THE MACROECONOMIC CONSEQUENCES OF A EUROPEAN MONETARY UNION

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ABSTRACT

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This dissertation develops a two-country model of a monetary union, analyzing fully the linkages between the countries by specifying structural equations for the goods, money and bond markets. Interdependencies arise through trade, the asset markets, and a common currency. The financing needs of the governments determine the supply of bonds; while the savings functions of the public and the monetary policy of the central bank determine demand. The central bank determines the supply of money in the world, but demand within each country determines its distribution. The model also includes a supply side for each economy based on an expectations augmented Phillips curve.

It is possible to trace the shifts in both aggregate demand and supply resulting from a change in fiscal and monetary policies. Because prices are not fixed, policies which affect aggregate demand and thus change inflation in one country cause a shift in aggregate supply in the other country. Past policies matter if they affect the relative current account balances, because these balances are reflected in the slope of each country's aggregate demand curve. Thus, given asymmetries in current account balances, the fiscal policy adopted by a country can have asymmetric effects on output. This result suggests that fiscal policies may cause friction among countries in a European monetary union, supporting arguments for fiscal policy convergence prior to the creation of a monetary union.

Next, policy interactions among the two fiscal authorities and the monetary authority are explored in a game theoretic setting. The fiscal authorities set targets for output and inflation in their own countries, while the monetary authority sets a target only for average inflation in the monetary union. Strategic interactions among the players is examined under coordination, a Nash game and a Stackelberg game. The central bank achieves its inflation target under noncooperation but not through coordination. The countries do not meet either of their targets under any of the games examined. A monetary union, in which the central bank does not care about the distribution of inflation across countries, may not bring welfare improvements for individual members.

DEDICATION

This dissertation is dedicated to my husband, Bill Garber

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CHAPTER 1: A MODEL OF A MONETARY UNION

I: Introduction

In December 1991, the leaders of the European Community (EC) met in Maastricht, Holland, to sign a treaty of economic and monetary union. This treaty formalized the intentions of the European Community to move toward full economic and monetary union, which began with the Single European Act of 1985. Initially, attention was focussed on the single market aspect of this Act: the elimination of all barriers to the free movement of people, goods and assets by 1992. It was not until the issuance of the "Report on Economic and Monetary Union in the Economic Community" (the Delors Report), in April 1989, that the issues of monetary union and coordination of economic policies came to the forefront.

The Delors Report envisioned a process of monetary union linked with the convergence of economic policies to be accomplished in three stages. Stage one would focus on the establishment of the single market through the integration of goods markets and financial markets. Stage two would see the establishment of the basic organizations needed for the functioning of an economic and monetary union, most important being the creation of the European System of Central Banks (ESCB), which would be given independence from the fiscal authorities of the member countries of the European Community and from the Community itself. In the final stage, exchange rates between member countries would be fixed, the ESCB would become the sole monetary policy authority for the EC, and ultimately a single currency would be adopted.

There has been much written concerning the process of establishing a monetary union, particularly with respect to the role of a central bank and the implementation of a common currency. While the process of creation of a monetary union will be a temporary one, the resultant union is expected to be permanent. Yet, there has been little written concerning the macroeconomics of the EC countries operating in a monetary union. One reason may be that until recently there have been doubts as to whether the final two stages would be implemented. Now however, there is little doubt that there will be a monetary union among most, if not all the EC countries, before the end of this decade. What remains unclear is the extent to which fiscal convergence will be mandated as part of the process of monetary union. The most important issue in this regard is whether members of the EC will provide the Community the ability to restrict the budget deficits of the national governments.

However this last issue is decided, one thing is clear: the European Community system can not be modelled as a typical federal system, such as that of the U.S. or Canada, nor as a standard open economy fixed exchange rate system. In the European Community, the members of the federation, the national governments, will continue to be the primary fiscal policy makers, controlling most of the revenues and expenditures of the Community. Thus, the interaction between the national governments through fiscal policies remains important, while the role of the central (i.e. EC) government is of minor importance. Even if the final step of the monetary union were the establishment of fixed exchange rates between the member countries, and not the adoption of a single currency, the establishment of an independent central banking system to set monetary policy for the entire EC would limit the applicability of the open economy fixed exchange rate model.

A model of a monetary union for the EC must capture the importance and interaction of fiscal policies across the member countries, the link between countries caused by the use of a common currency, and the restrictions imposed by an independent supranational central bank. This is done through the creation of a two country macroeconomic model, which has its roots in the Mundell-Fleming model. In order to depict more fully the nature of the linkages between the countries and the central bank, the model does not begin (as is standard) with the reduced form equations, but derives these equations based on a structural model of goods market, money market and bond market interaction. Furthermore the model developed in this chapter does not make the common assumption that output is demand determined, but instead develops a simple model of the supply side of the economy to capture both the demand and supply effects on output and inflation. This model is used to explore the interdependence among the countries and between the countries and the central bank in a monetary union and to analyze the policy interactions among them.

The second section of this chapter discusses the creation of a European monetary union. The third section examines issues in modelling a monetary union within the context of international macroeconomic models. The fourth section develops a two country model of a monetary union. The fifth section explains the linkages between the countries as captured in the aggregate supply and demand equations for each country. The sixth section gives the solution for equilibrium output and inflation in each country, and develops comparative statics to indicate how the policies of one country affect both countries within the monetary union. This section also examines how the existence of a monetary union changes the results of the standard two country open economy model. The final section presents the conclusions and indicates areas for further research.

Section II: Monetary Integration in the European Community

The original Treaty of Rome, which established the European Economic Community in 1957, gave little mention to monetary policy within the Community. The Treaty did set basic goals for the economic policies of the member countries: high employment, stable prices, maintenance of confidence in their currencies, and balance of payments equilibrium. It does make reference to coordination of monetary policies "to the full extent needed for the functioning of the common market (Louis, 1990, pp. 11-12). Nevertheless, no mention is given to the nature or merits of this coordination. Nor is there any reference to the need for monetary integration.

The first major step towards monetary integration occurred in 1969 when the governments of the member countries agreed that the Community should devise a plan for an economic and monetary union. The impetus for this agreement was the success of the Community in the 1960s, particularly the completion of the customs union and the development of the common agricultural policy (Baer and Padoa-Schioppa, 1989, p. 53). This agreement was formalized in the Werner Report, adopted in 1971. The Werner report envisioned the completion of a monetary and economic union within a decade. Such a union would be characterized by the centralization of economic policies, fixed exchange rates (possibly a single currency), and a Community system of central banks, based on the Federal Reserve System. The process towards the union began with the establishment of a margin system for exchange rate fluctuations for members' currencies, known as the "snake". This initial step was also one of the only steps made towards economic and monetary union, as by the mid-1970s the members of the EC lost their enthusiasm for closer integration. One primary reason for this change was the change in the international environment in the 1970s, particularly the collapse of the Bretton Woods system and the oil price shocks. Countries within the community disagreed with respect to the appropriate policy response to these shocks and saw exchange rate flexibility as a means to increase control over their domestic economies (Baer and Padoa-Schioppa, 1989, pp. 56-57).

The experience of high inflation and economic instability in the 1970s focussed the Community's attention once again on the need for policy coordination. In 1978 the European Monetary System (EMS) was created as a "scheme for the creation of closer monetary cooperation leading to a zone of monetary stability in Europe" (Louis, 1991, p. 19). The most important aspect of the EMS was the establishment in 1979 of the Exchange Rate Mechanism (ERM) which set narrow margins for exchange rate fluctuations between member countries.[1] At present all the member countries of the European Community, with the exception of Greece are members of the ERM. Although there were frequent readjustments of exchange rates between the member countries in the earlier years, these readjustments now occur infrequently.

The success of the EMS in bringing about nominal exchange rate stability for the member countries is undisputed. There is, however, much disagreement as to whether the EMS is also responsible for real exchange

[1] The bands are set at 2.5 percent for all countries, except Italy, Portugal, Spain and the United Kingdom which are allowed a 6 percent margin of fluctuation.

rate stability, and therefore a convergence of the inflation rates of the member countries. Inflation rates in the EMS countries have declined and converged since its establishment, but so have the inflation rates for all the industrialized countries. Given this common experience of disinflation throughout the industrialized world, there is no satisfactory way to determine the impact of the EMS on inflation rates in Europe.[2] Nonetheless, there are those who claim that participation in the EMS has made it easier for some countries to pursue disinflationary policies. Giavazzi and Giovannini (1990) argue that the constraint imposed by the exchange rate margins provided a justification for unpopular (i.e., disinflationary) domestic policies.

The EMS is also thought to be responsible for eliminating independent monetary policy within the Community. As noted in *The Wall Street Journal*: "new members in the mechanism and narrower divergence bands within it have highlighted the interdependence of European economies in general and monetary policy in particular" (Whitney, 1991). But this interdependence carries an asymmetric burden. The burden of adjustment is placed on the deficit countries whose currencies are at the bottom of the fluctuation band. No such burden falls on the countries whose currencies are near the top of the fluctuation band. These countries are "practically exonerated from correction of their external imbalances" (de

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[2] For more on this point, see Rudiger Dornbusch, 1991, Problems of European Monetary Integration, In A. Giovannini and C. Mayer, eds., *European Financial Integration*, New York: Cambridge University Press. Also, Susan M. Collins, Inflation and the EMS, *National Bureau of Economic Research Working Paper No. 2599*, May 1988.

Larosiere, 1990, p. 722).[3] Countries whose economies and consequently currencies are weak are limited in their ability to lower interest rates to promote growth, for to do so could weaken their currencies further, pushing them down to the bottom of the margin of fluctuation.

There is also some evidence that the EMS had an impact on the fiscal policies of the member countries. De Grauwe (1990) notes that while the decline in inflation rates in the 1980s was a characteristic of both EMS and non-EMS industrial economies, looking at output growth rates presents a different picture. The EMS countries, as a group, experienced a more pronounced slowdown in GDP growth than non-EMS countries, both within and outside of Europe, in the 1980s. De Grauwe states that this difference can be explained by looking at the fiscal policies of the EMS countries. Since 1982 the EMS countries have followed more deflationary fiscal policies than the non-EMS countries. Using a game-theoretic setup, he argues that countries in the EMS attach a large weight to external equilibrium, as measured by the Current Account balance. Although this has helped to stabilize exchange rates within the EMS, he shows that in the absence of cooperation countries in the EMS will react to an exogenous shock, which causes a deterioration in their current accounts, by restricting fiscal policy. Although cooperation would lead to less restrictive policies, and higher growth, de Grauwe states that "in the field of fiscal policies, the EMS countries have little incentive to cooperate" (de Grauwe, 1990, p.138).

[3] This asymmetry is fairly typical of fixed exchange rate systems and was a constant problem with the Bretton Woods system.

This more pronounced deceleration in economic growth and the increasing competition Europe faced from Japan and the United States were two important factors leading to the adoption of the Single European Act in 1985. The Single European Act called for the creation of an internal market in the EC, through the elimination of all barriers to the free flow of people, goods, services and capital. The process is to be completed by the end of 1992.

As the concept of and movement towards an internal market gained momentum, the issue of economic and monetary policy coordination once again received attention within the EC. In 1988, the European Council established the Committee for the Study of Economic and Monetary Union, chaired by Jacques Delors to examine this issue and to develop a program aimed at its implementation. In 1989 the Committee issued a report stating:

> Economic and monetary union in Europe would imply complete freedom of movement for persons, goods, services and capital, as well as irrevocably fixed exchange rates between national currencies and finally, a single currency. This, in turn would imply a common monetary policy and require a high degree of compatibility of economic policies and consistency in a number of other policy areas, particularly the fiscal field. These policies should be geared to price stability, balanced growth, converging standards of living, high employment and external equilibrium. (Committee for the Study of Economic and Monetary Union, 1989, p.17)

Monetary union would be characterized by a single currency, and the common management of monetary policy carried out through a European System of Central Banks (ESCB). Economic union would have four basic elements: 1) a single market; 2) measures aimed at strengthening the "market mechanism"; 3) common policies with respect to structural change and regional development; and, 4) macroeconomic policy coordination among member governments, to include binding rules for budgetary policies.

The achievement of economic and monetary union would occur in three stages. Stage one would aim at a greater convergence in economic performance among the member countries, through increased policy coordination. During this stage the internal market would be completed, all members of the EC would become participants in the ERM, and procedures would be established for budgetary policy coordination.[4] The details of the last two stages would be worked out during this first stage and the necessary Treaty amendments would be made to pave the way for economic and monetary union. The European Council, in June 1989 gave its approval for the commencement of the first stage on July 1, 1990 and established an intergovernmental conference to develop the last two stages. The first stage is scheduled to be completed in 1994.

Phase two involves the creation of the organizations necessary for full monetary union, most notably the ESCB. There is clear agreement on the structure of the new central banking system. It is to be comprised of a ruling council made up of 5-7 full time directors and the governor of each country's central bank.[5] The council is to be given full independence from the governments of the member countries and of the EC. During this phase, however the individual central banks will still set

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[4] The EC has adopted procedures for budgetary policy coordination. At present, the European Commission and finance ministers from all member countries, review each country's budget. These groups may publicly call for spending cuts by a member government, but their advice is non-binding.

[5] Even with the creation of a monetary union the member countries will retain their national central banks and thus appoint a governor of this bank. These banks, however, will not carry out independent national monetary policies, but will be incorporated into the central bank system. They will operate similar to the Landszentralbanken (literally: Land Central Banks) of the Bundesbank or the Regional Federal Reserve Banks.

national monetary policies. Also during this phase, countries are to achieve further economic convergence.

The final timetable for the start of phase three has not yet been established, but this phase is unlikely to occur before 1997. As seen at present, it is expected that in 1996, the European Commission and the ESCB will report to the Council of Finance Ministers (composed of the finance ministers from each member country) as to the progress countries have made towards economic convergence. Of particular importance is the convergence of interest rates and inflation rates, and exchange rate stability (i.e. the absence of realignments in the ERM). Based on this report, the member countries will decide if they are ready to move to full monetary union. The decision to move to a monetary union must be approved by all member countries, but participation of all countries in the monetary union is not required.[6]

As noted above, the participants in the monetary union will share a common currency, issued and controlled by a common, independent central bank. The central bank will be responsible for the formulation and implementation of the monetary policy for the entire Community (or members of the monetary union if these are less than the total EC countries). As formulated by the Delors Report, the primary objective of the central bank is to be price stability. To emphasize the weight which the central bank should place on achievement of this objective, the report states that only

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[6] At present the proposals range from a minimum of six to eight member states agreeing to participate in the monetary union, in order for it to be established. There is also some disagreement as to whether a country can be kept out of the monetary union if it has not achieved the required level of convergence. For a discussion of this issue, see "The Unpopularity of Two-Speed Europe", *The Economist*, September 14, 1991, page 89.

"subject to" this objective should the central bank "support the general economic policy set at the Community level" (Committee for the Study of Economic and Monetary Union, 1989, p. 25). To further emphasize the importance of price stability, and the independence of the central bank, the bank would be prohibited from the direct financing of government deficits.

While the basic structure of the monetary union has been agreed upon by the member countries of the EC, there is less agreement as to the degree of economic union that is to occur or that is necessary for monetary union - in particular, the coordination and control of fiscal policies. The Delors Report itself is equivocal on this point. For instance the report states:

> In the budgetary field, binding rules are required that would ... impose effective upper limits on budget deficits of individual member countries of the Community.

but then adds:

> although in setting these limits the situation of each member country might have to be taken into consideration. (Committee for the Study of Economic and Monetary Union, 1989, p.24)

At another point the Report states:

> Even after attaining economic and monetary union, the Community would continue to consist of individual nations with differing economic, social, cultural and political characteristics. The existence and preservation of this plurality would require a degree of autonomy in economic decision-making to remain with individual member countries and a balance to be struck between national and Community competences. (Committee for the Study of Economic and Monetary Union, 1989, p. 17)

There are two views on the need for fiscal convergence in a monetary union. The first argues that there is no need to establish binding rules for fiscal policy, as the monetary union will of its own accord lead to fiscal policy convergence. Cohen (1989) develops a model which supports this conclusion. He argues that monetary policy coordination if it is credible will "trigger the appropriate fiscal correction needed to make it sustainable" (Cohen, 1989, p. 304). Fiscal policy coordination might be welfare enhancing, but he notes it is not a prerequisite for monetary union.

The second view argues that binding rules with respect to the size of budget deficits are needed to ensure the proper functioning of a monetary union. This view is based on the premise that there would be a lack of fiscal restraint among the members of the monetary union, the result of which would be either pressure on the monetary authority to monetize the deficits and/or crowding out of investment within the Community due to an increase in the interest rate.[7] Fiscal laxity by some members is thus thought to create problems for the entire Community either through higher inflation or slower growth, which in turn could increase pressure within the fiscally sound countries to break away from the monetary union.

Although de Grauwe does not specifically discuss a monetary union, his conclusion that the EMS did not give incentives to coordinate fiscal policies can also be applied to the monetary union. In this case, however, one can argue that the lack of coordination will lead to too much fiscal restraint rather than too little. As noted in Buiter and Kletzer (1991), the asymmetry of constraints discussed on fiscal policies, "can only be rationalized through a belief that absent these constraints there

[7] Given the existence of perfect capital mobility and if assets are perfect substitutes, one can think of an interest rate prevailing for the entire EC.

would be a bias towards government deficits that are too large rather than too small." As de Grauwe points out, the opposite may in fact be true.

Section III: Modelling a Monetary Union

Modelling a monetary union, such as that expected to occur in the European Community, presents a problem because it does not fit into the mold of either a standard closed economy model, or an open economy fixed exchange rate model. The European Community system can not be modelled as a closed economy typical of federal systems, such as the United States or Canada, due to the difference in relative importance between the central government and regional authorities.[8] In modelling fiscal policy effects in a typical federal system one does not worry about the policy interactions and spillover effects among states, and between states and the central government. This is due to the dominant role of the central government as fiscal policy maker. Although states or regions in a federal system set taxation and revenue policies which have an impact upon the national economy, fiscal policy is still dominated by the actions of the federal government.

In the European Community, the members of the federation (the national governments) will continue to be the primary fiscal policy makers. Thus, the interactions of the regional players are of prime importance in modelling the monetary union. So, instead of ignoring the effects of fiscal policy decisions by the state governments (as in a closed economy model of the U.S.), the model of a European monetary union

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[8] There are those, however who seem to indicate that such a model is applicable. Both Cohen (1989) and Portes (1990) in discussing the issue of fiscal policy coordination note that one does not worry about this between states in the U.S. and thus it may not be a problem for the EC.

ignores the effects of fiscal policy decisions made by the Community government.

This distinction can be justified by looking at the relative sizes of fiscal expenditures by U.S. states, versus the "states" of the European Community. The expenditures of the U.S. federal government constitute around 20-25 percent of U.S. GDP, while the expenditures of even the most populous states (California and New York) are less than 2 percent of GDP, and the total expenditures of all fifty states are only about 13 percent of GDP. In contrast, the expenditures of the European Community government are at present only slightly more than 1 percent of the GDP of the EC and are not expected to exceed 3 percent of GDP (Lamfalussy, 1989, pp. 107 and 111). Furthermore, the European Community government, as the states in the U.S., have no means for active fiscal policy, as it must have a balanced budget.

A monetary union within the European Community also does not fit into the model of a fixed exchange rate system. First the use of a single currency permanently fixes (at one) nominal exchange rates between the member countries. Re-valuation or de-valuation of the exchange rate is not possible. More important is not the use of a single currency, but rather the common monetary policy which distinguishes the model of the European monetary union from that of an open economy fixed exchange rate model. Unlike the standard Mundell-Fleming fixed exchange rate model where monetary policy has no effects, in the model of a monetary union, the monetary policy actions of the central bank affect all countries in the union. This observation and the existence of separate fiscal policy makers within each country indicate that there is likely to be strategic behavior on the part of the policy makers within a monetary union. Thus, the model to be developed here must serve two purposes: 1) indicate the channels of influence across countries, and 2) address policy coordination issues.

Most open-economy models treating issues of interdependence and policy coordination between countries start from a reduced form aggregate demand model. Cohen and Wyplosz (1989) develop a macroeconomic model of a monetary union in which aggregate demand in each country is given by one variable which is directly controlled by the fiscal policy maker, and inflation is a choice variable of the central bank.[9] Cohen (1989), Bean (1985) and Pachecco (1985) use slightly more complex reduced form models to model coordination problems across countries. While these models are useful for treating certain issues, they have limited applicability to the issues treated in this chapter, as they do not adequately indicate the types of linkages between the countries. In the model developed here one can clearly distinguish the direct, spillover and feedback effects on aggregate demand and aggregate supply due to a change in one of the exogenous variables.

The model developed in this chapter is most similar to those of Oudiz and Sachs (1984), Sachs and Wyplosz (1984), Kole (1988), and Kenen (1989, 1990), all of which start from the explicit equations for the components of aggregate demand. Nevertheless, all of these models also have their limitations for modelling a monetary union. With the exception of Oudiz and Sachs, all of the models ignore the supply side of the economy. With the exception of Kole, they ignore interest-income terms in

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[9] This type of model is fairly common in game theory models of monetary and fiscal policy, see for example Cohen (1989) and de Grauwe (1990).

private absorption. This is a fairly typical restriction in open-economy model, as it simplifies the process of solving for aggregate demand and equilibrium output and prices. At the same time, however, in dynamic models it removes a link between the supply and demand side by removing inflation from the demand side. Likewise, it neglects the impact of last period's interest rate on this period's consumption. Further, Sachs and Wyplosz use a small country model and assume that prices are fully flexible so that output is fixed at its natural level. This eliminates the usefulness of active fiscal policy. Kole also assumes that output is fixed, and ignores the money market.

Although the degree of fiscal convergence which will be mandated by the European Community has not been settled, this does not hinder the ability to create a model of a monetary union. In fact, such a model can be used to resolve some of the issues relating to fiscal convergence, by indicating the nature of the spillover effects of fiscal policy in a monetary union, and addressing the issue of crowding out both internally and in other countries within the monetary union. These issues are addressed in this chapter.

Much attention has been focussed on the impact of the internal market on Europe and also on the mechanisms for creating a monetary union. Little attention however has been paid to the macroeconomic implications of a monetary union for the economies of the member countries of the Community. Cohen and Wyplosz present one of the few macroeconomic models of a monetary union. Their model, as discussed above, is too limited to be used to address the issues discussed in this chapter.[10]

Section IV: The Model

There are two countries, indexed by 1 and 2, which are of similar size. Each country maintains independent control over its fiscal policy, but the monetary policy for the two countries is controlled by an independent central bank.

The countries produce goods which are imperfect substitutes. Goods are traded freely between the two countries. There are no transportation costs, but preferences for goods may vary across the countries. The government and the private sector in each country demand both domestic and foreign goods.

Each government issues one period bonds which are bought by the residents of each country and the central bank. The bonds are perfect substitutes, and capital is perfectly mobile. Thus, the nominal interest rate prevailing in each country at all times is the world interest rate (i.e. $I_t = I_{1t} = I_{2t}$). There are no private issues of bonds, nor is there any capital accumulation.

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[10] Recently there have been models developed which focus on specific aspects of a monetary union. For example, Alesina and Grilli (1991) develop a simple game theoretic model to analyze the effect of the centralization of monetary policy on monetary policy in Europe. Since their focus is on the inflation-output trade off, the economy of the EC is modelled based on a single equation in which output is determined according to an expectations augmented Phillips curve (similar to equation (1) in this paper). As is common in many game theoretic models of monetary policy they ignore the demand side of the economy and also any interaction between a fiscal policy authority and the monetary policy authority.

There is a common currency, the ecu, which is issued by the central bank. Money creation is controlled solely by the central bank, through bond purchases.[11] At the end of each period, the governments repay their bonds plus interest. Thus, money is not held across periods. Since the real money supply is equal to the real value of bonds held by the central bank, the government makes its interest payments not in money but in goods. The central bank however, neither purchases goods, nor does it turn its profits over to the national governments. Therefore, one can assume that the goods payment of interest is immediately consumed by the central bank.[12]

All variables are measured in real terms. The deflator used to convert a country's nominal variables to real variables is its consumer price index. (See Table I for a listing of variables and coefficients.) All stock variables are measured at the start of the period, which is

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[11] Since the money supply is determined solely by the extent of bond purchases by the central bank, it is possible that the monetary authority might be constrained in its ability to increase the money supply. Given that this constraint is unlikely to be binding except in cases of hyperinflation, and given that the central bank's primary objective is price stability, it is assumed throughout that the constraint is nonbinding.

[12] This assumption is made for two reasons. First the seignorage issue in the European monetary union is at present unresolved. Nevertheless, it is unlikely to be left to the discretion of the central bank. Giving control over the allocation of seignorage to the central bank would give it control over not only monetary policy, but also provide it with the ability to conduct fiscal policy. That is, the central bank would have control over goods in the economy and could through its allocation of these goods directly influence the fiscal policy actions of the two governments. Furthermore, allowing the central bank to store the goods would give it the ability to dump them on the markets at any time and thus disrupt the fiscal actions or plans of the national governments. Secondly, given that one does not want to give the central bank control over goods, the only feasible alternative assumption is to have the central bank divide the seignorage equally between the two governments. This alternative was rejected as it would significantly complicate the model, without changing the nature of the results.

Table I: Notation

Variables:

a = real private domestic absorption

b^p = real private bond holdings

b = real bond issues by the goverment

b_m = real central bank bond holdings

g = real government spending

I = nominal world interest rate

$i = 1 + I$

m = real balances

nx = real net exports

p = domestic price index

p^c = consumer price index

R = real interest rate

$r = 1 + R$

R^e = expected real interest rate

t = real lump sum taxes

y = real output

y^d = real disposable income

$\bar{y}$ = optimal or natural level of real output.

π = inflation rate

π^e = expected inflation rate

d = government budget deficit

Parameters:

c = marginal propensity to consume

ϵ = private marginal propensity to import

ϵ_g = government marginal propensity to import

λ = income sensitivity of money demand

θ = interest sensitivity of money demand

η = substitution effect in net exports

ϕ = interest sensitivity of domestic absorption

α = output effect on domestic price inflation

γ = weight attached to domestic prices in cpi

Table II: Equations Underlying Country 1's Economy

Supply:

(1) $$\frac{p_{1,t}-p_{1,t-1}}{p_{1,t-1}} = \alpha\frac{y_{1,t}-\bar{y}}{\bar{y}} + \frac{E_{t-1}p_{1,t}-p_{1,t-1}}{p_{1,t-1}}, \qquad \alpha > 0$$

(2) $$p^c_{1,t} = \gamma p_{1,t} + (1-\gamma)p_{2,t}, \qquad \frac{1}{2} < \gamma < 1$$

(3) $$\pi_{1,t-1} = \frac{p^c_{1,t}-p^c_{1,t-1}}{p^c_{1,t-1}}$$

Demand:

(4) $$R^e_{1,t} = I_t - \pi^e_{1,t}$$

(5*a*) $$R_{1,t} = I_t - \pi_{1,t}$$ (5*b*) $$r_{1,t} = i_t - \pi_{1,t}$$

(6) $$\tilde{p}_t = \left(\frac{p^c_{2,t}}{p^c_{1,t}}\right)$$

(7) $$y_{1,t} = a_{1,t} + g_{1,t} + nx_{1,t}$$

(8) $$a_{1,t} = cy^d_{1,t} - \phi R^e_{1,t}, \qquad \frac{1}{2} < c < 1, \qquad 0 < \phi < 1$$

(9) $$y^d_{1,t} = y_{1,t} + r_{1,t-1}\, b^p_{1,t-1} - t_{1,t}$$

(10) $$nx_{1,t} = \epsilon a_{2,t}\tilde{p}_t + \epsilon_g g_{2,t}\tilde{p}_t - \epsilon a_{1,t} - \epsilon_g g_{1,t} + \eta\left(\frac{p_{2,t}}{p_{1,t}} - \frac{p_{1,t}}{p_{2,t}}\right),$$

$$0 < \epsilon < \frac{1}{2}, \qquad \epsilon_g \le \epsilon, \qquad \eta \ge 0$$

(11) $$g_{1,t} = t_{1,t} - r_{1,t-1}b_{1,t-1} + b_{1,t}$$

(12) $$m_{1,t} = \lambda y^d_{1,t} - \theta I_t, \qquad 0 < \lambda \le 1, \qquad 0 < \theta < 1$$

(13) $$b^p_{1,t} = y^d_{1,t} - a_{1,t}$$

indexed by t. Spending and portfolio decisions are made at the start of period t. Thus, I_t is the t to t+1 interest rate, and π_t is the t to t+1 inflation rate.

Country 1's economy:

The equations describing country 1's economy are listed in Table II. Equations (1)-(3) describe the supply side of the economy. Domestic price inflation, equation (1), is determined by the deviation of output from its natural level, and expected domestic price inflation. This gives a standard expectations augmented Phillips curve. The consumer price index is given by equation (2) as a weighted average of domestic and foreign prices. The weights are set in the initial period, as determined by the proportion of consumption consisting of domestic goods and imports, respectively.[13]

The demand side of the model, for country 1, is given by equations (4)-(13). Equations (4) and (5) are versions of the Fisher equation. They define the ex ante and ex post real interest rate, respectively. Equation (5a) is written with respect to the ex post net real interest rate while equation (5b) is written in terms of the ex post gross real interest rate. Equation (6) defines $\tilde{p}$ as the ratio of country 2's to country 1's consumer prices. This term is used to convert country 2's real variables, which appear in country 1's equations, into the same units as country 1's real variables. Thus, for example:

$$g_{2,t}\tilde{p}_t = \left(\frac{G_{2,t}}{p^c_{2,t}}\right)\left(\frac{p^c_{2,t}}{p^c_{1,t}}\right) = \frac{G_{2,t}}{p^c_{1,t}}$$

[13] As is the practice with consumer price indexes, these weights are not adjusted each period, but may be updated after a number of years. For the purpose of this paper, there is no updating.

where $G_{2,t}$ is nominal spending by the government of country 2. Therefore, all of the real variables in country 1's demand and supply equations are in terms of country 1's consumer price index, and all of the real variables in country 2's demand and supply equations are in terms of country 2's consumer price index.

Equations (7)-(11) describe the goods market. Equation (7) is the national income identity. Real income (output) is equal to real private domestic absorption, a_t, real government spending, g_t, and real net exports, nx_t. Real private domestic absorption, equation (8), depends positively on real disposable income, y_t^d, and negatively on the ex ante real interest rate, R_t^e. Real disposable income, y_t^d, is given by equation (9), as real income less taxes, t_t, (where taxes are lump sum), plus real interest earnings and the repayment of bonds bought by the private sector last period (where these two terms are by definition the gross real interest earnings on private holdings of bonds, $r_{t-1}b^p_{t,1}$).[14] Real net exports, real exports minus real imports, are given by equation (10). They are positively related to country 2's private and government purchases of country 1's goods, and negatively related to country 1's

---

[14] The term $r_{1,t-1}b^p_{1,t-1}$ is derived as follows:

$$b^p_{1,t-1}\left(\frac{p^c_{1,t-1}}{p^c_{1,t}}\right) + I_{t-1}b^p_{1,t-1}\left(\frac{p^c_{1,t-1}}{p^c_{1,t}}\right) = (1+I_{t-1})\,b^p_{1,t-1}\left(\frac{p^c_{1,t-1}}{p^c_{1,t}}\right)$$

$$= \left(\frac{i_{t-1}}{1+\pi_{1,t-1}}\right) b^p_{1,t-1}$$

$$= r_{1,t-1}\, b^p_{1,t-1}$$

$b^p_{1,t-1}$ denotes private bond holdings deflated by period t-1 consumer prices. Thus, to determine the real time t value of period t-1's bond purchases, it is necessary to multiply this term by the ratio of period t-1 to period t's consumer prices. The third step follows from the definition of inflation as given by equation (3).

private and government purchases of country 2's goods. As neither the private marginal propensity to consume, ϵ, nor the government's marginal propensity to consume, ϵ_g, is a function of relative prices, the parameter η is added to capture the price substitution effect. Therefore, an increase in the relative price of country 1's goods will, ceteris paribus, decrease real net exports, while conversely, an increase in the relative price of country 2's goods will increase real net exports. The government budget constraint is given by equation (11). Real government spending, g_t, is constrained by the real interest payments and repayments of last period's bonds (as noted above, these two combined give by definition the gross real interest payments on bonds, $r_{t-1}b_{t-1}$),[15] less tax revenues, t_t, and new bond issues, b_t. In each period the government can choose, at most, two of the three contemporaneous variables: government spending, taxes, and/or bond issues. The creation of an independent central bank removes the ability of the government to finance a deficit through money creation. Equations (12) and (13) describe the asset markets. Demand for real balances, equation (12), depends positively on real disposable income, via the transactions motive, and negatively on the nominal interest rate. Thus, an increase in the interest rate on government bonds will decrease the demand for money. As noted above, since the bonds issued by each country are perfect substitutes, there is only one nominal interest rate. The savings function is given by equation (13). All saving is through bond holdings and real private bond demand is determined

[15] For the derivation of the term $r_{1,t-1}b_{1,t-1}$ see footnote 14.

by the difference between real disposable income and real private domestic absorption.[16]

Country 2's Economy:

Given the symmetry between the two countries, the equations modelling country 2's economy, which are given in Table III, are basically the same as those for country 1. It is important to remember, however, that the variables for country 2's economy are deflated by its own consumer price index. Thus, to compare variables across the two countries one must use equation (6) to transform country 2's variables into variables deflated by country 1's consumer price index, and equation (19) to transform country 1's variables into variables deflated by country 2's consumer price index. Given this caveat, there are only two differences between the models of the two economies. First, in equation (18), which determines domestic price inflation in country 2, the natural level of output is divided by $\tilde{p}$. This is necessary because $\bar{y}$ is measured in units of country 1's consumer price index and thus must be converted into units

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[16] Equation (13) can also be used to derive the balance of payments equation. Substituting equations (8) and (9) into equation (13) yields:

$$
\begin{aligned}
b^{p}_{1,t} &= a_{1,t} + g_{1,t} + nx_{1,t} + r_{1,t-1}b^{p}_{1,t-1} - t_{1,t} - a_{1,t} \\
&= (g_{1,t} - t_{1,t}) + nx_{1,t} + r_{1,t-1}b^{p}_{1,t-1}
\end{aligned}
$$

Using equation (11) to substitute out for government spending net of taxes and rearranging, yields:

$$(b^{p}_{1,t} - b_{1,t}) - r_{1,t-1}\,(b^{p}_{1,t-1} - b_{1,t-1}) = nx_{1,t}$$

Now, replacing the bond variables with the individual components of bond demand, equations (28), results in the balance of payments equation:

$$
\begin{aligned}
&b_{11,t}+b_{21,t}\tilde{p}_t-b_{11,t}-b_{12,t}-b_{1m,t} \\
&\quad - r_{1,t-1}(b_{11,t-1}+b_{21,t-1}\tilde{p}_{t-1}-b_{11,t-1}-b_{12,t-1}-b_{1m,t-1}) = nx_{1,t}
\end{aligned}
$$

$$b_{21,t}\tilde{p}_t-b_{12,t}-b_{1m,t} = nx_{1,t}+r_{1,t-1}(b_{21,t-1}\tilde{p}_{t-1}-b_{12,t-1}-b_{1m,t-1})$$

of country 2's prices, through the division by $\bar{p}$. The second difference is that in country 2's net export equation, η enters with a negative sign. This again points out the symmetry between the two countries, as an increase in country 2's relative prices will decrease net exports for country 2 while increasing net exports for country 1.

Market Equilibrium Conditions:

The conditions for equilibrium in the bond, money, and goods markets are presented in Table IV. The bonds issued by the government of country i (i=1,2) are held by three groups of agents: the public in country i, the public in country j, and the central bank. Equation (28) shows the equality between the supply of bonds issued by country 1, and the demand for these bonds, broken down by type of demander. Equation (29) presents the same information for country 2. Equation (30) is the world bond market equilibrium condition. It states that the world demand for bonds is equal to the supply of bonds by the governments of country 1 and country 2.

The actual holdings by the residents in the two countries of the bonds issued by each government, is impossible to determine. This follows from the bonds being perfect substitutes and from the assumption of perfect capital mobility. Only by placing restrictions on the preferences of the individual bondholders (e.g. bondholders prefer to hold x% of their portfolio in their home country's bonds) can exact holdings be determined.[17] The central bank's holding of each bond is determined through the money market restrictions, as explained below.

Equation (31) gives the money market clearing condition, stating that the supply of real balances by the central bank is equal to the combined demand of the two countries. The supply of real balances is determined by the central bank's purchases of each government's bonds, as given by equation (32). The next two equations, (33) and (34), determine the central bank's holdings of each country's bond. For simplicity, it is assumed that the central bank buys half of its bonds from country 1 and half from country 2.[18] These four equations indicate that while the initial distribution of real balances between the countries is determined by the central bank, the ultimate distribution is determined by demand conditions in each country. Equation (35) presents the dynamic of the money supply. Since both countries issue only one period bonds, the increase in real balances in each period is determined by the difference between new bond purchases and the gross return on old bonds. The exclusion of the interest earnings on bonds from this equation is due to the assumption, noted above, that the central bank consumes these earnings.

Equation (36) gives the goods market clearing condition for either country. Real disposable income less domestic absorption and the

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[17] Such restrictions would change the magnitude of the interest rate effects due to an increase in bond issues by the governments. Furthermore, the magnitude of such effects could differ depending on which government issued the bonds if the residents of both countries prefer one countries bonds over the other. This in turn could change the fiscal policy effects derived in this model. Thus, an interesting extension of this paper would be to look at what happens if individuals do have strong preferences for their home country's bonds.

[18] Relaxing this assumption will not affect the model.

Table III: Equations Underlying Country 2's Economy

Supply:

(14) $$\frac{p_{2,t}-p_{2,t-1}}{p_{2,t-1}} = \alpha \frac{y_{2,t}-\dfrac{\bar{y}}{\tilde{p}_t}}{\dfrac{\bar{y}}{\tilde{p}_t}} + \frac{E_{t-1}p_{2,t}-p_{2,t-1}}{p_{2,t-1}}, \qquad \alpha > 0$$

(15) $$p^c_{2,t} = \gamma p_{2,t} + (1-\gamma)p_{1,t}, \qquad \frac{1}{2} < \gamma < 1$$

(16) $$\pi_{2,t-1} = \frac{p^c_{2,t}-p^c_{2,t-1}}{p^c_{2,t-1}}$$

Demand:

(17) $$R^e_{2,t} = I_t - \pi^e_{2,t}$$

(18a) $$R_{2,t} = I_t - \pi_{2,t}$$ (18b) $$r_{2,t} = i_t - \pi_{2,t}$$

(19) $$\frac{1}{\tilde{p}_t} = \frac{p^c_{1,t}}{p^c_{2,t}}$$

(20) $$y_{2,t} = a_{2,t} + g_{2,t} + nx_{2,t}$$

(21) $$a_{2,t} = cy^d_{2,t} - \phi R^e_{2,t}, \qquad \frac{1}{2} < c < 1, \quad 0 < \phi < 1$$

(22) $$y^d_{2,t} = y_{2,t} + r_{2,t-1}\, b^p_{2,t-1} - t_{2,t}$$

(23) $$nx_{2,t} = \epsilon a_{1,t}\frac{1}{\tilde{p}_t} + \epsilon_g g_{1,t}\frac{1}{\tilde{p}_t} - \epsilon a_{2,t} - \epsilon_g g_{2,t} - \eta\left(\frac{p_{2,t}}{p_{1,t}}-\frac{p_{1,t}}{p_{2,t}}\right)\frac{1}{\tilde{p}_t}$$

$$0 < \epsilon < \frac{1}{2}, \quad \epsilon_g \leq \epsilon \quad \eta \geq 0$$

(24) $$g_{2,t} = t_{2,t} - r_{2,t-1}b_{2,t-1} + b_{2,t}$$

(25) $$m_{2,t} = \lambda y^d_{2,t} - \theta i_t \qquad 0 < \lambda \leq 1, \quad 0 < \theta < 1$$

(26) $$b^p_{2,t} = y^d_{2,t} - a_{2,t}$$

Table IV: Equilibrium Conditions[19]

Bond Market:

(28) $b_{1,t} = b_{11,t} + b_{12,t} + b_{1m,t}$

(29) $b_{2,t} = b_{21,t} + b_{22,t} + b_{2m,t}$

(30) $b_t = b_{1,t} + b_{2,t}\,\tilde{p}_t$

Money Market:

(31) $m_t = m_{1,t}^d + m_{2,t}^d\,\tilde{p}_t$

(32) $m_t = b_{1m,t} + b_{2m,t}\,\tilde{p}_t = b_{m,t}$

(33) $b_{1m,t} = \frac{1}{2}\,m_t = \frac{1}{2}\,b_{m,t}$

(34) $b_{2m,t} = \frac{1}{2}\,m_t\,\frac{1}{\tilde{p}_t} = \frac{1}{2}\,b_{m,t}\,\frac{1}{\tilde{p}_t}$

(35) $$m_t - m_{t-1} = b_{m,t} - b_{m,t-1}$$
$$= (b_{1m,t} - b_{1m,t-1}) + (b_{2m,t}\,\frac{1}{\tilde{p}_t} - b_{2m,t-1}\,\frac{1}{\tilde{p}_{t-1}})$$

Goods Market:

(36) $y_{i,t}^d - a_{i,t} - d_{i,t} = ca_{i,t} \qquad i = 1,\ 2$

19 All world variables are deflated by country 1's price index. This is done for simplicity. It is also possible to use the world consumer price index (a weighted average of the two consumer price indices) to deflate world variables.

government deficit in each country must be equal to its real current account balance.

Assumptions:

In accordance with the idea that the countries have different preferences for goods produced in each country, it is assumed that $\gamma > 1/2$. This indicates that the residents of each country prefer their own goods to foreign goods. The marginal propensity to consume domestic goods out of disposable income, c, is assumed to be greater than 1/2, and the marginal propensity to consume imported goods, ϵ, is restricted to be less than 1/2.[20] These two assumptions correspond to the preference for home goods over foreign goods in each country. Furthermore, the government's marginal propensity to import, ϵ_g, is constrained to be not greater than the private marginal propensity to import, ϵ.[21]

Section V: Aggregate Supply and Aggregate Demand

Using equations (1)-(3) and (14)-(16) one can derive the aggregate supply equation for country 1:[22]

$$(37)\quad y_{1,t} = \bar{y}\left[1 + \frac{\gamma_2}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,(\pi^{\bullet}_{2,t-1} - \pi_{2,t-1})\right] - \bar{y}\left[\frac{\gamma_3}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,(\pi^{\bullet}_{1,t-1} - \pi_{1,t-1})\right]$$

---

[20] This restriction follows from the condition that: 1 = mpc+mpm+mps.

[21] This restriction is made since in practice a large portion of government spending goes towards the salaries of government workers, and governments generally do not have a weaker preference for domestically produced goods over foreign goods than do their citizens.

[22] See Appendix A for the derivation.

where:

$$\gamma_1 = \frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1} + (1-\gamma)p_{2,t-1}}, \qquad \gamma_2 = \frac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1} + (1-\gamma)p_{2,t-1}}$$

Using the same set of equations to solve for the aggregate supply equation for country 2 yields:

$$(38) \quad y_{2,t} = \frac{\bar{y}}{\tilde{p}_t}\left[1 + \frac{\gamma_4}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}(\pi^e_{1,t-1} - \pi_{1,t-1})\right] - \frac{\bar{y}}{\tilde{p}_t}\left[\frac{\gamma_1}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}(\pi^e_{2,t-1} - \pi_{2,t-1})\right]$$

where:

$$\gamma_3 = \frac{\gamma p_{2,t-1}}{\gamma p_{2,t-1} + (1-\gamma)p_{1,t-1}}, \qquad \gamma_4 = \frac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1} + (1-\gamma)p_{1,t-1}}$$

Each country's aggregate supply is determined by the natural level of output, the period t-1 to t expected change in the consumer price index and the period t-1 to t actual change in the consumer price index in both countries. Prices in country 2 influence supply in country 1 (and vice versa) through their effect on consumer prices in each country.

Given the restrictions on the weights in the consumer price indices of the two countries, it is possible to determine the sign of the coefficients on the inflation terms in each aggregate supply equation. Since $\gamma > 1/2$, it follows that:

$$\gamma_1\gamma_3 - \gamma_2\gamma_4 > 0$$

An increase in a country's own inflation rate, holding inflationary expectations constant, has a positive effect on output, which implies that the short-run aggregate supply curve is upward sloping in inflation-output space. The effects on short-run aggregate supply in country 1 of a change in the other variables in the short-run aggregate supply equation are shown in Table V. An increase in inflation in one country decreases aggregate supply in the other country. An increase in inflationary expectations (formulated in period t-1) in one country has a negative effect on short-run aggregate supply in that country, but a positive effect on short-run aggregate supply in the other country. Furthermore, an increase in the natural level of output has a positive, unitary effect on the actual level of output.

Table V: Aggregate Supply Effects

	Effect on Aggregate Supply
Increase in π_{2t}	-
Increase in $\pi^e_{1,t-1}$	-
Increase in $\pi^e_{2,t-1}$	+

The process of solving for aggregate demand indicates clearly the links between the two countries, through the goods, money and bond markets. The world interest rate is found by solving for equilibrium in the money market. This in turn provides a direct link between the LM curves in the two countries. Factors which affect money demand in one country will have an impact on money demand in the other country. Likewise, an increase in the money supply equally affects the LM curve in each country. The interest rate also links the two bond markets. Although individuals are indifferent between holding bonds issued by the governments of country 1 or country 2, the net bond demand (holdings less issues) in each country is shown below to have a direct impact on output.

Goods market linkages occur through trade and these (as shown below) have a significant impact on aggregate demand in each country.[23]

The first step in solving for aggregate demand in each country is the derivation of own output as an explicit function of output in the other country, and other exogenous variables.[24] The result of this process for country 1 is given below:

$$
\begin{aligned}
(39)\quad y_{1,t} = {} & \left[\frac{[(1-\epsilon_g) - (1-\epsilon)c]2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta) + \lambda\phi}\right] g_{1,t} \\
& + \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] r_{1,t-1}\,(b^{p}_{1,t-1} - b_{1,t-1}) \\
& + \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] b_{1,t} + \left[\frac{(1-\epsilon)2c\theta\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^{e}_{1,t} \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] y_{2,t}\,\tilde{p}_t \\
& + \left[\frac{(\epsilon_g - \epsilon c)2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] g_{2,t}\,\tilde{p}_t \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] r_{2,t-1}\,(b^{p}_{2,t-1} - b_{2,t-1})\,\tilde{p}_t \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] b_{2,t}\,\tilde{p}_t + \left[\frac{2\epsilon\phi\theta}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^{e}_{2,t}\,\tilde{p}_t \\
& + \left[\frac{\phi}{2\theta - (1-\epsilon)2c\theta + \lambda\phi}\right] b_{m,t}
\end{aligned}
$$

Equation (39) allows one to determine the direct and indirect demand linkages between the two countries, a process which is lost when one begins with a reduced form demand equation.

---

[23] For simplicity, throughout the remainder of this paper, it is assumed that $\eta=0$. The subsequent analysis would be unchanged by a relaxation of this restriction. Only if there is a large divergence in prices between the two countries will the substitution effect be important in altering the results.

[24] See Appendix A for the derivations.

The coefficients on the variables in equation (39) show the direct effects of these variables on aggregate demand in country 1. For example, real government expenditures in country 2 directly affect country 1, to the extent that they increase country 1's exports, and to the extent that they increase the world interest rate. The export effect is itself comprised of two parts: a positive effect due to the increase in spending on imports by the government in country 2, as measured by ϵ_g, and a negative effect resulting from a decline in private spending on imports in country 2 which occurs given that the increase in government spending was financed by an increase in taxes.[25] This latter effect is measured by ϵc. Thus the overall direct impact of an increase in government spending by country 2 on aggregate demand in country 1 is indeterminate.

Given the restrictions on the equations in the model, it is possible to sign the coefficients on five of the ten variables in equation (39). Real expenditures by country 1's government, g_{1t}, have a positive direct effect on aggregate demand in country 1. The direct effect of an increase in inflationary expectations in either country on aggregate demand in country 1 is positive. An increase in inflationary expectations in country 1 causes an increase in its domestic absorption, due to the decrease in saving (or increase in investment) resulting from the decrease in the real interest rate. An increase in inflationary expectations in country 2 causes an increase in country 1's exports due to the increase in consumption by the residents of country 2. An increase in the bond holdings of the central bank, b_{mt} also has a positive direct effect on

---

[25] Since the government's budget constraint given in equation (11) must be met, an increase in g_{1t} holding b_{1t} constant implies that taxes, t_{1t} are increased.

aggregate demand in country 1. This last term indicates that an increase in real balances in the world economy, will directly increase aggregate demand by shifting out the LM curve. As mentioned above, the initial impact is the same in both countries.

The term y_{2t} in equation (39) captures the indirect effects of the variables on output in country 1. A change in any of the exogenous or predetermined variables in equation (39) not only has a direct effect on output in country 1 through the linkages between the goods, money and bond markets in the two countries but also has an indirect effect due to the impact of the change on output in country 2. Returning to the example of an increase in government spending by the government of country 2, this not only directly affects country 1, as explained above, but also affects country 1 through its impact on demand in country 2. An increase in spending by the government in country 2 will increase income in country 2 and this in turn will have spillover effects on country 1. Thus, there are two channels of influence: a direct one through the initial effects on the markets and an indirect one through the impact on one's own output which in turn works through the market linkages to affect each country. Note that this is true for both changes in the other country and changes in one's own country. An increase in spending by the government of country 1 directly increases output in country 1, and also has a spillover effect on output in country 2, which in turn feeds back into aggregate demand in country 1.

It is important to note that, given the restrictions imposed so far, the sign of the coefficient on y_{2t} in equation (39) is indeterminate. Thus the overall impact of these feedback effects on demand in country 1 is unknown.

Solving for output in country 2 as an explicit function of output in country 1, and substituting the resultant equation into equation (39), is the final step in the solution for aggregate demand for country 1. The aggregate demand equation for country 1 which results from this procedure is:

$$\begin{aligned}(40)\quad y_{1,t} = {}& A_1\, g_{1,t} + A_2\, b_{1,t} + A_3\, \pi^{e}_{1,t} + A_4\, g_{2,t}\tilde{p}_t + A_5\, b_{2,t}\tilde{p}_t + A_6\, \pi^{e}_{2,t}\tilde{p}_t \\ &+ A_7\, b_{m,t} + A_8\, r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) \\ &+ A_9\, r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1} + A_{10}\, r_{1,t-1}b_{m,t-1}\end{aligned}$$

Likewise, the final form of the aggregate demand equation for country 2 is:

$$\begin{aligned}(41)\quad y_{2,t} = {}& A_1\, g_{2,t} + A_2\, b_{2,t} + A_3\, \pi^{e}_{2,t} + A_4\, g_{1,t}\left(\frac{1}{\tilde{p}_t}\right) + A_5\, b_{1,t}\left(\frac{1}{\tilde{p}_t}\right) \\ &+ A_6\, \pi^{e}_{1,t}\left(\frac{1}{\tilde{p}_t}\right) + A_7\, b_{m,t}\left(\frac{1}{\tilde{p}_t}\right) + A_8\, r_{2,t-1}(b^{p}_{2,t-1} - b_{2,t-1}) \\ &+ A_9\, r_{2,t-1}(b^{p}_{1,t-1} - b_{1,t-1})\left(\frac{1}{\tilde{p}_{t-1}}\right) + A_{10}\, r_{1,t-1}b_{m,t-1}\left(\frac{1}{\tilde{p}_t}\right)\end{aligned}$$

where the coefficients A_1 - A_{10} are given in Table VI. The coefficients on the own and foreign variables in each country are the same.

Using equations (5) and (18), and combining the predetermined bond variables into one term, the aggregate demand equations (40) and (41) can then be rewritten as follows:

$$\begin{aligned}(42)\quad y_{1,t} = {}& A_1\, g_{1,t} + A_2\, b_{1,t} + A_3\, \pi^{e}_{1,t} + A_4\, g_{2,t}\tilde{p}_t + A_5\, b_{2,t}\tilde{p}_t + A_6\, \pi^{e}_{2,t}\tilde{p}_t \\ &+ A_7\, b_{m,t} + i_{t-1}Y_1 - \pi_{1,t-1}Y_1\end{aligned}$$

$$(43)\quad y_{2,t} = A_1\, g_{2,t} + A_2\, b_{2,t} + A_3\, \pi^{\bullet}_{2,t} + A_4\, g_{1,t}\left(\frac{1}{\bar{p}_t}\right) + A_5\, b_{1,t}\left(\frac{1}{\bar{p}_t}\right)$$

$$+ A_6\, \pi^{\bullet}_{1,t}\left(\frac{1}{\bar{p}_t}\right) + A_7\, b_{m,t}\left(\frac{1}{\bar{p}_t}\right) + i_{t-1}Y_2 - \pi_{2,t-1}Y_2$$

where:

$$Y_1 = A_8(b^p_{1,t-1}-b_{1,t-1}) + A_9(b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} + A_{10}b_{m,t-1}$$

$$Y_2 = A_8(b^p_{2,t-1}-b_{2,t-1}) + A_9(b^p_{1,t-1}-b_{1,t-1})\left(\frac{1}{\bar{p}_{t-1}}\right) + A_{10}b_{m,t-1}\left(\frac{1}{\bar{p}_{t-1}}\right)$$

The slope of country 1's aggregate demand curve is $-1/Y_1$, and the slope of country 2's aggregate demand curve is given by $-1/Y_2$. Thus, in order for the aggregate demand curve in each country to be downward sloping[26], Y_i must be >0. It is not sufficient (nor necessary) that A_8, A_9 and A_{10} all be positive, since the terms:

$$(b^p_{i,t-1} - b_{i,t-1}) \qquad i=1,2$$

can be positive, negative, or zero. If this term is positive then country i was a net lender last period, and thus it had a current account surplus. If this term is negative country i was a net debtor last period, and thus it had a current account deficit. If this term is zero, it was neither a net borrower nor a net lender and thus its current account was in balance.

Table VII lists the nine possible cases to consider in analyzing the sign of Y_1, and the restrictions which each case imposes on the real bond

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[26] It is not necessary for stability that the aggregate demand curve be downward sloping. If both the aggregate demand curve and the aggregate supply curves are upward sloping, stability requires that the aggregate demand curve be steeper than the aggregate supply curve. Note, however, that one would expect the aggregate demand curve to be downward sloping since both the wealth effect through bonds and the income effect through money indicate that an decrease in inflation raises output.

Table VI: Coefficients in the Aggregate Demand Equation

$$A_1 = 1 - \frac{e_g}{1-c+2ce} > 0$$

$$A_2 = \frac{2\theta c(c-1+e-2ce) + \lambda\phi(1-2c+4ce)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}$$

$$A_3 = \frac{2\phi\theta(c-1+e-2ce)+\lambda\phi^2(2e-1)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)} > 0$$

$$A_4 = \frac{e_g}{1-c+2ce} > 0$$

$$A_5 = \frac{\lambda\phi-2ce\theta}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)} < 0$$

$$A_6 = \frac{\phi(\lambda\phi-2e\lambda\phi-2e\theta)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}$$

$$A_7 = \frac{\phi}{2(\theta-c\theta+\lambda\phi)} > 0$$

$$A_8 = \frac{2c\theta(c-1+e-2ce)+c(2e\lambda\phi-\lambda\phi)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)} > 0$$

$$A_9 = \frac{c(\phi\lambda-2e\phi\lambda-2e\theta)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}$$

$$A_{10} = \frac{\lambda\phi}{2(\theta-c\theta+\lambda\phi)} > 0$$

Table VII: Restrictions on Bond Holdings

Possible Cases		*Restrictions*
$b^{p}_{1,t-1}-b_{1,t-1} > 0$	$(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} > 0$	$b_{m,t-1} < 0$
$b^{p}_{1,t-1}-b_{1,t-1} > 0$	$(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} = 0$	$b_{m,t-1} < 0$
$b^{p}_{1,t-1}-b_{1,t-1} > 0$	$(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} < 0$	$b_{21,t-1}\tilde{p}_{t-1} > b_{12,t-1}$
$b^{p}_{1,t-1}-b_{1,t-1} < 0$	$(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} > 0$	$b_{21,t-1}\tilde{p}_{t-1} < b_{12,t-1}$
$b^{p}_{1,t-1}-b_{1,t-1} < 0$	$(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} = 0$	$b_{21,t-1}\tilde{p}_{t-1} < b_{12,t-1}$
$b^{p}_{1,t-1}-b_{1,t-1} < 0$	$(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} < 0$	$-\frac{1}{2}b_{m,t-1} < b_{21,t-1}\tilde{p}_{t-1}-b_{12,t-1} < \frac{1}{2}b_{m,t-1}$
$b^{p}_{1,t-1}-b_{1,t-1} = 0$	$(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} > 0$	$b_{m,t-1} < 0$
$b^{p}_{1,t-1}-b_{1,t-1} = 0$	$(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} = 0$	$b_{m,t-1} = 0$
$b^{p}_{1,t-1}-b_{1,t-1} = 0$	$(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} < 0$	$b_{21,t-1}\tilde{p}_{t-1} > b_{12,t-1}$

holdings.[27] Four of the cases can be eliminated because they imply that real bond holdings by the central bank are non-positive, indicating a non-positive real money supply. The remaining cases all impose restrictions on the relative sizes of the "cross-border" bonds, that is bonds issued by country 2's government which are held by the residents of country 1 (b_{21}), and bonds issued by country 1's government which are held by the residents of country 2 (b_{12}). When country 1 is not a net borrower, the amount of country 2's bonds held by the residents of country 1 must be greater than the amount of country 1's bonds held by the residents of country 2. Likewise when country 2 is not a net borrower, the amount of country 1's bonds held by the residents of country 2 must be greater than the amount of country 2's bonds held by the residents of country 1. The only case left to consider occurs when both countries are net borrowers.[28] In this

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[27] The analysis for this is the same for determining the slope of country 2's aggregate demand curve, the only difference being that the variables are measured in terms of country 2's price index. See Appendix B for the derivation of the restrictions presented in Table VII.

[28] The ability of both of the countries in a two country world to be net borrowers arises because of the presence of the common independent central bank. In this system the sum of the balance of payment accounts of two countries does not equal zero, but instead equals the central bank's holdings of bonds less its real interest earnings from last period's holdings of bonds. This can be shown as follows:

$$
\begin{aligned}
bop_{1,t} + bop_{2,t} &= nx_{1,t} + r_{1,t-1}(b_{21,t-1}\tilde{p}_{t,1} - b_{12,t-1} - b_{1m,t-1}) \\
&\quad + b_{12,t} + b_{1m,t} - b_{21,t}\tilde{p}_t + nx_{2,t}\tilde{p}_t \\
&\quad + r_{1,t-1}\left(b_{12,t-1}\frac{1}{\tilde{p}_{t-1}} - b_{21,t-1} - b_{2m,t-2}\frac{1}{\tilde{p}_{t-1}}\right)\tilde{p}_{t-1} \\
&\quad + b_{21,t}\tilde{p}_t + b_{2m,t} - b_{12,t} \\
&= b_{1m,t} + b_{2m,t} - r_{1,t-1}(b_{1m,t-1} + b_{2m,t-1}) \\
&= b_{m,t} - r_{1,t-1}b_{m,t-1}
\end{aligned}
$$

instance the difference between the cross border bond holdings of country 1 and country 2 is restricted to be between $-(1/2)b_m$ and $+(1/2)b_m$.

These cases have broader implications beyond determining the slope of the aggregate demand curve. They imply that both countries can not be net lenders, but both can be net borrowers. Given the link between this status and the fiscal policy stance of the governments, this result indicates that the framers of the European monetary union may indeed have more reason to be concerned with the size of fiscal deficits rather than surpluses. Furthermore, given that both countries abilities to be net borrowers is constrained by the willingness of the central bank to lend to them, it is likely that the pursuit of expansionary fiscal policies by both governments will come into conflict with the objectives of the central bank.

Another implication of these cases is that the ability of one country alone to be a net borrower is constrained by the willingness of the residents of the foreign country to be lenders. This indicates that even if the central bank is unwilling to finance the expansionary policies of a country, it can still pursue these policies as long as the residents of the other country are willing to lend to it. That is, given the lack of capital controls between the two countries, as long as the residents of one country are willing to finance the deficit spending of the other, neither the central bank nor the other government will be able to prevent or constrain the expansionary policies of one government. The central bank will still be able to constrain one government to the extent that it is willing to use restrictive monetary policies which will affect both countries. There are several issues to consider in this regard. The first is the relative impact of restrictive monetary polices on both countries, versus the impact of the expansionary fiscal policies on both countries. The second is the relative impact of the two policies on the country pursuing expansionary fiscal policies. A third issue is the willingness of the "errant" country to challenge the central bank and to engender the disdain of its neighbors. All of these factors are likely to be significant in determining policy outcomes. The first two issues are considered further below.[29]

Returning to the issue of determining the conditions needed to ensure that aggregate demand is downward sloping, rewrite Y_1 as follows:

$$(44)\quad Y_1 = A_8(b_{11,t-1}+b_{21,t-1}\tilde{p}_{t-1}-b_{11,t-1}-b_{12,t-1}-b_{1m,t-1}) + A_9(b_{22,t-1}\tilde{p}_{t-1}+b_{12,t-1}-b_{21,t-1}\tilde{p}_{t-1}-b_{22,t-1}\tilde{p}_{t-1}-b_{2m,t-1}) + A_{10}b_{m,t-1}$$

Canceling terms and making use of equations (33) and (34), equation (44) can be rewritten:

$$(45)\quad Y_1 = A_8(b_{21,t-1}\tilde{p}_{t-1}-b_{12,t-1}-\frac{1}{2}b_{m,t-1}) + A_9(b_{12,t-1}-b_{21,t-1}\tilde{p}_{t-1}-\frac{1}{2}b_{m,t-1}) + A_{10}b_{m,t-1}$$

Based on the restrictions on the bond terms, as given by Table VII, and noting from Table VI that A_8 and A_{10} are both positive, sufficient conditions for Y_1 to be positive are:

$$(46)\quad A_{10} - A_9 > 0 \qquad\qquad (47)\quad A_{10} - A_8 > 0$$

[29] Another issue which is outside the realm of this model is whether such expansionary policies will cause the residents of the two countries to view the bonds of the government in question as risky, and require a risk premium, which will drive a wedge between interest rates on the bonds. Such a premium depends on the belief that the government in question is likely to default on its debt.

A sufficient condition, in turn, to ensure that the inequalities (46) and (47) are indeed satisfied is that:

$$(48) \quad \lambda\phi > 2c\theta$$

This condition will be met if the interest sensitivity of domestic absorption is large and the interest sensitivity of money demand is small. This same condition is sufficient to ensure that the aggregate demand curve in country 2 is also downward sloping.[30] This condition is also useful in determining the signs of some of the coefficients on the aggregate demand variables, given in Table VI. These coefficients indicate the overall effect of changes in the policy and exogenous variables on aggregate demand in each country.

The direct, feedback and overall effects of an increase in the variables in the aggregate demand equation for country 1 are given in Table VIII.[31] The direct effects correspond to the effects captured in equation (39) which were analyzed above. The feedback effects correspond to the change in aggregate demand in country 1 resulting from a change in output in country 2. In the case of country 1's own variables these feedback effects result from the spillover effects on country 2's aggregate demand. Thus, for instance, an increase in real expenditures by country 1's government directly increases aggregate demand in country 1, as shown in Figure 1. The increase in government expenditures has a spillover effect on country 2, increasing its aggregate demand, through an

[30] Note also that this condition is sufficient for determining that the sign of the coefficient on y_{2t} in equation (39) is negative. This implies that the feedback effect on aggregate demand in country 1 from an increase in country 2's output is negative. See the earlier discussion of this point in the text.

[31] In determining these effects it was assumed that inequality (48) was satisfied. The results for country 2 are analogous.

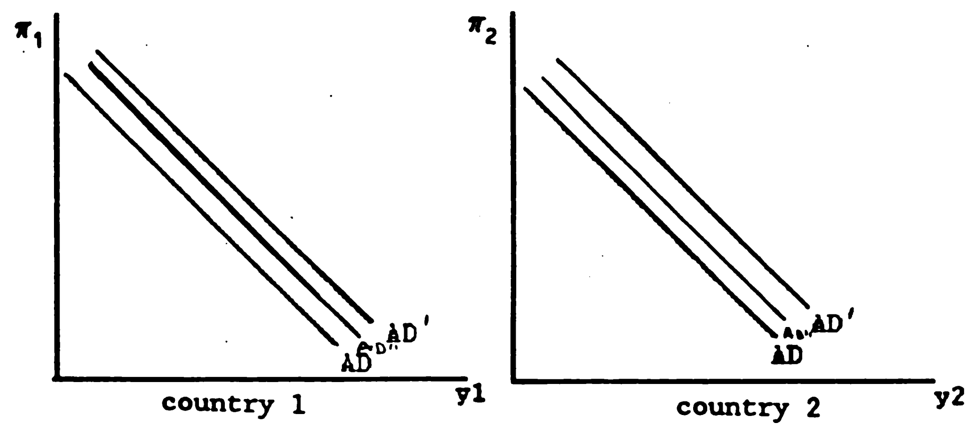

Figure 1
Effect of an Increase in Government Spending
by Country 1 on Demand in Country 1 and Country 2

increase in exports by country 2 to country 1. This increase in aggregate demand in country 2 pushes up the world interest rate (as did the increase in country 1) which has a crowding out effect, and thus aggregate demand in country 1 decreases. Although this feedback effect is negative, it is not strong enough to offset the initial increase in aggregate demand in country 1. This increase in country 1's aggregate demand and its effect on interest rates also feeds back into country 2's aggregate demand, causing a diminishing of the original spillover effect (Figure 1). In both countries, the overall effect of an increase in country 1's government expenditures is an increase in aggregate demand.

Table VIII: Aggregate Demand Effects

	Direct Effect	Feedback Effect	Overall Effect
Increase in g_{1t}	+	-	+
Increase in b_{1t}	-	+	±
Increase in π^e_{1t}	+	-	+
Increase in g_{2t}	+	-	+
Increase in b_{2t}	-	+	-
Increase in π^e_{2t}	+	-	±
Increase in b_{mt}	+	-	+

The effect of an increase in the bonds issued by country 1 on its aggregate demand is indeterminate. If the marginal propensity to import, ϵ, is small (less than ½) then A_2 will be unambiguously positive, and thus an increase in bond issues increases aggregate demand. The effect of this increase on country 2 is clear: demand in country 2 will decline as a result of an increase in bond issues by country 1.

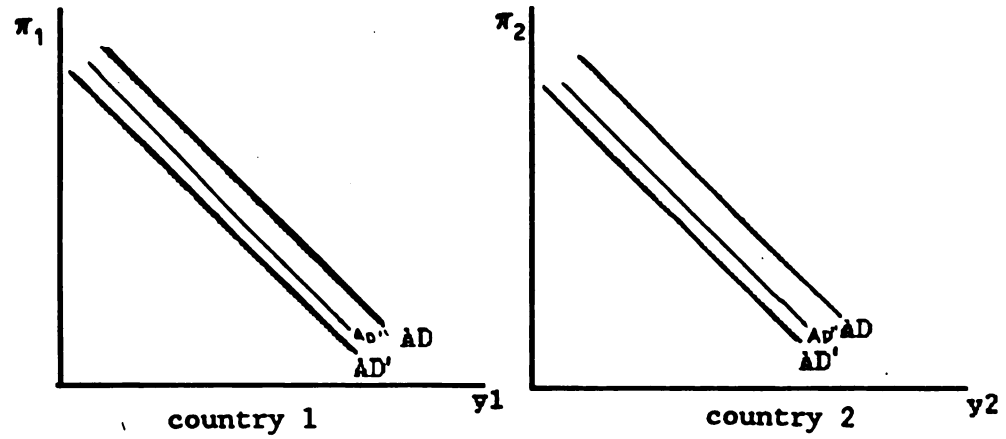

Figure 2
Effect of an Increase in Bond Issues by Country 1
on Demand in Country 1 and Country 2

The increase in bond issues by country 1 raises the world interest rate which produces a negative spillover effect in country 2. If aggregate demand decreases in country 1, this negative spillover effect is offset slightly, but the overall effect is a decrease in aggregate demand in country 2 (Figure 2). If aggregate demand in country 1 increases as a result of the increase in bond issues then the negative spillover effect in country 2 is strengthened by the feedback effect (Figure 3). In either case, an increase in bond issues by country 1 has an unambiguously negative effect on aggregate demand in country 2.

An increase in the inflationary expectations in country 1 will have a positive direct effect on its aggregate demand and a positive spillover effect on aggregate demand in country 2. The subsequent increase in the world interest rate as a result of these changes will cause aggregate demand to shift back in both countries (Figure 4). In country 1 the overall effect will still be positive, but in country 2 it is indeterminate. (Two possible outcomes for country 2 are given in Figure 4.) The different possible outcomes in the two countries occur because the initial demand effect in country 1 was stronger than that in country 2: the spillover effects are weaker than the direct effects.

An increase in the real bond holdings of the central bank increases aggregate demand in both countries. This comes directly from the increase in real balances' shifting out the LM curve. Likewise an increase in last period's real bond holdings of the central bank will increase aggregate demand in both countries this period.

If country 1 was a net lender last period, and the real interest rate, $r_{1,t-1}$, is positive, then it receives a net income transfer this period which increases domestic absorption and thus has a positive effect

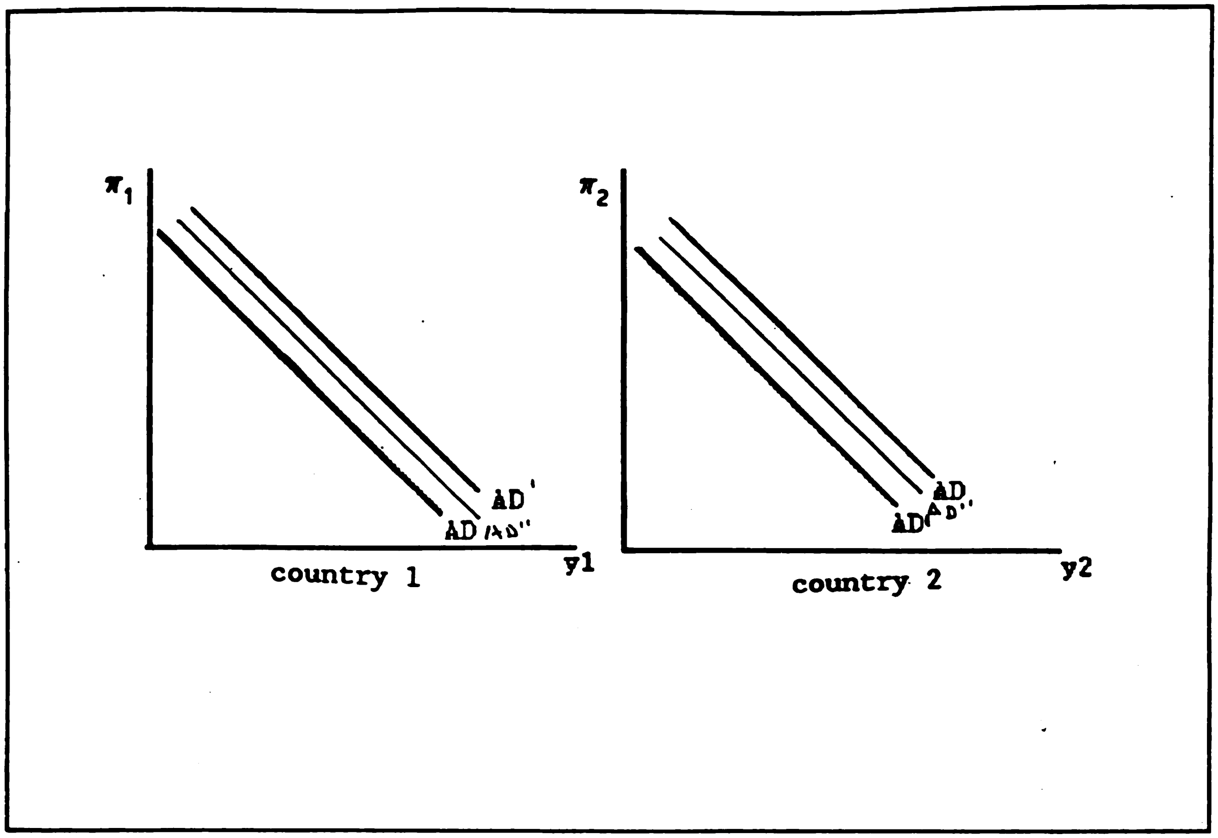

Figure 3
Effect of an Increase in Bond Issues by Country 1
on Demand in Country 1 and Country 2

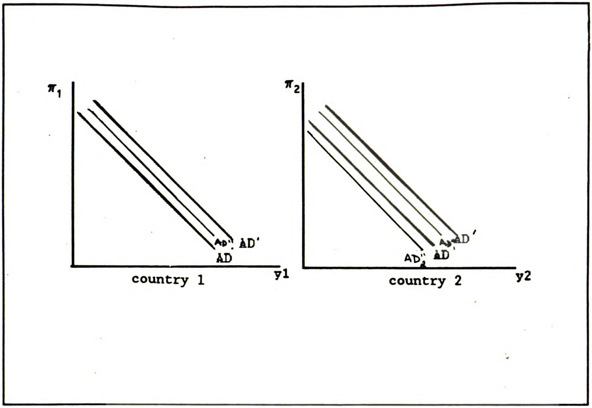

Figure 4
Effect of an Increase in Inflationary Expectations in Country 1 on Demand in Country 1 and Country 2

on aggregate demand. If, however, country 1 was a net borrower last period, and given that the real interest rate is positive, then it has a net income outflow this period and therefore the effect on aggregate demand is negative. The effect of country 1's being a net borrower or lender last period on country 2's aggregate demand this period is ambiguous.

Section VI: Equilibrium Inflation and Output

The equilibrium inflation and output equations for country 1 and country 2 are given in Tables IX and X. These were derived using the aggregate demand and supply equations discussed in Section V.[32]

Comparative Statics:

In the standard closed economy model an increase in government spending increases aggregate demand and so has a positive effect on output and prices. An increase in bonds issued by the government causes a contraction in aggregate demand by pushing up interest rates and thereby decreasing investment. Thus output and prices decline. The overall effect of a bond financed increase in government spending on output and prices is generally positive; the crowding out of investment which occurs is not complete. Expansionary monetary policy carried out through open market purchases increases output and inflation in the closed economy, by shifting out the LM curve and thus increasing aggregate demand. An increase in inflationary expectations will also increase output and prices in the closed economy model.

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[32] See Appendix C for these derivations.

Table IX: Equilibrium Inflation

$$
\begin{aligned}
\pi_{1,t-1} = & \left[\frac{A_1 [\alpha^2 \Upsilon_2 \tilde{p}_t (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_4 \alpha\gamma_2\bar{y}}{\Omega_1} \right] g_{1,t} \\
& + \left[\frac{A_2 [\alpha^2 \Upsilon_2 \tilde{p}_t (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_5 \alpha\gamma_2\bar{y}}{\Omega_1} \right] b_{1,t} \\
& + \left[\frac{A_3 [\alpha^2 \Upsilon_2 \tilde{p}_t (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_6 \alpha\gamma_2\bar{y}}{\Omega_1} \right] \pi^{e}_{1,t} \\
& + \left[\frac{A_4 [\alpha^2 \Upsilon_2 \tilde{p}_t (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_1 \alpha\gamma_2\bar{y}}{\Omega_1} \right] g_{2,t}\tilde{p}_t \\
& + \left[\frac{A_5 [\alpha^2 \Upsilon_2 \tilde{p}_t (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_2 \alpha\gamma_2\bar{y}}{\Omega_1} \right] b_{2,t}\tilde{p}_t \\
& + \left[\frac{A_6 [\alpha^2 \Upsilon_2 \tilde{p}_t (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_3 \alpha\gamma_2\bar{y}}{\Omega_1} \right] \pi^{e}_{2,t}\tilde{p}_t \\
& + \left[\frac{\alpha^2 \Upsilon_2 \tilde{p}_t (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_1\bar{y} + \alpha\gamma_2\bar{y}}{\Omega_1} \right] A_7 b_{m,t} \\
& + \left[\frac{\alpha^2 \Upsilon_1 \Upsilon_2 \tilde{p}_t (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\Upsilon_1\gamma_1\bar{y} + \alpha\Upsilon_2\tilde{p}_t\gamma_2\bar{y}}{\Omega_1} \right] i_{t-1} \\
& + \left[\frac{\bar{y}(\alpha\gamma_3\Upsilon_2\tilde{p}_t + \bar{y})}{\Omega_1} \right] \pi^{e}_{1,t-1} \\
& - \left[\frac{\alpha\bar{y}\,\gamma_2\Upsilon_2\tilde{p}_t}{\Omega_1} \right] \pi^{e}_{2,t-1} \\
& - \left[\frac{\alpha\bar{y}[\alpha\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \gamma_1\bar{y} + \gamma_2\bar{y}]}{\Omega_1} \right]
\end{aligned}
$$

Table IX (continued)

$$
\begin{aligned}
\pi_{2,t-1} = {} & \left[\frac{A_1 \left(\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_4 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{2,t} \\
& + \left[\frac{A_2 \left(\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_5 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{2,t} \\
& + \left[\frac{A_3 \left(\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_6 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^{e}_{2,t} \\
& + \left[\frac{A_4 \left(\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_1 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{1,t} \\
& + \left[\frac{A_5 \left[\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_2 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{1,t} \frac{1}{\tilde{p}_t} \\
& + \left[\frac{A_6 \left(\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_3 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^{e}_{1,t} \frac{1}{\tilde{p}_t} \\
& + \left[\frac{\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} + \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] A_7 b_{m,t} \frac{1}{\tilde{p}_t} \\
& + \left[\frac{\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha \Upsilon_2 \gamma_3 \frac{\bar{y}}{\tilde{p}_t} + \alpha \Upsilon_1 \frac{1}{\tilde{p}_t} \gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] i_{t-1} \\
& - \left[\frac{\frac{\bar{y}}{\tilde{p}_t} \left(\alpha\gamma_4 \Upsilon_1 \frac{1}{\tilde{p}_t} + \frac{\bar{y}}{\tilde{p}_t} \right)}{\Omega_2} \right] \pi^{e}_{1,t-1} \\
& + \left[\frac{\alpha \frac{\bar{y}}{\tilde{p}_t} \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t}}{\Omega_2} \right] \pi^{e}_{2,t-1} \\
& - \left[\frac{\alpha \frac{\bar{y}}{\tilde{p}_t} \left(\alpha \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \frac{\bar{y}}{\tilde{p}_t} (\gamma_3 + \gamma_4) \right)}{\Omega_2} \right]
\end{aligned}
$$

Table IX (continued)

where:

$$\Omega_1 = \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\overline{y}(\gamma_1 Y_1 + \gamma_3 Y_2 \tilde{p}_t) + \overline{y}^2$$

$$\Omega_2 = \alpha^2 Y_1 \frac{1}{\tilde{p}_t} Y_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\overline{y}}{\tilde{p}_t}(\gamma_1 Y_1 \frac{1}{\tilde{p}_t} + \gamma_3 Y_2) + \frac{\overline{y}^2}{\tilde{p}_t^2}$$

Table X: Equilibrium Output

$$
\begin{aligned}
y_{1,t} = & \left[\frac{A_1[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_4\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] g_{1,t} \\
& + \left[\frac{A_2[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_5\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] b_{1,t} \\
& + \left[\frac{A_3[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_6\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] \pi^e_{1,t} \\
& + \left[\frac{A_4[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_1\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] g_{2,t}\tilde{p}_t \\
& + \left[\frac{A_5[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_2\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] b_{2,t}\tilde{p}_t \\
& + \left[\frac{A_6[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_3\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] \pi^e_{2,t}\tilde{p}_t \\
& + \left[\frac{\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2 - \alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] A_7 b_{m,t} \\
& + \left[\frac{\alpha\Upsilon_1\Upsilon_2\tilde{p}_t\bar{y}(\gamma_3-\gamma_2) + \Upsilon_1\bar{y}^2}{\Omega_1}\right] i_{t-1} \\
& - \left[\frac{\bar{y}\,\Upsilon_1\,(\alpha\gamma_3\Upsilon_2\tilde{p}_t + \bar{y})}{\Omega_1}\right] \pi^e_{1,t-1} \\
& + \left[\frac{\alpha\bar{y}\gamma_2\Upsilon_1\Upsilon_2\tilde{p}_t}{\Omega_1}\right] \pi^e_{2,t-1} \\
& + \frac{\alpha\Upsilon_1\bar{y}[\alpha\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \bar{y}(\gamma_1+\gamma_2)]}{\Omega_1}
\end{aligned}
$$

Table X (continued)

$$y_{2,t} = \left[\frac{A_1 \left(\alpha\gamma_1\Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}^2} \right) - A_4 \alpha\gamma_4\Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{2,t}$$

$$+ \left[\frac{A_2 \left(\alpha\gamma_1\Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} \right) - A_5 \alpha\gamma_4\Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{2,t}$$

$$+ \left[\frac{A_3 \left(\alpha\gamma_1\Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} \right) - A_6 \alpha\gamma_4\Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{2,t}$$

$$+ \left[\frac{A_4 \left(\alpha\gamma_1\Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} \right) - A_1 \alpha\gamma_4\Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{1,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[\frac{A_5 \left(\alpha\gamma_1\Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} \right) - A_2 \alpha\gamma_4\Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{1,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[\frac{A_6 \left(\alpha\gamma_1\Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}^2} \right) - A_3 \alpha\gamma_4\Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{1,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[\frac{\alpha\gamma_1\Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} - \alpha\gamma_4\Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] A_7 b_{m,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[\frac{\alpha\Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t} (\gamma_1 - \gamma_4) + \Upsilon_2 \frac{\bar{y}^2}{\tilde{p}_t^2}}{\Omega_2} \right] i_{t-1}$$

$$+ \left[\frac{\frac{\bar{y}}{\tilde{p}_t} \Upsilon_2 \left(\alpha\gamma_4\Upsilon_1 \frac{1}{\tilde{p}_t} + \frac{\bar{y}}{\tilde{p}_t} \right)}{\Omega_2} \right] \pi^e_{1,t-1}$$

$$- \left[\frac{\alpha \frac{\bar{y}}{\tilde{p}_t} \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2}{\Omega_2} \right] \pi^e_{2,t-1}$$

$$+ \frac{\alpha\Upsilon_2 \frac{\bar{y}}{\tilde{p}_t} \left(\alpha\Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \frac{\bar{y}}{\tilde{p}_t} (\gamma_3 + \gamma_4) \right)}{\Omega_2}$$

In a model of a large open economy with perfect capital mobility and fixed exchange rates, both the fiscal and monetary policy actions of a country also have an effect on the rest of the world. Expansionary fiscal policy, regardless of how it is financed, has an ambiguous effect on output and prices abroad. The increased spending at home increases aggregate demand abroad through trade effects. At the same time, however, the increase in the world interest rate dampens demand overseas. A bond financed fiscal expansion will increase the magnitude of the latter effect. In large open economy models it is uncertain as to whether the trade effect or the interest rate effect dominates. Buiter (1988) states that if the countries are of similar size, then a fiscal expansion at home will always have a positive effect on world output and inflation. In the large country case, a monetary expansion at home has a positive effect on output and inflation abroad.

As noted previously, standard open economy models focus on the demand side of the economy. Even where a supply side is modelled, these models ignore connections between the demand and supply side of economies. In the model developed in this chapter countries are linked on the demand side through the goods market and assets market. Thus, policies which affect aggregate demand in one country lead to spillover effects on demand in the other country. Because of the market linkages, changes in inflation in one country affect the other country. This link comes through the supply side of the economies. Thus, a change in inflation in one country will lead to spillover effects on supply in the other country.

The comparative statics for the model of a monetary union developed in this chapter are given in Tables XI and XII. The first column in each table gives the comparative statics which result from focussing only on demand effects.[33] Tax financed expansionary fiscal policy will increase output and inflation in both countries. The increase at home will be larger than the increase abroad. Increases in bond issues will decrease output and inflation abroad, but may have positive or negative effects on output and inflation at home. Bond financed fiscal policy will increase output and inflation at home, but will decrease output and inflation abroad. Expansionary monetary policy will increase output and inflation in both countries. These results may be weakened or reversed through the inclusion of supply effects (see column three in Table XI and XII). Determining the conditions under which supply effects weaken or offset demand effects requires an analysis of the spillover demand effects and the magnitude of the supply effects.

Determining the Comparative Statics:

For every exogenous variable in the equilibrium output and inflation equations (with the exception of the period t-1 expected inflation variables) there are two separate effects that determine the overall effect of a change in that variable on output and inflation.[34] A change

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[33] As shown in Section IV, the change in aggregate demand in country 1 resulting from a change in one of its own (policy or expectational) variables includes both the initial demand effect and the feedback effect on demand, due to the change in output in country 2. The effect on country 2's aggregate demand (from a change in one of country 1's variables) includes both direct spillover effect and the feedback effects, due to the change in output in country 1. The analysis for equilibrium output and inflation, in this section, considers only the overall change in demand, and does not look at the component effects.

[34] The analysis concentrates on the effects as given by the numerators of the coefficients on the variables in the output and inflation equations. There are a two reasons for this emphasis. First, given that $\gamma>1/2$, and Y_1 and Y_2 are both positive, then Ω_i is positive. Therefore, determining the comparative static effects of a change in one of the exogenous variables is equivalent to determining the sign of the numerator of the exogenous variable in question. Also, it is the (continued...)

in an exogenous variable has a direct effect on aggregate demand. A change in a country's aggregate demand changes its inflation rate. The feedback effect captures the subsequent impact of this change in inflation on aggregate supply in the other country.[35] In most cases the aggregate supply effects reinforce the inflation effects of the change in aggregate demand, but weaken the output effects.

There are two potential sources for indeterminacy in signing the comparative static effects. First, as noted in Table VI, the overall demand effects on a country of changes in bond issues by either country and changes in inflationary expectations in the other country are indeterminate, which in turn presents a source of ambiguity in determining the equilibrium output and inflation effects. Second, as noted above, in many cases the feedback effects work to offset the effect of a change in aggregate demand on output. The overall effect depends on the direction of the changes in aggregate demand and aggregate supply and the magnitudes of these two changes.

An important determinant of the effects of policies on output and inflation is the steepness of the slope of a country's aggregate demand curve. The flatter the slope of the aggregate demand curve the larger is the effect of a shift in aggregate supply on output and the smaller is the effect of a shift in aggregate supply on inflation. The slope of the aggregate demand curve $-(1/Y_i)$, is determined by last period's current account balance of a country. If a country was a net debtor last period

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[34](...continued)
behavioral parameters in the numerator which are driving the aggregate demand and supply effects.

[35] As shown by equations (37) and (38), an increase in inflation in country j will decrease aggregate supply in country i.

Table XI: Output Effects in Country 1

	Direct Effect	Feedback Effect	Total Effect
Increase in g_{1t}	+	-	+
Increase in b_{1t}	+ (A_2>0) - (A_2<0)	+ +	+ ±
Increase in π^e_{1t}	+ +	+ (A_6<0) - (A_6>0)	+ +
Increase in g_{2t}	+	-	±
Increase in b_{2t}	- -	- (A_2>0) + (A_2<0)	- -
Increase in π^e_{2t}	+ (A_6>0) - (A_6<0)	- -	± -
Increase in b_{mt}	+	-	+
Increase in i_{t-1}	+	-	+
Increase in π^e_{1t-1}		-	-
Increase in π^e_{2t-1}		+	+

Table XII: Inflation Effects in Country 1

	Direct Effect	Feedback Effect	Total Effect
Increase in g_{1t}	+	+	+
Increase in b_{1t}	+ (A_2>0) - (A_2<0)	- -	± -
Increase in π^e_{1t}	+ +	+ (A_6>0) - (A_6<0)	+ +
Increase in g_{2t}	+	+	+
Increase in b_{2t}	- -	- (A_2<0) + (A_2>0)	- -
Increase in π^e_{2t}	+ (A_6>0) - (A_6<0)	+ +	+ ±
Increase in b_{mt}	+	+	+
Increase in i_{t-1}	+	+	+
Increase in π^e_{1t-1}		+	+
Increase in π^e_{2t-1}		-	-

(having a current account deficit) its aggregate demand curve will be relatively steep. If a country was a net creditor last period (having a current account surplus) its aggregate demand curve will be relatively flat. An increase in inflation reduces the real interest earnings on private holdings of bonds, thereby reducing disposable income (equations 9 and 22). An increase in inflation also reduces the government's real interest payments on bonds, thereby reducing the government deficit (equations 11 and 24). Since the aggregate demand curve for a country is downward sloping regardless of whether it is a net creditor or a net debtor, an increase in inflation produces a net negative effect on output (the output effect of the inflation benefit to the government can not fully offset the output effect of the inflation cost to the private sector). As expected, a net creditor nation is hurt more by inflation than a net debtor nation, which is reflected in the flatter aggregate demand curve for a net creditor and the steeper aggregate demand curve for a net debtor.

The importance of the slopes of the aggregate demand curves for determining the comparative statics can be illustrated by examining the three types of policy effects which are present in the model: 1) expansionary policies which have positive effects on aggregate demand in both countries; 2) expansionary policies which have positive effects on aggregate demand at home, but negative spillover effects on aggregate demand; and, 3) expansionary policies which have negative effects on aggregate demand in both countries.

Policies which fall into the first category lead to an increase in inflation abroad which causes a decline in aggregate supply at home. The supply effect strengthens the positive own demand effect on inflation but weakens the demand effect on output. As shown in Figure 5, if a country is a net creditor the supply effect is less likely to offset the demand effect on output. Whereas, if a country is a net debtor the supply effect is more likely to offset the demand effect on output. This follows from the fact that inflation is less harmful (in output terms) to a net debtor than to a net creditor.

Policies which fall into the second category lead to a reduction in inflation abroad (due to negative demand spillovers) which causes an increase in aggregate supply at home. Since the home demand effect is positive, the supply effect strengthens the demand effect on output but weakens the demand effect on inflation. In this case, as shown in Figure 6, a net creditor country is more likely to experience an overall decrease in inflation than a net debtor. Also, the output effect is larger for a net creditor than for a net debtor. This result follows from the greater benefit of a decline in inflation to a net creditor than to a net debtor.

The third category covers policies which lead to a reduction in inflation abroad which in turn results in an increase in aggregate supply at home. Since these policies decrease aggregate demand at home, the positive supply effect further reduces inflation while offsetting some (if not all) of the reduction in output resulting from the demand effect. Due to the slope of the aggregate demand curve, a net creditor county is more likely to experience an increase in output than a net debtor country (Figure 7). As stated above, this is because a decrease in inflation provides a greater benefit to a net creditor country than to a net debtor country.

Policies pursued by one country will have differing effects on the two countries due to differences in the demand effects in the countries

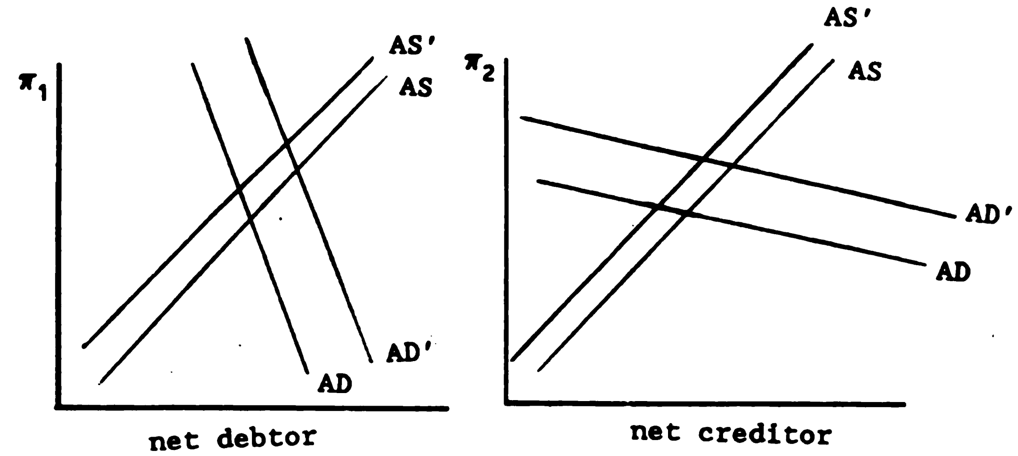

Figure 5
Net Debtor/Net Creditor Effects
With Positive Demand Spillovers

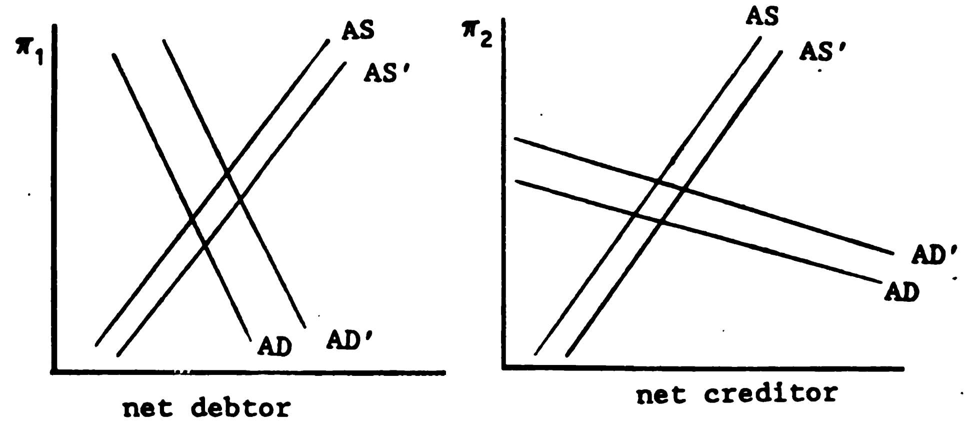

Figure 6
Net Debtor/Net Creditor Effects
With Negative Demand Spillovers

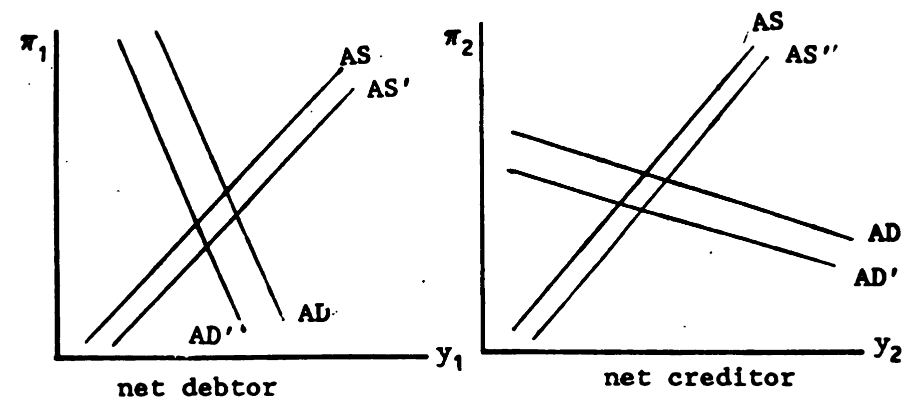

Figure 7
Net Debtor/Net Creditor Effects
Decreases in Home and Foreign Demand

and due to differences in the past behavior of the two countries. The more similar the past policies of the two countries, the more similar the current account balances of the two and thus the more similar the slopes of their aggregate demand curves. If the countries are symmetric[36] the two aggregate demand curves have the same slope. In this case an expansionary policy pursued by country 1 has the same effect on country 2 as an expansionary policy pursued by country 2 has on country 1. If the countries are not symmetric this does not hold. For example, if country 1 is a net creditor and country 2 is a net debtor, an expansionary fiscal policy pursued by country 1 may increase output in country 2, but an expansionary fiscal policy pursued country 2 may decrease output in country 1. In general policies which increase aggregate demand in both countries are less likely to have positive output spillovers if the country pursuing the policies is a net debtor and the other country is a net creditor.

Policy Effects: Country 1 is a Net Debtor, Country 2 is a Net Creditor[37]

Since A_1 is positive a tax financed increase in government spending in country 1 has a positive effect on its own aggregate demand, and since A_4 is positive the increase in spending raises aggregate demand in country 2. This policy fits into category 1, given above. Inflation in country 1 increases which reduces aggregate supply in country 2. Likewise, inflation in country 2 increases which reduces aggregate supply in country

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[36] Symmetry arises if $b_{12,t-1} = b_{21,t-1}$.

[37] The analysis in this section concentrates on the effects of changes in policies by country 1 on the two countries. The analysis is the same with respect to changes in country 2's policy variables. See appendix D for the derivation of the comparative static results given in this section.

1. Therefore, in both countries, the effect on inflation is compounded but the effect on output is reduced, Figure 8.

Since country 1 is a net debtor the slope of its aggregate demand curve is relatively steep which acts to limit the effect on output of a reduction in aggregate supply. Thus, the expansionary fiscal policy increases inflation and output in country 1. The output effect on country 2 is less likely to be positive. Given that $A_4<A_1$, aggregate demand increases less in country 2 than in country 1. Thus, the direct effect on output is smaller. Also, since country 2 is a net creditor, the slope of its aggregate demand curve is relatively flat (inflation is more harmful to country 2 than it is to country 1). The decrease in aggregate supply exacerbates inflation and so causes a large (relative to country 1) reduction in output. It is possible that the negative supply effect on output more than offsets the positive demand effect on output, and output decreases in country 2. Given that inflation definitely increases, expansionary fiscal policy undertaken by country 1 can lead to stagflation in country 2. The greater the asymmetry between the countries, the greater the differences in the output effects.

To determine the effect of a change in real bond issues by the government of country 1 on its equilibrium inflation rate and output it is necessary to consider two possible own aggregate demand effects. As noted in section V, the sign of the coefficient A_2 is indeterminate. An increase in bond issues may increase or decrease own aggregate demand.

If A_2 is positive then an increase in bond issues by country 1 increases its aggregate demand, as shown in Figure 9. The effect of the bond issues on country 2's aggregate demand is unambiguously negative.[38] The increase in inflation in country 1 causes a decrease in aggregate supply in country 2, while the decrease in inflation in country 2 causes an increase in aggregate supply in country 1. This policy fits into category 2. Thus, for country 1 output increases while inflation may increase or decrease. The overall effect on output is large relative to the effect on inflation. For country 2, the supply effect further decreases output, and acts to reverse (although not completely) the decline in inflation resulting from the decrease in demand.[39] The decrease in output in country 2 will be large relative to the decrease in inflation.

If $A_2<0$, then the increase in bonds issued by the government of country 1, decreases aggregate demand in country 1, as shown in Figure 10. Since $A_5<0$, aggregate demand in country 2 also declines. This fits into category 3. The decline in inflation in country 1, increases aggregate supply in country 2, while the decline in inflation in country 2 produces a similar effect on aggregate supply in country 1. The supply effect strengthens the demand effect on reducing inflation, while working to offset the decline in output following from a decline in demand. In both countries inflation decreases. The decrease in aggregate demand is greater in country 2 than in country 1 ($|A_5|>|A_2|$) which in turn produces a larger decline in inflation in country 2. Therefore, the increase in

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[38] This negative demand effect occurs, as explained in section V, because the interest rate spillover effect outweighs the income spillover effect.

[39] As shown in Appendix D, the overall effect on inflation in country 2 is clearly negative since $|A_5|>|A_2|$ and $\gamma_3>\gamma_4$. These two inequalities ensure that the numerator of the coefficient on b_1 in country 2's equilibrium inflation equation is negative.

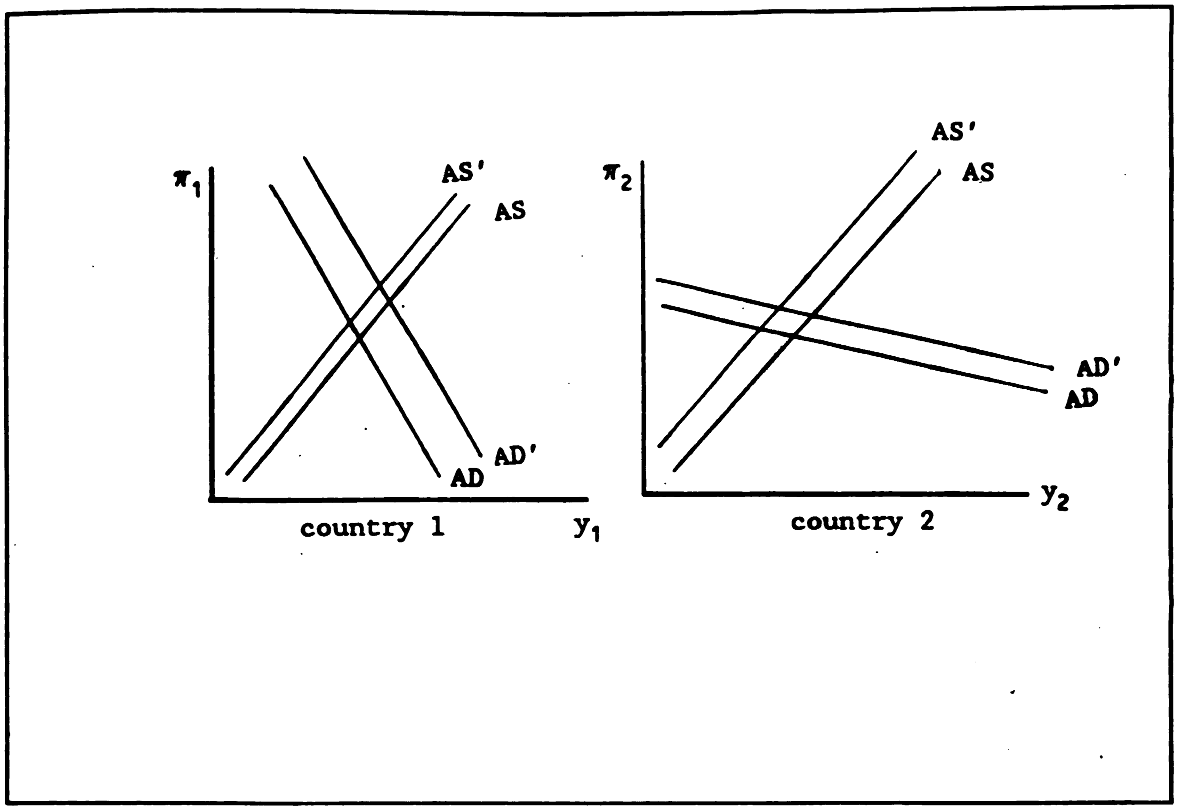

Figure 8
Increase in Government Spending By Country 1
$Y_2 > Y_1$

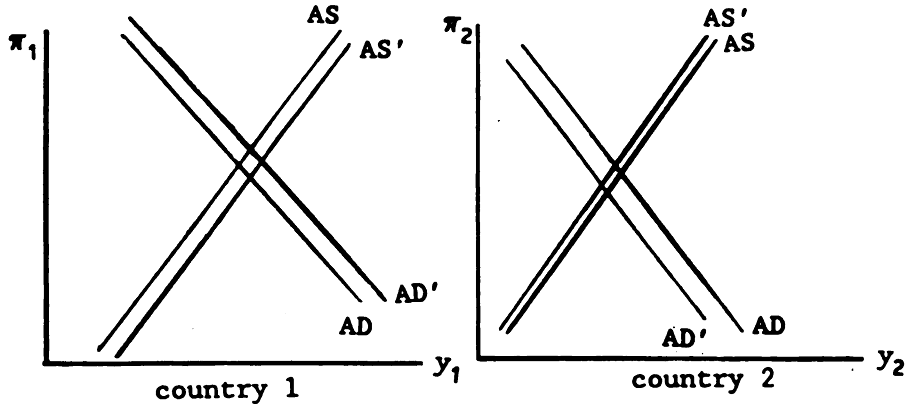

Figure 9
Increase in Bond Issues By Country 1
$A_2 > 0$

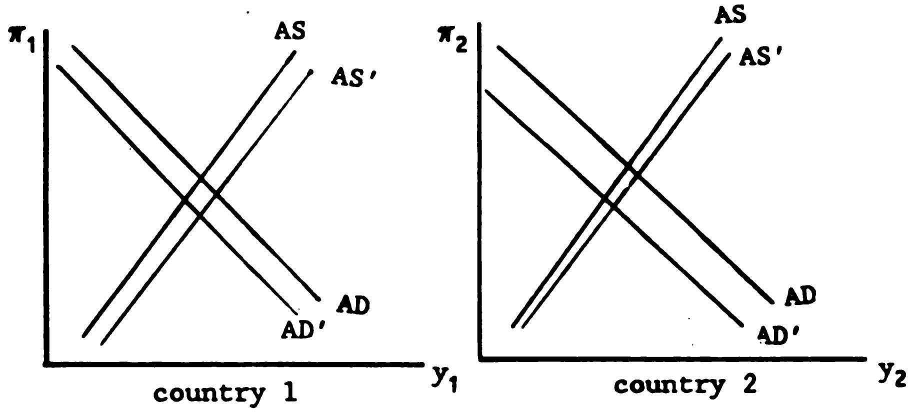

Figure 10
Increase in Bond Issues By Country 1
$A_2<0$

aggregate supply is greater in country 1 than in country 2. Given the differences in the relative magnitudes of the supply and demand effects (Figure 10), the shift in aggregate supply in country 2 is not strong enough to offset the demand induced decrease in output. In country 1, however, it is possible that the supply effect more than fully offsets the demand effect and output may increase.

A bond financed fiscal expansion in country 1 ($\Delta b_1 = \Delta g_1$) has a positive effect on inflation and output in country 1. The effects on output and inflation in country 2 are indeterminate.

Expansionary monetary policy resulting in an increase in real balances, holding the overall stock of bond issues fixed,[40] has an identical effect on aggregate demand in the two countries. The increase in aggregate demand in country 1 increases its inflation rate which causes a decrease in aggregate supply in country 2. Likewise, the increase in aggregate demand in country 2 causes a decrease in aggregate supply in country 1. Thus, the aggregate supply effects further increase inflation in both countries, but work to offset the demand induced increases in output in both countries. In both countries the overall effect is an increase in inflation and output. However, the increase in inflation and output is greater in country 1 than in country 2, as shown in Figure 11. This result follows from the fact that country 1 is a net debtor and country 2 is a net creditor. The slope of the aggregate demand curve in country 1 is steep relative to that in country 2. Thus, as explained above, a shift in aggregate supply has a relatively greater effect on inflation in country 1 and a relatively greater effect on output in

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[40] If the overall stock of bonds remains fixed expansionary monetary policy requires that private bond holdings decrease.

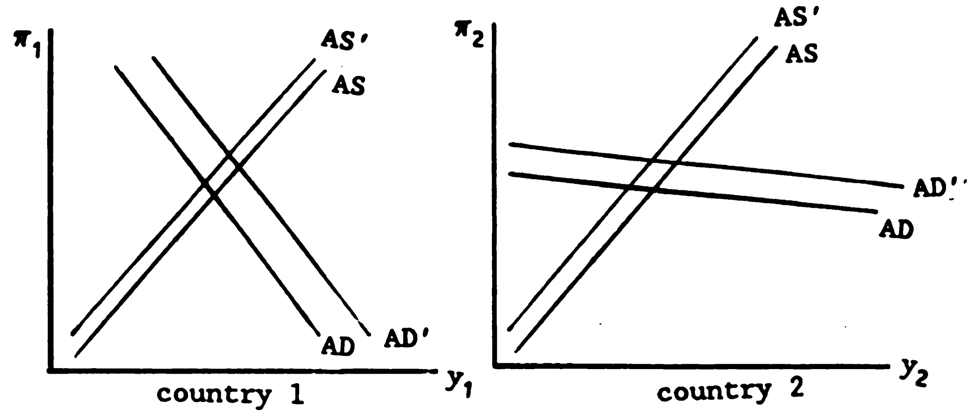

Figure 11
Increase in Bond Holdings By the Central Bank
$Y_2 > Y_1$

country 2. Since the supply effect and demand effect both lead to an increase in inflation, the overall inflation rate is higher in country 1. Since the change in supply reduces the positive demand effect on output, the overall change in output is greater in country 1.

Expansionary monetary policy accompanied by a bond financed fiscal expansion in country 1, but not in country 2,[41] has a positive effect on output and inflation in country 1, but an indeterminate effect on output and inflation in country 2.

Policy Effects: Country 1 is a Net Creditor, Country 2 is a Net Debtor

A tax financed increase in government spending in country 1 increases output and inflation in country 1. Since country 1 is a net creditor the slope of its aggregate demand curve is relatively flat which acts to limit the effect on inflation of a reduction in aggregate supply, but increases the effect on output, as shown in Figure 12. Thus, the output and inflation effects, although positive are smaller than in the case where country 1 is a net debtor. Given that country 2 is a net debtor the slope of its aggregate demand curve is relatively steep. The steepness of the slope limits the effect of a reduction in aggregate supply on output. Thus, expansionary fiscal policy conducted by country 1 has a positive effect on output and inflation in country 2. These effects are greater than in the case where country 2 is a net creditor.

If A_2 is positive an increase in real bond issues by the government of country 1 increases its output while inflation may increase or

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[41] The implicit assumption in this analysis is that the increase in the bond supply in country 1 is met by an increase in the central bank's demand for country 1's bonds. In country 2, however, the increased bond demand by the central bank is met through a reduction in private holdings of country 2's bonds.

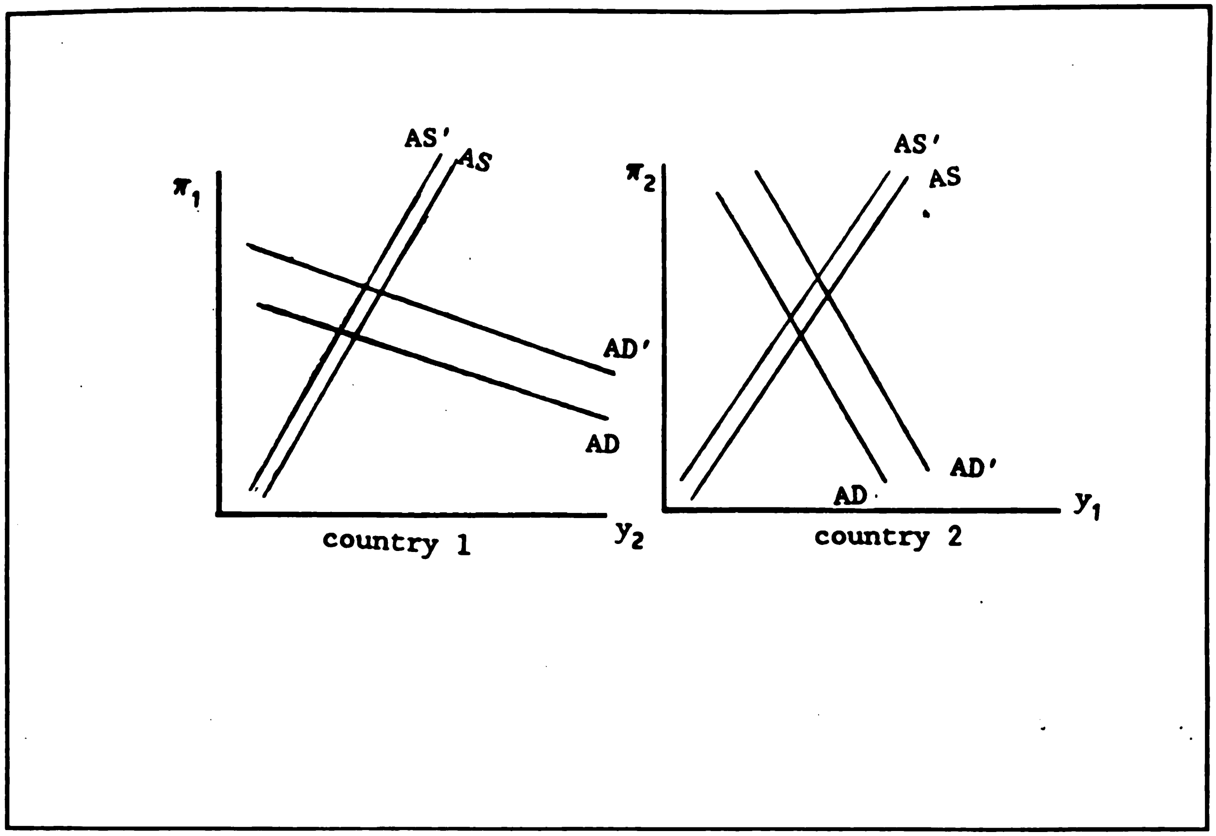

Figure 12
Increase in Government Spending By Country 1
$Y_1 > Y_2$

decrease. The overall effect on output and inflation is smaller than in the case where country 1 is a net debtor. For country 2, output and inflation decrease and these declines are larger than in the case where country 2 is a net debtor.

If A_2<0 aggregate demand decrease and aggregate supply increases in both countries, which decreases inflation in both countries. Given that country 2 is a net debtor and country 1 is a net creditor, the decrease in inflation has a greater effect on the level of output in country 1. In country 2, the supply effect is not strong enough to overcome the demand effect and output declines. In country 1, however, it is possible that the supply effect more than fully offsets the demand effect and output may increase.

A bond financed fiscal expansion in country 1 has a positive effect on inflation and output in country 1, but these increases are less than in the case where country 1 is a net debtor. The effects on output and inflation in country 2 remain indeterminate.

Expansionary monetary policy results in an increase in inflation and output in both countries. Although the nature of the effects are the same as in the case where country 1 is a net debtor and country 2 a net creditor, the magnitudes are reversed. When country 1 is a net creditor and country 2 is a net debtor, the increase in inflation and output is greater in country 2 than in country 1. The slope of the aggregate demand curve in country 2 is steep relative to that in country 1, as shown in Figure 13. Thus, a shift in aggregate supply has a relatively greater effect on inflation in country 2 and a relatively greater effect on output in country 1.

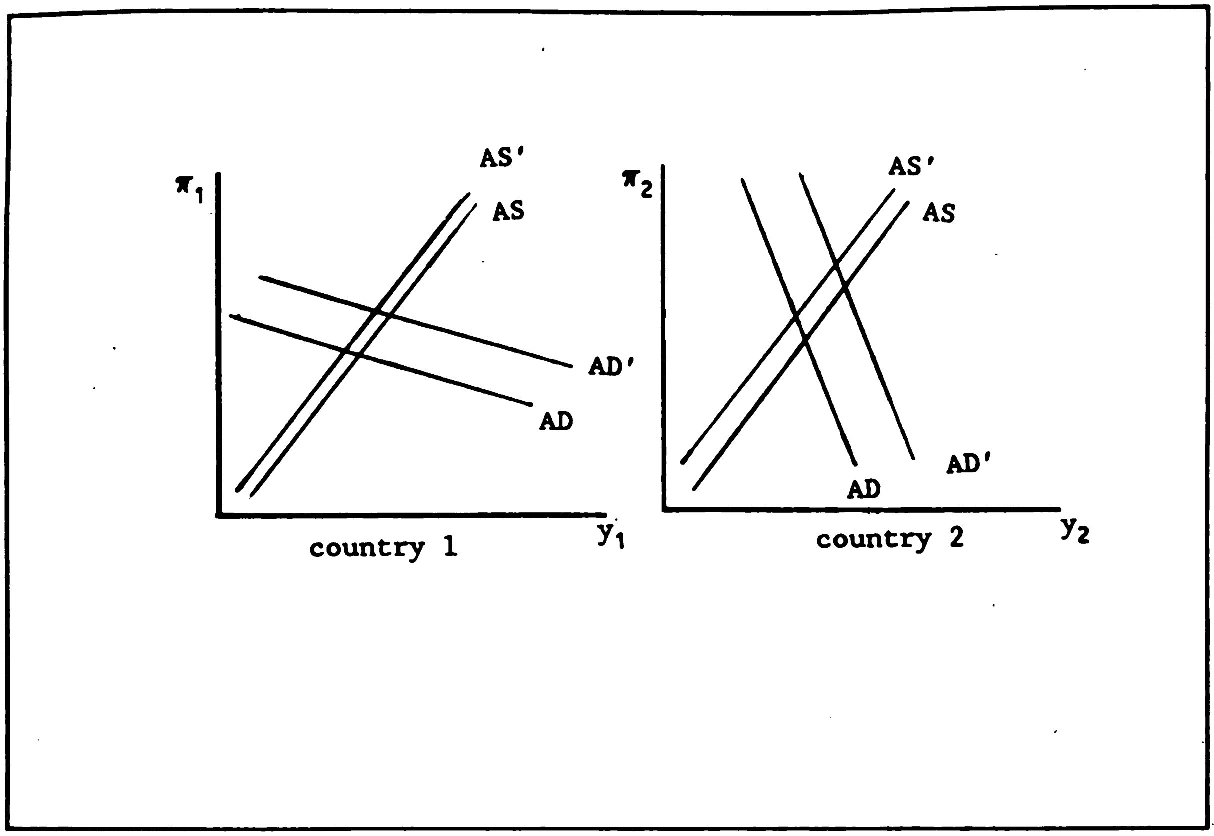

Figure 13
Increase in Bond Holdings By the Central Bank
$Y_1 > Y_2$

Expansionary monetary policy accompanied by a bond financed fiscal expansion in country 1, but not in country 2, has a positive effect on output and inflation in country 1. The increases in inflation and output, however, are smaller than in the case where country 1 is a net debtor. The effect on output and inflation in country 2 remains indeterminate.

In sum, if a country is a net debtor any expansionary policies which it enacts or which are undertaken by the central bank have a greater effect on its inflation rate and level of output than if it is a net creditor. If a country is a net debtor expansionary policies undertaken by the other country are more likely to have positive effects on its output and inflation rate than if it is a net creditor.

These results differ from the standard open economy models due not only to the inclusion of supply effects, but also the addition of interest earnings in the aggregate demand equations for the two countries. The inclusion of supply effects are important because they may work to offset the effects of a change in demand on output and/or inflation. Therefore, the supply effects can change the results one would obtain by solely concentrating on the demand side of economies. The addition of interest earnings in the aggregate demand equations is the means by which the net debtor/creditor status of a country affects the model. As shown above, this status determines the steepness/flatness of the slope of the demand curve which weakens or strengthen the effects of a shift in aggregate supply on output and inflation.

Changes in Inflationary Expectations:

An increase in the expected (t to t+1) inflation rate in country 1 has a positive effect on aggregate demand in country 1, but an indeterminate effect on aggregate demand in country 2. A_6 measures this latter effect. If $A_6>0$, then the expected increase in inflation in country 1 has a positive spillover effect on country 2's aggregate demand, as shown in Figure 14. The increase in inflation in country 1, resulting from the shift in its aggregate demand curve, has a negative spillover effect on aggregate supply in country 2. The increase in inflation in country 2 which occurs as a result of both the aggregate demand and aggregate supply effects, causes a decline in aggregate supply in country 1, thereby reinforcing the inflation effect in that country, but diminishing the output effect. The overall effect on country 1's output depends on the relative slopes of the aggregate demand curves in the two countries.[42] If country 1 is a net debtor and country 2 is a net creditor then the aggregate demand curve in country 2 is relatively flat and country 1's aggregate demand curve is relatively steep (Figure 14). In this case output in country 1 will definitely increase, but output in country 2 may decrease. If, however, country 2 is the net debtor and country 1 is the net creditor then aggregate demand curve in country 1 is relatively flat while the aggregate demand curve in country 2 is relatively steep (Figure 15). Under these conditions output in country 1 still increases, but it is less than in the case where country 1 is a net debtor. In country 2, the inflation effect is also positive, as is the effect on output.

If $A_6<0$ the increase in inflationary expectations in country 1 has a negative spillover effect on aggregate demand in country 2. The resulting decrease in inflation in country 2 causes an increase in

[42] Since $A_3>A_6$, the condition on the slopes is sufficient to determine the output effect.

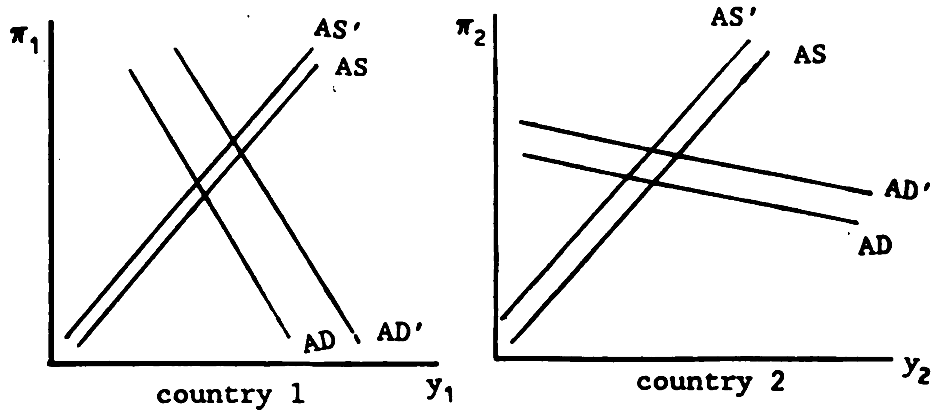

Figure 14
Increase in Inflation Expectations By Country 1
$A_6>0$, $Y_2>Y_1$

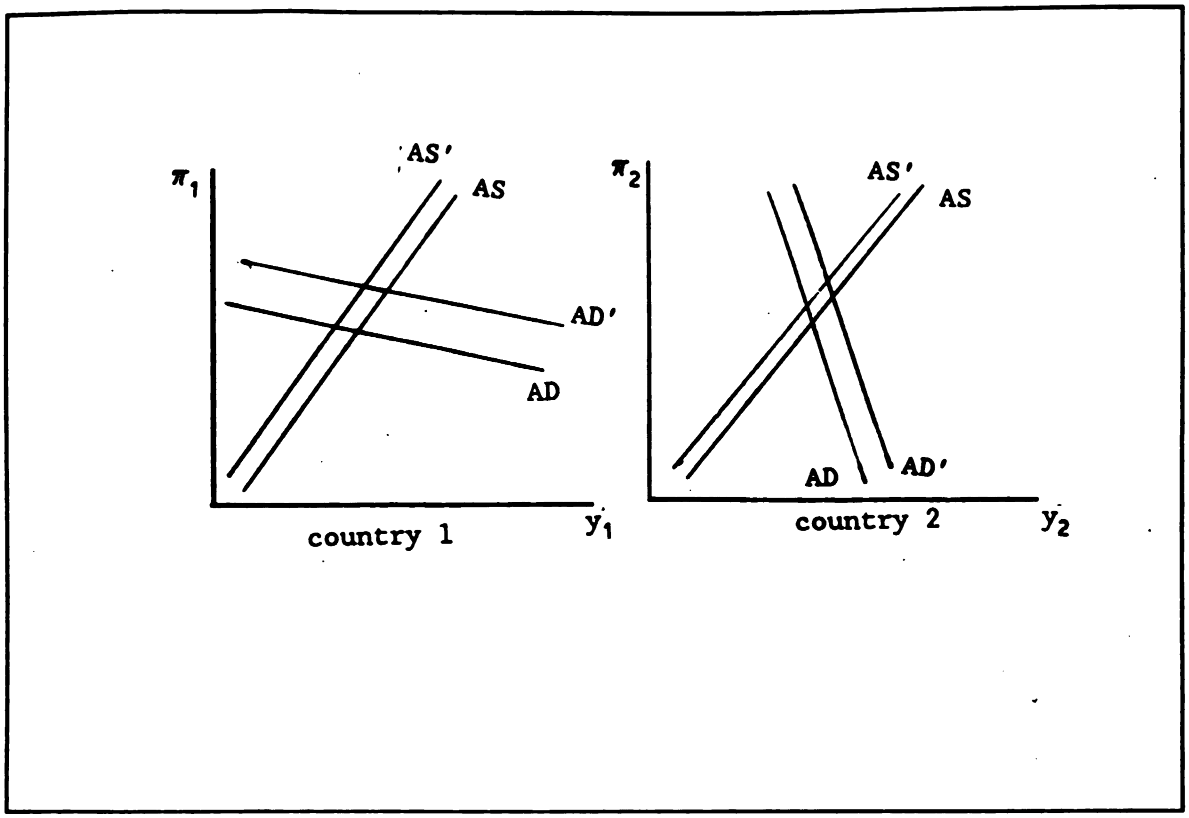

Figure 15
Increase in Inflation Expectations By Country 1
$A_6>0$, $Y_1>Y_2$

aggregate supply in country 1. This increase in supply strenghtens the output efect in country 1, but diminishes (although doesn't offset) the inflation effect, as shown in Figure 16. Aggregate supply in country 2 decreases which further decreases output. The overall effect on inflation is indeterminate.

An increase in inflationary expectations in country 1, in period t-1, decrease aggregate supply in that country in period t,[43] but increase aggregate supply in country 2. Thus, output decreases in country 1 but inflation increases in country 1, while in country 2, output increases and inflation decreases.

Interest Rate Effects:

The effect of an increase in last periods nominal interest rate is unambiguously positive for both countries with respect inflation and output. An increase in last period's interest rate leads to an increase in the nominal interest earnings on bonds and thus increases domestic absorption, which raises aggregate demand, and therefore, increases both equilibrium output and inflation.

Section VII: Conclusion

This chapter develops a two-country model of a monetary union. In order to analyze fully the linkages between the countries, the model specifies structural equations for the goods, money and bond markets in each country. Interdependencies arise through trade, through the asset markets, and through the existence of a common currency. The world supply of bonds is determined by the financing needs of the two governments,

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[43] The effect on aggregate demand occurred in period t-1.

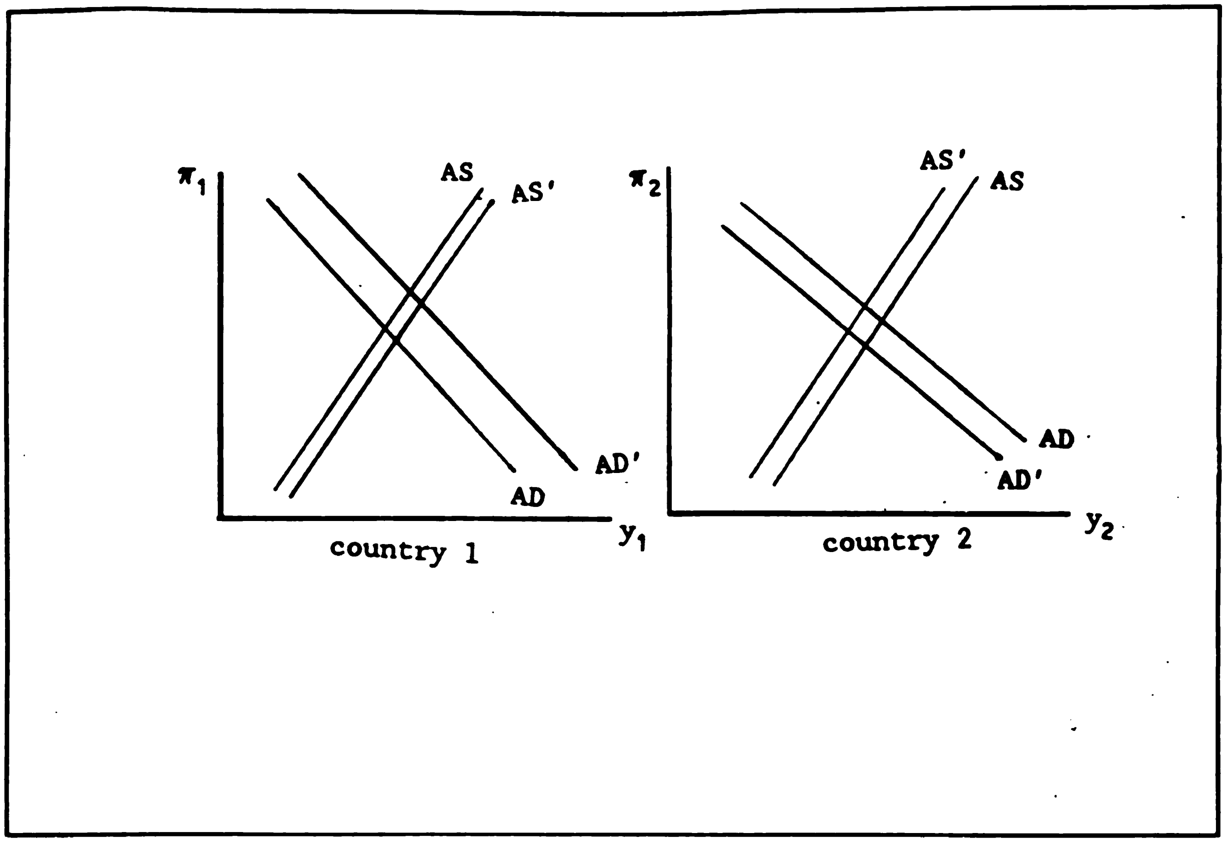

Figure 16
Increase in Inflation Expectations By Country 1
$A_6<0$

while demand is determined by the savings functions of the residents in each country and the monetary policy of the central bank. The bonds are perfect substitutes and capital is fully mobile so changes in the supply of bonds change the world interest rate. The central bank determines the amount of money in the world, but the demand within each country determines the distribution of money. This model also includes a supply side for each economy based on an expectations augmented Phillips curve.

Using this model it is possible to trace the shifts in both aggregate demand and aggregate supply resulting from a change in the following exogenous variables: government spending by each country, bond issues by each country, inflation expectations in each country, and bond holdings of the central bank. The change in an exogenous variable in one country directly affects domestic absorption in that country which leads to a shift in aggregate demand. Due to the trade and financial linkages between the two countries, a change in an exogenous variable in one country has a spillover effect on the other country, producing a shift in aggregate demand. The changes in aggregate demand in each country are accompanied by changes in the world interest rate which lead to a negative feedback effect on aggregate demand in each country.

The feedback effects, therefore, shift aggregate demand in the opposite direction to the initial effects. With respect to government spending (either at home or abroad), inflationary expectations at home, and the bond holdings of the central bank, the initial demand effects are weakened but not overcome by the feedback demand effects. With respect to bond issues (either at home or abroad) and inflationary expectations abroad, the overall direction of the shift in aggregate demand is indeterminate. An advantage of this model over the reduced form aggregate demand models, more typical in the literature, is that one can see where indeterminacies arise in signing the effects of changes in the exogenous variables on aggregate demand.

Because prices are not fixed in this model, and because consumer prices in each country depend on domestic prices in both countries, changes in inflation in one country result in a shift in aggregate supply in the other country. This feedback effect on aggregate supply is absent from most open economy macro-models, particularly those analyzing policy coordination, because they typically assume that output is demand determined.

The overall effect of a change an exogenous variable on output and inflation depends on the direction and magnitude of the shift in aggregate supply and aggregate demand. Indeterminacies in signing some of the comparative statics arise as a result of an indeterminacy in signing the effect on aggregate demand and/or as a result of a change in aggregate supply which works to offset the demand effect on output or inflation. Neglecting the aggregate supply effects can provide misleading results.

Standard open economy models also tend to eliminate interest earnings from the aggregate demand specification. In the model developed in this chapter, the inclusion of interest earnings on bonds in both the domestic absorption equation and the government budget constraint introduces the net debtor/creditor status of a country into the aggregate demand curve. This makes past policies a determinant of the effects of current policies pursued by either country on both output and inflation. A net debtor country is less adversely affected by inflation than a net creditor. Because of this difference, the aggregate demand curve for a net creditor country is relatively flat and the aggregate demand curve for a net debtor country is relatively steep. The slope of the aggregate demand curve influences the effects of shifts in aggregate supply on inflation and output. In general a net debtor is more likely to experience an increase in output following expansionary policies undertaken by either country's fiscal authority or by the monetary authority. A net debtor is also more likely to have a sharper increase in inflation than a net creditor as a result of such policies.

Specifically, a fiscal expansion (either tax or bond financed) in one country increases output and inflation in that country. The effect on the other country is ambiguous. If the expansionary country was a net debtor last period while the other country was a net creditor last period then it is more likely that the expansionary fiscal policies pursued by the one country cause stagflation in the other country. Expansionary monetary policies have a positive effect on output and inflation in both countries, but with greater effects on the net debtor. An increase in inflationary expectations in one country increases output and inflation in that country, but has an ambiguous effect on output and inflation abroad.

These results suggest that asymmetries in past policies, reflected in asymmetries in the current account balances of countries, and the continuation of asymmetric fiscal policies can be a source of friction among the countries in a monetary union. Looking at the countries that will comprise the European Monetary Union, it is clear that such asymmetries do exist. In the 1980s France, Italy and Greece had persistent current account deficits, while the Federal Republic of Germany, the Netherlands, Belgium and Luxembourg had persistent current account surpluses. The United Kingdom started the decade with a current account surplus (due to its oil exports), but since 1986 it has run current account deficits. In 1989 the current account deficit of the United Kingdom was equal to 4.1 percent of its GDP, while the current account surplus of the Federal Republic of Germany was equal to 4.4 percent of its GDP.

There is evidence that the creation of the European Monetary Union will increase these asymmetries. Artis and Bayoumi (1991) found that the increase in capital integration in the world economy in the 1980s corresponded with growing capital account imbalances. An increase in capital mobility reduces the external constraints on borrowing. Thus, the European Monetary Union, which is to be characterized by full capital mobility, is likely to increase the level and persistence of current account imbalances among its member countries. Differences in preferences for consumption versus saving among countries are more easily maintained when countries only need to concern themselves with a solvency constraint and not an external constraint.

Given the existence of asymmetries, there is the potential for conflict to arise within a monetary union over the fiscal policies pursued by the member countries of a monetary union Thus, this chapter lends support to the argument that fiscal policy convergence should be addressed before the creation of a monetary union in Europe.

CHAPTER 2: A GAME THEORETIC ANALYSIS OF THE EUROPEAN MONETARY UNION

Section I: Introduction

The European Monetary Union (EMU) is to consist of an independent supranational central bank which will design and implement monetary policy for the European Community (EC) and, twelve member countries which will maintain their national control over fiscal policies. Policies adopted by the fiscal authorities and set by the monetary authority will affect output and inflation in all countries within the monetary union. Furthermore, the interaction of these policies will determine their overall effect on output and inflation in each member country.

This chapter examines the strategic interaction among the national governments and the central bank in the model of a monetary union developed in chapter 1. Strategic interaction among policymakers has previously been analyzed in a game theoretic setting, to determine the possible gains from policy coordination. The structure of the game developed in this chapter draws on this previous literature in the formulation of the objective functions of the policymakers. The fiscal policy authorities attempt to minimize fluctuations in their countries' consumer prices while at the same time meeting an output target. The countries are not altruistic, they are concerned only with their own output level and inflation rate. The central bank is concerned solely with minimizing price fluctuations in the monetary union. It looks only at the overall inflation rate and is not concerned with the distribution of inflation across the two countries.

The interaction among the three policy authorities, given their different policy objectives, is considered within the framework of a noncooperative game. The analysis is based solely on single shot games where the policy decisions by the three players are taken at the start of the game and can not be revised. The results therefore can be seen as relevant to the impact effects of policy decisions within a monetary union.

This paper differs from previous analyses of the macroeconomic effects of a monetary union in two ways. First, it explicitly models the separation of monetary and fiscal policy decisions which is to occur under the framework established for the European Monetary Union. Other models either allocate policy decisions to a single European authority, or maintain all policy decisions at the national level.[1] Second, there is no restriction that inflation rates across the member countries in a monetary union be equalized.

Three types of games are considered: 1) The cooperative game, in which the preferences of the three policymakers are given equal weight, is derived as a benchmark. 2) Nash games in which decisions by all three players are made simultaneously, with each player adopting the best strategy assuming that other players actions are fixed. 3) Stackelberg games in which decisions by the players are made sequentially. Two Stackelberg games are modelled. In the first, the central bank is the Stackelberg leader, incorporating the reaction functions of the fiscal policy authorities into its reaction function. The two governments play a Nash game against each (moving simultaneously), both taking the actions of the central bank as given. In the second Stackelberg game modelled in this paper, the two countries act as Stackelberg leaders and the central

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[1] See section II for a discussion of these papers.

bank as the Stackelberg follower. In this game, both countries incorporate the reaction function of the central bank into their own reaction functions. The countries then play a Nash game against each other, and finally, the central bank moves taking the actions of the two governments as given.

Section II of this chapter examines the structure of the European Monetary Union in terms of the potential for strategic policy interactions, and discusses previous work on policy games which are used as a model for the work presented here. Section III develops the structure of the game which is then used in sections IV-VII to analysis the cooperative game, the Nash game and the two Stackelberg games. The final section presents the conclusions.

Section II: Strategic Interaction in the European Monetary Union

The treaty establishing the intention of the European Community to develop a monetary union was signed by the member countries of the EC, in Maastricht, the Netherlands, in December 1991. Despite the recent setbacks to this treaty, the leaders of the member countries have maintained their support for the creation of a European Monetary Union. Under the treaty agreement, the most notable feature of the EMU will be the European System of Central Banks (ESCB) which will establish a single currency and set monetary policy for the monetary union. The structure of this system is similar to the U.S. Federal Reserve System. The countries will maintain their national central banks, but all monetary policy powers will reside with the ruling council of the ESCB. The ruling council will be comprised of 5-7 full-time directors and the governors of the country central banks, who are to represent the interests of the Community and not their individual countries. The full-time directors will be responsible for the day-to-day operations of monetary policy. The framers of this central bank system hope that it will effectively remove the influence of the national governments over monetary policy. They also hoped to minimize pressure from the national governments on the central bank by prohibiting the central bank from monetizing the debt of any or all countries within the monetary union.

The national governments will maintain control over their fiscal policies. There are no binding rules for policies once entry into the monetary union is achieved.[2] There will be informal discussions of policies among the member countries, but the treaty does not place constraints on national fiscal policies for the member countries. In fact, due in part to the insistence of the British government, the treaty emphasizes national sovereignty.

The monetary policy decisions made by the ESCB will affect output and inflation in each of the member countries. Likewise, the policy decisions made by the national fiscal authorities will have spillover effects on all of the countries.[3] For policy to work most effectively the policy makers need to have similar goals for output and inflation and

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[2] Countries will have to meet convergence requirements with respect to inflation, interest rates, government debt and government deficits before entry is permitted. As established at Maastricht: a country's inflation rate can be no more than 1.5 percentage points above the average inflation rate for the 3 countries with the lowest rates. Long-term interest rates on government securities can be no more than 2 percentage points above the average of the 3 countries with the lowest rates. The government deficit can be no more than 3 percent of a country's GDP, and the government debt can be no more than 60 percent of a country's GDP. At present only 3 of the twelve EC members meet both the debt and deficit requriments.

[3] See chapter 1 for a discussion of these effects.

similar views with respect to the appropriate mix of policies to achieve these goals.

The strategic interaction between policymakers has been studied both in the context of a single country and in an international framework. The literature on monetary policy games had its origins in the rules versus discretion controversy: should monetary policy be tied to a rule (e.g. a fixed growth rate of money) or should policymakers be free to use their discretion in determining monetary policy? Kydland and Prescott (1977) and Calvo (1978) argued that discretionary policy is suboptimal, using the idea of time (dynamic) inconsistency. To be time consistent implies that given a chance to change policy at some future point in time, policymakers must have no incentive to do so. If a policy is not time consistent, rational agents will realize this and act accordingly.

Barro and Gordon (1983a) developed the standard objective function used in the macroeconomic policy game literature, basing their work on an example in Kydland and Prescott. The players in the game have preferences over unemployment (or output) and inflation, which are given by the following objective function:

$$V_t = -a(U_t-U^*)^2 - b(\pi_t-\pi^*)^2 \qquad a,b>0$$

where U is the unemployment rate, π is the inflation rate and, a * denotes a target variable. The use of a quadratic form implies that welfare decreases at an increasing rate as unemployment and inflation depart from their target levels.[4] The ratio a/b represents the tradeoff between meeting the unemployment versus the inflation target.

Andersen and Schneider (1986) model the strategic interaction between fiscal and monetary policy in a framework similar to Barro and Gordon. The two policymakers have preferences over output and inflation, and dislike deviations of output or inflation from their desired levels. The loss functions representing these preferences are given below:

$$V_f = -\left(\frac{a_f}{2}\right)(y-y_f^*)^2 - \left(\frac{b_f}{2}\right)(\pi-\pi_f^*)^2$$

$$V_m = -\left(\frac{a_m}{2}\right)(y-y_m^*)^2 - \left(\frac{b_m}{2}\right)(\pi-\pi_m^*)^2$$

where y is the level of output, π is the inflation rate, a * denotes the target of the policymaker, a/2 is the weight given to meeting the output target, b/2 is the weight given to meeting the inflation target, and the subscripts f and m refer to the fiscal and monetary policymakers, respectively.

In this model, the policymakers may differ in their target rates for inflation and output, and in the relative weight given to achieving these targets. The fiscal policymaker is assumed to place more emphasis on output goals than inflation goals, whereas the monetary policymaker does the opposite, and the fiscal policymaker has higher target values for output and inflation than does the monetary policymaker. Output and inflation are functions of fiscal and monetary policy instruments (f and

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[4] Barro and Gordon (and most of the literature) assume that $\pi^*=0$. A positive target rate for inflation could result from some optimal level of seigniorage.

m). Andersen and Schneider examine the interaction between the two policy makers under cooperation[5], Nash behavior and Stackelberg behavior.

The cooperative solution gives a Pareto Optimal outcome, but each player gains if she can deviate from the agreement while the other player maintains the agreement. Thus, unless players are bound to the agreement this outcome will not be sustainable.

In the Nash equilibrium each policymaker simultaneously chooses the value of her policy instrument to minimize her own loss function, taking the action of the other player as given. The Nash equilibrium solution is Pareto inefficient. Equilibrium output exceeds the target output for the fiscal policymaker and the equilibrium inflation rate is below the target rate for the monetary policymaker. The fact that output is always too high and inflation too low in the Nash equilibrium illustrates the benefits of coordinated policies. Non-coordination results in fiscal policy which is too contractionary and monetary policy which is too expansionary. The Stackelberg solution where the fiscal policymaker acts as the leader and the monetary policymaker acts as the follower also results in a Pareto inefficient outcome.

If the monetary authority is only concerned about inflation, then the Stackelberg equilibrium, where the fiscal authority acts as leader, is

[5] The concept of cooperation used in macroeconomic policy games is different from that used in the game theory bargaining literature. In the former, the cooperative solution is found by minimizing a weighted average of the players' loss functions, where the weights depend on the relative influence of the players over the centralized policymaker. In the latter, cooperation requires minimizing a weighted average of the loss functions subject to the constraint that no player can do worse than the noncooperative outcome.

the same as the cooperative equilibrium.[6] In the Nash equilibrium solution the central bank meets its inflation target, but the equilibrium is Pareto inefficient. If each policymaker only has one target than the Nash equilibrium and the Stackelberg equilibrium are the same as the cooperative equilibrium: $\pi=\pi^*$ and $y=y^*$.

The model developed and results obtained by Andersen and Schneider are relevant to the strategic interaction in the European Monetary Union in that their model is one of the few which examines policy interactions between fiscal and monetary authorities. Their model, by its focus on a single country, does not allow one to examine the linkages between economies and the effect of these linkages on policy interactions. To gain an insight into this side of the EMU, one must look to the literature on policy games between countries.

In two country games there is always a single policymaker for each country, and in general there is only one policy tool (monetary policy) which is controlled by each policymaker. The purpose of this literature is to determine if there are gains from policy coordination and what is the nature of these gains, i.e., what is it about the non-cooperative equilibrium which leads to inefficiencies?

Much of this literature focusses on problems arising due to floating exchange rates. McKibbin and Sachs (1988) develop a two country, floating exchange rate model in which each country attempts to minimize a loss function which depends on output, consumer price inflation, and the fiscal deficit. The loss functions are assumed to be identical and the countries are symmetric. In this model each country has two policy variables: the

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[6] Anderson and Schneider do not discuss the case in which the monetary authority acts as Stackelberg leader.

money supply and the fiscal deficit. An ISLM reduced form model determines output and real balances. Domestic prices are assumed to be fixed, so that consumer prices depend only upon the exchange rate.

McKibbin and Sachs assume that each country starts at a position of full employment and a balanced budget, but each is experiencing inflation. In the Nash game each country hopes to appreciate its currency vis à vis the other country's currency, in order to reduce its consumer price inflation. Thus each country adopts a contractionary monetary policy and an expansionary fiscal policy. The latter policy is chosen to exploit the anti-inflationary gains from the appreciation of its currency. The countries, however, are unable to both have a strong currency, so the exchange rate is unchanged as is inflation.

Under the cooperative solution each country can maintain its balanced budget and full employment, but must live with the inflation. In comparison, the Nash solution results in fiscal deficits which are too high and output which is too low, without any compensating decrease in inflation.

McKibbin and Sachs also consider the case in which the exchange rate between the two countries is fixed and the world money growth rate is set at a global optimum. Each country retains only its fiscal policy instrument. In this case, if there are symmetric inflationary shocks then the Nash equilibrium will be the same as the cooperative equilibrium. The fixed exchange rate eliminates the possibility of using a fiscal expansion to appreciate the exchange rate and reduce consumer price inflation. Thus fiscal policy remains unchanged and under both the Nash and the cooperative equilibrium, the countries remain at full employment with balanced budgets, but must accept the inflation. If shocks are asymmetric, efficiency requires a change in the exchange rate which neither solution brings about.

Bean (1985) uses a model similar to McKibbin and Sachs. In this paper each country only has one instrument to achieve its targets. The result that the Nash equilibrium is inefficient relative to the cooperative equilibrium is maintained. In the Nash game, given an initial inflationary shock, each country attempts to use its policy to appreciate the exchange rate. The exchange rate and inflation is unchanged, but output falls.

Oudiz (1985) develops a model closely related to the Bean, and McKibbin and Sachs papers. This paper, however, analyzes not only the cooperative and Nash solutions, but also the Stackelberg game. In this game, the equilibrium the policies adopted are less contractionary than in the Nash game. This result follows from the fact that the leader, knowing the reaction function of the follower, realizes that there is an incentive to compete in deflationary policies to the detriment of both countries.

De Grauwe (1990) analyzes fiscal policy interaction in the EMS. In this model, there are two countries each with preferences over output and the current account balance:

$$L_i = (y_i - y_i^*)^2 + \theta_i (B_i - B_i^*)^2$$

The output level and the current account balance for each country are determined by the following reduced form equations:

$$y_i = a_i x_i + b_i x_j + z_i$$

(12) $$B_i = -h_i x_i + k_j x_j + c_i z_w$$

where:

x_i is the fiscal policy instrument for country i
z_i is an exogenous disturbance affecting output in country i
z_w is an exogenous disturbance, originating in the rest of the world, which affects both countries' current account balance.

De Grauwe assumes that there is a Mundell-Fleming model underlying these reduced form equations, and that there is limited capital mobility between the two countries. These two assumption ensure that the reduced form parameters in equations (11) and (12) are all positive.

The Nash equilibrium solution results in a greater loss (lower level of utility) than the cooperative equilibrium, even if there are no disturbances. Analyzing the case where an exogenous disturbance results in a deterioration of the current account balance for both countries, de Grauwe finds that the Nash solution has a deflationary bias. Both countries react to the disturbance by adopting a contractionary fiscal policy. In the cooperative outcome, spending is reduced less and utility is higher than in the Nash game. The Stackelberg game also results in a lower level of utility than the cooperative solution, but it results in a better outcome than the Nash game. De Grauwe also finds that in Stackelberg game, the follower achieves a higher level of utility than the leader.

If one of the countries only cares about its output target and not its current account balance, then it will always be able to use its policy to reach its output target. Thus, de Grauwe concludes, this country will have no incentive to cooperate with the other country.[7] In the Stackelberg game, if the country with two targets (output and current account balance) acts as the leader it will have a smaller loss than in the Nash solution.

The usefulness of the international policy game literature for understanding the likely results of strategic interaction among policymakers in the European Monetary Union is limited by its concentration on single policymakers within each country. It does not capture the interaction between fiscal and monetary policies, nor does it shed light on any possible results when the goals of these authorities are different. To fully capture the policy interactions inherent in the EMU one needs model which incorporates the fiscal and monetary aspects of the Andersen and Schneider model with the inter-country links present in the international policy game literature.

There are few papers that analyze the policy interactions in a model based on the European Monetary Union and none which capture these two aspects which make a monetary union inherently different from a closed economy model with independent fiscal and monetary policy makers or an open economy fixed exchange rate model in which policies enacted by one country have cross-country effects.

Cohen and Wyplosz (1989), and Alesina and Grilli (1991) are among the few papers which have developed models to specifically analyze the potential policy benefits from monetary union. The former paper finds that policy coordination is not optimally achieved through monetary

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[7] Actually, given that this country is indifferent between the cooperative solution and the Nash solution, it should be indifferent to cooperation. Thus, the cooperative solution should not be ruled out.

integration, while the latter finds that the benefits of monetary union depend upon the preferences of the central bank versus the governments, and the economic similarities of the countries.

Cohen and Wyplosz develop a two country model of a monetary union. Each country sets targets for aggregate demand, output, and inflation. Aggregate demand is the fiscal policy instrument of the government, and inflation is the monetary policy instrument of the government. The trade balance for each country with respect to a third country is related to output and aggregate demand through the following relationship:

$$V_t = (Q_t - A_t)(1 - z_t)$$

where Q is output, A is aggregate demand and, z_t is the log of the real exchange rate for the monetary union vis á vis a third country.

Although each country maintains an independent monetary policy, Cohen and Wyplosz claim, given the existence of a monetary union, each country will enact an optimal monetary policy and there will be no difference in inflation rates between the two countries.[8]

The Nash solution is inefficient because each country neglects the effect of its fiscal policy actions on the trade balance of the other country. Since only the inflation rate is set efficiently, each country remains free to determine its trade balance vis á vis the rest of the world. Given symmetric shocks, the policies the two countries adopt will be identical. However, they both fail to realize that the other country will react identically. Therefore, the trade balance of the monetary

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[8] The paper does not explain how the countries arrive at an optimal monetary policy in the absence of coordination.

union, and so the real exchange rate with respect to the third country, is not determined efficiently. In the presence of asymmetric shocks, the monetary union is inefficient because it does not allow for differences in the inflation rates in the two countries.

Alesina and Grilli begin by examining policy decisions under the assumption that monetary union is characterized by complete economic and political union. In this case one can think of the members of the monetary union comprising a single country. The European Central Bank sets monetary policy (chooses the inflation rate) in order to minimize its loss function which depends upon the European inflation rate, π_E, and the deviation of European output, x_E, from its target level:

$$L_E = \frac{1}{2}E\left[\pi_E^2 + b(x_E-\overline{x_E})^2\right]$$

Output is determined by an expectations augmented Phillips curve relationship:

$$x_E = (\pi_E-\pi_E^e) + e, \quad e\sim(0,\sigma_e^2)$$

The natural level of output is assumed to be zero, and the target level of output is assumed to be above the natural rate.

The structure of the game is the same as that developed in Barro and Gordon (1983a). The public sets expectations for inflation. Given these expectations, if the central bank has an incentive to increase output through surprise inflation, it will do so. The public, however, having rational expectations anticipates this action. Thus the time consistent policy results in:

$$\pi_E = b\overline{x_E} - \frac{1}{1+b}e$$

$$x_E = \frac{1}{1+b}e$$

which implies, as is standard in such models, that in equilibrium inflation is too high, but there is no gain in output.

Next, Alesina and Grilli examine the case where political unification is incomplete. The central bank still sets policy, but the member countries of the monetary union evaluate this policy based on country specific loss functions:

$$L^i = \tfrac{1}{2}E\left[\pi_E^2 + b(y_i-\overline{y_i})^2\right]$$

where the output for country i is determined as follows:

$$y_i = (\pi_E-\pi_E^e)+\mu_i, \quad \mu_i\sim(0,\sigma_\mu^2)$$

There is no difference in inflation rates among the member countries of the monetary union.

The policy decision for the central bank is unchanged. Thus, the loss for each member country can be determined by substituting the time consistent inflation rate into the country specific output and loss functions:

$$L^i = \tfrac{1}{2}E\left[\left(b\overline{x_E}-\frac{b}{1+b}e\right)^2 + \beta_i\left(\mu_i-\frac{b}{1+b}e - \overline{y_i}\right)^2\right]$$

Finally, Alesina and Grilli determine the time consistent inflation policy under the assumption that each country sets its own policies. This is the pre-monetary union case. In this case,

$$\pi_i = \beta_i\overline{y_i} - \frac{\beta_i}{1+\beta_i}\mu_i$$

$$y_i = \frac{1}{1+\beta_i}\mu_i$$

and so the loss for country i is

$$L_N^i = \frac{1}{2}E\left[\left(\beta_i \overline{y_i} - \frac{\beta_i}{1+\beta_i}\mu_i\right)^2 + \beta_i\left(\frac{1}{1+\beta_i}\mu_i - \overline{y_i}\right)^2\right]$$

Alesina and Grilli compare the loss function, given equilibrium output and inflation, for the pre-monetary union case with the loss function for monetary union with incomplete political unification to determine the gains from monetary union.[9] If there are no economic differences between the countries then the gains from a monetary union depend on the differences between the central bank's and the individual countries' preferences. If the central bank is more conservative, so that it places more weight on the inflation target and less weight on the output target than does country i ($b<\beta_i$), then the monetary union will result in a higher level of utility (lower loss) for country i. In the absence of any differences in preferences, the benefit of the monetary union will depend on economic differences among the countries. If there are differences between the variance of output in country i and European output, the monetary union will result in a lower level of utility for that country. If output in country i has the same variance as European output, and there are no difference in preferences, the gain from monetary union will depend on the correlation between the shocks to output. The smaller the correlation the smaller will be the gain from monetary union to any country i. From the perspective of country i, if the correlation

[9] In making this comparison the authors assumed that the output shock, μ_i, is the same when the central bank sets monetary policy as it is when the individual countries set monetary policy. Also, they assumed that central bank's output target is the same as each country's output target.

is small, the central bank will always be "over or under stabilizing" output.

Since there is only one policymaker in either the monetary union or the individual country case, this paper cannot be used to determine the policy implications for a monetary union in which fiscal and monetary policies are not centralized. To do this it is necessary to consider a monetary union in which the member countries maintain control over fiscal policy, while the central bank assumes control over monetary policy.

Section III: Structure of the Game

The two governments set targets for output and inflation in their own countries. Because monetary policy is controlled by the independent central bank each government possesses only one instrument, fiscal policy, which it can manipulate to reach its targets. The fiscal policy which each government chooses can be tax financed or bond financed. The central bank sets a target only for inflation. This assumption is in keeping with the notion that the primary goal of the central bank is price stability and subject only to meeting this goal is the central bank to support general economic policy set at the Community level.[10] The inflation target set by the central bank is for the monetary union as a whole. The central bank is not concerned with the distribution of output or inflation across the monetary union, only the average output level and inflation rate.[11] The central bank also has one instrument, the money supply,

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[10] Reference is made to this point in both the Delors Report (1989) and a report by the European Commission in 1990.

[11] It is possible that the central bank will be concerned with the distribution of inflation across countries. The assumption that it is not (continued...)

which it can manipulate through bond purchases from the government to attempt to reach its target.

The two fiscal policy variables and the monetary policy variable affect output and inflation in both countries. These effects are known by all three parties. The system of reduced form equations representing this relationship is given below:

$$(1) \qquad y_{1,t} = B_{11,t}\, f_{1,t} + B_{12,t}\, f_{2,t} + B_{13,t}\, b_{m,t} + B_{14,t}$$

$$(2) \qquad y_{2,t} = B_{21,t}\, f_{1,t} + B_{22,t}\, f_{2,t} + B_{23,t}\, b_{m,t} + B_{24,t}$$

$$(3) \qquad \pi_{1,t} = C_{11,t}\, f_{1,t} + C_{12,t}\, f_{2,t} + C_{13,t}\, b_{m,t} - C_{14,t}$$

$$(4) \qquad \pi_{2,t} = C_{12,t}\, f_{1,t} + C_{22,t} f_{2,t} + C_{13,t} b_{m,t} - C_{24,t}$$

where $y_{1,t}$ and $y_{2,t}$ are the levels of real output for country 1 and country 2; $f_{1,t}$ and $f_{2,t}$ are the fiscal policy instruments of country 1 and country 2; $b_{m,t}$ is the monetary policy instrument of the central bank; and, $B_{14,t}$, $B_{24,t}$, $C_{14,t}$, and $C_{24,t}$ are constants.[12] These reduced form equations were developed in Chapter 1.[13]

Each government has two means of financing fiscal policy: through taxes or through bond issues. If a government adopts a tax financed

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[11](...continued) is used here to make the objective function of the central bank as simple as possible, and to determine if, given the objective functions of the individual countries, such a specification is consistent with both average an individual price stability.

[12] Because all of the analysis in this chapter involves static games the remainder of the chapter will drop the time subscripts, except where it is confusing to do so (i.e. in the case of a lagged variable).

[13] These parameters may look slightly different from those in chapter 1 for two reasons. First, to keep the notation simple, it is assumed that all real variables are found by deflating nominal variables by country 1's prices. This adjustment is made in the parameters for country 2. Second, as discussed below, the governments have two possible means of financing spending. The parameters for a bond financed fiscal policy, are found by combining the government spending and bond variables given in chapter 1.

TABLE I[14]

Coefficients in Output Equations: Tax Financed Fiscal Policy

$$B_{11} = \frac{A_1[\alpha\gamma_3\Upsilon_2\tilde{p}\,\bar{y} + \overline{y^2}] - A_4\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1} > 0$$

$$B_{12} = \left[\frac{A_4[\alpha\gamma_3\Upsilon_2\tilde{p}\,\bar{y} + \overline{y^2}] - A_1\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right]\tilde{p} > 0,\ < 0$$

$$B_{13} = \frac{[\alpha\gamma_3\Upsilon_2\tilde{p}\,\bar{y} + \overline{y^2} - \alpha\Upsilon_1\gamma_2\bar{y}]\ A_7}{\Omega_1} > 0$$

$$B_{14} = \frac{\alpha\Upsilon_1\bar{y}\ [\alpha\Upsilon_2\tilde{p}(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \bar{y}(\gamma_1+\gamma_4)]}{\Omega_1} > 0$$

$$B_{21} = \left[\frac{A_4\ (\alpha\gamma_1\Upsilon_1\bar{y} + \overline{y^2}) - A_1\alpha\gamma_4\Upsilon_2\tilde{p}\,\bar{y}}{\Omega_1}\right]\tilde{p} > 0,\ < 0$$

$$B_{22} = \frac{A_1\ (\alpha\gamma_1\Upsilon_1\bar{y} + \overline{y^2}) - A_4\alpha\gamma_4\Upsilon_2\tilde{p}\,\bar{y}}{\Omega_1} > 0$$

$$B_{23} = \frac{[\alpha\gamma_1\Upsilon_1\bar{y} + \overline{y^2} - \alpha\gamma_4\Upsilon_2\tilde{p}\,\bar{y}]\ A_7}{\Omega_1} > 0$$

$$B_{24} = \frac{\alpha\Upsilon_2\tilde{p}\,\bar{y}\,(\alpha\Upsilon_1(\gamma_1\gamma_3 - \gamma_2\gamma_4) - \bar{y}(\gamma_3+\gamma_4))}{\Omega_1} > 0$$

[14] The conditions for signing the parameters in this table, and the next three tables, are derived in Appendix D.

TABLE II
Coefficients in Inflation Equations: Tax Financed Fiscal Policy

$$C_{11} = \frac{A_1\left[\alpha^2\Upsilon_2\bar{p}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}\right] + A_4\alpha\gamma_2\bar{y}}{\Omega_1} > 0$$

$$C_{12} = \left[\frac{A_4\left[\alpha^2\Upsilon_2\bar{p}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}\right] + A_1\alpha\gamma_2\bar{y}}{\Omega_1}\right]\bar{p} > 0$$

$$C_{13} = \frac{\left[\alpha^2\Upsilon_2\bar{p}(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_1\bar{y} + \alpha\gamma_2\bar{y}\right]A_7}{\Omega_1} > 0$$

$$C_{14} = \frac{\alpha\bar{y}\left[\alpha\Upsilon_2\bar{p}(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \bar{y}(\gamma_1 + \gamma_2)\right]}{\Omega_1} > 0$$

$$C_{21} = \left[\frac{A_4\left[\alpha^2\Upsilon_1(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\bar{y}\right] + A_1\alpha\gamma_4\bar{y}}{\Omega_1}\right]\bar{p} > 0$$

$$C_{22} = \frac{A_1\left[\alpha^2\Upsilon_1(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\bar{y}\right] + A_4\alpha\gamma_4\bar{y}}{\Omega_1} > 0$$

$$C_{23} = \frac{\left[\alpha^2\Upsilon_1(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3\bar{y} + \alpha\gamma_4\bar{y}\right]A_7}{\Omega_1} > 0$$

$$C_{24} = \frac{\alpha\bar{y}\left(\alpha\Upsilon_1(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \bar{y}(\gamma_3+\gamma_4)\right)}{\Omega_1} > 0$$

TABLE III
Coefficients in Output Equations: Bond Financed Fiscal Policy

$$B_{11} = \frac{(A_1+A_2)\,[\alpha\gamma_3\Upsilon_2\tilde{p}\,\bar{y} + \overline{y^2}] - (A_4+A_5)\,\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1} > 0$$

$$B_{12} = \left[\frac{(A_4+A_5)\,[\alpha\gamma_3\Upsilon_2\tilde{p}\,\bar{y} + \overline{y^2}] - (A_1+A_2)\,\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right]\tilde{p} >0, \quad < 0$$

$$B_{13} = \frac{[\alpha\gamma_3\Upsilon_2\tilde{p}\,\bar{y} + \overline{y^2} - \alpha\Upsilon_1\gamma_2\bar{y}]\ A_7}{\Omega_1} > 0$$

$$B_{14} = \frac{\alpha\Upsilon_1\bar{y}\ [\alpha\Upsilon_2\tilde{p}(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \bar{y}(\gamma_1 + \gamma_4)]}{\Omega_1} > 0$$

$$B_{21} = \left[\frac{(A_4+A_5)\left(\alpha\gamma_1\Upsilon_1\bar{y} + \overline{y^2}\right) - (A_1+A_2)\,\alpha\gamma_4\Upsilon_2\bar{y}}{\Omega_1}\right]\tilde{p} >0, \quad < 0$$

$$B_{22} = \frac{(A_1+A_2)\left(\alpha\gamma_1\Upsilon_1\bar{y} + \overline{y^2}\right) - (A_4+A_5)\,\alpha\gamma_4\Upsilon_2\tilde{p}\,\bar{y}}{\Omega_1} > 0$$

$$B_{23} = \frac{\left[\alpha\gamma_1\Upsilon_1\bar{y} + \overline{y^2} - \alpha\gamma_4\Upsilon_2\tilde{p}\,\bar{y}\right] A_7}{\Omega_1} > 0$$

$$B_{24} = \frac{\alpha\Upsilon_2\tilde{p}\,\bar{y}\left(\alpha\Upsilon_1(\gamma_1\gamma_3 - \gamma_2\gamma_4) - \bar{y}(\gamma_3+\gamma_4)\right)}{\Omega_1} > 0$$

TABLE IV
Coefficients in Inflation Equations: Bond Financed Fiscal Policy

$$C_{11} = \frac{(A_1+A_2)\left[\alpha^2\Upsilon_2\bar{p}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}\right] + (A_4+A_5)\alpha\gamma_2\bar{y}}{\Omega_1} > 0$$

$$C_{12} = \left[\frac{(A_4+A_5)\left[\alpha^2\Upsilon_2\bar{p}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}\right] + (A_1+A_2)\alpha\gamma_2\bar{y}}{\Omega_1}\right]\bar{p} > 0, \ < 0$$

$$C_{13} = \frac{\left[\alpha^2\Upsilon_2\bar{p}(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_1\bar{y} + \alpha\gamma_2\bar{y}\right] A_7}{\Omega_1} > 0$$

$$C_{14} = \frac{\alpha\bar{y}\left[\alpha\Upsilon_2\bar{p}(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \bar{y}(\gamma_1 + \gamma_2)\right]}{\Omega_1} > 0$$

$$C_{21} = \left[\frac{(A_4+A_5)\left[\alpha^2\Upsilon_1(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\bar{y}\right] + (A_1+A_2)\alpha\gamma_4\bar{y}}{\Omega_1}\right]\bar{p} > 0, \ < 0$$

$$C_{22} = \frac{(A_1+A_2)\left[\alpha^2\Upsilon_1(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\bar{y}\right] + (A_4+A_5)\alpha\gamma_4\bar{y}}{\Omega_1} > 0, \ < 0$$

$$C_{23} = \frac{\left[\alpha^2\Upsilon_1(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3\bar{y} + \alpha\gamma_4\bar{y}\right] A_7}{\Omega_1} > 0$$

$$C_{24} = \frac{\alpha\bar{y}\left(\alpha\Upsilon_1(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \bar{y}(\gamma_3 + \gamma_4)\right)}{\Omega_1} > 0$$

fiscal policy then the f_i notation of this chapter is equivalent to g_i in Chapter 1. If the government adopts a bond financed fiscal policy then the f_i notation used in this chapter is equivalent to g_i+b_i in Chapter 1. Tables I and II list the output and inflation equation parameters, respectively, when the governments adopt tax financed fiscal policies. Tables III and IV list the parameters when the governments adopt bond financed fiscal policies. As discussed in Chapter 1, one country's fiscal policy can have negative spillover effects on the other country's output when past policies of the two countries were highly divergent and continue to be so. It is therefore not possible that the fiscal policies of both countries produce negative spillover effects (i.e. if $B_{12}<0$ then $B_{21}>0$ and if $B_{21}<0$ then $B_{12}>0$). Under a tax financed fiscal policy an increase in government spending by one country always causes inflation in both countries, whereas under a bond financed fiscal policy an increase in government spending may cause a decrease in inflation in the other country (C_{12} or $C_{21} < 0$).[15]

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[15] As derived in Chapter 1, Ω_i is positive as is $(\gamma_1\gamma_3-\gamma_2\gamma_4)$. Thus the signs of the C parameters will depend on the A terms. A_1 measures the effect of a change in government spending by a country on its own aggregate demand, and A_4 measures the effect of a change in government spending by a country on aggregate demand in the other country. These terms are both positive, so under a tax financed fiscal expansion, inflation in both countries will increase ($C_{ii},C_{ij}>0$).

A_2 measures the effect of a change in bond issues by a country on its own aggregate demand, while A_5 measures the effect of a change in bond issues by a country on aggregate demand in the other country. An increase in bond issues, holding government spending fixed, results in a decrease in taxes, which in turn stimulates aggregate demand. At the same time, an increase in bond issues will increase the interest rate which decreases aggregate demand. A_2 may be positive or negative, depending on the relative strengths of these two effects. The interest rate effect is always stronger than the consumption effect in the other country, so A_5 is negative.

$A_1+A_2>0$ and $A_1+A_2>|A_4+A_5|$. Thus, under a bond financed fiscal expansion $C_{ii}>0$, but C_{ij} may be positive or negative.

As shown below, changing the assumption on the signs of the parameters has an effect on the strategic interaction between the two countries. There is no effect, however, on the strategic interactions between the governments and the central bank since an increase in government spending, ceteris paribus, always has a positive effect on inflation in the monetary union as a whole, and an increase in the money supply, ceteris paribus, always increases output and inflation in both countries.

Given the system of equations determining output and inflation in each country, the governments and the central bank attempt to meet their targets. Formally, each attempts to minimize a loss function. The loss functions for the two governments are

(5) $$L_1 = \beta_1(y_1 - y_1^*)^2 + \nu_1(\pi_1 - \pi_1^*)^2$$

(6) $$L_2 = \beta_2(y_2 - y_2^*)^2 + \nu_2(\pi_2 - \pi_2^*)^2 \;;$$

the loss function for the central bank is

(7) $$L_m = \left(\frac{\pi_1+\pi_2}{2} - \pi_m^* \right)^2 ,$$

where y_1^* and y_2^* are the output targets for country 1 and country 2; π_1^* and π_2^* are the inflation targets for country 1 and country 2; and, π_m^* is the inflation rate target for the central bank. β_i and ν_i are the weights which government i (i = 1,2) places on meeting its output target and its inflation target.

All three players are assumed to have the same inflation target which is taken to be zero. The two governments, however, may have different targets for output. The quadratic nature of the loss functions implies that deviations on either side of the targets produces an equal loss to the policymaker.[16] In the case of the inflation target this means that deflation and inflation are seen as equally bad from the perspective of the policymakers. In the case of the output targets this quadratic formulation assumes each government weights undershooting and overshooting its output target equally in terms of its effect on the policymaker's welfare.

If $\beta_i > \nu_i$ country i cares relatively more about meeting its output goal than its inflation goal. If $\beta_i < \nu_i$ country i cares relatively more about meeting its inflation goal than its output goal. In general it is assumed that $\beta_i > \nu_i$, $\beta_1 \neq \beta_2$ and $\nu_1 \neq \nu_2$.

The three objective functions are known by all of the policymakers. Each policymaker selects the level of her policy instrument (government spending or the money supply) to minimize her loss function taking the actions of the other policymakers as given. Once actions are taken they cannot be revised, thus this is a one-shot game.

Substituting equations (1)-(4) into the loss functions, equations (5)-(7) give the minimization problem facing each policymaker.

$$(8) \qquad \min_{f_1} L_1 = \beta_1(B_{11}f_1 + B_{12}f_2 + B_{13}b_m + B_{14} - y_1^*)^2 + \nu_1(C_{11}f_1 + C_{12}f_2 + C_{13}b_m - C_{14} - \pi_1^*)^2$$

$$(9) \qquad \min_{f_2} L_2 = \beta_2(B_{21}f_1 + B_{22}f_2 + B_{23}f_3 + B_{24} - y_1^*)^2 + \nu_2(C_{21}f_1 + C_{22}f_2 + C_{23}b_m - C_{24} - \pi_2^*)^2$$

---

[16] As stated previously this formulation of the objective functions is standard in the macroeconomic game theory literature.

$$\text{(10)} \qquad \min_{b_m} L_m = \left(\frac{(C_{11} + C_{21})}{2} f_1 + \frac{(C_{21} + C_{22})}{2} f_2 \right) + \left(\frac{(C_{13} + C_{23})}{2} b_m - \frac{(C_{14} + C_{24})}{2} - \pi_m^* \right)^2$$

Section IV: The Cooperative Solution[17]

One possible scenario for policymaking in a monetary union is for the central bank and the fiscal authorities to coordinate their policies. This can be modelled as choosing the three policy variables (f_1, f_2, and b_m) to minimize a weighted average of the three loss functions given by equations (8)-(10):

$$\text{(11)} \qquad \min_{f_1, f_2, b_m} L = \lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_m$$

Assuming that each loss function is weighted equally, the minimization problem given by equation (11) results in the following three equations:

$$\text{(12)} \qquad f_1 = \frac{A_0 + A_1 b_m + A_2 f_2 + A_3 y_1^* + A_4 y_2^*}{D_0}$$

$$\text{(13)} \qquad f_2 = \frac{E_0 + E_1 b_m + A_2 f_1 + E_3 y_1^* + E_4 y_2^*}{D_1}$$

$$\text{(14)} \qquad b_m = \frac{H_0 + A_1 f_1 + E_1 f_2 + H_3 y_1^* + H_4 y_2^*}{D_2}$$

where the parameters are defined in Table V.

These three equations can be used to solve for the cooperative equilibrium. Substituting equation (13) into equation (12) gives f_1 as a function of b_m:

---

[17] As noted earlier, this concept of cooperation differs from that in the game theory literature on bargaining. In the context of the European Monetary Union cooperation is equivalent to allowing the European Commission of the European Community determine the monetary and fiscal policies for the EMU.

$$(15) \qquad f_1 = \frac{(A_0D_1+A_2E_0)+(A_1D_1+A_2E_1)\,b_m+(A_3D_1+A_2E_3)\,y_1^*+(A_4D_1+A_2E_4)\,y_2^*}{D_0D_1-A_2^2}$$

Substituting equation (13) into equation (14) gives b_m as a function of f_1:

$$(16) \qquad b_m = \frac{(D_1H_0+E_0E_1)+(D_1A_1+A_2E_1)\,f_1+(E_3E_1+D_1H_3)\,y_1^*+(E_1E_4+D_1H_4)\,y_2^*}{D_1D_2-E_1^2}$$

Likewise, substituting equation (12) into (13) gives f_2 as a function of b_m:

$$(17) \qquad f_2 = \frac{(D_0E_0+A_0A_2)+(A_1A_2+D_0E_1)\,b_m+(A_2A_3+D_0E_3)\,y_1^*+(A_2A_4+D_0E_4)\,y_2^*}{D_0D_1-A_2^2}$$

and substituting equation (12) into (14) gives b_m as a function of f_2:

$$(18) \qquad b_m = \frac{(D_0H_0+A_0A_1)+(A_1A_2+D_0E_1)\,f_2+(A_1A_3+D_0H_3)\,y_1^*+(A_1A_4+D_0H_4)\,y_2^*}{D_0D_2-A_1^2}$$

Equations (15) and (16) are used to derive the cooperative equilibrium solutions for f_1 and b_m:

$$(19) \qquad \begin{aligned} f_1 = {} & \frac{(D_1D_2-E_1^2)\,A_0+(D_1H_0+E_0E_1)\,A_1+(E_1H_0+D_2E_0)\,A_2}{D_0D_1D_2-A_2^2D_2-A_1^2D_1-2A_1A_2E_1-D_0E_1^2} \\ & + \frac{(D_1D_2-E_1^2)\,A_3+(E_1E_3+D_1H_3)\,A_1+(D_2E_3+E_1H_3)\,A_2}{D_0D_1D_2-A_2^2D_2-A_1^2D_1-2A_1A_2E_1-D_0E_1^2}\,y_1^* \\ & + \frac{(D_1D_2-E_1^2)\,A_4+(D_2E_4+E_1H_4)\,A_2+(E_1E_4+D_1H_4)\,A_1}{D_0D_1D_2-A_2^2D_2-A_1^2D_1-2A_1A_2E_1-D_0E_1^2}\,y_2^* \end{aligned}$$

$$
(20)\qquad
\begin{aligned}
b_m = {} & \frac{(D_1H_0+E_0E_1)D_0+(E_0A_1-A_2H_0)A_2+(A_1D_1+A_2E_1)A_0}{D_0D_1D_2-A_2^2D_2-A_1^2D_1-2A_1A_2E_1-D_0E_1^2} \\
& + \frac{(A_1D_1+A_2E_1)A_3+(A_1E_3-A_2H_3)A_2+(E_1E_3+D_1H_3)D_0}{D_0D_1D_2-A_2^2D_2-A_1^2D_1-2A_1A_2E_1-D_0E_1^2}\, y_1^{*} \\
& + \frac{(A_1D_1+A_2E_1)A_4+(A_1E_4-A_2H_4)A_2+(E_1E_4+D_1H_4)D_0}{D_0D_1D_2-A_2^2D_2-A_1^2D_1-2A_1A_2E_1-D_0E_1^2}\, y_2^{*}
\end{aligned}
$$

Equations (17) and (18) are used to derive the cooperative equilibrium solution for f_2:

$$
(21)\qquad
\begin{aligned}
f_2 = {} & \frac{(D_2E_0+E_1H_0)D_0+(A_2D_2+A_1E_1)A_0+(A_2H_0-A_1E_0)A_1}{D_0D_1D_2-A_2^2D_2-A_1^2D_1-2A_1A_2E_1-D_0E_1^2} \\
& + \frac{(A_2D_2+A_1E_1)A_3+(D_2E_3+E_1H_3)D_0+(A_2H_3-A_{13_3})A_1}{D_0D_1D_2-A_2^2D_2-A_1^2D_1-2A_1A_2E_1-D_0E_1^2}\, y_1^{*} \\
& + \frac{(A_2D_2+A_1E_1)A_4+(D_2E_4+E_1H_4)D_0+(A_2H_4-A_1E_4)A_1}{D_0D_1D_2-A_2^2D_2-A_1^2D_1-2A_1A_2E_1-D_0E_1^2}\, y_2^{*}
\end{aligned}
$$

These equilibrium values for the policy variables can be substituted into equations (1)-(4) to solve for the cooperative equilibrium output and inflation in each country. In the cooperative equilibrium outcome, except in the symmetric case, neither country reaches its targets for output nor for inflation. Nor does the central bank attain its inflation target.

If the countries are symmetric then the cooperative equilibrium yields the optimal outcome for all players. Inflation in both countries (and thus the community inflation rate) is zero, and both countries meet their output targets. This results follows from the fact that in the symmetric case the countries choose the same fiscal policy level. Thus, output levels and inflation rates are the same in the two countries.

TABLE V
Coefficients for Cooperative Equilbrium

$$A_0 = 4\beta_1 B_{11} B_{14} + 4\beta_2 B_{21} B_{24} - 4\nu_1 C_{11} C_{14} - 4\nu_2 C_{21} C_{24} - (C_{11}+C_{21})(C_{14}+C_{24})$$

$$A_1 = -(4\beta_1 B_{11} B_{13} + 4\beta_2 B_{21} B_{23} + 4\nu_1 C_{11} C_{13} + 4\nu_2 C_{21} C_{23} + (C_{11}+C_{21})(C_{13}+C_{23}))$$

$$A_2 = -(4\beta_1 B_{11} B_{12} + 4\beta_2 B_{21} B_{22} + 4\nu_1 C_{11} C_{12} + 4\nu_2 C_{21} C_{22} + (C_{11}+C_{21})(C_{12}+C_{22}))$$

$$A_3 = 4\beta_1 B_{11}$$

$$A_4 = 4\beta_2 B_{21}$$

$$D_0 = 4\beta_1 B_{11}^2 + 4\beta_2 B_{21}^2 + 4\nu_1 C_{11}^2 + 4\nu_2 C_{21}^2 + (C_{11}+C_{21})^2$$

$$D_1 = 4\beta_1 B_{12}^2 + 4\beta_2 B_{22}^2 + 4\nu_1 C_{12}^2 + 4\nu_2 C_{22}^2 + (C_{12}+C_{22})^2$$

$$D_2 = 4\beta_1 B_{13}^2 + 4\beta_2 B_{23}^2 + 4\nu_1 C_{13}^2 + 4\nu_2 C_{23}^2 + (C_{13}+C_{23})^2$$

$$E_0 = 4\beta_1 B_{12} B_{14} + 4\beta_2 B_{22} B_{24} - 4\nu_1 C_{12} C_{14} - 4\nu_2 C_{22} C_{24} - (C_{12}+C_{22})(C_{14}+C_{24})$$

$$E_1 = -(4\beta_1 B_{12} B_{13} + 4\beta_2 B_{22} B_{23} + 4\nu_1 C_{12} C_{13} + 4\nu_2 C_{22} C_{23} + (C_{12}+C_{22})(C_{13}+C_{23}))$$

$$E_3 = 4\beta_1 B_{12}$$

$$E_4 = 4\beta_2 B_{22}$$

$$H_0 = 4\beta_1 B_{13} B_{14} + 4\beta_2 B_{23} B_{24} - 4\nu_1 C_{13} C_{14} - 4\nu_2 C_{23} C_{24} - (C_{13}+C_{23})(C_{14}+C_{24})$$

$$E_3 = 4\beta_1 B_{13}$$

$$E_4 = 4\beta_2 B_{23}$$

Given this, monetary policy can be applied to meeting the inflation target, which is the same for the monetary union and for the two countries, and fiscal policy can be applied to meeting the output target for the two countries. As discussed in chapter 1, asymmetries currently exist among the members of the European Community, and are likely to continue to exist, if not increase, with the removal of capital controls leading up to the establishment of a monetary union.

Section V: The Nash Game

Differentiating equation (8) with respect to the choice variable f_1, gives the first order condition for the fiscal authority of country 1. Solving this first order condition for f_1 gives the Nash reaction function for country 1, equation (22). Differentiating equation (9) with respect to the choice variable f_2, gives the first order condition for the fiscal authority of country 2. Solving this first order condition for f_2 gives the Nash reaction function for country 2, equation (23). Likewise, differentiating equation (10) with respect to the choice variable b_m, gives the first order condition for the monetary authority. Solving this first order condition for b_m gives the Nash reaction function for the central bank, equation (24).

$$(22) \qquad f_1 = \frac{\beta_1 B_{11}(y_1^* - B_{14} - B_{12}f_2 - B_{13}b_m) + \nu_1 C_{11}(C_{14} - C_{12}f_2 - C_{13}b_m)}{\beta_1 B_{11}^2 + \nu_1 C_{11}^2}$$

$$(23) \qquad f_2 = \frac{\beta_2 B_{22}(y_2^* - B_{24} - B_{21}f_1 - B_{23}b_m) + \nu_2 C_{22}(C_{24} - C_{21}f_1 - C_{23}b_m)}{\beta_2 B_{22}^2 + \nu_2 C_{22}^2}$$

$$(24) \qquad b_m = \frac{(C_{14} + C_{24}) - (C_{11} + C_{21})f_1 - (C_{12} + C_{22})f_2}{C_{13} + C_{23}}$$

Reaction Functions When Spillover Effects are Positive:

Because there are three policy variables the reaction functions do not form lines, but surfaces. If the spillover effects are positive the reaction functions form triangular surfaces, as shown in Figures 1, 2 and 3.

The reaction function for country 1, equation (22), gives the fiscal policy variable for country 1, as a function of fiscal policy actions taken by country 2 and the monetary policy adopted by the central bank. As shown in Figure 1, if

$$b_m = \frac{\beta_1 B_{11} y_1^* + \nu_1 C_{11} C_{14} - \beta_1 B_{11} B_{14}}{\beta_1 B_{11} B_{13} + \nu_1 C_{11} C_{13}} \quad \text{and} \quad f_2 = 0$$

then $f_1 = 0$, point Q_1. If $b_m = 0$ and $f_2 = 0$ then

$$f_1 = \frac{\beta_1 B_{11} y_1^* + \nu_1 C_{11} C_{14} - \beta_1 B_{11} B_{14}}{\beta_1 B_{11}^2 + \nu_1 C_{11}^2}$$

as shown by point R_1. If

$$f_2 = \frac{\beta_1 B_{11} y_1^* + \nu_1 C_{11} C_{14} - \beta_1 B_{11} B_{14}}{\beta_1 B_{11} B_{12} + \nu_1 C_{11} C_{12}} \quad \text{and} \quad b_m = 0$$

then $f_1 = 0$, as shown by point S_1.

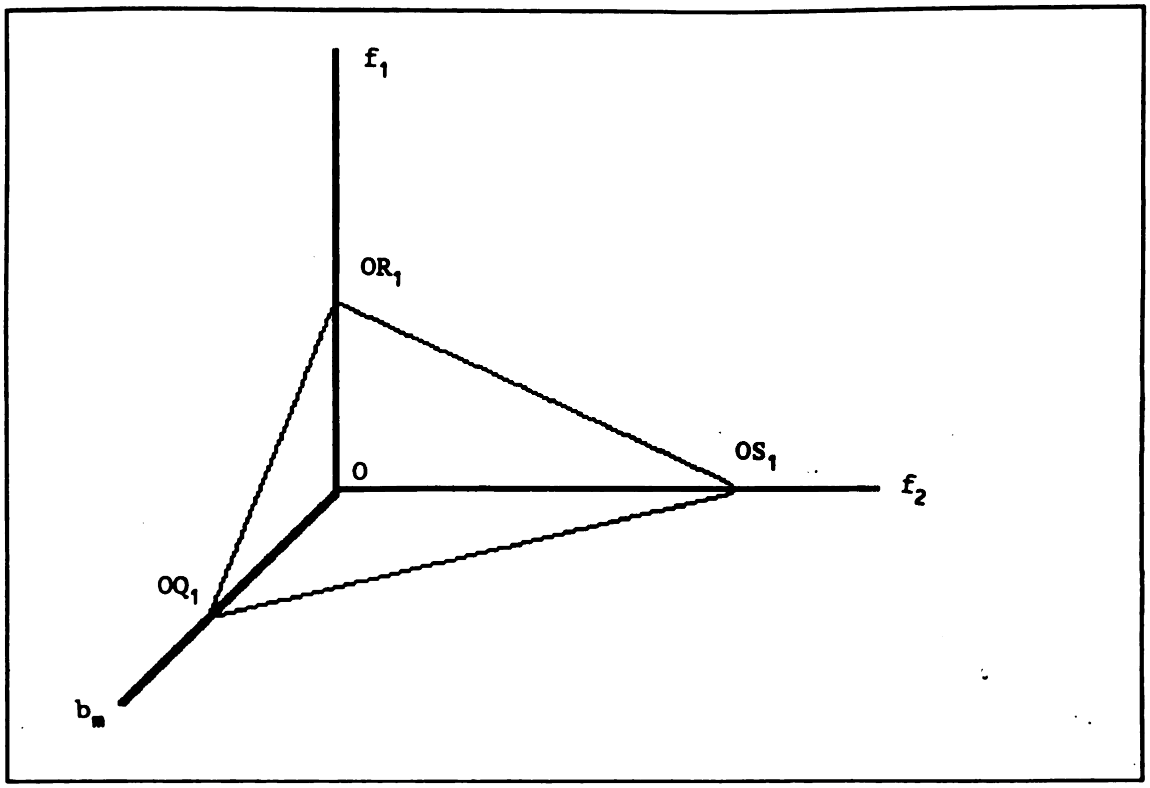

Figure 1
Country 1's Reaction Function

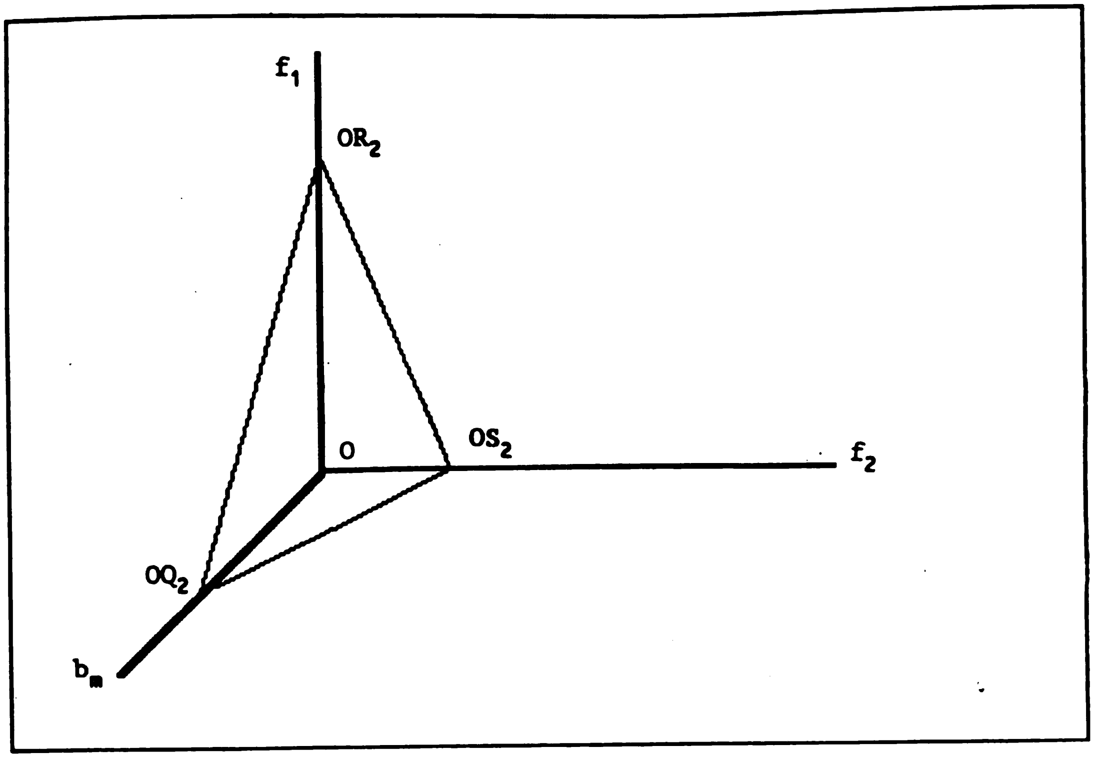

Figure 2
Country 2's Reaction Function

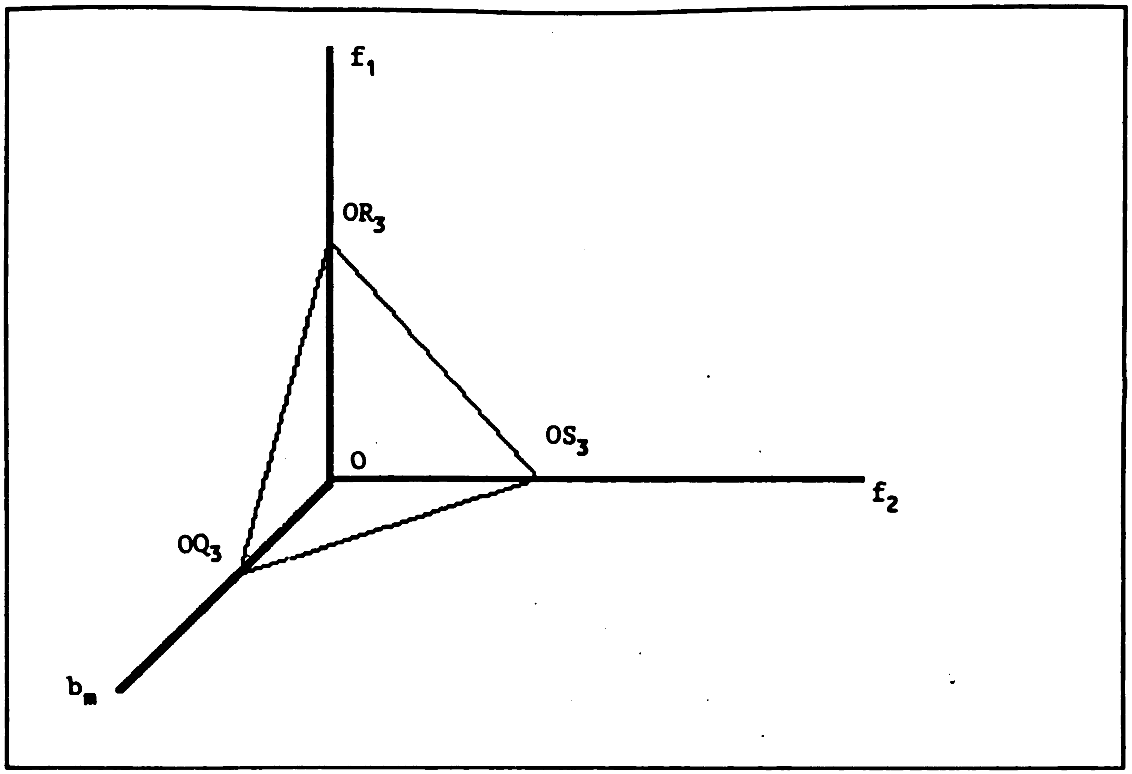

Figure 3
Central Bank's Reaction Function

Given that country 1's own fiscal policy has a greater effect on its output and inflation than do country 2's fiscal policy actions, ($B_{11}>B_{12}$ and $C_{11}>C_{12}$), it follows that $\overline{OS}_1>\overline{OR}_1$. Since the actions of the central bank have a greater effect on country 1's output and inflation than do country 2's fiscal policy actions, ($B_{13}>B_{12}$ and $C_{13}>C_{12}$), it follows that $\overline{OS}_1>\overline{OQ}_1$. The fiscal policy adopted by country 1 has a greater effect on its output than do the actions of the central bank, but monetary policy has a greater effect on country 1's inflation rate than do its fiscal policy actions, ($B_{11}>B_{13}$ but $C_{13}>C_{11}$). Thus, if country 1 places a higher priority on achieving its output target than on achieving its inflation target ($\beta_1>v_1$) it follows that $\overline{OQ}_1>\overline{OR}_1$. So, the relationship among the three intersection points is: $\overline{OS}_1>\overline{OQ}_1>\overline{OR}_1$.

The reaction function for country 2, equation (23), gives the fiscal policy actions for country 2 as a function of the actions taken by country 1, and the monetary policy adopted by the central bank. As shown in Figure 2, if

$$b_m = \frac{\beta_2 B_{22} y_2^* + v_2 C_{22} C_{24} - \beta_2 B_{22} B_{24}}{\beta_2 B_{22} B_{23} + v_2 C_{22} C_{23}} \quad \text{and} \quad f_1 = 0$$

then $f_2=0$, point Q_2. If $b_m = 0$ and

$$f_1 = \frac{\beta_2 B_{22} y_2^* + v_2 C_{22} C_{24} - \beta_2 B_{22} B_{24}}{\beta_2 B_{21} B_{22} + v_2 C_{21} C_{22}}$$

then $f_2=0$, point R_2. If $f_1 = 0$ and $b_m = 0$ then

$$f_2 = \frac{\beta_2 B_{22} y_2^* + v_2 C_{22} C_{24} - \beta_2 B_{22} B_{24}}{\beta_2 B_{22}^2 + v_2 C_{22}^2}$$

as shown by S_2.

Given that country 2's own fiscal policy has a greater effect on its output and inflation than do country 1's fiscal policy actions, ($B_{22}>B_{21}$ and $C_{22}>C_{21}$), it follows that $\bar{O}\bar{R}_2>\bar{O}\bar{S}_2$. Since the actions of the central bank have a greater effect on country 2's output and inflation than do country 1's fiscal policy actions, ($B_{23}>B_{21}$ and $C_{23}>C_{21}$), it follows that $\bar{O}\bar{R}_2>\bar{O}\bar{Q}_2$. The fiscal policy adopted by country 2 has a greater effect on its output than do the actions of the central bank, but monetary policy has a greater effect on country 2's inflation rate than do its fiscal policy actions, ($B_{22}>B_{23}$ but $C_{23}>C_{21}$). Therefore, if country 2 places a higher priority on achieving its output target than on achieving its inflation target, ($\beta_2>\nu_2$), it follows that $\bar{O}\bar{Q}_2>\bar{O}\bar{S}_2$. The overall relationship among the three intersection points, in Figure 2, is: $\bar{O}\bar{R}_2>\bar{O}\bar{Q}_2>\bar{O}\bar{S}_2$.

The reaction function for the central bank, equation (24), gives its monetary policy decisions as a function of the fiscal policy actions taken by country 1 and country 2. As shown in Figure 3, if

$$f_1=0 \quad \text{and} \quad f_2=0 \quad \text{then}$$

$$b_m=\frac{C_{14}+C_{24}}{C_{13}+C_{23}}$$

as shown by point Q_m. If

$$f_1=\frac{C_{14}+C_{24}}{C_{11}+C_{21}} \quad \text{and} \quad f_2=0$$

then $b_m=0$, point R_m. If

$$f_1=0 \quad \text{and} \quad f_2=\frac{C_{14}+C_{24}}{C_{12}+C_{22}}$$

then $b_m=0$, point S_m.

Given that monetary policy actions have a greater effect on inflation than do comparable fiscal policy actions of either country,

($C_{13}>C_{11}$ and $C_{23}>C_{22}$), it follows that $\overline{OR}_m>\overline{OQ}_m$ and $\overline{OS}_m>\overline{OQ}_m$.[10] The position of length of line segment $\overline{OR}_m$ relative to line segment $\overline{OS}_m$ depends upon the net debtor relationship of the two countries. If country 1 had a larger current account deficit than country 2[11] then $C_{11}>C_{22}$ and $C_{21}>C_{12}$ and thus $\overline{OS}_m>\overline{OR}_m$. If country 2 had the larger current account deficit than country 1[12] then $C_{22}>C_{11}$ and $C_{12}>C_{21}$, thus $\overline{OR}_m>\overline{OS}_m$.

If both countries had a current account deficit last period, and the value of their deficits were equal in real terms, then the countries are symmetric.[13] In this case, $\overline{OR}_m=\overline{OS}_m$. Looking at Figures 1 and 2, given symmetry, line segment $\overline{OQ}_1=\overline{OQ}_2$, $\overline{OR}_1=\overline{OS}_2$, and $\overline{OR}_2=\overline{OS}_1$.

Reaction Functions When Spillover Effects are Negative:

If the spillover effects are negative then the shape of the reaction function changes for the country experiencing the negative spillover effects.[14]

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[10] The effects of monetary versus fiscal policy on output are not relevant since the central bank places weight only on reaching its inflation target. Thus it reacts solely to changes in the inflation rate and not to changes in output.

[11] Country 2 may have been a net debtor or a net creditor last period.

[12] Country 1 could have been a net debtor or a net creditor last period.

[13] If the current account balances of the two countries were equally in deficit last period then, $Y_1=Y_2\tilde{p}_t$, which ensures that: $B_{11}=B_{22}$, $B_{12}=B_{21}$, $B_{13}=B_{23}$, $B_{14}=B_{24}$ $C_{11}=C_{22}$, $C_{12}=C_{21}$, $C_{13}=C_{23}$, and $C_{14}=C_{24}$.

[14] As noted above it is only possible for negative spillover effects to be generated by one country. The country generating the negative spillover effects still benefits from positive spillover effects, and thus has well behaved (i.e. triangular) reaction surface. The central bank can be affected by negative spillover effects caused by bond financed fiscal policy. The effect, however, is never strong enough to change the shape of its reaction surface. This can be verified by examining equation (24). If the countries are symmetric, there cannot be negative spillover effects.

Consider the case where an increase in government spending by country 2, ceteris paribus, causes a decrease in output in country 1, (B_{12}<0). The reaction functions for country 2 and the central bank remain as depicted in Figures 2 and 3. However, the reaction surface for country 1 changes from that shown in Figure 1. If f_2=0 no negative spillover effects can arise and thus the line segment Q_2R_2 remains the same as shown in Figure 1. If, however, $f_2 \neq 0$, the shape of the reaction function changes from that depicted in Figure 1, no longer remaining triangular. The new reaction surface is shown in Figure 4.

An increase in government spending by country 2 will cause country 1 to react by increasing its spending (holding b_m fixed). This occurs because country 1 acts to offset the negative spillover effect on its output. Mathematically this is shown by taking the partial derivative of f_1 (equation 10) with respect to f_2.

$$\frac{\delta f_1}{\delta f_2} = -\frac{\beta_1 B_{11} B_{12} + v_1 C_{11} C_{12}}{\beta_1 B_{11}^2 + v_1 C_{11}^2} > 0$$

Since B_{12}<0, if country 1 places more weight on meeting its output goal than its inflation goal ($\beta_1 > v_1$) then an increase in f_2 will lead to an increase in f_1. In the case where an increase in spending by country 2 decreases inflation in country 1 (C_{12}<0) then country 1 will react to this increase by increasing its own spending under any assumptions about the relative weights placed on its two targets.[15] This result occurs because country 2's action will help country 1 in meeting its inflation target but will move it away from its output target. Thus the inflation

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[15] As noted above C_{12}<0 can only occur if fiscal policy is bond financed, not if it is tax financed.

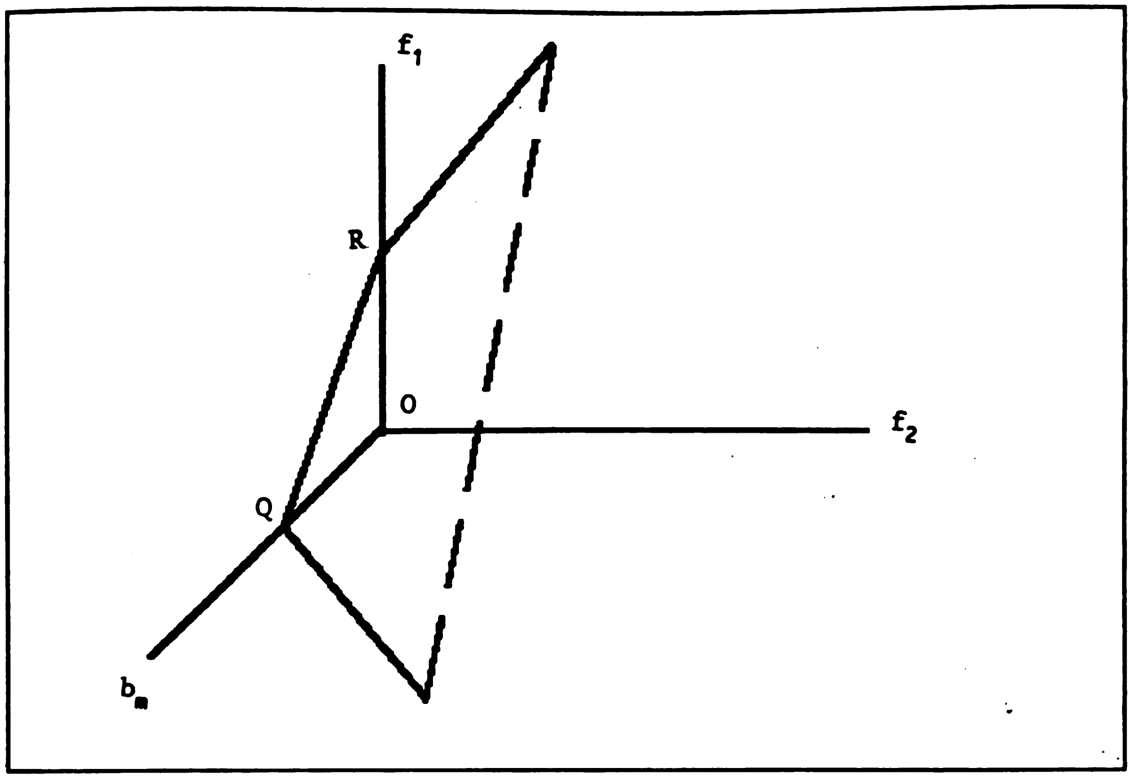

Figure 4
Country 1's Reaction Function
when $B_{12}<0$

experienced by country 1 due to an increase in its expenditures will be partially offset by the effect of country 2's policy, whereas the decrease in output in country 1, as a result of country 2's policy, will be more than offset by a comparable increase in spending by country 1.

It is also true that $[(\delta f_1)/(\delta f_2)]<1$. This result follows from the fact that an increase in spending by country 1 has a greater effect on its output (in absolute value terms) than the spillover effect resulting from an increase in spending by country 2.[16]

Given the negative spillover effect, f_1 will only remain fixed when government spending in country 2 increases if there is a compensating increase in bond purchases by the central bank. Mathematically, this can be shown by taking the partial derivative of b_m with respect to f_2 in country 1's reaction function, equation (10).

$$\frac{\delta b_m}{\delta f_2} = -\frac{\beta_1 B_{11} B_{12} + \nu_1 C_{11} C_{12}}{\beta_1 B_{11} B_{13} + \nu_1 C_{11} C_{13}} > 0$$

The analysis is the same as for the partial derivative of f_1 with respect to f_2. Likewise $[(\delta b_m)/(\delta f_2)]<1$.

Next, consider the case where an increase in government spending by country 1 has a negative spillover effect on output in country 2. The reaction surfaces for country 1 and the central bank are the same as those depicted in Figures 1 and 3. For country 2, if $f_1=0$ the negative spillover effect can not arise and thus the line segment Q_2S_2 remains the same as that shown in Figure 2. If $f_1\neq0$ the shape of the reaction function changes, according to the new reaction surface as shown in Figure 5.

[16] As shown in Chapter 1, $A_1>A_4$, and $(A_1+A_2)>(A_4+A_5)$. Thus it follows that $B_{11}>B_{12}$.

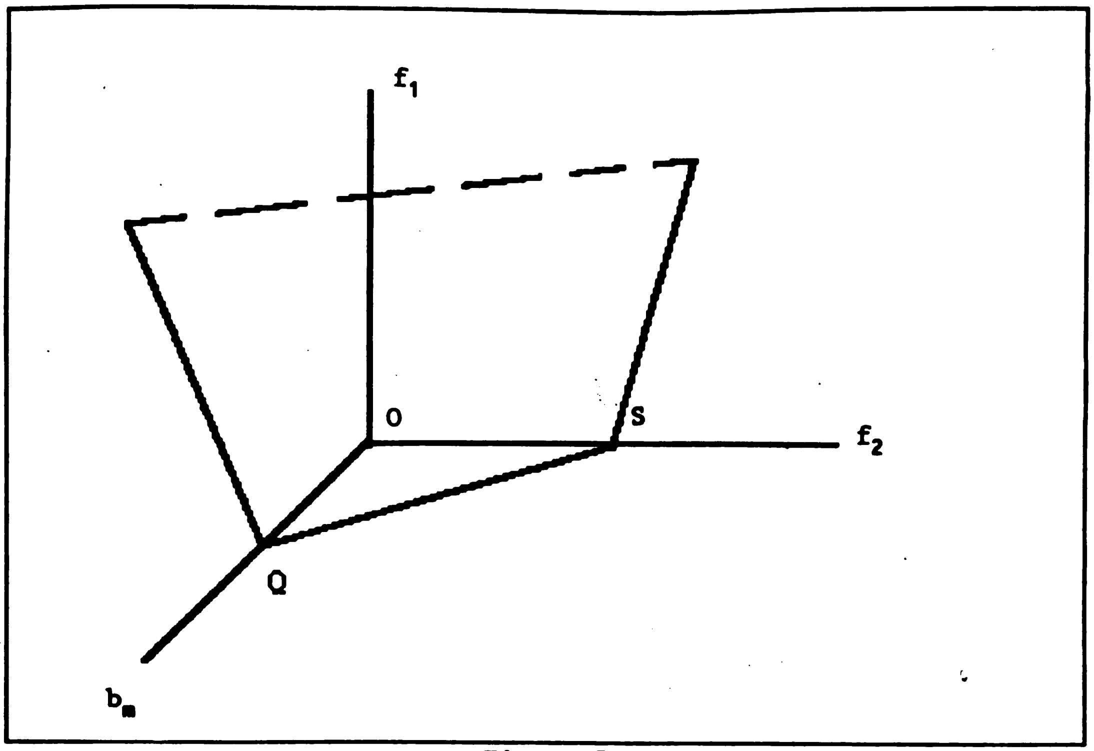

Figure 5
Country 2' Reaction Function
when $B_{21}<0$

The analysis underlying this reaction surface is the same as that given above for country 1. An increase in government spending by country 1 will cause country 2 to react by increasing its spending (holding b_m fixed).

$$\frac{\delta f_2}{\delta f_1} = -\frac{\beta_2 B_{21} B_{22} + \nu_2 C_{21} C_{22}}{\beta_2 B_{22}^2 + \nu_2 C_{22}^2} > 0 \quad \text{and} \quad \frac{\delta f_2}{\delta f_1} < 1$$

Since $B_{21}<0$, if country 2 places more weight on meeting its output goal than its inflation goal ($\beta_2>\nu_2$) then an increase in f_1 will lead to an increase in f_2. In the case where an increase in spending by country 1 decreases inflation in country 2 ($C_{21}<0$) then country 2 will react to this increase by increasing its own spending under any assumptions about the relative weights placed on its two targets.

Given the negative spillover effect, f_2 will only remain fixed when government spending in country 1 increases if there is a sufficient increase in bond purchases by the central bank.

$$\frac{\delta b_m}{\delta f_1} = -\frac{\beta_2 B_{21} B_{22} + \nu_2 C_{21} C_{22}}{\beta_2 B_{22} B_{23} + \nu_2 C_{22} C_{23}} > 0 \quad \text{and} \quad \frac{\delta b_m}{\delta f_1} < 1$$

The reaction surfaces shown in Figure 4 and Figure 5 are not become unbounded. Both are bounded by the governments' budget constraints. Government spending by both countries is constrained by total wealth within the monetary union.

Solving for a Nash Equilibrium:

Using the reaction functions given by equations (22)-(24) it is possible to solve for each policy variable as a function of only one of the other policy variables. Substituting equation (22) into equation (10) gives f_1 as a function of b_m and b_m as a function of f_1:

(25) $$f_1 = \frac{K_1 + M_1 b_m - H_1 y_1^* + J_1 y_2^*}{D_1}$$

(26) $$b_m = \frac{K_3 + F_3 f_1 + J_3 y_2^*}{D_2}$$

Next, these two equations can be used to solve for the Nash equilibrium level of f_1:

(27) $$f_1 = \frac{(D_2 K_1 + K_3 M_1) - (D_2 H_1) y_1^* + (D_2 J_1 + J_3 M_1) y_2^*}{D_1 D_2 - F_3 M_1}$$

Substituting equation (22) into (10) gives f_2 as a function of b_m:

(28) $$f_2 = \frac{K_2 + M_2 b_m - H_2 y_1^* + J_2 y_2^*}{D_1}$$

Finally, substituting equation (10) into equation (24) gives b_m as a function of f_2:

(29) $$b_m = \frac{K_4 + F_3 f_2 + J_4 y_2^*}{D_3}$$

Equations (28) and (29) can be used to solve for the Nash equilibrium level of f_2

(30) $$f_2 = \frac{(D_3 K_2 + K_4 M_2) - (D_3 H_2) y_1^* + (J_4 M_2 - D_3 J_2) y_2^*}{D_1 D_3 + F_3 M_2}$$

and the Nash equilibrium level of b_m

$$(31) \qquad b_m = \frac{(D_1K_4 + F_3K_2) - (F_3H_2)y_1^* + (F_3J_2 + D_1J_4)y_2^*}{D_1D_4 + F_3M_2}$$

where the parameters are defined in Table VI.

Using the Nash equilibrium values for f_1, f_2, and b_m, equations (28)-(30), it is possible to solve for the equilibrium values of inflation and output for each country. The inflation rate for the monetary union $(\pi_1+\pi_2)/2$ is zero in the Nash equilibrium. Thus, the central bank meets its inflation target. Neither country, however, is able to achieve its inflation target nor its output target ($y_1 \neq y_1^*$, $y_2 \neq y_2^*$).

To understand why the central bank is able to reach its inflation target in the Nash equilibrium, it is useful to examine its reaction function. This reaction function, given by equation (24), also defines the bliss space for the central bank[17], that is, the combinations of polices (f_1, f_2, and b_m) whereby the central bank achieves its inflation target. Formally this bliss space is found by setting

$$(32) \qquad \pi_1 + \pi_2 = \pi^* = 0$$

Substituting for π_1 and π_2 from equations (3) and (4) gives

$$(33) \qquad 0 = (C_{11}+C_{21})f_1 + (C_{12}+C_{21})f_2 + (C_{13}+C_{23})b_m - (C_{14}+C_{24})$$

which can be solved for b_m to obtain

$$(34) \qquad b_m = \frac{(C_{14} + C_{24}) - (C_{11} + C_{21})f_1 - (C_{12} + C_{21})f_2}{C_{13} + C_{23}}$$

[17] This corresponds to a bliss point which can be determined in a two person, two variable game.

TABLE VI
Coefficients For Nash Equilibrium

$$D_1 = \beta_1\beta_2B_{11}B_{22}(B_{12}B_{21} - B_{11}B_{22}) + \beta_2\nu_1B_{22}C_{11}(B_{21}C_{12} - B_{22}C_{11})$$
$$+ \beta_1\nu_2B_{11}C_{22}(B_{12}C_{21} - B_{11}C_{22}) + \nu_1\nu_2C_{11}C_{22}(C_{12}C_{21} - C_{11}C_{22})$$

$$D_2 = \beta_2B_{22}[B_{23}(C_{12}+C_{22}) - B_{22}(C_{13}+C_{23})] + \nu_2C_{22}[C_{12}C_{23} - C_{13}C_{22}]$$

$$D_3 = \beta_2B_{22}[B_{23}(C_{11}+C_{21}) - B_{21}(C_{13}+C_{23})] + \nu_2C_{22}[C_{11}C_{23} - C_{13}C_{21}]$$

$$F_3 = \beta_2B_{22}[B_{22}(C_{11}+C_{21}) - B_{21}(C_{12}+C_{22})] + \nu_2C_{22}[C_{11}C_{22} - C_{12}C_{21}]$$

$$H_1 = \beta_1B_{11}(\beta_2B_{22}^2 + \nu_2C_{22}^2)$$

$$H_2 = \beta_1B_{11}(\beta_2B_{21}B_{22} + \nu_2C_{21}C_{22})$$

$$J_1 = \beta_2B_{22}(\beta_1B_{11}B_{12} + \nu_1C_{11}C_{12})$$

$$J_2 = \beta_2B_{22}(\beta_1B_{11}^2 + \nu_1C_{11}^2)$$

$$J_3 = \beta_2B_{22}(C_{12} + C_{22})$$

$$J_4 = \beta_2B_{22}(C_{11} + C_{21})$$

$$K_1 = \beta_1\beta_2B_{11}B_{22}(B_{14}B_{22} - B_{12}B_{24}) - \beta_2\nu_1B_{22}C_{11}(B_{24}C_{12} + B_{22}C_{14})$$
$$+ \beta_1\nu_2B_{11}C_{22}(B_{14}C_{22} + B_{12}C_{24}) + \nu_1\nu_2C_{11}C_{22}(C_{12}C_{24} - C_{14}C_{22})$$

$$K_2 = \beta_1\beta_2B_{11}B_{22}(B_{11}B_{24} - B_{14}B_{21}) + \beta_2\nu_1B_{22}C_{11}(B_{24}C_{11} + B_{21}C_{14})$$
$$- \beta_1\nu_2B_{11}C_{22}(B_{14}C_{21} + B_{11}C_{24}) + \nu_1\nu_2C_{11}C_{22}(C_{14}C_{21} - C_{11}C_{24})$$

$$K_3 = -\beta_2B_{22}[B_{22}(C_{14}+C_{24}) + B_{24}(C_{12}+C_{22})] + \nu_2C_{22}[C_{12}C_{24} - C_{14}C_{22}]$$

$$K_4 = -\beta_2B_{22}[B_{21}(C_{14}+C_{24}) + B_{24}(C_{11}+C_{21})] + \nu_2C_{22}[C_{11}C_{24} - C_{14}C_{21}]$$

$$M_1 = \beta_1\beta_2B_{11}B_{22}(B_{13}B_{22} - B_{12}B_{23}) + \beta_2\nu_1B_{22}C_{11}(B_{22}C_{13} - B_{23}C_{12})$$
$$+ \beta_1\nu_2B_{11}C_{22}(B_{13}C_{22} - B_{12}C_{23}) + \nu_1\nu_2C_{11}C_{22}(C_{13}C_{22} - C_{12}C_{23})$$

$$M_2 = \beta_1\beta_2B_{11}B_{22}(B_{11}B_{23} - B_{13}B_{21}) + \beta_2\nu_1B_{22}C_{11}(B_{23}C_{11} - B_{21}C_{13})$$
$$+ \beta_1\nu_2B_{11}C_{22}(B_{11}C_{23} - B_{13}C_{21}) + \nu_1\nu_2C_{11}C_{22}(C_{11}C_{23} - C_{13}C_{21})$$

Since equation (34) is the same as equation (24), in a Nash equilibrium $\pi_1+\pi_2-\pi^*=0$. Given that the central bank always achieves its target in a Nash equilibrium, but can not do so under the cooperative solution, it will have no incentive to cooperate. Cooperation reduces the utility (increases the loss) for the central bank.

Since the central bank achieves the same level of utility (zero loss) at all points on its reaction function, this function can be used to convert the three-dimensional problem into a two-dimensional one. Substituting the central bank's reaction function into the reaction function for country 1 yields:

$$f_1 = \frac{[\beta_1 B_{11}(y_1^*-B_{14})+\nu_1 C_{11}C_{14}](C_{13}+C_{23}) - (\beta_1 B_{11}B_{13}+\nu_1 C_{11}C_{13})(C_{14}+C_{24})}{(\beta_1 B_{11}^2+\nu_1 C_{11}^2)(C_{13}+C_{23}) - (\beta_1 B_{11}B_{13}+\nu_1 C_{11}C_{13})(C_{11}+C_{21})}$$

$$+\frac{[(\beta_1 B_{11}B_{13}+\nu_1 C_{11}C_{13})(C_{12}+C_{22}) - (C_{13}+C_{23})(\beta_1 B_{11}B_{12}+\nu_1 C_{11}C_{12})]}{\beta_1 B_{11}^2+\nu_1 C_{11}^2)(C_{13}+C_{23}) - (\beta_1 B_{11}B_{13}+\nu_1 C_{11}C_{13})(C_{11}+C_{21})} f_2$$

Geometrically, this indicates the intersection of the two functions given in Figure 3 and Figure 1. Substituting the reaction function for the central bank into the reaction function for country 2 gives

$$f_2 = \frac{[\beta_2 B_{22}(y_2^*-B_{24})+\nu_2 C_{22}C_{24}](C_{13}+C_{23}) - (\beta_2 B_{22}B_{23}+\nu_2 C_{22}C_{23})(C_{14}+C_{24})}{(\beta_2 B_{22}^2+\nu_2 C_{22}^2)(C_{13}+C_{23}) - (\beta_2 B_{22}B_{23}+\nu_2 C_{21}C_{23})(C_{12}+C_{22})}$$

$$+\frac{[(\beta_2 B_{22}B_{23}+\nu_2 C_{22}C_{23})(C_{11}+C_{21}) - (C_{13}+C_{23})(\beta_2 B_{21}B_{22}+\nu_2 C_{21}C_{22})]}{\beta_2 B_{22}^2+\nu_2 C_{22}^2)(C_{13}+C_{23}) - (\beta_2 B_{22}B_{23}+\nu_2 C_{22}C_{23})(C_{12}+C_{22})} f_1$$

which indicates the intersection of the reaction functions shown in Figure 3 and Figure 2. The resulting functions are shown in Figure 6. Both functions are upward sloping regardless of whether the spillover effects of fiscal policy are negative or positive. In the case of positive spillover effects a country will react to an increase in foreign government spending by decreasing its own spending. An increase in foreign government spending will also cause the central bank to react, reducing its bonds purchases (adopting restrictive monetary policy) to fully counteract the average inflationary effects of the fiscal action. Because the central bank's policy has a negative spillover effect which more than offsets the positive fiscal policy spillover effect, the country in question will react by increasing its own level of fiscal expenditures. In the case of negative spillover effects, an increase in fiscal expenditures by one country will cause an increase in expenditures by the other country to compensate for the negative spillover effect on output.

Since at every point on the functions shown in Figure 6, the central bank is at a bliss point, the intersection of these two functions, which gives the Nash equilibrium, must also be a bliss point for the central bank. However, as noted above, the Nash equilibrium is not optimal from the perspective of either country. Only if the countries are symmetric will there be price stability as an average in the monetary union and across both countries. Under symmetry the countries also meet their output targets.

If the countries are not symmetric neither country achieves price stability. In this case, since the overall price level in the monetary union is stable, it follows that one country must experience inflation while the other country must experience deflation. If the inflation rates were the same in both countries last period and the fiscal policies are the same this period, $f_1=f_2$, then the net debtor country will be the inflationary country and the net creditor country will be the deflationary country. If the inflation rates were the same in both countries last period and this period the net debtor country adopts a more expansionary

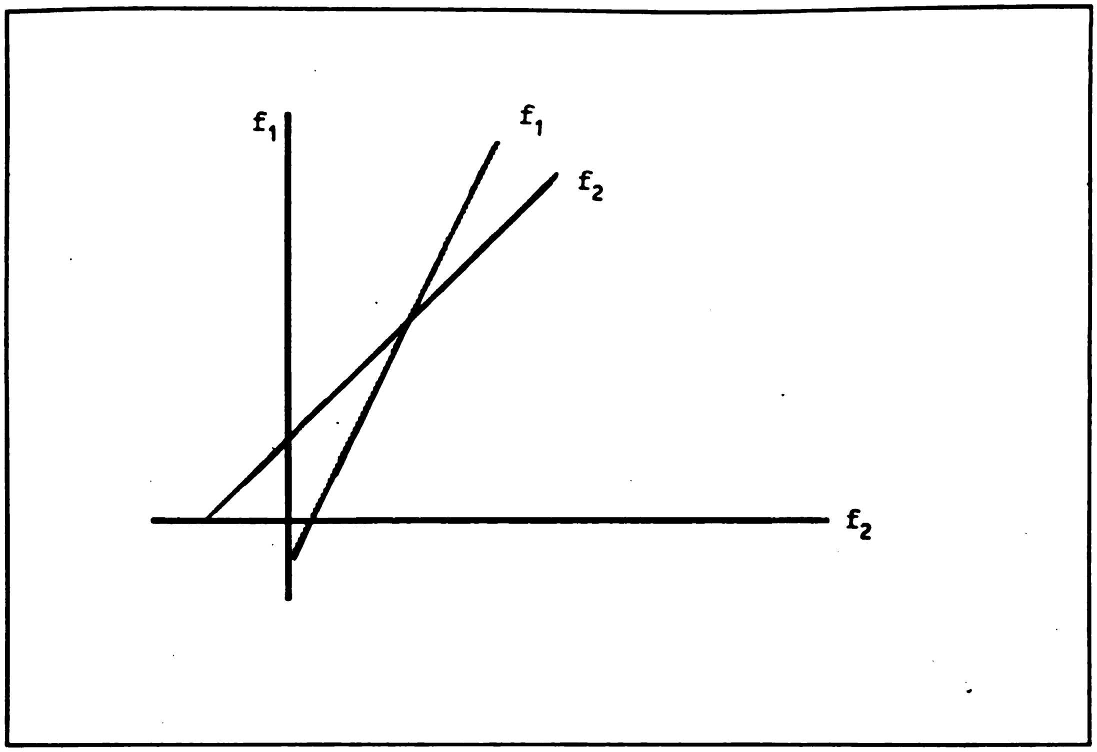

Figure 6
Reaction Functions After Substituting Out b_m

fiscal policy than the net creditor country, then the net debtor country will be the inflationary country and the net creditor country will be the deflationary country.[18]

Section VI: Stackelberg Game With the Central Bank As Leader

In the Stackelberg game with the central bank as the Stackelberg leader, the policy objectives of the three players are unchanged. Each player aims to minimize a loss function given by equations (8)-(10). This game differs from the Nash equilibrium because the central bank, moving first, is able to anticipate the fiscal policy decisions which the countries will take in reaction to its monetary policy decision. The central bank knows the reaction functions of the two governments and incorporates them into its objective function. The aim of the central bank is to minimize its loss function, subject to the impact of the fiscal policy reactions of the governments on inflation within the monetary union.

$$\min_{b_m} L_m = \left(\frac{\pi_1+\pi_2}{2}\right)^2 \tag{25}$$

subject to:

[18] As shown in chapter 1, expansionary policy tends to have a greater effect on inflation in a net debtor country than in a net creditor country.

$$\pi_1 = \left[\frac{\beta_1 B_{11}(y_1^* - B_{14} - B_{12}f_2 - B_{13}b_m) + \nu_1 C_{11}(C_{14} - C_{12}f_2 - C_{13}b_m)}{\beta_1 B_{11}^2 + \nu_1 C_{11}^2}\right] C_{11}$$

$$+ \left[\frac{\beta_2 B_{22}(y_2^* - B_{24} - B_{21}f_1 - B_{23}b_m) + \nu_2 C_{22}(C_{24} - C_{21}f_1 - C_{23}b_m)}{\beta_2 B_{22}^2 + \nu_2 C_{22}^2}\right] C_{12}$$

$$+ C_{13}b_m - C_{14}$$

$$\pi_2 = \left[\frac{\beta_1 B_{11}(y_1^* - B_{14} - B_{12}f_2 - B_{13}b_m) + \nu_1 C_{11}(C_{14} - C_{12}f_2 - C_{13}b_m)}{\beta_1 B_{11}^2 + \nu_1 C_{11}^2}\right] C_{21}$$

$$+ \left[\frac{\beta_2 B_{22}(y_2^* - B_{24} - B_{21}f_1 - B_{23}b_m) + \nu_2 C_{22}(C_{24} - C_{21}f_1 - C_{23}b_m)}{\beta_2 B_{22}^2 + \nu_2 C_{22}^2}\right] C_{22}$$

$$+ C_{23}b_m - C_{24}$$

As the Stackelberg leader the central bank is able to choose the area on its reaction surface which minimizes its loss function. This can be shown mathematically by minimizing the loss function, given in equation (14) with respect to b_m, which yields the first order condition:

$$(\pi_1 + \pi_2)\left(\frac{\delta\pi_1}{\delta b_m} + \frac{\delta\pi_2}{\delta b_m}\right) = 0$$

where π_1 and π_2 are defined above. The term $[(\delta\pi_1/\delta b_m)+(\delta\pi_1/\delta b_m)]$ is not a function of the policy variables. Thus the first order condition can be rewritten as:

(26) $$\pi_1 + \pi_2 = 0$$

Since π_1 and π_2 incorporate the reaction functions of the two governments, equation (26) indicates that the central bank will be able achieve its inflation target taking into account the reactions of the governments.

Solving the first order condition for b_m gives:

(27) $$b_m = \frac{A_1y_1^* + A_2y_2^* - A_3f_1 - A_4f_2 - A_5}{D_4}$$

where the parameters are defined in Table VII.

Equation (27) and the reaction functions for the two governments, equations (22) and (23) give a system of three equations in terms of the three policy variables. Solving this system of equation gives the Stackelberg equilibrium with the central bank in the role of Stackelberg leader. The solution is:

$$f_1 = \frac{K_1(D_1D_4+A_4M_2) - M_1(A_5D_1+A_4K_2)}{D_1(D_1D_4+A_3M_1+A_4M_2)}$$
$$+ \frac{M_1(A_1D_1-A_4H_2) - H_1(D_1D_4+A_4M_2)}{D_1(D_1D_4+A_3M_1+A_4M_2)} y_1^*$$
$$+ \frac{M_1(A_2D_1+A_4J_2) + J_1(D_1D_4+A_4M_2)}{D_1(D_1D_4+A_3M_1+A_4m_2)} y_2^*$$

$$f_2 = \frac{K_1(D_1D_4+A_3M_1) - M_2(A_5D_1+A_3K_1)}{D_1(D_1D_4+A_3M_1+A_4M_2)}$$
$$+ \frac{M_2(A_1D_1+A_3H_1) + H_2(D_1D_4+A_3M_1)}{D_1(D_1D_4+A_3M_1+A_4M_2)} y_1^*$$
$$+ \frac{M_1(A_2D_1-A_3J_1) - J_2(D_1D_4+A_3M_1)}{D_1(D_1D_4+A_3M_1+A_4m_2)} y_2^*$$

$$b_m = -\frac{(A_5D_1+A_3K_1+A_4K_2)}{(D_1D_4+A_3M_1+A_4M_2)}$$
$$+ \frac{(A_1D_1+A_3H_1-A_2H_2)}{(D_1D_4+A_3M_1+A_4M_2)} y_1^*$$
$$+ \frac{(A_2D_1-A_3J_1+A_4J_2)}{(D_1D_4+A_3M_1+A_4m_2)} y_2^*$$

As expected the central bank is able to reach its target so that the average inflation rate in the monetary union is zero, but in the absence of symmetry neither country will have price stability. Also, neither

TABLE VII
Coefficients for Stackelberg Equilbrium When Central Bank Is Leader

$$A_1 = \beta_1 B_{11}(C_{11}+C_{21})(\beta_2 B_{22}^2+\nu_2 C_{22}^2)$$

$$A_2 = \beta_2 B_{22}(C_{12}+C_{22})(\beta_1 B_{11}^2+\nu_1 C_{11}^2)$$

$$\begin{aligned} A_3 = {} & (C_{12}+C_{22})(\beta_1\beta_2 B_{11}^2 B_{21}B_{22} + \beta_2\nu_1 B_{21}B_{22}C_{11}^2 \\ & + \beta_1\nu_2 B_{11}^2 C_{21}C_{22} + \nu_1\nu_2 C_{11}^2 C_{21}C_{22}) \end{aligned}$$

$$\begin{aligned} A_4 = {} & (C_{11}+C_{21})(\beta_1\beta_2 B_{11}B_{12}B_{22}^2 + \beta_2\nu_1 B_{22}^2 C_{11}C_{12} \\ & + \beta_1\nu_2 B_{11}B_{12}C_{22}^2 + \nu_1\nu_2 C_{11}C_{12}C_{22}^2) \end{aligned}$$

$$\begin{aligned} A_5 = {} & \beta_1\beta_2 B_{11}B_{22}[(B_{11}B_{22}(C_{14}+C_{24}) + B_{11}B_{24}(C_{12}+C_{22}) + B_{14}B_{22}(C_{11}+C_{21})] \\ & + \beta_2\nu_1 B_{22}C_{11}[(B_{24}C_{11}(C_{12}+C_{22}) + B_{22}(C_{11}C_{24}-C_{14}C_{21})] \\ & + \beta_1\nu_2 B_{11}C_{22}[B_{14}C_{22}(C_{11}+C_{21}) + B_{11}(C_{14}C_{22}-C_{12}C_{24})] \\ & + \nu_1\nu_2 C_{11}C_{22}(C_{14}C_{21}C_{22} + C_{11}C_{12}C_{24}) \end{aligned}$$

$$\begin{aligned} D_4 = {} & \beta_1\beta_2 B_{11}B_{22}[(B_{13}B_{22}(C_{11}+C_{21}) + B_{11}B_{23}(C_{12}+C_{22}) + B_{11}B_{22}(C_{13}+C_{23})] \\ & + \beta_2\nu_1 B_{22}C_{11}[(B_{23}C_{11}(C_{12}+C_{22}) + B_{22}(C_{13}C_{21}-C_{11}C_{23})] \\ & + \beta_1\nu_2 B_{11}C_{22}[B_{11}(C_{12}C_{23}-C_{13}C_{22}) + B_{13}C_{22}(C_{11}+C_{21})] \\ & + \nu_1\nu_2 C_{11}C_{22}(C_{13}C_{21} + C_{12}C_{23}) \end{aligned}$$

country will meet its output target. Thus, as in the Nash equilibrium the central bank meets it target but the two countries, in general, do not meet either their output or inflation targets. Furthermore, the central bank once again has no incentive to cooperate.

Section VII: Stackelberg Game With the Governments As Leaders

In the Stackelberg game where the governments act as leader, the policy objectives of the three players, once again, remain unchanged. In this game the two governments move simultaneously, but they move before the central bank, and thus can anticipate the central bank's actions. Each government incorporates the reaction function of the central bank into its objective function, and minimizes this revised function with respect to its policy variable (f_i).

$$y_1 = B_{14} + B_{11}f_1 + B_{12}f_2 + \frac{B_{13}[(C_{14}+C_{24}) - (C_{11}+C_{21})f_1 - (C_{12}+C_{22})f_2]}{(C_{13}+C_{23})}$$

$$y_2 = B_{24} + B_{21}f_1 + B_{22}f_2 + \frac{B_{23}[(C_{14}+C_{24}) - (C_{11}+C_{21})f_1 - (C_{12}+C_{22})f_2]}{(C_{13}+C_{23})}$$

$$\pi_1 = C_{11}f_1 + C_{12}f_2 - C_{14} + \frac{C_{13}[(C_{14}+C_{24}) - (C_{11}+C_{21})f_1 - (C_{12}+C_{22})f_2]}{(C_{13}+C_{23})}$$

$$\pi_2 = C_{22}f_1 + C_{22}f_2 - C_{24} + \frac{C_{23}[(C_{14}+C_{24}) - (C_{11}+C_{21})f_1 - (C_{12}+C_{22})f_2]}{(C_{13}+C_{23})}$$

Solving the first-order conditions, which result from these minimizations, for f_1 and f_2 yields:

(28)
$$f_1 = \frac{H_0 + H_1 y_1^* + H_2 f_2}{D_5}$$

(29)
$$f_2 = \frac{H_3 + H_4 y_2^* + H_5 f_1}{D_6}$$

Equations (28) and (29) in conjunction with the central bank's reaction function, equation (24) can be used to solve for the Stackelberg equilibrium values of the policy variables.

$$f_1 = \frac{H_2H_3 + H_0D_6 + H_1D_6y_1^* + H_2H_4y_2^*}{D_5D_6 - H_2H_5}$$

$$f_2 = \frac{H_3D_5 + H_0H_5 + H_1H_5y_1^* + H_4D_5y_2^*}{D_5D_6 - H_2H_5}$$

$$\begin{aligned} b_m = \frac{(C_{14}+C_{24})}{(C_{13}+C_{23})} &- \frac{(C_{11}+C_{21})(H_2H_3+H_5D_6) - (C_{12}+C_{22})(H_3D_5+H_0H_4)}{(C_{13}+C_{23})(D_5D_6-H_2H_5)} \\ &- \frac{H_1[(C_{11}+C_{21})D_6 + (C_{12}+C_{22})H_5]}{(C_{13}+C_{23})(D_5D_6-H_2H_5)}\, y_1^* \\ &- \frac{H_4[(C_{11}+C_{21})H_2 + (C_{12}+C_{22})D_5]}{(C_{13}+C_{23})(D_5D_6-H_2H_5)}\, y_2^* \end{aligned}$$

where the parameters are defined in Table VIII.

For the Central Bank, the result of this game does not differ from the previous two games. Average inflation within the monetary union is zero, so the central bank achieves its target. The two governments remain unable to meet their output targets, and as in the other two games, do not achieve price stability unless the countries are symmetric. Thus, as in the Nash equilibrium and the Stackelberg equilibrium with the central bank as leader, the central bank has no incentive to cooperate.

TABLE VIII
Coefficients for Stackelberg Equilibrium When Governments Are Leaders

$$D_5 = 2\beta_1 \ [2(B_{13}C_{11}-B_{11}C_{13})(B_{11}C_{23}-B_{13}C_{21}) - (B_{13}C_{11}-B_{11}C_{23})^2$$
$$- (B_{13}C_{21}-B_{11}C_{23})^2 \ ] - 2\nu_1(C_{13}C_{21}-C_{11}C_{23})^2$$

$$D_6 = 2\beta_2 \ [(B_{23}C_{12}-B_{22}C_{13})^2 - (B_{23}C_{22}-B_{22}C_{23})^2$$
$$- (B_{23}C_{22}-B_{22}C_{23})(B_{23}C_{12}-B_{22}C_{13}) \ ] - 2\nu_2(C_{13}C_{22}-C_{12}C_{23})^2$$

$$H_0 = 2\beta_1 B_{11}(C_{13}+C_{23})[B_{13}(C_{14}+C_{24})+B_{14}(C_{13}+C_{23})]$$
$$- 2\beta_1 B_{13}(C_{11}+C_{21})[B_{14}(C_{13}+C_{23})+B_{13}(C_{11}+C_{21})]$$
$$+ 2\nu_1(C_{13}C_{21}-C_{11}C_{23})(C_{14}C_{23}-C_{13}C_{24})$$

$$H_1 = 2\beta_1(C_{13}+C_{23})[B_{13}(C_{11}+C_{21})-B_{11}(C_{13}+C_{23})]$$

$$H_2 = 2\beta_1[B_{11}B_{12}(C_{13}+C_{23})^2 + B_{13}^2(C_{11}+C_{21})(C_{12}+C_{22})]$$
$$- 2\beta_1 B_{13}(C_{13}+C_{23})[B_{11}(C_{12}+C_{22}) + B_{12}(C_{11}+C_{21})]$$
$$+ 2\nu_1(C_{11}C_{23}-C_{13}C_{21})(C_{12}C_{23}-C_{13}C_{22})$$

$$H_3 = 2\beta_2 B_{22}(C_{13}+C_{23})[B_{23}(C_{14}+C_{24})+B_{24}(C_{13}+C_{23})]$$
$$- 2\beta_2 B_{23}(C_{12}+C_{22})[B_{24}(C_{13}+C_{23})+B_{23}(C_{12}+C_{22})]$$
$$+ 2\nu_2(C_{23}C_{12}-C_{22}C_{13})(C_{24}C_{13}-C_{23}C_{14})$$

$$H_4 = 2\beta_2(C_{13}+C_{23})[B_{23}(C_{12}+C_{22})-B_{22}(C_{13}+C_{23})]$$

$$H_5 = 2\beta_2[B_{22}B_{21}(C_{13}+C_{23})^2 + B_{23}^2(C_{12}+C_{22})(C_{11}+C_{21})]$$
$$- 2\beta_2 B_{23}(C_{13}+C_{23})[B_{22}(C_{11}+C_{21}) + B_{21}(C_{12}+C_{22})]$$
$$+ 2\nu_2(C_{22}C_{13}-C_{23}C_{12})(C_{21}C_{13}-C_{23}C_{11})$$

VIII: Conclusion

In the framework of a monetary union, towards which the European Community is moving, fiscal and monetary policies are decentralized, with the policies being controlled by independent institutions. There are two key features of this structure. The central bank makes monetary policy decisions for the entire union and thus is concerned with the average inflation rate in the union, not the distribution of inflation across countries. Fiscal policy decisions are made by the member countries. These countries are concerned with the impact of these decisions on their own country, and not the effect such decisions have on other countries in the monetary union. The countries are also concerned with the effect of monetary policy decisions on their own country and not the average effect on the union.

Given this decentralization of policy decisions, and the different concerns of the decision makers, it is useful to analyze the policy decisions in the form of a game between the central bank and the two governments. The two governments attempt to use fiscal policy to meet their output and inflation goals. The countries may differ in both the goals they set and the weights attached to achieving one goal over the other. The central bank uses monetary policy to target the inflation rate for the monetary union. The central bank does not target output.

This chapter analyzes three possible strategic interactions among the players: A cooperative game, a Nash game, and a Stackelberg game. In the Nash and Stackelberg game the central bank is able to meet its inflation target: average inflation is zero in the monetary union. This result follows from the central bank's use of monetary policy to concentrate on only one goal. The two countries in general are unable to meet either their output goals or their inflation goals. Only if the countries are symmetric will they meet their inflation targets. In the symmetric case they will also meet their output targets. Symmetry, however, is does not characterize the European Community, nor is it likely to be achieved through monetary union.

The standard result found in international policy games, that policy coordination is welfare enhancing does not hold in the game developed in this chapter. In the cooperative game, monetary policy can not be used solely for meeting the inflation goal of the central bank. The preferences of each country are weighted equally with those of the central bank. This introduces complications for the use of monetary policy because the countries are willing to accept some inflation in order to move closer to their output goals. Thus, in the cooperative solution the central bank is unable to meet its inflation target. Since the central bank does not meet its target in the cooperative game, but is able to do so in both the Nash and Stackelberg games, it will have no incentive to cooperate with the fiscal authorities in formulating policy.

The results of this chapter indicate a potential source of friction between the countries in a monetary union and the central bank. One of the reasons why the European Community countries have been receptive to the idea of monetary union is that it is expected to provide them with low inflation. This chapter indicates that even in a monetary union in which the only goal of the central bank is price stability, the member countries may not benefit from the achievement of this goal. Although the central bank is able to achieve a zero average inflation rate for the Community, the individual countries do not achieve price stability.

These results indicate that the countries may not benefit from a central bank which is concerned only with average price stability but not with the distribution of inflation across countries. The alternative would be for the central bank to set targets both for average inflation and inflation in each country. In this case its loss function becomes:

$$L_m = \omega_1\pi_1^2 + \omega_2\pi_2^2 + \omega_3(\pi_1+\pi_2)^2$$

Since, as explained in chapter 1, the central bank is unable to aim its policies at individual countries, it can not achieve both price stability in one country and an average price stability. Thus, the central bank will have to decide how to weight the various objectives. Placing country specific targets into its loss function may make the central bank more susceptible to pressures from individual countries, weakening the independence of the central bank and creating a potential source of friction among the countries, something the advocates of monetary union have hoped to avoid. Subsequent research will attempt to more fully examine these issues by determining the effect of the objective function given above on the ability of the central bank to achieve its targets.

APPENDICES

APPENDIX A

SOLVING FOR AGGREGATE SUPPLY AND DEMAND

This appendix uses the equations in Tables I and II to derive the solutions for aggregate supply and aggregate demand for country 1, as given by equations (37) and (40), respectively, in the text. The derivations of the aggregate supply and demand equations for country 2, are discussed where they differ from those for country 1.

Solving for Aggregate Supply:

Lagging equation (2) yields:

$$(A1) \quad p^{c}_{1,t-1} = \gamma p_{1,t-1} + (1-\gamma) p_{2,t-1}$$

Substituting equations (2) and (A1) into equation (4) yields:

$$(A2) \quad \pi_{1,t-1} = \frac{\gamma (p_{1,t}-p_{1,t-1})}{\gamma p_{1,t-1}+(1-\gamma) p_{2,t-1}} + \frac{(1-\gamma)(p_{2,t}-p_{2,t-1})}{\gamma p_{1,t-1}+(1-\gamma) p_{2,t-1}}$$

which can be rewritten as:

$$(A3) \quad \pi_{1,t-1} = \left(\frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma) p_{2,t-1}}\right)\left(\frac{p_{1,t}-p_{1,t-1}}{p_{1,t-1}}\right) + \left(\frac{(1-\gamma) p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma) p_{2,t-1}}\right)\left(\frac{p_{2,t}-p_{2,t-1}}{p_{2,t-1}}\right)$$

Rewriting equation (14) as:[1]

$$(A4) \quad \frac{p_{2,t} - p_{2,t-1}}{p_{2,t-1}} = \alpha \left(\frac{y_{2,t}\, \tilde{p}_t - \bar{y}}{\bar{y}}\right) + \frac{E_{t-1}p_{2,t}-p_{2,t-1}}{p_{2,t-1}}$$

[1] This is done so that in the aggregate supply equation for country 1, all real variables will be measured in the same units.

Substituting equations (1) and (A4) into equation (A3) yields:

$$\text{(A5)} \quad \pi_{1,t-1} = \left(\frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1} + (1-\gamma)p_{2,t-1}}\right)\left[\alpha\,\frac{y_{1,t}-\bar{y}}{\bar{y}} + \frac{E_{t-1}p_{1,t} - p_{1,t-1}}{p_{1,t-1}}\right]$$
$$+ \left(\frac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1} + (1-\gamma)p_{2,t-1}}\right)\left[\alpha\,\frac{y_{2,t}\,\tilde{p}_t-\bar{y}}{\bar{y}} + \frac{E_{t-1}p_{2,t} - p_{2,t-1}}{p_{2,t-1}}\right]$$

Next, solving equations (2) and (15) for $p_{1,t}$ and $p_{2,t}$, respectively:

$$p_{1,t} = \frac{p^c_{1,t}}{\gamma} - \frac{(1-\gamma)p_{2,t}}{\gamma}\,, \qquad p_{2,t} = \frac{p^c_{2,t}}{\gamma} - \frac{(1-\gamma)p_{1,t}}{\gamma}$$

which after some algebra yields:

$$\text{(A6)}\ p_{1,t} = \frac{\gamma}{2\gamma-1}p^c_{1,t} - \frac{1-\gamma}{2\gamma-1}p^c_{2,t}\,, \qquad \text{(A7)}\ p_{2,t} = \frac{\gamma}{2\gamma-1}p^c_{2,t} - \frac{1-\gamma}{2\gamma-1}p^c_{1,t}$$

Taking expectations at t-1 of equations (A6) and (A7) gives:

$$\text{(A8)} \quad E_{t-1}p_{1,t} = \frac{\gamma}{2\gamma-1}E_{t-1}p^c_{1,t} - \frac{1-\gamma}{2\gamma-1}E_{t-1}p^c_{2,t}$$

$$\text{(A9)} \quad E_{t-1}p_{2,t} = \frac{\gamma}{2\gamma-1}E_{t-1}p^c_{2,t} - \frac{1-\gamma}{2\gamma-1}E_{t-1}p^c_{1,t}$$

Lagging equations (A6) and (A7), and using equations (A8) and (A9), it is possible to rewrite:

$$\frac{E_{t-1}p_{1,t} - p_{1,t-1}}{p_{1,t-1}} \qquad \text{and,} \qquad \frac{E_{t-1}p_{2,t} - p_{2,t-1}}{p_{2,t-1}}$$

as:

$$\pi^{\bullet}_{1,t-1}\left(\frac{\gamma p^c_{1,t-1}}{\gamma p^c_{1,t-1} - (1-\gamma)p^c_{2,t-1}}\right) - \pi^{\bullet}_{2,t-1}\left(\frac{(1-\gamma)p^c_{2,t-1}}{\gamma p^c_{1,t-1} - (1-\gamma)p^c_{2,t-1}}\right)$$

and:

$$\pi^{\bullet}_{2,t-1}\left(\frac{\gamma p^c_{2,t-1}}{\gamma p^c_{2,t-1} - (1-\gamma)p^c_{1,t-1}}\right) - \pi^{\bullet}_{1,t-1}\left(\frac{(1-\gamma)p^c_{1,t-1}}{\gamma p^c_{2,t-1} - (1-\gamma)p^c_{1,t-1}}\right)$$

respectively.

Making these substitutions into equation (A5) and solving for $y_{1,t}$, yields:

$$(A10)\quad y_{1,t} = \frac{\gamma_1 + \gamma_2}{\gamma_1}\,\bar{y} - \frac{\bar{y}}{\alpha\gamma_1}\left(\gamma_1\tilde{\gamma}_1^c - \gamma_2\tilde{\gamma}_4^c\right)\pi_{1,t-1}^{\bullet} + \frac{\bar{y}}{\alpha\gamma_1}\left(\gamma_1\tilde{\gamma}_2^c - \gamma_2\tilde{\gamma}_3^c\right)\pi_{2,t-1}^{\bullet}$$
$$+ \frac{\bar{y}}{\alpha\gamma_1}\,\pi_{1,t-1} - \frac{\gamma_2}{\gamma_1}\,y_{2,t}\tilde{p}_t$$

where:

$$\gamma_1 = \frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1} + (1-\gamma)p_{2,t-1}}, \qquad \gamma_2 = \frac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1} + (1-\gamma)p_{2,t-1}}$$

$$\tilde{\gamma}_1^c = \frac{\gamma p_{1,t-1}^c}{\gamma p_{1,t-1}^c - (1-\gamma)p_{2,t-1}^c}, \qquad \tilde{\gamma}_2^c = \frac{(1-\gamma)p_{2,t-1}^c}{\gamma p_{1,t-1}^c - (1-\gamma)p_{2,t-1}^c}$$

$$\tilde{\gamma}_3^c = \frac{\gamma p_{2,t-1}^c}{\gamma p_{2,t-1}^c - (1-\gamma)p_{1,t-1}^c}, \qquad \tilde{\gamma}_4^c = \frac{(1-\gamma)p_{1,t-1}^c}{\gamma p_{2,t-1}^c - (1-\gamma)p_{1,t-1}^c}$$

Following the same procedure for country 2 yields:

$$(A11)\quad y_{2,t} = \frac{\gamma_3 + \gamma_4}{\gamma_3}\left(\frac{\bar{y}}{\tilde{p}_t}\right) - \frac{1}{\alpha\gamma_3}\left(\frac{\bar{y}}{\tilde{p}_t}\right)\left(\gamma_3\tilde{\gamma}_3^c - \gamma_4\tilde{\gamma}_2^c\right)\pi_{2,t-1}^{\bullet}$$
$$+ \frac{1}{\alpha\gamma_3}\left(\frac{\bar{y}}{\tilde{p}_t}\right)\left(\gamma_3\tilde{\gamma}_4^c - \gamma_4\tilde{\gamma}_1^c\right)\pi_{1,t-1}^{\bullet} + \frac{1}{\alpha\gamma_3}\left(\frac{\bar{y}}{\tilde{p}_t}\right)\pi_{2,t-1}$$
$$- \frac{\gamma_4}{\gamma_3}\left(\frac{y_{1,t}}{\tilde{p}_t}\right)$$

where:

$$\gamma_3 = \frac{\gamma p_{2,t-1}}{\gamma p_{2,t-1} + (1-\gamma)p_{1,t-1}}, \qquad \gamma_4 = \frac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1} + (1-\gamma)p_{1,t-1}}$$

Substituting equation (A11) into equation (A10):

$$(A12)\quad y_{1,t} = \left(-\frac{\gamma_2}{\gamma_1}\right)\left[\frac{\gamma_3+\gamma_4}{\gamma_3}\bar{y} - \frac{\bar{y}}{\alpha}\left(\frac{\gamma_3\tilde{\gamma}_3^c-\gamma_4\tilde{\gamma}_2^c}{\gamma_3}\right)\pi^{\bullet}_{2,t-1} + \frac{\bar{y}}{\alpha}\left(\frac{\gamma_3\tilde{\gamma}_4^c-\gamma_4\tilde{\gamma}_1^c}{\gamma_3}\right)\pi^{\bullet}_{1,t-1}\right]$$

$$+\left(-\frac{\gamma_2}{\gamma_1}\right)\left[\frac{\bar{y}}{\alpha\gamma_3}\pi_{2,t-1} - \frac{\gamma_4}{\gamma_3}y_{1,t}\right] + \frac{\gamma_1+\gamma_2}{\gamma_1}\bar{y} - \frac{\bar{y}}{\alpha}\left(\frac{\gamma_1\tilde{\gamma}_1^c-\gamma_2\tilde{\gamma}_4^c}{\gamma_1}\right)\pi^{\bullet}_{1,t-1}$$

$$+\frac{\bar{y}}{\alpha}\left(\frac{\gamma_1\tilde{\gamma}_2^c-\gamma_2\tilde{\gamma}_3^c}{\gamma_1}\right)\pi^{\bullet}_{2,t-1} + \frac{\bar{y}}{\alpha\gamma_1}\pi_{1,t-1}$$

and solving for y_{1t} gives:

$$(A13)\quad y_{1,t} = \bar{y}\left[1 + \frac{\gamma_3}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\pi_{1,t-1} - \frac{\tilde{\gamma}_1^c}{\alpha}\pi^{\bullet}_{1,t-1}\right]$$

$$-\bar{y}\left[\frac{\gamma_2}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\pi_{2,t-1} - \frac{\tilde{\gamma}_2^c}{\alpha}\pi^{\bullet}_{2,t-1}\right]$$

Next note the following:

$$(A14)\quad \tilde{\gamma}_1^c = \frac{\gamma p^c_{1,t-1}}{\gamma p^c_{1,t-1} - (1-\gamma)p^c_{2,t-1}}$$

$$= \frac{\gamma^2 p_{1,t-1} + (1-\gamma)\gamma p_{2,t-1}}{\gamma^2 p_{1,t-1} + (1-\gamma)\gamma p_{2,t-1} - (1-\gamma)\gamma p_{2,t-1} - (1-\gamma)^2 p_{1,t-1}}$$

$$= \frac{\gamma^2 p_{1,t-1} + (1-\gamma)\gamma p_{2,t-1}}{(2\gamma-1)p_{1,t-1}}$$

$$(A15)\quad \tilde{\gamma}_2^c = \frac{(1-\gamma)p^c_{2,t-1}}{\gamma p^c_{1,t-1} - (1-\gamma)p^c_{2,t-1}}$$

$$= \frac{(1-\gamma)\gamma p_{2,t-1} + (1-\gamma)^2 p_{1,t-1}}{\gamma^2 p_{1,t-1} + (1-\gamma)\gamma p_{2,t-1} - (1-\gamma)\gamma p_{2,t-1} - (1-\gamma)^2 p_{1,t-1}}$$

$$= \frac{(1-\gamma)\gamma p_{2,t-1} + (1-\gamma)^2 p_{1,t-1}}{(2\gamma-1)p_{1,t-1}}$$

$$(A16)\quad \frac{\gamma_2}{\gamma_1\gamma_3-\gamma_2\gamma_4} = \frac{\dfrac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}}{\left(\dfrac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-2}}\right)\left(\dfrac{\gamma p_{2,t-1}}{\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}}\right)}$$

$$- \frac{\dfrac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}}{\left(\dfrac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}\right)\left(\dfrac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1}(1-\gamma)p_{1,t-1}}\right)}$$

$$= \left(\frac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}\right)\left(\frac{[\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}]\,[\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}]}{\gamma^2 p_{1,t-1}p_{2,t-1} - (1-\gamma)^2 p_{1,t-1}p_{2,t-1}}\right)$$

$$= \frac{(1-\gamma)\gamma p_{2,t-1}^2 + (1-\gamma)^2 p_{1,t-1}p_{2,t-1}}{(2\gamma-1)p_{1,t-1}p_{2,t-1}}$$

$$= \frac{(1-\gamma)\gamma p_{2,t-1}+(1-\gamma)^2 p_{1,t-1}}{(2\gamma-1)p_{1,t-1}}$$

and:

$$(A17)\quad \frac{\gamma_3}{\gamma_1\gamma_3-\gamma_2\gamma_4} = \frac{\dfrac{\gamma p_{2,t-1}}{\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}}}{\left(\dfrac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-2}}\right)\left(\dfrac{\gamma p_{2,t-1}}{\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}}\right)}$$

$$- \frac{\dfrac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}}{\left(\dfrac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}\right)\left(\dfrac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1}(1-\gamma)p_{1,t-1}}\right)}$$

$$= \left(\frac{\gamma p_{2,t-1}}{\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}}\right)\left(\frac{[\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}]\,[\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}]}{\gamma^2 p_{1,t-1}p_{2,t-1} - (1-\gamma)^2 p_{1,t-1}p_{2,t-1}}\right)$$

$$= \frac{\gamma^2 p_{1,t-1}p_{2,t-1} + (1-\gamma)\gamma p_{2,t-1}^2}{(2\gamma-1)p_{1,t-1}p_{2,t-1}}$$

$$= \frac{\gamma^2 p_{1,t-1}+(1-\gamma)\gamma p_{2,t-1}}{(2\gamma-1)p_{1,t-1}}$$

Equation (A14) and equation (A17) are identical as are equations (A15) and (A16), which proves that:

$$\tilde{\gamma}_1^c = \frac{\gamma_3}{\gamma_1\gamma_3-\gamma_2\gamma_4}, \qquad \tilde{\gamma}_2^c = \frac{\gamma_2}{\gamma_1\gamma_3-\gamma_2\gamma_4}$$

Thus, the aggregate supply curve for country 1 can be written as:

$$(A18) \quad y_{1,t} = \bar{y}\left[1 + \frac{\gamma_3}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\,(\pi_{1,t-1} - \pi_{1,t-1}^{\bullet})\right] - \bar{y}\left[\frac{\gamma_2}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,(\pi_{2,t-1} - \pi_{2,t-1}^{\bullet})\right]$$

To solve for aggregate supply in country 2, substitute equation (A10) into equation (A11):

$$\begin{aligned}(A19) \quad y_{2,t} = {} & \left(-\frac{\gamma_4}{\gamma_3}\right)\left[\frac{\gamma_1+\gamma_2}{\gamma_1}\,\frac{\bar{y}}{\tilde{p}_t} - \frac{\bar{y}}{\tilde{p}_t\alpha}\left(\left[\frac{\gamma_2\tilde{\gamma}_4^c}{\gamma_1} - \tilde{\gamma}_1^c\right]\pi_{1,t-1}^{\bullet} + \left[\tilde{\gamma}_2^c - \frac{\gamma_2\tilde{\gamma}_3^c}{\gamma_1}\right]\pi_{2,t-1}^{\bullet}\right)\right. \\ & + \left(-\frac{\gamma_4}{\gamma_3}\right)\left[\frac{\bar{y}}{\tilde{p}_t\alpha\gamma_1}\,\pi_{1,t-1} - \frac{\gamma_2}{\gamma_1}\,y_{2,t}\right] + \frac{\gamma_3+\gamma_4}{\gamma_3}\,\frac{\bar{y}}{\tilde{p}_t} \\ & - \frac{\bar{y}}{\tilde{p}_t\alpha}\left(\frac{\gamma_4\tilde{\gamma}_2^c}{\gamma_3} - \tilde{\gamma}_3^c\right)\pi_{2,t-1}^{\bullet} - \frac{\bar{y}}{\tilde{p}_t\alpha}\left(\tilde{\gamma}_4^c - \frac{\gamma_4\tilde{\gamma}_1^c}{\gamma_3}\right)\pi_{1,t-1}^{\bullet} \\ & + \frac{\bar{y}}{\tilde{p}_t\alpha\gamma_3}\,\pi_{2,t-1}\end{aligned}$$

and solving for y_{2t} gives:

$$\begin{aligned}(A20) \quad y_{2,t} = {} & \frac{\bar{y}}{\tilde{p}_t}\left[1 + \frac{\gamma_1}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\,\pi_{2,t-1} - \frac{\tilde{\gamma}_3^c}{\alpha}\left(\frac{2\gamma-1}{\gamma}\right)\pi_{2,t-1}^{\bullet}\right] \\ & - \frac{\bar{y}}{\tilde{p}_t}\left[\frac{\gamma_4}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,\pi_{1,t-1} - \frac{\tilde{\gamma}_4^c}{\alpha}\left(\frac{2\gamma-1}{\gamma}\right)\pi_{1,t-1}^{\bullet}\right]\end{aligned}$$

Next note the following:

(A21) $$\tilde{\gamma}_3^c = \frac{\gamma p_{2,t-1}^c}{\gamma p_{2,t-1}^c - (1-\gamma)p_{1,t-1}^c}$$

$$= \frac{\gamma^2 p_{2,t-1} + (1-\gamma)\gamma p_{1,t-1}}{\gamma^2 p_{2,t-1} + (1-\gamma)\gamma p_{1,t-1} - (1-\gamma)\gamma p_{1,t-1} - (1-\gamma)^2 p_{2,t-1}}$$

$$= \frac{\gamma^2 p_{2,t-1} + (1-\gamma)\gamma p_{1,t-1}}{(2\gamma-1)p_{2,t-1}}$$

(A22) $$\tilde{\gamma}_4^c = \frac{(1-\gamma)p_{1,t-1}^c}{\gamma p_{2,t-1}^c - (1-\gamma)p_{1,t-1}^c}$$

$$= \frac{(1-\gamma)\gamma p_{1,t-1} + (1-\gamma)^2 p_{2,t-1}}{\gamma^2 p_{2,t-1} + (1-\gamma)\gamma p_{1,t-1} - (1-\gamma)\gamma p_{1,t-1} - (1-\gamma)^2 p_{2,t-1}}$$

$$= \frac{(1-\gamma)\gamma p_{1,t-1} + (1-\gamma)^2 p_{2,t-1}}{(2\gamma-1)p_{2,t-1}}$$

(A23) $$\frac{\gamma_1}{\gamma_1\gamma_3-\gamma_2\gamma_4} = \frac{\dfrac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}}{\left(\dfrac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-2}}\right)\left(\dfrac{\gamma p_{2,t-1}}{\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}}\right)}$$

$$- \frac{\dfrac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}}{\left(\dfrac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}\right)\left(\dfrac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1}(1-\gamma)p_{1,t-1}}\right)}$$

$$= \left(\frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}\right)\left(\frac{[\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}]\,[\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}]}{\gamma^2 p_{1,t-1}p_{2,t-1} - (1-\gamma)^2 p_{1,t-1}p_{2,t-1}}\right)$$

$$= \frac{\gamma^2 p_{1,t-1}p_{2,t-1} + (1-\gamma)\gamma p_{1,t-1}^2}{(2\gamma-1)p_{1,t-1}p_{2,t-1}}$$

$$= \frac{\gamma^2 p_{2,t-1} + (1-\gamma)\gamma p_{1,t-1}}{(2\gamma-1)p_{2,t-1}}$$

and:

$$(A24)\quad \frac{\gamma_4}{\gamma_1\gamma_3-\gamma_2\gamma_4} = \frac{\dfrac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}}}{\left(\dfrac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-2}}\right)\left(\dfrac{\gamma p_{2,t-1}}{\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}}\right)}$$

$$= \frac{\dfrac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}}}{\left(\dfrac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}\right)\left(\dfrac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1}(1-\gamma)p_{1,t-1}}\right)}$$

$$= \left(\frac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}}\right)\left(\frac{[\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}]\,[\gamma p_{2,t-1}+(1-\gamma)p_{1,t-1}]}{\gamma^2 p_{1,t-1}p_{2,t-1} - (1-\gamma)^2 p_{1,t-1}p_{2,t-1}}\right)$$

$$= \frac{(1-\gamma)\gamma p_{1,t-1}^2 + (1-\gamma)^2 p_{1,t-1}p_{2,t-1}}{(2\gamma-1)p_{1,t-1}p_{2,t-1}}$$

$$= \frac{(1-\gamma)\gamma p_{1,t-1}+(1-\gamma)^2 p_{2,t-1}}{(2\gamma-1)p_{2,t-1}}$$

Equations (A21) and (A23) are the same as are equations (A22) and (A24) which proves that:

$$\tilde{\gamma}_3^c = \frac{\gamma_1}{\gamma_1\gamma_3-\gamma_2\gamma_4}, \qquad \tilde{\gamma}_4^c = \frac{\gamma_4}{\gamma_1\gamma_3-\gamma_2\gamma_4}$$

Thus the aggregate supply curve for country 2 can be written as:

$$y_{2,t} = \frac{\bar{y}}{\tilde{p}_t}\left[1 + \frac{\gamma_1}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\,(\pi_{2,t-1} - \pi^{*}_{2,t-1})\right]$$

$$- \frac{\bar{y}}{\tilde{p}_t}\left[\frac{\gamma_4}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,(\pi_{1,t-1} - \pi^{*}_{1,t-1})\right]$$

<u>Solving for Aggregate Demand:</u>

Adding equations (12) and (25) yields:

$$(A25)\quad m_{1,t} + m_{2,t}\tilde{p}_t = \lambda(y^d_{1,t} + y^d_{2,t}\tilde{p}_t) - 2\theta I_t$$

Substituting equation (31) into equation (A25):

$$(A26) \quad b_{m,t} = \lambda(y_{1,t}^{d} + y_{2,t}^{d}\tilde{p}_t) - 2\theta I_t$$

and solving for i_t:

$$(A27) \quad I_t = \frac{\lambda}{2\theta}(y_{1,t}^{d} + y_{2,t}^{d}\tilde{p}_t) - \frac{1}{2\theta}b_{m,t}$$

Substituting equation (4) into equation (8), and equation (17) into equation (21) yields:

$$(A28) \quad a_{1,t} = cy_{1,t}^{d} - \phi I_t + \phi\pi_{1,t}^{e}$$

$$(A29) \quad a_{2,t} = cy_{2,t}^{d} - \phi I_t + \phi\pi_{2,t}^{e}$$

Substituting equation (A27) into (A28):

$$(A30) \quad a_{1,t} = cy_{1,t}^{d} - \frac{\lambda\phi}{2\theta}(y_{1,t}^{d} + y_{2,t}^{d}\tilde{p}_t) + \frac{\phi}{2\theta}b_{m,t} + \phi\pi_{1,t}^{e}$$

and substituting equation (A27) into (A29):

$$(A31) \quad a_{2,t} = cy_{2,t}^{d} - \frac{\lambda\phi}{2\theta}(y_{2,t}^{d} + y_{1,t}^{d}\frac{1}{\tilde{p}_t}) + \frac{\phi}{2\theta}b_{m,t}\frac{1}{\tilde{p}_t} + \phi\pi_{2,t}^{e}$$

Substituting equation (10) into equation (7):

$$(A32) \quad y_{1,t} = (1-\epsilon)a_{1,t} + (1-\epsilon_g)g_{1,t} + \epsilon a_{2,t}\tilde{p}_t + \epsilon_g g_{2,t}\tilde{p}_t + \eta\left(\frac{p_{2,t}}{p_{1,t}} - \frac{p_{1,t}}{p_{2,t}}\right)$$

Substituting equations (A29) and (A30) into equation (A32):

$$\begin{aligned}(A33) \quad y_{1,t} = {} & (1-\epsilon)\left[(c-\frac{\lambda\phi}{2\theta})y_{1,t}^{d} - \frac{\lambda\phi}{2\theta}y_{2,t}^{d}\tilde{p}_t + \frac{\phi}{2\theta}b_{m,t} + \phi\pi_{1,t}^{e}\right] \\ & + \epsilon\left[(c-\frac{\lambda\phi}{2\theta})y_{2,t}^{d}\tilde{p}_t - \frac{\lambda\phi}{2\theta}y_{1,t}^{d} + \frac{\phi}{2\theta}b_{m,t} + \phi\pi_{2,t}^{e}\tilde{p}_t\right] \\ & + (1-\epsilon_g)g_{1,t} + \epsilon_g g_{2,t}\tilde{p}_t\end{aligned}$$

Collecting terms:

$$(A34)\quad y_{1,t} = \left[(1-e)c - \frac{\lambda\phi}{2\theta}\right] y_{1,t}^{d} - \left[\frac{\lambda\phi}{2\theta} - ec\right] y_{2,t}^{d}\tilde{p}_t + \frac{\phi}{2\theta}b_{m,t} + (1-e)\phi\pi_{1,t}^{e} + e\phi\pi_{2,t}^{e}\tilde{p}_t + (1-e_g)g_{1,t} + e_g g_{2,t}\tilde{p}_t$$

Substituting equations (9) and (22) into equation (A36):

$$(A35)\quad y_{1,t} = \left[(1-e)c - \frac{\lambda\phi}{2\theta}\right]\left(y_{1,t} + r_{1,t-1}b_{1,t-1}^{p} - t_{1,t}\right) - \left[\frac{\lambda\phi}{2\theta} - ec\right]\left(y_{2,t} + r_{2,t-1}b_{2,t-1}^{p} - t_{2,t}\right)\tilde{p}_t + \frac{\phi}{2\theta}b_{m,t} + (1-e)\phi\pi_{1,t}^{e} + e\phi\pi_{2,t}^{e}\tilde{p}_t + (1-e_g)g_{1,t} + e_g g_{2,t}\tilde{p}_t$$

Solving equation (12) for t_{1t} and equation (25) for t_{2t}, and substituting the resulting equations into equation (A35) yields:

$$(A36)\quad y_{1,t} = \left[(1-e)c - \frac{\lambda\phi}{2\theta}\right]\left(y_{1,t} + r_{1,t-1}b_{1,t-1}^{p} - g_{1,t} - r_{1,t-1}b_{1,t-1} + b_{1,t}\right) - \left[\frac{\lambda\phi}{2\theta} - ec\right]\left(y_{2,t} + r_{2,t-1}b_{2,t-1}^{p} - g_{2,t} - r_{2,t-1}b_{2,t-1} + b_{2,t}\right)\tilde{p}_t + \frac{\phi}{2\theta}b_{m,t} + (1-e)\phi\pi_{1,t}^{e} + e\phi\pi_{2,t}^{e}\tilde{p}_t + (1-e_g)g_{1,t} + e_g g_{2,t}\tilde{p}_t$$

Collecting terms:

$$(A37)\quad \left[\frac{2\theta - (1-e)2c\theta + \lambda\phi}{2\theta}\right]y_{1,t} = \left[(1-e)c - \frac{\lambda\phi}{2\theta}\right]r_{1,t-1}\left(b_{1,t-1}^{p} - b_{1,t-1}\right) + \left[(1-e_g) - (1-e)c + \frac{\lambda\phi}{2\theta}\right]g_{1,t} + \left[(1-e)c - \frac{\lambda\phi}{2\theta}\right]b_{1,t} + (1-e)\phi\pi_{1,t}^{e} + \left(ec - \frac{\lambda\phi}{2\theta}\right)y_{2,t}\tilde{p}_t + \left(ec - \frac{\lambda\phi}{2\theta}\right)r_{2,t-1}\left(b_{2,t-1}^{p} - b_{2,t-1}\right)\tilde{p}_t + \left(e_g - ec + \frac{\lambda\phi}{2\theta}\right)g_{2,t}\tilde{p}_t + \left(ec - \frac{\lambda\phi}{2\theta}\right)b_{2,t}\tilde{p}_t + \left(\frac{\phi}{2\theta}\right)b_{m,t} + e\phi\pi_{2,t}^{e}\tilde{p}_t$$

Collecting terms:

$$\text{(A34)}\quad y_{1,t} = \left[(1-\epsilon)c - \frac{\lambda\phi}{2\theta}\right] y^d_{1,t} - \left[\frac{\lambda\phi}{2\theta} - \epsilon c\right] y^d_{2,t}\tilde{P}_t + \frac{\phi}{2\theta}b_{m,t} + (1-\epsilon)\phi\pi^{e}_{1,t} + \epsilon\phi\pi^{e}_{2,t}\tilde{P}_t + (1-\epsilon_g)\,g_{1,t} + \epsilon_g g_{2,t}\tilde{P}_t$$

Substituting equations (9) and (22) into equation (A36):

$$\text{(A35)}\quad y_{1,t} = \left[(1-\epsilon)c - \frac{\lambda\phi}{2\theta}\right]\left(y_{1,t} + r_{1,t-1}b^P_{1,t-1} - t_{1,t}\right) - \left[\frac{\lambda\phi}{2\theta} - \epsilon c\right]\left(y_{2,t} + r_{2,t-1}b^P_{2,t-1} - t_{2,t}\right)\tilde{P}_t + \frac{\phi}{2\theta}b_{m,t} + (1-\epsilon)\phi\pi^{e}_{1,t} + \epsilon\phi\pi^{e}_{2,t}\tilde{P}_t + (1-\epsilon_g)\,g_{1,t} + \epsilon_g g_{2,t}\tilde{P}_t$$

Solving equation (12) for t_{1t} and equation (25) for t_{2t}, and substituting the resulting equations into equation (A35) yields:

$$\text{(A36)}\quad y_{1,t} = \left[(1-\epsilon)c - \frac{\lambda\phi}{2\theta}\right]\left(y_{1,t} + r_{1,t-1}b^P_{1,t-1} - g_{1,t} - r_{1,t-1}b_{1,t-1} + b_{1,t}\right) - \left[\frac{\lambda\phi}{2\theta} - \epsilon c\right]\left(y_{2,t} + r_{2,t-1}b^P_{2,t-1} - g_{2,t} - r_{2,t-1}b_{2,t-1} + b_{2,t}\right)\tilde{P}_t + \frac{\phi}{2\theta}b_{m,t} + (1-\epsilon)\phi\pi^{e}_{1,t} + \epsilon\phi\pi^{e}_{2,t}\tilde{P}_t + (1-\epsilon_g)\,g_{1,t} + \epsilon_g g_{2,t}\tilde{P}_t$$

Collecting terms:

$$\begin{aligned}\text{(A37)}\quad \left[\frac{2\theta - (1-\epsilon)2c\theta + \lambda\phi}{2\theta}\right]y_{1,t} = {} & \left[(1-\epsilon)c - \frac{\lambda\phi}{2\theta}\right]r_{1,t-1}\left(b^P_{1,t-1} - b_{1,t-1}\right) \\ & + \left[(1-\epsilon_g) - (1-\epsilon)c + \frac{\lambda\phi}{2\theta}\right]g_{1,t} \\ & + \left[(1-\epsilon)c - \frac{\lambda\phi}{2\theta}\right]b_{1,t} + (1-\epsilon)\phi\pi^{e}_{1,t} \\ & + \left(\epsilon c - \frac{\lambda\phi}{2\theta}\right)y_{2,t}\tilde{P}_t \\ & + \left(\epsilon c - \frac{\lambda\phi}{2\theta}\right)r_{2,t-1}\left(b^P_{2,t-1} - b_{2,t-1}\right)\tilde{P}_t \\ & + \left(\epsilon_g - \epsilon c + \frac{\lambda\phi}{2\theta}\right)g_{2,t}\tilde{P}_t + \left(\epsilon c - \frac{\lambda\phi}{2\theta}\right)b_{2,t}\tilde{P}_t \\ & + \left(\frac{\phi}{2\theta}\right)b_{m,t} + \epsilon\phi\pi^{e}_{2,t}\tilde{P}_t\end{aligned}$$

Solving equation (A37) for y_{1t} gives country 1's aggregate demand as a function of aggregate demand in country 2:

$$
\begin{aligned}
(A38)\quad y_{1,t} = {} & \left[\frac{[(1-\epsilon_g) - (1-\epsilon)c]2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta) + \lambda\phi}\right] g_{1,t} \\
& + \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] r_{1,t-1}\ (b^p_{1,t-1} - b_{1,t-1}) \\
& + \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] b_{1,t} + \left[\frac{(1-\epsilon)2c\theta\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^{e}_{1,t} \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] y_{2,t}\tilde{p}_t \\
& + \left[\frac{(\epsilon_g - \epsilon c)2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] g_{2,t}\tilde{p}_t \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] r_{2,t-1}\ (b^p_{2,t-1} - b_{2,t-1})\tilde{p}_t \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] b_{2,t}\tilde{p}_t + \left[\frac{2\epsilon\phi\theta}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^{e}_{2,t}\tilde{p}_t \\
& + \left[\frac{\phi}{2\theta - (1-\epsilon)2c\theta + \lambda\phi}\right] b_{m,t}
\end{aligned}
$$

Equivalently, solving for y_{2t} in terms of y_{1t} yields:

$$
\begin{aligned}
(A39)\quad y_{2,t} = {} & \left[\frac{[(1-\epsilon_g) - (1-\epsilon)c]2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta) + \lambda\phi}\right] g_{2,t} \\
& + \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] r_{2,t-1}\ (b^p_{2,t-1} - b_{2,t-1}) \\
& + \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] b_{2,t} + \left[\frac{(1-\epsilon)2c\theta\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^{e}_{2,t} \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] y_{1,t}\frac{1}{\tilde{p}_t} \\
& + \left[\frac{(\epsilon_g - c\epsilon)2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] g_{1,t}\frac{1}{\tilde{p}_t} \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] r_{1,t-1}\ (b^p_{1,t-1} - b_{1,t-1})\frac{1}{\tilde{p}_t} \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] b_{1,t}\frac{1}{\tilde{p}_t} \\
& + \left[\frac{2\epsilon\phi\theta}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^{e}_{1,t}\frac{1}{\tilde{p}_t} \\
& + \left[\frac{\phi}{2\theta - (1-\epsilon)2c\theta + \lambda\phi}\right] b_{m,t}\frac{1}{\tilde{p}_t}
\end{aligned}
$$

Substituting equation (A39) into equation (A38) yields:

$$\text{(A40)}\quad y_{1,t} = \left[1 - \frac{2e_g\theta}{\Lambda}\right] g_{1,t} + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} b_{1,t} + \frac{(1-e)2\phi\theta}{\Lambda} \pi^e_{1,t}$$

$$+ \frac{2ec\theta - \lambda\phi}{\Lambda} b_{2,t}\tilde{p}_t + \frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda} g_{2,t}\tilde{p}_t + \frac{2e\phi\theta}{\Lambda} \pi^e_{2,t}\tilde{p}_t$$

$$+ \frac{\phi}{\Lambda} b_{m,t} + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^p_{1,t-1} - b_{1,t-1})$$

$$+ \frac{2ce\theta - \lambda\phi}{\Lambda} r_{2,t-1}(b^p_{2,t-1} - b_{2,t-1})\tilde{p}_t$$

$$+ \left[\frac{2e\phi\theta - \lambda\phi}{\Lambda}\right] \{\left(1 - \frac{\beta e_g}{\Lambda}\right) g_{2,t}\tilde{p}_t + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} b_{2,t}\tilde{p}_t$$

$$+ \frac{(1-e)2\phi\theta}{\Lambda} \pi^e_{2,t} + \frac{2ce\theta - \lambda\phi}{\Lambda} y_{1,t} + \frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda} g_{1,t}$$

$$+ \frac{2ce\theta}{\Lambda} b_{1,t} + \frac{2ce\theta}{\Lambda} \pi^e_{1,t} + \frac{\phi}{\Lambda} b_{m,t}$$

$$+ \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} r_{2,t-1}(b^p_{2,t-1} - b_{2,t-1})\tilde{p}_t$$

$$+ \frac{2ce\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^p_{1,t-1} - b_{1,t-1})\}$$

where:

$$\Lambda = (1 - (1-e)c)2\theta + \lambda\phi$$

Next note the following:

$$\text{(A41)}\quad r_{2,t-1}(b^p_{2,t-1} - b_{2,t-1})\tilde{p}_t = \frac{i_{t-1}}{1+\pi_{2,t-1}}(b^p_{2,t-1} - b_{2,t-1})\tilde{p}_t$$

$$= (i_{t-1})\left(\frac{p^c_{2,t-1}}{p^c_{2,t}}\right)\left(\frac{p^c_{2,t}}{p^c_{1,t}}\right)(b^p_{2,t-1} - b_{2,t-1})$$

$$= (i_{t-1})\left(\frac{p^c_{2,t-1}}{p^c_{2,t}}\right)\left(\frac{p^c_{2,t}}{p^c_{1,t}}\right)\left(\frac{p^c_{1,t-1}}{p^c_{1,t-1}}\right)(b^p_{2,t-1} - b_{2,t-1})$$

$$= (i_{t-1})\left(\frac{p^c_{2,t-1}}{p^c_{1,t-1}}\right)\left(\frac{p^c_{1,t-1}}{p^c_{1,t}}\right)(b^p_{2,t-1} - b_{2,t-1})$$

$$= r_{1,t-1}\,(b^p_{2,t-1} - b_{2,t-1})\,\tilde{p}_{t-1}$$

Thus (A30) can be rewritten as:

$$(A42)\quad y_{1,t} = \left[1 - \frac{2e_g\theta}{\Lambda}\right] g_{1,t} + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} b_{1,t} + \frac{(1-e)2\phi\theta}{\Lambda} \pi^{e}_{1,t}$$
$$+ \frac{2ec\theta - \lambda\phi}{\Lambda} b_{2,t}\tilde{p}_t + \frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda} g_{2,t}\tilde{p}_t + \frac{2e\phi\theta}{\Lambda} \pi^{e}_{2,t}\tilde{p}_t$$
$$+ \frac{\phi}{\Lambda} b_{m,t} + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1})$$
$$+ \frac{2ce\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1}$$
$$+ \left[\frac{2e\phi\theta - \lambda\phi}{\Lambda}\right] \left\{\left(1 - \frac{\beta e_g}{\Lambda}\right) g_{2,t}\tilde{p}_t + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} b_{2,t}\tilde{p}_t\right.$$
$$+ \frac{(1-e)2\phi\theta}{\Lambda} \pi^{e}_{2,t} + \frac{2ce\theta - \lambda\phi}{\Lambda} y_{1,t} + \frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda} g_{1,t}$$
$$+ \frac{2ce\theta}{\Lambda} b_{1,t} + \frac{2ce\theta}{\Lambda} \pi^{e}_{1,t} + \frac{\phi}{\Lambda} b_{m,t}$$
$$+ \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1}$$
$$\left. + \frac{2ce\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1})\right\}$$

Given that:

$$b^{p}_{1,t-1} = b_{11,t-1} + b_{21,t-1}\tilde{p}_{t-1}, \qquad b^{p}_{2,t-1} = b_{22,t-1} + b_{12,t-1}\frac{1}{\tilde{p}_{t-1}}$$

$$b_{1,t-1} = b_{11,t-1} + b_{12,t-1} + b_{1m,t-1}, \qquad b_{2,t-1} = b_{22,t-1} + b_{21,t-1} + b_{2m,t-1}$$

the term:

$$\frac{(1-e)2c\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) + \frac{2ce\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1}$$

can be rewritten as:

$$(A43)\quad \frac{(1-e)2c\theta}{\Lambda} r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) + \frac{2ce\theta}{\Lambda} r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1}$$
$$+ \frac{\lambda\phi}{\Lambda} r_{1,t-1} b_{m,t-1}$$

Likewise the term:

$$\frac{2ce\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1}$$

can be rewritten as:

$$(A44)\quad \frac{2ce\theta}{\Lambda} r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) + \frac{(1-e)2c\theta}{\Lambda} r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1} + \frac{\lambda\phi}{\Lambda} r_{1,t-1} b_{m,t-1}$$

Substituting equations (A43) and (A44) into equation (A42) yields:

$$\begin{aligned}
(A45)\quad y_{1,t} = {} & \left[1 - \frac{2e_g\theta}{\Lambda}\right] g_{1,t} + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} b_{1,t} + \frac{(1-e)2\phi\theta}{\Lambda} \pi^{*}_{1,t} \\
& + \frac{2ec\theta - \lambda\phi}{\Lambda} b_{2,t}\tilde{p}_t + \frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda} g_{2,t}\tilde{p}_t + \frac{2e\phi\theta}{\Lambda} \pi^{*}_{2,t}\tilde{p}_t \\
& + \frac{\phi}{\Lambda} b_{m,t} + \frac{(1-e)2c\theta}{\Lambda} r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) \\
& + \frac{2ce\theta}{\Lambda} r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1} + \frac{\lambda\phi}{\Lambda} r_{1,t-1} b_{m,t-1} \\
& + \left[\frac{2e\phi\theta - \lambda\phi}{\Lambda}\right] \Big\{ \left(1 - \frac{\beta e_g}{\Lambda}\right) g_{2,t}\tilde{p}_t + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} b_{2,t}\tilde{p}_t \\
& \quad + \frac{(1-e)2\phi\theta}{\Lambda} \pi^{*}_{2,t} + \frac{2ce\theta - \lambda\phi}{\Lambda} y_{1,t} + \frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda} g_{1,t} \\
& \quad + \frac{2ce\theta}{\Lambda} b_{1,t} + \frac{2ce\theta}{\Lambda} \pi^{*}_{1,t} + \frac{\phi}{\Lambda} b_{m,t} \\
& \quad + \frac{(1-e)2c\theta}{\Lambda} r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1} \\
& \quad + \frac{2ce\theta}{\Lambda} r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) + \frac{\lambda\phi}{\Lambda} r_{1,t-1} b_{m,t-1} \Big\}
\end{aligned}$$

Combining terms and solving for y_{1t} gives:

$$
\begin{aligned}
(A46)\quad y_{1,t} = {} & \left[\left(1 - \frac{2e_g\theta}{\Lambda}\right) + V\left(\frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda}\right)\right]\frac{1}{\Psi}\, g_{1,t} \\
& + \left[\frac{(1-e)2c\theta - \lambda\phi}{\Lambda} + V^2\right]\frac{1}{\Psi}\, b_{1,t} \\
& + \left[\frac{(1-e)2\phi\theta}{\Lambda} + V\,\frac{2e\phi\theta}{\Lambda}\right]\frac{1}{\Psi}\, \pi^{\bullet}_{1,t} \\
& + \left[V\left(1 + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda}\right)\right]\frac{1}{\Psi}\, b_{2,t}\tilde{\beta}_t \\
& + \left[\frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda} + V\left(1 - \frac{2e_g\theta}{\Lambda}\right)\right]\frac{1}{\Psi}\, g_{2,t}\tilde{\beta}_t \\
& + \left[\frac{2e\phi\theta}{\Lambda} + V\,\frac{(1-e)2\phi\theta}{\Lambda}\right]\frac{1}{\Psi}\pi^{\bullet}_{2,t}\tilde{\beta}_t + \left[\frac{\phi}{\Lambda}(1 + V)\right]\frac{1}{\Psi}\, b_{m,t} \\
& + \left[\frac{(1-e)2c\theta}{\Lambda} + V\,\frac{2ce\theta}{\Lambda}\right]\frac{1}{\Psi}\, r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) \\
& + \left[\frac{2ce\theta}{\Lambda} + V\,\frac{(1-e)2c\theta}{\Lambda}\right]\frac{1}{\Psi}\, r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{\beta}_{t-1} \\
& + \left[\frac{\lambda\phi}{\Lambda} + V\,\frac{\lambda\phi}{\Lambda}\right]\frac{1}{\Psi}\, r_{1,t-1}b_{m,t-1}
\end{aligned}
$$

where:

$$V = \frac{2ce\theta - \lambda\phi}{\Lambda}$$

$$\Psi = \frac{\Lambda^2 - (2ce\theta - \lambda\phi)^2}{\Lambda^2}$$

Which can then be simplified to give:

$$
\begin{aligned}
(A47)\quad y_{1,t} = {} & \left(1 - \frac{e_g}{1-c+2ce}\right) g_{1,t} \\
& + \frac{2\theta c(c-1+e-2ce) + \lambda\phi(1-2c+4ce)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}\, b_{1,t} \\
& + \frac{2\phi\theta(c-1+e-2ce) + \lambda\phi^2(2e-1)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}\, \pi^{\bullet}_{1,t} \\
& + \frac{e_g}{1-c+2ce}\, g_{2,t}\tilde{\beta}_t + \frac{\lambda\phi-2ce\theta}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}\, b_{2,t}\tilde{\beta}_t \\
& + \frac{\phi(\lambda\phi-2e\lambda\phi-2e\theta)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}\, \pi^{\bullet}_{2,t}\tilde{\beta}_t + \frac{\phi}{2(\theta-c\theta+\lambda\phi)}\, b_{m,t} \\
& + \frac{2c\theta(c-1+e-2ce) + c(2e\lambda\phi-\lambda\phi)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}\, r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) \\
& + \frac{c(\phi\lambda-2e\phi\lambda-2e\theta)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}\, r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{\beta}_{t-1} \\
& + \frac{\lambda\phi}{2(\theta-c\theta+\lambda\phi)}\, r_{1,t-1}b_{m,t-1}
\end{aligned}
$$

This in turn can be written as in the form of equation (37) in the text:

$$
\begin{aligned}
y_{1,t} = {} & A_1\, g_{1,t} + A_2\, b_{1,t} + A_3\, \pi^{e}_{1,t} + A_4\, g_{2,t}\tilde{p}_t + A_5\, b_{2,t}\tilde{p}_t + A_6\, \pi^{e}_{2,t}\tilde{p}_t \\
& + A_7\, b_{m,t} + A_8\, r_{1,t-1}(b^{p}_{1,t-1} - b_{1,t-1}) \\
& + A_9\, r_{1,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1} + A_{10}\, r_{1,t-1} b_{m,t-1}
\end{aligned}
$$

where the coefficients A_1 to A_{10} are defined in Table VI in chapter 1.

The solution process for $y_{2,t}$ is the same as for $y_{1,t}$. Thus, the definitions of the coefficients are the same.

APPENDIX B

RESTRICTIONS ON THE NET BORROWING/LENDING STATUS OF THE TWO COUNTRIES

This appendix examines the nine possible combinations with respect to the net borrowing/lending status of the two countries. Each combination places restrictions on the bond holdings of either the central bank, or of private individuals in the two countries. Based on these restrictions, four of the combinations can be eliminated because they require the bond holdings of the central bank to be non-positive, which implies that the real money supply is non-positive.

The nine possible cases, as given in Chapter 1, Table VII, are:

(1) $b^p_{1,t-1}-b_{1,t-1} > 0 \quad (b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} > 0$

(2) $b^p_{1,t-1}-b_{1,t-1} > 0 \quad (b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} = 0$

(3) $b^p_{1,t-1}-b_{1,t-1} > 0 \quad (b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} < 0$

(4) $b^p_{1,t-1}-b_{1,t-1} < 0 \quad (b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} > 0$

(5) $b^p_{1,t-1}-b_{1,t-1} < 0 \quad (b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} = 0$

(6) $b^p_{1,t-1}-b_{1,t-1} < 0 \quad (b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} < 0$

(7) $b^p_{1,t-1}-b_{1,t-1} = 0 \quad (b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} > 0$

(8) $b^p_{1,t-1}-b_{1,t-1} = 0 \quad (b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} = 0$

(9) $b^p_{1,t-1}-b_{1,t-1} = 0 \quad (b^p_{2,t-1}-b_{2,t-1})\bar{p}_{t-1} < 0$

If

$$b^p_{i,t-1} - b_{i,t-1} > 0$$

then country i was a net creditor in period t-1, and if

$$b^p_{i,t-1} - b_{i,t-1} < 0$$

then country i was a net debtor in period t-1. As shown below, in the model developed in this paper one country must be a net debtor. The other country can be a net creditor or be neither a net borrower nor a net creditor. To show this, first note that:

$$(B1) \qquad b^p_{1,t-1} - b_{1,t-1} = b_{11,t-1} + b_{21,t-1}\tilde{p}_{t-1} - b_{11,t-1} - b_{12,t-1} - b_{1m,t-1}$$
$$= b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} - \frac{1}{2}b_{m,t-1}$$

$$(B2) \quad (b^p_{2,t-1} - b_{2,t-1})\tilde{p}_{t-1} = b_{22,t-1}p_{t-1} + b_{12,t-1} - b_{22,t-1}\tilde{p}_{t-1} - b_{21,t-1}\tilde{p}_{t-1} - b_{2m,t-1}$$
$$= b_{12,t-1} - b_{21,t-1}\tilde{p}_{t-1} - \frac{1}{2}b_{m,t-1}$$

Using equations (B1) and (B2) each of the nine cases can be rewritten to determine the restrictions on bond holdings.

<u>Case 1:</u>

Rewriting:

$$b^p_{1,t-1} - b_{1,t-1} > 0 \qquad (b_{2,t-1} - b^p_{2,t-1})\tilde{p}_{t-1} > 0$$

as:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} > \frac{1}{2}b_{m,t-1}$$
$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) > \frac{1}{2}b_{m,t-1}$$

Both of these inequalities can only hold if $b_{m,t-1} < 0$. This implies a negative real money supply. Thus, case 1 is not feasible.

<u>Case 2:</u>

Rewriting:

$$b^p_{1,t-1} - b_{1,t-1} > 0 \qquad (b_{2,t-1} - b^p_{2,t-1})\tilde{p}_{t-1} = 0$$

as:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} > \frac{1}{2}b_{m,t-1}$$
$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) = \frac{1}{2}b_{m,t-1}$$

implies that:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} > -(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1})$$
$$\Rightarrow$$
$$b_{21,t-1}\tilde{p}_{t-1} > b_{12,t-1}$$

This last inequality only holds if $b_{m,t-1} < 0$. Thus case 2 is infeasible.

Case 3:

Rewriting:

$$b^{p}_{1,t-1} - b_{1,t-1} > 0 \qquad (b_{2,t-1} - b^{p}_{2,t-1})\tilde{p}_{t-1} < 0$$

as:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} > \frac{1}{2}b_{m,t-1}$$
$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) < \frac{1}{2}b_{m,t-1}$$

implies that:

$$(B3) \quad -(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) < \frac{1}{2}b_{m,t-1} < b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}$$

Since $b_{m,t-1}$ must be positive, the inequality given by (B3) will only be met if

$$b_{21,t-1}\tilde{p}_{t-1} > b_{12,t-1}$$

Case 4:

Rewriting:

$$b^{p}_{1,t-1} - b_{1,t-1} < 0 \qquad (b_{2,t-1} - b^{p}_{2,t-1})\tilde{p}_{t-1} > 0$$

as:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} < \frac{1}{2}b_{m,t-1}$$
$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) > \frac{1}{2}b_{m,t-1}$$

implies that:

$$(B4)\quad (b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) < \frac{1}{2}b_{m,t-1} < -(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1})$$

Since $b_{m,t-1}$ must be positive, the inequality given by (B4) will only be met if

$$b_{21,t-1}\tilde{p}_{t-1} < b_{12,t-1}$$

Case 5:

Rewriting:

$$b^{p}_{1,t-1} - b_{1,t-1} < 0 \qquad (b_{2,t-1} - b^{p}_{2,t-1})\tilde{p}_{t-1} = 0$$

as:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} < \frac{1}{2}b_{m,t-1}$$

$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) = \frac{1}{2}b_{m,t-1}$$

implies that:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} < -(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1})$$

$$\Rightarrow \quad b_{21,t-1}\tilde{p}_{t-1} < b_{12,t-1}$$

Case 6:

Rewriting:

$$b^{p}_{1,t-1} - b_{1,t-1} < 0 \qquad (b_{2,t-1} - b^{p}_{2,t-1}) < 0$$

as:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} < \frac{1}{2}b_{m,t-1}$$

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} > -\frac{1}{2}b_{m,t-1}$$

implies that:

$$-\frac{1}{2}b_{m,t-1} < (b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) < \frac{1}{2}b_{m,t-1}$$

Case 7:

Rewriting:

$$b^{p}_{1,t-1} - b_{1,t-1} = 0 \qquad (b_{2,t-1} - b^{p}_{2,t-1})\tilde{p}_{t-1} > 0$$

as:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} = \frac{1}{2}b_{m,t-1}$$

$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) > \frac{1}{2}b_{m,t-1}$$

implies that:

$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) > b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}$$

$$\Rightarrow \quad b_{12,t-1} > b_{21,t-1}\tilde{p}_{t-1}$$

This last inequality only holds if $b_{m,t-1} < 0$. Thus case 7 is infeasible.

Case 8:

Rewriting:

$$b^{p}_{1,t-1} - b_{1,t-1} = 0 \qquad (b_{2,t-1} - b^{p}_{2,t-1})\tilde{p}_{t-1} = 0$$

as:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} = \frac{1}{2}b_{m,t-1}$$

$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) = \frac{1}{2}b_{m,t-1}$$

implies that:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} = -(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1})$$

$$\Rightarrow \quad b_{21,t-1}\tilde{p}_{t-1} = b_{12,t-1}$$

This equation only holds if $b_{m,t-1} = 0$. Thus case 8 is infeasible.

Case 9:

Rewriting:

$$b_{1,t-1}^{p} - b_{1,t-1} = 0 \qquad (b_{2,t-1} - b_{2,t-1}^{p}) < 0$$

as:

$$b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1} = \frac{1}{2}b_{m,t-1}$$

$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) < \frac{1}{2}b_{m,t-1}$$

implies that:

$$-(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) < (b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}$$

$$\Rightarrow$$

$$b_{21,t-1}\tilde{p}_{t-1} > b_{12,t-1}$$

APPENDIX C

SOLVING FOR EQUILIBRIUM INFLATION AND OUTPUT

This appendix uses the Aggregate Supply and Demand equations for country 1 and country 2 to solve for the each country's equilibrium inflation rate and output level as given by Tables IX and X in chapter 1.

Country 1's and country 2's aggregate supply equations are as follows:

$$(C1) \quad y_{1,t} = \bar{y}\left[1 + \frac{\gamma_3}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,(\pi_{2,t-1} - \pi^{e}_{2,t-1})\right] - \bar{y}\left[\frac{\gamma_2}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,(\pi_{1,t-1} - \pi^{e}_{1,t-1})\right]$$

$$(C2) \quad y_{2,t} = \frac{\bar{y}}{\tilde{p}_t}\left[1 + \frac{\gamma_1}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,(\pi_{2,t-1} - \pi^{e}_{2,t-1})\right] - \frac{\bar{y}}{\tilde{p}_t}\left[\frac{\gamma_4}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,(\pi_{1,t-1} - \pi^{e}_{1,t-1})\right]$$

where the coefficients:

$$\gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_4$$

are defined in the text.

The aggregate demand equations for country 1 and country 2 are:

$$(C3) \quad y_{1,t} = A_1\, g_{1,t} + A_2\, b_{1,t} + A_3\, \pi^{e}_{1,t} + A_4\, g_{2,t}\tilde{p}_t + A_5\, b_{2,t}\tilde{p}_t + A_6\, \pi^{3}_{2,t}\tilde{p}_t + A_7\, b_{m,t} + i_{t-1}\, Y_1 - \pi_{1,t-1} Y_1$$

$$(C4)\quad y_{2,t} = A_1\, g_{2,t} + A_2\, b_{2,t} + A_3\, \pi^{e}_{2,t} + A_4\, g_{1,t}\frac{1}{\tilde{p}_t} + A_5\, b_{1,t}\frac{1}{\tilde{p}_t}$$
$$+ A_6\, \pi^{e}_{1,t}\frac{1}{\tilde{p}_t} + A_7\, b_{m,t}\frac{1}{\tilde{p}_t} + i_{t-1}\Upsilon_2 - \pi_{2,t-1}\Upsilon_2$$

where the coefficients A_1 to A_{10} are defined as in the text, and:

$$\Upsilon_1 = A_8(b^{p}_{1,t-1}-b_{1,t-1}) + A_9(b^{p}_{2,t-1}-b_{2,t-1})\tilde{p}_{t-1} + A_{10}b_{m,t-1}$$

$$\Upsilon_2 = A_8(b^{p}_{2,t-1}-b_{2,t-1}) + A_9(b^{p}_{1,t-1}-b_{1,t-1})\left(\frac{1}{\tilde{p}_{t-1}}\right) + A_{10}b_{m,t-1}\left(\frac{1}{\tilde{p}_{t-1}}\right)$$

Substituting equation (C3) into (C1) gives $\pi_{1,t-1}$ as a function of the pre-determined, exogenous and policy variables, and $\pi_{2,t-1}$:

$$(C5)\quad \pi_{1,t-1} = \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_1\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\bar{y}}\left(A_1 g_{1,t}+A_2 b_{1,t}+A_3\pi^{e}_{1,t}+A_4 g_{2,t}\tilde{p}_t \right)$$
$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_1\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\bar{y}}\left(A_5 b_{2,t}\tilde{p}_t+A_6\pi^{e}_{2,t}\tilde{p}_t+A_7 b_{m,t} \right)$$
$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_1\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\bar{y}}\left(\Upsilon_1 i_{t-1}-\bar{y} \right)$$
$$+ \frac{\bar{y}}{\Upsilon_1\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\bar{y}}\left[\gamma_3\pi^{e}_{1,t-1} + \gamma_2(\pi_{2,t-1}-\pi^{e}_{2,t-1}) \right]$$

Rewrite equation (C2) as follows:

$$(C6)\quad y_{2,t}\,\tilde{p}_t = \bar{y}\left[1 + \frac{\gamma_1}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\ (\pi_{2,t-1} - \pi^{e}_{2,t-1}) \right]$$
$$- \bar{y}\left[\frac{\gamma_4}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\ (\pi_{1,t-1} - \pi^{e}_{1,t-1}) \right]$$

This is done so that after substituting equation (C4) into (C6) all the variables in the equation for $\pi_{2,t-1}$ are denominated in units of country 1's prices, as follows:

$$
\begin{aligned}
(C7)\quad \pi_{2,t-1} = {} & \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\tilde{p}_t\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)+\gamma_1\bar{y}}\left(A_1 g_{2,t}\tilde{p}_t+A_2 b_{2,t}\tilde{p}_t+A_3\pi^e_{2,t}\tilde{p}_t\right) \\
& + \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\tilde{p}_t\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)+\gamma_1\bar{y}}\left(A_4 g_{1,t}+A_5 b_{1,t}+A_6\pi^e_{1,t}+A_7 b_{m,t}\right) \\
& + \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\tilde{p}_t\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)+\gamma_1\bar{y}}\left(\Upsilon_2 i_{t-1}-\bar{y}\right) \\
& + \frac{\bar{y}}{\Upsilon_2\tilde{p}_t\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)+\gamma_1\bar{y}}\left[\gamma_1\pi^e_{2,t-1}+\gamma_4(\pi_{1,t-1}-\pi^e_{1,t-1})\right]
\end{aligned}
$$

Substituting (C7) into (C5) to solve for $\pi_{1,t-1}$, the equilibrium inflation rate in country 1, yields the result given on the next page:

$$(C8)\qquad \pi_{1,t-1} = \left[\frac{A_1[\alpha^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_4\alpha\gamma_2\bar{y}}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] g_{1,t}$$

$$+ \left[\frac{A_2[\alpha^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_5\alpha\gamma_2\bar{y}}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] b_{1,t}$$

$$+ \left[\frac{A_3[\alpha^2\Upsilon_2\tilde{p}_T(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_6\alpha\gamma_2\bar{y}}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] \pi^e_{1,t}$$

$$+ \left[\frac{A_4[\alpha^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_1\alpha\gamma_2\bar{y}}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] g_{2,t}\tilde{p}_t$$

$$+ \left[\frac{A_5[\alpha^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_2\alpha\gamma_2\bar{y}}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] b_{2,t}\tilde{p}_t$$

$$+ \left[\frac{A_6[\alpha^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_3\alpha\gamma_2\bar{y}}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] \pi^e_{2,t}\tilde{p}_t$$

$$+ \left[\frac{\alpha^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y} + \alpha\gamma_2\bar{y}}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] A_7 b_{m,t}$$

$$+ \left[\frac{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\Upsilon_1\gamma_1\bar{y} + \alpha\Upsilon_2\tilde{p}_t\gamma_2\bar{y}}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] i_{t-1}$$

$$+ \left[\frac{\bar{y}\,(\,\alpha\gamma_3\Upsilon_2\tilde{p}_t + \bar{y}\,)}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] \pi^e_{1,t-1}$$

$$- \left[\frac{\alpha\bar{y}\gamma_2\Upsilon_2\tilde{p}_t}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right] \pi^e_{2,t-1}$$

$$- \left[\frac{\alpha\bar{y}[\alpha\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\bar{y} + \gamma_2\bar{y}]}{\alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1\Upsilon_1+\gamma_3\Upsilon_2\tilde{p}_t) + \overline{y^2}} \right]$$

Equation (C8) gives equilibrium inflation in country 1, with all variables deflated by country 1's price index.

In order to calculate country 2's equilibrium inflation rate, with all the variables deflated by country 2's consumer price index, it is necessary to multiply both sides of equation (C1) by $(1/\tilde{p})$:

$$(C9)\quad \frac{y_{1,t}}{\tilde{p}_t} = \frac{\bar{y}}{\tilde{p}_t}\left[1 + \frac{\gamma_3}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}(\pi_{2,t-1} - \pi^e_{2,t-1})\right] - \frac{\bar{y}}{\tilde{p}_t}\left[\frac{\gamma_2}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}(\pi_{1,t-1} - \pi^e_{1,t-1})\right]$$

Now, substituting (C3) into (C9) to solve for $\pi_{1,t-1}$ as a function of the pre-determined, exogenous and policy variables, and $\pi_{2,t-1}$, yields:

$$(C10)\quad \pi_{1,t-1} = \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\frac{Y_1}{\tilde{p}_t}\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\frac{\bar{y}}{\tilde{p}_t}}\left(A_1 g_{1,t}\frac{1}{\tilde{p}_t} + A_2 b_{1,t}\frac{1}{\tilde{p}_t} + A_3\pi^e_{1,t}\frac{1}{\tilde{p}_t}\right)$$
$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\frac{Y_1}{\tilde{p}_t}\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\frac{\bar{y}}{\tilde{p}_t}}\left(A_3\pi^e_{1,t}\frac{1}{\tilde{p}_t} + A_4 g_{2,t} + A_5 b_{2,t}\right)$$
$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\frac{Y_1}{\tilde{p}_t}\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\frac{\bar{y}}{\tilde{p}_t}}\left(A_6\pi^e_{2,t} + A_7 b_{m,t}\frac{1}{\tilde{p}_t} + \frac{Y_1}{\tilde{p}_t}i_{t-1} - \frac{\bar{y}}{\tilde{p}_t}\right)$$
$$+ \frac{\frac{\bar{y}}{\tilde{p}_t}}{\frac{Y_1}{\tilde{p}_t}\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\frac{\bar{y}}{\tilde{p}_t}}\left[\gamma_3\pi^e_{1,t-1} + \gamma_2(\pi_{2,t-1} - \pi^e_{2,t-1})\right]$$

Likewise substituting (C4) into (C2) gives $\pi_{2,t-1}$ as a function of the pre-determined, exogenous and policy variables, and $\pi_{1,t-1}$:

$$(C11)\quad \pi_{2,t-1} = \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)+\gamma_1\frac{\bar{y}}{\tilde{p}_t}}\left(A_1 g_{2,t}+A_2 b_{2,t}+A_3\pi^e_{2,t}\right)$$

$$+\frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)+\gamma_1\frac{\bar{y}}{\tilde{p}_t}}\left(A_4 g_{1,t}\frac{1}{\tilde{p}_t}+A_5 b_{1,t}\frac{1}{\tilde{p}_t}+A_6\pi^e_{1,t}\frac{1}{\tilde{p}_t}\right)$$

$$+\frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)+\gamma_1\frac{\bar{y}}{\tilde{p}_t}}\left(A_7 b_{m,t}\frac{1}{\tilde{p}_t}+\Upsilon_2 i_{t-1}-\frac{\bar{y}}{\tilde{p}_t}\right)$$

$$+\frac{\frac{\bar{y}}{\tilde{p}_t}}{\Upsilon_2\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)+\gamma_1\frac{\bar{y}}{\tilde{p}_t}}\left[\gamma_1\pi^e_{2,t-1}+\gamma_4(\pi_{1,t-1}-\pi^e_{1,t-1})\right]$$

Substituting equation (C10) into (C11) one can solve for $\pi_{2,t-1}$, the equilibrium inflation rate in country 2, as given below:

$$(C12)\quad \pi_{2,t-1} = \left[\frac{A_1\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right]+A_4\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]g_{2,t}$$

$$+\left[\frac{A_2\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right]+A_5\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]b_{2,t}$$

$$+\left[\frac{A_3\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right]+A_6\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]\pi^e_{2,t}$$

$$+\left[\frac{A_4\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right]+A_1\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]g_{1,t}\frac{1}{\tilde{p}_t}$$

$\pi_{2,t-1}$ continued:

$$+\left[\frac{A_5\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right]+A_2\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]b_{1,t}\frac{1}{\tilde{p}_t}$$

$$+\left[\frac{A_6\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right]+A_3\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]\pi^{\bullet}_{1,t}\frac{1}{\tilde{p}_t}$$

$$+\left[\frac{\left(\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}+\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}\right)A_7}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]b_{m,t}\frac{1}{\tilde{p}_t}$$

$$+\left[\frac{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\Upsilon_2\gamma_3\frac{\bar{y}}{\tilde{p}_t}+\alpha\Upsilon_1\frac{1}{\tilde{p}_t}\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]i_{t-1}$$

$$-\left[\frac{\frac{\bar{y}}{\tilde{p}_t}\left(\alpha\gamma_4\ \Upsilon_1\frac{1}{\tilde{p}_t}+\frac{\bar{y}}{\tilde{p}_t}\right)}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2 p(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]\pi^{\bullet}_{1,t-1}$$

$$+\left[\frac{\alpha\frac{\bar{y}}{\tilde{p}_t}\ \gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}\right]\pi^{\bullet}_{2,t-1}$$

$$-\frac{\alpha\frac{\bar{y}}{\tilde{p}_t}\left(\alpha\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4)+\frac{\bar{y}}{\tilde{p}_t}(\gamma_3+\gamma_4)\right)}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4)+\alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2)+\frac{\bar{y}^2}{\tilde{p}_t^2}}$$

All real variables in equation (C12) are deflated by country 2's consumer price index.

Substituting equation (C8) into (C3) and combining terms gives the equilibrium output equation for country 1:

$$
\begin{aligned}
(C13)\quad y_{1,t} = & \left[\frac{A_1[\Omega_1 - \alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) - \Upsilon_1\alpha\gamma_1\bar{y}] - A_4\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] g_{1,t} \\
& + \left[\frac{A_2[\Omega_1 - \alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) - \Upsilon_1\alpha\gamma_1\bar{y}] - A_5\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] b_{1,t} \\
& + \left[\frac{A_3[\Omega_1 - \alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) - \alpha\Upsilon_1\gamma_1\bar{y}] - A_6\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] \pi^e_{1,t} \\
& + \left[\frac{A_4[\Omega_1 - \alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) - \alpha\Upsilon_1\gamma_1\bar{y}] - A_1\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] g_{2,t}\tilde{p}_t \\
& + \left[\frac{A_5[\Omega_1 - \alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) - \alpha\Upsilon_1\gamma_1\bar{y}] - A_2\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] b_{2,t}\tilde{p}_t \\
& + \left[\frac{A_6[\Omega_1 - \alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) - \alpha\Upsilon_1\gamma_1\bar{y}] - A_3\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] \pi^e_{2,t}\tilde{p}_t \\
& + \left[\frac{\Omega_1 - \alpha^2\Upsilon_1\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) - \alpha\Upsilon_1\gamma_1\bar{y} - \alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] A_7 b_{m,t} \\
& + \left[\frac{\Upsilon_1\Omega_1 - \alpha^2\Upsilon_1^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) - \alpha\Upsilon_1^2\gamma_1\bar{y} - \alpha\Upsilon_1\Upsilon_2\tilde{p}_t\gamma_2\bar{y}}{\Omega_1}\right] i_{t-1} \\
& - \left[\frac{\Upsilon_1\bar{y}(\alpha\gamma_3\Upsilon_2\tilde{p}_t + \bar{y})}{\Omega_1}\right] \pi^e_{1,t-1} \\
& - \left[\frac{\alpha\Upsilon_1\Upsilon_2\tilde{p}_t\bar{y}\gamma_2}{\Omega_1}\right] \pi^e_{2,t-1} \\
& + \frac{\alpha\Upsilon_1\bar{y}(\alpha\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \bar{y}(\gamma_1+\gamma_2))}{\Omega_1}
\end{aligned}
$$

where:

$$\Omega_1 = \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1 Y_1+\gamma_3 Y_2\tilde{p}_t) + \bar{y}^2$$

Making use of this definition of Ω_1, equation (C13) can be simplified to arrive at the equilibrium output equation for country 1, as given in chapter 1:

$$
\begin{aligned}
(C14) \quad y_{1,t} = {} & \left[\frac{A_1[\alpha\gamma_3 Y_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_4\alpha Y_1\gamma_2\bar{y}}{\Omega_1}\right] g_{1,t} \\
& + \left[\frac{A_2[\alpha\gamma_3 Y_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_5\alpha Y_1\gamma_2\bar{y}}{\Omega_1}\right] b_{1,t} \\
& + \left[\frac{A_3[\alpha\gamma_3 Y_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_6\alpha Y_1\gamma_2\bar{y}}{\Omega_1}\right] \pi^e_{1,t} \\
& + \left[\frac{A_4[\alpha\gamma_3 Y_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_1\alpha Y_1\gamma_2\bar{y}}{\Omega_1}\right] g_{2,t}\tilde{p}_t \\
& + \left[\frac{A_5[\alpha\gamma_3 Y_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_2\alpha Y_1\gamma_2\bar{y}}{\Omega_1}\right] b_{2,t}\tilde{p}_t \\
& + \left[\frac{A_6[\alpha\gamma_3 Y_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_3\alpha Y_1\gamma_2\bar{y}}{\Omega_1}\right] \pi^e_{2,t}\tilde{p}_t \\
& + \left[\frac{\alpha\gamma_3 Y_2\tilde{p}_t\bar{y} + \bar{y}^2 - \alpha Y_1\gamma_2\bar{y}}{\Omega_1}\right] A_7 b_{m,t} \\
& + \left[\frac{\alpha Y_1 Y_2\tilde{p}_t\bar{y}(\gamma_3-\gamma_2) + Y_1\bar{y}^2}{\Omega_1}\right] i_{t-1} \\
& - \left[\frac{Y_1\bar{y}(\alpha\gamma_3 Y_2\tilde{p}_t + \bar{y})}{\Omega_1}\right] \pi^e_{1,t-1} \\
& - \left[\frac{\alpha Y_1 Y_2\tilde{p}_t\bar{y}\gamma_2}{\Omega_1}\right] \pi^e_{2,t-1} \\
& + \frac{\alpha Y_1\bar{y}(\alpha Y_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \bar{y}(\gamma_1+\gamma_2))}{\Omega_1}
\end{aligned}
$$

Substituting equation (C12) into (C4) and combining terms gives the equilibrium output equation for country 2:

$$
\begin{aligned}
(C15)\quad y_{2,t} = {} & \left[\frac{A_1 \left[\Omega_2 - \alpha^2 Y_1 \frac{1}{\bar{p}_t} Y_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 Y_2 \frac{\bar{y}}{\bar{p}_t}\right] - A_4 \alpha\gamma_4 Y_2 \frac{\bar{y}}{\bar{p}_t}}{\Omega_2} \right] g_{2,t} \\
& + \left[\frac{A_2 \left[\Omega_2 - \alpha^2 Y_1 \frac{1}{\bar{p}_t} Y_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 Y_2 \frac{\bar{y}}{\bar{p}_t}\right] - A_5 \alpha\gamma_4 Y_2 \frac{\bar{y}}{\bar{p}_t}}{\Omega_2} \right] b_{2,t} \\
& + \left[\frac{A_3 \left[\Omega_2 - \alpha^2 Y_1 \frac{1}{\bar{p}_t} Y_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 Y_2 \frac{\bar{y}}{\bar{p}_t}\right] - A_6 \alpha\gamma_4 Y_2 \frac{\bar{y}}{\bar{p}_t}}{\Omega_2} \right] \pi^{\bullet}_{2,t} \\
& + \left[\frac{A_4 \left[\Omega_2 - \alpha^2 Y_1 \frac{1}{\bar{p}_t} Y_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 Y_2 \frac{\bar{y}}{\bar{p}_t}\right] - A_1 \alpha\gamma_4 Y_2 \frac{\bar{y}}{\bar{p}_t}}{\Omega_2} \right] g_{1,t} \frac{1}{\bar{p}_t} \\
& + \left[\frac{A_5 \left[\Omega_2 - \alpha^2 Y_1 \frac{1}{\bar{p}_t} Y_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 Y_2 \frac{\bar{y}}{\bar{p}_t}\right] - A_2 \alpha\gamma_4 Y_2 \frac{\bar{y}}{\bar{p}_t}}{\Omega_2} \right] b_{1,t} \frac{1}{\bar{p}_t} \\
& + \left[\frac{A_6 \left[\Omega_2 - \alpha^2 Y_1 \frac{1}{\bar{p}_t} Y_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 Y_2 \frac{\bar{y}}{\bar{p}_t}\right] - A_3 \alpha\gamma_4 Y_2 \frac{\bar{y}}{\bar{p}_t}}{\Omega_2} \right] \pi^{\bullet}_{1,t} \frac{1}{\bar{p}_t} \\
& + \left[\frac{\Omega_2 - \alpha^2 Y_1 \frac{1}{\bar{p}_t} Y_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 Y_2 \frac{\bar{y}}{\bar{p}_t} - \alpha\gamma_4 Y_2 \frac{\bar{y}}{\bar{p}_t}}{\Omega_2} \right] A_7 b_{m,t} \frac{1}{\bar{p}_t} \\
& + \left[\frac{Y_2 \Omega_2 - \alpha^2 Y_1 \frac{1}{\bar{p}_t} Y_2^2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha Y_2^2 \gamma_3 \frac{\bar{y}}{\bar{p}_t} - \alpha Y_1 \frac{1}{\bar{p}_t} Y_2 \gamma_4 \frac{\bar{y}}{\bar{p}_t}}{\Omega_2} \right] i_{t-1} \\
& + \left[\frac{\frac{\bar{y}}{\bar{p}_t} Y_2 \left(\alpha\gamma_4 Y_1 \frac{1}{\bar{p}_t} + \frac{\bar{y}}{\bar{p}_t}\right)}{\Omega_2} \right] \pi^{\bullet}_{1,t-1} \\
& - \left[\frac{\frac{\bar{y}}{\bar{p}_t} \alpha\gamma_1 Y_1 \frac{1}{\bar{p}_t} Y_2}{\Omega_2} \right] \pi^{\bullet}_{2,t-1} \\
& - \frac{\alpha \frac{\bar{y}}{\bar{p}_t} Y_2 \left(\alpha Y_1 \frac{1}{\bar{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \frac{\bar{y}}{\bar{p}_t} (\gamma_3 + \gamma_4)\right)}{\Omega_2}
\end{aligned}
$$

where:

$$\Omega_2 = \alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\bar{y}}{\tilde{p}_t}\left(\gamma_1\frac{\Upsilon_{11}}{\tilde{p}_t}+\gamma_3\Upsilon_2\right) + \frac{\bar{y}^2}{\tilde{p}_t^2}$$

Making use of this definition of Ω_2, equation (C15) can be simplified to arrive at the equilibrium output equation for country 2 given in the text, as shown below:

$$(C16)\quad y_{2,t} = \left[\frac{A_1\left(\alpha\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}\frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}}\right) - A_4\alpha\gamma_4\Upsilon_2\frac{\bar{y}}{\tilde{p}_t}}{\Omega_2}\right]g_{2,t}$$

$$+\left[\frac{A_2\left(\alpha\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}\frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t}\right) - A_5\alpha\gamma_4\Upsilon_2\frac{\bar{y}}{\tilde{p}_t}}{\Omega_2}\right]b_{2,t}$$

$$+\left[\frac{A_3\left(\alpha\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}\frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t}\right) - A_6\alpha\gamma_4\Upsilon_2\frac{\bar{y}}{\tilde{p}_t}}{\Omega_2}\right]\pi^{e}_{2,t}$$

$$+\left[\frac{A_4\left(\alpha\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}\frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t}\right) - A_1\alpha\gamma_4\Upsilon_2\frac{\bar{y}}{\tilde{p}_t}}{\Omega_2}\right]g_{1,t}\frac{1}{\tilde{p}_t}$$

$$+\left[\frac{A_5\left(\alpha\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}\frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t}\right) - A_2\alpha\gamma_4\Upsilon_2\frac{\bar{y}}{\tilde{p}_t}}{\Omega_2}\right]b_{1,t}\frac{1}{\tilde{p}_t}$$

$$+\left[\frac{A_6\left(\alpha\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}\frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}}\right) - A_3\alpha\gamma_4\Upsilon_2\frac{\bar{y}}{\tilde{p}_t}}{\Omega_2}\right]\pi^{e}_{1,t}\frac{1}{\tilde{p}_t}$$

$$+\left[\frac{\alpha\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}\frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t} - \alpha\gamma_4\Upsilon_2\frac{\bar{y}}{\tilde{p}_t}}{\Omega_2}\right]A_7 b_{m,t}\frac{1}{\tilde{p}_t}$$

$$+\left[\frac{\alpha\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2\frac{\bar{y}}{\tilde{p}_t}(\gamma_1-\gamma_4) + \Upsilon_2\frac{\bar{y}^2}{\tilde{p}_t^2}}{\Omega_2}\right]i_{t-1}$$

$$+\left[\frac{\frac{\bar{y}}{\tilde{p}_t}\Upsilon_2\left(\alpha\gamma_4\Upsilon\frac{1}{\tilde{p}_t} + \frac{\bar{y}}{\tilde{p}_t}\right)}{\Omega_2}\right]\pi^{e}_{1,t-1}$$

$y_{2,t}$ continued:

$$+\left[\frac{\frac{\bar{y}}{\tilde{p}_t}\,\alpha\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2}{\Omega_2}\right]\pi^{e}_{2,t-1}$$

$$-\ \frac{\alpha\frac{\bar{y}}{\tilde{p}_t}\,\Upsilon_2\left(\alpha\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4)\ +\ \frac{\bar{y}}{\tilde{p}_t}(\gamma_3+\gamma_4)\right)}{\Omega_2}$$

APPENDIX D

COMPARATIVE STATICS

This appendix uses the inflation and output equations for country 1 (Chapter 1, Tables IX and X), to derive the signs of the comparative statics given in Chapter 1, Tables XI and XII.

<u>Effect on inflation of a change in one of the exogenous variables.</u>

The denominator of each coefficient is Ω_1. Given that Y_2, Y_1, α, γ_i, and $\bar{y}$ are positive, the sign of the denominator depends upon the sign of the term $(\gamma_1\gamma_3 - \gamma_2\gamma_4)$.

Determining the sign of $(\gamma_1\gamma_3 - \gamma_2\gamma_4)$:

$$(\gamma_1\gamma_3 - \gamma_2\gamma_4) = \left(\frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1} + (1-\gamma)p_{2,t-1}}\right)\left(\frac{\gamma p_{2,t-1}}{\gamma p_{2,t-1} + (1-\gamma)p_{1,t-1}}\right)$$
$$- \left(\frac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1} + (1-\gamma)p_{2,t-1}}\right)\left(\frac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1} + (1-\gamma)p_{1,t-1}}\right)$$
$$= \left(\frac{(2\gamma - 1)\ p_{1,t-1}\ p_{2,t-1}}{(\gamma p_{1,t-1} + (1-\gamma)p_{2,t-1})\ (\gamma p_{2,t-1} + (1-\gamma)p_{1,t-1})}\right)$$

Since $\gamma > 1/2$ this term is positive. Therefore, the denominator of each coefficient is positive. Determining the sign of the coefficients on the exogenous variables in the inflation equation thus, becomes a matter of determining the sign of the numerators of these coefficients.

Since $(\gamma_1\gamma_3 - \gamma_2\gamma_4)>0$, the signs of the numerators of the first seven coefficients in the equilibrium inflation equation for country 1, depend upon the signs of the aggregate demand coefficients (A_1 through A_7). These seven coefficients are examined below.

a) Real expenditures by the government of country 1:

$$\frac{A_1[\alpha^2\Upsilon_2\beta_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_4\alpha\gamma_2\bar{y}}{\Omega_1}$$

Since $A_1 > 0$ and $A_4 > 0$ the numerator is positive. Thus an increase in own government expenditures will increase inflation in country 1.

b) Bond issues by the government of country 1:

$$\frac{A_2[\alpha^2\Upsilon_2\beta_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_5\alpha\gamma_2\bar{y}}{\Omega_1}$$

$A_5 < 0$ but A_2 may be positive or negative. If $A_2 < 0$, then the numerator is negative. If $A_2 > 0$ then the sign of the numerator is indeterminate. This result follows since $|A_5| > A_2$.

c) Inflationary expectations in country 1:

$$\frac{A_3[\alpha^2\Upsilon_2\beta_T(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_6\alpha\gamma_2\bar{y}}{\Omega_1}$$

$A_3 > 0$ but A_6 may be positive or negative. If $A_6 > 0$, then the numerator is positive. If $A_6 < 0$, the numerator is also positive. To prove this note that if:

$$(D1) \quad A_3\alpha\gamma_1\bar{y} > A_6\alpha\gamma_2\bar{y}$$

then the sign of the numerator is positive. Given that $\gamma > 1/2$ it follows that $\gamma_1 > \gamma_2$. It also true that $A_3 > |A_6|$. Thus the inequality given by (D1) holds. Therefore, an increase in inflationary expectations in country 1 will increase inflation in country 1.

d) Real expenditures by the government of country 2:

$$\frac{A_4[\alpha^2\Upsilon_2\bar{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_1\alpha\gamma_2\bar{y}}{\Omega_1}$$

Since $A_1 > 0$ and $A_4 > 0$ the numerator is positive. Thus, an increase in government expenditures by country 2 will increase inflation in country 1.

e) Bond issues by the government of country 2:

$$\frac{A_5[\alpha^2\Upsilon_2\bar{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_2\alpha\gamma_2\bar{y}}{\Omega_1}$$

$A_5 < 0$ but, as noted above, A_2 may be positive or negative. If $A_2 < 0$, then the numerator is negative. If $A_2 > 0$ then the sign of the numerator is negative if:

$$(D2) \quad A_5\alpha\gamma_1\bar{y} > A_2\alpha\gamma_2\bar{y}$$

As noted above $\gamma_1 > \gamma_2$, and since $|A_5| > A_2$, the inequality given in (D2) does hold. Thus, an increase in bond issues by country 2 will decrease inflation in country 1.

f) inflationary expectations in country 2:

$$\frac{A_6[\alpha^2\Upsilon_2\bar{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_3\alpha\gamma_2\bar{y}}{\Omega_1}$$

$A_3 > 0$, but A_6 may be positive or negative. If $A_6 > 0$, then the numerator of this coefficient is positive. If $A_6 < 0$ then the sign of the numerator is indeterminate. This follows since $A_3 > |A_6|$. Thus, an increase in inflationary expectations in country 2 will increase inflation in country 1 if $A_6 > 0$, but the effect on inflation is indeterminate if $A_6 < 0$.

g) **Bond holdings of the central bank:**

$$\frac{A_7[\alpha^2\Upsilon_2\beta_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y} + \alpha\gamma_2\bar{y}]}{\Omega_1}$$

Given that $A_7 > 0$ an increase in bond holdings by the central bank will increase inflation in country 1.

The remaining four coefficients do not contain aggregate demand parameters. The signs of the numerators of these coefficients are all positive as shown below:

h) **last period's nominal interest rate:**

$$\frac{\alpha^2\Upsilon_1\Upsilon_2\beta_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\Upsilon_1\gamma_1\bar{y} + \alpha\Upsilon_2\beta_t\gamma_2\bar{y}}{\Omega_1}$$

Since $(\gamma_1\gamma_3 - \gamma_2\gamma_4)>0$ the numerator is definitely positive. An increase in last period's nominal interest rate will increase inflation in country 1.

i) **last period's inflationary expectations in country 1:**

$$\frac{\bar{y}\,(\alpha\gamma_3\Upsilon_2\beta_t + \bar{y})}{\Omega_1}$$

All the terms in the numerator are positive, thus, an increase in last period's inflationary expectations in country 1 will increase inflation in country 1 this period.

j) **last period's inflationary expectations in country 2:**

$$\frac{\alpha\bar{y}\gamma_2\Upsilon_2\beta_t}{\Omega_1}$$

The numerator is positive as all of the terms are positive. Taking account of the negative sign attached to this coefficient, an increase in last period's inflationary expectations in country 2 will decrease inflation in country 1 this period.

h) constant term

$$\frac{\alpha\bar{y}\,[\alpha\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\bar{y} + \gamma_2\bar{y}\,]}{\Omega_1}$$

Since $(\gamma_1\gamma_3 - \gamma_2\gamma_4)>0$ the numerator is positive. Taking account of the negative sign attached to this coefficient, the constant term is thus negative.

<u>Effect on output of a change in one of the exogenous variables.</u>

The denominator of each coefficient is Ω_1. As shown above, this term is positive. Thus, determining the sign of the coefficients on the exogenous variables in the output equation becomes a matter of determining the sign of the numerators of these coefficients.

a) Real expenditures by the government of country 1:

$$\frac{A_1\,[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2\,] - A_4\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}$$

$A_1 > 0$ and $A_4 > 0$. Given that $\gamma>\frac{1}{2}$, $\gamma_3>\gamma_2$. If

$$(D3) \quad \Upsilon_2\tilde{p}_t > \Upsilon_1$$

then the numerator is definitely positive. A necessary condition for the inequality in (D3) to hold is for country 1 to have had a larger current account deficit last period than that of country 2. A sufficient condition is that country 1 was a net debtor and country 2 was a net debtor last period.

If this the inequality given by (D3) does not hold then a sufficient condition for the numerator to be positive is:

$$(D4)\quad A_1\bar{y} > A_4\alpha\gamma_2 Y_1$$

Making note of the fact that $A_4=(1-A_1)$, (D4) can be rewritten as:

$$(D5)\quad \bar{y} > \frac{1-A_1}{A_1}\alpha\gamma_2 Y_1$$

the terms: $\frac{1-A_1}{A_1}$ and γ_2 are both less than one. Thus, the inequality given by (D5) can be reduced to:

$$(D6)\quad \bar{y} > \alpha Y_1$$

Substituting the expression for Y_1 in (D6) yields:

$$(D7)\quad \bar{y} > \alpha[(2A_{10}-A_9-A_8)b_{1m,t-1} + (A_8-A_9)(b_{21,t-1}\tilde{p}_{t-1}-b_{12,t-1})]$$

Since, as shown in appendix B, $b_{21,t-1}\tilde{p}_{t-1} \geq b_{1m,t-1}$, the inequality given by (D7) can be reduced to:

$$(D8)\quad \bar{y} > 2\alpha(A_{10}-A_9)(b_{21,t-1}-b_{12,t-1})$$

This condition should be met as long as α is not too large. Thus, an increase in own government expenditures will increase output in country 1.

b) Bond issues by the government of country 1:

$$\frac{A_2[\alpha\gamma_3 Y_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_5\alpha Y_1\gamma_2\bar{y}}{\Omega_1}$$

$A_5 < 0$ but A_2 may be positive or negative. If $A_2 > 0$, then the numerator is positive. If $A_2 < 0$ and country 1 was a net debtor while country 2 was a net creditor last period then Y_2 will be large relative to Y_1. In this case the sign of the numerator is likely to be negative. If $A_2 < 0$ and country 1 was a net creditor while country 2 was a net debtor last period then Y_1 will be large relative to Y_2. In this case the sign of the numerator is indeterminate.

c) Inflationary expectations in country 1:

$$\frac{A_3\,[\alpha\gamma_3 Y_2 \tilde{p}_t \overline{y} + \overline{y^2}\,] - A_6 \alpha Y_1 \gamma_2 \overline{y}}{\Omega_1}$$

$A_3 > 0$ but A_6 may be positive or negative. If $A_6 < 0$, then the numerator is positive. If $A_6 > 0$, since $A_3 > |A_6|$, the numerator will be positive if the inequality given by (D8) holds. Therefore, an increase in inflationary expectations in country 1 will increase inflation in country 1.

d) Real expenditures by the government of country 2:

$$\frac{A_4\,[\alpha\gamma_3 Y_2 \tilde{p}_t \overline{y} + \overline{y^2}\,] - A_1 \alpha Y_1 \gamma_2 \overline{y}}{\Omega_1}$$

Since $A_1 > 0$ and $A_4 > 0$ the numerator is positive. If country 1 was a net debtor while country 2 was a net creditor last period then Y_2 will be large relative to Y_1. In this case the sign of the numerator is likely to be negative. If country 1 was a net creditor while country 2 was a net debtor last period then Y_1 will be large relative to Y_2. In this case the sign of the numerator is indeterminate.

e) Bond issues by the government of country 2:

$$\frac{A_5[\alpha\gamma_3 Y_2 \beta_t \bar{y} + \bar{y}^2] - A_2 \alpha Y_1 \gamma_2 \bar{y}}{\Omega_1}$$

$A_5 < 0$ but A_2 may be positive or negative. If $A_2 > 0$, then the numerator is negative. If $A_2 < 0$ then since $|A_5| > A_2$, if the inequality given in (D3) holds, the numerator is negative. If this inequality does not hold, then a sufficient condition for the numerator to be positive is given by (D8).

f) inflationary expectations in country 2:

$$\frac{A_6[\alpha\gamma_3 Y_2 \beta_t \bar{y} + \bar{y}^2] - A_3 \alpha Y_1 \gamma_2 \bar{y}}{\Omega_1}$$

$A_3 > 0$, but A_6 may be positive or negative. If $A_6 < 0$, then the numerator of this coefficient is negative. If $A_6 > 0$, then since $A_3 > |A_6|$, then the sign of the numerator is indeterminate. If country 1 was a net debtor while country 2 was a net creditor last period then Y_2 will be large relative to Y_1. In this case the sign of the numerator is likely to be negative. If country 1 was a net creditor while country 2 was a net debtor last period then Y_1 will be large relative to Y_2. In this case the sign of the numerator is indeterminate.

g) Bond holdings of the central bank:

$$\frac{A_7[\alpha\gamma_3 Y_2 \beta_t \bar{y} + \bar{y}^2 - \alpha Y_1 \gamma_2 \bar{y}]}{\Omega_1}$$

Given that $A_7 > 0$ if the inequality given by (D3) holds then the numerator is positive. If this inequality does not hold, then a sufficient condition for the numerator to be positive is given by (D8).

h) last period's nominal interest rate:

$$\frac{\alpha \Upsilon_1 \Upsilon_2 \bar{p}_t \bar{y}(\gamma_3 - \gamma_2) + \bar{y}}{\Omega_1}$$

Since $\gamma > \frac{1}{2}$, the term $(\gamma_3 - \gamma_2) > 0$. This ensures that the numerator is positive, so an increase in last period's nominal interest rate will increase output in country 1 this period.

i) last period's inflationary expectations in country 1:

$$\frac{\bar{y}\,\Upsilon_1(\alpha \gamma_3 \Upsilon_2 \bar{p}_t + \bar{y})}{\Omega_1}$$

All the terms in the numerator are positive. Taking into account the negative sign, an increase in last period's inflationary expectations in country 1 will decrease output in country 1 this period.

j) last period's inflationary expectations in country 2:

$$\frac{\alpha \bar{y}\,\gamma_2 \Upsilon_1 \Upsilon_2 \bar{p}_t}{\Omega_1}$$

The numerator is positive as all of the terms are positive. An increase in last period's inflationary expectations in country 2 will increase output in country 1 this period.

h) constant term

$$\frac{\alpha \Upsilon_1 \bar{y}\,[\alpha \Upsilon_2 \bar{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \gamma_1\bar{y} + \gamma_2\bar{y}]}{\Omega_1}$$

Since $(\gamma_1\gamma_3 - \gamma_2\gamma_4) > 0$ the numerator is positive, thus the constant term is positive.

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