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TESTING OF ~~THE ONE-DIMENSIONAL~~ INFILTRATION
EQUATION ON SOME MICHIGAN SOILS

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TESTING OF THE ONE-DIMENSIONAL INFILTRATION EQUATION ON SOME MICHIGAN SOILS

By

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ABSTRACT

TESTING OF THE ONE-DIMENSIONAL INFILTRATION
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By

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The infiltration and movement of water into soil is of great importance and concern to mankind and particularly to agriculture. It is important from an economic point of view to maximize crop productivity resulting from rainfall or irrigation and to manage the associated processes of infiltration, evapotranspiration and drainage wisely. To assess these processes accurately, a discription of the physics involved is helpful and should be documented whenever possible.

In this investigation, several columns of Metea and Spinks sandy loam soils were wetted to preselected distances and/or preselected periods of time. The horizontal and vertical soil-moisture distribution profiles were evaluated for soil columns with slightly different bulk densities. Soil moisture characteristic curves were obtained from capillary rise and filter paper experiments run on both soils.

Solutions of the one-dimensional Richards' equation were obtained by a numerical method (FINDIT) using a finite-difference, iterative technique. The technique, contrary to some earlier solutions, is extremely accurate for both short and long periods of time. The infiltration time, the soil-moisture diffusivity $D(\Theta)$ and conductivity $K(\Theta)$ were required for solving the equation, $D(\Theta)$ being derived from the

horizontal wetting profiles and $K(\Theta)$ from the differences between the horizontal and vertical profiles.

The one-dimensional infiltration equation of Richards' was tested by comparing experimental infiltration profiles with calculated profiles. Generally, good agreement was obtained particularly when considering the variations in bulk density and temperature, experimental error, etc. in these experiments. Although satisfactory agreement was obtained for Metea and Spinks data sets, a second data set for Metea disagreed considerably with the experimental results.

To my parents

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LIST OF SYMBOLS

Symbols	Meaning
A	transmissivity (cm/min)
c	arbitrary constant
D	soil-moisture diffusivity (cm^2/min)
$D(\Theta)$	D as a function of Θ
g	acceleration due to gravity (cm/sec^2)
h	pressure head or suction (cm of water)
i	cumulative infiltration (cm)
i, j	summation indices
K	unsaturated hydraulic conductivity (cm/min)
$K(\Theta_s)$	saturated hydraulic conductivity (cm/min)
$K(\Theta)_i$	calculated conductivity for a specified moisture content (cm/min)
K_s	measured saturated conductivity (cm/min)
K_{sc}	calculated saturated conductivity (cm/min)
K_i	hydraulic conductivity at moisture content Θ_i (cm/min)
$K(\Theta)$	K as a function of Θ
K_n	hydraulic conductivity at initial moisture content
k	intrinsic permeability
M	matching factor
m	number of increments of Θ from dryness to saturation

Symbols	Meaning
n	number of pore classes up to water content of interest
q	flux density (cm^3/cm^2 min)
qx, qy, qz	flux densities in the x-, y- and z-directions (cm^3/cm^2 min)
R	largest pore size which remains full of water
r_1, r_2,---, r_n	radii of n equal classes of soil porosity
S	sorptivity ($cm/min^{1/2}$)
t	infiltration time (min)
t_o	total infiltration time (min)
$t^{1/2}$	square root of time
V_o, $\partial i/\partial t$	infiltration rate (cm^3/cm^2 min)
V_x, V_y, V_z	Darcy velocities in x-, y- and z-directions (cm^3/cm^2 min)
x	wetting distance in vertical direction (cm)
x'	wetting distance in horizontal direction (cm)
y	vertical component of flow attributed to gravity
Θ_o, Θ_s	volumetric moisture content at saturation (cm^3/cm^2)
Θ, Θ_v	volumetric moisture content (cm^3/cm^2)
Θ_m, Θ_g	gravimetric moisture content (cm^3/g)
Θ_n, Θ_i	initial moisture content (cm^3/cm^2)
$\Theta(x, t)$	Θ as a function of x and t
$\Delta H/L$	hydraulic gradient
∇H	hydraulic gradient in three-dimensional space

Symbols	Meaning
dH/dx	hydraulic gradient in one-dimensional space
$\partial h/\partial x$, $\partial h/\partial y$, $\partial h/\partial z$	hydraulic gradients in x-, y- and z-directions
$\partial \Theta/\partial t$	volumetric moisture content rate of change with time
$\partial K/\partial z$	hydraulic conductivity gradient
Ψ	suction head (cm) or capillary potential
$\nabla \Psi$	matric suction gradient in three-dimensional space
$\partial \Psi/\partial x$	matric suction gradient
$\frac{\partial \Theta}{\partial x}$'	volumetric moisture content gradient
∇	vector differential operator
$\nabla.$	divergence
λ	Boltzmann transformation ($cm/min^{1/2}$)
$\lambda(\Theta)$	λ as a function of Θ
χ, Ψ, ω,---, fm	parameters which are single-valued functions of Θ
ℓ	density of water (g/cc)
η	absolute water viscosity (poise = g/cm sec)
Φ	soil porosity
γ	surface tension (dynes/cm = g/sec^2)
$f(\rho)\delta r$	partial area occupied by pores of radii ρ to $\rho + \delta r$
$f(\sigma)\delta r$	Partial area of pores with radii σ to $\sigma + \delta r$
$\int$	integral sign (operator)

INTRODUCTION

The movement of water into and through soil is of great importance and concern to agriculture. Knowledge of the changes in soil water content due to the influence of rainfall or irrigation and the resulting infiltration, evapotranspiration and drainage is necessary for good land management. To predict water content changes accurately within the profile, mathematical equations describing these processes are helpful and should be used whenever possible. The complex nature of the porous media and the water held within the pores makes it difficult to specify directly the forces acting on that water. The description of soil-water movement depends not only on the forces residing in the soil but also on the amount of water present. These forces are related to the total water potential which in turn can be divided to its four components: (1) gravitational potential, which relates to position in the gravitational field with respect to an arbitrary reference elevation; (2) matric potential, which relates to adsorption forces between solid surfaces and the amount of water present, including the effect of cohesive forces between water molecules; (3) osmotic potential, which relates to forces of attraction between ions and water molecules; and (4) pneumatic potential, which relates to forces arising from unequal pressures in the gaseous phase.

Understanding the mechanism by which water moves under unsaturated conditions into and through soil is extremely

important for promoting good soil-plant relationships. The water movement through the porous media may also be considered as diffusion phenomena, and analysis achieved by applying diffusion theory. Diffusion may occur in both liquid and gaseous phases, the solid matrix determining the diffusion path length and the cross-sectional area available for diffusion. Diffusive flow of water under unsaturated conditions through porous media has been known and studied for a long period of time, however, relatively few experimental investigations for testing this theory have been published.

The inflow , storage and redistribution of moisture in the soil profile after an irrigation or rainfall event require knowledge of both soil wetting and drainage processes. Knowledge of the infiltration rate is necessary for good irrigation system design and maximization of the water absorption capacity of the soil. Such conservation of our water resource will increase in importance as our population increases.

Quantitative measurements of the rate at which a diffusion process occurs in soil are usually expressed in terms of both diffusivity and conductivity coefficients, both are applicable to soil water movement. Therefore, the measurement of these parameters is quite necessary if flow, distribution and storage of moisture within or drainage from the soil are to be rigorously analyzed.

The mathematical equation for describing water movement in this study was derived by Richards (1931). The equation is a combination of the equation of continuity and Darcy's equation utilizing gravitational and matric potentials as driving forces. Since this equation is a non-linear partial differential equation, it is not readily solvable; however, its solution was achieved by using the finite difference, iterative method (FINDIT). The procedure required the knowledge of the soil moisture characteristics, the wetting profiles of horizontal and vertical flow regimes, the wetting times and the initial and boundary conditions. A microcomputer was required to solve the equation by the necessary procedure.

The flow system considered in this study was semi-infinite with water applied at one end of a homogenous soil column. The semi-infinite condition required that water never reach the end of the column

I. Objectives

The objectives of this study were:

1). To examine how well an existing mathematical equation described the water movement under unsaturated conditions.

2). To collect infiltration data on two of Michigan soils, Metea (Arenic Hapludalfs; sandy over loamy, mixed, mesic) sandy loam and Spinks (Psammentic Hapludalfs; sandy, mixed, mesic) sandy loam and to calculate the respective soil moisture diffusivity and conductivity functions for wetting processes.

3). To verify the computations with other experimental evidence.

LITERATURE REVIEW

I. Theoretical Background

1.1. Water Movement under Unsaturated Conditions

Over a century ago, in 1856, a French scientist with the name of Henri Darcy paved the way to understanding fluid flow through porous media by introducing an equation which described water flow through saturated sand beds. This equation showed that the flux density of water flow through saturated sand beds is directly proportional to the hydraulic gradient. The equation can be presented as:

$$q = K\Delta H/L \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where q is the flux density, the volume of water flowing through a unit-cross-sectional area per unit time, K the hydraulic conductivity and $\Delta H/L$ the hydraulic gradient or driving force. The assumptions in the above equations are that the soil volume considered is sufficiently large relative to individual soil pores and microscopic heterogeneities permit the averaging of velocity and potential over the cross sectional area of the soil (Hillel, 1980).

Equation (1) does not totally satisfy our understanding of water flow through porous media, particularly when flow is unsteady. Hillel (1980) pointed out that when the flux changes with time or the media is non-uniform, the hydraulic head may not decrease linearly along the flow direction. The variation in the hydraulic head gradient and/or the conductivity forces investigators to use more exact and generalized expressions of Darcy's law. The expression must be in a differential form to allow for change in the gradient, flux, and conductivity values for localized regions that comprise the soil system. By considering Poiseuille's law, Slichter (1899) derived an equation in which Darcy's law is included in a general form for saturated porous media. The three-dimensional macroscopic differential equation is:

$$q = -K\nabla H \quad . . . . (2)$$

where ∇H is the gradient of the hydraulic head in three-dimensional space. The negative sign which proceeds the right-hand side of equation (2) is required because the sign of the driving force is negative resulting in a positive product. Equation (2) may be written in a one-dimensional form as:

$$q = -K\ dH/dx \quad . . . . (3)$$

where dH is the change in hydraulic head along a streamline segment dx. By considering $q=v$ (Kirkham and Powers, 1972) and v as a vector, having both magnitude and direction, equation (3) expanded into the y and z directions becomes:

$$V_x = -K\ \partial h/\partial x \quad . . . . (4a)$$

$$V_y = -K\ \partial h/\partial y \quad . . . . (4b)$$

$$V_z = -K\ \partial h/\partial z \quad . . . . (4c)$$

In the above three expressions, a partial derivative has been used rather than a total derivative to show that one variable may change independently of the other two.

By combining equation (3) with the equation of continuity, Slichter (1899) derived an equation for flow of water through saturated media. This equation is analogous to heat, electricity and diffusion flow equations known as Fourier's law, Ohm's law and Fick's law, respectively. It will be discussed at some length in connection with unsaturated flow.

1.2. Water Movement Under Unsaturated Conditions

Understanding water movement under saturated conditions is quite important in the area of irrigation, drainage and infiltration phenomenon. The simplest type of fluid flow exists when porous media is saturated, that is, all pores are filled with the same fluid, a condition which seldom if ever occurs in agricultural soil. For this reason fundamental and mathematical concepts of unsaturated flow as well as saturated flow must be considered as a continuum.

The infiltration of water into and moving through unsaturated porous media is quite complex and difficult to describe quantitatively. Water movement under unsaturated conditions is impeded not only by the fact that water is moving in partially filled pores, but is further impeded by entrapped soil air and gases. The relationship between soil moisture and soil moisture potential, a topic which will be discussed later,

may be further complicated by hysteresis (Hillel, 1980). Neither soil water conductivity nor water potential associated with unsaturated flow are easily measured in all ranges of interest (Baver, 1972). For these reasons and others, the formulation and solution of unsaturated flow problems very often require the use of indirect methods of analysis, based upon approximations or numerical techniques (Hillel, 1980). In subsequent sections the fundamental concepts and development of diffusion theory in unsaturated porous media are discussed.

1.2.1. The Development of Diffusion Theory for Unsaturated Flow Systems

By combining the equation of continuity with Darcy's law, Richards (1931) extended Slichter's equation to unsaturated flow. The equation of continuity is a statement of the principle of conservation of matter and may be written for unsaturted porous medium as:

$$\frac{\partial \Theta}{\partial t} = -\nabla .\ q \quad \ldots\ldots\ldots\ldots \quad (5)$$

where $\partial\Theta/\partial t$ is the time rate of change of the volumetric water content, Θ, ∇ is the vector differential operator, representing the three-dimensional gradient in space. The (∇.) product is mathematically called the divergence and designated div. Therefore, equation (5) also can be written as follows:

$$\frac{\partial \Theta}{\partial t} = -\mathrm{div}\ q \quad \ldots\ldots\ldots\ldots \quad (5a)$$

and

$$\frac{\partial\Theta}{\partial t} = -(\frac{\partial qx}{\partial x} + \frac{\partial qy}{\partial y} + \frac{\partial qz}{\partial z}) \quad . \; . \; . \; . \; . \; . \; . \quad (5b)$$

where qx, qy, qz are the fluxes in the x-, y- and z- directions, respectively. From Richard's assumption that Darcy law is valid for unsaturated flow, the hydraulic conductivity is now a function of the matric suction head or soil water potential, Ψ, [i.e., $K = K(\Psi)$] and is commonly called the unsaturated conductivity and in the older literature "the capillary conductivity" (Richards, 1952). Therefore, equation (2) becomes:

$$q = -K(\Psi)\nabla H \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

where ∇H is the hydraulic head gradient, which may include both suction and gravitational components for vertical flow. Also if the unsaturated conductivity is assumed to be a single-valued function of Θ [i.e., $K = K(\Theta)$] equation (6) becomes:

$$q = -K(\Theta)\nabla H \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6a)$$

as used by Nielsen and Biggar (1961). Substituting equation (6) in equation (5) yields:

$$\frac{\partial\Theta}{\partial t} = \nabla . \; \{K(\Psi)\nabla H\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

which is the general equation of water flow in unsaturated soil. Remembering that H, the hydraulic head, is the sum of the pressure head (or its negative, the suction head Ψ) and the gravitational head equation (7) becomes:

$$\frac{\partial \Theta}{\partial t} = -\nabla.\{K(\Psi)\ \nabla\ (\Psi - z)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

By considering ∇z as zero for horizontal flow and unity for vertical flow, equation (8) may be written as:

$$\frac{\partial \Theta}{\partial t} = -\nabla.\{K(\Psi)\ \nabla\ \Psi\} + \frac{\partial K}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

for vertical flow. If horizontal flow is to be considered only, the last term on the right-hand side of equation (9) is omitted giving:

$$\frac{\partial \Theta}{\partial t} = \nabla.\{K(\Psi)\ \nabla\ \Psi\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

or, in a one-dimensional horizontal system:

$$\frac{\partial \Theta}{\partial t} = \frac{\partial}{\partial x'}\ \{K(\Psi)\ \frac{\partial \Psi}{\partial x'}\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

Equations (10) and (11) are nonlinear partial differential equations. Their solutions, which will be discussed elsewhere in this study, depend upon the initial and boundary conditions. Thus, problems involving these equations are frequently called boundary value problems (Ashcroft and Hanks, 1980). Because these equations readily can be connected to diffusion type equations with a transmission coefficient $D(\Theta)$, their diffusion nature will become evident in the next section.

II. Soil Moisture Diffusivity and Conductivity Functions and their Measurements

The soil moisture diffusivity function, $D(\Theta)$, and conductivity functions, $K(\Theta)$, must be measured or known to determine the ability of a soil to transmit water. Modeling soil water flow consisting of infiltration, drainage and redistribution, require knowledge of these coefficients. However, their measurement is complicated by the fact that they are not only a function of soil moisture content, but are also dependent on its moisture history. Therefore, it is quite possible to have the same moisture content under a wetting and drying condition, but yet have different diffusivities and conductivities. This phenomenon is known as hysteresis.

Bearing in mind that hysteresis is most evident in the water content-pressure relationships of wetting and drying processes, Childs and Collis-George (1948) introduced the following equations:

$$K(\Theta)\ \frac{\partial\Psi(\Theta)}{\partial x'} = K(\Theta)\frac{\partial\Psi(\Theta)}{\partial\Theta}\ \frac{\partial\Theta}{\partial x'} = D(\Theta)\ \frac{\partial\Theta}{\partial x'} \quad . \ . \ . \ . \quad (12)$$

Here the matric suction gradient $\partial\Psi/\partial x'$ is expanded by using the chain rule, under the assumption that there is a unique relationship between Ψ and Θ. In this case, the water content gradient becomes the driving force instead of the hydraulic potential gradient. Combining equations (11) and (12) gives:

$$\frac{\partial \Theta}{\partial t} = \frac{\partial}{\partial x'} \{D(\Theta) \frac{\partial \Theta}{\partial x'}\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

The reciprocal of the term $\partial \Psi / \partial \Theta$ is called the "specific moisture capacity" and is analogous to the specific heat in the theory of heat flow. It is also the first derivative of the soil-moisture characteristic curve at any particular value of Θ (Klute, 1952). Similarly, combining equations (9) and (12) results in:

$$\frac{\partial \Theta}{\partial t} = \frac{\partial}{\partial x} \{D(\Theta) \frac{\partial \Theta}{\partial x}\} - \frac{\partial k}{\partial x} \quad . \; . \; . \; . \; . \; . \; . \quad (14)$$

used for describing vertical flow upward by capillarity from a water table or downward infiltration through the soil surface.

2.1. Methods for Determining Soil Moisture Diffusivity and Conductivity

Determination of $D(\Theta)$ has been underway for a long period of time. Darcy's law has long been the basis for measuring saturated hydraulic conductivity, $K(\Theta_S)$, in soils. The constant and falling-head permeameters are the most common methods of measuring $K(\Theta_S)$ in the laboratory. Klute (1965) has given details of the constant head method, making it quite easy and straight forward for anyone who is interested in measuring the saturated hydraulic conductivity of soil. However, the measurements of $D(\Theta)$ and $K(\Theta)$ under unsaturated conditions are more difficult to achieve and can be grouped into three basic classes: (1) steady-state, (2) transient and (3) pore-size analysis.

2.1.1. Steady-state Method

This method is restricted entirely to the measurement of the hydraulic conductivity under unsaturated conditions which mandates a constancy in water content, tension (matric potential) and flux with time. Thus if water flow in the vertical direction into soil has reached equilibrium the value of the unsaturated conductivity is numerically equal to the flux density of water application; i.e. equation (1) becomes $q = K$ (Ashcroft and Hanks, 1980). Richards (1931) developed an apparatus used for such steady-state measurement of unsaturated flow. Later, Richards and Moore (1952) refined and improved Richard's method. Elrick and Bowman (1964) used this improved method for measuring unsaturated conductivities in steady-state analysis. Similarly, Nielsen and Biggar (1961) constructed a simple apparatus from stock materials to accomplish the same objectives. Klute (1965) has given more details on many aspects of this method in part one of the Methods of Soil Analysis.

By combining $K(\Theta)$ with data from either the desorption or absorption moisture characteristics as depicted in equation (12), values of $D(\Theta)$ may be calculated. Attention must be given to the wetting history of the soil because values of $D(\Theta)$ will change if calculations are made using the wetting characteristics as opposed to the drying characteristic.

2.1.2. Transient Methods

Because water content and hydraulic potential remain constant at each point in the flow system, steady-state flow rates do not change with time. In unsteady-state (transient) flow, these conditions do not exist (Ashcroft and Hanks, 1980) and the flux density and the volumetric water content change with distance and time, respectively. Due not only to its complexity but also because of greater similarity to actual field conditions, transient state flow has been under intensive investigation and development by many investigators. Several techniques have been proposed for determining soil moisture diffusivity in wetting and drying of soils. Diffusivity in wetting of a soil column may be determined by measuring the moisture contents along the flow axis at a given time. On the other hand, diffusivity of drying soil requires the determination of outflow data with time under a specified suction or pressure change.

Because $D(\Theta)$ derived from outflow processes is not sufficiently accurate for inflow processes such as infiltration, $D(\Theta)$ from such techniques will be recognized but not discussed in any details here. Gardner (1956) was the first to measure $D(\Theta)$ and $K(\Theta)$ using outflow of a pressure membrane device. By taking impedance of the porous plate into account, which was a serious limitation of Gardner's method, Miller and Elrick (1958) calculated conductivities up to 3.5 times larger than Gardner's values. Further modifications of the outflow method were made by Rijtema (1959) and by Kunze and Kirkham (1962). Elrick (1963) tested the outflow method and found that the experimental outflow did not agree with the theory close to saturation. Deoring (1965) measured soil moisture diffusivities of 5 different soils by using a number of methods, including the one-step method proposed by Gardner (1962). In comparison to other methods, he found the one-step method to be reasonably accurate. Jackson et al. (1965) calculated the unsaturated hydraulic conductivity from outflow data and compared the results with the methods of Childs and Collis-George (1950), Marshall (1958) and Millington and Quirk (1960). Their conclusion and those of Bruce and Klute (1963) suggested that the experimental results did not agree with that predicted by the theory. Rawlins and Gardner (1963) presented data which showed that D is not a unique function of Θ. Failure of uniqueness implies that either the potential function, $\Psi(\Theta)$ or the hydraulic conductivity functions, $K(\Theta)$, or both are not unique functions of Θ. The magnitude of these errors must be determined before the diffusion equation can be used reliably in modeling of soil water movements.

Bruce and Klute (1956) introduced the advance of a wetting front method and calculated values of diffusivity by applying their data to equation (13). Since equation (13) is a non-linear partial differential equation, Bruce and Klute used the Boltzman transformation $\lambda = xt^{1/2}$ to transform equation (13) to an ordinary differential equation. This transformation assumes that the moisture content (Θ) is a function of a variable (λ) dependent on distance (x) and ($t^{1/2}$). This transformation allows one to calculate the diffusivity function, $D(\Theta)$, directly from the moisture content distribution curve. The initial and boundary conditions for equations (13) and (14) considered for water infiltrating into a semi-infinite, homogenous soil with uniform initial moisture content are:

$$\Theta(x, t) = \Theta_n \text{ for } x' \text{ and } x > o \quad t = o \quad . \; . \; . \; . \; . \; (14a)$$

$$\Theta(x, t) = \Theta_o \text{ for } x' \text{ and } x = o \quad t > o \quad . \; . \; . \; . \; . \; (14b)$$

$$\Theta(x, t) = \Theta_n \text{ for } x' \text{ and } x \to \infty \quad t > o \quad . \; . \; . \; . \; . \; (14c)$$

where Θ_n is the initial moisture content, Θ_o is the moisture content at saturation, x' and x are the horizontal and vertical distances, respectively, and t is the time. By substituting the variable λ in equation (13), integrating with respect to λ and solving for $D(\Theta)_{x'}$, equation 13 yields:

$$D(\Theta)_{x'} = -\frac{1}{2} \left(\frac{d\lambda}{d\Theta}\right)_{\Theta x'} \int_{\Theta_i}^{\Theta_{x'}} \lambda \, d\Theta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

or in terms of x' and t at constant t:

$$D(\Theta)_{x'} = -\frac{1}{2t} \left(\frac{dx'}{d\Theta}\right)_{\Theta x'} \int_{\Theta_i}^{\Theta_{x'}} x' \, d\Theta \quad . \; . \; . \; . \; . \; . \; . \quad (16)$$

which can be evaluated in terms of $D(\Theta)x'$ from the plot of Θ vs λ or Θ vs. x', respectively.

Bruce and Klute point out that if soil in a column is not homogeneous, variations of Θ, $K(\Theta)$ and $D(\Theta)$ will exist along the column. They indicated that $D(\Theta)$ increases with moisture content with a maximum value near saturation. Childs and Collis-George (1950) also observed this phenomenon when they calculated the diffusivity from soil moisture absorption characteristics.

Since there is no standard against which the diffusivity values can be checked, Bruce and Klute concluded that the only way $D(\Theta)$ can be checked for correctness is to use the function in calculating the soil moisture profile following varying periods of infiltration. These calculated profiles can then be compared with experimentally observed distributions for the same materials.

Even though this method requires a column of homogenous soil with uniform moisture content, it is much simpler to obtain a series of diffusivity values with the Bruce and Klute method than with other methods. It appears to be quite sensitive to temperature change. Stockinger et al. (1965) observed that the temperature effect on the advance of a wetting front is quite pronounced. They concluded that the variable $\lambda = (xt^{-1/2})$ is temperature dependent. Jackson (1963) measured the soil-moisture diffusivity for three different soils at five different temperatures. He found that the surface tension to viscosity ratio is the dominant factor influencing the temperature dependence of soil moisture diffusivity. Nielsen et al. (1962) have suggested that heat evolvement from wetting soil could account for non-isothermal conditions and consequential failure between experiment and theory. Nielsen and Biggar rigorously examined the conditions: (1) that Darcy's law is valid for unsaturated flow and (2) the $\lambda(\Theta) = xt^{-1/2}$ relationship holds when using oil and water for wetting soil columns horizontally at different negative pressures. Their results show deviations for a linear relationship between x and $t^{1/2}$ for water entry pressures more negative than, but not at -2 mb. They concluded that the values of diffusivity calculated depend upon the boundary condition at which the water enters the column and, therefore, these same D values would not be appropriate for the solution of the diffusion equation for other boundary conditions. In his criticism of Nielsen et al. (1962) Peck (1964) states that Nielsen et al. used initially air-free water but which subsequently became air-saturated during later portions of the experiment, confounding the results. It has been established by (Christiansen, 1944) that the variation in the concentration of air dissolved in the water can alter the hydraulic conductivity of a porous material considerably.

2.1.3. Pore-size Analysis Methods

Childs and Collis-George (1950), Marshall (1958), and Millington and Quirk (1959) have calculated the unsaturated conductivity from capillary-tube analogy and the distribution of soil pore sizes. These methods are based on the assumptions that a soil contains distinct pores of various radii which are randomly distributed in soil, and that when adjacent planes or sections of the soil are brought into contact with water the overall hydraulic conductance across the plane depends statistically upon the number of pairs of superposed pores, and geometrically upon their configurations.

Childs and Collis-George (1950) used an equation for intrinsic permeability, k, given as:

$$k = M \sum_{\rho=o}^{\rho=R} \sum_{\rho=o}^{\rho=R} \sigma^2 f(\rho) \delta r f(\sigma) \delta r \quad . \; . \; . \; . \; . \; . \; . \qquad (17)$$

where M is a matching factor obtained by matching calculated and experimental values of the permeability at a certain point, $f(\rho)\delta r$ is the cross-sectional area corresponding to the range of pores ρ to $\rho + \delta r$, $f(\sigma)\delta r$ is the area corresponding to the range of pores σ to $\sigma + \delta r$, and R is the largest pore size of interest that remains full of water.

Marshall's (1958) equation, adapted from Childs and Collis-George (1950) can be written as:

$$k = \frac{\Phi^2 n^{-2}}{8} (r_1^2 + 3r_2^2 + 5r_3^2 + \text{---} + (2_{n-1})r_n^2) \quad . \; . \; . \; . \quad (18)$$

where r_1, r_2,---,r_n are radii of n equal classes of soil porosity, Φ. From capillary rise considerations the equation $r = (\frac{2\gamma}{\ell g h}$, is substituted in terms of h into equation (18) giving:

$$k = (\frac{2\gamma}{\ell g})^2 \frac{\Phi^2 n-2}{8} (h_1^{-2} + 3h_2^{-2} + 5h_3^{-2} + \text{---} + (2_{n-1})h_n^{-2} \qquad (19)$$

where γ is the surface tension; h, negative head; ℓ, density of water; g, gravitational acceleration constant and r_1 corresponds to h_1, $h_1 < h_2$, etc. Equation (19) gives intrinsic permeability (cm^2) for various suction values. Unsaturated hydraulic conductivity is obtained from $K(\Theta) = \ell g k/\eta$, where η is fluid vescosity. By substituting $K(\Theta)$ in equation (19) one obtains:

$$K(\Theta) = (\frac{\ell g}{\eta})(\frac{2\gamma}{\ell g})^2 \frac{\Phi^2 n^{-2}}{8} (h_1^2 + 3h_2^2 + 5h_3^2 + \text{---} + (2_{n-1})h_n^2 \quad (20)$$

The accuracy of this method depends upon the accuracy of measurement of pore-size distributions and will be affected by swelling materials as Marshall (1958) points out. Millington and Quirk (1960) replaced Φ^2 in Marshall's equation (20) by $\Phi^{4/3}$ and n became the total number of porosity classes. The calculation, based upon Poiseuille's law, does not require a matching factor; however, from a practical point of view a matching factor is needed to adjust the computed and measured conditivities at saturation (Hillel, 1980). Most of the investigations used the ratio of observed permeability to calculated permeability at saturation as the matching factor. Kunze et al. (1968) further simplified equation (20) by using Φ instead of Φ^2 or $\Phi^{4/3}$ as water filled porosity and not the total porosity. Their equation can be written as:

$$K(\Theta) = \frac{K_s}{K_{sc}} \quad \frac{30h^{-2}\gamma^{-2}\Phi}{\ell gh} \sum_{j=i}^{n} (2_{j+1}-2_i)\frac{1}{h_j^2} \quad . \; . \; . \; . \; . \; . \quad (21)$$

where i=1, 2, ----n., $K(\Theta)$; is calculated conductivity for a specified moisture content, (K_s/K_{sc}) is the matching factor which is the ratio of measured saturated conductivity to calculated saturated conductivity. The other terms have been defined. They observed good agreement between experimental and calculated conductivities especially at lower moisture contents. Jackson (1972) simplified the formula to:

$$K_i = K_s \left(\frac{\Theta_i}{\Theta_s}\right)^c \sum_{j=i}^{m} \left[(2_{j+1}-2_i)\Psi_j^{-2}\right] \Big/ \left[\sum_{j=1}^{m} (2_{j-1})\Psi_j^{-2}\right] \qquad (22)$$

where K_i is the hydraulic conductivity at moisture content Θ_i, m is the number of increments of Θ from dryness to saturation, j and i are summation indices, and c is an arbitrary constant.

Since this method is based upon the capillary hypothesis, it can be expected to apply more to coarse-grained than to fine-grained soils, as the latter might exhibit phenomena such as film flow and ionic effects unaccounted for in the simple theory. Another complication arises where the soil is strongly aggregated and two types of flow occur within and between aggregates (Hillel, 1980). Finally, for the $K(\Theta)$ values to be applicable to infiltration theory, these equations must be applied to adsorption as opposed to desorption moisture characteristics.

III. The Development of Infiltration Theory and its Solutions

3.1. Theoretical Development

Infiltration is the entry of water into the soil surface and a consequent wetting of the soil. Physically, it is a common phenomena encountered in agriculture and hydrology. Experimental and theoretical work directed toward achieving a satisfactory understanding of water movement through soils has been carried on for more than a half century; however, it was only recently that a well known series of papers on infiltration theory development and formulation was published by Philip (1955, 1957a, 1957b, 1957c, 1957d, 1957e, 1957f, 1958).

In his first two papers (1955, 1957a), Philip introduced a numerical solution of diffusion type equations with diffusivity concentration-dependent. Horizontal and vertical infiltration satisfying initial and boundary conditions are considered in these papers. Given the assumption that the diffusivity versus moisture content relationship is known, Philip (1955) solved equation (13) for horizontal flow subjected to (14a), (14b) and (14c). Philip used the Boltzman transformation $\lambda(\Theta) = xt^{-1/2}$ in equation (13) along with the $D(\Theta)$ function and found, after several mathematical iterations, a stable relationship between λ and Θ.

Even though both Philip (1955) and the Bruce and Klute (1956) methods evolved from equation (13), the objectives pursued were entirely different. Given a D versus Θ relationship, Philip calculated the value of x for any Θ and t in horizontal soil. On the other hand, Bruce and Klute (1956) used equation (13) to find $D(\Theta)$ from given values of x, Θ and t covering the entire range. From his work on horizontal infiltration Philip (1957e) proposed a new physical property of porous media which he called sorptivity, S, defined as a measure of the capacity of soil to absorb liquids by capillarity.

Philip (1957a) extended his work on horizontal infiltration to include vertical infiltration by substracting equation (13) from equation (14). Writing $y = x-x'$, the result is:

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial \Theta} \left(D \frac{\partial \Theta}{\partial x}, \frac{\partial \Theta}{\partial x}\right) + \frac{\partial K}{\partial \Theta} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23)$$

with D and K single-valued functions of Θ, and subject to the following conditions:

$$\Theta(x, t) = \Theta_o \quad \text{for } y=o, \; t>o \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23a)$$

and

$$\Theta(x, t) = \Theta_n \quad \text{for } x' \rightarrow \infty \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23b)$$

where y is the vertical component of flow attributed to gravity, positive downward. Using a technique of successive approximation, Philip (1957a) found a solution expressed in a power series of $t^{1/2}$:

$$x = \lambda t^{1/2} + \chi t + \Psi t^{3/2} + \omega t^{4/2} + \text{---} + fm(\Theta) t^{m/2} \qquad (24)$$

where λ, χ, Ψ, ω,---, $fm(\Theta)$ are single-valued functions of Θ, and subject to conditions (23a) and (23b). Equation (24) provides a theoretical formula for obtaining values of x versus θ, useful for comparing calculated and experimental wetting distances in vertical columns.

The solution of equation (24) is extremely accurate at short times; however, for long times it fails to converge and hence is inaccurate. To avoid that, Philip (1957c) used a matching procedure to empirically link the short time solution with that of long time. Kunze and Nielsen (1982) compared their solution with that of Philip using data for the Yolo light clay soil and found remarkable agreement without using a matching procedure in both short and long time. Their procedure, is a two-term solution of Richard's equation for one dimensional, vertical infiltration obtained by a finite-difference, iterative method (FINDIT). This procedure will be used by the author for comparisons of calculated and experimental data.

In his theory of infiltration, Philip (1957d) compared his theoretical moisture distribution curves with experimental curves of Bodman and Coleman (1944). In their experiments, Bodman and Coleman divided the soil moisture profile into four zones: a saturated zone, a transition zone, a transmission zone and a wetting zone. Philip (1957d) critically examined the basic assumptions of his mathematical analysis and found that his analysis predicted all the zones except the transition zone. He explains that the diffusivity is not a unique function of moisture content in this zone because of air entrapment near the surface, and therefore, his analysis could not predict the transition zone.

3.1.1. Cumulative Infiltration

Cumulative infiltration is the volume of water that moves into the surface of the soil profile over a specified time. Philip (1957b) describes this quantity of water as:

$$i = \int_{\Theta_n}^{\Theta_o} x d\Theta + K_n t \quad . . . . \quad (25)$$

where i is cumulative infiltration, and K_n is the conductivity at the initial moisture content Θ_n. The integral term in equation (25) can be found by integrating equation (24) with respect to Θ to give:

$$\int_{\Theta_n}^{\Theta_o} x d\Theta = t^{1/2} \int_\lambda + t \int_\chi + t^{3/4} \int_\omega + \text{---} + t^{m/2} \int_{fm} \qquad (26)$$

where:

$$\int_\lambda = \int_{\Theta_n}^{\Theta_o} \lambda d\Theta,$$

$$\int_\chi = \int_{\Theta_n}^{\Theta_o} \chi d\Theta,$$

$$\int_\Psi = \int_{\Theta_n}^{\Theta_o} \Psi d\Theta, \qquad \text{etc.}$$

and when summed yields:

$$i = t^{1/2}\int_{\lambda} + t(K_n + \int_{\chi}) + t^{3/2}\int_{\psi} + t^2\int_{\omega} + --- + t^{m/2}\int_{fm} \quad (27)$$

By reducing equation (27) to only its two terms, Philip attempted to describe an all-encompassing, simple, general infiltration equation which seems well suited to the needs of applied hydrology. This equation in reduced form as given by Philip (1957e) is:

$$i = St^{1/2} + At \quad \dots\dots\dots\dots\dots \quad (28)$$

where A is a constant, not well defined but related to the saturated hydraulic conductivity. This equation gave good results when tested for goodness of fit in experimental examples and was found to be superior when compared to other infiltration equations which were either completely unacceptable or only moderately acceptable.

3.1.2. Infiltration Rate

The infiltration rate can be obtained by differentiating equation (27) with respect to t and setting $Vo = \partial i/\partial t$ to give:

$$V_o = \frac{\partial i}{\partial t} = \frac{1}{2}t^{-\frac{1}{2}}\int_{\lambda} + (K_n + \int_{\chi}) + \frac{3}{2}t^{\frac{1}{2}}\int_{\psi} + 2t\int_{\omega} + --- + \frac{m}{2}t^{\frac{m}{2}-1}\int_{fm} \quad (29)$$

The infiltration rate can also be obtained by differentiating equation (28) with respect to t giving:

$$V_o = \frac{1}{2}St^{-\frac{1}{2}} + A \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

One serious limitation of both equations (28) and (30) is that both S and A are generally treated as constants whose values depend upon Θ_n and Θ_o. Kunze and Nielsen (1982) were able to show that A is time dependent and increases to the hydraulic conductivity, its maximum value, as time approaches infinity.

Philip (1957f, 1958a) also studied the influence of the initial moisture content and the water depth (h) on the infiltration rate, cumulative infiltration, the moisture profile shape and the advancing rate of the wetting front. The infiltration rate decreased while the advance of the wetting front increased at higher initial moisture content. He also found that as h increased the infiltration rate and cummulative infiltration increased by about 2 percent per cm of h, but as time increased the effect of h on the infiltration rate diminished and ultimately was negligible.

Philip also observed an increase in the depth of the saturated zone with larger h, the former persisted as time increased occupying an increasingly larger fraction of the total wetted profile. He pointed out that the moisture content-distance gradient in the unsaturated part of the profile becomes relatively steeper with increasing h.

In his last paper in this series, Philip (1958a) introduced a new aspect of tension-saturation zone in the area of infiltration. He defined the state of tension-saturation to be that of a medium in which the volumetric moisture content is equal to that at $\Psi = \Psi_o$, but in which Ψ assumes a non-zero negative value. Philip (1958a) further indicated that the term "saturation" is reserved for media in which the hydrostatic pressure is more than zero while "tension-saturation" is for media with the same moisture content as saturated, but the hydrostatic pressure is less than zero. In conclusion, he emphasized the importance of the $K(\Theta)$ and $\Psi(\Theta)$ functions for characterizing soils hydrologically.

3.2. Testing the Infiltration Equations by Numerical Analysis

The numerical analysis of the infiltration equations has been studied by many investigators. (Philip, 1955, 1957a; Klute, 1952; Ashcroft et al., 1962; Hanks and Bowers, 1961, 1963; Klute et al., 1965; Whisler and Klute, 1965; and others) have given numerical solutions for horizontal and/or vertical infiltration processes. Their efforts have contributed substantially to our understanding of soil water processes. However, these numerical solutions are only interesting exercises on the computer if not tested against real data from the laboratory or the field. For this reason and others, Youngs tested Philip's theory by using his equation to calculate moisture profiles in the laboratory for homogeneously packed glass beads and slate dust. Good agreement between theory and experiment was found. Nielsen et al. (1961) tested the same theory in the field with Monona and Ida silt loam soils and found good agreement between the calculated and measured profiles in spite of the failure of the soil-water system to fully satisfy the assumed boundary conditions of equation (14). They also observed that the experimental and calculated profiles for the Monona soil were in better agreement than those of Ida soil. Even though the shape of the wetting front was adequately predicted by the theory for both soils, the depth of penetration, on the other hand, was predicted correctly only for Monona.

Gupta and Stapel (1964) conducted vertical infiltration experiments on Greenvill silt loam soil using a small positive head. Using the finite difference technique and Philip's procedure in their solution, they found a satisfactory agreement between the theory and the experiment for drier portions of the moisture profile but poor agreement in the saturated zone. However, as higher conductivities in the saturated and transition zones were considered, better agreement was found. Using the procedure of Hanks and Bowers (1962), Green et al. (1964) solved the moisture flow equation for boundary conditions corresponding approximately to those existing for infiltration into a field soil. They found the calculated and measured infiltration rates in good agreement. Another field experiment was conducted on Panoche clay loam by Nielsen (1965) to measure soil water movement during infiltration and redistribution. He found that in order to model the infiltration correctly for four irrigation treatments, accurate determination of the K vs. Θ relationship and other soil parameters had to be established. Rubin and

Steinhardt (1963), on the other hand, compared the experimental results with mathematical analysis for infiltration and soil-moisture contents and found poor agreement.

By using a finite-difference, iterative (FINDIT) method proposed by Kunze and Nielsen (1982) for calculating soil moisture profiles of Columbia silt loam and Hesperia sandy loam, Kunze and Nielsen (1983) compared the results with the experimental infiltration data of Nielsen et al. (1962). Fair agreement was obtained for both soils, the lack of better agreement was attributed to the nature of respective conductivity functions. Their calculation for soil moisture profiles was based upon using integrated mean values of D and K over a range of time periods and Θ divisions to get accurate and predictable soil moisture profiles. Their method reduced the calculations for infiltration to a two-term algebraic equation partitioned conveniently into matric and gravitational components and gives an asymptotic relationship between the infiltration rate and the saturated conductivity as time approaches infinity. They question the need of always using the diffusion lip procedure proposed by Philip (1955) which is mathematically taxing.

IV. Soil Moisture Potential Function

A fundamental property of soil is its ability to retain water in the fabric as soil moisture. Soil particles will hold a film of moisture against strong extraction forces. At any point below saturation, soil moisture is under a tension analogous to the tension in a liquid held by capillarity in a tube (Gardner, 1937). This capillary tension increases from zero in a completely saturated soil to a very large value in air-dry soil. If a water table exists below the soil surface, water moves upward by capillarity. The tension at any point within the liquid above the water table is equal to the height of the water above the water table (Gardner, 1937). A measure of the moisture-holding power over a range of capillary tension not only furnishes a measure of the capacity of the soil for water storage but gives an index of the soil properites as Gardner (1937) pointed out.

The filter-paper method of measuring water tension or potential gradually evolved in Europe and the United States and is one of several methods being used by the scientific community. Hansen (1926) working at the University of Copenhagen used blotting paper as a carrier of sugar solutions. The water potential of the soil was estimated by determining the osmotic potential of the sugar solution which had the same vapor pressure as the soil sample under investigation. Stocker (1930) used a similar procedure with a large number of sugar solution concentrations for better accuracy. Gardner (1934) improved the method by using a single strip of blotting paper soaked in salt solution and then measured for weight as an index of potential. The filter paper method was proposed and reported in the United States by Gardner (1937) to overcome the limited range of other methods of measuring soil water potential.

The filter paper method is based upon the assumption that the water potential of moist soil and filter paper in contact with the soil will be the same at equilibrium. The method further assumes that if the soil sample is large compared to the filter paper, the water potential of the soil will be essentially the same before and after it is placed in contact with filter paper. Since filter paper can be obtained with highly uniform quality, it should be possible to estimate the water potential of a soil from the gravimetric determination of the water content of the filter paper in equilibrium with the soil (Al-Khafaf and Hanks, 1974). McQueen and Miller (1968) modified the procedure proposed by Gardner (1937) to eliminate some hazards and difficulties and adapted its use to routine gravimetric soil moisture determinations. They concluded that the method is versatile, accurate, convenient and economical and is effective over the entire tension range from .001 bars to 1,500 bars. They also concluded that moisture tension may be determined by this method with an accuracy that is comparable to or better than the accuracy of other methods with limited ranges.

Preliminary evaluation of the McQueen and Miller (1968) method was done by Al-Khafaf and Hanks (1974). They used salt solutions, thermocouple psychrometers, pressure plates and soil columns in their calibration of the method. They found that the predicted water potential was influenced by the type of contact of the soil with the filter paper and suggested that one filter paper be placed beneath the soil (good contact for liquid flow and vapor flow) and one filter paper be placed above the soil not in physical contact (allowing vapor flow only). Al-Khafaf and Hanks (1974) found problems with contact between the filter paper and soil sample, temperature at equilibrium and temperature variation during equilibrium. They found that the absolute temperature was not too important but temperature variations with time had a large effect on the predicted soil water potential.

EXPERIMENTAL PROCEDURE AND ANALYSIS

I. Materials and Methods

1.1. Materials

Two Michigan soils[1], the Metea sandy loam (Arenic Hapludalfs; sandy over loamy, mixed, mesic) and the Spinks sandy loam (Psammentic Hapludalfs; sandy, mixed, mesic) were investigated in this study. The A-horizon, of the Metea soil is a dark, sandy loam, approximately 10 cm thick. Permeability is very rapid in the upper part of this soil and moderate in the lower part. The water holding capacity of this series is described as moderate. The A-horizon, of the Spinks series is dark brown, sandy loam, 25 cm thick. Permeability of this soil series is described as rapid or moderately rapid. The water holding capacity of this series is low.

Disturbed samples of both soils were taken from the Michigan State University Soils Research Farm in East Lansing, located in the north central portion of Ingham County between 42 and 43 latitude and 84 and 85 longitude. Sampling for both soils were taken from the A-horizon between 0 and 10 cm depth.

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1 Soil survey of Ingham County, Michigan, United States Department of Agriculture and Soil Conservation Service in cooperation with Michigan Agricultural Experiment Station. 1977.

The particle densities and particle size distributions of both soils are shown in Table 1.

1.2. Preparation of the Flow System

1.2.1. Sample preparation

Soil samples were evenly spread over laboratory benches to obtain air-dryness and later screened through 1 mm and 2 mm sieves. The screenings were used in an attempt to pack columns of high and low bulk densities.

1.2.2. Fquipment

The infiltration apparatus used for both horizontal and vertical flow is shown in Figure (1). The water supply system contains a constant-head burette of 250 ml capacity and divisions of 1.00 ml facilitating measurement of inflow. The soil column consists of 2-cm and 1-cm sections of clear glass tubing 3.60 cm in diameter taped together to form a 1-meter column. Alignment of the column was maintained with a trapezoidal-shaped notch cut into a block of wood. The cover for the notch was transparent plastic sheeting with a meter stick attached and framed-in by strong sheet metal. The cover was fastened to the block with magnets embedded therein to confine the soil column, and facilitate measurement of the wetting front. Positions of the wetting front and its rate of movement were determined through

Table 1. Physical properties of soil materials used.

Soil	Depth (cm)	Particle density (g/cc)	Particle size distribution † per cent by weight @		
			sand	silt	clay
			(2.00-0.05mm)	(0.05-0.002mm)	(<0.002mm)
Metea	0-10	2.627	71.36	22.5	6.14
Spinks	0-10	2.634	71.36	21.0	7.64

† Obtained by using Hydrometer-Bouyoucos method.
@ The average of two replications.

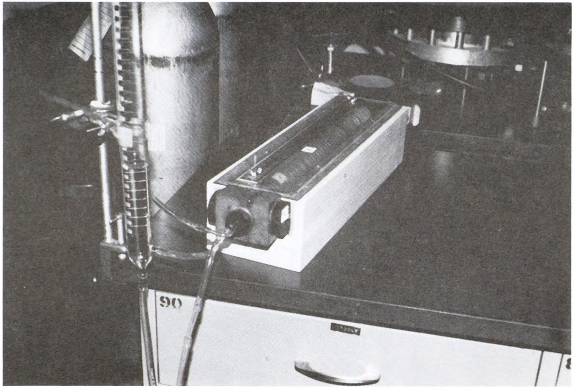

Figure 1. Experimental apparatus used for both horizontal and vertical water movement.

the transparent window by noting the progress of the wetting front relative to the meter.

1.2.3. Packing of the Soil Column

Twenty or more, 2-cm wide glass sections were fastened to each other by tape followed by an additional 30-sections of 1-cm width. The sections were numbered and arranged in sequential order forming a cylinder of 70 or more cm in length. The column was packed in a vertical position and closed from the bottom with a rubber stopper. To obtain a uniform bulk density, soil was added to the column through a special packing device.

The packing device was designed by Dr. A.J. Corey of Colorado State University for packing sand. A sketch of the device and modifications added to serve our purpose is shown in Fig (2). The device consisted of a copper cylinder, 3.2 cm inside diameter and 10 cm length, connected to a smaller cylinder of the same materials of 90 cm length and 2.0 cm inside diameter. Two screens roughly 8 cm apart and perpendicular to the axial dimension were attached at the lower end of each cylinder. The screen mesh was such that the soil particles would pass through without clogging. The upper part of the smaller tube was capped by a disk, with four individual and equally spaced openings of 2 mm inside diameter, to permit soil to feed continuously into the device. A supply funnel was connected to the top of the smaller tube and kept full with air-dry soil. Some soil movement in the funnel was maintained with a small, electrical, kitchen mixer.

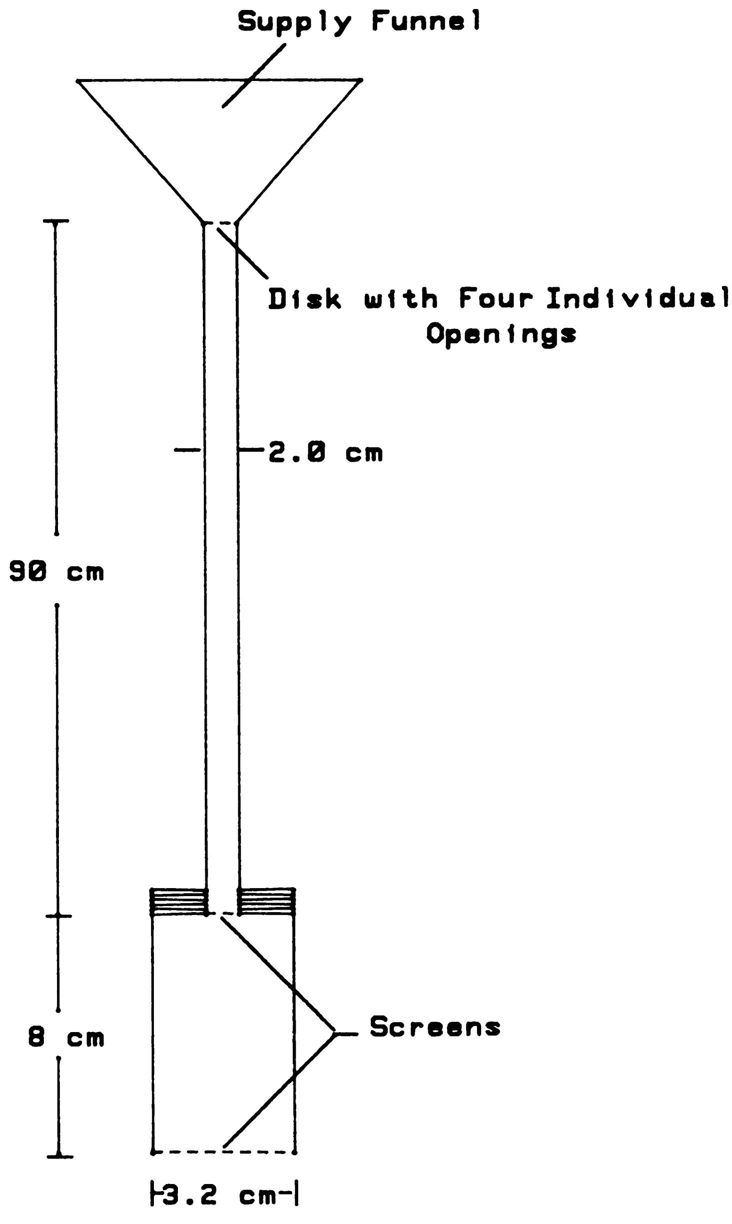

Figure 2. Schematic diagram of the soil column packer.

Before packing commenced, the bottom screen was positioned approximately 12 cm above the rubber stopper enclosing the end of the empty cylinder. This position between the soil surface and the packing device was maintained by lowering the column with a jack throughout the filling process. A rotating motion applied to the column manually was helpful in getting uniform distributions of falling soil. Experience showed that to obtain uniform density the distance between the bottom screen of the packer and the top of the packed soil in the column had to be maintained at a distance of approximately 12 cm at all times.

The purpose of the packing device was to maintain a continuous and uniform flow of soil from the source into the column and through the height of fall and striking the screens, obtain a homogenous distribution of soil across the entire column surface area. The random distribution of particles across the entire column area contributes significantly to obtaining a column with uniform bulk density.

When the column was filled, the upper part of the column was plugged with a small amount of glass wool and a rubber stopper. If further consolidation of the soil columns was found to be necessary, the column was dropped on each stoppered end from a height of 3 cm 50 times. By increasing the number of drops to 100 for each end a slightly greater bulk density was obtained. Soil was added to both ends during the latter compacting process to keep the column filled and consolidation in effect. Sectioning of the column showed that the bulk density was more variable at each end than in the middle; hence a 10-cm section was removed from the end to be wetted initially.

To remove the tape from the soil column the later was placed in a vertical position inside the notched block. By using a special device shown in Figure (3), the column was wedged tightly against a plate to allow removal of the tape without disturbing the soil in the column. Once the tape was removed, the soil column which remained in the notched block, was covered with the transparent cover and the meter scale adjusted to the $x = 0$ point.

1.3. Infiltration Method

Two soil columns were prepared for each soil analysis, one for vertical and the other for horizontal infiltration. Both were prepared by the same procedure as outlined earlier to achieve maximum uniformity of soil bulk density within and between columns. The air entry pressure was maintained at -2 mb for both vertical and horizontal wetting. The pressure of water entering the soil columns was controlled by a fritted glass bead plate (Nielsen and Phillips, 1958) placed at $x = 0$ required for the solution of the equations (13) and (14) as specified in (14a, 14b and 14c). To start an infiltration trial, the ceramic plate was filled with water and the desired pressure (-2 mb) applied to the plate before it was placed in contact with the end of the column.

Wetting agents used for these studies were deaerated tap water and distilled water saturated with $CaSO_4$ following

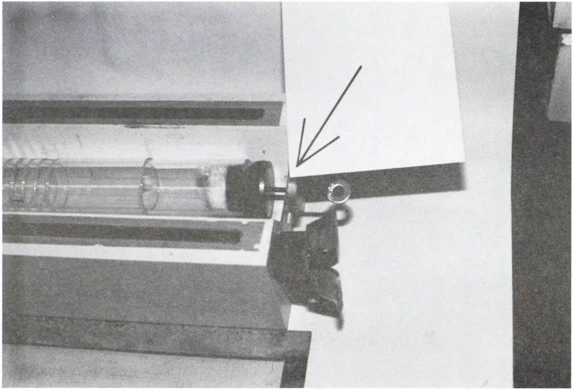

Figure 3. A special device used for adjusting the contact between the soil surface and the plate.

deaeration. Measurements of time and distance from the water source were taken simultaneously and commenced the instant contact was established between the wetted plate and the soil column. Water entering the columns was concurrently measured in the constant-head burette. When the flow had proceeded for desired time or distance, the water supply was severed and the column quickly segmented into its 2 cm and 1 cm sections. To minimize continued water movement after termination sectioning commenced at the wetting front constituting the 1-cm sections of the soil column. The water content of each section was determined gravimetrically (Θ_g) and converted to volumetric values (Θ_v) by using the average bulk density of the entire column.

Measurement of wetting front distances vs. the square root of time (x vs. $t^{1/2}$) and soil moisture distribution profiles (Θ vs x) were made for both vertical and horizontal columns. Soil moisture diffusivity and conductivity values were obtained by using a computer analysis procedure developed by Dr. Raymond J. Kunze from the Department of Crop and Soil Sciences of Michigan State University. The computer used by the author for the analysis was a Hewlett-Packard 9845B microcomputer with internal tape drives, thermal printer and a HP 7470A plotter.

II. Estimation of Soil Water Potential

Soil water potential values needed specifically for K(Θ) analysis were determined by suction wetting in the high potential range and the filter paper method (Gardner, 1937) in the low potential range. The latter is largely the method of McQueen and Miller (1968) and Al-Khafaf and Hanks (1974), but with modifications in an attempt to eliminate some difficulties.

The method is based upon the fact that if a sample of soil and filter paper are placed in contact, and one of them is moistened, water will pass from the moist medium to the dry medium until equilibrium is attained. If the capillary tension curve has been determined for filter paper, the tension for the soil is readily found by reference to the filter paper curve. A point on the absorption moisture characteristic of the soil may be established from the known equilibrium tension value and the measured water content of the soil.

2.1. Apparatus and Supplies

Beside the equipment required for gravimetric soil moisture determinations, the following were needed: (a) an analytical balance accurate to 0.001 g; (b) constant temperature chamber (24.5 C); (c) filter paper-Schleicher and Schuell No. 589 white Ribbon; (d) pentochlorophenol "Dowcide-7" 5.0 mg/ml in 95% Ethanol; (e) petri-dishes 150 x 15 mm; and (f) plastic electrical

tape to seal petri-dishes during periods of establishing moisture equilibrium.

2.1.1. Procedure

Predetermined weights of moistened soil required to fill a 150 x 15 mm petri-dish at a desired bulk density were packed uniformly. One-half of the moistened soil was placed in the dish and covered with two 9-cm filter papers sandwiched between two 12.5-cm all treated with "Dowcide-7". The rest of the moistened soil was then added, the cover placed on the dish and the entire dish sealed with plastic, insulating tape. The samples were placed in a constant temperature chamber and allowed to equilibrate for a week. After equilibrium was achieved, the smaller (9-cm) filter papers were removed and their moisture content accurately determined with an analytical balance. The moisture content also was determined on samples of soil in the nearness of the filter papers.

To avoid decomposition by soil organisms, the filter papers were pre-treated with "pentochlorophenol" in ethanol and allowed to air dry overnight (McQueen and Miller, 1968).

The gravimetric moisture percentages for the two 9-cm papers were averaged and from the moisture content-tension curve of Al-Khafaf and Hanks, (1974) the tension of both the filter paper and the soil were determined. The soil moisture characteristic curve, particularly for low potential values, was obtained by plotting the calculated tension values against the respective

soil moisture contents. At high potential values, the soil moisture characteristic curve was determined from capillary rise data. Several soil columns with slightly different bulk densities were prepared by the procedure outlined earlier and placed upright in contact with a free water table at the lower end facilitated by a fritted glass bead plate. The water level was adjusted to the lowest height of the soil inside the column and allowed to equilibrate for three to four weeks. After equilibrium was achieved, the columns were cut into 2 cm sections and the gravimetric and volumetric moisture content determined. The latter values were obtained by using the average bulk density of the entire column. The moisture contents and associated heights above the water table were plotted to form the soil moisture characteristics curve.

RESULTS AND DISCUSSION

Three of six experimental analyses, two on Metea sandy loam and one on Spinks sandy loam soil with the corresponding numerical simulations, are presented here. The numerical results were obtained using the computer analysis (FINDIT) proposed by Kunze and Nielsen (1982). Further developments such as generating the K function from infiltration profiles is new and has not been published. This investigation is to be one of several tests for these procedures. All experimental data obtained for both soils can be found in the Appendix.

I. Horizontal and Vertical Water Movement with Different Bulk Densities

Six examples of the relationship between the advance of the wetting front and the square root of time in horizontal flow with Metea sandy loam and Spinks sandy loam are shown in Figures 4-6. Each pair of packed soil columns was to fall into a specified bulk density range. Packing effort and soil aggregate size were to effect such changes in bulk density. Wetting distances were plotted as a function of time to observe the effect of bulk density on the water movement. The figures indicate that as the bulk density increased the slope (λ) of the associated distance-time$^{1/2}$ line decreased. All curves seem to give a small positive intercept if a straight line is drawn through the data by eye or fitted statistically. The phenomenon has not been

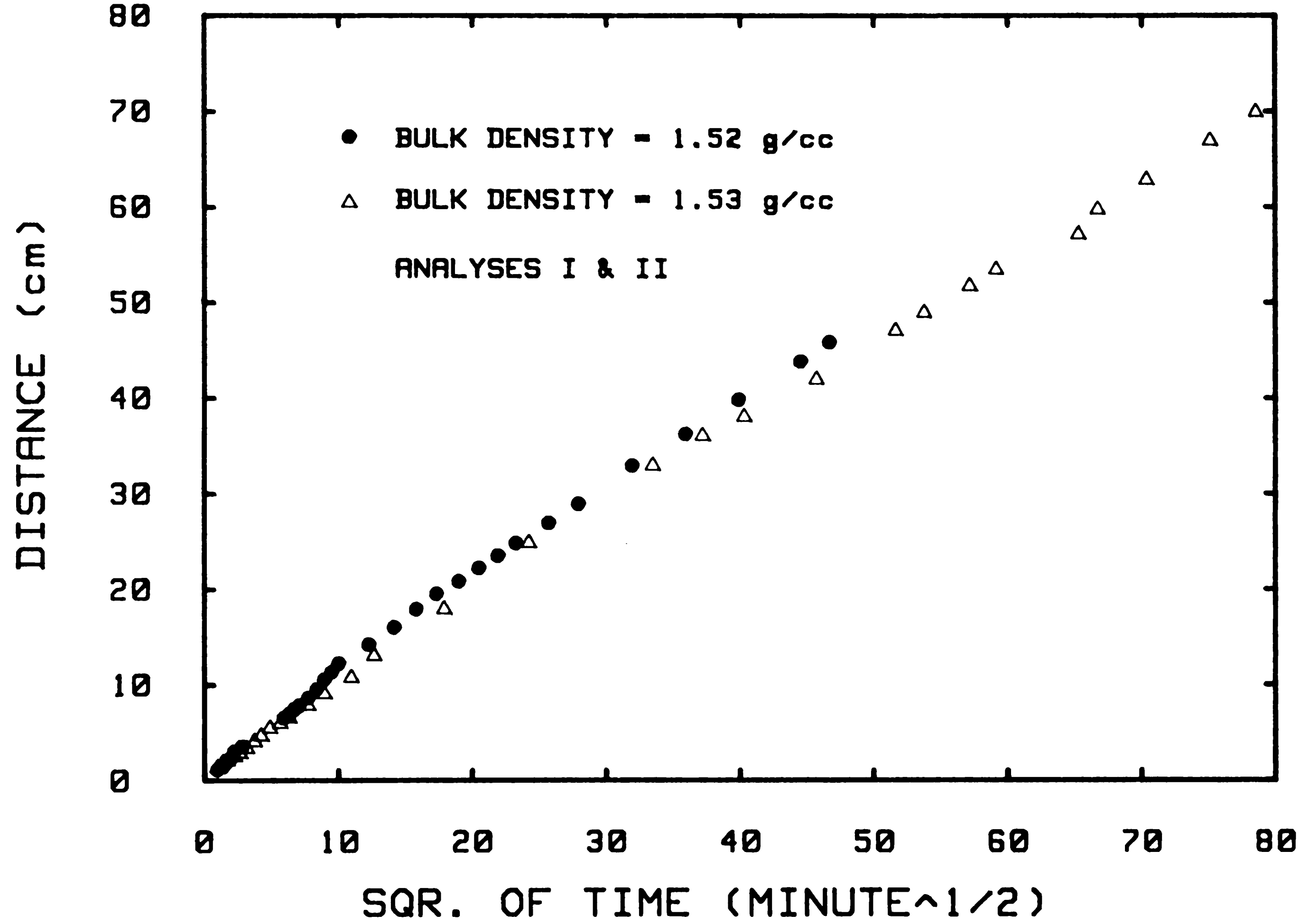

Figure 4. Distance to the wetting front versus the square root of time for Metea sandy loam at indicated bulk densities, comprising analyses I and II.

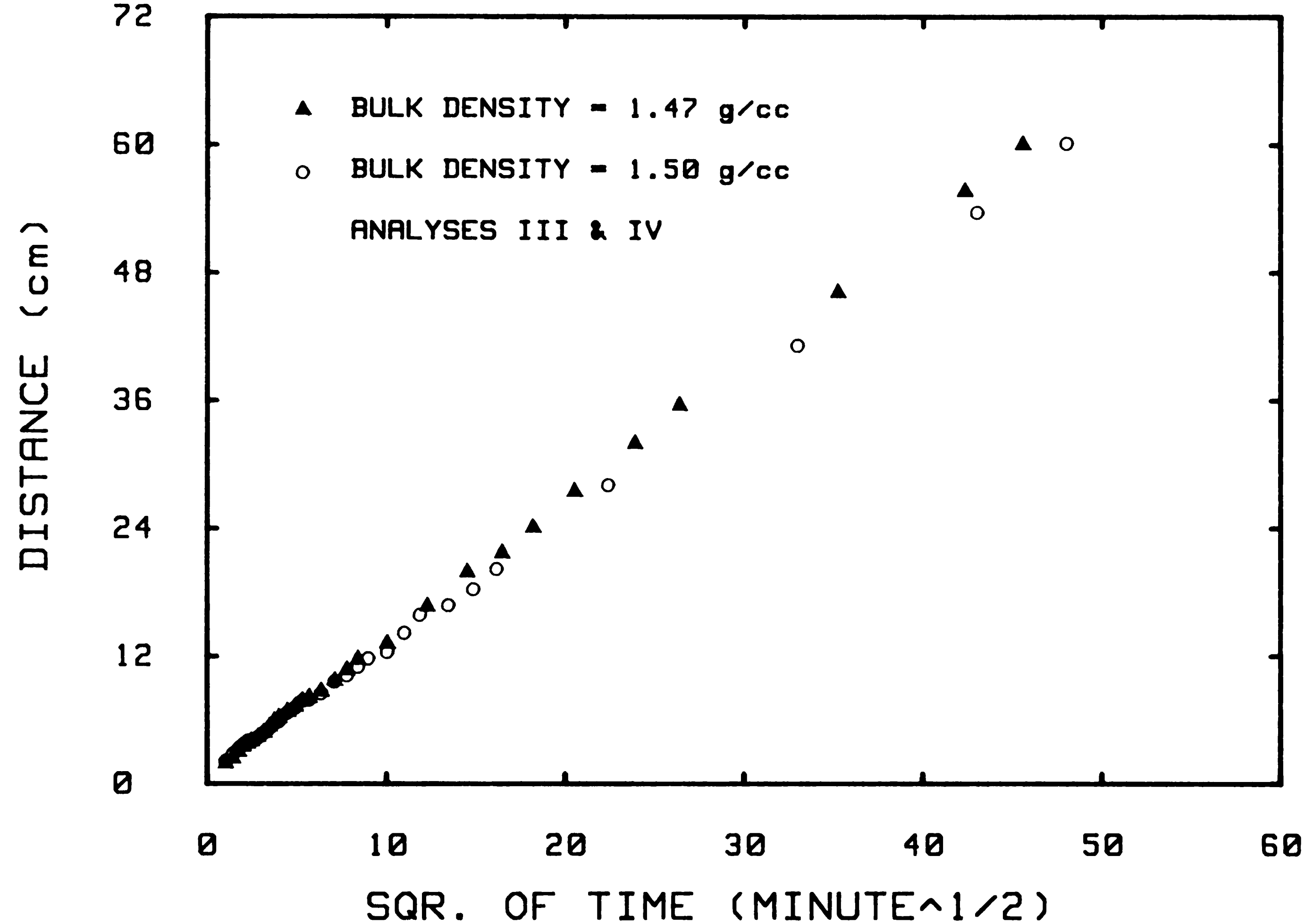

Figure 5. Distance to the wetting front versus the square root of time for Metea sandy loam at indicated bulk densities, comprising analyses III and IV.

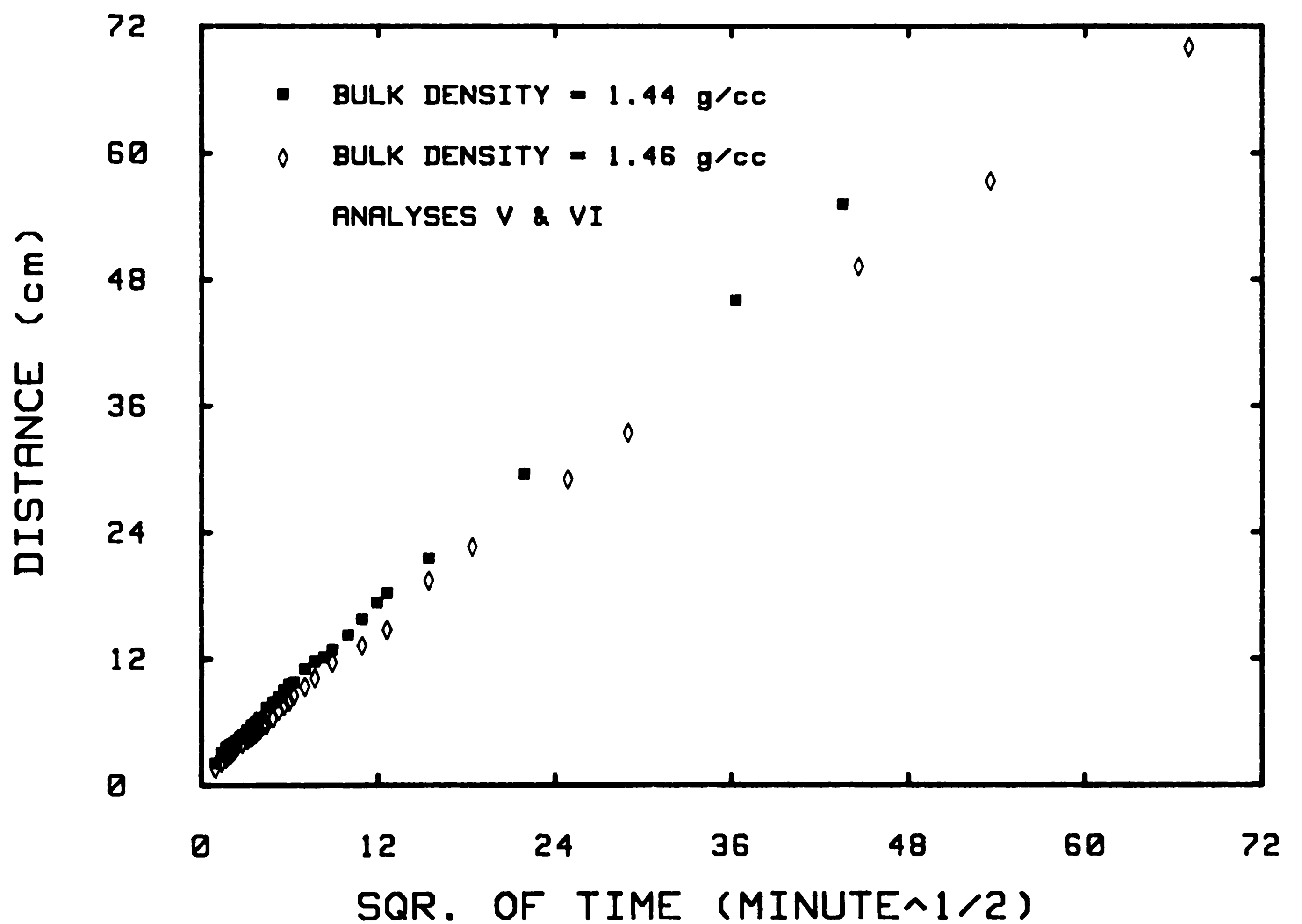

Figure 6. Distance to the wetting front versus the square root of time for Spinks sandy loam at indicated bulk densities, comprising analyses V and VI.

accounted for by theory. Such deviations were reported by Kirkham and Feng (1949); Biggar and Taylor (1960); Nielsen et al. (1962); Jackson (1963); and Peck (1964) and are attributed to inertial forces, bulk density variation within the column, temperature and experimental error. Since these data were collected in the laboratory with $\pm$ 3°c temperature variation, the temperature and the bulk density variation were likely contributors to experimental uncertainty and distortion of the desired λ-$t^{1/2}$ relationship. In any case, the non-linearity of λ with time is evident from these six curves.

Statistical data showing the degree of reproducibility of bulk density for segments within both horizontal and vertical soil columns are given in Tables 2 and 3. Actual standard deviations and the coefficients of variation within the column would be expected to be much smaller if the additional error introduced by sectioning the column were discounted. By increasing the number of drops (See Method and Materials) from 50 to 100 a slightly larger bulk density was obtained for each column of Metea soil but the opposite was true for Spinks soil (see Table 2 or 3). The effect of bulk density on water intake and distance moved is quite noticeable as shown in Table 4. In general, more water entered the soil and the rate of advance of the wetting-front was faster in the columns with lower bulk densities. No explanation can be given why columns wetted with tap water resulted in larger measured bulk densities than those wetted with distilled water saturated with $CaSO_4$. Student's t-tests were performed on horizontal and vertical columns wetted

Table 2. Characterization of soil columns used for horizontal flow.

Soil	Number of drops	Mean bulk density (g/cc)	Standard deviation	Coefficient of variation
Metea	50	1.519*	±.0693	4.56
	100	1.532	±.0599	3.91
	50	1.474*	±.0620	4.21
	100	1.497	±.0480	3.21
Spinks	50	1.460*	±.0638	4.37
	100	1.437	±.0646	4.50

* Soil columns used in analysis.

Table 3. Characterization of soil columns used for vertical flow.

Soil	Number of drops	Mean bulk density	Standard deviation	Coefficient of variation
Metea	50	1.525*	± .0677	4.44
	100	1.528	± .0638	4.18
	50	1.469*	± .0458	3.12
	100	1.482	± .0397	2.68
Spinks	50	1.498*	± .0743	4.96
	100	1.443	± .0629	4.36

* Soil columns used in analyses.

Table 4. Parameters describing one-dimension infiltration experiments with the Metea and Spinks sandy loam soils.

Soil	Wetting solution	Infiltration type	Bulk density (g/cc)	Time (min)	Distance (cm)	Total amount of water (cm^3)
Metea	Deairated tap water	Horizontal	1.519*	2175	45.80	148.50
		Vertical	1.525*	2270	70.00	250
		Horizontal	1.532	6167	70.00	280
		Vertical	1.528	2690	68.00	255
	Deairated distilled water sat. with $CaSO_4$	Horizontal	1.474*	2076	60.10	210
		Vertical	1.469*	827	70.10	250
		Horizontal	1.497	2305	60.10	204
		Vertical	1.482	840	67.10	242
Spinks		Horizontal	1.460*	4500	70.00	269
		Vertical	1.498*	2020	70.00	268
		Horizontal	1.437	1897	55.10	214
		Vertical	1.443	1188	64.20	245

* Soil columns used for D and K analysis

with each type of water. The measured mean differences were found to be highly significant at the one percent level of probability. Furthermore, non-significant differences were found when the same test was performed on pairs of horizontal and vertical columns used in any of the specified analyses.

A way of testing the assumption that there is a unique relationship between any λ and Θ is implied in the Boltzmann transformation, $\lambda(\Theta) = xt^{-1/2}$, explained by Nielsen et al. (1962). A unique λ_n-Θ_n relationship is present if a straight line is obtained by ploting λ_n values versus (t/t_o) where t_o is a total time. Such a plot is shown graphically in Figure 7. Even though these data were obtained for water entering the soil columns slightly below atmospheric pressure (-2 mb), the uniqueness of λ_n versus Θ_n seems to be questionable particularly at short time. Nielsen et al. (1962) obtained larger line curvatures at more negative water entry pressures. The non-linearity of these data indicate λ is not a constant for a specific soil moisture given in $\lambda(\Theta) = xt^{-1/2}$. The volume of water which had entered the unit cross-sectional area of two Metea and one Spinks soil column is shown in Figure 8. The flow equation (13) subjected to the Boltzmann transformation, initial and boundary conditions (14a, b and c) predicts a linear relationship between cumulative volume versus the square root of time expressed as $i = St^{1/2}$ where i is accumulated infiltration and S is sorptivity, Philip (1957). Two soils gave a satisfactory agreement with the theory, however, Analysis III shown in Figure 8 (Metea, BD = 1.47 g/cc) seems to disagree with

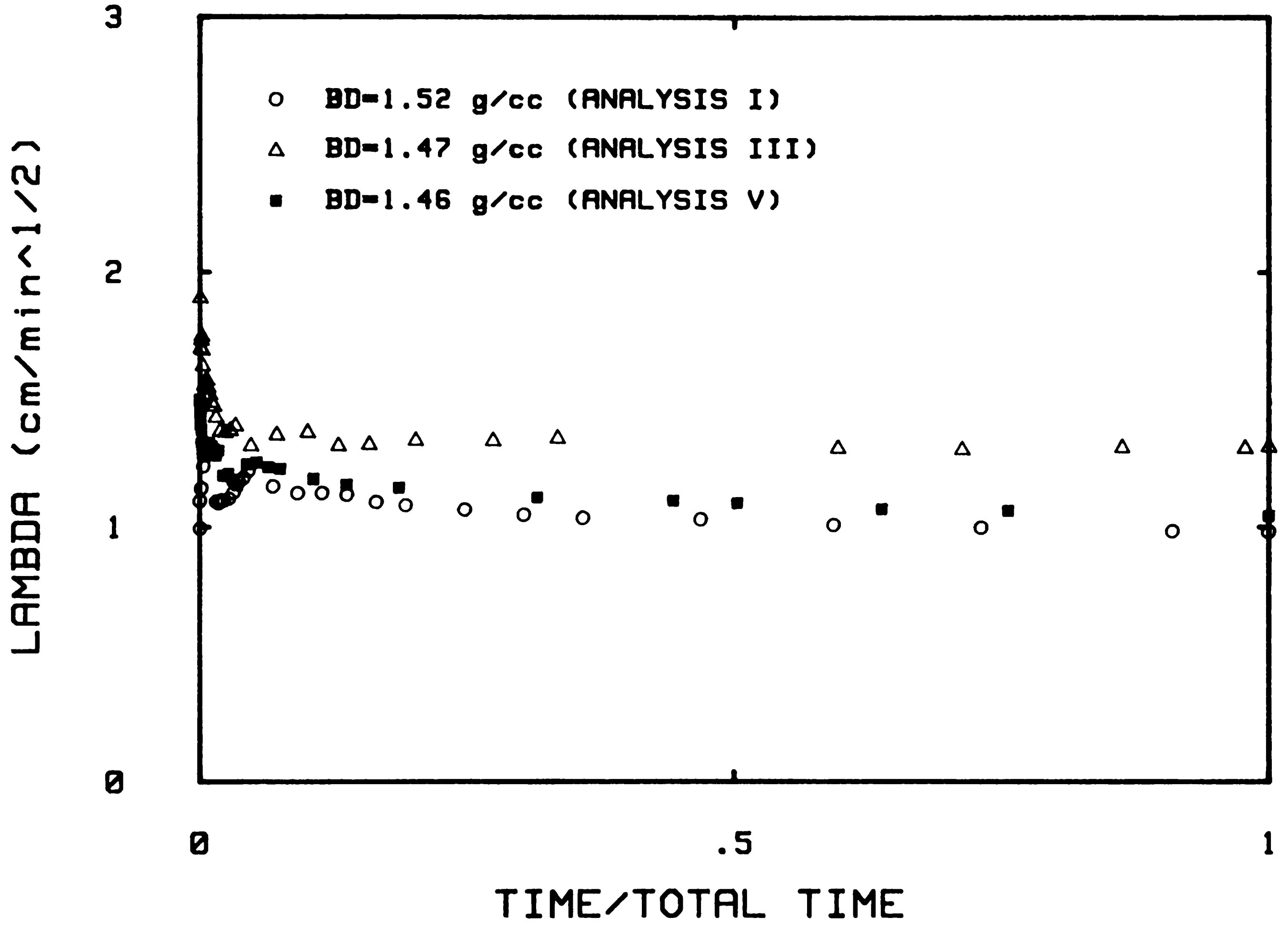

Figure 7. Values of lambda (λ) determined by visual distance to the wetting front divided by the square root of time for both Metea and Spinks sandy loam at indicated bulk densities.

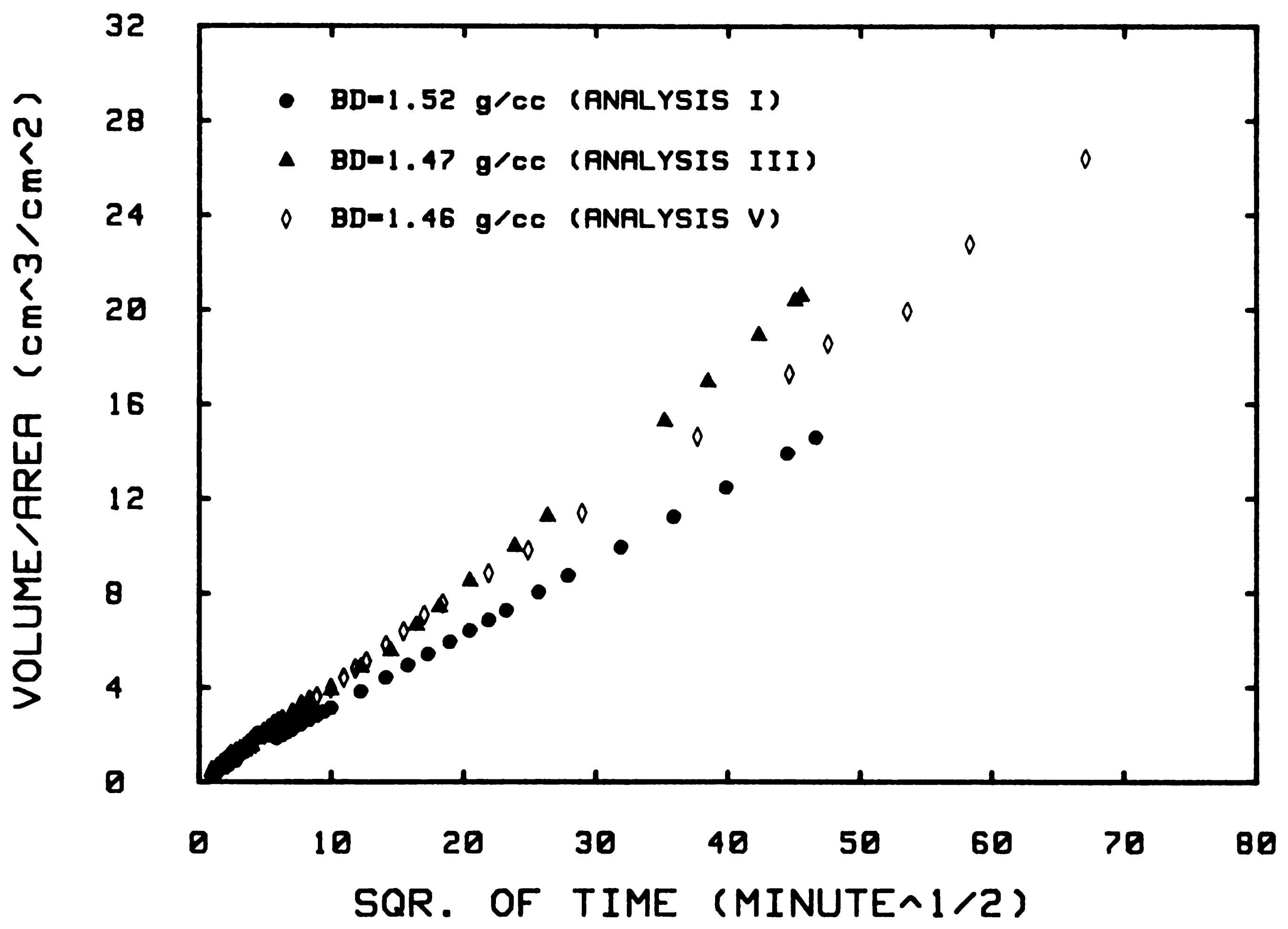

Figure 8. Cumulative water volume versus the square root of time for Metea and Spinks sandy loam at indicated bulk densities.

theory by giving a curvelinear relationship and increasing sorptivity with time. The bulk density variation within the soil column, temperature and experimental error may be part of the problem as was indicated earlier. The other part would be whether the Boltzmann transformation and it is application is always valid. This question is beyond the scope of this research. Finally, the issue is further confounded by consideration of data in Figures 7 and 8. The most acceptable analysis in Figure 7, Analysis III, is the worst in Figure 8 and vice versa for the other two data sets.

Three of the six pairs of horizontal and vertical columns were chosen to be presented as model data for D and K analysis, designated with "*" in Table 4.

II. Calculated Soil-Moisture Diffusivity and Conductivity Functions

The soil-moisture diffusivity and conductivity functions, $D(\Theta)$ and $K(\Theta)$, are calculated for several assumptions. The assumptions are necessary to address the incompatibilities resulting from bulk density differences, temperature variations, etc. between horizontal and vertical infiltration pairings. These assumptions were incorporated into the computer analysis procedure for calculating $D(\Theta)$ and $K(\Theta)$. Calculated distances were compared with experimentally measured distances and the degree of fit was judged to be the criterion for testing of both theory and the FINDIT procedure. Three assumptions were considered: (1) the horizontal wetting distances, associated diffusivity values and the moisture characteristic curve for a given soil are accurate. Accordingly, the FINDIT procedure analyzes the vertical infiltration data and generates conductivity values but not necessarily for the original vertical profile; (2) the measured horizontal and vertical profiles are accurate, but the measured moisture characteristic is questionable. If tension values generated from (D/K) functions are incompatible with measured tension values, FINDIT changes the measured tension values in the moisture characteristic to conform to those generated by the program; (3) the vertical profile and the moisture characteristic are accurate, but the horizontal profile and the associated diffusivity values are questionable. The procedure changes the diffusivity values which, in turn, will

alter the horizontal profile sufficiently enabling the program to fit the vertical profile. The assumptions can also be presented in the following abbreviated mathematical formulas: (1) $K = DC$; (2) $K = DC^*$; (3) $K = D^*C$ where K and D are the conductivity and diffusivity, respectively, and C the soil moisture capacity, the latter being the slope ($\partial\Theta/\partial\Psi$) of the soil moisture characteristic curve at any specific moisture content. The symbol "*" following a parameter means that it was allowed to change according to conditions specified earlier.

To test these assumptions, the first data set, referred to as Analysis (I), was prepared for computer input. These data consisted of smoothed horizontal and vertical wetting distances at specified water contents. Both scattered (experimental) and smoothed data for the Metea sandy loam are given in Figure 9. The smoothed profiles and the associated time values are the necessary input required for generating the diffusivity and conductivity functions. Based on the distance $-t^{1/2}$ relationship for any moisture content within the horizontal profile at 2175 minutes, the horizontal wetting distances for any moisture contents can be calculated for any other time including 2270 minutes. Repeated solution of equations (13) and (14) subject to conditions (14a), (14b) and (14c) are required to finalize the generation of the D and K functions for the time values specified.

The calculation of both functions is based on the vertical infiltration of 2270 minutes; however, the computer first uses the given horizontal infiltration time of 2175 minutes and its

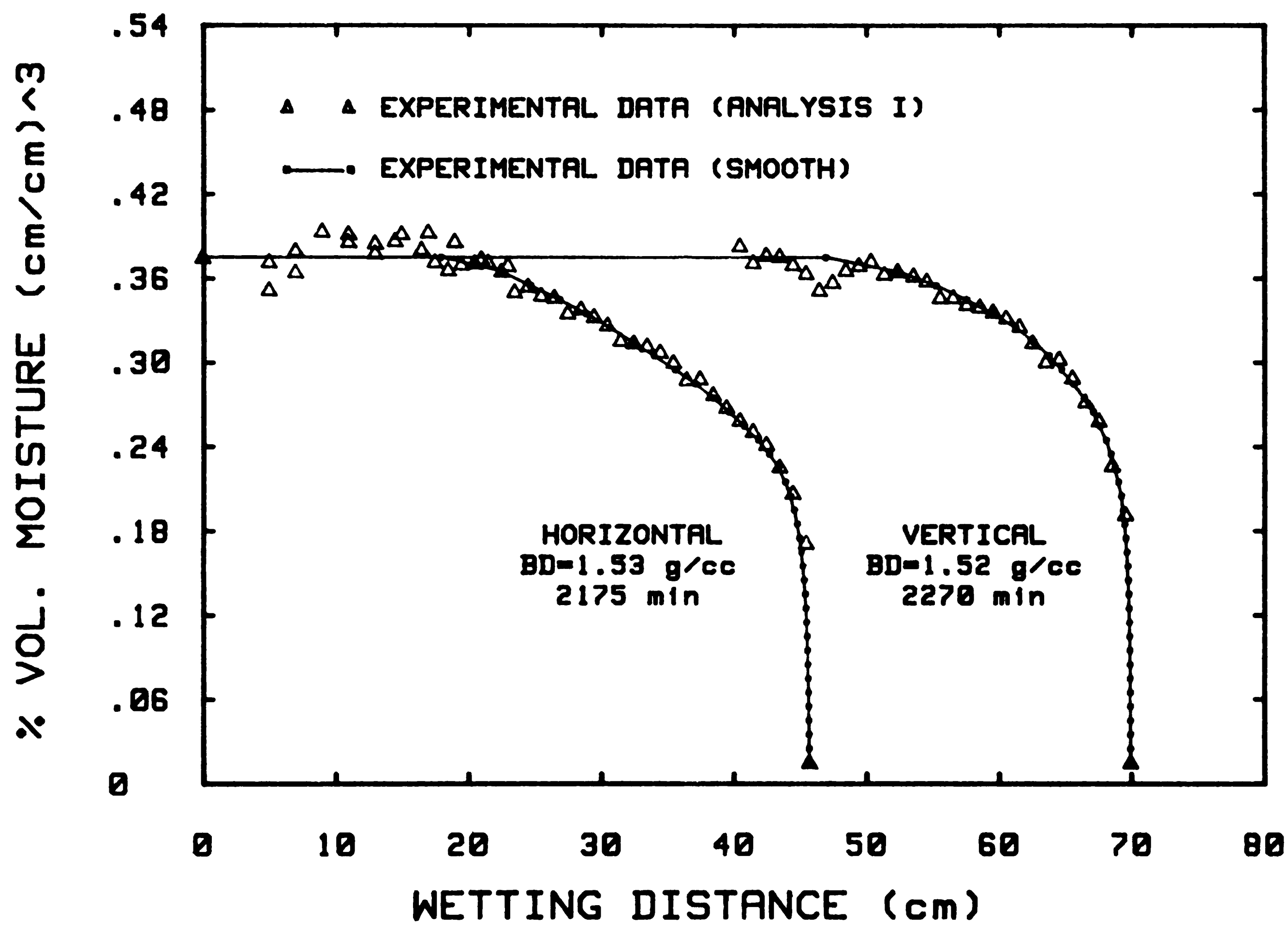

Figure 9. Experimental and smoothed soil moisture profiles for Metea sandy loam at indicated bulk densities and infiltration times.

associated profile to calculate D values. Alternately, using vertical time, it then generates the 2270 minute horizontal profile, the K values from the differences between the horizontal and vertical profiles and the adsorption soil moisture characteristic from D and K values given in Figure 12. The soil-moisture diffusivity and unsaturated hydraulic conductivity functions obtained by such analyses for given assumptions are presented graphically in Figures 10 and 11. Both curves follow the general trend of D and K increasing with increasing moisture content and terminate with maximum values for D and K near or at saturations.

To make the calculated vertical profile fit the measured vertical profile, small changes were required in the horizontal and vertical profiles resulting in changes of D, K and Ψ functions as can be seen in Figures 10, 11 and 12 in this section and in Figures 18 and 19 in the subsequent section. As Figure 10 shows, the change in the diffusivity function is distributed evenly over the entire moisture range, but for conductivity and (Ψ) the change occurs only in the higher moisture content range. This suggests that K and Ψ are interactive and one is dependent on the other. No other matching factor or approximations were required for generating these functions using the FINDIT procedure.

To check if calculated hydraulic conductivities agree in general with measured saturated hydraulic conductivities, a saturated hydraulic conductivity experiment was conducted on both soils and results compared. The results given in Table 5 appear

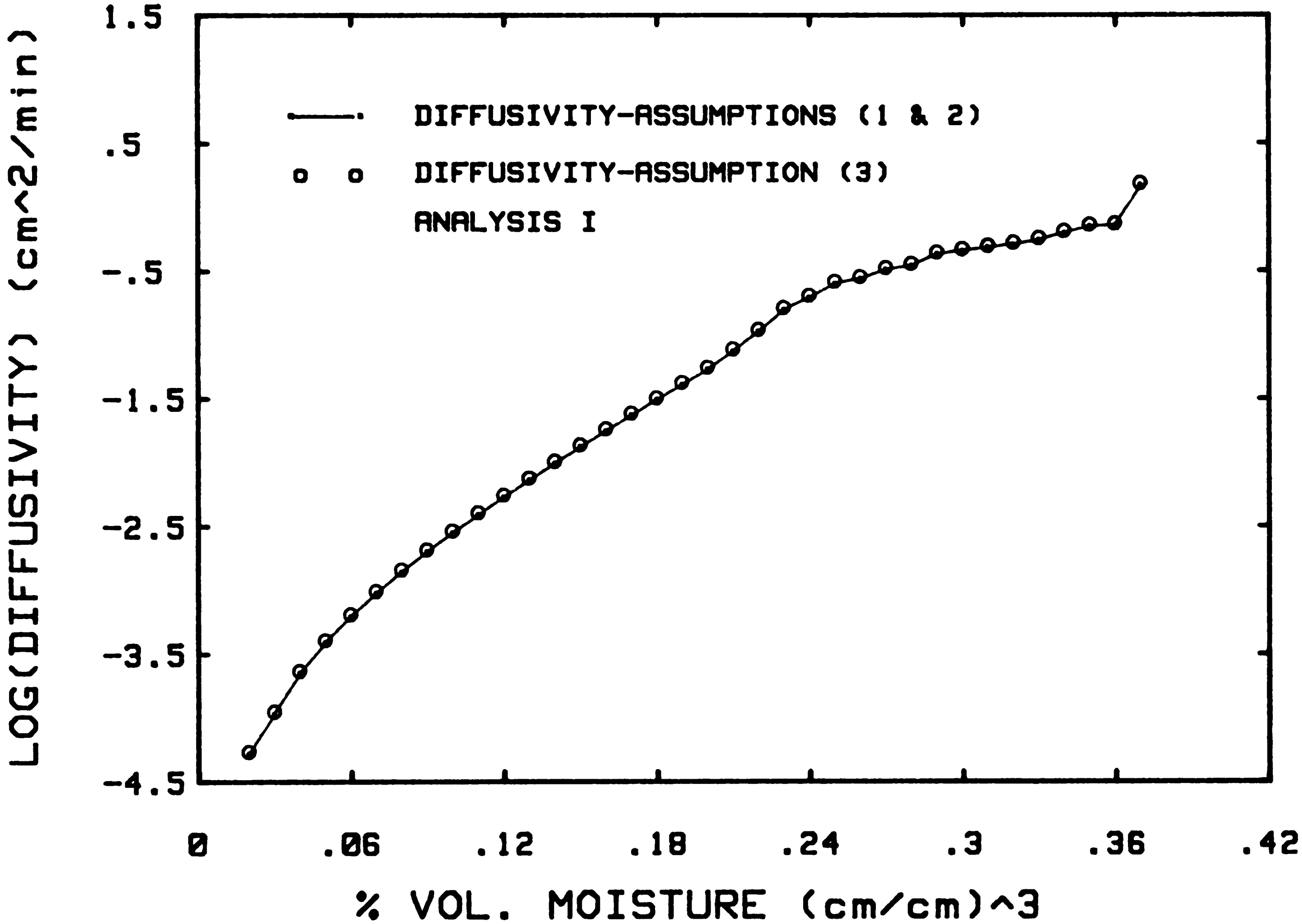

Figure 10. Calculated soil-moisture diffusivity D(Θ) using both assumptions (1 & 3) for Metea sandy loam.

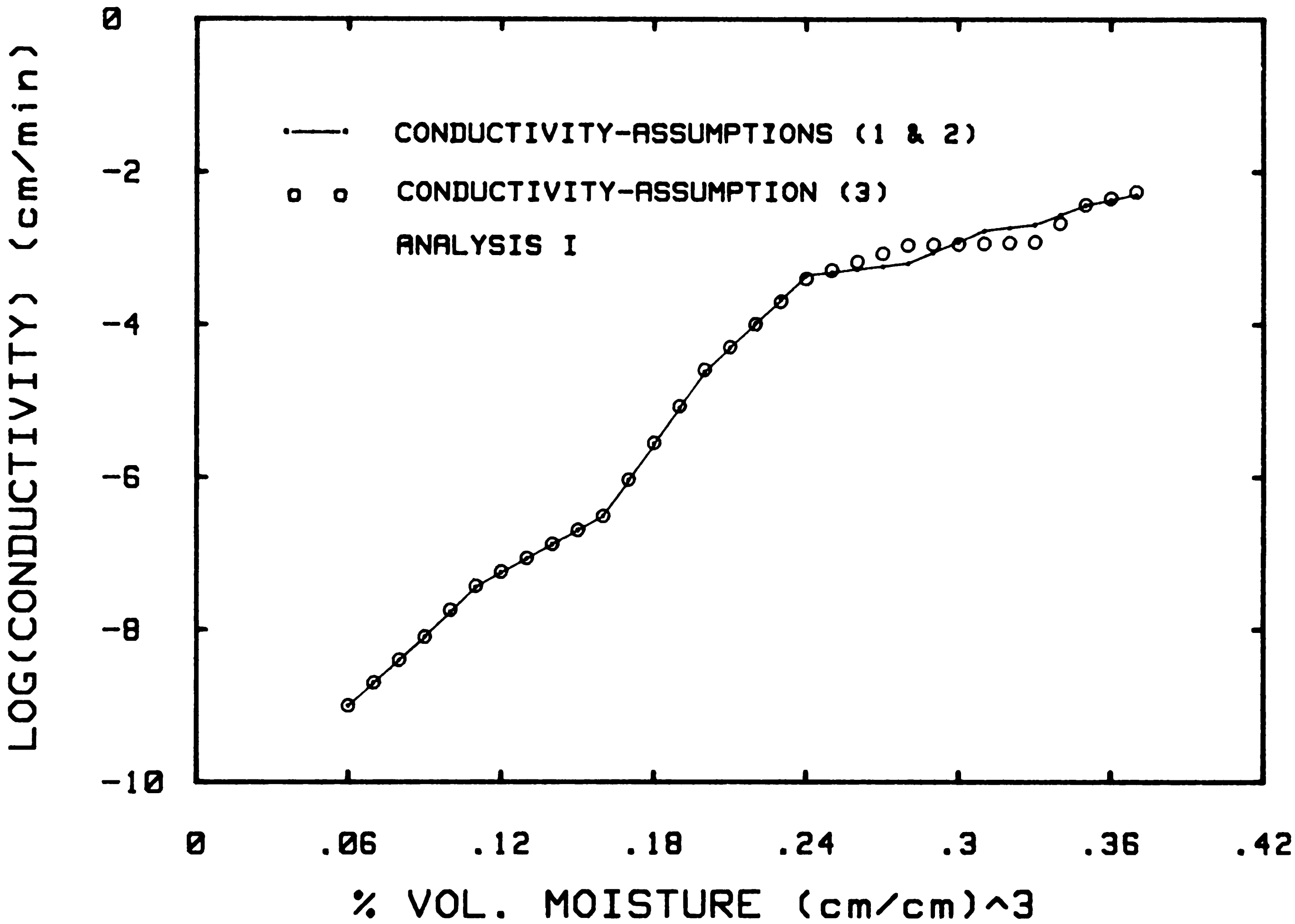

Figure 11. Calculated hydraulic conductivity using both assumptions (1 & 3) for Metea sandy loam.

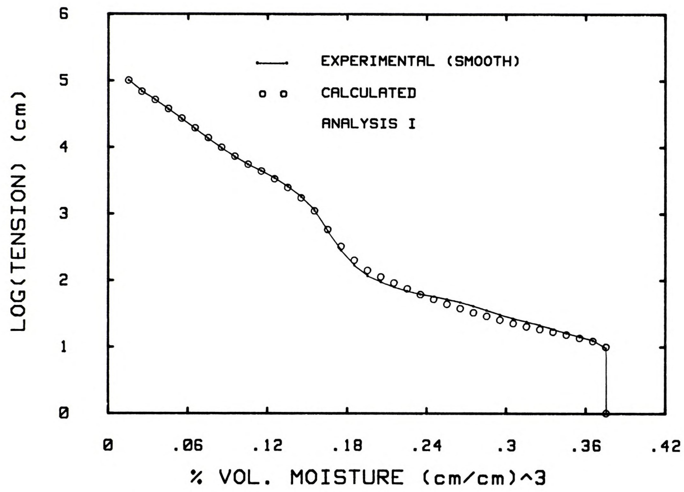

Figure 12. Experimental and calculated soil moisture characteristics using assumption (2) for Metea sandy loam.

Table 5. Comparison between measured and calculated saturated hydraulic conductivities for Metea and Spinks soils.

Soil	Analysis	Average bulk density (g/cc)	Measured saturated hydraulic conductivity (cm/min)	Average bulk density (g/cc)	Calculated saturated hydraulic conductivity (cm/min)
	I			1.525 @	.00643 ††
Metea		1.50 †	.00928 †		
	III			1.472 @	.01653 ††
Spinks	V	1.47 †	.01367 †	1.480 @	.00656 ††

@ Average of paired horizontal and vertical columns
† Average of five replications
†† Calculated under assumptions (2 & 3)

to be in fair agreement for both soils; however, the saturated conductivities in the infiltration experiments were definitely different. Consider that the vertical infiltration time for the wetting front to reach 70cm was 827 minutes in Analysis III compared to 2270 minues in Analysis I. This suggests that the former has a larger saturated conductivity as shown in Table 5. Also, tap water was used for measuring saturated hydraulic conductivities given in Table 5 and on Metea (Analysis I) whereas distilled water saturated with $CaSO_4$ was used on Metea (Analysis III) and Spinks (Analysis V).

The influence of bulk density on the diffusivity and conductivity functions shown in Figures 13 and 14, suggest that as the bulk density decreased both diffusivity and conductivity increased. However, this was not true when diffusivity functions from the Metea and Spinks soils were compared as shown in Figure 15. Mixed results were obtained when the conductivity functions for the same soils were compared as shown in Figure 16. The general shape of these functions and their respective soil moisture profiles suggest that the solution of the equations considered herein were equally accurate for all times and different soils.

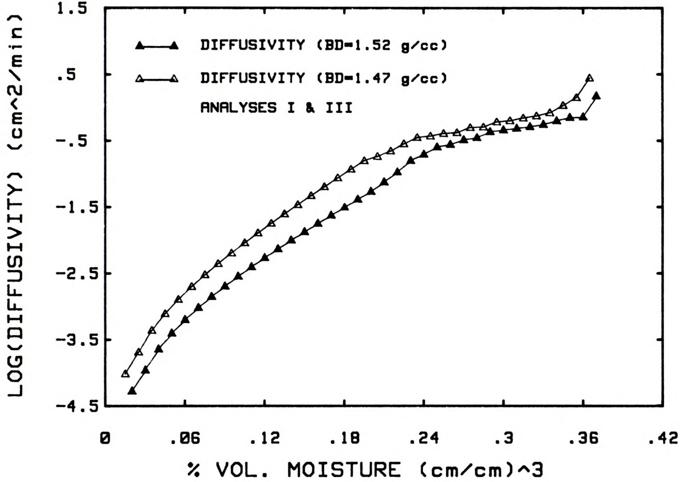

Figure 13. Calculated soil-moisture diffusivities using assumption (1) for Metea sandy loam at indicated bulk densities.

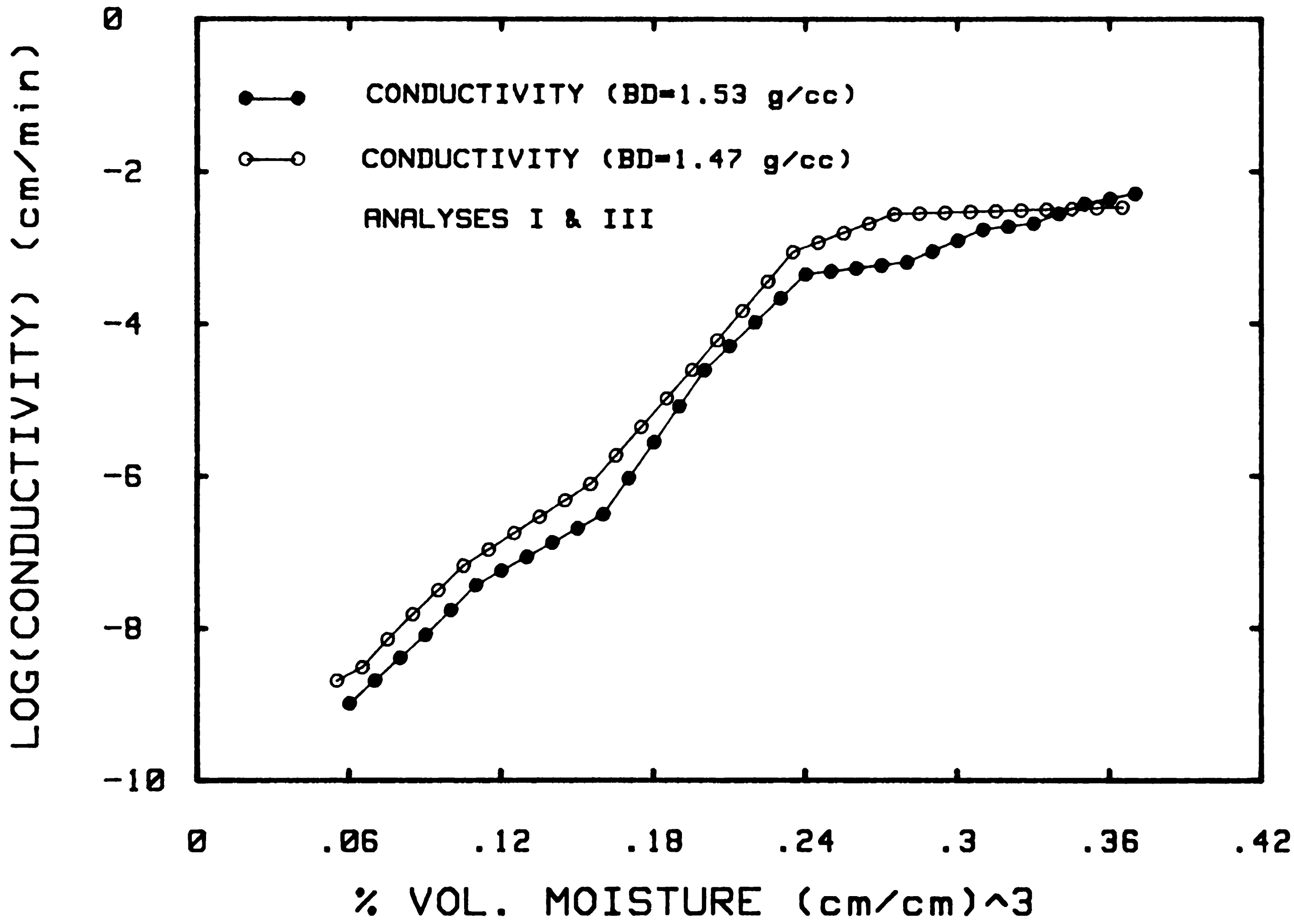

Figure 14. Calculated hydraulic conductivities using assumption (1) for Metea sandy loam at indicated bulk densities.

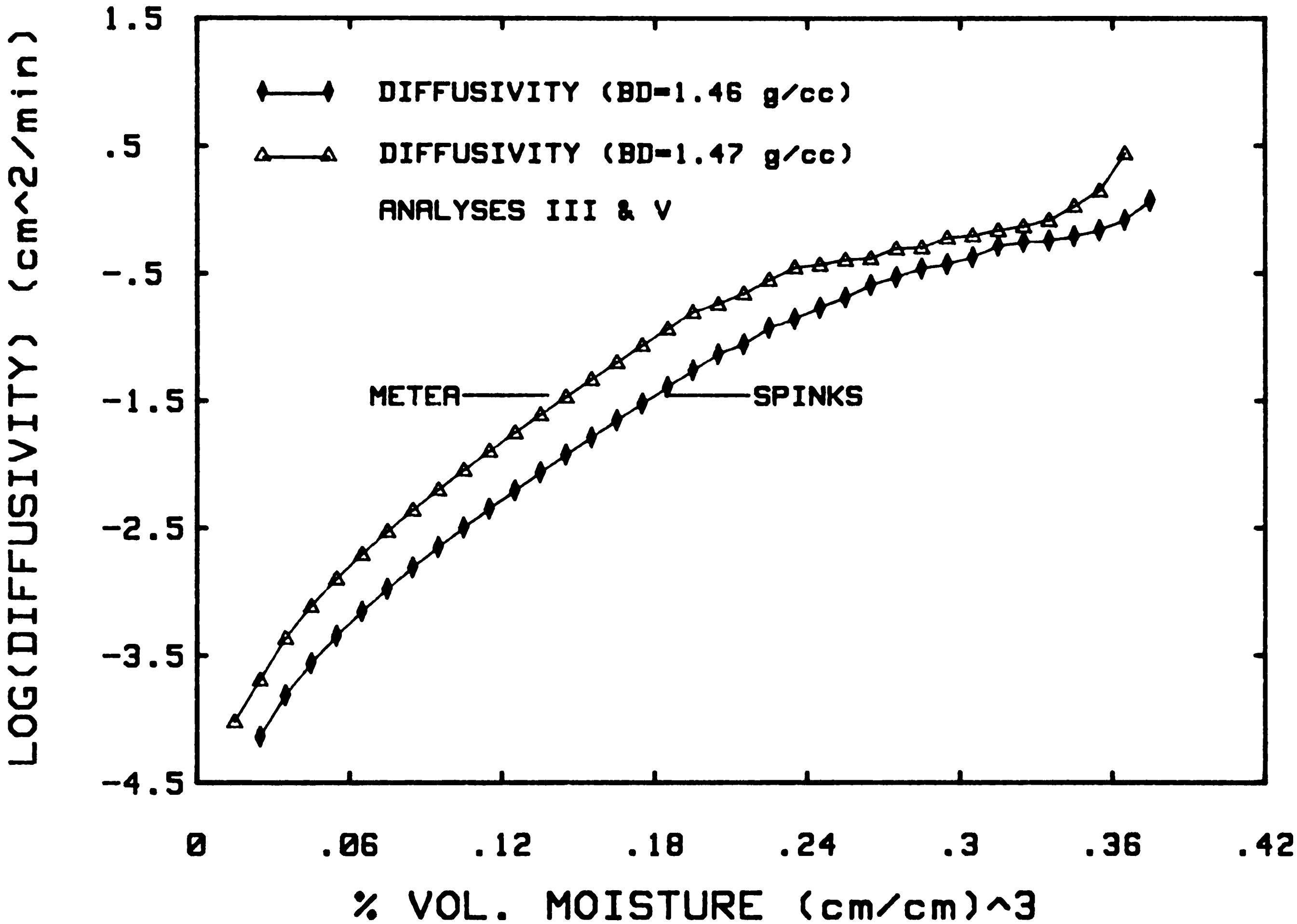

Figure 15. Calculated soil-moisture diffusivities using assumption (1) for Spinks sandy loam and Metea sandy loam at indicated bulk densities.

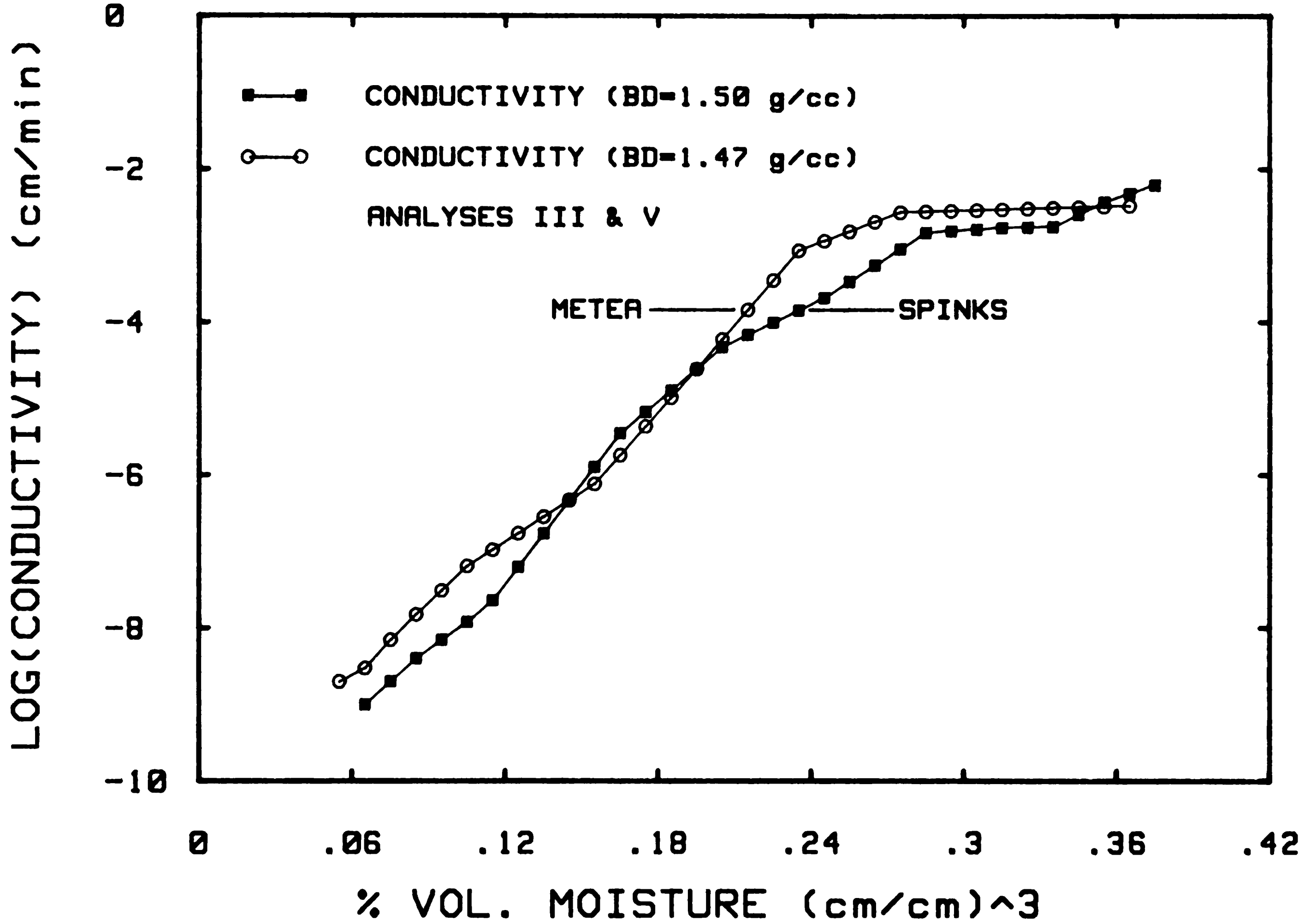

Figure 16. Calculated hydraulic conductivities using assumption (1) for Spinks and Metea sandy loam at indicated bulk densities.

III. Experimental and Calculated Soil-Moisture Profiles

The smoothed experimental horizontal and vertical infiltration profiles obtained under specified assumptions for Metea and Spinks sandy loam soils are presented in Figures 17-20. The calculation of these profiles and the associated D and K functions are independent processes and can not be done simultaneously for either horizontal or vertical infiltration analysis. Even though the computer procedure gives simultaneous output of both functions and the respective moisture profiles, the D and K functions are generated first during the interplay process and then used as input for solving equation (13) and (14) again to generate the respective moisture profiles for other time values.

The success of the flow equation in describing soil moisture profiles is evaluated here by comparing the predicted distributions with those determined experimentally. For assumption (1, K = DC), the agreement between the calculated and experimental vertical profiles seems to be quite satisfactory in Metea and Spinks soils as shown in Figures 17 and 20, respectively; however, the differences between the measured and calculated distances for the Metea soil analysis III, are considerably larger (see also Table 6). The differences in measured and calculated vertical profiles are believed to relate to the compatibility between horizontal and vertical profile pairs. In the Spinks soil, the difference in bulk density is

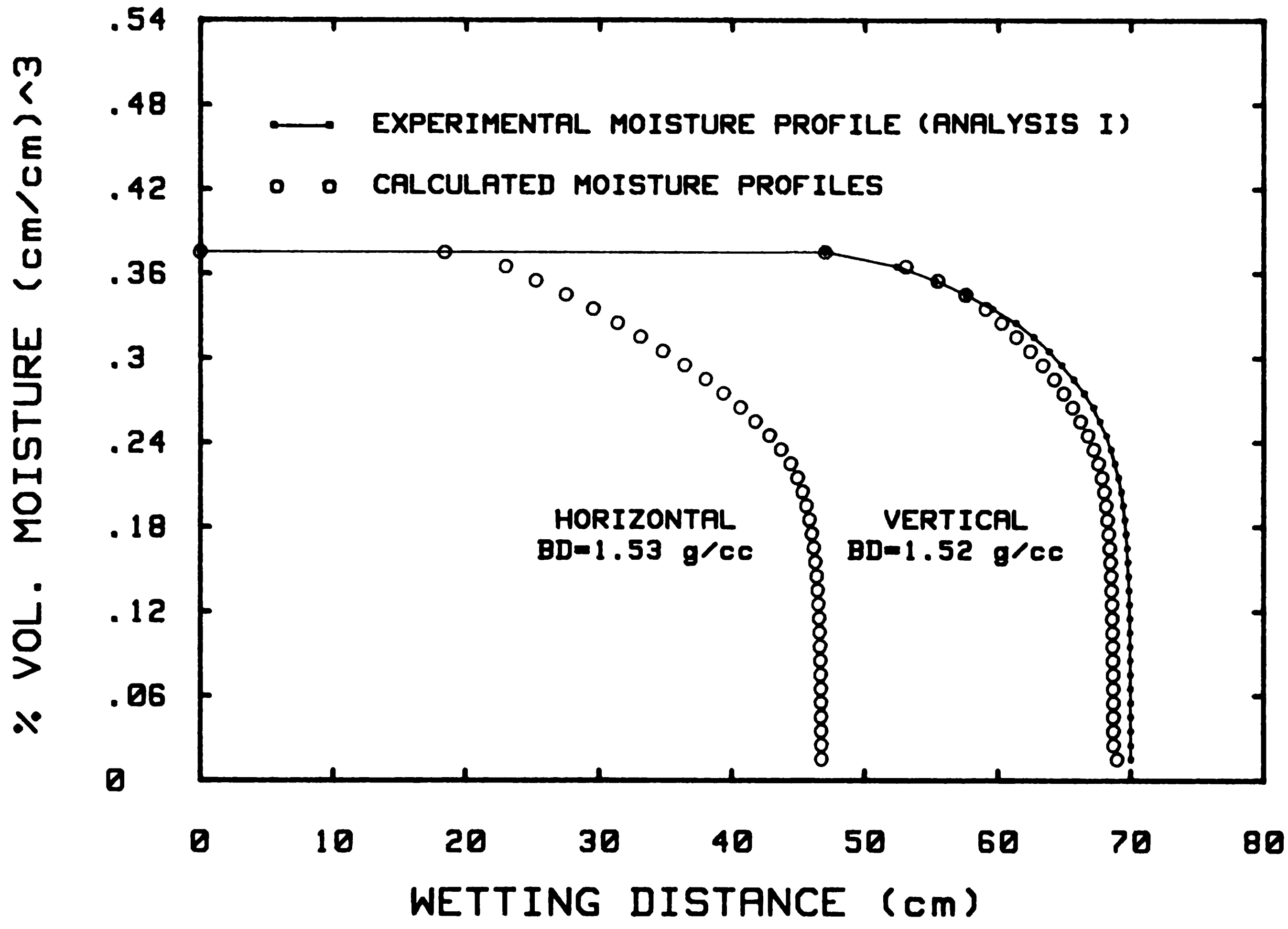

Figure 17. Horizontal and vertical soil moisture profiles obtained for Metea sandy loam soil using assumption (1) at indicated bulk densities and 2270 minutes.

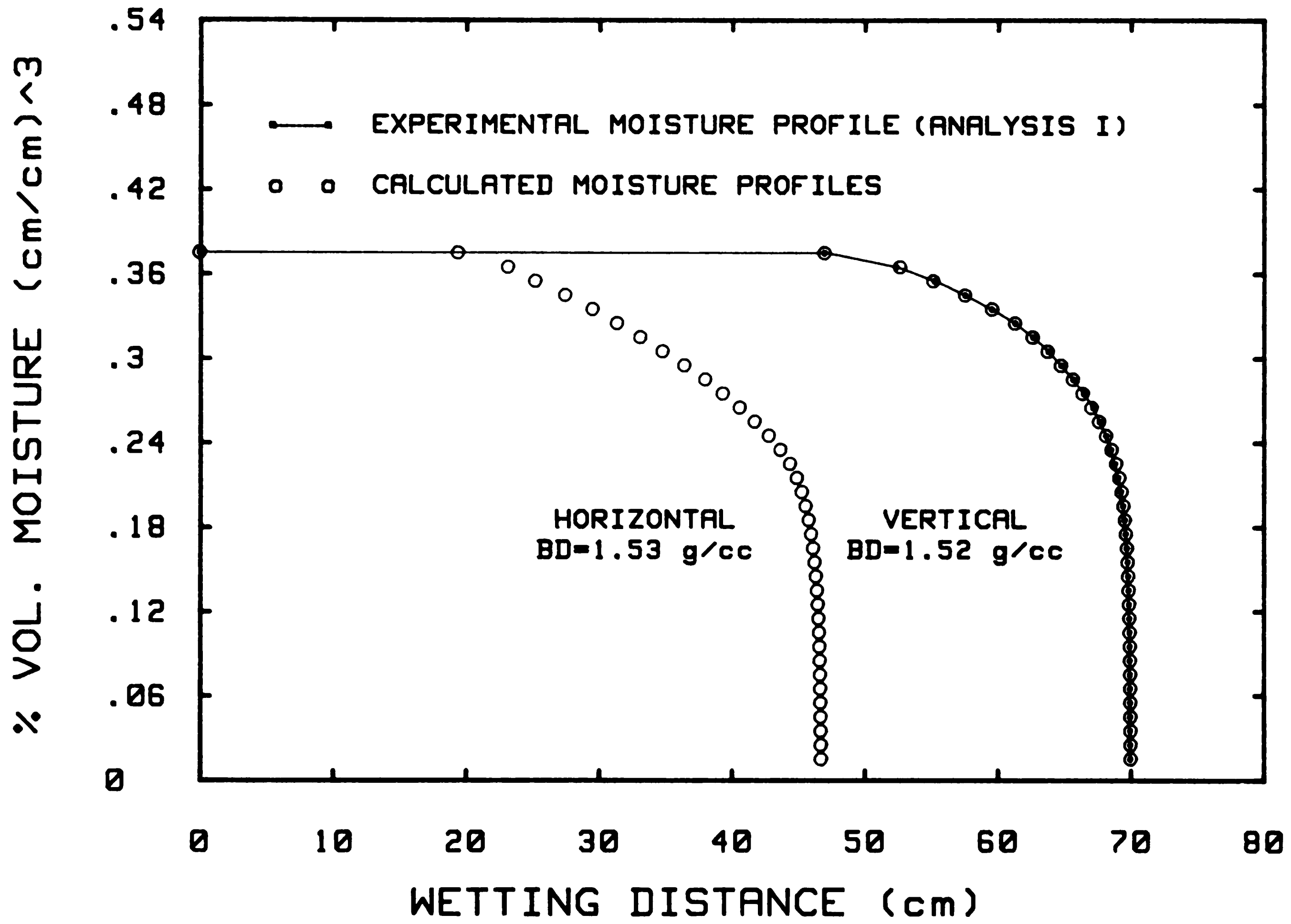

Figure 18. Horizontal and vertical soil moisture profiles obtained for Metea sandy loam soil using assumption (2) at indicated bulk densities and 2270 minutes.

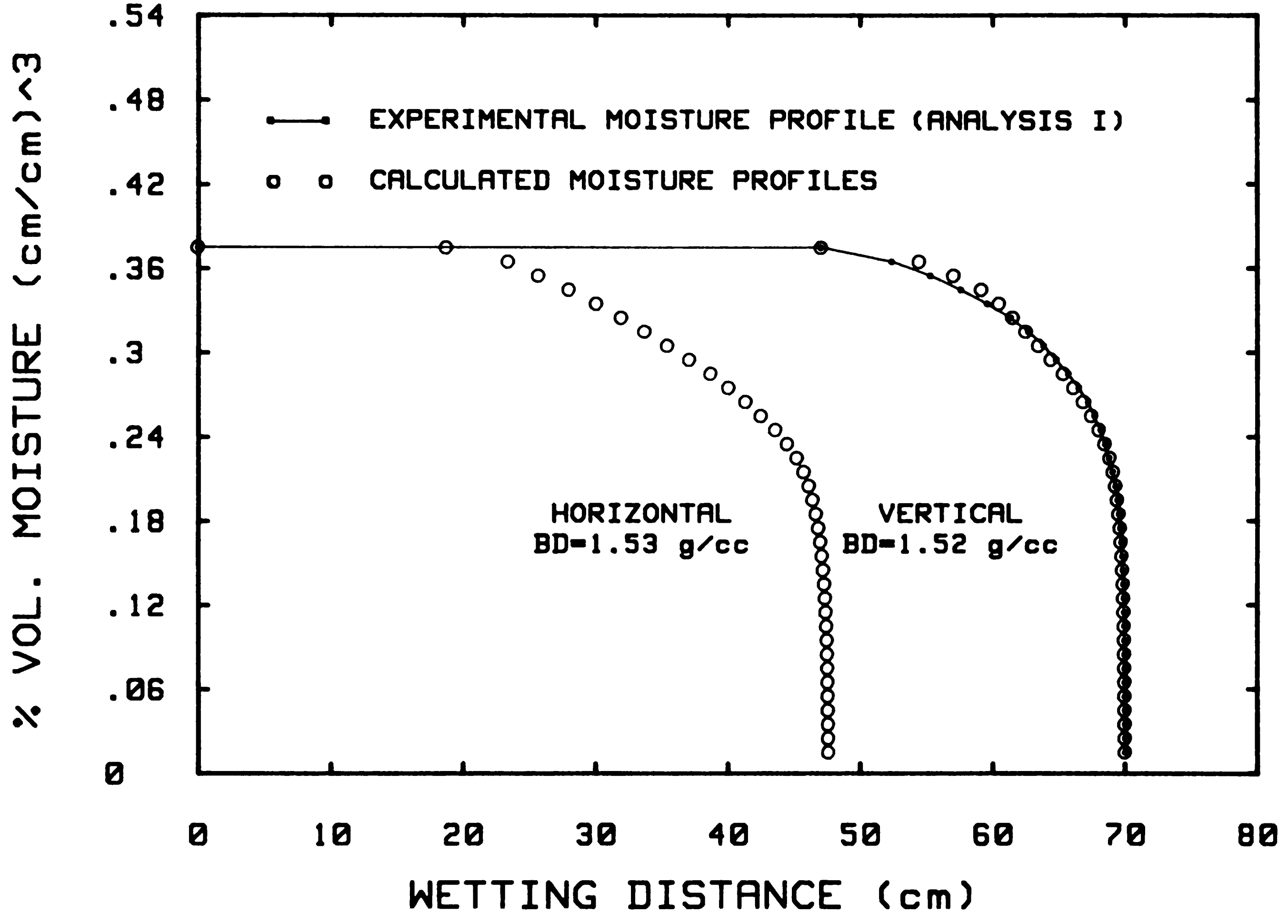

Figure 19. Horizontal and vertical soil moisture profiles obtained for Metea sandy loam soil using assumption (3) at indicated bulk densities and 2270 minutes.

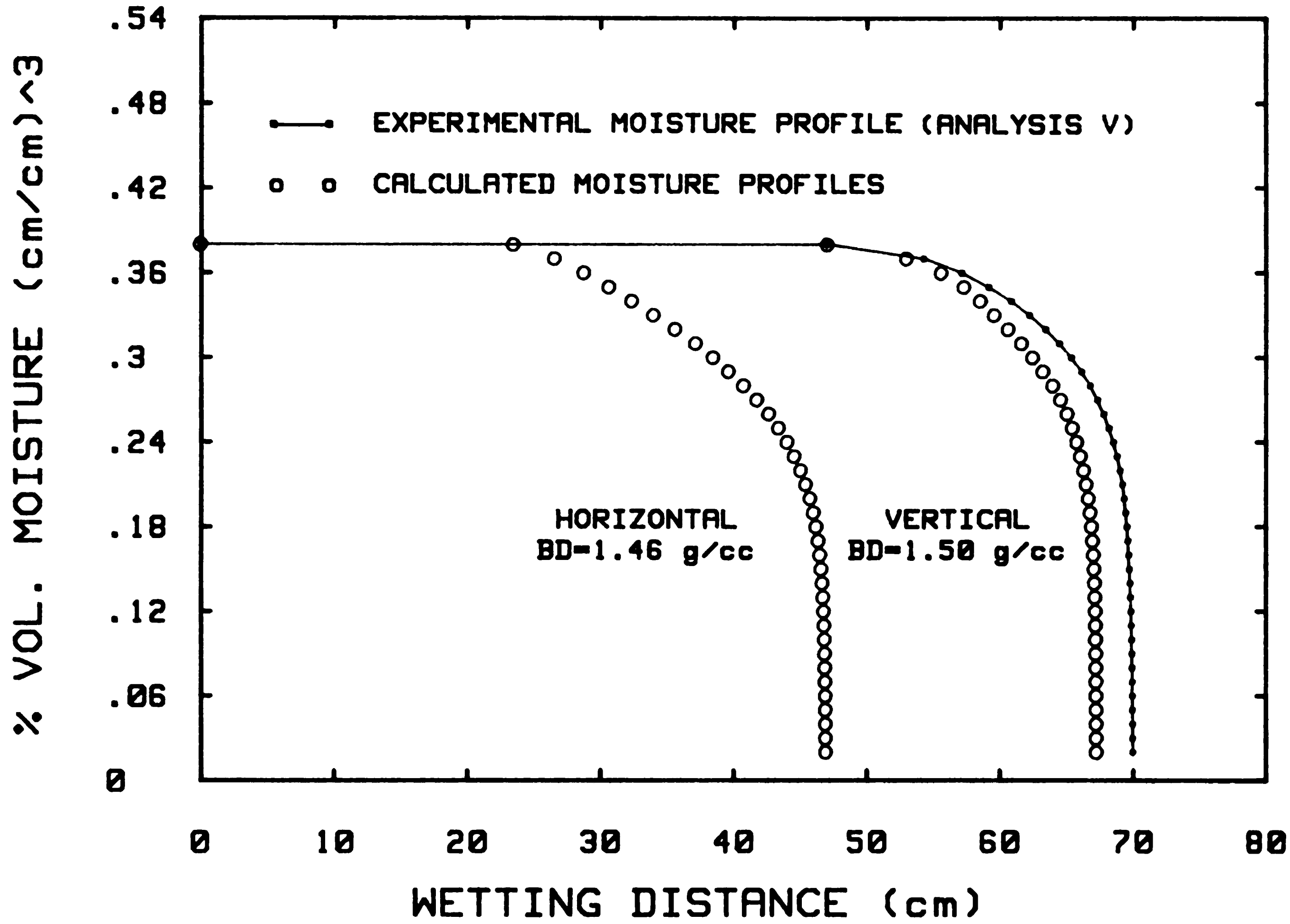

Figure 20. Horizontal and vertical soil moisture profiles obtained for Spinks sandy loam soil using assumption (1) at indicated bulk densities and 2020 minutes.

Table 6. Infiltration time and its relation to Θ-straight and maximum wetting distances.

Soil	Analysis	Infiltration time Horizontal (min)	Infiltration time vertical (min)	Θ-straight Horizontal (cm)	Θ-straight vertical (cm)	Maximum vertical dist. measured (cm)	Maximum vertical dist. calculated (cm)	Diff. (cm)
Metea	I	2175	2270	18.39	47.00	70.00	68.69	1.31
	III	2076	827	17.67	49.00	70.10	58.84	11.26
Spinks	V	4500	2020	23.20	47.00	70.00	67.23	2.77

equal to .04 g/cc while in Metea (I) it is equal to .01 g/cc; however, this is not to preclude the possibility of other experimental errors. Metea (III) on the other hand, showed a difference of 11.26 cm, a significant lack of agreement, between predicted and observed vertical profiles (Table 6). This discrepancy can be attributed to the long horizontal infiltration time of 2076 minutes versus the short vertical infiltration time of 827 minutes, resulting in relatively small D and K values. Once D values are established, K values have a definite upper bound because of the K = DC relationship. Furthermore, if the boundary and initial conditions assumed for the flow system did not actually exist as a result of experimental error, discrepancies between predicted and experimental infiltration profiles could be anticipated.

The calculated profiles could be also in error if the Θ-straight choice for either the horizontal or vertical moisture profiles was uncertain. The concept of a Θ-straight, the distance (x_o) from the water source to unsaturated zone, was introduced by Philip (1958), observed by (Nielsen et al. 1962; Jackson, 1963; and Davidson et al., 1963) and discussed by Kunze and Nielsen.[1] From the concept of capillary rise it is established that all homogeneous soils will develop a Θ-straight if wetted at zero or small negative pressure. Both soils used here exhibit a Θ-straight in their moisture profiles, shown in Figure 9 and Figures 17-20.

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1 Private communications from authors.

Excellent agreement between the calculated and measured soil moisture profiles was obtained for Metea Analysis I under assuptions 2 and 3 which are presented in Figures 18 and 19. Similar agreement was obtained for Metea Analysis III and Spinks Analysis V but presenting them here would have been redundant.

IV. Demonstration of Generated Data for Modeling Other Flow System

It is quite possible now to use generated D and K data as a form of computer input and the proposed FINDIT procedure for modeling other flow systems. As was indicated earlier, once D values are obtained for a given soil at a specific time, then the flow equation may be solved for any other time to generate new profiles. This is particularly true with horizontal profiles, in fact all horizontal moisture profiles given in Figures 17-20 are generated profiles, and may be applied to vertical profiles as well under the assumptions discussed.

This procedures requires only D and K values as input for each soil. No matching factors or further approximation are required for solving the flow equation. Changing the boundary and/or initial conditions to test their effect on the output is also part of this procedure. This technique is and continues to be useful in further testing of Equations (13) and (14) and sometimes is useful to exploit a particular soil character. Once D and K functions are in hand, it is equally simple to demonstrate capillary rise for a variety of initial and boundary conditions; however, it is an investigation in itself and is outside the bounds of this study.

SUMMARY AND CONCLUSIONS

An experimental study was conducted on the Metea (Arenic Hapludalfs; sandy over loamy, mixed, mesic) and Spinks (Psammentic Hapludalfs; sandy, mixed, mesic) sandy loam soils to investigate the validity of Richards' equation for describing one-dimensional flow of water in soil. A numerical simulation technique, FINDIT, was used for obtaining the solution of the Richards' equation by a finite difference, iterative procedure. The flow system considered in this study was semi-infinte with water applied at one end of a homogenous, uniform, rigid, porous material packed in columns. The semi-infinite condition requires that the wetting front never reach the end of the column.

To solve the flow equation, the horizontal and vertical soil moisture profiles and soil moisture characteristic were used in the computer analysis. To circumvent variations in bulk density between column pairs, temperature differences during infiltrations, experimental errors, etc., three assumptions were invoked for solving the infiltration equation under unsaturated conditions. These assumptions improved the output of the moisture profiles and their respective D and K functions. Results for two experimental runs on Metea soil and one on Spinks soil are presented and discussed.

Satisfactory agreement between experimental data and theoretical calculations was obtained when assumption (1) was considered for Metea Analysis I and Spinks Analysis V. The same

comparison for Metea Analysis III produced only fair agreement.

When the computer matched the given ψ and D values according to assumption (2) and (3), respectively, excellent agreement was obtained between experimental and calculated vertical profiles. A comparison of D and K functions obtained under assumption (1) with those obtained under assumptions (3) suggests that the variation due to bulk density, temperature and experimental errors can be circumvented. Improvement in packing and sectioning of soil columns resulting in more uniform bulk densities within columns should lead to more conclusive results.

It may be concluded that the infiltration equation and its solution considered herein can be used to describe water movement through unsaturated porous materials. However, further improvement in the analysis and rigorous testing of some assumptions on which the equation is based is suggested. The validity of the Boltzmann transformation is questioned on the basis of data presented. In spite of the limitations of the experimental technique, the FINDIT procedure was found to be quite satisfactory for solving water flow problems in the soils tested.

RECOMMENDATIONS FOR FURTHER STUDIES

From the results of this study further investigations of water flow through porous media under unsaturated conditions are recommended. Particularly, emphasis in the following areas should be considered:

1. Continued improvement for getting rapid, reproducible means of packing soil columns with uniform bulk density for further testing of the flow equations.

2. Work should be continued with capillary rise and measurements of the absorption moisture characteristics. These results combined with the vertical and horizontal flow data are basic to generating reliable diffusivity and conductivity functions for a specific soil.

3. More intensive use should be made of modern technology, specifically the computer, for solving unsaturated water flow problems and thereby realizing saving of time, money and manpower. Continued development of computer techniques in these investigations will be extremely helpful and is highly recommended.

4. Testing of the (FINDIT) procedure on very fine to very coarse textured soils should be of value.

5. Research should continue on diffusion theory to examine its validity for a wide range of initial and boundary conditions and improve its applicability to the water movement under unsaturated conditions.

6. Applying what we know theoretically and from laboratory measurements to field problems is extremely important and should be considered in the future work.

APPENDIX

EXPERIMENTAL DATA

Table 7. Horizontal infiltration data on Metea sandy loam at 2175 minutes. Analysis I.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% volume (Θ)	% Ave. Volume (Θ)
1.00	21.34	1.34	28.52	32.42
3.00	22.62	1.60	36.21	34.37
5.00	24.47	1.58	38.58	37.18
7.00	25.00	1.59	39.69	37.98
9.00	25.91	1.56	40.44	39.36
11.00	25.42	1.56	39.62	38.62
13.00	24.89	1.56	38.78	37.82
14.50	25.47	1.46	37.10	38.69
16.50	25.05	1.54	38.51	38.06
17.50	24.47	1.58	38.76	37.18
18.50	24.10	1.48	35.72	36.62
19.50	24.35	1.58	38.56	37.00
20.50	24.42	1.52	37.14	37.10
21.50	24.45	1.54	37.76	37.15
22.50	24.03	1.58	37.99	36.51
23.50	23.07	1.50	34.49	35.05
24.50	23.32	1.62	37.86	35.44
25.50	22.90	1.41	32.19	34.79
26.50	22.80	1.51	34.47	34.65
27.50	22.07	1.56	34.39	33.53
28.50	22.27	1.42	31.73	33.83
29.50	21.91	1.59	34.82	33.28
30.50	21.51	1.43	30.85	32.68
31.50	20.78	1.46	30.36	31.57
32.50	26.67	1.57	32.40	31.40
33.50	20.53	1.51	31.07	31.19
34.50	20.24	1.48	29.94	30.76
35.50	19.76	1.47	29.02	30.02
36.50	18.94	1.53	28.92	28.78
37.50	19.00	1.64	31.24	28.86
38.50	18.24	1.46	26.61	27.71
39.50	17.65	1.51	26.71	26.82

Table 7...continued

40.50	17.04	1.39	23.64	25.89
41.50	16.52	1.56	25.83	25.10
42.50	15.91	1.48	23.48	24.18
43.50	14.87	1.45	21.52	22.59
44.50	13.62	1.54	20.99	20.70
45.50	11.28	1.578	17.78	17.13
45.74				1.50†

@ Arithmetic mean = 1.519 g/cc
Standard deviation = .0693
Coefficient of variation = 4.561

† Initial moisture content (Θ_n)

Table 8. Vertical infiltration data on Metea sandy loam at 2270 minutes. Analysis I.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volume (Θ)	% Ave. Volume (Θ)
1.00	21.69	1.56	33.84	33.08
3.00	22.60	1.59	36.02	34.46
5.00	23.07	1.62	37.30	35.18
7.00	23.91	1.60	38.20	36.46
9.00	20.57	1.62	33.30	31.36
11.00	25.70	1.55	39.85	39.19
13.00	25.27	1.60	40.38	38.52
15.00	25.70	1.53	39.25	39.19
17.00	25.78	1.54	39.76	39.30
19.00	25.34	1.54	39.14	38.63
21.00	24.52	1.55	37.88	37.38
23.00	24.21	1.61	38.89	36.92
25.00	25.31	1.49	37.79	38.59
27.00	25.02	1.52	37.92	38.15
29.00	25.97	1.43	37.20	39.60
31.00	25.38	1.48	37.65	38.70
33.00	24.87	1.49	37.03	37.92
35.00	25.25	1.45	36.55	38.51
37.00	25.80	1.49	38.34	39.34
39.00	25.37	1.42	35.93	38.68
40.50	25.14	1.62	40.60	38.33
41.50	24.36	1.49	36.23	37.14
42.50	24.71	1.49	36.80	37.68
43.50	24.66	1.49	36.36	37.60
44.50	24.28	1.53	37.04	37.01
45.50	23.86	1.66	39.64	36.38
46.50	23.07	1.41	32.44	35.18
47.50	23.45	1.46	34.29	35.75
48.50	24.01	1.43	34.37	36.60
49.50	24.23	1.51	36.56	36.94
50.40	24.43	1.56	38.10	37.24
51.40	23.82	1.47	35.11	36.32
52.40	23.95	1.46	34.89	36.52

Table 8...continued

53.60	23.74	1.52	36.11	36.20
54.60	23.50	1.50	35.26	35.83
55.60	22.72	1.49	33.79	34.65
56.60	22.73	1.52	34.53	34.66
57.60	22.41	1.44	32.19	34.16
58.60	22.29	1.56	34.69	33.99
59.60	22.06	1.46	32.24	33.63
60.60	21.79	1.53	33.38	33.22
61.60	21.37	1.47	31.32	32.59
62.60	20.61	1.64	33.90	31.42
63.60	19.71	1.41	27.85	30.06
64.60	19.86	1.62	32.07	30.28
65.60	18.97	1.59	30.17	28.92
66.60	17.85	1.49	26.62	27.22
67.60	16.97	1.57	26.65	25.87
68.60	14.88	1.54	22.90	22.68
69.60	12.58	1.64	21.08	19.19
70.00				1.15†

@ Arithmetic mean = 1.525 g/cc
Standard deviation = .0677
Coefficient of variation = 4.44

† Initial moisture content (Θ_n)

Table 9. Horizontal infiltration data on Metea sandy loam at 6167 minutes. Analysis II.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volume (Θ)	% Ave. Volume (Θ)
1.00	20.86	1.61	33.68	31.68
3.00	21.59	1.52	32.80	33.08
5.00	22.51	1.61	36.14	34.49
7.00	23.28	1.59	37.01	35.67
9.00	24.53	1.59	38.90	37.59
11.00	25.70	1.57	40.39	39.38
13.00	25.40	1.60	40.62	38.92
15.00	26.40	1.50	40.31	40.45
17.00	26.02	1.54	39.96	39.87
19.00	26.09	1.51	39.39	39.97
21.00	25.16	1.56	39.18	38.55
23.00	25.37	1.51	38.40	38.40
25.00	25.13	1.49	37.49	38.49
27.00	24.57	1.53	37.58	37.65
29.00	24.74	1.47	36.31	37.90
31.00	24.02	1.50	36.08	36.80
33.00	24.11	1.53	36.88	36.94
35.00	24.31	1.47	35.66	37.24
36.09	23.55	1.54	36.35	36.35
39.00	23.74	1.48	35.24	36.36
40.50	23.04	1.52	34.99	35.29
41.50	22.82	1.55	35.42	34.97
42.60	23.39	1.54	35.99	35.85
43.50	22.16	1.49	33.08	33.94
44.50	27.12	1.38	37.43	41.55
45.50	22.03	1.50	33.15	33.75
46.50	21.16	1.52	32.14	32.41
47.50	20.86	1.47	30.68	31.97
48.50	21.40	1.51	32.37	32.79
49.50	18.90	1.65	31.14	28.96
50.50	20.47	1.53	31.24	31.36
51.50	20.39	1.44	29.41	31.24
52.50	20.27	1.63	33.08	31.06

Table 9...continued

53.50	19.84	1.45	28.85	30.39
54.50	19.10	1.54	29.45	29.27
55.50	18.81	1.56	29.34	28.82
56.50	17.84	1.47	26.20	27.33
57.50	18.01	1.57	28.21	27.59
58.50	17.19	1.46	25.02	26.34
59.50	17.44	1.52	26.57	26.71
60.50	16.63	1.55	25.83	25.49
61.50	16.35	1.55	25.38	25.05
62.50	14.89	1.49	22.18	22.81
63.60	15.71	1.69	26.53	24.06
64.60	14.83	1.53	22.63	22.72
65.60	15.89	1.62	25.67	24.35
66.60	13.48	1.45	19.55	20.66
67.60	12.92	1.64	21.21	19.80
68.60	11.86	1.48	17.51	18.18
69.60	10.71	1.57	16.71	16.42
70.00				1.05†

@ Arithmetic mean = 1.532 g/cc
Standard deviation = .0599
Coefficient of variation = 3.90

† Initial moisture content (Θ_n)

Table 10. Vertical infiltration data on Metea sandy loam at 2690 minutes. Analysis II.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volume (Θ)	% Ave. Volume (Θ)
1.00	21.70	1.62	35.15	33.17
3.00	22.88	1.61	36.94	34.96
5.00	25.80	1.55	39.97	39.42
7.00	25.63	1.53	39.20	39.17
9.00	25.21	1.53	38.51	38.52
11.00	25.61	1.54	39.44	39.13
13.00	24.24	1.61	38.93	37.04
15.00	26.01	1.47	38.30	39.74
17.00	26.99	1.49	40.28	41.28
19.00	27.01	1.51	40.75	41.28
21.00	27.43	1.47	40.28	41.91
23.00	27.14	1.51	41.05	41.47
25.00	27.08	1.54	41.62	41.39
27.00	26.85	1.51	40.66	41.03
29.00	25.68	1.52	39.00	39.24
31.00	26.48	1.56	41.19	40.46
33.00	21.43	1.60	34.34	32.75
35.00	26.67	1.54	41.16	40.56
37.00	30.03	1.44	43.26	45.90
39.00	25.25	1.48	37.49	38.58
40.50	25.06	1.57	39.19	38.30
41.50	25.18	1.47	36.94	38.49
42.50	25.24	1.53	38.58	38.58
43.50	24.77	1.44	35.54	37.85
44.50	24.64	1.56	38.49	37.66
45.50	23.69	1.51	35.89	36.20
46.50	23.45	1.60	37.61	35.84
47.50	22.16	1.46	32.29	33.87
48.50	22.69	1.74	39.47	34.68
49.50	23.34	1.64	38.23	35.66
50.40	23.27	1.51	35.13	35.56
51.40	22.76	1.55	35.39	34.78
52.40	22.72	1.51	34.22	34.72

Table 10...continued

53.50	22.95	1.48	34.00	35.07
54.60	22.60	1.69	38.12	34.53
55.60	22.01	1.49	32.87	33.63
56.60	21.26	1.51	32.09	32.50
57.60	20.10	1.60	32.23	30.72
58.60	19.84	1.46	28.88	30.31
59.60	20.91	1.54	32.13	31.95
60.60	19.82	1.47	29.09	30.29
61.60	19.13	1.49	28.50	29.23
62.60	19.53	1.51	29.54	29.84
63.60	18.80	1.49	28.00	28.73
64.60	17.20	1.45	24.95	26.28
65.60	16.19	1.49	24.04	24.74
66.60	13.92	1.46	20.34	21.27
67.60	12.50	1.50	18.78	19.10
68.00				1.05†

@ Arithmetic mean = 1.528 g/cc
Standard deviation = .0638
Coefficient of variation = 4.18

† Initial moisture content (Θ_n)

Table 11. Horizontal infiltration data on Metea sandy loam at 2076 minutes. Analysis III.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volume (Θ)	% Ave. Volume (Θ)
1.00	22.07	1.57	34.54	32.54
3.00	21.16	1.57	33.19	31.20
5.00	22.66	1.57	35.50	33.40
7.00	24.07	1.52	36.71	35.71
9.00	24.70	1.53	37.87	36.41
11.00	24.70	1.54	37.93	36.41
13.00	24.94	1.58	39.53	36.77
15.00	25.25	1.53	38.53	37.22
17.00	24.90	1.56	38.86	36.70
19.00	25.12	1.46	36.79	37.03
21.00	26.28	1.50	39.42	38.75
23.00	26.34	1.44	37.90	38.83
25.00	25.86	1.42	36.79	38.13
27.00	24.95	1.43	35.76	36.78
29.00	25.30	1.45	36.71	37.30
31.00	25.14	1.44	36.08	37.06
33.00	24.96	1.40	34.97	36.79
35.00	23.40	1.40	32.76	34.49
37.00	23.93	1.44	34.47	35.28
39.00	23.18	1.44	33.31	34.17
40.50	22.86	1.46	33.38	33.70
41.50	21.70	1.42	30.85	32.00
42.50	22.70	1.42	32.24	33.46
43.50	23.90	1.44	34.37	35.23
44.50	21.67	1.44	31.14	31.95
45.50	20.60	1.41	29.11	30.37
46.50	19.36	1.46	28.20	28.55
47.50	18.46	1.35	24.91	27.22
48.50	18.81	1.40	26.27	27.74
49.50	18.62	1.60	29.82	27.45
50.50	17.46	1.46	25.51	25.74
51.50	17.26	1.54	26.53	25.44
52.50	17.60	1.58	27.77	25.95

Table 11...continued

53.50	17.17	1.45	24.87	25.31
54.50	15.79	1.47	23.24	23.28
55.50	15.20	1.45	22.03	22.41
56.50	14.43	1.45	20.94	21.28
57.50	13.48	1.49	20.07	19.87
58.50	12.12	1.46	17.65	17.87
59.50	10.05	1.44	14.46	14.81
60.10				1.00†

@ Arithmetic mean = 1.474 g/cc
Standard deviation = .0620
Coefficient of variation = 4.20

† Initial moisture content (Θ_n)

Table 12. Vertical infiltration data on Metea sandy loam at 827 minutes. Analysis III.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volume (Θ)	% Ave. Volume (Θ)
1.00	22.52	1.47	33.20	33.07
3.00	22.62	1.56	35.22	33.22
5.00	23.62	1.53	36.22	34.69
7.00	24.57	1.57	38.51	36.11
9.00	25.29	1.46	37.03	37.14
11.00	25.31	1.51	38.14	37.17
13.00	25.46	1.48	37.59	37.40
15.00	25.17	1.54	38.74	36.97
17.00	25.79	1.45	37.50	37.87
19.00	25.52	1.51	38.65	37.48
21.00	26.31	1.46	38.30	38.63
23.00	26.64	1.46	38.76	39.12
25.00	26.18	1.48	38.86	38.45
27.00	26.54	1.48	39.21	38.97
29.00	25.75	1.41	36.42	37.82
31.00	26.33	1.46	38.38	38.66
33.00	25.75	1.48	38.10	37.82
35.00	25.90	1.47	39.97	38.03
37.00	25.73	1.43	36.89	37.78
39.00	25.44	1.43	36.31	37.36
40.50	25.87	1.49	38.45	37.99
41.50	25.72	1.43	36.84	37.77
42.50	25.77	1.45	37.29	37.85
43.50	25.03	1.48	36.97	36.76
44.50	25.14	1.48	37.28	36.92
45.50	25.37	1.41	35.89	37.26
46.50	25.38	1.51	38.31	37.27
47.50	25.44	1.44	36.57	37.36
48.50	24.78	1.42	35.21	36.39
49.50	24.46	1.42	34.66	35.92
50.50	24.25	1.50	36.46	35.62
51.50	25.03	1.48	37.09	36.77
52.50	25.15	1.49	37.39	36.94

Table 12...continued

53.50	27.39	1.42	38.81	40.23
54.50	21.16	1.43	30.34	31.07
55.50	23.55	1.36	31.97	34.58
56.50	23.31	1.50	34.97	34.24
57.50	24.09	1.45	34.83	35.37
58.50	23.01	1.46	33.53	33.80
59.50	29.66	1.40	41.46	43.55
60.50	22.67	1.57	35.58	33.30
61.50	21.76	1.45	31.52	31.96
62.50	21.57	1.43	30.83	31.67
63.50	20.17	1.41	28.49	29.62
64.50	19.66	1.44	28.39	28.87
65.50	19.58	1.52	29.70	28.76
66.50	18.45	1.55	28.51	27.10
67.50	16.42	1.48	24.34	24.12
67.90	15.65	1.47	23.00	23.00
69.01	12.93	1.47	19.00	19.00
69.72	9.52	1.47	14.00	14.00
70.10				1.00†

@ Arithmetic mean = 1.469 g/cc
Standard deviation = .0458
Coefficient of variation = 3.12

† Initial moisture content (Θ_n)

Table 13. Horizontal infiltration data on Metea sandy loam at 2305 minutes. Analysis IV.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volume (Θ)	% Ave. Volume (Θ)
1.00	22.56	1.56	35.21	33.78
3.00	22.11	1.49	32.85	33.11
5.00	22.94	1.56	35.80	34.34
7.00	23.31	1.55	36.06	34.90
9.00	24.46	1.56	38.21	36.62
11.00	25.34	1.54	38.96	37.94
13.00	25.25	1.59	40.19	37.81
15.00	25.63	1.49	38.28	38.38
17.00	26.46	1.50	39.66	39.61
19.00	26.56	1.44	38.36	39.76
21.00	25.22	1.50	37.73	37.76
23.00	25.02	1.47	36.86	37.46
25.00	25.26	1.46	36.94	37.83
27.00	24.38	1.48	36.06	36.50
29.00	24.31	1.48	35.91	36.40
31.00	23.77	1.47	34.95	35.59
33.00	23.22	1.48	34.39	34.77
35.00	23.15	1.46	33.89	34.67
37.00	22.55	1.44	32.53	33.76
39.00	21.96	1.44	31.68	32.88
40.50	21.83	1.49	32.48	32.69
41.50	21.80	1.49	32.38	32.64
42.50	21.40	1.42	30.41	32.04
43.50	20.74	1.45	30.10	31.05
44.50	21.09	1.51	31.93	31.58
45.50	20.97	1.40	29.41	31.39
46.50	19.96	1.49	29.78	29.89
47.50	20.88	1.47	30.68	31.26
48.50	18.61	1.53	28.54	27.87
49.50	19.05	1.49	28.38	28.52
50.50	18.49	1.56	28.79	27.68
51.50	17.54	1.62	28.37	26.27
52.50	17.46	1.52	26.47	26.14

Table 13...continued

53.50	17.21	1.47	25.36	25.76
54.50	16.27	1.49	24.24	24.37
55.50	14.97	1.52	22.82	22.41
56.50	14.48	1.42	20.54	21.67
57.50	13.34	1.54	20.56	19.97
58.50	12.12	1.54	18.74	18.28
59.50	10.37	1.50	15.53	15.53
60.10				.90†

@ Arithmetic mean = 1.497 g/cc
Standard deviation = .0480
Coefficient of variation = 3.21

† Initial moisture content (Θ_n)

Table 14. Vertical infiltration data on Metea sandy loam at 840 minutes. Analysis IV.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volume (Θ)	% Ave. Volume (Θ)
1.00	22.35	1.45	32.49	33.13
3.00	23.39	1.51	35.42	34.66
5.00	24.59	1.55	38.17	36.44
7.00	24.80	1.54	38.12	36.75
9.00	25.45	1.45	36.79	37.72
11.00	25.17	1.53	38.49	37.30
13.00	25.18	1.49	37.64	37.32
15.00	25.97	1.51	39.27	38.49
17.00	26.12	1.49	38.84	38.71
19.00	25.75	1.47	37.72	38.16
21.00	26.09	1.42	37.15	38.66
23.00	26.46	1.49	39.42	39.21
25.00	25.52	1.48	37.68	37.81
27.00	25.94	1.44	37.31	38.44
29.00	25.79	1.49	38.35	38.22
31.00	26.12	1.45	37.93	38.72
33.00	26.02	1.50	39.03	38.56
35.00	26.30	1.47	38.61	38.97
37.00	25.85	1.45	37.48	38.30
39.00	25.55	1.45	37.05	37.86
40.50	25.47	1.48	37.75	37.74
41.50	25.08	1.46	36.64	37.17
42.50	25.03	1.46	36.50	36.10
43.50	25.11	1.47	36.97	37.22
44.50	24.63	1.52	37.38	36.51
45.50	25.00	1.54	38.47	37.05
46.50	25.05	1.43	35.73	37.13
47.50	24.69	1.50	36.96	36.59
48.50	24.21	1.47	35.69	35.88
49.50	24.56	1.51	37.21	36.40
50.50	24.56	1.46	35.85	36.40
51.50	24.21	1.49	36.09	35.88
52.50	23.93	1.50	35.81	35.46

Table 14...continued

53.50	23.82	1.47	34.96	35.30
54.50	23.21	1.43	33.11	34.40
55.50	23.46	1.52	35.76	34.77
56.50	22.85	1.46	33.28	33.87
57.50	22.93	1.50	34.33	33.99
58.50	21.73	1.35	29.38	32.20
59.50	21.72	1.59	34.48	32.19
60.50	21.08	1.52	32.13	31.24
61.50	20.73	1.47	30.45	30.72
62.50	19.88	1.47	29.25	29.46
63.50	19.15	1.50	28.69	28.38
64.50	17.98	1.50	26.92	26.64
65.50	16.28	1.48	24.14	24.12
66.50	13.44	1.48	19.91	19.91
67.10				.90†

@ Arithmetic mean = 1.482 g/cc
Standard deviation = .0397
Coefficient of variation = 2.68

† Initial moisture content (Θ_n)

Table 15. Horizontal infiltration data on Spinks sandy loam at 4500 minutes. Analysis V.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% volume (Θ)	% Ave. Volume (Θ)
1.00	24.88	1.42	35.43	36.33
3.00	25.06	1.51	37.83	36.59
5.00	25.46	1.51	38.52	37.18
7.00	25.53	1.52	38.84	37.28
9.00	27.10	1.52	41.31	39.57
11.00	28.41	1.52	43.08	41.49
13.00	27.66	1.49	41.09	40.38
15.00	26.21	1.58	41.52	38.27
17.00	26.78	1.53	40.86	39.11
19.00	26.74	1.52	40.67	39.05
21.00	27.25	1.44	39.13	39.80
23.00	26.93	1.48	39.79	39.33
25.00	27.40	1.42	38.89	40.01
27.00	27.18	1.45	39.44	39.70
29.00	27.10	1.39	37.65	39.57
31.00	26.66	1.46	38.98	38.94
33.00	26.02	1.43	37.32	38.00
35.00	26.19	1.40	36.79	38.24
37.00	26.13	1.43	37.44	38.16
39.00	25.20	1.41	35.59	36.80
40.50	25.23	1.39	35.09	36.85
41.50	24.26	1.53	37.03	35.42
42.50	24.36	1.43	34.87	35.58
43.50	23.83	1.38	32.78	34.79
44.50	24.18	1.43	34.58	35.30
45.50	24.05	1.42	34.13	35.12
46.50	23.12	1.42	32.73	33.76
47.50	23.44	1.41	33.12	34.23
48.50	22.77	1.51	34.47	33.25
49.50	21.90	1.57	34.39	31.98
50.50	21.70	1.36	29.50	31.69
51.50	21.91	1.32	28.83	32.00
52.50	22.65	1.52	34.49	33.07

Table 15...continued

53.50	21.67	1.42	30.74	31.64
54.50	21.61	1.45	31.25	31.55
55.50	21.15	1.51	32.01	30.88
56.50	20.52	1.37	28.18	29.97
57.50	20.54	1.45	29.70	29.99
58.50	19.60	1.53	29.91	28.61
59.50	19.39	1.42	27.45	28.32
60.50	19.44	1.52	29.51	28.38
61.50	18.69	1.45	27.11	27.30
62.50	19.66	1.38	27.04	28.71
63.50	17.04	1.50	25.52	24.88
64.50	17.44	1.55	27.07	25.47
65.50	16.47	1.51	24.88	24.05
66.50	15.96	1.40	22.40	23.30
67.50	14.85	1.53	22.66	21.69
68.50	13.54	1.56	21.07	19.77
69.50	10.53	1.36	14.26	15.37
70.00				1.00†

@ Arithmetic mean = 1.460 g/cc
Standard deviation = .0638
Coefficient of variation = 4.37

† Initial moisture content (Θ_n)

Table 16. Vertical infiltration data on Spinks sandy loam at 2020 minutes. Analysis V.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volume (Θ)	% Ave. Volume (Θ)
1.00	23.45	1.60	37.43	35.14
3.00	24.04	1.55	37.31	36.01
5.00	23.73	1.57	37.22	35.55
7.00	23.52	1.60	37.53	35.24
9.00	25.26	1.50	37.87	37.85
11.00	25.16	1.57	39.57	37.69
13.00	24.62	1.59	39.24	36.89
15.00	24.42	1.58	38.55	36.58
17.00	24.73	1.54	38.15	37.05
19.00	25.09	1.56	39.04	37.60
21.00	24.67	1.55	38.15	36.96
23.00	25.35	1.54	39.12	37.97
25.00	25.29	1.54	38.98	37.88
27.00	25.06	1.56	39.06	37.55
29.00	25.17	1.50	37.81	37.72
31.00	25.85	1.48	38.38	38.74
33.00	26.00	1.49	38.64	38.96
35.00	25.93	1.56	40.48	38.85
37.00	25.85	1.39	36.06	38.74
39.00	25.93	1.49	38.57	38.85
40.50	25.23	1.57	39.59	37.80
41.50	26.00	1.44	37.53	38.96
42.50	25.60	1.50	38.28	38.35
43.50	25.36	1.46	36.99	38.00
44.50	25.46	1.54	39.30	38.14
45.50	25.18	1.50	37.67	37.73
46.50	26.03	1.49	38.80	39.00
47.50	25.30	1.37	34.73	37.91
48.50	25.86	1.52	39.37	38.74
49.50	24.82	1.40	34.76	37.18
50.50	24.86	1.61	40.05	37.24
51.50	24.66	1.47	36.29	36.95
52.50	25.02	1.46	36.50	37.48

Table 16...continued

53.50	24.21	1.55	37.56	36.27
54.50	23.86	1.53	36.55	35.75
55.50	24.25	1.40	33.87	36.33
56.50	23.50	1.41	33.18	35.22
57.50	23.98	1.50	35.92	35.93
58.50	23.60	1.45	34.12	35.36
59.50	23.65	1.48	34.97	35.44
60.50	22.71	1.44	32.64	34.02
61.50	22.75	1.50	34.24	34.09
62.50	21.66	1.48	32.02	32.46
63.50	22.03	1.45	31.83	33.00
64.50	20.81	1.50	31.24	31.18
65.50	20.26	1.45	29.41	30.35
66.50	18.73	1.51	28.31	28.07
67.50	17.85	1.45	25.93	26.75
68.50	15.80	1.54	24.34	23.67
69.50	12.89	1.18	15.21	19.31
70.00				1.00†

@ Arithmetic mean = 1.498 g/cc
Standard deviation = .0743
Coefficient of variation = 4.96

† Initial moisture content (Θ_n)

Table 17. Horizontal infiltration data on Spinks sandy loam at 1897 minutes. Analysis VI.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volume (Θ)	% Ave. Volume (Θ)
1.00	24.78	1.49	36.84	35.60
3.00	24.45	1.50	36.76	35.12
5.00	25.02	1.53	38.32	35.94
7.00	25.31	1.53	38.65	36.36
9.00	26.42	1.51	39.74	37.70
11.00	26.96	1.49	40.14	38.74
13.00	26.83	1.57	42.23	38.55
15.00	47.01	1.52	41.18	38.81
17.00	27.20	1.47	40.01	39.08
19.00	27.06	1.43	38.61	38.87
21.00	27.06	1.46	39.42	38.87
23.00	27.41	1.45	39.74	39.38
25.00	27.07	1.39	37.64	38.90
27.00	26.20	1.36	35.71	37.64
29.00	25.79	1.41	36.36	37.06
31.00	25.78	1.39	35.93	37.05
33.00	25.30	1.37	34.71	36.34
35.00	25.24	1.40	35.42	36.26
37.00	25.29	1.40	33.97	34.90
39.00	23.86	1.35	32.23	34.27
40.50	23.79	1.42	33.88	34.17
41.50	23.27	1.46	34.01	33.44
42.50	22.42	1.32	29.70	32.21
43.50	22.61	1.44	32.59	32.48
44.50	21.76	1.34	29.18	31.26
45.50	21.64	1.45	31.38	31.09
46.50	20.67	1.46	30.27	29.70
47.50	20.54	1.33	27.26	29.52
48.50	20.11	1.41	28.37	28.90
49.50	18.29	1.54	28.10	26.27
50.50	18.49	1.45	26.83	26.56
51.50	17.75	1.35	23.99	25.50
52.50	17.03	1.43	24.36	24.47

Table 17... continued

53.50	15.57	1.39	21.58	22.36
54.50	12.95	1.45	18.73	18.61
55.10				1.00†

@ Arithmetic mean = 1.437 g/cc
Standard deviation = .0646
Coefficient of variation = 4.50

† Initial moisture content (Θ_n)

Table 18. Vertical infiltration data on Spinks sandy loam at 1188 minutes. Analysis VI.

Wetting distance (cm)	% Mass (Θ_m)	Bulk density @ (g/cc)	% Volumne (Θ)	% Ave. Volume (Θ)
1.00	24.33	1.57	38.15	35.12
3.00	25.45	1.50	38.22	36.73
5.00	26.12	1.50	39.05	37.70
7.00	26.02	1.50	38.99	37.56
9.00	27.24	1.49	40.62	39.32
11.00	26.93	1.50	40.29	38.87
13.00	26.64	1.49	39.77	38.46
15.00	27.11	1.50	40.70	39.13
17.00	27.86	1.47	40.81	40.21
19.00	27.79	1.40	38.95	40.10
21.00	27.72	1.45	40.10	40.01
23.00	28.41	1.42	40.38	41.00
25.00	28.03	1.40	39.34	40.46
27.00	38.14	1.42	39.89	40.61
29.00	28.77	1.44	41.53	41.52
31.00	28.39	1.39	39.57	40.97
33.00	28.10	1.38	38.81	40.56
35.00	28.12	1.42	40.03	40.59
37.00	28.06	1.38	38.68	40.50
39.00	28.38	1.41	40.13	40.97
40.50	27.68	1.40	38.80	39.96
41.50	27.25	1.35	36.71	39.33
42.50	26.86	1.46	39.23	38.76
43.50	28.72	1.40	40.23	41.46
44.50	27.56	1.42	39.20	39.77
45.50	27.64	1.37	37.87	39.89
46.50	26.90	1.46	39.14	38.82
47.50	27.02	1.40	37.91	39.00
48.50	26.04	1.37	35.60	37.59
40.50	27.12	1.53	41.57	39.14
50.50	26.07	1.49	38.84	37.63
51.50	25.83	1.55	40.05	37.28
52.50	25.42	1.36	34.49	36.70

Table 18...continued

53.50	24.83	1.43	35.61	35.83
54.50	24.81	1.34	33.36	35.82
55.50	24.08	1.51	36.36	34.76
56.50	24.03	1.39	33.34	34.69
57.50	23.38	1.41	32.88	33.74
58.50	22.41	1.55	34.69	32.34
59.50	22.04	1.38	30.38	39.81
60.50	20.66	1.60	32.98	29.82
61.50	19.88	1.45	28.74	28.70
62.50	17.66	1.42	25.09	25.50
63.50	14.06	1.44	20.29	20.29
64.20				1.00†

@ Arithmetic mean = 1.443 g/cc
Standard deviation = .0629
Coefficient of variation = 4.36

† Initial moisture content (Θ_n)

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