This is to certify that the

dissertation entitled

Agency, Consequence and Morality

presented by

Philip Robert West

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Philosophy

Herbert E. Hendry
Major professor

Date 1/21/88

AGENCY, CONSEQUENCE AND MORALITY

By

Philip Robert West

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Philosophy

1987

ABSTRACT

AGENCY, CONSEQUENCE AND MORALITY

By

Philip Robert West

This is principally a formal semantics for moral obligation, forbiddance and permission. Central notions of the semantics are that of an interpretation and that of truth condition; the latter is explained in terms of the former. Preliminary to the discussion of the moral concepts, however, formal semantics for tense, alethic modality and the bringing–it–about–that relation between agents and the states of affairs they bring about are presented. Obligation, forbiddance and permission are then defined relative to agents and what they bring about. More specifically, agents are said to be obliged, forbidden or permitted to bring certain states of affairs about just in case sanctioning is an inevitable consequence of what they bring about or fail to bring about. Following this, various classes of interpretations for moral ascriptions are related to one another and several deontic paradoxes are considered. Then in terms of the formal apparatus, related issues are discussed, including egoism, consequentialism, collective action, collective obligation, and several forms of determinism involving natural law and divine action. It is argued that for Christian theists, the appropriate metaethical view is egoistic and consequentialistic.

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ACKNOWLEDGEMENTS

The completion of my dissertation is not due exclusively to my own efforts, but also to those of others. I am especially grateful to my wife, Ellen Marie, whose regular encouragement and sacrifices made the completion possible. Thanks also to my parents, Robert and Dolores West, for teaching me both by word and example to cling to what is true. I also thank James M. Grier, Jr., my first professor of philosophy, whose high academic standards and instruction have greatly influenced my work. I owe special gratitude to my dissertation director, Professor Herbert Hendry. Discussions of the project with him resulted in many improvements and preempted many errors; to say nothing of his influence on my understanding of the philosophical task. I also appreciate the remarks of Professor Richard Hall, who carefully read the manuscript and offered many helpful suggestions. Thanks also to Professor Harold Walsh, whose discussions of collective action and collective obligation stimulated the developments of Chapter Five. Finally, thanks go to my entire dissertation committee, Herbert Hendry, Richard Hall, Harold Walsh and Bruce Miller, professors in the Department of Philosophy, and William Whallon, Professor in the English Department, of Michigan State University. Of course, even though my debts of gratitude are great, responsibility for the opinions espoused in the dissertation is my own.

Philip R. West 1987, Grand Rapids

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INTRODUCTION

1. *RECONSTRUCTIVE ANALYSIS AND THE MORAL CONCEPTS*

This is a constructional analysis of the concepts of moral obligation, moral permission and moral forbiddance. It will be taken for granted that concepts are to be construed as properties or relations and that concepts are best clarified by a display of the behavior of the linguistic expressions with which they are associated. For example, if the concept of caninity is associated with the predicate "is a dog", analysis of the concept progresses as understanding of the role of the sentences in which "is a dog" occurs improves. In a constructional analysis, the concepts endemic to the pretheoretical conceptual apparatus targeted for analysis are clarified by the articulation of more orderly substitutes or reconstructions for them. The concepts targeted for analysis are sometimes called *explicanda* and their reconstructed counterparts, *explicata*[1]. The *explicata* are presented via linguistic expressions whose behavior is more exact than that of their ordinary language counterparts. Their exactness is improved by specifying the syntactic rules for the formation of sentences involving the substitute expressions and by detailing the linguistic conventions for their appropriate use, thereby eliminating, as much as possible, indeterminate or conflicting conventions.

Philosophers are especially interested in the influence these expressions have on the truth or falsehood of the sentences in which they appear, and most important to logicians, in how the relation of entailment impinges on these locutions. Where the moral concepts are the focus of attention, the

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This is a constructional analysis of the concepts of moral obligation, moral permission and moral forbiddance. It will be taken for granted that concepts are to be construed as properties or relations and that conceptual clarification is a display of the behavior of the linguistic expressions with which they are associated. For example, if the concept of animality is associated with the predicate "is a dog," analysis of the concept proceeds as understanding of the role of the sentences in which "is a dog" occurs interrelates. In a constructional analysis, the concepts chosen to be pretheoretical conceptual apparatus targeted for analysis are clarified by the articulation of more orderly substitutes or reconstructions for them. The concepts targeted for analysis are sometimes called explicanda and their reconstructed counterparts, explicata. The explicata are presented via linguistic expressions whose behavior is more exact than that of their ordinary language counterparts. Their exactness is improved by specifying the syntactic rules for the formation of sentences involving the substitute expressions and by detailing the linguistic conventions for their application, thereby eliminating, as much as possible, indeterminate or conflicting connotations.

Philosophers are especially interested in the notions these expressions bear on the truth or falsity of the sentences in which they appear, and most important to logicians is how the relation of entailment among these sentences. Where the moral concepts are the focus of interest, the products of constructive analysis are often called metaethical theories or deontic logics.

Truth and falsehood are semantic notions and so when an analysis focusses on giving an account of the conditions under which sentences containing certain expressions are true or false, it is said to be a semantics for the concepts the expressions are associated with. What follows is such a truth conditional semantics for the moral concepts. The several tasks involved in the presentation will be displayed in greater detail shortly.

Although deontic logics ostensibly agree in their focus on the concepts of morality, substantial differences regarding what is being analysed confuse assessment of these systems and make comparing systems with one another perilous. Some deontic logics, for example, are offered as analyses of behavior guiding rules. Accordingly, sentences of the type "p is obligatory", usually translated Op, are taken to have a prescriptive or imperative communicative role. Deontic systems of this sort are occasionally classified as studies of the logic of normative systems. Because uttering an imperative and uttering a moral ascription are appropriate in similar circumstances, this view is not devoid of attraction. For instance, "I forbid you to climb on the table", or "It is not permissible for you to climb on the table" and the imperative "Do not climb on the table" seem to stand or fall together respecting the propriety or impropriety of their utterance. But, in some deontic theories, prescriptive rules are not the center of attention. Sentences of the type "p is obligatory" are taken instead to have an assertive or declarative role.

This difference over the status of prescriptive rules in metaethical analysis apparently involves conflicting views in the philosophy of language. On the one hand, some regard information conveyance as only one among [illegible] of constitutive [illegible] can be [illegible] [illegible] logic.

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This disagreement over the status of prescriptive rules in moral semantic analysis apparently involves conflicting views in the philosophy of language. On the one hand, some regard indicative utterances as [illegible] several philosophically important roles language has. They evidently believe that exclusive attention to the truth value of an utterance, or to what a sentence asserts, is dangerously narrow and ignores other kinds of performance values of utterance besides assertion and denial. Moreover, utterances about what is obligatory, forbidden or permissible, although declarative in grammatical mood, might be imperatives and not declaratives at all, and if so, discussing their truth conditions is a mistake, since they have none.

On the other hand, opponents of the forgoing position seem to believe that unless utterances can be construed as assertions or denials, they are not amenable to philosophical analysis and that assertions and denials are not properly understood until their truth conditions are laid bare. To hold this view, one need not deny that linguistic utterance has no function but assertion or denial, but merely that other functions are not of value when it comes to philosophical analysis. According to this view, there is no *logic* of commands, strictly speaking, if commands have no truth value. The analysis presented here is in line with this second view.

More specifically, the view espoused here is obviously similar to Bohnert's suggestion that imperatives are really declaratives in disguise. He recommends, for example, that the imperative, "Do X" might be construed as the assertion, "If you do not do X, such and such harm will befall you", or "If you are to escape this harm, you will do X"[2]. Bohnert's suggestion shows a way to construe imperatives as declaratives and can be contrasted with the rule analysis of moral ascriptions according to which the declarative moral ascriptions are in effect treated as imperatives.

The similarity between the analysis here and Bohnert's suggestion is that the declaratives he sees as filling the role of imperatives are like the [illegible] philosophers who think that [illegible] that exclusive attention to the truth value of an utterance [illegible] whether it asserts, is dangerously narrow and ignores other kinds of performative values of utterances besides assertion and denial. Moreover, utterances about what is obligatory, forbidden or permissible, although declarative in grammatical mood, might be imperatives and not declaratives at all and if so, discussing their truth conditions is a mistake, since they lack none.

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The similarity between the analysis here and Lobnert's suggestion is that the declaratives be seen as fulfilling the role of imperatives [illegible] declaratives that will be treated herein as filling the role of the moral ascriptions. The moral ascription "Agent α is obliged to do X" will be treated as equivalent to the declarative "Something undesirable will happen to α if he fails to do X". And this similarity in turn makes manifest a similarity between the view here presented and that of A.R. Anderson who suggested that sentences of the type "It is forbidden that p" be identified with sentences of the type "If p is the case then punishment is necessary"[3]. Since Anderson's simplification of deontic logic identifies fulfilling obligation with escaping sanction or penalty, views that resemble his in this respect are sometimes called escapist. This label seems fitting for the presentation to follow, although the differences with the Andersonian simplification are several.

It is possible, of course, to give a truth conditional analysis of sentences of the type Op while pursuing a rule analysis of the moral concepts. Hintikka, for example, recommends that Op is true just in case it is true in all deontically perfect alternative possible worlds, viz., that Op is true just in case it is true in all the worlds in which the rules being analysed are universally fulfilled[4]. Once the list of rules is identified, the characterization of deontically perfect worlds in terms of the fulfillment of these rules seems relatively inoffensive (given that possible worlds are not themselves a problem). If one wants to find these deontically perfect worlds, so to speak, one simply locates those in which no agent ever commits a sin of commission or omission according to the rules. This is not to be understood as a suggestion that obligation and forbiddance are always relative to appropriately organised positive behavioral rules since one might hold Hintikka's view and some sort of natural law theory according to which moral obligations and forbiddances are identified with prescriptions built in to obligations that will be treated here as among the [illegible]

ascriptions. The moral ascription "[illegible] is obliged to do X" will be treated as equivalent to the declarative "Something undesirable will happen to [illegible] if he fails to do X". And this similarity in turn makes manifest a similarity between the view here presented and that of A.R. Anderson who suggested that sentences of the type "it is forbidden that p" be equated with sentences of the type "if p is the case then punishment is necessary." Since Anderson's simplification of deontic logic identifies fulfilling obligation with escaping sanction or penalty, views that resemble his in this respect are sometimes called escapist. This label seems fitting for the presentation to follow, although the differences with the Andersonian simplification are several.

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The analysis here applies to moral ascriptions Quine's view, that truth and falsehood are fundamental in the hierarchy of concepts. Moral ascriptions are assertions or denials and as such are true or false. The logician's work is to chase truth and falsehood up the grammatical tree, as Quine says[5]. Moreover, imperatives and rules play no role in the truth conditions here presented as they do in Hintikka's semantics. And it should be noted that ascriptions of obligation or prohibition are not herein handled as declarations of approval or disapproval according to the emotivist suggestion.

2. THE ASSESSMENT OF DEONTIC LOGICS

The evaluation of constructive analyses involves both systematic and intuitive criteria. Systematic success depends on the simplicity and clarity with which *explicata* are presented and relative to the manner in which the *explicata* are related to other concepts in the broader pretheoretical conceptual apparatus that is the ultimate object of reconstruction. An analysis that fully defines its *explicata*, or successfully reduces them to other notions, is systematically superior to one that does not. It is considered a virtue to minimize the number of fundamental or unanalysable concepts.

The intuitive success of a constructive analysis depends on the degree to which the conceptual behavior of its *explicata* harmonizes with the intuitions associated with the conceptual behavior of its *explicanda*. Intuitive success decreases with the degree to which intuitions are violated by the

[illegible]

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THE EVALUATION OF ANALYSES

The evaluation of constructive analyses involves both systematic and intuitive criteria. Systematic success depends on the simplicity and clarity with which concepts are presented and relative to the manner in which the concepts are related to other concepts in the broader pretheoretical conceptual structure that is the ultimate object of reconstruction. An analysis that fully defines its explicata or successively reduces them to other notions is systematically superior to one that does not. It is considered a virtue to minimize the number of fundamental or unanalysable concepts.

The intuitive success of a constructive analysis depends on the degree to which the conceptual behavior of its explicata harmonizes with the intuitions associated with the conceptual behavior of its explicanda. Intuitive success decreases with the degree to which intuitions are violated by the replacement of the *explicanda* with the system's *explicata*. Possible violations are of two sorts. Either some intuition captured by the *explicanda* is omitted in the substitution or some intuition unrelated to the *explicanda* is included in the behavior of the *explicata*. The traditional deontic paradoxes highlight these intuitive disvalues.

But it is at this juncture that a failure to elucidate the philosophy of language motivating the development of a system welcomes confusion. A deontic system is deemed intuitively weak to the extent that its declaration of theses is contrary to generally accepted beliefs. A weakness of this sort occurs where ϕ is a thesis of the system and where it is a generally accepted belief that ϕ (or the ordinary language counterpart of ϕ) is false, or where $\neg\phi$ is entailed by sentences generally believed to be true. But when one is asked whether ϕ is counterintuitive where ϕ is an ascription of obligation, forbiddance or permission, one faces the dilemma of conflicting judgments depending on how ϕ is to be understood. Is one to ask whether the imperative associated with ϕ is part of a system of prescriptive rules actual or possible, or whether this imperative is a natural law? Is one to ask whether or not one would agree to strive to obey this imperative, or whether members of a community would agree to this imperative as a commonly shared guide to behavior? Is one to question one's preferences about the truth or falsehood of ϕ? Questions like these make it difficult to say whether the situations hightlighted by some of the famous deontic paradoxes are paradoxical and this is especially a problem when the paradoxes associated with conflicting obligations are discussed.

In an overall assessment of a constructive analysis, systematic and intuitive strengths are weighed against one another. An analysis having great systematic power that violates the most persistent intuitions associated [illegible] of two sorts: either some intuition captured by the explicata is omitted in the constitution or some intuition unrelated to the explicata is included in the behavior of the explicata. The traditional deontic paradoxes highlight these intuitive dissatisfactions.

But it is at this juncture that a failure to elucidate the philosophy of language motivating the development of a system welcomes confusion. A deontic system is deemed intuitively weak to the extent that its behavior is contrary to generally accepted beliefs. Weaknesses of the sort occur where Q is a thesis of the system and where it is a generally accepted belief that Q (or the ordinary language counterpart of Q) is false, or where Q is entailed by sentences generally believed to be true. But when one is asked whether Q is counterintuitive where Q is an ascription of obligation, forbiddance or permission, one faces the dilemma of conflicting judgments depending on how Q is to be understood. Is one to ask whether the imperative associated with Q is part of a system of prescriptive rules actual or possible, or whether this imperative is a natural law? Is one to ask whether or not one would agree to strive to obey this imperative, or whether members of a community would agree to this imperative as a community directive to behavior? Is one to question one's preferences about the truth or falsehood of Q? Questions like these make it difficult to say whether the situations highlighted by some of the famous deontic paradoxes are paradoxical and this is especially a problem when the paradoxes associated with conflicting obligations are discussed.

In an overall assessment of a constructive analysis, structural and intuitive strengths are weighed against one another. An analysis having great systematic power that violates the most persistent intuitions associated with the *explicanda* is overall not much of a success. On the other hand, an analysis that violates few intuitions but has meager systematic development is not much of an overall success either. It is expected that the analysis here will violate more intuitions than do the well–known deontic systems presented heretofore. These violations diminish if the philosophy of language motivations behind the view here are kept in mind, but of course, these positions might themselves be repulsive. Moreover, it is apparent that certain metaethical beliefs are held by many and these metaethical beliefs influence judgments of intuitive strength and weakness when it comes to deontic theses and arguments. Here the metaphilosophical question whether the philosopher's task is descriptive, i.e., he should merely clarify the concepts embedded in the pretheoretical apparatus, or prescriptive, i.e., he can sometimes challenge accepted belief and offer what he takes to be a better alternative, comes to the surface. This analysis is a challenge and it is hoped that the presentation of the view will improve its intuitive strength as a viable metaethical position. Constructivists usually believe that alternative reconstructions for a notion should be offered (there might or might not be a best one) and this analysis is intended as one of the several possible alternatives.

Since reconstructions of the moral notions are metaethical, the sentences of deontic theories are metalinguistic. They are about ascriptions of the moral properties. Ordinarily, a metalinguistic theory describes the language its sentences are about, its object language, and subsequently makes assertions about the properties of its object language sentences. Suppose the object of analysis is a concept *F*, where this concept is associated with an n–place predicate $F\alpha_0...\alpha_n$. Then, according to Tarski's assertion that ascriptions of truth are metalinguistic assertions about object language with the concepts is certain not [illegible] of a success [illegible] the other [illegible] analysis that violates few intuitions but has greater systematic coherence is not much of an overall success either. It is expected that the analysis here will violate more intuitions than do the well-known deontic systems presented heretofore. Those violations diminish if the philosophy of language motivations behind the view here are kept in mind, but of course, these positions might themselves be repulsive. Moreover, it is apparent that certain metaethical beliefs are held by many and these metaethical beliefs influence judgments of intuitive strength and weakness when it comes to deontic theses and arguments. Here the metaphilosophical question whether the philosopher's task is descriptive, i.e., he should merely clarify the concepts embedded in the [illegible] apparatus, or prescriptive, i.e., he can sometimes challenge accepted belief and offer what he takes to be a better alternative, comes to the surface. This analysis is a challenge and it is hoped that the presentation of the view will improve its intuitive strength as a viable metaethical position. Constructivists usually believe that alternative reconstructions for a notion should be offered (there might or might not be a best one) and this analysis is intended as one of the several possible alternatives.

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A second task of deontic systems is the elucidation of the entailment relation as it impinges on sentences including F. As an example, consider the metaethical theory using "ϕ" as a variable ranging over sentences of its object language and also recognizing "is permissible" and "is obligatory" among the expressions of its object language. A sentence of this theory might be "'ϕ is obligatory' entails 'ϕ is permissible'". This metalinguistic assertion ascribes the relation of entailment to two sentences of the object language. If ϕ and ψ are sentences, ϕ is said to entail ψ just in case any conditions making ϕ true also make ψ true.

A third task of deontic systems is the demarcation of deontic truth. If logical truths are sentences that remain true under all grammatically acceptable replacements of their nonlogical expressions, then deontic truths are sentences true under all such replacements of their nonlogical or nondeontic expressions. Obviously, this guarantees that the set of logical truths is a subset of the set of deontic truths. Like truth, deontic truth is a property of sentences and a deontic system's declaration that some sentence ϕ is a thesis is tantamount to the declaration that ϕ is deontically true.

When logicians study the logic of some notion *F* they are attempting to ascertain not only the truth conditions of sentences containing F, but also how the truth conditions of these relate to the truth conditions of sentences containing other predicates. Specifically, they want to determine whether sentences containing F entail or are entailed by these sentences containing other predicates. For example, if a metaethical theory claims that 'ϕ is [illegible] theories of X should state the truth conditions [illegible] for sentences involving F. Hence, one task of deontic logic is to display the truth conditions for sentences ascribing the moral properties and this is the focus of the semantic developments herein.

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Accordingly, a fourth task for deontic logics is to relate the moral concepts to one another. Obligation, forbiddance and permission are ostensibly all of the moral sort. The question of whether these three exhaust the list of moral concepts could be discussed at some length. Many systems present obligation, forbiddance and permission as a redundant list so that all three may be expressed where only one need be reconstructed as basic assuming some version of negation. Systems of this kind might be called morally monistic if the three are taken to exhaust the moral concepts. The system herein presented, DBC[6], is, on the contrary, morally pluralistic in its denial that the multiple expressions for the moral concepts are redundant with respect to one another. This pluralism diminishes the systematic value of DBC.

A fifth task of deontic systems is to relate the moral notions to concepts that are not of the moral sort. In approaching this aspect of theory development, one ordinarily indicates one's position regarding whether what ought to be the case, or what is obligatory, entails or is entailed by what is the case or by what is necessarily or possibly the case. One's response to these issues usually involves a confrontation with naturalism, which we may identify with any view according to which the moral concepts are not to be treated as primitive in reconstruction, but are to be defined or reduced to concepts which are neither moral, axiological, nor in any respect normative.

It has been popular to display a similarity between the concepts of moral obligation and permission, and the alethic modal concepts, necessity obligatory entails it is permissible that [illegible] in terms of entailment, the truth conditions for sentences containing the predicate "is obligatory" relate to sentences containing the predicate "is permissible."

Accordingly, a fourth task for deontic logics is to relate the moral concepts to one another. Obligation, forbiddance and permission are ostensibly all of the moral sort. The question of whether these three exhaust the list of moral concepts could be discussed at some length. Many systems present obligation, forbiddance and permission as a redundant list so that all three may be expressed where only one need be reconstructed as basic, assuming some version of negation. Systems of this kind might be called morally monistic if the three are taken to exhaust the moral concepts. The system herein presented, Lds 5, is, on the contrary, morally pluralistic in its denial that the multiple expressions for the moral concepts are redundant with respect to one another. This pluralism diminishes the systematic value of Lds.

A fifth task of deontic systems is to relate the moral notions to concepts that are not of the moral sort. In approaching this aspect of theory development, one ordinarily indicates one's position regarding whether what ought to be the case, or what is obligatory, entails or is entailed by what is the case or by what is necessarily or possibly the case. One's response to these issues usually involves a confrontation with naturalism which we may identify with any view according to which the moral concepts are not to be treated as primitive in reconstruction, but are to be defined or reduced to concepts which are neither moral, axiological, nor in any respect normative.

It has been popular to display a similarity between the concepts of moral obligation and permission, and the alethic modal concepts necessity and possibility, a similarity discussed by medieval logicians[7]. One such similarity is said to be evident in comparing certain definitions of possibility which are given in terms of necessity and negation, with a *prima facie* acceptable definition of permission which is given in terms of obligation and negation; "ϕ is possible" is equivalent to "Not necessarily not ϕ" in many modal logics, and "ϕ is permissible" is apparently equivalent to "It is not obligatory that not ϕ". Some theorists are content to allow recognition of this analogy to complete the fifth task. Apparently, these hope that the systematic analogy between the moral concepts and the alethic modalities will display a "logic" of the moral notions in the sense that their place in the conceptual apparatus will be shown to be as rule governed and thus as "rational" as are the alethic modalities in some weak system of modal logic and that the deontic system's rules governing entailment will comport with intuitions about validity in moral argument. Moreover, theories developed in this way do not risk offending metaethical deontologists who believe that the moral concepts are basic in the conceptual apparatus and cannot be reductively defined via concepts not in the moral concept family.

In my judgment the disanalogies between the deontic and the alethic modalities erodes the alethic/deontic analogy as a satisfactory resolution for the fifth task. For example, I do not believe that permission can be defined in terms of obligation and negation as has been suggested, whether or not possibility can be defined in terms of necessity and negation. Moreover, the validity of many arguments from nonmoral premises to moral conclusions, arguments that are fairly typical, cannot be evaluated according to analyses in which recognizing this analogy is all that can be said about the relation between the moral and nonmoral concepts. The sixth task of deontic logic is involved here, to produce a notion of validity that agrees with ordinary and possibility, a similarity instanced by modal notions. The other similarity is said to be evident in comparing certain definitions of possibility which are given in terms of necessity and negation with a prima facie acceptable definition of permission which is given in terms of obligation and negation. "X is possible" is equivalent to "Not necessarily not X" in many modal logics and "X is permissible" is apparently equivalent to "It is not obligatory that not X". Some theorists are content to allow recognition of this analogy to complete the fifth task. Apparently, these hope that the systematic analogy between the moral concepts and the alethic modalities will [illegible] a "loss" of the moral notions in the sense that their place in the conceptual apparatus will be shown to be as rule governed and thus as "rational" as are the alethic modalities in some weak system of modal logic and that the deontic systems' rules governing entailment will comport with intuitions about validity in moral argument. Moreover, theories developed in this way do not risk offending metaethical deontologists who believe that the moral concepts are basic in the conceptual apparatus and cannot be reductively defined via concepts not in the moral concept family.

In my judgment the dissimilarities between the deontic and the alethic modalities erodes the alethic-deontic analogy as a satisfactory resolution for the fifth task. For example, I do not believe that permission can be defined in terms of obligation and negation as has been suggested, whether or not possibility can be defined in terms of necessity and negation. Moreover, the validity of many arguments from nonmoral premises to moral conclusions, arguments that are fairly typical, cannot be evaluated according to analyses in which recognizing this analogy is all that can be said about the relation between the moral and nonmoral concepts. The sixth task of deontic logic is involved here: to produce a notion of validity that accords with common intuitions about the reliability of arguments with moral ascriptions for conclusions. It will be seen that eschewing reductive definition for the moral concepts results in both systematic and intuitive weakness in this department.

3. *EGOISM, CONSEQUENTIALISM AND POSSIBLE WORLDS*

Instead of attending to the apparent analogy between the deontic and the alethic modalities, DBC analyses the moral concepts and attends to the rules of moral argument by reducing the moral to certain nonmoral concepts. Although it depends on the alethic modalities in its reductive definition of the moral concepts, it does not depend on an analogy between the deontic and the alethic modalities. Moreoever, it is intended that no moral concepts appear in the definienda of the moral concepts given here. DBC is supposed to be morally reductive and intended as naturalistic. The reductivism and the naturalism can be rejected in certain unintended understandings of the formalization and thus the formalization has some virtues independent of the metaethical positions that motivate its presentation.

Although DBC is morally pluralistic it reduces all three of the moral concepts to the same nonmoral concepts and because of this its reductivism offsets the systematic infelicity of its moral pluralism. The nonmoral notions the moral ones are reduced to in DBC are action and consequence, specifically, consequence in terms of sanction or penalty. Intuitively, the attempt to escape sanction and the attempt to keep obligation are the same attempts. As Prior has suggested, escapism is the logical basis of ethics[8]. Granted, it might be argued that sanction is itself a normative or moral concept, and that escapism is not naturalistic after all. This argument will in cases also. [illegible] conclusions. It will be seen that [illegible] reductive definition for the moral concepts results in both systematic and intuitive weakness in [illegible] of particular [illegible]

4. [illegible] AND POSSIBLE WORLDS

Instead of attending to the apparent analogy between the deontic and the alethic modalities, DK analyses the moral concepts and attends to the rules of moral argument by reducing the moral to certain nonmoral concepts. Although it depends on the alethic modalities in its reductive definition of the moral concepts, it does not depend on an analogy between the deontic and the alethic modalities. Moreover, it is intended that no moral concepts appear in the definienda of the moral concepts given here. DK is supposed to be morally reductive and intended as naturalistic. The reductivism and the naturalism can be rejected in certain unintended understandings of the formalization and thus the formalization has some virtues independent of the metaethical positions that motivate its presentation.

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Rather than recognising object language sentences of the form $O\phi$ where the obligation operator O attaches to sentences to form new sentences (as is popular among monadic deontic systems), DBC recognises object language sentences of the form $O\alpha\phi$ where α denotes an agent and ϕ a state of affairs. Whereas in monadic deontic systems, $O\phi$ type sentences are to be read as translations for "ϕ is obligatory", sentences of the type $O\alpha\phi$ in DBC are read, "α is obliged to bring it about that ϕ". In other words, obligation, forbiddance and permission are construed as relations between the actions of an agent and the states of affairs that are consequences of these actions rather than as a property of states of affairs *simpliciter*.

But, DBC does not define obligation only in terms of what agents *actually* bring about. As most agents are painfully aware, they fail to bring about certain states of affairs that they are obliged to bring about. For example, it is common to say that Mitchell was obliged to turn so as to avoid a collision even though he failed to do so. To account for situations like this, DBC defines the moral concepts as properties, not only relative to what an agent actually brings about, but relative to what an agent possibly brings about. This does not imply that DBC is developed under assumption of the kantian principle that agents are only obliged to do what they *can* do, for reasons that will be clear later. It is only to say that apparently the truth conditions of deontic ascriptions cannot be described without taking these possible but nonactual states of affairs into account. The analysis requires not only the logic of action sentences but presupposes the logic of the alethic modalities.

A word should be said here about the DBC construal of modality. There has been considerable discussion about possible worlds of late, Rather than presenting deontic language sentences of the form Oφ, where the obligation operator O attaches to sentences to form new sentences as is popular among monadic deontic systems, LDK recognizes object language sentences of the form a:φ where a denotes an agent and φ a state of affairs. Whereas in monadic deontic systems Oφ type sentences are to be read as translations for "φ is obligatory", sentences of the type Oaφ in LDK are read "a is obliged to bring it about that φ." In other words, obligation, forbiddance and permission are conceived as relations between the actions of an agent and the states of affairs that are consequences of those actions rather than as a property of states of affairs simpliciter.

But, LDK does not define obligation only in terms of what agents actually bring about. As most agents are fallible, as we are, they fail to bring about certain states of affairs that they are obliged to bring about. For example, it is common to say that Zutphil was obliged to turn so as to avoid a collision even though he failed to do so. To account for situations like this, LDK defines the moral concepts as properties, not only relative to what an agent actually brings about, but relative to what an agent possibly brings about. This does not imply that LDK is developed under assumption of the Kantian principle that agents are only obliged to do what they can do, for reasons that will be clear later. It is only to say that apparently the truth conditions of deontic ascriptions cannot be described without taking these possible but nonactual states of affairs into account. The analysis requires not only the logic of action sentences but presupposes the logic of the alethic modalities.

A word should be said here about the LDK's commitment to modality. There has been considerable discussion about possible worlds of late especially as the postulation of these entities is very fruitful in the analysis of the alethic modalities, as well as in the discussion of other concepts recalcitrant to analysis. Those who believe possible worlds exist and use them for analytical purposes, possible worlds theorists, customarily take sentences of the form "Possibly ϕ" as true just in case ϕ is true in at least one possible world and "Necessarily ϕ" as true just in case ϕ is true in every possible world. In contrast, Aristotle and Diodorus of Chronus analysed the modalities in terms of the temporal relation between moments or intervals. According to Diodorus, "Necessarily ϕ" is true just in case ϕ is now true and always will be true and "Possibly ϕ" is true just in case ϕ is either now true or will be true at some future moment. According to Aristotle "Necessarily ϕ" is true just in case ϕ always has been true, is now true, and is always going to be true, "Possibly ϕ" is true just in case ϕ is either true now, was true at some earlier moment or is going to be true at some future moment. DBC analyses alethic modality in temporal terms similar to these. As such it is a form of actualism. Its analysis of alethic modality makes no reference to nonactual possible worlds which have no temporal relation to the actual world. In other words, it is motivated by the belief that there is only one possible world, the actual one. The details of the view will be explained in more detail when the substantive theory behind the formal semantics is discussed in Chapter One.

Not only is it assumed that the deontic modalities cannot be clarified without recourse to the alethic modalities but also that action itself cannot be clarified without this recourse. What an agent does at a moment genuinely influences the course of history. The counterfacual assertion that what is the case would be other than it is had some agent not acted as he did is apparently true and thus action ascription like obligation ascription [illegible] the alethic modalities as well as in the discussion of other [illegible] is dominant in analysis. Those who believe possible worlds exist and use them for analytical purposes, possible worlds theorists, customarily take sentences of the form "Possibly q" as true just in case q is true in at least one possible world and "Necessarily q" as true just in case q is true in every possible world. In contrast, Aristotle and Diodorus of Chronus analyzed the modalities in terms of the temporal relation between moments or intervals. According to Diodorus, "Necessarily q" is true just in case q is now true and always will be true and "Possibly q" is true just in case q is either now true or will be true at some future moment. According to Aristotle "Necessarily q" is true just in case q always has been true, is now true, and is always going to be true. "Possibly q" is true just in case q is either true now, was true at some earlier moment or is going to be true at some future moment. This analyses alethic modality in temporal terms similar to those. As such it is a form of actualism. Its analysis of alethic modality makes no reference to nonactual possible worlds which have no temporal relation to the actual world. In other words, it is motivated by the belief that there is only one possible world, the actual one. The details of this view will be explained in more detail when the substantive theory behind the formal semantics is discussed in Chapter One.

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A second concept in DBC's explication of the deontic concepts is consequence–of–action. Consequentialism is the metaethical doctrine that actions have their moral properties exclusively relative to their outcomes or results. For example, a form of consequentialism, utilitarianism, declares that an action is obligatory just in case its omission creates a worse overall state of affairs in terms of the pain and pleasure of all agents than its commission. In comparison, DBC relativizes obligation to the action of the singular agent and limits the consequences that determine moral status to those properties of this agent that are consequences of his act i.e., an agent α is obliged to bring it about that ϕ only if α's having a certain property results from α's bringing ϕ about. This brands the analysis as egoistic.

[illegible]

A second concept in [illegible] is [illegible] of the deontic concepts [illegible] consequences-of-action. Consequentialism is the metaethical doctrine that actions have their moral properties exclusively relative to their outcomes or results. For example, a form of consequentialism, utilitarianism, declares that an action is obligatory just in case its omission creates a worse overall state of affairs in terms of the pain and pleasure of all sentients than its commission. In consequentialism, [illegible] relativizes obligation to the action of the standard agent and limits the consequences that determine moral status to those properties of the action that are consequences of his act. [illegible] an agent x is obliged to [illegible] that [illegible] only if [illegible] having a certain property results from [illegible] of action. This counts the analysis as egoistic

CHAPTER ONE

TENSE AND MODALITY

1. *TRUTH, TENSE AND DETERMINISM*

The deontic theories of Mally, von Wright, and their immediate successors were axiomatically presented. Atomic expressions and rules governing the construction of well–formed formulas were presented first, then the axioms and rules of inference characteristic of the theory. Theses were proved by deducing them from the axioms and rules of inference. At the time, an intuitive explanation of what the theory's object language sentences were about constituted the only semantic aspect of the presentation; no precise description of the domains in which these sentences are satisfied was included. More recently, however, greater attention has been given to developing a formal semantics for moral ascriptions in order to remedy this deficiency. The domains in which moral ascriptions are satisfied are described in set theoretical vocabulary and are called interpretations. In semantically presented theories of this type, sentences are proved to be theses by a clear manifestation that they are satisfied in every permissible interpretation instead of offering a deduction of the sentences from axioms. In similar fashion, it is established that a sentence is not a thesis by showing that there is some interpretation in which it fails to be universally satisfied, viz., by displaying a countermodel. Since DBC is semantically presented the proofs herein are in terms of universal satisfaction in permissible interpretations. In the text, a dagger (†) preceding a sentence or rule indicates that a corresponding proof or countermodel is in Appendix C.

CHAPTER ONE

TENSE AND MODALITY

A. [illegible]

The deontic theories of Mally, von Wright, and their immediate successors were axiomatically presented. Atomic expressions and rules governing the construction of well formed formulas were presented first, then the axioms and rules of inference constituting the theory. Theorems were proved by deriving them from the axioms and rules of inference. At the time, an intuitive explanation of what the theory's object language sentences were about constituted the only semantic aspect of the presentation; no precise description of the domains in which these sentences are satisfied was included. More recently, however, greater attention has been given to developing a formal semantics for moral assertions in order to remedy this deficiency. The domains in which moral assertions are satisfied are described in set theoretical vocabulary and are called interpretations. In semantically presented theories of this type, sentences are proved to be theses by a clear demonstration that they are satisfied in every permissible interpretation instead of offering a deduction of the sentences from axioms. In similar fashion, it is established that a sentence is not a thesis by showing that there is some interpretation in which it fails to be necessarily satisfied, viz., by displaying a countermodel. Since FKX is semantically presented the proofs herein are in terms of universal satisfaction in permissible interpretations. In the text, a dagger (†) preceding a sentence or rule indicates that a corresponding proof or countermodel is in Appendix [illegible]

The notational conventions for fundamental atomic expressions and rules governing the formation of well–formed formulas and sentences of DBC resemble those of *Principia Mathematica*. Syntactic information is preceded by an outline indicator beginning with an integer between 1 and 6 inclusive. For example, 1.1.2 and 3.2 mark syntactic information. Similarly, indicators beginning with integers between 7 and 9 inclusive mark semantic information. Those beginning with 12 mark theses and those beginning with 13 mark sentences that are not theses.

The presentation of the system is broken up into sections corresponding to the alteration of focus in the discursive explanation. For example, the operators, well–formed formulas and truth conditions for action ascriptions are presented in Chapter Two where action is discussed. However, operators, well–formed formulas and truth conditions for moral ascriptions make no appearance until Chapter Five, where the moral concepts are discussed. In most cases, the articulation of a section presupposes previously presented sections and so it is not intended that the chapters be read out of order. Each section will procede by first identifying atomic formulas and syntactic rules governing the formation of well–formed formulas, where the latter are intended as translations for the types of ordinary sentences under consideration. This will be followed by a discussion of the elements of interpretations that are intended to depict semantic characteristics of the locutions being examined and a specification, in terms of these interpretations, of truth conditions for the well–formed formulas. Finally, where appropriate, theses and nontheses related to the issues being discussed are presented.

Obviously, this scattering of the formal presentation can be confusing. Thus, Appendix A contains a sequential presentation of the formal system

The notational conventions for intensional atomic expressions, for rules governing the formation of well formed formulas and theorems resemble those of *Fragment Nine*. Syntactic information is provided by an outline indicator beginning with an integer between 1 and 6 inclusive. For example, 1.1.2 and 5.2 mark syntactic information. Similarly, notations beginning with integers between 7 and 9 inclusive mark semantic information. Those beginning with 12 mark theses and those beginning with 13 mark sentences that are not theses.

The presentation of the system is broken up into sections corresponding to the alteration of focus in the iterative explanation. For example, the operators, well-formed formulas and truth conditions for action descriptions are presented in Chapter Two where action is discussed. However, operators, well-formed formulas and truth conditions for moral ascriptions make no appearance until Chapter Five, where the moral concepts are discussed. In most cases, the articulation of a section presupposes previously presented sections and so it is not intended that the chapters be read out of order. Each section will proceed by first identifying atomic formulas and syntactic rules governing the formation of well formed formulas, where the latter are intended as translations for the types of ordinary sentences under consideration. This will be followed by a discussion of the elements of interpretations that are intended to depict semantic characteristics of the locutions being examined and a specification, in terms of these interpretations, of truth conditions for the well-formed formulas. Finally, where appropriate, theses and nontheses related to the issues being discussed are presented.

Obviously, this scattering of the formal presentation can be confusing. Thus, Appendix A contains a sequential presentation of the formal system

DBC based on outline numbers. In this appendix the commentary of the main text is omitted. Appendix B lists all theses and nontheses discussed, again minus the commentary.

We begin the presentation by discussing the truth-functional part of the theory. First presented are the atomic formulas. These constitute the basic signs of the system, or its lexicon. The analogy with typical English lexicons cannot be pressed too far, of course, inasmuch as these have words as their basic units and, for example, an isolated English word is usually neither true nor false, as are some of the atomic formulas below. Furthermore, the grouping indicators like the English "(" and ")", for example, are not like words in English, although they also are among the atomic formulas of DBC.

1. Atomic Formulas:

1.2 Atomic Sentences: Lower case p through r with or without subscripts.

1.3 Connectives:

1.3.1 Truth Functional Connectives: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\top$ and $\perp$.

1.3.2 Grouping Indicators: (,) and /.

Next are the formulas and well-formed formulas. To return to the dictionary analogy, the formulas are sequences of atomic formulas some of which, like some sequences of English words, do not make a sentence, for example, "sleeping out dog" and sequences that do, for example "Dogs sleep". DBC formulas that compose what are like grammatically acceptable sentences are called well-formed formulas. 3.1–3.3 can be thought of as giving the grammatical rules for well-formed formula (wff) formation in DBC.

... of the connectives in the object language ... more ... is omitted ... operators ... all those that mentions them ... and indicates the connectives.

We begin the presentation by discussing the truth-functional part of the theory. First presented are the atomic formulas. These constitute the basic signs of the system or its lexicon. The analogy with typical English lexicons cannot be pressed too far, of course, inasmuch as these have words as their basic units and, for example, an isolated English word is usually neither true nor false, as are some of the atomic formulas. Likewise parentheses, the grouping indicators like the English "(" and ")" for example, are not like words in English although they also are among the atomic formulas of LTF.

1 Atomic Formulas:

1.1 Atomic Sentences: lower case p through r with or without subscripts.

1.2 Connectives:

1.2.1 Truth Functional Connectives: ~, ∧, ∨, ⊃, and ≡

1.2.2 Grouping Indicators: (and)

Next are the formulas and well-formed formulas. To return to the dictionary analogy, the formulas are sequences of atomic formulas some of which, like some sequences of English words, do not make a sentence. (Compare "Sleeping cat dog" and sequences that do, for example "The dog sleeps".) The formulas that compose what are like grammatical sentences are called well-formed formulas (wffs). We ... to assume the grammatical rules for well-formed formula (wff) formation in

LTF:

2. Formulas: Finite sequences of atomic formulas.

3. Well–Formed Formulas (wffs):

3.1 If ϕ is an atomic sentence, ϕ is well–formed.

3.2 If ϕ and ψ are wffs, then the following are well–formed:

3.2.1 $\neg\phi$

3.2.2 $(\phi\wedge\psi)$

3.2.3 $(\phi\vee\psi)$

3.2.4 $(\phi\rightarrow\psi)$

3.2.5 $(\phi\leftrightarrow\psi)$.

3.3 $\top$ and $\bot$ are well–formed.

We turn now to the semantic notions, the most important of which for our purposes are that of an interpretation and that of an evaluation. The explanation of evaluations will be postponed until later in this chapter when semantics section 9 is presented. Our attention can be given first to explaining interpretations.

An interpretation is a set theoretically described structure, here a quintuple, in terms of which evaluations will be articulated. The definition of interpretations circumscribes the sorts of structures that constitute appropriate domains in terms of which truth conditions for the wffs of DBC can be delineated. This definition is given via sets and functions.

The first coordinate of the interpretations is a set M. M is to be thought of as the set of moments, or temporal stages. There is no connection between the members of this set and specific units of chronometric duration such as seconds, hours or decades. The members of M are temporally atomic as far as the theory is concerned, not temporally atomic as far as some practical chronometric scheme is concerned.

3. Formulas: the sets of atomic formulas

4. Well-Formed Formulas (wffs)

4.1 If φ is an atomic formula, φ is well formed.

4.2 If φ and ψ are wffs, then the following are well formed:

4.2.1 ¬φ

4.2.2 (φ∧ψ)

4.2.3 (φ∨ψ)

4.2.4 (φ→ψ)

4.2.5 (φ↔ψ)

4.3 φ and ψ are well formed.

We turn now to the semantic notions, the most important of which for our purposes are that of an interpretation and that of an evaluation. The explanation of evaluations will be postponed until later in this chapter when semantics section is presented. Our attention can be given first to explaining interpretations.

An interpretation is a set theoretically described structure, a quadruple, in terms of which evaluations will be articulated. The definition of interpretations characterizes the sorts of structures that contain appropriate domains in terms of which truth conditions for the wffs of [illegible] can be discussed. This definition is given via sets and functions.

The first coordinate of the interpretations is a set M, and is to be thought of as the set of moments or temporal stages. There is no connection between the members of this set and specific units of chronometric duration such as seconds, hours or decades. The members of M are temporally atomic as far as the theory is concerned, not temporally atomic as far as some practical chronometric scheme is concerned.

The second coordinate is *m*. *m* is to be thought of as the present temporal stage, or the interval that we call "now". *m* is, of course, a member of the set of temporal stages, M.

The third coordinate is $\lessdot$. Intuitively $\lessdot$ is the earlier–than relation between temporal stages. We will adopt the convention of writing $n \lessdot m$ for, "n has the relation $\lessdot$ to m"; intuitively, "n is earlier than m".

The fourth coordinate is the multi–purpose function *f*. Part of its work, for example, is to assign truth values to atomic sentences at temporal stages. The roles of this function will be discussed one part at a time as various issues are discussed. The fifth is function *g* which intuitively takes a moment, an agent and a wff as its arguments and gives the axiological value for the agent at the moment of the situation that would be the case if the wff were true. In other words, if m is January 1, 399 B.C., and if a is the agent Socrates, and the wff p is the translation for "Socrates discusses philosophy" then $g(m,a,p)$ is to be thought of as the value in terms of goodness at m relative to Socrates of the possible situation wherein Socrates discusses philosophy.

The formal presentation is as follows:

7. An Interpretation of DBC is an ordered quintuple $\mathfrak{A} = \langle M, m, \lessdot, f, g \rangle$ where

7.1 M is a nonempty set.

7.2 $m \in M$.

7.3 $\lessdot$ is a binary relation on M.

7.4 *f* is a function such that

7.4.1 where ϕ is an atomic sentence and $x \in M$, $f(x,\phi) \in \{0,1\}$, i.e., for each member of M, *f* assigns either 0 or 1 to every atomic sentence. If $f(x,\phi) = 1$ then ϕ is satisfied in $\mathfrak{A}$ at x or, intuitively, true at x, and if

$f(x,\phi)=0$, ϕ is not satisfied in $\mathfrak{A}$ at x or false at x.

It is to be noticed that the interpretation apparatus of DBC allows for distinct members of M, intuitively, distinct temporal stages, to share identical sets of true atomic sentences. That is to say, it is acceptable that $(\phi)(f(m,\phi)=f(n,\phi))$ where ϕ is an atomic sentence, even if $m \neq n$.

The relation $\ll$ is to be thought of as the earlier–than relation between moments. Thus, the atomic sentences of DBC are not necessarily eternal sentences, not necessarily eternally true or eternally false. It is acceptable that "Socrates is sitting" be true at one moment and false at some earlier or later moment. Accordingly, the evaluation apparatus of DBC is temporally chauvinistic and the atomic sentences are to be thought of as being preceded by the temporally indexical expression "now" or "at the present moment".

In view of the plausibility of the view that for any three moments, x, y and z, if x is earlier than y and y is earlier than z then x is earlier than z, transitivity is required of $\ll$.

7.3.1 $(x \ll y \wedge y \ll z) \rightarrow x \ll z$.

Next, it is required of $\ll$ that it be left–linear. In other words, if we think of each moment as a point, and if we think of the points earlier than the present as to the left of the present moment, then imposing left–linearity on $\ll$ is like requiring that for any moment we choose, if we connect it with each of its earlier moments by a line, we will end up with a single linear sequence. Each moment in the sequence must be either earlier or later than some other moment in the sequence.

7.3.2 $(x \ll y \wedge z \ll y) \rightarrow (x \ll z \vee z \ll x)$.

Imposing this characteristic on the earlier–than relation is motivated by two beliefs. The first is that the past cannot be changed. Once it has

..., is not satisfied in M at ... .

It is to be noted that the interpretation apparatus of T(L) allows for distinct members of M, intuitively distinct temporal states, to share certain sets of true atomic sentences. That is to say, it is acceptable that $f(m,\sigma) = f(n,\sigma)$ where σ is an atomic sentence, even if $m \neq n$.

The relation ≪ is to be thought of as the earlier-than relation between moments. Thus the atomic sentences of T(L) are not necessarily eternal sentences, not necessarily eternally true or eternally false. It is acceptable that "Socrates is sitting" be true at one moment and false at some earlier or later moment. Accordingly, the evaluation apparatus of T(L) is temporally chauvinistic and the atomic sentences are to be thought of as being preceded by the temporally indexical expression "now" or "at the present moment".

In view of the plausibility of the view that for any three moments x, y and z, if x is earlier than y and y is earlier than z then x is earlier than z, transitivity is required of ≪:

7.3 i $(x \ll y \wedge y \ll z) \supset x \ll z$

Next it is required of ≪ that it be left-linear. In other words, if we think of each moment as a point, and if we think of the points earlier than the present as to the left of the present moment, then imposing left-linearity on ≪ is the requirement that for any moment we choose, if we connect it with each of its earlier moments by a line, we will end up with a single linear sequence. Each moment in the sequence must be either earlier or later than some other moment in the sequence.

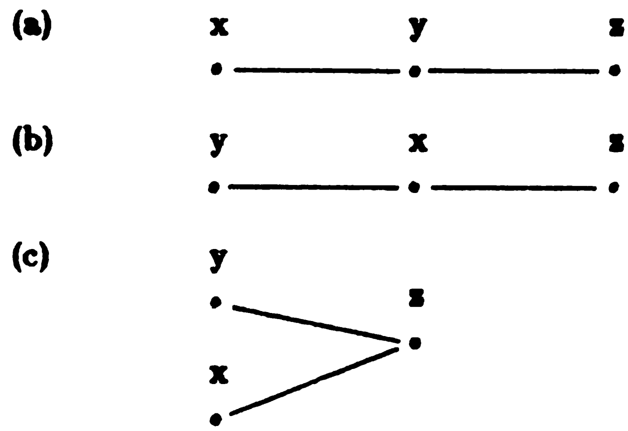
7.3 ii $(x \ll z \wedge y \ll z) \supset (x \ll y \vee x = y \vee y \ll x)$

Imposing this characteristic on the earlier-than relation is motivated by two beliefs. The first is that the past cannot be changed. Once it ...

been the case that ϕ, no one can bring it about that it is false that ϕ used to be the case. The requirement 7.3.2 orders all past possible worlds on a line from the present to the earliest world or to eternity past if there is no earliest world. Consider Figure 1.

FIGURE 1

LEFT–LINEARITY



In this illustration y, for example, is earlier than z just in case it is connected to z by a line and to the left of z. Condition 7.3.2 endorses orderings (a) and (b), but excludes (c).

The second belief is that the future, unlike the past, presents alternatives. Since right–linearity is not imposed, the temporal relation might branch into the future. From the perspective of the present there are several future courses of events, and not at most one, which right–linearity would require. For example, if right–linearity is not required, there might be some sequence of moments later than the present in which "The Pentagon is blue" is never true, other sequences containing some moment where it is. More metaphysically stated, the belief that the temporal relation been the case that [illegible] to be the case. The requirement Z.2 orders all past possible worlds in a line from the present to the earliest world or to eternity past if there is no earliest world. Consider Figure 1.

FIGURE 1
LEFT LINEARITY

In this illustration y, for example, is earlier than x just in case it is connected to x by a line and to the left of x. Condition Z.2 endorses orderings (a) and (b) but excludes (c).

The second belief is that the future, unlike the past, presents alternatives. Since right-linearity is not imposed, the temporal relation might branch into the future. From the perspective of the present there are several future courses of events, and not at most one, which right linearity would require. For example, if right-linearity is not required, there might be some sequence of moments later than the present in which "The Cold War is over" is never true, other sequences containing some [illegible] where it is [illegible] the belief that the [illegible] can branch toward the future is ordinarily associated with indeterminism regarding some future events.

In Chapter 9 of *De Interpretatione*, Aristotle, following his indeterministic inclinations, suggested that sentences like "There will be a sea–fight tomorrow" are at present neither true nor false. Many have taken this as a rejection of the principle of bivalence; the rule that truth and falsehood are mutually exclusive and jointly inclusive as far as truth value is concerned. Aristotle's worries over determinism stem from his belief that what agents do makes a difference in the way things are going to turn out.

Aristotle's intuitions cannot be taken lightly by any analysis of the moral notions or the concept of action. He seems to find it difficult to imagine that ascriptions of obligation, forbiddance or permission have any point if actions have no influence on the future course of events at all, and many theorists since have had the same difficulty. It is intuitively plausible that at each moment, a host of genuinely possible scenarios are presented to the agent and that some of these can be actualised by his choices and actions.

Semantic arrangements that impose both left and right linearity on the earlier–than relation are labelled deterministic, and those that like DBC allow for right–branching are considered indeterministic. This is not to say that right–branching in the temporal relation rules out all beliefs that might be legitimately called deterministic, but this matter will not be discussed until Chapter Six. Figure 2 depicts right branching.

[illegible] the future [illegible] as [illegible] with [illegible] some future events.

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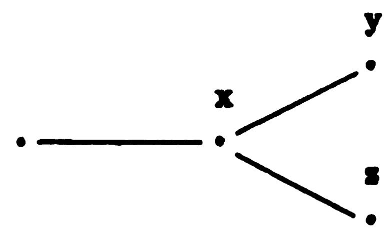
FIGURE 2

RIGHT BRANCHING



Following the diagramming conventions of Figure 1, this illustration depicts the preservation of left but not right linearity. x is earlier than both y and z but y and z do not have the earlier–than relation to one another.

Various attempts have been made to preserve bivalence for sentences of the form "It is going to be the case that ϕ" in right–branching semantics. The best of these attempts owes to A. N. Prior's presentation of the Ockhamist tense system OT in *Past, Present and Future*. In terms of the semantics of DBC, Prior can be taken as suggesting there that truth value for sentences be relative not only to a moment but relative to some left and right linear sequence of these moments containing the moment of truth valuation[9]. The earlier–than relation need not be right–linear although the left and right linear sequences having a role in evaluation are. These left and right linear sequences will hereafter be called histories.

Let "h" be a name for a history, and "h" be a variable ranging over histories.

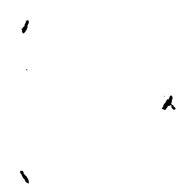
7.5 h is a history in interpretation $\mathfrak{I}$ iff:

i) $\mathit{h} \subseteq M$,

ii) $(x \neq y \wedge x \in \mathit{h} \wedge y \in \mathit{h}) \rightarrow (x \lessdot y \vee y \lessdot x)$,

FIGURE 2

RIGHT BRANCHING



Following the illustrating conventions of Figure 1, this illustration depicts the preservation of left but not right linearity: x is earlier than both y and z but y and z do not have the earlier-than relation to one another.

Various attempts have been made to preserve bivalence for sentences of the form "It is going to be the case that φ" in right-branching semantics. The best of these attempts owes to A. N. Prior's presentation of the Ockhamist tense system OT in *Past, Present and Future*. In terms of the semantics of T×W, Prior can be taken as suggesting there that truth value for sentences be relative not only to a moment but relative to some left and right linear sequence of those moments containing the moment of truth valuation. The earlier-than relation need not be right-linear although the left and right linear sequences having a role in evaluation are. These left and right linear sequences will hereafter be called histories.

Let "h" be a name for a history and "ĥ" be a variable ranging over histories.

7.5 **h** is a history in interpretation 𝔐 iff:

(i) $h \subseteq M$

(ii) $\forall x \forall y ((x \in h \wedge y \in h) \rightarrow (x < y \vee y < x \vee x = y))$

iii) $x \in h \rightarrow (y)(y \lessdot x \rightarrow y \in h)$ and

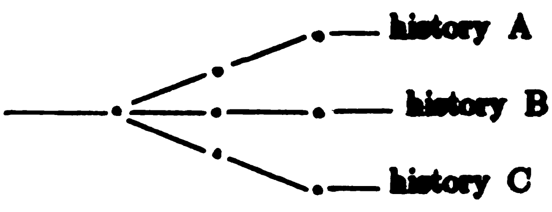
iv) $(x \in h \wedge (\exists y)(x \lessdot y)) \rightarrow (\exists y)(x \lessdot y \wedge y \in h)$.

Condition ii) requires that the relation $\lessdot$ as it joins members of h is left and right linear. iii) requires that for each moment x in the history, all moments earlier than x also be members of the history. One might say that this requires that histories be maximal as far as earlier moments are concerned. iv) requires that unless there are no moments later than x, there must be some moment later than x in the history. In other words, iv) requires that a history be maximal as far as later moments are concerned.

Suppose, for example, that it might come to be the case at some later situation that Meyers is sitting. Then there is a succession of possible moments later than the actual one including one at which "Meyers is sitting" is true. A sequence of moments each of which is earlier than or later than every other moment is like a pathway through the past and one of the possible futures as displayed in Figure 3.

FIGURE 3

HISTORIES



Why relativize evaluations to histories? The problem, as mentioned previously, centers on truth conditions for sentences of the type "It is going to be the case that ϕ", where the future truth of ϕ is a contingent matter.

(III) [illegible] and

(IV) [illegible]

Condition (III) requires that the relation < as it [illegible] members of a history be [illegible] and total linear. (IV) requires that for each moment x in the history, all moments earlier than x are members of the history. One might say that this requires that histories be maximal as far as earlier moments are concerned. (V) requires that unless there are no moments later than x, there must be some moment later than x in the history. In other words, (V) requires that a history be maximal as far as later moments are concerned.

Suppose, for example, that it might come to be the case at some later moment that Xerxes is strong. Then there is a succession of possible moments later than the actual one including one at which "Xerxes is strong" is true. A sequence of moments each of which is earlier than or later than every other moment is like a pathway through the past and one of the possible futures as displayed in Figure 3.

FIGURE 3

HISTORIES

history A

history B

history C

Why relativize evaluations to histories? The problem [illegible] previously centres on truth conditions for sentences of the type "It will [illegible] to be the case that [illegible]", where the future tense [illegible]

To return to Aristotle's example, it seems that today, (or yesterday, or the day before) it is (or was) indeterminate whether or not there will be a sea–battle tomorrow (or two days hence, etc.). Now, supposing the temporal relation branches after the present, if the evaluation apparatus does not take histories into account, there seems to be no way to depict truth conditions for "It is going to be the case that there is a sea–battle". If we say on the one hand that this sentence is true just in case "There is a sea–battle" is true in one of the future moments in one of the histories containing the present, we have depicted conditions more appropriate for the weaker claim "Possibly it is going to be the case that there is a sea–battle" than for the unmodalized sentence with which we began. On the other hand, if we say that this is true just in case "There is a sea–battle" is true in some moment in every history containing the present, we have depicted conditions more appropriate for the stronger "Necessarily it is going to be the case that there is a sea–battle" than the weaker unmodalized sentence with which we began.

From the viewpoint of eternity, only one of the histories containing the present moment will be materialized. But which one of these will be materialized is at present, not determined. Hence, Prior suggests that truth conditions be relativized to one of the histories. "It is going to be the case that there is a sea–battle" is true relative to some history h just in case "There is a sea–battle" is true in some future moment in h. It is to be remembered that at present, there is nothing special about any one of the histories that contain the present. In other words, any history of evaluation is to be thought of as merely *prima facie* the future. Which history the course of events will follow is at present not only unknown to us, it is not fixed. This procedure has the disvalue of importing not only the temporal to return to Aristotle's example, it seems that today (or yesterday, or in any instance) it is (or was) indeterminate whether or not there will be a sea-battle tomorrow (or two days hence, etc.). Now, supposing the temporal relation branches after the present, if the evaluation of statements does not take histories into account, there seems to be no way to depict truth condition for "It is going to be the case that there is a sea-battle." If we say on the one hand that this sentence is true just in case "There is a sea-battle" is true in one of the future moments in one of the histories containing the present, we have depicted conditions more appropriate for the weaker claim "Possibly it is going to be the case that there is a sea-battle" than for the unqualified sentence with which we began. On the other hand, if we say that this is true just in case "There is a sea-battle" is true in some moment in every history containing the present, we have depicted conditions more appropriate for the stronger "Necessarily it is going to be the case that there is a sea-battle" than the weaker unqualified sentence with which we began.

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We are now in a position to introduce truth conditions for atomic sentences and truth functional compounds as follows:

9. Evaluation: Where ϕ is a wff, let ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}$ abbreviate "the truth value of ϕ at world m relative to history h containing m under interpretation $\mathfrak{A}$".

9.1 Where ϕ is an atomic sentence, ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=t$ if $f(m,\phi)=1$, otherwise it is f.

9.2 Where ϕ is a wff, ${}^{h}_{m}[\![\neg\phi]\!]^{\mathfrak{A}}=t$ if ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}\neq t$, otherwise it is f.

9.3 Where ψ is also a wff, ${}^{h}_{m}[\![(\phi\wedge\psi)]\!]^{\mathfrak{A}}=t$ if both ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=t$ and ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=t$, otherwise it is f.

9.4 ${}^{h}_{m}[\![(\phi\vee\psi)]\!]^{\mathfrak{A}}=t$ if either ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=t$ or ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=t$, otherwise it is f.

9.5 ${}^{h}_{m}[\![(\phi\rightarrow\psi)]\!]^{\mathfrak{A}}=t$ if either ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=f$ or ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=t$, otherwise it is f.

9.6 ${}^{h}_{m}[\![(\phi\leftrightarrow\psi)]\!]^{\mathfrak{A}}=t$ if either ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=t$ and ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=t$, or ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=f$ and ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=f$, otherwise it is f.

$\top$ and $\bot$ are the constants for truth and falsehood respectively and their truth conditions are

9.7 ${}^{h}_{m}[\![\top]\!]^{\mathfrak{A}}=t$ for every m and h.

9.8 ${}^{h}_{m}[\![\bot]\!]^{\mathfrak{A}}=f$ for every m and h.

[illegible]

... problems ... difficulties seem less severe than the problems in systems that do not follow ... In any event, the matter will not here be further pursued and Lukas's suggestion will be taken as correct.

We are now in a position to introduce truth conditions for atomic sentences and truth functional compounds as follows.

(Evaluation) Where φ is a wff, let $V_{M}(\varphi, w, h)$ denote the truth value of φ at world w relative to history h contained in model interpretation M.

(1) If φ is an atomic sentence, $V_{M}(\varphi, w, h) = 1$ iff ... $= 1$; otherwise it is 0.

(2) Where φ is $\neg\psi$, $V_{M}(\varphi, w, h) = 1$ iff $V_{M}(\psi, w, h) \neq 1$; otherwise it is 0.

(3) If φ is $\psi \wedge \chi$, $V_{M}(\varphi, w, h) = 1$ iff both $V_{M}(\psi, w, h) = 1$ and $V_{M}(\chi, w, h) = 1$; otherwise it is 0.

(4) $V_{M}(\psi \vee \chi, w, h) = 1$ iff either $V_{M}(\psi, w, h) = 1$ or $V_{M}(\chi, w, h) = 1$; otherwise it is 0.

(5) $V_{M}(\psi \rightarrow \chi, w, h) = 1$ iff either $V_{M}(\psi, w, h) = 0$ or $V_{M}(\chi, w, h) = 1$; otherwise it is 0.

(6) $V_{M}(\psi \leftrightarrow \chi, w, h) = 1$ iff either $V_{M}(\psi, w, h) = 1$ and $V_{M}(\chi, w, h) = 1$, or $V_{M}(\psi, w, h) = 0$ and $V_{M}(\chi, w, h) = 0$; otherwise it is 0.

1 and 0 are the constants for truth and falsehood respectively and their truth conditions are

(7) $V_{M}(1, w, h) = 1$ for every m and h.

(8) $V_{M}(0, w, h) = 0$ for every m and h.

In order to accomodate tensed sentences the following supplements are required:

1.4 Tense Operators: P and F,

and to the wff list are added:

3.2.6 Pϕ

3.2.7 Fϕ.

The evaluation apparatus is enlarged as follows:

9.9 ${}^{h}_{m}[\![P\phi]\!]^{\mathfrak{A}} = t$ if $(\exists x)(x < m \wedge {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}} = t)$, otherwise it is $\mathfrak{f}$.

9.10 ${}^{h}_{m}[\![F\phi]\!]^{\mathfrak{A}} = t$ if $(\exists x)(m < x \wedge {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}} = t)$, otherwise it is $\mathfrak{f}$.

Intuitively, Pϕ translates ordinary sentences of the type "It used to be the case that ϕ". These are true just in case there is some moment earlier than the moment of evaluation in which ϕ is true. Fϕ translates "It is going to be the case that ϕ" and sentences having this form are true just in case there is some moment in history h later than that of evaluation in which ϕ is true. The similarities between this apparatus and that of Prior's tense system OT, as interpreted by Thomason[10], guarantee that the theses of OT are among the theses of DBC.

2. *ALETHIC MODALITY*

In the introduction, it was mentioned that DBC resembles the actualist versions of alethic modality, as proposed, for example, by Aristotle and Diodorus, which base alethic modality on temporal relations. This distinguishes the analysis here from the possible worlds analyses of modality offered by David Lewis or Saul Kripke, for example. There can be little doubt that possible worlds semantics has been very productive in the analysis

[illegible]

[illegible]

[illegible]

[illegible] the wff that are not [illegible]

[illegible]

[illegible]

The evaluation of formulas is carried out as follows:

(v) [illegible] = 1 iff [illegible]; otherwise it is 0.

(vi) [illegible] = 1 iff [illegible]; otherwise it is 0.

Intuitively, F translates ordinary sentences of the type "It will be the case that φ." These are true just in case there is some moment later than the moment of evaluation in which φ is true. P translates "It was the case that φ" and sentences having this form are true just in case there is some moment in history h later than that of evaluation in which φ is true. The similarities between this apparatus and that of Prior's tense system [illegible] as interpreted by Thomason [illegible] that the theses of [illegible] are among the theses of [illegible].

[illegible] MODALITY

In the introduction it was mentioned that [illegible] extends the versions of alethic modality as proposed, for example, by Arthur Prior, treatments which base alethic modality on temporal notions. This distinguishes the analysis here from the possible worlds analyses of modality offered by David Lewis or Saul Kripke, for example. There can be little doubt that possible worlds semantics has been very productive in the analysis not only of modal discourse, but in the treatment of other concepts recalcitrant to analysis. Moreover, describing the characteristics of certain sorts of sets and functions is, ontologically speaking, a relatively harmless mathematical exercise, and possible worlds semantics involves this sort of exercise. But when one applies these characteristics philosophically, when one claims that certain of these set–theoretically described objects are to be thought of as possible worlds, one is committed to the existence of possible worlds. If one sets out the truth conditions for certain sorts of sentences in terms of possible worlds, some account is due, explaining how these possible worlds are to be classified ontologically. According to Alvin Plantinga, for example, possible worlds exist, and they are abstract sorts of objects. According to David Lewis, on the other hand, a possible world is no more an abstract object than is the world we live in, with its nonabstract trees, tables, etc. As far as existence is concerned, the actual world and all the possible worlds are on a par. Of possible worlds, Lewis says,

> There are countless other worlds, other very inclusive things. Our world consists of us and all our surroundings, however remote in time and space; just as it is one big thing having lesser things as parts, so likewise do other worlds have lesser other–worldly things as parts. The worlds are something like remote planets; except that most of them are much bigger than mere planets, and they are not remote. Neither are they nearby. They are not at any spatial distance whatever from here. They are not far in the past or future, nor for that matter near; they are not at any temporal distance whatever from now. They are isolated: there are no spatiotemporal relations at all between things that belong to different worlds. Nor does anything that happens at one world cause anything to happen at another. Nor do they overlap; they have no parts in common...[11]

And it looks to me as if Lewis' modal realism is the proper ontological

[illegible]

[illegible]

[illegible]

corelative to a possible worlds semantics. Possible worlds semantics commits one to including possible worlds in one's ontology, and a possible world is the same sort of thing that the actual world is. In other words, a view like Plantinga's, according to which a possible world is an abstract object is to be rejected.

Lewis says that a possible world is temporally complete. Objects or states of affairs that are temporally related to the present moment are included in the possible world we call "actual" from our perspective. Socrates drinking hemlock, for example, is part of the actual world, although this event is not presently occurring from the temporally chauvinistic viewpoint. In terms of typical right–branching tense logic semantics, this is tantamount to the view that distinct possible worlds are simply distinct tree–like clusters of histories. If the earlier–than relation does not branch in either direction, this is tantamount to the view that distinct possible worlds are like distinct histories.

In the DBC analysis of modal sentences, no reference is made to moments in histories which have no temporal relation to the moment of evaluation. There are acceptable interpretations which contain distinct moments having no temporal relation to one another, and so, the semantics does not rule out a modal realistic view according to which multiple possible worlds exist. But, no reference is made to these multiple worlds in the truth conditions for modal sentences here. The semantics, in other words, does not commit one to modal realism. One need only believe that there is but one possible world, the actual one.

In some discussions of the alethic modalities, a world x is said to be possible relative to a world y just in case the histories of x and y are alike in a specified respect. In DBC the truth conditions for modal sentences are

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

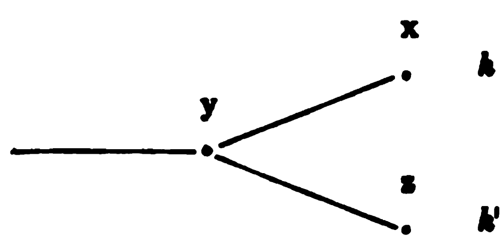
articulated in terms of this similary between histories for two moments where no reference is made to possible worlds other than the actual one. This is accomplished by introducing a relation R, for relative possibility between worlds such that where Rx,y, x is to be thought of as possible relative to y. The conditions specify that one moment is possible relative to another just in case both moments share identical sequences of past moments. x is possible relative to y just in case the sequence of possible worlds leading up to y is identical to the sequence leading up to x. So, we add to the semantics the following condition:

7.3.3 $Rx,y \leftrightarrow \{z : z < x\} = \{z : z < y\}$.

Notice that differences of order are ruled out, viz., if z is earlier than z' in the sequence leading up to x, then it must also be earlier than z' in the sequence leading up to y. Consider the following figure.

FIGURE 4

IDENTICAL PASTS



Here Rx,z. y is the next earliest moment relative to x and to z and so there is no distinction between the sequence of worlds leading up to x and that leading up to z.

But $<$ might be left dense. That is to say, there might be no possible

world *next* earliest relative to x or z. The conditions for R do not entail left discreteness as it would appear from Figure 4, indeed, nothing in the formal semantics makes it such that the relation $\lessdot$ must be discrete if there are distinct moments possible relative to one another. It might be that the set of earlier moments shared by x and z is larger than the set of natural numbers. Even in this case, the sets of earlier moments might be identical because they share the same members and their order in terms of $\lessdot$ might be identical so that Rx,z without there being a *next earliest* possible world relative to x or z. Density in cases like this is difficult if not impossible to diagram, but acceptable as a characteristic of $\lessdot$ nonetheless.

As is customary, the box, $\Box$, will serve as a sentence operator, that when prefixed to a wff, gives another wff. Where ϕ is a wff, $\Box\phi$ is to be read, "It is necessarily the case that ϕ". Thus we add to the syntax:

1.6 Modal Operator: $\Box$.

The corresponding wff is:

3.2.9 $\Box\phi$,

and the truth conditions for sentences of this form are:

9.11 $\frac{h}{m}[\![\Box\phi]\!]^{\mathfrak{A}} = t$ if $(x)(h)(Rx,m \rightarrow \frac{h}{x}[\![\phi]\!]^{\mathfrak{A}} = t)$, otherwise it is f.

The truth conditions for sentences of the form $\Diamond\phi$, which translate "It is possibly the case that ϕ" are introduced by means of a definitional thesis. Hereafter let $\models\phi$ abbreviate $\frac{h}{m}[\![\phi]\!]^{\mathfrak{A}} = t$ in every permissible interpretation $\mathfrak{A}$ for every m and h, viz., ϕ is a thesis.

Df$\Diamond$ $\models \Diamond\phi \leftrightarrow \neg\Box\neg\phi$.

Since R is an equivalence relation, the theorems of the Lewis modal system S5 are among the theses of DBC. Also it is to be noted that

13.1 $\not\models P\phi \rightarrow \Box P\phi$.

This seems to counter the intuition that what used to be the case cannot be

[illegible]

(11) [illegible]

The [illegible] will be

[illegible]

and the truth conditions for sentences of this form are

[illegible]

The truth conditions for sentences of the form [illegible] which [illegible]

[illegible]

since [illegible] is an equivalence relation [illegible]

[illegible]

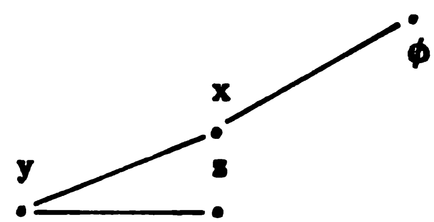
[illegible] seems to commit us [illegible] notion that what used to be the case [illegible] changed and thus necessarily used to be the case. But consider past tensed sentences that refer to events yet future, for example, "It used to be the case that it is going to be the case in 1999 that the U.S. has no navy". Unless the ϕ contains no sentences referring in this way to states of affairs that are future relative to the moment of evaluation, thesishood should be ruled out. And in DBC it is, as the obvious countermodels show[12].

It should also be noted that in order for $\Box F\phi$ to be true, ϕ must be true at some later moment in every history containing moments possible relative to m. This produces a problem if sentences of the form $\Diamond F\phi$ translate "It is possibly going to be the case that ϕ". $\Diamond F\phi$ can be true at m in cases where there is a moment future relative to some moment possible relative to m but distinct from m in which ϕ is true. In other words, the future tense possibilitation, "It is possibly going to be the case that ϕ", can be true at m even if ϕ is not a future truth in any history containing m. The histories including a moment earlier than m determine the future possibilities at m. Unless another primitive operator for future tense possibilitations is introduced, $\Diamond F\phi$ is to be thought of as translating something like "Relative to the past , it is possibly going to be the case that ϕ" rather than "Relative to the present, it is possibly going to be the case that ϕ". The counterintuitive situation is depicted in Figure 5.

[illegible]

FIGURE 5

FUTURE POSSIBILITY



Suppose y is the next earliest moment relative to x and z. It is permissible that at z, $\Diamond F\phi$ be true without ϕ being true in any of the moments later than z.

CHAPTER TWO

ACTION

1. BRINGING SOMETHING ABOUT

What is the relation between agents and their actions? Intuitively, the actions of each agent narrow the possibilities at each moment. If Smith now brings it about that the lamp on his desk is on, Smith's actions guarantee that the lamp on his desk is on. Smith guaranteeing that this is the case is tantamount to Smith preventing it from being the case that the lamp on his desk is off. What Smith now brings about can be identified with the contribution he makes to the way things now are. In other words, when Smith brings something about at m, he guarantees that the history that materializes must be among certain of all the histories containing m and he guarantees that certain histories will not be materialized. And it seems plausible to explain his influence on the course of events in this way. If the set of all x such that Rx,m is taken to be the set of moments simultaneously possible relative to m, and if truth at all of these moments is identified with simultaneous necessary truth, it seems fitting that what Smith brings about at m be identified with the set of moments (or the set of histories containing these moments) his actions guarantee or necessitate at m.

To return to our example, let us suppose that the lamp on Smith's desk is on. Furthermore, suppose that Smith brings this about. If the present moment is *m*, there is some moment, say n, simultaneous with the present at which the lamp on Smith's desk is off. In other words, that his

CHAPTER TWO

MOTION

[illegible]

[illegible] at each instant it now [illegible]

[illegible] the lamp on his desk is on [illegible]

[illegible] the lamp on his desk is on [illegible] that [illegible]

[illegible] the case that the lamp [illegible]

desk is on. [illegible] What [illegible] seems can be identified with [illegible]

[illegible] the way things now are. In other words [illegible]

[illegible] something about [illegible] that the [illegible]

[illegible] of all [illegible] constituents [illegible] and to

positions that certain [illegible] will not be [illegible] And it seems

[illegible] the course of events in this way. If the

[illegible]

[illegible] of all these moments [illegible]

[illegible] identified with [illegible]

[illegible] the set of moments for the [illegible]

[illegible] constituents [illegible]

To return to our example, let us suppose that the lamp on [illegible]

desk is on [illegible]

[illegible]

desk lamp is off is possible and that it is on is contingent. But Smith guarantees that it is on and in so doing guarantees that it is not off. In all the moments assigned to him at m, which do not include all the possible moments at m, the lamp is on. In all the rest of these possible moments it is off. In line with this account, it also seems fitting to think of the actions of an agent at a moment as associated with an action assignment at that moment as follows.

The syntax of DBC is enlarged to include names which are a species of terms. These names are to be thought of as the names of agents. The other species of terms are variables, which will be introduced later.

1.1 Terms

1.1.1 Names: Lower case letters a through e with or without subscripts.

For example, if a is a name, a might be thought of as the name for the agent Socrates. To depict agency the operation of the function f is enlarged so that it takes an agent name and a moment m and assigns to this agent at m a subset of the set of moments possible relative to m. Intuitively, the set assigned by f to agent α and moment m, is the set of moments possible relative to m that the actions of α determine. In other words, what α brings about at m is a sort of conditional necessity at m. What α brings about at m is what is necessary at m given what α does or has done. The corresponding operation of f is as follows:

7.4.2 Where α is a name and $x \in M$, $f(m,\alpha) = \Delta$ where

(i) $\Delta \subseteq \{y{:}Ry,x\}$,

viz., Δ is a subset of the set of moments simultaneously possible relative to x.

The subset of simultaneous possible moments so assigned at moment

The subset of simultaneously possible moments ... moment

(ii) ... is a subset of the set of moments simultaneously possible relative to

2.7.2 ... is a moment ... where

... The corresponding operation of λ is as follows:

... what is possible at m ... is a set of conditional ... at m ...

moments possible relative to m ... the set of ... operations ...

... assigned by λ to ... at m ...

this ... is a subset of the set of ... possible relative to ...

... For example, ... a moment ...

... lower case letters ... with or without

... subscripts

... which will be interpreted ...

... the names of agents ...

The ... which ... moment ...

... is a moment ... with ...

... In line with ... also ... of the ...

... assigned to ... at m, which ... possible ...

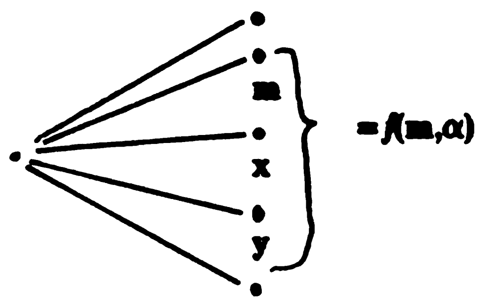
m, regardless of the agent, must include the present moment from the vantage point of m, namely m itself. This comports with the view that if the actions of each agent constitute a narrowing of the range of possibilities, then this narrowing includes the present actual situation. So we add to the conditions of 7.4.2:

(ii) $m \in \Delta$.

Consider the situation of Figure 6.

FIGURE 6

ACTION ASSIGNMENT



This diagram depicts the action assignment relative to α at moment m. The actions of agent α narrow the possibilities at m to the histories containing m, x, or y.

In order that no distinct agents have the same action assignment at a given moment we add:

(iii) $\alpha \neq \beta \rightarrow f(m,\alpha) \neq f(m,\beta)$

Action ascriptions in DBC have the form B$\alpha\phi$ where B is the two-place action operator, α is an agent, and ϕ is what α brings about. It is intended that wffs of the type B$\alpha\phi$ be read intuitively as "α brings it

[illegible]

[illegible]

the actions of each agent constitute a narrowing of the [illegible]

then this narrowing [illegible] the present actual situation [illegible] so we add to [illegible]

conditions of 7.4.2.

(a) m [illegible]

Consider the situation of Figure [illegible]

Figure [illegible]

[illegible]

m

[illegible] (m) =

[illegible]

This diagram depicts the action [illegible] relative [illegible] at moment m. The actions of [illegible] narrow the possibilities at m to the histories containing m [illegible]

In order that [illegible] have the same action [illegible] at [illegible] moment we need

[illegible] (m) [illegible]

When [illegible] in [illegible] have the form [illegible] where [illegible] the two place action operator [illegible] and [illegible] is what it brings about [illegible] is required that [illegible] of the type [illegible] be read intuitively as [illegible] about that ϕ". Accordingly the syntax of DBC is enlarged to include the actions operator:

1.7 Action Operator: B.

Where α is a term, the wff list may be expanded to include:

3.2.8 $B\alpha\phi$

The truth conditions for action acriptions are given in two parts. In condition (i) $B\alpha\phi$ is true at m only if ϕ is true at every member of the set $f(m,\alpha)$ which represents the set of possible moments made necessary at m by α's actions. But, in order for α's contribution to what happens at m to be depicted in the truth conditions, the *exclusion* of possible moments that α's actions are associated with is also required, and this exclusion is accomplished in condition (ii). Thus, the set $\{x{:}Rx,m \wedge x \notin f(m,\alpha)\}$ in (ii) represents the set of possible moments that the actions of α *prevent* from being the case at m. When α brings about ϕ he guarantees that ϕ is the case and in so doing he prevents $\neg\phi$ from being the case.

Truth conditions for wffs of this form are as follows:

9.12 $^{h}_{m}[\![B\alpha\phi]\!]^{\mathfrak{A}} = t$ iff

(i) $(x)(h)(x \in f(m,\alpha) \rightarrow {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}} = t)$ and

(ii) $(x)(h)((x \in \{x{:}Rx,m \wedge x \notin f(m,\alpha)\} \rightarrow {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}} = f))$.

Without condition (ii), $\models \Box\phi \rightarrow B\alpha\phi$, viz., every agent brings about any state of affairs that is necessarily the case, that 2+2=4, etc., which is counterintuitive.

More should be added to explain what is meant by the "narrowing" of possibilities associated with the actions of each agent. Consider the sentences ϕ and ψ. Suppose at moment m there are four types of x such that Rx,m. Where x is a moment of type 1, $(h)(x \in h \rightarrow {}^{h}_{x}[\![(\phi \wedge \psi)]\!]^{\mathfrak{A}} = t)$; where x is of type 2, $(h)(x \in h \rightarrow {}^{h}_{x}[\![(\neg\phi \wedge \psi)]\!]^{\mathfrak{A}} = t)$; where x is type 3, $(h)(x \in h \rightarrow {}^{h}_{x}[\![(\phi \wedge \neg\psi)]\!]^{\mathfrak{A}} = t)$

[illegible]

[illegible]

[illegible]

Where [illegible] is a form of "will" that is to be [illegible] in mind:

[illegible]

The truth conditions for such sentences are given in two parts. In condition (i) [illegible] is true at m only if [illegible] is true at every member of the set [illegible] which represents the set of possible moments made necessary at m by [illegible] actions. But, in order for [illegible] contradiction to what happens at m to be [illegible] in the truth conditions, the exclusion of possible moments that [illegible] are associated [illegible] with is also required, and this exclusion is accomplished in condition (ii) [illegible] the set [illegible] represents the set of possible moments that the actions of [illegible] prevent from being the case at m. When [illegible] brings about [illegible] he guarantees that [illegible] is the case and in so doing he prevents [illegible] from being the case.

Truth conditions for "will"s of this form are as follows:

(i) [illegible] = 1 iff

(ii) [illegible] = 1 and

(iii) [illegible] = 1)

Without condition (iii) [illegible], every agent brings about any state of affairs that is necessarily the case, that 2+2=4 etc., which is counterintuitive.

Note should be added to explain what is meant by the "narrowing" of possibilities associated with the actions of each agent. Consider the sentence [illegible]. Suppose at moment m there are four types of x such that [illegible]. Where x is a moment of type 1, [illegible] where x is of type 2, [illegible] where x is type 3, [illegible] and where x is of type 4, $(h)(x \in h \rightarrow {}^{h}_{x}[\![(\neg\phi \wedge \neg\psi)]\!]^{x} = t)$. The situation might be depicted as follows:

1	2	3	4	types of moments
ϕ	$\neg\phi$	ϕ	$\neg\phi$	
ψ	ψ	$\neg\psi$	$\neg\psi$	

Since R is an equivalence relation, m is of one of these types. And if the set of moments possible relative to m contains moments of all 4 types one might say this set is a ϕ–ψ–gamut, since as far as the truth and falsehood of ϕ and ψ are concerned, the moments possible relative to m span the entire series. If {x:Rx,m} is a ϕ–ψ–gamut there are eight possible different $f(m,\alpha)$ assignments for each α respecting types 1 through 4. What α might bring about at m relative to the eight is depicted in the following diagram. $[i_j,\ldots i_k]$ indicates that a row considers moments that are of each type i_k listed, but not any other types. For example [4,2,1] indicates that moments of types 4, 2, 1 are under consideration but not moments of type 3.

$f(m,\alpha)=$	α's actions	degrees
[1]	$B\alpha(\phi \wedge \psi)$	(i)
[1,2]	$B\alpha\psi$	(ii)
[1,3]	$B\alpha\phi$	(ii)
[1,4]	$B\alpha(\phi \leftrightarrow \psi)$	(ii)
[1,2,3]	$B\alpha(\phi \vee \psi)$	(iii)
[1,2,4]	$B\alpha(\phi \rightarrow \psi)$	(iii)
[1,3,4]	$B\alpha(\psi \rightarrow \phi)$	(iii)
[1,2,3,4]	$B\alpha(\phi \vee \neg\phi)$	(iv)

The narrowing associated with agent action is represented by the [illegible] of type 4 (that [illegible]) [illegible] can be depicted as follows.

Types of moments	1	2	3	4
	φ	φ	$\neg\varphi$	$\neg\varphi$
	ψ	$\neg\psi$	ψ	$\neg\psi$

Since R is an equivalence relation, m is of one of these types. The set of moments possible relative to m contains moments of all 4 types one might say this m is a φ-ψ-moment since as far as the truth and falsehood of φ and ψ are concerned, the moments possible relative to m span the entire space. If [illegible] is a φ-ψ-moment there are eight possible different [illegible] for each m respective types 1 through 4. What is true being about at m relative to the event is depicted in the following diagram [illegible] indicates that a row contains moments that are of each type listed, but not any other types. For example (1,2,4) indicates that moments of types 1, 2, 4 are under consideration but not moments of type 3.

Row	[illegible]	degrees
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
{1,2,3,4}	[illegible]	[illegible]

The diagram associated with agent action is represented as [illegible] column labelled "degree". These degrees can be associated with the columns of a truth table for each sentence above where Bα is detached. For example, an action of degree (i), which might be considered the strongest ((ii) the next strongest, etc.) can be associated with the truth table for (φ∧ψ) which has only *one* row where the formula takes the value t. Similarly, the tables for what is brought about under degree (ii) have but two rows where the formula takes the value t. Consider the following:

φ ψ	(φ∧ψ)	(φ↔ψ)	φ	ψ	(φ∨ψ)	(φ→ψ)	(ψ→φ)	(φ∨¬φ)
t f	t	t	t	t	t	t	t	t
t f			t		t		t	t
f t				t	t	t		t
f f		t				t	t	t
degrees of strength	(i)	(ii)			(iii)			(iv)

In column (iv) the weakest sort of action is considered. If at m α performs an action of degree (iv) relative to some φ and ψ, then α does not narrow the course of events at all at m. In order for an action to be this weak, ƒ(m,α) must equal the set of all x that are possible relative to m. Moreover, that α performs an action of degree (iii) relative to φ and ψ entails that Bα(φ∨ψ)∨Bα(φ→ψ)∨Bα(ψ→φ). That α performs an action of degree (i) relative to φ and ψ entails Bα(φ∧ψ), etc.

Notice that

13.2 $\not\models \Box\phi \rightarrow B\alpha\phi$.

And, since the set {x:Rx,m∧x∉ƒ(m,α)} might be empty, and since an agent might be assigned every moment possible relative to m, Bαφ∧□φ might be true at m. And in fact, this would have to be the case where α performs [illegible] These actions can be associated with [illegible] truth table for each sentence above where [illegible] example, an action of the sort which might be considered the [illegible] (in the next strategy [illegible]) can be associated with the truth table for [illegible] which has only one row where the formula takes the value t. Similarly, the tables for what is brought about under scheme (a) have but two rows where the formula takes the value t. Consider the following:

[illegible]

In column (v) the weak sort of action is considered. If at m α performs an action of the sort [illegible] relative to some φ and ψ then α does not narrow the course of events at all at m. In order for an action to be this weak, [illegible] must equal the set of all x that are possible relative to m. Moreover, that α performs an action of the sort (iii) relative to φ and ψ entails that [illegible]. That α performs an action of the sort [illegible] relative to φ and ψ entails both [illegible], etc.

Notice that

[illegible]

and since the [illegible] might be [illegible], and since an agent might be assigned every moment possible relative to m, [illegible] might be [illegible], and in fact this would have to be the case where α performs [illegible] any degree (iv) action relative to any ϕ and ψ. It is to be remembered that since the number of possible moments relative to any moment might not be finite, the notion of narrowing is merely an intuitive one.

The action theory section of DBC is enhanced by considering rules and theses that are analogous to well–known modal rules and theses. For example $B\alpha\phi \rightarrow \neg B\alpha\neg\phi$ is analogous to the modal sentence usually labelled "D", $\Box\phi \rightarrow \neg\Box\neg\phi$, or if $\Diamond\phi \leftrightarrow \neg\Box\neg\phi$, to $\Box\phi \rightarrow \Diamond\phi$. Hence, $B\alpha\phi \rightarrow \neg B\alpha\neg\phi$ will be labelled D_a since it is identical with D if each occurrence of $B\alpha$ (for any agent α) is replaced with $\Box$. Similarly, T_a is $B\alpha\phi \rightarrow \phi$ in analogy with T, $\Box\phi \rightarrow \phi$, etc.

The rule RE_a holds in DBC.

$$\dagger RE_a \quad \frac{\models \phi \leftrightarrow \psi}{\models B\alpha\phi \leftrightarrow B\alpha\psi}$$

Intuitively, one would expect that one brings about the logical consequences of what one brings about. But the rule RM_a does not hold.

$$\dagger RM_a \quad \frac{\models \phi \rightarrow \psi}{\models B\alpha\phi \rightarrow B\alpha\psi}.$$

This rule does hold if condition (ii) is dropped from the truth conditions 9.12 for action ascriptions. Here we are faced with two counterintuitive situations. Eliminating condition (ii) allows the counterintuitive $\Box\phi \rightarrow B\alpha\phi$ to be a thesis. Keeping (ii) eliminates rule RM_a. Although I have no strong reasons for favoring one alternative to the other, hereafter, it will be assumed that (ii) remains in the conditions.

Another thesis worth noticing is:

12.1 $\quad \models \neg B\alpha\bot$.

In other words, all agents fail to bring about the impossible, which we would expect. This thesis has importance in the proofs of Chapter Three.

[illegible] in the number of possible moments relative to [illegible] moment might [illegible] times, the notion of narrowing is merely an intuitive one.

The action theory section of LTA is enriched by connectives, rules and theorems that are analogous to well-known modal rules and theorems. For example [illegible] is analogous to the modal sentence usually [illegible] [illegible] be [illegible] since it is identical with [illegible] each occurrence of [illegible] for any agent [illegible] is replaced with [illegible]. Similarly, T [illegible] is analogous with [illegible].

The rule [illegible] holds in LTA:

[illegible]

Intuitively, one would expect that one brings about the logical consequences of what one brings about. But the rule [illegible] does not hold:

[illegible]

This rule does hold if condition (ii) is dropped from the truth conditions for action ascriptions. Here we are faced with two counterintuitive alternatives. The [illegible] condition (ii) allows [illegible] counterintuitive [illegible] rule [illegible]. Although I have no strong reasons for favoring one alternative to the other, hereafter, it will be assumed that (ii) remains in the conditions.

Another theorem worth noting is:

[illegible]

In other words, all agents fail to bring about the tautologies, which we would expect. [illegible] This has importance in the proofs of Chapter Three.

However,

13.3 $\not\models \neg B\alpha\top$

does not hold, and since an agent might be assigned every moment possible at a moment, $B\alpha\top$ might be true.

Similarly N_a and rule RN_a do not hold.

N_a $\not\models B\alpha(\phi \vee \neg\phi)$

RN_a $\not\frac{\models \phi}{\models B\alpha\phi}$.

Because of this there is no analogy between the action ascription theses of DBC and the normal alethic modal systems since in all of the latter, RM, N, and RN hold.

Regarding the passage of B over conjunction, C_a holds.

$\dagger C_a$ $\models (B\alpha\phi \wedge B\alpha\psi) \rightarrow B\alpha(\phi \wedge \psi)$.

But surprisingly, M_a does not.

$\dagger M_a$ $\not\models B\alpha(\phi \wedge \psi) \rightarrow (B\alpha\phi \wedge B\alpha\psi)$.

One would expect that if one brings about the conjunction of ϕ and ψ that one also brings about ϕ and brings about ψ. But not so. Again, the source of the problem is the second section of the truth conditions for action ascriptions (9.12 (ii)). Suppose $\phi \wedge \psi$ is true in all the moments assigned to α at m and that $\phi \wedge \psi$ is false in the rest of the possible moments relative to m because ϕ is false in all of them. This makes the antecedent of M_a true. Suppose also that ψ is logically true. Since ψ is not false in all possible moments not assigned to α at m, $B\alpha\psi$ is false, making the consequent of M_a false, and providing a countermodel.

Analogous to modal thesis K, K_a is a thesis.

$\dagger K_a$ $\models B\alpha(\phi \rightarrow \psi) \rightarrow (B\alpha\phi \rightarrow B\alpha\psi)$.

That is, if one brings about the conditional with ϕ for antecedent and ψ for

[illegible]

does not hold and since an agent can be ascribed every moment possible to a moment [illegible] might be true

unlikely N and rule I N do not hold.

[illegible]

Because of this there is no analogy between the action ascription theory of [illegible] and the normal modal systems since in all of the latter both [illegible] and [illegible] hold.

Regarding the passage to a lower conjunction, N both

[illegible]

but surprisingly N does not.

[illegible]

One would expect that if one brings about the conjunction of φ and ψ then one also brings about φ and brings about ψ. But not so. Again the source of the problem is the second section of the truth conditions for action ascriptions [illegible]. Suppose [illegible] is true in all the moments assigned to a at m and that [illegible] is false in the rest of the possible moments relative to m because ψ is false in all of them. This makes the antecedent of N true. Suppose also that φ is logically true. Since ψ is not false in all possible moments not assigned to a at m, [illegible] false, making the consequent of [illegible] false and producing a counterexample.

Analogous to modal thesis K, N is a thesis:

[illegible]

That is, if one brings about the conditional with φ as antecedent and ψ as [illegible] consequent, then if one also brings about the antecedent, one brings about the consequent. Also, T_a is a thesis.

$T_a \qquad \models B\alpha\phi \rightarrow \phi.$

Obviously, T_a entails D_a

$D_a \qquad \models B\alpha\phi \rightarrow \neg B\alpha\neg\phi.$

It also follows from T_a that the more general

12.2 $\qquad \models B\alpha\phi \rightarrow \neg(\exists s)Bs\neg\phi$

holds.

As P, RP and O hold in all normal KD systems of modal logic, so P_a, RP_a and O_a hold in DBC.

$P_a \qquad \models \neg B\alpha(\phi \wedge \neg\phi)$

$RP_a \qquad \dfrac{\models \phi}{\models \neg B\alpha\neg\phi}$

$O_a \qquad \models \neg B\alpha\neg\phi \vee \neg B\alpha\phi.$

Earlier, it was noticed that against ordinary intuitions, it is not a thesis that the logical consequences of what α brings about are also brought about by α. The principle that does hold, 12.3, indicates that the logical consequences of what α brings about do occur even though their occurrence is not necessarily attributable to α's actions.

12.3 $\qquad \dfrac{\models \phi \rightarrow \psi}{\models B\alpha\phi \rightarrow \psi.}$

It is also worth noting that:

13.4 $\qquad \not\models (B\alpha\phi \vee B\alpha\psi) \rightarrow B\alpha(\phi \vee \psi).$

This is counterintuitive. The countermodel is as follows: suppose $B\alpha\phi$ is true, making the antecedent of 13.4 true. And suppose that there is at least one moment possible relative to the moment of evaluation at which ϕ is false. If ψ is logically true, then $(\phi \vee \psi)$ must be true at this moment outside the action assignment to α at the moment of evaluation and thus consequent. This if one also holds about the antecedent, one knows about the consequent [illegible] T is a thesis:

T [illegible]

Obviously T entails I:

D [illegible]

It also follows from I that the more general

I.2 [illegible]

holds.

T, KT and [illegible] hold in all normal systems of modal logic, so T, KT, and [illegible] hold in [illegible].

I [illegible]

KT [illegible]

[illegible]

Earlier it was noticed that agents ordinarily [illegible], it is not a truism that the logical consequences of what a knows about are also known about by a. The principle that does hold, I.2, indicates that the logical consequences of what a knows about do occur even though their occurrence is not necessarily attributable to a's actions.

[illegible]

It is also worth noting that

I.3 [illegible]

This is counterintuitive. The counterintuitive is as follows: suppose [illegible] making the antecedent of [illegible] true. And suppose that there is at least one moment possible relative to the moment of evaluation at which [illegible]. If [illegible] is known true, then [illegible] must be true at this moment [illegible] action is sufficient to [illegible] at the moment of evaluation and the

$B\alpha(\phi v\psi)$ must be false, making the consequent of 13.4 false. Also note that:

13.5 $\not\models B\alpha(\phi v\psi)\rightarrow(B\alpha\phi v B\alpha\psi)$.

The theses 4_a, 5_a and B_a are not theses either.

The following two theses might be thought of as rules that with MP allow for the elimination of $B\alpha$:

12.4 $\models\Diamond B\alpha\phi\rightarrow\Diamond\phi$ and

12.5 $\models\Box B\alpha\phi\rightarrow\Box\phi$.

According to the former what is possibly brought about is possible. And according to the latter what is necessarily brought about is necessary. But consider

12.6 $\models(B\alpha\Box\phi\wedge\Diamond\psi\wedge\Diamond\neg\psi)\rightarrow\neg B\alpha\psi$

Suppose, for example that ϕ is necessarily the case at m. If α brings $\Box\phi$ about at m, then at m, α must be assigned every moment possible relative to m. If not, $\Box\phi$ would not be false in all the possible moments not assigned to α at m and thus condition 9.12 (ii) would be violated and $B\alpha\Box\phi$ would be false. If so, then for any contingent sentence, say ψ, there must be some moment of all those possible at m, where ψ is false and hence it is impossible for ψ to be true at all the moments assigned to α at m. In other words, those who bring about what is necessarily the case do not bring about anything contingent. Bringing about what is necessarily the case is not a sign of strength but of weakness. It follows from $B\Box\phi$ and T_a that $\Box\phi$. And $\Box\phi$ is true at m only if ϕ is true in every moment possible relative to m. Hence in order for α to bring this about, he must be assigned all possible moments. If so, he does not do any narrowing at all. For similar reasons, the rule of passage

12.7 $\models B\alpha\Box\phi\leftrightarrow\Box B\alpha\phi$.

holds.

be ◇(V[illegible]) must be false, again the consequent of 13.4 false. Also note that

$$13.5 \quad [illegible]$$

The theses 4, 5 and 13 are not theses either.

The following two theses might be thought of as rules that with Vi-sion for the elimination of Ex:

$$12.4 \quad \vdash \; [illegible] \; Ex\varphi \quad \text{and}$$

$$12.5 \quad \vdash \; [illegible] \; Ex\Box\varphi$$

According to the former what is possibly brought about is possible. And according to the latter what is necessarily brought about is necessary. But consider

$$12.6 \quad \vdash \; [illegible]$$

Suppose, for example, that φ is necessarily the case at m. If α brings about at m, then at m, α must be as[illegible] and every moment possible relative to m. If not, φ would not be false in all the possible moments not assigned to α at m and thus condition 9.12 (ii) would be violated and Exφ would be false. If so, then for any contingent sentence, say ψ, there must be some moment of all those possible at m, where ψ is false and hence it is impossible for ψ to be true at all the moments assigned to α at m. In other words, those who bring about what is necessarily the case do not bring about anything contingent. Bringing about what is necessarily the case is not a sign of strength but of weakness. It follows from 13.6 and T, that φ. And φ is true at m only if φ is true in every moment possible relative to m. Hence in order for α to bring this about, he must be assigned all possible moments. If so, he does not do any narrowing at all. For similar reasons, the rule of passage

$$12.7 \quad \vdash \; [illegible] \; \varphi \to Ex\varphi$$

fails.

Concerning the iteration of action ascription, which will play an important role in Chapter Four when coercion is considered and in Chapter Six when God's power is discussed consider:

13.6 $\nvDash B\alpha B\beta\phi \rightarrow B\alpha\phi$.

In other words that α brings it about that β brings it about ϕ does not entail that α brings ϕ about.

2. *AGENCY UNIQUENESS*

Here the question of the uniqueness of action arises. It is commonly believed that the actions of each agent at a moment are in some sense unique to him. Accordingly, we might add supplementary action operators to DBC that are more unique in acribing action than is B.

First, one might consider *temporal* repeatability or recurrence. Thus far, we have introduced wffs of the type $P\phi$ for ordinary sentences of the sort, "It used to be the case that ϕ" and $F\phi$ for ordinary sentences of the sort "It is going to be the case that ϕ". The temporally unique action operator we want to discuss can be introduced more efficiently if we add the well–known operators H and G for sentences of the form, "It always has been the case that ϕ" and "It is always going to be the case that ϕ" respectively. The definitional theses are as follows:

$$H\phi \leftrightarrow (\phi \wedge \neg P \neg \phi)$$

and

$$G\phi \leftrightarrow (\phi \wedge \neg F \neg \phi).$$

According to DBC an action can be performed by the same agent at temporally diverse moments. $B\alpha\phi$, $PB\alpha\phi$ and $FB\alpha\phi$ can all be true at the [illegible] which will [illegible] an [illegible] role in Chapter Four when coercion is considered and in Chapter [illegible] when God's power is discussed consider:

[illegible]

In other words that [illegible] brings it about that [illegible] brings it about [illegible] does not entail that [illegible] brings [illegible] about.

[illegible]

Here the question of the uniqueness of action arises. It is commonly believed that the actions of each agent at a moment are in some sense unique to him. Accordingly, we might add supplementary action operators to [illegible] that are more unique in ascribing action than is [illegible].

First, one might consider temporal necessitability or [illegible]. Thus far we have introduced ways of the type [illegible] for ordinary sentences of the sort, "It used to be the case that [illegible]" and [illegible] for ordinary sentences of the sort "It is going to be the case that [illegible]". The temporally unique action operator we want to discuss can be introduced more efficiently if we add the well-known operators H and G for sentences of the form, "It always has been the case that [illegible]" and "It is always going to be the case that [illegible]", respectively. The definitional theses are as follows:

[illegible]

and

[illegible]

According to [illegible] an action can be performed by the same agent at temporally discrete moments. [illegible] and [illegible] can all be true at the same interval. This agrees with the intuition that one can bring about today the same things one brought about yesterday. Moreover, there is no stipulation, as far as DBC is concerned, against HBαφ or GBαφ. But, if one desires that temporal uniqueness be a prerequisite for action ascription, and there are intuitions that favor this arrangement, this imposition can be associated with the following definitional thesis for an additional action operator B':

$$\models B'\alpha\phi \leftrightarrow (B\alpha\phi \wedge H\neg B\alpha\phi \wedge G\neg B\alpha\phi).$$

If B'αφ, α's bringing φ about is *temporally unique*. In other words, α's bringing φ about is temporally unique just in case α now brings it about but never did so before and will never do so again.

Another question about uniqueness is whether or not distinct agents can bring about the same state of affairs. It is permitted in DBC that for distinct agents α and β, Bαφ and Bβφ. This agrees with the intuition that, in some sense, agent α might be doing the same thing as agent β. The lack of agent to agent uniqueness, hereafter "social uniqueness", can be remedied by adding identity to the expressions of DBC in some acceptable way and by adding a definitional thesis for a new operator B" such that

$$\models B''\alpha\phi \leftrightarrow (B\alpha\phi \wedge \neg(\exists x)(x \neq \alpha \wedge Bx\phi)).$$

In other words α's bringing φ about is socially unique just in case α brings φ about at the world of evaluation and no distinct agent brings it about at this juncture.

A higher degree of uniqueness can be associated with a third action operator B‴ defined as follows:

$$\models B'''\alpha\phi \leftrightarrow (B'\alpha\phi \wedge \neg(\exists x)(x \neq \alpha \wedge (Bx\phi \vee PBx\phi \vee FBx\phi))).$$

In other words, α's bringing φ about in this socially and temporally unique way requires that no one else bring this about ever before or after, and that [illegible] with the intuition that one can bring about [illegible] some things one brought about yesterday. Moreover, there is no [illegible] conflation, as far as EBL is concerned against things or errors. But, if one desires that temporal uniqueness be a prerequisite for action ascription, and there are intuitions that favor this arrangement, this imposition can be associated with the following definitional thesis for an additional action operator B.

[illegible]

i.e. a's bringing p about is temporally unique. In other words, a's bringing p about is temporally unique just in case a now brings it about but never did so before and will never do so again.

Another question about uniqueness is whether or not distinct agents can bring about the same state of affairs. It is permitted in EBL that for distinct agents a and b, [illegible]. This agrees with the intuition that, in some sense, agent a might be doing the same thing as agent b. The lack of agent uniqueness, however "social uniqueness", can be remedied by adding identity to the expressions of EBL in some acceptable way and by adopting a definitional thesis for a new operator B' such that

[illegible]

In other words a's bringing p about is socially unique just in case a brings p about at the world of evaluation and no distinct agent brings it about at this juncture.

A third degree of uniqueness can be associated with a third action operator B'' defined as follows:

[illegible]

In other words, a's bringing p about in this world is socially and temporally unique [illegible] requires that no one else brings this about ever before or after, and [illegible]

α himself never before and never after brings it about.

The uniqueness of action has been a matter of concern since von Wright proposed his first monadic deontic system in 1951. He wanted the variables p,q... etc. which were being operated on by the obligation operator O (sentences of the form Op are well–formed) to range over act–types and not over act–tokens. Apparently the variable p, for example, might stand for "a murdering" or "a stealing" or something similar. In an obligation ascription one might say that "Stealing is obligatory", for instance. By treating the moral notions this way von Wright was in effect claiming that it is inappropriate to say of an act–token that it is obligatory, forbidden, etc. Since it is normally the case that published sets of rules refer only to act types and not act–tokens, von Wright's limitation might be fruitful in the analysis of normative systems, but it is not clear that it applies in the analysis of the moral concepts. So, although the question of whether the moral ascriptions relate to act types or act individuals is an interesting matter, DBC simply sidesteps it. It has no variables ranging over acts or act types at all. Sentences of the form $B\alpha\phi$ can be associated intuitively either with a socially or temporally unique action of α's or with a type of act α performs. And what an agent brings about can be obligatory whether or not the bringing about is unique in any of the respects thus far discussed.

In order to quantify over agents the following additions are made to the system. The variables range over agent names:

1.1.2 Variables: Lower case letters *s* through *z*, with or without subscripts.

1.5 Quantifier Key: $\exists$

The grammatical formation of quantified sentences involves the distinction between well–formed formulas and sentences.

[illegible] and never [illegible].

The uniqueness of action has been a matter of concern since von Wright proposed his first deontic system in 1951. He noted the variables p, q, etc. which were to be operated on by the obligation operator O, sentences of the form Op are well-formed, to range over act-types and not over act-tokens. Apparently the variable p, for example, might stand for "a murdering" or "a stealing" or something similar. In an obligation description one might say that "stealing is obligatory", for instance. In treating the moral notions this way von Wright was in effect claiming that it is inappropriate to say of an act-token that it is obligatory, forbidden, etc. since it is normally the case that public codes of rules refer only to act types and not act-tokens. Von Wright's limitation might be fruitful in the study of normative systems, but it is not clear that it applies to the analysis of the moral concepts. So, although the question of whether the moral ascriptions relate to act types or act individuals is an interesting matter, I simply sidestep it. It has no variables ranging over acts or act types at all. Sentences of the form [illegible] can be associated intuitively either with a socially or temporally unique action of [illegible] or with a type of act [illegible] performs. And what an agent brings about can be ambiguously whether or not the bringing about is unique in any of the respects thus far discussed.

In order to quantify over agents the following additions are made to the system. The variables range over agent names.

1.1. Variables: lower case letters x through z, with or without subscripts.

1.2. Quantifier: ( )

The grammatical formation of quantified sentences involves the distinction between well-formed formulas and sentences.

3.2.10 Where x is a variable and ϕ is a wff, $(\exists x)\phi$ is well–formed.

4. Freedom and Bondage: An occurrence of a variable x is free (bound) in a formula ψ iff it falls within no (some) well–formed part $(\exists x)\phi$ of ψ.

Thus in the formula $(Bsp \wedge (\exists y)Byq)$, the occurrence of *s* is free and the occurrences of *y* are bound. The former, because there is no well–formed part beginning with $(\exists s)$ of which *s* is a part; the latter, because the last occurrence of *y* falls within the well formed part Byq of $(\exists y)Byq$.

5. Sentences: Well–formed formulas having no free occurrences of variables are sentences.

E.g., $(Bxp \wedge (\exists y)Byq)$ is well–formed but not a sentence.

Adopting the following conventions simplifies the presentation. Let $\phi\alpha/\beta$ be the result of replacing each free occurrence of variable α in the wff ϕ with the name β.

6. Instances: Where ϕ is a wff, α a variable and β a name, $\phi\alpha/\beta$ is an instance of $(\exists x)\phi$.

The formula Bap is an instance of $(\exists s)Bsp$, for example.

Truth conditions for quantified sentences require the addition of the notion of a variant interpretation. Interpretations are variants of one another relative to a name, say a_4. Remember that for every interpretation $\mathfrak{A}$, *f* assigns to each name, like a_4, and moment x, a set of moments possible relative to x. Suppose $f(m,a_4)=\Delta$ in $\mathfrak{A}$. Also, suppose there is an interpretation $\mathfrak{B}$ that is just like $\mathfrak{A}$ except for the following possible difference. Whereas $f(m,a_4)=\Delta$ in $\mathfrak{A}$, f' in $\mathfrak{B}$ is such that $f'(m,a_4)\neq\Delta$. In other words, at some moment shared by $\mathfrak{A}$ and $\mathfrak{B}$ a_4 might be assigned one set in $\mathfrak{A}$ and a distinct set in $\mathfrak{B}$. It should be remembered that any

[illegible]

well-formed.

4. Freedom and bondage. An occurrence of a variable α is free (bound) in a formula φ iff it falls within no (some) well-formed part (α)ψ of φ.

Thus in the formula (y)(Fx ∨ Gy) the occurrence of x is free and the occurrences of y are bound. The former, because there is no well formed part beginning with (x) of which x is a part; the latter, because each occurrence of y falls within the well formed part (y)(Fx ∨ Gy).

5. Sentences. Well-formed formulas in which no occurrences of variables are free are sentences.

E.g., (y)(Fx ∨ Gy) is well formed but not a sentence.

Notation. The following conventions simplify the presentation. Let φ(β/α) be the result of replacing each free occurrence of variable α in the wff φ with a name β.

6. Instances. Where (α)φ is a wff, α a variable and β a name, φ(β/α) is an instance of (α)φ.

The formula (Fa ∨ Gy) is an instance of (x)(Fx ∨ Gy), for example.

Truth conditions for quantified sentences require the addition of the notion of a variant interpretation. Interpretations are variants of one another relative to a name, say a_i. Remember that for every interpretation ℑ, ℑ assigns to each name, like a_i, and moment x, a set of moments possible relative to x. Suppose ℑ(a_i) = z in ℑ. Then suppose there is an interpretation ℑ' that is just like ℑ except for the following possible difference. Whereas ℑ(a_i) = z in ℑ, ℑ' in ℑ' is such that ℑ'(a_i) = [illegible]. In other words, at some moment denoted by ℑ and ℑ', a_i might be assigned w in ℑ and a different set in ℑ'. It should be remembered that [illegible]

interpretation is an α–variant of itself for any name α. Accordingly we add to DBC the following:

> **11. Variant Interpretations: Where $\mathfrak{A} = \langle M, m, \lessdot, f, g \rangle$ and $\mathfrak{B} = \langle M', m', \lessdot', f', g' \rangle$ are interpretations of DBC, $\mathfrak{A}$ and $\mathfrak{B}$ are β–variants iff $M = M'$, $m = m'$, $\lessdot = \lessdot'$, $g = g'$ and where β is a name, f and f' differ at most from one another in what they assign to β at some moment, where β is a name.**

The names of DBC can be enumerated $a \ldots e, a_0 \ldots e_0 \ldots a_n \ldots e_n$ and this allows one to speak of an earliest or nth name of DBC. Thus to the evaluation procedure add:

> **9.13 Where x is a variable, ϕ a formula and $(\exists x)\phi$ a sentence, $\models_m [\![(\exists x)\phi]\!]^{\mathfrak{A}} = t$ iff $\models_m [\![\phi x/\beta]\!]^{\mathfrak{B}} = t$ under some β–variant $\mathfrak{B}$ of $\mathfrak{A}$ where β is the earliest name not appearing in ϕ.**

3. BASIC ACTION AND TRIAL

Action theorists have puzzled over the distinction between basic and mediate actions. Suppose White fires a revolver in the direction of his father, the projectile injures his father, and several days afterward, his father dies. In these circumstances, it is legitimate at some moment after the shooting and dying to assert that White killed his father or that he brought it about that his father died. Since these assertions involve tense, and since DBC wffs are to be thought of as temporally chauvinistic, it is important to establish the appropriate translation of the original ascription into DBC

interpretation [illegible] an assertion of itself for any name [illegible] we need to [illegible] the following:

11 *Variant Interpretations*: Where [illegible] and [illegible] are interpretations of [illegible] and [illegible] are β-variants iff $M = M'$, $m = m'$, [illegible], $q = q'$, and where β is a name, V and V' differ at most from one another in what they assign to [illegible] at some moment where β is a name.

The names of [illegible] can be enumerated as $e_1, e_2, \ldots, e_n$ and this allows one to speak of an earliest or nth name of [illegible]. Thus to the provisions [illegible] we add:

12 Where x is a variable, ϕ a formula and [illegible] a sentence [illegible] iff [illegible] $= t$ under some a-variant $\mathfrak{J}'$ of $\mathfrak{J}$ where a is the earliest name not appearing in ϕ.

[illegible] TENSE AND TIME

Logicians [illegible] have puzzled over the distinction between past and [illegible] Suppose White fires a revolver in the direction of his father, the projectile injures his father and several days afterward his father dies. In these circumstances, it is legitimate at some moment after the shooting and [illegible] to assert that White killed his father or that he [illegible] that his father died. Since these assertions involve tense and [illegible] to be thought of as temporally [illegible], it is important to establish the appropriate transition of the original assertion into [illegible] notation. Let p_1 translate "White's father expires". Where a is the name of the agent White, we might translate, Bap_1, "White brings it about that his father expires".

If the expiration of White's father is preceded by the firing of the revolver and if the expiring and the firing are causally related one might ask whether the firing is a basic action or if some previous event is more fundamental as an action. Indeed, one can deny that White brought the dying about at all in favor of some view that something he brought about at an earlier moment caused his father to die. But such a denial is unusual and ascriptions of agency in cases like these are customary. Assuming presently popular views of neurophysiology, tracing causally sufficient events antecedent to the dying leads from the firing to the flexing of certain muscles in White's arm to neurophysiological events in White's brain and perhaps to his beliefs, purposes, and choices (these might be neurophysiological events as well). Where, in this regressive causal sequence, does the analysis of action stop? The earliest event in the series, if there is an earliest event, is basic.

When it is said that b brings it about that ϕ, it is assumed that b is the originator who produces the truth of ϕ. Without some notion of origination, it is difficult to imagine what place agency would have at all. Hence, it seems that $B\alpha\phi$ ascribes agency in terms of the products of this originating event. What Jones brings about produces or guarantees or causes it to be the case that certain sentences are true. A reductivist might say that what α does is identical with the sum of what he guarantees.

Assuming for purposes of discussion that in causal production, there is a lapse of time between the producing and what is produced, the assignment $f(m,\alpha)$ might be thought of as the set of possible moments that *must* include [illegible] White we might translate [illegible] White [illegible]

In the explanation of White's behavior as produced by the [illegible] of [illegible] desire and of the [illegible] and the [illegible] are usually related one to [illegible] whether the [illegible] is a basic action or a [illegible] previous event [illegible] as an action. [illegible] one can deny that White brought [illegible] about at all in [illegible] or some [illegible] that something he brought about in an earlier [illegible] caused his failure to [illegible], but such a denial is unusual and [illegible] of action in cases like this are customary. [illegible] however, popular views of [illegible], tracing causality [illegible] events [illegible] to the [illegible] back from the [illegible] to the [illegible] of certain [illegible] to neurophysiological events in White's brain and [illegible] to his beliefs, purposes and choices, [illegible] neurophysiological events as well. [illegible] in this regress causal [illegible] does [illegible] of action stop? The earliest event in the series, if there is an earliest event, is basic.

When it is [illegible] that [illegible] brings it about that q, it is assumed that [illegible] the [illegible] the truth of q. [illegible] some notion of [illegible] it is difficult to imagine what place agency would have at all. [illegible] it seems [illegible] in terms of the products of [illegible] events. What does bring about products or [illegible] it to be the case that certain [illegible] are true. A [illegible] might say that what [illegible] does is identical with the sum of what he [illegible].

[illegible] for purposes of [illegible] that in cases of production there [illegible] a lapse of time between the producing and what is produced, the assertion [illegible] of as the set of [illegible] that [illegible] the present moment because of what α did at some moment before m. If $\{x:Rx,m\}$ is the set of moments simultaneously possible relative to m, $\{x:x \in f(m,\alpha)\}$ is the set of moments simultaneously possible relative to m, conditional on the action of α at some moment earlier than m.

Action ascription is a brand of conditional necessity. For example, relative to an agent α, what α brings about at m is just what is necessarily true given what α previously did. Where there is a time lapse between acting and what one brings about, ϕ might be thought of as a type of consequence of α's action. If the lapse between the acting and the occurrence of ϕ is great, it seems intuitively impossible to say whether or not α brings ϕ about or instead the say that the occurrence of ϕ is merely a result of something α brought about. If the lapse is very small, it is difficult or impossible to decide which event made necessary by α's action is the act, whether the decision to pull the trigger or the firing of the pistol is the act, for example, or both.

It is to be noted here that where $\{x:Rx,m\}$ is to be taken as the set of moments simultaneously possible relative to m, $\{x:x \in f(m,\alpha)\}$ is to be taken as the set of moments simultaneously necessary relative to m conditional on what α did at some earlier moment and not conditional on what anyone else did or does. In Chapter Five we consider collective agency where the conditional necessity involved includes the actions of more than one agent.

In conclusion, it is important to recognise that there is nothing in principle preempting any action, including a basic one, and including a mental act if these can be brought about, from moral ascription. In other words, there is nothing in the DBC analysis of action to keep it from being the case that a mental act can be obligatory or forbidden. Of course, there [illegible] of what x does at some moment before m [illegible]

[illegible] $Ax(m')$ is the set of moments simultaneously possible [illegible]

$\{x : Am\}$ is the set of moments simultaneously possible relative to m

conditional on the act of x at some moment earlier than m.

What was just shown is a kind of conditional necessity. [illegible] relative to an agent x, what x brings about at m is just what is necessarily true given what x previously did. Where there is a time lapse between acting and what one brings about, Q might be thought of as a result or consequence of x's action. If the lapse between the acting and the occurrence of Q is short, it seems intuitively impossible to say whether or not x brings Q about or instead the say that the occurrence of Q is merely a result of something x brought about. If the lapse is very small it is difficult or impossible to decide which event made necessary by x's action is the act: whether the decision to pull the trigger or the firing of the pistol is the act, for example [illegible].

It is to be noted here that where $\{x : Ax\,m\}$ is to be taken as the set of moments simultaneously possible relative to m, $\{x : Ax(m')\}$ is to be taken as the set of moments simultaneously necessary relative to m conditional on what x did at some earlier moment and not conditional on what anyone else did or does. In Chapter Five we consider collective action where the conditional necessity involved includes the actions of more than one agent.

In conclusion, it is important to recognize that there is nothing in principle precluding any action, including a basic one, and including a mental act, if there can be brought about from moral ascription. In other words, there is nothing in the [illegible] analysis of action to keep it from [illegible] the case that a mental act can be obligatory or forbidden. Of course, there is some relation between our epistemic limitations and what we are ordinarily accustomed to in the regular application of prescription and sanction, but there is nothing about the analysis of action that limits in this fashion. α might be obliged to bring ϕ about even if α does not know about this obligation.

Another metaethically important limitation in the DBC analysis of action is that it seems to have no way to represent trial actions that are unsuccessful. This is a deficiency in the analysis if agent's are prohibited from *trying* to bring about certain events, whether they succeed or not. For example, a robber is forbidden from trying to shoot the teller at the bank even if his revolver misfires and even if the sanction applied to trial of this sort is not as severe as that applied in the case of success.

Positive rules sometimes forbid trials. How can these cases be handled according to DBC? One possibility is to consider intentions as brought about by the agents who have them and by considering certain intentions as forbidden even if what is intended is not realized. The robber is prohibited from intending to shoot the teller even if his revolver misfires. This direction might be coupled with the view that unsuccessful actions are not actions and thus cannot be forbidden or obligatory. Another line of remedy is in the view that every unsuccessful action, even though it is not an action, does involve some successful action and it is the successful actions that are prohibited even though rules sometimes speak of intentions. Accordingly, one might believe that in spite of the misfiring of the robber's revolver, he did successfully point the revolver at the teller and successfully tripped the firing mechanism of the revolver, and he might thus be forbidden from performing these actions even if he cannot be forbidden from trying to do something.

is some tension between open-ended limitations and what we ordinarily considered to be the proper application of prescription and sanction, but there is nothing about the analysis of action that limits in this fashion what might be obliged to bring about even if one does not know about the of action.

Another metaethically important limitation in the Fitch analysis of action is that it seems to have no way to represent trial actions that are unsuccessful. This is a deficiency in the analysis if agents are prohibited from trying to bring about certain events whether they succeed or not. For example, a robber is forbidden from trying to shoot the teller at the bank even if his revolver misfires and even if the sanction applied to trial of this sort is not as severe as that applied in the case of success.

Codes and rules sometimes forbid trials. How can these cases be handled according to Fitch? One possibility is to consider intentions as brought about by the agents who have them and by considering certain intentions as forbidden even if what is intended is not realized. The robber is prohibited from intending to shoot the teller even if his revolver misfires. This direction might be coupled with the view that unsuccessful actions are not actions and thus cannot be forbidden or obligatory. Another line of remedy is in the view that every unsuccessful action, even though it is not an action, does involve some successful action and it is the successful actions that are prohibited even though rules sometimes speak of intentions. Accordingly, one might believe that in spite of the misfiring of the robber's revolver, he did successfully point the revolver at the teller and successfully tripped the firing mechanism of the revolver, and he might thus be forbidden from performing these actions even if he cannot be forbidden from trying to do something.

4. *ACTIONS DE DICTO AND DE RE*

The two types of sentences under consideration here have to do with the passage of B over existential quantification. Discussions of the logic of belief and the logic of the alethic modalities often distinguish between *de dicto* and *de re*. In epistemic logic, for example, it is acknowledged that one might believe *de dicto* that the inventor of bifocals is six feet tall but not believe *de re* that The inventor of bifocals is six feet tall. Believing *de dicto* that the inventor of bifocals is six feet tall is a belief that "The inventor of bifocals is six feet tall" is true. Believing *de re* that the inventor of bifocals is six feet tall is a belief that some individual, who satisfies the definite description "The inventor of bifocals", is six feet tall. If Jones believes *de dicto* that the inventor of bifocals is six feet tall, it is not legitimate to infer that someone exists who is believed by Jones to be six feet tall. If on the other hand Jones believes *de re* that the inventor of bifocals is six feet tall this may be inferred.

With action a similar distinction can be made. Suppose that by tampering with an aspirin bottle Jones brings it about that "Somebody or other is poisoned by Jones" is true. From this it seems plausible that it may not be inferred that someone exists who is such that "He is poisoned by Jones" is true of that someone. Jones' action in this case is like *de dicto* action. Jones brings it about that some sentence of the form "x is poisoned" is true but his action is unspecific about which one of these he makes true. On the other hand it might be that Jones does bring it about that White is poisoned. From the latter one might infer that someone exists such that this someone is poisoned by Jones. This kind of action is *de re*.

One might hope that in agreement with the plausibility of these

The two types of sentences under consideration here have to do with the scope of [illegible] over existential quantification. Discussions of the logic of belief and the logic of the action operators often distinguish between de dicto and de re. In epistemic logic, for example, it is acknowledged that one might believe de dicto that the inventor of bifocals is six feet tall but not believe de re that the inventor of bifocals is six feet tall. Believing de dicto that the inventor of bifocals is six feet tall is a belief that "The inventor of bifocals is six feet tall" is true. Believing de re that the inventor of bifocals is six feet tall is a belief that some individual who satisfies the definite description "The inventor of bifocals", is six feet tall. If Jones believes de dicto that the inventor of bifocals is six feet tall, it is not legitimate to infer that someone exists who is believed by Jones to be six feet tall. If on the other hand Jones believes de re that the inventor of bifocals is six feet tall this may be inferred.

With action a similar distinction can be made. Suppose that by tampering with an oxygen bottle Jones brings it about that "Somebody or other is poisoned by Jones" is true. From this it seems plausible that it may not be inferred that someone exists who is such that "He is poisoned by Jones" is true of that someone. Jones' action in this case is like de dicto action. Jones brings it about that some sentence of the form "X is poisoned" is true but his action is incomplete about which one of these he makes true. On the other hand it might be that Jones does bring it about that Smith is poisoned. From the last one may infer that someone exists such that that someone is poisoned by Jones. This kind of action is de re.

One might hold that in agreement with the plausibility of [illegible] inferences, 13.7 would be a thesis, but it is not.

13.7 $\nvDash ((\exists x)B\alpha\phi \rightarrow B\alpha(\exists x)\phi)$

That 13.8 is not a thesis, on the other hand, is what one would expect.

†13.8 $\nvDash (B\alpha(\exists x)\phi \rightarrow (\exists x)B\alpha\phi)$.

Thesishood for other Barcan–like formulas is as follows. For tense operators G and H introduced by the usual definitional theses $G\phi \leftrightarrow \neg F \neg \phi$ and $H\phi \leftrightarrow \neg P \neg \phi$:

12.8 $\vDash (\exists x)H\phi \rightarrow H(\exists x)\phi$.

12.9 $\vDash (\exists x)G\phi \rightarrow G(\exists x)\phi$.

But

13.9 $\nvDash H(\exists x)\phi \rightarrow (\exists x)H\phi$

and

13.10 $\nvDash G(\exists x)\phi \rightarrow (\exists x)G\phi$.

And for necessitations:

12.10 $\vDash (\exists x)\Box\phi \rightarrow \Box(\exists x)\phi$.

And

13.11 $\nvDash \Box(\exists x)\phi \rightarrow (\exists x)\Box\phi$.

CHAPTER THREE

CONSEQUENCES AND SANCTION

1. CONSEQUENCES

The analysis of the moral concepts proposed here is consequentialistic. It is suggested, for example, that "α is forbidden from bringing it about that ϕ" has truth conditions identical with "A consequence of α's bringing ϕ about is α being sanctioned". Given the semantic apparatus of DBC, what truth conditions are appropriate for sentences of the form "ψ is a consequence of ϕ"? Since "If ϕ then ψ" sometimes conveys this information, $\phi \rightarrow \psi$ is a possible translation schema.

But, suppose ψ is a type of consequential state of affairs determinative of forbiddance according to metaethical consequentialism. Accordingly, suppose that forbiddance is introduced as follows: $F\alpha\phi$ ("α is forbidden from bringing it about that ϕ") is equivalent to $B\alpha\phi \rightarrow \psi$ ("ψ is a consequence of α bringing ϕ about"). From the denial of $F\alpha\phi$, $B\alpha\phi$ follows by truth functional inference, and $\neg F\alpha\phi \rightarrow B\alpha\phi$ would be a thesis. In other words, α's not being prohibited from doing something entails that he does it. α keeps himself busy doing all the things that are not sins of commission. Similarly, if the consequentialist recognizes $O\alpha\phi$, ("α is obliged to bring ϕ about") as equivalent to $\neg B\alpha\phi \rightarrow \psi$, then from the denial of $O\alpha\phi$, $\neg B\alpha\phi$ follows and $\neg O\alpha\phi \rightarrow \neg B\alpha\phi$ is a thesis. In this case, that α is not obliged to do something entails that he does not do it. Perhaps α is stubborn. He only does what he is obliged to do. This situation is intuitively intolerable.

CHAPTER THREE

CONFLICTS AND SANCTIONS

It was mentioned in the introduction that agents are not obliged or forbidden relative to acts they actually fail to perform or actually perform, but relative to what they possibly fail to perform or possibly perform. Thus, rather than the conditional $\phi \rightarrow \psi$ translating "ψ is a consequence of ϕ" the strict conditional $\Box(\phi \rightarrow \psi)$ is an improvement. Perhaps if the relation of ψ being a consequence of ϕ is temporally extended, it would not be inappropriate to consider this relation between ϕ and ψ a natural descriptive law where ϕ is considered a cause of ψ. In the previous chapter, we introduced the well–known tense operators G and H where wffs of the types $G\phi$ and $H\phi$ are to translate ordinary sentences like "It is always going to be the case that ϕ" and "It always has been the case that ϕ". Now, since natural laws are eternally true, we can add another more extensive temporal operator E, where wffs of the type $E\phi$ are to translate sentences such as "It is eternally the case the ϕ" and this new operator can be introduced by the definitional thesis:

$$\models E\phi \leftrightarrow (H\phi \wedge G\phi).$$

That is, ϕ is eternally the case iff it always has been the case and it is always going to be the case that ϕ. We might then consider wffs of the form $E\Box(\phi \rightarrow \psi)$ as ascribing natural regularity to the succession of ϕ by ψ. ϕ can be considered a sufficient causal condition for ψ or ψ a necessary causal condition for ϕ. If a lapse of time is required between the consequence and what produces it, $E\Box(\phi \rightarrow F\psi)$ can be considered as translating "ψ is a consequence of ϕ". When different forms of determinism are discussed in Chapter Six, $E\Box(\phi \rightarrow \psi)$ will be treated as expressing this lawful situation, the issue of temporal span between cause and effect left undiscussed. Also, notice that the paradoxes of strict implication, that $\models \Box(\psi \rightarrow \phi)$ in case $\models \phi$ and that $\models \Box(\phi \rightarrow \psi)$ in case $\models \neg\phi$, remain problematic

but the matter will not be further pursued.

2. SANCTION

The consequence of interest where the obligations, forbiddances or permissions of an agent α are concerned involves the application of some sanction to α and this apparently must include the occurrence of something evil for α. Thus, it seems appropriate to make use of the semantic structure Rescher has proposed for ascriptions of preference in "Semantic Foundations for the Logic of Preference"[13], but with revisions.

Rescher bases his truth conditions for preference ascriptions on two functions. The first assigns to each possible world a real number value. This real number value intuitively represents the preference value of each possible world. For example, if world w_1 is assigned a higher number than w_2, w_1 is preferred or preferable to w_2. Call these the k–values of each world. Suppose we are interested in a wff of the type, "p is preferable to q", where p and q are atomic sentences or truth–functional compounds of these. This is where the second function comes into play. It gives as its value the average (arithmetical mean) of all the k–values of all possible worlds at which p is true, similarly for q (or any truth–functional compound). "p is preferable to q" is true just in case the average of all k–values of all possible worlds where p is true is greater than the average of all k–values of possible worlds where q is true[14].

But there is a problem. The number of possible worlds might be infinite. Suppose the k–values of possible worlds where p is true are

CHAPTER [illegible]

The concept of [illegible] W [illegible]

beliefs [illegible] of an agent [illegible] the specification of [illegible]

[illegible] and this approach must include the concept of [illegible]

[illegible] This [illegible] seems appropriate to make use of the semantic structure [illegible]

[illegible] has proved [illegible] for specification of preference in [illegible] Computational [illegible]

[illegible] logic of preference [illegible] with respect [illegible]

However, [illegible] has [illegible] main conditions for preference [illegible] on [illegible]

[illegible] The first assigns to each possible world a real number value [illegible]

[illegible] number [illegible] represents the preference value of each [illegible]

possible w[illegible] For example, if world w [illegible] a higher number [illegible]

w [illegible] preferred to w [illegible] then the k-values of [illegible]

[illegible] w [illegible] of the type [illegible] is [illegible]

[illegible] and q [illegible] some [illegible] truth-functional combinations of [illegible]

[illegible] is where the second method comes into play. [illegible]

[illegible] the average (arithmetic mean) of all the k-values of all possible [illegible]

[illegible] in which p is true [illegible] for any truth-functional [illegible]

[illegible] p is preferred to q [illegible] in the case of the [illegible]

k-values of all possible worlds where p is true is greater than the average [illegible]

k-values of possible worlds where q is true.

But there is a problem [illegible] the number of possible worlds [illegible]

[illegible]

1,2,3,...n where n is some infinite number and the k–values of possible worlds where q is true is 4,5,6...n. If so it is questionable whether there *are* averages to be compared. Since Rescher does not require that the number of possible worlds be finite, his second function is not well–defined. Unfortunately, he does not mention this problem in the article cited.

Since I do not know how to avoid this problem for DBC without limiting the number of moments possible at any moment to a finite number, I will simply sidestep this issue as far as formal description is concerned, while making use of the intuitions that make Rescher's account so plausible. In the explanation of the coordinates of permissible interpretations in Chapter Two, the function g was briefly introduced. g takes moments, names, and wffs as arguments and assigns a real number as value. For example, it might take moment m, b (the name for Socrates) and q (the wff translating "Athens is ruled by slaves") as arguments and assign some numerical value. Intuitively, the number g(m,b,q) represents the axiological standing of the truth of "Athens is ruled by slaves" at moment m relative to Socrates. The greater the number, the higher on Socrates's scale of values at m is the rule of Athens by slaves (at m).

The function g is to be thought of as similar to Rescher's second function. Of course, relativising the axiological value to agents, is something Rescher's account does not do, but the adaptation is straightforward. The real numbers the function assigns to an agent α, moment m and wff ϕ are to be thought of as the average numerical axiological values of all the moments possible relative to m at which ϕ is true, relative to agent α. In other words, the numbers g assigns are to be thought of as depicting the average axiological values we are at a loss to depict in the formal apparatus.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Hence we add to DBC:

7.4.3 Where α is an agent, $m \in M$, ϕ is a wff and k is a real number, $g(m,\alpha,\phi) = k$.

But, speaking of goodness on an agent's scale of goodness is notably ambiguous. We consider only some of the alternatives. On the one hand this goodness might have to do with what α prefers or desires. In other words, the scale might be an ordering of what is good *to* an agent. If this direction is taken, the names for agents seem appropriate for *personal* agents who are customarily said to have obligations rather than impersonal agents. Of course, what is good to an agent might be bad *for* him, which is the second sort of goodness that might be involved. Under this construal, goodness relative to α as what is beneficial for α whether α prefers it or not. Cigarettes are good *to* some people but bad *for* them. In other words, α might prefer circumstances that are not in his best interest either from the perspective of another observer, of an absolute standard for human goodness, or from his own point of view. And the goodness depicted by the quantitative value measure could assume any of these perspectives.

It is at this juncture that the issue of naturalism arises for any escapist view. It is perhaps more helpful, if instead of talking about naturalism, we discuss the matters at hand in terms of reductionism. It is possible to define one concept, say F, in terms of another, G, let us suppose, as, for example, when possibilitations such as $\Diamond\phi$ are defined in terms of their equivalence to $\neg\Box\neg\phi$. In this case, it is said that F has been reduced to G. As far as the alethic modalities are concerned many theorists agree that it makes no difference whether one defines possibility in terms of necessity or vice versa. Under either sort of construal, the alethic modal concepts are reducible to one. But many agree that it is (at present)

unavoidable to treat at least one of the two as primitive, that is, there is no better understood concept H distinct from possibily or necessity in terms of which these two can be defined. In distinguishing naturalism from nonnaturalism one can think of the moral and the axiological concepts as belonging to the same family. Let us say that all of these concepts are *M* concepts. Nonnaturalists usually maintain that at least one of the *M* concepts is primitive. That is, there is no non–*M* concept in terms of which all *M* concepts can be defined.

There are variations of course. Some might maintain that the moral concepts can all be reduced to certain axiological concepts. For example, utilitarians usually define the moral characteristics in terms of goodness. They are moral–concept–reductivists, so to speak. But they might or might not be axiological–concept–reductivists. Although they reduce the moral to the axiological concepts, it might be that there are still no nonmoral, nonaxiological concepts in terms of which the axiological concepts can be defined.

In one of Prior's discussions of the Andersonian simplification of deontic logic[15] according to which a state of affairs is forbidden just in case it necessitates the sanction, he denied that the escapist view was unavoidably naturalistic. He argued that the simplification depicted a situation in which the sanction was perfectly applied, that is, where sanctioning occurred just in case it was deserved. And since desert is itself a moral notion, the simplification does not analyse the moral notions in terms of nonmoral ones, it is not naturalistic. In our terms, it does not reduce all *M* concepts to some non–*M* concepts.

Whether or not Prior's arguments are acceptable, the mention of goodness or evil in sanctioning invites the reductivist versus nonreductivist

[illegible]

be a [illegible] concept is distinct from goodness or badness in terms of which these two can be defined. In [illegible] naturalism or non-naturalism one can think of the moral and the axiological concepts as belonging to the same family. Let us say that all of these concepts are M concepts. Nonnaturalists usually maintain that at least one of the M concepts is primitive. That is, there is no non-M concept in terms of which all M concepts can be defined.

There are variations of course. Some might maintain that the moral concepts can all be reduced to certain axiological concepts. For example [illegible] naturalists usually define the moral characteristics in terms of goodness. They are [illegible] [illegible] so to speak. But they ought [illegible] not be axiological-concept [illegible]. Although they reduce the moral to the axiological concepts, it might be that there are still no nonmoral nonaxiological concepts in terms of which the axiological concepts can be defined.

In view of Moore's discussions of the [illegible] [illegible] of [illegible] theory [illegible] according to which a state of affairs is [illegible] just in case it [illegible] the situation, he [illegible] that the earlier view was [illegible] naturalistic. He argued that the [illegible] [illegible] a situation in which the [illegible] was [illegible] that is, [illegible] something [illegible] in case it was [illegible]. That [illegible] is itself a moral notion [illegible] [illegible] does not analyze the moral notion in terms of nonmoral ones, is not naturalistic. In our terms, it does not reduce all M concepts to the non-M concepts.

[illegible] [illegible] arguments are [illegible] the notion of goodness or evil [illegible] [illegible] controversy at a different place. Whereas Prior believed that the Achilles heel of reductivistic construals of the Andersonian simplification was found in judging *how* the sanction applies according to the simplification, the heel might also be located in the intended construal of the *evil* involved in sanction regardless of how the sanction is applied. That is to say, the moral concepts might all be reducible to the axiological concepts, but the latter might remain primitive.

How could this happen? If the preference calculus associated with function *g* is taken in the first sense having to do with what an agent desires or prefers and if the necessity of application is free of the notion of desert, then the DBC analysis of the moral notions is wholly reductivistic in the sense that *all* of its *M* concepts can be reduced to non–*M* concepts, since what is desired is a fact about the way things are, and actual desire can be described without mentioning normative desire. In this case the reductivistic metaethical view espoused does not rest on axiological concepts treated as primitive. R. B. Perry, for example, offers such a position when he argues that the good is any object of any interest where interest has to do with what is the case and not with what ought to be the case. On the contrary, if the evil calculated has to do with normative standards, if it can only be described by what ought to be the case and not by what is the case, then metaethical naturalism is possible, but metaaxiological nonnaturalism is unavoidable.

At present the several alternatives can be left undiscussed as long as it is recognized that the issue of naturalism can arise at several places. DBC is motivated by a metaethically and metaaxiologically reductivist perspective and these matters will be discussed again in Chapter Six which mentions the role of God in matters ethical and axiological.

[illegible]

[illegible] of normative [illegible] unsatisfactory [illegible] trying how the sanction applies according to the interpretation [illegible] must also be located in the internal content of the very notion [illegible] sanction [illegible] of how the sanction is applied. [illegible] to say, the norms [illegible] concepts [illegible] all be [illegible] to the [illegible] concepts [illegible] the latter might remain normative.

How could this happen? If the precise content associated with a function φ is taken in the first sense having to do with what an agent desires or prefers and if the notion of application is free of the norm [illegible] then the first analysis of the moral notions is wholly reductive [illegible] the sense that all of its M-concepts can be reduced to non-M concepts since what is desired is a fact about the way things are and actual [illegible] can be described in a non-normative [illegible] In this case the [illegible] theoretical view [illegible] does not rest on [illegible] concepts treated as primitive. [illegible] for example, offers such a position wherein [illegible] good is any object of any interest which [illegible] has [illegible] do with what is the case and not with what ought to be the case. On the contrary, if the [illegible] has to do with normative [illegible] it can only be described by what ought to be the case and not by what is the case, then [illegible] naturalism is possible but [illegible] [illegible] is [illegible]

At [illegible] the several alternatives can be left unresolved as [illegible] concerned [illegible] the issue of naturalism can arise at several places [illegible] is motivated by a [illegible] and [illegible] perspectives and these [illegible] will be discussed again in [illegible] the role of [illegible] in matters ethical and [illegible]

The wffs for which truth conditions are to be given are of the form $A\alpha\phi/\psi$ and represent ordinary sentences of the form, "ϕ is better for/to α than ψ". The corresponding axiological operator is A and the new sentences can accordingly be thought of as translations for "ϕ is higher on the axiological scale of α than is ψ"[16]. The additions to DBC syntax are:

1.8 Axiological Operator: A.

Where α is a term, and ϕ and ψ are wffs,

3.2.11 $A\alpha\phi/\psi$.

Truth conditions for the new axiological ascriptions are as follows:

9.14 $\models^{h}_{m}([A\alpha\phi/\psi])^{\mathfrak{A}} = t$ if $g(m,\alpha,\phi) > g(m,\alpha,\psi)$, otherwise it is f.

The evil involved in the sanctioning event is a state of affairs that is brought about. For example, it is an evil for Johnson that the tree in his front yard falls on him, but in order to count the falling of the tree as a sanction, it seems that the falling must be brought about. Let p_8 = "The oak in Johnson's front yard falls on him". Then Johnson is sanctioned only if $(\exists x)Bxp_8$. In the case of what Rescher calls first order preference, Johnson prefers $\neg p_8$ to p_8. Where d denotes Johnson, we may translate $Ad\neg p_8/p_8$. Since these are cumbersome locutions we will adopt the following notational convention:

DfS $\models S\alpha \leftrightarrow (\exists x)(Bx\phi \wedge A\alpha\neg\phi/\phi)$.

In other words, α suffers sanctioning just in case ϕ is lower in α's axiological ranking than is $\neg\phi$ and someone brings it about that ϕ. ϕ must be a first order evil for α, in Rescher's terms. It should be noted here that the DBC apparatus is not able to account for certain conditions that might attach to sanction ascriptions. For example, if I unknowingly bring about something that is evil to someone else, it seems that I have not sanctioned him. We might want to consider the case where, for example, my neighbor brings

… the set … other conditions are … of the form

… and replace ordinary sequences of the form …

… The causal only axiomatic operator is J and the new sentences

can be … by the analogue of as translations for … is …

… are

1.8 … Causal Operator J

… is a form … and ϕ are wffs.

8.2.11 … J …

Their conditions for the new & … are as follows:

… $V(J \ldots) = 1$ iff … otherwise it is …

The … involved in the sanctioned event is a state of affairs that is

brought about. For example, it is an evil for Johnson that the tree in his front yard fell on him, but in order to count the falling of the tree as a sanction it seems that the tree must be brought about. Let p_s = "The oak in Johnson's front yard falls on him." Then Johnson is sanctioned only if … In the case of what … fault or other … Johnson … Where ϕ denotes … we may translate … are … we adopt the following notation convention:

…

In other words, … saying that in case … is lower in … axiom cannot hold … and someone brings it about that … must be a … other evil for … It should be noted here that the … is not able to account for certain conditions that might attach to sanction ascriptions. For example, if I unknowingly bring about something that is evil to someone else, it seems that I have not sanctioned him … want to … the case where, for example, …

about some evil relative to me, and he does this as a punishment to me for trimming the grass around the fire hydrant. Have I been sanctioned in this case if I have no knowledge that my trimming is the action for which punishment is forthcoming, or if I have no knowledge of what the punishment is for, or if there is no set of rules, known or unknown to me, according to which the trimming of grass around fire hydrants is forbidden? These matters are important to sanction ascription, although they are not included in the truth conditions for sanction ascription in DBC. Nevertheless, it is intended that no moral or axiological concept has been smuggled into the analysis at this point of systematic insufficiency. Sanction, whatever the epistemic conditions of the sanctioner or the victim, is to be understood as free of moral or axiological overtones, as far as the intended application of DBC is concerned.

In Chapter Six we consider what Rescher calls differential goodness since this is involved in prudential considerations. In Chapter Five we consider the possibility of collective action ascription. For example, it seems plausible that Johnson's neighbors could cooperate in bringing it about that the tree falls on him and for the falling to count as sanctioning in this case, even though no one of them alone brings this about. But for the time being, singular agent action will do.

Since it is common to several preference logics to have among the theses or axioms requirements that preference be irreflexive, transitive, and antisymmetric, we note that the following are theses:

†12.11 $\vdash A\alpha\phi/\psi \rightarrow \neg A\alpha\psi/\phi$.

This thesis declares that the goodness relation is antisymmetric and can be read "If ϕ is better for α than ψ then it is not the case that ψ is better for α than ϕ". It follows from this that goodness is irreflexive:

[illegible]

[illegible] I have no knowledge that my thinking is the action for which [illegible] phenomenal is forthcoming or if I have no knowledge of what its phenomenon is for or if there is no set of rules known or unknown [illegible] according to which the outcome of [illegible] [illegible] whatever the epistemic conditions of the subject or the victim is [illegible] as far as the internal [illegible] is concerned.

In Chapter six we consider what Hardin calls differential [illegible] some [illegible] is involved in [illegible]. In Chapter [illegible] we consider the possibility of collective action [illegible]. For example it seems plausible that [illegible] could cooperate in bringing it about that the tree [illegible] on him and for the failure to count as something in this case even though no one of them [illegible] brings this about [illegible] but for the [illegible] action will do.

[illegible] it is common to several [illegible] to have among the [illegible] of actions [illegible] are these.

[illegible]

Like these doctrines that one [illegible] is [illegible] and [illegible] that it is better for [illegible] than [illegible] it is not the case that [illegible] from this that goodness is [illegible].

$$12.12 \quad \models \neg A\alpha\phi/\phi.$$

Also,

$$\dagger 12.13 \quad \models (A\alpha\phi/\rho \wedge A\alpha\rho/\psi) \rightarrow A\alpha\phi/\psi.$$

This thesis declares that the goodness relation is transitive and can be read "If ϕ is better for α than ρ and ρ is better for α than ψ, then ϕ is better for α than ψ".

Finally, we should note a possible alteration in the notion of variant interpretation, which was introduced on page 48. According to 11 a β-variant interpretation of $\mathfrak{A}$ is simply an interpretation just like $\mathfrak{A}$ with the possible exception that the variant assigns a different set of moments to β at a moment than $\mathfrak{A}$ does. This variation might be expanded to allow for variation in the assignment of preference values. $\mathfrak{A}$ and $\mathfrak{B}$ might be considered β-variants not only if they differ at most in what set of moments their f functions assign to β at some moment m, but also if they possibly differ in the number their g functions assign to β at m.

This alteration would apparently not alter the list of theses and nontheses related to quantification mentioned on page 53. But, it would be fruitful to pursue this alternative relative to issues of collective preference, since according to utilitarian metaethical views, the consequences determinining the moral status of an action have to do with the preferences of all agents and not merely the preferences of the agent performing the action. But I have not yet given this issue the attention it deserves.

CHAPTER FOUR

OBLIGATION, FORBIDDANCE AND PERMISSION

1. MORAL PROPERTIES AND MODEL CLASSIFICATION

In order to include object language ascriptions of obligation, forbiddance and permission the following syntactic supplements are necessary.

1.9 Deontic Operators: O, F and P.

The Corresponding wffs are:

3.2.12 $O\alpha\phi$

3.2.13 $F\alpha\phi$

3.2.14 $P\alpha\phi$.

These are to translate sentences of the form "α is obliged to bring it about that ϕ", "α is forbidden from bringing it about that ϕ, and "α is permitted to bring it about that ϕ", respectively. Truth conditions are displayed by definitional theses. For obligation ascriptions:

DfO: $\models O\alpha\phi \leftrightarrow \Box(\neg B\alpha\phi \rightarrow FS\alpha)$.

That is to say, α is obliged to bring it about that ϕ just in case a consequence of α's failure to bring it about that ϕ is that α is going to be sanctioned. α's failure makes α's suffering sanction unpreventable. Notice that obligation is defined in terms of the sins of omission one is guilty of by failing to keep obligation rather than in terms of the positive fulfillment of obligation. This distinguishes the semantics of DBC from those in which the truth conditions of obligation sentences are in terms of kept obligations

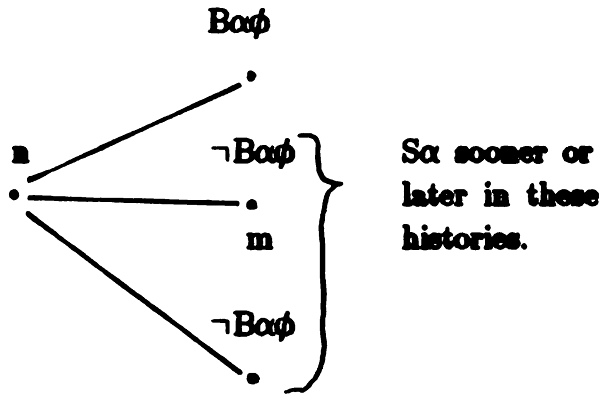
CHAPTER FOUR

OBLIGATION, FORBIDDANCE, AND PERMISSION

instead of broken ones. Hintikka, for example, recommends that $O\phi$ be true just in case ϕ is true in all the possible worlds where the obligation to do ϕ is kept.

FIGURE 7

OBLIGATION



Supposing n is the next earliest moment relative to m, $O\alpha\phi$ is true at m since in all histories containing moments possible relative to m in which $B\alpha\phi$ is true, α will be sanctioned.

In Chapter Two it was noticed that there is an analogy between notorious modal theses and DBC action ascription theses and labelling of theses took advantage of this. For example the action theory analogue of modal thesis D was labelled D_a. In keeping with these conventions this analogy will be apparent in this chapter. For example, K_o, the obligation ascription $O\alpha(\phi\rightarrow\psi)\rightarrow(O\alpha\phi\rightarrow O\alpha\psi)$ is simply K, $\Box(\phi\rightarrow\psi)\rightarrow(\Box\phi\rightarrow\Box\psi)$, where each occurrence of $O\alpha$ (for any agent α) is replaced with $\Box$. As in previous chapters, "†" preceding the label indicates that a proof or countermodel is to

be found in Appendix C. The rule RE_o holds in DBC:

$$\dagger RE_o \quad \frac{\models \phi \leftrightarrow \psi}{\models O\alpha\phi \leftrightarrow O\alpha\psi}$$

But the rule:

$$13.12 \quad \frac{\not\models \phi \rightarrow \psi}{\not\models O\alpha\phi \rightarrow O\alpha\psi}$$

does not. Contrary to expectations, the logical consequences of what is obligatory are not necessarily obligatory. Only what is equivalent to what is obligatory is also obligatory. The reason for this lies with action ascription. Failing to bring ϕ about does not entail failing to bring ψ about even if ϕ entails ψ. Thus, one might be unpreventably punished in the event of failing to bring ϕ about without being unpreventably punished for failing to bring ψ about.

C_o is a thesis:

$$\dagger C_o \quad \models (O\alpha\phi \wedge O\alpha\psi) \rightarrow O\alpha(\phi \wedge \psi).$$

But surprisingly, M_o is not:

$$\dagger M_o \quad \not\models O\alpha(\phi \wedge \psi) \rightarrow (O\alpha\phi \wedge O\alpha\psi).$$

One would expect that if α is obliged to bring the conjunction $\phi \wedge \psi$ about then α is also obliged to bring ϕ about and to bring ψ about. The reason again is in the characteristics of action ascriptions. The countermodel is possible since $B\alpha(\phi \wedge \psi)$ does not entail $B\alpha\phi$.

Neither of the following hold:

$$N_o \quad \not\models O\alpha(\phi \vee \neg\phi)$$

$$RN_o \quad \frac{\not\models \phi}{\not\models O\alpha\phi.}$$

Because of this, there can be no analogy between the obligation ascription theses of DBC and the normal modal systems since M, N, and RN all hold therein. In the weaker classical modal systems, K is not a thesis

unless M, N, C, and RE hold in them. But in DBC, even though M_o and N_o are not theses and RN_o does not hold, K_o does.

$\dagger K_o \qquad \models O\alpha(\phi \to \psi) \to (O\alpha\phi \to O\alpha\psi)$.

But the following analogues to the well-known modal theses do not:

$B_o \qquad \not\models \phi \to O\alpha\neg O\alpha\neg\phi$

$4_o \qquad \not\models O\alpha\phi \to O\alpha O\alpha\phi$

$5_o \qquad \not\models \neg O\alpha\neg\phi \to O\alpha\neg O\alpha\neg\phi$.

That 4_o is not a thesis rules out expanding obligation ascriptions to iterated obligation ascriptions. And since

13.13 $\qquad \not\models O\alpha O\alpha\phi \to O\alpha\phi$

is not either, iterated obligation ascriptions cannot be reduced.

The truth conditions for forbiddance ascriptions are also presented by means of a definitional thesis:

DfF: $\qquad \models F\alpha\phi \leftrightarrow \Box(B\alpha\phi \to FS\alpha)$.

α is forbidden from bringing it about that ϕ just in case a consequence of α bringing ϕ about is α's being sanctioned in the future. Like the definition for obligation, the truth conditions for forbiddances are given in terms of sins associated with doing what one is prohibited from doing. Whereas obligation ascriptions are true relative to sins of *omission*, forbiddance ascriptions are true relative to sins of *commission*. This distinction comports with that between the two sorts of action constraints represented in obligations and forbiddances, the one constraining refrainment from action and the other constraining action. Like the conventions of labelling heretofore, labels for forbiddance ascriptions will be followed by a subscripted "f". The rule RE_f holds:

$$RE_f \qquad \frac{\models \phi \leftrightarrow \psi}{\models F\alpha\phi \leftrightarrow F\alpha\psi}.$$

And similar to obligation ascriptions:

13.14 $\dfrac{\vDash\phi\rightarrow\psi}{\vDash F\alpha\phi\rightarrow F\alpha\psi}$

does not. This is obvious since $B\alpha\phi$ does not entail $B\alpha\psi$ even if ψ is a logical consequence of ϕ. It is possible that α cannot escape if he brings ϕ about but can if he brings ψ about and he can bring ϕ about without bringing ψ about at the same time.

M_f, N_f and RN_f do not hold:

M_f $\nvDash F\alpha(\phi\wedge\psi)\rightarrow(F\alpha\phi\wedge F\alpha\psi)$

N_f $\nvDash F\alpha(\phi\vee\neg\phi)$

RN_f $\dfrac{\vDash\phi}{\vDash F\alpha\phi.}$

But the rule

12.14 $\dfrac{\vDash\phi}{\vDash F\alpha\neg\phi.}$

holds. In other words, it is universally forbidden that $\neg\phi$ be brought about where ϕ is a thesis. Since no one ever violates this forbiddance, the rule is relatively unoffensive.

Furthermore, neither C_f, K_f, B_f, 4_f nor 5_f are theses:

C_f $\nvDash(F\alpha\phi\wedge F\alpha\psi)\rightarrow F\alpha(\phi\wedge\psi)$

K_f $\nvDash F\alpha(\phi\rightarrow\psi)\rightarrow(F\alpha\phi\wedge F\alpha\psi)$

B_f $\nvDash\phi\rightarrow F\alpha\neg F\alpha\neg\phi$

4_f $\nvDash F\alpha\phi\rightarrow F\alpha F\alpha\phi$

5_f $\nvDash\neg F\alpha\neg\phi\rightarrow F\alpha\neg F\alpha\neg\phi$

Permission is defined in terms of forbiddance:

DfP $\vDash P\alpha\phi\leftrightarrow\neg F\alpha\phi.$

In other words, α is permitted to bring ϕ about just in case α is not forbidden from bringing ϕ about. α is permitted to bring ϕ about just in

case the action of bringing ϕ about is not a sin of commission for α.

In Chapter Three, because of the problems that result if $\phi\rightarrow\psi$ is taken as a translation for "ψ is a consequence of ϕ, the translation $\Box(\phi\rightarrow\psi)$ was preferred. There are two theses that resemble the theses eliminated by this move, namely:

12.15 $\models\Box B\alpha\phi\rightarrow O\alpha\phi$, and

12.16 $\models\Box\neg B\alpha\phi\rightarrow F\alpha\phi$.

In other words, anything that α necessarily brings about α is obliged to bring about and anything that α necessarily fails to bring about α is forbidden to bring about. Since these obligations and forbiddances can never be violated, there seems to be little offense involved.

Also, it should be noted that

12.17 $\models F\alpha\bot$

is also a thesis, that is to say, everyone is always forbidden from doing the impossible.

The system can be strengthened by the addition of requirements and some of these improve the similarity between DBC and the traditional monadic systems of deontic logic. The following notational conventions will enhance the pending discussion. If {DBC} is the set of all acceptable interpretations of DBC, $\{\Theta:(m)(h)\models^h_m([\phi])\Theta=t\}$ is the subset of {DBC} containing just those interpretations of DBC where ϕ is universally true, viz., the set of models of ϕ. Let $\Theta\models\phi$ abbreviate "Each member of Θ is a model for ϕ".

Among the interpretations of DBC are those in which the avoidance of sanction in the future is impossible. It might be believed that under these conditions there is no distinction between what is obligatory and what is not, what is forbidden and what is not and hence that these conditions must be [illegible] is obtained and what is not, and from this these conditions in [illegible]

conditions there is no difficulty in [illegible] which is obtainable and which [illegible]

solution in the form is necessary. It must be [illegible] that under the [illegible]

Among the most useful sets of [illegible] those in which the [illegible]

for C_p.

the set of indices of [illegible] each member of [illegible]

[illegible]

[illegible]

[illegible] the permanent [illegible] is the set of all [illegible]

mutually disjoint sets of [illegible]. The following theorem characterizes [illegible]

some of these indices, the elements between [illegible] and the permanent

The solution can be accomplished by the addition of [illegible] or

[illegible]

[illegible]

$1 \cdot 1$ [illegible]

Now it should be noted that

be obtained, there seems to be little else involved.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

eliminated in the definitions of obligation and forbiddance. In the *forbiddance–escapist* interpretations an agent α is forbidden from doing something only if future escape from sanction for α is possible. The set of these interpretations, hereafter FE, are characterized as follows:

$$FE \models F\alpha\phi \rightarrow \Diamond \neg FS\alpha.$$

Similar is the set of *obligation–escapist* interpretations, hereafter OE, in which an agent α is obliged to do something only if α escaping sanction is a future possibility.

$$OE \models O\alpha\phi \rightarrow \Diamond \neg FS\alpha$$

In these interpretations, because of thesis 12.17 and MP,

$$12.18 \quad \models \Diamond \neg FS\alpha$$

also holds. In other words, every agent is granted the future possibility of escaping sanction. This implies that in these interpretations there is no last moment, that is to say, every $m \in M$ is such that $(\exists x) m \lessdot x$ in these interpretations.

Deontic logicians also discuss the principle that no one is obliged to bring about the impossible state of affairs. In monadic deontic systems, systems where O attaches to wffs to form new wffs, the thesishood of $\neg O(\phi \wedge \neg\phi)$ is discussed. Corresponding to the claim among monadic systems that $\neg O(\phi \wedge \neg\phi)$ is a thesis is the set of *consistent* interpretations for DBC, hereafter LC.

$$LC \models \neg O\alpha(\phi \wedge \neg\phi).$$

In these interpretations no one is obliged to bring about the logically impossible state of affairs. It should be noted that the principle that no one is obliged to bring about logical falsehoods is distinct from the principle that obligations do not conflict. In monadic systems the latter is associated with the sentence $O\phi \rightarrow \neg O \neg\phi$ or the equivalent $\neg(O\phi \wedge O\neg\phi)$. Associated with

this principle is the set of ***obligation–consistent*** interpretations of DBC, hereafter OC:

$$OC \models O\alpha\phi \rightarrow \neg O\alpha\neg\phi.$$

If α is obliged to bring ϕ about then α is not obliged to bring $\neg\phi$ about.

In monadic systems the obligation sentence analogous to D, $\Box\phi \rightarrow \Diamond\phi$ viz., $O\phi \rightarrow P\phi$, is well–known. Moreover, it has been shown that D_o is not among the theses for DBC. But D_o is characteristic of the set of ***deontically–consistent*** interpretations, called DC:

$$DC \models O\alpha\phi \rightarrow P\alpha\phi.$$

In these α is obliged to bring ϕ about only if α is permitted to bring ϕ about, or not forbidden from bringing ϕ about. In other words in these models, the obligation to bring ϕ about entails that it is not a sin of commission to bring it about. Whereas in OC, obligations cannot conflict, in DC, obligations cannot conflict with forbiddances.

Another much discussed issue among deontic theorists is the kantian principle that ought implies can, that one is obliged to do only what one is able to do. This principle has been associated with the thesis characteristic of the OC interpretations above. But, one might associate this principle with the two sorts of interpretations that follow. The first relates the kantian principle to obligation and the second to forbiddance. The set of ***obligation–kantian*** interpretations (OK) have the following characteristic thesis:

$$OK \models O\alpha\phi \rightarrow \Diamond B\alpha\phi.$$

α is obliged to do something only if it is possible for α to do it. That α is obliged to do something entails that he *can* do it.

Forbiddance–kantian interpretations (FK) are kantian as far as forbiddance is concerned:

[illegible] about [illegible] obligations [illegible]

[illegible]

(OC) [illegible]

It is obliged to bring y about than it is not obliged to bring [illegible] about.

In addition, it turns the obligation which [illegible] to [illegible] xiv, [illegible] is well known. Moreover, it has been shown that it is not among [illegible] theses for [illegible] characteristic of the [illegible] deontic [illegible] called [illegible]

(OC) [illegible]

In [illegible] is obliged to bring y about only if [illegible] is permitted to bring it about, it is not forbidden to bring [illegible] about. In other words, in these models, the obligation to [illegible] about entails that it is not a [illegible] [illegible] to bring it about [illegible], obligations cannot conflict [illegible] obligations cannot conflict with obligations.

Another much discussed principle among deontic theorists is the principle that ought implies can: that one is obliged to do only what one is able to do. This principle has been associated with the [illegible] of the [illegible] interpretations above. But one might associate the principle with the two sorts of interpretations that follow. The first relates the notion of obligation to [illegible] and the second to [illegible]. The [illegible] of [illegible] common interpretations [illegible] have the following characteristics:

(OK) [illegible]

[illegible] is obliged to do something only if it is possible for [illegible] to do it. [illegible] obliged to do something only if [illegible] can do it.

Agent-bound interpretations [illegible] are [illegible] as [illegible]

[illegible] obligations is [illegible]

$$FK \models F\alpha\phi \rightarrow \Diamond \neg B\alpha\phi.$$

α is forbidden from bringing ϕ about only if it is possible that α refrain from bringing ϕ about. Here, that α is forbidden from doing something entails that α *can* refrain from so doing.

The principles discussed heretofore center on conflicts an agent might encounter relative to his obligations or between his obligations and his forbiddances, being obliged to bring about the logically impossible, being obliged to bring about both some state of affairs and its contradictory, escape from sanction being impossible, being obliged to do what one cannot do, or being forbidden from doing what one cannot help doing. Nothing has been said about the temporal relativity of obligations and forbiddances. An interpretation can meet all of the foregoing requirements without ruling out $O\alpha\phi \wedge PO\alpha\neg\phi$. *Prima facie* some obligations change over time. For example, it might be asserted that males are obliged to register for conscriptive military service when they reach age eighteen but not before. In *omnitemporal–obligation* (OO) interpretations no obligations are temporally relative. What α is obliged to do α is eternally obliged to do. Supposing that $E\phi$ is equivalent to $(\phi \wedge \neg P\neg\phi \wedge \neg F\neg\phi)$, viz., $E\phi$ translates "It is eternally the case that ϕ", consider the following set of interpretations:

$$OO \models O\alpha\phi \rightarrow EO\alpha\phi.$$

Related to this are the *omnitemporal–forbiddance* (OF) interpretations which require that α be forbidden from doing something only if α is eternally so forbidden.

$$FO \models F\alpha\phi \rightarrow EF\alpha\phi.$$

One might think of the intersection of OO and OF as the temporally absolute interpretations.

Another controversy focuses on whether obligations are relative or [illegible] is forbidden from bringing about only if it is [illegible] from bringing about. [illegible] that [illegible] is forbidden from doing [illegible] entails that a can refrain from so doing.

The principle [illegible] imposes a constraint on [illegible] in relation to his obligations or between his obligations and his prohibitions, being obliged to bring about the logically impossible [illegible] obliged to bring about both some state of affairs and its contradictory, escape from satisfaction being impossible, being obliged to do what one cannot do or being forbidden from doing what one cannot help doing. So much has been said about the temporal relativity of obligations and prohibitions. A [illegible] interpretation can meet all of the foregoing requirements without ruling out [illegible] some obligations change over time. For example, it might be asserted that males are obliged to register for compulsory military service when they reach age eighteen but not before. [illegible] interpretations [illegible] obligations are temporally relative. What [illegible] is obliged to do [illegible] is currently obliged to do. Suppose that [illegible] is equivalent to [illegible] translates [illegible] currently the case that [illegible] consider the following set of interpretations:

(OC) [illegible]

Related to this are the [illegible] (FC) interpretations which require that [illegible] is forbidden from doing something only if it is currently forbidden.

(FC) [illegible]

One might think of the intersection of (OC) and (FC) as the temporally absolute interpretations.

A rather controversial [illegible] focusses on whether [illegible] absolute as far as different agents are concerned; whether what is obligatory or forbidden for α can differ from what β is obliged or forbidden from doing where $\alpha \neq \beta$. For example, one might argue that what a judge is obliged to do differs from what an ordinary citizen is obliged to do. Thus, if α is a judge then α has obligations he would not otherwise have if he were not. In the *obligation-fair* (OF) interpretations, this sort of relativity is ruled out as far as obligations are concerned. In these an agent is obliged to do something only if everyone else is obliged to do it as well:

$$OF \models (\exists x)Ox\phi \rightarrow (x)Ox\phi.$$

Related to these is the set of *forbiddance-fair* (FF) interpretations in which α is forbidden from doing something only if everyone is so forbidden.

$$FF \models (\exists x)Fx\phi \rightarrow (x)Fx\phi.$$

In the discussion of agent freedom it is sometimes claimed that one is obliged to bring about some state of affairs only if he is not coerced into bringing it about. Coercion has already been mentioned in Chapter Two and there it was asserted that coercion and necessity are distinct in the system. As in the case of kantian interpretations there are two sorts of interpretations associated with the principle that obligation or forbiddance entails freedom from coercion. The thesis characteristic of the set of *anticoercive* interpretations (OA) comports with the first of these alternative principles:

$$OA \models O\alpha\phi \rightarrow \neg(\exists x)Bx\neg B\alpha\phi.$$

α is obliged to bring ϕ about only if it is not the case that there is someone who coerces α to omit bringing ϕ about. One might say that α is excused from an obligation if there is someone who prevents α from keeping this obligation. Similarly in the set FA (*forbiddance-anticoercive*) one is forbidden from doing something only if he is not coerced into doing it:

$FA \vDash F\alpha\phi \rightarrow \neg(\exists x)BxB\alpha\phi$,

According to traditional monadic systems, obligation, forbiddance and permission are interdefinable. Ordinarily $P\phi \leftrightarrow \neg O \neg \phi$ and $F\phi \leftrightarrow \neg P\phi$. Thus far, permission has been defined in terms of forbiddance.

12.19 $\quad \vDash P\alpha\phi \rightarrow \neg O\alpha\neg\phi$

viz., this is a thesis of DBC. But this falls short of the interdefinability of $P\alpha\phi \leftrightarrow \neg O\alpha\neg\phi$. In order to attain this consider the set of *deontically–traditional* interpretations (DT) characterized by the following thesis:

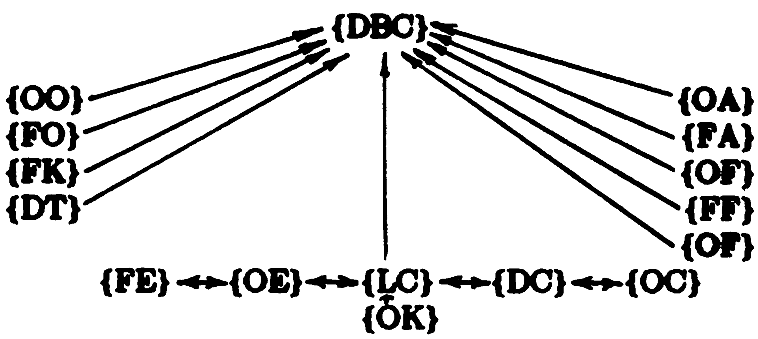
$DT \vDash P\alpha\phi \leftrightarrow \neg O\alpha\neg\phi$.

In other words, α is permitted to bring ϕ about if and only if α is not obliged to bring it about that $\neg\phi$.

The relations between all of the forgoing sets of interpretations is presented in Figure 8.

FIGURE 8

CLASSIFICATIONS



{P}→{Q} indicates that if ϕ is a thesis in P interpretations it is also a thesis in Q interpretations.

Surprisingly, the sets of FE, OE, LC, DC and OC interpretations are

identical as far as thesishood is concerned. Only the set of OK interpretations is stronger than all of these in that if ϕ is a thesis in OK interpretations, it must be a thesis in all of the five.

2. *DEONTIC PARADOXES*

In DT interpretations, obligation, forbiddance, and permission are interdefinable so that only one of the three need be primitive. Similar to the definitional thesis of traditional monadic systems, $P\alpha\phi \leftrightarrow \neg O\alpha\neg\phi$ is a thesis of DT interpretations. This systematically simplifies the elucidation of the moral concepts, but there is a disadvantage. Let p_3= "Jones dies" and for agent a, Fap_3, "a is forbidden from bringing it about that Jones dies". In DT interpretations $F\alpha\phi \leftrightarrow O\alpha\neg\phi$ is a thesis, and from this together with Fap_3 it follows that $O\alpha\neg p_3$, which is to say, a is obliged to bring it about that Jones does not die.

But this is too strong. One can be forbidden from killing Jones (bringing it about the he dies) without at the same time being obliged to bring it about that Jones does not die. Under some circumstances White is forbidden from killing Jones, but does it follow from this that White is also obliged to bring it about that Jones does not die? Of course, White could go out of his way to see to it that Jones does not die, and he would be commended for such heroics, but he is not obliged to be a hero.

Forbiddances and obligations are fundamentally different sorts of behavioral constraints. Forbiddances constrain agents to refrain from certain actions, obligations constrain them to perform certain actions. But, according to traditional interdefinability the sin of omitting to bring ϕ about is no

[illegible]

[illegible]

[illegible] that only one of the [illegible] premises [illegible]

[illegible] about that [illegible] from this fact [illegible]

[illegible] this comes does not [illegible]

[illegible]

different than the sin of bringing $\neg\phi$ about. In "Saints and Heroes", Urmson[17] noticed that most metaethical analyses are not able to assess the moral status of works of supererogation, deeds of moral heroism. Surprisingly, in DT interpretations, everyone who is forbidden from killing, for example, and keeps his obligations is a saint. There is no mere keeping of obligations in this case without supererogatory performance. One might call this the paradox of supererogation which apart from some revision or reinterpretation stands opposed to traditional interdefinability.

The escapist deontic theories that follow Anderson's simplification of the deontic notions are often attacked by showing that the Good Samaritan paradox is a problem for them. The paradox rests on two premises that read something like this: 1) if the good Samaritan helps Jones who was robbed then Jones was robbed and 2) it is forbidden that Jones is robbed. Suppose p = "The Good Samaritan helps Jones" and q = "Jones was robbed". Then according to monadic systems 1) is translated $(p \wedge q) \rightarrow q$ and 2) is translated Fq. According to the rule:

$$\frac{\phi \rightarrow \psi}{F\psi \rightarrow F\phi}$$

it follows from the translations of 1) and 2) that $F(p \wedge q)$, viz., it is forbidden that the Good Samaritan helps Jones who was robbed.

But this paradox cannot arise in DBC. Let q_1 = "Jones is robbed" and p_1 = "Jones is helped". Let a_1 denote the Good Samaritan. Then 1) can be translated $Ba_1p_1 \wedge Pq_1$. (If the robbery is brought about by an unnamed agent then $Ba_1p_1 \wedge P(\exists x)Bxq_1$.) Supposing robbing is forbidden for everyone, 2) can be translated $(x)Fxq_1$. And from this the paradoxical conclusion obviously does not follow.

Several of the other notorious paradoxes often mentioned in the [illegible] ... there is no more ... of obligations in this case without ... interpretation ... [illegible]

... interpretation seems opposed to traditional metaethics.

[illegible] ... the deontic logics are often attacked by showing that the Good Samaritan paradox is a problem for them. The paradox rests on two premises that read something like this: (1) If the Good Samaritan helps Jones who was robbed ... and (2) it is ... that Jones is robbed ... The Good Samaritan helps ... robber." ... (1) is translated ... (2) is ... According to the rule:

[illegible]

It follows from the truth tables of (1) and (2) that ... it is ... that the Good Samaritan helps Jones who was robbed.

[illegible]

assessment of deontic theories do not arise in DBC since, as is the case with the Good Samaritan Paradox, translation preempts them. These are the paradoxes named for Ross and Aqvist, and the Robber's and Victim's paradoxes[18]. But other deontic paradoxes, particularly those involving a possible conflict of obligations, such as Plato's and Sartre's paradox cannot be so easily dispensed with. These conflicts will be discussed in Chapter Six.

3. *DEONTOLOGISM, DIVINE SANCTION AND MORAL ARGUMENT*

The escapism of DBC is intended to be thoroughly consequentialistic. According to the consequentialist the moral properties of an action are identical with certain conquential properties it has, viz., with certain properties it produces or those involved in its fallout. Herein, the consequences determining the moral properties of an act are properties of the agent who does it, viz., his being sanctioned. Moreover, these properties have to do with the best interests of the agent regarding his good and evil and this makes the analysis not only consequentialistic but egoistic. "α is obliged to bring ϕ about" is equivalent to "It is necessarily the case that α's failure to bring ϕ about makes it unpreventable that α will be sanctioned".

In comparison, according to deontological theories, the moral properties of an action cannot be fully determined by the consequential properties it has. A thoroughgoing deontologist believes that sentences about an action's consequences are neither entailed by nor do they entail sentences about its moral status. Hybrid positions result when one of the two aforementioned entailments is denied. Both divine command views and intuitionism are popular forms of deontology. According to the former one is obliged to

[illegible]

[illegible] be easily dispensed with [illegible]

[illegible]

The example [illegible] is intended to be thoroughly consequ[illegible] according to the consequentialist the moral properties of an action [illegible] with certain consequential [illegible] with certain properties it produces [illegible] involved in its [illegible]. Herein [illegible] consequences [illegible] the moral properties of an act are properties of the agent who does it [illegible] this being so [illegible] properties have to do with the best interests of the agent [illegible] his good and [illegible] and this makes the analysis not only consequentialist but egoist [illegible] obliged to bring [illegible]

[illegible] failure to bring [illegible] it will be satisfactory [illegible] actions according to deontological theories, the moral properties of an action cannot be fully determined by its consequential properties [illegible] that actions about an action [illegible] consequences are not the sole [illegible] moral [illegible] result when one of the two alternatives [illegible] is done [illegible] and [illegible] either form of deontology. According to the latter [illegible] bring ϕ about if and only if God commands it, whether sanction or reward is a consequence of the action or not. According to a moral–property–platonistic version of intuitionism, if $O\alpha\phi$ is true, its truth is not related to the truth of any other sentence at all. Thus, it cannot be inferred on the basis of some other kind of sentence which is known to be true and some sort of moral "vision" is required so that the truth of moral ascriptions is possible. It is from this position in moral epistemology that the name "intuitionism" comes.

Deontological views do not fair well in their account of ordinary arguments which treat obligation, forbiddance, or permission ascriptions as entailed by sentences about resulting good or evil. For example, it is typical for one to hear arguments where Smith is obliged to do something because he will be punished if he does not. Not all moral arguments are of this form, of course. Many of them are casuistic in that by analogy they attempt to show that a certain action is of a type of action all instances of which are obligatory. The weakness of deontologism, however, emerges most clearly in the former case. And it should be remembered that one of the tasks of a deontic theory is to offer a procedure for distinguishing valid from invalid moral arguments that violates as little as possible the intuitions associated with validity and invalidity when it comes to moral argument.

Suppose the conclusion to such an argument is $O\alpha\phi$. It is not uncommon for evidence for such a conclusion to include information about the consequences of α bringing ϕ about or forbearing from the same. In many arguments these consequences are specified in terms of impending sanction. If these are valid arguments, an at least partial consequentialism is required. I will suggest that deontologism should be jettisoned altogether.

In the ensuing presentation, I have several goals in mind. By using

[illegible]

[illegible] property palpably [illegible] version of [illegible]

[illegible] on the basis of some other kind of subject which is known to be true [illegible]

[illegible]

several examples of moral argument from the tradition of the Christian scriptures, I want to show, first, that those inside the biblical tradition act as if their concepts of obligation, forbiddance, and prudence are no different than those of their interlocutors, both those who are inside the tradition and those who are not. And although the examples are all biblical, I think that obvious similarities of moral argumentation outside the biblical tradition indicate that there is nothing conceptually unusual about the biblical ones. In other words, the moral concepts do not have one role for Christian theists and another for nonchristians or atheists.

Second, by these examples, I aim to show that thoroughgoing consequentialism and egoism are correct as metaethical views. Again that the examples come from the biblical tradition should be taken as a recommendation to Christian thinkers that deontologism is improper for them. And the similarity between these examples and extra–biblical moral argument should be taken as a recommendation that deontologism is improper for everyone, theistic beliefs notwithstanding. Third, the examples indicate that it is not divine command but divine sanction that is obligation generating and thus divine command views should be rejected.

According to pure deontologism, sentences about reward are independent of sentences about obligation or forbiddance. In light of the preponderance of consequential evidence for moral conclusions one might favor a more moderate form of deontologism that allows sentences about consequences to entail moral ascriptions but does not allow moral ascriptions to entail those about consequences.

But both moderate and extreme deontologism are rejected in the following examples. In the Scriptures, God's commands are regularly accompanied by threats of punishment on disobedience and promises of

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

reward for obedience. These threats or promises are often presented as apparent evidence for, as implying the commands. Strictly, since imperatives like "Do x!" are neither true nor false, there can be no evidence for their truth or falsehood. But if it is assumed for purposes of argument that "Do x!" is tantamount to the declarative, "x is obligatory", as Bohnert has suggested, future tense claims about sanction or reward may serve as evidence for these. If biblical writers use future tense assertions of this sort in this way, there is *prima facie* indication that they are at least not pure deontologists in their view of the moral concepts, since sanction and punishment are consequences of action.

If pure deontologism is correct, then supposing God's commands to be in line with what is moral, obeying God's commands is obligatory even if no bad consequences result from disobedience. Even if God was not able or simply decided not to carry out his threats and promises, keeping the commands would still be obligatory.

First consider the argument of I Corinthians 15.

> For if the dead are not raised, then Christ has not been raised either. And if Christ has not been raised, your faith is futile; you are still in your sins. Then those also who have fallen asleep in Christ are lost. If only for this life we have hope in Christ, we are to be pitied more than all men. ... If the dead are not raised, "Let us eat and drink, for tomorrow we die". (I Corinthians 15:16–18,32b)[19]

That is to say, if there is no resurrection and no future reward or punishment after this life, no one is obliged to make the sacrifices Christian discipleship demands. It is to be noticed here that making these sacrifices is not seen by Paul as supererogatory. He is talking about the basic sacrifices

involved in making one a Christian. It is clear from other passages in the pauline corpus that Paul believed that God does distribute some reward and mete out some punishments in this present age. But apparently he does not believe these rewards beneficial enough nor these punishments severe enough to establish the obligatory status of the commands of Christ. "Keeping Christ's commands is obligatory" implies "Keeping Christ's commands results in the future rewards he has promised and not keeping them results in the punishments he threatened". If the falsehood of these claims about consequences can dissolve obligation, a pure deontology for metaethics is out of the question. In terms of the DBC analysis, Paul's argument is understood as the assertion that $\neg\Box(\neg B\alpha\phi \rightarrow FS\alpha)$ entails $\neg O\alpha\phi$. By contraposition this is tantamount to the claim that $O\alpha\phi$ entails $\Box(\neg B\alpha\phi \rightarrow FS\alpha)$. This is entailed by DfO in DBC and is just the entailment that even moderate deontology denies.

Many find it difficult to endorse the validity of Paul's argument. Paul's view is too selfish, too immature and too childish to be correct. But, this rejection of Paul's position is apparently based on acceptance of a view of moral development which seems to me deficient. I will discuss this view later on.

Secondly, that so many of the biblical arguments have moral ascriptions for conclusions suggests that these conclusions are entailed by sentences about punishment. Consider Exodus 20:4–6:

> You shall not make for yourself an idol in the form of anything in heaven above or on the earth below. You shall not bow down to them or worship them; for I the LORD your God, am a jealous God, punishing the children for the sin of the fathers to the third and fourth generation of

[illegible]

[illegible] Kant's view is too severe, too inhumane and too … to be correct … the rejection of Kant's position is apparently based on acceptance of … [illegible] … I will discuss these … [illegible]

[illegible] … many of the Bible's … [illegible] … that these concepts are … about punishment … [illegible]

> … not make for yourself an idol in the form of anything in heaven above or on the earth below … You shall not bow down to them or worship them; for I the LORD your God am a jealous God, punishing the children for the sin of the fathers to the third and fourth generation …

> those who hate me, but showing love to thousands who love me and keep my commandments.

Here divine punishment is apparently sufficient to establish the obligation to abstain from idolatry. This is an argument pattern frequently repeated in the Scriptures and because of the similarity noticed between the Exodus 20 structure and the more general suzerainty–vassal treaty form common in the ancient near east, an argument pattern that was not unique to the Hebrews. The writers of the Old Testament are intent on establishing that God not only is displeased with certain actions and commands abstention from them, but also that he will actually carry out his threats to punish those who do not abstain. These writers persist in proclaiming that God can and will do what he threatens. Why should Jehovah be served rather than idols? Idols can produce no *results* either by way of sanction or reward, they say. Jehovah, on the other hand, is without fail able to reward and punish as he wills. He is the only God to be feared and obeyed.

> Before me no god was formed, nor will there be one after me. I, even I, am the LORD, and apart from me there is no savior. I have revealed and saved and proclaimed– I, and not some foreign god among you. You are my witnesses", declares the LORD, "that I am God. Yes, and from ancient days I am he. No one can deliver out of my hand. When I act, who can reverse it?" (Isaiah 43:10b–13)

> All who make idols are nothing, and the things they treasure are worthless. Those who would speak up for them are blind; they are ignorant, to their own shame. Who shapes a god and casts an idol, which can profit him nothing? (Isaiah 44:9,10)

[illegible]

the same publication is apparently [illegible] to extend [illegible]

in [illegible] This is an argument [illegible]

the scope [illegible] and because of the similarity [illegible]

[illegible]

[illegible] that was [illegible]

the writers of the [illegible]

[illegible]

but also that [illegible]

not abstain [illegible] these writers [illegible] in [illegible]

[illegible]

[illegible] to reward [illegible] they [illegible]

[illegible] to reward and punish [illegible]

[illegible] and obeyed

[illegible] will there be [illegible] and [illegible] I have [illegible] Yes [illegible]

[illegible]

Christian deontologists have noticed the regular proximity of sentences about divine command to sentences threatening divine punishment or promising divine reward consequent on disobedience or obedience. But some of them claim that these sentences about sanction or reward are not evidence for moral ascriptions but motivation generators to encourage behavior is accord with obligation. The motivation generators are superfluous for the morally mature. This view comports with the position favored by deontologists, Christian belief notwithstanding.

These views gain plausibility by comparing infantile moral development with moral maturity. As a child does not have his will shaped simply by being told that something is his duty or contrary to his duty and needs the incentive of sanction and reward, so sanction and reward that are ostensibly part of biblical morality condescend to an immature level of moral understanding. Promises of sanction or reward are not premises in moral argument, but motivators encouraging the abstentions or actions required by the moral ascriptions. It is sometimes suggested that the Old Testament discussions of morality are superceded by the superior presentation of morality in the New Testament. But if the New Testament presents a superior form of morality and of moral development, there is reason to reject the view of moral development we have been discussing.

Consider Hebrews 12:2. In this morally didactic passage Jesus' supreme sacrificial act at the crucifixion is held up as an example of moral maturity. There it says:

> Let us fix our eyes on Jesus, the author and perfecter of our faith, who for the joy set before him endured the cross, scorning its shame, and sat down at the right hand of the throne of God. (Hebrews 12:2)

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The joy offered motivated Jesus' decision to endure the pain of the crucifixion and it seems that the joy offered was to be his. This motivation is consequentialistic and what is more, since it has to do with the long term interests of Christ, it is egoistic as well. His followers are urged to be motivated like he was, and there is no indication that he or they would be infantile by being so motivated.

Of course, the typical deontological view is not eliminated by these passages. Paul's argument in I Corinthians might simply be declared invalid and the passage of Hebrews 12 rejected as appropriate moral instruction. Indeed, one might believe that most of the biblical writers hold an infantile view of morality since they insist on bringing up threats and promises with great regularity. I find nothing compelling in this. Moreover, there is little in the biblical tradition that indicates any other metaethical view than one like the consequentialistic egoistic one we have been advocating, although careful treatment of individual passages is required to establish this.

The escapist is not left with no account of moral development even if he does not agree with the typical deontologist's account. An escapist account is as follows. According to the typical deontologist's account, in moral infancy, one is motivated toward morally acceptable behavior by sanction and reward. Moral maturity is reached when one recognizes that these cannot be evidence for moral conclusions and they are not needed as motivators. The morally mature are motivated to duty simply because it is duty. The escapist can agree that something changes in the course of moral development. But what changes is the immediacy of gratification or punishment. Being motivated by postponed gratification or the fear of postponed punishment comes with moral maturity, not the elimination of

gratification or fear altogether. This is not to say that the *longer* the postponement the greater the moral maturity. Moral maturity is reached, I suggest, when one is able to postpone the fulfillment of gratification until the next life; when one fears not merely the evils of the present life, but the punishments of the next. This is what makes the sacrifice of the cross supremely moral. Jesus is driven to it by the promise of future reward, reward in the life of the coming age, even though the immediate consequences, the consequences for life in the present from Jesus' temporal perspective before the crucifixion, were painful.

In all of this, there is a blending of prudential and moral concerns. Discussing the relation of these two will be postponed until Chapter Six.

4. *ALTERNATIVE SEMANTICS*

The escapist elements of DBC can be avoided by several alterations in the semantics. As it stands, the set $\{x{:}Rx,m\}$ is to be thought of as the set of moments possible relative to m. Let an additional function h be added, such that for every α and m, $\mathit{h}(\alpha,m)=\Delta$ where $\Delta \subseteq \{x{:}Rx,m\}$. Intuitively, Δ is the set of deontic alternatives to m in which α keeps all of his obligations and refrains from his forbiddances. Truth conditions can be as follows:

DfO* $\quad {}^{h}_{m}[\![O\alpha\phi]\!]^{\mathfrak{A}}=t \quad$ if $\quad (x)(h)((x\in h \wedge x\in \mathit{h}(\alpha,m))\rightarrow {}^{h}_{x}[\![B\alpha\phi]\!]^{\mathfrak{A}}=t)$, otherwise it is f.

And for forbiddance ascriptions:

DfF* $\quad {}^{h}_{m}[\![F\alpha\phi]\!]^{\mathfrak{A}}=t \quad$ if $\quad (x)(h)((x\in h \wedge x\in \mathit{h}(\alpha,m))\rightarrow {}^{h}_{x}[\![\neg B\alpha\phi]\!]^{\mathfrak{A}}=t)$, otherwise it is f.

Under this construal $O\alpha\phi \rightarrow P\alpha\phi$ and $O\alpha\phi \rightarrow \neg P\alpha\neg\phi$ are theses. Consideration

[illegible]

poetic moment [illegible] the moral maturity [illegible]

[illegible] st. When one is able to postpone the fulfilment of gratification until the next life. When one [illegible] not merely the [illegible] of the present [illegible] but the [illegible] This is what [illegible] the sacrifice of the [illegible] supremely moral [illegible] Jesus is driven to it by the promise of future [illegible] reward [illegible] of the coming [illegible] the more [illegible] for life in the present realm [illegible] temporal [illegible] before the conditions were painful.

In [illegible] there is a binding of intuition and moral concern. Just what the relation of these two will be postponed until chapter [illegible]

[illegible]

The [illegible] can be [illegible] by several [illegible]

[illegible]

of other systematic features of this semantic arrangement appears fruitful but is not pursued here.

A word should be said about conditional obligation. In the previous discussion of OE and FE interpretations, a way of imposing temporal absolutism was presented. Obligations and forbiddances are eternal in these interpretations. Of course, all examples of apparent temporal relativity could be treated as conditional on properties of the agent. For example, the obligation to register for conscriptive military service could be conditional on the agent being male and eighteen. Also, the OF and FF interpretations present a way to eliminate agent–to–agent difference of obligation and forbiddance. Again, instead of requiring that all obligations and forbiddances be absolute in this way, one might speak of obligations and forbiddances as conditional on certain properties of the agent. If α is a judge then α is obliged or forbidden in a specified way. In order to accomodate conditional obligations of both kinds one might add predicates $F_0 \ldots F_n$ to the syntax where formulas of the form $F_n\alpha$ are well formed, where α is the name of an agent. Sentences of the form $F_8\alpha$ translate ordinary sentences of the form "Agent α is F_8".

In typical monadic treatments of conditional obligation novel sentences of the form $O\phi/\psi$ are rendered into English as "ϕ is obligatory given that ψ". The DBC counterpart would be $O\alpha\phi/\psi$ which can be rendered "α is obliged to bring it about that ϕ given that ψ". Where the obligation is conditional on a property belonging to α, ψ would be of the form $*F_n\alpha$ where * might be a temporal operator P or F. Of course, this does not dispense with the difficulties associated with attempts to define conditionalization in terms of the expressions already in DBC[20].

[illegible]

[illegible]

[illegible]

[illegible]

CHAPTER FIVE

COLLECTIVE ACTION AND COLLECTIVE OBLIGATION

1. OBLIGATION HOLISM

Groups of agents are sometimes said to have obligations. One might say, for example, that Ford Motor Company is obliged to manufacture only relatively safe automobiles, that Tom and Dick are obliged to cooperate with one another in caring for their aged mother, etc. Referring to such social groups as "collectives" philosophers have asked whether ascriptions of collective obligation are anything more than abbreviations for conjoined *singular* agent obligation ascriptions.

Answers to this question are apparently motivated by the polarization between individualists and collectivists in social science methodology. Two issues are preeminent in the dispute. The first concerns the *terms* describing collective entities, the second, social science *laws* and their relation to the laws of other sciences[21]. The controversy over the first issue is whether there are any collective properties that cannot be defined either in terms of the behavior of individuals composing the collective or in terms of the relations between these individuals. An affirmative response to this question usually stands or falls with an affirmative response to the question, "Are there any collective persons?" The denial of undefinably collective properties is called *methodological individualism* and its opponent, *metaphysical holism*. In believing that collectives and their properties are other than constructs of

CHAPTER FIVE

COLLECTIVE ACTION

AND COLLECTIVE OBLIGATIONS

observable objects and their properties, holists deny empiricism and traditionally they are hostile to liberal individualism as well[22].

It has been suggested that the rational status of outlooks as broad as empiricism or liberal individualism is beyond assessment. Although I disagree I will not argue the point here. At any rate, it will be clear that both empiricism and individualism motivate my analysis. How the discussion of collective action and collective obligation relates to holism is plain. That collective obligations or actions are not individually definable is a sufficient (but not necessary) condition for the truth of metaphysical holism. Throughout, by "action holism", I denote the view that some collective actions are not individually definable and by "obligation holism" the view that some collective obligations are not thus definable.

The second issue in the collectivist/individualist debate is over whether the generalizations of the social sciences ought to be thought of as expressible as generalizations of nonsocial sciences in the way the generalizations of chemistry are thought of by some, viz., as, in principle, expressible as generalizations of physics. According to most individualists, social science generalizations ought to be treated as if they are expressible as constructs of psychology's generalizations. Also, in addition to the terminological dispute of the first issue, the second attends to the stochastic elements of social science generalizations. It raises the question of whether social scientists ought to eschew, or eliminate as much as possible the stochastic vocabulary of social science generalizations. The discussion following does not address this second cluster of questions.

Negative and affirmative answers to obligation holism have been offered and used in curious ways. Manuel Velasquez, for example, claims that philosophers holding to irreducible corporate responsibility are

unwittingly allying with a new form of totalitarianism[23]. During the Nuremburg trials collective obligation was a weapon of the prosecution to bring guilt on as many Germans as possible. More recently, the defenders of Lt. William Calley attempted his exoneration by arguing that he was no more than a member of a larger collective which was, in their reckoning, the locus of the obligations violated in that infamous massacre at My Lai[24]. Any axe grinding of this sort I might wish to indulge in will be done elsewhere.

In the following, a truth conditional semantics is attempted for sentences attributing actions and obligations to collectives. The central question is whether the truth conditions of these sentences are reducible to the truth conditions of sentences attributing actions, obligations or relations to singular agents. The main question in terms of the system DBC is whether sentences such as $O\alpha\phi$, where α names a collective, are reducible to some sentence in which no collective names and only singular person names appear.

Some distinguish between types of collectives. For example, a government or corporation might have obligations whereas a random collection, such as a mob, might not. Virginia Held [25] argues that "random" collectives do in some cases have obligations. John Ladd suggests that "formal organizations" (e.g. corporations) that persist in their identity, even under the substitution of individuals, are a new type of collective entity. He claims that whereas earlier discussions of individual freedom versus state power and authority could see the point of contact in relation to a single sovereign power, contemporary discussions cannot because authority and responsibility are now diffused through "formal organizations". "...their power and authority over us is continuous and blurred"[26].

But I am principally interested in determining whether *any* collective exists, acts or has obligations. In keeping with the current literature on collective responsibility which is more occupied with the moral status of the corporation than that of any other type of collective, corporations will serve as exemplary collectives at the crucial juncture of my argument. Furthermore, collective purpose and desire are more explicitly articulated in the case of corporations than in the case of other sorts of collectives and this makes them lucid examples. It might be contested that this is unfair and that tribes and families are more likely than corporations to have status as moral agents when the quantity and generality of shared belief is taken into account. Perhaps this is so. But there is no deliberate unfairness on this point. And I believe that variation in examples does not mitigate the success of the argument.

As mentioned earlier, according to consequentialist metaethics, an action is right or wrong exclusively in virtue of its consequences. "α is obliged to bring it about that ϕ" is equivalent to "Necessarily, if α fails to bring it about that ϕ, then ψ will be the case". Different specifications of ψ distinguish different consequentialisms from one another. Hence consequentialists might begin contemplating collective obligation by asking whether the specified ψ *can* result from a collective act as distinct from the acts of singular agents. My version of consequentialism is *escapist*, and asserts that someone is obliged to bring ϕ about just in case he will be penalized if he does not. Sanction is the outcome of an action determining its moral status. Since escapism defines obligation in terms of sanction, collective obligation is compatible with escapism only if a collective can be sanctioned.

As we have seen in Chapter Two, sanction is ascribed relative to the victim's preference or good. "Morris is sanctioned" can be true only if Morris' preferences are frustrated or if Morris suffers some evil. Whether or not collective obligation is compatible with the escapist view of DBC will depend on how preference, good and evil are construed. There are on the whole two cases to consider. First, if preference is construed as a conscious desire then the compatibility depends on whether or not a collektive agent has desires that sanction frustrates. This is not to say that collectives must have the same desires as singular agents. Collective desires, if there are any, are likely to differ from singular agent desires. "Collective β is sanctioned" is true only if β endures something contrary to its desires. If no collective has desire, then no collective can be sanctioned and thus no collective can have obligations. Second, the preference ordering of function g introduced in Chapter Two, might be taken instead as an ordering of states of events according to the benefits they involve for something, and an entity need not have desire to have goods in this sense. We might list possible worlds according to their benefits to a petunia, and in this sense, the petunia might have goods and thus might be sanctioned and is a candidate for obligations. These two possibilities will be discussed in turn.

In line with the first construal, Peter French attempts to show that some types of collectives do have desires, intentions and goals and that some collectives are moral agents. He claims that corporations have intentions by virtue of a corporate internal decision structure (CID).

> Every corporation has an internal decision structure. CID structures have two elements of interest to us here: (1) an organizational or responsibility flow chart that delineates stations and levels within the corporate power structure and (2) incorporate decision recognition rule(s) (usually embedded in

> something called "corporation policy"). The CID Structure is the personnel organization for the exercise of the corporation's power with respect to its ventures, and as such its primary function is to draw experience from various levels of the corporation into a decision-making and ratification process. When operative and properly activated, the CID Structure accomplishes a subordination and synthesis of the intentions and acts of various biological persons into a corporate decision. [27]

If, as French claims, corporations have intentions and desires then they have some mental states. And the definition of mental states might determine whether corporations have them. It seems that mental states are linked to sensory input and behavioral output. According to functionalism, a recent doctrine in the philosophy of psychology, mental states are functional states of an organism that have sensory input as arguments and behavioral output as values. Pursuing French's suggestion, it might be said that corporations have both sensory input (by means of the individual agents that make it up) and behavioral output (by what its members together bring about or what the corporation itself brings about). And a functionalist definition for collective mental states is as plausible as any. But, functionalism is too broad. Consider Ned Block's counterexample:

> Suppose we convert the government of China to functionalism, and we convince its officials that it would enormously enhance their international prestige to realize a human mind for an hour. We provide each of the billion people in China...with a specially designed two way radio that connects them in the appropriate way to other persons and to the artificial body mentioned in the previous example. ... It is not at all obvious that the China-body system is physically impossible. It could be functionally equivalent to you for a short time, say an hour.[28]

Block's counterexample to functionalism, if successful, also serves as a

[illegible]

[illegible] power. What [illegible] to its [illegible] sometimes [illegible] function is to draw experience from various levels [illegible] cooperation [illegible] a decision-making and [illegible] When [illegible] accomplishes a [illegible] function and [illegible] various [illegible] persons [illegible]

It [illegible] humans have [illegible] and [illegible] mental states, and the [illegible] of mental [illegible] might [illegible] that mental states [illegible] input and behavioral output [illegible] according to functionalists [illegible] in the philosophy of psychology, mental states are [illegible] as [illegible] in that they [illegible] and both [illegible] as [illegible] functional [illegible] it might be said that [illegible] have both sensory input, the inputs of the [illegible] that [illegible] and behavioral output (i.e. what it's [illegible] together bring about [illegible] itself [illegible] about [illegible] functional [illegible] mental states [illegible] as [illegible] functionalism [illegible] based [illegible] and [illegible] common [illegible]

Suppose we consider the possession of a kind of functional [illegible] and we [illegible] that [illegible] we [illegible] that [illegible] to realize a human mind for an [illegible] We provide each of one billion people in China with a special [illegible] to other persons [illegible] in [illegible] previous examples [illegible] that [illegible] China [illegible] is [illegible] impossible [illegible] it could [illegible] functionally equivalent to [illegible] a [illegible] mind

[illegible]

counterexample to the view that collectives can have mental states. In Thomas Nagel's article "What is it Like to be a Bat", Nagel says there is something that it is to *be* a bat. What it is like to *be* a bat cannot be captured by the most complete functional description imaginable. The qualitative "feel" of mental states is left out[29].

For our purposes, the point of Block's China counterexample and Nagel's argument is that for some x, if there is nothing it is like to be x, then it is false that x has mental states. Clearly there is nothing it is like to *be* the China mind. There *is* something it is like to be a bat, even though we can approach this feeling only by imagining ourselves to turn into bats, which is not the same thing. But is there something it is like to *be* a corporation? For example, is there something it feels like for a corporation to want some end? Is there something it is like to *be* a tribe, a family or a platoon? This is difficult for me to imagine. I can imagine being a bat, but I cannot imagine being a tribe. And, without this quality the mentality of collective entities is at best suspect. If collectives do not have mental states and hence do not have desires and intentions, they cannot be sanctioned. Thus according to the first construal of the preference ranking, collectivies do not have obligations on the escapist view.

But according to the second construal of preference where it is taken as good for α whether or not α prefers, there is no apparent objection to collective obligation. Fining Chrysler Corporation 100 million dollars is bad for Chrysler Corporation, and the corporation can be sanctioned and can have obligations.

But, it is difficult to detach sanction from the notion that it always involves the frustration of some preference or desire of the victim. Moreover, it seems to me that in sanctioning situations, goods and evils for objects that

[illegible]

[illegible]

[illegible] that it is to be a bat? What is it like to be a [illegible]

[illegible]

[illegible] of mental states is left out.

[illegible] point of [illegible]

[illegible] is that for some [illegible] there is nothing it is like to be [illegible]

[illegible] has mental states [illegible]

[illegible] There is something it is like [illegible]

[illegible] this feeling [illegible]

[illegible] same thing [illegible] that is [illegible] it is like to [illegible]

[illegible] is there something [illegible] feels like for a [illegible]

[illegible] Is there something it is like to be a table? [illegible]

[illegible] difficult for me to imagine. I can imagine being a [illegible]

[illegible] being a table [illegible] and without [illegible]

[illegible] it isn't clear that they do not have mental states

[illegible] do not have [illegible] and their [illegible] cannot be [illegible]

Thus [illegible] to the first conjunct of the [illegible] rather [illegible]

not [illegible] on [illegible] view.

[illegible] to the [illegible] of [illegible] where [illegible]

as good for [illegible] whether or not [illegible] physical [illegible]

[illegible]

[illegible] and the [illegible] can be the same [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

have no mental states are always relative to the preferences of objects that do. What makes a 100 million dollar fine bad for Chrysler Corporation? What comes to mind is the frustration of desire that comes to individuals who own stock and prefer the largest profits, individuals whose salary depends on the financial condition of the corporation, individuals who strongly dislike the unfavorable publicity such a fine will generate, etc. So the second option is not as attractive as it appears at first. In my judgment, neither is very attractive.

2. *REPRESENTATIVE VICTIMS*

Although the nonexistence of collective mental states damages one version of obligation holism, there is another way of understanding collective obligations where an escapist view is espoused. It might be argued that there are collective *actions* that cannot be defined in terms of singular agent actions. If so, collective entities might exist and bring about certain states of affairs although they have no desires or intentions in so doing. Even if collectives have no intentions or desires of their own, and although they cannot themselves be sanctioned, singular agents who act as their representatives can be. If each executive board member of Ford Motor Company can be sanctioned for something the Ford Motor Company brings about, then perhaps Ford Motor Company has obligations after all.

The plausibility of this move is related to the plausibility of variation in obligation relative to social role. A soldier, Skinner, killing an enemy soldier, Carpenter, in a time of war is not sanctioned for it. If Skinner kills Carpenter when neither of them are wearing uniforms and when their countries are not at war, sanction might very well be applied to Skinner.

[illegible]

[illegible]

[illegible]

[illegible]

Similarly, a guardian might be sanctioned for not providing appropriately for a child whereas others are not sanctioned for failing to provide for this child. Obligations can vary even within the life of singular agents depending on changes in the recognisable social roles they play. Most important for our discussion, some social roles involve one individual suffering a penalty for what someone else has done.

Representative obligation has plenty of opponents. Against it H.D. Lewis says, "If I were asked to put forward an ethical principle of particular certainty, it would be that no one can be responsible, in the properly ethical sense, for the conduct of another"[30]. The certainty of the principle is not clear to all. W. H. Walsh, for example, in "Pride, Shame, and Responsibility" attends to very obvious cases where agents take pride in, feel shame for, or bear responsibility for what others bring about[31]. A father, for example, is often punished for his child's action. In some cases this transfer is made because the father has deeper pockets than his child and legal redress at the child's expense is not feasible. And some would call this an example of legal, but not moral, responsibility.

Moreover, historical examples of one person bearing the penalty for the deeds of another are plentiful, although such vicarious penalizing is often considered morally primitive. In orthodox Christian soteriology, substitutionary sanction is very important and it seems that there are ways to distinguish acceptable from nonacceptable versions of representative sanction without rejecting the whole idea as primitive.

Assuming the plausibility of the representative victim version of oblilgation wholism, suppose Jones is sanctioned for what Smith brings about. If this is *representative* sanction there must be some relation constituting the representation between Jones and Smith, for example, that Jones volunteers

to suffer Smith's sanction after Jones learns about it. Or that Jones is picked by a lottery to suffer for what Smith did since Smith cannot be located. Or perhaps Jones is the nearest relative of Smith so that custom applies the sanction to Jones when Smith cannot be penalized. Whatever this relation is, suppose we add $Q\alpha\beta$ to the wffs of DBC where Q represents the representative relation between α and β and $Q\alpha\beta$ translates sentences of the form "β is representative for α". Note that there can be more than one representative for α, or none for that matter. DfO could be revised as DfO+:

$$\text{DfO+} \quad O\alpha\phi \leftrightarrow \Box(\neg B\alpha\phi \rightarrow (\exists x)(Q\alpha x \wedge FSx)).$$

In other words, α is obliged to bring it about that ϕ just in case it is necessary that if α fails to do so then some represeative of α will be sanctioned". Under this construal, one might hold to the obligation holist view—collectives can have obligations—while denying that collectives can be sanctioned.

3. *ACTION HOLISM*

What the obligation holist has yet to show, given his pursuit of the representative sanction route, is that action holism is defensible, viz., that there are collectives among the agents. There are *prima facie* cases of irreducibly collective actions. For example,

1) Tom, Dick and Harry carried the piano upstairs.

This sentence is not equivalent to

2) Tom carried the piano up the stairs and Dick carried the piano upstairs and Harry carried the piano upstairs.

The piano cannot be carried by Tom alone, Dick alone, or Harry

alone, but together they can carry it. Bringing it about that a piano is carried upstairs seems to require collective agency. No doubt, there are other ostensibly collective actions that do not require the mention of physical possibility, for example,

3) Tom, Dick and Harry carried the pencil upstairs.

Obviously a singular agent is likely to be *able* to carry a pencil upstairs but *prima facie* the truth conditions of 3) are distinct from those of 4);

4) Tom carried the pencil upstairs and Dick carried the pencil upstairs and Harry carried the pencil upstairs.

Gerald Massey has suggested that the validity of certain arguments involving reference to collectives is not captured by boolean predicate logic and that this logic is insufficient (even as sentential logic alone is insufficient to assess the validity of some arguments that are properly assessed by elementary logic). His examples of inferences not captured by boolean predicate logic are:

(A1) Tom and Dick carried the piano upstairs.
Dick and Tom carried the piano upstairs.

(A2) Tom, Dick and Harry are shipmates.
Dick, Harry and Tom are shipmates.

(A3) Tom, Dick and Harry are shipmates.
Tom and Harry are shipmates.[32]

Such predicates as "carried the piano upstairs" and "manufactures passenger vehicles" are not usually ascribed to singular agents. Yet these predicates are syntactically indistinguishable from "carried a pencil upstairs" and "made a hook rug" which are. Massey notices problems with two kinds of argument. First, with those involving collective actions (A1) and second, those involving the relation between persons within a collective (A2) and

(A3). I do not plan to treat inferences of the second type. With respect to inferences of the first type, DBC offers a solution to Massey's difficulty. (His revision, called mereological predicate logic, which allows, for example, the fusion of Dick and Harry to have a property in common is very interesting, but will not here be discussed).

A slight revision in the syntax of DBC allows for collective action ascriptions assuming that any acting collective can be denoted by listing each singular agent involved. Let 3.2.8 be replaced by 3.2.8* as follows: Where each α_i is a name

3.2.8* $B\alpha_0 \ldots \alpha_n \phi$.

For example, if "a" translates "Tom" and "b" translates "Dick" and "p" translates "The piano is carried upstairs", then the DBC sentence for 1) is "PBabp".

Revision in the semantics is also required. Let 9.12 be replaced by 9.12* as follows:

9.12* Where α_i is a name and ϕ is a wff, ${}^{h}_{m}[\![B\alpha_0 \ldots \alpha_n \phi]\!]^{\mathfrak{A}} = t$ iff

(i) $(\Delta = f(m,\alpha_0) \cap \ldots \cap f(m,\alpha_n) \rightarrow (x)(h)(x \in \Delta \rightarrow {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}} = t))$ and

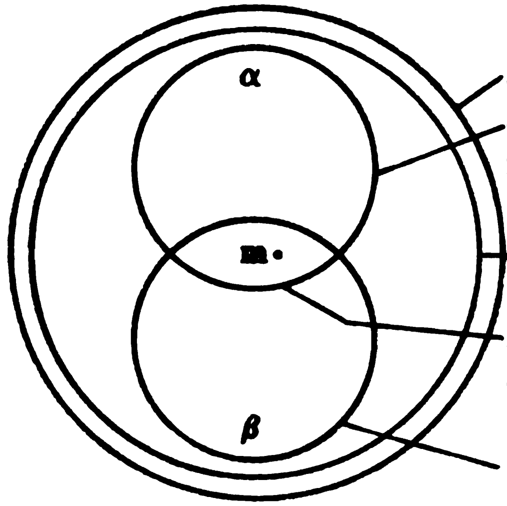
(ii) $(\Gamma = \{x{:}Rx,m \wedge x \notin \Delta\} \rightarrow (x)(h)(x \in \Gamma \rightarrow {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}} = f))$.

The function f has been designed to depict the bringing–it–about–that relation between agents and states of affairs as a narrowing of alternative scenarios that can come to pass. The sum narrowing takes into account the states of affairs brought about by all agents at a moment. See Figure 9.

All inferences such as (A1) are valid in the revised DBC, which shall hereafter be called DBC*. In order for Massey's criticism to be lethal to DBC* there must be some sequence of terms $\ldots,\alpha_j,\ldots,\alpha_k,\ldots$ such that where ϕ is a wff

$$\not\models (B\ldots,\alpha_j,\ldots,\alpha_k,\ldots \rightarrow B\ldots,\alpha_k,\ldots,\alpha_j,\ldots).$$

FIGURE 9



But this is impossible since the set theoretic intersection that forms Δ in the condition is order blind.

But action individualists in effect assert that the truth conditions for any sentence of the form $B\alpha_0 ... B\alpha_n \phi$ are the same as $(\exists\psi_0)...(\exists\psi_n)(B\alpha_0\psi_0 \wedge ... \wedge B\alpha_n\psi_n)$. For example, that the truth conditions for "Dick and Harry bring it about that the piano is carried up the stairs" need not mention the fusion of Dick and Harry but only the singular agent Dick and the singular agent Harry. Intuitively, we might think of the ψ_i as detailing the bodily movements or physical forces brought about by Tom and Dick. Here a practical difficulty emerges. What are these sentences? Can action individualism be maintained without them? In ordinary discourse there are some collective action ascriptions for which there are no singular agent ascriptions of the requisite sort.

But the plausibility of action individualism is similar to the plausibility of reductions of chemistry to physics. To my knowledge, no one has established a physical translation for every chemical sentence. The

[illegible]

[illegible]

[illegible] of luck and finally but only [illegible]

[illegible] What are these [illegible]

[illegible] for which there are [illegible]

[illegible]

motivation for showing that the chemical–to–physical translations are possible was systematic. And if part of the philosopher's task is to elucidate the relations between the sentences in different scientific theories, the reduction of chemistry to physics was valuable. In favor of individualism it should be noted that proponents of this view do not claim that collective action sentences have no useful function or that they should be banished from the discourse of social science. But they do claim that they should be banished, if possible, from a philosophical reconstruction and it appears that simplicity and parsimony are served by action individualism more than by action holism. Perhaps, the analogy between action individualism and chemical/physical reduction is too sanguine. Perhaps there is less epistemic value in showing how the individualist dispenses with practically valuable collection action ascriptions, than in showing how chemistry can be reduced to physics.

There is another practical problem. In ordinary discourse, we are accustomed to using individual names for individual agents. But, especially in the case of large corporations, can we name all the singular agents involved in an ostensibly collective action? Which agents are to be included on the list $\alpha_0 \ldots \alpha_n$ when we are considering the actions of General Motors Corporation? This is especially complicated since the list of employees, stock holders, etc. changes daily. The solution to this difficulty is like the solution to the last. In principle, it seems that there is no reason why the list of singular agents involved is impossible although very difficult or perhaps impossible to ascertain. This problem and the last indicate that a limit is being reached in human epistemic capacity when individualism is maintained. A decision relative to the individualist issue seems to involve a choice between two disvalues. On the one hand the individualist risks postulating

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truth conditions which are out of human epistemic reach. The truth conditions either hold or they do not, even if none of us can tell whether they do. On the other, the holist violates ordinary scruples against the existence of irreducibly collective agents, or worse yet, collective persons with mental states and all. It seems to me that there is hope for less offense in the individualist direction, and this hope is more attractive than violating the dictates of parsimony.

Among the theses for collective action ascriptions is:

12.20 $\models \neg B\alpha_0 \ldots \alpha_n \bot$,

since the intersection involved in the truth conditions for collective action ascription cannot be empty. Also it is to be noted that

13.15 $\not\models B\alpha\beta_0 \ldots \beta_n\phi \rightarrow B\alpha\phi$,

viz., that α together with $\beta_0 \ldots \beta_n$ bring it about that ϕ does not entail that α alone brings ϕ about. Also,

13.16 $\not\models B\alpha\beta_0 \ldots \beta_n\phi \rightarrow \neg B\alpha\phi$.

In other words, it does not follow that if α together with the cooperation of $\beta_0 \ldots \beta_n$ brings ϕ about, then α does not bring ϕ about alone. Furthermore,

13.17 $\not\models B\alpha\phi \rightarrow \neg B\alpha\beta_0 \ldots \beta_n\phi$.

That is to say, the possibility is not ruled out that α alone brings ϕ about and α in cooperation with $\beta_0 \ldots \beta_n$ brings ϕ about. Moreover, the collective version of RE_a holds and RM_a does not, as is the case with singular agent ascriptions.

CHAPTER SIX

THEISM, VOLUNTARISM AND DETERMINISM

1. ACTION POWER

It is widely believed that there are several agents, each uniquely contributing to the way things turn out. Any position regarding determinism ought to give some account of this belief and in the process explain the interplay between action ascriptions and descriptive natural law and between the actions of distinct agents. In Chapter Three, some attention was given to coercion and in Chapter Five cooperative actions involving several agents were considered. In this chapter, the interplay is center stage, especially where divine and human actions might conflict.

In the semantic structure of DBC, the moment m of evaluation is, from its own persective, the present moment. The set $\{x{:}Rx{,}m\}$, is to be thought of as having the moments possible relative to m as members. It is in terms of this set that the truth conditions for sentences of the form $\Box\phi$ are presented in Chapter One. The subset $\{x{:}x \in f(m,\alpha)\}$ represents the set of moments necessary at m given the actions of agent α, or moments that actual history must pass through because of what α does. Characteristics of this set along with truth conditions for sentences of the form $B\alpha\phi$ were explored in Chapter Two and object language sentences of the form $B\alpha\phi$ were taken as translations of ordinary sentences such as "α brings it about that ϕ". In Chapter Five the feasibility of collective action ascription was considered. There the set $\{x{:}x \in f(m,\alpha_1) \cap ... \cap f(m,\alpha_k)\}$ where each α_k is a

[illegible]

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[illegible]

[illegible]

name, was to be thought of as the set of moments necessary at m given the actions of agents $\alpha_1 ... \alpha_k$. Sentences of the form $B\alpha\beta\phi$ were taken as translations of "α and β bring it about that ϕ". If the list $\alpha_1 ... \alpha_k$ includes a name for every agent, the set represents the set of moments possible at m given what every agent does. In other words, $B\alpha_1 ... \alpha_k\phi$ is a translation for "ϕ is the product of all actions". These situations are depicted in Figure 10.

If we take the freedom of quantifying over wffs of DBC, we can see how various metaphysical positions can be expressed in the notation of DBC. According to some metaphysicians, the following is true:

$$\text{PU} \qquad (\phi)(\phi \rightarrow (\exists x_0)...(\exists x_n)Bx_0...x_n\phi)$$

unless ϕ contains some wff of the type $B\beta\phi$ as a well–formed part. One might call this weak personalism or pluralistic voluntarism. According to this view, everything that is the case, apart from actions themselves, springs from agency. This view rules out randomness if what is true randomly is not brought about.

But in order for an event to be deemed random, not only is it necessary that the event not be brought about, but also it must be the case that the event is not caused according to a natural law. If $E\Box(\psi \rightarrow \phi)$ holds only where there is a natural law according to which ϕ is caused by ψ, one might consider the following thesis which seems to express the major claim of natural law determinism. This thesis combines DBC notation with ordinary English.

LU $(\phi)(\phi \rightarrow (\exists\psi)(\psi$ is a descriptive natural regularity $\wedge E\Box(\psi \rightarrow \phi))$.

In other words, everything that is the case is determined by some descriptive natural regularity ψ.

The theses PU and LU resemble those that follow. According to the first, called PE (this resembles PU), at least some events spring from action.

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[illegible]

[illegible]

[illegible]

[illegible]

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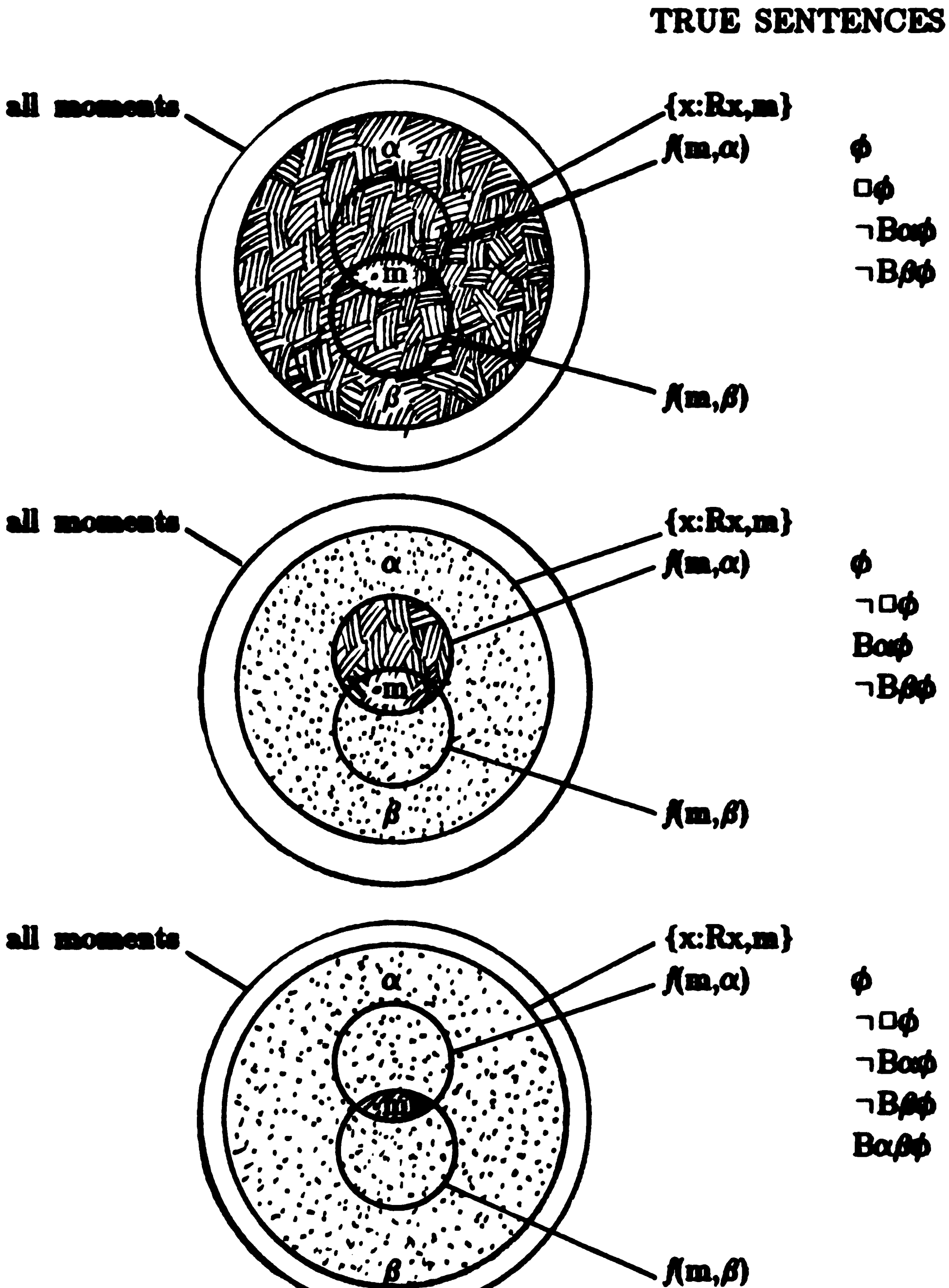
FIGURE 10

ACTION, COLLECTIVE ACTION AND POSSIBILITY

Suppose ϕ is true in cross–hatched regions and false in dotted regions.



PE $(\exists\phi)(\phi \wedge (\exists x_0)...(\exists x_n)Bx_0...x_n\phi)$.

Again it seems that wffs of the form $B\beta\psi$ should be kept from ϕ in order to protect the plausibility of the view. According to the second, called LE, there are some natural laws.

LE $(\exists\phi)(\phi \wedge (\exists\psi)(\psi$ is a descriptive natural regularity $\wedge E\Box(\psi \rightarrow \phi)$.

Neither PU nor PE is incompatible with LU. In other words, one could believe that everything or some things spring from action and at the same time believe that actions are a subset of things caused by natural law. This gives priority to natural law over action. Hence some would want to hold PU and deny LE.

But the latter view counters the popular belief that agency has impersonal descriptive natural law as a context and that agents can use lawful connections as means in the accomplishment of certain ends, but these lawful connections are themselves not subject to change on the basis of agent initiative. If a view in greater agreement with this popular belief is desired sentences of the form ϕ of LU could be restricted so that they contain no well–formed parts of the type $B\alpha\phi$. Or one might simply embrace LE and PE.

Another view advanced by some theists is that

T1 $(\exists x)(\phi)E(\phi \rightarrow Bx\phi)$.

If there is but one x like this, we have the claim of some monotheists that God brings about all that is the case. But according to DBC, some constraints on ϕ seem in order. There are some sentences, $2+2=4$, for example, which are necessarily true, that is, they are true at every possible moment. In order to bring $2+2=4$ about, God must be assigned every possible moment at every moment. It follows from T1 that $\phi \rightarrow \Box\phi$ is a thesis. If we suppose not, then there must be some moment m and sentence

[illegible]

ψ such that at some moment n possible relative to m, ψ is the case and some possible moment n' possible relative to m at which ψ is not the case. But in order for it to be the case at m that God brings about every sentence, including the necessitations, he must be assigned every moment possible relative to m. If so, it will be false at m that God brings ψ about, since in one of the moments he is assigned at m, namely n', ψ is false. In other words, T1 rules out the belief in contingency altogether. No event and no action is contingent. Even the action of God in bringing everything about is necessary.

The problem T1 poses can be seen by attending to the DBC thesis:

12.21 $(\phi)(\psi)((\Box\phi \wedge B\alpha\phi) \rightarrow \neg(\exists\psi)(\Diamond\psi \wedge \Diamond\neg\psi \wedge B\alpha\psi))$.

In other words, at any moment, any agent who brings something necessary about, does not bring anything contingent about. Some weakening in T1 is more in keeping with the assertion of strong theistic voluntarists such as:

T2 $(\exists x)(\phi)E((\Diamond\phi \wedge \Diamond\neg\phi) \rightarrow Bx\phi)$.

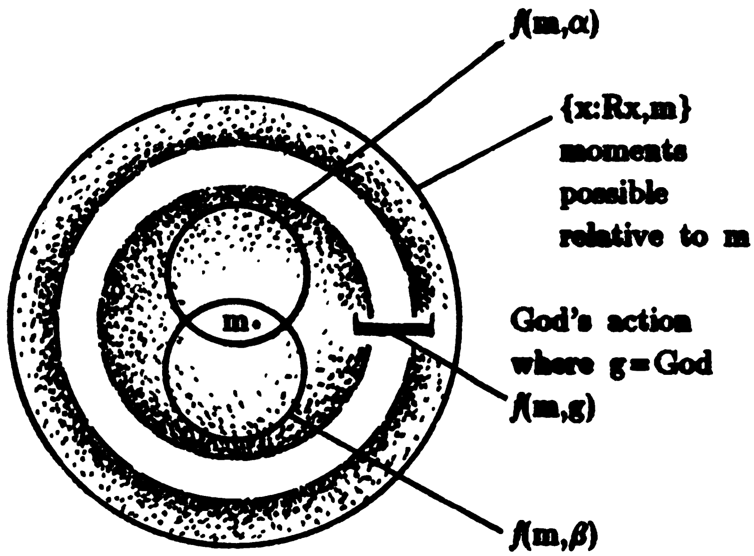
Here God brings about all that is contingent. This includes actions so that according to this view, God brings about all contingent actions. But since

12.22 $(\phi)((\Diamond\phi \wedge \Diamond\neg\phi \wedge B\alpha\phi) \rightarrow \neg(\exists\psi)(\Box\psi \wedge B\alpha\psi))$,

if God brings about any contingent state of affairs, he brings about nothing necessary and thus gives up any control over natural law if these are necessarily the case.

But consider another alternative. Assuming diagramming conventions as depicted in Figure 10, we might suppose that at each moment the picture is something like that depicted in Figure 11.

FIGURE 11



In this depiction we might think of God's actions as narrowing the course of events with variation according to the several possibilities labelled ƒ(m,g).

Perhaps God does not determine what tie I wear at m. But, he might. He might determine what tie I wear but not what kind of sandwich I have for lunch. The variation stops short of God determining what is necessarily the case, which has strange consequences as we have seen, but the strength of God's action can vary. Perhaps it varies from time to time. Perhaps God narrows the course history follows, but does not bring about everything about that (short of necessity) happens. This seems a plausible alternative according to the DBC semantics and is deserving of more careful consideration than I am able to give it here.

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[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

2. *ETHICS, DIVINE POWER AND PREVENTABILITY*

Consider how these matters impinge on obligation and prohibition. If God's command to α to bring ϕ about is sufficient for $\Box(\neg B\alpha\phi \rightarrow FS\alpha)$ then one might say God controls this obligation. Most Christian theists believe this, even if they hold a deontological view of metaethics. If T2 is also embraced, God also controls which obligations are kept and which broken. A similar situation holds for forbiddances. But this is intolerable for theists and nontheists alike; for theists because it seems to make God guilty for sins (although as we have seen $BgB\alpha\phi$ does not entail $Bg\phi$ and as is obvious $Bg\neg B\alpha\phi$ does not entail $\neg Bg\phi$) and for nontheists because T2 allows too much divine control over actions.

It might be argued that the DBC analysis of moral property ascriptions is, especially apart from theistic considerations, too strong. Suppose α fails to bring ϕ about. This can be a sin of commission only if the application of sanction to α is unpreventable *simpliciter*. In other words, sanction's future occurrence must be unstoppable, not only as far as α's powers of action are concerned, but as far as anyone's or any combination of agent actions is concerned. But sanctioning is something that fallible agents do, and because of the real occurrence of tried actions that fail, it might be that someone tries to sanction α in the event of α's failure, but somehow or other α escapes. If he can get away with not bringing ϕ about, that is, if he can escape necessary punishment as a consequence of his failure to bring ϕ about, then he is not obliged to bring it about.

If this situation is taken to indicate that the DBC analysis as it stands is too strong, some remedy is possible. One might weaken the unpreventability requirements of the analysis thus far presented. Instead of

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$O\alpha\phi$ being equivalent to $\Box(\neg B\alpha\phi \rightarrow FS\alpha)$ as has been suggested, it can be treated as equivalent to $\Box(\neg B\alpha\phi \rightarrow F\neg B\alpha\neg S\alpha)$. According to the latter, α is obliged to bring ϕ about just in case *he* cannot prevent his own sanction in the event of his failure, although it might fail to come about for some other reason. Perhaps Baker will not be sanctioned for failing to bring ϕ about and yet Baker is obliged to bring ϕ about. Baker cannot do anything to prevent the sanction, but he might get out of it nonetheless. This still leaves it the case that if α can himself escape future sanction in the event that he fails to bring ϕ about, he is not obliged to do ϕ.

3. PRUDENCE AND CONFLICTING OBLIGATIONS

In *A Theory of Justice*, Rawls claims that the distinguishing feature of a metaethical theory is the relation it espouses between the right and the good[38]. Utilitarian theories, for example, assert that an action is obligatory just in case it results in more good than its omission. As we mentioned previously, Prior believed that the escapism of the Andersonian simplification was not naturalistic because the necessity involved in the application of sanction involved the moral notion of desert, viz., α is forbidden to bring ϕ about if and only if α ought to be sanctioned in the event that he brings ϕ about. The weakening considered in the last section weakens the necessity of sanction definitive of obligation and forbiddance. And this weakening might be preferred by one who wants to avoid Prior's argument against the Andersonian simplification.

But if it is not, there is still a way of maintaining naturalism within the DBC analysis. Suppose McClelland robs the bank and that he is not punished for it. It seems that McClelland was morally forbidden to do this

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[illegible]

[illegible]

[illegible]

even though he got away with it. He ought to be punished, even though he is not. If a naturalistic view is behind the escapist metaethical doctrine, one might say one of two things. First, if one is a theist, one might maintain that God will punish this deed in the next life even if McClelland escapes in this one, and so it is indeed morally forbidden, even though it looks as if McClelland escapes. He will not ultimately escape. Kant, for example, in the second *Critique*, because he was bothered by the inequity involved in the good and evil consequences evident in this life, argues that this circumstance requires the postulation of a continued life after this one, and for similar reasons there is a supreme being who will see that punishments and rewards commensurate with obedience are meeted out. This seems like a plausible view for theists. Of course, if the standard by which God "appropriately" allocates punishment and reward is not naturalistically understood, naturalism fails even here.

On the other hand, one might take the intuitively more difficult route and suggest that our intuitions behind the belief that McClelland is morally forbidden to rob the bank are mistaken. His escape from sanction erodes his moral forbiddance, so to speak. But the intuitions about McClelland's deed are not entirely misguided. Someone else, or some collective, ought to have sanctioned McClelland for his deed. McClelland does not have the moral forbiddance because someone else does not keep his obligation to sanction. The obligation to sanction, or the obligation to bring it about that McClelland does not escape if he robs the bank, might be considered a second order obligation. McClelland does not have the first order fobiddance because someone else does not keep the second order obligation.

In both of these positions, there is considerable intuitive offense. The offense comes when three beliefs are attractive. First, if it is desirable to

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disbelieve that God exists, particularly, a God who interferes in the desires and preferences of humans and has ability to sanction; secondly, if the belief is attractive that death is final in that there is no life after this one in which the apparent inequities are resolved; and thirdly, if it is attractive to believe that the best sanctioning machinery allows people to get away with doing or omitting to do what they are surely forbidden or obliged to do, then, escapism as a naturalistic metaethical theory is unattractive indeed as compared to deontologism. It looks to me as if some believe that people are obliged to act as if there is a God when of course there might not be and if obligation is to be countenanced at all without presupposing theism, deontology must be the only option. This looks to me like having one's cake and eating it too.

What about prudential concerns? In Chapter Three, Rescher's semantics for preference was introduced. There, it was suggested that in a sanctioning situation $S\alpha \leftrightarrow (\exists x)(Bx\phi \wedge A\alpha\neg\phi/\phi)$, viz., α is sanctioned just in case someone brings it about that α suffers ϕ where he would prefer $\neg\phi$. In Rescher's terms, the truth of ϕ must be a first order evil for α. Sentences of the form $A\alpha\phi/\psi$ are true just in case the possible moments in which ϕ is true are higher on α's axiological scale than those in which ψ is true. Moreover these locutions may be taken as declarations that ϕ is *first order* better for α than is ψ. In this comparison $\mathfrak{g}(m,\alpha,\phi)$ must be larger than $\mathfrak{g}(m,\alpha,\psi)$ in order for ϕ to be first order better for α than ψ.

In his article Rescher presents another sort of good which he calls "differential preference"[34]. To adapt Rescher's position for DBC we might add another function $\bigstar$ such that $\bigstar(m,\alpha,\phi) = \mathfrak{g}(m,\alpha,\phi) - \mathfrak{g}(m,\alpha,\neg\phi)$. Intuitively, if $\mathfrak{g}(m,\alpha,\phi) > \mathfrak{g}(m,\alpha,\neg\phi)$, then at m, α has differential preference for ϕ over $\neg\phi$. New sentences for differential preference ascriptions of the

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type $A^{\star}\alpha\phi/\psi$ might be added with truth conditions as follows:

$$9.15^{\star} \quad {}^{h}_{m}[\![A^{\star}\alpha\phi/\psi]\!]^{\mathfrak{A}} = t \text{ iff } \star(m,\alpha,\phi,) \rightarrow \star(m,\alpha,\psi).$$

These sentences assert that ϕ is a greater differential good to α than is ψ. In other words, α has a higher differential preference for ϕ than for ψ. Perhaps the best illustration will involve the serious issue of conflicts of obligation. Thus far, we are to understand that sanctioning is not the exclusive province of particular persons or particular institutions. Hence, Wilson might be sanctioned from one direction if he moves in with his lover, and sanctioned from another if he does not. Supposing the sanction is unpreventable in both cases, Wilson has a conflict of what he is obliged to do and what he is forbidden to do. He cannot win. Suppose, "Wilson's parents cut him off from their will" is translated p and "Wilson will loose his job" is translated q. Suppose also that b names Wilson and that r translates "Wilson moves in with his lover". $\Box(Bbr \rightarrow F(\exists x)(Bxp \wedge Ab\neg p/p)$ is true and hence Fbr is too. $\Box(\neg Bbr \rightarrow F(\exists x)(Bxq \wedge Ab\neg q/q)$ is true and hence Obr is. Wilson has a genuine conflict. But suppose Wilson's parents are very wealthy, and he can get another job if he looses this one. Loosing the job, in other words, is much less of a disvalue to Wilson than being written out of his parent's will. Suppose there are four moments possible at m with the following values for $g(m,b,\phi)$.

moment	true sentences	g value
m	$p \wedge q$	2
m'	$p \wedge \neg q$	10
n	$\neg p \wedge q$	100
n'	$\neg p \wedge \neg q$	110

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Here $A^{\star}b\neg p/\neg q$. In other words, although Wilson has the conflict between forbiddance and obligation, it is clear that in the interests of prudence, he ought not to move in with his lover. Here the well noticed ambiguity of "ought" comes to the fore. Sometimes by saying that Wilson ought to do x, it is being asserted that Wilson is obliged to do x. The same sentence might assert that it is in Wilson's best interest to do x, whether he is obliged to do so or not. This sort of conflict between obligation or fobiddance and prudence is common, and especially important to some theists.

For example, in Acts 5, the apostle Peter is threatened by the Sanhedrin with some sort of punishment if he preaches in the name of Jesus. But he also fears God, and the punishment of God if he does not preach (or the reward of God if he does, or both) make him respond that he ought to obey God rather than men[35]. I take it that Peter believed that it was prudent to risk the punishment of the Sanhedrin rather than disobey God.

Christian theists sometimes face the sort of conflict that Peter did and during the times of most severe persecution for them, it is amazing that the fear of God should make them endure the most tortuous sanctions. The point of this as far as DBC is concerned is not to argue that Christians have a different concept of obligation or prudence from nonchristians or atheists. I think they have the same one. Both are or should be egoists and consequentialists.

Here, of course, I offend not only many nontheists, but many theists as well if I hold a naturalistic view of the good. And I do not hope to engage in both of these intellectual battles here. I prefer a view of the good according to which it is always related to interest and need not be related to any property outside of interest. This sounds like R. B. Perry's view in

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General Theory of Value[36]. Most theists will want to know how ascriptions of goodness to God can be so construed. In other words, how can believers ascribe goodness to God if goodness is understood in this way? Others will want to know how I explain the apparent similarity between the goods of one agent and the goods of another. But I do no more than raise these clouds of dust here, without showing how they are to be settled.

NOTES

[1]See Rudolf Carnap [1950] *Logical Foundations of Probability* (London: Routledge and Kegan Paul) Chapter 1 and Appendix J for explanations of explication.

[2]H. G. Bohnert [1945] "The Semiotic Status of Commands", *Philosophy of Science* 12:302.

[3]A. R. Anderson [1956] "The Formal Analysis of Normative Systems". Reprinted in N. Rescher (ed.), *The Logic of Decision and Action* [1966] (Pittsburgh: University of Pittsburgh Press) pages 147–213.

[4]Jaako Hintikka [1981] "Some Main Problems of Deontic Logic". Printed in R. Hilpinen (ed.) *Deontic Logic: Introductory and Systematic Readings* [1981] (Dordrecht: D. Reidel Pub. Co.) pages 59–104.

[5]W. V. Quine [1986] *Philosophy of Logic*, Second Edition, (Cambridge: Harvard University Press) page 35.

[6] "D" for "deontic", "B" for the bringing–it–about–that relation, and "C" for consequentialism.

[7]S. Knuuttila [1981]. "The Emergence of Deontic Logic in the Fourteenth Century". Printed in R. Hilpinen (ed.) *New Studies in Deontic Logic* [1981] (Dordrecht: D. Reidel) pages 225–250.

[8]A. N. Prior [1958] "Escapism the Logical Basis of Ethics". Printed in A.I. Melden (ed.) *Essays in Moral Philosophy* (Seattle: University of Washington Press) pages 135–159.

[9]A. N. Prior [1967] *Past, Present and Future*. (Oxford: Clarendon Press). See pages 122–128.

[10]R. A. Thomason [1970] "Indeterminist Time and Truth–Value Gaps", *Theoria* 36:264–281.

[11]David Lewis [1986] *On the Plurality of Worlds* (Oxford: Basil Blackwell), page 2.

[12]A. N. Prior [1967] *Past Present and Future* (Oxford: Clarendon Press). See pages 226–227.

[13]N. Rescher [1966] "Semantic Foundations for the Logic of Preference". Printed in N. Rescher (ed.) *The Logic of Decision and Action* (Pittsburgh: University of Pittsburgh Press) pages 37–62.

[14]N. Rescher *loc cit* page 44.

[15]A. N. Prior [1958] "Escapism: The Logical Basis of Ethics". Printed in A. I. Melden (ed.) *Essays in Moral Philosophy* [1958] (Seattle: University of Washington Press) pages 135–159.

[16]N. Rescher, *loc cit*, page 46.

[17]J. O. Urmson [1958] "Saints and Heroes" Printed in A.I. Melden (ed.) *Essays in Moral Philosophy* (Seattle: University of Washington Press) pages 198–216.

[18]A. Al–Hibri [1978] *Deontic Logic: A Comprehensive Appraisal and a New Proposal* (Washington: University Press of America) surveys the deontic paradoxes and provides bibliographic information.

[19]All quotations from the Bible are from the *New International Version* published by Zondervan Corporation, Grand Rapids, 1978.

[20]B. F. Chellas [1980] *Modal Logic: An Introduction* (Cambridge: Cambridge University Press) discusses some of these difficulties in Chapter Six.

[21]M. Brodbeck [1958] "Methodological Individualisms: Definition and Reduction" *Philosophy of Science* 25:1–22. Page 1.

[22]M. Brodbeck, *loc cit*, page 3.

[23]M. G. Velasquez [1983] "Why Corporations are not Morally Responsible for Anything They Do" *Business and Professional Ethics Journal* 2:(no. 3) 1–18. Page 16.

[24]H. Fain [1972] "Some Moral Infirmities of Justice". Printed in P. French (ed.) *Individual and Collective Responsibility* (Cambridge: Schenkman Pub. Co.) pages 19–34. Page 19.

[25]V. Held [1970] "Can a Random Collection of Individuals Be Morally Responsible?" *The Journal of Philosophy* 67: 471–481.

[26]J. Ladd [1970] "Morality and the Ideal of Rationality of Formal Organizations" *Monist* 54:488–516. Page 489.

[27]P. French [1979] "The Corporation as a Moral Person" *American Philosophical Quarterly* 16: 206–215. Page 212.

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APPENDICES

Appendix A

DBC

1 **Atomic Formulas**

1.1 **Terms**

1.1.1 **Names: Lower case letters a through e with or without subscripts.**

1.1.2 **Variables: Lower case letters *x* through *z* with or without subscripts.**

1.2 **Atomic Sentences: Lower case letters p through r with or without subscripts.**

1.3 **Connectives**

1.3.1 **Truth Functional Connectives: ¬, ∧, ∨, →, ↔, ⊤ and ⊥.**

1.3.2 **Grouping Indicators: (,) and /.**

1.4 **Tense Operators: P and F.**

1.5 **Quantifier Key: ∃**

1.6 **Modal Operator: □.**

1.7 **Action Operator: B.**

1.8 **Axiological Operator: A.**

1.9 **Deontic Operators: O, F and P.**

2\. **Formulas: Finite sequences of atomic formulas.**

3\. **Well–Formed Formulas (wffs)**

3.1 **If ϕ is an atomic sentence, ϕ is well–formed.**

3.2 **If ϕ and ψ are wffs, x a variable and α a term, then the following are well–formed:**

3.2.1 $\neg\phi$

3.2.2 $(\phi \wedge \psi)$

3.2.3 $(\phi \vee \psi)$

3.2.4 $(\phi \rightarrow \psi)$

3.2.5 $(\phi \leftrightarrow \psi)$

3.2.6 $P\phi$

3.2.7 $F\phi$

3.2.8 $B\alpha\phi$

3.2.8* $B\alpha_0 \ldots \alpha_n\phi$

3.2.9 $\Box\phi$

3.2.10 $(\exists x)\phi$

3.2.11 $A\alpha\phi/\psi$

3.2.12 $O\alpha\phi$

3.2.13 $F\alpha\phi$

3.2.14 $P\alpha\phi$

3.3 $\top$ and $\bot$ are well–formed.
Nothing else is well–formed.

4. Freedom and Bondage: An occurrence of a variable x is free (bound) in a formula ψ iff it falls within no (some) well–formed part $(\exists x)\phi$ of ψ.

5. Sentences: Well–formed formulas having no free occurrences of variables are sentences.

6. Instances: Where ϕ is a wff, α a variable and β a name, $\phi\alpha/\beta$ is an instance of $(\exists x)\phi$.

7. An Interpretation of DBC is an ordered quintuple $\mathfrak{A} = \langle M, m, \lessdot, f, g\rangle$ where;

7.1 M is a nonempty set.

7.2 $m \in M$.

7.3 $\lessdot$ is a binary relation on M.

7.3.1 $(x \lessdot y \wedge y \lessdot z) \rightarrow x \lessdot z$.

7.3.2 $((x \lessdot y \wedge z \lessdot y) \rightarrow (x \lessdot z \vee z \lessdot x))$.

7.3.3 $Rx,y \leftrightarrow \{z : z \lessdot x\} = \{z : z \lessdot y\}$

7.4 f is a function such that:

7.4.1 where ϕ is an atomic sentence and $x \in M$, $f(x,\phi) \in \{0,1\}$.

7.4.2 Where $i \in P$ and $x \in M$, $f(m,i) = \Delta$ where
(i) $\Delta \subseteq \{y : Ry,x\}$,
(ii) $m \in \Delta$.
(iii) $\alpha \neq \beta \rightarrow f(m,\alpha) \neq f(m,\beta)$.

7.4.3 Where α is a name, $m \in M$ and k is a real number, $g(m,\alpha,\phi) = k$.

7.5 h is a history in interpretation $\mathfrak{A}$ iff:
i) $h \subseteq M$,
ii) $(x \neq y \wedge x \in h \wedge y \in h) \rightarrow (x \lessdot y \vee y \lessdot x)$.
iii) $x \in h \rightarrow (y)(y \lessdot x \rightarrow y \in h)$
iv) $(x \in h \wedge (\exists y)(x \lessdot y)) \rightarrow (\exists y)(x \lessdot y \wedge y \in h)$.

8. Variant Interpretations: Where $\mathfrak{A} = \langle M, m, \lessdot, f, g\rangle$ and $\mathfrak{B} = \langle M', m', \lessdot', f', g'\rangle$ are interpretations of DBC, $\mathfrak{A}$ and $\mathfrak{B}$ are β–variants iff $M = M'$, $m = m'$, $\lessdot = \lessdot'$ and where β is a name, f and f' differ at most in what they assign to β for any $x \in M$.

9. Evaluation: Where ϕ is a wff, let ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}$ abbreviate "the truth value of ϕ at world m relative to history h containing m under interpretation $\mathfrak{A}$".

9.1 Where ϕ is an atomic sentence, ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=t$ if $\mathit{f}(m,\phi)=1$, otherwise it is $\mathfrak{f}$.

9.2 Where ϕ is a wff, ${}^{h}_{m}[\![\neg\phi]\!]^{\mathfrak{A}}=t$ if ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}\neq t$, otherwise it is $\mathfrak{f}$.

9.3 Where ψ is also a wff, ${}^{h}_{m}[\![(\phi\wedge\psi)]\!]^{\mathfrak{A}}=t$ if both ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=t$ and ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=t$, otherwise it is $\mathfrak{f}$.

9.4 ${}^{h}_{m}[\![(\phi\vee\psi)]\!]^{\mathfrak{A}}=t$ if either ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=t$ or ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=t$, otherwise it is $\mathfrak{f}$.

9.5 ${}^{h}_{m}[\![(\phi\rightarrow\psi)]\!]^{\mathfrak{A}}=t$ if either ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=\mathfrak{f}$ or ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=t$, otherwise it is $\mathfrak{f}$.

9.6 ${}^{h}_{m}[\![(\phi\leftrightarrow\psi)]\!]^{\mathfrak{A}}=t$ if either ${}^{h}_{m}[\![\phi]\!]^{\mathfrak{A}}=t$ and ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=t$, or ${}^{h}_{m}[\![\ \phi]\!]^{\mathfrak{A}}=\mathfrak{f}$ and ${}^{h}_{m}[\![\psi]\!]^{\mathfrak{A}}=\mathfrak{f}$, otherwise it is $\mathfrak{f}$.

9.7 ${}^{h}_{m}[\![\top]\!]^{\mathfrak{A}}=t$ for every m and h.

9.8 ${}^{h}_{m}[\![\bot]\!]^{\mathfrak{A}}=\mathfrak{f}$ for every m and h.

9.9 ${}^{h}_{m}[\![P\phi]\!]^{\mathfrak{A}}=t$ if $(\exists x)(x\lessdot m\wedge{}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=t)$, otherwise it is $\mathfrak{f}$.

9.10 ${}^{h}_{m}[\![F\phi]\!]^{\mathfrak{A}}=t$ if $(\exists x)(m\lessdot x\wedge{}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=t)$, otherwise it is $\mathfrak{f}$.

9.11 ${}^{h}_{m}[\![\Box\phi]\!]^{\mathfrak{A}}=t$ if $(x)(Rx,m\rightarrow{}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=t)$, otherwise it is $\mathfrak{f}$.

9.12 ${}^{h}_{m}[\![B\alpha\phi]\!]^{\mathfrak{A}}=t$ iff

(i) $(x)(h)(x\in\mathit{f}(m,\alpha)\rightarrow{}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=t)$ and

(ii) $(x)(h)((x\in\{x{:}Rx,m\wedge x\notin\mathit{f}(m,i)\}\rightarrow{}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=\mathfrak{f}))$.

9.12* Where α_i is a name and ϕ is a wff, ${}^{h}_{m}[\![B\alpha_0\ldots\alpha_n\phi]\!]^{\mathfrak{A}}=t$ iff

(i) $(\Delta=\mathit{f}(m,\alpha_0)\cap\ldots\cap\mathit{f}(m,\alpha_n)\rightarrow(x)(h)(x\in\Delta\rightarrow{}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=t))$

(ii) $(\Gamma=\{x{:}Rx,m\wedge x\notin\Delta\}\rightarrow(x)(h)(x\in\Gamma\rightarrow{}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=\mathfrak{f}))$.

9.13 Where x is a variable, ϕ a formula and $(\exists x)\phi$ a sentence, $[\![(\exists x)\phi]\!]^{\mathfrak{A}}_{m}=t$ iff $[\![\phi x/\beta]\!]^{\mathfrak{B}}_{m}=t$ under some β–variant $\mathfrak{B}$ of $\mathfrak{A}$ where β is the earliest name not appearing in ϕ.

9.14 ${}^{h}_{m}[\![A\alpha\phi/\psi]\!]^{\mathfrak{A}}=t$ if $\mathit{g}(m,\alpha,\phi)>\mathit{g}(m,\alpha,\psi)$, otherwise it is $\mathfrak{f}$.

9.15 Where $\bigstar(m,\alpha,\phi)=\mathit{g}(m,\alpha,\phi)-\mathit{g}(m,\alpha,\neg\phi)$, ${}^{h}_{m}[\![A^{\bigstar}\alpha\phi/\psi]\!]^{\mathfrak{A}}=t$ if $\bigstar(m,\alpha,\phi)>\bigstar(m,\alpha,\psi)$, otherwise it is $\mathfrak{f}$.

Df$\Diamond$ $\models\Diamond\phi\leftrightarrow\neg\Box\neg\phi$.

DfS $\models S\alpha\leftrightarrow(\exists x)(Bx\phi\wedge A\alpha\neg\phi/\phi)$.

DfO $\models O\alpha\phi\leftrightarrow\Box(\neg B\alpha\phi\rightarrow FS\alpha)$.

DfF $\models F\alpha\phi\leftrightarrow\Box(B\alpha\phi\rightarrow FS\alpha)$.

DfP $\models P\alpha\phi\leftrightarrow\neg F\alpha\phi$.

DfO* ${}^{h}_{m}[\![O\alpha\phi]\!]^{\mathfrak{A}}=t$ if $(x)(h)((x\in h\wedge x\in\mathit{h}(\alpha,m))\rightarrow{}^{h}_{x}[\![B\alpha\phi]\!]^{\mathfrak{A}}=t)$, otherwise it is $\mathfrak{f}$.

DfF* ${}^{h}_{m}[\![F\alpha\phi]\!]^{\mathfrak{A}}=t$ if $(x)(h)((x\in h\wedge x\in\mathit{h}(\alpha,m))\rightarrow{}^{h}_{x}[\![\neg B\alpha\phi]\!]^{\mathfrak{A}}=t)$, otherwise it is $\mathfrak{f}$.

DfO+ $O\alpha\phi\leftrightarrow\Box(\neg B\alpha\phi\rightarrow(\exists x)Q\alpha x\wedge FSx)$.

APPENDIX B

THESES AND NONTHESES

12.1 $\models \neg B\alpha \bot$

12.2 $\models B\alpha\phi \to \neg(\exists x)Bx\neg\phi$

12.3 $\dfrac{\models \phi \to \psi}{\models B\alpha\phi \to \psi}$

12.4 $\models \Diamond B\alpha\phi \to \Diamond\phi$

12.5 $\models \Box B\alpha\phi \to \Box\phi$

12.6 $\models (B\alpha\Box\phi \wedge \Diamond\psi \wedge \Diamond\neg\psi) \to \neg B\alpha\psi$

12.7 $\models B\alpha\Box\phi \leftrightarrow \Box B\alpha\phi$

12.8 $\models (\exists x)H\phi \to H(\exists x)\phi$

12.9 $\models (\exists x)G\phi \to G(\exists x)\phi$

12.10 $\models (\exists x)\Box\phi \to \Box(\exists x)\phi$

†12.11 $\models A\alpha\phi/\psi \to \neg A\alpha\psi/\phi$

†12.12 $\models \neg A\alpha\phi/\phi$

†12.13 $\models (A\alpha\phi/\rho \wedge A\alpha\rho/\psi) \to A\alpha\phi/\psi$

12.14 $\dfrac{\models \phi}{\models F\alpha\neg\phi}$

12.15 $\models \Box B\alpha\phi \to O\alpha\phi$, **and**

12.16 $\models \Box\neg B\alpha\phi \to F\alpha\phi$

12.17 $\models F\alpha\bot$

12.18 $FK \models \Diamond\neg FS\alpha$

12.19 $\models P\alpha\phi \to \neg O\alpha\neg\phi$

12.20 $\models \neg B\alpha_0 \ldots \alpha_n \bot$

12.21 $\models (\phi)(\psi)((\Box\phi \wedge B\alpha\phi) \to \neg(\exists\psi)(\Diamond\psi \wedge \Diamond\neg\psi \wedge B\alpha\psi))$

12.22 $\models (\phi)((\Diamond\phi \wedge \Diamond\neg\phi \wedge B\alpha\phi) \to \neg(\exists\psi)(\Box\psi \wedge B\alpha\psi))$

†RE_a $\dfrac{\models \phi \leftrightarrow \psi}{\models B\alpha\phi \leftrightarrow B\alpha\psi}$

†RM_a $\dfrac{\models \phi \to \psi}{\models B\alpha\phi \to B\alpha\psi}$

†C_a $\models (B\alpha\phi \wedge B\alpha\psi) \to B\alpha(\phi \wedge \psi)$

†K_a $\models B\alpha(\phi \to \psi) \to (B\alpha\phi \to B\alpha\psi)$.

T_a $\models B\alpha\phi \to \phi$

D_a $\models B\alpha\phi \to \neg B\alpha\neg\phi$

P_a $\models \neg B\alpha(\phi \wedge \neg\phi)$

RP_a $\dfrac{\models \phi}{\models \neg B\alpha\neg\phi}$

O_a	$\models \neg B\alpha\neg\phi \vee \neg B\alpha\phi$
†RE_o	$\dfrac{\models \phi\leftrightarrow\psi}{\models O\alpha\phi\leftrightarrow O\alpha\psi}$
†C_o	$\models (O\alpha\phi \wedge O\alpha\psi)\rightarrow O\alpha(\phi\wedge\psi)$
†K_o	$\models O\alpha(\phi\rightarrow\psi)\rightarrow(O\alpha\phi\rightarrow O\alpha\psi)$
RE_γ	$\dfrac{\models \phi\leftrightarrow\psi}{\models F\alpha\phi\leftrightarrow F\alpha\psi}$
FE	$\models F\alpha\phi\rightarrow\Diamond\neg FS\alpha$
OE	$\models O\alpha\phi\rightarrow\Diamond\neg FS\alpha$
LC	$\models \neg O\alpha(\phi\wedge\neg\phi)$
OC	$\models O\alpha\phi\rightarrow\neg O\alpha\neg\phi$
DC	$\models O\alpha\phi\rightarrow P\alpha\phi$
OK	$\models O\alpha\phi\rightarrow\Diamond B\alpha\phi$
FK	$\models F\alpha\phi\rightarrow\Diamond\neg B\alpha\phi$
OO	$\models O\alpha\phi\rightarrow EO\alpha\phi$
FO	$\models F\alpha\phi\rightarrow EF\alpha\phi$
OF	$\models (\exists x)Ox\phi\rightarrow(x)Ox\phi$
FF	$\models (\exists x)Fx\phi\rightarrow(x)Fx\phi$,
OA	$\models O\alpha\phi\rightarrow\neg(\exists x)Bx\neg B\alpha\phi$
FA	$\models F\alpha\phi\rightarrow\neg(\exists x)BxB\alpha\phi$,
DT	$\models P\alpha\phi\leftrightarrow\neg O\alpha\neg\phi$
13.1	$\not\models P\phi\rightarrow\Box P\phi$
13.2	$\not\models \Box\phi\rightarrow B\alpha\phi$
13.3	$\not\models \neg B\alpha\bot$
13.4	$\not\models (B\alpha\phi\vee B\alpha\psi)\rightarrow B\alpha(\phi\vee\psi)$
13.5	$\not\models B\alpha(\phi\vee\psi)\rightarrow(B\alpha\phi\vee B\alpha\psi)$
13.6	$\not\models B\alpha B\beta\phi\rightarrow B\alpha\phi$
13.7	$\models (\exists x)B\alpha\phi\rightarrow B\alpha(Ex)\phi$
†13.8	$\not\models (B\alpha(\exists x)\phi\rightarrow(\exists x)B\alpha\phi)$
13.9	$\not\models H(\exists x)\phi\rightarrow(\exists x)H\phi$ and
13.10	$\not\models G(\exists x)\phi\rightarrow(\exists x)G\phi$
13.11	$\not\models \Box(\exists x)\phi\rightarrow(\exists x)\Box\phi$
13.12	$\dfrac{\not\models \phi\rightarrow\psi}{\not\models O\alpha\phi\rightarrow O\alpha\psi}$
13.13	$\not\models O\alpha O\alpha\phi\rightarrow O\alpha\phi$
13.14	$\dfrac{\not\models \phi\rightarrow\psi}{\not\models F\alpha\phi\rightarrow F\alpha\psi}$

13.15	$\not\models B\alpha\beta_0 \ldots \beta_n \phi \rightarrow B\alpha\phi,$
13.16	$\not\models B\alpha\beta_0 \ldots \beta_n \phi \rightarrow \neg B\alpha\phi.$
13.17	$\not\models B\alpha\phi \rightarrow \neg B\alpha\beta_0 \ldots \beta_n \phi$
N_a	$\not\models B\alpha(\phi \vee \neg\phi)$
RN_a	$\dfrac{\not\models \phi}{\not\models B\alpha\phi}$
$\dagger M_a$	$\not\models B\alpha(\phi \wedge \psi) \rightarrow (B\alpha\phi \wedge B\alpha\psi).$
$\dagger M_o$	$\not\models O\alpha(\phi \wedge \psi) \rightarrow (O\alpha\phi \wedge O\alpha\psi)$
N_o	$\not\models O\alpha(\phi \vee \neg\phi)$
RN_o	$\dfrac{\not\models \phi}{\not\models O\alpha\phi}$
B_o	$\not\models \phi \rightarrow O\alpha\neg O\alpha\neg\phi$
4_o	$\not\models O\alpha\phi \rightarrow O\alpha O\alpha\phi$
5_o	$\not\models \neg O\alpha\neg\phi \rightarrow O\alpha\neg O\alpha\neg\phi$
M_f	$\not\models F\alpha(\phi \wedge \psi) \rightarrow (F\alpha\phi \wedge F\alpha\psi)$
N_f	$\not\models F\alpha(\phi \vee \neg\phi)$
RN_f	$\dfrac{\not\models \phi}{\not\models F\alpha\phi}$
C_f	$\not\models (F\alpha\phi \wedge F\alpha\psi) \rightarrow F\alpha(\phi \wedge \psi)$
K_f	$\not\models F\alpha(\phi \rightarrow \psi) \rightarrow (F\alpha\phi \wedge F\alpha\psi)$
B_f	$\not\models \phi \rightarrow F\alpha\neg F\alpha\neg\phi$
4_f	$\not\models F\alpha\phi \rightarrow F\alpha F\alpha\phi$
5_f	$\not\models \neg F\alpha\neg\phi \rightarrow F\alpha\neg F\alpha\neg\phi$

APPENDIX C

PROOFS AND COUNTERMODELS

Several notational conventions simplify the proofs and countermodels. Let $(x)\alpha[\![\phi]\!]^{\mathfrak{A}}_{m}=t$ abbreviate $(x)(h)((x\in h\wedge x\in f(m,\alpha))\rightarrow {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=t)$. Let $(x)\bar{\alpha}[\![\phi]\!]^{\mathfrak{A}}_{m}=t$ abbreviate $(x)(h)((x\in h\wedge x\in\{y{:}Ry,m\}\wedge x\notin f(m,\alpha))\rightarrow {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=t)$. Let $(\exists x)\alpha[\![\phi]\!]^{\mathfrak{A}}_{m}=t$ abbreviate $(\exists x)(\exists h)((x\in h\wedge x\in f(m,\alpha))\wedge {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=t)$. And let $(\exists x)\bar{\alpha}[\![\phi]\!]^{\mathfrak{A}}_{m}=t$ abbreviate $(\exists x)(\exists h)((x\in h\wedge x\in\{y{:}Ry,m\}\wedge x\notin f(m,\alpha))\wedge {}^{h}_{x}[\![\phi]\!]^{\mathfrak{A}}=t)$. The same conventions apply where t is replaced with $\mathfrak{f}$.

RE_a

$$\frac{\models\phi\leftrightarrow\psi}{\models B\alpha\phi\leftrightarrow B\alpha\psi.}$$

Proof. Suppose $\models\phi\leftrightarrow\psi$. If RE_a does not hold then for some interpretation $\mathfrak{A}$ there is an m and h such that either (i) ${}^{h}_{m}[\![B\alpha\phi\wedge\neg B\alpha\psi]\!]^{\mathfrak{A}}=t$ or (ii) ${}^{h}_{m}[\![\neg B\alpha\phi\wedge B\alpha\psi]\!]^{\mathfrak{A}}=t$. If (i) is the case then $(x)\alpha[\![\phi]\!]^{\mathfrak{A}}_{m}=t$ and $(x)\bar{\alpha}[\![\phi]\!]^{\mathfrak{A}}_{m}=\mathfrak{f}$. But since $\models\phi\leftrightarrow\psi$ by assumption, $(x)\alpha[\![\psi]\!]^{\mathfrak{A}}_{m}=t$ and $(x)\bar{\alpha}[\![\psi]\!]^{\mathfrak{A}}_{m}=\mathfrak{f}$ as well, and it follows from this that if (i) is the case then ${}^{h}_{m}[\![B\alpha\psi]\!]^{\mathfrak{A}}=t$. But this contradicts i). The situation is similar in case (ii) is true, QED.

RM_a

$$\frac{\models\phi\rightarrow\psi}{\not\models B\alpha\phi\rightarrow B\alpha\psi.}$$

Suppose $\phi\rightarrow\psi$ is $(p\wedge q)\rightarrow p$, which is a thesis. Further, suppose $(x)\alpha[\![(p\wedge q)]\!]^{\mathfrak{A}}=t$ and $(x)\bar{\alpha}[\![(p\wedge q)]\!]^{\mathfrak{A}}=\mathfrak{f}$. By truth conditions 9.12 it follows that ${}^{h}_{m}[\![(B\alpha\phi]\!]^{\mathfrak{A}}=t$. Since $(x)\alpha[\![(p\wedge q)]\!]^{\mathfrak{A}}=t$, $(x)\alpha[\![p]\!]^{\mathfrak{A}}=t$ as well by truth function. But if $(\exists x)\bar{\alpha}[\![p]\!]^{\mathfrak{A}}=t$, which is not incompatible with

$(x)\bar{\alpha}[\![(p \wedge q)]\!]^{\mathfrak{A}}=f$ above, it follows from conditions 9.12 that ${}^{h}_{m}[\![B\alpha\phi]\!]^{\mathfrak{A}}=f$. $\mathfrak{A}$ is a countermodel, QED.

C_a $$\models (B\alpha\phi \wedge B\alpha\psi) \rightarrow B\alpha(\phi \wedge \psi).$$

Proof. Suppose ${}^{h}_{m}[\![B\alpha\phi \wedge B\alpha\psi)]\!]^{\mathfrak{A}}=t$. By truth function it follows that ${}^{h}_{m}[\![B\alpha\phi]\!]^{\mathfrak{A}}=t$ and ${}^{h}_{m}[\![B\alpha\psi)]\!]^{\mathfrak{A}}=t$. From the former by conditions 9.12 it follows that $(x)\alpha[\![\phi]\!]^{\mathfrak{A}}=t$ and $(x)\bar{\alpha}[\![\phi]\!]^{\mathfrak{A}}=f$ and from the latter that $(x)\alpha[\![\psi]\!]^{\mathfrak{A}}=t$ and $(x)\bar{\alpha}[\![\phi]\!]^{\mathfrak{A}}=f$. This guarantees that $(x)\alpha[\![(\phi \wedge \psi)]\!]^{\mathfrak{A}}=t$ and $(x)\bar{\alpha}[\![(\phi \wedge \psi)]\!]^{\mathfrak{A}}=f$ and from this by conditions 9.12 it follows that ${}^{h}_{m}[\![B\alpha(\phi \wedge \psi)]\!]^{\mathfrak{A}}=t$ as well, QED.

M_a $$\not\models B\alpha(\phi \wedge \psi) \rightarrow (B\alpha\phi \wedge B\alpha\psi).$$

Proof. Suppose that $(x)\alpha[\![(\phi \wedge \psi)]\!]^{\mathfrak{A}}=t$ and $(x)\bar{\alpha}[\![(\phi \wedge \psi)]\!]^{\mathfrak{A}}=f$ guaranteeing the truth of the antecedent ${}^{h}_{m}[\![B\alpha(\phi \wedge \psi)]\!]^{\mathfrak{A}}$. Suppose that $(\exists x)(\bar{\alpha}[\![\phi]\!]^{\mathfrak{A}}=t \wedge \bar{\alpha}[\![\psi]\!]^{\mathfrak{A}}=f)$. This makes ${}^{h}_{m}[\![B\alpha\phi]\!]^{\mathfrak{A}}=f$. From this it follows that ${}^{h}_{m}[\![(B\alpha\phi \wedge B\alpha\psi)]\!]^{\mathfrak{A}}=f$ which makes $\mathfrak{A}$ a countermodel, QED.

K_a $$\models B\alpha(\phi \rightarrow \psi) \rightarrow (B\alpha\phi \rightarrow B\alpha\psi).$$

In the proof there are two cases to consider. i) First, assume that $f(m,\alpha) \neq \Lambda$. If ${}^{h}_{m}[\![B\alpha(\phi \rightarrow \psi)]\!]^{\mathfrak{A}}=t$, then $(x)\bar{\alpha}[\![\phi \rightarrow \psi]\!]^{\mathfrak{A}}=f$. But if so $(x)\bar{\alpha}[\![\phi]\!]^{\mathfrak{A}}=t$ by truth function and it follows from this by conditions 9.12 that ${}^{h}_{m}[\![B\alpha\phi]\!]^{\mathfrak{A}}=f$, guaranteeing that the consequent of K_a, ${}^{h}_{m}[\![B\alpha\phi \rightarrow B\alpha\psi]\!]^{\mathfrak{A}}$ is true thus ruling out a countermodel in the first case. ii) But, second, assume that $f(m,\alpha)=\Lambda$. In this event, the truth of the antecedent of K_a,

$\frac{h}{m}[\![B\alpha(\phi\rightarrow\psi)]\!]^{\mathfrak{A}}=t$, guarantees that $(x)\bar{\alpha}[\![\phi\rightarrow\psi]\!]^{\mathfrak{A}}=f$ by conditions 9.12. A countermodel in this second case requires $\frac{h}{m}[\![B\alpha\phi]\!]^{\mathfrak{A}}=t$ and from this it follows by 9.12 that $(x)\bar{\alpha}[\![\phi]\!]^{\mathfrak{A}}=f$. But if so, $(x)\bar{\alpha}[\![\phi\rightarrow\psi]\!]^{\mathfrak{A}}=t$ which is impossible. Q.E.D.

13.8 $\quad\quad \not\models((\exists x)B\alpha\phi\rightarrow B\alpha(\exists x)\phi.$

Suppose that $\frac{h}{m}[\![(\exists x)B\alpha\phi]\!]^{\mathfrak{A}}=t$. By condition 9.13, it follows that $\frac{h}{m}[\![B\alpha\phi x/\beta]\!]^{\mathfrak{B}}=t$ for some β–variant $\mathfrak{B}$ of $\mathfrak{A}$. If so, $(x)\alpha[\![\phi x/\beta]\!]^{\mathfrak{B}}=t$ and $(x)\bar{\alpha}[\![\phi x/\beta]\!]^{\mathfrak{B}}=f$ and hence $(x)\alpha[\![(\exists x)\phi]\!]^{\mathfrak{B}}=t$. But suppose $(\exists x)\bar{\alpha}[\![\phi x/\beta]\!]^{\mathfrak{C}}=t$ for some β–variant $\mathfrak{C}$ of $\mathfrak{A}$. It follows that $\neg(x)\bar{\alpha}[\![B\alpha(\exists x)\phi]\!]^{\mathfrak{B}}=f$ and hence $\frac{h}{m}[\![B\alpha(\exists x)\phi]\!]^{\mathfrak{A}}=f$ making $\mathfrak{A}$ a countermodel, QED.

RE_o

$$\frac{\models\phi\leftrightarrow\psi}{\models O\alpha\phi\leftrightarrow O\alpha\psi.}$$

Proof. Suppose that $\models\phi\leftrightarrow\psi$ and $\frac{h}{m}[\![O\alpha\phi]\!]^{\mathfrak{A}}=t$. By DfO, $\frac{h}{m}[\![\Box(\neg B\alpha\phi\rightarrow FS\alpha)]\!]^{\mathfrak{A}}=t$ also. Hence, i) $(x)(h)(Rx,m\rightarrow\frac{h}{x}[\![\neg B\alpha\phi\rightarrow FS\alpha]\!]^{\mathfrak{A}}=t)$. Assume that $A=\{x{:}Rx,m\wedge\frac{h}{x}[\![\neg B\alpha\phi]\!]^{\mathfrak{A}}=f\}$ and that $B=\{x{:}Rx,m\wedge\frac{h}{x}[\![FS\alpha]\!]^{\mathfrak{A}}=t\}$. Since $\models\phi\leftrightarrow\psi$ it follows from RE_a that $(x)(h)(x\in A\rightarrow\frac{h}{x}[\![\neg B\alpha\psi]\!]^{\mathfrak{A}}=f)$ and thus by truth function that $(x)(h)(x\in A\rightarrow\frac{h}{x}[\![\neg B\alpha\psi\rightarrow FS\alpha]\!]^{\mathfrak{A}}=t)$. It also follows that $(x)(h)(x\in B\rightarrow\frac{h}{x}[\![\neg B\alpha\psi\rightarrow FS\alpha]\!]^{\mathfrak{A}}=t)$. Because of the truth conditions for conditionals together with i), $A\cup B=\{x{:}Rx,m\}$ and hence $(x)(h)(x\in\{x{:}Rx,m\}\rightarrow\frac{h}{x}[\![\neg B\alpha\psi\rightarrow FS\alpha]\!]^{\mathfrak{A}}=t)$. This makes $\frac{h}{m}[\![\Box(\neg B\alpha\psi\rightarrow FS\alpha)]\!]^{\mathfrak{A}}$ true so that $\frac{h}{m}[\![O\alpha\phi]\!]^{\mathfrak{A}}=t$ only if $\frac{h}{m}[\![O\alpha\psi]\!]^{\mathfrak{A}}=t$. By replacing ψ with ϕ, the same procedure as that followed above shows that $\frac{h}{m}[\![O\alpha\psi]\!]^{\mathfrak{A}}=t$ only if $\frac{h}{m}[\![O\alpha\phi]\!]^{\mathfrak{A}}=t$, QED.

C_o $$\models (O\alpha\phi \wedge O\alpha\psi) \rightarrow O\alpha(\phi \wedge \psi).$$

Proof. Suppose not. Then ${}^{h}_{m}[\![O\alpha\phi \wedge O\alpha\psi]\!]^{\mathfrak{A}} = t$ and ${}^{h}_{m}[\![O\alpha(\phi \wedge \psi)]\!]^{\mathfrak{A}} = \mathfrak{f}$ for some $\mathfrak{A}$, h and m. From the falsehood of the latter it follows from DfO that ${}^{h}_{m}[\![\Box(\neg B\alpha(\phi \wedge \psi) \rightarrow FS\alpha]\!]^{\mathfrak{A}} = \mathfrak{f}$. Hence, $(\exists x)(Rx,m \wedge {}^{h}_{x}[\![\neg B\alpha(\phi \wedge \psi)]\!]^{\mathfrak{A}} = t \wedge {}^{h}_{x}[\![FS\alpha]\!]^{\mathfrak{A}} = \mathfrak{f})$. If ${}^{h}_{x}[\![\neg B\alpha(\phi \wedge \psi)$ then by truth function and C_a either ${}^{h}_{x}[\![\neg B\alpha\phi]\!]^{\mathfrak{A}} = t$ or ${}^{h}_{x}[\![\neg B\alpha\psi]\!]^{\mathfrak{A}} = t$ and hence either i) ${}^{h}_{x}[\![\neg B\alpha\phi \rightarrow FS\alpha]\!]^{\mathfrak{A}} = t$ or ii) ${}^{h}_{x}[\![\neg B\alpha\psi \rightarrow FS\alpha]\!]^{\mathfrak{A}} = t$. If i) is true then ${}^{h}_{m}[\![O\alpha\phi]\!]^{\mathfrak{A}} = \mathfrak{f}$, contradicting the assumption that ${}^{h}_{m}[\![O\alpha\phi \wedge O\alpha\psi]\!]^{\mathfrak{A}} = t$. But if ii) is true then ${}^{h}_{m}[\![O\alpha\psi]\!]^{\mathfrak{A}} = \mathfrak{f}$ contradicting the same assumption, QED.

M_o $$\not\models O\alpha(\phi \wedge \psi) \rightarrow (O\alpha\phi \wedge O\alpha\psi).$$

Proof. Suppose that in $\mathfrak{A}$ there are only three possible moments relative to m, m, m', and m". And further assume there are but three histories in $\mathfrak{A}$ distinguished by containing three moments, $m \in h$, $m' \in h'$ and $m'' \in h''$. At moment m, ${}^{h}_{m}[\![\phi \wedge \psi \wedge \neg FS\alpha]\!]^{\mathfrak{A}} = t$, at moment m', ${}^{h'}_{m'}[\![\neg\phi \wedge \psi \wedge FS\alpha]\!]^{\mathfrak{A}} = t$ and at moment m", ${}^{h''}_{m''}[\![\phi \wedge \neg\psi \wedge FS\alpha]\!]^{\mathfrak{A}} = t$. The action assignments at the three moments is as follows: $f(m,\alpha) = \{m\}$, $f(m',\alpha) = \{m',m''\}$ and $f(m'',\alpha) = \{m',m''\}$. The antecedent of M_o must be true in this situation since at m, ${}^{h}_{m}[\![B\alpha(\phi \wedge \psi)]\!]^{\mathfrak{A}} = t$, which makes the antecedent of M_o false. At m', ${}^{h'}_{m'}[\![FS\alpha]\!]^{\mathfrak{A}} = t$ and at m", ${}^{h''}_{m''}[\![FS\alpha]\!]^{\mathfrak{A}} = t$ making the consequent of M_o true at both moments. Thus, ${}^{h}_{m}[\![\Box(\neg B\alpha(\phi \wedge \psi) \rightarrow FS\alpha)]\!]^{\mathfrak{A}} = t$ from which ${}^{h}_{m}[\![O\alpha(\phi \wedge \psi)]\!]^{\mathfrak{A}} = t$ follows. But, ${}^{h}_{m}[\![\neg B\alpha\phi]\!]^{\mathfrak{A}} = t$ and ${}^{h}_{m}[\![FS\alpha]\!]^{\mathfrak{A}} = \mathfrak{f}$ making ${}^{h}_{m}[\![\Box(\neg B\alpha\phi \rightarrow FS\alpha)]\!] = \mathfrak{f}$. From this it follows that ${}^{h}_{m}[\![O\alpha\phi]\!]^{\mathfrak{A}} = \mathfrak{f}$ from which the falsehood of the consequent of M_o follows, QED.

K_o $\qquad\qquad \models O\alpha(\phi\rightarrow\psi)\rightarrow(O\alpha\phi\rightarrow O\alpha\psi)$.

Proof. Supposing $\mathfrak{A}$ at m and h is a countermodel for K_o, it must be the case that ${}^{h}_{m}[\![O\alpha(\phi\rightarrow\psi)]\!]^{\mathfrak{A}}=t$, ${}^{h}_{m}[\![O\alpha\phi)]\!]^{\mathfrak{A}}=t$ and ${}^{h}_{m}[\![O\alpha\phi)]\!]^{\mathfrak{A}}=f$. In order for ${}^{h}_{m}[\![O\alpha\phi)]\!]^{\mathfrak{A}}=f$ there must be some n and h', such that Rn,m and ${}^{h'}_{n}[\![B\alpha\psi]\!]^{\mathfrak{A}}=f$ and ${}^{h'}_{n}[\![\neg FS\alpha]\!]^{\mathfrak{A}}=t$. There are two cases to consider in the falsehood of ${}^{h'}_{n}[\![B\alpha\psi]\!]^{\mathfrak{A}}$. i) It might be that $(\exists x)\alpha[\![\neg\psi]\!]^{\mathfrak{A}}=t$. In order to maintain the truth of ${}^{h}_{m}[\![O\alpha\phi]\!]^{\mathfrak{A}}$ of thesis K_o, it must be the case that ${}_{n}[\![B\alpha\phi]\!]^{\mathfrak{A}}=t$ since ${}^{h'}_{n}[\![\neg FS\alpha]\!]^{\mathfrak{A}}=t$, and thus $(x)\alpha[\![\phi]\!]^{\mathfrak{A}}=t$ and $(x)\bar{\alpha}[\![\phi]\!]^{\mathfrak{A}}=f$. Bu so, ${}^{h'}_{n}[\![B\alpha(\phi\rightarrow\psi)]\!]^{\mathfrak{A}}=f$ since at the x such that ${}^{h''}_{x}[\![\neg\psi]\!]^{\mathfrak{A}}=t$ and ${}^{h''}_{x}[\![\phi]\!]^{\mathfrak{A}}=t$, and since ${}^{h'}_{n}[\![\neg FS\alpha]\!]^{\mathfrak{A}}=t$, ${}^{h}_{m}[\![O\alpha(\phi\rightarrow\psi)]\!]^{\mathfrak{A}}=f$ contrary to the assumption. ii) it might be that $(\exists x)\bar{\alpha}[\![\psi]\!]^{\mathfrak{A}}=t$. But if so, ${}_{n}[\![B\alpha(\phi\rightarrow\psi)]\!]^{\mathfrak{A}}$ will be false as well and since ${}^{h'}_{n}[\![\neg FS\alpha]\!]^{\mathfrak{A}}=t$, ${}^{h}_{m}[\![O\alpha(\phi\rightarrow\psi)]\!]^{\mathfrak{A}}$ will be false as well, again contradicting the assumption, QED.

The following proofs show relations between different interpretations of DBC as depicted in Figure 8.

$\{LC\}\twoheadrightarrow\{OE\}$.

Proof. If $\mathfrak{A}$ at m and h is a countermodel, then for some agent a, ${}^{h}_{m}[\![\diamond\neg FSa]\!]^{\mathfrak{A}}=f$. Hence, ${}^{h}_{m}[\![\Box FS\alpha]\!]^{\mathfrak{A}}=t$. But since $\models\neg(\phi\wedge\neg\phi)$, $\models\neg B\alpha(\phi\wedge\neg\phi)$ since $(\phi\wedge\neg\phi)$ must be true at the moment of evaluation in order for a sentence of the form $B\alpha(\phi\wedge\neg\phi)$ to be true there, and this is impossible. Hence, ${}^{h}_{m}[\![B\alpha(\phi\wedge\neg\phi)]\!]^{\mathfrak{A}}=t$ and this makes ${}^{h}_{m}[\![\Box(\neg B\alpha(\phi\wedge\neg\phi)\rightarrow FS\alpha]\!]^{\mathfrak{A}}=t$ which by definition is equivalent to ${}^{h}_{m}[\![Oa(\phi\vee\neg\phi)]\!]$. But this makes the thesis of LC interpretations false. A countermodel is impossible, QED.

$\{OE\} \rightarrow \{LC\}$.

If not then i) $^h_m[\![O\beta(\phi\wedge\neg\phi)]\!]^{\mathfrak{A}}=t$ for some $\mathfrak{A}$, h, m, and β. Since $\neg B\alpha(\phi\wedge\neg\phi)$, $^h_m[\![\Box FS\beta]\!]^{\mathfrak{A}}=t$ as well. But if $\models O\alpha\phi\rightarrow\Box\neg FS\alpha$, then by i) and MP, $^h_m[\![\Diamond\neg FS\beta]\!]^{\mathfrak{A}}=t$. This is impossible, QED.

$\{FE\} \rightarrow \{LC\}$.

Proof. Since $\models\neg B\alpha(\phi\wedge\neg\phi)$, so is $F\alpha(\phi\wedge\neg\phi)$. If $F\alpha\phi\rightarrow\Diamond\neg FS\alpha$, then it follows that $\models\Diamond FS\alpha$. But if so, $\models\neg O\alpha(\phi\wedge\neg\phi)$, QED.

$\{OC\} \rightarrow \{LC\}$.

Proof. Suppose not. Then there is some $\mathfrak{A}$, m, and h such that $^h_m[\![O\alpha(\phi\wedge\neg\phi)]\!]^{\mathfrak{A}}=t$, viz., $^h_m[\![\Box(\neg B\alpha(\phi\wedge\neg\phi)\rightarrow FS\alpha)]\!]^{\mathfrak{A}}=t$. But $\models\neg B\alpha(\phi\wedge\neg\phi)$ and so for every x such that Rx,m, $^h_x[\![FS\alpha]\!]^{\mathfrak{A}}=t$. The thesis of OC, $O\alpha\phi\rightarrow\neg O\alpha\neg\phi$, requires then that $^h_m[\![O\alpha(\phi\wedge\neg\phi)\rightarrow\neg O\alpha\neg(\phi\wedge\neg\phi)]\!]^{\mathfrak{A}}=t$ and since the antecedent is true, i) $^h_m[\![\neg O\alpha\neg(\phi\wedge\neg\phi)]\!]^{\mathfrak{A}}$ must be true as well. It follows from this by thesis DfO that $^h_m[\![\neg\Box(\neg B\alpha\neg(\phi\wedge\neg\phi)\rightarrow FS\alpha)]\!]^{\mathfrak{A}}=t$. Since it has already been shown that $^h_x[\![FS\alpha]\!]^{\mathfrak{A}}=t$ for every x such that Rx,m, $^h_m[\![\Box(\neg B\alpha\neg(\phi\wedge\neg\phi)\rightarrow FS\alpha)]\!]^{\mathfrak{A}}=t$, viz., $^h_m[\![O\alpha\neg(\phi\wedge\neg\phi)]\!]^{\mathfrak{A}}$. But this contradicts i), QED.

$\{LC\} \rightarrow \{OC\}$.

Proof. If not then $^h_m[\![O\alpha\phi\wedge O\alpha\neg\phi]\!]^{\mathfrak{A}}=t$ for some $\mathfrak{A}$, m, h and α.

$\models B\alpha\phi \vee \neg B\alpha\phi$ by truth function. And for all x such that Rx,m, and ${}^{h}_{x}[\![B\alpha\phi]\!]^{\mathfrak{A}}=t$ thesis D_a requires that, ${}^{h}_{x}[\![\neg B\alpha\neg\phi]\!]^{\mathfrak{A}}=t$ as well. But since ${}^{h}_{m}[\![O\alpha\neg\phi]\!]^{\mathfrak{A}}=t$, ${}^{h}_{x}[\![FS\alpha]\!]^{\mathfrak{A}}=t$ also. But, for all y such that Ry,m and ${}^{h}_{y}[\![\neg B\alpha\phi]\!]^{\mathfrak{A}}=t$ the assumed ${}^{h}_{m}[\![O\alpha\phi]\!]^{\mathfrak{A}}=t$ requires that, ${}^{h}_{y}[\![FS\alpha]\!]^{\mathfrak{A}}=t$ also. But since $\{z:z=x\vee z=y\}=\{x':Rx',m\}$, ${}^{h}_{m}[\![\Box FS\alpha]\!]^{\mathfrak{A}}=t$. Because $\models\neg B\alpha(\phi\wedge\neg\phi)$, ${}^{h}_{m}[\![\Box(\neg B\alpha(\phi\wedge\neg\phi)\to FS\alpha)]\!]^{\mathfrak{A}}=t$ and thus by DfO, ${}^{h}_{m}[\![O\alpha(\phi\wedge\neg\phi)]\!]^{\mathfrak{A}}=t$, contradicting the thesis for LC, QED.

$\{OK\}\to\{LC\}$.

Suppose $\models O\alpha\phi\to\Box B\alpha\phi$. If $\not\models\neg O\alpha\bot$ then ${}^{h}_{m}[\![\Box(\neg Bb\bot\to FSb]\!]^{\mathfrak{A}}=t$ for some $\mathfrak{A}$, m, h, and b, viz., ${}^{h}_{m}[\![O\alpha\bot]\!]^{\mathfrak{A}}=t$. But it follows from the OK thesis and MP that ${}^{h}_{m}[\![\Diamond B\bot]\!]^{\mathfrak{A}}=t$. But this is impossible since $\models\neg B\alpha\bot$, QED.

$\{DC\}\to\{LC\}$.

Suppose that ${}^{h}_{m}[\![O\alpha\bot]\!]^{\mathfrak{A}}=t$ for some $\mathfrak{A}$, m, h, and α. It follows from $\models O\alpha\phi\to P\alpha\phi$ that ${}^{h}_{m}[\![P\alpha\bot]\!]^{\mathfrak{A}}=t$, viz., ${}^{h}_{m}[\![\neg F\alpha\bot]\!]^{\mathfrak{A}}=t$. According to DfF, i) ${}^{h}_{m}[\![\Diamond(B\alpha\bot\to FS\alpha]\!]^{\mathfrak{A}}=\mathfrak{f}$. But since $\models\neg B\alpha\bot$, ${}^{h}_{x}[\![B\alpha\bot\to FS\alpha]\!]=t$ for every x such that Rx,m and hence ${}^{h}_{m}[\![\Diamond(B\alpha(\phi\wedge\neg\phi)\to FS\alpha]\!]=t$ which contradicts i), QED.

12.11 $\models A\alpha\phi/\psi \rightarrow \neg A\alpha\psi/\phi$.

Proof. Suppose not. Then $^{h}_{m}[\![A\alpha\phi/\psi]\!]^{\mathfrak{A}} = t$ and $^{h}_{m}[\![A\alpha\psi/\phi]\!]^{\mathfrak{A}} = \mathfrak{f}$ for some $\mathfrak{A}$, m, and h. From the former it follows that $\mathfrak{g}(m,\alpha,\phi) > \mathfrak{g}(m,\alpha,\psi)$ and from the latter that the denial of this is true, which is impossible, QED.

12.13 $\models (A\alpha\phi/\psi \wedge A\alpha\psi/\rho) \rightarrow A\alpha\phi/\rho$.

Proof. Suppose $^{h}_{m}[\![A\alpha\phi/\psi]\!]^{\mathfrak{A}} = t$ and $^{h}_{m}[\![A\alpha\psi/\rho]\!]^{\mathfrak{A}} = t$ which makes the antecedent of 12.13 true. It follows that $\mathfrak{g}(m,\alpha,\phi) > \mathfrak{g}(m,\alpha,\psi)$ and thus that $\mathfrak{g}(m,\alpha,\psi) > \mathfrak{g}(m,\alpha,\rho)$. This makes it impossible that $^{h}_{m}[\![A\alpha\phi/\rho]\!]^{\mathfrak{A}} = \mathfrak{f}$, QED.