EFFECT OF MULTIPLE DROPS ON THE DAMAGE BOUNDARY CURVE

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ABSTRACT

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This study investigated the effect of multiple drops on the so called "damage boundary curve". The damage done to a product by a shock is dependent on the shock pulse shape, its duration, and on the number of drops. Since handling of a product involves several drops of varying severity, both the product and the package are likely to suffer cumulative damage in the process even though each individual drop might involve much lower accelerations and velocity changes than the critical values indicated on the damage boundary curve. The effects of multiple drops were checked for two different products, light bulbs and bricks, and the damage boundary curves were developed and analyzed. The evidence showed that the number of previous drops before damage greatly altered the damage boundary curve. Hence, a product-package system is likely to accumulate damage with each drop and this factor should be included in the construction of the damage boundary curve.

In dedication to my late father, Dr. K. Kirpal Singh,
whose love and guidance has inspired me beyond words.

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LIST OF SYMBOLS

Symbol	Notation
m	mass of critical element
k	spring constant of critical element
M	mass of bulk
K	spring constant of bulk
G_M	maximum deceleration of mass M
W	weight of product
h	drop height
τ	duration of shock
$G_{max,a}$	maximum deceleration for 'a'
$f_{n,a}$	natural frequency of 'a'
A^*_{max}	amplification factor for square wave deceleration
A_{max}	amplification factor for half sine wave deceleration
A	input square wave acceleration
ΔV	velocity change
A_c	critical acceleration level
ΔV_c	critical velocity change

BACKGROUND

The idea behind the 'damage boundary curve' (DBC) is to characterize the fragility of products in relation to shocks incurred during handling and distribution (8). This method is a more improved measure of fragility over the former method where maximum accelerations or 'g-level' alone was used to describe fragility of a product. The method generally used to determine damage boundary curve (see Appendix B) assumes that damage to the product takes place only if the input shock exceeds the critical levels indicated on the DBC and that these levels are not a function of the number of drops prior to damage.

If we look into the theoretical development of an ideal DBC (Appendix A) we see that the product is modelled as a spring-mass system in series with a critical element (c.e.) which is also treated as a spring-mass system. Damage to the product is assumed to occur when the acceleration level of the critical element exceeds some limiting value for the element. Implied in the derivation of the DBC is that the model and its parameters remain constant throughout a series of drops as long as the c.e. is not damaged. Intuition however suggests that each drop tends to weaken the springs in this model, and eventually they fail as a result of elastic fatigue. It is a well known fact that the "cushion curves" developed by companies which produce such cushions as Ethafoam* etc., reflect changes in cushion performance with an increasing number of drops (see

* Trademark of The Dow Chemical Company

Figure 15,16). After each drop, the cushion's protective capability decreases and the shock levels transmitted to the cushioned product increase. It is reasonable to expect therefore, that such a degradation in strength also takes place in the product itself.

Now, product-package systems in normal distribution environments usually encounter several shocks of varying magnitude, and even though each of these shocks may be lower than the critical value, the cumulative effect may lead to damage of the product.

This study accomplishes two things:

1) It shows that a product can sustain damage due to repetitive shocks even if these shock levels are lower than those indicated on a conventional DBC.

2) It decribes the effect of multiple drops on the development of damage boundary curves for simple products like light bulbs and bricks.

INTRODUCTION

The generally accepted method of determining the damage boundary curve for a product is listed as ASTM Test Method D-3332-77, 'Mechanical Shock Fragility of Products Using Shock Machines' (8). It states, that the fragility of a product depends on three parameters of the shock pulse; shock pulse shape, shock pulse velocity change and shock pulse maximum faired acceleration. The purpose of this study is to include the effect of multiple drops on the damage boundary curve derived using the above method.

During the initial phase of this experiment, a fixture was developed for which damage is defined as the movement of a bolt through a slot. This setup models what happens to products containing parts fastened with bolts or rivets. It was observed that multiple drops damage the fixture as much as a more severe single shock.

Products such as light bulbs and bricks were then subjected to similar handling. The cumulative effect of each drop, however small the shock, is highly evident and this eventually leads to the failure of filaments in light bulbs and to the splitting of bricks. This effect is also pronounced in agricultural products like fruits and vegetables. Each drop adds to spoilage and decay in these products even though it may be of a severity below the critical level indicated on a conventional damage boundary curve.

DESIGN OF EXPERIMENTS

DEFINITION OF DAMAGE

It is very important to understand what the term 'damage' means and how it differs depending on the nature of the product. In general, damage is said to be diminished goodness, soundness or value of a product. For a light bulb, the breaking of a filament is damage to the whole light bulb, even though the glass remains intact. Similarly, for electronic equipment, excessive deformation of critical conponents may induce a short circuit and this renders the equipment useless. The breaking up of solids in food products may render them unsaleable and this is also considered to be damage. In most cases, damage to a product is usually a result of excessive internal stresses induced by shocks and inertial resistance. These take place when the product undergoes sudden deceleration of high magnitude (shocks) resulting in an almost instantaneous change in velocity.

It was experimentally shown by Kornhauser (4) that no damage to a product occurs until both a critical value of velocity change and a critical value of acceleration are exceeded. This leads to a damage boundary region in which the product is damaged. (see the derivation of the Damage Boundary Curve in Appendix A) The damage boundary curve for a product is determined using a shock machine. (see the procedure for this in Appendix B)

Damage Boundary Curves were determined for both single drop[1] and multiple drop treatments for different products to investigate the effect of multiple drops on the evolution of the conventional Damage Boundary Curve.

[1]As it is impossible to determine the critical drop height for damage in a single drop for each specimen apriori, a drop height at which damage to most specimens occured on the first drop was chosen as the definition for the single drop treatment. Naturally, it may take more than one drop in some cases due to sample variation. This is still considered a single drop treatment.

THE EFFECT OF MULTIPLE DROPS ON BOLTS IN SLOTS

For the initial stage of this experiment the fixture shown in Figure 1 was developed. It was made from a mild steel plate 0.125 inches thick which was bent to the required shape. A slot was cut at each end. In the center, a mild steel shaft 1.25 inches in diameter was clamped to the slotted plates using two bolts and washers. The plate was then screwed on to a base plate and the whole fixture was fastened to the table of the shock machine.

Initially, the bolts were tightened when the shaft was at the uppermost position as shown in Figure 1. Damage to this fixture was defined as the travel of the shaft from the top position to the bottom position inside the slot. It was experimentally shown that this could be accomplished not only by a single shock but by much less severe multiple shocks. The successive travel of the shaft and bolts is shown in Figure 2. Different shock levels were obtained by varying the drop heights on the shock machine using the 2ms plastic programmers. It was observed that it took eight to ten drops at a 10 inch drop height to produce the same damage as obtained from a single shock at a 30 inch drop height. Hence, damage in this case is a function of both shock level and the number of drops.

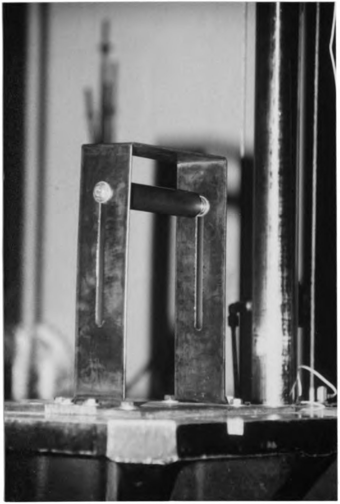

Figure 1. Bolt in slotted fixture.

Figure 2. Effect of multiple drops on bolt in slotted fixture.

MULTIPLE DROPS ON LIGHT BULBS

The effect of multiple drops on damage was also observed in light bulbs. The bulbs used for this experiment were 60 watt clear light bulbs made by the same company and bought in a single lot. A fixture consisting of two plastic sockets mounted on a plywood base was used to hold the bulbs. This fixture could be bolted down on the bed of the shock machine, allowing two bulbs to be tested at the same time. The fixture used is shown in Figure 3.

In the first part of this test a drop height was chosen using the 2ms plastic programmers so that the bulbs were damaged on the first drop. Damage to the bulb was defined as the breaking of the filament. (see Appendix A for the definition and discussion of a c.e.) The initial position of the filament is shown in Figure 4. A batch of bulbs was chosen at random and each bulb was dropped from a height of 30 inches. The number of drops required to break the filament in the bulb was recorded (Table 1) to check for sample variation.

In the second part, a much lower drop height of 22 inches was chosen and the bulbs were subjected to the same treatment. Again, the number of drops required to break the filaments as a result of multiple drops was recorded. (see Table 2) The filaments stretched (deformed) as a result of each drop (see Figure 5) until they eventually failed as shown in Figure 6.

It must however be remembered that the shock input to the table is much more than that transmitted to the filament, because the natural

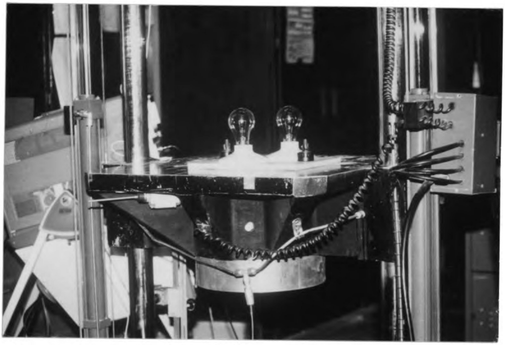

Figure 3. Fixture to hold light bulbs.

Figure 4. Initial position of filament before damage.

Figure 5. Extension of filament as a result of multiple drops.

Figure 6. Fail ure of filament due to multiple drops.

frequency of the filament (determined to be around 30 Hz) is much lower than the frequency of the shock input and this results in an amplification factor lower than one.

Similar tests were done using the gas programmer and varying the gas pressure to determine the critical acceleration values. Knowing the critical acceleration and the critical velocity change, the damage boundary curves were constructed for single and multiple drops. The results are shown in Tables 1,2,3, and 4.

MULTIPLE DROPS ON BRICKS

In this part, cement bricks were subjected to multiple drops and the damage boundary curves were determined. The bricks were mounted on the fixture shown in Figure 7. The fixture was designed so that the maximum stress concentration occured at the center of the brick and as a result of excessive inertial stresses, the bricks split in half. Even though bricks don't have identifiable critical elements, as do light bulbs, the same procedure was used to determine the damage boundary curve for this type of product.

The experimental procedures used were analogous to those used for light bulbs. A drop height of 11 inches was selected to produce damage in bricks as a result of a single shock using the 2ms plastic programmers. Lower drop heights of 7 inches were then used and the bricks were subjected to repetitive drops. The number of drops required to split them was recorded. In the latter case, each of the drops led to the propagation of micro-cracks that resulted in the final failure as shown in Figure 8.

Similar tests to those used for light bulbs were done to determine the critical acceleration values for single and multiple drops for bricks using the gas programmers. The results are shown in Tables 5,6,7 and 8.

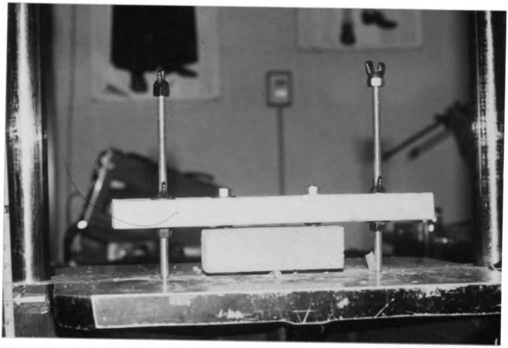

Figure 7. Fixture to hold cement bricks.

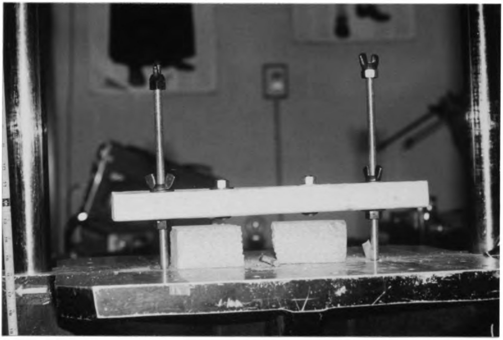

Figure 8. Failure of bricks.

Table 1. Critical Velocity Change Measurement for Single Drops: Light Bulbs

Sample Number	Drop height[a] (inches)	Velocity change[b] in/sec.	Number of drops required for damage
1	30	270	1
2	30	270	1
3	30	270	1
4	30	270	2
5	30	270	1
6	30	270	4
7	30	270	1
8	30	270	2
9	30	270	1
10	30	270	2
11	30	270	1

[a]Tests performed on the MTS Model 2424 Shock Machine in the laboratory of the School of Packaging at Michigan State University for 2ms half sine programmers, to determine velocity change measurement.

[b]The values used were determined from calibration tables for this machine determined by Goff and Twede (3).Errors associated with the above velocity change are unavailable.

Table 2. Critical Velocity Change Measurement for Multiple Drops: Light Bulbs

Sample Number	Drop height[a] (inches)	Velocity change[b] in/sec.	Number of drops required for damage
1	22	212	4
2	22	212	6
3	22	212	9
4	22	212	7
5	22	212	10
6	22	212	6
7	22	212	8
8	22	212	9
9	22	212	6
10	22	212	9
11	22	212	8

[a]Tests performed on the MTS Model 2424 Shock Machine in the laboratory of the School of Packaging at Michigan State University for 2ms half sine programmers, to determine velocity change measurements.

[b]The values used were determined from calibration tables for this machine determined by Goff and Twede (3).

Table 3. Critical Acceleration Measurements for Single Drops: Light Bulbs

Sample Number	Gas pressure[a] (psi)	Drop height inches	Critical[b] Acceleration g's	Number of drops required for damage
1	1800	48	370	2
2	1800	48	370	1
3	1800	48	370	3
4	1800	48	370	2
5	1800	48	370	1
6	1800	48	370	2
7	1800	48	370	1

[a]Tests performed on the MTS Model 2424 Shock Machine in the laboratory of the School of Packaging at Michigan State University.

[b]The values used were determined from calibration tables for this machine determined by Goff and Chatman (1).

Table 4. Critical Acceleration Measurements for Multiple Drops: Light Bulbs

Sample Number	Gas pressure[a] (psi)	Drop height inches	Critical[b] Acceleration g's	Number of drops required for damage
1	1000	48	180	13
2	1000	48	180	17
3	1000	48	180	16
4	1000	48	180	18
5	1000	48	180	19
6	1000	48	180	14
7	1000	48	180	22

[a]Tests performed on the MTS Model 2424 Shock Machine in the laboratory of the School of Packaging at Michigan University.

[b]The values used were determined from calibration tables for this machine determined by Goff and Chatman (1).

Table 5. Critical Velocity Change Measurements for Single Drops: Cement Bricks

Sample Number	Drop height[a] inches	Velocity change[b] in/sec.	Number of drops required for damage
1	11	130	1
2	11	130	1
3	11	130	2
4	11	130	1
5	11	130	1
6	11	130	1
7	11	130	1

[a]Tests performed on the MTS Model 2424 Shock Machine in the laboratory of the School of Packaging at Michigan State University for 2ms half sine programmers, to determine velocity change measurements.

[b]The values were determined from calibration tables for this machine determined by Goff and Twede (3).

Table 6. Critical Velocity Change Measurement for Multiple Drops: Cement Bricks

Sample Number	Drop height[a] inches	Velocity change[b] in/sec.	Number of drops required for damage
1	7	98	8
2	7	98	4
3	7	98	8
4	7	98	8
5	7	98	5
6	7	98	9
7	7	98	6

[a]Tests performed on the MTS Model 2424 Shock Machine in the laboratory of the School of Packaging at Michigan State University for 2ms half sine programmers, to determine velocity change measurements.

[b]The values were determined from calibration tables for this machine determined by Goff and Twede (3).

Table 7. Critical Accelereation Measurements for Single Drops: Cement Bricks

Sample Number	Gas pressure[a] (psi)	Drop height inches	Critical[b] Acceleration g's	Number of Drops required for damage
1	1500	30	235	1
2	1500	30	235	2
3	1500	30	235	1
4	1500	30	235	1
5	1500	30	235	1

[a]Tests performed on the MTS Model 2424 Shock Machine in the laboratory of the School of Packaging at Michigan State University.

[b]The values used were determined from calibration tables for this machine determined by Goff and Chatman (1).

Table 8. Critical Acceleration Measurements for Multiple Drops: Cement Bricks

Sample Number	Gas pressure[a] (psi)	Drop height inches	Critical[b] Acceleration g's	Number of drops required for damage
1	1300	24	190	8
2	1300	24	190	5
3	1300	24	190	6
4	1300	24	190	7
5	1300	24	190	8

[a]Tests performed on the MTS Model 2424 Shock Machine in the laboratory of the School of Packaging at Michigan State University.

[b]The values used were determined from calibration tables for this machine determined by Goff and Chatman (1).

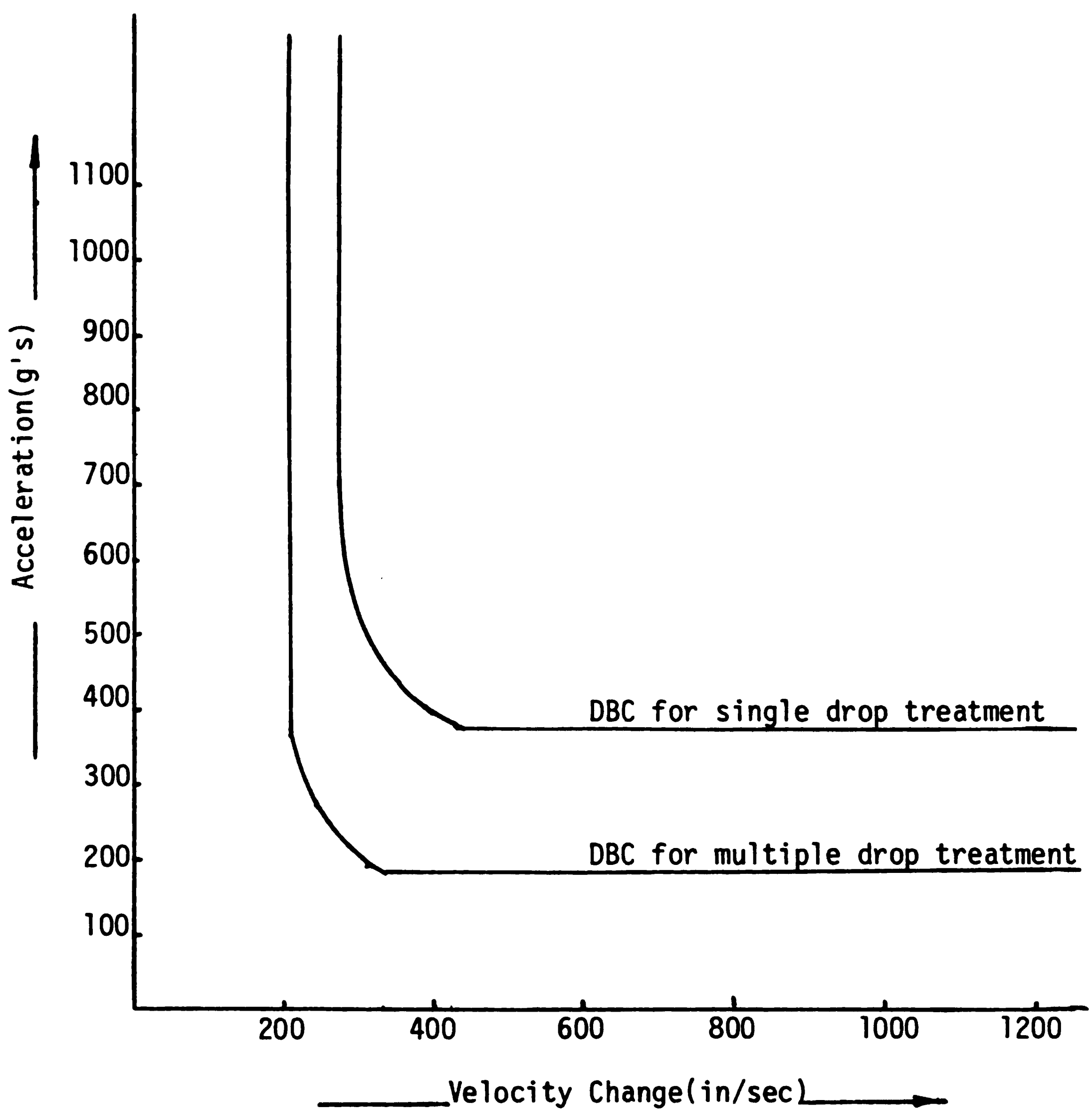

Figure 9. Damage Boundary Curves for light bulbs.

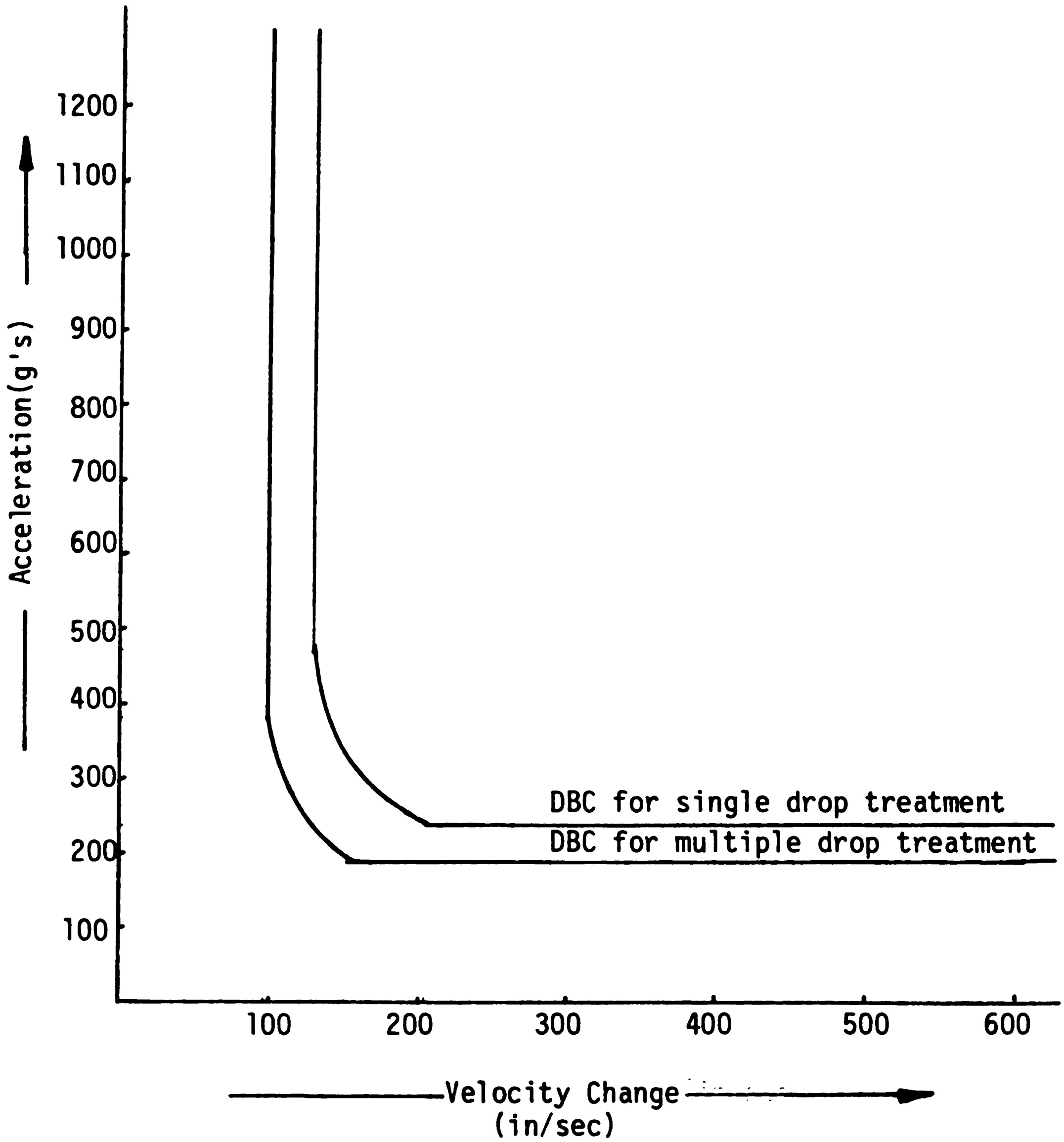

Figure 10. Damage Boundary Curves for cement bricks.

ANALYSIS OF DATA

We see that the bulbs require a shock input with ΔV_c = 270 in/sec and A_c = 370 g's for a single drop treatment to produce damage (From Tables 1 and 3). On the other hand for a multiple drop treatment, there is a decrease in these values and we get damage at lower values of ΔV_c = 212 in/sec and A_c = 180 g's (From Tables 2 and 4). This clearly indicates that the product suffers damage at lower shock inputs as a result of the cumulative effects of the multiple drops.

The same reduction in strength occurs for cement bricks which show damage at ΔV_c = 130 in/sec and A_c = 235 g's (From Tables 5 and 7) for single drop treatment as compared to ΔV_c = 98 in/sec and A_c = 190 g's (From Tables 6 and 8) for multiple drop treatments.

Light Bulbs:

a) ΔV_c measurement; From Tables 1 and 2

Mean number of drops for single drop treatment, $X_a = \frac{17}{11} = 1.54$

Standard deviation for single drop treatment, $S_a = \left[\frac{35-26}{10}\right]^{\frac{1}{2}} = 0.94$

Mean number of drops for multiple drop treatment, $X_b = \frac{82}{11} = 7.45$

Standard deviation for multiple drop treatment, $S_b = \left[\frac{644-610.5}{10}\right]^{\frac{1}{2}} = 1.83$

b) A_c measurement; From Tables 3 and 4

$$X_a = \frac{12}{7} = 1.71$$

$$S_a = \left[\frac{24-20.4}{6}\right]^{\frac{1}{2}} = 0.77$$

$$X_b = \frac{119}{7} = 17.0$$

$$S_b = \left[\frac{2079-2023}{6}\right]^{\frac{1}{2}} = 3.06$$

Cement Bricks:

a) ΔV_c measurement; From Tables 5 and 6

$$X_a = \frac{8}{7} = 1.14$$

$$S_a = \left[\frac{10-9.09}{6} \right]^{\frac{1}{2}} = 0.39$$

$$X_b = \frac{48}{7} = 6.85$$

$$S_b = \left[\frac{350-328}{6} \right]^{\frac{1}{2}} = 1.91$$

b) A_c measurement; From Tables 7 and 8

$$X_a = \frac{6}{5} = 1.2$$

$$S_a = \left[\frac{8.4-7.2}{4} \right]^{\frac{1}{2}} = 0.55$$

$$X_b = \frac{32}{5} = 6.4$$

$$S_b = \left[\frac{210-204.8}{4} \right]^{\frac{1}{2}} = 1.14$$

The fact that each of the standard deviations for the above cases is small in comparison to the mean for that case indicates that damage is in fact correlated to the number of drops (it is not a random process) and the experiment used to deduce this fact provides results which are repeatable.

CONCLUSIONS/RECOMMENDATIONS

Damage boundary curves were determined for both single drop and multiple drop treatments for light bulbs (Figure 9) and cement bricks (Figure 10). It was observed that for multiple drops, the critical velocity change and critical acceleration values were lower than those for a single drop treatment. This means that the entire damage boundary curve shifts towards the origin of the coordinate axis, thereby widening the damage zone. Therefore a package system designed to protect a fragile product using the values from a conventional DBC may still suffer damage due to repeated shocks at lower shock levels.

The conventional DBC gives an estimate of the damage zone for a single drop only and therefore needs to be developed for multiple drop situations independently. The number of drops required for this treatment depends on the type of handling encountered in the distribution environment. As part of this newly developed DBC the number of drops used in its development should be mentioned, in a manner similar to that for cushion curves (see Figures 15,16). In this way, the user of such a curve may select that particular DBC which has been developed for the number of drops likely to be encountered in the handling environment.

APPENDIX

APPENDIX A

DERIVATION OF THE DAMAGE BOUNDARY CURVE

Review of shock amplification:

Let 'm' and 'k' represent the mass and spring constant of the critical element (c.e.). Also, let 'M' and 'K' be the mass and spring constant of the bulk mass (see Figure 11).

It is assumed that

$$m << M$$

$$\text{and} \quad k << K$$

Now if this system is dropped from a height 'h', (see Figure 11) mass 'M' experiences a sinusoidal deceleration characterized by

$$G_M = \left[\frac{2hK}{W} \right]^{\frac{1}{2}} \tag{A-1}$$

The duration of shock is

$$\tau = \pi \left[\frac{W}{Kg} \right]^{\frac{1}{2}} \tag{A-2}$$

Mass 'm', the critical element, undergoes in general aperiodic (τ nonexistant) motion characterized by

$$G_{max,\ c.e.} = A_{max} \times G_{max,M} \tag{A-3}$$

More detail can be found in Mindlin (5).

The familiar shock amplification factor A_{max} is a measure of the extent to which the shock received by M is transmitted to the critical element and is a function of the frequency ratio $\frac{f_{n,c.e.}}{f_{n,M}}$ only.

$$\text{where} \quad f_{n,c.e.} = \frac{1}{2\pi} \left[\frac{k}{m} \right]^{\frac{1}{2}} \tag{A-4}$$

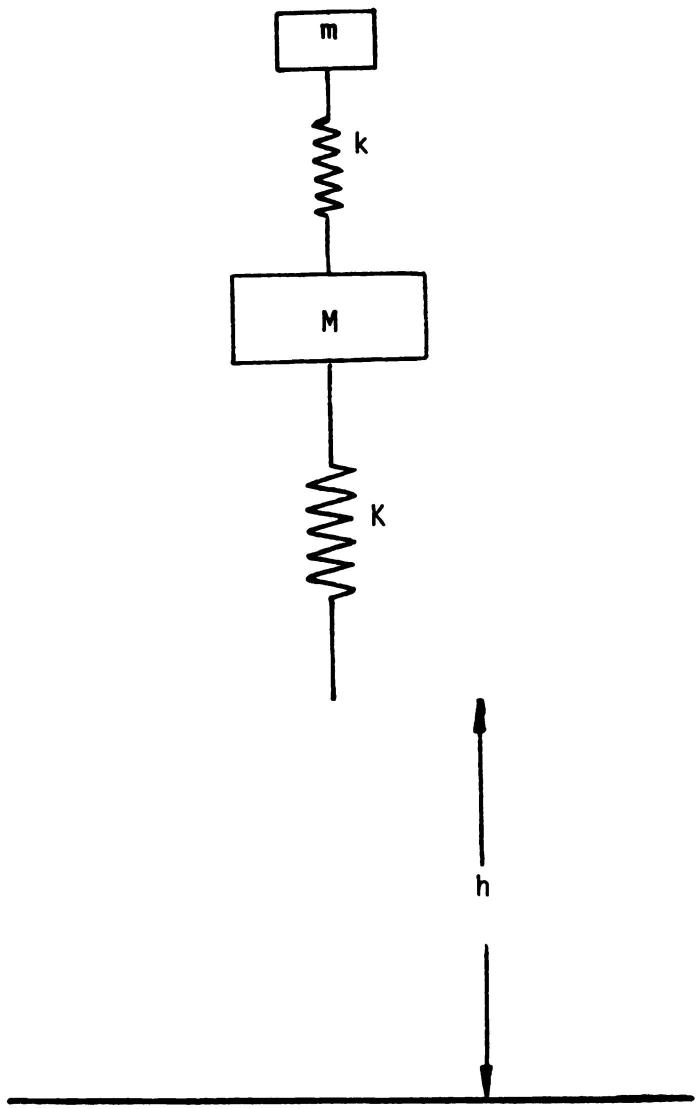

Figure 11. Spring-mass model for product-package system.

$$\text{and} \quad f_{n,M} = \frac{1}{2\pi}\left[\frac{K}{M}\right]^{\frac{1}{2}} \qquad \text{(A-5)}$$

Criticism of the Shock Amplification Analysis:

Spring K usually represents a 'cushion' which in reality is a viscoelastic non linear element of the system and which is only approximately modelled by $F = Kx$. Spring k on the other hand usually represents an elastic but fragile element of the system which is adequately described by $f = kx$. The shock amplification factor is likely to be in error due mainly to the assumptions associated with the cushion. What is needed is a means by which a more realistic shock may be imposed on M. A shock machine accomplishes this; the cushion (spring K) is removed from the system and M is rigidly mounted to the table of a shock machine. The table is then dropped from a height h onto a programmable decelerator -- an ideal shock machine has the capability of programming the decelerator for any pulse shape, level and duration. If such a machine were to be programmed for a half sine deceleration pulse of magnitude $G_{max} = \left[\frac{2hK}{W}\right]^{\frac{1}{2}}$ and of duration $\tau = \pi\left[\frac{W}{Kg}\right]^{\frac{1}{2}}$, then the shock delivered to M would be identical to that produced by the linear spring K and the critical element would respond as described earlier by equation (A-3) with A_{max} given by

$$A_{max} = \begin{bmatrix} \dfrac{2(f_1/f_2)\cos(\pi f_1/2f_2)}{1-(f_1/f_2)^2} & \text{when } f_1 < f_2 \\ \\ \dfrac{(f_1/f_2)}{(f_1/f_2)-1}\sin\dfrac{2n\pi}{(f_1/f_2)+1} & \text{when } f_1 > f_2 \end{bmatrix} \qquad \text{(A-6)}$$

$f_1 = f_{n,\ c.e.}$ (natural frequency of c.e.)

$f_2 = f_{n,\ bulk}$ (natural frequency of bulk)

as shown by Newton (6).

If now the shock machine were to be programmed for a square wave deceleration of amplitude 'A' and duration τ, which incidentally is the worst type of input in terms of shock amplification (6), the critical element would experience a shock amplified A^*_{max} times the input shock where

$$A^*_{max} = \frac{G_{max,c.e.}}{G_{max,input}} = \begin{bmatrix} 2\sin\left(\frac{\pi f_1}{2 f_2}\right) & f_1 \le f_2 \\ 2 & f_1 \ge f_2 \end{bmatrix} \qquad \text{(A-7)}$$

where f_2 is frequency of input shock and if τ is the duration of the half square wave;

$$f_1 = \text{natural frequency of c.e.}$$

$$f_2 = \frac{1}{2\tau}$$

The amplification factors for half sine and square wave inputs can be graphically described as shown in Figure 12. The Damage Boundary Curve introduced by Newton (6) is a plot of the curve with 'A' (input square wave acceleration) on the ordinate versus ΔV (velocity change) on the abscissa axis.

The natural frequency of the critical element, f_1, and the shock level to the critical element, $A_{c.e.}$, are assumed known for purposes of discussion.

From A-7, $$A^*_{max} = \frac{\text{maximum c.e. acceleration}}{\text{input square wave acceleration}} = \frac{A_{c.e.}}{A}$$

and $\Delta V = A\tau$ for a square wave input; therefore the above relations for A^*_{max} can be written as;

$$A_{c.e.} = \begin{bmatrix} 2A\sin\left(\frac{\pi f \Delta V}{A}\right) & \frac{f\,\Delta V}{A} \le \frac{1}{2} \\ 2A & \frac{f\,\Delta V}{A} \ge \frac{1}{2} \end{bmatrix} \qquad \text{(A-8)}$$

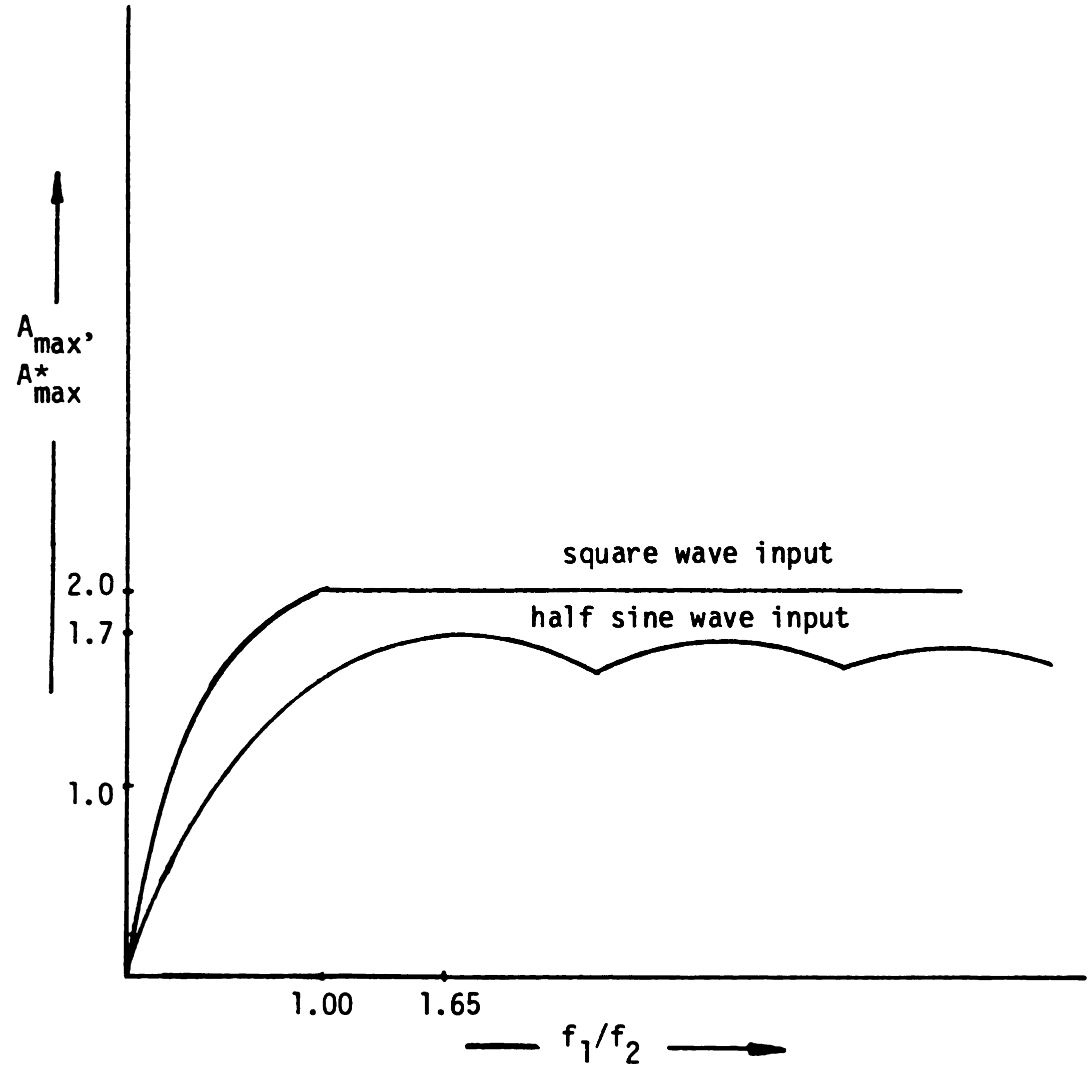

Figure 12. Amplification factors for square wave and half sine wave shock pulses.

where $f = f_1 = f_{n,c.e.}$

$$f_2 = \frac{1}{2\tau} = \frac{A}{2\Delta V}$$

The 'damage boundary region' described by Kornhauser (4) is that region plotted on A versus ΔV axis for which $A_{c.e.}$ reaches its maximum tolerable level. These values are described as;

$$\begin{aligned} 2A \sin\left(\frac{\pi f \Delta V}{A}\right) &> A_{c.e.} \qquad \text{when} \quad \frac{\Delta V}{A} \leq \frac{1}{2f} \\ 2A &> A_{c.e.} \qquad \text{when} \quad \frac{\Delta V}{A} \geq \frac{1}{2f} \end{aligned} \tag{A-9}$$

The boundary of the damage region for $\Delta V \leq \frac{A}{2f}$ is

$$\Delta V = \frac{A}{\pi f} \arcsin\left(\frac{A_{c.e.}}{2A}\right) \tag{A-10}$$

Some points in the A versus ΔV plane which lie on the boundary of the region described by these inequalities are:

1. $A = \frac{A_{c.e.}}{2}$ when $\Delta V = \frac{A_{c.e.}}{2\pi f} \arcsin(1) = \frac{A_{c.e.}}{4f}$

2. $A = \frac{A_{c.e.}}{(3)^{\frac{1}{2}}}$ when $\Delta V = \frac{A_{c.e.}}{(3)^{\frac{1}{2}}\pi f} \arcsin\frac{(3)^{\frac{1}{2}}}{2} = \frac{A_{c.e.}}{3(3)^{\frac{1}{2}} f}$

3. $A = A_{c.e.}$ when $\Delta V = \frac{A_{c.e.}}{\pi f} \arcsin\left(\frac{1}{2}\right) = \frac{A_{c.e.}}{6f}$

4. When A approaches infinity, ΔV approaches $\frac{A.A_{c.e.}}{\pi f.2A} = \frac{A_{c.e.}}{2\pi f}$

because for a very small angle, z, arcsin(z)=z.

Hence there are two bounds for this curve,

(1) 'A' cannot fall below $\frac{A_{c.e.}}{2}$

(2) 'ΔV' cannot fall below $\frac{A_{c.e.}}{2\pi f}$

These results hold for $\Delta V \leq \frac{1}{2f}.A$. For $\Delta V \geq \frac{1}{2f}.A$,'A' is not a function of ΔV, rather 'A' is a constant and is equal to $\frac{A_{c.e.}}{2}$.

The plot of the damage boundary curve with these points indicated is shown in Figure 13.

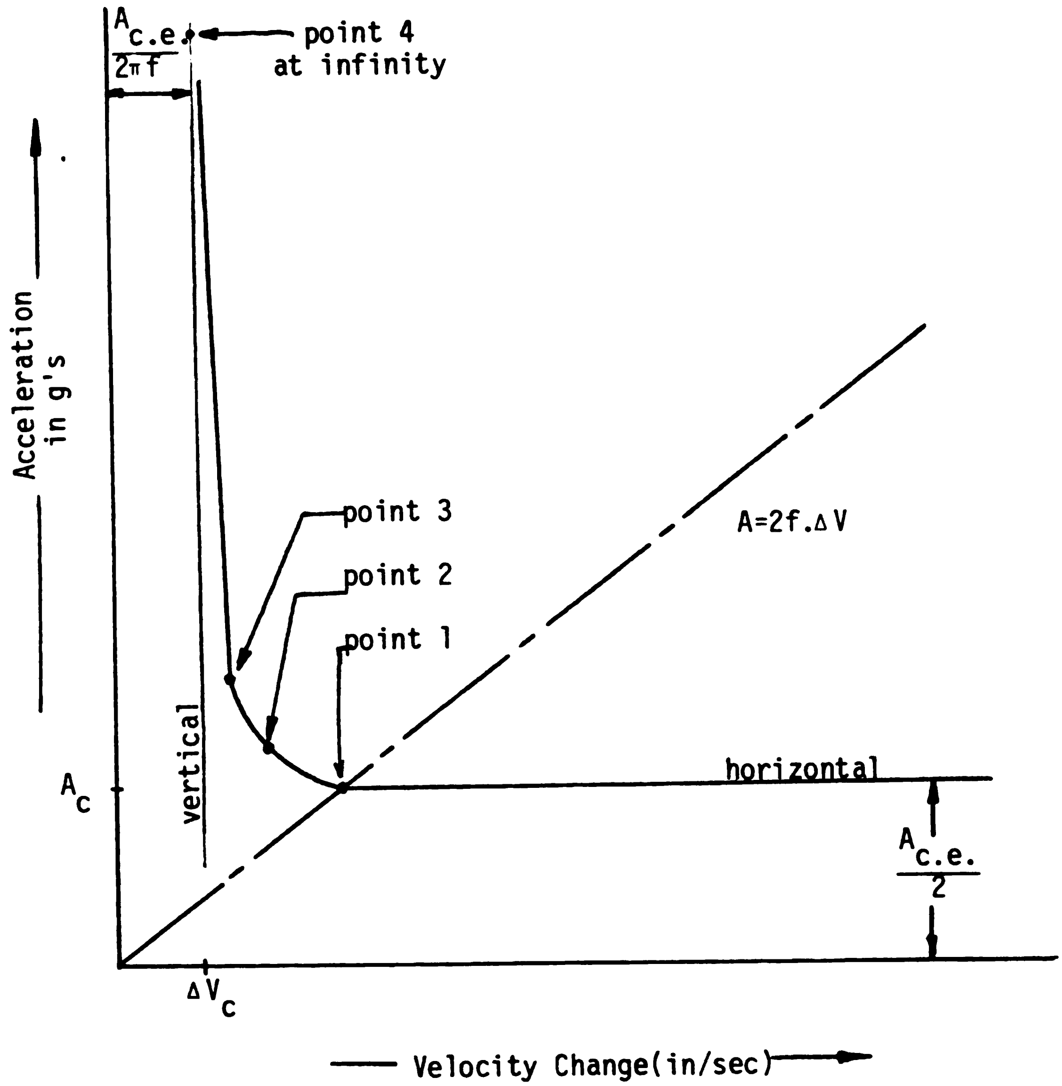

Figure 13. Ideal Damage Boundary Curve.

The important characteristics are:

1. The left hand portion of the curve is not really vertical- it is however very steep. But conventional damage boundary curves assume this portion is vertical from point 3 upwards and is located $\Delta V = \frac{A_{c.e.}}{2\pi f}$ units from the A-axis.

2. For point 1, $\Delta V = \frac{A_{c.e.}}{4f} = \frac{A_{c.e.}}{2\pi f} \cdot \frac{\pi}{2}$ or point 1 is 1.57 times the ΔV coordinate of point 3.

3. The critical value of the input acceleration, call it A_c, determined experimentally is actually $\frac{A_{c.e.}}{2}$

Therefore $$A_{c.e.} = 2A_c \tag{A-11}$$

4. Similarly for experimentally measured critical velocity change, ΔV_c, we actually get a measure of $\frac{A_{c.e.}}{2\pi f}$ from which the natural frequency of the c.e. can be determined,

$$f_{n,c.e.} = \frac{A_{c.e.}}{2\pi \Delta V_c} = \frac{A_c}{\pi \Delta V_c} \tag{A-12}$$

Thus the critical element properties, $A_{c.e.}$ and f_n, can be determined directly from the damage boundary curve information, A_c and ΔV_c.

APPENDIX B

PROCEDURE FOR DETERMINING THE DAMAGE BOUNDARY CURVE

This procedure for determining damage boundary curves was developed by Goff and Pierce (2) and is described here briefly. It is similar to the ASTM standard for fragility measurements.

Procedure:

1. A test item is clamped to the table of the shock machine.

2. A series of half sine pulses of increasing velocity change are applied to the product until damage occurs (see Figure 14).

3. The vertical line on the damage boundary curve is determined by the shock level just before damage. (The third drop in this example)

4. A series of rectangular pulses of constant velocity change and increasing peak acceleration is applied to a new sample until damage occurs. The velocity change should be at least 1.57 times greater than that determined in Step 3.

5. The horizontal line on the damage boundary curve is determined by the shock level, just before damage. (The ninth drop in this example)

6. A rounded curve is drawn between the values of ($\Delta V_c, 2A_c$) and ($\frac{\pi}{2}\Delta V_c, A_c$).

7. The damage boundary curve is shown in Figure 14.

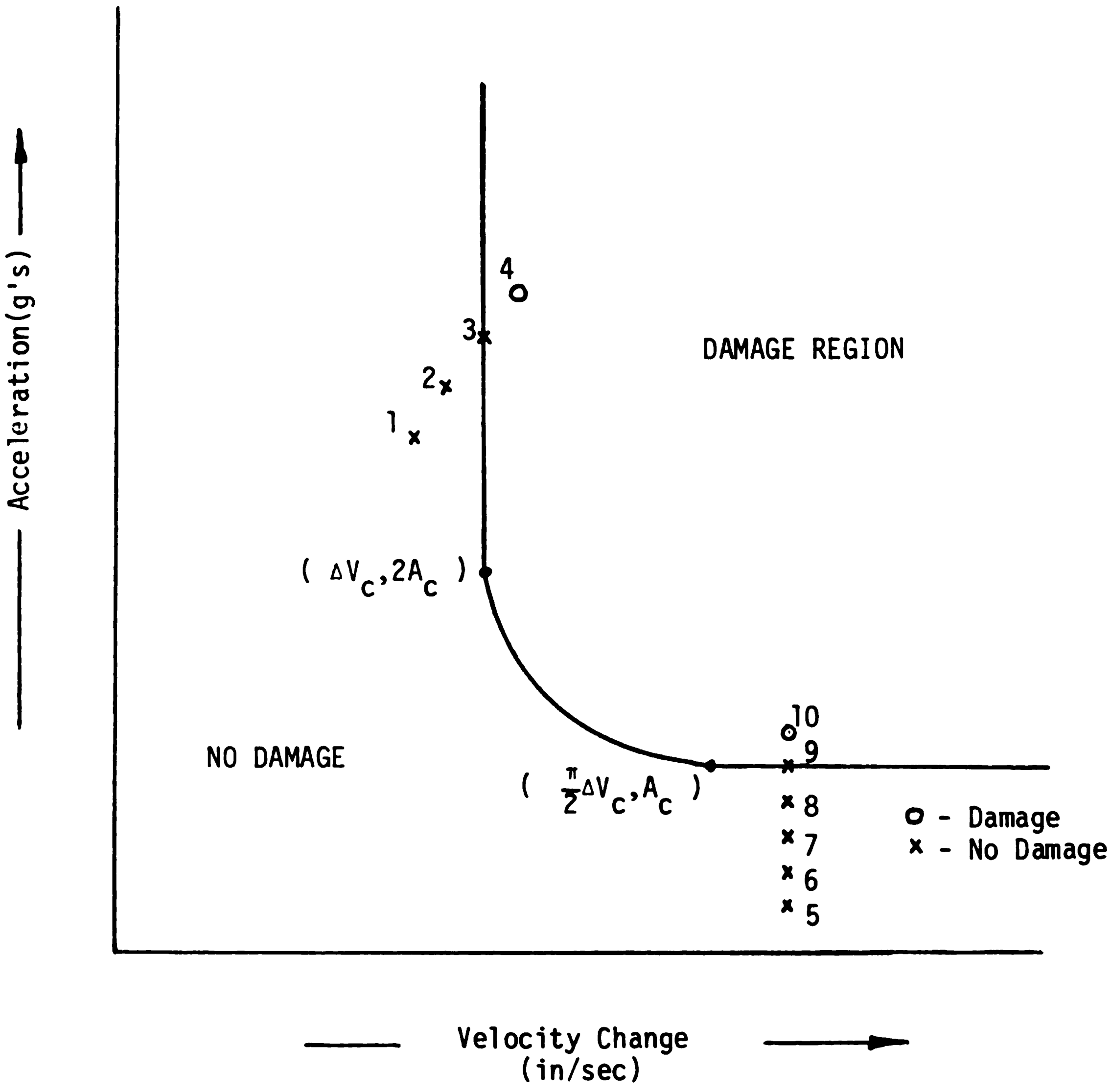

Figure 14. Method for constructing Damage Boundary Curves.

APPENDIX C

EFFECT OF MULTIPLE IMPACTS ON CUSHION PERFORMANCE

The number of drops greatly influences the performance of cushion materials. This is evident in the dynamic cushion curves (7) for Ethafoam[1] 220 Polyethylene Foam (see Figures 15,16).

The cushion curves for a single impact (Figure 15) show lower deceleration values for the same static loading (product weight per unit cushion area) indicating better protection when compared with those for multiple impacts (Figure 16) which have higher decelerations for the same thickness of foam. Thus we see the cushion material becomes progressively degraded with each drop and thereby becomes less useful as a protective device.

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[1]Trademark of The Dow Chemical Company

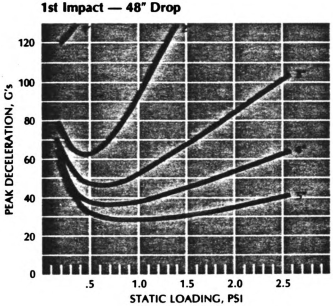

Figure 15. Cushion curves*for single impact.

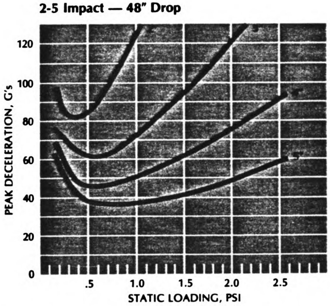

Figure 16. Cushion curves* for multiple impacts.

*Courtesy of Dow Chemical Company.

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