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By

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ABSTRACT

COMPLIANT CONTROL OF CONSTRAINED ROBOT MANIPULATORS

By

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A new method for the compliant control of robot manipulators is presented. We formulate the compliant control problem mathematically employing the framework of constrained Hamiltonian systems. We then derive nonlinear control expressions for the force and the motion on the constraint surface. The control strategy consists of the sum of two nonlinear controls: the force part and the motion part. The force control restricts (the end effector of) the manipulator to the constraint surface and the motion control steers (the end effector of) the manipulator along a specified path on the constraint surface. The derivations reveal conditions that define the class of constraint surfaces allowable in the formulations. Two examples are then given to illustrate the formulation and the methodology.

We then consider the feedback of error signals with respect to a desired position, velocity and force. We also allow for uncertainty in modeling and for external disturbances. We show that the introduced nonlinear feedback controls specify the dynamics of the manipulator onto the constraint surface.

Finally, we present computer simulations which verify the analytical formulation and the stabilization control.

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CHAPTER 1

INTRODUCTION

Currently most robots are used for very limited tasks which is usually characterized by position-to-position movements, e.g., pick-and-place, spot welding and spray painting. Other essential but complicated tasks involve contact with the manipulator's environment, e.g., inserting a pin into a hole, assembling, plasma welding, contour following, deburring, grinding, etc., see [1, 2]. Such contact usually results in the generation of external forces acting on the end effector of the manipulator. External contact forces such as the ones introduced by constraint surfaces always modify the dynamical behavior of a manipulator. Consequently, issues of appropriate modeling and of effective new control strategies arise.

Compliant control is concerned with the control of a robot manipulator in contact with its environment, see [3-7]. The end effector of the manipulator first converges to the constraint surface at a specified position generating a specified force upon contact. Then, the end effector moves along a desired path on the surface while maintaining a desired contact force profile (along this path). Thus, compliant motion calls for the input torque to achieve tracking for a specified path on the constraint surface, and with a specified contact force.

In principle, such tracking is possible because the movement of the end effector is limited to a submanifold on the constraint surface and consequently frees some components of the input torque to control the contact force upon the surface. However, the nonlinearity of the governing dynamics as well as the constraint equations could potentially make the control process difficult if not impossible. The difficulty may translate mathematically to the presence of singularities at some points on the constraint surface or to the lack of well-posedness of the governing system of dynamic equations with algebraic constraints.

We choose to formulate the problem in the joint space. An advantage of this choice is that the constraint now applies to the joint angles directly; consequently the constraint applies to the links of the manipulator as oppose to its end effector as it is the case in the task space formulation. This makes it convenient for the control action which manipulators the torques to control the angles directly. Another advantage is that once the class of allowable constraints characterized by our formulation is identified in the joint space, the simpler and direct use of the forward kinematic transformation would provide the corresponding class of the allowable constraint surfaces in the joint space. We remark, however, that determining useful class of surfaces, from the view point of applications, is a nontrivial research problem that still needs investigation.

The control process we envision may take the following steps. The end effector is first steered to a point on the constraint surface using, e.g., the linear feedback control strategies reported in [8-11]. In addition, one must also guarantee that at the final (desired) position on the surface, a specified force (normal to the constraint surface) is generated. Once the end effector is located at a specific position and with a specified force, one may then apply compliant control strategies to generate or to track a desired path with a desired contact force profile. Some results on compliant control have been reported in [3-7].

In this work, we propose a control strategy which consists of the sum of two nonlinear controls. One control restricts (the end effector of) the manipulator to the constraint surface; this control represents the force control part. The other control steers (the end effector of) the manipulator along a specified path on the constraint surface; this control represents the position control part. Then we show that these nonlinear controls can be supplied by the input torque vector at the joints. Specifically, we give an expression for the (physical) torque which would generate the desired nonlinear controls. (It is possible to include the dynamics of the actuators and consider the actuator voltages as the physical inputs.)

We employ the geometric tools of symplectic Hamiltonian systems in setting up our framework. Although these tools have been used in [3], our emphasis is quite different: we assume that the amplitude (modulo a multiplicative constant) of the desired force is given as a function defined on the constraint surface; then we derive the control required to maintain that desired force. We also derive the second component of the control strategy which generates desired paths or trajectories on the constraint surface. The derivations require that the constraint surfaces satisfy conditions in terms of a matrix of Poisson brackets being nonsingular. These conditions in fact specify the class of constraint surfaces allowable in our formulation. We remark that the mathematical formulation and the main computational tool, namely, the Poisson bracket, are not adopted here because of personal inclination and preference for complexity. They are adopted here because of necessity. They represent the only accurate, precise way of decomposing the vector field of the dynamics of the nonlinear system (the manipulator) at every point onto the constraint surface into two components: an orthogonal component and a tangential component to the constraint surface. In addition, it should be recognized that the theoretical framework provides guidance and deep insights into how to properly devise and apply the control strategies.

As a next step, we apply, in addition, a linear controller to the resulting compliant system. This linear controller takes advantage of feeding back error signals, with respect to a desired position, velocity and force. For given constraint equations, the additional linear controller may depend on the variables of all joints.

The organization of this dissertation is as follows. In chapter 2, previous works on the compliant control of robot manipulators are briefly presented. The methodologies in these works classify the compliant control into passive mechanical compliance and into active compliance. It is observed that these previous works have not rigorously taken into account the dynamic equations governing the constrained robot manipulator, nor have they substantiated analytically that the desired position and desired force trajectories can be simultaneously achieved. Furthermore, we describe examples of mathematical modeling of constrained robot systems [12] which are described via singular differential equations, i.e., differential equations and algebraic equations. See [12-17], for example. We then describe the principal problem of compliant control. In chapter 3, we are concerned with the modeling of a manipulator employing a Hamiltonian structure, both free of contact and in contact with a constraint surface. In Chapter 4, the main results are derived: an expression for a (nonlinear) force control on the contact surface; then, a control expression for compliant motion (on the contact surface). In chapter 5, we compute the required physical torque which would supply the two derived control expressions. In chapter 6, we present two examples to illustrate the methodology. In Chapter 7, an additional linear (feedback) controller is introduced to the resulting compliant model for our two examples. The linear feedback controller achieves the stabilization of the overall compliance-controlled robot system. In Chapter 8, computer simulations are presented which verify the effectiveness of the overall nonlinear control strategies introduced. Finally, chapter 9 provides summary, discussion and conclusions of this dissertation.

CHAPTER 2

LITERATURE REVIEW

Compliant motion means that the position and velocity of a robot manipulator are constrained by the task: the robot interacts with its environment. Examples of compliant motion are contour following, grinding, cutting, drilling, inserting, fastening, assembly related tasks, etc.. Figure 2.1 shows some examples of compliant motions.

There are two primary methods for producing compliant motion: a passive mechanical compliance built into the manipulator, and an active compliance implemented into the (software) control loop, often called force control. Passive compliance, if successful, offers some performance advantages, but the force control methods offer the advantage of programmability.

Passive compliance

Figure 2.2 shows a typical example of a passive mechanical compliance using the center compliance linkage [18, 19] mounted to the end-effector of a robot manipulator. This center compliance linkage allows the robot to give a little rotation when it encounters the object to be inserted.

For example, there may be a jam if the peg is not aligned in the direction of z when the peg is to be inserted into the hole. This effect of misalignment of a piece part and the robot may result in failure of the task process and in damage to the robot manipulator and the piece. The only way to free it is to rotate the peg towards the axis if the hole while the peg is in contact with the hole. The center compliance linkage provides a sideway movement compensated for by the linkage such that the peg rotates about the x direction.

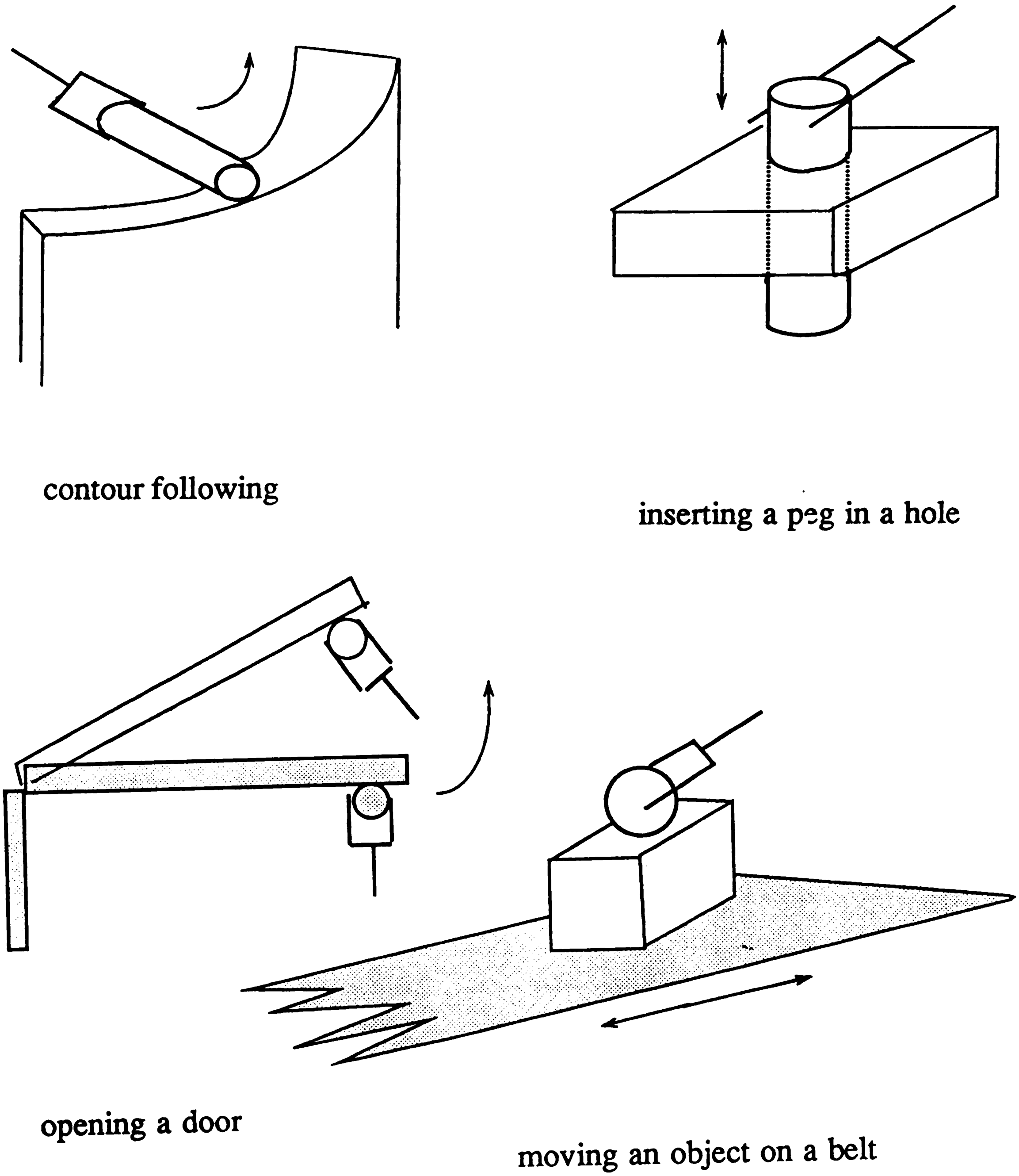

Figure 2.1. Compliant motions

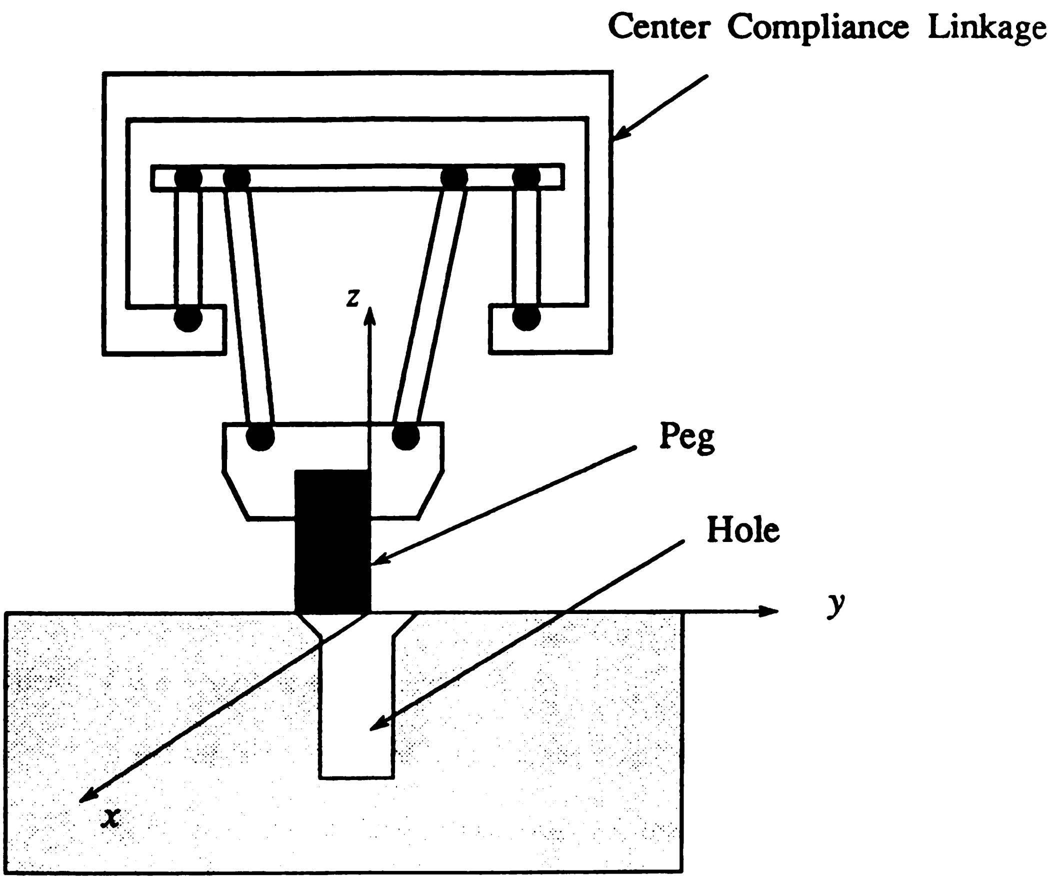

Figure 2.2. A peg being inserted into a hole

Force control

In the force control the problem arises as how to relate the requirements of a task to the motions required and forces anticipated so that force-motion strategies could be formulated. Various force control techniques have been proposed, but development of an underlying theory of these techniques has not materialized. Most of these techniques are motivated by concepts and ideas from linear systems and they ignore the (nonlinear) dynamics of the manipulator and its interaction with the constraint. In this section, the main efforts in the development of force control are briefly delineated.

In what follows the torque at the joints τ related to the expression of the force f vie the equation: $\tau = -J^T(q)f$, where $J(q)$ is the Jacobian of the kinematic function relation joint angles to task q coordinates.

1) The generalized spring method

This method feeds back force information through a stiffness matrix to a position control for generating force and can be modeled by a matrix version of Hook's law as [20-22],

$$f = K(p - p_0)$$

where f is a vector of forces and torques,

p is a vector of position and orientation of the manipulator; $(x,y,z,\theta_x,\theta_y,\theta_z)^T$,

p_0 is a vector of desired position and orientation, which is supplied by the planning system or the user program,

K is the stiffness matrix, which relates forces observed at the effector to deviations from the desired position. This stiffness matrix can be chosen to optimize the performance of a particular task.

In the case of inserting a peg in a hole, the stiffness matrix K can be selected in such a way that the peg is complying with forces along the x– and y–axes and with torques about x– and y–axes, and follows a trajectory straight down the middle of the hole. The K matrix for this task have been postulated to be in the form:

$$K = \begin{bmatrix} k_s & 0 & 0 & 0 & 0 & 0 \\ 0 & k_s & 0 & 0 & 0 & 0 \\ 0 & 0 & k_l & 0 & 0 & 0 \\ 0 & 0 & 0 & k_s & 0 & 0 \\ 0 & 0 & 0 & 0 & k_s & 0 \\ 0 & 0 & 0 & 0 & 0 & k_l \end{bmatrix},$$

with k_s a small number and k_l a large number. The motions corresponding to k_l (z, θ_z) are position-controlled, while for k_s are force-controlled. Note that the center compliant device can be characterized as a passive generalized spring.

2) The generalized damper method

This generalized damper method is similar in form, to the generalized spring method but it assumes a velocity controller instead of a position controller. This method can be modeled by the relation [23-25],

$$f = B(v - v_0),$$

where v is a vector of velocity and angular velocity of the manipulator; $(\dot{x},\dot{y},\dot{z},\dot{\theta}_x,\dot{\theta}_y,\dot{\theta}_z)^T$,

v_0 is a vector of desired velocity and angular velocity, which is input from the planning system or user program,

B is the damping matrix which have negative damping coefficient.

In early examples, no contact force will be maintained when assembly is completed except in the insert direction velocity $(\dot{z})$. In other words, the feedback gain should be large in the direction $(\dot{z}, \dot{\theta}_z)$ where the task is expected to yield and low $(\dot{x},\dot{y},\dot{\theta}_x\dot{\theta}_y)$ where the task is expected to resist deflection; this is the same as before.

3) The impedance control

It is a generalization of the previous two methods as in [26]. Now

$$f = K(p - p_0) + B(v - v_0).$$

4) The explicit force control

Unlike the previous methods, this one has a desired force input rather than position or velocity input [27];

$$\delta f = F - F_d,$$

where δf is the deviation in forces and torques of the manipulators from prescribed forces and torques,

F is a vector of actual forces and torques, and

F_d is a vector of desired forces and torques.

5) The hybrid force-position control

A hybrid controller commands force along certain degrees of freedom, and position along the remaining degrees of freedom. The task degrees of freedom is the number of independent coordinates required to specify completely the position and orientaion of a system: translation along each of the three axes, and rotation about each of three axes. For given tasks, these degrees of freedom are specified in the form of compliance, allowing the user to define which are position-controlled and which are force-controlled.

As in the case of a peg in a hole, translation of, and rotation about, the z–axis are position controlled, while the other degrees of freedom are force controlled.

5.1) Discrete method [28,29]

The discrete method works as follows. Each programming instruction is written in primitives defined at a lower level. The lowest level of a hierarchical structure provides the interface with the sensors and actuators of a robot manipulator. The user constructs strings of programming instruction for given tasks, then each instruction executes control strategy by combining input from higher level code and from sensors to provide signals for the actuators.

For the case of inserting a peg into a hole, the programming instructions are given by

```
MOVE TO D WITH
  [ FORCE X=0
    FORCE Y=0
    TORQUE X=0
    TORQUE Y=0 ],
```

where D is the goal position of the peg, i.e., move to D until no force is detected and rotate until no torque is detected. As shown in the programming instructions, the compliance axes are the translation and rotation of x and y in the task coordinate system. Therefore, the non-compliance axes are the translation and rotation of z.

5.2) Continuous method

The basic idea of this method is based on coordinated continuous axis motion by selection of a vector which would determine which world or hand axes are to be force controlled and which are to be position controlled.

For example, a hybrid controller is proposed in [5] as

$$\tau_i = \sum_{j=1}^{N} \{\Gamma_{ij}[s_j \Delta f_j] + \psi_{ij}[(1-s_j)\Delta x_j]\}$$

where τ_i : torque applied by the i^{th} actuator,

Δf_j : force error in the j^{th} degree of freedom,

Δx_j : position error in the j^{th} degree of freedom,

Γ_{ij} : force compensation function (PI),

ψ_{ij} : position compensation function (PID),

s_j : component of compliance selection vector that selects which world or hand axes is to be force controlled or which is to be position controlled.

This method deals with the situation where the position of the end-effector must be controlled in certain directions and the force must be controlled in the remaining other directions by the selection vector. Note that the constraint equations and their complement orthogonal equations in the task space do not transform into orthogonal pair in the joint space. Consequently, this method can not in actuality decouple the control of force and position.

We remark that this method is conceptually much clearer than the previously described ones; however, it still lacks an analytically sound base due to the unavoidable interaction between the force and position components in the torque elements.

For the application of robot manipulators to complex tasks which can not be defined solely in terms of the motion of the end-effector, it is necessary to properly formulate the compliance problem which would include the dynamic equations of the robot manipulators as well as their constraint equations.

Singular Differential Equation in Constrained Robot Systems

Mathematical models of a robot system which is in contact with its environment naturally give rise to a mathematical system of differential equations with algebraic equations. The latter can be viewed as a singular system of differential equations. Following [12], we show examples of modeling singular differential equations of robot system configurations

I) A Robot Manipulator Gripping a Load

The end-effector of a robot manipulator grips a load as shown in Figure 2.3.

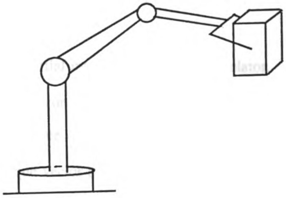

Figure 2.3. A Robot Manipulator Gripping a Load

The vector form of the equation of motion of the system shown in Figure 2.3 are $n+m$ differential equations with m algebraic equations in the $n+2m$ unknowns q, x and f. It is given by

$$\begin{bmatrix} M(q) & 0 & 0 \\ 0 & mI & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \ddot{x} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} T - G(q,\dot{q}) + J^T(q)f \\ -f - mg \\ M(q) - x \end{bmatrix},$$

where $x \in R^m$ is the position vector of the load in the fixed workspace coordinate system,

$q \in R^n$ is the vector robot joint angles ($\dot{q}$, $\ddot{q}$ denote the velocity and acceleration of q, respectively),

$f \in R^m$ is the contact force, in the task coordinates, between the robot and the load,

$M(q)$ is the inertial matrix of a robot manipulator,

$G(q,\dot{q})$ is a vector function which characterizes the Coriolis, centrifugal and gravitational force of the robot,

T is the torque vector at the joints,

$J(q) = \dfrac{\partial K(q)}{\partial q}$ is the Jacobian matrix, here $K(q)$ is a vector function representing the direct kinematic relation of the robot: $x = K(q)$.

II) A Robot with its End in Contact with a Rigid Constraint Surface

Consider the end effector of robot contact with a rigid constraint surface as shown in Figure 2.4.

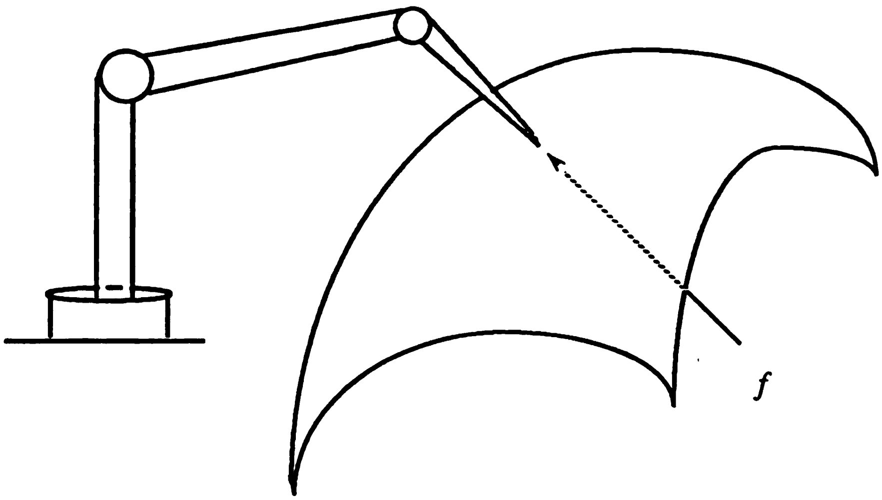

Figure 2.4. A Robot with its End in Contact with a Rigid Constraint Surface

The dynamic equation of Figure 2.4 can be represented in vector form as,

$$\begin{bmatrix} M(q) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} -G(q,\dot{q}) + T + J^T Q^T(K(.))\lambda \\ \Phi(K(q)) \end{bmatrix}$$

where $f = Q^T(x)\lambda$ is the contact force vector; here $Q(x) = \dfrac{\partial\Phi(x)}{\partial x}$ is a gradient of the constraint function ($\Phi(x) = 0$), and λ is a scalar multiplier for the constraint function.

Here, we have n differential equations and 1 algebraic equation in the n+1 unknowns q and λ.

III) Two Robots Gripping a Load

Consider the case that two robot manipulators grip a load as shown in Figure 2.5.

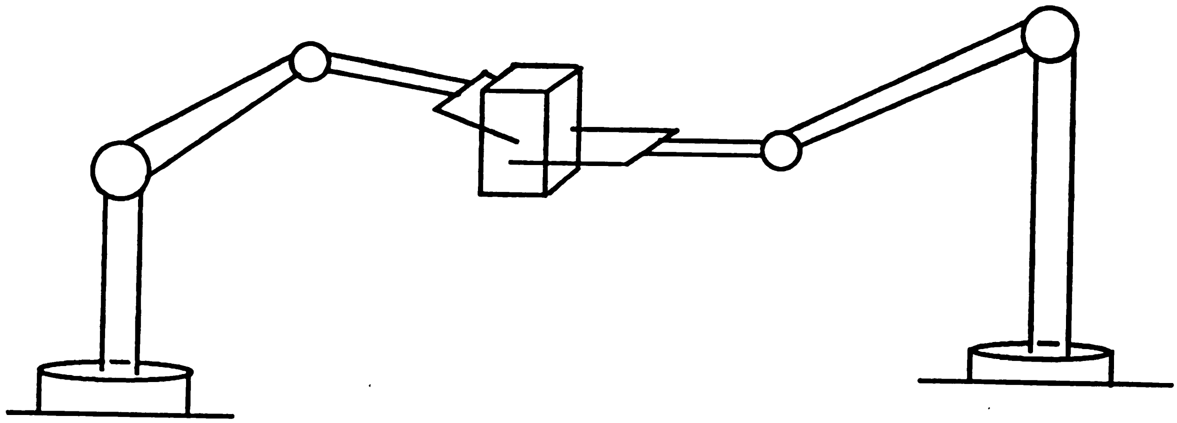

Figure 2.5. Two Robots Gripping a Load

The dynamic equation is given in vector form as

$$\begin{bmatrix} M_1(q_1) & 0 & 0 & 0 & 0 \\ 0 & M_2(q_2) & 0 & 0 & 0 \\ 0 & 0 & mI & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{x} \\ \dot{f}_1 \\ \dot{f}_2 \end{bmatrix} = \begin{bmatrix} T_1 - G_1(q_1,\dot{q}_1) + J_1^T(q_1)f_1 \\ T_2 - G_2(q_2,\dot{q}_2) + J_2^T(q_2)f_2 \\ -f_1 - f_2 - mg \\ M_1(q_1) - x \\ M_2(q_2) - x \end{bmatrix},$$

where the dimensions of x, q_1, and q_2 are m, n_1, and n_2, respectively.

Here, we have n_1+n_2+m differential equations and $2m$ algebraic equations in the n_1+n_2+3m unknowns q_1, q_2, x, f_1 and f_2.

To apply control to the constrained system where its dynamics occur within a $(2n-m)$-dimensional state space due to m-dimensional constraint equations, one may carry out a reduction of the dimension of the overall system. The reduction amounts to solving the constraint equations, i.e., obtaining expressions of m state variables in terms of the remaining variables. If such a reduction can be carried out, then reduced dynamic equations are obtained. In [4], for instance, a nonlinear canonical transformation based on the local implicit function theorem is presented. By using the implicit function theorem, the reduced dynamic system may become nonphysical at points where the nonlinear canonical transformation becomes singular. At such impasse points the reduced system is not well posed. In the following, we pursue analysis using the Hamiltonian framework for constrained dynamic systems which is valid globally. We are then able to formulate the compliance control problem in a globally sense.

CHAPTER 3

MODELING MANIPULATORS AS HAMILTONIAN SYSTEMS

3.1 Free Hamiltonian Systems on manifolds

Let M be a $2n$-dimensional differential manifold and let w^2 be a symplectic structure on M. Denote the symplectic manifold as the pair (M,w^2). See [30-33] for more discussion of symplectic manifolds.

For finite dimensional manifolds, w^2 is given in local coordinates by

$$w^2 = dq^i \Lambda dp^i \qquad i = 1,\ldots, n, \tag{3.1}$$

where the notation Λ represents the wedge (or the exterior) product.

Let $H : M \rightarrow R$ be a scalar function defined on the symplectic manifold. Then $dH : M \rightarrow T^*M$ is a differential 1-form on M, where dH is the (locally) differential of H and T^*M is the cotangent bundle of M. The function H is called Hamiltonian function. These geometric notions will be needed for the development later.

Now let the local canonical coordinates on the $2n$-dimensional symplectic manifold be $(q^1, \ldots, q^n; p^1, \ldots, p^n) := (q\ ;p)$. Denote the Hamiltonian vector field associated with the Hamiltonian H by X_H. The equation that associates a unique vector field X_H to H by w^2 is

$$i_{X_H} w^2 = dH, \tag{3.2}$$

where $i_{X_H} w^2$ is interior product (the contraction of X and w of 2-form w^2 with a vector field X). Let the tangential vector $X \in TM$, where TM is the tangent bundle of M, be in the local coordinates;

$$X = \sum_{i=1}^{n} \left(A^i \frac{\partial}{\partial q^i} + B^i \frac{\partial}{\partial p^i} \right), \tag{3.3}$$

then

$$i_{X_H} dq^i = A^i \tag{3.4}$$

$$i_{X_H} dp^i = B^i,$$

and for w^2

$$i_{X_H} w^2 = \sum_{i=1}^{n} i_{X_H}(dq^i \wedge dp^i) = \sum_{i=1}^{n} \left((i_{X_H} dq^i) dp^i - (i_{X_H} dp^i) dq^i \right) \tag{3.5}$$

$$= \sum_{i=1}^{n} \left(A^i dp^i - B^i dq^i \right) = dH.$$

This equals

$$dH = \sum_{i=1}^{n} \left(\frac{\partial H}{\partial q^i} dq^i + \frac{\partial H}{\partial p^i} dp^i \right) \tag{3.6}$$

if and only if $A^i = \frac{\partial H}{\partial p^i}$ and $B^i = -\frac{\partial H}{\partial q^i}$. Therefore

$$X = \sum_{i=1}^{n} \left(\frac{\partial H}{\partial p^i} \frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i} \frac{\partial}{\partial p^i} \right). \tag{3.7}$$

The formula for X_H is thus given by

$$X_H = \left(\frac{\partial H}{\partial p}, - \frac{\partial H}{\partial q} \right) = J \text{ grad } H \tag{3.8}$$

where $J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$.

Thus, the Hamiltonian equations are

$$\dot{q}^i = \frac{\partial H}{\partial p^i} \tag{3.9}$$

$$\dot{p}^i = -\frac{\partial H}{\partial q^i}.$$

This is a local representation of the Hamiltonian vector field X_H defined by means of w^2 in the natural coordinates which describe the behavior of the system.

The free Hamiltonian function of a general n degrees of freedom mechanical manipulator is obtained by deriving the kinetic energy, denoted by T, and the potential energy, denoted by V, as follows. The kinetic energy is given by

$$T(q\ ,\ \dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q}, \tag{3.10}$$

where q and $\dot{q}$ are the $n\times 1$ angular (or linear) position and velocity, respectively. $M(q)$ is the $n\times n$ inertial matrix which is known to be symmetric and positive definite, and hence it is invertible. The generalized momentum p is defined as follows [34]

$$p = \left[\frac{\partial T}{\partial \dot{q}}\right]^T = M(q)\dot{q}. \tag{3.11}$$

Thus the kinetic energy may equivalently be expressed as

$$T = \frac{1}{2}p^T M(q)^{-1} p. \tag{3.12}$$

Thus the free Hamiltonian is defined as

$$\begin{aligned} H(q,p) &= T(q,p) + V(q) \\ &= \frac{1}{2}p^T M(q)^{-1}p + V(q). \end{aligned} \tag{3.13}$$

3.2 Hamiltonian Systems with Constraints

Now we place constraints on the free Hamiltonian system. The constraints modify the equations of motion and generate new forces.

Let the $2m$ constraints be defined by the smooth functions $\Phi^j : M \to R$, $j = 1, ..., 2m$ as

$$\Phi^j(q, p) = 0, \qquad j = 1, ..., 2m \tag{3.14}$$

Assume that 0 is a regular point of (3.14), see [31], then the constraint "surface" is a $2n-2m$ submanifold of M given by

$$C = \{(q, p) \in T^*M \mid \Phi^1(q, p) =, ..., = \Phi^{2m}(q, p) = 0 \} \tag{3.15}$$

Furthermore, we require that the square ($2m$ x $2m$) matrix defined by

$$C_{\Phi\Phi} = [(\Phi^{ij})] = [\{\Phi^i, \Phi^j\}] \tag{3.16}$$

be nonsingular at every point on C, where $\{\ ,\ \}$ denotes the Poisson bracket. Eqns (3.15, 3.16) in fact specify the class of allowable constraint surfaces in this formulation.

The submanifold C is a symplectic (sub-)manifold with the induced symplectic form $w^2|C$, which is the restriction of the symplectic form w^2 to C. Recall that the Hamiltonian vector fields on a symplectic manifold form a Lie algebra under the Poisson bracket operation.

To satisfy the constrained Hamiltonian system, it is necessary that a corresponding force of constraint be added to the system in the sense of the d'Alembert's principle [see 22]. Let $Q : M \to T^*M$ be the resultant force, then motion is given by

$$i_{X_H}w^2 = dH + Q \tag{3.17}$$

$$d\Phi(X_H) = 0, \qquad j=1,..,2m,$$

where $Q = \sum_{j=1}^{2m}\lambda_j d\Phi^i$ and λ_j is the Lagrangian multiplier which in turn is proportional to the amplitude of the force along the gradient of the j-th constraint function evaluated at point (q, p), see [15]. In local coordinates, $i_{X_H}w^2$ is given by

$$i_{X_H} = \sum_{i=1}^{n} \left(\frac{\partial H}{\partial q^i}dq^i + \frac{\partial H}{\partial p^i}dp^i + \sum_{j=1}^{2m}\lambda_j\frac{\partial \Phi^j}{\partial q^i}dq^i + \sum_{j=1}^{2m}\lambda_j\frac{\partial \Phi^j}{\partial p^i}dp^i \right) \tag{3.18}$$

Therefore

$$X_H = J \text{ grad } H_\lambda, \tag{3.19}$$

where

$$H_\lambda = H + \sum_{j=1}^{2m}\lambda_j \, \Phi^j(q, p) \tag{3.20}$$

Thus the equations of motion are $(i = 1, ..., n)$

$$\dot{q}^i = \frac{\partial H}{\partial p^i} + \sum_{j=1}^{2m}\lambda_j\frac{\partial \Phi^j}{\partial p^i} \tag{3.21i}$$

$$\dot{p}^i = -\frac{\partial H}{\partial q^i} - \sum_{j=1}^{2m}\lambda_j\frac{\partial \Phi^j}{\partial q^i} \qquad i=1,...,n \tag{3.21ii}$$

$$d\Phi = 0 \qquad\qquad j=1,...,2m \tag{3.21iii}$$

The geometric meaning of (3.21iii) is as follows: let $\gamma : t \rightarrow \gamma(t) = (q^i(t), p^i(t))$ be on M, then the motion satisfies the constrains if $d\Phi^i = 0$. Also we rewrite (3.21iii) as

$$0 = \frac{\partial H_\lambda}{\partial \lambda_j} = \Phi^j(q, p) \,. \tag{3.21iv}$$

By the symplectic orthogonal decomposition [33], the tangent space T^*M can be split with respect to the constraint surface C as shown in Figure 3.1;

$$TT^*M_x = TC_x \oplus (TC_x)^{\perp} \text{ for every } x \in C. \tag{3.22}$$

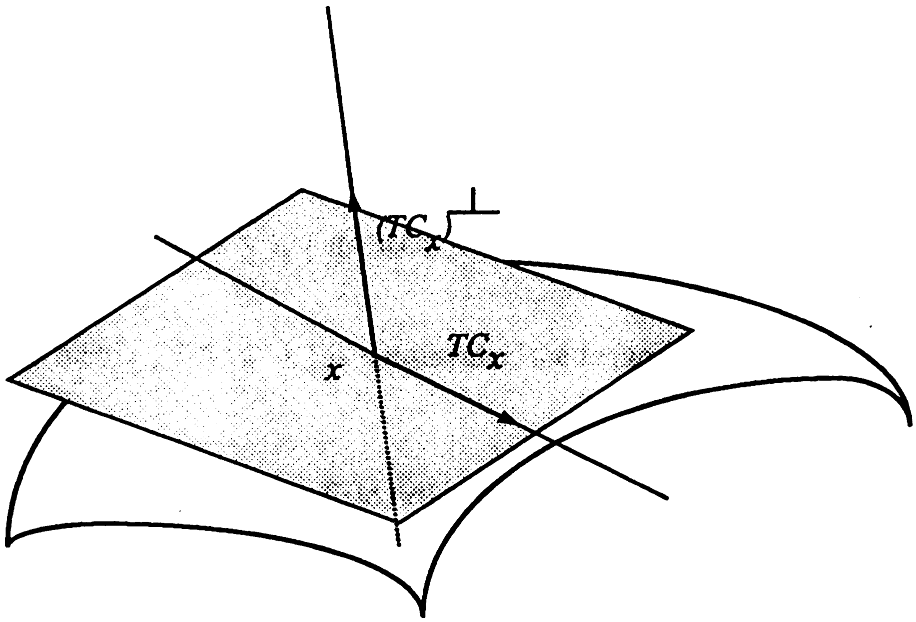

Figure 3.1. The symplectic orthogonal space

Note that the the Hamiltonian vector field X_H is generally not tangent to C.

The following two Lemmas are available in the literature. We reproduce them here in a form which is appropriate to our subsequent results.

LEMMA 1 ([3,17]) : Let the Lagrangian multipliers be the vector $\lambda = [\lambda_1, \ldots, \lambda_{2m}]^T$. If

$$\lambda = C_{\Phi\Phi}^{-1} C_{H\Phi}, \tag{3.23}$$

where $C_{H\Phi} = [\ \{H, \Phi^1\}\ , \ldots, \{H, \Phi^{2m}\}\ ]^T$, and $C_{\Phi\Phi}$ is the $(2m \times 2m)$ matrix given in (3.16), then the vector field of the constrained Hamiltonian is the vector field of the free Hamiltonian restricted to the constraint submanifold C, namely,

$$X_{H_\lambda} = X_{H|C} \tag{3.24}$$

Proof: X_{H_λ} must be tangent to C. Thus, we have for $i = 1, ..., 2m$

$$0 = \{ H_\lambda , \Phi^i \} = \{H + \sum_{j=1}^{2m}\lambda_j\Phi^j , \Phi^i\}.$$

The Poisson bracket operation is bilinear and skew symmetric, consequently one obtains

$$\{ H, \Phi^i \} = \sum_{j=1}^{2m}\lambda_j \{ \Phi^i, \Phi^j \} .$$

Since the matrix [{ Φ^i,Φ^j }] is nonsingular, the lemma follows.

LEMMA 2 ([3]) : The vector field orthogonal to $X_{H|C}$ is given by

$$(X_{H|C})^{\perp} = \sum_{j=1}^{2m}\lambda_j\phi^j \tag{3.25}$$

where $\phi^j, j = 1, ..., 2m$ are vector fields associated with the constraint functions $\Phi^j(q,p), j=1,...,2m$.

Proof: Since, for all j, Φ^j is zero on C, its corresponding vector field restricted to C (i.e., $\phi^j \mid C$) is zero. Consequently, $\phi^j, j=1,...,2m$, form a basis for $(TC)^{\perp}$, the orthogonal complement to the tangent bundle of the submanifold C.

The Lemma 1 and 2 can be graphically shown in Figure 3.2.

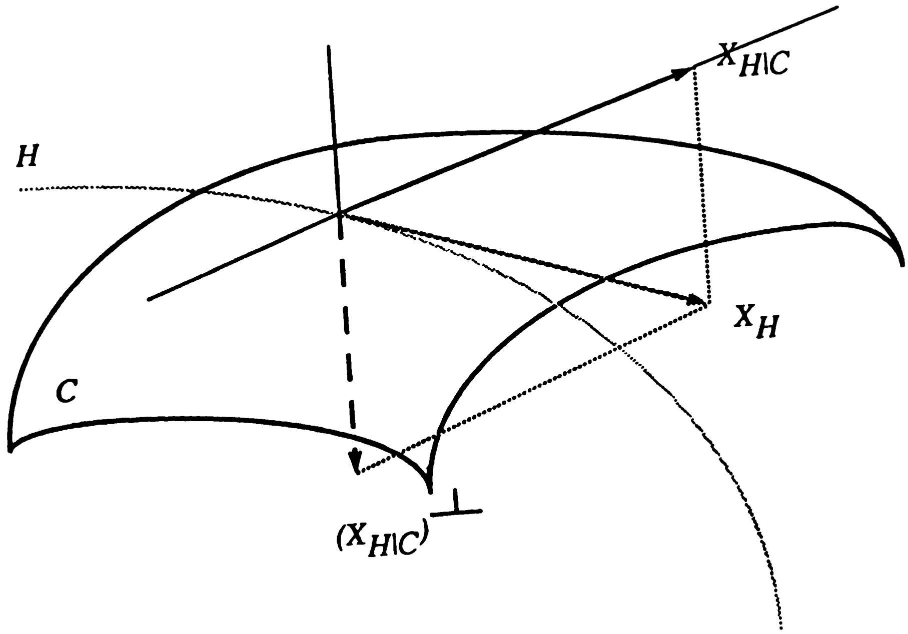

Figure 3.2. The decomposition of Hamiltonian vector X_H

CHAPTER 4

THE CONTROL OF CONSTRAINED HAMILTONIAN SYSTEMS

4.1 The Control of The Contact Force

For a given set of constraint equations and under the conditions of Lemma 1, the Lagrangian multiplier vector (λ) can be determined uniquely by (3.12). In turn, the corresponding (natural) contact forces are also determined at every point (q, p). These contact forces are the natural response of the surface to (the end effector of) the manipulator.

Here our focus and interest is in the converse: we want to specify a desired Lagrangian multiplier vector (λ^*), and consequently specify the (desired) normal force at the point of contact on the submanifold C. Our viewpoint is motivated by applications, e.g., cutting, or grinding, etc., where it is desired to achieve contact at a specific (normal) force.

Let the desired constrained Hamiltonian system $(M, w^2, H, C_{\lambda^*})$ be

$$H_{\lambda^*}(q, p) = H(q,p) + \sum_{j=1}^{2m} \lambda^*_j \, \Phi^j(q,p), \tag{4.1}$$

where λ^*_j, j=1,...,2m, are any desired functions of q and p.

We choose functions $\Theta^j(q,p)$, j=1,...,2r for some integer r, defined on T^*M with the following properties

(i) $\quad \Theta^j(q,p) = 0, \quad j=1,\ldots,2r \qquad (4.2)$

(ii) the differential 1-forms $d\Theta^j$ are linearly independent at every point on (equivalently 0 is a regular point)

$$E = \{(q,p) \in T^*M \mid \Theta^1(q,p) = \ldots = \Theta^{2r}(q,p) = 0\}, \tag{4.3}$$

where E is the $2n-2r$ smoothly embedded submanifold of T^*M. Then E is a symplectic manifold with symplectic form $w^2|E$.

(iii) The square matrix

$$E_{\Theta\Theta} = [(\Theta^{ij})] = [\{\Theta^i , \Theta^j\}] \tag{4.4}$$

is nonsingular at every point on E.

The conditions (4.3) and (4.4) specify the class of E surfaces allowable by the formulation. It is now required that the submanifold C ($2n-2m$ dimensional) be embedded into the submanifold E ($2n-2r$ dimensional) or vise versa. For the case of $m<r$ the orthogonal submanifold of C must be a subset of the orthogonal submanifold of E, consequently we can choose any λ^* defined on C. On the other hand, the control surface E is a subset of the constraint surface C, consequently the flexibility of steering the position and velocity on C is restricted to the subset E. For $m>r$ the converse is true.

The Hamiltonian of this new constrained (Hamiltonian) system is

$$H_{\hat{u}*} = H_{\lambda*} + \sum_{j=1}^{2r} \hat{u}_j \Theta^j. \tag{4.5}$$

As in section 2.2, the tangent space of T^*M can be split at every point on the submanifold E as follows

$$TT^*M_x = TE_x \oplus (TE_x)^\perp .$$

Then the Hamiltonian vector field $X_{H_{\lambda^*}}$ and $X_{H_{u^*}}$ have the decompositions

$$X_{H_{\lambda^*}}(T^*M)_x = X_{H_{\lambda^*}}(C_x) \oplus (X_{H_{\lambda^*}}(C_x))^\perp$$

$$X_{H_{g*}}(T^*M)_x = X_{H_{g*}}(E_x) \oplus (X_{H_{g*}}(E_x))^{\perp} \ , \tag{4.6}$$

where $X_{H_{\lambda *}}(C_x) \in TC_x$ and $X_{H_{g*}} \in TE_x$

LEMMA 3:

$$X_{H_{\lambda *}}(C_x) = X_{H_{\lambda *}|C_x} \tag{4.7}$$

$$X_{H_{g*}}(E_x) = X_{H_{g*}|E_x}$$

Proof: For any vector $v \in TC_x$, we have

$$w^2(X_{H_{\lambda *}}(C_x),v) = w^2(X_{H_{\lambda *}}(TT^*M_x),v) - w^2(\ (X_{H_{\lambda *}}(C_x))^{\perp},v)$$

$$= \langle dH_{\lambda *},v\rangle = \langle dH_{\lambda *}|C,v\rangle, \qquad \text{for all } \ x.$$

Similarly, for $\hat{v} \in TE_x$.

LEMMA 4: Suppose that the input vector is given as

$$\hat{u} = E_{\Theta\Theta}^{-1}\ E_{H\Theta}, \tag{4.8}$$

where $\hat{u}=[\hat{u}_1,\ldots\ldots,\hat{u}_{2r}]^T$, $E_{\Theta\Theta}$ is as defined in (3.4), and

$$E_{H\Theta} = [\ \{\ H_{\lambda *},\Theta^1\ \}\ ,\ldots\ldots,\ \{\ H_{\lambda *},\Theta^{2r}\ \}\ ]^T.$$

Then the vector field of the total Hamiltonian is the vector field of the desired constrained Hamiltonian restricted to E, specifically,

$$X_{H_g} = X_{H_{\lambda *}|E}\ . \tag{4.9}$$

Moreover, the orthogonal vector field to $X_{H_{\lambda *}|E}$ is given by

$$(X_{H_{\lambda *}|E})^{\perp} = \sum_{j=1}^{2r} \hat{u}_j \theta^j, \tag{4.10}$$

where θ^j is the vector field associated with $\Theta^j(q,p)$

Proof: Along E, we have for all $\Theta^i, i=1,...,2r$

$$0 = \{ H_{\hat{u}*}, \Theta^i \} = \{ H_{\lambda*} + \sum_{j=1}^{2r} \hat{u}_j \Theta^j, \Theta^i \}$$

$$= \{ H_{\lambda*}, \Theta^i \} - \sum_{j=1}^{2r} \hat{u}_j \{ \Theta^i, \Theta^j \} .$$

Consequently, if $\{ \Theta^i, \Theta^j \}$ is nonsingular, eqn. (4.8) follows.

We now summarize our results in the following theorem.

THEOREM 1: Let r be equal to m and let the input constraint equations be identically zero on C. For a desired λ^* defined on C, which corresponds to a desired (normal) force, the corresponding input control vector is given by (4.8)

Proof: Since the submanifolds E and C (see Figure 4.1) are now identical, the vector field generated by the input constraint equations affects only the directions normal to C (Lemma 3). The rest of the proof follows from the previous lemma.

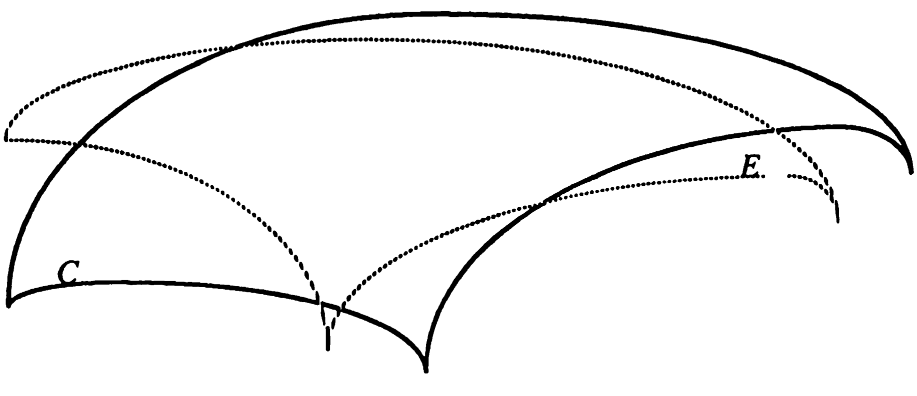

Figure 4.1. The (constraint) surfaces C and E

Now we consider a geometric interpretation of the force control. By the effect of the input $\hat{u}$, the Hamiltonian energy level is changed from H to $H + \sum_{j=1}^{2m} \hat{u}_j \Phi^j$ as shown Figure 4.2. The decomposition of the new Hamiltonian vector field can be presented on the constraint surface as before. The orthogonal vector of the new Hamiltonian vector field is corresponding to the desired Lagrangian multiplier vector.

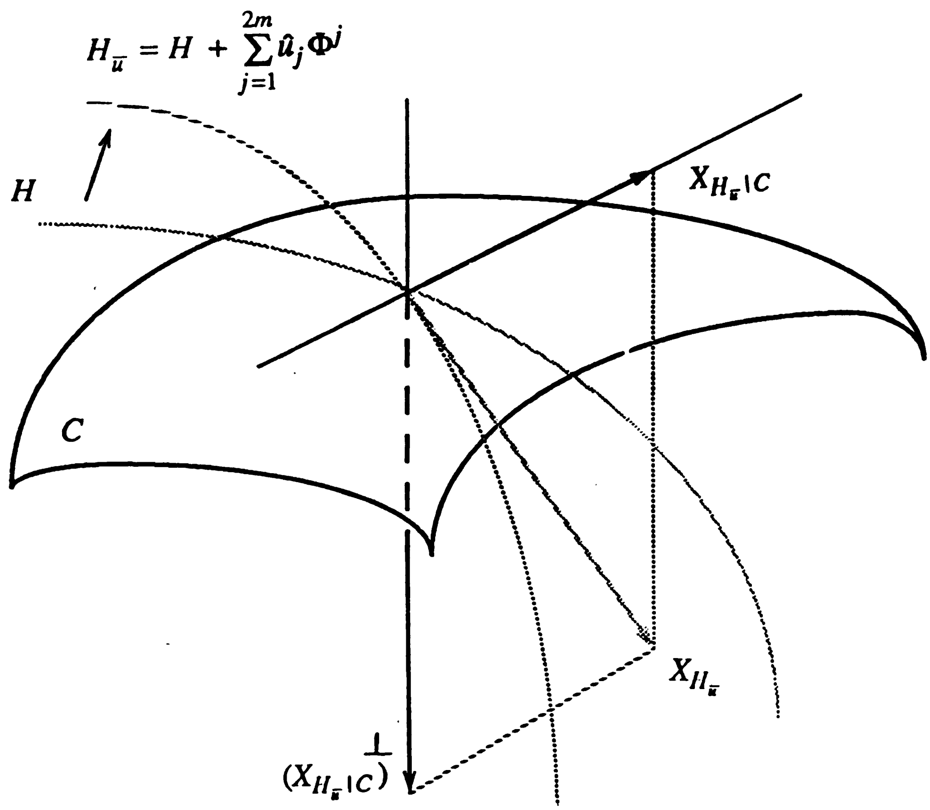

Figure 4.2. The decomposition of the constraint Hamiltonian vector

Remark: When the submanifolds E and C are identical, the expression of control (4.8) simplifies to

$$\hat{u} = C_{\Phi\Phi}^{-1}[\ \{ H_{\lambda*}, \Phi^1 \}\ , \ldots , \{ H_{\lambda*}, \Phi^{2m} \}\ ]^T = C_{\Phi\Phi}^{-1}[C_{H\Phi} - C_{\Phi\Phi}\lambda^*]$$

$$= C_{\Phi\Phi}^{-1} C_{H\Phi} - \lambda^*$$

This simpler expression simplifies the computations considerably.

4.2 The Control of Compliant Motion

The end effector is acted upon by the input expression given in Theorem 1 to exert the desired contact force at any point on the constraint surface. Now we consider the scenario where it is desired that the end effector moves along a specified path on the constraint surface, while maintaining a desired contact force profile. This scenario describes the so-called compliant motion control. To develop a control that achieves compliance, we need to augment the previous force control by a position (and a velocity) control onto the constraint surface.

Construct the functions $\Psi^j(q,p) : T^*M \rightarrow R\ j=1,...,2l$ such that $\Psi^j(q,p)=0$ is defined. Then as before the Hamiltonian for the augmented constrained system is given by

$$H_T(q,p,\lambda,\hat{u},\tilde{u}) = H_{\hat{u}} + \sum_{j=1}^{2l} \tilde{u}_j \Psi^j, \tag{4.11}$$

where $(\tilde{u}_1,....,\tilde{u}_{2l})$ is another input control vector which directs or steers the vector field on the constraint surface C.

We know that $H_{\hat{u}}$ is a constrained Hamiltonian which has λ^* on C maintained by the input $\hat{u}$. The constraint equations ($\Psi^j(q,p)=0$) must be chosen such that the input $\tilde{u}$ affects only directions tangent to C. Thus we define the submanifold

$$S = \{ \ (q,p) \in T^*M \mid \Psi^1(q,p)=...=\Psi^{2l}(q,p)=0^- \}, \tag{4.12}$$

where the differential 1-forms of $\Psi^j(q,p)$ are linearly independent on S (i.e., 0 is a regular point) and

$$S_{\Psi\Psi} = [(\Psi^{ij})] = [\ \{ \ \Psi^i, \Psi^j \ \} \ ] \tag{4.13}$$

is nonsingular at every point on S. Then S is a symplectic manifold with the symplectic form $w^2|S$. S must be of co-dimension $2n-2m$ (i.e. the dimension of S must equal to the co-dimension of C) and be a submanifold *transversal* to C as (at every point along the desired path on C) shown in Figure 4.3. Hence $l=n-m$.

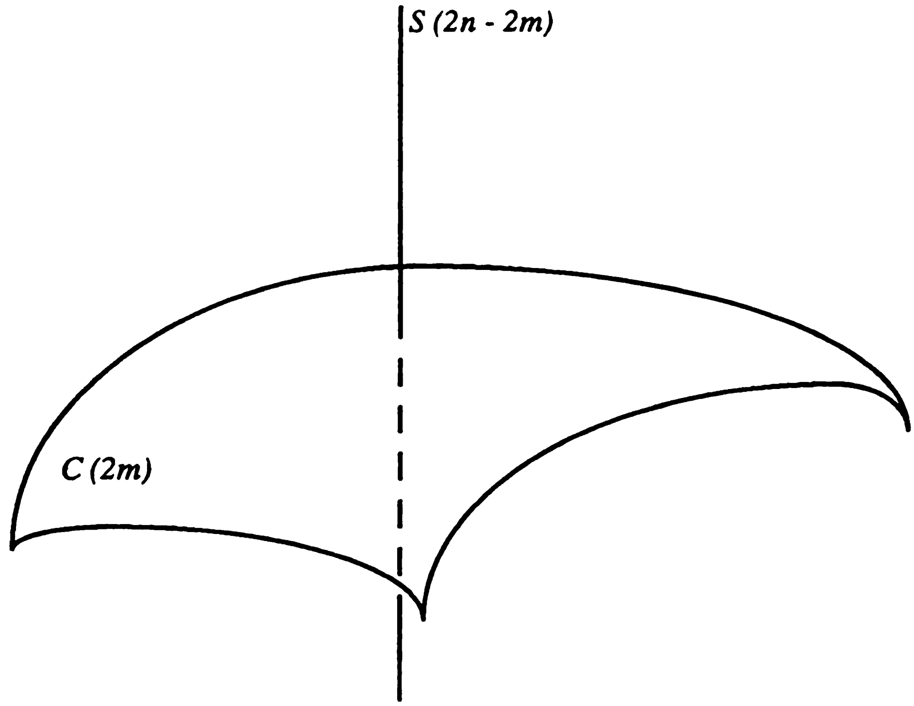

Figure 4.3. The (constraint) surfaces C and S

We consequently obtain the following lemma

LEMMA 5: Let the desired Hamiltonian for the compliant motion on C be H^*. If $\tilde{u}$ is given by

$$\tilde{u} = S_{\Psi\Psi}^{-1}\, S_{H\Psi} \tag{4.14}$$

where $\tilde{u}=[\tilde{u}^1, \ldots, \tilde{u}^{2n-2m}]^T$, $S_{\Psi\Psi} = [(\Psi^{ij})] = [\{\Psi^i,\Psi^j\}]$ and

$$S_{H\Psi} = [\ \{\ H - H^*, \Psi^1\ \}\ , \ldots , \{\ H - H^*, \Psi^{2n-2m}\ \}\ ]^T$$

Then the Hamiltonian vector field of H becomes identically the Hamiltonian vector field of H^*.

Proof: On S $\Psi^j(q,p) = 0$. At every point on the constraint surface C, the tangent to the compliant control surface S, namely TS, is orthogonal to TC. Thus $d\Psi^j$ j=1,...,2l, form a basis of TC. Along a given path on C, the following condition is satisfied for all $\Psi^i(q,p)$, i=1,....,2n−2m,

$$\{H^*, \Psi^i\} = \{H_T, \Psi^i\}$$

$$= \{H, \Psi^i\} + \{\sum_{j=1}^{2m}(\lambda_j + \hat{u}_j)\Phi^j, \Psi^i\} + \{\sum_{j=1}^{2l}\tilde{u}_j\Psi^j, \Psi^i\}.$$

The second term on the right-hand side vanishes since the Poisson bracket $\{\Phi^j , \Psi^i\}$ is always zero for all indices. Hence

$$-\{\sum_{j=1}^{2n-2m} \tilde{u}_j\Psi^j , \Psi^i\} = \{H - H^* , \Psi^i\}, \text{ or}$$

$$\sum_{j=1}^{2n-2m} \tilde{u}_j\{\Psi^i , \Psi^j\} = \{H - H^* , \Psi^i\} ,$$

which gives equation (4.14).

Remark:

Note that the control input $\tilde{u}$ modifies the total Hamiltonian H_T so that the desired path on C is an integral curve of the resulting Hamiltonian H^*.

Moreover, the orthogonal vector field to TS is given by

$$(X_{H|S})^{\perp} = \sum_{j=1}^{2n-2m} \tilde{u}_j\psi^j. \tag{4.15}$$

We now summarize this result in the form of a theorem.

THEOREM 2: On C, the Hamiltonian H_T has the desired Lagrangian multipliers and its trajectories are steered along a desired path on C, provided that the inputs $\hat{u}$ and $\tilde{u}$ are given by (4.8) and (4.14) respectively, (see Figure 4.4).

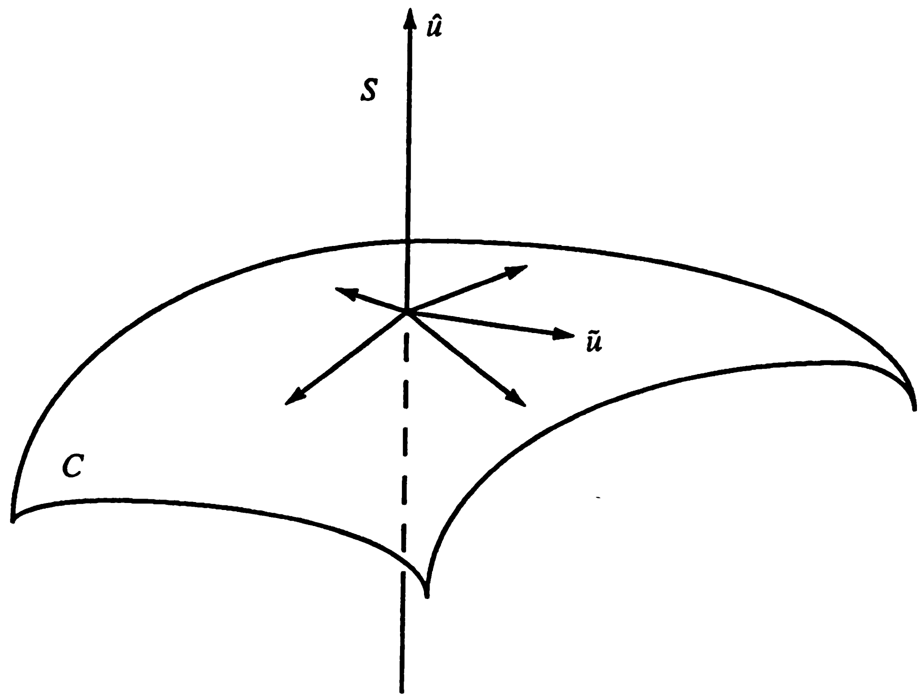

Figure 4.4. The force control $\hat{u}$ and the motion control $\tilde{u}$

CHAPTER 5

THE REQUIRED TORQUE INPUT

The control expressions we have derived <u>must</u> be generated by the input torque at the joints of the manipulator, otherwise the controls we have derived can not be applied in practice. In this section, we show that it is indeed possible to generate the proposed control expressions from the input torques.

The Hamiltonian of the overall compliant system (H_T) is given by

$$H_T = H + \sum_{j=1}^{2m}(\lambda_j + \hat{u}_j)\Phi^j + \sum_{j=1}^{2n-2m} \tilde{u}_j \Psi^j, \tag{5.1}$$

and the corresponding differential equations are

$$\dot{q}^i = \frac{\partial H}{\partial p^i} + \sum_{j=1}^{2m}(\lambda_j + \hat{u}_j)\frac{\partial \Phi^j}{\partial p^i} + \sum_{j=1}^{2n-2m} \tilde{u}_j \frac{\partial \Psi^j}{\partial p^i} \quad i=1,\ldots,n \tag{5.2i}$$

$$\dot{p}^i = -\frac{\partial H}{\partial q^i} - \sum_{j=1}^{2m}(\lambda_j + \hat{u}_j)\frac{\partial \Phi^j}{\partial q^i} - \sum_{j=1}^{2n-2m} \tilde{u}_j \frac{\partial \Psi^j}{\partial q^i} \quad i=1,\ldots,n \tag{5.2ii}$$

The vector form of (5.2) can be written as

$$\dot{q} = \frac{\partial H}{\partial p} + A_1(\hat{u} + \lambda) + B_1\tilde{u} \tag{5.3i}$$

$$\dot{p} = -\frac{\partial H}{\partial q} - A_2(\hat{u} + \lambda) - B_2\tilde{u} \tag{5.3ii}$$

where

$$A_1 = \begin{bmatrix} \frac{\partial\Phi^1}{\partial p^1} & \cdots & \frac{\partial\Phi^{2m}}{\partial p^1} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial\Phi^1}{\partial p^n} & \cdots & \frac{\partial\Phi^{2m}}{\partial p^n} \end{bmatrix} \quad A_2 = \begin{bmatrix} \frac{\partial\Phi^1}{\partial q^1} & \cdots & \frac{\partial\Phi^{2m}}{\partial q^1} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial\Phi^1}{\partial q^n} & \cdots & \frac{\partial\Phi^{2m}}{\partial q^n} \end{bmatrix} \tag{5.4}$$

$$B_1 = \begin{bmatrix} \frac{\partial\Psi^1}{\partial p^1} & \cdots & \frac{\partial\Psi^{2n-2m}}{\partial p^1} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial\Psi^1}{\partial p^n} & \cdots & \frac{\partial\Psi^{2n-2m}}{\partial p^n} \end{bmatrix} \quad B_2 = \begin{bmatrix} \frac{\partial\Psi^1}{\partial q^1} & \cdots & \frac{\partial\Psi^{2n-2m}}{\partial q^1} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial\Psi^1}{\partial q^n} & \cdots & \frac{\partial\Psi^{2n-2m}}{\partial q^n} \end{bmatrix}$$

5.1 Special constraint functions

In this case Φ and Ψ are not functions of p; they are functions only of the position q. This corresponds to a physical constraint on the position alone, e.g. surfaces, obstacles, etc.. In this case, equation (4.3) specializes to

$$\dot{q} = \frac{\partial H}{\partial p} \tag{5.5i}$$

$$\dot{p} = -\frac{\partial H}{\partial q} - A_2(\hat{u} + \lambda) - B_2\tilde{u} \tag{5.5ii}$$

Consequently, the expression for the required physical torque can be read immediately from (5.5ii) as

$$\tau = -A_2\hat{u} - B_2\tilde{u} \tag{5.6}$$

5.2 General Constraint functions

We allow the most general constraint functions for which there corresponds an input torque at the manipulator's joints. The derivations proceed as follows. From (5.3i), after multiplying through by $M(q)$, we get

$$M(q)\dot{q} = p + M(q)A_1(\hat{u} + \lambda) + M(q)B_1\tilde{u}. \tag{5.7}$$

Now take the derivative of (5.7) with respect to time and substitute for $\dot{p}$ from (5.3ii). Then, re-arranging terms, we get

$$\begin{aligned} M(q)\ddot{q} + \dot{M}(q)\dot{q} + \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} \\ &= (-A_2 + \dot{M}(q)A_1 + M(q)\dot{A}_1)\lambda + M(q)A_1\dot{\lambda} \\ &+ (-A_2 + \dot{M}(q)A_1 + M(q)\dot{A}_1)\hat{u} + M(q)A_1\dot{\hat{u}} \\ &+ (-B_2 + \dot{M}(q)A_1 + M(q)\dot{B}_1)\tilde{u} + M(q)B_1\dot{\tilde{u}} \end{aligned} \tag{5.8}$$

Eqn. (5.8) represents the 2nd order dynamic equations of the 2n-degree of freedom manipulator. The left-hand side of (5.8) represent the model of the manipulator. On the right-hand side, the first two terms containing λ and $\dot{\lambda}$ are due to the presence of the constraint surface, while all of the subsequent terms ought to be generated by the physical torque at the joints. Therefore, we specify the physical torque at the joints formally by

$$\begin{aligned} \tau = {} & (-A_2 + \dot{M}(q)A_1 + M(q)\dot{A}_1)\hat{u} + M(q)A_1\dot{\hat{u}} \\ & + (-B_2 + \dot{M}(q)A_1 + M(q)\dot{B}_1)\tilde{u} + M(q)B_1\dot{\tilde{u}} \end{aligned} \tag{5.9}$$

Note that the dots over the expressions in (4.9) denote differentiation with respect to time. In the special case when the constraint and its orthogonal complement constraint equations are not functions of p, i.e.,

$$\Phi(q) = 0$$

$$\Psi(q) = 0.$$

Then $A_1 = B_1 = \dot{A}_1 = \dot{B}_1 = 0$. Hence the torque expression specializes to the case in subsection (5.1), namely,

$$\tau = -A_2\hat{u} - B_2\tilde{u} \tag{5.10}$$

CHAPTER 6

APPLICATION OF THE CONTROL LAWS TO EXAMPLE MODELS

The applications are performed on the industrial three-joint revolute manipulator shown in Figure 6.1; this type of manipulator is used in industry. The joint axes for this manipulator are derived from the Denavit - Hartenberg specification [2] which is an efficient formulation of the forward and inverse kinematics. The approach assumes the model is a series of rigid bodies.

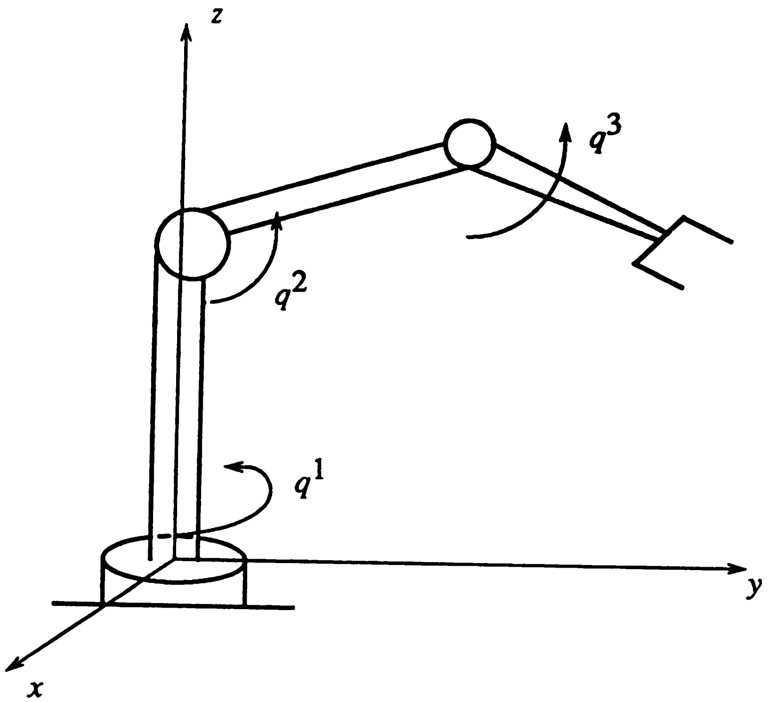

Figure 6.1. A 3 degree of freedom manipulator

The kinematic energy of each link can be easily evaluated as follows [35]

$$T_i = \frac{1}{2}m_i v^2_{c.m._i} + \frac{1}{2}(\overline{I}_{\overline{x}_i}{}^{P}\, w^2_{\overline{x}_i} + \overline{I}_{\overline{y}_i}{}^{P}\, w^2_{\overline{x}_i} + \overline{I}_{\overline{z}_i}{}^{P}\, w^2_{\overline{x}_i}) \tag{6.1}$$

where m_i is the mass of link i, $v_{c.m._i}$ is the velocity of link i at the center of mass, and $\overline{x}_i$, $\overline{y}_i$ and $\overline{z}_i$ are, respectively, distances from the x_{i-1}, y_{i-1} and z_{i-1} to the center of mass [see Figure 6.2]. The variables $\overline{I}_{\overline{x}_i}{}^{P}$, $\overline{I}_{\overline{y}_i}{}^{P}$ and $\overline{I}_{\overline{z}_i}{}^{P}$ denote the principle inertia momenta at the center of mass along the instantaneous directions $\overline{x}_i$, $\overline{y}_i$ and $\overline{z}_i$. The variables $w_{\overline{x}_i}$, $w_{\overline{y}_i}$ and $w_{\overline{y}_i}$ denote the respective angular velocities along the instantaneous directions $\overline{x}_i$, $\overline{y}_i$ and $\overline{z}_i$ relative to the inertial reference space denoted by x_0, y_0 and z_0.

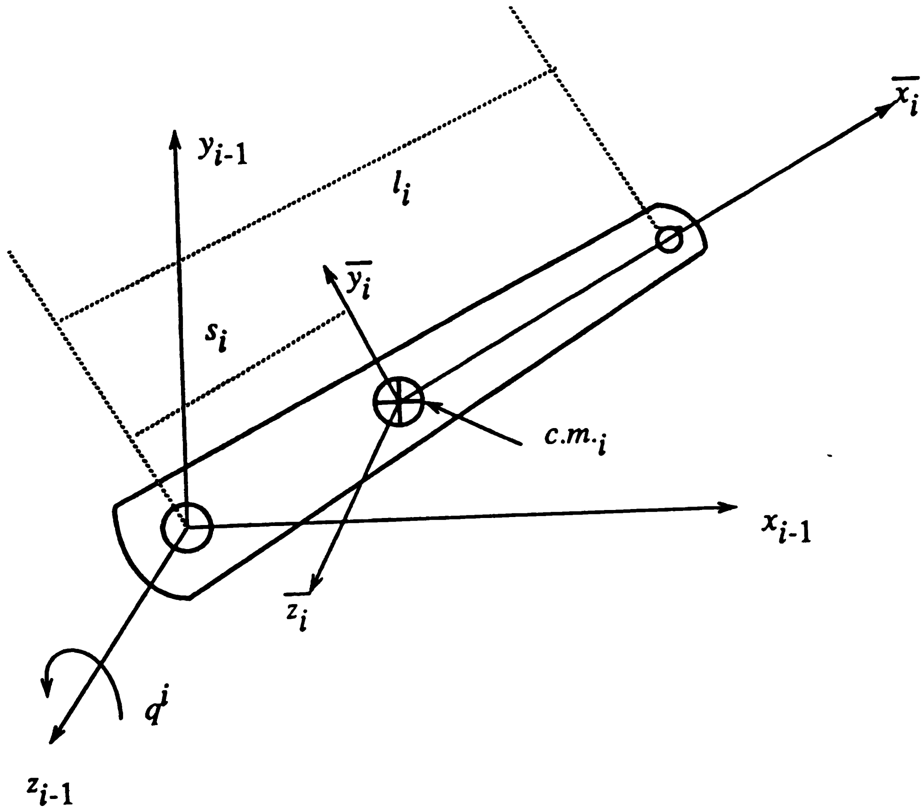

Figure 6.2. Physical constants of link i

The kinetic energy of the manipulator is given by

$$T = \frac{1}{2}\dot{q}^T M(q)\dot{q}, \tag{6.2}$$

where

$$M(q) = \begin{bmatrix} r_{11}(q) & 0 & 0 \\ 0 & r_{22}(q) & r_{23}(q) \\ 0 & r_{23}(q) & r_{33}(q) \end{bmatrix}, \tag{6.3}$$

and the the elements of $M(q)$ are given by

$$r_{11}(q) = a_6 + a_3(\sin(q^2))^2 + a_4(\sin(q^2 + q^3))^2 - 2a_5\cos(q^2)\sin(q^2 + q^3)$$

$$r_{22}(q) = a_7 - 2a_5\sin(q^3)$$

$$r_{23}(q) = a_8 - a_5\sin(q^3)$$

$$r_{33}(q) = a_8.$$

The above constant values are given by

$$a_1 = \overline{I}_{\overline{x}_2}{}^P - \overline{I}_{\overline{y}_2}{}^P$$

$$a_2 = \overline{I}_{\overline{x}_3}{}^P - \overline{I}_{\overline{y}_3}{}^P$$

$$a_3 = a_1 - m_2 s_2^2 - m_3 l_2^2$$

$$a_4 = m_3 s_3^2 + a_2$$

$$a_5 = m_2 l_2 s_3$$

$$a_6 = m_2 s_2^2 + m_3 l_2^2 + \overline{I}_{\overline{z}_1}{}^P + \overline{I}_{\overline{y}_2}{}^P + \overline{I}_{\overline{y}_3}{}^P$$

$$a_7 = m_2 s_2^2 + m_3 s_3^2 + m_2 l_2^2 + \overline{I}_{\overline{z}_2}{}^P + \overline{I}_{\overline{z}_3}{}^P$$

$$a_8 = m_3 s_3^2 + \overline{I}_{\overline{z}_3}{}^P.$$

The kinetic energy as a function of p is described as

$$T = \frac{1}{2} p^T M(q)^{-1} p$$

$$= \frac{1}{2r_{11}(q)}(p^1)^2 + \frac{1}{2r_{44}(q)}[r_{33}(q)(p^2)^2 + r_{22}(p^3)^2 \tag{6.4}$$

$$- 2r_{23}(q)(p^2)(p^3)]$$

where $r_{44}(q) = r_{22}(q)r_{33}(q) - r_{23}(q)^2$.

And the potential energy is given by

$$V = (s_1m_1 + l_1m_2 + l_2m_3)g + a_{10}\sin(q^2) + a_{11}\sin(q^2 + q^3) \tag{6.5}$$

where a_{10}=$(s_2m_2 + l_2m_3)g$ and a_{11}=s_3m_3g. Two examples are now provided to illustrate the actions of the control laws when the manipulator is subjected to certain constraints. In the first example, we choose the simplest (from the viewpoint of computation) constraint equations, and their complement orthogonal constraint equations; the constraint are along the natural coordinates. In the second example, we illustrate the construction of the complement orthogonal constraint equations for a practical constraint equations. The two examples illustrate the actions of the introduced control laws.

Example 1:

Let the constraint equations be

$$\Phi^1 = q^2 \tag{6.6}$$

$$\Phi^2 = p^2$$

then the complement orthogonal constraint equations are chosen to be the rest of the coordinates, i.e. (q^1, q^3, p^1, p^3). Specifically, the orthogonal constraints are

$$\begin{aligned}
\Psi^1 &= q^1 \\
\Psi^2 &= q^3 \\
\Psi^3 &= p^1 \qquad\qquad (6.7)\\
\Psi^4 &= p^3,
\end{aligned}$$

where the above constraints satisfy the following equalities

$$\{\Phi^i\ ,\ \Psi^j\} = 0 \qquad i=1,2 \text{ and } j=1,2,3,4$$

and $S_{\Psi\Psi}$ is nonsingular.

The matrix $C_{\Phi\Phi}$ and $S_{\Psi\Psi}$ are given by

$$C_{\Phi\Phi} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$S_{\Psi\Psi} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}. \qquad (6.8)$$

The total Hamiltonian is given by

$$H_T = H + (\lambda_1 + \hat{u}_1)\Phi^1 + (\lambda_2 + \hat{u}_2)\Phi^2 \qquad (6.9)$$

$$+\ \tilde{u}_1\Psi^1 + \tilde{u}_2\Psi^2 + \tilde{u}_3\Psi^3 + \tilde{u}_4\Psi^4.$$

Hence the corresponding dynamic equations are given by

$$\begin{bmatrix} \dot{q}^1 \\ \dot{q}^2 \\ \dot{q}^3 \\ \dot{p}^1 \\ \dot{p}^2 \\ \dot{p}^3 \end{bmatrix} = \begin{bmatrix} \frac{\partial H}{\partial p^1} \\ \frac{\partial H}{\partial p^2} \\ \frac{\partial H}{\partial p^3} \\ -\frac{\partial H}{\partial q^1} \\ -\frac{\partial H}{\partial q^2} \\ -\frac{\partial H}{\partial q^3} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 + \hat{u}_1 \\ \lambda_2 + \hat{u}_2 \\ \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \\ \tilde{u}_4 \end{bmatrix}. \tag{6.10}$$

Let the desired Hamiltonian for force control be

$$H_{\lambda*} = H + \lambda_1{*}\Phi^1 + \lambda_2{*}\Phi^2. \tag{6.11}$$

Then, the force control inputs are calculated according to formula (4.8) as

$$\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} = C_{\Phi\Phi}^{-1} \begin{bmatrix} \{ H_{\lambda*} , \Phi^1 \} \\ \{ H_{\lambda*} , \Phi^2 \} \end{bmatrix} \tag{6.12}$$

$$= \begin{bmatrix} -\{ H , \Phi^2 \} & -\lambda_1{*} \\ \{ H , \Phi^1 \} & -\lambda_2{*} \end{bmatrix},$$

and the control inputs for the compliant motion (on the constraint submanifold) are calculated according to formula (4.14) as

$$\begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \\ \tilde{u}_4 \end{bmatrix} = S_{\Psi\Psi}{}^{-1} \begin{bmatrix} \{ H - H{*} , \Psi^1 \} \\ \{ H - H{*} , \Psi^2 \} \\ \{ H - H{*} , \Psi^3 \} \\ \{ H - H{*} , \Psi^4 \} \end{bmatrix} \tag{6.13}$$

$$= \begin{bmatrix} -\{ H - H^* , \Psi^3 \} \\ -\{ H - H^* , \Psi^4 \} \\ \{ H - H^* , \Psi^1 \} \\ \{ H - H^* , \Psi^2 \} \end{bmatrix} .$$

Thus the augmented control system equations are "modified" to

$$\begin{aligned} \dot{q}^1 &= \frac{\partial H^*}{\partial p^1} \\ \dot{q}^2 &= \lambda_2 - \lambda_2{}^* \\ \dot{q}^3 &= \frac{\partial H^*}{\partial p^3} \\ \dot{p}^1 &= -\frac{\partial H^*}{\partial q^1} \\ \dot{p}^2 &= -(\lambda_1 - \lambda_1{}^*) \\ \dot{p}^3 &= -\frac{\partial H^*}{\partial q^3} . \end{aligned} \tag{6.14}$$

We drive the following relation for the Poisson bracket;

$$\begin{aligned} \{ \ \{ H , F \} G , Q \} &= G \{ \ \{ H , F \} \ , Q \} \\ &\quad + \{ H , F \} \ \{ G , Q \} . \end{aligned} \tag{6.15}$$

proof: By definition of Poisson bracket

$$\begin{aligned} &\{ \ \{ H , F \} G , Q \} \\ &= \sum_{i=1}^{n} \frac{\partial}{\partial q^i} (\{ H , F \} G) \frac{\partial Q}{\partial p^i} - \frac{\partial}{\partial p^i} (\{ H , F \} G) \frac{\partial Q}{\partial q^i} \\ &= \sum_{i=1}^{n} \frac{\partial}{\partial q^i} (\{ H , F \}) G \frac{\partial Q}{\partial p^i} + \ \{ H , F \} \ \frac{\partial G}{\partial q^i} \frac{\partial Q}{\partial p^i} \\ &\qquad - \frac{\partial}{\partial p^i} (\{ H , F \}) G \frac{\partial Q}{\partial q^i} - \ \{ H , F \} \ \frac{\partial G}{\partial p^i} \frac{\partial Q}{\partial q^i} \end{aligned}$$

$$= G \{ \{ H , F \} , Q \} + \{ H , F \} \{ G , Q \}$$

We now take the Poisson bracket of (in (6.9)) H_T and Φ^1. For the $\{ H_T , \Phi^1 \} = 0$, we have the following relation;

$$\begin{aligned}
0 = & \{ H , \Phi^1 \} \\
& + (\lambda_1 - \lambda^*_1) \{ \Phi^1 , \Phi^1 \} - \{ \{ H , \Phi^2 \} \Phi^1 , \Phi^1 \} \\
& + (\lambda_2 - \lambda^*_2) \{ \Phi^2 , \Phi^1 \} + \{ \{ H , \Phi^1 \} \Phi^2 , \Phi^1 \} \\
& - \{ \{ H - H^* , \Psi^3 \} \Psi^1 , \Phi^1 \} \\
& - \{ \{ H - H^* , \Psi^4 \} \Psi^2 , \Phi^1 \} \\
& + \{ \{ H - H^* , \Psi^1 \} \Psi^3 , \Phi^1 \} \\
& + \{ \{ H - H^* , \Psi^2 \} \Psi^4 , \Phi^1 \}
\end{aligned} \tag{6.16}$$

Using the relation of (6.15), (6.16) results in

$$\lambda_2 = \lambda^*_2 .$$

Similarly, we obtain $\lambda_1 = \lambda^*_1$ from the requirement $\{ H_T , \Phi^2 \} = 0$.

Observe that from $\lambda_1 = \lambda^*_1$ and $\lambda_2 = \lambda^*_2$, (6.14) reduces to

$$\begin{aligned}
\dot{q}^1 &= \frac{\partial H^*}{\partial p^1} \\
\dot{q}^2 &= 0 \\
\dot{q}^3 &= \frac{\partial H^*}{\partial p^3} \\
\dot{p}^1 &= -\frac{\partial H^*}{\partial q^1} \\
\dot{p}^2 &= 0 \\
\dot{p}^3 &= -\frac{\partial H^*}{\partial q^3} ,
\end{aligned} \tag{6.17}$$

which represents the dynamics on the constraint surface along the desired path.

Example 2:

Now we consider a practical constraint: if joint 2 is fixed and the rest of the joints are allowed to move. Then the constraint surface (or region) becomes the inside of a donut shape with its center located at $(0,0,l_1)$ as shown in Figure 6.3.

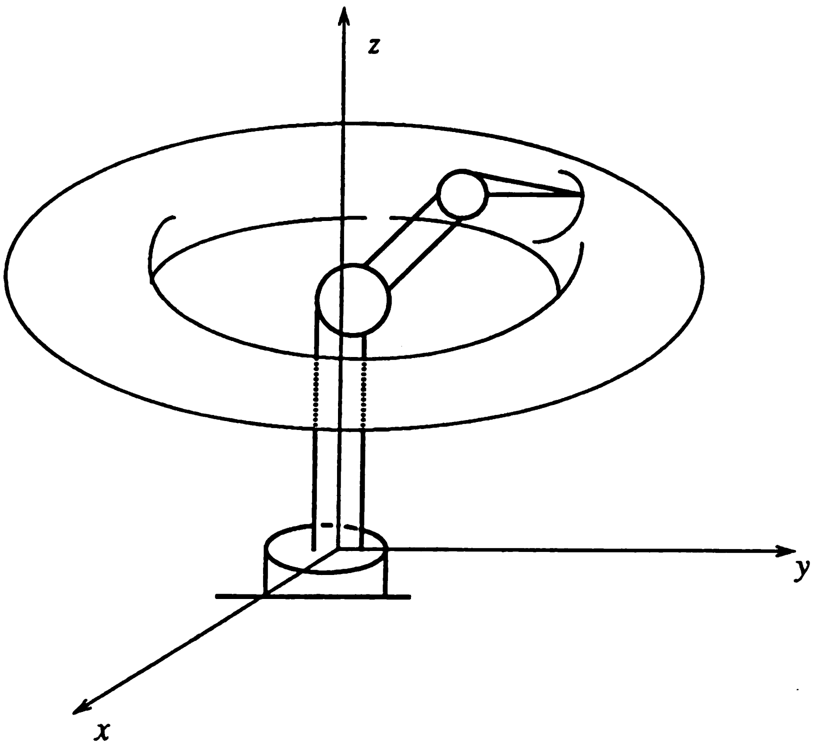

Figure 6.3. The constraint surface of Example 2

This constraint results in the equation

$$q^2 = 0 \tag{6.18}$$

Eqn. (6.18) implies that $\dot{q}^2 = 0$, which in the (q, p)-coordinates reads

$$r_{33}(q)p^2 - r_{23}(q)p^3 = 0. \tag{6.19}$$

Thus the constraint functions are

$$\Phi^1 = q^2$$
$$\Phi^2 = r_{33}(q)p^2 - r_{23}(q)p^3. \tag{6.20}$$

The complement orthogonal constraint functions Ψ^1, Ψ^2, Ψ^3 and Ψ^4 are to be conveniently defined.

Consider the differential of the constraint equations and their orthogonal-complement constraint equations.

$$J = \begin{bmatrix} \dfrac{\partial \Phi^1}{\partial x^j} \\ \dfrac{\partial \Phi^2}{\partial x^j} \\ \dfrac{\partial \Psi^1}{\partial x^j} \\ \dfrac{\partial \Psi^2}{\partial x^j} \\ \dfrac{\partial \Psi^3}{\partial x^j} \\ \dfrac{\partial \Psi^4}{\partial x^j} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_1 & 0 & b_2 & b_3 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & c_3 & 0 & 0 & c_5 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & c_7 & c_8 & 0 & 0 & c_{10} \end{bmatrix}, \tag{6.21}$$

where we used $x^1=q^1, x^2=q_2, x^3=q^3, x^4=p^1, x^5=p^2, x^6=p^3$ and where

$$b_1 = a_5\cos(q^3)p^3$$
$$b_2 = a_8 \tag{6.22}$$
$$b_3 = -(a_8 - a_5\sin(q^3)) \, .$$

Now given b_j's, any quantities c_i's which satisfy the equations

$$b_1c_5 - b_2c_2 - b_3c_3 = 0$$
$$b_1c_{10} - b_2c_7 - b_3c_8 = 0, \tag{6.23}$$

would define an appropriate complement orthogonal constraint equations. Consequently, the following relations are satisfied

$$\{ \Phi^i , \Psi^j \} = 0 \quad i=1,2 \text{ and } j=1,2,3,4.$$

The condition that $S_{\Psi\Psi}$ is nonsingular is given by

$$c_3 c_{10} - c_5 c_8 \neq 0. \tag{6.24}$$

Now we may choose the c_j's as follows

$$c_2 = -b_3 \quad c_3 = b_2$$

$$c_7 = b_1 \quad c_{10} = b_2 .$$

Hence the matrix $C_{\Phi\Phi}$ and $S_{\Psi\Psi}$ are given by

$$C_{\Phi\Phi} = \begin{bmatrix} 0 & b_2 \\ -b_2 & 0 \end{bmatrix}$$

$$S_{\Psi\Psi} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & b_2^2 \\ -1 & 0 & 0 & 0 \\ 0 & -b_2^2 & 0 & 0 \end{bmatrix} \tag{6.25}$$

The dynamic equations now read

$$
\begin{bmatrix} \dot{q}^1 \\ \dot{q}^2 \\ \dot{q}^3 \\ \dot{p}^1 \\ \dot{p}^2 \\ \dot{p}^3 \end{bmatrix} = \begin{bmatrix} \frac{\partial H}{\partial p^1} \\ \frac{\partial H}{\partial p^2} \\ \frac{\partial H}{\partial p^3} \\ -\frac{\partial H}{\partial q^1} \\ -\frac{\partial H}{\partial q^2} \\ -\frac{\partial H}{\partial q^3} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & b_2 & 0 & 0 & 0 & 0 \\ 0 & b_3 & 0 & 0 & 0 & b_2 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & b_3 & 0 & -b_1 \\ 0 & -b_1 & 0 & -b_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 + \hat{u}_1 \\ \lambda_2 + \hat{u}_2 \\ \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \\ \tilde{u}_4 \end{bmatrix}, \tag{6.26}
$$

where the force control inputs are

$$
\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} = C_{\Phi\Phi}^{-1} \begin{bmatrix} \{ H_{\lambda *} , \Phi^1 \} \\ \{ H_{\lambda *} , \Phi^2 \} \end{bmatrix} = \begin{bmatrix} -\frac{1}{b_2} \{ H , \Phi^2 \} - \lambda_1{}^* \\ \frac{1}{b_2} \{ H , \Phi^1 \} - \lambda_2{}^* \end{bmatrix}, \tag{6.27}
$$

and the compliant motion control inputs are

$$
\begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \\ \tilde{u}_4 \end{bmatrix} = S_{\Psi\Psi}{}^{-1} \begin{bmatrix} \{ H - H^* , \Psi^1 \} \\ \{ H - H^* , \Psi^2 \} \\ \{ H - H^* , \Psi^3 \} \\ \{ H - H^* , \Psi^4 \} \end{bmatrix} \tag{6.28}
$$

$$
= \begin{bmatrix} -\{ H - H^* , \Psi^3 \} \\ -\frac{1}{b_2^2} \{ H - H^* , \Psi^4 \} \\ \{ H - H^* , \Psi^1 \} \\ \frac{1}{b_2^2} \{ H - H^* , \Psi^2 \} \end{bmatrix}.
$$

The augmented control system thus becomes

$$\dot{q}^1 = \frac{\partial H^*}{\partial p^1} \tag{6.29.i}$$

$$\dot{q}^2 = b_2(\lambda_2 - \lambda_2^*) \tag{6.29.ii}$$

$$\dot{q}^3 = \frac{\partial H^*}{\partial p^3} + b_3(\lambda_2 - \lambda_2^*) - \frac{b_3}{b_2}\frac{\partial H^*}{\partial p^2} \tag{6.29.iii}$$

$$= \frac{\partial H^*}{\partial p^3} + b_3(\lambda_2 - \lambda_2^*)$$

$$\dot{p}^1 = -\frac{\partial H^*}{\partial q^1} \tag{6.29.iv}$$

$$\dot{p}^2 = -(\lambda_1 - \lambda_1^*) + \frac{b_3}{b_2}\frac{\partial H^*}{\partial q^3} - \frac{b_1}{b_2}\frac{\partial H^*}{\partial p^3} \tag{6.29.v}$$

$$= -(\lambda_1 - \lambda_1^*) - \frac{\partial H^*}{\partial q^2}$$

$$\dot{p}^3 = -\frac{\partial H^*}{\partial q^3} - b_1(\lambda_2 - \lambda_2^*) + \frac{b_1}{b_2}\frac{\partial H^*}{\partial p^2} \tag{6.29.vi}$$

$$= -\frac{\partial H^*}{\partial q^3} - b_1(\lambda_2 - \lambda_2^*),$$

where we have used the following relations for (6.29.iii),(6.29.v) and (6.29.vi), respectively,

$$0 = \{ H^* , \Phi^1 \} = \frac{\partial H^*}{\partial p^2},$$

$$0 = \{ H^* , \Phi^2 \} = b_2\frac{\partial H^*}{\partial q^2} + b_3\frac{\partial H^*}{\partial q^3} - b_1\frac{\partial H^*}{\partial p^3} . \tag{6.30}$$

As before, the Poisson bracket $\{ H_T , \Phi^1 \} = 0$ results in

$$
\begin{aligned}
0 = \; & \{ H , \Phi^1 \} \\
& + (\lambda_1 - \lambda^*_1) \{ \Phi^1 , \Phi^1 \} - \frac{1}{b_2} \{ \{ H , \Phi^2 \} \; \Phi^1 , \Phi^1 \} \\
& + (\lambda_2 - \lambda^*_2) \{ \Phi^2 , \Phi^1 \} + \frac{1}{b_2} \{ \{ H , \Phi^1 \} \; \Phi^2 , \Phi^1 \} \\
& - \{ \{ H - H^* , \Psi^3 \} \; \Psi^1 , \Phi^1 \} \\
& - \frac{1}{b_2^2} \{ \{ H - H^* , \Psi^4 \} \; \Psi^2 , \Phi^1 \} \\
& + \{ \{ H - H^* , \Psi^1 \} \; \Psi^3 , \Phi^1 \} \\
& + \frac{1}{b_2^2} \{ \{ H - H^* , \Psi^2 \} \; \Psi^4 , \Phi^1 \}
\end{aligned}
\tag{6.31}
$$

Using the relation of (6.15), the (6.31) becomes as

$$\lambda_2 = \lambda^*_2$$

and we obtain the $\lambda_1 = \lambda^*_1$ from $\{ H_T , \Phi^2 \} = 0$.

From $\lambda_1 = \lambda_2^*$, $\lambda_2 = \lambda_2^*$, Eqns. (6.29) reduce to

$$
\begin{aligned}
\dot{q}^1 &= \frac{\partial H^*}{\partial p^1} \\
\dot{q}^2 &= 0 \\
\dot{q}^3 &= \frac{\partial H^*}{\partial p^3} \\
\dot{p}^1 &= -\frac{\partial H^*}{\partial q^1} \\
\dot{p}^2 &= -\frac{\partial H^*}{\partial q^2} \\
\dot{p}^3 &= -\frac{\partial H^*}{\partial q^3}
\end{aligned}
\tag{6.32}
$$

CHAPTER 7

STABILIZATION ON THE CONSTRAINT SURFACE FOR THE EXAMPLE MODELS

Example 1

Let H be the Hamiltonian of the actual model and $\tilde{H}$ the mathematical Hamiltonian. Now choose the force inputs as

$$\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} = \begin{bmatrix} -\{\ \tilde{H}\ ,\ \Phi^2\ \}\ \ -\lambda^*{}_1 + e_{\lambda_1} + k_{p5}(p^2 - p^{2*}) + k_{i5}e^2 \\ \{\ \tilde{H}\ ,\ \Phi^1\ \}\ \ -\lambda^*{}_2 + e_{\lambda_2} - k_{p2}(q^2 - q^{2*}) - k_{i2}e^2 \end{bmatrix}, \tag{7.1}$$

where $e_{\lambda_i} = \bar{\lambda}_i - \lambda^*{}_i$, i=1,2 and $\bar{\lambda}$ is the measured value from a force sensor. And choose the control inputs for the compliant motion as

$$\begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \\ \tilde{u}_4 \end{bmatrix} = \begin{bmatrix} -\{\ \tilde{H}\ ,\ \Psi^3\ \}\ \ + k_{p4}(p^1 - p^{1*}) + k_{i4}e_4 \\ -\{\ \tilde{H}\ ,\ \Psi^4\ \}\ \ + k_{p6}(p^3 - p^{3*}) + k_{i6}e_6 \\ \{\ \tilde{H}\ ,\ \Psi^1\ \}\ \ - k_{p1}(q^1 - q^{1*}) - k_{i1}e_1 \\ \{\ \tilde{H}\ ,\ \Psi^2\ \}\ \ - k_{p3}(q^3 - q^{3*}) - k_{i3}e_3 \end{bmatrix}, \tag{7.2}$$

where

$$\dot{e}_1 = q^1 - q^{1*}$$

$$\dot{e}_2 = q^2 - q^{2*}$$

$$\dot{e}_3 = q^3 - q^{3*}$$

$$\dot{e}_4 = p^1 - p^{1*}$$

$$\dot{e}_5 = p^2 - p^{2*}$$

$$\dot{e}_6 = p^3 - p^{3*}.$$

Then, the augmented control system including the control inputs (7.1) and (7.2) are given by

$$\begin{aligned}
\dot{q}^1 &= -k_{p1}(q^1 - q^{1*}) - k_{i1}e_1 + m_1 \\
\dot{q}^2 &= -k_{p2}(q^2 - q^{2*}) - k_{i2}e_2 + m_2 \\
\dot{q}^3 &= -k_{p3}(q^3 - q^{3*}) - k_{i3}e_3 + m_3 \\
\dot{p}^1 &= -k_{p4}(p^1 - p^{1*}) - k_{i4}e_4 + m_4 \\
\dot{p}^2 &= -k_{p5}(p^2 - p^{2*}) - k_{i5}e_5 + m_5 \\
\dot{p}^3 &= -k_{p6}(p^3 - p^{3*}) - k_{i6}e_6 + m_6 \\
\dot{e}_1 &= q^1 - q^{1*} \\
\dot{e}_2 &= q^2 - q^{2*} \\
\dot{e}_3 &= q^3 - q^{3*} \\
\dot{e}_4 &= p^1 - p^{1*} \\
\dot{e}_5 &= p^2 - p^{2*} \\
\dot{e}_6 &= p^3 - p^{3*},
\end{aligned} \tag{7.3}$$

where each m_i, i=1,..,6, is a mismatch function given as

$$m_1 := \frac{\partial H}{\partial p^1} - \frac{\partial \tilde{H}}{\partial p^1}$$

$$m_2 := \frac{\partial H}{\partial p^2} - \frac{\partial \tilde{H}}{\partial p^2}$$

$$m_3 := \frac{\partial H}{\partial p^3} - \frac{\partial \tilde{H}}{\partial p^3}$$

$$m_4 := -\left(\frac{\partial H}{\partial q^1} - \frac{\partial \tilde{H}}{\partial q^1}\right) \tag{7.4}$$

$$m_5 := -\left(\frac{\partial H}{\partial q^2} - \frac{\partial \tilde{H}}{\partial q^2}\right)$$

$$m_6 := -\left(\frac{\partial H}{\partial q^3} - \frac{\partial \tilde{H}}{\partial q^3}\right).$$

Let the Liapunov candidate function be

$$\begin{aligned} E = {} & \frac{1}{2}(q^1 - q^{1*})^2 + \frac{1}{2}(q^2 - q^{2*})^2 + \frac{1}{2}(q^3 - q^{3*})^2 \\ & + \frac{1}{2}(p^1 - p^{1*})^2 + \frac{1}{2}(p^2 - p^{2*})^2 + \frac{1}{2}(p^3 - p^{3*})^2 \\ & + \frac{1}{2}k_{i1}e_1^2 + \frac{1}{2}k_{i2}e_2^2 + \frac{1}{2}k_{i3}e_3^2 + \frac{1}{2}k_{i4}e_4^2 + \frac{1}{2}k_{i5}e_5^2 + \frac{1}{2}k_{i6}e_6^2, \end{aligned} \tag{7.5}$$

where k_{ik}, $k = 1,..,6$ are positive constant values. The derivative of E along the trajectories is calculated to be

$$\begin{aligned} \dot{E} = {} & -k_{p1}(q^1 - q^{1*})^2 - k_{p2}(q^2 - q^{2*})^2 - k_{p3}(q^3 - q^{3*})^2 \\ & - k_{p4}(p^1 - p^{1*})^2 - k_{p5}(p^2 - p^{2*})^2 - k_{p6}(p^3 - p^{3*})^2 \\ & + m_1(q^1 - q^{1*}) + m_2(q^2 - q^{2*}) + m_3(q^3 - q^{3*}) \\ & + m_4(p^1 - p^{1*}) + m_5(p^2 - p^{2*}) + m_6(p^3 - p^{3*}). \end{aligned} \tag{7.6}$$

Over any compact region in the state space, the mismatch quantities m_i, $i = 1,...,6$ are bounded. Assume that each m_i, i=1,...,6 is bound above by a (positive) constant $\overline{m}_i$, i.e. $|m_i| \le \overline{m}_i$. Now consider the hypercube:

$$R = \{(q\ ,p) \mid |q^i - q^{i*}| \le \frac{\overline{m}_i}{k_{pi}}\ , |p^i - p^{i*}| \le \frac{\overline{m}_{3+i}}{k_{p3+i}},\ i\text{=1,2,3}\} \tag{7.7}$$

Then outside the hypercube R, we have

$$\dot{E} \le - k_{p1}(q^1 - q^{1*})^2 - k_{p2}(q^2 - q^{2*})^2 - k_{p3}(q^3 - q^{3*})^2 \tag{7.8}$$

$$- k_{p4}(p^1 - p^{1*})^2 - k_{p5}(p^2 - p^{2*})^2 - k_{p6}(p^3 - p^{3*})^2 \le 0.$$

Consequently, all initial conditions of the system equation (7.3) converge to the hypercube R defined by (7.7). Moreover, if $m_i \neq k_{pi}(q^i - q^{i*})$, $m_{3+i} \neq k_{p3+i}(p^i - p^{i*})$, i=1,2,3, then $\dot{E} = 0$ if and only if $q^i = q^{i*}$ and $p^i = p^{i*}$, i=1,2.3.

Example 2

Choose the force inputs and the control inputs for the compliant motion as

$$\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{b_2} \{ \tilde{H} , \Phi^2 \} - \lambda^*_1 + e_{\lambda_1} + \hat{u}_1^* \\ \dfrac{1}{b_2} \{ \tilde{H} , \Phi^1 \} - \lambda^*_2 + e_{\lambda_2} + \hat{u}_2^* \end{bmatrix}, \tag{7.9}$$

and

$$\begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \\ \tilde{u}_4 \end{bmatrix} = \begin{bmatrix} - \{ \tilde{H} , \Psi^3 \} + \tilde{u}_1^* \\ -\dfrac{1}{b_2^2} \{ \tilde{H} , \Psi^4 \} + \tilde{u}_2^* \\ \{ \tilde{H} , \Psi^1 \} + \tilde{u}_3^* \\ \dfrac{1}{b_2^2} \{ \tilde{H} , \Psi^2 \} + \tilde{u}_4^* \end{bmatrix} \tag{7.10}$$

where

$$\hat{u}_1^* = k_{p5}(p^2 - p^{2*}) + k_{i5}e_5 + \frac{b_3}{b_2}[k_{p6}(p^3 - p^{3*}) + k_{i6}e_6]$$

$$+ \frac{b_1}{b_2}[k_{p3}(q^3 - q^{3*}) + k_{i3}e_3]$$

$$\hat{u}_2^* = - \frac{1}{b_2}[k_{p2}(q^2 - q^{2*}) + k_{i2}e_2]$$

$$\tilde{u}_1^* = k_{p4}(p^1 - p^{1*}) + k_{i4}e_4$$

$$\tilde{u}_2^* = \frac{1}{b_2}[k_{p6}(p^3 - p^{3*}) + k_{i6}e_6] + \frac{b_1}{b_2^2}[k_{p2}(q^2 - q^{2*}) + k_{i2}e_2]$$

$$\tilde{u}_3^* = - [k_{p1}(q^1 - q^{1*}) + k_{i1}e_1]$$

$$\tilde{u}_4{}^* = -\frac{1}{b_2}[k_{p3}(q^3 - q^{3*}) + k_{i3}e_3] + \frac{b_3}{b_2^2}[k_{p2}(q^2 - q^{2*}) + k_{i2}e_2]$$

$$\dot{e}_1 = q^1 - q^{1*}$$

$$\dot{e}_2 = q^2 - q^{2*}$$

$$\dot{e}_3 = q^3 - q^{3*}$$

$$\dot{e}_4 = p^1 - p^{1*}$$

$$\dot{e}_5 = p^2 - p^{2*}$$

$$\dot{e}_6 = p^3 - p^{3*}.$$

The above linear control inputs ($\hat{u}_1{}^*$, $\hat{u}_2{}^*$, $\tilde{u}_1{}^*$, $\tilde{u}_2{}^*$, $\tilde{u}_3{}^*$, $\tilde{u}_4{}^*$) can be obtained using the Liapunov function approach; for a Liapunov candidate function such as (7.5), substitute (7.9) and (7.10) into the derivative of E and then choose the linear control inputs which would make $\dot{E}$ negative semidefinite. The augmented control system equations using the control inputs (7.9) and (7.10) are given by

$$\dot{q}^1 = m_1 + \tilde{u}_3{}^*$$

$$\dot{q}^2 = m_2 + b_2\hat{u}_2{}^*$$

$$\dot{q}^3 = m_3 + b_3\hat{u}_2{}^* + u_2\tilde{u}_4{}^*$$

$$\dot{p}^1 = m_4 - \tilde{u}_2{}^*$$

$$\dot{p}^2 = m_5 - \hat{u}_1{}^* + b_3\tilde{u}_2{}^* - b_1\tilde{u}_4{}^*$$

$$\dot{p}^3 = m_6 - b_1\hat{u}_2{}^* - b_2\tilde{u}_2{}^* \qquad (7.11)$$

$$\dot{e}_1 = q^1 - q^{1*}$$

$$\dot{e}_2 = q^2 - q^{2*}$$

$$\dot{e}_3 = q^3 - q^{3*}$$

$$\dot{e}_4 = p^1 - p^{1*}$$

$$\dot{e}_5 = p^2 - p^{2*}$$

$$\dot{e}_6 = p^3 - p^{3*},$$

where m_i, i=1,..,6, is a mismatch function defined as before.

Let the Liapunov candidate function be as before and take the derivative along trajectories. Then $\dot{E}$ becomes identically (7.8) under the same assumption (7.7).

CHAPTER 8

SIMULATION

Computer simulations are performed for Example (2) discussed earlier to examine the effectiveness of the control algorithms introduced. The parameters of the manipulator are set as in Table 1. For the computer simulation of the dynamic equations, a fourth-order Runge-Kutta method is used on a VAX 11/780/8600 Ultrix computer system.

Link	Mass(Kg)	Length(m)	$\bar{I}_{\bar{x}_i}{}^{P}$ (Kgm^2)	$\bar{I}_{\bar{y}_i}{}^{P}$ (Kgm^2)	$\bar{I}_{\bar{z}_i}{}^{P}$ (Kgm^2)
1	15	0.8			3.2
2	10	0.7	0.41	1.63	1.63
3	7	0.5	0.21	0.84	0.84

Table 1. The physical parameters of the 3 d.o.f. manipulator

There are two possible ways to obtain the desired values; one is to obtain the desired Hamiltonian for the given desired force and motion if possible, and the other is to obtain the vector field corresponding to the desired Hamiltonian as

$$\frac{\partial H^*}{\partial p} = \dot{q}^* \tag{8.1}$$

$$-\frac{\partial H^*}{\partial q} = \dot{p}^* = M\ddot{q}^* + \dot{M}\dot{q}^* \ . \tag{8.2}$$

To simulate the performance of the robot manipulator, the desired path is chosen as

$$q^{1*} = 0.02t \; (rad)$$

$$q^{2*} = 0.0 \; (rad)$$

$$q^{3*} = 0.01t \; (rad) \tag{8.3}$$

$$\dot{q}^{1*} = 0.02 \; (rad/sec)$$

$$\dot{q}^{2*} = 0.0 \; (rad/sec)$$

$$\dot{q}^{3*} = 0.01 \; (rad/sec),$$

and the desired Lagrangian multipliers as

$$\lambda^*_1 = 0.02 \tag{8.4}$$

$$\lambda^*_2 = 0.0$$

Figures (8.1-3) and Figures (8.4-6) depict the result of the control in achieving position and velocity tracking, respectively, for the desired values with initial condition set at (q^1=0.0, q^2=0.0, q^3=0.0, $\dot{q}^1$=0.02 $\dot{q}^2$=0.0, $\dot{q}^3$=0.01). Figures (8.7-12) display the corresponding torques ($\hat{u}_1$, $\hat{u}_2$ $\tilde{u}_1$,..,$\tilde{u}_4$). Figure (8.13 and 8.14) show the resulting Lagrangian multipliers.

In this simulation, the integration frequency is taken as f = 100,000. Even if the initial conditions are the same as the desired values, we see a diversion of the trajectory in Figure 8.4. This indicates that the overall system is unstable. To overcome the instability effect of numerical errors, we must take advantage of feedback of error signals.

For the control inputs (7.9) and (7.10), Figures (8.15-26) depict the result of the control in achieving position and velocity tracking and the corresponding torques with the initial condition set at (q^1=0.1, q^2=0.0, q^3=0.1, $\dot{q}^1$=0.0 $\dot{q}^2$=0.0, $\dot{q}^3$=0.0). In this simulation, the integration frequency is f = 60 and the feedback gains are k_{pk} = 5.0 and k_{ik} = 5.0 k = 1,..,6.

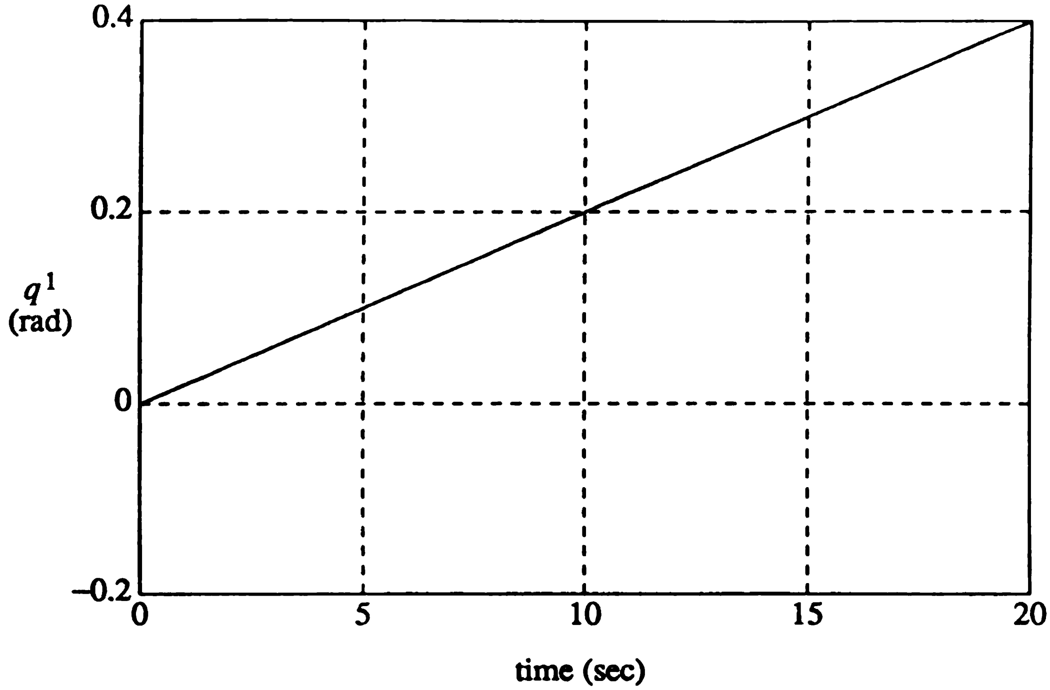

Figure 8.1. Plot of the (generalized) angle q^1 versus time, Example 2

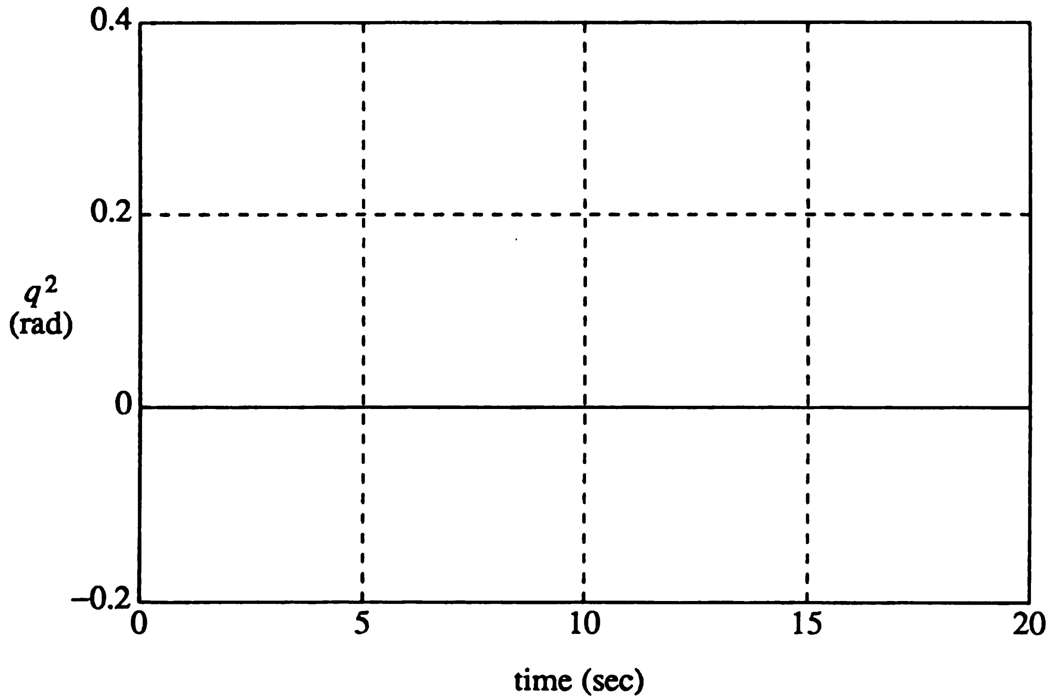

Figure 8.2. Plot of the (generalized) angle q^2 versus time, Example 2

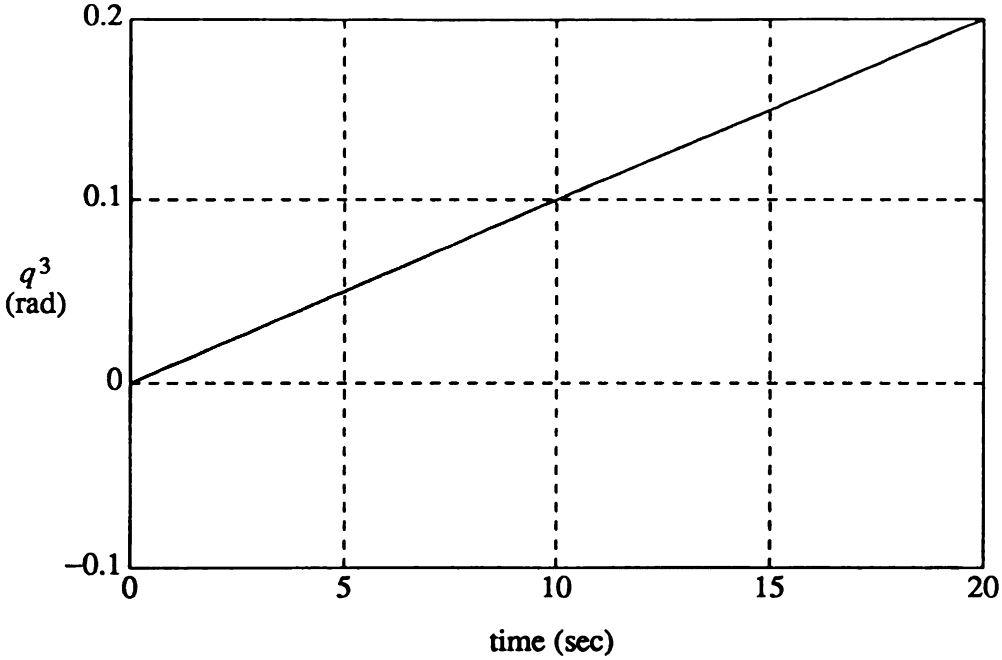

Figure 8.3. Plot of the (generalized) angle q^3 versus time, Example 2

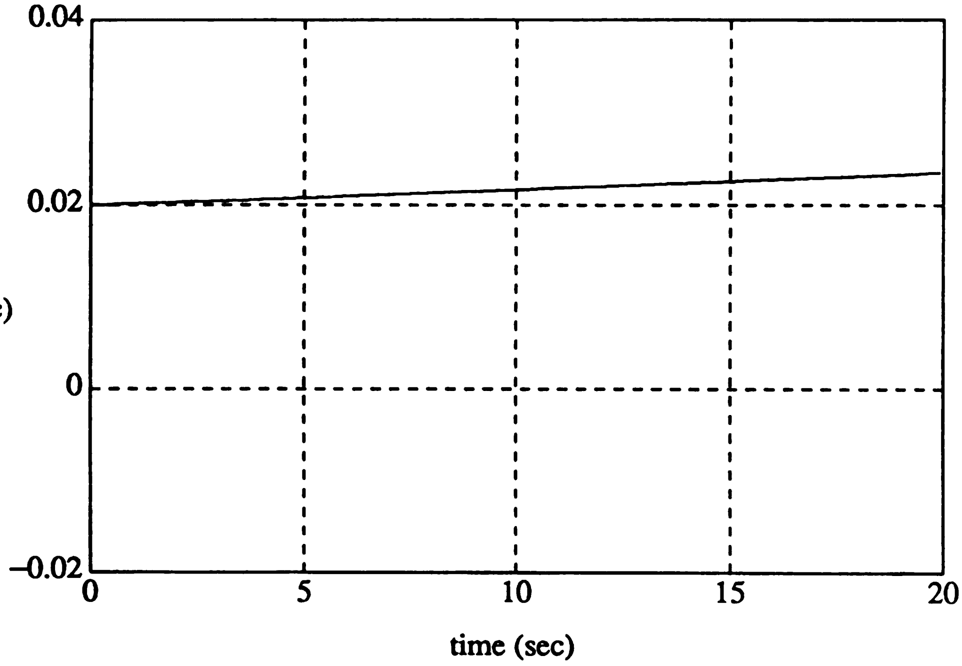

Figure 8.4. Plot of the (generalized) velocity $\dot{q}^1$ versus time, Example 2

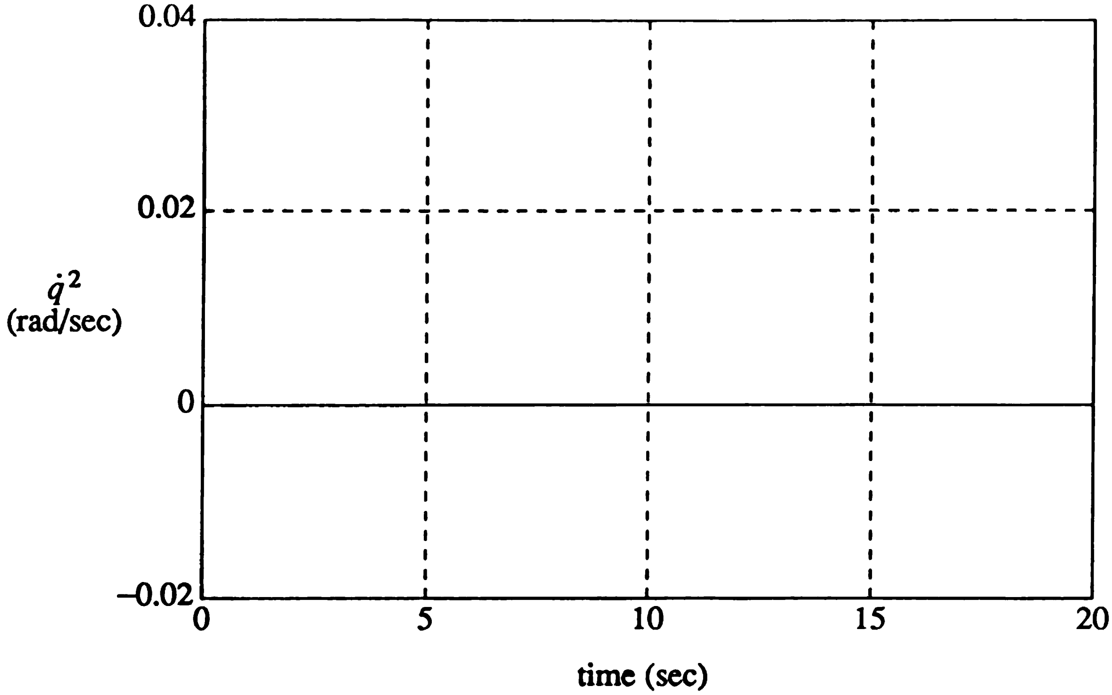

Figure 8.5. Plot of the (generalized) velocity $\dot{q}^2$ versus time, Example 2

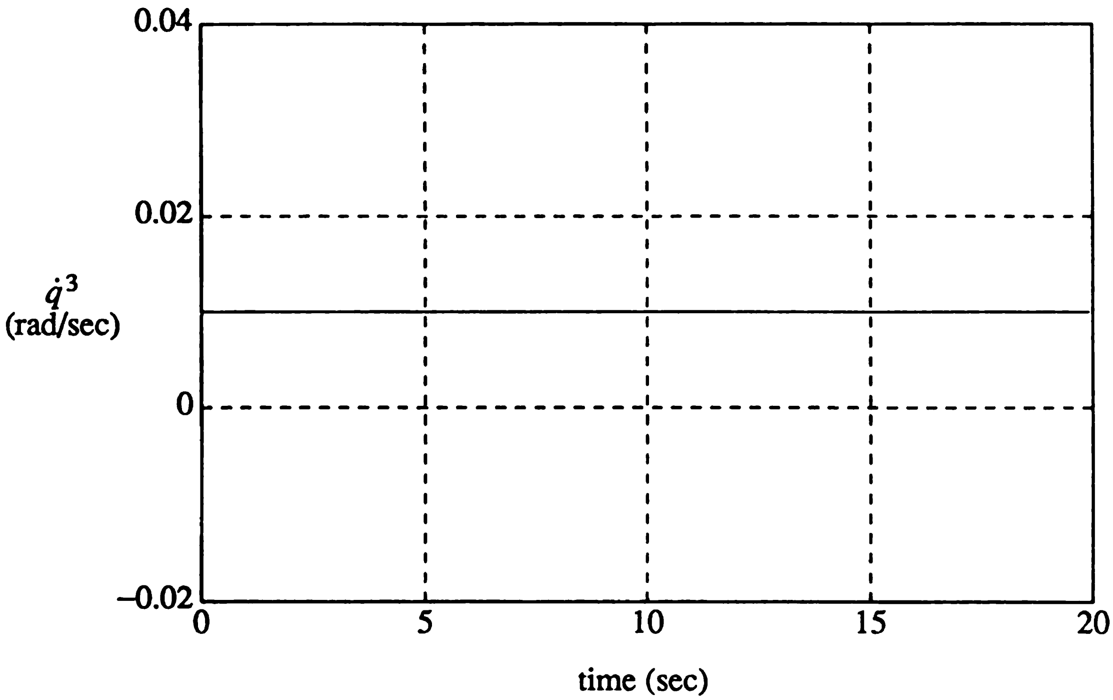

Figure 8.6. Plot of the (generalized) velocity $\dot{q}^3$ versus time, Example 2

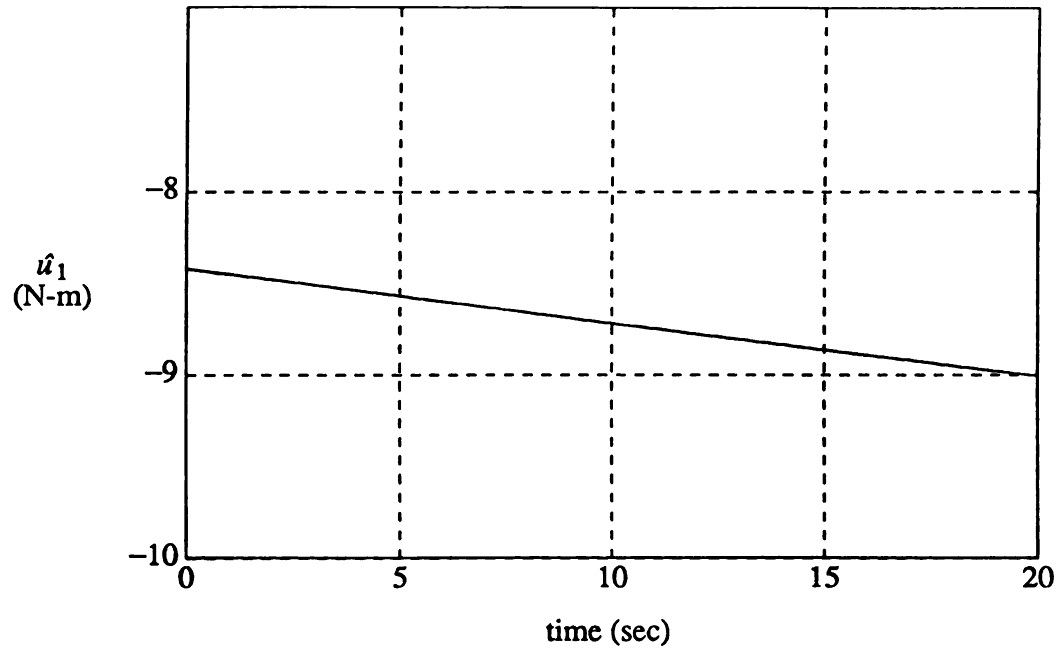

Figure 8.7. Plot of the force control $\hat{u}_1$ versus time, Example 2

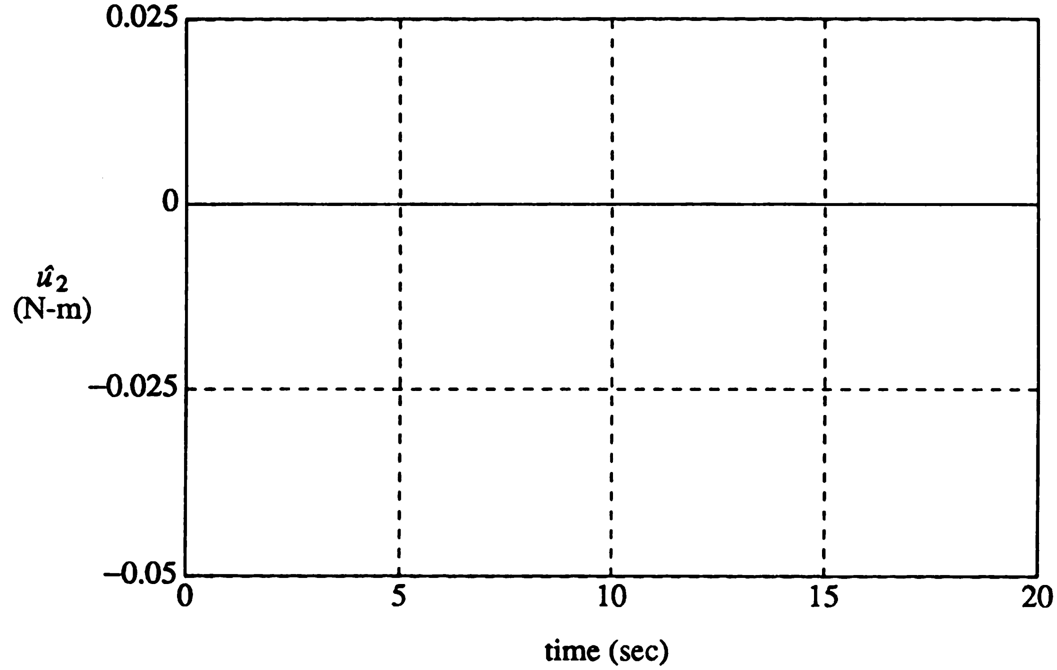

Figure 8.8. Plot of the force control $\hat{u}_2$ versus time, Example 2

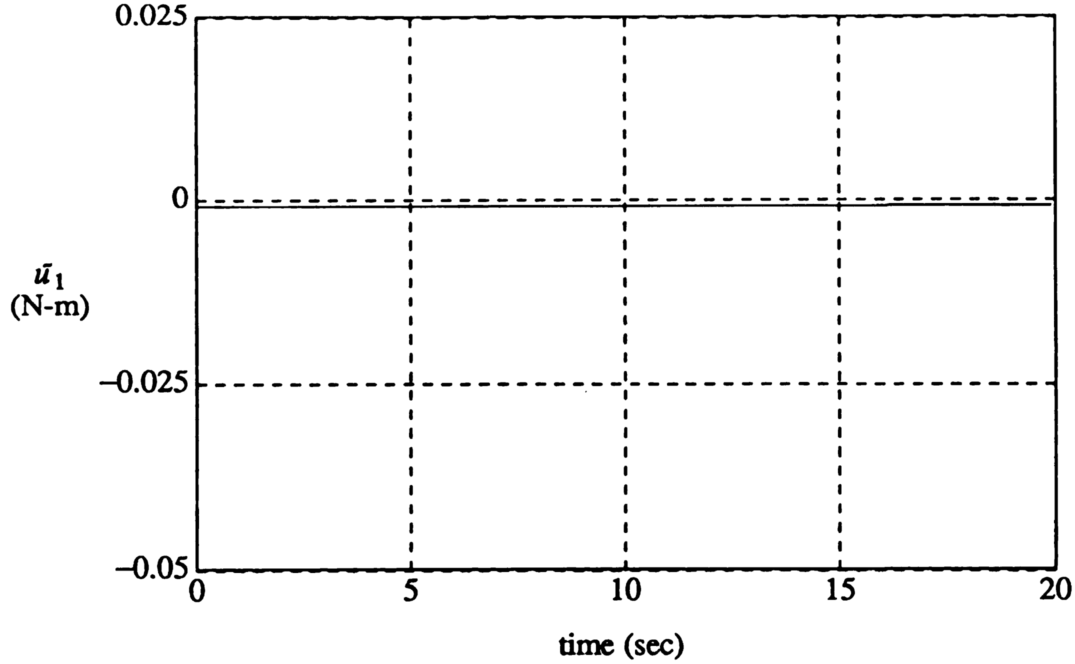

Figure 8.9. Plot of the motion control $\bar{u}_1$ versus time, Example 2

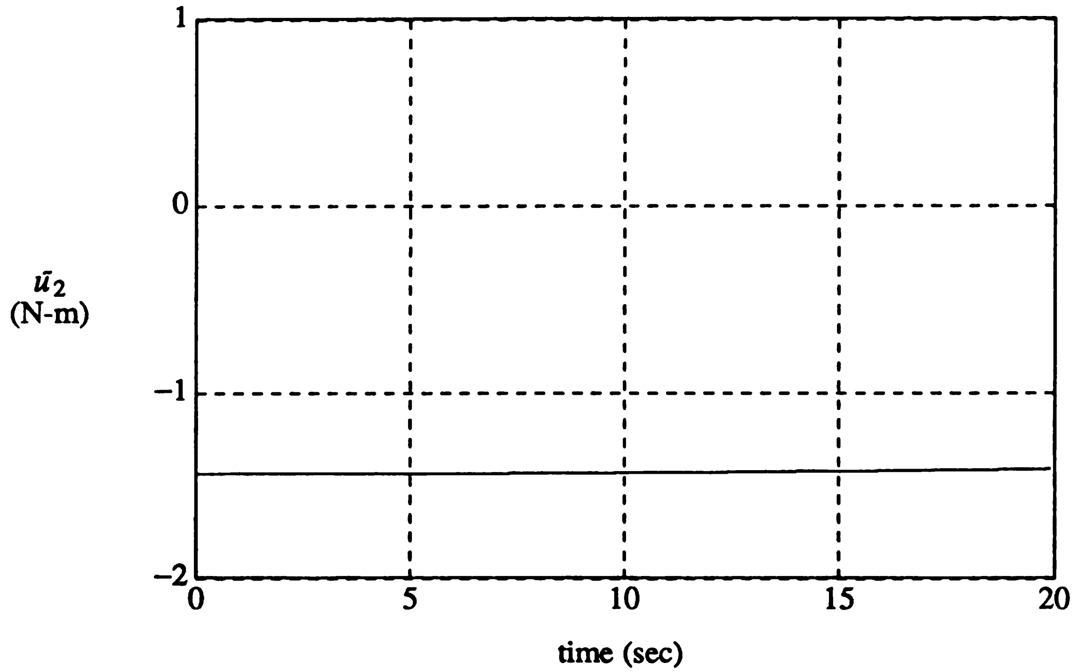

Figure 8.10. Plot of the motion control $\bar{u}_2$ versus time, Example 2

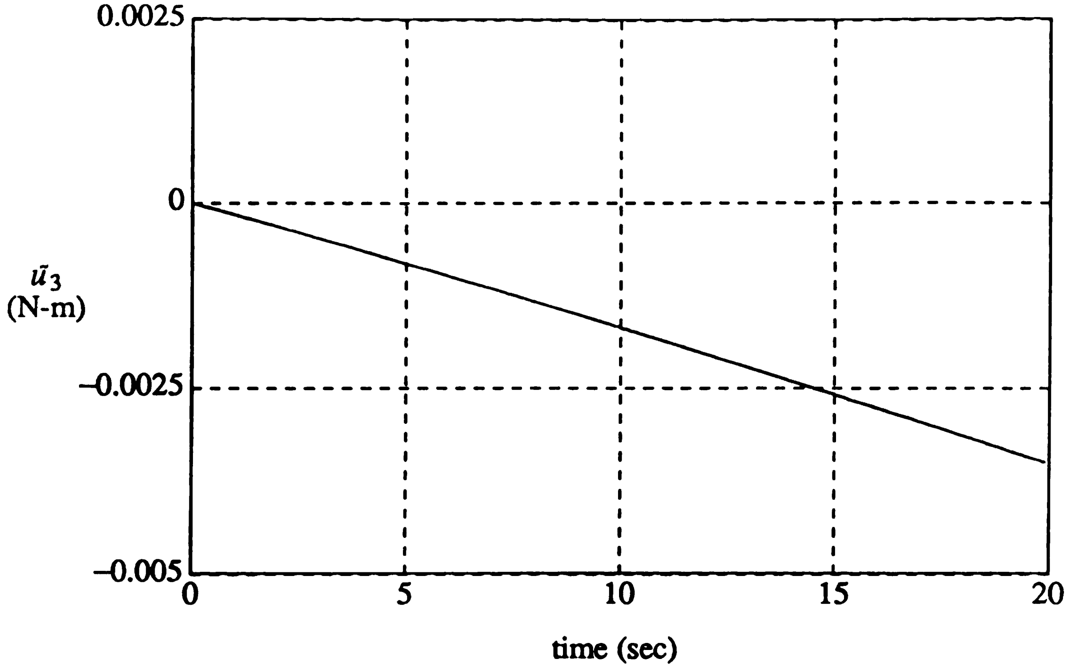

Figure 8.11. Plot of the motion control $\bar{u}_3$ versus time, Example 2

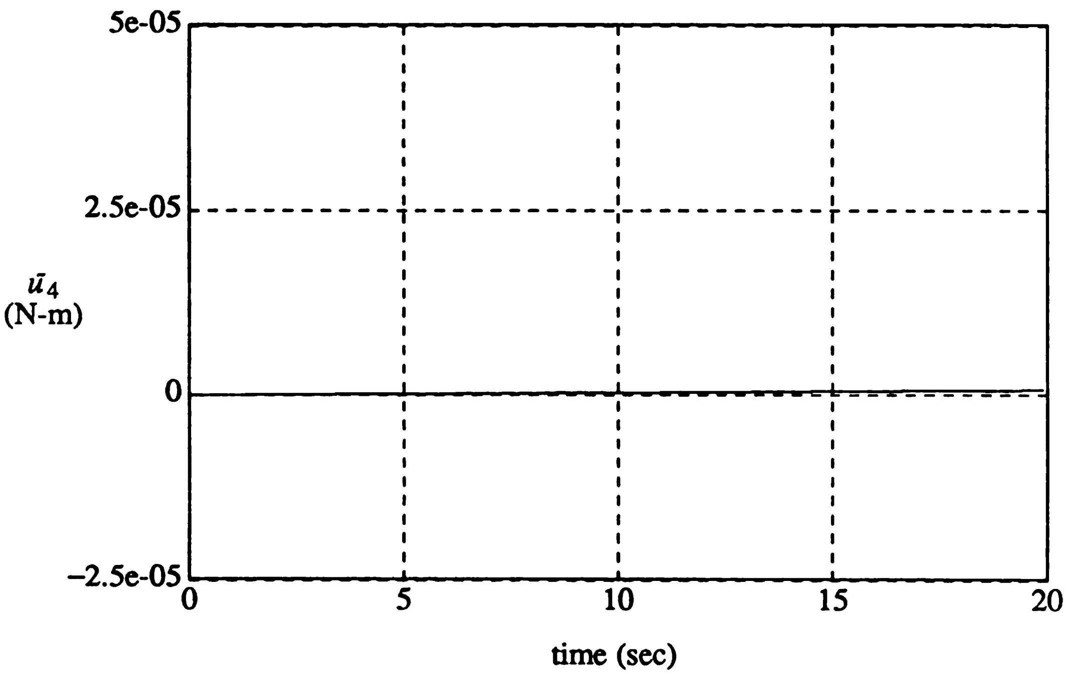

Figure 8.12. Plot of the motion control $\bar{u}_4$ versus time, Example 2

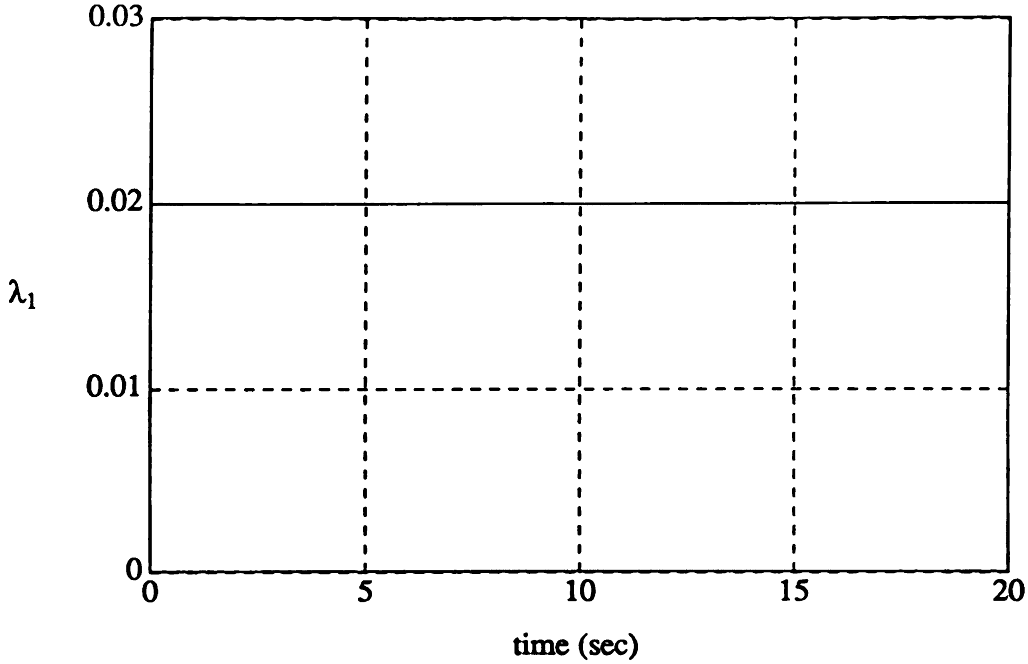

Figure 8.13. Plot of the Lagrangian multiplier λ_1 versus time, Example 2

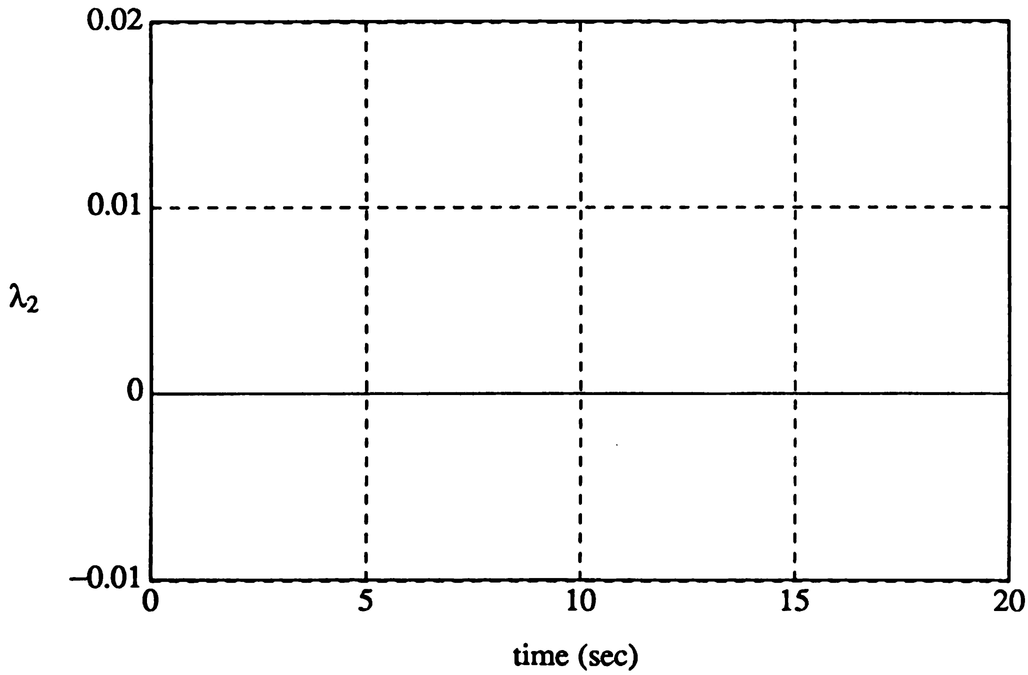

Figure 8.14. Plot of the Lagrangian multiplier λ_2 versus time, Example 2

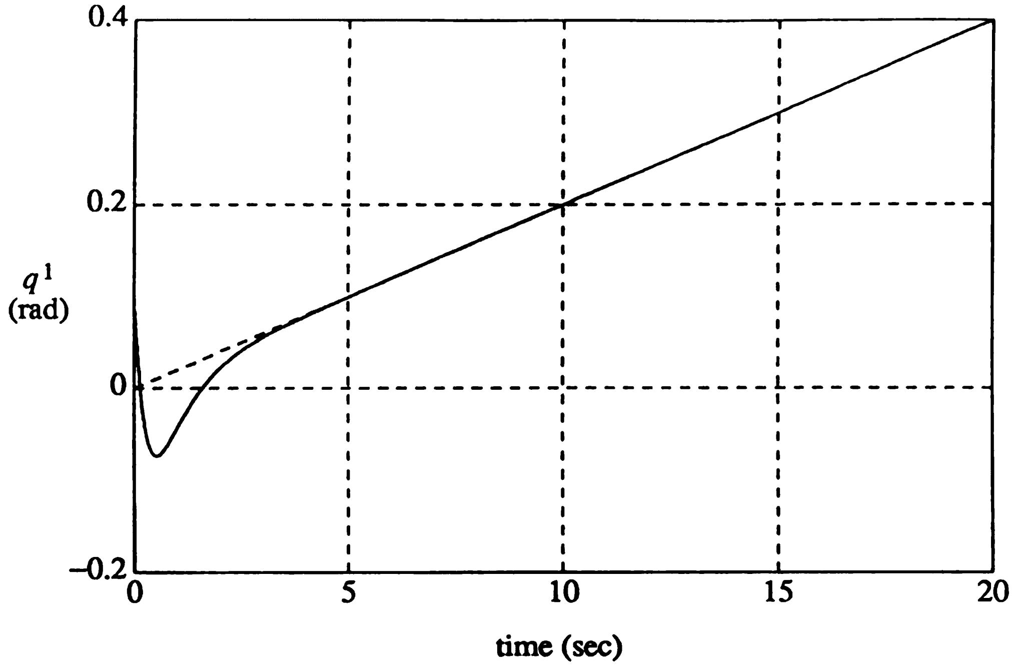

Figure 8.15. Plot of the (generalized) angle q^1 versus time, Example 2 (the overall nonlinear control)

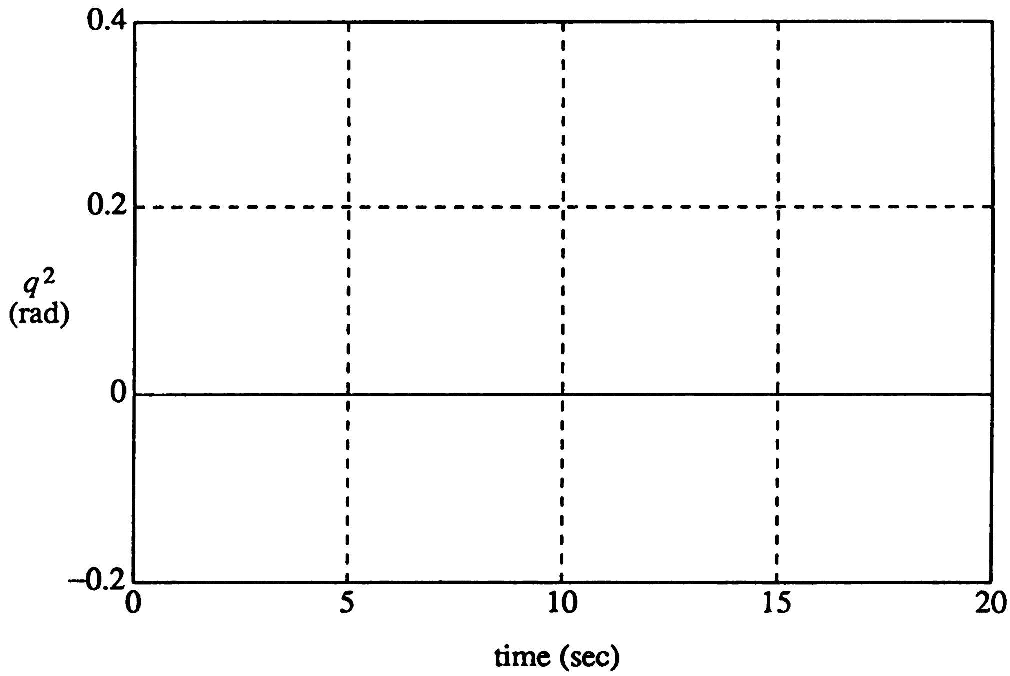

Figure 8.16. Plot of the (generalized) angle q^2 versus time, Example 2 (the overall nonlinear control)

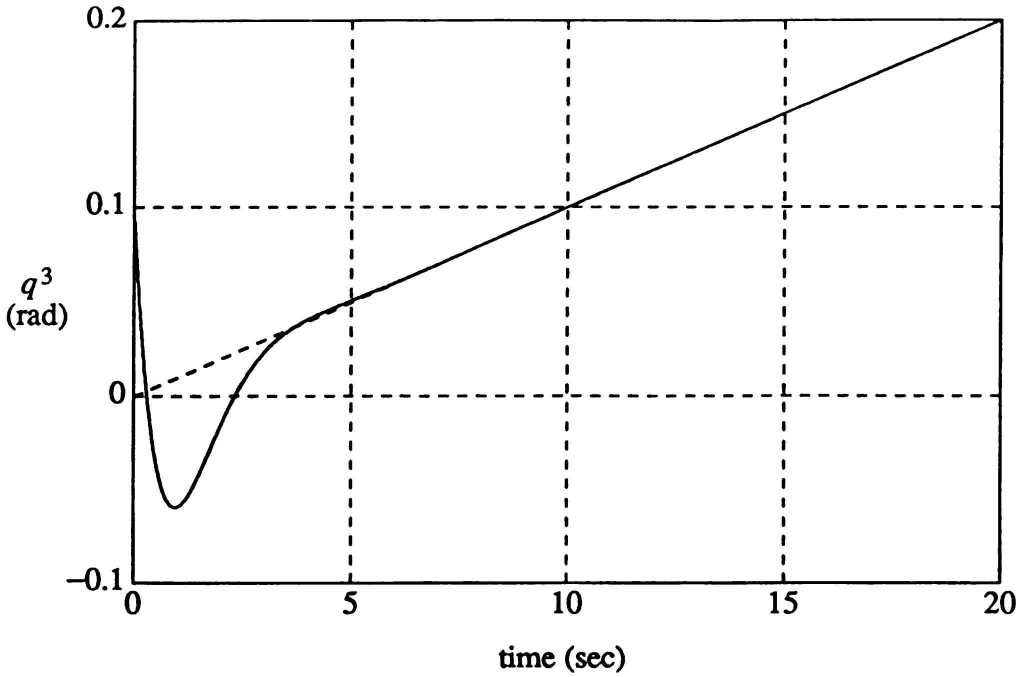

Figure 8.17. Plot of the (generalized) angle q^3 versus time, Example 2 (the overall nonlinear control)

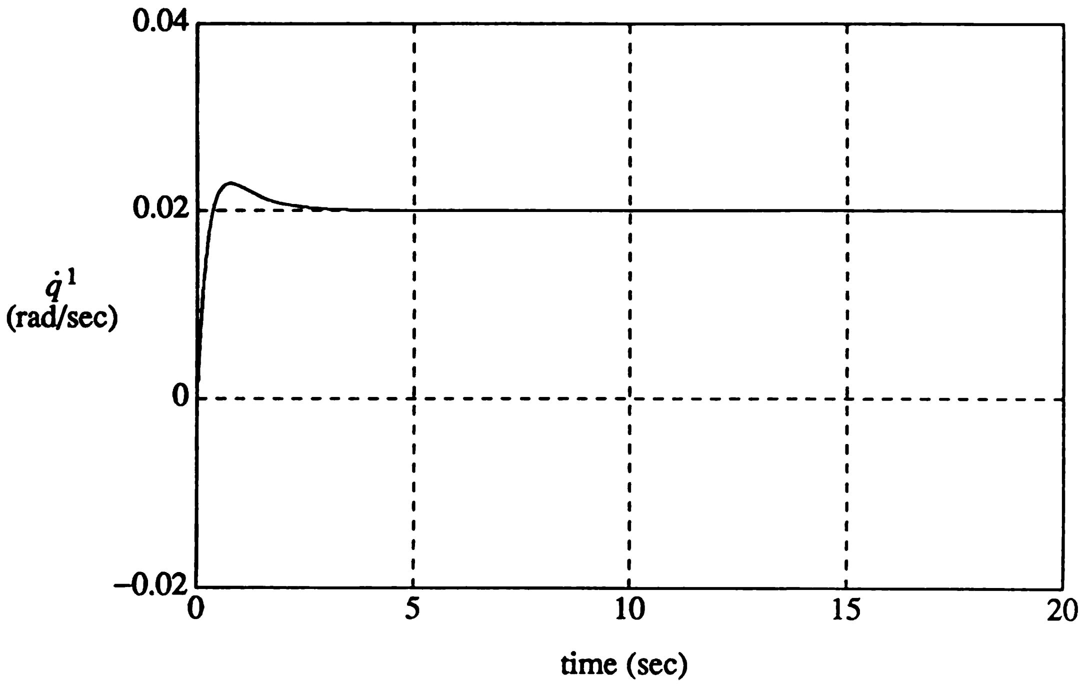

Figure 8.18. Plot of the (generalized) velocity $\dot{q}^1$ versus time, Example 2 (the overall nonlinear control)

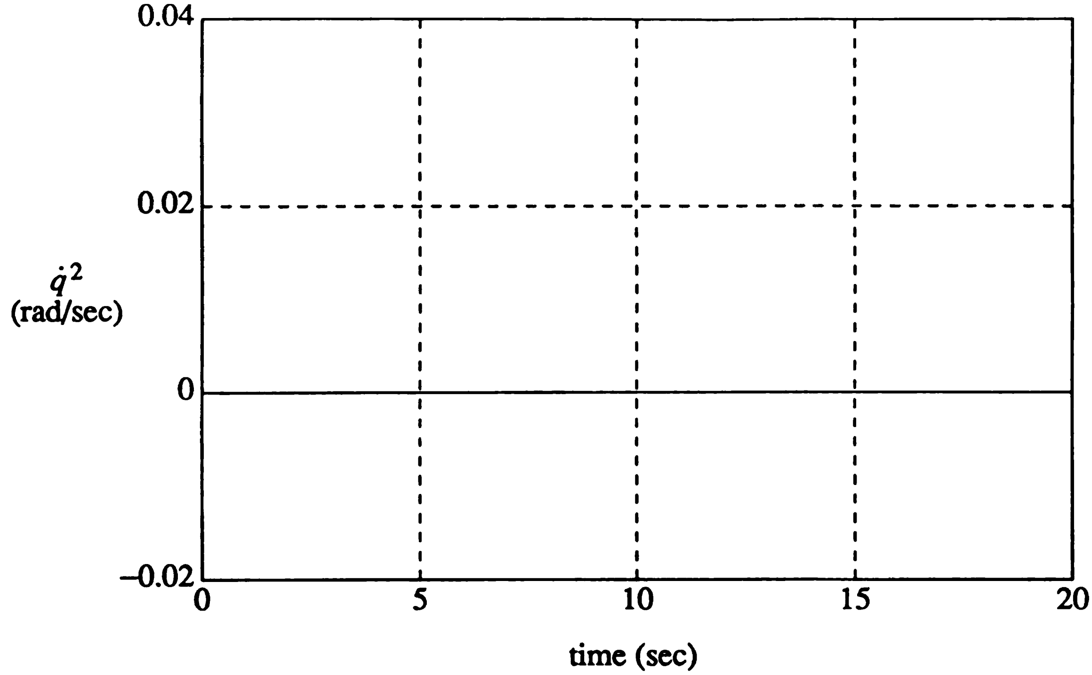

Figure 8.19. Plot of the (generalized) velocity $\dot{q}^2$ versus time, Example 2 (the overall nonlinear control)

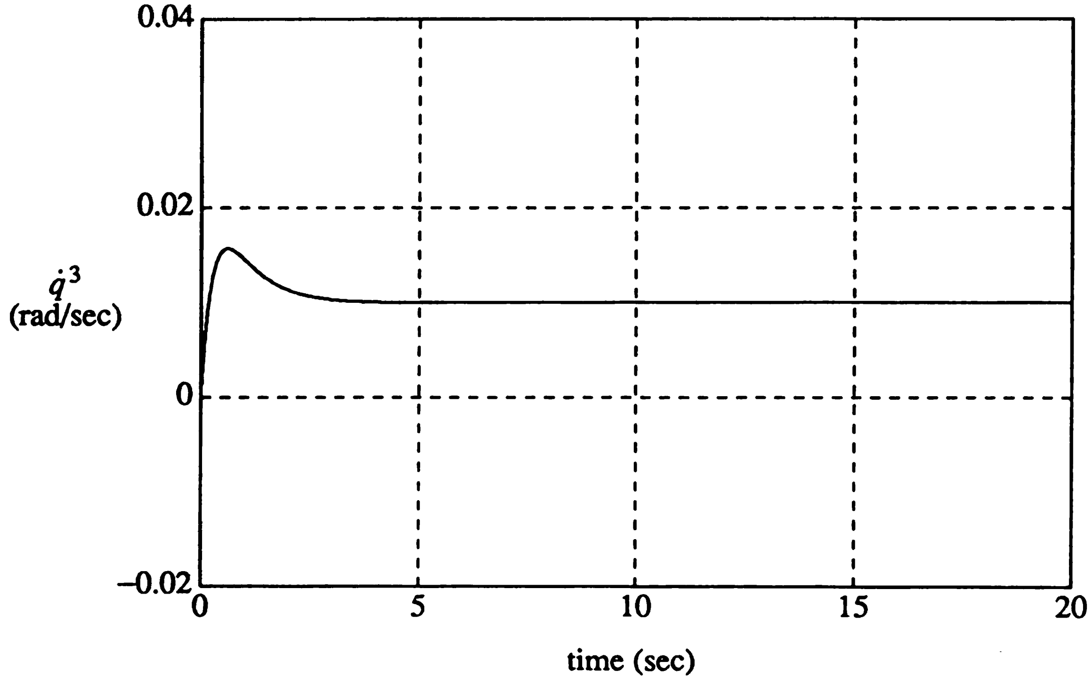

Figure 8.20. Plot of the (generalized) velocity $\dot{q}^3$ versus time, Example 2 (the overall nonlinear control)

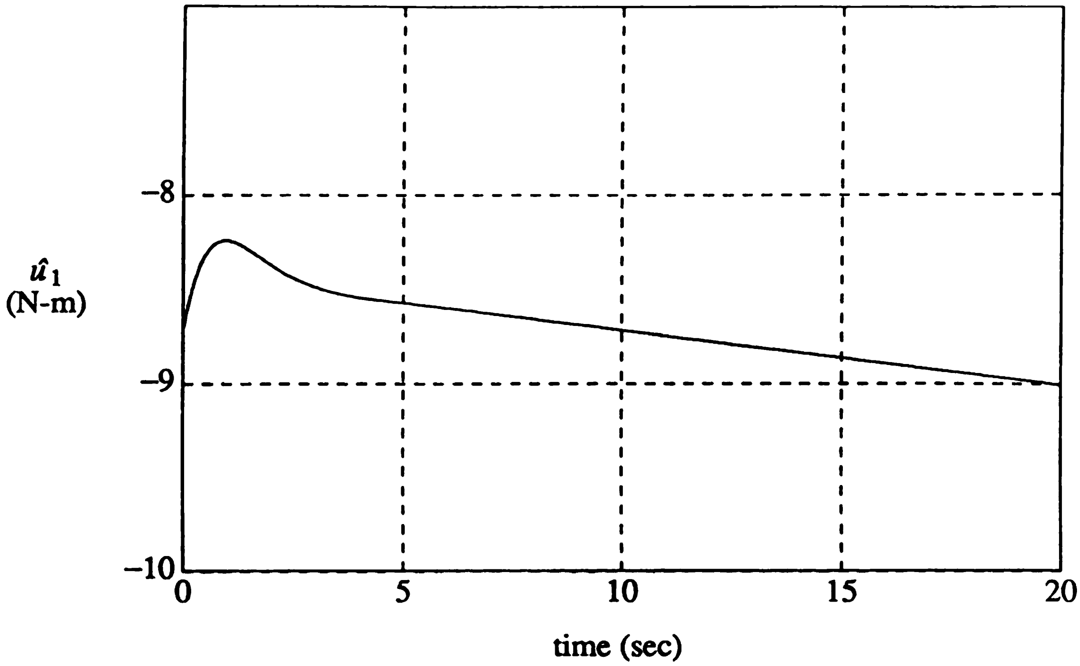

Figure 8.21. Plot of the force control $\hat{u}_1$ versus time, Example 2 (the overall nonlinear control)

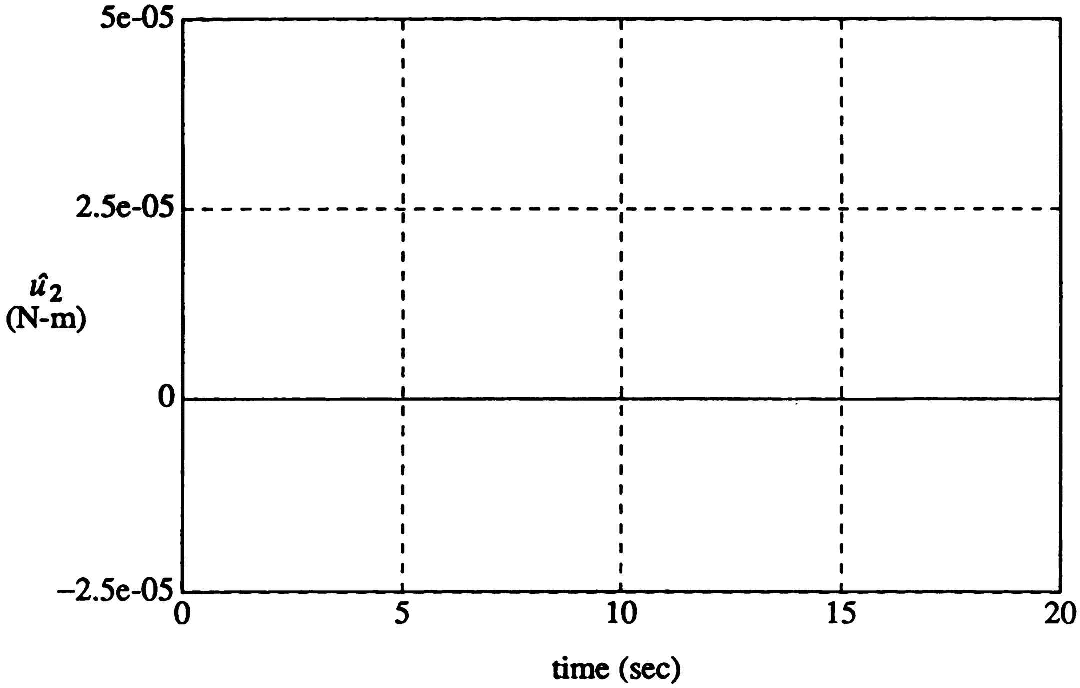

Figure 8.22. Plot of the force control $\hat{u}_2$ versus time, Example 2 (the overall nonlinear control)

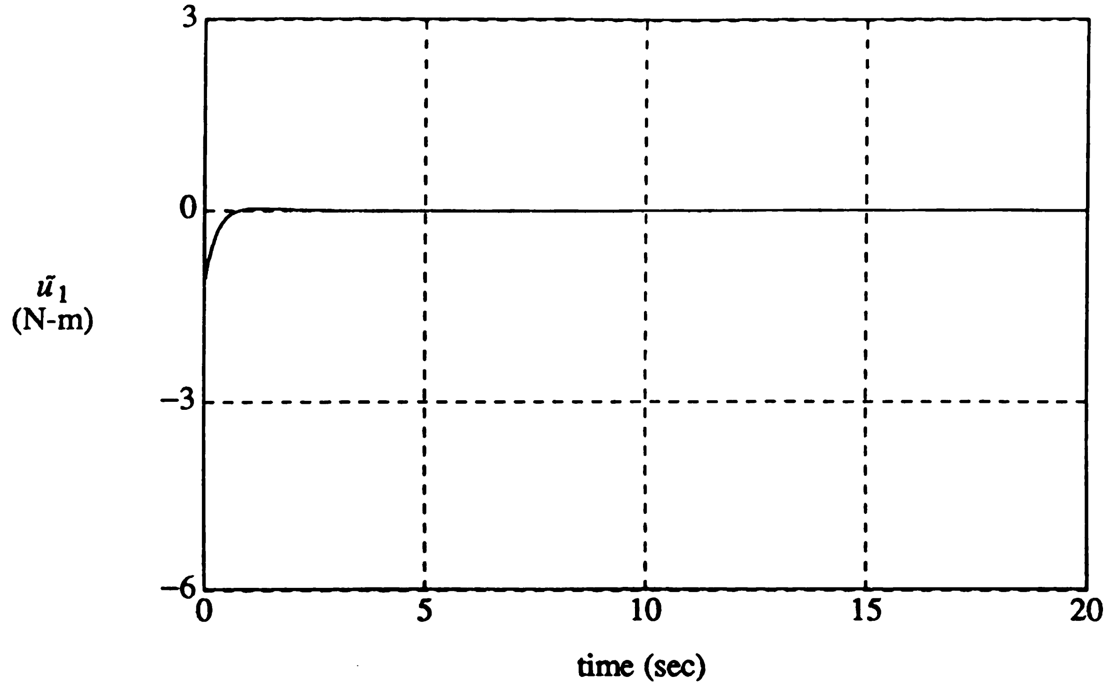

Figure 8.23. Plot of the motion control $\tilde{u}_1$ versus time, Example 2 (the overall nonlinear control)

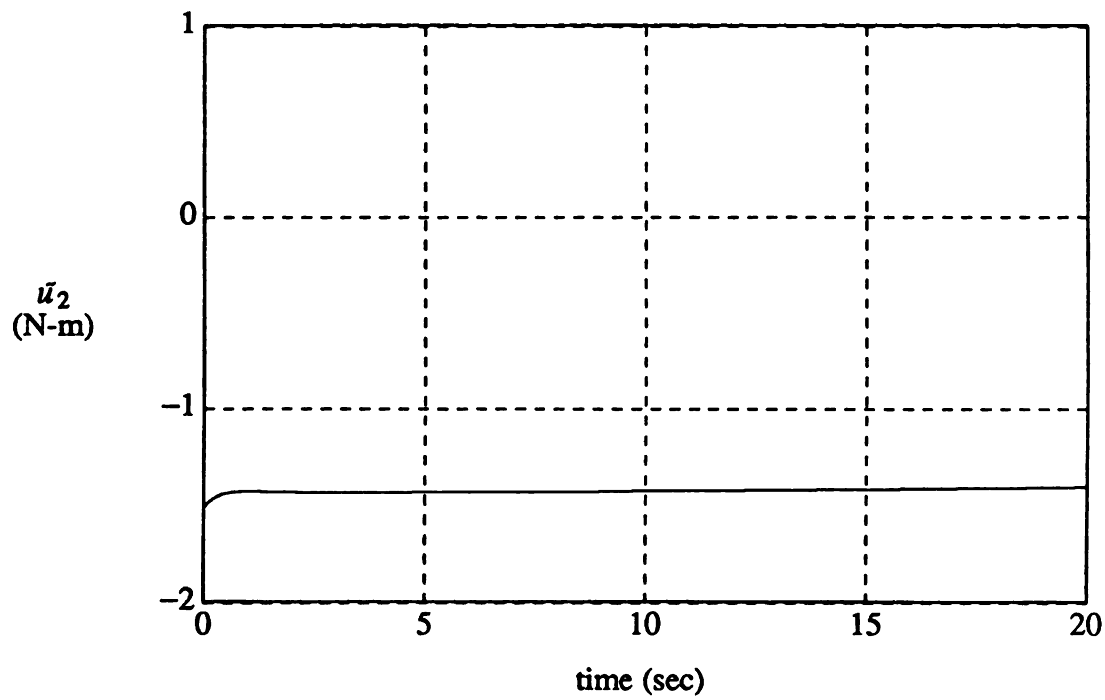

Figure 8.24. Plot of the motion control $\tilde{u}_2$ versus time, Example 2 (the overall nonlinear control)

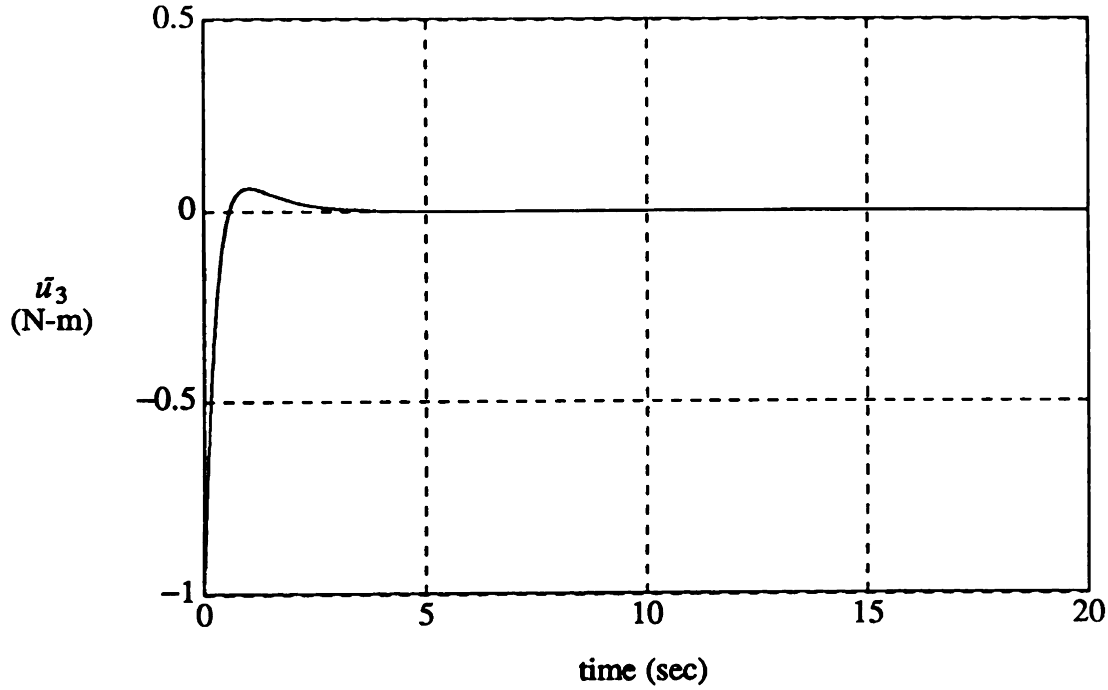

Figure 8.25. Plot of the motion control $\tilde{u}_3$ versus time, Example 2 (the overall nonlinear control)

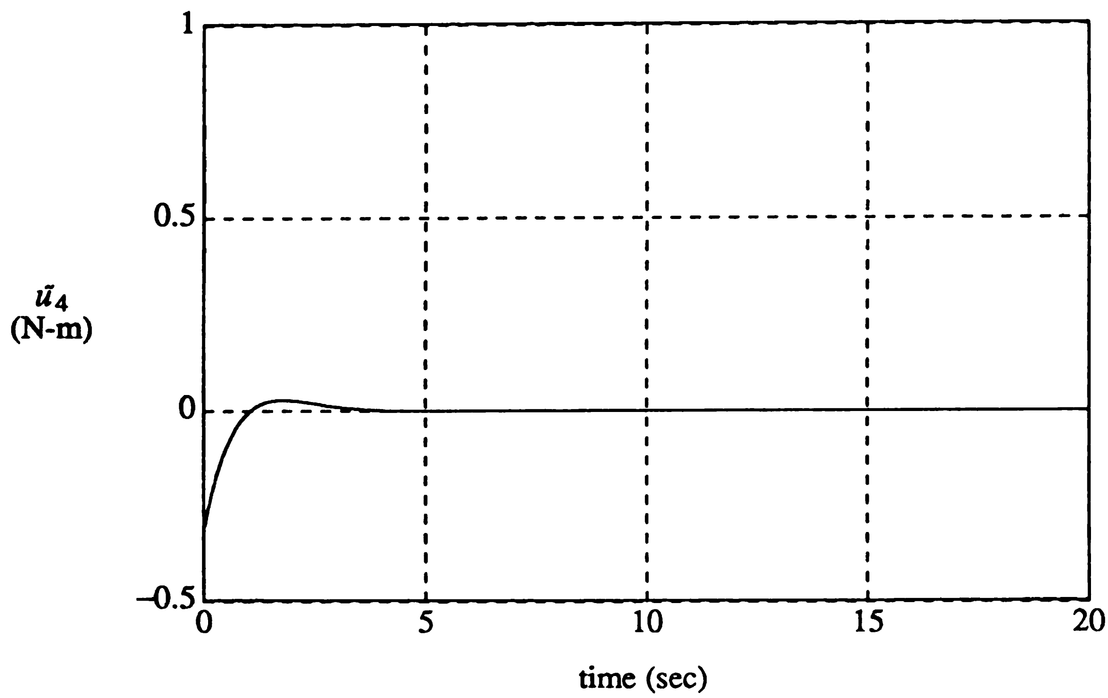

Figure 8.26. Plot of the motion control $\tilde{u}_4$ versus time, Example 2 (the overall nonlinear control)

CHAPTER 9

SUMMARY, DISCUSSION AND CONCLUSIONS

The geometric tools of symplectic Hamiltonian systems are used to develop a compliant control of constrained robot manipulators. To formulate the compliant control of constrained robot manipulators, we first analyze the geometric characteristics of singular differential equations which represent the governing system of dynamic equations with algebraic constraints. The analysis employs a constrained symplectic Hamiltonian systems.

Based on the analysis, we propose a control strategy which consists of the sum of two nonlinear controls, i.e., the force control part and the position control part. By these two nonlinear controls, the desired force and the desired position trajectories can be realized simultaneously, that is, the force control restricts (the end effector of) the manipulator to the constraint surface and at the same time the position control steers (the end effector of) the manipulator along a specified path on the constraint surface. Such controls are possible because the vector field is decomposed into a normal and a tangential components with respect to the constraints surface.

Consequently, we are able to control the normal component of the vector field at every point on the constraint surface and thereby solely modify the force effect. Analogously, we are able to control the tangential component of the vector field at the same point on the constraint surface and solely modify the position or motion of the manipulator (constrained to the constraint surface). The conditions resulting from our formulation in fact specify the class of constraint surfaces allowable. The derived conditions on the constraint surfaces are given in term of a matrix of Poisson brackets being nonsingular.

The effort in the formulation phase focuses on guidance and deep insights into how to properly devise and apply the control strategies even if this formulation does not include error signals of a desired position, velocity or force in order to counteract the effect of unmodeled dynamics and disturbances. As a final phase, we applied a linear controller to the resulting model to achieve attractivity of the constraint surface and to take into account the effect of disturbances and unmodeled dynamics.

The force of contact is a function of the state of the system. However, it may appear to qualitatively represent the dynamics of the contact force near the constraint surface.

One approach to modeling the force may be to modify the algebraic constraint equations to convert them to singularly perturbed differential equations. These equations act as a model for the dynamics of the contact force for when the end effector is near or onto the constraint surface to account for the chattering behavior frequently observed in applications. The basic feature of the model is that the rate of change of the amplitude of the force at some point, normalized with respect to the force amplitude itself, is (negatively) proportional to the algebraic constraint evaluated at that point ($\varepsilon \dot{f} = -\Phi$ where ε is small number). The physical meaning of this model is explained as follows. The rate of change of the contact force is related to the constraint function. The end effector is acted upon by the input torques to exert force on the surface. In response, the constraint surface generates a reaction force. The rate of change of the force equals zero, thus having a constant force value, if the end effector is positioned on the surface; it becomes positive, thus leading to an increase in the force value, if the end effector were positioned beneath the surface; and it becomes negative, thus leading to a decrease of the value of the force, were the end effector positioned above the surface.

Improved models should be derived for the force near or in contact with the surface based on phenomena arising between the end-effector of manipulator and its environment, and it may also include the dynamics of the material of the constraint surface itself.

After investigating the modeling problem, one may apply a controller, such as a singular perturbed controller, which characterizes a smooth approach from and off the constraint surface at the contact point. Then the manipulator may follow the desired path on the constraint surface with the desired force trajectories without changing controllers on the constraint surface.

Compliant control of constrained robot manipulators is well behind vision in both theory and level of applications. So more effort is needed to identify and solve basic theoretical problems that take into account the dynamic equations of the constrained robot manipulator and the constraint equations.

On some of these points, this dissertation provides some insight into the dynamic behavior of constrained robot manipulators as they relate to their environment, and also it provides guidance onto how to simultaneously realize the desired position and force trajectories onto a given constraint surface.

To be stabilized on the constraint surface, we have discussed the addition of a linear controller to example models. This stabilization is valid only on the constraint surface. Thus, there are other challenging issues, such as the development of more sophisticated algorithms capable of stabilizing the constrained system near and onto the constraint surface. Such algorithms may include the dynamics generated due to the material of the constraint surface itself.

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