ABSTRACT

DEMAND ANALYSIS FOR COMMERCIAL
FERTILIZER IN THE UNITED STATES,
BY STATES

by Eldon A. Reiling

One of the major factors explaining productivity of American agriculture is the use of commercial fertilizer. Yet, the economic and noneconomic variables related to the level of fertilizer use are little understood. The purpose of this study was to relate and measure the effects of these variables on fertilizer use. Models were constructed for each of the principle fertilizer nutrients - nitrogen, phosphate and potash.

The annual quantity of each nutrient applied per acre of cropland in each of the 48 continental states was the variable to be explained. Explanatory variables included the "real" price of the nutrient, "real" prices of the most important fertilizer consuming crops in the preceding year, average "real" farm income from the preceding year, a proxy variable for technological change and farmers' awareness of fertilizer response, and a proxy variable for differences among states in fertilizer productivity and other factors.

The basic data were both time-series and cross-section in nature. They represented the fifteen year period 1950 through 1964 with observations on each state giving a total of 720 observations. The estimated model is a typical covariance model fitted with least squares procedures.

A major effort was made to develop state indexes of nutrient

price for nitrogen and phosphate. Five alternative methods for constructing the indexes were considered and the resulting indexes are included and evaluated in the study.

The major conclusions of the study are:

1. The fertilizer nutrient price is an important factor in explaining the increased fertilizer consumption. Potash and nitrogen price - both lower over the period studied - explain more than 20 percent of the increase in nutrient consumption.

2. Net farm income as a variable representing the firm's expenditure restriction is a restrictive factor in all models. Easing of this restriction increases expenditures for all three nutrients.

3. Technological change and increased acceptance of fertilizer by farmers is the most important consideration in explaining increased consumption of nitrogen, phosphate and potash. But the economic variables are relatively more important in explaining increased nitrogen consumption.

4. To demonstrate the use of the models for state estimates, projections for Michigan were made. By 1980, it is estimated that Michigan will be consuming 187,000 tons of nitrogen, 172,000 tons of phosphate and 167,000 tons of potash annually.

5. After considering five basic price index constructions for nitrogen and phosphate on theoretical and empirical grounds, the chain indexes were selected for use in the nutrient models. The estimated state nutrient indexes warn against generalizations about fertilizer price from the aggregate index. For although the aggregate index is relatively stable. the aggregate is currently made up of two diverging elements and one stable element.

6. A variation of the usual coefficient of determination, R^2,

is suggested as an improved measure of multicollinearity. Each major predetermined variable is regressed on the remaining predetermined variables. This measure identifies which variables are collinear and is independent of the matrix size.

7. The Durbin-Watson statistic and the VonNeumann-Hart ratio do not provide a sufficient test for serial correlation for models combining time-series and cross-section data. Depending upon the way the data are arranged for the problem, the test refers to either the successive differences over time or successive differences between elements of the cross-section. Both tests are necessary but may still not be sufficient.

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INTRODUCTION

The Problem Setting

Commercial fertilizer, as one of the major inputs in the production of farm crops, has increased in importance both absolutely and relative to other inputs in recent years. In 1964 fertilizer expenditures constituted from 6 to 21 percent of the total cost of crop production[1] depending on crops. As a percent of total inputs on commercial farms, fertilizer consumption doubled in the U.S. during the period 1947-49 to 1960-62.

During the year ended June 30, 1964, fertilizers consumed in the continental United States contained 10,351,814 tons of primary plant nutrients (nitrogen, available phosphate and potash). All three increased relative to 1963: nitrogen, 12.2 percent; available phosphate, 9.5 percent; and potash, 9.0 percent. But these increases were not evenly distributed geographically. The East North Central Region, the West North Central Region and the West South Central Region increased the consumption of nitrogen (N) by 25.8 percent, 12.7 percent and 9.5 percent respectively, while the New England Region consumed only 5.5 percent more than in 1963. Likewise, the consumption of available phosphate (referred to as P_2O_5 or phosphoric acid) increased most in the East and West North Central Regions (14.1 percent and 12.3 percent), but

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[1]U.S.D A., Economic Research Service, Farm Costs and Returns; Commercial Farms by Type, Size and Location, Agriculture Information Bulletin No. 230 (June 1964). Crop expense includes 1) fertilizer and lime, 2) other crop expenditures, 3) machinery, 4) one half or all the labor depending on the amount of livestock and 5) taxes.

increased only slightly in the South Atlantic and East South Central Regions (.4 percent and 1.3 percent). Changes in potash (K_2O) consumption ranged from a 2.3 percent decline in the Mountain Region to a 20.2 percent increase in the West North Central Region.

During the 1950-64 period the consumption of the three nutrients expanded at varying rates. While the 1964 consumption of nitrogen was 454 percent of the 1950 consumption, phosphate and potash were 173 and 250 percent of their 1950 levels. Regional expansion varies even more: primary nutrient consumption increased .8 percent in New England and 564 percent in the West North Central Region between 1950 and 1964.

Changes in consumption of this magnitude suggest changes in related variables. Nitrogen price fell 10 percent; potash price fell 6 percent; and phosphate price rose 20 percent. Comparison of these fertilizer prices with the 12 percent increase in the Index of Prices Paid for Production Items (including fertilizer) suggests fertilizer prices, on the average, decreased relative to other production items. Part of this change is explained by use of a more highly concentrated fertilizer. Between 1950 and 1964 the average nutrient content of fertilizer increased from 22.0 percent to 34.2 percent. The need to understand the reasons for these changes prompts this particular study.

The factors effecting this rapid change in the consumption pattern can be categorized as economic, technological and educational. These categories are not mutually exclusive and all will be considered. But economic variables will be emphasized in this study. The following types of questions are of prime interest:

1. What is the relationship between nutrient price changes and consumption of each fertilizer nutrient in each state and in the aggregate?

2. What is the relationship between crop or product price changes and each fertilizer nutrient consumption?

3. What conditional projections can be made of future consumption?

Objectives

It is within the framework of the above questions that this study is formulated.

The primary objectives of this study are to identify, describe, quantify and analyze the factors affecting the demand for commercial fertilizer.

More specific objectives include:

1. To develop price indexes for nitrogen and available phosphate necessary in the models of each nutrient.

2. To estimate parameters of economic models describing the market relationships for each nutrient.

3. To forecast consumption of fertilizer under specified prices.

4. To compare the results of this study with those of earlier studies.

Method

This study begins by reviewing previous studies and by examining the economic relationships impinging on farmers' decisions regarding fertilizer use. This leads to the development of appropriate economic and statistical models. The lack of adequate data on nutrient prices by states led to the development of a series of indexes of nutrient prices. Separate economic models are estimated for each of the three primary nutrients; all regress the quantity of nutrient per acre on various economic variables, dummy variables for each state, and various time variables. All models combine cross-section and time-series data: data from 48 states and the 15 year period 1950 through 1964 combine to give 720 observations. These covariance models, by considering the wide variation in resources, technology and environment among states, obtain estimates relevant to the individual states. The covariance models are estimated by ordinary least squares.

Review of Literature

Nonfarm agriculture inputs all have generally been neglected in economic research. But fertilizer is somewhat an exception. As early as 1927, E. E. Vail[2] related U.S. fertilizer consumption to various factors and found that the lagged value of cotton per acre and tobacco per acre contributed most to an explanation of the variations of fertilizer consumption.

A. L. Mehring and B. T. Shaw[3] published a model in 1944 which implied that farmers spend a constant proportion of their income on fertilizers. Their model, using the period 1911 through 1943, explained 93 percent of the total variation for that period.[4] But since 1950 the estimates fall considerably below the actual consumption.

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[2] E. E. Vail, "Prices of Fertilizer Materials and Factors Affecting the Fertilizer Tonnage" (Unpublished Ph.D. thesis, Cornell University, Ithaca, N.Y., 1927).

[3] A. L. Mehring and B. T. Shaw, "Relationship Between Farm Income and Farmer's Expenditure for Fertilizer and a Forecast of the Commercial Demand for Fertilizer in 1944 and 1945, by States", American Fertilizer, (c. 1944).

[4] Equate explained variation with R^2 where:

$$R^2 = SSR/SST$$

$$SSR = \text{regression sum of squares} = \Sigma(\hat{Y} - \bar{Y})^2$$

$$SST = \text{total sum of squares} = \Sigma(Y - \bar{Y})^2$$

During the late 1950's Griliches undertook an extensive fertilizer research project testing the hypothesis that the decline in the "real" price of fertilizer largely explains the great increase in consumption of fertilizer in the U.S. He developed several models: one cross-sectional and several time-series. In all, price of nutrient is hypothesized to be the most important variable.

The first model[5], which is an equation linear in logarithms of the variables, argues that fertilizer use is a function of the "real" price of fertilizer, the price paid for fertilizer relative to the prices received for farm crops, and an adjustment factor proportional to the amount of "disequilibrium". The amount of disequilibrium is defined as the difference between the logarithm of actual and desired level of use. He fitted this model to national data for the years 1911-56. Then he divided this period into two sub-periods, 1911-33 and 1934-56, to test for changes in coefficients over time. Since the explanation fits both periods equally well Griliches concluded that the tremendous increase in fertilizer consumption can be interpreted as a movement along a given production function in response to changing relative prices. He argues that the technical change has occurred not in agriculture but in the fertilizer industry.

As a further test of the hypothesis that fertilizer use can largely be explained by the decline in the "real" price of fertilizer

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[5] Zvi Griliches, "The Demand for Fertilizer: An Economic Interpretation of a Technical Change", Journal of Farm Economics, Vol. XL, No. 3, (Aug. 1958), pp. 591-606.

Griliches[6] utilized the same variables but disaggregated to the nine census regions. Rather than simply summing the nitrogen, phosphoric acid and potash content of the fertilizers as in the first model, the various nutrients were weighted by their relative prices before aggregating. He fitted this revised model to regional data for the years 1931-56 utilizing two sets of price data. Griliches found some regional differences: (1) the regions with historically more fertilizer experience adjust faster to changes in price than those with less, and (2) the demand for fertilizer is more price elastic, in the long run, in regions with low fertilizer use.

As a test of the time series models Griliches[7] developed a cross-sectional model using data from the 1954 Census of Agriculture. Again fertilizer is viewed as a function of fertilizer price relative to prices received. But in addition, the price of fertilizer relative to labor, fertilizer price relative to land and the average percent content of nitrogen in the soil contribute to the explanation. The form of the equation is the same as the one used in time series analysis: linear in the logarithms of the variables. The results show labor as a complement and land as a substitute for fertilizer. This model explained between 75 and 90 percent of the interstate variation.

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[6] Zvi Griliches, "Distributed Lags Disaggregation, and Regional Demand Functions for Fertilizer", Journal of Farm Economics, Vol. XII, No. 1, (Feb. 1959), pp. 90-102.

[7] Zvi Griliches, "The Demand for Fertilizer in 1954: An Interstate Study", Journal of the American Statistical Association, Vol. 54, (June 1959), pp. 377-84.

Yeh and Heady[8] (1958) employed numerous algebraic functional forms. But the main ones were linear in logarithms, and fitted to data from 1926-56, omitting 1944-50. Their logarithmic models for total commercial fertilizer consumption, and for consumption of each nutrient included the following independent variables: (1) ratio of current fertilizer price index to the general wholesale price index, (2) average of the crop price index lagged one year relative to the general wholesale price index, (3) all cash receipts from farming lagged one year, (4) cash receipts from crops and government payments lagged one year, (5) total acreage of cropland, (6) time, (7) time squared, and (8) an income fraction, indicating trends in income over the previous three years.

Results from the regional models, which explained more than 90 percent of the variation in fertilizer consumption, show an elasticity of demand with respect to fertilizer price greater in regions which have increased use the most in recent years. Contrast this with Griliches' conclusion that regions with historically more fertilizer experience adjust faster to changes in price than those with less.

With the objective of improving predictive methods and explaining economic relationships, Brake[9] (1959) disaggregated and concentrated his attention on two historically different regions: the East North Central

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[8]E. O. Heady and M. H. Yeh, "National and Regional Demand Functions for Fertilizer", Journal of Farm Economics, Vol. 41, (May 1959), pp. 332-48.

[9]John R. Brake, "Prediction of Fertilizer Consumption in Two Regions of the United States", (Unpublished Ph.D. thesis, North Carolina State College, Raleigh, 1959).

and the South Atlantic. Both are heavy consumers of fertilizer, yet their historical consumption patterns differ considerably. Predictive variables used in the study can be grouped into five general classes: (1) product price, (2) fertilizer price, (3) price of associated inputs, (4) fertilizer acreage, and (5) capital restriction. Data for the years 1930-58 are used in models of three different forms: linear, first differences and distributed lag. To test predictive performance Brake tested the various models for the decade of the '50's. Some of the models demonstrate a high degree of stability with respect to the coefficients over the time period and the recommended models all explained more than 95 percent of the total variation.

Heady and Tweeten[10] (1962) update and expand the study reported by Heady and Yeh. Total fertilizer tonnage and total nutrient quantity were estimated separately for the nutrients N, P_2O_5 and K_2O. Independent variables can be grouped into: (1) fertilizer price, (2) index of price for land, (3) cash receipts, (4) acres of cropland, (5) time, and (6) assets on the farm. Deflation, where used, was by crop prices. Both linear and logarithmic forms were experimented with but only the logarithmic is reported and as in the earlier report by Heady and Yeh, the models yield high R^2's.

Most of the studies reviewed above utilized time series data. All of them encountered serious problems of multicollinearity. Because of this, avoidance of multicollinearity was emphasized in constructing

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[10]E. O. Heady and L. G. Tweeten, Resource Demand and Structure of the Agricultural Industry (Ames, Ia., Iowa State University Press, 1963).

models for this study. More will be said on this in Chapter II.

Organization of the Study

Chapter II specifies and rationalizes the models. Chapter III discusses and develops alternative nutrient price indexes for use in the models. Chapter IV presents and analyzes the results of the models. Chapter V demonstrates the applicability of the models in forecasting. And Chapter VI is a summary with recommendations and conclusions.

CHAPTER II

THE ECONOMIC MODEL

Introduction

Chapter I briefly surveyed the fertilizer market both as it varies from state to state and the changes occurring since 1950. In view of this as well as the earlier studies of fertilizer demand and some understanding of the economic framework within which the farmer makes decisions regarding fertilizer use, this chapter develops the general model used in this study. The formulation considers both economic theory and the characteristics of the fertilizer market within the data restrictions.

Preliminary Structure

The demand for commercial fertilizer is derived from the demand for farm crops. The quantity demanded of fertilizer, as any input, depends on the price of the input, the price of the product and the price of close complements or substitutes in production. Treating the supply of fertilizer equally crudely, we say the supply depends on the price of fertilizer, the prices of inputs (raw chemicals, machinery and labor) and some concept of a production function. To consider possible modifications it will be convenient to represent them symbolically. The following notation will be used:

Q_F^D = quantity of fertilizer demanded

Q_F^S = quantity of fertilizer supplied

P_F = price of fertilizer

S_1 = a set of exogenous variables which affect the quantity of fertilizer demanded and the price of the fertilizer

S_2 = a set of exogenous variables affecting the supply of fertilizer and its price.

The relations discussed above can now be summarized as:

$$Q_F^D \text{ , } P_F \text{ ; } S_1$$
$$Q_F^S \text{ , } P_F \text{ ; } S_2$$
$$Q_F^D = Q_F^S$$

A colon may be read "depends on"; a semicolon may be read "appear in relation with"; and a comma may be read "and". Consider the variables to the left of the colon or semicolon as endogenous.

Before further specifying the model, variables presented as endogenous must be examined to see if, in fact, they are endogenous. Without question, the quantity of fertilizer demanded is determined within the system. But the price of fertilizer may be exogenously determined. And if this is true, then the structure outlined above is altered to:

$$Q_F^D \text{ ; } S_1' \text{ , where } S_1' = S_1 + P_F$$
$$Q_F^S \text{ ; } S_2' \text{ , where } S_2' = S_2 + P_F$$
$$Q_F^D = Q_F^S$$

Whether the price of the fertilizer is considered endogenous or exogenous depends on those relations which generate the data.

The price of fertilizer paid by the farmer consists of charges for the fertilizer itself plus the costs of distribution. Since fertilizer has a low value per unit of weight, the transportation share of the price is often quite large. This is especially true of phosphate and potash which, for the most part, must be shipped from Florida and New Mexico.[11] Markam's[12] estimate of production costs per ton of ordinary super phosphate (20 percent available P_2O_5) for the Midwest in 1950 attributed 63 percent of the farmer's final price to distribution costs, 17 percent to raw material costs and 20 percent to fabrication costs. This estimate of distribution costs is probably too high for the latter part of the period included in this study since the average nutrient content increased from 22 percent in 1950 to 34.2 percent in 1964. But the point remains, distribution costs constitute a large part of the total delivered price of fertilizer. And freight rates controlled by the Interstate Commerce Commission are very stable.

Another characteristic of the phosphate and potash sectors contributing to stable prices is a contracting arrangement between primary producers and their customers (processing and mixing plants). These contracts are signed around the beginning of the fiscal year. They serve, not as rigid legal contracts, but as rough indicators to

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[11] U.S. Dept. of Interior, Bureau of Mines, Minerals Yearbook 1963, p. 913: "Approximately 90 percent of the domestic (potash) production came from mines in the Carlsbad, New Mexico area with California and Utah furnishing the bulk of the remainder." Canadian shipments were also very important. p. 878: "Florida's production of marketable rock (phosphate) amounted to 74 percent of total domestic output: the Western States (Wyoming, Idaho and Utah) accounted for 14 percent and Tennessee, for 12 percent."

[12] Jesse W. Markham, The Fertilizer Industry, Study of an Imperfect Market (Nashville: Vanderbilt University Press, 1958), pp. 150-52. Markham, using a specific example of Florida rock phosphate (less than 3 percent available P_2O_5), estimates the average cost of transportation to equal 75 percent of the delivered price.

the primary producers of next year's consumption and as aids in scheduling production. Although the contracted quantity is not rigidly adhered to, the price normally is. This does not eliminate price variation at retail, but certainly restricts it.

The productive capacity of an industry indirectly measures the pressure on price. Product prices of industries with insufficient capacity tend to rise, while those with excess capacity tend to fall or remain stable. Using the estimated supply of each nutrient available for domestic consumption as an approximation of the industry's capacity,[13] the period since 1950 generally indicates excess capacity as shown in Appendix I. Estimated nitrogen supply (capacity) available for domestic use exceeded consumption in all but one year since 1950. During this same period the estimated domestic phosphate supply (capacity) fell short of consumption three times. Estimated potash supplies generally deviate less from actual consumption than either nitrogen or phosphate and estimated deficits occur more frequently. Five times in 15 years the actual consumption exceeded the estimated domestic supply (capacity) but, as in phosphate, periods of excess capacity (estimated supply greater than consumption) preceded the periods of shortage.

Further evidence to suggest excess capacity in the fertilizer industry is suggested in a recent study by the Federal Reserve Bank of Chicago.[14] This study found that "....since the end of World War II....

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[13]U.S.D.A., Agricultural Stabilization and Conservation Service, The Fertilizer Situation (annual issues, 1950-64).

[14]Federal Reserve Bank of Chicago, "Commercial Fertilizer and Agricultural Production", Business Conditions (Sept. 1965), pp. 7-12.

the margin of unused capacity has increased substantially."

Closely related to the estimated supply (capacity) is the size of inventory. Stocks held by producers on December 31, 1962, represented 12 percent of the total K_2O sales for the year ended June 30, 1963; 47 percent of the available P_2O_5 total; and 31 percent, 6 percent and 34 percent of anhydrous ammonia, ammonium nitrate and ammonium sulfate totals respectively.[15] Data available at both the producer and retail levels for these nitrogen commodities suggest fertilizer inventory estimates could easily double if both retail and producer inventories were included. Generalization from these limited nitrogen data suggests that stocks available January 1 of the fiscal year are in some cases greater than the sales in the remaining 6 months of the fertilizer year. This, of course, varies from year to year and for the different fertilizer materials, but does indicate a large inventory and limited pressure on prices.

All the above arguments for stable prices neglect the effect of retail level margins. Observation of historical data reveals that fertilizer prices at retail vary only slightly within a fertilizer year (the fiscal year)[16] thus confirming the arguments for stable prices within the year.

No one of these arguments conclusively shows that prices of the individual nutrients are exogenously determined. Nitrogen

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[15] U.S. Dept. of Interior, op. cit., p. 884, p. 917.
U.S.D.A., op. cit., Fertilizer Situation (March 1964), pp. 10-11.

[16] U.S.D.A., Statistical Reporting Service, Agricultural Prices (April and Sept. issues).

capacity exceeded consumption requirements in the past, and current expansion rates suggest this pattern will continue. Phosphate and potash prices include substantial transportation charges in addition to the contracted prices which contribute to stable prices; and all the reported nutrient prices are stable during the fertilizer year. For these reasons the nutrient prices originally regarded as endogenous are now considered predetermined. This revision implies that the study can be focused on the demand equation of the form:

$$Q_F^D \, ; \, S_1' \, , \text{ where } S_1' = S_1 + P_F$$

The variables in S_1' are considered as exogenous or predetermined and will be discussed in the next section.

Variables of the General Demand Equation

To find which variables belong in S_1, the set of exogenous variables which affect the quantity of fertilizer demanded, we appeal to economic theory. The theory specifies that the quantity of an input demanded depends upon the price of the factor, price of the product, price of close substitutes and complements and the marginal physical product of the input. This set of variables divides into variables which refer to movements along the demand curve and those which shift the demand.

The variable related to movements along the demand curve is the nutrient price. The U.S. Department of Agriculture publishes[17] a general U.S. index of fertilizer price and state level data of specific forms of N, P_2O_5 and K_2O as well as mixtures of the three,

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[17] _Ibid._

but no price indexes for the individual nutrients. These state price data and state quantity data[18] provide the raw materials for the construction of state price indexes for each nutrient. Because of problems encountered in constructing the indexes and because it is really a major but subordinate study, the procedure and results are presented in Chapter III.

Several factors cause shifts in the demand curve of an input. Chief among these shifters is the price of the product. As the price of the product changes the MVP of the nutrient changes proportionately and the quantity of input increases or decreases depending on the direction of the initial change. Selection of the most appropriate crop price is based on data published in the Census of Agriculture.[19] From the census it is possible to assess the relative importance of each crop (in every state) as a consumer of fertilizer. Prices of crops consuming the most fertilizer become "the" product prices for the purposes of analysis. With few exceptions states consume most of the fertilizer on one or two crops. In states where similar amounts of fertilizer are consumed on two crops both crops are included in the models. The exceptions to this criterion are states applying a major share of the fertilizer on fruit and vegetable crops. Since neither fruit nor vegetable price indexes are available, a field crop price is used even though the crop is not the most

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[18]W. Scholl, et.al., Consumption of Commercial Fertilizers and Primary Plant Nutrients in the U.S., U.S.D.A., Soil and Water Conservation Research Division, Agricultural Research Service (Annual Issues).

[19]U.S. Dept. of Commerce, Bureau of Census, 1959 Census of Agriculture, Vol. IV.

important consumer of fertilizer.

The question of which year's crop price is relevant to the model remains open. A study of farmers' attitudes toward the use of fertilizer by the National Plant Food Institute[20] provides some insight into the farmer's decision-making process. In response to the question "When did you first start thinking about using (analysis) on your 1956 (selected crop)?" slightly more than half of them named some month earlier than August 1, 1955. And of those farmers generally using rates close to the recommended level, 64 percent reported thinking about their present analysis prior to August 1, 1955. This, along with the fact that over 70 percent of the total fertilizer purchases occur in the January-June period implies that the crop prices during the decision period are the most suitable price for the models. Therefore, the average annual price for the preceding crop is used in the models.

A second shifter of the demand curve is changes in price of closely related substitutes and complements. If the related good is a substitute, then an increase in its price causes an increase in the consumption of fertilizer. Conversely, an increase in the price of a complement causes a decrease in the consumption of fertilizer. Exclusion of other inputs, viz., machinery, labor and land, does not imply independence between fertilizer and these inputs, but, rather, implies that the relationship is not close. Exclusion of these related inputs as independent variables assumes that farmers respond little to marginal price movements. Prices of the related inputs are not

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[20]National Plant Food Institute, A Study of Farmers' Attitudes Toward the Use of Fertilizer (Washington, D.C., 1958), p. 53, p. 14 of Appendix A. The survey was taken in 1956.

completely ignored in the models, however, because deflation, which is discussed later in the chapter, by the Index of Prices Paid for Production Items considers relative prices.

A second reason for excluding the prices of other inputs is the lack of state data. Use of available national data would entail assuming that the same prices hold for all states. And the estimates of fertilizer nutrient price indexes for states developed in Chapter III clearly suggest this assumption is questionable. For example, the 1964 index of nitrogen price in New Jersey and California equal 108.6 and 70.8 respectively (with 1950 = 100.0 in both cases). If prices of related inputs have changed in the same manner, then assuming equal prices for all states introduces a specification error. But, at the same time, omitting the variables introduces a specification error. Therefore, a choice must be made between the specification error of omitting the variables and thus attributing their effects to temporal variation; and the specification error of including them (all states equal), thus getting spurious results across states. In all models the prices of related variables are excluded.

The model, as it is specified, ignores the complementarity between the three nutrients. The study referred to earlier by the National Plant Food Institute found that farmers who purchased fertilizer decide on the analysis and amount using trial and error slightly more frequently than by accepting recommendations of soil tests. But farmers classified as those "using fertilizer close to the standards for most economical operation for his type of crop and soil as compared to the best practice of agronomy" decide more frequently on the basis of soil tests than trial and error.[21] But in

[21] _Ibid._, pp. 52-60.

either case, relative nutrient prices have not played an important role in the decision and for this reason are not included in the model.

Another shifter of the demand function for a factor of production is the expenditure restriction (cost constraint). This is analogous to the budget constraint of consumer demand theory. It is assumed that for any given expenditure for factors of production farmers intend to maximize profit. Those farmers who have no expenditure restriction purchase inputs until the last unit of factor purchased is worth in production just what it cost - and when each factor is purchased at this rate, then profit is maximized. Employing any more units of the factor will add less value to the output than it costs to produce. But there is another group of farmers - those with an expenditure restriction - who are unable or unwilling to purchase inputs to this point. In the National Plant Food Institute study[22] 28 percent of the farmers interviewed said they would not borrow money for fertilizer purchases (when available at a reasonable rate of interest). Twelve percent said they probably would not. This means that for about forty percent of the farmers their expenditure restriction (for fertilizer) is their income and other assets.

Secondly, there is a group of farmers who are unable to utilize credit to buy fertilizer. For this group their fertilizer expenditure restriction is also conditioned by their income and asset position.

A third group is made up of those farmers who do use credit to buy fertilizer (in the study referred to above 49 percent of the purchasers utilized credit). But, of course, the price and amount of the credit available is conditioned by their income and asset position.

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[22]Ibid., p. 117.

To bring the expenditure restriction into the model, experiments were conducted using alternately average gross farm income and average net farm income (both from the previous year) among the independent variables. The two variables contribute almost equally to explaining variation in fertilizer use, but the ratio of the estimated coefficient to its standard error was greater for the net income variable. Because of this, and the intuitive appeal of net income as the expenditure restriction, only net farm income is considered in the models presented in Chapter IV. The net farm income of the previous year serves as a proxy variable for measuring expenditure restrictions: as a rough measure of the ability to pay cash and the availability of credit. Inclusions of more than one year's income, weighted either arbitrarily or by the equation, would refine the measure but this added refinement is not considered in the study.

The last variable to be considered as a demand shifter is technological change. We can ignore the technical progress in the production of fertilizer because it is reflected in the price of the nutrients. But significant technological developments in the production of farm crops need consideration in the model. Since 1950 hybrid plants have improved; fertilizer placement is better; and machinery and other factors have improved. But for none of these are good measurements available. The use of time as representative of these changes may properly cause some concern. These specific factors mentioned are part of a set of influences whose net effect during the period of observation has fairly steadily increased the efficiency of converting inputs into crops. Another factor, not included in the set of technological changes, which has been changing steadily over the period is the knowledge or awareness farmers have about the effects of fertilizer. For example, the percentage of total corn acres fertilized in the U.S. increased from 60 percent in 1954 to 63.7 percent in 1959. If this five-year increase is assumed representative of the period studied, then there were about 11 percent more corn acres fertilized in 1964 than in 1950. Extrapolating the percent of total crops and pasture fertilized from the same period results in an estimated 5.1 percent increase between 1950 and 1964 of the total crops and pastures fertilized. Concomitantly, awareness of the effects of higher rates of application increased the quantity used per acre.

Because there is no way of specifying this set of factors - both the technological changes and farmers' awareness - several alternative forms of the time variable are explored. Various forms include the exclusion of time, time as a linear variable, two quadratic forms of time, and time as a set of 0,1 dummy variables. The linear form of time takes the values 1 through 15 with 1950 equal to 1. The first quadratic form is a set of 2 variables, one linear and the second, the square of the linear. The second quadratic form of time is a squared term and takes the values 1 through 225 with 1950 equal 1, 1951 equal 4, etc. And the set of dummy variables, a variable for each of the 14 years 1951 through 1964, take the value 0 or 1 depending on the year being observed. The mechanics of this set will be discussed later in the chapter with the actual model.

The variables discussed above - the price of the nutrient, price of the product (crop price) and the average net farm income - are the economic variables included in the models. Time, in several forms, is also included as a measure of technological changes and the awareness of fertilizer response by farmers. Just how these variables are included in the models is considered in the next section.

Specification Considerations

Fertilizer has been considered in a very general way until now. The commodity, fertilizer, consists of primary plant nutrients, secondary plant nutrients, trace elements and a carrier which is often limestone or an inert material. For different fertilizer materials, the concentration of nutrients varies considerably: from less than 2 percent available phosphate in rock phosphate to over 80 percent elemental nitrogen in anhydrous ammonia. Simply considering fertilizer as a whole would mean treating a ton of fertilizer with a high concentration of nutrients equal to one with a low concentration. It also would mean ignoring the increase in concentration from 22 percent in 1950 to more than 34 percent in 1964: an increase in the real quantity of fertilizer. While the secondary elements (calcium, magnesium and sulfur) and the trace elements (copper, zinc, boron, manganese and iron) have increased in importance since 1950, they accounted for less than 5 percent of the total fertilizer tonnage consumed in the fiscal year 1964. When considered on a nutrient basis this 5 percent reduces considerably since the concentration of secondary and trace elements is much less than of primary elements. While the secondary nutrients are becoming more important as the technical understanding of their function increases and as an increasing number of soils respond, they are not considered in this study. The primary nutrients (nitrogen, N; available phosphate, P_2O_5; and potash, K_2O) are the concern of this study.

The description of the diverse consumption patterns of the different nutrients in the introductory chapter suggests the estimation of the three primary nutrients separately. Disaggregation of the three increases both the possibility of good explanation and the relevance of the individual estimates. The individual estimates can later be merged to yield a more aggregate description of the market.

Because of the diversity of agriculture in the U.S. and the ways in which this diversity affects the consumption of fertilizer, the model developed in this chapter must be flexible enough to take this into consideration. One of the objectives is to estimate the parameters which describe the fertilizer consumption of each state. The covariance model, using both time-series and cross-section data offers the most potential.[23] This model, in addition to estimating the economic parameters mentioned earlier, is capable of estimating a parameter for each state: a constant term. These parameters are viewed as measuring the effect of unobservable characteristics that are peculiar to a state over time such as climate, soils and types of cropping. And statistical tests can be performed to determine the significance of estimating a constant for each state. The cross-section and time-series model also provides the opportunity to estimate parameters common to a set of states, but not the entire country.

The characteristics of the data designate states as the measurement unit for the study. Consumption, price and income data are available or can be constructed for each state. But because states

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[23]The models are a variation of those suggested by C. Hildreth, Preliminary Considerations Regarding Time Series and/or Cross-Section Studies, Cowles Commission Discussion Paper: Statistics No. 333 (July 1949), and C. Hildreth, Combining Cross-Section Data and Time Series, Cowles Commission Paper: Statistics No. 347 (May 1950).

vary markedly in size and rate of application the consumption (quantity) data are transformed to pounds of nutrient per harvested acre (harvested acreage of 59 major crops). Another argument for the covariance model is larger sample size - in this study 48 states and 15 years giving 720 observations - which reduces sampling errors and makes tests of significance more powerful.

This study examines the period 1950 through 1964. Earlier periods were excluded because (1) generation of nutrient price data prior to 1950 presents serious difficulties, (2) the World War II and immediate post-war periods were adjustment periods, and (3) the combined time-series and cross-section model permits relatively shorter time periods than the time series alone.

The combined time-series and cross-section model of this study utilizes more information and at the same time provides a means of avoiding multicollinearity. Earlier studies of the demand for fertilizer, except Griliches' one cross-sectional study, all used time series and all confronted multicollinearity problems. Johnston[24] and Theil[25] point out that the solution to problems of multicollinearity lies in the acquisition of new data which will break the multicollinearity deadlock. Because time-series data are, in general, multicollinear, combined cross-section and time-series offers an alternative procedure.

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[24] J. Johnston, Econometric Methods (New York: McGraw-Hill Book Co., 1963), p. 207.

[25] H. Theil, Economic Forecasts and Policy (Amsterdam: North Holland Publishing Co., 1962), p. 217.

Chipman[26] argues that where the matrix of cross-section and the matrix of time-series data are complementary the combined cross-section and time-series model is the natural procedure to employ to avoid multicollinearity.

Even where multicollinearity exists, unbiased forecasts can be made. But successful forecasts with multicollinear variables require (1) perpetuation of a stable dependency relationship between the dependent and independent variables and (2) the perpetuation of a stable interdependency relationship within the set of explanatory variables.

Where the intent of the model is explanation, the multicollinearity reveals itself by increasing variance of the parameter estimates. Arbitrary rules established as a criteria for defining unacceptable collinearity frequently break down in practice. The most common rules of thumb constrain simple correlation (r) to less than .9 and often to less than .6. This is often qualified by arguing that intercorrelation is harmful only if the simple correlation between explanatory variables exceeds the multiple correlation coefficient ($r_{ij} \geq R$). Farrar and Glauber refute both these rules of thumb with, "complete multicollinearity - i.e., perfect singularity - with a set of explanatory variables is quite consistent with very small simple correlations between members of X (the set of explanatory variables). A set of dummy variables whose non-zero elements accidentally exhaust the sample space

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[26]John S. Chipman, "On Least Squares With Insufficient Observations", Journal of the American Statistical Association, Vol. 59, No. 38, (December 1964), pp. 1078-1112, especially 1100-1102.

is an obvious and aggrevatingly common example."[27] Combining time-series and cross-section data eliminates most of the pairwise inter-correlation between economic variables in the set of explanatory variables of this study. To measure other than pairwise collinearity one can use the determinant of the X'X matrix. Where this determinant is based on a normalized matrix it takes a value between zero and one. Values approaching zero warn of a singular matrix and a value of one would mean an orthogonal set of independent variables.

But this is not a very good measure. True, it warns when multicollinearity exists, but it provides no meaningful measure of the problem. It ignores the fact that the determinant tends toward zero as the size of the matrix increases and it fails to recognize that collinearity between some variables is more serious than between others. And it cannot identify which variables are multicollinear. An alternative measure is the multiple correlation coefficient, R^2, where each "important" explanatory variable is regressed on the remaining variables of the X matrix. This is not a regression in the usual sense because it immediately violates the assumption of X being a fixed set, but is merely an exploitation of R^2's property: percent of the variance of one variable explained by a set of variables. This measure of multicollinearity will be presented in Chapter IV with the results of the models.[28]

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[27] D. E. Farrar and R. R. Gauber, Multicollinearity in Regression Analysis: The Problem Revisited (Sloan School of Management, Mass. Institute of Technology, Cambridge, 1964), Vol. 105-64, p. 23.

[28] I am indebted to William Ruble for discussions of the problems of measuring multicollinearity.

Another consideration in specifying the model is whether to use nominal or real prices and incomes. To use nominal prices and incomes implicitly assumes either (1) uniform price changes for the economic variables or (2) that farmers respond to nominal prices and not real prices. Tests of uniform price changes reveal that while the Index of Prices Paid for Production Items increased 31 index points from 1949 to 1964 the Index of Prices Paid by Farmers increased 58.6 index points, or nearly double. Because of this difference and the fact that economic theory suggests consumers and producers react to real prices, the economic variables in the models are deflated. The question of the appropriate deflator is confronted by asking the question - what deflator will transform the variable to real terms? The price of the nutrient is deflated by the Index of Prices Paid for Production Items. That is, "real" is here considered to be how much does the fertilizer nutrient cost relative to the alternative inputs; or how much of the other inputs will a unit of fertilizer buy? The crop price was deflated by the same index because it is assumed that farmers consider the relative prices of the product and inputs when deciding the quantity of inputs to buy. Since both consumption and production have a claim on net farm income, the expenditure restriction, the Index of Prices Paid by Farmers is used as the deflator. In all the deflation the actual deflator is the arithmetic average of the February, March and April indexes.[29]

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[29]Actual deflators listed in Appendix II.

The General Model

The nitrogen, phosphate and potash models all follow the same general format. All consider the same economic variables, state constants and forms of time. The differences are in the way states are combined in the nutrient price variables and in the crop price variables. Here, a general model is presented to aid discussion.

To consider the variables and characteristics of specification described above more fully and to consider possible modifications it will be convenient to expand the notation used early in the chapter and represent the relations symbolically. The set of exogenous variables (S_1') affecting the quantity of fertilizer demanded includes:

$P^F_{t,s}$ = index price of the nutrient in t and s

where t refers to the years 1950-1964
s refers to the 48 continental states
F indexes the nutrient in the model
F is designated as: N, nitrogen; P, available P_2O_5; and K, potash

$P^c_{t,s}$ = price of crop (product) in t and s

where c indexes the particular crop and region: crops are either corn, cotton, wheat, hay or potatoes (Some crops, say corn, are the most important consumers of fertilizer in many states but because of the environmental differences between states, the response is not expected to be equal and therefore, the model is specified with two or more corn price variables: each for a different set of states.)

$Y_{t,s}$ = average net farm income in t and s

$T_{t,*}$ = time variables for t

where * means constant for all states

$S_{*,s}$ = dummy variables (0,1) for states

where * means constant for all years

The endogenous variable is:

$Q^F_{t,s}$ = quantity of fertilizer nutrient consumed per harvested acre in year t and state s

The discussion can now be summarized in the following relations:

$$Q^F_{t,s} = \alpha + \sum^{g} \beta_g P^F_{t,s} + \beta Y_{t-1,s} + \sum^{k} \beta_k P^c_{t-1,s} + \sum^{i} \beta_i S_{*,s}$$

$$+ \sum^{j} \beta_j T_{t,*} + U_{t,s}$$

where g = 6 different nutrient price variables

k = 14 different crop price variables

i = 1...47, state constant terms

j = 1...14, time constant terms

The variables of the equation can best be examined in sets since all but the net income variable (Y_{t-1}) are some form of synthetic variable. First consider the set of variables $\sum^{g} \beta_g P_{t,s}$, the six nutrient price variables. Each of these six variables is used over a region. For example, in the nitrogen model the first of the six nitrogen price variables $\beta_{83} P^N_{t,s}$ pertains to the states Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New York, New Jersey and Pennsylvania. The estimated parameter, $\hat{\beta}_{83}$, is relevant only to these states. A description of the way the variable is included may help to clarify its meaning. Each observation of the dependent variable is associated with a fixed set of independent variables - six of these independent variables are nutrient price variables. Let the observation on Q be the 6th year, 1955, of the period and the state of Maine. Then the value of the first of the six nutrient price variables ($P^N_{1955,\ Maine}$) will be the 1955 nitrogen price in Maine. The value of the second of the six nutrient price variables will be zero (non-zero values will be entered in the second variable when the Q^N of each state in the second region of nitrogen price is being observed). The remaining four variables will also be zero.

The second set of variables in the equation is lagged net farm income ($\beta Y_{t-1,s}$) and for each observed value of Q^N there is an associated non-zero value of Y_{t-1}.

The third set of 14 (13 for K_2O) variables ($\overset{k}{\Sigma} \beta_k P^c_{t-1,s}$) is synthetic variables constructed much like the set of nutrient price variables. They have the value zero or the actual value of the variable. Let us extend the example used for the nutrient price variables. The 1959 census shows that Maine applied 77 percent of the total nitrogen consumed on potatoes and 14 percent on hay. Using the criterion established earlier, potato price is "the" product price in Maine. Let variable 10 be the price of potatoes in Maine and Rhode Island. In the example, where the observation is 1955 of Maine, the first independent variable is the price of nitrogen in Maine 1955, variables 84 through 88 are zero, variable 67 is the 1954 net farm income in Maine (because income is a variable lagged one year), variables 8 and 9 are zero and variable 10 ($\beta_{10}\ P^{potatoes}_{1954,Maine}$) is the 1954 price of potatoes.

The fourth and fifth sets of variables are synthetic variables in the more common zero-one form. The fourth set ($\overset{i}{\Sigma} \beta_i\ S_{*,s}$) consists of 47 zero-one dummy variables. These variables affect only the level of the line. To avoid a singular matrix one state, Michigan, is included in the overall constant term (α) for the equation. To further extend the example considered above the variable for Maine (X_{18}) will take the value one when the state's observations are read and zero for all other observations. The other variables, X_{19} through X_{64}, will take the value zero when Maine's observations are read.

The fifth set of variables ($\sum^{j} \beta_j T_{t,*}$) is zero-one variables for the years 1951 through 1964. Looking at the year 1955 (X_{72}) again, this variable will take the value 1 when the observations for 1955 are read and zero for all other years.

This example using the year 1955 and the state of Maine demonstrates the flexibility of the covariance model with time-series and cross-section data. Other models which differ from the model above only in the form of the time variable are presented in Appendix VI.

Estimation by ordinary least squares of the above model produces best linear unbiased estimates if the following assumptions are satisfied.

1. $E(U_{t,s}) = 0$, where E means expected value
2. $E(U_{t,s}\ U'_{t,s}) = \sigma^2 I_n$
3. X is a set of fixed numbers
4. X has rank $\geq$ number of parameters

The general linear model assumes first, the expected value of the error term equals zero (1). This is not a crucial assumption; for if $E(U_{t,s}) = k$, where $K \neq 0$, then the estimate of the overall constant term (α) will have a bias of k. The estimated coefficients of the independent variables remain unaffected by this bias.

Secondly, the model assumes equal variance for all observations and no serial correlation of the error terms; that is, $E(U_{t,s}\ U'_{t,s}) = \sigma^2 I_n$. This assumption of serial independence is crucial to the general linear model. Although violations of it still yield unbiased estimates of α and β_i, application of the usual least-squares formulas for the sampling variances of the regression coefficients is likely to seriously underestimate these variances.[30] Violation of this assumption also invalidates the t and F tests as they are normally used in the regression model. A third consequence of violating this assumption is predictions with large variances.

To test for interdependence of the error terms the Durbin-Watson[31] d statistic is offered, but the meaning of the statistic will be discussed in Chapter IV.

The third assumption requires that in repeated sampling, the sole source of variation in the dependent variable is variation in the error term. Or, to rephrase it, violation of the assumption that X is a set of fixed numbers means dependence exists between the error term and the explanatory variables. If the assumption does not hold, application of ordinary least squares yields biased and inconsistent estimates of the parameters α and β_i.

The fourth assumption specifies that the rank of the matrix of explanatory variables be at least as great as the number of parameters: a mathematical property required for a solution.

The general model outlined above will be more precisely specified in Chapter IV where the resulting estimates are discussed. The notation developed in Chapter II pertains to Chapters IV and V as well.

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[30] Johnson, op. cit., p. 179.

[31] J. Durbin and G. S. Watson, "Testing for Serial Correlation in Least Squares Regression, I and II", Biometrika, Vol. 37, (1950), pp. 409-28 and Vol. 38, (1951), pp. 159-77. An alternative test modifying the d statistic has been developed by H. Theil and A. L. Nagar, "Testing the Independence of Regression Disturbances", Journal of the American Statistical Association, Vol. 56, (1961), pp. 793-806.

But since Chapter III is a subordinate study developing nutrient price indexes for the models outlined above, no attempt is made to avoid some duplicate notation.

CHAPTER III

INDEXES OF PRICES PAID FOR FERTILIZER NUTRIENTS

Introduction

The models developed in Chapter II call for state prices of fertilizer nutrients. To use the currently available national aggregate fertilizer price index as the individual state price implicitly assumes that state price patterns closely resemble the national pattern. This chapter (1) examines the current fertilizer price index, (2) discusses alternative construction from both abstract and empirical viewpoints and (3) presents estimates of alternative fertilizer nutrient price indexes for each state.

The chapter is divided into two parts. Part one discusses alternative weights available for fertilizer indexes and presents reasons for preferring one of several alternative formulas (constructions) to another. Part two examines estimates of five indexes of phosphate price and nine of nitrogen price in light of the considerations of part one and the data restrictions.

Part I

A brief description of the Department of Agriculture's fertilizer price index will facilitate comparisons later in the chapter. Since its development, the index has used a Laspeyres type of construction linked back to the base period 1910-14. Two weight revisions have been made; the first changing the weights from 1924-29 to 1937-41 and the second revising the weights from 1937-41 to 1955.[32]

As an alternative to this Laspeyres construction, consider a Paasche, a Fisher Ideal, a chain and a chain of Fisher Ideals. Which of these five, or variations of these, best suits economic studies of fertilizer? For investigations into fertilizer demand or resource allocation those indexes using current year quantities, that is, Paasche, Fisher Ideal, and the chain of Fisher Ideals, will probably be ruled

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[32]The following formulas were used for computation for the period 1910-35 (1910-14 = 100):

$$\frac{\Sigma P_i Q_{'24-'29}}{(1/5) \sum\limits_{j=1910}^{1914} \Sigma P_j Q_{'24-'29}} \cdot 100$$

where: PQ extends over all items in the index
i (or j) designates the date

for the period March 1935 to September 1952:

$$\frac{\Sigma P_{m35} Q_{'24-'29}}{(1/5) \sum\limits_{j=1910}^{1914} \Sigma P_j Q_{'24-'29}} \cdot \frac{\Sigma P_i Q_{'37-'41}}{\Sigma P_{m35} Q_{'37-'41}} \cdot 100$$

where: P_{m35} = price in March 1935

$$\frac{\Sigma P_{m35} Q_{'24-'29}}{(1/5) \sum\limits_{j=1910}^{1914} \Sigma P_j Q_{'24-'29}} \cdot \frac{\Sigma P_{s52} Q_{'37-'41}}{\Sigma P_{m35} Q_{'37-'41}} \cdot \frac{\Sigma P_i Q_{'55}}{\Sigma P_{s52} Q_{'55}} \cdot 100$$

where: s52 = September 1952
"Q's" = total quantity weights

out because these constructions will not yield the current year observation.[33] For this reason, the paper considers their theoretical characteristics very little, but the resultant series are included so that comparisons can be made with the Laspeyres and the chain indexes. Since the chain index suffers no data restrictions it will be considered as the most likely alternative to the Laspeyres.[34]

Before evaluating arguments for various constructions the question of weighting will be considered in terms of the purpose of the index. For economic analysis researchers need the price per unit of input as it is used in production. For fertilizer this is the price per unit of nutrient applied to the soil. Fertilizer price data are

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[33] U.S.D.A., "Agricultural Prices and Parity", Major Statistical Series of the U.S. Department of Agriculture, Vol. I, Ag. Handbook No. 118 (August 1957), p. 39. "However, the use of a formula such as Fisher's Ideal...is impracticable, since data as to the quantities of goods purchased for use as weights, are available only at considerable intervals, and never on a 'given-year basis' in time to use for current calculations."

[34] The construction of the chain index is:

$$D_o = \frac{\overset{k}{\Sigma} P_o Q_o}{\overset{k}{\Sigma} P_o Q_o} = 1$$

$$D_1 = (D_o) \frac{\overset{k}{\Sigma} P_1 Q_o}{\overset{k}{\Sigma} P_o Q_o}$$

$$D_2 = (D_1) \frac{\overset{k}{\Sigma} P_2 Q_1}{\overset{k}{\Sigma} P_1 Q_1}$$

$$\vdots$$

now collected from dealers in such a way that costs of application are not likely to be included in the price reported. Hence, this is not the "best" price, but because application charges are based on acreage and application rates per acre vary, the basic data can only be improved with considerable increase in data collection cost. But increased information can be reaped from the present data. In the current index all fertilizers are weighted equally: a ton of 16-0-0 equals a ton of 80-0-0. If various fertilizers are weighted by their respective nutrient content then the resulting index would be a price index of fertilizer nutrients rather than a price index of fertilizer. And assuming the entrepreneur bases his decision on the price per unit of nutrient, the nutrient weighted index would be more appropriate.

The alternative forms or constructions of the index must also be considered. The defense of the presently used Laspeyres construction rests primarily on three arguments. These will be considered in turn.

First, the Laspeyres index is easy to conceptualize; it indicates what a given bundle of goods costs today relative to an earlier day. Mudgett[35] and more recently Sawhill[36] emphasize that easy understandability of a tool of science is a poor criteria unless all other things are equal. Easy understandability is even less relevant as an argument for the index of fertilizer price because it is linked periodically anyway. And once a base-period index is linked (even once) there is no conceptual difference between the base-weighted

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[35]Bruce D. Mudgett, Index Numbers (New York: John Wiley and Sons, Inc., 1951), p. 72.

[36]John C. Sawhill, A Chain Index Versus a Fixed-Base Index for Measuring Changes in the Cost of Living, Paper presented at December 1964 Social Science Meetings.

index and the chain index. In both, a complex multiplicative relationship exists, as compared to a ratio in the Laspeyres index.

A second argument supporting the Laspeyres index is that it's easier to compute. This is unquestionable. And this was undoubtedly of major importance in the early development of the index. But with modern computing facilities the increased cost of computing an alternative form is negligible.

Third, all the major indexes of the Federal government except the import-export index and the construction index are Laspeyre indexes. The validity of this argument is, that, like all institutions, the index should be changed only with good reason.

In a general discussion of the merits of various weighting systems Mudgett[37] points out that weights used in the index must be causally related to prices being used. He argues, "...if quantities are to reflect the importance of P_1 or P_0 they must not go beyond the historical record of the actual quantities which this group of persons purchased at the prices of the market." Mudgett's argument is unassailable on logical grounds and is one of the strongest arguments for the chain index. Closely related to this point is the idea that few price indexes are interested only in prices. If this were the issue then a simple price relative would suffice. The issue more nearly concerns the cost of production (an input in this case) or the cost of living for indexes of consumption goods. The chain index is much more sensitive to changes in either price or quantity than the Laspeyres. However, if a Laspeyres index extends over a long period then an

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[37]Mudgett, op. cit., p. 20.

argument can be made (opposing Mudgett) for weights based on an average or representative period. That is, if for some reason, economic or natural, consumption of the index commodities varies greatly from year to year then to use any one year incorrectly assumes that the single year is representative. The chain index avoids this, first, by using only historically realized P's and Q's, and second, by changing the weights every period. Mudgett's arguments clearly suggest chain indexes on theoretical grounds.

A second point in favor of the chain index is the weighting scheme. Both the Laspeyres and the chain index use weights; but in the chain index revisions are made each period while the Laspeyres index weights are revised only periodically. Changes in the weights of either index bring about changes in the index. And as one would expect, the longer the interval between weight changes, the further the revised weights deviate from their base period and, hence, the greater the index change. While changes in the chain index may be greater than the Laspeyres between revisions of the Laspeyres, these changes in the chain are reflections of the current relative importance of index components. Interpretations of price indexes, therefore, should consider the construction of the index. That is, a fluctuation in the base-weighted index in the year of a weight revision consists of changes in the current year and some part of the accumulated change since the last revision.

Fluctuations in the price index resulting from changes in weights decrease in prominence as the aggregation of goods increases. In an index with many components the fluctuations are damped due to "averaging" while indexes with few components are affected considerably

by the rapid acceptance of a new component. Contrasted with the Laspeyres' the acceptance of a new commodity by consumers will be reflected when it occurs in the chain index. The Department of Agriculture has, in the past, argued that new commodities should be added only after they become well-established.

> At the time new commodities first become available, they are usually more or less in the developmental stage and they are often relatively high in price. As they move into mass production and consumption, their prices tend to decline relative to other prices. Hence, if a commodity were introduced into the index at an early stage in development with a weight representative of its importance after stabilization, a downward bias would thereby be introduced into the index.[38]

This argument assumes the base-weighted index to be the best and then rationalizes a weighting procedure. But look at the introduction of a new commodity in a different light. A new commodity is consumed only because it is (or entrepreneurs think it is) a cheaper substitute than those originally included in the group. Here, cheaper means a lower marginal cost. Depending on the substitutability of the new goods for the original goods and on the relative prices, the consumption pattern may vary considerably but definitely in favor of the new goods. Depending upon these factors, the base-period weights will be in lesser or greater error. Admittedly the entrance of a new good into a base-weighted index depresses current prices of other goods in the index and thereby tends to lower the index. But the index would be lowered even more, if the weights were also changed because the level of relative importance would be taken into account.

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[38]U.S.D.A., Ag. Handbook No. 118, Vol. I, op. cit., p. 58.

Part II

The need for price indexes of plant nutrients became apparent in Chapter II. And since the question of which index construction is proper (in this case) remains unsettled, and the marginal cost of computing is small, several alternative procedures are considered. Any gross generalizations from this small sample would be improper, but much can be learned from it. Indexes for two primary fertilizer nutrients, nitrogen (N) and phosphate (P_2O_5) are constructed. Since farmers consume about 80 percent of all potash (K_2O) as potassium chloride (KCL) and there is no evidence of significant change in this pattern, it is assumed that the price per unit of K_2O as it is found in KCL is representative of all K_2O. Data collected and published by the Federal Government are used since no other source provides comparable data for individual states and the period 1950 to date.

Price Data

The Statistical Reporting Service collects price data from fertilizer dealers twice a year: April and September. Mail-in questionnaires are sent on or about the 12th of the month and tabulation begins on or about the 18th. Dealers are asked to report the prices (not a range of prices) on the 15th of the month excluding state sales tax. Included is the following special notice:[39]

> IMPORTANT
>
> The price reported should be for the grade, quality and kind of each item MOST COMMONLY SOLD to farmers in your community.

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[39] U.S.D.A., Statistical Reporting Service, Form No. CE 5-209, "H-N.C." (1964).

Price data currently collected include several mixed fertilizers, four specified nitrogen materials, four specified phosphate materials and one potash material.

The April data were preferred to September data for two reasons. First, most of the fertilizer is sold during the spring months which makes the April price more relevant. More than 70 percent of the annual sales occur in the January-June period. For example, in the South Atlantic Region 81 percent of the 1963 fiscal year consumption took place during this period. Secondly, within a fiscal year, the September and April prices vary only slightly. The nature of the fertilizer industry partially explains this. Both potash and phosphate sectors utilize contracts between the primary producers (quarrying) and their customers (mixing plants or other processors). These contracts are signed around the beginning of the fiscal year. They serve, not as rigid legal contracts, but as rough indicators to the primary producers of next year's consumption and as aids in scheduling production. The scheduling of production is especially important in the phosphate sector because a curing or processing period of 30-45 days is required to convert the raw material to usable forms. Although the nitrogen sector of the industry does not utilize a contracting procedure there appears to be no greater price variation between the two annually reported prices than in the phosphate and potash prices.

Quantity Data

U.S.D.A. publications furnish the commercial fertilizer consumption data.[40] These data are for 15 principle mixtures, 8 nitrogen

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[40] Schöll, op. cit.

materials, 3 phosphate materials and 1 potash material. Because the data are, for the most part, composed of shipments rather than consumption, some small error due to annual inventory fluctuations is built in.

Estimated Indexes

From the available data outlined above, only the prices and quantities of fertilizer materials used for direct application were utilized to build the indexes. Mixtures of fertilizer materials were excluded. The justifications for excluding mixtures in construction of the index are: first, mixtures consist of the same materials used in direct application. The only difference then amounts to the price of mixing which, for purposes of argument, let us assume, remains constant over the period of the index. The ratio of mixtures to direct application nitrogen decreased 15 percent while this ratio for phosphate increased 6 percent since 1950. Since the price of mixing constitutes such a small part of the price of a ton of fertilizer, even a change in the ratio as great as 15 percent will have a negligible effect.[41]

Secondly, the only way of estimating the price of each of the three nutrients in mixtures is by a regression procedure which adds considerable cost to the construction of the index.[42]

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[41]For example: If the cost of a ton of fertilizer is $50 and the cost of mixing is $5, then a 15 percent change in mixing cost yields a 1.6 percent change in the index. If the cost of mixing is $1, the change is .4 percent. Based on a limited survey, the $1 per ton figure must be regarded as most realistic.

[42]One possible procedure:
1. A regression problem for each region must be run because the mixtures vary widely between regions to suit the different areas.

The actual formulas for the five alternative phosphate indexes are in Appendix III and the nine nitrogen indexes are in Appendix IV. The indexes are briefly:

Phosphate A: Laspeyres with base weight 1950 throughout

B: Paasche with current weight 1962 throughout

C: Fisher ideal using A and B

D: Chain

E: Chain of Fisher ideals

Nitrogen F: Chain

G: Chain of Fisher ideals (using annually revised weights for both the Laspeyres and Paasche indexes)

H: Laspeyres with 3 commodities weighted at 1950, 2 commodities weighted at 1955 - linked in 1956 i.e., Laspeyres with 3 commodities through 1955 and 5 commodities after 1955. The two added commodities have 1955 weights.

I: same as H but all weights revised in 1955

J: same as I through 1960 when all weights were revised - linked in 1961

K: Paasche with 3 commodities weighted at 1955 for '50... '55, 5 commodities weighted at 1962 for '46...'62 - linked in 1956

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2. The estimated coefficient, say for nitrogen, must then be multiplied by "the price" of mixtures.
3. "The price" of mixtures is the weighted (by quantity) average price of mixtures.
4. Steps 1, 2 and 3 would be repeated for each year.
5. This resulting price would then be weighted by the quantity of elemental N consumed in the respective state and utilized with other p's and q's of directly applied phosphated potash materials in the final index.

Note: Some data problems will be incurred in doing the above. Because some series are incomplete it would necessitate combining states to gain sufficient degrees of freedom.

L: same as K through 1955, 5 commodities weighted at '60 for '56...'60, 5 commodities weighted at '62 for '61, '62 - linked in 1956, 1961

M: Fisher ideal of I and K

N: Fisher ideal of J and L

Unfortunately none of these series replicates the U.S.D.A. construction. The Laspeyres index of phosphate prices most closely resembles the U.S.D.A. construction. The nitrogen indexes intentionally include weight changes to capture the consumption shifts of the mid-fifties. Weights in all these series were changed and linked in 1956 (or more frequently).

Results of the Indexes

These indexes demonstrate many of the issues suggested in the first section. The two fertilizer sectors are quite diverse. The phosphate sector has had gradual changes in price with the consumption pattern of rock phosphate, super phosphate (grades less than 22 percent available P_2O_5) and super phosphate (grades more than 22 percent available P_2O_5) shifting toward the higher analysis goods. Nitrogen consumption, on the other hand, has shifted markedly since the introduction of anhydrous ammonia and urea during the mid-fifties. Both these high analysis fertilizers have relatively low cost per unit of nitrogen.

In addition to the variation in the nutrient price,changes in consumption patterns have been far from uniform in the different regions of the U.S. For example, since anhydrous ammonia is particularly well adapted for side dressing row crops, adoption in the corn belt and cotton belt occurs more rapidly than in other regions.

With this rough view of the two sectors in mind many of the resulting indexes will be more easily understood. One look at the five

United States indexes for available P_2O_5 (Table 1) makes one point very clear: under stable conditions all constructions yield very similar results. After thirteen years less than two percent separates the final estimates of the commonly accepted indexes. Nearly four index points separate the Laspeyres and the Chain of Fisher Ideals. The latter was constructed to gain the advantages of the Fisher Ideal (using t and t-1 years) and the chain index but the only characteristic displayed consistently in this small sample is a downward bias. The only state showing much variation in the P_2O_5 index is Illinois with a difference between indexes of 5 percent; but then it was the only state with a large shift in consumption patterns.

A second criterion for evaluating indexes is the yearly change in an index. Under the stable conditions of the phosphate sector this criterion, as temporal change, indicates no preference for one index over another.

But nitrogen, a more volatile sector, subjects the indexes to much more stringent tests. The introduction of high analysis materials substantially increased nitrogen consumption patterns during the period studied. The nine series (Table 2) for the United States yield 1962 estimates ranging from 91.6 to 104.8 or a difference of more than 13 index points. Surprisingly, the chain index (F) after 13 years is greater than the Laspeyres (H,I,J) and less than the Paasche (K,L). Proofs based on indifference curves[43] show the Laspeyres construction to be biased upward and the Paasche to be biased downward. The reversed

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[43]F. E. Croxton and D. J. Cowden, Applied General Statistics Englewood Cliffs, N.J., Prentice-Hall, Inc., 1956), p. 409. For a more complete proof see: Sawhill, op. cit., pp. 6-11.

Table 1. Price Index of Available P_2O_5 - United States[a]

Year \ Series[b]	A	B	C	D	E
1950	100.0	100.0	100.0	100.0	100.0
1951	106.9	107.2	107.1	106.9	106.8
1952	109.1	109.4	109.2	109.2	102.3
1953	111.3	112.3	111.8	111.1	111.1
1954	113.3	114.7	114.0	113.0	113.0
1955	113.9	114.9	114.4	113.5	113.5
1956	112.9	113.5	113.2	112.4	112.3
1957	113.9	113.8	113.8	113.0	112.8
1958	117.9	117.7	117.8	116.8	116.7
1959	117.9	117.5	117.7	116.6	116.5
1960	118.4	118.0	118.2	117.1	116.9
1961	120.5	119.9	120.2	119.0	118.8
1962	120.0	119.2	119.6	118.3	116.2
1963				120.8	
1964				119.9	

[a]1950 = 100

[b]A = Laspeyres: base weight, 1950
B = Paasche: current weight, 1962 weights throughout
C = Fisher Ideal
D = Chain
E = Chain of Fisher Ideals

order of the two indexes can apparently be attributed to the change in the index weights and linkage in 1956 because index movements of Laspeyres (I) and Paasche (K) are generally consistent with expectation except for the year of the weight changes. Comparison of yearly changes (Table 3) shows that series I and K differ in direction and size between '55 and '56. In other observations the direction is usually the same with the Laspeyres showing slightly greater increases in periods of price increase and smaller decreases in periods of price decline which more nearly concur with expectation.

The Paasche series (K,L) suggest an additional weakness of current weighted indexes. The procedure for the introduction or exit of commodities inherently causes wide fluctuation in the index. And the longer the period the greater this fluctuation. So that whenever the index covers a period when commodities enter or drop from the index, it must either ignore the change or discontinue.

Table 2. Price Index of Nitrogen - United States[a]

Year \ Series[b]	F	G	H	I	J	K	L	M	N
1950	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1951	105.7	102.7	105.7	105.7	105.7	103.9	103.9	104.8	104.8
1952	109.7	106.6	109.8	109.8	109.8	108.0	108.0	108.9	108.9
1953	111.9	108.7	119.9	119.9	119.9	109.6	109.6	110.7	110.7
1954	113.7	110.5	109.9	109.9	109.9	107.2	107.2	108.5	108.5
1955	110.5	107.6	110.1	110.1	110.1	104.5	104.5	107.3	107.3
1956	107.7	104.6	110.4	107.4	107.4	120.0	115.4	113.5	113.5
1957	102.3	99.3	104.8	101.7	101.7	114.1	109.5	107.7	105.5
1958	103.1	99.9	105.2	101.3	101.3	114.3	109.9	107.6	105.5
1959	100.8	97.8	102.8	99.1	99.1	111.8	107.6	105.3	103.2
1960	98.0	95.0	100.1	96.2	96.2	108.5	104.5	102.2	100.3
1961	98.7	95.6	100.7	96.7	96.8	109.1	109.1	102.7	102.8
1962	94.6	91.6	96.8	92.4	92.9	104.8	104.5	98.2	98.5
1963	91.3								
1964	89.9								

[a] 1950 = 100

[b] F = Chain
G = Chain of Fisher Ideals
H = Laspeyres '50 weights through '55, '50 and '55 weights through '62
I = Laspeyres '50 weights through '55, '55 weights through '62
J = Laspeyres '50 weights through '55, '55 weights through '60, '60 weights through '62
K = Paasche '55 weights through '55, '62 weights through '62
L = Paasche '55 weights through '55, '60 weights through '60, '62 weights through '62
M = Fisher Ideal, $\sqrt{IK}$
N = Fisher Ideal, $\sqrt{JL}$

The relative position of the chain index was mentioned in conjunction with the base and current weighted indexes but the expected

Table 3. Annual Changes in Nitrogen Indexes - United States[a]

Year \ Series[b]	F	G	H	I	J	K	L	M	N
1951-1950	+5.7	+2.7	+5.7	+5.7	+5.7	+3.9	+3.9	+4.8	+4.8
52- 51	+4.0	+3.9	+4.1	+4.1	+4.1	+4.1	+4.1	+4.1	+4.1
53- 52	+2.2	+2.1	+2.1	+2.1	+2.1	+1.6	+1.6	+1.8	+1.8
54- 53	+1.8	+1.8	-2.0	-2.0	-2.0	-2.4	-2.4	-2.2	-2.2
55- 54	-3.5	-2.9	+ .2	+ .2	+ .2	-2.7	-2.7	-1.2	-1.2
1956-1955	-2.8	-3.0	+ .3	-2.7	-2.7	+15.5	+10.9	+6.2	+6.2
57- 56	-5.4	-5.3	-5.6	-5.7	-5.7	-5.9	-5.9	-5.8	-8.0
58- 57	+ .8	+ .6	+ .4	- .4	- .4	+ .2	+ .4	- .1	0.0
59- 58	-2.3	-2.1	-2.4	-2.2	-2.2	-2.5	-2.3	-2.3	-2.3
60- 59	-2.8	-2.8	-2.7	-2.9	-2.9	-3.3	-3.1	-3.1	-2.9
1961-1960	+ .7	+ .6	+ .6	+ .5	+ .6	+ .6	+4.6	+ .5	+2.5
62- 61	-4.1	-4.0	-3.9	-4.3	-3.9	-4.3	-4.6	-4.5	-4.3
63- 62	-3.3								
64- 63	-1.4								

[a] 1950 = 100

[b] F = Chain
G = Chain of Fisher Ideals
H = Laspeyres '50 weights through '55, '50 and '55 weights through '62
I = Laspeyres '50 weights through '55, '55 weights through '62
J = Laspeyres '50 weights through '55, '55 weights through '60, '60 weights through '62
K = Paasche '55 weights through '55, '62 weights through '62
L = Paasche '55 weights through '55, '60 weights through '60, '62 weights through '62
M = Fisher Ideal, $\sqrt{IK}$
N = Fisher Ideal, $\sqrt{JL}$

position of the chain was ignored. A discussion of index numbers often neglects the expected value of the chain index relative to the "true" index. Mitchell[44] argued that the chain index biases upward relative to the fixed-base index. That is, increases in the base-weighted index would be reflected as smaller drops in the chain index. Mitchell

[44] Wesley C. Mitchell, The Making and Use of Index Numbers, Part I, U.S. Dept. of Labor, Bulletin of the Bureau of Labor Statistics No. 284 (Oct. 1921), pp. 86-91.

constructed his argument on the nonsymmetric property of percentage change: a given absolute increase yields a greater percentage change than an equal absolute decrease from the higher position. Conclusions different than Mitchell's are yielded by indifference curve analysis. The same indifference curve argument demonstrating upward bias of the Laspeyres also, because of analogous construction, demonstrates upward bias of the chain index between any two adjacent years. But the bias after year two, when both indexes are the same, is always less than for the base-weighted index. Inclusion of revised weights each period re-evaluates the relative importance of the individual commodities and therefore modifies the bias over a series. This, however, does not quite solve the question of bias for series, for beyond the comparison of consecutive years the multiplicative factor in the chain index clouds the issue.

Implications for Fertilizer Price Indexes

The results shown in Tables 1 and 2 strongly suggest that economic questions asked about specific nutrients will be better answered if the price of the nutrient is utilized rather than a combined fertilizer price. Using the chain index as an example, the 1962 price index of phosphate is 118.3 compared with 94.6 for nitrogen: a difference of 23.7 index points. And neglect of a 20 percent difference between the price of nutrients entails erroneous answers to all but the most general aggregative type questions. This error will be aggravated further if the yield response per pound is greater for the cheaper nutrients.

In addition to the price variation between nutrients the indexes vary as much between geographic regions. Table 4 shows the 1962 index value (chain index) for nine regions. The index price of available phosphate varies from 108 to 128 while the index of nitrogen price varies from 86 to 104. This geographic variation is nearly as great as variation between the nutrients. Disaggregation of regions to states further demonstrates the point: price of P_2O_5 varies from 134 in Mississippi to 103 in Illinois and price of N varies from 104 in Maine to 81 in California. This wide variation must not be mistaken as differences due to transportation, taxation and other such factors. Admittedly, changes in these factors since 1950 are reflected in the index, but most of these differences were established before 1950.

Table 4. Index of nitrogen and phosphate price in 1962, by region[a]

Region[b]	Chain index value for 1962 P_2O_5	N
N. England (Maine)	115.8	103.6
M. Atlantic (New York)	125.2	104.1
E. North Central	108.3	102.7
W. North Central	113.1	96.1
S. Atlantic (North Carolina)	126.1	99.4
E. South Central	128.0	101.4
W. South Central	121.9	97.9
Mountain	107.6	90.1
Pacific	116.7	86.3

[a]1950 = 100 for all regions

[b]Where regional estimates were not made, a representative state is considered.

To compare the current U.S.D.A. index of fertilizer prices, a weighted average of the chain indexes for available P_2O_5 and N and the price of KCL (set 1950=100) was constructed (Table 5). The

Table 5. Index of fertilizer price - United States, 1950-64

Year	Currently published USDA index[a]	Chain index[b]	--Year-to-year change-- USDA	Chain
1950	100.0	100.0	+5.6	+4.5
1951	105.6	104.5	+2.7	+2.0
1952	108.3	106.5	+ .8	+ .9
1953	109.1	107.4	+ .6	- .2
1954	109.7	107.2	-2.1	-1.5
1955	107.6	105.7	-2.0	-2.0
1956	105.6	103.7	+ .6	-1.0
1957	106.2	102.7	0.0	+1.9
1958	106.2	104.6	- .6	-1.3
1959	105.6	103.3	0.0	-1.1
1960	105.6	102.2	+1.3	+1.4
1961	106.9	103.6	- .7	-1.4
1962	106.2	102.2	- .6	- .6
1963	105.6	101.6	- .7	-2.9
1964	104.9	98.7		

[a]Currently published U.S.D.A. index is based on 1910-14 = 100. But for purposes of comparison 1950 was set = 100.

[b]The aggregate chain index is the weighted average of the nutrient indexes. Using the chain indexes for N and P_2O_5 and price KCL adjusted to 1950 = 100 for K_2O. The quantity of N, available P_2O_5 and K_2O were used as weights.

resulting aggregate index is listed with the currently published index and a comparison of their year-to-year changes. The two indexes differ by about 6 percent after 15 years. Whether this difference is due to the method of calculation - the formula - or due to the use of nutrient content of fertilizer rather than total tonnage of fertilizer is a speculative matter. The ratio of nutrients to total tonnage of fertilizer increased 55 percent from 1950 to 1964. Both constructions are affected by this; but only one revision of the U.S.D.A. index weights suggests an upward bias relative to the annually revised chain index. More important than the increase in ratio of nutrients to total

fertilizer are the changes in the combination of nutrients purchased. The combination of nutrients shifted to the relatively cheaper goods. Since the U.S.D.A. revision to 1955 weights when fertilizer nutrient consumption was divided into 31.6 percent nitrogen, 37.8 percent available phosphate and 30.6 percent potash, nitrogen has increased to 41.9 percent while available phosphate and potash fell to 32.2 percent and 25.9 percent respectively. Disregard of this shift by the U.S.D.A. index offers a partial explanation of the difference. While the effects of multiplicative relationships of the chain are indefinite, the chain index is preferred even at this aggregate level primarily because of the utilization of more market data.

Summary

From this small study of index numbers several conclusions can be drawn. First, the index of fertilizer price is improved by replacement of the current index by one of fertilizer nutrient price. This reduces the index number problems normally associated with quality changes. A second point also refers specifically to the fertilizer indexes. Where the price and consumption patterns of the three nutrients are changing rapidly a significant amount of information is hidden in an aggregate index. A divergence of twenty percent in the nitrogen and phosphate indexes is undisclosed in an aggregate index which appears almost stable. Wide regional variations are also undisclosed in the aggregate figure.

Comparison of results in the nitrogen and phosphate sectors shows that the form of the index makes little difference under stable conditions. But when unstable prices and quantities characterize the

index goods, estimates of the four indexes vary widely. The Paasche and the Fisher Ideal are seldom of importance because of the usual lack of current quantity data. And even when current data exist, the problems of changing weights introduce noneconomic fluctuations into the index. The best index then, is either a Laspeyres or a chain. The chain with weights changed annually measures neither the varying cost of a constant amount of goods nor the varying amount of goods which a dollar will buy. But the importance of price fluctuations depends largely upon the accompanying changes in the quantities of goods bought. To reflect these changes, the weights of the Laspeyres must be revised frequently. And the revised weights must be representative. But the chain index, using historically realized prices and quantities, avoids these arbitrary decisions by using market data.

No statistical tests are available to measure the amount of bias in either the chain or the Laspeyres indexes. Because of this, the arguments for each index must be considered in view of the individual commodity or set of commodities being studied. In the case of fertilizer nutrient prices, the chain index more accurately reflects the actual price movements. Appendix V presents the estimated chain indexes for each state. To generalize to all commodities from this sample of two fertilizer prices would be presumptuous, but the behavior of the chain index in these two diverse fertilizer sectors suggests that more frequent use of chain indexes should be considered.

CHAPTER IV

QUANTITATIVE RESULTS

Introduction

Chapter III discussed the development of the fertilizer nutrient price indexes used in this study. This chapter is presented in five sections. The first section discusses characteristics common to the models of all three nutrients. The second section specifies the preferred nitrogen model (Model N-1) and presents the estimates of Model N-1's parameters. Because of the large number of synthetic variables, parameters are tested individually and by sets for level of significance. A measure of the relative importance of the economic variables and time variables in explaining the change in fertilizer consumption between 1950 and 1964 in each state completes the section. The third section presents the preferred phosphate model (Model P-1) and discusses it following the same format as the nitrogen section. The fourth section, potash, again follows the same format. As a note of caution, the specified models of the three nutrients are deceivingly similar. All the state constants and time constants have the same variable number throughout, but the state groupings of the nutrient price variables are not the same in every model. And the crop price variable may include different crops or different sets of states or possibly both. The fifth section discusses some problems common to all the nutrient models and the sixth section makes comparisons of the model estimates.

Selection of the Nutrient Models

The criteria used to select preferred models from those models discussed in Chapter II includes (1) $\bar{R}^2$, the percent of variation of the dependent variable explained by the independent variables adjusted for degrees of freedom[45], (2) estimated standard error of the disturbances[46], (3) the significance of the estimated regression line[47], (4) the significance of the individual regression coefficients[48], and (5) the correctness of the signs of the economic

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[45]

$$\bar{R}^2 = \frac{\hat{\sigma}_y^2 - \hat{\sigma}_u^2}{\hat{\sigma}_y^2} = 1 - (1-R^2)\,\frac{T-P-1}{T-1}$$

where $\hat{\sigma}_y^2 = \frac{\sum^{i}(Y_i-\bar{Y})^2}{T-1}$

$\hat{\sigma}_u^2 = \frac{\sum^{i}(Y_i-\hat{Y})^2}{T-P-1}$ = estimated standard error of disturbances

T = total number of observations

P = number of independent variables

[46]Defined in footnote 45 above.

[47]$F = \frac{RSS/d.f.}{ESS/d.f.}$

RSS = regression sum of squares

ESS = error sum of squares

[48]

$$t = \frac{\hat{\beta}_i - 0}{\left(\sqrt{\Sigma u_{t,s}^2} \;\Big/\; T-P\right)\sqrt{a_{ii}}}$$

where a_{ii} is the ith element in the principal diagonal of $(X'X)^{-1}$

variables according to economic theory. These criteria were, in turn, tempered by a judgment since maximization of all five was not always possible.

The models selected for the nutrients are all "form free" with respect to time. In experimenting with various forms of the time variable, it soon became evident that multicollinear relationships exist: changing the form of the time variable changes the estimated coefficients of other variables. For example, excluding time as a variable in the nitrogen model (Model N-2) results in estimated coefficients of the economic variables more than twice the size of the coefficients for the same economic variables in Model N-3 where time is included as a linear variable. Since changes in many of the economic variables are associated with time, leaving time unspecified (time constants) tends to purify both the economic and time variables. That is, the time constants pick up only that effect attributable to the individual year and the economic fluctuations which happen to move together over time are picked up by the economic variables.

Predictions using the "form free" models are more difficult than models specifying time as linear or quadratic or something else. In the "form free" models, judgments based on expectations of the market must replace the assumption that the future is a functional extrapolation of time. But this puts the burden of projection on the investigator where it really should be.

Before discussing the results of the models another point needs clarifying. In Chapter II, it was argued that the prices of other inputs could be excluded as independent variables because (1) they are neither close substitutes nor close complements and (2) only national data are available. Validity of the second point rests on a generalization from the estimated indexes of fertilizer nutrient price which revealed differences of more than 15 percent between individual state indexes and the U.S. index. Using the national data for related inputs would ignore such differences and, in doing so, would introduce specification error. But Chapter II also argued in favor of deflation - and because no state indexes are available, by a national index. To relieve any uneasiness let us examine the national index. The indexes of Prices Paid for Production Items and the Prices Paid by Farmers used for deflating[49] consist of a composite of components representing the general U.S. price level. In any given state, particular components in these indexes may be higher or lower than the national level of the same component as the fertilizer indexes demonstrate. But because of the limited importance of any one component in the index, the set of state components (a hypothetical state index analogous to the national index) is expected to deviate much less from the national index than individual components. And since the intent of the deflation in these models is to transform the nominal state prices to real state prices, the national index appears to be the best deflator available. To insure that deflation is appropriate, all the models were estimated with undeflated data. Although the percent explained changed very little, the estimated variance of the parameter estimates increased for more than one-half the variables. Therefore, the deflated models were considered more appropriate.

The nitrogen, phosphate and potash models will be discussed

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[49]See Appendix I.

in the next three sections. Models N-3, P-3 and K-3 are presented in Appendix VI along with the estimated parameters: and Models N-2,4,5; P-2,4,5 and K-2,4,5 are specified in Appendix VII.

Nitrogen Model

In this section the model postulated as representing the demand for nitrogen is discussed.

Model N-1 (time constants)

$$Q^N_{t,s} = \alpha + \sum^{g} \beta_g P^N_{t,s} + \beta_{67} Y_{t-1,s} + \sum^{k} \beta_k P^c_{t-1,s} \sum^{i} \beta_i S_{*,s}$$

$$+ \sum^{j} \beta_j T_{t,*} + U_{t,s}$$

$\bar{R}^2$ = .9361 Standard error of residuals = 7.61

F = 129.55

where:

α = the overall constant term

g = variables 83 through 88

X_{83} = price of nitrogen in Me., N.H., Vt., Mass., R.I., Conn., N.Y., N.J. and Pa.

X_{84} = price of nitrogen in O., Ind., Ill., Mich., Wisc., Minn., Ia., Mo., N.D., S.D., Neb. and Kan.

X_{85} = price of nitrogen in Del., Md., Va., W.Va., N.C., S.C., Ga., Fla., Ky., Tenn., Ala. and Miss.

X_{86} = price of nitrogen in Ark., La., Okla. and Tex.

X_{87} = price of nitrogen in Mont., Id., Wyo., Colo., N.Mex., Ariz., Ut. and Nev.

X_{88} = price of nitrogen in Wash., Ore. and Calif.

X_{67} = average net farm income

k = variables 4 through 17

X_4 = price of hay in N.H., Vt., Mass. and Conn.

X_5 = price of corn in Wyo., Colo. and Ut.

X_6 = price of corn in O., Ind., Ill., Ia. and Mo.

X_7 = price of corn in Del., Md., Va. and W.Va.

X_8 = price of corn in Ky., Tenn., Ala. and Miss.

X_9 = price of cotton in S.C., Tenn., Miss., Ark., La., Okla. and Tex.

X_{10} = price of potatoes in Me. and R.I.

X_{11} = price of wheat in Kan. and Okla.

X_{12} = price of wheat in Id., Nev., Wash. and Ore.

X_{13} = price of cotton in N.Mex., Ariz. and Calif.

X_{14} = price of wheat in N.D. and Mont.

X_{15} = price of corn in N.Y., N.J. and Pa.

X_{16} = price of corn in Mich., Wisc., Minn., S.D. and Neb.

X_{17} = price of corn in N.C., S.C., Ga. and Fla.

i = variables 18 through 64 = state constants (listed in Table 9) Each state is associated with a 0-1 dummy variable which is a rough measure of differences in cropping practices, and environmental differences.

j = variables 68 through 81 = time constant for each year: 1951 through 1964
Each year is associated with a 0-1 dummy variable.

Because of the size of the model the parameter estimates will be presented and discussed in groups. The dummy variables in the model provide a natural division into variables relating to price of nitrogen, income restriction, crop price variables, state constants and time constants.

The six nitrogen price variables are considered first. Table 6 shows the estimated coefficients along with their standard errors and the calculated value of the t statistic. The table can be read as follows: a decrease of one index point (deflated) in the index of nitrogen price in the New England states results in a 1.4 pounds per acre increase in nitrogen consumption in those states. Similarly, a one index point decline in the North Central Region increases nitrogen consumption .7 pounds.

Table 6.[a] Estimated regression coefficients of nutrient price variables and net income variable - Model N-1

Variable	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient	t statistic
X(83)	-1.417	.375	-3.77**[b]
X(84)	- .721	.261	-2.76*[c]
X(85)	-1.809	.352	-5.14**
X(86)	- .892	.342	-2.61*
X(87)	-1.229	.278	-4.42**
X(88)	-2.265	.318	-7.11**
where nitrogen price index, 1950 = 100.0			
X(67)	+ .684	.103	6.58**
where income is in hundreds of dollars			

[a]Estimates in all tables referring to the parameters are carried to three decimal places. Admittedly, this implies much more precise measurement than is actually the case. But rounding is postponed to the final estimates.

[b]* means the null hypothesis (β_i=0) is rejected at .01 level of significance.

[c]** means the null hypothesis (β_i=0) is rejected at .001 level of significance.

The estimated nitrogen price coefficients of Model N-1 all have negative signs as expected and all have very small standard errors of estimate.

The response to nitrogen price changes in different regions follows no definite pattern, but states which have historically consumed relatively high amounts of fertilizer tend to respond more to price changes than states consuming less. Namely, the Northeast and Southeast states respond more than the North Central and South Central states. This is not surprising. First, states which have historically used more fertilizer are probably more aware of the response to fertilizer and second, states which have historically used more fertilizer generally apply more per acre and therefore apply fertilizer at more nearly the economic optimum rate. Later in the chapter another measurement of response, price elasticities, will be discussed and compared with the other nutrients.

The form specified for the time variable influences the estimates markedly. Excluding time as a variable in Model N-2 increased the estimates of the nutrient price coefficients in all but the North Central Region. This is not too surprising since nitrogen price declined over much of the period as consumption was increasing. This, along with the expected inverse relation between price and quantity combine to make the large coefficient estimates. Model N-3 which specifies time as a linear variable, produces estimated price coefficients which are much smaller and, incidentally, not significant at the .05 level in four of the six estimates. Model N-4, a quadratic equation with time and time squared terms, produces estimates nearly identical to Model N-3. In Model N-5 the size of the estimates decreases even farther and two become positive.

In contrast to the instability of the estimated nutrient price coefficients in the four models, the estimated coefficient of net farm income shown (X_{67}) in Table 6 remains very stable in all the models, very significant, and with the expected sign. The hypothesis that

expenditure restrictions are unimportant (the parameter equal to zero) is rejected at something less than the .001 level of significance. An increase in real net income of $100 per farm will be associated with a .7 pounds per acre increase in nitrogen consumption. It is clear that although this variable is highly significant in the statistical sense, only a small part of the expanded consumption can be attributed to it because real farm incomes have increased little[50] since 1949.

The remaining set of economic variables is the crop prices. Table 7 gives the estimates of the crop price coefficients with their standard error and the calculated t statistic. The units of measure represent what is considered as meaningful changes that can be expected to enter into production decisions. For example, hay prices range from about $20 to $40 per ton so $1 is selected as the unit of measure. Potatoes, wheat and corn prices range from $1 to $2 per bushel so $.10 is selected as the unit of measure. Cotton ranges from $.25 to $.40 per pound so $.01 is the selected unit of measure. If these units of measure are unsatisfactory the estimates and their standard errors can be multiplied by a preferred constant without affecting the t statistic. The reader should be reminded that the range in prices discussed is in money terms and that the variables are in real (deflated) terms.[51] As another example consider the estimated coefficient of variable X_4. This is to be read: a one dollar increase in the real price of hay in year t-1 in the states of New Hampshire, Vermont, Massachusetts or

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[50]Average real net farm income per farm in the U.S. increased from $939 in 1949 to $1331 in 1963.

[51]Indexes used for deflation are presented in Appendix II.

Connecticut will be associated with a .738 pounds per acre increase in consumption of nitrogen in these states in year t.

Table 7. Estimated regression coefficients of crop price variables - Model N-1

Variable	Unit of measure (real $)	Estimated regression coefficient, $\hat{B}$	Standard error regression coefficient	t statistic
X(4)	1.00/ton hay	.738	.823	.90
X(5)	.10/bu. corn	6.212	1.493	4.16**[b]
X(6)	.10/bu. corn	1.197	1.257	.95
X(7)	.10/bu. corn	.147	1.609	.09
X(8)	.10/bu. corn	.710	1.340	.53
X(9)	.01/lb. cotton	1.570	.665	2.36**
X(10)	.10/bu. potatoes	-1.095	.678	-1.62
X(11)	.10/bu. wheat	6.878	2.374	2.90**
X(12)	.10/bu. wheat	3.000	2.045	1.47
X(13)	.01/lb. cotton	-.138	.827	-.17
X(14)	.10/bu. wheat	6.841	2.541	2.69**
X(15)	.10/bu. corn	2.990	1.572	1.90*[a]
X(16)	.10/bu. corn	4.286	1.318	3.25**
X(17)	.10/bu. corn	-12.164	1.556	-7.82**

[a] *means the null hypothesis (B_i=0) is rejected at the .10 level of significance.

[b] ** means the null hypothesis (B_i=0) is rejected at the .05 level of significance.

The estimated crop price coefficients have the expected sign in 11 of the 14 cases. Of the three with incorrect sign two are not significant at the .10 level, but the corn price coefficient (X_{17}) for North Carolina, South Carolina, Georgia and Florida is incorrect and very significant. No explanation for this has yet been found and the problem is present again in the phosphate and potash models. The estimated coefficients are not as highly significant as the nutrient prices and net income variables but this is not surprising. The decision to buy fertilizer is affected by expected crop prices and the lagged price is a poor proxy for the expected price and is, therefore, less than the expenditure restriction, which is measured at the time of purchase and the price which the buyer observes at decision time. Nevertheless, one of the coefficients of crop price is significant at the .10 level and five (excluding the variable which is significant but has the wrong sign) are significant at the .05 level.

Constants for each state (variables $X_{18}...X_{64}$) were included to take into consideration the diversity among states. Casual observation of individual states reveals differences in type of agriculture, the productivity of soils, level of technology and many environmental factors. As expected the estimated coefficients (Table 8) differ greatly in size and sign with 33 of the 47 states significant at .05 level or lower. The test referred to is of the null hypothesis ($\beta_i = 0$) which, because of the construction of the model, really tests whether the estimated state constant terms differ from the overall constant term which includes the state of Michigan.

Table 8. Estimated regression coefficients of state constants for 48 states - Model N-1

Variable	Estimated regression coefficient, $\hat{\beta}$	Standard error regression coefficient	t statistic
Mich.[a]	0.0	0.0	na[b]
X(18) Me.	77.351	13.844	5.59
X(19) N.H.	40.122	17.753	2.26
X(20) Vt.	36.317	17.576	2.07
X(21) Mass.	64.747	18.791	3.45
X(22) R.I.	107.985	14.716	7.34
X(23) Conn.	61.369	18.990	3.23
X(24) N.Y.	34.958	15.270	2.29
X(25) N.J.	60.005	15.478	3.88
X(26) Pa.	33.739	15.170	2.22
X(27) O.	17.613	7.459	2.36
X(28) Ind.	19.791	7.406	2.67

Table 8--Continued.

Variable	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient	t statistic
X(29) Ill.	9.163	7.508	1.22
X(30) Wisc.	-8.763	2.845	-3.08
X(31) Minn.	-9.512	2.832	-3.36
X(32) Ia.	5.304	7.458	.71
X(33) Mo.	14.994	7.515	2.00
X(34) N.D.	-49.754	19.950	-2.49
X(35) S.D.	-13.577	2.946	-4.61
X(36) Neb.	-8.426	2.885	-2.92
X(37) Kan.	-41.149	18.119	-2.27
X(38) Del.	75.531	14.638	5.16
X(39) Md.	71.700	13.864	5.17
X(40) Va.	132.420	13.723	9.95
X(41) W.Va.	57.615	14.449	3.99
X(42) N.C.	155.435	12.832	12.11
X(43) S.C.	135.266	15.539	8.71
X(44) Ga.	153.446	12.696	12.09
X(45) Fla.	253.723	12.820	19.79
X(46) Ky.	66.930	12.830	5.22
X(47) Tenn.	51.833	14.994	3.46
X(48) Ala.	86.996	12.636	6.89
X(49) Miss.	70.985	14.691	4.83
X(50) Ark.	14.496	15.026	.96
X(51) La.	34.291	14.769	2.32
X(52) Okla.	-51.169	21.051	-2.43
X(53) Tex.	3.843	14.368	.27
X(54) Mont.	-26.931	21.129	-1.27
X(55) Id.	14.051	16.452	.85
X(56) Wyo.	-5.312	12.349	-.43
X(57) Colo.	-3.402	12.056	-.28
X(58) N.Mex.	35.475	14.648	2.42
X(59) Ariz.	35.475	15.771	3.97
X(60) Ut.	-4.872	12.802	-.38
X(61) Nev.	-4.058	16.266	-.25
X(62) Wash.	67.167	17.159	3.91
X(63) Ore.	69.601	17.517	3.97
X(64) Calif.	112.411	16.639	6.76

[a]To avoid singularity of the X'X matrix one state (Michigan) is arbitrarily included in the overall constant term. Because of this construction all other state constant terms should be interpreted as ($\pm$) deviations from the estimated overall constant ($\hat{\alpha}$=6.264). Statistical tests can be interpreted as tests of significant differences between any particular state and Michigan.

[b]na means not applicable.

Next, consider the time constants (Table 9), the zero-one dummy variables for the years 1951 through 1964. The year 1950 is included in the overall constant to avoid singularity of the X'X matrix. As would be expected from examination of historical data and the estimated coefficient of the linear time variable in Model N-3 (1.986) the estimated coefficients of the time constants trend upward. Only once is this general trend broken: 1956-1958. In 1956, the inauguration of the Acreage Reserve Program reduced acreage of cotton, peanuts, rice, tobacco, wheat and corn substantially.[52] Assuming that normal production on cropland in the program equals cropland not in the program, total crop production was reduced 5 percent.[53]

The decrease in the size of the estimated coefficient for 1957 demonstrates the sensitivity of the model. And the advantage of utilizing "form free" time constants is also demonstrated; for this

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[52]As a measure of the effect of the program consider the planted acreage of cotton, wheat and corn.

Year	Harvested acreage: Upland Cotton 1,000 A.	Wheat 1,000 A.	Corn 1,000 A.
1955	16,928	47,290	81,100
1956	15,615	49,768	78,200
1957	13,558	43,754	73,900
1958	11,849	53,047	74,500
1959	15,090	51,781	84,400

Source: R. P. Christensen and R. O. Aines, Economic Effects of Acreage Control Programs in the 1950's, U.S.D.A., Agricultural Economic Report No. 18 (1962), pp. 11, 14, 16.

[53]Ibid., p. 26.

change in farm policy remains unrevealed in the alternative models.

Table 9. Estimated regression coefficients of time constants for the years 1951 through 1964 - Model N-1

Variable	Estimated regression coefficient, $\hat{\beta}$	Standard error regression coefficient	t statistic
X(68) 1951	-4.573	1.957	-2.34
X(69) 1952	-1.554	1.763	.88
X(70) 1953	7.062	1.665	4.24
X(71) 1954	9.954	1.782	5.59
X(72) 1955	11.502	1.675	6.87
X(73) 1956	11.242	1.637	6.87
X(74) 1957	9.434	1.617	5.83
X(75) 1958	11.916	1.667	7.15
X(76) 1959	11.784	1.887	6.24
X(77) 1960	14.672	2.011	7.30
X(78) 1961	17.091	2.036	8.39
X(79) 1962	18.236	2.104	8.67
X(80) 1963	18.445	2.266	8.14
X(81) 1964	21.814	2.319	9.41
overall constant term, $\hat{\alpha}$ = 6.264			

This set of coefficients with their low standard errors and large size obviously contribute considerably to the total explanation. The amount explained by the individual elements will be considered in more detail later.

We have been discussing the various variables in sets. But the significance has been discussed only of the elements of the set. That is, each β_i was tested with the null hypothesis ($\beta_i = 0$). However, four of these sets of variables are some form of dummy variable. The first set, the nutrient prices, are dummy variables which change the slope of the demand equation depending on the region under consideration. The second set, the crop prices, similarly are made up of zeros and observed values depending on whether the crop is considered most important for the state in question. The third set of dummy variables, the state constants, consists of zero-one variables which affect only the level of the regression line. The fourth set, the time constants, likewise are zero-one variables and affect only the level of the line. Quite often in work with dummy variables the question of significance is really relevant only to the variables in the set: the significance of individual elements has little meaning. In Model N-1 some of the estimates are of interest by themselves and others only as members of the set. Different readers may prefer one or the other and therefore both are presented. But generally there is little interest in the individual estimates of 0,1 variables, and more interest in the economic variables. Table 10 presents the F statistic[54] used to test whether particular sets of variables exercise any significant influence upon the dependent variable. Capacity limitations of the computer confined these tests to models other than the Time Constants Model (N-1). Since estimates of the Time Linear Model (N-3) most closely approximate those of N-1, the tests are made using Model N-3

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[54]This is a test to ascertain whether the composite effect of several variables in an analysis is statistically significant. The test statistic is:

$$F = \frac{(ESS - ESS_R)/(df - df_R)}{ESS/df}$$

where: ESS = error sum of squares without any restriction

ESS_R = error sum of squares with the restriction; the restriction being the set of variables specified.

df and df_R are the associated degrees of freedom and the results are then inferred to Model N-1. To test the set of time constants, Models N-1 and N-2 (time excluded) are utilized.

Table 10. Evaluation of the contribution attributable to sets of variables[a] - Model N-1 and Model N-3

Variables included in the set[b]		F statistic	Degrees of freedom[c]	Level of significance	$\bar{R}^2$ [d]
Model N-3					
X(83)...X(88)	Nitrogen price	7.470	6;650	.001	.9304
X(4)...X(17)	Crop prices	9.564	14;650	.001	.9225
X(5)...X(8), X(15)...X(17)	Corn price	16.164	7;650	.001	.9237
X(11), X(12), X(14)	Wheat price	5.600	3;650	.001	.9329
X(9), X(13)	Cotton price	2.358	2;650	.10	.9341
X(18)...X(64)	State constants	80.401	47.650	.001	.5827
X(18)...X(23)	New England	42.385	6;650	.001	.9095
X(27)...X(33)	Corn Belt	7.944	7;650	.001	.9295
X(40)...X(49)	SE & ESC States	249.963	10;650	.001	.6866
X(54)...X(61)	Rocky Mountain	13.118	8;650	.001	.9247
X(62)...X(64)	West Coast	24.100	3;650	.001	.9274
Model N-1					
X(68)...X(81)	Time constants	21.658	14;637	.001	.9078

[a]The restriction consists of specifying certain variables equal to zero. That is, does the inclusion of specified sets of variables significantly increase the explanation?

[b]Variables are defined on pp. 59, 60.

[c]Degrees of freedom are listed with d.f. associated with numerator given first; d.f. associated with denominator second.

[d]This is the $\bar{R}^2$ when the specified variables are excluded from Model N-3 or Model N-1. E.g., specifying the nitrogen price equal to zero results in $\bar{R}^2$ of .9304 in Model N-3.

From Table 10 it is clearly seen that although individual elements of

sets are sometimes insignificant the entire set does contribute to the explanation. The set of nutrient price variables is not of particular interest since all nutrient price variables in the unrestricted case had correct sign and were significant, but note that the set of variables adds significantly to the explained variation. The crop price variables offer a more interesting case because collectively they have a statistically significant effect on the dependent variable, even though it was impossible to obtain significant results for each of them separately.

But the F statistic only determines whether or not the variable or set of variables adds to the explanation. For policy decisions the amount of the total estimate attributable to each variable or set of variables is equally important. That is, we need to know: (1) whether the variable is significantly different from zero and (2) if it is different from zero, what is the effect of its inclusion. That is, what percent of the change in the dependent variable can be attributed to the various independent variables? To evaluate the importance of any one variable consideration must be given to the size (absolute value) of the estimated coefficient, its associated standard error and the unit of measure of the independent variable. For example, is the unit of measure tons or pounds? In addition to these three considerations, all of which are normally known, the range of values assumed by the variable must be known. If the variable changes little from one observation to the next, and a small estimated coefficient is associated with a large standard error (possibly including zero), then the variable probably contributes little to the estimated change between the two observations. But if the variable assumes a wide range of values between observations and the estimated coefficient is large with small standard error, then this variable contributes much toward the explanation of change in the dependent variable. Table 11 demonstrates a method of measuring the importance of the variables using Michigan as an example.[55] The algebraic equivalent of Table 11 is:

$$\hat{Y} = \hat{a}_{84}(X_{84_{1963}} - X_{84_{1950}}) + \hat{a}_{67}(X_{67_{1963}} - X_{81_{1950}})$$
$$+ \hat{a}_{16}(X_{16_{1963}} - X_{16_{1949}}) + \hat{a}_{81}(X_{81_{1964}} - X_{81_{1950}})$$
$$+ \hat{a}_{0}(X_{0_{1964}} - X_{0_{1950}})$$

or $24.231 = (7.858) + (.087) + (-5.528) + (21.814) + (0)$

The estimated change in nitrogen consumption is the algebraic sum of the factors on the right of the equation. The ratio of each of these factors with the sum provides a measure of the change in consumption between 1950 and 1964 explained by that factor. An alternative is to sum the absolute values which is the procedure used in Table 11. Using the absolute value preserves the relative importance of the variables; it has the satisfying property of summing to 100 percent; and it avoids confusion about signs (both the change in the variable and the coefficient can be either positive or negative). A similar example using South Carolina demonstrates the procedure for states with two crop prices. Table 12 summarizes the results for all

[55]The units of measure used earlier in the interpretation section have been changed to the more common cents per bushel, etc., to reduce the possibility of errors in arithmetic operations. To make this change the coefficient and the variable were multiplied by a constant and its reciprocal respectively.

Table 11. A method of measuring the importance of variables in the estimating equation - Model N-1

Michigan	Deflated raw data[a]				Observation 15		
	Observation 1		Observation 15		minus	$\hat{\beta}$	$\Delta\hat{Y}_{xi}$ [b]
Variable	1949	1950	1963	1964	Observation 1		
X(84) Nitrogen price		.419		.310	-.109	-72.089	7.858
X(67) Net income	803.817		816.274		12.457	.007	.087
X(16) Corn price	.524			.395	-.129	42.855	-5.528
X(81) Time constant		0.000		1.000	1.000	21.814	21.814
X(0) Overall constant		1.000		1.000	0.000	6.264	0.000
							24.231[c]
					sum of the absolute values =		35.287

Contribution of	Percent
X(84) Nitrogen price	22.3 = 7.858 / 35.287
X(67) Net income	.2
X(16) Corn price	15.7
X(81) Time	61.8

Table 11--Continued.

South Carolina Variable	Deflated raw data[a] Observation 1 1949	1950	Observation 15 1963	1964	Observation 15 minus Observation 1	$\hat{Q}$	$\Delta\hat{Y}_{xi}$ [b]
X(85) Nitrogen price		.419		.364	-.055	-180.933	9.951
X(67) Net income	528.548		803.119		274.571	.007	1.922
X(17) Corn price	.536		.471		-.065	-121.644	7.907
X(9) Cotton price	1.225		1.213		-.012	1.570	-.018
X(81) Time constant		0.000		1.000	1.000	21.814	21.814
X(0) Overall constant		1.000		1.000	0.000	6.264	0.000
X(43) State constant		1.000		1.000	0.000	155.435	0.000
							41.576[d]
					sum of the absolute values =		41.612

Contribution of	Percent
X(85) Nitrogen price	23.9
X(67) Net income	4.6
X(17) Corn price	19.0
X(9) Cotton price	0.0
X(81) Time	52.4

[a] Units of measure used in this table are: X(84), X(85), nitrogen price index deflated by the Index of Prices Paid for Production Items; X(67), average net farm income in dollars deflated by Index of Prices Paid; X(16), X(17) and X(9), corn and cotton prices in cents per bushel and pound respectively deflated by the Index of Prices Paid for Production Items.

[b] $\Delta\hat{Y}_{xi}$ = the change in the estimated Y due to the xi.

[c] Actual change in the quantity of nitrogen consumed = 24.973.

[d] Actual change in the quantity of nitrogen consumed = 32.398.

Table 12. Percent of the change in nitrogen consumption between 1950 and 1964 explained by the economic variables and time - Model N-1

State	Percent explained by: Price of nitrogen	Price of main crop	Price of secondary crop	Net farm income	Time
	------Percent------				
Me.	18.3	8.9		8.7	64.0
N.H.	22.2	1.8		6.3	69.7
Vt.	17.1	6.7		5.1	71.1
Mass.	24.1	2.3		0.7	72.9
R.I.	17.5	1.0		8.5	73.0
Conn.	25.3	7.6		1.1	66.0
N.Y.	15.2	6.7		7.4	70.8
N.J.	10.4	5.2		0.4	84.0
Pa.	28.3	6.1		0.7	64.9
O.	16.5	5.5		0.9	77.0
Ind.	16.3	4.5		6.6	72.6
Ill.	13.3	4.7		8.9	73.1
Mich.	22.3	15.7		0.2	61.8
Wisc.	10.7	10.4		3.4	75.5
Minn.	8.8	16.7		4.4	70.1
Ia.	17.4	5.8		5.4	71.3
Mo.	17.4	3.6		3.9	75.2
N.D.	16.2	17.8		11.0	55.0
S.D.	7.9	16.4		12.4	63.3
Neb.	15.1	13.7		4.7	66.4
Kan.	16.3	18.6		4.8	60.3
Del.	14.9	0.4		3.2	81.5
Md.	30.0	0.3		1.4	68.3
Va.	36.8	0.2		3.1	59.9
W.Va.	14.2	0.3		3.9	81.5
N.C.	22.9	21.7*[a]		4.2	51.2
S.C.	23.8	18.9*	0.5	4.6	52.2
Ga.	25.1	18.2*		6.8	48.0
Fla.	25.8	27.6*		2.7	43.8
Ky.	34.0	2.0		4.6	59.4
Tenn.	31.3	2.4	0.2	0.9	65.1
Ala.	26.4	1.9		8.0	63.7
Miss.	33.4	0.8	1.2	11.7	53.0
Ark.	12.7	1.5		15.4	70.4
La.	23.5	1.5		8.9	66.1
Okla.	25.0	13.4	3.3	1.6	56.7
Tex.	31.5	3.4		1.5	63.6
Mont.	17.9	16.7		14.0	51.3
Id.	20.3	6.4		2.7	70.6
Wyo.	16.0	18.6		11.9	53.5

Table 12--Continued.

State	Percent explained by Price of nitrogen	Price of main crop	Price of secondary crop	Net farm income	Time
		------Percent------			
Colo.	24.5	13.8		5.3	56.4
N.Mex.	39.1	1.9		11.4	49.3
Ariz.	22.2	0.0		44.1	33.7
Ut.	20.3	15.9		4.0	59.9
Nev.	26.4	6.7		20.2	46.8
Wash.	39.6	7.1		0.4	52.9
Ore.	37.5	9.0		2.5	50.9
Calif.	55.2	0.1		11.0	33.7
U.S.	22.6	8.1	1.3	6.6	62.5

[a]* means the estimated coefficient of Model N-1 has incorrect sign and is significant at the .05 level.

states. The decline in price of nitrogen explains about 10 to 40 percent of the increase in nitrogen consumption since 1950, depending upon the particular state in question. States with marked declines in nitrogen price generally attribute a greater share of the increased consumption to this decline. Changes in net farm income explain little of the increase in nitrogen consumption as is true of changes in the crop price. Time, which includes technological change and the increased awareness of farmers explains more than half the increased consumption.

Phosphate Model

The model postulated as representing the demand for available phosphate is:

Model P-1 (time constants)

$$Q^{p}_{t,s} = \alpha + \sum^{g} \beta_g P^{p}_{t,s} + \beta Y_{t-1,s} + \sum^{k} \beta_k P^{c}_{t-1,s} + \sum^{i} \beta_i S_{*,s}$$

$$+ \sum^{j} \beta_j T_{t,*} + U_{t,s}$$

$\bar{R}^2 = .9799$ Standard error of residuals = 4.72

F = 428.24

where:

α = the overall constant term

g = variables X_{83} through X_{88}

X_{83} = price of phosphate in Me., N.H., Vt., Mass., R.I., Conn., N.Y., N.J. and Pa.

X_{84} = price of phosphate in O., Ind., Ill., Mich., Wisc., Minn., Ia., Mo., N.D., S.D., Neb. and Kan.

X_{85} = price of phosphate in Del., Md., Va., W.Va., N.C., S.C., Ga., Fla., Ky., Tenn., Ala. and Miss.

X_{86} = price of phosphate in Ark., La., Okla. and Tex.

X_{87} = price of phosphate in Mont., Id., Wyo., Colo., N.Mex., Ariz., Ut. and Nev.

X_{88} = price of phosphate in Wash., Ore. and Calif.

X_{67} = average net farm income

k = variables X_4 through X_{17} = crop prices

X_4 = price of hay in N.H., Vt., Mass., R.I. and Conn.

X_5 = price of cotton in Ariz. and Calif.

X_6 = price of corn in O., Ind., Ill., Ia. and Mo.

X_7 = price of corn in Del., Md., Va. and W.Va.

X_8 = price of corn in Ky., Tenn., Ala. and Miss.

X_9 = price of cotton in S.C., Ga., Ala., Miss., Ark., La. and Tex.

X_{10} = price of potatoes in Me.

X_{11} = price of wheat in Kan. and Okla.

X_{12} = price of hay in Wash., Ore. and Calif.

X_{13} = price of hay in N.Y., N.J. and Pa.

X_{14} = price of hay in Id., Wyo., Colo., N.Mex., Ut. and Nev.

X_{15} = price of wheat in N.D. and Mont.

X_{16} = price of corn in Mich., Wisc., Minn., S.D. and Neb.

X_{17} = price of corn in N.C., S.C., Ga. and Fla.

i = variables X_{18} through X_{64} = constant for each state except Michigan which is included in α, the overall constant (States, with their variable number are listed in Table 15)

j = variables X_{68} through X_{81} = time constant for each year: 1951 through 1964. The overall constant, α, includes 1950.

The same format used in presenting and discussing the nitrogen model will be followed for the phosphate model.

Table 13 presents the estimated coefficients of phosphate price in Model P-1 along with the standard errors of the coefficients and the calculated value of the t statistic. Four of the six regional coefficients have the correct sign, but only one is significantly different from zero (.05 level): the South East and East South Central Region. This region's response to price changes may be more sensitive and consistent because more nearly equilibrium quantities of phosphate are consumed. Historically this region has had more experience with fertilizer; many of these states were applying more fertilizer in 1950 than others fifteen years later. States outside this region may be less sensitive to price changes (in the range experienced in this period) because the marginal return to fertilizer remains above the marginal cost.[56]

Table 13. Estimated regression coefficients of phosphate price variables and the net income variable - Model P-1

Variable	Estimated regression coefficient, $\hat{\beta}$	Standard error regression coefficient	t statistic
X(83)	-.410	.233	-1.76*[a]
X(84)	.472	.229	2.06**[b]
X(85)	-.648	.189	-3.42**
X(86)	-.514	.296	-1.73*
X(87)	.035	.219	.16
X(88)	-.143	.372	-.38
where phosphate price index, 1950 = 100.0			
X(67)	.180	.065	2.75*
where income is in hundreds of dollars			

[a]* means the null hypothesis (β_i=0) is rejected at .10 level of significance.

[b]** means the null hypothesis (β_i=0) is rejected at .05 (or lower) level of significance.

Changing the specification of the time variable in the phosphate models affects the estimated coefficients differently than

[56]An argument based on a study of all fertilizer is relevant to phosphate as well: D. B. Ibach and R. C. Lindbert, The Economic Position of Fertilizer Use in the United States, U.S.D.A., Agriculture Information Bulletin No. 202 (1958), p. 8. The estimated marginal return per $1.00 input to all fertilizer at the 1954 average level of use was $3.06 for corn and $3.40 for all intertilled crops.

More recent (preliminary - based on 1963 rates of application) estimates still indicate the marginal return to all fertilizer is greater than two dollars.

identical changes in the nitrogen models. Appendix VI shows the estimated coefficients of Model P-2. The phosphate price coefficients in Model P-1 are nearly the same as those in P-3 which are greater than the estimates of Model P-2. In nitrogen, the reverse was true.

As in the set of phosphate price variables, the range of crop price movements is insufficient in most regions to affect the consumption of phosphate. Table 14 shows that the price of hay in the New England states (Maine, New Hampshire, Vermont, Massachusetts, Rhode Island and Connecticut) and the price of corn in Delaware, Maryland, Virginia and West Virginia significantly affect the consumption of phosphate. The price of corn in the South East is again significant but with the unexpected sign. With the limited number of significant estimates, it appears a bit hollow to speak of correct signs but never-the-less, eleven of the fourteen estimates have the correct sign.

The estimated state constant terms in Table 15 show that in addition to accounting for differences in soils, crops, climatic and other environmental factors, the estimates consider the relative rate of phosphate consumption in different states.

The set of time constants, included to pick up changes in phosphate consumption due to the year are shown in Table 16. These estimates follow nearly the same pattern traced by the time constants in Model N-1: an upward trend broken by a slight decline in 1954, 1956 and 1958. But the time constants of Model P-1 are only about one-half those of Model N-1.

To test whether sets of variables influence the dependent variable the F statistic (referred to in footnote 54, page 69) for several sets is presented in Table 17. Computer capacity and programming

Table 14. Estimated regression coefficients of crop price variables - Model P-1

Variable	Unit of measure (real $)	Estimated regression coefficient, $\hat{\alpha}$	Standard error regression coefficient	t statistic
X(4)	1.00/ton hay	2.925	.460	6.36**[b]
X(5)	.01/lb. cotton	-.393	.627	-.63
X(6)	.10/bu. corn	.687	.698	.98
X(7)	.10/bu. corn	-2.273	1.033	-2.20**
X(8)	.10/bu. corn	-1.942	.909	-2.14**
X(9)	.01/lb. cotton	.600	.449	1.34
X(10)	.10/bu. potatoes	1.094	.752	1.46
X(11)	.10/bu. wheat	2.378	1.403	1.69*[a]
X(12)	1.00/ton hay	.175	.649	.27
X(13)	1.00/ton hay	.431	.387	1.12
X(14)	1.00/ton hay	-.066	.522	-.13
X(15)	.10/bu. wheat	1.907	1.536	1.24
X(16)	.10/bu. corn	1.622	.737	2.20**
X(17)	.10/bu. corn	-6.609	.947	-6.98**

[a]* means the null hypothesis (α_i=0) is rejected at the .10 level of significance.

[b]** means the null hypothesis (α_i=0) is rejected at the .05 (or lower) level of significance.

restrictions prevent applying this test to Model P-1, but the test was applied to Model P-3, where time is entered as a linear variable. From these, inferences can be made to Model P-1. All the sets, taken as a complete set, contribute significantly to the explained variation in phosphate consumption. But this is not always true of the subsets: particularly the wheat and cotton price variables. The presence of large government price support programs which increased price stability of these two crops during the period studied may explain the lack of significance. Of the state constants only the Pacific states of Washington and Oregon and California add little to the explained changes in phosphate consumption.

Table 15. Estimated regression coefficients of state constants for 48 states - Model P-1

Variable	Estimated regression coefficient, $\hat{\alpha}$	Standard error regression coefficient	t statistic
Mich.[a]	0.0	0.0	na[b]
X(18) Me.	66.536	14.163	4.698
X(19) N.H.	5.891	15.065	.391
X(20) Vt.	8.220	14.976	.549
X(21) Mass.	30.837	15.736	1.960
X(22) R.I.	70.533	15.523	4.544
X(23) Conn.	30.940	15.829	1.955
X(24) N.Y.	49.231	14.310	3.440
X(25) N.J.	91.034	15.060	6.045
X(26) Pa.	52.489	14.539	3.610
X(27) O.	6.892	4.619	1.492
X(28) Ind.	5.792	4.586	1.263
X(29) Ill.	-6.986	4.730	-1.477
X(30) Wisc.	-12.736	1.782	-7.146
X(31) Minn.	-17.028	1.749	-9.738
X(32) Ia.	-12.648	4.630	-2.732
X(33) Mo.	-6.583	4.718	-1.395
X(34) N.D.	-29.892	12.465	-2.398
X(35) S.D.	-24.169	1.918	-12.603
X(36) Neb.	-23.666	1.821	-12.994
X(37) Kan.	-31.315	10.889	-2.876
X(38) Del.	84.046	15.414	5.452
X(39) Md.	85.776	15.216	5.637
X(40) Va.	207.399	15.111	13.725
X(41) W.Va.	51.481	15.334	3.357
X(42) N.C.	117.065	14.518	8.064
X(43) S.C.	101.503	16.202	6.265
X(44) Ga.	101.752	15.907	6.397
X(45) Fla.	201.814	14.127	14.286
X(46) Ky.	73.983	14.793	5.001
X(47) Tenn.	70.988	15.285	4.644
X(48) Ala.	82.316	16.516	4.984
X(49) Miss.	54.302	16.651	3.261
X(50) Ark.	29.666	18.739	1.583
X(51) La.	38.155	18.573	2.054
X(52) Okla.	12.764	19.813	.644
X(53) Tex.	25.511	18.020	1.416
X(54) Mont.	-12.086	18.095	-.668
X(55) Id.	5.090	14.182	.359
X(56) Wyo.	-.834	14.150	-.059
X(57) Colo.	1.126	13.993	.081

Table 15--Continued.

Variable	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient	t statistic
X(58) N.Mex.	8.545	13.923	.614
X(59) Ariz.	25.514	16.757	1.523
X(60) Ut.	5.848	13.740	.426
X(61) Nev.	-1.787	13.827	-.129
X(62) Wash.	14.309	21.808	.656
X(63) Ore.	17.368	21.855	.795
X(64) Calif.	32.063	23.687	1.354

[a]To avoid singularity of the X'X matrix one state (Michigan) is arbitrarily included in the overall constant term. Because of this construction all other state constant terms should be interpreted as deviations ($\pm$) from the estimated overall constant ($\hat{a}$=-9.317). Statistical tests can be interpreted as tests of significant differences between any particular state and Michigan.

[b]na means not applicable.

The contribution of each variable to the explanation of total change is examined for each state in Table 18. Again, as in the nitrogen model, the price and time variables contribute most to the explanation with only few exceptions. The importance of time is even greater than in the nitrogen model.

Table 16. Estimated regression coefficients of time constants for the years 1951 through 1964 - Model P-1

Variable	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient	t statistic
X(68) 1951	.539	1.189	.45
X(69) 1952	1.271	1.135	1.12
X(70) 1953	2.330	1.021	2.28
X(71) 1954	2.067	1.030	2.01
X(72) 1955	3.801	1.026	3.71
X(73) 1956	3.584	1.063	3.37
X(74) 1957	5.224	1.002	5.21
X(75) 1958	4.916	1.042	4.72
X(76) 1959	6.549	1.040	6.30
X(77) 1960	8.403	1.050	8.00
X(78) 1961	9.980	1.077	9.26
X(79) 1962	11.768	1.056	11.15
X(80) 1963	13.088	1.044	12.54
X(81) 1964	14.131	1.042	13.56

overall constant term, $\hat{\alpha} = 6.264$

Table 17. Evaluation of the contribution attributable to sets of variables[a] - Model P-1 and Model P-3

Variables included in the set[b]		F statistic	Degrees of freedom[c]	Level of significance	$\bar{R}^2$ [d]
Model P-3					
X(83)...X(88)	Phosphate price	5.454	6;650	.001	.9787
X(4)...X(17)	Crop prices	8.183	14;650	.001	.9764
X(6)...X(8), X(16), X(17)	Corn price	12.312	5;650	.001	.9778
X(4), X(12), X(14)	Hay price	10.977	4;650	.001	.9783
X(5), X(9)	Cotton price	1.343	2;650	[e]	.9795
X(11), X(15)	Wheat price	2.720	2;650	.10	.9794
X(18)...X(64)	State constants	349.949	47;650	.001	.4978
X(18)...X(23)	New England	282.589	6;650	.001	.9268
X(27)...X(33)	Corn Belt	38.869	7;650	.001	.9713
X(40)...X(49)	SE & ESC States	1,071.692	10;650	.001	.6474
X(54)...X(61)	Rocky Mountain	8.167	8;650	.001	.9777
X(62)...X(64)	West Coast	2.445	3;650	.10	.9794
Model P-1					
X(68)...X(81)	Time constants	26.372	14;637	.001	.9689

[a]Sets of variables are restricted (essentially excluded from the equation) to test whether they significantly improve explanation.

[b]Variables are defined on pp. 77, 78.

[c]Degrees of freedom are listed with d.f. associated with numerator given first; d.f. associated with denominator second.

[d]This is the $\bar{R}^2$ when the specified variables are excluded from Model P-1 or Model P-3.

[e]Not significant at the .10 level.

Table 18. Percent of the change in phosphate consumption between 1950 and 1964 explained by the economic variables and time - Model P-1

State	Percent explained by				
	Price of phosphate	Price of main crop	Price of secondary crop	Net farm income	Time
	------Percent------				
Me.	2.9	16.4		4.6	76.2
N.H.	0.2	13.4		3.3	83.0
Vt.	2.1	35.2		1.9	60.8
Mass.	2.6	15.6		0.4	81.5
R.I.	2.3	7.0		4.4	86.3
Conn.	4.5	24.2		0.5	70.8
N.Y.	11.3	4.1		3.7	80.9
N.J.	14.5	7.2		0.2	78.1
Pa.	7.1	7.9		0.4	84.6
O.	10.1*[a]	5.3		0.4	84.1
Ind.	0.6*	4.9		3.7	90.8
Ill.	12.8*	4.5		4.2	78.4
Mich.	12.5*	11.3		0.1	76.1
Wisc.	7.1*	6.8		1.7	84.5
Minn.	4.6*	11.4		2.3	81.7
Ia.	4.1*	6.2		2.9	86.8
Mo.	10.7*	3.5		1.9	83.9
N.D.	5.2*	10.7		6.8	77.3
S.D.	5.1*	11.6		6.6	76.6
Neb.	2.5*	10.2		2.7	84.6
Kan.	2.4*	13.4		2.8	81.3
Del.	20.1	9.0*		1.2	69.7
Md.	11.4	8.0*		0.7	80.0
Va.	11.3	7.3*		1.8	79.6
W.Va.	12.6	7.4*		1.7	78.3
N.C.	19.2	20.6*		2.1	58.1
S.C.	27.6	16.3*	0.3	2.1	53.7
Ga.	9.9	20.2*	1.5	5.0	63.4
Fla.	13.9	25.9*		11.2	49.0
Ky.	8.2	45.7*		1.5	44.6
Tenn.	28.5	9.7*		0.4	61.4
Ala.	14.1	8.9*	1.4	4.0	71.6
Miss.	26.0	4.1*	0.8	6.1	63.0
Ark.	15.5	1.0		7.3	76.2
La.	13.2	1.1		4.8	80.9
Okla.	0.3	11.0		1.1	87.5
Tex.	1.0	3.0		1.0	95.0
Mont.	0.7	11.1		9.5	78.7
Id.	0.4	0.4		1.7	97.5
Wyo.	0.0	0.8		8.9	90.3

Table 18--Continued.

State	Percent explained by: Price of phosphate	Price of main crop	Price of secondary crop	Net farm income	Time
	------Percent------				
Colo.	0.9	0.7		3.9	94.4
N.Mex.	1.0	0.9		9.1	89.0
Ariz.	0.2	0.0		36.5	63.2
Ut.	0.2	0.6		2.8	96.3
Nev.	0.4	19.1		12.8	67.6
Wash.	1.5	2.7		0.3	95.6
Ore.	4.9	2.5		2.0	90.6
Calif.	1.4	1.0	1.4	12.1	84.0
U.S.	7.7	9.8	1.1	4.3	78.1

[a]* means the estimated coefficient of Model P-1 has incorrect sign and is significant at the .05 level.

Potash Model

The model postulated as representing the demand for potash (K_2O) is:

Model K-1 (time constants)

$$Q^k_{t,s} = \alpha + \sum^g \beta_g P^k_{t,s} + \beta Y_{t-1,s} + \sum^k \beta_k P^c_{t-1,s} + \sum^i \beta_i S_{*,s} + \sum^j \beta_j T_{t,*} + U_{t,s}$$

$\bar{R}^2 = .9569$ Standard error of residuals = 8.05

F = 197.97

where:

α = the overall constant term

g = variables X_{83} through X_{88} = potash price

X_{83} = price of potash in Me., N.H., Vt., Mass., R.I., Conn., N.Y., N.J. and Pa.

X_{84} = price of potash in O., Ind., Ill., Mich., Wisc., Minn., Ia. and Mo.

X_{85} = price of potash in Del., Md., Va., W.Va., N.C., S.C., Ga. and Fla.

X_{86} = price of potash in Ky., Tenn., Ala., Miss., Ark. and La.

X_{87} = price of potash in N.D., S.D., Neb., Kans., Okla., Tex., Mont., Id., Wyo., Colo., N.Mex., Ariz., Ut. and Nev.

X_{88} = price of potash in Wash., Ore. and Calif.

X_{67} = average net farm income

k = variables X_4 through X_{16} = crop prices

X_4 = price of hay in N.H., Vt., Mass., R.I. and Conn.

X_5 = price of potatoes in Me.

X_6 = price of corn in Ohio, Ind., Ill., Ia. and Mo.

X_7 = price of corn in Del., Md., Va. and W.Va.

X_8 = price of corn in Ky., Tenn., Ala. and Miss.

X_9 = price of cotton in S.C., Ga., Miss., Ark., La., Okla. and Tex.

X_{10} = price of potatoes in N.D., Mont., Id., Wyo., Colo., Ut., and Nev.

X_{11} = price of wheat in Kan.

X_{12} = price of cotton in N.Mex., Ariz. and Calif.

X_{13} = price of corn in N.C., S.C., Ga. and Fla.

X_{14} = price of hay in Wash. and Ore.

X_{15} = price of corn in N.Y., N.J. and Pa.

X_{16} = price of corn in Mich., Wisc., Minn., S.D. and Neb.

i = variables X_{18} through X_{64} = constants for each state except Michigan which is included in the overall constant

j = variables X_{68} through X_{81} = time constant for each year: 1951 through 1964. The overall constant, α, includes 1950.

Again, the results will be presented in the same format as the other two nutrient models.

Table 19 presents the estimated coefficients of potash price variables in Model K-1 with their standard errors and the calculated value of the t statistic. Note that state groupings for the potash price variables differ considerably from Models N-1 and P-1. Several of the Western states are grouped together because of the limited consumption by those states. As in the phosphate model, the states with historically higher level of potash consumption respond to price changes as expected. That is, price and consumption of potash vary inversely. But in all other regions consumption varies directly with price. Recall that the observations are states and not individual farms. And while it may be illogical for a firm in equilibrium to employ more of an input whose price is rising, it may not be illogical for a state to consume more potash even though price is rising. This can occur when some farmers increase consumption from zero to something greater than zero and (or) when they increase consumption from less than an optimum profit rate to a higher rate. And the states which have historically used little potash are those with the positive coefficient estimates.

The estimated coefficients for the crop prices and their standard errors are shown in Table 20. The hay price variable in the New England states is significant and both cotton price variables are significant. But, the other variables have either incorrect sign or are not significant.

Table 19. Estimated regression coefficients of potash price variable and the net income variable - Model K-1

Variable	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient	t statistic
X(83)	-1.761	.852	-2.07*[a]
X(84)	.121	.503	.24
X(85)	-7.451	.971	-7.68*
X(86)	-1.347	.997	-1.35
X(87)	1.249	.271	4.61*
X(88)	1.080	1.434	.75
where potash price is the price per ton of K_2O in muriate of potash, deflated by the Index of Prices Paid for production items: the deflated price of K_2O			
X(67)	.345	.108	3.19*
where income is in hundreds of dollars			

[a]* means the null hypothesis (a_i=0) is rejected at .05 level of significance

The state constant terms for Model K-1 presented in Table 21 differ from the state constants of Model P-1 mainly in size. Treating the constants as deviations from the level of consumption in Michigan supports the utilization of state constants as a means of catching differences among states. Thirty-one of the constants differ significantly from the overall constant at the .05 level.

The estimated time constants in Table 22 follow the same pattern as the time constants for the two nutrients above: an upward trend broken in 1956 and again slightly in 1959.

The sets of variables discussed in Tables 19, 20, 21 and 22 are tested as sets, using the F test described earlier. Table 23 presents the test statistics and the respective levels at which the null hypothesis (i.e., the set of variables contribute nothing to the explanation) is rejected. All major sets of variables - the potash price variables, crop price variables, state constants and time constants significantly influence the dependent variable. But, as in the phosphate model, some crop prices do not significantly improve the explained variation in the quantity of potash consumed. The $\bar{R}^2$ under the hypothesis is affected most by the set of state constants and it is apparent that the subset of states consisting of the Southeastern states contribute a disproportionate share.

Table 20. Estimated regression coefficients of crop price variables - Model K-1

Variable	Unit of measure (real $)	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient	t statistic
X(4)	1.00/ton hay	2.360	.778	3.03*[a]
X(5)	.10/bu. potatoes	.770	1.289	.60
X(6)	.10/bu. corn	.668	1.276	.52
X(7)	.10/bu. corn	-10.060	1.612	-6.24*
X(8)	.10/bu. corn	-3.745	1.404	-2.67*
X(9)	.01/lb. cotton	1.484	.675	2.20*
X(10)	.10/bu. potatoes	.017	.501	.03
X(11)	.10/bu. wheat	2.168	3.304	.66
X(12)	.01/lb. cotton	1.743	.853	2.04*
X(13)	.10/bu. corn	-23.953	1.567	-15.28*
X(14)	1.00/ton hay	1.471	1.127	1.31
X(15)	.10/bu. corn	-.527	1.619	-.33
X(16)	.10/bu. corn	.481	1.308	.37

[a]* means the null hypothesis (a_i=0) is rejected at the .05 level of significance.

Table 24 shows the contribution of nutrient price, crop price (or prices) income and the time constant to the change in potash consumption from 1950 to 1964. The importance of a particular variable, say price of potash, varies considerably from state to state. Even within the Southeast and East South Central Regions where price of potash is very significant the percent of the change in consumption explained by nutrient price varies from 7 to 52 percent. In general, however, crop prices and net income contribute less to the explanation than the time variable in all but the Southeast and Arizona where time contributes less than 50 percent of the explanation.

Table 21. Estimated regression coefficients of state constants for 48 states - Model K-1

Variable	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient	t statistic
Mich.[a]	0.0	0.0	na[b]
X(18) Me.	76.029	24.306	3.128
X(19) N.H.	15.314	23.846	.642
X(20) Vt.	14.601	23.657	.617
X(21) Mass.	36.036	23.676	1.522
X(22) R.I.	77.823	23.606	3.297
X(23) Conn.	37.279	23.690	1.574
X(24) N.Y.	43.517	22.107	1.969
X(25) N.J.	88.807	22.264	3.989
X(26) Pa.	50.290	22.544	2.231
X(27) O.	.849	8.003	.106
X(28) Ind.	5.298	8.047	.658
X(29) Ill.	-13.466	8.145	-1.653
X(30) Wisc.	-7.914	2.951	-2.682
X(31) Minn.	-18.943	2.998	-6.318
X(32) Ia.	-22.376	7.996	-2.798
X(33) Mo.	-13.912	8.114	-1.715
X(34) N.D.	-49.343	12.205	-4.043
X(35) S.D.	-51.685	11.812	-4.376
X(36) Neb.	-52.039	11.974	-4.346
X(37) Kan.	-63.974	26.607	-2.404
X(38) Del.	241.591	23.770	10.164
X(39) Md.	237.206	23.687	10.014
X(40) Va.	359.924	23.860	15.085
X(41) W.Va.	210.355	23.931	8.790
X(42) N.C.	329.620	26.194	12.584
X(43) S.C.	290.945	25.783	11.284
X(44) Ga.	292.389	25.936	11.274
X(45) Fla.	443.091	24.941	17.765
X(46) Ky.	60.544	22.075	2.743
X(47) Tenn.	56.391	21.496	2.623

Table 21--Continued.

Variable	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient	t statistic
X(48) Ala.	50.208	23.652	2.123
X(49) Miss.	24.873	22.879	1.087
X(50) Ark.	3.478	24.180	.144
X(51) La.	5.072	24.094	.211
X(52) Okla.	-60.334	13.751	-4.388
X(53) Tex.	-61.863	13.941	-4.437
X(54) Mont.	-51.141	12.868	-3.974
X(55) Id.	-47.571	12.149	-3.916
X(56) Wyo.	-50.826	12.373	-4.108
X(57) Colo.	-48.563	12.341	-3.935
X(58) N.Mex.	-70.323	16.184	-4.345
X(59) Ariz.	-85.257	17.438	-4.889
X(60) Ut.	-46.945	12.388	-3.790
X(61) Nev.	-51.626	12.576	-4.105
X(62) Wash.	-55.607	34.623	-1.606
X(63) Ore.	-54.810	34.739	-1.578
X(64) Calif.	-63.662	34.137	-1.865

[a]To avoid singularity of the X'X matrix one state (Michigan) is arbitrarily included in the overall constant term. Because of this construction all other state constant terms should be interpreted as deviations ($\pm$) from the estimated overall constant ($\hat{a}$=8.065). Statistical tests can be interpreted as tests of significant differences between any particular state and Michigan.

[b]na means not applicable.

Table 22. Estimated regression coefficients of time constants for the years 1951 through 1964 - Model K-1

Variable	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient	t statistic
X(68) 1951	-.143	2.149	-.06
X(69) 1952	1.830	2.095	.87
X(70) 1953	5.794	1.798	3.22
X(71) 1954	6.073	1.804	3.37
X(72) 1955	9.042	1.882	4.81
X(73) 1956	6.159	1.832	3.36
X(74) 1957	8.326	1.900	4.38
X(75) 1958	8.627	1.960	4.40
X(76) 1959	8.311	2.097	3.96
X(77) 1960	9.617	2.180	4.41
X(78) 1961	11.543	2.117	5.45
X(79) 1962	13.990	2.010	6.96
X(80) 1963	16.067	2.099	7.65
X(81) 1964	16.443	2.055	8.00

overall constant term, $\hat{\alpha} = 8.065$

Table 23. Evaluation of the contribution attributable to sets of variables[a] - Model K-1 and Model K-3

Variables included in the set[b]		F statistic	Degrees of freedom[c]	Level of signifi- cance	$\bar{R}^2$ [d]
Model K-3					
X(83)...X(88)	Potash price	15.745	6;651	.001	.9506
X(4)...X(16)	Crop prices	21.572	13;651	.001	.9389
X(6)...X(8), X(13), X(15), X(16)	Corn price	42.419	6;651	.001	.9400
X(4), X(14)	Hay price	4.711	2;651	.01	.9560
X(5), X(10)	Potato price	.178	2;651	[e]	.9566
X(9), X(12)	Cotton price	4.063	2;651	.05	.9560
X(18)...X(64)	State constants	151.420	47;651	.001	.5153
X(18)...X(23)	New England	98.698	6;651	.001	.9176
X(27)...X(33)	Corn Belt	21.981	7;651	.001	.9467
X(40)...X(49)	SE & ESC States	450.520	10;651	.001	.6603
X(54)...X(61)	Rocky Mountain	3.488	8;651	.001	.9551
X(62),..X(64)	West Coast	1.490	3;651	[e]	.9563
Model K-1					
X(68)...X(81)	Time constants	9.463	14;638	.001	.9488

[a]Sets of variables are restricted (essentially excluded from the equation) to test whether they significantly improve explanation.

[b]Variables are defined on pp. 87-89.

[c]Degrees of freedom are listed with d.f. associated with numerator given first; d.f. associated with denominator second.

[d]This is the $\bar{R}^2$ when the specified variables are excluded from Model K-1 and Model K-3.

[e]Not significant at the .10 level.

Table 24. Percent of the change in potash consumption between 1950 and 1964 explained by the economic variables and time - Model K-1

State	Percent explained by				
	Price of potash	Price of main crop	Price of secondary crop	Net farm income	Time
	------Percent------				
Me.	14.9	9.2		5.5	70.5
N.H.	14.1	8.3		3.8	73.8
Vt.	11.7	24.6		2.5	61.2
Mass.	25.4	8.7		0.4	65.5
R.I.	23.7	3.8		4.5	68.0
Conn.	22.0	14.5		0.6	63.0
N.Y.	4.9	1.9		5.2	88.0
N.J.	5.2	1.3		0.3	93.2
Pa.	21.4	1.7		0.4	76.5
O.	1.4	4.9		0.6	93.1
Ind.	2.8	4.0		4.6	88.6
Ill.	3.5	4.1		6.0	86.4
Mich.	2.9	3.5		0.2	93.4
Wisc.	3.1	1.9		2.4	92.7
Minn.	4.8	3.2		3.2	88.9
Ia.	4.7	5.2		3.7	86.4
Mo.	2.6	3.2		2.7	91.5
N.D.	4.7*[a]	0.1		9.7	85.6
S.D.	4.5*	3.2		9.2	83.0
Neb.	4.8*	2.7		3.6	88.8
Kan.	4.4*	10.5		3.7	81.4
Del.	28.5	23.2*		1.1	47.3
Md.	30.2	19.0*		0.6	50.2
Va.	45.9	13.7*		1.1	39.2
W.Va.	45.7	14.1*		1.1	39.1
N.C.	8.9	46.8*		2.0	42.3
S.C.	27.7	34.1*	0.4	1.8	36.0
Ga.	15.3	39.1*	2.0	4.0	39.5
Fla.	36.2	35.7*		6.4	21.7
Ky.	54.3	8.5*		1.6	35.6
Tenn.	54.6	9.4*		0.3	35.7
Ala.	52.5	7.5*	1.5	2.6	36.0
Miss.	42.0	4.9*	1.3	5.8	46.0
Ark.	18.4	1.9		8.8	70.9
La.	12.1	2.3		6.1	79.5
Okla.	13.4*	5.8		1.3	79.5
Tex.	29.7*	4.3		0.9	65.0
Mont.	32.8*	0.1		9.0	58.0
Id.	31.5*	0.1		1.5	67.0
Wyo.	33.4*	0.1		7.5	59.0

Table 24--Continued.

State	Percent explained by				
	Price of potash	Price of main crop	Price of secondary crop	Net farm income	Time
	------Percent------				
Colo.	27.9*	0.0		3.7	68.4
N.Mex.	26.1*	4.1		8.1	61.7
Ariz.	21.4*	0.1		33.5	45.0
Ut.	23.3*	0.0		2.8	73.9
Nev.	26.2*	0.1		14.5	59.2
Wash.	12.4	14.8		0.3	72.5
Ore.	10.2	14.5		2.1	73.2
Calif.	11.9	4.6		13.1	70.4
U.S.	20.0	8.9	1.3	4.4	66.4

[a]* means the estimated coefficient of Model K-1 has incorrect sign and is significant at the .05 level.

Some Problems Common to All Nutrient Models

Before comparing the estimates of the three nutrient models, the measures of serial correlation and multicollinearity discussed in Chapter II will be considered. First, look at the problem of multicollinearity. The pairwise correlation between predetermined variables is considered first. Among the economic variables the highest simple correlation is .58, .69 and .82 in Models N-1, P-1 and K-1 respectively. But the simple correlations between state constants and other variables runs as high as .80, .95 and .99 for the three models. One aim in selecting the covariance model was to reduce the problem of multicollinearity, but these levels of correlation suggest that the model has not met the objective. However, these high correlations result from the use of dummy variables and not from the model. The high correlations between economic variables are between the regional nutrient price and the main crop price where the states included in the two variables coincide. When response to changes in nutrient price is assumed to be similar in all states - one nutrient price variable - the highest simple correlations are .31, .37 and .32. The high correlations between state constants and other variables is likewise a result of the dummy variables. For example, the .95 correlation found in Model P-1 is between the state constant and the price of potatoes in Maine and the .99 correlation in Model K-1 is between the state constant for Kansas and the price of wheat in Kansas.

A measure of multicollinearity proposed in Chapter II, the coefficient of multiple correlation, is presented in Table 25. Economic variables selected from the set of independent variables of the nitrogen, phosphate and potash models presented above are regressed upon the remaining independent variables or a subset of them.

The coefficient of multiple correlation provides a reasonable measure of multicollinearity. The simple correlations of equation (1) between the dependent variable (the price of nitrogen) and the independent variables (net farm income and state constants) are all .205 or less; yet the multiple correlation is .407. The highest simple correlation between net farm income and any of the independent variables in equation (2) is .770: the association between net farm income and the state constant for Arizona. In equation (3) the highest simple correlation is .395. Chapter II discussed rules of thumb based on simple correlation as a measure of multicollinearity. For example, if the simple correlation is greater than .6, then the rule suggests that multicollinearity is a problem. Examination of these few nitrogen model variables clearly implies that high multicollinearity is not inconsistent with low simple correlations. In fact, simple correlation usually underestimates the degree of multicollinearity.

Table 25. The coefficient of multiple correlation as a measure of multicollinearity

Equation	Dependent variable	Independent variable	R^2
		------Variables from nitrogen models------	
1.	X(2):	X(67), X(4)...X(64)	.407
2.	X(67):	X(83)...X(88), X(4)...X(66)	.890
3.	X(67):	X(83)...X(88), X(4)...X(17)	.494
4.	X(16):	X(83)...X(88), X(4)...X(15), X(17)...X(66), X(67)	.974
		------Variables from phosphate models------	
5.	X(2):	X(67), X(4)...X(64)	.412
6.	X(83):	X(84)...X(88), X(67), X(4)...X(66)	.997
7.	X(84):	X(83), X(85)...X(88), X(67), X(4)...X(17)	.978
8.	X(67):	X(83)...X(88), X(4)...X(66)	.888
9.	X(67):	X(83)...X(88), X(4)...X(17)	.727
10.	X(16):	X(83)...X(88), X(4)...X(15), X(17)...X(66), X(67)	.970
		------Variables from potash models------	
11.	X(2):	X(67), X(4)...X(16), X(18)...X(64)	.483
12.	X(67):	X(83)...X(88), X(4)...X(16), X(18)...X(64)	.883
13.	X(67):	X(83)...X(88), X(4)...X(16)	.493
14.	X(16):	X(83)...X(88), X(67), X(4)...X(15), X(18)...X(64)	.972

In equation (5) X_2 is the price of phosphate in the U.S. Equation (7) disperses this one variable over six synthetic variables. The resulting R^2's of these two equations demonstrate, too well, the risk of multicollinearity with dummy variables. Intuition might suggest a lower R^2 for equation (7) than for equation (5) since the number of independent variables is less. But the dispersion of X_2 into six regional variables instead, increases the R^2. Obviously a choice between more refined variables (refined in the sense that they are less general) and wider variances of the estimates is necessary.

In this study, the six nutrient price variables were preferred because a single variable implies an equal response to a 1 unit change in price throughout the country. This is unlikely when the consumption per acre varies from less than 10 pounds to well over 150 pounds.

The coefficient of multiple correlation, where one of the independent variables is regressed on other independent variables, remains only a measure of multicollinearity; a measure which has advantages over the usual measures - simple correlation and size of the determinant. The basic question of what level of multicollinearity is tolerable remains with the researcher after consideration of the particular variables involved.

The Durbin-Watson[57] statistic - testing for serial correlation in the residuals - was computed for the three nutrient models: the statistic for Model N-1 was .731; Model P-1, .903 and Model K-1, .782. Unfortunately neither axis of the Durbin-Watson table of critical values extends far enough to test these statistics. Tables for the VonNeuman-Hart statistic are similarly too small. And even if the tables were large enough, some question remains as to the relevance of these tests in models combining time-series and cross-section data. To avoid possible confusion of terminology serial correlation is regarded as correlation between a series of residuals and that same series lagged one or more observations. The Durbin-Watson statistic, as it is normally used, assumes that serial correlation reveals itself in consecutive observations. For time series this assumption would seldom be incorrect. But in models combining time series and cross-section data, confirming the absence of serial correlation between U_t and U_{t-1} is not sufficient. Those non-random considerations affecting the system

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[57] Durbin and Watson, op. cit., pp. 159-177.

can be associated either with time or with the cross-section. For example, in this model non-random factors may be acting over time or across states so that the absence of serial correlation where the statistic is based on U_t and U_{t-1} is a necessary but not sufficient test. Confirmation of no serial correlation over time (U_t and U_{t-1}) does not exclude the possibility of serial correlation between states (U_t and U_{t-15}). Further, the tests must be expanded to consider all possible combinations of adjacent states. It is because of these several considerations that the relevancy of the Durbin-Watson test as it is normally used is questioned for use in combined time-series and cross-section models.

A Comparison of the Three Models

Several characteristics are available for comparing the models for the three primary nutrients. Comparisons here will be based on $\bar{R}^2$, standard error of the residuals, relative importance of the variables in explaining the changes in nutrient consumption between 1950 and 1964, size and significance of variables and the price elasticities for each state. There is little difference in the percentage explained of the variation of the individual nutrients. The $\bar{R}^2$'s are .9361, .9799 and .9569 for nitrogen, phosphate and potash respectively. The estimated standard errors of residuals are 7.61, 4.72 and 8.05.

Tables 12, 18 and 24 provide some measure of the relative importance of different economic variables and time in explaining the changes in consumption of each nutrient between 1950 and 1964. Comparison of the nutrient price variables in the three nutrient models clearly shows that changes in nitrogen price have contributed more to the explanation of changes in nitrogen consumption than either phosphate or potash price in their models. In all models variation between states qualifies generalizations but the average contribution of nitrogen price is 23 percent; phosphate, 8 percent; and potash, 20 percent. Price of the main crop contributes less than nutrient price in all models with little variation between models. Comparison of average contribution is refrained from because of the number of states in which the crop price coefficient has an unexpected and significant sign. Secondary crop price is consistently of little importance in all the models. Net farm income, as mentioned earlier, makes a small contribution to the total explanation. The average percentage explained by the expenditure restriction is 7 percent, 4 percent and 5 percent in Models N-1, P-1 and K-1. And this brings us to time, the proxy variable representing technological change and increased "awareness" of farmers. In all the models, technological changes and changes in farmers' attitudes toward fertilizer generally contribute more to the total explanation than any other variable. The average percentage explained by the time variable is 63 percent, 78 percent and 66 percent in Models N-1, P-1 and K-1.

A graph (Figure 1) of the estimated coefficients of time constants provides another comparison of the three models. Although levels of the estimates differ, the general pattern is similar for all three. All demonstrate rapid increases over time and give no evidence of a downturn in the future.

Price elasticities for each nutrient in each state are computed at the mean. The price elasticity of nitrogen is generally the highest with potash next and phosphate the least elastic. Table 26 shows all

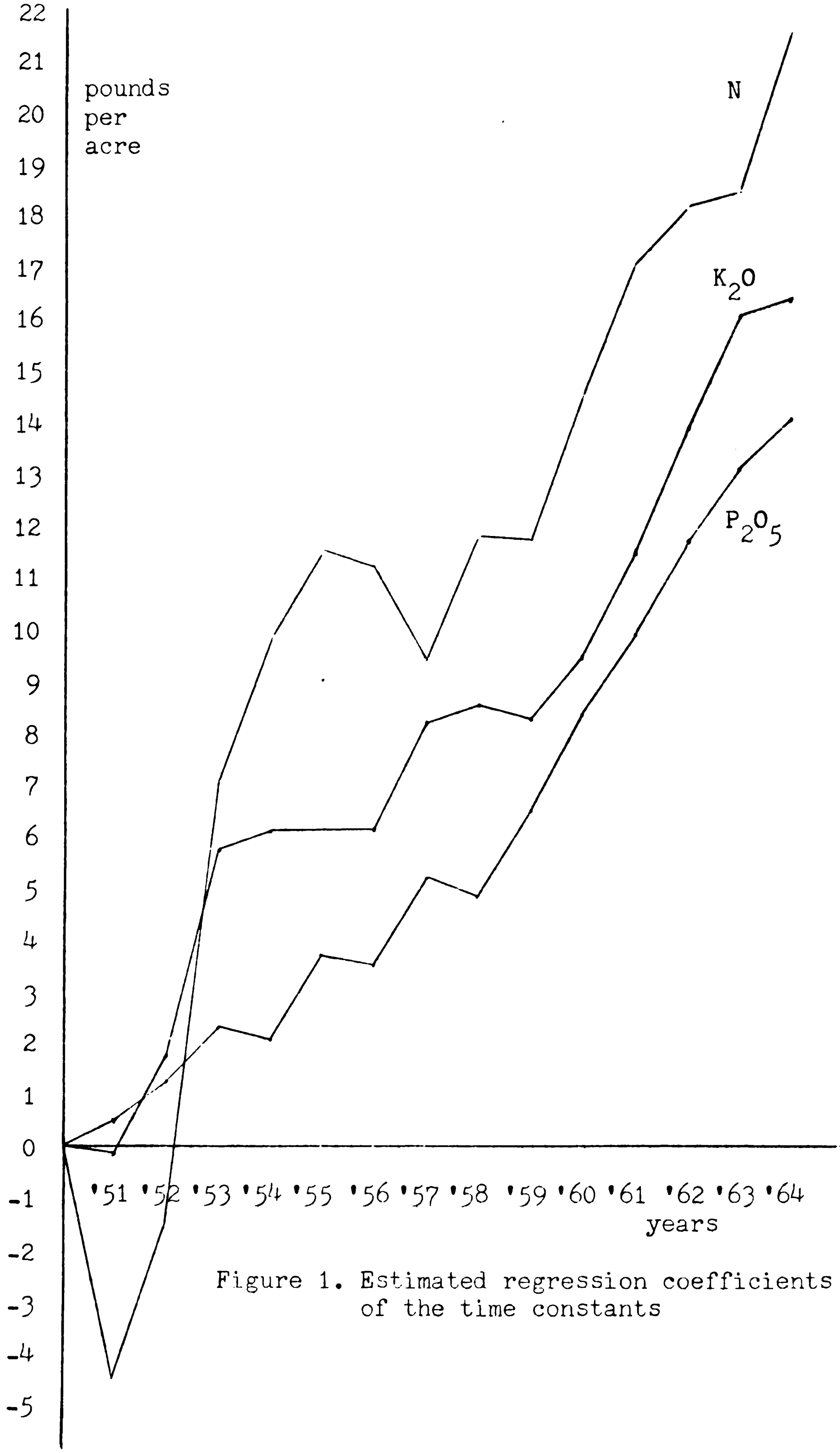

Figure 1. Estimated regression coefficients of the time constants

Table 26. Price elasticities of primary plant nutrients by states, computed at the mean

Nutrient / State	Nitrogen	Phosphate	Potash
Me.	-1.663	-.333	-.806
N.H.	-5.266	-.820	-2.528
Vt.	-11.457	-.859	-2.983
Mass.	-1.456	-.345	-.925
R.I.	-.946	-.183	-.469
Conn.	-1.425	-.328	-.838
N.Y.	-3.649	-.639	-1.947
N.J.	-1.300	-.256	-.590
Pa.	-3.924	-.575	-1.607
O.	-2.089	.718*[a]	.098
Ind.	-1.533	.730*	.077
Ill.	-2.600	1.318*	.180
Mich.	-2.117	.783*	.112
Wisc.	-6.220	1.511*	.160
Minn.	-6.333	2.170*	.445
Ia.	-3.393	2.025*	.512
Mo.	-1.944	1.377*	.223
N.D.	-23.053	5.712*	179.830*
S.D.	-30.909	19.408*	663.832*
Neb.	-2.560	6.537*	125.000*
Kan.	-4.594	3.822*	48.422*
Del.	-3.352	-.734	-3.850
Md.	-3.229	-.683	-4.403
Va.	-.828	-.169	-1.057
W.Va.	-34.554	-4.004	-37.291
N.C.	-1.577	-.563	-3.197
S.C.	-1.787	-.713	-3.555
Ga.	-1.584	-.634	-3.315
Fla.	-.452	-.190	-.845
Ky.	-3.688	-.837	-1.014
Tenn.	-3.073	-1.094	-1.140
Ala.	-1.715	-.591	-.709
Miss.	-1.552	-1.501	-1.694
Ark.	-1.739	-1.972	-1.829
La.	-.848	-1.120	-1.662
Okla.	-7.548	-3.388	17.283*
Tex.	-2.637	-2.484	11.430*
Mont.	-42.961	.596	1151.681*
Id.	-4.467	.174	110.358*
Wyo.	-19.355	.403	489.475*

Table 26--Continued.

Nutrient State	Nitrogen	Phosphate	Potash
Colo.	-8.786	.274	54.716*
N.Mex.	-3.208	.103	71.456*
Ariz.	-.638	.051	12.484*
Ut.	-4.315	.156	72.933*
Nev.	-10.161	.401	104.850*
Wash.	-3.530	-.755	8.078
Ore.	-3.625	-.576	7.803
Calif.	-1.291	-.243	3.004

[a]* means the estimated coefficient of nutrient price has incorrect sign and is significant at .05 or .10 level.

the estimates and (*) indicates those which are both significant and have incorrect sign. With the exception of the three states using the most nitrogen, the elasticities are all greater than one implying that further reductions in the price of nitrogen will result in net gains for both farmers and manufacturers of nitrogen. The elasticities of potash price vary more over the country. In the Southeast and East South Central Regions the response to price changes is greater than for changes in nitrogen price while in the Northeast the pattern of price response is the opposite. The estimated elasticities for potash price in the Plains and Mountain States must essentially be disregarded since the change in consumption per acre in some states since 1950 has been less than one pound. This is explained by the fact that this region inherently has sufficient or excess potash.

The estimates of phosphate price elasticities are often less than one. It is surprising that the model has done as well as it has. During the 15 years of the study both price and consumption have increased and yet the estimated coefficient of price is either negative or not significant.

The estimated elasticities of Table 26 clearly indicate that farmers are responsive to price changes and with the exception of response to potash price changes in the Plains and Mountain States, the responses conform to intuitive tests of reasonableness.

Summary

To summarize, this chapter has presented the models in greater detail along with their parameter estimates. The differences between estimates in the three models substantiate the arguments presented in Chapter II for different structures in the three nutrient markets. Estimated coefficients reveal technological change and the changes in farmers' attitude toward fertilizer contribute heavily to the total explanation of the changes in consumption. But price is also important, especially in the nitrogen and potash markets.

Estimates of future consumption utilizing data from other studies in conjunction with the parameter estimates of this study are presented in Chapter V.

CHAPTER V

APPLICATION OF THE MODELS

Introduction

Chapter I raised questions about the effect of changes in nutrient prices, crop prices, income and technology on the consumption of fertilizer in the United States. To answer these questions models for the three primary nutrients were developed and estimated using combined time-series and cross-section data for the period 1950 through 1964. This chapter demonstrates how the estimated parameters of the model can be used to make conditional projections of future consumption.

Projections

The model is designed to facilitate individual state estimates. But estimates will not be presented for all states in this paper - only Michigan will be used to demonstrate the procedure. Projections are limited to Michigan primarily because of the necessary expansion of the study entailed in collecting estimates of the future values of exogenous variables. And secondly, assumptions of exogenous variables acceptable to a large part of the readers are scarce goods. Therefore, tables for individual state estimates are arranged so that the reader need only supply his own assumed values for the exogenous variables and sum a row of products to arrive at the final estimate for any state.

The estimates needed from sources outside the model are (1) total harvested acreage, (2) price of the nutrient, either nitrogen, phosphate or potash, (3) price of the state's most important fertilizer consuming crop, and for some states the price of a second crop, (4) average net farm income, (5) some estimate of the rate of technological change and (6) expectations about inflation. Estimates of nutrient prices are based primarily upon the price indexes developed in Chapter III (reported by states in Appendix V) and outside information of expected changes in the fertilizer industry which affect price of nutrient. Estimates of technological change, as it relates to fertilizer consumption, depend heavily upon the estimates generated by the model and a judgment factor. With these estimates in hand the model can be used to make the state projection.

Before presenting the general projection table an example using Michigan will be presented. This example is then used in the projection table to demonstrate the table's use.

Projections for Michigan depend largely on Project '80 - A Look at Michigan's Rural Potential in 1980.[58] Estimated average net farm income for 1980 is $3,571[59] and estimated cropland harvested is 5.8 million acres.[60] Prices of crops are not expected to advance as fast as the 1.5 percent projected annual increase of the general price

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[58] John Ferris, et al., Project '80 - A Look at Michigan's Rural Potential in 1980 (East Lansing, Agriculture Experiment Station, Michigan State University, 1965).

[59] K. T. Wright, "Trends in Agricultural Output and Earnings, and Projections on Farm Characteristics, Income and Earnings", Project '80 - A Look at Michigan's Rural Potential in 1980 (East Lansing, Agricultural Experiment Station, Michigan State University, August 1965), p. 65.

[60] Ibid., p. 71.

level. The index of prices paid is also expected to increase at a rate less than the general price level.[61]

The estimates of nitrogen consumption in Michigan will be developed using Model N-1.[62]

$$Q^{N}_{1980,Mich.} = \hat{\alpha} + \hat{\beta}_{84} P^{N}_{1980,Mich.} + \hat{\beta}_{67} Y_{1979,Mich.}$$

$$+ \hat{\beta}_{16} P^{Corn}_{1979,Mich.} + \hat{\beta}_{i} T_{1980}$$

$$= (6.264) - (72.089) P^{N}_{1980,Mich.} + (.007) Y_{1979,Mich.}$$

$$+ (42.855) P^{Corn}_{1979,Mich.} + (44)$$

$$= 6.264 - 72.089 (.214) + .007 (955)$$

$$+ 42.855 (.398) + 44$$

$$= 6.264 - 15.427 + 6.685 + 17.056 + 44$$

$$= 58.578 \text{ pounds per acre}$$

With 5.8 million acres harvested this is equivalent to 170,000 tons of elemental nitrogen for Michigan in 1980.

Where do the estimates of exogenous variables enter? Starting with the price of nitrogen in Michigan, the price per unit of nutrient is assumed to continue falling. The shift to anhydrous ammonia, a

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[61] L. V. Manderscheid, et al., "Income and Price Levels", Project '80 - A Look at Michigan's Rural Potential in 1980 (East Lansing, Agricultural Experiment Station, Michigan State University, March 1965), p. 3.

[62] Defined on pp. 59-60 of Chapter IV.

very cheap source of nitrogen on a per unit basis, will continue and may account for 50 percent of nitrogen sales by 1980. This shift to anhydrous is assumed to reduce the price index of nitrogen to 68: approximately a 26 percent decline relative to the 1964 level of 84.3. Deflation of the projected 1980 nitrogen index of 68 by the expected Index of Prices Paid for Production Items (assuming 1 percent annual inflation) yields the index of the real price of nitrogen in Michigan in 1980: .214. Average net farm income is assumed to be $3,500 in 1979, $71 less than Wright's estimate for 1980. This appears to be a money income estimate based on 1964 prices which must be deflated before entering the model. Since Manderscheid estimates that the Index of Prices Paid will increase less than the 1.5 percent annual rate of the general price level, a 1 percent rate of inflation is assumed. This yields an estimated real income of $955 in 1979.[63] The real price of corn is assumed unchanged in 1980. That is, the price of corn and the index of prices paid for production items are expected to increase by the same percentage between 1964 and 1980. The increase in nitrogen applied per acre due to technological changes and increased acceptance by farmers is assumed to be 44 pounds. Inspection of Figure 1 on page 103 clearly indicates that fitting a

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[63] The index of prices paid for production items (February, March and April average) for 1964 was 271.66. The index of prices paid at that time was 313.33. Which rate of inflation one assumes has a considerable effect on the final estimate. For example, assuming 1.5 percent inflation for 16 years increases the prices paid index from 313.33 to 397.62; 1 percent inflation corresponds to a 1980 index of 1.1726 (313.33) = 367.41; 1.25 percent corresponds to 313.33 (1.2199) = 382.23; 1.75 percent corresponds to 1.3199 (313.33) = 413.56 and; 2 percent corresponds to (313.33) (1.3728) = 430.14. Real income associated with these rates of inflation range from $955 with 1 percent inflation to $814 with 2 percent inflation.

least square line of the time constants excluding 1950 and 1951 would yield an estimate greater than 44 pounds per acre. Examination of the last seven years, the period since 1957, suggests an even greater increase.[64] Michigan's estimated consumption of nitrogen in 1980 is 58.6 pounds per acre or 170,000 tons for the state. This implies a 109 percent increase relative to the 1964 consumption.

The estimation procedure used for Michigan can be generalized for all states as can be demonstrated in Table 27. Michigan is found in Table 27-B. Starting at the left, the state constant for Michigan is 0 because Michigan is included in the overall constant. The values for all other states are listed in the table. The second column is the estimated overall constant - the same for all states, 6.264. The third column is the product of the estimated real income in the future period and the estimated income coefficient, .007. The fourth column is the product of the coefficient of nitrogen price (-72.09) and the estimated real nutrient price. The fifth and sixth columns are not relevant to Michigan: the estimated coefficients equal zero. The seventh column is the product of the estimated real price of corn and the estimated crop price coefficient, 42.86. The eighth column requires the estimated increase in nitrogen consumption due to technological change and increased acceptance by farmers. So, with the exception of the estimated overall constant term, each column with an X in it

[64]Likewise, utilizing the estimated increase in nitrogen consumption due to time from Model N-3 suggests an increase of 58.5 pounds attributable to this single factor. The utilization of estimates from two different models is inappropriate and leads to erroneous conclusions. In making forecasts for individual states any of the models in the study can be used but not some combination of them.

Table 27-A. Projection table: nitrogen

State	State constant $\hat{\beta}_{18} \ldots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{83} P^N_{t,s}$	$\hat{\beta}_{84} P^N_{t,s}$	hay $\hat{\beta}_4 P_{t-1,s}$	corn $\hat{\beta}_6 P_{t-1,s}$	potato $\hat{\beta}_{10} P_{t-1,s}$	corn $\hat{\beta}_{15} P_{t-1,s}$	Tech.[a]
		6.264	+.007( )	-141.67( )	-72.09( )	+.738( )	+11.97( )	-10.95( )	+29.90( )	+( )
Me.	77.35	x	x	x				x		x
N.H.	40.12	x	x	x		x				x
Vt.	36.32	x	x	x		x				x
Mass.	64.75	x	x	x		x				x
R.I.	107.99	x	x	x				x		x
Conn.	61.37	x	x	x		x				x
N.Y.	34.96	x	x	x					x	x
N.J.	60.00	x	x	x					x	x
Pa.	33.74	x	x	x					x	x

Table 27-B. Projection table: nitrogen

State	State constant $\hat{\beta}_{18} \ldots \hat{\beta}_{64}$	$\hat{\alpha}$ 6.264	$\hat{\beta}_{67} Y_{t-1,s}$ +.007( )	$\hat{\beta}_{84} P^{N}_{t,s}$ -72.09( )	$\hat{\beta}_{6} P^{corn}_{t-1,s}$ +11.97( )	$\hat{\beta}_{11} P^{wheat}_{t-1,s}$ +68.78( )	$\hat{\beta}_{14} P^{wheat}_{t-1,s}$ +68.41( )	$\hat{\beta}_{16} P^{corn}_{t-1,s}$ +42.86( )	Tech.[a] +( )
O.	17.61	x	x	x	x				x
Ind.	19.79	x	x	x	x				x
Ill.	9.16	x	x	x	x				x
Mich.		x	x	x				x	x
Wisc.	-8.76	x	x	x				x	x
Minn.	-9.51	x	x	x				x	x
Ia.	5.30	x	x	x	x				x
Mo.	14.99	x	x	x	x				x
N.D.	-49.75	x	x	x			x		x
S.D.	-13.58	x	x	x				x	x
Neb.	-8.43	x	x	x				x	x
Kan.	-41.15	x	x	x		x			x

Table 27-C. Projection table: nitrogen

State	State constant $\hat{\beta}_{18} \cdots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{85} P^{N}_{t,s}$	$\hat{\beta}_{7} P^{corn}_{t-1,s}$	$\hat{\beta}_{8} P^{corn}_{t-1,s}$	$\hat{\beta}_{9} P^{cotton}_{t-1,s}$	$\hat{\beta}_{17} P^{corn}_{t-1,s}$	Tech.[a]
		6.264	+.007( )	-180.93( )	+1.47( )	+7.10( )	+1.57( )	-121.64( )	+( )
Del.	75.53	x	x	x	x				x
Md.	71.70	x	x	x	x				x
Va.	132.47	x	x	x	x				x
W.Va.	57.62	x	x	x	x				x
N.C.	155.43	x	x	x				x	x
S.C.	135.27	x	x	x			x	x	x
Ga.	153.45	x	x	x				x	x
Fla.	253.72	x	x	x				x	x
Ky.	66.93	x	x	x		x			x
Tenn.	51.83	x	x	x		x	x		x
Ala.	87.00	x	x	x		x			x
Miss.	70.98	x	x	x		x	x		x

Table 27-D. Projection table: nitrogen

State	State constant $\hat{\beta}_{18} \ldots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{86} P^{N}_{t,s}$	$\hat{\beta}_{87} P^{N}_{t,s}$	$\hat{\beta}_{5} P^{corn}_{t-1,s}$	$\hat{\beta}_{9} P^{cotton}_{t-1,s}$	$\hat{\beta}_{11} P^{wheat}_{t-1,s}$	$\hat{\beta}_{14} P^{wheat}_{t-1,s}$	Tech.[a]
		6.264	+.007( )	-89.18( )	-122.92( )	+62.12( )	+1.57( )	+68.78( )	+68.41	+( )
Ark.	14.50	x	x	x			x			x
La.	34.29	x	x	x			x			x
Okla.	-51.17	x	x	x			x	x		x
Tex.	3.84	x	x	x			x			x
Mont.	-26.93	x	x		x				x	x

Table 27-E. Projection table: nitrogen

State	State constant $\hat{\beta}_{18} \ldots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{87} P^{N}_{t,s}$	$\hat{\beta}_{88} P^{N}_{t,s}$	$\hat{\beta}_{5} P^{corn}_{t-1,s}$	$\hat{\beta}_{12} P^{wheat}_{t-1,s}$	$\hat{\beta}_{13} P^{cotton}_{t-1,s}$	Tech.[a]
		6.264	+.007( )	-122.92( )	-226.50( )	+62.12( )	+30.00( )	-.14( )	+( )
Id.	14.05	x	x	x			x		x
Wyo.	-5.31	x	x	x		x			x
Colo.	-3.40	x	x	x		x			x
N.Mex.	35.47	x	x	x				x	x
Ariz.	62.54	x	x	x				x	x
Ut.	-4.87	x	x	x		x			x
Nev.	-4.06	x	x	x			x		x
Wash.	67.17	x	x		x		x		x
Ore.	69.60	x	x		x		x		x
Calif.	112.41	x	x		x			x	x

[a]Estimated values of the time constants:

1951 = -4.57	1956 = 11.24	1961 = 17.09
1952 = -1.55	1957 = 9.43	1962 = 18.24
1953 = 7.06	1958 = 11.92	1963 = 18.45
1954 = 9.95	1959 = 11.78	1964 = 21.81
1955 = 11.50	1960 = 14.67	

requires an estimate of the exogenous variable. The state estimate is found by algebraically summing the products of the relevant columns yielding an estimate of 58.578 pounds per acre. To estimate total tonnage for the state multiply the estimated per acre rate by the projected harvested acreage and divide this product by 2,000. For Michigan this yields a projected tonnage of 170,000 tons of elemental nitrogen in 1980.

Changing the assumed increase in consumption due to technology and awareness to 50 pounds per acre (leaving all other estimates of nutrient price, income, etc., unchanged) boosts the projection to 187,000 tons. And the higher estimate appears to be the more realistic; it assumes the average rate of increase since 1957 continues rather than the average rate of increase since 1950.

The projections for phosphate and potash consumption follow the same general format. Table 28 is constructed similar to Table 27. Again Michigan will be used to demonstrate the use of the table in making projections. Some of the assumptions used in projecting nitrogen consumption are relevant for phosphate as well. Real income will again be assumed to be $955 and the real price of corn is assumed unchanged. Although the price of phosphate has increased 27 percent since 1950, the price relative to the prices paid for production items has increased less. This trend is expected to be curtailed to a similar rate of increase for the two for two reasons. One, there will be a continued shift to higher analysis phosphate fertilizers such as ammonium phosphate. And secondly, the development of new sources of rock phosphate in North Carolina precludes shortages of raw materials before 1980. If the 1950-1964 average change in

Table 28-A. Projection table: phosphate

State	State constant $\hat{\beta}_{18} \cdots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{83} P^{P}_{t,s}$	$\hat{\beta}_{4} P^{hay}_{t-1,s}$	$\hat{\beta}_{10} P^{potato}_{t-1,s}$	$\hat{\beta}_{13} P^{hay}_{t-1,s}$	Tech.[a]
		-9.317	+.002( )	-40.97( )	+2.92( )	+10.94( )	+.43( )	+( )
Me.	66.54	x	x	x		x		x
N.H.	5.89	x	x	x	x			x
Vt.	8.22	x	x	x	x			x
Mass.	30.84	x	x	x	x			x
R.I.	70.53	x	x	x	x			x
Conn.	30.94	x	x	x	x			x
N.Y.	49.23	x	x	x			x	x
N.J.	91.03	x	x	x			x	x
Pa.	52.49	x	x	x			x	x

Table 28-B. Projection table: phosphate

State	State constant $\hat{\beta}_{18} \cdots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{84} P^{P}_{t,s}$	$\hat{\beta}_{6} P^{corn}_{t-1,s}$	$\hat{\beta}_{11} P^{wheat}_{t-1,s}$	$\hat{\beta}_{15} P^{wheat}_{t-1,s}$	$\hat{\beta}_{16} P^{corn}_{t-1,s}$	Tech.[a]
		-9.317	+.002( )	+47.24( )	+6.87( )	+23.77( )	+19.07( )	+16.22( )	+( )
O.	6.89	x	x	x	x				x
Ind.	5.79	x	x	x	x				x
Ill.	-6.99	x	x	x	x				x
Mich.		x	x	x				x	x
Wisc.	-12.74	x	x	x				x	x
Minn.	-17.03	x	x	x				x	x
Ia.	-12.65	x	x	x	x				x
Mo.	-6.58	x	x	x	x				x
N.D.	-29.89	x	x	x			x		x
S.D.	-24.17	x	x	x				x	x
Neb.	-23.67	x	x	x				x	x
Kan.	-31.31	x	x	x		x			x

Table 28-C. Projection table: phosphate

State	State constant $\hat{\beta}_{18} \cdots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{85} P^{P}_{t,s}$	$\hat{\beta}_{7} P^{corn}_{t-1,s}$	$\hat{\beta}_{8} P^{corn}_{t-1,s}$	$\hat{\beta}_{9} P^{cotton}_{t-1,s}$	$\hat{\beta}_{17} P^{corn}_{t-1,s}$	Tech.[a]
		-9.317	+.002( )	-64.83( )	-22.73( )	-19.42( )	+.60( )	-66.09( )	+( )
Del.	84.05	x	x	x	x				x
Md.	85.78	x	x	x	x				x
Va.	207.40	x	x	x	x				x
W.Va.	51.48	x	x	x	x				x
N.C.	117.07	x	x	x				x	x
S.C.	101.50	x	x	x			x	x	x
Ga.	101.75	x	x	x			x	x	x
Fla.	201.81	x	x	x				x	x
Ky.	73.98	x	x	x		x			x
Tenn.	70.99	x	x	x		x			x
Ala.	82.32	x	x	x		x	x		x
Miss.	54.30	x	x	x		x	x		x

Table 28-D. Projection table: phosphate

State	State constant $\hat{\beta}_{18} \cdots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{86} P^{P}_{t,s}$	$\hat{\beta}_{87} P^{P}_{t,s}$	$\hat{\beta}_{9} P^{cotton}_{t-1,s}$	$\hat{\beta}_{11} P^{wheat}_{t-1,s}$	$\hat{\beta}_{15} P^{wheat}_{t-1,s}$	Tech.[a]
		-9.317	+.002( )	-51.36( )	+3.48( )	+.6( )	+23.77( )	+19.07( )	+( )
Ark.	29.67	x	x	x		x			x
La.	38.15	x	x	x		x			x
Okla.	12.76	x	x	x			x		x
Tex.	25.51	x	x	x		x			x
Mont.	-12.09	x	x		x			x	x

Table 28-E. Projection table: phosphate

State	State constant $\hat{\beta}_{18} \cdots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{87} P^{P}_{t,s}$	$\hat{\beta}_{88} P^{P}_{t,s}$	$\hat{\beta}_{5} P^{cotton}_{t-1,s}$	$\hat{\beta}_{12} P^{hay}_{t-1,s}$	$\hat{\beta}_{14} P^{hay}_{t-1,s}$	Tech.[a]
		-9.317	+.002( )	+3.48( )	-14.30( )	-.39( )	+.18( )	-.07( )	+( )
Id.	5.09	x	x	x				x	x
Wyo.	- .83	x	x	x				x	x
Colo.	1.13	x	x	x				x	x
N.Mex.	8.55	x	x	x				x	x
Ariz.	25.51	x	x	x		x			x
Ut.	5.85	x	x	x				x	x
Nev.	-1.79	x	x	x				x	x
Wash.	14.31	x	x		x		x		x
Ore.	17.37	x	x		x		x		x
Calif.	32.06	x	x		x	x	x		x

[a]Estimated values of the time constants:

1951 = .54	1956 = 3.58	1961 = 9.98
1952 = 1.27	1957 = 5.22	1962 = 11.77
1953 = 2.33	1958 = 4.92	1963 = 13.09
1954 = 2.07	1959 = 6.55	1964 = 14.13
1955 = 3.80	1960 = 8.40	

consumption of phosphate remains unchanged the estimate for this variable is 28.3 pounds per acre. But if the average increase of the most recent 6 years is the more relevant, then the estimate is 38 pounds. The latter appears more realistic, but the example will be done with the 28.3 pound estimate to give something of a range.

Using Table 28-B and the estimated values of the exogenous variables above, find the algebraic sum of the Michigan row. Substituting the above estimates for the X's yields:

$$Q^{P}_{1980,\text{Mich.}} = -9.317 + .002\ (955) + 47.24\ (.47) + 16.22\ (.398) + 28.3$$

$$= 49.55 \text{ pounds per acre}$$

or 143,000 tons of P_2O_5 in 1980 in Michigan

If the higher estimate of the technology and farmer acceptance variable is utilized, the projected consumption increases to 172,000 tons. The latter appears to be the more realistic estimate.

The potash estimates are made in the same manner as the nitrogen and phosphate estimates. Again, the estimated real income and real crop price variables are the same. The price of potash (represented by muriate of potash) has fallen relative to the Prices Paid for Production Items Index over the period studied. But for the past ten years the ratio has been quite stable. Because of this stability and the rapidly increasing capacity, especially in Saskatchewan, the relative price is expected to remain stable until 1980. If anything, some decreases in the ratio may be expected. How should technology be treated? The average increase from 1950 through 1964 attributable to technology is 1.09 pounds per year yielding an estimated

32.7 pounds in 1980. But again, as in the phosphate model, the average annual increase in the five year period since 1959 is much greater: 1.62 pounds per year. Expected potash consumption based on these assumptions are found by substituting these estimates for the X's in Table 29-B.

$$Q^K_{1980,Mich.} = 8.06 + .003\ (955) + .12\ (20.982) + 4.81\ (.398) + 32.7 = 48.06 \text{ pounds per acre}$$

This is equivalent to 139,000 tons for the state.

Using the higher estimate for the technology variable yields a forecast of 167,000 tons.

The estimates until now have been concerned only with a single nutrient in individual states. To get the expected rate of consumption of all fertilizer for the individual state, sum the state's nutrient estimates. Similarly, the national estimate of, say, nitrogen is found by summing the individual state estimates. And the expected consumption of the three fertilizer nutrients for the U.S. is the sum of the estimated state consumption of the three.

Michigan's Project '80 estimates[65] of fertilizer consumption ranged from 150,000 to 170,000 tons for nitrogen, 135,000 to 164,000 tons for available phosphate and 160,000 to 195,000 tons of potash - the two estimates of the range being arrived at by two different methods. The preferred estimates from the models of this study are

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[65]Robert E. Lucas, et al., "Grains, Beans and Farm Supply", Project '80 - A Look at Michigan's Rural Potential in 1980 (East Lansing, Agricultural Experiment Station, Michigan State University, July 1965).

Table 29-A. Projection table: potash

State	State constant $\hat{\beta}_{18} \cdots \hat{\beta}_{64}$	$\hat{\alpha}$ +8.06	$\hat{\beta}_{67} Y_{t-1,s}$ +.003( )	$\hat{\beta}_{83} P^{K}_{t,s}$ -1.76( )	$\hat{\beta}_{4} P^{hay}_{t-1,s}$ +2.36( )	$\hat{\beta}_{5} P^{potato}_{t-1,s}$ +7.70( )	$\hat{\beta}_{15} P^{corn}_{t-1,s}$ -5.27( )	Tech.[a] +( )
Me.	76.03	x	x	x		x		x
N.H.	15.31	x	x	x	x			x
Vt.	14.60	x	x	x	x			x
Mass.	36.04	x	x	x	x			x
R.I.	77.82	x	x	x	x			x
Conn.	37.28	x	x	x	x			x
N.Y.	43.52	x	x	x			x	x
N.J.	88.81	x	x	x			x	x
Pa.	50.29	x	x	x			x	x

Table 29-B. Projection table: potash

State	State constant $\hat{\beta}_{18} \ldots \hat{\beta}_{64}$	$\hat{\alpha}$	$\hat{\beta}_{67} Y_{t-1,s}$	$\hat{\beta}_{84} P^{K}_{t,s}$	$\hat{\beta}_{87} P^{K}_{t,s}$	$\hat{\beta}_{6} P^{corn}_{t-1,s}$	$\hat{\beta}_{10} P^{potato}_{t-1,s}$	$\hat{\beta}_{11} P^{wheat}_{t-1,s}$	$\hat{\beta}_{16} P^{corn}_{t-1,s}$	Tech.[a]
		+8.06	+.003( )	+.12( )	+1.25( )	+6.68( )	+.17( )	+21.68( )	+4.81( )	+( )
O.	.85	x	x	x		x				x
Ind.	5.30	x	x	x		x				x
Ill.	-13.47	x	x	x		x				x
Mich.		x	x	x					x	x
Wisc.	-7.91	x	x	x					x	x
Minn.	-18.94	x	x	x					x	x
Ia.	-22.38	x	x	x		x				x
Mo.	-13.91	x	x	x		x				x
N.D.	-49.34	x	x		x		x			x
S.D.	-51.69	x	x		x				x	x
Neb.	-52.04	x	x		x				x	x
Kan.	-63.97	x	x		x			x		x

Table 29-C. Projection table: potash

State	State constant $\hat{\beta}_{18} \ldots \hat{\beta}_{64}$	$\hat{\alpha}$ +8.06	$\hat{\beta}_{67} Y_{t-1,s}$ +.003( )	$\hat{\beta}_{85} P^K_{t,s}$ -7.45( )	$\hat{\beta}_{86} P^K_{t,s}$ -1.35( )	$\hat{\beta}_7 P^{corn}_{t-1,s}$ -100.60( )	$\hat{\beta}_8 P^{corn}_{t-1,s}$ -37.45( )	$\hat{\beta}_9 P^{cotton}_{t-1,s}$ +1.48( )	$\hat{\beta}_{13} P^{corn}_{t-1,s}$ -239.53( )	Tech.[a] +( )
Del.	241.59	x	x	x		x				x
Md.	237.21	x	x	x		x				x
Va.	359.92	x	x	x		x				x
W.Va.	210.35	x	x	x		x				x
N.C.	329.62	x	x	x					x	x
S.C.	290.95	x	x	x				x	x	x
Ga.	292.39	x	x	x				x	x	x
Fla.	443.09	x	x	x					x	x
Ky.	60.54	x	x		x		x			x
Tenn.	56.39	x	x		x		x			x
Ala.	50.21	x	x		x		x			x
Miss.	24.87	x	x		x		x	x		x
Ark.	3.48	x	x		x			x		x
La.	5.07	x	x		x			x		x

Table 29-D. Projection table: potash

State	State constant $\hat{\beta}_{18} \cdots \hat{\beta}_{64}$	$\hat{\alpha}$ +8.06	$\hat{\beta}_{67} Y_{t-1,s}$ +.003( )	$\hat{\beta}_{87} P^{K}_{t,s}$ +1.25( )	$\hat{\beta}_{88} P^{K}_{t,s}$ +1.08( )	$\hat{\beta}_{9} P^{cotton}_{t-1,s}$ +1.48( )	$\hat{\beta}_{10} P^{potato}_{t-1,s}$ +.17( )	$\hat{\beta}_{12} P^{cotton}_{t-1,s}$ +1.74( )	$\hat{\beta}_{14} P^{hay}_{t-1,s}$ +1.47( )	Tech.[a] +( )
Okla.	-60.33	x	x	x		x				x
Tex.	-61.86	x	x	x		x				x
Mont.	-51.14	x	x	x			x			x
Id.	-47.57	x	x	x			x			x
Wyo.	-50.83	x	x	x			x			x
Colo.	-48.56	x	x	x			x			x
N.Mex.	-70.32	x	x	x				x		x
Ariz.	-85.26	x	x	x				x		x
Ut.	-46.95	x	x	x			x			x
Nev.	-51.63	x	x	x			x			x
Wash.	-55.61	x	x		x				x	x
Ore.	-54.81	x	x		x				x	x
Calif.	-63.66	x	x		x			x		x

[a]Estimated values of the time constants:

1951 = -.14	1956 = 6.16	1961 = 11.54
1952 = 1.83	1957 = 8.33	1962 = 13.99
1953 = 5.79	1958 = 8.63	1963 = 16.07
1954 = 6.07	1959 = 8.31	1964 = 16.44
1955 = 9.04	1960 = 9.62	

187,000 tons, 172,000 tons and 167,000 tons of nitrogen, phosphate and potash respectively. These estimates exceed the Project '80 estimates for nitrogen and phosphate and fall in the lower part of the range for potash. Estimates for 1980 from this study exceed the 1964 level of consumption of nitrogen by 106 percent; phosphate consumption is estimated to be up 29 percent and potash consumption is estimated to be up 32 percent.

The estimates produced by these models must be recognized as merely estimates: each supported by strong assumptions. The major assumption is that the structure which generated the economic data of the observation period remains unchanged through the period being forecast. If structural changes are expected, say, changes in cropping patterns, then some consideration can be given this change in the proxy variable representing technology and farmer's awareness of fertilizer. For example, if a shift from small grain to corn is apparent and corn is more heavily fertilized than small grain, the only means of considering this in these models is by increasing the estimate of technology's proxy variable.

All the estimates of this chapter ignore the variances of the parameter estimates. While the 1980 estimates made are the "best" estimates of the models, the confidence in each of them is dependent upon the variances associated with $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\sigma}_u$. These variances are combined in the standard error of forecast. That is:

$$S_F = \hat{\sigma}_u \left(1 + \frac{1}{n} + \sum^{i} \sum^{j} C_{ij}(X_i - \bar{X}_i)(X_j - \bar{X}_j)\right)^{\frac{1}{2}}$$

where: $\hat{\sigma}_u$ = estimated standard error of disturbances

C_{ij} = element of the X'X matrix

The first term, 1, refers to the estimated standard error of disturbances, the estimated variation of a single observation about the true regression line. The second term 1/n, considers variance in the level of the regression line at the means of the X's. The third term considers the variances associated with the B's. Clearly, consideration of observations deviating widely from the mean value of the X's leads to greater standard errors of forecast than observations near the means. Long-run projections such as those made above are based on data which often deviate widely from the mean and, if so, the "best" estimates must be recognized as having large standard errors. Since no C_{ij}'s were saved in computing the regression estimates, no standard errors of forecast were made in this study.

Summary

This chapter demonstrates the procedure for making conditional forecasts of fertilizer nutrient consumption in individual states using this study's models. To assist in forecasting consumption in individual states, data such as nutrient prices, per acre consumption rates and other related data are included in the appendices to give an improved historical view of the fertilizer market.

CHAPTER VI

SUMMARY, RECOMMENDATIONS AND CONCLUSIONS

Summary

Problem

Commercial fertilizer has increased in importance both absolutely and relative to other inputs as an explanatory factor in the rapid increase in agricultural productivity. Between 1950 and 1964 nitrogen consumption increased 354 percent while phosphate increased 73 percent and potash increased 150 percent. Concomitantly changes were occurring in fertilizer use technology, farmer's attitudes toward fertilizer were changing, and economic variables such as fertilizer prices, crop prices and farm incomes were changing. In addition, the effect of these changes differs greatly from region to region within the U.S.

Quantified information about the effect of such changes and their relative importance in each state improves evaluation of future changes in these factors.

Method

This study measured the effects of some of the possible changes in economic and noneconomic variables related to the fertilizer market. Separate models were fitted for each of the three primary nutrients; each regressed the quantity of nutrient per acre on various economic variables, dummy variables for each state and dummy variables for each year of the study. Estimation of nutrients separately necessitated the construction of price indexes for nitrogen and phosphate by states. Muriate of potash price was used as an index of potash price. The analysis relies on covariance models estimated by ordinary least squares. Each model combines cross-section and time-series data: data from the 48 continental states and the 15 year period 1950 through 1964 combine to give 720 observations. By design of the models, all yield estimates for the individual states.

In each model the endogenous variable is the pounds of nutrient per acre; the exogenous variables are the price of the nutrient relative to the Index of Prices Paid for Production Items, the price of the most important fertilizer consuming crop lagged one year relative to the same index, the average net farm income of the previous year relative to the Index of Prices Paid by Farmers, a dummy variable for each state and a dummy variable for each year. State grouping in the nutrient price variables differs between models as does the crop price considered most important for each nutrient.

Results

The results will be summarized by first reviewing the nutrient price indexes and then discussing the nutrient models. Detailed results (of the indexes and models) for individual states are presented in Chapters III, IV and V.

Five basic index constructions with various weights were considered in developing price indexes for nitrogen and phosphate: Laspeyres, Paasche, Fisher Ideal, Chain and Chain of Fisher Ideals. Comparison of the results shows that the form of the index makes little difference under stable conditions as experienced in the phosphate sector. But when rapidly changing prices and quantities characterize the index goods, estimates of the various constructions vary widely. A difference of 13 index points separated the Paasche and Laspeyres indexes after 13 years.

Comparing the indexes of nitrogen and phosphate (chain indexes for both) demonstrates the danger of using a general fertilizer index to answer questions about individual nutrients. The indexes of nitrogen and phosphate price differ by 30 index points in 1964 with base of 1950 = 100. In addition to the variation in price between nutrients, the price variation between states of either nutrient is also more than 30 index points.

Comparing a weighted average of the chain indexes for available phosphate and nitrogen and price of muriate of potash (set 1950 = 100) with the current U.S.D.A. index of fertilizer prices shows the chain index is about 6 percent less after 15 years.

The covariance model selected for each of the three nutrients was estimated by ordinary least squares. Each is "form free" in the sense that the variable representing technological change and farmers' attitudes toward fertilizer is left unspecified. Models N-1, P-1 and K-1 yield adjusted coefficients of determination ($\bar{R}^2$) of .94, .98 and .96 respectively with standard errors of the residuals for the same three models of 7.61, 4.72 and 8.05.

In the nitrogen model nutrient price was significant in all regions. And the price elasticity of demand calculated for each state generally exceeds 1 in absolute value. While neither the phosphate nor potash models give all correct and significant signs, all three nutrient prices in each of the models when considered as the set of regional nutrient prices, were statistically significant at the .001 level. State's estimated elasticities of potash tend to be less than nitrogen but greater than phosphate. Variation in nitrogen price and potash price contribute 23 percent and 20 percent to the explained increase in consumption of the two nutrients while phosphate price contributes only 8 percent.

Crop prices of the previous year are responsible for 8 percent of the increased consumption of nitrogen and 10 percent of phosphate. Of the increased potash consumption, however, about 9 percent can be attributed to changes in crop prices. Secondary crops which were considered in only a few states explain about 1 percent of the increased nutrient consumption in those states.

Net farm income, as a determinant of fertilizer consumption, is consistently significant, yet in no model can a large part of the increased consumption of nutrients be attributed to it. Seven percent of the increase in nitrogen consumption, 4 percent of the phosphate and 5 percent of the potash consumption are attributable to income.

For all models, time, the proxy variable representing technological improvements and farmers' changing attitude toward fertilizer contributes more to the increased nutrient consumption than any other variable.

Projections of state consumption of fertilizer nutrients are possible from each model. Michigan was used to demonstrate which data are required and the method of using them. Expected Michigan consumption of nitrogen is 187,000 tons, phosphate is 172,000 tons and potash is 167,000 tons. These estimates are 206 percent, 129 percent and 132 percent of the 1964 level of consumption.

Recommendations

As this study progressed, the need for further research in several areas became evident.

While reviewing earlier fertilizer studies and studying the trends in fertilizer consumption it became evident that the three nutrients were really different and to treat them as a single good could lead to erroneous conclusions. To treat them separately, price data for each nutrient were accumulated in an index. Continuation of these series appears prerequisite to further detailed studies in the area and to an accurate appraisal of policy alternatives. Another need for data is in the area of related inputs, especially machinery and pesticides. Both are areas of growing importance but data restrictions limit extensive research.

The problem of multicollinearity was undoubtedly responsible for many of the poor (large variances) crop price estimates. Use of dummy variables in the study suggest questioning the relationship between dummy variables and multicollinearity. Clearly, the intercorrelation varied inversely with the number of states involved. Where only one or two states were involved, as in the price of some crops, intercorrelation was serious. Where several states were included the problem decreased. And where dummy variables were used only sparingly, the problem - at least as revealed in the simple correlation between independent variables - was nonexistent.

Long-run projections can be made with these models but it is well known that the variances associated with the estimates increase as projections extend beyond the range of the original data. Better long-run estimates may have been arrived at if the study had been extended to a second stage. By regarding the estimated state constants as the dependent variable and regressing this on such non-economic variables as percentage of class I land, class II land, weather and percentage of cropland being fertilized, a better understanding of the differences between states may result. And from these relationships better long-run projections might be possible.

Finally, information on the non-farm use of commercial fertilizer was of importance to this study. Unfortunately the information desired was seldom available. Data on the quantity used by the non-farm sector have been estimated as high as 10 percent of the total consumption. Similarly data on the non-farm price are also unknown. Information about the non-farm market would be extremely helpful both to the industry and to the researcher.

Conclusions

The major conclusions of this study are as follows:

1. Perhaps one of the most conclusive results was obtained when tests were made to determine the significance of the estimated state constants as a set. In all models the hypothesis of no difference could be rejected at less than the .001 level. In general, the coefficient of determination increased 35-48 percent when the coefficients were estimated for each state. From this result alone, it would seem that the combined model used in this study should be seriously considered whenever adequate data are available.

2. Multicollinearity, a problem encountered in time-series

studies of fertilizer is overcome by the combined time-series and cross-section model. But the problem is then reintroduced with the widespread use of dummy variables. A variation of the usual coefficient of determination, R^2, is suggested as an improved measure of multicollinearity. Each major predetermined variable is regressed on the remaining predetermined variables. This measure overcomes the problems encountered with other measures such as the degree of simple correlation or the size of the determinant. The tolerance by the researcher of multicollinearity is then analogous to the tolerance of certain levels of acceptance in hypothesis testing: both become value judgments.

3. The Durbin-Watson statistic and the VonNeuman-Hart ratio do not provide a sufficient test for serial correlation for models utilizing both time-series and cross-section data. Depending upon the way the data are set up for the problem the test refers to either the successive differences over time or successive differences between elements of the cross-section. Both tests are necessary but may still not be sufficient. For example, in this study after tests were made over time and for ordered elements in the cross-section, tests must still be made on any state adjacent to more than one state - which would be most of them. That is, tests for all possible combinations of "adjacency".

4. The currently used method of estimating price indexes should be reconsidered. Where data are available or can be developed at relatively low cost the chain index should be seriously considered. In addition, generalizations about fertilizer prices must be made with an awareness that although the aggregate of three indexes is relatively stable, the aggregate is currently made up of two diverging

elements and one stable one. And the pattern of price change for any one element is not uniform over the country. If the Laspeyres construction is continued, then more frequent revisions of weights are needed to avoid drawing inferences from other than economic phenomenon.

5. The nitrogen price elasticity of demand estimates which generally exceed 1 in absolute value imply that expected further price reductions of nitrogen will benefit farmers and possibly nitrogen producers. The benefit to nitrogen producers depends on the costs of the increased production relative to the increased total revenue. Shifts to higher analysis phosphate materials are likely to curtail the price increases of phosphate nutrient making phosphate relatively more attractive as an input. Potash prices are expected to remain stable relative to prices of other inputs.

6. The fertilizer nutrient price, relative to the cost of other inputs is an important factor in explaining the increased consumption of fertilizer. Potash price and nitrogen price - both relatively lower - explain more than 20 percent of the increased nutrient consumption.

7. Net farm income as a variable representing expenditure restriction and credit restriction is shown by this study to be a restrictive factor in all models. Easing of this restriction increases expenditures for all three nutrients: the greatest effect is on nitrogen. This conclusion might be due to the stability of net farm income over the time period studied in conjunction with farmers' attitude toward the use of credit for fertilizer. Future periods when income might be more variable and attitudes might change could alter this conclusion.

8. Changes in crop prices tend to affect the consumption of nitrogen, phosphate and potash. But generalizations are hazardous here, states and regional responses differ widely.

9. A proxy variable for technological change and increased acceptance by farmers is the most important consideration in explaining increased nutrient consumption. But in the nitrogen model economic variables are relatively more important than in the other models. Phosphate consumption has increased despite increasing prices so that farmer acceptance and technology have been much more important factors. This conclusion assumes that the time variable is an appropriate proxy for technology.

10. By 1980, Michigan will be consuming 187,000 tons of nitrogen, 172,000 tons of phosphate and 167,000 tons of potash annually. The exact quantity consumed will depend on (1) the price of the nutrient relative to the price paid for all production items, (2) the net farm income relative to the prices paid by farmers, (3) the price of the most important fertilizer consuming crops and (4) the most important determinant, the changes in fertilizer related technology and the changes in farmer attitudes toward fertilizer. Chapter V outlines the procedure for making similar estimates for other states.

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Appendix I, Table 1. Estimated supply[a] of fertilizer nutrients versus actual consumption (1,000 tons)

Year ending June 30	N		Available P_2O_5		K_2O	
	Estimated domestic supply	Consumption	Estimated domestic supply	Consumption	Estimated domestic supply	Consumption
1950	1,250	956	2,200	1,930	1,150	1,070
1951	1,250	1,171	1,921	2,086	1,300	1,337
1952	1,375	1,366	2,100	2,180	1,515	1,545
1953	1,585	1,584	2,465	2,250	1,850	1,704
1954	1,916	1,790	2,325	2,222	1,830	1,767
1955	2,126	1,897	2,286	2,264	1,841	1,834
1956	2,303	1,875	2,359	2,228	1,931	1,836
1957	2,265	2,065	2,352	2,280	1,874	1,895
1958	2,400	2,263	2,235	2,276	1,871	1,900
1959[b]	2,770	2,618	2,664	2,530	2,251	2,151
1960	3,071	2,686	2,761	2,552	2,253	2,112
1961[b]	3,082	2,973	2,722	2,622	2,055	2,123
1962	3,269	3,303	2,938	2,781	2,581	2,220
1963[b]	3,982	3,870	3,171	3,044	2,644	2,456
1964	4,462	4,342	3,492	3,333	2,807	2,677

[a]U.S.D.A., Agricultural Stabilization and Conservation Service, The Fertilizer Situation (annual reports released early in the calendar year.

[b]Fertilizer supply estimates are revised estimates.

Appendix II, Table 1. Price indexes used for deflation

Year[a]	Index of prices paid by farmers[b]	Index of prices paid for production items
1949	254.66	240.66
1950	250.00	238.66
1951	280.66	273.33
1952	289.33	280.66
1953	279.00	260.66
1954	278.66	257.33
1955	278.33	256.66
1956	275.33	247.66
1957	285.66	258.00
1958	292.66	262.66
1959	298.00	267.33
1960	300.33	267.00
1961	302.00	267.66
1962	306.00	269.00
1963	311.66	273.66
1964	313.33	271.66

[a] Arithmetic average for February, March and April indexes, 1910-14 = 100.

[b] Commodities and services, interest, taxes and wage rates.

Source: U.S.D.A. Statistical Reporting Service, Agricultural Prices (monthly issues).

APPENDIX III

Formulas for Phosphate Price Indexes

Series A - Laspeyres

$$A = \frac{\sum^{k} P_i Q_o}{\sum^{k} P_o Q_o}$$

where: i = 1950...1962
o = 1950
P = price
Q = quantity of available P_2O_5
k = phosphate rock, super phosphate grades under 22 percent, superphosphate grades over 22 percent

Series B - Paasche

$$B = \frac{\sum^{k} P_i Q_n}{\sum^{k} P_n Q_n}$$

where: n = 1962

Series C - Fisher Ideal

$$C = \sqrt{A \quad B}$$

Series D - Chain

$$D_o = \frac{\sum^{k} P_o Q_o}{\sum^{k} P_o Q_o}$$

$$D_1 = (D_o) \frac{\sum^{k} P_1 Q_o}{\sum^{k} P_o Q_o}$$

$$D_2 = (D_1) \frac{\sum^{k} P_2 Q_1}{\sum^{k} P_1 Q_1}$$

$$\vdots$$

Series E - Chain of Fisher Ideal

$$E_o = 1$$

$$E_1 = (E_o)\sqrt{\left(\frac{\sum^k P_1 Q_o}{\sum^k P_o Q_o}\right)\left(\frac{\sum^k P_1 Q_1}{\sum^k P_o Q_1}\right)}$$

$$E_2 = (E_1)\sqrt{\left(\frac{\sum^k P_2 Q_1}{\sum^k P_1 Q_1}\right)\left(\frac{\sum^k P_2 Q_2}{\sum^k P_1 Q_2}\right)}$$

$$\vdots$$

Note: Available P_2O_5 in rock phosphate, superphosphate grades under 22% and superphosphate grades over 22% is estimated to be .003, .45, .18.

APPENDIX IV

Formulas for Nitrogen Price Index

Series F - Chain

$$F_o = \frac{\sum^{l} P_o\ Q_o}{\sum^{l} P_o\ Q_o}$$

where:
i = 1950...1962
o = 1950
P = price
Q = quantity of N
l = three commodities: sodium nitrate ammonia nitrate, ammonium sulphate
m = l plus anhydrous ammonia and urea

$$F_1 = (F_o)\ \frac{\sum^{l} P_1\ Q_o}{\sum^{l} P_o\ Q_o}$$

$$\vdots$$

$$F_6 = (F_5)\ \frac{\sum^{m} P_6\ Q_5}{\sum^{m} P_5\ Q_5}$$

$$F_7 = (F_6)\ \frac{\sum^{m} P_7\ Q_6}{\sum^{m} P_6\ Q_6}$$

$$\vdots$$

Series G - Chain of Fisher Ideals

$$G_o = 1$$

$$G_1 = (G_o)\ \sqrt{\frac{\sum^{l} P_1\ Q_1}{\sum^{l} P_o\ Q_1}}$$

$$G_2 = (G_1)\ \sqrt{\left(\frac{\sum^{l} P_2\ Q_1}{\sum^{l} P_1\ Q_1}\right)\left(\frac{\sum^{l} P_2\ Q_2}{\sum^{l} P_1\ Q_2}\right)}$$

$$\vdots$$

$$G_6 = (G_5) \sqrt{\left(\frac{\sum^{m} P_6 Q_5}{\sum^{m} P_5 Q_5}\right)\left(\frac{\sum^{m} P_6 Q_6}{\sum^{m} P_5 Q_6}\right)}$$

Series H - Laspeyres

$$H_j = \frac{\sum^{l} P_i Q_o}{\sum^{l} P_o Q_o}$$ where: $j = 1950...1955$

$$H_k = (H_{55}) \frac{\sum^{m} P_i Q_a}{\sum^{m} P_a Q_a}$$ where: $k = 1956...$
a = weights are 1950 quantities for commodities in l and 1955 quantities for anhydrous ammonia and urea

Series I - Laspeyres

$$I_j = H_j$$

$$I_k = (H_{55}) \frac{\sum^{m} P_i Q_c}{\sum^{m} P_c Q_c}$$ where: c = 1955 weights for m

Series J - Laspeyres

$$J_j = H_j$$

$$J_k = I_k$$ where: $k = 1956...1960$

$$J_s = (I_{60}) \frac{\sum^{m} P_i Q_d}{\sum^{m} P_d Q_d}$$ where: d = 1960 weights for m
$s = 1961...$

Series K - Paasche

$$K_j = \frac{\sum^{l} P_i \, Q_c}{\sum^{l} P_c \, Q_c} (b)$$

where: b is an adjustment factor to standardize index to year 1950 = 1

$$K_k = (K_{55}) \frac{\sum^{m} P_i \, Q_n}{\sum^{m} P_n \, Q_n}$$

where: n = 1962 weights

Series L - Paasche

$$L_j = K_j$$

$$L_k = (K_{55}) \frac{\sum^{m} P_i \, Q_d}{\sum^{m} P_d \, Q_d}$$

$$L_s = (L_{60}) \frac{\sum^{m} P_i \, Q_n}{\sum^{m} P_n \, Q_n}$$

Series M - Fisher Ideal

$$\sqrt{I \quad K}$$

Series N - Fisher Ideal

$$\sqrt{J \quad L}$$

Appendix V, Table 1. Index[a] of prices paid by farmers for nitrogen (N), by states and United States, 1950-1964

Yr. / State	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964
Me.	100.0	105.5	116.5	111.2	116.1	115.9	111.4	109.1	110.9	106.4	104.7	106.4	103.6	102.4	101.8
N.H.	100.0	106.4	117.0	111.8	116.9	116.6	111.5	109.1	110.7	108.7	105.8	104.9	103.2	101.1	100.5
Vt.	100.0	106.8	119.2	119.2	119.3	118.7	118.8	113.5	115.1	113.3	109.5	109.4	106.2	105.8	103.8
Mass.	100.0	106.7	116.4	113.4	120.4	110.9	109.0	104.8	106.9	105.6	104.2	104.6	102.5	101.0	99.8
R.I.	100.0	107.1	118.6	116.5	124.5	115.8	111.5	107.6	110.8	109.6	107.9	107.8	106.5	105.2	103.9
Conn.	100.0	107.0	117.3	113.1	120.7	110.4	105.9	101.6	106.2	104.5	103.1	103.5	100.4	99.4	97.8
N.Y.	100.0	108.6	114.1	115.8	117.4	115.9	110.1	104.9	109.4	107.7	106.5	105.8	104.1	104.9	104.9
N.J.	100.0	103.9	111.9	113.7	117.0	115.2	110.9	108.4	111.3	109.4	108.2	109.4	109.9	108.5	108.6
Pa.	100.0	107.4	111.9	115.6	118.0	113.7	109.8	104.8	108.5	105.4	103.1	102.9	101.3	98.2	95.7
O.	100.0	117.3	119.7	123.3	129.8	125.5	117.8	115.8	113.4	109.5	108.6	110.7	106.5	101.7	96.2
Ind.	100.0	116.5	115.7	122.2	128.2	121.6	112.0	104.4	103.6	105.8	105.4	104.8	101.6	101.4	95.4
Ill.	100.0	110.4	112.5	115.8	123.5	119.7	114.0	110.5	111.6	110.1	109.9	107.9	101.8	98.7	98.8
Mich.	100.0	107.8	115.0	115.6	114.8	116.2	113.7	108.5	106.9	100.9	104.9	104.0	96.7	91.6	84.3
Wisc.	100.0	108.5	112.3	115.0	118.6	118.6	112.4	112.1	108.6	116.6	113.7	112.1	109.4	106.9	102.1
Minn.	100.0	107.7	109.0	112.7	118.9	114.1	111.0	107.3	110.2	106.9	104.5	104.7	101.1	98.3	103.5
Ia.	100.0	111.6	116.6	118.1	122.9	117.9	115.3	108.6	107.5	106.1	108.4	103.5	100.6	100.3	93.8
Mo.	100.0	105.3	110.4	110.1	117.6	110.4	104.7	102.3	109.0	105.3	102.5	102.5	98.0	97.6	94.9
N.D.	100.0	106.6	106.6	112.7	113.8	116.7	113.3	109.8	111.4	106.8	95.8	92.5	93.1	89.0	89.5
S.D.	100.0	112.2	110.7	115.9	121.6	119.4	114.7	110.6	113.4	112.7	103.5	107.9	104.8	103.1	103.4
Neb.	100.0	107.9	115.3	119.0	125.2	111.7	102.3	95.6	99.6	95.9	92.3	92.5	92.1	88.6	95.2
Kan.	100.0	108.4	112.4	112.0	122.7	114.2	112.9	104.1	106.3	101.2	100.3	97.4	98.3	96.8	91.6
Del.	100.0	108.2	114.7	120.7	125.9	125.8	121.1	121.1	119.6	117.4	114.3	110.3	109.8	105.4	107.9
Md.	100.0	104.4	113.7	112.0	118.2	114.5	108.1	105.1	102.0	102.0	100.8	99.6	101.1	97.8	99.5
Va.	100.0	104.5	108.8	109.2	108.6	107.8	105.8	101.5	99.2	94.8	95.8	97.4	98.9	97.8	93.7
W.Va.	100.0	106.6	115.3	117.2	117.6	115.2	112.7	108.4	108.7	103.9	104.8	108.9	108.7	108.8	108.1
N.C.	100.0	103.2	108.3	107.6	101.5	102.4	102.1	97.7	99.7	97.4	96.6	99.4	99.4	101.3	99.1
S.C.	100.0	107.3	111.3	113.5	108.6	107.2	106.2	98.4	102.7	99.4	100.5	101.3	100.3	99.5	98.8
Ga.	100.0	103.8	110.3	109.9	105.7	103.9	98.9	94.1	96.5	94.8	95.0	96.3	95.2	97.2	96.6

Appendix V, Table 1--Continued.

Yr. / State	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964
Fla.	100.0	102.4	105.3	110.0	110.5	106.3	103.3	96.6	100.6	97.4	98.5	96.5	98.3	96.5	94.6
Ky.	100.0	108.5	112.4	117.1	116.6	111.6	104.8	104.1	105.2	103.4	101.2	100.6	98.5	96.4	95.2
Tenn.	100.0	106.4	112.5	107.1	111.9	112.6	109.1	103.1	107.5	104.5	105.1	106.4	102.2	100.3	98.1
Ala.	100.0	104.0	108.6	110.3	109.2	109.3	101.7	96.7	100.8	99.5	98.4	99.2	98.9	98.7	100.2
Miss.	100.0	104.7	108.4	109.4	110.1	106.4	104.5	99.7	100.4	100.0	98.5	101.4	99.6	99.0	93.2
Ark.	100.0	107.8	112.5	112.1	115.2	109.8	107.9	102.5	106.2	107.3	104.8	105.4	105.5	107.0	101.8
La.	100.0	106.7	112.2	113.8	114.8	108.9	102.9	98.0	103.2	99.7	99.7	101.4	99.3	96.4	90.1
Okla.	100.0	110.8	119.4	114.6	124.9	116.0	103.2	102.7	103.2	103.9	100.9	97.8	91.7	90.8	84.5
Tex.	100.0	108.5	111.9	113.5	112.4	110.2	102.9	103.6	111.4	103.9	95.5	98.0	86.7	77.9	80.9
Mont.	100.0	102.6	114.0	114.0	114.3	110.2	108.3	102.8	98.8	102.7	101.3	106.7	100.8	97.1	97.1
Id.	100.0	107.9	111.2	114.4	118.2	112.9	112.4	105.6	100.7	103.3	102.1	106.4	101.6	102.7	100.0
Wyo.	100.0	113.8	116.7	121.3	116.6	115.8	113.8	108.9	115.3	113.9	107.9	112.0	105.3	99.5	99.4
Colo.	100.0	106.5	107.5	109.4	116.7	114.9	108.9	102.4	106.8	103.0	102.0	98.4	95.4	96.0	92.8
N.Mex.	100.0	105.6	108.7	108.1	111.4	108.6	101.2	98.5	98.7	94.9	89.0	92.9	88.8	78.6	75.4
Ariz.	100.0	106.4	113.1	121.6	113.2	111.0	103.1	92.9	92.6	86.0	84.7	86.3	84.6	81.5	82.0
Ut.	100.0	106.0	107.6	109.5	113.6	108.4	100.8	94.3	94.2	97.5	94.5	97.0	98.9	97.3	97.5
Nev.	100.0	100.0	100.0	101.4	103.2	98.7	94.4	85.7	86.0	89.1	86.8	86.9	87.3	86.6	86.6
Wash.	100.0	107.2	109.3	111.3	113.1	113.7	112.7	105.8	103.6	100.5	97.9	98.1	98.1	93.6	94.1
Ore.	100.0	110.2	112.3	115.0	117.4	113.4	109.9	106.4	106.1	106.2	98.7	100.8	101.1	94.6	94.6
Calif.	100.0	104.7	107.9	109.5	113.6	108.3	107.6	101.5	100.1	93.7	84.7	82.9	81.0	72.8	70.8
U.S.	100.0	105.7	109.7	111.9	113.7	110.5	107.7	102.3	103.1	100.8	98.0	98.7	94.6	91.3	89.9

[a]Index is a chain index estimated using Series F, specified in Appendix IV.

Data Sources: U.S.D.A., Agricultural Research Service, Consumption of Commercial Fertilizer and Primary Plant Nutrients in the United States (Annual Issues).
U.S.D.A., Statistical Reporting Service, Agricultural Prices (April and September issues).

Appendix V, Table 2. Index[a] of prices paid by farmers for available phosphate (P_2O_5), by states and United States, 1950-1964

Yr. / State	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964
Me.	100.0	108.8	111.8	113.2	111.8	111.8	111.7	110.3	114.7	111.7	113.2	114.7	115.8	117.5	117.5
N.H.	100.0	105.6	111.3	114.2	117.0	105.7	102.8	107.1	114.2	108.5	112.8	114.2	112.8	114.2	114.2
Vt.	100.0	105.7	108.6	111.4	117.1	108.5	108.5	108.5	114.3	115.7	118.5	115.7	118.0	118.5	117.1
Mass.	100.0	106.0	115.0	118.0	124.1	112.2	115.1	115.1	116.7	113.7	112.2	116.7	117.5	118.4	116.9
R.I.	100.0	102.9	107.1	110.0	117.1	106.0	108.7	110.1	111.5	111.5	111.5	111.5	111.4	115.7	111.4
Conn.	100.0	105.9	113.2	116.1	123.5	113.7	108.3	112.6	114.2	113.8	116.3	119.2	118.5	119.9	119.9
N.Y.	100.0	103.2	108.0	111.2	116.1	116.2	116.1	117.8	122.0	123.0	124.6	124.7	125.2	127.1	126.9
N.J.	100.0	105.1	108.5	108.5	115.3	116.2	116.6	117.0	122.5	118.8	123.7	124.2	126.7	123.8	131.2
Pa.	100.0	106.6	109.9	113.2	116.5	115.4	116.1	118.0	122.1	119.1	120.4	122.3	121.4	121.0	121.7
O.	100.0	105.4	109.1	112.0	113.5	115.9	116.9	121.9	124.4	123.0	119.5	121.4	120.3	124.4	123.7
Ind.	100.0	108.3	110.4	108.7	111.0	115.5	113.5	115.5	118.6	117.3	118.9	117.4	115.7	109.8	113.3
Ill.	100.0	101.6	102.6	101.7	103.2	104.6	102.4	103.7	109.2	104.8	106.1	106.7	102.5	102.6	100.6
Mich.	100.0	109.1	110.0	110.0	113.0	111.1	109.3	110.4	119.5	116.7	116.8	124.2	120.8	122.2	127.1
Wisc.	100.0	110.5	112.1	110.6	107.7	111.5	110.4	108.9	112.0	110.0	111.5	112.5	109.4	108.2	107.0
Minn.	100.0	109.8	108.7	114.0	117.9	117.8	116.5	116.5	117.9	118.1	119.3	121.8	120.7	120.6	118.5
Ia.	100.0	109.5	110.7	111.3	112.7	109.9	107.7	106.6	110.5	112.2	111.6	112.8	110.3	110.8	109.9
Mo.	100.0	103.9	106.2	105.5	106.7	106.8	103.0	103.6	105.2	103.8	105.4	104.4	104.4	103.7	103.4
N.D.	100.0	105.5	105.7	109.8	116.5	108.3	105.6	105.6	108.3	108.3	109.7	108.3	111.1	111.1	108.3
S.D.	100.0	107.7	107.7	110.8	108.2	107.1	103.3	103.3	107.1	103.2	103.2	108.4	108.4	108.4	108.4
Neb.	100.0	104.6	111.6	119.4	117.0	109.2	105.3	105.4	109.7	109.7	111.0	112.4	109.6	109.4	111.4
Kan.	100.0	109.9	111.5	115.6	116.7	114.0	111.0	112.7	115.7	109.9	112.8	114.2	112.8	112.8	111.3
Del.	100.0	110.6	114.0	117.5	121.1	121.5	124.9	126.0	123.5	122.8	121.5	129.2	129.2	132.6	130.9
Md.	100.0	107.1	112.4	114.1	119.1	119.3	120.8	119.7	121.6	117.9	123.1	123.2	125.5	122.9	122.3
Va.	100.0	105.0	110.7	109.8	113.8	114.2	112.4	113.8	112.1	110.8	113.0	118.8	119.4	121.0	122.2
W.Va.	100.0	102.7	110.7	113.7	117.4	117.7	114.7	112.6	115.7	115.0	118.9	124.6	123.6	126.7	123.4

Appendix V, Table 2--Continued.

Yr. / State	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964
N.C.	100.0	110.8	112.0	112.0	112.2	112.4	114.3	116.4	114.4	116.4	118.5	124.6	126.1	135.9	133.4
S.C.	100.0	109.4	114.2	118.8	119.0	114.4	114.4	119.2	119.3	126.1	124.2	125.0	120.6	142.0	144.2
Ga.	100.0	106.4	112.9	110.8	111.6	112.0	112.0	111.9	112.1	117.5	121.4	124.6	124.6	127.2	123.0
Fla.	100.0	110.1	110.1	114.5	113.5	110.7	107.1	107.7	109.0	109.7	112.7	110.2	113.5	117.1	97.1
Ky.	100.0	105.2	108.4	108.4	108.3	111.7	111.0	114.5	115.6	116.2	118.6	122.2	123.7	124.3	124.6
Tenn.	100.0	108.5	109.4	113.5	114.2	124.8	124.8	121.0	124.0	124.5	129.6	134.4	142.6	141.6	141.4
Ala.	100.0	117.1	115.0	115.0	114.0	114.4	112.2	120.1	118.7	116.7	119.1	123.9	119.8	128.0	125.5
Miss.	100.0	112.5	110.6	110.6	114.8	117.3	117.3	117.6	122.9	124.8	128.6	134.2	133.9	137.1	138.4
Ark.	100.0	109.5	109.5	112.1	118.5	119.2	119.0	125.2	125.7	129.0	129.2	127.9	124.8	128.2	128.9
La.	100.0	113.9	111.3	111.1	116.3	120.8	118.4	117.2	119.1	125.5	122.2	122.2	128.0	123.4	126.4
Okla.	100.0	105.9	106.4	107.3	110.1	113.0	109.0	114.2	118.6	119.2	118.2	117.7	116.3	117.1	113.6
Tex.	100.0	109.3	110.8	112.9	114.0	115.5	116.4	113.0	118.6	116.8	115.3	117.5	117.8	119.2	114.6
Mont.	100.0	104.4	107.4	113.2	116.2	116.2	120.6	120.6	125.0	125.0	126.5	129.4	125.0	125.0	123.5
Id.	100.0	108.2	108.2	116.5	117.1	117.3	117.0	117.3	121.8	126.0	124.6	123.3	121.8	119.2	118.0
Wyo.	100.0	102.7	111.1	111.5	112.5	113.9	116.8	115.4	118.2	119.7	118.2	119.7	116.8	115.4	114.0
Colo.	100.0	103.0	105.5	110.6	113.3	113.4	107.9	109.3	114.7	114.8	113.4	113.3	110.7	106.1	102.8
N.Mex.	100.0	97.6	106.3	107.0	106.9	106.6	103.4	100.9	106.0	105.1	104.0	103.2	102.1	104.2	101.7
Ariz.	100.0	101.0	103.5	108.9	108.2	110.4	113.7	113.2	113.8	113.1	109.9	113.6	113.3	108.7	117.9
Ut.	100.0	95.8	97.9	98.4	103.5	105.4	106.8	107.1	111.9	113.6	112.5	111.7	112.3	111.5	111.2
Nev.	100.0	94.8	97.4	97.4	102.6	105.2	106.5	106.5	111.7	113.0	111.7	110.4	111.7	106.5	106.5
Wash.	100.0	107.0	110.0	112.3	113.7	119.4	117.3	120.2	123.1	120.6	118.6	120.6	120.6	119.9	117.9
Ore.	100.0	107.2	107.0	111.0	113.5	112.3	112.3	113.9	120.2	120.2	119.5	123.7	126.7	128.7	128.4
Calif.	100.0	104.1	108.2	103.8	101.9	102.6	105.1	107.1	113.4	113.9	112.1	113.0	114.7	116.0	118.4
U.S.	100.0	106.9	109.2	111.1	113.0	113.5	112.4	113.0	116.8	116.6	117.1	119.0	118.3	120.8	119.9

[a]Index is a chain index estimated using Series D, specified in Appendix III.

Data Sources: U.S.D.A., Agricultural Research Service, Consumption of Commercial Fertilizer and Primary Plant Nutrients in the United States (Annual Issues).
U.S.D.A., Statistical Reporting Service, Agricultural Prices (April and September issues).

Appendix 7, Table 3. Real price[a] of potash (K_2O) paid by farmers, by states

Yr. / State	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964
Me.	26.82	23.78	23.52	25.32	25.65	24.94	25.44	25.19	25.51	25.06	24.35	25.03	25.65	24.85	25.03
N.H.	26.82	23.42	23.16	24.94	25.26	24.94	25.44	25.19	25.51	25.06	24.35	25.03	25.65	24.85	25.03
Vt.	26.82	23.42	23.16	24.94	25.65	24.94	25.44	25.19	25.51	25.06	24.35	25.03	25.65	24.85	25.03
Mass.	26.82	23.42	23.16	24.55	25.65	24.55	24.23	24.03	23.99	22.82	22.47	23.16	24.16	23.39	23.19
R.I.	26.82	23.42	23.16	24.17	25.26	23.77	24.23	23.64	23.61	22.07	22.47	23.16	24.16	23.39	23.56
Conn.	26.82	23.42	23.16	24.17	25.26	23.77	24.23	23.64	23.61	22.07	22.47	23.16	24.16	23.39	23.56
N.Y.	23.05	20.49	21.02	23.21	23.94	23.42	22.85	22.52	22.73	23.08	21.42	22.12	23.05	22.47	22.53
N.J.	23.05	22.32	21.02	23.21	23.94	23.42	22.85	22.52	22.73	23.08	21.42	22.12	23.05	22.47	22.53
Pa.	25.14	24.15	23.52	23.21	23.94	23.42	22.85	22.52	22.73	23.08	21.42	22.12	23.05	22.47	22.53
O.	22.63	19.76	18.88	20.72	22.54	21.04	22.21	21.71	21.32	19.83	19.85	19.80	20.82	20.46	20.61
Ind.	23.05	20.49	19.24	21.87	20.21	19.87	19.79	19.38	18.85	18.14	17.98	18.49	19.33	18.64	18.77
Ill.	24.30	21.22	21.02	22.25	20.21	19.87	19.58	18.99	19.04	18.14	18.73	19.05	18.96	18.64	18.77
Mich.	25.14	24.15	23.52	22.64	22.54	20.65	21.80	22.48	21.32	20.95	20.97	21.30	20.45	21.56	20.98
Wisc.	25.14	23.05	22.80	23.79	22.54	22.60	23.02	22.48	21.70	20.57	20.23	20.55	21.19	20.46	20.61
Minn.	27.24	24.88	24.59	23.02	23.32	19.48	19.79	19.38	19.42	18.70	18.54	18.68	19.33	20.83	19.88
Ia.	27.24	26.34	24.23	24.17	24.09	19.48	19.79	19.38	19.42	18.70	18.54	18.68	19.33	19.37	19.88
Mo.	23.05	21.95	22.80	23.02	20.60	19.48	19.79	19.38	19.42	18.70	18.54	18.68	19.33	19.00	19.14
N.D.	27.65	25.61	25.65	26.47	27.20	22.60	23.02	22.09	21.70	20.95	20.23	20.92	20.82	20.46	20.25
S.D.	27.65	25.61	25.65	26.47	27.20	22.60	23.02	22.09	21.70	20.95	20.23	20.92	20.82	20.46	20.25
Neb.	27.65	25.61	25.65	26.47	27.20	22.60	23.02	22.09	21.70	20.95	20.23	20.92	20.82	20.46	20.25
Kan.	27.24	25.61	25.65	26.86	23.32	22.21	22.61	21.71	21.32	20.57	19.85	20.55	20.45	20.10	19.88
Del.	23.05	21.22	21.73	23.02	22.93	22.60	23.42	22.48	22.46	21.32	20.97	21.30	21.93	22.29	21.72
Md.	23.05	20.85	21.38	23.02	22.93	22.60	22.42	22.48	22.46	21.32	20.97	21.30	21.93	22.29	21.72
Va.	24.30	21.22	21.73	23.02	22.93	22.60	23.42	22.48	22.46	21.32	20.97	21.30	21.94	22.29	21.72
W.Va.	24.30	21.22	21.73	23.02	22.93	22.60	23.42	22.48	22.46	21.32	20.97	21.30	21.93	22.29	21.72
N.C.	23.46	20.85	20.31	22.25	22.54	22.21	22.61	22.87	22.84	22.44	22.85	23.16	23.79	24.12	23.93
S.C.	23.05	20.12	20.31	21.10	20.99	20.65	21.80	20.93	20.56	20.20	20.23	20.55	20.82	20.83	21.35
Ga.	22.21	21.22	19.60	21.10	20.60	21.04	21.80	21.71	21.32	20.57	21.35	21.67	21.56	21.56	21.35

Appendix V, Table 3--Continued.

Yr. / State	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964
Fla.	24.30	20.85	19.60	22.25	21.76	21.82	22.21	20.54	20.56	20.20	20.97	21.30	21.19	21.19	20.61
Ky.	24.72	23.78	23.87	24.55	23.71	22.60	22.61	21.32	20.94	20.57	20.60	21.30	21.56	21.19	21.35
Tenn.	24.72	21.95	21.38	22.64	22.54	21.82	21.80	21.32	20.94	20.20	20.23	20.92	21.19	21.19	21.35
Ala.	23.88	21.22	19.95	21.87	21.37	21.43	21.80	21.32	20.56	20.20	20.23	20.55	20.82	20.46	20.61
Miss.	22.63	19.39	18.88	19.95	20.60	19.87	19.79	20.16	20.18	19.83	19.48	19.80	20.45	20.46	20.61
Ark.	23.05	19.76	18.53	20.33	19.82	19.87	20.59	19.77	19.80	19.83	19.10	19.43	19.70	19.73	19.88
La.	21.37	18.66	19.24	20.72	20.60	19.87	20.19	20.16	19.80	19.08	19.10	19.43	20.45	19.37	19.51
Okla.	23.05	20.85	20.67	21.87	20.99	20.26	20.59	20.16	20.94	21.32	21.72	20.92	20.82	20.46	20.98
Tex.	24.72	21.22	21.38	22.64	22.15	22.60	22.61	21.71	20.56	19.83	19.85	19.43	20.07	19.00	19.14
Mont.	28.07	26.34	26.37	27.62	24.09	22.99	23.42	22.48	22.08	21.32	20.60	21.30	21.19	20.83	20.61
Id.	26.82	23.78	23.16	26.09	25.26	22.99	23.42	22.48	22.08	21.32	20.60	21.30	21.19	20.83	20.61
Wyo.	28.07	26.34	26.37	27.62	24.09	22.99	23.42	22.48	22.08	21.32	20.60	21.30	21.19	20.83	20.61
Colo.	25.98	24.88	24.23	26.86	23.32	22.99	23.42	22.48	22.08	21.32	20.60	21.30	21.19	20.83	20.61
N.Mex.	24.72	21.22	21.38	22.64	22.15	22.60	22.61	21.71	20.56	19.83	19.85	19.43	20.07	19.00	19.14
Ariz.	27.24	23.78	23.52	24.55	24.09	24.55	24.63	23.64	22.46	21.70	21.72	21.30	21.93	20.83	20.98
Ut.	25.14	20.12	21.38	24.55	21.37	24.55	24.63	23.64	22.46	21.70	21.72	21.30	21.93	20.83	20.98
Nev.	26.82	23.05	23.16	24.55	24.09	24.55	24.63	23.64	22.46	21.70	21.72	21.30	21.93	20.83	20.98
Wash.	25.14	21.95	22.09	24.17	23.71	23.85	24.71	23.72	23.30	22.89	22.92	22.87	22.75	22.36	22.53
Ore.	24.72	21.95	21.73	24.55	24.48	23.92	24.79	23.80	23.38	22.97	23.00	22.94	22.83	22.44	22.60
Calif.	23.05	20.12	20.31	21.48	21.37	21.66	22.45	21.55	21.17	20.80	20.82	20.77	20.67	20.32	20.47

[a]The real price of K_2O is: the April (or previous Sept. if April is not reported) price of muriate of potash (reported in Agricultural Prices) divided by the arithmetic average of February, March and April's Index of Prices Paid for Production Items (listed in Appendix II) times 100. Note: where state prices were unreported in either April or the previous Sept., the price of muriate of potash in a bordering state was used.

Data Source: U.S.D.A. Statistical Reporting Service, Agricultural Prices (April and September issues).

APPENDIX VI

Models N-3, P-3 and K-3 and Their Parameter Estimates

Model N-3

Specification of Model N-3 closely resembles that of Model N-1 (specified on pp. 59, 60). The difference is in the treatment of the time variables. A linear time variable is substituted for the set of time constants.

Model N-3 (time linear):

$$Q^{N}_{t,s} = \alpha + \sum^{g} \beta_g P^{N}_{t,s} + \beta Y_{t-1} + \sum^{k} \beta_k P^{c}_{t-1,s} + \sum^{i} \beta_i S_{*,s}$$

$$+ \beta T + U_{t,s}$$

$\bar{R}^2 = .9343$ Standard error of residuals = 7.72

$F = 149.26$

where:

Variables other than T are defined on pp. 59, 60.

T is linear time variable with 1950 = 1, 1951 = 2, 1952 = 3,...

The estimated coefficients are listed in Appendix VI, Table 1.

Appendix VI, Table 1. Estimated regression coefficients of variables in Model N-3 (time linear)

Variable	Unit of measure (real $)	Estimated regression coefficient, $\hat{\beta}$	Standard error regression coefficient
X(0)		-26.593	9.099
X(83) PN-N.Eng.		-42.169	27.378
X(84) PN-NCR		-0.097	19.356
X(85) PN-SE&ESC		-79.298	26.002
X(86) PN-WSC		-19.614	29.640
X(87) PN-Mount.		-52.168	22.480
X(88) PN-Pac.		-171.287	28.728
X(67) Net income	1.00	.006	.000
X(4) Hay price	1.00/ton	.727	.803
X(5) Corn price	.10/bu.	6.238	1.433
X(6) Corn price	.10/bu.	.726	1.169
X(7) Corn price	.10/bu.	-.221	1.513
X(8) Corn price	.10/bu.	.685	1.288
X(9) Cotton price	.01/lb.	.878	.626
X(10) Potato price	.10/bu.	-.996	.677
X(11) Wheat price	.10/bu.	6.251	2.338
X(12) Wheat price	.10/bu.	3.293	1.950
X(13) Cotton price	.01/lb.	-1.267	.783
X(14) Wheat price	.10/bu.	7.020	2.476
X(15) Corn price	.10/bu.	2.919	1.510
X(16) Corn price	.10/bu.	4.136	1.224
X(17) Corn price	.10/bu.	-12.211	1.471
X(18) Me.		64.038	
X(19) N.H.		27.313	
X(20) Vt.		22.181	
X(21) Mass.		52.565	
X(22) R.I.		93.719	
X(23) Conn.		49.828	
X(24) N.Y.		22.493	
X(25) N.J.		47.524	
X(26) Pa.		21.969	
X(27) O.		17.135	
X(28) Ind.		20.460	
X(29) Ill.		9.996	
X(30) Wisc.		-10.174	
X(31) Minn.		-9.881	
X(32) Ia.		6.434	
X(33) Mo.		16.857	
X(34) N.D.		-51.167	
X(35) S.D.		-14.610	

Appendix VI, Table 1--Continued.

Variable	Unit of measure (real $)	Estimated regression coefficient, $\hat{\beta}$	Standard error regression coefficient
X(36) Neb.		-7.148	
X(37) Kan.		-36.888	
X(38) Del.		61.623	
X(39) Md.		61.176	
X(40) Va.		123.267	
X(41) W.Va.		45.267	
X(42) N.C.		144.842	
X(43) S.C.		132.528	
X(44) Ga.		143.226	
X(45) Fla.		243.777	
X(46) Ky.		54.551	
X(47) Tenn.		48.136	
X(48) Ala.		75.620	
X(49) Miss.		68.559	
X(50) Ark.		23.239	
X(51) La.		43.994	
X(52) Okla.		-37.976	
X(53) Tex.		13.863	
X(54) Mont.		-27.704	
X(55) Id.		11.803	
X(56) Wyo.		-6.574	
X(57) Colo.		-3.109	
X(58) N.Mex.		52.902	
X(59) Ariz.		82.070	
X(60) Ut.		-4.033	
X(61) Nev.		-2.283	
X(62) Wash.		71.581	
X(63) Ore.		73.435	
X(64) Calif.		136.089	
Mich.[a]		0.0	
X(65) Time		1.953	0.120

[a]To avoid singularity of the X'X matrix one state (Michigan is arbitrarily included in the overall constant term. Because of this construction all other state constant terms should be interpreted as (±) deviations from the estimated overall constant ($\hat{\alpha}$=-26.593).

Model P-3

Specification of Model P-3 closely resembles that of Model P-1 (specified on pp. 77, 78). Only the treatment of the time variable is different.

Model P-3 (time linear):

$$Q^{P}_{t,s} = \alpha + \sum^{g} \gamma_g P^{p}_{t,s} + \beta Y_{t-1,s} + \sum^{k} \gamma_k P^{c}_{t-1,s} + \sum^{i} \gamma_i S_{*,s}$$

$$+ \beta T + U_{t,s}$$

$\bar{R}^2 = .9795$ Standard error of residuals = 4.76

$F = 499.59$

where:

Variables other than T are defined on pp. 77, 78.

T is a linear time variable with 1950 = 1, 1951 = 2, 1952 = 3,...

The estimated coefficients are listed in Appendix VI, Table 2.

Appendix VI, Table 2. Estimated regression coefficients of variables in Model P-3 (time linear)

Variable	Unit of measure (real $)	Estimated regression coefficient, $\hat{\beta}$	Standard error regression coefficient
X(0)		-3.315	10.055
X(83) PP-N.Eng.		-64.710	20.081
X(84) PP-NCR		24.340	20.316
X(85) PP-SE&ESC		-70.250	17.740
X(86) PP-WSC		-78.319	28.234
X(87) PP-Mount.		-16.901	20.091
X(88) PP-Pac.		-30.550	36.119
X(67) Net income	1.00	.002	.001
X(4) Hay price	1.00/ton	2.811	.441
X(5) Cotton price	.01/lb.	-.445	.617
X(6) Corn price	.10/bu.	.853	.668
X(7) Corn price	.10/bu.	-1.981	.978
X(8) Corn price	.10/bu.	-1.671	.869
X(9) Cotton price	.01/lb.	.612	.418
X(10) Potato price	.10/bu.	1.057	.751
X(11) Wheat price	.10/bu.	2.575	1.377
X(12) Hay price	1.00/ton	.410	.641
X(13) Hay price	1.00/ton	.632	.372
X(14) Hay price	1.00/ton	.098	.512
X(15) Wheat price	.10/bu.	2.197	1.519
X(16) Corn price	.10/bu.	1.858	.704
X(17) Corn price	.10/bu.	-6.269	.905
X(18) Me.		67.986	
X(19) N.H.		8.376	
X(20) Vt.		10.774	
X(21) Mass.		33.900	
X(22) R.I.		73.149	
X(23) Conn.		33.982	
X(24) N.Y.		49.508	
X(25) N.J.		90.464	
X(26) Pa.		52.406	
X(27) O.		7.395	
X(28) Ind.		5.941	
X(29) Ill.		-7.737	
X(30) Wisc.		-13.214	
X(31) Minn.		-16.901	
X(32) Ia.		-12.864	
X(33) Mo.		-7.169	
X(34) N.D.		-31.683	
X(35) S.D.		-24.876	

Appendix VI, Table 2--Continued.

Variable	Unit of measure (real $)	Estimated regression coefficient,[a]	Standard error regression coefficient
X(36) Neb.		-24.205	
X(37) Kan.		-31.945	
X(38) Del.		76.034	
X(39) Md.		77.788	
X(40) Va.		199.290	
X(41) W.Va.		43.419	
X(42) N.C.		108.831	
X(43) S.C.		93.182	
X(44) Ga.		93.370	
X(45) Fla.		193.334	
X(46) Ky.		66.115	
X(47) Tenn.		63.301	
X(48) Ala.		74.362	
X(49) Miss.		46.443	
X(50) Ark.		32.993	
X(51) La.		41.287	
X(52) Okla.		13.972	
X(53) Tex.		28.165	
X(54) Mont.		-13.957	
X(55) Id.		3.690	
X(56) Wyo.		-2.685	
X(57) Colo.		-1.137	
X(58) N.Mex.		5.602	
X(59) Ariz.		25.109	
X(60) Ut.		3.562	
X(61) Nev.		-4.391	
X(62) Wash.		10.475	
X(63) Ore.		13.565	
X(64) Calif.		28.158	
Mich.[a]		0.0	
X(65) Time		1.070	0.058

[a]To avoid singularity of the X'X matrix one state (Michigan is arbitrarily included in the overall constant term. Because of this construction all other state constant terms should be interpreted as (±) deviations from the estimated overall constant ($\hat{\alpha}$=3.315).

Model K-3

Model K-3 is the same as Model K-1 (specified on pp. 87-89) with the exception of the time variables. In Model K-1 no form of time is specified but in Model K-3 time is entered as a linear variable.

Model K-3 (time linear):

$$Q^K_{t,s} = \alpha + \sum^{g} \beta_g P^K_{t,s} + \beta Y_{t-1,s} + \sum^{k} \beta_k P^c_{t-1,s} + \sum^{i} \beta_i S_{*,s}$$

$$+ \beta T + U_{t,s}$$

$\bar{R}^2 = .9564$ Standard error of residuals = 8.09

F = 233.20

where:

Variables other than T are defined on pp. 87-89.

T is a linear time variable where 1950 = 1, 1951 = 2, 1953 = 3,...

The estimated coefficients are listed in Appendix VI, Table 3.

Appendix VI, Table 3. Estimated regression coefficients of variables in Model K-3 (time linear)

Variable	Unit of measure (real $)	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient
X(0)		2.099	11.039
X(83) PK-N.Eng.		-1.115	.773
X(84) PK-NCR		.216	.480
X(85) PK-SE		-6.659	.898
X(86) PK-S.Cent.		-.988	.945
X(87) PK-Pl.&Mt.		1.391	.264
X(88) PK-Pac.		1.683	1.393
X(67) Net income	1.00	.004	.001
X(4) Hay price	1.00/ton	2.062	.743
X(5) Potato price	.10/bu.	.643	1.282
X(6) Corn price	.10/bu.	1.153	1.212
X(7) Corn price	.10/bu.	-9.521	1.553
X(8) Corn price	.10/bu.	-2.999	1.361
X(9) Cotton price	.01/lb.	1.330	.626
X(10) Potato price	.10/bu.	-.152	.473
X(11) Wheat price	.10/bu.	3.083	3.280
X(12) Cotton price	.01/lb.	1.642	.819
X(13) Corn price	.10/bu.	-22.463	1.521
X(14) Hay price	1.00/ton	1.487	1.112
X(15) Corn price	.10/bu.	-.036	1.572
X(16) Corn price	.10/bu.	.962	1.241
X(18) Me.		64.852	
X(19) N.H.		7.169	
X(20) Vt.		6.146	
X(21) Mass.		29.117	
X(22) R.I.		71.041	
X(23) Conn.		30.571	
X(24) N.Y.		30.653	
X(25) N.J.		75.662	
X(26) Pa.		37.064	
X(27) O.		.904	
X(28) Ind.		5.461	
X(29) Ill.		-13.467	
X(30) Wisc.		-7.954	
X(31) Minn.		-18.780	
X(32) Ia.		-22.479	
X(33) Mo.		-13.846	
X(34) N.D.		-47.638	
X(35) S.D.		-52.758	

Appendix VI, Table 3--Continued.

Variable	Unit of measure (real $)	Estimated regression coefficient, $\hat{a}$	Standard error regression coefficient
X(36) Neb.		-53.352	
X(37) Kan.		-69.715	
X(38) Del.		225.425	
X(39) Md.		221.200	
X(40) Va.		343.750	
X(41) W.Va.		194.133	
X(42) N.C.		308.162	
X(43) S.C.		272.798	
X(44) Ga.		274.248	
X(45) Fla.		422.960	
X(46) Ky.		53.115	
X(47) Tenn.		49.147	
X(48) Ala.		45.235	
X(49) Miss.		20.256	
X(50) Ark.		2.680	
X(51) La.		4.333	
X(52) Okla.		-57.129	
X(53) Tex.		-58.678	
X(54) Mont.		-48.856	
X(55) Id.		-45.544	
X(56) Wyo.		-49.044	
X(57) Colo.		-46.602	
X(58) N.Mex.		-67.660	
X(59) Ariz.		-83.649	
X(60) Ut.		-44.750	
X(61) Nev.		-49.653	
X(62) Wash.		-65.450	
X(63) Ore.		-64.660	
X(64) Calif.		-71.045	
Mich.[a]		0.0	
X(65) Time		1.193	0.111

[a]To avoid singularity of the X'X matrix one state (Michigan is arbitrarily included in the overall constant term. Because of this construction all other state constant terms should be interpreted as ($\pm$) deviations from the estimated overall constant ($\hat{\alpha}$=2.099).

APPENDIX VII

Other Models Considered in the Study

In addition to the time constants model specified in the text and the time linear models specified in Appendix VI, three other models were considered in the study. To avoid repetition the general notation developed in Chapter II will be used. That is, F represents each of the primary nutrients: nitrogen, phosphate and potash. The three alternative models differ from Model F-1 only in the form of the time variable. Model F-2 excludes time as a variable. And although the inclusion of time significantly improves percentage explained, all the nutrient models without a time variable have $\bar{R}^2$ greater than .90 (N-2, .9078; P-2, .9689; K-2, .9488).

Two models specify time as a quadratic. In Model F-4, two variables, time (linear) and time square are specified. That is, one variable enters as 1950 = 1, 1951 = 2, 1952 = 3, etc., and another variable (T^2) enters as 1950 = 1, 1951 = 4, 1952 = 9, etc. Model F-5 specifies one variable (T^2) as the form of the time variable. T^2 enters as 1950 = 1, 1951 = 4, 1952 = 9, etc. All the models regarding time as a quadratic yield $\bar{R}^2$'s greater than .90.