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A SIMULATION ANALYSIS OF INTERFERENCE INDUCED BY A FREQUENCY HOPPING SIGNAL TO AN FM SIGNAL

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A SIMULATION ANALYSIS OF INTERFERENCE INDUCED BY A FREQUENCY HOPPING SIGNAL TO AN FM SIGNAL

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ABSTRACT

A SIMULATION ANALYSIS OF INTERFERENCE INDUCED BY A FREQUENCY HOPPING SIGNAL TO AN FM SIGNAL

By

Ghulam H. Raz

There is a growing interest in utilizing Spread Spectrum communication techniques for civilian applications. As we enter to the twenty first century more bandwidth will be required for growing communication industry. Unfortunately the existing bands are limited and most of them are already occupied by different licensees. Most of these users are utilizing conventional communication systems. One possible way to utilize the spectrum more efficiently is to overlay spread spectrum systems on top of conventional system. Two important implementations of spread spectrum are popular at the present time, namely direct sequence (DS) and frequency hopping (FH) spread spectrum. These two techniques will produce some interference to amplitude modulation (AM) and frequency modulation (FM) systems. Therefore one can consider four interference cases: i) DS to AM, ii) DS to FM, iii) FH to AM, iv) FH to FM. Frequency hopping interference to FM systems is the most severe case. Consequently this study consider FH interference to FM systems. The FM receiver presented adapts a zero-crossing scheme for demodulation.

A mathematical analysis of an FM zero-crossing is presented. It has been shown theoretically that the FM signal can be detected from its zero-crossings by a low pass

filter. A mathematical model for this detection has been developed, and expressions for every component of the FM zero-crossing detection is presented.

In the ideal zero-crossing technique for FM demodulation which is considered, there is no need to differentiate the modulated signal. Samples of the received signal are stored in memory, and are immediately processed by an appropriate microprocessor.

To demonstrate the potential compatibility of spread spectrum and conventional FM, a detection simulation study was conducted and algorithms developed for the detection. The results of the simulation confirm that as one gets closer to ideal zero-crossing, the performance of the detector improves remarkably. The algorithms developed were used for simulating FM detection in the presence of white Gaussian noise and FH signals. It is shown for the simple case simulated that FH as an interfering signal induces less distortion to FM system than the background noise normally present. The conclusion is that for many applications conventional FM systems could coexists with appropriately designed frequency hopping spread spectrum systems.

DEDICATION

I dedicate this dissertation to:

my parents

my teachers

and

my family

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CHAPTER 1

INTRODUCTION

1.1 Spread Spectrum:

A new class of communication system named spread spectrum has been emerging in recent years. "Spread spectrum is a means of transmission in which the signal occupies a bandwidth in excess of the minimum necessary to send the information; the band spread accomplished by means of a code which is independent of data, and a synchronized reception with the code at the receiver is used for despreading and subsequent data recovery." [1]

A block diagram of a general transmitter-receiver spread spectrum system is shown in figure 1.1. From this diagram one can notice the following two important observations:

1) Modulation and demodulation are independent of the type of spread spectrum used. This will become more clear as one discusses different types of spread spectrum modulation.

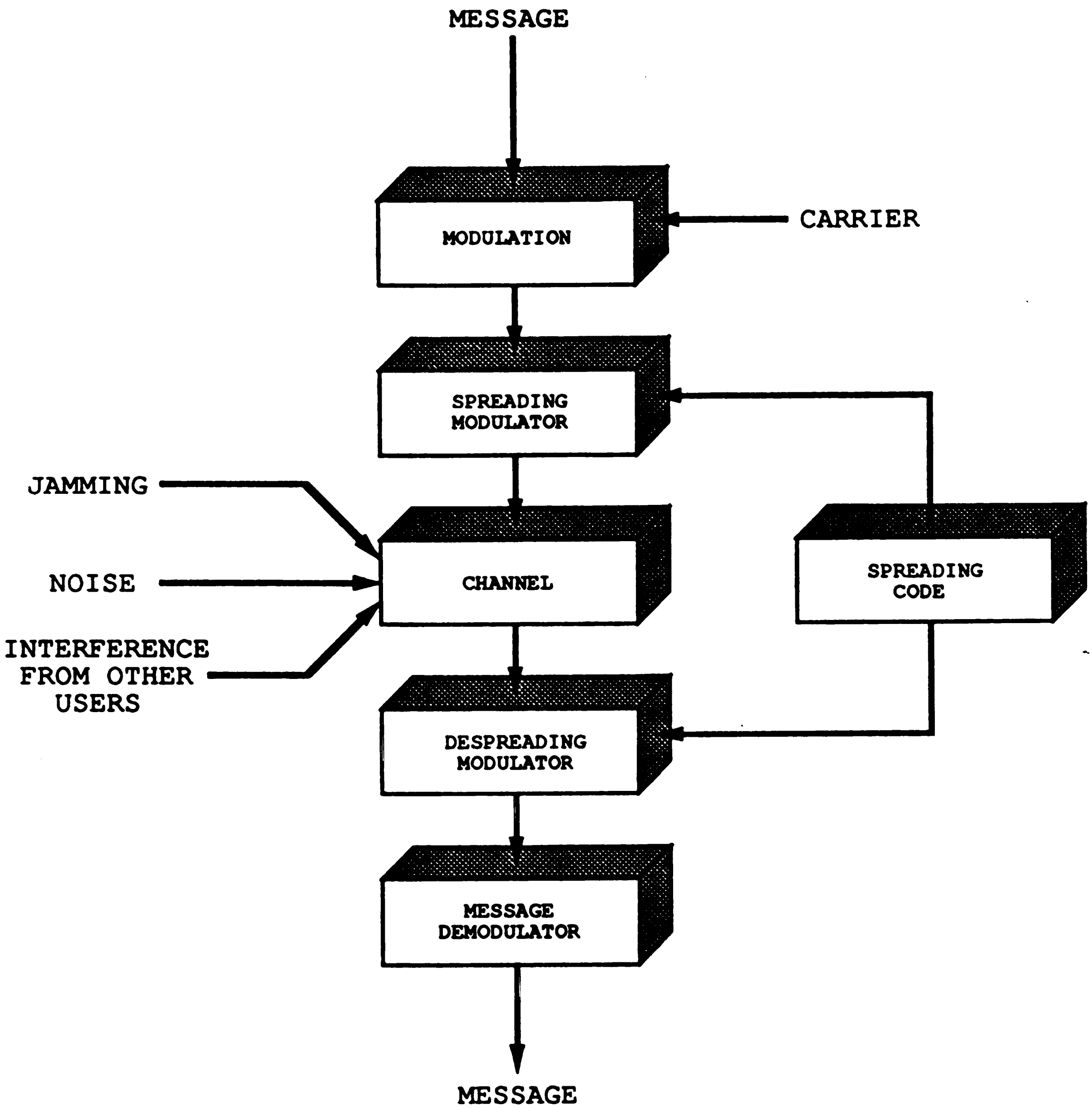

Figure 1.1 Spread spectrum transmitter receiver

2) The spreading modulator and demodulator both are using the same code. Thereforethe code in transmitter and receiver should be in synchrony, otherwise one has to use an acquisition scheme until synchrony is achieved. A discussion and generation of these codes see [2] and [3].

Using spread spectrum techniques one can spread the power across a wide band of frequencies. As a result the power density (watt/Hz) is very low, and make it impossible to detect the signal without the knowledge of the spreading code. Thus the most important application of this technique has been in the military. The military system designers combined these techniques with the rapid evolution in integrated circuit technology and made it possible for the development of extremely wide band direct sequence and frequency hopping spread spectrum systems.

Bandwidth spreading by direct modulation of a data modulated carrier by a wideband spreading signal or code is called direct sequence (DS) spread spectrum [4].

In a Frequency Hopping (FH) signal, the frequency is constant in each time chip, but changes from chip to chip. If the hopping rate is greater than the bit message rate, it is called fast hopping. The number of frequencies over which the signal may hop is usually a power of 2 [5].

There are many reasons for spreading the spectrum. Some of these are:

1) Anti Jamming

2) Anti Interference

3) Low probability of Intercept

4) Multiple user random access communications
with selective addressing capability

5) High resolution ranging

6) Accurate Universal Timing

The above features of spread spectrum are used primarily by military communication systems. However, there has been a growing interest in utilizing spread spectrum techniques for civilian systems for example mobile radio communications, packet radio, radio telephony, amateur radio, timing and some specialized applications in satellites. The suggested application which received considerable attention in published literature is that of cellular mobile radio [6].

1.2 Historical Background:

Military usage of spread spectrum signal is well established. In many military applications there is no alternative to spread spectrum.
The civil applications of spread spectrum are different in character, and represent relatively new areas of investigation. The details of some of these cases are examined in this chapter. The purpose of this section is to present some of the historical background and the growing importance of this area of communication.

Preliminary discussions on a possible study of non-government uses of spread spectrum techniques was first held between Mr. Jack Robinson of the FCC (Federal Communication Commission) and Dr. Feisal Keblawi of the MITRE Corporation. After their meeting on the subject, a proposal was prepared, and after coordination with the FCC's Chief Scientist, Dr. Stephen Lukasik, a contract was awarded. Dr. Michael

Marcus of the FCC's office of science and technology was named as the coordinator. The MITRE corporation prepared the following document.

"Potential Use of Spread Spectrum Techniques in Non-Government Applications",By Walter C. Scale, December 1980, MTR - 80W335 CONTRACT SPONSOR:Federal Communications Commission CONTACT:FCC-0320 Project 14570 [7].

The document which is 215 pages long consists of 7 sections and 3 appendices. A list of individuals who had contributed to the study is given in Footnote [1].

As a result of the MITRE Report, the FCC issued a notice of inquiry, Docket 81-413. In the matter of "authorization of spread spectrum and other wideband emissions not presently provided for in the FCC Rules and Regulations."

This inquiry notice raised many questions in communication circles. The most important questions which are related and, in reality motivated this study will be repeated with their answers later.

At the April 1982 Communication Theory Workshop (Wickenberg, Arizona) the communication theory committee of the IEEE Communication Society decided to generate an IEEE submission to the FCC Docket No. 81-413. A drafting committee had

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Footnote 1. A list of contributors to the MITRE report

Yarsolav Kaminsky	MITRE	Joe Fee	MITRE
Jim Morrell	MITRE	George Steelman	MITRE
Art Smith	MITRE	Mary Ann Davis	MITRE
Bill Zeiner	MITRE	Bob Noyer	MITRE
Richard Horn	MITRE	Paul Ebert	COMSAT
Aaron Weinberg	Stanford Telecom	Richard Tell	EPA/Las Vegas
Ray W. Nettleton	Michigan State University	George Cooper	Purdue University
Jerry Silverman	Magnavox	Charles Cahn	Magnavox
Paul Goodman	Bell Lab/Holmdel	Thijs de Haas	DoC, NTIA
Carl S. Mathews	DoC, Maritime	John Juroshek	DoC, NTIA
Samuel Musa	DoD	Don J. Torrieri	DoD

been formed which was led by Ray Nettleton of Michigan State University. At the June ICC'82 (in Philadelphia, PA) the communication theory committee received the draft which was prepared by Ray Nettleton [8], and subsequently established a "Review Committee," led by Don Schilling of the City College of New York. After two iterations between the Nettleton and Schilling committees a final report was generated, entitled "Before the Federal Communications Commission, Washington, DC 20554 General Docket No. 81-143. In the matter of: Authorization of spread spectrum and other wideband emission not presently provided for in the FCC Rules and Regulations." The report met the deadline set by FCC and later on was published in the IEEE communication magazine entitled "FCC Testimony."

In the remainder of this section the questions which are related to this study and raised by FCC are repeated along with a summary of reply comments which were provided by IEEE Communication Society. For details the reader is referred to The FCC Testimony [9].

Question: What services can be accommodated by the use of spread spectrum system? Can they be implemented by band overlay or will they require dedicated frequency allocations?

Answer: The answer to the above question is provided in two parts. Part I lists the recommended potential applications and Part II lists the applications which were not recommended.

I. Potential Applications

a. Low Power Density Application:

This category which could cover a distance of less than one kilometer in open terrain includes almost all transmission applications that do not require an FCC license at that time. Some of the applications are as follows: Biomedical and other telemetry; remote control of domestic and other appliances and machinery; wireless telephone and microphone; walkie-talkie; wireless intercom tracking; homing and trailing; intrusion alarms; wireless data terminals, and radio location in urban areas. If the total power spectral density of all simultaneous users within a given interference zone is limited to levels below the ambient noise level these systems could be overlaid onto existing services.

b. Multiple Access Systems:

Multi users each with low duty cycles at ranges up to 10 Km. Representative applications are: mobile telephone and data networks, pagers telemetry, a "citizen's band," radio location and automatic status reporting devices such as proposed for law enforcement and tracking fleets. If the power spectral densities are higher than ambient noise then the use of overlay would be undesirable. With appropriate constraints overlay is always technically feasible, the question remains whether or not it would be desirable.

Another area that overlay may be seriously considered is in wide industrial scientific, medical (ISM) bands where the other uses of the spectrum are not for communication purposes, and only a minimal set of rules relating to interference would be required.

II. Applications Which are not Recommended:

Any spread spectrum overlay on an existing service that generates a high level of spectral interference power.

Services that would require a dedicated band for spread spectrum use, for example most multiple channel links that numerous transmitters are collocated.

Question: In the case of band overlay, should a spread spectrum system be required to operate on a noninterference basis with conventional system?

Answer: Overlay will always cause perceptible interference to a narrowband service. The interference would be smaller if an overlay is to be used to exploit an underutilized existing service.

Question: What narrowband receiver characteristic should be considered in determining the interference potential of spread spectrum systems to conventional narrowband emissions? Do these characteristics affect the possibility of having spread spectrum systems and conventional systems coexist in the same frequency band?

Answer: The response of a narrowband receiver to a spread spectrum interference depends on both the narrowband receiver characteristics and the nature of spread spectrum signal. Two important characteristics of the narrowband are: bandwidth image rejection, capture ratio.

Question: Should each wideband modulation technique be considered on its own merits as to the spectrum use and efficiency?

Answer: Yes.

Question: Will special test equipment be necessary to evaluate spread spectrum emission?

Answer: Yes.

Question: Can systems which use different wideband modulation method be evaluated by the same measurement techniques?

Answer: No.

Question: Will transmitter duty cycle affect the measurement technique and results?

Answer: Yes.

Question: If the commission chooses to authorize spread spectrum systems, will detailed technical standards be needed?

Answer: Yes.

There were some general questions about the cost of the new system, such as cordless telephone, and questions with lengthy answers which were not reported here, however the above questions and answers clearly show that both the industry and FCC were interested in utilizing the spread spectrum in civilian applications at that time.

In early 1984 FCC proposed to allow spread spectrum utilization to operate on any range of frequencies above 70 MHZ without any restriction on their occupied bandwidth but with limited radiation. Dr. Michael Marcus of the FCC was assigned to receive replies and comments about the further notice of inquiry.

On May 9, 1985 the FCC adopted new regulations regarding authorization of spread spectrum and other wideband emission which did not previously exist in the Rules and Regulation of FCC.

The new Rules and Regulation appeared in the FCC 85-245, 35747 document which is 20 pages long. Some of the new Rules and Regulations which supports the relevance of this study regarding frequency hopping spread spectrum are as follows:

1) Spread spectrum system may be operated in the 902-928 MHZ, 2400 - 2483.5 MHZ and 5725 - 5850 MHZ frequency bands subject to:

(a) Maximum peak output power of 1 watt.

(b) RF output power outside these bands over any 100 KHZ must be 20 dB bellow that in any 100 KHZ bandwidth within the band of operation.

(c) They will be operated on a noninterference basis to any other operations.

(d) For frequency hopping systems at least 75 hopping frequencies, separated by at least 25 KHZ shall be used. The average occupancy time on any frequency shall not be greater than .4 second within a 30 second period. The maximum bandwidth of the hopping channel is 25 KHZ.

2) Spread spectrum transmitters may be operated on public safety frequencies between 37 and 952 MHZ, subject to:

(a) Output power of 2 watts.

(b) At least 20 hopping frequencies shall be used.

(c) The average occupancy time on any frequency shall not be greater than .1 sec. in every 2 sec.

The new Rules and Regulations clearly show that FCC acknowledges the utilization of spread spectrum in civilian applications.

Recently FCC released another document GE docket No. 89-354 in the matter of amending the Rules and Regulations of 1985. This document, which was adopted on June 14, 1990 and released on July 9 and became effective August 24, 1990, relaxed some of the conditions which were previously imposed in 1985. The conditions which affect frequency hopping are as follows:

i) Frequency hopping systems operating in 902-928 MHZ band shall use at least 50 hopping (previously 75) frequencies. The maximum allowed 20 dB bandwidth is set to 500 KHZ (previously 100 KHZ). The average time of occupancy can not be greater than .4 seconds within a 20 second period.

ii) Frequency hopping systems operating in the 2400-2483.5 MHz and 5725-5850

MHz bands shall use at least 75 hopping frequencies. The maximum 20 dB bandwidth of the hopping channel is set to 1 MHz. The average time of occupancy on any frequency can be as much as .4 seconds within a 30 second period.

This new amendment has further promoted the use of spread spectrum in public domain applications. At the present time some of the proposed applications have already been realized, such as cordless telephone and microphone etc, and active research is conducted toward implementation or improvement of new spread spectrum systems for civilian applications [10]-[17].

Recently three prominent investigators in spread spectrum technology, namely Donald Schilling, Raymond Pickholtz, and Laurence Milstein, reexamined spread spectrum applications in public domain in an article " Spread Spectrum Goes Commercial" [18]. The article reflects the great potential of spread spectrum in peace time after so many years of cold war. The frequency spectrum is extremely congested, and spread spectrum is going to provide more utilization of the available spectrum and permit sharing the existing spectrum. The article also discusses many aspects of spread spectrum, including overlay and spectrum sharing with conventional systems. The article addresses many points that are being considered in this study.

In summary the recent regulation of FCC, the research activities in this field, and most important of all the recent article by three famous professors in this field reveals that this study is worthwhile. Hopefully this study will enhance spectrum sharing and overlaying, as well as additional investigation of spread spectrum technology.

1.3 Spectrum Management:

Although there are several justifications for using spread spectrum communication systems, there are some draw- backs also. The main problem which will be caused by spread spectrum to narrow band system is the problem of interference. Spread spectrum produces some interference to existing conventional narrowband AM and FM communication systems. Therefore there will be a spectrum efficiency dilemma. To manage this problem there are two basic methods for implementing spread spectrum. The first method involves locating and allocating the existing spectrums by reserving some band of frequencies to be used by spread spectrum users. In this approach narrowband systems would be prohibited from operating in the same band. The second approach would allow spread spectrum systems to operate in the same band as conventional narrowband systems. This approach will allow the spread spectrum signals to overlay the signal on top of the existing narrowband signals.

The first method, reserving a band for spread spectrum, is not feasible or at least is not an easy task to accomplish. If spread spectrum proved to be more efficient than the existing narrow band system, it would gradually phase out the narrow band; and this is an effort toward that task. Therefore a concise study of the coexistence of these two systems is crucial.

1.4 Problem Identification:

The second method, which is overlaying spread spectrum signals on top of current existing systems, will also cause complications. In lower frequencies where the band is crowded, spread spectrum signals produce interference and, as a result, distortion.

The only frequencies for which spread spectrum overlay seems feasible are above 800 MHz; since in this portion of the band block allocation is wide, spread spectrum signals would overlay fewer services. In any circumstance in this application, one should deal with interferences between narrowband and spread spectrum signals.

1.4.1 Literature Review:

Besides the growing demand for spectrum resources, which was discussed in section 1.2, there is still enough room for more users. An FCC report [6] indicates that as few as 48% of the 25-47 MHz land mobile channels monitored in Los Angeles had a very "high occupancy" (defined as at least 60 percent peak hour message occupancy).

By overlaying spread spectrum on existing narrowband systems, it might be possible to improve the overall efficiency of spectrum management. In any kind of band sharing or overlay between spread spectrum and narrowband systems, three distinct types of interferences will arise.

1) Interference to spread spectrum signals from other spread spectrum signals: This type of interference was the topic of investigation in many papers, for example [6] and [19]-[35]. This kind of interference mainly occurs in multiple-access applications of spread spectrum. To study this type of interference one needs to specify the type of application and spread spectrum technique which will be utilized.

2) Interference to spread spectrum systems from conventional broadcasting systems: The survivability of spread spectrum signals in bands of conventional signals operations depends on the following parameters:

I) Number of users in the conventional band.

II) The maximum power which is utilized by different users.

III) The frequency spacing of the conventional signals.

IV) The specific spread spectrum technique.

In direct sequence systems for example the interfering (conventional) signals are attenuated by an amount equal to the receiver's processing gain. This has been studied in [1], [2],[36] and [37]. Slow frequency hopping systems can be distorted severely by a number of conventional interferers at widely separated frequencies than by a single strong narrow band interferer [38].

3) Interference to conventional services from spread spectrum signals:

This kind of interference was studied experimentally for some applications. The elementary studies on band-sharing between spread spectrum and television signals were published in 1978 by Ormandroyd [39] and Juroshek [40].

Ormandroyd used a black and white TV for this experiment. He found that spread spectrum signals generated by modulating an AM signal with a direct sequence pseudonoise code caused much less degradation than a conventional narrowband AM signal as an interfering signal.

Juroshek found that direct sequence spread spectrum signals produce about the same amount of interference to color television as narrowband FM signals of the same energy level, provided that spread spectrum bandwidth is about 2 MHz or less. For spread spectrum bandwidth greater than 6 MHz, spread spectrum should cause little interference relative to narrow band FM signals simply because of the TV receiver's ability to attenuate interference outside of its nominal 6 MHz pass band.

A frequency hopping signal can produce short bursts of relatively unattenuated interference in a narrowband receiver. A concise comprehensive summary of methods for analyzing spread spectrum interference to conventional receiver is given in [41].

None of the above works addressed the third kind of interference mathematically. Therefore mathematical modeling and simulation of these interferences is the purpose of this study.

1.4.2 Problem Statement:

There has been a more receptive attitude by the regulating agencies toward the possibility of examining the coexistence of spread spectrum system with the narrowband system. This has been reported by the Federal Communication Commission (FCC), (General Docket No. 81-413 Sept. 15, 1981). In response to the FCC inquiry a reply has been provided by the IEEE communication theory committee [42].

Most spread spectrum communication systems utilize the entire band for there applications. An interesting application, which is also spectrally efficient, is when spread spectrum overlays on top of narrowband signals. This process could be for various reasons. The most important one are as follows:

1) Utilization of more frequency.

2) Employing for emergency services.

3) Using jamming in a hostile environment.

These kind of applications are not considered seriously. Any kind of spread spectrum overlay or coexistence must be studied very carefully, to the extent that it either phases out the narrowband completely, or such that it coexists with the narrowband system on a permanent basis. Therefore the topic of this study is to determine whether the coexistence of spread spectrum with the existing narrowband is possible; and, if it is, what would be the tolerance of the narrowband system in the presence of spread spectrum?

CHAPTER II

INTERLEAVING

2.1 Introduction:

In this chapter the desired AM and FM signals, together with their receiver models are presented . The signal models of direct sequence and frequency hopping systems are also presented. Considering these signal sets one gets four cases of interference which are:

1) Interferences of DS to AM

2) Interferences of DS to FM

3) Interferences of FH to AM

4) Interferences of FH to FM

From the study of the input and output signals and interferences in the presence of additive noise one can derive expressions for input signal to noise and interference ratio ($SNIR_I$), and output signal to noise and interference ratio ($SNIR_o$). Finally from the study of their signal to noise and interference ratios, one can determine the worst interference case among the four cases, and in the next chapter a simulation study of the worst case will be performed.

2.2 Interference Effects of DS System on AM System:

The system models in this case are composed of the following subsystems.

2.2.1 Receiver Model:

The AM receiver model consists of a predetection band pass filter, an envelope detector, and a post detection low-pass filter. The band-pass filter has a bandwidth of $2w_m$ and the low-pass filter has a band width of w_m, where w_m is the band width of the message signal. A block diagram of the model is given in Figure 2.1.

2.2.2 Signal Model

The received signal is composed of three signals; the desired AM signal s(t), the white Gaussian noise n(t) and a set of direct sequence spread spectrum signals i(t).

The desired signal s(t) can be represented by

$$s(t) = A_m(t)\cos(w_c t+\Theta) \tag{2.1}$$

where $A_m(t)$ is the amplitude and Θ (which is a constant) represents the phase angle, $w_c=2\pi f_c$ is the angular frequency of the carrier, of the AM signal.

The noise n(t) is assumed to be additive white Gaussian noise (WGN). The direct sequence signal i(t) can be described in the form of

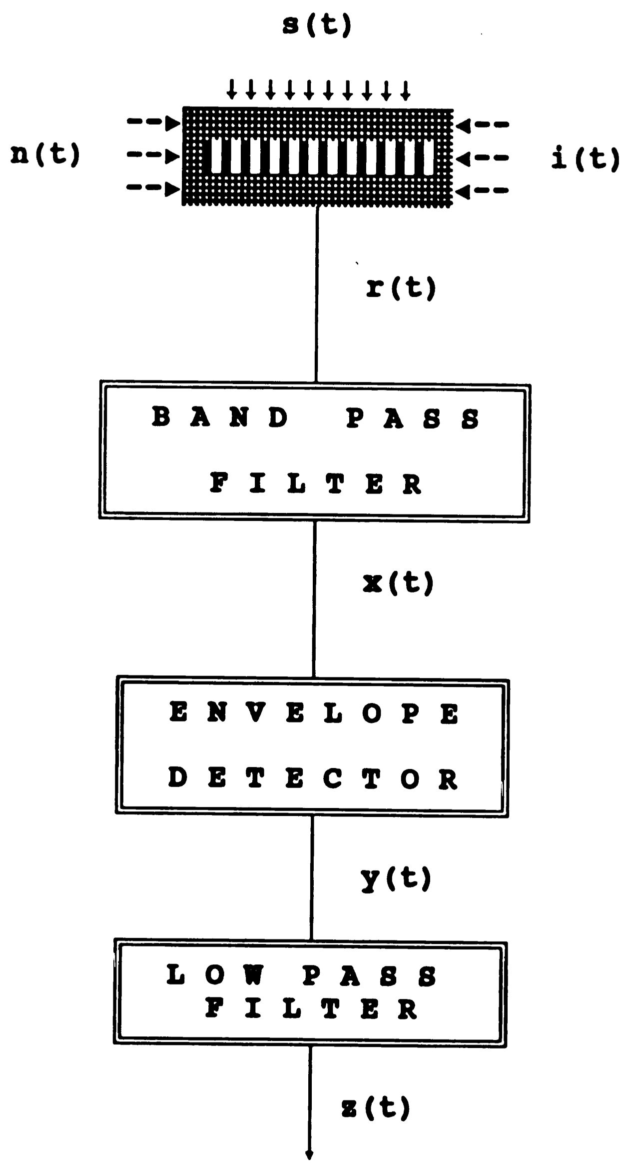


Figure 2.1 AM receiver Model

$$i_{DS}=\sum_{j=1}^{u} I_j d_j(t)\, PN_j(t) \sin(\omega_{jt}+\theta_j) \tag{2.2}$$

where u is the number of users, I_j is the signal amplitude, w_j is the angular frequency and Θ_j is the phase angle for the jth user, $d_j(t)$ is the digital data (message) carried by the jth user and $PN_j(t)\epsilon\{1,-1\}$ is its pseudo-noise code sequence. Furthermore $d_j(t)\epsilon\{1,-1\}$ over the code signal duration T_c for all j, j=1,2,3,...u. The statistical properties of interfering signal i(t) are derived in Appendix A, and are as follows:

$$\overline{i_j(t)}=0 \tag{2.3a}$$

$$\overline{i_j^2(t)}=I_j \tag{2.3b}$$

where the overbar denotes the ensemble average. These expressions play an important role in the derivation of the signal to noise and interference ratios.

2.2.3 System Analysis:

Consider the worst interference case. In this case $w_j=w_c$ for all j, j=1,...,u., thus the interfering power at the output of the predetection band-pass filter is maximum. This kind of communication will only happen in a hostile situation i.e. in deliberate jamming circumstance. If the spread spectrum system designer uses the guard band of AM for spread spectrum users, or in other words interleaving the spread spectrum between two conventional AM signals, only the side lobes of the neighboring signal will contribute some distortion. This idea which could also be applied to FM signals is shown in Figure 2.2. Thus the output power of the band-pass filter will be reduced. Since this power reduction is due to interfering signals, meaningful communication is possible.

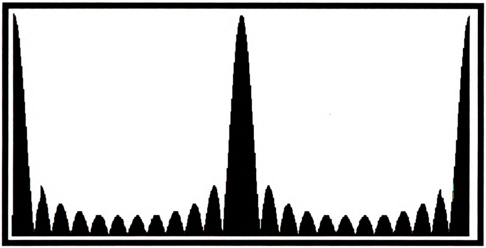


Figure 2.2 The overlay concept demonstrated by the spectra of three adjacent channel

Now consider the AM receiver step by step. The predetection band-pass filter has a bandwidth just sufficient for passing the desired signal, therefore it suppresses the out of band components of noise and interfering signal. As a result one gets the desired signal, narrowband noise and narrowband interference at the output of the band-pass filter. Using in phase and quadrature representation of each of these narrowband signals one can write x(t) the output of band pass filter as follows:

$$\begin{aligned} x(t) = {} & [A_m(t) + n_c(t) + \sum_{j=1}^{u} i_{c_j}(t)]\cos(\omega_c t + \theta) \\ & - [n_s(t) + \sum_{j=1}^{u} i_{s_j}(t)]\sin(\omega t + \theta) \end{aligned} \tag{2.4}$$

where $n_c(t)$ and $n_s(t)$ are the in phase and quadrature component of noise and $i_{cj}(t)$ and $i_{sj}(t)$ are the in phase and quadrature components of interference signal for $j=1,\ldots,u$. Assuming that n(t), $i_j(t)$ and s(t) are independent random variables one can write the following expression for signal to noise and interference ratio (SNIR) at the input of the detector.

$$SNR_I=\frac{\frac{1}{2}\overline{A_m^2}}{\frac{1}{2}(\overline{n_c^2}+\overline{n_s^2})+\frac{1}{2}\sum_{j=1}^{u}(\overline{i_{c_j}^{\,2}}+\overline{i_{s_j}^{\,2}})} \tag{2.5}$$

or

$$SNR_I=SNR_I'\frac{1}{1+\sum_{j=1}^{u}(\overline{i_{c_j}^2}+\overline{i_{s_j}^2})/(\overline{n_c^2}+\overline{n_s^2})} \tag{2.6}$$

where $SNIR_I$' is the input signal to noise ratio, and SNR_I' is the usual signal to noise ratio at the input of the AM detector. If one denotes the interference to noise ratio by INR, then (see Appendix A for details and an expression for INR)

$$SNIR_I=SNR_I'\frac{1}{1+INR}\,. \tag{2.7}$$

where SNR_I' is the ordinary input signal to noise ratio of the AM system. The key parameter in this expression is INR. In Appendix A an expression for INR has been derived $INR=IT_cu/N_o$. The parameters in this expression are: the noise power N_o, number of users u, interfering signal's power I, and code duration T_c. From Appendix A one gets the following expression.

$$SNIR_I=\frac{SNR_I'}{1+IT_cU/N_o}=\frac{SNR_I'}{1+INR} \tag{2.8}$$

as INR $\longrightarrow$ 0, $SNIR_I$ $\longrightarrow$ SNR_I'.

Similarly an expression of signal to noise and interference ratio for output of AM system ($SNIR_o$) have been derived, the result of which is

$$SNR_O = \frac{SNR_O'}{1+INR} \tag{2.9}$$

where SNR_o' is the ordinary signal to noise ratio of the AM system.

The detection gain G is given as follows:

$$G = \frac{SNIR_O}{SNIR_I}$$

or

$$G = \frac{(SNR_O')\,(1+INR)}{(1+INR)\,(SNR_I')} = \frac{SNR_O'}{SNR_I'} = G'. \tag{2.10}$$

For instance the detection gain (G) in the presence of interference can be regarded as the detection gain (G') in the presence of Gaussian noise.

In the worst possible case which is all direct sequence spread spectrum signals transmit on top of an AM system, that is $w_j = w_c$ for all j, then INR will be very large. As a result one gets a very small $SNIR_o$, and possibly loss of the signal.

In conclusion, for one user the performance is like noise [43]; for a large number of users the signal will be lost if $w_j = w_c$ for all j. However if one uses interleaving overlay properly the coexistence of direct sequence spread spectrum and AM system is feasible. For a mathematical discussion see Appendix B.

2.3 Interference Effects of DS on FM System:

The receiver in this case is an FM receiver, and is composed of a predetection band-pass filter, a limiter discriminator, and a post detection low pass filter. The band-pass and low pass filters have a bandwidth of $2w_m(1+\beta)$ and w_m respectively, where β is the modulation index of the FM signal. The discriminator output is proportional to the time derivative of the phase angle of the input to the discriminator. A block diagram of the FM receiver is given in Figure 2.3.

2.3.1 Signal Model:

The received signal is composed of three signals, the desired FM signal s(t), white Gaussian noise n(t) and a set of direct sequence spread spectrum signals i(t).

The desired FM signal can be represented by

$$s(t) = A\cos[w_c t + \phi_m(t)] \tag{2.11}$$

where A, w_c and $\phi_m(t)$ are the amplitude, the carrier frequency, and the phase angle of the FM signal. For FM modulation system $\phi_m(t)$ has usually the following form

$$\phi_m(t) = w_{max}\int_{-\infty}^{t} m(\tau)\,d\tau \tag{2.12}$$

where w_{max} is the maximum instantaneous frequency deviation, m(t) is the base band modulating waveform, or the message signal. The direct sequence signal is modeled in the previous section. The system analysis is now considered.

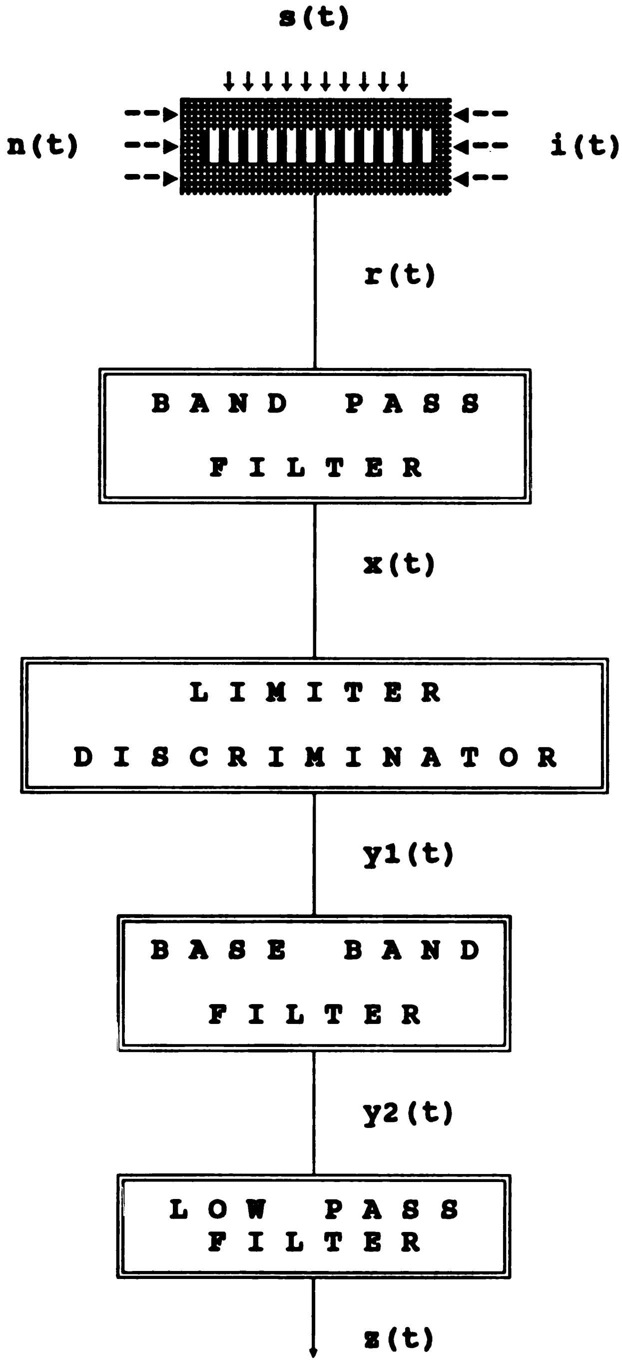


Figure 2.3 FM receiver Model

2.3.2 System Analysis:

Like the AM case the worst interfering case occurs when $w_j = w_c$ for all j's, $j=1,\ldots,u$, because the interfering power at the output of the predetection band-pass filter is maximum.

Again the solution is interleaving overlay. Since the interfering signal's full power will be filtered as described in the AM case, only the side-lobes of the neighboring signal will contribute some distortion. Since FM operate in the MH_z range, in this case one will have enough space in the guard bands for interleaving spread spectrum signals. The above technique is shown in Figure 2.2.

The predetection band-pass filter of FM has a bandwidth just sufficient for passing the desired signal, therefore it suppresses the out of band component of noise and interference. As a result one gets the desired signal, narrowband noise and narrowband interferences. Using the in phase and quadrature representation of each of the narrowband signals one can obtain the following expression at the output of predetection filter [38].

$$\begin{aligned} x(t) &= [A+n_c(t)+\sum_{j=1}^{u} i_{c_j}(t)]\cos[w_c t+\phi_m(t)] \\ &\quad -[n_s(t)+\sum_{j=1}^{u} i_{s_j}(t)]\sin[w_c t+\phi_m(t)] \\ &= R(t)\cos[w_c t+\phi_m(t)+\Phi(t)] \end{aligned} \tag{2.13a}$$

with

$$R(t)=\sqrt{[A+n_c(t)+\sum_{j=1}^{u} i_{c_j}(t)]^2+[n_s(t)+\sum_{j=1}^{u} i_{s_j}(t)]^2} \tag{2.13b}$$

and

$$\Phi(t)=\tan^{-1}\frac{n_s(t)+\sum_{j=1}^{u} i_{s_j}(t)}{A+n_c(t)+\sum_{j=1}^{u} i_{c_j}(t)}. \qquad (2.13c)$$

Using this representation one can determined an expression for $SNIR_I$ and $SNIR_o$. These expressions have been obtained in Appendix C. The results are:

$$SNIR_I=\frac{SNR_I'}{1+INR} \qquad (2.14)$$

and

$$SNIR_o=\frac{SNR_o'}{1+INR} \qquad (2.15)$$

where SNR_I' and SNR_o' are the ordinary SNR for input and output of an FM system and INR is the same as AM system except that there should be some more interference due to digital data and discriminator of FM.

In conclusion one can observe that the form for SNIR of FM is similar to SNIR of AM, however the interference which will be produced by discriminator will reduce the output $SNIR_o$ of FM.

2.4 Interference Effects of FH on AM and FM Systems:

In this section a study of the interference effects of frequency hopping spread spectrum on AM and FM systems is considered. The AM and FM signals have been modeled in the previous sections, therefore one can start the analysis by modeling the frequency hopping signal.

The frequency hopping signal i(t) can be described in the following form:

$$i(t) = \sum_{j=1}^{u} I_j d_j(t) \cos[w_j(t) + \theta_j] \qquad \tau \le t < k\tau \tag{2.16}$$

where $w_j(t)$ in this case is given by the following expression [44] and [45].

$$w_j(t) = w_c + h_j(t) \bullet \delta w \tag{2.17}$$

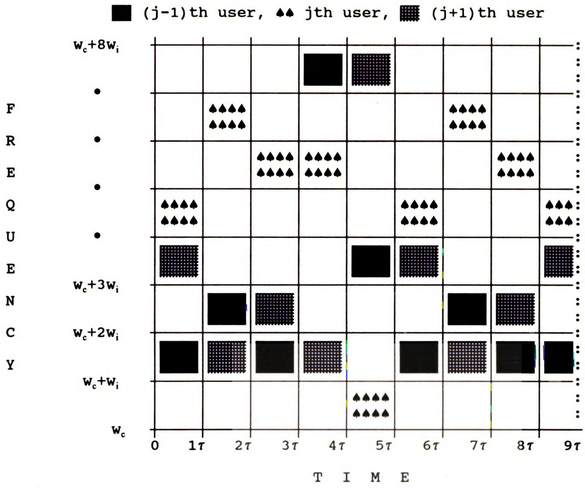
$$j = 1,\ldots,u; \quad (k-1)\tau \le t < k\tau$$

where δw is the minimum frequency shift, and $h_j(t)$ is an integer for jth user such that no other user can have that same integer in a given interval of time $t \in [(k-1)\tau, k\tau]$ with τ is the hopping duration or hopping width, $k \in [1,K]$ with $K = L = T/\tau$, and T is the period of one complete hopping pattern. Thus $h(t) = h(t+T)$ and L is the number of slots for hopping.

Consider Figure 2.4 as an example. In this example only three users are considered, namely (j-1)th, jth, and (j+1)th users. The following values are chosen for different parameters:

$$w_c = 50 \text{ MHz}, \ \tau = 1 \text{ ms}, \ w_i = 10 \text{ kHz}, \ T = 8 \text{ ms}$$

thus $L=8$. Therefore, the function h(t) will be repeated each 8 ms. The function $h_j(t)$ is determined pseudorandomly by a PN sequence. Figure 2.4 shows the periodic nature of $h_{j-1}(t)$, $h_j(t)$, and $h_{j+1}(t)$.



τ = 1 ms, w_c = 50 MHz, w_i = 10 KHz

$k\tau<t<(k+1)\tau$	k=0	k=1	k=2	k=3	k=4	k=5	k=6	K=L=8
$h_{j-1}(t)$	1	2	1	7	3	1	2	1
$h_j(t)$	4	6	5	5	0	4	6	5
$h_{j+1}(t)$	3	1	2	1	7	3	1	2

Figure 2.4 Hopping pattern for three users

The statistical properties of i(t) for the frequency hopping signal are similar to the direct sequence statistics and are derived in Appendix D, the results of which are given as follows:

$$\overline{i(t)}=0 \tag{2.18a}$$

$$\overline{i^2(t)}=I. \tag{2.18b}$$

Using the above statistical results one can write the following expressions for input and output signal to noise and interferences ratios,

$$SNIR_I=\frac{SNR_I'}{1+\sum_{j=1}^{u}(\overline{i_{c_j}^2}+\overline{i_{s_j}^2})/(\overline{n_c^2}+\overline{n_s^2})} \tag{2.19}$$

$$SNIR_o=\frac{SNR_o'}{1+\sum_{j=1}^{u}(\overline{i_{c_j}^2}+\overline{i_{s_j}^2})/(\overline{n_c^2}+\overline{n_s^2})} \tag{2.20}$$

where SNR_I' and SNR_o' are the ordinary SNR. If the system is an AM system, the SNR terms are for AM; and if the system is an FM system, the SNR terms represent the SNR for the FM system. Thus as far as mathematical analysis is concerned one gets the same expression in both cases, but one should remember that these results are for average SNIR's, the instantaneous SNIR is different in each case, and will be studied by simulation. The details of a mathematical consideration is given in Appendix B. In Appendix E a signaling scheme for DS and FH is considered. Based on this inference and observations of Appendix B one may come to a conclusion which is given in the next section.

2.5 Conclusion:

It is shown in Appendix E that the time interval for a signal set is important for frequency hopping only, and direct sequence is independent of these intervals. Thus the main difference between DS and FH is the hopping function h(t), which causes an unavoidable interference.

As a conclusion one can make the following statement: "The worst kind of spread spectrum interference to conventional systems is the interference of FH systems to FM systems". To see the behavior of this type of interference, a simulation study of this case is considered in the next chapter.

CHAPTER III

MODELING AND SIMULATION OF FM DETECTION BY ZERO-CROSSING

3.1 Introduction:

In this chapter a signal model of frequency hopping spread spectrum system and FM system will be considered. The demodulation of FM signals in the presence of interfering FH and noise, have been presented also. The FM receiver will adapt a zero crossing scheme for demodulation. This technique, which has existed in the communication system literature for quite some time [49]-[53], has emerged as a major technique for frequency discrimination and spectrum analysis [54]-[58].

The zero crossing technique will be a convenient method of demodulation that will be used with the new generation of micro-processors, especially with those customized digital signal processing chips which have fast Fourier transform (FFT) capability.

In the zero crossing technique of FM demodulation there is no need to differentiate the modulated carrier signal. The entire blocks of limiter and discriminator will be replaced by a simple zero crossing sensor; however, samples of this information need to be stored in a block of memory. The data which is stored in the memory can be processed with appropriate digital signal processors.

This technique can be applied to satellite communication systems as well as many other data communication systems, such as packet switching and any time division multiple access systems.

To demonstrate the zero crossing technique, several algorithms have been examined. When compared with the ideal zero crossing algorithm, the result of the simulation confirmed that as one gets closer to ideal zero-crossing, the performance of the detector improves greatly.

3.2 The Zero - Crossing Demodulation of an FM Signal:

A discussion of the demodulation of an FM signal by zero-crossing techniques is given in some communication texts, for example [5], [59] and [60]. In the following discussion a unified representation of the signal, demodulation and a practical algorithm for demodulation is summarized.

The FM signal is usually expressed as follows:

$$s(t) = A\cos[w_c t + \beta \int_0^t m(\tau)d\tau] \tag{3.1}$$

Where m(t) is the derivative of the message signal, and ß is a measure of the modulation index of the FM signal. Thus one can write s(t) as follows:

$$s(t) = A\cos[\theta(t)] \tag{3.2}$$

with

$$\theta(t) = w_c t + \beta \int_0^t m(\tau)d\tau \tag{3.3}$$

The message m(t) can be recovered from a knowledge of the zero crossing as follows: Let t_1 and t_2 be the times of the two adjacent zero-crossing of s(t) as shown in Figure 3.1.

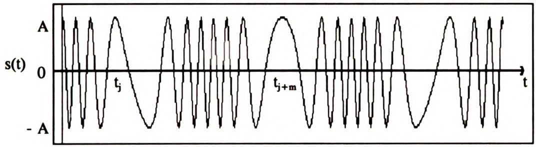


Figure 3.1 An FM signal and its zero-crossings instant

From the above diagram one can write

$$\begin{aligned} \Theta(t_2)-\Theta(t_1) &= \pi \\ \Theta(t_3)-\Theta(t_2) &= \pi \\ \Theta(t_4)-\Theta(t_3) &= \pi \\ &\vdots \end{aligned} \tag{3.4}$$

The above observation can be graphed as shown in Figure 3.2.

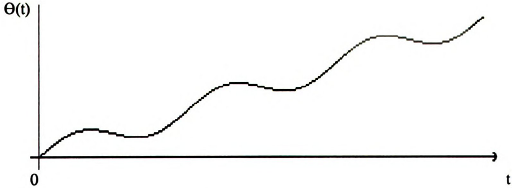


Figure 3.2 A plot of Θ(t) versus t

Let us concentrate at the two adjacent arbitrary zero crossing at t_1 and t_2 of Figure 3.2.

$$\Theta(t_2)-\Theta(t_1)=(m+1)\pi-m\pi=\pi \tag{3.5}$$

or

$$\pi=w_c t_2+\beta\int_0^{t_2} m(\tau)d\tau-w_c t_1-\beta\int_0^{t_1} m(\tau)d\tau \tag{3.6}$$

Thus

$$\pi=w_c(t_2-t_1)+\beta\int_{t_1}^{t_2} m(\tau)d\tau \tag{3.7}$$

Assuming that the FM carrier frequency w_c is much higher than the message's largest frequency w_m (which is usually the case). Then in the interval of $[t_1, t_2]$, m(t) can be assumed as a constant and can be factored out of integration, thus

$$\pi=w_c(t_2-t_1)+\beta\, m(t)(t_2-t_1) \tag{3.8}$$

$$t_1<t<t_2$$

or

$$[w_c+\beta\, m(t)](t_2-t_1)=\pi \tag{3.8}$$

but the term

$$w_c+\beta\, m(t)=d(s(t))/dt=w_i(t) \tag{3.9}$$

where w_i is the instantaneous frequency of the FM signal. Therefore one gets:

$$w_i = w_c + \beta m(\tau) = \frac{\pi}{t_2 - t_1} \tag{3.10}$$

or

$$f_i(t) = f_c + \frac{\beta}{2\pi} m(t) = \frac{1}{2(t_2 - t_1)} \tag{3.11}$$

If the number of zero crossing in an interval of T is n_T, then one gets

$$n_T \doteq \frac{T}{t_2 - t_1} \tag{3.12}$$

or

$$t_2 - t_1 \doteq \frac{T}{n_T} \tag{3.13}$$

with

$$\frac{1}{f_c} < T < \frac{1}{f_m} \tag{3.14}$$

thus

$$f_i = f_c + \frac{\beta}{2\pi} m(t) \approx \frac{n_T}{2T} \tag{3.15}$$

or

$$m(t) \approx \frac{2\pi}{\beta}\left(\frac{n_T}{2T} - f_c\right) \tag{3.16}$$

This shows that the message m(t) can be recovered from the number of zero crossing n_T. Other terms in this expression include a dc bias and possibly amplitude distortion which can be adjusted electronically.

In conclusion, one can recover m(t) from averaging The n_T's as shown in Figure 3.3.

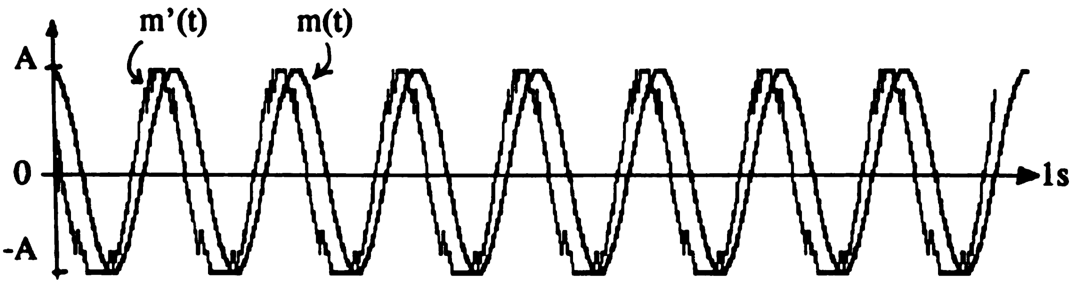


Figure 3.3 The message signal m'(t) is recovered by a moving average scheme. m(t) is plotted for reference

In the above discussion one can actually recover the FM signal from its zero-crossings by moving average method. Averaging is in reality a low-pass filtering process; therefore, one can recover the FM signal from its zero-crossing by a low pass filter.

A rigorous mathematical analysis of why one can recover the FM signal from its zero-crossings by a low pass filter is presented in chapter 4. However, in the following sections the algorithm and methods which are presented are based on the results of chapter 4.

The zero crossing demodulation technique is also considered in [5], [59] and [60] and the realization is proposed by a four step algorithm. This algorithm together with other algorithms are discussed in section 3.4.

3.3 The Simulation Model:

In chapter two it was stated that the worst type of interference which is considered in this study is the interference of a frequency hopping spread spectrum signal with an FM signal. It has also been established that the FM signal can be recovered from the zero crossings of the received FM signal, thus an FM demodulator in the presence of a frequency hopping interfering signal and noise is considered. Since there is a growing interest in zero crossing detection of signals and their spectra, this scheme is also adapted for the simulation model of this study. Thus the demodulation of an FM signal by zero crossing in the presence of frequency hopping interference and noise is modelled.

A block diagram of the simulation model is given in Figure 3.4. After the FM signal s(t) and the frequency hopping interfering signal i(t), and the noise signal n(t) are received in the antenna, they are added together and are applied to the first stage of the model. The first stage is composed of a zero crossing scheme which will be discussed in more detail in the next section. This stage simply estimates the location of the zero's of incoming signal. To illustrate the technique let us consider a tone-modulated FM signal alone and process this signal point by point in the model. Consider an FM signal s(t) as follows:

$$s(t) = A\cos(w_c t + 5\int_0^t m(\tau)d\tau) \qquad (3.17)$$

for A=20, and m1(t)=5cos10πt one gets a signal set at point 1 and point 2 of the simulation model as shown in Figures 3.5a and 3.5b respectively.

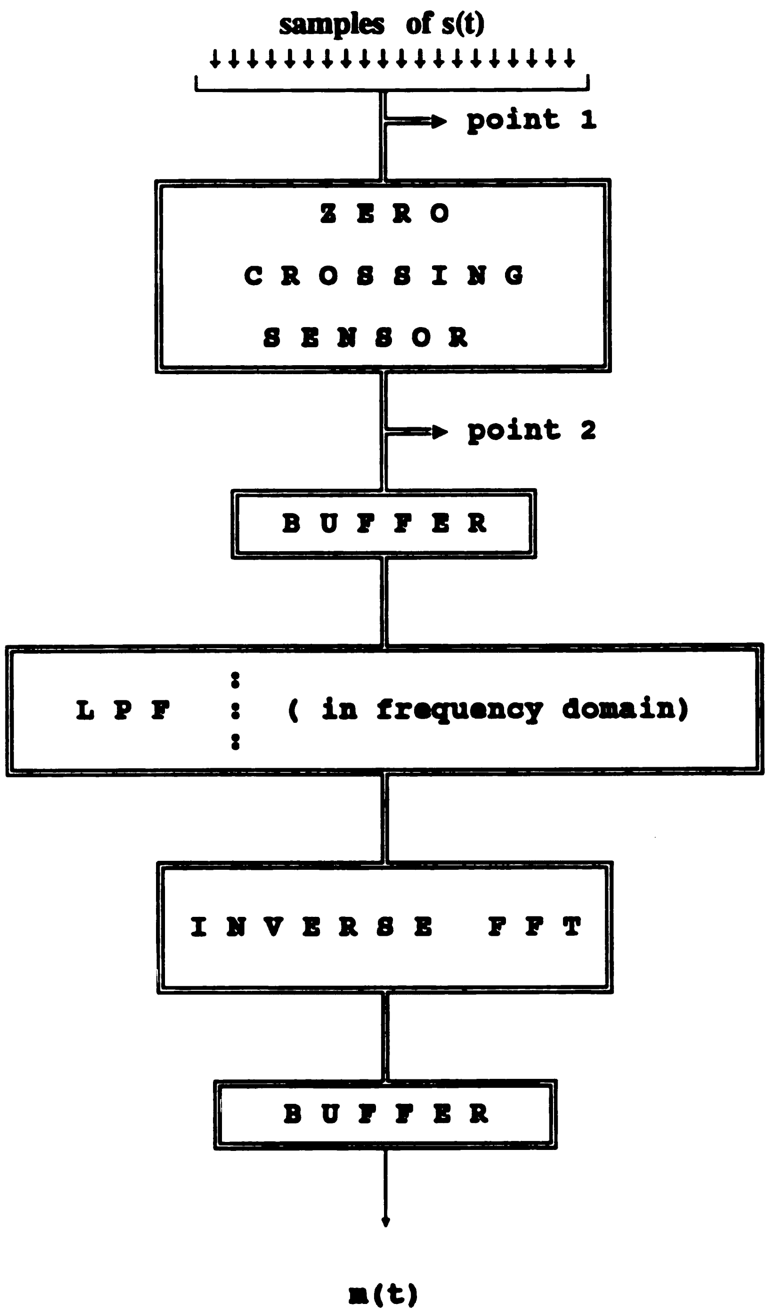


Figure 3.4 Block diagram of the simulation process

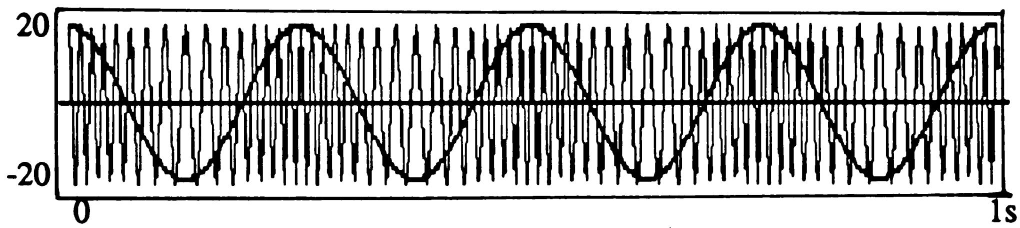


Figure 3.5a The received signal s(t)

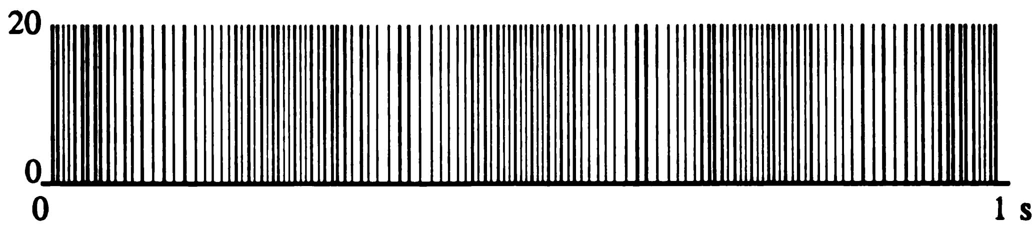


Figure 3.5b The zero-crossing function z(t)

The next stage of the model consists of a Buffer, where an array of 512 points is stored, and then it is used to calculate the FFT in the next stage. Stage III finds the fast Fourier transform of the corresponding zero crossing array. This stage involves low pass filtering in frequency domain, and that is the reason for calculating FFT of the zero-crossing of the signal. After FFT have been calculated, the lower frequency components could be used for low pass filtering, and the high frequency components could be ignored or simply would not be calculated. Hence, the above scheme is very suitable for VLSI digital signal processing applications. For example one can program the chip for calculating the FFT for lower frequencies, and that will serve as a low pass filter in the frequency domain. Consequently this stage could be considered as two steps. In step one the FFT is calculated, and in the second step it is low pass filtered. The result of FFT stage gives the spectrum of the appropriate signal. This spectrum is given in Figure 3.6.

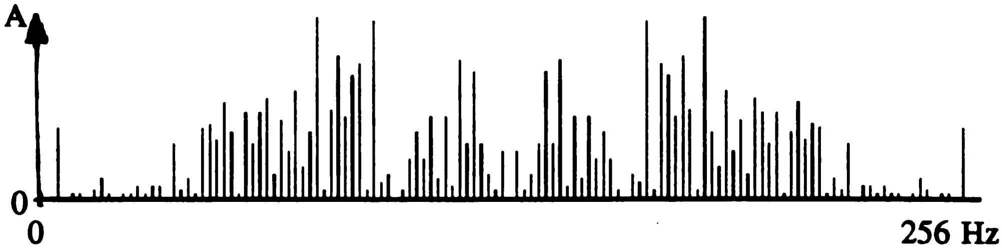


Figure 3.6 Z(f) The spectrum of z(t)

The last stage simply calculates the inverse Fourier Transform (IFFT) of the filtered spectra or the partial FFT. As one expects it should return the message m(t). To compare the original signal m(t) with m(t) a plot of these signals is given in Figure 3.7.

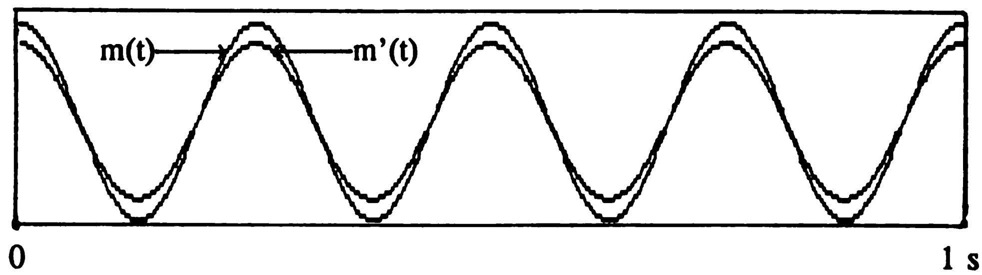


Figure 3.7 A plot of m(t) and m'(t)

In summary the simulation model determines the zero crossing of an FM signal using an array of length 512, and then it calculates the FFT of the zero crossing signal and passes it through a low pass filter. Finally it calculates its inverse FFT (IFFT) which is the approximation to the desired signal. In the next section, different algorithms have been developed to determine the zero crossing of the incoming FM signal, and the rest of the simulation process will not be changed. There will be some distortion and amplitude scaling in the output, since this type of distortion is very common in practice and have been examined. The simulation model is also adjusted to include these type of distortions. The simulation in the presence of noise and interference is considered in section 3.5. In the next section different algorithms for zero crossings detection are examined.

3.4 Zero Crossings Detection Algorithms:

In this section some algorithms for zero crossing detections are presented. Each algorithm approximates the location of the zero crossings of the FM signal in the interval of $(0,2\pi)$, however none of the algorithm determines the exact locations of the zero crossings. The first algorithm has existed in the literature for quite some time; the second and the third algorithms have been proposed by the author; and the last algorithm, which is called an ideal zero crossing detector, is also considered. Each of these algorithms have been tested by the simulation model. The results are reported through some pictorial diagrams of the original signal, and the corresponding simulation outputs. A comparison of the results shows that the third and fourth algorithms are the ones that should be considered for the future. If the zero crossing is going to be handled by hardware the third algorithm is suitable. However, if some application permits a software handling of the data, the fourth algorithm is the one which is most suitable. Now these four algorithms are presented according to their order of improvement toward ideal zero crossing detection.

Algorithm I:

This algorithm is based on the discussion of section 3.2 and it is proposed in [5] and [59]. One can refer to this as a classical algorithm. This algorithm can be realized by a four step procedure as follows:

I) Half wave rectify the received FM signal s(t).

II) Differentiate to accentuate the zero crossing points.

III) Rectify once again to eliminate the negative pulses due to the negative going zero crossings and generates z(n).

IV) Pass the signal through a low pass filter.

The above algorithm is applied to an FM signal $s_1(t)$, which is a single tone modulation.

$$s_1(t)= 20\cos[2\pi 128000t+25\pi\int_0^t \cos(2\pi 5000\tau d\tau) \tag{3.18}$$

Notice that if one consider t in seconds, the FM signal $s_1(t)$ will have a carrier frequency of 64 Hz. However, if one consider t in microseconds, then the FM signal will have a carrier frequency of 128 MHz. For simplicity of discussion (without loss of generality) a period of 1 ms have been considered. The above signal $s_1(t)$ is sampled with a sampling frequench of 512 Hz, and $s_1(n)$ is obtained. The signal $s_1(n)$, 512 samples of $s_1(t)$ is processed by all algorithms.

The result of the simulation when it applied to $s_1(n)$ is given in Figure 3.8. Obviously the above classical algorithm is working, but it can not recover the message completely. For example there is some distortion. The MSE (mean square error) for this algorithm is 8.71298 V^2/sec.

In the following algorithm the above method is modified to improve the zero-crossing technique. This algorithm determines the zero crossings more closely to the theoretical ones.

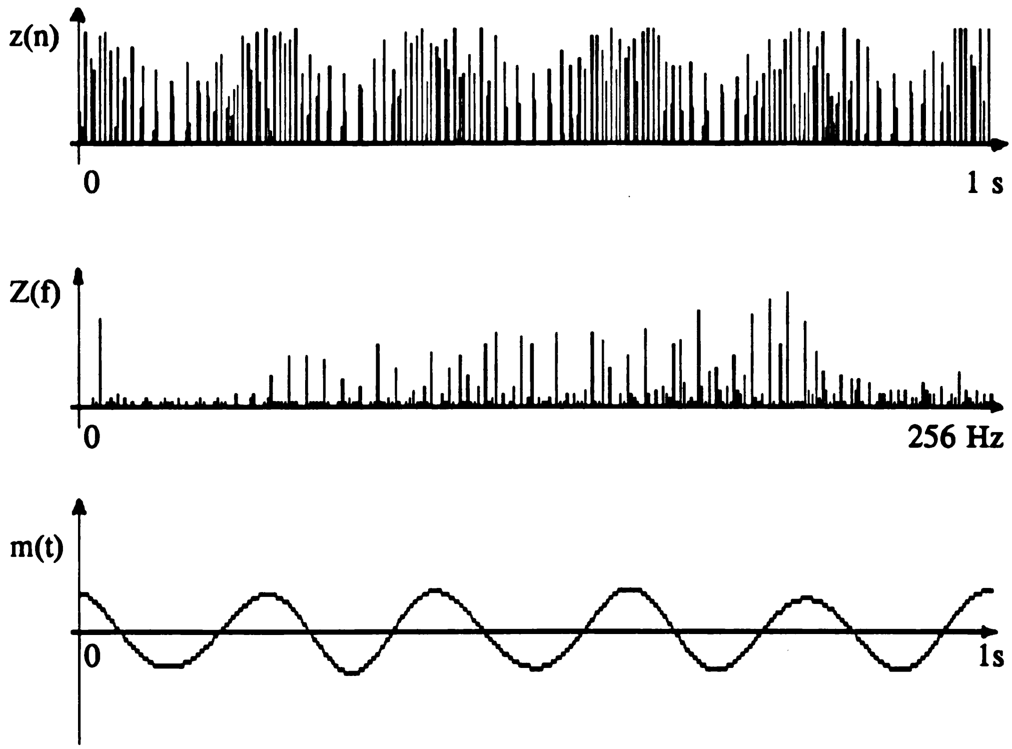


Figure 3.8 The message signal is recovered by Algorithm I

Algorithm II:

In this algorithm the first stage which is a half wave rectifier is preceded by a limiter, but this limiter is not similar to the classic limiter which was in use for FM demodulators. This limiter uses a comparator to increase the level of a signal which falls above some predetermined level a_L as follows:

$$L(n) = \begin{cases} A_L sgn(s(n)) & |s(n)| \geq a_L \\ s(n) & |s(n)| < a_L \end{cases} \tag{3.19}$$

where A_L is in the order of 1 to 5 volts and a_L is a fraction of a volt, and sgn is the sign function.

This algorithm is also applied to the FM signal $s_1(n)$ in order to detect m(t). The result of the simulation shows an improvement compared to the first algorithm. The MSE for this algorithm is 6.9826 V^2/sec. The output signal which is recovered by algorithm II is shown in Figure 3.9.

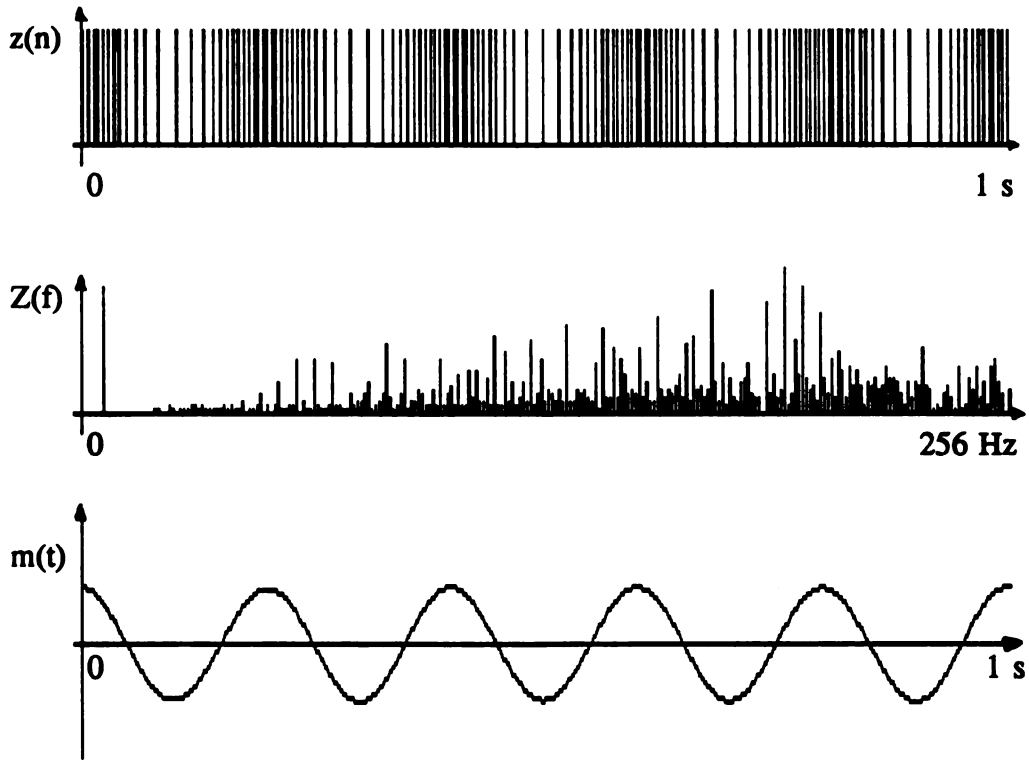


Figure 3.9 The message signal is recovered by Algorithm II

Comparing Figure 3.9 with Figure 3.8 and their MSE, one can conclude that Algorithm II demonstrates an improvement compared to algorithm I. However, algorithm III will bring further improvement.

Algorithm III:

In this algorithm the limiter which existed in algorithm II is preserved, but the first half wave rectifier is removed. Algorithm III has the following 4 steps:

1) Limiter (As described in algorithm II)

2) Differentiator

3) Half Wave Rectifier, generates z(n)

4) Low Pass Filter

The result of the simulation when applied to the same signal set $s_1(n)$ is given in Figure 3.10. The MSE in this case is 3.0626 V^2/sec.

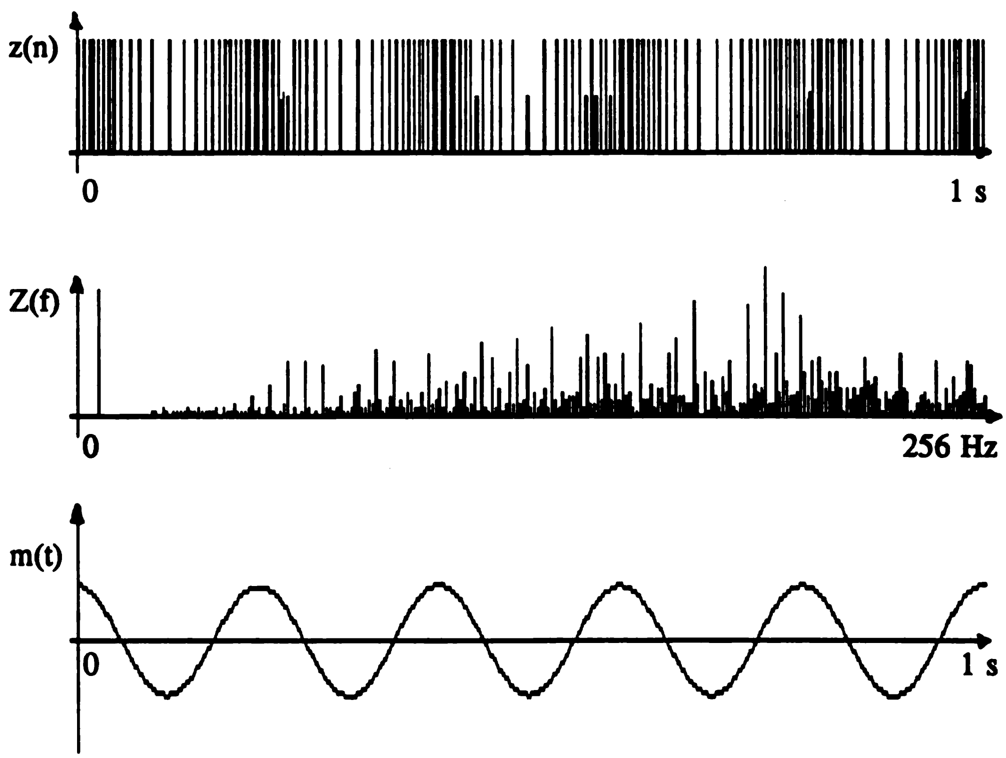


Figure 3.10 The message signal is recovered by Algorithm III

By a careful study of Figures 3.8 - 3.10 and their MSE, one can conclude that algorithm III is superior to algorithms I and II. The final algorithm is a simple zero - crossing scheme. Due to the nature of the algorithm it is referred as the ideal zero crossing algorithm.

Algorithm IV: (Ideal Zero-Crossing)

In this algorithm, two adjacent signal samples are multiplied. If the result is negative, this indicates a zero crossing has occurred. Thus we have the following 2 step algorithm:

1) detect the zero-crossing by examining the product of two adjacent samples as follows:

$$s(t) = \begin{cases} 0 & s(t)s(t+1) \geq 0 \\ A_L & s(t)s(t+1) < 0 \end{cases} \tag{3.19}$$

2) low pass filter the result.

A simulation result of this algorithm is shown in Figure 3.11. The MSE for this algorithm is 0.02161 V^2/sec, a drastic improvement. In order to compare these algorithms, the quantitative results or MSE results are summarized in Table 3.1.

Table 3.1 MSE (in V^2/sec) results for single tone modulation

Algorithm I	Algorithm 2	Algorithm 3	Algorithm 4
8.71298	6.9826	3.0626	0.02161

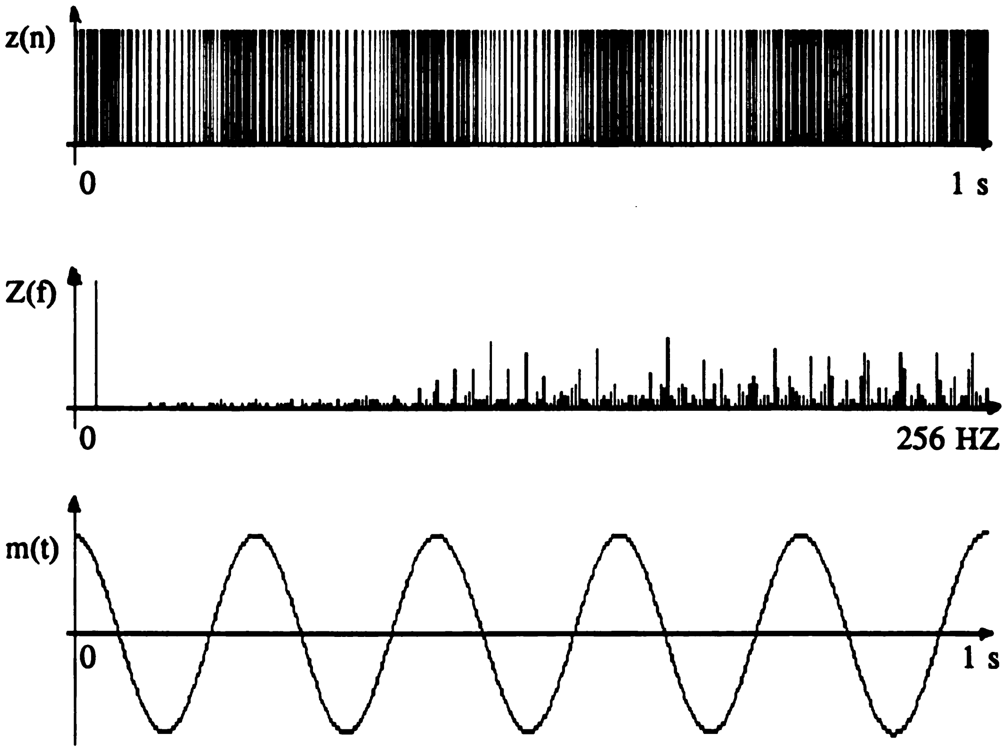


Figure 3.11 The message signal is recovered by Algorithm IV

In all of these simulations, the signal samples which were obtained by specific algorithms were processed in the frequency domain; for example the spectra of the signals were obtained by taking FFT of the zero crossing samples, and then low passed filtered in frequency domain. Finally by taking inverse FFT (IFFT) the message signal is detected.

Since the low pass filtering is performed in frequency domain, it is more appropriate to find the FFT of the signal up to the band width of the low pass filter. This will save a great amount of time in the realization process of the system.

3.5 Multi-Tone FM Signal:

In the previous section the FM signal was generated by a single tone message signal. In this section the FM signal $s_2(t)$ is generated by a multi-tone message signal m(t) as follows:

$$s_2(t) = 20\cos(w_c t + \beta \int_0^t m(\tau)d\tau) \tag{3.20}$$

where

$$m(t) = \sum_{i=1}^{3} a_i \sin(2\pi f_i t) \tag{3.21}$$

with a_i and f_i being random numbers uniformly distributed in their range of variation. For illustration purposes the following values have been considered.

a_1=2 V; a_2=3 V; a_3=5 V; f_1=4 kHz; f_2=6 kHz and f_3=8 kHz

The amplitudes are measured in volts while the frequencies are in kilo Hertz. Although the choice may be random, the above values has been chosen to illustrate the multi-tone modulation and demodulation of the FM signal $s_2(t)$. These values give a reasonable amplitude and frequencies to compare this case with the single-tone situation.

The four algorithms which were introduced in the previous section have been applied to the multi-tone signal set $s_2(n)$, (512 samples of $s_2(t)$) and the results of the simulation are given in Figures 3.12-3.15. The MSE for these cases have been calculated and summarized in Table 3.2.

Table 3.2 MSE results for multi-tone modulation

Algorithm 1	Algorithm 2	Algorithm 3	Algorithm 4
11.7085	10.8202	4.9352	0.0267

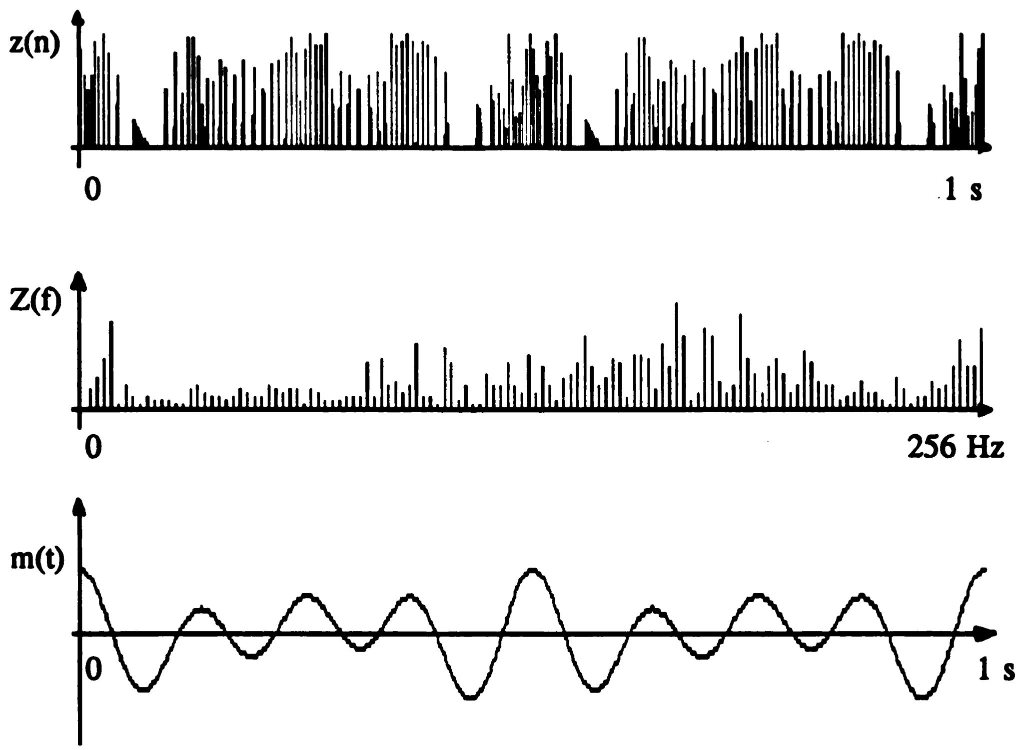


Figure 3.12 The multi-tone signal is recovered by Algorithm I

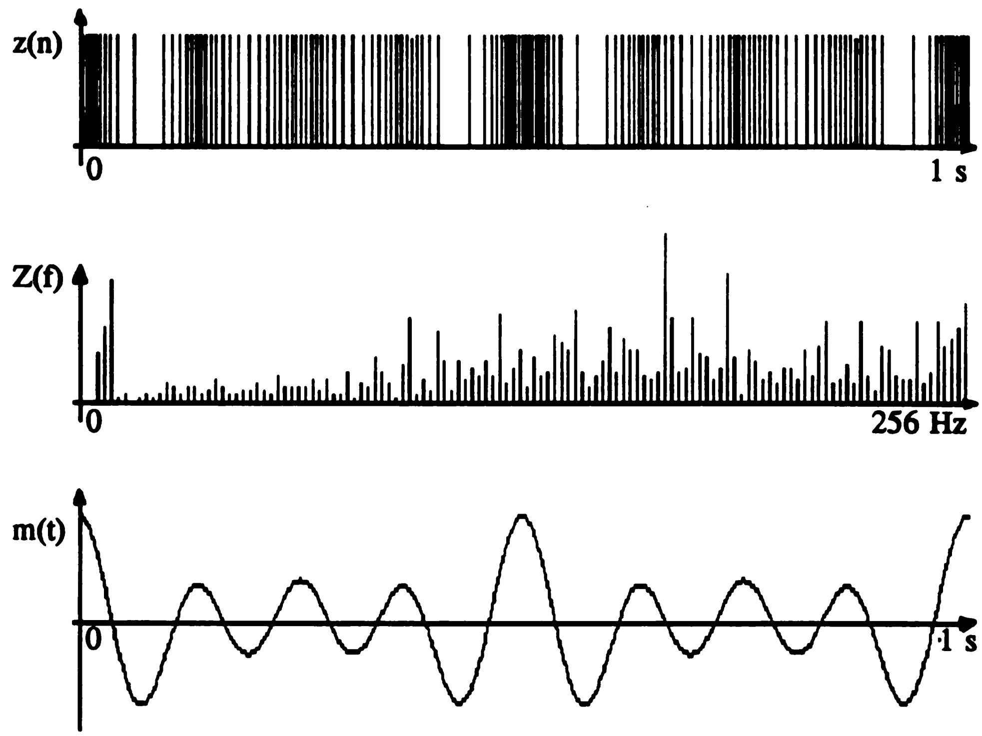


Figure 3.13 The multi-tone signal is recovered by Algorithm II

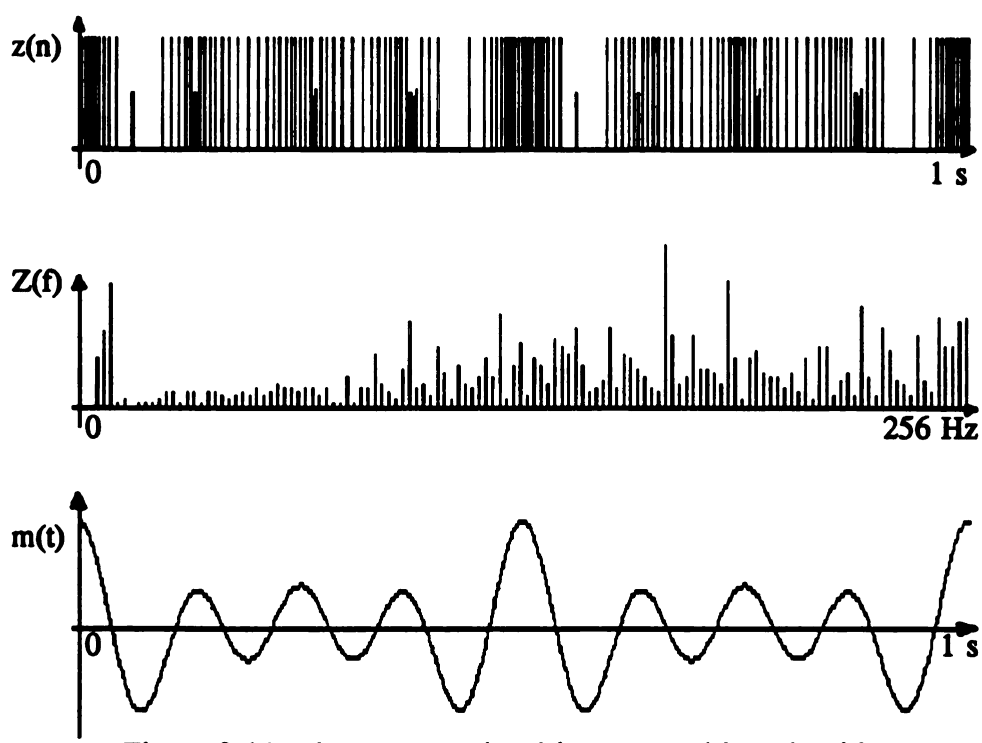


Figure 3.14 The message signal is recovered by Algorithm III

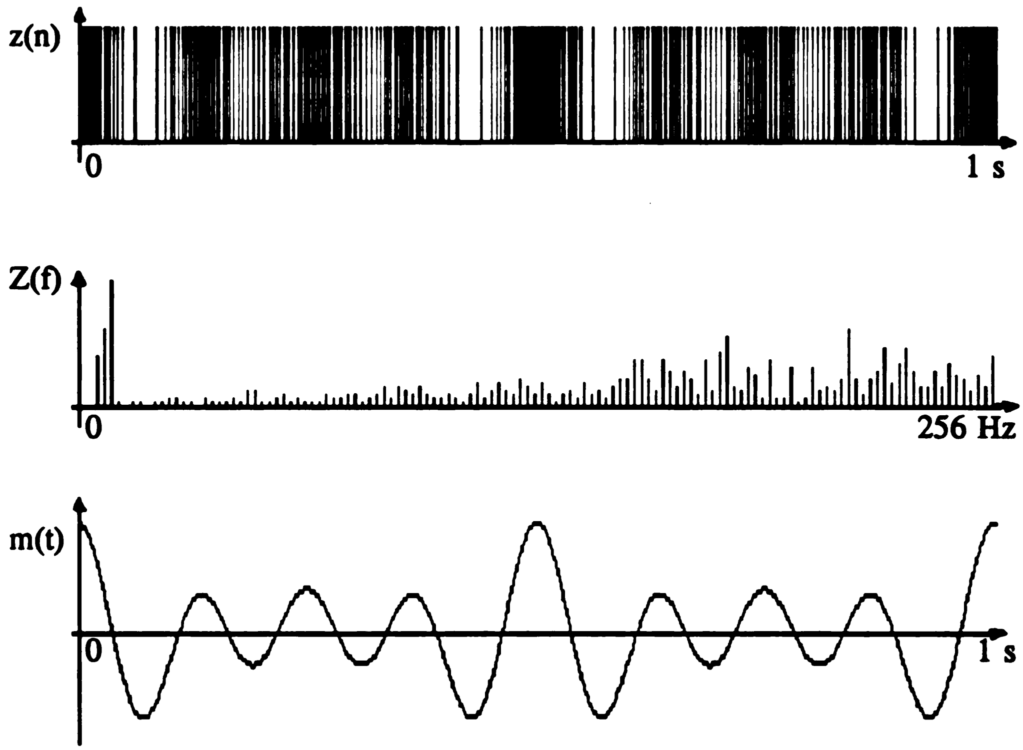


Figure 3.15 The multi tone signal is recovered by Algorithm IV

These results confirm the previous inferences about these algorithms, and furthermore these exhibit the demodulation of the FM signal in a more realistic manner. In the following section the demodulation of the FM signal in the presence of noise and frequency hopping interfering signal is considered.

3.6 The Frequency Hopping and Noise Signals:

Although the frequency hopping signal set was given in previous chapter, in this section the noise and the frequency hopping signals are considered for the simulation purpose.

3.6.1 The Noise Signal:

The noise signal is considered to be Gaussian, where each seed is obtained from a uniform distribution in the interval of [-1,1], and then by a Box-Muller transformation the Gaussian or Normal deviates are obtained, [62] and [63]. The resulting noise has a mean of zero and a variance of unity.

The noise signal which is generated by the above scheme is given in Figure 3.16a. The spectra of the above noise is given in Figure 3.16b. As one expects the noise has components at every frequency.

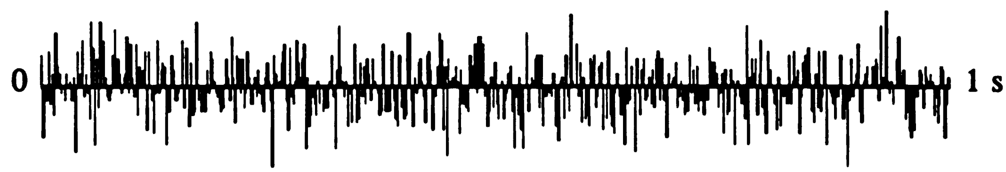


Figure 3.16a The noise signal n(t)

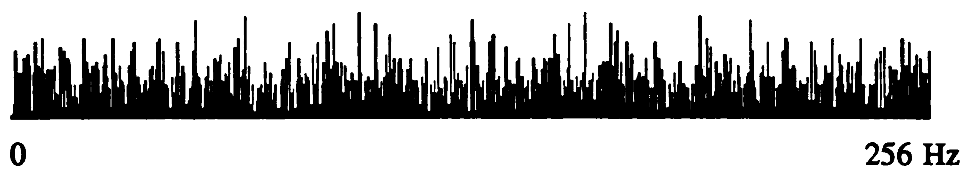


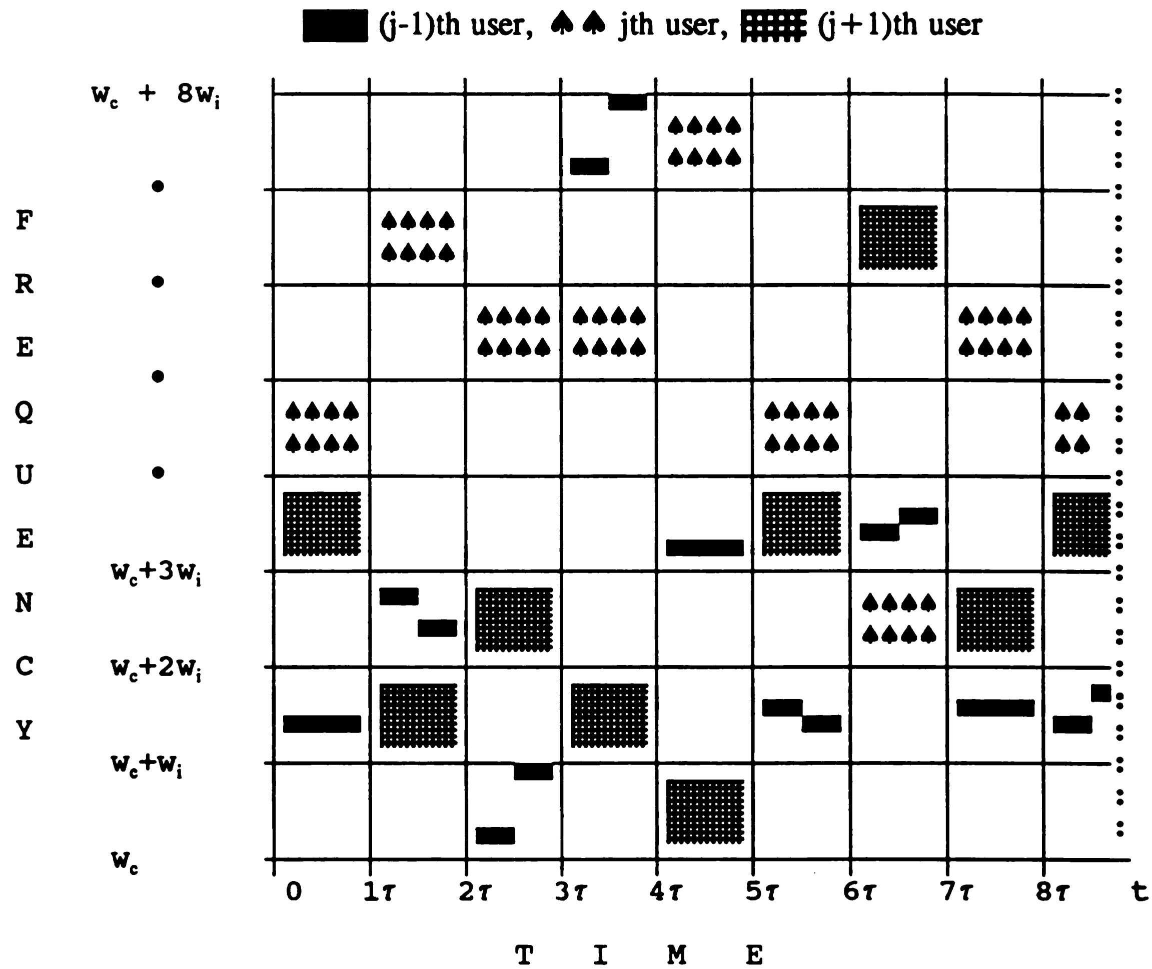
Figure 3.16b The amplitude spectrum of noise N(f)

3.6.2 Frequency Hopping Signal:

Frequency Hopping unlike AM and FM can be represented in many ways. The most popular representation which is M-ary Frequency shift keying MFSK has been adapted for the simulation model, [4] and [61]. In the M-ary frequency shift keying representation of the frequency hopping signal (FH/MFSK), the data bits determine the increment frequency to be added to the carrier frequency. Thus in a FH/MFSK the increment of frequency is added to the carrier frequency which is determined by a PN code. Thus in this system it is very difficult for an intentional jammer to jam the communication system. One can represent the above modulation system by the following mathematical expression for interfering frequency hopping signal i(t).

$$i(t) = A\cos\{[w(t)+d(t)dw]t\} \quad (3.22)$$

where $w(t)=w_c+h(t)\bullet w_i$ with h(t) is the hopping function which could be generated by a PN sequence, and w_i is the hopping increment to be multiplied by h(t) and then added to the carrier frequency w_c. The frequency w_c in this case is the carrier frequency of the FM system. $d(t)\epsilon\{0,1\}$ is the digital data which regulates the M frequencies that each group of bits determines. Thus dw is the increment of frequency which separates two adjacent frequencies in the M-ary frequency shift keying. This scheme of signalling can be demonstrated by a frequency versus time diagram as shown in Figure 3.17. Notice that as the time proceeds the frequency will change in each half period of the hopping chirp due to the pattern of the data, and at the end of each hop increment the PN sequence determines the next hopping slot by h(t), and again in every hopping slot there are two frequency increments which are determined by d(t). A common practice is to choose the symbol rate R_s for the MSK signal as an integer multiple of the hopping rate R_h or vice versa. Thus if R_s is an integer multiple of R_h, several bits could be transmitted in each frequency hopping. This option is adapted for the simulation model and in each hopping slot two of the M-ary frequencies are transmitted. Since each M-ary frequency is determined by two bits (4 frequencies), a total of four bits is transmitted in each frequency hopping slot.



τ= 1 ms, w_c = 128 KHz, w_i = 4 KHz, dw = 1 KHz

$k\tau<t<(k+1)\tau$	k=0	k=1	k=2	k=3	k=4	k=5	k=6	k=L=8
$h_j(t)$	1	2	0	7	3	1	3	1

Figure 3.17 Frequency hopping with M-ary scheme of signaling

One can notice that all the slots of the frequency hopping are not used, thus by using PN sequences of large length one can utilize coding scheme to achieve multiple access. This technique is known as code division multiple access (CDMA) which has applications in military and satellite communication systems.

3.6.3 Digital Data and PN Sequence:

The digital data are random in nature, since they usually come from the output of a pulse code modulation (PCM) system or simply from an analog to digital (A/D) converter. Thus the digital data are considered to be random bits of zero's and ones. The pseudonoise sequences are generated by specific polynomials via shift registers. These polynomials are known as primitive polynomials [64]. The following recursive relation, which is obtained from the primitive polynomial $1+X^3+X^4$, is adapted to generate the corresponding pseudonoise sequence for the simulation model.

$$PN(i) = PN(i-3) \cdot PN(i-4) \tag{3.23}$$

with the initial loading of the register as follows:

$$PN(0)=-1;\ PN(1)=-1;\ PN(2)=1;\ PN(3)=-1 \text{ and } PN(4)=1 \tag{3.24}$$

Based on the above digital data and PN sequence the following digital data and PN sequence is obtained

digital data: 1 0 1 0 1 0 0 1 0 1 1 1 1 0 1 1 1 0 1 0 1

PN sequence: 0 0 1 0 1 0 0 0 0 1 1 1 0 1 1 0 0 1 0 1 0 0 0 0 1 1 1 0 1 1 0 0 1

Now each pair of the digital binary data is grouped as 00, 01,10, and 11 to determine the four decimal numbers 0,1,2, and 3. One of these numbers would be multiplied by dw, and dw is considered to be 1 KHz in this model. Thus the frequency within the M-ary changes from 0 to 4 KHz. Since Rs is an integer multiple of R_b, each three bits of the PN sequence is grouped to determine $2^3=8$ different hopping slots out of 16 possible slots. Thus the hopping starts with the binary coded decimal of the four bits, which could be any value from 000B=OD to 111b=7D where B denotes the binary and D stands for decimal in this case. From the total of 8 possible hopping slots only 5 of them have been used by the simulation model. The hopping increment is considered to be 4 KHz. A frequency hopping signal and its spectrum are shown in Figure 3.18.

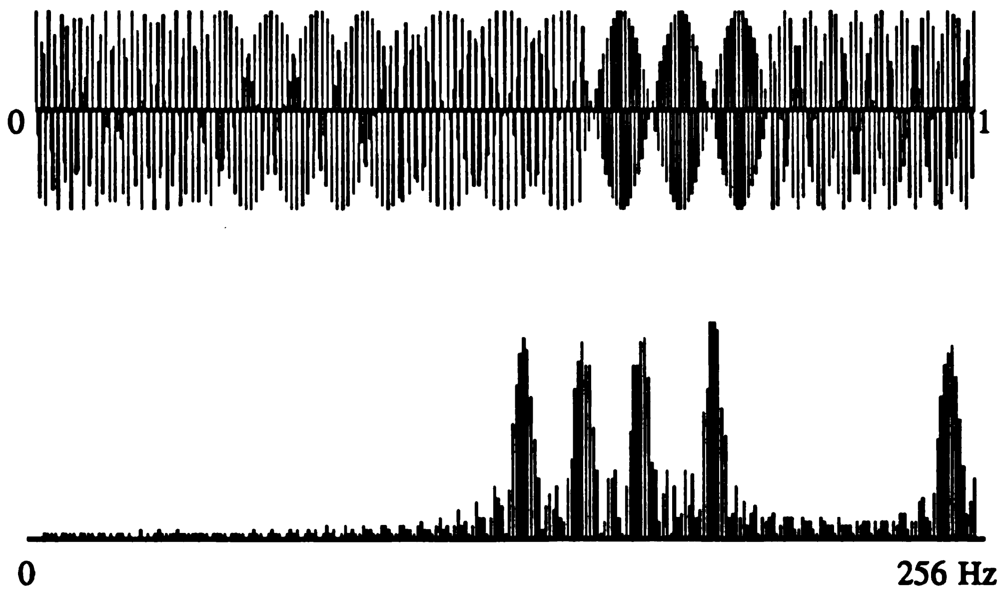


Figure 3.18 The Frequency Hopping signal and its spectrum

The frequency separations of the hopping scheme are obvious in the spectrum of the frequency hopping signal. In the next section the interfering FH signal has been added to the FM signal, and the results have been compared.

3.7 FM Detection in the Presence of Noise and FH:

In this section the noise and frequency hopping signals which have been modeled in the previous section will be added to the FM signal. Three cases have been considered. In order to compare the various results indisputably, in all of these cases the FM signal is considered to be a tone modulated signal.

I. A single tone FM in the presence of noise only.

II. A single tone FM in the presence of FH only.

In all of these cases an amplitude of 20 is considered for the FM signal.

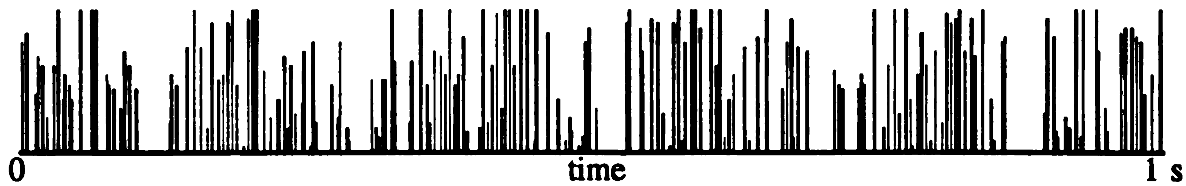
3.7.1 FM signal in the presence of Noise:

In this case while the amplitude of the FM signal is fixed, the amplitude of the noise is changed to different levels, namely 10, 15,and 20. The corresponding simulation result for the four algorithms of zero crossings are shown in Figures 3.19A - 3.22C. A simulation study of MSE is also conducted, and the result of simulation is reported in Table 3.3.

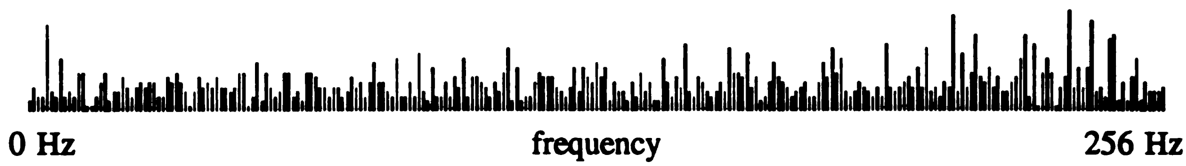
Table 3.3 MSE results for a single tone modulation in the presence of noise

	ALG I	ALG II	ALG III	ALG IV
5 V	9.153	6.988	3.071	0.004
10 V	10.120	7.954	4.255	0.417
15 V	10.977	9.826	7.348	3.572
20 V	11.329	11.349	10.153	8.142

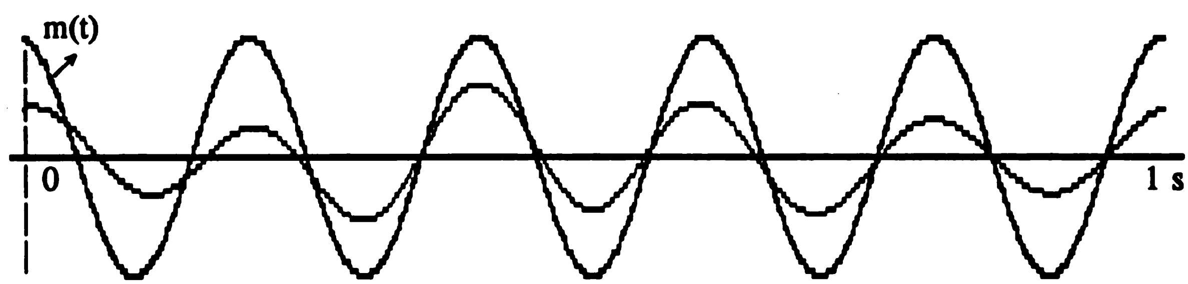
One can observe from these figures or Table 3.2 that as the noise level becomes larger more distortion results. From these Figures and Table 3.2 algorithm III an IV appears to perform noticeably better for this example



a) The zero-crossing function z(t)



b) Z(f) the spectrum of z(t)



c) The message m(t) and m'(t)

Figure 3.19a The message signal is recovered by ALG. 1

(the noise level=10)

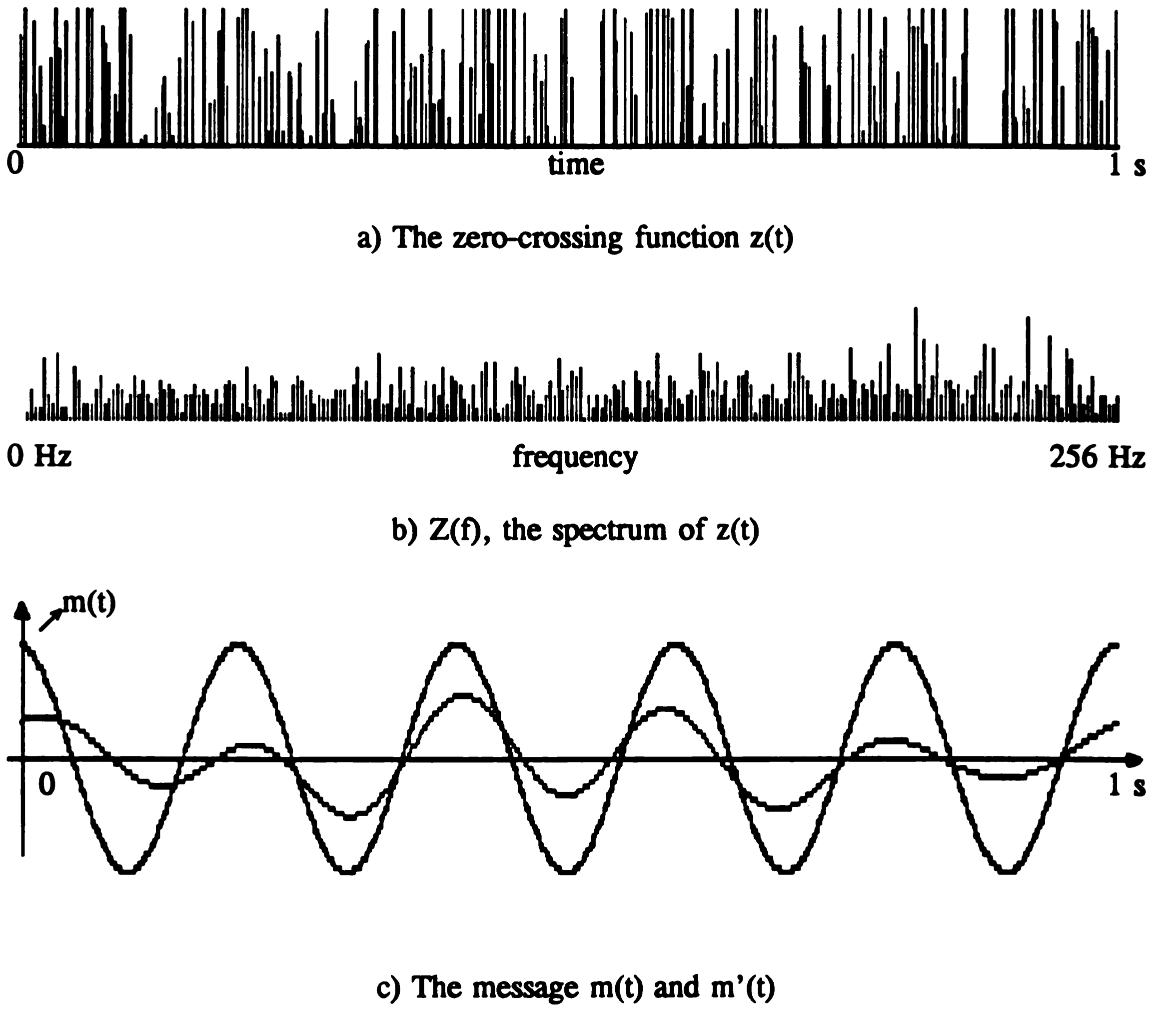
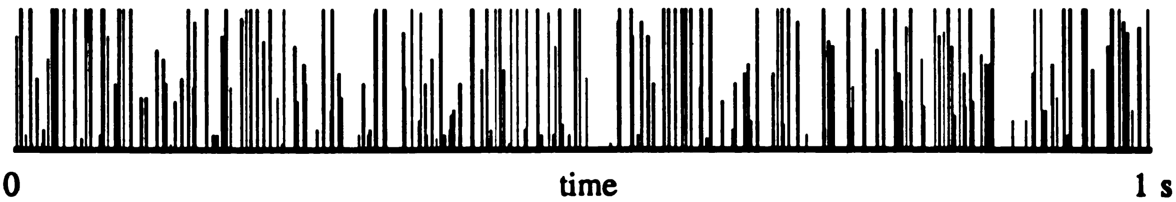
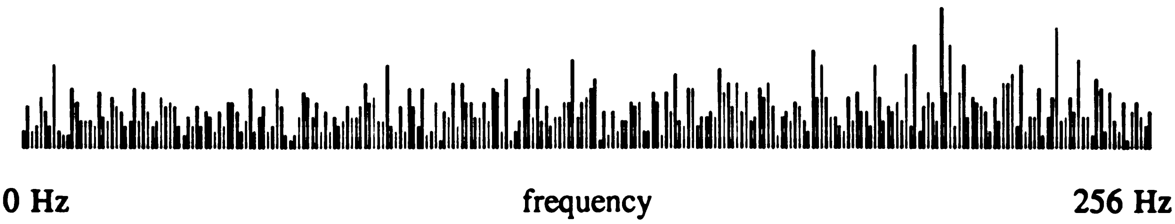


Figure 3.19b The message signal is recovered by ALG. 1

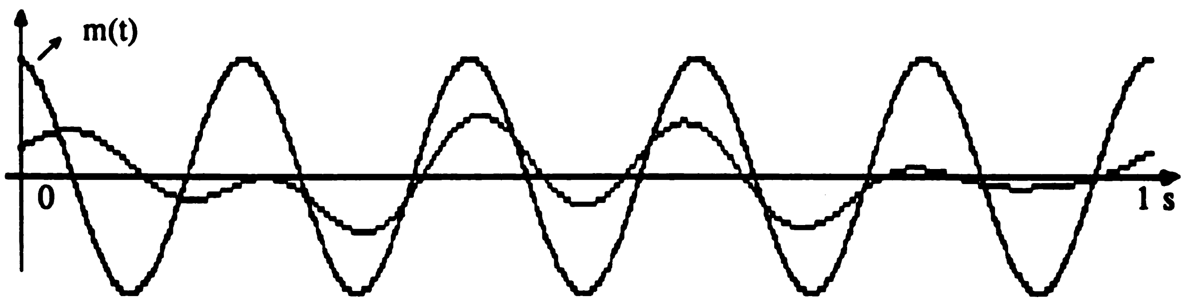
(the noise level=15)



a) The zero-crossing function z(t)



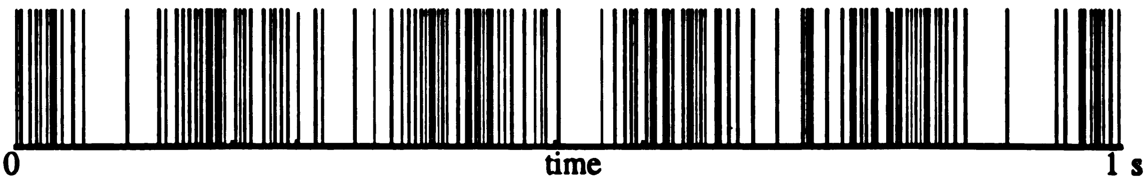
b) Z(f), the spectrum of z(t)



c) The message m(t) and m'(t)

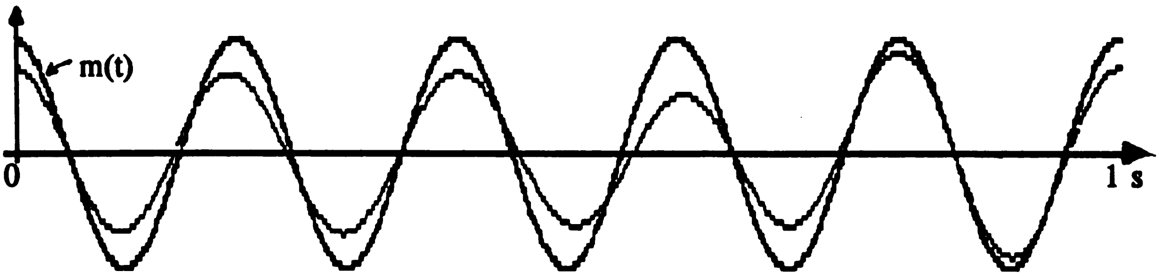
Figure 3.19c The message signal is recovered by ALG. 1

(the noise level=20)



a) The zero-crossing function z(n)

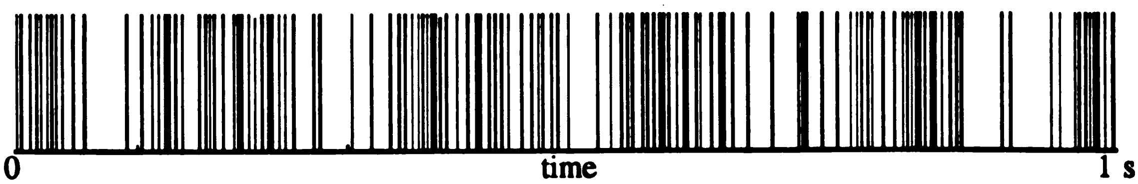
b) Z(f)



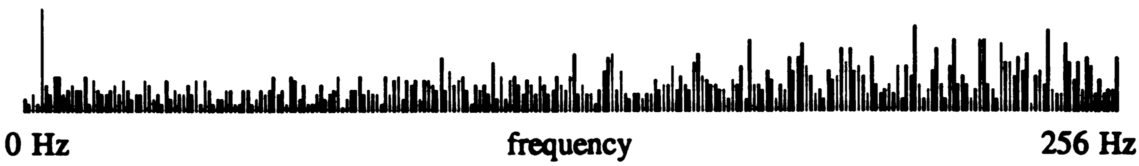
c) The message m(t) and m'(t)

Figure 3.20a The message signal is recovered by ALG. 2

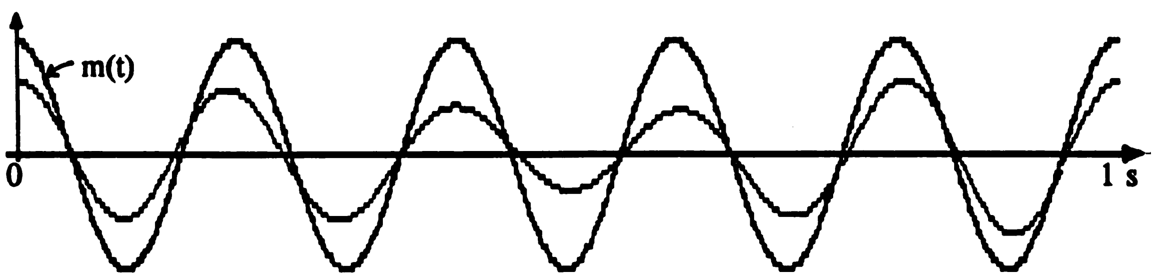
(the noise level=10)



a) The zero-crossing function z(n)



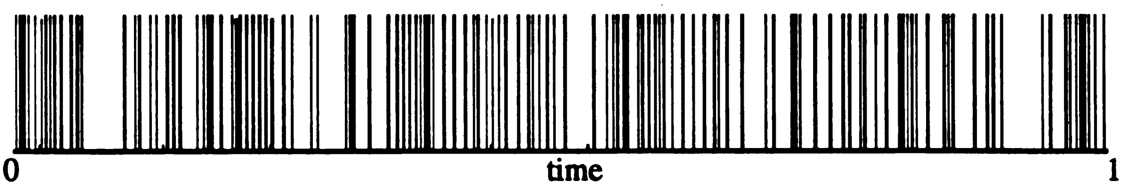
b) Z(f)



c) The message m(t) and m'(t)

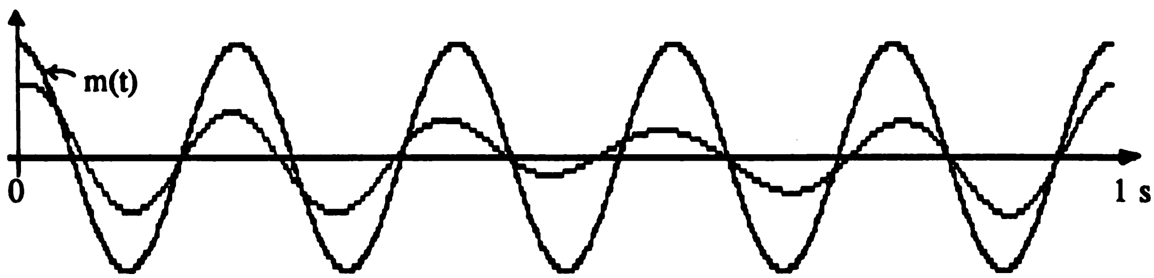
Figure 3.20b The message signal is recovered by ALG. 2

(the noise level=15)



a) The zero-crossing function z(n)

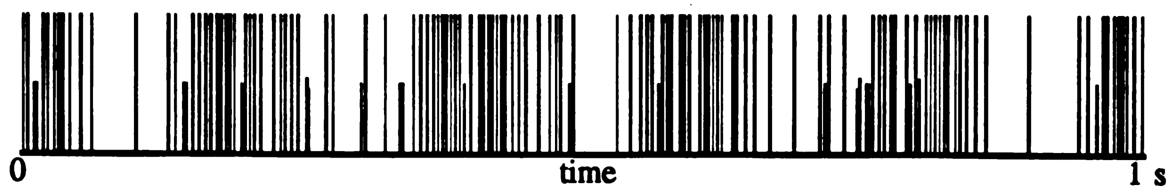
b) Z(f)



c) The message m(t) and m'(t)

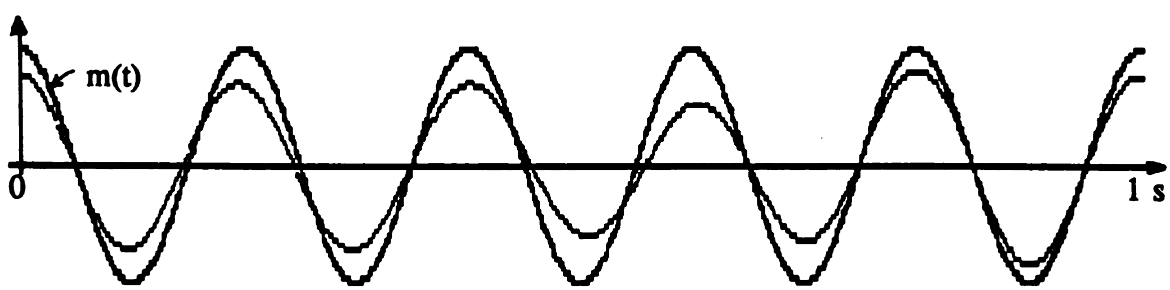
Figure 3.20c The message signal is recovered by ALG. 2

(the noise level=20)



a) The zero-crossing function z(n)

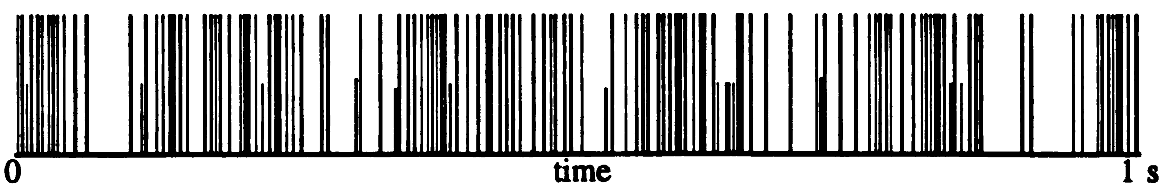
b) Z(f)



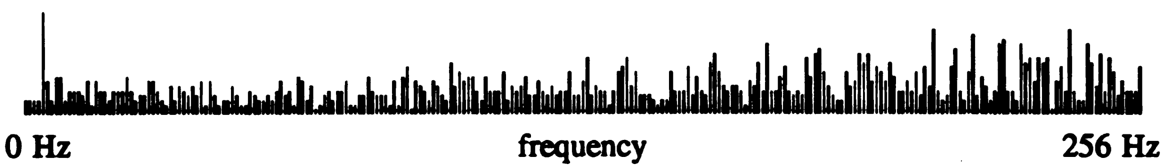
c) The message m(t) and m'(t)

Figure 3.21a The message signal is recovered by ALG. 3

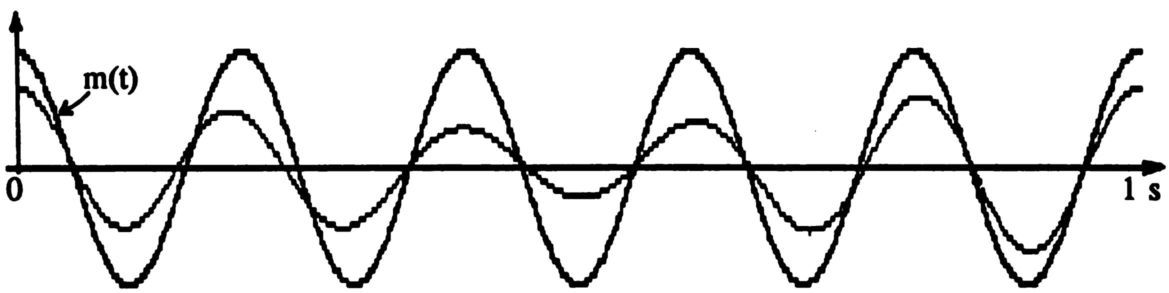
(the noise level=10)



a) The zero-crossing function z(n)



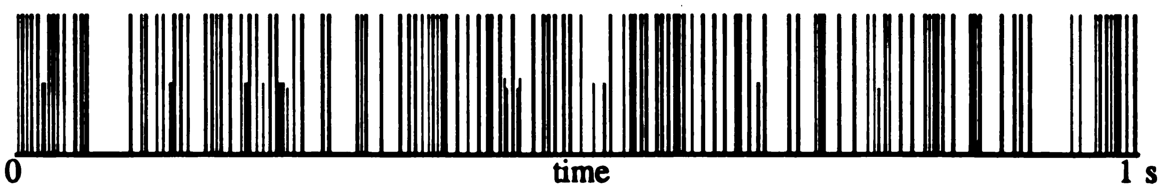
b) Z(f)



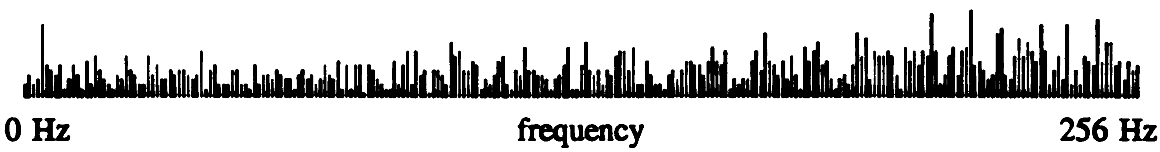
c) The message m(t) and m'(t)

Figure 3.21b The message signal is recovered by ALG. 3

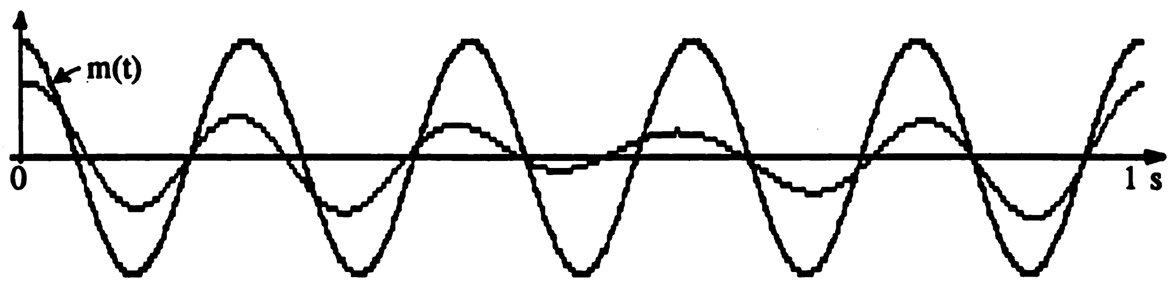
(the noise level=15)



a) The zero-crossing function z(n)



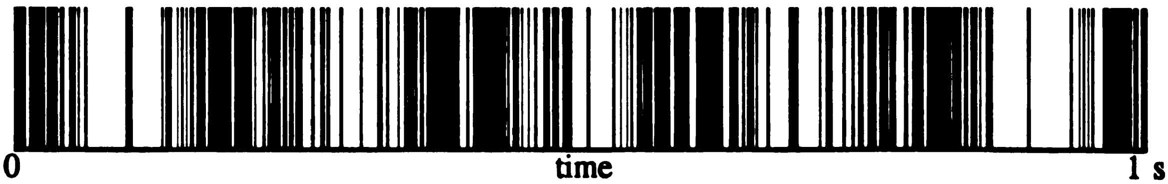
b) Z(f)



c) The message m(t) and m'(t)

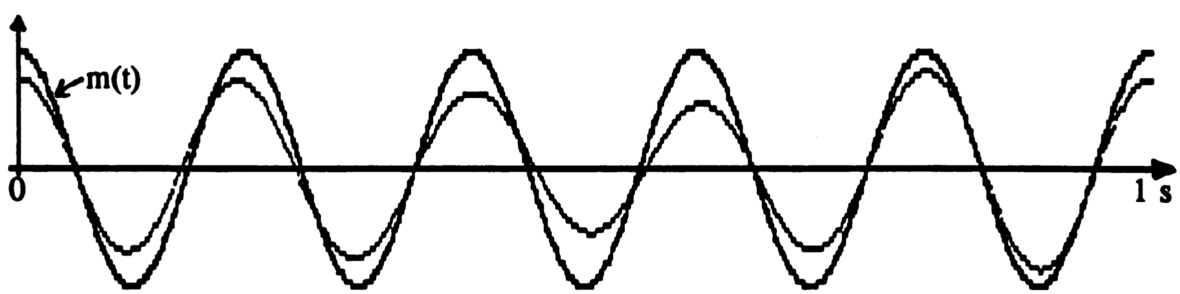
Figure 3.21 The message signal is recovered by ALG. 3

(the noise level=20)



a) The zero-crossing function z(n)

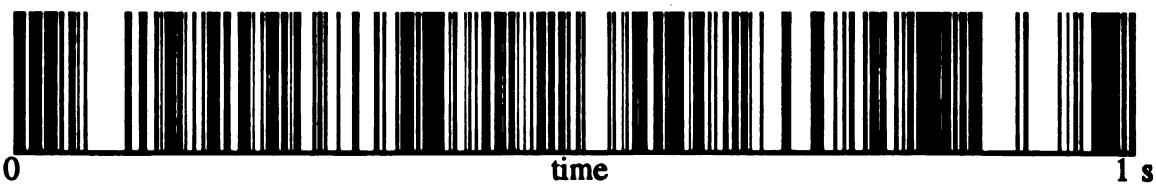
b) Z(f)



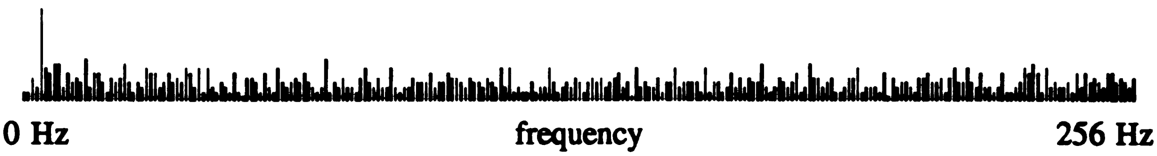
c) The message m(t) and m'(t)

Figure 3.22a The message signal is recovered by ALG. 4

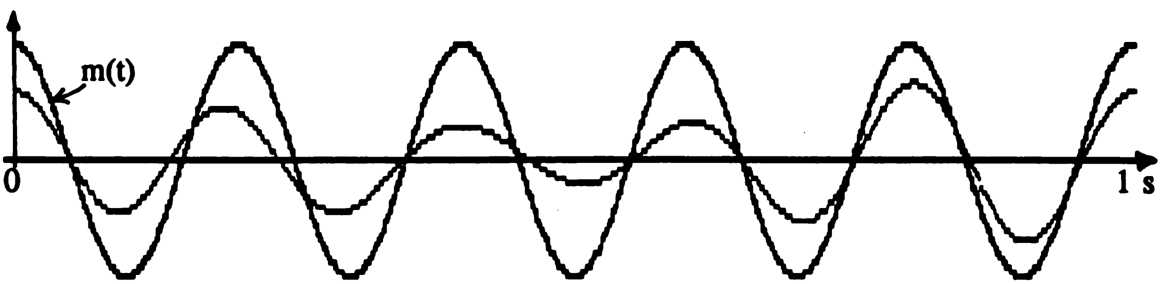
(the noise level=10)



a) The zero-crossing function z(n)



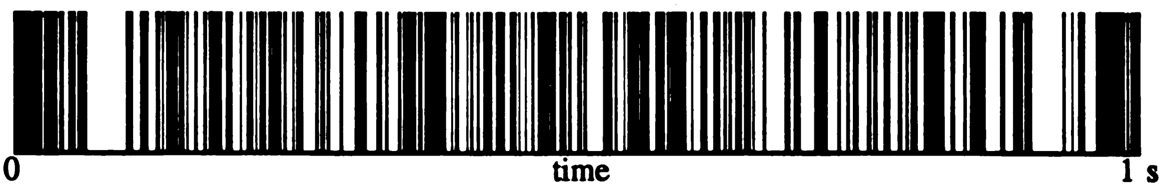
b) Z(f)



c) The message m(t) and m'(t)

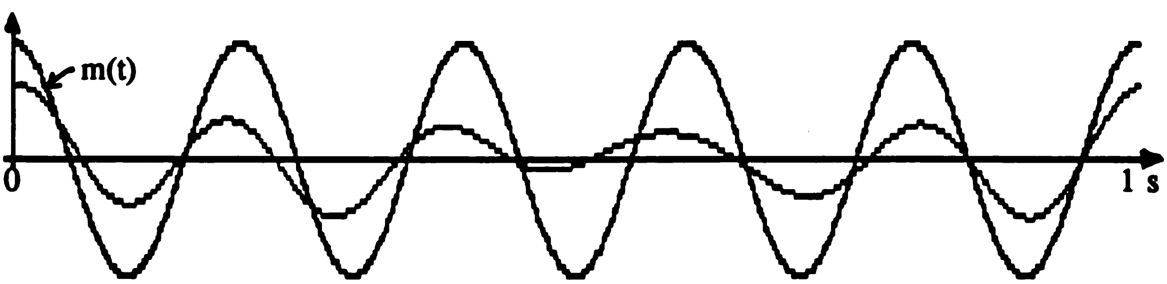
Figure 3.22b The message signal is recovered by ALG. 4

(the noise level=15)



a) The zero-crossing function z(n)

b) Z(f)



c) The message m(t) and m'(t)

Figure 3.22c The message signal is recovered by ALG. 4

(the noise level=20)

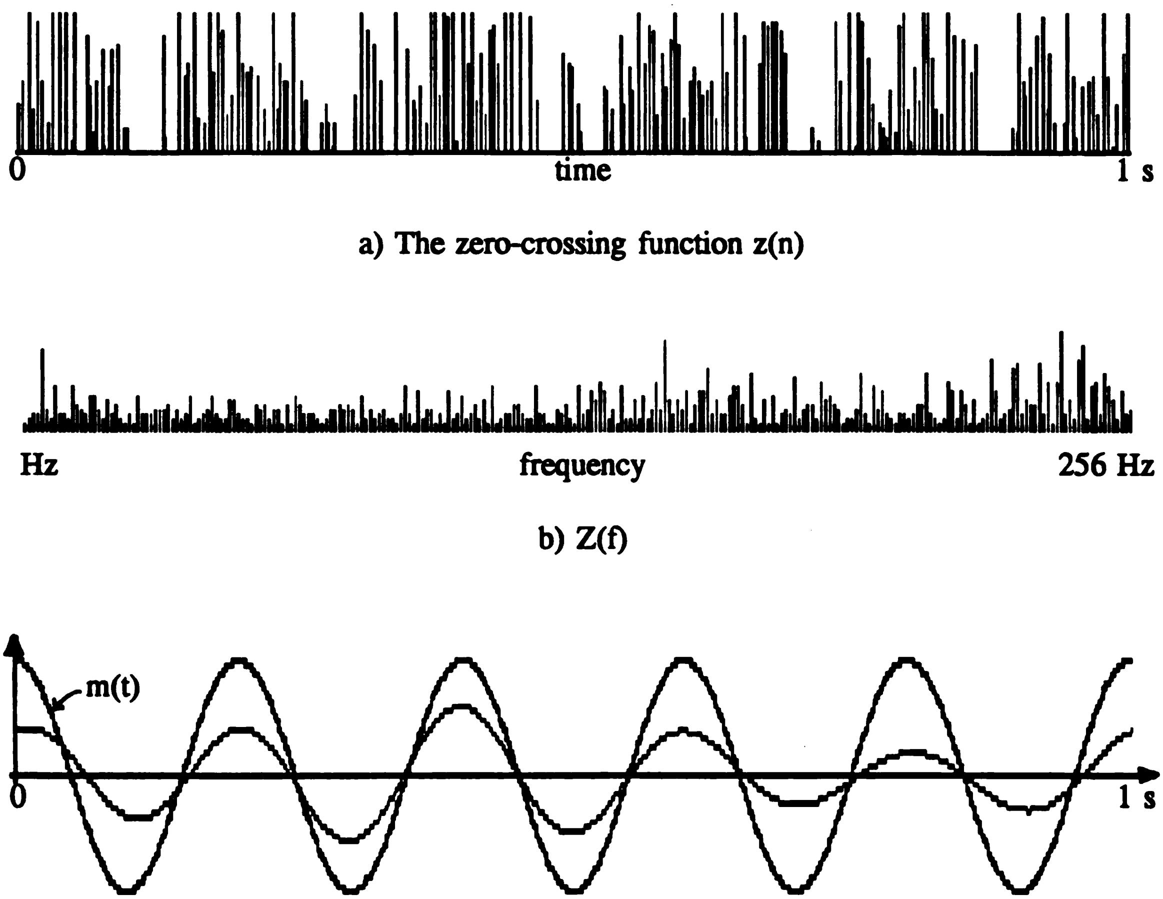
3.7.2 FM Signal in the Presence of FH:

In this case while the amplitude of FM is fixed at 20, the frequency hopping signal's amplitude is changed to 10, 15, and 20. The corresponding simulation results are shown in Figures 3.23a - 3.26c, and the quantitave results are tabulated in Table 3.4.

Table 3.3 MSE results for a single tone modulation in the presence of FH interference

	ALG I	ALG II	ALG III	ALG IV
5 V	8.418	6.962	3.043	0.0016
10 V	8.677	7.113	3.135	0.1047
15 V	9.182	8.112	4.445	0.8610
20 V	9.670	9.424	7.098	3.4313

From a study of the above figures and tables, one can see that the distortion which is caused by the frequency hopping is less than the noise for the same level of frequency hopping and noise level. One can notice that the simulation results are applicable to the multi-tone modulation as well. The single tone has been considered to demonstrate the different algorithms geometrically. The simulation model thus has been used to test the four algorithms. It showed that the FM signal can be detected in the presence of noise and FH signal.



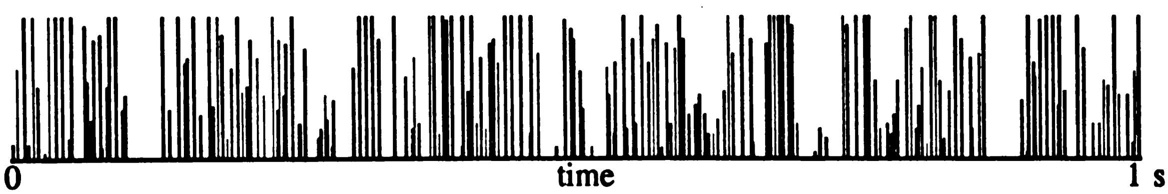
a) The zero-crossing function z(n)

b) Z(f)

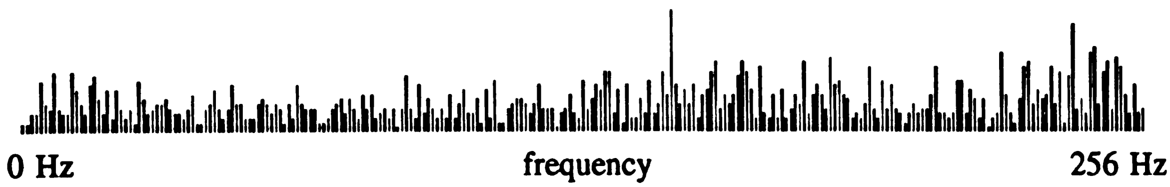
c) The message m(t) and m'(t)

Figure 3.23a The message signal is recovered by ALG. 1

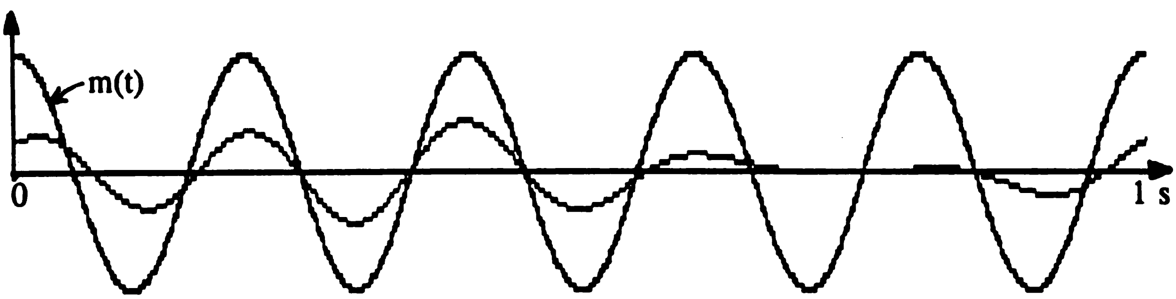
(the FH interference level = 10)



a) The zero-crossing function z(n)



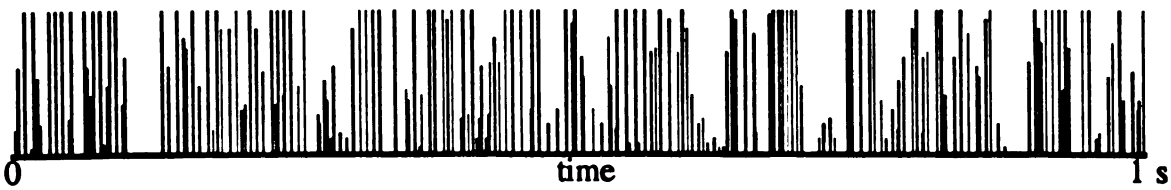
b) Z(f)



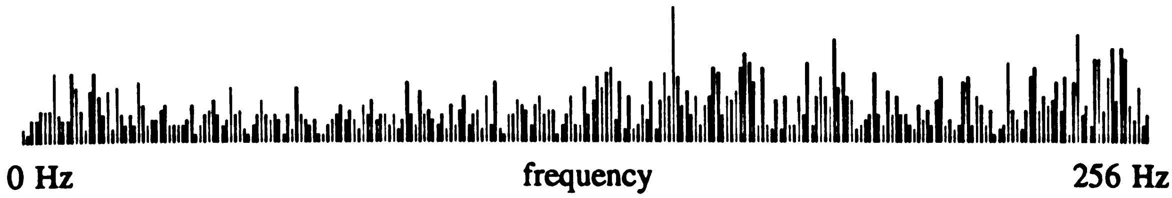
c) The message m(t) and m'(t)

Figure 3.23b The message signal is recovered by ALG. 1

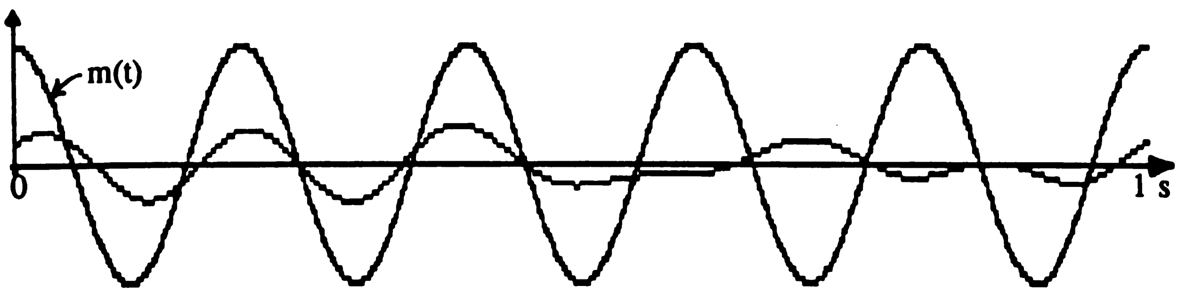
(the FH interference level=15)



a) The zero-crossing function z(n)



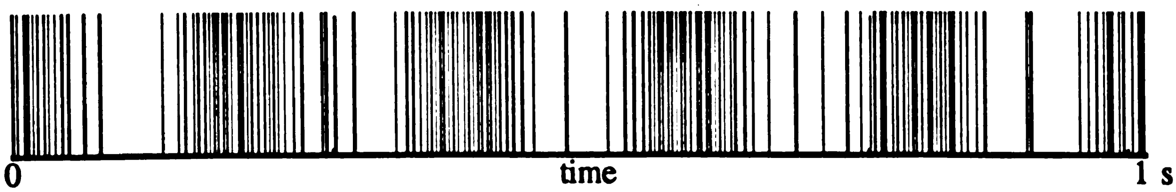
b) Z(f)



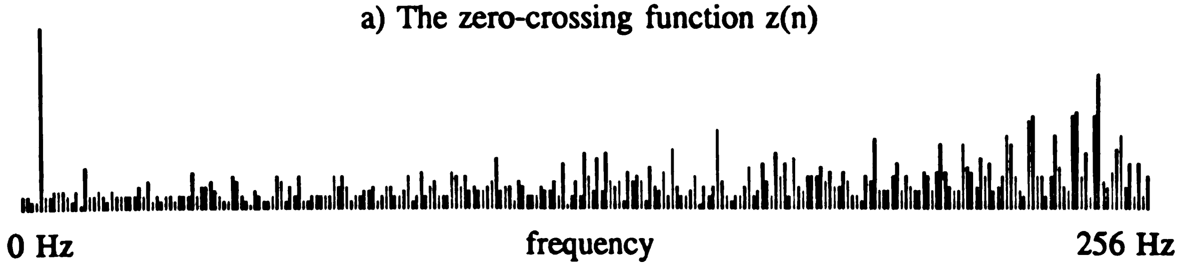
c) The message m(t) and m'(t)

Figure 3.23c The message signal is recovered by ALG. 1

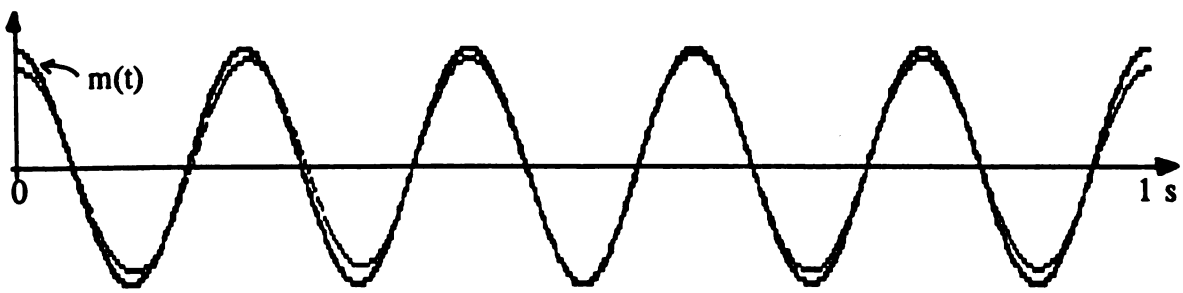
(the FH interference level=20)



a) The zero-crossing function z(n)



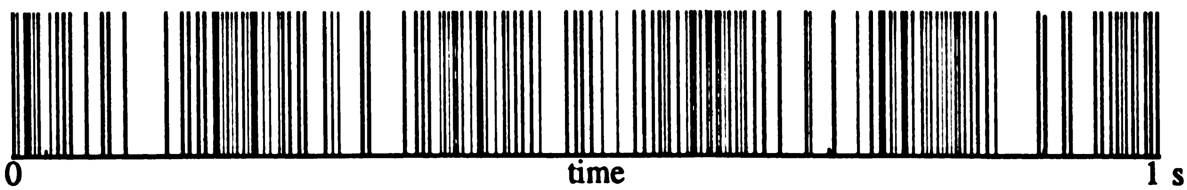
b) Z(f)



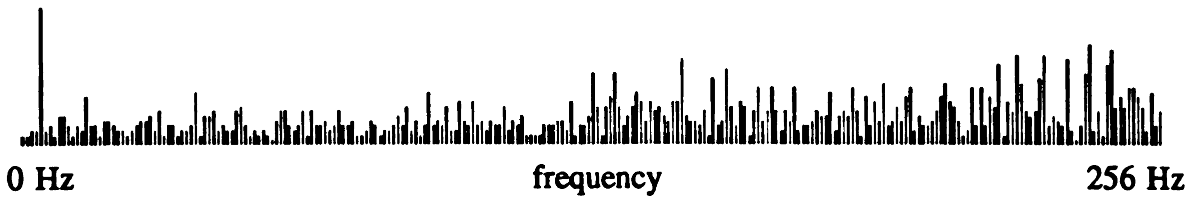
c) The message m(t) and m'(t)

Figure 3.24a The message signal is recovered by ALG. 2

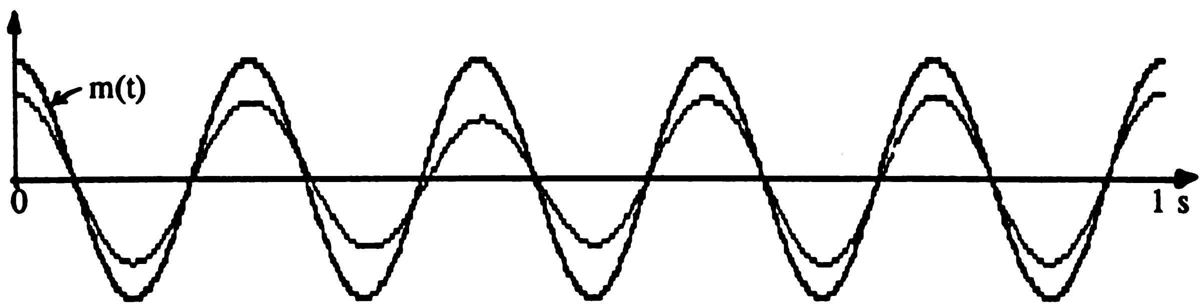
(the FH interference level = 10)



a) The zero-crossing function z(n)



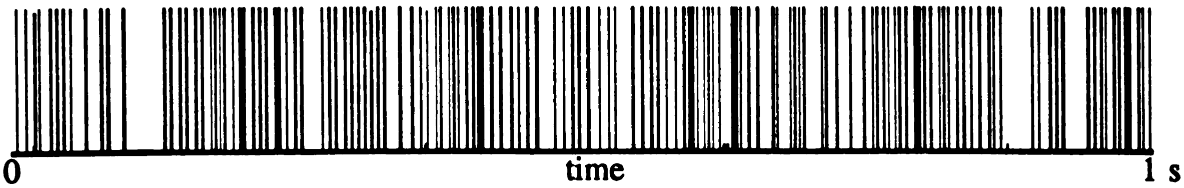
b) Z(f)



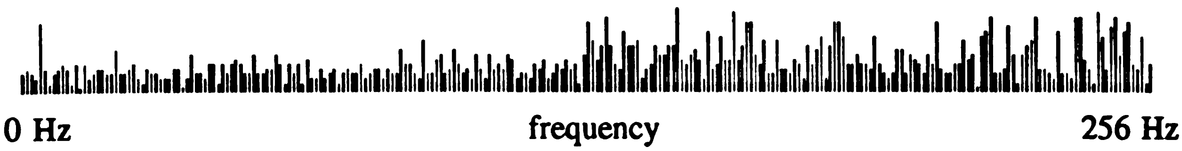
c) The message m(t) and m'(t)

Figure 3.24b The message signal is recovered by ALG. 2

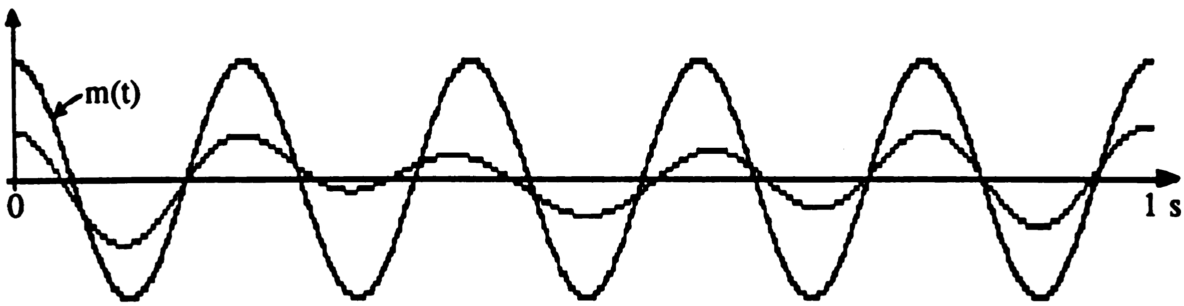
(the FH interference level=15)



a) The zero-crossing function z(n)



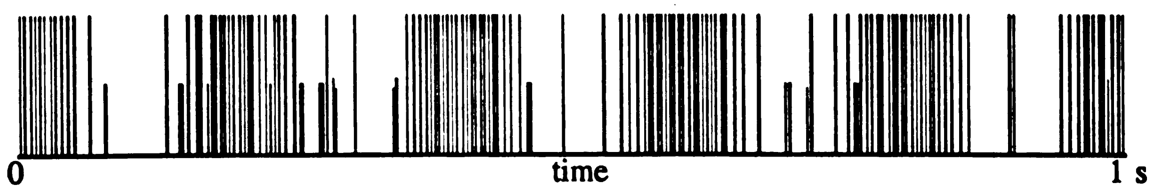
b) Z(f)



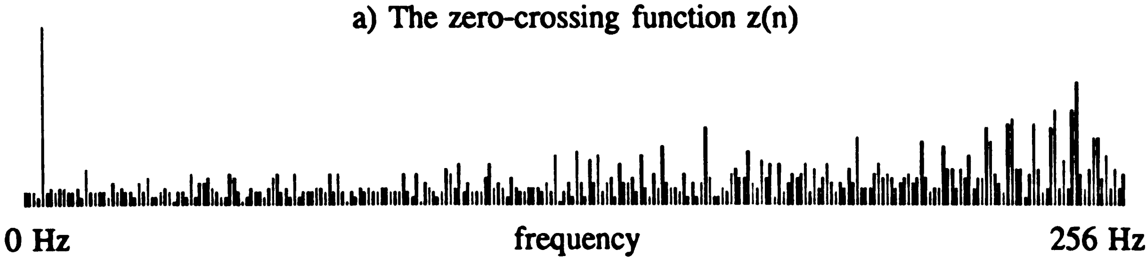
c) The message m(t) and m'(t)

Figure 3.24c The message signal is recovered by ALG. 2

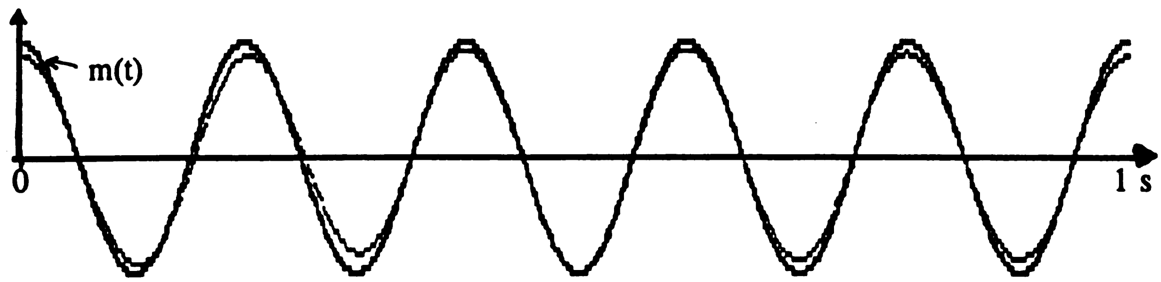
(the FH interference level=20)



a) The zero-crossing function z(n)



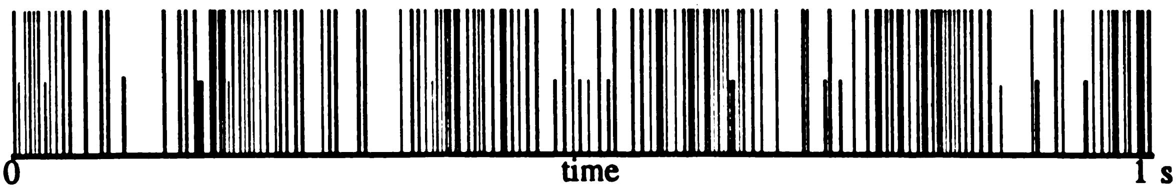
b) Z(f)



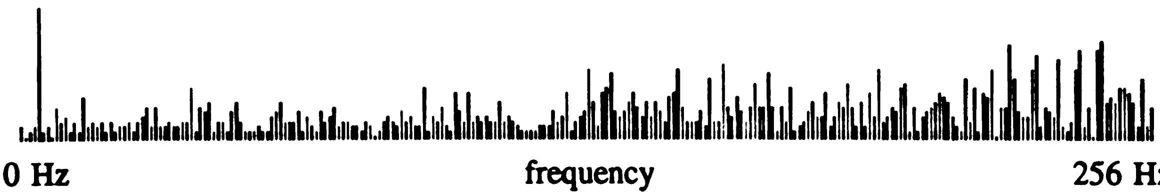
c) The message m(t) and m'(t)

Figure 3.25a The message signal is recovered by ALG. 3

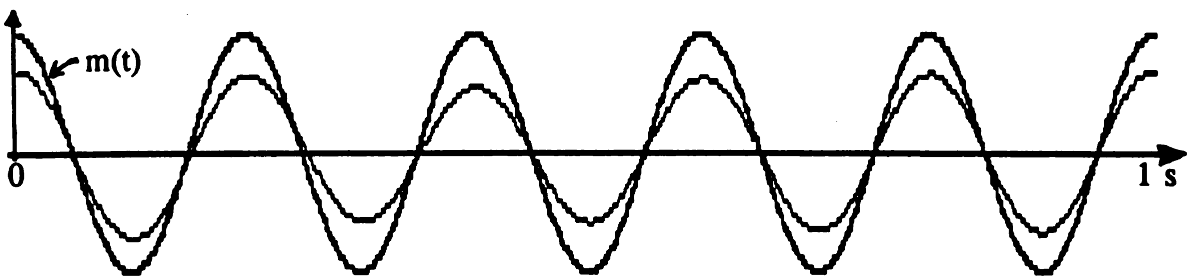
(the FH interference level=10)



a) The zero-crossing function z(n)



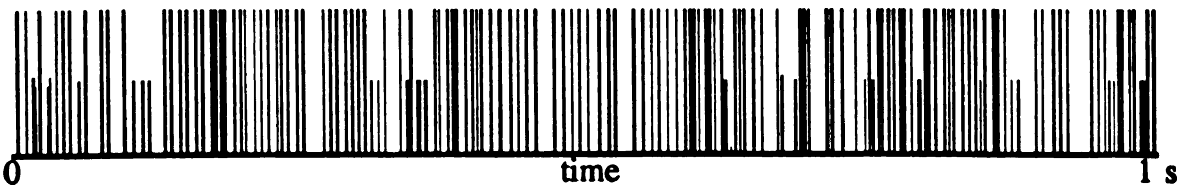
b) Z(f)



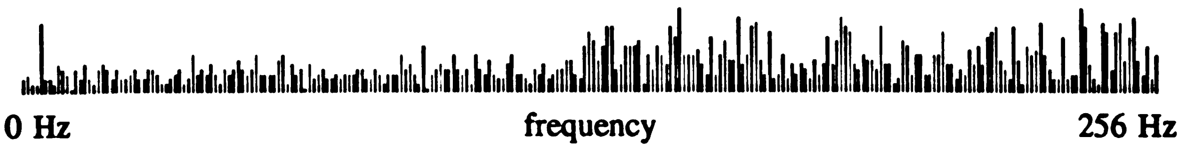
c) The message m(t) and m'(t)

Figure 3.25b The message signal is recovered by ALG. 3

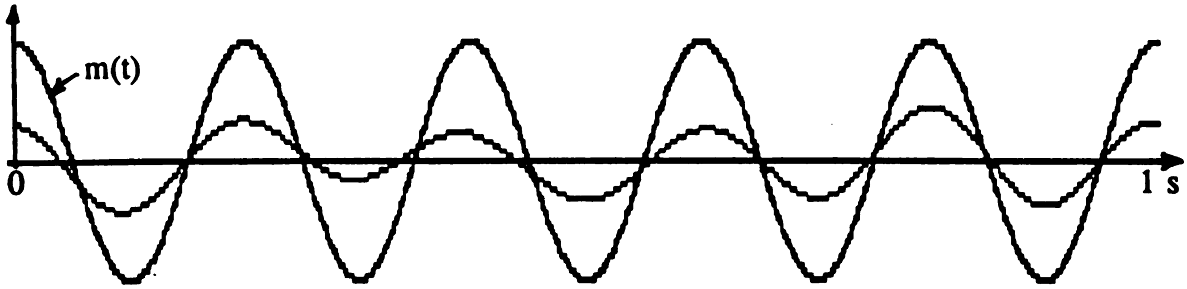
(the FH interference level=15)



a) The zero-crossing function z(n)



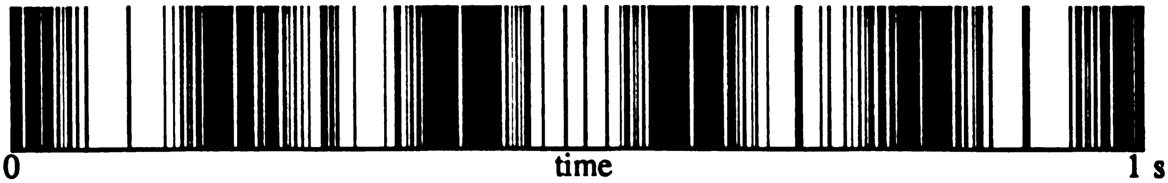
b) Z(f)



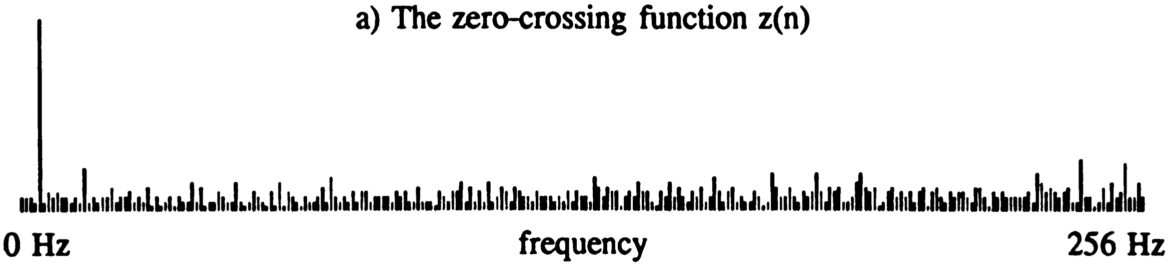
c) The message m(t) and m'(t)

Figure 3.25c The message signal is recovered by ALG. 3

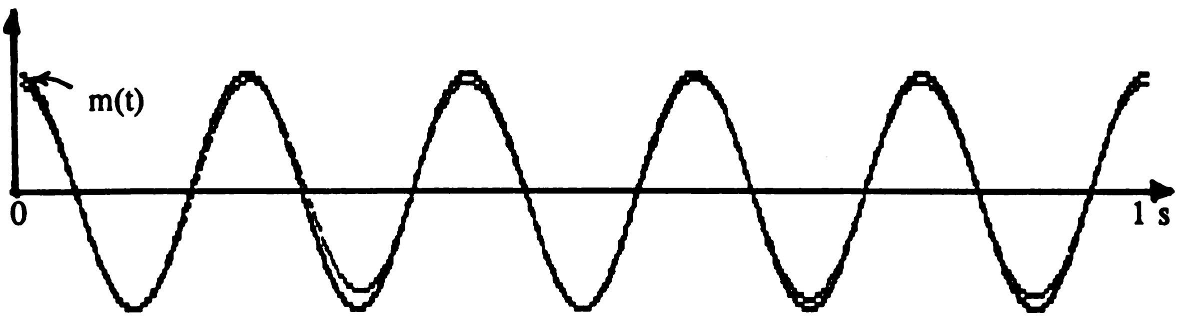
(the FH interference level=20)



a) The zero-crossing function z(n)



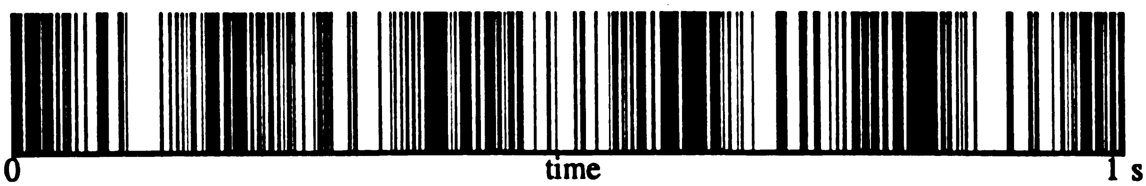
b) Z(f)



c) The message m(t) and m'(t)

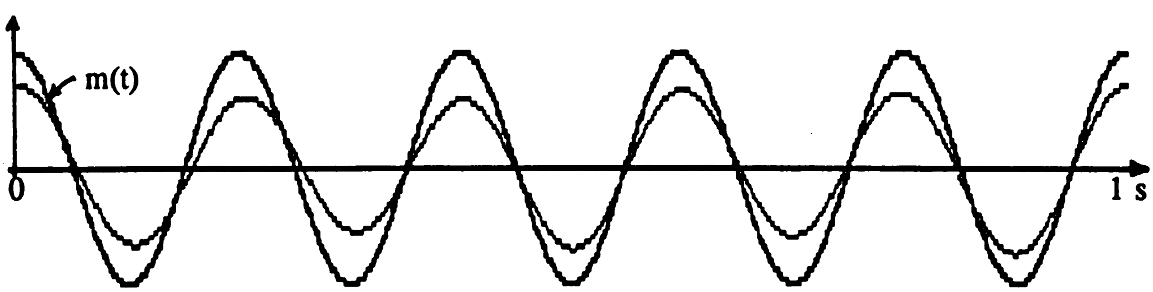
Figure 3.26a The message signal is recovered by ALG. 4

(the FH interference level = 10)



a) The zero-crossing function z(n)

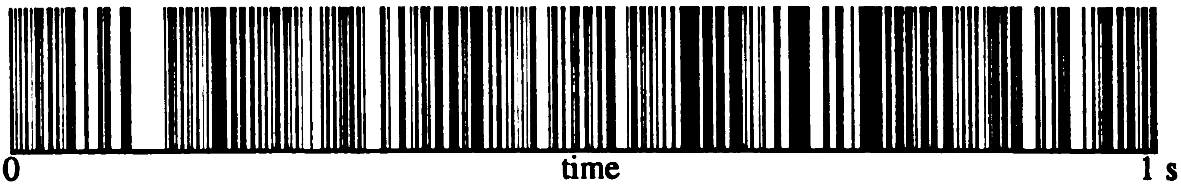
b) Z(f)



c) The message m(t) and m'(t)

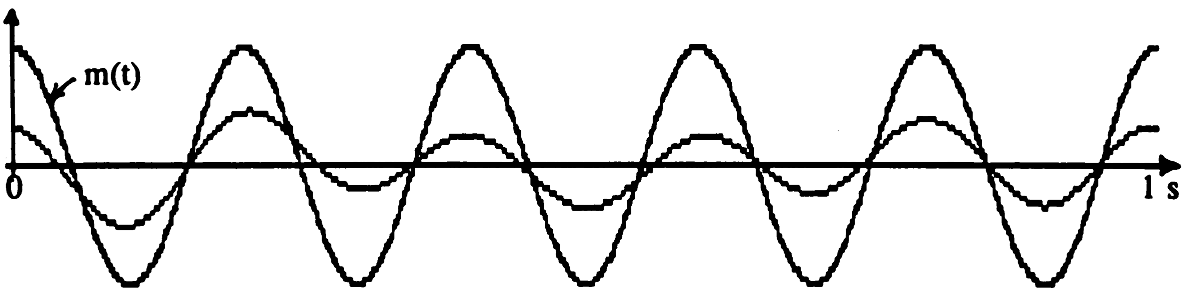
Figure 3.26b The message signal is recovered by ALG. 1

(the FH interference level=15)



a) The zero-crossing function z(n)

b) Z(f)



c) The message m(t) and m'(t)

Figure 3.26c The message signal is recovered by ALG. 4

(the FH interference level=20)

From the above simulation figures, tables and from the discussions in the beginning of this chapter, one can conclude that the FM signal can be detected in the presence of noise and interfering frequency hopping signal i(t) by a zero crossing scheme. The zero crossing schemes that perform best under the test conditions are algorithm III and IV which were developed in this study. If FM detection is going to be performed by utilizing a hard-ware system, then algorithm III would be the proper choice, and if the FM detection is considered by employing a software system, then algorithm IV would be a suitable candidate. Furthermore from the detection of signal in the presence of noise and frequency hopping interfering signal one can conclude that the FM system performs better in the presence of frequency hopping than that of the noise. In the next section different performance measures such as signal to noise ratio (SNR), signal to interference ration (SIR), and signal to distortion ratio (SDR) are considered.

3.8 System's Performance Measure:

A standard performance measure for Analog Communication systems such as FM systems are signal to noise ratio (SNR), signal to interference ratio (SIR), signal to noise and interference ratio (SNIR), and signal to distortion ratio (SDR), [65]. The above mentioned measures are adapted for this model also.

3.8.1 Signal to Noise Ratio:

Signal to noise ratio is the ratio of the signal power to the noise power. The signal to noise ratio must be calculated at the input of the detector as well as at the output of the detector, then a plot of the output signal to noise ratio (SNRO) versus the input signal to noise ratio (SNRI) will determine the performance of the system. In this simulation study the signal to noise ratios are calculated for all different algorithms. Since the signal power is usually very large compared to the noise power the signal to noise ratio is measured in dB. Before plotting different signal to noise or interference ratio, a brief discussion of the calculation procedures is given.

3.8.2 Calculation of SNR and SIR:

Considering the FM signal $s_1(t)$, the signal's power in the input of the receiver can be determined by the amplitude of the signal. In this case $A^2/2 = (20)^2/2 = 200$ watts, where A is the amplitude of the FM signal. This value is the theoretical value of the power for the FM signal. But in simulation the input power of the signal s_1 can be calculated as follows:

$$S_i=\sum_{i=1}^{N} s_i(i)^2/N \qquad (3.25)$$

where S_i represents the input average power, and $s_1(i)$ is the same as $s_1(t)$ with t is replaced by i/N for simulation. This power has been calculated for an amplitude of 20 and the result is 200.78475 watts which is very close to the theoretical value. This value is more realistic, since $A^2/2$ is actually the power of a pure sinusoidal signal, and it is only an approximation to the real power of the FM signal. It has been also observed that the power of an FM signal depends on the carrier and the message frequencies. Thus this method of calculation is also utilized for calculation of the noise power and the interfering frequency hopping signals power. The input noise power N_i and the input interfering frequency hopping power I_i have been calculated as follows:

$$N_i=\sum_{k=1}^{N} n(k)^2/N \tag{3.26}$$

and

$$I_I=\sum_{k=1}^{N} i(k)^2/N \tag{3.27}$$

where n(k) and i(k) are the simulation expression for n(t) and i(t) respectively, with t replaced by k/N in their corresponding expressions.

The output powers are calculated as follows:

First, the FM signal is detected in the absence of noise and the interfering frequency hopping signal. This signal is actually the message signal $m_1(t)$ and it is denoted by $m_d(t)$, the detected message signal at the out put of the demodulator. Thus the power of the FM signal at the output of the detector S_o is

$$S_o=\sum_{i=1}^{N} m_d(i)^2/N \tag{3.28}$$

where $m_d(i)$ is the simulation expression of $m_d(t)$ with t is replaced by i/N in the simulation process.

The noise is then added to the FM signal at the input of the detector and the corresponding output is the output signal plus noise at the output of the detector SN_o, giving the following expression for SN_o:

$$SN_o = S_o + N_o \tag{3.29}$$

where N_o is the output noise at the output of the zero crossing detector in the absence of FM signal. Thus the output noise N_o is calculated as follows:

$$N_o=\sum_{i=1}^{N}[n_o(i)-m_d(i)]^2/N \tag{3.30}$$

where $n_o(i)$ and $m_d(i)$ are the simulation expressions for the detected message and noise at the output of the detector.

Similarly if frequency hopping interfering signal is added to the FM signal, then the output interfering power I_o is

$$I_o=\sum_{j=1}^{N}[si_o(j)-m_d(t)]^2/N \tag{3.31}$$

where $si_o(t)$ is the output detected FM signal in the presence of the frequency hopping interfering signal. Finally if noise and interference are present, the corresponding output signal is denoted by sni and the corresponding output power of noise plus interference is NI_o which is given by the following sum

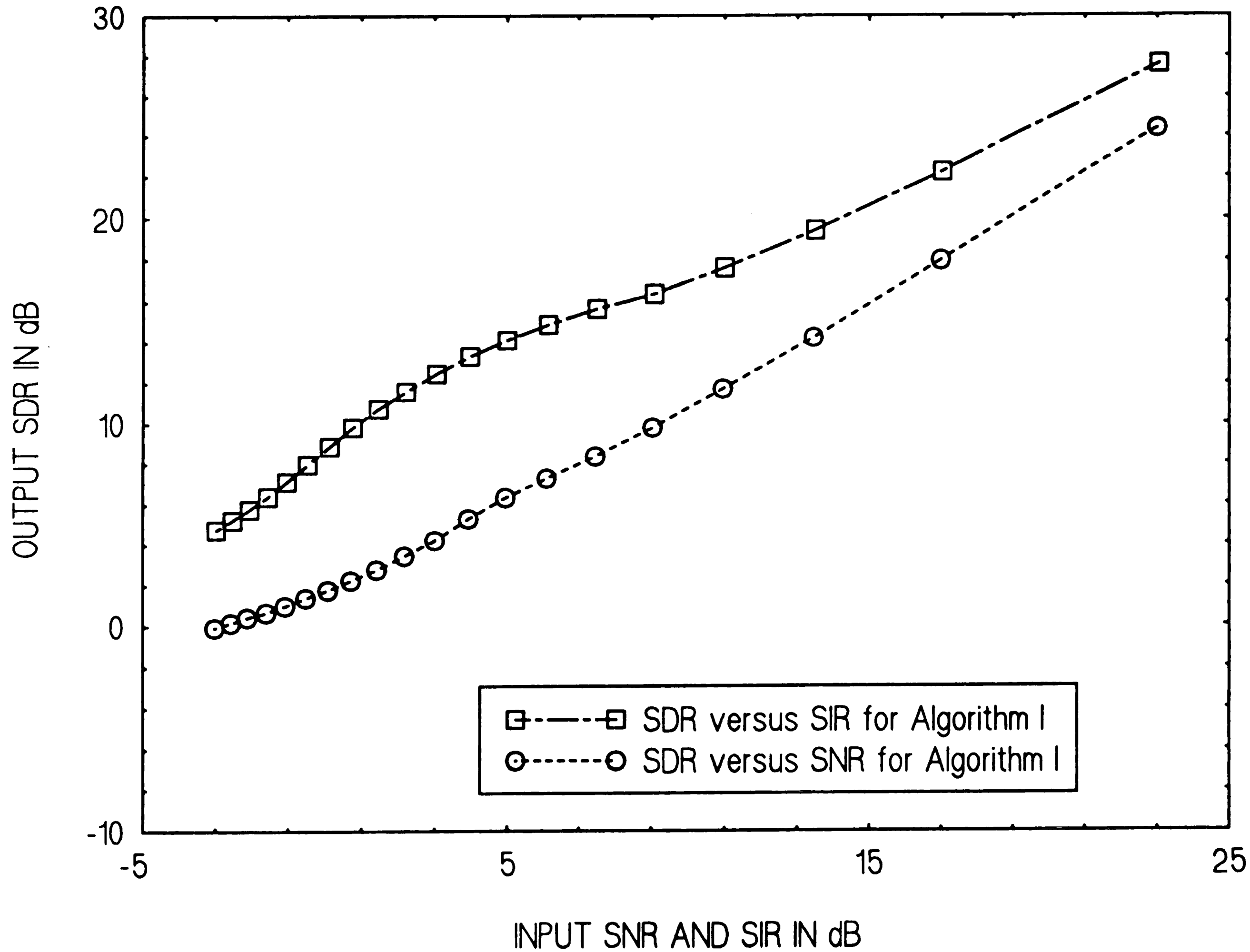
$$NI_o=\sum_{j=1}^{N}[sni_o(j)-m_d(j)]^2/N \tag{3.32}$$

Having these parameters calculated, the different signal to noise ratios, signal to interference ratios and signal to noise and interference ratios at the input and output of the detector are obtained as follows:

SNRI	The input signal to noise ratio
SNRO	The output signal to noise ratio
SIRI	The input signal to interference ratio
SIRO	The output signal to interference ratio
SNIRI	The signal to noise and interference ratio at the input
SNIRO	The signal to noise and interference ratio at the output

each of the above parameters is obtained in dB's, for example SNRI is $10\log_{10}(S_i/N_i)$. The output parameters such as SNRO, SIRO and SNIRO are categorically referred as signal to distortion ratio SDR [65]. The parameter SDR can be used to determine and compare the performance of the system under different disturbances. Thus this parameter is also adapted for this study, to compare the distortion due to noise and frequency hopping interfering signal. The plots of SDR are given in Figures. 3.27-3.30. Notice that these parameters are plotted for different algorithms. Once again, one can come to the same conclusion that algorithm 3 and 4 are performing better than other algorithms. By comparing the SDR's one can also conclude that the distortion which is caused by frequency hopping interference is less than the distortion which caused by the noise. Therefore the distortion effects of frequency hopping signal is less than the distortion which is caused by the noise. To see the performance of the FM signal in the presence of FH signal and noise, a simultaneous performance is considered.

Figure 3.27 SDR versus input SNR and SIR for Algorithm I



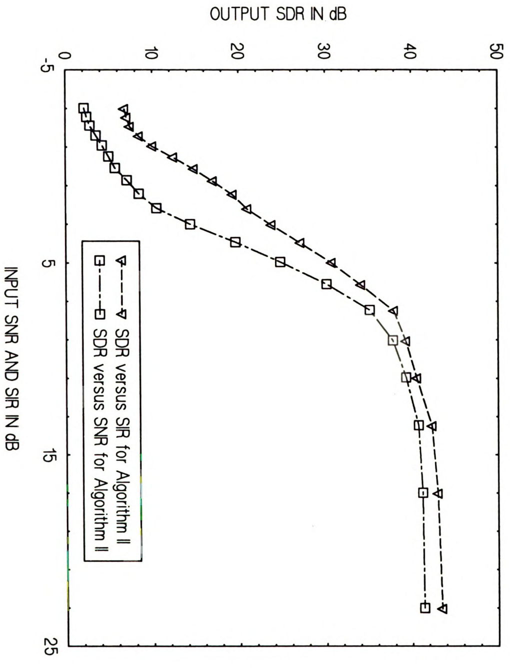


Figure 3.28 SDR versus input SNR and SIR for Algorithm II

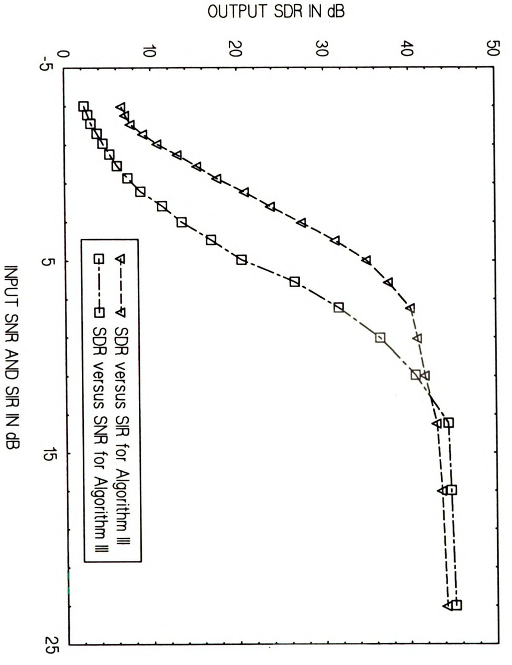
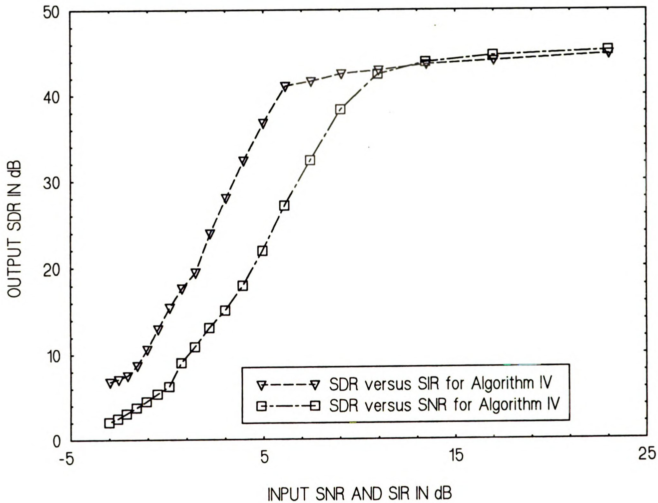


Figure 3.29 SDR versus input SNR and SIR for Algorithm III

Figure 3.30 SDR vrsus input SNR and SIR for Algorithm IV



3.8.3 Simultaneous Performance:

In the previous section the performance of the system through averaging signal to noise and interference have been examined. By a plot of these parameters it was concluded that best algorithms are algorithm III and IV, and the noise induces more distortion than frequency hopping signal to the FM system which was considered. Although those calculations and plots determine the overall performance of a system, they fail to reveal other system behavior, such as group delay or phase shifts. With the help of advancements in minicomputers and their graphics capability one can determine the simultaneous behavior of the signal in the presence of different level of noise or interference. This is referred as the simultaneous performance measure.

In average performance measure one determines the behavior of the system in one complete period of the signal for example over a length of 512 points. In the simultaneous performance one can see the behavior of the system point by point pictorially. The simultaneous performance is actually a summary or combination of several figures, in one figure. One can think of it as a multi channel oscilloscope, which shows the effects of different levels of additive noise or interference to a signal. The simultaneous performance of the algorithms in the presence of different levels of noise or FH interference is shown in Figures 3.31-3.34. These Figures demonstrate the amplitude and phase behaviors of the recovered signal in the presence of noise and/or interference.

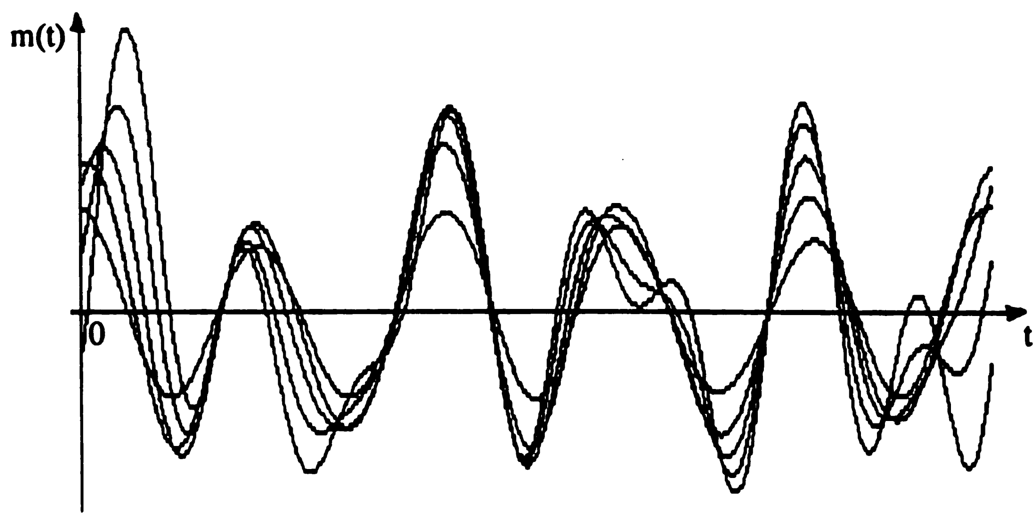


Fig. 3.31a Simultaneous performance of m(t) in the presence of noise

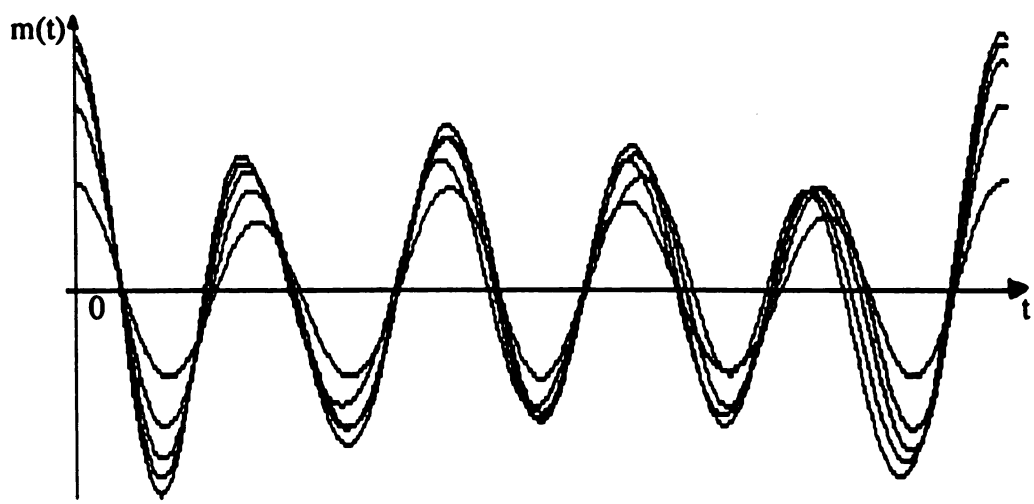


Fig. 3.31b A Simultaneous performance of m(t) in the presence of FH

Figure 3.31 The performance of the message under Algorithm I

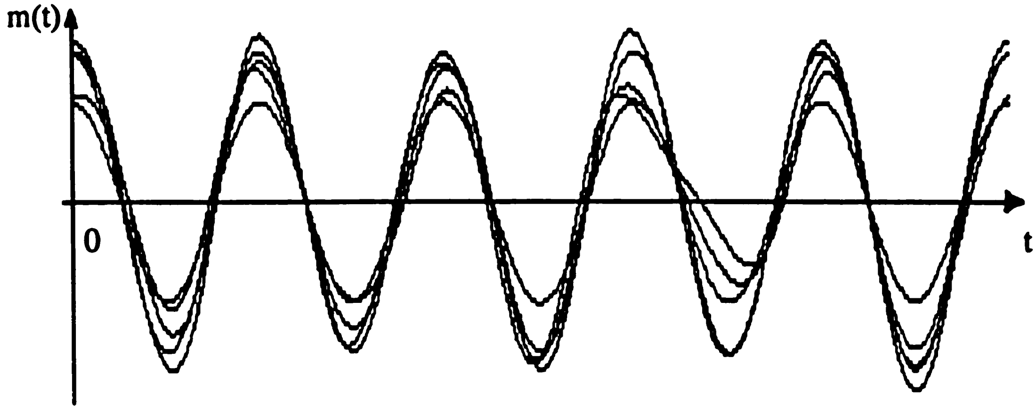


Fig. 3.32a Simultaneous performance of m(t) in the presence of noise

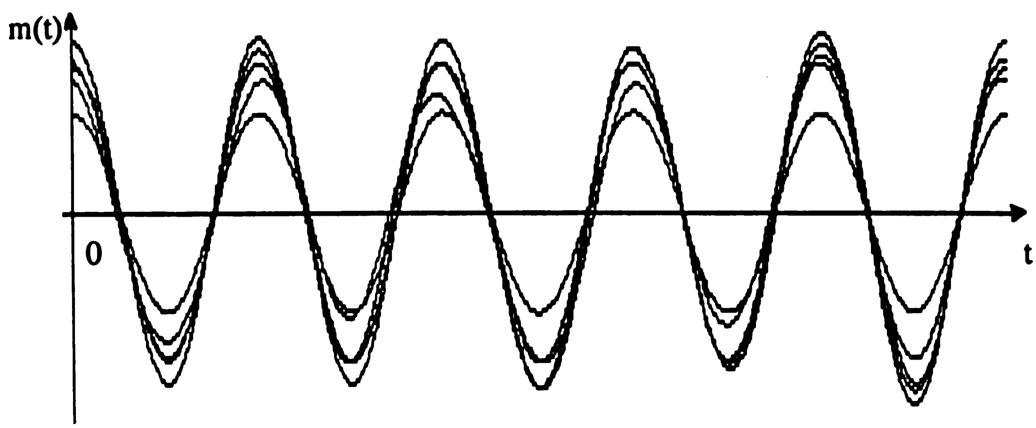


Fig. 3.32b Simultaneous performance of m(t) in the presence of FH

Figure 3.32 The performance of the message under Algorithm II

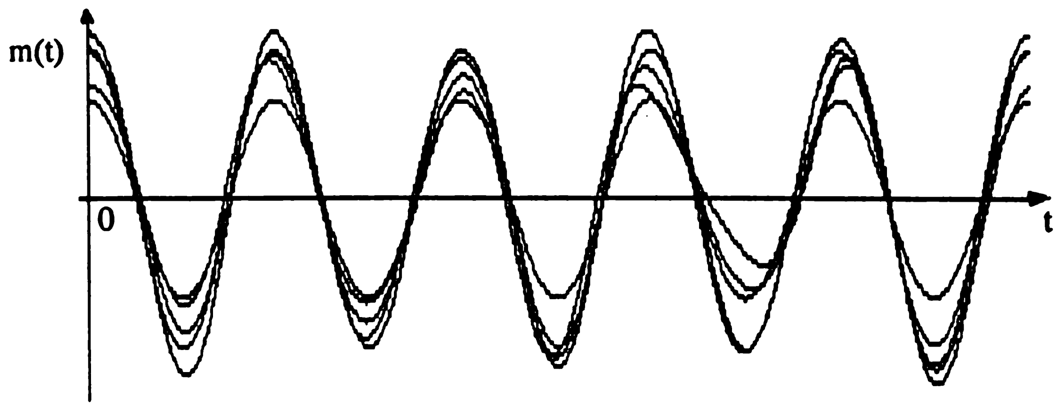
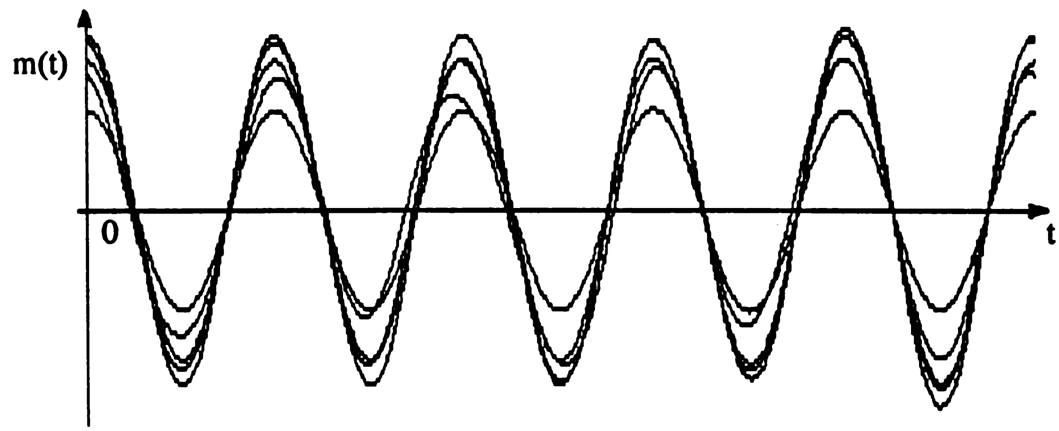


Fig. 3.33a Simultaneous performance of m(t) in the presence of noise



Figu. 3.33b Simultaneous performance of m(t) in the presence of FH

Figure 3.33 The performance of the message under Algorithm III

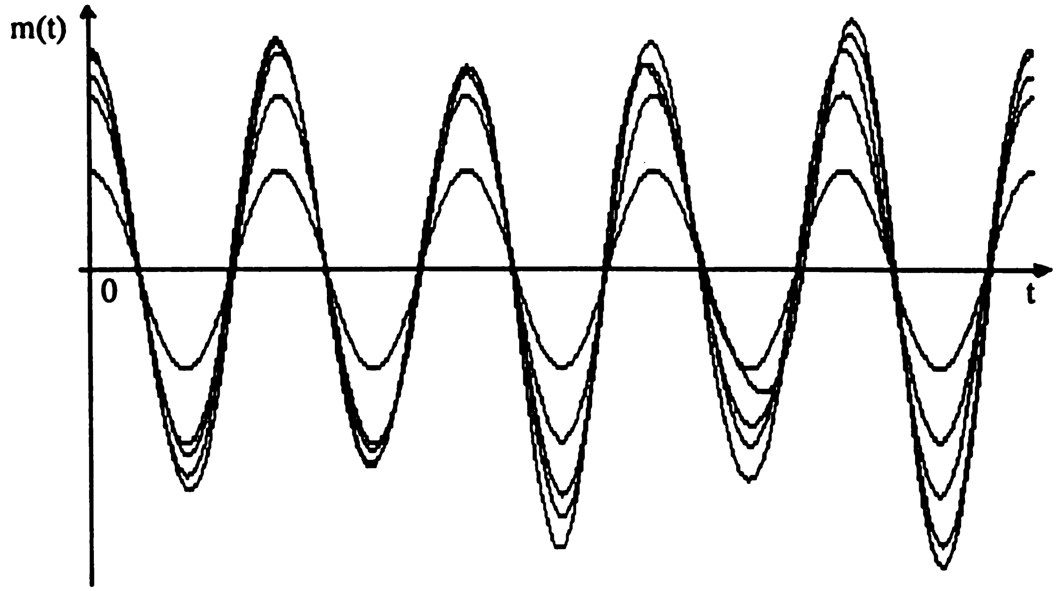


Fig. 3.34a Simultaneous performance of m(t) in the presence of noise

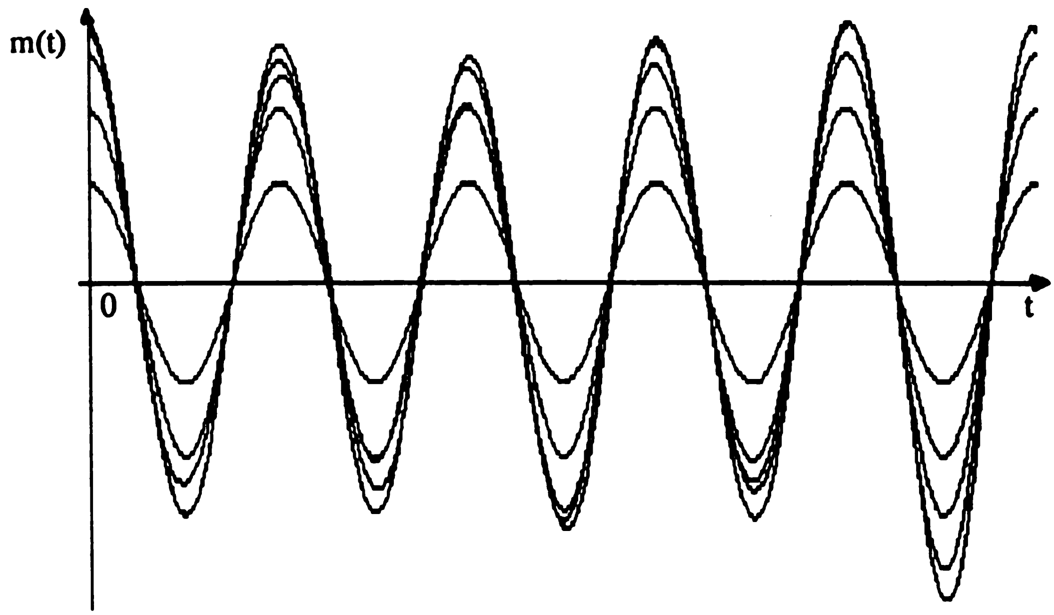


Fig. 3.34b Simultaneous performance of m(t) in the presence of FH

Figure 3.34 The performance of the message under Algorithm IV

Accordingly these figures confirm the result which were obtained by the averaged performance measure. Furthermore they reveal the phase distortions which were not obvious in the preceding figures.

In summary, the simultaneous performance shows the amplitude and phase distortions, together with group delays that exists in the output of the detector. All of these graphs and studies show that the FM signal could coexist with a frequency hopping interfering signal. This is possible by a careful system design which manages the bands of each system carefully.

3.9 Conclusion:

The demodulation of an FM signal by zero crossing scheme has been considered. The proposed algorithm which exist in the literature has been examined. Recovering the signal by frequency domain methods showed a better result. The frequency domain detection consists of an ideal low pass filter, followed by taking the inverse Fourier transform (IFFT) of the filtered signal. The output is the recovered version of the message signal which was employed for generation of the FM signal. The frequency domain detection procedure is applied to all algorithms. The result of the simulation shows that these algorithm bring more improvement as they get closer to the ideal zero-crossings.

The result of the simulation also showed that algorithm III and IV are the best schemes for applications. Furthermore, it has been observed that algorithm III is suitable for hardware applications, and algorithm IV which is ideal zero-crossing could be adapted for software applications. The above observations have been confirmed by a study of performance measure such as SNR, SIR and SDR. Finally the simultaneous performance of the system has been plotted point by point. The result of the study showed that the distortion that resulted from frequency hopping was less than that of noise of the same amplitude. The conclusion is that the zero-crossing method of demodulation should emerge as a major alternative method of detection for the decades to come. Since FH causes less distortion to FM system compared to the noise, FM could coexist with a frequency hopping spread spectrum system.

CHAPTER IV

THE SPECTRUM OF FM ZERO-CROSSING

4.1 Introduction:

The idea of FM detection by zero-crossing has existed in the literature for quite sometime [59], and it has been used in a number of applications. In this part of the study it is verified that the FM signal can be detected by low pass filtering its zero-crossings.

In this analysis an FM signal s(t) is considered. The signal x(t) is constructed from the ratio of the signal s(t) and its absolute value $|s(t)|$. The function x(t) and s(t) have the same zero-crossings. However, the signal x(t) is a non-uniform square wave, which is similar to a hard limited version of s(t). The zero-crossing instants are determined by taking the derivative of the signal x(t). Since the function x(t) is a non-uniform square wave, its derivative y(t) is a non-uniform sequence of δ-functions. consequently every zero-crossing instant t_k is marked by positive amplitude or negative amplitude $\delta(t_k)$ or -$\delta(t_k)$. The low-pass filtering of the signal y(t) is considered next. Low-pass filtering in frequency domain approximates integration or averaging in time domain. Since y(t) is an alternating sequence of positive and negative δ functions the result of low-pass filtering would be zero. Accordingly it is necessary to mark the zero-crossing instants by positive amplitude δ-functions only. Thus z(t) which is the square of y(t) is considered. z(t) has a variety of different spectral components. It will be shown in the next section that one of the components is the amplitude scaled version of the message signal m(t). This signal could easily be recovered by a low-pass filter.

In the following sections a mathematical analysis of the FM zero-crossing has been presented, followed by an example.

4.2 A Mathematical Discussion of FM Zero-Crossings:

In the following discussion an FM signal s(t) is considered, the zero-crossings of this signal are investigated using a trigonometric power series expansion. Subsequently an example is presented to demonstrate the theory.

Let the FM signal s(t) be defined as follows:

$$s(t) = A\sin(w_c t+\beta\int_0^t m(\tau)\,d\tau) \tag{4.1}$$

or simply

$$s(t) = A\sin[\theta(t)] \tag{4.2}$$

with

$$\theta(t) = w_c t+\beta\int_0^t m(\tau)\,d\tau \tag{4.3a}$$

and

$$\theta'(t) = w_c+\beta m(t) \tag{4.3b}$$

Where β is the frequency deviation, and m(t) is the message signal. The FM signal s(t) which is generated by modulating signal m(t) is shown in figure 4.1.

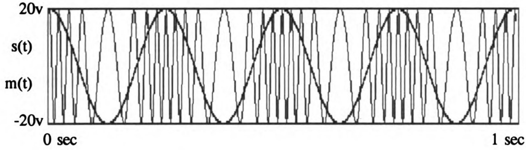


Figure 4.1 The FM signal s(t) is superimposed on m(t)

Consider the function x(t) which is constructed from s(t) as follows:

$$x(t) = \frac{s(t)}{|s(t)|} \tag{4.4}$$

This function changes the FM signal s(t) to a normalized square wave type FM signal. However, the zero-crossings of s(t) and x(t) are the same, as shown in figures 4.1 and 4.2.

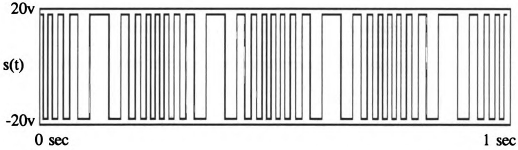


Figure 4.2 The function 20 x(t) marks the zero-crossings of s(t)

Now one can consider x(t) in the following form.

$$x(t) = \frac{A\sin\theta(t)}{|A\sin\theta(t)|} = \frac{\sin\theta(t)}{|\sin\theta(t)|} \tag{4.5}$$

since $|\sin\Theta(t)| = [1-\cos^2\Theta(t)]^{1/2}$; thus x(t) can be written as follows:

Expanding $[1 - \cos^2\Theta(t)]^{-1/2}$ in a series approximation [66] one gets the following

$$x(t) = \frac{\sin\theta(t)}{[1-\cos^2\theta(t)]^{\frac{1}{2}}} \tag{4.6}$$

expression for x(t)

$$x(t) = \sin\theta(t)\left[1+\sum_{k=1}^{N} \frac{(2k-1)o!}{(2k)e!}\cos^{2k}\theta(t)\right] \tag{4.7}$$

where (2k-1)o! = 1•3•5•...•(2k-1); product of odd numbers up-to 2k-1.

(Thus o!, or odd factorial can be defined as the product of odd numbers, and similarly e!, or even factorial is the product of even numbers could be expressed as follows: (2k)e! = 2•4•6•...•(2k); product of even numbers up-to 2k.)

(note: Zero factorial is equal to one by definition.)

Since x(t) is the square wave version of the FM signal s(t), by taking the derivative of x(t) one gets nonuniform (in period) samples of δ-functions. Accordingly y(t)=x'(t) can locate the zero-crossings of x(t), and consequently it also designates the zero-crossings of the FM signal s(t). Figure 4.3 shows the function y(t) which is constructed from x(t), and it clearly marks the zero-crossings of x(t) and s(t).

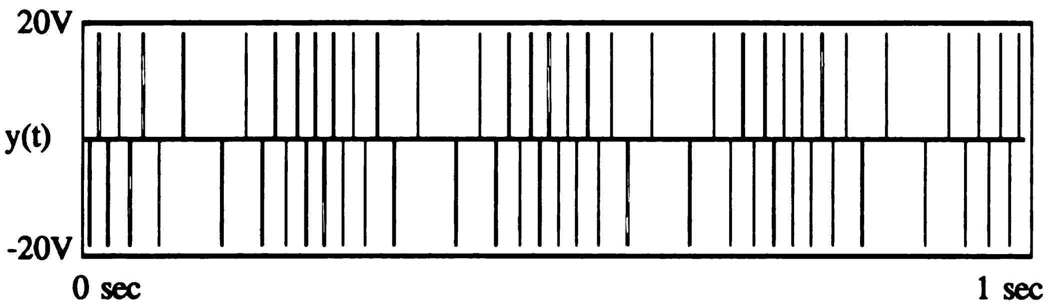


Figure 4.3 The function y(t) marks the bipolar zero-crossings of s(t)

By taking the derivative of x(t), one gets y(t) as follows:

$$
\begin{aligned}
y(t) = {} & \theta'(t)\cos\theta(t)\left[1+\sum_{k=1}^{N}\frac{(2k-1)o!}{(2k)e!}\cos^{2k}\theta(t)\right] \\
& -\theta'(t)\sin^{2}\theta(t)\left[\sum_{k=1}^{N}\frac{(2k-1)o!}{(2k-2)e!}\cos^{2k-1}\theta(t)\right]
\end{aligned}
\tag{4.8}
$$

After substituting $1-\cos^2\Theta(t)$ for $\sin^2\Theta(t)$, multiplying the cosine terms, regrouping the terms, and finally factoring $\Theta'(t)$, y(t) can be expressed in equation (4.9).

$$
\begin{aligned}
y(t) = {} & \theta'(t)\left[\cos\theta(t)+\sum_{k=1}^{N}\frac{(2k-1)o!}{(2k)e!}\cos^{2k+1}\theta(t)\right] \\
& -\theta'(t)\left[\sum_{k=1}^{N}\frac{(2k-1)o!}{(2k-2)e!}\cos^{2k-1}\theta(t)\right] \\
& +\theta'(t)\left[\sum_{k=1}^{N}\frac{(2k-1)o!}{(2k-2)e!}\cos^{2k+1}\theta(t)\right]
\end{aligned}
\tag{4.9}
$$

From the equation (4.9) it is obvious that each term is composed of an odd power of $\cos\Theta(t)$. The coefficient of each cosine term can be calculated separately. For instance the coefficient of $\cos\Theta$ itself is $(1-1)=0$; and the coefficient of any other arbitrary odd power of cosine is zero except the last term. Consider an arbitrary term, for example the rth term. In this case one must find C_r the coefficient of the term $\cos^{2r-1}\Theta$ when $k=r<N$. The coefficient C_r can be found from equation (4.9), and it is given in equation (4.10).

$$C_r = \frac{[2(r-1)-1]o!}{(2r-2)e!} - \frac{(2r-1)o!}{(2r-2)e!} + \frac{[2(r-1)-1]o!}{[2(r-1)-2]e!}$$
$$= \frac{(2r-3)o! - (2r-1)o! + [(2r-3)o!](2r-2)}{(2r-2)e!} \qquad (4.10)$$
$$= \frac{(2r-3)o![1-(2r-1)+2r-2]}{(2r-2)e!} = 0$$

Thus the coefficients of $\cos^{2k-1}\Theta(t)$ is zero for all $k<N$. Now consider the Nth term, the case for which $k=N$. In this case the power of cosine is $2k+1$, and it has only positive coefficients. Thus the coefficient of the last term C_N or the coefficient of $\cos^{2N+1}\Theta$ is given by equation (4.11)

$$C_N = \frac{(2N-1)o!}{(2N)e!} + \frac{(2N-1)o!}{(2N-2)e!} = \frac{(2N-1)o! + (2N-1)o!(2N)}{(2N)e!}$$
$$= \frac{(2N-1)o!(1+2N)}{(2N)e!} = \frac{(2N+1)o!}{(2N)e!} \qquad (4.11)$$

Therefore the coefficient of the last term survives. Thus

$$y(t) = x'(t) = \theta'(t)\frac{(2N+1)o!}{(2N)e!}\cos^{2N+1}\theta(t) \qquad (4.12)$$

One can notice that y(t) converges slowly to its ideal version, which is a sequence of δ-function non-uniformly spaced in time. The value of C_N shows that the convergence is slow. Nonetheless, theoretically (by applying the ratio test) the coefficient C_N of the cosine term in the above equation approaches infinity as N approaches infinity.

Since y(t) becomes very large as N becomes very large, the above function y(t) becomes a non-uniformly spaced distribution of δ-functions, and the locations of these δ-functions mark the zero-crossings of the FM signal s(t). The function y(t) is composed of positive and negative amplitudes (δ-functions). Furthermore, the positive and negative δ-functions alternate. Consequently, one can not extract the message signal m(t) from this signal by using a low-pass filter. This is due to characteristic of the low pass filter. Low-pass filtering in the frequency domain is equivalent to integration or averaging in the time domain. However, it is possible to take the absolute value of y(t) or the square of y(t) to mark the unipolar δ-functions at the zero-crossing of the FM signal s(t), and then low pass the resulting signal. Since squaring y(t) is easier to deal with mathematically, it is considered next. Accordingly squaring y(t) one gets z(t) as follows:

$$z(t) = y^2(t) = [\theta'(t)]^2 \frac{[(2N+1)o!]^2}{[(2N)e!]^2} \cos^{4N+2}\theta(t) \qquad (4.13a)$$

which after substituting the value of $\Theta'(t)$ becomes

$$z(t) = [w_c^2 + 2\beta w_c m(t) + \beta^2 m^2(t)] \frac{[(2N+1)o!]^2}{[(2N)e!]^2} \cos^{4N+2}\theta(t) \qquad (4.13b)$$

Figure 4.4 shows the function z(t) which is generated from y(t).

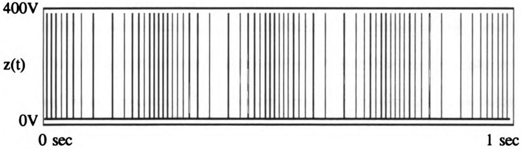


Figure 4.4 The function z(t) marks the zero-crossings of s(t) unipolarly

Since the power of cosine becomes even, the series has a dc value and the cosines are all of even powers. To find the exact value of the dc component, and the coefficients of different components involving m(t) and $m^2(t)$, the term $\cos^{4N+2}\Theta(t)$ is expanded as follows: [66]

$$\cos^{4N+2}\theta = \frac{1}{2^{4N+2}}\left\{\sum_{k=0}^{2N+1} 2\binom{4N+2}{k}\cos 2(2N+1+k)\theta + \binom{4N+2}{2N+1}\right\} \tag{4.14}$$

After utilizing equations (4.14), one can identifies a number of different signal sets in equation (4.13). The signals that one can identify are:

I. A large dc value

II. The message signal m(t).

III. DSB AM type signals, with the message signals m(t) and $m^2(t)$.

IV. FM signals with a carrier frequency of at least $2w_c$.

Using the above relations, one can extract the message by using a dc blocking capacitor and a low-pass filter. The recovered signal will be

$$2w_c\beta \frac{1}{2^{2(2N+1)}}\left(\frac{(2N+1)o!}{(2N)e!}\right)^2\binom{4N+2}{2N+1}m(t) \tag{4.15}$$

Thus the recovered message is an amplitude scaled version of m(t), and could be adjusted to an appropriate level electronically. Notice that one also gets a large dc component which involves w_c^2. However, the dc component could be blocked by a capacitor, or simply ignored in software applications. To compare the coefficients of different signals which are expressed in equation (4.13), in the following section different spectral components of z(t) are considered.

4.3 The Spectral Components of z(t):

The above signal z(t) which marks the zero-crossings of the FM signal has been considered to show mathematically the extraction of the message signal from a complicated FM modulated wave. However, as it was discussed in the simulation process one need not proceed as above. It is possible to obtain the zero-crossings by a simple comparison of two adjacent samples as described in the simulation process. In summary the above mathematical discussion showed that the spectrum of the message signal is located in the lower portion of the frequency band of the corresponding FM zero-crossing waveform, and as a result one can recover the message signal from its zero-crossings waveform by using a low-pass filter. Beside the message signal and the d.c. components which were mentioned one also gets a variety of FM signals with different carrier frequencies, and double sideband (DSB) AM types of waveforms. Using equation (4.14) one can determine the coefficients of each of the signal components. To illustrate this method of demodulation and compare the coefficients of different components, an approximation for $N=3$ will be presented next. In this approximation the least value of N is chosen to make the mathematical discussion simple.

4.3.1 A Three Term Approximation of x(t):

In this section an expansion of x(t) for $N=3$ is considered, and then the function y(t) and z(t) are obtained. In the next section different components of z(t) are discussed. Let $\Theta(t)=\Theta$ (for convenience) in equation 4.7, and expand it for $N=3$ as follows:

$$x(t) = \sin\theta\left(1+\frac{1}{2}\cos^2\theta+\frac{1\cdot 3}{2\cdot 4}\cos^4\theta+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos^6\theta\right) \tag{4.16}$$

Once again notice that x(t) is the square wave version of the FM signal. by taking the derivative of x(t) one can mark the zero-crossings of x(t), and therefore the zero-crossings of the corresponding signal s(t). Thus y=x'(t) becomes

$$y(t) = \theta'\cos\theta\left(1+\frac{1}{2}\cos^2\theta+\frac{3}{8}\cos^4\theta+\frac{15}{48}\cos^6\theta\right) \tag{4.17}$$

$$+\sin\theta\left[0+\cos\theta(-\sin\theta)\theta'+\frac{3}{2}\cos^3\theta(-\sin\theta)\theta'+\frac{15}{8}\cos^5\theta(-\sin\theta)\theta'\right]$$

factoring Θ', replacing $-\sin^2\Theta$ with $\cos^2\Theta - 1$, and multiplying the cosine terms one gets the following value for y(t)

$$y(t) = \theta'\left(\cos\theta+\frac{1}{2}\cos^3\theta+\frac{3}{8}\cos^5\theta+\frac{15}{48}\cos^7\theta\right)$$

$$-\theta'\left(-\cos\theta-\frac{3}{2}\cos^3\theta-\frac{15}{8}\cos^5\theta+\cos^3\theta+\frac{3}{2}\cos^5\theta+\frac{15}{8}\cos^7\theta\right)$$

$$=\theta'\left[0\cdot(\cos\theta+\cos^3\theta+\cos^5\theta)+\frac{105}{48}\cos^7\theta\right] \tag{4.18}$$

or simply

$$y(t) = x'(t) = \frac{105}{48}\theta'(t)\cos^7\theta(t) \tag{4.19}$$

squaring y(t) one gets z(t) as follows:

$$z(t) = y^2(t) = \left(\frac{105}{48}\right)^2[\theta'(t)]^2\cos^{14}\theta(t) \tag{4.20}$$

substituting the value of $\Theta'(t)$ and expanding $\cos^{14}\Theta$ in terms of multiple angles [66], one obtains:

$$z(t)=\left(\frac{105}{48}\right)^2(w_c^2+2\beta w_c m(t)+\beta^2 m^2(t))\frac{1}{2^{14}}\left[\binom{14}{7}+\sum_{k=0}^{6}2\binom{14}{k}\cos2(7-k)\theta\right] \quad (4.21)$$

The above signal z(t) is composed of a variety of different signals, such as a large d.c., the message signal,the square of the message signal, FM signals with high frequency carrier, and DSB AM type signal with the FM signals being its carrier. The spectrum of z(t), and the recovered version of m(t) from z(t) by low pass filtering are shown in Figures 4.5 and 4.6

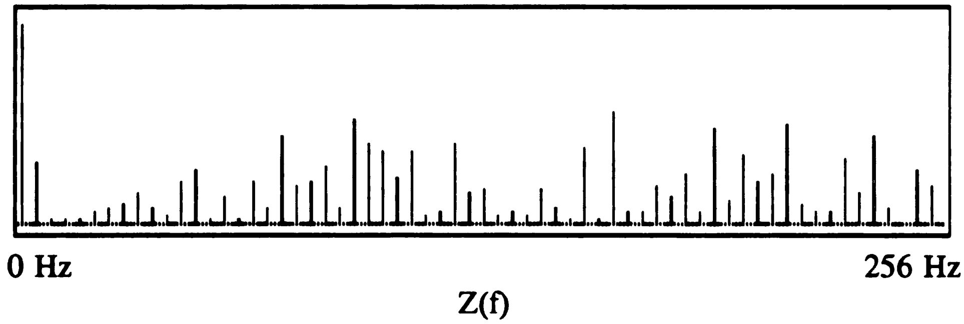


Figure 4.5 The amplitude spectrum of z(t)

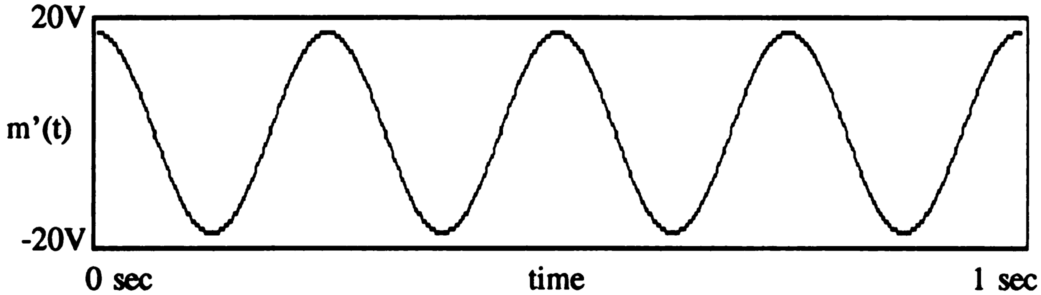


Figure 4.6 m'(t) the recovered version of m(t)

To comprehend this signal, an expression for each component has been calculated, and presented in the following section.

4.3.2 Decomposition of z(t) in its Components:

The signal z(t) is a very complicated signal. In this section a classification of different components of the function z(t) is presented. considering the expression for z(t) in equation (4.21), One can identify the following components from z(t).

1) a dc component with the following magnitude

$$w_c^2 \left(\frac{105}{48}\right)^2 \left(\frac{1}{2^{14}}\right) \binom{14}{7} \tag{4.22}$$

the above dc value is easily blocked by a capacitor in hardware implementation or simply ignored in software applications.

2) the detected message signal with amplitude scale as follows:

$$2\beta w_c \left(\frac{105}{48}\right)^2 \left(\frac{1}{2^{14}}\right) \binom{14}{7} m(t) \tag{4.23}$$

One can easily recover this signal by a low-pass filter, and then magnitude scale it in hardware or software. The filter must exclude the zero frequency in order to block the dc component. This scheme is also used in simulation study of chapter 3.

3) a term involving the square of the message signal given as follows:

$$\beta^2 \left(\frac{105}{48}\right)^2 \left(\frac{1}{2^{14}}\right) \binom{14}{7} m^2(t) \tag{4.24}$$

the above term which involve the square of the message signal can induce some interference in the band of the message signal, specially if the message signal is composed of frequencies with some harmonics and their multiples. Nonetheless, by comparing the coefficient of m(t) with that of $m^2(t)$ one can notice that the ratio of their coefficients is $2w_c/\beta$. Since this ratio is very large the interference introduced by $m^2(t)$ is negligible.

4) seven term FM signals, containing higher carrier frequencies given as follows:

$$2w_c^2\left(\frac{105}{48}\right)^2\left(\frac{1}{2^{14}}\right)\binom{14}{7}\sum_{k=0}^{6}\binom{14}{k}\cos[2(7-k)\theta] \tag{4.25}$$

The above signals are centered around $2w_c$ up-to $7w_c$. Therefore, all of the above signal will be in the upper portion of the spectrum. However if the value of β is large it could cause some interference very small in magnitude.

5) seven terms involving components, at multiple of w_c, multiplied by m(t). The mathematical expression for these terms are given as follows:

$$4\beta w_c\left(\frac{105}{48}\right)^2\left(\frac{1}{2^{14}}\right)\binom{14}{7}\, m(t)\sum_{k=0}^{6}\binom{14}{k}\cos[2(7-k)\theta] \tag{4.26}$$

The above equation represents a set of double sideband AM type of the message signal m(t). However, it is not an ordinary double sideband AM modulation, since the carrier portion consist of frequency modulated signal rather than $\cos w_c t$. The last components which are given by equation (4.27) are similar to the above scenario, except for the message signal which is replaced by $m^2(t)$.

The critical point in the above deliberation is that the spectrum of each of those signals is centered around $2w_c$ or higher. Consequently, they are not inducing any

$$2\beta^2\left(\frac{105}{48}\right)^2\left(\frac{1}{2^{14}}\right)\binom{14}{7}\; m(t)^2\sum_{k=0}^{6}\binom{14}{k}\cos[2(7-k)\theta] \qquad (4.27)$$

interference to the lower portion of the band which the message signal is located. In the case of $m^2(t)$ it has been noticed that the amplitude of the m(t) component is much higher than that of $m^2(t)$. Thus one can deduce that, the message signal which is modulated by an FM scheme can be recovered from the zero-crossings of the corresponding FM signal by low-pass filtering the zero-crossing signal z(t). The above argument is illustrated in Figure 4.7, where the multi-tone signal m'(t) is recovered from the zero-crossing function z(t) by low-pass filtering.

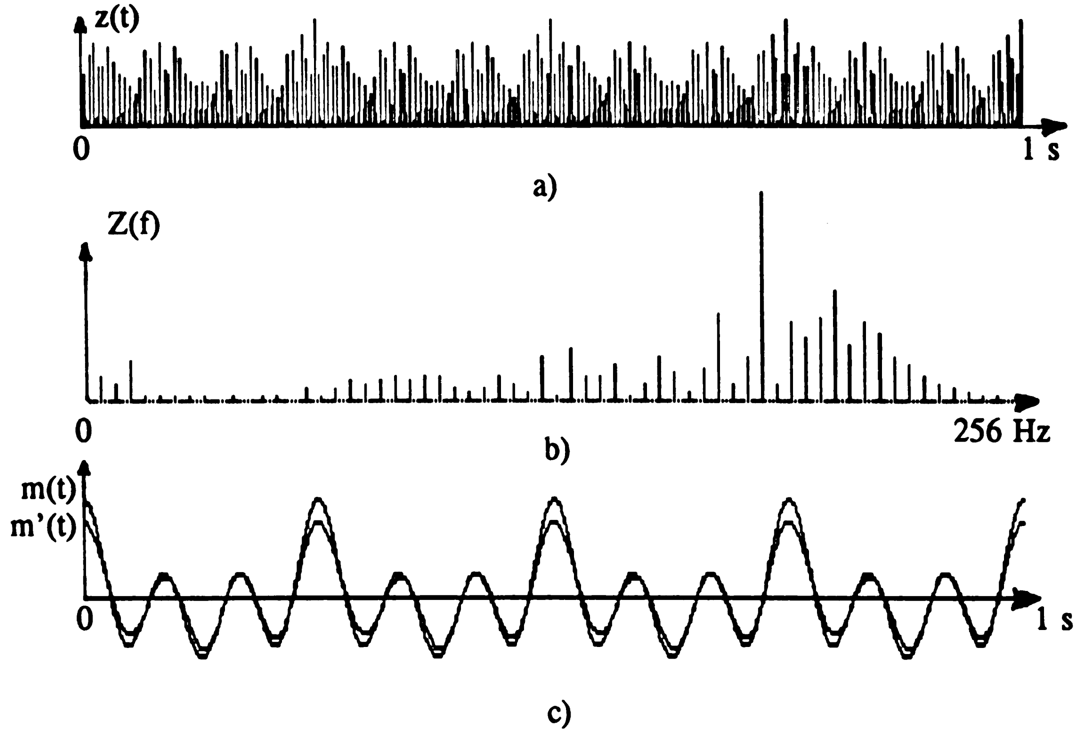


Figure 4.7 a) The function z(t)
b) Z(f), the amplitude spectrum of z(t)
c) The message signal m(t) and its recovered version

In the next section a modified version of z(t) is considered to show that the interference induced by $m^2(t)$ is caused by squaring the signal. Therefore, the interference expected from the spectral component involving $m^2(t)$ could be avoided.

4.4 A Practical Aspect of FM Zero-Crossings:

In the above discussion of zero-crossings, z(t) is considered to designates the unipolar zero-crossing instants of the function x(t). $z(t)=y^2(t)$ as given in equation (13). z(t) is mainly composed of multiplication of two functions, $[\Theta'(t)]^2$, and $\cos^{4N+2}\Theta(t)=[\cos^{2N+1}\Theta(t)]^2$. Since $\Theta'(t)=w_c+\beta m(t)$ and $w_c>>\beta m(t)$ for almost all practical purpose, thus, one can consider $z_1(t)$ the absolute value of y(t) as follows:

$$z_1(t)=[w_c+\beta m(t)]\frac{(2N+1)o!}{(2N)e!}|\cos^{2N+1}\theta(t)| \qquad (4.28)$$

As far as zero-crossings are concerned the function $|\cos^{2N+1}\Theta|$ and $\cos^{4N+2}\Theta$ mark the same unipolar zero-crossing instants, but the absolute value of the cosine function is not tabulated. Thus it was easy to consider the square of y(t) rather than its absolute value. Nevertheless, the absolute value of the cosine functions has dc value. Furthermore, the two approaches has the same zero-crossings, and the only difference is the amplitude of the corresponding zero-crossings. In this last approach the function $z_1(t)$ is marking the zero-crossing instants without amplitude distortion which is induced by the squaring process. This last observation shows that the FM zero-crossing signal $z_1(t)$ contains the following signals:

1) A large dc component which involves w_c

2) The message signal with a spectrum which is located in the lower portion of the

zero-crossing spectrum. Thus it could be recovered by a low-pass filter

3) A variety of FM signals, with amplitude w_c and their spectrum centered at integer multiples of w_c. Thus they could be filtered out by a low-pass filter.

4) A variety of AM type signals with FM signal serving as their carrier, thus these signals also have their spectrum centered at integer multiples of w_c. Thus it could also be filtered out by a low-pass filter as shown in Figure 4.8.

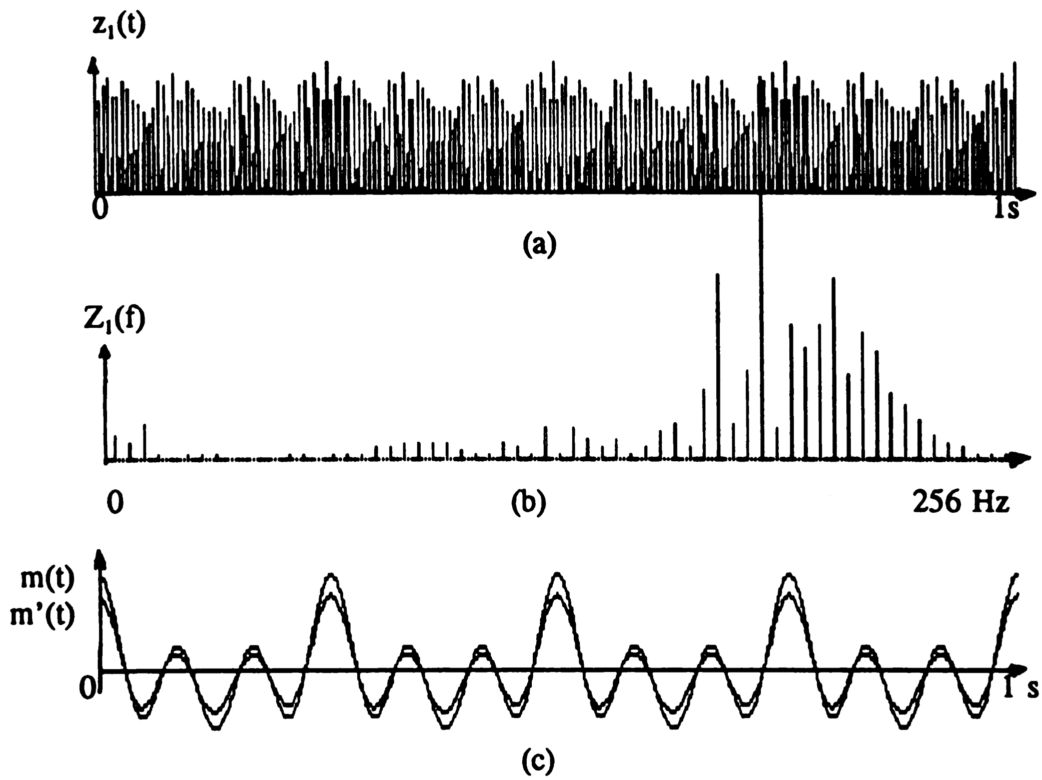


Figure 4.8 a) The signal $z_1(t)$
b) $Z_1(f)$, the amplitude spectrum of $z_1(t)$
c) The message signal m(t) and its recovered version

An important observation that one can make from the above consideration is that $z_1(t)$ guarantees that the lower portion of the spectrum does not have any spectral component involving $m^2(t)$. In conclusion one can recover the message signal m(t) by a low-pass filter from the zero-crossing of the FM signal which is modulated by the corresponding message signal m(t). Furthermore, there is no interference due to the square of the message signal $m^2(t)$.

4.5 Summary and Conclusion:

A mathematical study of FM zero-crossing has been considered in this chapter. A square wave type frequency modulation signal x(t) was desired from the corresponding FM signal s(t). It has been observed that the FM signal and its corresponding square wave signal have the same zero-crossings. Subsequently, a power series expansion of the x(t) was obtained. To mark the zero-crossings of the non-uniform square wave, its derivative y(t) was considered. The function y(t) marked the zero-crossings of the FM signal with either positive or negative large values (positive or negative δ-functions in the limit). Although y(t) marked the zero-crossing instants, the value of the function fluctuated between positive values and negative values. As a result one can not recover the message signal from y(t). Thus the function z(t), the square of y(t) was considered.

The function z(t) marked the zero-crossings of the FM signal in a unipolar manner. This function is composed of a variety of components. The spectrum of the message signal is located in the lower portion of the spectrum z(t). Therefore, it is possible to separate it from the rest of the spectrum. It has been shown that there would be a negligible interference due to the squaring process. Finally a practical aspect of the FM was considered and the function $z_1(t)$ has been constructed. This function showed that in practice the interference which was induced by the squaring process could be eliminated. In conclusion: It has been shown mathematically that the FM signal can be recovered using a low-pass filter from the spectrum of its zero-crossings.

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Interleaving Overlay:

The interference effects of spread spectrum modulation on the performance of conventional AM and FM system has been investigated. A mathematical description of all signals has been presented. A study of signal to noise ratio and signal to interference ratio for different signal sets has been considered. An expression for signal to nose ratio at the input and output of AM receiver has been obtained. It has been shown analytically that when the interference to noise ratio approaches zero, the signal to noise ratio approaches the signal to noise ratio in the absence of interference. It has also been shown analytically that the interference to noise ratio is a function of the bit duration τ [Appendix B].

In studying DS-FM system interference, the tolerance of an FM system over an AM system has been shown analytically, and an expression for signal to noise ratio has been obtained. It has been observed that the degradation of signal to noise ratio in this FM case would be larger than for AM, but due to the wide bandwidth of FM and interleaving overlay it could be tolerated. It was noticed that the average signal to noise ratio for FH-AM would be the same as the signal to noise ratio for DS-AM. However there is be an important difference in their instantaneous value, because in this case the instantaneous value would be deterministically non stationary, due to FH signalling pattern.

The problem of intersymbol interferences has been touched upon [Appendix E], and the solutions proposed as follows:

1) To reduce intersymbol interference, the data bit duration time should be an integer multiple of PN bits duration time.
2) One should use MSK signaling instead of non return to zero to prevent generation of impulse type interference.

In the final stage of the signal to noise ratio analysis it has been observed that direct sequence produces click type interference to FM system, which is induced by digital data and PN code. Since in frequency hopping signaling scheme, one can not combine the digital data and PN code, the click type interference induced by frequency hopping to FM system would be the worst type of interference.

Thus in the first portion of this study it has been shown that the worst type of interference is the interference of FH spread spectrum to FM system. Therefore this type of interference analysis was chosen for a simulation study.

5.2 Zero-crossing and Simulation:

The demodulation of an FM signal by zero crossing scheme has been considered in chapter 3. The proposed algorithm which exists in the literature has been examined. The detection of signals by frequency domain methods appear to show a better result. The frequency domain detection consists of an ideal low pass filter, followed by an inverse Fourier transformation (IFFT) of the filtered signal. Three new algorithms have been developed in chapter 3. The simulation procedure was applied to all four algorithms. The result of the simulation shows that these algorithms brings further improvement as it gets closer to the ideal zero-crossings.

The result of the simulation also showed that algorithms III and IV show the best promise. Since algorithm III is composed of a limiter, differentiator, and half wave rectifier, algorithm III is suitable for hardware applications. Similarly algorithm IV which is the ideal zero-crossing is suitable for software application. This algorithm is composed of a simple comparator, comparing of two adjacent samples. Thus it is simple to compare the samples which are stored in the memory or exist in a port by a software technique. Therefore algorithm IV would be used for software applications in years to come. The above observations have been confirmed by a study of performance measure such as SNR, SIR and SNIR. A new type of performance measure has been introduced. This new measure, which is called the instantaneous signal to noise and/or interference ratio, demonstrates the performance of the system point by point. The result of these performance measures showed that the distortions which were induced by frequency hopping is less than that of the noise for the same amplitude. The conclusion is that the zero-crossing method of demodulation will emerge as an alternative method of detection. Furthermore, it has been shown for the simple case simulated that FH as an interfering signal induces less distortion to the FM system compared with noise. Therefore frequency modulation system could coexist with the frequency hopping spread spectrum system. Finally in chapter 4 a mathematical analysis of the FM zero-crossings has been considered. It has been shown theoretically that the FM signal can be detected from its zero crossings by a low pass filter. This analysis also revealed that there is some spectrum overlap if the message frequency is composed of a frequency and its multiple. However, it was shown mathematically that the amplitude of that interference is very small.

5.3 Areas of Further Study and Recommendations:

The research which was reported in this study consists of two parts. The first part of this study was devoted to the problem of interleaving overlay, and the coexistence of different spread spectrum signals with AM and FM systems. Since this concept is considerably new in the field of communications, it raises a number of questions. Some of the questions are:

1. In this study only one interfering frequency hopping signal has been considered. What if one consider more than one, and possibly heavy loads (many users)?
2. There might be some other types of frequency sharing, and coexistence. A first step is to analyze and study the corresponding system similar to this study.
3. The analysis which is considered in this study was based on just two systems at one time. It would be desirable if a more general system with more variables variable could be considered for interference and overlay studies. For example a study of many different types of modulation occurring at the same time.
4. The FCC has expressed its concern about overlaid systems. The FCC has also expressed its concern about the use of spread spectrum in the public domain. As the modern technology advances, and expands, adequate research will be conducted in this area. The FCC will consider the results of these type of studies, and certainly field test the corresponding situation before issuing any

license.

The second portion of this research was the simulation of the FM modulation in the presence of frequency hopping interference and noise. The FM zero-crossing which was adapted for this study has existed in the literature and application. However, questions to be answered or explored are as follows:

1. The different algorithms that have been developed and examined by simulation, need to be verified experimentally as well. The experimental outcomes together with the simulation results will determine the adaption of a particular technique for a particular application.
2. In this study only one user of frequency hopping has been used. A study of several user and perhaps considerable load would be of interest.
3. The zero-crossing demodulation could be adapted for microprocessor based application, such as video cassette recorder and other FM detection systems which operate in the time (clock) range of a microprocessor. For high frequency applications one needs to translate the FM signal to the IF band by using a mixer and then detection could be performed by a zero-crossing algorithm.
4. The concerns and questions which were expressed regarding interleaving overlay in items 3 and 4 of the previous segment also holds for this segment.

5.4 Final Remarks:

The concept of interleaving overlay, which is a new idea in modern communication technology has been examined. For the purpose of this study there was no concern about industrial limitations such as cost, components, geophysical limitation, and finally FCC rules and regulations. Consequently the real credit will belong to those individuals who actually implement these ideas in the field. However, this study attempts to demonstrate that the idea has potential benefit.

APPENDICES

APPENDIX A

SIGNAL TO NOISE AND INTERFERENCE RATIO FOR AM

In this appendix an expression for signal to noise and interference ratio for an AM signal in the presence of direct sequence spread spectrum interference signal and WGN is obtained. Furthermore, an expression for interference to noise ratio is also derived. The study shows that the gain of the AM detector is not changed due to interference.

Consider an AM signal r(t) which is received at the input of the receiver as shown in figure A.1.

$$r(t) = s(t) + n(t) + i(t) \tag{A.1}$$

where s(t) is the transmitted AM signal, n(t) is the additive white Gaussian noise (WGN), and i(t) is the interference due to direct sequence spread spectrum signal.

The desired AM signal s(t) can be represented as follows:

$$s(t) = A_m(t)\cos(w_c t+\Theta) \tag{A.2}$$

Where A_m is the amplitude of the AM signal, w_c is the carrier frequency, and Θ is the phase shift of the AM signal s(t).

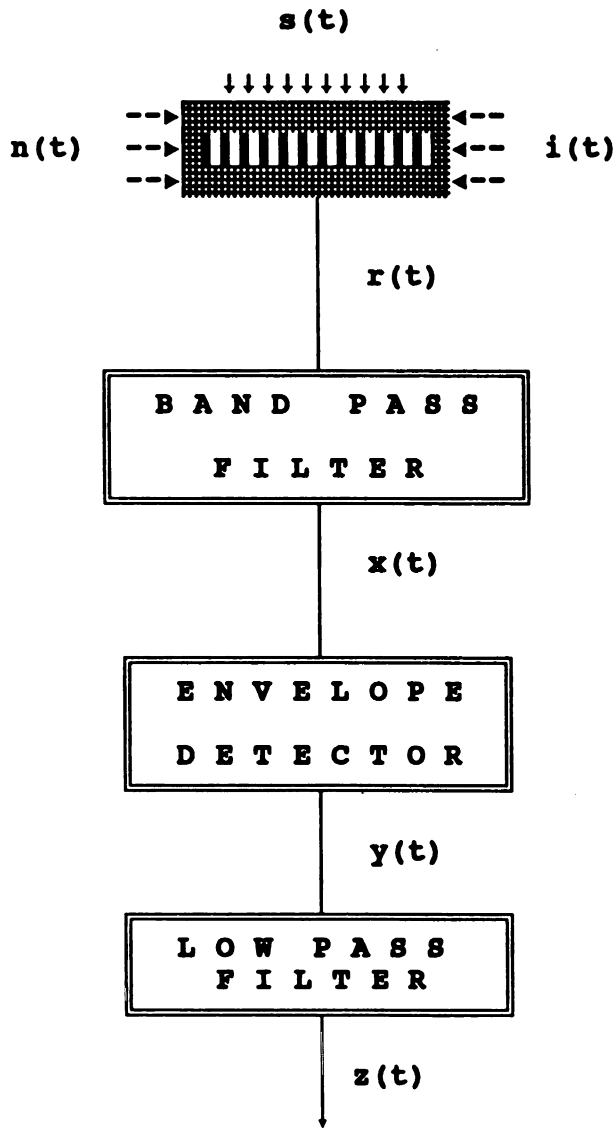


Figure A.1 AM receiver Model

The DS spread spectrum signal can be described in the following form: [4]

$$i_{DS}(t) = \sum_{j=1}^{u} I_j d_j(t)\, PN_j(t) \sin(w_j t+\theta_j) \tag{A.3}$$

where u is the number of users, I_j is the signal amplitude, w_j is the angular frequency and Θ_j is the phase angle for the jth user, $d_j(t)$ is the digital data (message) carried by the jth user and $PN_j(t)\epsilon\{1,-1\}$ is its pseudo-noise code sequence. Furthermore $d_j(t)\epsilon\{1,-1\}$ over the code (chip) duration T_c for all j, j=1,2,3,...u.

Assuming that the noise is additive white Gaussian noise with zero mean and two sided power spectral density of $N_o/2$, one gets the following expressions for noise at the output of band-pass filter [67].

$$\overline{n_c(t)} = \overline{n_s(t)} = 0 \tag{A.4a}$$

$$\overline{n_c^2(t)} = \overline{n_s^2(t)} = 2w_m N_o \tag{A.4b}$$

where $n_c(t)$ is the in-phase and $n_s(t)$ is quadrature component of the noise n(t) at the output of band-pass filter, and $2w_m$ is the bandwidth of the band-pass filter. On the other hand using statistics of the random data, one gets the following statistical properties for the interfering signal i(t) [4].

$$\overline{i_{c_j}(t)} = \overline{i_{s_j}(t)} = 0 \tag{A.5}$$

$$\overline{i_{c_j}^2(t)} = \overline{i_{s_j}^2(t)} = \int_{-\infty}^{\infty} S_j(f)\, |H_{BPF}|^2 df = 2 I w_m T_c \tag{A.6}$$

for all j, j=1,...,u

where $i_{cj}(t)$ and $i_{sj}(t)$ representing the in-phase and quadrature components of the interfering signal $i_j(t)$, $2w_m$ is the bandwidth of the band pass filter, and u is the number of spread spectrum users. In worst case that all users of direct sequence spread spectrum are transmitting in w_c, the following expression can be written for the input signal to noise plus interference ratio (SNR_I).

$$SNR_I = \frac{\frac{1}{2}\overline{A_m^2}}{\frac{1}{2}(\overline{n_c^2}+\overline{n_s^2})+\frac{1}{2}(\overline{i_{cj}^2}+\overline{i_{sj}^2})} = \frac{\frac{1}{2}\overline{A_m^2}}{2w_m N_o + 2w_m I T_c u} \tag{A.7}$$

or

$$SNR_I = \frac{\frac{1}{2}\overline{A_m^2}/(2w_m N_o)}{1+\frac{IT_c u}{N_o}} = \frac{SNR_I'}{1+\frac{IT_c u}{N_o}} = \frac{SNR_I'}{1+INR} \tag{A.8}$$

where SNR_I' is the SNR of the ordinary AM system in the absence of interference and INR is the interference to noise ratio thus $INR = IT_c u/N_o$.

Using the in-phase and quadrature components for noise and interference signals, the input to the envelope detector of Figure A.1, x(t) can be written as follows:

$$\begin{aligned} x(t) &= [A_m(t)+n_c(t)+\sum_{j=1}^{u} i_{c_j}(t)]\cos(\omega_c t+\theta) \\ &\quad -[n_s(t)+\sum_{j=1}^{u} i_{s_j}(t)]\sin(\omega t+\theta) \end{aligned} \tag{A.9}$$

The next stage of the AM demodulation is the envelope detector as shown in Figure A.1, it simply detects the envelope of the AM signal. If the SNR_I is high enough one can approximate y(t) as follows:

$$y(t) \approx A_m(t)+n_c(t)+\sum_{j=1}^{u} i_{c_j}(t) \tag{A.10}$$

y(t) will pass through a low pass filter. This filter further suppresses all high frequency components which are higher than the desired message bandwidth. Thus the output of this filter which is shown in Figure A.1 is simply given by the following expression.

$$z(t)=A_m(t)+n_c(t)+\sum_{j=1}^{u} i_{c_j}(t) \tag{A.11}$$

Using the above expression, the signal to noise ratio at the output of post detection (SNR_O) is

$$SNR_o=\frac{\overline{A_m^2}-\overline{A_m}^2}{(\overline{n_c^2}-\overline{n_c}^2)+\sum_{j=1}^{u}(\overline{i_{c_j}^2}-\overline{i_{c_j}}^2)}$$

which after a simple manipulation becomes

$$SNR_o=\frac{SNR_o'}{1+\frac{\sum_{j=1}^{u}(\overline{i_{c_j}^2}-\overline{i_{c_j}}^2)}{\overline{n_c^2}-\overline{n_c}^2}}=\frac{SNR_o'}{1+INR} \tag{A.12}$$

where SNR_O' is the signal to noise ratio at the output of the AM receiver in the absence of interferences, and INR is the interference to noise ratio as before. The detection gain is given by the following expression

$$G=\frac{SNR_O}{SNR_I}=\frac{\frac{SNR_O'I}{1+INR}}{\frac{SNR_I'}{1+INR}}=\frac{SNR_O'}{SNR_I'}=G' \tag{A.13}$$

where G' is the detection gain in the absence of interference. Thus G = G', for example the detection gain in the presence of interference is the same is the same as the detection gain in the absence of interference. Furthermore, from the above relationships the interference to noise ratio (INR) is defined as follows:

$$INR=\frac{IT_c u}{N_o} \quad \text{(A.14)}$$

APPENDIX B

INTERLEAVING OVERLAY

In this appendix a mathematical discussion of interleaving overlay is presented. The results exhibit that by using the spectrum of the spread spectrum signal one can design a system to reduce the interference induced by spread spectrum signals.

To show the interleaving overlay mathematically, consider a general waveform at the front of the receiver. This waveform includes amplitude and frequency modulations as its special cases. The waveform could be given by the following expression

$$s(t) = A(t)\cos[w_o t + \Theta_o(t)] \tag{B.1}$$

in the above expression when $\Theta(t) = \Theta_o = \text{constant}$, s(t) represents an AM signal, and when $A(t) = A = \text{constant}$ s(t) represents an FM signal. A general form of the spread spectrum interference signal could be written as follows

$$I(t) = \sum_{j=1}^{u} d_j(t)\cos[w_j(t)\,t + \theta_j] \tag{B.2a}$$

by using trigonometry it becomes

$$I(t) = B(t)\cos[w_j(t)\,t + \theta] \tag{B.2b}$$

The second line of the above equation lumps all interferences into a single complex interference.

Consider AM and FM demodulations. For AM coherent demodulation $\Theta(t)=\Theta_o$ a constant, then A(t) will be recovered by multiplying $x(t)=s(t)+I(t)$ by $2\cos(w_o t+\Theta_o)$ and then low-pass filtering the result. In this case the double frequency terms will be removed and the following expression will be obtained at the output of the LPF.

$$A(t)+I(t)\cos(w_2(t)+\Theta_2) \qquad \text{(B.3)}$$

where $w_2=w_I-w_o$ and $\Theta_2=\Theta_I-\Theta_o$. Now if $w_2=0$ then $w_I=w_o$ it becomes similar to jamming, however by using interleaving overlay this situation will be avoided, and in this case $|w_o-w_I|>B/2$, where B is the bandwidth of the filter. Therefore for a reliable communication one has to overlay the interfering spread spectrum signal in the guard band portions of the AM system, otherwise there will be distortion and even jamming.

To understand the system behavior in this situation, consider I(t) which is given by equation (B.2a). By considering only one of the interferences for example $i_j(t)=d_j(t)\cos(w_j(t)t+\Theta_j)$ {assuming that Θ_j is uniformly distributed in $[o,2\pi]$ and $Pr(d_j(t)=1)=Pr(d_j=-1)=1/2$ and the random variable $d_j(t)$ and Θ_j are independent events} the mean and variance of $i_j(t)$ can be evaluated as follows:

$$E[i_j(t)] = E[d_j(t)]E\{\cos[w_j(t)t+\Theta_j]\}=0 \qquad \text{(B.4)}$$

where E is the expectation operator. Since $E[d_j(t)]=0$ and $E\{\cos[w_j(t)t+\Theta_j]\}=0$, therefore $E[i_j(t)]=0$.

To find $E[i_j^2(t)]$, first find the autocorrelation function of $i_j(t)$

$$R_j(\tau) = E[i_j(t+\tau)i_j(t)]$$

$= E[d_j(t+\tau)\cos(w_jt+w_j\tau+\Theta_j)\ d_j(t)\cos(w_jt+\Theta_j)]$

$= E[d_j(t+\tau)d_j(t)]E[\cos(w_jt+w_j\tau+\Theta_j)\cos(w_jt+\Theta_j)]$

$= \frac{1}{2}R_{dj}(\tau)E[\cos w_j\tau+\cos(2w_jt+w_j\tau+2\Theta_j)]$

$$= \frac{1}{2}R_{dj}(\tau)\cos w_j\tau \qquad \text{(B.5)}$$

where $R_{dj}(\tau)$ is the autocorrelation of $d_j(t)$, if $d_j(t)$ represents a semirandom binary transmission $R_{dj}(\tau)=1$; if $d_j(t)$ represents a random binary transmission $R_{dj}(\tau)$ is given as follows [48].

$$R_{dj}(t) = \begin{cases} 1-\frac{|\tau|}{T} & \tau \leq T \\ 0 & \tau > T \end{cases} \qquad \text{(B.6)}$$

where T in this case is the bit duration time.

In any case the following expression is obtained

$$E[i_j^2(t)] = \frac{1}{2}R_{dj}(0)\cos 0 = \frac{1}{2}R_{dj}(0) \qquad \text{(B.7)}$$

In case of semirandom binary transmission one gets [48]

$$E[i_j^2(t)] = 1/2 \qquad \text{(B.8)}$$

and in the case of random binary transmission the result will not be changed, since $\tau=0<T$ in this application. Therefore the following expression for signal to interference ratio (SIR) is obtained.

$$SIR= \frac{\overline{A(t)^2}}{\overline{I^2(t)}} \qquad \text{(B.9)}$$

where

$$\overline{i^2(t)} = E[I^2(t)] = E[\sum_{j=1}^{u} i_j(t)^2] \quad \text{(B.10)}$$

Since each $i_j(t)$ is independent and is identically distributed one gets the following

$$I^2(t) = \sum_{j=1}^{u} R_{dj}(0) = \frac{u}{2} \quad \text{(B.11)}$$

and

$$SIR = \frac{\overline{2A^2(t)}}{u} \quad \text{(B.12)}$$

Therefore the reduction in SNR will be from 0 dB (overlaying) to 10log(u/2), in cases that each user transmits with different amplitudes for example A_j; j=1,...,u. Then the reduction is $10\log(\frac{1}{2}\Sigma A_j)$;j=1,2,3,...u. For example for 10 users with unit power the SNR will be reduced by 101og5
or about 7dB.

An important conclusion will be obtained by finding the spectral density of the interference. Since the autocorrelation functions are known from equation (B.5), and the spectral density is the Fourier transform of autocorrelation [48], the following important relationship is obtained

$$S_j(f) = \frac{1}{4}[S_{dj}(f-f_j)+S_{dj}(f+f_j)] \quad \text{(B.13)}$$

where $S_j(f)$ is the Fourier transform of the autocorrelation of the jth user $R_j(\tau)$, and $S_{dj}(f)$ is the Fourier transform of the autocorrelation of the digital data $d_j(\tau)$. From the above relationship one can clearly see the importance of overlay. For instance, one can overlay by choosing f_j's properly, or the relationship simply suggests that the system designer can assign the carrier frequency of the jth user f_j to the guardbands of the narrowband systems. In the case of a complicated form of interference, for example, the sum of several interfering signals, the following system of inequalities are necessary for reliable communication

$$|w_o - w_j| > \tfrac{1}{2}B \; ; \; j=1,\ldots,u \tag{B.14}$$

Now consider the FM case, in this case $A(t)=A$ (a constant). A block diagram of this system is given in Figure B.1. One can transform the output of the predetection filter $x(t)$ to a form suitable for FM detection. Using the in phase and quadrature for the signals, $x(t)$ can be written as follows

$$\begin{aligned} x(t) &= \{A+B(t)\cos[w_2 t+\Theta_2(t)]\}\cos[w_o t+\Theta_o(t)] \\ &\quad - \{B(t)\sin[w_2 t+\Theta_2(t)]\}\sin[w_o t+\Theta_o(t)] \end{aligned} \tag{B.15}$$

where $w_2 = w_I - w_o$ and $\Theta_2 = \Theta_I - \Theta_o$.

Using more trigonometry one can write

$$x(t) = R(t)\cos[w_o t+\Theta_o(t)+\Phi(t)] \tag{B.16}$$

where

$$R(t) = \{A^2+B^2(t)+2AB(t)\cos[w_2 t+\Theta_2(t)]\}^{1/2} \tag{B.17}$$

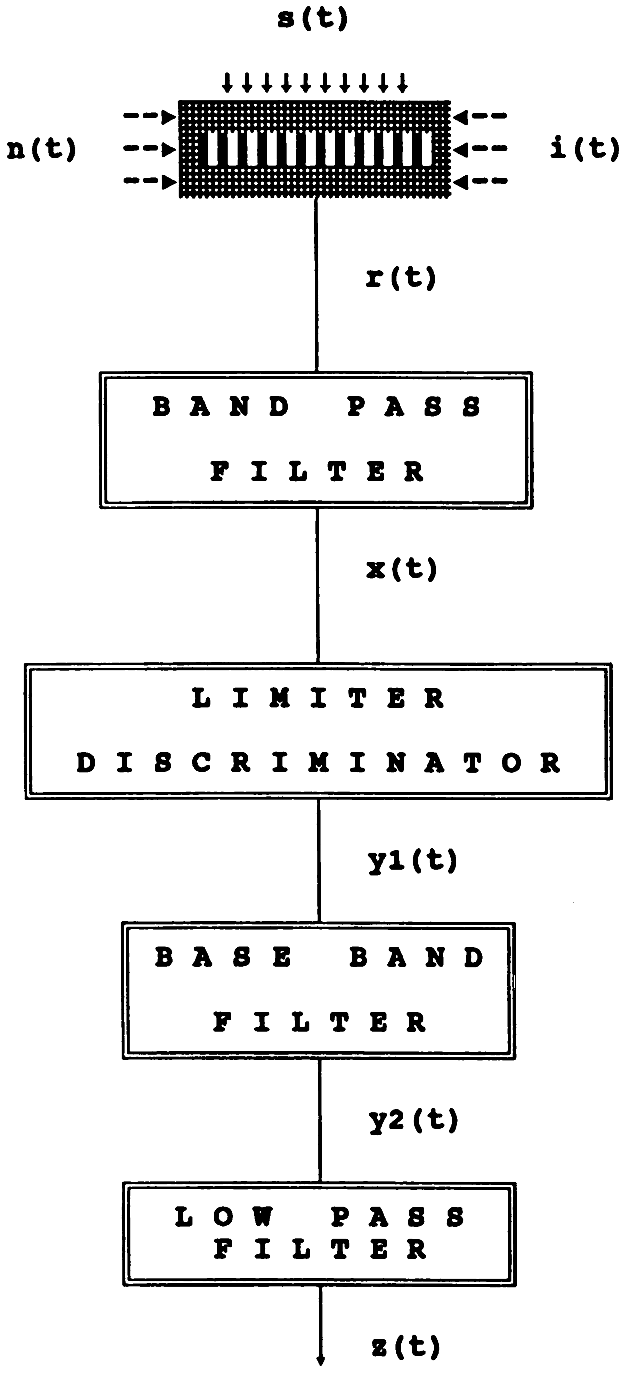


Figure B.1 FM receiver Model

and

$$\phi(t) = \tan^{-1}\frac{B(t)\sin[w_2 t+\theta_2(t)]}{A+B(t)\cos[w_2 t+\theta_2(t)]} \tag{B.18}$$

In an FM system the message is recovered from the derivative of the phase function, therefore the output y(t) is given by the following expressions

$$y(t) = \theta_0'(t) + \Phi'(t) \tag{B.19}$$

$$y(t) = \theta_0'(t) + \frac{(w_2+\theta_2')\,[AB(t)\cos(w_2 t+\theta_2(t)) + B^2(t)]}{A^2+B^2(t)+2AB(t)\cos[w_2(t)+\theta_2(t)]}$$

where f'(t)=df/dt. Expanding this expression in a Taylor series and assuming that A/2 > B(t) one gets the following two term approximation

$$\begin{aligned} y(t) &= \theta_0' + (W_2+\theta_2')\frac{B(t)}{A}\cos[w_2 t+\theta_2'(t)] \\ &- \frac{B(t)}{A}\cos[2w_2 t+2\theta_2(t)] \end{aligned} \tag{B.20}$$

From the above expression it follows that with increase of w_2 the interference term increases. For example the best performance will be achieved when one overlays the interfering signal in the top of FM signal ($w_2=0$, $w_1=w_0$), this situation shows the tolerance of the FM systems that the AM system is lacking. Now consider the terms $\Theta_2(t)$ and B(t) which involve discontinuous functions and their derivatives, for example data $d_j(t)$ and code $PN_j(t)$. On the one hand one should have $w_2=0$ in order to overlay on top of FM signal. On the other hand if w_2 increase the interfering term is also increase. To get out of this dilemma one should consider one more stage of FM receiver which is the post detection filter. In this case one will realize that if overlay has been utilized and w_2 is large enough, for instance $|w_1 - w_o| = w_2 = B'/2$ where B' is the bandwidth of the post detection filter, then in this case almost all of the interference will be filtered out and reliable communication is possible.

In the case that w_2 is not large enough, for instance $w_2 < B'/2$ but $2w_2 > B'/2$, the last term of equation (B.20) will be filtered, and one gets the following expression

$$y(t) \approx \theta_0' + (w_2 + \theta_2' [B(t)/A] \cos(w_2 t + \theta_2) \tag{B.21}$$

since $A > 2B$. Assuming Θ to be a constant the determining factor of interference would be w_2 itself. Therefore the reduction in SNR will be from 0 dB (w_2 is small) to approximately 10 logw_2 dB. For large value of w_2 it could be as large as 40 dB which could happen in wideband FM.

In summary if w_2 is small one can overlay over an FM signal, and the reduction in SNR would be small. For large values of w_2 SNR will be reduced further. If w_2 is large enough the filter will take care of the loss in SNR [68].

APPENDIX C

SIGNAL TO NOISE AND INTERFERENCE RATIO OF FM

In this appendix an expression for signal to noise and interference for FM is derived. The receiver in this case is an FM receiver, and is composed of a predetection band-pass filter, a limiter discriminator, and a post detection low pass filter. The band-pass and low pass filters have a bandwidth of $2w_m(1+\beta)$ and w_m respectively, where β is the modulation index of the FM signal. The discriminator output is proportional to the time derivative of the phase angle of the input to the discriminator. A block diagram of the FM receiver is given in Figure B.1.

Similar to the AM case the received signal is composed of three signals, the desired FM signal s(t), white Gaussian noise n(t) and a set of direct sequence spread spectrum signals i(t). The desired FM signal can be represented by

$$s(t) = A\cos[w_c t + \phi_m(t)] \tag{c.1}$$

where A, w_c and $\phi_m(t)$ are the amplitude, the carrier frequency, and the phase angle of the FM signal. For the FM modulation system $\phi_m(t)$ usually has the following form

$$\phi_m(t) = W_{max}\int_{-\infty}^{t} m(\tau)\, d\tau \tag{C.2}$$

where w_{max} is the maximum instantaneous frequency deviation, m(t) is the base band modulating waveform, or the message signal. The direct sequence signal is modeled in Appendix A. Assuming that s(t), n(t), and $i_j(t)$ are independent random variables, the input signal to noise ratio can be written as before (Appendix A equation (A.8)) as follows:

$$SNR_I = SNR_I' \frac{1}{1+\frac{\overline{i_{cj}^2}+\overline{i_{sj}^2}}{\overline{n_c^2}+\overline{n_s^2}}} \tag{C.3}$$

or

$$SNR_I = SNR_I' \frac{1}{1+INR} \tag{C.4}$$

where SNR'_I is the predetection SNR in the absence of interference, and INR is the interference to noise ratio as in the case of AM system. Thus

$$SNR_I \longrightarrow SNR'_I \text{ as } INR \longrightarrow 0 \tag{C.5}$$

All inferences that have been made for the AM case are also valid for FM predetection, thus that development is not repeated here. Instead, the discriminator is considered, which is quite different from the AM detection stage.

Consider the signal x(t), the output of the predetection in Figure B.1. Using the in phase and quadrature representation of each of the narrowband signals one can obtain the following expression at the output of predetection filter [38].

$$x(t) = [A+n_c(t)+\sum_{j+1}^{u} i_{cj}(t)]\cos[w_c t+\phi_m(t)]$$
$$-[n_s(t)+\sum_{j=1}^{u} i_{sj}(t)]\sin[w_c(t)+\phi_m(t)] \qquad \text{(C.6a)}$$
$$= R(t)\cos[w_c t+\phi_m(t)+\psi(t)]$$

with

$$R(t) = \sqrt{[A+n_c(t)+\sum_{j=1}^{u} i_{cj}(t)]^2+[n_s(t)+\sum_{j=1}^{u} i_{sj}(t)]^2} \qquad \text{(C.6b)}$$

and

$$\psi(t) = \arctan\frac{n_s(t)+\sum_{j=1}^{u} i_{s1}(t)}{A+n_c(t)+\sum_{j=1}^{u} i_{cj}(t)} \qquad \text{(C.6c)}$$

Using this representation the discriminator output can be expressed as follows:

$$y(t) = \frac{1}{2\pi}\frac{d}{dt}[\phi_m(t)+\psi(t)] \qquad \text{(C.7)}$$

The lowpass filter suppresses all components higher than the desired message bandwidth. Thus the output will be

$$Z(t) = Z_m(t) + Z_n(t) + Z_i(t) \qquad \text{(C.8)}$$

with $Z_m(t)$ is the desired message

$$Z_m(t) = \frac{1}{2\pi}\frac{d}{dt}\Phi_m(t) \tag{C.9}$$

or

$$Z_m(t) = \frac{d}{dt}\int_0^t m(\tau)\,d\tau = m(t) \tag{C.10}$$

where m(t) is the message, and $Z_n(t)$ and $Z_i(t)$ are the unavoidable contribution of noise and direct sequence signals. $Z_i(t)$ will contain a collection of δ-functions at points where $d_j(t)PN_j(t)$ changes its polarity, this is in the case of nonreturn to zero data format, this could be improved by using minimum shift keying or raised cosine signaling format [46].

Although one can improve the system performance in this case, however in the worst case two kinds of distortion will occur, one due to phase and another due to code and digital data. Thus in this type of interference two kinds of distortions will be noticed. Similar to AM case, the output signal to noise ratio is given by the following expression

$$SNR_O = \frac{\overline{z_m^2} - \overline{z_m}^2}{\overline{z_n^2} - \overline{z_n}^2 + \sum_{j=1}^{u}\left(\overline{z_{ij}^2} - \overline{z_{ij}}^2\right)}$$

$$SNR_O = SNR_O' \frac{1}{1 + \frac{\sum_{j=1}^{u} \overline{z_{ij}^2} - \overline{z_{ij}}^2}{\overline{z_n^2} - \overline{z_n}^2}} = SNR_O' \frac{1}{1 + INR} \tag{C.11}$$

where SNR_o' is the signal to noise ratio at the output of the receiver in the absence of interferences. The statistics of noise and interference at the output of discriminator is given as follows: [4] and [65]

$$\overline{Z_n^2} = \frac{2N_0 w_m^3}{3A^2} \quad ; \quad \overline{Z_i^2} = \frac{2IT_c w_m^3}{3A^2} \tag{C.12}$$

Using equations (C.3) and (C.11) the following value for the gain G and G' is obtained, where G' is the gain without interference.

$$G = \frac{SNR_O}{SNR_I} = \frac{\frac{SNR_O'}{1+INR}}{\frac{SNR_I'}{1+INR}} = \frac{SNR_O'}{SNR_I'} = G' \tag{C.13}$$

Thus similar to AM case, G = G' and $INR = IT_c u/N_o$.

Appendix D

STATISTICS OF THE INTERFERING SIGNALS

In this Appendix the mean and variance of the direct sequence signal and frequency hopping signals have been obtained. The derivations involves the in-phase and quadrature components of the pertinent signals.

Consider $i_c(t)$ and $i_s(t)$ the in-phase and the quadrature components in the two systems (direct sequence and frequency hopping). For simplicity consider only one user, for example the jth user; the generalization to multiple users is straightforward. Any interfering signal could be written in the following general form.

$$i_j(t)=\sqrt{2}a(t)\cos[w_jt+\theta(t)] \tag{D.1}$$

where for direct sequence a(t)=d(t)PN(t) and Θ is a random variable uniform in $[0,2\pi]$. Furthermore d(t) is random sequence of ±1 with E[d(t)]=0, where E denotes the expectation operator. Consider i(t) in terms of in-phase and quadrature components as follows:

$$i(t)=\sqrt{2}x(t)\cos w_jt-\sqrt{2}y(t)\sin w_jt \tag{D.2}$$

with

$$x(t) = d(t)PN(t)\cos\Theta \;\; ; \;\; y(t) = d(t)PN(t)\sin\Theta \tag{D.3}$$

In the above expression d(t) and Θ are stochastic processes and random variables respectively. Furthermore assume they are independent of each other, thus

$$E(x) = E[d(t)]\, E[PN(t)]\, E(\cos\Theta) = 0$$

$$E(y) = E[d(t)]\, E[PN(t)]\, E(\sin\Theta) = 0 \qquad (D.4)$$

since $E[d(t)] = 0$; therefore $E[i(t)] = 0$, and the variance of i(t) is

$$Var[i(t)] = E\{[i(t)-E(i(t))]^2\} \qquad (D.5)$$

$$= E[i^2(t)]-E[i(t)]^2$$

$$= E[i^2(t)]$$

thus

$$Var[i(t)] = E[2a^2(t)\cos^2(wt+\Theta)]$$

$$= E[2\bullet 1\bullet \cos^2(wt+\Theta)] \qquad (D.6)$$

since

$$a^2(t) = d^2(t)\cdot PN^2(t) = 1 \qquad (D.7)$$

Using uniform distribution for Θ one can write the

following expression

$$Var[i(t)] = \frac{2}{\pi}\int_0^{\pi} \cos^2(w_j t+\theta)\, d\theta = 1 \qquad (D.8)$$

Now consider the frequency hopping signal. In this case one can have $a(t)=d(t)$ and $w_j=w_o+h(t)\delta W$, where h(t) is the hopping function, and δw is the hopping increment. Since $E(d(t))=0$ the expectation of the interference signal is also zero or $E(i(t))=0$ and the variance becomes unity, as shown:

$$Var[i(t)] = E\{2a^2\bullet \cos^2[w_j t+\Theta]\} = 1 \qquad (D.9)$$

as expected by inspection.

APPENDIX E

SIGNALING SCHEMES FOR DS AND FH

In this appendix a study of the mathematical differences between the different types of signaling are considered. The discussion shows the difference between direct sequence and frequency hopping interference. As a result, one can compare the interference case among the four different types that have been proposed.

Consider DS and FH systems for a single user, the generalization to multi user is straight forward.

The Direct sequence signal can be written as follows:

$$i(t)_{DS} = \sqrt{2}d(t)PN(t)\cos(w_o t+\theta) \tag{1a}$$

or

$$i(t)_{DS} = \sqrt{2}\sin[w_{ot}+\theta+d(t)PN(t)\pi/2] \tag{E.1b}$$

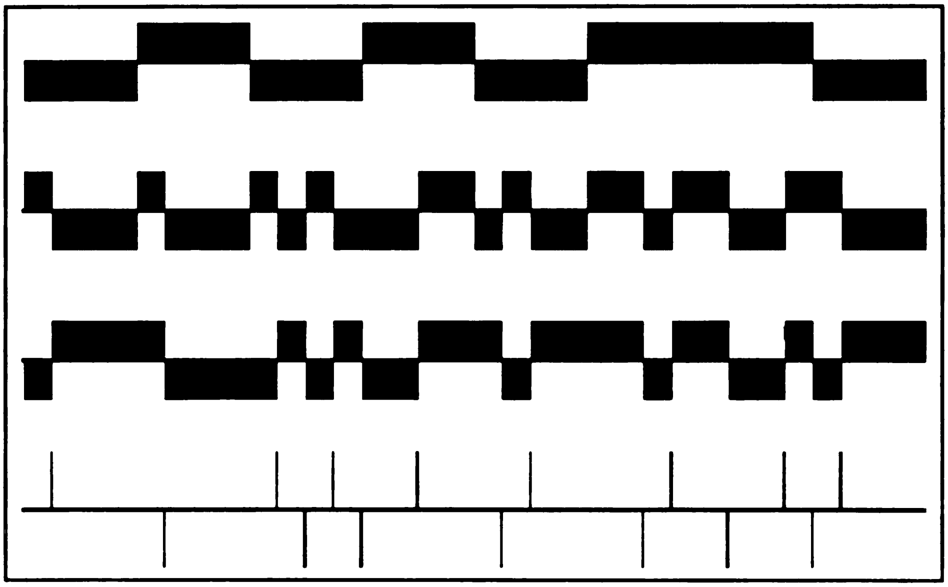
$$(k-1)\tau \leq t < k\tau$$

and for frequency hopping one can have

$$i(t)_{FH} = \sqrt{2}\sin[w_o t+h(t)\Delta wt+\theta+d(t)\pi/2] \tag{E.2}$$

$$(k-1)\tau \leq t < k\pi$$

In order to prevent intersymbol interference the period of the PN sequence should be some integer multiple of the data bit duration. Therefore d(t)PN(t) is a random sequence of $\{\pm 1\}$ rectangular waves and the derivative of these type of signals is not continuous as shown in figure E.1.



a) Digital data d(t) b) PN sequence PN(t)
c) d(t)PN(t) e) d{[D(t)PN(t)]}/dt

Figure E.1 Digital data D(t) and PN sequence PN(t)

As shown in figure E.1-d the derivatives are not defined when the random sequence changes its signs. Denoting these random points by an impulse function, one can write an expression for this sequence as follows:

$$\frac{d}{dt}[d(t)PN(t)] = \sum_{k=-\infty}^{\infty} a_k \delta(t-k) \tag{E.3}$$

where $a_k \in \{\pm 1\}$ if there is a sign change and zero otherwise.

In the multi-user case the situation becomes worse since one gets the sum of several sequences of delta functions. Mathematically one gets the following expression.

$$\frac{d}{dt}\sum_{j=1}^{u} d_j(t)PN_j(t)=\sum_{j=1}^{u}\sum_{k=-\infty}^{\infty} a_{jk}\delta(t-k) \qquad \text{(E.4)}$$

with $a_{jk} \in \{\pm 1\}$ if there is a sign change for jth user, and zero otherwise.

The difficulty is associated with the rectangular signaling of the channel bits. If one uses a minimum shift keying (MSK) signal for the channel bits, the discontinuity which was caused by rectangular signaling could be removed. An expression for MSK is as follows [46].

$$s(t)= \cos[w_0 t+b_k(t)\frac{\pi t}{2T}\psi_k] \qquad \text{(E.5)}$$

where $b_K(t) \in \{-1,1\}$ and $\phi_K \in [0,\pi]$.

This signalling scheme has many important properties [46] and [47]. The property which is significant in this study is the fact that there is phase continuity in the RF carrier at the bit transition instant. Thus one can prevent phase discontinuity which was caused by rectangular signaling. For instance the function is not only continuous, but its derivative is also continuous. Therefore this kind of signaling can suppress the delta functions, or at least it smoothers the process.

The above solution is also true for frequency hopping, but in this case an additional function h(t) has to be included. This is a discontinuous function; these discontinuities can not be removed. The derivative of this function is a sequence of delta functions which is deterministically periodic in nature. Thus for FM systems there are two kinds of interference due to frequency hopping. One due to random data d(t), and the other due to the hopping function h(t). This frequency hopping interference appears to have a larger impact than direct sequence interference. Consequently, one can conclude that for the cases considered, frequency hopping interference to FM presents the worst interference case. As a result the interference induced by FH spread spectrum to an FM signal becomes the subject of this study.

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