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SCATTERING OF ARBITRARILY-POLARIZED EM WAVES BY
A DISCONTINUITY IN A GROUNDED DIELECTRIC SHEET
AND PROPOAGATION OF EM PULSES EXCITED
BY AN ELECTRIC DIPOLE IN CONDUCTING MEDIA

presented by

Jiming Song

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Electrical Engineering

Major professor

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By

Jiming Song

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ABSTRACT

SCATTERING OF ARBITRARILY-POLARIZED EM WAVES BY A DISCONTINUITY IN A GROUNDED DIELECTRIC SHEET AND PROPAGATION OF EM PULSES EXCITED BY AN ELECTRIC DIPOLE IN CONDUCTING MEDIA

By

Jiming Song

There are two parts in this thesis. In Part I, the two-dimensional problem of the interaction between an arbitrarily-polarized electromagnetic (EM) wave and a dielectric discontinuity in a grounded dielectric sheet is analyzed. An electric field integral equation (EFIE) is derived based on the dyadic Green's function for the field produced by induced polarization currents in the discontinuity region. Impressed electric fields consist of either plane waves incident from space above the dielectric sheet or surface waves supported by that sheet; both TE and TM polarizations are considered. The EFIE is solved using Galerkin's method with pulse and sinusoidal/piecewise-sinusoidal (SPS) basis functions. Computation of the inverse Fourier transform representation of the Green's function is performed by integration along the real axis, along the branch cuts directly, and along the branch cuts with a variable change. It is found that the excitation of the surface-wave mode reduces the back scattered radiation field for TE-polarized plane wave incidence. The reflected power for both TE- and TM-polarized surface-wave incidence is calculated, and it is found that the numerical results agree very well with those of existing studies. The EFIE is also solved iteratively using successive approximations. These approximate results agree very well with the direct MoM (Method of Moments) solutions to the EFIE.

In Part II, the exact solutions are given for the transient EM fields excited by an electric dipole antenna with an impulsive current in a conducting medium. There exists an optimum waveform for the antenna current which can generate an EM pulse with a maximum intensity at a particular distance from the antenna. It is found that the EM fields of an EM pulse excited by an antenna in a conducting medium can be divided into two parts. The first part is an impulse wave which propagates with the speed of light ($1/\sqrt{\mu\epsilon}$) and decays exponentially. The second part builds up gradually and propagates slowly, and more importantly this part attenuates as an inverse power of distance which is a much slower rate than an exponential decay.

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To my wife

Weiqun

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PART I
SCATTERING OF ARBITRARILY-POLARIZED EM
WAVES BY A DISCONTINUITY IN A GROUNDED
DIELECTRIC SHEET

CHAPTER 1

INTRODUCTION

Scattering of an electromagnetic (EM) wave by a conducting body can be controlled or modified by coating the body with a dielectric sheet. However, when there is a discontinuity in the dielectric sheet, undesirable EM wave scattering can occur.

If the interaction of an incident EM wave with a discontinuity in a grounded dielectric sheet is determined, and the scattering of that EM wave excited by the discontinuity can be analyzed accurately, it may be possible to mitigate the undesirable scattering due to the discontinuity. It may also provide information useful for the selection of an appropriate dielectric coating for the purpose of controlling the EM wave scattering by a metallic body. On the other hand, the problem of surface wave scattering from discontinuities or gaps along dielectric waveguides is also very important, because accurate knowledge of scattered waves permits the design and development of optical and millimeter-wave components. A discontinuity problem arises in many forms such as in the splicing of two dielectric waveguides or in inter-device coupling of millimeter and optical integrated circuits.

The theory of planar waveguides may be found in [1]-[4]. There are discrete surface-wave modes and continuous radiation modes supported by dielectric waveguides. Because of the importance of dielectric waveguide discontinuities for many applications in optical and millimeter-wave circuits, various analytical methods for the study of discontinuities along dielectric-slab waveguides have been developed [5]-[31], and most of them

assume that fundamental TE and TM surface-wave-mode incidence in their studies. A few results have been reported for higher order surface-wave incidence and plane wave incidence.

A literature review covering the analysis of discontinuities in planar waveguides was given by Liu and Chew [30] recently. One difficulty encountered in the analysis of planar waveguide discontinuity problems is the treatment of the radiation modes, which have a continuous spectrum. Various types of mode matching methods have been applied. For small steps, Marcuse [5] derived a simple approximation; Clarricoats and Sharpe [6] neglected the continuous radiation modes; Hockman and Sharpe [7] proposed the use of a first-order variational solution which neglected the backward radiation loss at the step. For large abrupt step discontinuities, [16] introduced a metallic enclosure to the original waveguide structure in order to discretize the radiation spectrum; the Laguerre transform has been used in [8], [19]; and [15] approximated the waveguide structure in the transverse direction so as to discretize the continuous radiation modes.

In addition to the various types of mode matching methods, variational approaches have been applied successfully. Morishita et al. [12] formulated a least-squares boundary residual method. The Ritz-Galerkin (RG) variational approach has been used by Rozzi [10] to analyze a step discontinuity. A successive iteration method has been proposed by Gelin et al. [13], [14], and has been improved by Morita [31] recently. Hirayama and Koshiba [24] have developed a combination of the finite element method (FEM) and boundary element method (BEM). Chung and Chen [20]-[22] proposed a partial variational principle for the analysis of the waveguide discontinuity problem.

In addition to the above methods, the beam-propagation method (BPM) [25]-[29] has been proposed to calculate the propagation of waveguide modes through waveguide discontinuities. Recently, Liu and Chew [30] proposed an eigenmode propagation method (EPM).

One of the most powerful and commonly used techniques to analyze planar waveguide discontinuity problems is the integral equation approach [32]-[38]. An electric field integral equation (EFIE) can be derived based on the dyadic Green's function for the electric field produced by induced polarization currents in the discontinuity region [35]-[38]. The EFIE is solved by the Method of Moments (MoM) [32]-[34]. This method has several advantages over other existing approaches, viz. (1) the impressed electric field can consist of either plane waves incident from space above the dielectric sheet or surface waves supported by that sheet, (2) it is conceptually exact, and (3) it is believed to be computationally less cumbersome than the finite-element and related methods.

Recently, Moore and Ling [39] [40] proposed to analyze the electromagnetic scattering from an isolated conductor-backed dielectric gap using a boundary integral approach and a spectral integral approach. To facilitate numerical implementation, the specular solution and the surface wave contribution are removed from the boundary integral equation. Consequently the discretization domain of the infinite structure can be reduced to a localized region near the gap. Numerical diffraction coefficients and surface wave launch coefficients are extracted from the scattered field.

In this thesis, the two-dimensional (2-D) problem of arbitrarily-polarized electromagnetic wave interactions with a dielectric discontinuity in a grounded dielectric sheet is analyzed. An electric field integral equation (EFIE) is derived based on the Green's function for the field produced by induced polarization currents in the discontinuity region. Impressed electric fields consist of either plane waves incident from space above the dielectric sheet or surface waves supported by that sheet; both TE and TM polarizations are considered. The EFIE is solved using Galerkin's method with pulse and sinusoidal/piecewise-sinusoidal (SPS) basis functions. Computation of the inverse Fourier transform representation of the electric Green's function is performed by integration along real axis, along the branch cuts directly, and along the branch cuts with a variable change. The reflected power for both TE- and TM-polarized surface wave incidence is calculated, and

it is found that the numerical results agree very well with those of existing studies. The EFIE is also solved iteratively using successive approximations. First, both the zeroth- and the first-order approximations (leading terms of the Neumann series) for the total electric field in the discontinuity region are derived when a TE- or TM-polarized plane wave is incident upon a narrow gap in grounded sheet. Then, the first- and second-order approximations to the scattered field are obtained by using the steepest-descents method. These approximate results agree very well with the direct MoM solutions to the EFIE.

There are ten chapters in this part. Chapter 1 gives an introduction for this research and a literature survey.

Chapter 2 presents a description of the problem to be solved and the derivation of the EFIE. Details on the derivation of the dyadic Green's functions and the electric field in the unperturbed dielectric sheet with TE- and TM-polarized plane wave incidence are given in Appendices.

In Chapter 3, the complex analysis is applied to the spectral integral representation of the Green's function to identify the propagation-mode spectrum in the dielectric slab waveguide. Then, those Green's functions are rewritten for the field in a source-free region in terms of integrals along branch cuts and residues at surface-wave poles. Finally, some observations on the Sommerfeld-integral representation of the dyadic Green's function in the source region will be discussed.

Chapter 4 presents the derivations of formulas to calculate the surface waves excited, and the far-zone field is obtained by using the saddle-point method. Then, MoM is applied to transform the integral equations into a matrix equation.

In Chapter 5, the EFIE is solved iteratively by using successive approximations. First, the zeroth- and the first-order approximations for the total electric field in the discontinuity region are derived when a TE- or TM-polarized plane wave is incident upon a grounded sheet with a narrow gap. Then, the first- and second-order approximations to the scattered

field are obtained by using the steepest-descent method. These approximate results agree very well with the direct MoM solutions to the EFIE.

The EFIE is solved numerically using Galerkin's method in Chapter 6 for TE illumination. Computation of the inverse Fourier transform representation of the electric Green's function is performed by integration along the real axis, along the branch cuts directly, and along the branch cuts with a variable change. It is found that the integration along the branch cuts directly requires the least computation time, and allows a separation of the field into bound surface-wave modes and radiation modes.

For TM illumination, the EFIE is solved using Galerkin's method with pulse and sinusoidal/piecewise-sinusoidal (SPS) basis functions in Chapters 7 and 8, respectively. Some numerical techniques are also discussed in these two chapters.

Chapter 9 presents the numerical results for both TE- and TM-polarized surface-wave incidence and compares them with those of existing studies. Next, the electric field distributions, radiation patterns, and back scattering width are quantified for TE- and TM-polarized plane wave incidence. Finally, the back scattering width for a lossy dielectric sheet is calculated.

In the final chapter, Chapter 10, the research accomplishments are summarized. Conclusions are drawn from the collected results of this part.

CHAPTER 2

PROBLEM DESCRIPTION AND GREEN'S FUNCTION

2.1. INTRODUCTION AND GEOMETRY

In this chapter the Electric Field Integral Equation (EFIE) will be derived. First, the problem is described and decomposed by using the principle of superposition. Then, the electric field without the discontinuity and the Green's function are found to form the EFIE. Some details will be given in the appendices.

The configuration of electromagnetic (EM) fields scattered from the discontinuity in a grounded dielectric sheet is described geometrically in Figure 2.1(a). Region 1 is a half-space dielectric medium with permittivity ϵ_1 and permeability μ_0 (it is usually air). Region 2 represents dielectric sheet of complex permittivity ϵ_2 and permeability μ_0 with a discontinuity region of permittivity ϵ_d and permeability μ_0 . Region 3 is a perfect conductor.

When a TE- or TM-polarized plane wave is incident on the sheet, if there is no discontinuity, the scattered EM field is the reflected field which is easily derived. If there is a discontinuity, the problem becomes quite complicated. Since this problem can not be solved analytically, an Electric Field Integral Equation (EFIE) will be derived in the next several sections, and is solved numerically in the following chapters.

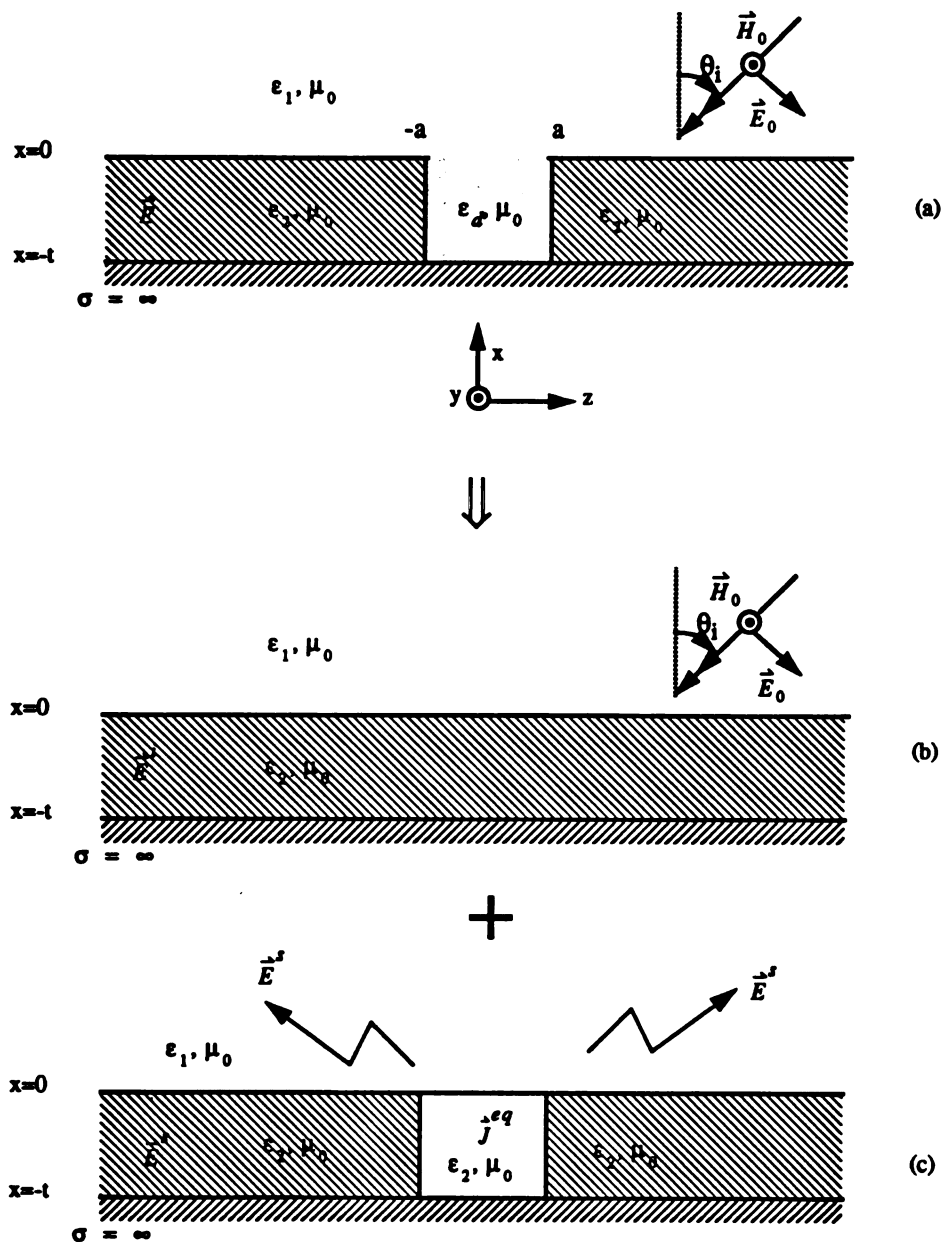


Figure 2.1. Problem description and decomposition.

2.2. PROBLEM DECOMPOSITION AND ELECTRIC FIELD INTEGRAL EQUATION (EFIE)

The total electric field $\vec{E}$ can be decomposed into the electric field $\vec{E}^i$ without the discontinuity and the scattered electric field $\vec{E}^s$ maintained by the equivalent current $\vec{J}^{eq}$ in the discontinuity region (Figure 2.1) by using the principle of superposition. Then

$$\vec{E} = \vec{E}^i + \vec{E}^s \quad (2.2.1)$$

If the x-axis is normal to the horizontal sheet interfaces while the waveguiding z-axis is parallel to them, then when a plane wave is incident (the propagation direction is in x-z plane), a y-invariant electric field is excited by the induced current. The scattered electric field $\vec{E}^s$ can be written as:

$$\vec{E}^s(x, z) = \int_{LCS} \bar{G}(x, z|x', z') \bullet \vec{J}^{eq}(x', z') dx' dz' \quad (2.2.2)$$

where LCS designates the longitudinal cross section of the discontinuity region and $\bar{G}(x, z|x', z')$ is the electric dyadic Green's function.

The equivalent current $\vec{J}^{eq}$ is identified by adding and subtracting the displacement current of the unperturbed waveguide in the Ampere's-law Maxwell equation [harmonic time dependence $\exp(j\omega t)$ implied but suppressed throughout] to obtain

$$\begin{aligned} \nabla \times \vec{H}(x, z) &= \vec{J}^e(x, z) + j\omega[\epsilon_d(x, z) - \epsilon_2] \vec{E}(x, z) + j\omega\epsilon_2 \vec{E}(x, z) \\ &= \vec{J}^e(x, z) + \vec{J}^{eq}(x, z) + j\omega\epsilon_2 \vec{E}(x, z) \end{aligned} \quad (2.2.3)$$

where $\vec{J}^e(x, z)$ is the impressed electric current, which maintains an impressed incident field $\vec{E}^i(x, z)$ without the discontinuity, and the induced current $\vec{J}^{eq}(x, z)$, nonvanishing only in the discontinuity region, is expressed in terms of the total electric field $\vec{E}$ in that region as

$$\vec{J}^{eq}(x, z) = j\omega[\epsilon_d(x, z) - \epsilon_2]\vec{E}(x, z) = j\omega\delta n^2(x, z)\epsilon_0\vec{E}(x, z) \quad (2.2.4)$$

where

$$\delta n^2(x, z) = [\epsilon_d(x, z) - \epsilon_2]/\epsilon_0 = \epsilon_{dr}(x, z) - \epsilon_{2r} \quad (2.2.5)$$

with $\epsilon_{dr} = \epsilon_d/\epsilon_0$, and $\epsilon_{2r} = \epsilon_2/\epsilon_0$. Rearranging $\vec{E} = \vec{E}^i + \vec{E}^s$ leads to the EFIE:

$$\vec{E}(x, z) - j\omega\epsilon_0 \int_{LCS} \delta n^2(x', z') \vec{G}(x, z|x', z') \bullet \vec{E}(x', z') dx' dz' = \vec{E}^i(x, z) \quad (2.2.6)$$

In the next two sections, the electric field $\vec{E}^i$ without the discontinuity and the dyadic Green's function $\vec{G}(x, z|x', z')$ will be derived.

2.3. ELECTRIC FIELD WITHOUT THE DISCONTINUITY

It is easy to find the electric field $\vec{E}^i$ when there is no discontinuity. The details are given in Appendices A and B for TE- and TM-polarized plane wave incidence, respectively. The final results are

TE-polarized plane wave incidence:

Region 1: $0 < x < \infty$

$$E_{1y}^i = E_0 [e^{jk_1 x \cos \theta_i} + R e^{-jk_1 x \cos \theta_i}] e^{jk_1 z \sin \theta_i} \quad (2.3.1)$$

Region 2: $-t < x < 0$

$$E_{2y}^i = E_0 \frac{2\eta_2 \cos \theta_i \sin [k_2 (x + t) \cos \theta_r] e^{jk_1 z \sin \theta_i}}{\eta_2 \cos \theta_i \sin (k_2 t \cos \theta_r) - j\eta_1 \cos \theta_r \cos (k_2 t \cos \theta_r)} \quad (2.3.2)$$

where E_0 is the amplitude of the incident plane wave, R is the reflection coefficient and is given in Appendix A, and

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$$k_i = \omega \sqrt{\epsilon_i \mu_0} \quad i = 1, 2 \quad (2.3.3)$$

$$\eta_i = \sqrt{\mu_0 / \epsilon_i} \quad i = 1, 2 \quad (2.3.4)$$

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{k_2}{k_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \quad (2.3.5)$$

Here θ_i is the incident angle in region 1 and θ_r is the refracted angle in region 2.

TM-polarized plane wave incidence:

Region 1: $0 < x < \infty$

$$\vec{E}_1^i = -2E_0 \{ \hat{x} \sin \theta_i \cos [k_1 x \cos \theta_i + \phi_e] - \hat{z} j \cos \theta_i \sin [k_1 x \cos \theta_i + \phi_e] \} e^{j[k_1 z \sin \theta_i - \phi_e]} \quad (2.3.6)$$

$$H_{1y}^i = \frac{2E_0}{\eta_1} \cos [k_1 x \cos \theta_i + \phi_e] e^{j[k_1 z \sin \theta_i - \phi_e]} \quad (2.3.7)$$

Region 2: $-t < x < 0$

$$\vec{E}_2^i = -E_0 \frac{2\eta_2 \cos \theta_i \{ \hat{x} \sin \theta_r \cos [k_2 (t+x) \cos \theta_r] - \hat{z} j \cos \theta_r \sin [k_2 (t+x) \cos \theta_r] \} e^{jk_1 z \sin \theta_i}}{\eta_1 \cos \theta_i \cos [k_2 t \cos \theta_r] + j\eta_2 \cos \theta_r \sin [k_2 t \cos \theta_r]} \quad (2.3.8)$$

$$H_{2y}^i = \frac{2E_0 \cos \theta_i \cos [k_2 (t+x) \cos \theta_r] e^{jk_1 z \sin \theta_i}}{\eta_1 \cos \theta_i \cos [k_2 t \cos \theta_r] + j\eta_2 \cos \theta_r \sin [k_2 t \cos \theta_r]} \quad (2.3.9)$$

where

$$\tan \phi_e = \frac{\eta_2 \cos \theta_r}{\eta_1 \cos \theta_i} \tan [k_2 t \cos \theta_r] \quad (2.3.10)$$

2.4. 2 - D DYADIC GREEN'S FUNCTION FOR A GROUNDED DIELECTRIC MATERIAL SHEET

The development of general 3-D electric dyadic Green's functions for layered structures has been presented by Bagby and Nyquist [36]. The electric field Green's dyads are found in terms of associated Hertzian potential Green's dyads, developed by an extension of Sommerfeld's classic method [41]. The 3-D Hertzian potential dyadic Green's function was shown to have scalar components expressible as 2-D spatial frequency integrals of the Sommerfeld type. In this development, the analysis in [36] is specialized for the 2-D structure of Figure 2.1(c). Using the symmetry, where the fields are invariant with respect to y , the Green's function can be expressed in 1-D spectral integral representation [42] [43]. In general, these electric fields can be written as:

$$\vec{E}_1(x, z) = \int_{LCS} \vec{G}_{12}(x, z|x', z') \bullet \vec{J}(x', z') dx' dz' \quad (2.4.1)$$

$$\vec{E}_2(x, z) = \int_{LCS} \vec{G}_{22}(x, z|x', z') \bullet \vec{J}(x', z') dx' dz' \quad (2.4.2)$$

where LCS designates the longitudinal cross section of the discontinuity region, and the dyadic Green's functions $\vec{G}_{12}(x, z|x', z')$ and $\vec{G}_{22}(x, z|x', z')$ represent that the source point (x', z') is in region 2, the field point (x, z) is in region 1 and region 2, respectively. Since TE or TM illumination can excite TE or TM fields only, then they will be considered separately. For TE-polarized wave incidence, a y -invariant electric field, having only y -component, is excited by the induced y -component of current. On the other hand, for TM-polarized wave incidence, a y -invariant electric field, having only x - and z -components, is excited by the induced x - and z -components of current. Thus, the dyadic Green's functions have five components

$$\vec{G}_{i2}(x, z|x', z') = \sum_{\alpha=x, z} \sum_{\beta=x, z} \hat{\alpha} G_{i2\alpha\beta}(x, z|x', z') \hat{\beta} + \hat{y} G_{i2yy}(x, z|x', z') \hat{y} \quad i = 1, 2 \quad (2.4.3)$$

In Appendix C, the Green's function for TE excitation [$G_{i2yy}(x, z|z', z')$] is derived by finding the Hertzian potential first. In Appendix D, the 2-D Green's functions for TM excitation of a grounded dielectric sheet when the current source is in region 2 is derived by a direct field approach. Maxwell's equations are first transformed into the spectral domain, then the wave equations are solved for the longitudinal component of electric field; the traverse components of EM fields are then determined from that longitudinal component. By matching the boundary conditions, unknown spectral amplitudes of EM fields are determined. Finally, the spectral EM fields are inverse transformed into space domain. The five components of the dyadic Green's functions are

$$G_{12xx}(x, z|x', z') = -\frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{e^{-p_1 x}}{2\pi\bar{\Delta}_e} \zeta^2 \cosh[p_2(t+x')] e^{j\zeta(z-z')} d\zeta \quad (2.4.4)$$

$$G_{12xz}(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{e^{-p_1 x}}{2\pi\bar{\Delta}_e} jp_2 \zeta \sinh[p_2(t+x')] e^{j\zeta(z-z')} d\zeta \quad (2.4.5)$$

$$G_{12zx}(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{e^{-p_1 x}}{2\pi\bar{\Delta}_e} jp_1 \zeta \cosh[p_2(t+x')] e^{j\zeta(z-z')} d\zeta \quad (2.4.6)$$

$$G_{12zz}(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{e^{-p_1 x}}{2\pi\bar{\Delta}_e} p_1 p_2 \sinh[p_2(t+x')] e^{j\zeta(z-z')} d\zeta \quad (2.4.7)$$

$$G_{12yy}(x, z|x', z') = -jk_1 \eta_1 \int_{-\infty}^{\infty} \frac{e^{-p_1 x}}{2\pi\bar{\Delta}_h} \sinh[p_2(t+x')] e^{j\zeta(z-z')} d\zeta \quad (2.4.8)$$

$$\bar{G}_{22}(x, z|x', z') = \hat{x}\hat{x} \frac{j\eta_2}{k_2} \delta(\hat{p} - \hat{p}') + \bar{G}_{22}^R(x, z|x', z') \quad (2.4.9)$$

$$\begin{aligned} G_{22xx}^R(x, z|x', z') = & \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{-\zeta^2 e^{j\zeta(z-z')}}{4\pi p_2} \{ e^{-p_2|x-x'|} + e^{-p_2(2t+x+x')} \\ & - 2 \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} e^{-p_2 t} \cosh[p_2(t+x)] \cosh[p_2(t+x')] \} d\zeta \end{aligned} \quad (2.4.10)$$

$$G_{22xz}^R(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{j\zeta e^{j\zeta(z-z')}}{4\pi} \{ \operatorname{sgn}(x-x') e^{-p_2|x-x'|} - e^{-p_2(2t+x+x')} \\ - 2 \frac{p_1 - p_2 \epsilon_1/\epsilon_2}{\tilde{\Delta}_e} e^{-p_2 t} \cosh[p_2(t+x)] \sinh[p_2(t+x')] \} d\zeta \quad (2.4.11)$$

$$G_{22zx}^R(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{j\zeta e^{j\zeta(z-z')}}{4\pi} \{ \operatorname{sgn}(x-x') e^{-p_2|x-x'|} + e^{-p_2(2t+x+x')} \\ + 2 \frac{p_1 - p_2 \epsilon_1/\epsilon_2}{\tilde{\Delta}_e} e^{-p_2 t} \sinh[p_2(t+x)] \cosh[p_2(t+x')] \} d\zeta \quad (2.4.12)$$

$$G_{22zz}^R(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{p_2 e^{j\zeta(z-z')}}{4\pi} \{ e^{-p_2|x-x'|} - e^{-p_2(2t+x+x')} \\ + 2 \frac{p_1 - p_2 \epsilon_1/\epsilon_2}{\tilde{\Delta}_e} e^{-p_2 t} \sinh[p_2(t+x)] \sinh[p_2(t+x')] \} d\zeta \quad (2.4.13)$$

$$G_{22yy}^R(x, z|x', z') = -jk_2\eta_2 \int_{-\infty}^{\infty} \frac{e^{j\zeta(z-z')}}{4\pi p_2} \{ e^{-p_2|x-x'|} - e^{-p_2(x+x'+2t)} \\ + 2 \frac{p_2 - p_1}{\tilde{\Delta}_h} e^{-p_2 t} \sinh[p_2(x+t)] \sinh[p_2(x'+t)] \} d\zeta \quad (2.4.14)$$

where

$$\operatorname{sgn}(x-x') = \begin{cases} 1 & x > x' \\ -1 & x < x' \end{cases} \quad (2.4.15)$$

$$p_i = \sqrt{\zeta^2 - k_i^2} \quad i = 1, 2 \quad (2.4.16)$$

$$\tilde{\Delta}_e = p_1 \cosh(p_2 t) + p_2 (\epsilon_1/\epsilon_2) \sinh(p_2 t) \quad (2.4.17)$$

$$\tilde{\Delta}_h = p_2 \cosh(p_2 t) + p_1 \sinh(p_2 t) \quad (2.4.18)$$

$\tilde{\Delta}_e = 0$ is the eigenvalue equation of TM-even modes, and $\tilde{\Delta}_h = 0$ is the eigenvalue equation of TE-odd modes.

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Note that to properly ensure waves which decay as they propagate, the appropriate branch of the square root which determines p_i from (2.4.16) is that which demands

$$\operatorname{Re} \{p_i\} > 0 \quad \operatorname{Im} \{p_i\} > 0 \quad i = 1, 2 \quad (2.4.19)$$

where $\operatorname{Re} \{ \dots \}$ designates the real part of the quantity within the braces, $\operatorname{Im} \{ \dots \}$ designates the imaginary part.

These Green's functions satisfy the reciprocity theorem, i.e.

$$\left. \begin{aligned} G_{22\alpha\beta}(x, z|x', z') &= G_{22\beta\alpha}(x', z'|x, z) \\ \alpha &= x, y, z \quad \beta = x, y, z \end{aligned} \right\} \quad (2.4.20)$$

It is clear that all Green's functions are functions of $(z - z')$. These Green's functions can be thus rewritten as

$$\left. \begin{aligned} G_{i2\alpha\beta}(x, z|x', z') &= G_{i2\alpha\beta}(x|x', z - z') \\ \alpha &= x, y, z \quad \beta = x, y, z \quad i = 1, 2 \end{aligned} \right\} \quad (2.4.21)$$

Therefore, EFIE in Section 2.2 in the discontinuity region is rewritten as:

$$\vec{E}_2(x, z) - j \frac{k_0}{\eta_{0LCs}} \int \delta \pi^2(x', z') \vec{G}_{22}(x|x', z - z') \bullet \vec{E}_2(x', z') dx' dz' = \vec{E}_2^i(x, z) \quad (2.4.22)$$

where

$$k_0 = \sqrt{\epsilon_0 \mu_0} \quad (2.4.23)$$

$$\eta_0 = \sqrt{\mu_0 / \epsilon_0} \quad (2.4.24)$$

The electric field without the discontinuity and the dyadic Green's functions have been derived and the EFIE is formed. These Green's functions are expressed as Sommerfeld integrals along the real axis in the ζ -plane. In the next chapter, the complex analysis is applied to rewrite these Green's functions in terms of integrals along branch cuts and residues of surface-wave poles.

CHAPTER 3

PROPAGATION MODE SPECTRUM IN DIELECTRIC SLAB WAVEGUIDE

3.1. INTRODUCTION

In this chapter, the complex analysis is applied to the spectral integral representation of the Green's function to identify the propagation-mode spectrum in dielectric slab waveguide. Then, these Green's functions for the field in a source-free region will be rewritten in terms of integrals along branch cuts and residues at surface-wave poles. Finally, some observations on the Sommerfeld-integral representation of the dyadic Green's function in the source region will be discussed.

The functions $p_i = \sqrt{\zeta^2 - k_i^2}$ ($i = 1, 2$) are double-value functions because, in taking the square root of a number, two values are possible. Then, $\pm k_1, \pm k_2$ are branch points. After some algebraic manipulation, $\bar{G}_{22}^R(x|x', z - z')$ can be rewritten as

$$G_{22xx}^R(x|x', z - z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \{ p_1 [\sinh [p_2 (|x - x'| - \iota)] + \sinh [p_2 (x + x' + \iota)]] - \frac{\epsilon_1}{\epsilon_2} p_2 [\cosh [p_2 (|x - x'| - \iota)] + \cosh [p_2 (x + x' + \iota)]] \} \frac{\zeta^2 e^{j\zeta(z - z')}}{4\pi p_2 \bar{\Delta}_e} d\zeta \quad (3.1.1)$$

$$G_{22xz}^R(x|x', z - z') = \int_{-\infty}^{\infty} \{ p_1 [\operatorname{sgn}(x - x') \cosh [p_2 (|x - x'| - \iota)] - \cosh [p_2 (x + x' + \iota)]] \}$$

$$-\frac{\epsilon_1}{\epsilon_2} p_2 [\operatorname{sgn} (x - x') \sinh [p_2 (|x - x'| - t)] - \sinh [p_2 (x + x' + t)]] \} \frac{j\zeta e^{j\zeta(z-z')}}{4\pi\bar{\Delta}_e} \frac{j\eta_2}{k_2} d\zeta \quad (3.1.2)$$

$$G_{22zx}^R(x|x', z - z') = \int_{-\infty}^{\infty} \{ p_1 [\operatorname{sgn} (x - x') \cosh [p_2 (|x - x'| - t)] + \cosh [p_2 (x + x' + t)]] - \frac{\epsilon_1}{\epsilon_2} p_2 [\operatorname{sgn} (x - x') \sinh [p_2 (|x - x'| - t)] + \sinh [p_2 (x + x' + t)]] \} \frac{j\zeta e^{j\zeta(z-z')}}{4\pi\bar{\Delta}_e} \frac{j\eta_2}{k_2} d\zeta \quad (3.1.3)$$

$$G_{22zz}^R(x|x', z - z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \{ p_1 [- \sinh [p_2 (|x - x'| - t)] + \sinh [p_2 (x + x' + t)]] + \frac{\epsilon_1}{\epsilon_2} p_2 [\cosh [p_2 (|x - x'| - t)] - \cosh [p_2 (x + x' + t)]] \} \frac{p_2 e^{j\zeta(z-z')}}{4\pi\bar{\Delta}_e} d\zeta \quad (3.1.4)$$

$$G_{22yy}^R(x|x', z - z') = -jk_2\eta_2 \int_{-\infty}^{\infty} \{ p_2 [- \sinh [p_2 (|x - x'| - t)] + \sinh [p_2 (x + x' + t)]] + p_1 [\cosh p_2 (t - |x - x'|) - \cosh p_2 (t + x + x')] \} \frac{e^{j\zeta(z-z')}}{4\pi p_2 \bar{\Delta}_h} d\zeta \quad (3.1.5)$$

The integrands in the Green's functions $\bar{G}_{12}(x|x', z - z')$ and $\bar{G}_{22}(x|x', z - z')$ are even functions of p_2 . Hence, $\pm k_2$ are removable branch points and only branch points at $\pm k_1$ are significant. This observation can be generalized to a n-layered environment (n=3 in this problem, the third layer is the perfect conductor). Only $\pm k_1$ and $\pm k_n$ are branch points, as discussed by Chew [44].

A double-valued function is represented on the complex plane with the help of Riemann sheets [44]. Two Riemann sheets can be assigned to a single double-valued com-

plex function. On one of the Riemann sheets, p_1 assumes just the opposite sign to the value on the other sheet. The branch cut separates these sheets.

In the integrations along the real axis to calculate the Green's functions $\bar{G}_{12}(x|x', z-z')$ and $\bar{G}_{22}(x|x', z-z')$, $Re\{p_i\} > 0$, $Im\{p_i\} > 0$, $i = 1, 2$ are chosen to satisfy the radiation condition that leads to outward-propagating and attenuating waves. Let the integration contour reside on the top Riemann sheet, then $Re\{p_1\} > 0$ and $Im\{p_1\} > 0$ along the real ζ axis on the top Riemann sheet. When the contour is deformed to the infinite semicircle in the upper half plane (UHP) (not crossing the branch cut), $Re\{p_1\} > 0$ should be chosen to force the integrand of $\bar{G}_{12}(x|x', z-z')$ vanish on the infinite semicircle. Hence, it is expedient to define a top Riemann sheet on the complex ζ plane on which $Re\{p_1\} > 0$. The corresponding bottom Riemann sheet is then defined by $Re\{p_1\} < 0$, and the branch cut that separates these sheets is defined by $Re\{p_1\} = 0$. To find such a branch cut, first assume that $k_1 = k_{1r} + jk_{1i}$, $\zeta = \zeta_r + j\zeta_i$ so that k_1 and ζ are both complex numbers. Then

$$p_1^2 = \zeta^2 - k_1^2 = (\zeta_r^2 - \zeta_i^2) - (k_{1r}^2 - k_{1i}^2) + j2(\zeta_r\zeta_i - k_{1r}k_{1i}) \quad (3.1.6)$$

In order for $Re\{p_1\} = 0$, the conditions are that

$$Im\{p_1^2\} = 0 \quad Re\{p_1^2\} < 0 \quad (3.1.7)$$

which leads to

$$\zeta_r\zeta_i = k_{1r}k_{1i} \quad \zeta_r^2 - \zeta_i^2 < k_{1r}^2 - k_{1i}^2 \quad (3.1.8)$$

Consequently, the above equations define rectangular hyperbolas that pass through the branch points $\pm k_1$ as shown in Figure 3.1.

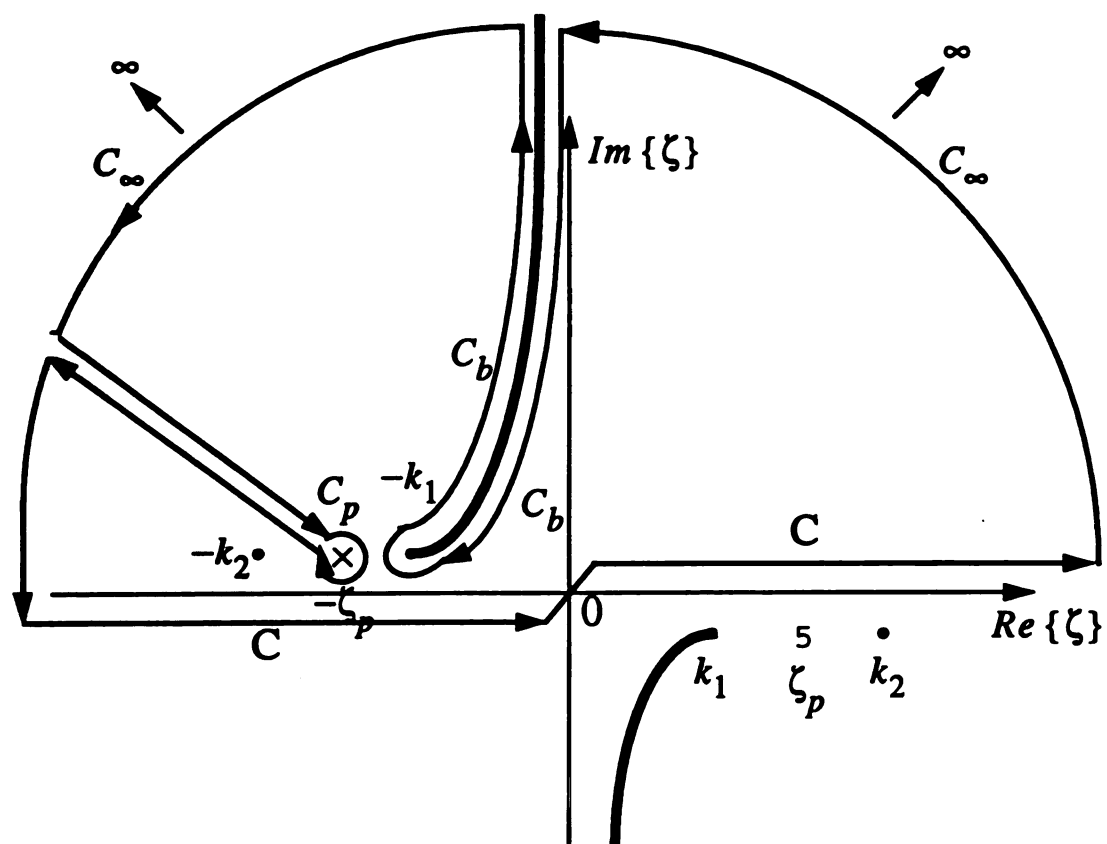


Figure 3.1. Branch cut and integration contour deformation on the top Riemann sheet ($Re\{p_1\} > 0$).

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3.2. CONTOUR DEFORMATION

Using the relations

$$\int_{-\infty}^{\infty} d\zeta e^{j\zeta(z-z')} f(\zeta) = \int_{-\infty}^{\infty} d\zeta e^{j\zeta|z-z'|} f(\zeta) \quad (3.2.1)$$

when $f(\zeta)$ is an even function of ζ , and

$$\int_{-\infty}^{\infty} d\zeta e^{j\zeta(z-z')} f(\zeta) = \operatorname{sgn}(z-z') \int_{-\infty}^{\infty} d\zeta e^{j\zeta|z-z'|} f(\zeta) \quad (3.2.2)$$

when $f(\zeta)$ is an odd function of ζ , the integrations in $\bar{G}_{22}^R(x|x', z-z')$ can be rewritten as:

$$G_{22\alpha\beta}^R(x|x', z-z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta \quad (3.2.3)$$

where

$$g_{xx} = \frac{-\zeta^2}{4\pi p_2} \left\{ e^{-p_2|x-x'|} + e^{-p_2(2t+x+x')} - 2 \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_e} e^{-p_2 t} \cosh[p_2(t+x)] \cosh[p_2(t+x')] \right\} \quad (3.2.4)$$

$$g_{xz} = \operatorname{sgn}(z-z') \frac{j\zeta}{4\pi} \left\{ \operatorname{sgn}(x-x') e^{-p_2|x-x'|} - e^{-p_2(2t+x+x')} - 2 \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_e} e^{-p_2 t} \cosh[p_2(t+x)] \sinh[p_2(t+x')] \right\} \quad (3.2.5)$$

$$g_{zx} = \operatorname{sgn}(z-z') \frac{j\zeta}{4\pi} \left\{ \operatorname{sgn}(x-x') e^{-p_2|x-x'|} + e^{-p_2(2t+x+x')} + 2 \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_e} e^{-p_2 t} \sinh[p_2(t+x)] \cosh[p_2(t+x')] \right\} \quad (3.2.6)$$

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$$g_{zz} = \frac{p_2}{4\pi} \{ e^{-p_2|x-x'|} - e^{-p_2(2t+x+x')} + 2 \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} e^{-p_2 t} \sinh [p_2(t+x)] \sinh [p_2(t+x')] \} \quad (3.2.7)$$

$$g_{yy} = -\frac{k_2^2}{4\pi p_2} \{ e^{-p_2|x-x'|} - e^{-p_2(2t+x+x')} + 2 \frac{p_2 - p_1}{\bar{\Delta}_h} e^{-p_2 t} \sinh [p_2(t+x)] \sinh [p_2(t+x')] \} \quad (3.2.8)$$

Since the $g_{\alpha\beta}$ are even functions of p_2 , only the $\pm k_1$ are branch points. The integration contour is chosen as shown in Figure 3.1 using the branch cut which was introduced in the last section. The integrand in (3.2.3) is analytic and Cauchy's theorem for the contour integral leads to

$$\int_{-\infty}^{\infty} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta = - \left\{ \int_{C_-} + \int_{C_b} + \int_{C_p} \right\} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta \quad (3.2.9)$$

where C_- designates the infinite semicircle, C_b designates the contour along the branch cut, and C_p designates the small circle around the surface wave pole at $-\zeta_p$, which is the solution of the eigenvalue equation of TM-even modes or TE-odd modes.

In any source-free region, $x \neq x'$ and $z \neq z'$, it can be shown that the $g_{\alpha\beta}$ exponentially decay when $|\zeta| \rightarrow \infty$ and the integral along the infinite semicircle C_- vanishes. Thus

$$\int_{C_-} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta = 0 \quad (3.2.10)$$

and

$$\int_{-\infty}^{\infty} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta = - \int_{C_b} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta - \int_{C_p} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta \quad (3.2.11)$$

The original integral is equal to integration along the branch cut augmented by the surface-wave pole integral. The branch cut integral represents the continuous radiation spectrum, while the pole contribution represents the discrete surface-wave modes.

Applying the residue theorem gives the integration around the surface-wave poles as

$$\int_{\zeta_r} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta = -2\pi j \text{Res} \{ g_{\alpha\beta} e^{j\zeta|z-z'|} \} \Big|_{\zeta = -\zeta_r} \quad (3.2.12)$$

where $\text{Res} \{ \dots \}$ designates the residue of the quantity within the braces at the given point.

Let's discuss the integral along the branch cut. There are two components to the integration path along the branch cut as shown in Figure 3.2. One is the right-lower part from $j\infty$ to $-k_1$ in the complex ζ plane. Another is the left-upper part from $-k_1$ to $j\infty$. $\zeta_r \zeta_i = k_{1r} k_{1i}$ and $p_1^2 = (\zeta_r^2 - \zeta_i^2) - (k_{1r}^2 - k_{1i}^2) < 0$ along the branch cut, such that $p_1 = \pm j\rho$, where $\rho > 0$. Thus the following conditions prevail:

- i) On the right-lower part, $\zeta_r \zeta_i - k_{1r} k_{1i} \rightarrow 0^+$, then $\arg \{p_1^2\} \equiv \pi$ and $\arg \{p_1\} \equiv \pi/2$, where $\arg \{ \dots \}$ designates the phase angle. Therefore, $p_1 = j\rho$ is chosen.
- ii) On the left-upper part, $\zeta_r \zeta_i - k_{1r} k_{1i} \rightarrow 0^-$, then $\arg \{p_1^2\} \equiv -\pi$ and $\arg \{p_1\} \equiv -\pi/2$. Therefore, $p_1 = -j\rho$ is chosen.

Along the integration path following the branch cut,

$$\zeta^2 = \zeta_r^2 - \zeta_i^2 + 2j\zeta_r \zeta_i = p_1^2 + k_{1r}^2 - k_{1i}^2 + 2jk_{1r} k_{1i} = k_1^2 - \rho^2 \quad (3.2.13)$$

To satisfy the radiation condition,

$$\text{Re} \{ \zeta \} < 0 \quad \text{and} \quad \text{Im} \{ \zeta \} > 0 \quad (3.2.14)$$

should be chosen. Let $\zeta = j\alpha$, then

$$\alpha^2 = \rho^2 - k_1^2 \quad (3.2.15)$$

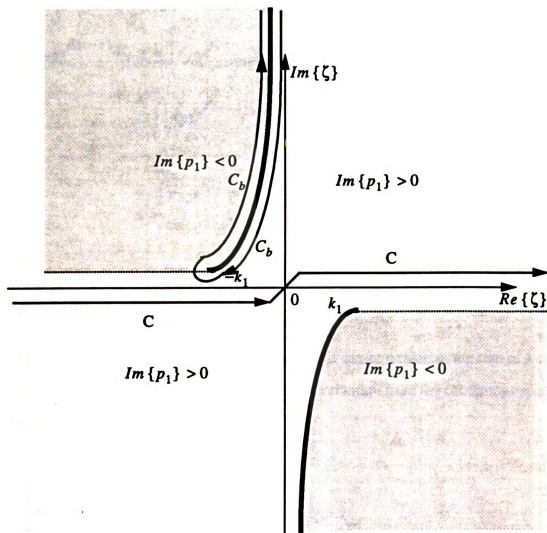


Figure 3.2. The path of integration along the branch cut on the top Riemann sheet ($Re\{p_1\} > 0$). $Im\{p_1\} < 0$ in the shadow region and $Im\{p_1\} > 0$ in other region.

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Changing variable ζ to ρ leads to

$$d\zeta = j d\alpha = \frac{j\rho}{\alpha} d\rho \quad (3.2.16)$$

Then the integration along the branch cut can be written as

$$\begin{aligned} \int_{C_1} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta &= \left\{ \int_{-\infty}^0 g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1=j\rho} + \int_0^{\infty} g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1=-j\rho} \right\} \frac{j\rho}{\alpha} d\rho \\ &= \int_0^{\infty} \left[g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1=-j\rho} - g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1=j\rho} \right] \frac{j\rho}{\alpha} d\rho \end{aligned} \quad (3.2.17)$$

$$= \int_{-\infty}^{\infty} g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1=-j\rho} \frac{j\rho}{\alpha} d\rho \quad (3.2.18)$$

where

$$\alpha = \sqrt{\rho^2 - k_1^2} \quad (3.2.19)$$

Hence, the original integration along the real axis can be written as the sum of a contribution from the surface-wave-mode pole and a contribution from the continuous spectrum radiation modes in source-free region, i.e.

$$\begin{aligned} \int_{-\infty}^{\infty} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta &= 2\pi j Res \{ g_{\alpha\beta} e^{j\zeta|z-z'|} \} \Big|_{\zeta=-\zeta_p} \\ &\quad - \int_0^{\infty} \left[g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1=-j\rho} - g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1=j\rho} \right] \frac{j\rho}{\alpha} d\rho \end{aligned} \quad (3.2.20)$$

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3.3. THE SPATIAL AND EIGENFUNCTION REPRESENTATIONS OF DYADIC GREEN'S FUNCTION IN SOURCE REGION

The dyadic Green's function in source region has been studied by numerous workers [45]-[54], some of them write the Green's function as the sum of principal volume integral and a correcting term, which is determined solely from the geometry of the principal volume. Recently, the principal-volume method of expressing the spatial representation of the dyadic Green's function was reviewed by Chew [55]. The Green's function in the Sommerfeld-integral representation derived in the last chapter is valid in source region. But, the complex analysis in the last section is invalid when $x = x'$ and $z = z'$, since the integrand becomes unbounded when $|\zeta| \rightarrow \infty$. Thus, a delta singularity may be missed on the right side of (3.2.20) at points in source region, and the equation should be

$$\begin{aligned} \int_{-\infty}^{\infty} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta = C_{\alpha\beta} \delta(\vec{\rho} - \vec{\rho}') + 2\pi j \text{Res} \{ g_{\alpha\beta} e^{j\zeta|z-z'|} \} \Big|_{\zeta = -\zeta_p} \\ - \int_0^{\infty} \left[g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1 = -j\rho} - g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1 = j\rho} \right] \frac{j\rho}{\alpha} d\rho \end{aligned} \quad (3.3.1)$$

for all $\vec{\rho}$ and $\vec{\rho}'$, where $\vec{\rho} = \hat{x}x + \hat{z}z$, and $C_{\alpha\beta}$ is an unknown constant. One way to find the constant is by integrating over variables x', z' in a small cross section S_s which surrounds the field point (x, z) , i.e.

$$\begin{aligned} C_{\alpha\beta} = \lim_{S_s \rightarrow 0} \int_{S_s} dx' dz' \left\{ \int_{-\infty}^{\infty} g_{\alpha\beta} e^{j\zeta|z-z'|} d\zeta - 2\pi j \text{Res} \{ g_{\alpha\beta} e^{j\zeta|z-z'|} \} \Big|_{\zeta = -\zeta_p} \right. \\ \left. + \int_0^{\infty} \left[g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1 = -j\rho} - g_{\alpha\beta} e^{j\zeta|z-z'|} \Big|_{p_1 = j\rho} \right] \frac{j\rho}{\alpha} d\rho \right\} \end{aligned} \quad (3.3.2)$$

The surface integral of a single term on the right side of the above equation is dependent on the geometry of the small surface. Therefore, it seems that $C_{\alpha\beta}$ is nonunique. However, a combination of these integrals still yields a unique value. Hence, the Green's function in (3.3.1) is unique.

Since it is needed to do integrals of all Green's function terms to find the constant, it is not elementary and the procedure is dependent on the radiation mode spectrum. Thus, Chew's [44], [55] Fourier representation of Green's function in unbounded region will be used to find some relations, then, the delta singularity term in source region can be obtained using these relations.

The electric field is expressed in terms of the dyadic Green's function by

$$\vec{E}(\vec{r}) = -j\omega\mu \int_V d\vec{r}' \vec{G}(\vec{r}, \vec{r}') \bullet \vec{J}(\vec{r}') \quad (3.3.3)$$

In an unbounded region, the dyadic Green's function is the solution of

$$\nabla \times \nabla \times \vec{G}(\vec{r}, \vec{r}') - k_0^2 \vec{G}(\vec{r}, \vec{r}') = \vec{I} \delta(\vec{r} - \vec{r}') \quad (3.3.4)$$

where

$$\vec{I} = \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z} \quad (3.3.5)$$

Fourier transforming (3.3.4) in the 3-D spatial $\vec{r}$ variable yields

$$-\vec{k} \times \vec{k} \times \vec{g}(\vec{k}, \vec{r}') - k_0^2 \vec{g}(\vec{k}, \vec{r}') = \vec{I} e^{-j\vec{k} \bullet \vec{r}'} \quad (3.3.6)$$

with $\vec{k} = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z$. The above can be inverted to yield [56]

$$\vec{g}(\vec{k}, \vec{r}') = \frac{\vec{I}k_0^2 - \vec{k}\vec{k}}{k_0^2(k^2 - k_0^2)} e^{-j\vec{k} \bullet \vec{r}'} \quad (3.3.7)$$

where $k^2 = k_x^2 + k_y^2 + k_z^2$. Consequently, the dyadic Green's function in space domain can be written as

$$\vec{G}(\vec{r}, \vec{r}') = \frac{1}{(2\pi)^3} \int \int \int_{-\infty}^{\infty} d\vec{k} \frac{\vec{I}k_0^2 - \vec{k}\vec{k}}{k_0^2(k^2 - k_0^2)} e^{j\vec{k} \bullet (\vec{r} - \vec{r}')} \quad (3.3.8)$$

where $d\vec{k} = dk_x dk_y dk_z$.

The above Fourier representation of $\bar{G}(\vec{r}, \vec{r}')$ is a nonconvergent integral in the classical sense because the magnitude of the integrand tends to a constant when $|\vec{k}| \rightarrow \infty$. Chew [55] divided the integrand into two terms, one term is a constant and its inverse Fourier transformation is a delta function. The integral of the second term can be evaluated using Jordan's lemma and Cauchy's theorem. Consequently,

$$\bar{G}(\vec{r}, \vec{r}') = -\frac{\hat{x}\hat{x}}{k_0^2} \delta(\vec{r} - \vec{r}') + \frac{j}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\vec{k}_s \frac{\bar{k}_0^2 - \vec{k}_0 \vec{k}_0}{k_0^2 k_{0x}} e^{j\vec{k}_s \cdot (\vec{r} - \vec{r}') + jk_{0x}|x - x'|} \quad (3.3.9)$$

where $\vec{k}_s = \hat{y}k_y + \hat{z}k_z$, $d\vec{k}_s = dk_y dk_z$, $k_{0x} = \sqrt{k_0^2 - k_y^2 - k_z^2}$, $\vec{r}_s = \hat{y}y + \hat{z}z$, and

$$\vec{k}_0 = \hat{x}k_{0x} \text{sgn}(x - x') + \hat{y}k_y + \hat{z}k_z \quad (3.3.10)$$

The Green's function in y-invariant 2-D problem is

$$\bar{G}(\vec{p} - \vec{p}') = \int_{-\infty}^{\infty} \bar{G}(\vec{r}, \vec{r}') dy' \quad (3.3.11)$$

where $\vec{p} = \hat{x}x + \hat{z}z$, and it is noted that

$$\int_{-\infty}^{\infty} e^{jk_y(y - y')} dy' = 2\pi \delta(k_y) \quad (3.3.12)$$

Thus,

$$\bar{G}(\vec{p}, \vec{p}') = -\frac{\hat{x}\hat{x}}{k_0^2} \delta(\vec{p} - \vec{p}') + \frac{j}{4\pi} \int_{-\infty}^{\infty} dk_z \frac{\bar{k}_0^2 - \vec{k}_0 \vec{k}_0}{k_0^2 k_{0x}} e^{jk_z(z - z') + jk_{0x}|x - x'|} \quad (3.3.13)$$

where $k_{0x} = \sqrt{k_0^2 - k_z^2}$, and $\vec{k}_0 = \hat{x}k_{0x} \text{sgn}(x - x') + \hat{z}k_z$.

$$\bar{i}k_0^2 - \hat{k}_0 \hat{k}_0 = \hat{x}\hat{x}k_z^2 + \hat{z}\hat{z}k_{0x}^2 + \hat{y}\hat{y}k_0^2 - (\hat{x}\hat{z} + \hat{z}\hat{x})k_{0x}k_z \operatorname{sgn}(x - x') \quad (3.3.14)$$

then, applying the relation used in the beginning of the last section to (3.3.13) yields

$$k_0^2 \bar{G}(\vec{p}, \vec{p}') = -\hat{x}\hat{x}\delta(\vec{p} - \vec{p}') + \int_{-\infty}^{\infty} d\zeta e^{j\zeta|z-z'|} \bar{g} \quad (3.3.15)$$

$$\bar{g} = \frac{\hat{x}\hat{x}\zeta^2 - \hat{z}\hat{z}p^2 + \hat{y}\hat{y}k_0^2 - (\hat{x}\hat{z} + \hat{z}\hat{x})jp\zeta \operatorname{sgn}(x - x') \operatorname{sgn}(z - z')}{4\pi p} e^{-p|x-x'|} \quad (3.3.16)$$

It is noted that k_z is changed to ζ , and k_{0x} is changed to jp , where $p = \sqrt{\zeta^2 - k_0^2}$. On the other hand, interchanging the roles of x and z in (3.3.13), the dyadic Green's function can be written as

$$\bar{G}(\vec{p}, \vec{p}') = -\frac{\hat{z}\hat{z}}{k_0^2} \delta(\vec{p} - \vec{p}') + \frac{1}{4\pi} \int_{-\infty}^{\infty} dk_x \frac{\bar{i}k_0^2 - \hat{k}_0 \hat{k}_0}{k_0^2 \alpha} e^{jk_x(x-x') - \alpha|z-z'|} \quad (3.3.17)$$

where $\alpha = \sqrt{k_x^2 - k_0^2}$, and $\hat{k}_0 = \hat{x}k_x + \hat{z}j\alpha \operatorname{sgn}(z - z')$. Similarly, after some operations, (3.3.17) becomes

$$\begin{aligned} k_0^2 \bar{G}(\vec{p}, \vec{p}') &= -\hat{z}\hat{z}\delta(\vec{p} - \vec{p}') \\ &+ \int_{-\infty}^{\infty} dk_x \frac{-\hat{x}\hat{x}\alpha^2 + \hat{z}\hat{z}k_x^2 + \hat{y}\hat{y}k_0^2 - (\hat{x}\hat{z} + \hat{z}\hat{x})jk_x\alpha \operatorname{sgn}(x - x') \operatorname{sgn}(z - z')}{4\pi\alpha} e^{jk_x|x-x'| - \alpha|z-z'|} \end{aligned} \quad (3.3.18)$$

$$= -\hat{z}\hat{z}\delta(\vec{p} - \vec{p}') - \int_{-\infty}^{\infty} \bar{g} e^{j\zeta|z-z'|} \bigg|_{\substack{\zeta = j\alpha \frac{jk_x}{\alpha} \\ p = -jk_x}} dk_x \quad (3.3.19)$$

Hence, an important formula is obtained,

$$-\hat{x}\hat{x}\delta(\vec{p} - \vec{p}') + \int_{-\infty}^{\infty} d\zeta e^{j\zeta|z-z'|} \bar{g} = -\hat{z}\hat{z}\delta(\vec{p} - \vec{p}') - \int_{-\infty}^{\infty} \bar{g} e^{j\zeta|z-z'|} \bigg|_{\substack{\zeta = j\alpha \frac{jk_x}{\alpha} \\ p = -jk_x}} dk_x \quad (3.3.20)$$

In fact, the second term on the right side of the above equation is the integral along the branch cut of p .

Now, let's apply this formula to our problem. First, the Sommerfeld-integral representation of the dyadic Green's function is rewritten as

$$\begin{aligned}\bar{G}_{22}(x|x', z-z') &= \hat{x}\hat{x}\frac{j\eta_2}{k_2}\delta(\vec{\rho}-\vec{\rho}') + \bar{G}_{22}^R(x|x', z-z') \\ &= \frac{j\eta_2}{k_2}\left\{\hat{x}\hat{x}\delta(\vec{\rho}-\vec{\rho}') - \int_{-\infty}^{\infty} d\zeta e^{j\zeta|z-z'|} [\bar{g}_1 + \bar{g}_2]\right\}\end{aligned}\quad (3.3.21)$$

where

$$\bar{g}_1 = \frac{\hat{x}\hat{x}\zeta^2 - \hat{z}\hat{z}p_2^2 + \hat{y}\hat{y}k_2^2 - (\hat{x}\hat{z} + \hat{z}\hat{x})jp_2\zeta\operatorname{sgn}(x-x')\operatorname{sgn}(z-z')}{4\pi p_2} e^{-p_2|x-x'|} \quad (3.3.22)$$

and $\bar{g}_2$ is a second part of the integrand of the Green's function. $\bar{g}_1$ and $\bar{g}_2$ are not even functions of p_2 , but $\bar{g}_1 + \bar{g}_2$ is even in p_2 . Applying complex analysis on $\bar{g}_1$ and $\bar{g}_2$ separately yields

$$\begin{aligned}\frac{k_2}{j\eta_2}\bar{G}_{22}(x|x', z-z') &= \hat{x}\hat{x}\delta(\vec{\rho}-\vec{\rho}') - \int_{-\infty}^{\infty} d\zeta e^{j\zeta|z-z'|}\bar{g}_1 - \int_{-\infty}^{\infty} d\zeta e^{j\zeta|z-z'|}\bar{g}_2 \\ &= \hat{z}\hat{z}\delta(\vec{\rho}-\vec{\rho}') + \int_{-\infty}^{\infty} \bar{g}_1 e^{j\zeta|z-z'|} \bigg|_{\substack{\zeta=j\alpha \\ p_2=-jk_x}} \frac{jk_x}{\alpha} dk_x + \int_{-\infty}^{\infty} \bar{g}_1 e^{j\zeta|z-z'|} \bigg|_{\substack{\zeta=j\alpha \\ p_1=-j\rho}} \frac{j\rho}{\alpha} d\rho \\ &\quad + \int_{-\infty}^{\infty} \bar{g}_2 e^{j\zeta|z-z'|} \bigg|_{\substack{\zeta=j\alpha \\ p_2=-jk_x}} \frac{jk_x}{\alpha} dk_x + \int_{-\infty}^{\infty} \bar{g}_2 e^{j\zeta|z-z'|} \bigg|_{\substack{\zeta=j\alpha \\ p_1=-j\rho}} \frac{j\rho}{\alpha} d\rho \\ &\quad + 2\pi j \operatorname{Res} \{ [\bar{g}_1 + \bar{g}_2] e^{j\zeta|z-z'|} \} \bigg|_{\zeta=-\zeta_p}\end{aligned}\quad (3.3.23)$$

Since $\bar{g}_1 + \bar{g}_2$ is an even function of p_2 , then

$$\int_{-\infty}^{\infty} \bar{g}_1 e^{j\zeta|z-z'|} \Big|_{\substack{\zeta=j\alpha \\ p_2=-jk_x}} \frac{jk_x}{\alpha} dk_x + \int_{-\infty}^{\infty} \bar{g}_2 e^{j\zeta|z-z'|} \Big|_{\substack{\zeta=j\alpha \\ p_2=-jk_x}} \frac{jk_x}{\alpha} dk_x = 0 \quad (3.3.24)$$

Therefore,

$$\begin{aligned} \bar{G}_{22}(x|x', z-z') &= \hat{x}\hat{x} \frac{j\eta_2}{k_2} \delta(\hat{\rho} - \hat{\rho}') + \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \bar{g} e^{j\zeta|z-z'|} d\zeta \\ &= \hat{x}\hat{x} \frac{j\eta_2}{k_2} \delta(\hat{\rho} - \hat{\rho}') + \frac{j\eta_2}{k_2} 2\pi j \text{Res} \{ \bar{g} e^{j\zeta|z-z'|} \} \Big|_{\zeta=-\zeta_c} \\ &\quad - \frac{j\eta_2}{k_2} \int_0^{\infty} \left[\bar{g} e^{j\zeta|z-z'|} \Big|_{p_1=-j\rho} - \bar{g} e^{j\zeta|z-z'|} \Big|_{p_1=j\rho} \right] \frac{j\rho}{\alpha} d\rho \end{aligned} \quad (3.3.25)$$

3.4. SUMMARY OF GREEN'S FUNCTION

$$\begin{aligned} \bar{G}_{22}(x|x', z-z') &= \hat{x}\hat{x} \frac{j\eta_2}{k_2} \delta(\hat{\rho} - \hat{\rho}') + \bar{G}_{22}^R(x|x', z-z') \\ &= \hat{x}\hat{x} \frac{j\eta_2}{k_2} \delta(\hat{\rho} - \hat{\rho}') + \bar{G}_{22}^{Pole}(x|x', z-z') + \bar{G}_{22}^{Rad}(x|x', z-z') \end{aligned} \quad (3.4.1)$$

where the superscript “*R*” designates the integration along real axis, “*Pole*” designates the contribution from the surface wave which is the integration about surface-wave poles, and “*Rad*” designates the contribution from radiation modes arising from integration along branch cuts. The contribution by term $\hat{x}\hat{x} \frac{j\eta_2}{k_2} \delta(\hat{\rho} - \hat{\rho}')$ is the same as the delta singularity term in the eigenfunction representation of the dyadic Green's function in rectangular waveguides and cavities derived by some authors [45]-[47], [49].

$\bar{G}_{22}^R(x|x', z-z')$ was given in Chapter 2, and

$$G_{22xx}^{Pole}(x|x', z-z') = \frac{j\eta_2 j\zeta_p^2 p_1 \cos[q_2(t+x)] \cos[q_2(t+x')]}{k_2 \bar{\Delta}'_e(\zeta_p) \sin(q_2 t)} e^{-j\zeta_p |z-z'|} \quad (3.4.2)$$

$$G_{22xz}^{Pole}(x|x', z-z') = \text{sgn}(z-z') \frac{j\eta_2 \zeta_p p_1 \cos[q_2(t+x)] \sin[q_2(t+x')]}{k_2 \bar{\Delta}'_e(\zeta_p) \sin(q_2 t)} e^{-j\zeta_p |z-z'|} \quad (3.4.3)$$

$$G_{22zx}^{Pole}(x|x', z-z') = -\text{sgn}(z-z') \frac{j\eta_2 \zeta_p p_1 \sin[q_2(t+x)] \cos[q_2(t+x')]}{k_2 \bar{\Delta}'_e(\zeta_p) \sin(q_2 t)} e^{-j\zeta_p |z-z'|} \quad (3.4.4)$$

$$G_{22zz}^{Pole}(x|x', z-z') = \frac{j\eta_2 j q_2 p_1 \sin[q_2(t+x)] \sin[q_2(t+x')]}{k_2 \bar{\Delta}'_e(\zeta_p) \sin(q_2 t)} e^{-j\zeta_p |z-z'|} \quad (3.4.5)$$

$$G_{22yy}^{Pole}(x|x', z-z') = -j k_2 \eta_2 \frac{\sin[q_2(t+x)] \sin[q_2(t+x')]}{\bar{\Delta}'_h(\zeta_p) \sin(q_2 t)} e^{-j\zeta_p |z-z'|} \quad (3.4.6)$$

where ζ_p is the solution of $\bar{\Delta}'_e(\zeta) = 0$ or $\bar{\Delta}'_h(\zeta) = 0$, and

$$p_1 = \sqrt{\zeta_p^2 - k_1^2} \quad (3.4.7)$$

$$q_2 = \sqrt{k_2^2 - \zeta_p^2} \quad (3.4.8)$$

$$\bar{\Delta}'_e(\zeta) = p_1 \cos(q_2 t) - q_2 (\epsilon_1 / \epsilon_2) \sin(q_2 t) \quad (3.4.9)$$

$$\bar{\Delta}'_e(\zeta) = \zeta \left\{ \frac{\cos(q_2 t)}{p_1} \left[1 + \frac{\epsilon_1}{\epsilon_2} p_1 t \right] + \frac{\sin(q_2 t)}{q_2} \left[p_1 t + \frac{\epsilon_1}{\epsilon_2} \right] \right\} \quad (3.4.10)$$

$$\bar{\Delta}'_e(\zeta_p) = \zeta_p \frac{\cos(q_2 t)}{p_1 q_2^2} \left[k_2^2 - k_1^2 + p_1 t \left(\frac{\epsilon_2}{\epsilon_1} p_1^2 + \frac{\epsilon_1}{\epsilon_2} q_2^2 \right) \right] \quad (3.4.11)$$

When $\mu_1 = \mu_2 = \mu_0$

$$\bar{\Delta}'_e(\zeta_p) = \zeta_p \frac{\cos(q_2 t)}{p_1 q_2^2} \frac{\epsilon_2}{\epsilon_1} \left(1 - \frac{\epsilon_1}{\epsilon_2} \right) \left[k_1^2 (1 - p_1 t) + p_1 t \zeta_p^2 \left(1 + \frac{\epsilon_1}{\epsilon_2} \right) \right] \quad (3.4.12)$$

$$\bar{\Delta}_h(\zeta) = j [q_2 \cos(q_2 t) + p_1 \sin(q_2 t)] \quad (3.4.13)$$

$$\bar{\Delta}'_h(\zeta) = \frac{j\zeta}{p_1 q_2} (1 + p_1 t) [q_2 \sin(q_2 t) - p_1 \cos(q_2 t)] \quad (3.4.14)$$

$$\bar{\Delta}'_h(\zeta_p) = \frac{j\zeta_p (1 + p_1 t)}{p_1 \sin(q_2 t)} \quad (3.4.15)$$

The five components of $\bar{G}_{22}^{Rad}(x|x', z-z')$ are

$$G_{22xx}^{Rad}(x|x', z-z') = \frac{j\eta_2}{k_2} \int_0^\infty \frac{\alpha \rho C_e(\rho)}{2\pi q_2} \cos[q_2(t+x)] \cos[q_2(t+x')] e^{-\alpha|z-z'|} d\rho \quad (3.4.16)$$

$$G_{22xz}^{Rad}(x|x', z-z') = \text{sgn}(z-z') \frac{j\eta_2}{k_2} \int_0^\infty \frac{\rho C_e(\rho)}{2\pi} \cos[q_2(t+x)] \sin[q_2(t+x')] e^{-\alpha|z-z'|} d\rho \quad (3.4.17)$$

$$G_{22zx}^{Rad}(x|x', z-z') = -\text{sgn}(z-z') \frac{j\eta_2}{k_2} \int_0^\infty \frac{\rho C_e(\rho)}{2\pi} \sin[q_2(t+x)] \cos[q_2(t+x')] e^{-\alpha|z-z'|} d\rho \quad (3.4.18)$$

$$G_{22zz}^{Rad}(x|x', z-z') = -\frac{j\eta_2}{k_2} \int_0^\infty \frac{\rho q_2 C_e(\rho)}{2\pi \alpha} \sin[q_2(t+x)] \sin[q_2(t+x')] e^{-\alpha|z-z'|} d\rho \quad (3.4.19)$$

$$G_{22yy}^{Rad}(x|x', z-z') = -jk_2 \eta_2 \int_0^\infty \frac{\rho C_h(\rho)}{2\pi \alpha q_2} \sin[q_2(t+x)] \sin[q_2(t+x')] e^{-\alpha|z-z'|} d\rho \quad (3.4.20)$$

where

$$\alpha = \sqrt{\rho^2 - k_1^2} = \begin{cases} \sqrt{\rho^2 - k_1^2} & k_1 < \rho < \infty \\ j\sqrt{k_1^2 - \rho^2} & 0 < \rho < k_1 \end{cases} \quad (3.4.21)$$

$$q_2 = \sqrt{k_2^2 + \alpha^2} = \sqrt{k_2^2 - k_1^2 + \rho^2} \quad (3.4.22)$$

$$C_e(\rho) = \frac{2\varepsilon_1\varepsilon_2\rho q_2}{(\rho\varepsilon_2)^2 \cos^2(q_2 t) + (\varepsilon_1 q_2)^2 \sin^2(q_2 t)} \quad (3.4.23)$$

$$C_h(\rho) = \frac{2\rho q_2}{q_2^2 \cos^2(q_2 t) + \rho^2 \sin^2(q_2 t)} \quad (3.4.24)$$

CHAPTER 4

CALCULATION OF SCATTERED FIELD AND METHOD OF MOMENTS (MoM) SOLUTION TO THE EFIE

4.1. INTRODUCTION

Once the electric fields in the discontinuity region have been obtained, the scattered electric fields $\vec{E}_i^s$ ($i = 1, 2$) in both regions may be determined by

$$\vec{J}(x, z) = j\omega\epsilon_0\delta n^2(x, z)\vec{E}_2(x, z) = j\frac{k_0}{\eta_0}\delta n^2(x, z)\vec{E}_2(x, z) \quad (4.1.1)$$

$$\begin{aligned} \vec{E}_i^s(x, z) &= \int_{LCS} \vec{G}_{i2}(x|x', z-z') \bullet \vec{J}(x', z') dx' dz' \\ &= \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') \vec{G}_{i2}(x|x', z-z') \bullet \vec{E}_2(x', z') dx' dz' \end{aligned} \quad (4.1.2)$$

Both the near and far-zone fields can be calculated exactly through numerical integration along the real ζ axis in (4.1.2). But this is numerically quite time-consuming and leads to little physical insight. This method can be used to compute the scattered field at a specific point or to validate the results from other more efficient approximation methods. It is not suited for radiation pattern calculation. In the far-zone field and radiation pattern calculations, the spatial parameter z has a very large dynamic range, and terms like $e^{j\zeta z}$ oscillate rapidly. Highly oscillatory integrands make accurate and rapidly convergent numerical integration difficult to achieve.

In the last chapter, Cauchy's theorem is used for contour integrals to deform the original real-line inversion integral contour for the transform domain Green's function. The right choice of contour deformation revealed that the integral can be decomposed into two parts, the contributions from integration along surface-wave pole exclusions and along the branch cuts. Thus, the scattered field has two parts. One part is the field of the discrete surface-wave modes, and the other is the field of radiation modes.

In the next section, the residue theorem will be used to evaluate the field of the surface-wave mode. But, the integration along the branch cuts is also quite time-consuming. When x and z have a very large dynamic range, since terms like $e^{j\zeta z}$, $e^{-j\rho x}$ and $e^{\pm jq_z x}$ oscillate rapidly, accurate integration is difficult to achieve and convergence is slow. Thus, the far-zone field will be calculated approximately by using the saddle-point method in section 4.3.

In the last part of this chapter, the Method of Moments (MoM) is applied to transform the integral equations into a matrix equation. Using general basis and testing functions, formulas are derived to calculate the matrix elements, forcing term and scattered field. These will be used in the pulse Galerkin's method and the sinusoidal/piecewise sinusoidal Galerkin's methods in Chapters 6, 7, and 8.

4.2. EVALUATION OF THE ELECTRIC FIELD OF SURFACE-WAVE MODES

Now, let's apply Cauchy's theorem to (4.1.2). The contribution from the surface-wave mode is evaluated using the residue theorem as

$$\vec{E}_i^{ss}(x, z) = \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') \vec{G}_{i2}^{Pole}(x|x', z-z') \bullet \vec{E}_2(x', z') dx' dz' \quad (4.2.1)$$

where the superscript “*ss*” represents the surface-wave part of the scattered field, and $\bar{G}_{22}^{Pole}(x|x', z-z')$ was given in the last chapter and, similarly, $\bar{G}_{12}^{Pole}(x|x', z-z')$ is derived as

$$G_{12xx}^{Pole}(x|x', z-z') = \frac{j\eta_2}{k_2} \frac{j\zeta_p^2}{\bar{\Delta}'_e(\zeta_p)} \cos[q_2(t+x')] e^{-p_1x} e^{-j\zeta_p|z-z'|} \quad (4.2.2)$$

$$G_{12xz}^{Pole}(x|x', z-z') = \text{sgn}(z-z') \frac{j\eta_2}{k_2} \frac{\zeta_p q_2}{\bar{\Delta}'_e(\zeta_p)} \sin[q_2(t+x')] e^{-p_1x} e^{-j\zeta_p|z-z'|} \quad (4.2.3)$$

$$G_{12zx}^{Pole}(x|x', z-z') = \text{sgn}(z-z') \frac{j\eta_2}{k_2} \frac{-p_1\zeta_p}{\bar{\Delta}'_e(\zeta_p)} \cos[q_2(t+x')] e^{-p_1x} e^{-j\zeta_p|z-z'|} \quad (4.2.4)$$

$$G_{12zz}^{Pole}(x|x', z-z') = \frac{j\eta_2}{k_2} \frac{jp_1q_2}{\bar{\Delta}'_e(\zeta_p)} \sin[q_2(t+x')] e^{-p_1x} e^{-j\zeta_p|z-z'|} \quad (4.2.5)$$

$$G_{12yy}^{Pole}(x|x', z-z') = -\frac{jk_1\eta_1}{\bar{\Delta}'_h(\zeta_p)} \sin[q_2(t+x')] e^{-p_1x} e^{-j\zeta_p|z-z'|} \quad (4.2.6)$$

“+” will be used to represent the surface-wave field propagating along the +*z* direction for $z > \text{all } z'$, and “-” to represent the surface-wave field propagating along the -*z* direction for $z < \text{all } z'$.

TE-polarized wave incidence:

The electric field of excited TE-odd surface-wave modes has y-component only

$$E_{1y}^{ss\pm}(x, z) = \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') G_{12yy}^{Pole} E_{2y} dx' dz' = A_h^\pm \sin(q_2 t) e^{-p_1x} e^{\mp j\zeta_p z} \quad (4.2.7)$$

$$E_{2y}^{ss\pm}(x, z) = \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') G_{22yy}^{Pole} E_{2y} dx' dz' = A_h^\pm \sin[q_2(t+x)] e^{\mp j\zeta_p z} \quad (4.2.8)$$

where

$$A_h^\pm = \frac{k_0^2}{\bar{\Delta}'_h(\zeta_p) \sin(q_2 t)} \int_{LCS} \delta n^2(x', z') E_{2y}(x', z') \sin[q_2(t + x')] e^{\pm j\zeta_p z'} dx' dz' \quad (4.2.9)$$

and p_1 , q_2 , and $\bar{\Delta}'_h(\zeta_p)$ were given in the last chapter. It is noted that the electric fields in two regions satisfy the boundary condition

$$E_{2y}^{ss\pm} \Big|_{x=0} = E_{1y}^{ss\pm} \Big|_{x=0} \quad (4.2.10)$$

TM-polarized wave incidence:

The electric fields of excited TM-even surface-wave modes in both regions have x-, z-components

$$\begin{aligned} E_{1z}^{ss\pm}(x, z) &= \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') [G_{12zx}^{Pole} E_{2x} + G_{12zz}^{Pole} E_{2z}] dx' dz' \\ &= A_e^\pm e^{-p_1 x} e^{\mp j\zeta_p z} \end{aligned} \quad (4.2.11)$$

$$\begin{aligned} E_{1x}^{ss\pm}(x, z) &= \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') [G_{12xx}^{Pole} E_{2x} + G_{12xz}^{Pole} E_{2z}] dx' dz' \\ &= \mp \frac{j\zeta_p}{p_1} A_e^\pm e^{-p_1 x} e^{\mp j\zeta_p z} \end{aligned} \quad (4.2.12)$$

$$\begin{aligned} E_{2z}^{ss\pm}(x, z) &= \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') [G_{22zx}^{Pole} E_{2x} + G_{22zz}^{Pole} E_{2z}] dx' dz' \\ &= \frac{A_e^\pm}{\sin(q_2 t)} \sin[q_2(t + x)] e^{\mp j\zeta_p z} \end{aligned} \quad (4.2.13)$$

$$\begin{aligned} E_{2x}^{ss\pm}(x, z) &= \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') [G_{22xx}^{Pole} E_{2x} + G_{22xz}^{Pole} E_{2z}] dx' dz' \\ &= \mp \frac{j\zeta_p}{q_2} \frac{A_e^\pm}{\sin(q_2 t)} \cos[q_2(t + x)] e^{\mp j\zeta_p z} \end{aligned} \quad (4.2.14)$$

where

$$A_e^\pm = \frac{\epsilon_0}{\epsilon_2} \frac{p_1}{\tilde{\Delta}'_e(\zeta_p)} \int_{LCS} \delta n^2(x', z') \{ \pm \zeta_p \cos[q_2(t+x')] E_{2x}(x', z') - j q_2 \sin[q_2(t+x')] E_{2z}(x', z') \} e^{\pm j \zeta_p z'} dx' dz' \quad (4.2.15)$$

and p_1 , q_2 , and $\tilde{\Delta}'_e(\zeta_p)$ were given in the last chapter.

Thus, the electric fields of TM-even surface-wave modes excited in the two regions are

$$\vec{E}_1^{ss\pm}(x, z) = A_e^\pm \left\{ \hat{x} \frac{\mp j \zeta_p}{p_1} + \hat{z} \right\} e^{-p_1 x} e^{\mp j \zeta_p z} \quad (4.2.16)$$

$$\vec{E}_2^{ss\pm}(x, z) = \frac{A_e^\pm}{\sin(q_2 t)} \left\{ \hat{x} \frac{\mp j \zeta_p}{q_2} \cos[q_2(t+x)] + \hat{z} \sin[q_2(t+x)] \right\} e^{-p_1 x} e^{\mp j \zeta_p z} \quad (4.2.17)$$

and they satisfy the boundary condition

$$E_{2z}^{ss\pm} \big|_{x=0} = E_{1z}^{ss\pm} \big|_{x=0} \quad (4.2.18)$$

4.3. ASYMPTOTIC EVALUATION OF THE FAR-ZONE FIELD

In order to calculate the far-zone field more efficiently, the saddle-point method (also known as the method of steepest descent) of approximate integration will be used [1] [44].

Let's write $\bar{G}_{12}(x|x', z-z')$ as

$$G_{12\alpha\beta}(x|x', z-z') = \frac{j\eta_2}{k_2} I_{\alpha\beta} \quad (4.3.1)$$

where

$$I_{\alpha\beta} = \int_{-\infty}^{\infty} g_{\alpha\beta}(\zeta) e^{-p_1 x} e^{j \zeta z} d\zeta \quad (4.3.2)$$

with

$$g_{xx}(\zeta) = -\frac{\zeta^2}{2\pi\bar{\Delta}_e} \cosh [p_2 (t + x')] e^{-j\zeta z'} \quad (4.3.3)$$

$$g_{xz}(\zeta) = \frac{jp_2\zeta}{2\pi\bar{\Delta}_e} \sinh [p_2 (t + x')] e^{-j\zeta z'} \quad (4.3.4)$$

$$g_{zx}(\zeta) = \frac{jp_1\zeta}{2\pi\bar{\Delta}_e} \cosh [p_2 (t + x')] e^{-j\zeta z'} \quad (4.3.5)$$

$$g_{zz}(\zeta) = \frac{p_1p_2}{2\pi\bar{\Delta}_e} \sinh [p_2 (t + x')] e^{-j\zeta z'} \quad (4.3.6)$$

$$g_{yy}(\zeta) = -\frac{k_2^2}{2\pi\bar{\Delta}_h} \sinh [p_2 (t + x')] e^{-j\zeta z'} \quad (4.3.7)$$

To implement the saddle-point method, the integral of (4.3.2) must first be placed in an appropriate standard form by transforming to polar coordinates in both the (x, z) and (p, ζ) planes (where $p_1 = jp$). Let

$$x = r \cos \theta \quad (4.3.8)$$

$$z = r \sin \theta \quad (4.3.9)$$

$$\zeta = -k_1 \sin \phi \quad (4.3.10)$$

$$p_1 = jk_1 \cos \phi \quad (4.3.11)$$

$$\phi = \sigma + j\eta \quad (4.3.12)$$

$$e^{-p_1 x} e^{j\zeta z} = e^{-jk_1 r \cos(\phi - \theta)} \quad (4.3.13)$$

with polar coordinates as defined in Figure 4.1.

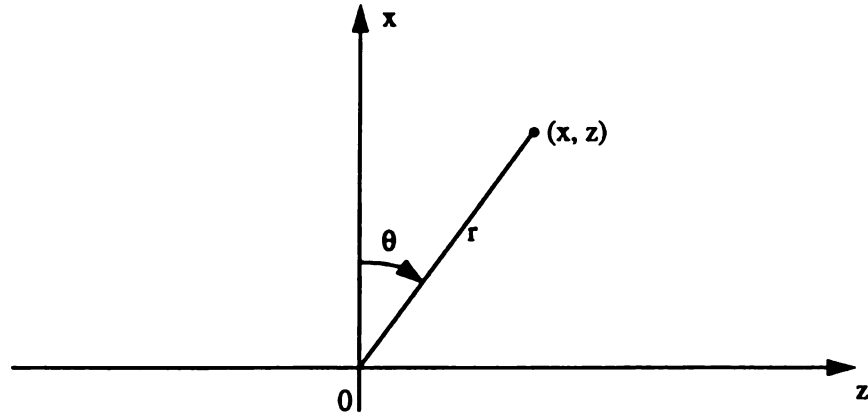


Figure 4.1. Polar coordinates in the x-z plane.

Eqs. (4.3.10) and (4.3.11) represent a mapping of the complex ζ plane into a strip of the complex ϕ plane. The two sheets of the Riemann surface map into a connected strip of width 2π along the σ axis. From (4.3.10)-(4.3.12), it is found that

$$\zeta = \beta + j\alpha = -k_1 \sin(\sigma + j\eta) \quad (4.3.14)$$

$$\beta = -k_1 \sin \sigma \cosh \eta \quad (4.3.15)$$

$$\alpha = -k_1 \cos \sigma \sinh \eta \quad (4.3.16)$$

$$p_1 = j\rho = k_1 \sin \sigma \sinh \eta + jk_1 \cos \sigma \cosh \eta \quad (4.3.17)$$

This mapping is illustrated in Figure 4.2. On the proper sheet of the Riemann surface (top sheet), $\operatorname{Re}\{p_1\} > 0$, and hence $\sin \sigma \sinh \eta > 0$. The four quadrants of the proper and improper branches of the ζ plane map into the regions designated P_i and I_i ($i = 1, 2, 3, 4$), respectively, in Figure 4.2.

The original contour of integration along the real ζ axis denoted by C passes from $(\alpha = 0, \beta = -\infty)$ to $(\alpha = 0, \beta = \infty)$. Since

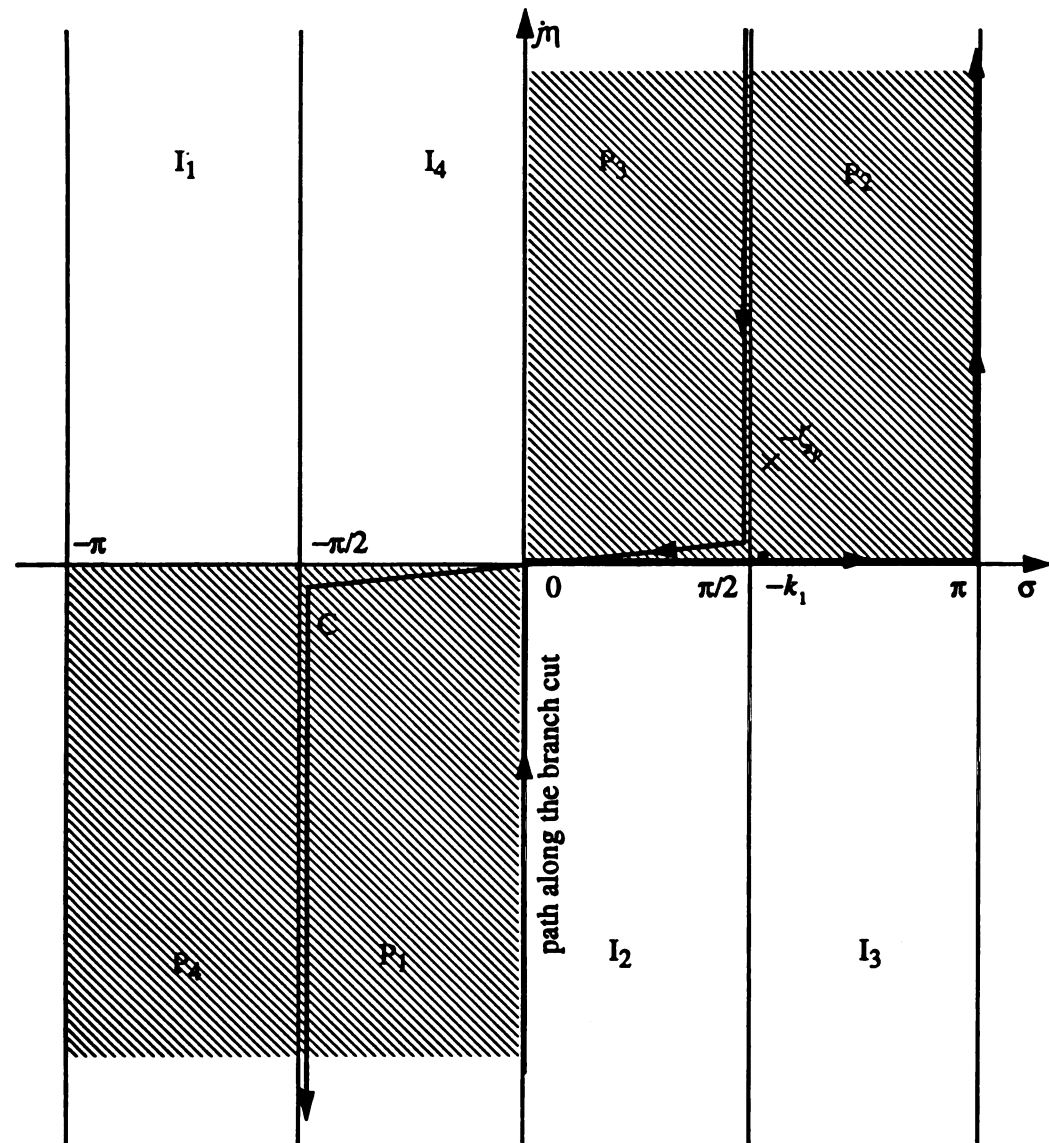


Figure 4.2. Mapping of the two Riemann sheets of the ζ plane onto a strip of the $\phi = \sigma + j\eta$ plane. The crosshatched regions are the proper Riemann sheet (top sheet).

$$(\alpha = 0, \beta = -\infty) \leftrightarrow (\sigma = \pi/2, \eta = \infty) \quad (4.3.18)$$

$$(\alpha = 0, \beta = \infty) \leftrightarrow (\sigma = -\pi/2, \eta = -\infty) \quad (4.3.19)$$

the integral in (4.3.2) now becomes

$$I_{\alpha\beta} = - \int_{\pi/2 + j\infty}^{-\pi/2 - j\infty} k_1 \cos \phi g_{\alpha\beta} (-k_1 \sin \phi) e^{-jk_1 r \cos(\phi - \theta)} d\phi \quad (4.3.20)$$

Let's consider the following generic integral

$$I(\lambda) = \int_C h(t) e^{\lambda \mathcal{V}(t)} dt \quad (4.3.21)$$

If $f(t)$ has a stationary point at $t = t_0$, implying $f'(t_0) = 0$, then

$$\begin{aligned} f(t) - f(t_0) &\approx \frac{(t - t_0)^2}{2} f''(t_0) = \frac{1}{2} F R^2 e^{j(s + 2\delta)} \\ &= \frac{1}{2} F R^2 \cos(s + 2\delta) + \frac{j}{2} F R^2 \sin(s + 2\delta) \end{aligned} \quad (4.3.22)$$

where

$$f''(t_0) = F e^{js} \quad (4.3.23)$$

$$t - t_0 = R e^{j\delta} \quad (4.3.24)$$

On the path of $s + 2\delta = \pm\pi$, the function $f(t) - f(t_0)$ is purely real and negative, hence

$$e^{\lambda \mathcal{V}(t)} \approx e^{-\lambda F R^2 / 2 + \lambda \mathcal{V}(t_0)} \quad (4.3.25)$$

Notice that along this path, $e^{\lambda \mathcal{V}(t)}$ has a constant phase, hence, it is also called the constant-phase path. Furthermore $e^{\lambda \mathcal{V}(t)}$ has a maximum at $R = 0$, or $t = t_0$, and is exponentially small along the path passing away from $R = 0$, or away from $t = t_0$. But along

the path $s + 2\delta = 0, 2\pi$, $e^{\lambda \mathcal{V}(t)} = e^{\lambda FR^2/2 + \lambda \mathcal{V}(t_0)}$ becomes exponentially large. Therefore, the function $e^{\lambda \mathcal{V}(t)}$, when $\lambda \rightarrow \infty$, looks like a saddle at the point $t = t_0$ on the complex t plane. Hence, the constant-phase path, on which the function $e^{\lambda \mathcal{V}(t)}$ descends steeply away from its value at the saddle-point, is known as the steepest-descent path.

With the change of variable in (4.3.24), (4.3.21) becomes

$$I(\lambda) = e^{\lambda \mathcal{V}(t_0) + j\delta} \int_C h(t_0 + R e^{j\delta}) e^{-\lambda FR^2/2} dR \quad (4.3.26)$$

The path of integration can be deformed from C to the steepest-descent path and paths A and B in Figure 4.3. The contributions to the integral over the paths A and B are vanishingly small at infinity. However, in deforming the contour, the contributions due to any singularities enclosed between two paths (captured during the path deformation) need to be included. This leads to

$$I(\lambda) = I_p + e^{\lambda \mathcal{V}(t_0) + j\delta} \int_{-\infty}^{\infty} h(t_0 + R e^{j\delta}) e^{-\lambda FR^2/2} dR \quad (4.3.27)$$

where I_p is the contribution from the singularities enclosed between the original integration path and the steepest-descent path.

When $\lambda \rightarrow \infty$, $e^{-\lambda FR^2/2}$ is exponentially small away from $R = 0$ on the steepest-descent path. This implies that most of the contribution to the integral is from around $R = 0$. Hence

$$I(\lambda) \approx I_p + h(t_0) e^{\lambda \mathcal{V}(t_0) + j\delta} \int_{-\infty}^{\infty} e^{-\lambda FR^2/2} dR = I_p + \sqrt{\frac{2\pi}{\lambda F}} h(t_0) e^{\lambda \mathcal{V}(t_0) + j\delta} \quad (4.3.28)$$

In the integral of (4.3.20),

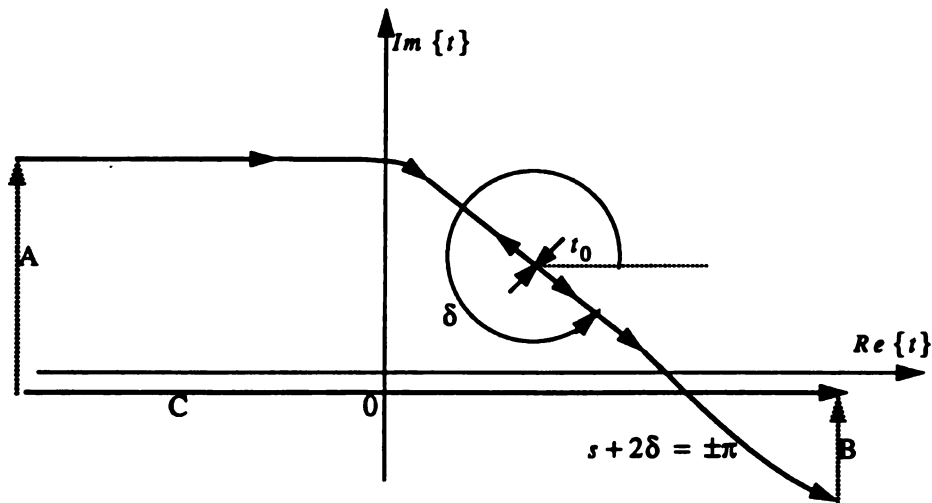


Figure 4.3. The original integration path and the steepest-descent path

$$\lambda = k_1 r \quad (4.3.29)$$

$$f(\phi) = -j \cos(\phi - \theta) \quad (4.3.30)$$

$$f'(\phi) = j \sin(\phi - \theta) \quad (4.3.31)$$

$$f''(\phi) = j \cos(\phi - \theta) \quad (4.3.32)$$

From $f'(\phi_0) = 0$, the saddle-point is obtained

$$\phi_0 = \theta \quad (4.3.33)$$

Thus,

$$f''(\theta) = j \quad F = 1 \quad s = \pi/2 \quad (4.3.34)$$

On the steepest-descent path,

$$\arg\{\phi\} = \delta = -\frac{s}{2} \pm \frac{\pi}{2} = -\frac{\pi}{4} \pm \frac{\pi}{2} \quad (4.3.35)$$

From Figure 4.4, it is clear that $\delta = -3\pi/4$ should be chosen. Thus, the integral (4.3.20) becomes

$$I_{\alpha\beta} = I_{\alpha\beta}^{Pole} + I_{\alpha\beta}^{SDC} \quad (4.3.36)$$

where $I_{\alpha\beta}^{Pole}$ is the contribution from the integration along the poles which are enclosed between the original integration path and the steepest-descent path, and $I_{\alpha\beta}^{SDC}$ is the contribution from the integration along the steepest-descent contour with

$$I_{\alpha\beta}^{SDC} = k_1 \cos \theta g_{\alpha\beta} (-k_1 \sin \theta) \sqrt{\frac{2\pi}{k_1 r}} e^{-j(k_1 r - \pi/4)} \quad (4.3.37)$$

There are two kinds of the singularities, the surface-wave poles of finite number (poles on the proper Riemann sheet) and the leaky modes of infinite number (poles on the improper Riemann sheet) [1]. When θ is small, no singularities captured. When θ is large, since the field of the surface-wave mode will exponentially decay as $e^{-p_1 x} = e^{-p_1 r \cos \theta}$, then, if $\theta \neq \pi/2$, $e^{-p_1 r \cos \theta} \rightarrow 0$, as $r \rightarrow \infty$. Thus, it is not needed to consider the contribution from the surface-wave mode in the far-zone field for $|\theta| < \pi/2$.

It is assumed that a leaky mode pole ζ_L could be captured. The electric field of the leaky mode can be written as

$$\begin{aligned} \vec{E}_1^L &= A_L \vec{E}_0 e^{-j\rho_L x - j\zeta_L z} = A_L \vec{E}_0 e^{-jk_1 r \cos(\phi_L - \theta)} \\ &= A_L \vec{E}_0 e^{-k_1 r \sin(\sigma_L - \theta) \sinh \eta_L} e^{-jk_1 r \cos(\sigma_L - \theta) \cosh \eta_L} \end{aligned} \quad (4.3.38)$$

where

$$-\zeta_L = k_1 \sin \phi_L = k_1 \sin(\sigma_L + j\eta_L) \quad (4.3.39)$$

Since $(\sigma_L - \theta) < 0$ and $\eta_L < 0$ for any leaky pole captured, then $\vec{E}_1^L \rightarrow 0$, as $r \rightarrow \infty$. This is also true for $\theta = \pm\pi/2$. Thus, the far-zone field ($|\theta| < \pi/2$) can be approximated entirely by the radiation field arising from the integration along the steepest-descent path.

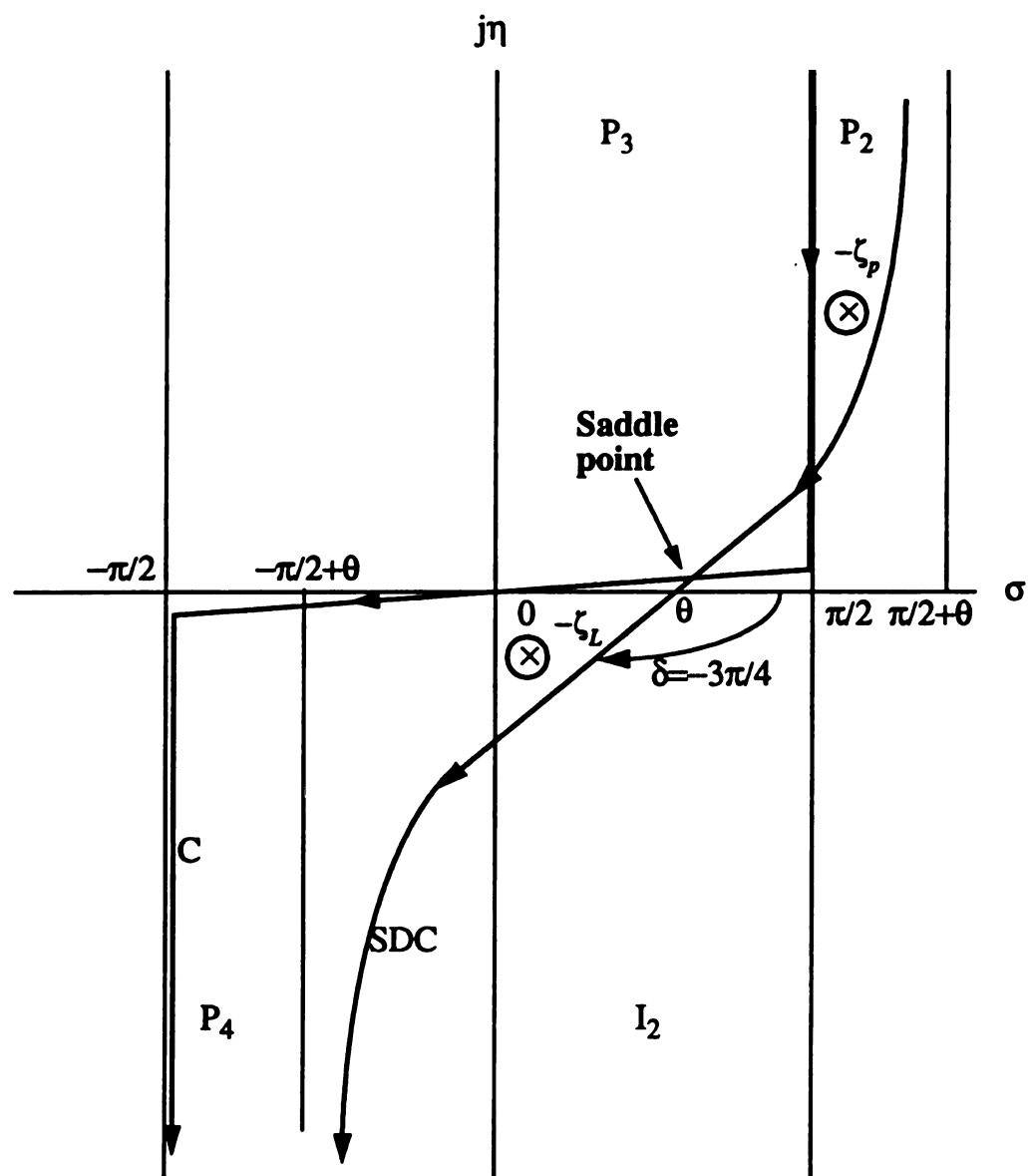


Figure 4.4. Deformation of contour C into steepest-descent contour (SDC).

$$\vec{E}_1^{sr}(x, z) = \frac{jk_0}{\eta_0} \int_{LCS} \delta \pi^2(x', z') \vec{G}_{12}^{SDC}(x|x', z-z') \bullet \vec{E}_2(x', z') dx' dz' \quad (4.3.40)$$

where the superscript “*sr*” represents the radiative component of the scattered field, and

$$G_{12\alpha\beta}^{SDC}(x|x', z-z') = \frac{j\eta_2}{k_2} k_1 \cos\theta g_{\alpha\beta} (-k_1 \sin\theta) \sqrt{\frac{2\pi}{k_1 r}} e^{-j(k_1 r - \pi/4)} \quad (4.3.41)$$

Leaky mode poles arise from solutions of the eigenvalue equation which occur on the improper Riemann sheets, they are also called improper modes [1]-[3], [57]-[59]. The corresponding EM fields do not satisfy the radiation condition at $|x| = \infty$. They are not included in a complete spectral representation used to describe a general field in the guiding layer and its surround. They are parts of an approximation to the continuous radiation spectrum. But, some of these modes can play an important part in the asymptotic description of the radiation field when the distance between source and field points is not very large. The EM fields of leaky modes decrease exponentially along the interface direction (z direction in our problem), and increase exponentially along the direction which is normal to the interface, but they decrease exponentially along any radius from the source within their domain of existence. Thus leaky modes exist near source and near the interface only. Leaky modes lead to a net power flow away from the interface, which is why they are called leaky modes.

Cassedy and Cohn [60] calculated the amplitude of the electric field of the leaky wave due to a line source above a grounded dielectric slab, and found that the amplitude decreases as the distance from the source increases when θ is near $\pm\pi/2$. They also verified the existence of the leaky-wave mode experimentally.

Thus, the EM fields of leaky modes are negligible in the far-zone field, and that field can be calculated by the saddle point method. Let's simplify the formula to calculate the far-zone field

TE-polarized wave incidence:

$$E_{1y}^{sr}(x, z) = \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') G_{12yy}^{SDC} E_{2y} dx' dz' = B_h(\theta) \sqrt{\frac{2\pi}{k_1 r}} e^{-j(k_1 r - \pi/4)} \quad (4.3.42)$$

This is a cylindrical wave polarized in the y-direction, and where

$$B_h(\theta) = \frac{jk_0^2 k_1 \cos \theta}{2\pi \tilde{\Delta}_h(-k_1 \sin \theta)} \int_{LCS} \delta n^2(x', z') E_{2y}(x', z') \sin[q_2(t+x')] e^{jk_1 z' \sin \theta} dx' dz' \quad (4.3.43)$$

with

$$q_2 = \sqrt{k_2^2 - k_1^2 \sin^2 \theta} \quad (4.3.44)$$

$$\tilde{\Delta}_h(-k_1 \sin \theta) = jq_2 \cos(q_2 t) - k_1 \cos \theta \sin(q_2 t) \quad (4.3.45)$$

TM-polarized wave incidence:

$$\begin{aligned} E_{1z}^{sr}(x, z) &= \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') [G_{12zx}^{SDC} E_{2x} + G_{12zz}^{SDC} E_{2z}] dx' dz' \\ &= B_e(\theta) \cos \theta \sqrt{\frac{2\pi}{k_1 r}} e^{-j(k_1 r - \pi/4)} \end{aligned} \quad (4.3.46)$$

$$\begin{aligned} E_{1x}^{sr}(x, z) &= \frac{jk_0}{\eta_0} \int_{LCS} \delta n^2(x', z') [G_{12xx}^{SDC} E_{2x} + G_{12xz}^{SDC} E_{2z}] dx' dz' \\ &= -B_e(\theta) \sin \theta \sqrt{\frac{2\pi}{k_1 r}} e^{-j(k_1 r - \pi/4)} \end{aligned} \quad (4.3.47)$$

where

$$\begin{aligned} B_e(\theta) &= \frac{\epsilon_0}{\epsilon_2} \frac{jk_1^2 \cos \theta}{2\pi \tilde{\Delta}_e(-k_1 \sin \theta)} \int_{LCS} \delta n^2(x', z') \{jk_1 \sin \theta \cos[q_2(t+x')] E_{2x}(x', z') \\ &\quad + q_2 \sin[q_2(t+x')] E_{2z}(x', z')\} e^{jk_1 z' \sin \theta} dx' dz' \end{aligned} \quad (4.3.48)$$

with

$$q_2 = \sqrt{k_2^2 - k_1^2 \sin^2 \theta} \quad (4.3.49)$$

$$\bar{\Delta}_e(-k_1 \sin \theta) = j k_1 \cos \theta \cos(q_2 t) - \frac{\epsilon_1}{\epsilon_2} q_2 \sin(q_2 t) \quad (4.3.50)$$

Combining (4.3.46) and (4.3.47) yields

$$\vec{E}_1^{sr}(r, \theta) = B_e(\theta) \sqrt{\frac{2\pi}{k_1 r}} e^{-j(k_1 r - \pi/4)} \{ \hat{z} \cos \theta - \hat{x} \sin \theta \} \quad (4.3.51)$$

This is a cylindrical wave polarized in the θ -direction.

If the electric field $\vec{E}_1^{ss}$ of the surface-wave mode is calculated separately, the total far-zone field is

$$\vec{E}_1^s(x, z) = \vec{E}_1^{ss\pm}(x, z) + \vec{E}_1^{sr}(r, \theta) \quad (4.3.52)$$

for r large and $|\theta| \leq \pi/2$.

4.4. THE SCATTERED POWER AND BACK-SCATTERING WIDTH

The power of the surface-wave modes and radiation modes can be calculated by using the Poynting's theorem as follows

TE-polarized wave incidence:

$$E_y = \begin{cases} A_h^\pm \sin[q_2(x+t)] e^{\mp j \zeta_p z} & -t < x < 0 \\ A_h^\pm \sin(q_2 t) e^{-p_1 x} e^{\mp j \zeta_p z} & 0 < x < \infty \end{cases} \quad (4.4.1)$$

$$H_x = \frac{1}{j\omega\mu_0} \frac{\partial E_y}{\partial z} = \mp \frac{\zeta_p}{k_0 \eta_0} E_y \quad (4.4.2)$$

$$\begin{aligned}
P_z &= -\frac{1}{2} Re \left\{ \int_{-t}^{\infty} E_y H_x^* dx \right\} = \pm \frac{1}{2} \frac{Re \{ \zeta_p \}}{k_0 \eta_0} \int_{-t}^{\infty} E_y E_y^* dx \\
&= \pm \frac{\zeta_{pr}}{4k_0} \frac{|A_p^\pm|^2}{\eta_0} e^{\pm 2\zeta_{pz}} \left[\frac{|\sin(q_2 t)|^2}{p_{1r} t} + \frac{\sinh(2q_2 t)}{2q_{2i} t} - \frac{\sin(2q_{2r} t)}{2q_{2r} t} \right] t
\end{aligned} \quad (4.4.3)$$

where

$$\zeta_p = \zeta_{pr} + j\zeta_{pi} \quad (4.4.4)$$

$$p_1 = p_{1r} + jp_{1i} \quad (4.4.5)$$

$$q_2 = q_{2r} + jq_{2i} \quad (4.4.6)$$

If the two regions are lossless, all parameters are real. Then, (4.4.3) is simplified to

$$P_z = \pm \frac{1}{4} \frac{\zeta_p}{k_0} \frac{|A_h^\pm|^2}{\eta_0} \left(1 + \frac{1}{p_{1i} t}\right) t \quad (4.4.7)$$

The equivalent scattering width σ_s of the discontinuity associated with the scattered surface wave is defined as

$$\sigma_s = \frac{P_z}{|E_0|^2 / 2\eta_1} = \frac{1}{2} \frac{\zeta_p}{k_1} \left[\left| \frac{A_h^+}{E_0} \right|^2 + \left| \frac{A_h^-}{E_0} \right|^2 \right] \left(1 + \frac{1}{p_{1i} t}\right) t \quad (4.4.8)$$

The radiation field was given in the last section, and the total radiated power is

$$\begin{aligned}
P_r &= \frac{1}{2\eta_1} \lim_{r \rightarrow \infty} \int_{-\pi/2}^{\pi/2} |E_{1y}^{sr}(r, \theta)|^2 r d\theta \\
&= \frac{1}{2\eta_1} \frac{2\pi}{k_1} \int_{-\pi/2}^{\pi/2} |B_h(\theta)|^2 d\theta
\end{aligned} \quad (4.4.9)$$

The equivalent scattering width of the discontinuity associated with the scattered radiation field may be defined as follows

$$\sigma(\theta) = \lim_{r \rightarrow \infty} 2\pi r \left| \frac{E_{1y}^{sr}(r, \theta)}{E_0} \right|^2 \quad (4.4.10)$$

and some operations lead to

$$\sigma(\theta) = \frac{(2\pi)^2}{k_1} \left| \frac{B_h(\theta)}{E_0} \right|^2 \quad (4.4.11)$$

The back scattering width σ_B is defined as [61]

$$\sigma_B = \sigma(\theta_i) \quad (4.4.12)$$

where θ_i is the incidence angle of the incident plane wave.

TM-polarized wave incidence:

$$E_x = \begin{cases} \mp \frac{j\zeta_p}{q_2} \frac{A_e^\pm}{\sin(q_2 t)} \cos[q_2(x+t)] e^{\mp j\zeta_p z} & -t < x < 0 \\ \mp \frac{j\zeta_p}{p_1} A_e^\pm e^{-p_1 x} e^{\mp j\zeta_p z} & 0 < x < \infty \end{cases} \quad (4.4.13)$$

$$H_y = \pm \frac{\omega \epsilon}{\zeta_p} E_x \quad (4.4.14)$$

$$P_z^\pm = \frac{1}{2} \operatorname{Re} \left\{ \int_{-t}^{\infty} E_x H_y^* dx \right\}$$

$$= \pm \omega \frac{|A_e^\pm|^2}{4} e^{\pm 2j\zeta_p z} \operatorname{Re} \left\{ \zeta_p \left[\frac{\epsilon_1^*}{|p_1|^2 p_{1,t}} + \frac{\epsilon_2^*}{|q_2 \sin(q_2 t)|^2} \left(\frac{\sinh(2q_{2,t} t)}{2q_{2,t}} + \frac{\sin(2q_{2,t} t)}{2q_{2,t}} \right) \right] \right\} \quad (4.4.15)$$

If the two regions are lossless, all parameters are real. Then, (4.4.15) can be simplified to

$$P_z^\pm = \pm \frac{|A_e^\pm|^2}{4\eta_2} \frac{k_2 \zeta_p t}{[q_2 \sin(q_2 t)]^2} \left\{ 1 + \frac{\epsilon_1 \epsilon_2 (k_2^2 - k_1^2)}{p_1 t [(\epsilon_1 q_2)^2 + (\epsilon_2 p_1)^2]} \right\} \quad (4.4.16)$$

and when $\mu_1 = \mu_2 = \mu_0$, this can be reduced to

$$P_z^\pm = \pm \frac{|A_e^\pm|^2}{4\eta_2} \frac{k_2 \zeta_p t}{[q_2 \sin(q_2 t)]^2} \left\{ 1 + \frac{k_1^2}{p_1 t [\zeta_p^2 (1 + \epsilon_1/\epsilon_2) - k_1^2]} \right\} \quad (4.4.17)$$

Similarly, the equivalent scattering width σ_s of the discontinuity, associated with the scattered surface wave, is defined as

$$\sigma_s \equiv \frac{P_z}{|E_0|^2/2\eta_1} = \frac{\epsilon_2}{2\epsilon_1} \frac{k_1 \zeta_p t \left[\left| \frac{A_e^+}{E_0} \right|^2 + \left| \frac{A_e^-}{E_0} \right|^2 \right]}{[q_2 \sin(q_2 t)]^2} \left\{ 1 + \frac{k_1^2}{p_1 t [\zeta_p^2 (1 + \epsilon_1/\epsilon_2) - k_1^2]} \right\} \quad (4.4.18)$$

The radiation field was given in the last section, and the total radiated power is

$$\begin{aligned} P_r &= \frac{1}{2\eta_1} \lim_{r \rightarrow \infty} \int_{-\pi/2}^{\pi/2} \left| \vec{E}_1^{sr}(r, \theta) \right|^2 r d\theta \\ &= \frac{1}{2\eta_1} \frac{2\pi}{k_1} \int_{-\pi/2}^{\pi/2} |B_e(\theta)|^2 d\theta \end{aligned} \quad (4.4.19)$$

The equivalent scattering width of the discontinuity, associated with the scattered radiation field, may be defined as follows

$$\sigma(\theta) \equiv \lim_{r \rightarrow \infty} 2\pi r \left| \frac{\vec{E}_1^{sr}(r, \theta)}{E_0} \right|^2 \quad (4.4.20)$$

and subsequent calculations lead to

$$\sigma(\theta) = \frac{(2\pi)^2}{k_1} \left| \frac{B_e(\theta)}{E_0} \right|^2 \quad (4.4.21)$$

The back-scattering width σ_B is defined as [61]

$$\sigma_B = \sigma(\theta_i) \quad (4.4.22)$$

where θ_i is the incidence angle of the incident plane wave.

The formulas to calculate amplitudes of the surface wave which is excited and the cylindrical radiative wave were given in the last section. In the next section, these formula will be rewritten after the unknown field is expanded in a chosen set of basis functions $\{e_n\}$.

4.5. MoM SOLUTION TO THE EFIE

In Chapter 2, the polarization electric field integral equation (2.4.22) is derived. Now, it is rewritten as

$$\vec{E}_2(x, z) - j \frac{k_0}{\eta_{0LCS}} \int \delta n^2(x', z') \vec{G}_{22}(x|x', z-z') \bullet \vec{E}_2(x', z') dx' dz' = \vec{E}_2^i(x, z) \quad (4.5.1)$$

where

$$\delta n^2(x, z) = n_d^2(x, z) - n_2^2 = [\epsilon_d(x, z) - \epsilon_2] / \epsilon_0 \quad (4.5.2)$$

The EFIE for the unknown electric field in the discontinuity region induced by an incident wave will be solved using the standard MoM technique [32]-[34]. The unknown discontinuity field $\vec{E}_2(x, z)$ is first expanded in an appropriately chosen set of basis functions $\{e_n\}$

$$E_{2\beta}(x, z) = \sum_{n=1}^N a_{\beta n} e_{\beta n}(x, z) \quad \beta = x, y, z \quad (4.5.3)$$

where $a_{\beta n}$ are the unknown expansion coefficients. The EFIE for $E_{2\beta}(x, z)$ is discretized by substituting the above expansion in terms of unknown $a_{\beta n}$. Then, rather than forcing (4.5.1) to be satisfied for all (x, z) , it is multiplied by a set of N testing functions $t_{\alpha m}(x, z)$ and the inner products are taken:

$$\sum_{\beta=x,y,z} \sum_{n=1}^N A_{mn}^{\alpha\beta} a_{\beta n} = F_{\alpha m} \quad \begin{matrix} \alpha = x, y, z \\ m = 1, 2, \dots, N \end{matrix} \quad (4.5.4)$$

where

$$A_{mn}^{\alpha\beta} = \delta_{\alpha\beta} \int_S t_{\alpha m}(x, z) e_{\alpha n}(x, z) dx dz - j \frac{k_0}{\eta_0} \int_S t_{\alpha m}(x, z) \left[\int_{S'} \delta n^2(x', z') G_{22\alpha\beta}(x|x', z-z') e_{\beta n}(x', z') dx' dz' \right] dx dz \quad (4.5.5)$$

and the forcing vector F_{α} is

$$F_{\alpha m} = \int_S t_{\alpha m}(x, z) E_{2\alpha}^i(x, z) dx dz \quad (4.5.6)$$

where S, S' are the longitudinal cross section of the discontinuity region parameterized in field/source coordinates. Equation (4.5.4) can be written as a $3N$ by $3N$ matrix equation with a appropriate partitioning:

$$\begin{bmatrix} A^{xx} & 0 & A^{xz} \\ 0 & A^{yy} & 0 \\ A^{zx} & 0 & A^{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \quad (4.5.7)$$

where the $A^{\alpha\beta}$ are N by N matrices, and a_{α}, F_{α} are column N -vectors. The above matrix equation can be divided into two independent matrix equations

$$\begin{bmatrix} A^{yy} \end{bmatrix} \begin{bmatrix} a_y \end{bmatrix} = \begin{bmatrix} F_y \end{bmatrix} \quad (4.5.8)$$

for TE-polarized wave incidence, and

$$\begin{bmatrix} A^{xx} & A^{xz} \\ A^{zx} & A^{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_z \end{bmatrix} = \begin{bmatrix} F_x \\ F_z \end{bmatrix} \quad (4.5.9)$$

for TM-polarized wave incidence.

The Green's function was given in the last two chapters in different forms. In fact, impressed electric field $\vec{E}_2^i$ can be either a plane wave incident from space above the dielectric sheet or a surface wave supported by that sheet. The electric field of a TE- or TM-polarized plane wave without the discontinuity was derived in section 2.3 and Appendices A and B. The electric field of an incident surface wave propagating along the -z direction is given by

$$E_y^i(x, z) = \begin{cases} E_0 \sin [q_2(x+t)] e^{j\zeta_p z} & -t < x < 0 \\ E_0 \sin (q_2 t) e^{-p_1 x} e^{j\zeta_p z} & 0 < x < \infty \end{cases} \quad (4.5.10)$$

for TE-odd surface-wave incidence, and

$$\vec{E}_1^i(x, z) = E_0 \left\{ \hat{x} \frac{j\zeta_p}{p_1} + \hat{z} \right\} e^{-p_1 x} e^{j\zeta_p z} \quad (4.5.11)$$

$$\vec{E}_2^i(x, z) = \frac{E_0}{\sin (q_2 t)} \left\{ \hat{x} \frac{j\zeta_p}{q_2} \cos [q_2(t+x)] + \hat{z} \sin [q_2(t+x)] \right\} e^{-p_1 x} e^{j\zeta_p z} \quad (4.5.12)$$

for TM-even surface-wave incidence.

Since the unknown electric field is expanded in a chosen set of basis functions in (4.5.3), then the amplitudes of the surface-wave and radiation modes excited can be calculated by

$$A_h^\pm = \frac{k_0^2}{\bar{\Delta}'_h(\zeta_p) \sin (q_2 t)} \sum_{n=1}^N \int_S \delta n^2(x', z') a_{yn} e_{yn}(x', z') \sin [q_2(t+x')] e^{\pm j\zeta_p z'} dx' dz' \quad (4.5.13)$$

$$B_h(\theta) = \frac{jk_0^2 k_1 \cos \theta}{2\pi \bar{\Delta}_h (-k_1 \sin \theta)} \sum_{n=1}^N \int_S \delta n^2(x', z') a_{yn} e_{yn}(x', z') \sin[q_2(t+x')] e^{jk_1 z' \sin \theta} dx' dz' \quad (4.5.14)$$

for TE-polarized wave incidence, and

$$A_e^\pm = \frac{\epsilon_0 p_1}{\epsilon_2 \bar{\Delta}'(\zeta_p)} \sum_{n=1}^N \int_S \delta n^2(x', z') \{ \pm \zeta_p \cos[q_2(t+x')] a_{xn} e_{xn}(x', z') \\ - j q_2 \sin[q_2(t+x')] a_{zn} e_{zn}(x', z') \} e^{\pm j \zeta_p z'} dx' dz' \quad (4.5.15)$$

$$B_e(\theta) = \frac{\epsilon_0 j k_1^2 \cos \theta}{\epsilon_2 2\pi \bar{\Delta}_e (-k_1 \sin \theta)} \sum_{n=1}^N \int_S \delta n^2(x', z') \{ j k_1 \sin \theta \cos[q_2(t+x')] a_{xn} e_{xn}(x', z') \\ + q_2 \sin[q_2(t+x')] a_{zn} e_{zn}(x', z') \} e^{jk_1 z' \sin \theta} dx' dz' \quad (4.5.16)$$

for TM-polarized wave incidence.

The integral equation (4.5.1) has now been reduced to MoM matrix equation (4.5.4). In Chapters 6, 7 and 8, the Galerkin's method with pulse and sinusoidal/piecewise sinusoidal basis functions are applied to establish the matrices $A^{\alpha\beta}$ and the forcing vectors F_α

CHAPTER 5

NARROW GAP APPROXIMATIONS

5.1. INTRODUCTION

An alternative approach to solving the EFIE is to apply an iterative procedure. But a prior assessment of the convergence of the solution cannot be made. It is clear that the total electric field in the discontinuity region for a narrow gap will converge to the electric field in the gap when the width of the gap becomes zero, which is called the electric field in a zero-width gap. Using boundary conditions at the gap edges, it is found that the x- and y-components of the electric field in a zero-width gap are the same as the corresponding components of the incident field without the discontinuity, but the z-component should be

$$E_{2z} = E_{2z}^i \epsilon_2 / \epsilon_d \quad (5.1.1)$$

since the normal component of the electric displacement vector is continuous.

In the next section, the integral equation and successive approximations (the Neumann series) will be reviewed. Then, the first- and the second-order approximations of the back scattering width will be found in Sections 5.3 and 5.4 when a TE- or TM-polarized plane wave is incident upon a grounded sheet with a narrow gap. Finally, these approximate results are compared with the direct MoM EFIE solutions.

5.2. INTEGRAL EQUATION AND SUCCESSIVE APPROXIMATIONS

The integral equation [62]-[65]

$$\phi(x) = f(x) + \lambda \int_a^b K(x|x') \phi(x') dx' \quad (5.2.1)$$

is called an *inhomogeneous Fredholm equation of the second kind*, where $K(x|x')$ and $f(x)$ are known functions, a and b are the fixed points at which $\phi(x)$ satisfies boundary conditions, and λ is a numerical parameter. The quality $K(x|x')$ is called *kernel of the integral equation*. A kernel is *symmetric* if $K(x|x') = K(x'|x)$.

A *Fredholm equation of the first kind* has the general form

$$f(x) = \lambda \int_a^b K(x|x') \phi(x') dx' \quad (5.2.2)$$

Fredholm integral equations (5.2.1) and (5.2.2) involve definite integrals. When the limits are variable, the corresponding equations are called *Volterra equations* rather than Fredholm equations. If $f(x) = 0$, the integral equation (5.2.1) is *homogeneous*.

The integral equation (5.2.1) can be solved by many methods. One is the Method of Moments which was applied to EFIE in the last chapter. Now, let's introduce the method of successive approximations (Picard iteration) [65].

For the zeroth-order approximation let us take the right-hand side of equation (5.2.1):

$$\phi_0(x) = f(x) \quad (5.2.3)$$

Let us substitute the zeroth-order approximation in the right-hand side of equation (5.2.1) and take the resulting expression as the first-order approximation:

$$\phi_1(x) = f(x) + \lambda \int_a^b K(x|x') \phi_0(x') dx' \quad (5.2.4)$$

Let us again substitute the first-order approximation in the right-hand side of equation (5.2.1), and so on. In general, if the n -th approximation $\phi_n(x)$ has been obtained in this way, then the $(n+1)$ -th approximation is taken to be the result of substituting $\phi_n(x)$ in the right-hand side of equation (5.2.1). In this manner, successive approximations are determined from the recurrence relation

$$\phi_{n+1}(x) = f(x) + \lambda \int_a^b K(x|x') \phi_n(x') dx' \quad (5.2.5)$$

If the successive approximations tend uniformly to a certain limit, then this limit must certainly be a solution of equation (5.2.1); if this limit does not exist, then the use of the method of successive approximations evidently does not have any meaning.

Examination of the speed of convergence of the successive approximations leads to the following theorem [64]:

If

$$\int_a^b |K^2(x|x')| dx' \leq C_1 \quad C_1 = \text{constant} \quad (5.2.6)$$

then successive approximations converge uniformly for all values of λ lying inside the circle of

$$|\lambda| \leq \frac{1}{B} \quad B^2 = \int_a^b \int_a^b |K^2(x|x')| dx dx' \quad (5.2.7)$$

The limit of successive approximations is the solution of equation (5.2.1), and this solution is unique.

It is assumed that the cross section of the discontinuity region is rectangular ($|z| \leq a$, $-t \leq x \leq 0$) and the permittivity ϵ_d is a constant. Let's consider the scattered electric field,

$$\vec{E}_1(x, z) = \frac{jk_0}{\eta_0} \Delta n^2 \int_{LCS} \vec{G}_{12}(x|x', z-z') \cdot \vec{E}_2(x', z') dx' dz' \quad (5.2.8)$$

where $\Delta n^2 = (\epsilon_d - \epsilon_2) / \epsilon_0$, and $\vec{E}_2(x, z)$ is the solution of the EFIE,

$$\vec{E}_2(x, z) = \vec{E}_2^i(x, z) + j \frac{k_0}{\eta_0} \Delta n^2 \int_{LCS} \vec{G}_{22}(x|x', z-z') \cdot \vec{E}_2(x', z') dx' dz' \quad (5.2.9)$$

According to the definitions, it is found that equation (5.2.9) is an inhomogeneous Fredholm equation of the second kind, and the kernel is symmetric.

In the last chapter, the dyadic Green's function is derived as

$$\begin{aligned} \vec{G}_{22}(x|x', z-z') &= \hat{x}\hat{x} \frac{j\eta_2}{k_2} \delta(\hat{\rho} - \hat{\rho}') + \vec{G}_{22}^R(x|x', z-z') \\ &= \hat{z}\hat{z} \frac{j\eta_2}{k_2} \delta(\hat{\rho} - \hat{\rho}') + \vec{G}_{22}^{Pole}(x|x', z-z') + \vec{G}_{22}^{Rad}(x|x', z-z') \end{aligned} \quad (5.2.10)$$

The delta function in the first equation of (5.2.10) looks like the correction term with an infinitely thin disk-shaped principal volume with the disk axis in x-direction. The delta function in the second equation of (5.2.10) looks like the correction term with an infinitely thin disk-shaped principal volume with the disk axis in z-direction. Thus, the Green's function expressed in the second equation of (5.2.10) will be used when the discontinuity region is a narrow gap. Substituting (5.2.10) in equation (5.2.9) yields

$$\vec{E}_2(x, z) = \vec{E}_2^i(x, z) - \hat{z} \frac{\epsilon_d - \epsilon_2}{\epsilon_2} E_{2z}(x, z) + j \frac{k_0}{\eta_0} \Delta n^2 \int_{LCS} [\vec{G}_{22}^{Pole} + \vec{G}_{22}^{Rad}] \cdot \vec{E}_2(x', z') dx' dz' \quad (5.2.11)$$

Rearranging above equation yields

$$\vec{E}_2(x, z) = \vec{C} \cdot \{ \vec{E}_2^i(x, z) + j \frac{k_0}{\eta_0} \Delta n^2 \int_{LCS} [\vec{G}_{22}^{Pole} + \vec{G}_{22}^{Rad}] \cdot \vec{E}_2(x', z') dx' dz' \} \quad (5.2.12)$$

where

$$\vec{C} = \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}\epsilon_2/\epsilon_d \quad (5.2.13)$$

Using the iterative method of successive approximations, the zeroth-order approximation to the total electric field in the discontinuity region is found to be

$$\vec{E}_2^0(x, z) = \vec{C} \cdot \vec{E}_2^i(x, z) = \hat{x}E_{2x}^i + \hat{y}E_{2y}^i + \hat{z}E_{2z}^i\epsilon_2/\epsilon_d \quad (5.2.14)$$

and the first-order approximation,

$$\vec{E}_2^1(x, z) = \vec{C} \cdot \{ \vec{E}_2^i(x, z) + j \frac{k_0}{\eta_0} \Delta n^2 \int_{LCS} [\vec{G}_{22}^{Pole} + \vec{G}_{22}^{Rad}] \cdot \vec{E}_2^0(x', z') dx' dz' \} \quad (5.2.15)$$

and the $(n+1)$ -th order approximation,

$$\vec{E}_2^{n+1}(x, z) = \vec{C} \cdot \{ \vec{E}_2^i(x, z) + j \frac{k_0}{\eta_0} \Delta n^2 \int_{LCS} [\vec{G}_{22}^{Pole} + \vec{G}_{22}^{Rad}] \cdot \vec{E}_2^n(x', z') dx' dz' \} \quad (5.2.16)$$

and so on. The $(n+1)$ -th order approximation to the scattered electric field in region 1 can be calculated by

$$\vec{E}_1^{n+1}(x, z) = \frac{jk_0}{\eta_0} \Delta n^2 \int_{LCS} \vec{G}_{12}(x|x', z-z') \cdot \vec{E}_2^n(x', z') dx' dz' \quad (5.2.17)$$

It is noted that the zeroth-order approximation for the total electric field in the discontinuity region is the same as the electric field in the zero-width gap obtained in the beginning of this chapter by using the boundary conditions directly, but they are different from the results obtained by using the method of successive approximations to equation (5.2.9) directly.

Whether the successive approximations converge or not depends on the value $(\epsilon_d - \epsilon_2)/\epsilon_0$ and other parameters of the geometry. When the discontinuity region is a narrow gap, the contribution from the integral term in (5.2.16) is small, and the successive approximations converges fast. Thus, in the next two sections, the zeroth- and the first-order approximations of the total electric field in the discontinuity region will be found first, then the first- and the second-order approximations of the far-zone field are calculated. Finally, these results are compared with the direct MoM solutions of EFIE.

5.3. NARROW GAP APPROXIMATIONS FOR TE-POLARIZED PLANE WAVE INCIDENCE

When a TE-polarized plane wave is incident upon the grounded dielectric sheet with a discontinuity, since $G_{22yy} = G_{22yy}^R = G_{22yy}^{Pole} + G_{22yy}^{Rad}$, then the first-order approximation of the total electric field in the discontinuity region is calculated by

$$E_{2y}^1(x, z) = E_{2y}^i(x, z) + j \frac{k_0}{\eta_0} \Delta n^2 \int_{-a}^a \int_0^0 G_{22yy}(x|x', z-z') E_{2y}^i(x', z') dx' dz' \quad (5.3.1)$$

where $E_{2y}^i(x, z)$ and $G_{22yy}(x|x', z-z')$ are given in Chapter 2, and can be rewritten as

$$G_{22yy}(x|x', z-z') = -jk_2\eta_2 \int_{-\infty}^{\infty} \frac{e^{j\zeta(z-z')}}{4\pi p_2} d\zeta \{ e^{-p_2|x-x'|} - e^{-p_2(x+x'+2t)} + 2 \frac{p_2 - p_1}{\Delta_h} e^{-p_2 t} \sinh[p_2(x+t)] \sinh[p_2(x'+t)] \} \quad (5.3.2)$$

$$E_{2y}^i(x, z) = \frac{2jE_0}{C_h(\theta_i)} e^{-j\phi_h} \cos\theta_i \sin[q_{2\theta_i}(t+x)] e^{jk_1 z \sin\theta_i} \quad (5.3.3)$$

where

$$p_i = \sqrt{\zeta^2 - k_i^2} \quad (i = 1, 2) \quad (5.3.4)$$

$$\tilde{\Delta}_h = p_2 \cosh(p_2 t) + p_1 \sinh(p_2 t) \quad (5.3.5)$$

$$q_{2\theta_i} = k_2 \cos \theta_r = \sqrt{k_2^2 - k_1^2 \sin^2 \theta_i} \quad (5.3.6)$$

$$\phi_h = \tan^{-1} \left[\frac{k_1 \cos \theta_i}{q_{2\theta_i}} \tan(q_{2\theta_i} t) \right] \quad (5.3.7)$$

$$C_h(\theta_i) = [\cos^2 \theta_i + \cos^2(q_{2\theta_i} t) (\epsilon_2 - \epsilon_1) / \epsilon_1]^{1/2} \quad (5.3.8)$$

When $\theta_i \rightarrow 90^\circ$,

$$E_{2y}^i(x, z) = E_0 \begin{cases} 2 \sin[q_{2\theta_i}(t+x)] e^{j k_1 z} & q_{2\theta_i} t = n\pi + \pi/2 \\ 0 & \text{otherwise} \end{cases} \quad (5.3.9)$$

This resonance phenomenon is explained as follows:

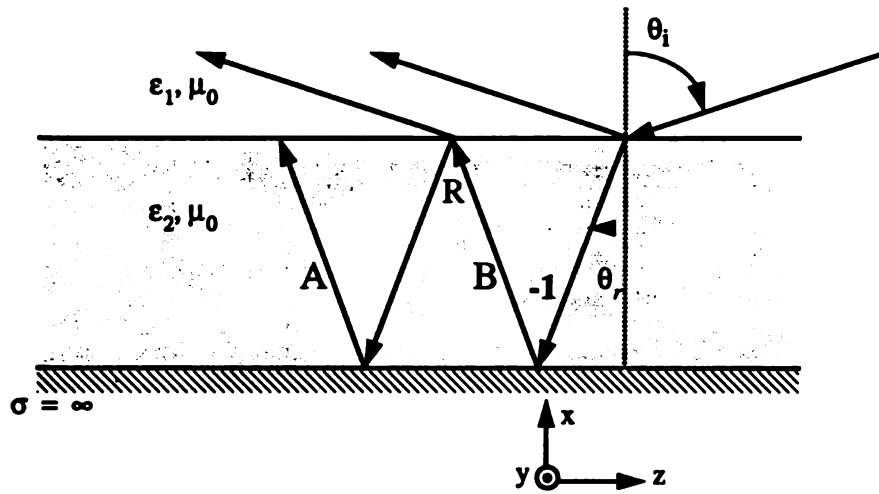


Figure 5.1. The explanation of the resonance phenomenon when $\theta_i \rightarrow 90^\circ$.

Since the equivalent wave impedance of TE-polarized plane wave is [1]

$$Z = \eta / \cos \theta \quad (5.3.10)$$

then the reflection coefficient of electric field at the interface between region 2 and 1 is given by

$$R = \frac{Z_1 - Z_2}{Z_1 + Z_2} = \frac{\eta_1 \cos \theta_r - \eta_2 \cos \theta_i}{\eta_1 \cos \theta_r + \eta_2 \cos \theta_i} = 1 \quad \text{if } \theta_i \rightarrow 90^\circ \quad (5.3.11)$$

and the transmitting coefficient from region 1 to region 2 becomes zero when $\theta_i \rightarrow 90^\circ$. Thus, the total field is not zero only when the phase difference between points A and B shown in Figure 5.1 is $2n\pi$.

Substituting E_{2y}^I in (5.3.1), the first-order approximation of the total electric field in the discontinuity region, and using the relations of

$$\int e^{ax} \sin(bx) dx = \frac{a \sin(bx) - b \cos(bx)}{a^2 + b^2} e^{ax} \quad (5.3.12)$$

$$\int \sinh(ax) \sin(bx) dx = \frac{a \cosh(ax) \sin(bx) - b \sinh(ax) \cos(bx)}{a^2 + b^2} \quad (5.3.13)$$

yield

$$\begin{aligned} E_{2y}^I = & \frac{2jE_0 \cos \theta_i e^{-j\phi_h}}{C_h(\theta_i)} \left\{ \sin[q_{2\theta_i}(t+x)] e^{jk_{1z} \sin \theta_i} + \frac{\Delta n^2 k_0^2 a t}{\pi} \int_{-\infty}^{\infty} \frac{d\zeta}{p_2} \left\{ \frac{1}{t(p_2^2 + q_{2\theta_i}^2)} \right. \right. \\ & \left. \left[p_2 \sin[q_{2\theta_i}(t+x)] - e^{-p_2 t} [p_2 \sin(q_{2\theta_i} t) + q_{2\theta_i} \cos(q_{2\theta_i} t)] \sinh[p_2(t+x)] \right] \right. \\ & \left. \left. + \frac{p_2 - p_1}{\Delta_h} f_1(q_{2\theta_i}, p_2) \sinh[p_2(t+x)] \right\} \frac{\sin[a(k_1 \sin \theta_i - \zeta)]}{a(k_1 \sin \theta_i - \zeta)} e^{j\zeta z} \right\} \quad (5.3.14) \end{aligned}$$

where

$$f_1(x, y) = \frac{y \sin(tx) (1 + e^{-2ty}) - x \cos(tx) (1 - e^{-2ty})}{2t(x^2 + y^2)} \quad (5.3.15)$$

From (5.2.17), the second-order approximation of the scattered electric field in region 1 is found to be

$$E_{1y}^{s2}(x, z) = \Delta n^2 k_0^2 \int_{-a-t}^a \int_0^\infty \int_{-\infty}^\infty \frac{\sinh[p_2(t+x')]}{2\pi\tilde{\Delta}_h} e^{j\zeta(z-z')} e^{-p_1x} d\zeta E_{2y}^1(x', z') dx' dz' \quad (5.3.16)$$

The integration with respect to ζ can be approximated by the saddle-point method, which is introduced in the proceeding chapter. Substituting (5.3.14) into (4.3.42) leads to

$$E_{1y}^{s2}(r, \theta) = \Delta n^2 k_0^2 k_1 \cos\theta \sqrt{\frac{2\pi}{k_1 r}} e^{-j(k_1 r - \pi/4)} \int_{-a-t}^a \int_0^\infty \frac{j \sin[q_{2\theta}(t+x')]}{2\pi\tilde{\Delta}_h(-k_1 \sin\theta)} e^{jk_1 z' \sin\theta} E_{2y}^1(x', z') dx' dz' \quad (5.3.17)$$

where

$$q_{2\theta} = \sqrt{k_2^2 - k_1^2 \sin^2\theta} \quad (5.3.18)$$

$$\tilde{\Delta}_h(-k_1 \sin\theta) = jq_{2\theta} \cos(q_{2\theta}t) - k_1 \cos\theta \sin(q_{2\theta}t) \quad (5.3.19)$$

Substituting (5.3.14) in (5.3.17) leads to

$$E_{1y}^{s2}(r, \theta) = -E_0 \frac{\Delta n^2 k_0^2 k_1 \cos\theta}{\sqrt{2\pi} C_h(\theta_i)} \frac{e^{-j(k_1 r - \pi/4)}}{\sqrt{k_1 r}} \frac{4at \cos\theta_i e^{-j\phi_h}}{\tilde{\Delta}_h(-k_1 \sin\theta)} f(\theta, \theta_i)$$

where

$$f(\theta, \theta_i) = \frac{\sin[k_1 a (\sin\theta + \sin\theta_i)]}{k_1 a (\sin\theta + \sin\theta_i)} f_2(q_{2\theta_i}, q_{2\theta}) + \frac{\Delta n^2 k_0^2 a t}{\pi} \int_{-\infty}^\infty \frac{d\zeta}{p_2} \left\{ \frac{1}{t(p_2^2 + q_{2\theta_i}^2)} \right. \\ \left. [p_2 f_2(q_{2\theta_i}, q_{2\theta}) - [p_2 \sin(q_{2\theta_i}t) + q_{2\theta_i} \cos(q_{2\theta_i}t)] f_1(q_{2\theta}, p_2)] \right\}$$

$$+ \frac{p_2 - p_1}{\bar{\Delta}_{h1}} f_1(q_{2\theta}, p_2) f_1(q_{2\theta}, p_2) \} \frac{\sin[a(k_1 \sin \theta_i - \zeta)]}{a(k_1 \sin \theta_i - \zeta)} \frac{\sin[a(k_1 \sin \theta + \zeta)]}{a(k_1 \sin \theta + \zeta)} \quad (5.3.20)$$

with

$$f_2(x, y) = \frac{1}{2} \left\{ \frac{\sin[(x-y)t]}{(x-y)t} - \frac{\sin[(x+y)t]}{(x+y)t} \right\} \quad (5.3.21)$$

$$\bar{\Delta}_{h1} = [p_2(1 + e^{-p_2 t}) + p_1(1 - e^{-2p_2 t})] / 2 \quad (5.3.22)$$

If only the first term of $f(\theta, \theta_i)$ is kept, the result is the first-order approximation. From the definition, the second-order approximation of the equivalent scattering width of the discontinuity associated with the scattered radiated field is found to be

$$\sigma(\theta) = \lim_{r \rightarrow \infty} 2\pi r \left| \frac{E_{1y}^s(r, \theta)}{E_0} \right|^2 = \frac{|f(\theta, \theta_i)|^2}{k_1} \left[\frac{4\Delta n^2 k_0^2 a t \cos \theta \cos \theta_i}{C_h(\theta) C_h(\theta_i)} \right]^2 \quad (5.3.23)$$

where the relation

$$|\bar{\Delta}_h(-k_1 \sin \theta)|^2 = q_{2\theta}^2 \cos^2(q_{2\theta} t) + k_1^2 \cos^2 \theta \sin^2(q_{2\theta} t) = k_1^2 C_h^2(\theta) \quad (5.3.24)$$

has been used.

It is assumed that region 1 and the discontinuity region are air ($\epsilon_1 = \epsilon_d = \epsilon_0$) throughout in the following numerical results. MoM solution of EFIE is given in the next chapter.

Figure 5.2 shows the normalized back scattering width calculated by the first-order approximation and the MoM EFIE solutions as functions of frequency at various incidence angles θ_i when a TE-polarized plane wave is incident upon a grounded sheet with a narrow air gap ($2a = 0.06''$). It is observed that two methods agree with each other very well. This is because the air gap is very thin, $2a/\lambda_0 = 0.09$ at $f = 18\text{GHz}$. It is also ob-

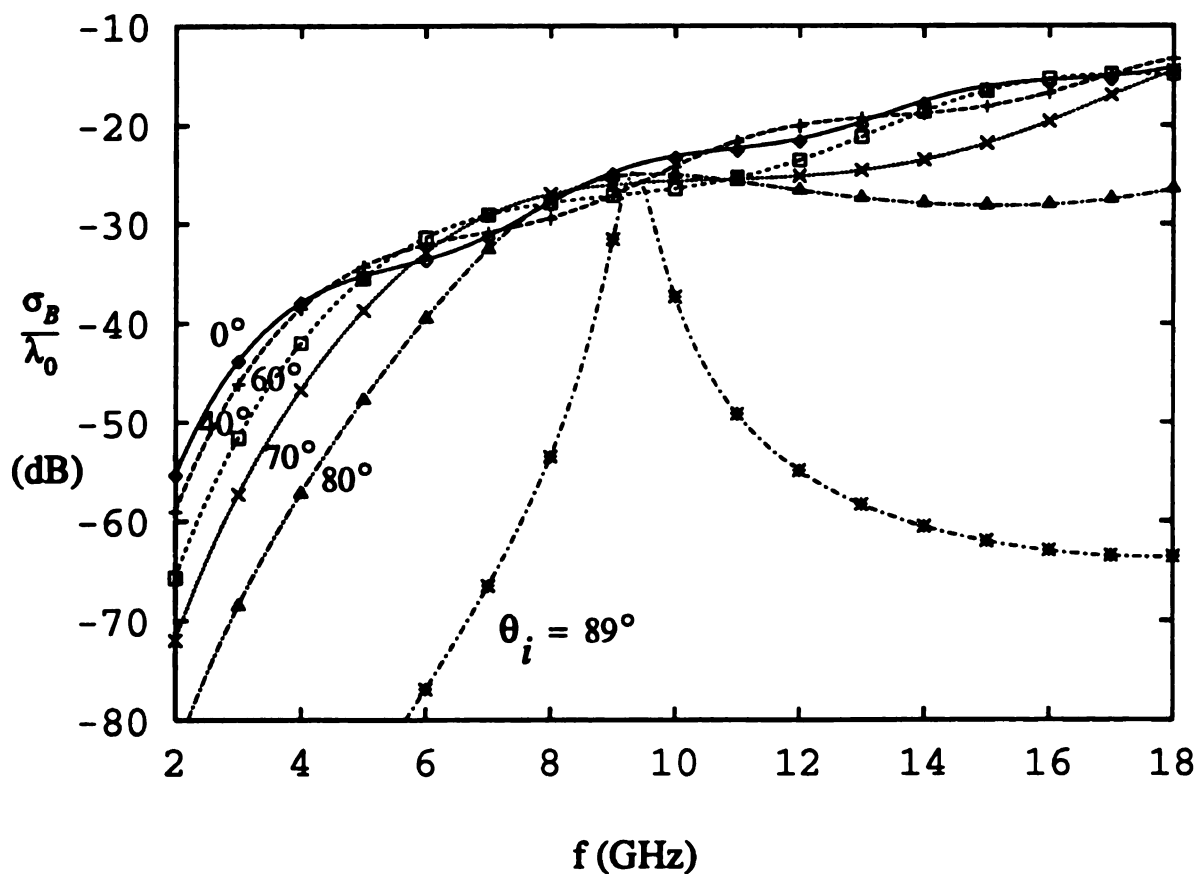


Figure 5.2. Normalized back scattering width as a function of frequency at various incidence angles θ_i when a TE plane wave is incident upon a grounded sheet with a narrow air gap. Lines -- First-order approximation, Points -- MoM solutions of EFIE. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.06''$, $t = 1''$, $N_z = 2$ to 8, and $N_x = 5$ to 30)

served that there is a peak near $f = 9\text{GHz}$ when $\theta_i = 89^\circ$. It is the resonance phenomenon discussed earlier.

Figure 5.3 plots the normalized back scattering width calculated by the first- and the second-order approximations and the MoM EFIE solutions as functions of frequency at the incidence angle of $\theta_i = 60^\circ$ with $2a = 1''$. It is found that the difference between the first-order approximation and the MoM solutions becomes significant as the frequency increases. However, the second-order approximation agrees very well with the MoM solutions up to $f = 12\text{GHz}$ ($2a/\lambda_0 = 1$).

Since the higher order approximations are proportional to powers of the factor $\Delta n^2 k_0^2 a t$, the smaller that factor is, the more accurate the second-order approximation will be. Because $|\Delta n^2| = 0.1$ for the parameters given in Figures 5.2 and 5.3, the second-order approximation is valid even the gap is not thin. If the relative permittivity ϵ_2/ϵ_0 of region 2 increases and the second-order approximation is kept to be valid, the width of the air gap should be reduced. But when the factor $\Delta n^2 k_0^2 a t$ is not small, the successive approximations may not converge, in fact, the second-order approximation may be worse than the first-order approximation as shown in Figure 5.4. From Figure 5.4, it is observed that the second-order approximation agrees better than the first-order approximation with the MoM EFIE solutions for low frequencies case, but when the frequency becomes high, the first-order approximation becomes better than the second-order approximation.

5.4. NARROW GAP APPROXIMATIONS FOR TM-POLARIZED PLANE WAVE INCIDENCE

Similar to the TE-polarized plane wave case, the zeroth- and the first-order approximations of the total electric field in the discontinuity region can be found first, then the first- and the second-order approximations of the scattered field in region 1 are calculated for the TM-polarized plane wave case. However, the Green's functions for TM excitation

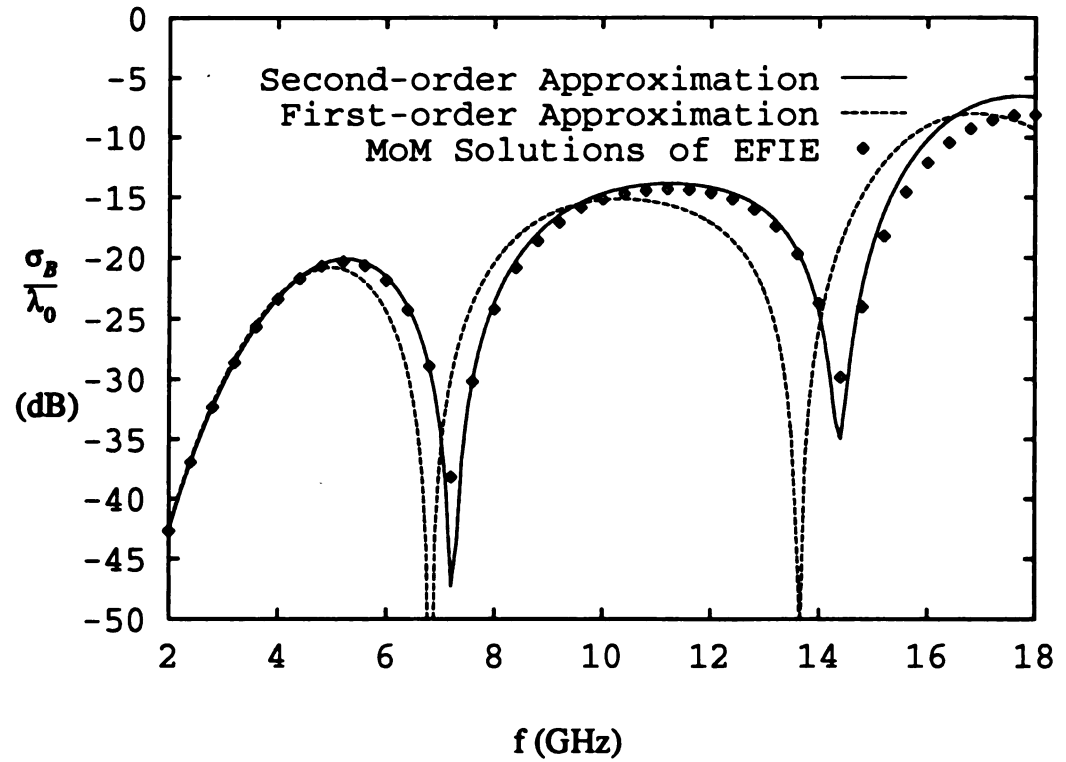


Figure 5.3. Normalized back scattering width as a function of frequency at the incidence angle of $\theta_i = 60^\circ$ when a TE plane wave is incident upon a grounded sheet with a narrow air gap. Dashed line -- First-order approximation, Solid line -- Second-order approximation, Points -- MoM solutions of EFIE. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 1''$, $t = 1''$, $N_z = 5$ to 20, and $N_x = 5$ to 20)

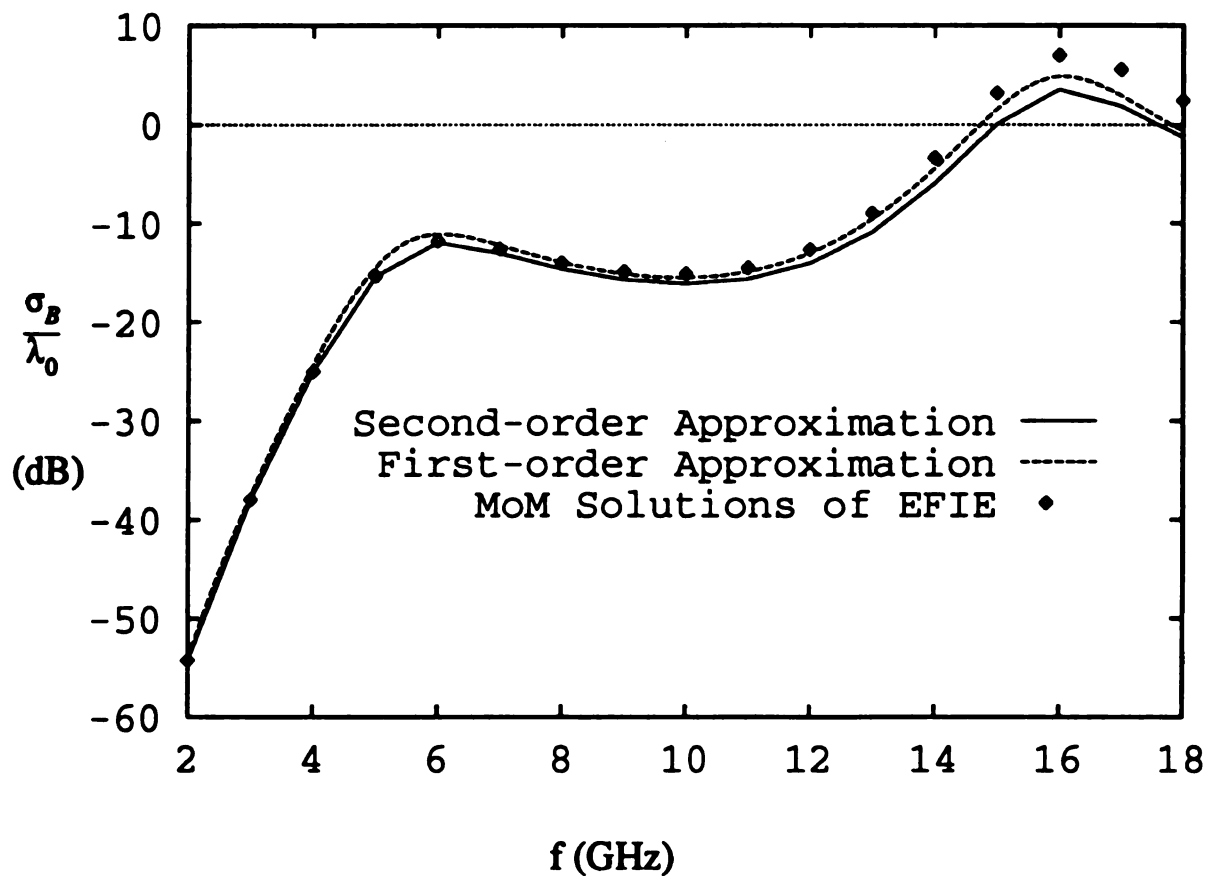


Figure 5.4. Normalized back scattering width as a function of frequency at the incidence angle of $\theta_i = 60^\circ$ when a TE plane wave is incident upon a grounded sheet with a narrow air gap. Dashed line -- First-order approximation, Solid line -- Second-order approximation, Points -- MoM solutions of EFIE. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 2$, $2a = 0.15''$, $t = 0.5''$, $N_z = 5$ to 10 , and $N_x = 5$ to 20)

have four components, and it is very difficult to obtain higher order approximations. Thus, only the first-order approximation of the far-zone field is calculated using the zeroth-order approximation of the total electric field in the discontinuity region (the electric field in a zero-width gap). The zeroth-order approximation of the total electric field in the discontinuity region is given by

$$\vec{E}_2^0(x, z) = \hat{x}E_{2x}^i(x, z) + \hat{z}E_{2z}^i(x, z)\epsilon_2/\epsilon_d \quad (5.4.1)$$

where $E_{2x}^i(x, z)$ and $E_{2z}^i(x, z)$ are given in Chapter 2, and can be rewritten as

$$\vec{E}_2^i(x, z) = -\frac{2E_0 \cos \theta_i \epsilon_1}{C_e(\theta_i) \epsilon_2} e^{-j\phi_e} \left\{ \hat{x} \sin \theta_i \cos [q_{2\theta_i}(t+x)] - \hat{z} j \frac{q_{2\theta_i}}{k_1} \sin [q_{2\theta_i}(t+x)] \right\} e^{jk_1 z \sin \theta_i} \quad (5.4.2)$$

where

$$q_{2\theta_i} = k_2 \cos \theta_r = \sqrt{k_2^2 - k_1^2 \sin^2 \theta_i} \quad (5.4.3)$$

$$\phi_e = \tan^{-1} \left[\frac{\epsilon_1}{\epsilon_2} \frac{q_{2\theta_i}}{k_1 \cos \theta_i} \tan (q_{2\theta_i} t) \right] \quad (5.4.4)$$

$$C_e(\theta_i) = \left[\cos^2 \theta_i \cos^2 (q_{2\theta_i} t) + [\epsilon_1 q_{2\theta_i} / (\epsilon_2 k_1)]^2 \sin^2 (q_{2\theta_i} t) \right]^{1/2} \quad (5.4.5)$$

When $\theta_i \rightarrow 90^\circ$,

$$\vec{E}_2^i(x, z) = E_0 \begin{cases} -2 \frac{\epsilon_1}{\epsilon_2} [\hat{x} \cos [q_{2\theta_i}(t+x)] - \hat{z} j (q_{2\theta_i}/k_1) \sin [q_{2\theta_i}(t+x)]] e^{jk_1 z} & q_{2\theta_i} t = n\pi \\ 0 & \text{otherwise} \end{cases} \quad (5.4.6)$$

This resonance phenomenon has the same physical meanings as the resonance phenomenon for the TE-polarized plane wave incidence. The differences are that the equivalent

wave impedance of TM-polarized plane wave being $Z = \eta \cos \theta$, and the reflection coefficient of electric field at the interface becoming $R = -1$ when $\theta_i \rightarrow 90^\circ$.

In the last chapter, the far-zone field is derived using the Saddle-point method. Substituting the zeroth-order approximation of the total electric field in the discontinuity region in (4.3.51) yields the first-order approximation of the far-zone field

$$\vec{E}_1^{s1}(r, \theta) = B_e(\theta) \sqrt{\frac{2\pi}{k_1 r}} e^{-j(k_1 r - \pi/4)} \{ \hat{z} \cos \theta - \hat{x} \sin \theta \} \quad (5.4.7)$$

where

$$B_e(\theta) = \frac{\epsilon_0}{\epsilon_2} \frac{j \Delta n^2 k_1^2 \cos \theta}{2\pi \tilde{\Delta}_e(-k_1 \sin \theta)} \int_{-a}^0 \int \{ j k_1 \sin \theta \cos [q_{2\theta}(t+x')] E_{2x}^i(x', z') + (\epsilon_2/\epsilon_d) q_{2\theta} \sin [q_{2\theta}(t+x')] E_{2z}^i(x', z') \} e^{jk_1 z' \sin \theta} dx' dz' \quad (5.4.8)$$

with

$$q_{2\theta} = \sqrt{k_2^2 - k_1^2 \sin^2 \theta} \quad (5.4.9)$$

$$\tilde{\Delta}_e(-k_1 \sin \theta) = j k_1 \cos \theta \cos(q_{2\theta} t) - (\epsilon_1/\epsilon_2) q_{2\theta} \sin(q_{2\theta} t) \quad (5.4.10)$$

Substituting (5.4.2) in (5.4.8), and doing some integrations yield

$$B_e(\theta) = \frac{2}{\pi} E_0 \left(\frac{\epsilon_1}{\epsilon_2} \right)^2 e^{-j\phi_e} \frac{\Delta n^2 \cos \theta \cos \theta_i k_0^2 a t}{C_e(\theta_i) \tilde{\Delta}_e(-k_1 \sin \theta)} k_1 f(\theta, \theta_i) \quad (5.4.11)$$

where

$$f(\theta, \theta_i) = \left\{ \sin \theta \sin \theta_i f_1(q_{2\theta}, q_{2\theta_i}) - \frac{\epsilon_2 q_{2\theta} q_{2\theta_i}}{\epsilon_d k_1^2} f_2(q_{2\theta}, q_{2\theta_i}) \right\} \frac{\sin [k_1 a (\sin \theta + \sin \theta_i)]}{k_1 a (\sin \theta + \sin \theta_i)} \quad (5.4.12)$$

with

$$f_1(x, y) = \frac{1}{2} \left\{ \frac{\sin[(x-y)t]}{(x-y)t} + \frac{\sin[(x+y)t]}{(x+y)t} \right\} \quad (5.4.13)$$

$$f_2(x, y) = \frac{1}{2} \left\{ \frac{\sin[(x-y)t]}{(x-y)t} - \frac{\sin[(x+y)t]}{(x+y)t} \right\} \quad (5.4.14)$$

From the definition, the first-order approximation of the equivalent scattering width of the discontinuity associated with the scattered radiated field is found to be

$$\sigma(\theta) = \lim_{r \rightarrow \infty} 2\pi r \left| \frac{\tilde{E}_1^{s1}(r, \theta)}{E_0} \right|^2 = \frac{|f(\theta, \theta_i)|^2}{k_1} \left[4 \frac{\Delta n^2 \cos \theta \cos \theta_i k_0^2 a t}{C_e(\theta) C_e(\theta_i)} \left(\frac{\epsilon_1}{\epsilon_2} \right)^2 \right]^2 \quad (5.4.15)$$

where the relation

$$|\tilde{\Delta}_e(-k_1 \sin \theta)|^2 = (\epsilon_1 q_{2\theta} / \epsilon_2)^2 \sin^2(q_{2\theta} t) + k_1^2 \cos^2 \theta \cos^2(q_{2\theta} t) = k_1^2 C_e^2(\theta) \quad (5.4.16)$$

has been used.

It is assumed that region 1 and the discontinuity region are air ($\epsilon_1 = \epsilon_d = \epsilon_0$) throughout in the following numerical results. The MoM solutions of EFIE are calculated by the pulse Galerkin's method given in Chapter 7.

Figure 5.5 shows the normalized back scattering width calculated by the first-order approximation and the MoM EFIE solutions as functions of frequency at various incidence angles θ_i when a TM-polarized plane wave is incident upon a grounded sheet with a narrow air gap ($2a = 0.06''$). It is observed that two methods agree with each other very well. It is also observed that the normalized back scattering width becomes almost zero at some discrete frequencies when $\theta_i = 50^\circ$. This phenomena can be explained from the behavior of the term $f(\theta_p, \theta_i)$. Since the air gap is very thin ($2a/\lambda_0 = 0.09$ at $f = 18\text{GHz}$), then the second term $\sin(2k_1 a \sin \theta_i) / (2k_1 a \sin \theta_i)$ in $f(\theta_p, \theta_i)$ is almost unity. After some simplifications, the first term of $f(\theta_p, \theta_i)$ ($\epsilon_d = \epsilon_1$) can be reduced to

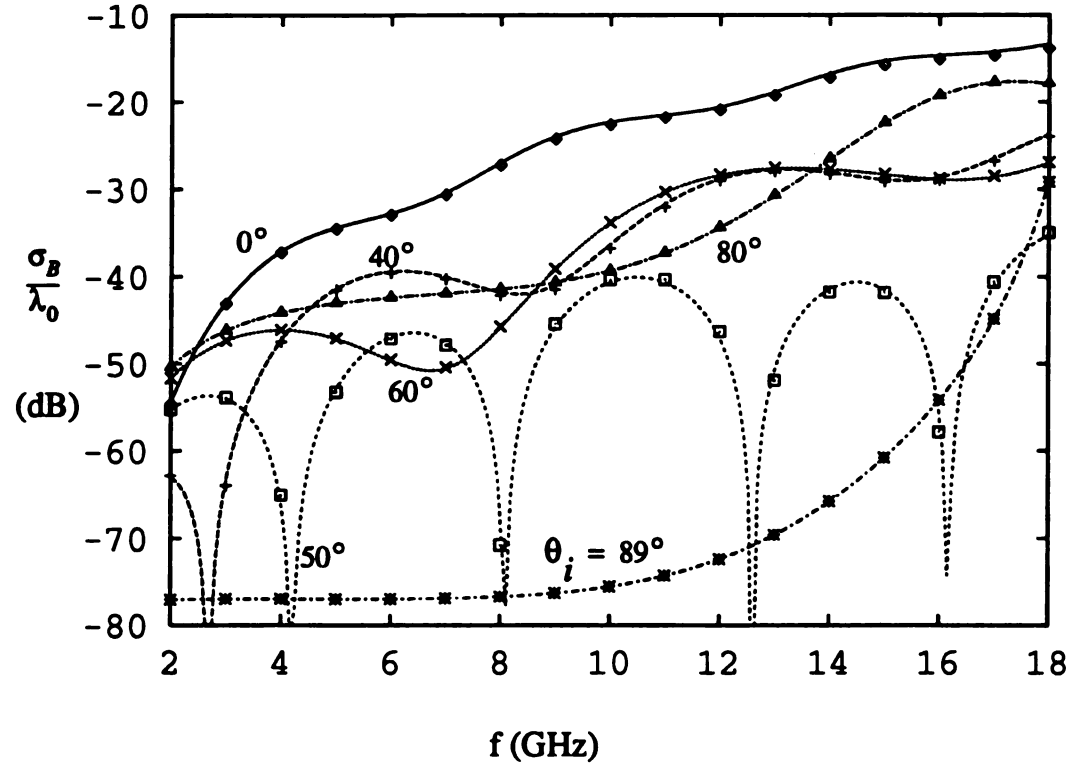


Figure 5.5. Normalized back scattering width as a function of frequency at various incidence angles θ_i when a TM plane wave is incident upon a grounded sheet with a narrow air gap. Lines -- First-order approximation, Points -- MoM solutions of EFIE. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.06''$, $t = 1''$, $N_z = 2$ to 8 , and $N_x = 5$ to 30)

$$\begin{aligned} \sin^2 \theta_i f_1(q_{2\theta_i}, q_{2\theta_i}) - (\epsilon_2/\epsilon_1) (q_{2\theta_i}/k_1)^2 f_2(q_{2\theta_i}, q_{2\theta_i}) = \\ \{ (n_2^2 + 1) \sin^2 \theta_i - n_2^4 + [n_2^4 - (n_2^2 - 1) \sin^2 \theta_i] \sin(2k_2 t \cos \theta_r) / (2k_2 t \cos \theta_r) \} / 2 \end{aligned} \quad (5.4.17)$$

where $n_2^2 = \epsilon_2/\epsilon_1$. Let the right-hand side of the above equation equal to zero, then

$$\frac{\sin(2k_2 t \cos \theta_r)}{2k_2 t \cos \theta_r} = \frac{n_2^4 - (n_2^2 + 1) \sin^2 \theta_i}{n_2^4 - (n_2^2 - 1) \sin^2 \theta_i} \quad (5.4.18)$$

When θ_i increases from 0° to 90° , the right-hand side of the above equation decreases from 1 to $(n_2^4 - n_2^2 - 1)/(n_2^4 - n_2^2 + 1)$. Thus, given θ_i and ϵ_2/ϵ_1 , (5.4.18) may have no solution, one solution, several solutions, or infinite solutions for t/λ_0 .

Figure 5.6 plots the normalized back scattering width calculated by the first-order approximation and the MoM EFIE solutions as functions of frequency at the incidence angle of $\theta_i = 60^\circ$ with $2a = 0.3''$. It is found that the difference between the first-order approximation and the MoM EFIE solutions becomes large as the frequency increases.

5.5. MORE NUMERICAL RESULTS FOR A NARROW AIR GAP

Figures 5.7 and 5.8 show the normalized back scattering width calculated by the first-order approximation and the MoM EFIE solutions as functions of frequency at various incidence angles θ_i when a TE- and a TM-polarized plane wave is incident upon a grounded sheet with a narrow air gap ($2a = 0.1''$), respectively.

Figures 5.9 and 5.10 replot the normalized back scattering width calculated by the MoM EFIE solutions as functions of incidence angle θ_i at various frequencies when a TE-polarized plane wave is incident upon a grounded sheet with a narrow air gap with $2a = 0.06''$, and $0.1''$, respectively.

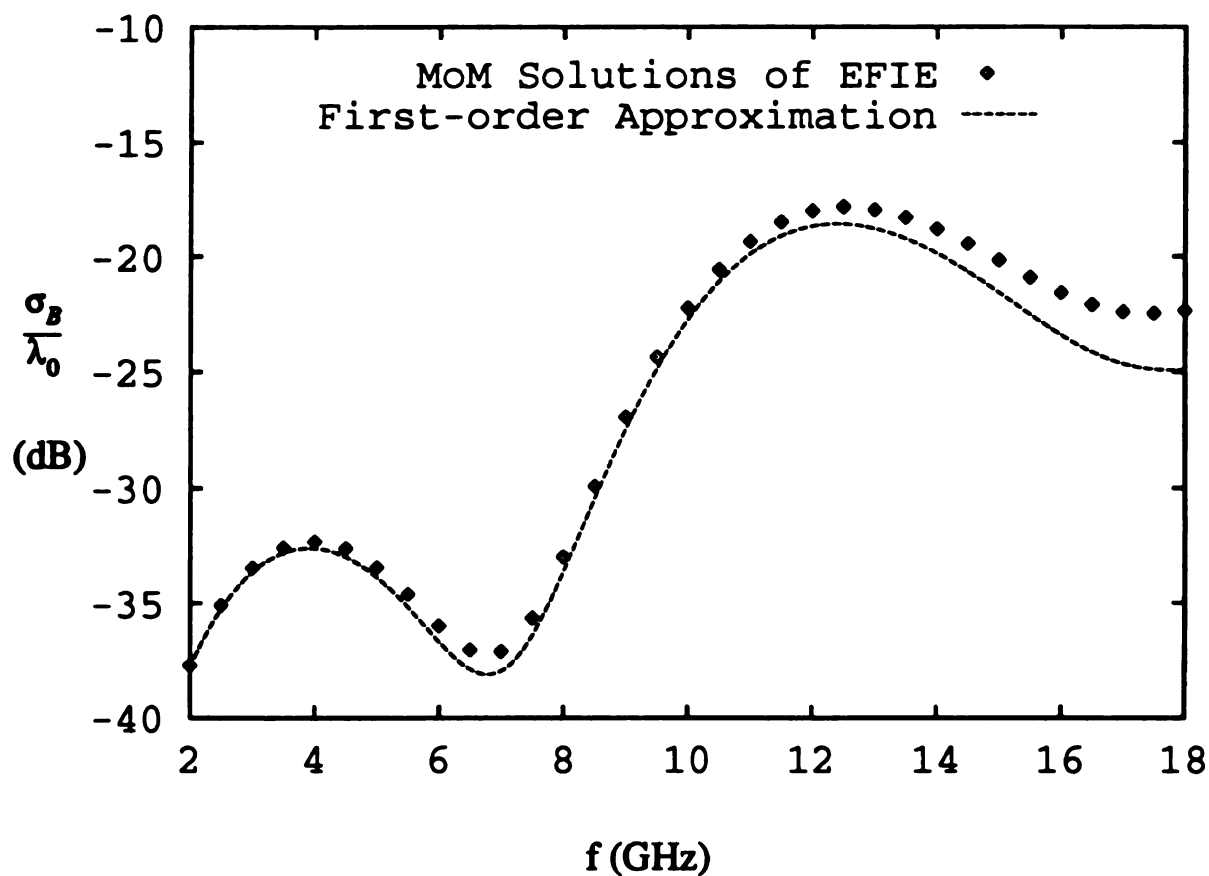


Figure 5.6. Normalized back scattering width as a function of frequency at the incidence angle of $\theta_i = 60^\circ$ when a TM plane wave is incident upon the grounded sheet with a narrow air gap. Line -- First-order approximation, Points -- MoM solutions of EFIE. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.3''$, $t = 1''$, $N_z = 3$ to 8 , and $N_x = 5$ to 30)

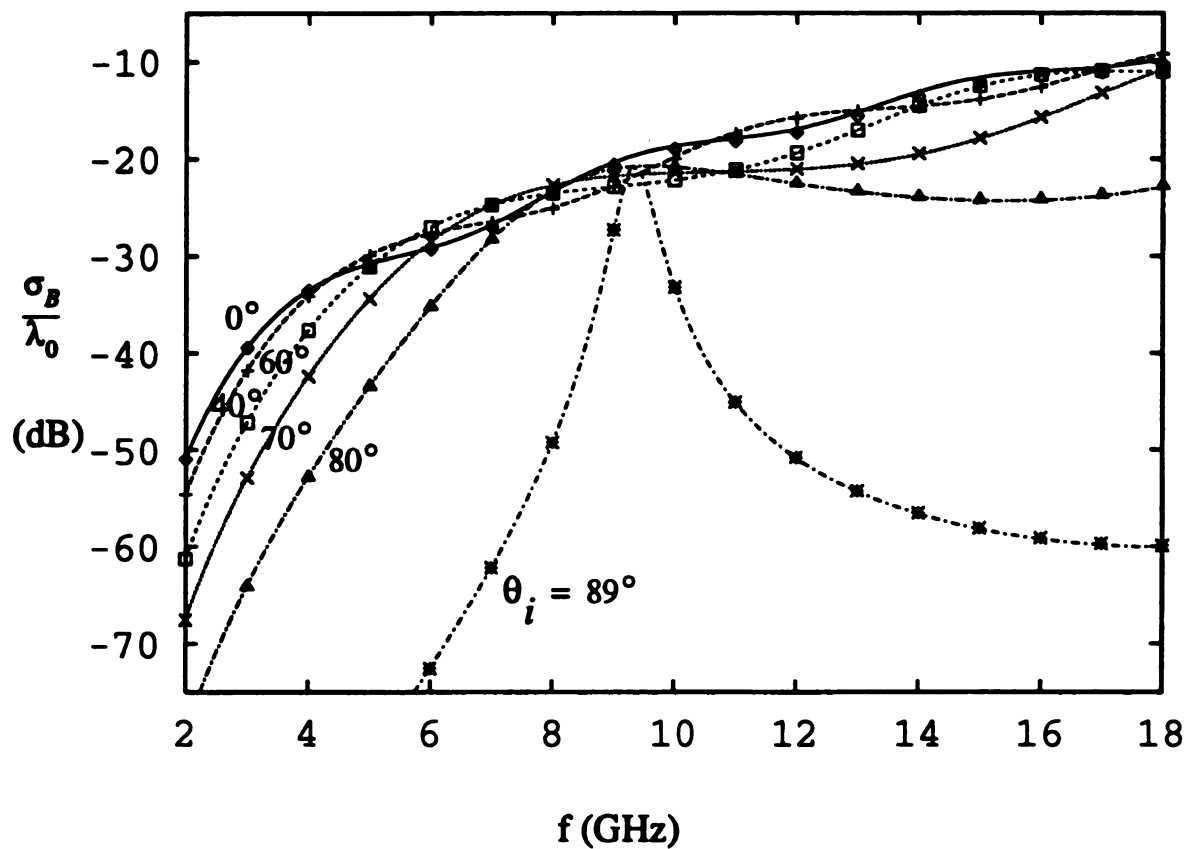


Figure 5.7. Normalized back scattering width as a function of frequency at various incidence angles θ_i when a TE plane wave is incident upon a grounded sheet with a narrow air gap. Lines -- First-order approximation, Points -- MoM solutions of EFIE. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.1''$, $t = 1''$, $N_z = 2$ to 8 , and $N_x = 5$ to 30)

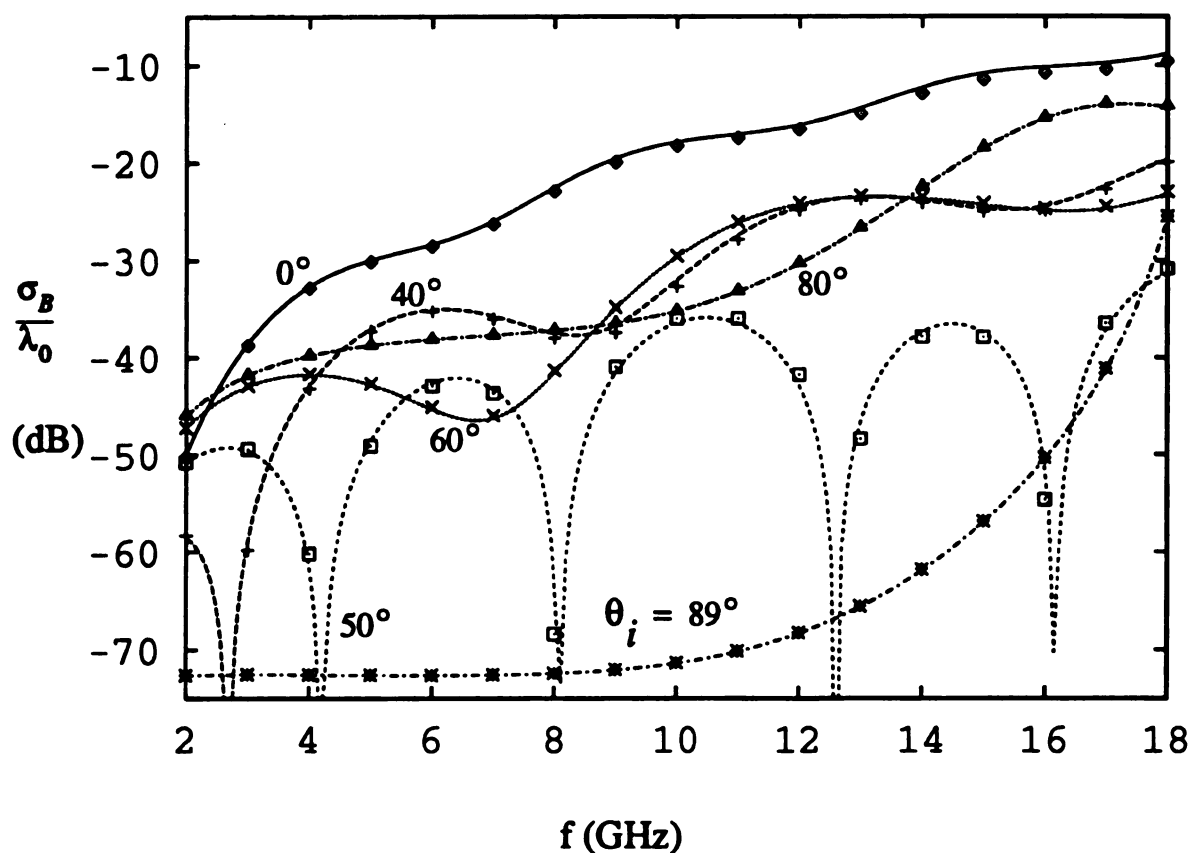


Figure 5.8. Normalized back scattering width as a function of frequency at various incidence angles θ_i when a TM plane wave is incident upon a grounded sheet with a narrow air gap. Lines -- First-order approximation, Points -- MoM solutions of EFIE. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.1''$, $t = 1''$, $N_z = 2$ to 8 , and $N_x = 5$ to 30)

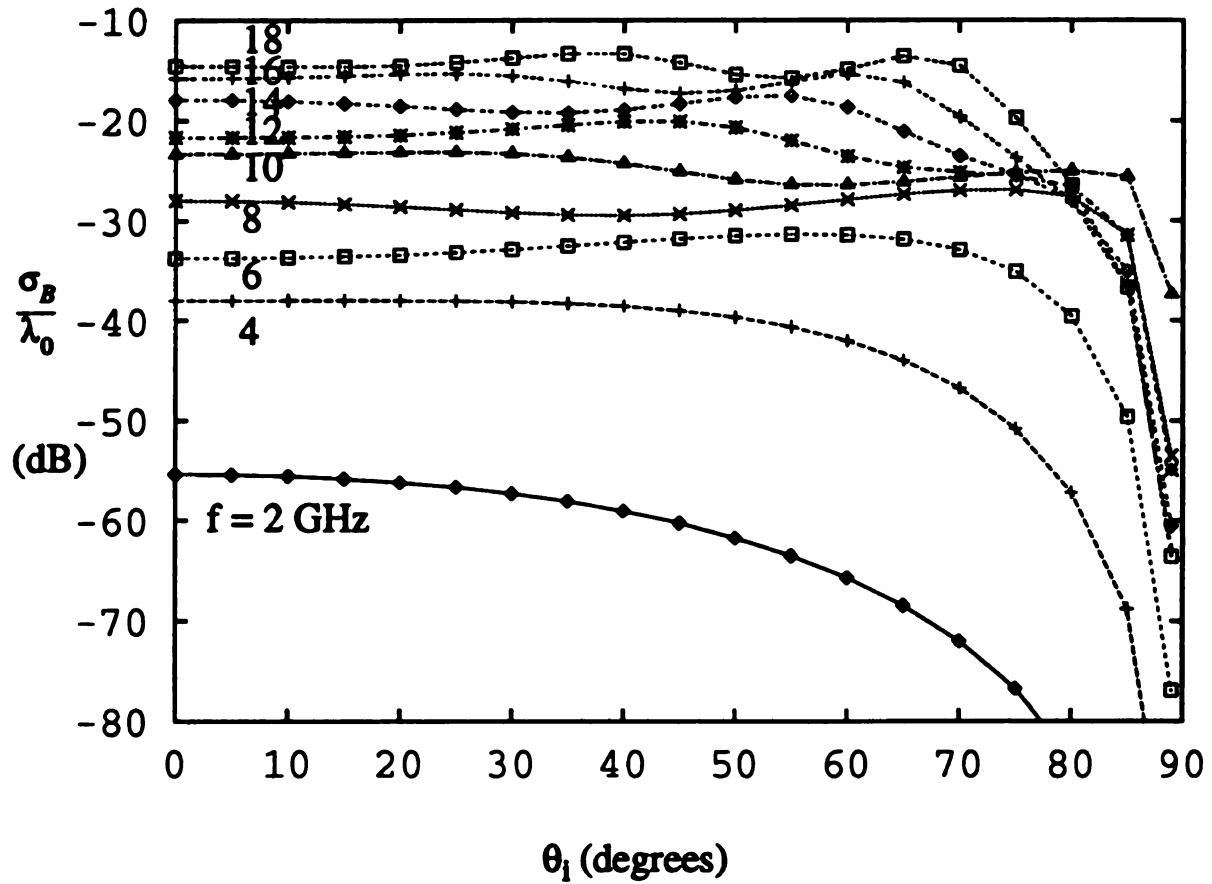


Figure 5.9. Normalized back scattering width as a function of incidence angle θ_i at various frequencies when a TE plane wave is incident upon a grounded sheet with a narrow air gap. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.06''$, $t = 1''$, $N_z = 2$ to 8, and $N_x = 5$ to 30)

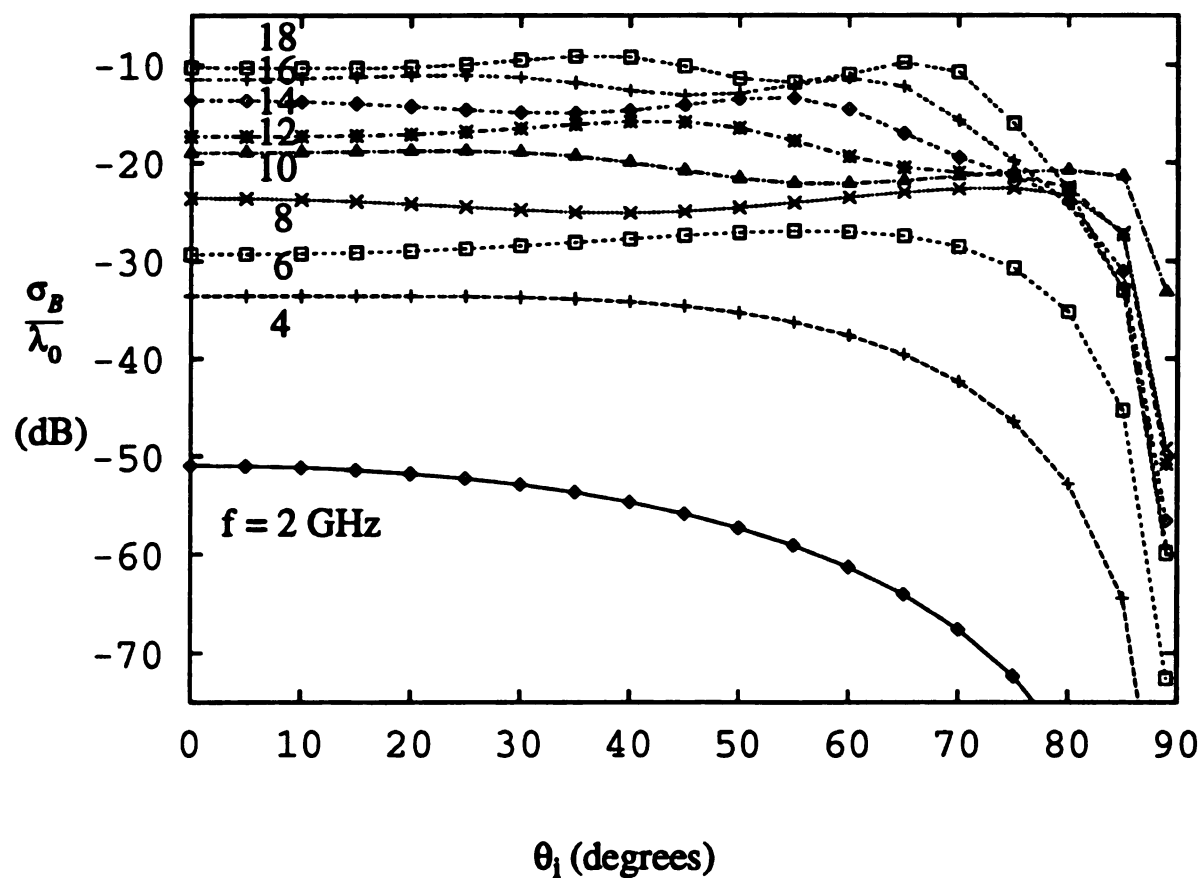


Figure 5.10. Normalized back scattering width as a function of incidence angle θ_i at various frequencies when a TE plane wave is incident upon a grounded sheet with a narrow air gap. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.1''$, $t = 1''$, $N_z = 2$ to 8, and $N_x = 5$ to 30)

Figures 5.11 and 5.12 replot the normalized back scattering width calculated by the MoM EFIE solutions as functions of incidence angle θ_i at various frequencies when a TM-polarized plane wave is incident upon a grounded sheet with a narrow air gap with $2a = 0.06''$, and $0.1''$, respectively.

Comparing Figure 5.2 with Figure 5.7, Figure 5.5 with Figure 5.8, Figure 5.9 with Figure 5.10, and Figure 5.11 with Figure 5.12, it is found that the shape of curves are almost identical in the two corresponding figures, in which only $2a$ is changed from $0.06''$ to $0.1''$. In the first-order approximations of the back scattering width for both TE- and TM-polarized plane wave incidence, the term involves $2a$ is $[(2k_0a)^2 \sin(2k_1a \sin\theta_i) / (2k_1a \sin\theta_i)]^2$. Because the air gap is thin, the term $\sin(2k_1a \sin\theta_i) / (2k_1a \sin\theta_i)$ is nearly equal to 1. $[\sin(2k_1a \sin\theta_i) / (2k_1a \sin\theta_i) = 0.85$, when $\theta_i = 90^\circ$, $2a = 0.1''$, and $f = 18\text{GHz}$] Thus, the back scattering width is approximately proportional to $(2k_0a)^2$. But when the air gap is not thin, the term $\sin(2k_1a \sin\theta_i) / (2k_1a \sin\theta_i)$ is small, and the curve shape of the back scattering width vs. frequency or incident angle will be changed, as shown in Figure 5.3.

In this chapter, the total electric field in the discontinuity region is approximated with the electric field in a zero-width gap, and then the first- and the second-order iterative approximations to the far-zone field for a narrow air gap are calculated. The approximate results agree very well with the MoM EFIE solutions. The first-order approximation to the far-zone field is expressed in a closed form in terms of elementary functions, and the second-order approximation is expressed as an integral representation. However, the MoM requires calculation of the matrix elements by integration followed by solution of linear equations. Thus, the approximate method is much computationally efficient than the MoM for a narrow gap.

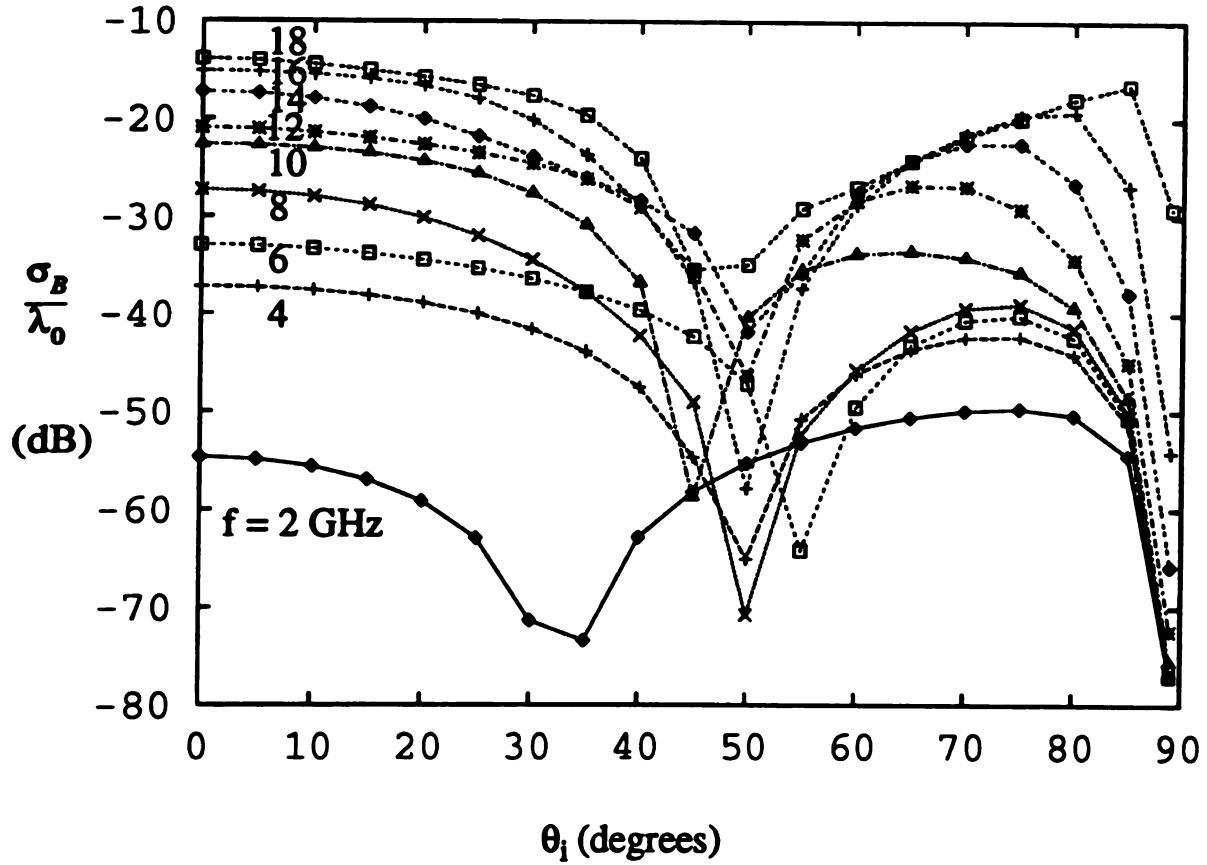


Figure 5.11. Normalized back scattering width as a function of incidence angle θ_i at various frequencies when a TM plane wave is incident upon a grounded sheet with a narrow air gap. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.06''$, $t = 1''$, $N_z = 2$ to 8, and $N_x = 5$ to 30)

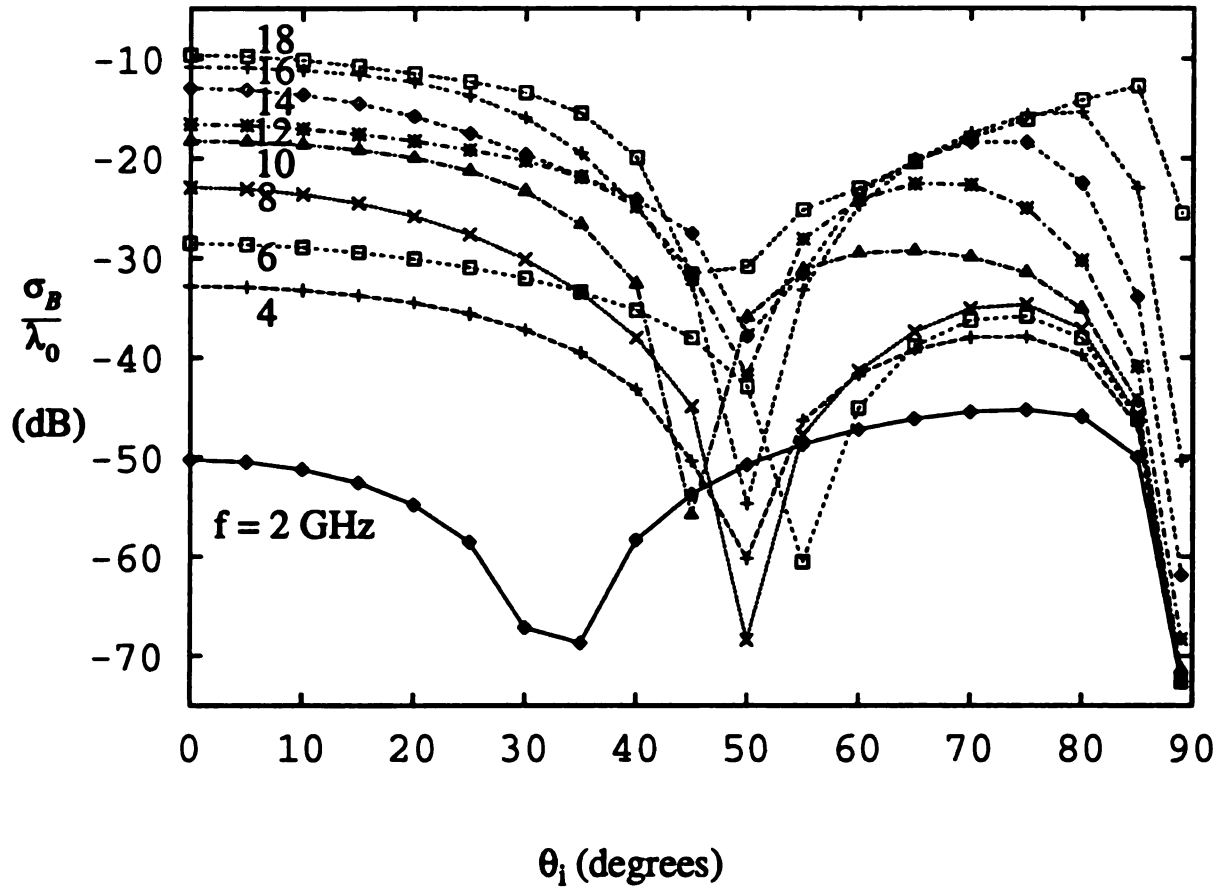


Figure 5.12. Normalized back scattering width as a function of incidence angle θ_i at various frequencies when a TM plane wave is incident upon a grounded sheet with a narrow air gap. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.1''$, $t = 1''$, $N_z = 2$ to 8, and $N_x = 5$ to 30)

CHAPTER 6

PULSE GALERKIN'S SOLUTIONS FOR TE-POLARIZED WAVE INCIDENCE

6.1. INTRODUCTION

In Chapter 4, the Method of Moments (MoM) is applied to transform the electric field integral equation (EFIE) into matrix equations for general basis and testing functions. Here, a pulse Galerkin's implementation is applied to establish the matrix elements A_{mn}^{yy} and the forcing vectors F_{ym} for TE-polarized wave incidence. First, formulas to calculate the matrix elements and forcing term will be derived when both basis and testing functions are 2-D pulse functions. Then, computation of the inverse Fourier transform representation of the electric Green's function is performed by integration along real axis, along the branch cuts directly, and along the branch cuts with a variable change. Finally, these integration methods will be compared with each other by checking the power conservation.

6.2. PULSE GALERKIN'S SOLUTIONS

It is assumed that the cross section of the discontinuity region is rectangular ($|z| \leq a$, $-t \leq x \leq 0$) (Figure 6.1) and the permittivity ϵ_d is a constant. For TE-polarized wave incidence, the EFIE in the discontinuity region becomes

$$E_{2y}(x, z) - j \frac{k_0 \Delta n^2}{\eta_0} \int_{-a}^a \int_{-t}^0 G_{22yy}(x|x', z-z') E_{2y}(x', z') dx' dz' = E_{2y}^i(x, z) \quad (6.2.1)$$

for $|z| \leq a, -t \leq x \leq 0$

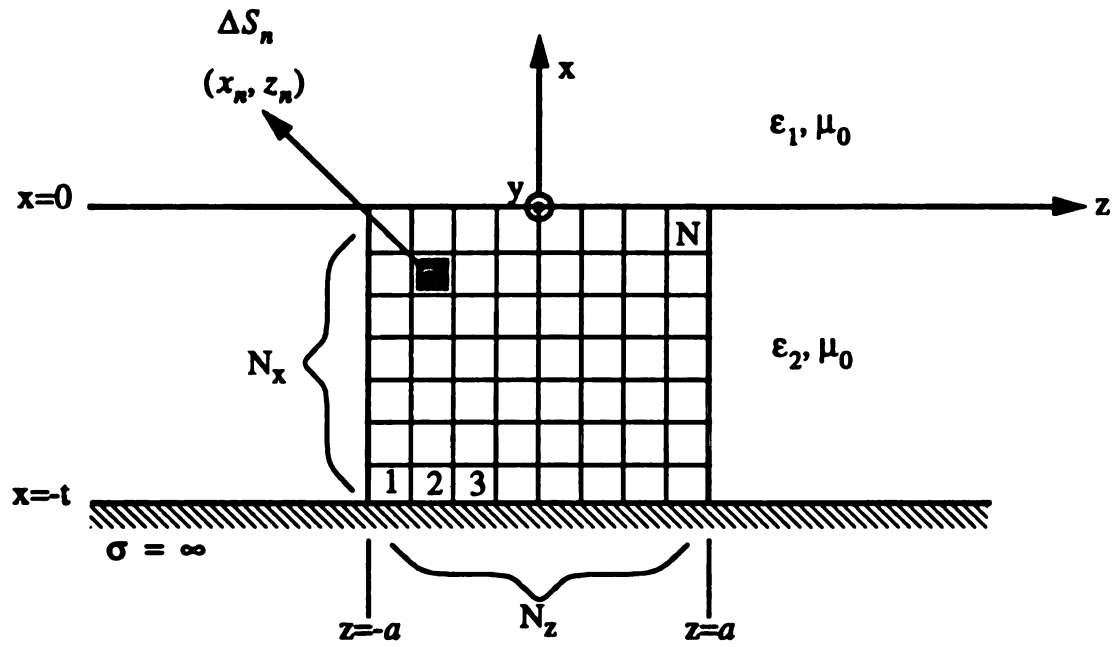


Figure 6.1. Applying Pulse Galerkin's Method to the EFIE in the discontinuity region.

where

$$\Delta n^2 = n_d^2 - n_2^2 = (\epsilon_d - \epsilon_2) / \epsilon_0 \quad (6.2.2)$$

In Figure 6.1, the discontinuity region is partitioned into $N = N_x N_z$ equal rectangular elements ΔS_n centered at (x_n, z_n) , where N_x, N_z are total numbers of partitions along x and z , respectively. The partitioning is defined such that

$$\int_{-a}^a \int_{-t}^0 f(x, z) dx dz = \sum_{n=1}^N \int_{\Delta S_n} f(x, z) dx dz \quad (6.2.3)$$

From Figure 6.1, it is found that

$$\Delta S_n \Leftrightarrow |x - x_n| < \Delta x/2, |z - z_n| < \Delta z/2 \quad (6.2.4)$$

$$x_n = -t + \Delta x/2 + [L_x(n) - 1] \Delta x \quad (6.2.5)$$

$$z_n = -a + \Delta z/2 + [L_z(n) - 1] \Delta z \quad (6.2.6)$$

where

$$L_x(n) = 1 + INT\{(n-1)/N_z\} \quad (6.2.7)$$

$$L_z(n) = n - INT\{(n-1)/N_z\} N_z \quad (6.2.8)$$

$$\Delta x = t/N_x \quad (6.2.9)$$

$$\Delta z = 2a/N_z \quad (6.2.10)$$

and $INT\{\dots\}$ designates the integer part of the quantity within the braces.

Let's apply pulse Galerkin's method to the EFIE. The basis functions $e_{yn}(x, z)$ and the testing functions $t_{yn}(x, z)$ are chosen to be the 2-D pulse functions defined as

$$P_n(x, z) = \begin{cases} 1 & (x, z) \in \Delta S_n \\ 0 & \text{otherwise} \end{cases} \quad (6.2.11)$$

Substituting the expressions for the basis functions and the testing functions in the formulas for A_{mn}^{yy} and F_{ym} [(4.5.5) and (4.5.6)] gives

$$\sum_{n=1}^N A_{mn}^{yy} a_{yn} = F_{ym} \quad m = 1, 2, \dots, N \quad (6.2.12)$$

where

$$F_{ym} = \frac{1}{\Delta x \Delta z} \int_{\Delta S_m} E_{2y}^i(x, z) dx dz \quad (6.2.13)$$

$$A_{mn}^{yy} = \delta_{mn} - j \frac{k_0 \Delta n^2}{\eta_0 \Delta x \Delta z} \int_{\Delta S_m} dx dz \int_{\Delta S_n} G_{22yy}(x|x', z-z') dx' dz' \quad (6.2.14)$$

Note that $A_{mn}^{yy}/(\Delta x \Delta z)$ and $F_{ym}/(\Delta x \Delta z)$ are newly defined as A_{mn}^{yy} and F_{ym} , respectively, and $E_{2y}^i(x, z)$, $G_{22yy}(x|x', z-z')$ are given in Chapters 2 to 4.

Let's evaluate the above spatial integrals required in spectral integral representation of MoM matrix elements. Note that ΔS_m and ΔS_n are defined by (6.2.4). Substituting $G_{22yy}(x|x', z-z') = G_{22yy}^R(x|x', z-z')$ in (6.2.14), where $G_{22yy}^R(x|x', z-z')$ is given in (3.2.3) and (3.2.8), and using

$$\int_{z_m - \Delta z/2}^{z_m + \Delta z/2} dz \int_{z_n - \Delta z/2}^{z_n + \Delta z/2} e^{j\zeta|z-z'|} dz' = \begin{cases} 2 [1 - \cos(\zeta \Delta z)] e^{j\zeta|z_m - z_n|} / \zeta^2 & z_m \neq z_n \\ 2 (1 + j\zeta \Delta z - e^{j\zeta \Delta z}) / \zeta^2 & z_m = z_n \end{cases} \quad (6.2.15)$$

$$\int_{x_m - \Delta x/2}^{x_m + \Delta x/2} dx \int_{x_n - \Delta x/2}^{x_n + \Delta x/2} e^{-p_2|x-x'|} dx' = \begin{cases} 2 [\cosh(p_2 \Delta x) - 1] e^{-p_2|x_m - x_n|} / p_2^2 & x_m \neq x_n \\ 2 (e^{-p_2 \Delta x} - 1 + p_2 \Delta x) / p_2^2 & x_m = x_n \end{cases} \quad (6.2.16)$$

$$\int_{x_m - \Delta x/2}^{x_m + \Delta x/2} e^{-p_2 x} dx = 2 \sinh(p_2 \Delta x/2) e^{-p_2 x_m} / p_2 \quad (6.2.17)$$

$$\begin{aligned}
& \int_{x_m - \Delta x/2}^{x_m + \Delta x/2} dx \int_{x_n - \Delta x/2}^{x_n + \Delta x/2} \sinh [p_2 (x + t)] \sinh [p_2 (x' + t)] dx' \\
&= \frac{2 [\cosh (p_2 \Delta x) - 1]}{p_2^2} \sinh [p_2 (x_m + t)] \sinh [p_2 (x_n + t)] \\
&= \frac{\cosh (p_2 \Delta x) - 1}{p_2^2} \{ -\cosh [p_2 (x_m - x_n)] + \cosh [p_2 (2t + x_m + x_n)] \} \quad (6.2.18)
\end{aligned}$$

yield

$$\begin{aligned}
A_{mn}^{yy} = & \delta_{mn} - \frac{k_0^2 \Delta x \Delta z}{\pi} \Delta n^2 \int_{-\infty}^{\infty} g_{mn}(\zeta) \frac{d\zeta}{p_2} \left\{ \phi_{mn}(\zeta) + \frac{\cosh (p_2 \Delta x) - 1}{(p_2 \Delta x)^2} \{ -e^{-p_2 (2t + x_m + x_n)} \right. \\
& \left. + \frac{p_2 - p_1}{\Delta_h} e^{-p_2 t} [-\cosh [p_2 (x_m - x_n)] + \cosh [p_2 (2t + x_m + x_n)]] \} \right\} \quad (6.2.19)
\end{aligned}$$

where

$$g_{mn}(\zeta) = \begin{cases} [1 - \cos (\zeta \Delta z)] e^{j\zeta |z_m - z_n|} / (\zeta \Delta z)^2 & z_m \neq z_n \\ (1 + j\zeta \Delta z - e^{j\zeta \Delta z}) / (\zeta \Delta z)^2 & z_m = z_n \end{cases} \quad (6.2.20)$$

$$\phi_{mn}(\zeta) = \begin{cases} [\cosh (p_2 \Delta x) - 1] e^{-p_2 |x_m - x_n|} / (p_2 \Delta x)^2 & x_m \neq x_n \\ (e^{-p_2 \Delta x} - 1 + p_2 \Delta x) / (p_2 \Delta x)^2 & x_m = x_n \end{cases} \quad (6.2.21)$$

It is noted that the integrand in (6.2.19) depends only upon functions of $|x_m - x_n|$, $2t + x_m + x_n$, and $|z_m - z_n|$. Thus, the matrix elements can be rewritten as:

$$A_{mn}^{yy} = \delta_{mn} - \frac{k_0^2 \Delta z (\Delta x)^2}{\pi} \Delta n^2 \left[B^{yy} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{|x_m - x_n|}{\Delta x} \right) - B^{yy} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{2t + x_m + x_n}{\Delta x} \right) \right] \quad (6.2.22)$$

where

$$B^{yy}(l, s) = \int_{-\infty}^{\infty} g_l(\zeta) f_s^{yy}(\zeta) d\zeta \quad (6.2.23)$$

with

$$g_l(\zeta) = \begin{cases} [1 - \cos(\zeta \Delta z)] e^{j\zeta l \Delta z} / (\zeta \Delta z)^2 & l = 1, 2, \dots \\ (1 + j\zeta \Delta z - e^{j\zeta \Delta z}) / (\zeta \Delta z)^2 & l = 0 \end{cases} \quad (6.2.24)$$

$$f_0^{yy}(\zeta) = \frac{1}{(p_2 \Delta x)^2} + \frac{1}{(p_2 \Delta x)^3} \left\{ e^{-p_2 \Delta x} - 1 - \frac{p_2 - p_1}{\bar{\Delta}_h} e^{-p_2 t} [\cosh(p_2 \Delta x) - 1] \right\} \quad (6.2.25)$$

$$f_s^{yy}(\zeta) = \frac{\cosh(p_2 \Delta x) - 1}{(p_2 \Delta x)^3} \left\{ e^{-p_2 s \Delta x} - \frac{p_2 - p_1}{\bar{\Delta}_h} e^{-p_2 t} \cosh(p_2 s \Delta x) \right\} \quad (6.2.26)$$

(6.2.22) allows a dramatic reduction in the amount of effort needed to fill the moment method matrix. There are $N^2 = N_x^2 N_z^2$ matrix elements, and $N^2/2$ matrix elements should be calculated because $A_{mn}^{yy} = A_{nm}^{yy}$. But in (6.2.22), since

$$\left. \begin{aligned} x_n &= -t + \Delta x/2 + [L_x(n) - 1] \Delta x \\ z_n &= -a + \Delta z/2 + [L_z(n) - 1] \Delta z \end{aligned} \right\} \quad (6.2.27)$$

where

$$\left. \begin{aligned} L_x(n) &= 1 + INT \left\{ \frac{n-1}{N_z} \right\} \\ L_z(n) &= n - INT \left\{ \frac{n-1}{N_z} \right\} N_z \end{aligned} \right\} \quad (6.2.28)$$

and n is from 1 to N , then $L_x(n)$ has N_x different values from 1 to N_x and $L_z(n)$ has N_z different values from 1 to N_z . Thus

$$l = \frac{|z_m - z_n|}{\Delta x} = 0, 1, \dots, N_z - 1 \quad (6.2.29)$$

$$s = \frac{|x_m - x_n|}{\Delta x} = 0, 1, \dots, N_x - 1 \quad (6.2.30)$$

$$s = \frac{x_m + x_n + 2t}{\Delta x} = 1, 2, \dots, 2N_x - 1 \quad (6.2.31)$$

There are N_z l 's and $2N_x$ s 's. Therefore, only $2N_x N_z$ integrations are needed to compute. Evidently, much computer time can be conserved. For example, if $N_x = N_z = 10$, then $N^2/2 = 5000$, but $2N_x N_z = 200$.

SUMMARY:

$$\sum_{n=1}^N A_{mn}^{yy} a_{yn} = F_{ym} \quad m = 1, 2, \dots, N \quad (6.2.32)$$

$$A_{mn}^{yy} = \delta_{mn} - \frac{k_0^2 \Delta z (\Delta x)^2}{\pi} \Delta n^2 \left[B^{yy} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{|x_m - x_n|}{\Delta x} \right) - B^{yy} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{2t + x_m + x_n}{\Delta x} \right) \right] \quad (6.2.33)$$

where

$$B^{yy}(l, s) = \int_{-\infty}^{\infty} g_l(\zeta) f_s^{yy}(\zeta) d\zeta \quad (6.2.34)$$

with

$$g_l(\zeta) = \begin{cases} [1 - \cos(\zeta \Delta z)] e^{j\zeta l \Delta z} / (\zeta \Delta z)^2 & l = 1, \dots, N_z - 1 \\ (1 + j\zeta \Delta z - e^{j\zeta \Delta z}) / (\zeta \Delta z)^2 & l = 0 \end{cases} \quad (6.2.35)$$

$$f_0^{yy}(\zeta) = \frac{1}{(p_2 \Delta x)^2} + \frac{1}{(p_2 \Delta x)^3} \left\{ e^{-p_2 \Delta x} - 1 - \frac{P_2 - P_1}{\bar{\Delta}_h} e^{-p_2 t} [\cosh(p_2 \Delta x) - 1] \right\} \quad (6.2.36)$$

$$f_s^{yy}(\zeta) = \frac{\cosh(p_2 \Delta x) - 1}{(p_2 \Delta x)^3} \left\{ e^{-p_2 s \Delta x} - \frac{P_2 - P_1}{\bar{\Delta}_h} e^{-p_2 t} \cosh(p_2 s \Delta x) \right\} \quad (6.2.37)$$

for $s = 1, 2, \dots, 2N_x - 1$.

The forcing vector F_{ym} is computed in the same way to be

$$F_{ym} = E_0 \frac{2\eta_2 \cos\theta_i \sin[k_2(x_m + t) \cos\theta_r]}{\eta_2 \cos\theta_i \sin[k_2 t \cos\theta_r] - j\eta_1 \cos\theta_r \cos[k_2 t \cos\theta_r]} \frac{\sin(\cos\theta_r k_2 \Delta x/2)}{\cos\theta_r k_2 \Delta x/2} \frac{\sin(\sin\theta_i k_1 \Delta z/2)}{\sin\theta_i k_1 \Delta z/2} e^{jk_1 z_m \sin\theta_i} \quad (6.2.38)$$

for TE-polarized plane wave incidence, and

$$F_{ym} = E_0 \sin[q_2(t + x_m)] \frac{\sin(q_2 \Delta x/2)}{q_2 \Delta x/2} \frac{\sin(\zeta_p \Delta z/2)}{\zeta_p \Delta z/2} e^{j\zeta_p z_m} \quad (6.2.39)$$

for TE-odd surface-wave mode incidence.

Since the unknown electric field in the discontinuity region is expanded into a set of 2-D pulse functions, then the amplitudes of the excited surface wave and radiation wave are obtained by substituting the pulse functions in (4.5.13) and (4.5.14).

$$A_h^\pm = \frac{k_0^2 \Delta x \Delta z \Delta n^2}{\bar{\Delta}'_h(\zeta_p) \sin(q_2 t)} \frac{\sin(q_2 \Delta x/2)}{q_2 \Delta x/2} \frac{\sin(\zeta_p \Delta z/2)}{\zeta_p \Delta z/2} \sum_{n=1}^N a_{yn} \sin[q_2(t + x_n)] e^{\pm j\zeta_p z_n} \quad (6.2.40)$$

$$B_h(\theta) = \frac{jk_0^2 k_1 \Delta x \Delta z \Delta n^2 \cos\theta}{2\pi \bar{\Delta}_h(-k_1 \sin\theta)} \frac{\sin(q_2 \Delta x/2)}{q_2 \Delta x/2} \frac{\sin(\sin\theta k_1 \Delta z/2)}{\sin\theta k_1 \Delta z/2} \sum_{n=1}^N a_{yn} \sin[q_2(t + x_n)] e^{jk_1 z_n \sin\theta} \quad (6.2.41)$$

where $\bar{\Delta}'_h(\zeta_p)$ and $\bar{\Delta}_h(-k_1 \sin\theta)$ are given in Chapter 4.

6.3. NUMERICAL INTEGRATION ALONG REAL AXIS

It is noted that $f_s^{\gamma\gamma}(\zeta)$ in (6.2.36) and (6.2.37) is an even function of ζ , and $g_l(\zeta)$ can be written as the sum of an even function and an odd function of ζ , i.e.

$$g_l(\zeta) = \begin{cases} [1 - \cos(\zeta\Delta z)] [\cos(l\zeta\Delta z) + j\sin(l\zeta\Delta z)] / (\zeta\Delta z)^2 & l = 1, \dots, N_z - 1 \\ \{ [1 - \cos(\zeta\Delta z)] + j[\zeta\Delta z - \sin(\zeta\Delta z)] \} / (\zeta\Delta z)^2 & l = 0 \end{cases} \quad (6.3.1)$$

Since the integral from $-\infty$ to ∞ in (6.2.34) is symmetric, then it can be rewritten as

$$B^{\gamma\gamma}(l, s) = \int_0^{\infty} g_l^e(\zeta) f_s^{\gamma\gamma}(\zeta) d\zeta \quad (6.3.2)$$

where

$$g_l^e(\zeta) = 2 \cos(l\zeta\Delta z) [1 - \cos(\zeta\Delta z)] / (\zeta\Delta z)^2 \quad l = 0, 1, \dots, N_z - 1 \quad (6.3.3)$$

Since $f_s^{\gamma\gamma}(\zeta)$ involves hyperbolic sine and cosine functions, it is prone to numerical difficulty. As the integration variable in (6.3.2) increases toward infinitely, both the sinh and cosh functions become unbounded. In addition, it is difficult to ascertain the convergence properties of the integral. Both of these problems can be overcome if the integrands are written in terms of exponential functions. After some algebraic manipulation, the results become

$$f_0^{\gamma\gamma}(\zeta) = \frac{1}{(p_2\Delta x)^2} - \frac{1 - e^{-p_2\Delta x}}{(p_2\Delta x)^3} \left\{ 1 + (1 - e^{-p_2\Delta x}) \frac{p_2 - p_1}{\bar{\Delta}_{h1}} e^{-p_2(2t - \Delta x)} \right\} \quad (6.3.4)$$

$$f_s^{\gamma\gamma}(\zeta) = \frac{(1 - e^{-p_2\Delta x})^2}{2(p_2\Delta x)^3} \left\{ e^{-p_2(s-1)\Delta x} - \frac{p_2 - p_1}{\bar{\Delta}_{h1}} [e^{-p_2[2t - (s+1)\Delta x]} + e^{-p_2[2t + (s-1)\Delta x]}] \right\} \quad (6.3.5)$$

where

$$\bar{\Delta}_{h1} = 2\bar{\Delta}_h e^{-p_2 z'} = p_2 (1 + e^{-2p_2 z'}) + p_1 (1 - e^{-2p_2 z'}) \quad (6.3.6)$$

CONVERGENCE OF THE MoM MATRIX ELEMENT INTEGRALS

Before undertaking the numerical integrations of equation (6.3.2), it is quite helpful to anticipate the rate of convergence of the integrals. Of interest is the behavior of the integrand $g_i^e(\zeta) f_s^y(\zeta)$ as $\zeta \rightarrow \infty$.

$$f_s^y(\zeta) \rightarrow \begin{cases} 1/\zeta^2 & s = 0 \\ 1/\zeta^3 & s = 1 \\ e^{-c\zeta\Delta x}/\zeta^3 & \text{otherwise} \end{cases} \quad (6.3.7)$$

where $c \geq 1$. Then

$$g_i^e(\zeta) f_s^y(\zeta) \rightarrow \begin{cases} [1 - \cos(\zeta\Delta z)] \cos(l\zeta\Delta z) / \zeta^4 & s = 0 \\ [1 - \cos(\zeta\Delta z)] \cos(l\zeta\Delta z) / \zeta^5 & s = 1 \\ [1 - \cos(\zeta\Delta z)] \cos(l\zeta\Delta z) e^{-c\zeta\Delta x} / \zeta^5 & \text{otherwise} \end{cases} \quad (6.3.8)$$

Hence, the integration $\int_0^\infty g_i^e(\zeta) f_s^y(\zeta) d\zeta$ will converge quickly.

INTEGRATION THROUGH SURFACE-WAVE-POLE SINGULARITIES

Since the presence of the loss in region 2 makes k_2 a complex number, then all the surface-wave poles of the integrand are not along the real axis. But, if the loss becomes small or vanishes, the surface-wave poles will shift to the real axis, and thus reside within the domain of integration. Therefore, pole extraction will be used to deal with the surface-wave-pole singularities.

In (6.3.2), the integrand becomes infinite when

$$\tilde{\Delta}_h(\zeta) = p_2 \cosh(p_2 t) + p_1 \sinh(p_2 t) = 0 \quad (6.3.9)$$

which leads to TE surface-wave poles at $\zeta = \pm \zeta_p$. Since

$$p_i = \sqrt{\zeta^2 - k_i^2} \quad i = 1, 2 \quad (6.3.10)$$

and p_2 is changed to

$$p_2 = jq_2 \quad q_2 = \sqrt{k_2^2 - \zeta^2} \quad (6.3.11)$$

Using above definition, $\tilde{\Delta}_h(\zeta) = 0$ becomes

$$\tilde{\Delta}_h(\zeta) = j[q_2 \cos(q_2 t) + p_1 \sin(q_2 t)] = 0 \quad (6.3.12)$$

Rearranging leads to

$$\tan(q_2 t) = -q_2/p_1 \quad (6.3.13)$$

This is the well-known TE-odd mode eigenvalue equation in dielectric slab waveguide. When the frequency is greater than the cut off frequency, it has proper root. The roots of (6.3.12) or (6.3.13) can be numerically obtained by the Newton-Raphson method, which requires the derivative of $\tilde{\Delta}_h(\zeta)$ with respect to ζ .

$$\tilde{\Delta}'_h(\zeta) = \frac{d}{d\zeta} \tilde{\Delta}_h(\zeta) = \frac{j\zeta}{p_1 q_2} (1 + p_1 t) [q_2 \sin(q_2 t) - p_1 \cos(q_2 t)] \quad (6.3.14)$$

The function $\tilde{\Delta}_h(\zeta)$ can be approximated by a Taylor's series of first degree near $\zeta = \pm \zeta_p$. Hence it can be written as

$$\tilde{\Delta}_h(\pm \zeta_p) = 0 \quad (6.3.15)$$

$$\tilde{\Delta}_h(\zeta) \equiv \tilde{\Delta}'_h(\pm \zeta_p) (\zeta \mp \zeta_p) \quad (6.3.16)$$

Let's consider the integral (Figure 6.2).

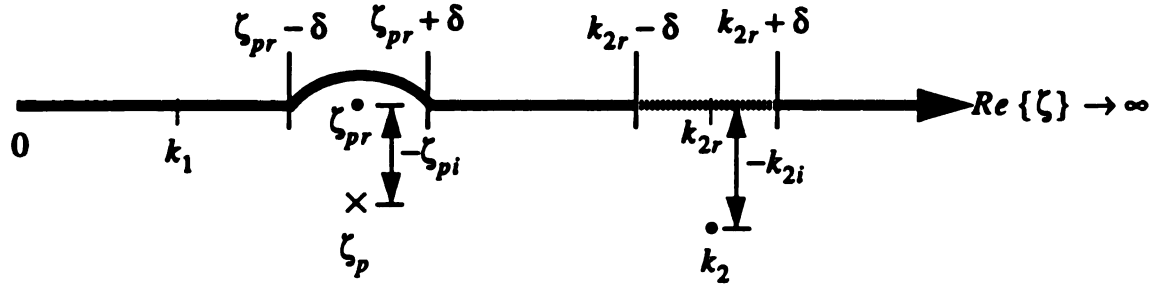


Figure 6.2. Integration along the real axis.

$$\int_{\zeta_{pr}-\delta}^{\zeta_{pr}+\delta} \frac{f(\zeta)}{\bar{\Delta}_h(\zeta)} d\zeta \quad (6.3.17)$$

where $\zeta_p = \zeta_{pr} + j\zeta_{pi}$, ζ_{pr} and ζ_{pi} are the real and imaginary parts of ζ_p , respectively. δ is a small number. Making use of the approximation for the $\bar{\Delta}_h(\zeta)$ in (6.3.16), and $f(\zeta) \equiv f(\zeta_p)$, the integral is evaluated as

$$\int_{\zeta_{pr}-\delta}^{\zeta_{pr}+\delta} \frac{f(\zeta)}{\bar{\Delta}_h(\zeta)} d\zeta = \frac{f(\zeta_p)}{\bar{\Delta}'_h(\zeta_p)} \int_{\zeta_{pr}-\delta}^{\zeta_{pr}+\delta} \frac{d\zeta}{\zeta - \zeta_p} = 2j \frac{f(\zeta_p)}{\bar{\Delta}'_h(\zeta_p)} \operatorname{atan} \frac{\delta}{\zeta_{pi}} \quad (6.3.18)$$

Therefore, the integration of (6.3.2) from the vicinity of the surface-wave pole is

$$\int_{\zeta_{pr}-\delta}^{\zeta_{pr}+\delta} g_l^e(\zeta) f_s^{\gamma p}(\zeta) d\zeta = j \frac{2g_l^e(\zeta_p) f_s^{\gamma p}(\zeta_p)}{\bar{\Delta}'_h(\zeta_p)} \operatorname{atan} \frac{\delta}{\zeta_{pi}} \quad (6.3.19)$$

where

$$f_s^{\gamma p}(\zeta) = \frac{j}{\Delta x} \frac{1 - \cos(q_2 \Delta x)}{(q_2 \Delta x)^2} \frac{\cos(q_2 s \Delta x)}{\sin(q_2 l)} \quad (6.3.20)$$

Using (6.3.14) and (6.3.15) leads to

$$\tilde{\Delta}'_h(\pm\zeta_p) = \pm \frac{j\zeta_p}{p_1} \frac{1+p_1 t}{\sin(q_2 t)} \quad (6.3.21)$$

The result of (6.3.19) is right only when ζ_{pi} is small, i.e. for small loss or lossless dielectric. If ζ_{pi} is large, since δ is small, then

$$\int_{\zeta_{pr}-\delta}^{\zeta_{pr}+\delta} g_l^e(\zeta) f_s^{yy}(\zeta) d\zeta \approx 2g_l^e(\zeta_{pr}) f_s^{yy}(\zeta_{pr}) \delta \quad (6.3.22)$$

INTEGRATION IN THE VICINITY OF k_2

Let's consider the integrands of (6.3.2) in the vicinity of $k_2 = k_{2r} + jk_{2i}$ (Figure 6.2)

$$\int_{k_{2r}-\delta}^{k_{2r}+\delta} g_l^e(\zeta) f_s^{yy}(\zeta) d\zeta \quad (6.3.23)$$

From (6.2.36) and (6.2.37), it is found that $\zeta = k_2(p_2 = 0)$ is a branch point, and p_2 occurs in its denominator. But the behavior of the x-relative part of integrand in (6.2.19) as $\zeta \rightarrow k_2(p_2 \rightarrow 0)$ is

$$\begin{aligned} \frac{\phi_{mn}(\zeta)}{p_2 \Delta x} + \frac{\cosh(p_2 \Delta x) - 1}{(p_2 \Delta x)^3} \left\{ -e^{-p_2(2t+x_m+x_n)} + \frac{p_2 - p_1}{\Delta_h} e^{-p_2 t} \{ -\cosh[p_2(x_m - x_n)] + \cosh[p_2(2t+x_m+x_n)] \} \right\} \\ \rightarrow f_s^{yyk}(k_2) \Big|_{s=\frac{|x_m-x_n|}{\Delta x}} - f_s^{yyk}(k_2) \Big|_{s=\frac{x_m+x_n+2t}{\Delta x}} \end{aligned} \quad (6.3.24)$$

where

$$f_s^{yyk}(\zeta) = -\frac{s}{2} + \frac{p_1 \Delta x}{1+p_1 t} \frac{s^2}{4} \quad s = 1, 2, \dots, 2N_x - 1 \quad (6.3.25)$$

$$f_0^{yyk}(\zeta) = -1/6 \quad (6.3.26)$$

Then, $\zeta = k_2(p_2 = 0)$ does not lead to a zero in the denominator of the integrand of

$$(6.2.19). \text{ The integral } \int_{k_{2r}-\delta}^{k_{2r}+\delta} g_l^e(\zeta) f_s^{yy}(\zeta) d\zeta \text{ can be replaced by } \int_{k_{2r}-\delta}^{k_{2r}+\delta} g_l^e(\zeta) f_s^{yyk}(\zeta) d\zeta.$$

It does not change the final result in (6.2.22). This leads to

$$\begin{aligned} \int_{k_{2r}-\delta}^{k_{2r}+\delta} g_l^e(\zeta) f_s^{yy}(\zeta) d\zeta &\Rightarrow \int_{k_{2r}-\delta}^{k_{2r}+\delta} g_l^e(\zeta) f_s^{yyk}(\zeta) d\zeta \\ &\approx 2\delta g_l^e(k_2) f_s^{yyk}(k_2) \quad \text{for small } k_{2i} \end{aligned} \quad (6.3.27)$$

If k_{2i} is large, the integration of (6.3.2) can be replaced by

$$\int_{k_{2r}-\delta}^{k_{2r}+\delta} g_l^e(\zeta) f_s^{yy}(\zeta) d\zeta \approx 2\delta g_l^e(k_{2r}) f_s^{yy}(k_{2r}) \quad (6.3.28)$$

SUMMARY

The integration of (6.3.2) along the real axis shown in Figure 6.2 can be written as the sum of five terms

$$\int_0^{\infty} g_l^e(\zeta) f_s^{yy}(\zeta) d\zeta = \left\{ \int_0^{\zeta_{pr}-\delta} + \int_{\zeta_{pr}-\delta}^{\zeta_{pr}+\delta} + \int_{\zeta_{pr}+\delta}^{k_{2r}-\delta} + \int_{k_{2r}-\delta}^{k_{2r}+\delta} + \int_{k_{2r}+\delta}^{\infty} \right\} g_l^e(\zeta) f_s^{yy}(\zeta) d\zeta \quad (6.3.29)$$

The second and fourth terms are discussed above. Romberg integration method [66] is used in the first and third terms. The fifth term is computed by using the convergence form [(6.3.4) and (6.3.5)]. The convergence is found to be relatively rapid.

6.4. NUMERICAL INTEGRATION ALONG BRANCH CUTS

In the last section, the matrix elements are expressed as the integrals along the real axis. There are two ways to express the matrix elements in terms of integrals along branch cuts and residues at surface-wave poles. One is substituting

$$G_{22yy}(x|x', z-z') = G_{22yy}^{Pole}(x|x', z-z') + G_{22yy}^{Rad}(x|x', z-z') \quad (6.4.1)$$

in (6.2.14) and doing the same simplifications in the last section. Other way is applying complex analysis to (6.2.34), and rewriting the integral along the real axis in terms of integrals along branch cuts and residues at surface-wave poles. Two ways give us the same results as follows:

$$B^{yy}(l, s) = \int_{-\infty}^{\infty} g_l(\zeta) f_s^{yy}(\zeta) d\zeta = - \int_{C_p} g_l(\zeta) f_s^{yy}(\zeta) d\zeta - \int_{C_r} g_l(\zeta) f_s^{yy}(\zeta) d\zeta \quad (6.4.2)$$

Residues at surface-wave poles are

$$\begin{aligned} \int_{C_p} g_l(\zeta) f_s^{yy}(\zeta) d\zeta &= -2\pi j \text{Res} \{ g_l(\zeta) f_s^{yy}(\zeta) \} \Big|_{\zeta = -\zeta_p} \\ &= -2\pi j \frac{g_l(-\zeta_p) f_s^{yyP}(-\zeta_p)}{\bar{\Delta}'_h(-\zeta_p)} = 2\pi j \frac{g_l(-\zeta_p) f_s^{yyP}(\zeta_p)}{\bar{\Delta}'_h(\zeta_p)} \end{aligned} \quad (6.4.3)$$

where $\bar{\Delta}'_h(\pm\zeta_p)$ and $f_s^{yyP}(\zeta_p)$ are given in the last section, and

$$g_l(-\zeta_p) = \begin{cases} [1 - \cos(\zeta_p \Delta z)] e^{-j\zeta_p l \Delta z} / (\zeta_p \Delta z)^2 & l = 1, \dots, N_z - 1 \\ (1 - j\zeta_p \Delta z - e^{-j\zeta_p \Delta z}) / (\zeta_p \Delta z)^2 & l = 0 \end{cases} \quad (6.4.4)$$

Integrals along the branch cut can be written as

$$\int_{C_r} g_l(\zeta) f_s^{yy}(\zeta) d\zeta = \int_0^{\infty} g_l(j\alpha) f_s^{yyb}(\rho) \frac{j\rho}{\alpha} d\rho \quad (6.4.5)$$

where

$$g_l(j\alpha) = \begin{cases} \frac{1}{2} [e^{-\alpha(l-1)\Delta z} - 2e^{-\alpha l \Delta z} + e^{-\alpha(l+1)\Delta z}] / (\alpha \Delta z)^2 & l = 1, 2, \dots, N_z - 1 \\ (e^{-\alpha \Delta z} + \alpha \Delta z - 1) / (\alpha \Delta z)^2 & l = 0 \end{cases} \quad (6.4.6)$$

$$f_s^{yb}(\rho) = jC_h(\rho) \frac{1 - \cos(q_2 \Delta x)}{(q_2 \Delta x)^3} \cos(q_2 s \Delta x) \quad s = 0, 1, \dots, 2N_x - 1 \quad (6.4.7)$$

with

$$C_h(\rho) = \frac{2\rho q_2}{q_2^2 \cos^2(q_2 t) + \rho^2 \sin^2(q_2 t)} \quad (6.4.8)$$

and

$$\alpha = \sqrt{\rho^2 - k_1^2} \quad (6.4.9)$$

$$q_2 = \sqrt{k_2^2 - k_1^2 + \rho^2} \quad (6.4.10)$$

Let's discuss the behavior of the integrand in (6.4.5). When $\rho \rightarrow \infty$,

$$C_h(\rho) \rightarrow 2 \quad (6.4.11)$$

$$g_l(j\alpha) \rightarrow \begin{cases} 1/\rho & l = 0 \\ 1/\rho^2 & l = 1 \\ e^{-c\rho \Delta x} / \rho^2 & \text{otherwise} \end{cases} \quad (6.4.12)$$

where $c \geq 1$. Thus,

$$g_l(j\alpha) f_s^{yb}(\rho) \rightarrow \begin{cases} C_h(\rho) / \rho^4 & l = 0 \\ C_h(\rho) / \rho^5 & l = 1 \\ C_h(\rho) e^{-c\rho \Delta x} / \rho^5 & \text{otherwise} \end{cases} \quad (6.4.13)$$

It is observed that the integrands of the integral along the branch cuts and along real axis go to zero with a same rate (at least $1/\rho^4$ or $1/\zeta^4$) when the integral variable is large.

SPECIAL CASE: $Im\{k_1\} = 0$

If region 1 is lossless, then $k_{1i} = 0$ and $k_1 = k_{1r}$. The equations defining the branch cut become

$$\zeta_r \zeta_i = 0 \quad \zeta_r^2 - \zeta_i^2 < k_1^2 \quad (6.4.14)$$

Thus, the branch cuts are decomposed to include a contour on the real ζ axis and a contour on the imaginary ζ axis as shown in Figure 6.3. By the same argument as given in Chapter 3, p_1 should be chosen as positive pure imaginary along the right and lower parts and negative pure imaginary along the upper and left parts.

A. Integration along the Branch Cut Directly

If the integration is along the branch cut directly, then

$$\int_{C_1} g_l(\zeta) f_s^{yy}(\zeta) d\zeta = \int_{-k_1}^0 g_l(\zeta) f_s^{yyr}(\zeta) d\zeta + \int_0^{\infty} g_l(j\alpha) f_s^{yyi}(\alpha) j d\alpha \quad (6.4.15)$$

In the first term on the right side of above equation,

$$f_s^{yyr}(\zeta) = jC_h(\zeta) \frac{1 - \cos(q_2 \Delta x)}{(q_2 \Delta x)^3} \cos(q_2 s \Delta x) \quad s = 0, 1, \dots, 2N_x - 1 \quad (6.4.16)$$

with

$$C_h(\zeta) = \frac{2q_1 q_2}{q_2^2 \cos^2(q_2 t) + q_1^2 \sin^2(q_2 t)} \quad (6.4.17)$$

and

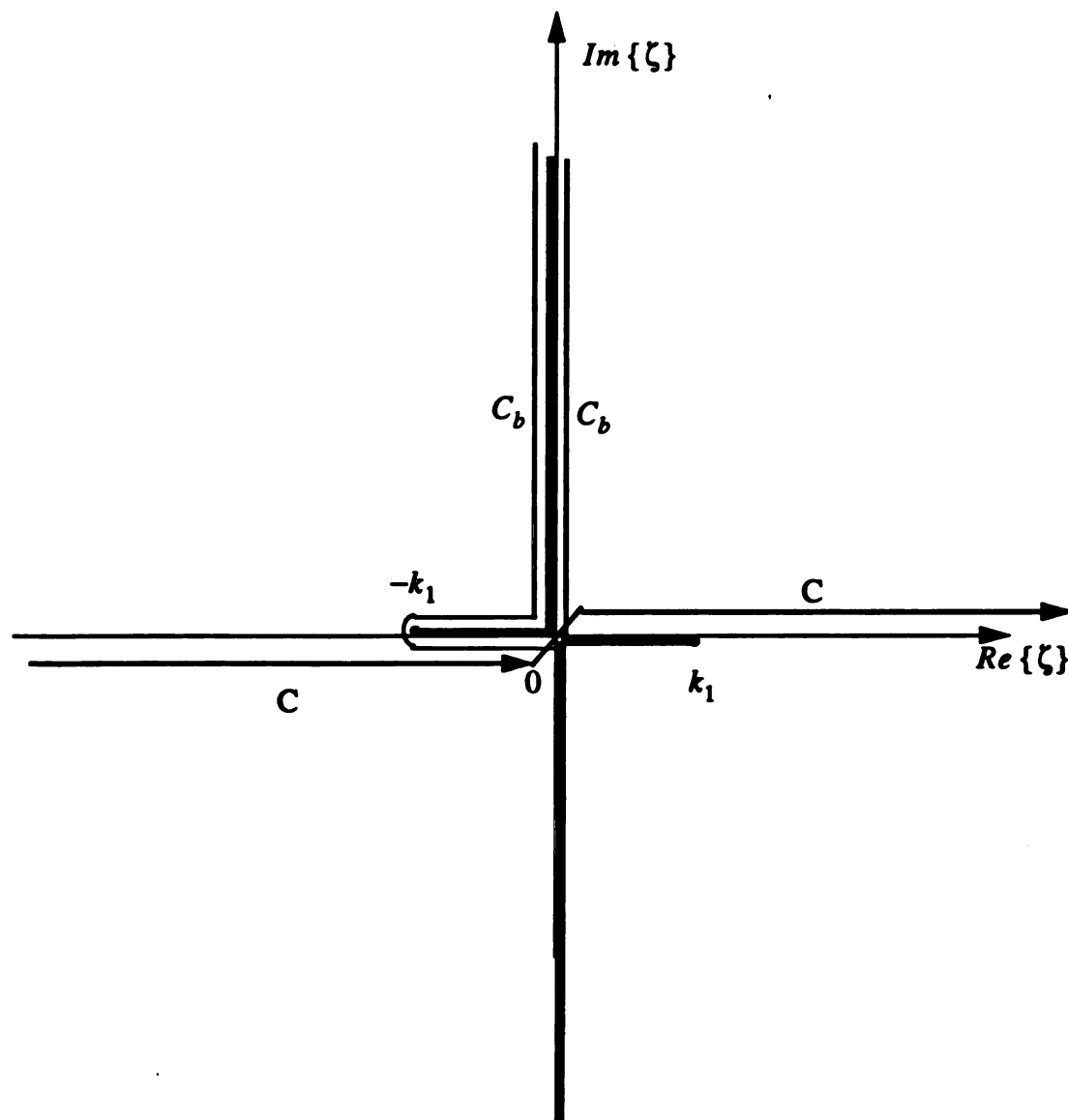


Figure 6.3. The path of integration along the branch cut on the top Riemann sheet when region 1 is lossless.

$$q_1 = \sqrt{k_1^2 - \zeta^2} \quad (6.4.18)$$

$$q_2 = \sqrt{k_2^2 - \zeta^2} \quad (6.4.19)$$

In the second term, ζ has been changed to $j\alpha$, and where

$$f_s^{yyi}(\alpha) = jC_h(\alpha) \frac{1 - \cos(q_2 \Delta x)}{(q_2 \Delta x)^3} \cos(q_2 s \Delta x) \quad s = 0, 1, \dots, 2N_x - 1 \quad (6.4.20)$$

with

$$C_h(\alpha) = \frac{2q_1 q_2}{q_2^2 \cos^2(q_2 t) + q_1^2 \sin^2(q_2 t)} \quad (6.4.21)$$

and

$$q_1 = \sqrt{k_1^2 + \alpha^2} \quad (6.4.22)$$

$$q_2 = \sqrt{k_2^2 + \alpha^2} \quad (6.4.23)$$

B. Integration along the Branch Cut with a Variable Change

Let's change the variables in the integrals in eq. (6.4.15). In the first term ($-k_1 < \zeta < 0$)

$$\zeta = -\sqrt{k_1^2 - \rho^2} \quad (0 < \rho < k_1) \quad (6.4.24)$$

and

$$\alpha = \sqrt{\rho^2 - k_1^2} \quad (k_1 < \rho < \infty) \quad (6.4.25)$$

in the second term ($0 < \alpha < \infty$).

Substituting (6.4.24) and (6.4.25) into (6.4.15) leads to

$$\int_{C_1} g_l(\zeta) f_s^{yb}(\zeta) d\zeta = \int_0^{\infty} g_l(j\alpha) f_s^{yb}(\rho) \frac{j\rho}{\alpha} d\rho \quad (6.4.26)$$

where $g_l(j\alpha)$ and $f_s^{yb}(\rho)$ are given in (6.4.6) and (6.4.7), respectively, and

$$\alpha = \sqrt{\rho^2 - k_1^2} = \begin{cases} \sqrt{\rho^2 - k_1^2} & k_1 < \rho < \infty \\ j\sqrt{k_1^2 - \rho^2} & 0 < \rho < k_1 \end{cases} \quad (6.4.27)$$

The result in (6.4.26) is the same as that in (6.4.5). But (6.4.5) is the general case, and (6.4.26) is derived under the condition that region 1 is lossless.

Note that the integrand of (6.4.26) has a singularity at $\rho = k_1$. The integral is rewritten as the sum of three terms

$$\int_0^{\infty} g_l(j\alpha) f_s^{yb}(\rho) \frac{j\rho}{\alpha} d\rho = \left\{ \int_0^{k_1-\delta} + \int_{k_1-\delta}^{k_1+\delta} + \int_{k_1+\delta}^{\infty} \right\} g_l(j\alpha) f_s^{yb}(\rho) \frac{j\rho}{\alpha} d\rho \quad (6.4.28)$$

Since $\alpha = \sqrt{\rho^2 - k_1^2}$, the second term is integrable. Some manipulations lead to

$$\begin{aligned} & \int_0^{\infty} g_l(j\alpha) f_s^{yb}(\rho) \frac{j\rho}{\alpha} d\rho \\ &= \frac{1-j}{2} f_s^{yb}(k_1) \sqrt{2\delta k_1} + \left\{ \int_0^{k_1-\delta} + \int_{k_1+\delta}^{\infty} \right\} g_l(j\alpha) f_s^{yb}(\rho) \frac{j\rho}{\alpha} d\rho \end{aligned} \quad (6.4.29)$$

The accuracy of this approximation depends upon δ . The smaller δ is, the more accurate the integral, but it requires greater computation time.

6.5. COMPARISON OF THE INTEGRATION METHODS

In the last two sections, several integration methods have been introduced to compute the inverse Fourier transform. The total power with surface wave incidence will be computed using each of these integration methods to compare these integration methods.

There is a gap in a grounded dielectric sheet as shown Figure 6.4. A TE_1 surface wave is incident upon the discontinuity from right to left. Then, a transmitted TE_1 surface wave will propagate to the left, a reflected TE_1 surface wave will propagate to the right, and radiated waves will be excited in space. It is assumed that both regions are lossless ($\epsilon_{1r} = 1$ and $\epsilon_{2r} = 4$) and the thickness of the gap is a constant ($t/\lambda_0 = 0.25$, only the TE_1 surface-wave mode exists, where λ_0 is the wavelength in free space). Thus, the total of the transmitted power, reflected power and radiated power should be equal to the incident power and should not depend upon the width $2a$ of the gap.

The Method of Moments is applied to obtain numerical results for this problem. First, the moment matrix is filled by using the above integration methods, and the linear equations are solved for the total electric field in the discontinuity. Then, the reflected and transmitted surface wave power and the radiated power are computed by the formulas derived in Section 6.2. Finally, the total power is compared with the incident power.

The numerical results of the total power vs. the width $2a/\lambda_0$ of the gap are shown in Figure 6.5 with three integration methods. The integration along the real axis is introduced in section 6.3 with $\delta/k_0 = 10^{-3}$. The methods of integration along the branch cut are introduced in the last section, and $\delta/k_0 = 10^{-4}$ and $\delta/k_0 = 10^{-5}$ in the method with a variable change. It is observed from the figure that the relative error of numerical results is increased when the width is increased for all three methods.

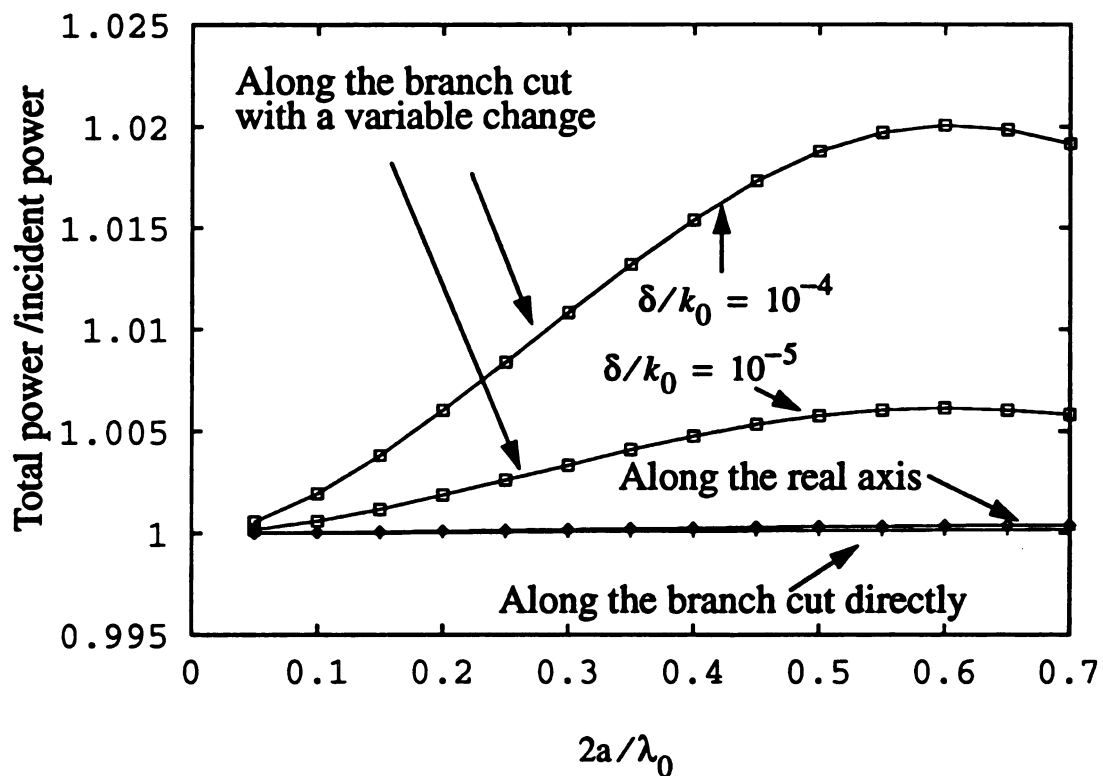


Figure 6.5. The numerical total power normalized by the incident power with TE₁ surface wave incidence vs. the width of the discontinuity. It is computed by three integration methods: along the real axis, along the branch cut with/without a variable change. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $t/\lambda_0 = 0.25$, $N_x = 13$ and $N_z = 4$ to 17)

The relative computation times of these methods are

Integration along the real axis	100%
Integration along the branch cut directly	27%
Integration along the branch cut with a variable change ($\delta/k_0 = 10^{-4}$)	50%
Integration along the branch cut with a variable change ($\delta/k_0 = 10^{-5}$)	105%

Comparing the curves in Figure 6.5 and above results, it is observed that the integration along the branch cut directly is the best in the three methods. It is also found that the sum of the transmitted power, reflected power and radiated power is almost equal to the incident power (the relative difference between the incident power and the total power computed by integration along the branch cut directly is less than 0.02%).

Convergence of the MoM solution is checked by comparing results when the partition numbers N_x and N_z of the discontinuity region are increased. The results are presented in Figure 6.6, where the convergence is evident.

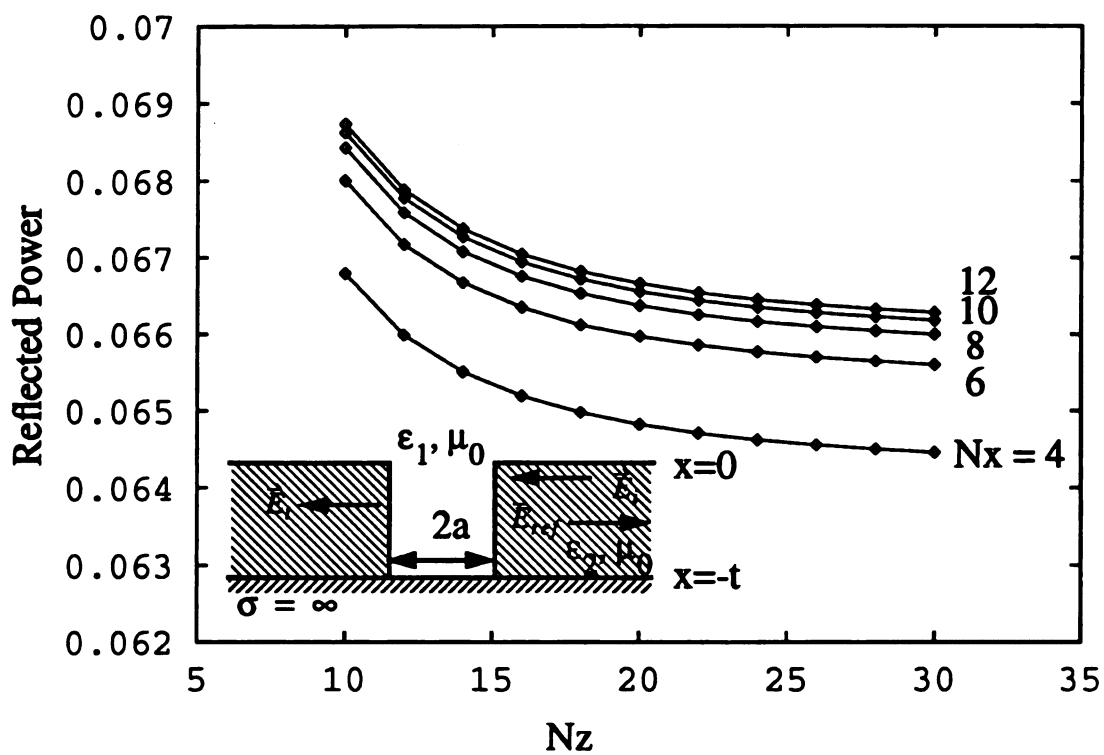


Figure 6.6. Convergence check on the reflected power computed with different N_x and N_z when a TE_1 surface wave is incident upon an air gap in the grounded sheet. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 2.1316$, $2a/\lambda_0 = 1$, and $k_0 t = 2.5$)

CHAPTER 7

PULSE GALERKIN'S SOLUTIONS FOR TM-POLARIZED WAVE INCIDENCE

7.1. INTRODUCTION

In Chapter 4, the Method of Moments (MoM) is applied to transform the electric field integral equation (EFIE) into matrix equations for general basis and testing functions. Here, a pulse Galerkin's implementation is applied to establish the matrix elements $A_{mn}^{\alpha\beta}$ and the forcing vectors $F_{\alpha m}$ for TM-polarized wave incidence. First, formulas to calculate the matrix elements and forcing term will be derived when both basis and testing functions are 2-D pulse functions. Then some numerical integral techniques in the contour integrations along real axis and along the branch cuts will be discussed. Finally, these methods will be compared with each other by checking their convergencies.

7.2. PULSE GALERKIN'S SOLUTIONS

It is assumed that the cross section of the discontinuity region is rectangular ($|z| \leq a$, $-t \leq x \leq 0$) (Figure 6.1) and the permittivity ϵ_d is a constant. For TM-polarized wave incidence, the EFIE in the discontinuity region becomes

$$\vec{E}_2(x, z) - j \frac{k_0 \Delta n^2}{\eta_0} \int_{-a}^a \int_{-t}^0 \vec{G}_{22}(x|x', z-z') \cdot \vec{E}_2(x', z') dx' dz' = \vec{E}_2^i(x, z) \quad (7.2.1)$$

for $|z| \leq a, -t \leq x \leq 0$

where

$$\Delta n^2 = n_d^2 - n_2^2 = (\epsilon_d - \epsilon_2) / \epsilon_0 \quad (7.2.2)$$

The discontinuity region is partitioned into $N = N_x N_z$ equal rectangular elements ΔS_n centered at (x_n, z_n) , where N_x, N_z are total numbers of partitions along $\hat{x}$ and $\hat{z}$, respectively. The partitioning is defined such that

$$\int_{-a}^a \int_{-t}^0 f(x, z) dx dz = \sum_{n=1}^N \int_{\Delta S_n} f(x, z) dx dz \quad (7.2.3)$$

From Figure 6.1, it is found that

$$\Delta S_n \Leftrightarrow |x - x_n| < \Delta x / 2, |z - z_n| < \Delta z / 2 \quad (7.2.4)$$

$$x_n = -t + \Delta x / 2 + [L_x(n) - 1] \Delta x \quad (7.2.5)$$

$$z_n = -a + \Delta z / 2 + [L_z(n) - 1] \Delta z \quad (7.2.6)$$

where

$$L_x(n) = 1 + INT \{ (n - 1) / N_z \} \quad (7.2.7)$$

$$L_z(n) = n - INT \{ (n - 1) / N_z \} N_z \quad (7.2.8)$$

and $INT \{ \dots \}$ designates the integer part of the quantity within the braces. Δx and Δz have the same definitions as in Section 6.2.

Let's apply pulse Galerkin's method to the EFIE. The basis functions $e_{\alpha n}(x, z)$ and the testing functions $t_{\alpha n}(x, z)$ are chosen to be 2-D pulse functions defined as

$$P_n(x, z) = \begin{cases} 1 & (x, z) \in \Delta S_n \\ 0 & \text{otherwise} \end{cases} \quad (7.2.9)$$

Substituting the expressions for the basis functions and the testing functions in the formula for $A_{mn}^{\alpha\beta}$ and $F_{\alpha n}$ [(4.5.5) and (4.5.6)] gives

$$F_{\alpha m} = \frac{1}{\Delta x \Delta z} \int_{\Delta S_m} E_{2\alpha}^i(x, z) dx dz \quad (7.2.10)$$

$$A_{mn}^{\alpha\beta} = \delta_{\alpha\beta} \delta_{mn} - j \frac{k_0 \Delta n^2}{\eta_0 \Delta x \Delta z} \int_{\Delta S_m} dx dz \int_{\Delta S_n} G_{22\alpha\beta}(x|x', z-z') dx' dz' \quad (7.2.11)$$

Note that $A_{mn}^{\alpha\beta}/(\Delta x \Delta z)$ and $F_{\alpha m}/(\Delta x \Delta z)$ are newly defined as $A_{mn}^{\alpha\beta}$ and $F_{\alpha m}$, respectively, and $E_{2\alpha}^i(x, z)$, $G_{22\alpha\beta}(x|x', z-z')$ are given in Chapters 2 to 4.

Let's evaluate the above spatial integrals required in spectral integral representation of MoM matrix elements. Note that ΔS_m and ΔS_n are defined by (7.2.4). Substituting $\bar{G}_{22}(x|x', z-z') = \hat{x}\hat{x}(\hat{\rho} - \hat{\rho}')j\eta_2/k_2 + \bar{G}_{22}^R(x|x', z-z')$ in (7.2.11), where $\bar{G}_{22}^R(x|x', z-z')$ is given in (3.2.3), and using

$$\int_{z_m - \Delta z/2}^{z_m + \Delta z/2} dz \int_{z_n - \Delta z/2}^{z_n + \Delta z/2} e^{j\zeta|z-z'|} dz' = \begin{cases} 2[1 - \cos(\zeta\Delta z)] e^{j\zeta|z_m - z_n|}/\zeta^2 & z_m \neq z_n \\ 2(1 + j\zeta\Delta z - e^{j\zeta\Delta z})/\zeta^2 & z_m = z_n \end{cases} \quad (7.2.12)$$

$$\begin{aligned} & \int_{z_m - \Delta z/2}^{z_m + \Delta z/2} dz \int_{z_n - \Delta z/2}^{z_n + \Delta z/2} \text{sgn}(z-z') e^{j\zeta|z-z'|} dz' \\ &= 2 \text{sgn}(z_m - z_n) [1 - \cos(\zeta\Delta z)] e^{j\zeta|z_m - z_n|}/\zeta^2 \end{aligned} \quad (7.2.13)$$

$$\int_{x_m - \Delta x/2}^{x_m + \Delta x/2} dx \int_{x_n - \Delta x/2}^{x_n + \Delta x/2} e^{-p_2|x-x'|} dx' = \begin{cases} 2[\cosh(p_2\Delta x) - 1] e^{-p_2|x_m - x_n|}/p_2^2 & x_m \neq x_n \\ 2(e^{-p_2\Delta x} - 1 + p_2\Delta x)/p_2^2 & x_m = x_n \end{cases} \quad (7.2.14)$$

$$\begin{aligned} & \int_{x_m - \Delta x/2}^{x_m + \Delta x/2} dx \int_{x_n - \Delta x/2}^{x_n + \Delta x/2} \text{sgn}(x-x') e^{-p_2|x-x'|} dx' \\ &= 2 \text{sgn}(x_m - x_n) [\cosh(p_2\Delta x) - 1] e^{-p_2|x_m - x_n|}/p_2^2 \end{aligned} \quad (7.2.15)$$

$$\int_{x_m - \Delta x/2}^{x_m + \Delta x/2} e^{-p_2 x} dx = 2 \sinh(p_2 \Delta x/2) e^{-p_2 x_m} / p_2 \quad (7.2.16)$$

with

$$\operatorname{sgn}(z_m - z_n) = \begin{cases} 1 & z_m > z_n \\ 0 & z_m = z_n \\ -1 & z_m < z_n \end{cases} \quad (7.2.17)$$

yield

$$\begin{aligned} A_{mn}^{xx} = & \left(1 + \frac{\varepsilon_0}{\varepsilon_2} \Delta n^2\right) \delta_{mn} + \frac{\varepsilon_0 \Delta x \Delta z}{\varepsilon_2 \pi} \Delta n^2 \int_{-\infty}^{\infty} \frac{\zeta^2 g_{mn}(\zeta) d\zeta}{p_2} \{-\phi_{mn}(\zeta) \\ & + \frac{\cosh(p_2 \Delta x) - 1}{(p_2 \Delta x)^2} \left\{ -e^{-p_2(2t+x_m+x_n)} + \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_e} e^{-p_2 t} [\cosh[p_2(x_m - x_n)] + \cosh[p_2(2t+x_m+x_n)]] \right\} \} \end{aligned} \quad (7.2.18)$$

$$\begin{aligned} A_{mn}^{zz} = & \delta_{mn} + \frac{\varepsilon_0 \Delta x \Delta z}{\varepsilon_2 \pi} \Delta n^2 \int_{-\infty}^{\infty} p_2 g_{mn}(\zeta) d\zeta \left\{ \phi_{mn}(\zeta) + \frac{\cosh(p_2 \Delta x) - 1}{(p_2 \Delta x)^2} \{-e^{-p_2(2t+x_m+x_n)} \right. \\ & \left. + \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_e} e^{-p_2 t} [-\cosh[p_2(x_m - x_n)] + \cosh[p_2(2t+x_m+x_n)]] \} \right\} \end{aligned} \quad (7.2.19)$$

$$\begin{aligned} A_{mn}^{xz} = & \frac{\varepsilon_0 \Delta x \Delta z}{\varepsilon_2 \pi} \Delta n^2 \operatorname{sgn}(z_m - z_n) \int_{-\infty}^{\infty} j \zeta g_{mn}(\zeta) d\zeta \left\{ \operatorname{sgn}(x_m - x_n) \phi_{mn}(\zeta) \right. \\ & \left. + \frac{\cosh(p_2 \Delta x) - 1}{(p_2 \Delta x)^2} \left\{ -e^{-p_2(2t+x_m+x_n)} - \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_e} e^{-p_2 t} [-\sinh[p_2(x_m - x_n)] + \sinh[p_2(2t+x_m+x_n)]] \right\} \right\} \end{aligned} \quad (7.2.20)$$

$$A_{zx}^{mn} = A_{xz}^{nm} \quad (7.2.21)$$

where

$$g_{mn}(\zeta) = \begin{cases} [1 - \cos(\zeta \Delta z)] e^{j\zeta|z_m - z_n|} / (\zeta \Delta z)^2 & z_m \neq z_n \\ (1 + j\zeta \Delta z - e^{j\zeta \Delta z}) / (\zeta \Delta z)^2 & z_m = z_n \end{cases} \quad (7.2.22)$$

$$\phi_{mn}(\zeta) = \begin{cases} [\cosh(p_2 \Delta x) - 1] e^{-p_2|x_m - x_n|} / (p_2 \Delta x)^2 & x_m \neq x_n \\ (e^{-p_2 \Delta x} - 1 + p_2 \Delta x) / (p_2 \Delta x)^2 & x_m = x_n \end{cases} \quad (7.2.23)$$

It is noted that the integrands in (7.2.18-20) depend upon functions of $|x_m - x_n|$, $2t + x_m + x_n$, and $|z_m - z_n|$. Thus, the matrix elements can be rewritten as:

$$A_{mn}^{xx} = \left(1 + \frac{\epsilon_0}{\epsilon_2} \Delta n^2\right) \delta_{mn} + \frac{\epsilon_0 \Delta z}{\epsilon_2 \pi} \Delta n^2 \left[B^{xx}\left(\frac{|z_m - z_n|}{\Delta z}, \frac{|x_m - x_n|}{\Delta x}\right) + B^{xx}\left(\frac{|z_m - z_n|}{\Delta z}, \frac{2t + x_m + x_n}{\Delta x}\right) \right] \quad (7.2.24)$$

$$A_{mn}^{zz} = \delta_{mn} + \frac{\epsilon_0 \Delta z}{\epsilon_2 \pi} \Delta n^2 \left[B^{zz}\left(\frac{|z_m - z_n|}{\Delta z}, \frac{|x_m - x_n|}{\Delta x}\right) - B^{zz}\left(\frac{|z_m - z_n|}{\Delta z}, \frac{2t + x_m + x_n}{\Delta x}\right) \right] \quad (7.2.25)$$

$$A_{mn}^{xz} = \operatorname{sgn}(z_m - z_n) \frac{\epsilon_0 \Delta z}{\epsilon_2 \pi} \Delta n^2 \left[-\operatorname{sgn}(x_m - x_n) B^{xz}\left(\frac{|z_m - z_n|}{\Delta z}, \frac{|x_m - x_n|}{\Delta x}\right) + B^{xz}\left(\frac{|z_m - z_n|}{\Delta z}, \frac{2t + x_m + x_n}{\Delta x}\right) \right] \quad (7.2.26)$$

with

$$B^{xz}(l, 0) = B^{xz}(0, s) = 0 \quad (7.2.27)$$

and where

$$B^{\alpha\beta}(l, s) = \int_{-\infty}^{\infty} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta \quad (7.2.28)$$

with

$$g_l(\zeta) = \begin{cases} [1 - \cos(\zeta\Delta z)] e^{j\zeta l\Delta z} / (\zeta\Delta z)^2 & l = 1, \dots, N_z - 1 \\ (1 + j\zeta\Delta z - e^{j\zeta\Delta z}) / (\zeta\Delta z)^2 & l = 0 \end{cases} \quad (7.2.29)$$

$$f_0^{xx}(\zeta) = -\frac{\zeta^2}{p_2^2} + \frac{\zeta^2}{p_2^2} \frac{1}{p_2\Delta x} \left\{ 1 - e^{-p_2\Delta x} + \frac{p_1 - p_2\varepsilon_1/\varepsilon_2}{\bar{\Delta}_e} e^{-p_2t} [\cosh(p_2\Delta x) - 1] \right\} \quad (7.2.30)$$

$$f_0^{zz}(\zeta) = 1 + \frac{1}{p_2\Delta x} \left\{ -1 + e^{-p_2\Delta x} - \frac{p_1 - p_2\varepsilon_1/\varepsilon_2}{\bar{\Delta}_e} e^{-p_2t} [\cosh(p_2\Delta x) - 1] \right\} \quad (7.2.31)$$

$$f_0^{xz}(\zeta) = 0 \quad (7.2.32)$$

$$f_s^{xx}(\zeta) = \frac{\zeta^2}{p_2^2} \frac{\cosh(p_2\Delta x) - 1}{p_2\Delta x} \left\{ -e^{-p_2s\Delta x} + \frac{p_1 - p_2\varepsilon_1/\varepsilon_2}{\bar{\Delta}_e} e^{-p_2t} \cosh(p_2s\Delta x) \right\} \quad (7.2.33)$$

$$f_s^{zz}(\zeta) = \frac{\cosh(p_2\Delta x) - 1}{p_2\Delta x} \left\{ e^{-p_2s\Delta x} - \frac{p_1 - p_2\varepsilon_1/\varepsilon_2}{\bar{\Delta}_e} e^{-p_2t} \cosh(p_2s\Delta x) \right\} \quad (7.2.34)$$

$$f_s^{xz}(\zeta) = \frac{j\zeta}{p_2} \frac{\cosh(p_2\Delta x) - 1}{p_2\Delta x} \left\{ -e^{-p_2s\Delta x} - \frac{p_1 - p_2\varepsilon_1/\varepsilon_2}{\bar{\Delta}_e} e^{-p_2t} \sinh(p_2s\Delta x) \right\} \quad (7.2.35)$$

for $s = 1, 2, \dots, 2N_x - 1$.

It is observed that when $\zeta \rightarrow \pm\infty$, $f_0^{xx}(\zeta) \rightarrow -1$, and $f_0^{zz}(\zeta) \rightarrow 1$. Using [67]

$$\int_0^\infty \frac{\sin^2(ax)}{x^2} \cos(2bx) dx = \begin{cases} (a-b)\pi/2 & b < a \\ 0 & b > a \end{cases} \quad (7.2.36)$$

yields

$$\int_{-\infty}^\infty g_l(\zeta) d\zeta = 2 \int_0^\infty \frac{1 - \cos(\zeta\Delta z)}{(\zeta\Delta z)^2} \cos(\zeta l\Delta z) d\zeta = \begin{cases} \pi/\Delta z & l = 0 \\ 0 & l \neq 0 \end{cases} \quad (7.2.37)$$

SUMMARY:

Rearranging the matrix equations leads to

$$\sum_{\beta=x,z} \sum_{n=1}^N A_{mn}^{\alpha\beta} a_{\beta n} = F_{\alpha m} \quad \alpha = x, z \quad m = 1, 2, \dots, N \quad (7.2.38)$$

$$A_{mn}^{xx} = \delta_{mn} + \frac{\epsilon_0 \Delta z}{\epsilon_2 \pi} \Delta n^2 \left[B^{xx} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{|x_m - x_n|}{\Delta x} \right) + B^{xx} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{2t + x_m + x_n}{\Delta x} \right) \right] \quad (7.2.39)$$

$$A_{mn}^{zz} = \left(1 + \frac{\epsilon_0}{\epsilon_2} \Delta n^2 \right) \delta_{mn} + \frac{\epsilon_0 \Delta z}{\epsilon_2 \pi} \Delta n^2 \left[B^{zz} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{|x_m - x_n|}{\Delta x} \right) - B^{zz} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{2t + x_m + x_n}{\Delta x} \right) \right] \quad (7.2.40)$$

$$A_{mn}^{xz} = \operatorname{sgn}(z_m - z_n) \frac{\epsilon_0 \Delta z}{\epsilon_2 \pi} \Delta n^2 \left[-\operatorname{sgn}(x_m - x_n) B^{xz} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{|x_m - x_n|}{\Delta x} \right) + B^{xz} \left(\frac{|z_m - z_n|}{\Delta z}, \frac{2t + x_m + x_n}{\Delta x} \right) \right] \quad (7.2.41)$$

$$A_{zx}^{mn} = A_{xz}^{nm} \quad (7.2.42)$$

with

$$B^{xz}(l, 0) = B^{xz}(0, s) = 0 \quad (7.2.43)$$

and where

$$B^{\alpha\beta}(l, s) = \int_{-\infty}^{\infty} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta \quad (7.2.44)$$

with

$$g_l(\zeta) = \begin{cases} [1 - \cos(\zeta \Delta z)] e^{j\zeta l \Delta z} / (\zeta \Delta z)^2 & l = 1, \dots, N_z - 1 \\ (1 + j\zeta \Delta z - e^{j\zeta \Delta z}) / (\zeta \Delta z)^2 & l = 0 \end{cases} \quad (7.2.45)$$

$$f_0^{xx}(\zeta) = -\frac{k_2^2}{p_2^2} + \frac{\zeta^2}{p_2^2 p_2 \Delta x} \left\{ 1 - e^{-p_2 \Delta x} + \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} e^{-p_2 t} [\cosh(p_2 \Delta x) - 1] \right\} \quad (7.2.46)$$

$$f_0^{zz}(\zeta) = \frac{1}{p_2 \Delta x} \left\{ -1 + e^{-p_2 \Delta x} - \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} e^{-p_2 t} [\cosh(p_2 \Delta x) - 1] \right\} \quad (7.2.47)$$

$$f_0^{xz}(\zeta) = 0 \quad (7.2.48)$$

$$f_s^{xx}(\zeta) = \frac{\zeta^2 \cosh(p_2 \Delta x) - 1}{p_2^2 p_2 \Delta x} \left\{ -e^{-p_2 s \Delta x} + \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} e^{-p_2 t} \cosh(p_2 s \Delta x) \right\} \quad (7.2.49)$$

$$f_s^{zz}(\zeta) = \frac{\cosh(p_2 \Delta x) - 1}{p_2 \Delta x} \left\{ e^{-p_2 s \Delta x} - \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} e^{-p_2 t} \cosh(p_2 s \Delta x) \right\} \quad (7.2.50)$$

$$f_s^{xz}(\zeta) = \frac{j\zeta \cosh(p_2 \Delta x) - 1}{p_2 p_2 \Delta x} \left\{ -e^{-p_2 s \Delta x} - \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} e^{-p_2 t} \sinh(p_2 s \Delta x) \right\} \quad (7.2.51)$$

for $s = 1, 2, \dots, 2N_x - 1$. Note that matrices B^{xx} and B^{zz} , and functions $f_0^{xx}(\zeta)$ and $f_0^{zz}(\zeta)$ have been redefined.

The forcing vector F_{am} is computed in the same way to be

$$\begin{aligned} \hat{x}F_{xm} + 2F_{zm} = E_0 & \frac{\hat{x} \sin \theta_r \cos[k_2(t + x_m) \cos \theta_r] - 2j \cos \theta_r \sin[k_2(t + x_m) \cos \theta_r]}{\eta_1 \cos \theta_i \cos[k_2 t \cos \theta_r] + j \eta_2 \cos \theta_r \sin[k_2 t \cos \theta_r]} \\ & \frac{\sin(\cos \theta_r k_2 \Delta x / 2) \sin(\sin \theta_i k_1 \Delta z / 2)}{\cos \theta_r k_2 \Delta x / 2 \sin \theta_i k_1 \Delta z / 2} 2\eta_2 \cos \theta_i e^{jk_1 z_m \sin \theta_i} \end{aligned} \quad (7.2.52)$$

for TM-polarized plane wave incidence, and

$$\begin{aligned} \hat{x}F_{xm} + 2F_{zm} = E_0 & \{ \hat{x} j \zeta_p \cos[q_2(t + x_m)] + 2q_2 \sin[q_2(t + x_m)] \} \\ & \frac{\sin(q_2 \Delta x / 2) \sin(\zeta_p \Delta z / 2)}{q_2 \Delta x / 2 \zeta_p \Delta z / 2} \frac{e^{j\zeta_p z_m}}{q_2 \sin(q_2 t)} \end{aligned} \quad (7.2.53)$$

for TM-even surface-wave mode incidence.

Since the unknown electric field in the discontinuity region is expanded into a set of 2-D pulse functions, then the amplitudes of the excited surface wave and radiation wave are obtained by substituting the pulse functions in (4.5.15) and (4.5.16).

$$A_e^\pm = \frac{\epsilon_0 4p_1 \Delta n^2 \sin(q_2 \Delta x/2) \sin(\zeta_p \Delta z/2)}{\epsilon_2 \zeta_p q_2 \tilde{\Delta}'_e(\zeta_p)} \sum_{n=1}^N \{ \pm \zeta_p \cos[q_2(t+x_n)] a_{xn} - j q_2 \sin[q_2(t+x_n)] a_{zn} \} e^{\pm j \zeta_p z_n} \quad (7.2.54)$$

$$B_e(\theta) = \frac{\epsilon_0 2j \Delta n^2 k_1 \cos \theta \sin(q_2 \Delta x/2) \sin(\sin \theta k_1 \Delta z/2)}{\epsilon_2 \pi q_2 \tilde{\Delta}_e(-k_1 \sin \theta)} \sum_{n=1}^N \{ j k_1 \sin \theta \cos[q_2(t+x_n)] a_{xn} + q_2 \sin[q_2(t+x_n)] a_{zn} \} e^{j k_1 z_n \sin \theta} \quad (7.2.55)$$

where $\tilde{\Delta}'_e(\zeta_p)$ and $\tilde{\Delta}_e(-k_1 \sin \theta)$ are given in Chapter 4.

Equations (7.2.39-42) allow a dramatic reduction in the amount of effort needed to fill the matrix. There are $4N^2 = 4N_x^2 N_z^2$ matrix elements, and $2N^2$ matrix elements should be calculated by integrations because $A_{mn}^{\alpha\beta} = A_{nm}^{\beta\alpha}$. But applying equations (7.2.39-42), only about $6N_x N_z$ integrations are needed to compute. Evidently, much computing time can be conserved. For example, if $N_x = N_z = 10$, then $2N^2 = 20000$, but $6N_x N_z = 600$.

7.3. NUMERICAL INTEGRATION ALONG REAL AXIS

It is noted that $f_s^{xx}(\zeta)$ and $f_s^{zz}(\zeta)$ in (7.2.44) are even functions of ζ , $f_s^{xz}(\zeta)$ is an odd function of ζ , and $g_l(\zeta)$ can be written as the sum of an even function and an odd function of ζ , i.e.

$$g_l(\zeta) = \begin{cases} [1 - \cos(\zeta\Delta z)] [\cos(l\zeta\Delta z) + j\sin(l\zeta\Delta z)] / (\zeta\Delta z)^2 & l = 1, \dots, N_z - 1 \\ \{ [1 - \cos(\zeta\Delta z)] + j[\zeta\Delta z - \sin(\zeta\Delta z)] \} / (\zeta\Delta z)^2 & l = 0 \end{cases} \quad (7.3.1)$$

Since the integral from $-\infty$ to ∞ in (7.2.44) is symmetric, then it can be rewritten as

$$B^{xx}(l, s) = \int_0^{\infty} g_l^e(\zeta) f_s^{xx}(\zeta) d\zeta \quad (7.3.2)$$

$$B^{zz}(l, s) = \int_0^{\infty} g_l^e(\zeta) f_s^{zz}(\zeta) d\zeta \quad (7.3.3)$$

$$B^{xz}(l, s) = \int_0^{\infty} g_l^o(\zeta) f_s^{xz}(\zeta) d\zeta \quad (7.3.4)$$

where

$$g_l^e(\zeta) = 2 \cos(l\zeta\Delta z) [1 - \cos(\zeta\Delta z)] / (\zeta\Delta z)^2 \quad l = 0, 1, \dots, N_z - 1 \quad (7.3.5)$$

$$g_l^o(\zeta) = 2j \sin(l\zeta\Delta z) [1 - \cos(\zeta\Delta z)] / (\zeta\Delta z)^2 \quad l = 1, \dots, N_z - 1 \quad (7.3.6)$$

Since $B^{xz}(l, 0) = B^{xz}(0, s) = 0$, then no definition for $g_0^o(\zeta)$ is necessary.

Since $f_s^{\alpha\beta}(\zeta)$ involves hyperbolic sine and cosine functions, it is prone to numerical difficulty. As the integration variable in (7.3.2) to (7.3.4) increase toward infinitely, both the sinh and cosh functions become unbounded. In addition, it is difficult to ascertain the

convergence properties of the integral. Both of these problems can be overcome if the integrands are written in terms of exponential functions. After some algebraic manipulation, the results become

$$f_0^{xx}(\zeta) = -\frac{k_2^2}{p_2^2} + \frac{\zeta^2}{p_2^2} \frac{1 - e^{-p_2 \Delta x}}{p_2 \Delta x} \left\{ 1 + (1 - e^{-p_2 \Delta x}) \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_{e1}} e^{-p_2 (2t - \Delta x)} \right\} \quad (7.3.7)$$

$$f_0^{zz}(\zeta) = \frac{1 - e^{-p_2 \Delta x}}{p_2 \Delta x} \left\{ -1 - (1 - e^{-p_2 \Delta x}) \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_{e1}} e^{-p_2 (2t - \Delta x)} \right\} \quad (7.3.8)$$

$$f_0^{xz}(\zeta) = 0 \quad (7.3.9)$$

$$f_s^{xx}(\zeta) = \frac{\zeta^2}{p_2^2} \frac{(1 - e^{-p_2 \Delta x})^2}{2p_2 \Delta x} \left\{ -e^{-p_2 (s-1) \Delta x} + \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_{e1}} [e^{-p_2 [2t - (s+1) \Delta x]} + e^{-p_2 [2t + (s-1) \Delta x]}] \right\} \quad (7.3.10)$$

$$f_s^{zz}(\zeta) = \frac{(1 - e^{-p_2 \Delta x})^2}{2p_2 \Delta x} \left\{ e^{-p_2 (s-1) \Delta x} - \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_{e1}} [e^{-p_2 [2t - (s+1) \Delta x]} + e^{-p_2 [2t + (s-1) \Delta x]}] \right\} \quad (7.3.11)$$

$$f_s^{xz}(\zeta) = \frac{j\zeta}{p_2} \frac{(1 - e^{-p_2 \Delta x})^2}{2p_2 \Delta x} \left\{ -e^{-p_2 (s-1) \Delta x} - \frac{p_1 - p_2 \varepsilon_1 / \varepsilon_2}{\bar{\Delta}_{e1}} [e^{-p_2 [2t - (s+1) \Delta x]} - e^{-p_2 [2t + (s-1) \Delta x]}] \right\} \quad (7.3.12)$$

where

$$\bar{\Delta}_{e1} = 2\bar{\Delta}_e e^{-p_2 t} = p_1 (1 + e^{-2p_2 t}) + p_2 (1 - e^{-2p_2 t}) \varepsilon_1 / \varepsilon_2 \quad (7.3.13)$$

CONVERGENCE OF THE MoM MATRIX ELEMENT INTEGRALS

Before undertaking the numerical integrations of equations (7.3.2-4), it is quite helpful to anticipate the rate of convergence of the integrals. Of interest is the behavior of the integrand $g_l(\zeta)f_s^{\alpha\beta}(\zeta)$ as $\zeta \rightarrow \infty$.

$$f_s^{\alpha\beta}(\zeta) \rightarrow \begin{cases} 1/\zeta & s = 0 \text{ for } \alpha\beta = xx, zz \\ s = 1, 2N_x - 1 \text{ for all } \alpha\beta's \\ e^{-c\zeta\Delta x}/\zeta & \text{otherwise} \end{cases} \quad (7.3.14)$$

where $c \geq 1$. Then

$$g_l(\zeta)f_s^{\alpha\beta}(\zeta) \rightarrow \begin{cases} \begin{cases} \cos(l\zeta\Delta z) \\ j\sin(l\zeta\Delta z) \end{cases} [1 - \cos(\zeta\Delta z)]/\zeta^3 & s = 0 \text{ for } \alpha\beta = xx, zz \\ s = 1, 2N_x - 1 \text{ for all } \alpha\beta's \\ \begin{cases} \cos(l\zeta\Delta z) \\ j\sin(l\zeta\Delta z) \end{cases} [1 - \cos(\zeta\Delta z)] e^{-c\zeta\Delta x}/\zeta^3 & \text{otherwise} \end{cases} \quad (7.3.15)$$

Since there are some periodic functions in the integrand $g_l(\zeta)f_s^{\alpha\beta}(\zeta)$, it is integrated one by one period when ζ is large in the program. Using

$$\int \frac{\cos x}{x^m} dx = \frac{\sin x}{x^m} + m \int \frac{\sin x}{x^{m+1}} dx \quad (7.3.16)$$

yields

$$\int_{x_0}^{x_0+2\pi} \frac{\cos x}{x^m} dx = \frac{\sin x_0}{x_0^m} \left[\left(\frac{x_0}{x_0+2\pi} \right)^m - 1 \right] + m \int_{x_0}^{x_0+2\pi} \frac{\sin x}{x^{m+1}} dx \quad (7.3.17)$$

When $x_0 \rightarrow \infty$,

$$\left(\frac{x_0}{x_0+2\pi} \right)^m - 1 \approx -\frac{2m\pi}{x_0} \quad (7.3.18)$$

Thus, the right side of (7.3.17) will go to zero at least as $1/x_0^{m+1}$. There is a similar result

for $\int_{x_0}^{x_0+2\pi} \frac{\sin x}{x^m} dx$. However, $\left\{ \frac{\cos(l\zeta\Delta z)}{j \sin(l\zeta\Delta z)} \right\} [1 - \cos(\zeta\Delta z)]$ in $g_l(\zeta)$ is not a single pe-

riodic function, the period of $\cos \zeta\Delta z$ is l times of the period of $\cos(l\zeta\Delta z)$ and $\sin(l\zeta\Delta z)$. If they are written in terms of several single periodic functions as

$$2 \cos(l\zeta\Delta z) [1 - \cos(\zeta\Delta z)] = 2 \cos(l\zeta\Delta z) - \cos[(l-1)\zeta\Delta z] - \cos[(l+1)\zeta\Delta z] \quad (7.3.19)$$

$$2 \sin(l\zeta\Delta z) [1 - \cos(\zeta\Delta z)] = 2 \sin(l\zeta\Delta z) - \sin[(l-1)\zeta\Delta z] - \sin[(l+1)\zeta\Delta z] \quad (7.3.20)$$

$g_l(\zeta)$ can be expressed as the sum of three single periodic functions, and the integrations

can be carried out separately. Hence, the integration $\int_0^\infty g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta$ will convergence quickly.

INTEGRATION THROUGH SURFACE-WAVE-POLE SINGULARITIES

Since the presence of the loss in region 2 makes k_2 a complex number, then all the surface wave poles of the integrand are not along the real axis. But, if the loss becomes small or vanishes, the surface-wave poles will shift to the real axis, and thus reside within the domain of integration. Therefore, pole extraction will be used to deal with the surface-wave-pole singularities.

Let $\zeta_p = \zeta_{pr} + j\zeta_{pi}$, where ζ_{pr} and ζ_{pi} are the real and imaginary parts of ζ_p , respectively, and ζ_p is a solution of TM-even mode eigenvalue equation $\tilde{\Delta}_e = 0$. After the same analysis in the last chapter, it is found that

$$\int_{\zeta_{pr}-\delta}^{\zeta_{pr}+\delta} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta = j \frac{2g_l(\zeta_p) f_s^{\alpha\beta p}(\zeta_p)}{\bar{\Delta}'_e(\zeta_p)} \operatorname{atan} \frac{\delta}{\zeta_{pi}} \quad (7.3.21)$$

where

$$f_s^{xsp}(\zeta_p) = -\left(\frac{\zeta_p}{q_2}\right)^2 \frac{p_1}{\sin(q_2 t)} \frac{\sin^2(q_2 \Delta x/2)}{q_2 \Delta x/2} \cos(sq_2 \Delta x) \quad (7.3.22)$$

$$f_s^{zsp}(\zeta_p) = -\frac{p_1}{\sin(q_2 t)} \frac{\sin^2(q_2 \Delta x/2)}{q_2 \Delta x/2} \cos(sq_2 \Delta x) \quad (7.3.23)$$

$$f_s^{xzp}(\zeta_p) = -\frac{j\zeta_p}{q_2} \frac{p_1}{\sin(q_2 t)} \frac{\sin^2(q_2 \Delta x/2)}{q_2 \Delta x/2} \sin(sq_2 \Delta x) \quad (7.3.24)$$

with

$$p_1 = \sqrt{\zeta_p^2 - k_1^2} \quad (7.3.25)$$

$$q_2 = \sqrt{k_2^2 - \zeta_p^2} \quad (7.3.26)$$

Equation (7.3.21) is valid only when ζ_{pi} is small, i.e. for small loss or lossless dielectric. If ζ_{pi} is large, δ is small, then

$$\int_{\zeta_{pr}-\delta}^{\zeta_{pr}+\delta} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta \approx 2g_l(\zeta_{pr}) f_s^{\alpha\beta}(\zeta_{pr}) \delta \quad (7.3.27)$$

INTEGRATION IN THE VICINITY OF k_2

Let's consider the integrands of (7.3.2-4) in the vicinity of $k_2 = k_{2r} + jk_{2i}$ (Figure 6.2). From (7.2.46-51), it is noted that $\zeta = k_2(p_2 = 0)$ is a branch point, and p_2 occurs in their denominators. But the behavior of $f_s^{\alpha\beta}(\zeta)$ as $\zeta \rightarrow k_2(p_2 \rightarrow 0)$ is

$$f_0^{xx}(k_2) = 1 + (k_2 \Delta x)^2 \{1/3 - [t/\Delta x + \varepsilon_1/(\varepsilon_2 p_1 \Delta x)]\} / 2 \quad (7.3.28)$$

$$f_0^{zz}(k_2) = -1 \quad (7.3.29)$$

$$f_0^{xz}(k_2) = 0 \quad (7.3.30)$$

$$f_s^{xx}(k_2) = (k_2 \Delta x)^2 \{s - [t/\Delta x + \varepsilon_1/(\varepsilon_2 p_1 \Delta x)]\} / 2 \quad (7.3.31)$$

$$f_s^{zz}(k_2) = 0 \quad (7.3.32)$$

$$f_s^{xz}(k_2) = -jk_2 \Delta x / 2 \quad (7.3.33)$$

Then, $\zeta = k_2 (p_2 = 0)$ does not lead to a zero in the denominators of the integrands.

Thus $f_s^{\alpha\beta}(\zeta)$ can be replaced by $f_s^{\alpha\beta}(k_2)$ in the vicinity of k_2 .

SUMMARY

The integrations of (7.3.2-4) along the real axis shown in Figure 6.2 can be written as the sum of four terms

$$\int_0^{\infty} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta = \left\{ \int_0^{\zeta_{pr}-\delta} + \int_{\zeta_{pr}-\delta}^{\zeta_{pr}+\delta} + \int_{\zeta_{pr}+\delta}^{2k_{2r}} + \int_{2k_{2r}}^{\infty} \right\} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta \quad (7.3.34)$$

The second term is discussed above. Romberg integration method [66] is used in the first and third terms. The fourth term is computed by using the convergence form [eqs. (7.3.7-12)] and dividing $g_l(\zeta)$ into the sum of three single periodic functions. The convergence is found to be relatively rapid.

7.4. NUMERICAL INTEGRATION ALONG BRANCH CUTS

In the last section, the matrix elements are expressed as the integrals along the real axis. There are two ways to express the matrix elements in terms of integrals along branch cuts and residues at surface-wave poles. One is substituting

$$\bar{G}_{22}(x|x', z-z') = \hat{z}\hat{z}' \frac{j\eta_2}{k_2} \delta(\vec{\rho} - \vec{\rho}') + \bar{G}_{22}^{Pole}(x|x', z-z') + \bar{G}_{22}^{Rad}(x|x', z-z') \quad (7.4.1)$$

in (7.2.11) and doing the same simplifications in the last section. Other way is applying complex analysis to (7.2.44), and rewriting the integral along the real axis in terms of integrals along branch cuts and residues at surface-wave poles. Two ways give us the same results as follows:

$$B^{\alpha\beta}(l, s) = \int_{-\infty}^{\infty} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta = - \int_{C_l} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta - \int_{C_p} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta \quad (7.4.2)$$

Residues at surface-wave poles are

$$\begin{aligned} \int_{C_p} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta &= -2\pi j \text{Res} \{ g_l(\zeta) f_s^{\alpha\beta}(\zeta) \} \Big|_{\zeta = -\zeta_p} \\ &= -2\pi j \frac{g_l(-\zeta_p) f_s^{\alpha\beta p}(-\zeta_p)}{\bar{\Delta}'_e(-\zeta_p)} = 2\pi j \frac{g_l(-\zeta_p) f_s^{\alpha\beta p}(-\zeta_p)}{\bar{\Delta}'_e(\zeta_p)} \end{aligned} \quad (7.4.3)$$

where $f_s^{\alpha\beta p}(\zeta_p)$ is given in the last section, and

$$g_l(-\zeta_p) = \begin{cases} [1 - \cos(\zeta_p \Delta z)] e^{-j\zeta_p l \Delta z} / (\zeta_p \Delta z)^2 & l = 1, \dots, N_z - 1 \\ (1 - j\zeta_p \Delta z - e^{-j\zeta_p \Delta z}) / (\zeta_p \Delta z)^2 & l = 0 \end{cases} \quad (7.4.4)$$

Integrals along the branch cut can be written as

$$\int_{C_l} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta = \int_0^{\infty} g_l(j\alpha) f_s^{\alpha\beta b}(\rho) \frac{j\rho}{\alpha} d\rho \quad (7.4.5)$$

where

$$g_l(j\alpha) = \begin{cases} \frac{1}{2} [e^{-\alpha(l-1)\Delta z} - 2e^{-\alpha l \Delta z} + e^{-\alpha(l+1)\Delta z}] / (\alpha \Delta z)^2 & l = 1, 2, \dots, N_z - 1 \\ (e^{-\alpha \Delta z} + \alpha \Delta z - 1) / (\alpha \Delta z)^2 & l = 0 \end{cases} \quad (7.4.6)$$

$$f_s^{xxb}(\rho) = j \frac{\alpha^2 C_e(\alpha)}{q_2^2} \frac{\sin^2(q_2 \Delta x / 2)}{q_2 \Delta x / 2} \cos(sq_2 \Delta x) \quad (7.4.7)$$

$$f_s^{zzb}(\rho) = -j C_e(\alpha) \frac{\sin^2(q_2 \Delta x / 2)}{q_2 \Delta x / 2} \cos(sq_2 \Delta x) \quad (7.4.8)$$

$$f_s^{xzb}(\rho) = j \frac{\alpha C_e(\alpha)}{q_2} \frac{\sin^2(q_2 \Delta x / 2)}{q_2 \Delta x / 2} \sin(sq_2 \Delta x) \quad (7.4.9)$$

with

$$C_e(\rho) = \frac{2\rho q_2 \epsilon_1 / \epsilon_2}{\rho^2 \cos^2(q_2 t) + (q_2 \epsilon_1 / \epsilon_2)^2 \sin^2(q_2 t)} \quad (7.4.10)$$

and

$$\alpha = \sqrt{\rho^2 - k_1^2} \quad (7.4.11)$$

$$q_2 = \sqrt{k_2^2 - k_1^2 + \rho^2} \quad (7.4.12)$$

Let's discuss the behavior of the integrand in (7.4.5). When $\rho \rightarrow \infty$,

$$C_e(\rho) \rightarrow \frac{4\epsilon_1 / \epsilon_2}{1 + (\epsilon_1 / \epsilon_2)^2 + [1 - (\epsilon_1 / \epsilon_2)^2] \cos(2\rho t)} \quad (7.4.13)$$

$$g_l(j\alpha) \rightarrow \begin{cases} 1/\rho & l = 0 \\ 1/\rho^2 & l = 1 \\ e^{-c\rho \Delta x} / \rho^2 & \text{otherwise} \end{cases} \quad (7.4.14)$$

where $c \geq 1$. Thus,

$$g_l(j\alpha)f_s^{\alpha\beta b}(\rho) \rightarrow \begin{cases} C_e(\rho)/\rho^2 & l = 0 \text{ for } \alpha\beta = xx, zz \\ C_e(\rho)/\rho^3 & l = 1 \\ C_e(\rho)e^{-c\rho\Delta x}/\rho^3 & \text{otherwise} \end{cases} \quad (7.4.15)$$

It is observed that the integrands of the integral along the branch cut go to zero slower than the integrands of the integrals along real axis as shown in (7.3.15). Furthermore, there is an oscillating function in the denominator of $C_e(\rho)$. The integrands in (7.4.5) cannot be written in terms of several single periodic functions. Thus, the integration along the branch cuts will take more time than the integration along real axis when the integral variable is large.

SPECIAL CASE: $Im\{k_1\} = 0$

If region 1 is lossless, then $k_{1i} = 0$ and $k_1 = k_{1r}$. The equations defining the branch cut become

$$\zeta_r \zeta_i = 0 \quad \zeta_r^2 - \zeta_i^2 < k_1^2 \quad (7.4.16)$$

Thus, the branch cuts are decomposed to include a contour on the real ζ axis and a contour on the imaginary ζ axis. By the same argument as given in Chapters 3 and 6, the integration along the branch cut directly is found to be

$$\int_{C_b} g_l(\zeta)f_s^{\alpha\beta}(\zeta)d\zeta = \int_{-k_1}^0 g_l(\zeta)f_s^{\alpha\beta r}(\zeta)d\zeta + \int_0^{\infty} g_l(j\alpha)f_s^{\alpha\beta i}(\alpha)jd\alpha \quad (7.4.17)$$

In the first term on the right side of the above equation,

$$f_s^{xxr}(\zeta) = -\frac{j\zeta^2 C_e(\zeta)}{q_2^2} \frac{\sin^2(q_2\Delta x/2)}{q_2\Delta x/2} \cos(sq_2\Delta x) \quad (7.4.18)$$

$$f_s^{zzr}(\zeta) = -jC_e(\zeta) \frac{\sin^2(q_2\Delta x/2)}{q_2\Delta x/2} \cos(sq_2\Delta x) \quad (7.4.19)$$

$$f_s^{xzz}(\zeta) = \frac{\zeta C_e(\zeta)}{q_2} \frac{\sin^2(q_2\Delta x/2)}{q_2\Delta x/2} \sin(sq_2\Delta x) \quad (7.4.20)$$

with

$$C_e(\zeta) = \frac{2q_1q_2\varepsilon_1/\varepsilon_2}{q_1^2\cos^2(q_2t) + (q_2\varepsilon_1/\varepsilon_2)^2\sin^2(q_2t)} \quad (7.4.21)$$

and

$$q_1 = \sqrt{k_1^2 - \zeta^2} \quad (7.4.22)$$

$$q_2 = \sqrt{k_2^2 - \zeta^2} \quad (7.4.23)$$

In the second term, ζ has been changed to $j\alpha$, and where

$$f_s^{xxi}(\alpha) = j \frac{\alpha^2 C_e(\alpha)}{q_2^2} \frac{\sin^2(q_2\Delta x/2)}{q_2\Delta x/2} \cos(sq_2\Delta x) \quad (7.4.24)$$

$$f_s^{zzi}(\alpha) = -jC_e(\alpha) \frac{\sin^2(q_2\Delta x/2)}{q_2\Delta x/2} \cos(sq_2\Delta x) \quad (7.4.25)$$

$$f_s^{xzi}(\alpha) = j \frac{\alpha C_e(\alpha)}{q_2} \frac{\sin^2(q_2\Delta x/2)}{q_2\Delta x/2} \sin(sq_2\Delta x) \quad (7.4.26)$$

with

$$C_e(\alpha) = \frac{2q_1q_2\varepsilon_1/\varepsilon_2}{q_1^2\cos^2(q_2t) + (q_2\varepsilon_1/\varepsilon_2)^2\sin^2(q_2t)} \quad (7.4.27)$$

and

$$q_1 = \sqrt{k_1^2 + \alpha^2} \quad (7.4.28)$$

$$q_2 = \sqrt{k_2^2 + \alpha^2} \quad (7.4.29)$$

Let's change the variables in the integrals in (7.4.17). In the first term ($-k_1 < \zeta < 0$)

$$\zeta = -\sqrt{k_1^2 - \rho^2} \quad (0 < \rho < k_1) \quad (7.4.30)$$

and

$$\alpha = \sqrt{\rho^2 - k_1^2} \quad (k_1 < \rho < \infty) \quad (7.4.31)$$

in the second term ($0 < \alpha < \infty$). Substituting (7.4.30) and (7.4.31) into (7.4.17) leads to the integration along the branch cut with a variable change

$$\int_{C_1} g_l(\zeta) f_s^{\alpha\beta}(\zeta) d\zeta = \int_0^\infty g_l(j\alpha) f_s^{\alpha\beta b}(\rho) \frac{j\rho}{\alpha} d\rho \quad (7.4.32)$$

where $g_l(j\alpha)$ and $f_s^{\alpha\beta b}(\rho)$ are given in (7.4.6-9), respectively, and

$$\alpha = \sqrt{\rho^2 - k_1^2} = \begin{cases} \sqrt{\rho^2 - k_1^2} & k_1 < \rho < \infty \\ j\sqrt{k_1^2 - \rho^2} & 0 < \rho < k_1 \end{cases} \quad (7.4.33)$$

The result in (7.4.32) is the same as that in (7.4.5). But (7.4.5) is the general case, and (7.4.32) is derived under the condition that region 1 is lossless.

7.5. COMPARISON OF THE INTEGRATION METHODS

In the last two sections, several integration methods have been introduced to compute the inverse Fourier transform. The convergence of the MoM solution using each of these integration methods will be checked to compare these methods.

Let's consider a grounded dielectric sheet with a gap as shown Figure 2.1. A TM_0 surface wave is incident upon the discontinuity from right to left. It is assumed that both regions are lossless and the dielectric sheet is so thin that only TM_0 surface-wave mode exists. Then, a transmitted TM_0 surface wave will propagate to the left, a reflected TM_0 surface wave will propagate to the right, and radiated waves will be excited in space. Thus, the total of the transmitted power, reflected power and radiated power should be equal to the incident power.

The Method of Moments (MoM) is applied to obtain numerical results for this problem. First, the moment matrix is filled by using the above integration methods, and the linear equations are solved for the total electric field in the discontinuity region. Then, the reflected and transmitted surface wave power and the radiated power are computed. Finally, the total power is compared with the incident power.

Convergence of the MoM solution is checked by comparing results when the partition numbers N_x and N_z of the discontinuity region are increased. The results are presented in Figure 7.1, where the convergence is evident. Two integration methods are used to calculate the matrix elements in Figure 7.1. One is the integration along real axis. Other one is the integration along the branch cuts, where B^{xx} and B^{xz} are calculated by integration along the branch cuts with a variable change, and B^{zz} is calculated by integration along the branch cuts directly. The integration along the branch cut takes 4 times more of computation time compared with the integration along real axis. The deviation between the numerical total power and the incident power is about 0.01% for the parameters given in Figure 7.1. It is also found that numerical results calculated by integration along the branch cut may not converge for some parameters. Therefore, the integration along real axis is more efficient for the case of TM-polarized wave incidence.

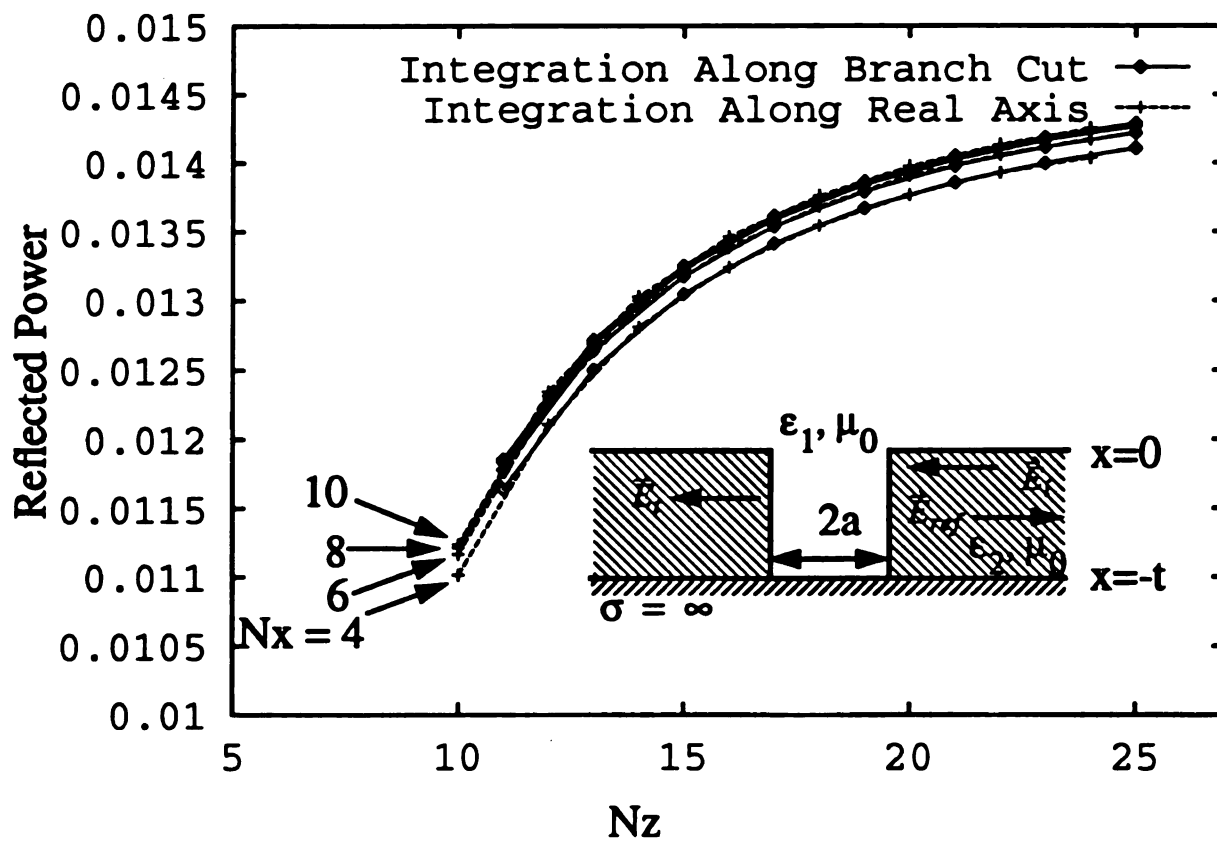


Figure 7.1. Convergence check on the reflected power computed with different N_x and N_z calculated by integration along real axis and integration along branch cut when a TM_0 surface wave is incident upon an air gap in the grounded sheet. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 2.1316$, $2a/\lambda_0 = 1$, and $k_0 t = 2.5$)

CHAPTER 8

SINUSOIDAL AND PIECEWISE-SINUSOIDAL (SPS) GALERKIN'S SOLUTIONS FOR TM-POLARIZED WAVE INCIDENCE

8.1. INTRODUCTION

To choose basis functions and testing functions is required in applying the Method of Moments (MoM) to solve the EFIE. In Galerkin's method, both basis and testing functions are the same. It is inevitable that certain basis functions give faster convergence than others, which means less computational effort for a given accuracy since one has a smaller matrix to calculate and invert. There are two classes of basis functions [33]. The first class will be basis functions defined and non-zero over the entire domain except possibly for a set of measure zero. This class of basis functions will be denoted entire-domain bases. Contrasted with this class is the second class whereby the basis functions are defined within the domain but vanish over part of that domain. The latter class will be denoted sub-domain bases. While the use of bases spanning the whole domain can result in rapid convergence, the computation of the matrix elements is usually not efficient in terms of computer time. On the other hand, the sub-domain basis functions usually afford efficient matrix element computation, but can require a large number of such functions to adequately describe the problem and obtain convergence.

The functions in the pulse Galerkin's method of the last two chapters are sub-domain basis functions, and are also called piecewise basis functions. It is found that it is not difficult to derive the formulas to calculate the matrix elements by using pulse functions, but

a large number of equations is needed to describe the problem. The numbers of partitions in both x - and z -directions should be about 15 to 20 per wavelength in the dielectric sheet. When the gap is large, the number of linear equations ($2N = 2N_x N_z$ for TM-polarized wave incidence) becomes very large. Recalling that the computer solution time for N linear equations is approximately proportional to N^3 , therefore, the pulse Galerkin's method is quite time-consuming for a large gap.

In this chapter, sinusoidal and piecewise-sinusoidal (SPS) Galerkin's methods are applied to solve the EFIE for TM-polarized wave incidence. First, the appropriate sinusoidal and piecewise-sinusoidal functions will be introduced. Then, the formulas to calculate the matrix elements are derived by using the SPS Galerkin's methods. Finally, some numerical integration techniques will be discussed, and the convergence of the SPS Galerkin's and the pulse Galerkin's methods will be compared with each other.

8.2. SINUSOIDAL FUNCTIONS AND PIECEWISE-SINUSOIDAL FUNCTIONS

It is assumed that the cross section of the discontinuity region is rectangular ($|z| \leq a$, $-t \leq x \leq 0$) and the permittivity ϵ_d is a constant. Since it is 2-D problem, the 2-D basis functions $e_{\alpha n}(x, z)$ and testing functions $t_{\alpha n}(x, z)$ should be functions of variables x and z . It is also assumed that they are the product of an x -dependent function and a z -dependent function.

$$e_{\alpha n}(x, z) = t_{\alpha n}(x, z) = X_{\alpha n_x}(x) Z_{\alpha n_z}(z) \quad (8.2.1)$$

where the $X_{\alpha n_x}(x)$ will be chosen as sinusoidal functions, which are entire-domain bases, and $Z_{\alpha n_z}(z)$ will be chosen as piecewise-sinusoidal functions, which are sub-domain bases, with

$$n = (n_x - 1)N_z + n_z \quad (8.2.2)$$

where n_x varies from 1 to N_x , and n_z from 1 to N_z .

It is evident that the closer the basis functions $e_{\alpha n}(x, z)$ resemble the total electric field distributions $E_{\alpha}(x, z)$ in the discontinuity region, the better the convergence. It is noted that

$$E_z(x, z)|_{x=-t} = 0 \quad (8.2.3)$$

and $\nabla \cdot \vec{E}(x, z) = 0$, which requires

$$\frac{\partial}{\partial x} E_x(x, z) \Big|_{x=-t} = -\frac{\partial}{\partial z} E_z(x, z) \Big|_{x=-t} = 0 \quad (8.2.4)$$

Thus, it is expedient to choose

$$X_{zn_x}(x, z) \Big|_{x=-t} = 0 \quad (8.2.5)$$

and

$$\frac{\partial}{\partial x} X_{zn_x}(x, z) \Big|_{x=-t} = 0 \quad (8.2.6)$$

In the original EFIE, $E_x(x, z)$ and $E_z(x, z)$ have definitions only in region $|z| \leq a$, $-t \leq x \leq 0$. Let's extend them to periodic functions of x at a given z as shown in Figure 8.1. It is observed that the extended $E_x(x, z)$ and $E_z(x, z)$ are continuous at the boundaries $x = 0, -t$, and have continuous first derivatives at $x = -t$. Since $E_x(x, z)$ is an even function of x with a period $2t$, it can be expanded in the Fourier cosine series

$$E_x(x, z) = \sum_{n=0}^{\infty} a_{xn}(z) \cos \frac{n\pi x}{t} \quad (8.2.7)$$

where

$$a_{xn}(z) = \frac{2}{\delta_n t} \int_{-t}^0 E_x(x, z) \cos \frac{n\pi x}{t} dx \quad (8.2.8)$$

with

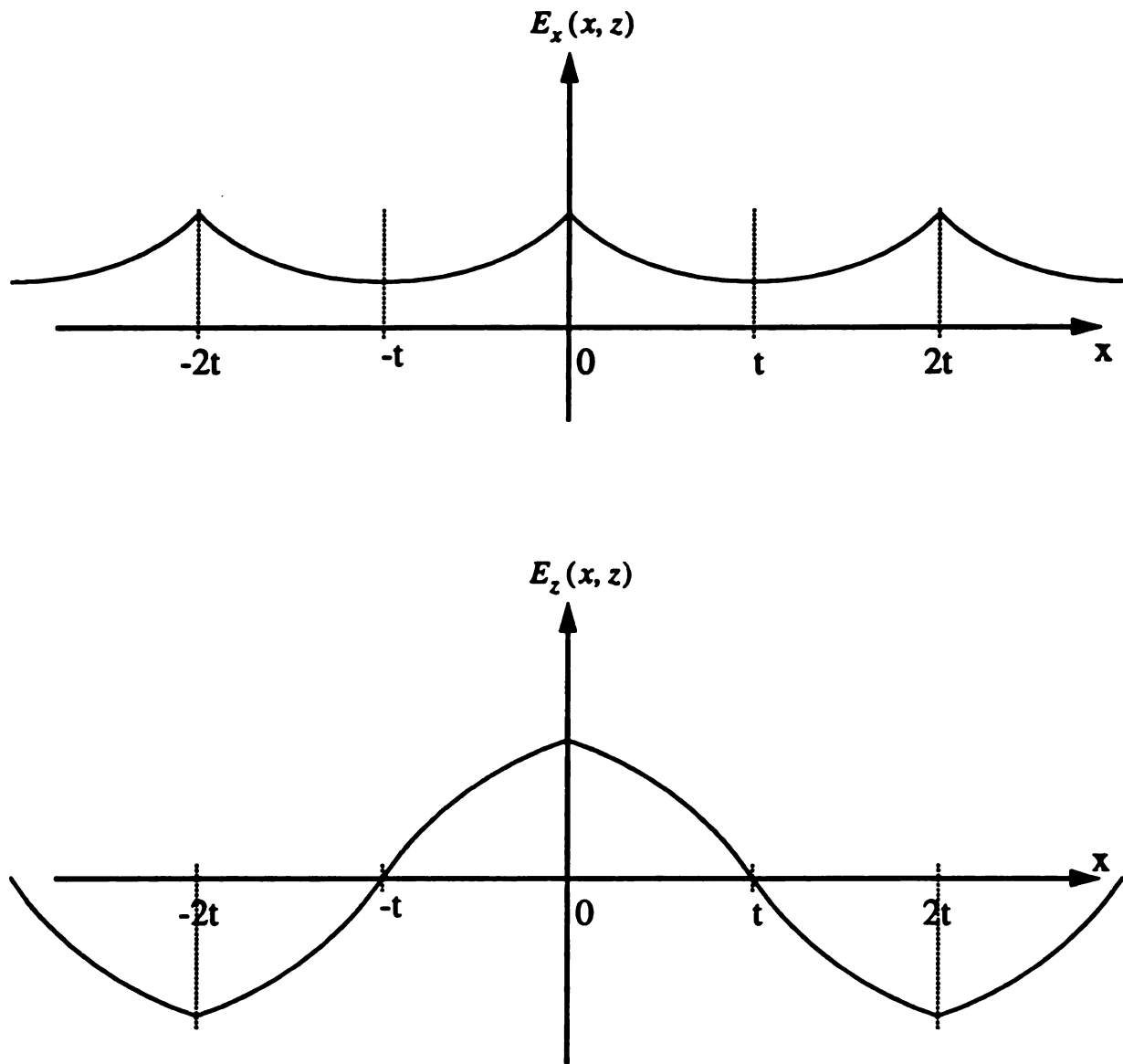


Figure 8.1. Extend $E_x(x, z)$ and $E_z(x, z)$ to periodic functions of x at a given z .

$$\delta_n = \begin{cases} 2 & n = 0 \\ 1 & n \geq 1 \end{cases} \quad (8.2.9)$$

Similarly, $E_z(x, z)$ is an even function of x with a period $4t$, so it can be expanded in terms of the Fourier cosine series

$$E_z(x, z) = \sum_{n=0}^{\infty} a_{zn}(z) \cos \frac{n\pi x}{2t} \quad (8.2.10)$$

where

$$a_{zn}(z) = \frac{1}{\delta_n t} \int_{-2t}^0 E_z(x, z) \cos \frac{n\pi x}{2t} dx \quad (8.2.11)$$

Changing the variable x to $y = t + x$, the above equation leads to

$$a_{zn}(z) = \frac{1}{\delta_n t} \int_{-t}^t E_z(y - t, z) \left[\cos \frac{n\pi y}{2t} \cos \frac{n\pi}{2} + \sin \frac{n\pi y}{2t} \sin \frac{n\pi}{2} \right] dy \quad (8.2.12)$$

Since $E_z(-x, z) = E_z(x, z) = -E_z(2t + x, z)$, i.e. $E_z(t - y, z) = -E_z(t + y, z)$ is an odd function of y , then

$$a_{zn}(z) = \begin{cases} \frac{2 \sin(n\pi/2)}{t} \int_0^t E_z(y - t, z) \sin \frac{n\pi y}{2t} dy & n = 1, 3, 5, \dots \\ 0 & n = 0, 2, 4, \dots \end{cases} \quad (8.2.13)$$

Thus, $E_z(x, z)$ can be expanded as

$$E_z(x, z) = \sum_{n=1}^{\infty} a_{zn}(z) \cos \frac{(2n-1)\pi x}{2t} \quad (8.2.14)$$

Consequently, the x -dependent basis functions are obtained

$$\left. \begin{aligned} X_{xn_x}(x) &= \cos \frac{(n_x - 1) \pi x}{t} \\ X_{zn_x}(x) &= \cos \frac{(2n_x - 1) \pi x}{2t} \end{aligned} \right\} \quad (8.2.15)$$

for $n_x = 1, 2, \dots, N_x$. It is clear that the above basis functions satisfy conditions (8.2.5) and (8.2.6).

The discontinuity region is partitioned into $N_z - 1$ equal sub-domain along the z-direction as shown in Figure 8.2. It is observed that

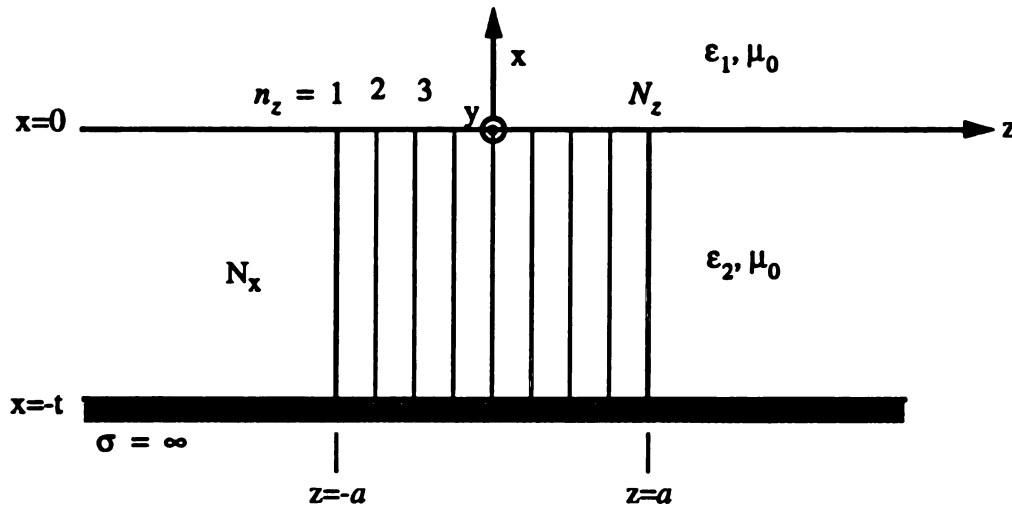


Figure 8.2. Applying sinusoidal and piecewise-sinusoidal (SPS) Galerkin's Method to EFIE in the discontinuity region.

$$z_{n_z} = -a + (n_z - 1) \Delta z \quad (8.2.16)$$

where

$$\Delta z = 2a / (N_z - 1) \quad (8.2.17)$$

Recall that in the solution of Hallen's integral equation for the current distribution on a linear antenna [68], only $N_z - 2$ piecewise-sinusoidal functions are needed since the currents are zero at both ends of the antenna, where $N_z - 1$ is the number of partitions. However, since the electric field $E_x(x, z)$ and $E_z(x, z)$ are non-zero at boundaries $z = \pm a$, then we can not use just $N_z - 2$ piecewise-sinusoidal functions to expand these fields but should add one half piecewise-sinusoidal function at each end as shown in Figure 8.3.

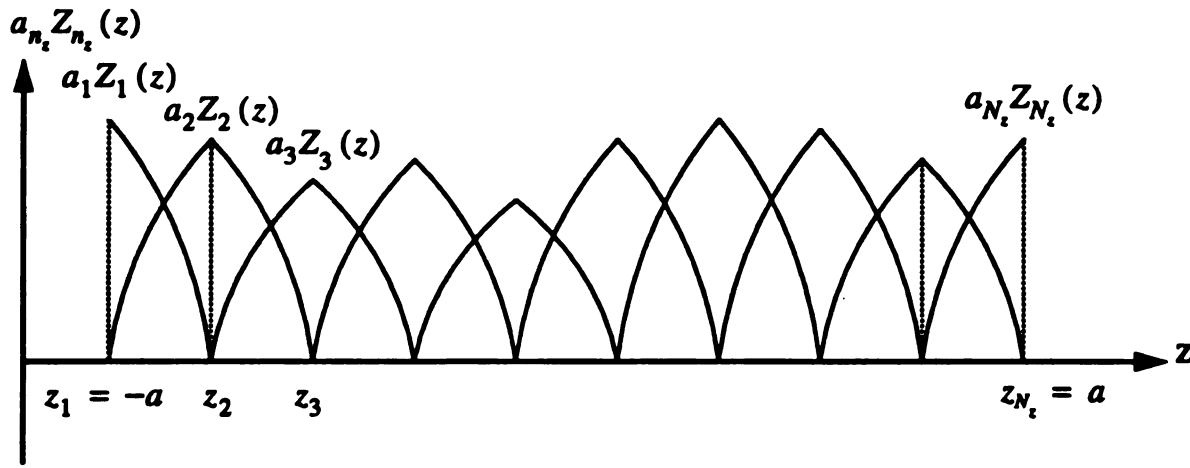


Figure 8.3. Piecewise-sinusoidal functions

Both $Z_{x_{n_z}}(z)$ and $Z_{z_{n_z}}(z)$ are chosen as the piecewise-sinusoidal functions

$$Z_{x_{n_z}}(z) = Z_{z_{n_z}}(z) = Z_{n_z}(z) \quad (8.2.18)$$

From Figure 8.3, it is evident that

$$Z_1(z) = \begin{cases} \frac{\sin[k(z_2 - z)]}{\sin(k\Delta z)} & z_1 < z < z_2 \\ 0 & \text{otherwise} \end{cases} \quad (8.2.19)$$

$$Z_{N_z}(z) = \begin{cases} \frac{\sin [k(z - z_{N_z-1})]}{\sin(k\Delta z)} & z_{N_z-1} < z < z_{N_z} \\ 0 & \text{otherwise} \end{cases} \quad (8.2.20)$$

$$Z_{n_z}(z) = \begin{cases} \frac{\sin [k(z - z_{n_z-1})]}{\sin(k\Delta z)} & z_{n_z-1} < z < z_{n_z} \\ \frac{\sin [k(z_{n_z+1} - z)]}{\sin(k\Delta z)} & z_{n_z} < z < z_{n_z+1} \\ 0 & \text{otherwise} \end{cases} \quad (8.2.21)$$

for $n_z = 2, 3, \dots, N_z - 1$, and where k will be k_0 for numerical results in free space. $Z_{n_z}(z)$ has been normalized by $\sin k\Delta z$ to let the maximum is 1.

It is observed that since the piecewise-sinusoidal functions are continuous in the domain $|z| < a$, then SPS Galerkin's method may converge faster than pulse Galerkin's method, because the pulse functions are discontinuous in the same domain.

8.3. SPS GALERKIN'S SOLUTIONS

In this section, the formulas to calculate the matrix elements and the forcing term will be derived by using SPS Galerkin's method. Substituting the expressions for the basis and testing functions in the formulas to calculate $A_{mn}^{\alpha\beta}$ and $F_{\alpha m}$ [(4.5.5) and (4.5.6)] gives

$$F_{\alpha m} = \frac{1}{i\Delta z} \int_{-a}^a \int_{-t}^0 E_{2\alpha}^i(x, z) X_{\alpha m_z}(x) Z_{m_z}(z) dx dz \quad (8.3.1)$$

$$A_{mn}^{\alpha\beta} = \frac{\delta_{\alpha\beta}}{i\Delta z} \int_{-a}^a Z_{m_z}(z) Z_{n_z}(z) dz \int_{-t}^0 X_{\alpha m_z}(x) X_{\alpha n_z}(x) dx$$

$$-j \frac{k_0 \Delta n^2}{\eta_0 i \Delta z} \int_{-a}^a \int_{-t}^0 dx dz X_{\alpha m_z}(x) Z_{m_z}(z) \int_{-a}^a \int_{-t}^0 G_{22\alpha\beta}(x|x', z-z') X_{\beta n_z}(x') Z_{n_z}(z') dx' dz' \quad (8.3.2)$$

where

$$m = (m_x - 1)N_z + m_z \quad (8.3.3)$$

$$n = (n_x - 1)N_z + n_z \quad (8.3.4)$$

Note that $A_{mn}^{\alpha\beta}/(t\Delta z)$ and $F_{\alpha m}/(t\Delta z)$ are newly defined as $A_{mn}^{\alpha\beta}$ and $F_{\alpha m}$, respectively, and where $\Delta n^2 = n_d^2 - n_2^2 = (\epsilon_d - \epsilon_2)/\epsilon_0$, and $E_{2\alpha}^i(x, z)$, $G_{22\alpha\beta}(x|x', z - z')$ are given in Chapters 2 to 4.

Let's evaluate the above spatial integrals required in spectral-integral representations of MoM matrix elements. The first term of (8.3.2) is evaluated first.

$$\int_{-t}^0 X_{xm_x}(x) X_{xn_x}(x) dx = \int_{-t}^0 \cos \frac{(m_x - 1)\pi x}{t} \cos \frac{(n_x - 1)\pi x}{t} dx = \frac{t}{2} \delta_{m_x, n_x} \quad (8.3.5)$$

$$\int_{-t}^0 X_{zm_x}(x) X_{zn_x}(x) dx = \int_{-t}^0 \cos \frac{(2m_x - 1)\pi x}{2t} \cos \frac{(2n_x - 1)\pi x}{2t} dx = \frac{t}{2} \delta_{m_x, n_x} \quad (8.3.6)$$

$$\begin{aligned} \frac{\sin^2(k\Delta z)}{\Delta z} \int_{-a}^a Z_{m_z}(z) Z_{n_z}(z) dz &= \begin{cases} \frac{1}{2} \left[1 - \frac{\sin(2k\Delta z)}{2k\Delta z} \right] & m_z = n_z = 1, N_z \\ 1 - \frac{\sin(2k\Delta z)}{2k\Delta z} & m_z = n_z = 2, \dots, N_z - 1 \\ \frac{1}{2} \left[\frac{\sin(k\Delta z)}{k\Delta z} - \cos(k\Delta z) \right] & |m_z - n_z| = 1 \\ 0 & \text{otherwise} \end{cases} \\ &= \frac{1}{2} \left\{ \left[1 - \frac{\sin(2k\Delta z)}{2k\Delta z} \right] \tilde{\delta}_{m_z} \delta_{m_z, n_z} + \left[\frac{\sin(k\Delta z)}{k\Delta z} - \cos(k\Delta z) \right] \delta_{|m_z - n_z|, 1} \right\} \end{aligned} \quad (8.3.7)$$

where

$$\delta_m = \begin{cases} 2 & m = 1 \\ 1 & m > 1 \end{cases} \quad (8.3.8)$$

$$\delta_{mn} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \quad (8.3.9)$$

$$\bar{\delta}_{m_z} = \begin{cases} 1 & m_z = 1, N_z \\ 2 & m_z = 2, \dots, N_z - 1 \end{cases} \quad (8.3.10)$$

Substituting $\bar{G}_{22}(x|x', z-z') = \hat{x}\hat{x}\delta(\vec{\rho} - \vec{\rho}')j\eta_2/k_2 + \bar{G}_{22}^R(x|x', z-z')$ in (8.3.2),

where $\bar{G}_{22}^R(x|x', z-z')$ is given in (2.4.10-13), it is found that

$$\begin{aligned} A_{mn}^{xx} = & \left(1 + \frac{\epsilon_0}{\epsilon_2} \Delta n^2\right) \delta_{m_z} D_{mn} + \frac{\epsilon_0 t \Delta z}{\epsilon_2 4\pi} \Delta n^2 \int_{-\infty}^{\infty} d\zeta \frac{\zeta^2}{p_2} g_{m_z n_z}(\zeta) \int_{-t}^0 \int_{-t}^0 \{ -e^{-p_2|x-x'|} \\ & - e^{-p_2(2t+x+x')} + \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} 2e^{-p_2 t} \cosh[p_2(t+x)] \cosh[p_2(t+x')] \} \\ & \cos \frac{(m_x - 1)\pi x}{t} \cos \frac{(n_x - 1)\pi x'}{t} \frac{dx dx'}{t^2} \end{aligned} \quad (8.3.11)$$

$$\begin{aligned} A_{mn}^{zz} = & D_{mn} + \frac{\epsilon_0 t \Delta z}{\epsilon_2 4\pi} \Delta n^2 \int_{-\infty}^{\infty} d\zeta p_2 g_{m_z n_z}(\zeta) \int_{-t}^0 \int_{-t}^0 \{ e^{-p_2|x-x'|} \\ & - e^{-p_2(2t+x+x')} + \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} 2e^{-p_2 t} \sinh[p_2(t+x)] \sinh[p_2(t+x')] \} \\ & \cos \frac{(2m_x - 1)\pi x}{2t} \cos \frac{(2n_x - 1)\pi x'}{2t} \frac{dx dx'}{t^2} \end{aligned} \quad (8.3.12)$$

$$\begin{aligned} A_{mn}^{xz} = & \frac{\epsilon_0 t \Delta z}{\epsilon_2 4\pi} \Delta n^2 \int_{-\infty}^{\infty} d\zeta j\zeta g_{m_z n_z}(\zeta) \int_{-t}^0 \int_{-t}^0 \{ \text{sgn}(x-x') e^{-p_2|x-x'|} \\ & - e^{-p_2(2t+x+x')} - \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}_e} 2e^{-p_2 t} \cosh[p_2(t+x)] \sinh[p_2(t+x')] \} \\ & \cos \frac{(m_x - 1)\pi x}{t} \cos \frac{(2n_x - 1)\pi x'}{2t} \frac{dx dx'}{t^2} \end{aligned} \quad (8.3.13)$$

$$A_{mn}^{zx} = A_{nm}^{xz} \quad (8.3.14)$$

where

$$D_{mn} = \frac{\delta_{m,n}}{4 \sin^2(k\Delta z)} \left\{ \left[1 - \frac{\sin(2k\Delta z)}{2k\Delta z} \right] \bar{\delta}_{m_i} \delta_{m_i n_i} + \left[\frac{\sin(k\Delta z)}{k\Delta z} - \cos(k\Delta z) \right] \delta_{|m_i - n_i|, 1} \right\} \quad (8.3.15)$$

$$g_{m_i n_i}(\zeta) = \int_{-a}^a \int_{-a}^a e^{j\zeta(z-z')} Z_{m_i}(z) Z_{n_i}(z') \frac{dz dz'}{(\Delta z)^2} \quad (8.3.16)$$

Now, the integrations with respect to x and x' are evaluated. Using

$$\int e^{\alpha x} \cos(bx) dx = \frac{e^{\alpha x} [\alpha \cos(bx) + b \sin(bx)]}{\alpha^2 + b^2} \quad (8.3.17)$$

yields

$$\begin{aligned} & \int_{-t}^0 \int_{-t}^0 e^{-p_2|x-x'|} \{ \text{sgn}(x-x') \}^1 \cos(ax) \cos(bx') dx dx' \\ &= \int_{-t}^0 \left[\int_{-t}^x e^{-p_2(x-x')} \cos(bx') dx' \pm \int_x^0 e^{-p_2(x-x')} \cos(bx') dx' \right] \cos(ax) dx \\ &= \frac{1}{p_2^2 + b^2} \int_{-t}^0 \{ \mp p_2 e^{p_2 x} - e^{-p_2(t+x)} [p_2 \cos(bt) - b \sin(bt)] \\ &\quad + p_2 \cos(bx) + b \sin(bx) \mp [-p_2 \cos(bx) + b \sin(bx)] \} \cos(ax) dx \\ &= \frac{1}{p_2^2 + b^2} \left\{ \frac{1}{p_2^2 + a^2} \{ \mp p_2 [p_2 - e^{-p_2 t} [p_2 \cos(at) - a \sin(at)]] - [p_2 \cos(bt) - b \sin(bt)] \right. \\ &\quad \left. [-p_2 e^{-p_2 t} - [-p_2 \cos(bt) - b \sin(bt)]] \} + (1 \pm 1) p_2 \left[\frac{\sin[(b-a)t]}{2(b-a)} + \frac{\sin[(b+a)t]}{2(b+a)} \right] \right. \\ &\quad \left. - (1 \mp 1) b \left[\frac{1 - \cos[(b-a)t]}{2(b-a)} + \frac{1 - \cos[(b+a)t]}{2(b+a)} \right] \right\} \quad (8.3.18) \end{aligned}$$

Then,

$$\begin{aligned}
\Phi_{mn}^{0xx} &= \frac{1}{2} \int_{-t}^0 \int_{-t}^0 e^{-p_2|x-x'|} \cos \frac{(m-1)\pi x}{t} \cos \frac{(n-1)\pi x'}{t} dx dx' \\
&= \frac{p_2 t \delta_m \delta_{mn}}{(p_2 t)^2 + [(m-1)\pi]^2} - \frac{(p_2 t)^2 [1 + (-1)^m e^{-p_2 t}] [1 + (-1)^{m+n}]}{[(p_2 t)^2 + [(m-1)\pi]^2] [(p_2 t)^2 + [(n-1)\pi]^2]} \quad (8.3.19)
\end{aligned}$$

$$\begin{aligned}
\Phi_{mn}^{0zz} &= \frac{1}{2} \int_{-t}^0 \int_{-t}^0 e^{-p_2|x-x'|} \cos \frac{(2m-1)\pi x}{2t} \cos \frac{(2n-1)\pi x'}{2t} dx dx' \\
&= \frac{p_2 t \delta_{mn}}{(p_2 t)^2 + [(2m-1)\pi/2]^2} \\
&\quad - \frac{(p_2 t)^2 - (-1)^{m+n} (2m-1)(2n-1)\pi^2/4 - p_2 t e^{-p_2 t} [(2m-1)(-1)^m + (2n-1)(-1)^n] \pi/2}{[(p_2 t)^2 + [(2m-1)\pi/2]^2] [(p_2 t)^2 + [(2n-1)\pi/2]^2]} \quad (8.3.20)
\end{aligned}$$

$$\begin{aligned}
\Phi_{mn}^{1xz} &= \frac{1}{2} \int_{-t}^0 \int_{-t}^0 e^{-p_2|x-x'|} \operatorname{sgn}(x-x') \cos \frac{(m-1)\pi x}{t} \cos \frac{(2n-1)\pi x'}{2t} dx dx' \\
&= \frac{1}{(p_2 t)^2 + [(2n-1)\pi/2]^2} \left\{ -\frac{2(2n-1)^2}{(2n-1)^2 - 4(m-1)^2} + \frac{p_2 t [1 + (-1)^m e^{-p_2 t}] [p_2 t + (-1)^{m+n} (2n-1)\pi/2]}{(p_2 t)^2 + [(m-1)\pi]^2} \right\} \quad (8.3.21)
\end{aligned}$$

Using the following integral formulas,

$$\frac{1}{t} \int_{-t}^0 \cosh [p_2(t+x)] \cos \frac{(m-1)\pi x}{t} dx = \frac{p_2 t \sinh(p_2 t)}{(p_2 t)^2 + [(m-1)\pi]^2} \quad (8.3.22)$$

$$\frac{1}{t} \int_{-t}^0 \cosh [p_2(t+x)] \cos \frac{(2m-1)\pi x}{2t} dx = \frac{p_2 t \sinh(p_2 t) - (-1)^m (2m-1)\pi/2}{(p_2 t)^2 + [(2m-1)\pi/2]^2} \quad (8.3.23)$$

$$\frac{1}{t} \int_{-t}^0 \sinh [p_2(t+x)] \cos \frac{(m-1)\pi x}{t} dx = \frac{p_2 t [\cosh(p_2 t) + (-1)^m]}{(p_2 t)^2 + [(m-1)\pi]^2} \quad (8.3.24)$$

$$\frac{1}{t} \int_{-t}^0 \sinh [p_2 (t+x)] \cos \frac{(2m-1) \pi x}{2t} dx = \frac{p_2 t \cosh (p_2 t)}{(p_2 t)^2 + [(2m-1) \pi/2]^2} \quad (8.3.25)$$

$$\frac{1}{t} \int_{-t}^0 e^{-p_2 (t+x)} \cos \frac{(m-1) \pi x}{t} dx = \frac{-p_2 t [e^{-p_2 t} + (-1)^m]}{(p_2 t)^2 + [(m-1) \pi]^2} \quad (8.3.26)$$

$$\frac{1}{t} \int_{-t}^0 e^{-p_2 (t+x)} \cos \frac{(2m-1) \pi x}{2t} dx = \frac{-p_2 t e^{-p_2 t} + (-1)^m (2m-1) \pi/2}{(p_2 t)^2 + [(2m-1) \pi/2]^2} \quad (8.3.27)$$

the remaining integrations with respect to x and x' can be evaluated. Consequently, it is found that

$$A_{mn}^{xx} = \left(1 + \frac{\epsilon_0}{\epsilon_2} \Delta n^2\right) \delta_{m_x} D_{mn} + \frac{\epsilon_0 t \Delta z}{\epsilon_2 4\pi} \Delta n^2 \int_{-\infty}^{\infty} d\zeta \frac{\zeta^2}{p_2} g_{m_x n_x}(\zeta) [-\Phi_{m_x n_x}^{0xx} + \Phi_{m_x n_x}^{2xx}] \quad (8.3.28)$$

$$A_{mn}^{zz} = D_{mn} + \frac{\epsilon_0 t \Delta z}{\epsilon_2 4\pi} \Delta n^2 \int_{-\infty}^{\infty} d\zeta p_2 g_{m_x n_x}(\zeta) [\Phi_{m_x n_x}^{0zz} + \Phi_{m_x n_x}^{2zz}] \quad (8.3.29)$$

$$A_{mn}^{xz} = \frac{\epsilon_0 t \Delta z}{\epsilon_2 4\pi} \Delta n^2 \int_{-\infty}^{\infty} d\zeta j \zeta g_{m_x n_x}(\zeta) [\Phi_{m_x n_x}^{1xz} + \Phi_{m_x n_x}^{2xz}] \quad (8.3.30)$$

where

$$\Phi_{mn}^{2xx} = \frac{(p_2 t)^2 \{-[(-1)^m + e^{-p_2 t}] [(-1)^n + e^{-p_2 t}] + (1 - e^{-2p_2 t})^2 (p_1 - p_2 \epsilon_1 / \epsilon_2) / \bar{\Delta}_{e1}\}}{[(p_2 t)^2 + [(m-1) \pi]^2] [(p_2 t)^2 + [(n-1) \pi]^2]} \quad (8.3.31)$$

$$\Phi_{mn}^{2zz} = \frac{-[p_2 t e^{-p_2 t} + (-1)^m (2m-1) \pi/2] [p_2 t e^{-p_2 t} + (-1)^n (2n-1) \pi/2] + (p_2 t)^2 (1 + e^{-2p_2 t})^2 (p_1 - p_2 \epsilon_1 / \epsilon_2) / \bar{\Delta}_{e1}}{[(p_2 t)^2 + [(2m-1) \pi/2]^2] [(p_2 t)^2 + [(2n-1) \pi/2]^2]} \quad (8.3.32)$$

$$\Phi_{mn}^{2xz} = \frac{-p_2 t [(-1)^m + e^{-p_2 t}] [p_2 t e^{-p_2 t} + (-1)^n (2n-1) \pi/2] - (p_2 t)^2 (1 - e^{-4p_2 t}) (p_1 - p_2 \epsilon_1 / \epsilon_2) / \bar{\Delta}_{e1}}{[(p_2 t)^2 + [(m-1) \pi]^2] [(p_2 t)^2 + [(2n-1) \pi/2]^2]} \quad (8.3.33)$$

with

$$\bar{\Delta}_{e1} = 2\bar{\Delta}_e e^{-p_2 t} = p_1 (1 + e^{-2p_2 t}) + p_2 (1 - e^{-2p_2 t}) \varepsilon_1 / \varepsilon_2 \quad (8.3.34)$$

It is observed that the leading terms of $(\zeta^2/p_2) \Phi_{m_x m_x}^{0xx}$ and $p_2 \Phi_{m_x m_x}^{0zz}$ are constant when $\zeta \rightarrow \pm\infty$, they can be consequently rewritten as

$$(\zeta^2/p_2) \Phi_{m_x n_x}^{0xx} = \frac{1}{t} \{ \delta_{m_x} \delta_{m_x n_x} + \Phi_{m_x n_x}^{1xx} \} \quad (8.3.35)$$

$$p_2 \Phi_{m_x n_x}^{0zz} = \frac{1}{t} \{ \delta_{m_x n_x} + \Phi_{m_x n_x}^{1zz} \} \quad (8.3.36)$$

with

$$\Phi_{mn}^{1xx} = \frac{[(k_2 t)^2 - [(m-1)\pi]^2] \delta_m \delta_{mn}}{(p_2 t)^2 + [(m-1)\pi]^2} - \frac{p_2 \zeta^2 t^3 [1 + (-1)^m e^{-p_2 t}] [1 + (-1)^{m+n}]}{[(p_2 t)^2 + [(m-1)\pi]^2] [(p_2 t)^2 + [(n-1)\pi]^2]} \quad (8.3.37)$$

$$\begin{aligned} \Phi_{mn}^{1zz} &= \frac{[(2m-1)\pi/2]^2 \delta_{mn}}{(p_2 t)^2 + [(2m-1)\pi/2]^2} \\ &\quad - \frac{p_2 t \{ (p_2 t)^2 - (-1)^{m+n} (2m-1)(2n-1)\pi^2/4 - p_2 t e^{-p_2 t} [(2m-1)(-1)^m + (2n-1)(-1)^n] \pi/2 \}}{[(p_2 t)^2 + [(2m-1)\pi/2]^2] [(p_2 t)^2 + [(2n-1)\pi/2]^2]} \end{aligned} \quad (8.3.38)$$

Inverting the constant terms leads to

$$\begin{aligned} &\frac{\varepsilon_0 t \Delta z}{\varepsilon_2 4\pi} \Delta n^2 \int_{-\infty}^{\infty} \frac{d\zeta}{t} \delta_{m_x} \delta_{m_x n_x} g_{m_x n_x}(\zeta) \\ &= \frac{\varepsilon_0 \Delta z}{\varepsilon_2 4\pi} \Delta n^2 \delta_{m_x} \delta_{m_x n_x} \int_{-a}^a \int_{-a}^a Z_{m_x}(z) Z_{n_x}(z') \frac{dz dz'}{(\Delta z)^2} \int_{-\infty}^{\infty} d\zeta e^{j\zeta(z-z')} \\ &= \frac{\varepsilon_0 \Delta z}{\varepsilon_2 2} \Delta n^2 \delta_{m_x} \delta_{m_x n_x} \int_{-a}^a \int_{-a}^a Z_{m_x}(z) Z_{n_x}(z') \frac{dz dz'}{(\Delta z)^2} \delta(z-z') \\ &= \frac{\varepsilon_0 \Delta n^2}{\varepsilon_2 2} \delta_{m_x} \delta_{m_x n_x} \int_{-a}^a Z_{m_x}(z) Z_{n_x}(z) \frac{dz}{\Delta z} \end{aligned}$$

$$= \frac{\epsilon_0}{\epsilon_2} \Delta n^2 \delta_{m_x} D_{mn} \quad (8.3.39)$$

Thus, A_{mn}^{xx} and A_{mn}^{zz} have the new forms

$$A_{mn}^{xx} = \delta_{m_x} D_{mn} + \frac{\epsilon_0}{\epsilon_2} \frac{\Delta z}{4\pi} \Delta n^2 \int_{-\infty}^{\infty} d\zeta g_{m_x n_x}(\zeta) \left[-\Phi_{m_x n_x}^{1xx} + \Phi_{m_x n_x}^{2xx} \zeta^2 t/p_2 \right] \quad (8.3.40)$$

$$A_{mn}^{zz} = \left(1 + \frac{\epsilon_0}{\epsilon_2} \Delta n^2 \right) D_{mn} + \frac{\epsilon_0}{\epsilon_2} \frac{\Delta z}{4\pi} \Delta n^2 \int_{-\infty}^{\infty} d\zeta g_{m_x n_x}(\zeta) \left[\Phi_{m_x n_x}^{1zz} + p_2 t \Phi_{m_x n_x}^{2zz} \right] \quad (8.3.41)$$

It is noted that since all Φ 's are even functions of ζ and the integrals from $-\infty$ to ∞ in (8.3.40-41) and (8.3.30) are symmetric, then

$$A_{mn}^{xx} = \delta_{m_x} D_{mn} + \frac{\epsilon_0}{\epsilon_2} \frac{\Delta z}{4\pi} \Delta n^2 \int_0^{\infty} d\zeta g_{m_x n_x}^e(\zeta) \left[-\Phi_{m_x n_x}^{1xx} + \Phi_{m_x n_x}^{2xx} \zeta^2 t/p_2 \right] \quad (8.3.42)$$

$$A_{mn}^{zz} = \left(1 + \frac{\epsilon_0}{\epsilon_2} \Delta n^2 \right) D_{mn} + \frac{\epsilon_0}{\epsilon_2} \frac{\Delta z}{4\pi} \Delta n^2 \int_0^{\infty} d\zeta g_{m_x n_x}^e(\zeta) \left[\Phi_{m_x n_x}^{1zz} + p_2 t \Phi_{m_x n_x}^{2zz} \right] \quad (8.3.43)$$

$$A_{mn}^{xz} = -\frac{\epsilon_0 t \Delta z}{\epsilon_2 4\pi} \Delta n^2 \int_0^{\infty} d\zeta \zeta g_{m_x n_x}^o(\zeta) \left[\Phi_{m_x n_x}^{1xz} + \Phi_{m_x n_x}^{2xz} \right] \quad (8.3.44)$$

where

$$g_{m_x n_x}^e(\zeta) = 2 \operatorname{Re} \{ g_{m_x n_x}(\zeta) \} = 2 \int_{-a}^a \int_{-a}^a \cos [\zeta (z - z')] Z_{m_x}(z) Z_{n_x}(z') \frac{dz dz'}{(\Delta z)^2} \quad (8.3.45)$$

$$g_{m_x n_x}^o(\zeta) = 2 \operatorname{Im} \{ g_{m_x n_x}(\zeta) \} = 2 \int_{-a}^a \int_{-a}^a \sin [\zeta (z - z')] Z_{m_x}(z) Z_{n_x}(z') \frac{dz dz'}{(\Delta z)^2} \quad (8.3.46)$$

Let's evaluate the integrations with respect to z and z' . After defining

$$\begin{aligned}
g_n(\zeta) &= \int_{-a}^a e^{j\zeta z} Z_n(z) \frac{dz}{\Delta z} \\
&= \frac{k e^{j\zeta z_n}}{\Delta z (k^2 - \zeta^2) \sin(k\Delta z)} \begin{cases} e^{j\zeta\Delta z} - j(\zeta/k) \sin(k\Delta z) - \cos(k\Delta z) & n = 1 \\ 2 [\cos(\zeta\Delta z) - \cos(k\Delta z)] & 2 \leq n \leq N_z - 1 \\ e^{-j\zeta\Delta z} + j(\zeta/k) \sin(k\Delta z) - \cos(k\Delta z) & n = N_z \end{cases} \quad (8.3.47)
\end{aligned}$$

then

$$g_{m_z n_z}(\zeta) = g_{m_z}(\zeta) g_{n_z}(-\zeta) \quad (8.3.48)$$

and the real and imaginary parts of $g_{m_z n_z}(\zeta)$ are divided into two groups as follows

$$g_{m_z n_z}^e(\zeta) = \begin{cases} g_l^{e1}(\zeta) & l = m_z - n_z \quad m_z = 1, N_z \text{ or } n_z = 1, N_z \\ g_l^{e2}(\zeta) & l = m_z - n_z \quad \text{otherwise} \end{cases} \quad (8.3.49)$$

$$g_{m_z n_z}^o(\zeta) = \begin{cases} g_l^{o1}(\zeta) & l = m_z - n_z \quad m_z = 1, N_z \text{ or } n_z = 1, N_z \\ g_l^{o2}(\zeta) & l = m_z - n_z \quad \text{otherwise} \end{cases} \quad (8.3.50)$$

$g_l^{e1}(\zeta)$, $g_l^{e2}(\zeta)$, $g_l^{o1}(\zeta)$, and $g_l^{o2}(\zeta)$ are given in the summary below.

SUMMARY

$$\sum_{\beta=x,z} \sum_{n=1}^N A_{mn}^{\alpha\beta} a_{\beta n} = F_{\alpha m} \quad \begin{matrix} \alpha = x, z \\ m = 1, 2, \dots, N \end{matrix} \quad (8.3.51)$$

$$A_{mn}^{xx} = \delta_{m_z} D_{mn} + \frac{\epsilon_0 \Delta z}{\epsilon_2 4\pi} \Delta n^2 B_{m_z n_z}^{xx}(m_z, n_z) \quad (8.3.52)$$

$$A_{mn}^{zz} = \left(1 + \frac{\epsilon_0}{\epsilon_2} \Delta n^2\right) D_{mn} + \frac{\epsilon_0 \Delta z}{\epsilon_2 4\pi} \Delta n^2 B_{m_z n_z}^{zz}(m_z, n_z) \quad (8.3.53)$$

$$A_{mn}^{xz} = -\frac{\epsilon_0 \Delta z}{\epsilon_2 4\pi} \Delta n^2 \operatorname{sgn}(m_z - n_z) B_{m_z n_z}^{xz}(m_z, n_z) \quad (8.3.54)$$

$$A_{zx}^{mn} = A_{xz}^{nm} \quad (8.3.55)$$

$$B_{m_x n_x}^{\alpha\beta}(m_z, n_z) = \begin{cases} B_{m_x n_x}^{\alpha\beta 1}(l) & l = |m_z - n_z| \quad m_z = 1, N_z \text{ or } n_z = 1, N_z \\ B_{m_x n_x}^{\alpha\beta 2}(l) & l = |m_z - n_z| \quad \text{otherwise} \end{cases} \quad (8.3.56)$$

$$B_{m_x n_x}^{xxi}(l) = \frac{2k^2}{[\Delta z \sin(k\Delta z)]^2} \int_0^\infty d\zeta g_l^{ei}(\zeta) \Phi_{m_x n_x}^{xx} \quad (8.3.57)$$

$$B_{m_x n_x}^{zzi}(l) = \frac{2k^2}{[\Delta z \sin(k\Delta z)]^2} \int_0^\infty d\zeta g_l^{ei}(\zeta) \Phi_{m_x n_x}^{zz} \quad (8.3.58)$$

$$B_{m_x n_x}^{xzi}(l) = \frac{2k^2}{[\Delta z \sin(k\Delta z)]^2} \int_0^\infty d\zeta g_l^{oi}(\zeta) \Phi_{m_x n_x}^{xz} \quad (8.3.59)$$

$i = 1, 2$ in (8.3.57-59), and where

$$m = (m_x - 1)N_z + m_z \quad (8.3.60)$$

$$n = (n_x - 1)N_z + n_z \quad (8.3.61)$$

$$D_{mn} = \frac{\delta_{m_x n_x}}{4 \sin^2(k\Delta z)} \left\{ \left[1 - \frac{\sin(2k\Delta z)}{2k\Delta z} \right] \bar{\delta}_{m_z} \delta_{m_z n_z} + \left[\frac{\sin(k\Delta z)}{k\Delta z} - \cos(k\Delta z) \right] \delta_{|m_z - n_z|, 1} \right\} \quad (8.3.62)$$

with

$$\delta_m = \begin{cases} 2 & m = 1 \\ 1 & m > 1 \end{cases} \quad (8.3.63)$$

$$\delta_{mn} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \quad (8.3.64)$$

$$\bar{\delta}_{m_z} = \begin{cases} 1 & m_z = 1, N_z \\ 2 & m_z = 2, \dots, N_z - 1 \end{cases} \quad (8.3.65)$$

and

$$\Phi_{m_x n_x}^{xx} = -\Phi_{m_x n_x}^{1xx} + \Phi_{m_x n_x}^{2xx} \zeta^2 t / p_2 \quad (8.3.66)$$

$$\Phi_{m_x n_x}^{zz} = \Phi_{m_x n_x}^{1zz} + p_2 t \Phi_{m_x n_x}^{2zz} \quad (8.3.67)$$

$$\Phi_{m_x n_x}^{xz} = \zeta t [\Phi_{m_x n_x}^{1xz} + \Phi_{m_x n_x}^{2xz}] \quad (8.3.68)$$

$$\Phi_{mn}^{1xx} = \frac{[(k_2 t)^2 - [(m-1)\pi]^2] \delta_m \delta_n}{(p_2 t)^2 + [(m-1)\pi]^2} - \frac{p_2 \zeta^2 t^3 [1 + (-1)^m e^{-p_2 t}] [1 + (-1)^{m+n}]}{[(p_2 t)^2 + [(m-1)\pi]^2] [(p_2 t)^2 + [(n-1)\pi]^2]} \quad (8.3.69)$$

$$\Phi_{mn}^{1zz} = -\frac{[(2m-1)\pi/2]^2 \delta_{mn}}{(p_2 t)^2 + [(2m-1)\pi/2]^2} - \frac{p_2 t \{ (p_2 t)^2 - (-1)^{m+n} (2m-1)(2n-1)\pi^2/4 - p_2 t e^{-p_2 t} [(2m-1)(-1)^m + (2n-1)(-1)^n] \pi/2 \}}{[(p_2 t)^2 + [(2m-1)\pi/2]^2] [(p_2 t)^2 + [(2n-1)\pi/2]^2]} \quad (8.3.70)$$

$$\Phi_{mn}^{1xz} = \frac{1}{(p_2 t)^2 + [(2n-1)\pi/2]^2} \left\{ -\frac{2(2n-1)^2}{(2n-1)^2 - 4(m-1)^2} + \frac{p_2 t [1 + (-1)^m e^{-p_2 t}] [p_2 t + (-1)^{m+n} (2n-1)\pi/2]}{(p_2 t)^2 + [(m-1)\pi]^2} \right\} \quad (8.3.71)$$

$$\Phi_{mn}^{2xx} = \frac{(p_2 t)^2 \{ -[(-1)^m + e^{-p_2 t}] [(-1)^n + e^{-p_2 t}] + (1 - e^{-2p_2 t})^2 (p_1 - p_2 \varepsilon_1 / \varepsilon_2) / \bar{\Delta}_{e1} \}}{[(p_2 t)^2 + [(m-1)\pi]^2] [(p_2 t)^2 + [(n-1)\pi]^2]} \quad (8.3.72)$$

$$\Phi_{mn}^{2zz} = \frac{-[p_2 t e^{-p_2 t} + (-1)^m (2m-1)\pi/2] [p_2 t e^{-p_2 t} + (-1)^n (2n-1)\pi/2] + (p_2 t)^2 (1 + e^{-2p_2 t})^2 (p_1 - p_2 \varepsilon_1 / \varepsilon_2) / \bar{\Delta}_{e1}}{[(p_2 t)^2 + [(2m-1)\pi/2]^2] [(p_2 t)^2 + [(2n-1)\pi/2]^2]} \quad (8.3.73)$$

$$\Phi_{mn}^{2xz} = \frac{-p_2 t [(-1)^m + e^{-p_2 t}] [p_2 t e^{-p_2 t} + (-1)^n (2n-1)\pi/2] - (p_2 t)^2 (1 - e^{-4p_2 t}) (p_1 - p_2 \varepsilon_1 / \varepsilon_2) / \bar{\Delta}_{e1}}{[(p_2 t)^2 + [(m-1)\pi]^2] [(p_2 t)^2 + [(2n-1)\pi/2]^2]} \quad (8.3.74)$$

$$g_l^{e1}(\zeta) (k^2 - \zeta^2)^2$$

$$= \begin{cases} [\cos(\zeta \Delta z) - \cos(k \Delta z)]^2 + [\sin(\zeta \Delta z) - (\zeta/k) \sin(k \Delta z)]^2 & l = 0 \\ 2[\cos(\zeta \Delta z) - \cos(k \Delta z)] \{ [\cos(\zeta \Delta z) - \cos(k \Delta z)] \cos(l \zeta \Delta z) + [\sin(\zeta \Delta z) - (\zeta/k) \sin(k \Delta z)] \sin(l \zeta \Delta z) \} & 0 < l < N_z - 1 \\ \{ [\cos(\zeta \Delta z) - \cos(k \Delta z)]^2 - [\sin(\zeta \Delta z) - (\zeta/k) \sin(k \Delta z)]^2 \} \cos(l \zeta \Delta z) + 2[\cos(\zeta \Delta z) - \cos(k \Delta z)] [\sin(\zeta \Delta z) - (\zeta/k) \sin(k \Delta z)] \sin(l \zeta \Delta z) & l = N_z - 1 \end{cases} \quad (8.3.75)$$

$$g_l^{o1}(\zeta) (k^2 - \zeta^2)^2 = \begin{cases} 0 & l = 0 \\ \begin{aligned} & 2 [\cos(\zeta \Delta z) - \cos(k \Delta z)] \{ [\cos(\zeta \Delta z) - \cos(k \Delta z)] \sin(l \zeta \Delta z) \\ & - [\sin(\zeta \Delta z) - (\zeta/k) \sin(k \Delta z)] \cos(l \zeta \Delta z) \} \end{aligned} & 0 < l < N_z - 1 \\ \begin{aligned} & \{ \{ [\cos(\zeta \Delta z) - \cos(k \Delta z)]^2 - [\sin(\zeta \Delta z) - (\zeta/k) \sin(k \Delta z)]^2 \} \sin(l \zeta \Delta z) \\ & - 2 [\cos(\zeta \Delta z) - \cos(k \Delta z)] [\sin(\zeta \Delta z) - (\zeta/k) \sin(k \Delta z)] \cos(l \zeta \Delta z) \} \end{aligned} & l = N_z - 1 \end{cases} \quad (8.3.76)$$

$$g_l^{e2}(\zeta) = \frac{4 \cos(l \zeta \Delta z)}{(k^2 - \zeta^2)^2} [\cos(\zeta \Delta z) - \cos(k \Delta z)]^2 \quad (8.3.77)$$

$$g_l^{o2}(\zeta) = \frac{4 \sin(l \zeta \Delta z)}{(k^2 - \zeta^2)^2} [\cos(\zeta \Delta z) - \cos(k \Delta z)]^2 \quad (8.3.78)$$

$l = 0, \dots, N_z - 3$ in (8.3.77-78).

The forcing vector $F_{\alpha m}$ is computed in the same manner to be

$$\hat{x}F_{xm} + \hat{z}F_{zm} = E_0 \frac{2g_{m_i}(k_1 \sin \theta_i) \eta_2 \cos \theta_i k_2 t \cos \theta_r}{\eta_1 \cos \theta_i \cos[k_2 t \cos \theta_r] + j \eta_2 \cos \theta_r \sin[k_2 t \cos \theta_r]} \\ \left\{ \hat{x} \frac{\sin \theta_r \sin[k_2 t \cos \theta_r]}{(k_2 t \cos \theta_r)^2 - [(m_x - 1)\pi]^2} + \hat{z} \frac{j \cos \theta_r \cos[k_2 t \cos \theta_r]}{(k_2 t \cos \theta_r)^2 - [(2m_x - 1)\pi/2]^2} \right\} \quad (8.3.79)$$

for TM-polarized plane wave incidence, and

$$\hat{x}F_{xm} + \hat{z}F_{zm} = E_0 g_{m_i}(\zeta_p) \left\{ \hat{x} \frac{j \zeta_p t}{(q_2 t)^2 - [(m_x - 1)\pi]^2} - \hat{z} \frac{(\epsilon_1/\epsilon_2) q_2^2 t / p_1}{(q_2 t)^2 - [(2m_x - 1)\pi/2]^2} \right\} \quad (8.3.80)$$

for TM-even surface-wave mode incidence, and where

$$g_n(\zeta) = \frac{k e^{j\zeta z_n}}{\Delta z (k^2 - \zeta^2) \sin(k\Delta z)} \begin{cases} e^{j\zeta\Delta z} - j(\zeta/k) \sin(k\Delta z) - \cos(k\Delta z) & n = 1 \\ 2 [\cos(\zeta\Delta z) - \cos(k\Delta z)] & 2 \leq n \leq N_z - 1 \\ e^{-j\zeta\Delta z} + j(\zeta/k) \sin(k\Delta z) - \cos(k\Delta z) & n = N_z \end{cases} \quad (8.3.81)$$

Since the unknown electric field in the discontinuity region is expanded in sets of sinusoidal functions and piecewise-sinusoidal functions, then the amplitudes of the surface and radiation waves excited are obtained by substituting the expanded field in (4.5.15) and (4.5.16).

$$A_e^\pm = \frac{\epsilon_0 p_1 q_2 t^2 \Delta z \Delta n^2}{\epsilon_2 \tilde{\Delta}'_e(\zeta_p)} \sum_{n_x=1}^{N_x} \left\{ \pm \frac{\zeta_p \sin(q_2 t)}{(q_2 t)^2 - [(n_x - 1)\pi]^2} \sum_{n_z=1}^{N_z} a_{xn} g_{n_z}(\pm \zeta_p) \right. \\ \left. + \frac{j q_2 \cos(q_2 t)}{(q_2 t)^2 - [(2n_x - 1)\pi/2]^2} \sum_{n_z=1}^{N_z} a_{zn} g_{n_z}(\pm \zeta_p) \right\} \quad (8.3.82)$$

$$B_e(\theta) = \frac{\epsilon_0 j \Delta n^2 k_1^2 t \Delta z \cos \theta}{\epsilon_2 2\pi \tilde{\Delta}_e(-k_1 \sin \theta)} q_2 t \sum_{n_x=1}^{N_x} \left\{ \frac{j k_1 \sin \theta \sin(q_2 t)}{(q_2 t)^2 - [(n_x - 1)\pi]^2} \sum_{n_z=1}^{N_z} a_{xn} g_{n_z}(k_1 \sin \theta) \right. \\ \left. - \frac{q_2 \cos(q_2 t)}{(q_2 t)^2 - [(2n_x - 1)\pi/2]^2} \sum_{n_z=1}^{N_z} a_{zn} g_{n_z}(k_1 \sin \theta) \right\} \quad (8.3.83)$$

where $\tilde{\Delta}'_e(\zeta_p)$ and $\tilde{\Delta}_e(-k_1 \sin \theta)$ are given in Chapter 4.

Equation (8.3.56) allows a reduction in the amount of effort needed to fill the matrix. There are $4N^2 = 4N_x^2 N_z^2$ matrix elements, and $2N^2$ matrix elements should be calculated by integrations because $A_{mn}^{\alpha\beta} = A_{nm}^{\beta\alpha}$. But applying equation (8.3.56), only about $4N_x^2 N_z$ integrations are needed to compute [the properties $B_{m_x n_x}^{xxi}(l) = B_{n_x m_x}^{xxi}(l)$ and $B_{m_x n_x}^{zzi}(l) = B_{n_x m_x}^{zzi}(l)$ have been used]. The number of required integrations in the SPS Galerkin's method are more than times the $6N_x N_z$ integrations required in the pulse Galerkin's method for a given N_x and N_z . However, in the next section, it will be observed that

N_x and N_z in the SPS Galerkin's method will be much smaller than the N_x and N_z in the pulse Galerkin's method. Evidently, much computer time and the memory to store the matrix elements can be conserved.

8.4. NUMERICAL INTEGRATION ALONG REAL AXIS

In the last chapter, it is found that the integrations along the real axis in the pulse Galerkin's method converge faster than the integrations along the branch cut. When the complex analysis is applied to the matrix elements derived by the SPS Galerkin's method, it is found that the integrands in the integrals along the branch cut approach zero slower than those in integrals along the real axis when the integration variables are large. Thus, the real axis integrations will be used in the SPS Galerkin's method.

The $\Phi_{m_x n_x}^{\alpha\beta}$ in integrands of (8.3.57-59) are written in terms of exponential functions. It is easy to find the behavior of the integrands $\Phi_{m_x n_x}^{\alpha\beta} g_l^{si}(\zeta)$ ($\alpha, \beta = x, z$, $s = e, o$, and $i = 1, 2$) as $\zeta \rightarrow \infty$

$$\Phi_{m_x n_x}^{\alpha\beta} \rightarrow 1/\zeta \quad (8.4.1)$$

$$g_l^{s1}(\zeta) \rightarrow \begin{cases} 1/\zeta^2 & l = 0, N_z - 1 \\ 1/\zeta^3 & 0 < l < N_z - 1 \end{cases} \quad (8.4.2)$$

$$g_l^{s2}(\zeta) \rightarrow 1/\zeta^4 \quad (8.4.3)$$

It is noted that $g_l^{si}(\zeta)$ is not a single periodic function, consequently it is written as the sum of several such functions

$$\begin{aligned} [k^2 - \zeta^2]^2 g_0^{e1}(\zeta) &= 1 + \cos^2(k\Delta z) + (\zeta/k)^2 \sin^2(k\Delta z) - 2\cos(k\Delta z) \cos(\zeta\Delta z) \\ &\quad - 2(\zeta/k) \sin(k\Delta z) \sin(\zeta\Delta z) \end{aligned} \quad (8.4.4)$$

$$[k^2 - \zeta^2]^2 g_{N_z-1}^{e1}(\zeta) = \cos[(N_z - 3)\zeta\Delta z] - 2\cos(k\Delta z) \cos[(N_z - 2)\zeta\Delta z]$$

$$\begin{aligned}
& + [\cos^2(k\Delta z) - (\zeta/k)^2 \sin^2(k\Delta z)] \cos[(N_z - 1)\zeta\Delta z] \\
& + 2(\zeta/k) \sin(k\Delta z) \{ \cos(k\Delta z) \sin[(N_z - 1)\zeta\Delta z] - \sin[(N_z - 2)\zeta\Delta z] \} \quad (8.4.5)
\end{aligned}$$

$$\begin{aligned}
[k^2 - \zeta^2]^2 g_{N_z-1}^{o1}(\zeta) &= \sin[(N_z - 3)\zeta\Delta z] - 2\cos(k\Delta z) \sin[(N_z - 2)\zeta\Delta z] \\
& + [\cos^2(k\Delta z) - (\zeta/k)^2 \sin^2(k\Delta z)] \sin[(N_z - 1)\zeta\Delta z] \\
& - 2(\zeta/k) \sin(k\Delta z) \{ \cos(k\Delta z) \cos[(N_z - 1)\zeta\Delta z] - \cos[(N_z - 2)\zeta\Delta z] \} \quad (8.4.6)
\end{aligned}$$

$$\begin{aligned}
[k^2 - \zeta^2]^2 g_l^{e1}(\zeta) &= \cos[(l - 2)\zeta\Delta z] - 3\cos(k\Delta z) \cos[(l - 1)\zeta\Delta z] \\
& + [1 + 2\cos^2(k\Delta z)] \cos[l\zeta\Delta z] - \cos(k\Delta z) \cos[(l + 1)\zeta\Delta z] \\
& - (\zeta/k) \sin(k\Delta z) \{ \sin[(l - 1)\zeta\Delta z] - 2\cos(k\Delta z) \sin[l\zeta\Delta z] + \sin[(l + 1)\zeta\Delta z] \} \quad (8.4.7)
\end{aligned}$$

$$\begin{aligned}
[k^2 - \zeta^2]^2 g_l^{o1}(\zeta) &= \sin[(l - 2)\zeta\Delta z] - 3\cos(k\Delta z) \sin[(l - 1)\zeta\Delta z] \\
& + [1 + 2\cos^2(k\Delta z)] \sin[l\zeta\Delta z] - \cos(k\Delta z) \sin[(l + 1)\zeta\Delta z] \\
& + (\zeta/k) \sin(k\Delta z) \{ \cos[(l - 1)\zeta\Delta z] - 2\cos(k\Delta z) \cos[l\zeta\Delta z] + \cos[(l + 1)\zeta\Delta z] \} \quad (8.4.8)
\end{aligned}$$

for $l = 1, 2, \dots, N_z - 2$, and

$$\begin{aligned}
[k^2 - \zeta^2]^2 g_l^{e2}(\zeta) &= \cos[(l - 2)\zeta\Delta z] - 4\cos(k\Delta z) \cos[(l - 1)\zeta\Delta z] \\
& + 2[1 + 2\cos^2(k\Delta z)] \cos[l\zeta\Delta z] - 4\cos(k\Delta z) \cos[(l + 1)\zeta\Delta z] + \cos[(l + 2)\zeta\Delta z] \quad (8.4.9)
\end{aligned}$$

$$\begin{aligned}
[k^2 - \zeta^2]^2 g_l^{o2}(\zeta) &= \sin[(l - 2)\zeta\Delta z] - 4\cos(k\Delta z) \sin[(l - 1)\zeta\Delta z] \\
& + 2[1 + 2\cos^2(k\Delta z)] \sin[l\zeta\Delta z] - 4\cos(k\Delta z) \sin[(l + 1)\zeta\Delta z] + \sin[(l + 2)\zeta\Delta z] \quad (8.4.10)
\end{aligned}$$

for $l = 0, 1, \dots, N_z - 3$. Finally, the periodic integrand terms can be integrated separately for large ζ .

When $\zeta\Delta z$ is small, and $\zeta \approx k$,

$$\cos(\zeta \Delta z) - \cos(k \Delta z) = -(\zeta^2 - k^2) (\Delta z)^2 / 2 \quad (8.4.11)$$

If it is calculated numerically by using $\cos(\zeta \Delta z) - \cos(k \Delta z)$ directly, some significant digits will be lost. Thus,

$$\cos(\zeta \Delta z) - \cos(k \Delta z) = -2 \sin[(\zeta + k) \Delta z / 2] \sin[(\zeta - k) \Delta z / 2] \quad (8.4.12)$$

and

$$\begin{aligned} \sin(\zeta \Delta z) - (\zeta/k) \sin(k \Delta z) \\ = 2 \cos[(\zeta + k) \Delta z / 2] \sin[(\zeta - k) \Delta z / 2] + (1 - \zeta/k) \sin(k \Delta z) \end{aligned} \quad (8.4.13)$$

will be used for small ζ .

The points $\zeta = k$, $\zeta = k_2$, $(p_2 t)^2 + [(m_x - 1) \pi]^2 = 0$, and $(p_2 t)^2 + [(2n_x - 1) \pi / 2]^2 = 0$ do not lead poles of the integrand, even though $\zeta - k$, p_2 , $(p_2 t)^2 + [(m_x - 1) \pi]^2$, and $(p_2 t)^2 + [(2n_x - 1) \pi / 2]^2$ occur in its denominators. Numerical problems occur, however, if the integration is along real axis exactly. Furthermore, if the loss in region 2 becomes small or vanishes, the surface-wave poles will shift to locations along the real axis. Although the pole extraction can be used to deal with the surface-wave pole singularities, the integration will be calculated, for small ζ , along a deformed path above the real axis as shown in Figure 8.4.

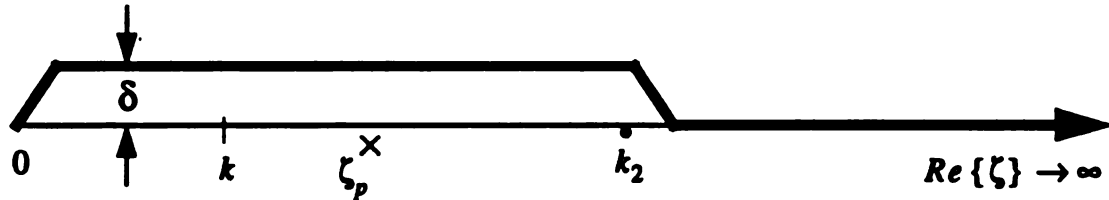


Figure 8.4. Integration along real axis in the SPS Galerkin's method.

It is important to choose an appropriate δ . If δ is too small, there are numerical difficulties in the vicinities of ζ_p , k , etc. If δ is too large, it is impossible to obtain enough significant digits in calculating $\cos [l\zeta\Delta z]$ and $\sin [l\zeta\Delta z]$. By our experience,

$$e^{-\delta 2a} < 10^{-6} \quad (8.4.14)$$

and

$$\delta/k_0 > 0.1 \quad (8.4.15)$$

should be chosen using double-precision variables, where $2a$ is the width of the gap. The above two equations can be combined to

$$0.1 < \delta/k_0 < \lambda_0/a \quad (8.4.16)$$

Let's consider a grounded dielectric sheet with a gap as shown Figure 2.1. A TM_0 surface wave is incident upon the discontinuity from right to left. It is assumed that the dielectric sheet is so thin that only TM_0 surface-wave mode exists. Then, a transmitted TM_0 surface wave will propagate to the left, a reflected TM_0 surface wave will propagate to the right, and radiated waves will be excited in space. Figure 8.5 shows the numerical convergence of the transmitted power for TM_0 surface-wave incidence, calculated by SPS and pulse Galerkin's methods as the partition numbers N_x and N_z of the discontinuity region are increased. It is observed that the SPS Galerkin's method converges much faster than the pulse method. To achieve the same accuracy, N_x and N_z in the SPS Galerkin's method are only about one third of N_x and N_z in the pulse method for the parameters given in Figure 8.5. Thus, the number of linear equations implicated in the SPS Galerkin's method is only one ninth of that in the corresponding pulse method. It is also found that the relative difference between the numerical total power and the incident power is about 0.001%. Therefore, SPS Galerkin's method can save much memory space and computer time.

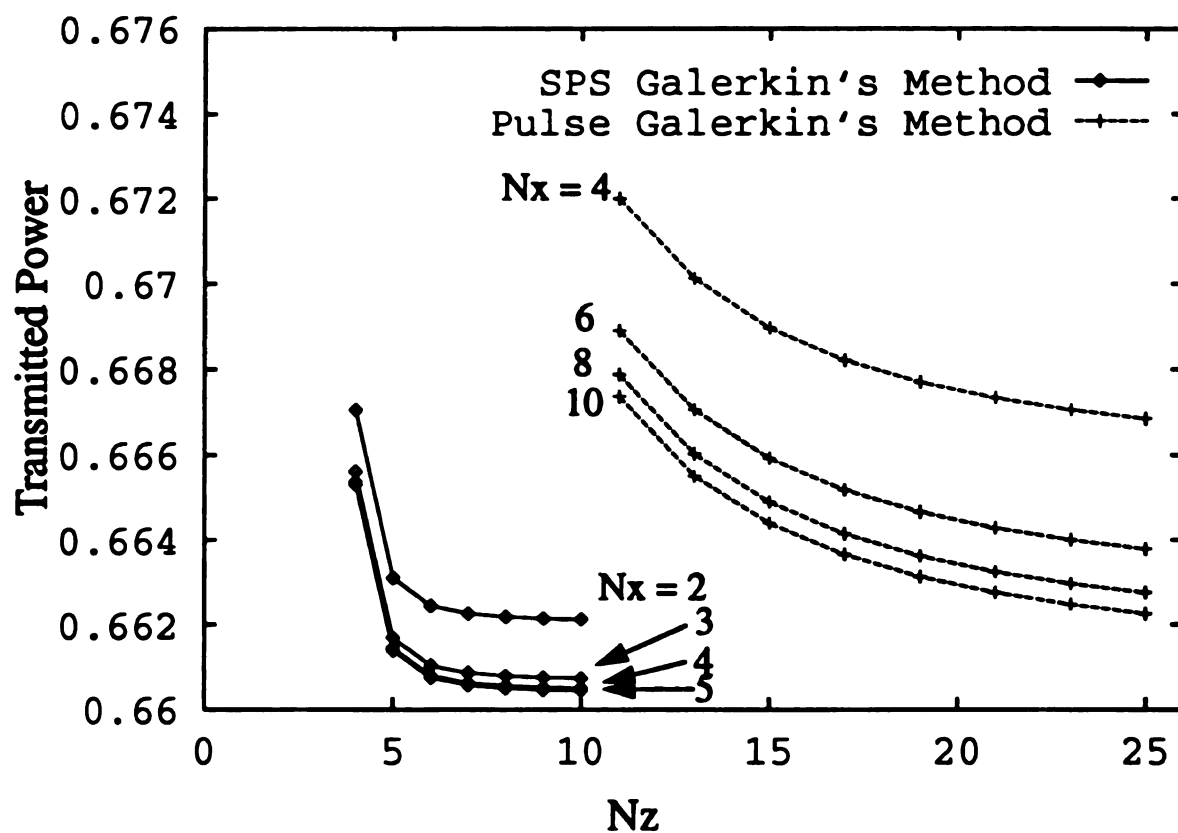


Figure 8.5. Convergence check on the transmitted power computed with different N_x and N_z by sinusoidal and piecewise-sinusoidal (SPS) Galerkin's method and pulse Galerkin's method when a TM_0 surface wave is incident upon an air gap in the grounded sheet. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 2.1316$, $2a/\lambda_0 = 1$, and $k_0t = 2.5$)

CHAPTER 9

NUMERICAL RESULTS

9.1. INTRODUCTION

FORTRAN programs have been written to implement the MoM solution for the interaction of EM waves with a discontinuity in a grounded dielectric sheet for both pulse and SPS Galerkin's methods. The width and thickness are normalized by the wavelength in free space.

For the chosen parameters of the dielectric sheet the roots of the eigenvalue equation are found by the Newton-Raphson method [66]. Next, the moment matrix is filled by using integration along the real axis or along the branch cuts, then the linear equations are solved for the total electric field in the discontinuity region by using a standard numerical technique. Finally, the excited surface wave and the far-zone field are calculated. Since the equivalent scattering width has the dimension of length, it is normalized by the wavelength in free space.

The numerical results for both TE- and TM-polarized surface-wave incidence will be presented and compared with those of existing studies. Then, the electric field distributions, radiation patterns, and back scattering width for TE- and TM-polarized plane wave incidence will be given in Sections 9.3 and 9.4, respectively. Finally, the back scattering width for a lossy dielectric sheet will be calculated.

9.2. NUMERICAL RESULTS FOR SURFACE-WAVE INCIDENCE

It is assumed that region 1 and the discontinuity region are air ($\epsilon_{1r} = \epsilon_{dr} = 1$) throughout in the following numerical calculations. There are some results for TE- and TM-polarized surface-wave incidence to compare different integration methods and check the convergence. Now, more numerical results for surface-wave incidence will be given.

Let's consider a grounded dielectric sheet with a gap as shown Figure 6.5. If a TE_1 surface wave is incident upon the discontinuity from right to left, a transmitted TE_1 surface wave will propagate to the left, a reflected TE_1 surface wave will propagate to the right, and a radiated wave will be excited in space. Figure 9.1 shows the reflected power, indicated by the solid line, when a TE_1 surface wave is incident upon an air gap in the grounded sheet. It is observed that when the width of the air gap increases, the reflected power oscillates and converges to a constant. When the width is infinite, the problem becomes that of power reflected by an abruptly ended grounded sheet. This case was studied by Shigesawa and Tsuji [19] by the mode-expansion method and their results are indicated by the dashed line. The asymptotic value of our results for an infinite width gap agrees very well with reference [19]. Figure 9.2 plots the asymptotic reflected power of the MoM solution of EFIE and the reflected power computed by Shigesawa and Tsuji [19] by the mode-expansion method when a TE_1 surface wave propagates through a step discontinuity in a grounded sheet as a function of the thickness ratio. Some deviations are observed only when the power reflection is very small.

In Figure 9.3, the radiated power, transmitted power, and reflected power normalized by the incident power are plotted as functions of the width of the discontinuity for $k_0 t = 1$ and $\epsilon_{2r} = 5$. It is found that the sum of the normalized transmitted power, reflected power and radiated power is almost equal to 1.

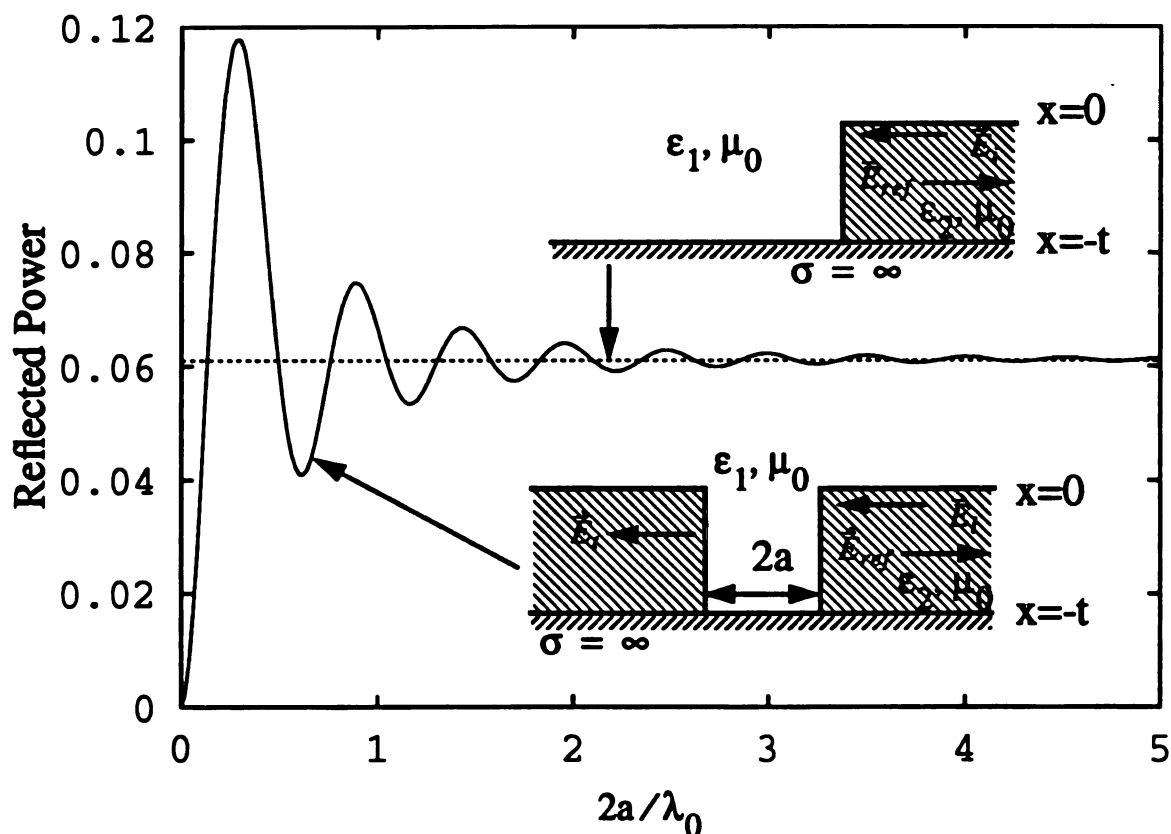


Figure 9.1. Comparison of the reflected power computed with MoM when a TE_1 surface wave is incident upon an air gap in the grounded sheet (solid line) and that computed by Shigesawa and Tsuji [19] by the mode-expansion method when a TE_1 surface wave is incident upon an abruptly ended grounded sheet (dashed line). ($\epsilon_{2r} = 2.1316$, $k_0 t = 2.5$, $N_x = 10$, and $N_z = 2$ to 100)

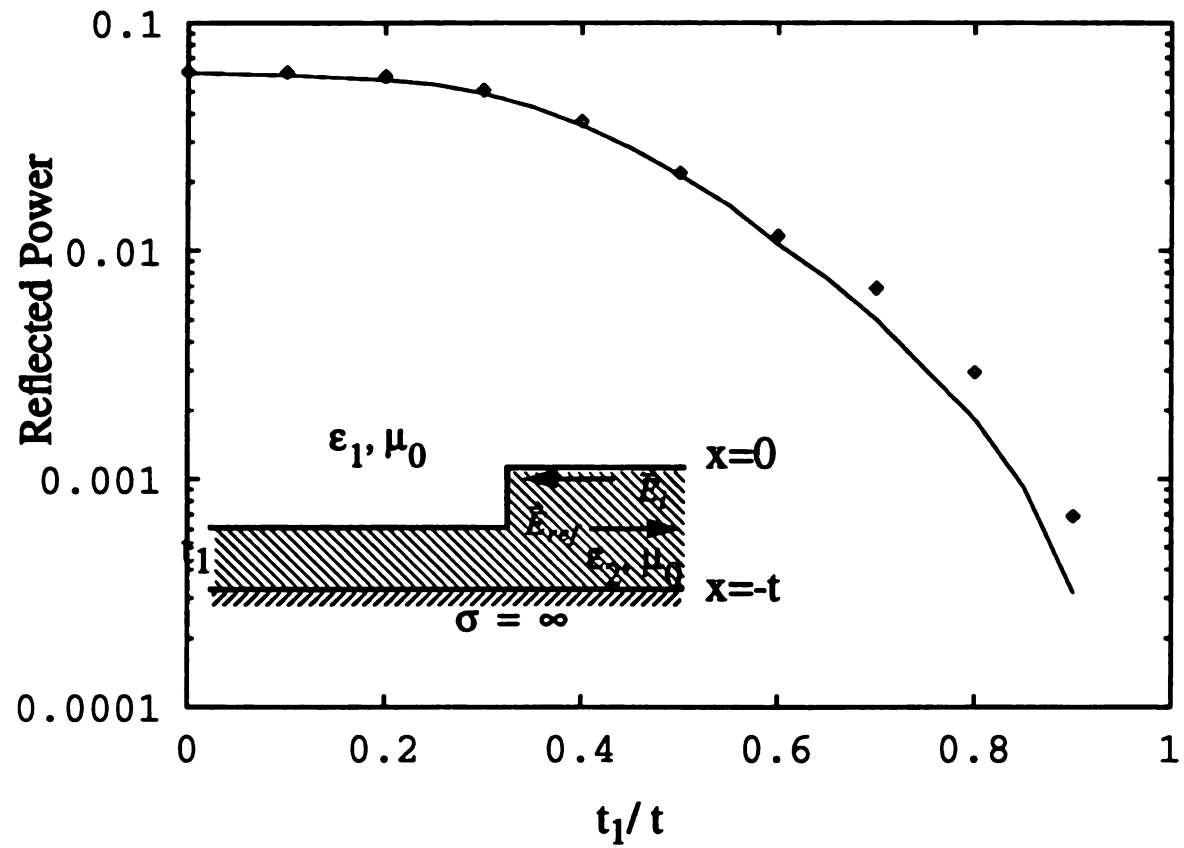


Figure 9.2. The reflected power as a function of t_1/t when a TE_1 surface wave propagates through a step discontinuity in the grounded dielectric sheet computed by MoM (points) and by the mode-expansion method (line) [19]. ($\epsilon_{2r} = 2.1316$)

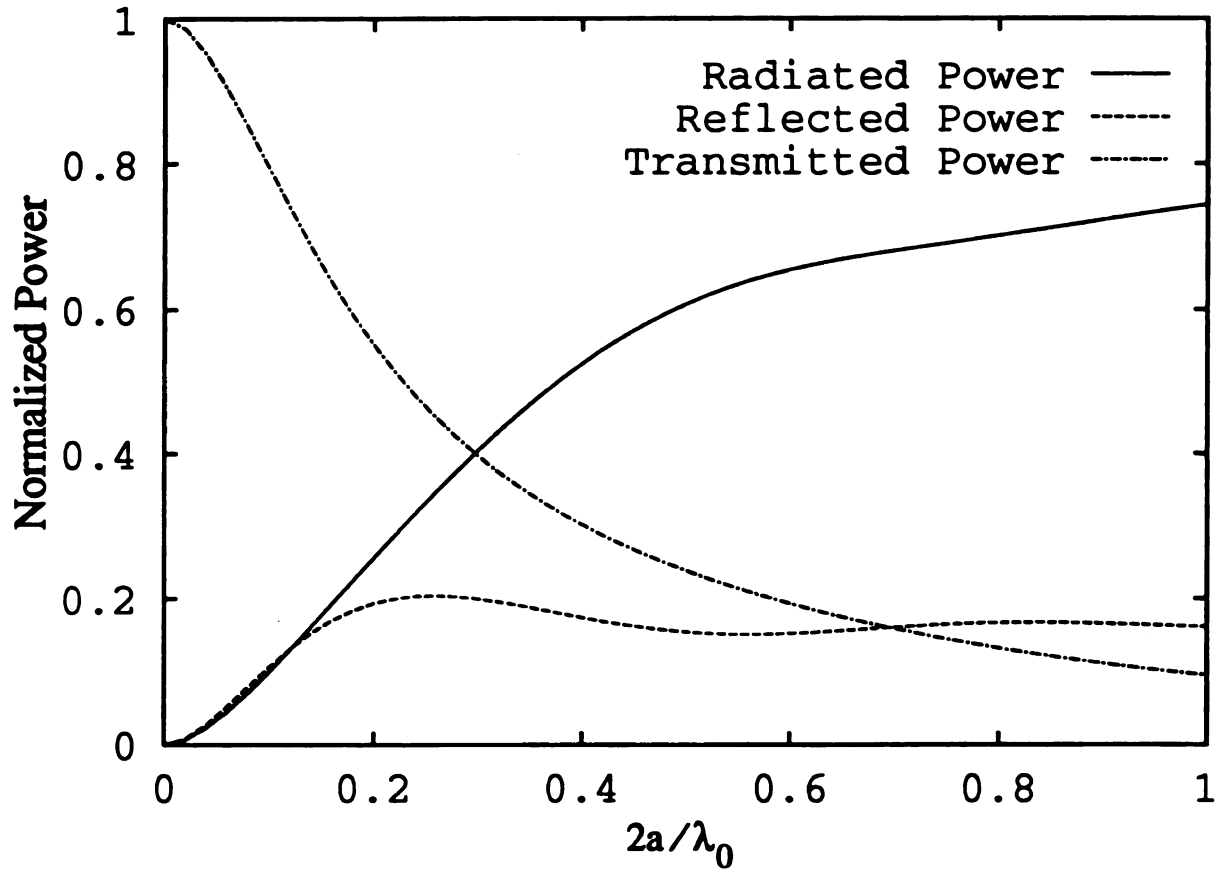


Figure 9.3. The radiated power, transmitted power, and reflected power normalized by the incident power as functions of the width of the discontinuity when a TE_1 surface wave is incident upon an air gap in the grounded sheet. ($\epsilon_{2r} = 5$, $k_0 t = 1$, $N_x = 8$, and $N_z = 2$ to 50)

Since only TM-even surface-wave modes can be excited in a grounded sheet, a TM-even surface wave incidence will be considered. Figure 9.4 shows the reflected power, indicated by the solid line, when a TM_0 surface wave is incident upon an air gap in the grounded sheet. It is observed that when the width of the air gap increases, the reflected power oscillates and converges to a constant. When the width is infinite, the problem becomes that of power reflected by an abruptly ended grounded sheet, which was studied by Shigesawa and Tsuji [19] by the mode-expansion method and their results are indicated by the dashed line. The asymptotic value of our results for an infinite width gap agrees very well with reference [19]. Thus, to study the power reflected by an abruptly ended grounded sheet for surface-wave incidence, we can first calculate the power reflected by a finite width gap with MoM is calculated, then, find the asymptotic value for an infinite width gap.

When the thickness of the dielectric sheet increases or the permittivity increases, more TM-even surface-wave modes can be excited. When TM_0 surface wave is incident from right to left in an abruptly ended grounded sheet (see the geometry in Figure 9.5), a reflected TM_0 surface wave will propagate to the right, a radiated wave will be excited in space, and TM-even higher order surface-wave modes will be excited and propagate to the right if the higher order modes can be supported by that sheet. Figure 9.5 plots the amplitudes of the reflection coefficient a_0 and the coupling coefficient a_2 to TM_2 mode computed by SPS Galerkin's method (points) and by Neuman series [14] (lines) as functions of $k_0 t$ when a TM_0 surface wave is incident in an abruptly ended grounded sheet for $\epsilon_{2r} = 20$. It is found that our results agrees very well with reference [14]. Gelin et al. [14] derived coupled integral equations on discrete and continuous wave amplitudes of the modal fields at the discontinuity. These coupled integral equations are solved by an iterative procedure, namely the Neuman series. Figure 9.6 shows the amplitudes of the reflection coefficient a_2 and the coupling coefficient a_0 to TM_0 mode computed by SPS Galerkin's method (points) and by Neuman series [14] (lines) as functions of $k_0 t$ when a TM_2 surface

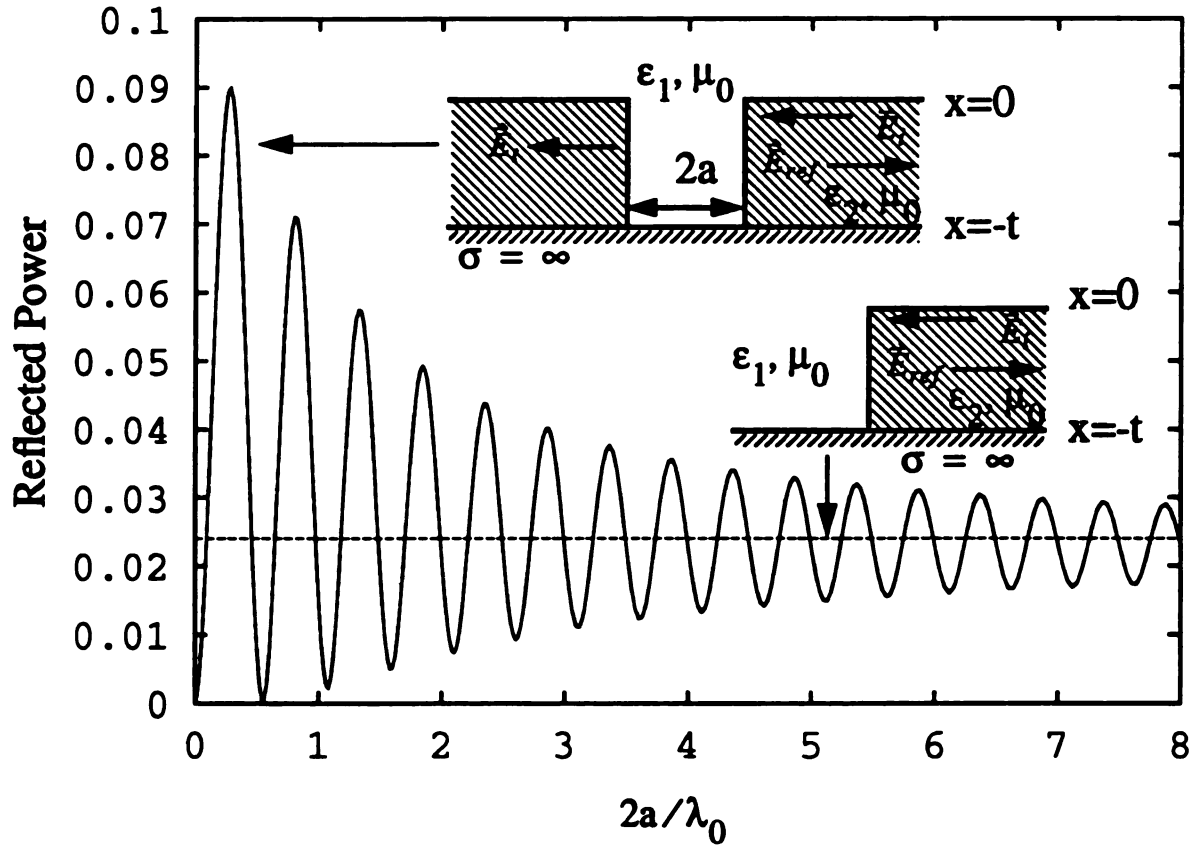


Figure 9.4. Comparison of the reflected power computed with MoM when a TM_0 surface wave is incident upon an air gap in the grounded sheet (solid line) and that computed by Shigesawa and Tsuji [19] by the mode-expansion method when a TM_0 surface wave is incident in an abruptly ended grounded sheet (dashed line). ($\epsilon_{2r} = 2.1316$, $k_0 t = 2.5$, SPS Galerkin's method with $N_x = 4$ and $N_z = 2$ to 62)

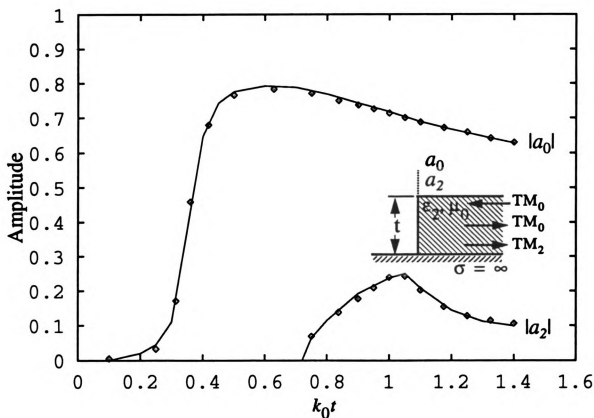


Figure 9.5. Amplitudes of the reflection coefficient a_0 and the coupling coefficient a_2 to TM_2 mode computed by SPS Galerkin's method (points) and by Neuman series [14] (lines) as functions of $k_0 t$ when a TM_0 surface wave is incident in an abruptly ended grounded sheet for $\epsilon_{2r} = 20$.

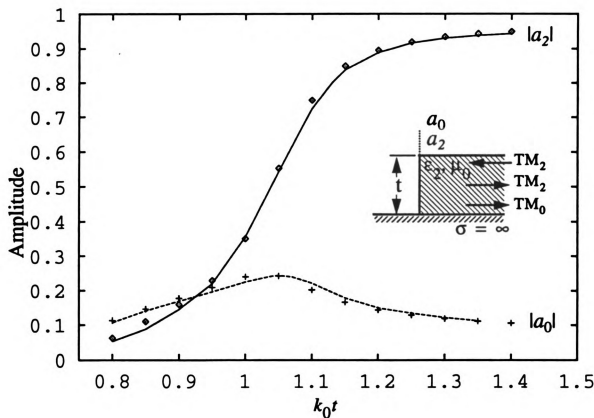


Figure 9.6. Amplitudes of the reflection coefficient a_2 and the coupling coefficient a_0 to TM_0 mode computed by SPS Galerkin's method (points) and by Neuman series [14] (lines) as functions of $k_0 t$ when a TM_2 surface wave is incident in an abruptly ended grounded sheet for $\epsilon_{2r} = 20$.

wave is incident in an abruptly ended grounded sheet. It is also found that our results agree very well with the reference [14]. It is noted that power conservation in [14] is verified to an accuracy of better than 1%, while our results on power conservation has an accuracy of better than 0.1% for the parameters given in Figures 9.5 and 9.6.

In Figure 9.7, the radiated power, transmitted power, and reflected power normalized by the incident power are plotted as functions of the width of the discontinuity for $k_0 t = 1$ and $\epsilon_{2r} = 5$. It is also found that the sum of the normalized transmitted power, reflected power and radiated power is almost equal to 1.

In this section, the computer programs are tested by using surface wave incidence, and find that our results agree very well with those of existing studies. The problem of surface wave scattering by a discontinuity along a dielectric waveguide is very important, because a good understanding of this problem is essential in the design and development of optical and millimeter-wave components. A discontinuity problem arises in many forms such as in the splicing of two dielectric waveguides or in inter-device coupling of millimeter and optical integrated circuits.

In the next two sections, numerical results for TE- and TM-polarized plane wave incidences will be given.

9.3. NUMERICAL RESULTS FOR TE-POLARIZED PLANE WAVE INCIDENCE

It is clear that when the discontinuity is narrow ($2a/\lambda_0 \ll 1$), the total electric field in the discontinuity should converge to the incident field for TE-polarized plane wave incidence. This phenomenon is shown in Figures 9.8 and 9.9.

Figures 9.10 and 9.11 show the spatial intensity distribution of the normalized total electric field in the discontinuity when a TE-polarized plane wave is incident upon the

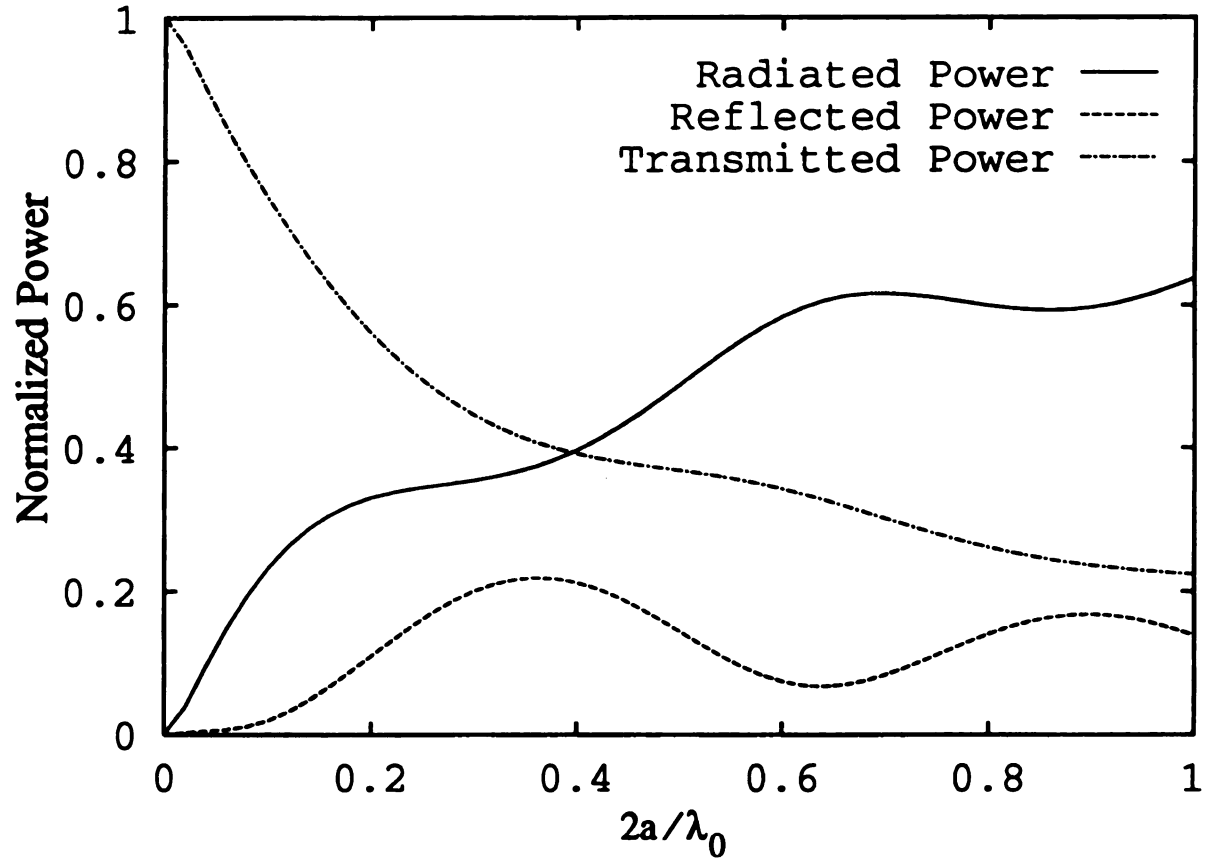


Figure 9.7. The radiated power, transmitted power, and reflected power normalized by the incident power as functions of the width of the discontinuity when a TM_0 surface wave is incident upon an air gap in the grounded sheet. ($\epsilon_{2r} = 5$, $k_0 t = 1$, SPS Galerkin's method with $N_x = 3$ and $N_z = 3$ to 20)

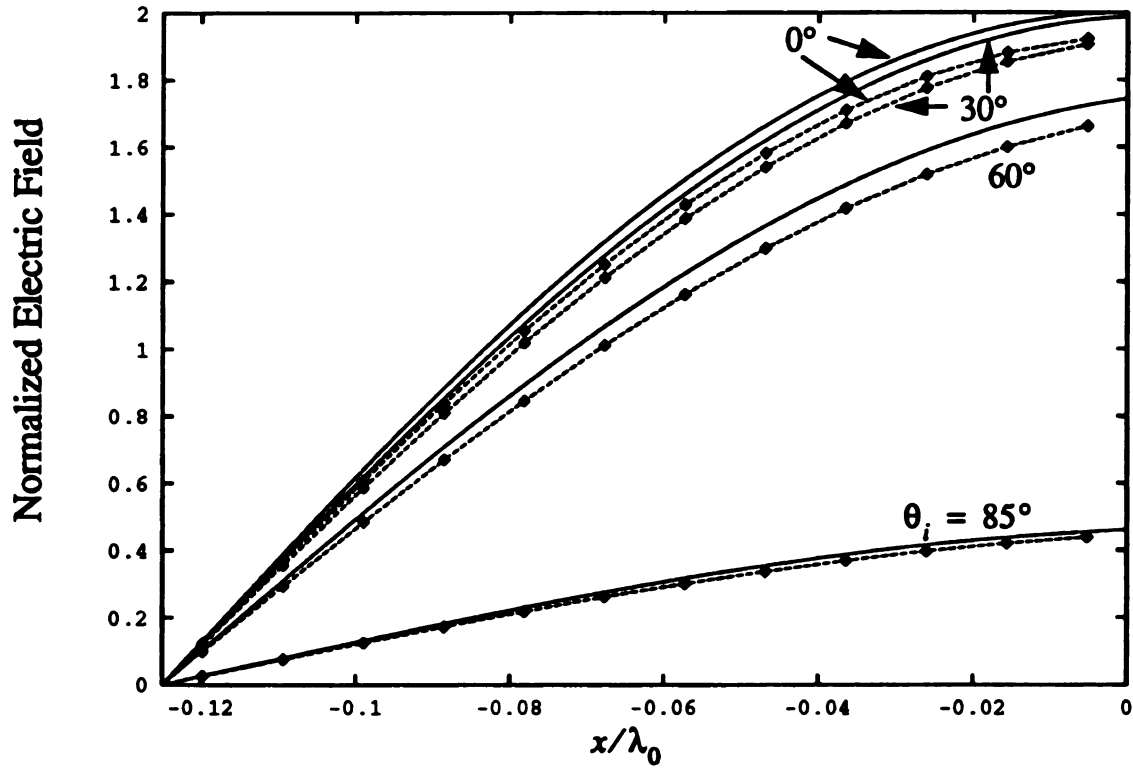


Figure 9.8. Comparison of the normalized total electric field amplitude $|E_{2y}/E_0|$ (dashed line) and the incident field $|E_{2y}^i/E_0|$ (solid line) when a TE-polarized plane wave is incident upon a narrow gap in a grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.01$, $N_z = 1$, $t/\lambda_0 = 0.125$ and $N_x = 12$)

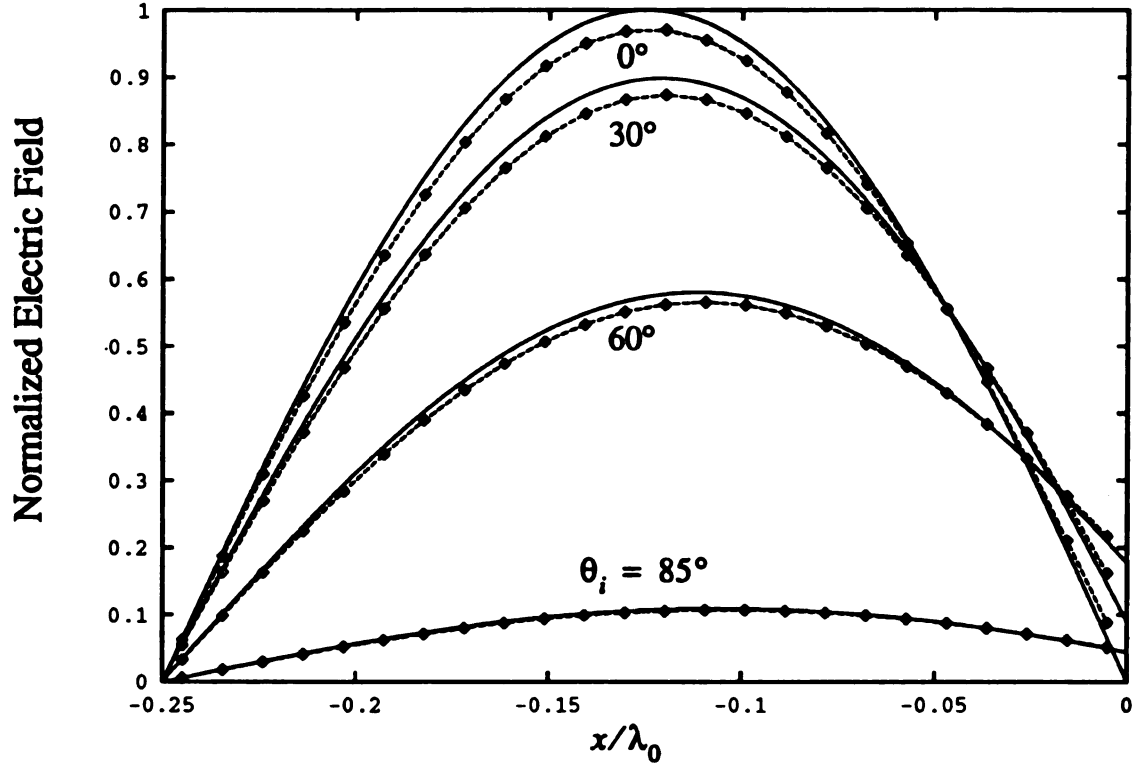


Figure 9.9. The comparison of the normalized total electric field amplitude $|E_{2y}/E_0|$ (dashed line) and the incident field $|E_{2y}^i/E_0|$ (solid line) when a TE-polarized plane wave is incident upon a narrow gap in a grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.01$, $N_z = 1$, $t/\lambda_0 = 0.25$ and $N_x = 24$)

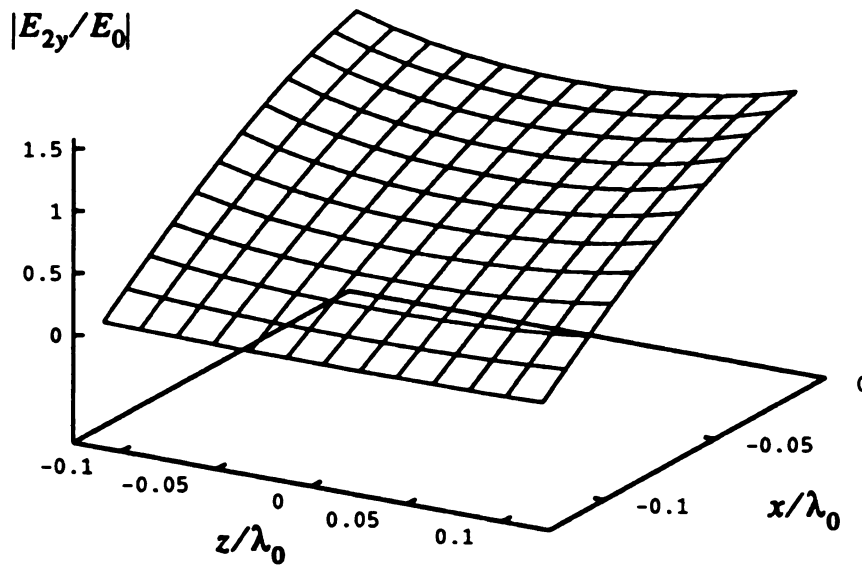


Figure 9.10. Spatial intensity distribution of the normalized total electric field in the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet normally ($\theta_i = 0^\circ$). ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.25$, $N_z = 13$, $t/\lambda_0 = 0.125$ and $N_x = 12$)

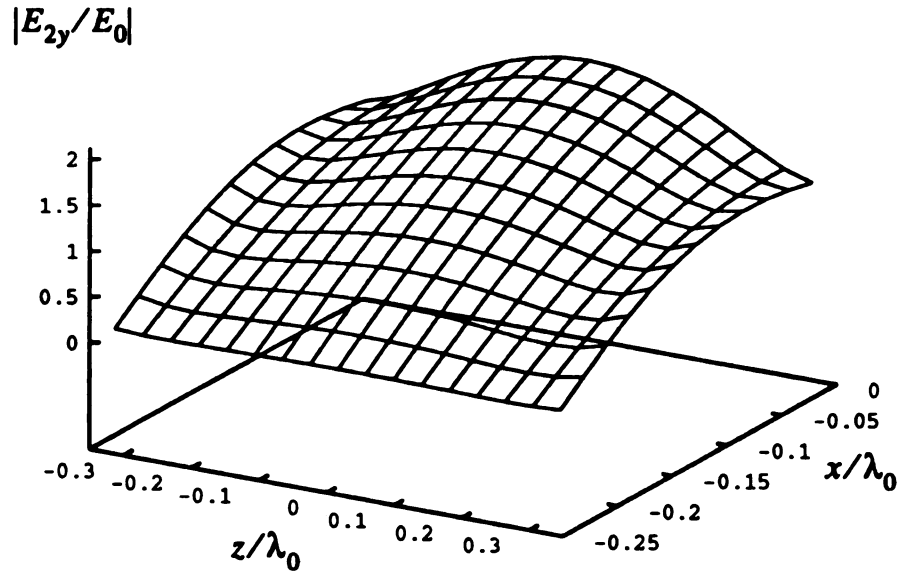


Figure 9.11. Spatial intensity distribution of the normalized total electric field in the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet normally ($\theta_i = 0^\circ$). ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.7$, $N_z = 17$, $t/\lambda_0 = 0.25$ and $N_x = 12$)

sheet normally ($\theta_i = 0^\circ$). It is found that the field distribution is symmetric with respect to the $z = 0$ plane.

Figures 9.12 and 9.13 show the normalized radiation patterns of the radiated far field when a plane wave is incident upon the discontinuity at various incidence angles θ_i .

The normalized back scattering width and the equivalent scattering width of the excited surface-wave mode as functions of the thickness t/λ_0 of the discontinuity are shown in Figures 9.14 and 9.15, respectively. Comparing these two figures, it is observed that if t is less than a critical value t_c , no surface-wave mode is excited and the back scattering width increases as the thickness increases. On the other hand, if t is greater than the critical value t_c , the surface-wave-mode is excited and the back scattering width decreases slowly as the thickness increases. This is expected since only TE-odd modes can be excited in a grounded sheet. Since the cut-off thickness of the TE_n mode is [1]

$$\frac{t_{cn}}{\lambda_0} = \frac{n}{4\sqrt{\epsilon_{2r}\mu_{2r} - \epsilon_{1r}\mu_{1r}}} \quad n = 1, 3, 5, \dots \quad (9.3.1)$$

it is observed that the lowest TE-odd mode is the TE_1 and the cut-off thickness is $t_{c1}/\lambda_0 = 0.1443375$ for the parameters given in Figures 9.14 and 9.15.

Figures 9.16 and 9.17 show the back scattering width as functions of the width of the discontinuity for $t/\lambda_0 = 0.125$ and $t/\lambda_0 = 0.25$, respectively. When $t/\lambda_0 = 0.25$, the TE_1 surface-wave mode can be excited. The normalized equivalent scattering width of the excited TE_1 mode is shown in Figure 9.18.

9.4. NUMERICAL RESULTS FOR TM-POLARIZED PLANE WAVE INCIDENCE

It was found that when the discontinuity of the dielectric sheet is narrow ($2a/\lambda_0 \ll 1$), the total electric field in the discontinuity converges to the electric field in a zero-width

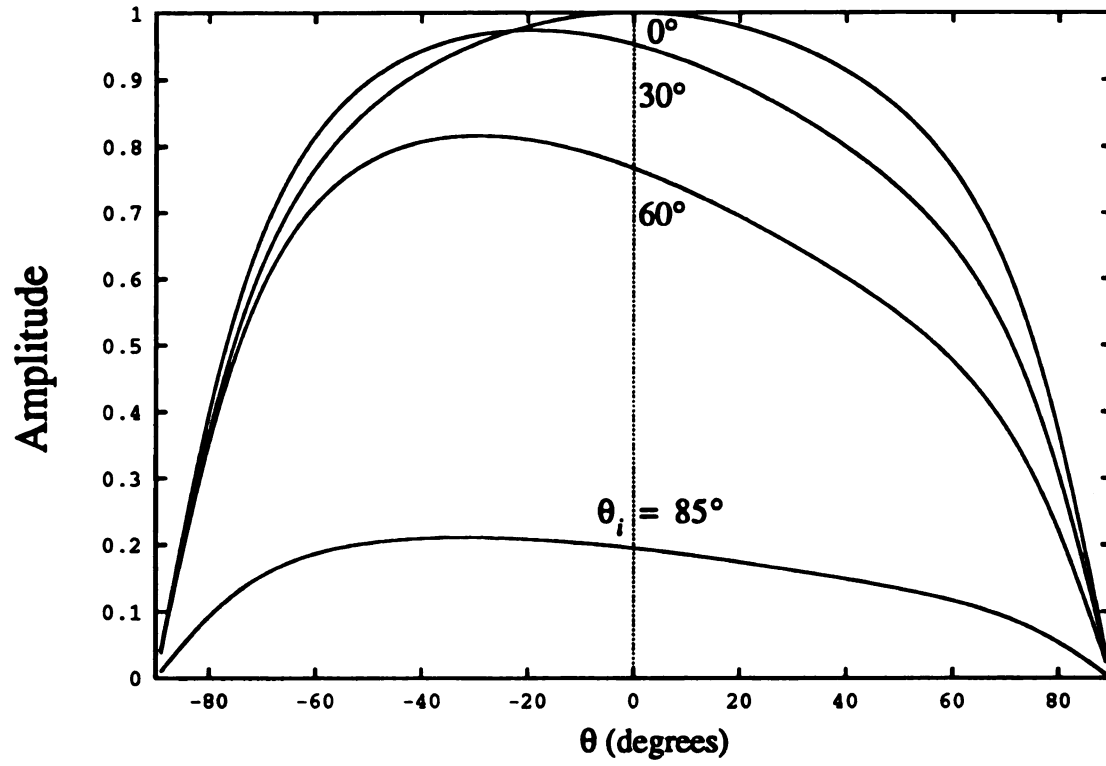


Figure 9.12. Radiation patterns of the far field when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.25$, $N_z = 13$, $t/\lambda_0 = 0.125$ and $N_x = 12$)

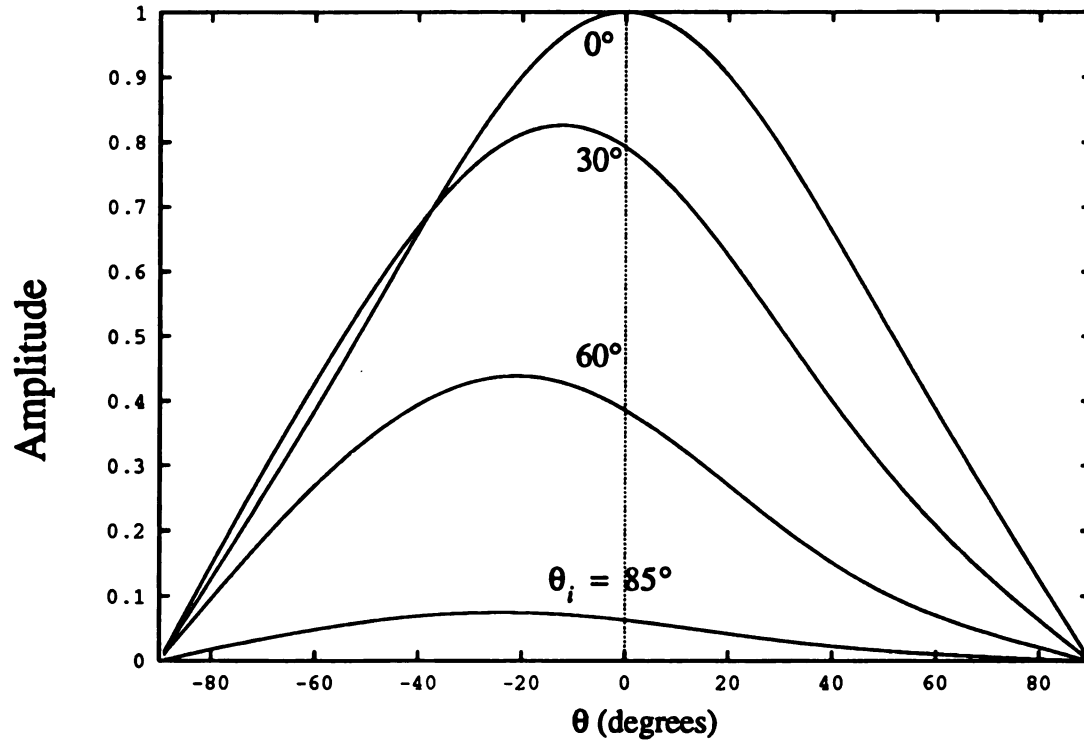


Figure 9.13. Radiation patterns of the far field when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.7$, $N_z = 17$, $t/\lambda_0 = 0.25$ and $N_x = 12$)

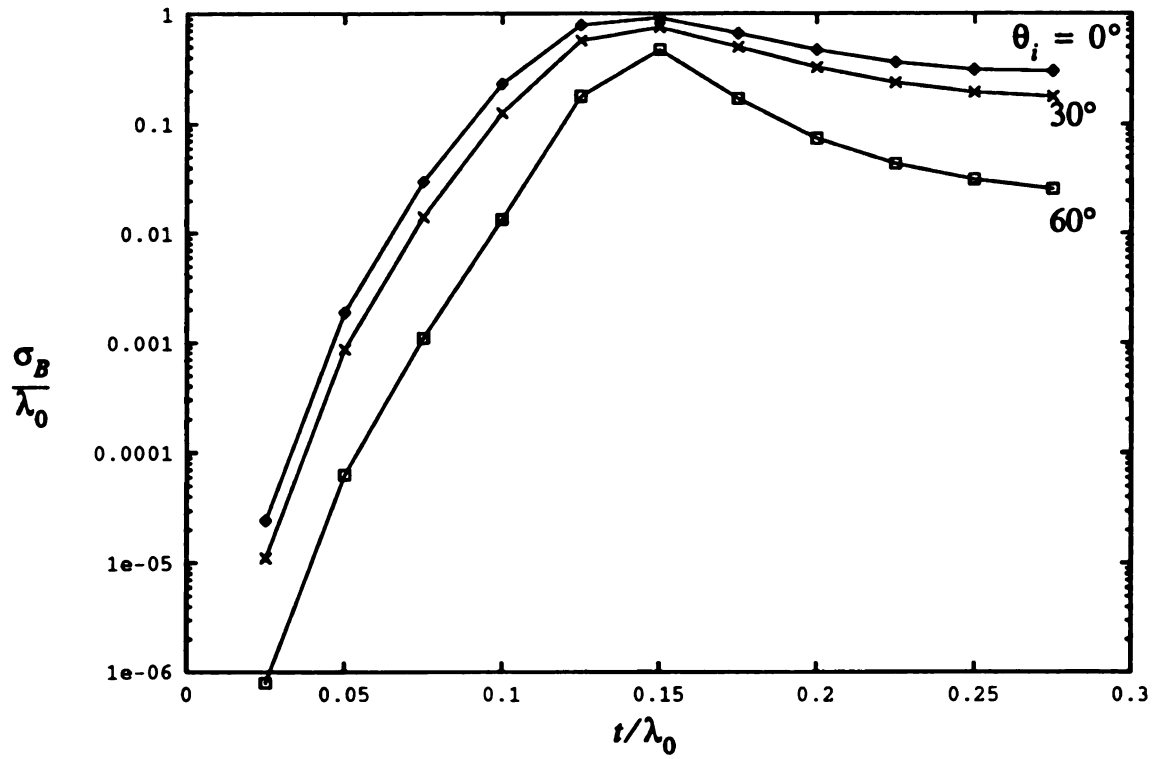


Figure 9.14. Normalized back scattering width as a function of the thickness t/λ_0 of the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.25$, $N_z = 13$, and $N_x = 4$ to 14)

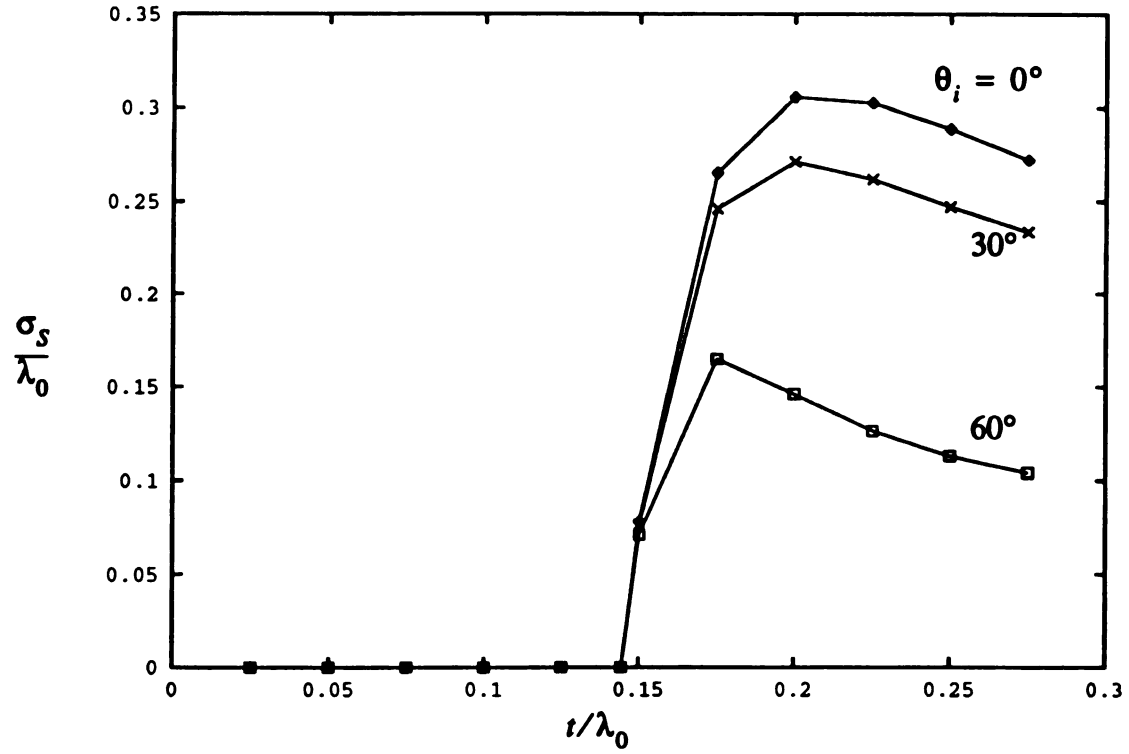


Figure 9.15. Normalized equivalent scattering width of the surface-wave mode excited as a function of the thickness t/λ_0 of the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.25$, $N_z = 13$, and $N_x = 4$ to 14)

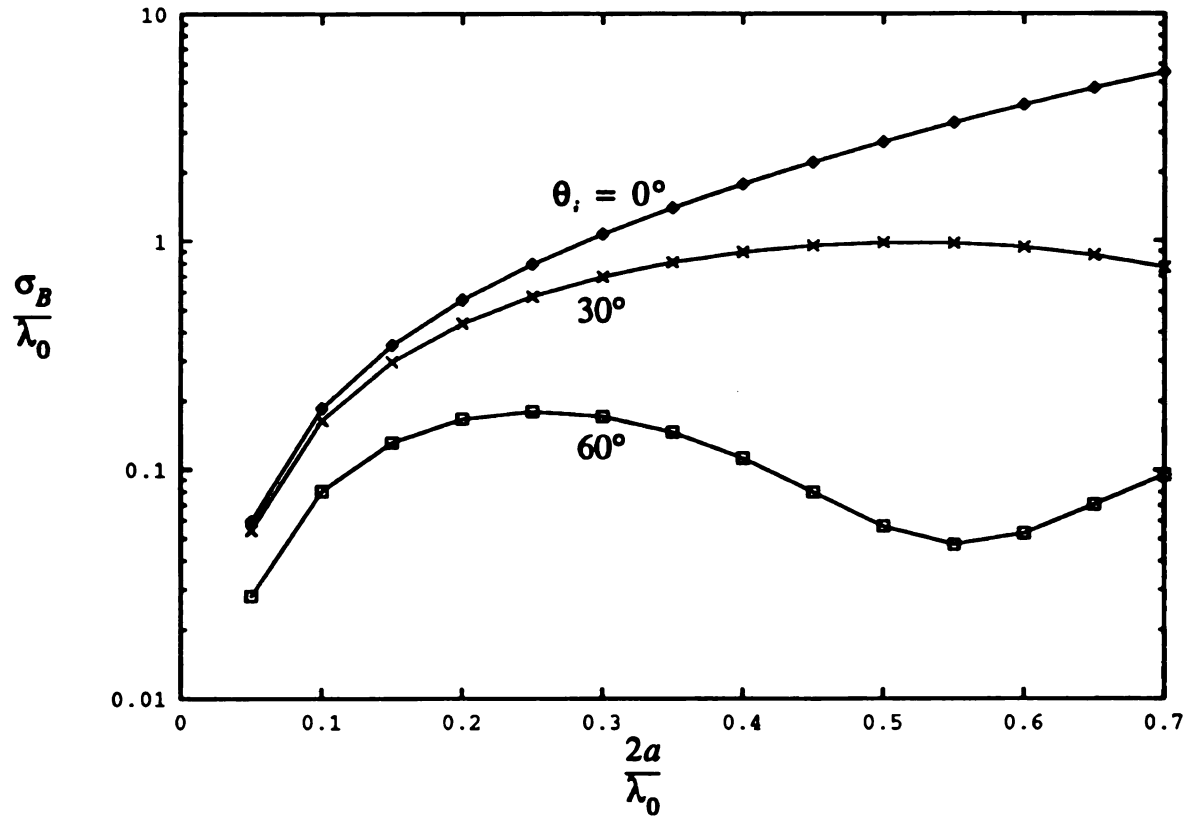


Figure 9.16. Normalized back scattering width as a function of the width $2a/\lambda_0$ of the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $t/\lambda_0 = 0.125$, $N_x = 13$, and $N_z = 4$ to 17)

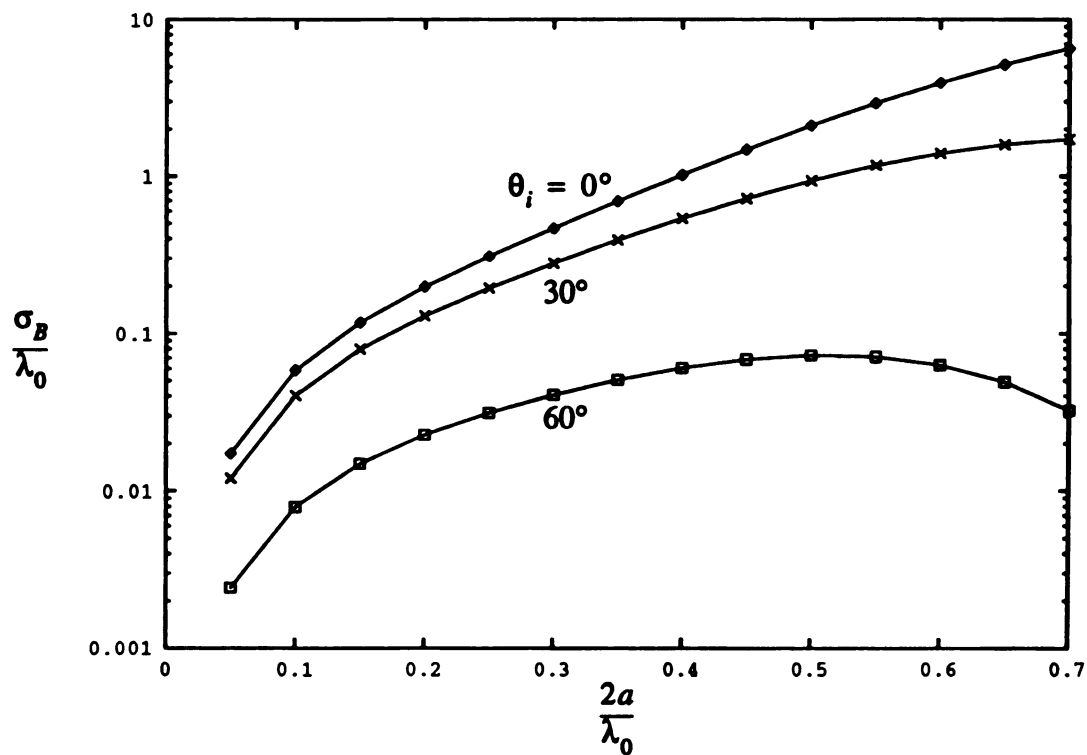


Figure 9.17. Normalized back scattering width as a function of the width $2a/\lambda_0$ of the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $t/\lambda_0 = 0.25$, $N_x = 13$, and $N_z = 4$ to 17)

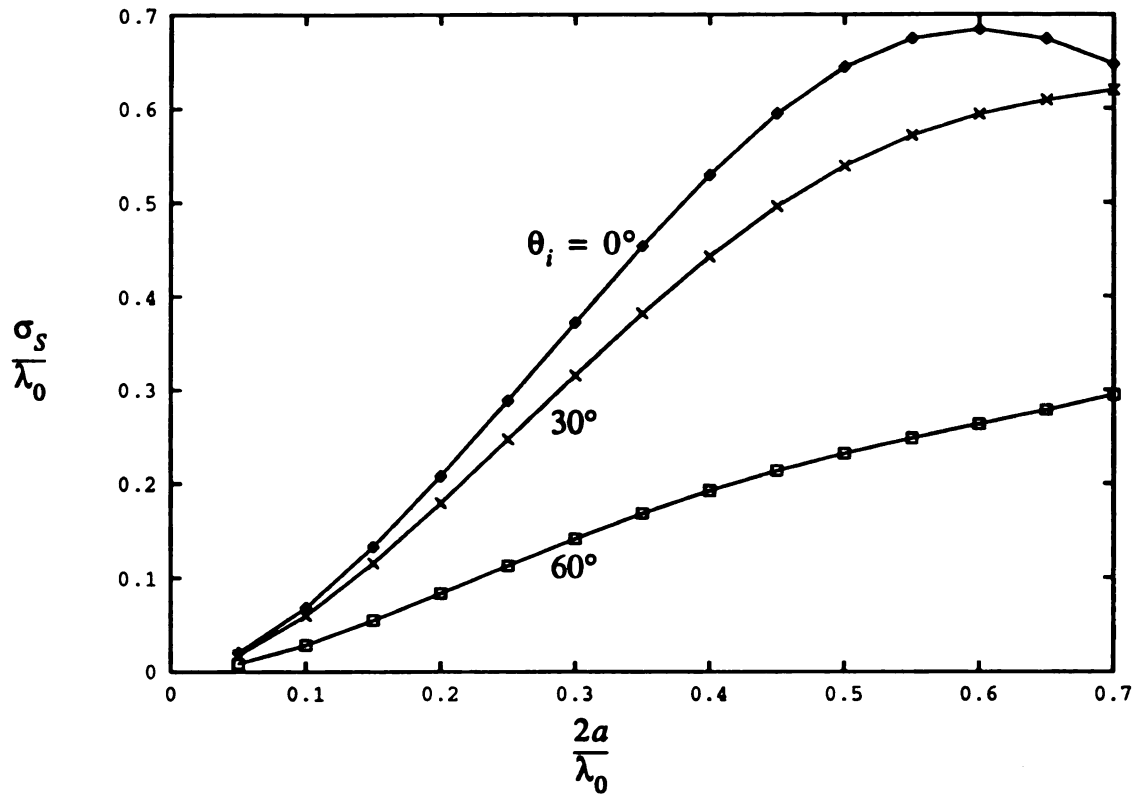


Figure 9.18. Normalized equivalent scattering width of the surface-wave mode excited as a function of the width $2a/\lambda_0$ of the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $t/\lambda_0 = 0.25$, $N_x = 13$, and $N_z = 4$ to 17)

gap which was defined in Chapter 5. This phenomenon is shown in Figures 9.19 and 9.20. These two figures plot the normalized amplitudes of the x - and z -components of the total electric field (points) at the center of a narrow gap ($z = 0$) and the electric field in a zero-width gap (solid line) when a TM-polarized plane wave is incident upon the gap in a grounded sheet at various incidence angles θ_i , respectively. It is noted that the x -component of the incident electric field becomes zero when a TM-polarized plane wave is incident upon the sheet normally ($\theta_i = 0^\circ$).

Figures 9.21 and 9.22 show the spatial intensity distributions of the x - and z -components, respectively, of the total electric field in the discontinuity normalized by the amplitude of the incident electric field when a TM-polarized plane wave is incident upon the sheet normally ($\theta_i = 0^\circ$). It is found that the field distribution is symmetric with respect to the $z = 0$ plane, and the x -component of the total electric field is not zero.

Figure 9.23 plots the normalized radiation patterns of the radiated far-zone field when a plane wave is incident upon the discontinuity at various incidence angles θ_i .

The normalized back scattering width and the equivalent scattering width of the excited surface-wave mode as functions of the thickness t/λ_0 of the discontinuity when a TM-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i are shown in Figures 9.24 and 9.25, respectively. Comparing these two figures, it is observed that as the sheet thickness increases the equivalent scattering width of the excited surface-wave mode increases while the normalized back scattering width does not decrease. In the last section, it is found that the excited surface wave will significantly alter the magnitude and pattern of the scattered field for a TE-polarized plane wave incidence.

Figure 9.26 shows the back scattering width as a function of the width of the discontinuity for $t/\lambda_0 = 0.125$ when a TM-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i .

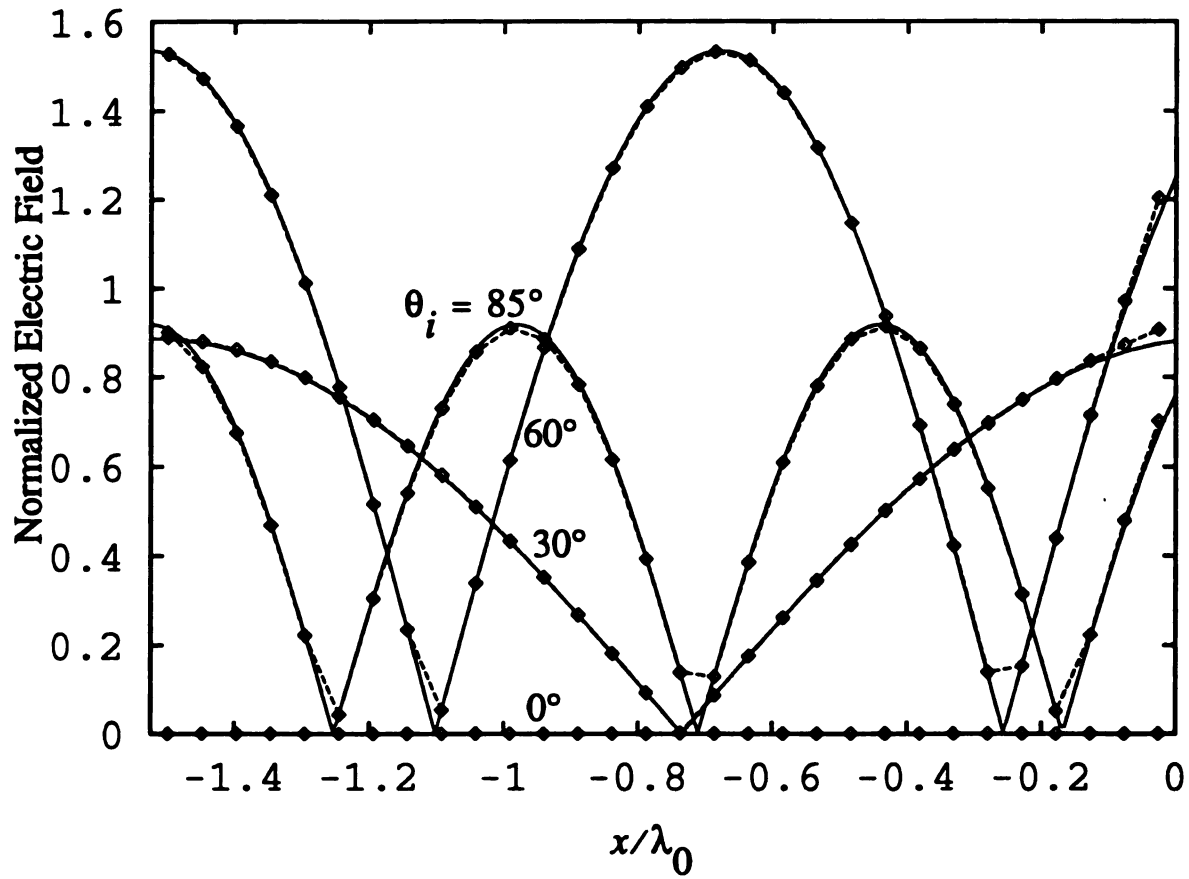


Figure 9.19. The comparison of the normalized amplitude of x-component of the total electric field $|E_{2x}/E_0|$ (points) at the center of a narrow gap ($z = 0$) and the electric field in a zero-width gap $|E_{2x}^i/E_0|$ (solid line) when a TM-polarized plane wave is incident upon the gap in a grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.1''$, $t = 1''$, $f = 18\text{GHz}$, Pulse Galerkin's method with $N_z = 3$ and $N_x = 30$)

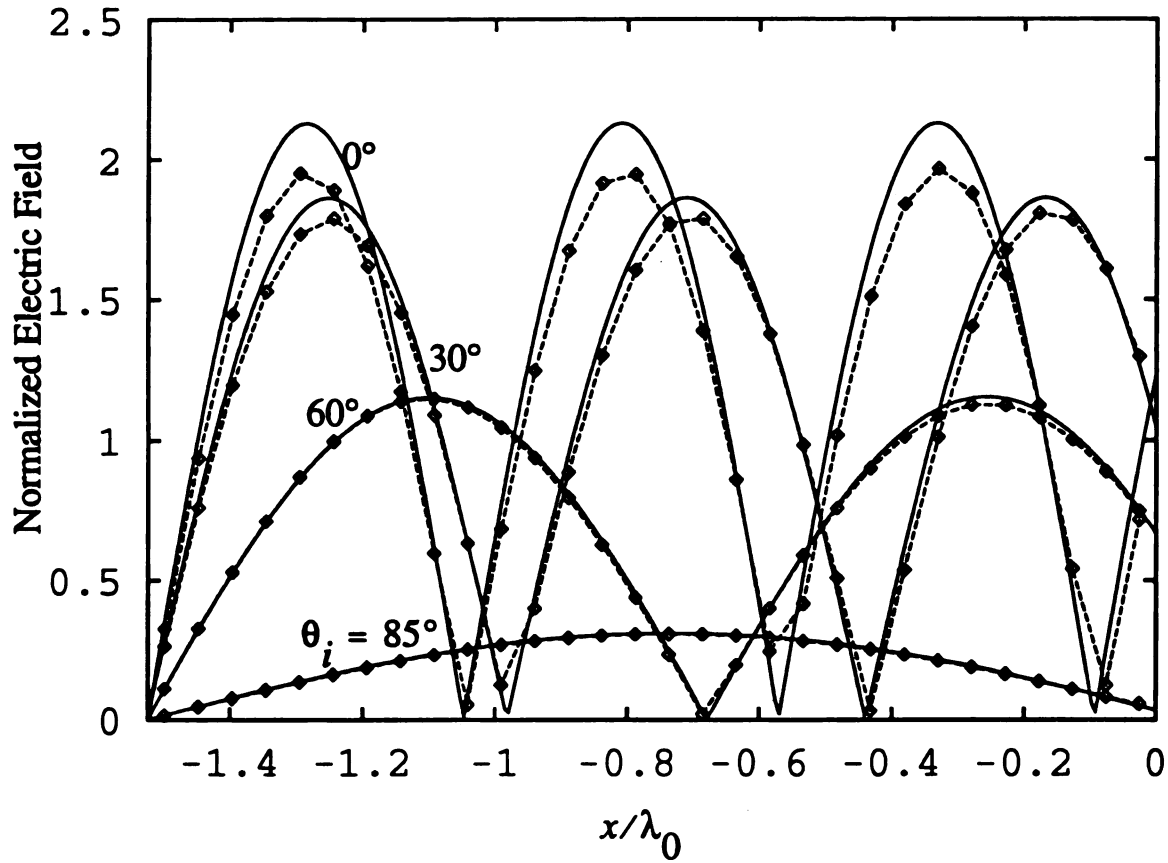


Figure 9.20. The comparison of the normalized amplitude of z-component of the total electric field $|E_{2z}/E_0|$ (points) at the center of a narrow gap ($z = 0$) and the electric field in a zero-width gap $|(\epsilon_{2r}/\epsilon_{dr}) E_{2z}^i/E_0|$ (solid line) when a TM-polarized plane wave is incident upon the gap in a grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 1.1$, $2a = 0.1\lambda$, $t = 1\lambda$, $f = 18\text{GHz}$, Pulse Galerkin's method with $N_z = 3$ and $N_x = 30$)

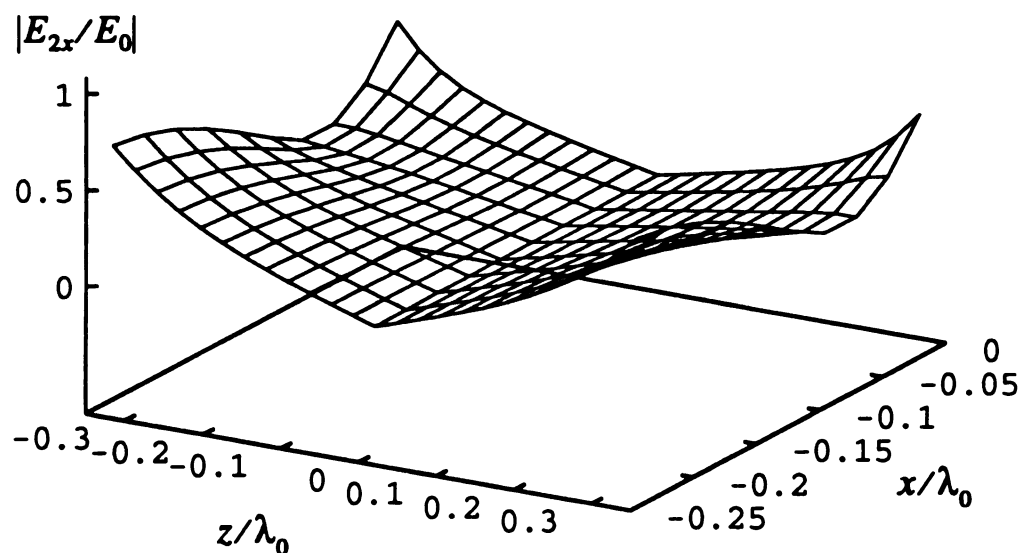


Figure 9.21. Spatial intensity distribution of the x-component of the normalized total electric field in the discontinuity when a TM-polarized plane wave is incident upon the grounded sheet normally ($\theta_i = 0^\circ$). ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.7$, $t/\lambda_0 = 0.25$, Pulse Galerkin's method with $N_x = 10$ and $N_z = 25$)

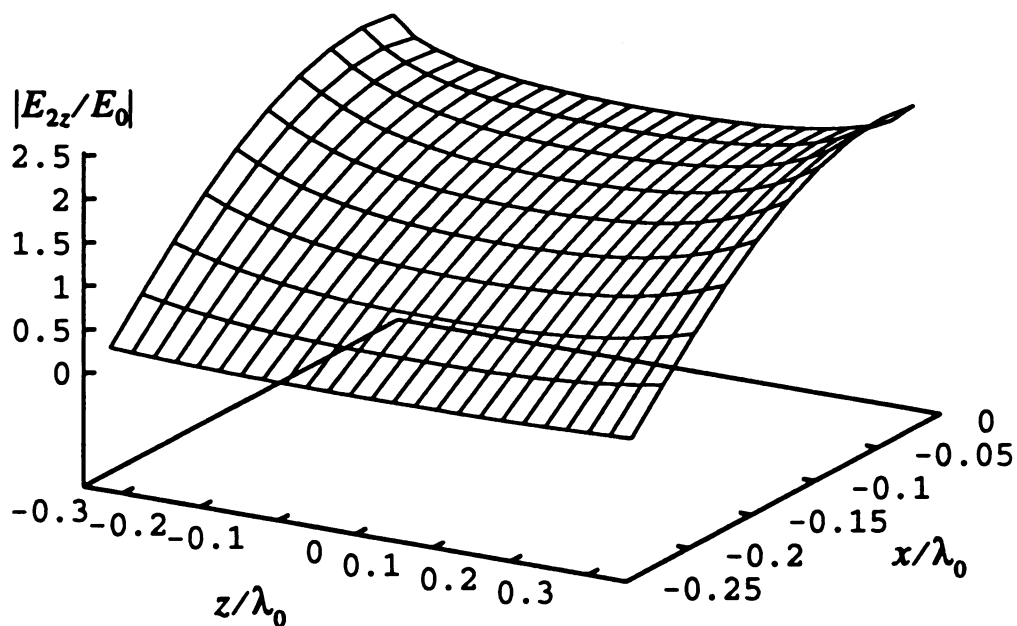


Figure 9.22. Spatial intensity distribution of the z -component of the normalized total electric field in the discontinuity when a TM-polarized plane wave is incident upon the grounded sheet normally ($\theta_i = 0^\circ$). ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.7$, $t/\lambda_0 = 0.25$, Pulse Galerkin's method with $N_x = 10$ and $N_z = 25$)

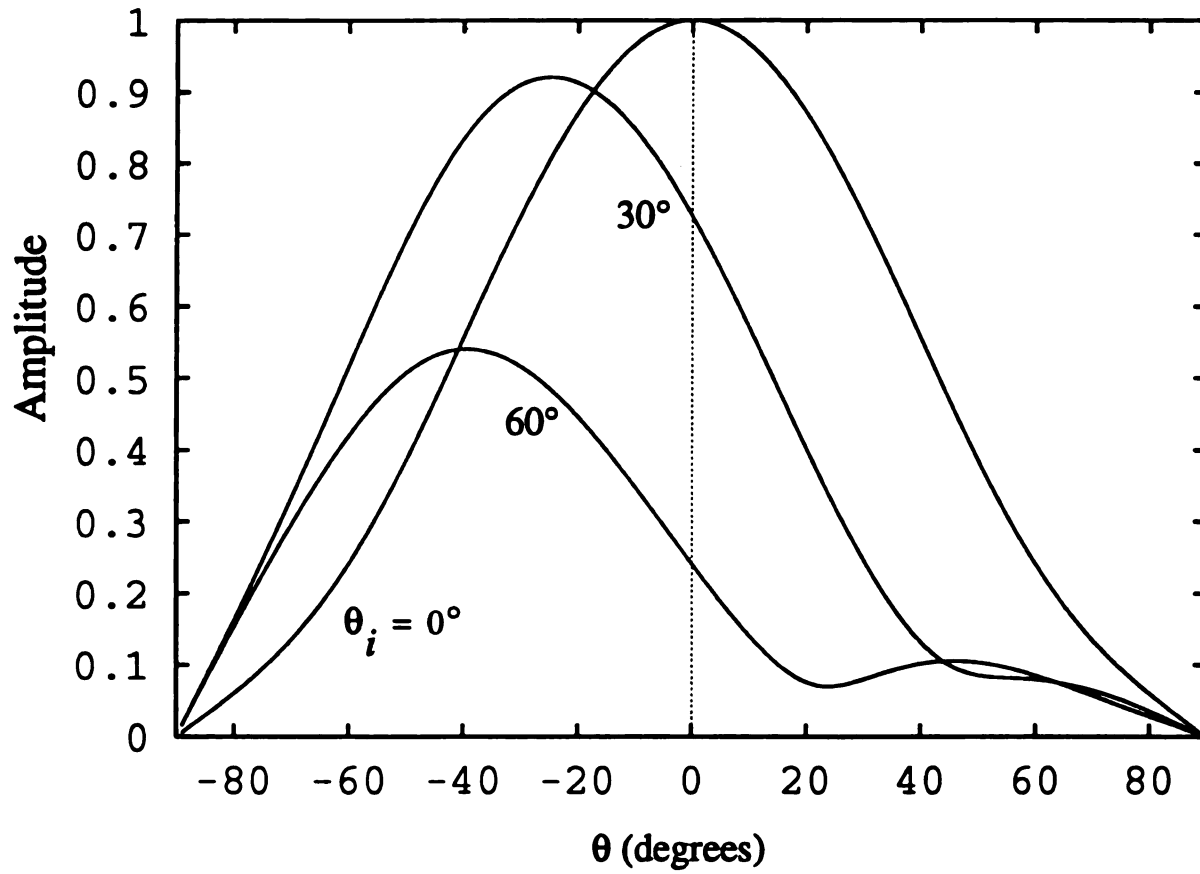


Figure 9.23. Radiation patterns of the far-zone field when a TM-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.7$, $t/\lambda_0 = 0.25$, Pulse Galerkin's method with $N_x = 10$ and $N_z = 25$)

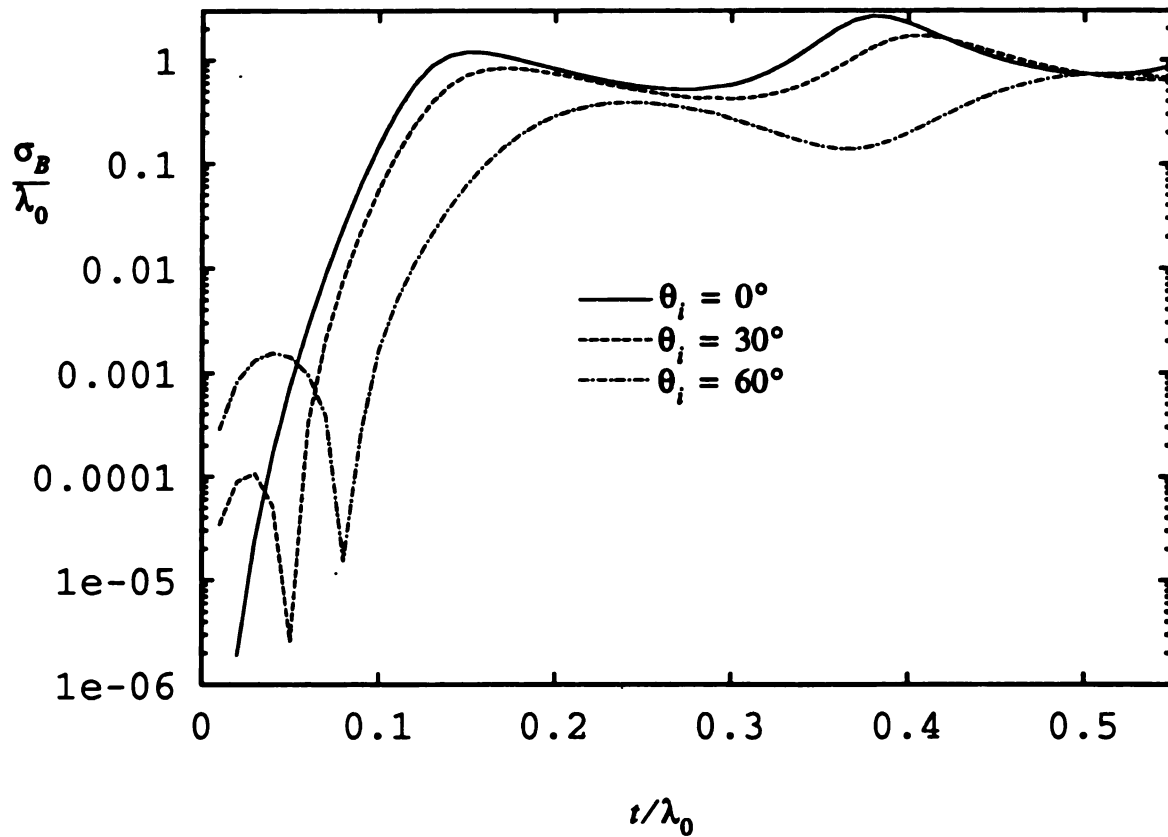


Figure 9.24. Normalized back scattering width as functions of the thickness t/λ_0 of the discontinuity when a TM-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.125$, Pulse Galerkin's method with $N_z = 8$ and $N_x = 4$ to 25)

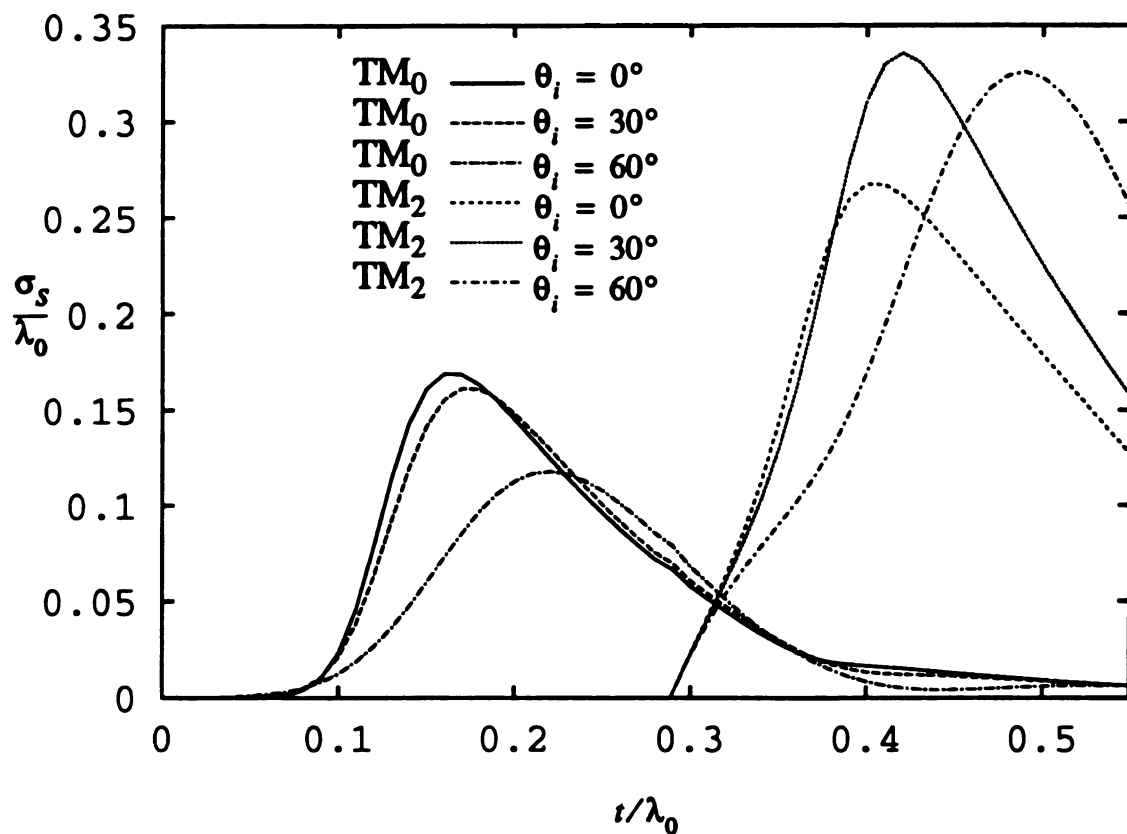


Figure 9.25. Normalized equivalent scattering width of the surface-wave modes excited as functions of the thickness t/λ_0 of the discontinuity when a TM-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $2a/\lambda_0 = 0.125$, Pulse Galerkin's method with $N_z = 8$ and $N_x = 4$ to 25)

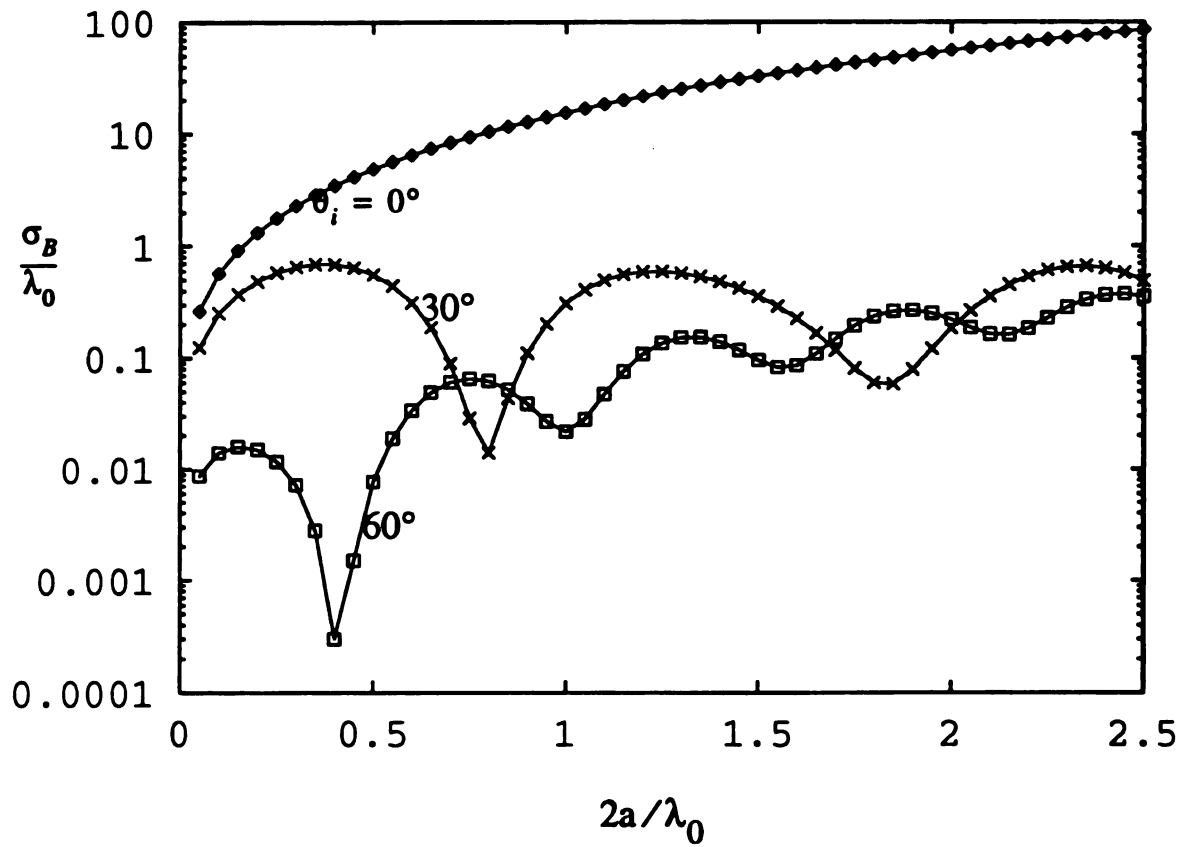


Figure 9.26. Normalized back scattering width as a function of the width $2a/\lambda_0$ of the discontinuity when a TM-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . ($\epsilon_{1r} = \epsilon_{dr} = 1$, $\epsilon_{2r} = 4$, $t/\lambda_0 = 0.125$, SPS Galerkin's method with $N_x = 3$ and $N_z = 3$ to 40)

9.5. NUMERICAL RESULTS FOR THE LOSSY DIELECTRIC SHEET

The various surface-wave mode fields are orthogonal to each other, as well as orthogonal to the continuous radiation modes. The different spectral components of continuous modes are orthogonal too. The orthogonality properties may be established using the reciprocity theorem with the hypothesis that the total radiation field satisfies the radiation condition [1] [69] [70]. Now, let's discuss the orthogonality properties for power in the lossy dielectric slab.

The total radiation electric field (only TE modes will be discussed) can be written as

$$E_{yR}(x, z) = \int_0^{\infty} A(\rho) e_y(x, \rho) e^{-j\beta(\rho)z} d\rho \quad (9.5.1)$$

Hence $e_y(x, \rho)$ satisfies

$$\left\{ \frac{\partial^2}{\partial x^2} + [k^2(x) - \beta^2(\rho)] \right\} e_y(x, \rho) = 0 \quad (9.5.2)$$

where

$$\beta(\rho) = \begin{cases} \sqrt{k_1^2 - \rho^2} & 0 < \rho < k_1 \\ -j\sqrt{\rho^2 - k_1^2} & k_1 < \rho < \infty \end{cases} \quad (9.5.3)$$

$$k^2(x) = \begin{cases} k_1^2 = \omega^2 \mu_0 \epsilon_0 & 0 < x < \infty \\ k_2^2 = \omega^2 \mu_0 \epsilon'_2 + j\omega^2 \mu_0 \epsilon''_2 & -t < x < 0 \end{cases} \quad (9.5.4)$$

The EM fields of the TE_n surface-wave mode can be written as

$$E_{yn}(x, z) = e_{yn}(x) e^{-j\beta_n z} \quad (9.5.5)$$

$$H_{xn}(x, z) = \frac{1}{j\omega\mu_0} \frac{\partial}{\partial z} E_{yn}(x, z) = -\frac{\beta_n}{\omega\mu_0} e_{yn}(x) e^{-j\beta_n z} \quad (9.5.6)$$

$$\left\{ \frac{\partial^2}{\partial x^2} + [k^2(x) - \beta_n^2] \right\} e_{yn}(x, \rho) = 0 \quad (9.5.7)$$

Now, let's evaluate one interaction term

$$\begin{aligned} P_z &= \frac{1}{2} \text{Re} \left\{ \int_{-t}^{\infty} \hat{z} \cdot (\vec{E}_R \times \vec{H}_n^*) dx \right\} \\ &= \frac{1}{2} \text{Re} \left\{ -\frac{\beta_n^*}{\omega\mu_0} e^{j\beta_n^* z} \int_0^{\infty} A(\rho) e^{-j\beta(\rho)z} d\rho \int_{-t}^{\infty} e_y(x, \rho) e_{yn}^*(x) dx \right\} \end{aligned} \quad (9.5.8)$$

Subtracting the multiplication of (9.5.2) and the complex conjugate of $e_{yn}(x)$ from the multiplication of $e_y(x, \rho)$ and the complex conjugate of (9.5.7) leads to

$$\begin{aligned} &[k^{*2}(x) - k^2(x) - \beta_n^{*2} + \beta^2(\rho)] e_{yn}^*(x) e_y(x, \rho) \\ &= \frac{\partial}{\partial x} \left[e_{yn}^*(x) \frac{\partial}{\partial x} e_y(x, \rho) - e_y(x, \rho) \frac{\partial}{\partial x} e_{yn}^*(x) \right] \end{aligned} \quad (9.5.9)$$

Integrating above equation respect x from -t to ∞ and making some manipulations lead to

$$\begin{aligned} &[\beta^2(\rho) - \beta_n^{*2}] \int_{-t}^{\infty} e_{yn}^*(x) e_y(x, \rho) dx - 2j\omega^2\mu_0 \int_{-t}^{\infty} \epsilon''(x) e_{yn}^*(x) e_y(x, \rho) dx \\ &= \left[e_{yn}^*(x) \frac{\partial}{\partial x} e_y(x, \rho) - e_y(x, \rho) \frac{\partial}{\partial x} e_{yn}^*(x) \right] \Big|_{-t}^{\infty} \end{aligned} \quad (9.5.10)$$

Since the slab is grounded at $x = -t$ and the electric fields should be bounded, then

$$x = -t \quad e_{yn}(x) = 0 \quad e_y(x, \rho) = 0 \quad (9.5.11)$$

$$x \rightarrow \infty \quad e_{yn}(x) \rightarrow 0 \quad \frac{\partial}{\partial x} e_{yn}(x) \rightarrow 0 \quad (9.5.12)$$

This leads to

$$\int_{-\infty}^{\infty} e^{*}_{yn}(x) e_y(x, \rho) dx = \frac{2j\omega^2 \mu_0 \int_{-\infty}^{\infty} \epsilon''(x) e^{*}_{yn}(x) e_y(x, \rho) dx}{\beta^2(\rho) - \beta_n^{*2}} \quad (9.5.13)$$

and

$$P_z = \frac{1}{2} Re \left\{ -\frac{\beta_n^{*}}{\omega \mu_0} e^{j\beta_n^{*} z} \int_0^{\infty} A(\rho) e^{-j\beta(\rho) z} d\rho \frac{2j\omega^2 \mu_0 \int_{-\infty}^{\infty} \epsilon''(x) e^{*}_{yn}(x) e_y(x, \rho) dx}{\beta^2(\rho) - \beta_n^{*2}} \right\} \quad (9.5.14)$$

It is observed that if both regions are lossless [$\epsilon''(x) = 0$], then $P_z = 0$ because $\beta(\rho) \neq \beta_n^{*}$. The surface-wave modes and radiation modes are then orthogonal in the power relation. Thus the total scattered power is the sum of the radiated power and the surface-wave power. On other hand, if $\epsilon''(x) \neq 0$, i.e. one or both regions are lossy, then $P_z \neq 0$. If $\epsilon''(x)$ is small and the surface-wave mode is not near the branch cuts, $P_z \approx 0$, and they are almost orthogonal in power. But, since the surface-wave mode may be near the branch cuts, i.e. $\beta^2(\rho) - \beta_n^{*2}$ is small for some ρ 's, then $P_z \neq 0$.

Therefore, if the dielectric slab is lossy, although the surface-wave modes and radiation mode fields are orthogonal, the modes are not orthogonal in power. The total power cannot be calculated as the sum of the radiated power and the surface-wave power.

Figure 9.27 plots the loci of the z-axis propagation constant of β_1 for the TE_1 surface-wave mode along a lossy sheet. It is observed that the TE_1 mode becomes a fast surface wave ($Re\{\beta_1\} < k_1 = k_0$) when the thickness decreases. Shevchenko and Richmond et al. discuss this in [59] [71].

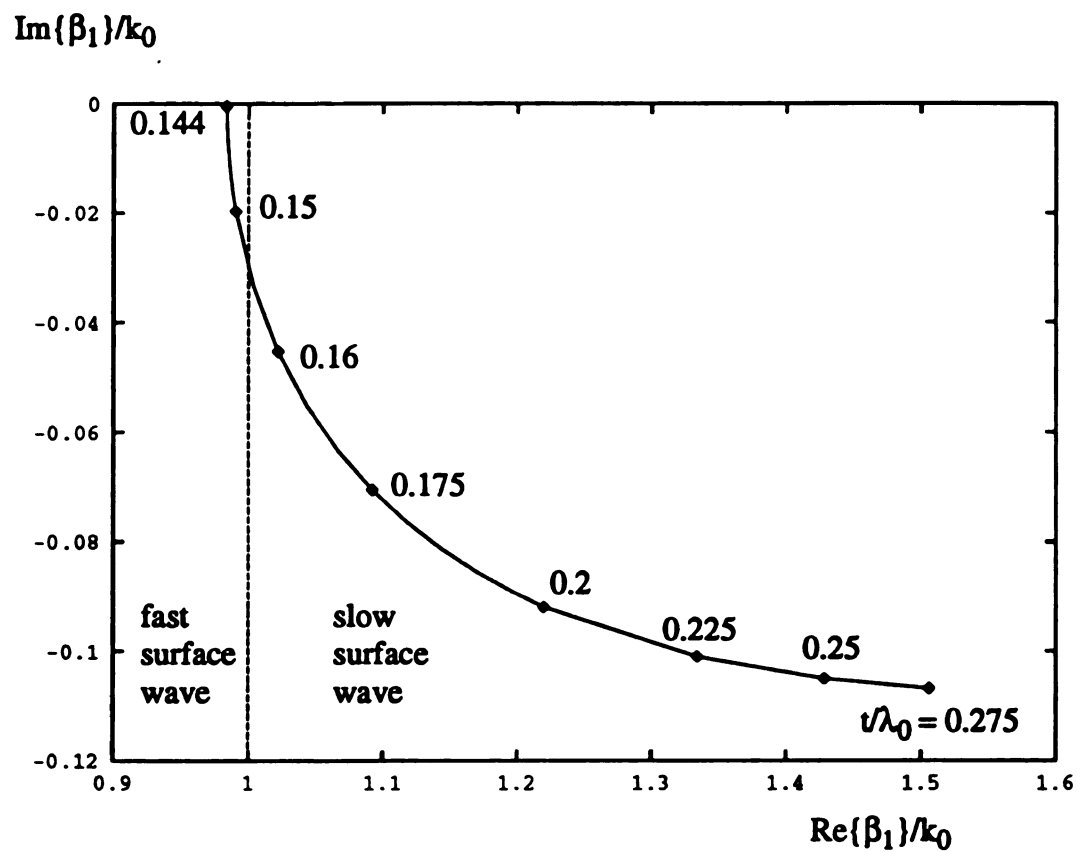


Figure 9.27. The loci of the z-axis propagation constant of β_1 for the TE_1 surface-wave mode along a lossy sheet ($\epsilon_{1r} = 1$, $\epsilon_{2r} = 4 - j0.4$).

The normalized back scattering width as functions of the width $2a/\lambda_0$ of the discontinuity for $\epsilon_{2r} = 4$ and $\epsilon_{2r} = 4 - j0.4$ are shown in Figures 9.28 and 9.29 when a TE-polarized plane wave is incident upon the discontinuity in a grounded dielectric slab with $t/\lambda_0 = 0.125$ and $t/\lambda_0 = 0.25$, respectively. The back scattering width of the lossy dielectric slab is less than that of the perfect dielectric slab.

Figures 9.30 and 9.31 show the normalized back scattering width as functions of the thickness t/λ_0 of the discontinuity for $\epsilon_{2r} = 4$ and $\epsilon_{2r} = 4 - j0.4$ when $2a/\lambda_0 = 0.25$ and $2a/\lambda_0 = 0.5$, respectively. It is observed that the back scattering width of the lossy dielectric slab is less than that of the perfect dielectric slab.

Figure 9.32 shows the normalized back scattering width as a function of the thickness t/λ_0 of the discontinuity for $\epsilon_{2r} = 4$ and $\epsilon_{2r} = 4 - j0.4$ when a TM-polarized plane wave is incident upon the discontinuity in a grounded dielectric sheet with $2a/\lambda_0 = 0.25$. It is observed that the back scattering width of the lossy dielectric sheet is less than that of the perfect dielectric sheet.

Finally, the normalized back scattering width as a function of the width $2a/\lambda_0$ of the discontinuity for $\epsilon_{2r} = 4$ and $\epsilon_{2r} = 4 - j0.4$ is shown in Figure 9.33 when $t/\lambda_0 = 0.25$. The back scattering width of the lossy dielectric sheet is found to be less than that of the perfect dielectric sheet.

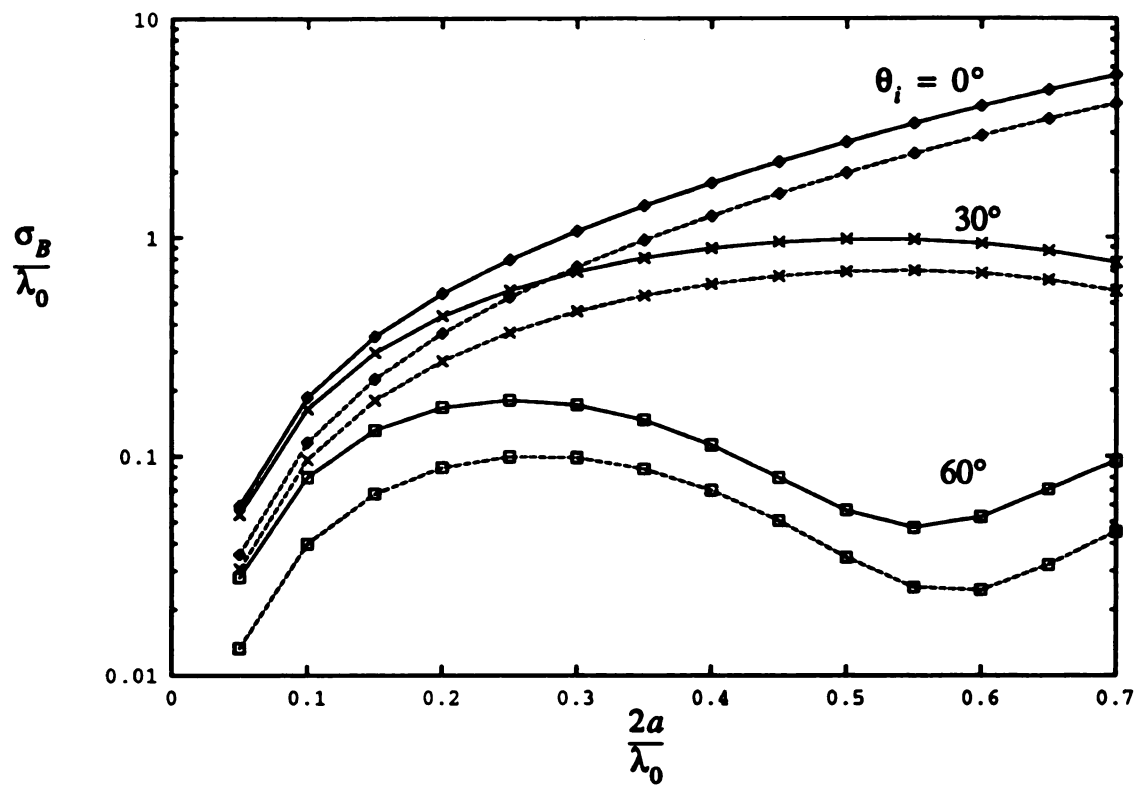


Figure 9.28. Normalized back scattering width as a function of the width $2a/\lambda_0$ of the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . Solid line: $\epsilon_{2r} = 4$, Dashed line: $\epsilon_{2r} = 4 - j0.4$. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $t/\lambda_0 = 0.125$, $N_x = 13$, and $N_z = 4$ to 17)

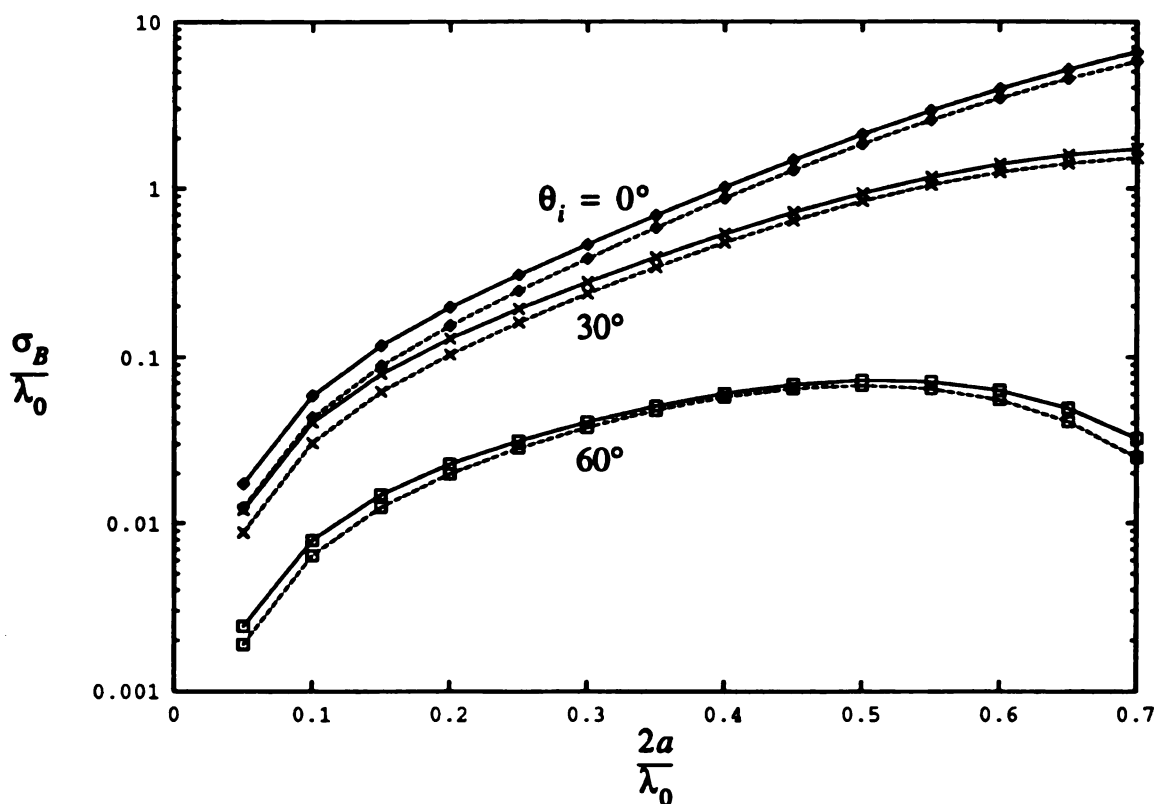


Figure 9.29. Normalized back scattering width as a function of the width $2a/\lambda_0$ of the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . Solid line: $\epsilon_{2r} = 4$, Dashed line: $\epsilon_{2r} = 4 - j0.4$. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $t/\lambda_0 = 0.25$, $N_x = 13$, and $N_z = 4$ to 17)

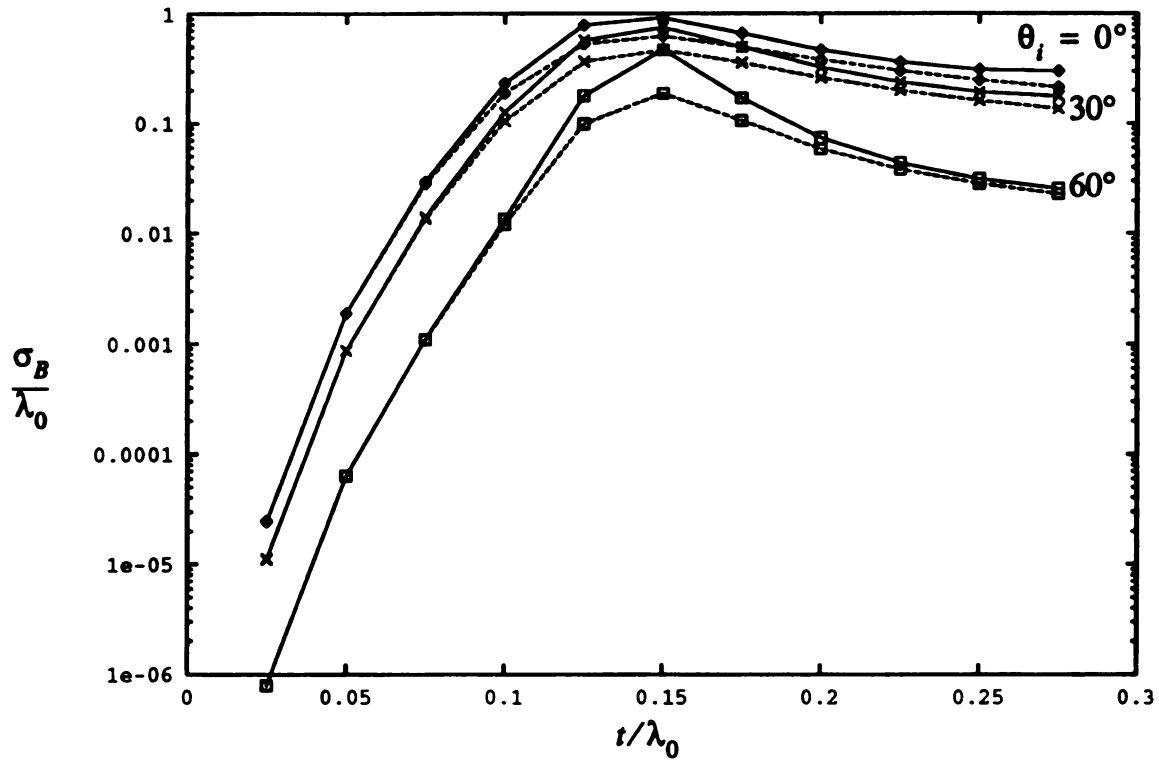


Figure 9.30. Normalized back scattering width as a function of the thickness t/λ_0 of the discontinuity when a TE plane wave is incident upon the grounded sheet at various incidence angles θ_i . Solid line: $\epsilon_{2r} = 4$, Dashed line: $\epsilon_{2r} = 4 - j0.4$. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $2a/\lambda_0 = 0.25$, $N_z = 13$, and $N_x = 4$ to 14)

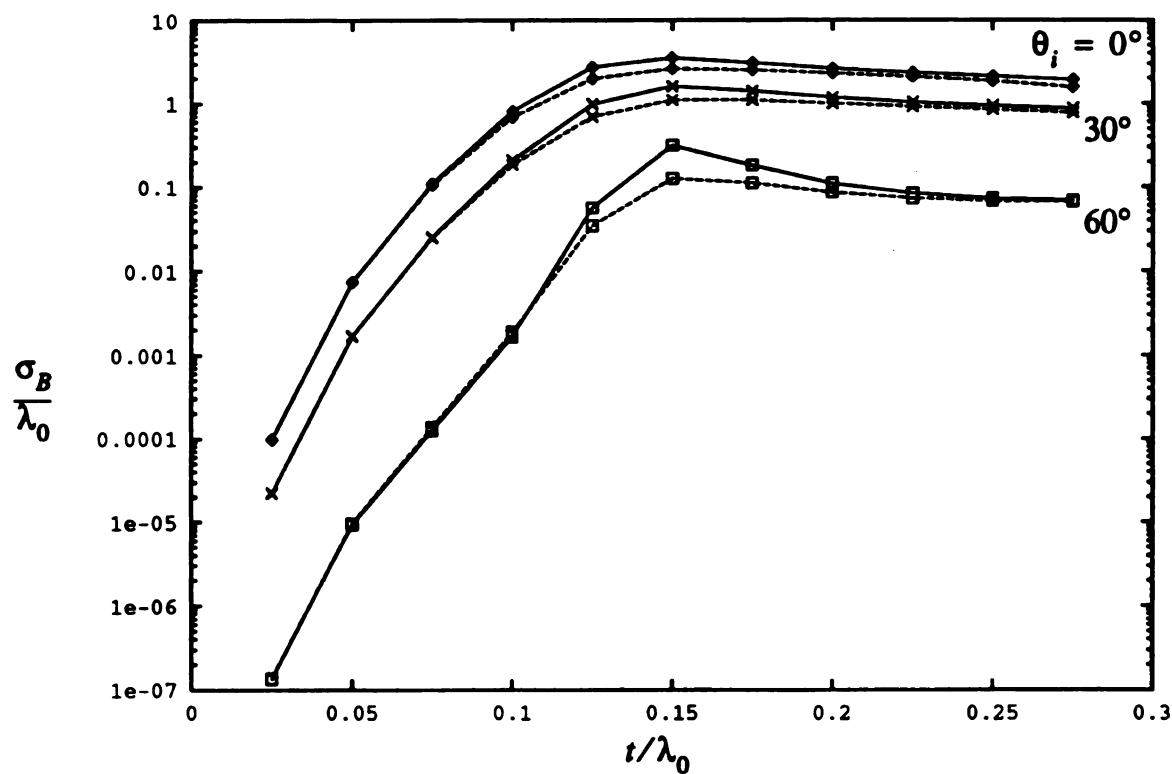


Figure 9.31. Normalized back scattering width as a function of the thickness t/λ_0 of the discontinuity when a TE-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . Solid line: $\epsilon_{2r} = 4$, Dashed line: $\epsilon_{2r} = 4 - j0.4$. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $2a/\lambda_0 = 0.5$, $N_z = 13$, and $N_x = 4$ to 14)

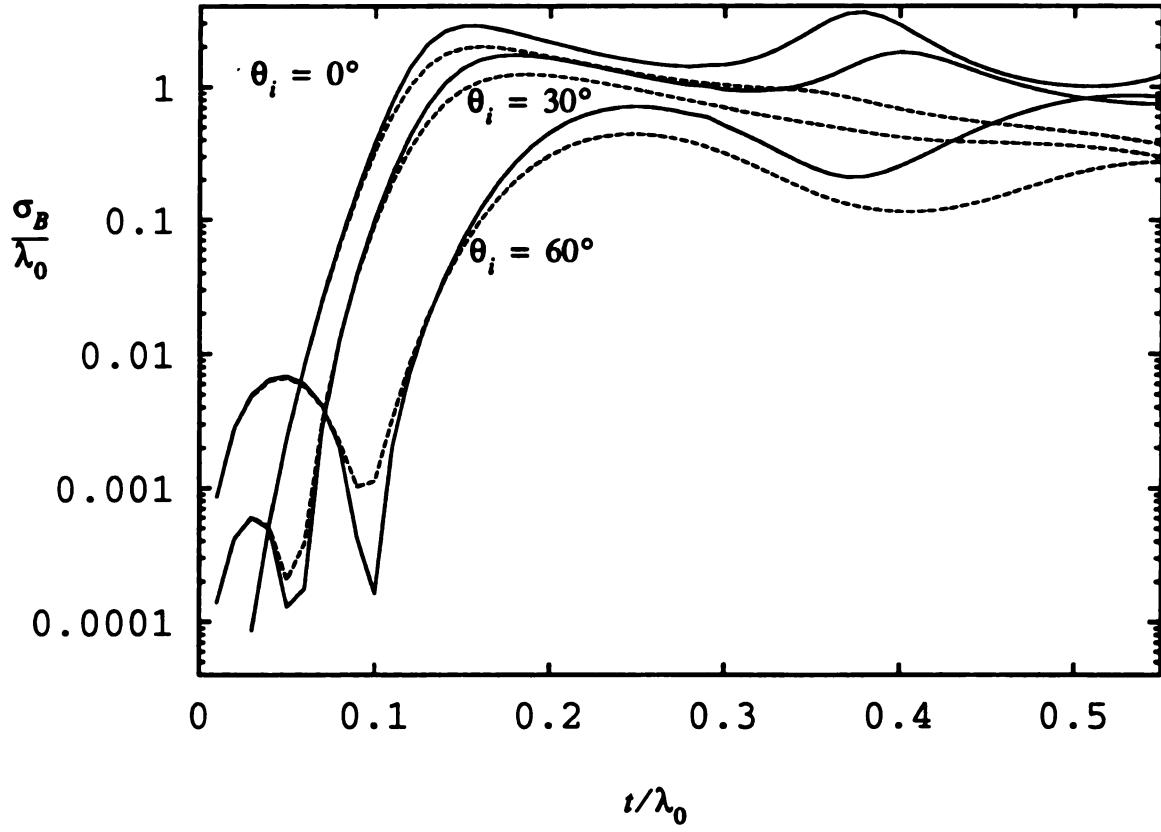


Figure 9.32. Normalized back scattering width as functions of the thickness t/λ_0 of the discontinuity when a TM-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . Solid line: $\epsilon_{2r} = 4$, Dashed line: $\epsilon_{2r} = 4 - j0.4$. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $2a/\lambda_0 = 0.25$, Pulse Galerkin's method with $N_z = 10$ and $N_x = 4$ to 25)

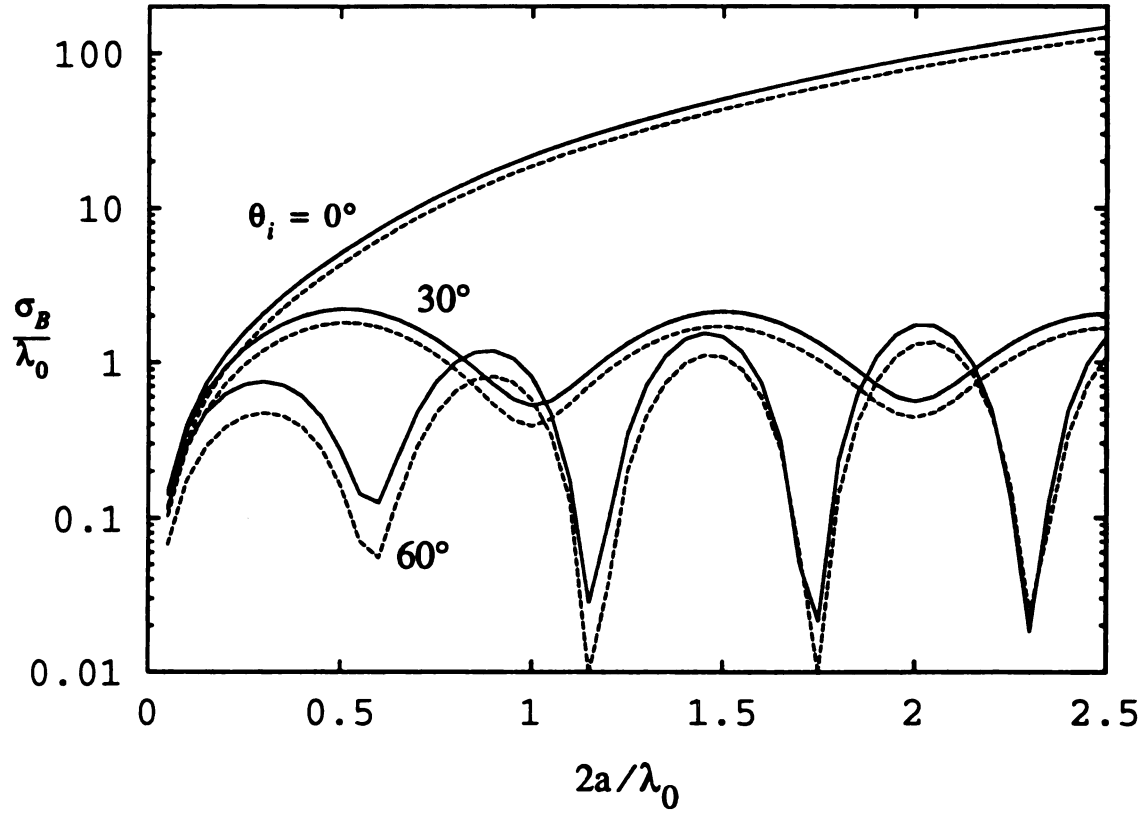


Figure 9.33. Normalized back scattering width as a function of the width $2a/\lambda_0$ of the discontinuity when a TM-polarized plane wave is incident upon the grounded sheet at various incidence angles θ_i . Solid line: $\epsilon_{2r} = 4$, Dashed line: $\epsilon_{2r} = 4 - j0.4$. ($\epsilon_{1r} = \epsilon_{dr} = 1$, $t/\lambda_0 = 0.25$, SPS Galerkin's method with $N_x = 5$ and $N_z = 3$ to 40)

CHAPTER 10

CONCLUSIONS

The scattering of TM-polarized EM waves by a dielectric discontinuity in an infinite grounded dielectric sheet has been analyzed. An electric field integral equation (EFIE) has been derived based on the Green's function for the electric field produced by induced polarization currents in the discontinuity region.

The EFIE is solved numerically using Galerkin's method for TE-polarized wave incidence. Computation of the inverse Fourier transform representation of the electric Green's function is performed by integration along real axis, along the branch cuts directly, and along the branch cuts with a variable change. It is found that the integration along the branch cuts directly requires the least computation time, and allows a separation of the field into bound surface-wave modes and radiation modes. Numerical calculations of the scattering pattern and the back scattering width are performed when a TE-polarized plane wave is incident upon the discontinuity in the grounded sheet. It is found that the discontinuity causes the excitation of surface-wave modes in the sheet and the excitation of the surface-wave mode reduces the back scattered radiation field.

The EFIE is solved using Galerkin's method with pulse and sinusoidal/piecewise-sinusoidal (SPS) basis functions for TM-polarized wave incidence. The latter afford a great savings in memory requirements and computation time. It is found that the excitation of the surface-wave modes do not reduce the back scattered radiation field for TM-polarized plane wave incidence. On the other hand, the EFIE is also solved iteratively by using suc-

cessive approximations (the Neumann series). Both the zeroth- and the first-order approximations for the total electric field in the discontinuity region are found when a TE- or TM-polarized plane wave is incident upon a grounded sheet with a narrow gap. Then, the first- and second-order approximations to the scattered field are calculated by applying the steepest-descents method. These approximate results agree very well with the direct MoM solutions to the EFIE.

It is found that the back scattering width is a minimum when $(2a/\lambda_0) \sin\theta_i = n/2$ for TM-polarized plane wave incidence, and for TE-polarized plane wave incidence for the thickness $t < t_{c1}$, where t_{c1} is the cut-off thickness of the TE_1 mode.

The reflected power for both TE- and TM-polarized surface-wave incidence is calculated too, and it is found that the numerical results agree very well with those of existing studies.

APPENDICES

APPENDIX A

ELECTRIC FIELD WITHOUT THE DISCONTINUITY FOR TE-POLARIZED PLANE WAVE INCIDENCE

From Figure A.1, it is found that the EM field in the absence of the discontinuity consists of two plane waves. Then, the electric field in the two regions can be written as:

Region 1: $0 < x < \infty$

$$E_{1y}^i = E_0 e^{jk_1(x \cos \theta_i + z \sin \theta_i)} + E_0 R e^{-jk_1(x \cos \theta_i - z \sin \theta_i)} \quad (\text{A. 1})$$

Region 2: $-t < x < 0$

$$E_{2y}^i = B E_0 \sin [k_2(x+t) \cos \theta_r] e^{jk_1 z \sin \theta_i} \quad (\text{A. 2})$$

it is noted that the boundary condition $E_y = 0$ at $x=-t$ has been used in (A.2), and where E_0 is the amplitude of the incident plane wave, R and B are unknown coefficients, and

$$k_i = \omega \sqrt{\epsilon_i \mu_0} \quad i = 1, 2 \quad (\text{A. 3})$$

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{k_2}{k_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \quad (\text{A. 4})$$

where θ_i is the incident angle in region 1 and θ_r is the refracted angle in region 2.

The magnetic field is obtained from Maxwell's equation

$$\vec{H}^i = -\frac{1}{j\omega\mu_0} \nabla \times \vec{E}^i \quad (\text{A. 5})$$

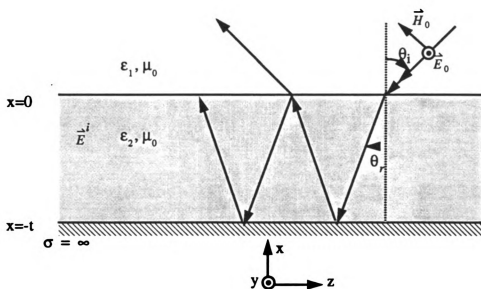


Figure A.1. TE-polarized plane wave incidence.

Then,

$$H_{1z}^i = -\frac{1}{j\omega\mu_0} \frac{\partial E_{1y}^i}{\partial x} = -\frac{\cos\theta_i}{\eta_1} [E_0 e^{jk_1(x\cos\theta_i + z\sin\theta_i)} - E_0 R e^{-jk_1(x\cos\theta_i - z\sin\theta_i)}] \quad (\text{A. 6})$$

$$H_{2z}^i = -\frac{1}{j\omega\mu_0} \frac{\partial E_{2y}^i}{\partial x} = -\frac{\cos\theta_t}{j\eta_2} B E_0 \cos[k_2(x+t)\cos\theta_t] e^{jk_1 z \sin\theta_i} \quad (\text{A. 7})$$

where

$$\eta_i = \sqrt{\mu_0/\epsilon_i} \quad i = 1, 2 \quad (\text{A. 8})$$

The boundary conditions that E_y^i and H_z^i are continuous at $x=0$ gives

$$E_{1y}^i|_{x=0} = E_{2y}^i|_{x=0} \Rightarrow 1 + R = B \sin[k_2 t \cos\theta_t] \quad (\text{A. 9})$$

$$H_{1z}^i|_{x=0} = H_{2z}^i|_{x=0} \Rightarrow \frac{\cos\theta_i}{\eta_1} (1-R) = \frac{\cos\theta_r}{j\eta_2} B \cos[k_2 t \cos\theta_r] \quad (\text{A. 10})$$

The two equations (A.9) and (A.10) have the solutions:

$$R = \frac{\eta_2 \cos\theta_i \sin(k_2 t \cos\theta_r) + j\eta_1 \cos\theta_r \cos(k_2 t \cos\theta_r)}{\eta_2 \cos\theta_i \sin(k_2 t \cos\theta_r) - j\eta_1 \cos\theta_r \cos(k_2 t \cos\theta_r)} \quad (\text{A. 11})$$

$$B = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i \sin(k_2 t \cos\theta_r) - j\eta_1 \cos\theta_r \cos(k_2 t \cos\theta_r)} \quad (\text{A. 12})$$

SUMMARY:

Region 1: $0 < x < \infty$

$$E_{1y}^i = E_0 [e^{jk_1 x \cos\theta_i} + R e^{-jk_1 x \cos\theta_i}] e^{jk_1 z \sin\theta_i} \quad (\text{A. 13})$$

Region 2: $-t < x < 0$

$$E_{2y}^i = E_0 \frac{2\eta_2 \cos\theta_i \sin[k_2 (x+t) \cos\theta_r] e^{jk_1 z \sin\theta_i}}{\eta_2 \cos\theta_i \sin(k_2 t \cos\theta_r) - j\eta_1 \cos\theta_r \cos(k_2 t \cos\theta_r)} \quad (\text{A. 14})$$

APPENDIX B

ELECTRIC FIELD WITHOUT THE DISCONTINUITY FOR TM- POLARIZED PLANE WAVE INCIDENCE

The total EM fields without the discontinuity can be written as the sum of two plane waves as shown in Figure B.1.:

Region 1: $0 < x < \infty$

$$E_{1x}^i = -E_0 \sin \theta_i [e^{jk_1(x \cos \theta_i + z \sin \theta_i)} + R_1 e^{-jk_1(x \cos \theta_i - z \sin \theta_i)}] \quad (\text{B.1})$$

$$E_{1z}^i = E_0 \cos \theta_i [e^{jk_1(x \cos \theta_i + z \sin \theta_i)} - R_1 e^{-jk_1(x \cos \theta_i - z \sin \theta_i)}] \quad (\text{B.2})$$

$$H_{1y}^i = \frac{E_0}{\eta_1} [e^{jk_1(x \cos \theta_i + z \sin \theta_i)} + R_1 e^{-jk_1(x \cos \theta_i - z \sin \theta_i)}] \quad (\text{B.3})$$

Region 2: $-t < x < 0$

$$E_{2x}^i = -BE_0 \sin \theta_r [e^{jk_2(x \cos \theta_r + z \sin \theta_r)} + R_2 e^{-jk_2(x \cos \theta_r - z \sin \theta_r)}] \quad (\text{B.4})$$

$$E_{2z}^i = BE_0 \cos \theta_r [e^{jk_2(x \cos \theta_r + z \sin \theta_r)} - R_2 e^{-jk_2(x \cos \theta_r - z \sin \theta_r)}] \quad (\text{B.5})$$

$$H_{2y}^i = \frac{BE_0}{\eta_2} [e^{jk_2(x \cos \theta_r + z \sin \theta_r)} + R_2 e^{-jk_2(x \cos \theta_r - z \sin \theta_r)}] \quad (\text{B.6})$$

where E_0 is the amplitude of the incident plane wave, R_1 , R_2 , and B are unknown coefficients, and where

$$k_i = \omega \sqrt{\epsilon_i \mu_i} \quad i = 1, 2 \quad (\text{B.7})$$

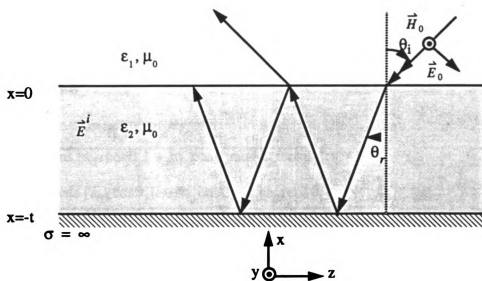


Figure B.1. TM-polarized plane wave incidence.

$$\eta_i = \sqrt{\mu_i / \epsilon_i} \quad i = 1, 2 \quad (\text{B.8})$$

and θ_i is the incidence angle in region 1 and θ_r is the angle of refraction in region 2.

The boundary conditions at $x=0, -t$ are applied to find the unknowns. The condition that $x=-t$ is a perfectly conducting plane gives

$$E_{2z}^i = 0 \Rightarrow R_2 = e^{-2jk_2 t \cos \theta_r} \quad (\text{B.9})$$

The conditions that E_z^i and H_y^i are continuous at $x=0$ give

$$E_{1z}^i \Big|_{x=0} = E_{2z}^i \Big|_{x=0} \Rightarrow E_0 \cos \theta_i [1 - R_1] e^{jk_1 z \sin \theta_i} = e^{-jk_2 t \cos \theta_r} 2jB E_0 \cos \theta_r \sin [k_2 t \cos \theta_r] e^{jk_2 z \sin \theta_r} \quad (\text{B.10})$$

$$H_{1y}^i \Big|_{x=0} = H_{2y}^i \Big|_{x=0} \Rightarrow$$

$$\frac{E_0}{\eta_1} [1 + R_1] e^{jk_1 z \sin \theta_i} = e^{-jk_2 t \cos \theta_r} 2B \frac{E_0}{\eta_2} \cos [k_2 t \cos \theta_r] e^{jk_2 z \sin \theta_r} \quad (\text{B.11})$$

The two equations (B.10) and (B.11) have the solutions:

$$k_1 \sin \theta_i = k_2 \sin \theta_r \quad (\text{B.12})$$

$$B = \frac{\eta_2 \cos \theta_i e^{jk_2 t \cos \theta_r}}{\eta_1 \cos \theta_i \cos [k_2 t \cos \theta_r] + j\eta_2 \cos \theta_r \sin [k_2 t \cos \theta_r]} \quad (\text{B.13})$$

$$R_1 = \frac{\eta_1 \cos \theta_i \cos [k_2 t \cos \theta_r] - j\eta_2 \cos \theta_r \sin [k_2 t \cos \theta_r]}{\eta_1 \cos \theta_i \cos [k_2 t \cos \theta_r] + j\eta_2 \cos \theta_r \sin [k_2 t \cos \theta_r]} = e^{-j2\phi_e} \quad (\text{B.14})$$

where

$$\tan \phi_e = \frac{\eta_2 \cos \theta_r}{\eta_1 \cos \theta_i} \tan [k_2 t \cos \theta_r] \quad (\text{B.15})$$

SUMMARY:

Region 1: $0 < x < \infty$

$$\vec{E}_1^i = -2E_0 \{ \hat{x} \sin \theta_i \cos [k_1 x \cos \theta_i + \phi_e] - \hat{z} j \cos \theta_i \sin [k_1 x \cos \theta_i + \phi_e] \} e^{j[k_1 z \sin \theta_i - \phi_e]} \quad (\text{B.16})$$

$$H_{1y}^i = \frac{2E_0}{\eta_1} \cos [k_1 x \cos \theta_i + \phi_e] e^{j[k_1 z \sin \theta_i - \phi_e]} \quad (\text{B.17})$$

Region 2: $-t < x < 0$

$$\vec{E}_2^i = -E_0 \frac{2\eta_2 \cos \theta_i \{ \hat{x} \sin \theta_r \cos [k_2 (t+x) \cos \theta_r] - \hat{z} j \cos \theta_r \sin [k_2 (t+x) \cos \theta_r] \} e^{jk_1 z \sin \theta_i}}{\eta_1 \cos \theta_i \cos [k_2 t \cos \theta_r] + j\eta_2 \cos \theta_r \sin [k_2 t \cos \theta_r]} \quad (\text{B.18})$$

$$H_{2y}^i = \frac{2E_0 \cos \theta_i \cos [k_2 (t+x) \cos \theta_r] e^{jk_1 z \sin \theta_i}}{\eta_1 \cos \theta_i \cos [k_2 t \cos \theta_r] + j\eta_2 \cos \theta_r \sin [k_2 t \cos \theta_r]} \quad (\text{B.19})$$

APPENDIX C

2-D GREEN'S FUNCTION FOR TE EXCITATION OF GROUNDED DIELECTRIC MATERIAL SHEET

C.1. NOTATION

Some words about notation here might be helpful. As a convention, upper case letters denote space domain quantities, while their transform domain counterparts are designated by lower case letters. The symbol j denotes the elementary imaginary number.

C.2. HERTIZAN POTENTIAL REPRESENTATION OF 2-D TE EM FIELD

Consider the structure depicted in Figure C.1. The current source $\vec{J}(\vec{r}) = \hat{y}J_y(x, z)$ radiates into the two regions, and generates electric Hertzian potential in each region. The relations that relate the EM fields to the Hertz potentials are

$$\vec{E} = k^2 \vec{\Pi} + \nabla (\nabla \cdot \vec{\Pi}) \quad (C.2.1)$$

$$\vec{H} = j\omega\epsilon \nabla \times \vec{\Pi} \quad (C.2.2)$$

where

$$k = \sqrt{\epsilon\mu} \quad (C.2.3)$$

The Hertzian potentials satisfy the following Helmholtz equation

$$\nabla^2 \vec{\Pi} + k^2 \vec{\Pi} = -\frac{\vec{P}}{\epsilon} = -\frac{\vec{J}}{j\omega\epsilon} \quad (C.2.4)$$

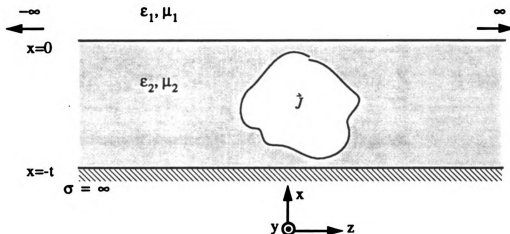


Figure C.1. Geometrical configuration. $\vec{J}(\vec{r}) = \hat{y}J_y(x, z)$ for 2-D TE excitation.

Since the current $\vec{J}$ has only a y -component, the Hertzian potential $\vec{\Pi}$ has only a y -component. Therefore

$$\vec{\Pi} = \hat{y}\Pi_y(x, z) \quad (\text{C.2.5})$$

$$\vec{E} = \nabla k^2 \Pi_y(x, z) \quad (\text{C.2.6})$$

$$\vec{H} = j\omega\epsilon \left[-\hat{x}\frac{\partial}{\partial z}\Pi_y(x, z) + \hat{z}\frac{\partial}{\partial x}\Pi_y(x, z) \right] \quad (\text{C.2.7})$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2 \right] \Pi_y(x, z) = -\frac{J_y(x, z)}{j\omega\epsilon} \quad (\text{C.2.8})$$

C.3. SPECTRAL-DOMAIN POTENTIAL REPRESENTATION

Equation (C.2.8) can be solved for the potential by Fourier transforming on spatial variables tangential to the layer interfaces. Axial uniformity along the z -axis [i.e., $\epsilon \neq \epsilon(z)$ and $\mu \neq \mu(z)$] prompts Fourier transformation on that variable. Define the axial transform pair $F(x, z) \leftrightarrow f(x, \zeta)$ as:

$$F(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x, \zeta) e^{j\zeta z} d\zeta \quad (\text{C.3.1})$$

$$f(x, \zeta) = \int_{-\infty}^{\infty} F(x, z) e^{-j\zeta z} dz \quad (\text{C.3.2})$$

Taking Fourier transforms of equations (C.2.6) through (C.2.8) yields

$$\dot{e}(x, \zeta) = jk^2 \pi_y(x, \zeta) \quad (\text{C.3.3})$$

$$\dot{h}(x, \zeta) = j\omega\epsilon \left[-xj\zeta \pi_y(x, \zeta) + x \frac{\partial}{\partial x} \pi_y(x, \zeta) \right] \quad (\text{C.3.4})$$

$$\left[\frac{\partial^2}{\partial x^2} - p^2 \right] \pi_y(x, \zeta) = -\frac{j_y(x, \zeta)}{j\omega\epsilon} \quad (\text{C.3.5})$$

where

$$p^2 = \zeta^2 - k^2 \quad (\text{C.3.6})$$

is wavenumber parameter.

C.4. DECOMPOSITION INTO PRIMARY AND REFLECTED WAVES

Since region 1 is source-free, the homogeneous wave equation

$$\left[\frac{\partial^2}{\partial x^2} - p_1^2 \right] \pi_{1y}(x, \zeta) = 0 \quad (\text{C.4.1})$$

has a solution

$$\pi_{1y}(x, \zeta) = A(\zeta) e^{-p_1 x} \quad (\text{C.4.2})$$

Note that the solution satisfy the radiation condition, i.e. it is an outward travelling, decaying potential wave with $\text{Re}\{p_1\} > 0$ and $\text{Im}\{p_1\} > 0$, where

$$p_1 = \sqrt{\zeta^2 - k_1^2} \quad (\text{C.4.3})$$

and $Re \{ \dots \}$ designates the real part of the quantity within the braces, $Im \{ \dots \}$ designates the imaginary part.

The total potential in region 2 is the sum of a primary wave π_{2y}^p excited by j_y in an unbounded layer region of the layer and the source free reflected wave π_{2y}^r in the layer. These potentials must satisfy

$$\left[\frac{\partial^2}{\partial x^2} - p_2^2 \right] \pi_{2y}^p(x, \zeta) = -\frac{j_y(x, \zeta)}{j\omega\epsilon_2} \quad (C.4.4)$$

$$\left[\frac{\partial^2}{\partial x^2} - p_2^2 \right] \pi_{2y}^r(x, \zeta) = 0 \quad (C.4.5)$$

where

$$p_2 = \sqrt{\zeta^2 - k_2^2} \quad (C.4.6)$$

π_{2y}^r is found directly from the homogeneous equation as

$$\pi_{2y}^r(x, \zeta) = B(\zeta) e^{-p_2 x} + C(\zeta) e^{p_2 x} \quad (C.4.7)$$

Before solving eq. (C.4.4), the general inhomogeneous second order equation

$$\left[\frac{\partial^2}{\partial x^2} - p^2 \right] \psi(x, \zeta) = -s(x, \zeta) \quad (C.4.8)$$

is considered. Fourier transforming on the x variable gives

$$\psi(x, \zeta) \leftrightarrow \tilde{\psi}(\xi, \zeta) \quad s(x, \zeta) \leftrightarrow \tilde{s}(\xi, \zeta) \quad (C.4.9)$$

$$\psi(x, \zeta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\psi}(\xi, \zeta) e^{j\xi x} d\xi \quad (C.4.10)$$

$$s(x, \zeta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{s}(\xi, \zeta) e^{j\xi x} d\xi \quad (C.4.11)$$

Substituting (C.4.10) and (C.4.11) into (C.4.8) yields

$$\tilde{\psi}(\xi, \zeta) = \frac{\bar{s}(\xi, \zeta)}{\xi^2 + p^2} = \frac{\bar{s}(\xi, \zeta)}{(\xi - jp)(\xi + jp)} \quad (\text{C.4.12})$$

Note that this spectral solution has simple pole singularities at $\xi = \pm jp$. Taking the inverse Fourier transform yields

$$\begin{aligned} \psi(x, \zeta) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\bar{s}(\xi, \zeta)}{(\xi - jp)(\xi + jp)} e^{j\xi x} d\xi \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dx' s(x', \zeta) \int_{-\infty}^{\infty} \frac{e^{j\xi(x-x')}}{(\xi - jp)(\xi + jp)} d\xi \\ &= \int_{-\infty}^{\infty} s(x', \zeta) g_{\zeta}^p(x|x') dx' \end{aligned} \quad (\text{C.4.13})$$

where $g_{\zeta}^p(x|x')$ is the Green's function for the forced principal wave

$$g_{\zeta}^p(x|x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\xi(x-x')}}{(\xi - jp)(\xi + jp)} d\xi \quad (\text{C.4.14})$$

To perform the inverse transform, a deformation of the real line integration is used and Cauchy's theorem is applied to an upper half plane closure (UHP) for $x > x'$ and to a lower half plane closure (LHP) for $x < x'$. This yields

$$g_{\zeta}^p(x|x') = \frac{e^{-p|x-x'|}}{2p} \quad (\text{C.4.15})$$

and with $\text{Re}\{p\} > 0$ and $\text{Im}\{p\} > 0$.

The primary wave solution is obtained by comparing eqs. (C.4.4) and (C.4.8)

$$\pi_{2y}^p(x, \zeta) = \int_{-\infty}^0 \frac{j_y(x', \zeta)}{j\omega\epsilon_2} g_{\zeta}^p(x|x') dx' \quad (\text{C.4.16})$$

where

$$g_{\zeta}^p(x|x') = \frac{e^{-p_2|x-x'|}}{2p_2} \quad (\text{C.4.17})$$

and with $\text{Re}\{p_2\} > 0$ and $\text{Im}\{p_2\} > 0$.

SUMMARY OF SPECTRAL SOLUTION:

$$\pi_{1y}(x, \zeta) = A(\zeta) e^{-p_1 x} \quad (\text{C.4.18})$$

$$\pi_{2y}(x, \zeta) = \int_{-t}^0 \frac{j_y(x', \zeta)}{j\omega\epsilon_2} g_{\zeta}^p(x|x') dx' + B(\zeta) e^{-p_2 x} + C(\zeta) e^{p_2 x} \quad (\text{C.4.19})$$

Here $A(\zeta)$, $B(\zeta)$ and $C(\zeta)$ are unknown coefficients which will be determined by application of the boundary conditions.

C.5. IMPLEMENTATION OF BOUNDARY CONTIONS TO DETERMINE UNKNOWN SPECTRAL AMPLITUDES

The boundary conditions that $E_y = 0$ at the surface of conductor and E_y, H_z are continuous at the interface between two regions give

$$E_y|_{x=-t} = 0 \rightarrow e_y|_{x=-t} = 0 \rightarrow \pi_{2y}|_{x=-t} = 0 \quad (\text{C.5.1})$$

$$E_y|_{x=0^+} = E_y|_{x=0^-} \rightarrow e_y|_{x=0^+} = e_y|_{x=0^-} \rightarrow k_1^2 \pi_{1y}|_{x=0} = k_2^2 \pi_{2y}|_{x=0} \quad (\text{C.5.2})$$

$$H_z|_{x=0^+} = H_z|_{x=0^-} \rightarrow h_z|_{x=0^+} = h_z|_{x=0^-} \rightarrow \epsilon_1 \frac{\partial \pi_{1y}}{\partial x} \Big|_{x=0} = \epsilon_2 \frac{\partial \pi_{2y}}{\partial x} \Big|_{x=0} \quad (\text{C.5.3})$$

Substituting (C.4.18) and (C.4.19) allows us to solve for the spectral amplitudes

$$A(\zeta) = \frac{F e^{p_2 t}}{\Delta} \quad (\text{C.5.4})$$

$$C(\zeta) = \frac{F}{2e^{-p_2 t} \Delta} \left[\left(\frac{k_1}{k_2} \right)^2 - \frac{\epsilon_1 p_1}{\epsilon_2 p_2} \right] \quad (\text{C.5.5})$$

$$B(\zeta) = -e^{-2p_2 t} \left[C(\zeta) + \int_{-t}^0 \frac{j_y(x', \zeta)}{j\omega\epsilon_2} \frac{e^{-p_2 x'}}{2p_2} dx' \right] \quad (C.5.6)$$

where

$$F = \int_{-t}^0 \frac{j_y(x', \zeta)}{j\omega\epsilon_2} \frac{e^{-p_2 t}}{p_2} \sinh[p_2(x' + t)] dx' \quad (C.5.7)$$

$$\tilde{\Delta} = \left(\frac{k_1}{k_2} \right)^2 \cosh(p_2 t) + \frac{\epsilon_1 p_1}{\epsilon_2 p_2} \sinh(p_2 t) \quad (C.5.8)$$

C.6. POTENTIAL SOLUTION IN EITHER REGION

Substituting the spectral amplitudes (C.5.4)-(C.5.6) into the transform domain representation of the potentials (C.4.18) and (C.4.19) gives

$$\pi_{1y}(x, \zeta) = \int_{-t}^0 \frac{j_y(x', \zeta)}{j\omega\epsilon_2} g_{12}(x|x') dx' \quad (C.6.1)$$

$$\pi_{2y}(x, \zeta) = \int_{-t}^0 \frac{j_y(x', \zeta)}{j\omega\epsilon_2} g_{22}(x|x') dx' \quad (C.6.2)$$

where

$$g_{12}(x|x') = \frac{e^{-p_1 x} \sinh[p_2(x' + t)]}{p_2 \tilde{\Delta}} \quad (C.6.3)$$

$$g_{22}(x|x') = g_{22}^p(x|x') + g_{22}^r(x|x') \quad (C.6.4)$$

$$g_{22}^p(x|x') = \frac{e^{-p_2|x-x'|}}{2p_2} \quad (C.6.5)$$

$$g_{22}^r(x|x') = -e^{-p_2(x+x'+2t)} + \left[\left(\frac{k_1}{k_2} \right)^2 - \frac{\epsilon_1 p_1}{\epsilon_2 p_2} \right] \frac{e^{-p_2 t}}{p_2 \tilde{\Delta}} \sinh[p_2(x+t)] \sinh[p_2(x'+t)] \quad (C.6.6)$$

C.7. SPACE-DOMAIN FIELDS AND GREEN'S FUNCTIONS

Applying the relation between the electric field and Hertzian potentials, the electric field in the spectral domain is obtained

$$e_{1y}(x, \zeta) = k_1^2 \pi_{1y}(x, \zeta) = \frac{-jk_1 \eta_1 \epsilon_1}{\epsilon_2} \int_{-t}^0 j_y(x', \zeta) g_{12}(x|x') dx' \quad (C.7.1)$$

$$e_{2y}(x, \zeta) = k_2^2 \pi_{2y}(x, \zeta) = -jk_2 \eta_2 \int_{-t}^0 j_y(x', \zeta) g_{22}(x|x') dx' \quad (C.7.2)$$

where

$$\eta_i = \sqrt{\frac{\mu_i}{\epsilon_i}} \quad i = 1, 2 \quad (C.7.3)$$

Taking the inverse Fourier transform of $e_y(x, \zeta)$ and noting that j_y is the Fourier transform of J_y yields

$$\begin{aligned} E_{1y}(x, z) &= \frac{-jk_1 \eta_1 \epsilon_1}{\epsilon_2} \int_{-t}^0 dx' \int_{-\infty}^{\infty} dz' J_y(x', z') \int_{-\infty}^{\infty} g_{12}(x|x') \frac{e^{j\zeta(z-z')}}{2\pi} d\zeta \\ &= \int_{LCS} J_y(x', z') G_{12}(x|x', z-z') dx' dz' \end{aligned} \quad (C.7.4)$$

$$\begin{aligned} E_{2y}(x, z) &= -jk_2 \eta_2 \int_{-t}^0 dx' \int_{-\infty}^{\infty} dz' J_y(x', z') \int_{-\infty}^{\infty} g_{22}(x|x') \frac{e^{j\zeta(z-z')}}{2\pi} d\zeta \\ &= \int_{LCS} J_y(x', z') G_{22}(x|x', z-z') dx' dz' \end{aligned} \quad (C.7.5)$$

where LCS designates the longitudinal cross section of the discontinuity region, and

$$G_{12}(x|x', z-z') = \frac{-jk_1 \eta_1 \epsilon_1}{\epsilon_2} \int_{-\infty}^{\infty} g_{12}(x|x') \frac{e^{j\zeta(z-z')}}{2\pi} d\zeta \quad (C.7.6)$$

$$G_{22}(x|x', z-z') = -jk_2 \eta_2 \int_{-\infty}^{\infty} g_{22}(x|x') \frac{e^{j\zeta(z-z')}}{2\pi} d\zeta \quad (C.7.7)$$

C.8. SUMMARY OF THE GREEN'S FUNCTIONS

$$G_{12}(x|x', z-z') = -\frac{jk_1\eta_1\varepsilon_1}{\varepsilon_2} \int_{-\infty}^{\infty} \frac{e^{-p_1x} \sinh[p_2(x'+t)] e^{j\zeta(z-z')}}{2\pi p_2 \bar{\Delta}} d\zeta \quad (\text{C.8.1})$$

$$G_{22}(x|x', z-z') = G_{22}^p(x|x', z-z') + G_{22}^r(x|x', z-z') \quad (\text{C.8.2})$$

$$G_{22}^p(x|x', z-z') = -jk_2\eta_2 \int_{-\infty}^{\infty} \frac{e^{-p_2|x-x'|} e^{j\zeta(z-z')}}{4\pi p_2} d\zeta \quad (\text{C.8.3})$$

$$G_{22}^r(x|x', z-z') = -jk_2\eta_2 \left\{ - \int_{-\infty}^{\infty} \frac{e^{-p_2(x+x'+2t)} e^{j\zeta(z-z')}}{4\pi p_2} d\zeta \right. \\ \left. + \int_{-\infty}^{\infty} \left[\left(\frac{k_1}{k_2} \right)^2 - \frac{\varepsilon_1 p_1}{\varepsilon_2 p_2} \right] \frac{e^{-p_2 t} e^{j\zeta(z-z')}}{2\pi p_2 \bar{\Delta}} \sinh[p_2(x+t)] \sinh[p_2(x'+t)] d\zeta \right\} \quad (\text{C.8.4})$$

where

$$p_i = \sqrt{\zeta^2 - k_i^2} \quad i = 1, 2 \quad (\text{C.8.5})$$

$$\bar{\Delta} = \left(\frac{k_1}{k_2} \right)^2 \cosh(p_2 t) + \frac{\varepsilon_1 p_1}{\varepsilon_2 p_2} \sinh(p_2 t) \quad (\text{C.8.6})$$

When $\mu_1 = \mu_2 = \mu_0$, the above Green's functions can be rewritten as

$$G_{12}(x|x', z-z') = -jk_1\eta_1 \int_{-\infty}^{\infty} \frac{e^{-p_1x} \sinh[p_2(x'+t)] e^{j\zeta(z-z')}}{2\pi \bar{\Delta}_h} d\zeta \quad (\text{C.8.7})$$

$$G_{22}(x|x', z-z') = -jk_2\eta_2 \int_{-\infty}^{\infty} \frac{e^{j\zeta(z-z')}}{4\pi p_2} d\zeta \{ e^{-p_2|x-x'|} - e^{-p_2(x+x'+2t)} \\ + 2 \frac{p_2 - p_1}{\bar{\Delta}_h} e^{-p_2 t} \sinh[p_2(x+t)] \sinh[p_2(x'+t)] \} \quad (\text{C.8.8})$$

where

$$\bar{\Delta}_h = p_2 \cosh(p_2 t) + p_1 \sinh(p_2 t) \quad (\text{C.8.9})$$

APPENDIX D

2-D GREEN'S FUNCTION FOR TM EXCITATION OF GROUNDED DIELECTRIC MATERIAL SHEET

In deriving the Green's function for TE excitation in Appendix C, the electric Hertzian potential in each region is found first, then the EM fields and the Green's function. Here, the Green's function for TM excitation is derived by a direct field approach.

D.1. NOTATION

Some words about notation here might be helpful. As a convention, upper case letters denote space domain quantities, while their transform domain counterparts are designated by lower case letters. The symbol j denotes the elementary imaginary number.

D.2. TRANSFORM DOMAIN MAXWELL EQUATIONS AND GENERAL GUIDED WAVE THEORY

Consider the structure depicted in Figure D.1. The current source $\vec{J}(\vec{r}) = \hat{x}J_x(x, z) + \hat{z}J_z(x, z)$ radiates into the two regions, and Maxwell's equations are:

$$\left. \begin{aligned} \nabla \cdot \vec{E} &= \rho/\epsilon \\ \nabla \times \vec{E} &= -j\omega\mu\vec{H} \\ \nabla \times \vec{H} &= \vec{J} + j\omega\epsilon\vec{E} \\ \nabla \cdot \vec{H} &= 0 \end{aligned} \right\} \quad (D.2.1)$$

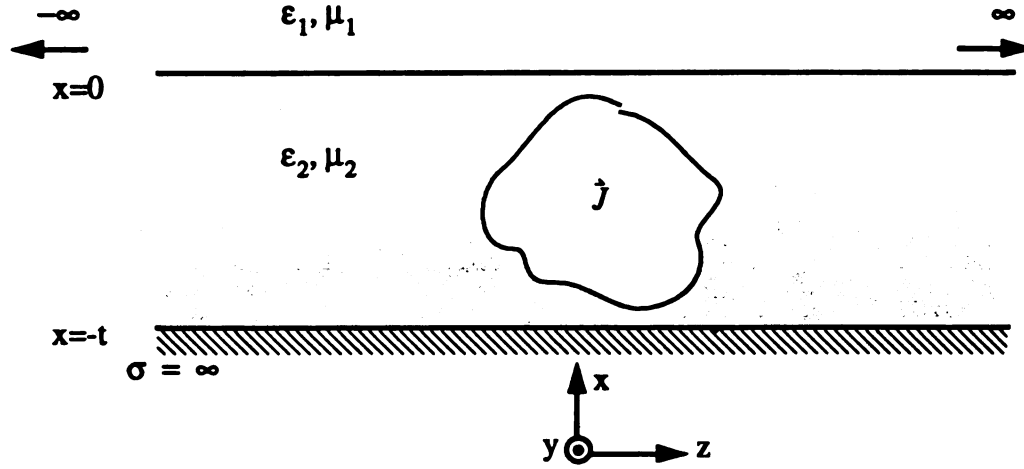


Figure D.1. Geometrical configuration. $\hat{J}(\vec{r}) = \hat{x}J_x(x, z) + \hat{z}J_z(x, z)$ for 2-D TM excitation.

Maxwell's equations can be rewritten in spectral domain by Fourier transforming on spatial variables tangential to the layer interfaces. Axial uniformity along the z -axis [i.e., $\epsilon \neq \epsilon(z)$ and $\mu \neq \mu(z)$] prompts Fourier transformation on that variable. Define the axial transform pair $F(x, z) \leftrightarrow f(x, \zeta)$ as:

$$F(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x, \zeta) e^{j\zeta z} d\zeta \quad (\text{D.2.2})$$

$$f(x, \zeta) = \int_{-\infty}^{\infty} F(x, z) e^{-j\zeta z} dz \quad (\text{D.2.3})$$

Taking Fourier transforms of equations (D.2.1) yields

$$\left. \begin{aligned} (\nabla_t + \hat{z}j\zeta) \cdot \vec{e} &= \rho/\epsilon \\ (\nabla_t + \hat{z}j\zeta) \times \vec{e} &= -j\omega\mu\vec{h} \\ (\nabla_t + \hat{z}j\zeta) \times \vec{h} &= \vec{j} + j\omega\epsilon\vec{e} \\ (\nabla_t + \hat{z}j\zeta) \cdot \vec{h} &= 0 \end{aligned} \right\} \quad (\text{D.2.4})$$

where

$$\nabla_t = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} \quad (\text{D.2.5})$$

The EM fields are written as the sum of transverse and longitudinal components

$$\left. \begin{aligned} \vec{e} &= \vec{e}_t + \hat{z}e_z \\ \vec{h} &= \vec{h}_t + \hat{z}h_z \end{aligned} \right\} \quad (\text{D.2.6})$$

The transverse components are derived from the longitudinal components by

$$\left. \begin{aligned} \vec{e}_t &= -\frac{1}{p^2} [j\zeta\nabla_t e_z + j\omega\mu\hat{z} \times \nabla_t h_z + j\omega\mu\vec{j}_t] \\ \vec{h}_t &= -\frac{1}{p^2} [-j\omega\epsilon\hat{z} \times \nabla_t e_z + j\zeta\nabla_t h_z - j\zeta\hat{z} \times \vec{j}_t] \end{aligned} \right\} \quad (\text{D.2.7})$$

where

$$p^2 = \zeta^2 - k^2 \quad (\text{D.2.8})$$

is a wavenumber parameter and

$$k = \omega\sqrt{\epsilon\mu} \quad (\text{D.2.9})$$

The longitudinal components of EM fields satisfy the following Helmholtz equations

$$\left. \begin{aligned} \nabla_t^2 e_z - p^2 e_z &= -\frac{\zeta}{\omega\epsilon} (\nabla_t + \hat{z}j\zeta) \cdot \vec{j} + j\omega\mu j_z \\ \nabla_t^2 h_z - p^2 h_z &= -\hat{z} \cdot [(\nabla_t + \hat{z}j\zeta) \times \vec{j}] \end{aligned} \right\} \quad (\text{D.2.10})$$

D.3. EM FIELDS OF Y-INVARIANT TM MODES

For y-invariant TM fields $h_z = 0$, $\frac{\partial}{\partial y} = 0$ and

$$\left. \begin{aligned} \hat{j} &= \hat{x}j_x + \hat{z}j_z \\ \hat{e} &= \hat{x}e_x + \hat{z}e_z \\ \hat{h} &= \hat{y}h_y \end{aligned} \right\} \quad (\text{D.3.1})$$

the transverse components are related to e_z by

$$\left. \begin{aligned} e_x &= -\frac{1}{p^2} \left(j\zeta \frac{\partial e_z}{\partial x} + j\omega\mu j_x \right) \\ h_y &= \frac{1}{p^2} \left(j\omega\epsilon \frac{\partial e_z}{\partial x} + j\zeta j_x \right) \end{aligned} \right\} \quad (\text{D.3.2})$$

where e_z is the solution of wave equation

$$\frac{\partial^2 e_z}{\partial x^2} - p^2 e_z = -\frac{\zeta}{\omega\epsilon} \frac{\partial j_x}{\partial x} - \frac{jp^2 j_z}{\omega\epsilon} \quad (\text{D.3.3})$$

D.4. DECOMPOSITION INTO PRIMARY AND REFLECTED WAVES

Since region 1 is source-free, the homogeneous wave equation

$$\left[\frac{\partial^2}{\partial x^2} - p_1^2 \right] e_{1z}(x, \zeta) = 0 \quad (\text{D.4.1})$$

for total field e_{1z} has a solution

$$e_{1z}(x, \zeta) = A(\zeta) e^{-p_1 x} \quad (\text{D.4.2})$$

Note that the solution satisfies the radiation condition, i.e. it is an outward travelling, decaying wave with $\text{Re}\{p_1\} > 0$ and $\text{Im}\{p_1\} > 0$, where

$$p_1 = \sqrt{\zeta^2 - k_1^2} \quad (\text{D.4.3})$$

and $Re \{ \dots \}$ designates the real part of the quantity within the braces, $Im \{ \dots \}$ designates the imaginary part.

The total EM fields in region 2 are the sum of a primary wave excited by j_x and j_z in an unbounded region of that sheet and source-free reflected waves in the same sheet. They satisfy

$$\left[\frac{\partial^2}{\partial x^2} - p_2^2 \right] e_{2z}^p(x, \zeta) = -\frac{\zeta}{\omega \epsilon} \frac{\partial j_x}{\partial x} - \frac{j p^2 j_z}{\omega \epsilon} \quad (D.4.4)$$

$$\left[\frac{\partial^2}{\partial x^2} - p_2^2 \right] e_{2z}^r(x, \zeta) = 0 \quad (D.4.5)$$

where

$$p_2 = \sqrt{\zeta^2 - k_2^2} \quad (D.4.6)$$

e_{2z}^r is obtained directly from the homogeneous equation as

$$e_{2z}^r(x, \zeta) = B(\zeta) e^{-p_2 x} + C(\zeta) e^{p_2 x} \quad (D.4.7)$$

In Appendix C, it is found that the general inhomogeneous second order equation

$$\left[\frac{\partial^2}{\partial x^2} - p^2 \right] \psi(x, \zeta) = -s(x, \zeta) \quad (D.4.8)$$

has a solution

$$\psi(x, \zeta) = \int_{-\infty}^{\infty} s(x', \zeta) g_{\zeta}^p(x|x') dx' \quad (D.4.9)$$

where $g_{\zeta}^p(x|x')$ is the Green's function for the forced principal wave

$$g_{\zeta}^p(x|x') = \frac{e^{-p|x-x'|}}{2p} \quad (D.4.10)$$

and with $Re \{ p \} > 0$ and $Im \{ p \} > 0$.

The primary wave solutions are obtained by comparing (D.4.4) and (D.4.8), and some operations lead to

$$e_{2z}^p(x, \zeta) = -\frac{1}{j\omega\epsilon_2} \int_{-i}^0 [j\zeta p_2 j_x \operatorname{sgn}(x-x') + p_2^2 j_z] \frac{e^{-p_2|x-x'|}}{2p_2} dx' \quad (\text{D.4.11})$$

$$e_{2x}^p(x, \zeta) = -\frac{1}{j\omega\epsilon_2} \int_{-i}^0 [-\zeta^2 j_x + j\zeta p_2 j_z \operatorname{sgn}(x-x')] \frac{e^{-p_2|x-x'|}}{2p_2} dx' - \frac{j_x(x, \zeta)}{j\omega\epsilon_2} \quad (\text{D.4.12})$$

$$h_{2y}^p(x, \zeta) = \int_{-i}^0 [j\zeta j_x + p_2 j_z \operatorname{sgn}(x-x')] \frac{e^{-p_2|x-x'|}}{2p_2} dx' \quad (\text{D.4.13})$$

where

$$\operatorname{sgn}(x-x') = \begin{cases} 1 & x > x' \\ -1 & x < x' \end{cases} \quad (\text{D.4.14})$$

and with $\operatorname{Re}\{p_2\} > 0$ and $\operatorname{Im}\{p_2\} > 0$.

SUMMARY OF SPECTRAL SOLUTION:

$$e_{1z}(x, \zeta) = A(\zeta) e^{-p_1 x} \quad (\text{D.4.15})$$

$$e_{1x}(x, \zeta) = -\frac{j\zeta}{p_1^2} \frac{\partial e_{1z}}{\partial x} = \frac{j\zeta}{p_1} A(\zeta) e^{-p_1 x} \quad (\text{D.4.16})$$

$$h_{1y}(x, \zeta) = \frac{j\omega\epsilon_1}{p_1^2} \frac{\partial e_{1z}}{\partial x} = -\frac{j\omega\epsilon_1}{p_1} A(\zeta) e^{-p_1 x} \quad (\text{D.4.17})$$

$$e_{2z}(x, \zeta) = e_{2z}^p(x, \zeta) + B(\zeta) e^{-p_2 x} + C(\zeta) e^{p_2 x} \quad (\text{D.4.18})$$

$$e_{2x}(x, \zeta) = e_{2x}^p(x, \zeta) + \frac{j\zeta}{p_2} [B(\zeta) e^{-p_2 x} - C(\zeta) e^{p_2 x}] \quad (\text{D.4.19})$$

$$h_{2y}(x, \zeta) = h_{2y}^p(x, \zeta) - \frac{j\omega\epsilon_2}{p_2} [B(\zeta) e^{-p_2 x} - C(\zeta) e^{p_2 x}] \quad (\text{D.4.20})$$

Here $A(\zeta)$, $B(\zeta)$ and $C(\zeta)$ are unknown coefficients which will be determined by application of the boundary conditions.

D.5. IMPLEMENTATION OF BOUNDARY CONDITIONS TO DETERMINE UNKNOWN SPECTRAL AMPLITUDES

The boundary conditions that $E_z = 0$ at the conductor surface and E_z, H_y are continuous at the interface between two regions give

$$E_z|_{x=-t} = 0 \rightarrow e_{2z}|_{x=-t} = 0 \quad (\text{D.5.1})$$

$$E_z|_{x=0^+} = E_z|_{x=0^-} \rightarrow e_{1z}|_{x=0^+} = e_{2z}|_{x=0^-} \quad (\text{D.5.2})$$

$$H_y|_{x=0^+} = H_y|_{x=0^-} \rightarrow h_{2y}|_{x=0^+} = h_{2y}|_{x=0^-} \quad (\text{D.5.3})$$

Substituting equations from (D.4.15) to (D.4.20) allows us to solve for the spectral amplitudes

$$A(\zeta) = -\frac{j\zeta V_x^c + p_2 V_z^c}{\bar{\Delta}} p_1 p_2 \quad (\text{D.5.4})$$

$$B(\zeta) e^{-p_2 x} + C(\zeta) e^{p_2 x} = [-j\zeta V_x^- + p_2 V_z^-] p_2 e^{-p_2(2t+x)} - \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}} (j\zeta V_x^c + p_2 V_z^c) p_2 e^{-p_2 t} \sinh[p_2(t+x)] \quad (\text{D.5.5})$$

$$B(\zeta) e^{-p_2 x} - C(\zeta) e^{p_2 x} = [-j\zeta V_x^- + p_2 V_z^-] p_2 e^{-p_2(2t+x)} + \frac{p_1 - p_2 \epsilon_1 / \epsilon_2}{\bar{\Delta}} (j\zeta V_x^c + p_2 V_z^c) p_2 e^{-p_2 t} \cosh[p_2(t+x)] \quad (\text{D.5.6})$$

where

$$\left. \begin{aligned} V_x^- &= \int_{-t}^0 \frac{j_x(x', \zeta)}{j\omega\epsilon_2} \frac{e^{-p_2 x'}}{2p_2} dx' \\ V_z^- &= \int_{-t}^0 \frac{j_z(x', \zeta)}{j\omega\epsilon_2} \frac{e^{-p_2 x'}}{2p_2} dx' \end{aligned} \right\} \quad (\text{D.5.7})$$

$$\left. \begin{aligned} V_x^c &= \int_{-t}^0 \frac{j_x(x', \zeta)}{j\omega\epsilon_2} \frac{\cosh[p_2(x' + t)]}{p_2} dx' \\ V_z^s &= \int_{-t}^0 \frac{j_z(x', \zeta)}{j\omega\epsilon_2} \frac{\sinh[p_2(x' + t)]}{p_2} dx' \end{aligned} \right\} \quad (\text{D.5.8})$$

$$\bar{\Delta} = p_1 \cosh(p_2 t) + p_2 (\epsilon_1/\epsilon_2) \sinh(p_2 t) \quad (\text{D.5.9})$$

D.6. SPACE-DOMAIN FIELDS AND GREEN'S FUNCTIONS

Substituting the spectral amplitudes (D.5.4)-(D.5.6) into the transform domain representations of the EM fields (D.4.15)-(D.4.20) gives

$$e_{1z} = -\frac{1}{j\omega\epsilon_2} \frac{p_1 e^{-p_1 x}}{\bar{\Delta}} \int_{-t}^0 \{ j\zeta j_x \cosh[p_2(t+x')] + p_2 j_z \sinh[p_2(t+x')] \} dx' \quad (\text{D.6.1})$$

$$e_{1x} = \frac{j\zeta}{p_1} e_{1z} \quad (\text{D.6.2})$$

$$\begin{aligned} e_{2z} &= -\frac{1}{2j\omega\epsilon_2} \int_{-t}^0 \{ [j\zeta j_x \operatorname{sgn}(x-x') + p_2 j_z] e^{-p_2|x-x'|} \\ &\quad - [-j\zeta j_x + p_2 j_z] e^{-p_2(2t+x+x')} + 2 \frac{p_1 - p_2 \epsilon_1/\epsilon_2}{\bar{\Delta}} \sinh[p_2(t+x)] \\ &\quad e^{-p_2 t} [j\zeta j_x \cosh[p_2(t+x')] + p_2 j_z \sinh[p_2(t+x')]] \} dx' \end{aligned} \quad (\text{D.6.3})$$

$$\begin{aligned} e_{2x} &= -\frac{1}{2j\omega\epsilon_2 p_2} \int_{-t}^0 \{ [-\zeta^2 j_x + j\zeta p_2 j_z \operatorname{sgn}(x-x')] e^{-p_2|x-x'|} \\ &\quad - [\zeta^2 j_x + j\zeta p_2 j_z] e^{-p_2(2t+x+x')} - 2 \frac{p_1 - p_2 \epsilon_1/\epsilon_2}{\bar{\Delta}} \cosh[p_2(t+x)] \\ &\quad e^{-p_2 t} [-\zeta^2 j_x \cosh[p_2(t+x')] + j\zeta p_2 j_z \sinh[p_2(t+x')]] \} dx' - \frac{j_x}{j\omega\epsilon_2} \end{aligned} \quad (\text{D.6.4})$$

These results are rewritten in Green's function form as

$$\dot{e}_1(x, \zeta) = \int_{-t}^0 \bar{g}_{12}(x|x') \bullet \dot{j}(x', \zeta) dx' \quad (D.6.5)$$

$$\dot{e}_2(x, \zeta) = \int_{-t}^0 \bar{g}_{22}(x|x') \bullet \dot{j}(x', \zeta) dx' \quad (D.6.6)$$

where

$$g_{12xx}(x|x') = \frac{1}{j\omega\epsilon_2} \frac{e^{-p_1 x}}{\bar{\Delta}} \zeta^2 \cosh[p_2(t+x')] \quad (D.6.7)$$

$$g_{12xz}(x|x') = \frac{1}{j\omega\epsilon_2} \frac{e^{-p_1 x}}{\bar{\Delta}} (-j\zeta p_2) \sinh[p_2(t+x')] \quad (D.6.8)$$

$$g_{12zx}(x|x') = \frac{1}{j\omega\epsilon_2} \frac{e^{-p_1 x}}{\bar{\Delta}} (-j\zeta p_1) \cosh[p_2(t+x')] \quad (D.6.9)$$

$$g_{12zz}(x|x') = \frac{1}{j\omega\epsilon_2} \frac{e^{-p_1 x}}{\bar{\Delta}} (-p_1 p_2) \sinh[p_2(t+x')] \quad (D.6.10)$$

$$g_{22xx}(x|x') = -\frac{\delta(x-x')}{j\omega\epsilon_2} + \frac{\zeta^2}{2j\omega\epsilon_2 p_2} \{ e^{-p_2|x-x'|} + e^{-p_2(2t+x+x')} \\ - 2 \frac{p_1 - p_2\epsilon_1/\epsilon_2}{\bar{\Delta}} e^{-p_2 t} \cosh[p_2(t+x)] \cosh[p_2(t+x')] \} \quad (D.6.11)$$

$$g_{22xz}(x|x') = -\frac{\zeta}{2\omega\epsilon_2} \{ \operatorname{sgn}(x-x') e^{-p_2|x-x'|} - e^{-p_2(2t+x+x')} \\ - 2 \frac{p_1 - p_2\epsilon_1/\epsilon_2}{\bar{\Delta}} e^{-p_2 t} \cosh[p_2(t+x)] \sinh[p_2(t+x')] \} \quad (D.6.12)$$

$$g_{22zx}(x|x') = -\frac{\zeta}{2\omega\epsilon_2} \{ \operatorname{sgn}(x-x') e^{-p_2|x-x'|} + e^{-p_2(2t+x+x')} \\ + 2 \frac{p_1 - p_2\epsilon_1/\epsilon_2}{\bar{\Delta}} e^{-p_2 t} \sinh[p_2(t+x)] \cosh[p_2(t+x')] \} \quad (D.6.13)$$

$$\begin{aligned}
g_{22zz}(x|x') = & -\frac{p_2}{2j\omega\epsilon_2} \{ e^{-p_2|x-x'|} - e^{-p_2(2t+x+x')} \\
& + 2\frac{p_1 - p_2\epsilon_1/\epsilon_2}{\bar{\Delta}} e^{-p_2t} \sinh[p_2(t+x)] \sinh[p_2(t+x')] \} \quad (D.6.14)
\end{aligned}$$

Taking the inverse Fourier transform of $\vec{e}(x, \zeta)$ and noting that $\vec{J}$ is the Fourier transform of $\vec{J}$ yields

$$\vec{E}_1(x, z) = \int_{LCS} \vec{G}_{12}(x, z|x', z') \bullet \vec{J}(x', z') dx' dz' \quad (D.6.15)$$

$$\vec{E}_2(x, z) = \int_{LCS} \vec{G}_{22}(x, z|x', z') \bullet \vec{J}(x', z') dx' dz' \quad (D.6.16)$$

where *LCS* designates the longitudinal cross section of the discontinuity region, and

$$\vec{G}_{12}(x, z|x', z') = \int_{-\infty}^{\infty} \bar{g}_{12}(x|x') \frac{e^{j\zeta(z-z')}}{2\pi} d\zeta \quad (D.6.17)$$

$$\vec{G}_{22}(x, z|x', z') = \int_{-\infty}^{\infty} \bar{g}_{22}(x|x') \frac{e^{j\zeta(z-z')}}{2\pi} d\zeta \quad (D.6.18)$$

D.7. SUMMARY OF THE GREEN'S FUNCTIONS

$$G_{12xx}(x, z|x', z') = -\frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{e^{-p_1x}}{2\pi\bar{\Delta}} \zeta^2 \cosh[p_2(t+x')] e^{j\zeta(z-z')} d\zeta \quad (D.7.1)$$

$$G_{12xz}(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{e^{-p_1x}}{2\pi\bar{\Delta}} j\zeta p_2 \sinh[p_2(t+x')] e^{j\zeta(z-z')} d\zeta \quad (D.7.2)$$

$$G_{12zx}(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{e^{-p_1x}}{2\pi\bar{\Delta}} j\zeta p_1 \cosh[p_2(t+x')] e^{j\zeta(z-z')} d\zeta \quad (D.7.3)$$

$$G_{12zz}(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{e^{-p_1 x}}{2\pi\tilde{\Delta}} p_1 p_2 \sinh[p_2(t+x')] e^{j\zeta(z-z')} d\zeta \quad (D.7.4)$$

$$G_{22xx}(x, z|x', z') = \frac{j\eta_2}{k_2} \delta(\vec{p} - \vec{p}') + \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{-\zeta^2}{4\pi p_2} \{ e^{-p_2|x-x'|} + e^{-p_2(2t+x+x')} \\ - 2 \frac{p_1 - p_2 \epsilon_1/\epsilon_2}{\tilde{\Delta}} e^{-p_2 t} \cosh[p_2(t+x)] \cosh[p_2(t+x')] \} e^{j\zeta(z-z')} d\zeta \quad (D.7.5)$$

$$G_{22xz}(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{j\zeta}{4\pi} \{ \operatorname{sgn}(x-x') e^{-p_2|x-x'|} - e^{-p_2(2t+x+x')} \\ - 2 \frac{p_1 - p_2 \epsilon_1/\epsilon_2}{\tilde{\Delta}} e^{-p_2 t} \cosh[p_2(t+x)] \sinh[p_2(t+x')] \} e^{j\zeta(z-z')} d\zeta \quad (D.7.6)$$

$$G_{22zx}(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{j\zeta}{4\pi} \{ \operatorname{sgn}(x-x') e^{-p_2|x-x'|} + e^{-p_2(2t+x+x')} \\ + 2 \frac{p_1 - p_2 \epsilon_1/\epsilon_2}{\tilde{\Delta}} e^{-p_2 t} \sinh[p_2(t+x)] \cosh[p_2(t+x')] \} e^{j\zeta(z-z')} d\zeta \quad (D.7.7)$$

$$G_{22zz}(x, z|x', z') = \frac{j\eta_2}{k_2} \int_{-\infty}^{\infty} \frac{p_2}{4\pi} \{ e^{-p_2|x-x'|} - e^{-p_2(2t+x+x')} \\ + 2 \frac{p_1 - p_2 \epsilon_1/\epsilon_2}{\tilde{\Delta}} e^{-p_2 t} \sinh[p_2(t+x)] \sinh[p_2(t+x')] \} e^{j\zeta(z-z')} d\zeta \quad (D.7.8)$$

where

$$p_i = \sqrt{\zeta^2 - k_i^2} \quad i = 1, 2 \quad (D.7.9)$$

$$\eta_i = \sqrt{\frac{\mu_i}{\epsilon_i}} \quad i = 1, 2 \quad (D.7.10)$$

$$\tilde{\Delta} = p_1 \cosh(p_2 t) + p_2 (\epsilon_1/\epsilon_2) \sinh(p_2 t) \quad (D.7.11)$$

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PART II
PROPAGATION OF EM PULSES EXCITED
BY AN ELECTRIC DIPOLE IN CONDUCTING MEDIA

CHAPTER 1

INTRODUCTION

The purpose of this research is to analyze the propagation characteristics of electromagnetic (EM) pulse in conducting media, and to understand the effects of the pulse waveform, the antenna dimensions and other factors on the EM pulse propagation. The interaction of an EM pulse with a conducting shell in seawater will also be studied. The results of this study may lead to some applications of EM pulses in an underground or seawater environment.

It is well known that a sinusoidal EM wave suffers a strong attenuation of exponential decaying nature when it propagates in a conducting medium. However, an EM pulse excited by an antenna in a conducting medium may not attenuate as rapidly and may propagate over a moderate distance from the antenna because it contains a wide band of low frequency components. Existing studies on the propagation of a transient EM wave in a conducting medium are either incomplete or approximate in nature [1]-[6].

In this part of the thesis, the exact solutions for transient EM fields excited by an electric dipole antenna with an impulsive current in a conducting medium are found. Thus, by convolution, the propagation of EM pulses excited by the antenna with currents of arbitrary waveforms can be evaluated. The optimum waveform for the antenna current which can generate an EM pulse with a maximum intensity at a particular distance from the antenna has been found. It is found that the EM fields of an EM pulse excited by an antenna in a conducting medium can be divided into two parts. The first part is an impulse

wave which propagates with the speed of light ($1/\sqrt{\mu\epsilon}$) and decays exponentially. The second part of the EM field builds up gradually and propagates slowly. More importantly, this second part of the EM field attenuates as an inverse power of distance (between the third and the fifth inverse power of distance depending on the waveform of the antenna excitation current), which is a much slower rate than exponential decay. This wave behaves somewhat like a “diffusion wave”.

Computational methods for obtaining numerical results are discussed. EM pulses excited by a dipole antenna with various antenna currents in a conducting medium are obtained. The optimum antenna current which generates an EM pulse with a maximum intensity at a particular distance is derived. Numerical results are presented when the conducting medium is seawater.

There are seven chapters in this part. Chapter 1 gives an introduction for this research.

Chapter 2 presents the derivations of the exact time domain solutions for the electric and magnetic fields of the impulse radiation of an electric dipole antenna in conducting media. Some special cases and several computational methods are discussed.

In Chapter 3, the EM pulses in conducting media generated by an electric dipole antenna excited by various excitation currents are studied. The optimum waveform for the antenna current which can generate an EM pulse with a maximum intensity at a particular distance from the antenna is derived.

Chapter 4 presents an estimation of the maximum strength of an EM pulse which can be excited by a dipole antenna with a given finite EM pulse energy.

In Chapter 5, the effects of antenna size on the propagation of EM pulses excited in conducting media is studied.

How well an EM pulse can penetrate into a conducting shell in seawater is discussed in Chapter 6. In the final chapter, Chapter 7, the research accomplishments are summarized.

CHAPTER 2

EM PULSES EXCITED BY AN ELECTRIC DIPOLE ANTENNA WITH AN IMPULSIVE CURRENT IN CONDUCTING MEDIA

In this chapter the electromagnetic (EM) pulse radiated by an electric dipole antenna which is excited by a pulse antenna current and immersed in conducting media of infinite extent will be studied. Since the nature of the EM pulse radiated by the antenna in conducting media is dependent on the antenna excitation current, the EM pulse radiated by the antenna with an impulsive current will be determined. This EM pulse may be called the impulse radiation of a dipole antenna in conducting media. After this impulse radiation is obtained, the EM pulses generated by the same antenna with different antenna excitation currents can be easily obtained through the convolution of the impulse radiation with the antenna excitation currents.

First, the exact solutions in the time domain for the electric and magnetic fields of the impulse radiation of an electric dipole antenna in conducting media will be derived. Then some special cases will be reduced. Finally, several computational techniques will be discussed.

2.1. EXACT SOLUTIONS FOR THE EM FIELDS OF IMPULSE RADIATION MAINTAINED BY AN ELECTRIC DIPOLE ANTENNA IN CONDUCTING MEDIA

Consider an electric dipole antenna immersed in the conducting media of infinite extent and excited by an impulsive current. The dipole antenna located at the origin of a

spherical coordinate system and oriented in the z-direction has a small length dl and is excited by a current $I f(t)$ where I is the amplitude and $f(t)$ the temporal function of the antenna current as shown in Figure 2.1. Constant conductivity σ and permittivity ϵ and the free-space permeability μ are assumed for the conducting media.

Our aim is to find the electric and magnetic fields radiated by the dipole antenna using Laplace transform techniques.

The current density of the dipole antenna can be expressed as

$$\vec{j}_e(\vec{r}, t) = 2\delta(\vec{r}) Idl f(t) \quad (2.1.1)$$

or

$$\vec{J}_e(\vec{r}, s) = 2\delta(\vec{r}) Idl F(s) \quad (2.1.2)$$

in Laplace transform, and where

$$\left. \begin{aligned} F(s) &= \int_0^{\infty} f(t) e^{-st} dt \\ F(s) &\Leftrightarrow f(t) \end{aligned} \right\} \quad (2.1.3)$$

The electric and magnetic fields maintained by the dipole antenna are related to the antenna current by Maxwell's equations (in Laplace transform) as follows:

$$\nabla \times \vec{E}(\vec{r}, s) = -s\mu \vec{H}(\vec{r}, s) \quad (2.1.4)$$

$$\begin{aligned} \nabla \times \vec{H}(\vec{r}, s) &= \sigma \vec{E}(\vec{r}, s) + s\epsilon \vec{E}(\vec{r}, s) + \vec{J}_e(\vec{r}, s) \\ &= s(\epsilon + \sigma/s) \vec{E}(\vec{r}, s) + \vec{J}_e(\vec{r}, s) \\ &= s\epsilon^* \vec{E}(\vec{r}, s) + \vec{J}_e(\vec{r}, s) \end{aligned} \quad (2.1.5)$$

where $\epsilon^* = \epsilon + \sigma/s$ is the complex permittivity, and $\vec{E}(\vec{r}, s)$, $\vec{H}(\vec{r}, s)$ are the single-sided Laplace transforms of the electric and magnetic fields, respectively.

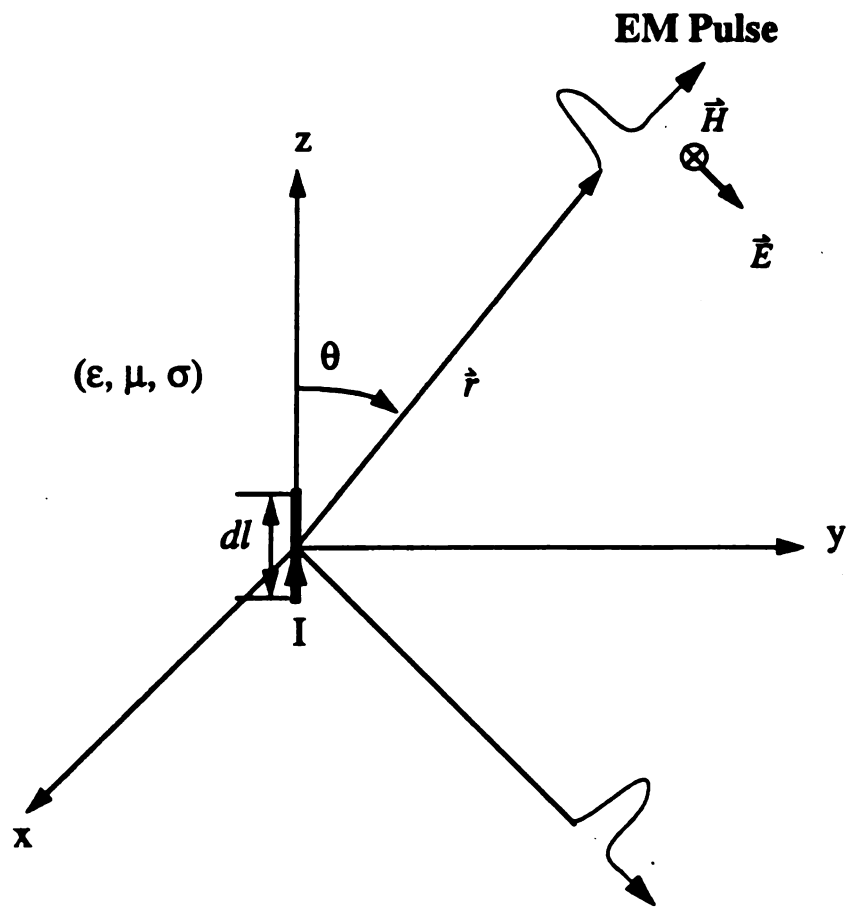


Figure 2.1. An EM pulse generated by an electric dipole antenna in conducting media.

The electric and magnetic fields can be expressed as the vector potential $\vec{A}(\vec{r}, s)$ and the scalar potential $\Phi(\vec{r}, s)$:

$$\vec{H}(\vec{r}, s) = \frac{1}{\mu} \nabla \times \vec{A}(\vec{r}, s) \quad (2.1.6)$$

$$\begin{aligned} \vec{E}(\vec{r}, s) &= -\nabla \Phi(\vec{r}, s) - s\vec{A}(\vec{r}, s) \\ &= \frac{1}{s\epsilon^* \mu} \nabla \times \nabla \times \vec{A}(\vec{r}, s) - \frac{1}{s\epsilon^*} \vec{J}_e(\vec{r}, s) \end{aligned} \quad (2.1.7)$$

$\vec{A}(\vec{r}, s)$ and $\Phi(\vec{r}, s)$ are related by the Lorentz's condition

$$\nabla \cdot \vec{A} = -s\mu\epsilon^* \Phi \quad (2.1.8)$$

The vector potential $\vec{A}(\vec{r}, s)$ satisfies the Helmholtz equation:

$$\nabla^2 \vec{A}(\vec{r}, s) - s^2 \mu\epsilon^* \vec{A}(\vec{r}, s) = -\mu \vec{J}_e(\vec{r}, s) \quad (2.1.9)$$

It is easy to find the solution of inhomogeneous wave equation in an unbounded region.

$$\begin{aligned} \vec{A}(\vec{r}, s) &= \frac{\mu}{4\pi} \int_V \frac{e^{-\sqrt{s^2 \mu\epsilon^*} R}}{R} \vec{J}_e(\vec{r}', s) dV \\ &= \frac{\mu}{4\pi} \int_V \frac{e^{-\sqrt{s^2 \mu\epsilon + s\mu\sigma} R}}{R} \vec{J}_e(\vec{r}', s) dV \end{aligned} \quad (2.1.10)$$

where

$$R = |\vec{r} - \vec{r}'| = \text{distance between the observation point } \vec{r} \text{ and source point } \vec{r}'.$$

With the dipole antenna current given in (2.1.2), the vector potential has only a z-component and is given by

$$\vec{A}(\vec{r}, s) = \hat{z} \frac{\mu I dl}{4\pi} \frac{e^{-\sqrt{s^2 \mu\epsilon + s\mu\sigma} r}}{r} F(s) \quad (2.1.11)$$

From $\vec{A}(\vec{r}, s)$, the field components are obtained:

$$H_{\phi}(\vec{r}, s) = \frac{Idl}{4\pi r^2} [1 + \gamma(s)r] e^{-\gamma(s)r} \sin\theta F(s) \quad (2.1.12)$$

$$E_r(\vec{r}, s) = \frac{Idl}{2\pi(\sigma + s\epsilon)r^3} [1 + \gamma(s)r] e^{-\gamma(s)r} \cos\theta F(s) \quad (2.1.13)$$

$$E_{\theta}(\vec{r}, s) = \frac{Idl}{4\pi(\sigma + s\epsilon)r^3} [1 + \gamma(s)r + \gamma^2(s)r^2] e^{-\gamma(s)r} \sin\theta F(s) \quad (2.1.14)$$

where

$$\gamma(s) = \sqrt{s^2\mu\epsilon + s\mu\sigma} \quad (2.1.15)$$

Let's consider the case of the impulse radiation [7]. $\vec{a}(\vec{r}, s)$, $\vec{e}(\vec{r}, s)$ and $\vec{h}(\vec{r}, s)$ will be used to indicate the vector potential, the electric field and magnetic field of the impulse radiation, respectively.

For impulse antenna excitation current,

$$f(t) = \delta(t) \quad F(s) = 1 \quad (2.1.16)$$

$$\vec{a}(\vec{r}, s) = \hat{z} \frac{\mu Idl}{4\pi} \frac{e^{-\gamma(s)r}}{r} = a_z(r, s) \hat{z} \quad (2.1.17)$$

$$h_{\phi}(\vec{r}, s) = \frac{Idl}{4\pi r^2} [1 + \gamma(s)r] e^{-\gamma(s)r} \sin\theta \quad (2.1.18)$$

$$e_r(\vec{r}, s) = \frac{Idl}{2\pi(\sigma + s\epsilon)r^3} [1 + \gamma(s)r] e^{-\gamma(s)r} \cos\theta \quad (2.1.19)$$

$$e_{\theta}(\vec{r}, s) = \frac{Idl}{4\pi(\sigma + s\epsilon)r^3} [1 + \gamma(s)r + \gamma^2(s)r^2] e^{-\gamma(s)r} \sin\theta \quad (2.1.20)$$

Next, before inverting Laplace transform to obtain $\vec{a}(\vec{r}, t)$, $\vec{h}(\vec{r}, t)$, and $\vec{e}(\vec{r}, t)$, some notations will be defined first.

$$\text{Wave Velocity} \quad v = 1/\sqrt{\epsilon\mu} \quad (2.1.21)$$

$$\text{Relaxation Time} \quad \tau_0 = \epsilon/\sigma \quad (2.1.22)$$

$$\text{Attenuation Constant} \quad \alpha = \sqrt{\mu/\epsilon\sigma} \quad (2.1.23)$$

v , τ_0 and α are related by

$$\alpha v \tau_0 = 1 \quad (2.1.24)$$

Using the tables of Laplace transforms [8] [9] yields

$$e^{-x\sqrt{as^2+bs+c}} \Leftrightarrow e^{-\frac{b}{2\sqrt{a}}x} \delta(t - \sqrt{a}x) + \frac{x\sqrt{d/ae}^{-\frac{b}{2a}t} I_1(\sqrt{t^2 - ax^2}\sqrt{d/a})}{\sqrt{t^2 - ax^2}} u(t - \sqrt{a}x) \quad (2.1.25)$$

where

$$d = (b/2)^2 - ac \quad (2.1.26)$$

This leads to

$$a_z(r, t) = \frac{\mu Idl}{4\pi} \left\{ \frac{e^{-\alpha r/2}}{r} \delta(t - r/v) + \frac{\alpha e^{-t/(2\tau_0)} I_1[\sqrt{t^2 - r^2/v^2}/(2\tau_0)]}{2\sqrt{t^2 - r^2/v^2}} u(t - r/v) \right\} \quad (2.1.27)$$

where

$\delta(t - r/v)$ is the Dirac- δ function.

$u(t - r/v)$ is the Heaviside step function.

$I_1(x)$ is the modified Bessel function.

Maxwell's equations in the time domain will be applied to obtain $h_\phi(\vec{r}, t)$, rather than directly inverting $h_\phi(\vec{r}, s)$.

$$\begin{aligned}\vec{h}(\vec{r}, t) &= \frac{1}{\mu} \nabla \times \vec{a}(\vec{r}, t) \\ &= -\frac{\sin\theta}{\mu} \frac{\partial}{\partial r} a_z(r, t) \hat{\phi}\end{aligned}\quad (2.1.28)$$

$$\begin{aligned}h_\phi(\vec{r}, t) &= -\frac{\sin\theta}{\mu} \frac{\partial}{\partial r} a_z(r, t) \\ &= \frac{Idl\sin\theta}{4\pi} \left\{ e^{-\alpha r/2} \left[\delta(t-r/v) (1 + \alpha r/2 + \alpha^2 r^2/8) / r^2 + \frac{1}{vr} \delta'(t-r/v) \right] \right. \\ &\quad \left. + e^{-t/(2\tau_0)} I_2 \left[\sqrt{t^2 - r^2/v^2} / (2\tau_0) \right] \frac{\alpha^2 r u(t-r/v)}{4v(t^2 - r^2/v^2)} \right\}\end{aligned}\quad (2.1.29)$$

where

$$\delta'(x) = \frac{d}{dx} \delta(x) \quad (2.1.30)$$

Let's define

$$G_1(r, s) \equiv [1 + \gamma(s)r] e^{-\gamma(s)r} \quad (2.1.31)$$

and then compare $G_1(r, s)$ with $h_\phi(\vec{r}, s)$. In view of equations (2.1.18), (2.1.29) and (2.1.31), the inverse of $G_1(r, s)$ is found to be

$$\begin{aligned}g_1(r, t) &= r^2 \left\{ e^{-\alpha r/2} \left[\delta(t-r/v) (1 + \alpha r/2 + \alpha^2 r^2/8) / r^2 + \frac{1}{vr} \delta'(t-r/v) \right] \right. \\ &\quad \left. + e^{-t/(2\tau_0)} I_2 \left[\sqrt{t^2 - r^2/v^2} / (2\tau_0) \right] \frac{\alpha^2 r u(t-r/v)}{4v(t^2 - r^2/v^2)} \right\}\end{aligned}\quad (2.1.32)$$

The electric field in the time domain can be obtained as follows:

$e_r(\vec{r}, s)$ in (2.1.19) is rewritten as

$$e_r(\vec{r}, s) = \frac{Idl \cos \theta}{2\pi r^3} G_2(r, s) G_1(r, s) \quad (2.1.33)$$

where

$$G_2(r, s) = \frac{1}{\sigma + s\varepsilon} = \frac{1}{\varepsilon} \frac{1}{s + \sigma/\varepsilon} = \frac{1}{\varepsilon} \frac{1}{s + 1/\tau_0} \quad (2.1.34)$$

and its inverse Laplace transform,

$$g_2(r, t) = \frac{1}{\varepsilon} e^{-t/\tau_0} u(t) \quad (2.1.35)$$

Applying the convolution theorem of Laplace transform

$$G_2(r, s) G_1(r, s) \Leftrightarrow g_1(r, t) * g_2(r, t) = \int_0^t g_1(r, t - \tau) g_2(r, \tau) d\tau \quad (2.1.36)$$

to (2.1.33) leads to

$$\begin{aligned} e_r(\vec{r}, t) &= \frac{Idl \cos \theta}{2\pi r^3} \int_0^t g_1(r, t - \tau) g_2(t, \tau) d\tau \\ &= \frac{Idl \cos \theta}{2\pi \varepsilon} u(t - r/v) \left\{ e^{-[t - r/(2v)]/\tau_0} (1 - \alpha r/2 + \alpha^2 r^2/8) / r^3 \right. \\ &\quad \left. + \frac{\alpha^2}{4v} \int_{r/v}^t e^{-(t-\tau/2)/\tau_0} I_2 \left[\sqrt{\tau^2 - r^2/v^2} / (2\tau_0) \right] \frac{d\tau}{\tau^2 - r^2/v^2} \right\} \quad (2.1.37) \end{aligned}$$

Similarly, $e_\theta(\vec{r}, s)$ in (2.1.20) can be rewritten as

$$e_\theta(\vec{r}, s) = \sin \theta s a_z(r, s) + \frac{Idl \sin \theta}{4\pi r^3} G_1(r, s) G_2(r, s) \quad (2.1.38)$$

Then,

$$\begin{aligned}
e_{\theta}(\vec{r}, t) &= \sin\theta \frac{\partial}{\partial t} a_z(r, t) + \frac{Idl \sin\theta}{4\pi r^3} \int_0^t g_1(r, t-\tau) g_2(r, \tau) d\tau \\
&= \frac{\mu Idl \sin\theta}{4\pi} \left\{ e^{-\alpha r/2} \left[\frac{\alpha}{8\tau_0} \delta(t-r/v) + \frac{1}{r} \delta'(t-r/v) \right] \right. \\
&\quad + \frac{\alpha}{4\tau_0} e^{-t/2\tau_0} \frac{u(t-r/v)}{t^2 - r^2/v^2} \left\{ t I_2 \left[\sqrt{t^2 - r^2/v^2} / (2\tau_0) \right] - \sqrt{t^2 - r^2/v^2} I_1 \left[\sqrt{t^2 - r^2/v^2} / (2\tau_0) \right] \right\} \left. \right\} \\
&\quad + \frac{Idl \sin\theta}{4\pi \epsilon} u(t-r/v) \left\{ e^{-[t-r/(2v)]/\tau_0} (1 - \alpha r/2 + \alpha^2 r^2/8) / r^3 \right. \\
&\quad \left. + \frac{\alpha^2}{4v} \int_{r/v}^t e^{-(t-\tau/2)/\tau_0} I_2 \left[\sqrt{\tau^2 - r^2/v^2} / (2\tau_0) \right] \frac{d\tau}{\tau^2 - r^2/v^2} \right\} \quad (2.1.39)
\end{aligned}$$

The electric and magnetic fields, $e_r(\vec{r}, t)$, $e_{\theta}(\vec{r}, t)$ and $h_{\phi}(\vec{r}, t)$, in the time domain given in equations (2.1.37), (2.1.39) and (2.1.29) are considered to be original and new. Existing results available in literature on this subject are either incomplete or approximate in nature. For example, similar solutions of $a_z(\vec{r}, t)$ are given by W. C. Chew [10]. J. R. Wait [2] [4] reported the electric field of a dipole carrying a step-function current. P. I. Richards [3] derived the EM fields excited by an electric dipole, but his results were expressed in terms of vector operators and he did not give components of the EM fields in explicit form.

2.2. SOME SPECIAL CASES AND DISCUSSIONS

In this section, the general solutions will be reduced to some special cases. From the solutions of the special cases, valuable physical insights about the propagation and excitation of an EM pulse in a conducting medium are obtained.

I). Lossless Medium

In lossless medium, $\sigma = 0$, $\tau_0 \rightarrow \infty$, and $\alpha = 0$. It is easy to approximate (2.1.27), (2.1.29), (2.1.37) and (2.1.39) to the following results:

$$a_z(r, t) = \frac{\mu I dl}{4\pi r} \delta(t - r/v) \quad (2.2.1)$$

$$h_\phi(\vec{r}, t) = \frac{I dl \sin\theta}{4\pi r^2} \left[\delta(t - r/v) + \frac{r}{v} \delta'(t - r/v) \right] \quad (2.2.2)$$

$$e_r(\vec{r}, t) = \frac{I dl \cos\theta}{2\pi \epsilon r^3} u(t - r/v) \quad (2.2.3)$$

$$e_\theta(\vec{r}, t) = \frac{\mu I dl \sin\theta}{4\pi r} \delta'(t - r/v) + \frac{I dl \sin\theta}{4\pi \epsilon r^3} u(t - r/v) \quad (2.2.4)$$

They are well known electromagnetic fields generated by an electric dipole with an impulsive current in free space. The electric field behaves as an electrostatic field because the impulsive current ends up as static charges at the tips of the dipole.

II). Low Loss Medium ($\alpha r \ll 1$)

The conductivity σ is very small in this case. With $\alpha r \ll 1$, (2.1.27), (2.1.29), (2.1.37) and (2.1.39) can be approximated as

$$a_z(r, t) = \frac{\mu I dl}{4\pi r} e^{-\alpha r/2} \delta(t - r/v) \quad (2.2.5)$$

$$h_\phi(\vec{r}, t) = \frac{I dl \sin\theta}{4\pi r^2} \left[\delta(t - r/v) + \frac{r}{v} e^{-\alpha r/2} \delta'(t - r/v) \right] \quad (2.2.6)$$

$$e_r(\vec{r}, t) = \frac{I dl \cos\theta}{2\pi \epsilon r^3} u(t - r/v) e^{-t/\tau_0} \quad (2.2.7)$$

$$e_{\theta}(\vec{r}, t) = \frac{\mu I dl \sin \theta}{4\pi r} e^{-\alpha r/2} \left[\delta'(t - r/v) + \frac{\alpha r}{8\tau_0} \delta(t - r/v) \right] + \frac{I dl \sin \theta}{4\pi \epsilon r^3} u(t - r/v) e^{-t/\tau_0} \quad (2.2.8)$$

These fields decay exponentially but slowly because of $\alpha r \ll 1$. The finite conductivity of the medium will dissipate the electric charges at the dipole tips into the medium.

III). Late Time Approximation ($t \gg r/v$ and $t \gg \tau_0$)

The asymptotic expression of the modified Bessel function is [11]:

$$I_n(x) \rightarrow \frac{e^x}{\sqrt{2\pi x}} \left(1 - \frac{4n^2 - 1}{8x}\right) \quad \text{as } x \rightarrow \infty \quad (2.2.9)$$

Then for $t \gg r/v$ and $t \gg \tau_0$, the following approximations can be obtained

$$\sqrt{t^2 - r^2/v^2} = t - r^2/(2v^2 t) \quad (2.2.10)$$

$$I_1\left[\sqrt{t^2 - r^2/v^2}/(2\tau_0)\right] = \sqrt{\tau_0/(\pi t)} e^{\frac{t}{2\tau_0}} e^{-\frac{r^2}{4v^2 t \tau_0}} [1 - 3\tau_0/(4t)] \quad (2.2.11)$$

$$I_2\left[\sqrt{t^2 - r^2/v^2}/(2\tau_0)\right] = \sqrt{\tau_0/(\pi t)} e^{\frac{t}{2\tau_0}} e^{-\frac{r^2}{4v^2 t \tau_0}} [1 - 15\tau_0/(4t)] \quad (2.2.12)$$

$a_z(r, t)$ in (2.1.27) then becomes

$$a_z(r, t) = \frac{I dl}{\pi^{3/2} \sigma r^3} \left[\frac{a}{2\sqrt{t}}\right]^3 e^{-\frac{a^2}{4t}} \quad (2.2.13)$$

$$= \frac{I dl}{2\pi \sigma r^3} \left[\frac{a}{2\sqrt{t}}\right]^3 \text{erf}'\left(\frac{a}{2\sqrt{t}}\right) \quad (2.2.14)$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy \quad (2.2.15)$$

$$\operatorname{erf}'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \quad (2.2.16)$$

$$a^2 = \frac{r^2}{v^2 \tau_0} = \alpha r \frac{r}{v} = (\alpha r)^2 \tau_0 = \sigma \mu r^2 \quad (2.2.17)$$

On application of (2.2.10) and (2.2.12), magnetic field $h_\phi(\dot{r}, t)$ is approximated as

$$h_\phi(\dot{r}, t) = \frac{2Idl \sin \theta}{\pi^{3/2} \mu \sigma r^4} \left[\frac{a}{2\sqrt{t}} \right]^5 e^{-\frac{a^2}{4t}} \quad (2.2.18)$$

$$= \frac{Idl \sin \theta}{\pi \mu \sigma r^4} \left[\frac{a}{2\sqrt{t}} \right]^5 \operatorname{erf}'\left(\frac{a}{2\sqrt{t}}\right) \quad (2.2.19)$$

Using (2.1.37) and (2.2.12) yields

$$e_r(\dot{r}, t) = \frac{Idl \cos \theta}{8\pi^{3/2}} \frac{\mu \sqrt{\sigma \mu}}{\tau_0} \int_{r/v}^t e^{-\frac{t-\tau}{\tau_0}} \tau^{-5/2} e^{-\frac{a^2}{4\tau}} d\tau \quad (2.2.20)$$

Let

$$t - \tau = x\tau_0 \quad (2.2.21)$$

then

$$\begin{aligned} e_r(\dot{r}, t) &= \frac{Idl \cos \theta}{8\pi^{3/2}} \mu \sqrt{\sigma \mu} \int_0^{(t-r/v)/\tau_0} e^{-x} (t - x\tau_0)^{-5/2} e^{-\frac{a^2}{4(t-x\tau_0)}} dx \\ &= \frac{Idl \cos \theta}{8\pi^{3/2}} \mu \sqrt{\sigma \mu} \int_0^{(t-r/v)/\tau_0} e^{-x} \left\{ \sum_{n=0}^{\infty} g^{(n)}(t) \frac{(-x\tau_0)^n}{n!} \right\} dx \end{aligned} \quad (2.2.22)$$

$$\begin{aligned}
&= \frac{Idl \cos \theta}{8\pi^{3/2}} \mu \sqrt{\sigma \mu} \left\{ \sum_{n=0}^{\infty} g^{(n)}(t) \frac{(-\tau_0)^n}{n!} \int_0^{\infty} e^{-x} x^n dx \right\} \\
&= \frac{Idl \cos \theta}{8\pi^{3/2}} \mu \sqrt{\sigma \mu} \left\{ \sum_{n=0}^{\infty} g^{(n)}(t) \frac{(-\tau_0)^n}{n!} \int_0^{\infty} e^{-x} x^n dx \right\} \\
&= \frac{Idl \cos \theta}{8\pi^{3/2}} \mu \sqrt{\sigma \mu} \left\{ \sum_{n=0}^{\infty} g^{(n)}(t) (-\tau_0)^n \right\} \\
&= \frac{Idl \cos \theta}{8\pi^{3/2}} \mu \sqrt{\sigma \mu} [g(t) - \tau_0 g'(t) + \tau_0^2 g''(t) + \dots] \tag{2.2.23}
\end{aligned}$$

where

$$g(t) = t^{-5/2} e^{-\frac{a^2}{4t}} \tag{2.2.24}$$

$$g^{(n)}(t) = \frac{d^n}{dt^n} g(t) \tag{2.2.25}$$

$$g'(t) = \left[-\frac{5}{2} \frac{1}{t} + \frac{a^2}{4t^2} \right] t^{-5/2} e^{-\frac{a^2}{4t}} \tag{2.2.26}$$

Since

$$\left| \frac{-\tau_0 g'(t)}{g(t)} \right| = \left| \frac{5\tau_0}{2t} - \frac{a^2 \tau_0}{4t^2} \right| = \left| \frac{5\tau_0}{2t} - \frac{(r/v)^2}{4t^2} \right| \ll 1 \tag{2.2.27}$$

and it can be shown that

$$\left| \frac{(-\tau_0)^n g^{(n)}(t)}{g(t)} \right| \ll 1 \tag{2.2.28}$$

for $n \geq 2$, then

$$\begin{aligned}
e_r(\vec{r}, t) &= g(t) \frac{Idl \cos \theta}{8\pi^{3/2}} \mu \sqrt{\sigma \mu} \\
&= \frac{4Idl \cos \theta}{\pi^{3/2} \mu \sigma^2 r^5} \left[\frac{a}{2\sqrt{t}} \right]^5 e^{-\frac{a^2}{4t}} \\
&= \frac{2Idl \cos \theta}{\pi \mu \sigma^2 r^5} \left[\frac{a}{2\sqrt{t}} \right]^5 \operatorname{erf}'\left(\frac{a}{2\sqrt{t}}\right)
\end{aligned} \tag{2.2.29}$$

Next, let's approximate the θ -component of electric field. Using the same approximations used in the beginning of this section yields

$$tI_2\left[\sqrt{t^2 - r^2/v^2}/(2\tau_0)\right] - \sqrt{t^2 - r^2/v^2}I_1\left[\sqrt{t^2 - r^2/v^2}/(2\tau_0)\right] = \sqrt{\frac{\tau_0}{\pi t}} \left[\frac{r^2}{2tv^2} - 3\tau_0 \right] e^{\frac{t}{2\tau_0}} e^{-\frac{a^2}{4t}} \tag{2.2.30}$$

This leads to

$$e_\theta(\vec{r}, t) = \frac{4Idl \sin \theta}{\pi^{3/2} \mu \sigma^2 r^5} \left[\frac{a^2}{4t} - 1 \right] \left[\frac{a}{2\sqrt{t}} \right]^5 e^{-\frac{a^2}{4t}} \tag{2.2.31}$$

$$= \frac{Idl \sin \theta}{2\pi \sigma r^3} \frac{\partial}{\partial t} \left[\left[\frac{a}{2\sqrt{t}} \right]^3 \operatorname{erf}'\left(\frac{a}{2\sqrt{t}}\right) \right] + \frac{Idl \sin \theta}{\pi \mu \sigma^2 r^5} \left[\frac{a}{2\sqrt{t}} \right]^5 \operatorname{erf}'\left(\frac{a}{2\sqrt{t}}\right) \tag{2.2.32}$$

IV). Wavefront Approximation

Now, let's discuss the wavefront behavior. In this case, $t - r/v$ is very small. i.e.

$$\sqrt{t^2 - r^2/v^2} \ll 2\tau_0 \tag{2.2.33}$$

Thus, the small argument approximation of the modified Bessel functions [11] will be used.

$$I_1(x) = \frac{x}{2}, \quad I_2(x) = \frac{1}{2} \left(\frac{x}{2}\right)^2 \quad \text{as } x \rightarrow 0 \quad (2.2.34)$$

$$tI_2\left[\sqrt{t^2 - r^2/v^2}/(2\tau_0)\right] - \sqrt{t^2 - r^2/v^2}I_1\left[\sqrt{t^2 - r^2/v^2}/(2\tau_0)\right] = \frac{t^2 - r^2/v^2}{4\tau_0} \left(\frac{t}{8\tau_0} - 1\right) \quad (2.2.35)$$

Then the EM fields can be approximated as

$$a_z(r, t) = \frac{\mu Idl}{4\pi} \left[e^{-\alpha r/2} \frac{1}{r} \delta(t - r/v) + e^{-t/2\tau_0} \frac{\alpha}{8\tau_0} u(t - r/v) \right] \quad (2.2.36)$$

$$h_\phi(\vec{r}, t) = \frac{Idl \sin \theta}{4\pi} \left\{ e^{-\alpha r/2} \left[\delta(t - r/v) (1 + \alpha r/2 + \alpha^2 r^2/8) / r^2 + \frac{1}{vr} \delta'(t - r/v) \right] \right. \\ \left. + e^{-t/2\tau_0} \frac{\alpha^3 r}{128\tau_0} u(t - r/v) \right\} \quad (2.2.37)$$

$$e_r(\vec{r}, t) = \frac{Idl \cos \theta}{2\pi\epsilon} u(t - r/v) e^{-[t - r/(2v)]/\tau_0} \\ \left\{ (1 - \alpha r/2 + \alpha^2 r^2/8) / r^3 + [e^{(t - r/v)/(2\tau_0)} - 1] \alpha^3 / 64 \right\} \quad (2.2.38)$$

$$e_\theta(\vec{r}, t) = \frac{\mu Idl \sin \theta}{4\pi} \left\{ e^{-\alpha r/2} \left[\frac{\alpha}{8\tau_0} \delta(t - r/v) + \frac{1}{r} \delta'(t - r/v) \right] \right. \\ \left. + \frac{\alpha}{16\tau_0^2} e^{-t/(2\tau_0)} \left(\frac{t}{8\tau_0} - 1 \right) u(t - r/v) \right\} \\ + \frac{Idl \sin \theta}{4\pi\epsilon} u(t - r/v) e^{-[t - r/(2v)]/\tau_0} \\ \left\{ (1 - \alpha r/2 + \alpha^2 r^2/8) / r^3 + [e^{(t - r/v)/(2\tau_0)} - 1] \alpha^3 / 64 \right\} \quad (2.2.39)$$

V). Normalized Fields

The above results can be rewritten in the following forms for normalization.

$$\bar{h}_\phi(r, s) \equiv \frac{\pi\mu\sigma r^4}{\sin\theta l dl} h_\phi(\dot{r}, s) = \frac{a^2}{4} [1 + \gamma(s)r] e^{-\gamma(s)r} \quad (2.2.40)$$

$$\bar{e}_r(r, s) \equiv \frac{\pi\mu\sigma^2 r^5}{\cos\theta l dl} e_r(\dot{r}, s) = \frac{a^2}{2} \frac{1 + \gamma(s)r}{1 + s\tau_0} e^{-\gamma(s)r} \quad (2.2.41)$$

$$\bar{e}_\theta(r, s) \equiv \frac{\pi\mu\sigma^2 r^5}{\sin\theta l dl} e_\theta(\dot{r}, s) = \frac{a^2}{4} \frac{1 + \gamma(s)r + \gamma^2(s)r^2}{1 + s\tau_0} e^{-\gamma(s)r} \quad (2.2.42)$$

Let

$$T = \frac{t}{r/v} \quad (2.2.43)$$

and

$$R = \alpha r \quad (2.2.44)$$

For $t > r/v$, i.e. $T > 1$, (2.1.29), (2.1.37) and (2.1.39) can be normalized to

$$\bar{h}_\phi(r, t) \equiv \frac{\pi\mu\sigma r^4}{\sin\theta l dl} h_\phi(\dot{r}, t) = \frac{R^3}{16} \frac{1}{T^2 - 1} I_2[\sqrt{T^2 - 1}(R/2)] e^{-RT/2} \quad (2.2.45)$$

$$\begin{aligned} \bar{e}_r(r, t) &\equiv \frac{\pi\mu\sigma^2 r^5}{\cos\theta l dl} e_r(\dot{r}, t) \\ &= \frac{R^2}{2} (1 - R/2 + R^2/8) e^{-R(T-1/2)} + \frac{R^4}{8} \int_1^T \frac{e^{-R(T-\tau/2)} I_2[\sqrt{\tau^2 - 1}(R/2)]}{\tau^2 - 1} d\tau \end{aligned} \quad (2.2.46)$$

$$\begin{aligned} \bar{e}_\theta(r, t) &\equiv \frac{\pi\mu\sigma^2 r^5}{\sin\theta l dl} e_\theta(\dot{r}, t) \\ &= \frac{R^2}{4} (1 - R/2 + R^2/8) e^{-R(T-1/2)} + \frac{R^4}{16} \int_1^T \frac{e^{-R(T-\tau/2)} I_2[\sqrt{\tau^2 - 1}(R/2)]}{\tau^2 - 1} d\tau \end{aligned}$$

$$+ \frac{R^4}{16} e^{-RT/2} \frac{1}{T^2 - 1} [TI_2[\sqrt{T^2 - 1}(R/2)] - \sqrt{T^2 - 1}I_1[\sqrt{T^2 - 1}(R/2)]] \quad (2.2.47)$$

The late time approximation ($t \gg r/v$ and $t \gg \tau_0$) can be simplified to

$$\bar{h}_\phi(r, t) = \frac{2}{\sqrt{\pi}} \left[\frac{a}{2\sqrt{t}} \right]^5 e^{-\frac{a^2}{4t}} \quad (2.2.48)$$

$$\bar{e}_r(r, t) = \frac{4}{\sqrt{\pi}} \left[\frac{a}{2\sqrt{t}} \right]^5 e^{-\frac{a^2}{4t}} \quad (2.2.49)$$

$$\bar{e}_\theta(r, t) = \frac{4}{\sqrt{\pi}} \left[\frac{a^2}{4t} - 1 \right] \left[\frac{a}{2\sqrt{t}} \right]^5 e^{-\frac{a^2}{4t}} \quad (2.2.50)$$

Now, let's discuss the waveform of the late time approximation. When

$$t = a^2/4 = \sigma\mu r^2/4, \quad (2.2.51)$$

$$\bar{e}_\theta(r, t) = 0 \quad (2.2.52)$$

When

$$t = \frac{a^2}{9 + \sqrt{41}} = 0.065\sigma\mu r^2, \quad (2.2.53)$$

$\bar{e}_\theta(r, t)$ has a maximum of 3.981, and when

$$t = \frac{a^2}{9 - \sqrt{41}} = 0.385\sigma\mu r^2, \quad (2.2.54)$$

$\bar{e}_\theta(r, t)$ reaches a minimum of -0.140. On the other hand, at

$$t = a^2/10 = 0.1\sigma\mu r^2, \quad (2.2.55)$$

$\bar{e}_r(r, t)$ has a maximum of 1.831, and $\bar{h}_\phi(r, t)$ has a maximum of 0.915.

For seawater with $\epsilon_r=80$, $\mu=\mu_0$ and $\sigma=4$ s/m,

$$\tau_0 = \epsilon/\sigma = 1.77 \times 10^{-10} \text{ s} \quad (2.2.56)$$

$$v = c/\sqrt{\epsilon_r} = 3.35 \times 10^7 \text{ (m/s)} \quad (2.2.57)$$

At the distance of $r=1$ m, $a^2 = 5.03 \times 10^{-6} \text{ s}$, when at $r=1$ km, $a^2 = 5.03 \text{ s}$.

The maximum point of $\tilde{e}_\theta(r, t)$ occurs at $t = 0.065a^2$, thus, the propagation velocity v_0 of this maximum point in the waveform can find to be

$$v_0 = \frac{\partial r}{\partial t} = \frac{1}{0.13\sigma\mu r} \quad (2.2.58)$$

For seawater, at the distance of $r=100$ m from the antenna, $v_0=1.53 \times 10^4 \text{ m/s}$ and at $r=1$ km, $v_0=1.53 \times 10^3 \text{ m/s}$. This implies that the maximum point of the $\tilde{e}_\theta(r, t)$ component of the EM pulse propagates with a velocity inversely proportional to the conductivity and the distance from the antenna.

The exact time domain solutions for the electric and magnetic fields maintained in conducting media by an electric dipole antenna with an impulsive current are given in section 2.1. Mathematical expressions for these EM fields are quite complicated. In this section, the exact solutions are reduced to simple forms in lossless and low loss medias and approximated for the wavefront structure and late time waveforms. From these results, there are the following observations. The EM fields of the EM pulse excited by an electric dipole with an impulsive current in conducting media consist of two parts. The first part is an impulse wave with a wavefront structure which propagates with the speed of light ($1/\sqrt{\mu\epsilon}$) and decays exponentially. This part of the EM fields is negligibly small except at an immediate vicinity of the antenna and can be ignored for practical purposes. The second part of the EM fields is the late time waveform. It builds up slowly and propagates slowly with a decreasing speed, which is in the same order as the acoustic speed in seawater.

ter at a distance of 1000 m away from the antenna. More importantly, this part of the EM fields decays as an inverse power of distance (the order depends on the shape of the excitation current and it is the fifth for the impulsive current), which is a much slower rate than the exponential decay. This wave behaves somewhat like a “diffusion wave” in nature. In fact, this is the only significant EM fields which can be measured at a distance from the antenna when the conducting medium is seawater.

2.3. SEVERAL COMPUTATIONAL METHODS AND NUMERICAL RESULTS

In this section, several computational methods will be discussed and compared with each other. Since the EM fields have singularities at $t = r/v$ for the case of the impulsive radiation, the fields at $t > r/v$ will be computed. For simplicity, only the θ -component of electric fields is discussed in some cases.

I). Computing Exact Solutions

Section 2.2 presents the normalized exact time domain solutions for the electric and magnetic fields maintained in conducting media by an electric dipole antenna with an impulsive current. Equations (2.2.45), (2.2.46) and (2.2.47) can be used to compute these fields. However, the mathematical expressions for these fields are quite complicated. For a given time t , it is needed to integrate some functions from r/v to t . These functions involve modified Bessel functions with real arguments, and they converge slower than an FFT algorithm which will be introduced later. At the end of this section, the results computed by these methods will be presented.

II). Inverse Laplace Transform by Integrating around the Branch Cut

In this part, only the θ -component of electric fields will be discussed for brevity.

The Laplace transform of the normalized θ -component of electric fields is given in (2.2.42) as

$$\bar{e}_\theta(r, s) \equiv \frac{\pi \mu \sigma^2 r^5}{\sin \theta l dl} e_\theta(\vec{r}, s) = \frac{a^2}{4} \frac{1 + \gamma(s)r + \gamma^2(s)r^2}{1 + s\tau_0} e^{-\gamma(s)r} \quad (2.3.1)$$

where

$$\gamma(s) = [\epsilon \mu s^2 + \mu \sigma s]^{1/2} = \frac{1}{v} [s^2 + s/\tau_0]^{1/2} = \frac{1}{v} [s(s + 1/\tau_0)]^{1/2} \quad (2.3.2)$$

Taking the inverse Laplace transform of $\bar{e}_\theta(r, s)$ to obtain the time domain solution $\bar{e}_\theta(r, t)$ yields

$$\bar{e}_\theta(r, t) = \frac{1}{2\pi j} \int_{\alpha_0 - j\infty}^{\alpha_0 + j\infty} \bar{e}_\theta(r, s) e^{st} ds \quad (2.3.3)$$

Since $\bar{e}_\theta(r, s)$ has a branch cut between $s = 0$ and $s = -1/\tau_0$ due to $\gamma(s)$, an integration path over the closed contours in the complex s -plane as indicated in the Figure 2.2 will be considered. Because $\bar{e}_\theta(r, s)$ does not have any pole within the contour, then

$$\oint_C \bar{e}_\theta(r, s) e^{st} ds = \int_{\alpha_0 - j\infty}^{\alpha_0 + j\infty} \bar{e}_\theta(r, s) e^{st} ds + \int_{C_-} \bar{e}_\theta(r, s) e^{st} ds + \int_{C_+} \bar{e}_\theta(r, s) e^{st} ds = 0 \quad (2.3.4)$$

based on Cauchy's integral theorem.

First, let's show that the second term of the right hand side of (2.3.4) vanishes.

$$\int_{C_-} \bar{e}_\theta(r, s) e^{st} ds = \frac{a^2}{4} \int_{C_-} \frac{1 + \gamma(s)r + \gamma^2(s)r^2}{1 + s\tau_0} e^{-\gamma(s)r + st} ds \quad (2.3.5)$$

Let

$$s = \rho e^{j\theta} \quad (2.3.6)$$

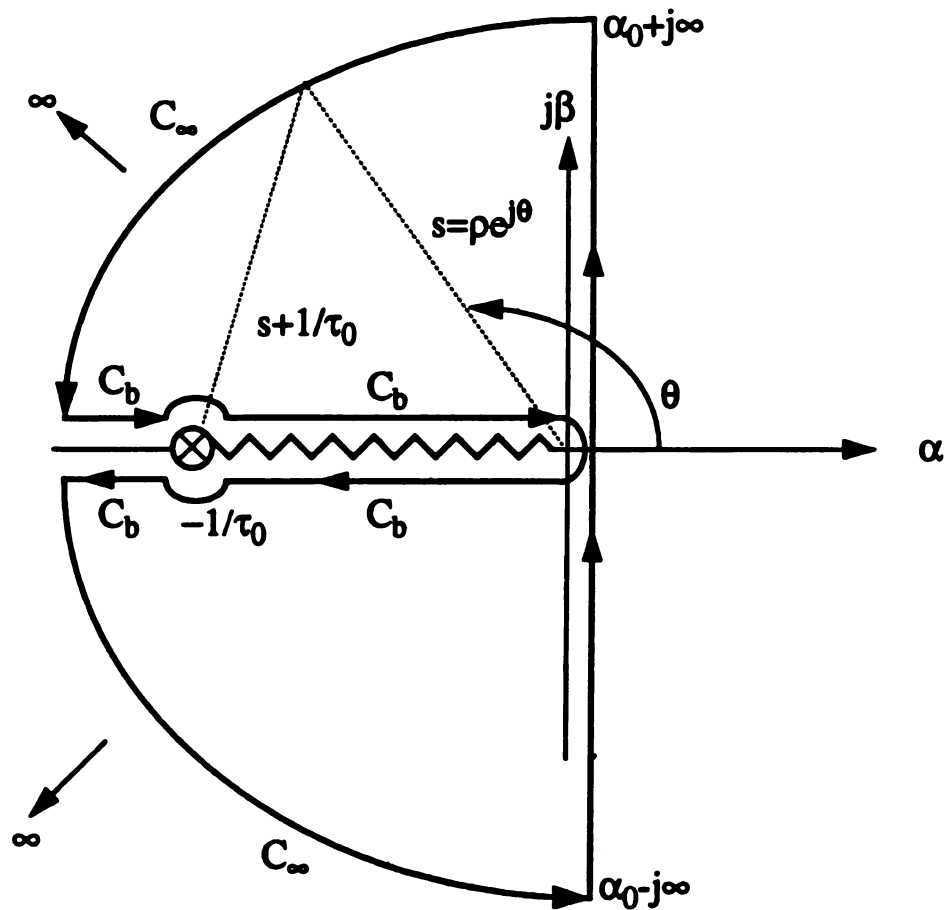


Figure 2.2. Integration path for inverting Laplace transform to obtain time domain electric field and magnetic field of an EM pulse propagating in conducting media.

when $\rho \rightarrow \infty$,

$$\begin{aligned}
 -\gamma(s)r + st &= -\frac{r}{v} [s(s + 1/\tau_0)]^{1/2} + st \\
 &= -\frac{r}{v} \rho e^{j\theta} \left(1 + \frac{e^{-j\theta}}{\rho\tau_0}\right)^{1/2} + \rho t e^{j\theta} \\
 &\approx -\frac{r}{v} \rho e^{j\theta} \left(1 + \frac{e^{-j\theta}}{2\rho\tau_0}\right) + \rho t e^{j\theta} \\
 &= \left(t - \frac{r}{v}\right) \rho \cos\theta - \frac{r}{2v\tau_0} + j\left(t - \frac{r}{v}\right) \rho \sin\theta
 \end{aligned} \tag{2.3.7}$$

Now, $\alpha_0 \rightarrow 0$, $\pi/2 \leq |\theta| \leq \pi$ and $\cos\theta \leq 0$. Therefore, when $t > r/v$,

$$\operatorname{Re}\{-\gamma(s)r + st\} = (t - r/v) \rho \cos\theta \rightarrow -\infty \tag{2.3.8}$$

and

$$e^{-\gamma(s)r + st} \rightarrow 0 \tag{2.3.9}$$

Then

$$\int_{C_-} \tilde{e}_\theta(r, s) e^{st} ds = 0 \tag{2.3.10}$$

Substituting the above result into (2.3.4) yields

$$\tilde{e}_\theta(r, t) = \frac{1}{2\pi j} \int_{\alpha_0 - j\infty}^{\alpha_0 + j\infty} \tilde{e}_\theta(r, s) e^{st} ds = -\frac{1}{2\pi j} \int_{C_+} \tilde{e}_\theta(r, s) e^{st} ds \tag{2.3.11}$$

The path C_b indicated in the Figure 2.3, consists of 7 parts.

$$C_b = C_{b1}^{up} + C_{b1}^{low} + C_{b2}^{up} + C_{b2}^{low} + C_{b3}^{up} + C_{b3}^{low} + C_{b4} \tag{2.3.12}$$

Let

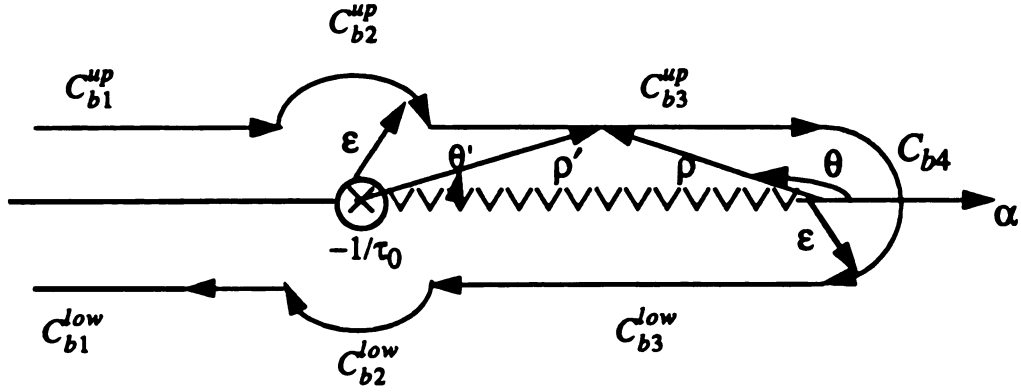


Figure 2.3. The detail of the integration path C_b indicated in Figure 2.2.

$$s = \rho e^{j\theta} \quad (2.3.13)$$

and

$$s + 1/\tau_0 = \rho' e^{j\theta'} \quad (2.3.14)$$

then,

$$\gamma(s) = \frac{1}{v} \sqrt{\rho \rho'} e^{j[(\theta + \theta')/2]} \quad (2.3.15)$$

The integrations along $C_{b1}^{up} + C_{b1}^{low} + C_{b2}^{up} + C_{b2}^{low} + C_{b4}$ are easily carried out:

$$\frac{1}{2\pi j} \left(\int_{C_{b1}^{up}} \tilde{e}_\theta(r, s) e^{st} ds + \int_{C_{b1}^{low}} \tilde{e}_\theta(r, s) e^{st} ds \right) = 0 \quad (2.3.16)$$

$$\frac{1}{2\pi j} \left(\int_{C_{b2}^{up}} \tilde{e}_\theta(r, s) e^{st} ds + \int_{C_{b2}^{low}} \tilde{e}_\theta(r, s) e^{st} ds \right) = -\frac{a^2}{4\tau_0} e^{-t/\tau_0} \quad (2.3.17)$$

$$\frac{1}{2\pi j} \int_{C_{b4}} \tilde{e}_\theta(r, s) e^{st} ds = 0 \quad (2.3.18)$$

Now, let's take greater care for the integrations along the branch cut between $s = 0$ and $s = -1/\tau_0$.

For C_{b3}^{up} , s from $-1/\tau_0$ to 0 , then $\theta = \pi$, $\theta' = 0$, $\rho' = 1/\tau_0 - \rho$ and $ds = -d\rho$.

This leads to

$$\gamma(s) = \frac{1}{v} \sqrt{\rho(1/\tau_0 - \rho)} e^{j\pi/2} = \frac{j}{v} \sqrt{\rho(1/\tau_0 - \rho)} \quad (2.3.19)$$

For C_{b3}^{low} , s from 0 to $-1/\tau_0$, then $\theta = -\pi$, $\theta' = 0$, $\rho' = 1/\tau_0 - \rho$ and $ds = -d\rho$.

This yields

$$\gamma(s) = \frac{1}{v} \sqrt{\rho(1/\tau_0 - \rho)} e^{-j\pi/2} = -\frac{j}{v} \sqrt{\rho(1/\tau_0 - \rho)} \quad (2.3.20)$$

Therefore,

$$\begin{aligned} \bar{e}_\theta(r, t) &= \frac{a^2}{4\tau_0} e^{-\frac{t}{\tau_0}} - \frac{1}{2\pi j} \left\{ \int_{-1/\tau_0}^0 \bar{e}_\theta(r, s) e^{st} ds + \int_0^{-1/\tau_0} \bar{e}_\theta(r, s) e^{st} ds \right\} \\ &= \frac{a^2}{4\tau_0} e^{-\frac{t}{\tau_0}} + \frac{a^2}{4\pi} \int_0^{1/\tau_0} \frac{e^{-\rho t}}{1 - \rho\tau_0} \{ [1 - \rho(1/\tau_0 - \rho) r^2/v^2] \sin \left[\frac{r}{v} \sqrt{\rho(1/\tau_0 - \rho)} \right] \\ &\quad - \frac{r}{v} \sqrt{\rho(1/\tau_0 - \rho)} \cos \left[\frac{r}{v} \sqrt{\rho(1/\tau_0 - \rho)} \right] \} d\rho \end{aligned} \quad (2.3.21)$$

With the change of variables of $y = \rho t$, $\rho = y/t$, $d\rho = dy/t$, above equation can be rewritten as

$$\begin{aligned} \bar{e}_\theta(r, t) &= \frac{a^2}{4\tau_0} e^{-\frac{t}{\tau_0}} + \frac{a^2}{4\pi t} \int_0^{t/\tau_0} \frac{e^{-y}}{1 - \tau_0 y/t} \{ [1 - y(t/\tau_0 - y) r^2/(vt)^2] \sin \left[\frac{r}{v} \sqrt{(t/\tau_0 - y)y} \right] \\ &\quad - \frac{r}{vt} \sqrt{(t/\tau_0 - y)y} \cos \left[\frac{r}{v} \sqrt{(t/\tau_0 - y)y} \right] \} dy \end{aligned} \quad (2.3.22)$$

This integration will have the major contribution from the region where y is small.

Now, let's derive the late time waveform from the integration around the branch cut.

When $t \gg \tau_0$, (2.3.22) can be reduced to

$$\bar{e}_\theta(r, t) = \frac{a^2}{4\pi t} \int_0^\infty e^{-y} \left[(1 - ya^2/t) \sin \sqrt{ya^2/t} - \sqrt{ya^2/t} \cos \sqrt{ya^2/t} \right] dy \quad (2.3.23)$$

Let

$$ya^2/t = x^2 \quad (2.3.24)$$

then $dy = 2xdx/a^2$. (2.3.23) becomes

$$\bar{e}_\theta(r, t) = \frac{1}{2\pi} \int_0^\infty e^{-x^2/a^2} \left[(1 - x^2) \sin x - x \cos x \right] x dx \quad (2.3.25)$$

The series expression is used to compute above integral.

$$\sin x = \sum_{n=0}^\infty (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad (2.3.26)$$

$$\cos x = \sum_{n=0}^\infty (-1)^n \frac{x^{2n}}{(2n)!} \quad (2.3.27)$$

$$\begin{aligned} (1 - x^2) \sin x - x \cos x &= (1 - x^2) \sum_{n=0}^\infty (-1)^n \frac{x^{2n+1}}{(2n+1)!} - x \sum_{n=0}^\infty (-1)^n \frac{x^{2n}}{(2n)!} \\ &= -4 \sum_{n=0}^\infty (-1)^n x^{2n+3} \frac{(n+1)^2}{(2n+3)!} \end{aligned} \quad (2.3.28)$$

$$\bar{e}_\theta(r, t) = -\frac{2}{\pi} \int_0^\infty e^{-\frac{t}{a^2} x^2} \sum_{n=0}^\infty (-1)^n x^{2(n+2)} \frac{(n+1)^2}{(2n+3)!} dx \quad (2.3.29)$$

Using the relation

$$\int_0^{\infty} x^{2n} e^{-ax^2} dx = \frac{(2n-1)!!}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}} \quad (2.3.30)$$

yields

$$\begin{aligned} \tilde{e}_\theta(r, t) &= -2 \sqrt{\frac{a^2}{t} \frac{1}{\pi}} \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)^2}{(2n+3)!} \frac{(2n+3)!!}{2^{n+3}} \left[\frac{a^2}{t} \right]^{n+2} \\ &= -\frac{4}{\sqrt{\pi}} \left[\frac{a^2}{4t} \right]^{1/2} \sum_{n=0}^{\infty} (-1)^n \frac{n+1}{n!} \left[\frac{a^2}{4t} \right]^{n+2} \\ &= \frac{4}{\sqrt{\pi}} \left[\frac{a^2}{4t} \right]^{5/2} \left\{ -\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left[\frac{a^2}{4t} \right]^n + \frac{a^2}{4t} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \left[\frac{a^2}{4t} \right]^m \right\} \\ &= \frac{4}{\sqrt{\pi}} \left[\frac{a^2}{4t} \right]^{5/2} \left[\frac{a^2}{4t} - 1 \right] e^{-\frac{a^2}{4t}} \end{aligned} \quad (2.3.31)$$

This result is the same as the late time approximation from the exact time domain solutions given in Section 2.2.

III). Inverse Fourier Transform by Integrating along Imaginary Axis

Since conducting media are lossy, $\tilde{e}_\theta(r, s)$ doesn't have any poles in the right half plane and on the imaginary axis (it has been checked from the expression given in Section 2.2), then inverse Laplace transform can be changed to inverse Fourier transform.

$$\begin{aligned} \tilde{e}_\theta(r, t) &= \frac{1}{2\pi j} \int_{\alpha_0 - j\infty}^{\alpha_0 + j\infty} \tilde{e}_\theta(r, s) e^{st} ds \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{e}_\theta(r, j\omega) e^{j\omega t} d\omega \end{aligned} \quad (2.3.32)$$

= Inverse Fourier Transform of $\tilde{e}_\theta(r, j\omega)$.

Since $\tilde{e}_\theta(r, t)$ should be a real function of t , then $\tilde{e}_\theta^*(r, j\omega) = \tilde{e}_\theta(r, -j\omega)$. Hence, the infinite integral can be converted into a semi-infinite integral as

$$\tilde{e}_\theta(r, t) = \frac{1}{\pi} \text{Re} \left\{ \int_0^\infty \tilde{e}_\theta(r, j\omega) e^{j\omega t} d\omega \right\} \quad (2.3.33)$$

IV). Inverse Fourier Transform by FFT Algorithm

The inverse Laplace transform is changed to an inverse Fourier transform. The integral of (2.3.33) can be evaluated to obtain the EM fields at a particular time t . However, if the values of the EM fields at many t 's are needed to obtain the waveforms of the fields, FFT algorithm (Fast Fourier Transform [12]) is the best method for this purpose. Thus the inverse FFT will be used to evaluate the integral.

Let's present some numerical results now. The conducting medium is seawater throughout in the following numerical calculations.

The θ -component of electric fields of the EM pulse generated by an electric dipole with an impulsive current are computed using the exact solutions, FFT algorithm and the late time approximation at a distance of 1 m from the antenna. The results are shown in Figure 2.4. It is observed that the numerical results computed with FFT algorithm are very accurate and close to the exact solutions.

The numerical integration around the branch cut has been evaluated also. However, it is found that the numerical integration around the branch cut suffers very slow conver-

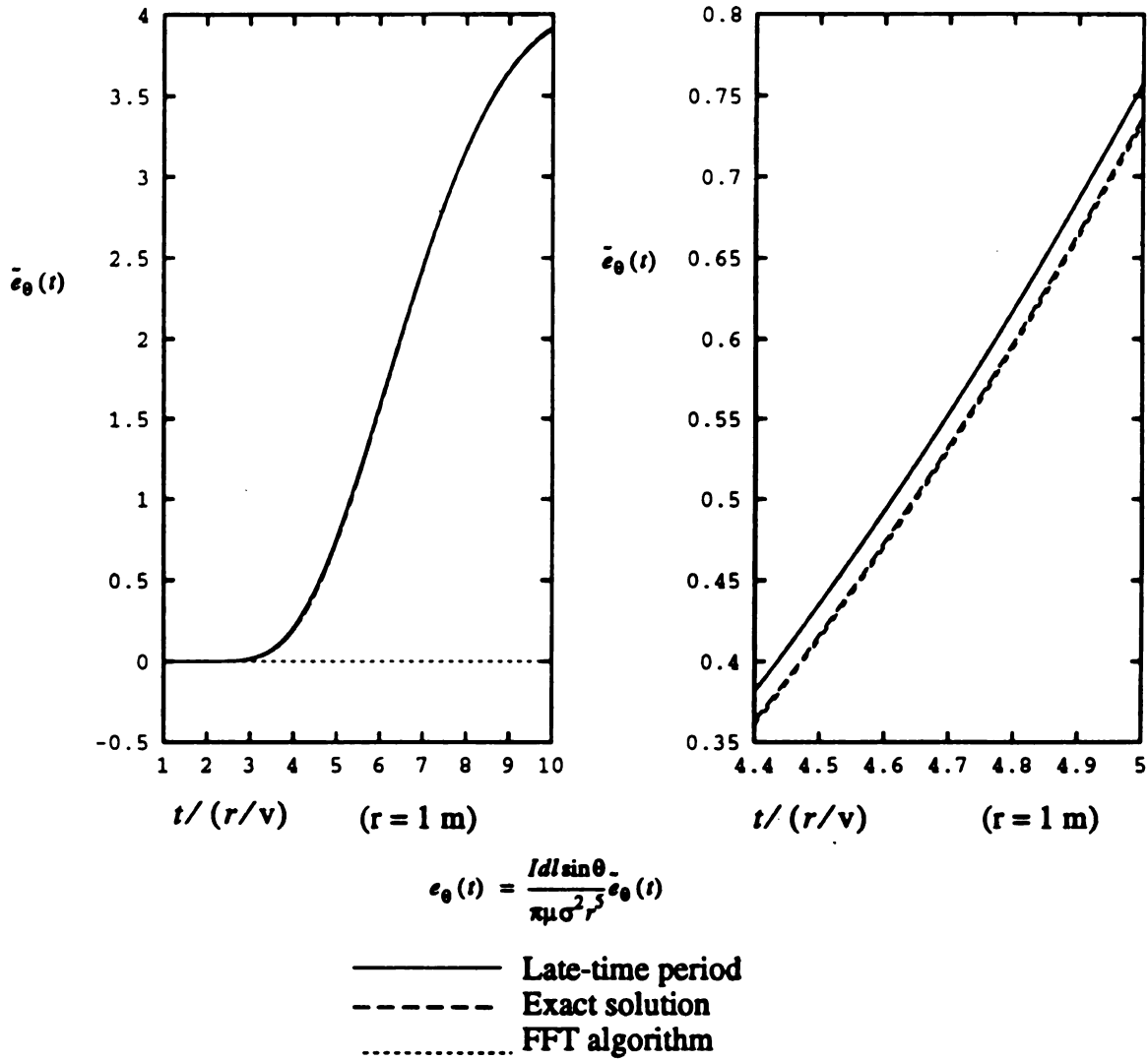


Figure 2.4. Comparison the numerical results (using FFT algorithm) of the electric field (θ -component) maintained by an electric dipole with an impulsive current with the corresponding exact solution and the late-time approximation ($t \gg r/v$ and $t \gg \tau_0$) of the exact solution.

gence, and, the total computation time for the branch cut integration is much greater than that using FFT algorithm.

Figures 2.5 and 2.6 show the comparison of the numerical results of the θ -component of electric field with the corresponding exact solution of the same quantity under the condition of $t \gg r/v$ and $t \gg \tau_0$ at $r=0.2$ m and 1 m, respectively. An excellent agreement between numerical results and the exact solutions is observed in Figure 2.6 ($r=1$ m). It is noted that the EM pulse at $t = r/v$, which may be significant only at an immediate vicinity of the antenna, is very difficult to obtain by the numerical integration because of its delta function behavior. For any other points away from the antenna this impulse wave is negligibly small.

The late time waveforms of the electric and magnetic fields maintained by an electric dipole with an impulsive current in the late time period ($t \gg r/v$ and $t \gg \tau_0$) are given in Figures 2.7 and 2.8. The plots of waveforms for these fields at a distance of 10 m are shown as functions of time in Figure 2.7. The same fields at a distance of 100 m are shown in Figure 2.8 as functions of time. Comparing these two figures, it is observed that the waveforms of electric and magnetic fields change as they propagate away from the antenna.

From the expressions of the late time waveforms, it is found that the EM fields depend on $t/(\sigma\mu r^2)$ only. Figure 2.9 shows the EM fields as functions of the normalized time $[t/(\sigma\mu r^2)]$.

Now that the numerical integration method by FFT algorithm and the late time approximation for the EM pulse have been found to be valid and accurate, FFT algorithm will be used to evaluate the numerical integration and the late time approximation will be used in the further analysis.

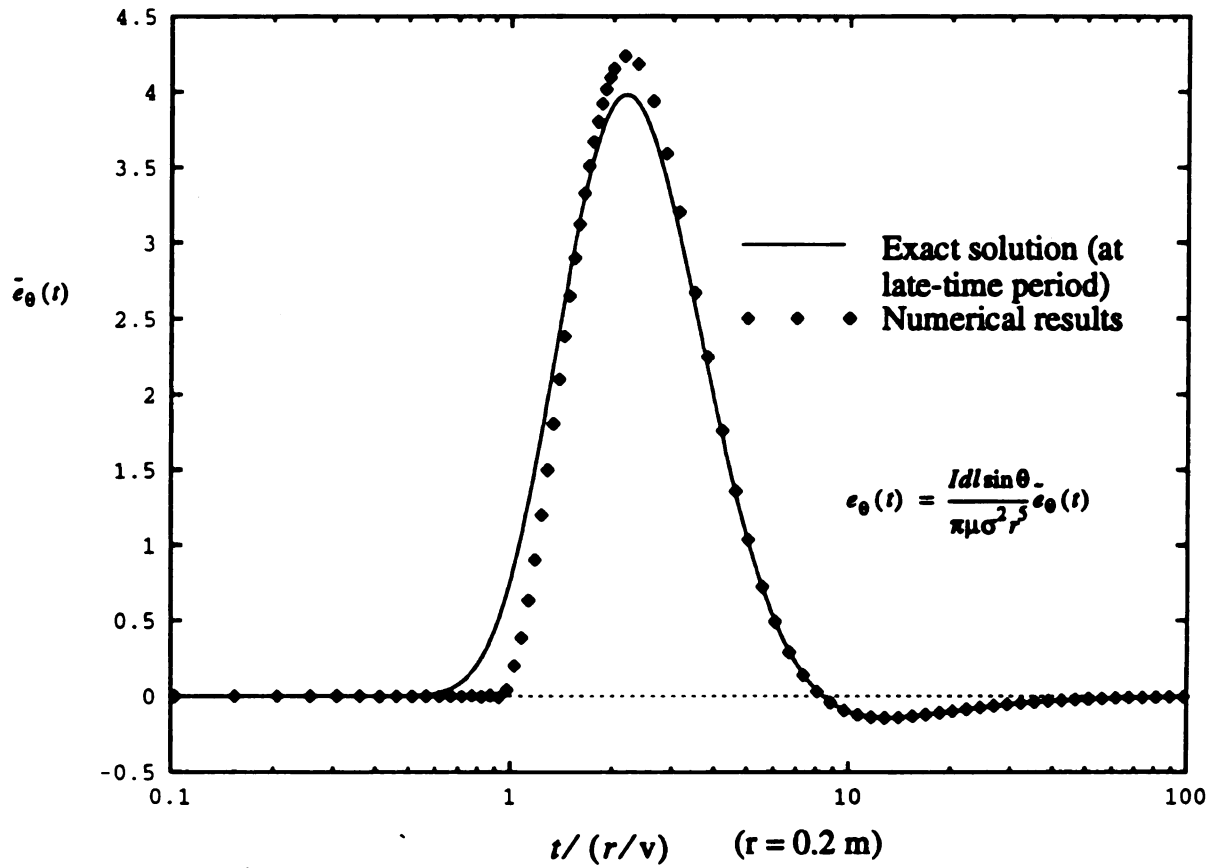


Figure 2.5. Comparison the numerical results (using FFT algorithm) of the electric field (θ -component) maintained by an electric dipole with an impulsive current with the corresponding exact solution of the same quality at the late-time approximation ($t \gg r/v$ and $t \gg \tau_0$). ($r=0.2 \text{ m}$)

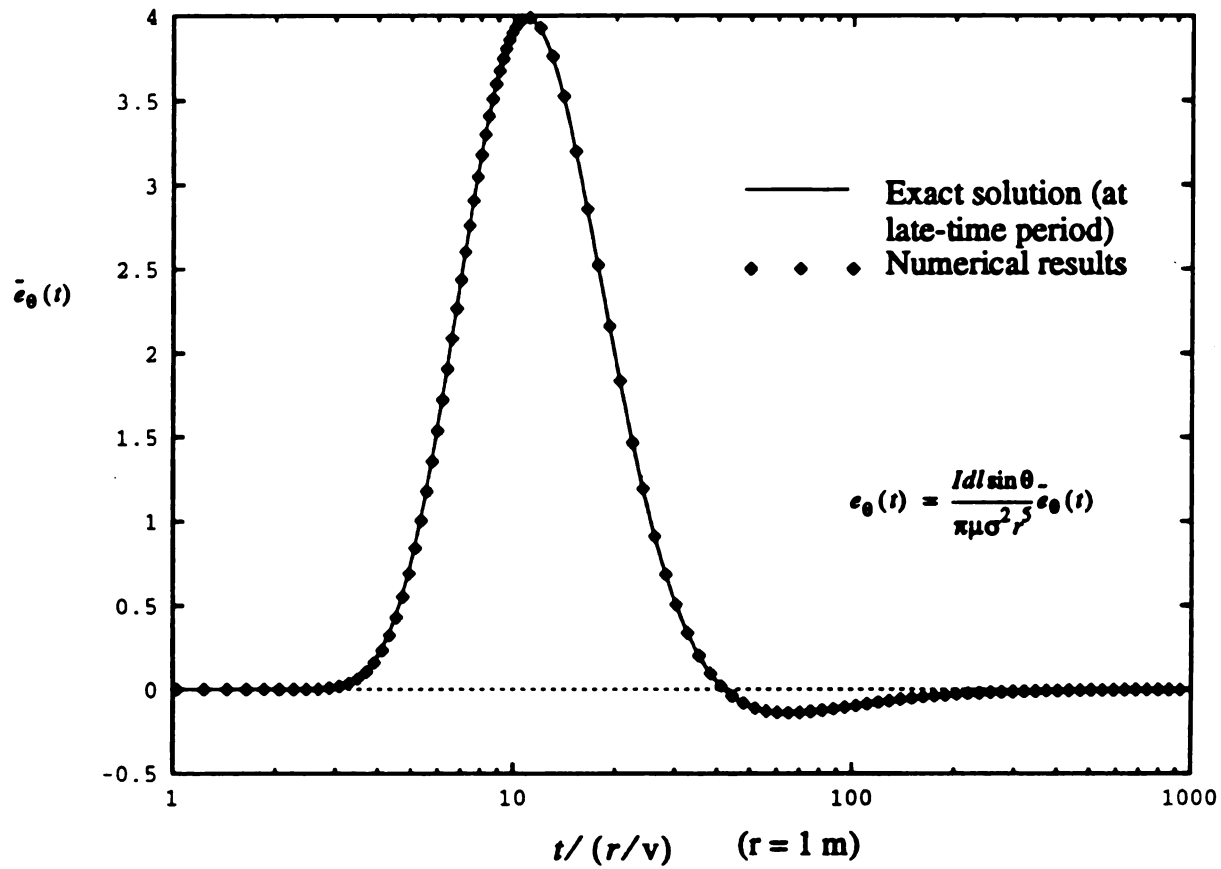


Figure 2.6. Comparison the numerical results (using FFT algorithm) of the electric field (θ -component) maintained by an electric dipole with an impulsive current with the corresponding exact solution of the same quality at the late-time approximation ($t \gg r/v$ and $t \gg \tau_0$). ($r=1 \text{ m}$)

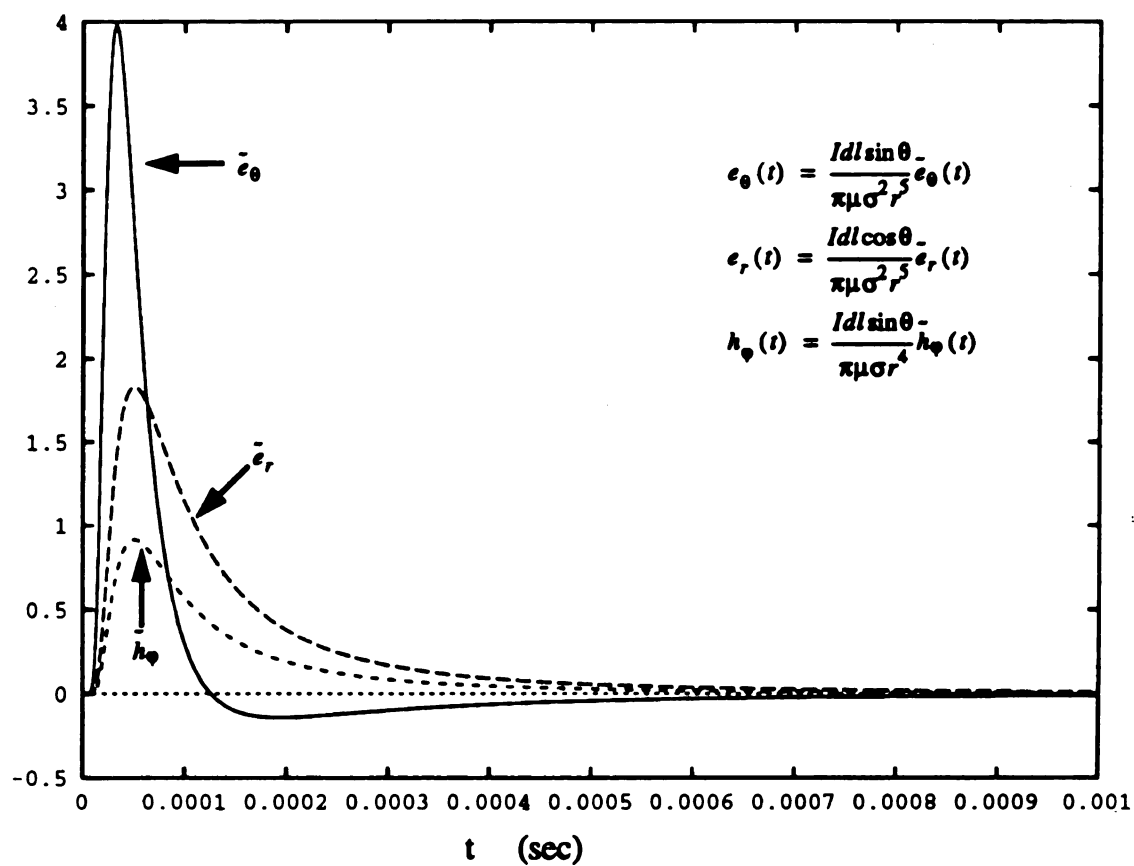


Figure 2.7. Electric and magnetic fields maintained by an electric dipole with an impulsive current at a distance of 10 m in seawater.

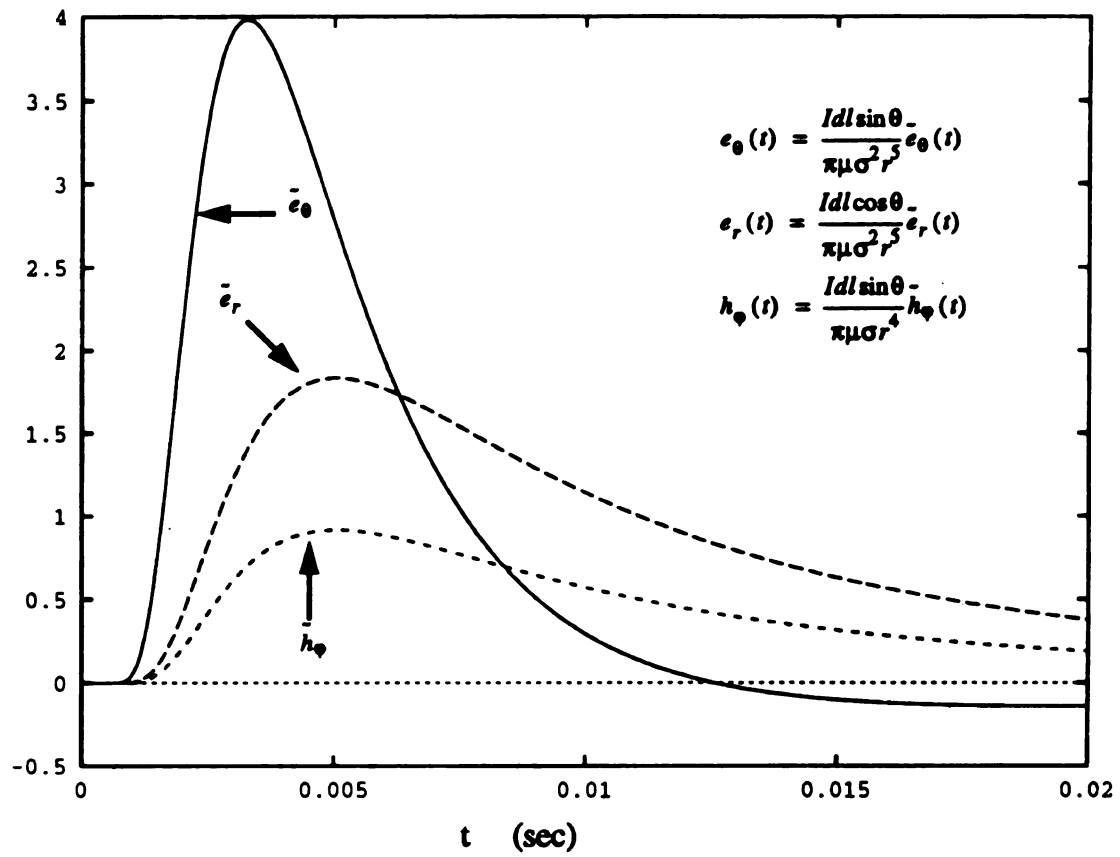


Figure 2.8. Electric and magnetic fields maintained by an electric dipole with an impulsive current at a distance of 100 m in seawater.

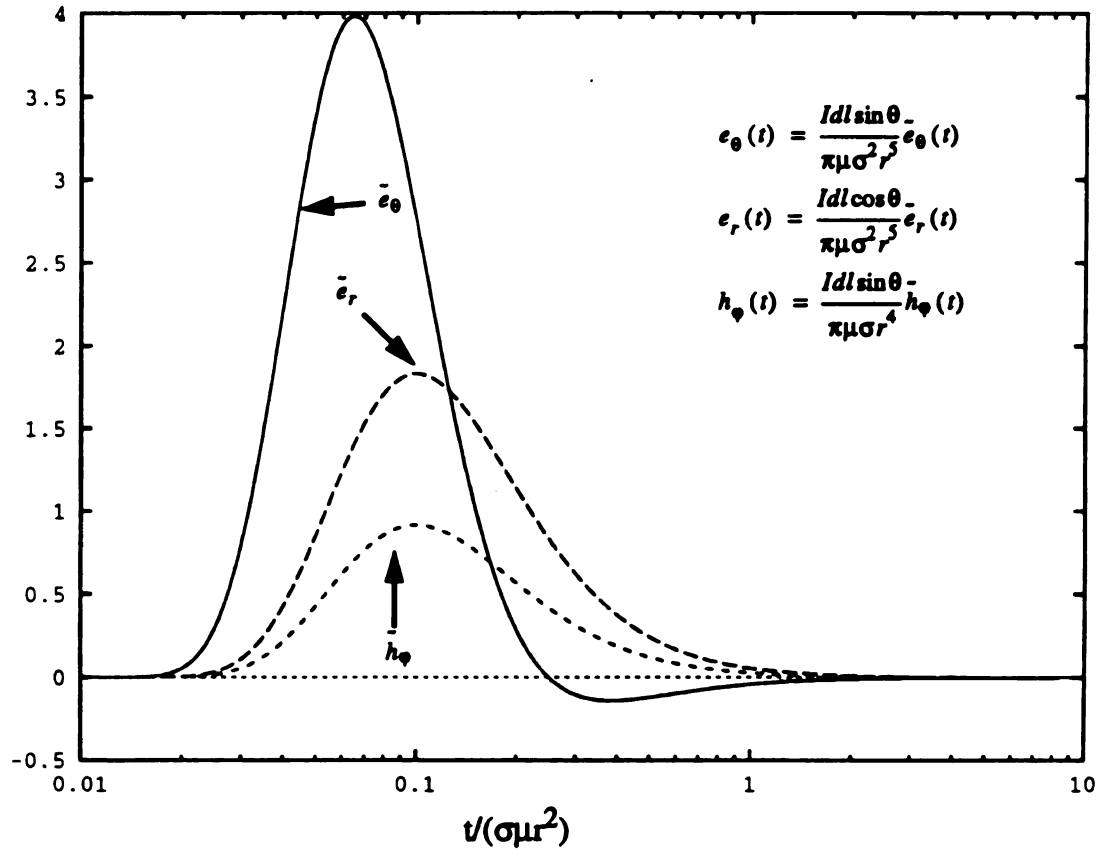


Figure 2.9. Electric and magnetic fields maintained by an electric dipole with an impulsive current as functions of normalized time $a^2 = \sigma \mu r^2$ in seawater.

CHAPTER 3

EM PULSES RADIATED BY AN ELECTRIC DIPOLE ANTENNA WITH VARIOUS EXCITATION CURRENTS

In the last chapter, the EM pulse generated by an electric dipole antenna excited by an impulsive current, or the impulse radiation of an electric dipole in conducting media, is derived. In this chapter, the EM pulses in conducting media generated by an electric dipole antenna excited by various excitation currents will be studied. Since the EM pulse can be obtained by convolving the impulse radiation with the antenna excitation current, the convolution theorem will be used to determine the EM pulse excited by an electric dipole with a step excitation current. However, it is difficult to use the convolution theorem directly to compute the EM pulse excited by an electric dipole with an arbitrary excitation current. Thus, FFT algorithm will be used to compute the EM pulses excited by an electric dipole with other excitation currents.

3.1. EM FIELDS OF EM PULSES GENERATED BY AN ELECTRIC DIPOLE WITH ARBITRARY EXCITATION CURRENTS

The Laplace transforms of the electric and magnetic fields of an EM pulse maintained by a current source $\vec{J}_e(\vec{r}, t) = \hat{z}\delta(\vec{r}) Idl f(t)$ are given by (2.1.12), (2.1.13) and (2.1.14), and can be rewritten as

$$H_\phi(\vec{r}, s) = \frac{Idl}{4\pi r^2} [1 + \gamma(s)r] e^{-\gamma(s)r} \sin\theta F(s) = \frac{Idl \sin\theta}{\pi r^2} \tilde{H}_\phi(r, s) \quad (3.1.1)$$

$$E_r(\vec{r}, s) = \frac{Idl}{2\pi(\sigma + s\varepsilon)r^3} [1 + \gamma(s)r] e^{-\gamma(s)r} \cos\theta F(s) = \frac{Idl \cos\theta}{\pi\sigma r^3} \tilde{E}_r(r, s) \quad (3.1.2)$$

$$\begin{aligned} E_\theta(\vec{r}, s) &= \frac{Idl}{4\pi(\sigma + s\varepsilon)r^3} [1 + \gamma(s)r + \gamma^2(s)r^2] e^{-\gamma(s)r} \sin\theta F(s) \\ &= \frac{Idl \sin\theta}{\pi\sigma r^3} \tilde{E}_\theta(r, s) \end{aligned} \quad (3.1.3)$$

where $F(s)$ is the Laplace transform of $f(t)$, and

$$\tilde{H}_\phi(r, s) = \frac{F(s)}{4} [1 + \gamma(s)r] e^{-\gamma(s)r} = \frac{1}{a^2} \tilde{h}_\phi(r, s) F(s) \quad (3.1.4)$$

$$\tilde{E}_r(r, s) = \frac{F(s)}{2} \frac{1 + \gamma(s)r}{1 + s\tau_0} e^{-\gamma(s)r} = \frac{1}{a^2} \tilde{e}_r(r, s) F(s) \quad (3.1.5)$$

$$\tilde{E}_\theta(r, s) = \frac{F(s)}{4} \frac{1 + \gamma(s)r + \gamma^2(s)r^2}{1 + s\tau_0} e^{-\gamma(s)r} = \frac{1}{a^2} \tilde{e}_\theta(r, s) F(s) \quad (3.1.6)$$

where

$$a^2 = \sigma\mu r^2 \quad (3.1.7)$$

$\tilde{h}_\phi(r, s)$, $\tilde{e}_r(r, s)$ and $\tilde{e}_\theta(r, s)$ are the normalized Laplace transforms of the EM fields of an EM pulse excited by an electric dipole with an impulsive current and are given in (2.2.40), (2.2.41) and (2.2.42).

Applying the convolution theorem, the EM fields in the time domain are:

$$H_\phi(\vec{r}, t) = \int_0^t h_\phi(\vec{r}, \tau) f(t - \tau) d\tau = \frac{Idl \sin\theta}{\pi r^2} \tilde{H}_\phi(r, t) \quad (3.1.8)$$

$$E_r(\vec{r}, t) = \int_0^t e_r(\vec{r}, \tau) f(t - \tau) d\tau = \frac{Idl \cos\theta}{\pi\sigma r^3} \tilde{E}_r(r, t) \quad (3.1.9)$$

$$E_{\theta}(\vec{r}, t) = \int_0^t \vec{e}_{\theta}(\vec{r}, \tau) f(t-\tau) d\tau = \frac{Idl \sin \theta}{\pi \sigma r^3} \tilde{E}_{\theta}(r, t) \quad (3.1.10)$$

and

$$\tilde{H}_{\phi}(r, t) = \int_0^t \tilde{h}_{\phi}(r, \tau) f(t-\tau) \frac{d\tau}{a^2} \quad (3.1.11)$$

$$\tilde{E}_r(r, t) = \int_0^t \tilde{e}_r(r, \tau) f(t-\tau) \frac{d\tau}{a^2} \quad (3.1.12)$$

$$\tilde{E}_{\theta}(r, t) = \int_0^t \tilde{e}_{\theta}(r, \tau) f(t-\tau) \frac{d\tau}{a^2} \quad (3.1.13)$$

where $\tilde{h}_{\phi}(r, t)$, $\tilde{e}_r(r, t)$ and $\tilde{e}_{\theta}(r, t)$ are the normalized EM fields in the time domain excited by an electric dipole with an impulsive current.

3.2. THE EM FIELDS OF AN EM PULSE EXCITED BY AN ELECTRIC DIPOLE WITH A HEAVISIDE STEP-FUNCTION EXCITATION CURRENT

The Heaviside step-function excitation current is expressed as:

$$f(t) = u(t) = \begin{cases} 1 & t > 0 \\ 0 & t \leq 0 \end{cases} \quad (3.2.1)$$

$$F(s) = 1/s \quad (3.2.2)$$

Thus, the EM fields of the EM pulse are:

$$\tilde{H}_{\phi}(r, t) = \int_0^t \tilde{h}_{\phi}(r, \tau) \frac{d\tau}{a^2} \quad (3.2.3)$$

$$\tilde{E}_r(r, t) = \int_0^t \tilde{e}_r(r, \tau) \frac{d\tau}{a^2} \quad (3.2.4)$$

$$\bar{E}_\theta(r, t) = \int_0^t \bar{e}_\theta(r, \tau) \frac{d\tau}{a^2} \quad (3.2.5)$$

Now, let's discuss the late time waveform of the EM pulse. Equations (2.2.48), (2.2.49) and (2.2.50) give the late time waveforms of the normalized EM fields of the EM pulse excited by an electric dipole with an impulsive current. Then, at $t \gg r/v$ and $t \gg \tau_0$, the EM fields of the EM pulse excited by an electric dipole with a step-function excitation current are:

$$\bar{H}_\phi(r, t) = \frac{2}{\sqrt{\pi}} \int_0^t \left[\frac{a}{2\sqrt{\tau}} \right]^5 e^{-\frac{a^2}{4\tau}} \frac{d\tau}{a^2} \quad (3.2.6)$$

Let's make a variable change,

$$x = \frac{a}{2\sqrt{\tau}} \quad (3.2.7)$$

then,

$$\tau = \frac{a^2}{4x^2} \quad (3.2.8)$$

and

$$d\tau = -\frac{a^2}{2x^3} dx \quad (3.2.9)$$

Substituting (3.2.8) and (3.2.9) into (3.2.6) yields

$$\bar{H}_\phi(r, t) = \frac{1}{\sqrt{\pi}} \int_{a/(2\sqrt{t})}^{\infty} x^2 e^{-x^2} dx$$

$$\begin{aligned}
&= \frac{1}{2\sqrt{\pi}} \left[\int_{a/(2\sqrt{t})}^{\infty} e^{-x^2} dx + \frac{a}{2\sqrt{t}} e^{-\frac{a^2}{4t}} \right] \\
&= \frac{1}{4} \left[1 - \operatorname{erf}\left(\frac{a}{2\sqrt{t}}\right) + \frac{a}{2\sqrt{t}} \operatorname{erf}'\left(\frac{a}{2\sqrt{t}}\right) \right]
\end{aligned} \tag{3.2.10}$$

Similarly, (3.2.4) and (3.2.5) can be evaluated as

$$\begin{aligned}
\tilde{E}_r(r, t) &= \frac{4}{\sqrt{\pi}} \int_0^t \left[\frac{a}{2\sqrt{\tau}} \right]^5 e^{-\frac{a^2}{4\tau}} \frac{d\tau}{a^2} \\
&= \frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{a}{2\sqrt{t}}\right) + \frac{a}{2\sqrt{t}} \operatorname{erf}'\left(\frac{a}{2\sqrt{t}}\right) \right]
\end{aligned} \tag{3.2.11}$$

$$\begin{aligned}
\tilde{E}_\theta(r, t) &= \frac{2}{\sqrt{\pi}} \int_0^t \left[\left[\frac{a}{2\sqrt{\tau}} \right]^5 e^{-\frac{a^2}{4\tau}} + \frac{1}{2} \frac{\partial}{\partial \tau} \left(\left[\frac{a}{2\sqrt{\tau}} \right]^3 e^{-\frac{a^2}{4\tau}} \right) \right] \frac{d\tau}{a^2} \\
&= \frac{1}{4} \left\{ 1 - \operatorname{erf}\left(\frac{a}{2\sqrt{t}}\right) + \left(\frac{a}{2\sqrt{t}} + 2 \left[\frac{a}{2\sqrt{t}} \right]^3 \right) \operatorname{erf}'\left(\frac{a}{2\sqrt{t}}\right) \right\}
\end{aligned} \tag{3.2.12}$$

The plots of the waveforms for these EM fields [(3.2.10), (3.2.11) and (3.2.12)] are shown in Figure 3.1 as functions of the normalized time, t/a^2 .

It is noted that the same results have been reported by J. R. Wait [1] [13] with the approximation that the displacement current is negligible compared with the conduction current in conducting media.

The total current density in seawater is given by

$$\vec{J} = (\sigma + s\epsilon) \vec{E} \tag{3.2.13}$$

When the frequency is 0.9 GHz, the displacement current $j\omega\epsilon\vec{E}$ is equal to the conduction current $\sigma\vec{E}$ in amplitude. Thus, when the frequency is lower than 0.1 GHz, the displace-

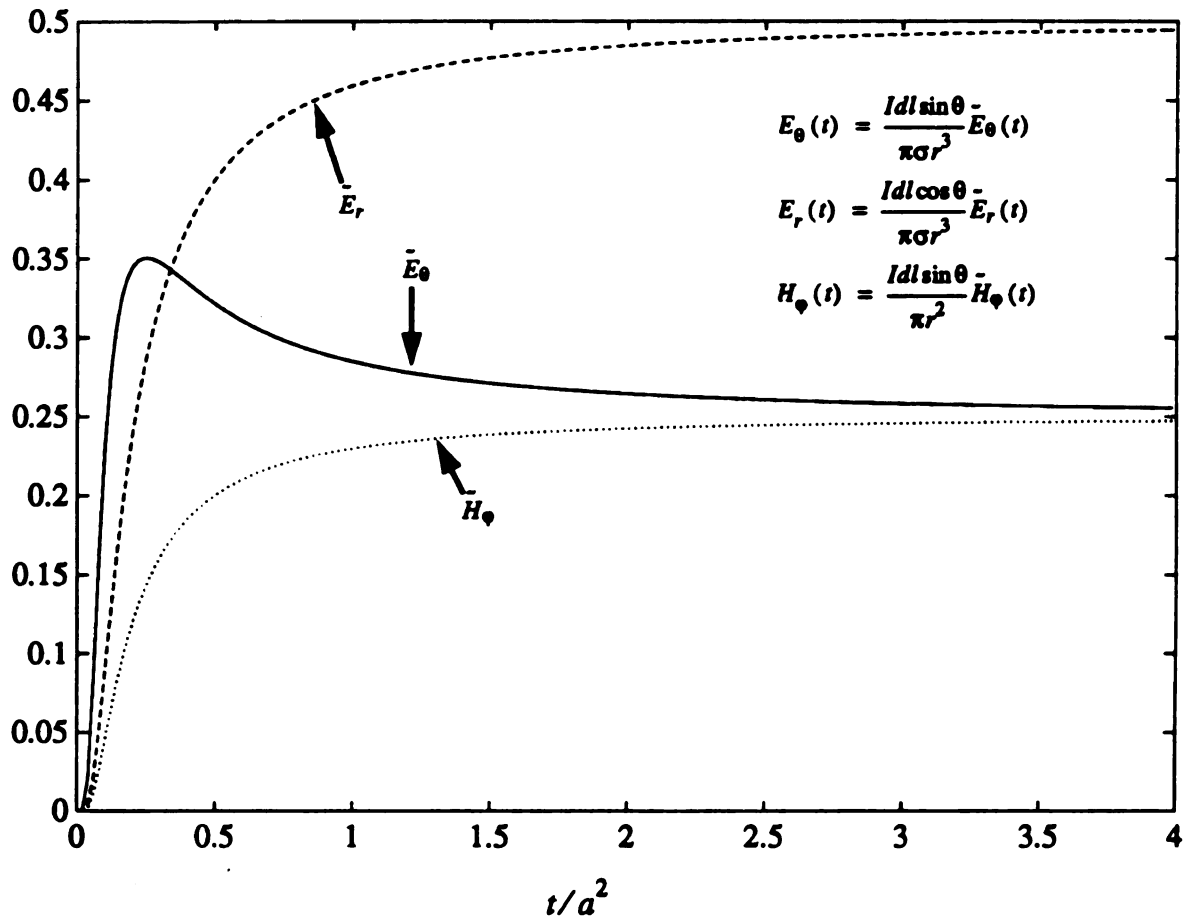


Figure 3.1. Electric and magnetic fields maintained by an electric dipole with a step-function excitation current as functions of normalized time ($a^2 = \sigma \mu r^2$) in seawater.

ment current in seawater becomes negligible. Under this condition the propagation constant then becomes

$$\gamma^2(s) = s^2 \epsilon \mu + s \sigma \mu \approx s \sigma \mu \quad (3.2.14)$$

The normalized Laplace transforms of the EM fields of the EM pulse excited by an electric dipole with a step-function excitation current are:

$$\tilde{H}_\phi(r, s) = \frac{1}{4s} (1 + \sqrt{s \sigma \mu} r) e^{-\sqrt{s \sigma \mu} r} \quad (3.2.15)$$

$$\tilde{E}_r(r, s) = \frac{1}{2s} (1 + \sqrt{s \sigma \mu} r) e^{-\sqrt{s \sigma \mu} r} \quad (3.2.16)$$

$$\tilde{E}_\theta(r, s) = \frac{1}{4s} (1 + \sqrt{s \sigma \mu} r + s \sigma \mu r^2) e^{-\sqrt{s \sigma \mu} r} \quad (3.2.17)$$

To invert these Laplace transforms is easier than to invert the Laplace transforms of the EM fields without neglecting the displacement current like what is done in the last chapter.

Wait [4] calculated the transient magnetic field of a step-function excited electric dipole in a homogeneous conducting earth. His results showed in a quantitative manner that the displacement current influences only the early portion of the waveform.

The late time approximation and the neglect of the displacement current lead to the same physical effect. As time grows, the low frequency component of the EM pulse becomes more dominant. Thus the displacement current can be ignored. Physically, the conducting medium behaves like a low-pass filter to an EM pulse. The spectra of the EM fields of an EM pulse at a distance of 100 m in seawater excited by an electric dipole with an impulsive current are shown in Figure 3.2 as functions of frequency. It is observed that the θ -component of the electric fields at 200 Hz is reduced to be less than -3dB, relative to the d. c. value.

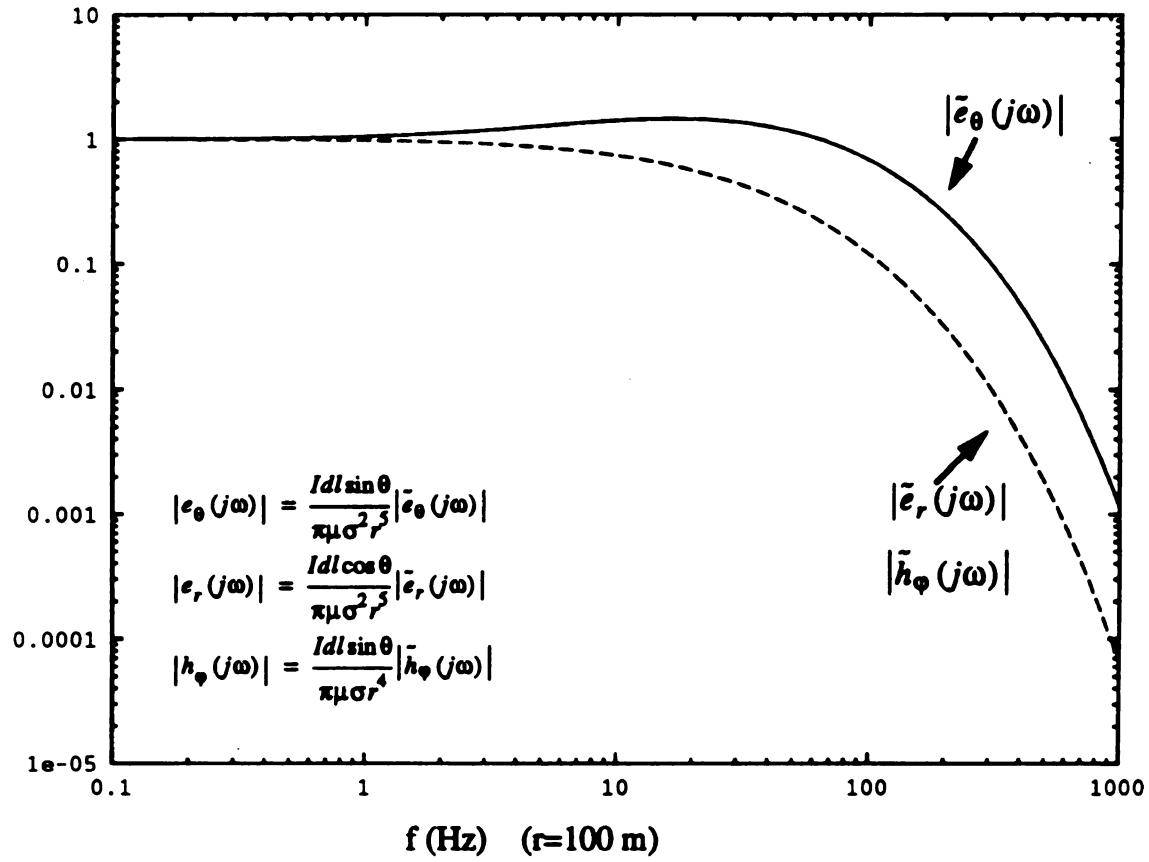


Figure 3.2. The amplitudes of Fourier transforms of electric and magnetic fields maintained by an electric dipole with an impulsive current at a distance of 100 m in seawater.

3.3. OPTIMAL EXCITATION CURRENT

In this section, an optimal excitation current for an electric dipole to maintain the maximum θ -component of the electric field in seawater will be synthesized.

The excitation current $f(t)$ should be causal, and it assumed that $f(t)$ is of finite duration. Then

$$f(t) = 0 \quad \text{for } t < 0 \text{ and } t > T_0. \quad (3.3.1)$$

To compare the EM fields excited by various excitation currents, it should be assumed that the total energy of the EM pulse is the same in each case:

$$\int_0^{T_0} f^2(t) dt = \int_0^{T_0} f^2(t) dt = A \quad (3.3.2)$$

where A is a constant.

Now, let's estimate $\tilde{E}_\theta(r, t)$ at given (r, T) , where $T > T_0$.

$$\begin{aligned} |\tilde{E}_\theta(r, T)| &= \left| \int_0^T \tilde{e}_\theta(r, \tau) f(T - \tau) \frac{d\tau}{a^2} \right| \\ &\leq \frac{1}{a^2} \left[\int_0^T \tilde{e}_\theta^2(r, \tau) d\tau \right]^{1/2} \left[\int_0^T f^2(T - \tau) d\tau \right]^{1/2} \end{aligned} \quad (3.3.3)$$

$$\begin{aligned} &= \frac{1}{a^2} B^{1/2}(r, T) \left[\int_0^T f^2(\tau') d\tau' \right]^{1/2} \\ &= \frac{A^{1/2}}{a^2} B^{1/2}(r, T) \end{aligned} \quad (3.3.4)$$

where

$$B(r, T) = \int_0^T \tilde{e}_\theta^2(r, \tau) d\tau \quad (3.3.5)$$

The well-known Cauchy-Schwarz inequality

$$\left| \sum_{i=1}^n x_i y_i \right| \leq \left[\sum_{i=1}^n |x_i|^2 \right]^{1/2} \left[\sum_{i=1}^n |y_i|^2 \right]^{1/2} \quad (3.3.6)$$

has been used in (3.3.3). If

$$f(t) = C \tilde{e}_\theta(r, T-t) u(t) \quad (3.3.7)$$

where C is a constant, then the inequality of (3.3.3) can be replaced by an equality. Thus, for given r and T , $\tilde{E}_\theta(r, T)$ will be a maximum when the waveform of the excitation current is the inverse of the impulse radiation. To satisfy (3.3.2), C should be

$$C^2 = \frac{A}{B(r, T)} \quad (3.3.8)$$

and

$$\tilde{E}_\theta(r, T) = \frac{C}{a^2} B(r, T) = \frac{A^{1/2}}{a^2} B^{1/2}(r, T) \quad (3.3.9)$$

When T increases, $\tilde{E}_\theta(r, T)$ increases also. Figure 3.3 plots $\frac{B(r, T)}{B(r, \infty)}$ as a function of normalized time $T/(\sigma \mu r^2)$. It is observed that at $T = W = a^2/4$, $[\tilde{e}_\theta(r, W) = 0]$, $B(r, W) = 0.992B(r, \infty)$ and $\tilde{E}_\theta(r, W) = 0.996\tilde{E}_\theta(r, \infty)$. Thus,

$$f(t) = C \tilde{e}_\theta(r, W-t) u(t) u(W-t) \quad (3.3.10)$$

will be used as the optimal excitation current. Since W depends on the distance r , we can only design an optimal excitation current for maintaining a maximum EM fields at a

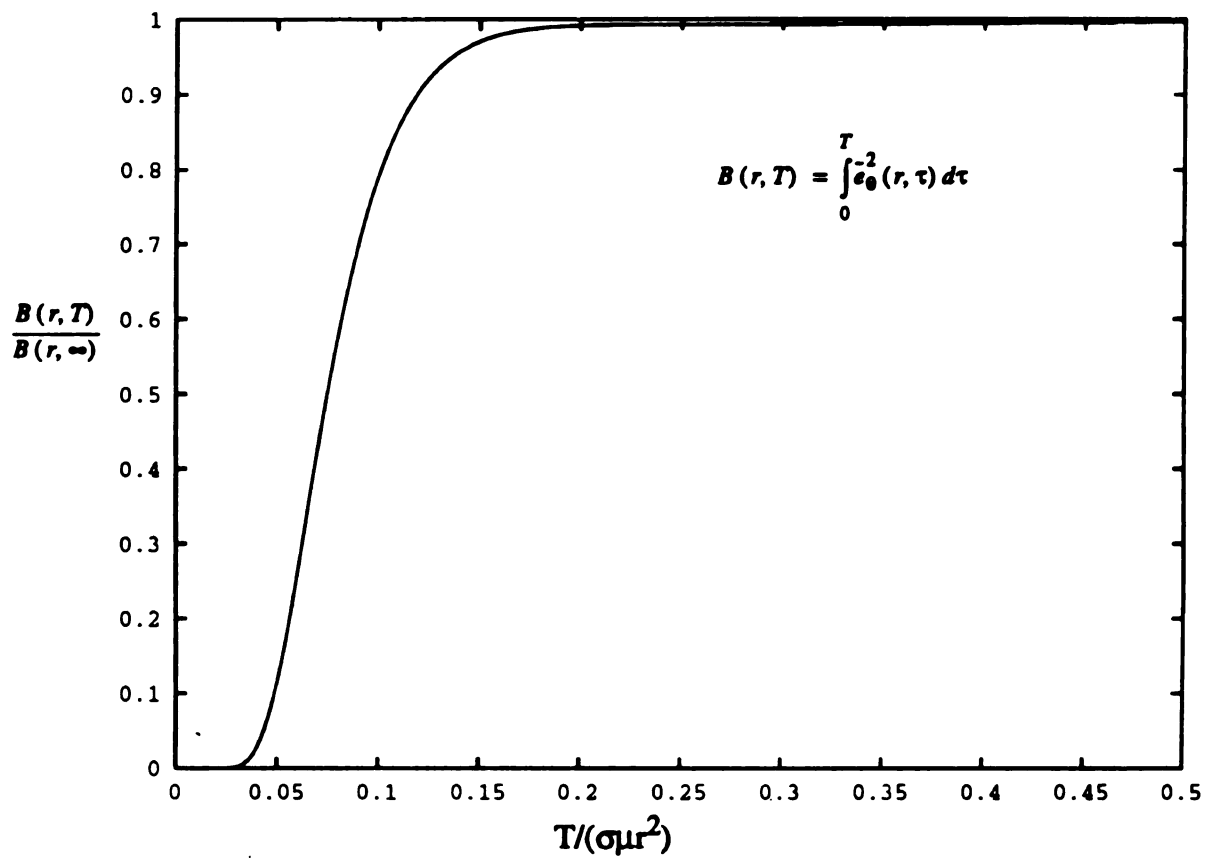


Figure 3.3. The total energy of the θ -component of electric field maintained by an electric dipole antenna excited by an impulsive current in seawater.

particular distance. In the next section, the EM pulse excited by an electric dipole with the optimal excitation current at $r=100$ m and that with other excitation currents will be presented.

3.4. NUMERICAL RESULTS OF EM FIELDS OF EM PULSES EXCITED BY AN ELECTRIC DIPOLE WITH VARIOUS EXCITATION CURRENTS

Four excitation currents will be considered, (1) optimum excitation current, (2) Gaussian excitation current, (3) Exponentially decaying excitation current and (4) rectangular excitation current, each having the same amplitude and the total energy as the optimum excitation current.

I). Optimum Excitation Current

In the last section, it has shown that the optimum excitation current of the electric dipole should have a waveform which is the inverse of the impulse radiation waveform. Then, the optimum excitation current to generate a maximum θ -component of the electric field at a distance of 100 m in seawater should have a form:

$$f(t) = \frac{1}{0.995\sqrt{\pi}} \left(\frac{W}{W-t}\right)^{5/2} \left[\frac{W}{W-t} - 1\right] e^{-\frac{W}{W-t}} u(t) u(W-t) \quad (3.4.1)$$

where

$$W = a^2/4 = (\sigma\mu r^2)/4 = 1.26 \times 10^{-2} s \quad (3.4.2)$$

and the coefficient of $f(t)$ is chosen so that $f(t)$ has a maximum of 1 at

$W-t = a^2/(9 + \sqrt{41})$. Then

$$A = \int_0^W f^2(t) dt = 3ms \quad (3.4.3)$$

The other excitation currents should have the same amplitude and the total energy as that of the optimum excitation current.

II). Gaussian Excitation Current

$$f(t) = e^{-\left[\frac{t-5W}{2W}\right]^2} \quad (3.4.4)$$

where

$$W = \frac{3ms}{\sqrt{2\pi}} \quad (3.4.5)$$

it is found that (3.4.4) has a maximum at $t = 5W$ with

$$f(5W) = 1 \quad (3.4.6)$$

and

$$\int_0^{\infty} f^2(t) dt = \int_{-\infty}^{\infty} e^{-2\left[\frac{\tau}{2W}\right]^2} d\tau = \sqrt{2\pi}W = 3ms \quad (3.4.7)$$

III). Exponentially Decaying Excitation Current

$$f(t) = e^{-t/W} u(t) \quad (3.4.8)$$

where

$$W = 6ms \quad (3.4.9)$$

(3.4.8) has a maximum $f(0) = 1$ and

$$\int_0^{\infty} f^2(t) dt = \frac{W}{2} = 3ms \quad (3.4.10)$$

IV). Rectangular Excitation Current

$$f(t) = \begin{cases} 1 & 0 < t < W \\ 0 & t \leq 0, t \geq W \end{cases} \quad (3.4.11)$$

where

$$W = 3ms \quad (3.4.12)$$

These four excitation currents, as shown in Figure 3.4, are then used to convolve with the impulse radiation of the electric dipole to produce four E_θ fields at a distance of 100 m. FFT algorithm is applied to compute these fields. The four E_θ fields produced by these four different excitation currents at a distance of 100 m are shown in Figure 3.5. It is observed that the optimum excitation current (designed for a distance of 100 m) indeed produces a maximum E_θ field as compared with the E_θ fields produced by other excitation currents.

If the same optimum excitation current (designed for a distance of 100 m) is used to generate the E_θ field at a distance of 10 m, the generated E_θ field is not larger than the E_θ fields produced by three other excitation currents. This phenomenon is depicted in Figure 3.6. This indicates that an optimum excitation current will generate a maximum EM field at a specified distance only. To generate a maximum EM field at a different distance, a different excitation current will be needed.

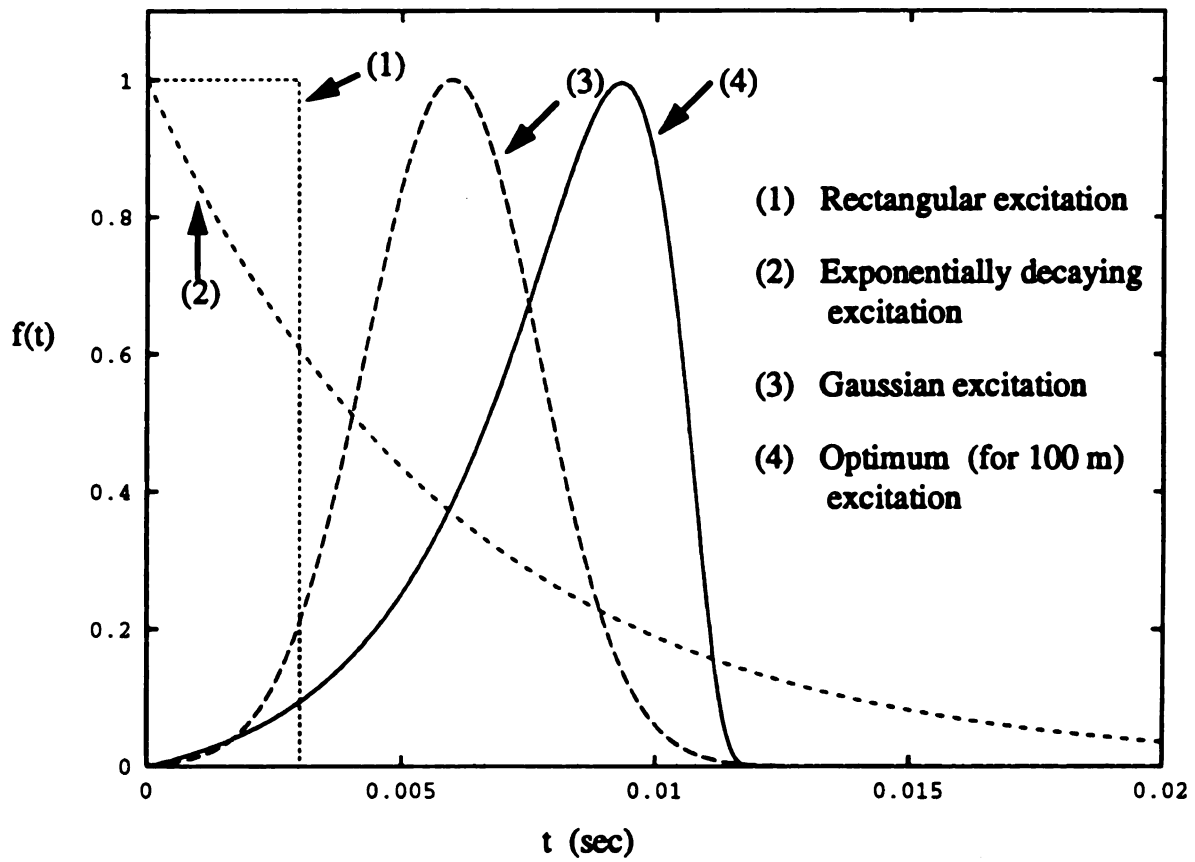


Figure 3.4. Various forms of excitation currents for an electric dipole including rectangular, exponentially decaying, Gaussian and optimum (designed for a distance of 100 m) excitation.

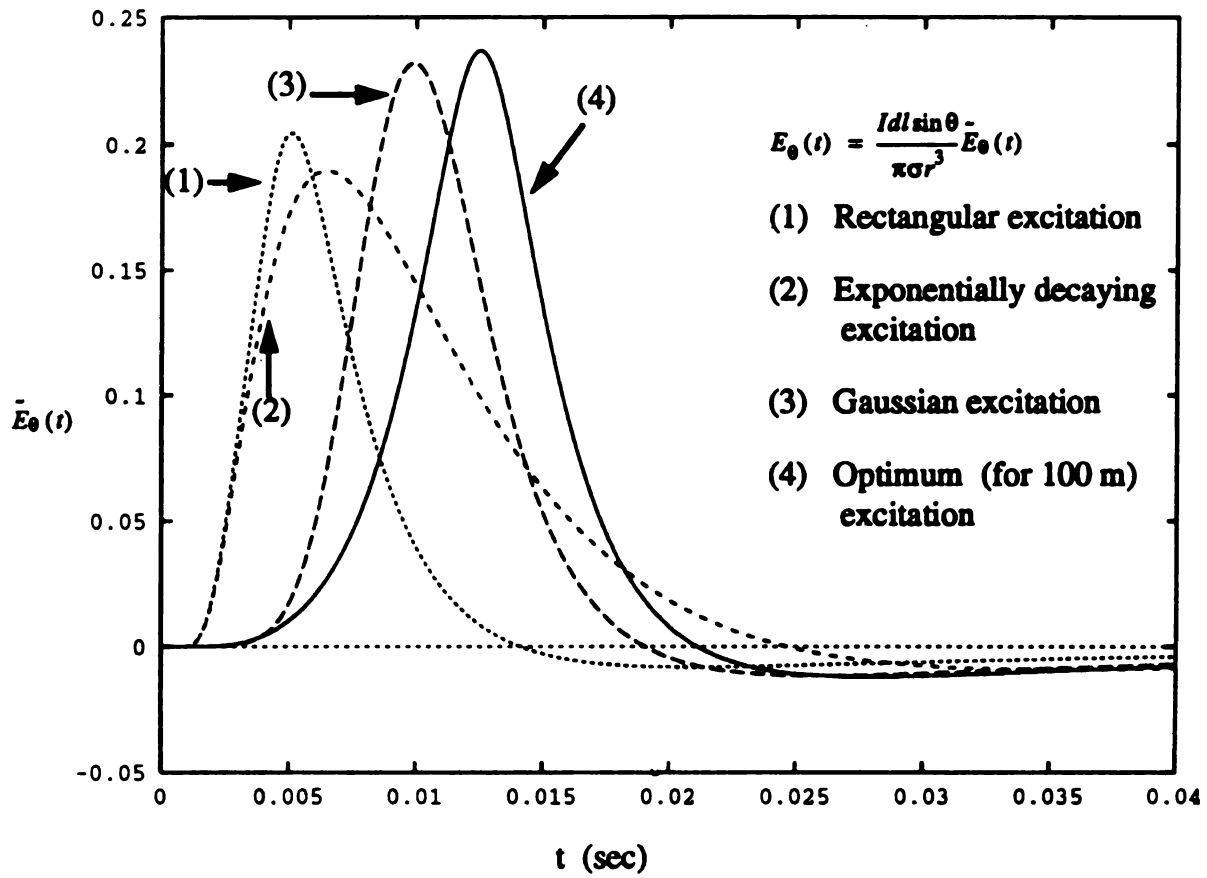


Figure 3.5. The θ components of electric fields maintained by an electric dipole with various excitation currents at a distance of 100 m in seawater.

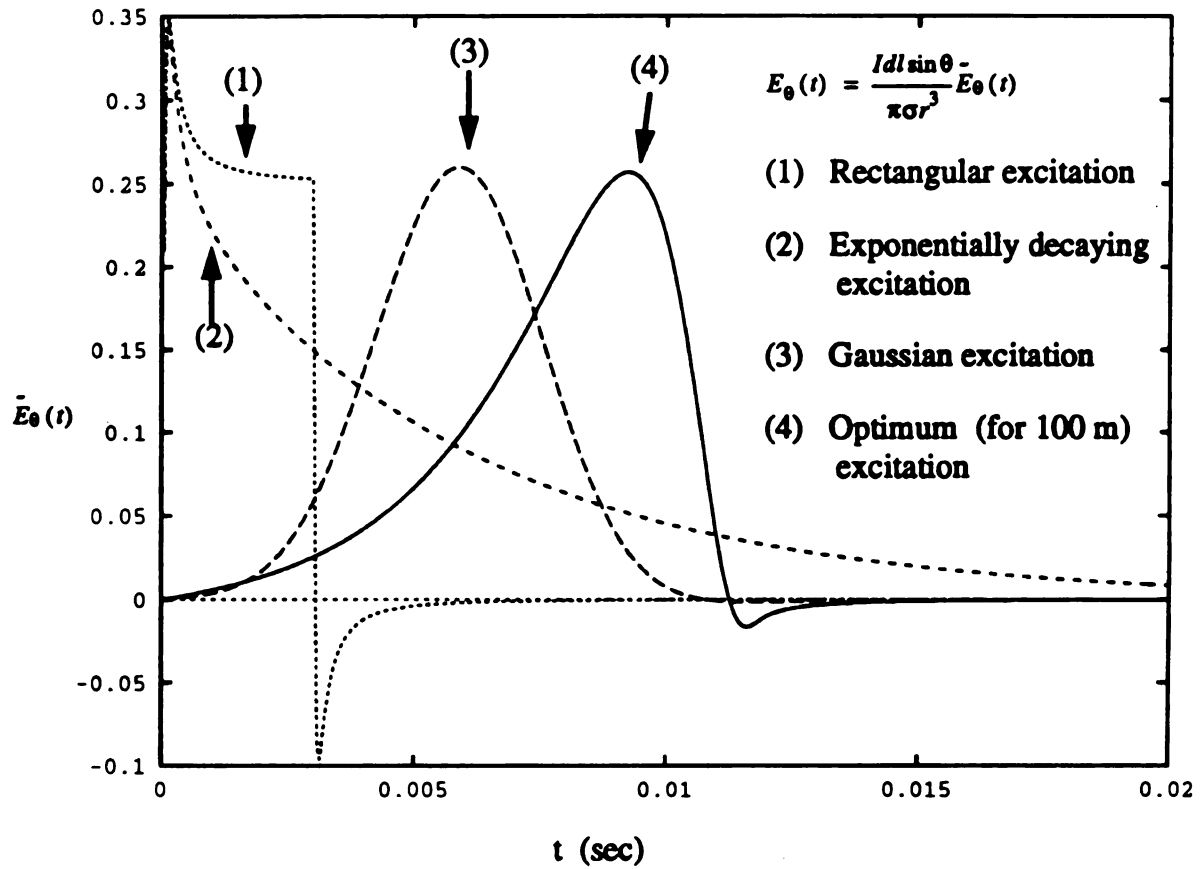


Figure 3.6. The θ components of electric fields maintained by an electric dipole with various excitation currents at a distance of 10 m in seawater.

CHAPTER 4

EM PULSES EXCITED BY AN ELECTRIC DIPOLE ANTENNA WITH A FIXED TOTAL EMP ENERGY IN SEAWATER

4.1. PARAMETERS OF AN ELECTRIC DIPOLE ANTENNA GENERATING AN EM PULSE WITH A FIXED EMP ENERGY

To estimate the maximum strength of an EM pulse which can be excited by a dipole antenna with a given finite EMP (EM pulse) energy, the following assumptions are made. First, the antenna current has the optimum excitation waveform $f(t)$ with a duration of about 12 ms as shown in Figure 3.4, which is designed to yield a maximum electric field at a distance of 100 m from the antenna. Secondly, the total energy of the EM pulse propagating across a spherical surface of 1 m radius surrounding the antenna has 1 Megajoule. In this section, we aim to find the parameter Idl , the product of the antenna current and the antenna length, of the electric dipole.

The optimum excitation current to generate a maximum E_θ (the θ -component of the electric field) at a distance of 100 m from the antenna has a waveform which is given in chapter 3:

$$f(t) = \frac{1}{0.995\sqrt{\pi}} \left[\frac{T}{T-t} \right]^{3/2} \left[\frac{T}{T-t} - 1 \right] e^{-\frac{T}{T-t}} u(t) u(W-t) \quad (4.1.1)$$

where

$$T = \frac{a_{100}^2}{4} = \frac{\sigma \mu r^2}{4} \Big|_{r=100m} = 1.26 \times 10^{-2} \text{ sec} \quad (4.1.2)$$

The total energy of the optimum excitation current is proportional to:

$$A = \int_0^{\infty} f^2(t) dt = \int_0^T f^2(t) dt = 3ms$$

The EM fields can be obtained by applying the convolution theorem. This leads to

$$E_{\theta}(\vec{r}, t) = \frac{Idl \sin \theta}{\pi \sigma r^3} \bar{E}_{\theta}(r, t) = \frac{Idl \sin \theta}{\pi \sigma r^3} \int_0^t \bar{e}_{\theta}(r, \tau) f(t - \tau) \frac{d\tau}{a_r^2} \quad (4.1.3)$$

Since $\bar{e}_{\theta}(r, t)$, $\bar{h}_{\phi}(r, t)$ have a duration of about $10a_r^2$ and $a_r^2 = 5.03 \times 10^{-6} \text{ sec} \ll T$ at $r = 1m$, then $f(t)$ can be considered as a constant in the duration when $r \leq 1m$. Thus,

$$\begin{aligned} E_{\theta}(\vec{r}, t) &= \frac{Idl \sin \theta}{\pi \sigma r^3} f(t) \int_0^t \bar{e}_{\theta}(r, \tau) \frac{d\tau}{a_r^2} \\ &= \frac{Idl \sin \theta}{\pi \sigma r^3} \frac{f(t)}{a_r^2} \int_0^{\infty} \bar{e}_{\theta}(r, \tau) d\tau \\ &= \frac{Idl \sin \theta}{\pi \sigma r^3} \frac{f(t)}{a_r^2} \bar{e}_{\theta}(r, s) \Big|_{s=0} \\ &= \frac{Idl \sin \theta}{4\pi \sigma r^3} f(t) \end{aligned} \quad (4.1.4)$$

Similarly,

$$H_{\phi}(\vec{r}, t) = \frac{Idl \sin \theta}{4\pi r^2} f(t) \quad (4.1.5)$$

On application of (4.1.4) and (4.1.5), the r-component of Poynting vector is found to be

$$P_r(\vec{r}, t) = E_{\theta}(\vec{r}, t) H_{\phi}(\vec{r}, t) = \frac{(Idl)^2 \sin^2 \theta}{16\pi^2 \sigma r^5} f^2(t) \quad (4.1.6)$$

The total power propagating across a spherical surface of radius r is easily carried out:

$$W(r, t) = \oint P_r(\vec{r}, t) dS = \int_0^\pi P_r(\vec{r}, t) 2\pi r^2 \sin\theta d\theta = \frac{(Idl)^2 \dot{f}^2(t)}{6\pi\sigma r^3} \quad (4.1.7)$$

Because of the presence of the $1/r^3$ factor in (4.1.7), the power radiated from an electric dipole in dissipative media behaves singular near $r = 0$. In practice, the dipole antenna is of finite size, thus it is reasonable to compute the total radiated power as the power propagating across the spherical surface of 1 m radius surrounding the antenna.

By integrating the total power with time, the total energy propagating across the spherical surface of radius r surrounding the antenna is obtained as:

$$E_0 = \int_0^\infty W(r, t) dt = \frac{(Idl)^2}{6\pi\sigma r^3} \int_0^\infty \dot{f}^2(t) dt = \frac{(Idl)^2}{6\pi\sigma r^3} A \quad (4.1.8)$$

It is assumed that the total energy is 1 Megajoule, $E_0 = 1 \times 10^6 \text{ Joules}$, when $r = 1 \text{ m}$ and $A = 3 \text{ ms}$, then

$$Idl = \left[\frac{6\pi\sigma r^3 E_0}{A} \right]^{1/2} = 1.58 \times 10^5 (A \cdot m) \quad (4.1.9)$$

The amplitude of the total electric field is obtained from its r - and θ - components as

$$E = [E_\theta^2 + E_r^2]^{1/2} = \frac{Idl}{\pi\sigma r^3} [\sin^2\theta \tilde{E}_\theta^2 + \cos^2\theta \tilde{E}_r^2]^{1/2} \quad (4.1.10)$$

The maximum electric field can be calculated as

$$E_{max} = \begin{cases} \frac{Idl}{\pi\sigma r^3} |\tilde{E}_r| & \text{for } |\tilde{E}_\theta| \leq |\tilde{E}_r| \\ \frac{Idl}{\pi\sigma r^3} |\tilde{E}_\theta| & \text{for } |\tilde{E}_\theta| > |\tilde{E}_r| \end{cases} \quad (4.1.11)$$

At a given particular distance, $\tilde{E}_\theta(r, t)$ and $\tilde{E}_r(r, t)$ are computed. Then E_{max} is determined from the values of $|\tilde{E}_\theta(r, t)|_{max}$ and $|\tilde{E}_r(r, t)|_{max}$.

4.2. NUMERICAL RESULTS OF THE EM FIELDS OF AN EM PULSE GENERATED BY AN ELECTRIC DIPOLE ANTENNA WITH THE OPTIMUM EXCITATION CURRENT AND WITH A FINITE EMP ENERGY IN SEAWATER

In the last section, two assumptions are made to estimate the EM fields of an EM pulse which is excited by an electric dipole antenna with a given finite EMP energy in seawater. First, the antenna current has the optimum excitation waveform which is designed to yield a maximum electric field at a distance of 100 m from the antenna. Secondly, the total energy of the EM pulse propagating across a spherical surface of 1 m radius surrounding the antenna has 1 Megajoule. Then the parameter Idl of the electric dipole antenna for generating such an EM pulse is calculated. With this information, the real values of the EM fields of the EM pulse can be calculated, rather than the normalized values of the EM fields as computed in the last two chapters.

First, the waveforms of an optimally excited EMP are calculated as it propagates away from the antenna. When this EMP propagates away from the antenna, its amplitude decreases, its waveform changes and its propagation velocity is reduced. To depict the behavior of this propagating EMP, the waveforms of the EMP at the distances of 10 m, 50 m, and 100 m away from the antenna are plotted as functions of time in Figure 4.1. It is noted that the scale for the amplitude of the electric field of the EMP indicated in the vertical coordinate is different for each plot.

The maximum electric field of the EM pulse propagating away from the antenna is calculated as a function of distance from the antenna and the results are shown in Figure 4.2. The waveform of the EM pulse changes greatly as it propagates away from the antenna, and the strength of its electric field attenuates roughly as $1/r^3$ between 10 to 100 m range

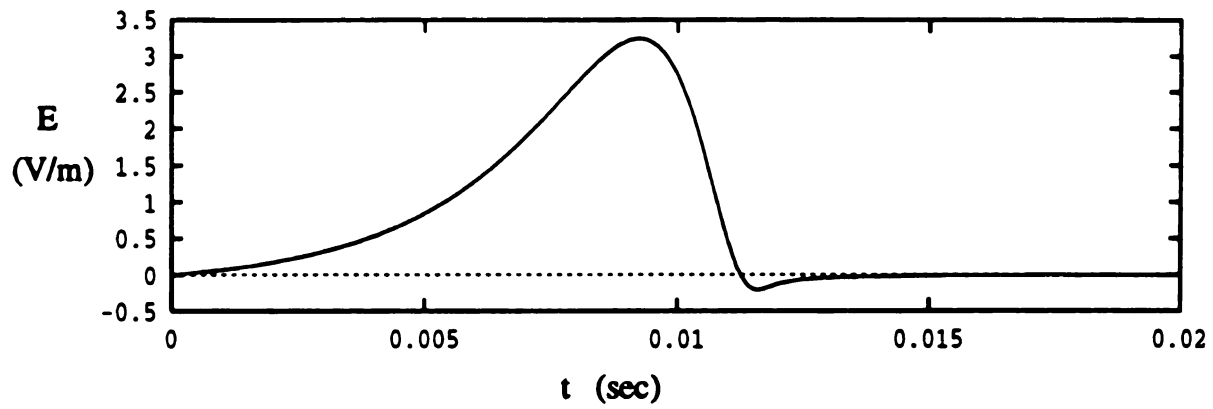
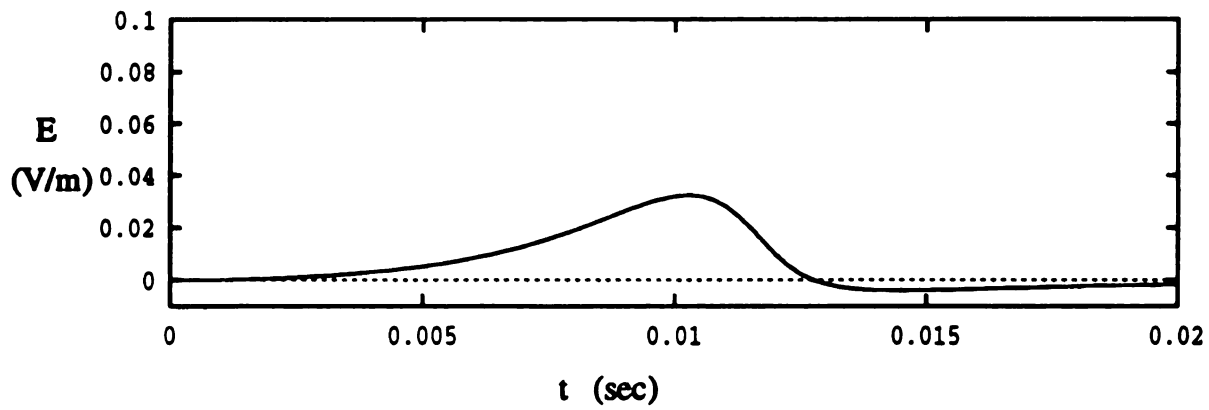
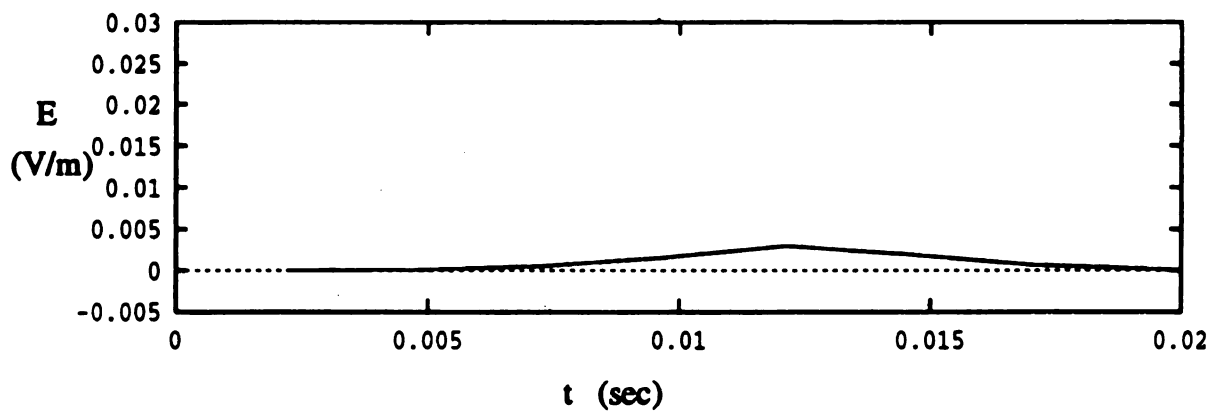
(a) $r = 10$ m(b) $r = 50$ m(c) $r = 100$ m

Figure 4.1. The waveforms of the electric field of an optimally excited EM pulse at the distance of 10 m, 50 m, and 100 m away from the antenna plotted as functions of time.

Intensity of Maximum Electric Field (V/m)

0

0.0

1e

Figure

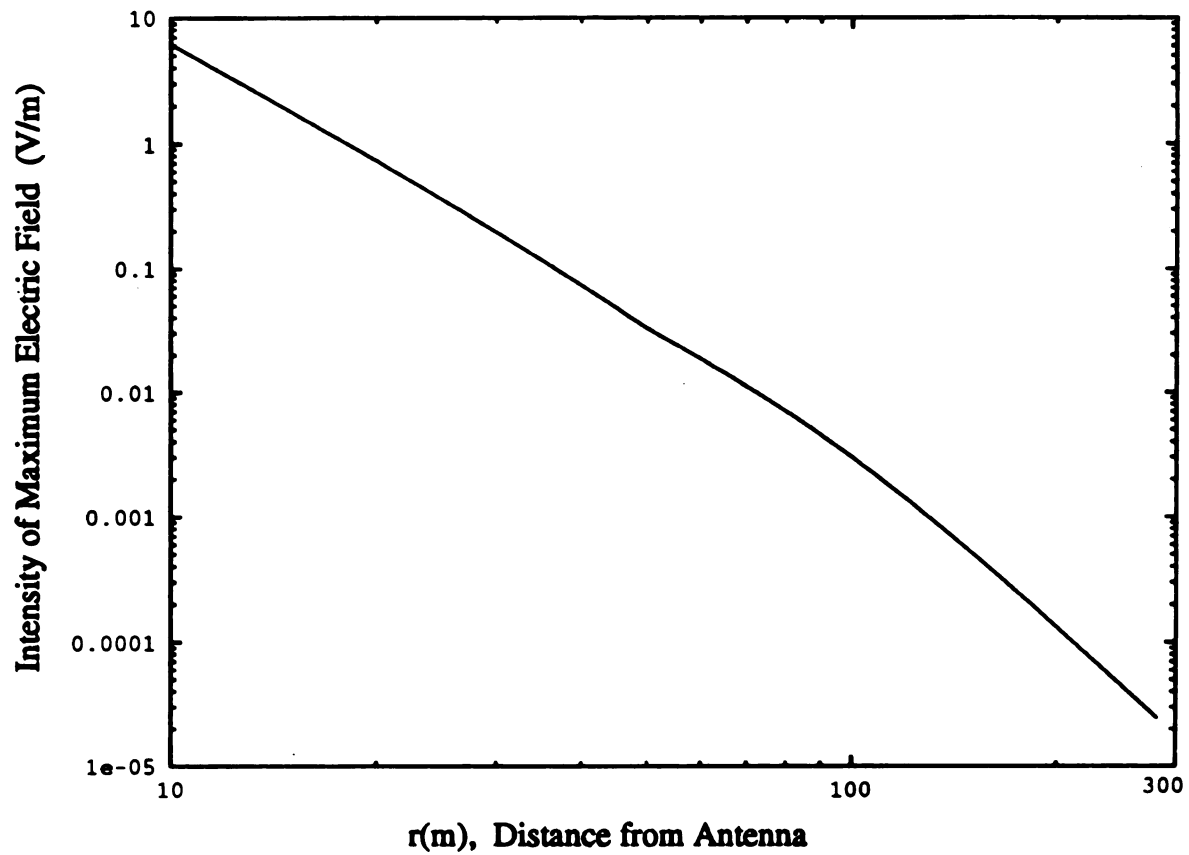


Figure 4.2. Intensity of the maximum electric field as a function of distance, which is excited by a dipole antenna with optimum current excitation (with the waveform of Figure 3.4) and input EM pulse energy of 1 Megajoule.

and as $1/r^5$ between 100 to 1000 m range, where r is the distance from the antenna. As indicated in Figure 4.2, the strength of the electric field at 100 m from the dipole antenna is in the order of mV/m. This intensity appears small because an energy of 1 Megajoule possessed by an EM pulse is rather small and the loss suffered by the pulse in seawater is extremely great. It is pointed out that if the EM pulse were to attenuate exponentially, the field intensity at a distance of 100 m from the antenna would be completely negligible. An electric field of mV/m level may not be strong enough to damage an electronic device, but it is very strong to be detected by any sensing devices.

Another observation is pointed out that if the duration of the excitation current waveform is narrowed to the order of nanosecond, the power density of the EM pulse will be greatly enhanced in the vicinity of the antenna. However, because of the narrow waveform of the EM pulse, it will attenuate roughly as $1/r^5$ instead of $1/r^3$ as the case of the optimum excitation waveform. As the result, the field intensity of this EM pulse at the distance of 100 m will also be in the same order of mV/m.

Next, let's discuss Poynting vector $\vec{P}$ which represents the power flow of the EM pulse.

$$\begin{aligned}\vec{P} &= \vec{E} \times \vec{H} = \frac{(Idl)^2}{\pi^2 \sigma r^5} [\sin^2 \theta \vec{E}_\theta(r, t) \vec{H}_\phi(r, t) \hat{r} - \cos \theta \sin \theta \vec{E}_r(r, t) \vec{H}_\phi(r, t) \hat{\theta}] \\ &= \frac{(Idl)^2}{\pi^2 \sigma r^5} [\tilde{P}_r \hat{r} + \tilde{P}_\theta \hat{\theta}]\end{aligned}\quad (4.2.1)$$

where

$$\tilde{P}_r(r, \theta, t) = \sin^2 \theta \vec{E}_\theta(r, t) \vec{H}_\phi(r, t) \quad (4.2.2)$$

$$\tilde{P}_\theta(r, \theta, t) = -\sin\theta \cos\theta \tilde{E}_r(r, t) \tilde{H}_\phi(r, t) \quad (4.2.3)$$

From Figure 2.9 in chapter 2, it is found that $\tilde{h}_\phi(r, t) = \frac{1}{2}\tilde{e}_r(r, t) \geq 0$ for $t > t_0$ and $t > r/v$. This leads to:

$$\tilde{H}_\phi(r, t) = \frac{1}{2}\tilde{E}_r(r, t) \quad (4.2.4)$$

and

$$\tilde{E}_r(r, t) \tilde{H}_\phi(r, t) = \frac{1}{2}\tilde{E}_r^2(r, t) \geq 0 \quad (4.2.5)$$

Since $\tilde{e}_\theta(r, t)$ changes the sign at $t = a^2/4$ from positive to negative, and if $f(t)$ is a finite duration function, $\tilde{E}_\theta(r, t)$ changes the sign also when t is large (compared with the duration of excitation current). Thus,

for small t , $\tilde{E}_\theta(r, t) \tilde{H}_\phi(r, t) > 0$ and $P_r > 0$

for large t , $\tilde{E}_\theta(r, t) \tilde{H}_\phi(r, t) < 0$ and $P_r < 0$

When $P_r < 0$ at large t , it means the energy flows in the $-\hat{r}$ direction. Physically, it can be interpreted as follows. As the EM pulse propagates away from the antenna, the induced conduction current in conducting media will generate secondary EM pulses which scatter in both $+\hat{r}$ and $-\hat{r}$ directions. At large t , a scattered EM pulse propagates back toward $r = 0$. The Poynting vectors showing this phenomenon are approximately depicted in Figure 4.3.

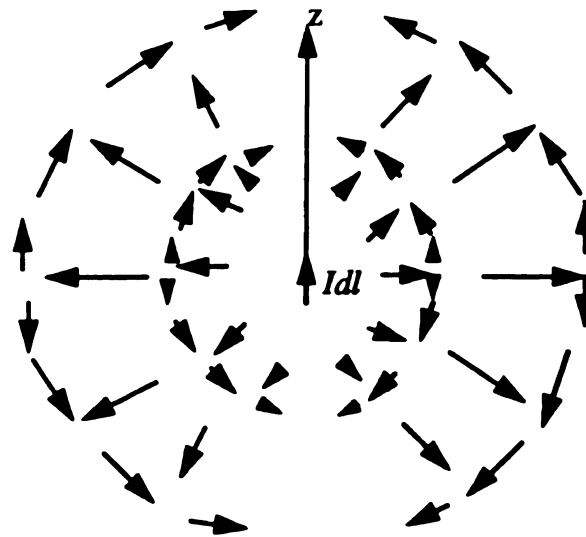
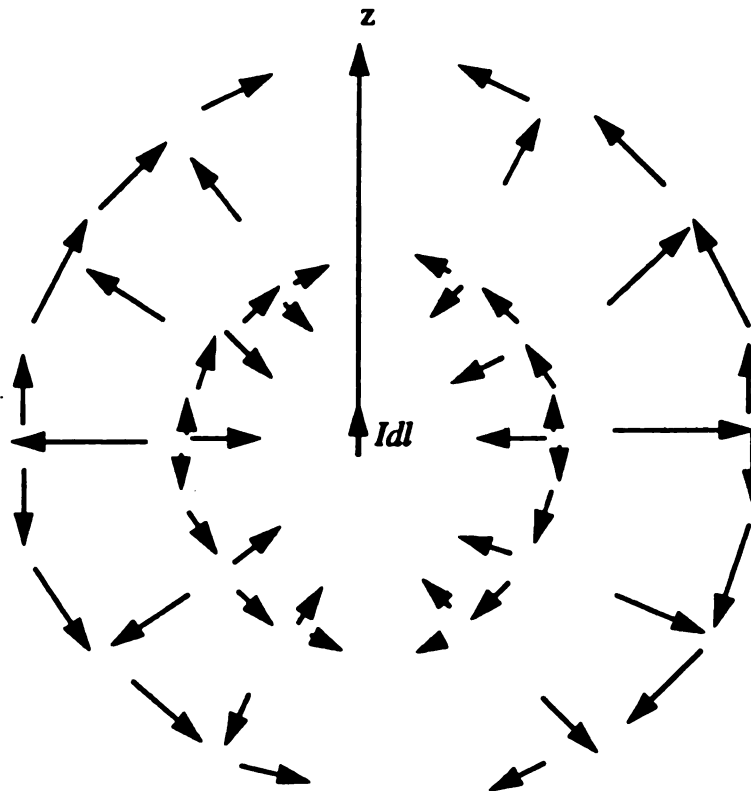
(a) t is small(b) t is large

Figure 4.3. Poynting vectors maintained by an electric dipole in seawater at two different instants: (a) t is small: (b) t is large (compare to the duration of the excitation current).

CHAPTER 5

EFFECTS OF ANTENNA SIZE ON THE PROPAGATION OF EM PULSES EXCITED IN SEAWATER

So far it has assumed the antenna to be short and of electric dipole type. Since the EM pulse excited by an antenna in seawater depends on the waveform of the antenna excitation current, it may also depend on the antenna size. It is to be determined whether an EM pulse excited by a long antenna will attenuate slower than that excited by a short dipole antenna.

To study the EM pulse excited by a long antenna in seawater, it is assumed that the antenna is a cylindrical antenna of length L located along the z -axis as shown in Figure 5.1. It is also assumed that the displacement current in seawater is negligible and only the late time waveform will be considered. To simplify the analysis, it is assumed that the antenna is excited by a step-function excitation current with an amplitude of I amperes. Under these assumptions, components of the electric field of the EM pulse in Laplace transform at the point (r, z) can be given as:

$$E_z(r, z, s) = I \int_A^B \left[P(R) + \frac{\partial^2}{\partial l \partial z} Q(R) \right] dl \quad (5.1)$$

$$E_r(r, z, s) = I \int_A^B \left[\frac{\partial^2}{\partial l \partial r} Q(R) \right] dl \quad (5.2)$$

where

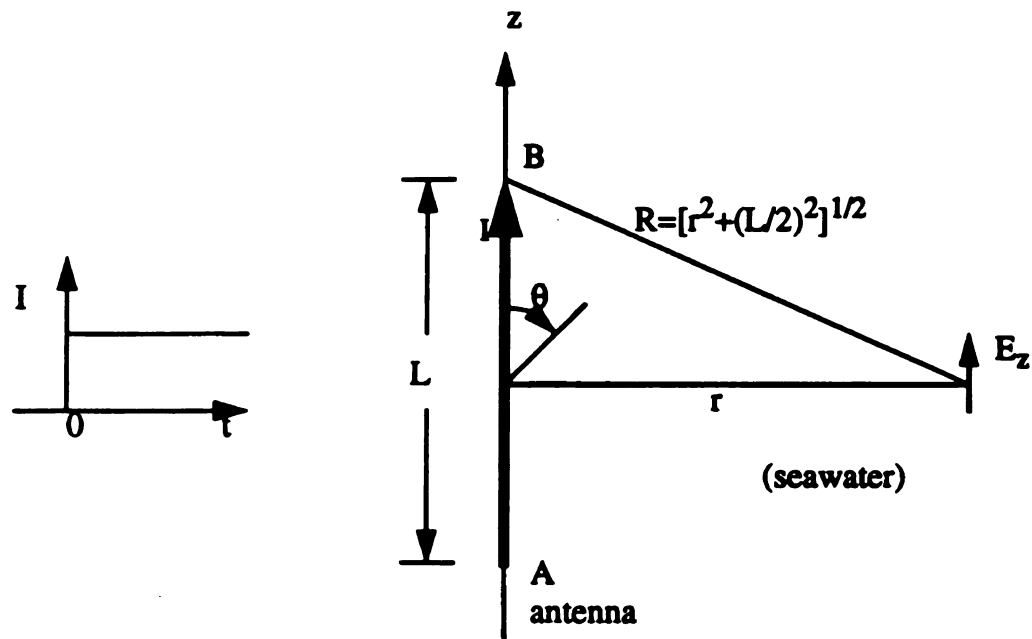


Figure 5.1. The electric field of the EM pulse generated by an antenna of finite length with a step-function excitation current in seawater.

$$P(R) = \frac{\mu}{4\pi} \frac{e^{-\sqrt{\sigma\mu s} R}}{R} \quad (5.3)$$

$$Q(R) = \frac{e^{-\sqrt{\sigma\mu s} R}}{4\pi\sigma s R} \quad (5.4)$$

and

$$R = [(l-z)^2 + r^2]^{1/2} \quad (5.5)$$

Wait [13] obtained the electric field of the EM pulse in time domain by inverting these Laplace transforms as:

$$\begin{aligned} \frac{E_z(r, z, t)}{I} = & -\frac{1}{4\pi\sigma} \{ 2\beta^2 e^{-(\beta r)^2} [\operatorname{erf}[\beta(B-z)] - \operatorname{erf}[\beta(A-z)]] \\ & + M(A) - M(B) \} u(t) \end{aligned} \quad (5.6)$$

$$\frac{E_r(r, z, t)}{I} = \frac{1}{4\pi\sigma} [N(A) - N(B)] u(t) \quad (5.7)$$

where

$$M(l) = \frac{z-l}{R^3} \{ 1 - \operatorname{erf}[\beta R(l)] + \beta R(l) \operatorname{erf}'[\beta R(l)] \} \quad (5.8)$$

$$N(l) = \frac{r}{R^3} \{ 1 - \operatorname{erf}[\beta R(l)] + \beta R(l) \operatorname{erf}'[\beta R(l)] \} \quad (5.9)$$

$$R(l) = [(l-z)^2 + r^2]^{1/2} \quad (5.10)$$

and

$$\beta^2 = \frac{\sigma\mu}{4t} \quad (5.11)$$

The electric field of the EM pulse excited by an antenna of length L with step-function excitation current in seawater at a distance r from the antenna in the broadside direction ($\theta = 90^\circ$) can be reduced from eqs. (5.6) and (5.7) with $z=0$, and $A = -B = -L/2$. This leads to:

$$\begin{aligned} \frac{E_z(r, t)}{I} = & -\frac{1}{4\pi\sigma} \left\{ 4\beta^2 e^{-(\beta r)^2} \operatorname{erf}\left(\frac{1}{2}\beta L\right) \right. \\ & \left. + \frac{L}{R^3} [1 - \operatorname{erf}(\beta R) + \beta R \operatorname{erf}'(\beta R)] \right\} u(t) \end{aligned} \quad (5.12)$$

$$\frac{E_r(r, t)}{I} = 0 \quad (5.13)$$

where

$$R = [r^2 + (L/2)^2]^{1/2} \quad (5.14)$$

In the limit of $L \rightarrow \infty$ or for an infinitely long antenna, the electric field of the EM pulse becomes:

$$\frac{E_z(r, t)}{I} = -\frac{(\beta r)^2}{\pi\sigma r^2} e^{-(\beta r)^2} u(t) \quad (5.15)$$

This expression implies that the peak of the EM pulse travels away from the antenna and attenuates with a rate of $1/r^2$.

On the other hand, in the limit of $L \rightarrow 0$ or an electric dipole case, the electric field of the excited EM pulse is given as

$$\frac{E_z(r, t)}{I} = -\frac{L}{4\pi\sigma r^3} \left\{ 1 - \operatorname{erf}(\beta r) + [\beta r + 2(\beta r)^3] \operatorname{erf}'(\beta r) \right\} u(t) \quad (5.16)$$

This result is the same as the electric field of EM pulse excited by an electric dipole with step-function excitation current in conducting media given in chapter 3 with $\theta = 90^\circ$. It also shows that the propagated EM pulse attenuates with a rate of $1/r^3$.

Based on these findings, an EM pulse excited by an antenna of finite length with a step-function excitation current should attenuate with a rate somewhere between $1/r^2$ and $1/r^3$. It is pointed out that this attenuation rate is only true when the antenna is excited by a step-function antenna current. This type of excitation current is not practical and it is adopted merely to simplify the analysis.

A numerical example is given in Figure 5.2 for the antenna is in seawater, where the normalized electric field of the excited EM pulse in the broadside direction, $-E_z/I$, is shown as a function of the distance r for three antennas: (1) an infinitely long antenna, (2) an antenna of 20 m long and (3) an antenna of 1 m long, each carrying a step-function excitation current with an amplitude of I amperes. It is shown in Figure 5.2 that the excited EM pulse attenuates like $1/r^2$ for antenna (1) and like $1/r^3$ for antenna (3). For antenna (2), the EM pulse attenuates like $1/r^2$ for $r < 10$ m and like $1/r^3$ for $r > 10$ m. The amplitude of the EM pulse for the antenna (2) for the case of $r > 10$ m is simply 20 times higher than that of antenna (3) because the former is 20 times longer than the latter. It is important to observe that by increasing the antenna length from 1 m to 20 m, it does not reduce the attenuation rate of the excited EM pulse traveling beyond the distance of $r > 10$ m. The only change is the increase in the amplitude of the EM pulse due to the increase of antenna length.

In conclusion, increasing the antenna length does not seem to reduce the attenuation rate of the excited EM pulse; it can only increase the amplitude of the EM pulse.

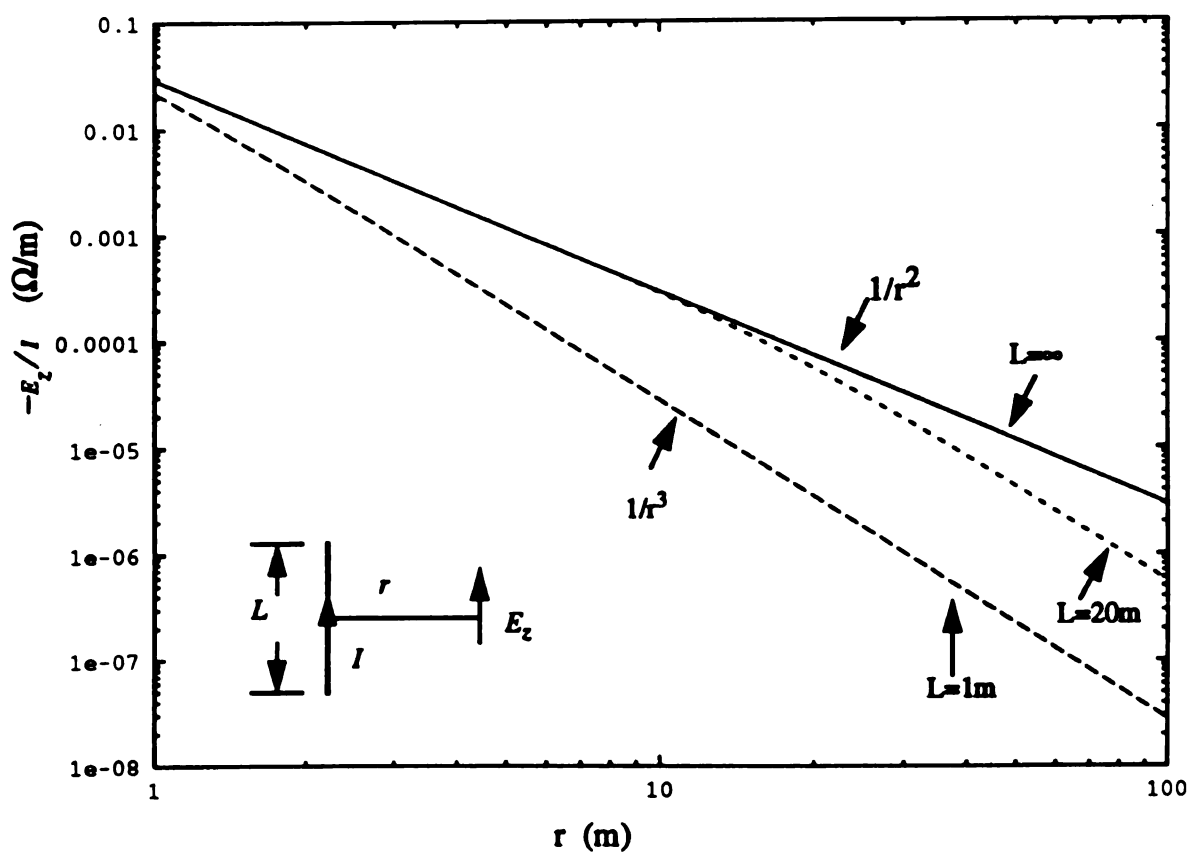


Figure 5.2. The normalized electric field of the EM pulse excited by cylindrical antennas of various lengths with a step-function excitation current in seawater in the broadside direction as functions of the distance from the antenna.

CHAPTER 6

INTERACTION OF AN EM PULSE WITH A CONDUCTING CYLINDRICAL SHELL IN SEAWATER

In this chapter, the interaction of an EM pulse with a conducting cylindrical shell in seawater will be studied. A propagating EM pulse is assumed to be incident upon an infinite conducting cylindrical shell with an outer radius b and a shell thickness $(b - a)$ as shown in Figure 6.1. We aim to determine the penetration of EM fields into the interior space of the shell.

6.1. THE EM FIELDS IN THE INTERIOR SPACE OF THE SHELL

Consider an infinitely long, conducting cylindrical shell in seawater, with the geometry as shown in Figure 6.1. A cylindrical coordinate system is chosen with the z axis coinciding with the axis of the cylindrical shell. The whole space is divided into three regions: Region 1, the interior space of the shell, is assumed to be free space. Region 2, the shell, is assumed to be a metallic conductor. Region 3, the exterior space of the shell, is seawater.

The incident EM pulse is generated by an electric dipole antenna which is located at a distance of 100 m away from the cylinder and is driven by an optimum excitation current with the total EMP energy of 1 Megajoule. The optimum excitation current is designed to yield a maximum electric field at the surface of the cylinder. For simplicity, the electric field of the incident EMP at the vicinity of the shell is assumed to be a plane wave and po-

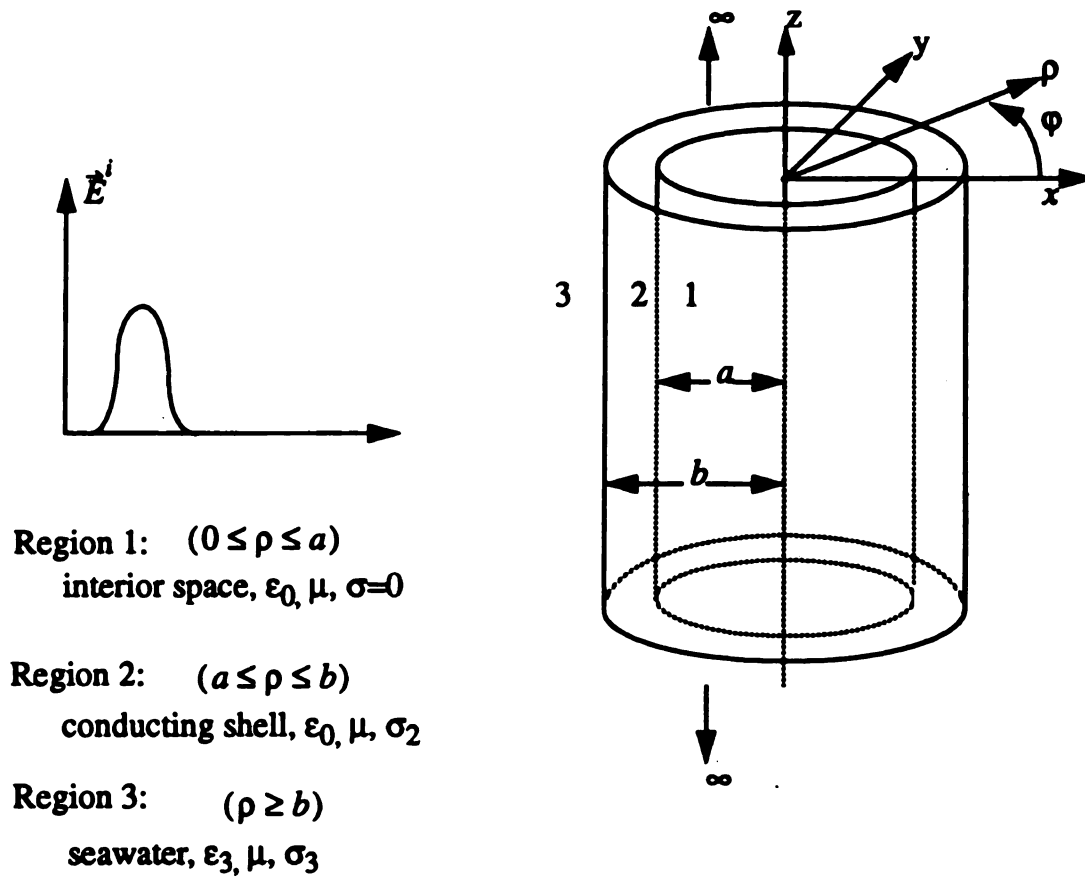


Figure 6.1. An EM pulse is incident upon a conducting, cylindrical shell in seawater.

larized in the direction of the cylinder axis. The incident plane wave strikes the shell surface at the instant of $t = 0$, and the waveform of the incident electric field can be written as:

$$\vec{E}^i(\vec{\rho}, t) = \hat{z}E[t - (x + b)/v_0]u[t - (x + b)/v_0] \quad (6.1.1)$$

where $E(t)$ is the waveform function of the incident electric field, $u(t)$ is the Heaviside step-function and v_0 is the propagating velocity of the EMP in seawater.

The Laplace transform of $\vec{E}^i(\vec{\rho}, t)$ is:

$$\vec{E}^i(\vec{\rho}, s) = \hat{z}E(s)e^{-\gamma_0(x+b)} \quad (6.1.2)$$

where $\gamma_0 = s/v_0$ and $E(s)$ is the single-sided Laplace transform of the $E(t)$.

In cylindrical coordinates, for the TM-polarized excitation, the incident electric field has only a z-component.

$$\vec{E}^i(\vec{\rho}, s) = \hat{z}E_z^i(\rho, \varphi, s) \quad (6.1.3)$$

The magnetic field is decomposed into ρ and φ components:

$$\vec{H}^i(\vec{\rho}, s) = \hat{\rho}H_\rho^i(\rho, \varphi, s) + \hat{\varphi}H_\varphi^i(\rho, \varphi, s) \quad (6.1.4)$$

The components of $\vec{H}^i$ can be related to $\vec{E}^i$ through Maxwell's equations.

Since the incident electric field has only a z-component, then the scattered electric field also has a z-component only due to the simple geometry. The scattered electric field E_z satisfies the wave equation:

$$\nabla^2 E_z - \gamma^2 E_z = 0 \quad (6.1.5)$$

where

$$\gamma^2 = s^2 \mu \epsilon + s \mu \sigma \quad (6.1.6)$$

Once E_z is known, the magnetic fields $\vec{H}$ can be obtained from Maxwell's equations as:

$$H_\rho = -\frac{1}{s\mu} \frac{1}{\rho} \frac{\partial E_z}{\partial \phi} \quad (6.1.7)$$

$$H_\phi = \frac{1}{s\mu} \frac{\partial E_z}{\partial \rho} \quad (6.1.8)$$

To facilitate the process of matching boundary conditions, the incident fields are expanded into infinite cylindrical wave functions:

$$\begin{aligned} E_z^i &= E(s) e^{-\gamma_0 b} e^{-\gamma_0 \rho \cos \phi} \\ &= E(s) e^{-\gamma_0 b} \sum_{n=0}^{\infty} (-1)^n \delta_n I_n(\gamma_0 \rho) \cos(n\phi) \end{aligned} \quad (6.1.9)$$

$$H_\phi^i = \frac{\gamma_0}{s\mu_3} E(s) e^{-\gamma_0 b} \sum_{n=0}^{\infty} (-1)^n \delta_n I_n(\gamma_0 \rho) \cos(n\phi) \quad (6.1.10)$$

To derive eqs. (6.1.9) and (6.1.10) the following relation was used:

$$e^{z \cos \theta} = \sum_{n=0}^{\infty} \delta_n I_n(z) \cos(n\theta) \quad (6.1.11)$$

where $I_n(z)$ denotes the first kind of modified Bessel function and Neumann's number is

$$\delta_n = \begin{cases} 1 & n = 0 \\ 2 & n > 0 \end{cases} \quad (6.1.12)$$

The EM fields including the scattered fields in the three regions are expressed using the following cylindrical harmonic expansions:

EM fields in region 3 ($\rho > b$):

$$E_z = E_z^i + E(s) e^{-\gamma_0 b} \sum_{n=0}^{\infty} (-1)^n \delta_n a_n(s) K_n(\gamma_3 \rho) \cos(n\varphi) \quad (6.1.13)$$

$$H_\varphi = H_\varphi^i + \frac{\gamma_3}{s\mu} E(s) e^{-\gamma_0 b} \sum_{n=0}^{\infty} (-1)^n \delta_n a_n(s) K'_n(\gamma_3 \rho) \cos(n\varphi) \quad (6.1.14)$$

where $K_n(z)$ is the second kind of modified Bessel function.

EM fields in region 2 ($a < \rho < b$):

$$E_z = E(s) e^{-\gamma_0 b} \sum_{n=0}^{\infty} (-1)^n \delta_n [b_n(s) I_n(\gamma_2 \rho) + c_n(s) K_n(\gamma_2 \rho)] \cos(n\varphi) \quad (6.1.15)$$

$$H_\varphi = \frac{\gamma_2}{s\mu} E(s) e^{-\gamma_0 b} \sum_{n=0}^{\infty} (-1)^n \delta_n [b_n(s) I'_n(\gamma_2 \rho) + c_n(s) K'_n(\gamma_2 \rho)] \cos(n\varphi) \quad (6.1.16)$$

EM fields in region 1 ($\rho < a$):

$$E_z = E(s) e^{-\gamma_0 b} \sum_{n=0}^{\infty} (-1)^n \delta_n d_n(s) I_n(\gamma_1 \rho) \cos(n\varphi) \quad (6.1.17)$$

$$H_\varphi = \frac{\gamma_1}{s\mu} E(s) e^{-\gamma_0 b} \sum_{n=0}^{\infty} (-1)^n \delta_n d_n(s) I'_n(\gamma_1 \rho) \cos(n\varphi) \quad (6.1.18)$$

Here the complex wave numbers are defined as:

$$\gamma_i = \sqrt{s^2 \mu \epsilon_i + s \mu \sigma_i} \quad i = 1, 2, 3 \quad (6.1.19)$$

The free-space permeability μ is assumed for the three regions.

Four independent boundary conditions on the two interfaces are:

$$E_z|_{\rho=a^-} = E_z|_{\rho=a^+} \quad (6.1.20)$$

$$H_{\Phi}|_{\rho=a^-} = H_{\Phi}|_{\rho=a^+} \quad (6.1.21)$$

$$E_z|_{\rho=b^-} = E_z|_{\rho=b^+} \quad (6.1.22)$$

$$H_{\Phi}|_{\rho=b^-} = H_{\Phi}|_{\rho=b^+} \quad (6.1.23)$$

From eqs. (6.1.20)-(6.1.23), four simultaneous equations for the four unknowns, $a_n(s)$, $b_n(s)$, $c_n(s)$ and $d_n(s)$, are obtained by enforcing boundary conditions and using the orthogonality of cosine functions. The matrix form of these equations is

$$\begin{bmatrix} -K_n(\gamma_3 b) & I_n(\gamma_2 b) & K_n(\gamma_2 b) & 0 \\ -\gamma_3 K'_n(\gamma_3 b) & \gamma_2 I'_n(\gamma_2 b) & \gamma_2 K'_n(\gamma_2 b) & 0 \\ 0 & I_n(\gamma_2 a) & K_n(\gamma_2 a) & -I_n(\gamma_1 a) \\ 0 & \gamma_2 I'_n(\gamma_2 a) & \gamma_2 K'_n(\gamma_2 a) & -\gamma_1 I'_n(\gamma_1 a) \end{bmatrix} \begin{bmatrix} a_n(s) \\ b_n(s) \\ c_n(s) \\ d_n(s) \end{bmatrix} = \begin{bmatrix} I_n(\gamma_0 b) \\ \gamma_0 I'_n(\gamma_0 b) \\ 0 \\ 0 \end{bmatrix} \quad (6.1.24)$$

The solutions of these linear equations give $d_n(s)$ as:

$$d_n(s) = \frac{1}{a} \frac{[\gamma_0 I'_n(\gamma_0 b) K_n(\gamma_3 b) - \gamma_3 I_n(\gamma_0 b) K'_n(\gamma_3 b)]}{D_n(s)} \quad (6.1.25)$$

where

$$\begin{aligned} D_n(s) = & [\gamma_3 I_n(\gamma_2 b) K'_n(\gamma_3 b) - \gamma_2 I'_n(\gamma_2 b) K_n(\gamma_3 b)] [\gamma_2 I_n(\gamma_1 a) K'_n(\gamma_2 a) - \gamma_1 I'_n(\gamma_1 a) K_n(\gamma_2 a)] \\ & - [\gamma_3 K_n(\gamma_2 b) K'_n(\gamma_3 b) - \gamma_2 K'_n(\gamma_2 b) K_n(\gamma_3 b)] [\gamma_2 I_n(\gamma_1 a) I'_n(\gamma_2 a) - \gamma_1 I'_n(\gamma_1 a) I_n(\gamma_2 a)] \end{aligned} \quad (6.1.26)$$

The following Wronskin

$$W\{K_n(z), I_n(z)\} = K_n(z) I'_n(z) - K'_n(z) I_n(z) = 1/z \quad (6.1.27)$$

has been used to simplify (6.1.25). Therefore, the EM fields penetrated into the interior space of the shell are given in eqs. (6.1.17), (6.1.18), (6.1.25) and (6.1.26).

6.2. EM FIELDS AT THE CYLINDER AXIS

To determine how well an EM pulse can penetrate into the conducting shell, the electric field induced by the EM pulse at the cylinder axis will be calculated.

At the cylinder axis, $\rho = 0$, since [11]

$$I_n(0) = \begin{cases} 1 & n = 0 \\ 0 & n > 0 \end{cases} \quad (6.2.1)$$

and

$$I_0(z) = I_1(z) \quad (6.2.2)$$

$$K_0(z) = -K_1(z) \quad (6.2.3)$$

then, the electric field at the cylinder axis is simplified to:

$$E_z(s) = \frac{E(s)}{a} e^{-\gamma_0 b} \frac{[\gamma_3 I_0(\gamma_0 b) K_1(\gamma_3 b) + \gamma_0 I_1(\gamma_0 b) K_0(\gamma_3 b)]}{D(s)} \quad (6.2.4)$$

where

$$\begin{aligned} D(s) = & [\gamma_3 I_0(\gamma_2 b) K_1(\gamma_3 b) + \gamma_2 I_1(\gamma_2 b) K_0(\gamma_3 b)] [\gamma_2 I_0(\gamma_1 a) K_1(\gamma_2 a) + \gamma_1 I_1(\gamma_1 a) K_0(\gamma_2 a)] \\ & + [\gamma_3 K_0(\gamma_2 b) K_1(\gamma_3 b) - \gamma_2 K_1(\gamma_2 b) K_0(\gamma_3 b)] [\gamma_2 I_0(\gamma_1 a) I_1(\gamma_2 a) - \gamma_1 I_1(\gamma_1 a) I_0(\gamma_2 a)] \end{aligned} \quad (6.2.5)$$

The time domain solution of $E_z(t)$ can be obtained by taking inverse Laplace transform of $E_z(s)$.

6.3. NUMERICAL RESULTS

Sections 6.1 and 6.2 present the derivations of the Laplace transform of the electric field at the axis of a conducting, cylindrical shell excited by an electric dipole antenna

which is located at a distance of 100 m away from the shell and is driven by an optimum excitation current with the total EMP energy of 1 Megajoule. The Laplace transform of the electric field given in (6.2.4) and (6.2.5) involves the second kind of modified Bessel functions. K_0 and K_1 go to infinity as $s \rightarrow 0$ [11]. But, it can be shown that $E_z(s)$ does not go to infinity as $s \rightarrow 0$, and

$$E_z(s)|_{s=0} = E(s)|_{s=0} \quad (6.3.1)$$

This means that when an electric field penetrates into a conducting shell, the dc-component of the electric field at the center of the shell in the direction of the cylinder axis is the same as the dc component of the incident electric field. Thus, no singularity is at $s = 0$, the FFT algorithm can be used to invert $E_z(j\omega)$ to obtain the time domain solution. In the calculation, the conducting shell is assumed to be made of iron.

Figure 6.2 shows the electric field (normalized to the incident electric field) at the central axis of the shell, with an outer radius of 27 cm (about 10.5-inch), induced by the incident EMP as a function of the shell thickness. From the figure, if the shell thickness is 1/8-inch (about 3.2 mm), the electric field at the central axis of the shell is about 1/10 of the incident electric field in intensity. The magnitude of the penetrated EMP into the shell interior is about 10% of that of the incident EMP. In Figure 6.2, it also shows that the penetration of an EMP into the shell can increase or decrease if the shell thickness is reduced or increased.

Figure 6.3 shows the electric field (normalized to the incident electric field) at the central axis of the shell, with a shell thickness of 3 mm (about 1/8-inch), induced by the incident EMP as a function of shell outer radius. From this figure, for an outer radius of 10.5-inch (about 27 cm), the induced electric field at the central axis of the shell is again about 10% of the incident electric field, the same result as shown in Figure 6.2. A somewhat

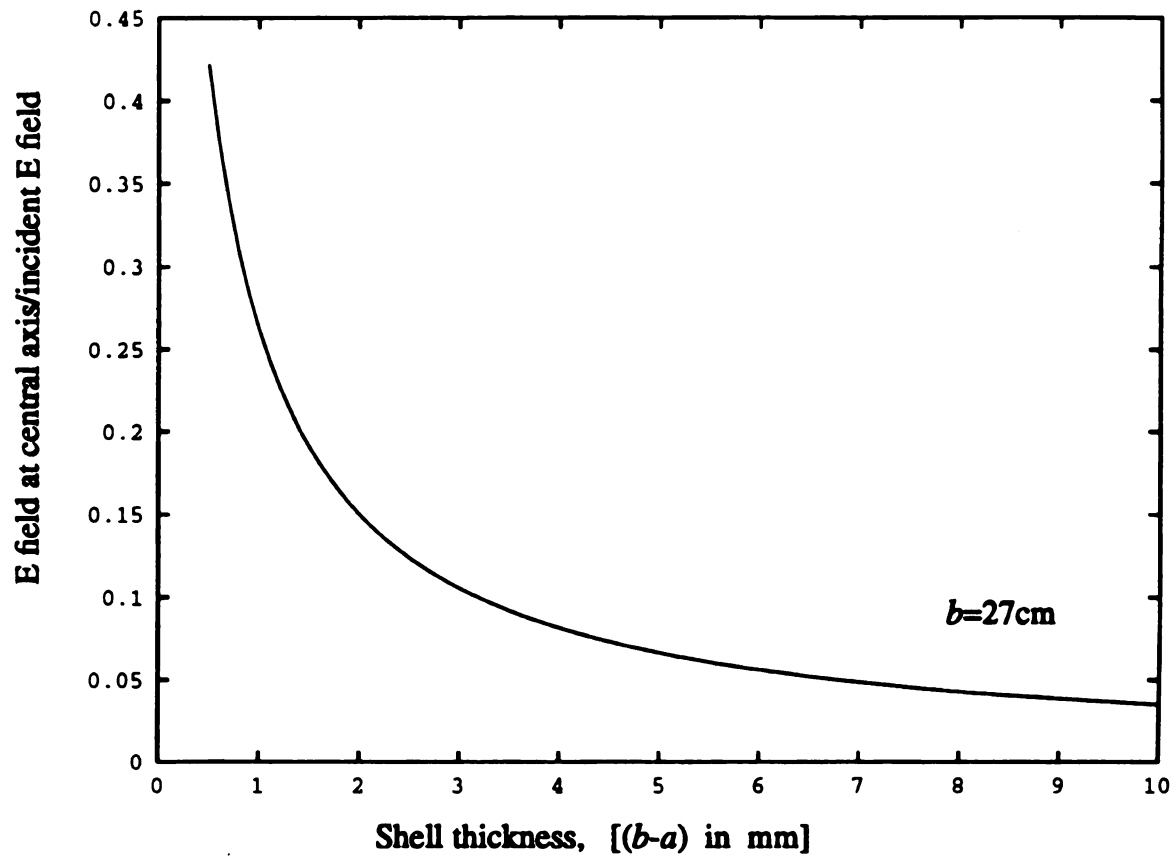


Figure 6.2. Electric field at the central axis of an infinite cylindrical conducting shell induced by an incident EMP as a function of shell thickness. The EMP has an electric field parallel to the cylindrical axis and is excited by a dipole antenna which is located 100 m away from the shell and is driven by an optimum excitation current with the total EMP energy of 1 Megajoule.

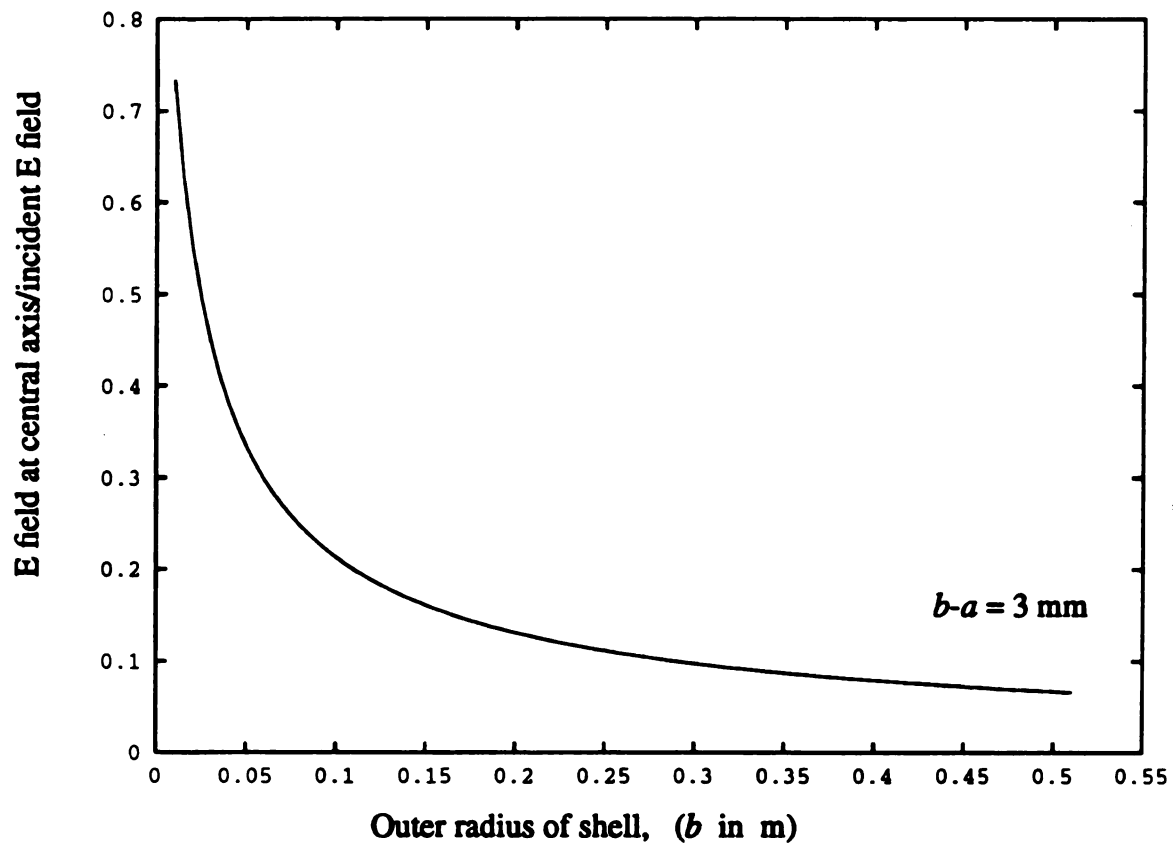


Figure 6.3. Electric field at the central axis of an infinite cylindrical conducting shell induced by an incident EMP as a function of shell outer radius. The EMP has an electric field parallel to the cylindrical axis and is excited by a dipole antenna which is located 100 m away from the shell and is driven by an optimum excitation current with the total EMP energy of 1 Megajoule.

interesting result from Figure 6.3 is that the penetration of EMP can be enhanced if the outer radius of the shell is reduced.

Figure 6.4 shows the waveforms of the electric field of incident EMP and the induced electric field at the central axis of the shell as functions of time. The dimensions of the shell are indicated in the figure. It is observed that the waveform of the incident electric field is quite different from that of the induced electric field inside the shell. This phenomenon is due to the fact that as the incident EMP penetrates through the conducting shell it suffers a strong attenuation and dispersion. Higher frequency components of the penetrated EM fields suffer stronger attenuation and dispersion.

In summary, for a rough estimate when an EMP is incident on a conducting cylindrical shell (with a diameter of 21-inch and a thickness of 1/8-inch) in seawater, the intensity of penetrated EMP at the central axis of the shell is about 10% of that of the incident EMP.

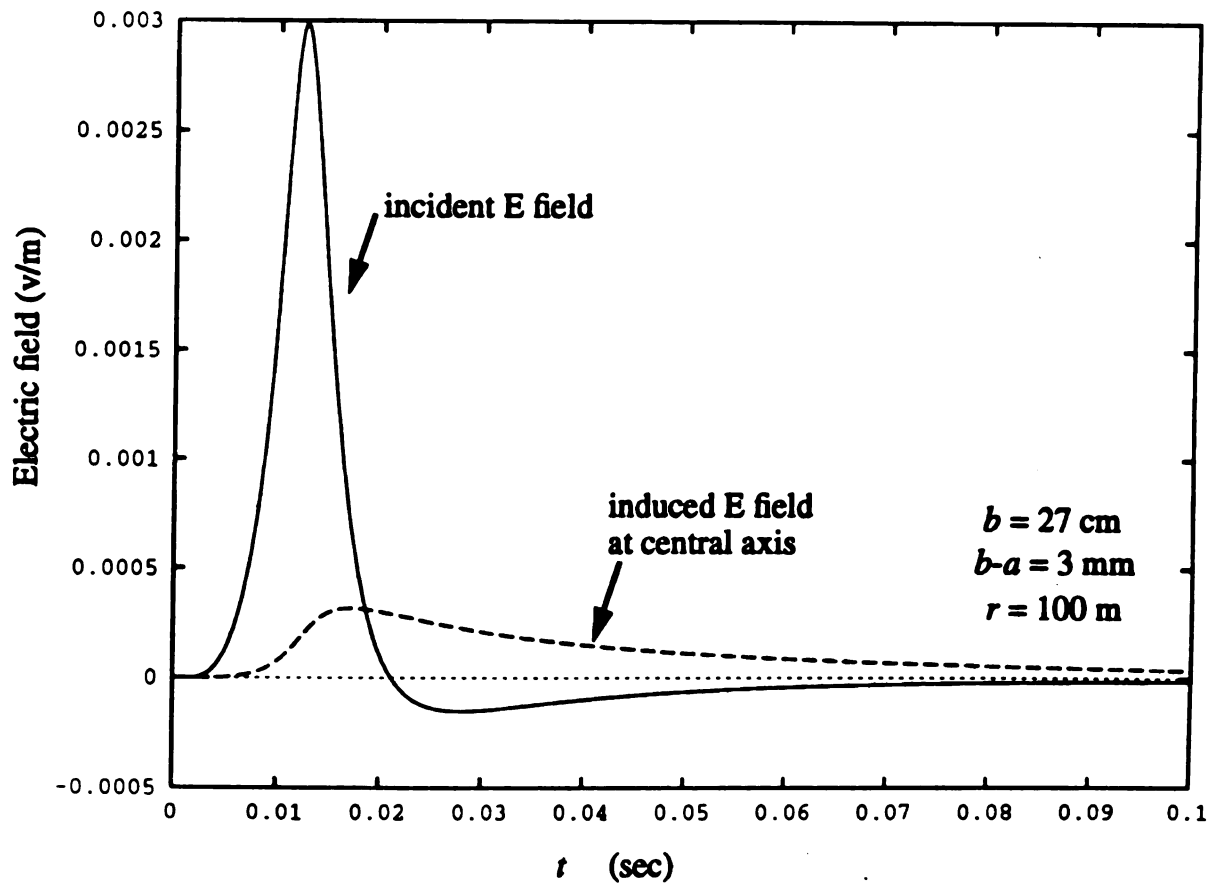


Figure 6.4. Incident electric field of an EMP and induced electric field at the central axis of an infinite cylindrical conducting shell as functions of time. The EMP is excited by a dipole antenna which is located 100 m away from the shell and is driven by an optimum excitation current with the total EMP energy of 1 Megajoule.

CHAPTER 7

CONCLUSIONS

The propagation of an EM pulse in conducting media has been studied. The exact solution for an EM pulse excited by a dipole antenna with an impulsive current has been derived. There exists an optimum waveform for the antenna current to excite an EM pulse with a maximum strength at a particular distance. It is found that a properly excited EM pulse can propagate slowly in seawater with a much lower attenuation rate than the exponential decay, and this EM pulse behaves like a "diffusion wave". The penetration of an EM pulse into a conducting shell in seawater has also been studied.

The findings of the present study may lead to some applications in an environment such as seawater or underground.

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