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DEVELOPMENT OF AN APPROXIMATE ACOUSTIC POINT
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THE DEVELOPMENT OF AN APPROXIMATE ACOUSTIC POINT SOURCE FOR ACOUSTICS TESTING

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ABSTRACT

THE DEVELOPMENT OF AN APPROXIMATE ACOUSTIC POINT SOURCE FOR ACOUSTICS TESTING

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The point source is a powerful modeling tool in many scientific and engineering fields. In acoustics, the point source is an infinitesimal object that displaces the surrounding fluid equally in all directions. The point source is a powerful modeling tool because in linear systems analysis, which includes acoustics, the principle of superposition allows complex sources of excitation to be modeled by a collection of simple sources, or point sources. The point source can also be a powerful tool in experimentation, and models which use simple sources can be more easily verified. A physical approximation to a point source for experimental work is developed here. The acoustics theory dealing with point sources is discussed, and acoustic intensity is introduced. The acoustic exciter is presented, and the physics of the exciter are compared with the physics of the point source. An analytical model is developed for design optimization and source calibration. The experimental techniques used in the model verification and parameter identification are presented. Sound intensity comparisons with conventional speakers clearly show that the acoustic exciter is an excellent approximation to the point source.

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INTRODUCTION

An omnidirectional acoustic source is needed for experimental verification of acoustic models and principles. The ideal point source is a basic building block in acoustics analysis and modeling because any source of sound can be modeled as a set of ideal point sources. Point sources are also used in acoustic finite element models, where the driving term at an interior acoustic node is either zero or the volume acceleration of a point volume source at that node. Similarly, the principle of acoustic reciprocity is defined in terms of ideal point volume sources, and this is the primary motivation for this work (Fahy, 1992). An ideal point source cannot be physically realized, because it has zero physical extent. The development of a physical approximation to the ideal point source is the subject of the paper. The source must have finite physical size, but can approximate a point source in the far field by having an omnidirectional intensity pattern there.

The design problem is the development of an acoustic source that approximates an ideal point source in the far field, has a known volume velocity output function, and is easily constructed from ordinary components. The volume velocity output function must be known because the volume velocity of a point source is not an easily measured quantity. In the mechanical domain, excitation is provided by a shaker, and that excitation is measured with a force transducer. In acoustics, there is no such thing as an "acoustic force transducer," so the output of the source must be inferred from some easily measured input quantity. A candidate source must be constructed, modeled, and tested to indicate the performance of the design. The design process involves exercises in system modeling, acoustic intensity measurement and analysis, as well as parameter identification.

This paper demonstrates the methods and principles used during the development of the acoustic exciter. The ideal point source is examined first, and then the acoustic exciter is shown to be a good approximation to the point source. The principles and methods used in the development of the system model are presented. The experimental techniques used and the methods used to process the data are also presented in detail.

THE IDEAL POINT SOURCE, AN ACOUSTIC MONOPOLE

The ideal acoustic monopole emits acoustic power equally in all directions from an infinitesimally small point in space. The ideal point volume source can in theory be realized by a very small sphere with a periodically changing radius (Fig. 1). This model of a point source is valid only outside the region of the sphere, and is an exact model there.

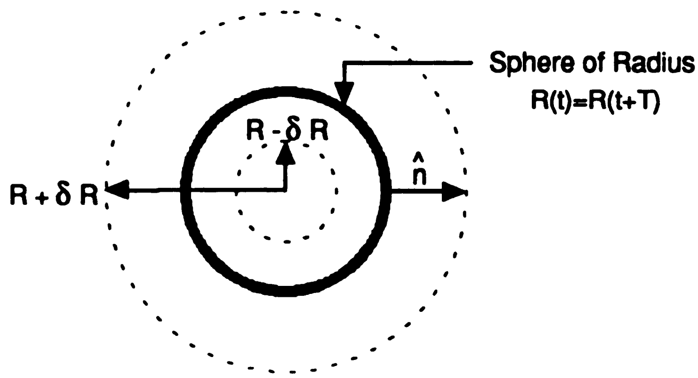

Figure 1. The Radially Oscillating Sphere; an Ideal Point Volume Source Model

Two important concepts when describing an acoustic source are efficiency, and symmetry. Efficiency describes how well the source converts energy into acoustic power flow, and symmetry deals with how that acoustic power flow is distributed in space. An efficient source is a volume source that does not have any surface motions that are "out of phase." When surface motions are "out of phase," efficiency is reduced, and the acoustic input to the system under test is also reduced.

The ideal point source, or acoustic monopole, is 100% efficient and symmetric. The source is 100% efficient because every part of the surface of the radially oscillating sphere has the same velocity outward or inward, and so every part of the surface has "in phase" motions. The source is 100% symmetric because there are surface velocities along every imaginable axis through the center of the source.

The unenclosed speaker is not a volume source; as the cone moves back and forth, the volume occupied by the speaker does not change. An unenclosed speaker has two surfaces, the front and the back. As the front of the speaker cone moves "out," the back of the speaker cone moves "in." (Fig. 2) This causes acoustic power to flow entirely from one surface to the other, and results in extremely low efficiency. The unenclosed speaker is not symmetric because the resulting power flow field is like that of a dipole.

Enclosed speakers are highly directional, and therefore unsuitable as point sources (Fig. 2). While the efficiency of an enclosed speaker is high, the needed symmetry is not present. The enclosed speaker has one emitting surface, and that surface occupies a small percentage of the total source surface area. If the entire speaker and enclosure were an emitting surface, with no "out of phase" cone motions, it would be suitable as a point source.

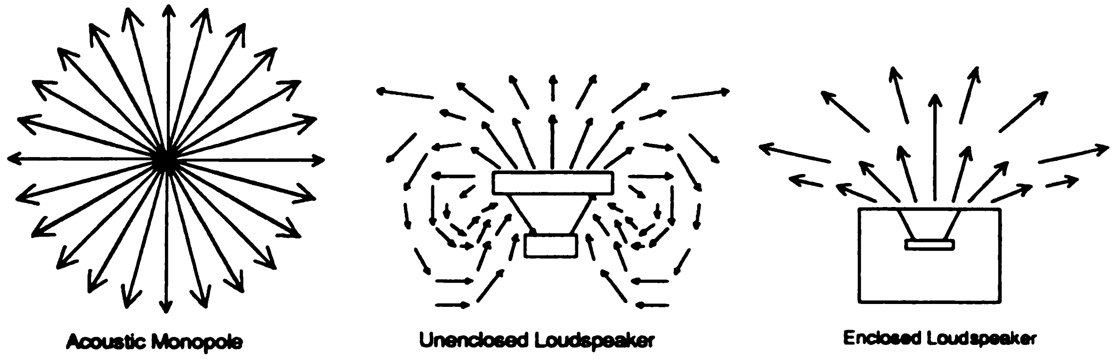

Figure 2. Intensity Field of a Loudspeaker Compared with an Acoustic Monopole

THE ACOUSTIC EXCITER

The acoustic exciter must imitate the mechanics of a radially oscillating sphere. A typical speaker cannot do this because of the high degree of directionality mentioned above. Two speakers could do this if they were pointed in opposite directions, driven such that the cones would move symmetrically, and the space between them sealed. If the speakers are placed face to face, this would result in a large emission area percentage. The acoustic exciter is just two identical speakers, bolted face to face on a spacer, and driven in series (Fig. 3a, 3b). It was constructed using two Radio Shack 4" woofers, and a 1/2" aluminum spacer. The two speaker cones, moving symmetrically, approximate the physics of a radially oscillating sphere (Fig. 4). As the two cones move away from one another (Thick line), the net volume occupied by the acoustic exciter increases, and as they move together (dashed line), the volume decreases. Because the percentage of emitting area is near 100%, the acoustic exciter should be a good point source candidate. The intensity field around the source will be sampled so that the quality of the point source approximation can be determined.

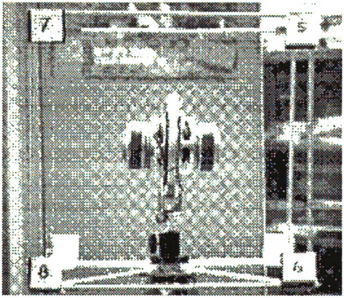

Figure 3a. Acoustic Exciter and Intensity Measurement Guide (Photograph)

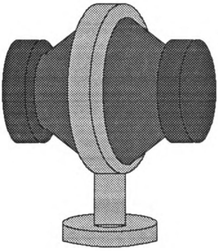

Figure 3b Sketch of Acoustic Exciter

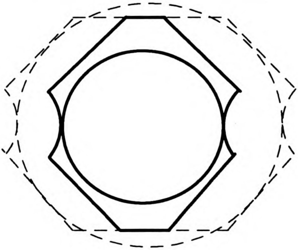

Figure 4. Speaker Cones Moving to Imitate the Radially Oscillating Sphere

ACOUSTIC INTENSITY TECHNIQUES

Acoustic intensity is a popular technique for determining acoustic energy propagation patterns, because intensity is a measure of the source's input to the acoustic field surrounding it. Sound pressure is a measure of the acoustic field's response to the source's input, and depends on both the source input as well as the system under test. The intensity field around the source will be sampled at several locations on the surface of an imaginary cube, so that the radiation pattern can be determined. Volume velocity will later be shown to be determined from these intensity measurements.

Acoustic intensity is measured using two phase calibrated microphones that are separated by a small spacer. More information on intensity can be found in the acoustics literature by Brüel and Kjær. Three orthogonal intensity component measurements must be made at each point in order to obtain the full spatial intensity vector.

MODELING THE ACOUSTIC EXCITER

An analytical model for the acoustic exciter is needed for prediction of source output, because the volume velocity of the source cannot be measured at test time. A model would also be needed for future design optimization. Depending on the frequency range of interest and the sound power levels needed, the correct choice of design variables can be critical to the device's performance. The design variables are the speakers, and the thickness of the spacer. The spacer thickness can be chosen arbitrarily, but speaker parameters discussed in this section are a consequence of the choice of speakers.

The acoustic exciter is modeled as a two degree of freedom dynamic system, where the degrees of freedom are the cone displacements. Each speaker individually is essentially a one degree of freedom spring-mass-dashpot system driven by an electromagnetic coil (Radcliffe, Gogate 1992). The stiffness, damping, and mass parameters for the cone are k_{cone} R_{cone} and m_{cone}, respectively. The electrical resistance and electromagnetic constant of the coil are R_{coil}, and bl, respectively. When the two speakers are placed together, and the air cavity between them is sealed, the air cavity acts like a stiff spring, with stiffness k_{air} (Fig. 5). The single input to the acoustic exciter is the applied voltage, e(t). The outputs of the model are the cone displacement and velocity, x(t), and v(t).

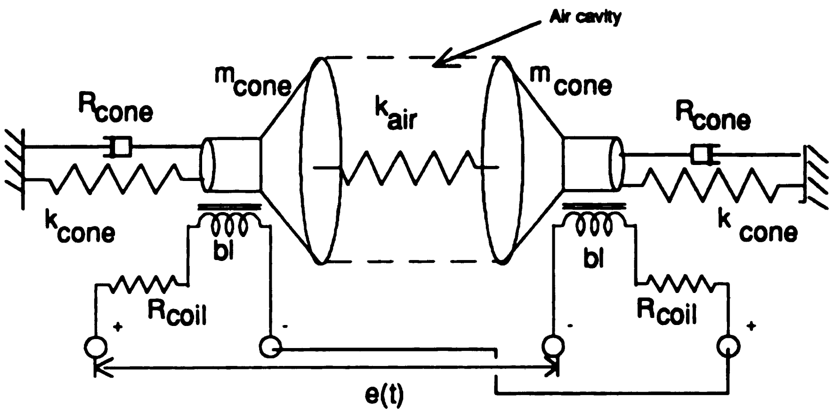

Figure 5. Schematic of Acoustic Exciter System Model

The equations of motion of the speaker cones (2) are derived from a force balance on each speaker cone, where the cones move horizontally with respect to gravity, as well as a voltage sum around the current loop (Appendix A). The equation can be reduced to one degree of freedom assuming that the cone motion is symmetric. This assumption is valid if the speakers are exactly identical, or if the frequency range of interest is low enough that small amounts of asymmetry between the two speakers do not invalidate the assumption of symmetric motion. Dissipative and stiffness terms have been collected such that $k_{eff} = k_{cone} + 2 \cdot k_{air}$, and $R_{eff}=R_{cone}+bl^2/R_{coil}$. R_{eff} is the total equivalent mechanical viscous friction coefficient.

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_{eff}}{m_{cone}} & -\dfrac{R_{eff}}{m_{cone}} \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} \dfrac{E(t)}{2} \cdot \begin{matrix} 0 \\ \dfrac{bl}{R_{coil} \cdot m_{cone}} \end{matrix} \end{bmatrix} \tag{2}$$

Further assuming that only steady state solutions are needed, the Laplace transformation is used to solve for cone velocity response $V(s)=\mathcal{L}[v(t)]$ due to applied voltage, $E(s)=\mathcal{L}[e(t)]$ (3).

$$\frac{V(s)}{E(s)} = \frac{s \cdot bl}{2R_{coil} \cdot (m_{cone} \cdot s^2 + R_{eff} \cdot s + k_{eff})} \tag{3}$$

The desired solution should be for volume velocity, $Q(s)=\mathcal{L}[q(t)]$, so the cone velocity solution is multiplied by the effective area of the convex surface of the backs of the speaker cones (4).

$$\frac{Q(s)}{E(s)} = \frac{s \cdot bl \cdot A_{back}}{2R_{coil} \cdot (m_{cone} \cdot s^2 + R_{eff} \cdot s + k_{eff})} \tag{4}$$

Equation 4 is the system transfer function model for the acoustic exciter, and it will be correlated to the measured transfer function for the purpose of identifying the model parameters.

IDENTIFICATION OF MODEL PARAMETERS

Three parameter groups can be identified by fitting the simplified model (5) to the measured volume velocity response function (Fig. 6). The parameter groups, (6) can then be used to solve for each parameter.

$$T(s) = \frac{k_1 s}{s^2 + k_2 s + k_3} \tag{5}$$

$$\{k\} = \left\{ \frac{bl \cdot A_{back}}{2 \cdot R_{coil} \cdot m_{cone}}, \ \frac{R_{cone} + bl^2 / R_{coil}}{2 \cdot R_{coil} \cdot m_{cone}}, \ \frac{k_{cone} + 2 \cdot k_{air}}{2 \cdot R_{coil} \cdot m_{cone}} \right\} \tag{6}$$

The seven model parameters must be determined to correlate the model to the measured response. This can be achieved by curve fitting the simplified model (5) to the data in a least squares sense. Doing so will identify only three parameter groups, k_1, k_2, k_3, because (4) must be simplified so that there are the minimum number of independent parameters.

Four other equations or parameters must be identified with other testing or supplied information so that the identification problem will have a unique solution. Sometimes, the speaker manufacturer supplies many of the model parameters, but it is assumed here that only the free air resonance frequency, and the electrical resistance of the coil are provided.

One equation results from the free air resonance constraint provided by the manufacturer. The speaker manufacturer usually gives a free air resonance frequency for the speaker. If this is not provided, it can be found by measuring the cone velocity frequency response of a single speaker. Knowing the resonance frequency, f_0, sets up a needed constraint (7) on the solution of the identification problem.

$$\sqrt{\frac{k_{cone}}{m_{cone}}} = 2\pi f_0 \qquad (7)$$

Three parameters can be determined directly from lab testing. The three parameters are R_{coil}, k_{air}, and bl. The coil resistance can be either measured with an ohmmeter, or is provided by the manufacturer. The air stiffness can be computed by merely knowing the volume of the air cavity, the speed of sound, and the density of air (8) (Radclifffe, Gogate 1992). S_D is effective area through which the cone face acts upon the air cavity, and is estimated to be the area of the cone face computed using a diameter that is 87% of the advertised diameter (Beranek, 1986).

$$k_{air} = \frac{S_D \cdot \rho \cdot c^2}{Vol_{air}} \qquad (8)$$

The electromagnetic constant, bl, was found by determining the best linear relationship between applied DC current, and zero displacement force generated at the cone. The parameter bl is the ratio of force generated at the cone per unit of current, and has units of N/A; it is also the ratio of back emf to cone velocity, which yields units of (sV)/m.

EXPERIMENTAL RESULTS

Intensity was measured at twenty-four locations on the surface of an imaginary cube 14x14x14 inches wide, inside of which was centered the acoustic exciter This cube can be seen in Figure 3a. Those twenty-four intensity vectors were used to compute volume velocity (9), and were compared with those of a point source.

The volume velocity output of a noise source is determined from intensity measurements, provided that the measurements are made over the entire measurement surface (9). The index, j, is the index of the intensity measurements and associated areas.

$$Q = \oint \bar{v} \cdot \bar{n} dA = \oint \frac{1}{p}(p\bar{v} \cdot \bar{n}) dA = \oint \frac{1}{p} \bar{I} \cdot \bar{n} dA = \sum \frac{1}{p_j}\left(\bar{I}_j \cdot \hat{n}_j\right) A_j \qquad (9)$$

The simplified exciter model (5) was correlated to the measured volume velocity by minimizing the model error in a least squares sense (Fig. 6).

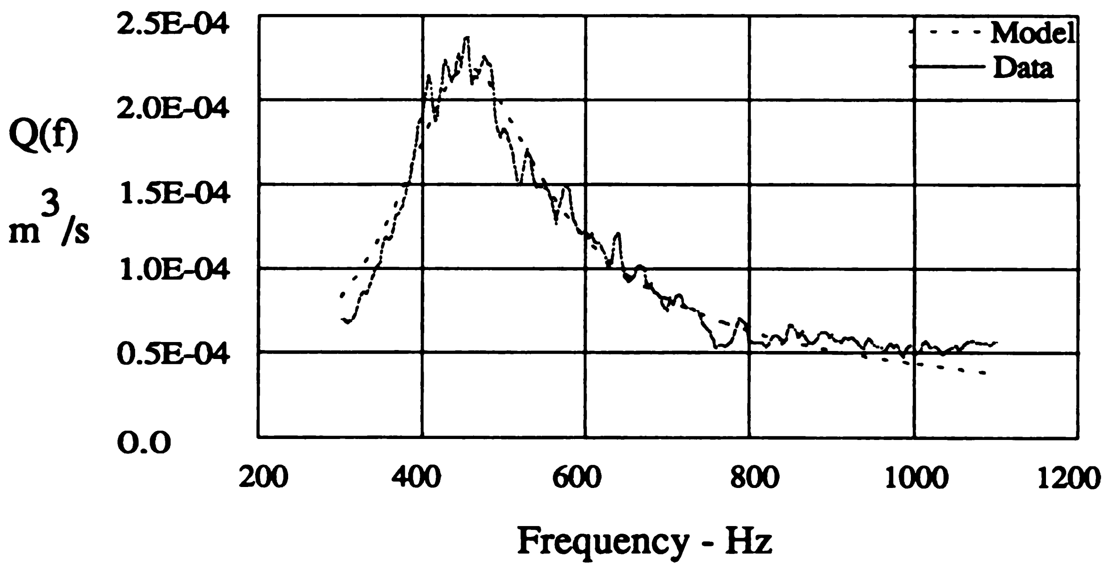

Figure 6. Measured, Analytical Volume Velocity Frequency Response Function

Fitting the model (5) to the data yields the values of the parameter groups, {k}. Using these values, as well as the four other constraints discussed in the parameter identification section, the values for the seven parameters were determined (Table 1).

Table 1. List of model parameters for a pair of 4" woofers and a 0.5" spacer

Parameter	Value to Correlate Model	Parameter Name and Units
R_{coil}	8 Ω	Electrical Resistance of Speaker, Ohms
R_{cone}	4.64 Ns/m	Viscous friction of Speaker, N*s/m
bl	3.75 N/A	Electromagnetic Constant, N/A
k_{cone}	836 N/m	Stiffness of Speaker Cone, N/m
A_{back}	0.000374 m^2	Effective Area of back of cones
k_{air}	2.5589E+04 N/m	Stiffness of Acoustic Cavity, N/m
m_{cone}	0.000395 Kg	Mass of Speaker Cone, kg

The needed calibration function is then $T(s)=0.2222/(s^2+1014s+8.235E+6)$. This function is used to infer volume velocity output based on frequency and applied voltage.

Many times, acousticians use box speakers to approximate a point source using the assumption that if the speaker is much smaller than the system under test, the approximation will be a good one. This assumption will not hold if the system under test is relatively small, and the sound power levels required are large. If the system is small, the box speaker will need to be even smaller, and the ability to create a large amount of acoustical energy is reduced. For comparison purposes, four radial intensity measurements were made surrounding a Radio Shack miniature box speaker (Fig. 7-8).

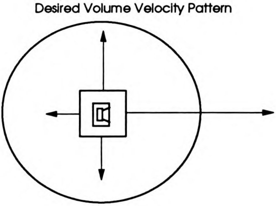

Figure 7. Radial Intensity Components (800 Hz) for Box Speaker

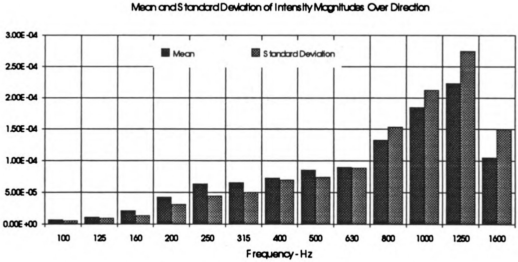

Figure 8. Mean, Standard Deviation of Intensity Vector Magnitudes for Box Speaker

The intensity data shows that the acoustic exciter approximates an ideal point source with a high degree of accuracy (Fig. 9-11). A vertical projection of the second picture in Figure 9 was taken to clarify the results (Fig. 10). The comparison with a typical box speaker shows that the acoustic exciter is a far better point source than an ordinary speaker.

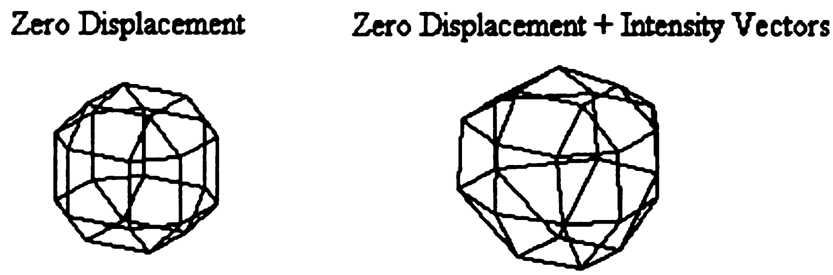

Figure 9. Wire Frame Animation of Intensity Vector "Displacements" at 800Hz

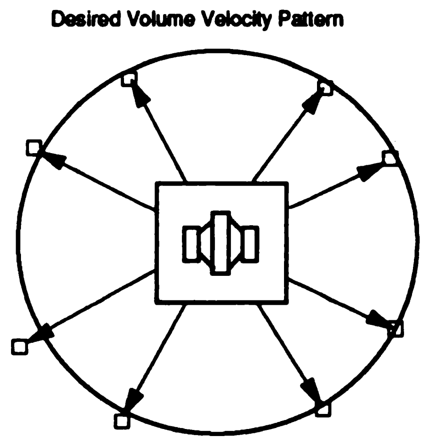

Figure 10. Vertical Projection of Data Depicted by Figure 9

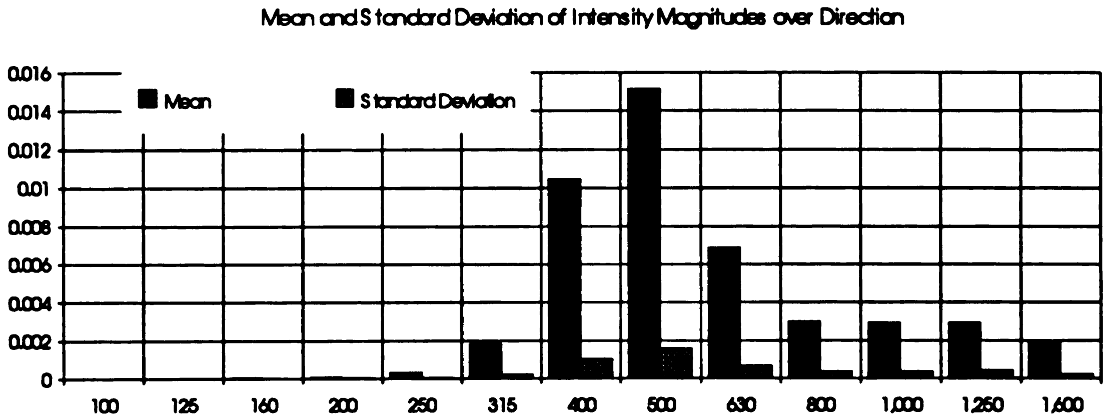

Figure 11. Mean, Standard Deviation of Intensity Magnitudes for Acoustic Exciter

Comparing the intensity data for the box speaker and the acoustic exciter shows that the acoustic exciter is omnidirectional, and the box speaker is not. Figure 11 shows that the acoustic exciter is a very good point source. The standard deviation of the acoustic exciter's intensity magnitudes is about 10% of the mean value (Fig. 9), and the exciter can be said to be within 20% of spherical at 95% confidence. The standard deviation of the box speaker's magnitudes are around 100% of the mean (Fig. 8), and the box speaker may not be considered a point source without moving a great distance from the speaker..

CONCLUSIONS

A device that approximates an acoustical point source is needed for experimental verification of acoustic reciprocity (Fahy, 1992). This acoustical point source must emit acoustical power equally in all directions, and also must be calibrated for volume velocity because it cannot be measured during the test. The acoustic exciter developed in this paper was designed and built to fulfill this need for a point source. An analytical model for the acoustic exciter was developed for source calibration and future design optimization. Acoustic intensity measurements were taken around the acoustic exciter, and used to

Acoustic intensity measurements were taken around the acoustic exciter, and used to determine the volume velocity calibration. These measurements were then used to identify the model parameters and to verify the model.

It was also determined that the acoustic exciter is a much better point source than a conventional loudspeaker. The acoustic exciter approximates a point source within 20% at 95% confidence; considering that box speakers have been used to approximate the point source, this is a great improvement.

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Appendix A

APPENDIX A - DERIVATION OF ACOUSTIC EXCITER SYSTEM EQUATIONS

First, a schematic of the acoustic exciter using ideal engineering elements is constructed (Fig. A1)

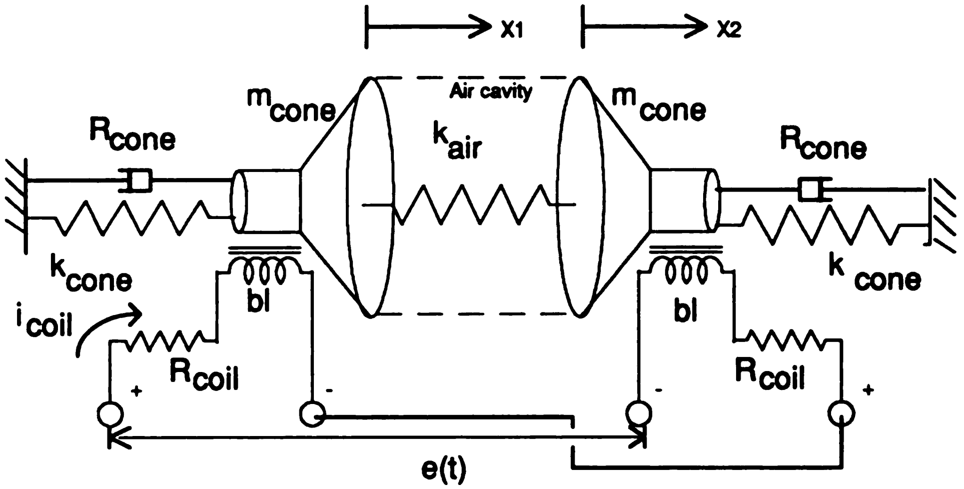

Figure A1. Schematic of acoustic exciter using ideal engineering elements.

Ignoring the electrical domain, the equations of motion for the cones are derived from applying Newton's laws to the cones (A1)

$$\begin{bmatrix} m_{cone} & 0 \\ 0 & m_{cone} \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} R_{cone} & 0 \\ 0 & R_{cone} \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}+\begin{bmatrix} k_{cone}+k_{air} & -k_{air} \\ -k_{air} & k_{cone}+k_{air} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad \text{(A1)}$$

In (A1), the force terms that drive the system, **f**, are equal to bl*i_{coil}. The variable i_{coil} can be determined from Kirchoff's loop law for voltages (A2), where V_{sp1} and V_{sp2} are the back emf of speaker 1 and speaker 2, respectively.

$$e(t)+2i_{coil}R_{coil}+V_{sp1}+V_{sp2}=0 \quad \text{(A2)}$$

The back emf, V_{sp1} and V_{sp2}, are equal to bl*$\dot{x}_1$, and bl*$\dot{x}_2$, respectively (A3).

$$e(t)+2i_{coil}R_{coil}+bl\cdot\dot{x}_1-bl\cdot\dot{x}_2=0 \quad \text{(A3)}$$

The sign of the second back emf term in (A3) is negative because the current passes through the speakers in opposite directions, because of the way in which the terminals are wired.

The current is found by solving (A3), and the force terms in (A1) are written in terms of bl, Rcoil, $\dot{x}_1$, and $\dot{x}_2$. The coefficients of $\dot{x}_1$, and $\dot{x}_2$ are collected into the damping matrix of (A1), and the system of equations is determined (A4).

$$\begin{bmatrix} m_{cone} & 0 \\ 0 & m_{cone} \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} R_{cone} + \dfrac{bl^2}{2R_{coil}} & -\dfrac{bl^2}{2R_{coil}} \\ -\dfrac{bl^2}{2R_{coil}} & R_{cone} + \dfrac{bl^2}{2R_{coil}} \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_{cone} + k_{air} & -k_{air} \\ -k_{air} & k_{cone} + k_{air} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{bl \cdot e(t)}{2R_{coil}} \\ \dfrac{bl \cdot e(t)}{2R_{coil}} \end{bmatrix} \quad \text{(A4)}$$

Performing an eigenvalue-eigenvector analysis on the left hand side of (A4) reveals that the eigenvectors are $[1\ -1]^t$, and $[1\ 1]^t$. This means that the force vector on the right hand side of (A4) is orthogonal to the unison mode of vibration, and the opposition mode only will be excited ($x_1 = -x_2$). This simplification allows an order reduction for the model, and substituting $x_1 = -x_2$ into (A4) results in a one DOF system of equations (A5).

$$m_{cone}\ddot{x} + \left(R_{cone} + \frac{bl^2}{R_{coil}}\right)\dot{x} + \left(k_{cone} + 2k_{air}\right)x = \left(\frac{bl}{2R_{coil}}\right)e(t) \quad \text{(A5)}$$

The model variables need to be defined so that velocity may be found, so the substitution of $v=\dot{x}$ into (A5) gives the equations of motion in state space form (A6), where the dissipative and stiffness terms in (A5) have been simplified into R_{eff} and k_{eff}.

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_{eff}}{m_{cone}} & -\dfrac{R_{eff}}{m_{cone}} \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{bl}{2R_{coil} \cdot m_{cone}} \end{bmatrix} e(t) \quad \text{(A6)}$$

The acoustic exciter will always operate at steady state, that is, e(t) will always be periodic, so the solution for v is carried out using the Laplace transformation (A7).

$$s\begin{bmatrix} X(s) \\ V(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_{eff}}{m_{cone}} & -\dfrac{R_{eff}}{m_{cone}} \end{bmatrix} \begin{bmatrix} X(s) \\ V(s) \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{bl}{2R_{coil} \cdot m_{cone}} \end{bmatrix} E(s) \tag{A7}$$

Solving (A7) for V(s) gives the acoustic exciter's transfer function, T(s)=V(s)/E(s) (A8).

$$T(s) = \frac{V(s)}{E(s)} = \frac{bl}{2R_{coil}} \cdot \frac{s}{m_{cone}s^2 + R_{eff}s + k_{eff}} \tag{A8}$$