I,L+1) .LT. 0.0) THEN
                    IF (HICE2(I,L) .EQ. 0.0) THEN
                       DELI2(I,L+1) = ABS(DELI2(I,L+1))
                       HICE2(I,L+1) = HICE2(I,L) + DELI2(I,L+1)
                    ELSE
                       HICE2(I,L+1) = HICE2(I,L) + DELI2(I,L+1)
                       IF (HICE2(I,L+1) .LT. 0.0) THEN
                          HICE2(I,L+1) = 0.0
                       END IF
                    END IF
                 ELSE
                    HICE2(I,L+1) = HICE2(I,L) + DELI2(I,L+1)
                 END IF
```

```
C
               IF (HICE2(I,L+1) .LT. (0.5*ZINCRE)) THEN
                  TEMP(I,J,K,L+1) = 32.0
               END IF
C
            END IF
         ELSE
            HICE2(I,L+1) = HICE2(I,L)
         END IF
C
C
      ELSE IF (CHOICE .EQ. 2) THEN
         IF (HICE2(I,L) .GT. (0.5*ZINCRE)) THEN
C
            AKMAJ2 = TIMSTP / (DNSOIL * CPFUSE * AMOIST * XINCRE *
     +YINCRE)
            AK00 = AMASOL * CPSOIL / TIMSTP
            DELI2(I,L+1) = AKMAJ2 *
     +         (AK00 * (TEMP(I,J,K,L+1) - TEMP(I,J,K,L)))
C
            HICE2(I,L+1) = HICE2(I,L) + DELI2(I,L+1)
C
            IF (HICE2(I,L+1) .GT. (0.5*ZINCRE)) THEN
               IF (HICE2(I,L+1) .LT. (1.5*ZINCRE)) THEN
                  IF (TEMP(I,J,K,L+1) .LT. 32.0) THEN
                     TEMP(I,J,K,L+1) = 32.0
                  END IF
               ELSE
                  WRITE(8,*)' ICE FORMATION IS GREATER THAN OR EQUAL
     + TO 3/2 ZINCRE'
                  GO TO 820
               END IF
            END IF
         END IF
C
C
```

```
      ELSE IF (CHOICE .EQ. 3) THEN
         IF (TEMP(I,J,K,L+1) .LT. 32.0) THEN
C
            IF (HICE1(I,L) .LE. (0.5*ZINCRE)) THEN
               AKMAJ2 = TIMSTP / (2.0 * DNSOIL * CPFUSE * AMOIST *
     +XINCRE * YINCRE)
               AK00 = AMASOL * CPSOLC / TIMSTP
               DELI1(I,L+1) = AKMAJ2 *
     +          (AK00 * (TEMP(I,J,K,L+1) - TEMP(I,J,K,L)))
C
               IF (DELI1(I,L+1) .LT. 0.0) THEN
                  IF (HICE1(I,L) .EQ. 0.0) THEN
                     DELI1(I,L+1) = ABS(DELI1(I,L+1))
                     HICE1(I,L+1) = HICE1(I,L) + DELI1(I,L+1)
                  ELSE
                     HICE1(I,L+1) = HICE1(I,L) + DELI1(I,L+1)
                     IF (HICE1(I,L+1) .LT. 0.0) THEN
                        HICE1(I,L+1) = 0.0
                     END IF
                  END IF
               ELSE
                  HICE1(I,L+1) = HICE1(I,L) + DELI1(I,L+1)
               END IF
C
               IF (HICE1(I,L+1) .LT. (0.5*ZINCRE)) THEN
                  TEMP(I,J,K,L+1) = 32.0
               END IF
            END IF
         ELSE
            HICE1(I,L+1) = HICE1(I,L)
         END IF
C
C
      ELSE IF (CHOICE .EQ. 4) THEN
         IF (HICE1(I,L) .GT. (0.5*ZINCRE)) THEN
C
```

```
            AKMAJ2 = TIMSTP / (DNSOIL * CPFUSE * AMOIST * XINCRE *
     +YINCRE)
            AK00 = AMASOL * CPSOIL / TIMSTP
            DELI1(I,L+1) = AKMAJ2 *
     +         (AK00 * (TEMP(I,J,K,L+1) - TEMP(I,J,K,L)))
C
            IF (HICE1(I,L+1) .GT. (0.5*ZINCRE)) THEN
               IF (HICE1(I,L+1) .LT. (1.5*ZINCRE)) THEN
                  IF (TEMP(I,J,K,L+1) .LT. 32.0) THEN
                     TEMP(I,J,K,L+1) = 32.0
                  END IF
               ELSE
                  WRITE(8,*)' ICE FORMATION IS GREATER THAN OR EQUAL
     + TO 3/2 ZINCRE'
                  GO TO 820
               END IF
            END IF
         END IF
C
C
      ELSE IF (CHOICE.EQ.5 .OR. CHOICE.EQ.6 .OR. CHOICE.EQ.9 .OR.
     +         CHOICE.EQ.11 .OR. CHOICE.EQ.12 .OR. CHOICE.EQ.13 .OR.
     +         CHOICE.EQ.16 .OR. CHOICE.EQ.17 .OR. CHOICE.EQ.20 .OR.
     +         CHOICE.EQ.22) THEN
C if temperature is less than 32 degrees, set temperature to 32 degrees
         IF (TEMP(I,J,K,L+1) .LT. 32.0) THEN
            TEMP(I,J,K,L+1) = 32.0
         END IF
C
C
      ELSE IF (CHOICE.EQ.23 .OR. CHOICE.EQ.25 .OR. CHOICE.EQ.26 .OR.
     +         CHOICE.EQ.28 .OR. CHOICE.EQ.29 .OR. CHOICE.EQ.31 .OR.
     +         CHOICE.EQ.32 .OR. CHOICE.EQ.34 .OR. CHOICE.EQ.35 .OR.
     +         CHOICE.EQ.37) THEN
C if temperature is less than 32 degrees, set temperature to 32 degrees
         IF (TEMP(I,J,K,L+1) .LT. 32.0) THEN
```

```
            TEMP(I,J,K,L+1) = 32.0
          END IF
C
C
        END IF
C
C
340                 CONTINUE
C
330               CONTINUE
C
320             CONTINUE
C
C
C **********************************************************************
C reset tube entrance temperature to current ambient temperature
C **********************************************************************
C
        TEMP(1,2,TUBEHT,L+1) = TATM
C
C **********************************************************************
C considering the plane of symmetry around the tube in the y-z plane
C - fill temp(i,1,k,l+1) with temp(i,3,k,l+1)
C **********************************************************************
C
        DO 400 A = 2,XNUM+1
          DO 410 B = 2,ZNUM+1
            TEMP(A,1,B,L+1) = TEMP(A,3,B,L+1)
410       CONTINUE
400     CONTINUE
C
C **********************************************************************
C match end wall temperature to satisfy the insulation condition
C **********************************************************************
C
        TEMP(1,1,TUBEHT,L+1) = TEMP(2,1,TUBEHT,L+1)
```

```
      TEMP(1,3,TUBEHT,L+1) = TEMP(2,3,TUBEHT,L+1)
      TEMP(1,2,TUBEHT-1,L+1) = TEMP(2,2,TUBEHT-1,L+1)
      TEMP(1,2,TUBEHT+1,L+1) = TEMP(2,2,TUBEHT+1,L+1)
C
C ********************************************************************
C calculate the temperature of air in the tube
C ********************************************************************
C
      AKMAJ1 = XINCRE / (AMASAR * CPAIR * VELCTY)
      AH1 = 2.0 * H * XINCRE * ZINCRE
      AH2 = 2.0 * H * XINCRE * YINCRE
      DO 420 I = 2,XNUM+1
         TEMP(I,2,TUBEHT,L+1) = AKMAJ1 *
     +(AH1 * (TEMP(I-1,1,TUBEHT,L+1) - TEMP(I-1,2,TUBEHT,L+1)) +
     + AH1 * (TEMP(I-1,3,TUBEHT,L+1) - TEMP(I-1,2,TUBEHT,L+1)) +
     + AH2 * (TEMP(I-1,2,TUBEHT-1,L+1) - TEMP(I-1,2,TUBEHT,L+1)) +
     + AH2 * (TEMP(I-1,2,TUBEHT+1,L+1) - TEMP(I-1,2,TUBEHT,L+1))) +
     + TEMP(I-1,2,TUBEHT,L+1)
420   CONTINUE
C
C ********************************************************************
C test for temperature stability - if stable terminate program
C ********************************************************************
C
      TDIFF = ABS(TEMP(XNUM+1,2,TUBEHT,L+1) - TEMP(XNUM+1,2,TUBEHT,L
     +))
      IF (TDIFF .LT. TDIFFS) GO TO 800
C
C ********************************************************************
C move l+1 temperatures to l for next loop
C ********************************************************************
C
      DO 430 A = 2,XNUM+1
         DO 440 B = 1,YNUM+2
            DO 450 C = 2,ZNUM+1
               TEMP(A,B,C,L) = TEMP(A,B,C,L+1)
```

```
450           CONTINUE
440         CONTINUE
430       CONTINUE
C
C *********************************************************************
C move l+1 ice formation to l for next loop
C *********************************************************************
C
          DO 460 A = 2,XNUM+1
             HICE1(A,L) = HICE1(A,L+1)
             HICE2(A,L) = HICE2(A,L+1)
460       CONTINUE
C
C
310         CONTINUE
C
C *********************************************************************
C write out the temperature of the soil elements for every TIMLOP
C *********************************************************************
C
          WRITE(8,*)' TEMPERATURE OF NODES FOR EACH X-INCREMENT :'
          WRITE(8,10)
          DO 600 I = 2,XNUM+1
             WRITE(8,50)I
50           FORMAT(1X,' X = ',I2)
             WRITE(8,*)
             WRITE(8,90)
90           FORMAT(1X,'K',4X,'J = 2',5X,'J = 3',5X,'J = 4',5X,'J = 5',
     +5X,'J = 6',5X,'J = 7')
             DO 500 K = ZNUM+1,1,-1
                WRITE(8,40)K,TEMP(I,2,K,L),TEMP(I,3,K,L),
     +TEMP(I,4,K,L),TEMP(I,5,K,L),TEMP(I,6,K,L),TEMP(I,7,K,L)
40              FORMAT(1X,I2,6F11.7)
500          CONTINUE
C
             WRITE(8,*)
```

```
            WRITE(8,91)
91          FORMAT(1X,'K',4X,'J = 8',5X,'J = 9',4X,'J = 10',4X,'J = 11'
     +,4X,'J = 12',4X,'J = 13')
            DO 510 K = ZNUM+1,1,-1
               WRITE(8,40)K,TEMP(I,8,K,L),TEMP(I,9,K,L),
     +TEMP(I,10,K,L),TEMP(I,11,K,L),TEMP(I,12,K,L),
     +TEMP(I,13,K,L)
510         CONTINUE
C
            WRITE(8,*)
            WRITE(8,92)
92          FORMAT(1X,'K',3X,'J = 14',4X,'J = 15',4X,'J = 16',4X,
     +'J = 17')
            DO 520 K = ZNUM+1,1,-1
               WRITE(8,41)K,TEMP(I,14,K,L),TEMP(I,15,K,L),
     +TEMP(I,16,K,L),TEMP(I,17,K,L)
41             FORMAT(1X,I2,4F11.7)
520         CONTINUE
C
            WRITE(8,*)
600      CONTINUE
C
C **********************************************************************
C write out the temperature of air in the tube for every TIMLOP
C **********************************************************************
C
         WRITE(8,*)' TEMPERATURE OF AIR IN THE TUBE :'
         WRITE(8,10)
         DO 700 I = 2,11
            WRITE(8,51)TEMP(I,2,TUBEHT,L+1),TEMP(I+10,2,TUBEHT,L+1),
     +TEMP(I+20,2,TUBEHT,L+1),TEMP(I+30,2,TUBEHT,L+1),
     +TEMP(I+40,2,TUBEHT,L+1)
51          FORMAT(1X,5F11.7)
700      CONTINUE
C
C **********************************************************************
```

```
C write out the error in temperature difference
C ********************************************************************
C
        WRITE(8,*)
        WRITE(8,52)TDIFF
52      FORMAT(1X,' ERROR IS ',F12.8)
C
C ********************************************************************
C calculate time for every TIMLOP
C ********************************************************************
C
        HOURS = TIMSTP * TIMLOP * T
        WRITE(*,53)TEMP(XNUM+1,2,TUBEHT,L+1)
        WRITE(8,53)TEMP(XNUM+1,2,TUBEHT,L+1)
53      FORMAT(1X,' OUTPUT TEMPERATURE OF TUBE IS ',F11.7)
        WRITE(*,54)HOURS
        WRITE(8,54)HOURS
54      FORMAT(1X,' TIME IS ',F12.8,' HOURS')
C
C ********************************************************************
C write ice formation for every TIMLOP
C ********************************************************************
C
        WRITE(8,*)
        WRITE(8,*)' THICKNESS OF ICE FORMED BELOW THE TUBE:'
        DO 900 I = 2,11
           WRITE(8,30)HICE2(I,L),HICE2(I+10,L),HICE2(I+20,L),
     +HICE2(I+30,L),HICE2(I+40,L)
30         FORMAT(1X,5F10.8)
900     CONTINUE
C
        WRITE(8,*)
        WRITE(8,*)' THICKNESS OF ICE FORMED ABOVE THE TUBE:'
        DO 910 I = 2,11
           WRITE(8,31)HICE1(I,L),HICE1(I+10,L),HICE1(I+20,L),
     +HICE1(I+30,L),HICE1(I+40,L)
```

```
31          FORMAT(1X,5F10.8)
910     CONTINUE
C
C
300     CONTINUE
C
C ********************************************************************
C if maximum time step is reached write out message
C ********************************************************************
C
        WRITE(8,*)
        WRITE(8,10)
        WRITE(8,10)
        WRITE(*,*)' MAXIMUM TIME EXCEEDED'
        WRITE(8,*)' MAXIMUM TIME EXCEEDED'
        GO TO 810
C
C ********************************************************************
C if stable temperature is achieved write out message
C ********************************************************************
C
800     WRITE(8,*)
        WRITE(8,10)
        WRITE(8,10)
        WRITE(*,*)' OUTPUT TEMPERATURE IS STABLE'
        WRITE(8,*)' OUTPUT TEMPERATURE IS STABLE'
        HOURS = (TIMSTP * (T-1) * TIMLOP) + (TIMSTP * P)
        WRITE(*,55)TEMP(XNUM+1,2,TUBEHT,L+1)
        WRITE(8,55)TEMP(XNUM+1,2,TUBEHT,L+1)
55      FORMAT(1X,' OUTPUT TEMPERATURE OF TUBE IS ',F11.7)
        WRITE(*,56)HOURS
        WRITE(8,56)HOURS
56      FORMAT(1X,' TIME IS ',F12.8,' HOURS')
C
        WRITE(8,*)' TEMPERATURE OF AIR IN THE TUBE:'
        WRITE(8,10)
```

```
      DO 750 I = 2,11
         WRITE(8,51)TEMP(I,2,TUBEHT,L+1),TEMP(I+10,2,TUBEHT,L+1),
     +TEMP(I+20,2,TUBEHT,L+1),TEMP(I+30,2,TUBEHT,L+1),
     +TEMP(I+40,2,TUBEHT,L+1)
750   CONTINUE
C
      WRITE(8,*)
      WRITE(8,*)' THICKNESS OF ICE FORMED BELOW THE TUBE:'
      DO 760 I = 2,11
         WRITE(8,30)HICE2(I,L+1),HICE2(I+10,L+1),HICE2(I+20,L+1),
     +HICE2(I+30,L+1),HICE2(I+40,L+1)
760   CONTINUE
C
      WRITE(8,*)
      WRITE(8,*)' THICKNESS OF ICE FORMED ABOVE THE TUBE:'
      DO 770 I = 2,11
         WRITE(8,30)HICE1(I,L+1),HICE1(I+10,L+1),HICE1(I+20,L+1),
     +HICE1(I+30,L+1),HICE1(I+40,L+1)
770   CONTINUE
      WRITE(8,*)
      GO TO 820
C
C
810   WRITE(*,*)' CONTINUE? (YES = 1)'
      READ(*,*)ANS
      IF (ANS .EQ. 1) GO TO 20
820   WRITE(*,*)' PROGRAM TERMINATED'
      WRITE(8,*)' PROGRAM TERMINATED'
      CLOSE(8)
      STOP
      END
```

APPENDIX C

DERIVATION OF HEAT TRANSFER EQUATIONS

Appendix C

DERIVATION OF HEAT TRANSFER EQUATIONS

There are several basic heat transfer equations which describe the heat transfer mechanism in the models.

One-Dimensional Model

1. Energy balance on an element of air within the tube, assuming steady state is as follows (derivation of equation 3.1):

$$\frac{m_a c_{pa}}{\Delta x} u \, (T^k_{i+1,0} - T^k_{i,0}) = h \, A \, (T^k_{i,1} - T^k_{i,0})$$

or

$$\rho_a \pi r_1^2 \, c_{pa} \, u \, (T^k_{i+1,0} - T^k_{i,0}) = h \, 2\pi r_1 \Delta x \, (T^k_{i,1} - T^k_{i,0})$$

Solving for $T^k_{i+1,0}$:

$$T^k_{i+1,0} = \frac{2h \, \Delta x}{\rho_a c_{pa} r_1 u} (T^k_{i,1} - T^k_{i,0}) + T^k_{i,0}$$

2. Energy balance on the soil element is as follows (derivation of equation 3.5):

$$\frac{m_s c_{ps}}{\Delta t}(T_{i,j}^{k+1} - T_{i,j}^{k}) = k_{s1}\frac{2\pi(r_j - \frac{\Delta r}{2})\Delta x}{\Delta r}(T_{i,j-1}^{k} - T_{i,j}^{k})$$
$$+ k_{s2}\frac{2\pi(r_j - \frac{\Delta r}{2})\Delta x}{\Delta r}(T_{i,j+1}^{k} - T_{i,j}^{k})$$

Solving for $T_{i,j}^{k+1}$:

$$T_{i,j}^{k+1} = \frac{\Delta t}{m_s c_{ps}}\left[k_{s1}\frac{2\pi(r_j - \frac{\Delta r}{2})\Delta x}{\Delta r}(T_{i,j-1}^{k} - T_{i,j}^{k}) + k_{s2}\frac{2\pi(r_j + \frac{\Delta r}{2})\Delta x}{\Delta r}(T_{i,j+1}^{k} - T_{i,j}^{k}) \right] + T_{i,j}^{k}$$

Here, heat transfer in the x-direction is taken to be negligible since it takes so long for temperature to change with x.

3. When the node borders on the tube, the equation for the soil element is used, as in number 2, with the substitution of the appropriate subscripts and application of the boundary condition (derivation of equation 3.2). The equation used as in number 2 is as follows:

$$\frac{m_s c_{ps}}{\Delta t} (T^{k+1}_{i,1} - T^k_{i,1}) = k_{s1}\frac{2\pi(r_1 - \frac{\Delta r}{2})\Delta x}{\Delta r} (T^k_{i,0'} - T^k_{i,1})$$

$$+ k_{s2}\frac{2\pi(r_1 - \frac{\Delta r}{2})\Delta x}{\Delta r} (T^k_{i,2} - T^k_{i,1}) \qquad \text{(C.1)}$$

where $T^k_{i,0'}$ is an image point. Applying and numerically translating the boundary condition given by equation 2.4 results the following:

$$k_s A \frac{T^k_{i,2} - T^k_{i,0'}}{2\Delta r} = hA\ (T^k_{i,1} - T^k_{i,0})$$

or

$$k_{s1} 2\pi(r_1 - \frac{\Delta r}{2})\Delta x \frac{T^k_{i,1} - T^k_{i,0'}}{2\Delta r} +$$

$$k_{s2} 2\pi(r_1 + \frac{\Delta r}{2})\Delta x \frac{T^k_{i,2} - T^k_{i,1}}{2\Delta r} = h\ 2\pi r_1 \Delta x\ (T^k_{i,0} - T^k_{i,1})$$

Solving for $T^k_{i,0'}$:

$$T^k_{i,0'} = \frac{\Delta r}{k_{s1} 2\pi(r_1 - \frac{\Delta r}{2})\Delta x} \Big[h\ 4\pi r_1 \Delta x\ (T^k_{i,0} - T^k_{i,1})$$

$$+ k_{s2}\frac{2\pi(r_1 + \frac{\Delta r}{2})\Delta x}{\Delta r} (T^k_{i,2} - T^k_{i,1}) \Big]$$

$$+ T^k_{i,1}$$

Substituting this image point into equation C.1, the following equation is thus obtained:

$$\frac{m_s c_{ps}}{\Delta t} (T_{i,1}^{k+1} - T_{i,1}^{k}) = h\ 4\pi r_1 \Delta x\ (T_{i,0}^{k} - T_{i,1}^{k}) + 2k_{s2} \frac{2\pi(r_1 + \frac{\Delta r}{2})\Delta x}{\Delta r} (T_{i,2}^{k} - T_{i,1}^{k})$$

where $m_s = \rho_s \Delta V_{element}$

$$= \rho_s \pi\ [(r_1 + \frac{\Delta r}{2})^2 - r_1^{\ 2}]\Delta x$$

$$= \rho_s \pi\ (r_1 + \frac{\Delta r}{4})\Delta r \Delta x$$

Solving for $T_{i,1}^{k+1}$:

$$T_{i,1}^{k+1} = \frac{\Delta t}{\rho_s \pi (r_1 + \frac{\Delta r}{4})\Delta r \Delta x c_{ps}} \left[h\ 4\pi r_1 \Delta x\ (T_{i,0}^{k} - T_{i,1}^{k}) + k_{s2} \frac{4\pi(r_1 + \frac{\Delta r}{2})\Delta x}{\Delta r} (T_{i,2}^{k} - T_{i,1}^{k}) \right] + T_{i,1}^{k}$$

4. When ice formed at the boundary of the tube is less than $\Delta r/2$, the derivation of equation 3.3 follows:

$$\frac{\rho_s \pi (r_1 + \frac{\Delta r}{4}) \Delta r \Delta x c_{ps}}{\Delta t} (T_{i,1}^{k+1} - T_{i,1}^{k})$$
$$= \text{Release of Latent Heat}$$

where,

$$\text{Release of Latent Heat} = \rho_s LM \; 2\pi R \; \Delta x \; \frac{\Delta r_{ice}}{\Delta t}$$

Solving for Δr_{ice}:

$$\Delta r_{ice} = \frac{\Delta t}{\rho_s LM \; 2\pi R \; \Delta x} \left[\frac{\rho_s c_{ps}}{\Delta t} \pi (r_1 + \frac{\Delta r}{4}) \Delta r \Delta x \; (T_{i,1}^{k+1} - T_{i,1}^{k}) \right]$$

5. When ice is formed at the next soil element, the derivation of equation 3.4 follows:

$$\frac{m_s c_{ps}}{\Delta t} (T_{i,2}^{k+1} - T_{i,2}^{k}) = \text{Release of Latent Heat}$$

where,

$$m_s = \rho_s \Delta V_{element}$$
$$= \rho_s \pi \; [(r_2 + \frac{\Delta r}{2})^2 - (r_2 - \frac{\Delta r}{2})^2] \; \Delta x$$
$$= \rho_s 2\pi r_2 \Delta r \Delta x$$

$$\text{Release of Latent Heat} = \rho_s LM \; 2\pi R \; \Delta x \; \frac{\Delta r_{ice}}{\Delta t}$$

Solving for Δr_{ice}:

$$\Delta r_{ice} = \frac{\Delta t}{\rho \text{ LM } 2\pi R \text{ } \Delta x} \left[\frac{\rho_s c_{ps}}{\Delta t} 2\pi r_2 \Delta r \Delta x \text{ } (T^{k+1}_{i,2} - T^{k}_{i,2})\right]$$

Two-Dimensional Model

1. The steady state energy balance on the air element in the tube is as follows (derivation of equation 3.6):

$$\begin{aligned} \frac{m_a c_{pa}}{\Delta x} u \text{ } (T^{l}_{i+1,0,0} - T^{l}_{i,0,0}) &= 2h \text{ } \Delta x \Delta z \text{ } (T^{l}_{i,-1,0} - T^{l}_{i,0,0}) \\ &+ 2h \text{ } \Delta x \Delta z \text{ } (T^{l}_{i,1,0} - T^{l}_{i,0,0}) \\ &+ 2h \text{ } \Delta x \Delta y \text{ } (T^{l}_{i,0,-1} - T^{l}_{i,0,0}) \\ &+ 2h \text{ } \Delta x \Delta y \text{ } (T^{l}_{i,0,1} - T^{l}_{i,0,0}) \end{aligned}$$

Solving for $T^{l}_{i+1,0,0}$:

$$\begin{aligned} T^{l}_{i+1,0,0} = \frac{\Delta x}{m_a c_{pa} u} \Big[& 2h \text{ } \Delta x \Delta z \text{ } (T^{l}_{i,-1,0} - T^{l}_{i,0,0}) \\ & + 2h \text{ } \Delta x \Delta z \text{ } (T^{l}_{i,1,0} - T^{l}_{i,0,0}) \\ & + 2h \text{ } \Delta x \Delta y \text{ } (T^{l}_{i,0,-1} - T^{l}_{i,0,0}) \\ & + 2h \text{ } \Delta x \Delta y \text{ } (T^{l}_{i,0,1} - T^{l}_{i,0,0}) \Big] \\ + T^{l}_{i,0,0} & \end{aligned}$$

2. Energy balance on the soil element is as follows (derivation of equation 3.20):

$$\frac{m_s c_{ps}}{\Delta t} (T^{l+1}_{i,j,k} - T^{l}_{i,j,k}) = k_{s1} \frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,j-1,k} - T^{l}_{i,j,k}) + k_{s2} \frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,j+1,k} - T^{l}_{i,j,k}) + k_{s3} \frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,j,k-1} - T^{l}_{i,j,k}) + k_{s4} \frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,j,k+1} - T^{l}_{i,j,k})$$

Solving for $T^{l+1}_{i,j,k}$:

$$T^{l+1}_{i,j,k} = \frac{\Delta t}{m_s c_{ps}} \left[k_{s1} \frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,j-1,k} - T^{l}_{i,j,k}) + k_{s2} \frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,j+1,k} - T^{l}_{i,j,k}) + k_{s3} \frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,j,k-1} - T^{l}_{i,j,k}) + k_{s4} \frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,j,k+1} - T^{l}_{i,j,k}) \right] + T^{l}_{i,j,k}$$

As in the one-dimensional model, heat transfer in the x-direction is also neglected since it takes so long for temperature to change with x.

3. When the node borders on the center of the bottom face of the tube, the equation obtained in number 2 is used, with the proper substitution of the appropriate subscripts

and boundary condition (derivation of equation 3.7). The following equation as in number 2 is obtained:

$$\frac{m_s C_{ps}}{\Delta t} (T^{l+1}_{i,0,-1} - T^{l}_{i,0,-1}) = k_{s1}\frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,-1,-1} - T^{l}_{i,0,-1})$$
$$+ k_{s2}\frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,1,-1} - T^{l}_{i,0,-1})$$
$$+ k_{s3}\frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,0,-2} - T^{l}_{i,0,-1})$$
$$+ k_{s4}\frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,0',0'} - T^{l}_{i,0,-1}) \qquad \text{(C.2)}$$

where $T^{l}_{i,0',0'}$ is an image point. Applying and numerically translating the boundary condition given by equation 2.8, the following equation results:

$$-k_s \frac{T^{l}_{i,0',0'} - T^{l}_{i,0,-2}}{2\Delta z} = h\,(T^{l}_{i,0,-1} - T^{l}_{i,0,0})$$

or

$$-k_{s4} \frac{T^{l}_{i,0',0'} - T^{l}_{i,0,-1}}{2\Delta z} - k_{s3} \frac{T^{l}_{i,0,-1} - T^{l}_{i,0,-2}}{2\Delta z}$$
$$= h\,(T^{l}_{i,0,-1} - T^{l}_{i,0,0})$$

Solving for $T^{l}_{i,0',0'}$:

$$T^{l}_{i,0',0'} = \frac{2h\,\Delta z}{k_{s4}} (T^{l}_{i,0,0} - T^{l}_{i,0,-1})$$

$$+ \frac{k_{s3}}{k_{s4}} (T^{l}_{i,0,-2} - T^{l}_{i,0,-1}) + T^{l}_{i,0,-1}$$

Substituting the image point into equation C.2, the following equation is thus obtained:

$$\frac{m_s c_{ps}}{\Delta t} (T^{l+1}_{i,0,-1} - T^{l}_{i,0,-1}) = k_{s1}\frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,-1,-1} - T^{l}_{i,0,-1})$$
$$+ k_{s2}\frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,1,-1} - T^{l}_{i,0,-1})$$
$$+ 2k_{s3}\frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,0,-2} - T^{l}_{i,0,-1})$$
$$+ 2h\ \Delta x \Delta y\ (T^{l}_{i,0,0} - T^{l}_{i,0,-1})$$

Solving for $T^{l+1}_{i,0,-1}$:

$$T^{l+1}_{i,0,-1} = \frac{\Delta t}{m_s c_{ps}} \Big[k_{s1}\frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,-1,-1} - T^{l}_{i,0.-1})$$
$$+ k_{s2}\frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,1,-1} - T^{l}_{i,0,-1})$$
$$+ 2k_{s3}\frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,0,-2} - T^{l}_{i,0,-1})$$
$$+ 2h\ \Delta x \Delta y\ (T^{l}_{i,0,0} - T^{l}_{i,0,-1})\Big]$$
$$+ T^{l}_{i,0,-1}$$

4. When ice formed below the tube is less than $\Delta z/2$, the derivation of equation 3.8 is as follows:

$$\frac{m_s c_{pc}}{\Delta t} (T^{l+1}_{i,0,-1} - T^{l}_{i,0,-1}) = \text{Release of Latent Heat}$$

where,

$$\text{Release of Latent Heat} = 2\rho_s LM\ \Delta x \Delta y\ \frac{\Delta h_{ice2}}{\Delta t}$$

Solving for Δh_{ice2}:

$$\Delta h_{ice2} = \frac{\Delta t}{2\rho_s LM\ \Delta x \Delta y} \left[\frac{m_s c_{pc}}{\Delta t} (T^{l+1}_{i,0,-1} - T^{l}_{i,0,-1})\right]$$

5. Energy balance on the soil element at the bottom corner of the tube (derivation of equation 3.10) is as follows:

Internal Distributed Source of Energy	=	Conduction Heat Rates along Four Different Lanes from Neighboring Nodes	+	Total Convection Rate along Two Half-Lanes in the y and z directions

or

$$\frac{3m_s c_{ps}}{4\Delta t} (T^{l+1}_{i,1,-1} - T^{l}_{i,1,-1}) = k_{s1}\frac{\Delta x \Delta z}{2\Delta y} (T^{l}_{i,0,-1} - T^{l}_{i,1,-1})$$
$$+ k_{s2}\frac{\Delta x \Delta z}{\Delta y} (T^{l}_{i,2,-1} - T^{l}_{i,1,-1})$$
$$+ k_{s3}\frac{\Delta x \Delta y}{\Delta z} (T^{l}_{i,1,-2} - T^{l}_{i,1,-1})$$

$$+ k_{s4}\frac{\Delta x\Delta y}{2\Delta z}\,(T^{l}_{i,1,0} - T^{l}_{i,1,-1})$$

$$+ h\;\Delta x\;\frac{\Delta z}{2}\,(T^{l}_{i,0,0} - T^{l}_{i,1,-1})$$

$$+ h\;\Delta x\;\frac{\Delta y}{2}\,(T^{l}_{i,0,0} - T^{l}_{i,1,-1})$$

Solving for $T^{l}_{i,1,-1}$:

$$T^{l+1}_{i,1,-1} = \frac{2\Delta t}{3m_s c_{ps}}\Big[k_{s1}\frac{\Delta x\Delta z}{\Delta y}\,(T^{l}_{i,0,-1} - T^{l}_{i,1,-1})$$

$$+ 2k_{s2}\frac{\Delta x\Delta z}{\Delta y}\,(T^{l}_{i,2,-1} - T^{l}_{i,1,-1})$$

$$+ 2k_{s3}\frac{\Delta x\Delta y}{\Delta z}\,(T^{l}_{i,1,-2} - T^{l}_{i,1,-1})$$

$$+ k_{s4}\frac{\Delta x\Delta y}{\Delta z}\,(T^{l}_{i,1,0} - T^{l}_{i,1,-1})$$

$$+ h\;\Delta x\Delta z\;(T^{l}_{i,0,0} - T^{l}_{i,1,-1})$$

$$+ h\;\Delta x\Delta y\;(T^{l}_{i,0,0} - T^{l}_{i,1,-1})\Big]$$

$$+ T^{l}_{i,1,-1}$$

6. Energy balance on the soil element at the bottom corner of the tube when ice formed is less than $\Delta z/2$ (derivation of equation 3.11) is as follows:

$$\frac{3m_s c_{ps}}{4\Delta t}\,(T^{l+1}_{i,1,-1} - T^{l}_{i,1,-1}) = \text{Release of Latent Heat}$$

where,

$$\text{Release of Latent Heat} = \rho_s LM\;\Delta x\;\left(\frac{\Delta y}{2} + \frac{\Delta z}{2} + 2h_{ice2} + \Delta h_{ice2}\right)\frac{\Delta h_{ice2}}{\Delta t}$$

Solving for $T^{l+1}_{i,1,-1}$:

$$T^{l+1}_{i,1,-1} = \frac{2\Delta t}{3m_s c_{ps}} \left[\rho_s LM \, \Delta x \, (\Delta y + \Delta z + 4h_{ice2} + 2\Delta h_{ice2}) \frac{\Delta h_{ice2}}{\Delta t} \right] + T^{l}_{i,1,-1}$$