This is to certify that the

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THREE DIMENSIONAL VIBRATION ISOLATION
USING ELASTIC AXES

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ABSTRACT

THREE DIMENSIONAL VIBRATION ISOLATION USING ELASTIC AXES

By

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Design for three dimensional vibration isolation is an important part of the design of all motor and pump mounting systems. A three dimensional vibration isolation system is analyzed here using elastic axes. Optimization is performed to align the elastic axes to a design specified coordinate system. Partial decoupling of the compliance of this six degree of freedom system has been accomplished. The performance of the optimized isolation system is measured through the magnitude of translational and rotational vibration. The normalized RMS response for the original and optimized designs were compared using real automobile engine inertia data. Two methods of realigning elastic axes with specified coordinate system were developed although results show that this alignment did not improve the vibration isolation of the system significantly. Response decoupling was found to be very sensitive to small errors in elastic alignment.

to my parents

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NOMENCLATURE

[H]	6 by 6 matrix partitioned into 3 by 3 submatrices	
C.G.	Center of Gravity	
DA	Angle difference between two axes	
EA	Elastic Axis	
i	$\sqrt{-1}$	
TR	Translation from the C.G.	
ω	Frequency	
A	Flexibility matrix,	6 by 1
M	Mass/ Inertia matrix,	6 by 6
C	Viscous damping coefficient matrix,	6 by 6
D	Structural Damping coefficient matrix,	6 by 6
K	Stiffness coefficient matrix,	6 by 6
f	Force/Torque vector,	6 by 1
F	Magnitude of Force/Torque vector,	6 by 1
Q	Transformation matrix,	6 by 6
$\mathfrak{R}$	Rotational transformation matrix,	6 by 6
R	Elastic center rotation matrix,	3 by 3
$\mathfrak{T}$	Translational transformation matrix,	6 by 6
T	Translation matrix,	3 by 3
x	Displacement/Rotation vector,	6 by 1
X	Magnitude of Displacement/Rotation vector,	6 by 1

INTRODUCTION

Vibration isolation in engineering has a variety of applications; pumps, electric motors, air compressors, and so on. For example, isolation of automobile engine vibration from the body structure is very important to current customer acceptance. The effects of forces generated by the vibration of these machines can be minimized by proper isolator design. Some progress has been made with approaches which include; the active damper [Crosby and Karnopp (1973)], materials for vibration control [Nashif (1973)], effect of on-off damper for isolation [Rakheja and Sankar (1987)], liquid spring design [Winiarz (1986)], and passive load control dampers [Eckbald (1985)].

The effect of aligning elastic axes to excitation forces and moments on vibration isolation will be analyzed. It has been observed that engine mount designers believe that aligning elastic axes to the excitation helps vibration isolation. There has not been any previously published research on vibration isolation using elastic axes. The objectives of this study are to determine whether; i) elastic axes can be aligned to a fixed coordinate system of a designer's choice, ii) response of the system can be decoupled by aligning the elastic axes, and iii) aligning elastic axes can help the vibration isolation.

A computer simulation program of automobile engine and mount dynamics, ENGSIM II, was developed at the A. H. CASE Center for Computer-Aided Design at Michigan State University [Spiekermann (1982)]. A modification, ENGSIM III, has been developed to run on Macintosh Computers with the additional capability to optimize the position and orientation of the elastic axes of a mount design.

The elastic axes of a compliant system are a coordinate system which decouples the system's flexibility matrix, the inverse of the stiffness matrix. Changing the mount characteristics changes the location of the elastic axes. These may change until they coincide with the reference coordinate system. In this work, the external forces and moments are applied along one of the reference coordinate directions. This investigation determined how the decoupling resulting from elastic axes alignment with the applied forces and moments affected the translational and rotational response.

The mount design problem discussed here is the minimization of displacement and rotation of the vibration isolation model. The displacements and rotations of the mounts are the linear transformation of the displacement and rotational responses of the center of gravity. The forces transmitted through the mounts are proportional to the displacements/rotations and the velocities of the mounts. The minimization of the forces transmitted through the compliant mounts of a rigid body is the primary concern for mount designers. Only the displacements response at the center of gravity is analyzed and demonstrated in this study.

RIGID BODY VIBRATION ISOLATION MODEL

This study's vibration isolation model is a rigid body with six degrees of freedom supported on compliant mounts. The six degrees of freedom are translations along, and rotations around, each of the three orthogonal coordinate axes. The four compliant mounts are modeled as springs and dampers to simulate the general automobile engine (Figure 1). The mounts used in this analysis have linear stiffness and a combination of viscous and structural damping. The XYZ reference coordinate is the primary fixed rectangular coordinate with its origin at the center of gravity (C.G.) and is placed so that the positive Y axis is along the crankshaft from engine rear to front.

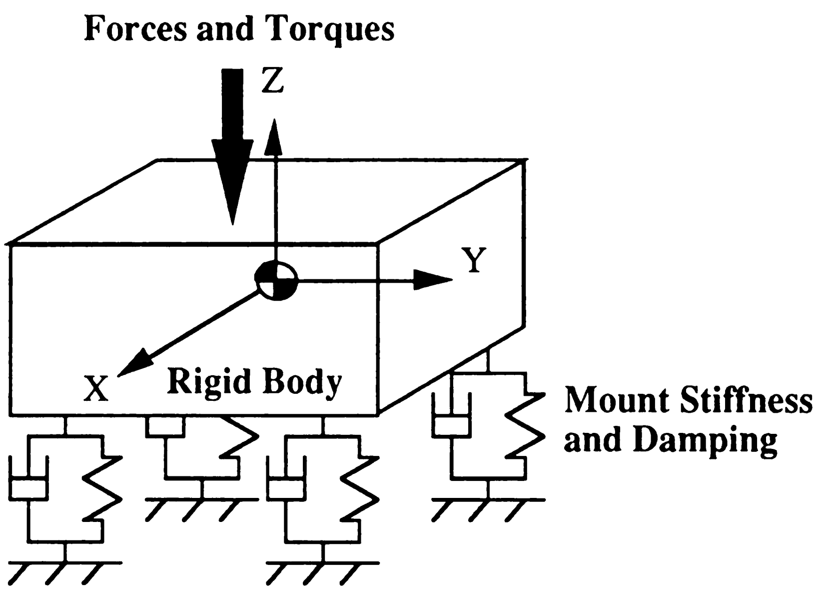

Figure 1 Vibration isolation model on four compliant mounts

The damped forced vibration problem is formulated. The three-dimensional equations of motion for a rigid body on compliant mounts have six degrees of freedom.

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x + i\,\mathbf{D}x = \mathbf{f} \tag{1}$$

where $\mathbf{x}^T = \{x, y, z, \theta_x, \theta_y, \theta_z\}$ is the displacement/torsion vector, **M** is the inertia matrix, **C** is the viscous damping matrix, **K** is the stiffness matrix, **D** is the structural damping matrix, $\mathbf{f}^T = \{f_x, f_y, f_z, \tau_x, \tau_y, \tau_z\}$ is the operating forces and moments vector, and $i = \sqrt{-1}$. This equation is used for the frequency response calculation later in this paper.

The effect of damping on natural frequencies and mode shapes is assumed negligible. This common assumption [Rao and Gupta (1985)] simplifies the procedures for determining natural frequencies and mode shapes. With this assumption, the eigenvalue problem can be solved from the homogeneous form of the undamped system equations.

$$\mathbf{M}\ddot{x} + \mathbf{K}x = 0 \tag{2}$$

This equation is used to find the eigenvalues which are the square of the natural frequencies of this system, and the eigenvectors which represent the mode shapes at each of those natural frequencies.

CONCEPTS

Frequency Response

Calculation of the frequency response of the vibration isolated mass predicts its vibration characteristics. For harmonic excitation,

$$\mathbf{f} = \mathbf{F}\, e^{i\omega t}. \tag{3}$$

The system response, $\{\mathbf{x}\}$ is assumed to be of the same form as the harmonic excitation.

$$\mathbf{x} = \mathbf{X}\, e^{i\omega t} \tag{4}$$

Substitution of equation (3) and (4) into equation (1) yields a linear equation.

$$[\,[\,\mathbf{K} - \omega^2 \mathbf{M}\,] + i\,[\,\mathbf{D} + \omega \mathbf{C}\,]\,]\,\mathbf{X} = \mathbf{F} \tag{5}$$

The frequency response, **X**, can be obtained by solving this complex linear equation for the selected range of frequency, ω, using the LINPACK subroutine ZGESL. The response is a complex value with the magnitude equal to the square root of the sum of the squares of the real and imaginary parts. The magnitude of this response is the displacement and torsion of the system at the center of gravity.

Elastic Axes

The ideal elastic axes form an orthogonal coordinate system in which the only displacement or rotational response to an applied force or torque is in the same direction as the degree of freedom in which the input force or torque is directed. The center of elasticity (C.E.) is the origin of this coordinate system. In the elastic axes coordinate system, the response will be pure decoupled translational modes and pure decoupled rotational modes. This ideal definition of the elastic axes which fully decouples the flexibility matrix can only occur in planar analysis. Although elastic axis for planar problems can be easily found by determining the coordinate system in which the flexibility matrix is diagonal, the search for the elastic axes for three dimensional problem with six degrees of freedom can be achieved through partial decoupling of the flexibility matrix.

The elastic axes are found by using the general flexibility matrix, **A**, which is the inverse of the stiffness matrix, **K**.

$$\mathbf{x} = \mathbf{K}^{-1}\,\mathbf{f} = \mathbf{A}\,\mathbf{f} \tag{6}$$

In this representation, a force **f**, expressed in the reference coordinate system causes a deflection, **x**. The investigation to find the elastic axes will assume only that the flexibility matrix, **A**, is known in the reference coordinate system of the analysis.

Eigenvector based, modal decoupling of the flexibility matrix does not yield its elastic axes. Although it is always possible to use the eigenvectors to diagonalize the flexibility matrix, the eigenvectors do not define a physical, orthogonal, coordinate system [Hall and Woodhead (1965)]. A physical transformation method, consisting of a combination of rotational and translational transformations defines the elastic axes. This set of transformation is found through the solution of the set of simultaneous equations (see page 8,9).

Two Dimensional Elastic Axes

Planar motion is described by two translations in a plane and a rotation about an axis normal to the plane. Two dimensional, planar elastic axes define coordinates such that a force is applied along one of the axes in the plane generates only a translational along that axis and a torque applied around the axis normal to the plane generates only a rotation around that axis.

Planar problems have a flexibility matrix given by

$$\mathbf{A} = \mathbf{K}^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{7}$$

The flexibility matrix, **A**, is symmetric so that full decoupling requires only three off-diagonal terms: a_{12}, a_{13}, and a_{23} be made equal to zero. These three elastic axes conditions can be met through two independent translations and one rotation . Analytically, the result is three independent linear equations for the two translations and one rotation. After the three coordinate transformations, the planar flexibility matrix, **A** takes the form

$$\mathbf{A}_{C.E.} = \mathbf{K}^{-1} = \begin{bmatrix} a^*_{11} & 0 & 0 \\ 0 & a^*_{22} & 0 \\ 0 & 0 & a^*_{33} \end{bmatrix} \tag{8}$$

Three Dimensional Elastic Axes

Full decoupling of a three dimensional flexibility matrix through coordinate transformation is not possible. Each coordinate transformation can introduce only a single pair of symmetric, off-diagonal zeros. The six by six, three dimensional, flexibility matrix has fifteen pairs of off-diagonal terms. Only six, independent, coordinate transformations are possible so that the definition of three dimensional elastic axes is a compromise and only partially decouples the flexibility matrix.

The one widely accepted choice for the three dimensional problem maximizes decoupling between the translational and rotational flexibility. Maximized decoupling can be obtained by diagonalizing the off diagonal sub-matrices [Fox (1977)]. Writing the six by six flexibility matrix as

$$\mathbf{A} = \mathbf{K}^{-1} = \left[\begin{array}{c|c} \mathbf{U} & \mathbf{W}^T \\ \hline \mathbf{W} & \mathbf{V} \end{array}\right] \tag{9}$$

the off diagonal submatrices **W** and $\mathbf{W}^T$ will contain the coupling terms. Although **W** cannot be made to vanish, it can be made diagonal.

If a stiffness matrix, referenced to some coordinate system, is known, then it can be referenced to a new coordinate system by a congruent transformation [Fox (1976)]. The coordinates of the transformation are measured and the translation is performed first by

$$\mathbf{K}' = \mathbf{Q}\ \mathbf{K}\ \mathbf{Q}^T \tag{10}$$

and $\mathbf{Q} = \mathfrak{R}\ \mathfrak{T}$

where $\mathfrak{R}$ is rotation transformation matrix and $\mathfrak{T}$ is translation transformation matrix.

$$\mathfrak{R} = \left[\begin{array}{c|c} \mathbf{R} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{R} \end{array}\right] \tag{11}$$

where **R** is the Euler angle three by three matrix and

$$\mathfrak{T} = \left[\begin{array}{c|c} \mathbf{1} & \mathbf{0} \\ \hline \mathbf{T} & \mathbf{1} \end{array}\right] \tag{12}$$

where

$$\mathbf{T} = \begin{bmatrix} 0 & -P_3 & P_2 \\ P_3 & 0 & -P_1 \\ -P_2 & P_1 & 0 \end{bmatrix} \tag{13}$$

P_i are locations of the old system measured in the new system. By inverting Equation (8) the transformation matrix is obtained

$$\mathbf{A'} = (\mathfrak{R}^T)^{-1} (\mathfrak{T}^T)^{-1} \mathbf{A} \, \mathfrak{T}^{-1} \mathfrak{R}^{-1} \tag{14}$$

Equation (14) can be solved to make the off-diagonal elements of submatrices **W** and **WT** zeros. The center of elasticity {P1, P2, P3} and the elastic rotation matrix, **R**, for this problem are obtained from the solution of the equation. The partial decoupling for this problem, the maximal diagonalization form, used here is equation (15).

$$\mathbf{A'} = \mathbf{A}_{C.E.} = \mathbf{K}^{-1} = \left[\begin{array}{ccc|ccc} * & * & * & * & 0 & 0 \\ * & * & * & 0 & * & 0 \\ * & * & * & 0 & 0 & * \\ \hline * & 0 & 0 & * & * & * \\ 0 & * & 0 & * & * & * \\ 0 & 0 & * & * & * & * \end{array}\right] \tag{15}$$

where $*$ are usually non-zero terms.

In this three dimensional diagonalization method, if a force is applied along one of the coordinate axes, the only resultant rotation will be around that axis combined with a translation along a direction which is not one of the coordinate axes. If a torque is applied around one of the coordinate axes, the only resultant translation will be along that axis combined with a rotation about a direction which is in general not one of the coordinate axes. To investigate the presence of the displacement response decoupling, the excitation torque is applied along one of the reference axes when elastic axes are realigned to coincide the reference axes.

Optimization

The optimization procedure locates the elastic axes each time minimizing the difference between the elastic axes and coordinate axes. A set of mount design parameters is sought which aligns the elastic axes to a coordinate system of the rigid body while avoiding large design changes. The design parameters are changed to minimize a penalty function that becomes smaller as the design criteria are met. The penalty function, P(E), is of the form

$$P(E) = a\,S(E) + b\,L(E) \tag{16}$$

where E is a vector of normalized design parameters, S(E) is a scalar size-of-change penalty function which becomes large when design changes begin to exceed prescribed limits, and L(E) is a scalar elastic axes penalty function which is large when elastic axes are far from the desired coordinate axes. The scalars a and b indicate the relative importance of the size-of-change penalty as compared to the elastic axes penalty. A single size of change and two different methods of expressing the elastic axes penalty function are introduced below. Using the IMSL subroutine ZXMIN, a locally optimal set of design changes, E, is found which define a local minimum of penalty function, P(E) [IMSL (1980)].

The size-of-change penalty used is expressed by [Spiekermann (1985)]

$$S(E) = \sum_{j=1}^{N} S_j(E_j) \tag{17}$$

$$\text{where:} \quad S_j = \begin{cases} (E_j - A)^2 & \text{for } E_j > A \\ 0 & \text{for } E_j \le A \end{cases}$$

This penalty conforms with common design situations, where large design changes correspond to increased real cost to produce the vibration isolation system. In some design situations, small design changes are possible which have no associated real cost. The size-of-change penalty function includes this situation through a zero penalty for small design changes less than some value A, with larger changes penalized.

The geometrical penalty function optimizes the elastic axes location through penalizing the physical differences in angular orientation and origin position between the elastic axes and the reference coordinate system. Since the angle (DA) and translation (TR) of the elastic center are obtained by computing the elastic axes, they are a function of design parameters. First step in determining the minimum angles is to find the angles between each one of the elastic axes and the X, Y, and Z axes. After obtaining three angles for each of elastic axes, smallest matching axes corresponding to each elastic axes can be found by comparing these angles numerically so that the sum of the square of those angle differences becomes the minimum (Appendix B). The penalty function expression becomes

$$L(E) = DA(EA_1 - X)^2 + DA(EA_2 - Y)^2 + DA(EA_3 - Z)^2 + c*TR \qquad (18)$$

where $DA(EA_1\text{-}X)$, $DA(EA_2\text{-}Y)$, and $DA(EA_3\text{-}Z)$ are the smallest possible angles between the elastic axes, EAs, and XYZ. TR is the translation of the center of elasticity, and c is scalar weight factor which indicates relative importance compared to DA terms.

The numerical penalty function optimizes elastic axes location by penalizing non-zero off-diagonal terms of the submatrix in the six by six flexibility matrix (Eq. 15). This method is a shortcut to align the elastic axes with numerical efficiency, although it does not necessarily guarantee elastic axes alignment except in the limit.

$$\mathbf{A} = \mathbf{K}^{-1} = \left[\begin{array}{c|c} & \begin{matrix} \ddots & * & * \\ * & \ddots & * \\ * & * & \ddots \end{matrix} \\ \hline & \\ & \end{array}\right] \qquad (19)$$

where: $*$ = off diagonal terms of submatrix

In this case the penalty function is replaced as a function of elements of **A**.

$$L(E) = a_{15}^2 + a_{16}^2 + a_{24}^2 + a_{26}^2 + a_{34}^2 + a_{35}^2 \qquad (20)$$

where each a_{ij} is an element of the six by six flexibility matrix. Since the flexibility matrix is symmetric in this study, only the upper off diagonal terms are included in the penalty function.

EXAMPLE PROBLEM AND RESULT

This example problem used previously published engine/mount data from General Motors Corporation [Spiekermann (1983)]. A torque around the Y axes, imitating real engine torque around the crank shaft, is applied by setting force/moment vector elements, **F**, zero except the Y axis torque. A simple unit force vector, $\mathbf{F}^T=\{0, 0, 0, 0, 1.0, 0\}$, was used to reduce the complexity and make the results easy to visualize. The example was used to test the decoupling of the system and whether the elastic axes decoupling helps the vibration isolation.

Four different result cases are investigated in this example problem. Case 1 is the results using original system input data. Case 2 to 4 are results for various methods of realignment of elastic axes (Table 1).

Table 1 Classification of the Example Cases

	Optimization Method	Design Parameter Change
Case 1	None	None
Case 2	Geometrical	All (coord., stiffness, orientation)
Case 3	Numerical	All (coord., stiffness, orientation)
Case 4	Numerical	Only orientation of the Mounts

The results for these cases include the elastic center rotation matrix (**R**), the location of center of elasticity (C.E.), and the frequency response plot of the system. The combined result of the location of the C.E. and the elastic center rotation matrix determines the alignment of the elastic axes. The RMS of the C.E. coordinate gives the translational alignment of the elastic axes to the center of reference coordinates (Table 2). The elastic center rotation matrix determines the angular alignment of elastic axes (Table 3). The diagonal elements of the elastic center rotation matrix give the proximity of the elastic axes to the XYZ reference coordinate. The closer the absolute values of these elements is to 1.0,

the closer the alignment is to the reference coordinate system. More detailed optimized results for these cases are placed in Appendix A which includes the optimized flexibility matrices to help visualize the decoupled condition.

Table 2 Center of Elasticity Measured from C.G. (m)

	X	Y	Z	RMS
Case 1	0.150992	0.005021	-0.045411	0.157758
Case 2	-0.000002	0.000002	0.000001	0.000003
Case 3	-0.001752	-0.005120	-0.015521	0.016437
Case 4	0.069853	0.019008	-0.027679	0.077504

Table 3 Elastic Center Rotation Matrices for Original and Optimized

Case 1	$\mathbf{R}_{original} = \begin{bmatrix} 0.751179 & -0.654768 & -0.083723 \\ 0.642215 & 0.695601 & 0.322024 \\ -0.152614 & -0.295666 & 0.943022 \end{bmatrix}$
Case 2	$\mathbf{R}_{optimized} = \begin{bmatrix} -0.999973 & 0.007219 & 0.001090 \\ -0.007220 & -0.999974 & -0.000456 \\ 0.001087 & -0.000464 & 0.999999 \end{bmatrix}$
Case 3	$\mathbf{R}_{optimized} = \begin{bmatrix} 0.985475 & 0.018507 & -0.168807 \\ 0.014039 & -0.999520 & -0.027623 \\ -0.169238 & 0.024852 & -0.985262 \end{bmatrix}$
Case 4	$\mathbf{R}_{optimized} = \begin{bmatrix} 0.989988 & 0.053136 & -0.130765 \\ 0.137865 & -0.984798 & 0.171045 \\ -0.152614 & -0.165375 & 0.976547 \end{bmatrix}$

The frequency response plots determine the quality of vibration isolation. Figure 2 is the frequency response displacement plot of the original system (Case 1) calculated in three reference coordinate directions when a unit torque is applied about Y axis. The plot shows the highest peak of 4.0E-5 (m) at the frequency of 6.42 Hz in Z direction.

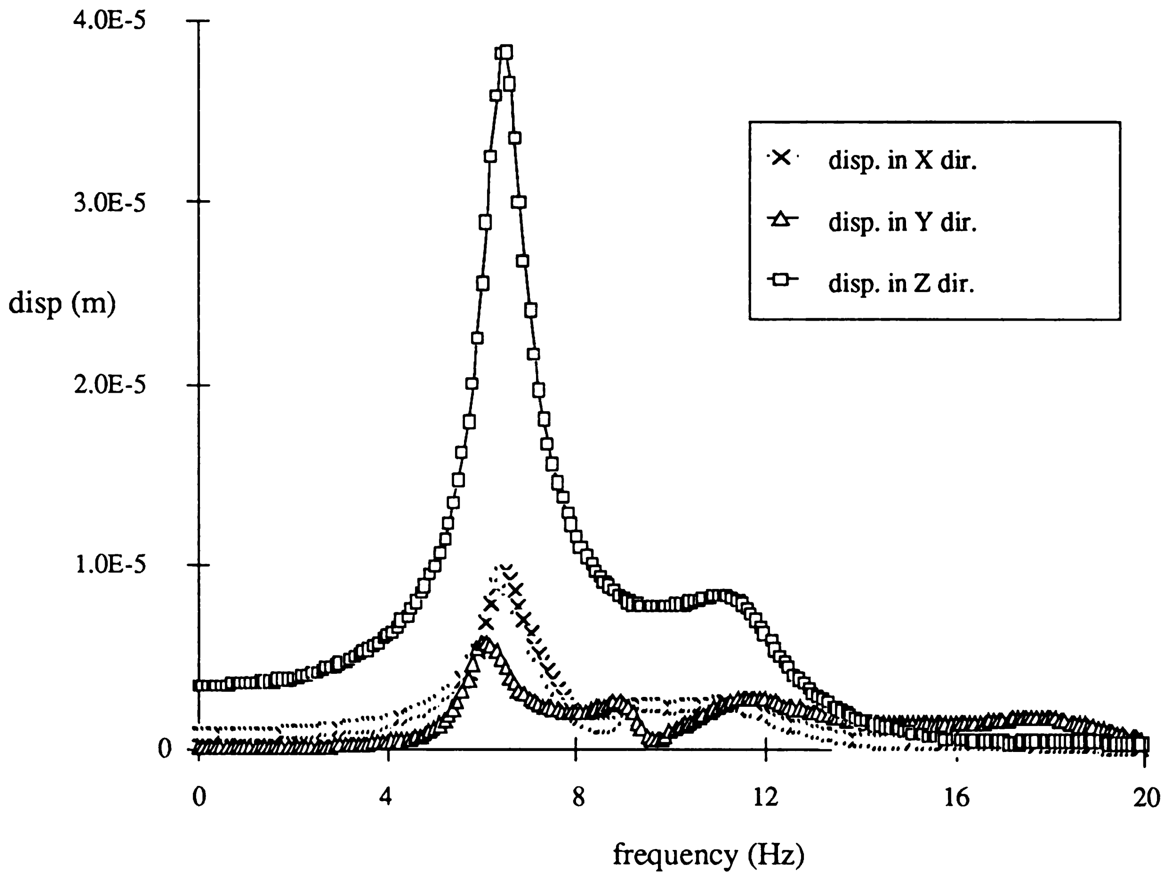

Figure 2 Frequency response plot of original system, for a unit torque (τ_y=1.0 N m) applied along Y axes @ C.G. (Case 1)

Figure 3 shows the realigned frequency response using physical location of the elastic axes, angle and translation, with all design parameters allowed to change (Case 2). In this plot, there is no sign of response decoupling, even though the elastic axes are aligned very closely to the reference coordinate system. The plot shows the response peak of 7.0E-6 m at 10.28 Hz in Y direction.

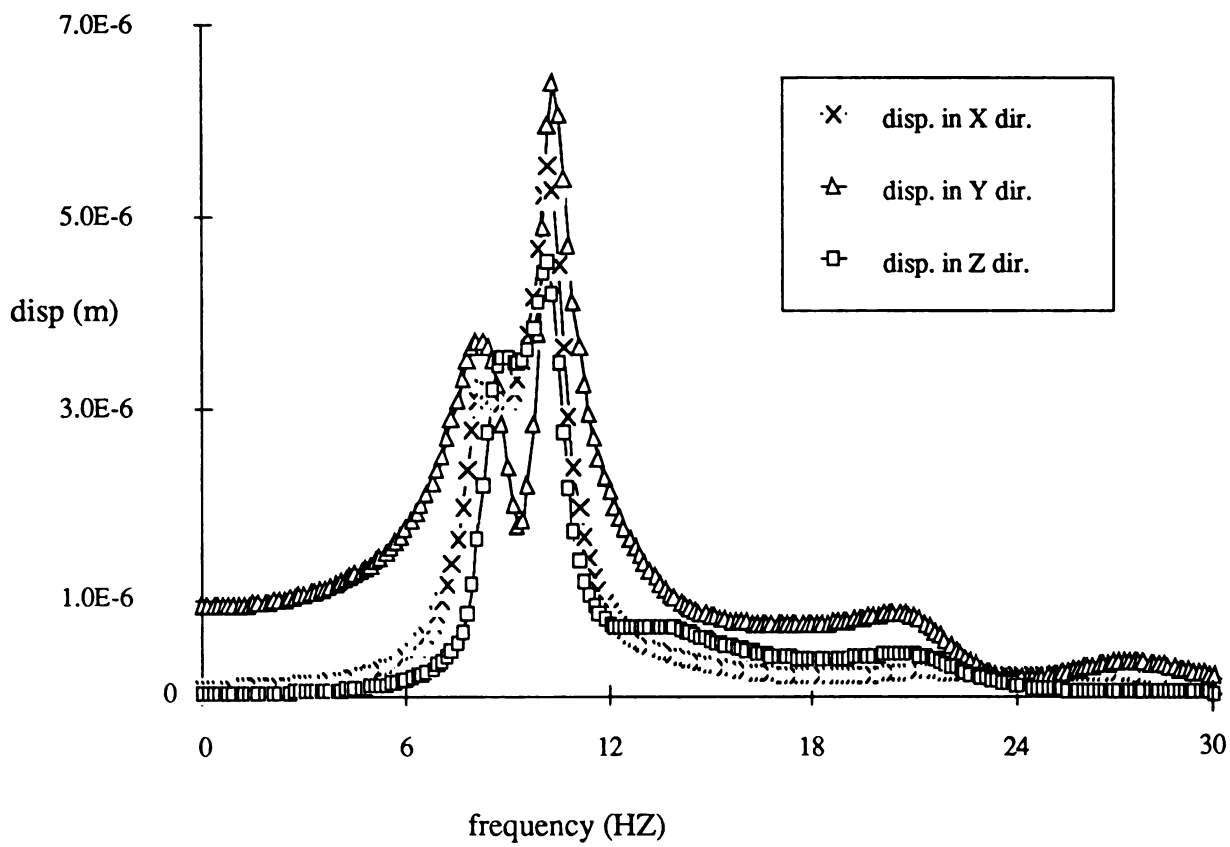

Figure 3 Frequency response plot of geometrically realigned system using Angle & Translation with all design parameters allowed to change, for a unit torque (τ_y=1.0 Nm) applied along Y axis @ C.G. (Case 2)

Figure 4 shows the realigned frequency response using off-diagonal terms of the submatrices of the flexibility matrix with all design parameters allowed to change (Case 3). This plot shows the remarkable decoupling of the response and displacement in X and Z direction has been greatly reduced. The highest peak of the response has been reduced to 5.0E-6 (m) at 9.36 Hz in Y direction.

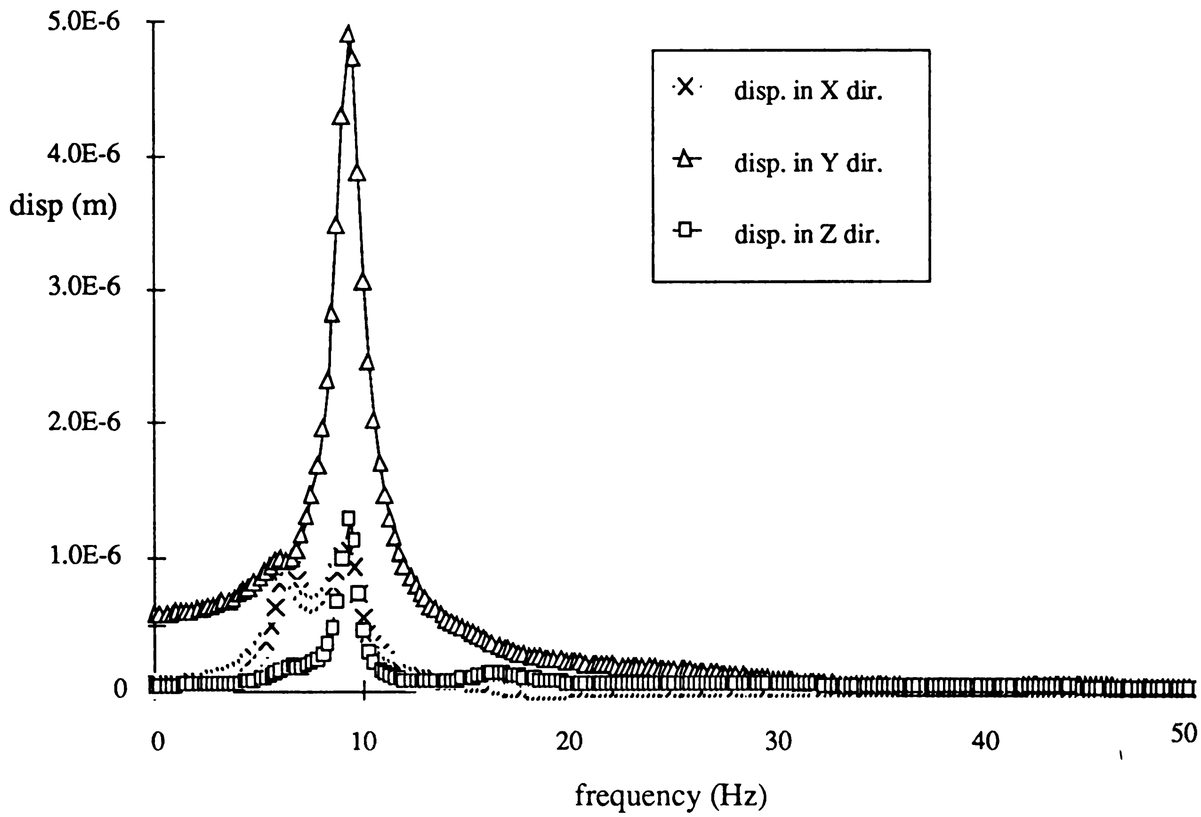

Figure 4 Frequency response plot of numerically realigned system using off-diagonal terms of **A** with all design parameters allowed to change for a unit torque (τ_y=1.0 N m) applied along Y axis @ C.G. (Case 3)

The optimization procedure was constrained by requiring the design parameters to fall within a prescribed range. Since this was a theoretical investigation, large design changes were allowed (Table A.2.1,A.3.1 in Appendix A) which may not be acceptable for real automobile mounts. Nearly all the design parameters were changed by the optimization (Case 2 and 3). In Case 3, coordinates were changed from 37 to 1230 mm, stiffness from 10.99 to 50.18 percent of the original stiffness, and orientation from 3.24 to 188.58 degrees. The design changes used in the previous realigned examples are too large to be practical in automobile industry. Another realignment example with only mount orientation parameters allowed to change was investigated for this reason (Case 4). Figure 5 shows the numerically realigned frequency response using the off-diagonal term penalty function while allowing only mount orientation change. The result does not show any improvement in response decoupling.

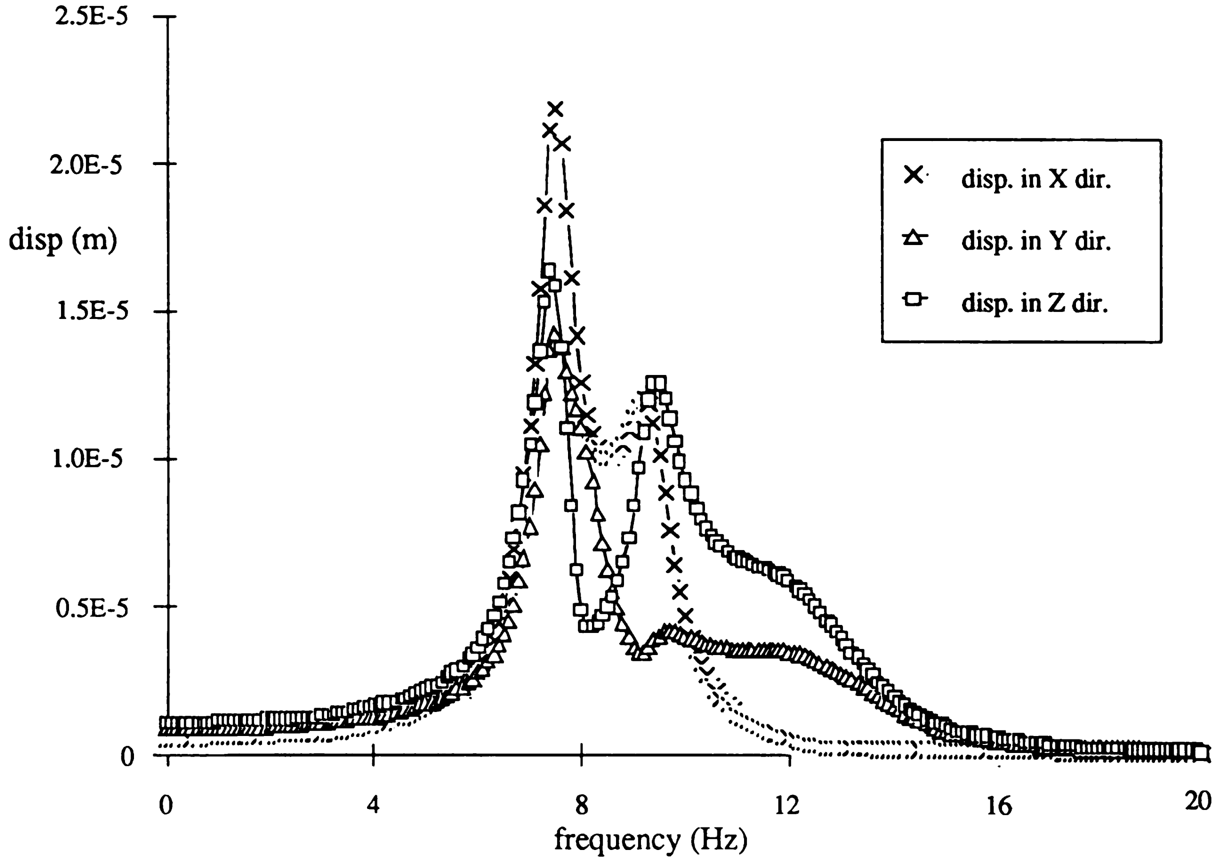

Figure 5 Frequency response plot of numerically realigned system using off-diagonal terms of **A**, with only orientation of the mounts is allowed to change, for unit torque (τ_y= 1.0 Nm) applied along Y axis @ C.G. (Case 4)

Figure 6 shows the RMS value of the X, Y, and Z displacement for each test case to demonstrate the total magnitude of the vibration. All the realigned designs shows the

improved results in reducing the total displacement of the system. The RMS plot shows that the total magnitude of the vibration can be reduced with only mount orientation change, but not reduced as large as the model realigned with all the parameters allowed to change.

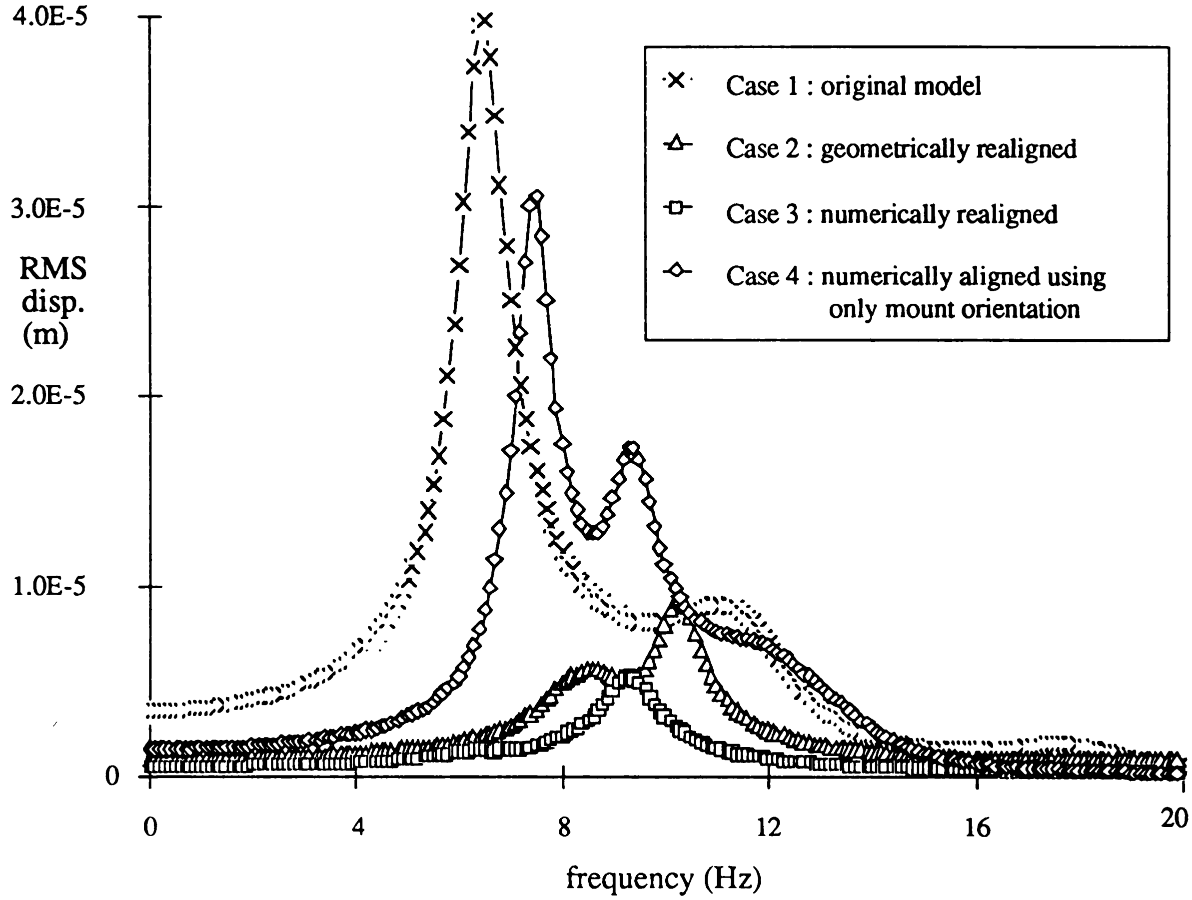

Figure 6 RMS displacement response plot for various methods of elastic axes realignment

If the static response is too large, the design may not be feasible due to limited space or material property constraints. Static response should also be constrained to meet the design requirements. The static displacement response in XYZ direction (ω=0.0) of Case 2, and 3 were minimized except the displacement in Y direction. All the realigned designs have smaller static RMS displacement than original system.

Table 4 Static Displacement Response of Example Cases for τ_y = 1.0 N m

	disp. in X dir.	disp. in Y dir.	disp. in Z dir.	RMS
Case 1	7.5842E-7	0.1325E-7	34.261E-7	35.091E-7
Case 2	0.8136E-7	9.2886E-7	0.2160E-7	9.3266E-7
Case 3	0.5520E-7	5.7619E-7	0.4227E-7	5.8037E-7
Case 4	5.1521E-7	8.4921E-7	10.539E-7	14.482E-7

Natural frequency is a important factor for the vibration isolation. When the excitation frequency is near one of the natural frequencies of the system, both the rigid body displacement and the forces transmitted through the mounts can be large. One way to avoid resonance is to remove the natural frequency from the desirable frequency range. In present designs, automobile have idle speed range of 600-780 RPM (10 to 13 Hz). An optimal design should avoid this natural frequency range. In the result of Case 3, the natural frequencies between 10 and 13 Hz were all removed from the idling frequency range of the engine.

Table 5 Natural Frequencies for the Original and Realigned System (Hz)

Case 1	5.94	6.42	7.67	9.11	11.54	18.06
Case 2	8.11	8.54	10.28	13.55	21.28	27.39
Case 3	6.21	9.25	9.36	16.23	28.06	45.96
Case 4	6.69	7.46	7.97	9.36	12.17	13.63

The effectiveness of the vibration isolation in this problem is determined by the normalized RMS displacement response to an input torque as compared to the original design. Normalized RMS response is considered because large stiffness of the mounts can easily reduce the vibratory displacement while the forces transmitted through the mounts are still large. The normalized response is obtained by dividing frequency response by static response.

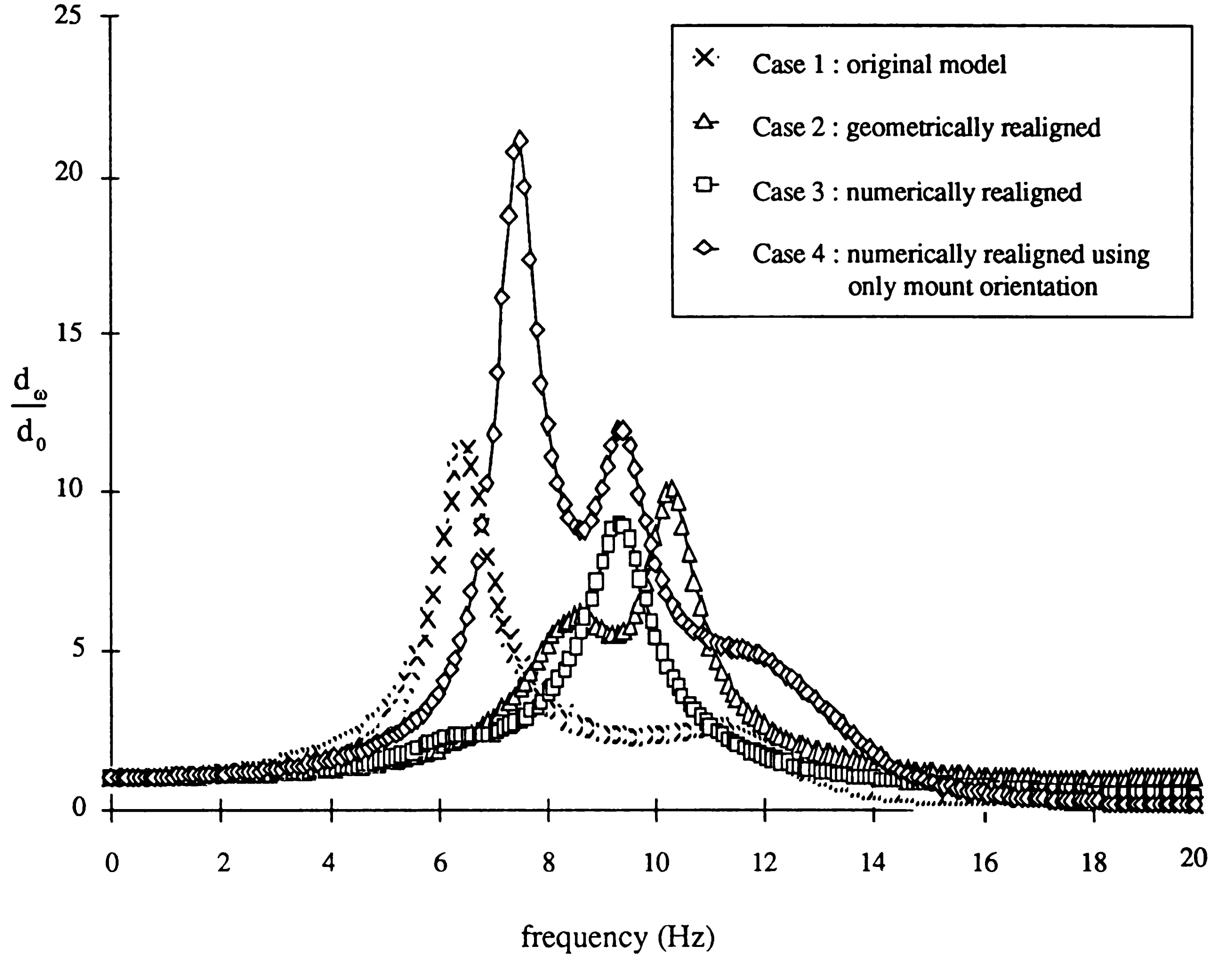

Figure 7 Normalized RMS displacement response plot for various methods of elastic axes realignment

Case 2 and Case 3 show only small reduction of the normalized peak response compared to original design, Case 1. The result also shows that changing only mount orientation cannot make any improvement in vibration isolation. None of the realigned elastic axes designs show significant improvement in vibration isolation.

SUMMARY

An optimization of a three-dimensional vibration isolation system for partial decoupling of the system's compliance has been demonstrated. This thesis has shown that elastic axes can be aligned to a fixed coordinate system of a designer's choice. Two different approaches of aligning elastic axes to a fixed coordinate were investigated: 1) using physical location of elastic axes and 2) using off-diagonal terms of flexibility matrix. Both of these approaches could decouple the static response, and only the latter approach could decouple the dynamic response when the elastic axes are aligned to an excitation force. The normalized RMS plot showed that this mothod could reduce only small amount of the vibration of the system. The plot showed that the proper mount design changes can reduce the original normalized response peak of 11.32 down to 9.01 (Case 3). This 20% reduction of the normalized displacement is not significant to mount designers and can hardly be said to help the vibration isolation.

The results showed that the more accurate decoupling of the flexibility matrix does not necessarily mean decoupling the response of the system. Elastic axes of Case 2 aligned much closer to the reference coordinate than any other cases (Table 2 and 3), but the response did not show any improvement in decoupling the response (Figure 3). The clear explanation about this cause has not been found yet, and the investigation of this reason is left as a further study of this problem.

Factors not addressed in this thesis are torsional response, and numerical sensitivity of the results. Since the results are only displacements of the C.G. instead of forces transmitted through each mounts, it does not guarantee that this optimization of elastic axes is the solution to the minimization of the forces through the mounts. Future topics include development of a program that computes the forces transmitted through each mount.

APPENDICES

APPENDIX A - More Results for Example Problems

CASE 1 Original System

Table A.1.1 Original Engine System Input

Mass (kg) 225.40

Local Coordinates (m)

	X	Y	Z
C.G.	0.0000	0.0000	0.0000
monnt 1	-0.1866	-0.2893	-0.0100
mount 2	0.4334	-0.2993	-0.1600
mount 3	-0.0916	0.1707	-0.0850
mount 4	0.4534	0.1657	-0.1000

Mount stiffness (N/m)

	compression	lateral	fore/aft
mount 1	203667.	30733.	43733.
mount 2	160167.	115050.	49619.
mount 3	219167.	439334.	102583.
mount 4	225207.	440334.	116083.

Mount orientation (deg)

	theta X	theta Y	theta Z
mount 1	0.0	-30.0	0.0
mount 2	0.0	-41.0	180.0
mount 3	0.0	-70.0	0.0
mount 4	0.0	-48.0	180.0

Table A.1.2 Flexibility Matrix of Original System

$$\mathbf{A}_{Ref} = \mathbf{K}_{Ref}^{-1} = \begin{bmatrix} 1.857 & 0.068 & -0.096 & -0.440 & -0.736 & -0.561 \\ & 1.645 & 0.085 & -3.333 & 0.003 & -1.735 \\ & & 2.259 & -0.391 & 3.440 & -0.236 \\ & & & 37.477 & 2.338 & -1.639 \\ & & & & 25.293 & -1.163 \\ & & & & & 9.545 \end{bmatrix} \times 10^{-6}$$

CASE 2 Optimized with all design parameters change
(using DA and TR)

Table A.2.1 Optimized System and Changes

Coordinates (m)

	X		Y		Z	
mount 1	.593	.779	-.521	-.231	.460	.470
mount 2	.404	-.029	.645	.945	-.780	-.620
mount 3	-.491	-.399	-.314	-.484	-.396	-.311
mount 4	-.078	-.532	-.001	-.167	.109	.209
	NEW VALUE	CHG	NEW VALUE	CHG	NEW VALUE	CHG

Mount Stiffness (N/m)

	compression		lateral		fore/aft	
mount 1	276676.	35.85%	49584.	61.34%	75350.	72.30%
mount 2	157891.	-1.42%	227030.	97.33%	44380.	-10.56%
mount 3	232747.	6.20%	496946.	13.11%	123355.	20.25%
mount 4	219866.	-2.37%	362468.	-17.68%	168123.	44.83%
	NEW VALUE	% CHG	NEW VALUE	% CHG	NEW VALUE	% CHG

Mount Orientation (deg)

	THETAX		THETAY		THETAZ	
mount 1	9.72	9.72	-159.60	-129.60	-66.07	-66.07
mount 2	115.11	115.11	-56.69	-15.69	212.03	32.03
mount 3	238.14	238.14	-45.09	24.91	252.34	252.34
mount 4	-345.62	-345.62	26.42	74.42	155.82	-24.18
	NEW VALUE	CHG	NEW VALUE	CHG	NEW VALUE	CHG

Table A.2.2 Optimized Flexibility Matrix

Flexibility matrix in reference coordinates

$$\mathbf{A}_{Ref} = \mathbf{K}_{Ref}^{-1} = \begin{bmatrix} 1.486 & -0.250 & -0.178 & -0.352 & 0.009 & 0.000 \\ & 1.313 & -0.112 & 0.009 & 0.937 & 0.001 \\ & & 1.412 & 0.000 & 0.001 & -0.765 \\ & & & 3.202 & 0.954 & 1.477 \\ & & & & 5.486 & 0.372 \\ & & & & & 8.575 \end{bmatrix} \times 10^{-6}$$

Flexibility matrix in principal elastic axis coordinates

$$\mathbf{A}_{C.E.} = \mathbf{K}_{C.E.}^{-1} = \begin{bmatrix} 1.490 & -0.249 & 0.177 & -0.352 & 0.000 & 0.000 \\ & 1.310 & 0.113 & 0.000 & 0.937 & 0.000 \\ & & 1.411 & 0.000 & 0.000 & -0.765 \\ & & & 3.185 & 0.937 & -1.468 \\ & & & & 5.500 & -0.385 \\ & & & & & 8.578 \end{bmatrix} \times 10^{-6}$$

CASE 3 **Optimized with all design parameters change**
(using off-diagonal terms of **A**)

Table A.3.1 Optimized System and Changes

Coordinates (m)

	X NEW VALUE	X CHG	Y NEW VALUE	Y CHG	Z NEW VALUE	Z CHG
mount 1	-.610	-.423	-.476	-.187	.307	.317
mount 2	.396	-.037	-.727	-.428	-.676	-.516
mount 3	-1.322	-1.230	.531	.360	1.098	1.183
mount 4	.520	.066	.479	.314	-.413	-.313

Mount Stiffness (N/m)

	compression NEW VALUE	compression % CHG	lateral NEW VALUE	lateral % CHG	fore/aft NEW VALUE	fore/aft % CHG
mount 1	284917.	39.89%	42994.	39.89%	61180.	39.89%
mount 2	203768.	27.22%	146369.	27.22%	63126.	27.22%
mount 3	109194.	-50.18%	218887.	-50.18%	51109.	-50.18%
mount 4	249957.	10.99%	488727.	10.99%	128840.	10.99%

Mount Orientation (deg.)

	theta X NEW VALUE	theta X CHG	theta Y NEW VALUE	theta Y CHG	theta Z NEW VALUE	theta Z CHG
mount 1	3.24	3.24	-218.58	-188.58	54.20	54.20
mount 2	51.24	51.24	-68.24	-27.24	142.20	-37.80
mount 3	-58.50	-58.50	-136.27	-66.27	51.53	51.53
mount 4	84.01	84.01	-128.67	-80.67	267.01	87.01

Table A.3.2 Optimized Flexibility Matrix

<u>Flexibility matrix in reference coordinates</u>

$$\mathbf{A}_{Ref} = \mathbf{K}_{Ref}^{-1} = \begin{bmatrix} 2.825 & -0.336 & 0.134 & 0.354 & 0.048 & -0.005 \\ & 1.366 & -0.042 & -0.046 & -0.582 & 0.014 \\ & & 1.293 & 0.024 & -0.030 & 0.362 \\ & & & 3.509 & 0.102 & -2.230 \\ & & & & 2.406 & 0.460 \\ & & & & & 3.554 \end{bmatrix} \times 10^{-6}$$

<u>Flexibility matrix in principle elastic axis coordinates</u>

$$\mathbf{A}_{C.E.} = \mathbf{K}_{C.E.}^{-1} = \begin{bmatrix} 2.724 & 0.325 & -0.388 & 0.340 & 0.000 & 0.000 \\ & 1.372 & -0.104 & 0.000 & -0.583 & 0.000 \\ & & 1.386 & 0.000 & 0.000 & 0.376 \\ & & & 4.253 & 0.064 & 2.100 \\ & & & & 2.431 & 0.519 \\ & & & & & 2.785 \end{bmatrix} \times 10^{-6}$$

<u>Case 4</u> **Optimized with only mount orientation change**
(using off-diagonal terms of **A**)

Table A.4.1 Optimized System and Changes

Mount Orientation (deg)

	theta X		theta Y		theta Z	
mount 1	-25.51	-25.51	-240.44	-210.44	98.89	98.89
mount 2	27.49	27.49	-27.19	13.81	181.05	1.05
mount 3	-82.13	-82.13	-135.08	-65.08	-103.80	-103.80
mount 4	29.95	29.95	20.78	68.78	128.87	-51.13
	NEW VALUE	CHG	NEW VALUE	CHG	NEW VALUE	CHG

Table A.4.2 Optimized Flexibility Matrix

$$\mathbf{A}_{Ref} = \mathbf{K}_{Ref}^{-1} = \begin{bmatrix} 1.661 & -0.118 & 0.323 & -0.217 & 0.4860 & 0.319 \\ & 1.596 & 0.166 & -0.286 & -0.858 & -1.128 \\ & & 1.487 & -0.140 & 1.015 & 0.571 \\ & & & 30.357 & 4.374 & -7.895 \\ & & & & 19.908 & -3.572 \\ & & & & & 15.084 \end{bmatrix} \times 10^{-6}$$

APPENDIX B - Optimization Using Elastic Axes

This appendix presents the elastic axes analysis and detailed example results for determining location of the elastic axes. Elastic axes coordinates as a vector can be found by multiplying the three by three elastic center rotation matrix, **R**, to the reference coordinate system. The following shows the way to find the one of three elastic axes

$$EA_1 = \text{Elastic Axis (X)} = \mathbf{R}\begin{Bmatrix} X \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} XX' \\ XY' \\ XZ' \end{Bmatrix} \tag{B.1}$$

By adding these coordinate to the center of elasticity, the transformed coordinates of the location of elastic axis can be found. Since the final purpose is the angle difference, only vector form of the elastic axes (equation B.1) is needed to calculate the angle difference. Simple trigonometry is used to determine the angle-difference, DA, between EA_1 and three X, Y, and Z coordinate (equation B.2,3,4).

$$DA(EA_1 - X) = \cos^{-1}\left(\frac{X * XX'}{\sqrt{X^2 * \sqrt{XX'^2 + XY'^2 + XZ'^2}}} \right) \tag{B.2}$$

$$DA(EA_1 - Y) = \cos^{-1}\left(\frac{Y * XY'}{\sqrt{X^2 * \sqrt{XX'^2 + XY'^2 + XZ'^2}}} \right) \tag{B.3}$$

$$DA(EA_1 - Z) = \cos^{-1}\left(\frac{Z * XZ'}{\sqrt{X^2 * \sqrt{XX'^2 + XY'^2 + XZ'^2}}} \right) \tag{B.4}$$

The rest of the angles can be obtained by the same manner. Once all the angles are obtained, three angles per one elastic axes, the matching angles should be found to minimize the penalty function. The following shows the example run to calculate the angle

difference and to find the smallest matching axes. The following is the result from original model.

```
> Choose one of the options 'by entering two letters
ANIMATE MODE-(AM) STATIC DEFLECT-(SD) RESTART ENGSIM----
(RS)
MOUNT FORCES-(MF) FREQ RESPONSE--(FR) RESTART NEW INPUT-
(NU)
CHG NORM------CN) OPTIM PARAMS---(OP) SAVE ENGSIM FILE--
(EF)
ELASTIC AXIS-(EA)                     QUIT--------------
(QU)
EA

  CENTER OF ELASTICITY   (Measured from the C.G.)
        X          Y          Z
     .150992     .005021   -.045411

  ELASTIC CENTER ROTATION MATRIX
     .751179   -.654768   -.083723
     .642215    .695601    .322024
    -.152614   -.295666    .943022

  TRANSLATION OF THE ELASTIC CENTER (m)     .15775

  3 ANGLES BETWEEN #1 ELASTIC AXIS & X-Y-Z COORD (rad)
      .72095      .87341     1.41758

  3 ANGLES BETWEEN #2 ELASTIC AXIS & X-Y-Z COORD (rad)
      .85692      .80154     1.27064

  3 ANGLES BETWEEN #3 ELASTIC AXIS & X-Y-Z COORD (rad)
     1.48698     1.24293      .33920

  CLOSEST 3 ANGLES BETWEEN 3 ELASTIC AXIS AND X-Y-Z
  COORD. (rad)
      .33920      .72095      .80154
```

APPENDIX C - Using ENGSIM III

This appendix presents an example run of the rigid body engine dynamics simulation program, ENGSIM III, which was used for main analysis discussed in this thesis. Because ENGSIM II was developed to run on Prime Computers and ENGSIM III was modified to run on Macintosh Computer, some of the ENGSIM II capabilities were deleted. One of main capability deleted is the graphic capability, since the MacFortran II does not support graphics. Since the clear explanation to use this software has been already introduced in the thesis of the writer of ENGSIM II, the user's manual is not repeated and the thesis is placed in reference list. Only the parts which related to this study are demonstrated here.

The following is the input file used for the simulation and also shows the format of the file.

```
------------ INPUT1 -----------------
***************************
Oldsmobile engine test stand configuration.
**
************************
Mass of engine.  Kilogram mass and kilogram force are
numerically equal.
**
225.4
************************
Engine center of gravity coordinates.  (X Y Z  Meters)
**
1.4366 .0793 .51
************************
Number of engine mounts.
**
4
************************
Engine mount coordinates. (Meters) (X Y Z mount #1, X Y Z
mount #2 etc.)
**
1.250  -.21   .500
1.870  -.220  .35
1.345   .25   .425
```

```
1.89    .245  .410
************************
Mount Stiffness.
** Compression Lateral Fore/Aft (N/m) ThetaX ThetaY ThetaZ
(Degrees)
203667.  30733.  43733.  0.  -30.  0.
160167. 115050.  49619.  0.  -41.  180.
219167. 439334. 102583.  0.  -70.  0.
225207. 440334. 116083.  0.  -48.  180.
******************
Engine mass moment of inertia matrix.  (N-M-SEC2)
**
15.80 -0.80   .9
-0.80 11.64 -3.2
  .90 -3.2  15.69
******************
Direction Cosine Angles to Principal Inertia Axis  (Degrees)
**
17.87  73.91  82.43
100.52 31.07 118.87
104.28 64.19  30.03
******************
Mount viscous damping.  Compression Lateral Fore/Aft   (N-
sec/M)
**
 'A' 100. 110. 120.
 'A' 130. 140. 150.
 'A' 160. 170. 180.
 'A' 190. 200. 210.
******************
Mount structural damping.  Compression Lateral Fore/Aft
(N/M)
**
 'A'  4000.  5000.  6000.
 'A'  7000.  8000.  9000.
 'A' 10000. 11000. 12000.
 'A' 13000. 14000. 15000.
************************
Number of cradle mounts. (enter 0 for no cradle)
**
6
************************
Cradle mount coordinates. (Meters) (X Y Z mount #1, X Y Z
mount #2 etc.)
**
1.125 -.541  .511
1.125  .541  .511
2.041 -.4265 .3495
2.041  .4265 .3495
2.168 -.584  .3495
2.168  .584  .3495
************************
Cradle stiffness (N/M) (X Y Z mount #1,X Y Z mount #2 etc.)
**
```

```
144000 280000 400000 0 0 0
144000 280000 400000 0 0 0
250000 520000 950000 0 0 0
250000 520000 950000 0 0 0
250000 520000 950000 0 0 0
250000 520000 950000 0 0 0
*************************
EOF
```

The following is the optimization design change input file used in the main part of this thesis (Case 2 and 3 of the Example problems) which shows the amount of design changes allowed in the simulation. In this file "A" means absolute value change and "%" means percentage of the value change.

```
---------- INPUTOP5 -------------
------------------------------
Demo Optimization input file, Changes all possible
design
---
------------------------------
 Number of engine mounts.
---
 4
------------------------------
 Mount location of optimization parameters
---
------------------------------
 Changes associated with E=1 (% or A)(XYZ values)
---
     'A'    0.50    0.50    0.50
     'A'    0.50    0.50    0.50
     'A'    0.50    0.50    0.50
     'A'    0.50    0.50    0.50
------------------------------
 Starting E
---
        0.00    0.00    0.00
        0.00    0.00    0.00
        0.00    0.00    0.00
        0.00    0.00    0.00
------------------------------
 Minimum E without penalty
---
        0.00    0.00    0.00
        0.00    0.00    0.00
```

```
        0.00     0.00     0.00
        0.00     0.00     0.00
------------------------------
 Maximum E without penalty
---
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
------------------------------
 Mount stiffness optimization parameters
---
------------------------------
 Changes associated with E=1 (% or A)(XYZ values)
---
     '%'   100.000   100.000   100.000
     '%'   100.000   100.000   100.000
     '%'   100.000   100.000   100.000
     '%'   100.000   100.000   100.000
------------------------------
 Starting E
---
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
------------------------------
 Minimum E without penalty
---
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
------------------------------
 Maximum E without penalty
---
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
------------------------------
 Mount orientation optimization parameters
---
------------------------------
 Changes associated with E=1 (% or A)(XYZ values)
---
     'A'  100.000  100.000  100.000
     'A'  100.000  100.000  100.000
     'A'  100.000  100.000  100.000
     'A'  100.000  100.000  100.000
------------------------------
 Starting E
---
        0.00     0.00     0.00
```

```
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
------------------------------
 Minimum E without penalty
---
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
------------------------------
 Maximum E without penalty
---
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
        0.00     0.00     0.00
EOF
```

A sample run of ENGSIM III is presented next. Program prompts and output are indented and user typed answers shown underlined.

```
          EEEE  N   N  GGGG  SSSS  I  M   M   3333333
          E     NN  N  G     S     I  MM MM    3 3 3
          EEEE  N N N  G GG  SSSS  I  M M M    3 3 3
          E     N  NN  G  G     S  I  M   M    3 3 3
          EEEE  N   N  GGGG  SSSS  I  M   M   3333333

          Michigan State University

     RIGID BODY ENGINE DYNAMICS OPTIMIZATION/SIMULATION


** Rev. 3.0 **



    > ENTER NAME OF FILE WITH MOUNT GEOMETRY, STIFFNESS, &
      DAMPING
INPUT1
```

```
WEIGHT (NEWTONS)  2208.92        MASS (KILOGRAMS)   225.40

COORDINATES(METERS) GLOBAL                          LOCAL
             X        Y        Z         X        Y        Z
 C.G.      1.4366   .0793   .5100     .0000   .0000   .0000
 MOUNT 1   1.2500  -.2100   .5000    -.1866  -.2893  -.0100
 MOUNT 2   1.8700  -.2200   .3500     .4334  -.2993  -.1600
 MOUNT 3   1.3450   .2500   .4250    -.0916   .1707  -.0850
 MOUNT 4   1.8900   .2450   .4100     .4534   .1657  -.1000

MOUNT STIFFNESS  (NEWTONS/METER)
        COMPRESSION LATERAL FORE/AFT THETAX    THETAY   THETAZ
 MOUNT 1 203667.  30733. 43733. .0 .0  -30.0  -.5       .0   .0
 MOUNT 2 160167. 115050. 49619. .0 .0  -41.0  -.7  180.0  3.1
 MOUNT 3 219167. 439334 102583. .0 .0  -70.0  -1.2      .0   .0
 MOUNT 4 225207. 440334 116083. .0 .0  -48.0  -.8  180.0  3.1

MOUNT DAMPING  (N-sec/M)   VISCOUS                    STRUCTURAL
  COMPRESSION LATERAL FORE/AFT   COMPRESSION LATERAL FORE/AFT
 MOUNT 1     100.0  110.0   120.0      4000.0   5000.0   6000.0
 MOUNT 2     130.0  140.0   150.0      7000.0   8000.0   9000.0
 MOUNT 3     160.0  170.0   180.0     10000.0  11000.0  12000.0
 MOUNT 4     190.0  200.0   210.0     13000.0  14000.0  15000.0

    > DO YOU WANT TO CHANGE ANY OF THESE VALUES ENTER Y OR N
N

    > ENTER COMPREHENSIVE LEVEL OF OUTPUT (MINIMUM= 1
      MAXIMUM= 4)
1

  MASS MATRIX EQUALS...
    225.40       .00       .00       .00       .00       .00
       .00    225.40       .00       .00       .00       .00
       .00       .00    225.40       .00       .00       .00
       .00       .00       .00     15.80       .80      -.90
       .00       .00       .00       .80     11.64      3.20
       .00       .00       .00      -.90      3.20     15.69

  STIFFNESS MATRIX EQUALS...
557431.60        -.02  -2276.35   -6247.89  18656.93  33880.79
     -.02  1025451.00      -.01  100092.11       .01 203532.37
 -2276.35        -.01 562794.40   10993.51 -77328.33   6247.89
 -6247.89   100092.11  10993.51   37070.46  -4011.25  23970.00
 18656.93         .01 -77328.33   -4011.25  51183.29   4730.49
 33880.79   203532.37   6247.89   23970.00   4730.49 148587.11

    > Choose the MODE SHAPE NORMALIZATION method
      MASS---(MA)    STIFFNESS---(ST)    LARGEST DOF---(LD)
LD
```

```
  THE MODE SHAPES ARE ...    (normalized to largest DOF)
     X     -.002      .014     -.060     -.024     -.009      .002
     Y      .033     -.002     -.017      .046     -.011      .028
     Z      .001     -.057     -.016     -.004      .031      .004
ThetaX     -.215      .014     -.030      .103      .018      .074
ThetaY     -.029     -.142     -.006      .014     -.247     -.097
ThetaZ     -.010     -.002      .057     -.116     -.017      .226

  NATURAL FREQUENCIES ... (CYCLES/SEC)
       5.94       6.42       7.67       9.11      11.54      18.06


    > Choose one of the options 'by entering two letters
    ANIMATE MODE-(AM) STATIC DEFLECT-(SD) RESTART ENGSIM----
(RS)
    MOUNT FORCES-(MF) FREQ RESPONSE--(FR) RESTART NEW INPUT-
(NU)
    CHG NORM-----(CN) OPTIM PARAMS---(OP) SAVE ENGSIM FILE--
(EF)
    ELASTIC AXIS-(EA)                     QUIT--------------
(QU)

OP
OP> ENTER LOWER & UPPER FREQ LIMITS FOR UNDESIRABLE RANGE
10 13
OP> ENTER # OF SIG DIGITS (3 OR LESS)
3
OP> ENTER MAX NUMBER OF FUNCTION CALLS (500 OR SO)
2000
OP> ENTER SCALE FACTORS FOR SIZE OF CHG & FREQ PENALTIES
0.01 0 0 1000000000000
OP> SHOULD THE 3 PRINCIPAL MOUNT STIFFNESSES
    CHANGE INDEPENDENTLY--(I)  MAINTAIN CONSTANT RATIOS--(C)
C
OP> ARE YOU ENTERING THE OPTIMIZATION PARAMETERS
       FROM A FILE (FILE) or INTERACTIVELY (INTER)
FILE
OP> ENTER NAME OF INPUT FILE TO BE READ IN
INPUTOP5


                           OPTIMIZATION PARAMETER TABLE
COORDINATES  LOCAL
          X                       Y                       Z
MOUNT 1 .500 ( .0 .0 .0 ) .500 ( .0 .0 .0 ) .500 ( .0 .0 .0 )
MOUNT 2 .500 ( .0 .0 .0 ) .500 ( .0 .0 .0 ) .500 ( .0 .0 .0 )
MOUNT 3 .500 ( .0 .0 .0 ) .500 ( .0 .0 .0 ) .500 ( .0 .0 .0 )
MOUNT 4 .500 ( .0 .0 .0 ) .500 ( .0 .0 .0 ) .500 ( .0 .0 .0 )
E=1  START MIN  MAX  E=1  START MIN  MAX  E=1  START MIN  MAX

MOUNT STIFFNESS
```

```
                COMPRESSION                          LATERAL
FORE/AFT
MOUNT 1 100.0%(   .0    .0    .0 )
MOUNT 2 100.0%(   .0    .0    .0 )
MOUNT 3 100.0%(   .0    .0    .0 )
MOUNT 4 100.0%(   .0    .0    .0 )
          E=1  START MIN  MAX      E=1  START MIN  MAX      E=1
START MIN  MAX

MOUNT ORIENTATION
                  THETAX                       THETAY
THETAZ
MOUNT 1 100. ( .0 .0 .0 ) 100. ( .0 .0 .0 ) 100. ( .0 .0 .0 )
MOUNT 2 100. ( .0 .0 .0 ) 100. ( .0 .0 .0 ) 100. ( .0 .0 .0 )
MOUNT 3 100. ( .0 .0 .0 ) 100. ( .0 .0 .0 ) 100. ( .0 .0 .0 )
MOUNT 4 100. ( .0 .0 .0 ) 100. ( .0 .0 .0 ) 100. ( .0 .0 .0 )
        E=1 START MIN MAX E=1 START MIN MAX E=1 START MIN MAX

OP> DO YOU WANT TO CHANGE ANY OF THESE VALUES ENTER Y OR N
N
OP> OPTIMIZATION IN PROGRESS

    CODE = 131, ITERATIONS EXCEEDED  2000

FINAL FUNCTION VALUE IS  0.24635D+00
TO MOVE FREQUENCIES OUT OF THE RANGE     10.00 TO     13.00

OBTAINED IN 0.20370D+04 ITERATIONS
OP>

COORDINATES  (M)                             LOCAL
                    X                          Y                  Z
MOUNT 1   -.610   -.423       -.476   -.187         .307   .317
MOUNT 2    .396   -.037       -.727   -.428        -.676  -.516
MOUNT 3 -1.322  -1.230         .531    .360        1.098  1.183
MOUNT 4    .520    .066        .479    .314         -.41  -.313
       NEW VALUE   CHG     NEW VALUE   CHG     NEW VALUE   CHG

MOUNT STIFFNESSS  (N/M)
            COMPRESSSION                      LATERAL
FORE/AFT
MOUNT 1  284917.  39.89%    42994.  39.89%     61180.   39.89%
MOUNT 2  203768.  27.22%   146369.  27.22%     63126.   27.22%
MOUNT 3  109194. -50.18%   218887. -50.18%     51109.  -50.18%
MOUNT 4  249957.  10.99%   488727.  10.99%    128840.   10.99%
    NEW VALUE    % CHG  NEW VALUE    % CHG  NEW VALUE    % CHG

MOUNT ORIENTATION  (DEG)
            THETAX                        THETAY
THETAZ
MOUNT 1     3.24     3.24    -218.58  -188.58     54.20    54.20
MOUNT 2    51.24    51.24     -68.24   -27.      142.20   -37.80
MOUNT 3   -58.50   -58.50    -136.27   -66.27     51.53    51.53
```

```
MOUNT 4   84.00   84.01     -128.67   -80.67   267.01    87.01
    NEW VALUE   CHG        NEW VALUE    CHG    NEW VALUE   CHG

OP> Choose one of the options by entering the two letters.
    FREQS & MODES----(FM) CHG NORM-----(CN) ANIMATE MODES---
(AM)
    OP PARAMETERS----(PA) RESTART OP---(RO) RESTART W/ZEROS-
(RZ)
    SAVE ENGSIM FILE-(EF) SAVE OP FILE-(OF) QUIT OP---------
(QU)
QU



WEIGHT (NEWTONS)  2208.92        MASS (KILOGRAMS)   225.40
COORDINATES  (METERS)    GLOBAL
LOCAL
        X         Y         Z          X        Y         Z
 C.G.      1.4366   .0793    .5100      .0000    .0000     .0000
 MOUNT 1  1.2500  -.2100    .5000     -.6095   -.4760     .3067
 MOUNT 2  1.8700  -.2200    .3500      .3963   -.7272    -.6764
 MOUNT 3  1.3450   .2500    .4250   -1.3220    .5312    1.0983
 MOUNT 4  1.8900   .2450    .4100      .5196    .4795    -.4126

MOUNT STIFFNESS  (NEWTONS/METER)
  COMPRESSION LATERAL FORE/AFT THETAX      THETAY      THETAZ
MOUNT 1 284917. 42994.  61180.  3.2  1   -218.6 -3.8  54.2 .9
MOUNT 2 203768.146369.  63126. 51.2  9    -68.2 -1.2 142.2 2.5
MOUNT 3 109194 218887.  51109.-58.5-1.0 -136.3 -2.4  51.5  .9
MOUNT 4 249957.488727. 128840. 84.0 1.5 -128.7 -2.2 267.0 4.7

MOUNT DAMPING  (N-sec/M)      VISCOUS
STRUCTURAL
COMPRESSION LATERAL FORE/AFT    COMPRESSION LATERAL FORE/AFT
 MOUNT 1    100.0     110.0    120.0      4000.0   5000.0  6000.0
 MOUNT 2    130.0     140.0    150.0      7000.0   8000.0  9000.0
 MOUNT 3    160.0     170.0    180.0     10000.0  11000.0 12000.0
 MOUNT 4    190.0     200.0    210.0     13000.0  14000.0 15000.0

    > DO YOU WANT TO CHANGE ANY OF THESE VALUES ENTER Y OR N
N

    > ENTER COMPREHENSIVE LEVEL OF OUTPUT          (MINIMUM=
1  MAXIMUM= 4)
1

  MASS MATRIX EQUALS...
    225.40       .00       .00       .00       .00       .00
       .00    225.40       .00       .00       .00       .00
       .00       .00    225.40       .00       .00       .00
       .00       .00       .00     15.80       .80      -.90
       .00       .00       .00       .80     11.64      3.20
       .00       .00       .00      -.90      3.20     15.69
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  STIFFNESS MATRIX EQUALS...
  374450.5 101313.6  -22271.4  -62917.7   27244.1  -40643.5
  101313.6 850849.8   41437.2  -47587.8  219098.5  -65773.8
  -22271.4  41437.2  823767.9 -103689.6   55011.2 -156180.8
  -62917.7 -47587.8 -103689.6  514269.9  -99492.5  346190.2
   27244.1 219098.5   55011.2  -99492.5  498592.9 -133433.7
  -40643.5 -65773.8 -156180.8  346190.2 -133433.7  531961.5

    > Choose the MODE SHAPE NORMALIZATION method
      MASS---(MA)    STIFFNESS---(ST)    LARGEST DOF---(LD)
LD

  THE MODE SHAPES ARE ...    (normalized to largest DOF)
    X      .065     -.015     -.002      .002      .001     -.001
    Y     -.014     -.055     -.034      .002     -.006     -.004
    Z      .006      .033     -.057     -.005      .002     -.003
ThetaX     .009     -.004      .002     -.167     -.128      .139
ThetaY     .004      .026      .017      .028     -.217     -.206
ThetaZ    0.000      .013     -.020      .172     -.064      .183

  NATURAL FREQUENCIES ... (CYCLES/SEC)
      6.21       9.25       9.36     16.23      28.06      45.96

    > Choose one of the options 'by entering two letters
    ANIMATE MODE-(AM) STATIC DEFLECT-(SD) RESTART ENGSIM----
(RS)
    MOUNT FORCES-(MF) FREQ RESPONSE--(FR) RESTART NEW INPUT-
(NU)
    CHG NORM-----(CN) OPTIM PARAMS---(OP) SAVE ENGSIM FILE--
(EF)
    ELASTIC AXIS-(EA)                     QUIT--------------
(QU)
EA

  CENTER OF ELASTICITY    (Measured from the C.G.)
        X           Y           Z
     -.001752    -.005120    -.015521

  ELASTIC CENTER ROTATION MATRIX
      .985475     .018507    -.168807
      .014039    -.999520    -.027623
     -.169238     .024852    -.985262

  TRANSLATION OF THE ELASTIC CENTER (m)     .01644

  3 ANGLES BETWEEN #1 ELASTIC AXIS & X-Y-Z COORD (rad)
       .17065      1.55676      1.40074

  3 ANGLES BETWEEN #2 ELASTIC AXIS & X-Y-Z COORD (rad)
      1.55229       .03099      1.54594

  3 ANGLES BETWEEN #3 ELASTIC AXIS & X-Y-Z COORD (rad)
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      1.40118      1.54317       .17190

  CLOSEST 3 ANGLES BETWEEN 3 ELASTIC AXIS AND X-Y-Z COORD.
(rad)
    .03099       .17065       .17190

  X Y Z  ON TORQUE AXIS FROM C.G.
    -.33       5.25      -1.09

    > Choose one of the options 'by entering two letters
    ANIMATE MODE-(AM) STATIC DEFLECT-(SD) RESTART ENGSIM----
(RS)
    MOUNT FORCES-(MF) FREQ RESPONSE--(FR) RESTART NEW INPUT-
(NU)
    CHG NORM-----(CN) OPTIM PARAMS---(OP) SAVE ENGSIM FILE--
(EF)
    ELASTIC AXIS-(EA)                     QUIT--------------
(QU)
FR

  MASS MATRIX (SECOND ORDER)
    225.4        .0        .0        .0        .0        .0
       .0     225.4        .0        .0        .0        .0
       .0        .0     225.4        .0        .0        .0
       .0        .0        .0      15.8        .8       -.9
       .0        .0        .0        .8      11.6       3.2
       .0        .0        .0       -.9       3.2      15.7

  VISCOUS DAMPING MATRIX (SECOND ORDER)
     643.2      -4.1       5.7       9.9      43.0     -27.0
      -4.1     612.1     -11.1     -37.3      -4.5    -124.7
       5.7     -11.1     604.7      28.2     137.4      -5.4
       9.9     -37.3      28.2     490.3      84.4     333.9
      43.0      -4.5     137.4      84.4     751.3    -119.7
     -27.0    -124.7      -5.4     333.9    -119.7     605.1

  STRUCTRUAL DAMPING MATRIX (SECOND ORDER)
   40322.8    -413.8     571.0     993.5    2400.6   -3854.5
    -413.8   37206.1   -1112.7   -1837.4    -453.5   -6375.0
     571.0   -1112.7   36471.1    3974.5    7649.1    -540.1
     993.5   -1837.4    3974.5   29854.3    5733.2   20662.7
    2400.6    -453.5    7649.1    5733.2   48281.2   -7584.5
   -3854.5   -6375.0    -540.1   20662.7   -7584.5   37631.8

  STIFFNESS MATRIX (SECOND ORDER)
   374450.5  101313.6  -22271.4  -62917.7   27244.1  -40643.5
   101313.6  850849.8   41437.2  -47587.8  219098.5  -65773.8
   -22271.4   41437.2  823767.9 -103689.6   55011.2 -156180.8
   -62917.7  -47587.8 -103689.6  514269.9  -99492.5  346190.2
    27244.1  219098.5   55011.2  -99492.5  498592.9 -133433.7
   -40643.5  -65773.8 -156180.8  346190.2 -133433.7  531961.5

ENTER FORCE VECTOR ( Fx Fy Fz Tx Ty Tz )
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0 0 0 0 1 0
ENTER THE FREQUENCY RANGE YOU WANT TO SEE (HZ)
0 20
FREQUENCY RESPONSE IS ON PROCESS ...

    > Choose one of the options 'by entering two letters
    ANIMATE MODE-(AM) STATIC DEFLECT-(SD) RESTART ENGSIM----
(RS)
    MOUNT FORCES-(MF) FREQ RESPONSE--(FR) RESTART NEW INPUT-
(NU)
    CHG NORM-----(CN) OPTIM PARAMS---(OP) SAVE ENGSIM FILE--
(EF)
    ELASTIC AXIS-(EA)                     QUIT--------------
(QU)
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