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Junsoo Lee

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Peter Schmidt
Major professor

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By

Junsoo Lee

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ABSTRACT

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Whether observed series contains a unit root or not has profound implications in macroeconomics. While the existing unit root tests have mostly been based on one variant or another of the Dickey-Fuller (1979, hereafter DF) tests, they do not survive in the presence of strongly autocorrelated errors in terms of <u>both</u> size and power of the tests. Thus, a recent survey paper by Campbell and Perron (1991) suggests the necessity of finding new tests which alleviate the size distortion problem, while retaining good power properties.

This dissertation suggests new IV tests which have surprisingly small size distortions in the presence of strongly autocorrelated errors, and are more powerful than other tests of similar size. Therefore, this remarkable performance of the new IV tests may well provide the solution to the necessity of finding robust tests.

This dissertation shows that their improved performance is driven from adapting an appropriate testing procedure. The new IV tests are based on the Schmidt-Phillips (1990, hereafter SP) tests, not variants of the DF tests. Thus this suggests that there are perceivable advantages to

operating in the SP framework rather than in the DF framework.

The most distinguished difference between the DF and SP framework lies in the parameterization. It has been argued by SP that the DF parameterization is "clumsy" in the sense that it handles exogenous variables in a potentially confusing ways. This argument has been made more visible from in-depth discussion (in Chapter 5) on the parameterization issue. The problem of the clumsy DF parameterization gets more serious especially in the presence of exogenous structural break. Specifically, a suitably modified SP tests allowing for a structural change reverse the results of Perron (1989) who finds that most of the Nelson-Plosser series are trend stationary if allowance is made for a structural break. We show that the contrary results of Perron arise not from the presence of an exogenous structural break in the DGP, but from the way in which the DF tests allow for such a break. In this dissertation, some new theoretical findings on the asymptotic behavior of the SP and DF tests in the presence of structural break are also provided to clarify this issue.

Dedicated to my parents

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CHAPTER 1

CHAPTER 1

INTRODUCTION

Since the pioneering work of Nelson and Plosser (1982), which found that most U.S. macroeconomic time series are nonstationary, there has been a vast amount of work done on the macroeconomic and statistical implications of unit roots, and on the question of how to test whether observed series contains a unit root. From a macroeconomic point of view, unit roots imply long-run persistence, in the sense that the effects of random shocks on macroeconomic variables are not just temporary. Many economists have argued that theories designed to explain the properties of economic time series with unit roots should necessarily be different from theories designed to explain the properties of stationary time series. From a statistical (or econometric) point of view, correct treatment of the stationary or nonstationary nature of the data is necessary for meaningful statistical inference, owing to the spurious regression phenomenon pointed out by Granger and Newbold (1974) and Phillips (1986). Thus, for both economic and statistical reasons, it is important to be able to distinguish a series with a unit root from a stationary series. A considerable elaboration of this point, along with a general survey of the unit root literature, is given by Campbell and Perron (1991). See also Diebold and Nerlove (1990).

Efforts to distinguish a unit root from stationarity have most often taken the form of the test of the null hypothesis of a unit root against the alternative of stationarity. While there is an enormous literature on the problem of testing for a unit root, most of the existing unit roots tests are variants of the Dickey and Fuller (hereafter, DF) tests provided by Fuller (1976) and Dickey and Fuller (1979). The DF unit root tests are based on the following regressions:

(1) $$y_t = \beta y_{t-1} + \epsilon_t$$

(2) $$y_t = \alpha + \beta y_{t-1} + \epsilon_t$$

(3) $$y_t = \alpha + \beta y_{t-1} + \delta t + \epsilon_t$$

for $t = 1,\ldots,T$. In each case, the unit root hypothesis is $\beta = 1$. Two types of test statistics are defined from each regression equation. One is the normalized coefficient test statistic $T(\hat{\beta}-1)$, where $\hat{\beta}$ is the OLS estimate of β; this yields the DF statistics $\hat{\rho}$, $\hat{\rho}_\mu$ and $\hat{\rho}_\tau$ from regressions (1), (2) and (3), respectively. The other type of statistic is the usual t-statistic for testing the hypothesis $\beta = 1$, which yields the DF statistics $\hat{\tau}$, $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$ corresponding to the same three regressions.

The DF regressions differ in what they assume (or allow) with respect to level and deterministic trend. The regression (1) does not allow for non-zero level or trend under the alternative, though the initial value (y_0) is effectively the level of the series under the null. The regression (2) allows for linear deterministic trend (drift) under the null, since the solution for y_t contains the deterministic trend term αt, but it allows only for non-zero level under the alternative, since then y is stationary around the value $\alpha/(1-\beta)$. As a result, tests derived from regression (2) are inconsistent against trend-stationary alternatives; see West (1987). Similarly, regression (3) allows for non-zero level and trend under the null, even when $\delta = 0$, but implies nonlinear trend under the null when $\delta \neq 0$; under the alternative $\delta \neq 0$ simply allows for linear trend.

This thesis will largely be concerned with extensions of an alternative set of unit root tests suggested by Schmidt and Phillips (1990, hereafter, SP). Their tests are based on the parameterization

(4) $$y_t = \psi + \xi \cdot t + X_t, \quad X_t = \beta X_{t-1} + \epsilon_t,$$

which was also used by Bhargava (1986). The unit root corresponds to $\beta = 1$. SP criticize the DF parameterization as "clumsy", because the meaning of the parameters α and δ in (2) and (3) is different under the null and the alternative. This type of difficulty does not arise in the SP parameterization. The parameterization allows for non-zero level and trend under both the null hypothesis and the alternative hypothesis; ψ and ξ in (4) always represent level and deterministic trend respectively, independently of whether $\beta = 1$ or not. Since the distributions of the Schmidt-Phillips tests do not depend on the nuisance parameters ψ and ξ, the test statistics require just one tabulation for each statistic independently of level or time trend. This is in contrast to the DF tests, since the distributions of $\hat{\rho}_\mu$ and $\hat{\tau}_\mu$ under the null depend on the values of α in (2), while the distributions of $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$ under the null depend on the values of δ in (3).

The Schmidt-Phillips test statistics are derived from the LM test of the hypothesis $\beta = 1$ in equation (4). The restricted MLE's of ξ and $\psi_x (= \psi + X_0)$ are given by: $\tilde{\xi} = (y_T - y_1) / (T-1)$, and $\tilde{\psi}_x = y_1 - \tilde{\xi}$. Then the test statistics are given from the following regression:

(5) $$\Delta y_t = \text{constant} + \phi \tilde{S}_{t-1} + \text{error}$$

where $\tilde{S}_{t-1} = y_{t-1} - \tilde{\psi}_x - \tilde{\xi}(t-1)$. The test statistics are defined by:

(6) $$\tilde{\rho} = T\tilde{\phi}$$

(7) $$\tilde{\tau} = \text{t-statistic for the hypothesis } \phi=0.$$

Since $\Delta\tilde{S}_t = \Delta y_t - \tilde{\xi}$, the $\tilde{\rho}$ and $\tilde{\tau}$ tests are not affected if we make the dependent variable in (5) $\Delta\tilde{S}_t$ instead of Δy_t; that is, we can also derive $\tilde{\rho}$ and $\tilde{\tau}$ from the regression

(8) $$\Delta\tilde{S}_t = \text{constant}' + \phi \tilde{S}_{t-1} + \text{error}.$$

In Chapter 3, we show that the population intercept (constant') in (8) equals zero, and we consider the modified SP tests that set the intercept to zero in running regression (8). We derive asymptotic distributions for these modified tests, and we compare their powers in finite samples to the powers of the DF and SP tests.

Chapters 2 and 4 deal with the problem of autocorrelated errors in the DF or SP regressions. Since the tabulated distributions for the test statistics assume the errors in the model are i.i.d., they are affected by the presence of autocorrelated errors even asymptotically. This problem is commonly dealt with in two ways.

First, the so-called augmented Dickey-Fuller (hereafter ADF) tests accommodate error autocorrelation by adding lagged differences in the variable to the regression. Said and Dickey (1984, 1985) showed that, if the number of lagged differences is suitably chosen, the ADF statistics have the same asymptotic distribution as the original statistics would have under iid errors, so the usual tabulations apply, at least asymptotically. We consider similarly augmented versions of the SP tests, in which lagged values of $\Delta\tilde{S}$ (or Δy) are added to the regression (5). We presume, though we do not present the proof, that the result equivalent to the Said-Dickey result holds, so that asymptotically the usual SP tabulations are correct.

Second, Phillips (1987) and Phillips and Perron (1988) derive the asymptotic distributions of the DF statistics under assumptions that allow for a wide class of weakly dependent and heterogeneous errors, and they provide transformed (corrected) versions of the statistics that have the same asymptotic distribution as the original DF statistics would have under iid errors. Thus, for these so-called Phillips-Perron (hereafter PP) statistics, the usual DF tabulations apply, at least asymptotically. SP provide the corresponding transformed versions of the SP tests.

While the ADF tests and the PP tests are valid asymptotically, they do not necessarily perform very well in finite samples of reasonable size. Simulation evidence presented by Godfrey and Tremayne (1988), Phillips and Perron (1988), Schwert (1989) and Kim and Schmidt (1990) has shown that the uncorrected DF tests reject the null hypothesis (when it is true) too often in the presence of negative autocorrelation and too seldom in the presence of positive autocorrelation. These size distortions can be quite considerable. The PP corrected tests perform somewhat better than the uncorrected tests, but still suffer from considerable size distortions even for surprisingly large sample sizes. The ADF tests also perform somewhat better than the uncorrected tests, with the extent of the improvement depending on the number of augmentations. With a small number of augmentations, the behavior of the ADF tests is similar to the behavior of the PP tests noted above. With a sufficiently large number of augmentations, the size of the augmented tests becomes more or less correct, even for cases in which the errors are strongly autocorrelated. However, this result is less optimistic than it might first seem, because the simulation results in Chapter 2 indicate that the augmented tests with many augmentations have almost no power.

In Chapter 2 of this thesis, we present simulation results for the SP tests that are similar to the simulation results that others have presented for the DF tests. The performance of the augmented and corrected SP tests is not very different from the performance of the augmented and corrected DF tests, as summarized above. This is a discouraging result.

Therefore it seems that no tests survive in the presence of strongly autocorrelated errors in terms of both size and power of the tests. The tests with correct size have poor power and the tests with high power have serious size distortions. This problem is also discussed in the recent survey paper on the unit root literature by Campbell and Perron (1991). They assert that none of the variants of the DF tests seems to solve the problem, and suggest the necessity of finding new tests which modify the PP procedure in such a way as to alleviate the size problem, while retaining good power properties.

In Chapter 4, we consider unit root tests based on instrumental variables (IV) estimation. Hall (1989) and Pantula and Hall (1991) have considered IV tests that are based on IV estimation of the DF regressions (1) - (3). They assume moving average (MA) errors, and the instrument for y_{t-1} is y_{t-k}, where k is chosen to be at least as large as one plus the order of the MA process. These IV tests are valid asymptotically in the presence of autocorrelation of MA form. We propose similar IV versions of the SP statistics, where the instrument for $\tilde{S}_{t-1}$ in (5) is $\tilde{S}_{t-k}$, and where again k is chosen to be at least as large as one plus the order of the MA process for the errors. Hall's simulation results in finite samples indicate size distortions that are less than those of the ADF or PP tests, though they are still substantial. Our simulation results are more optimistic. IV versions of the SP tests have surprisingly smaller size distortions, and they are more powerful than other tests of similar size. The new IV tests therefore show that it is still possible to improve the performance of the unit root tests when the appropriate testing procedure is used, and this improvement seems to be implicitly related to the parameterization issue discussed above.

Despite the difference in parameterization between the DF and SP tests, however, in practice their implications have not been very different. Specifically, both the DF and SP tests have given the similar conclusions when they are applied to the Nelson-Plosser data. Both tests, like a large number of other tests, have failed to reject the null hypothesis of a unit root in many macroeconomic time series. A contrary conclusion is asserted by Perron (1989), who finds most of the US macroeconomic series to be trend-stationary if allowance is made for a structural break at the time of the Great Depression. He shows that the DF tests are biased toward accepting the null when a structural break occurs in the data, and his modified tests which allow for such a change reject the null hypothesis of a unit root for eleven out of the fourteen Nelson-Plosser time series.

In Chapter 5, we consider Perron's analysis in the SP setting. We argue that the clumsy way in which the DF parameterization handles explanatory variables, like intercept and trend, becomes more serious as we add additional explanatory variables, like a structural break dummy. Using modified versions of the SP tests which allow for the same structural break reverses Perron's results. Thus, the distinction between the two parameterizations becomes more important when additional exogenous variables are considered. It is shown that though both SP and DF tests are affected by the structural break if the alternative hypothesis of stationarity is true, the asymptotic distributions of the SP tests are not affected if the null of a unit root is true. Chapter 5 deals with this parameterization issue in depth, and provides theoretical and empirical results. These results support the argument of SP that the DF parameterization is "clumsy" in the sense that it handles exogenous variables in a potentially confusing way.

The structure of the dissertation is as follows. Chapter 2 investigates the finite performance of the SP tests in the presence of autocorrelation. Chapter 3 considers modified SP tests based on the fact that the constant term in regression (5) equals zero in the population. Chapter 4 provides new IV tests based on the SP framework, and shows their remarkable performance in the presence of autocorrelated errors. Chapter 5 examines the parameterization issue closely, with new theoretical findings on the asymptotic behavior of the SP and DF tests in the structural break model and with an empirical application to the Nelson-Plosser data. Chapter 6 gives our concluding remarks.

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CHAPTER 2

CHAPTER 2

FINITE SAMPLE PERFORMANCE OF SCHMIDT-PHILLIPS UNIT ROOT TESTS IN THE PRESENCE OF AUTOCORRELATION

1. INTRODUCTION

This chapter considers a new unit root test proposed recently by Schmidt and Phillips (1990) (hereafter, SP). The tabulated distributions for their test statistics assume that the errors in their model are iid. The SP tests can be made asymptotically robust to autocorrelation by a Phillips-Perron (hereafter, PP) type correction of Phillips and Perron (1988), or by augmenting the regression that yields the test statistics like the augmented Dickey-Fuller tests (hereafter, ADF) of Said and Dickey (1984, 1985). This chapter investigates the finite sample performance of these extended versions of the SP tests. It is basically found that the corrected and augmented SP tests behave very much like the corresponding PP-corrected and ADF tests, in terms of size distortions and power under autocorrelated errors.

2. TEST STATISTICS

The SP tests employ the parameterization:

(1) $$y_t = \psi + \xi t + X_t, \qquad X_t = \beta X_{t-1} + \epsilon_t.$$

The unit root corresponds to $\beta = 1$. As discussed in Chapter

1, this parameterization is different from the parameterization of Dickey and Fuller (1979, hereafter DF) tests. It consistently allows for non-zero level and trend under both the null hypothesis and the alternative hypothesis; ψ and ξ always represent level and deterministic trend respectively, independently of whether $\beta = 1$ or not.

The SP test statistics are derived from the LM test of the hypothesis $\beta = 1$ in the equation (1). The restricted MLE's of ξ and $\psi_x (= \psi + X_0)$ are given by $\tilde{\xi} = (y_T - y_1)/(T-1)$, and $\tilde{\psi}_x = y_1 - \tilde{\xi}$. Then the test statistics are derived from the following regression:

(2) $$\Delta y_t = \text{constant} + \phi\tilde{S}_{t-1} + \text{error}$$

where $\tilde{S}_{t-1} = y_{t-1} - \tilde{\psi}_x - \tilde{\xi}(t-1)$. The test statistics are defined by:

(3) $$\tilde{\rho} = T\tilde{\phi}$$

(4) $$\tilde{\tau} = \text{t-statistic for the hypothesis } \phi = 0.$$

The corrected test statistics $Z(\tilde{\rho})$ and $Z(\tilde{\tau})$ are designed to allow for a wide class of weakly dependent and heterogeneous errors by employing correction term which is the ratio of the estimates for the innovation variance σ_ϵ^2 and the long-run variance σ^2, respectively. Specifically, define $\omega^2 = \sigma_\epsilon^2/\sigma^2$ and

(5) $$Z(\tilde{\rho}) = \tilde{\rho}/\hat{\omega}^2$$

(6) $$Z(\tilde{\tau}) = \tilde{\tau}/\hat{\omega}$$

where $\hat{\omega}^2 = \hat{\sigma}_\epsilon^2/\hat{\sigma}^2$. A choice of the truncation lag parameter ℓ is required for $\hat{\sigma}^2$; see Phillips (1987) or Schmidt and Phillips (1990) for details.

The augmented versions of the test statistics are derived from the following regression, which includes lagged differences of $\tilde{S}_t$ in the regressors:

$$(7) \qquad \Delta y_t = \text{constant} + \phi\tilde{S}_{t-1} + \sum_{j=1}^{p}\delta_j\Delta\tilde{S}_{t-j} + \text{error}.$$

The coefficient test statistic Aug($\tilde{\rho}$) is defined as $T\tilde{\phi}$, and the statistic Aug($\tilde{\tau}$) is the t-statistic for $\phi = 0$, where both are based on regression (7). Note that including Δy_{t-j} instead of $\Delta\tilde{S}_{t-j}$ will lead to the same statistics, since $\Delta\tilde{S}_{t-j} = \Delta y_{t-j} + \tilde{\xi}$ and $\tilde{\xi}$ is absorbed into the intercept. To capture the autocorrelation structure of the errors, it is required to choose the number of augmentations properly. If the error (ϵ_t) is AR(p), then p augmentations are necessary in (7) for the t-test to be asymptotically valid. If the error is MA or ARMA, validity of the test requires that the number of augmentations rise with sample size, though more slowly than sample size; for example, p can be taken to be proportional to $T^{1/4}$.

For comparison, our simulations also consider the DF test statistics $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$, the PP test statistics $Z(\hat{\rho}_\tau)$ and $Z(\hat{\tau}_\tau)$ and the ADF test statistics Aug($\hat{\rho}_\tau$) and Aug($\hat{\tau}_\tau$).

3. EXPERIMENT DESIGN

To examine the sensitivity of the test statistics to autocorrelated errors, the data generating process will allow AR(1) disturbances as well as MA(1) disturbances. Thus we consider the ARMA(1,1) error process.

$$(8) \qquad \epsilon_t = \rho\epsilon_{t-1} + u_t + \theta u_{t-1}$$

where u_t are iid with mean zero and variance σ_u^2 and $|\rho| < 1$, $|\theta| < 1$. The parameters ψ, ξ and initial value X_0 are assumed to be zero, whereas we assume $\sigma_u^2 = 1$. Schmidt and Phillips (1990) and DeJong et al. (1989) showed that the distributions of the test statistics under the null hypothesis are independent of ξ, ψ, X_0 and σ_u^2. Under the alternative, $X_0^* = X_0/\sigma_u$ is relevant but the other parameters are still irrelevant. When the autoregressive (AR) parameter ρ is zero, the data will follow an ARIMA(0,1,1) process. Then the construction of the data is exactly the same as in Schwert (1987, 1989), Phillips and Perron (1988), and Hall (1989). Schwert (1987) discusses the reasons to believe that an economic time series contains MA disturbances, rather than being a pure AR process. Nicholls, Pagan and Terrell (1975) have provided a survey on the use of moving average errors in modelling of economic time series. On the other hand, when the moving average (MA) parameter θ is zero, the data will follow an AR(1) process with autoregressive errors, as considered also by

Kim and Schmidt (1990). In the simulations, process (8) is initialized by drawing an initial value of u, setting the initial value of ϵ equal to the initial value of u, and then drawing and discarding 20 values of u and ϵ (to remove possible effects of these initial values) before drawing the values that are actually used.

All the simulation results are calculated using 20,000 replications, except that 10,000 replications are used for the simulations with sample sizes equal to or greater than 500. A 95% confidence interval around a rejection rate of .05 is calculated as approximately [.047,.053] with 20,000 replications and [.046,.054] with 10,000 replications. The random numbers were generated using the subroutine GASDEV/RAN3 of Press, Flannery, Teukolsky and Vetterling (1986).

4. SIMULATION RESULTS

The first simulation is performed on the basic SP test statistics $\tilde{\rho}$ and $\tilde{\tau}$ and the DF tests statistics $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$, none of which correct for the autocorrelation. Table 1 shows the 5% sizes of these tests (i.e., the proportions of rejections using 5% critical values) under MA errors. Of course, no size distortion is found when the MA parameter θ is zero. But when θ is negative, the sizes are close to one, which implies too many rejections of the (true) null hypothesis of a unit root. It is a curiosity that the problem becomes more severe as the sample size grows. Conversely, when θ is positive, the sizes are less than the nominal size 5%, resulting in too few rejections of the null hypothesis. The same patterns are found in Table 2 for the case of AR errors. These results for the DF tests $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$ are already known, from the results of Schwert (1989), Phillips and Perron (1988) and Kim and Schmidt (1990). The results in Tables 1 and 2 indicate that the severity of the problem is about the same for the SP test statistics $\tilde{\rho}$ and $\tilde{\tau}$ as for the DF test statistics $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$.

The next question is whether there is any significant improvement from using the corrected and the augmented versions of the tests. These simulations are executed for values of -.8, -.5, .0, .5 and .8 for each of the parameters θ and ρ, and for sample sizes 50, 100, 500 and 1000. The number of truncation lags (ℓ) used for estimating the long-run error variances was chosen in the same manner as Schwert (1989). That is; $\ell4 = \text{int}[4(T/100)^{1/4}]$ and $\ell12 = \text{int}[12(T/100)^{1/4}]$, where 'int' stands for integer. The values of $\ell4$ are 3, 4, 6 and 7, and the values of $\ell12$ are 10, 12, 18 and 21 for T = 50, 100, 500 and 1000 respectively. Tables 3 and 4 present the simulation results along with those for the PP tests, quoted from Schwert for comparison. It is interesting how similar the results for the SP tests are to the results for the corresponding PP tests. The sizes of the corrected SP tests $Z(\tilde{\tau})$ and $Z(\tilde{\rho})$

are very close to the sizes of the PP tests $Z(\hat{t}_\tau)$ and $Z(\hat{\rho}_\tau)$. This similarity is consistent for each set of values of θ, T and ℓ. Comparing the sizes of $Z(\tilde{t})$ and $Z(\tilde{\rho})$ with the sizes of $\tilde{t}$ and $\tilde{\rho}$, which do not use correction terms, we can see a small improvement. For example, when T = 100 and θ = -.5, the sizes of the statistics $Z(\tilde{t})$ and $Z(\tilde{\rho})$ are .713 and .708, which are slightly less than the sizes of $\tilde{t}$ and $\tilde{\rho}$, .796 and .800 respectively. The improvement is larger for larger sample size. For example, when T = 1000 and θ = -.5, the sizes of $Z(\tilde{t})$ and $Z(\tilde{\rho})$ are .433 and .428, which are much smaller than the sizes of $\tilde{t}$ and $\tilde{\rho}$, .860 and .861 respectively. Such an improvement can be found in the PP tests as well. As a whole, we can conclude that the corrected versions of the tests are not very accurate in the presence of significant autocorrelation. They still show most of the problems that the uncorrected tests showed.

Simulation results on the augmented test statistics $Aug(\tilde{t})$ and $Aug(\tilde{\rho})$ are also presented in Tables 3 and 4 along with simulation results of the ADF tests for comparison. The performance of the augmented tests depends strongly on the number of augmentations. The simulation results indicate that the performance of the augmented SP t-test statistic $Aug(\tilde{t})$ is more or less same as that of the ADF t-test statistic $Aug(\hat{t}_\tau)$, and these augmented t-tests with enough augmentations are generally more accurate than the corrected tests. But, this result is less optimistic under the alternative hypothesis, as we see later in this section. Meanwhile, the augmented coefficient test is not as good as the augmented t-test. The augmented coefficient test shows a similar pattern as $Z(\tilde{t})$ and $Z(\tilde{\rho})$.

Simulation results under AR errors, as presented in Table 5, show similar patterns as under MA errors. The test statistics $Z(\tilde{t})$ and $Z(\tilde{\rho})$ result in too many rejections of the true null hypothesis with negative AR errors and too few rejections with positive AR errors. $Aug(\tilde{\rho})$ under AR errors shows similar problems as under MA errors. But $Aug(\tilde{t})$ seems to be relatively accurate, since the size of $Aug(\tilde{t})$ is close to nominal size with enough augmentations even when the AR parameter is close to -1.

Now, let us look at the powers of the tests when $\beta \neq 1$. Simulation results for T = 100 and $\beta = 0.9$ are given in Tables 6 and 7. For positively autocorrelated errors ($\theta > 0$ or $\rho > 0$), most tests, except $Aug(\tilde{t})$ and $Z(\tilde{\rho})$, tend to accept the null hypothesis of a unit root when it is not true. This tendency of decreased power of the tests does not disappear in large samples. For negatively autocorrelated errors ($\theta < 0$ or $\rho < 0$), the power of $Aug(\tilde{t})$, the only test which is acceptably accurate under the null hypothesis, is not very good. Note that the power of $Aug(\tilde{t})$ decreases considerably with more augmentations. For example, when $\theta = -0.8$, the power of $Aug(\tilde{t})$ is 0.551 for $\ell = 4$, but it is 0.106 for $\ell = 12$. That is, more augmentations make the test better under the null hypothesis, but worse under the alternative. Thus adding all these lags has diminished the ability of the test to reject anything. This fact is also true for the ADF test statistic Aug($\hat{\tau}_\tau$).

A final question of some interest, though not of immediate practical importance, is why the corrected tests $Z(\tilde{\rho})$ and $Z(\tilde{\tau})$ are not more accurate. The corrections depend on an estimate of the nuisance parameter $\omega^2 = \sigma_\epsilon^2/\sigma^2$, and one may suspect that the denominator of this fraction, the long run variance σ^2, is difficult to estimate accurately. It is therefore possible that the poor performance of the corrected tests just reflects difficulty in estimating ω^2. However, it is also possible that the problem is simply slow convergence of the statistics to their asymptotic distributions. In order to distinguish between these two explanations of the poor performance of the corrected tests, we follow Kim and Schmidt (1990) in considering the corrected tests that are based on the true value of ω^2. Because ω^2 is unknown, these are not feasible tests in actual applications, but we can consider them in our simulations. For the MA(1) process with parameter θ, we have $\omega^2 = (1+\theta^2)/(1+\theta)^2$, and for the AR(1) process with parameter ρ, we have $\omega^2 = (1-\rho)^2/(1-\rho^2)$. We will denote the corrected versions of $\tilde{\rho}$ and $\tilde{\tau}$, using the true value of ω^2, by $Z^*(\tilde{\rho})$ and $Z^*(\tilde{\tau})$.

Tables 8 and 9 give the 5% sizes of the corrected tests that use the true value of ω^2, for T = 100 and for the MA and AR errors, respectively. They also give the mean values of the estimates of ω, ω^2 and σ_ϵ^2 and the corresponding biases (mean values minus true values). The bias results indicate that we do indeed estimate ω^2 poorly when there is substantial autocorrelation, and especially when there is substantial negative autocorrelation. For example, when θ = -0.8 in Table 8, the true value of ω^2 is 41, while the mean value of estimated ω^2 is less than one. These substantial differences between true and estimated ω^2 lead to substantial differences between $Z(\tilde{\rho})$ and $Z^*(\tilde{\rho})$ and between $Z(\tilde{\tau})$ and $Z^*(\tilde{\tau})$. However, it is still true that the corrected tests based on the true values of ω^2 do not perform very well. In the MA case, $Z^*(\tilde{\rho})$ and $Z^*(\tilde{\tau})$ are reasonably accurate for $\theta \geq 0$ but not for $\theta < 0$. In the AR case, $Z^*(\tilde{\tau})$ is reasonably accurate for $\rho \leq 0$ but not for $\rho >$ 0, while $Z^*(\tilde{\rho})$ is never very satisfactory. This must reflect a very slow convergence of these statistics to their asymptotic distributions. The poor performance of the corrected tests even when the nuisance parameter ω^2 is assumed known suggests the need for new tests, not just new estimates of the nuisance parameters.

5. SUMMARY AND CONCLUSION

This chapter has investigated the sensitivity to error autocorrelation of the new unit root tests proposed by

Schmidt and Phillips (1990). Unfortunately, the performance of the tests is found to be similar to that of the DF tests. Corrected versions of the tests, similar to the PP versions of the DF tests, reject too often under negative autocorrelation and too seldom under positive autocorrelation. Augmented versions of the tests, similar to the Said-Dickey versions of the DF tests, have similar problems. The augmented version of the t-test, irrespective of whether it is the augmented SP test or the ADF test, has much smaller size distortions than the other tests in the presence of negative autocorrelation, and is probably the best test overall, but it turns out that including the augmentation terms diminishes the power of the test considerably.

An alternative approach to the problem of testing for a unit root in the presence of autocorrelation has recently been advocated by Hall (1989), based on IV estimation of the DF regressions. IV versions of the SP tests are also possible, and results to be reported in Chapter 4 are more optimistic for these IV tests than for either the original SP tests or Hall's IV tests.

TABLE 1

5% Sizes of Schmidt-Phillips Tests and Dickey-Fuller Tests Under MA Errors

		S-P Tests		D-F Tests	
T	θ	$\tilde{\tau}$	$\tilde{\rho}$	$\hat{\tau}_\tau$	$\hat{\rho}_\tau$
50	-.8	.993	.993	.998	.999
	-.5	.708	.711	.702	.751
	.0	.049	.050	.048	.048
	.5	.002	.002	.010	.002
	.8	.001	.001	.009	.001
100	-.8	1.000	1.000	1.000	1.000
	-.5	.796	.800	.801	.832
	.0	.051	.052	.050	.051
	.5	.003	.003	.010	.003
	.8	.001	.001	.009	.001
500	-.8	1.000	1.000	1.000	1.000
	-.5	.849	.854	.845	.827
	.0	.049	.051	.048	.077
	.5	.002	.002	.008	.004
	.8	.001	.001	.008	.002
1000	-.8	1.000	1.000	1.000	1.000
	-.5	.860	.861	.851	.916
	.0	.050	.051	.046	.076
	.5	.002	.002	.008	.004
	.8	.001	.001	.007	.002

TABLE 2

5% Sizes of Schmidt-Phillips Tests and Dickey-Fuller Tests Under AR Errors

		S-P Tests		D-F Tests	
T	ρ	$\tilde{\tau}$	$\tilde{\rho}$	$\hat{\tau}_\tau$	$\hat{\rho}_\tau$
100	-.8	.957	.958	.956	.967
	-.5	.570	.574	.570	.611
	.0	.051	.052	.050	.051
	.5	.000	.000	.009	.000
	.8	.000	.000	.003	.000
500	-.8	.977	.977	.972	.980
	-.5	.589	.593	.585	.549
	.0	.049	.051	.048	.029
	.5	.000	.000	.008	.000
	.8	.000	.000	.000	.000

TABLE 3

5% Sizes of Corrected and Augmented t-tests Under MA Errors

T	θ	$Z(\tilde{\tau})$ $\ell 4$	$Z(\tilde{\tau})$ $\ell 12$	$Aug(\tilde{\tau})$ $\ell 4$	$Aug(\tilde{\tau})$ $\ell 12$	$Z(\hat{\tau}_\tau)$ $\ell 4$	$Z(\hat{\tau}_\tau)$ $\ell 12$	$Aug(\hat{\tau}_\tau)$ $\ell 4$	$Aug(\hat{\tau}_\tau)$ $\ell 12$
50	-.8	.991	.963	.292	.073	.999	1.000	.518	.045
	-.5	.679	.718	.082	.076	.669	.753	.099	.032
	.0	.057	.025	.058	.086	.056	.038	.045	.034
	.5	.006	.000	.071	.092	.013	.010	.033	.039
	.8	.005	.000	.110	.085	.010	.009	.020	.044
100	-.8	1.000	1.000	.330	.064	1.000	1.000	.568	.055
	-.5	.713	.840	.066	.054	.704	.831	.079	.039
	.0	.064	.045	.052	.059	.060	.050	.044	.040
	.5	.016	.001	.047	.060	.020	.011	.061	.040
	.8	.013	.000	.028	.057	.016	.009	.096	.043
500	-.8	1.000	1.000	.306	.054	1.000	1.000	.613	.057
	-.5	.533	.709	.054	.049	.545	.704	.065	.046
	.0	.058	.065	.055	.066	.057	.067	.049	.048
	.5	.032	.024	.050	.050	.030	.028	.042	.046
	.8	.030	.022	.036	.048	.027	.026	.029	.048
1000	-.8	1.000	1.000	.258	.049	.999	1.000	.350	.051
	-.5	.433	.600	.052	.049	.453	.600	.053	.047
	.0	.055	.061	.049	.048	.056	.063	.051	.046
	.5	.036	.035	.050	.048	.036	.037	.048	.049
	.8	.035	.033	.049	.065	.039	.038	.041	.051

Note: The numbers of augmentations are 3, 4, 6 and 7 for $\ell 4$ and 10, 12, 18 and 21 for $\ell 12$ when T = 50, 100, 500 and 1000 respectively. The simulation results for the Phillips-Perron tests and the ADF tests are quoted from Schwert (1989).

TABLE 4

5% Sizes of Corrected and Augmented Coefficient Tests Under MA Errors

T	θ	$Z(\tilde{\rho})$ $\ell 4$	$Z(\tilde{\rho})$ $\ell 12$	$Aug(\tilde{\rho})$ $\ell 4$	$Aug(\tilde{\rho})$ $\ell 12$	$Z(\hat{\rho}_\tau)$ $\ell 4$	$Z(\hat{\rho}_\tau)$ $\ell 12$	$Aug(\hat{\rho}_\tau)$ $\ell 4$	$Aug(\hat{\rho}_\tau)$ $\ell 12$
50	-.8	.990	.921	.813	.666	.999	.998	.858	.292
	-.5	.667	.707	.354	.414	.673	.776	.320	.283
	.0	.063	.024	.090	.153	.056	.024	.171	.273
	.5	.010	.001	.018	.063	.010	.001	.121	.259
	.8	.007	.000	.011	.035	.008	.000	.077	.251
100	-.8	1.000	1.000	.893	.747	1.000	1.000	.821	.408
	-.5	.708	.844	.363	.387	.707	.852	.220	.343
	.0	.067	.047	.064	.104	.061	.045	.143	.355
	.5	.020	.001	.012	.029	.016	.002	.171	.353
	.8	.017	.000	.003	.015	.015	.001	.238	.363
500	-.8	1.000	1.000	.939	.839	1.000	1.000	.711	.186
	-.5	.529	.711	.352	.356	.551	.719	.095	.152
	.0	.060	.067	.055	.066	.059	.068	.072	.151
	.5	.034	.025	.009	.012	.030	.025	.062	.147
	.8	.033	.023	.002	.004	.027	.023	.037	.149
1000	-.8	1.000	1.000	.937	.853	.999	1.000	.431	.114
	-.5	.428	.598	.347	.348	.455	.611	.070	.099
	.0	.055	.062	.054	.057	.055	.063	.065	.107
	.5	.036	.036	.008	.010	.035	.035	.062	.109
	.8	.035	.034	.003	.003	.035	.034	.046	.103

Note: The numbers of augmentations are 3, 4, 6 and 7 for $\ell 4$ and 10, 12, 18 and 21 for $\ell 12$ when T = 50, 100, 500 and 1000 respectively. The simulation results for the Phillips-Perron tests and the ADF tests are quoted from Schwert (1989).

TABLE 5

5% Sizes of Corrected Tests and Augmented Tests under AR Errors

T	ρ	t-tests $Z(\tilde{t})$ $\ell 4$	t-tests $Z(\tilde{t})$ $\ell 12$	t-tests Aug$(\tilde{t})$ $\ell 4$	t-tests Aug$(\tilde{t})$ $\ell 12$	Coefficients Tests $Z(\tilde{\rho})$ $\ell 4$	Coefficients Tests $Z(\tilde{\rho})$ $\ell 12$	Coefficients Tests Aug$(\tilde{\rho})$ $\ell 4$	Coefficients Tests Aug$(\tilde{\rho})$ $\ell 12$
100	-.8	.926	.973	.050	.057	.924	.975	.288	.338
	-.5	.479	.632	.051	.058	.475	.638	.203	.255
	.0	.064	.045	.052	.059	.067	.047	.064	.104
	.5	.004	.000	.053	.061	.006	.000	.003	.011
	.8	.000	.000	.055	.064	.000	.000	.000	.000
500	-.8	.804	.926	.502	.049	.801	.926	.287	.297
	-.5	.277	.439	.050	.050	.275	.441	.190	.204
	.0	.058	.065	.055	.066	.060	.067	.055	.066
	.5	.016	.014	.051	.049	.018	.016	.001	.002
	.8	.001	.004	.051	.050	.001	.005	.000	.000

Note: The numbers of augmentations are 4, 6 for $\ell 4$ and 12, 18 for $\ell 12$ when T = 100, 500 respectively.

TABLE 6

5% Powers of Tests Under MA Errors
(T = 100, β = .9)

		t-tests			Coefficients Tests		
θ	ℓ	$\tilde{\tau}$	$Z(\tilde{\tau})$	$A(\tilde{\tau})$	$\tilde{\rho}$	$Z(\tilde{\rho})$	$A(\tilde{\rho})$
-.8	4	1.000	1.000	.551	1.000	1.000	.953
	12		1.000	.106		1.000	.820
-.5	4	.991	.985	.226	.991	.985	.778
	12		.996	.113		.997	.733
0	4	.264	.315	.191	.270	.326	.294
	12		.232	.125		.234	.350
.5	4	.019	.095	.170	.019	.114	.071
	12		.005	.125		.005	.131
.8	4	.010	.077	.113	.010	.096	.025
	12		.001	.116		.001	.071

TABLE 7

5% Powers of Tests Under AR Errors
(T = 100, β = .9)

		t-tests			Coefficient Tests		
ρ	ℓ	$\tilde{t}$	$Z(\tilde{t})$	$A(\tilde{t})$	$\tilde{\rho}$	$Z(\tilde{\rho})$	$A(\tilde{\rho})$
-.8	4	1.000	1.000	.162	1.000	1.000	.678
	12		1.000	.109		1.000	.665
-.5	4	.947	.922	.181	.949	.921	.593
	12		.972	.117		.973	.603
0	4	.264	.315	.191	.270	.326	.294
	12		.232	.125		.234	.350
.5	4	.001	.028	.175	.011	.039	.024
	12		.000	.122		.000	.059
.8	4	.000	.001	.137	.000	.002	.000
	12		.000	.107		.000	.003

TABLE 8

5% Sizes of Tests Using True Error Variances and Bias of Estimates of Error Variances Under MA Errors (T = 100)

		5% Sizes		Bias					
θ	ℓ	$Z^*(\tilde{\tau})$	$Z^*(\tilde{\rho})$	$\hat{\omega}$	bias	$\hat{\omega}^2$	bias	$\hat{\sigma}_\epsilon^2$	bias
-.8	4	.000	.000	.94	-5.46	.88	-40.1	.97	-.67
	12	.000	.000	.83	-5.57	.71	-40.3	.97	-.67
-.5	4	.012	.000	1.05	-1.18	1.11	-3.89	1.03	-.20
	12	.012	.000	.90	-1.33	.81	-4.19	1.03	-.20
0	4	.051	.052	.98	-.02	.96	-.04	.93	-.07
	12	.051	.052	.98	-.02	.98	-.02	.93	-.07
.5	4	.051	.060	.81	.06	.66	.10	1.17	-.08
	12	.051	.060	.86	.11	.76	.20	1.17	-.08
.8	4	.050	.061	.78	.07	.61	.10	1.53	-.11
	12	.050	.061	.83	.12	.71	.20	1.53	-.11

TABLE 9

5% Sizes of Tests Using True Error Variances
and Bias of Estimates of Error Variances
Under AR Errors (T = 100)

		5% Sizes				Bias			
ρ	ℓ	$Z^*(\tilde{t})$	$Z^*(\tilde{\rho})$	$\hat{\omega}$	bias	$\hat{\omega}^2$	bias	$\hat{\sigma}_\epsilon^2$	bias
-.8	4	.049	.000	.89	-2.11	.81	-8.19	1.89	-.89
	12			.70	-2.30	.50	-8.50	1.89	-.89
-.5	4	.047	.020	1.08	-.65	1.18	-1.82	1.15	-.18
	12			.94	-.79	.09	-2.11	1.15	-.18
0	4	.051	.052	.98	-.02	.96	-.04	.93	-.07
	12			.98	-.02	.98	-.04	.93	-.07
.5	4	.065	.081	.71	.13	.51	.18	1.22	-.11
	12			.73	.15	.55	.22	1.22	-.11
.8	4	.115	.147	.56	.23	.32	.21	2.24	-.54
	12			.51	.18	.27	.16	2.24	-.54

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CHAPTER 3

CHAPTER 3

A MODIFICATION OF THE SCHMIDT-PHILLIPS UNIT ROOT TEST

1. INTRODUCTION

Schmidt and Phillips (1990, hereafter, SP) have proposed a unit root test based on the parameterization

(1) $$y_t = \psi + \xi t + X_t\ ,\quad X_t = \beta X_{t-1} + \epsilon_t.$$

The unit root restriction is $\beta = 1$, which they test according to the LM (score) principle. Specifically, define $\psi_X = \psi + X_0$, $\tilde{\xi} = \overline{\Delta y} = (y_T - y_1)/(T-1)$, $\tilde{\psi}_X = y_1 - \tilde{\xi}$, and $\tilde{S}_t = y_t - \tilde{\psi}_X - \tilde{\xi}t$; $\tilde{\psi}_X$ and $\tilde{\xi}$ are the restricted MLE's of the corresponding parameters, and $\tilde{S}_t$ is the residual from (1) with these estimates. SP consider the regression

(2) $$\Delta y_t = c_1 + \phi \tilde{S}_{t-1} + \text{error},\quad t = 2,\ldots,T,$$

which is estimated by least squares, yielding an estimate $\tilde{\phi}$. Then the test statistics $\tilde{\rho}$ and $\tilde{\tau}$ are defined as follows: $\tilde{\rho} = T\tilde{\phi}$, $\tilde{\tau}$ = usual t statistic for testing the hypothesis $\phi=0$.

Under the null hypothesis that $\beta = 1$, it is obvious from (1) that $\Delta y_t = \xi + \epsilon_t$, so that in (2) $\phi = 0$, $c_1 = \xi$ and the error equals ϵ_t. In particular, the intercept c_1 is nonzero unless $\xi = 0$. It is also clear from the definition of $\tilde{S}_t$ that $\Delta\tilde{S}_t = \Delta y_t - \tilde{\xi}$, so that the SP test can equally well be performed using the regression

(3) $\Delta\tilde{S}_t = c_2 + \phi\tilde{S}_{t-1} + \text{error}$, $t = 2,\ldots,T.$

The estimate of ϕ and its t-ratio will be unaffected by the change in dependent variable, while the estimates of c_1 and c_2 will differ by $\tilde{\xi}$.

The motivation for this chapter is the observation that the intercept c_2 in (3) equals zero (in the population). Thus we will consider new tests based on the regression

(4) $\Delta\tilde{S}_t = \phi\tilde{S}_{t-1} + \text{error}$, $t = 2,\ldots,T.$

In addition, we also consider the F-test of the null hypothesis that both c_2 and ϕ in (3) equal zero. The F-test statistic, to be denoted by $\bar{F}$, is defined in the usual manner. Intuitively, the parameter c_2 is redundant in (3), and setting it to zero could be expected to lead to a more powerful test. However, the comparison of the power of these tests to the power of the original SP tests or the DF tests will be seen to depend on the value of the (unobserved) initial conditions parameter $X_0^* = X_0/\sigma_\epsilon$.

2. THE NEW TESTS AND THEIR DISTRIBUTIONS

Let $\bar{\phi}$ be the least squares estimate of ϕ in (4). We define the new unit root test statistics $\bar{\rho} = T\bar{\phi}$ and $\bar{\tau}$ = usual t-statistic for testing the hypothesis $\phi = 0$ in (4). Also, as above, we define $\bar{F}$ to be the usual F statistic for testing the hypothesis $c_2 = \phi = 0$ in (3).

The basic parameters of the model are ψ, ξ, β, σ_ϵ, $X_0^* = X_0/\sigma_\epsilon$ and T. It is known from SP and from DeJong, Nankervis, Savin and Whiteman (1989) that the distributions of the SP tests $\tilde{\rho}$ and $\tilde{\tau}$ and of the DF tests $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$ are independent of ψ, ξ, σ_ϵ and X_0^*, and therefore depend only on T. The same is true of the tests proposed in this chapter. Under the null hypothesis that $\beta = 1$, it is easy to demonstrate that $\tilde{S}_t = \Sigma_{j=2}^{t}(\epsilon_j - \bar{\epsilon})$. Clearly this does not depend on the parameters ψ, ξ or X_0^*, and it has the same scale as σ_ϵ. The statistics $\bar{\rho}$, $\bar{\tau}$ and $\bar{F}$ are functions of $\tilde{S}_t$, $t = 2,\ldots,T$, so their distributions do not depend on ψ, ξ or X_0^*. Furthermore, the scale factor σ_ϵ cancels in all expressions for $\bar{\rho}$, $\bar{\tau}$ and $\bar{F}$. Thus their distributions under the null hypothesis depend only on T.

The finite sample distributions of the statistics depend on the assumption that the ϵ_t are iid normal, but their asymptotic distributions can be derived under weaker assumptions. We follow SP in assuming that the error sequence $\{\epsilon_t\}$ satisfies the regularity conditions of Phillips and Perron (1988, p. 336), and we define the nuisance parameters σ_ϵ^2 and σ^2 as in Phillips and Perron (p. 337). (Our σ_ϵ^2 is their σ_u^2. Under iid errors, $\sigma^2 = \sigma_\epsilon^2$ and σ_ϵ^2 is the variance of ϵ as above.) Define the Wiener process W(r) and the Brownian bridge $V(r) = W(r) - rW(1)$, for $r \in [0,1]$. It then follows (SP, Appendix 3) that

(5) $$T^{-1/2}\tilde{S}_{[rT]} \to \sigma V(r)$$

where [rT] is the integer part of rT, and where "$\to$" indicates weak convergence. Using (5), we can establish the asymptotic distributions for the statistics $\bar{\rho}$, $\bar{\tau}$ and $\bar{F}$:

(6a) $\bar{\rho} \to -(1/2)(\sigma_\epsilon^2/\sigma^2)[\int_0^1 V(r)^2 dr]^{-1}$

(6b) $\bar{\tau} \to -(1/2)(\sigma_\epsilon/\sigma)[\int_0^1 V(r)^2 dr]^{-1/2}$

(6c) $\bar{F} \to (1/8)(\sigma_\epsilon^2/\sigma^2)[\int_0^1 \underline{V}(r)^2 dr]^{-1}$

Here, $\underline{V}(r) = V(r) - \int_0^1 V(s)ds$ is a demeaned Brownian bridge. The proof is given in the Appendix. The asymptotic distributions of $\bar{\rho}$ and $\bar{\tau}$ are the same as the asymptotic distributions given for $\tilde{\rho}$ and $\tilde{\tau}$ in SP, except that in SP the Brownian bridge V(r) must be replaced by the demeaned Brownian bridge $\underline{V}(r)$. In SP, the Brownian bridge is demeaned by the inclusion of intercept in the regression (2) or (3). Here, as in SP, the simple manner in which the ratio of nuisance parameters (σ_ϵ/σ) enters the asymptotic distribution leads to very simple corrections for error autocorrelation. One needs simply to multiply $\bar{\rho}$ or $\bar{F}$ by a consistent estimate of $(\sigma_\epsilon^2/\sigma^2)^{-1}$, or $\bar{\tau}$ by a consistent estimate of $(\sigma_\epsilon/\sigma)^{-1}$. Estimation of the nuisance parameters σ_ϵ^2 and σ^2 is accomplished in exactly the same way as in SP or Phillips and Perron.

We obtain $\sigma_\epsilon^2 = \sigma^2$ under iid errors so that we can tabulate the critical values. Table 1 gives critical values for the new test statistics $\bar{\rho}$, $\bar{\tau}$ and $\bar{F}$, as a function of T.

These are calculated via simulation, as discussed in SP, section 3. The number of replications is 50,000. Note that $\bar{\rho}$ and $\bar{\tau}$ are lower tail tests if the alternative hypothesis is stationarity, while $\bar{F}$ is an upper tail test.

3. POWER OF THE TESTS

Under the alternative that $\beta \neq 1$, the powers of the SP tests $\tilde{\rho}$ and $\tilde{\tau}$ and of the DF tests $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$ are known to be independent of the parameters ψ, ξ and σ_ϵ, but they depend on β, T and the initial conditions parameter X_0*. If the errors are symmetrically distributed, they depend on X_0* only through $|X_0^*|$. The same is true of the powers of the tests proposed in this chapter. This is most easily seen from the SP (Appendix 2) representation of $\tilde{S}_t$:

$$(7) \qquad \tilde{S}_t = \sum_{j=2}^{t}(w_j - \bar{w}), \text{ where } w_j = \beta^{j-1}\beta X_0 + \epsilon_j + (\beta-1)\sum_{i=0}^{j-1}\beta^i\epsilon_{j-i}.$$

This depends on β, T and X_0*, and given these parameters, has the same scale as σ_ϵ; σ_ϵ then cancels from all expressions for $\bar{\rho}$, $\bar{\tau}$ and $\bar{F}$.

We now compare the power of the new tests $\bar{\rho}$, $\bar{\tau}$ and $\bar{F}$ with the power of the SP tests and DF tests. This is done via a Monte Carlo simulation, using essentially the same experimental design as in SP. The results of the experiment are given in Table 2. The results in Table 2 for the SP tests and for the DF tests are as in SP, while the results for this chapter's new tests ($\bar{\rho}$, $\bar{\tau}$ and $\bar{F}$) are new.

The power of the F-test $\bar{\bar{F}}$ is quite similar to the power of the SP test $\tilde{\rho}$ for all of our experiments. Therefore, we do not need any separate explanation on the power of the F-test. In the remainder of this section, the "new tests" will mean the new $\bar{\rho}$ and $\bar{\tau}$ tests.

Experiment 1 takes $\beta = 1$, T = 100, 200 and 500, and simply confirms that, apart from randomness, the sizes of the test are correct.

Experiment 2 considers the power of the tests, for $X_0* = 0$, T = 100, and $\beta = 1, .95, .90$ and .80. This variation in β induces a substantial variation in the power of the tests. From SP it is known that the SP tests are more powerful than the DF tests for these parameter values. The comparison of the new tests to the SP tests is more ambiguous. Basically, the new tests are more powerful than the SP tests when power is low (β = .95 and .90) and less powerful when power is high (β = .80). Experiments 3 and 4 provide similar results and essentially the same conclusions, but for T = 200 and T = 500 respectively.

We now turn to the effect of the initial conditions parameter X_0* on power. Because we use normal errors, which are symmetrically distributed, only $|X_0*|$ matters. Experiment 5 considers T = 100, β = .90, and X_0* = 0, -1, -2, -5 and -10, while Experiment 6 considers β = .95 instead of .90 (so that powers are lower). From SP it is known that the SP tests and the DF tests have powers that are monotonic in $|X_0*|$, though in opposite directions. Over the range we consider, the power of $\hat{\tau}_\tau$ increases with $|X_0*|$, while the powers of the other tests decrease as $|X_0*|$ increases. In both Experiment 5 and Experiment 6, we can see that the power of the $\bar{\rho}$ and $\bar{\tau}$ tests also decreases as $|X_0*|$ increases, but their power decreases faster than the power of the original SP tests. The new tests $\bar{\rho}$ and $\bar{\tau}$ are about equally powerful, and they are marginally more powerful than the SP tests or the DF tests when $|X_0*| = 0$, but less powerful when $|X_0*|$ is large. For example, the new $\bar{\tau}$ test is more powerful than the SP $\tilde{\tau}$ test for $|X_0*| \leq 1.7$ (approximately), and it is more powerful than the DF $\hat{\tau}_\tau$ test for $|X_0*| \leq 2.8$ (approximately). The above discussion is summarized in Figure 1.

If X_0* is treated as a fixed parameter, there is not much more to say. However, following SP, when $\beta \neq 1$ we can also think of X_0* as random and drawn from the stationary distribution of X_t/σ_ϵ, which is $N[0, 1/(1-\beta^2)]$. When X_0* is treated as random, we can calculate the probability that the new tests are more powerful than the others; for example, the probability that the new $\bar{\tau}$ test is more powerful than the SP $\tilde{\tau}$ test is about .54 and the probability that the new test is more powerful than the DF test $\hat{\tau}_\tau$ is about .78.

Experiments 7 and 8 treat X_0* as random and drawn from the stationary distribution of X_t/σ_ϵ, and therefore calculate the powers of the tests <u>not</u> conditional on X_0*.

For these experiments the number of replications used is 50,000. Experiment 7 has T = 100 while Experiment 8 has T = 500, and in each case a variety of values of β is considered so that power ranges from nominal size to nearly unity.

The basic result of these experiments is that the power of the tests proposed in this chapter is greater than the power of the SP or DF tests when power is low, but it is less than the power of the competing tests when power is high. However, comparing the new tests to the SP tests, the gain in power when power is low is small compared to the loss in power when power is higher.

4. CONCLUDING REMARKS

The calculation of the SP test statistics involves estimation of a redundant intercept, and the new tests $\bar{\rho}$ and $\bar{\tau}$ proposed in this chapter successfully eliminate the need to estimate it. This might be expected to lead to a gain in power. However, our Monte Carlo power calculations do not support this expectation. Treating the initial conditions term as fixed, the new tests are better than the SP tests when the initial condition term is small and worse when it is large, and neither the magnitude of the power differences nor the range over which the tests are better provides a strong basis for preferring the new tests to the SP tests or vice-versa. Treating the initial conditions term as random, the new tests are slightly more powerful than the SP tests when power is low, and moderately less powerful when power is high. These results would appear to support the general use of the original SP tests rather than the new tests.

TABLE 1

CRITICAL VALUES FOR $\bar{\tau}$

T	.01	.025	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.975	.99
25	-3.40	-3.01	-2.71	-2.37	-2.01	-1.78	-1.59	-1.43	-1.29	-1.15	-1.00	-0.84	-0.74	-0.66	-0.59
50	-3.29	-2.94	-2.66	-2.36	-2.02	-1.79	-1.61	-1.45	-1.30	-1.16	-1.02	-0.85	-0.74	-0.66	-0.59
100	-3.24	-2.89	-2.64	-2.35	-2.01	-1.79	-1.61	-1.46	-1.31	-1.17	-1.02	-0.85	-0.74	-0.66	-0.58
200	-3.19	-2.89	-2.63	-2.34	-2.01	-1.79	-1.61	-1.45	-1.31	-1.17	-1.02	-0.85	-0.74	-0.66	-0.59
500	-3.21	-2.90	-2.63	-2.34	-2.01	-1.79	-1.61	-1.45	-1.31	-1.17	-1.02	-0.85	-0.74	-0.66	-0.59
1000	-3.20	-2.89	-2.62	-2.34	-2.00	-1.78	-1.60	-1.45	-1.31	-1.17	-1.02	-0.85	-0.74	-0.66	-0.59

CRITICAL VALUES FOR $\bar{\rho}$

T	.01	.025	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.975	.99
25	-16.7	-14.1	-12.0	-9.82	-7.46	-6.02	-4.92	-4.07	-3.33	-2.68	-2.08	-1.47	-1.14	-0.93	-0.74
50	-18.3	-15.2	-12.8	-10.4	-7.79	-6.22	-5.08	-4.17	-3.39	-2.71	-2.09	-1.47	-1.12	-0.90	-0.71
100	-19.3	-15.7	-13.2	-10.6	-7.92	-6.30	-5.14	-4.21	-3.41	-2.72	-2.07	-1.45	-1.09	-0.88	-0.69
200	-19.5	-16.1	-13.5	-10.7	-8.00	-6.35	-5.14	-4.20	-3.40	-2.72	-2.07	-1.44	-1.08	-0.86	-0.69
500	-20.3	-16.5	-13.6	-10.8	-7.99	-6.33	-5.14	-4.21	-3.42	-2.71	-2.07	-1.43	-1.09	-0.87	-0.69
1000	-20.2	-16.5	-13.6	-10.8	-7.97	-6.31	-5.11	-4.17	-3.40	-2.71	-2.07	-1.45	-1.09	-0.88	-0.68

CRITICAL VALUES FOR $\bar{F}$

T	.01	.025	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.975	.99
25	0.48	0.57	0.66	0.80	1.04	1.27	1.51	1.78	2.10	2.50	3.05	4.00	4.96	6.01	7.45
50	0.47	0.56	0.66	0.82	1.06	1.29	1.53	1.80	2.11	2.49	3.02	3.93	4.84	5.73	6.85
100	0.46	0.56	0.67	0.82	1.07	1.30	1.54	1.80	2.11	2.48	2.98	3.81	4.65	5.49	6.56
200	0.47	0.56	0.67	0.82	1.07	1.30	1.54	1.80	2.10	2.47	2.97	3.80	4.60	5.38	6.48
500	0.47	0.56	0.67	0.82	1.06	1.29	1.53	1.78	2.08	2.45	2.94	3.78	4.59	5.34	6.38
1000	0.46	0.56	0.67	0.82	1.07	1.29	1.53	1.79	2.09	2.45	2.94	3.75	4.54	5.29	6.28

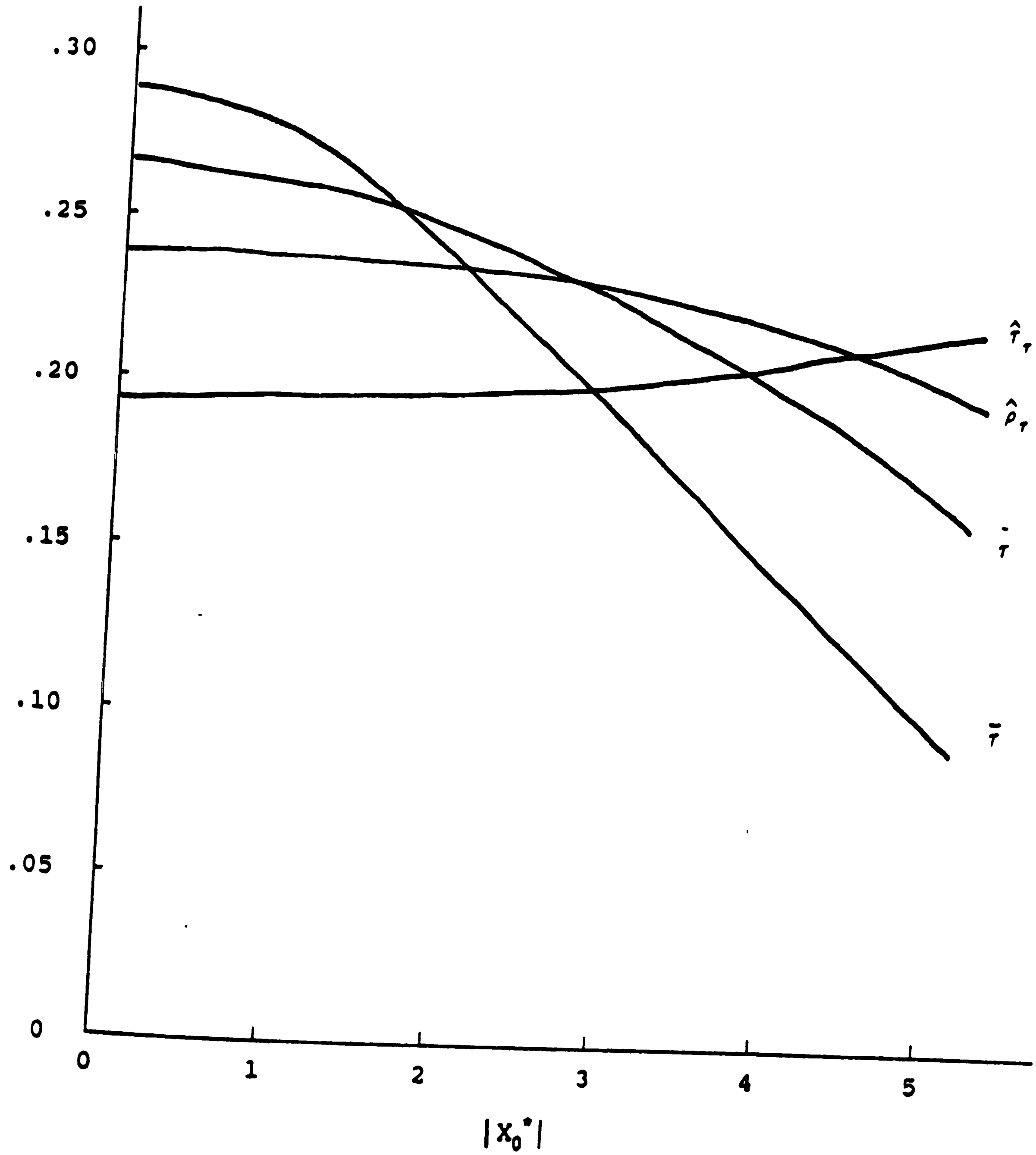
TABLE 2

SIZE AND POWER, 5% LOWER TAIL TESTS

Exp No.	T	β	X_0^*	$\hat{\tau}_\tau$	$\hat{\rho}_\tau$	$\tilde{\tau}$	$\tilde{\rho}$	$\bar{\tau}$	$\bar{\rho}$	$\bar{F}$
1	100	1	0	.048	.050	.051	.052	.050	.052	.051
	200			.048	.048	.048	.050	.050	.051	.051
	500			.051	.051	.049	.051	.050	.050	.049
2	100	1	0	.048	.050	.051	.052	.050	.052	.051
		.95		.082	.098	.105	.108	.113	.116	.108
		.90		.186	.239	.264	.270	.288	.294	.273
		.80		.644	.734	.759	.765	.719	.723	.767
3	200	1	0	.048	.048	.048	.050	.050	.051	.051
		.95		.178	.233	.254	.266	.290	.292	.269
		.90		.617	.724	.751	.763	.742	.744	.765
		.80		.999	1.00	.997	.997	.972	.973	.998
4	500	1	0	.051	.051	.049	.051	.050	.050	.050
		.95		.819	.897	.910	.914	.874	.874	.913
		.90		1.00	1.00	1.00	1.00	.993	.993	1.00
5	100	.90	0	.186	.239	.264	.270	.288	.294	.273
			-1	.188	.239	.260	.267	.276	.280	.268
			-2	.191	.234	.248	.252	.238	.242	.251
			-5	.211	.198	.161	.165	.095	.097	.165
			-10	.304	.120	.032	.033	.003	.003	.032
6	100	.95	0	.082	.098	.105	.108	.113	.116	.108
			-1	.083	.097	.104	.106	.113	.115	.105
			-2	.082	.095	.101	.104	.108	.111	.102
			-5	.083	.085	.086	.088	.082	.084	.089
7	100	.95	rd	.081	.090	.096	.098	.098	.100	.098
		.90		.187	.225	.238	.242	.236	.241	.244
		.85		.390	.464	.476	.483	.441	.446	.483
		.80		.647	.732	.722	.728	.634	.638	.725
		.70		.961	.982	.952	.954	.854	.857	.954
8	500	.99	rd	.081	.094	.095	.098	.100	.099	.096
		.98		.184	.225	.230	.237	.234	.234	.233
		.97		.374	.452	.455	.464	.433	.433	.460
		.95		.828	.892	.863	.869	.762	.762	.865
		.90		1.00	1.00	.998	.999	.973	.973	.998

FIGURE 1

POWER OF VARIOUS TESTS, T = 100, β = 0.9



APPENDIX

We employ the same functional limit theory and assume the same regularity conditions as Phillips and Perron (1988, p.336). First, we want to show the following results:

(A.1) $$T^{-2} \sum_{2}^{T} \tilde{S}_{t-1}{}^2 \rightarrow \sigma^2 \int_0^1 V(r)^2 dr$$

(A.2) $$T^{-1} \sum_{2}^{T} \tilde{S}_{t-1}\epsilon_t \rightarrow -(1/2)\sigma_\epsilon{}^2$$

Since $\tilde{S}_{t-1} = S_{t-1} - (t-2) - \bar{\epsilon} - \epsilon_1$ and $\Delta\tilde{S}_t = \epsilon_t - \bar{\epsilon}$ from (17) of Schmidt and Phillips (1990), we have:

(A.3) $$T^{-1} \sum_{2}^{T} \tilde{S}_{t-1}\Delta\tilde{S}_t = T^{-1} \sum_{2}^{T} \{S_{t-1}-(t-2)\bar{\epsilon}-\epsilon_1\}(\epsilon_t-\bar{\epsilon})$$

$$= T^{-1} \sum_{2}^{T} S_{t-1}\epsilon_t - T^{-1}\bar{\epsilon} \sum_{2}^{T} S_{t-1} - T^{-1}\bar{\epsilon} \sum_{2}^{T}(t-2)\epsilon_t$$

$$+ T^{-1}\bar{\epsilon}^2 \sum_{2}^{T}(t-2) - T^{-1} \sum_{2}^{T}\epsilon_1\epsilon_t + T^{-1}\bar{\epsilon} \sum_{2}^{T}\epsilon_1.$$

The following results follow (where summations are over t from 2 to T):

(i) $T^{-1} \Sigma S_{t-1}\epsilon_t \rightarrow \frac{1}{2}[\sigma^2 W(1)^2-\sigma_\epsilon{}^2]$

(ii) $T^{-1}\bar{\epsilon}\ \Sigma S_{t-1} = T^{-1/2}\bar{\epsilon}\ T^{-3/2}\ \Sigma S_{t-1} \rightarrow \sigma W(1)\cdot\sigma\int_0^1 W(r)dr$

(iii) $T^{-1}\bar{\epsilon}\ \Sigma(t-2)\epsilon_t = T^{-1/2}\bar{\epsilon}\ T^{-3/2}\ \Sigma t\epsilon_t - 2T^{-1/2}\bar{\epsilon}T^{-1}T^{1/2}\bar{\epsilon}$

$$\rightarrow -\sigma W(1)\sigma[\int_0^1 W(r)dr - W(1)]$$

(iv) $T^{-1}\bar{\epsilon}^2\ \Sigma(t-2) = (T^{1/2}\bar{\epsilon})^2 T^{-2}\ \Sigma(t-2) \rightarrow (1/2)[\sigma W(1)]^2$

and the last two terms cancel each other. Then, it is straightforward from above to see that

$$\text{(A.4)}\quad T^{-1}\sum_{2}^{T}\tilde{S}_{t-1}\Delta\tilde{S}_t \rightarrow \tfrac{1}{2}\sigma^2W(1)^2 - \tfrac{1}{2}\sigma_\epsilon^2 - \sigma^2W(1)\int_0^1 W(r)dr - \sigma^2W(1)^2$$

$$+ \sigma^2W(1)\int_0^1 W(r)dr + \tfrac{1}{2}\sigma^2W(1)^2$$

$$= -\tfrac{1}{2}\sigma_\epsilon^2$$

which proves (A.2). The result in (A.1) is similarly obtained as in Schmidt and Phillips (1990, p.23), while a standard Brownian bridge V(r) replaces a demeaned Brownian bridge $\underline{V}(r)$ used for the original SP test.

From the results in (A.1) and (A.2), it is straightforward to see that the results in (6a) and (6b) hold:

$$\text{(A.5)}\qquad \bar{\rho} = T\bar{\phi} = [T^{-1}\,\Sigma\tilde{S}_{t-1}\Delta\tilde{S}_t]/[T^{-2}\,\Sigma\tilde{S}_{t-1}^{\;2}]$$

$$\rightarrow -\tfrac{1}{2}(\sigma_\epsilon^2/\sigma^2)\,[\int_0^1 V(r)^2dr]^{-1}$$

$$\text{(A.6)}\qquad \bar{\tau} = T^{-1}\bar{\rho}\cdot[\sigma_\epsilon^2/\Sigma\tilde{S}_{t-1}^{\;2}]^{1/2}$$

$$\rightarrow -\tfrac{1}{2}(\sigma_\epsilon^2/\sigma^2)\,[1/\int_0^1 V(r)^2dr]^{-1}[\sigma^2\int_0^1 V(r)^2dr]^{1/2}\sigma_\epsilon^{-1}$$

$$= -\tfrac{1}{2}(\sigma_\epsilon/\sigma)\,[\int_0^1 V(r)^2dr]^{-1/2}.$$

The F-statistic for testing the joint hypothesis $c_2 = \phi = 0$ in (3) is given by:

$$\bar{F} = \frac{RSS/2}{ESS/(T-3)}$$

where

$$RSS = \sum_{2}^{T}[\bar{\Phi}(\tilde{S}_{t-1}-\bar{S})]^2$$

$$ESS = \sum_{2}^{T}[(\Delta\tilde{S}_t-\Delta\bar{S})-\bar{\Phi}(\tilde{S}_{t-1}-\bar{S})]^2$$

Using the result in (A.1), we have

$$\frac{1}{2}RSS = \frac{1}{2}\bar{\Phi}^2\sum_{2}^{T}(\tilde{S}_{t-1}-\bar{S})^2$$

$$= \frac{1}{2}(T\bar{\Phi})^2T^{-2}\sum_{2}^{T}(\tilde{S}_{t-1}-\bar{S})^2$$

$$\rightarrow \frac{1}{2}\left[-\frac{1}{2}\frac{\sigma_\varepsilon^2}{\sigma^2}\frac{1}{\int_0^1 \underline{Y}(r)^2dr}\right]^2\sigma^2\int_0^1\underline{Y}(r)^2dr$$

$$= \frac{\sigma_\varepsilon^4}{8\sigma^2}\frac{1}{\int_0^1\underline{Y}(r)^2dr}$$

$$\frac{ESS}{T} = \sum_{2}^{T}(\Delta\tilde{S}_t-\Delta\bar{S})^2\frac{1}{T} + (T\bar{\Phi})^2T^{-2}\sum_{2}^{T}(\tilde{S}_{t-1}-\bar{S})^2\frac{1}{T}$$

$$- 2T\bar{\Phi}\,T^{-1}\sum_{2}^{T}(\tilde{S}_{t-1}-\bar{S})(\Delta\tilde{S}_t-\Delta\bar{S})\frac{1}{T}$$

$$\rightarrow \sigma_\varepsilon^2$$

since the last two terms vanish asymptotically. Therefore the asymptotic distribution of the F-statistic in (6c) can be obtained from the ratio of the last two expressions above. □

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CHAPTER 4

CHAPTER 4

UNIT ROOT TESTS BASED ON INSTRUMENTAL VARIABLES ESTIMATION

1. INTRODUCTION

In this chapter, new tests for a unit root, based on instrumental variables (IV) estimation are developed. The tests are similar in spirit to the IV tests of Hall (1989) and Pantula and Hall (1990), but they are derived from the unit root tests of Schmidt and Phillips (1990) rather than from the Dickey-Fuller tests of Fuller (1976) and Dickey and Fuller (1979). The new tests perform remarkably well. In the presence of moving average errors, the tests are quite accurate under the null hypothesis of a unit root. Furthermore, their power against stationary alternatives is better than the power of other tests that are accurate under the null.

It is well known that the tabulations of the Dickey-Fuller (hereafter, DF) tests assume iid errors. When the errors are autocorrelated, the distributions of the test statistics are affected, even asymptotically. Subsequent research has suggested modifications of the DF tests that have the same asymptotic distributions under autocorrelated errors as the DF tests have in the presence of iid errors, so that the usual DF tabulations can be used. As discussed in Chapter 1, these modifications can be put roughly into three categories.

First, Said and Dickey (1984, 1985) have suggested augmented DF (hereafter, ADF) tests based on the Dickey-Fuller regression augmented with lagged differences of the dependent variable. The ADF tests were intended to accommodate autoregressive errors, and their size should also be correct asymptotically in the presence of moving average errors, if the number of augmentations increases with sample size at an appropriate rate. Second, Phillips (1987) and Phillips and Perron (1988) developed transformed tests (hereafter, PP tests) that are valid asymptotically in the presence of a wide class of weakly dependent and heterogeneous errors. Third, Hall (1989) and Pantula and Hall (1991) proposed IV tests that easily allow for moving average errors. They are based on the IV estimation of the Dickey-Fuller regression, where the instrument is the dependent variable with a lag greater than the order of the MA process of the errors. If there are ARMA errors, they can be handled by a combination of augmentations and IV estimation.

A considerable body of simulation evidence, including Phillips and Perron (1988), Schwert (1989) and Kim and Schmidt (1990), has documented serious problems with the finite sample properties of the PP and ADF tests. The PP tests reject the null too often in the presence of negatively autocorrelated errors and too seldom in the presence of positively autocorrelated errors. These size distortions are considerable even for rather large sample sizes, such as T = 1000. The size distortions of the ADF tests can be more or less removed by using a large enough number of augmentations, but then the tests have very little power against reasonable alternatives. Hall (1989) presents simulation evidence showing that the size distortions of the IV tests are less than those of the PP tests, though they are still considerable at reasonable sample sizes if the errors are strongly autocorrelated.

Schmidt and Phillips (1990, hereafter, SP) have proposed unit root tests and discussed the advantages of their tests relative to the DF tests. The SP tabulations also assume iid errors, and error autocorrelation can be handled in essentially the same ways as for the DF tests. Chapter 2 of this dissertation provides simulation evidence which shows that the augmented SP tests and the SP tests with PP-type corrections suffer from the same problems as the corresponding DF tests did. Specifically, the SP tests with PP-type corrections and the augmented SP tests with a small number of augmentations both suffer from very substantial size distortions. Furthermore, while the augmented SP tests with enough augmentations do not have large size distortions, they also have very little power.

This chapter provides IV tests based on the SP framework. Our simulations show that these tests have desirable operating characteristics; they have surprisingly small size distortions and are more powerful than other tests of similar size. This chapter provides an alternative interpretation of the IV tests, as PP-type tests but with unusual estimates of the relevant error variances. This interpretation may explain why IV versions of the SP tests outperform IV versions of the DF tests.

2. NEW IV TESTS

The SP tests are based on the following parameterization, as discussed in Chapter 1:

$$(1) \qquad y_t = \psi + \xi \cdot t + X_t \ , \quad X_t = \beta X_{t-1} + \epsilon_t \ , \quad t = 1,..,T.$$

The unit root hypothesis is $H_0 : \beta = 1$. The basic parameters are ψ, ξ, σ_ϵ and $X_0^* = X_0/\sigma_\epsilon$, where X_0 is the initial value of X. The only observable is y_t, t=1,..,T.

The SP tests are tests of $\beta = 1$ derived according to the LM (score) principle. Specifically, define $\psi_X = \psi + X_0$, $\tilde{\xi} = \overline{\Delta y} = (y_T - y_1)/(T-1)$, $\tilde{\psi}_X = y_1 - \tilde{\xi}$, and $\tilde{S}_t = y_t - \tilde{\psi}_X - \tilde{\xi}t$; $\tilde{\psi}_X$ and $\tilde{\xi}$ are the restricted MLE's of the corresponding parameters, and $\tilde{S}_t$ is the residual from (1) with these estimates. SP consider the regression

$$(2) \qquad \Delta y_t = c + \phi \tilde{S}_{t-1} + \text{error} \ , \quad t = 2,..,T \ ,$$

which is estimated by least squares, yielding an estimate $\tilde{\phi}$.

Then the SP statistics $\tilde{\rho}$ and $\tilde{\tau}$ are defined as follows: $\tilde{\rho} = T\tilde{\phi}$, $\tilde{\tau}$ = usual t statistic for testing the hypothesis $\phi = 0$.

In this chapter we actually employ a slightly modified version of the SP tests, discussed in Chapter 3. Since $\Delta\tilde{S}_t = \Delta y_t - \tilde{\xi}$, the dependent variable in (2) can equivalently be taken to be $\Delta\tilde{S}_t$ instead of Δy_t. However, if this is done, the population intercept is zero, so that we can consider the alternative regression:

(3) $$\Delta\tilde{S}_t = \phi\tilde{S}_{t-1} + \text{error} , \quad t = 2,..,T.$$

If this is estimated by least squares, yielding the estimate $\bar{\phi}$, the modified SP statistics are

(4) $$\bar{\rho} = T\cdot\bar{\phi}$$

(5) $\bar{\tau}$ = usual t-statistic for the hypothesis $\phi = 0$.

Define the "innovation variance" σ_ϵ^2 and the "long-run variance" σ^2 as in PP:

(6a) $$\sigma_\epsilon^2 = \lim_{T\to\infty} T^{-1} \sum_t \epsilon_t^2$$

(6b) $$\sigma^2 = \lim_{T\to\infty} T^{-1}(\sum_t \epsilon_t)^2 .$$

Then, assuming the regularity conditions of PP, p. 336, the asymptotic distributions for $\bar{\rho}$ and $\bar{\tau}$ (as given in Chapter 3) are:

(7a) $$\bar{\rho} \to -(1/2)(\sigma_\epsilon^2/\sigma^2)[\int_0^1 V(r)^2dr]^{-1}$$

(7b) $\tilde{\tau} \rightarrow -(1/2)(\sigma_\epsilon/\sigma)[\int_0^1 V(r)^2 dr]^{-1/2}$.

Here V(r) is a Brownian bridge: V(r) = W(r) - rW(1), where W(r) is a Wiener process. The asymptotic distributions in (7) are the same as those given for $\tilde{\rho}$ and $\tilde{\tau}$ by SP, except that in SP the Brownian bridge V(r) must be replaced by the demeaned Brownian bridge $\underline{V}(r) = V(r) - \int_0^1 V(s)ds$.

In this chapter we assume that the errors ϵ_t follow an MA(q) process:

(8) $$\epsilon_t = u_t + \sum_{j=1}^{q} \theta_j u_{t-j}$$

with u_t iid. We assume that the errors satisfy conditions 1 and 2 of Hall (1989, p. 51). We will discuss in most detail the special case q = 1, so that $\epsilon_t = u_t + \theta u_{t-1}$ with $|\theta| < 1$. For the MA(1) case, we note that $\sigma_\epsilon^2/\sigma^2 = (1+\theta^2)/(1+\theta)^2$.

We derive the IV versions of the (modified) SP tests by estimating (3) by instrumental variables estimation, where the instrumental variable is $\tilde{S}_{t-k}$, $k \geq q+1$. Let $\tilde{\phi}_{iv}$ be the IV estimate of ϕ, and define the consistent estimator $\tilde{\sigma}_\epsilon^2$ of the error variance σ_ϵ^2 as follows:

(9) $$\tilde{\sigma}_\epsilon^2 = \frac{1}{T}\sum_{t=k+1}^{T} (\Delta\tilde{S}_t - \tilde{\phi}_{iv}\tilde{S}_{t-1})^2 .$$

Then the test statistics are given by

$$(10a)\qquad \tilde{\rho}_{iv} = T\,\tilde{\phi}_{iv} = T\,\frac{\sum_{t=k+1}^{T} \tilde{S}_{t-k}\Delta\tilde{S}_{t}}{\sum_{t=k+1}^{T} \tilde{S}_{t-k}\tilde{S}_{t-1}}$$

$$(10b)\qquad \tilde{\tau}_{iv} = \frac{\tilde{\phi}_{iv}}{\sqrt{\dfrac{\tilde{\sigma}_{\varepsilon}^{2}\sum_{t=k+1}^{T} \tilde{S}_{t-k}^{2}}{\left(\sum_{t=k+1}^{T} \tilde{S}_{t-k}\tilde{S}_{t-1}\right)^{2}}}}\ .$$

Theorem 1: Assume that the data are generated by (1) with $\beta = 1$ and (8), and that the errors in (8) satisfy conditions 1 and 2 of Hall (1989, p. 51). Suppose that $\tilde{\phi}_{iv}$ is the IV estimate of ϕ from (3), based on the instrument $\tilde{S}_{t-k}$, $k \geq q+1$. Then,

$$(11)\qquad \tilde{\rho}_{iv} \rightarrow -\frac{1}{2\int_{0}^{1} V(r)^{2}dr}$$

$$(12)\qquad \tilde{\tau}_{iv} \rightarrow -\frac{\sigma}{\sigma_{\varepsilon}}\,\frac{1}{2\left(\int_{0}^{1} V(r)^{2}dr\right)^{\frac{1}{2}}}\ ,$$

where "$\rightarrow$" indicates weak convergence as $T \rightarrow \infty$.

Proof: Write $\tilde{\rho}_{iv}$ as

$$\tilde{\rho}_{iv} = T\tilde{\Phi}_{iv} = \frac{T^{-1}\sum_{t=k+1}^{T}\tilde{S}_{t-k}\Delta\tilde{S}_t}{T^{-2}\sum_{t=k+1}^{T}\tilde{S}_{t-k}\tilde{S}_{t-1}} .$$

The denominator converges weakly to $\sigma^2\int_0^1 V(r)^2 dr$ for $k \geq 0$; that is, regardless of whether or not $k \geq q+1$. However, the probability limit of the numerator depends on whether $k \geq q+1$. We show in Appendix 1 that

$$plim\ T^{-1}\sum_{t=k+1}^{T}\tilde{S}_{t-k}\Delta\tilde{S}_t = \begin{cases} -\sigma^2/2\ , & k \geq q+1 \\ -\sigma_\epsilon^2/2\ , & k \leq q\ . \end{cases}$$

For $k \geq q+1$, the asymptotic distribution given by (11) follows by joint convergence of the numerator and denominator. The proof for $\tilde{\tau}_{iv}$ follows exactly the same lines. □

Our Theorem 1 is very similar to Theorem 1 of Hall (1989, p. 51). But, the asymptotics are expressed in much simpler form than those of Hall's statistics.

Theorem 1 shows that the IV estimator successfully handles moving average errors. The intuition behind it is appealing. For $k \geq q+1$, $\tilde{S}_{t-k}$ is uncorrelated with ϵ_t and so a bias correction is asymptotically unnecessary. In other words, in estimating by IV we have taken advantage of the maintained assumption that $\mathrm{Cov}(\epsilon_t \epsilon_{t-j}) = 0$ for $j \geq q+1$. This restriction is not so easily exploited in the PP framework.

The proper choice of the order of moving average errors, q, is obviously important. As asserted in Hall (1989), the choice of q should ideally rely on the economic theory on which the model is based. If theory does not provide any information on q, one may empirically determine q; for instance, the Wald tests developed by Hall (1990) can be applied by using the IV residuals in (3). It would be safe to select q conservatively (i.e., higher q rather than lower), since choosing q too low causes a serious problem, but choosing q too high does not, at least asymptotically.

Theorem 1 indicates that the asymptotic distribution of the test statistic $\tilde{\rho}_{iv}$ does not depend on the nuisance parameters σ^2 and σ_ϵ^2, so that the $\tilde{\rho}_{iv}$ test is valid asymptotically in the presence of MA errors. This is similar to Hall's result for IV versions of the DF coefficient tests. Also similarly to Hall's t-test, the new IV t-test $\tilde{\tau}_{iv}$ still depends on the nuisance parameters. If we define $\lambda = \sigma_\epsilon/\sigma$, the asymptotic distribution of $\tilde{\tau}_{iv}$ involves the scale factor λ^{-1}. It is very interesting that the asymptotic distribution of the OLS-based SP t-test $\bar{\tau}$ (given in (7b)) involves the scale factor λ. These factors can be eliminated easily by multiplication, and so we define the statistic:

(13) $$\tilde{\tau}^2 = \bar{\tau} \cdot \tilde{\tau}_{iv} \; .$$

The asymptotic distribution of $\tilde{\tau}^2$ is therefore free of nuisance parameters, and is the same as the distribution (under iid errors) of the square of the SP statistic $\bar{\tau}$. Thus, unlike Hall's case, there is a simple way to obtain a nuisance-parameter-free t-statistic, without having to estimate MA parameters.

Interestingly, the $\tilde{\tau}^2$ statistic can be interpreted as a PP-corrected version of the SP statistic $\bar{\rho}$, but with an unusual estimate of the nuisance parameters. Recall that $\tilde{\sigma}_\epsilon^2$ is defined above in (9) as the error variance estimate for IV estimator of equation (3). Let $\bar{\sigma}_\epsilon^2$ be the corresponding estimate from OLS (that is, $\bar{\phi}$ replaces $\tilde{\phi}_{iv}$ in (9)). Then we have

$$(14)\qquad \tilde{\tau}^2 = \frac{\dfrac{\sum \tilde{S}_{t-1}\Delta\tilde{S}_t}{\sum \tilde{S}_{t-1}^2}}{\bar{\sigma}_\epsilon\sqrt{\dfrac{1}{\sum \tilde{S}_{t-1}^2}}} \cdot \frac{\dfrac{\sum \tilde{S}_{t-k}\Delta\tilde{S}_t}{\sum \tilde{S}_{t-k}\tilde{S}_{t-1}}}{\tilde{\sigma}_\epsilon\sqrt{\dfrac{\sum \tilde{S}_{t-k}^2}{\left(\sum \tilde{S}_{t-k}\tilde{S}_{t-1}\right)^2}}}$$

$$= \frac{T^{-1}\sum \tilde{S}_{t-k}\Delta\tilde{S}_t}{\tilde{\sigma}_\epsilon\bar{\sigma}_\epsilon} \cdot T\,\frac{\sum \tilde{S}_{t-1}\Delta\tilde{S}_t}{\sqrt{\left(\sum \tilde{S}_{t-1}^2\right)\left(\sum \tilde{S}_{t-k}^2\right)}}\ ,$$

where all summations are over t from k+1 to T. Notice the second equation. The second part of this expression is essentially $\bar{\rho}$. Furthermore, since $T^{-1}\sum_t \tilde{S}_{t-k}\Delta\tilde{S}_t$ converges in probability to $-\sigma^2/2$, the first part of the expression is a consistent estimate of $-0.5(\sigma^2/\sigma_\epsilon^2)$, and cancels the factor $\lambda^2 = \sigma_\epsilon^2/\sigma^2$ that appears in the asymptotic distribution of $\bar{\rho}$, as given by equation (7a) above. Thus the $\tilde{\tau}^2$ test is equivalent to a PP-corrected version of $\bar{\rho}$, where the estimate of λ^2 implicitly used is

$$(15) \qquad \tilde{\lambda}^2 = -\tilde{\sigma}_\epsilon \bar{\sigma}_\epsilon \,/\, 2T^{-1} \sum_{t=k+1}^{T} \tilde{S}_{t-k} \Delta \tilde{S}_t \,.$$

Interestingly, the $\tilde{\tau}^2$ test performs well, as we shall see in the next section. However, the estimate $\tilde{\lambda}^2$ in (15) is intuitively unappealing because it estimates σ^2 in an unconventional way, but it estimates σ_ϵ^2 in a conventional way (or, actually in two conventional ways, as $\tilde{\sigma}_\epsilon \bar{\sigma}_\epsilon$). We can easily consider estimating σ_ϵ^2 analogously to the way that σ^2 is estimated, in which case we have the consistent estimate of λ^2 given by

$$(16) \qquad \hat{\lambda}^2 = \sum_{t=k+1}^{T} \tilde{S}_{t-1} \Delta \tilde{S}_t \,/\, \sum_{t=k+1}^{T} \tilde{S}_{t-k} \Delta \tilde{S}_t \,.$$

Using this estimate, we can correct either the SP coefficient test $\bar{\rho}$ or the SP t-test $\bar{\tau}$. This gives

$$(17a) \qquad \tilde{\rho}_{pp} = \bar{\rho}/\hat{\lambda}^2$$

$$(17b) \qquad \tilde{\tau}_{pp}^{\,2} = \bar{\tau}^2/\hat{\lambda}^2 \,.$$

It should be noted that while $\lambda^2 > 0$, $\hat{\lambda}^2 < 0$ is possible. In fact, $\hat{\lambda}^2 < 0$ occurs with non-negligible probability when the errors are strongly negatively autocorrelated. This point will be discussed more in the next section. For now, we will simply observe that we consider $\tilde{\tau}_{pp}^2$ in (17b), rather than $\tilde{\tau}_{pp}$, to avoid taking the square root of $\hat{\lambda}^2$. Since the distribution of $\bar{\tau}$ is essentially completely negative, there is in any case no loss in information in squaring it.

3. PERFORMANCE OF THE IV TESTS

In this section we consider the finite sample performance of the new IV tests. We are interested both in their size under the null hypothesis, in the presence of moving average errors, and also in their power against stationary alternatives.

The model to be considered is the SP model (1), with the errors following the MA(1) process $\epsilon_t = u_t + \theta u_{t-1}$. The parameters of the problem are therefore ψ, ξ, σ_ϵ, $X_0^* = X_0/\sigma_\epsilon$, θ and T. From SP and from DeJong et al. (1989) it is known that the distributions of the SP and DF statistics under the null do not depend on ψ, ξ, σ_ϵ or X_0^*. The same is easily seen to be true of Hall's and our IV statistics, and so the only relevant parameters under the null are θ and T. The distributions of the test statistics under the alternative ($\beta < 1$) also depend on β and X_0^*. If the errors are symmetrically distributed, they depend on X_0^* only through $|X_0^*|$.

We will investigate the size and power properties of the tests in a Monte Carlo simulation, with the data generating process being the model (1), as just described.

We will use random normal errors generated using the FORTRAN routines GASDEV/RAN3 of Press, Flannery, Tuekolsky and Vetterling (1986). For each sample, MA(1) errors are generated by initializing u and ϵ at zero, and then generating and discarding 20 observations. Our simulations use 20,000 replications, except that only 10,000 replications are used for sample size T = 1000. We consider sample sizes of T = 25, 50, 100, 200, 500 and 1000, and moving average parameters θ = 0, ±0.5 and ±0.8.

The statistics that we consider are the SP statistics $\bar{\rho}$ and $\bar{\tau}$, Hall's IV statistics $\hat{\rho}_{iv}$ and $\hat{\tau}_{iv}$, and the new IV statistics $\tilde{\rho}_{iv}$, $\tilde{\tau}_{iv}$, $\tilde{\tau}^2$, $\tilde{\rho}_{pp}$ and $\tilde{\tau}_{pp}{}^2$, as defined in this chapter in equations (10a), (10b), (13), (17a) and (17b), respectively. All IV tests use as their instruments the relevant dependent variable lagged two periods ($\tilde{S}_{t-2}$ for our tests and y_{t-2} for Hall's). We will also make some reference to previously-reported simulation results for the ADF and PP tests.

In Table 1 we present the critical values of the test statistics just listed. These are generated by a simulation with iid errors (θ = 0), using 50,000 replications. (The critical values for the $\bar{\rho}$ and $\bar{\tau}$ statistics agree with those presented in Chapter 3.) Asymptotically there should be simple relationships between the critical values of the various statistics. For example, the critical values for the $\tilde{\tau}^2$ and $\tilde{\tau}_{pp}{}^2$ statistics should be the square of the critical values for the $\bar{\tau}$ and $\tilde{\tau}_{iv}$ statistics, and this is more or less so for T = 1000. (For example, at the 5% level, 2.61^2 and 2.62^2 are nearly equal to 6.80.) Furthermore, the critical values for the $\tilde{\tau}^2$ and $\tilde{\tau}_{pp}{}^2$ statistics should be equal to minus one half of the critical values for the $\bar{\rho}$, $\tilde{\rho}_{iv}$ and $\tilde{\rho}_{pp}$ statistics, and again this is more or less so for T = 1000. (For example, at the 5% level, 6.80 is nearly equal to minus one half of -13.6, -13.7 and -13.5.) For smaller sample sizes these relationships hold less well, as would be expected. Although the justification for the IV tests under autocorrelated errors is only asymptotic, the use of critical values simulated at the appropriate sample size may still increase the accuracy of the tests, as we will see shortly.

Table 2 gives the size (proportion of rejections under the null) of the tests, at the 5% level, in the presence of MA(1) errors, when the "asymptotic" critical values (i.e., those of the modified SP statistics $\bar{\rho}$ and $\bar{\tau}$, or of the DF statistics $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$) are used. Similarly, Table 3 gives the size at the 5% level, in the presence of MA(1) errors, when the "finite sample" critical values (i.e., those from Table 1 for the appropriate value of T) are used. As noted above, the only two relevant parameters are θ and T. The asymptotic distributions of the $\bar{\rho}$, $\bar{\tau}$, $\tilde{\tau}_{iv}$ and $\hat{\tau}_{iv}$ test statistics depend on nuisance parameters and so we do not expect these tests to have correct size, even asymptotically, in the presence of autocorrelated errors. In fact, it is true in Tables 2 and 3 that all of these tests suffer from large size distortions, even for large values of T, except when $\theta = 0$. Accordingly, we will focus our attention on the performance of the $\tilde{\rho}_{iv}$, $\tilde{\tau}^2$, $\tilde{\rho}_{pp}$, $\tilde{\tau}_{pp}^2$ and $\hat{\rho}_{iv}$ tests, which should have correct size asymptotically in the presence of MA(1) errors.

Comparing the results in Tables 2 and 3, it is apparent that the sizes of the tests tend to be more nearly correct when the "finite sample" critical values are used (Table 3) than when the "asymptotic" critical values are used (Table 2). Using the "asymptotic" critical values in small samples causes $\tilde{\rho}_{iv}$ and $\hat{\rho}_{iv}$ to reject too often, and $\tilde{\tau}^2$, $\tilde{\rho}_{pp}$ and $\tilde{\tau}_{pp}^2$ to reject too seldom; see particularly the results for $T \leq 100$ and $\theta = 0$. We will therefore focus on the results using the "finite sample" critical values, as given in Table 3.

The main conclusion from the simulation results in Table 3 is that the new IV tests perform better than Hall's IV test. The $\tilde{\rho}_{pp}$ test appears to be the best overall. For example, when $\theta = -0.8$ and $T = 100$, the 5% sizes of the new statistics $\tilde{\rho}_{iv}$, $\tilde{\tau}^2$, $\tilde{\rho}_{pp}$ and $\tilde{\tau}_{pp}^2$ are 0.235, 0.125, 0.077 and 0.137, respectively, while the size of Hall's $\hat{\rho}_{iv}$ test is 0.446. A similar pattern is found for larger sample sizes, though, as Schwert (1989) and others have found, the improvement of the various tests is neither rapid nor monotonic in sample size. When $\theta = -0.5$, the size distortions are minimal for the $\tilde{\tau}^2$, $\tilde{\rho}_{pp}$ and $\tilde{\tau}_{pp}{}^2$ tests, so that serious size distortions occur only in the presence of a very large negative MA parameter. The $\tilde{\rho}_{pp}$ test tends to over-reject a bit when $\theta > 0$ and $T \leq 100$, but this problem is not very serious. If $\theta > 0$ is a reasonable possibility, the $\tilde{\rho}_{iv}$ and $\tilde{\tau}^2$ tests appear to be best, but their size distortions are clearly larger than those of the $\tilde{\rho}_{pp}$ test when $\theta < 0$.

To put the optimistic results for the new IV tests in perspective, it is instructive to compare them to the corresponding results for the PP and ADF tests. For example, for $T = 100$ and $\theta = -0.8$, Schwert (1989) reports size of 1.00 for the PP-corrected $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$ tests, with either four or twelve covariances used to estimate the long run variance. (Note that $\theta = -0.8$ in our notation is $\theta = 0.8$ in Schwert's notation.) For the ADF $\hat{\tau}_\tau$ test, he reports size of 0.568 with four augmentations and 0.055 with twelve augmentations. Chapter 2 reports similar results for the PP-corrected and augmented SP tests; for example, for $T = 100$ and $\theta = -0.8$, the size of the SP test is 0.330 for the SP $\tilde{\tau}$ test with four augmentations and 0.064 with twelve augmentations. Similar results obtain for other sample sizes. Our new IV tests clearly have better size properties than the corresponding PP tests or the ADF tests with only a few augmentations.

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We noted at the end of Section 2 the possibility that the nuisance parameter estimator $\hat{\lambda}^2$ could be negative. This outcome is related to the signs of all of the IV-type tests, and is worth a brief discussion. Let $\tilde{S}_t$ be as defined in Section 2, and let $\hat{S}_t$ be the residual in an OLS regression of y_t on [1,t]; the various SP statistics depend on the $\tilde{S}_t$, while the various DF statistics depend on the $\hat{S}_t$.

With a probability of essentially one, we have $\sum_t \tilde{S}_{t-1}\Delta\tilde{S}_t < 0$ and $\sum_t \hat{S}_{t-1}\Delta\hat{S}_t < 0$, which implies that the uncorrected statistics $\bar{\rho}$, $\bar{\tau}$, $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$ are all negative with a probability of essentially one. It is also usual to have $\sum_t \tilde{S}_{t-2}\Delta\tilde{S}_t < 0$ and $\sum_t \tilde{S}_{t-2}\tilde{S}_{t-1} > 0$, which implies $\tilde{\rho}_{iv} < 0$, $\tilde{\tau}_{iv} < 0$, $\tilde{\tau}^2 > 0$, $\hat{\lambda}^2 > 0$, $\tilde{\rho}_{pp} < 0$ and $\tilde{\tau}_{pp}{}^2 > 0$; and to have $\sum_t \hat{S}_{t-2}\Delta\hat{S}_t < 0$ and $\sum_t \hat{S}_{t-2}\hat{S}_{t-1} > 0$, which implies $\hat{\rho}_{iv} < 0$. We will call these the "usual signs" for these statistics. For example, with T = 100, and $\theta = 0$, all of these signs obtain in our simulations with probabilities in excess of 0.99, and many obtain in every replication of the simulation.

When the errors are strongly negatively autocorrelated, however, unusual signs sometimes appear. For example, with T = 100 and $\theta = -0.8$, we find $\sum_t \tilde{S}_{t-2}\Delta\tilde{S}_t > 0$ with frequency 0.385, which therefore yields $\hat{\lambda}^2 < 0$, $\tilde{\rho}_{pp} > 0$ and $\tilde{\tau}_{pp}{}^2 < 0$ with this same frequency. We have $\tilde{\rho}_{iv} > 0$, $\tilde{\tau}_{iv} > 0$ and $\tilde{\tau}^2 < 0$ with slightly lower frequency, 0.379, because occasionally (frequency = 0.007) we also have $\sum_t \tilde{S}_{t-1}\tilde{S}_{t-2} < 0$, and these cases tend to overlap with cases in which $\sum_t \tilde{S}_{t-2}\Delta\tilde{S}_t > 0$, so that the unusual signs cancel. Similarly, we have $\sum_t \hat{S}_{t-2}\Delta\hat{S}_t>0$ with frequency 0.418, but $\hat{\rho}_{iv} > 0$ with frequency only 0.360, because a few cases with $\sum_t \hat{S}_{t-2}\Delta\hat{S}_t > 0$ also have $\sum_t \hat{S}_{t-2}\hat{S}_{t-1} < 0$.

Because unit root tests are one-tail tests, an unusual sign for a statistic implies an acceptance of the null hypothesis. The occurrence of unusual signs when the errors are strongly negatively autocorrelated is therefore helpful, as it counteracts the tendency of unit root tests to reject the null too often. This is true for our new IV tests and for Hall's IV tests as well.

We now turn to the power of the tests against stationary alternatives. In addition to T and θ, the parameters β and X_0* are now relevant. We pick β = 0.9 and X_0* = 0 as reasonable values to consider. Some limited experimentation (not reported here) with other values of β and X_0* indicates that the power comparisons are not changed markedly by other plausible choices for these parameters.

Our results for power against the alternative β = 0.9 are given in Table 4. These are somewhat difficult to interpret because the tests with upward size distortions will tend to appear more powerful, and the tests with downward size distortions will tend to appear less powerful, than the tests with no size distortions. For example, for T = 25 and θ = -0.8, Hall's $\hat{\rho}_{iv}$ test appears to be more powerful than our $\tilde{\rho}_{pp}$ test, but this difference in power is in fact just the same as the difference in size; if properly size-corrected, neither test has appreciable power against $\beta = 0.9$ for T = 25. Thus, power comparisons are meaningless when there are size distortions. The simplest way to handle this problem appears to be to consider power when $\theta = 0$, in which case none of the tests we are considering suffers from appreciable size distortions. If this is done, we find in Table 4 that the powers of all of the tests are rather similar. Among the set of tests that are asymptotically robust to MA errors (i.e., $\tilde{\rho}_{iv}$, $\tilde{\tau}^2$, $\tilde{\rho}_{pp}$, $\tilde{\tau}_{pp}{}^2$ and $\hat{\rho}_{iv}$), there is no clear basis for preferring one to the other in terms of power when $\theta = 0$. There is some indication that the new IV tests are more powerful than Hall's $\hat{\rho}_{iv}$ test for $T \leq 200$, but this difference is not large. Furthermore, it is known from Schmidt and Phillips (1990) that the parameter value $X_0{}^* = 0$ favors SP tests over DF tests. However, it is noteworthy that these tests are just as powerful when $\theta = 0$ as the OLS-based SP $\bar{\rho}$ and $\bar{\tau}$ tests (or the SP $\tilde{\rho}$ and $\tilde{\tau}$ tests, or the DF $\hat{\rho}_{\tau}$ and $\hat{\tau}_{\tau}$ tests; see SP or DeJong et al. (1989) for the relevant power tabulations), which do not allow for MA errors. In other words, robustness in the presence of MA errors is purchased at no loss in power in the presence of iid errors. This is a strong argument in favor of the new IV tests.

A fairly clear-cut comparison exists between the powers of the new IV tests and the augmented DF and SP tests. Following Schwert (1989), define the numbers of augmentations $\ell 4$ = integer$[4(T/100)^{1/4}]$ and $\ell 12$ = integer$[12(T/100)^{1/4}]$, which yields $\ell 4$ = 4, 5, 6 and $\ell 12$ = 12, 14, 18 for T = 100, 200 and 500, respectively. When θ = 0, the power against β = 0.9 of the ADF t-test with $\ell 4$ augmentations is 0.118 for T = 100, 0.385 for T = 200 and 0.991 for T = 500. The $\tilde{\rho}_{pp}$ test is more powerful at all three sample sizes (with powers of 0.259, 0.691 and 0.992), as are our other asymptotically valid tests. This is so despite the fact that the ADF t-test with $\ell 4$ augmentations suffers from greater size distortions than the $\tilde{\rho}_{pp}$ test. The ADF t-test with $\ell 12$ augmentations is even less powerful; its power against β = 0.9 is only 0.058, 0.166 and 0.778, for T = 100, 200 and 500, respectively. Chapter 2 reports similar results for the augmented SP t-test. For example, for T = 100, β = 0.9 and θ = 0, the power of the augmented SP t-test is 0.191 for with $\ell 4$ augmentations and 0.125 with $\ell 12$ augmentations. Thus, if enough augmentations are used to make the size distortions of the augmented DF and SP tests small, the tests have very little power. Unlike the case for the IV tests, the robustness of the augmented tests in the presence of MA errors is purchased at the cost of a considerable loss in power. This is not surprising, since the augmented tests were really developed to be suitable in the presence of autoregressive errors, while the IV tests were specifically designed to be useful in the presence of MA errors; in particular, they take advantage of the fact that, with MA errors, error covariances at lags greater than the order of the MA process equal zero.

4. CONCLUSION

In this chapter we have provided new IV tests for a unit root. We extend to the Schmidt-Phillips (SP) testing framework the IV techniques that Hall (1989) and Pantula and Hall (1990) had previously applied in the Dickey-Fuller (DF) framework. Like Hall's IV tests, ours are asymptotically robust to MA errors. However, there are several advantages to operating in the SP framework rather than in the DF framework. Primary among these, in the present context, is that in the SP framework the nuisance parameters that reflect error autocorrelation enter only through scalar multiplication of the asymptotic distribution that would otherwise apply. As SP note, this makes Phillips-Perron type corrections much more straightforward for the SP tests than for the DF tests. As this chapter has shown, it also makes IV versions of the SP tests much more straightforward than IV versions of the DF tests. As a result we have been able to provide asymptotically robust IV statistics based <u>both</u> on the estimated coefficient and on the t-statistic. Furthermore, we have been able to interpret our IV statistics as Phillips-Perron corrected SP statistics, using unconventional but consistent (given MA errors) estimates of the nuisance parameters. Neither the t-statistic versions of the IV tests nor the Phillips-Perron correction interpretation appear to be easy in the DF framework.

Our tests have generally smaller finite sample size distortions than Hall's IV test. They are comparable in their accuracy to the augmented DF (or SP) statistics with a moderate to large number of augmentations. However, compared to the augmented tests, they are much more powerful against stationary alternatives. This constitutes the main argument in favor of using our new IV tests.

Many authors since Wichern (1973) have pointed out that, as the MA(1) parameter θ approaches minus one, the variable approaches stationarity (indeed, white noise) and so it is natural that unit root tests should have considerable size distortions with large negative MA parameters. Indeed, it may therefore be natural to think in terms of a trade-off between accuracy (lack of size distortion) and power. However, our results show that this viewpoint should not be taken too far. Some tests do dominate others, in the sense that they can have smaller size distortions with very highly autocorrelated errors <u>and</u> higher power with iid errors; greater accuracy under the null is not necessarily purchased with lower power under the alternative.

An important extension of our work is to allow for more general ARMA errors. Pantula and Hall (1991) have taken a step in this direction, in the DF framework, by using augmentations to handle the AR component of the errors and IV estimation to handle the MA component. However, this is awkward in the DF framework, because augmentations make the t-statistic test robust to AR errors, while IV estimation makes the coefficient test robust to MA errors. Making the coefficient test robust to the AR component of the errors requires multiplication by a function of the sum of the coefficients of the lagged difference regressors. However, in this chapter we have shown how to construct the IV t-statistic tests in the SP framework. Thus it is potentially straightforward to construct IV-based t-statistic tests that should be robust to ARMA errors. This will be the subject of future research.

TABLE 1

Critical Values

T	%	$\bar{\rho}$	$\bar{\tau}$	$\tilde{\rho}_{iv}$	$\tilde{\tau}_{iv}$	$\tilde{\tau}^2$	$\tilde{\rho}_{pp}$	$\tilde{\tau}_{pp}^2$	$\hat{\rho}_{iv}$
25	1	-16.7	-3.40	-25.9	-2.59	6.36	-11.8	7.20	-66.2
	5	-12.0	-2.71	-15.3	-2.26	4.99	-9.00	5.11	-34.3
	10	-9.82	-2.37	-11.6	-2.04	4.12	-7.49	4.14	-25.9
50	1	-18.3	-3.29	-22.1	-2.97	8.39	-15.0	8.48	-36.7
	5	-12.8	-2.66	-14.3	-2.47	5.90	-10.9	5.92	-25.6
	10	-10.4	-2.36	-11.2	-2.20	4.76	-8.96	4.75	-20.9
100	1	-19.3	-3.24	-20.9	-3.10	9.23	-17.2	9.23	-32.2
	5	-13.2	-2.64	-14.0	-2.55	6.36	-12.2	6.36	-23.1
	10	-10.6	-2.35	-11.0	-2.27	5.10	-9.88	5.10	-19.3
200	1	-19.5	-3.19	-20.2	-3.14	9.50	-18.3	9.49	-30.8
	5	-13.5	-2.63	-13.7	-2.58	6.56	-12.8	6.55	-22.4
	10	-10.7	-2.34	-10.9	-2.30	5.25	-10.4	5.26	-18.8
500	1	-20.3	-3.21	-20.5	-3.16	9.97	-19.6	9.97	-30.2
	5	-13.6	-2.63	-13.7	-2.61	6.74	-13.4	6.74	-22.1
	10	-10.8	-2.34	-10.9	-2.32	5.35	-10.6	5.35	-18.5
1000	1	-20.2	-3.20	-20.4	-3.17	10.0	-19.9	10.0	-29.8
	5	-13.6	-2.62	-13.7	-2.61	6.80	-13.5	6.80	-21.7
	10	-10.8	-2.34	-10.8	-2.32	5.37	-10.7	5.37	-18.2

TABLE 2

5% Sizes of the IV tests In the Presence of MA errors (Asymptotic Critical Values Used)

T	θ	$\bar{\rho}$	$\bar{\tau}$	$\tilde{\rho}_{iv}$	$\tilde{\tau}_{iv}$	$\tilde{\tau}^2$	$\tilde{\rho}_{pp}$	$\tilde{\tau}_{pp}^{2}$	$\hat{\rho}_{iv}$	$\hat{\tau}_{iv}$
25	-.8	.730	.726	.288	.004	.023	.007	.020	.580	.012
	-.5	.417	.413	.164	.001	.004	.005	.011	.422	.002
	.0	.049	.048	.093	.004	.000	.009	.009	.249	.004
	.5	.004	.004	.085	.044	.001	.022	.014	.216	.024
	.8	.002	.002	.085	.068	.001	.027	.015	.213	.034
50	-.8	.931	.931	.266	.003	.071	.029	.068	.601	.003
	-.5	.553	.552	.114	.003	.033	.016	.032	.244	.000
	.0	.048	.047	.067	.029	.023	.024	.024	.123	.015
	.5	.004	.004	.065	.141	.025	.037	.029	.115	.145
	.8	.002	.002	.066	.179	.024	.039	.030	.114	.199
100	-.8	.987	.987	.246	.003	.107	.062	.116	.466	.000
	-.5	.632	.628	.092	.002	.040	.026	.042	.155	.001
	.0	.051	.050	.062	.039	.036	.037	.036	.083	.036
	.5	.005	.005	.060	.185	.038	.046	.040	.078	.242
	.8	.002	.002	.060	.223	.038	.048	.041	.077	.301
200	-.8	.997	.997	.226	.002	.124	.093	.141	.365	.001
	-.5	.658	.656	.075	.001	.044	.036	.048	.108	.000
	.0	.051	.050	.054	.043	.042	.042	.042	.064	.041
	.5	.003	.003	.052	.201	.041	.045	.043	.063	.273
	.8	.002	.002	.052	.239	.041	.045	.042	.063	.331
500	-.8	1.00	1.00	.189	.000	.128	.111	.141	.290	.000
	-.5	.681	.678	.061	.000	.045	.042	.048	.076	.000
	.0	.052	.051	.052	.047	.047	.048	.047	.056	.046
	.5	.004	.004	.053	.212	.047	.050	.048	.055	.290
	.8	.002	.002	.054	.249	.047	.050	.048	.055	.349
1000	-.8	1.00	1.00	.160	.000	.119	.109	.131	.234	.000
	-.5	.686	.684	.055	.000	.046	.044	.048	.063	.000
	.0	.049	.048	.050	.049	.047	.048	.047	.053	.049
	.5	.003	.003	.050	.216	.047	.049	.047	.053	.297
	.8	.002	.002	.050	.254	.048	.049	.048	.053	.360

TABLE 3

**5% Sizes of the IV tests
In the Presence of MA errors
(Finite Sample Critical Values Used)**

T	θ	$\bar{\rho}$	$\bar{\tau}$	$\tilde{\rho}_{iv}$	$\tilde{\tau}_{iv}$	$\tilde{\tau}^2$	$\tilde{\rho}_{pp}$	$\tilde{\tau}_{pp}{}^2$	$\hat{\rho}_{iv}$	$\hat{\tau}_{iv}$
25	-.8	.730	.726	.241	.009	.079	.034	.070	.365	.030
	-.5	.417	.413	.121	.007	.048	.028	.048	.224	.007
	.0	.049	.048	.051	.049	.052	.050	.052	.050	.049
	.5	.004	.004	.042	.199	.057	.089	.067	.024	.267
	.8	.002	.002	.042	.248	.057	.097	.070	.021	.351
50	-.8	.931	.931	.245	.006	.108	.050	.102	.555	.005
	-.5	.553	.552	.095	.006	.055	.033	.053	.164	.001
	.0	.048	.047	.049	.049	.050	.050	.050	.052	.050
	.5	.004	.004	.047	.202	.052	.069	.057	.040	.282
	.8	.002	.002	.046	.244	.052	.072	.059	.040	.348
100	-.8	.987	.987	.235	.004	.125	.077	.137	.446	.001
	-.5	.632	.628	.081	.003	.052	.038	.054	.120	.001
	.0	.051	.050	.051	.050	.051	.050	.051	.051	.051
	.5	.005	.005	.051	.212	.054	.062	.056	.047	.295
	.8	.002	.002	.051	.253	.054	.063	.057	.046	.357
200	-.8	.997	.997	.223	.002	.137	.102	.154	.352	.002
	-.5	.658	.656	.071	.001	.051	.042	.055	.091	.000
	.0	.051	.050	.051	.049	.051	.051	.051	.050	.050
	.5	.003	.003	.048	.216	.050	.055	.052	.048	.299
	.8	.002	.002	.048	.256	.050	.055	.052	.048	.355
500	-.8	1.00	1.00	.187	.000	.131	.115	.147	.281	.000
	-.5	.681	.678	.059	.000	.049	.045	.051	.070	.000
	.0	.052	.051	.051	.050	.051	.051	.051	.051	.050
	.5	.004	.004	.051	.218	.052	.053	.052	.050	.302
	.8	.002	.002	.051	.257	.052	.054	.053	.050	.361
1000	-.8	1.00	1.00	.158	.000	.121	.109	.132	.230	.000
	-.5	.686	.684	.054	.000	.048	.045	.049	.061	.000
	.0	.049	.048	.048	.049	.048	.048	.048	.051	.050
	.5	.003	.003	.049	.213	.049	.050	.049	.050	.301
	.8	.002	.002	.049	.250	.049	.050	.049	.050	.364

TABLE 4

5% Powers of the IV tests (β=0.9)
In the Presence of MA errors

T	θ	$\bar{\rho}$	$\bar{\tau}$	$\tilde{\rho}_{iv}$	$\tilde{\tau}_{iv}$	$\tilde{\tau}^2$	$\tilde{\rho}_{pp}$	$\tilde{\tau}_{pp}^2$	$\hat{\rho}_{iv}$	$\hat{\tau}_{iv}$
25	-.8	.741	.738	.255	.011	.084	.035	.073	.366	.032
	-.5	.474	.470	.139	.007	.057	.032	.056	.252	.007
	.0	.066	.065	.066	.060	.066	.065	.066	.061	.044
	.5	.006	.006	.058	.245	.078	.118	.092	.030	.277
	.8	.003	.003	.057	.303	.077	.129	.096	.027	.374
50	-.8	.948	.947	.279	.007	.121	.054	.112	.604	.007
	-.5	.745	.744	.158	.008	.091	.052	.089	.256	.001
	.0	.113	.113	.108	.093	.106	.101	.105	.093	.060
	.5	.011	.011	.107	.358	.120	.147	.128	.077	.369
	.8	.007	.007	.107	.428	.117	.155	.130	.076	.458
100	-.8	.997	.997	.300	.005	.165	.100	.173	.619	.002
	-.5	.949	.948	.253	.009	.164	.119	.174	.331	.003
	.0	.293	.288	.268	.205	.264	.259	.264	.229	.131
	.5	.036	.035	.279	.656	.292	.319	.299	.223	.619
	.8	.021	.020	.279	.725	.292	.327	.303	.222	.710
200	-.8	1.00	1.00	.353	.007	.227	.169	.250	.585	.003
	-.5	.998	.998	.477	.013	.372	.319	.397	.577	.005
	.0	.744	.742	.698	.574	.695	.691	.696	.670	.419
	.5	.228	.226	.731	.942	.738	.750	.743	.679	.956
	.8	.156	.154	.733	.962	.740	.755	.746	.680	.977
500	-.8	1.00	1.00	.445	.006	.343	.298	.370	.578	.005
	-.5	1.00	1.00	.874	.045	.827	.804	.842	.937	.016
	.0	.994	.993	.989	.976	1.00	.992	.992	1.00	.996
	.5	.914	.912	.992	1.00	.992	.992	.992	1.00	1.00
	.8	.879	.877	.992	1.00	.992	.992	.992	1.00	1.00
1000	-.8	1.00	1.00	.525	.005	.445	.415	.468	.625	.007
	-.5	1.00	1.00	.989	.199	.985	.983	.986	.998	.100
	.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	.5	.997	.997	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	.8	.995	.995	1.00	1.00	1.00	1.00	1.00	1.00	1.00

APPENDIX

In this Appendix, we prove that, under the assumptions of Theorem 1,

$$\text{(A.1)} \qquad plim\, T^{-1} \sum_{t=k+1}^{T} \tilde{S}_{t-k} \Delta \tilde{S}_t = \begin{cases} -\sigma^2/2 , & k \geq q+1 \\ -\sigma_\epsilon^2/2 , & k \leq q . \end{cases}$$

Here q is the order of MA error process, σ^2 is the long-run variance, and σ_ϵ^2 is the innovation variance.

Since $\tilde{S}_{t-1} = S_{t-1} - (t-2)\bar{\epsilon} - \epsilon_1$ from (17) in Schmidt and Phillips (1990), where $S_t = \sum_{j=1}^{t} \epsilon_j$, and $\Delta\tilde{S}_t = \epsilon_t - \bar{\epsilon}$, we obtain

$$\text{(A.2)} \qquad T^{-1} \sum_{k+1}^{T} \tilde{S}_{t-k} \Delta \tilde{S}_t = T^{-1} \sum_{k+1}^{T} [S_{t-k} - (t-k-1)\bar{\epsilon} - \epsilon_1][\epsilon_t - \bar{\epsilon}]$$

$$= T^{-1} \sum_{k+1}^{T} S_{t-k}\epsilon_t - T^{-1} \sum_{k+1}^{T} (t-k-1)\bar{\epsilon}\epsilon_t - T^{-1} \sum_{k+1}^{T} S_{t-k}\bar{\epsilon} + T^{-1} \sum_{k+1}^{T} (t-k-1)\bar{\epsilon}^2 .$$

First of all, we can show that, for k ≥ q+1,

$$\text{(A.3)} \qquad T^{-1} \sum_{k+1}^{T} \sum_{j=1}^{k-1} \epsilon_{t-j}\epsilon_t$$

$$= 0.5 \left\{ T^{-1}[\epsilon_{k+1} + \cdots + \epsilon_T]^2 - T^{-1}[\epsilon_{k+1}^2 + \cdots + \epsilon_T^2] \right\}$$

$$- T^{-1} \sum_{k+1}^{T} \sum_{j=k}^{T} \epsilon_{t-j}\epsilon_t$$

$$\rightarrow 0.5\, [\sigma^2 - \sigma_\epsilon^2] .$$

Here, we have applied the a priori restriction, $E(\epsilon_{t-j}\epsilon_t) = 0$

for $j \geq k \geq q+1$, to show that

$$T^{-1} \sum_{k+1}^{T} \sum_{j=k}^{T} \epsilon_{t-j}\epsilon_t \rightarrow 0 .$$

This reflects the fact that error covariances of order higher than the order of the MA process should be zero. But, this *a priori* restriction cannot be imposed, when $k \leq q$.

Now, each term in (A.2) can be calculated as follows:

(i) The first term:

For $k \leq q$, $T^{-1} \sum_{k+1}^{T} S_{t-k}\epsilon_t \rightarrow 0.5\ \sigma^2[W(1)^2 - \sigma_\epsilon^2]$

from (A3.1) of SP (1990).

For $k \geq q+1$, $T^{-1} \sum_{k+1}^{T} S_{t-k}\epsilon_t = T^{-1} \sum_{k+1}^{T} S_{t-1}\epsilon_t - T^{-1} \sum_{j=1}^{k-1} \epsilon_{t-j}\epsilon_t$

$$\rightarrow 0.5\ [\sigma^2 W(1)^2 - \sigma_\epsilon^2] - 0.5\ [\sigma^2 - \sigma_\epsilon^2]$$

$$= 0.5\ \sigma^2[W(1)^2 - 1].$$

(ii) $T^{-1} \sum_{k+1}^{T} (t-k-1)\bar{\epsilon}\epsilon_t = (T^{1/2}\bar{\epsilon})\ T^{-3/2} \sum_{k+1}^{T} (t-k-1)\epsilon_t$

$$\rightarrow \sigma^2 W(1)^2 - \sigma^2 W(1)\int_0^1 W(r)\,dr$$

(iii) $T^{-1} \sum_{k+1}^{T} S_{t-k}\bar{\epsilon} = (T^{1/2}\bar{\epsilon})\ T^{-3/2} \sum_{k+1}^{T} S_{t-k}$

$$\rightarrow \sigma^2 W(1)\int_0^1 W(r)\,dr$$

(iv) $T^{-1} \sum_{k+1}^{T} (t-k-1)\bar{\epsilon}^2 = (T^{1/2}\bar{\epsilon})^2\ T^{-2}\ T(T-1)/2 + o_p(1)$

$$\rightarrow 0.5\ \sigma^2 W(1)^2$$

Therefore, for $k \geq q+1$,

$$T^{-1}\sum_{k+1}^{T}\tilde{S}_{t-k}\Delta\tilde{S}_t \rightarrow 0.5\sigma^2[W(1)^2-1] - \sigma^2 W(1)^2 +$$

$$\sigma^2 W(1)\int_0^1 W(r)dr - \sigma^2 W(1)\int_0^1 W(r)dr + 0.5\sigma^2 W(1)^2$$

$$= -\ 0.5\sigma^2\ ,$$

and for $k \leq q$,

$$T^{-1}\sum_{k+1}^{T}\tilde{S}_{t-k}\Delta\tilde{S}_t \rightarrow 0.5\sigma^2[W(1)^2-\sigma_\epsilon^2] - \sigma^2 W(1)^2 +$$

$$\sigma^2 W(1)\int_0^1 W(r)dr - \sigma^2 W(1)\int_0^1 W(r)dr + 0.5\sigma^2 W(1)^2$$

$$= -\ 0.5\sigma_\epsilon^2. \qquad \square$$

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CHAPTER 5

CHAPTER 5

STRUCTURAL CHANGE AND PARAMETERIZATION PROBLEMS IN UNIT ROOT TESTS

1. INTRODUCTION

Following the pioneering work of Nelson and Plosser (1982), a large number of studies have failed to reject the hypothesis of a unit root in many macroeconomic time series. These studies have mostly been based on one variant or another of the Dickey-Fuller (1979, hereafter, DF) tests. However, other tests generally give more or less the same results. Specifically, the unit root tests recently proposed by Schmidt and Phillips (1990, hereafter SP) lead to the same conclusions as the DF tests when they are applied to the Nelson-Plosser data.

A contrary conclusion is reached by Perron (1989), who finds that most of the Nelson-Plosser series are trend stationary if allowance is made for a structural break in 1929, at the beginning of the Great Depression. He shows that the DF tests are biased toward accepting the null when there is a structural change, and when he allows for such a change, he finds eleven of the fourteen Nelson-Plosser time series to be trend stationary. From this perspective, the persistent effects of the great crash make the series appear to have a unit root, when in fact they do not. He also reaches similar conclusions for more recent quarterly series, treating the oil price shock of 1973 as an exogenous structural break.

Banerjee, Lumsdaine and Stock (1990) and Zivot and Andrews (1990), among others, have criticized Perron for treating the structural break as exogenous and its time of occurrence as known a priori (independently of the data). They show that the asymptotic critical values for the unit root test must be modified if the structural change occurs at an unknown time, so that its placement is decided from the data. When this is done, the null hypothesis of a unit root is no longer rejected.

In this chapter Perron's result is also reversed, but in a different way. It is shown that his results disappear if we use a suitably modified version of the SP unit root tests instead of the DF tests. This chapter shows this result empirically for the Nelson-Plosser series, and also provides theoretical results that explain why these empirical results are not surprising. Those results support the argument advanced by SP that the DF parameterization is "clumsy", in the sense that it handles level, trend and other relevant variables in a needlessly complicated and potentially confusing way. Allowing for a one-time structural break affects the distributions of the DF statistics under the null, even asymptotically, and allowing for a break at an unknown time affects the asymptotic

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distribution even more. These difficulties simply do not arise in the SP framework. A one-time structural break does not affect the asymptotic distributions of the SP tests, and this is true irrespectively of whether the break is allowed for in the model or not. This constitutes a powerful reason for preferring SP tests to DF tests in the presence of possible structural change.

The plan of the chapter is as follows. In Section 2 the DF and SP parameterizations are presented, and the relevant asymptotic results for the SP tests in the presence of a possible structural break are provided. In Section 3 the SP tests are applied to the Nelson-Plosser data and Perron's conclusions are reversed. In Section 4 we consider and test the common factor restrictions that are implied by the SP model; they are not rejected. Section 5 gives the concluding remarks. Throughout this chapter, "$\rightarrow$" indicates weak convergence as $T \rightarrow \infty$.

2. THEORETICAL RESULTS

To allow for structural change, Perron (1989) considers three kinds of models. Here, we will examine only his first crash model, which allows for a one-time change in the level of the variable's time path. We consider Perron's crash model using the following data generating process (DGP):

(1) $$y_t = \delta_1 + \delta_2 t + \delta_3 DU_t + X_t \ , \quad X_t = \rho X_{t-1} + \epsilon_t.$$

where $DU_t = 1$ for $t \geq T_B+1$ and zero otherwise, and $D(TB)_t = 1$ for $t = T_B+1$ and zero otherwise, while T_B stands for the time period when the structural change breaks out. Note that $D(TB)_t = \Delta DU_t = DU_t - DU_{t-1}$. The unit root hypothesis is $\rho = 1$, and this is the null hypothesis to be tested. Note that this DGP is consistent with Perron's crash model:

(2a) Null $y_t = \mu + d \cdot D(TB)_t + y_{t-1} + v_t$

(2b) Alternative $y_t = \mu_1 + \beta \cdot t + (\mu_2 - \mu_1) \cdot DU_t + v_t$.

This is so, since equation (1) implies equation (2a) under the null ($\rho = 1$) with $\mu = \delta_2$, $d = \delta_3$, $y_0 = \delta_1 + X_0$, and $v_t = \epsilon_t$; and it implies equation (2b) under the alternative ($\rho < 1$) with $\mu_1 = \delta_1$, $\beta = \delta_2$, $(\mu_2 - \mu_1) = \delta_3$, and $v_t = X_t$. More generally, the DGP (1) implies:

(3) $y_t = \gamma_1 + \gamma_2 y_{t-1} + \gamma_3 t + \gamma_4 DU_t + \gamma_5 D(TB)_t + \epsilon_t$

where

(4a) $\gamma_1 = \delta_1(1-\rho) + \delta_2 \rho$

(4b) $\gamma_2 = \rho$

(4c) $\gamma_3 = \delta_2(1-\rho)$

(4d) $\gamma_4 = \delta_3(1-\rho)$

(4e) $\gamma_5 = \rho\delta_3$.

Also, note that equation (3) is Perron's nested equation (12) which includes the null and the alternative models. It is important to notice that a unit root imposes three re

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restrictions on (3); $\gamma_2 = 1$, $\gamma_3 = 0$, and $\gamma_4 = 0$. The first two restrictions are the usual unit root restrictions, and the last restriction is the common factor restriction (CFR) to be discussed in Section 4. Perron tests only the first unit root restriction ($\rho = 1$), ignoring the two additional restrictions. The missing restrictions can potentially lead to quite different inference, as discussed in Section 4. It could well be that the unit root hypothesis is true, but that the other restrictions do not hold, or vice-versa.

It is typical of the DF regressions that they include variables which are relevant under the alternative but not under the null. Here, the dummy variable (DU_t) representing changed level and the time trend variable are variables that belong in (2b) but not (2a), and thus are relevant in (3) under the alternative, but not under the null. The inclusion of these variables affects the asymptotic distributions of the DF-type unit root statistics, and so Perron's Theorem 2 shows that the asymptotic distributions of his test statistics depend on $\lambda = T_B/T$ even under the null.

This problem does not arise in the SP framework. We may consider the SP-type test in the structural change model by writing (1) as:

(5) $$y_t = \psi + Z_t\delta + X_t , \quad X_t = \rho X_{t-1} + \epsilon_t$$

where $\psi = \delta_1$, $Z_t = [t, DU_t]$, and $\delta = [\delta_2, \delta_3]'$. The unit root

restriction is $\rho = 1$. According to the LM (score) principle, the restricted MLE's of δ and $\psi_x (\equiv \psi + X_0)$ are given by:

(6a) $\tilde{\delta}$ = coefficients of the regression of Δy_t on ΔZ_t

(6b) $\tilde{\psi}_x = y_1 - Z_1\tilde{\delta}$.

Then the SP-type test statistics are obtained from the following regression:

(7) $\Delta y_t = \Delta Z_t \gamma + \phi \tilde{S}_{t-1}$ + error

where $\tilde{S}_t \equiv y_t - \tilde{\psi}_x - Z_t\tilde{\delta} = \sum_{j=2}^{t} (\Delta y_j - \Delta Z_j \tilde{\delta})$, $t = 2,..,T$. The test statistics are accordingly defined by:

(8a) $\check{\rho} = T \cdot \check{\phi}$

(8b) $\check{\tau}$ = t-statistic for the hypothesis $\phi = 0$.

See SP for more details.

To establish the asymptotic distributions of the SP-type tests that allow for structural break, we need the following assumption.

Assumption 1 The data are generated according to (5), and the innovations ϵ_t satisfy the regularity conditions of Phillips and Perron (1988, p.336).

Theorem 1 Suppose that Assumption 1 holds, and $\rho = 1$.

Let $\check{\rho}$ and $\check{\tau}$ be defined as in (8). Then,

$$(9a) \qquad \check{\rho} \rightarrow -\frac{1}{2}\frac{\sigma_\varepsilon^2}{\sigma^2}\frac{1}{\int_0^1 \underline{V}(r)^2 dr}$$

$$(9b) \qquad \check{\tau} \rightarrow -\frac{1}{2}\frac{\sigma_\varepsilon}{\sigma}\frac{1}{\left[\int_0^1 \underline{V}(r)^2 dr\right]^{1/2}}$$

where $\underline{V}(r) = V(r) - \int_0^1 V(r)dr$ is a demeaned Brownian bridge, and $V(r) = W(r) - rW(1)$ is a Brownian bridge; where $W(r)$ is the Wiener process on [0,1]; and where the nuisance parameters σ^2 and σ_ε^2 are defined as in Phillips and Perron (1988).

Proof: See Appendix.

Note that the above asymptotic distributions are the same as those of the usual SP tests (hereafter the "usual" SP or DF tests refer to those which do not allow for the break), as given in equations (20) and (21) of Schmidt and Phillips (1990). So, unlike the case in Perron's model, allowing for structural break at a single known time does not affect the asymptotic distributions of the test statistics under the null. The intuitive reason is that the SP-type tests are based on a regression in differences, and $\Delta DU_t = D(TB)_t$ equals one at only one point, so that its inclusion has no effect asymptotically. Perron's DF-type tests are based on a regression that includes DU_t in levels, and DU_t equals one for a constant fraction $(1-\lambda)$ of the sample even asymptotically.

Next, we turn to a question that was not asked in Perron's paper. What will be the effects on the asymptotic distributions of the usual DF and SP tests under the null hypothesis, if there occurs a one-time structural break, but it is ignored? The answer, perhaps surprisingly, is that there will be no effect on either the DF or SP-type tests.

Theorem 2 Suppose that Assumption 1 holds, and $\rho = 1$ so that the null hypothesis is true. Suppose that there occurs a structural break at time T_B, and that $T_B/T \to \lambda$ as $T \to \infty$. Then the asymptotic distributions of the usual DF and SP tests are unaffected by the structural break.

Proof: See Appendix.

Theorem 2 indicates that the asymptotic distributions under the null of both the usual DF and SP tests are not affected by the presence of a one-time structural break. Under the null hypothesis, it is unnecessary (asymptotically) to worry about the presence of a one-time structural break, and this is so for both the DF and the SP-type tests. Note that this result does not conflict with our Theorem 1 and Perron's Theorem 2, which deal with the question of whether asymptotics under the null are affected if one does allow for a structural break. The changes demonstrated by Perron in the null distribution of the DF tests arise not from the presence of a structural break in the DGP, but from the way in which the DF tests allow for such a break. The SP tests allow for the break differently, and their asymptotics under the null are unaffected by either the presence of a break or by allowance for such a break.

This raises an obvious question: why should we allow for a break in performing the SP tests? The answer is that we do so to increase the power of the tests. Perron shows, in his Theorem 1, that when the alternative of stationarity is true, the probability limit of the coefficient estimate of the lagged dependent variable in the usual DF regression depends on the magnitude of the structural change, and the coefficient estimate gets closer to one, as this magnitude increases. Thus, the usual DF tests will be biased toward accepting the null. This fact is also true for the SP tests. When the null is true, the usual SP tests are not affected by the presence of the break, but when the alternative is true, they are affected. The following theorem formally shows that the equivalent of Perron's Theorem 1 also holds for the usual SP tests.

__Theorem 3__ Suppose that Assumption 1 holds with $\rho < 1$. Suppose that a structural break occurs at time T_B, and that $T_B/T \to \lambda$ as $T \to \infty$. Define

$$\sigma_x^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} Var(X_t)$$

$$\gamma_1 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E(X_t X_{t-1})$$

$$\rho_1 = \gamma_1 / \sigma_x^2 .$$

Then, if $\tilde{\phi}$ is the coefficient of $\tilde{S}_{t-1}$ in the regression of Δy_t on $[1, \tilde{S}_{t-1}]$,

$$(10) \quad \tilde{\phi} \to \frac{\sigma_x^2(\rho_1 - 1)}{\sigma_x^2 + \frac{1}{3}(X_\infty^2 + X_1 X_\infty + X_1^2) + (\lambda^2 - \lambda + \frac{1}{3})\delta_3^2 + (X_\infty - X_1)(\lambda^2 - \frac{1}{3})\delta_3 + X_1\delta_3}$$

Proof: See Appendix.

Theorem 3 shows that when the alternative is true ($\rho < 1$), the asymptotic distribution of the estimated coefficient in the usual SP tests depends on the magnitude of the structural change. In particular, $\tilde{\phi} \to 0$ as $\delta_3 \to \infty$, which corresponds to a potential bias toward accepting the null. Thus, ignoring the break reduces power. Therefore it is better to allow for the structural break if one exists. However, note that whether or not we allow for the break is not a question of size of the SP tests. That is, if we put the break in the wrong place or if we do not consider the break at all, the size of the SP tests (under the null) is not affected asymptotically; only power is affected. This advantage does not hold for Perron's tests which are based on the DF framework.

3. EMPIRICAL RESULTS

In this section, we now apply the SP tests with structural change to the Nelson and Plosser time series analyzed by Perron. Specifically, we analyze the same eleven series to which Perron applied his crash model (A).

First, we calculate some critical values. Table 1 gives critical values calculated by simulation, using iid normal errors, for various relevant values of T and T_B. The simulation used 50,000 replications for each value of T and T_B. The FORTRAN subroutine program GASDEV/RAN3 of Press et al. (1986) has been used for generating the iid N(0,1) random numbers.

These critical values are almost same as those of the usual SP test statistics. For example, the 5% critical values of the usual SP statistics $\tilde{\rho}$ and $\tilde{\tau}$ are -17.5 and -3.06 respectively, when T = 100. These numbers are quite similar to -17.6 and -3.06, which are the critical values of the statistics $\check{\rho}$ and $\check{\tau}$ in (8), when T = 102 with T_B = 61. This similarity is to be expected from Theorem 1, for large sample sizes, and it is comforting to see in sample sizes found in the data we consider, since it reinforces our faith in the asymptotic theory.

To allow for autocorrelated errors, we consider the augmented versions of the SP tests which include the augmented terms $\Delta\tilde{S}_{t-j}$, j = 1,..,k in regression (5). We will consider the augmented t-test version only, not the coefficient version. The augmented tests critically depend on the choice of k. In this chapter, the values of k are determined as the same numbers that Perron selects, so that differences between our results and Perron's will not be due to differences in the choice of k.

As it was mentioned earlier, only the first crash model (A) of Perron is considered. The natural logarithm has been taken for all of the series except for the "interest rate" series. Table 2 presents the test results. They show that at the 5% level, we can reject the null hypothesis of a unit root only for two series: the "money stock" and "employment" series. At the 1% level, we cannot reject the null hypothesis of a unit root for any of the Nelson-Plosser time series considered here. The striking results of Perron disappear.

The overall results of the SP tests indicate that the "unit root" model is more relevant for the Nelson-Plosser macroeconomic time series than the "trend-stationary" model, contrary to Perron's results, even when we consider structural change in the model.

As the discussion of the last section indicates, we note, as a practical implication, that if the null is true, allowing for the break should change the DF test statistics, but should not change the SP test statistics much. This is exactly what we find in the data. The usual DF test statistics ignoring the break are quite different from the

DF test statistics allowing for the break. For example, when the same number of augmentations is chosen, the usual augmented DF t-statistics are -2.27 and -2.47 for the "real GNP" and "GNP deflator" series, while the augmented DF t-statistics allowing for the break are -5.03 and -4.04 for the same time series. On the other hand, for the same time series, the augmented SP statistics allowing for the break are -1.96 and -2.29, while the SP statistics ignoring the break are -1.91 and -2.23. Comparing the two sets of SP statistics, we note that they are not quite different. Thus the pattern in the DF and SP statistics with and without allowance for structural break is exactly as the asymptotic theory would predict, if the null were true.

4. COMMON FACTOR RESTRICTIONS

It is well known that AR models imply common factor restrictions (hereafter, CFR); for example, see the classic paper of Hendry and Mizon (1978). For instance, if $y_t = Z_t\delta + u_t$, $u_t = \rho u_{t-1} + \epsilon_t$, then

$$(11) \qquad y_t = \rho y_{t-1} + Z_t\delta + Z_{t-1}(-\rho\delta) + \epsilon_t.$$

Thus the coefficients of Z_{t-1} contain no new parameters. The coefficients of y_{t-1}, Z_t and Z_{t-1} are subject to nonlinear restrictions: $(\rho)(\delta) + (-\rho\delta) = 0$. These are CFR's, and they are testable. In general, if Z_t contains k variables, there are k CFR's.

The same principle applies, with some modification, to our model. Our equation (1), or (5), is an AR(1) model, and Perron's "nested" equation, our equation (3), is the unrestricted equation that corresponds to (11). In our model, Z_t contains two variables, t and DU_t. The time trend variable is special in the sense that it does not generate a CFR; given intercept and time trend, the lagged time trend variable is redundant (perfectly collinear). The variable DU_t does generate a CFR: the coefficient of DU_{t-1} must be equal to the negative of the coefficient of y_{t-1} times the coefficient of DU_t. Recall that $D(TB)_t = \Delta DU_t$, and note that (3) actually includes DU_t and $D(TB)_t$. This is equivalent to inclusion of DU_t and DU_{t-1}, but it changes the form of the CFR slightly, so that it becomes:

(12) $\gamma_2\gamma_4 - \gamma_5(1-\gamma_2) = 0.$

There are also two unit root restrictions:

(13) $\gamma_2 = 1, \quad \gamma_3 = 0.$

These restrictions transform the CFR in (12) into $\gamma_4 = 0$. This means that although the CFR is nonlinear, it becomes linear when the unit root restrictions are imposed. The validity of the CFR in its general form (12) does not depend on the unit root restrictions. However, we cannot test the CFR in (12) independently of the unit root restrictions, at least based on asymptotics, since γ_5 in equation (3) cannot be consistently estimated. The reason is that the regressor $D(TB)_t$ is asymptotically degenerate. Therefore, we settle for testing the joint restrictions of the unit root and the CFR. Thus we wish to test

(14) $$H_0 : \quad \gamma_2 = 1, \quad \gamma_3 = \gamma_4 = 0 .$$

Note that the SP test amounts to an LM test of $\gamma_2 = 1$, imposing the restriction $\gamma_3 = 0$ and the CFR. Perron's test is a test of $\gamma_2 = 1$, not imposing $\gamma_3 = 0$ or the CFR. In this section, we develop a generalized Wald statistic which considers the joint hypothesis (14) consisting of the unit root restriction and the CFR. To do so, we define:

(15) $$r(\psi) = [r_1(\psi), r_2(\psi), r_3(\psi)]'$$

where

$$r_1(\psi) = \gamma_2 - 1.0$$
$$r_2(\psi) = \gamma_3$$
$$r_3(\psi) = \gamma_4$$
$$\psi = [\gamma_2, \gamma_3, \gamma_4, \gamma_5].$$

Also define:

$$R = \frac{dr(\psi)}{d\psi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} .$$

and $X_t = [y_{t-1}^*, t^*, DU_t^*, \Delta DU_t^*]$, where the asterisk (*) represents deviations from means, and let X be the Tx4 matrix with t^{th} row X_t. Let $\tilde{\psi} = (\tilde{\gamma}_2, \tilde{\gamma}_3, \tilde{\gamma}_4, \tilde{\gamma}_5)$ be the OLS estimates in the regression of y_t on X_t, t=1,..,T, and let $r(\tilde{\psi})$ be $r(\psi)$ evaluated at $\tilde{\psi}$. Let $\hat{\sigma}_\epsilon^2$ be the usual error variance estimate from the regression of y_t on X_t. Then, the generalized Wald test statistic is defined as:

$$(16) \qquad \hat{\xi}_W = r(\tilde{\psi})[R\ \hat{\sigma}_\epsilon^2(X'X)^{-1}\ R']^{-1}r(\tilde{\psi})'\ .$$

<u>**Theorem 4**</u> Let $\{y_t\}$ be generated by (1) under the null hypothesis of a unit root ($\rho = 1$). Assume that Assumption 1 is satisfied. Suppose that there occurs a structural break at time T_B, and that $T_B/T \to \lambda$ as $T \to \infty$. Then, the asymptotic distribution of the Wald statistic in (16) follows:

$$(17) \qquad \xi_w \to \frac{\sigma^2}{\sigma_\epsilon^2} E' \left(\frac{H}{B}\right) E$$

where

$$H = \begin{bmatrix} \frac{3\lambda^2+3\lambda-1}{12} & H_1+\frac{H_2}{2} & \frac{H_1}{2}+\frac{H_2}{12\lambda-12\lambda^2} \\ & \frac{H_2^2}{\lambda-\lambda^2}-H_3 & -\frac{H_1H_2}{\lambda-\lambda^2}-\frac{H_3}{2} \\ & & \frac{H_1^2-\frac{H_3}{12}}{\lambda-\lambda^2} \end{bmatrix}$$

and

$$B = H_1^2 + H_1H_2 + H_2^2/(12\lambda-12\lambda^2) + H_3(-3\lambda^2+3\lambda-1)/12$$

$$H_1 = \int_0^1 rW(r)dr - 0.5\int_0^1 W(r)dr$$

$$H_2 = \int_0^\lambda W(r)dr - \lambda\int_0^1 W(r)dr$$

$$H_3 = \int_0^1 W(r)^2 - (\int_0^1 W(r)dr)^2$$

$$E = [F_1, F_2, F_3]'$$

$$F_1 = 0.5[W(1)^2-\sigma_\epsilon^2/\sigma^2] - W(1)\int_0^1 W(r)dr$$

$$F_2 = 0.5W(1) - \int_0^1 W(r)dr$$

$$F_3 = W(\lambda) - \lambda W(1)$$

and where $W(r)$ is a Brownian motion defined on [0,1]. □

Proof: See Appendix.

The theorem shows that the asymptotic distribution of the Wald test statistic depends on λ, σ^2, σ_ϵ^2 and T. We obtain $\sigma^2 = \sigma_\epsilon^2$ under iid errors so that we can tabulate the critical values, as presented in Table 3. These are the values that correspond to k = 0 in the table.

To allow for general innovations, one may use an augmented version of the Wald test by including the augmentation terms Δy_{t-j}, j = 1,..,k in (3). We may presume (though no proof will be given) that the standard asymptotic results on augmentations apply. Specifically, if the number of augmentations is made a suitable function of sample size (e.g., k proportional to $T^{1/4}$), the asymptotic distribution of the augmented test in the presence of innovations satisfying Assumption 1 is same as (16), the asymptotic distribution of the non-augmented test in the presence of iid innovations. This should be so because the relevant asymptotic covariance matrix is block-diagonal with respect to the main regressors and the augmented terms. In small samples, however, this distributional equivalence may not be true. The simulated critical values for different values of k show significant discrepancies in small samples. For example, when the sample size is T = 62 (with the break point T_B = 21, thus λ = .338), the simulated critical values under iid errors are 16.4, 31.2 and 35.2 at the 5% level for k = 0, 5 and 8 respectively. But when the sample size is large enough, such differences disappear, as our conjectured asymptotics predict. When T = 1000 (with T_B = 338, so λ = .338), the 5% critical values are 1.48, 1.42 and 1.37 for k = 0, 5 and 8, for example. Therefore, as long as the critical values are not same for the different values of k in small samples, we will use the critical values corresponding to the actual choice of k.

The results of the augmented Wald tests applied to the Nelson-Plosser data are given in Table 4. Again we use the same number of augmentations as Perron did. They show that at the 5% level the joint hypothesis is rejected only for the "nominal wage" and "industrial production" series. At the 1% level, we cannot reject the joint hypothesis for any of the time series except the "industrial production" series.

The slight difference between these results and the results of the SP-type tests in the last section is that rejections of the null hypothesis occur for different series using different tests. For the augmented Wald test, at the 5% level, the joint null hypothesis of a unit root and the CFR is rejected only for the "industrial production" and "nominal wage" series; the SP test does not reject the unit root null for these series. For the SP test, at the 5% level, the null hypothesis is rejected only for the "money stock" and "employment" series, for which the Wald test did not reject the joint null of a unit root and the CFR. Overall, the results of the augmented Wald test are similar to the results of the SP tests; most of the Nelson-Plosser time series are nonstationary. This is favorable evidence that the restrictions that the SP tests impose are in fact not rejected by the data.

The strong dependence of the critical values on k is a considerable disadvantage of the augmented Wald test. It presumably indicates that the relevant asymptotic theory does not apply well at the sample sizes found in the Nelson-Plosser data. From a more pragmatic viewpoint, the choice of critical values matters; using the "asymptotic" values for k = 0 would cause the Wald test to reject for most series. Therefore, we alternatively consider the "corrected" Wald test which is based on a Phillips-Perron type correction of Phillips and Perron (1988). Define S^2 and $S^2(\ell)$ as in (27) and (28) of SP, using $\hat{\epsilon}_t$ as the residuals from a unrestricted regression on (3); these are consistent estimates of the innovation variance σ_ϵ^2 and the long-run variance σ^2 that appear in (17). Define the corrected Wald test by:

$$(18) \qquad \hat{\xi}_W^* = \frac{S^2}{S^2(\ell)} r^*(\tilde{\psi})[R\, \hat{\sigma}_\epsilon^2 (X'X)^{-1} R']^{-1} r^*(\tilde{\psi})'$$

where

$$r^*(\tilde{\psi}) = r(\tilde{\psi}) - \tilde{\pi}\, i_3'\, D_T^*[R(X'X)R']$$

$$\tilde{\pi} = 0.5[S^2(\ell) - S^2]$$

$$i_3 = [1, 0, 0]'$$

$$D_T^* = \text{diagonal}[T, T^{3/2}, T^{1/2}].$$

Then the asymptotic distribution of the corrected Wald test is given in the next theorem.

<u>**Theorem 5**</u> Assume that the conditions in Theorem 4 are satisfied. Then, the asymptotic distribution of the corrected Wald test in (18) follows:

$$(19) \qquad \xi^*_w \to E^{*\prime}\left(\frac{H}{B}\right)E^*$$

where $E^* = [F_1^*, F_2, F_3]'$ with $F_1^* = 0.5W(1)^2 - 0.5 - W(1)\int_0^1 W(r)dr$, and with F_2, F_3, H and B as defined in Theorem 4.

Proof: See Appendix.

The theorem indicates that the asymptotic distribution of the corrected Wald test does not depend on σ^2 or σ_ϵ^2. Furthermore, it is the same as that of the uncorrected Wald test under iid errors. Thus, the critical values for k = 0 in Table 3 can be used.

The results of the corrected Wald test applied to the Nelson-Plosser data are also provided in Table 4. We supply the results for ℓ = 4 and 12, which are popular choices for data sets of this size. The results again indicate that we cannot reject the joint hypothesis of a unit root and the CFR at the 5% level for all of the Nelson-Plosser series except two: the "industrial production" and "GNP deflator" series. At the 1% level, the null is rejected only for the "industrial production" series. These results are basically similar to those of the augmented Wald tests with the minor difference that once again the null is rejected for different series using different tests. But, most of the time series considered here remain nonstationary. Once again the overall results agree with those provided by the

SP tests; namely, that most series appear to contain a unit root.

5. CONCLUDING REMARKS

In this chapter we have adapted the SP unit root test to allow for a one-time structural change. This is analogous to Perron's adaptation of the DF unit root tests. However, we find important advantages to operating in the SP framework rather than in the DF framework. Most importantly, allowing or not allowing for a one-time structural change does not affect the asymptotic distribution theory for SP statistics under the null hypothesis. As a result, the asymptotic validity of the SP tests is not affected by possibly incorrect placement of the structural break, or by allowance for a break when there is not a break (or vice versa). These findings reflect and reinforce the SP argument that the DF parameterization is clumsy, in the sense that explanatory variables (such as level, trend, or in this case a structural change dummy) are treated in a statistically inconvenient way.

When the modified SP tests are applied to the Nelson and Plosser data, Perron's striking results favoring the trend-stationary model disappear. Most U.S. time series now appear to have a unit root even when structural change is allowed for. In fact, the effect of allowing for structural change in the SP framework is minimal, and this is as predicted by the asymptotic theory if the null is true. Finally, the common factor restrictions implied by the SP model appear to hold in these data.

Phillips (1988) and others have argued that models (such as Perron's) that allow for exogenous structural change actually support the unit root hypothesis, in the sense that the innovation that is labelled as structural change is obviously persistent. That is, the exogeneity of the structural change can be argued to be questionable. Models that endogenize structural change have been considered in the DF framework by Zivot and Andrews (1990) and Banerjee et al. (1990). Similar extensions of the SP model with structural change are presumably possible, and this is a topic for further research.

TABLE 1

Critical Values of the Schmidt-Phillips Tests With Structural Change

T	T_B		$\check{\rho}$	$\check{\tau}$
62	21	1.0%	-23.49	-3.69
		2.5%	-20.22	-3.37
		5.0%	-17.52	-3.09
71	30	1.0%	-23.82	-3.68
		2.5%	-20.40	-3.36
		5.0%	-17.60	-3.08
82	41	1.0%	-24.29	-3.69
		2.5%	-20.68	-3.36
		5.0%	-17.81	-3.09
102	61	1.0%	-23.53	-3.66
		2.5%	-20.63	-3.32
		5.0%	-17.73	-3.06
111	70	1.0%	-24.26	-3.63
		2.5%	-20.54	-3.31
		5.0%	-17.83	-3.06

TABLE 2

Schmidt-Phillips Tests for a Unit Root
With Structural Change

Series	T	T_B	k	$\check{\tau}$
Real GNP	62	21	8	-1.90
Nominal GNP	62	21	8	-2.38
Real Per Capita GNP	62	21	7	-2.26
Industrial Production	111	70	8	-2.83
Employment	81	40	7	-3.24*
GNP Deflator	82	41	5	-2.23
CPI	111	70	2	-1.78
Nominal Wage	71	30	7	-3.04
Money Stock	82	41	6	-3.60**
Velocity	102	61	1	-1.74
Interest Rate	71	30	2	-1.12

** significant at the 2.5% level
* significant at the 5% level

TABLE 3

Critical Values of the Wald test

T	T_B	k	1%	2.5%	5%	10%
62	21	0	22.1	18.8	16.4	13.9
		5	42.9	36.4	31.4	26.6
		8	46.4	39.3	33.9	28.4
71	30	0	18.6	16.0	14.0	11.9
		5	34.5	29.9	26.2	22.3
		8	38.0	32.6	28.3	24.0
82	41	0	15.7	13.5	11.8	10.0
		5	28.7	24.8	21.5	18.3
		8	30.5	26.1	22.8	19.3
102	61	0	12.3	10.6	9.3	8.0
		5	21.3	18.7	16.5	14.1
		8	22.0	19.2	16.8	14.4
111	70	0	11.3	9.8	8.6	7.3
		5	19.6	17.0	15.0	12.8
		8	20.0	17.3	15.3	13.0

TABLE 4

Results of Wald Test for a Unit Root with the CFR

Series	T TB	Augmented		Corrected	
		k		$\ell=4$	$\ell=12$
Real GNP	62 21	8	26.90	9.99	8.61
Nominal GNP	62 21	8	30.13	13.25	15.08
Real Per Capita GNP	62 21	7	16.86	7.97	6.64
Industrial Production	111 70	8	33.18***	18.97***	18.40***
Employment	81 40	7	20.89	10.27	9.37
GNP Deflator	82 41	5	16.67	13.61**	13.64**
CPI	111 70	2	4.28	2.74	2.76
Nominal Wage	71 30	7	29.70*	12.73	12.86
Money Stock	82 41	6	18.67	10.00	10.39
Velocity	102 61	1	5.39	6.52	7.79
Interest Rate	71 30	2	7.52	9.94	10.19

*** significant at the 1% level
** significant at the 2.5% level
* significant at the 5% level

APPENDIX

ASYMPTOTICS

Proof of Theorem 1

Since $\tilde{S}_t \equiv y_t - \tilde{\psi}_x - Z_t\tilde{\delta}$, we have

$$\tilde{u}_t = \Delta\tilde{S}_t = \Delta y_t - \Delta Z_t\tilde{\delta} = \epsilon_t - \Delta Z_t(\tilde{\delta}-\delta)$$

$$\tilde{S}_t = \sum_{j=2}^{t}[\epsilon_j - \Delta Z_j(\tilde{\delta}-\delta)] = \sum_{j=2}^{t}\epsilon_j - (Z_t-Z_1)(\tilde{\delta}-\delta).$$

Letting $S_t = \sum_{j=2}^{t}\epsilon_j$ and [rT] be the integer part of rT, for r ϵ [0,1], we have

(A.1) $$T^{-1/2}\tilde{S}_{[rT]} = T^{-1/2}S_{[rT]} - ([rT]-1)T^{-1}\cdot T^{1/2}(\tilde{\delta}_2 - \delta_2)$$
$$- T^{-1}(DU_{[rT]} - DU_1)\cdot T^{1/2}(\tilde{\delta}_3 - \delta_3).$$

We consider the three terms on the right hand side of (A.1) separately. First,

(A.2) $$T^{-1/2}S_{[rT]} \to \sigma W(r),$$

a standard result. For the second term, note that $\Delta Z_t = [1, D(TB)_t]$, and $D(TB)_t$ equals zero except when $t = T_B+1$. Thus $(\tilde{\delta}_2 - \delta_2)$ is asymptotically equivalent to $\bar{\epsilon}$, and $T^{1/2}(\tilde{\delta}_2 - \delta_2) \to \sigma W(1)$. Therefore,

(A.3) $$([rT]-1)T^{-1}\cdot T^{1/2}(\tilde{\delta}_2 - \delta_2) \to \sigma r W(1) .$$

For the third term, we have

$$DU_{[rT]} - DU_1 \to \begin{cases} 0 & \text{if } r < \lambda \\ 1 & \text{if } r \geq \lambda \end{cases} .$$

We also have

$$(\tilde{\delta}_3 - \delta_3) = [D(TB)'M_1D(TB)]^{-1}D(TB)'M_1\epsilon$$

with D(TB) a Tx1 vector of zeros except for a one in position T_B+1, and with $M_1 = I - i(i'i)^{-1}i' = I - T^{-1}i'i$, where i is a vector of ones. Then,

$$D(TB)'M_1D(TB) = 1 - T^{-1}$$

$$D(TB)'M_1\epsilon = \epsilon_{TB+1} - \bar{\epsilon}$$

$$T^{1/2}(\tilde{\delta}_3 - \delta_3) = [1-T^{-1}]^{-1}T^{1/2}(\epsilon_{TB+1} - \bar{\epsilon})$$

$$\to - \sigma W(1) .$$

Thus,

(A.4) $$T^{-1}(DU_{[rT]} - DU_1)T^{1/2}(\tilde{\delta}_3 - \delta_3) \to 0$$

Combining (A.2), (A.3) and (A.4), we get

(A.5) $$T^{-1/2}\tilde{S}_{[rT]} \to \sigma[W(r) - rW(1)] = \sigma V(r),$$

where V(r) is a Brownian bridge. Then, similarly as in Schmidt and Phillips (1990, p. 24), we obtain

(A.6) $$T^{-2}\sum_{t=2}^{T}(\tilde{S}_{t-1}{}^2 - \bar{\tilde{S}})^2 \to \sigma^2 \int_0^1 \underline{V}(r)^2 dr$$

$$T^{-1}\sum_{t=2}^{T}(\tilde{S}_{t-1} - \bar{\tilde{S}})(\Delta\tilde{S}_t - \Delta\bar{\tilde{S}}) \rightarrow -0.5\sigma_\epsilon^2$$

where $\underline{V}(r) = V(r) - \int_0^1 V(r)dr$ is referred to as a demeaned Brownian bridge. Therefore, the result in (9a) can be obtained from the ratio of the above expressions, and the result in (9b) similarly can be obtained.

Proof of Theorem 2

The DGP is, similarly as in (1):

$$\text{(A.7)} \qquad y_t = \psi + \xi t + \delta_3 DU_t + X_t, \quad X_t = \rho X_{t-1} + \epsilon_t,$$

where $\rho = 1$ under the null. When the usual SP test statistics (ignoring the break) are calculated from data following this DGP, we have

$$\Delta y_t = \xi + \delta_3 D(TB)_t + \epsilon_t$$

$$\tilde{\xi} = \text{mean } \Delta y = \xi + T^{-1}\delta_3 + \bar{\epsilon}$$

and

$$\begin{aligned}
\tilde{S}_t &= (y_t - y_1) - \tilde{\xi}(t-1) \\
&= (X_t - X_1) + \delta_3 DU_t - (\tilde{\xi} - \xi)(t-1) \\
&= \sum_{j=2}^{t}\epsilon_j - (t-1)\bar{\epsilon} + \delta_3 DU_t - (t-1)\delta_3/T
\end{aligned}$$

Then,

$$T^{-1/2}\tilde{S}_{[rT]} = T^{-1/2}\sum_{j=2}^{[rT]}\epsilon_j - ([rT]-1)T^{-1}\cdot T^{1/2}\bar{\epsilon} + T^{-1/2}\delta_3 DU_t$$

$$- ([rT]-1)T^{-3/2}\delta_3 \ .$$

Noticing that the last two terms vanish as $T \to \infty$, we obtain

(A.8) $$T^{-1/2}\tilde{S}_{[rT]} \ \to \ \sigma[W(r) - rW(1)] \ .$$

This is the same result as we obtain from the usual SP tests when the DGP does not contain a break; see the equation just before (A3.1) in SP (Appendix 3, p. 24). Therefore, ignoring the break in the usual SP tests does not matter asymptotically under the null.

Next, we want to show that this result also holds for the DF tests, when the DGP is again (A.7) with $\rho = 1$. Define $W_t \equiv [1, t]$. Since the DF tests are based on the regression $y_t = \alpha + \gamma t + \rho y_{t-1} + \epsilon_t$, consider the projection residual:

$$\begin{aligned} \hat{S}_t &= \text{(residual from regression of } y_t \text{ on } W_t) \\ &= d\cdot\text{(residual from regression of } DU_t \text{ on } W_t) \\ &\quad + \text{(residual from regression of } X_t \text{ on } W_t) \ . \end{aligned}$$

(The properties of the DF tests will depend on the properties of these residuals, since the DF tests are obtained by a regression of $\hat{S}_t$ on $\hat{S}_{t-1}$.) Then consider,

(A.9) $$\begin{aligned} T^{-1/2}\hat{S}_t &= d\cdot T^{-1/2}\text{(residual from regression of } DU_t \text{ on } W_t) \\ &\quad + T^{-1/2}\text{(residual from regression of } X_t \text{ on } W_t). \end{aligned}$$

Notice that the second term converges to a demeaned and detrended Wiener process, shown in equation (24) in Kiatkowski et al. (1990, p.16) and in equation (16) in Park and Phillips (1988, p. 474). This is exactly same asymptotic result as we encounter in considering the usual DF test statistics which do not allow for the break. Therefore, to complete the proof, we only need to show that the first term vanishes asymptotically. To do so, define the convergence rate matrix,

$$R = \text{diagonal}[T^{-1/2},\ T^{-3/2}].$$

Then, the first term in (A.5) follows:

$$\text{(A.10)}\quad d\cdot T^{-1/2}DU_t - d\cdot W_t R(RW'WR)^{-1}RW'DU$$

$$= d\cdot T^{-1/2}DU_t - d\cdot(T^{-1/2},\ tT^{-3/2})(RW'WR)^{-1}(T^{-1}\sum_t DU_t,\ T^{-2}\sum_t tDU_t)'$$

Here, notice that the following limits exist:

$$(RW'WR) \rightarrow \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{pmatrix}$$

$$T^{-1}\sum_t DU_t \rightarrow \lambda$$

$$T^{-2}\sum_t tDU_t \rightarrow (1-\lambda^2)/2 \ .$$

But since $T^{-1/2}DU_t \rightarrow 0$, and $(T^{-1/2},\ tT^{-3/2}) \rightarrow 0$, the whole term in (A.10) vanish asymptotically.

Proof of Theorem 3

The DGP is (A.7) under the alternative hypothesis. The usual SP tests statistics are calculated from:

$$\tilde{S}_t = (y_t - y_1) - \tilde{\xi}(t-1)$$

$$= (X_t - X_1) + \delta_3 DU_t - (\tilde{\xi} - \xi)(t-1)$$

$$= (X_t - X_1) - (t-1)\overline{\Delta X} + \delta_3[DU_t - (t-1)/T] .$$

Define σ_x^2, γ_1 and ρ_1 as in the statements of the Theorem. Note that $\overline{\Delta X} = o_p(T^{-1})$, since $T\,\overline{\Delta X} = X_t - X_1 = o_p(1)$. Thus, $\overline{\Delta X}$ and $\sqrt{T}\,\overline{\Delta X}$ vanish asymptotically. Now, consider the following expression for the denominator of $\tilde{\phi}$:

(A.11) $$T^{-1}\sum_t \tilde{S}_t^2 = T^{-1}\sum_t [(X_t-X_1) - (t-1)\overline{\Delta X} + \delta_3(DU_t-(t-1)/T)]^2$$

$$= T^{-1}\sum_t [(X_t-X_1)-(t-1)\overline{\Delta X}]^2 + \delta_3^2 T^{-1}\sum_t [DU_t-(t-1)/T]^2$$

$$+ 2\delta_3 T^{-1}\sum_t [(X_t-X_1)-(t-1)\overline{\Delta X}][DU_t-(t-1)/T].$$

Evaluating each term of the above expression, we get:

(i) $$T^{-1}\sum_t [(X_t-X_1) - (t-1)\overline{\Delta X}]^2 = T^{-1}\sum_t (X_t-X_1)^2$$

$$+ T^{-1}\overline{\Delta X}^2 \sum_t (t-1)^2 - 2T^{-1}\overline{\Delta X} \sum_t tX_t$$

$$+ 2\overline{X}\overline{\Delta X} + 2X_1\overline{\Delta X}\, T^{-1}\sum_t t - 2X_1\overline{\Delta X}$$

$$\rightarrow (\sigma_x^2+X_1^2) + (X_\infty-X_1)^2/3 + 0 + 0 + X_1X_\infty - X_1^2 + 0$$

$$= \sigma_x^2 + (X_\infty^2 + X_1X_\infty + X_1^2)/3$$

where we have used $\sum_t t = T(T+1)/2$ and $\sum_t t^2 = T(T+1)(2T+1)/6$.

(ii) $\delta_3^2[T^{-1}\sum_t DU_t^2 + T^{-3}\sum_t (t-1)^2 - 2T^{-2}\sum_t DU_t(t-1)]$

$$\to \delta_3^2[(1-\lambda) + 1/3 - (1-\lambda^2)]$$

$$= (\lambda^2 - \lambda + 1/3)\delta_3^2$$

(iii) $2\delta_3 T^{-1}\sum_t (X_t-X_1)DU_t - 2\delta_3 T^{-1}\sum_t (t-1)\overline{\Delta X}\ DU_t$

$$- 2\delta_3 T^{-2}\sum_t (X_t-X_1)(t-1) + 2\delta_3\overline{\Delta X}T^{-2}\sum_t (t-1)^2$$

$$\to 0 - \delta_3(X_\infty-X_1)(1-\lambda^2) + X_1\delta_3 + (2\delta_3/3)(X_\infty-X_1)$$

$$= \delta_3(X_\infty-X_1)(-1/3+\lambda^2) + X_1\delta.$$

Thus, the whole expression in (A.11) follows:

(A.12) $$T^{-1}\sum_t \tilde{S}_t^2 \to \sigma_x^2 + (X_\infty^2+X_1X_\infty+X_1^2)/3 + (\lambda^2-\lambda+1/3)\delta_3^2$$

$$+ (X_\infty-X_1)(-1/3+\lambda^2)\delta_3 + X_1\delta_3 .$$

Note that the last three terms of the above expression do not exist for the denominator of $\tilde{\phi}$ when there is no break.

For the numerator of $\tilde{\phi}$, consider,

(A.13) $$T^{-1}\sum_t \tilde{S}_{t-1}\Delta\tilde{S}_t = T^{-1}\sum_t [(\Delta X_t-\overline{\Delta X}) + \delta_3(D(TB)_t-1/T)]\cdot[(X_{t-1}-X_1)$$

$$- (t-2)\overline{\Delta X} +\delta_3(DU_{t-1}-(t-2)/T)]$$

$$= T^{-1}\sum_t (\Delta X_t-\overline{\Delta X})[(X_{t-1}-X_1) - (t-2)\overline{\Delta X}]$$

$$+ \delta_3 T^{-1}\sum_t (\Delta X_t-\overline{\Delta X})[DU_{t-1}-(t-2)/T]$$

$$+ \delta_3 T^{-1}\sum_t [D(TB)_t-1/T][(X_{t-1}-X_1) - (t-2)\overline{\Delta X}]$$

$$+ \delta_3^2 T^{-1} \sum_t [D(TB)_t - 1/T][DU_{t-1} - (t-2)/T].$$

The first term is as follows:

$$T^{-1}\sum_t \Delta X_t (X_{t-1}-X_1) - T^{-1}\sum_t \overline{\Delta X}(X_t - X_1) - T^{-1}\sum_t \Delta X_t (t-2)\overline{\Delta X}$$
$$+ T^{-1}\sum_t \overline{\Delta X}^2 (t-2)$$
$$\rightarrow (\gamma_1 - \sigma_x^2) + 0 + 0 + 0 = \sigma_x^2(\rho_1 - 1) \ .$$

The other terms in (A.13) vanish asymptotically. Thus the whole expression in (A.13) follows:

$$\text{(A.14)} \qquad T^{-1}\sum_t \tilde{S}_{t-1}\Delta\tilde{S}_t \ \rightarrow \ \sigma_x^2(\rho_1 - 1) \ .$$

Thus, the asymptotic distribution of $\tilde{\phi}$ is given by the ratio of two expressions in (A.12) and (A.14).

$$\text{(A.15)} \quad \tilde{\phi} \rightarrow \frac{\sigma_x^2(\rho_1 - 1)}{\sigma_x^2 + \frac{1}{3}(X_\infty^2 + X_1 X_\infty + X_1^2) + (\lambda^2 - \lambda + \frac{1}{3})\delta_3^2 + (X_\infty - X_1)(\lambda^2 - \frac{1}{3})\delta_3 + X_1{}^{\prime}}$$

From above we see that $\tilde{\phi}$ is not a consistent estimator of $(\rho_1 - 1)$. Interestingly, this is so even when there is no break ($\delta_3 = 0$). Since $(\lambda^2 - \lambda + 1/3) > 0$, $\delta_3 \neq 0$ will tend to move the distribution of $\tilde{\phi}$ toward zero, and this is certainly so for large enough $|\delta_3|$.

Proof of Theorem 4

Consider the demeaned version of equation (3):

(A.16) $y_t^* = \alpha_2 y_{t-1}^* + \alpha_3 \cdot t^* + \alpha_4 DU_t^* + \alpha_5 \Delta DU_t^* + \epsilon_t^*$.

where $X_t = [y_{t-1}^*, t^*, DU_t^*, \Delta DU_t^*]$, while the asterisk (*) represents deviations from means. Also, X is the Tx4 matrix with t^{th} row X_t. Then consider the convergence rate matrices $D_T = \text{diagonal}[T, T^{3/2}, T^{1/2}, 1]$ and $D_T^* = \text{diagonal}[T, T^{3/2}, T^{1/2}]$. Notice that $RD_T^{-1} = D_T^{*-1}R$. So, we obtain:

(A.17) $$\hat{\xi}_W = r(\tilde{\psi})[RD_T^{-1}\{D_T^{-1}(X'X)D_T^{-1}\}^{-1}D_T^{-1}R']^{-1}r(\tilde{\psi})'/\hat{\sigma}_\epsilon^2$$

$$= r(\tilde{\psi})D_T^*[R\{D_T^{-1}(X'X)D_T^{-1}\}^{-1}R']^{-1}D_T^* r(\tilde{\psi})'/\hat{\sigma}_\epsilon^2 \ .$$

Note that the dummy variable $D(TB)_t$ does not affect the asymptotic distribution of the test statistic, since the matrix $D_T^{-1}(X'X)D_T^{-1}$ is asymptotically block diagonal with respect to the sub-matrices $D_T^{*-1}(X_1'X_1)D_T^{*-1}$ and $(X_2'X_2)$, where X_1 and X_2 are Tx3 and Tx1 matrix with t^{th} rows being $X_{1t} = [y_{t-1}^*, t^*, DU_t^*]$ and $X_{2t} = [D(TB)_t^*]$, respectively. That is, $D_T^{-1}(X_1'X_2) \to 0$ as $T \to \infty$. Therefore, $R[D_T^{-1}(X'X)D_T^{-1}]^{-1}R'$ can be replaced with the sub-matrix $[D_T^{*-1}(X_1'X_1)D_T^{*-1}]^{-1}$. (But it does not necessarily mean that we can exclude $D(TB)_t$ in equation (3).)

Also notice that under the null, $r(\tilde{\psi})D_T^*$ is same as $E_1'D_T^*$, where $E_1 = [\sum_t y_{t-1}^{*\prime}\epsilon^*, \sum_t t^{*\prime}\epsilon^*, \sum_t DU_t^{*\prime}\epsilon^*]'$. Then,

(A.18) $$\xi_W = \frac{1}{\hat{\sigma}_\epsilon^2}[T(\hat{\alpha}_2-1),\ T^{3/2}\hat{\alpha}_3,\ T^{1/2}\hat{\alpha}_4][D_T^{*-1}(X_1'X_1)D_T^{*-1}]^{-1}\begin{bmatrix} T(\hat{\alpha}_2-1) \\ T^{3/2}\hat{\alpha}_3 \\ T^{1/2}\hat{\alpha}_4 \end{bmatrix}$$

$$= (1/\hat{\sigma}_\epsilon^2)(D_T^{*-1}X_1'E_1)'[D_T^{*-1}(X_1'X_1)D_T^{*-1}]^{-1}(D_T^{*-1}X_1'E_1) \quad .$$

Denote

$$X_1'X_1 = \begin{bmatrix} f_0 & f_1 & f_2 \\ f_1 & d_{11} & d_{12} \\ f_2 & d_{12} & d_{22} \end{bmatrix} = \begin{bmatrix} \sum_t y_{t-1}^{*2} & \sum_t t^* y_{t-1}^* & \sum_t y_{t-1}^* DU_t^* \\ \cdot & \sum_t t^{*2} & \sum_t t^* DU_t^* \\ \cdot & \cdot & \sum_t DU_t^{*2} \end{bmatrix}$$

and

$$(X_1'X_1)^{-1} = \frac{1}{|D|} \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12} & X_{22} & X_{23} \\ X_{13} & X_{23} & X_{33} \end{bmatrix}$$

where $|D|$ is the determinant of $X_1'X_1$. Then,

$$(A.19) \quad [D_T^{*-1}(X_1'X_1)D_T^{*-1}]^{-1} = \frac{1}{T^{-6}|D|} \begin{bmatrix} T^{-4}X_{11} & T^{-7/2}X_{12} & T^{-9/2}X_{13} \\ T^{-7/2}X_{12} & T^{-3}X_{22} & T^{-4}X_{23} \\ T^{-9/2}X_{13} & T^{-4}X_{23} & T^{-5}X_{33} \end{bmatrix}$$

Now, consider the asymptotic results.

(a) $T^{-2}f_0 = T^{-2} \sum_t y_{t-1}^{*2} \rightarrow \sigma^2 H_3$

(b) $T^{-5/2}f_1 = T^{-5/2} \sum_t t^* y_{t-1}^* \rightarrow \sigma H_1$

(c) $T^{-3/2}f_2 = T^{-3/2} \sum_t y_{t-1}^* DU_t^* \rightarrow \sigma H_2$

(d) $T^{-3}d_{11} = T^{-3} \sum_t t^{*2} \rightarrow 1/12$

(e) $T^{-2}d_{12} = T^{-2} \sum_t t^* DU_t \rightarrow -0.5(1-\lambda)\lambda$

(f) $T^{-1}d_{22} = T^{-1} \sum_t DU_t^{*2} \to (1-\lambda)\lambda$

where H_1, H_2 and H_3 are defined in Theorem 4. Let $E_1 \equiv [e_1, e_2, e_3]'$. Then we also have the additional asymptotic results:

(g) $T^{-1}e_1 = T^{-1} \sum_t y_{t-1}^* \epsilon_t^* \to \sigma^2 F_1$

(h) $T^{-3/2}e_2 = T^{-3/2} \sum_t t^* \epsilon_t^* \to \sigma F_2$

(i) $T^{-1/2}e_3 = T^{-1/2} \sum_t DU_t^* \epsilon_t^* \to \sigma F_3$

where F_1, F_2 and F_3 are defined in Theorem 4. Then, we obtain for each term in (A.19):

(i) $T^{-6}|D| = -(T^{-2}d_{12})^2 T^{-2}f_0 + T^{-3}d_{11} \cdot T^{-1}d_{22} \cdot T^{-2}f_0$

$$- T^{-1}d_{22}(T^{-5/2}f_1)^2 + 2 \cdot T^{-2}d_{12} \cdot T^{-5/2}f_1 \cdot T^{-3/2}f_2$$

$$- T^{-3}d_{11}(T^{-3/2}f_2)^2$$

$$\to -(1-\lambda)\lambda\sigma^2 \cdot B$$

(ii) $T^{-4}X_{11} = -(T^{-2}d_{12})^2 + T^{-3}d_{11} \cdot T^{-1}d_{22}$

$$\to -(1-\lambda)\lambda\sigma^2[(-3\lambda^2+3\lambda-1)/(12\sigma^2)]$$

(iii) $T^{-7/2}X_{12} = -T^{-1}d_{22} \cdot T^{-5/2}f_1 + T^{-2}d_{12} \cdot T^{-3/2}f_2$

$$\to -(1-\lambda)\lambda\sigma^2[H_1/\sigma + H_2/(2\sigma)]$$

(iv) $T^{-9/2}X_{13} = T^{-2}d_{12} \cdot T^{-5/2}f_1 - T^{-3}d_{11} \cdot T^{-3/2}f_2$

$$\to -(1-\lambda)\lambda\sigma^2[H_1/(2\sigma) + H_2/(12(1-\lambda)\lambda\sigma)]$$

(v) $T^{-3}X_{22} = T^{-1}d_{22}\cdot T^{-2}f_0 - (T^{-3/2}f_2)^2$

$$\to -(1-\lambda)\lambda\sigma^2[H_2^2/((1-\lambda)\lambda) - H_3]$$

(vi) $T^{-4}X_{23} = -T^{-2}d_{12}\cdot T^{-2}f_0 + T^{-5/2}f_1\cdot T^{-3/2}f_2$

$$\to -(1-\lambda)\lambda\sigma^2[-H_3/2 - H_1H_2/((1-\lambda)\lambda)]$$

(vii) $T^{-5}X_{33} = T^{-3}d_{11}\cdot T^{-2}f_0 - (T^{-5/2}f_1)^2$

$$\to -(1-\lambda)\lambda\sigma^2[(H_1^2 - H_3/12)/((1-\lambda)\lambda)]$$

Then, we obtain from the above,

(A.20) $$[D_T^{*-1}(X_1'X_1)D_T^{*-1}]^{-1} \to \frac{H^*}{B} \quad .$$

where

$$H^* = \begin{bmatrix} \dfrac{3\lambda^2+3\lambda-1}{12\sigma^2} & \dfrac{H_1}{\sigma}+\dfrac{H_2}{2\sigma} & \dfrac{H_1}{2\sigma}+\dfrac{H_2}{\sigma(12\lambda-12\lambda^2)} \\ & \dfrac{H_2^2}{\lambda-\lambda^2}-H_3 & -\dfrac{H_1H_2}{\lambda-\lambda^2}-\dfrac{H_3}{2} \\ & & \dfrac{H_1^2-\dfrac{H_3}{12}}{\lambda-\lambda^2} \end{bmatrix}$$

Also, we have:

(A.21) $$D_T^{*-1}E_1 \to E^{\#} = [\sigma^2F_1, \sigma F_2, \sigma F_3]' \ .$$

Now, it is useful to note that

$$H^* = I_\sigma \cdot H \cdot I_\sigma .$$

where I_σ = diagonal$[1/\sigma, 1, 1]$. Also, note that $I_\sigma E^\# = \sigma E$, where $E = [F_1, F_2, F_3]'$. Then it is directly shown from (A.20) and (A.21) that

$$\hat{\xi}_W \rightarrow \frac{1}{\sigma_\epsilon^2} E^{\#'} (\frac{H^*}{B}) E^\# = \frac{\sigma^2}{\sigma_\epsilon^2} E' (\frac{H}{B}) E$$

which proves Theorem 4.

Proof of Theorem 5

Define $F_1^* = F_1 - \Gamma$, where $\Gamma = 0.5(\sigma^2 - \sigma_\epsilon^2)/\sigma^2$. Also define $E^* = [F_1^*, F_2, F_3]'$ such that $E = E^* + \Gamma \cdot i_3$ where $i_3 = (1, 0, 0)'$. Then it is easy to show that under iid errors the Wald test in (16) follows:

$$\text{(A.22)} \qquad \hat{\xi}_W \rightarrow E^{*\prime}(H/B)E^* .$$

Here, H, B and E^* do not depend on error variances, so that the above expression is free of the nuisance parameters. We want to show that the corrected Wald test follows the same asymptotic distribution in (A.22) as the uncorrected Wald test in (16) does under iid errors. First, we alternatively show that the asymptotic distribution in (A.22) of the uncorrected Wald test is expressed by:

$$\text{(A.23)} \qquad \hat{\xi}_W \rightarrow (\sigma^2/\sigma_\epsilon^2) \cdot E'(H/B)E$$

$$= (\sigma^2/\sigma_\epsilon^2)\cdot(E^*+\Gamma i_3)'(H/B)(E^*+\Gamma i_3)$$

$$= (\sigma^2/\sigma_\epsilon^2)\cdot E^{*\prime}(H/B)E^* + 2(\sigma^2/\sigma_\epsilon^2)\cdot[(\sigma^2-\sigma_\epsilon^2)/2\sigma^2]$$

$$i_3'(H/B)E^* + (\sigma^2/\sigma_\epsilon^2)\cdot[(\sigma^2-\sigma_\epsilon^2)/2\sigma^2]^2 i_3'(H/B)i_3 \ .$$

Consider the corrected Wald test in (18) as:

(A.24) $$\hat{\xi}^* = r(\tilde{\psi})^*[R\cdot\hat{\sigma}_\epsilon^2(X'X)^{-1}\cdot R']^{-1}r(\tilde{\psi})^{*\prime}$$

$$= \{r(\tilde{\psi})D_T^* - \tilde{\pi}i_3'[D_T^{*-1}(X_1'X_1)D_T^{*-1}]^{-1}\}\cdot[D_T^{*-1}(X_1'X_1)D_T^{*-1}]$$

$$\cdot\{D_T^*\cdot r(\tilde{\psi})' - \tilde{\pi}[D_T^{*-1}(X_1'X_1)D_T^{*-1}]^{-1}i_3\} / \hat{\sigma}_\epsilon^2$$

$$= [D_T^* r(\tilde{\psi})']'[D_T^{*-1}(X_1'X_1)D_T^{*-1}][D_T^* r(\tilde{\psi})'] / \hat{\sigma}_\epsilon^2$$

$$- 2\tilde{\pi}i_3'[D_T^* r(\tilde{\psi})'] / \hat{\sigma}_\epsilon^2$$

$$+ \tilde{\pi}^2 i_3'[D_T^{*-1}(X_1'X_1)D_T^{*-1}]^{-1}i_3 / \hat{\sigma}_\epsilon^2 \ .$$

Now, notice that the first term of the last equation is same as the expression for the "uncorrected" Wald test $\hat{\xi}_W$ in (16), and its asymptotic distribution is expressed in (A.23). Also notice that $\tilde{\pi} \to \pi = \Gamma\cdot\sigma^2$. Then the distribution of the second term is given by:

(A.25) $$-2\tilde{\pi}i_3'[D_T^* r(\tilde{\psi})']/\hat{\sigma}_\epsilon^2$$

$$\to -2\pi i_3'(H^*/B)E^\#/\sigma_\epsilon^2$$

$$= -(2/\sigma_\epsilon^2)[(\sigma^2-\sigma_\epsilon^2)/2]\cdot i_3'\cdot I_\sigma(H/B)I_\sigma E^\#$$

$$= -(1/\sigma_\epsilon^2)(\sigma^2-\sigma_\epsilon^2)\cdot i_3'(H/B)E^*$$

$$- (1/\sigma_\epsilon^2)(\sigma^2-\sigma_\epsilon^2)\cdot i_3'(H/B)[(\sigma^2-\sigma_\epsilon^2)/2\sigma^2]i_3 \ .$$

And, the third term is as follows:

$$\text{(A.26)} \qquad \tilde{\pi}^2 i_3'[D_T^{*-1}(X_1'X_1)D_T^{*-1}]^{-1}i_3'/\hat{\sigma}_\epsilon^2$$

$$\rightarrow [(\sigma^2-\sigma_\epsilon^2)/2\sigma^2]^2(\sigma^2/\sigma_\epsilon^2)\cdot i_3'\cdot(H/B)i_3.$$

Finally, from (A.23), (A.25) and (A.26), the asymptotic distribution of the whole expression in (A.24) is given by:

$$\hat{\xi}^* = r(\tilde{\psi})^*[R\cdot\hat{\sigma}_\epsilon^2(X'X)^{-1}\cdot R']^{-1}r(\tilde{\psi})^{*\prime}$$

$$\rightarrow (\sigma^2/\sigma_\epsilon^2)\cdot E^{*\prime}(H/B)E^*$$

$$+ (1/\sigma_\epsilon^2)\cdot(\sigma^2-\sigma_\epsilon^2)\cdot i_3'(H/B)E^*$$

$$+ (\sigma^2/\sigma_\epsilon^2)\cdot[(\sigma^2-\sigma_\epsilon^2)/2\sigma^2]^2\cdot i_3'(H/B)i_3.$$

$$- (1/\sigma_\epsilon^2)(\sigma^2-\sigma_\epsilon^2)\cdot i_3'(H/B)E^*$$

$$- 2(\sigma^2/\sigma_\epsilon^2)[(\sigma^2-\sigma_\epsilon^2)/2\sigma^2]^2\cdot i_3'(H/B)i_3$$

$$+ (\sigma^2/\sigma_\epsilon^2)[(\sigma^2-\sigma_\epsilon^2)/2\sigma^2]^2\cdot i_3'\cdot(H/B)i_3$$

$$= (\sigma^2/\sigma_\epsilon^2)\cdot E^{*\prime}(H/B)E^*.$$

Therefore, the asymptotic distribution of the corrected Wald test statistic $\hat{\xi}_W^*$ in (18) is the same as the distribution (which is expressed in (A.22)) of the uncorrected Wald test under iid errors:

$$(A.24) \quad \xi^*_w \to E^{*\prime}\left(\frac{H}{B}\right)E^*$$

which is free of nuisance parameters.

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CHAPTER 6

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CONCLUSION

Whether observed series contains a unit root or not has profound implications in macroeconomics. Thus, finding better unit root tests has been a hot issue in time series econometrics. However, many authors, including Schwert(1989), find that no unit root test perform satisfactorily in the presence of strongly autocorrelated errors, in terms of both size and power of the tests. The tests with correct size under the null have poor power under the alternative, and the tests with high power do not have correct size under the null. Therefore, a recent survey paper by Campbell and Perron (1991) stresses the necessity of finding new tests which alleviate the size distortion problem, while retaining good power properties.

While existing unit root tests have mostly been based on one variant or another of the Dickey-Fuller (1979) tests, other tests which are not based on the DF tests have been proposed recently by Schmidt and Phillips (1990). They employ a different parameterization. Unfortunately, Chapter 2 of this thesis finds that they also exhibit similar problems of size distortion or lower power in the presence of autocorrelated errors.

This dissertation, however, finds that significant advantages can be drawn from the SP parameterization. The new IV tests based on the SP framework have surprisingly small size distortions in the presence of strongly autocorrelated errors, and they are more powerful than other tests of similar size. This result may well provide an acceptable solution to the problem of finding new tests which are adequate in terms of both size and power of the tests, and it suggests that it is still possible to improve the performance of unit root tests when the appropriate testing procedure is employed.

The good performance of the new IV tests indicates tangible advantages to operating in the SP framework rather than in the DF framework. The SP tests have simpler asymptotic distributions than the DF tests, and the simple structure of the asymptotic distributions of the SP tests may account for their remarkably improved performance, since it may result in smaller finite sample bias terms.

The perceptible difference between the DF and SP frameworks lies in their respective parameterizations. It has been argued by SP that the DF parameterization is "clumsy", in the sense that it handles exogenous variables in a potentially confusing way. This argument has been elaborated in Chapter 5, in the presence of possible structural break. Specifically, a suitably modified SP test allowing for a structural change reverses the results of Perron (1989), who found most of the Nelson-Plosser series to be trend-stationary if allowance is made for a structural break. The simplicity of the SP parameterization compared to the DF parameterization is reflected in simpler properties of the SP tests under the null hypothesis. Both DF and SP tests are unaffected by a structural break if the null hypothesis of a unit root is true, while they are affected if the alternative hypothesis is true. However, if we allow for one time break under the null, the asymptotic distributions of the SP tests are unaffected by the presence of a break, although the asymptotic distributions of the DF tests are affected. Thus, if the null is true, allowing or not allowing for a break does not matter asymptotically in the SP tests. This result does not hold for the DF tests.

Therefore, this dissertation has addressed the importance of employing an appropriate parameterization in unit root testing procedures, and explained why the SP framework is superior to the DF framework.

The exogeneity of a structural change can be argued to be questionable, so the models that endogenize structural change can be extended from the SP framework, just as Banerjee et al. (1990) and Zivot and Andrews (1990) did in the DF framework. This remains as a topic for further research. Another extension of our work is to modify the new IV tests provided in Chapter 4 in order to allow for more general ARMA errors. Pantula and Hall (1991) have developed a coefficient test which allows for ARMA errors, in the DF framework. However, we have shown in Chapter 4 how to construct IV t-statistic tests in the SP framework. Thus it is potentially straightforward in the SP setting to construct IV-based t-statistic tests that should be robust to ARMA errors. This will also be the subject of future research.

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