This is to certify that the

thesis entitled

A CRITICISM OF METHODOLOGY IN
ALFRED NORTH WHITEHEAD'S AN ENQUIRY CONCERNING
THE PRINCIPLES OF NATURAL KNOWLEDGE

presented by

Rebecca A. Gardner

has been accepted towards fulfillment
of the requirements for

MA degree in Philosophy

Winston A. Wilkinson
Major professor

Date July 17, 1992

PLACE IN RETURN BOX to remove this checkout from your record.
TO AVOID FINES return on or before date due.

DATE DUE	DATE DUE	DATE DUE

A CRITICISM OF METHODOLOGY IN ALFRED NORTH WHITEHEAD'S <u>AN ENQUIRY CONCERNING THE PRINCIPLES OF NATURAL KNOWLEDGE</u>

By

Rebecca A. Gardner

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

MASTER OF ARTS

Department of Philosophy

1992

ABSTRACT

A CRITICISM OF METHODOLOGY IN ALFRED NORTH WHITEHEAD'S <u>AN ENQUIRY CONCERNING PRINCIPLES OF NATURAL KNOWLEDGE</u>

By

Rebecca A. Gardner

This study investigated the natural philosophy of Alfred North Whitehead in <u>An Enquiry Concerning the Principles of Natural Knowledge.</u> The method of extensive abstraction was the central tool employed by Whitehead to reconstruct the physical and temporal theories of the natural sciences. It is proposed that Whitehead's use of this method and his presupposition of Euclidean parallelism distort the results of his work and defeat the purpose of his enterprise. The entities derived from the method of extensive abstraction do not possess the logical properties necessary for their application to the mathematics of the physical sciences. Whitehead's entire project rests on the applicability of these abstract entities and without their use Whitehead's results are called into question.

ACKNOWLEDGMENTS

I would like to thank the members of my committee, for their graciousness and support, for being very giving of their time, and most of all, for helping make this project an enjoyable endeavor. I appreciate their individual contributions:

Dr. Peter Asquith, whose detailed comments and questions on an earlier draft made me rethink and clarify conceptual issues.

Dr. Mark Risjord, whose suggestions and comments on the foundations of mathematics helped make the issues come alive.

Dr. Winston Wilkinson, for his patience in reading revisions of revisions. His thoroughness has improved each one. I thank him also for his generosity with his time and resources. I thank him especially for the freedom to make this project my own.

I would like to thank a friend, Francesca Galarraga, for sharing her time, and especially her computer skills.

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

Historical Overview

The time in which we live our lives and the space which comprises our world weave together to form the basic fabric of our existence. Throughout history mankind has sought to understand these aspects of life. The passage of time and the existence of space were once attributed to deities or mythological creatures. Today the study of time and space falls in great part to the physical sciences. From the mathematics and philosophy of ancient Greece to our own time, numerous theories have attempted to explain the nature of the world and have been superseded by still other theories. The seventeenth century saw tremendous growth in the physical sciences and mathematics with the advent of Newton's Laws of Motion and Gravitation and its accompanying calculus. The next two centuries brought further developments in classical mechanics and mathematics. With the dawn of the twentieth century came the general acceptance of relativity theory and the introduction of quantum mechanics. However, the exact nature of time and space remain an enigma.

Aim of Paper

The philosophical career of Alfred North Whitehead consisted in part in an effort to extract an understanding of the world in which we live. Whitehead's notions of time and space (indeed, his very conception of reality) differ greatly from the science of his age. It is the purpose of this paper to examine the natural philosophy of Whitehead as it is presented in An Enquiry Concerning the Principles of Natural Knowledge.[1] In particular I will be concerned with Whitehead's use of the method of extensive abstraction and his theory of parallelism and the consequences of their application upon his philosophy of nature.[2]

The method of extensive abstraction is of particular interest because of its pervasive use by Whitehead as a tool for abstracting and illuminating the basic relations which occur among events in nature. Whitehead's use of the notion of parallelism, especially in his early works, will be shown to have unduly influenced the outcome of his natural philosophy in a direction contrary to current opinions of

---

[1]Alfred North Whitehead. An Enquiry Concerning the Principles of Natural Knowledge. The University Press: Cambridge, 1955. In the future references to this text will be inserted by page number in parantheses in the main body of this paper.

[2]The content of this paper will be strictly limited in its scope. I will deal with those aspects of events which are subject to the relation of extension and will not cover certain other aspects of Whitehead's philosophy, such as his Theory of Objects, which are of great importance but are not directly affected by the relation of extension. The technical terms referred to in this note will be explained in detail as this paper progresses.

the physical sciences. It is noted that Principles of Natural Knowledge is among the first in a series of Whitehead's works in the philosophy of science and that his notions have matured over time. It is, however, a worthwhile enterprise to examine Whitehead's ideas at the point of their conception. This prospective adds insight to the development of later works as well as being of interest in its own right.

Format of Paper

I will begin this paper with a detailed exposition of Whitehead's natural philosophy as it appears in An Enquiry Concerning The Principles of Natural Knowledge. Once this exposition has been completed I will embark upon a discussion of the influence of the method of extensive abstraction and the theory of parallelism on the work as a whole and on the theories of time and space in particular. It is my contention that the use of these theories result in a natural philosophy which is not consistent with current scientific beliefs and practices. I will present several critical arguments concerning these aspects of Whitehead's work and then add my own critical comments on these arguments.

PART I

EXPOSITION: AN ENQUIRY CONCERNING THE PRINCIPLES OF NATURAL KNOWLEDGE

CHAPTER 1
THE NATURAL WORLD

The Whole of Nature

The natural world, according to Whitehead, is not to be considered in terms of either absolute time or absolute space. Nor is it to be considered as being composed of matter in some continual process of shifting or rearrangement. Neither is it to be considered in an idealist fashion, as something which exists only as a form of thought or ideas. Instead of thinking of nature in any of these ways, Whitehead asks us to drop the materialistic point of view and adopt the perespective that what we experience are events. He considers this view to be both epistemologically and ontologically fundamental.[3]

---

[3]By indicating that the experiencing of events is both epistemologically and ontologically fundamental I mean that from an epistemological standpoint Whitehead justifies his view that the fundamental structure of nature is expressed by events and relations among events because such events and relations are shown to be observable in any experience of events. From an ontological standpoint the experiencing of events is said to be fundamental because it is through such an experience that the passage of nature becomes visible. Through abstraction from this passage of nature Whitehead shows the existence of those basic relations found among events. The meanings of the technical terms used in this brief explanation will be made clear in the development of this paper.

The notions of events, experience, and process are interrelated and their application in Whitehead's philosophy can be somewhat confusing. Events are the basic entities which make up nature. Experience is an essential aspect of events, in the sense that events have the ability to experience other events (through their relations). We (as a certain type of event) gain our knowledge of nature through our experiencing of other events. Process is the fundamental activity in which events are involved. This activity of events, their coming into being and passing out of being, is the progressive advance of nature. Each of these notions, in its own way, is a fundamental part of Whitehead's philosophy of nature. They can be viewed as the expression of nature from three different perspectives. Because an understanding of these notions is fundamental to my project, I will concentrate my attention on Part III of An Enquiry Concerning the Principles of Natural Knowledge, which offers an explanation of the experiencing of nature.[4]

Whitehead begins the development of his philosophical system with an explanation of nature which differs considerably from the way in which it is commonly conceived. Because experience is considered to be one of the fundamental elements of nature, questions arise as to what is meant by the term "experience" and how the more

---

[4] I will, however, refer to comments and definitions offered by Whitehead in other parts of the work.

complicated elements of nature are established from the notion of experience. Whitehead wants to distinguish his theory of nature from the classical Newtonian system of his time and so he chooses a vocabulary which is decidedly different from that generally employed in the description of nature. We must take note, then, of his definitions of terms as they are employed in the development of his natural theory.

Experience and Process

If we were to consider the experiencing of nature in terms of material things we would lose the Whiteheadian view. When we drop the traditional point of view and try to visualize nature as Whitehead does, we find that our very notion of nature must change. Ordinarily when we speak of experience we mean our experience of certain things: objects, physical properties, and even emotions. Whitehead's use of the term experience is something entirely different when considered in its fundamental form. He speaks of experiencing the progressive advance of nature (13, also Mays[5] 38).

Consider experience as the first derivative of process.[6] "Process" can be defined as that act of becoming

---

[5]W. Mays. The Philosophy of Whitehead. Macmillan: New York, 1959.

[6]By derivative I mean a subsequent level of complication in a series extending from the most simple to the most complex (4).

which is inherent in the creative advance of nature (14). Events come into existence and are then replaced by other events. Process is this basic activity. The experiencing of the process is not a necessary part of the creative advance of nature. In other words, the actuality of nature is not dependent upon its having been experienced.

We are now in a position to consider two equally correct perspectives. Both of these involve a discussion of process as the creative advance of nature. First, we could consider process to be an endless stream of events, i.e., the coming into existence and the passing out of existence of an infinite number of events.[7] The totality of these events compose the creative advance of nature. This first view of the term "process" involves events.

The second view exploits the idea of relations, where the notion of an event is expressed in terms of the interaction of relations (relationships) among one another. Ordinarily we think of relations as occurring between things, but Whitehead proposes the possibility that relations can interact among themselves and not, at a basic level, among material objects. This implies that events themselves can be considered complexes of relations interacting with other such complexes.

---

[7]Alfred North Whitehead. Concept of Nature. University Press: Cambridge, 1964. p. 14. Concept of Nature is a companion text to Principles of Natural Knowledge and will be used as such in this paper. Future reference to this text will be designated by CN.

When we considered process in the first sense, we arrived at a notion of events which is fundamental in the development of nature, prior even to any experiencing of nature and, so, prior to any experiencing of material entities. By adding to this view the second notion (that events can be conceived as a complex of interacting relations), we have a view of the creative advance of nature described in terms of events of a complex of interacting relations.

Relation of Extension

It is in the Whiteheadian style to seek that which is most fundamental and to build more complex systems from these fundamental notions. If we accept this method, our next step is to ask which relation is primary, in the sense of being an aspect of all events. For Whitehead, this is the relation of extension (61).

Extension among events can be described as consisting of four possible cases. Given events A and B, the relation of extension can produce the following states: either (i) A and B are identical; (ii) A contains B either in part or totally; (iii) B contains A either in part or totally; or (iv) A and B are entirely separate. It is obvious from this description that the relation of extension involves the concept of containing, and the relation of part to whole. Whitehead seems to use these notions in the ordinary sense.

The relation of extension, as it applies to a and b, is represented by aKb (to be read "a extends over b"), and aKb is to be interpreted as "b is a proper part of a". When we speak of the relation of extension with respect to events we say that the relation aKb means "event b is a part of event a". We find a certain similarity between this description of the relation of extension and the description given by classical set theory of a proper subset of a class, which is represented in set notation as: $b \subset a \;\&\; b \neq a$.

By applying the method of extensive abstraction[8] to events (utilizing the relation of extension) Whitehead defines a space-time framework which will provide a physical description of nature. The domain to which the method of extensive abstraction can be applied is the domain of events. By adding the relation of cogredience[9] to that of extension, it becomes possible for Whitehead to develop the notion of an extensive continuum.[10] It is from the events of the extensive continuum that the concepts of time and space are abstracted or derived. To consider time apart from space or space apart from time is itself an abstraction

---

[8]The method of extensive abstraction will be discussed fully below.

[9]This concept will be explained in more detail later in this paper. Let it suffice for now to say that cogredience indicates the here-now-present of all events which are simultaneous to a given event and, so, indicates the position of these events in a space-time continuum (70).

[10]The extensive continuum refers to a continuum of events (the totality of which gives nature as a whole) which is formed through the relation of extension.

from the concept of the whole of nature. It is important to remember, however, that although time and space can be considered in isolation from one another, in nature they are never separated and are dependent upon one another.

CHAPTER 2
THE DEVELOPMENT OF TEMPORALITY

Durations

At this point it becomes necessary to introduce some technical terminology. Let us begin with durations. A duration is an event and can be considered a piece of the temporal advance of nature. When we consider a duration we are speaking of nature as a whole, i.e., we perceive an entire complex of events which occurs simultaneous to our act of perceiving. This is all of nature at the time of our perception. This complex of events forms the background for our current perceptions and is called a "duration" (68). Although it would seem obvious that all events constituting a duration cannot be directly perceived by a percipient event, the relation of extension, inherent in all events, enables even the furthermost events in a duration to be indirectly perceived. Since extension is the basic relation involved in a duration, further explanation of the concept of a duration will be helpful in understanding the notion of extension.

A duration has a finite temporal dimension and also a physical dimension, which represents a spatial part of the extensive continuum that exists simultaneously to an event within the given duration. These two dimensions of a duration present the whole of nature taken from a particular perspective (that of some percipient event). A duration is infinitely divisible (each division creating another duration) and the totality of these is nature from this particular perspective. In the Whiteheadian sense, a duration is not an abstract stretch of time but is a temporal slab or section of nature.

Durations can vary in their degree of extension. If the notion of duration is to be of use in a temporal theory applicable to the natural sciences, we must be able to abstract[11] from our experience of durations a single moment of time. This is accomplished by considering sets of progressively diminishing durations until we reach an approximation of a time-less instant. Whitehead refers to this process as the method of extensive abstraction (104). Because durations form a continuum of temporal extension, it does not matter where we begin the application of the method of extensive abstraction to a series of durations.

A duration extends over (overlaps) other durations which are part of it, and, in turn, these durations extend over still other durations which are parts of them. It is in this sense that the series of durations mentioned above

---

[11]Whitehead's use of the term "abstraction" refers to a process, involving the relations of events, where he attends to certain relations while ignoring others. This results in a method of simplification, where the relations attended to illuminate an aspect of the entity being discussed. In this particular case, temporal extension is being abstracted from a duration, i.e., this temporal relation is being ignored. When the relation of temporal extension is left out of the consideration of a duration, we are left with an extensionless moment of time.

is established. By continuing the process of diminishment of a series of durations, we will eventually approach the ideal of an extensionless moment of time. It is ideal because an extensionless moment of time does not actually exist in nature. This ideal abstraction is quite valuable when we wish to examine spatial features of nature in the absence of temporal change. Whitehead refers to a moment as "a route of approximation to all of nature which has lost its (essential) temporal extension; thus it is nature under the aspect of a three-dimensional instantaneous space"(112).

A moment of time is established by the use of the method of extensive abstraction applied to some duration experienced from a particular standpoint. It is entirely within the realm of reason that from a particular standpoint (that of a some percipient event) indefinitely many different divisions of durations can be made and so we can abstract indefinitely many moments of time. The entire set of moments established from a particular standpoint form what Whitehead refers to as a "family" of moments (112-113). Analogously we may speak of families of durations from which these families of moments are derived.

Parallelism

Whitehead further explains the characteristics of these families of moments/durations by asserting that all moments/durations of one family are parallel to one another (113-114). This is to say that a family of parallel durations consists of all the durations parallel to a given duration, including that duration itself. Because any two members of a family of moments are parallel to one another, no duration or moment outside of a given family of moments/durations can be considered parallel to any member of the given family; a duration must either intersect the given family or be parallel to it. If a duration is parallel to the given family of durations/moments, it will be included in that family (113).

The concept of parallelism[12] is essential to Whitehead's notion of the abstraction of time and space from events. It seems, in fact, that without the assumption of Euclidean parallelism in his generation of the geometry of the physical universe Whitehead does not feel he can succeed

---

[12]Whitehead's use of the notion of parallelism is somewhat ambigious. At times he makes particular reference to the Euclidean geometry constructed by the use of entities he has derived through the use of the method of extensive abstraction. These entities require the property of parallelism as a basic part of their construction. It would be assumed in these cases that Whitehead is asserting Euclidean parallelism. Palter, in Whitehead's Philosophy of Science, states that "the particular concept of time which [Whitehead] formulates is called `parabolic' ...because it is assumed to lead ...to a `parabolic' (or Euclidean) concept of space. Whitehead suggests the possible usefulness to natural science of alternative types of time and space,...both hyperbolic and elliptic space and time" (Palter 60-61). This reference adds further evidence to the idea that the type of geometry Whitehead is reconstructing is Euclidean and so involves the notion of Euclidean parallelism.

at his task of explaining the nature of our observations. This statement will be considered in more detail later in this paper.

Whitehead's system of time and space is dependent on the assumption of parallelism. It is employed in the description of durations when Whitehead states that "a pair of durations both of which are parts of the same duration are called "parallel"; and also a pair of moments such that there are durations in which both inhere are called "parallel"" (113). The importance of the concept of non-intersection is evident in his statement that "two durations which do not intersect are parallel; and parallel moments which are not identical never intersect" (113). Throughout his discussion of durations, moments, and time-systems Whitehead continually refers to this type of parallelism. We will return again to a discussion of the concept of parallelism at the end of this paper.

Time-systems

A time-system contains all durations which are parallel one to another and thus all parallel moments associated with these durations (114). It is possible for there to exist more than one time-system becauee of the possibility that there can exist sets of durations which are not parallel to one another (see figure 1, p. 18). Indeed, this fact is vital to the Whiteheadian theory of relative motion and his explanation of physical change (176). A time-system is considered to be a family of parallel moments which are arranged in serial order. Whitehead establishes the following criteria for serial order:

> (i) A duration belonging to a time-system is `bounded' by a moment of the same time-system when each duration in which that moment inheres intersects the given duration and also intersects events separate from the given duration;
> (ii) Every duration has two such bounding moments, and every pair of parallel moments bound one duration of that time-system;
> (iii) A moment B of a time-system `lies between' two moments A and C of the same time-system when B inheres in the duration which A and C bound;
> (iv) This relation of `lying between' has the following properties which generate continuous serial order on each time-system, namely,
> (a) Of any three moments of the same time-system, one of them lies between the other two;
> (b) If the moment B lies between the moments A and C, and the moment C lies between the moments B and D, then B lies between A and D;
> (c) There are not four moments in the same time-system such that one of them lies between each pair of the remaining three;
> (d) The serial order among moments of the same time-system has the Cantor-Dedekind type of continuity'[13] (114-115).

---

[13]The Cantor-Dedekind continuum involves the key notions of "an infinite set and a nondenumerable set. Dedekind defined a set as infinite if, and only if, a one-one correspondence can be established between its members and the members of a proper subset of it. An infinite set is denumerable if, and only if, it can be mapped in a one-one manner on the set of natural numbers; otherwise it is indenumerable. A continuum is a nondenumerably infinite set, say K, the elements of which constitute a series that, apart from its serial order, also conforms to the following postulates, as given by E. V. Huntington in his <u>The Continuum</u> (1917): (1) If K_1 and K_2 are any two nonempty parts of K, such that every element of K belongs either to K_1 or K_2 and every element of K_1 precedes every elememt of K_2, then there is at least one element X in K such that any element that precedes X belongs to K_1, and every element that follows X belongs to K_2. (2)If a and b are elements of the class K and a precedes b, then there

Because the members of any given family of moments are all parallel to one another and because parallelism implies the properties of transitivity, symmetry, and reflexiveness,[14] these properties also apply to all members of a family of moments. In this way we are able to establish serial order within a time-system.

A moment has been defined as an approximation of an extensionless instant of time. It also has spatial implications. An ideal moment of time is considered to be

---

exists at least one element x in K such that a precedes x and x precedes b. The continuum is linear if in addition (3) the class K contains a denumerable subclass R in such a way that between any two elements of the class K there is an element of R. For example, the class of real numbers between 0 and 2 is a continuum. It is, moreover, a linear continuum with R the class of rational numbers and $\sqrt{2}$ an element of the continuum but not of R." S. Körner. "Continuity" in <u>The Encyclopedia of Philosophy</u>. Vol. 2. ed. Paul Edwards. Macmillan: New York, 1967. p. 206. This is a long and rather technical explanation. However, it will prove helpful when we consider certain objections leveled against Whitehead's notion of an extensive continuum.

[14]The properites of transitiveness, symmetry, and reflexiveness can be defined as follows:

1) Transitive Property:

R is said to be transitive on A if and only if whenever a first element [of A] is related to a second and the second is related to a third, then it must follow that the first and third are related - if this holds for all elements of A, then R has the transitive property.

2) Symmetric Property:

R is said to be symmetric on A if and only if a pair of elements in A is related via R, then it must follow that the reverse-order pair is also related via R in order that R have the symmetric property.

3) Reflexive Property:

R is said to be reflexive on A if and only if every element in the set must be related to itself via the relation R in order that R have the reflexive property.

These definitions were taken from John F Lucas. <u>Introduction to Abstract Mathematics</u>. Wadsworth Publishing Co.: Belmont, 1986. p. 157.

the whole of physical nature'[15] taken from some particular perspective. Here once again we find the inseparability of space and time. Whitehead derives his theory of space from his theory of time, that is, from durations, moments and time-systems.

<u>Figure 1</u>[16]

Durations and Time-Systems

> * The slab of nature forming a duration is limited in its temporal dimension and unlimited in its spatial dimensions. Thus it represents a finite time and infinite space. For example let the horizontal line represent the time; and assume nature to be spatially one-dimensional, so that an unlimited vertical line in the diagram represents space at an instant. Then the area between the unlimited parallel lines *AB* and *HG* represents a duration. Also the area between *CD* and *EF* represents another duration which is extended over by the duration bounded by *AB* and *HG*. But in fig. 7 we have assumed only one time-system, which is the Newtonian hypothesis. Suppose there are many time-systems and consider two such systems α and β. These are represented by two lines inclined to each other. A duration of time-system α is represented by the area between *AB* and *CD*, and a duration of time-system β is represented by the area between *EF* and *HK*. Two such durations necessarily intersect and also can neither completely extend over the other.
>
> These diagrams are crude illustrations of some properties of durations and are in many respects misleading as the sequel will show.

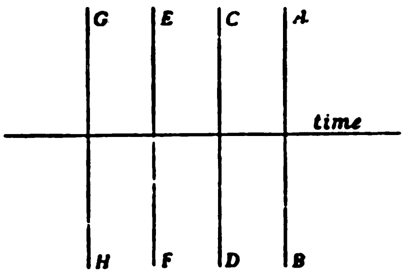
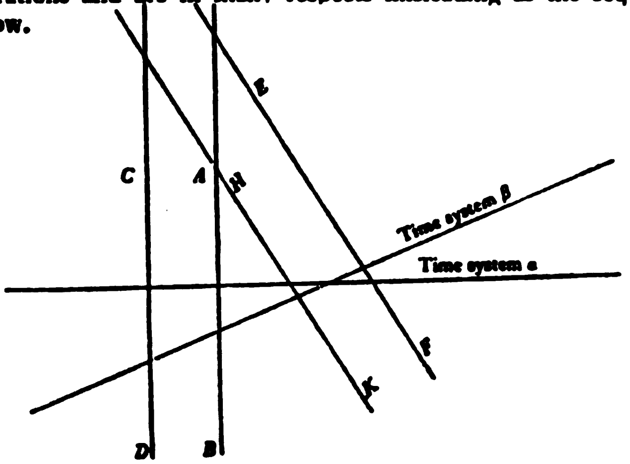


Fig. 7.



[15]The physical nature of space is meant to be that of instantaneous space. This is the space of all of nature taken at an instant of time. It is an ideal condition and does not occur in nature itself.

[16]Figure 1 is taken from <u>Principles of Natural Knowledge</u>, pp. 111-112.

CHAPTER 3
THE METHOD OF EXTENSIVE ABSTRACTION

Abstractive Sets

Let us now introduce the notions of "formative condition," "abstractive sets," "primes," "anti-primes," and the relations of "inhering," "adjoining," "covering," and "intersection." This discussion will help us to connect Whitehead's technical definitions with the explanation of events and durations given above and also demonstrate their relationship to the method of extensive abstraction.

In Principles of Natural Knowledge an explicit definition of "formative condition" is not given. However, the notion of formative condition plays an important role in the development of abstractive sets. A formative condition provides a starting point for the identification of particular groups of abstractive sets. Abstractive sets can be seen as a series of nested durations which tend to converge to some ideal approximation. Let us take, as an example, a moment.[17] There are a number of different abstractive sets which can converge toward the same moment and all such abstractive sets have some common feature which is called their formative condition. We can, then, say that the set of all abstractive sets having the same formative condition will converge toward a single unique moment. When

---

[17]Here the events which comprise the abstractive set are durations.

Whitehead speaks of some "formative condition σ" in respect to abstractive sets, σ-primes, σ-anti-primes, etc., the term "σ" refers to that common feature mentioned above.

An abstractive set is defined as a set of events for which the following conditions hold: "(i) of any two members one extends over the other and (ii) there is no event which is extended over by every event in the set" (104). Because the relation of extension is inherent in events, an abstractive set forms a series of events whose converging end "diminishes without limit" (104). This is to say that the series of events which form the abstractive set has no smallest member.

Primes and Anti-primes

An abstractive set is called "prime" whenever all the members of the abstractive set retain the formative condition σ and the abstractive set "is covered by every other abstractive...[set] satisfying the same condition σ" (106). The reference to "covered" means that every event in one set is extended over[18] at least partially by some other event in the other set (see figure 2, p. 23)(104).

When an abstractive set is referred to as being "anti-prime" it fulfills the following two conditions: "(i) it satisfies the condition σ and (ii) it covers every other abstractive...[set] satisfying the same condition σ" (106).

---

[18]To say event A extends over event B is to say A includes, at least in part, B.

The reference to "covering" means that given two abstractive sets every member of one set extends over some member of the other set (104).[19] An abstractive set is considered an "absolute anti-prime" when each set with the formative condition σ covers every abstractive set which covers it. It is assumed that in each case the formative condition σ is represented in every event in the abstractive sets.

The reference to "primes" and "anti-primes" is another way of expressing the notion that, in the case of an σ-prime abstractive set, the convergence which occurs is the sharpest with respect to the condition σ. In the case of an σ-anti-prime abstractive set, the convergence which occurs with respect to the condition σ is the broadest that can occur. This means that the neighborhood[20] established around the abstractive element produced by an "σ-prime" abstractive set is very narrow, while the corresponding neighborhood around the abstractive element produced by an "σ-anti-prime" abstractive set is quite broad in comparison (CN 88).

---

[19]One distinctive difference between prime and anti-prime abstractive sets is that a prime abstractive set is covered by every other abstractive set satisfying some particular formative condition. An anti-prime abstractive set covers every other abstractive set satisfying some particular formative condition. The properties of being covered by and covering are being emphasized here.

[20]Neighborhood is being used here as a topological term referring to the comparative area surrounding an abstractive element located in the extensive continuum.

Abstractive Elements

Now that we have defined abstractive sets, primes, and anti-primes we are in a position to discuss "abstractive elements". An abstractive element is an example of the ideal limit mentioned above in our discussion of durations and moments. Abstractive elements can be identified "with the whole set of abstractive sets which are K-equal[21] (see figure 2, p. 23) to any one of themselves"[22] (Palter 53). They are defined in terms of "primeness" and "anti-primeness" and are divided into two types, finite and infinite.

"A `finite abstractive element deduced from the formative condition σ' is the set of events of σ-primes where σ is a formative condition regular for primes" (108). A prime event (which possesses a particular formative condition σ and is then termed an σ-prime) is said to be regular "when (i) there are σ-primes and (ii) the set of abstractive...[sets] K-equal to any one assigned σ-prime is identical with the complete set of σ-primes" (107). Regular anti-primes are defined in an analogous manner.

---

[21]For abstractive sets or classes K-equal means that each infinite abstractive set or class in a set of such classes covers and is covered by every other abstractive class in that set (105).
[22]Robert M. Palter. <u>Whitehead's Philosophy of Science</u>. University of Chicago Press: Chicago, 1960.

Figure 2[23]

Properties of Covering and K-equality

[*Note.* Abstractive classes and the relation of 'covering' can be illustrated by spatial diagrams, with the same caution as to their possibly misleading character.

Consider a series of squares, concentric and similarly situated. Let the lengths of the sides of the successive squares, stated in order of diminishing size, be

$$h_1, h_2, \ldots h_n, \ldots.$$

Then each square extends over all the subsequent squares of the set. Also let

$$\underset{n \to \infty}{L}\ h_n = 0;$$

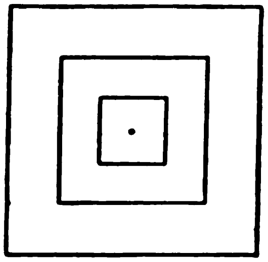


Fig. 5.

namely, let h_n tend to zero as n increases indefinitely. Then the set forms an abstractive class.

Again, consider a series of rectangles, concentric and similarly situated. Let the lengths of the sides of the successive rectangles, stated in order of diminishing size, be (a, h_1), (a, h_2),...(a, h_n),....

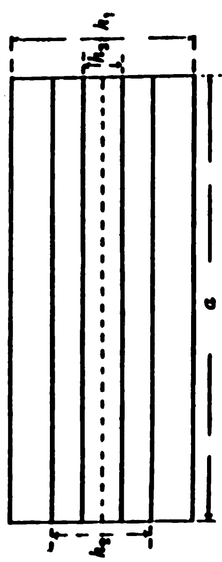


Fig. 6.

Thus one pair of opposite sides is of the same length throughout the whole series. Then each rectangle extends over all the subsequent rectangles. Let h_n tend to zero as n increases indefinitely. Then the set forms an abstractive class.

Evidently the set of squares converges to a point, and the set of rectangles to a straight line. Similarly, using three dimensions and volumes, we can thus diagrammatically find abstractive classes which converge to areas. If we suppose the centre of the set of squares to be the same as that of the set of rectangles, and place the squares so that their sides are parallel to the sides of the rectangles, then the set of rectangles covers the set of squares, but the set of squares does not cover the set of rectangles.

Again, consider a set of concentric circles with their common centre at the centre of the squares, and let each circle be inscribed in one of the squares, and let each square have one of the circles inscribed in it. Then the circles form an abstractive class converging to their common centre. The set of squares covers the set of circles and the set of circles covers the set of squares. Accordingly the two sets are *K*-equal.]

---

[23]Figure 2 is taken from Principles of Natural Knowledge, p. 105-106.

An "infinite abstractive element" is defined in the same manner as a "finite abstractive element" except σ-anti-primes are substituted for σ-primes. The generic term, "abstractive element," refers to that set of elements which contain all finite and infinite abstractive elements. This combined set "represents a set of equivalent routes of approximation guided by the condition that each route is to satisfy the condition σ" (109).

Whitehead also uses abstractive elements to define moments and durations. Those abstractive "elements of the greatest complexity...which can cover [they are anti-primes] elements of all types" will be moments (109). This designates a particular type of infinite set of abstractive elements (109). A moment includes all those abstractive elements in a duration which are formed in reference to some formative condition σ. A duration is any member of an absolute anti-prime, i.e., any member of an abstractive set which covers every abstractive set which covers it (111). In other words, these abstractive sets are K-equal.

A problem arises with this formulation of moments in terms of anti-primes because durations are not the only events which can satisfy the conditions for an anti-prime (Palter 56). Whitehead himself notes this problem in Note IV to the second edition of <u>Principles of Natural Knowledge</u>: "The attempt...to define a duration merely by means of its unlimitedness[24] is a failure. In a note to the Concept of Nature, I point out that there is an analogous unlimitedness through time, corresponding to the spatial unlimitedness of a duration" (204).

Because a duration is not the only event which can be considered an anti-prime, the application of the method of extensive abstraction to an anti-prime would not necessarily result in approximate convergence to a moment. This point proves to be a problem for Whitehead. Later, in Concept of Nature, he considers duration to be a primitive concept (CN 59). In this way he tries to avoid the problem of the derivation of moments from anti-prime abstractive sets.

Relations

When we consider relations such as "inhering," "adjoining," "junction," and "intersection," we are referring to different ways in which events or abstractive elements interact with one another (see figure 3, p. 27). An abstractive element is said to inhere in an event when it is extended over by some part of that event (109). An abstractive element is called a member of an event when it is a part of the set of abstractive elements which are particular to that event (109). Abstractive elements

[24]The term "unlimitedness" here refers to the property of an anti-prime abstractive set to cover all other abstractive sets which satisfy the same formative condition. This is another reference to an anti-prime establishing the broadest neighborhood around an abstractive element.

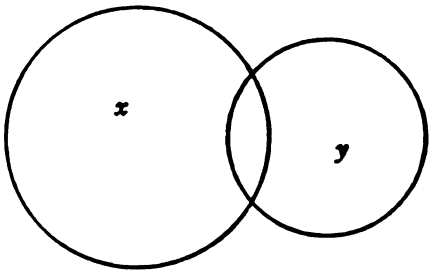
intersect if they are members of abstractive sets which cover each other (109). The relation adjoining is defined by Whitehead in the following manner: "Two events which are joined have that relation to each other necessary for the existence of one event which extends over them and over no extraneous events. Two events which are both separate and have a junction are said to be `adjoined'" (103).

Whitehead depicts the continuity of nature by considering the interaction of a variety of relations. These relations are expressed in technical terms which, for our purposes, need not be defined. It should be understood, however, that these relations are, among other things, used to explain the temporal and spatial boundaries of events and so are important to Whitehead's philosophy of nature. They are used in the development of the four-dimensional geometry of events and analogous relations are employed to develop three-dimensional geometry (103).

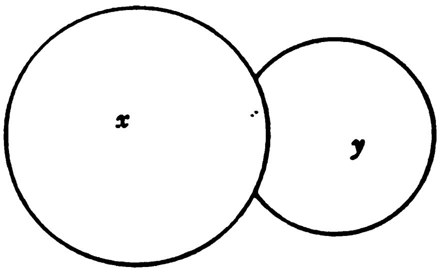
Let us take up anew the connection between abstractive elements, anti-primes and the temporal system abstracted from the events of the extensive continuum. As stated earlier Whitehead attempts to define moments in terms of abstractive elements and absolute anti-primes (110). But not every event is an absolute anti-prime (111). Those which are can be termed durations and only durations can

Figure 3[25]

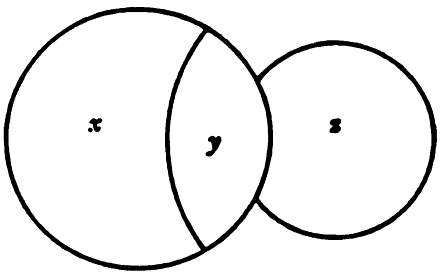
Properties: Junction, Adjunction, and Injunction



(i) Junction of intersecting events



(ii) Junction of separated events (or adjunction)



(iii) Injunction of events *x* and *y* (*z* is separated from *x* and adjoined to *y*).

FIG. 6.—Junction and injunction of events

[25]Figure 3 is taken from Palter, Whitehead's Philosophy of Science, p. 48.

extend over other durations. When we consider two time-systems, each consisting of parallel durations (and all the moments associated with a particular duration), it is necessarily true that the durations of these two time-systems will intersect. At this point we are able to introduce the spatial system of Whitehead's three-dimensional geometry.

CHAPTER 4
THE DEVELOPMENT OF SPATIALITY

Instantaneous Space

Whitehead begins the development of his philosophy of nature with the examination of durations and their corresponding time-systems. With a multiplicity of time-systems at his disposal, he is now able to establish a geometry of three-dimensional space as part of the four-dimensional world.[26] He does so by considering the various intersections of moments defined by series of durations. When two nonparallel moments intersect, a level is created (116). In the Whiteheadian system a level is analogous to the plane in Euclidean geometry. This new terminology is necessary because of Whitehead's need to contrast his philosophical system with that of the old classical system, where time and space were considered to be separate entities.

The result of the intersection of three nonparallel moments is termed a "rect" (116). It is considered to be analogous to the Euclidean straight line created from the intersection of two planes. The intersection of a rect and a level results in a "punct" (117), which is analogous to the Euclidean point. A punct can also be created by the

[26]The four-dimensional world refers to the conjoining of three-dimensional space with the dimension of temporality.

intersection of four distinct moments where "there is a common intersection which is neither a rect nor a level" (117). "A rect either intersects a moment in one punct, or is parallel to it, or contained in it" (118).

The characteristic of parallelism is essential in Whitehead's derivation of space, as it was in his derivation of time. We are forewarned that the characteristic of parallelism in the derivation of space is not to be separated from those properties to which it is applied (the properties of levels, rects, and puncts) (119). Rects, levels, and puncts are not natural entities but are logical notions and as such cannot constitute routes of approximation posited in sense-awareness[27] (Kneebone 349). They are, however, indirectly related to events (through their relationship with moments) and so provide tools for use in scientific study.[28]

Because a rect intersects a moment in only one punct, when we consider a whole time-system, it is clear that a rect will intersect each moment in that time-system only once per moment. The same can be said of levels. Since a level intersects a moment in only one rect, when we consider a whole time-system, a particular level will intersect each

---

[27]G. T. Kneebone. Mathematical Logic and the Foundations of Mathematics. D. Van Nostrand Co. Ltd.: London, 1963.
[28]Levels, rects, and puncts are similar to the ideal Euclidean point, straight line, and plane, in that they do not actually exist in nature. Nevertheless, they all remain valuable tools when applied to the task of understanding the physical aspects of nature.

moment of that time-system, creating a rect, once only per moment of the time-system. The puncts lying on a rect are arranged in a particular order and that order is determined by the order in which the rect intersects the moments of a time-system of which it is a part (119).

Levels, rects and puncts are used to construct the instantaneous space associated with moments. Instantaneous space, like the moment itself, represents an ideal to which we can only realize a close approximation. When we speak of the domain of instantaneous space with its associated levels, rects and puncts, we are also speaking of the space to which the classical mechanical laws of Newton apply. Whitehead insists that his system in no way violates these Newtonian laws but goes beyond them to show a connection of time and space which the Newtonian system cannot.

Through the use of levels, rects, and puncts Whitehead determines a complete three-dimensional geometry of instantaneous space (120). In a consideration of the various theories of relativity Whitehead considers the electromagnetic theory to be the more general (115).[29] This electromagnetic theory of relativity provides instantaneous

---

[29]Whitehead's use of the term "electromagnetic theory of relativity" is intended to imply opposition to the pre-Einsteinian conceptions of relativity. Whitehead asserts that the electromagnetic theory of relativity is that which exhibits the physical electromagnetic field which is the topic of physican science. He further asserts that it establishes that some pairs of durations are extended over by other durations but that some pairs are not (Palter 55).

space with "definitions of flatness, of straightness, of punctual position,...[of time order and spatial order], of perpendicularity, and of congruence, as well as of parallelism" (115). These abstract concepts all arise from "the nature of things as expressed by the fundamental relations of extension and cogredience" (115).

Although events themselves do not change (61), it is obvious that we observe changes in the physical state of nature. Such changes include change in position, in velocity, in motion, in mass, in density, in shape, in direction and/or rotation, etc. Whitehead needs to explain these changes in terms of events and relations among events. We must refer to four-dimensional space-time for a discussion of motion and change in velocity. Whitehead begins this process with the introduction of abstractive elements and event-particles.[30]

Event-particles

The intersection of time-systems introduces relative motion into Whitehead's system. Thus, an event-particle which is perceived as stationary in one time-system is perceived by some event in another time-system to be moving in a uniform motion, following some definite route. The converse of this statement is also true for Whitehead.

---

[30]Event-particles are simple events illuminated by the use of the method of extensive abstraction.

If we are to consider motion along a route of approximation within the extensive continuum, it is necessary to introduce the notion of event-particles. An "event-particle" is defined by Whitehead as "the route of approximation to an atomic event,[31] which is an ideal satisfied by no actual event" (121). Thus an event-particle is an event in its simplest form.

When Whitehead speaks of an event-particle following a route of approximation, he does not mean that the particle actually moves, intersecting durations, and following a given path of motion. Instead he reminds us that in the intersection of time-systems every moment intersects each duration of another time-system only once. By means of this intersection puncts are established. An apparent route is actually a series of puncts in sequential order, each punct associated with a particular moment, along some possible path. An event-particle is not a material-like entity traveling from punct to punct. Rather, there exists an event-particle which covers a punct at each moment along some route of approximation.

---

[31]An atomic event is an event in its simplest form. It is no longer divisible into other events. At this stage in Whitehead's development of his philosophy of nature he thought that all events were divisible. In this respect, an atomic event could only be an idealization not found in nature. In his later works Whitehead reformulated this notion of event and took the position that all events are atomic (non-divisible). The reference in this quote to an event-particle as an approximation of an atomic event refers to Whitehead's notion of event-particles as the simplest of all actual events involved in spatial representations.

Solids and Volumes

The move from event-particles to solids with density and volume is made by the employment of a particular type of "kinematic" route. Whitehead defines this route in the following manner:

> A "kinematic route" is a route (i) whose end-points are sequent and (ii) such that each moment, which in any time-system lies between the two moments covering the end-points, covers one and only one event-particle on the route, and (iii) all the event-particles of the route are so covered (125).

Event-particles which lie on a kinematic route represent a possible path for what Whitehead terms a "material particle".[32] He defines the term "solid" with a reference to "solid primes" (126). A solid prime is a prime whose formative condition is being a simple abstractive set[33] which "covers all event-particles shared in common by both boundaries of adjoining[34] events" (126). A solid, then, is the set of event-particles common to the boundaries

---

[32]"Although Whitehead is dealing with event-particles, he...stresses that each event-particle is related to other event-particles in the four-dimensional (physical) continuum by means of...elements of impetus or 'atomic physical fields'. They are the set of physical qualities referring to the gravitational and electro-magnetic fields in the neighborhood of E [where E refers to some definite event-particle in the four-dimensional manifold]....On his view, then, **material particles** are pervaded tracks of event-particles in the space-time continuum" (Mays 242). Bold type is added by this writer.

[33]A simple abstractive set is one in which there is no one event-particle at which all members of the converging end of the set have contact (123).

[34]For the definition of the term adjoining, refer to page 26 of this paper.

of two adjoining events. Such a solid is not equivalent to a physical entity. It is a locus of event-particles, i.e. it illustrates some quality of position in the space-time manifold. A solid which is co-momental[35] is termed a "volume" (126). A volume, then, is "conceived as a locus of event-particles" (127).

Whitehead indicates that a "concrete event...is defined by...the event-particles inhering in it, and such a set of event-particles defines only one event" (127). Such an event can be said to be "uniquely defined by the set of event-particles which form its boundary" (127). By a "concrete event," Whitehead means the locus of event-particles which defines that event we perceive as an ordinary physical entity, e.g., a chair.

To say that this physical entity is defined by the event-particles inhering in it is to say that the event which constitutes such a given set of event-particles is unique and that it represents what is commonly thought of as the volume of the entity (as a locus of event-particles); it is a cross section of that part of nature[36] which represents the locus of event-particles that designates the physical entity (the chair). To say that the physical entity, e.g., the chair, is an event "uniquely defined by the set of

---

[35]Co-momental in this definition means that the locus of event-particles defining the solid lies in a single moment.
[36]The moment which is associated with instantaneous space provides position for the locus of event-particles.

event-particles which form its boundary" is to recognize the particularity of that physical entity. That particular locus of event-particles inheres in the event which is that particular chair at that particular time.

Whitehead tells us that "the instantaneous volumes in instantaneous space which are the ideals of our sense-perception are volumes as abstractive elements" (CN 102). This is not intended to provide an explanation of how we perceive material objects but, rather, to explain how material objects can be seen as having designated position in the extensive continuum using the technical terms: abstractive sets, event-particles, primes, anti-primes, etc.

The Components of the Three Types of Space

In Principles of Natural Knowledge Whitehead discusses three different types of space. Only one of these three terms refers to that space which we perceive, i.e., instantaneous space. The other two types of space are required for the physical sciences, both the traditional Newtonian science and the newer view of relativity. It is, then, necessary for Whitehead to account for the types of space required in these sciences in his theory of spatiality. The three types of space discussed by Whitehead are 1)instantaneous space, 2)four-dimensional space-time, and 3)three-dimensional time-less space. The following chart (Palter 55) is helpful in distinguishing the elements in the geometries of these three types of space. Under the name of each type of space are listed those basic elements from which the geometries of these spaces can be constructed.

Table 1

Types of Space

Instantaneous space	Space-time	Time-less space
Punct	Event-particle	Point
Rect	Point-track, Null-track, Set of co-rect event-particles	Straight line
Level	Matrix, Set of co-level event-particles	Plane

Whitehead is interested in space abstracted from those events associated with our ordinary perceptions of the natural world. Through the use of the method of extensive abstraction he develops the notions of puncts, rects, and levels needed to express the geometry of three-dimensional space. Although these notions are ideal constructs, they are developed from durations/moments and so they retain, through this indirect relationship, the essential relations found in nature, i.e., those relations of perceivable events.[37] Because of his desire to show that the basic relations found in nature are preserved in these abstract entities, Whitehead describes events in terms of abstractive sets, abstractive elements, anti-primes, and primes and then defines puncts, rects, and levels using these technical terms.

Whitehead maintains that all three types of space should be connected in some manner to the perceivable events of the natural world. He feels this can be accomplished by defining the basic geometrical elements of four-dimensional space-time and three-dimensional time-less space in the same manner he defined the elements of instantaneous space. He employs the method of extensive abstraction and the relations of cogredience and parallelism to define and derive the basic elements of these other two types of space.

In the chart given above, the three groups of basic elements correspond with one, two, and three dimensional phenomena. Reading horizontally across the chart puncts, event-particles, and points are considered to be one dimensional. Rects, point-tracks, null-tracks, sets of co-rect event-particles and straight lines are two dimensional,

---

[37]The use of the method of extensive abstraction applied to abstractive sets of events represents a basic tool in Whitehead's natural philosophy. It is essential to Whitehead because he believes that the end products of this method retain just those relations which are found throughout the entire series of events in the abstractive sets.

and levels, matrices, sets of co-level event-particles, and planes are three dimensional. In basic Euclidean geometry, points are used to define straight lines and straight lines are used to define planes. A similar move, proceeding in the opposite direction, is used to define the components of the three types of space (for example, in three-dimensional instantaneous space, first levels are derived, then rects from levels, and finally puncts from both rects and levels).

<u>Four-dimensional Space-time</u>

As stated above, Whitehead needs to express the development of the elements of the three types of space in terms of abstractive sets, abstractive elements, and primes and anti-primes, in conjunction with various relations. By following this procedure he maintains that each of these types of space is related in some manner to the perceivable natural world and, therefore, each type of space retains the basic relations found in nature.

Event-particles have been defined in relation to particular types of abstractive sets (via routes). Point-tracks and null-tracks (necessary for the construction of four-dimensional space-time) have yet to be introduced. Point-tracks are defined by Whitehead as "the complete locus of event-particles thus defined by the indefinite prolongation of a station throughout its associated time-system..." (130). The definition is of little value without an understanding of the term "station". Any detailed description of the terms involved in the definition must show their connection to abstractive sets, abstractive elements, primes, and anti-primes. The derivation of these terms is somewhat complicated but it is necessary to consider them if we are to understand how the method of extensive abstraction relates to the development of the different types of space being discussed.

Let us briefly review some definitions already established. Event-particles have been defined in terms of absolute anti-primes which, in turn, are deduced from abstractive sets. Abstractive sets and absolute-anti-primes have also been defined in the preceding sections.[38] We have referred to puncts as the intersection of four nonparallel moments. Event-particles which cover puncts are said to have been given absolute position in the instantaneous space of the moment in which they lie (121). Here the term "absolute" is not used to indicate a position in absolute space but rather means "absolute position" relative only to that time-system in which the event-particle and the punct lie.

Event-particles are considered dense within any particular moment of a time-system; there is no punct in an instantaneous moment which is not covered by an event-particle and every position possible within a moment is

---

[38]For these definitions see, respectively, pages 20 and 21 of this paper.

designated by some punct. A rect, then, since it can be generated by the intersection of two levels in different time-systems, can be said to cover a series of puncts or event-particles. A rect can be considered a series of event-particles, that is, a rect can be seen as being composed of that series of event-particles which it intersects.

Routes and Stations

The importance of event-particles lies in the fact that they are able to connect one instantaneous space with another and so they allow the identification of one particular "point" at different times (Kneebone 350). This identification is achieved by means of "stations" which correspond to various event-particles. A station is itself a type of event-particle. As an intermediate step to an understanding of stations, we must again consider the meaning of the term "route".

If an event-particle is considered as an "abstractive element of atomic simplicity" (123), a route can be considered as the next advance "towards increasing complexity" (123). A route is defined by Whitehead as "the abstractive element deduced from a linear prime" (124). It is a class of events (a simple abstractive class) which consists of some possible finite set of event-particles, including end points (i.e., it covers two event-particles, p_1 and p_2, which are the extreme event-particles in the set), and "is such that no selection of the event-particles which it covers can be completely covered by another simple abstractive class, provided that the selection comprise all the event-particles covered by [the simple abstractive class]..." (124). The first part of the statement above indicates the "end-points" of a route are included as part of the route and the second part of the statement indicates that the abstractive class has "linear type [of] continuity"[39] (124).

A "linear prime" can now be defined as "an abstractive set which is prime in respect to the formative condition of (i) being covered by an assigned linear abstractive set which covers the two assigned end-points and (ii) being itself a linear abstractive set covering the same assigned end-points" (124). It must also be considered regular for primes, i.e., K-equal to itself.

The complex definitions given above indicate that when any two event-particles are designated as end-points (p_1 and p_2) of some series of event-particles, there can be found an indefinite number of linear segments connecting these end points. These linear segments constitute a route. A route

---

[39]Whitehead refers to linear type continuity as that type of continuity expressed by a route (as a linear segment). He also indicates that the continuity of events is derived from the continuity of routes, i.e., linear type continuity (124). The fundamental relation involved with this type of continuity is that of "lying-between" (holding for triads of points on the linear segment). Also associated with this relation is the property of continuous serial order for the particles on any route (125).

contains, i.e., covers, its own end-points and infinitely many other event-particles. This is due to the "linear type continuity" displayed by routes.

In order to connect the notion of routes to that of stations, another ingredient is needed. This ingredient is the relation of cogredience. Earlier in this paper I stated that cogredience indicates an event's being "here-present". This is to say that cogredience indicates position in the extensive continuum and so is vital to the derivation of space (since space is necessarily concerned with the property of position). The term "cogredience" is now defined as being "here throughout a duration" or "there throughout a duration" (128).

A stationary prime has been defined by those conditions required by the definition of all primes[40] and it is also cogredient to some particular duration. Therefore, "a station within a duration is the absolute abstractive element deduced from a stationary prime" (128). It is an "absolute abstractive element" because, with the inclusion of the relation of cogredience, the station is always "here", this route in a duration.

Whitehead tells us that "a station is a route [a linear segment including its own end-points]; and also every station in a duration intersects every moment of that

---

[40]The definition of a prime is given on page 20 of this paper.

duration...in one and only one event-particle and intersects no other moments of that time-system" (128). Any particular station can be indefinitely prolonged within the duration in which it resides.

Point-tracks and Null-tracks

A point-track, then, is defined by Whitehead as the complete locus of event-particles found along a station in some given time-system (129-130). Thus, a point-track can be said to follow an indefinitely prolonged station throughout the duration in which the station resides. The theory of parallelism can be observed in the notion of point-tracks. All point-tracks which are points in the space of some time-system are said to be parallel to one another[41] (131). Since point-tracks involve the relation of cogredience,[42] they can be said to be there-now in association with particular points of space within a time-system. Therefore, Whitehead concludes that "a complete family of parallel point-tracks is merely a complete family of points in the space of some time-system" (131).

---

[41] A "point-track has a unique association with the time-system in which the routes lying on it are stations. A point-track is called a "point" in the "space of its associated time-system." This space of a time-system is called "time-less" because its points have no special relation to any one moment of its associated time-system" (130).

[42] Point-tracks establish their connection with cogredience through the association of a point-track to some station.

The point-track in four-dimensional space-time is analogous to the rect in instantaneous space and to the straight line in three-dimensional time-less space. To complete the set of straight lines required by the geometry of four-dimensional space-time, we must include a set of event-particles which occupy what is termed "the set of null-tracks." The order of these null-tracks is derived from routes (131). Although Whitehead only briefly discusses null-tracks in Principles of Natural Knowledge, Robert Palter, in Whitehead's Philosophy of Science, discusses them in greater detail.

He states:

> ...a matrix [to be discussed below] is divided, relative to an event-particle in it, into four mutually exclusive regions...which may equally well be called rects or point-tracks - these loci (and all others parallel to either one of them) Whitehead terms "null-tracks". In any matrix there are two families of parallel null-tracks, exactly one member of each family passing through each event-particle in the matrix. Thus, through any given event-particle there is an infinity of null-tracks which form two three-dimensional "conical" surfaces whose mutual vertex is the given event-particle. The order of event-particles on a null-track is derived from its intersection with systems of non-co-momental parallel rects or of parallel point-tracks, or from the order of the event-particles on routes lying on the null-track (Palter 74-75).

Matrices

At this point we are in a position to introduce the notion of "matrix" in four-dimensional space-time. A matrix can be viewed as analogous to a level in three-dimensional instantaneous space and a plane in three-dimensional time-less space. The process of the generation of a matrix is similar to that of the generation of a level. In order to better understand the generation of a matrix, let us begin with a discussion about the generation of levels.

A level can be generated in several ways: 1) by considering some rect, r, and some event-particle, P, both of which lie in the same moment and by constructing a locus of event-particles lying on rects, where all the rects intersect r and pass through P. Included in this locus are the event-particles which lie on r and P itself. The resulting locus of event-particles represents a level (133). 2) Given the same rect, r, and event-particle, P, the same level can be generated by forming a locus of event-particles which lie on rects intersecting r, with the qualification that the rects intersecting r are parallel to some one rect which passes through P and intersects r (133).

A matrix can be generated in four-dimensional space-time in a similar manner. Matrices are defined in terms of rects, event-particles, and point-tracks. It should be noted that these terms have themselves been defined using the notions of abstractive sets, abstractive elements, primes, and anti-primes. Through this chain of development, levels and matrices are indirectly established by the method of extensive abstraction.

In the description of the generation of a level we employed rects and an event-particle which were co-momental, i.e., they lie in the same moment. Let us now consider a rect r and an event-particle P which are non-co-momental. We can construct a matrix by taking the locus of event-particles lying on rects which intersect rect, r, and pass through event-particle P. These include the event-particles lying on the rect parallel to r and passing through P.

A second method of construction of a matrix is analogous to the second method given above for the generation of a level. Here the matrix is obtained by "an event-particle P and a point-track p not passing through P (e.g., the event-particle p_2' and the point-track p_1...)" (Palter 73). See the diagram below (Palter 72).

The connection between levels, matrices, and the straight lines of time-less space is made by stating that every matrix contains sets of parallel point-tracks and any one of these sets can be considered "a locus of points[43] in the space[44] of some time-system. Such a locus of points is called a `straight line' in the space of the time-system" (136). Matrices can be associated with many different time-systems but there is only one straight line in each matrix associated with a particular time-system (136-137).

---

[43]See footnote 41, page 44 of this text, for a brief discussion of the derivation of points in time-less space.
[44]This is still four-dimensional space-time.

Figure 4

Construction of a matrix

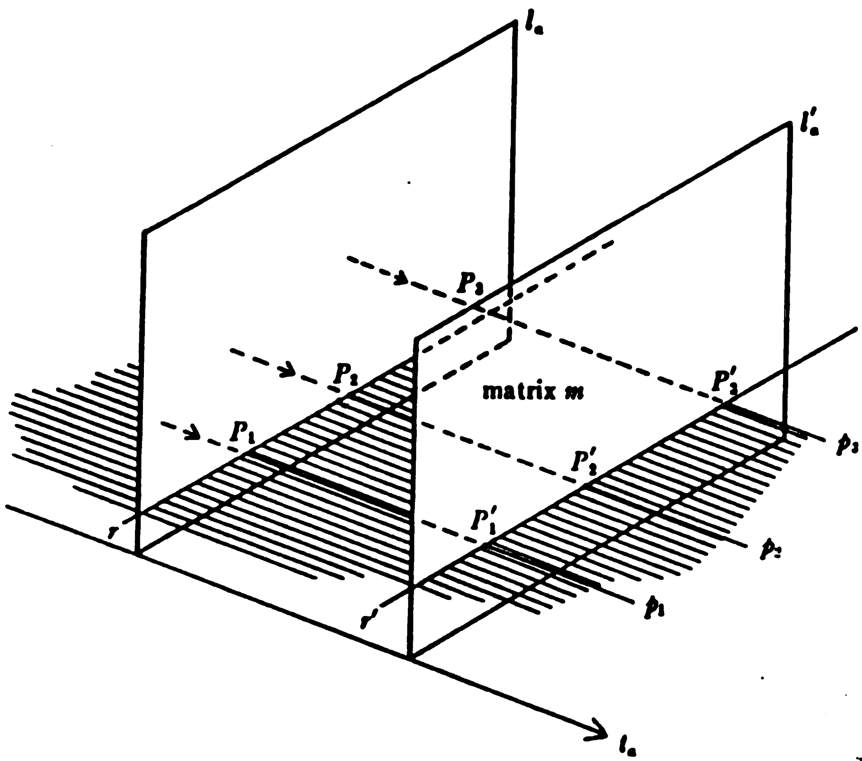


FIG. 13.—Point-tracks and points; matrices and straight lines. Each point-track p_n defines a point in time-less space of t_a; the set of parallel point-tracks p_n in the matrix m define a straight line in time-less space of t_a.

The principle of parallelism can and is applied to matrices. There exist families of parallel matrices and in any one time-system the straight lines associated with a family of parallel matrices are parallel to one another. Whitehead asserts that levels and matrices combine to make up what is equivalent to the complete set of planes in Euclidean geometry, and that they retain all the properties of this set (133). He does not, however, go into detail as to how one might prove this statement.

Three-dimensional Time-less Space

When we consider the third type of space discussed by Whitehead in Principles of Natural Knowledge, i.e., three-dimensional time-less space, we find a direct correlation between it and the instantaneous space previously discussed:

> There is an exact correlation between the time-less space of a time-system and any momentary space[45] of the same time-system. For any point of the momentary space is an event-particle which occupies one and only one point in time-less space; and any straight line of the momentary space is a rect which lies in one associated matrix including one straight line of the time-less space or (in other words) each straight line of the momentary space occupies a straight line of the time-less space (138).

Whitehead has attempted to reconstruct the properties of space and time by purely logical means, employing only these primitive notions provided by the immediate data of

[45]Momentary space is a reference to instantaneous space.

perception.[46] Time and space, thus constructed, contain no metrical properties. They only possess the relations of order and extension. The correlation between the different types of space which Whitehead discusses and his attempt to deduce these by the use of the method of extensive abstraction through the (not always direct) employment of definitions in terms of abstractive sets, abstractive elements, primes, and anti-primes emphasizes one of the main aims of this paper, i.e., to give a careful examination of the method of extensive abstraction and the implications of its use which arise in Whitehead's philosophy.

---

[46]These primitive notions include events and the relation of extension along with the property of cogredience.

CHAPTER 5
THE CONSTANTS OF EXTERNALITY

In Principles of Natural Knowledge Whitehead lists what he terms "the constants of externality." He uses these to define events in terms of their inherent properties and to indicate the relationship of these properties to the passage of nature (71). Externality and extension are seen as essential properties of events. It is an event's relationship to the other events which allows us to experience what we commonly refer to as nature. Through an examination of the properties illustrated in the constants of externality Whitehead hopes to highlight those basic concepts of nature which are presupposed in the physical sciences. He considers only those relations which are fundamental to the formulation of the concepts of time, space, and physical material.

Whitehead discusses six constants of externality. I will consider only the first three in this paper. The first three constants of externality have to do with specifications of qualities and discriminations among various entities. The first constant of externality states that the natural continuum apprehended through perception is really a "potentially definite complex of entities for knowledge" (74). By this Whitehead means that we are able to perceive and understand the passage of nature because that which we can know about nature is that which we can directly perceive. There are, however, other aspects of nature (beyond our direct apprehension) which can be **indirectly known** through an awareness of various relations existing between events.

The second constant of externality states that the relation of extension holds among events. This relation must obtain if events are to be considered as belonging to the order of nature.[47] The third constant of externality states that "an event as apprehended is related to a complete whole of nature which extends over it and is the duration associated with the percipient event of that duration" (77). This refers to our common perception of nature as having extension and that what we perceive at any given time is that which exists within the duration we (as a percipient event) presently occupy. Our perception represents that standpoint from which this particular duration is determined. It is here where we find the concept of a duration as a cross-section of nature.

Whitehead attempts to conduct his study based on observable nature. This aim would seem to give rise to a number of problems. An example of such problems is seen if

---

[47]It might be noted that there could be no knowledge of nature without an ordering of the events apprehended (temporal ordering as well as spatial ordering is being implied here). Such ordering is required for inferential judgments and without it we could not make the necessary connections between similar events required for the process of recognition.

we take another look at the description of events considered from the perspective of perception.

If the concept of a duration of an event relies upon the perception of that duration by some other event (the percipient event), then such a perception and the accompanying abstraction of time and space would seem to imply the necessity of some type of consciousness. Consciousness in a percipient event, it could be argued, would also imply that the temporal and spatial structures of nature have a subjective origin.[48]

I believe Whitehead would respond to this argument with an empathic denial that this would be the case. He would maintain that every event is capable of "feeling" all other events in its presence.[49] An event "feels" those relations which hold between itself and other events and also the relations which hold among other events not directly connected to itself. Such feeling is possible in even the least complex of events, the event-particle.

However, he would remind us that the relations which occur among events do not result from a process of abstraction but are a part of the structure of nature.

---

[48]They would be considered subjective in reference to the consciousness inherent in the percipient event.

[49]This view is expressed by Whitehead in those works beginning with his Process and Reality. Corrected Edition. ed. David Ray Griffin and Donald W. Sherburne. The Free Press: New York, 1978. Because of this, these views of the "feelings" of events is not applicable in this paper, although the foundations of these views are seen in Principles of Natural Knowledge and other early works.

Events are actual entities and they exist in, and only in, relation to one another. Temporal or spatial abstractions are established because they are inherent in this structure of nature. The recognition of these abstractions, apprehended by a consciousness, in no way influences the natural structure of events. This gives a brief explanation of what Whitehead means when he says that nature is closed to mind, i.e., the actual existence of nature does not depend on perception by some "knowing" subject. For this reason, it is not a problem that perception, implying the presence of consciousness, would interject a subjective element into the structure of nature. Perception is accomplished through those same relations which are inherent in all events and so in nature itself.

CHAPTER 6
THE THEORY OF PERCEPTION

Immediate Data of Perception

Whitehead requires that his project start with the immediate data of perception (5). In this statement "immediate data" refers to our ordinary perception of the natural world, e.g., our perception of physical objects, our sense of spatial distance and temporal extension through our perception of the passage of time. Since he considers the perception of this immediate data as a starting point for his investigation, Whitehead also needs to explain the general characteristics of perception in terms of events and relations of events. Only through an understanding the characteristics of perception can we claim any knowledge of the natural world.

From these immediately perceived data Whitehead develops by the method of extensive abstraction those basic notions we have been discussing, i.e., abstractive sets, abstractive elements, event-particles, levels, rects, puncts, etc. It should be noted that Whitehead does not claim that entities established by means of the method of extensive abstraction are actually perceivable. Whitehead requires only that we begin our study with perceivable data. However, these entities do retain the same basic relations as the data from which they were abstracted and as such, Whitehead claims that they are indirectly available for perception. This point will be explained more fully below.

Having introduced these points, let us begin to examine Whitehead's theory of perception. An understanding of what Whitehead means by the term perception is not obvious. It has been mentioned that Whitehead has the habit of taking common terms and giving them uncommon meanings. This is true with the term "perception." Its meaning has changed throughout the development of his early works.[80] Paul F. Schmidt, in Perception and Cosmology in Whitehead's Philosophy,[81] gives an exposition of this development. The following account consists basically of Schmidt's comments on the development of the meanings assigned to such terms as "perception," "awareness," "cognition," "consciousness," etc.

The Principle of Relativity

Beginning with the more complete version of Whitehead's theory of perception, I will move backwards through Whitehead's works until I reach its sketchy formulation in Principles of Natural Knowledge. In The Principle of

---

[80] Whitehead first begins his explication of the concept of perception in The Organization of Thought and it is developed systematically through Principles of Natural Knowledge, Concept of Nature, and is given a more complete formulation in The Principle of Relativity.

[81] Paul F. Schmidt. Perception and Cosmology in Whitehead's Philosophy. Rutgers University Press: New Brunswick, 1967.

Relativity[52] we find that Whitehead refers to perception with respect to nature, i.e., in terms of fact and factors. He tells us that "`perception' will be the name given to the consciousness of a factor when to full awareness cognition[53] of it as an entity is also superadded" (PREL 19). To understand this definition of perception, it is necessary to determine the meanings of the terms involved. Fact is defined as the totality of all that is (PREL 59). It is seen as the whole of infinite and inexhaustible nature. Factors are those elements (parts) of fact involved in the relationships occurring within fact.[54] Although factors are always found within fact, there is no set of factors which can completely exhaust fact. Awareness is then defined as a factor in relation to other factors. This entails that everything has awareness of everything else. It might be argued that this is an absurd statement but this is exactly what Whitehead means to imply. At the level of basic awareness each event (factor) is either directly or indirectly influenced by the relations of all other events.

---

[52]Alfred North Whitehead. The Principle of Relativity. University Press: Cambridge, 1922. Here after in this paper this work will be referred to as PREL. Schmidt's discussion of Whitehead's theory of perception as found in The Principle of Relativity is found on pages 39-41 in Perception and Cosmology in Whitehead's Philosophy.

[53]At this point in the development of Whitehead's theory of perception, cognition (thought) is considered to be a part of perception.

[54]The terminology is somewhat confusing unless it is remembered that fact is considered to be the whole of nature and factors represent some part or section of this whole.

In

pr

W

ur

av

pe

av

se

a

of

re

qu

fr

fa

o

p

f

a

f

r

c

In this sense they have awareness. Consciousness is a prerequisite for awareness of factors (events). For Whitehead "consciousness" is considered a primary or undefined term.

In the case of a percipient event much more than awareness is needed to say that the percipient event perceives. "Cognition" or "thought" is required, along with awareness, to focus upon a factor in such a way that it is seen as separate from other factors. It is the isolation of a particular factor (event) from all other factors by means of the awareness of this particular factor's particular relations. In this respect, cognition adds a limiting quality to awareness; it allows for the apprehension of a factor, either by disregarding its relations to other factors or by the apprehending of it in abstraction from its own relations. When cognition is added to awareness, perception is said to occur. "`Full awareness' occurs when factors are clearly apprehended and their mutual relations are jointly apparent" (Schmidt 40, PREL 64). "Perception" is defined as the combination of basic awareness and the recognition of a factor as some particular entity by means of cognition.

Concept of Nature

In Concept of Nature,[55] Whitehead replaces "perception" with "sense-perception," and "awareness" with "sense-awareness". Sense-awareness is considered to be awareness,[56] from which the consciousness of logical, aesthetic, and moral awareness of factors within fact has been eliminated. Sense-perception is sense-awareness to which cognition has been added. For this reason, sense-awareness is considered more basic than sense-perception, i.e., the former is necessary for the occurrence of the latter.

Sense-perception is the apprehension of all events which make up nature in respect to some duration. In an initial statement concerning the purpose of his work, Whitehead states that he is concerned with the objects of perceptual knowledge and not with the knower of this knowledge. He limits his study to this aspect of nature (vii). Nature, it has been stated, is "closed to mind,"[57] i.e., it does not require perception for its existence. Whitehead asserts that the natural sciences are interested in that which is apprehended by sense-awareness and not with cognition or sense-awareness itself. This exclusion of

---

[55]Schmidt's discussion of Whitehead's theory of perception as it appears in Concept of Nature is found on page 41 of Schmidt's work.
[56]Awareness here is defined as consciousness of some factor in contrast to its relationship to other factors in fact.
[57]For a discussion of the meaning implied by "closed to mind," see page 54 of this paper.

thought from nature does not mean the the two are separable. Rather, "[i]t means that in sense-perception nature is disclosed as a complex of entities whose mutual relations are expressible in thought without reference to mind, that is, without reference either to sense-awareness or thought" (CN 4-5). Nature, in this sense, is seen as one part of a larger system of complex relations.

Principles of Natural Knowledge

In Principles of Natural Knowledge[58] Whitehead's theory of perception is less refined than in the former works, although the same basic elements are evident. "Perception" is defined in terms of the diversification of nature into natural entities.[59] The presence of thought in perception is implied although not explicitly expressed. That Whitehead does at this time recognize thought as a part of perception is seen in his assertion that the vividness of perception depends on stimulation from both awareness and thought ("[Perception] is essentially apprehension of the becomingness of nature....But equally perception requires recognition" (98)). Those entities of nature which are made known through perception are "events," "percipient objects," "sense-objects," "perceptual objects," and "scientific

---

[58]Schmidt's discussion of Whitehead's theory of perception in Principles of Natural Knowledge is found on pages 41-42 in Schmidt's work.

[59]Whitehead discusses perception in terms of those entities involved in the process of perception rather than discussing the process itself.

objects" (60). A discussion of the individual properties of these entities is not needed to fulfill the purpose of this paper, although the nature of events and percipient objects are discussed in some detail in this paper.

In summery it is noted that a basic understanding of perception is required in any study of Whitehead's natural philosophy because perception (at least in the form of basic awareness of relations between events) is necessary for any explanation of events. Events can be defined in terms of their particular relations. In this sense, any interaction in nature depends upon an awareness of the relations of events by events. Without such awareness, Whitehead's notion of events would be reduced to something resembling the monads of Leibniz[60] (Leclerc 120-121).

This is a mere sketch of Whitehead's theory of perception. I have discussed how the concepts of awareness, cognition, perception, etc., developed in Whitehead's early works. It should be remembered that I began with Whitehead's more complete formulation of these concepts and then preceded backwards in the process of development, ending with the more rudimentary explanations presented in Principles of Natural Knowledge.

---

[60] Ivor Leclerc. "Whitehead and the Problem of Extension." in Alfred North Whitehead: Essays on His Philosophy. ed. George L. Kline. Englewood Cliffs: Prentice- Hall, Inc., 1963.

Re-examination of a Problem

We can now attempt to relate Whitehead's theory of perception as presented in Principles of Natural Knowledge to the problem raised earlier in this paper. This problem indicated that since perception seemed to require some type of consciousness, the necessity of such consciousness could interject a subjective element into Whitehead's interpretation of the basic relations of nature.[61] We can now conclude that not only is consciousness a necessary part of perception but so are awareness and cognition, although presence of the latter is only implied in Principles of Natural Knowledge. Whitehead believes he can sidestep the problem of introducing subjectivity into the interpretation of natural relations in the following manner.

Since nature is events, we are not dealing with two different kinds of "stuff" when we consider an entity in nature (a concrete event) and the observer of that entity. The percipient event (the observer) is itself a concrete event. The same relations are possible among all events. No problem arises in considering the perception of nature on one hand and nature itself on the other, because these are not different kinds of things (one subjective and the other objective). This view is seen as consistent with the

---

[61]The answer to this problem discussed earlier was formulated in terms of Whitehead's later works, in particular, Process and Reality.

totality of Whitehead's natural philosophy, but it has been an occasion of contention among Whitehead's critics.

The relationship between Whitehead's theory of perception and the method of extensive abstraction becomes evident when we consider that what is perceived (and, also, the perceiver) are events, including those objects[62] inherent in events, and the various relations which occur between events.[63] All three of these aspects of events are brought into focus through the application of the method of extensive abstraction.

I have not given a complete exposition of Whitehead's work but I have indicated several areas where problems might ultimately arise. Because of the complexity and scope of Whitehead's writings in natural philosophy, it has been necessary to limit the scope of my discussion and omit topics where points of contention might arise which could assist my thesis. Such areas include the problems Whitehead faces in explaining the measurement of space and time, and those problems arising from his notions of cogredience and significance.

---

[62]An explanation of Whitehead's theory of objects is not necessary for the purpose of this paper. Suffice it to say that objects make possible the permanence for the recognition of some "awareness" vital to cognition.

[63]For a statement of Whitehead's fundamental thesis concerning the importance of his theory of perception to his entire project and his insistence that a philosophy of nature be erected on a structure ultimately based on data immediately perceived, see p. 66 below (<u>Principles of Natural Knowledge</u>, pp. 12-13.)

PART II

POSSIBLE PROBLEMS WITH WHITEHEAD'S PHILOSOPHY

INTRODUCTION

I now end my exposition of Principles of Natural Knowledge and turn my attention to several problems related to the method of extensive abstraction and the relation of parallelism. It will be necessary during the course of this discussion to include some further explanation of certain notions in Whitehead's works. However, these will be related to the particular problems under consideration.

I intend to discuss three interrelated reasons why one might choose to reject Whitehead's natural philosophy. These reasons are related to specific topics in Whitehead's early works. These topics include: 1) Whitehead's fundamental thesis that natural philosophy should be concerned with the perceivable phenomena of the natural world, 2) the nature of Whitehead's extensive continuum of events, and 3) Whitehead's application of the relation of parallelism to his natural philosophy.

The particular problems I raise in respect to these topics are: 1) Whitehead appears unable to maintain the connection between the basic events derived by the method of extensive abstraction and the entities of the perceivable world. 2) The relations expressed in Whitehead's extensive continuum of events do not meet the requirements of his natural philosophy. 3) The assumption of Euclidean parallelism in Whitehead's early works directly influences the outcome of his view of nature.

CHAPTER 1
PROBLEMS WITH THE THEORY OF PERCEPTION

The Fundamental Thesis in Perception

Whitehead has asserted that natural philosophy should be based upon the immediate data of perception.

He states:

> The relatedness which is the subject of natural knowledge cannot be understood without reference to the general characteristics of perception. Our perception of natural events and natural objects is a perception from within nature, and is not an awareness contemplating all of nature impartially from without....[P]erception involves a percipient object, a percipient event, the complete event which is all of nature simultaneous with the percipient event, and the particular events which are perceived as parts of the complete event....The point here to be emphasized is that natural knowledge is knowledge from within nature, a knowledge "here within nature" and "now within nature," and is an awareness of the natural relations of one event in nature (namely, the percipient event) to the rest of nature. Also what is known is not barely the things but the relations of things, and not the relations in the abstract but specifically those things as related (12-13).

In this passage Whitehead expresses some of the fundamental notions which make up his natural philosophy. An integral part of this thesis is that our knowledge of nature is, without exception, tied to perception of nature by means of, in Whiteheadian terms, a "percipient event". On the surface this thesis appears trite. Without the involvement of some type of perception, knowledge of the natural world would be impossible.

It is noted, however, that the physical sciences deal routinely with entities which are not perceivable (for example, the electron or the nucleus of an atom). It is, however, assumed that these entities underlie the basic structure of nature. Nor are the notions of space and time, as they are used by the physical sciences, considered available for our common perception. Space itself is not perceived. It is the objects which appear to be located within space that are studied. Nor are individual instants of time perceived but time is considered with respect to the passage of events.

It is Whitehead's contention that the physical sciences separate natural entities into those things which are perceived and those things which are not. The physical science construct knowledge of the natural world through the use of both types of entities. If Whitehead is to preserve his thesis that a knowledge of nature must be founded upon that which is perceived, he must overcome this separation of types of entities established by the physical sciences. He must be able to bridge the gap between the perceived and the non-perceived.

One way he can accomplish this task is to demonstrate that all of nature is open to perception.[64] For Whitehead,

---

[64]The phrase "open to perception" is used by Whitehead to indicate those events or relations occurring between events or objects which are directly or indirectly preceivable (by some precepient event).

entities or experiential events of nature can be expressed in terms of basic relationships. These relations are consistent throughout all of nature. We can be said to perceive a non-observable entity if we understand that the relations involved in this non-observable entity are consistent in content and structure with the relations of these observable entities from which they are derived (abstracted).[65] Whitehead believes that he can illustrate this consistency through the application of the method of extensive abstraction to abstractive sets of events. He would say that non-observable entities are available to perception because the relations which are found in any abstractive set of events are consistent with the relations of the observable event from which the abstractive set is formed.

The Method of Extensive Abstraction and Mathematical Limiting

An argument could be raised against this theory by employing the nature of mathematical limiting. By showing that the method of extensive abstraction is a type of mathematical limiting, we could contend that this method, applied to an event, results in something that is not a

---

[65]The use of the term consistent might be considered somewhat ambigious. Here, it is to be understood in the same sense as it is consistent with perception to say that molecules have no color or electrons have no determinate position.

natural entity but is an idealized construct of some sort, i.e., it does not exhibit the same basic relations exhibited by an entity found in nature. If this were the case, Whitehead would have violated the requirements of his natural philosophy. Having started with a perceived event in nature he would have proceeded through the process of abstraction to an idealized construction which is not perceived (does not contain the same basic relations as the original entity). It would not be an event (a natural entity). To investigate this line of argument, we must consider both the process of mathematical limiting and the process exhibited by the method of extensive abstraction. I will begin with the notion of mathematical limiting.

One way to illuminate this mathematical method is with an example. In his book, Beyond Numeracy,[56] John Allen Paulos illustrates the principle of mathematical limiting in the following manner:

> Take a circle that is one foot in diameter and inscribe it in an equilateral triangle. Now inside this triangle inscribe a circle and then inside this smaller circle inscribe a square. Inside the square inscribe a still smaller circle, inside of which you next inscribe a regular pentagon. Continue with these nested inscribings, alternating between a circle and a regular polygon whose sides increase by one with each iteration. It's clear that the area of the inscribed figures decreases with each repetition, but what is the ultimate area achieved by this sequence of figures? At first glance it appears that it should be zero, the process leading only to an

[56]John Allen Paulos. Beyond Numeracy. Alfred A. Knopf: New York, 1991.

> isolated point. Remember, however, that as the number of sides of the polygons increases, they become more circular and after a while the process becomes, almost at least, one of placing a circle inside another circle with very little lost in the area from one step to the next....The limit of this procedure is a circle concentric with the original one and having a diameter of approximately one inch, 1/12th that of the original (Paulos 129).

The notion of mathematical limiting is not only applicable to geometrical instances, but also to the sum of a numerical series, "the area of a curved region, the asympototic limit of a curve or a sequence of curves...and a host of other, more esoteric mathematical constructs" (Paulos 131). For our purposes, we wish to isolate the basic principles of mathematical limiting in order to compare these with the basic principles at work in Whitehead's method of extensive abstraction.

In the quote given above, Paulos refers to the application of mathematical limiting to idealized constructs, that is, to idealized representations which are expressed mathematically. The process begins with a mathematical representation[67] of some occurrence and ends with an idealized approximation in the form of a numerical

---

[67]The term "mathematical representation" is intended to mean a mathematical expression which exhibits certain qualities of some natural occurrence or natural entity. The type of mathematical representation will vary with the particular case to which it is being applied. In the examples given here, an arithmetic mathematical representation is being given in the form of the numerical expression of area or volume.

value. This would seem to be true of all instances of mathematical limiting.

Now let us consider Whitehead's method of extensive abstraction. If we can show that the method of extensive abstraction is an instance of mathematical limiting, we will have proved that Whitehead has not overcome the problem of the perception of non-observable entities because the method of extensive abstraction must begin with some natural entity and end with some idealized mathematical construct rather than an event.

We have seen that Whitehead derives the concept of time from natural events by applying the method of extensive abstraction to durations. To show that the method of extensive abstraction is an instance of mathematical limiting we must prove that a duration can be expressed by a mathematical representation and that the result of the process is some idealized construct, in this case, an extensionless instant of time.

At first glance, it would seem that this is a case of mathematical limiting. A duration, it would seem, should be able to be expressed by some arithmetic mathematical representation and an instant of time is certainly be an idealized construct which is not found in nature. A closer examination will show, however, that there are several major flaws in this presentation. First, Whitehead would not agree that a duration can be given an arithmetic mathematical representation. It is admitted that an arbitrary numerical value could be assigned to some portion of a duration in such a way that the numerical value would decrease with the diminution of the series of durations. But this is not an arithmetic mathematical representation of a duration, because the numerical values do not express any property of the duration.

The method of extensive abstraction is being applied to an abstractive set of durations and an abstractive set has the following properties: 1) "of any two of its members one extends over the other, and [2] there is no event [duration] which is extended over by every event [duration] of the set" (104). The method of extensive abstraction differs from the mathematical process of limiting in this case because there is no actual numerical limit being expressed. Neither can we say that there is convergence towards a duration (an event) as a limit. Such a convergence would violate the second condition listed above since in an abstractive set of durations there is no smallest duration. If there were, it would necessarily be extended over by every event of the set and this constitutes a violation of the second condition.

Whitehead addresses this problem in the following passage:

> The properties of an abstractive class secure that its members form a series in which the predecessors extend over their successors, and that the extension of the members of the series (as we pass towards the "converging end" comprising the smaller members) diminishes without

> limit; so that there is no end to the series in this direction along it and the diminution of the extension finally excludes any assignable event. Thus any property of the individual events which survives throughout members of the series as we pass towards the converging end is a property belonging to an ideal simplicity which is beyond that of any assignable event. There is no one event which the series marks out, but the series itself is a route of approximation towards an ideal simplicity of "content" (104).

While the method of extensive abstraction may seem to converge toward some ideal limit in nature, in fact no such limit can be stated. What the method of extensive abstraction does do is to produce a series of progressively simpler natural events. In the case of durations this is a series in which the temporal extension progressively decreases.

If we were to assign numerical values to the members of this series, these values would converge to a mathematical limit. It is noted, however, that it is not the abstractive set which converges but the series of assigned numerical values. An abstractive set approaches or approximates an instant of time but there is no such entity existing in nature. This illustration has been concerned with durations but the same technique is used in establishing an approximation of a dimensionless point.

Whitehead's theory of perception applies to these considerations because it would seem reasonable to assume that the durations toward the end of an abstractive set are not perceived entities. In our previous discussion of perception it became obvious that Whitehead's use of the term "perception" is not that which is commonly accepted. If we accept his theory of perception, we must also accept that even the smallest of durations would be perceivable, not through sense-perception but indirectly, through awareness of the relatedness of these durations with the rest of nature (and so with the percipient event).

It can be seen that we have not succeeded to show that the method of extensive abstraction is an instance of mathematical limiting. Nor have we proven that Whitehead fails to show that even infinitely small events are perceivable. The discussion has, however, illuminated much about the procedure of the method of extensive abstraction and Whitehead's application of it.

A Review of the Method of Extensive Abstraction

Whitehead has intended the method of extensive abstraction to provide a means of reducing the relations exhibited by events to their greatest simplicity. In this way he hopes to illuminate the basic structure or framework of the events we perceive in nature. These are the same basic relations upon which the natural sciences depend for the expression of the laws of nature. Abstractive sets of events provide the vehicle for the simplification of relations exhibited by events when they are subjected to the method of extensive abstraction. An abstractive set is, in essence, a set of nested events, each lying (at least, in part) within another, each containing smaller and smaller events, i.e., the events form an infinite non-convergent series.

Since the larger events toward the beginning of the series are observable, even those toward the smaller end of the series are to be considered observable. These latter events have been simplified in such a way that they retain only those relations which are common to the whole series. In this manner, the abstractive set, taken as a whole series, is considered to be open to perception. If Whitehead is to maintain the central thesis of his natural philosophy (stated above) he must be able to prove that these assertions are true.

CHAPTER 2

CRITICISMS: THE METHOD OF EXTENSIVE ABSTRACTION

There have been a number of criticisms offered against Whitehead's use of the method of extensive abstraction. I will consider the criticisms of three authors, C. D. Broad,[64] Adolf Grünbaum,[65] and W. Mays,[66] in the following discussion. These authors present opinions for and against the applicability of Whitehead's method of extensive abstraction in the construction of his natural philosophy. They also comment on each other's opinions on this subject. After a discussion of these criticisms, I will offer my own comments on the applicability of the method of extensive abstraction to Whitehead's project.

Comments by C. D. Broad

We have been considering the use of Whitehead's method of extensive abstraction as it relates to his requirement that the natural sciences must be based on perceived entities. C. D. Broad, in Scientific Thought, discusses this aspect of Whitehead's work. He presents a comparison between the definition of a point, i.e., a point established

---

[64] C. D. Broad. Scientific Thought. Routledge & Kegan Paul LTD: London, 1953, cf. Chapter 1, pp. 26-52.

[65] Adolf Grünbaum. "Whitehead's Method of Extensive Abstraction." in British Journal for the Philosophy of Science. Nov. 1953. pp. 215-226.

[66] W. Mays. The Philosophy of Whitehead. Collier Books: New York, 1962. cf. Chapters 6 and 7, pp. 108-125.

through the use of the method of extensive abstraction, and the development of a definition for irrational numbers (Broad 38-52).

Broad begins his discussion by raising the following question: "Have we any right to believe that finite objects consist of parts of no magnitude, or that such parts, if they exist at all, will have relations in the least like those which hold between finite areas and volumes?" (Broad 37-38). He intends to show that a method of re-definition, applied to the commonly accepted definition of a point, will solve the problem of establishing volumes of physical objects from collections of dimensionless points. Broad contends that Whitehead's method of extensive abstraction allows this process of re-definition to accomplish its purpose (Broad 38-39).

Broad presents two criteria for the definition of a point, stated as follows: "...(i) that points must have to each other the kind of relations which geometry demands; and (ii) that points must have to finite areas and volumes such a relation that a reasonable sense can be given to the statement that such areas and volumes can be exhaustively analyzed into sets of points" (Broad 39). These criteria can be related to Whitehead's project[71] in the following manner: criterion (i) requires a theory of points adequate

---

[71]The project is to present an analysis of the relations of events in nature through the use of the method of extensive abstraction.

for use by the physical sciences; and criterion (ii) can be maintained by allowing the use of the method of extensive abstraction in the construction of points (where abstractive sets are diminished in such a way that the diminishing series is infinite and non-converging and the relation of extension is retained throughout the set). Even the smallest members of such a set are extended and this extension can be expressed in terms of area or volume.

Re-definitions: Irrationals and Integers

Broad provides support for his thesis by referring to the internal structure of a scientific definition.[72] He contends that the internal structure of a definition does not matter as long as the definition maintains all the logical relations involved in the entity under consideration. He uses the definition of irrational numbers as an example of the method of re-definition (Broad 42-44).

We call irrationals such as $\sqrt{2}$ or $\sqrt{3}$ numbers (just as we call integers such as 2 or 3 numbers) because these irrationals obey formal relations which are a subset of the formal relations which all numbers obey, including integers.[73] To illustrate his point, Broad considers the

---

[72]By the internal structure of a scientific definition, Broad is speaking of the explicit linguistic formulation of the definition. Broad refers to this as the inner nature of the term (Broad 39).

[73]It should be noted that integers obey some formal relations which irrationals do not. Some of the properties involved with the subset of formal relations which both integers and irrationals obey include the associative

operations of addition and multiplication as they are applied to both irrationals and integers. He points out that although these operations produce quite different kinds of entities when they are applied to irrationals and to integers, the logical properties expressed by both types of entities are the same. Broad asks us to accept on faith that this similarity in logical properties is maintained in

---

property, the commutative property, the existence of an identity element, and the existence of inverses of elements. These properties can be defined as follows:

Let $a, b, c, \in R$, where R represents the set of all real numbers (including integers and irrationals).

Associative Property under addition:
$a + (b + c) = (a + b) + c$.
Associative Property under multiplication:
$a(bc) = (ab)c$.
Identity Property under addition:
R contains an element 0 such that
$a + 0 = 0 + a = a$.
Identity Property under multiplication:
R contains an element 1 such that
$a \cdot 1 = 1 \cdot a = a$.
Inverse Property under addition:
For each $a \in R$ there exists a unique element $(-a) \in R$ so that
$a + (-a) = (-a) + a = 0$.
Inverse Property under multiplication:
For each nonzero $a \in R$ there exists a unique element $(1/a) \in R$ so that
$a(1/a) = (1/a)a = 1$.
Commutative Property under addition:
$a + b = b + a$.
Commutative Property under multiplication:
$ab = ba$.

These properties, since they apply to the set of all real numbers, apply to integers and irrationals alike. Other properties include those of closure, the distributive property for multiplication over addition and the trichotomy property. Karl J. Smith. Finite Mathematics. Scott, Foresman, and Co.: Glenview, Illinois, 1975. pp. 335-336.

all operations which can properly be applied to both irrationals or integers (Broad 40).

There is a striking difference between the internal structure of what is meant by an irrational and what is meant by an integer, but we refer to them both as numbers. This is because they obey the same subset of formal logical properties, i.e., we assign the term "number" to any entity which possesses these particular logical properties.

Re-definitions: Points

The same type of considerations can be applied to the definition of a point. Broad indicates that a point is often defined as the ultimate limit of some diminishing series of areas or volumes (Broad 46). This concept works quite well in mathematical considerations but it can appear paradoxical when we try to visualize how the combination of a collection of massless point-particles can constitute a volume or area. It seems counter-intuitive that when a collection of points are combined they can constitute a physical object. However, this notion is generally assumed in the applied physical sciences. It is the project of both Whitehead and Broad to overcome this apparent paradox.

The difficulty seems to arise from the traditional definition of a geometrical point. Broad attempts to show that a point can be defined in a different manner (different in internal structure from the traditional definition) but, at the same time, retain those relations required of a geometrical point for use in the physical sciences. This re-definition should make clear how volumes and areas can be constructed by combining collections of points. He intends to accomplish this feat through the use of Whitehead's method of extensive abstraction.

When the method of extensive abstraction was applied to a duration the result was a series of progressively diminishing durations with an increasing simplicity of relations.[74] The end product of this process was the approximation of a moment in time. This abstractive set of durations is infinite and results in a simplicity of relations expressed by an idealized instantaneous moment.

When we consider the spatial side of nature and apply the method of extensive abstraction to the space associated with an instantaneous moment, the result is diminishing series of volumes or areas. These series are also infinite and approach or approximate dimensionless points. Traditionally, a connection between such an infinite series and the point itself is never established.

Broad suggests a re-definition of the point (similar to the re-definition of irrational numbers discussed above) which will eliminate this problem. Instead of considering

---

[74]Refer to the section, The Method of Extensive Abstraction and Mathematical Limiting, for an explanation of what is meant by an increasing simplicity of relations. In particular, see the quote from Whitehead, p.72-73, in this paper.

the approximate limit of the series as determining a point, Broad suggests that we re-define a point so that it consists of the entire series of diminishing volumes. Although this seems intuitively incorrect, let us consider it in terms of the criteria Broad requires for the definition of a point. If this re-definition can fulfill these criteria, we are justified in considering it a definition of a point because it retains all the relations required of a point by the physical sciences.

The first criterion requires that a point fulfill those relations required by geometry.[75] Broad contends that the re-definition of a point meets this requirement, as well as the other geometrical entities constructed through the use of the method of extensive abstraction (i.e., lines, planes, etc.). He offers no proof for this assertion but asks us to take this point on trust (Broad 47). Instead of a proof he offers an example of how two different series of concentric circles generate two unique points. These two points can then be used to define a straight line connecting them. These constructions illustrate two of the basic properties required of geometrical points.

The second criterion asks: "Do points, straight lines, etc., really exist in the same sense as volumes, or are they

---

[75]Broad is referring to the logical relations necessary to construct a coherent physical geometry. It is assumed that the geometry is Euclidean and the logical relations are the postulates required by Eucild.

merely convenient and perhaps indispensable fictions?" (Broad 51).[76] Broad asserts that points are not merely fictions. They have a reality of their own which is different from that of volumes. He stresses that only particular entities can be directly perceived and that points are not particulars but represent classes of series of volumes. These series of volumes really do exist and the larger elements of the series can easily be perceived. Points themselves are to be considered logical functions of such series and they have existence in the sense that "they are determinate functions of real series of actually existing particulars" (Broad 51). This is to say that points are expressions of the relations which exist in such series. This is, Broad, asserts, exactly what Whitehead

---

[76]It should be noted that Broad's argument is not without critics. A number of questions could be raised against the points Borad makes in his argument. First, it might be questioned whether points really represent classes. It could be argued that they are the classes of a series of volumes. Second, it can be questioned whether or not a class of volumes will have all of the relevant properties of a point. Two points determine a line, but two sequences of volumes do not. The question can also be raised as to how a realism about points is supposed to follow from the construction given in Broad's argument? Broad also refers to a "determinate function" without giving us a clear definition for his use of this term. Finally, it might be asked, if Broad is a realist about points, then why does he present his re-definition in the first place? Why does he go to such lengths to construct them out of volumes? These points offer a number of reasons why Broad's arguments should not be accepted out of hand. I am indebted to Dr. Mark Risjord, Department of Philosophy, Michigan State University, for bring these objections to my attention.

intends to accomplish through his use of the method of extensive abstraction.

Criticism: Adolf Grünbaum

Broad's interpretation of Whitehead's method of extensive abstraction is not without critics. Adolf Grünbaum[77] addresses Broad's criteria for the definition of a point and attempts to show that even with the assumption of these criteria, Whitehead's method fails. Broad's criteria for the definition of a point state "(i)...points must have to each other the kind of relations which geometry demands; and (ii)...points must have to finite areas and volumes such a relation that a reasonable sense can be given to the statement that such areas and volumes can be exhaustively analyzed into sets of points" (Broad 39). Grünbaum believes that Whitehead's "...Method is vitiated by Zeno's mathematical paradox of plurality[78] because of an important violation of the two conditions stated by Broad..." (Grünbaum 216).

---

[77]Adolf Grünbaum. "Whitehead's Method of Extensive Abstraction." British Journal for the Philosophy of Science. Nov. 1953. pp. 215-226.

[78]"Zeno's mathematical paradox of plurality...[is intended] to show that since spatial and temporal intervals are extended, it is self-contradictory to regard them as aggregates of respectively unextended points and instants" (Grünbaum 216).

Denumerable and Non-denumerable Infinite Sets

Grünbaum's argument draws directly from the mathematical concept which states that, given the standard notion of additivity customarily assumed in mathematics, an aggregate of points can only be conceived as an interval of length if the number of points in the aggregate is non-denumerably infinite. If we were to consider a finite line segment as a series of denumerable[79] point-elements,[80] we find that we cannot define measurement of length of a line segment in terms of an aggregate of these point-elements. The measurement of length of such a finite line segment can be defined in terms of an aggregate of unit point-sets,[81] which are super-denumerably infinite.

Grünbaum's argument, presented briefly, states that an aggregate of any unit point-sets of a denumerable infinite set always results in a length of zero. He illustrates this point in the following passage:

---

[79]A denumerably infinite set of points is defined as an infinite set that can be placed in one-to-one correspondence with the set of natural numbers. It is said to be countably infinite. A super-denumerably infinite set of points is defined as a set of points which cannot be put into a 1-1 correspondence with the set of natural numbers. An example of a denumerable infinite set is the set of rational numbers found on the interval [0,1]. The set of points which make up the real number line on the same interval is non-denumerablly infinite.

[80]For a definition of "point-element," see the quote from Grünbaum below (p. 86 of this paper).

[81]Grünbaum discusses this point at length in his article, "A Consistent Conception Of The Extended Linear Continuum As An Aggregate Of Unextended Elements." Philosophy of Science. Oct. 1952. 19. pp. 288-306. p. 295.

> ...that length or extension is also defined as a property of point-sets rather than of individual points and that zero length is assigned to the unit set, i.e., to a set containing only a single point. While it is both logically correct and even of central importance to our problem that we treat a line interval of geometry as a set of point-elements, the definition of "length" renders it strictly incorrect to refer to such an interval as an "aggregate of unextended points." For the property of being unextended characterizes unit point-sets but is not possessed by their respective individual point-elements, just as temperature is a property only of aggregates of molecules and not of individual molecules. The entities which can therefore be properly said to be unextended are included in but are not members of the aggregate of points constituting a line interval. Accordingly, the line interval is a union of unextended unit point-sets and not an "aggregate of unextended points" (Grünbaum "Consistent" 295).

This theory asserts that the length of a finite line segment, for example, [0,1], cannot be defined in terms of an aggregate of individual points and even though measurement of length is defined for an aggregate of unit point-sets, this length will always be zero because this aggregate is denumerably infinite. We have not escaped Zeno's paradox. Grünbaum asserts, however, that a finite line segment actually consists of a non-denumerably infinite set of unit point-sets (Grünbaum 217). Measurement of length of an interval (consisting of such a set of non-denumerably infinite unit point-sets) is established by considering the aggregate of such unit point-sets and in practical application is established by the standard formula for determining the length of any finite line segment (i.e., $|b-a|$, on the interval [a,b]).

Grünbaum uses this theory against Whitehead's method of extensive abstraction by showing that any series of points Whitehead can derive from abstractive sets is necessarily denumerable.[82]

Grünbaum also asserts that the

> ...set-theoretical meaning of super-denumerability eludes all logically possible sensory exemplification, since any collection of non-overlapping three-dimensional regions of space is at most denumerably infinite....
>
> Thus the manner in which the super-denumerable kind of infinity is conceived shows clearly that sense-awareness cannot suggest the idea of a collection of perceptible regions whose cardinality exceeds [aleph zero], and, more significantly, that a fortiori, sense awareness cannot exhibit the actual existence of such a collection in sensed nature. And on Whitehead's assumptions, it is the latter condition which must be satisfied, if Cantor's actual super-denumerable infinities of points, which are required by modern geometry and physics for metrical consistency, are to be preserved through epistemological reconstruction (Grünbaum 217-218).

Thus, Grünbaum asserts, Whitehead's use of the method of extensive abstraction to construct points fails to comply with Broad's first criterion for points (that such points must be consistent with the principles of geometry). Since Whitehead's method cannot derive non-denumerable infinite sets of points, it does not provide entities applicable to

---

[82]Such a collection of abstractive sets is considered denumerable because the establishment of points by abstractive sets is an ongoing process and each point, as it is established, can be put into a 1-1 relationship with the natural numbers. For this reason any set of points developed from abstractive sets is always denumerable.

geometry in its modern form which requires such sets for its application (Grünbaum 218).

Differentiation Between Points

Grünbaum also attempts to prove that the method of extensive abstraction keeps Whitehead from meeting Broad's second criterion. He points out that "all the properties of abstractive sets which are essential to the Method are subsensory" (Grünbaum 223). Having established this point (which Whitehead himself concedes[83]), Grünbaum asks how we can know from sense perception that there exists two different abstractive sets which define two unique points. If we claim that such sets exist, how are we to establish their particular difference by sense-perception? Whitehead has given us no reason to believe that abstractive sets do not fall under Hume's characterization of the structure of appearance.[84] "Hume has shown that the structure of appearance is irremediably non-isomorphic with that which geometry attributes to space in virtue of the infinite divisibility of intervals" (Grünbaum 219). To say that the structure of appearance is non-isomorphic to that which geometry attributes to space is to say that the structure of

---

[83]Alfred North Whitehead. The Aims of Education and Other Essays. Macmillan: New York, 1929. pp. 176-177. This work was written in the same period of the other works under consideration and so I am taking the liberty of assuming that the views expressed in this work are consistent with the views expressed in the other works of this period.
[84]David Hume. A Treatise of Human Nature. 2nd. ed. Clarendon Press: Oxford, 1978. p. 27.

the space of our perception cannot be placed in a one-to-one relationship with the structure of the space developed by geometry.

Hume states:

> 'Tis therefore certain, that the imagination reaches a minimum, and may raise up to itself an idea, of which it cannot conceive any subdivision, and which cannot be diminished without total annihilation. When you tell me of the thousandth and ten thousandth part of a grain of sand, I have a distinct idea of these numbers and of their different proportions; but the images, which I form in my mind to represent the things themselves, are nothing different from each other, nor inferior to that image, by which I represent the grain of sand itself, which is suppos'd so vastly to exceed them. What consists of parts is distinguishable into them, and what is distinguishable is separate. But whatever we may imagine of the thing, the idea of a grain of sand is not distinguishable, nor separable into twenty, much less into ten thousand, or an infinite number of different ideas (27).

Hume is asserting that because our perceptions of space are at most limited to entities of finite area, we cannot even consider the possibility of distinguishing between such infinitesimal entities as two points. Whitehead's derivation of points is achieved through abstractive sets of diminishing areas. It would seem that sense perception provides no means for distinguishing between these abstractive sets which Whitehead needs to confer separate identity for points lying close to one another. The difference in identity between classes of sensible volumes defining a given point and a class defining another point separated from the first by a super-denumerable infinity of intervening points seems beyond the powers of the method of extensive abstraction. For this reason, Grünbaum concludes that Whitehead's method of extensive abstraction does not provide for the distinction in identity between points in the mathematics of physics (Grünbaum 220).

Continuity of Inclusion

Whitehead attempts to guarantee the existence of infinite series of subsensory events by simply claiming "continuity of inclusion" among the members of an abstractive set: "...every event contains every other event as parts of itself" (CN 76). Broad adds further support to Whitehead's position by saying of abstractive sets: "...ordinary perception makes us acquainted with their earlier and bigger terms, and the assumption that Space is continuous guarantees the [existence of the] later ones" (Broad 44). But this gives us no reason why we should not accept Hume's characterization of "the structure of appearance" as correct.

According to Grünbaum the argument from the continuity of inclusion (referred to above) "derives its semblance of plausibility wholly from a tacit appeal to the infinite divisibility which geo-chronometry[55] attributes to physical space-time (but not to sensed space-time!)" (Grünbaum 224). He further asserts that Broad begs the question if his

---

[55]Geo-chronometry refers to that branch of the physical sciences which deals with the measurement of space and time.

assertion that space is continuous is based on the strength of the Cantorean continuity[86] affirmed by geo-chronometry because that continuity is "defined on the basis of relations between non-perceptual point-elements which are postulated ad initio" (Grünbaum 224).

Since neither Whitehead nor Broad has been able to give a satisfactory defense of the principle of continuity of inclusion[87] Grünbaum insists that they both have failed in their project to show that elements established by abstractive sets are open to perception.

Assumptions: Grünbaum

It should be noted that Grünbaum bases his arguments against Whitehead's method of extensive abstraction on two assumptions. The first assumption, which is explicitly stated, is that Broad's analysis of the criteria for a definition of points is correct (Mays 118). The second assumption, which is implicitly assumed, is that the type of continuity upon which Whitehead's bases his theory of nature is that assumed by the classical theory of a finite line segment, i.e., that of the Cantor-Dedekind continuum (Mays 125).

---

[86]See footnote 13 on page 16-17 of this paper for an explanation of Cantorean continuity.

[87]The principle of the continuity of inclusion is required by condition (ii), proposed by Whitehead's criteria for abstractive sets because "...condition (ii) in the definition of "abstractive set" rules out a last, smallest member of the set and thereby requires an infinite membership for all abstractive sets" (Grünbaum 222, note 3).

I am willing to concede that Broad's criteria for the definition of a point seem consistent with Whitehead's project. It is also understandable why Grünbaum might assume that the type of continuity Whitehead attributes to his theory of nature is that of the classical theory of the finite line segment. In Whitehead's explanation of the relation of serial order, he asserts that "the serial order among moments of the same time-system has the Cantor-Dedekind type of continuity" (115). This is the continuity Grünbaum attributes to a finite line segment. If Whitehead had intended that the continuity expressed by the extensive continuum is of the Cantor-Dedekind type, it would be consistent with that continuity assumed by the physical sciences. However, if this were the case, it has been seen that it would be difficult for Whitehead to establish many of other the properties necessary for this type of continuity.

There are indications elsewhere in Whitehead's works that he intends another type of continuity. Hammerschmidt, in <u>Whitehead's Philosophy of Time</u>,** sees Whitehead's purpose as discarding the traditional conception of continuum "as a mere closely packed collection of discrete points" and replacing it with "the conception of the continuum as an exhibition of the interconnection of

---

**William W. Hammerschmidt. <u>Whitehead's Philosophy of Time</u>. King's Crown Press: New York, 1947.

regions, derived from their basic extensiveness" (Hammerschmidt 43).[89] V. Mays, in The Philosophy of Whitehead, indicates that the continuity which Whitehead describes can be considered to be close to that of the mathematical intuitionist (Mays 112-125). I will return shortly to these questions concerning Whitehead's conception of continuity and the extensive continuum.

Comments: V. Mays

Mays offers several comments on Grünbaum's criticisms against Whitehead. A part of Grünbaum's criticisms, Mays feels, arises because Whitehead is not clear about whether or not his method is "an exact description of some actual process of convergence" (Mays 118). The indication that the method of extensive abstraction arrives at an actual process of convergence is important to Grünbaum's argument that points established by the method of extensive abstraction do not represent unique, individually recognizable points. If such convergence did occur, each abstractive set would converge to zero and this would offer no help in distinguishing between different points (Grünbaum 220).

Grünbaum, it will be remembered, maintains that points established by the method of extensive abstraction are indistinguishable from any other point derived by the same

---

[89] We can find this notion in Whitehead's work's in several places, among these are Principles of Natural Knowledge, p.4, and Concept of Nature, p.59.

method. Points established by Whitehead's method would not display the same properties as points on a finite line segment. These latter points are considered to be unique and can be distinguished one from another. Grünbaum denies that these properties apply to points constructed through use of the method of extensive abstraction.

If Whitehead is to be successful in his project he needs to show that the method of extensive abstraction provides events derived from perceivable entities and that these are applicable to the methods used by the physical sciences. Grünbaum contends that Whitehead does not fulfill these requirements.

He also supposes that the method of extensive abstraction obliterates precision of meaning made possible in the statement of natural laws by use of real variables because the method of extensive abstraction fails to provide unique distinguishable points (Grünbaum 219). Whitehead admits that there is no way for his natural philosophy to discriminate between a space which possesses the compactness of a series of rational numbers (it is dense) and a space which possesses the compactness of the series of real numbers (it is continuous)[30] (Whitehead "Historical" 203). In this respect, Whiteheadian natural philosophy would have

---

[30]Alford North Whitehead. "Historical Changes." in <u>Essay's in Science and Philosophy</u>. Philosophical Library: New York, 1947. p. 203.

difficulty maintaining a strict distinction between real number variables.

Comments: Arguments Against the Method of Extensive Abstraction

Grünbaum's criticisms have lead us to another point of consideration, i.e., the problems involved with Whitehead's conception of the extensive continuum. Before discussing this point, I would like to comment on the criticisms which have been offered against Whitehead's method of extensive abstraction. It would seem that Grünbaum's criticisms raise a number of issues which Whitehead would have difficulty dismissing. Whitehead himself maintains that his theory cannot discriminate between denumerably infinite sets and non-denumerably infinite sets. This alone seems a crushing blow to Whitehead's assertion that the use of the method of extensive abstraction can establish geometrical entities (points, event-particles, etc.) which are applicable for use by the physical sciences. The only way, it would seem, for Whitehead to extract himself from this criticism would be to show that the type of continuity expressed in nature is not the continuity of the classical theory of a finite line segment (as Grünbaum assumes).

The idea that an entity can be considered perceivable (even when it is admittedly smaller than ordinary perception would allow), because it expresses the same relations as those expressed by larger perceivable entities of the same abstractive set, seems contrary to our notion of perception. Whitehead must show that the method of extensive abstraction does, in fact, isolate the relations which are consistent throughout an abstractive set. It is not obvious that the mere process of diminishment of a series of events would produce the required simplification of relations.

Whitehead's project is to express the spatial and temporal properties (those necessary to construct the geometry of the natural world) founded solely upon perceivable data. Even if one accepts that minute entities are perceivable (in the Whiteheadian sense, i.e., they express relations similar to those expressed by entities from which they were abstracted), there still remains the problem of the establishment of non-denumerably infinite sets. As stated above, the answer to this problem lies in the type of continuity Whitehead attributes to nature. I will table my discussion of this problem until we have achieved a greater understanding of the continuity expressed by the extensive continuum.

CHAPTER 3
PROBLEM OF THE
EXTENSIVE CONTINUUM

Extensive Continuum of Events

Whitehead offers few direct comments in his early works on the type of continuity he considers to be exhibited in nature. Events are seen as the primary entities which compose nature as a whole and, as such, are considered to be continuous with each other. This can be seen in Whitehead's references to the relation of extension as it applies to events. All events are extended over by other events and all events also extend over other events. Such extension enables us to perceive a continuity in nature. In order to better understand this type of continuity we might wish to examine those properties exhibited by the relations which comprise the extensive continuum.

As mentioned above, there has been some controversy as to the exact nature of Whitehead's extensive continuum. It has been suggested that Grünbaum considered the properties expressed by Whitehead's extensive continuum to be those properties of continuity expressed by the classical theory of a finite line segment. Other authors[86] have suggested that the properties of the extensive continuum might

---

[86]W. Mays and G. T. Kneebone deal quite extensively with the similarities between Whitehead's extensive continuum and the intuitionist's infinitely proceeding sequence. This will be explained in greater detail below.

resemble those properties of continuity expressed by the intuitionist's infinitely proceeding sequence.

It should be noted that much of the criticism discussed in this paper against Whitehead's natural philosophy has been grounded on the assumption of a particular kind of continuity which Whitehead was believed to have held. In a consideration of these criticisms it would be advantageous to explore those properties exhibited by Whitehead's extensive continuum as well as the properties of those other types of continuum mentioned above.

General Properties of a Continuum

Let me begin with a brief description of those properties generally attributed to a continuum. If we rule out the possibility of a discrete continuum, it could be stated that it is the nature of a continuum to be infinitely divisible and to have limitless extension (Hooper 205).[92] For example, if we consider the continuum of the classical theory of a finite line segment, the natural numbers can be divided in infinitely many ways because we can divide between any two numbers. The property of limitless extension is illustrated by considering that between any two points there is another. Indeed, between any two points there is an infinite number of distinct points. If we include discrete continua, we must modify these general

---

[92]Sidney E. Hooper. "Whitehead's Philosophy: Space, Time and Things." Philosophy, 18, 1943. pp. 204-230.

properties to that of being "gapless," which gives us the intuitive notion of being continuous. For the purposes of this paper, we only need to regard the general properties of a continuum to be infinitely divisible and extensionless.

Properties of the Extensive Continuum

The extensive continuum proposed by Whitehead arises out of the general character of the world. It is not to be conceived as an infinite receptacle into which all matter can be placed. This is Newton's notion of absolute space. It also should be noted that the extensive continuum does not involve shapes, dimensions, or measurability. Whitehead maintains that these characteristics are additional determinations which are to be associated with some particular duration (Hooper 208). Hooper states that the extensive continuum also has a necessary connection with "real" potentiality.[93] Thus, the extensive continuum can be seen as "...the primary determination of `order' of real potentiality arising out of the general character of the world" (Hooper 208).

Hooper continues with the following assertion:

> ...the extensive continuum is one vast relational complex providing a niche for all potential objectifications of actual ...[events], past, present and future. The future as well as the past and present are involved, because all

[93]To speak of "real" potentiality is to speak of some fact which is real in the sense of arising from the actual world and which expresses the potentiality of that which is to be derived through the progressive advance of nature.

> possible actual...[events] in the future must exemplify these determinations in their relations with the already actual world from which they arise. Thus the extensive continuum underlies the whole process of the world, and expresses the solidarity of all possible standpoints throughout this process....[T]he extensive continuum is a complex system of...[events] united by certain specific relationships, such as "whole and part," "overlapping of parts," "contact" and other relationships derived from these primary types....[T]here is involved in the notion of a continuum the two properties of indefinite divisibility and of unbounded extension. This is so for the reason that "there are always...[events] beyond...[events] and because ...non[event] is no boundary"" (Hooper 208-209).

The extensive continuum is regarded as potential in respect to its potentiality for division (Hooper 209). In the creative advance of nature, that which was previously potential in the extensive continuum becomes the initial stage of the development for new events, each event culminating in something unique and all its own. Every actual event can be thought of as being **somewhere** in the extensive continuum in virtue of its relationships with other events (i.e., it has definite position). In another sense, each event can be thought of as being **everywhere** throughout the extensive continuum because each event pervades or is related to all other events in the continuum (i.e., it has unlimited influence on other events through the relations it possesses).

The extensive continuum, then, lays claim to the relations of extension and divisibility. It also exhibits the relations of whole and part, overlapping of parts, and contact, among others. From these relations it has been seen that Whitehead derives the properties of lying between and serial order. It is obvious that the extensive continuum meets the general requirements for a continuum.

If the extensive continuum also possesses those properties of the classical theory of a finite line segment, then the criticisms Grünbaum launches against Whitehead's natural philosophy might be considered valid or, at least, they would gain in plausibility. Before we can attempt to determine the truth of this assertion, we must examine those properties of continuity expressed by this view of a finite line segment.

<u>Continuity of a Finite Line Segment</u>

The continuity expressed by the classical theory of a finite line segment is the continuity of the real number line. As mentioned above, Whitehead refers to the continuity of a serial order as that of the Cantor-Dedekind type. He offers this proclamation without any attempt at explanation or proof (Hammerschmidt 48).[94] This is clearly

---

[94]Hammerschmidt continues as follows:
"...[Whitehead] asserts that the numerical and analytic aspects of the theory of extensive abstraction can be validly interpreted in terms of the conventional system of points in a real continuum. If his assertion is justified, the continuum deduced from extensive abstraction permits the use of usual modern mathematical methods, and the metrical analysis is that of usual geometry. Once the elementry geometrical conceptions are defined by the abstractive sets, the new system and the conventional modern system will have

a reference to the continuity represented by the classical theory of a finite line segment.

Whitehead's project requires him to derive concepts applicable to entities used by the physical sciences. The classical notion of continuity applies to series of entities which are ordered by a relation with the properties of asymmetry, transitivity, and connectedness (Mays 109). Connectedness refers to the property that between any two members of a set, they are related to one another and the asymmetrical property refers to a set such that if one thing is so related to a second, then the second cannot be so related to the first.[95] Whitehead's definition of serial order establishes that the relation of "lying between" possesses the properties of transitivity[96] (iv-b), asymmetry (iv-c), and connectedness (iv-a).[97]

---

equivalent analytical properties" (Hammerschmidt 48).

[95] Irving M. Copi. Introduction to Logic. 7th ed. Macmillan: New York, 1986. p. 385.

[96] Transitivity, as a property of the relation of lying between, is triadic and can be expressed in set notation in the following way:

Let B be the relation of betweenness and x,y,z,u $\in$ A, where A is the set of all points on some (Euclidean) straight line, then

$$(x)(y)(z)(u)\ ((Bxyu\ \&\ Byzu) \rightarrow Bxyz)$$

Gerald Massey. Philosophy of Science. Macmillan: New York, p. 191.

[97] The references following the particular properties mentioned here refer to the sections of Whitehead's definition of serial order, found on p. 16 of this paper.

In "The Nature and Status of Time and Passage,"[30] Paul Weiss emphasizes that the moments of time also express a relational ordering which possesses the properties of connexity, asymmetry, and transitivity (Weiss 155).

> The relation of the moments is connexive; i.e., any pair of them whatsoever, if they are not in the relation of stretch and moment, are such that one is before the other (Weiss 155-156).
>
> The relation between the moments of time are asymmetrical; i.e., if a moment is before another, that other is not also before it (Weiss 156).
>
> The relation between the moments of time is transitive; i.e., if any moment precedes a second which precedes a third, the first precedes the third as well (Weiss 157).

This is the same type of serial ordering which was established by Whitehead. Because the moments of time are derived by the same procedure as points or event-particles lying along routes or on stations (by the method of extensive abstraction), it can be assumed that this same relation of serial ordering holds among points or event-particles along a route, point-track, or null-track (i.e., they express linear order).

Although we have established certain similarities between properties expressed by the extensive continuum and properties expressed by the classical theory of a finite line segment, problems arise if we press this similarity too far. One problem mentioned above concerns Whitehead's

---

[30]Paul Weiss. "The Nature and Status of Time and Passage." in Philosophical Essays for Alfred North Whitehead. Longmans, Green and Co.: London, 1936. pp. 153-173.

inability to produce non-denumerably infinite series of event-particles by the method of extensive abstraction.[39] The source of this problem lies in the fact that the points on a finite line segment are considered to be initially given (i.e., it is assumed to be an axiom of this branch of classical mathematics that the points of a finite line segment are not derived or constructed or established by division of an already existing line segment. They are assumed to be initially given in the concept of any finite line segment).

Whitehead's extensive continuum, on the other hand, can only express potentially infinite series of entities. This is due to the fact that the extensive continuum reflects the progressive advance of nature (i.e., all entities of nature are never completely established and so, as explained above, the set of all such entities must be considered to be denumerably infinite).

Another problem arising in a comparison between properties of a finite line segment and those of the extensive continuum lies in the fact that the continuum of a finite line segment consists of unique points or elements. By unique I mean that they are identifiably different (in location or order along the line segment). Event-particles

---

[39]It will be remembered that event-particles for Whitehead serve the same function as points in the conventional system of Euclidean geometry (33).

or points in the extensive continuum seem to lack this distinctness.[100]

This point may be illustrated in the following manner. Event-particles, established by the method of extensive abstraction, retain their character as events. Whitehead expresses diverse views about the essential character of events. On one hand, he states that an event is a distinct entity. Events never change, they are just what they are and only that (62). On the other hand, he asserts that events also have an indistinct side to their nature. "...[E]vents appear as indefinite entities without clear demarcations and with mutual relations which are of baffling complexity" (73).

Grünbaum seems to refer to this ambiguity concerning events when he insists that event-particles cannot be a substitute for determinate points on a continuum (Grünbaum 219). He calls to this problem the ambiguity of convergence. Thus, event-particles "provide no means for distinguishing a given point from a continuum of others" (Grünbaum 219). This is due to the fact that all abstractive sets converge to entities which are exactly alike and so, in the case of points, there is no way to differentiate between two separate points which lie close to each other in an interval (Grünbaum 220).

---

[100]See Grünbaum's argument, given above, for an explanation of this point.

Broad has tried to show that, by constructing neighboring series of abstractive sets, each of which defines a different event-particle in definite relation to one another, this problem is overcome. This procedure is that which Whitehead uses to establish serial order among events. This represents a possible solution to this problem raised by Grünbaum.

Even if there is a way to determine unique points in a continuum, there still remains the problem that any such series can never be anything but denumerable in nature. In this respect, Whitehead's extensive continuum does not maintain the similarity of its properties to those of a finite line segment.

In the course of these considerations we seem to have come to an _impasse_. Whitehead's extensive continuum does not express the same relational properties as a finite line segment and so we must question its applicability for use by the physical sciences. However, this applicability represents a major part of Whitehead's project. It should be noted that the greatest difficulty in establishing similarity in properties with a finite line segment seems to lie in the fact that the extensive continuum exhibits the property of infinite potentiality as an integral part of its character. Such potentiality is not a property of the classical theory of a finite line segment.

Intuitionist's Notion of Continuum

This property of potentiality expressed by Whitehead's extensive continuum has led Mays to suggest a similarity between the extensive continuum and the intuitionist's infinitely proceeding sequence. Mays specifically points to Brouwer's[101] intuitionist mathematics for a concept of continuity and continuum which might be similar to Whitehead's theory (Mays 112).[102] He expresses this in the following passages:

> It has often been pointed out that there is a radical difference between the continuity of our experience and mathematical continuity which applies only to series. By experiential continuity we refer to the fact that the perceptual field is given as a connected whole which cannot be split up into ultimate simple elements. Mathematical continuity, on the other hand, deals precisely with such elements, i. e., an infinite collection of individuals arranged in a certain order.
>
> It is clear from Whitehead's conception of the extensive continuum as exhibiting the properties of the inclusion, overlap and contact of regions, that he has not the classical mathematical continuum in mind. It is in any case difficult to see how he could conceive it as a closely packed infinite series of points, since he regards points as derivative from these extensive relationships. In this respect his position would seem in some ways to resemble the mathematical intuitionist's notion of the continuum, where the fundamental relationship is taken to be that of part to whole. In the mathematical continuum of Brouwer, for example, the individual real number

---

[101]Luitzen Egbert Jan Brouwer (1882-1966) founded the system of mathematical intuitionism.

[102]The complexities of the intuitionist mathematics is beyond the scope of this paper. Its importance in the present context lies in Mays' assertion of similarity between Whitehead's concept of continuum and the intuitionists' conception of continuum.

> is defined as "an infinite sequence of nested division intervals of increasing level"[103] (Mays 112).

Also:

> Brouwer...has argued that statements which concern all real numbers, i. e., all the values of a real variable, are to be interpreted in terms of the totality of natural numbers. He defines the individual place in the continuum not by a set but by a sequence of natural numbers created by free acts of choice. Whitehead's conception of the continuum as a connected system of regions rather than a series of discrete points brings him closer to the intuitionist's continuum, where the fundamental relation is that of part to whole rather than element to set (Mays 122).

After a brief outline of the intuitionist notion of continuum, we will be in a position to discuss those properties expressed in this notion of continuum and seek any similarities between these properties and those expressed in Whitehead's extensive continuum. The information presented concerning intuitionist mathematics has been drawn chiefly from Stephan Körner's The Philosophy of Mathematics[104] and G. T. Kneebone's Mathematical Logic and The Foundations of Mathematics.[105]

---

[103] H. Weyl, Philosophy of Mathematics and Natural Science, pp. 51-53.

[104] Stephan Körner. The Philosophy of Mathematics. Hutchinson University Library: London, 1960. cf. Chapters VI and VII.

[105] G. T. Kneebone. Mathematical Logic and The Foundations of Mathematics. D. Van Nostrand Co. Ltd.: London, 1963. cf. pp. 243-257 and pp. 341 - 352.

Basic Properties: Intuitionist Mathematics

In order to understand the intuitionist notion of continuum it will be necessary to consider a few of the basic principles upon which this mathematics is formulated. The primary principle of intuitionist mathematics is that only those concepts which can actually be constructed are considered as part of the mathematics. In practice intuitionist mathematics consists of "creat[ing] or construct[ing] mathematical objects....It is not [considered a part of this program] to show the legitimacy of these constructions by either logic or formalization. For they are legitimate in themselves, they are self-validating" (Körner 124-125).

The mathematics of the intuitionist is considered an intuitive or mental activity and is not based upon empirical evidence. The ability to construct a mathematical object in the mind by intuition alone is considered enough justification for its validation.

Properties of an Infinitely Proceeding Sequence

The aspect of intuitionist mathematics which is most applicable to our concerns is the notion of an infinitely proceeding sequence.[106] Such a sequence must be established

---

[106]Körner describes an infinitely proceeding sequence as "a sequence which can be continued ad infinitum no matter how the components of the sequence are determined, whether by law, free choice or what you will" (Körner 128).

by construction and the intuitionist establishes a real number generator[107] as one way to construct such a sequence.

The notion of an infinite set of elements expressed by a finite line segment is counter to the principles of intuitionism because the elements of a finite line segment are not constructed but are considered to be initially given. The construction of an infinitely proceeding sequence is never complete. It produces a potential infinity of entities. The intuitionist, in constructing the set of all real numbers, does so by a process which can produce only a potentially infinite set of real numbers.

Similarities of Properties

In this respect it is clear why Mays considered Whitehead's notion of the extensive continuum to be similar to that of the mathematical intuitionist. Both can provide only potential infinite series of entities rather than an actually existing infinite series. Whitehead's use of the method of extensive abstraction to derive points represents

---

[107]The term "real number generator" indicates a mathematical technique used by the intuitionist to produce an infinite series of real numbers. The technique was developed by Brouwer as a way of introducing the continuum into mathematics, by treating it not as a set of existent entities but as the potential generation of real numbers. Incorporated in this generation is sufficient freedom for the range of possible outcomes of the process of construction to correspond to what a classical mathematician would refer to as the totality of all real numbers. Basically, a real number generator consists of some specific requirement which limits the range of possibilities of construction in such a manner that real numbers are produced (Kneebone 250-253).

a type of construction where a continuing series of points or event-particles is produced. Since this set of entities is never complete, it can never represent an actual infinite set. In this way the method of extensive abstraction is similar to an "infinitely proceeding sequence," e.g., the series of numbers produced by a real number generator. Also, both Whitehead's extensive continuum and an infinitely proceeding sequence exhibit the relation of part to whole as an important aspect of their continuity (Mays 112).

Another property the two share is that of overlapping. This is because the mathematical intuitionist denies the law of excluded middle.[108] It is impossible to determine if

---

[108]The law of excluded middle asserts that any statement is either true or false (Copi 306). Intuitionist mathematics denies this law. For the intuitionist, there are only simple mathematical constructions and mathematical arguments. If these are valid this can be seen immediately, by direct intuition. However, for the intuitionist, negation ($\neg p$) is "also a record of a construction, and is really an affirmation which states: "I have effected in my mind a construction B which deduces a contradiction from the supposition that the construction A were brought to an end." The proposition "I have not effected a construction...." is of no interest to either the intuitionist or the classical mathematician. But whereas the classical mathematician admits "there exists a mathematical construction...", even if nobody has so far been able to effect it, such a proposition could from the intuitionist point of view only be an empty promise...[and] not a piece of mathematics" (Körner 132).

When we consider the intuitionist meaning of p and $\neg p$ we can see that the proposition (p or $\neg p$) is not a universally valid principle of mathematical logic. For the intuitionist, p or $\neg p$ are not the only possibilities available for a proposition, p. There is another possibility, because, in intuitionist logic, the proposition, $\neg\neg p \ne p$, is also true, where $\ne$ is defined as not coinciding. Only if a=b is contradictory, i.e., "only if we can effect a construction which deduces a pairs of numbers in an infinitely proceeding sequence overlap. This is because there is always a certain ambiguity about the discreteness of individual points on an interval developed by such a sequence(Mays 121). This possibility of the overlapping of points on an interval is similar to the properties displayed by points which are derived from events. Since events are necessarily bound up in the overlapping property of extension, points abstracted from them retain some degree of extension (and so, the property of overlapping also).

We have discussed several important similarities between the relational properties exhibited by Whitehead's extensive continuum and the properties exhibited by the intuitionist's infinitely proceeding sequence. These have included the property of potential infinities of entities, the relation of part to whole, and the property of overlapping. I refer to these as important similarities because each of these notions of continuum express these relational properties as a part of their basic constitution.

Differences Between Philosophical Systems

There also exist certain important differences in the primary objectives of these two systems. These differences extend beyond the properties displayed by the two types of continua; they reach well into the basic philosophies upon

---

contradiction from the supposition that a=b", are we entitled to assert that a and b do not coincide, i.e., $a \neq b$.

which these systems are based. Perhaps the most obvious difference lies in the fact that Whitehead's natural philosophy is integrally tied to his concept of perception. Such a basis in empiricism would be totally inappropriate for the mathematical intuitionist. Any mathematical construction of an intuitionist is based only on pure perception or intuition. It will be remembered that pure perception does not include perception in our common sense of the term (Körner 120).

Another difference can be seen in Whitehead's emphasis on logical principles to express the structure of nature. While the mathematical intuitionist has developed a logic consistent with his view of mathematics, it is quite different from the logic used by Whitehead and Russell in their work Principia Mathematica[109] as well as the logic expressed by Whitehead in later works (Körner 140).

Whitehead has insisted on the actuality of the extensive continuum (CN 59, CN 76). Whatever type of continuum Whitehead intends it to be, it must be is one which is representative of the reality of nature. The extensive continuum, through the actual existence of events, expresses the past, present, and future in the progressive advance of nature. These differences are related to areas of

[109]Bertrand Russell. Principia Mathematics. 3 vols., written with A. N. Whitehead. Cambridge, 1910-1913.

Whitehead's philosophy which are indispensable to his project as a whole.

A summary of the points we have established can be illustrated by noting that those areas of dissimilarity established between the extensive continuum and the classical theory of a finite line segment are just those areas where the properties of the extensive continuum are similar to those of the intuitionist's infinitely proceeding sequence. Where the infinity of elements which make up a finite line segment are considered to be initially given, those entities established by the extensive continuum and an infinitely proceeding sequence are only potentially infinite. The points of classical a finite line segment are unique and clearly individual while the entities of the extensive continuum and those of an infinitely proceeding sequence lack a degree of distinctness.

The mathematics based upon the properties of the real number system (the system represented by a finite line segment) is the mathematics traditionally and currently used by the physical sciences. It has been stated that "[r]egarding convergence and the theory of infinite series,... Brouwer's...[notion of] convergence leads to two quite different theories of which only the first is similar to the classical theory....The classical foundations of calculus, all the more the modern theory of real functions..., clearly become meaningless in this light."[110] Because the property of potentially infinite series pervades both Whitehead's natural philosophy and intuitionist mathematics neither is able to develop the classical system of mathematics which is generally used in the physical sciences.[111] This distinction between the properties of the real number system and those of Whitehead's extensive continuum is devastating to Whitehead's natural philosophy because a major part of Whitehead's project was to develop a natural philosophy which could be consistently applied by the physical sciences.

Conclusions: Extensive Continuum

The preceding considerations have been intended to illuminate ways in which the use of the method of extensive abstraction is a method for representing the basic framework of nature. We have considered similarities and dissimilarities between the properties exhibited by the extensive continuum and two types of continua whose properties are known and which play an important role in the mathematics.

---

[110] Abraham A. Fraenkel and Yehoshua Bar-hillel. Foundations of Set Theory. Amsterdam: North-Holland Publishing Co., 1958. p. 258.

[111] Because the property of potentiality is related to both infinities and infinitesimals in the mathematics of the intuitionist and Whitehead, it is understandable that the use of such potential infinite series in calculus, which is integral to the physical sciences, is inappropriate.

It has been established that the extensive continuum differs from each of these two types. When we compare the properties of the extensive continuum to those of a finite line segment, we find that there are certain essential properties which cannot be expressed by the extensive continuum. In the same manner, when we compare the properties of the extensive continuum to those of an infinitely proceeding sequence, we find that while many of the relational properties are similar, certain aspects of this latter type of continuum are counterproductive to major aspects of Whitehead's project. This is especially clear in Whitehead's emphasis on a natural philosophy which will provide the mathematical and conceptual tools necessary for the application of the physical sciences.

The discussion given above has shown that the properties exhibited by the extensive continuum are not those same relational properties exhibited by a classical view of the finite line segment or an infinitely proceeding sequence. It has also shown that the mathematics derived from the relational properties of the extensive continuum are not generally considered applicable for use in the physical sciences.

Conclusions: Method of Extensive Abstraction

The method of extensive abstraction enters into this discussion because this method is fundamental in the derivation of entities abstracted from the extensive continuum. This method, when applied to abstractive sets, provides a tool for Whitehead to illuminate the basic relations among events, which represent the very fabric of nature itself. If Whitehead's philosophy cannot produce those entities necessary for its success, the method of extensive abstraction also fails as a tool for providing these entities.

It seems obvious that there remains issues of concern involving the use of the method of extensive abstraction and doubts as to its effectiveness in providing the type of data Whitehead intended. We cannot, then, say with surety that the method of extensive abstraction is the effective tool needed for Whitehead's project.

With the relations which occur among the events and, in particular, those events which are durations, Whitehead intended to reconstruct the geometrical entities necessary to build a complete geometry of nature. Using the method of extensive abstraction as a vehicle for this project, he has tried to develop not only temporal entities but also spatial geometrical elements, such as levels, rects, puncts, event-particles, matrices, and finally, planes, lines, and points.

As a result of this discussion I conclude that Whitehead's project is not entirely wrong-headed, but suggest that the method of extensive abstraction needs re-working. I arrive at this conclusion because of the integral part the method of extensive abstraction plays in the derivation of such entities as points, event-particles, moments, routes, etc., all of which are necessary for the development of a mathematics, in particular, a geometry, applicable for use by the physical sciences. It has been shown that because of the problems of convergence with respect to abstractive sets and the development of potential infinities in the case of the derivation of points and event-particles, such entities are not fully applicable in the mathematics of modern geometry and physics.

CHAPTER 4
PROBLEMS WITH THE
RELATION OF PARALLELISM

In his early works[107] Whitehead assumed a four-dimensional flat space-time and proceeded in his development of those relations necessary to reconstruct the geometry of time and space from this point (Hammerschmidt 50). The use of the method of extensive abstraction on abstractive sets is not sufficient in itself for Whitehead to complete his project. The relation of cogredience and the notions of dimensionality and parallelism are also necessary. None of these can be developed through the application of the method of extensive abstraction to events and must to be assumed as primary elements in Whitehead's natural philosophy.

In Whitehead's later works (Process and Reality, in particular), he generalized his philosophical system for generating those elements necessary to produce the geometry of nature. One could insert particular instances of dimensionality and parallelism into the system and arrive at a completed geometry based on those elements inserted as variables. The resulting geometries would be Euclidean or non-Euclidean, depending on the types of dimensionality and parallelism inserted into the system (Hammerschmidt 69).

---

[107]Those works written before Science In the Modern World. Alfred North Whitehead. Science of the Modern World. The Free Press: New York, 1967. Original copyright: 1925.

This generality is not exhibited, however, in Whitehead's earlier works. The assumption of a four-dimensional flat space-time (three spatial dimensions, assumed to be perpendicular to one another and a temporal dimension united in such a manner as to create a four-dimensional space-time) and Euclidean parallelism have resulted in establishing the geometry of nature as Euclidean.

The notion of parallelism (and the type of parallelism represented) is vital to the development of Whitehead's natural philosophy. Because the notion of parallelism cannot be developed through the use of the method of extensive abstraction, it must be inserted into Whitehead's philosophical system as a presupposition. The type of parallelism to be inserted must be determined from our perceptions of nature.

Parallelism and Temporality

Whitehead thought that this derivation could be accomplished through our perception of durations. In the exposition of Principles of Natural Knowledge given above it was stated that Whitehead begins his development of his natural philosophy with the application of the method of extensive abstraction to temporal durations. The result of this application was a moment in time, itself a duration. A duration also exhibits a spatial representation of all events simultaneous to that particular duration. This is the space referred to when we say a moment represents all of nature at that instant of time.

A central characteristic of durations/moments is that all durations/moments parallel to one another determine a family. There is also a family of moments associated with every family of durations. These families constitute a time-system. The term parallel is assumed to be equivalent to the term non-intersecting (Kneebone 348). Whitehead considers the type of parallelism being implied here to be parabolic or Euclidean (113).

Parallelism and Spatiality

Whitehead develops a natural geometry with spatial elements which are introduced through a consideration of moment in time. This geometry of instantaneous space is developed from puncts, rects, and levels, all of which exhibit the parallelism assumed in connection with durations/moments.

From the development of instantaneous space, Whitehead continued to produce the geometry of four-dimensional space-time. This geometry is constructed from the elements of event-particles, point tracks, null tracks, and matrices, all of which exhibit the same type of parallelism assumed from the beginning (Euclidean parallelism).

The development of time-less space (that space necessary for the application of the physical sciences) proceeds in the same manner, based on the elements of points, straight lines, and planes, all of which also exhibit Euclidean parallelism.

Assumption of Euclidean Parallelism

From the brief synopsis of the exposition of Principles of Natural Knowledge given above, it can be seen that the notion of parallelism permeates almost every aspect of this work. It is also clear that the assumption of Euclidean parallelism must necessarily result in the development of geometries which are Euclidean. In later works Whitehead generalized his procedure so that a change in the variables of dimensionality and type of parallelism lead to geometries which can be other than Euclidean. This does not negate the fact that in Principles of Natural Knowledge the assumption of Euclidean parallelism is a major factor in Whitehead's deduction that the geometry of nature is Euclidean.

This difficulty is perhaps not as vital as those issues raised against the method of extensive abstraction but it does lead to the faulty deduction that the geometry of nature is Euclidean (a notion which at this time is not accepted in the physical sciences). Since the notion of parallelism is intrinsic in Whitehead's natural philosophy, the assumption of Euclidean parallelism and Whitehead's assertion that the geometry of space is Euclidean is problematic.

PART IV

CONCLUSION

The thesis of this paper has been to show that certain notions found in Whitehead's natural philosophy are faulty and that the application of these notions result in the apparent failure of Whitehead's entire project. These notions are the method of extensive abstraction and the assumption of Euclidean parallelism. I have examined the method of extensive abstraction with respect to Whitehead's theory of perception and also with respect to its role in the development of the extensive continuum. In both cases the use of the method of extensive abstraction has shown to produce questionable or faulty results. It has also been shown that the assumption of Euclidean parallelism in the development of Whitehead's geometry of nature results in a geometry that can be only be Euclidean.

It is my contention that Whitehead's natural philosophy is not entirely wrong-headed but rather that the use of the method of extensive abstraction and the assumption of Euclidean parallelism contort his early works so that the whole attempt appears to be a failure. It seems entirely reasonable that one could construct a natural philosophy in which empiricism and the notion of events (as represented by Whitehead) are primary. The development of such a philosophy would require methods as yet unformulated.

LITERATURE CITED

Broad, C.D. Scientific Thought. International Library of Psychology, Philosophy and Scientific Method. London: Routledge & Kegan Paul, 1953.

Copi, Irving M. Introduction To Logic. 7th edition. New York: Macmillan, 1986.

Fraenkel, Abraham A. and Yehoshua Bar-Hillel. Foundations of Set Theory. Studies in Logic and the Foundations of Mathematics. Amsterdam: North-Holland Publishing Company, 1958.

Grünbaum, Adolf. "A Consistent Conception of the Extended Linear Continuum As an Aggregate of Unextended Points." Philosophy of Science, 19 (October 1952): 288-306.

Grünbaum, Adolf. "Whitehead's Method of Extensive Abstraction." British Journal for the Philosophy of Science. (November 1953): 215-226.

Hammerschmidt, William W. Whitehead's Philosophy of Time. New York: King's Crown Press, 1947.

Hooper, Sidney E. "Whitehead's Philosophy: Space, Time and Things." Philosophy. 18 (1943): 204-230.

Hume, David. A Treatise of Human Nature. 1888. ed. L. A. Selby-Bigge. 2nd. edition. Oxford: Clarendon Press, 1978.

Kneebone, G.T. Mathematical Logic and the Foundations of Mathematics. London: D. Van Nostrand Company LTD., 1963.

Körner, Stephan. "Continuity." The Encyclopedia of Philosophy, Vol 2. 1967.

Körner, Stephan. The Philosophy of Mathematics. London: Hutchinson University Library, 1960.

Leclerc, Ivor. "Whitehead and the Problem of Extension." in Alfred North Whitehead: Essays on His Philosophy. ed. George L. Kline. Englewood Cliffs: Prentice-Hall, Inc., 1963.

Lucas, John F. Introduction to Abstract Mathematics. Belmont: Wadsworth Publishing Company, 1986.

Massey, Gerald. Philosophy of Science. New York: Macmillan, 1966.

Mays, W. The Philosophy of Whitehead. New York: Collier Books, 1962.

Palter, Robert M. Whitehead's Philosophy of Science. Chicago: University of Chicago Press, 1960.

Paulos, John Allen. Beyond Numeracy. New York: Alfred A. Knopf, 1991.

Russell, Bertrand, and Alfred North Whitehead. Principia Mathematica. 3 Vols. Cambridge: Cambridge University Press, 1912.

Schmidt, Paul F. Perception and Cosmology in Whitehead's Philosophy. New Brunswick: Rutgers University Press, 1967.

Smith, Karl J. Finite Mathematics. Glenview, Illinois: Scott, Foresman, and Company, 1975.

Weiss, Paul. "The Nature and Status of Time and Passage." in Philosophical Essays for Alfred North Whitehead. London: Longmans, Green and Company, 1936.

Weyl, H. Philosophy of Mathematics and Natural Science. Princeton: Princeton University Press, 1949.

Whitehead, Alfred North. An Enquiry Concerning the Principles of Natural Knowledge. 2nd. edition. 1925. Cambridge: Cambridge University Press, 1955.

Whitehead, Alfred North. Concept of Nature. 1920. Cambridge: Cambridge University Press, 1964.

Whitehead, Alfred North. "Historical Changes." in Essays in Science and Philosophy. New York: Philosophical Library, 1947.

Whitehead, Alfred North. The Aims of Education and Other Essays. New York: Macmillan, 1929.

Whitehead, Alfred North. The Organization of Thought. Philadelphia: J.B. Lippincott Company, 1917.

Whitehead, Alfred North. The Principle of Relativity. Cambridge: Cambridge University Press, 1922.

Whitehead, Alfred North. Process and Reality. 1929. Corrected Edition. ed. David Ray Griffin and Donald W. Sherburne. New York: The Free Press, 1978.

Whitehead, Alfred North. Science in the Modern World. 1925. New York: The Free Press, 1967.