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A PROBABILISTIC CHOICE MODEL FOR ANALYZING
THE DEMAND FOR FOOD IN SENEGAL

presented by

Aliou Diagne

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Eric W. Crawford
Major professor

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A PROBABILISTIC CHOICE MODEL FOR ANALYZING THE DEMAND FOR FOOD IN SENEGAL

By

Aliou Diagne

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ABSTRACT

A PROBABILISTIC CHOICE MODEL FOR ANALYZING

THE DEMAND FOR FOOD IN SENEGAL

By

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This paper develops a structural model of household food consumption in Senegal. The model is based on the assumption that the household does not maximize the utility of the raw food staples but instead maximizes the utility of the dishes derived by means of some technological transformation of these raw food staples.

The household maximization problem is solved to show that both the unconditional indirect utility and expenditure functions depend on the relative prices of the raw foods <u>only</u> through the costs of the dishes. Methods of estimation are discussed in detail, and the asymptotic distributions of the estimators are derived. The traditional model of food demand is shown to be a special case of this model, corresponding to the restriction of no dish choice effects. Means of testing for this restriction are provided. Finally, the elasticities of demand are derived and the policy implications of the model are discussed.

To My Parents, Brothers and Sister.

For their continuous, moral and material supports.

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Ousmane for

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CHAPTER 1

INTRODUCTION

Senegal's agricultural and food policies have been the focus of major studies in recent years. These studies have been motivated by the bad performance of Senegal's agricultural sector since the early seventies, and its increasing dependence on food imports to satisfy the consumption needs of its population.

A series of severe droughts, a high population growth rate, and inappropriate agricultural policies have been identified as the major causes of Senegal's chronic food deficits. But the agricultural sector plays an important role in the Senegalese economy, and its overall performance along with the level of cereal imports has a tremendous impact on the balance of payments and on government revenues, most efforts to solve the crisis have been directed toward designing adequate agriculture and food policies.

In short, the government is very concerned about having a agricultural policy that can:

1) Increase farm income.
2) Insure a high level of food self-sufficiency for the country.
3) Generate revenues for the government and contribute to reducing the balance of payments deficit.

So far, the policy followed by the government to achieve these goals has been to change the relative prices between locally produced cereals and imported ones - especially between millet/sorghum and imported

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imported rice - and to provide general producer price incentives to boost the production of export crops - mainly peanuts and cotton. However, food crops and export crops tend to compete for land and scarce resources. Thus there is an apparent conflict between food self sufficiency and increase in export earnings.

Many studies have been carried out to analyze the ways Senegal can achieve Increased food self-sufficiency. However, most of the studies have focused on the problems constraining the agricultural production, mainly because of lack of adequate data on food consumption and/or the urgent need to improve the living conditions of the rural people which constitute around 70% of Senegal's 6.5 million people.

Many studies emphasizes the infeasibilty (and economic costs) of the food self-sufficiency goal. Indeed, given Senegal's present and potential resource endowments, along with the consumption habits of its population, this goal is not achievable unless a miracle happens (see, for example, Martin, 1988). Thus, the concept of food security is the relevant one for Senegal.

One study that attempted to deal with food consumption is a World Bank policy study conducted in 1983 (Braverman et al. 1983). This study tried to link the supply side of the agricultural sector to the demand of food, by using a multimarket model based on a farm household model. Despite the poor data, which affected the reliability of the estimated structural parameters, the model gave some insights into policy outcomes (income changes, production changes, and sizes of the deficits) under different scenarios of producer and consumer relative prices for the major crops and food staples. However, this model (in our opinion), is weakened

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weakened by certain misspecifications of the nature of food demand in Senegal.

This paper is an attempt to contribute to the understanding of the consumption pattern underlying food demand in Senegal. Specifically, the paper develops a structural model of household food consumption in Senegal based on the assumption that the household does not maximize the utility of the raw food staples but instead maximizes the utility of the dishes derived by means of some technological transformation of these raw food staples. Previous studies of food demand in Senegal have focussed only on substitutability between cereals and particularly on the degree of substitutability between rice and millet (Ross, 1980a and 1980b; Josserand and Ross,1982). This model will incorporate other food items that are complements of cereals and that are important for the household when deciding which cereal to consume. But equally important, the model will incorporate information concerning how the nature of the different dishes consumed by the average Senegalese affects the degree of substitutability of the different cereals.

Furthermore, within the household production model this food consumption model is shown to be a structural model whose reduced form corresponds to the traditional system of food demand equations but depends explicitly on the household's tastes, consumption technology and habits. A set of estimable elasticities including the traditional ones can be derived from both the structural model and its reduced form. These elasticities have policy implications that depart from the traditional food policy so far followed by the government, which is

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based primarily on the manipulation of the relative prices of the different cereals.

The paper has six chapters including this introduction.

Chapter 2 discusses the socio-and microeconomic consumption behavior of the average Senegalese household. The nature of the consumption technology which guides the household strategies for coping with shortages and/or increases in price of certain basic food staples, is analyzed in detail. This discussion serves as a background and justification for the food consumption model analyzed in subsequent chapters. It also discusses the consequences of the separability assumption in the household's food consumption choices.

Chapter 3 reviews the household production model that will be our economic model for analyzing food demand in Senegal. It reviews the main economic type results of this model which are relevant for our food consumption model. It also discusses the major limitations for applying it to our case.

Chapter 4 presents and develops the mathematics of the probabilistic choice model (PCM) which will be used to estimate our model. A version of the fundamental axiom of the PCM is used to derive a utility function from an underlying preference ordering on the choice set (set of dishes). Duality theory is then used to derive the conditional and unconditional Marshallian demand functions. The restrictions implied by these demand functions are also investigated. One statistical consequence of viewing the household as choosing primarily among dishes rather than among raw foods is the presence of we call dish selection bias which introduces some biases on the coefficients estimates of the demand equations. It is also argued that the household's dish selection is the main reason why there are such a large number of "zero expenditures reported" usually found in food consumption surveys. Finally, the AIDS cost function (Deaton and Muellbauer, 1980) is used to present explicitly the model to be estimated.

Chapter 5 is concerned with methods of estimation. First, the normal distribution is used to derive explicit expressions of the conditional moments that correct for the selection bias resulting from the household's dish selection. Then maximum likelihood estimation and Heckman's two-stage method are discussed. The asymptotic properties of the proposed estimators are also analyzed.

Chapter 6 contains final remarks about the model, and ways of deriving elasticities of demand for the different raw foods used in policy analysis. Some measures of changes in household's tastes that can be used to evaluate implemented food policies are proposed. The chapter also indicates possible ways of extending the model to capture taste variations both across time and households, and help design future food policies.

CHAPTER 2

THE DETERMINANTS OF THE DEMAND FOR CEREALS IN SENEGAL

2.1 The Socio-Economics of Food Consumption in Senegal.

The feasibility of a food policy which consists of forcing urban dwellers to change their food consumption habits by setting imported food prices very high, has two major limiting factors. The first one is political. The government was forced to decrease in May 1988 the prices of the basic food staples in order to ease the social and political tensions that followed the February 1988 general elections, during which food prices were the popular rallying point for the opposition. The second limiting factor comes from the possibility for people to smuggle part of their needed supply of food from Gambia where prices are much lower. Indeed, it was estimated that at least 85,000 tons of rice (about 25% of yearly rice imports) was smuggled into Senegal in 1987 when prices were at CFA. 160 (see, for example, N'doye, Ouedraogo, and Goetz 1989 or Lambert and Diouf, 1987)[1].

But, more importantly, this policy may not be effective in inducing urban consumers to switch from imported cereals to locally produced ones because of cultural practices and also because cereals are consumed along with other complements (fish, meat, oil, vegetables, etc...) which are important for the household in deciding which cereals

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[1] N'doye, Ouedraogo, and Goetz (1989) estimated that the price differential between the smuggled rice and the official rice (of same quality) was up to 20% of the official price in some of the markets surveyed.

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to consume. The examination of the "food consumption matrix" constructed in Figure 2.1 is a first step toward understanding why prices of these complements are important to consider when evaluating consumer responses to changes in the relative prices between cereals. The first column of the figure shows the major dishes consumed by a Senegalese household. In the top row of the figure are presented the major basic ingredients used for the preparation of these dishes. The figure is read like a linear programming table with the difference that in place of the usual requirement coefficients we put signs to show the degree of substitutability of the food staple in the preparation of the corresponding dish. Had the transformation of the ingredients into dishes been linear for all dishes, the table would have been a true linear programming table. In general, for any given dish the degree of substitutability between two staples is measured by Allen's partial elasticity of substitution defined in the same way as in the substitutability between inputs in production theory. The dishes in the middle of the first column of Figure 2.1, couscous 1,2, and "lax" are historically the major dishes consumed in Senegal up to the early fifties when imported rice from the French colony Indochina, began to be preponderant in the urban diet. These dishes are still preponderant in rural Senegal (except maybe in Casamance).

FIGURE 2.1 : FOOD CONSUMPTION MATRIX

Food Staples →	Rice	Millet	Wheat	Fish	Meat	Green Salad	Other Vege-	Smoked and	Vege- table	Curdled Milk	Sugar

FIGURE 2.1 : FOOD CONSUMPTION MATRIX

Food Staples → Dishes ↓	Rice	Millet	Wheat	Fish	Meat	Green Salad	Other Vege-tables	Smoked and Dried Fish	Vege-table Oil	Curdled Milk	Sugar
"Cebbu Jen" * (Rice and Fish)	⊕			⊕			⊕	⊙	⊕		
"Niari Cinn" * (Rice and Sauce)	⊕			±	±		+	⊙	⊕		
Other Rice based dishes **	⊕			±	±		±	± ⊙	±	±	±
Couscous 1 **		⊕			⊕		+		⊕		
Couscous 2 **		⊕		±	±		+ ⊙	+	⊕		
"Lax" ▪ (Porridge)		⊕								⊕	⊕
Green Salad **			+	±	±	⊕	+		⊕		
Other Fish or ** Meat based dshs.			±	±	±		+		⊕		
Coffee, Milk and Bread ***			⊕								⊕

Legend:
⊕ stands for necessary complement
+ stands for secondary complement
± stands for substitutable complement
⊙ stands for weak complementarity

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** consumed in the evening
*** consumed in the morning
▪ consumed at midday, evening or morning

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But rural people are slowly adopting the urban style of consumption. The growing importance of rice in rural areas is usually explained by its relatively low price and easiness to prepare compared to millet which is time consuming and difficult to prepare. However, there two other factors at least equally important in influencing this shift toward rice in rural areas. The first one is the legitimate aspiration of rural people to diversify their diet. This fact probably explains one of the findings of the University of Michigan rural consumption study in Senegal in 1982. In this study, H. Josserand (1982), reported that in one village which was deficient in millet because of the drought, people were travelling far to other villages to buy millet while they could have easily bought rice in their same village. The other factor is the increasing availability of fish in rural markets. Indeed, in its preliminary survey of the marketing of fish in the interior regions of Senegal, C.R.O.D.T reported that the marketing of fish has been expanding at a steady rate (both in space and in time) since the early sixties; and before that time almost no fish could be found in rural markets. Some of the traders interviewed still remember the arrival of the first lot of fish in their market (Kebe et al. 1983). Since fish is the major complement of rice, one can easily understand why rice consumption is increasing in these areas. With respect to Senegalese agricultural and food policies, these factors point out the need to know to what extent increases in rural income (through agricultural price increases) will affect rice consumption.

One of the striking facts in the table is that among the major dishes, only one-third are based on rice. None of the other dishes use any rice.

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any rice. One might ask then, why rice is so important in the Senegalese diet? Part of the answer is that these two rice-based dishes are consumed every other day at midday in urban areas. This fact was confirmed by two consumption surveys in Dakar, one done by C. Ross (1978), and the other done by Abt. Inc. (1984), In both surveys 99% of the samples declared eating exclusively rice at midday.

Right now, the most common dish consumed in Senegal is "cebbu jenn". This dish is almost exclusively consumed at midday. As a source of protein, fish is an important complement of rice for this dish. Hence, its availability and its price are very important parameters in the decision of the average household to consume rice, and how much rice to use in a given dish. In other words, for a given dish of "cebbu jenn", the ratio between rice and fish depends to a large extent on the availability and the price of the latter.

This ratio is determined as follows: when fresh fish is not available or its price is high, the household generally has two other alternatives. Either it can buy a small quantity and/or a low quality of fish and use more rice for the midday dish to make up the caloric deficiency, or it can buy dried and smoked fish and use much more rice and vegetables. Given the high price of meat, this strategy is usually the one adopted by the household to deal with the fish shortage, although some high income households may substitute meat for fish.

Couscous is exclusively consumed in the evening; and meat along with fish - to a lesser extent - is an important complement of millet in the preparation of this dish. Thus, the decision of the household to prepare couscous depends primarily on the price of meat. For some poor househo

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households, or in some rural areas where meat is scarce, dried and/or smoked fish are substituted for meat.

So, even if the Senegalese household prefers to consume couscous in the evening, the high price of meat will prevent it from doing so. Instead, it tends to substitute a poor quality rice based dish for couscous. Meat (or fish) is also a complement to green salad and potatoes in another vegetable-based dish consumed in the evening. This relatively meat-intensive or fish-intensive dish is usually out of reach for the poor.

In any case, a rice-based dish in the evening is generally considered as an inferior alternative by the average household. But, faced with expensive substitute dishes, it tends to turn to cheap and very rice-intensive dishes for dinner.

Another factor that increases Senegalese consumption of rice - which has a kind of income effect - comes from the cultural practice that gives more importance to the midday meal compared to the evening meal, so that the daily budget share of this meal is very high. Hence, with a perfect inelastic demand for this rice-based meal, an increase in the price of rice and/or fish will merely erode the budget share of the evening meal. Then, with not enough left for the evening meal, the Senegalese household tends to consume a low cost rice-intensive dish instead of couscous, green salad, or other vegetable based-dishes which are far preferred for dinner.

This role of rice as preferred dish at midday and security dish in the evening, is so important for the urban household that the first food staple secured for a month of consumption is rice, bought in bags of 100

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kilograms. The strategy is to have enough rice to face all possible situations. This systematic behavior was confirmed by the consumption surveys cited earlier. In these surveys, only the very poor buy rice daily because their income does not allow them to buy the whole monthly rice requirement at once.

Another traditional dish that is losing its place in the Senegalese diet is "lax". This millet-intensive dish, prepared with curdled milk and sugar, used to be consumed (especially in the rural areas) at midday and for breakfast. Now, because of the high price of sugar and the scarcity of curdled milk since the drought, which decimated the livestock population in the mid seventies, this dish has been replaced by rice at midday, and by coffee and bread in the morning.

This brief, but relatively detailed discussion of the consumption behavior of the Senegalese household can help understand - at least partly - why rice imports have doubled between 1978 and 1987, despite a doubling of its retail price. This discussion also suggests some insight on why previous food demand studies in Senegal, which analyzed the degree of substitutability between rice and millet by using information on these two food staples only (thus ignoring the technical and taste constraints), embodied an incomplete food consumption model. The same criticisms apply to the policy makers' approach that views food consumption in Senegal simply as a problem of the relative price between rice and millet.

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2.2 Separability in the Food Consumption Choices

For modelling purposes, the starting point is to recognize that cereals are always consumed with other complementary food staples. The complete list of these complementary food staples may be relatively long, especially in urban areas. Even in some rural areas, the list is quite impressive, as reported by the 1982 rural consumption survey of the University of Michigan (Josserand and Ross, 1982).

Table 2.2 and Table 2.3, reproduced from the cited study, give the share of income spent on food and the amount spent on major food items in the three villages surveyed, respectively. A full interpretation of these tables in relation to the food consumption matrix is beyond the scope of this paper, but one may notice some interesting facts about the tables: (1) the correlation between non-farm income and diversity of the food basket; (2) the relatively high rice consumption in Thienthie - almost three times higher than in the other villages - which certainly cannot be explained only by its low millet harvest. Indeed the absence of fresh fish and meat in the diet points to the likely presence of substitution and income effects of the types described in Section 2.1. That is, the unavailability of fresh fish (and vegetables) is compensated for by a high ratio of rice to smoked fish in the midday dish, and the absence of meat leads to the replacement of couscous by low cost rice-intensive dish in the evening. Note also that the absence of milk (curdled, powdered or fresh) would rule out the consumption of the "lax" dish in Thienthie. Thus, in relation to our food consumption matrix, one can infer from Table 2.3 that the 27 households surveyed in Thienthie were consuming at that time almost exclusively the third,

TABLE 2.2

Percentage of money income spent on food.

MAY - AUGUST 1981.

(CFA Francs)

	FARM INCOME [a]	NON-FARM INCOME [a]	TOTAL INCOME [a]	FOOD PURCHASES [b]	PERCENTAGE OF INCOME SPENT ON FOOD
LAYABE	178,755	605,502	784,257	111,640	14%
SESSENE	679,245	194,237	873,482	61,271	7%
THENTHIE	237,379	109,440	346,819	149,230	43%

Notes: (a) Averaged over 3 months from year's total.
(b) Recorded May 15 - August 15, 1981.
No. of Households: Layabe, 24; Sessene, 24; Thienthie, 27.
Source: Josserand, P. and C.G. Ross. (1982).

LAYA
Millet
Rice
Cowpeas
Oil
Sugar
Coffee
Salt
Beef
Smoked
Bread
Chicken
Tea
Tomato
Maggi C
Onions
Fresh T
Fresh F
Diar
Hot Pep
Fresh M
Cabbage
Lalo
Guinea
Black P
Goatmea
Eggs
Dried F
Corn
Powder

Source:

TABLE 2.3

Amount Spent on Major Food Items,
May 15 - August 15, 1981, by Village (CFA Francs).

LAYABE		SESSENE		THIENTHIE	
Millet	17,400	Salt	18,300	Millet	57,190
Rice	16,535	Sugar	14,963	Rice	46,585
Cowpeas	12,545	Rice	5,030	Sugar	13,865
Oil	12,140	Smoked Fish	4,247	Oil	11,090
Sugar	11,508	Onions	3,789	Salt	10,660
Coffee	10,320	Hot Pepper	2,747	Smoked Fish	1,690
Salt	6,250	Beef	2,200	Fresh Tomatoes	1,395
Beef	2,870	Bread	1,925	Coffee	750
Smoked Fish	2,800	Oil	1,530	Tomato Paste	630
Bread	2,225	Fresh Fish	1,425	Onions	555
Chicken	2,100	Powder milk	1,200	Hot Pepper	295
Tea	1,900	Tomato Paste	1,145	Maggi Cubes	180
Tomato Paste	1,850	Dried Fish	1,090	Dried Fish	120
Maggi Cubes	1,415	Tea	800	Diar	10
Onions	1,395	Cabbage	470		
Fresh Tomatoes	1,165	Cowpeas	250		
Fresh Fish	1,126	Maggi Cubes	80		
Diar	980	Black Pepper	25		
Hot Pepper	785	Fresh Tomatoes	25		
Fresh Milk	540				
Cabbagè	475				
Lalo	450				
Guinea Sorrel	400				
Black Pepper	395				
Goatmeat	200				
Eggs	200				
Dried Fish	200				
Corn	170				
Powder Milk	125				

Source: Josserand, P. and C.G. Ross. (1982).

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fifth and last dishes in Table 2.1. Finally, one may hypothesize that the Layabe food basket is the closest to the urban household food basket because it has the highest non-farm income.

Since the fact that cereals are always consumed with other food complements cannot be ignored, some separability between these two groups of food must have been implicitly assumed in the previous studies which investigated the degree of substitutability between rice and millet, as well as by the policy makers who base their food policies - whose goal is to alter the food consumption behavior of the Senegalese household - almost exclusively on the relative price of rice and millet. Indeed, this methodology amounts to saying that the household's marginal rate of substitution between rice and millet (which is equal at internal equilibrium to the relative price of the two goods faced by all households) is independent of all other food staples. But the Leontief separation theorem (Deaton and Muellbauer, 1983; p. 136) tells us that this condition is equivalent to weak separability.

For a full discussion of the consequences of separability, we need a formal definition of the different types of separability. However, we will restrict ourselves to the three types commonly used: Hicksian separability, weak separability, and strong or additive separability. The consequences of the other types of separability (implicit separability, indirect separability, etc.) are basically the same (Varian, 1984; Deaton and Muellbauer, 1983).

The Hicksian separability or composite commodity theorem asserts that if we divide the household commodity bundle into two groups (cereals and other food complements in our case) such that the prices in one of th

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one of the groups (food complements) move together proportionally with the group price index t, then the demand for commodities (cereals) in the first group can be thought of as depending on only the individual cereal prices, the group price index t, and total expenditure or income. In our particular case, the food complements include staples such as fish, meat, vegetables etc., whose prices fluctuate daily and/or seasonally with very different time paths. Thus no group price index satisfying this theorem can be found. Even if we could find one, we would still need this group price index when assessing the substitutability between cereals. Hence, this separability by the price vector does not justify the neglect of the food complements.

The other type of separability which includes weak and strong separability as particular cases, is functional separability: This separability concerns the underlying preference ordering on the space of commodity bundles. It means that preferences over one subset of the space of commodity bundles (cereals in our case) are independent of the other commodity bundles (food complements) in the space. If this so called weak separability is true then, assuming local non satiation, the household's overall food utility function can be shown to be an increasing function of a sub-utility function of the cereals and of the level of consumption of the other food complements. That is we can write the overall utility as $U(V(X),Z)$ where $U(v,Z)$ is an increasing function of v, U is the overall utility function, V the sub-utility function of the cereals, and X and Z are the cereals and food complements vectors, respectively. The demand for cereals will be given then by maximization of the sub-utility function subject to the budget one of the groups (food complements) move together proportionally with the group price index t, then the demand for commodities (cereals) in the first group can be thought of as depending on only the individual cereal prices, the group price index t, and total expenditure or income. In our particular case, the food complements include staples such as fish, meat, vegetables etc., whose prices fluctuate daily and/or seasonally with very different time paths. Thus no group price index satisfying this theorem can be found. Even if we could find one, we would still need this group price index when assessing the substitutability between cereals. Hence, this separability by the price vector does not justify the neglect of the food complements.

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constraints. It will depend on the prices of cereals and on cereals expenditures. But cereals expenditures will depend in general on all prices and total income (Varian, 1984; p. 48). Thus, with weak separability alone we will need the prices of all food complements to estimate properly the demand for rice and millet.

However, if the utility function is homothetic then the food consumption decision of the household can be decomposed into two stages. In the first stage the household decides how much millet and rice to consume given the cereal price index. In the second stage it decides how much and which cereal to consume given their relative prices. But this two-stage budgeting is too restrictive since it implies that food complements altogether share the same type of relationship with respect to rice and millet(either substitute or complement). In other words, apart from income effects, all pairs of cereal/food complements are all together either substitutes or complements to the same degree. For instance, it would mean that the pairs of rice and fish, rice and meat, millet and fish, and millet and meat share the same type and degree of relationship. This is not certainly the case given the consumption technology of the Senegalese household. Even if this were true, the homotheticity condition imposes severe restrictions. It would imply that income elasticities of millet and rice are independent of the level of income and utility, and are equal to unity (Deaton and Muellbauer, 1984; pp. 142-167). This may not be empirically justified.

Finally, we come to the most restrictive (and most popular) type of separability: strong or additive separability. That is, in addition to weak separability, we assume that the overall utility of food is an

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additive function of the sub-utility of cereals and sub-utility of food complements. If so, then we may not need information on the food complements to estimate the price elasticities of millet and rice. Indeed all we need to determine the own and cross price elasticities of millet and rice are their respective expenditure elasticities and knowledge of one price elasticity (Deaton and Muellbauer, 1983). This seems the closest form of separability implicitly assumed in previous studies and by policy makers. However, this separability has at least three drastic theoretical consequences: (1) there can be no inferior good, (2) goods are only substitutes, never complements, and (3) in some cases own price elasticities are approximately equally proportional to expenditure elasticities. Empirically, in spite of its econometric convenience, all the tests so far reported have unanimously rejected the additivity assumption as "... too strong to be used in empirical work..." (Deaton and Muellbauer, 1983; p. 140). Furthermore, one logical consequence is that one cannot show substitutions between food staples by using this model.

This partial discussion of the consequences of separability shows the limits in generating degrees of freedom when estimating a food demand model. Thus for empirical work, an estimation of the (own, cross, and income) elasticities of millet and rice using data on only millet, rice, and disposable income, is not only inefficient, but will yield biased and inconsistent estimates - because of omitted variables belonging to the model. Also, the use of only relative price of millet and rice as a policy tool to assess household response to change in prices is misleading, at least on theoretical grounds.

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It is our opinion that the separability between food consumption and the consumption of other commodities (such as durable goods) and/or other production activities, especially in the case of a subsistence farmer (see Singh, Squire and Strauss; 1986), generally assumed for practical purposes, is a restriction strong enough to prevent us from putting further restrictions on the system of demand equations.

The inclusion of the major complementary food staples in the analysis of cereal demand in Senegal will certainly improve the reliability of the estimated elasticities. It implies no substantial additional costs in surveys like the one cited above.

The additional elasticities, though apparently of no great interest for policy makers, may potentially reveal unsuspected consumption linkages, and thus be worthy of attention by them. Furthermore, the ability to forecast the demand for these complementary food staples may be worthy of interest for private entrepreneurs involved in marketing these commodities.

From the foregoing discussion, it is clear that any separability assumption would imply making explicit or implicit assumptions about the existence of complementarities and substitutions among the raw foods. But the possibility of complementarity and substitution depends intrinsically on the way the household combines the raw food to prepare its dishes; that is upon the "consumption technology" of the household. This leads us to the household production model as a framework for analyzing the demand for food in Senegal.

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CHAPTER 3

FOOD CONSUMPTION WITHIN THE HOUSEHOLD PRODUCTION MODEL

We have seen in section 2.1 that the way the household combines the different ingredients puts important technical restrictions on the degree of substitutability between them. These technical restrictions which depend on tastes, are virtually independent of the relative prices of the food staples and thus are a valuable additional source of information if they are appropriately integrated into our food consumption model. This will improve the reliability of the estimates of the parameters.

This leads us to view the household as not maximizing the utility of the raw food as is assumed in the traditional model of food demand, but instead as maximizing the utility of the dishes obtained from a technological transformation of the same raw food, subject to technological and budget constraints. This view is not new since it is just an special case of the household production theory (see Gorman (1956); Morishima (1959); Becker (1965); Lancaster (1965); Muth (1966).) At this point, the utility maximizing behavior over the set of dishes is just a working hypothesis that must be theoretically justified. In the next chapter, we will see how such a utility function can be derived from a preference ordering describing the choice behavior of the household on the set of dishes in the same way the traditional utility function is derived from an underlying preference ordering over the set of basic

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The purpose of this brief review is to highlight the major (economic type) theoretical results from the household production theory, that we think are pertinent for modelling food consumption in Senegal. However, for practicality and reasons that will be clear later, our food consumption model will depart somewhat from this approach, while keeping the basic idea of viewing the household as maximizing the utility of its home produced nonmarketable goods obtained from a technological transformation of the inputs bought from the market.

According to this theory, to quote Pollak and Watcher (1975, p. 255) ". . . the household purchases "goods" on the market and combines them with time, in a "household production function" to produce "commodities." These commodities rather than the goods are the arguments of the household's utility function; market goods and time are not desired for their own sake, but only as inputs into the production of "commodities."

The approaches taken for modelling the basic idea of viewing the household as deriving utility from the home produced nonmarketable goods generally differ from author to author. The literature can be divided into two broad lines. One inspired by Becker (1965) emphasizes the role of time. The second line of work follows generally Lancaster's (1966) linear characteristic model.

In the first approach, the household is viewed as a production unit much like a firm (see Michael and Becker (1973) for a brief summary; F

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summary; Pollak and Watcher (1975) for a critique and further developments; and Barnett (1977) for econometric methods of estimation.)

An example of statement formulating the approach is as follows (Pollak and Watcher, 1975, p. 257):

> The household's preferences are represented by a utility function . . . defined over the commodity space. Goods are not desired for their own sake, but only because they are inputs for the production of commodities. In the household production model the household faces two types of constraints on its consumption opportunities: the budget constraint and the limitation imposed by its technology. Hence, the demand for goods and the demand for commodities both depend on goods prices, the household's income, its tastes, and its technology.

Formally, with respect to our particular case, the household food consumption problem can be formulated as maximizing the utility of dishes:

$$U(d_1, d_2, \ldots, d_k) \tag{3.1}$$

subject to the consumption technology constraint

$$d_i = f_i(x_{i1}, x_{i2}, \ldots, x_{in}) \qquad i = 1, 2, \ldots, k. \tag{3.2}$$

and the budget constraint

$$\sum_{i=1}^{k} C(i) \leq I \tag{3.3}$$

with

$$C(i) = \sum_{j=1}^{n} P_j \cdot x_{ij} + w\tau_i \tag{3.4}$$

where

U is the household utility function,

d_i, $i=1,\ldots,k$ are the set of k dishes consumed by the household, C(i) is t

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C(i) is the cost of dish i which includes the cost of raw food and the time preparation cost of the dish.

x_{ij} j=1,...,n. are the quantities of the n raw foods used in dish i,

P_j j=1,...,n. are the unit prices of the raw foods

r_i is the time preparation of dish i

w is the per unit cost of time .

In fact, this idea is quite old, dating back at least to A. Marshall (Michael and Becker, 1973). It was also suggested by J. R. Hicks in his "A Revision of Demand Theory" published in 1956 (Morishima, 1959). In this book, Hicks reportedly suggested that one should " . . . think of the consumer as choosing, according to his preferences between certain objectives, and then making decisions more or less as the entrepreneur decides, between alternative means of reaching those objectives."[2] The mathematical translation of Hick's "verbal theory" on that subject was the basis for Morishima's developments on intrinsic complementarity and separability of goods.

Early mathematical formulations and analysis of the second approach were done using linear programming methods (see, Stigler (1945); Gorman (1956); Morishima (1959)). These studies focused on analysis of the diet problem of the household, combining raw foods to

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[2]Quoted from M. Morishima, 1959; p. 1989.

get calories, protein, vitamins, etc. This model was later generalized (including all activities on the household) and extensively analyzed by Lancaster (1966) in his linear characteristic model. In the linear characteristic model, the consumption technology constraint is assumed to be of a linear type. The space of characteristics (which are the objects of choices in this model) is taken as a subspace of the Euclidean space.

More precisely, Lancaster assumes that "consumption is an activity in which goods, singly or in combination, are inputs and in which the output is a collection of characteristics. Utility or preference orderings are assumed to rank collection of characteristics and only to rank collections of good indirectly through the characteristics that they possess" (Lancaster 1966 p. 133). In summary, to quote Lancaster, there are three consequences (departing from the traditional consumer theory) that follows from this approach (Lancaster 1966 page 134).

1. The good, per se, does not give utility to the consumer; it possess characteristics, and these characteristics give rise to utility.

2. In general, a good will possess more than one characteristic, and many characteristics will be shared by more than one good.

3. Goods in combination may possess characteristics different from those pertaining to the goods separately

Similarities between these two approaches, could not have gone unnoticed. The following sentence quoted from Pollak and Watcher (1975 footnote 1) illustrates best these similarities.

"In Lancaster's model, good possess "characteristics;" these characteristics, which we can identify with Becker's commodities, are arguments of the household's utility function. Each unit of a good, for example, a "glass of

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orange juice" produces a vector of characteristics such as calories and vitamin C."

A complete and separate review of these two approaches is beyond the scope of this paper. Thus we will just mention some of the results from the linear characteristic model that are relevant to the household food consumption behavior that we are trying to model. (For details, see Lancaster's original paper or Lipsey and Rosenbluth). Since the essence of the two approaches are the same, it is possible to derive most of the results summarized below, by following Becker's approach.

In the linear characteristic framework, the household choice problem can be decomposed into two parts as in the case of a profit maximizing firm: (1) A private choice which consists of choosing the desired vector of characteristics, and (2) an efficiency choice which consists of finding the least cost combination of goods that yields the chosen vector of characteristics. The consumption technology constraint which gives rise to the efficiency choice is virtually independent of the household preference ordering of characteristics, and implies additional technological relationships between goods which are completely ignored in the traditional framework of demand analysis. These technological relationships induce a switching effect between goods called the "efficiency substitution effect" (Lancaster, 1966; pp. 140-149). Some of the consequences of this efficiency substitution effect are:

(1) Small changes in relative prices of the good may leave the household's efficient consumption point unchanged, whereas changes that are large enough make it switch completely from one good to another.

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(2) With a one-to-one correspondence between goods and activities, the efficiency substitution effect will result in a complete switch from consumption of one good to consumption of another.

(3) If there is no one-to-one relationship between goods and activities, that is some goods are used in more than one activity, then the efficiency substitution effect will simply result in less consumption of a good whose price rises, but not a complete disappearance of that good from consumption.

This model reveals a substantial amount of variations in the quantity of goods demanded by the household which cannot be explained by relative price and income variation alone.

Perhaps, the distinguishing features of this model compared to the conventional consumer theory is better illustrated by quoting Lancaster's contrasting examples showing the conclusion that one would get depending on which theory you are using (Lancaster, 1966, p. 155).

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This Theory	Conventional Theory
Wood will not be a close substitute for bread, since characteristics are dissimilar	No reason except "tastes" why they should not be close substitutes.
A red Buick will be a close substitute for a gray Buick	No reason why they should be any closer substitutes than wood and bread
Substitution (for example, butter and margarine) is frequently intrinsic and objective, will be observed in many societies under many market conditions	No reason why close substitutes in one context should be close substitutes in another
A good may be displaced from the market by new goods or by price changes	No presumption that goods will be completely displaced
The labor choice may have a marked occupational pattern	Labor-leisure choice determined solely by individual preferences; no pattern, other than between individuals, would be predicted
(Gresham's Law) A monetary asset may cease to be on the efficiency frontier, and will disappear from the economy	No ex ante presumption that any good or asset will disappear from the economy
An individual is completely unaffected by price changes that leave unchanged the portion of the efficiency frontier on which his choice rests	An individual is affected by changes in all prices
Some commodity groups may be intrinsic, and universally so	No presumption that commodities forming a group (defined by a break in spectrum of cross-elasticities) in one context will form a group in another context

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In empirical demand analysis using the traditional framework of consumer behavior, much of the frequently high unexplained variations are attributed to an unspecified "taste" variation component. This treatment of "taste" is unsatisfactory within the household production model, since the model makes clear the differences between variations inherent to the consumption technology constraint that faces the household and variations due to the household's preference ordering of its objects of choice. However, as pointed out by Deaton and Muellbauer (1984, p. 244), when many of the variables in the model are not observable or hardly measurable, it may be impossible to separate in practice the two sources of variation. This is perhaps one of the limitations of the household production model. Often, the scope theoretically covered includes a wide range of "unusual" variables such as health and medical care (Grossman, 1972, 1976), "environmental variable," (Michael and Becker, 1973); "seeing of a play" (Becker, 1965) etc., whose definitions and treatments are not operationally satisfactory.

For instance, marginal or (infinitesimal) differential methods are widely used to derive equilibrium conditions while some of the variables identified by Becker are intrinsically discrete.

In general, even if the linearity of the consumption technology is dropped, the choice set is assumed to be a subspace of the Euclidean space. But in most cases such as our food consumption model, this assumption is neither realistic nor practical. Here, the household's set of choices is postulated to be primarily the set of dishes available to it, and a dish embodies more than the characteristics of the raw foods used in

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foods used in it (per unit content of calory, protein, vitamin, etc., and individual flavor, taste, etc.). Indeed, a dish has other attributes that are independent of these characteristics (example, amount of time, energy and labor required for cooking it, taste, digestibility etc.). In addition, the household demand for a particular dish, depends on its cost which includes the price of the raw foods used in it, the cost of time, energy and labor required for cooking it, the household vector of characteristics and other unobservable variables. But with respect to the actual choice of the household, all we can observe is which dish is consumed by the household, by the household. That is to say that this choice set is inherently discrete. Quantifying the dishes consumed in a continuous manner can hardly be done without using some weighted combination of the raw foods used in the dishes, a procedure which would not be satisfactory for us. From the ongoing discussion, it is clear that our formulation is different from Lancaster's formulation in that we do not assume that the household maximizes the utility of the characteristics of the raw foods, but rather the utility of the dishes themselves. In other words, we assume that the household choice set is the set of dishes, not Lancaster's characteristics space. That makes a difference since the characteristics space is generally continuous, while our choice set is discrete. But still the basic relationship between inputs and the object of choices which are supposed to yield utility is the same and that is this basic relationship that we are interested in.

The inherent discreteness of the choice set not only prevents us from using differential methods to solve our utility maximization problem, but it also brings new features to the problem. In this case, corner solutions are the rule rather than the exception. This makes the efficiency substitution effect discussed earlier more pertinent.

With respect to our food consumption model this has the following consequences (see Deaton and Muellbauer, 1984; p. 252). First, At the household level, the derived demands for the individual raw foods are likely to be highly discontinuous (unstable) since small changes in relative prices can bring about large changes in quantities demanded of certain raw foods; and conversely, large changes in relative prices may leave the quantities demanded unchanged. For instance, in the food consumption matrix of table 2.1, if the relative share cost of meat in couscous 1 is very high, a small change in the price of meat may lead the household to drop this dish from its diet, and this will cause a large drop in its demand for millet. In the extreme case where couscous 2 and "lax" are not available for various reasons, the household switch to rice-based dishes and cut completely its demand for millet. Conversely, if the relative share cost of rice is low in the rice-intensive dishes, then it may require a big change in the price of rice before the household switches to non rice-based dishes.

Second, still the household level, with non-homothetic indifference maps, as the household budget shrinks (or expands), new corner solutions will appear with changes in the shape of the indifference curves. This will then make inferior goods as well as of Giffen goods more common. For example, an increase in the price of rice will shrink the overall household daily food budget. Given the relative inelasticity of demand of the midday rice-based meal, for reasons explained in section 2.1, this will reduce more the budget share for the evening meal, which can lead the household to switch more often from the costly couscous 1 to rice-intensive dishes. In this instance, rice would clearly be a Giffen good. Conversely, a rise in the household food budget will make couscous 1 more affordable, thus potentially increasing the demand for this dish at the expense of the rice-intensive dish. In this case, rice would be an inferior good.

While economists recognize the possibility of necessity goods being inferior, they usually dismiss the case of Giffen goods as intuitively contrary to "common sense". However, this "common sense" cannot be justified on theoretical grounds since the negativity of the Slustky substitution matrix implied by the theory of demand applies only to the Hicksian demand functions, not to the Marshallian demand functions which are the ones estimated. Despite of this fact, in empirical demand estimation, the presence of positive own price elasticities is generally attributed to misspecification of the demand equation. But we have seen that within the household production model the existence of Giffen goods is theoretically justified. Moreover, using the linear characteristics model, Lipsey and Rosenbluth (1971, pp. 151-158) have shown that for some consumption technologies, Giffen goods and more likely inferior goods would be commonplace for some range of income. Furthermore, after reviewing thirty empirical studies on demand measurement published between 1960 and 1970, they concluded that ". . . the frequency with which positive price coefficients are found leads us to believe that the existing evidence does not support " the economist's "conv

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"conventional wisdom" on Giffen goods (Lipsey and Rosenbluth; 1971; p. 159).

With respect to our particular model, we have seen that the relative share of costs of the raw foods in the different dishes are important parameters determining the different elasticities of substitution. In chapter 4 we will see formally, how these share cost variables enter in the systems of demand equations. Also, what has not been explicit so far in our analysis is the time cost of dish preparation. This exogenous variable is important in determining household choices among dishes. This is especially the case for couscous whose preparation is known to be difficult and time consuming. In general, the allocation of time among production and consumption activities is an essential parameter in the household production model (Becker, 1965).

From this discussion of the household production model, we see the importance of the "consumption technology" constraint in determining the household's response to changes in relative prices of the food staples. Thus, the lack of response by subsistence farmers to producer price incentives (a argument often advanced by some researchers), may have its analog with the food consumption behavior of the household. In this case, policy tools that tend to alter these consumption technology constraints, might be more effective than simple distortion of relative prices.

It is easily seen that the relationship between this food consumption model, given by equations (3.1), (3.2), and (3.3), and the traditional model of consumer demand is the one linking a structural model

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model to its reduced form. Since the dishes are (nonlinear) functions of the vector of raw foods, maximizing the utility of the dishes amounts to indirectly maximizing the utility of the raw foods subject to the budget constraint (3.4). Hence, solving indirectly the maximization problem might eventually yield the traditional system of demand functions for the food staples which can be adequately estimated by econometrically efficient methods, provided there is exact identification between the structural model and its reduced form. But, because the functions in (3.2) are generally highly nonlinear, the substitution leading to the reduced form is generally not feasible. Even so, the exact identification will be unlikely. In fact, overidentification is the normal case (Barnett, 1977), and it is well known that in cases of overidentification the estimation of the structural model is statistically more efficient. Even for forecasting purposes, where the inherently less informative reduced form is often enough, the reduced form forecasts may be more easily generated from the structural model, (see Barnett, 1977; footnotes 8, 13, 14 and 21 for a more complete discussion of the advantages in estimating the structural model directly).

In summary, the superiority of the structural model comes from its ability to separate the three sources of variation in observed household behavior: those due to changes in income and relative prices, those due to changes in the consumption technology, and those due to changes in preferences or tastes.

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structural maximization problem (given by equations (3.1) - (3.4)) which would yield a system of demand equations on the dishes (see, for example, Pollak and Watcher, 1975, for details on the additional assumptions needed):

$$(3.5) \qquad d_i = g_i(C, I) \qquad i=1,\ldots,k.$$

where $C = (C(1),\ldots,C(k))$ is the vector of costs of the dishes and I is taken here as income to simplify but in general a vector of household characteristics.

This system of demand equations would satisfy all the restrictions yielded by the neoclassical theory of demand and production: homogeneity, symmetry, non-negativity of the Slustky or substitution matrix, etc. In turn, these restrictions imply restrictions upon the response of the demands to variation in taste and technology (Barnett,1977).

To estimate the system of demands in (3.5), one would specify an error process and proceed to estimate the conditional means $E(d_i|C,I)$ and the conditional variances $var(d_i|C,I)$ which are sufficient for the econometric estimation of the system of demands in (3.5). This supposes of course that we can measure adequately the dishes d_i.

However, with the discreteness of our choice set this direct approach is not possible. But since in econometrics all we do is to estimate the first two moments of the conditional distribution of the endogenous variables[3], which are in our case the actual observable dish choices made by the household, the structural model is still estimable, although the testable neoclassical restrictions are not anymore immediate. If we recognize that the actual demand for a dish will depend on the characteristics of the raw foods (per unit content of calory, protein, vitamins etc., individual flavor and taste, etc.), the price vector of the raw foods, the opportunity cost of time, other observable and unobservable attributes of the dish (taste, digestibility, etc.), and the household vector of characteristics (income, household composition, etc.), then with a discrete endogenous variable, the conditional mean $E(d_i|\bar{X}_i,C,h_c)$ is given by $Prob(d_i=1)$, the probability of occurrence of the event that consists of dish i being chosen by the household. Here d_i is a dichotomous random variable that takes the value one if dish i is chosen and zero otherwise, $\bar{X}_i$ is the vector of the attributes of the dish (including the characteristics of the raw foods), C is the vector of costs of the dishes, and h_c is the vector of household characteristics. In summary, the structural model to be estimated is now

$$(3.6) \qquad E(d_i|\bar{X}_i,C,h_c) = Prob(d_i=1) = F[h_i(\bar{X}_i,C,h_c)] \quad i=1,\ldots,k.$$

where F is the distribution function of the multinomial probability distribution underlying the choice process, and h_i is an unknown function. This probability distribution depends in general on the

[3]In special cases like the normal distribution, these two moments are enough to completely determine the distribution.

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attributes of the dishes, the vector price of the raw foods, and the vector of characteristics of the household. We see from (3.6) that knowledge of the multinomial distribution function F and the exact functional form of the h_i will allow estimation of the demand for the different dishes by their estimated probability of being chosen.

The discreteness of the choice set also makes more likely the possibility of simultaneity and/or temporal correlation of the dish choice decisions. For instance, because of taste and diet diversification, the choice of today's dishes is affected by yesterday's choices, and will certainly affect tomorrow's choices. This might not have been a problem if the set of dishes available to the household were convex, since with a convex preference ordering the household could choose any convex combination of dishes to satisfy its taste and diet diversification needs.

Having the demand for the dishes given by their conditional means, which constitute the set of structural equations of the model, we can recover the reduced form demand equations for the different basic food staples. From this, we can get the usual system of demand elasticities (own, cross and income elasticities) which are of interest for policy makers.

Indeed, if x_{ij} is the amount of raw food j used in the preparation of dish i, then the demand X_j of raw food j given by its unconditional mean is:

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$$(3.7) \qquad X_j = E(X_{ij} \mid \bar{X}_i, C, h_c) = \sum_{i=1}^{k} x_{ij} \cdot \mathrm{Prob}(m_i = 1)$$
$$= \sum_{i=1}^{k} x_{ij} \cdot F[h_i(\bar{X}_i, C, h_c)]$$

where X_{ij} for $j=1,\ldots,n$ is a random variable taking the different values x_{ij} conditionally on i. The cross price and income elasticities can be calculated from equation (3.7) and they will all depend on the probability distribution function F.

From this development, we see that the probability distribution underlying the choice process plays an important role in this model. thus, its appropriate specification is essential for estimating this model. For this, we need to redefine precisely the nature of the household preference ordering, the objects of the choices and the properties of the choice set, and the decision rule that leads to these discrete choices. This leads us to the probabilistic choice or random utility model, originally developed by psychologists (Thurston, 1927; Luce, 1959 and 1965; Block and Marshak, 1960) and then made econometrically estimable by the pioneering work of D. McFadden, C. F. Manski and colleagues.

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CHAPTER 4

THE MATHEMATICS OF THE PROBABILISTIC CHOICE MODEL

As its title indicates, the purpose of this chapter is to work out in detail the mathematical framework within which the household food consumption behavior is analyzed. The organization of the chapter is as follows: In section 4.1, we discuss in detail all the assumptions that we are making to model the household food consumption behavior. We tried as much as possible to state clearly and rigorously all the assumptions. In section 4.2, we give a mathematical structure to the choice set of the household and then proceed to derive a utility function representing the household preference ordering. The remaining of the section is to link this utility to the random utility model. Because of its abstractness, we need to justify the relevance and usefulness of this axiomatic approach.

It is difficult to debate whether or not the household is maximizing the utility of the dish or the utility of the raw foods in it since utility is not observed. Indeed, utility is a pure and convenient creation of the analyst to represent the assumed household preference ordering over the choice set. But, we do observe the household choosing both among dishes and among raw foods, and Debreu has shown that with a minimum of assumptions about the household preference ordering over the space of raw foods, we can derive a ordinal utility function that represents this preference ordering. Our propose in this section is to show that based on a preference ordering over the space of

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dishes we can equally derive an ordinal utility function that represents the household preference ordering over the space of dishes. For some economists like Debreu, it is desired that any model explaining choices be based on the more basic concept of preference than of utility[4]. Having changed the structure of the household choice set from the familiar commodity space (which is assumed to have the "continuum" structure of the Euclidean space) to the inherently discrete structure of the set of dishes, we felt the need to explore the type of mathematical structure and assumptions required to represent the household preference ordering by a utility function. Furthermore, summarizing the point of view of some early critiques of probabilistic choice models, Luce (1965, pg. 337) wrote: "They suggest that we cannot hope to be completely successful in dealing with preferences until we include some mathematical structure over the set of outcomes that for example, permits us to characterize those outcomes that are simply substitutable for one another and those that are special cases of others. Such functional and logical relations among the outcomes seem to have sharp control over the preference."

Coming back to our problem, our main argument is that viewing the household as choosing primarily among dishes is more useful than viewing it as making its choices over the space of raw foods. Furthermore, if our view is correct, then, because of the constraints brought about by the "technology" of dishes and the discreteness of the choice process, using the traditional approach will introduce what I will refer

4. See Deaton and Muellbauer (1984) for a similar argument.

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hereafter as "dish selection bias." This point will be clear by the end of this chapter, and its statistical consequences will be discussed thoroughly in the chapter on estimation where we give methods for correcting for this "dish selection bias." It turns out that statistically, the traditional view is the special case of no "dish selection bias." Hence one can statistically test the validity of our argument. A testing procedure is proposed in the estimation chapter. Later, we will argue that this dish selection problem which has a censoring effect is the major explanation of the relatively large number of missing observations (observed zero quantity consumed for some food items) in almost all the food consumption surveys. The basic fact is that, you do not buy raw foods that you do not use in your "selected" dishes.

In section 4.3 we solve the household maximization problem and derive the conditional indirect utility functions. Section 4.4 goes in great detail (that some may find unnecessary) in deriving and discussing the proprieties of the different utility and expenditure functions (direct, indirect, conditional and unconditional), the demands for the dishes, and the conditional and unconditional demands for the raw foods. We felt the need for this discussion because of the nature of our choice system. It was not obvious to us that we will end up with the same properties and restrictions implied by the traditional demand theory. Given the importance attached on testing these restrictions in modern applied consumption analysis, we preferred this detailed discussion rather than the short sentence: "all the usual properties and restrictions apply." The last part of this section shows how the model explains the pro

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Finally, the purpose of the last section is to work out in details the implementation of our model, by using an specific functional form, namely the AIDS functional form.

4.1. The Structure of the Choice System: Definitions and Assumptions.

The following definitions and assumptions are needed to clarify the meaning and intuitions behind the fundamental axiom that will be given below, on which most of the results of the model are based. Most of the definitions, assumptions, and concepts have been discussed in the probabilistic choice literature (e.g., Manski, 1977; McFadden, 1981). Because we are mainly concerned with deriving an econometrically estimable model with precise and easily observable variables, however, the formulations and meanings are changed to fit our food consumption model better.

4.1.1. Definitions and Assumptions Related to the Consumption Units:

A.1. The population of individuals $\mathcal{P}$ is supposed to be partitioned into subsets of consumption units called households. The set of all households noted $\mathcal{S}$ constitutes the set of individual decision makers. Thus, it is a subset of $2^{\mathcal{P}}$ the power set of $\mathcal{P}$.

A.2. It is assumed that each household is described by a vector of observable and unobservable characteristics. More precisely, we assume that there exists a mapping

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ς: $\mathcal{S} \to R^k_+$ associating every household h with a vector of characteristics h_c, element of R^k_+. The range of ς is then called the characteristic space and is noted C. For instance, the coordinates of h_c may include income level, source of income, household size, number of adults, household "tastes", and other unobservable characteristics.

4.1.2. Definitions and Assumptions Related to the Object of Choices:

B.1. We assume as usual that the commodity space which includes the raw foods is the non-negative orthant of the Euclidean space R^p. Hence, the space of raw foods will be R^n_+ with $n<p$.

B.2. We assume that the household combines a subset of the raw foods to produce, by means of some technological transformation, a final product called a dish, and the set of all dishes is called the space of alternatives and noted A. More precisely, we assume that there exists a mapping $T: R^n_+ \to A$ associating every vector of raw foods r, with an alternative $a=T(r)$.

B.3. We assume that each dish or alternative is described by a vector of observable and unobservable attributes. More precisely, we assume that there exists a mapping $\mu: A \to R^q$ associating every alternative a with a vector of attributes x_a element of R^q. The range of μ is called the space of attributes and noted χ. For instance, the coordinates of x_a may include the characteristics of the individual raw food used in the dish (per unit content of calory, protein, vitamins, etc., flavor, taste, etc.), dish cooking time, amount of energy required for cooking the dish, dish taste, dish digestibility, etc.

B.4. The choice set is postulated to be not the space of raw foods, but rather the set A of all possible dishes.

B.5. We assume that each household has a preference ordering which completely orders the elements of A, each time it makes its consumption decision. This preference ordering which depends on the household's characteristics and the time the consumption decision is made, is assumed to be reflexive, transitive and complete, and is noted $\succeq$ (for $a \in A$ and $b \in A$ $a \succeq b$ means a is preferred to b).

B.6. The household is assumed to make its food consumption decisions repetitively and on a daily basis (We assumed that the household has three meals a day, and for each dish it cooks only one dish).

B.7. We assume that each time the household makes its consumption decision, there exists a finite choice set E contained in A from where it must select only one alternative. E is called the set of feasible alternatives available to the household at the time the choice is made. In general, E depends not only on the consumption technology and the household vector of

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characteristics, but also it depends on the time the choice is made.

B.8. It is assumed that the selection of an alternative is unpredictable because of unobservable household characteristics (e.g. "tastes") and attributes of the dishes. In that sense, the outcome of the household choice is random.

The above definitions and assumptions motivate the following adaptation of the axiom of the probabilistic choice model (P.C.M.).

4.1.3. Fundamental Axiom of the P.C.M. :

C.1. It is assumed that the choice set or set of alternatives is $B(R^d{}_+)$, the Borel σ-algebra of $R^d{}_+$, that is $A = B(R^d{}_+)$, with $d>n$.

C.2. $R^d{}_+ \in E$, and $(T^{-1}(E), E)$ is a measurable space where E is the feasible set and T is the mapping defined in B2.

C.3. There exists a probability measure P_E defined in E, such that $(T^{-1}(E), E, P_E)$ is a probability space with $\sum_{a \in E} P_E(a) = 1$. $P_E(a)$ measures the probability that the household chooses alternative a.

C.4 The household preference ordering $\succsim$ is described by:

$$a \succsim b \quad \text{if and only if } P_E(a) \geq P_E(b).$$

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Assumption C1 identifies a dish with a Borel set of $R^d{}_+$. This parallels the usual identification of the commodity space with the non-negative orthant of the Euclidean space. Indeed, given that a dish is a (technological) transformation of some vector of raw foods, C1 merely specifies the nature of this transformation which is a correspondence that associates each vector of $R^d{}_+$ with a subset of $R^d{}_+$ whose elements share a special relationship. Assumption C2 excludes $R^d{}_+$ from the feasible choice set although it is an alternative by C1. The measurability assumption is needed so that the following assumption C3 can make sense. Assumption C3 formalizes the randomness underlying the household choice process. The reasons for this randomness were discussed in B8. In some sense the postulated probability distribution measures the "chance" of a given dish of being selected or preferred by the household. Hence, in some sense, at least for the observer the dishes are events of the underlying choice process. This has some intuitive appeal given that the choice of a dish is observationally discrete and that for the econometrician the household's selection of a dish is an action, the consequence or outcome of which is an event. By this we do not mean that the choice action itself is random but rather the outcome of the choice process (which dish is selected by the household) is random for the observer. In summary, C3 says that the selection of a dish is an event with a probability of occurrence $P_E(a)$. The condition $\sum_{a \in E} PE(a) = 1$ is just to formalize the fact that the household must select one alternative from E. Assumption C4 is a modified version of a probabilistic definition of utility usually found in the probabi

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in the probabilistic choice literature[1] (see, for example, Luce (1965), Block and Marshak (1960), or Debreu 1958). $P_E(a) \geq P_E(b)$ is interpreted as "a is preferred to b " (Debreu, 1958, p. 440).

The probability distribution P_E depends indirectly through E on the consumption technology, the household characteristics and on time. But also it depends directly on time, since for an unchanging E, P_E may change from one decision to the next.

4.2 The Mathematical Structure of the Model.

4.2.1 Construction of a topology on the alternative space.

Suppose that the fundamental axiom holds, and let $E \subset A$ be the finite choice set given by the axiom. Define the function d_E from AxA into [0,1] as follows:

i) $d_E(a,b) = |P_E(a) - P_E(b)|$ if $a \in E$ and $b \in E$

ii) $d_E(a,b) = 1$ if $a \in E$ and $b \notin E$ or $a \notin E$ and $b \in E$

iii) $d_E(a,b) = 0$ if $a \notin E$ and $b \notin E$

Proposition: d_E is a pseudometric for A.

Proof - see Appendix A1.

Comments: We note that d_E is not a metric since for $a,b \in E$ $d_E(a,b) = 0$ implies $P_E(a) = P_E(b)$, which does not imply $a = b$, because two alternatives in the feasible choice set may well have the same probability of being selected yet have different attributes.

Given d_E, (A, d_E) is a pseudometric space. Hence d_E defines a topology in A, namely the topology of which base is the collection of all open balls $B(a,r) = \{ b \in A; d_E(a,b) < r \}$ for all $a \in A$ and $r > 0$. Let (A,

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F_E) the corresponding topological space. Because d_E is not a metric, (A, F_E) cannot be a Hausdorff topological space.

4.2.2. Existence of a utility function on the alternative space.

By definition a utility function on A is any real valued function that takes its values in R_+.

Theorem

Assume that the fundamental axiom holds, then there exists on $(A, d_E, \succcurlyeq)$ a continuous (ordinal) utility function that represents the household preference ordering $\succcurlyeq$.

Before proving the theorem, we need the following lemma.

Lemma:

Under the axiom, (A, d_E) is a perfectly separable pseudometric space.

Proof:

See Appendix A2.

Proof of the Theorem

By C4., the household's preference ordering $\succcurlyeq$ on A was defined as follows:

(4.2.2) $a \succcurlyeq b$ if and only if $P_E(a) \geq P_E(b)$

for all a and b $\in$ A

Then, $\succcurlyeq$ is a complete ordering on A (see Appendix A3 for the proof). Next, F_E, the topology corresponding to d_E is a natural topology for $\succcurlyeq$, that is, the sets

$$F_1 = \{ b \in A \; / \; a \succcurlyeq b \} \quad \text{and} \quad F_2 = \{ b \in A \; / \; b \succcurlyeq a \}$$

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are closed for all $a \in A$. This is the same as saying that the preference ordering $\succsim$ is continuous for F_E. See appendix A4 for the proof.

So, A is a completely ordered and perfectly separable topological space whose topology is natural for the ordering $\succsim$. Hence from a theorem of Debreu (1954, pp. 159-65), there exists on A, an ordinal continuous utility function U_E that preserves the preference ordering $\succsim$, that is

$$a \succsim b \text{ if and only if } U_E(a) \geq U_E(b) \quad \text{for all } a \text{ and } b \in A. \tag{4.2.3}$$

In summary, there exists a continuous ordinal utility function in the choice set A such that the alternative yielding the maximum utility has the best chance to be chosen by the household.

Since by definition a random variable on the measurable space (R^n_+, A) is any measurable real valued function defined in (R^n_+, A), one point that comes immediately to mind is the measurability of the utility function U_E. This would enable us to speak of U_E as a random utility function. However, this cannot make sense since the domain of U_E is A instead of R^n_+.

4.2.3. The Random Utility Model.

If in addition to the fundamental axiom one makes the standard assumption of utility maximization behavior, that is, the household always chooses the alternative yielding the maximum utility, then, from the observer's viewpoint, the alternative yielding the maximum utility corresponds to the one with the highest probability of being chosen.

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For a formal development of the random utility model, one can proceed as follows:

Let $U_E(A)$ be the range of U_E and consider the σ-field B_E generated by the collection of all half lines $]-\infty, U_E(a)]$ $a \in A$, such that $]-\infty, U_E(a)[\subset U_E(A)$. Then $(U_E(A), B_E)$ is a measurable space. Now, consider the σ-field A_E on A generated by U_E that is $A_E = \{ U_E^{-1}(B); B \in B_E \}$. Then by construction U_E is a measurable function from (A, A_E) onto $(U_E(A), B_E)$. In that sense, we would say that U_E is a random variable meaning that it is measurable with respect to the σ-field defined above. Then, based on our postulated probability space, we can construct a probability measure on A_E. More precisely, we would like to show that there exists a probability measure P defined on A_E such that $P[U_E(a) \geq U_E(b)] = P[\{ b \in A; U_E(a) \geq U_E(b) \}] = P_E(a)$ for all $a \in A$. (A, A_E, P) would be then a probability space with $P[U_E(a) \geq U_E(b)] = P_E(a)$ for all $a \in A$. For our purpose, we will take the existence of this probability space as a conjecture. In Appendix A5, we give some indications on how one may proceed to show this existence.

An alternative way to introduce randomness in the utility is to note that U_E depends implicitly on the household vector of characteristic h_c and on the attributes of the alternative's vector of attributes x_a, because both E and P_E depend on these. But all these characteristics and attributes cannot be completely known and observed. Only a finite number of them can be observed and measured (with some measurement errors). Then, one may want to partition the vectors of characteristics and attributes into two parts: one part observable and measurable, the other part not observable. That is the approach taken by Manski (1977). Formally, he wrote h_c and x_a as $h_c = (h_{co}, h_{cu})$; $x_a =$

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(x_{ao}, x_{au}) with h_{co} and x_{ao} observable but h_{cu} and x_{au} unobservable. This approach yields the random utility model where the source of randomness of the utility is attributed to the inability to observe all the household characteristics and goods attributes. Manski (1973) cited in Ben-Akiva et al. (1985) identified the following sources of randomness of the utility function:

1 -- unobserved attributes

2 -- unobserved taste variations

3 -- measurement errors and imperfect information

4 -- instrumental (or proxy) variables

For a detailed analysis of these sources of randomness, see Ben-Akiva et al. (1985, pp. 55-57).

4.3. Solution of the Household Maximization Problem.

4.3.1. The general solution.

Let $E=\{a_1, \ldots, a_m\}$ be the finite set of feasible alternatives, for $i=1,\ldots,m$ let d_i be the dichotomous random variable defined as follows:

$$(4.3.1) \qquad d_i = \begin{cases} 1 & \text{if } a_i \text{ is selected by the household} \\ 0 & \text{if not} \end{cases}$$

then we have :

$$(4.3.2) \qquad P\{d_i=1\} = P_E(a_i)$$
$$= P\{\, U_E(a_i) = \max U_E(a_j) \quad j=1,\ldots,m \,\}$$

This follows immediately from the conjecture defining P and the assumption of utility maximization, with utility maximized subject to the constraint defined by the set E of feasible alternatives. We note that

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that the d_i $i=1,\ldots,m$ are observable. Here i is an index for the alternative a_i, and since there is a one to one correspondence between a_i and i, we can just keep the index i standing for a_i. For instance, the finite set of feasible dishes of the household is designated as dish 1, dish 2, . . ., dish m.

Now, since E depends on the alternatives' vectors of attributes x_{a_i}, and costs C(i) $i=1,\ldots,m.$, and the household vector of characteristics h_c, a solution a_i of the maximization problem $\max\{ U_E(a_j)\ j=1,\ldots,m \}$ subject to $a_j \in E$, will be function of x_{a_i}, the vector of attributes of the chosen alternative, h_c, and a vector of costs of all the alternatives, noted v. That is we should have a demand function $a_i(x_{a_i},v,h_c)$ for a chosen alternative a_i such that

(4.3.3) $\quad U_E[a_i(x_{a_i},v,h_c)] = \max\{ U_E(a_j)\ j=1,\ldots,m. \}.$

Hence, when (x_{a_i},v,h_c) varies over $X \times R^m_+ \times C$, we have the conditional indirect utility function V_i (conditional on dish i being chosen) defined as:

$$X \times R^m_+ \times C \dashrightarrow R$$

$$V_i:\ (x_{a_i},v,h_c) \dashrightarrow V_i(x_{a_i},v,h_c) = U_E[a_i(x_{a_i},v,h_c)] = \max_{1 \le j \le m} U_E(a_j)$$

with

(4.3.4) $\quad P(d_i=1)=P\{ V_i(x_{a_i},v,h_c) = \max\{ U_E(a_j)\ j=1,\ldots,m\} \}.$

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and given that U_E is random in the sense defined above, V_i will also be random.

The randomness of V_i can also be derived by decomposing (x_{a_i}, h_c) into a observable component (x_{a_io}, h_{co}) and an unobservable component (x_{au}, h_{cu}). Then following Manski (1977) we can write:

$$(4.3.5) \qquad V_i(x_{a_i},v,h_c)=V_i(x_{a_io},v,h_{co}) + \epsilon_i(x_{a_iu},h_{cu})$$

where as usual, $\epsilon_i(x_{a_iu},h_{cu})$ is a random disturbance summarizing the unobservable components of the conditional indirect utility function.

Let $V_k(x_{a_i},v,h_c)$ $k=1,\ldots,m$ be the m conditional indirect utility functions, then we have:

$$(4.3.6) \qquad V_i(x_{a_i},v,h_c) = \max \{U_E(a_j),\ j=1,\ldots m.\}$$

if and only if $V_i(x_{a_i},v,h_c) \geq V_k(x_{a_i},v,h_c)$ for all $k \neq i$, $k=1,\ldots,m$.

Hence,

$$(4.3.7) \qquad P(d_i=1)=P\{ V_i(x_{a_i},v,h_c) \geq V_k(x_{a_i},v,h_c) \text{ for } k\neq i \text{ and } k=1,\ldots,m\}$$

This is the familiar formulation of the random utility model (McFadden, 1981).

4.3.2. <u>The structure of the feasible set of alternatives</u>:

Let x_{ij} be the amount of raw food j used in the preparation of dish i for $i=1,\ldots,m$ and $j=1,\ldots,m$. Hence $x_{ij}=0$ if no raw food j is used in dish i.

We can see that a priori, for a given dish i, x_{ij} depends in general on observable household characteristics (ex: household composition, household income etc.), as well as on unobservable household c

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household characteristics (ex: household "tastes", technical knowledge, etc...). It depends also on the characteristics of raw food j (per unit content of calorie, protein, vitamins, etc.), the relative prices of the raw foods used in the dish, and some other unobservable attributes (ex: the intrinsic contributing flavor of the raw food j with respect to dish i). For a given raw food j, x_{ij} depends generally among other things on the unobservable consumption technology. Hence, for a given dish i, the amount of raw food j used in dish i, which is the household's conditional quantity demanded of raw food j, is a real valued random variable X_{ij} with observable and unobservable components that is[5]:

$$X_{ij} = h_i(P_i, c) + \eta_{ij} \qquad (4.3.8)$$

where c is a vector of observable household characteristics that includes: household size, number of adults and women in the household, household income etc... η_{ij} is a random disturbance term. $Pi=(P_1, \ldots, P_{J_i})$ is the vector price of the J_i raw foods used in dish i, h_i is an unknown function of P_i and c.

Given i, one might expect h_i to be a nondecreasing function of the household size and number of adults for most of the raw foods. It is also expected to be a non increasing function of P_j for $j=1,\ldots,J_i$ (ceteris paribus of course). Hence, an appropriate specification of the functions h_i $i=1,\ldots,m$ would enable one to capture the economy of scale

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[5]. In general we will consider all the raw foods characteristics as unobservable to simplify the analysis although it is possible to observe some of them (ex: per unit content of calorie, protein, vitamins, etc.). If one is interested in some nutritional aspects of food consumption, he may consider measuring these nutritional contents of the individual raw foods and incorporate them into the model. The following results can be easily changed to accommodate these nutritional aspects.

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embodied in the household consumption technology. The precise functional form of the h_i's will be clear later, when we derive them as a solution of the household utility maximization problem.

In general, the only observable intrinsic attributes of the dishes are the amount of time and energy needed for their preparations.

Given the food prices and the consumption technology, the observable constraints facing the household when making its daily consumption choices are: (1) the household meal budget d(c) which depends on its vector of characteristics c, and (2) the time constraint[6] measured by the maximum total time available for dish preparation. The major components of the vector of characteristics c in d(c) are: the household income y and its expenditure on non food items e_N. But, the household size, its number of adults and other observable and unobservable characteristics are included in c as well. Hence, if we assume some degree of separability between food consumption and non food consumption, then we can write:

(4.3.9) $d = d(y,e_N,c)$

where c now stands for the components of the household characteristics other than income and expenditure on non food items.

One should expect $d(y,e_N,c)$ to be a nondecreasing function of income, household size and number of adults but a nonincreasing function of expenditure on nonfood items. However, since food is a necessity, beyond some range of income, $d(y,e_N,c)$ is probably insensitive to

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6. Since energy and time enter the model in a similar way, to simplify the analysis we will concentrate only on the time variable. Alternatively, one can think of our time variable as a bi-dimensional vector with coordinates time and energy.

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changes in income or expenditure on nonfood items. It is also likely that changes in $d(y,e_N,c)$ is less than proportional to changes in household size, and composition because of economy of scale inherent in the household consumption technology.

Hence, if we denote by τ_i the time preparation of dish i and by w the unit cost of time, the set of feasible alternatives can be rewritten as:

$$(4.3.10)\quad E = \{ a_i \in A;\ \sum_{j=1}^{J_i} x_{ij}.P_j + w\tau_i \le d(y - e_N,\ c)\quad i=1,\dots,m.\}$$

But if we let $\tau_i = x_{i0}$, $P_0 = w$, and noting that $d_i \in \{0,1\}$ for $i=1,\dots,m$ with

$$\sum_{i=1}^{m} d_i = 1 \qquad \text{(since only one dish is chosen)},$$

then the budget constraint can be rewritten as:

$$(4.3.11)\quad \sum_{i=1}^{m} \sum_{j=0}^{m} d_i x_{ij} P_j \le d$$

where we have put $d = d(y,e_N,c)$ to simplify the notation.

Introducing the variables $C(i,j) = x_{ij}P_j$ the share cost of food j in dish i for $j=0,\dots,J_i$ with $C(i,o) = x_{io}P_o = w\tau_i$ being the share cost of time for dish i $i=1,\dots,m$.

For $i=1,\dots,m$ let the total cost of dish i be:

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$$(4.3.12)\quad C(i) = \sum_{j=0}^{J_i} C(i,j)$$

where J_i is the number of raw foods used in dish i then the budget constraint can be rewritten as:

$$(4.3.13)\quad \sum_{i=1}^{m} \sum_{j=0}^{Ji} d_i \cdot x_{ij} \cdot P_j = \sum_{i=1}^{m} d_i \cdot \left(\sum_{j=0}^{J_i} C(i,j) \right) = \sum_{i=1}^{m} d_i \cdot c(i) \leq d$$

Hence

$$(4.3.14)\quad E = \{a_i \in A;\ \sum_{i=1}^{m} d_i c(i) \leq d \ \text{ with } d_i \in \{0,1\} \text{ and } \sum_{i=1}^{m} d_i = 1\}$$

This is clearly a linear budget constraint, with the cost of the dishes, the c(i) replacing the usual vector of prices. With this specification of E, the household maximization problem can be reformulated as:

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Subject to

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with $d_i \in \{0,1\}$; $\sum_{i=1}^{m} d_i = 1$; and $P_E(a_i) = P(d_i = 1)$.

Since by assumption, the household must select one dish from the finite set of dishes E, this problem has a unique solution.

Let $v=(c(1),\ldots,c(m))$ be the m-dimensional vector of dish costs. Define $q=v/d=(q_1,\ldots,q_m)$ as the vector of real costs of the dishes. Then, the unconditional indirect utility function is defined as

(4.3.16)

$$V(q) = \max_{1\le i\le m} \left\{ U_E(a_i);\ \sum_{i=1}^{m} d_i q_i \le 1;\ d_i \in \{0,1\};\ \sum_{i=1}^{m} d_i = 1;\ P_E(a_i) = P(d_i = 1) \right\}$$

$$= \max_{1\le i\le m} V_i(Pi/d)$$

where the V_i, $i=1,\ldots,m$. are the m conditional indirect utility functions, and $P_i=(P_o,\ldots,P_i)$ the price vector of the raw foods including the time cost of preparation of dish i.

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4.4 The Utility, Expenditure, and Demand Functions.

4.4.1. The direct conditional utility functions

One way to define the conditional direct utility would be to use the fact that, by assumption, each dish a_i is the result of some technological transformation T such that $a_i=T(x_i)$, where $x_i=(x_1,\ldots,x_{J_i})$, the vector of J_i raw foods used in the preparation of dish i.

Hence the direct utility of dish i can be written as

$$(4.4.1)\qquad U_E(a_i) = U_E(T(x_i)) = U_i(x_i)$$

Then, following McFadden (1981), for a selected alternative a_i, the household chooses x_i to maximize the conditional utility U_i subject to the budget constraint

$$c(i) = \sum_{j=0}^{Ji} c(i,j) = \sum_{j=0}^{Ji} x_{ij}\ P_j \leq d$$

The result would yield the conditional indirect utility

$$(4.4.2)\quad V_i(P_i,d)=\max_{x_i}(U_i(x_i);\quad c(i)=\sum_{j=0}^{Ji} x_{ij}P_j\leq d;\ x_{ij}\geq 0;\quad P_j\geq 0\ )$$

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For $i=1,\ldots,m$. we would get the m conditional indirect utility functions derived earlier with the unconditional indirect utility function given by:

$$(4.4.3) \qquad V(q) = \max_{1\leq i\leq m} V_i(P_i,d) = \max_{1\leq i\leq m} V_i^*(P_i^*,1)$$

where $q=(q_1,\ldots,q_m) = (c(1)/d,\ldots,c(m)/d)$ is the m-dimensional vector of real costs of the dishes and $P_i^* = Pi/d=(P_0/d,\ldots,P_i/d)$ is the vector of real prices of the raw foods and the real cost of time for $i=1,\ldots,m$.

Hence, we see that while the conditional indirect utilities depend only on the real prices of the raw food and on real cost of time, the unconditional indirect utility depend explicitly on the real costs of all the dishes. This gives a formal proof of the important fact already discussed in Chapter 2 and 3 that the variables relevant for the household consumption decisions are not the relative prices of the raw foods, but rather the relative costs of the dishes.

This derivation of the conditional direct and indirect utility functions, is probably the most natural and familiar way. Furthermore, if U_E and the technological transformation function T satisfy appropriate regularity conditions so that the direct conditional utility functions satisfy the usual following conditions:

<u>Conditions Du:</u> for $i=1,\ldots,m$. $U_i(x_i)$ is

(i) continuous

(ii) nondecreasing

(iii) subject to local nonsatiation

(iv) Quasi-concave

then the conditional indirect utility functions will satisfy the following conditions: (see, Diewert; 1977 and 1981)

<u>Conditions ID</u>: for i=1,...,m. $V_i(q_i)$ is

(i) Continuous

(ii) nonincreasing

(iii) subject to local nonsatiation

(iv) Quasi-convex for $P_i^* >> 0$

Furthermore, we have by duality

$$U_i(X_i) = \min_{Pi} \left\{ V_i(P_i, d): \quad C(i) = \sum_{j=0}^{J_i} x_{ij} P_j \leq d \ ; \quad P_j >> 0 \right\}$$

$$i=1,\ldots,m.$$

Conversely, if the conditional indirect utility functions satisfy conditions ID, then the direct conditional utility functions will satisfy conditions DU and we have

$$V_i(P_i, d) = \max_{x_i} \left\{ U_i(x_i): C(i) = \sum_{j=0}^{J_i} x_{ij} \cdot P_j \leq d; \quad x_{ij} \geq 0 \right\}$$

$$i=1,\ldots,m.$$

So the U_i i=1,...,m satisfying DU is dually equivalent to the V_i i=1,...,m. satisfying ID. (Diewert, 1977).

Since the conditions DU and ID are dually equivalent, one could alternatively derive the conditional direct utility functions from the conditional indirect utility functions which are already derived in (4.3.16) from the unconditional indirect utility function by

$$V(q)= \max\{ V_i(P_i,d) \quad i=1,\ldots,m \}$$

then postulating that the V_i $i=1,\ldots,m.$ satisfy the conditions ID, we can define the conditional direct utility functions by

$$(4.4.4) \qquad U_i(x_i) = \min_{Pi} \left\{ V_i(P_i,d) \colon C(i) = \sum_{j=0}^{J_i} x_{ij} P_j \leq d, \; P_j \gg 0 \right\}$$

$$i=1,\ldots,m.$$

and the U_i's will satisfy the conditions DU.

The advantage of this derivation, would be to avoid the use of the technological transformation functions T and to make any assumption about their regularity conditions.

If in addition to conditions ID, we assume that the m conditional indirect utility functions are differentiable with respect to P_i and d, then we have by Roy's identity, the conditional Marshallian demands given by:

$$(4.4.5)\quad X_{ij}(P_i,d) = -\frac{\partial V_i(P_i,d)/\partial P_j}{\partial V_i(P_i,d)/\partial d} \qquad j=0,\dots,Ji\ ; \quad i=1,\dots,m.$$

Since $d = d(y,e_N,C)$

The conditional Marshallian demands are of the form

$$X_{ij}(P_i,d) = h_i(P_i,d) + \eta_{ij} = h_i'(P_i,c) + \eta_{ij}$$
$$j=0,\dots,Ji. \text{ and } \quad i=1,\dots,m.$$

where η_{ij} is a disturbance term. This is exactly the functional dependence predicted for the conditional Marshallian demands in (4.6.1)

4.4.2. <u>The conditional expenditure functions:</u>

Given the conditional direct utility functions, we can define the corresponding expenditure functions by

$$(4.4.6)\qquad e_i(P_i,u) = \min_{xi} \{ c(i) = \sum_{j=0}^{J_i} x_{ij}P_j \ : \ U_i(x_i) \geq u \} \quad i=1,\dots,m.$$

One of the major advantages of working with the expenditure function is that it satisfies many regularity conditions yet requires only that the direct utility function be continuous and that the level of utility u belong to the range of the utility function. Indeed if the direct utility functions $U_i(x_i)$ are continuous and if u belongs to the range of U_i $i=1,\ldots,m$. then it can be shown (Diewert; 1977, 1981) that the corresponding expenditure functions have the following properties.

Properties EX:

For $i=1,\ldots, m$ $e_i(P_i, u)$ is:

(i) a nonnegative function that is, $e_i(P_i,u)\geq 0$ for any $Pi>>0$ and $u \in$ range of U_i.

(ii) linearly homogeneous in P_i that is, $e_i(kP_i,u)= ke_i(P_i,u)$ for any $P_i>>0$, $u \in$ range of U_i, and $k>0$.

(iii) non decreasing in P_i that is, $e_i(P^1{}_i,u)\geq e(P^0_i, u)$ for any $P^1_i>P^0_i$.

(iv) concave in P_i that is, $e_i(P_i,u)$ is a concave function of P_i for any $u \in$ range of U_i.

(v) continuous in P_i that is, $e_i(P_i,u)$ is a continuous function of P_i for any $u \in$ range of U_i.

(vi) non-decreasing in u that is, $e_i(P_i,u)$ is a non-decreasing function of u for any P_i.

(vii) continuous from below in u that is, for any u and any non-decreasing sequence $\{U_n,\ n\in N\}$ in the range of U_i such that lim $u_n=u$ then lim $e_i(P_i,u_n) = e_i(P_i, u)$ for any $P_i>>0$.

(viii) twice differentiable with respect to P_i except possibly at a set of Lebesgue measure zero (see Deaton and Muellbauer, 1984 p.40 and, Fuss and McFadden, 1978 for a proof).

If in addition the conditional direct utility functions satisfy conditions DU, then they can be recovered by

$$(4.4.7) \qquad U_i(x_i)=\max_u \left\{e_i(P_i,\ u)\leq P_i.x_i - \sum_{j=0}^{J_i} x_{ij}P_j \ \forall\ P_i\geq 0\right\} \quad i=1,\ldots,m.$$

As usual, we can apply Shephard's lemma to find the conditional Hicksian demand functions.

$$h_{ij}\ (P_i,\ u) = \frac{\partial e_i(P_i,u)}{\partial P_j} \quad j=0,\dots,J_i. \text{ and } \quad i=1,\dots,m.$$

Given our assumptions, the conditional Marshallian and Hicksian demands satisfy all the usual properties and restrictions which are: homogeneity of degree zero, adding up, symmetry and negativity of the Slustky matrix. They also satisfy a conditional Slustky equation.

4.4.3. Properties of the unconditional indirect utility and expenditure functions:

The unconditional indirect utility function was derived in (4.6.9) as

$$V(q) = \max_{1\le i\le m} V_i\ (P_i,d)$$

where $q=v/d=(c(1)/d,\dots,C(m)/d)$

For convenience, we may write it in the form

$$V(v,d) = \max_{1\le i\le m} V_i(P_i,d) \tag{4.4.8}$$

If all the conditional indirect utility functions satisfy the conditions ID, then the unconditional indirect utility V(v,d) will satisfy the usual regularity conditions of an indirect utility function with the vector of costs of the dishes replacing the vector of prices. If in addition the $V_i(P_i,d)$ are differentiable, then $V(v,d)$ will be differentiable, and the Marshallian demands for the dishes are given by the usual Roy's identity (McFadden, 1981; pp. 207-208).

As usual, we can define the unconditional expenditure function as:

$$(4.4.9)\quad e(v,u) = \min_{1\le i\le m} \left\{ \sum_{i=1}^{m} d_i c(i):\ \ U_E(a_i) \ge u;\ d_i \in \{0,1\};\ i=1,\ldots,m;\ \sum_{i=1}^{m} d_i = 1;\ \ P_E(a_i) = P\{d_i = 1\} \right\}$$

$$= \min \{e_i(P_i,u);\ i=1,\ldots,m.\}$$

Because U_E is continuous, $e(v,u)$ has the usual regularity conditions of an expenditure function that is, it satisfies the properties EX. with the vector of costs of the dishes replacing the vector of prices. In particular, it is twice differentiable, and using Shephard's lemma we can derive the Hicksian demands for the dishes by taking the partial derivative of $e(v,u)$ with respect to each dish cost.

Before investigating the properties of the demands for the dishes, we point out again the fact that while the conditional expenditures depend on the vector of prices and the time cost of dish preparation, the unconditional expenditure depends only on the vector cost of the dishes. Hence, it is affected by changes in prices only through these costs of dishes. In other words, using the statistician's language, the vector cost of the dishes is a sufficient statistic for changes in relative prices.

4.4.4. The Marshallian and Hicksian demands for the dishes:

Under our assumption of differentiability, the household's demand for the dishes are given by Roy's identity (McFadden, 1981 p. 208):

$$(4.4.10) \qquad d_i = D_i(v,d) = -\frac{\partial V(v,d)/\partial c(i)}{\partial V(v,d)/\partial d}$$

$$= \begin{cases} 1 \text{ if } V_i \geq V_k \text{ for } k \neq i \text{ and } k=1,\ldots,m. \\ 0 \text{ otherwise} \end{cases}$$

$i=1,\ldots,m.$

where $v=(c(1),\ldots,c(m))$ and $c(i)$=cost of dish i and $V_i = V_i(P_i,d)$ $i=1,\ldots,m.$

Similarly, and by duality, the Hicksian demands for the dishes are given by

$$(4.4.11) \qquad d_i = H_i(v,u) = \frac{\partial e(v,u)}{\partial c(i)} = \begin{cases} 1 \text{ if } e_i \leq e_k \text{ for } k \neq i;\ k=1,\ldots,m. \\ 0 \text{ otherwise} \end{cases}$$

where $e_i = e_i(P_i,u)$ $i=1,\ldots,m.$

And by duality, if the alternative a_i maximizes utility U_E at dish costs v^* and daily expenditure d^*, we have the identity:

$$(4.4.12) \qquad H_i(v,u^*) = D_i(v,\ e(v,u^*))$$

where $u^* = U_E(a_i)$

Because U_E is continuous and the demand functions are derived from utility maximization, the Marshallian and Hicksian demands for the dishes satisfy all the usual properties and restrictions of demand theory which are:

(i) Adding up: $\sum_{i=1}^{m} c(i)D_i(v,d) = \sum_{i=1}^{m} c(i)H_i(v,u) = d$

(ii) Homogeneity of degree zero in the vector of dish costs and total expenditure: $D_i(\theta v, \theta d) = Di(v,d)$, $H_i(\theta v, u) = H_i(v,u)$ for any positive scalar $\theta > 0$.

(iii) Symmetry of the substitution or Slutsky matrix:

the matrix $\left[\dfrac{\partial^2 e(v,u)}{\partial c(i) \partial c(j)} \right]_{\substack{1 \le i \le m \\ 1 \le j \le m}}$ is symetric

or equivalently, $\dfrac{\partial H_i(v,u)}{\partial C(j)} = \dfrac{\partial H_j(v,u)}{\partial C(i)}$ $\quad i=1,\ldots,m. \quad j=1,\ldots, m.$

(iv) Negativity of the substitution matrix:

$$\left[\frac{\partial^2 e(v,u)}{\partial C(i) \partial (j)} \right]_{\substack{1 \le i \le m \\ 1 \le j \le m}} \quad \text{is negative semi definite}$$

which implies that $\dfrac{\partial H_i(v,u)}{\partial C(i)} \leq 0$ for all i.

However, the Slutsky equations cannot be derived, because it would require differentiability of the Marshallian demands for the dishes, which is unlikely given the nature of our choice set.

So far, these testable restrictions on the demands for the dishes and the conditional demands for the raw foods are the only ones implied by the model.

4.4.5 The unconditional demands for the raw foods

In this part, we will focus only on the Marshallian demands. The unconditional Hicksian demands can be derived similarly.

If we introduce randomness in the conditional indirect utility functions by decomposing them into observable and unobservable components, then for i=1,...,m. we can write:

(4.4.13). $V_i(P_i,d,\epsilon_i) = V_i(P_i,d) + \epsilon_i \quad i=1,\ldots,m.$

Where ϵ_i is a disturbance term for the nonmeasurable components of the conditional indirect utility. Hence the conditional demands for the raw foods given by Roy's identity are:

$$(4.4.14) \qquad X_{ij} = -\frac{\partial V_i(P_i,d,\epsilon_i)/\partial P_j}{\partial V_i(P_i,d,\epsilon_i)/\partial d} = -\frac{\partial V_i(P_i,d)/\partial P_j}{\partial V_i(P_i,d)/\partial d} + \eta_{ij}$$

for $j=1,\ldots,J_i$ and $i=1,\ldots,m$.

Where η_{ij} is an error term standing for measurement error and unobservable variables.

It is clear from the derivation that the η_{ij} $j=1,\ldots,J_i$ are somehow connected to ϵ_i. Indeed, there are many reasons why the η_{ij} and ϵ_i are not independent. Dubin and McFadden (1984), Hausman (1985), and Duncan (1980) have given numerous reasons with models similar to ours. Without going into details, one reason in our food consumption model would be the fact that some unobservable household characteristics and/or unobservable alternatives' attributes may be correlated to unobservable attributes of some raw foods. For instances, if some dishes have increasing returns to scale with respect to household size, then, for a large household, the quantity demanded for some raw food is likely to be correlated with its dish choice.
Thus in general we will assume that η_{ij} and ϵ_i are jointly distributed.

More generally, let $\eta_i = (\eta_{i0},\ldots,\eta_{iJ_i})'$ $i=1,\ldots,m.$, $\eta = vec(\eta_1,\ldots,\eta_m)$,
and $\epsilon = (\epsilon_1, \ldots, \epsilon_m)$ $i=1,\ldots,m$.
with (η,ϵ) jointly distributed with density $f(\eta,\epsilon)$ and distribution function $F(\eta,\epsilon)$. With this notation, the conditional mean demand for raw food j $j=1,\ldots,J_i$ is given by:

(4.4.15) $E[X_{ij}|d_i=1] =$

$$E\left[\frac{-\partial Vi(P_i,d)/\partial P_j}{\partial V_i(P_i,d)/\partial d} + \eta_{ij} \;\middle|\; V_i + \epsilon_i \geq V_k+\epsilon_k \;\forall k\neq i\right]$$

or

$$(4.4.16)\quad E[X_{ij}\ |d_i = 1] = \frac{-\partial V_i(P_i,d)/\partial P_j}{\partial Vi(P_i,d)/\partial d}$$

$$+\ E(\eta_{ij}\ |\epsilon_k \le V_i - V_k + \epsilon_i\ \forall\ k \ne i)$$

where we have put $V_i = V_i(P_i,d)$ $i=1,\ldots,m$. to simplify the notation. Let A_i be the event $\{d_i=1\}$, that is

$$A_i = \{V_i + \epsilon_i \ge V_k + \epsilon_k,\ \ k \ne i \text{ and } k=1,\ldots,m.\}$$

then we have by definition of the conditional mean:

$$(4.4.17)\quad E(\eta_{ij}\ |A_i) = \frac{1}{P(A_i)} \times \int_{A_i} \eta_{ij} f(\eta,\epsilon)\,d\eta\,d\epsilon \qquad j=1,\ldots,J_i$$

where $\int_{A_i}$ is a multiple integral of order $m + \sum_{i=1}^{m} J_i$ and

$$d\eta = \prod_{i=1}^{m} \prod_{j=1}^{J_i} d\eta_{ij}\ ; \qquad d\epsilon = \prod_{i=1}^{m} d\epsilon_i$$

If we integrate out η and ϵ_i in (4.4.17), we can express the conditional mean as a function of the $V_i - V_k$; $k \ne i$ and $k=1,\ldots,m$. that is:

$$E(\eta_{ij}\ |A_i) = \lambda_j^i(V_i - V_1,\ldots,V_i - V_{i-1},\ V_i - V_{i+1},\ldots,V_i - V_m)$$

In the case where (η,ϵ) is bivariate, $\lambda^i_j(V_i-V_1,\ldots,V_i-V_m)$ is known as the hazard rate. To simplify the notation we will write V_i-V_k as $V^k{}_i$ for $k=1,\ldots,m$. and $k\neq i$.

In the discussion of estimation, in chapter 5, we will see how the expression $\lambda^i_j(.)$ can be simplified for some form of the joint distribution function F of (η,ϵ). For instance, if (η,ϵ) is distributed multivariate normal, a simplified expression of $\lambda^i_j(.)$ can be derived (Tallis, (1961); Johnson and Kotz, (1972); Duncan, (1982); and Amemiya, (1985)). When (η,ϵ) has the multivariate generalized extreme value (GEV) distribution, then $\lambda^i_j(.)$ can be integrated (McFadden, 1978).

The conditional mean demands for the raw foods are then:

$$(4.4.18)\quad E(X_{ij}\ |d_i=1) = -\frac{\partial V_i(P_i,d)|\partial P_j}{\partial Vi(P_i,d)\partial d}$$

$$+\ \lambda^i_j(V^1_i,\ldots,V^{i-1}_i,V^{i+1}_i,\ldots,V^m_i)$$

$$j=0,\ldots,J_i$$

Hence, the estimable equations of the conditional demands for the raw foods are:

$$(4.4.19) \quad X_{ij} = -\frac{\partial V_i(P_i,d)/\partial P_j}{\partial V_i(P_i,d)/\partial d} + \lambda_j^i(V_i^1,\ldots,V_i^{i-1},V_i^{i+1},\ldots,V_i^m) + \eta_{ij}^*$$

$$j=0,\ldots,J_i.$$

where $\eta_{ij}^* = X_{ij} - E[X_{ij}|d_i=1]$ is an error term (conditional on i) with $E[\eta_i^* j]=0$ and it can be shown that its variance is (see Appendix A6 for details):

$$(4.4.20) \quad \mathrm{Var}\ (\eta_{ij}^*) = E(\eta_{ij}^2 |\ d_i = 1) - [\lambda_j^i(V_i^1,\ldots,V_i^{i-1}, V_i^{i+1},\ldots, V_i^m)]^2$$

Again, a simplified expression for $\mathrm{Var}(\eta_i^* j)$ can be obtained with specific distributional form like the multivariate normal or the G.E.V distribution.

In practice, we do observe the X_{ij} the conditional quantities demanded for the raw foods. Hence, if we specify a functional form for the conditional indirect utility $V_i(P_i,d)$ and distribution function F for (η,ϵ), then equation (4.4.19) can be estimated. But, because $\lambda_j^i(.)$ is nonlinear in the parameters, the regression will be nonlinear regardless of the functional form of $V_i(P_i,d)$. In the next chapter, we will explore the different methods of estimation.

In general, we do not observe the unconditional quantities demanded for the raw foods because the raw foods used by the household are conditional to the dish selected. However, we can estimate these unconditional demands by their unconditional means given by:

$$(4.4.21) \quad E[X_j] = \sum_{i=1}^{m} E[X_{ij} \mid d_i=1]\cdot P(d_i=1) \qquad j=1,\ldots,J$$

where J is the total number of raw foods used in all dishes. Using the expression of $E[X_{ij} \mid d_i=1]$ in (4.11.7) and letting $P(d_i) = \pi_i$, we have:

$$(4.4.22) \quad E[X_j] = \sum_{i=1}^{m} \left\{ \left[\frac{\partial V_i(P_i,d)/\partial P_j}{\partial V_i(P_i,d)/\partial d} \right] + \lambda_j^i(V_i^1,\ldots,V_i^{i-1},V_i^{i+1},\ldots,V_i^m \right\} \cdot \pi_i$$

$$j=1,\ldots,J$$

These unconditional mean demands for the raw foods can be consistently estimated, provided the parameters in the equations of the conditional demands and the choice probabilities π_i are consistently estimated.

To summarize, the estimable equations for our food consumption model are:

$$(4.4.23) \quad \pi_i = P[d_i=1] = F(\eta=\infty,\epsilon_i=\infty,V_i^1+\epsilon_i,\ldots,V_i^{i-1}+\epsilon_i,V_i^{i+1}+\epsilon_i,\ldots,V_i^m+\epsilon_i)$$

$$i=1,\ldots,m.$$

$$(4.4.24)\quad X_{ij} = -\frac{\partial V_i(P_i,d)/\partial P_j}{\partial V_i(P_i,d)/\partial d} + \lambda_j^i(V_i^1,\ldots,V_i^{i-1},V_i^{i+1},\ldots,V_i^m) + \eta_{ij}^*$$

$$j=1,\ldots,J_i.$$

$$(4.4.25)\quad d=d(y,e_N,c) + \epsilon^*$$

(where ϵ^* is an error term).

From these 3 equations we can derive the unconditional mean demands

$$(4.4.26)\quad E[X_j] = \sum_{i=1}^{m}\left[\frac{\partial V_i(P_i,d)/\partial P_j}{\partial V_i(P_i,d)/\partial d}\right]\cdot\pi_i + \sum_{i=1}^{m}\lambda_j^i(V_i^1,\ldots,V_i^{i-1},V_i^{i+1},\ldots,V_i^m)\cdot\pi_i$$

$$j=1,\ldots,J$$

These four equations completely describe the food consumption model and they contain all the information needed to evaluate and predict accurately the household's response to changes in exogenous variables. Indeed, equation (4.4.23) gives the dish choice probabilities, (4.4.24) gives the conditional demands of the raw foods, (4.4.25) determines the meal budget as a function of household income, expenditure on nonfood items, and other household characteristics, and (4.4.26) gives the unconditional demand for any food staple consumed by the household. This latter equation, from which the usual elasticities of demand are derived, is of major interest for policy analysis.

It may be worth emphasizing that only the first three equations are relevant for the estimation procedure and that all the variables appearing in these equations are observable. We should also note the particular way that household characteristics (including income) enter the model: they influence food consumption decisions only through the meal budget.

Finally, if the conditional demands for the raw foods are independent of the dish choices (that is η_{ij} and ϵ_i are independent), then the second terms in equations (4.4.24) and (4.4.26) become zero. Hence this condition can be tested after estimating equation (4.4.24).

The next section will specify an explicit functional form for the $V_i(P_i,d)$. First, however, we want to summarize three major implications of the model: the first one is of an economic nature, namely, conditional on the distribution function F or $\lambda_j^i(.)$ having the desired properties, the usual restrictions implied by demand theory (adding up, homogeneity, symmetry and nonnegativity of the substitution matrix) apply only to the conditional demands for the raw foods and (partially) to the unconditional demands for the dishes. These restrictions can be tested after estimation or imposed before estimation. Beyond that, apparently the model implies no restrictions for the unconditional demands for the raw foods. This is an open question that can be investigated using the equations of the unconditional demands for the raw foods.

The second implication is of a statistical nature, namely that if the conditional demands for the raw foods are dependent on the dish choices, then estimations of food demand systems that do not incorporate information on the dish choices generally yield biased and inconsistent estimates.

The last implication is also of statistical nature, that is if household demands for food are conditional on the dishes it prepares, then the unconditional quantities demanded for the raw foods cannot in general be observed. However, they can be estimated using equation (4.4.26).

One important feature which turns out to simplify greatly the analysis of the model, is the fact that the vector price of all the raw foods P, and the meal budget d, is the same for all dishes. Only the dish preparation time τ_i is different from one dish to another. Hence the observable components of the conditional indirect utilities can be decomposed as:

$$(4.4.27) \qquad V_i(P_i,d) = V(P,d,\theta) + \gamma w\tau i \qquad i=1,\ldots,m$$

where w is the opportunity cost of time and (θ,γ) is the vector of parameters common to all dishes.

This parametrization with a common vector of observable attributes and the same vector of parameters for all alternatives, is standard in discrete choice models, and is made possible by putting all the alternative specific attributes in the error term ϵ_i.

It then follows:

$$(4.4.28) \qquad V_i^k = V_i(P_i,d) - V_k(P_i,d) = \gamma w\cdot(\tau_i-\tau_k) = \gamma r_i^k$$

for $k\neq i$ and $k=1,\ldots,m$.

where $r_i^k = w(\tau_i-\tau_k)$

and the conditional demands are given by:

$$(4.4.29)\qquad X_{ij} = \frac{\partial V(P,d,\theta)/\partial P_j}{\partial V(P,d,\theta)/\partial d} + \lambda_j^i(\gamma r_i^1,\ldots,\gamma r_i^{j-1},\gamma r_i^{j+1},\ldots,\gamma r_i^m) + \eta_{ij}$$

if food j is used in dish i

and

$$X_{ij} = 0 \qquad \text{if food j is not used in dish i}$$

And since

$$\sum_{i=1}^{m}\left[\frac{-\partial V_i(P_i,d)/\partial P_j}{\partial V_i(P_i,d)/\partial d}\right]\cdot\pi_i = \sum_{i=1}^{m}\left(\frac{-\partial V(P,d,\theta)/\partial P_j}{\partial V(P,d,\theta)/\partial d}\right)\cdot\pi_i$$

$$= \frac{\partial V(P,d,\theta)/\partial P_j}{\partial V(P,d,\theta)/\partial d}$$

(where the last equality follows from the fact that $\sum_{i=1}^{m}\pi_i = 1$)

the unconditional mean demands of the raw foods are:

$$(4.4.30)\qquad E(X_j) = \frac{\partial V(P,d,\theta)/\partial P_j}{\partial V(P,d,\theta)/\partial d}$$

$$+ \sum_{i=1}^{m} \lambda_j^i(\gamma r_i^1,\ldots,\gamma r_i^{i-1},\gamma r_i^{i+1},\ldots,\gamma r_i^m)\cdot\pi_i$$

$$j=1,\ldots,J$$

To further simplify the notation, define ξ_i and r_i as the two (m-1) dimensional vector whose k^{th} elements are $\xi_i^k=\epsilon_k-\epsilon_i$ and $r_i^k=w\cdot(\tau_i-\tau_k)$ respectively, for $k\neq i$. It can be shown that $\xi_i=A_i\epsilon$ where A_i is the (m-1)xm matrix with ones in the diagonal, minus ones in its i^{th} column, and zero everywhere else. That is:

$$A_i = \begin{bmatrix} 1 & 0\ldots0 & -1 & 0\ldots0 \\ 0 & 1\ldots0 & -1 & 0\ldots0 \\ . & \ldots\ldots & . & \ldots\ldots \\ . & \ldots\ldots & . & \ldots\ldots \\ . & \ldots\ldots & . & \ldots\ldots \\ 0 & 0 \quad\; 0 & -1 & 0\ldots1 \end{bmatrix}$$

So ξ_i is a random vector with mean $E(\xi_i)= A_iE(\epsilon) = 0$ and covariance matrix $\Omega_i= \text{cov}\,(A_i\epsilon) = A_i\Sigma_\epsilon A_i'$ where Σ_ϵ is the covariance matrix of ϵ.

With these notations and change in variables, it is easy to see that $\pi_i = P\{d_i=1\} = P\{\xi_i\leq\gamma r_i\} = F_{\xi_i}(\gamma ri)$ $i=1,\ldots,m$.
where F_{ξ_i} is the joint distribution function of ξ_i. To simplify the notation we will write it as F_i and omit the subscript ξ_i.

Hence, the estimable equations of the food consumption model can be compactly summarized as:

$$(4.4.31) \qquad \pi_i = P\{d_i=1\} = F_i(\gamma r_i) \qquad i=1,\ldots,m$$

$$(4.4.32) \qquad X_{ij} = -\frac{\partial V(P,d,\theta)/\partial P_j}{\partial V(P,d,\theta)/\partial d} + \lambda_j^i(\gamma r_i) + \eta_{ij}^* \qquad j=1,\ldots,m$$

$$(4.4.33) \qquad d = d(y,e_N,\ c) + \epsilon^*$$

From these three equations we can derive the unconditional mean demands.

$$(4.4.34) \quad E(X_j) = -\frac{\partial V(P,d,\theta)/\partial P_j}{\partial V(P,d,\theta)/\partial d} + \sum_{i=1}^{m} \lambda_j^i(\gamma r_i)\cdot F_i(\gamma r_i) \quad j=1,\ldots,J$$

where J_i is the number of raw foods used in dish i, and $J = \sum_{i=1}^{m} J_i$ is the number of all raw foods used by the household.

Note that we could write equation (4.4.34) in regression form as

$$(4.4.35) \quad X_j = -\frac{\partial V(P,d,\theta)/\partial Pj}{\partial V(P,d,\theta)/\partial d} + \sum_{i=1}^{m} \lambda_j^i \, (\gamma r_i)\cdot F_j(\gamma r_i) + \eta_j^*$$

$$j=1,\ldots,J.$$

where η_j^* is an error term with zero mean and variance implicitly defined. Had we been able to estimate (4.4.35), we would not need equation (4.4.32) to estimate the unconditional mean demands. But, equation (4.4.35) is useless because we cannot observe the X_j's which are the unconditional quantities demanded for the raw foods. Indeed, all we observe are the X_{ij} which are the raw food quantities demanded given a particular dish is selected.

We also note that if the dish choices were not correlated with the demands for the raw foods, we would have $\lambda_j^i(\gamma r_i)=0$, and $X_{ij}=X_j$ for all j; and Equations (4.4.32) and (4.4.34) would be identical and would correspond to the traditional system of demand equations. Then, they could be estimated separately without reference to equation (4.4.31) which defines the dish choices. Thus, it is clear that the traditional model of demand for food is a special case of this model, corresponding to the restriction that the demands for the raw foods are uncorrelated with the dish choices. The empirical validity of this restriction can be tested easily once the unrestricted model is estimated.

4.4.6 Dish selection as a cause for zero quantity reported

We have already noted that one of the implications of the model is that the unconditional quantities demanded cannot be observed. We shall argue here that this is probably the main reason why household food consumption surveys usually have a relatively large number of zero quantities observed.

In general, there are two levels of censoring. The first level come from the fact that each household can be considered as having a set of dishes that it prepares all year long. For each meal, the household picks one dish from this set. Hence, if a given raw food is not used in any of the dishes in the set, then the household will almost never buy this raw food. This type of censoring is important when the population surveyed is not homogenous (different ethnic groups, location, etc.) so that the set of dishes can be different from one household to the other.

The second level of censoring comes from the fact that the dishes in a given household set of dishes do not necessarily use the same type of raw foods as inputs. Hence, since the household selects only one dish for a given meal, then it will buy only the raw foods that must be used in the selected dish. This level of censoring is pertinent in cross section analysis because the surveys are generally one-shot surveys or at best of limited duration, so that data on some raw foods may be missing (zero quantity reported) because at the time of the survey, the household did not consume any dish using these raw foods. This fact must not be ignored.

These two sources of "zero quantity reported" should not be confused with an effective zero quantity consumed resulting from a conscious household decision not to buy a particular raw food (inessential for the selected dish) because of its price. An example of this is reported in the preliminary results of the ongoing food consumption survey conducted in Niger (Caputo et al., 1989) where some households said that for a given dish, there are some "inessential" raw foods usually used in the dish which they would buy only if they can afford them. In such a case, there is no missing observation; the quantity demanded is effectively zero.

The statistical consequences of selection bias (bias and inconsistency of the estimators and wrong inferences) can be severe and are well documented (see for example, Heckman (1976, 1979); Lee (1978, 1989); and Newey et al. (1990). Selection bias has the same effects as specification errors (Heckman, 1979). In many cases, differences in magnitude between the estimated coefficients under correction of the selection bias and the coefficients without correction can be very large. Sometimes, not only is the absence of selection bias rejected, but the latter coefficients have the wrong signs (example, Newey et al.

1990). This should be of concern when the estimated elasticities are used for policy recommendations.

In food demand analysis using a system approach with cross section data, one will necessarily have to deal with this censoring problem, because the system is constituted by the individual demands for all the raw foods and the observations are by household. Hence, theoretically for each sampling unit (household) we should have a complete system of demand equations. However, in practice almost for any observation unit (household) there will be many "zero quantity reported" since the household do not consume all the raw foods at the time the survey is conducted (unless the household is followed for a long time and the data aggregated).

Instead of asking the question: "why are zero quantities reported?", one can either remove from the sample those units with "missing observations", which may reduce severely the sample size (up to zero sample size if we have only a one-shot survey with no possibility for aggregation), or one may choose to put zero actual quantity demanded in the place of the "zero quantity reported," a procedure that will be incorrect. Another possibility is to rearrange the system by grouping the observations according to the raw foods and then use the seemingly unrelated regression (SUR) method with an unequal number of observations (see Schmidt, 1977), but this method ignores the behavioral implications of the complete system with its implied restrictions (for example, the adding up restrictions are not maintained).

All the above methods suffer from the problem of selection bias. This can jeopardize the reliability of the estimates.

When faced with missing observations, the critical question, to paraphrase Heckman (1979), is: why are the observations missing? The answer to this question will direct you to the appropriate method of estimation. In the next chapter, we will explore methods of estimating these models with selection bias.

4.5 Parametrization of the conditional indirect utility functions:

So far, all the derivations and results obtained do not depend on the functional forms of either the unconditional indirect utility or the probability distribution of (η,ϵ). The exact functional forms are always unknown, but an appropriate parametrization can give good approximations of them. One criterion for the choice of functional form is flexibility, that is, the ability of the chosen parametrization to allow for the maximum of substitutions among choices and yet be computationally feasible. Specifically, we want the chosen parametrization of the conditional indirect utility to allow all kinds of substitution among raw foods, and the probability distribution of (η,ϵ) to not restrict substitutions among dishes in the household's feasible choice set. The choice of an appropriate probability distribution for (η,ϵ) is taken up in the next chapter dealing with estimation. The remainder of this chapter is devoted to the choice of functional form for the conditional indirect utility.

Recent developments in estimation of production and demand functions emphasize the use of flexible functional forms to approximate, up to some degree of accuracy, any unknown but twice continuously differentiable indirect utility or expenditure function. By definition, a function is said to be second order flexible at some point if it contains enough free parameters so that its level, first derivatives and all of its second partial derivatives at that point coincide with those of the true function at the same point (Diewert and Wales, 1987).

Many flexible functional forms have been derived (see Diewert and Wales, 1987 for a brief review), among them, the best known are the translog (Christensen, Jorgenson, and Lau; 1971 and 1975), the generalized Leontief (Diewert, 1971) and the AIDS (almost ideal demand systems) (Deaton and Muellbauer; 1980 and 1984). These flexible functional forms differ mainly by the difficulties involved in estimating their parameters, and/or imposing or testing the restrictions implied by demand theory.

For simplicity and ease of exposition, we will use the AIDS functional form for the conditional indirect utility just to illustrate how the model can be parametrized. But, when implementing the estimation procedure, one may want to try other flexible functional forms and pick the one that fits best. However, before deriving the AIDS conditional demands, we will present a possible parametrization of the meal budget as a function of household characteristics.

4.5.1 Parametrization of the Meal Budget.

In section (4.3.2), we discussed how the household meal budget depended on income, nonfood expenditures and other household characteristics. We hypothesized that the meal budget is given by the function

$$d = d\ (y, e_N, c).$$

Following the discussion in section (4.3.2) on the extent to which each variable affects the meal budget d, we can postulate the following functional form:

(4.5.1) $$\log d = \alpha_0^* + \alpha_1^* \log(y) + \alpha_2^* \log(e_N) + \alpha_3^* \log z + C' \cdot \beta + \epsilon^*$$

where y is the household income level,

e_N is the household expenditure on nonfood items

z is the household size or the number of adult-equivalents in the household

C' is a vector of dummy variables representing other household characteristics (e.g., ethnicity, location etc.)

α_0^*, α_1^*, α_2^*, α_3^*, and β^* are unknown parameters; and ϵ^* is a disturbance term.

4.5.2 The AIDS conditional indirect utility functions:

The AIDS conditional expenditure function (Deaton and Muellbauer, 1984) is defined as:

(4.5.2) $$\text{Log } e(P,u,\theta) = a(P,\theta) + u \cdot b(P,\theta)$$

where $$a(P,\theta) = \alpha_0 + \sum_{k=1}^{J} \alpha_k \log P_k + \frac{1}{2} \sum_{k=1}^{J} \sum_{\ell=1}^{J} \gamma_{k\ell}^* \log P_k \cdot \log P_\ell$$

$$\log b(P,\theta) = \log\beta_0 + \sum_{K=1}^{J} \beta_k \log P_k$$

and α_k, β_k, and $\gamma^*_{k\ell}$ $k,\ell = 1,\ldots,J$ are parameters $P=(P_1,\ldots,P_J)$ is the vector price of the raw food.

By inverting $e(P,u)$ we get the conditional indirect utility function:

$$(4.5.3) \qquad V(P,d,\theta) = \frac{\log d - a(P,\theta)}{b(P,\theta)}$$

(omitting $\lambda_j^i(\gamma r_i)$ for now)

Then by Roy's identity we get the conditional Marshallian demands

$$(4.5.4) \qquad X_{ij} = -\frac{\partial V(P,d,\theta)/\partial P_j}{\partial V(P,d,\theta)/\partial d} = \frac{d}{P_j}\left[\alpha_j + \sum_{k=1}^{J} \gamma_{jk}\log P_k + \beta_j \log\frac{d}{P^x}\right]$$

or written in "dish shares" form

$$(4.5.5) \quad c(i,j) = P_j \cdot X_{ij} = d\left(\alpha_j + \sum_{k=1}^{J} \gamma_{jk}\log P_k + \beta_j \log\frac{d}{P^x}\right)$$

or in budget shares form

$$(4.5.6) \quad S_{ij} = \frac{P_j \cdot X_{ij}}{d} = \alpha_j + \sum_{k=1}^{J} \log P_k + \beta_j \log\frac{d}{P^x}$$

where $\log P^x = \alpha_0 + \sum_{k=1}^{J} \alpha_k \log P_k + \frac{1}{2} \sum_{k=1}^{J} \sum_{\ell=1}^{J} \gamma_{k\ell} \log P_k \cdot \log P_\ell$

P^x is defined to be a price index. S_{ij} is the meal budget share of food j.

The parameters $\gamma_{k\ell}$ are defined by:

$$\gamma_{k\ell} = \frac{1}{2} (\gamma_k^* + \gamma_\ell^*) = \gamma_{\ell k}$$

The theoretical restrictions on the conditional demands apply directly to the parameters (See Deaton and Muellbauer, 1980 and 1984 for details). They are for a given dish i:

Adding up: $\sum_{k=1}^{J} \alpha_k = 1$; $\sum_{k=1}^{J} \beta_k = 0$; $\sum_{k=1}^{J} \gamma_{kj} = 0$; for $j=1,\ldots,J$.

Homogeneity: $\sum_{k=1}^{J} \gamma_{jk} = 0$ for $j=1,\ldots,J$.

Symmetry: $\gamma_{k\ell} = \gamma_{\ell k}$ for $\ell,k = 1,\ldots,J$.

Negativity of the Substitution Matrix:

The matrix $T = (\psi_{k\ell})$ is negative semi-definite where

$$\psi_{k\ell} = \gamma_{k\ell} + \beta_k \beta_\ell [\log d - a(P,\theta)] - S_{i\ell}\delta_{k\ell} - S_{ik}S_{i\ell}$$
$$\text{for } k,\ell = 1,\ldots,J.$$

with $\delta_{\ell k}$ being the Kronecker delta that is unity if $\ell = k$ and zero otherwise.

For more details about the issues involved in estimating the AIDS model, the reader is referred to the original paper of Deaton and Muellbauer (1980).

The 3 forms of the conditional demands given in (4..5.4), (4.5.5) and (4.5.6) are equivalent. However, their relative merits depend on the setting of the data collection. Indeed, in many situations it is relatively difficult to get good measurements of the quantity X_{ij} of the different raw foods consumed by the household. This is especially true in developing countries where most often the households buy and use raw foods in a form lacking a well-defined unit of measurement. To deal with this problem, the enumerators in consumption studies need to know the unit of measurement of each household, then use some conversion factor. This of course introduces additional measurement errors in estimating equation (4.5.4).

In contrast, with equations (4.5.5) and (4.5.6) you need not know these quantities, but can directly use c(i,j), the monetary cost or dish share of the raw food j in the preparation of dish i. In most cases, this is more readily available and more accurately evaluated by the household, and potentially facilitates the task of both the household and the enumerator. Besides, this may be closer to the way the household allocates its meal budget among the different food items it needs to prepare its selected dish. In instances where it is easier to collect data on the quantities X_{ij}, the c(i,j) can be computed easily by getting the unit price either from the household or from the market.

In practice, before estimating (4.4.6), one usually approximate P^x by a known price index like Stone's price index defined by $\text{Log}P^x = \Sigma\, \omega_k \log P_k$ where the ω_k are weights. This was originally suggested by Deaton and Muellbauer (1980 and 1984). If this is done, then the Marshallian demand equations will be linear in the parameters.

Also, given a selected dish i, there is no reason that the conditional demand X_{ij} of a raw food j used in dish i should be influenced by the price of a raw food not used in dish i except through the price index P^x, and the dish choice effects $\lambda_j^i(\gamma r_i)$. Hence we can limit the list of prices appearing in the conditional Marshallian demands only to the prices of those raw foods used in the same dish. Equation (4.5.6) then becomes :

$$(4.5.7) \quad S_{ij} = \alpha_j + \sum_{k=1}^{J_i} \gamma_{jk} \mathrm{Log} P_k + \beta_j \log(d/p^x) = Z_i \cdot \theta_j$$
$$j=1,\ldots,J_i$$

where Z_i is the J_i+2 dimensional vector defined by: $Z_i=(1, \log P_1, \ldots, \log P_{J_i}, \log(d/px))'$ and θ_j is the J_i+2 dimensional vector of parameters defined by $\theta_j=(\alpha_j, \gamma_{j1}, \ldots, \gamma_{jJ_i}, \beta_j)'$.

The budget equation can also be written in vector form as:

$$(4.5.8) \quad \mathrm{Log}\ d = Z^{*\prime} \cdot \theta^* + \epsilon^*$$

where $\theta^*=(\alpha_0^*, \alpha_1^*, \alpha_2^*, \beta^{*\prime})'$ is the parameter vector and $Z^* = (1, \log(y), \log(e_N), z, C')'$ is the vector of household characteristics.

Given the above notations, the equations of the probabilistic choice (PC)-AIDS model for one household can be written as:

PC-AIDS Model

$$(4.5.9) \quad \pi_i = P\{d_i=1\} = Fi(\gamma r_i) \qquad i=1,\ldots,m.$$

$$(4.5.10)\quad S_{ij} = \frac{c(i,j)}{d} = Z_i' \cdot \theta_j + \lambda_j^i(\gamma r_i) + \eta_{ij}^* \qquad j=1,\ldots,J_i.$$

$$(4.5.11)\quad \log d = Z^{*'} \cdot \theta^* + \epsilon^*$$

$$(4.5.12)\quad E(X_j) = \sum_{i=1}^{m} F_i(\gamma r_i) \cdot \left[\frac{d}{P_j}\right] \cdot Z_i' \cdot \theta_j + \sum_{i=1}^{m} F_i(\gamma r_i) \cdot \lambda_j^i(\gamma r_i).$$

The equations in (4.5.10) can be rewritten in a more compact way by defining

$S_i=(S_{i1},\ldots,S_{iJ_i})'$ a $J_i \times 1$ vector

$\lambda^i(\gamma r_i) = (\lambda_1^i(\gamma r_i),\ldots,\lambda_{J_i}^i(\gamma r_i))'$ a $J_i \times 1$ vector

$\eta_i^* = (\eta_{i1}^*,\ldots,\eta_{iJ_i}^*)'$ a $J_i \times 1$ vector

and $\theta^i = (\theta_1',\ldots,\theta_{J_i}')'$ a $J_i(J_i+2) \times 1$ vector

then the J_i equations in (4.14.9) are equivalent to the following single vector representation:

$$(4.5.13)\quad S_i = (I_{J_i} \otimes Z_i')\theta_i + \lambda^i(\gamma r_i) + \eta_i^*$$

where I_{J_i} is the identity matrix of dimension J_i and $\otimes$ is the Kronecker product.

We can finally present the PC-AIDS equations in a intentionally suggestive order referring to how we think the household actually makes its food consumption decisions.

PC-AIDS.

$$\text{(4.5.14)} \quad \text{Log } d = Z^{*'} + \epsilon^{*}$$

$$\text{(4.5.15)} \quad \pi_i = P(d_i = 1) = Fi(\gamma r_i) \qquad i = 1, \ldots, m.$$

$$\text{(4.5.16)} \quad S_i = (I_{J_i} \otimes Z_i')\theta^i + \lambda^i(\gamma r_i) + \eta_i^*$$

$$\text{(4.5.17)} \quad E(X_j) = \sum_{i=1}^{m} F_i(\gamma r_i) \cdot \left[\frac{d}{P_j}\right] \cdot (Z_i'\theta_j) + \sum_{i=1}^{m} F_i(\gamma r_i) \cdot \lambda_j^i(\gamma r_i)$$

$$j = 1, \ldots, J$$

That is, the household determines first the meal budget according to its characteristics. Next, subject to the meal budget constraint, it chooses one dish in the set of feasible dishes according to its "tastes". Then, given the technology of the chosen dish, and the relative prices of the raw food needed for the dish, it determines how much of each raw food to buy. The final equation permits us to compute the unconditional mean demands for all the raw foods consumed by the household.

CHAPTER 5

ESTIMATION OF THE MODEL

This chapter is concerned with methods of estimating the model using the AIDS conditional indirect utility function. The use of the AIDS functional form is for simplicity, and is without loss of generality as long as the estimation methods and the asymptotic properties are concerned. The conclusions in this chapter will be valid for any other flexible functional form.

The organization of the chapter is as follows: In section 1, we derive the explicit expressions of $\lambda_j^i(\gamma r_i)$ and of the covariance matrix of η_i^*, under the assumption that ϵ and η are jointly normally distributed. Section 2 gives the likelihood function of a sample of N households and discusses briefly the maximum likelihood estimator of the parameters. In Section 3, we present a computationally feasible method of estimating the model; namely the two-stage Heckman method. This method yields consistent estimates and is the one generally used in problem of selectivity bias.

5.1. The Model Under Normality

Let the joint distribution of (ϵ, η) be normal with zero mean and covariance matrix $\mathrm{Cov}[(\epsilon, \eta)] = \Sigma$, a $(m + \sum_{i=1}^{m} J_i) \times (m + \sum_{i=1}^{m} J_i)$ dimensional matrix. We can partition Σ in blocks as:

$$\Sigma = \begin{bmatrix} \Sigma_\epsilon & \Sigma_{\epsilon\eta} \\ \Sigma_{\eta\epsilon} & \Sigma_\eta \end{bmatrix}$$

where

$$\Sigma_{\eta\epsilon} = [\Sigma_{\epsilon\eta_1} \ . \ . \ . \ \Sigma_{\epsilon\eta_m}] \quad \text{a} \quad m \times \sum_{i=1}^{m} J_i \quad \text{matrix}$$

and

$$\Sigma_{\epsilon\eta_i} = \begin{bmatrix} \Sigma_{\epsilon_1\eta_i} \\ \vdots \\ \Sigma_{\epsilon_m\eta_i} \end{bmatrix} = \begin{bmatrix} \Sigma_{\epsilon_1\eta_{i1}} & . \ . \ . & \Sigma_{\epsilon_1\eta_{iJ_i}} \\ \vdots & & \vdots \\ \Sigma_{\epsilon_m\eta_{i1}} & . \ . \ . & \Sigma_{\epsilon_m\eta_{iJ_i}} \end{bmatrix}$$

a $m \times J_i$ matrix For $i = 1,\ldots,m$.

with

$\Sigma_\epsilon = \text{cov}(\epsilon)$ a $m{\times}m$ matrix; $\Sigma_\eta = \text{cov}(\eta)$ a $\sum_{i=1}^{m} J_i \times \sum_{i=1}^{m} J_i$ matrix.

$\Sigma_{\eta\epsilon} = \Sigma'_{\epsilon\eta}$ is the matrix of covariances of η and ϵ.

$\Sigma_{\epsilon\eta_i} = \Sigma'_{\eta_i\epsilon}$ is the matrix of covariances of η_i and ϵ,

$\Sigma_{\epsilon_k\eta_i} = \Sigma'_{\eta_i\epsilon_k}$ is the matrix of covariances of η_i and ϵ_k

$\Sigma_{\epsilon_k\eta_{ij}} = \Sigma'_{\eta_{ij}\epsilon_k}$ is the matrix of covariances of ϵ_k and η_{ij}

For $i, k = 1,\ldots,m$., $j = 1,\ldots,J_i$

The reason for this partition will be clear later.

Recall from Chapter 4 that ξ_i and η^*_i were defined as:

$\xi_i = A_i\epsilon$

where

$$A_i = \begin{bmatrix} 1 & 0 \ldots 0 & -1 & 0 \ldots 0 \\ 0 & 1 \ldots 0 & -1 & 0 \ldots 0 \\ \vdots & \vdots \quad \vdots & \vdots & \vdots \quad \vdots \\ 0 & 0 \quad\; 0 & -1 & 0 \quad\; 1 \end{bmatrix}$$

a $(m-1)\times m$ matrix.

with $E\xi_i = A_i E\epsilon = 0$ and $\text{cov}(\xi_i) = \Omega_i = A_i \Sigma_\epsilon A_i'$

and $\eta_i^* = (\eta_{i1}^*, \ldots, \eta_{iJ_i}^*)$

where $\eta_{ij}^* = X_{ij} - E[X_{ij} | d_i = 1] \quad j = 1,\ldots,J_i$

with $E\eta_{ij}^* = 0$ and $\text{Var}[\eta_{ij}^*] = E[\eta_{ij}^{*2} \mid d_i = 1] - [\lambda_j^i(\gamma r_i)]^2$

Recall also that $\{d_i = 1\} = \{\xi_i < \gamma r_i\}$,

$\lambda_j^i(\gamma r_i) = E[\eta_{ij} \mid d_i=1] = E[\eta_{ij} \mid \xi_i < \gamma r_i]$ and

$$\lambda^i(\gamma r_i) = [\lambda_1^i(\gamma r_i), \ldots, \lambda_{Ji}^i(\gamma r_i)]' = E[\eta_i \mid \xi_i < \gamma r_i]$$

a $J_i \times 1$ dimensional vector.

In what follows, we shall derive the explicit expressions of $\lambda^i(\gamma r_i)$ and $\text{cov}(\eta_i^*)$.

Define: $R = E(\eta_i \xi_i').\text{cov}(\xi_i)^{-1}$ a $J_i \times (m-1)$ matrix.

and

$$Q = \text{Cov}(\eta_i) - RE(\xi_i \eta_i') \text{ a } J_i \times J_i \text{ matrix.}$$

then it can be shown that (See Amemiya, 1985, pp. 406-407 or Duncan, 1980, page 851):

$$(5.1.1) \quad \lambda^i(\gamma r_i) = E[\eta_i \mid \xi_i < \gamma r_i] = RE[\xi_i \mid \xi_i < \gamma r_i]$$

and

(5.1.2) $\quad \mathrm{Cov}(\eta_i^*) = R\ \mathrm{cov}(\xi_i \mid \xi_i < \gamma r_i)\ R' + Q$

This result is merely a multivariate generalization of the well-known facts about the conditional distribution of a random variable jointly normally distributed with another random variable. They can be obtained easily by noting that ξ_i and $\eta_i - R\xi_i$ are independent.

The conditional moments of $\xi_i | \xi_i < \gamma r_i$ were derived by Tallis (1961) in terms of the correlation matrix of ξ_i. Amemiya (1974) rewrote Tallis's results in terms of the covariance matrix of ξ_i. We will rewrite Amemiya's results in matrix notation in a way suitable to our model.

Using Amemiya's notation, let f_k^i be the marginal density of the k^{th} variable of ξ_i, $f^i_{(k)}$ the joint conditional density of the remaining m-2 variables given that the k^{th} variable of ξ_i is equal to $\gamma r^k{}_i$, $f^i_{k\ell}$ the joint marginal density of the k^{th} and ℓ^{th} variables of ξ_i, and $f^i_{(k\ell)}$ the joint conditional density of the remaining m-3 variables given that the k^{th} and ℓ^{th} variables of ξ_i are equal to γr_i^k and γr_i^ℓ respectively.

Also define:

$$F^i_{(k)} = F^i_{(k)}(\gamma r_{ik}) = \prod_{\ell \neq k}^{m-1} \int_{-\infty}^{\gamma r_i^\ell} f^i_{(k)}(\lambda) d\lambda$$

where r_{ik} means r_i without its k^{th} element

$$F^i_{(k\ell)} = F^i_{(k\ell)}(\gamma r_{ik\ell}) = \prod_{s \neq k,\ell}^{m-1} \int_{-\infty}^{\gamma r_i^s} f^i_{(k\ell)}(\lambda) d\lambda$$

where $r_{ik\ell}$ means r_i without its k^{th} and ℓ^{th} element.

let a^i be the m-1 dimensional vector whose k^{th} element is

$$a_k^i = a_k^i(\gamma r_i) = f_k^i(\gamma r_i^k)\ F_{(k)}^i(\gamma r_{ik})$$

and B^i the (m-1) × (m-1) dimensional matrix whose k^{th}, ℓ^{th} element is:

$$b_{k\ell}^i = b_{k\ell}^i(\gamma r_i) = f_{k\ell}^i(\gamma r_i^k,\ \gamma r_i^\ell)\ F_{(k\ell)}^i(\gamma r_{ik\ell})$$

To further simplify the notation, let the truncated variable

$\xi_i \mid \xi_i < \gamma r_i = W$ and

$F_i = F_i(\gamma r_i)$

With these notations, it can be easily verified that equations (2.7) and (2.8) in Amemiya (1974, pp. 1002-1003) can be respectively rewritten in matrix form as:

$$(5.1.3)\qquad EWW' = \Omega_i + \frac{1}{F_i}\,\Omega_i[D(\frac{\gamma r_i^k a_k^i - \omega_k^{i\prime} b_k^i}{\omega_{kk}^i}) + B^i]\Omega_i$$

$$(5.1.4)\qquad EW = \frac{1}{F_i}\,\Omega_i a^i$$

where $\Omega_i = A'_i\Sigma_\epsilon A_i = \mathrm{Cov}\ (\xi_i)$ was defined earlier. ω_k^i is the k^{th} column of Ω_i and $\omega^i{}_{kk}$ its k^{th} diagonal element. $D(\cdot)$ is a diagonal matrix for which the term in the parentheses is the k^{th} diagonal element. b_k^i is the k^{th} column of B^i.

From (5.1.3) and (5.1.4) we can get the covariance matrix of W by:

$$(5.1.5)\qquad \mathrm{Cov}\ (W) = EWW' - EW\cdot EW'$$

$$= \Omega_i + \Omega_i[\frac{1}{F_i}D(\frac{\gamma r_i^k a_k^i - \omega_k^i b_k^i}{\omega_{kk}^i}) + \frac{1}{F_i}\ B^i - \frac{1}{F_i^2}\ a^i a^{i\prime}]\Omega_i$$

Now with these notations we have from (5.1.1) and (5.1.2)

$$\lambda^{i}(\gamma r_{i}) = REW \quad \text{and} \quad Cov(\eta_{i}^{*}) = RCov(W)R' + Q$$

but

(5.1.6) $$R = E(\eta_{i}\xi_{i}')\Omega_{i}^{-1} = E[\eta_{i}\epsilon' A_{i}']\Omega_{i}^{-1} = \Sigma_{\eta_{i}\epsilon}A_{i}'\Omega_{i}^{-1}$$

and

(5.1.7) $$Q = \Sigma_{\eta_{i}} - RA_{i}\Sigma_{\epsilon\eta_{i}} = \Sigma_{\eta_{i}} - \Sigma_{\eta_{i}\epsilon}A_{i}'\Omega_{i}^{-1}A_{i}\Sigma_{\epsilon\eta_{i}} = \Sigma_{\eta_{i}} - R\Omega_{i}R'$$

hence using (5.1.4) and (5.1.6) we have

(5.1.8) $$\lambda^{i}(\gamma r_{i}) = \frac{1}{F_{i}}\Sigma_{\eta_{i}\epsilon}A_{i}'a^{i} \qquad \text{(a } J_{i} \times 1 \text{ vector)}$$

and using (5.1.5) and (5.1.7) we have

(5.1.9) $$Cov(\eta_{i}^{*}) = \Sigma_{i}^{*}$$

$$= \Sigma_{\eta_{i}} + \Sigma_{\eta_{i}\epsilon}A_{i}'[\frac{1}{F_{i}}D(\frac{\gamma r_{i}^{k}a_{k}^{i} - \omega_{k}^{i'i}b_{k}}{\omega_{kk}^{i}}) + \frac{1}{F_{i}}B^{i} - \frac{1}{F_{i}^{2}}a^{i}a^{i'}]A_{i}\Sigma_{\eta_{i}\epsilon}'$$

(a $J_{i} \times J_{i}$ matrix)

$\lambda_{j}^{i}(\gamma r_{i})$, the j^{th} element of $\lambda^{i}(\gamma r_{i})$, can be obtained from (5.1.8) and is given by:

(5.1.10) $$\lambda_{j}^{i}(\gamma r_{i}) = \frac{1}{F_{i}}[\sum_{\substack{k=1 \\ k\neq i}}^{m} a_{k}^{i}(\Sigma_{\eta_{ij}\epsilon_{i}} - \Sigma_{\eta_{ij}\epsilon_{k}})]$$

The expression of $\lambda_{j}^{i}(\gamma r_{i})$ is very interesting and open to interpretations with respect to how the different dishes in the household's choice set affect the conditional demands of the raw foods[2]. We will see later that only the term

$\delta^{j}_{ik} = \Sigma_{\eta_{ij}\epsilon_i} - \Sigma_{\eta_{ij}\epsilon_k}$ can be identified.

Note that the restriction that leads to the traditional model is:

(5.1.11) $\Sigma_{\eta_i\epsilon} A_i' a^i(\gamma r_i) = 0$

which leads to $\Sigma_{\eta_i\epsilon} A_i' = 0$ or $a^i(\gamma r_i) \in$ kernel of $\Sigma_{\eta_i\epsilon} A_i'$

5.2. Methods of Estimation Under Normality

5.2.1. Assumptions and Notations

In this section, in addition to the normality assumption, we assume a random sample of N households having the same set of dish choices. The case of different choice sets for different households can be handled by a slight modification of the procedure followed here (by allowing the number of dishes m to depend on the household), by the usual treatment of sample selection bias, or by stratifying the population according to the choice set and perform separate regressions for each section of the population.

Before rewriting the model in estimation format, we introduce the following binary variables and notations.

$d_{in} = 1$ if the n^{th} household selects dish i

$= 0$ otherwise

and

$P(d_{in} = 1) = F_{in}(\gamma_0 r_{in}) = E(d_{in})$ $i = 1,\ldots,m;\ m = 1,\ldots,N$

Where γ_0 is the time parameter value. Accordingly, we will add a subscript n to designate the n^{th} household, to all the relevant variables that vary from household to household. We will also add a subscript 0 to the parameters to differentiate between a true but unknown value of a parameter and a particular value in the parameter space. Sometimes when there is no confusion we will drop some of the subscripts and/or arguments to simplify the notation.

Given these specific points, we can write the estimable equations (leaving out the unconditional mean demand equations) of the P.C-AIDS model as:

(5.2.1) $\text{Log } d_n = Z_n^{*\prime} \theta_0^* + \epsilon_m^*$

(5.2.2) $P(d_{in} = 1) = F_{in}(\gamma_0 r_{in})$

(5.2.3)
$$s_{in} = (I_{Ji} \otimes Z'_{in}) \theta_0{}^i + \frac{1}{F_{in}(\gamma_0 r_{in})} \Sigma_{0\eta_i\epsilon} A'_i a^i(\gamma_0 r_{in}) + \eta_{in}^*$$

$n = 1, \ldots, N$

$i = 1, \ldots, m$

Where ϵ_m^* are iid random variables with zero mean and variance ν^2. The η_{in}^* are also independent for given i with zero means, and covariance matrix Σ_{in}^* given by (5.1.9), and g_{in} as density function F_{in} is the distribution function of $\xi_{in} = A_i \epsilon_n$ where the (ϵ_n, η_n) are iid normal across n with zero mean and covariance matrix Σ_0 given in Section 5.1.

All the other variables, parameters, and constants are defined as in Section (4.5.2) and (5.1). To further simplify the notation, let $\theta = (\theta^{1\prime}, \ldots, \theta^{m\prime})'$ and define:

(5.2.4) $\bar{s}_{in} = \bar{s}_{in}(Z_{in}, r_{in}, \theta_0^i, \gamma_0) = (I_{J_i} \otimes Z'_{in}) \delta_0^i + \lambda^i(\gamma_0 r_{in})$

Note that if ϵ_n^* is independent of ϵ_n and η_{in}^* for all n, then equations 5.2.1 can be estimated separately without loss of efficiency. That is what we will assume so that we can concentrate on the two other equations since (5.2.1) is a standard regression equation.

The following technical assumptions which are adapted from Amemiya (1985, p. 288) are needed for the consistency and asymptotic efficiency of the M.L.E. estimator to hold.

Assumption 5.2.1: The parameter space is an opened bounded subset of the Euclidean space.

Assumption 5.2.2: $\{r_{in}\}$ and $\{Z_{in}\}$ are respectively uniformly bounded in n for every i. Furthermore, the empirical distribution functions of $\{r_{in}\}$ and $\{Z_{in}\}$ converge to some distribution functions for every i.

Assumption 5.2.3: For each i, $\lambda_{\ell}(\Sigma^{*-1}_{0in})$, the largest characteristic roots of Σ^{*-1}_{0in} are uniformly bounded for all n. Furthermore,

$$\lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} r'_{in}r_{in} \quad \text{and} \quad \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} (I_{J_i} \otimes Z_{in})\, \Sigma^{*-1}_{in}(I_{J_i} \otimes Z'_{in})$$

are finite nonsingular for every i.

5.2.2. Maximum Likelihood Estimation (M.L.E.)

With these assumptions and notations, the likelihood of observing the n^{th} household selecting a particular dish along with the necessary quantities of raw foods is given by:

(5.2.5)

$$L(\theta,\gamma,\Sigma) = \prod_{i=1}^{m} \{g_{in}[S_{in} - \hat{S}_{in}(Z_{in}, r_{in}, \theta, \gamma)]\, F_{in}(\gamma r_{in})\}^{d_{in}}$$

But the conditional density g_{in} of η^*_{in} is given by:

$$(5.2.6) \qquad g_{in}(\eta^*_{in}) = \frac{1}{F_{in}(\gamma r_{in})} \times \int_{\xi_{in}=-\infty}^{\gamma r_{in}} f(\xi_{in}, \eta^*_{in})\, d\xi_i$$

$$= \frac{1}{F_{in}(\gamma r_{in})} f^*(\eta_{in}^*) \int_{\xi_{in}=-\infty}^{\gamma r_{in}} f(\xi_{in} | \eta_{in} = \eta_{in}^*) d\xi_i$$

where f is the joint density of (ξ_{in}, η_{in}) and f^* is the marginal density of η_{in}.

Using the expression of the conditional distribution of a random variable normally distributed, we get:

(5.2.7)

$$g_{in}(S_{in}=\bar{S}_{in}) = \frac{1}{F_{in}(\gamma r_{in})} \phi_{J_i}[S_{in}-(I_{J_i} \otimes Z'_{in})\theta_i, \Sigma_{\eta_i}]$$
$$\times \Phi_{m-1}[\gamma r_{in}; -(S_{in}-(I_{J_i} \otimes Z'_{in})\theta_i)'\Sigma_{\eta_i \epsilon} A'_i \Omega_i^{-1}, \Omega_i]$$

where $\phi_k(x,A)$ is the density of the k-dimensional random variable, normally distributed with zero mean, covariance matrix A^{-1}, and evaluated at the point x. $\Phi_k(x,\mu,A)$ is the cumulative distribution of the k-dimensional normal random variable with mean μ, covariance matrix A^{-1} and evaluated at the point x. Hence, the likelihood function of a random sample of N households is given by:

(5.2.8) $$L(\theta,\gamma,\Sigma) = \prod_{n=1}^{N} \prod_{i=1}^{m} \{\phi_{J_i}[S_{in}-(I_{Ji} \otimes Z'_{in})\theta_{ij} \Sigma_{\eta_i}]$$
$$\times \Phi_{m-1}[\gamma r_{in}, -(S_{in}-(I_{J_i} \otimes Z'_{in})\theta_i)'\Sigma_{\eta_i \epsilon} A'_i \Omega_i^{-1}, \Omega_i]\}^{d_{in}}$$

The log likelihood is then :

(5.2.9) $$\mathcal{L}(\theta,\gamma,\Sigma) = \sum_{n=1}^{N} \sum_{i=1}^{m} d_{in} \operatorname{Log} \phi(S_{in}-(I_{J_i} \otimes Z'_{in})\theta_i, \Sigma_{\eta i})$$

$$+\sum_{n=1}^{N}\sum_{i=1}^{m} d_{in} \operatorname{Log} \Phi(\gamma r_{in}, -S_{in}-(I_{J_i} \otimes Z'_{in})\theta_i)'\Sigma_{\eta_i\epsilon}A'_i\Omega_i^{-1}, \Omega_i)$$

$$= \mathcal{L}_1(\theta, \Sigma_{\eta_i}) + \mathcal{L}_2(\gamma, \theta, \Sigma_{\eta_i\epsilon}, \Sigma_{\epsilon_i})$$

The way the likelihood decomposes is very interesting and has some implications with respect to the solutions of the likelihood equation of Σ_{η_i}. Indeed, since the second term does not involve Σ_{η_i}, we can readily solve for the MLE of Σ_{η_i} in terms of the one for θ_i. We can then use the concentrated log-likelihood to solve numerically for the other parameters. Note also that if the dish choices are not relevant, that is if $m=1$, $d_{in}=1$ for all n and $\Sigma_{\eta_i\epsilon}=0$, then the second term of the log-likelihood is zero, and the first term reduces to the log-likelihood of the traditional model. This provides us with a means of testing directly the restriction implied by the traditional model by use of the likelihood ratio test or the Hausman test.

Σ has $\frac{1}{2}(m + \sum_{i=1}^{m} J_i)(m + \sum_{i=1}^{m} J_i + 1)$ free parameters. But it is rarely the case that all the parameters are identified. Indeed, it is known from multinomial probit estimation that in general only part of Σ_ϵ can be identified. (See, for example, Hausman and Wise, 1978). One usually has to normalize some elements of Σ_ϵ and/or use some type of parametrization like the one used by Hausman and Wise (1978). It will be seen also later that without prior restrictions on the elements of Σ not all the elements of $\Sigma_{\eta_i\epsilon}$ are identified; only the elements of $\Sigma_{\eta_i\epsilon}A'_i$ can be identified in general.

Given our assumptions, the above log-likelihood satisfies the usual regularity conditions so that the MLE of the identifiable parameters are consistent and asymptotically efficient with the asymptotic covariance matrix given by the Inverse of the information matrix.

The computation of the MLE's is done iteratively and involves the evaluation of a multivariate normal probability (which is an integral of dimension m-1) using numerical methods. Until recently, this was computationally unfeasible for $m \geq 3$. The next section will discuss some of the recent methods of approximating the integral. The method of iteration normally used, is the Newton-Raphson method and its variants. (See, for example, Amemiya, 1985, pp. 137-141 and page 274).

The difficulties involved with the computation of the MLE have led to a search for other computationally more feasible methods of estimation. The method usually used is the Heckman's two-stage method. This method consists of estimating first γ and Σ_ϵ by the probit MLE using equation 5.2.2 alone, and then performing an ordinary least squares on equation 5.2.3 after replacing the unknown parameters γ and Σ_ϵ by their probit MLE. However, the computational advantage of this method over the MLE which is more efficient is not obvious in our model since the probit MLE is obtained by iteration which also needs the evaluation of the m-1-dimensional multivariate normal integral. The only simplification would come from the reduction in the number of parameters that have to be simultaneously estimated.

5.2.3 Heckman's Two-Step Estimator

Heckman's method is attractive not only because of its possible computational advantage but also because it reduces our food consumption model to a simple problem of correcting what is referred in the literature as selection bias, when estimating the conditional demand equations. This selection bias problem arises whenever the dependent variable (the quantity of raw food) is conditionally observed according to some selection process (the dish choices). Without correcting for the selectivity bias, the least squares estimates of the parameters of the demand equations are biased and inconsistent. Heckman's method is a relatively simple way to get consistent estimates. Furthermore, with Heckman's method the assumption of joint normality of ϵ and η can be removed, since the derivation of the conditional moments involved in equation 5.2.3 depends only on the linearity of the conditional expectation of η given ϵ, the normality of ϵ, and the independence between ϵ and the regression residual of η on ϵ. (Lee, 1982; Johnson and Kotz, 1972, p. 70). The consequence of this is that the correction of the selectivity bias is insensitive to the distribution of η^* (Lee, 1982).

The two stages in Heckman's method are as follows:

Stage 1: Computation of the Probit MLE of γ and Σ_ϵ

In this stage, we maximize the probit likelihood function

$$\mathcal{L}^*(\gamma, \Sigma_\epsilon) = \sum_{n=1}^{N} \sum_{i=1}^{m} d_{in} \text{Log}\, \Phi_{m-1}(\gamma r_{in}, 0, A_i \Sigma_\epsilon A_i')$$

to find $\hat{\gamma}$ and $\hat{\Sigma}_\epsilon$ the probit MLE of γ and Σ_ϵ. (normalization of some elements of Σ_ϵ would be needed.)

$\hat{\gamma}$ and $\hat{\Sigma}_\epsilon$ are consistent. See, for example, Amemiya (1985, pp. 286-292) for its asymptotic distribution and other theoretical properties. All the remarks made in the previous section with respect to the computation of the MLE apply equally here. These computational difficulties has limited the use of the multinomial probit MLE in the past, despite its theoretical advantage over the multinomial logit (Hausman and Wise, 1978). However, recent progress has been made in developing computationally attractive methods of approximating the value of the multiple integral involved. Hausman and Wise (1978) reported that a series expansion method is feasible up to five alternatives in the choice set. An approximation method originally proposed by Clark in 1961 was refined by Daganzo and Sheffi (1977). They argued that their algorithm was computationally efficient. Finally, Lerman and Manski (1981), used Monte Carlo methods to approximate the multivariate normal integral. According to them, the method was feasible up to ten alternatives. More importantly, they reported having a computer program that lets you choose between the Monte Carlo method and Clark's method. This is an interesting feature because, although Clark's method dominated Monte Carlo in their experiments, in some other instances the former performed poorly. (Amemiya, 1985, p. 309).

In the process of getting $\hat{\gamma}$ and $\hat{\Sigma}_\epsilon$, $F_{in}(\hat{\gamma}r_{in})$ and $a_i(\hat{\gamma}r_{in})$ can be computed at the same time. Hence, from this first stage, we should be able to compute

$$h_i(\hat{\gamma r_{in}}) = \frac{a_i(\hat{\gamma r_{in}})}{F_{in}(\hat{\gamma r_{in}})} .$$

Stage 2: Getting Consistent Estimates of θ_{10}, $\Sigma^0_{\eta_i \epsilon} A_i$ and Σ^*_{0in}

In the second stage, we collect all the observations on the conditional demand equations for the raw foods corresponding to each dish. Then we can rewrite the system of conditional demand (5.2.3) by replacing the unknown values

$\dfrac{a_i(\gamma_0 r_{in})}{F_{in}(\gamma_0 r_{in})}$ by their estimates $h_i(\hat{\gamma r_{in}}) = \dfrac{a_i(\hat{\gamma r_{in}})}{F_{in}(\hat{\gamma r_{in}})}$

obtained from stage 1. We will then have :

$$(5.2.11) \quad S_{in} = (I_{J_i} \otimes Z'_{in})\theta^i + \Sigma_{\eta_i \epsilon} A'_i h_i(\hat{\gamma r_{in}}) + \tilde{\eta}_{in}$$

$$n=1,\ldots,N_i \qquad i = 1,\ldots,m$$

where:

N_i is the number of households having dish i in the sample. And

$$(5.2.12) \quad \tilde{\eta}_{in} = \eta^*_{in} + \Sigma_{\eta_i \epsilon} A'_i [h_i(\gamma r_{in}) - h_i(\hat{\gamma r_{in}})]$$

is the new disturbance term.

We can estimate the system (5.2.11) by ordinary least squares equation by equation to get consistent estimates of θ^i and $\Sigma_{\eta_i \epsilon} A'_i$. That is, for each dish i, we perform J_i O.L.S. using the J_i conditional demands with N_i observations for each variable. In total, we will perform

$\sum_{i=1}^{m} J_i$ O.L.S to cover all the dishes.

From (5.2.11), the j^{th} conditional demand corresponding to dish i is:

$$(5.2.13)\quad S^{i}_{jn} = Z'_{in}\theta_j + \sum_{\substack{\ell=1 \\ \ell\neq i}}^{m} \delta^{j}_{i\ell} h^{\ell}_{i}(\hat{\gamma r}_{in}) + \tilde{\eta}_{ijn} \qquad n = 1,\ldots, N_i$$

where:

$\delta^{j}_{i\ell} = \Sigma_{\eta_{ij}\epsilon_i} - \Sigma_{\eta_{ij}\epsilon_\ell}$ is the j^{th}, ℓ^{th} element of $\Sigma_{\eta_i\epsilon}A'_i$, and

$h^{\ell}_{i}(\hat{\gamma r}_{in}) = \dfrac{1}{F_{in}(\hat{\gamma r}_{in})} \times a^{\ell}_{i}(\hat{\gamma r}_{in})$ is the ℓ^{th} element of $h_i(\hat{\gamma r}_{in})$

It is clear from (5.2.13) that we cannot estimate all the $\Sigma_{\eta_{ij}\epsilon_\ell}$ for $i,\ell=1,\ldots,m$ only the differences $\delta^{j}_{i\ell}$ can be estimated. That is to say that only the elements of the matrix $\Sigma_{\eta_i\epsilon}A'_i$ $i=1,\ldots,m$ are identified.

Note also that for each equation we have $J_i + m + 1$ parameters to estimate. Thus for O.L.S. to be feasible for each i we should have $N_i \geq J_i+m+1$. Furthermore, for the consistency to apply, N_i should be large.

Provided this later condition is met, the second step of Heckman's procedure will yield consistent estimates of the

$$\sum_{i=1}^{m} J_i(J_i + m + 1)$$

conditional demand parameters for the raw food corresponding to all dishes. If one were interested in only getting consistent estimates, he may stop here. However, inferences based on the standard errors given by the regression output would be wrong. The reason for this is that (1) the disturbances of the regression in (5.2.13) are heteroskedastic - this can be seen from (5.2.12) - and (2) in (5,2,13) $\hat{\gamma}$ was estimated before performing least squares so that the covariance matrix of the O.L.S. estimator should take account of this fact. Although the exact covariance matrix of least squares is intractable, we can find its asymptotic distribution in exactly the same manner as in Amemiya (1985, pp. 369-70) or Heckman (1979). For this purpose, we write (5.2.13) in matrix notation as:

$$(5.2.14) \quad S_j^i = [Z_i, \hat{H}_i] \begin{bmatrix} \theta_j \\ \Delta_j^i \end{bmatrix} + \tilde{\eta}_{ij}$$

where S_j^i and $\tilde{\eta}_{ij}$ are the N_i dimensional vector the n^{th} element of which are respectively S_{jn}^i and $\tilde{\eta}_{ijn}$ Z_i is the $N_i \times (J_i + 2)$ dimensional matrix of the original regressors of the j^{th} conditional demand, the n^{th} row of which is Z_{in} $\hat{H}_i$ is the $N_i \times (m-1)$ matrix of the additional regressors - correcting for the selectivity bias - , the n^{th} row of which is

$\hat{h}_i' = h_i(\hat{\gamma}r_{in})'$. Finally, Δ_j^i is the j^{th} row of $\Sigma_{\eta_i \epsilon} A_i'$,

a m-1 vector of parameters, the ℓ^{th} element of which is $\delta_{i\ell}^j$ with $\ell \neq i$. To simplify further the notation we will put

$\hat{X}_i = (Z_i, \hat{H}_i)$ a $N_i \times (J_i+m+1)$ matrix,

and $\beta_j^i = (\theta_j', \Delta_j^{i\prime})'$ a $(J_i+m+1)\times 1$ vector.

Similarly, the expression of $\tilde{\eta}_{ij}$ can be found from (5.2.12) and is given by:

(5.2.15) $\tilde{\eta}_{ij} = \eta^*_{ij} + (H_i - \hat{H}_i)\Delta^i_j$

where η^*_{ij} is the N_i dimensional vector the n^{th} element of which is η^*_{ijn}

H_i is the $N_i \times (m-1)$ matrix the n^{th} row of which is $h'_i = h_i(\gamma r_{in})'$.

If we combine the J_i regression equations similar to (5.2.14), then for each dish i, we have a system of seemingly unrelated regression (S.U.R.) with heteroskedastic disturbances and correlation across equations depending on the observations.

The set of J_i regression equations can be written in S.U.R. format as one combined equation

(5.2.16) $S_i = (I_{J_i} \otimes \hat{X}_i)\beta_i + \tilde{\eta}_i$

where $S_i = (S^{i'}_1, \ldots, S^{i'}_{J_i})'$; $\tilde{\eta}_i = (\tilde{\eta}'_{i1}, \ldots, \tilde{\eta}'_{iJ_i})'$

$\beta_i = (\beta^{i'}_1, \ldots, \beta^{i'}_{J_i})'$ and I_{J_i} is the identity matrix of dimension J_i

Similarly, we can combine the J_i equations defining $\tilde{\eta}_i$ as:

(5.2.17) $\tilde{\eta}_i = \eta^*_i + [I_{J_i} \otimes (H_i - \hat{H}_i)]\Delta^i$

where $\eta^*_i = (\eta^{*'}_{i1}, \ldots, \eta'_{iJ_i})'$ and

$\Delta^i = (\Delta^{i'}_i, \ldots, \Delta^{i'}_{J_i})' = \text{vec}\,(\Sigma_{\eta_i\epsilon}A'_i)$

It can be shown that η^*_i and $(I_{J_i} \otimes (H_i - \hat{H}_i)]\Delta^i$ are uncorrelated (see Amemiya, 1985, p. 370). Hence we have

$$(5.2.18)\quad \mathrm{cov}(\tilde{\eta}_i) = \mathrm{cov}(\eta_i^*) + \mathrm{cov}[(I_{J_i} \otimes (H_i - \hat{H}_i)\Delta^i]$$

From the expression of $\Sigma_{in}^* = \mathrm{cov}(\eta_{in}^*)$ in (15.1.9) we get:

$$(5.2.19)\quad \mathrm{cov}(\eta_i^*) = \Sigma_{\eta_i} \otimes I_{N_i} + \Sigma_i^*$$

where Σ_i^* is the $N_iJ_i \times N_iJ_i$ square block matrix of $J_i \times J_i$ square submatrices of size $N_i$the j^{th}, k^{th} submatrix of which is

$D_{jk}(\Delta_j^{i'} \Lambda_{in}^* \Delta_k^i)$ a diagonal matrix of size N for which

the term in the parentheses is its n^{th} element;
with Λ_{in}^* given by:

$$\Lambda_{in}^* = \frac{1}{F_{in}} D\left(\frac{\gamma r_{in}^k a_{kn}^i - \omega_k^{i'} b_{kn}^i}{\omega_{kk}^i}\right) + \frac{1}{F_{in}} B_n^i - \frac{1}{F_{in}^2} a_n^i a_n^{i'}$$

where $D(\cdot)$ is as usual, the diagonal matrix for which the term in the parentheses is the k^{th} diagonal element. From (5.2.14) or (5.2.16), the O.L.S. estimates of

β_j^i $j=1,\ldots,J_i$ are:

$$(5.2.20)\quad \hat{\beta}_j^i = (\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i's_j^i = \beta_j^i + (\hat{X}_i'\hat{X}_i)^{-1}[\hat{X}_i'\eta_{ij}^* + \hat{X}_i'(H_i - \hat{H}_i)\Delta_j^i]$$

we can now derive the asymptotic distribution of $\hat{\beta}_j^i$ along the same lines as in Amemiya (1985, pp. 369-70).

Because of our assumptions and the consistency of the probit MLE, we have:

$$(5.2.21)\quad \underset{N_i\to\infty}{\mathrm{plim}} \frac{1}{N_i}\hat{X}_i'\hat{X}_i = \lim_{N_i\to\infty} \frac{1}{N_i}X_i'X_i \qquad \text{and}$$

(5.2.22) $N_i^{-\frac{1}{2}}\hat{X}_i'\eta_{ij}^* \xrightarrow{d} N(0, \lim_{N_i\to\infty} X_i'\Sigma_j^i X_i)$

where $X_i = (Z_i, H_i)$ and $\Sigma_j^i = \text{cov}(\eta_{ij}^*)$ is the diagonal matrix of size N_i the n^{th} element of which is the j^{th} diagonal element of Σ_{in}^*. From the expression of Σ_{in}^* in (15.1.9) or from (5.2.19) we get

(5.2.23) $\Sigma_j^i = \sigma_{jj} I_{N_i} + D(\Delta_j^{i'}\Lambda_{in}^*\Delta_j^i) = D(\sigma_{jj} + \Delta_j^{i'}\Lambda_{in}^*\Delta_j^i)$.

where σ_{jj} is the j^{th} diagonal element of Σ_{η_i}.

By a Taylor expansion of $h_i(\hat{\gamma} r_{in})$, the n^{th} column of $\hat{H}_i'$, around γ we have:

(5.2.24) $h_i(\hat{\gamma} r_{in}) - h_i(\gamma r_{in}) = \dfrac{\partial h_i(\gamma^* r_{in})}{\partial \gamma} (\hat{\gamma} - \gamma) + 0(\frac{1}{N_i})$

where γ^* lies between γ and $\hat{\gamma}$ and $\lim_{N_i\to\infty} 0(\frac{1}{N_i}) = 0$

Because of our assumptions and the consistency of $\hat{\gamma}$, we have:

(5.2.25) $\text{plim}_{N_i\to\infty} \dfrac{\partial h_i(\gamma^* r_{in})}{\partial \gamma} = \dfrac{\partial h_i(\gamma r_{in})}{\partial \gamma}$

Hence, $N_i^{-\frac{1}{2}}\hat{X}_i'(\hat{H}_i - H_i)\Delta_j^i$ has the same limit distribution as:

$$\left[\lim_{N_i\to\infty} \frac{1}{N_i} X_i' \frac{\partial H_i(\gamma r_{in})}{\partial \gamma} \Delta_j^i\right] N^{\frac{1}{2}}(\hat{\gamma} - \gamma).$$

By taking the partial derivative of h_i with respect to γ and using the notation in section 5.1 we get:

(5.2.26) $$\frac{\partial h_i(\gamma r_{in})}{\partial \gamma} = -\frac{1}{F_{in}^2} a_n^i \nabla F'_{in} r'_{in} - \frac{1}{F_{in}} D(\gamma r_i^k a_{kn}^i) + f^i \nabla F^i_{(k)} r'_{ik}$$

where $F_{in} = F_{in}(\gamma r_{in})$, ∇F is the gradient vector of F, f^i is the $(m-1)\times 1$ vector the k^{th} element of which is $f_k^i(\gamma r_i^k)$, and $D(\cdot)$ is, as usual the diagonal matrix of size m-1 for which the term in parentheses is the k^{th} diagonal element.

Noting that $\nabla F_{in} = a_n^i$ and $f^i \nabla F^{i'}_{(k)} r'_{ik} = B_n^i r'_{in} - D(b_{kk}^i r_i^k)$ we have after some algebra:

(5.2.27) $$\frac{\partial h_i(\gamma r_{in})}{\partial \gamma} = -\left[\frac{1}{F_{in}} D(\gamma a_{kn}^i + b_{kk}^i) - \frac{1}{F_{in}} B_n^i + \frac{1}{F_{in}^2} a_n^i a_n^{i'} \right] r'_{in}$$

where b_{kk}^i is the k^{th} diagonal element of B_n^i

Thus the n^{th} row of $\frac{\partial H_i(\gamma r_{in})}{\partial \gamma}$ is:

$$\frac{\partial h_i(\gamma r_{in})'}{\partial \gamma} = -r_{in}\left[\frac{1}{F_{in}} D(\gamma a_{kn}^i + b_{kk}^i) - \frac{1}{F_{in}} B_n^i + \frac{1}{F_{in}^2} a_n^i a_n^{i'}\right]$$

Hence, we have:

(5.2.28) $$\frac{\partial H_i(\gamma r_{in})}{\partial \gamma} = -R_i\left[\frac{1}{F_{in}} D(\gamma a_{kn}^i + b_{kk}^i) - \frac{1}{F_{in}} B_n^i + \frac{1}{F_{in}^2} a_n^i a_n^{i'}\right]$$

$$= -R_i \Lambda_{in}$$

where R_i is the $N_i \times (m-1)$ matrix of regressors in the probit equation, the n^{th} row of which is r_{in}; and to simplify the notation we have put

$$\Lambda_{in} = \frac{1}{F_{in}} D(\gamma a_{kn}^i + b_{kk}^i) - \frac{1}{F_{in}} B_n^i + \frac{1}{F_{in}^2} a_n^i a_n^{i'}$$

It follows from above that $N_i^{-\frac{1}{2}}\hat{X}_i'(\hat{H}_i - H_i)\Delta_j^i$ has same limit distribution as
$X_i'R_i\Lambda_{in}\Delta_j^i N^{-\frac{1}{2}}(\hat{\gamma} - \gamma)$ which converges to

$$N\,(0,\ \lim_{N_i\to\infty}[\frac{1}{N_i} X_i'R_i\Lambda_{in}\Delta_j^i] \times \Gamma \times \lim_{N_i\to\infty}[\frac{1}{N_i}\Delta_j^{i'}\Lambda_{in}R_i'X_i])$$

where Γ is the asymptotic variance of $N_i^{-\frac{1}{2}}(\hat{\gamma}-\gamma)$ from the probit MLE which is given in Amemiya (1985, pp. 288-89). With our notation, we can write Γ as

(5.2.29)

$$\Gamma = \lim_{N_i\to\infty} N_i[\sum_{n=1}^{N_i} \frac{1}{F_{in}} a_n^{i'} r_{in}' r_{in} a_n^i]^{-1} = \lim_{N_i\to\infty} N_i[\sum_{n=1}^{N_i} \frac{1}{F_{in}}(r_{in}a_n^i)^2]^{-1}$$

$$= \lim_{N_i\to\infty} N_i[a_n^{i'}R_i'D^{-1}(F_{in})R_i a_n^i]^{-1}$$

where $D^{-1}(F_{in})$ is the inverse of $D(F_{in})$ a diagonal matrix of size N_i with F_{in} as the n^{th} element. Finally, using the fact that η^*_{ij} and $\hat{X}_i'(H_i - \hat{H}_i)\Delta_j^i$ are uncorrelated, we conclude that $\hat{\beta}_j^i$, the O.L.S. of β_j^i, $j=1,\ldots,J_i$ are asymptotically normal with means β_j^i and asymptotic covariance matrices given by[3]:

$$(5.2.30)\quad \mathrm{Acov}(\beta_j^i) = \lim_{N_i\to\infty} (X_i'X_i)^{-1}X_i'[D(\sigma_{jj} + \Delta_j^{i'}\Lambda_{in}^*\Delta_j^i)$$

$$+ R_i\Lambda_{in}\Delta_j^i[a_n^{i'}R_i'D^{-1}(F_{in})R_i a_n^i]^{-1}\Delta_j^{i'}\Lambda_{in}R_i']X_i(X_i'X_i)^{-1}$$

A computable consistent estimate of $\mathrm{Acov}(\beta_j^i)$ can be obtained by dropping the sign $\lim_{N_i\to\infty}$ and replacing the unknown parameters by their consistent

It follows from above that $N_i^{-1/2}\hat{X}_i'(\hat{H}_i - H_i)\Delta_j^i$ has same limit distribution as
$X_i'R_i\Lambda_{in}\Delta_j^i N^{-1/2}(\hat{\gamma} - \gamma)$ which converges to

$$N\left(0,\ \lim_{N_i\to\infty}\left[\frac{1}{N_i} X_i'R_i\Lambda_{in}\Delta_j^i\right] \times \Gamma \times \lim_{N_i\to\infty}\left[\frac{1}{N_i}\Delta_j^{i'}\Lambda_{in}R_i'X_i\right]\right)$$

where Γ is the asymptotic variance of $N_i^{-1/2}(\hat{\gamma}-\gamma)$ from the probit MLE which is given in Amemiya (1985, pp. 288-89). With our notation, we can write Γ as

(5.2.29)

$$\Gamma = \lim_{N_i\to\infty} N_i\left[\sum_{n=1}^{N_i} \frac{1}{F_{in}} a_n^{i'} r_{in}' r_{in} a_n^i\right]^{-1} = \lim_{N_i\to\infty} N_i\left[\sum_{n=1}^{N_i} \frac{1}{F_{in}}(r_{in}a_n^i)^2\right]^{-1}$$

$$= \lim_{N_i\to\infty} N_i\left[a_n^{i'} R_i' D^{-1}(F_{in}) R_i a_n^i\right]^{-1}$$

where $D^{-1}(F_{in})$ is the inverse of $D(F_{in})$ a diagonal matrix of size N_i with F_{in} as the n^{th} element. Finally, using the fact that η_{ij}^* and $\hat{X}_i'(H_i - \hat{H}_i)\Delta_j^i$ are uncorrelated, we conclude that $\hat{\beta}_j^i$, the O.L.S. of β_j^i, $j=1,\ldots,J_i$ are asymptotically normal with means β_j^i and asymptotic covariance matrices given by[3]:

(5.2.30) $$\mathrm{Acov}(\beta_j^i) = \lim_{N_i\to\infty} (X_i'X_i)^{-1}X_i'\left[D(\sigma_{jj} + \Delta_j^{i'}\Lambda_{in}^*\Delta_j^i)\right.$$

$$\left. + R_i\Lambda_{in}\Delta_j^i\left[a_n^{i'}R_i'D^{-1}(F_{in})R_i a_n^i\right]^{-1}\Delta_j^{i'}\Lambda_{in}R_i'\right]X_i(X_i'X_i)^{-1}$$

A computable consistent estimate of $\mathrm{Acov}(\beta_j^i)$ can be obtained by dropping the sign $\lim_{N_i\to\infty}$ and replacing the unknown parameters by their consistent estimates. Consistent estimates of γ, $\omega^i_{k\ell}$, are respectively $\hat{\gamma}$ the probit MLE of γ and $\hat{\omega}^i_{k\ell}$ the k^{th}, ℓ^{th} element of $\hat{\Omega}_i$ probit MLE of Ω_i consistent estimates of Δ^i_j $j=1,\ldots,J$ $_i$are the O.L.S. estimates $\hat{\Delta}^i_j$. Then it remains to find a consistent estimate of σ_{jj}. It can be shown that

$$(5.2.31)\quad \hat{\sigma}_{jj} = \frac{1}{N_i}\sum_{n=1}^{N_i}\hat{\eta}^2_{ijn} - \hat{\Delta}^i_{j_i}\left[\frac{1}{N}\sum_{n=1}^{N_i}\hat{\Lambda}^*_{in}\right]\hat{\Delta}^i_j$$

is a consistent estimate of σ_{jj}. Where $\hat{\tilde{\eta}}_{ijn}$ are the O.L.S. residuals from the regression equation (5.2.14) and $\hat{\Lambda}^*_{in}$ is obtained from Λ^*_{in} after replacing γ by $\hat{\gamma}$. However, the computation of this estimate of Acov($\hat{\beta}^i_j$) is very cumbersome as it requires the numerical evaluation of many multivariate normal integral. A simpler consistent estimate of Acov($\hat{\beta}^i_j$) can be obtained by using the method of White (1980). Under this method, Acov($\hat{\beta}^i_j$) is estimated by:

$$(\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i' D(\hat{\tilde{\eta}}^2_{ijn})\hat{X}_i(\hat{X}_i'\hat{X}_i)^{-1}$$

where $D(\hat{\tilde{\eta}}^2_{ijn})$ is the diagonal matrix of size N_i, the n^{th} element of which is $\hat{\tilde{\eta}}^2_{ijn}$ the square of the n^{th} O.L.S. residual. White (1980) has proved the consistency of this estimator under general form of heteroskedasticity.

Because of the heteroskedasticity in each equation and the correlation across equations, we can get asymptotically more efficient estimators than O.L.S. by using weighted least squares (WLS) equation by equation or SUR using the regression equation in (5.2.16). SUR is in general more efficient than WLS because it uses the correlation across equations. Note that without the heteroskedastic variances and correlation, OLS, WLS, and SUR would yield the same estimator since for a given dish i, the regressors are the same in all J_i equations.

The WLS estimate of β_j^i can be computed using a consistent estimate of the asymptotic covariance matrix of $\tilde{\eta}_{ij}$ (Amemiya, 1985, p. 371). The asymptotic covariance matrix of $\tilde{\eta}_{ij}$ can be found from (5.2.15) and is given by the matrix within the outer square bracket in (5.2.30). A consistent estimate of this matrix can be obtained by following the procedure after (5.2.30). The WLS estimate of β_j^i will be given then by:

$$(5.2.33)\quad \hat{\beta}^i_{jWLS} = (\hat{X}_i'\hat{\Psi}_{ij}^{-1}\hat{X}_i)^{-1}\hat{X}_i'\hat{\Psi}_{ij}^{-1} s_j^i \qquad j=1,\ldots,J_i$$

where $\hat{\Psi}_{ij}$ is the consistent estimate of $ACov(\tilde{\eta}_{ij})$ the asymptotic covariance matrix of $\tilde{\eta}_{ij}$, obtained following the procedure outlined above. It can be shown in a way similar to the O.L.S. case, that $\hat{\beta}^i_{jWLS}$ is consistent and asymptotically normal with asymptotic covariance matrix given by:

$$(5.2.34)\quad Acov(\hat{\beta}^i_{jWLS}) = (X_i'[Acov(\tilde{\eta}_{ij})]^{-1}X_i)^{-1}$$

which can be estimated by $(\hat{X}_i'\hat{\Psi}_{ij}^{-1}\hat{X}_i)^{-1}$.

However, as in the OLS case, to avoid the computation of multivariate normal integrals involved in $\hat{\Psi}_{ij}$ we can use the method of White (1980) and estimate the WLS of $\beta^i{}_j$ and its asymptotic covariance matrix respectively by:

$$[\hat{X}_i' D^{-1}(\bar{\eta}^2_{ijn})\hat{X}_i]^{-1} \hat{X}_i' D^{-1}(\bar{\eta}^2_{ijn}) S^i_j \quad \text{and} \quad [\hat{X}_i' D^{-1}(\bar{\eta}^2_{ijn})\hat{X}_i]^{-1}$$

where $D^{-1}(\bar{\eta}^2_{ijn})$ is the inverse of $D(\bar{\eta}^2_{ijn})$.

To compute the SUR estimate of β_i in (5.2.16), we need a consistent estimate of ACov(η_i), the asymptotic covariance matrix of η_i from (5.2.18), (5.2.19) and after (5.2.25) we have:

$$(5.2.35) \quad \text{Acov}(\eta_i) = \Sigma_{\eta_i} \otimes I_{N_i} + \Sigma^*_i +$$

$$\lim_{N_i \to \infty} (I_{Ji} \otimes R_i \Lambda_{in}) \Delta^i (a_i' R_i' D^{-1}(F_{in}) R_i a^i_n)^{-1} \Delta^{i\prime} (I_{Ji} \otimes \Lambda_{in} R_i')$$

Acov(η_i) can be consistently estimated by replacing the unknown parameters γ and Ω_i by respectively $\hat{\gamma}$ and $\hat{\Omega}_i$ their probit estimate, and Δ^i_j j=1,....,J_i by $\hat{\Delta}^i_j$ the O.L.S. estimates obtained using least squares equation by equation. It can be shown that a consistent estimate of Σ_{η_i} is given by

$$(5.2.36) \quad \hat{\Sigma}_{\eta_i} = \hat{\Sigma}_i + \hat{\Delta}^*_i \left[\frac{1}{N_i} \sum_{n=1}^{N_i} \Lambda^*_{in}\right] \hat{\Delta}^{*\prime}_i$$

where $\hat{\Sigma}_i$ is the $J_i \times J_i$ matrix of O.L.S. residuals, the j^{th}, k^{th} element of which is

$\frac{1}{N_i} \sum_{n=1}^{N_i} \hat{\bar{\eta}}_{ijn} \hat{\bar{\eta}}_{ikn}$ and $\hat{\Delta}^*_i$ is the OLS estimate of $\Sigma_{\eta_i \epsilon} A_i'$

a $J_i \times (m-1)$ matrix, the j^{th} row of which is $\hat{\Delta}^i_j$.

The SUR estimate of β_i is then given by

$$(5.2.37)\quad \tilde{\beta}_i = [(I_{J_i} \otimes \hat{X}_i')\hat{\Psi}_i^{-1}(I_{J_i} \otimes \hat{X}_i)]^{-1}(I_{J_i} \otimes \hat{X}_i')\hat{\Psi}_i^{-1}S_i$$

where $\hat{\Psi}_i$ is the consistent estimate of $\mathrm{Acov}(\eta_i)$ obtained by following the procedure outlined after (5.2.35).

Again, it can be shown that $\tilde{\beta}_i$ is consistent and asymptotically normal with asymptotic covariance matrix given by

$$(5.2.38)\quad \mathrm{Acov}(\tilde{\beta}_i) = [(I_{J_i} \otimes \hat{X}_i')(\mathrm{Acov}(\eta_i))^{-1}(I_{J_i} \otimes \hat{X}_i)]^{-1}$$

which can be estimated by $[(I_{J_i} \otimes \hat{X}_i')\hat{\Psi}_i^{-1}(I_{J_i} \otimes \hat{X}_i)]^{-1}$

As in the previous cases, we can avoid the computation of the multivariate normal integral in $\hat{\Psi}_i$ by using White's method in the SUR context to estimate β_i and its asymptotic covariance matrix respectively by

$[(I_{J_i} \otimes \hat{X}_i')\hat{D}_i^{-1}(I_{J_i} \otimes \hat{X}_i)]^{-1}(I_{J_i} \otimes \hat{X}_i')\hat{D}_i^{-1}S_i$ and

$[(I_{J_i} \otimes \hat{X}_i')\hat{D}_i^{-1}(I_{J_i} \otimes \hat{X}_i)]^{-1}$. Where $\hat{D}_i$ is the $N_iJ_i \times N_iJ_i$ square block matrix of $J_i \times J_i$ diagonal submatrix of size N_i, the j^{th}, k^{th} diagonal submatrix of which is

$D_{jk}(\hat{\tilde{\eta}}_{ijn}\hat{\tilde{\eta}}_{ikn})$ where the term in the parentheses stands for the n^{th}

element of the matrix.

Finally we can test the significance of the selectivity bias or dish choice effects after the second stage of Heckman's method. The null hypothesis of no selectivity bias would be then:

$$H_0 \ : \ H_i \Delta_j^i = 0$$

A sufficient condition for H_0 to hold is: $\Delta_j^i = 0$. This leads to the following sequential test:

(1) test

$$H_1 \ : \ \Delta_j^i = 0$$

by using a standard F test of the significance of a subset of the parameter vector. Note that under H_1 the asymptotic covariance matrix of O.L.S. is $\sigma_{jj}(X'X)^{-1}$, an estimate of which is given by the regression output in most regression packages.

(2) If H_1 is not rejected then we can stop and conclude that the dish choice effects are not significant. If H_1 is rejected then we test $H_0 \ : \ H_i \Delta_j^i = 0$ by using the fact that

$$\text{(5.2.40)} \quad N_i^{-\frac{1}{2}}(\hat{H}_i - H_i)\hat{\Delta}_j^i \xrightarrow{d} N(0, \lim_{N_i \to \infty} (\frac{1}{N_i} R_i \Lambda_{in} \Delta_j^i) \Gamma (\frac{1}{N_i} \Delta_j^{i'} \Lambda_{in} R_i^i))$$

so that under H_0 we have

$$\text{(5.2.41)} \quad \hat{\Delta}_j^{i'} \hat{H}_i' (\hat{a}_n^{i'} R_i' D^{-1} (\hat{F}_{in}) R_i \hat{a}_n^i)[R_i \hat{\Lambda}_{in} \hat{\Delta}_j^i \hat{\Delta}_j^{i'} \hat{\Lambda}_{in} R_i']^+ \hat{H}_i \hat{\Delta}_j^i \xrightarrow{d} \chi_{\rho_i}$$

where the sign "+" stands for generalized inverse, ρ_i is the rank of the matrix within the square bracket of (5.2.41). The $\hat{a}^i_n$, $\hat{F}_{in}$ and $\hat{\Lambda}_{in}$ are obtained by replacing γ with $\hat{\gamma}$ in a^i_n, F_{in}, and Λ_{in} respectively. Note that under H_1 the limit distribution of the expression in (5.2.41) is degenerate. Before concluding the chapter, we should point out the possibility to simultaneously estimate θ_j, Δ^i_j and γ by using non linear least squares applied to 5.2.13 without the probit MLE $\hat{\gamma}$. See, Amemiya (1985, p. 372) for details. However, the potential gain (if any) in computational time and simplicity over the MLE, is not probably worth the loss of efficiency compared to the MLE which is always efficient. We finally conclude the chapter by summarizing the estimation procedure recommended for this model.

Given the computational cost involved in completely estimating the model, we recommend that one should first do the Heckman's two-stage and get the OLS estimates which are consistent. Then, since the additional gain in efficiency of the WLS, SUR and MLE estimators are mostly relevant only when selectivity bias is significant, one should test for this significance by following the testing procedure outlined above before proceeding further. If the selectivity bias is not statistically significant, we can content ourselves with the OLS estimates. Otherwise, we can use these consistent estimates to get the more efficient WLS or SUR estimates. Or alternatively, with minor changes in the program that computes the probit MLE $\hat{\gamma}$ we can use these consistent OLS estimates along with $\hat{\gamma}$ as starting values in the Newton-Raphson iteration that computes simultaneously the MLE of all the parameters. In this case, it can be shown (see Amemiya, 1985, pp. 138) that the estimator from the second round of the iteration is asymptotically efficient.

CHAPTER 6

CONCLUSIONS AND POLICY IMPLICATIONS OF THE MODEL.

As we were unsatisfied with the present models used to analyze the demand for food in Senegal, our purpose in this paper was to present an alternative model that represents more realistically the micro-behavior of the Senegalese household. Our experience in the Senegalese context made us feel that the consumption technology was equally if not more important than the relative prices of the raw foods in determining the demand for the cereals. Although this model was specifically designed with reference to the Senegalese context, it can be thought of as a general model of food demand having the traditional model as a special case. Indeed, almost every society has its own consumption technology that guides its selection of which raw foods to consume[7]. We have seen in the estimation procedure that correcting for this selection bias is necessary for the estimates of the demand parameters to be unbiased and consistent.

A major concern during this study was to have a realistic but workable model for forecasting food demand and for performing policy analysis. For this purpose, the relevant equations are the

[7] This model may not be applicable in a society with more complex food consumption habits (like in the United States or Europe) where for each meal, the household can cook more than one dish of varying sizes. In such a complex setting, the household can be seen as making not a discrete but a continuous choice among all the raw foods used in all the dishes in the feasible choice set. Even in this case, the fact that the household has a specific set of dishes that guides its choice of raw foods, if ignored, will introduce selection bias of the level 2 type discussed in section 4.4.6.

unconditional demands for the raw foods and the dish choice probabilities. The unconditional quantities demanded are not observable, but can be estimated. These unconditional demand equations given in (4.5.12) can be used for forecasting the demands for the individual raw foods. Since the dish choice probabilities depend only on time and do not depend on prices and expenditures, the easiest way of computing the elasticities of demand for each food is to compute the different conditional elasticities from the conditional demand equations corresponding to the dishes where the food is used. Then we can get the unconditional elasticities by a weighted average of these conditional elasticities where the weights are the dish choice probabilities. More precisely, let ϵ_{jki} be the elasticity of demand of raw food j with respect to the price of raw food k when they are both used in dish i (with $\epsilon_{jki}=0$ if one of them is not used in dish i) then the unconditional elasticity of demand of food j with respect to price k is:

$$(6.1) \qquad \epsilon_{jk} = \sum_{i=1}^{m} \epsilon_{jki} P(m_i=1) = \sum_{i=1}^{m} \epsilon_{jki} F_i(\gamma r_i)$$

which can be consistently estimated by

$$(6.2) \qquad \hat{\epsilon}_{jk} = \frac{1}{N} \sum_{i=1}^{m} N_i \epsilon_{jki}$$

The dish frequencies N_i/N are known to be strongly consistent estimates of the dish choice probabilities $P(d_i=1)$. The expenditure elasticities can be computed similarly. Then to compute the income elasticities and the other household characteristic elasticities, we use the estimated equation corresponding to (5.2.1), which gives expenditure as a function of household characteristics, and then apply the chain rule. For instance, the elasticity of income will be given by the product of the expenditure elasticity and the elasticity of expenditure with respect to income; that is $e_{jy} = e_{jd} \cdot e_{dy}$. Finally, we note that the elasticities with respect to dish preparation time will involve the density of the probability distribution.

One immediate policy implication of this model is the statistical consequences of the selectivity bias. As already noted, the dish selection bias can have serious consequences on the reliability of the estimated parameters. In some empirical examples (Newey et al., (1990)), the presence of selection bias if not corrected, yielded wrong signs for some of the parameters. This should be of major concern when one bases policy recommendations on the magnitudes and signs of the estimated price and income elasticities.

The policy implications of the model go beyond this statistical consequence. The fact that the dish choice probabilities do not depend on prices of the raw foods not only simplifies the computation of the elasticities but also has important policy implications. As long as the ordering of the different dishes by their relative costs is not reversed, then for a given level of utility and meal budget, changes in the relative prices of the raw foods cannot alter the dish choice probabilities which describe the structural consumption behavior of the household.

More precisely, we have from the equation giving the dish choice probabilities (ex: in (4.5.15)).

$\partial P(d_i=1)/_{\partial P_j} = 0$ for all prices P_j.

To support this result, we can cite a preliminary finding of the CEEMAT/CIRAD ongoing survey in Dakar (Kelly and Reardon, 1989). It is reported that in their sample, only the households with monthly income above CFA 100,000 consume couscous. Our interpretation of this finding is that the poorest households cannot afford this millet-based dish because of the high cost of the complements going into it.

Furthermore, these dish choice probabilities are important in determining the size of the unconditional elasticities. One consequence is that policies aimed at changing these choice probabilities may be more effective in changing the consumption behavior of the household than those that change the relative price of the raw foods. There are at least two forms these policies could take:

1. The dish probabilities can depend on an exogenous shift parameter θ in the control of the government so that $\partial F_i(\gamma r_i, \theta)/_{\partial\theta}$ measure the effects that changes in θ have on the household dish choices. θ can include all the factors that can be used to influence the structural behavior of the household (ex: generation of technology that reduces the processing costs of local cereals used in some dishes).
2. Increases in the number of dishes in the household choice set should decrease the dish choice probabilities. In particular, if the dish introduced is accepted and is maize-based rather than rice-based, this will decrease rice consumption.

Moreover, as noted at the beginning of chapter 4, the dish choice probabilities are time dependent, that is they are stochastic (Markov) processes. Hence their moments (variances, covariances and higher moments) can be used to evaluate the effectiveness of implemented food policies, by indicating the degree of change in household food preferences (dish choice probabilities) over time.

For example, we have seen that because of the consumption technology constraint, any policy aimed at changing substantially the demand for a particular raw food must necessarily change the relative size of the dish choice probabilities. That is for some dishes, we should have:

$$P_{t_1}(m_i-1) \lesseqgtr P_{t_2}(m_i-1)$$

where the subscripts t_1, and t_2 stand for the time when the choices are made. Hence these dish choice probabilities (how often a household consumes a particular dish) are good indicators of how the household has responded to a particular policy.

To be more precise, let $\pi_{it} = P_t(m_i-1) - F_{it}(\gamma r_i)$.

Then $\Delta\pi_{it} = \pi_{it} - \pi_{it-1}$ measures the change in the probability (between t and t-1) that dish i is consumed by the household. It can be interpreted as a partial measure (with respect to dish i) of the change in the "household's taste." Thus:

if $\Delta\pi_{it} > 0$ we would say that the household taste has changed in favor of dish i

if $\Delta\pi_{it} < 0$ we would say that the household taste has changed in defavor of dish i

if $\Delta\pi_{it} = 0$ we would say that the household taste did not change with respect to dish i.

If dish i is a newly introduced dish, then $\Delta\pi_{it}$ can be used to measure the rate of adoption or diffusion through time of the dish.

Since the sum of the dish choice probabilities is always 1, a change in the household's taste in favor of one dish necessarily implies a change away from at least one other dish. Hence a good measure of overall change in taste should take into account these correlations. This motivates our definition of the overall change in household's taste at time t by the covariance of $\Delta\pi_t$ where $\pi_t=(\pi_{1t},\ldots,\pi_{mt})$ is the vector of dish choice probabilities at time t.

$\text{cov}(\Delta\pi_t)$ is a square matrix of size m, with its diagonal elements measuring the variances of the dish choice probabilities and its off diagonal terms measuring the degree of substitution between dishes.

The household taste at time T_0 (a state variable) can be measured by:

$$\Gamma(T_0) = \sum_{t=-\infty}^{T_0} \text{cov}(\Delta\pi_t)$$

and the change in the household's taste (structural behavior) between T_0 and T_1 (say, after one or two years) is measured by:

$$\Gamma(T_0,T_1) = \Gamma(T_1) - \Gamma(T_0) = \sum_{t=T_0}^{T_1} \text{cov}(\Delta\pi_t)$$

which is a matrix of size m.

Based on survey data (discussed below), a measure can be constructed from $\Gamma(T_0,T_1)$ (ex: its trace or its determinant, or whether or not it is positive definite or negative definite), and this measure can be used to evaluate the effectiveness of food policies implemented between time T_0 and T_1. For example, if the trace of $\Gamma(T_0,T_1)$ is a large number, this will be an indication that the dish choice probabilities are changing through time. Conversely, a small value of the trace would indicate no change in the household's structural behavior, implying that the policy has no effect.

Furthermore, at any point in time, without knowing the actual demand for the raw foods, we can predict their values by using the dish choice frequencies and the previously estimated conditional quantities demanded for the raw food, and compute the unconditional quantity demanded for each raw foods. This should be a good approximation, because, given the dish technology constraints, the conditional quantities demanded (x_{ij} in the text) do not change very much (especially when prices are stable).

This also indicates that we may not need to conduct comprehensive (and expensive) surveys each time to collect the x_{ij}, for it is far easier to conduct a survey to estimate only the dish choice probabilities. Indeed, it is easy and takes only few minutes to fill out a questionnaire where the only question asked is which dishes the household has consumed at a particular day (breakfast, lunch and dinner). This kind of survey could be conducted all year long with a relatively large sample, with a more expensive survey (to reestimate and update the conditional x_{ij}) conducted every two, five or ten years on a smaller sample.

In this way, any ongoing food policy can be monitored continuously. Then, if it is clear that the dish choice probabilities are not changing, one can decide to discontinue the policy or identify why it is not working and make the relevant correction.

The above development gives a theoretical framework within which the ongoing experiments in ITA[8] (creation of new dishes, and promotion of dishes based on local cereals, etc.) can be analyzed and evaluated.

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[8] Institut de Technologie Alimentaire, a governmental food research institute located in Dakar.

This framework can also be used to perform ex-ante cost-benefit analysis to determine which policy to follow or which dish to promote based on their measured potential effects on the cereal balance of trade. The data needed for this type of analysis can be obtained by tracking a small sample of households who participate in the ITA experiments (market tests), and measuring how they respond over time to the policy under consideration. Data from this small sample can be extrapolated and used as an estimate of how the population (on average) will respond to the policy (and change of parameters of the policy) under consideration, if implemented on a large scale.

The framework can also be used to evaluate ex-ante the potential payoffs from investing in research that would create new dishes based on local cereals and that are potentially acceptable, or in research to generate technologies that can reduce the processing cost of some raw foods used in some dishes. For example, the cost of the research can include research development costs and market promotion of the dishes and the benefits would include gains from the increase (shift) in demand for local cereals which can be measured by the producer surplus, and a possible net gain in consumer surplus if there is an indirect shift in supply. The benefits would also include the possible saving in foreign exchange by the reduction in imported cereals.

Another area where the model could be useful is in forecasting the demand for specific food items usually consumed by the household. Indeed, given the diversity of the household food basket, in practice we cannot include all the food items used by the household separately in the system of demand equations. In other words, because of degrees of freedom and computational problems, there are some limits in disaggregating the household food basket. Consequently, for certain types of food or fine quality distinctions we cannot directly estimate individual elasticities or forecast demand. However, with this model we can have good forecasts for the demand of these individual raw foods. Indeed, since we can determine which dishes use the raw foods, the unconditional demand for these raw food j can be estimated by the quantity

$$\sum_{i=1}^{m} x_{ij} \cdot P(d_i=1)$$

where $P(d_i=1)$ $i=1,\ldots,m$ are the dish choice probabilities which can be estimated by $F(\hat{\gamma}r_i)$ or by N_i/N their frequencies, and x_{ij} is the amount of raw j used in dish i, an estimate of which can be obtained from expert opinion or by conducting quick interviews of households from which we compute the sample means of the observed x_{ij} (after appropriately correcting for the effect of household size).

These forecasts can be obtained for almost any raw food consumed in the country. Although this may not be of particular interest to policy makers (they are mainly concerned with the cereals), it can be of great value for the private entrepreneurs involved in the marketing of these raw foods.

Finally, the model can be used to test hypotheses about the food consumption behavior of particular sections of the population, or to evaluate some present micro/macro policies of the Senegalese government, the consumption effects of which are often overlooked. For example, the following hypotheses may be of interest:

1. Does the decrease in the supply of fish increase the demand for imported rice in the urban areas?

2. Does the increase in the supply of fish and/or farm income increase the demand for imported rice in rural areas?

While the second hypothesis might seem trivial knowing that rice and fish are complementary in the most consumed dish ("cebbu Jenn") in Senegal and that with an increase in income the rural household can be expected to want to diversify its diet, the answer to the second hypothesis may not be obvious. Given that rice and fish are the main components of the "cebbu Jenn" dish, the most common dish in urban areas, one would think that a decrease in the supply of fish will lead to a decrease in the demand for rice. This may not hold because the household tends to increase the coefficient of rice utilization for this important dish (to partly substitute for the calorie deficiency) whenever there is a shortage of fish.

As an example, there are two policies that we can analyze within the model: fishing policy and meat policy. The present fishing policy of the government consists of two components:

(1) An export subsidy to boost fish exports. The benefit of this policy is the generation of foreign exchange. The cost includes the monetary cost of the subsidy, the consumption costs coming from the shrinking of the domestic supply and indirect foreign exchange costs if the first hypothesis above is not rejected.

(2) Signing of agreements with industrialized countries that allow the fishing boats of these countries to fish in Senegalese waters. The benefit of these agreements is the monetary

compensation that the government gets. The costs include the same consumption costs due to the shrinking of the domestic supply, as well as domestic producer costs incurred by the traditional fishermen.

The government meat policy restricts the importation of meat by imposing high import tariffs on beef, chicken and other livestock products. The benefits of this policy are domestic meat producer gains (through higher prices). The costs are consumption costs because of supply limits, but also foreign exchange costs coming from increased rice demand, and millet producer costs because of shrinking millet demand. These two costs arise because meat is the main complement of millet in the couscous dish.

Of course, these policy questions can be addressed by using the traditional full system of demand equations, by incorporating all the relevant food complements of the cereals (fish, meat, vegetables etc.) so as to have a complete set of cross-price elasticities. However, there will still be missing information on the consumption technology constraints facing the household. We believe that besides the statistical problem of dish selection bias, these constraints are important in shaping the household's food consumption behavior. Thus, we argue that these policy problems are better addressed by using this model.

Perhaps we should again emphasize the fact that the problem of the food complements that we discussed in chapter 2, so far neglected in the analysis of food demand in Senegal, is different from the problem of dish selection bias that we particularly emphasize in this paper. This complementarity/substitutability problem is related to the general problem of separability. The separability problem is familiar to demand analysts; the severe restrictions it implies were reviewed in Chapter 2. In our particular example, an unjustified separability between cereals and food complements would lead to biased and inconsistent estimates of the price elasticities of the cereals. I addition, the dish selection bias, must be corrected for, in order to obtain unbiased and consistent price elasticity estimates. Viewing the household as making its choices primarily among dishes rather than among raw foods has policy implications. The household's choice of dishes leads to unstable and highly discontinuous demands for the individual raw foods because of the limited degree of substitution between raw foods allowed by the technologies of the different dishes.

For clarity and ease of exposition, regarding the estimation, Chapter 5 considered only a random sample of N households, that is, a pure cross section analysis. It would be more realistic to have observations across time for each household in the sample, so that we can take into account heterogeneity ("taste" differences across households), serial correlation, and state dependencies which model the correlations among the midday meal, evening meal and previous day's meals. Heterogeneity and serial correlation could be analyzed by using panel data methods while the state dependency which reveals the dynamic nature of the model could be analyzed using a third order Markov chain model and/or the state dependency model of Heckman (1981). This would enable us to estimate the long run equilibrium or steady state food consumption behavior of the household (Heckman 1981; Amemiya, 1985, pp. 351-54, 412-432).

APPENDICES

Appendix A1

d_E is a pseudometric

For d_E to be a pseudometric it should be positive (which is trivial by construction) and should satisfy the following three properties:

i) $\forall a \in A \qquad d_E(a,a) = 0$

ii) $\forall a,b \in A \qquad d_E(a,b) = d_E(b,a)$ (symmetry)

iii) $\forall a,b,c \in A \qquad d_E(a,c) \leq d_E(a,b) + d_E(b,c)$ (triangular inequality)

For i) we have:

$$\forall a \in A \; d_E(a,a) = |P_E(a) - P_E(a)| = 0 \text{ if } a \in E$$
$$= 0 \qquad \text{if } a \notin E$$

For ii) we have:

$$\forall a,b \in A \quad d_E(a,b) = |P_E(a)-P_E(b)| = |P_E(b)-P_E(a)| = d_E(b,a)$$
$$\text{if } a,b \in E$$

and $\quad d_E(a,b) = d_E(b,a) = 1$ or $0 \quad$ otherwise

For iii) we have:

if $a,b,c \in A$ then we have the following four cases:

1. $a,b,c \in E$ then

$$d_E(a,c) = |P_E(a) - P_E(c)| = |P_E(a)-P_E(b)+P_E(b)-P_E(c)|$$
$$\leq |P_E(a)-P_E(b)| + |P_E(b)-P_E(c)| = d_E(a,b) + d_E(b,c)$$

2. $a \notin E$, $b \in E$, and $c \in E$ then

$d_E(a,c)=1$, $d_E(a,b)=1$, and $d_E(b,c)=|P_E(b)-P_E(c)| \geq 0$. Hence,

$$d_E(a,c) \leq d_E(a,b) + d_E(b,c)$$

3. $a \notin E$, $b \notin E$, and $c \in E$ then

$d_E(a,c)=1$, $d_E(a,b)=0$, and $d_E(b,c)=1$. Hence,

$$d_E(a,c) = d_E(a,b) + d_E(b,c)$$

4. $a \notin E$, $b \notin E$, and $c \notin E$ then

$d_E(a,c)=0$, $d_E(a,b)=0$, and $d_E(b,c)=0$. Hence,

$$d_E(a,c) = d_E(a,b) + d_E(b,c)$$

Thus because of the symmetric role played by a, b, and c we conclude that

$$d_E(a,c) \leq d_E(a,b) + d_E(b,c) \quad \forall a,b,c \in A$$

$\square$

As already said, d_E is not a metric since $d_E(a,b)=0$ does not imply $a=b$. But we can construct a metric space based on d_E by considering the equivalence relation $\sim$ defined in A as follows:

$$a \sim b \text{ if and only if } P_E(a)=P_E(b)$$

this is indeed an equivalence relation that partitions A into classes of alternatives having the same probability of selection.

Denote by $S = A/\sim$ the quotient set of $\sim$ then we can define in S a metric $\tilde{d}_E$ by:

i) $\tilde{d}_E(\bar{a},\bar{b}) = |P_E(\bar{a})-P_E(\bar{b})| \quad \forall \bar{a},\bar{b} \in S$ such that $\bar{a} \subset E$, $\bar{b} \subset E$

ii) $\tilde{d}_E(\bar{a},\bar{b}) = 1$ if $\bar{a} \subset E$ and $\bar{b} \not\subset E$ or $\bar{a} \not\subset E$ and $\bar{b} \subset E$

iii) $\tilde{d}_E(\bar{a},\bar{b}) = 0$ if $\bar{a} \not\subset E$ and $\bar{b} \not\subset E$

then $(S,\tilde{d}_E)$ is a metric space thus is Hausdorff. $\square$

Appendix A2

Separability of (A, d_E)

Proposition: (A, d_E) is separable

Proof:

Since (A, d_E) is a pseudometric space, then it is separable if and only if it is second countable (Dudley, 1989, p. 25-26) that is if and only if it has a countable base. But A is (by assumption) the Borel σ-field of R^d_+. Hence the set 3 of all open balls of R^d_+ with radius 1/n n∈N and the centers of which are the elements of Qd_+ where Q is the set of positive rationales, is a countable base for the Borel σ-field A.

Let 3 = { B(z,1/n): z∈Qd_+, n∈N} we need to show that 3 is dense in A endowed with the pseudometric d_E.

Let a∈A and B(a,r) = {b∈A; d_E(a,b) <r } an open ball in A, but for any b∈A, b is union of elements of 3 thus B(a,r) contains at least one element of 3. So that 3∩B(a,r) ≠ Ø. And since the open balls B(a,r) a∈A constitute a base for the topology F_E corresponding to d_E, we conclude that 3 is dense in A. 3 is countable, hence (A,d_E) is separable. In fact, it is perfectly separable. Indeed the set of all open balls B(b^*, 1/n), b^*∈3, n∈N is a countable base for F_E. □

Appendix A3:

Proposition: $\succsim$ is a complete ordering

Proof:

1. reflexivity: the reflexivity comes from the triviality $P_E(a)=P_E(a)$ so that we have $a \succsim a$.

2. transitivity: let $a,b,c \in A$ then

$$a \succsim b \Leftrightarrow P_E(a) \geq P_E(b)$$

$$b \succsim c \Leftrightarrow P_E(b) \geq P_E(c)$$

this implies $P_E(a) \geq P_E(b) \geq P_E(c)$ which in turn implies $P_E(a) \geq P_E(c)$ so that $a \succsim c$

3. Completeness: the completeness comes trivially from the completeness of $\leq$ in [0,1] indeed for any $a,b \in A$ $P_E(a) \geq P_E(b)$ or $P_E(b) \geq P_E(a)$ since they are all element of [0,1]. Thus $\forall a,b \in A$ $a \succsim b$ or $b \succsim a$. $\square$

Appendix A4

Proposition: F_E is a natural topology for $\succsim$. That is: the sets

$F_1=\{b\in A\colon a \succsim b\}$ and $F_2=\{b\in A\colon b \succsim a\}$

are closed for all $a\in A$.

Proof:

We note that in showing the separability of (A,d_E) we have shown that $\mathcal{B}=\{B(z,\ 1/n)\colon z\in Q^d_+,\ n\in N\}$ is countable and dense in A. So that the set of all open balls $B(b^*,\ 1/n)$, $b^*\in\mathcal{B}$ is a countable base for F_E the topology corresponding to the pseudometric d_E. Thus, like for a metric space, a subset F of A is closed if and only if any convergent sequence of elements of F has its limit in F. (Schwartz; 1970, pp. 47-48)

So let $\{b_n\}_{n\in N}$ a sequence in F_1 such that $\lim\limits_{n\to\infty} b_n=b$

that is

$\lim\limits_{n\to\infty} d_E(b_n,b)=0$. Note that for this to hold, we must have $d_E(b_n,b)<1$ for a infinite number of b_n thus we have only 2 cases.

1) $b\notin E$ and there is $k\in N$ such that $b_n\notin E$ for all $n\geq k$ then

$d(b_n,b)=0$ and $P_E(b_n)=P_E(b)$ for all $n\geq k$. Hence

$b_n\in F_1 \iff a \succsim b_n \iff P_E(a)\geq P_E(b_n) = P_E(b)$

this implies that $a \succsim b \iff b\in F_1$

2) $b\in E$ and it exists $k\in N$ such that $b_n\in E'$ for all $n\geq k$

then in this case we have:

$d(b_n,b) = |P(b_n)-P(b)|$ and

$\lim\limits_{n\to\infty} d(b_n,b) = 0$ if and only if $\lim\limits_{n\to\infty} P(b_n) = P(b)$

but $b_n\in F_1$ implies $P(a) \geq P(b_n)$

$\Rightarrow P(a) \geq \lim\limits_{n\to\infty} P(b_n) = P(b) \Rightarrow$

or $P(a) \geq P(b)$ So that $b\in F_1$

Thus F_1 is closed

Similarly F_2 is closed by the same way

We conclude that F_E is a natural topology for $\mathfrak{D}$. $\square$

Appendix A5

In section 4.4 we made the following claim:

There exists a probability measure P on A_E such that (A, A_E, P) is a probability space with $P\{b\epsilon A: U_E(a)\geq U_E(b)\} = P_E(a)$ $\forall a\in A$.

This conjecture can be shown along the following lines.

First we will need the following standard theorem of measure theory

Theorem:

For any set X, and ring* A of subsets of X, any countably additive function μ from A into $[0, +\infty]$ extends to a measure on the σ-algebra $\mathcal{L}$ generated by A.

Proof: See Dudley (1989, p. 66-67).

Next we show that the collection $\mathcal{P}=\{ \{b: b\epsilon A; U_E(a)\geq u_E(b)\}: a\in A\}$ is a ring and then define on $\mathcal{P}$ the following countably additive function: $P\{b: b\epsilon A; U_E(a)\geq U_E(b)\} = P_E(a)$.

We then show that A_E is the σ-algebra generated by $\mathcal{P}$ and using the above theorem we extend P to A_E. The final step is then to show that the range of the extended P is in [0, 1] that is, $P^e(A)=1$ where P^e is the extension of P in A_E.

□

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* A collection $\mathcal{P}$ of subsets of X is called a ring iff $\emptyset\in A$ and for all A,B in A we have $A\cup B \in A$ and $B\setminus A \epsilon A$.

Appendix A6

Variance of η_{ij}^*

By definition $\eta^*_{ij} = X_{ij} - E(X_{ij}|d_i=1)$ is conditional to i. Hence

Var(η*ij) = $Var(X_{ij}|d_i=1) =$ E(X2ij$|d_i=1$) - [E(Xij$|d_i=1$)]2

To simplify the notation, let

$$\bar{X}_{ij} = \frac{\partial V_i(P_i,d)/\gamma P_j}{\partial V_i(P_i,d)/\partial d} \qquad \text{and}$$

$$\lambda = \lambda(V_i^1,\ldots,V_i^{i-1},\ V_i^{i+1},\ldots,\ V_i^m)$$

then

$$[E(X_{ij}|d_i=1)]^2 = \bar{X}_{ij}^2 + 2\bar{X}_{ij}\lambda + \lambda^2$$

and

$$E[X_{ij}^2|d_i=1] = E(\bar{X}_{ij}^2 + 2\eta_{ij}\bar{X}_{ij} + \eta_{ij}^2|d_i=1)$$

$$= \bar{X}_{ij}^2 + 2\bar{X}_{ij}E(\eta_{ij}|d_i=1) + E(\bar{\eta}_{ij}^2|d_i=1)$$

$$= \bar{X}_{ij}^2 + 2\bar{X}_{ij}\lambda + E(\bar{\eta}_{ij}^2|d_i=1)$$

hence

$$Var(\eta^*_{ij}) = \bar{X}_{ij}^2 + 2\bar{X}_{ij}\lambda + E(\bar{\eta}_{ij}^2|d_i=1) - \bar{X}_{ij}^2 - 2\bar{X}_{ij}\lambda - \lambda^2$$

$$= E[\eta_{ij}^2|d_i=1] - \lambda^2$$

□

ENDNOTES

1. Strictly speaking, there are two classes of probabilistic choice models: the constant utility model which is more frequently used by the psychologists, and the random utility model used mainly by the economists. A good summary of the differences between the two models is given by Tversky (1972, p. 341) in the following terms.

> Random utility models assume that the utility, or the value, of each alternative undergoes random fluctuations, and that the alternative with the highest momentary value is selected. Constant utility models, on the other hand, express choice as a probabilistic function of the (constant) scale value assigned to each of the alternatives. The two types of representations differ with respect to the locus of the probabilistic element in the choice process. Random utility representations attribute uncertainty to the determination of value, while constant utility representations attribute uncertainty to the decision rule. The two types of representations, however, are not incompatible: some (though not all) choice can be represented as either a random or a constant utility model.

2. We should point out that, in our knowledge, the compact expressions of $\lambda_j^i(\gamma r_i)$ and of the covariance matrix of η_i^* given in (5.1.8) - (5.1.10) (which are a multinomial/multivariate generalization of the standard dichotomous/univariate Tobit results) have not been derived yet. The closest forms found in the literature (see, for example Duncan (1980) or Amemiya (1985, p. 407)) are respectively expressions (5.1.1) and (5.1.2). For the conditional moments appearing in (5.1.1) and (5.1.2), the readers are usually referred to either Tallis (1961) (where they are given in a derivative forms not readily computable) or Amemiya (1973). We were able to derive these compact and more simple expressions only after rewriting Amemiya's results in matrix

form. Our expressions are much simpler to compute with a programming language like GAUSS.

3. This result generalizes the one of Amemiya (1985, p. 370) and Heckman (1979).

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