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THE EFFECT OF INTERFACE ON THERMO-MECHANICAL PROPERTIES OF COMPOSITES

By

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ABSTRACT

The Effect of Interface on Thermo-Mechanical Properties of Composites

By

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The effect of interface on local stress and displacement fields and thermo-mechanical properties of composites is studied. The inclusions are assumed to be uniformly but non periodically distributed in the matrix. The interface is varied theoretically by considering two models. The first one is the flexible interface model, in which the continuity of tractions at the interfaces is maintained but there exist jumps in the displacements, such that the jumps in the tangential and normal displacements are proportional to shear tractions and normal tractions, respectively. Two parameters are introduced to describe the degree of adhesion between inclusion and matrix. Specific interface condition can be simulated by proper selection of the two parameters. The second model describes the interface as a layer between the inclusion and the matrix. This layer, called interphase, has a given thickness and the thermo-mechanical properties different from those of the matrix and the inclusions. The elastic properties of the layer are assumed uniform

or variable. The perfect bond is assumed at both the matrix-interphase and interphase -inclusion interfaces.

For both of these interfacial representations, a unified approximate approach to evaluate the effective thermo-mechanical properties is used. Initially, the boundary value problem of the isolated inclusion embedded in the matrix is solved. Then, stress disturbance in the inclusion due to the presence of other inclusions is accounted for by using a successive iteration method (Mori and Wakashima, 1990) based on Mori-Tanaka theory (Mori and Tanaka, 1973). The successive iteration yields solutions that converge into closed forms under a certain condition. The analytical forms of the local stress and displacement fields and the effective properties are obtained. The latter are predicted by using the concept of the average strain in the composite. The approach is simple and can be easily extended to other boundary conditions and is valid for any shape. In the numerical results presented, the inclusions are assumed to be cylindrical or spherical in shape for simplicity. The influence of various mechanisms at the interface is studied and the results are compared with the perfect bonding case, bounds, as well as the other analytical results. It is shown that imperfect interface may have a significant effect on the local fields and the effective properties of composites.

DEDICATION

To my father, mother, brother, uncle Chose Ide

and my husband Kuailin Sun

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Finally, for all of my friends, I would like to thank them too: they helped the sun shine overhead during the last few years.

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TABLE OF CONTENTS

TABLE OF CONTENTS (CONT)

TABLE OF CONTENTS (CONT)

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (CONT)

PAGE

LIST OF FIGURES (CONT)

NOMENCLATURE

D - domain of composite

Ω - domain of inclusion

f - volume fraction of inclusions

ℓ - volume fraction of layers

a - radius of inclusion

b - outside radius of layer

d - outside radius of matrix

G - shear modulus

K - bulk modulus

E - Young's modulus

k - tangential bonding parameter

g - normal bonding parameter

n_i - normal vector

t_i - traction vector

u_i - displacement vector

$C_{ijk\ell}$ - stiffness tensor

$S_{ijk\ell}$ - compliance tensor

$T_{ijk\ell}$ - Eshelby's tensor

ν - Poisson's ratio

W - elastic strain energy

ϵ_{ij} - elastic strain

σ_{ij} - stress

Γ - G_f/G_m

Subscripts

c - composite

f - inclusion

ℓ - interphase (layer)

m - matrix

n - normal

t - tangential

0,1,2 .. - numbers of iteration

superscripts

c - composite

f - inclusion

ℓ - interphase (layer)

m - matrix

equiv - equivalent

s - phase (s - f, ℓ, m)

∞ - isolated inclusion

(0),(1),(2) .. - applied loading

CHAPTER 1

INTRODUCTION

The effectiveness of the bond between matrix and inclusion in transferring the load across the interface is one of the principal factors affecting the mechanical response of composite materials. Many theories have been developed to predict the mechanical behavior of composite materials, but most of them assume perfect bonding at the matrix-inclusion interface. However, experimental results clearly indicate that a more complex state exists at the interface between the constituents (Drzal, 1983, 1986, 1987). The imperfect contact due to the poor chemical bonding, the presence of microcracks due to the thermal loading, and other, may more accurately describe the condition at the interface. In order to increase understanding and provide guidance for material development, the mechanical models to describe interfacial characteristics need to be established. Since a precise description of the interface is complicated, in order to include its effect in the modeling of composite, it is necessary to introduce simplified interfacial models, which simulate the actual behavior.

In this dissertation, two terms will be used to describe the boundary between inclusions and the matrix: interphase and interface.

The "interphase" is a region in which the inclusion and matrix phases are chemically and/or mechanically combined. The interphase may be a diffusion zone, a nucleation zone, a chemical reaction zone, etc., or any combination of the above. An "interface" is a two-dimensional boundary separating distinct phases, such as inclusion, matrix, interphase, coating, etc. (Swain et al., 1989).

Since the control of interface behavior has become a key factor in developing composite materials, the understanding of its role is highly desirable. The influence of interfaces and interphases on the thermal and mechanical behavior of composite materials has been widely discussed in the literature, particularly in the last few years. Several books have been devoted to the subject. However, the work in this area is far from complete.

One of the analytical models of interface that appears in the literature is so called flexible interface model. In this model, the debonding between the constituents is simulated by a very thin ficticious layer having a spring-like behavior. At the interface, continuity of tractions is maintained but there exist jumps in the displacement, such that the jumps in the tangential and normal displacements are proportional to shear tractions and normal tractions, respectively. Consequently, two parameters are introduced that determine the degree of bonding between inclusion and matrix. Specific interface condition can be simulated by proper selection of the two parameters. The infinite values of the parameters imply vanishing of displacement jumps and therefore perfect bonding case; the zero values of the tangential debonding parameters imply vanishing of shear tractions at the interface and therefore pure sliding case; the zero values of the normal and tangential debonding parameters imply vanishing of tractions at the interface and therefore debonding case; any finite positive values of the interface parameters define the imperfect interface. This flexible interfacial model was employed by Jones and Whitter (1967), Mal and Bose (1975), Lene and Leguillon (1982), Benveniste (1984, 1985), Aboudi (1987), Steif and Hoysan (1986, 1987), Achenbach and Zhu (1989), Jasiuk and Tong (1989), Jasiuk et al. (1989), and Hashin (1990b), among others.

In an alternate model, the interface is described as a layer between the inclusion and the matrix. This layer, called interphase, has a given thickness and the interphase properties different from those of the matrix and the inclusion. The perfect bonding is assumed at both matrix-layer and layer-inclusion interfaces. The extreme case of perfect bonding at inclusions/matrix interface is obtained by decreasing the interphase thickness to zero, while the case of complete debonding is obtained by an interphase of infinitesimal thickness and material properties that approach zero. The moduli of the interphase may simultaneously represent the degree of bonding and the material properties of the region. Owing to the lack of definitive data, the interphase zone is often treated as a phase with uniform material properties which are different from those in the bulk matrix. Such a model might accurately decribe systems in which finishes or coatings are employed. The interphase model which has constant properties was used by Broutman and Agarwal (1974), Maurer et al. (1986, 1988), and Pagano and Tandon (1988), Jasiuk and Tong (1989), Benveniste et al. (1989), Chen et al. (1990), Tong and Jasiuk (1990 a, 1990 b), Maurer (1990), Sullvian and Hashin (1990), and others. However, in many composite systems, the interphase may have a gradient in resin properties. The composites with interphase which has property variation ware studied by Theocharis et al. (1985), Theocharis (1986), Sideridis (1988), Papanicolaou et al. (1989), Sottos et al. (1989), and Jayaraman et al. (1990) among others.

The primary motivation of the present work is to predict the thermoelastic properties of composites in order to increase the understanding how various interface conditions can influence the thermal and mechanical behavior of composite materials. The effect of interface is investigated by considering the two above mentioned models, i.e., the flexible interface model and the interphase model. The present study supplements the previous results in this area. For both of these interfacial representations, a unified approximate approach to evaluate the effective thermo-mechanical properties is used. Initially, the boundary value problem of the isolated inclusion embedded in the matrix is solved. The stress disturbance in the inclusion due to the presence of other inclusions is accounted for by using a successive iteration method (Mori and Wakashima, 1990) based on Mori-Tanaka theory (Mori and Tanaka, 1973). The successive iteration yields solutions that converge into closed forms under a certain condition. The analytical forms of the local stress and displacement fields and the effective properties are obtained, the latter are predicted by using the concept of the average strain in the composite. The approach is simple and can be easily extended to other boundary conditions and is valid for any shape. The present derivation applies for a composite consisting of uniformly but non-periodically distributed inclusions (with overlapping not allowed). In the numerical results presented, the inclusions are assumed to be cylindrical or spherical in shape for simplicity. The influence of various mechanisms at the interface is studied and the results are compared with the perfect bonding case, bounds, as well as the other analytical results. It is shown that imperfect interface may have a significant effect on the local fields and the effective properties of the composites.

CHAPTER 2

BACKGROUND: EFFECTIVE MEDIUM THEORIES

2.1 REFERENCES TO THE MAIN THEORETICAL MODELS

In the determination of effective properties of heterogeneous materials, a fundamental problem is the phase interaction. The problem of a single ellipsoidal inclusion embedded in an infinite body is easily solved by the use of Eshelby's (1957) equivalent inclusion method. The case of two ellipsoidal inclusions embedded in an infinite body was solved by Moschovidis and Mura (1975). However, there are seldom only one or two inclusions in a matrix. The case of finite concentration of inclusions is an extremely difficult problem due to the complex spatial distribution of the inclusions. It is very hard to find the exact solution for the stress field since the stresses will differ for every inclusion. Therefore, several simplified micromechanics models have been developed to account for the interaction between inclusions at high concentrations (Hashin, 1983). Among these are self-consistent scheme SCS (Budiansky, 1965; Hill, 1965), generalized self-consistent scheme GSCS or three phase model (Kerner, 1956; Christensen and Lo, 1979), composite spheres and cylinders model (Hashin, 1962; Hashin and Rosen,1964, Hashin, 1965), the differential scheme (McLaughlin, 1977) and the Mori-Tanaka method (Mori and Tanaka, 1973; Wakashima et al., 1974; Benveniste, 1987; Mori and Wakashima, 1990). Although there are many other micromechanics models to determine the effective properties of the composites than the ones listed, they usually are either of numerical nature involving series or finite element solutions, or they are of an empirical nature, or finally they involve grossly oversimplifying assumptions.

BOUNDS

Bounds for the effective elastic moduli of the composites are obtained by using variational methods. The minimum complementary energy theorem yields the lower bounds, while the minimum potential energy theorem yields the upper bounds. Method suitable for arbitrary phase geometry was given by Hashin and Shtrikman (1963), and it was generalized by Hill (1963), Walpole (1966), and others (Willis, 1977; Kroner, 1977). Bounds for the effective elastic moduli of particulate composites were given by Hashin (1962), while bounds for fiber composites with arbitrary transverse phase geometry were given by Hill (1964) and Hashin (1965). It is found that for the composites reinforced with spherical particles, the bounds on the bulk modulus coincide, so that the exact result for bulk modulus was obtained. In contrast to the situation with the bulk modulus, the bounds on the shear modulus do not coincide. Same phenomenon was observed for the fiber composite having circular cross sections, where the exact solutions for four of the elastic constants and the bounds for the fifth (transverse shear modulus) were obtained (Hashin, 1983).

2.2 DESCRIPTION OF THE MODELS

SELF-CONSISTENT SCHEME

In the most commonly used version of the self-consistent scheme (SCS) (Budiansky, 1965; Hill, 1965) it is assumed that an inclusion is embedded in a homogeneous body which has the unknown properties of the effective medium, Figure 2.1. This defines a boundary value problem which can be solved for an arbitrary ellipsoidal inclusion. The solution for an isolated ellipsoidal inclusion was given by Eshelby (1957). His most valuable result is that the strain and stress fields are uniform for the interior points (points inside the inclusion). The method has been extended to randomly oriented ellipsoidal inclusions by Wu (1966) and Walpole (1969) to investigate the effect of inclusion shape on the effective properties of the composites. They both found that for stiffer inclusions, the disk shape inclusions give most significant increase in the elastic modulus. Hill showed that the expressions derived by this method give reliable values at low inclusion volume fractions, reasonable values at intermediate volume fractions, and unreliable values at high ones when applied to composite materials. When the reinforcing particles are much stiffer than matrix, this method overestimates the effective moduli, while for particles much more compliant than the matrix, the effective moduli are underestimated.

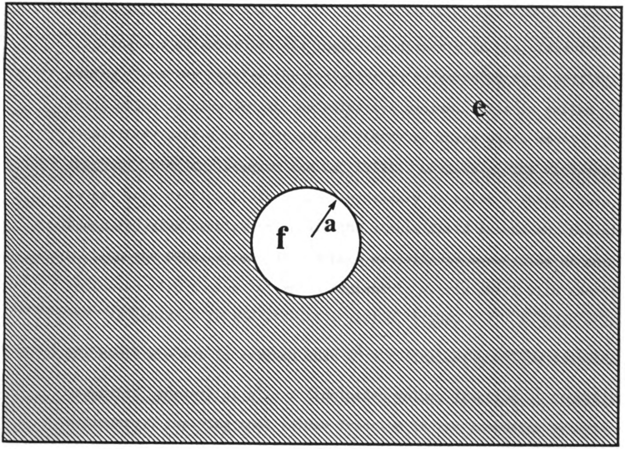

Figure 2.1 Self-consistent scheme

GENERALIZED SELF-CONSISTENT SCHEME

Instead of embedding the inclusion directly in the effective medium, one may imagine the inclusion to be embedded in a matrix shell which is embedded in the effective medium. This is called generalized self-consistent scheme (GSCS) or three phase model (Kerner, 1956; Christensen and Lo, 1979). Obviously, the mathematics is now more difficult since it is necessary to solve a three-phase boundary value problem to obtain the stress field around an inclusion.

In the GSCS, a composite sphere or cylinder consisting of an inclusion with radius "a" and a concentric matrix shell with radius "b", is embedded in the effective medium, Figure 2.2. In most works, the ratio $\eta = a/b$ is assumed that $\eta^3 = f$ (inclusion volume fraction of composite spheres) or $\eta^2 = f$ (inclusion volume fraction of composite cylinders), implying that volume fraction in the composite spheres or cylinders is the same as in the composite.

Using GSCS, Kerner (1956) obtained the exact solution for the effective bulk modulus of the composites with spherical inclusions. The solution for the effective shear modulus was given by Christensen and Lo (1979). The result for the effective shear modulus lies within Hashin-Shtrikman's (1963) upper and lower bounds.

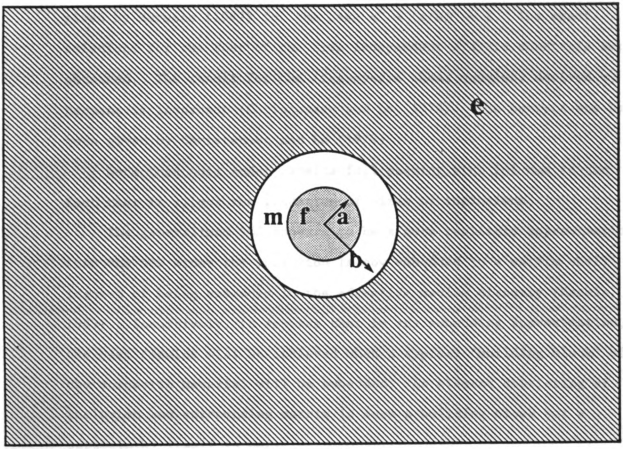

Figure 2.2 Generalized self-consistent scheme

The result for the composites with cylindrical inclusions was given by Hermans (1967) for the case $\eta^2 = (a/b)^2 = f$. The exact solutions for the four of the five independent elastic constants were obtained. The result for the (transverse shear modulus) derived by Hermans is incorrect (Christensen and Lo, 1979). The correct one has been given by Christensen and Lo (1979).

Note that this model permits full packing with $f \to 1$ due to the fact that it allows the gradation of sizes of inclusions.

The generalized SCS appears to be a more realistic approximation than the SCS since the inclusion is now embedded in a matrix shell instead of being embedded in the effective medium directly (Hashin, 1983). Intuitively, it appears that in any embedding approximation, the best results will be achieved when a typical "building block" of the composite material will be embedded. An element consisting of inclusion and surrounding matrix is such a building block but a particle by itself is not.

COMPOSITE SPHERES AND CYLINDERS MODEL

The composite spheres or cylinders model assumes gradation of sizes of spherical or cylindrical inclusions, such that a volume-filling configuration is obtained. Each individual composite sphere or cylinder has the same ratio of radii, a/b as seen in Figure 2.3. By using this model and minimum theorems of elasticity, the bounds for the effective elastic moduli of particulate (Hashin, 1962) and fiber composites (Hill, 1964; Hashin, 1965) were derived.

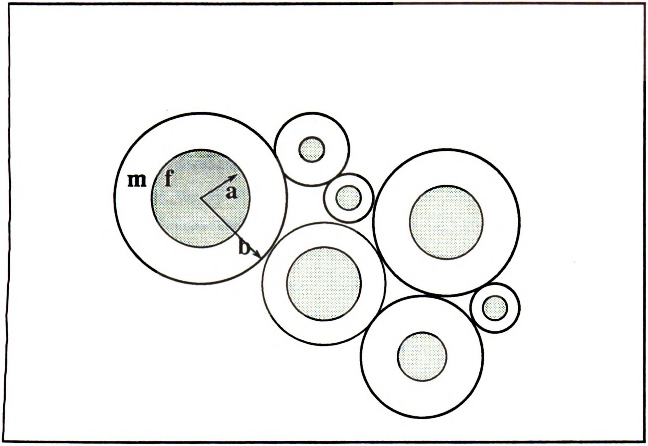

Figure 2.3 Composite spheres and cylinders model

It was found that for the composites reinforced with spherical particles, the bounds on the bulk modulus coincide and the results give the exact solution. In contrast to the situation with the bulk modulus, the bounds on the shear modulus do not coincide. The reason why the bounds do not coincide in the shear modulus problem is because a composite sphere cannot be simultaneously subjected to pure shear displacement and traction boundary conditions. The composite sphere model for shearing displacement boundary conditions leads to an upper bound for the shear modulus while the solution for shearing traction boundary conditions leads to a lower bound. The greater the disparity of the stiffness between the matrix and the inclusions, the greater is the gap between the bounds. Same phenomenon is observed for the composite cylinders model where the exact solutions for the four of the five independent elastic constants are obtained while the bounds of transverse shear modulus do not coincide, for the same reason as explained above. The solution for displacement boundary conditions leads to an upper bound for the shear modulus, while the solution for shearing traction boundary conditions leads to a lower bound.

DIFFERENTIAL SCHEME

The starting point of the differential scheme is the well known dilute suspension result for the effective modulus of a composite containing non-interacting inclusions. It is assumed that the addition of a small amount of particles to a composite will increase the effective modulus by a dilute concentration type expression with current effective modulus replacing the matrix modulus. The basic concept of the method is to view the composite as a sequence of dilute suspensions. The first inclusions which are added to the matrix are used to calculate the effective properties from dilute solutions. Next, that suspension is viewed as a homogeneous medium of those properties, to which a new increment of inclusions is added under assumed dilute conditions. The new effective properties are obtained from suitably modified form of dilute solutions. The process is continued up to the condition of full packing of the inclusion phase, i.e., $f \to 1$. Mathematically the process involves going to the limit where the increments of added inclusions become infinitesimal and a differential form results.

MORI-TANAKA THEORY

The concept of an average field (Mori and Tanaka, 1973) in inclusions and the surrounding matrix is another model to include the interaction between the inclusions. It is summarized here for completeness.

Let us denote the domain of the composite by D and the inclusions by Ω. $D\text{-}\Omega$ will denote the matrix. Assume there are many inclusions in the matrix. When $\sigma_{ij}^{(0)}$ is applied at infinity, the average total stress in the matrix is $\langle\sigma_{ij}^{(0)} + \sigma_{ij}\rangle_{D-\Omega}$, where σ_{ij} is the stress disturbance. The average stress in the inclusions is calculated as

$$\langle\sigma_{ij}^{(0)} + \sigma_{ij}\rangle_{\Omega} = \langle \sigma_{ij}^{(0)} + \sigma_{ij}\rangle_{D-\Omega} + \langle\sigma_{ij}^{\infty}\rangle_{\Omega} \tag{2.1}$$

where $<\sigma_{ij}^{\infty}>_{\Omega}$ is the average stress disturbance in a single inclusion present in an infinite medium. σ_{ij}^{∞} for perfect bonding case was obtained by Eshelby (1957). The above equation is obtained by adding an isolated inclusion into the matrix which is subjected to the applied stress $<\sigma_{ij}^{(0)} + \sigma_{ij}>_{D-\Omega}$ caused by the rest of the inclusions and the boundary surface of body D. The similar relation involving the stress disturbance only is

$$<\sigma_{ij}>_{\Omega} = <\sigma_{ij}>_{D-\Omega} + <\sigma_{ij}^{\infty}>_{\Omega} \tag{2.2}$$

Since the average stress disturbance must vanish,

$$f <\sigma_{ij}>_{\Omega} + (1-f) <\sigma_{ij}>_{D-\Omega} = 0 \tag{2.3}$$

where $f = \frac{\Omega}{D}$ is the volume fraction of inclusions. From (2.2) and (2.3), we obtain the average stress disturbance in the matrix

$$<\sigma_{ij}>_{D-\Omega} = - f <\sigma_{ij}^{\infty}>_{\Omega} \tag{2.4}$$

and

$$<\sigma_{ij}>_{\Omega} = (1 - f) <\sigma_{ij}^{\infty}>_{\Omega} \tag{2.5}$$

The work of Mori-Tanaka (1973) originally concerned with calculating the average internal stress in the matrix of a material containing precipitates with eigenstrains. It is exact for an elastically homogeneous body. It is also a good approximation when the volume fraction of inclusion is small. However, when the method is extended to the large volume fraction of inclusions, the basic equation to determine the elastic state of an inclusion must be modified from the original form given by Eshelby (1957). The modification is to include the interaction between inclusions. The modification was first given by Wakashima et al. (1974) who analyzed thermal expansion of composites. They used Eshelby's solution (1957) of an ellipsoidal inclusion and Mori-Tanaka's concept (1973) of average stress in the matrix. Other authors who employed this method are Taya and Chou (1981), Taya and Mura (1981), Weng (1984), Takao and Taya (1985), Tandon and Weng (1986a, 1986b), Takahashi and Chou (1988), Zhao et al. (1988), Luo and Weng (1987, 1989), Norris (1989), and others.

Following Wakashima et al. (1974), when an inhomogeneity-bearing body is subjected to a uniform change of temperature, the total strain in the inclusion $\langle\epsilon_{ij}\rangle_\Omega$ is taken as

$$\langle\epsilon_{ij}\rangle_\Omega = (1-f)T_{ijk\ell}\epsilon^*_{k\ell} + f\epsilon^*_{k\ell} \tag{2.6}$$

where $T_{ijk\ell}$ is Eshelby's tensor (Eshelby, 1957), which depends on the aspect ratios of the ellipsoidal inclusion and Poisson's ratio. f is the volume fraction of the inclusions and ϵ^*_{ij} is the equivalent eigenstrain, which can be found by using Eshelby's equivalent inclusion method.

The equivalent inclusion method, proposed by Eshelby (1957), states that the stress disturbance of an applied stress caused by an inhomogeneity Ω (a sub-domain which has different elastic moduli than those of the matrix) can be simulated by the stress field caused by an inclusion having the same elastic moduli as those of the matrix with a suitablly chosed eigenstrain ϵ^{*}_{ij} (stress-free strain, phase transformation strain or inelastic strain). The equivalency condition to determine the stress and strain disturbances in the inhomogeneities is given as

$$\sigma_{ij} = C^{m}_{ijk\ell}\{ (1-f)T_{k\ell pq}\epsilon^{*}_{pq} + f\epsilon^{*}_{ij} - \epsilon^{*}_{ij} \}$$

$$= C^{f}_{ijk\ell}\{ (1-f)T_{k\ell pq}\epsilon^{*}_{pq} + f\epsilon^{*}_{ij} \} \tag{2.7}$$

where $C^{m}_{ijk\ell}$ and $C^{f}_{ijk\ell}$ are the elastic stiffness tensors of the matrix and inclusion, respectively.

In a recent paper, Benveniste (1987) reexamined the average field method (Mori and Tanaka, 1973) and applied it to composites with large volume fraction of inclusions. By introducing the "concentration-factor" tensors, he presented the formulation to calculate the average elastic moduli of composites. The advantage of this approach is that the approximation affects only the boundary conditions of the modified dilute problem, but the problem itself can be solved exactly for the stress field in the inclusion, matrix and the respective interfaces, Figure 2.4. The result obtained is consistent with the one obtained by Weng (1984) where the effective behavior of multiphase composite has been studied by means of Mori-Tanaka's method in the framework of equivalent inclusion and eigenstrain concepts. The works following this approach are due to: Norris (1989), Benveniste et al. (1989), and Chen et al. (1990), among others.

Very recently another approach has been offered. Mori and Wakashima (1990) have examined the elastic state of an inhomogeneous body by introducing a successive iteration method. The method involves infinite series which converge to close forms, from which the pertinent quantities such as the average stresses in the inclusions and the matrix, the average stress disturbances in the inclusions, and the equivalent eignstrains in the inclusions are obtained. The works which follow this approach are: Tong and Jasiuk (1990a, 1990b), Shibata et al. (1990), and others.

In contrast to the approach of Wakashima et al. (1974), which is suitable for perfect bonding case only, both Benveniste's approach (1987) and Mori and Wakashima's approach (1990) are applicable to the imperfect bonding case also. The results obtained from the succesive iteration method (Mori and Wakashima, 1990) have been found to coincide with those given by Benveniste (1987) as indicated in the paper of Mori and Wakashima (1990), however, the approach is different.

It was showed by Weng (1984) that the Mori-Tanaka method yields consistent results when applied either with displacement or traction boundary conditions. Weng (1984) also indicated that the Mori-Tanaka method with spherical inclusions gives the Hashin-Shtrikman lower (upper) bound for the bulk and shear moduli when the inclusions are harder (softer). Zhao et al. (1988) proved that in the case of composite reinforced with cylindrical inclusions of circular cross-section, the five effective constants of composite derived from the Mori-Tanaka method coincide with Hill's (1964) and Hashin's (Hill-Hashin bound, arbitrary transverse phase geometry, 1965) lower bounds if the inclusions are the harder phase, and coincide with their upper bounds if the inclusions are the softer phase. Norris (1985) pointed out that randomly-oriented disk-shaped particles of the harder (softer) phase yield the Hashin-Shtrikman lower (upper) bounds. Benveniste (1987) proved that the bulk and shear moduli predicted by Mori-Tanaka for a two-phase composite with randomly-oriented ellipsoidal particles lie within the Hashin-Shtrikman bounds.

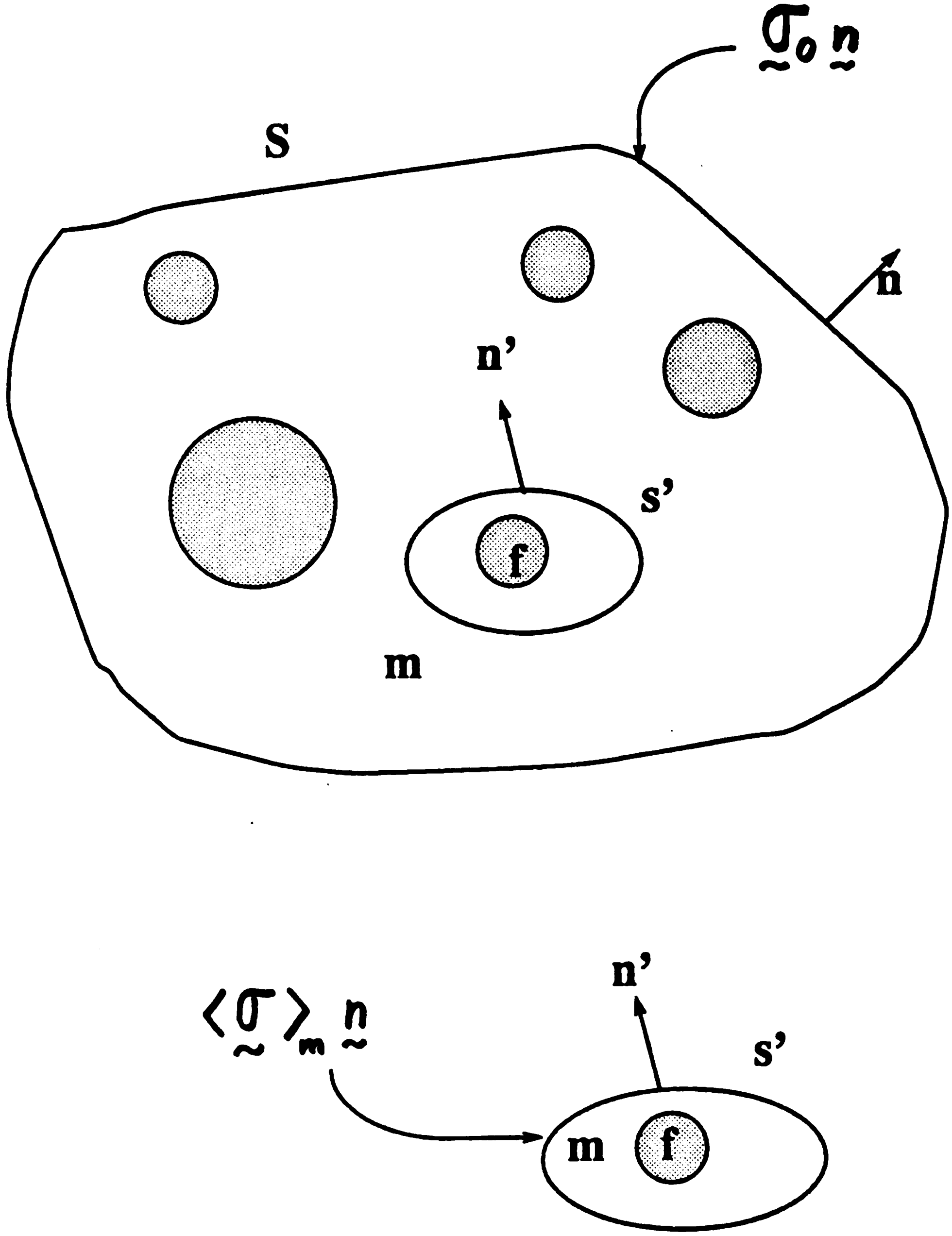

Figure 2.4 MORI-TANAKA THEORY

2.3 COMPARISON OF THE MODELS

Comparing the theoretical models mentioned above, several interesting observations have been found.

First, it is observed that, in all cases, all models recover dilute behavior (Christensen, 1990).

Secondly, it is also interesting to note that in all cases where the composite spheres and cylinders models yield closed form results, both GSCS model and Mori-Tanaka method give precisely the same results. As it was discussed by Christensen (1979), the physical meaning behind this is that in determining the properties of the equivalent homogeneous medium, the criterion to be used in composite spheres and composite cylinders models is that the repeating cells be replaced by the equivalent homogeneous material without changing the conditions of average stress and average strain. In considering an infinite medium of the composite spheres or composite cylinders models, one could replace all but one of the cells by the equivalent homogeneous medium to arrive at the GSCS model. Since the Mori-Tanaka theory is based on the average stress or strain in the matrix, it also answered the question of why it gives the same result for bulk modulus as the composite spheres model and all but transverse shear modulus as composite cylinders model.

The composite cylinders (unidirectional composite) or spheres (particulate composite) model does not yield a solution for the transverse shear modulus or shear modulus, whereas both the generalized self-consistent scheme and the Mori-Tanaka method do give the solution.

Another interesting phenomenon is found by Jasiuk et al. (1990) in their study of the elastic mouli of composites with rigid sliding inclusions. They indicated that when the flexible interface is considered, the self-consistent approach and differential scheme yield a solution for the bulk modulus which depends on the tangential sliding parameter $\bar{k}$. The other models such as composite spheres or cylinders model, generalized self-consistent model, and Mori-Tanaka method would give the result, which does not depent on $\bar{k}$, since they assume that symmetry is maintained for whole range of f.

Note that all of the models except the self-consistent scheme allow the volume fraction of inclusions up to $f \rightarrow 1$ and therefore require a wide distribution of inclusion size. As it was indicated by Christensen (1979), the self-consistent scheme is suitable for polycrystalline materials but not for composites. When applied to multi-phase media, it does not always cover the full range of volume fraction up to $f \rightarrow 1$. This is true particularly when there is a large mismatch in properties of the phases.

The differential scheme is not described by a single physical model, but rather by a hierarchy of models. As discussed by Norris (1985), the differential scheme involves an initial dilute suspension which is then "homogenized", after which a new dilute suspension is formed by inserting inclusions which are at least an order of magnitude larger than the initial inclusions. Then this second stage dilute suspension is homogenized and a third one is formed by inserting yet larger particles. This process is repeated until a limit is approached. As it was mentioned by Christensen (1990), this sequence will provide results very different from a single model involving very tightly packed inclusions. such as the generalized self-consistent scheme, the composite spheres or cylinders model, and the Mori-Takana method.

Comparing the Mori-Tanaka theory with the generalized self-consistent scheme, we found that the latter is simple in concept but complex in execution. When this model is used, it is necessary to solve a three-phase boundary value problem (for the flexible interface model) and four-phase boundary value problem (for the interphase model) to obtain the stresses in the composites. In contrast, the Mori-Tanaka method is mathematically more simple. The advantage of this approach is that the local stresses and displacements in a composite can be evaluated by using the solution of a single inclusion embedded in an infinite matrix, and inclusion interaction can be taken into account by using a successive iteration method (Mori and Wakashima, 1990) based on the average field theory (Mori and Tanaka, 1973). This method can treat any boundary conditions at inclusion-matrix interface provided that the solution for an isolated inclusion is known. This method is truly versatile, since hard, soft, or void inclusions of any geometrical shape can all be treated in a unified fashion. This is a reason why in this dissertation, the Mori-Tanaka method is used. The focus of this work is the study of the effect of interface on the effect thermo-mechanical properties of the composites.

CHAPTER 3

EFFECTIVE ELASTIC MODULI

3.1 DESCRIPTION OF THE METHOD

In this chapter, the method to predict the local stress and displacement fields and the effective elastic moduli of the composites reinforced with imperfectly bonded inclusions is described. First, the elastic field of a single inclusion embedded in a matrix and subjected to a uniform stress state at the remote boundaries of the matrix is solved by using linear elasticity theory. Next, the successive iteration method (Mori and Wakashima 1990) based on the average field theory (Mori and Tanaka 1973) is used to account for the interaction between the inclusions. This method, decribed by Mori and Wakashima (1990), is modified here to account for the effect of interface. Then, the overall elastic moduli are evaluated by equating the average strain in the effective medium (composite) and the average strain in the material with the inclusions subjected to equivalent eigenstrain. The advantage of this approach is that the local fields in the inclusion, the interphase and the adjacent volume of the matrix can be evaluated by using the solution of isolated inclusion.

The present method can be clearly demonstrated by considering a composite reinforced with spherical inclusions and subjected to an applied shear stress $\sigma_{12}^{(0)}$ at infinity. The case of inclusions having the shape of circular cylinder is treated in Appendices A and B.

3.2 ISOLATED INCLUSION SOLUTION

When an isolated inhomogeneity Ω in domain D is subjected to an applied stress $\sigma_{ij}^{(0)} = \sigma_0$ at infinity, the stress and displacement fields for the entire body can be obtained by using the following govening equations of linear elasticity:

a) equilibrium equations

$$\sigma_{ij,j}^{s} = 0 \tag{3.1}$$

b) Hooke's law

$$\sigma_{ij}^{s} = C_{ijk\ell}^{s} \epsilon_{k\ell}^{s} \tag{3.2}$$

c) strain-displacement relations

$$\epsilon_{ij}^{s} = \frac{1}{2} (u_{i,j}^{s} + u_{j,i}^{s}) \tag{3.3}$$

and the specified boundary conditions. The superscript s denotes the matrix (m) or inclusion (f). In the present work, the interface conditions are varied theoretically by considering two models, the flexible interface model and the interphase model.

3.2.1 The flexible interface model

The flexible interface model represents an interface as a continuous spring connecting the inclusion and the matrix, such that tractions at the interface are continuous, while the displacement components are discontinuous, with the displacement jumps being proportional to the tractions. The spring constants that one would choose in practice depend on the nature of interface. This model has the advantage of mathematical simplicity as it incorporates two parameters that can be adjusted appropriately. By varying the interfacial parameters and comparing the predicted results with the experimental results, it is possible to infer the quality of the interface in the composite. (Note that by adopting interphase model, several material parameters need to be appropriately selected). One could view this model as representing, for example, a thin inclusion coating or a series of cracks along the interface. The boundary conditions for the flexible interface model are:

a) continuity of tractions

$$[\sigma_{ij}]n_j=0 \tag{3.4}$$

b) tangential tractions proportional to the jump in tangential displacement

$$\sigma_{nt} = k\ [u_t] \tag{3.5}$$

c) normal tractions proportional to the jump in normal displacement

$$\sigma_{nn} = g\ [u_n] \tag{3.6}$$

where u and σ denote the displacement and stress at the interface. The subscripts nt and t refer to the tangential direction, while the superscripts nn and n refer to the normal direction. [ ] implies the jump of the bracketed expression across the interface. k and g are the spring constants which have dimension of stress divided by length. These spring constants represent the degree of bonding at the interface. Note that the classical case of perfect bonding is obtained from the limit case when $k \to \infty$ and $g \to \infty$, the case of pure sliding is reached when $k \to 0$ and $g \to \infty$, the case of completely debonded interface is obtained by setting $k \to 0$ and $g \to 0$, while any finite positive values of k and g represents the imperfectly bonded interface. However, this model needs to be used with caution. The imperfect bonding representation in the normal direction might be unrealistic because it infers that the behavior under normal tensile stresses is identical to its response under equal but compressive stresses. Numerical treatment of this model has been given by Achenbach and Zhu (1989), where they solved the mixed boundary value problem to avoid the radial overlap by using boundary element method.

In this dissertation, the numerical calculations for the effective shear modulus are limited to the case when only the jump in the tangential displacement is allowed while the continuity of normal displacements is maintained in order to avoid the overlapping of the material.

Elastic strain energy

When an isolated elastic inhomogeneity (inclusion) Ω_f in a domain D (composite) is subjected to an applied stress $\sigma_{ij}^{(0)}$ at infinity, the elastic strain energy is expressed as

$$W = \frac{1}{2} \int_D (\sigma_{ij}^{(0)} + \sigma_{ij}) \, (u_{i,j}^{(0)} + u_{i,j}) \, dV \tag{3.7}$$

where $u_i^{(0)}$ is the displacement caused by the applied stress $\sigma_{ij}^{(0)}$ in the situation when the inclusions are not present. σ_{ij} and u_i are the stress and displacement disturbances caused by the presence of the inclusion. The Equation (3.7) is rewritten as

$$\begin{aligned} W = {} & \frac{1}{2} \int_D \sigma_{ij}^{(0)} u_{i,j}^{(0)} \, dV + \frac{1}{2} \int_D \sigma_{ij}^{(0)} u_{i,j} \, dV \\ & + \frac{1}{2} \int_D \sigma_{ij} \, u_{i,j}^{(0)} \, dV + \frac{1}{2} \int_D \sigma_{ij} \, u_{i,j} \, dV \end{aligned} \tag{3.8}$$

Since Equation (3.8) involves complicated quadratic form integrations over the volumetric region, it is desirable to express (3.8) in terms of integrals in Ω_f or on $|\Omega_f|$, where only linear form integrations are involved. Here Ω_f denotes the domain of the inclusion, and $|\Omega_f|$ denotes the surface of the inclusion. By employing Gauss's theorem to the Equation (3.8), the third term in (3.8) vanishes, while the last term in (3.8) becomes as

$$\frac{1}{2}\int_D \sigma_{ij}\ u_{i,j}\ dV = -\ \frac{1}{2}\int_{|\Omega_f|} \sigma_{ij}\ [u_i]\ n_j\ dS \tag{3.9}$$

where n_j is the outward unit vector normal to $|\Omega_f|$. Then (3.8) becomes

$$\begin{aligned} W = \frac{1}{2}\int_D \sigma_{ij}^{(0)} u_{i,j}^{(0)}\ dV + \frac{1}{2}\int_{\Omega_f} \sigma_{ij}^{(0)} u_{i,j}\ dV \\ + \ \frac{1}{2}\int_{D-\Omega_f} \sigma_{ij}^{(0)} u_{i,j}\ dV - \frac{1}{2}\int_{|\Omega_f|} \sigma_{ij}\ [u_i]\ n_j\ dS \end{aligned} \tag{3.10}$$

The third term in (3.10) is rewritten as

$$\begin{aligned} \frac{1}{2}\int_{D-\Omega_f} \sigma_{ij}^{(0)} u_{i,j}\ dV = \frac{1}{2}\int_{D-\Omega_f} \sigma_{ij}\ u_{i,j}^{(0)}\ dV \\ = -\ \frac{1}{2}\int_{|\Omega_f|} \sigma_{ij} n_j u_i^{(0)}\ dS = -\ \frac{1}{2}\int_{\Omega_f} \sigma_{ij}\ u_{i,j}^{(0)}\ dV \end{aligned} \tag{3.11}$$

Finally, the elastic strain energy is given as

$$\begin{aligned} W = \frac{1}{2}\int_D \sigma_{ij}^{(0)} u_{i,j}^{(0)}\ dV + \frac{1}{2}\int_{\Omega_f} \sigma_{ij}^{(0)} u_{i,j}\ dV \\ -\ \frac{1}{2}\int_{\Omega_f} \sigma_{ij}\ u_{i,j}^{(0)}\ dV - \frac{1}{2}\int_{|\Omega_f|} \sigma_{ij}\ n_j\ [u_i]\ dS \end{aligned} \tag{3.12}$$

which can be rewritten as

$$W = \frac{1}{2}\int_{D} \sigma_{ij}^{(0)} u_{i,j}^{(0)}\, dV + \frac{1}{2}\int_{\Omega_f} \sigma_{ij}^{(0)} u_{i,j}^{f}\, dV$$

$$- \frac{1}{2}\int_{\Omega_f} \sigma_{ij}^{f}\, u_{i,j}^{(0)}\, dV - \frac{1}{2}\int_{|\Omega_f|} \sigma_{ij}\, n_j\, [u_i]\, dS \qquad (3.13)$$

where $\sigma_{ij}^{f} = \sigma_{ij}^{(0)} + \sigma_{ij}$ and $u_i^{f} = u_i^{(0)} + u_i$ are the total stress and the total displacement in the inclusions, respectively.

The elastic strain energy of the interface springs is given by

$$W^{spring} = \frac{1}{2}\int_{|\Omega_f|} \sigma_{ij}^{f}\, n_j\, [u_i]\, dS$$

$$= \frac{1}{2}\int_{|\Omega_f|} k\, [u_\theta]^2\, dS + \frac{1}{2}\int_{|\Omega_f|} g\, [u_r]^2\, dS \qquad (3.14)$$

The total elastic strain energy of the composites with boundary conditions given by this model is the summation of the energy of the elastic medium (matrix and inclusions) and the energy of the springs (Jasiuk and Tong, 1989). Expressed in terms of integrals in Ω_f or on $|\Omega_f|$, it becomes:

$$W^{c} = \frac{1}{2}\int_{D} \sigma_{ij}^{(0)} u_{i,j}^{(0)}\, dV + \frac{1}{2}\int_{\Omega_f} \sigma_{ij}^{(0)} u_{i,j}^{f}\, dV$$

$$- \frac{1}{2}\int_{\Omega_f} \sigma_{ij}^{f}\, u_{i,j}^{(0)}\, dV + \frac{1}{2}\int_{|\Omega_f|} \sigma_{ij}^{(0)} n_j\, [u_i]\, dS \qquad (3.15)$$

By employing Hooke's law, the elastic strain energy per unit volume is expressed as

$$\frac{W^c}{D} = \frac{1}{2}\,\sigma_{ij}^{(0)}\,S_{ijk\ell}^{m}\,\sigma_{k\ell}^{(0)} + \frac{1}{2}\,f\,\sigma_{ij}^{(0)}\,(S_{ijk\ell}^{f} - S_{ijk\ell}^{m})\,\frac{1}{\Omega_f}\int_{\Omega_f}\sigma_{k\ell}^{f}dV$$

$$+ \frac{1}{2}\,f\,\frac{1}{\Omega_f}\int_{|\Omega_f|}\sigma_{ij}^{(0)}\;n_j[u_i]dS \tag{3.16}$$

where $S_{ijk\ell}^{s}$ is the compliance of phase s, s = f,m.

Suppose the same work is done by the applied stress $\sigma_{ij}^{(0)}$ on the (non-sliding) inclusion with the eigenstrain ϵ_{ij}^{*} (uniform)

$$W^{equiv} = \frac{1}{2}\int_D \sigma_{ij}^{(0)}u_{i,j}\;dV + \frac{1}{2}\int_{\Omega_f}\sigma_{ij}^{(0)}\epsilon_{ij}^{*}\;dV \tag{3.17}$$

The elastic strain energy per unit volume stored in the equivalent homogeneous medium is

$$\frac{W^{equiv}}{D} = \frac{1}{2}\,\sigma_{ij}^{(0)}S_{ijk\ell}^{m}\,\sigma_{k\ell}^{(0)} + \frac{1}{2}\,f\,\sigma_{ij}^{(0)}\epsilon_{ij}^{*} \tag{3.18}$$

If we compare these two results, the idea of Eshelby's "equivalent inclusion method" (Eshelby, 1957) is employed. Thus, the expression for ϵ_{ij}^{*} is obtained:

$$\sigma_{ij}^{(0)}\epsilon_{ij}^{*} = \sigma_{ij}^{(0)} \{ (S_{ijk\ell}^{f} - S_{ijk\ell}^{m}) \frac{1}{\Omega_f} \int_{\Omega_f} \sigma_{k\ell}^{f} \, dV$$

$$+ \frac{1}{\Omega_f} \int_{|\Omega_f|} [u_i] n_j \, dS \} \tag{3.19}$$

As an example, let us consider a spherical inclusion in the matrix subjected an uniformly applied stress $\sigma_{12}^{(0)} = \sigma_0$ at infinity. Both inclusion and the matrix are assumed isotropic. The solution of Equations (3.1)-(3.6) can be expressed as:

$$\sigma_{ij}^{s}(\mathbf{x}) = W_{ij}^{s}(\mathbf{x})\sigma_0 \tag{3.20}$$

$$u_i^{s}(\mathbf{x}) = w_i^{s}(\mathbf{x})\sigma_0 \tag{3.21}$$

where $W_{ij}^{s}(\mathbf{x})$ and $w_i^{s}(\mathbf{x})$ are functions of $\mathbf{x}$ for phase s which are defined as concentration factors in the paper of Benveniste et al. (1989). Then, the only non-zero eigenstrain is $\epsilon_{12}^{*} = \epsilon_{21}^{*}$. If we define $\epsilon_0^{*} = 2\epsilon_{12}^{*}$, ϵ_0^{*} is expressed as:

$$\epsilon_0^{*} = \beta\sigma_0 \tag{3.22}$$

where

$$\beta = \left(\frac{1}{G_f} - \frac{1}{G_m} \right) \frac{1}{\Omega_f} \int_{\Omega_f} W_{12}^f(\mathbf{x}) dV$$

$$+ \frac{1}{\Omega_f} \int_{|\Omega_f|} \{ [w_1^m(\mathbf{x}) - w_1^f(\mathbf{x})] n_2 + [w_2^m(\mathbf{x}) - w_2^f(\mathbf{x})] n_1 \} dS \qquad (3.23)$$

The average stress in the isolated inclusion due to the applied stress σ_0 is

$$<\sigma_{12}>_{\Omega_f} = \frac{1}{\Omega_f} \int_{\Omega_f} \sigma_{12}^f \, dV$$

$$= \frac{1}{\Omega_f} \int_{\Omega_f} W_{12}^f(\mathbf{x}) \sigma_0 dV \qquad (3.24)$$

The average of the stress disturbance in the isolated inclusion due to σ_0 is

$$\Delta\sigma_{12} = \Delta\sigma_0 = <\sigma_{12}>_{\Omega_f} - \sigma_0 = \alpha\sigma_0 \qquad (3.25)$$

where

$$\alpha = \frac{1}{\Omega_f} \int_{\Omega_f} W_{12}^f(\mathbf{x}) dV - 1 \qquad (3.26)$$

In this example, α and β are scalars. However, in general, they will be in the tensor form.

3.2.2 Interphase model

Consider a composite system consisting of three components: fiber (f), layer (ℓ) and the matrix (m). The perfect bonding boundary conditions, which imply continuity of tractions and displacements, are assumed at the fiber-layer and layer-matrix interfaces. The elastic properties of constituents are distinct and the interlayer has a given thickness. In this representation, the degree of debonding or damage at the interface can be simulated by adjusting the elastic constants and the thickness of the interphase region. For example, a soft layer will imply the weak or damaged interface. It might be noted that this model is more realistic than the previous one, but algebraically it is more involved.

Elastic strain energy

The elastic strain energy for this case is obtained by using Equation (3.15) with the substitution of Ω_f by $\Omega_f+\Omega_\ell$ and the omission of last term:

$$W^c = \frac{1}{2}\int_D \sigma_{ij}^{(0)} u_{i,j}^{(0)} \, dV + \frac{1}{2}\int_{\Omega_f} \sigma_{ij}^{(0)} u_{i,j}^{f} \, dV + \frac{1}{2}\int_{\Omega_\ell} \sigma_{ij}^{(0)} u_{i,j}^{\ell} \, dV$$

$$- \frac{1}{2}\int_{\Omega_f} \sigma_{ij}^{f} \, u_{i,j}^{(0)} \, dV - \frac{1}{2}\int_{\Omega_\ell} \sigma_{ij}^{\ell} \, u_{i,j}^{(0)} \, dV \qquad (3.27)$$

where $u_i^{(0)}$ is the displacement due to $\sigma_{ij}^{(0)}$ in the absence of inclusions. σ_{ij}^{f} and u_i^{f} are the total stresses and displacements in the inclusion, respectively, and σ_{ij}^{ℓ} and u_i^{ℓ} are the total stresses and displacements in the layer, respectively. D is the volume of the composite, Ω_f and Ω_ℓ are the volumes of fiber and layer, respectively.

Then, by using Hooke's law, the expression for the elastic strain energy per unit volume becomes

$$\frac{W^c}{D} = \frac{1}{2}\,\sigma_{ij}^{(0)} S_{ijkl}^{m}\,\sigma_{kl}^{(0)} + \frac{1}{2}\,f\,\sigma_{ij}^{(0)}(S_{ijkl}^{f} - S_{ijkl}^{m})\,\frac{1}{\Omega_f}\int_{\Omega_f}\sigma_{k\ell}^{f}\,dV$$

$$+\ \frac{1}{2}\,\ell\,\sigma_{ij}^{(0)}(S_{ijkl}^{\ell} - S_{ijkl}^{m})\,\frac{1}{\Omega_\ell}\int_{\Omega_\ell}\sigma_{k\ell}^{\ell}\,dV \qquad (3.28)$$

where f is the volume fraction of inclusions, $f = \frac{\Omega_f}{D}$, and ℓ is the volume fraction of layers, $\ell = \frac{\Omega_\ell}{D}$.

Suppose the same work is done by the applied stress $\sigma_{ij}^{(0)}$ on the inclusion (consisting of inclusion and interphase) with a ficticious eigenstrain ϵ_{ij}^{*}

$$W^{equiv} = \frac{1}{2}\int_D \sigma_{ij}^{(0)} u_{i,j}^{(0)}\,dV + \frac{1}{2}\int_{\Omega_f+\Omega_\ell}\sigma_{ij}^{(0)}\epsilon_{ij}^{*}\,dV \qquad (3.29)$$

The elastic strain energy per unit volume stored in the equivalent homogeneous medium is

$$\frac{W}{D}^{equiv} = \frac{1}{2}\, \sigma_{ij}^{(0)} S_{ijk\ell}^{m}\, \sigma_{k\ell}^{(0)} + \frac{1}{2}\, (\, f + \ell\,)\, \sigma_{ij}^{(0)} \epsilon_{ij}^{*} \tag{3.30}$$

If we compare these two results, as it was done in previous section, the expression for ϵ_{ij}^{*} is obtained:

$$\sigma_{ij}^{(0)} \epsilon_{ij}^{*} = \sigma_{ij}^{(0)} \Big(\frac{f}{f + \ell} (S_{ijk\ell}^{f} - S_{ijk\ell}^{m}) \frac{1}{\Omega_f} \int_{\Omega_f} \sigma_{k\ell}^{f} dV$$

$$+ \frac{\ell}{f + \ell} (S_{ijk\ell}^{\ell} - S_{ijk\ell}^{m}) \frac{1}{\Omega_\ell} \int_{\Omega_\ell} \sigma_{k\ell}^{\ell} dV \tag{3.31}$$

As an example, let us again consider a spherical inclusion and let the applied stress $\sigma_{12}^{(0)} = \sigma_0$. Again, both the inclusion and the matrix are assumed isotropic. Then, the only non-zero eigenstrain is $\epsilon_{12}^{*} = \epsilon_{21}^{*}$. If we define $\epsilon_0^{*} = 2\epsilon_{12}^{*}$, ϵ_0^{*} is expressed as:

$$\epsilon_0^{*} = \beta \sigma_0 \tag{3.32}$$

where

$$\beta = \frac{f}{f + \ell} \left(\frac{1}{G_f} - \frac{1}{G_m} \right) \frac{1}{\Omega_f} \int_{\Omega_f} W_{12}^{f}(\mathbf{x}) dV$$

$$+ \frac{\ell}{f + \ell} \left(\frac{1}{G_\ell} - \frac{1}{G_m} \right) \frac{1}{\Omega_\ell} \int_{\Omega_\ell} W^{\ell}_{12}(\mathbf{x}) dV \tag{3.33}$$

The average stress in the isolated coated inclusion due to the applied stress σ_0 is

$$<\sigma_{12}>_{\Omega_f+\Omega_\ell} = \frac{1}{\Omega_f+\Omega_\ell} \left(\int_{\Omega_f} \sigma^{f}_{12} \; dV + \int_{\Omega_\ell} \sigma^{\ell}_{12} \; dV \right)$$

$$= \frac{1}{\Omega_f+\Omega_\ell} \left(\int_{\Omega_f} W^{f}_{12}(\mathbf{x}) dV + \int_{\Omega_\ell} W^{\ell}_{12}(\mathbf{x}) dV \right) \sigma_0 \tag{3.34}$$

The average of the stress disturbance in the isolated inclusion due to σ_0 is

$$\Delta\sigma_{12} = \Delta\sigma_0 = <\sigma_{12}>_{\Omega_f} - \sigma_0 = \alpha\sigma_0 \tag{3.35}$$

where

$$\alpha = \frac{1}{\Omega_f+\Omega_\ell} \left(\int_{\Omega_f} W^{f}_{12}(\mathbf{x}) dV + \int_{\Omega_\ell} W^{\ell}_{12}(\mathbf{x}) dV \right) - 1 \tag{3.36}$$

Note that here, α and β are given as scalars. However, in general, they will be in tensor form.

3.3. Successive iteration and effective elastic moduli of imperfectly bonded composites

In section 3.2 we obtained the zeroth-order solution, in which only a single inclusion is considered. The zeroth-order eigenstrain and the stress disturbance in the inclusion are the overestimates or underestimates of the real situation when the elastic stiffness of inclusion is greater or smaller than the one of the matrix (Mori and Wakashima, 1990). The reason for this is because the presence of other inclusions will reduce or increase the stress in the inclusion, since the other inclusions carry stress larger (if the inclusions are stiffer) or smaller (if the inclusions are softer) than σ_0 (the average stress of the matrix for the single inclusion case). In order to obtain the actual stress distribution and the effective properties of composites with finite concentration of inclusions, some correction should be made.

3.3.1 The flexible interface model

Equations (3.22) - (3.26) give the zeroth-order solution. If the total volume fraction of inclusions is f, these inclusions with the eigenstrain of the zeroth-order produce the average stress in the matrix. According to Mori-Tanaka's theory (1973), this average stress is

$$\sigma_{12}^{(1)} = \sigma_1 = -f\Delta\sigma_0 = -f\alpha\sigma_0 \tag{3.37}$$

Following Mori and Wakashima (1990), this average stress in the matrix acts as an applied loading and causes additional disturbance in the vicinity of inclusions. The first-order correction becomes

$$\sigma_1 = - f\alpha\sigma_0 \tag{3.38}$$

$$\Delta\sigma_1 = \alpha\sigma_1 = - f\alpha\Delta\sigma_0 = - f\alpha^2\sigma_0 \tag{3.39}$$

$$\epsilon_1^* = \beta\sigma_1 = - f\beta\Delta\sigma_0 = - f\alpha\epsilon_0^* \tag{3.40}$$

The above procedure is repeated infinite number of times. The n-th order correction is given as

$$\sigma_n = - f\Delta\sigma_{n-1} = (-f\alpha)^n\sigma_0 \tag{3.41}$$

$$\Delta\sigma_n = \alpha(-f\alpha)^n \sigma_0 \tag{3.42}$$

$$\epsilon_n^* = (-f\alpha)^n\epsilon_0^* \tag{3.43}$$

Then, the total equivalent eigenstrain is the sum of the eigenstrains from every iteration

$$\epsilon^* = \epsilon_0^* + \epsilon_1^* + \epsilon_2^* + \ldots$$

$$= \epsilon_0^*(1 - f\alpha + f^2\alpha^2 + \dots) \tag{3.44}$$

Note that, under the condition: $|f\alpha| < 1$, ϵ^* will converge to the closed form expression:

$$\begin{aligned} \epsilon^* &= \frac{1}{1 + f\alpha} \epsilon_0^* \\ &= \frac{1}{1 + f\alpha} \beta\sigma_0 \end{aligned} \tag{3.45}$$

The effective elastic moduli are defined by using the concept of the average strain in the composite, i.e.

$$S^c_{ijk\ell}\, \sigma^{(0)}_{k\ell} = S^m_{ijk\ell}\, \sigma^{(0)}_{k\ell} + f\, \epsilon^*_{ij} \tag{3.46}$$

Therefore, the effective shear modulus is given by

$$\frac{1}{G_c} = \frac{1}{G_m} + \frac{f}{1+f\alpha} \beta \tag{3.47}$$

where α and β are given by Equation (3.23) and (3.26).

The actual stresses and displacements (including interaction of inclusions) can be also estimated by using successive iteration method. For example, the actual stress component $\sigma_{ij}(\mathbf{x})$ and displacement component $u_i(\mathbf{x})$ in the phase s are

$$\sigma_{ij}^{s}(\mathbf{x}) = \sigma_{ij}^{(0)} + \Delta\sigma_{ij}^{(0)}(\mathbf{x}) + \sigma_{ij}^{(1)} + \Delta\sigma_{ij}^{(1)}(\mathbf{x}) + \sigma_{ij}^{(2)} + \Delta\sigma_{ij}^{(2)}(\mathbf{x}) + \ldots \tag{3.48}$$

and

$$u_{i}^{s}(\mathbf{x}) = u_{i}^{(0)} + \Delta u_{i}^{(0)}(\mathbf{x}) + u_{i}^{(1)} + \Delta u_{i}^{(1)}(\mathbf{x}) + u_{i}^{(2)} + \Delta u_{i}^{(2)}(\mathbf{x}) + \ldots \tag{3.49}$$

In this particular case

$$\begin{aligned} \sigma_{ij}^{s}(\mathbf{x}) &= W_{ij}^{s}(\mathbf{x})\ (\ \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \ldots\ ) \\ &= W_{ij}^{s}(\mathbf{x})\ \sigma_0(\ 1 - f\alpha + f^2\alpha^2 - f^3\alpha^3 + \ldots\ ) \\ &= \frac{1}{1 + f\alpha}\ \sigma_{ij}^{\infty s}(\mathbf{x}) \end{aligned} \tag{3.50}$$

and

$$\begin{aligned} u_{i}^{s}(\mathbf{x}) &= w_{i}^{s}(\mathbf{x})\ (\ \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \ldots\ ) \\ &= w_{i}^{s}(\mathbf{x})\ \sigma_0(\ 1 - f\alpha + f^2\alpha^2 - f^3\alpha^3 + \ldots\ ) \\ &= \frac{1}{1 + f\alpha}\ u_{i}^{\infty s}(\mathbf{x}) \end{aligned} \tag{3.51}$$

where $\sigma_{ij}^{\infty s}(\mathbf{x})$ and $u_{i}^{\infty s}(\mathbf{x})$ represent the stress and displacement fields of an elastic body containing an isolated inclusion. Note that the local stresses and local displacements given in Equations (3.50) and (3.51) are the product of the solutions of the isolated inclusion given by Equations (3.20) and (3.21) and the correction factor $1/(1+f\alpha)$, which accounts for the inclusion interaction. Note also that $\sigma_0/(1+f\alpha)$ is the average stress in the matrix (Mori and Wakashima, 1990). It might be noted that the Equations (3.50)-(3.51) are not the exact solutions of the stress and displacement fields, since the result assumes that the other inclusions are not very close to the given inclusion and they all exert an additional uniform stress on the inclusion of interest. However, the solution is a good approximation in the average sense since it takes into account the inclusion interaction. By using the average stress in the matrix, other inclusions are smeared out so that no detailed information about the distributions of inclusions is neccessary. Note that the exact solution for the stress field is very difficult due to complex spatial distributions of inclusions. Also the stresses will differ for every inclusion since the inclusions are non-periodically distributed.

3.3.2 Interphase model

The procedure here is similar to the derivation in the previous model. Equations (3.32) - (3.36) give the zero-order solution. If the total volume fraction of inclusions and coatings is $f+\ell$, these coated inclusions with the eigenstrain of the zeroth-order produce the average stress in the matrix. According to Mori-Tanaka's theory (1973) this stress is

$$\sigma_{12}^{(1)} = \sigma_1 = -(f+\ell)\ \Delta\sigma_0 = -(f+\ell)\ \alpha\sigma_0 \tag{3.52}$$

Following Mori and Wakashima (1990), this average stress in the matrix acts as an additional applied loading which causes the additional disturbance in the vicinity of fibers. The first-order correction becomes:

$$\sigma_1 = -\ (f+\ell)\ \Delta\sigma_0 = -(f+\ell)\ \alpha\sigma_0 \tag{3.53}$$

$$\Delta\sigma_1 = \alpha\ \sigma_1 = -\ (f+\ell)\ \alpha\ \Delta\sigma_0 = -\ (f+\ell)\ \alpha^2\sigma_0 \tag{3.54}$$

$$\epsilon_1^* = \beta\ \sigma_1 = -\ (f+\ell)\ \beta\ \Delta\sigma_0 = -\ (f+\ell)\ \alpha\ \beta\ \sigma_0 = -\ (f+\ell)\ \alpha\ \epsilon_0^* \tag{3.55}$$

The above procedure is repeated infinite number of times. The n-th order correction is given as

$$\sigma_n = -(f+\ell)\ \Delta\sigma_{n-1} = [\ -\ (f+\ell)\ \alpha\ ]^n\ \sigma_0 \tag{3.56}$$

$$\Delta\sigma_n = \alpha\ \sigma_n = -\ (f+\ell)\ \Delta\sigma_{n-1} = \alpha\ [-\ (f+\ell)\ \alpha\ ]^n\ \sigma_0 \tag{3.57}$$

$$\begin{aligned} \epsilon_n^* &= \beta\ \sigma_n = -\ (f+\ell)\ \beta\ \Delta\sigma_{n-1} = -\ (f+\ell)\ \alpha\ \beta\ \sigma_{n-1} \\ &= [\ -\ (f+\ell)\ \alpha\ ]^n\ \epsilon_0^* \end{aligned} \tag{3.58}$$

Then, the total equivalent eigenstrain is the sum of the eigenstrains from every iteration

$$\epsilon^* = \epsilon_0^* + \epsilon_1^* + \epsilon_2^* + \ldots$$
$$= \epsilon_0^* \{ 1 - (f+\ell)\alpha + (f+\ell)^2\alpha^2 + \ldots \} \qquad (3.59)$$

ϵ^* converges under the condition: $|(f+\ell)\alpha| < 1$.

$$\epsilon^* = \frac{1}{1 + (f+\ell)\alpha} \epsilon_0^*$$
$$= \frac{1}{1 + (f+\ell)\alpha} \beta\sigma_0 \qquad (3.60)$$

The effective elastic moduli are defined by using the concept of the average strain in the composite, i.e.

$$S^c_{ijk\ell}\, \sigma^{(0)}_{k\ell} = S^m_{ijk\ell}\, \sigma^{(0)}_{k\ell} + (f+\ell)\, \epsilon^*_{ij} \qquad (3.61)$$

Therefore, the effective shear modulus is

$$\frac{1}{G_c} = \frac{1}{G_m} + \frac{f+\ell}{1+(f+\ell)\alpha}\, \beta \qquad (3.62)$$

where α and β are given in the Equations (3.32) and (3.36). Again, in general, α and β are tensors.

The estimate of the actual stresses and displacements (including interaction of inclusions) can be also given by using successive iteration method. For example, the stress components $\sigma_{ij}(\mathbf{x})$ and displacement components $u_i(\mathbf{x})$ in the phase s are

$$\sigma^s_{ij}(\mathbf{x}) = \sigma^{(0)}_{ij} + \Delta\sigma^{(0)}_{ij}(\mathbf{x}) + \sigma^{(1)}_{ij} + \Delta\sigma^{(1)}_{ij}(\mathbf{x}) + \sigma^{(2)}_{ij} + \Delta\sigma^{(2)}_{ij}(\mathbf{x}) + \ldots \tag{3.63}$$

and

$$u^s_i(\mathbf{x}) = u^{(0)}_i + \Delta u^{(0)}_i(\mathbf{x}) + u^{(1)}_i + \Delta u^{(1)}_i(\mathbf{x}) + u^{(2)}_i + \Delta u^{(2)}_i(\mathbf{x}) + \ldots \tag{3.64}$$

In this particular case

$$\begin{aligned}\sigma^s_{ij}(\mathbf{x}) &= W^s_{ij}(\mathbf{x})\,(\sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \ldots)\\ &= W^s_{ij}(\mathbf{x})\,\sigma_0(1 - (f+\ell)\alpha + (f+\ell)^2\alpha^2 - (f+\ell)^3\alpha^3 + \ldots)\\ &= \frac{1}{1 + (f+\ell)\alpha}\,\sigma^{\infty s}_{ij}(\mathbf{x})\end{aligned} \tag{3.65}$$

$$\begin{aligned}u^s_i(\mathbf{x}) &= w^s_i(\mathbf{x})\,(\sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \ldots)\\ &= w^s_i(\mathbf{x})\,\sigma_0(1 - (f+\ell)\alpha + (f+\ell)^2\alpha^2 - (f+\ell)^3\alpha^3 + \ldots)\\ &= \frac{1}{1 + (f+\ell)\alpha}\,u^{\infty s}_i(\mathbf{x})\end{aligned} \tag{3.66}$$

where $\sigma_{ij}^{\infty s}(\mathbf{x})$ and $u_i^{\infty s}(\mathbf{x})$ represent the stress and displacement field of an elastic body containing an isolated coated inclusion. Note that the local stresses and local displacements given in Equations (3.65) and (3.66) are the products of solutions for the isolated inclusion given by Equations (3.20) and (3.21) and the correction factor $1/[1+(f+\ell)\alpha]$, which accounts for the inclusion interaction. Note also that $\sigma_0/[1+(f+\ell)\alpha]$ is the average stress in the matrix (Mori and Wakashima, 1990). Again, the Equations (3.65)-(3.66) are not the exact solutions of the stress and displacement fields, however, they are good approximation in the average sense since they take into account the inclusion interaction.

3.4 COMPARISON BETWEEN TWO MODELS

Comparing the two interfacial models we used, we can see that the flexible interface model simplifies the problem in that we need to specify only two parameters in order to describe the interface. Thickness and the moduli of the interphase need not be separately prescribed. Also, we only need solve a boundary value problems of two-phase materials. The interphase model is mathematically more involved since it is necesary to solve a boundary value problem of a three-phase material. Both thickness and moduli of the interlayer need be prescribed. But physically it seems to be a more realistic boundary condition.

Although the flexible interface model and the interphase model are mathematically different, they may represent the similar physical behavior of the bond between inclusion and matrix for the special case when the inclusions are coated with very thin and very compliant interphase layers (Hashin, 1990b). The conditions of this special case are quantitatively expressed by

$$\ell = 2t / a \ll 1 \tag{3.67}$$

$$K_\ell \, , \, G_\ell \ll K_f, \, G_f \tag{3.68}$$

where a is inclusion radius and t is an interphase thickness.

It is observed that the interface parameters of the flexible interface model can be simply related to interphase properties and geometry of the interphase model for this case. For example, the relation between interface parameters and interphase characteristics for cylindrical fiber composites is given by Hashin (1990b) as follows:

$$g = \frac{K_\ell + G_\ell}{t} \tag{3.69}$$

$$k = \frac{G_\ell}{t} \tag{3.70}$$

where K is transverse bulk modulus, G is transverse shear modulus, and substript ℓ denotes interphase.

CHAPTER 4

THERMAL STRESS AND THERMAL EXPANSION COEFFICIENTS

When a composite material is subjected to temperature change, thermal stresses are created due to the mismatch of thermal expansion coefficients. Large thermal stresses may develop in the interphase or at the interface during composite processing and cure shrinkage in thermosetting matrices, which may cause stress concentration and initiate yielding or debonding. If these stresses exceed the bond-strength of the inclusion/interphase interface or the interphase/matrix interface, the microcracks will form, and the local failure of the composite will occur. Thus, these thermal stresses may ultimately control the structural performance of composite. Therefore, for design purposes it is important to know and to control the magnitude of these stresses. Also it is important to know the overall thermal expansion coefficients.

In this dissertation, the effect of interface on thermal stresses and thermal expansion coefficients of composite is also investigated. The thermal stress and thermal expansion coefficients of a composite with perfectly bonded interfaces have been studied by Schapery (1968), Wakashima et al. (1974), Ishikawa et al. (1978), Uemura et al. (1979), Takahashi et al. (1984), Avery and Herakovich (1986), Hahn and Kim (1988), Bowles and Tompkins (1989), Dvorak and Chen (1989), and others. The stress field around coated reinforcement (inclusion) has been addressed by Mikata and Taya (1985, 1986), Luo and Weng (1987, 1989), Pagano and Tandon (1988), Sottos et al. (1989), Vedula et al.

(1988a), Hsueh et al. (1988), among others. The effective thermal expansion coefficients have been predicted by Pagano and Tandon (1988), Maurer et al. (1988), Vedula et al. (1988b), and Tong and Jasiuk (1990), and others. The composite with sliding interfaces was studied by Jasiuk et al. (1988). In these works, various methods were used to account for the inclusion interaction.

4.1 DESCRIPTION OF THE METHOD

The effect of interface on thermal stress and thermal expansion coefficients is studied here by using again the successive iteration method (Mori and Wakashima, 1990). In the analysis, two composite systems are used, i.e., the spherical particle composite and the cylindrical fiber composite. The cylindrical fibers are assumed to have a circular cross-section, and are aligned. Since the composites with spherical or circular cylindrical inclusions will not allow the sliding to happen along the interface under the uniform temperature change, the flexible interface model is not applicable here. Therefore, only the interphase model is used in this chapter.

4.2 ISOLATED INCLUSION SOLUTION

Consider a three phase composite consisting of coated inclusions uniformly distributed in the matrix. The inclusions, layers (coatings) and the matrix are assumed to be linearly elastic and isotropic. They have distinct material properties for each phase s ($s = f,\ell,m$): elastic constants $C^{s}_{ijk\ell}$ and thermal expansion coefficients α^{s}_{ij}. The perfect bonding conditions are assumed at the inclusion-layer and layer-matrix interfaces. In the notation used, the superscripts and subscripts f, ℓ and m refer to the inclusion, layer, and matrix, respectively.

When the composite is subjected to a uniform temperature change ΔT, the stress and displacement field for the entire body can be obtained by the following governing equations and the specified interface conditions:

a) equilibrium equations:

$$\sigma^s_{ij,j} = 0 \tag{4.1}$$

b) Hooke's law:

$$\sigma^s_{ij} = C^s_{ijk\ell}(\epsilon^s_{k\ell} - \alpha^s_{ij}\Delta T) \tag{4.2}$$

c) strain-displacement relations:

$$\epsilon^s_{ij} = \frac{1}{2}(u^s_{i,j} + u^s_{j,i}) \tag{4.3}$$

For the given interface model, we have a boundary value problem which can be solved by using linear elastic theory. The solution can be expressed as:

$$\sigma^{s}_{ij}(\mathbf{x}) = H^{s}_{ij}(\mathbf{x})\ \Delta T \tag{4.4}$$

$$u^{s}_{i}(\mathbf{x}) = h^{s}_{i}(\mathbf{x})\ \Delta T \tag{4.5}$$

where $H^{s}_{ij}(\mathbf{x})$, $h^{s}_{i}(\mathbf{x})$ are functions of $\mathbf{x}$ for phase s, which are defined as concentration factors in the paper of Benveniste et al. (1989).

Let us denote the volume of the composite by D and the volumes of the phases by Ω_s, where s = f, ℓ and m. The volumes Ω_f and Ω_ℓ are the sums of the volumes of all the inclusions and coatings, respectively, such that $\Omega_r = \sum_{i=1}^{N} \Omega_i$, where r = f, ℓ and N is the number of inclusions. Then, the total average strain in the composite is given as

$$\begin{aligned}\langle\epsilon_{ij}\rangle_D &= \frac{1}{D}\int_D \epsilon_{ij}dV \\ &= \frac{1}{D}\left(\int_{\Omega_f} \epsilon_{ij}dV + \int_{\Omega_\ell} \epsilon_{ij}dV + \int_{\Omega_m} \epsilon_{ij}dV\right)\end{aligned} \tag{4.6}$$

By employing Hooke's law

$$\sigma^{s}_{ij} = C^{s}_{ijk\ell}(\epsilon^{s}_{k\ell} - \alpha^{s}_{k\ell}\Delta T) \qquad (s = f,\ \ell,\ m) \tag{4.7}$$

the Equation (4.6) can be expressed as

$$<\epsilon_{ij}>_D = \frac{1}{D} \{ [S^f_{ijk\ell}] \int_{\Omega_f} \sigma^f_{k\ell} dV + [S^\ell_{ijk\ell}] \int_{\Omega_\ell} \sigma^\ell_{k\ell} dV$$

$$+ [S^m_{ijk\ell}] \int_{\Omega_m} \sigma^m_{k\ell} dV + \int_{\Omega_f} \alpha^f_{ij} \Delta T dV$$

$$+ \int_{\Omega_\ell} \alpha^\ell_{ij} \Delta T dV + \int_{\Omega_m} \alpha^m_{ij} \Delta T dV \} \qquad (4.8)$$

Since the volume average of the stress disturbance in D vanishes (Mura, 1987, pp. 334-335)

$$[S^m_{ijk\ell}] \int_D \sigma_{k\ell} \, dV = 0, \qquad (4.9)$$

then

$$[S^m_{ijk\ell}] \int_{\Omega_m} \sigma^m_{k\ell} dV = - [S^m_{ijk\ell}] \int_{\Omega_f} \sigma^f_{k\ell} dV$$

$$= - [S^m_{ijk\ell}] \int_{\Omega_\ell} \sigma^\ell_{k\ell} dV \qquad (4.10)$$

Substitution of Equation (4.7) into (4.5) yields

$$<\epsilon_{ij}>_D = \alpha^m_{ij} \Delta T + \frac{1}{D} \{ ([S^f_{ijk\ell}] - [S^m_{ijk\ell}]) \int_{\Omega_f} \sigma^f_{k\ell} dV$$

$$+ ([S^\ell_{ijk\ell}] - [S^m_{ijk\ell}]) \int_{\Omega_\ell} \sigma^\ell_{k\ell} dV \}$$

$$+ f(\alpha^f_{ij} - \alpha^m_{ij}) \Delta T + \ell(\alpha^\ell_{ij} - \alpha^m_{ij}) \Delta T \qquad (4.11)$$

where f and ℓ are the volume fractions of particles and layers, respectively, and $S^{s}_{ijk\ell}$ is the compliance tensor of phase s, s = f, ℓ, m.

Next, suppose the same total average strain is obtained when a homogeneous material is subjected to a uniform ficticious strain ϵ^{*}_{ij} (called eigenstrain by Eshelby, 1957) in the region originally occupied by the inclusion and the layer. Then

$$\langle\epsilon_{ij}\rangle_{D} = \alpha^{m}_{ij}\,\Delta T + (f+\ell)\epsilon^{*}_{ij} \tag{4.12}$$

Again, we employ the idea of Eshelby's (1957) "equivalent inclusion method". By equating (4.11) and (4.12), the "equivalent" eigenstrain ϵ^{*}_{ij} is obtained as

$$\begin{aligned} \epsilon^{*}_{ij} = \frac{1}{f+\ell}\,\frac{1}{D}\,\Big\{ & ([S^{f}_{ijk\ell}]-[S^{m}_{ijk\ell}]) \int_{\Omega_f} \sigma^{f}_{k\ell}\,dV \\ & + ([S^{\ell}_{ijk\ell}]^{-1}-[S^{m}_{ijk\ell}]^{-1}) \int_{\Omega_\ell} \sigma^{\ell}_{k\ell}\,dV \Big\} \\ & + \frac{f}{f+\ell}\,(\alpha^{f}_{ij} - \alpha^{m}_{ij})\Delta T + \frac{\ell}{f+\ell}\,(\alpha^{\ell}_{ij} - \alpha^{m}_{ij})\Delta T \end{aligned} \tag{4.13}$$

For a numerical example, let's consider a spherical coated inclusion. Then, the only non-zero eigenstrain is $\epsilon^{*}_{11} = \epsilon^{*}_{22} = \epsilon^{*}_{33} = \epsilon^{*}_{0}$. Introducing the expressions (4.4) into (4.13), we have

$$\epsilon_0^* = \eta \, \Delta T \tag{4.14}$$

where

$$\eta = \frac{f}{f+\ell}\left(\frac{1}{K_f} - \frac{1}{K_m}\right)\frac{1}{\Omega_f}\int_{\Omega_f} H_{11}^f(\mathbf{x})dV + \frac{\ell}{f+\ell}\left(\frac{1}{K_\ell} - \frac{1}{K_m}\right)\frac{1}{\Omega_\ell}\int_{\Omega_\ell} H_{11}^\ell(\mathbf{x})dV$$

$$+ \frac{f}{f+\ell}(\alpha^f - \alpha^m) + \frac{\ell}{f+\ell}(\alpha^\ell - \alpha^m) \tag{4.15}$$

The average stress in the isolated inclusion and layer due to a uniform temperature change in the composite is

$$\langle\sigma_{ij}\rangle_{\Omega_f+\Omega_\ell} = \frac{1}{\Omega_f+\Omega_\ell}\left(\int_{\Omega_f}\sigma_{ij}^f \, dV + \int_{\Omega_\ell}\sigma_{ij}^\ell \, dV\right) \tag{4.16}$$

Substituting the stress expressions given in (4.4) into (4.16) yields the following result:

$$\langle\sigma_{11}\rangle_{\Omega_f+\Omega_\ell} = \langle\sigma_{22}\rangle_{\Omega_f+\Omega_\ell} = \langle\sigma_{33}\rangle_{\Omega_f+\Omega_\ell}$$

$$= - \langle\sigma\rangle_{\Omega_f+\Omega_\ell} = \gamma \, \Delta T \tag{4.17}$$

where

$$\gamma = \frac{f}{f+\ell}\frac{1}{\Omega_f}\int_{\Omega_f} H_{11}^f(\mathbf{x})dV$$

$$+ \frac{\ell}{f+\ell}\frac{1}{\Omega_\ell}\int_{\Omega_\ell} H_{11}^\ell(\mathbf{x})dV \tag{4.18}$$

Note that the stresses given in (4.17) represent the disturbance due to temperature change. Let us define this disturbance as $\Delta\sigma_0$

$$\Delta\sigma_0 = \langle\sigma\rangle_{\Omega_f+\Omega_\ell} = \gamma\ \Delta T \tag{4.19}$$

Let us define the corresponding displacement disturbance by Δu_0. Note that γ and η are scalars here. However, in general, they would be tensors.

4.3 SUCCESIVE ITERATION AND EFFECTIVE THERMAL EXPANSION COEFFICIENTS

The above solution is the zeroth-order solution, in which only a single coated inclusion is considered. In order to estimate the actual stress distribution and the effective thermal expansion coefficients of composite with finite concentration of inclusions, some correction should be made. The argument is the same as given in section 3.3. If the total volume fraction of inclusions and coatings is $f+\ell$, these coated inclusions with the eigenstrain of the zeroth-order produce the average stress in the matrix. According to Mori-Tanaka's theory (1973) this average stress is

$$\sigma_{11}^{(1)} = \sigma_{22}^{(1)} = \sigma_{33}^{(1)} = \sigma_1 = -(f+\ell)\ \Delta\sigma_0 = -(f+\ell)\gamma\Delta T \tag{4.20}$$

Following Mori and Wakashima (1990), this average stress in the matrix acts as an applied loading and causes additional disturbance in the vicinity of inclusions. Therefore, it is necessary to solve the second boundary value problem involving the isolated inclusion subjected to hydrostatic stress $\sigma_{11}^{(1)} = \sigma_{22}^{(1)} = \sigma_{33}^{(1)} = \sigma_1$ at infinity. The stress and displacement fields in this case are of the same forms as expressed in (3.20) and (3.21) of section 3.2 except that σ_0 is replaced by σ_1.

$$\sigma_{ij}^{s}(\mathbf{x}) = W_{ij}^{s}(\mathbf{x})\sigma_1 \tag{4.21}$$

$$u_i^{s}(\mathbf{x}) = w_i^{s}(\mathbf{x})\sigma_1 \tag{4.22}$$

The elastic strain energy produced by the applied stress $\sigma_{ij}^{(1)}$ is

$$W^c = \frac{1}{2}\int_D \sigma_{ij}^{(1)}u_{i,j}^{(1)}\,dV + \frac{1}{2}\int_{\Omega_f}\sigma_{ij}^{(1)}u_{i,j}^{f}\,dV - \frac{1}{2}\int_{\Omega_f}\sigma_{ij}^{f}\,u_{i,j}^{(1)}\,dV$$
$$+ \frac{1}{2}\int_{\Omega_\ell}\sigma_{ij}^{(1)}u_{i,j}^{\ell}\,dV - \frac{1}{2}\int_{\Omega_\ell}\sigma_{ij}^{\ell}\,u_{i,j}^{(1)}\,dV \tag{4.23}$$

where $u_i^{(1)}$ is the displacement due to $\sigma_{ij}^{(1)}$ in the absence of inclusions. σ_{ij}^{f} and u_i^{f} are the total stresses and displacements in the inclusion, respectively, while σ_{ij}^{ℓ} and u_i^{ℓ} are the total stresses and displacements in the layer, respectively. D is the volume of the composite, Ω_f and Ω_ℓ are the volumes of fiber and layer, respectively.

Note that this expression is the same as given in Equation (3.27) except for superscripts (1) which replace superscripts (0).

Then, the expression for the elastic strain energy per unit volume becomes

$$\frac{W^c}{D} = \frac{1}{2}\,\sigma_{ij}^{(1)} s_{ijkl}^{m}\,\sigma_{kl}^{(1)} + \frac{1}{2}\,f\,\sigma_{ij}^{(1)}(S_{ijkl}^{f} - S_{ijkl}^{m})\,\frac{1}{\Omega_f}\int_{\Omega_f}\sigma_{k\ell}^{f}\,dV$$

$$+\ \frac{1}{2}\,\ell\,\sigma_{ij}^{(1)}(S_{ijkl}^{\ell} - S_{ijkl}^{m})\,\frac{1}{\Omega_\ell}\int_{\Omega_\ell}\sigma_{k\ell}^{\ell}\,dV \qquad (4.24)$$

Suppose the same work is done by the applied stress $\sigma_{ij}^{(1)}$ on the inclusion (consisting of inclusion and interphase) with ficticious eigenstrain ϵ_{ij}^{*}

$$W^{equiv} = \frac{1}{2}\int_D \sigma_{ij}^{(1)} u_{i,j}^{(1)}\,dV + \frac{1}{2}\int_{\Omega_f+\Omega_\ell}\sigma_{ij}^{(1)}\epsilon_{ij}^{*}\,dV \qquad (4.25)$$

The elastic strain energy per unit volume stored in the equivalent homogeneous medium is

$$\frac{W^{equiv}}{D} = \frac{1}{2}\,\sigma_{ij}^{(1)} S_{ijk\ell}^{m}\,\sigma_{k\ell}^{(1)} + \frac{1}{2}\,(\ f + \ell\ )\,\sigma_{ij}^{(1)}\epsilon_{ij}^{*} \qquad (4.26)$$

By equating (4.25) and (4.26), the expression for ϵ_{ij}^{*} is obtained:

$$\sigma_{ij}^{(1)}\epsilon_{ij}^{*} = \sigma_{ij}^{(1)} \left(\frac{f}{f+\ell} (S_{ijk\ell}^{f} - S_{ijk\ell}^{m}) \frac{1}{\Omega_f} \int_{\Omega_f} \sigma_{k\ell}^{f} dV \right.$$

$$\left. + \frac{\ell}{f+\ell} (S_{ijk\ell}^{\ell} - S_{ijk\ell}^{m}) \frac{1}{\Omega_\ell} \int_{\Omega_\ell} \sigma_{k\ell}^{\ell} dV \right) \qquad (4.27)$$

For the case of a spherical coated inclusion, the only non-zero eigenstrain is $\epsilon_{11}^{*} = \epsilon_{22}^{*} = \epsilon_{33}^{*} = \epsilon_{1}^{*}$. ϵ_{1}^{*} is expressed as:

$$\epsilon_{1}^{*} = \beta\sigma_{1} \qquad (4.28)$$

where

$$\beta = \frac{f}{f+\ell} \left(\frac{1}{K_f} - \frac{1}{K_m} \right) \frac{1}{\Omega_f} \int_{\Omega_f} W_{11}^{f}(\mathbf{x}) dV$$

$$+ \frac{\ell}{f+\ell} \left(\frac{1}{K_\ell} - \frac{1}{K_m} \right) \frac{1}{\Omega_\ell} \int_{\Omega_\ell} W_{11}^{\ell}(\mathbf{x}) dV \qquad (4.29)$$

The component of the average stress in the isolated coated inclusion is

$$\langle\sigma_{11}\rangle_{\Omega_f+\Omega_\ell} = \frac{1}{\Omega_f+\Omega_\ell} \left(\int_{\Omega_f} \sigma_{11}^{f}\, dV + \int_{\Omega_\ell} \sigma_{11}^{\ell}\, dV \right)$$

$$= \langle\sigma_{22}\rangle_{\Omega_f+\Omega_\ell} = \langle\sigma_{33}\rangle_{\Omega_f+\Omega_\ell}$$

$$= \frac{1}{\Omega_f+\Omega_\ell} \left(\int_{\Omega_f} W_{11}^{f}(\mathbf{x}) dV + \int_{\Omega_\ell} W_{11}^{\ell}(\mathbf{x}) dV \right) \sigma_1 \qquad (4.30)$$

The average of the stress disturbance in the isolated inclusion due to $\sigma_{11}^{(1)}$ is

$$\Delta\sigma_{11} = \Delta\sigma_1 = <\sigma_{11}>_{\Omega_f} - \sigma_1 = \lambda\sigma_1 \tag{4.31}$$

where

$$\lambda = \frac{1}{\Omega_f+\Omega_\ell} \left(\int_{\Omega_f} W_{11}^f(\mathbf{x})dV + \int_{\Omega_\ell} W_{11}^\ell(\mathbf{x})dV \right) -1 \tag{4.32}$$

Then, the second-order correction is similarly performed. Using again Mori-Tanaka's average field concept, the additional stress disturbance $\Delta\sigma_1$, causes the additional average stress σ_2 in the matrix, which acts as an applied loading. Therefore, as before

$$\sigma_2 = -(f+\ell)\ \Delta\sigma_1 = -(f+\ell)\lambda\sigma_1 \tag{4.33}$$

$$\Delta\sigma_2 = \lambda\ \sigma_2 = -\ (f+\ell)\ \lambda\ \Delta\sigma_1 = -\ (f+\ell)\ \lambda^2\sigma_1 \tag{4.34}$$

$$\begin{aligned} \epsilon_2^* &= \beta\ \sigma_2 = -\ (f+\ell)\ \beta\ \Delta\sigma_1 \\ &= -\ (f+\ell)\ \lambda\ \beta\ \sigma_1 = -\ (f+\ell)\lambda\ \epsilon_1^* \end{aligned} \tag{4.35}$$

The n-th order correction is given as

$$\sigma_n = -(f+\ell)\ \Delta\sigma_{n-1} = [-(f+\ell)\lambda]^n \sigma_1 \tag{4.36}$$

$$\Delta\sigma_n = \lambda\ \sigma_n = -\ (f+\ell)\ \lambda\ \Delta\sigma_{n-1} = \lambda[-\ (f+\ell)\lambda]^n \sigma_1 \tag{4.37}$$

$$\begin{aligned} \epsilon_n^* &= \beta\ \sigma_n = -\ (f+\ell)\ \beta\ \Delta\sigma_{n-1} = -\ (f+\ell)\ \lambda\ \beta\ \sigma_{n-1} \\ &= [-\ (f+\ell)\lambda]^n \epsilon_1^* \end{aligned} \tag{4.38}$$

The above procedure is repeated infinite number of times. Then, the total equivalent eigenstrain is the sum of the eigenstrains from every iteration

$$\begin{aligned} \epsilon^* &= \epsilon_0^* + \epsilon_1^* + \epsilon_2^* + \dots \\ &= \epsilon_0^* + \epsilon_1^* \ \{\ 1-\ (f+\ell)\lambda + (f+\ell)^2\lambda^2 + \dots\ \} \end{aligned} \tag{4.39}$$

Note that, under the condition: $|(f+\ell)\lambda| < 1$, ϵ^* will converge to

$$\epsilon^* = \epsilon_0^* + \frac{1}{1 + (f+\ell)\lambda}\ \epsilon_1^* \tag{4.40}$$

This closed form expression can be further simplified. Since

$$\epsilon_1^* = \beta\ \sigma_1 = -\ (f+\ell)\ \beta\ \Delta\sigma_0 = -\ (f+\ell)\ \beta\ \frac{\gamma}{\eta}\ \epsilon_0^* \tag{4.41}$$

by (4.15), (4.18), (4.29), (4.32), and

$$\frac{\gamma}{\eta} = \frac{\Delta\sigma^{(0)}}{\epsilon^{*(0)}} = \frac{\Delta\sigma^{(1)}}{\epsilon^{*(1)}} = \frac{\lambda}{\beta} \tag{4.42}$$

Therefore,

$$\begin{aligned}\epsilon^{*} &= \{1- \frac{(f+\ell)\lambda}{1 + (f+\ell)\lambda}\}\epsilon_0^{*} \\ &= \frac{1}{1 + (f+\ell)\lambda}\,\epsilon_0^{*} = \frac{\eta}{1 + (f+\ell)\lambda}\,\Delta T\end{aligned} \tag{4.43}$$

The stresses and displacements (including interaction of inclusions) can be estimated in the same way. For example, the local fields for phase s are:

$$\begin{aligned}\sigma_{ij}^{s}(\mathbf{x}) &= \Delta\sigma_{ij}^{(0)}(\mathbf{x}) + \sigma_{ij}^{(1)} + \Delta\sigma_{ij}^{(1)}(\mathbf{x}) \\ &\quad + \sigma_{ij}^{(2)} + \Delta\sigma_{ij}^{(2)}(\mathbf{x}) + \ \ldots\end{aligned} \tag{4.44}$$

and

$$\begin{aligned}u_i^{s}\ (\mathbf{x}) &= \Delta u_i^{(0)}(\mathbf{x}) + u_i^{(1)} + \Delta u_i^{(1)}(\mathbf{x}) \\ &\quad + u_i^{(2)} + \Delta u_i^{(2)}(\mathbf{x}) + \ \ldots\end{aligned} \tag{4.45}$$

Introducing the solutions from zeroth order and first order, we have :

$$\sigma_{ij}^{s}(\mathbf{x}) = H_{ij}^{s}(\mathbf{x})\ \Delta T + W_{ij}^{s}(\mathbf{x})\ (\sigma_1 + \sigma_2 + \sigma_3 + \ldots)$$

$$= H_{ij}^{s}(\mathbf{x})\ \Delta T + W_{ij}^{s}(\mathbf{x})\ \{\ 1 - (f+\ell)\lambda + (f+\ell)^2\lambda^2 + \ldots\}\sigma_1$$

$$= H_{ij}^{s}(\mathbf{x})\ \Delta T + W_{ij}^{s}(\mathbf{x})\ \frac{1}{1 + (f+\ell)\lambda}\ \sigma_1$$

$$= H_{ij}^{s}(\mathbf{x})\ \Delta T - W_{ij}^{s}(\mathbf{x})\ \frac{f+\ell}{1 + (f+\ell)\lambda}\ \gamma\ \Delta T \tag{4.46}$$

and

$$u_{i}^{s}(\mathbf{x}) = h_{i}^{s}(\mathbf{x})\ \Delta T + w_{i}^{s}(\mathbf{x})\ (\sigma_1 + \sigma_2 + \sigma_3 + \ldots)$$

$$= h_{i}^{s}(\mathbf{x})\ \Delta T + w_{i}^{s}(\mathbf{x})\ \{\ 1 - (f+\ell)\lambda + (f+\ell)^2\lambda^2 + \ldots\}\sigma_1$$

$$= h_{i}^{s}(\mathbf{x})\ \Delta T + w_{i}^{s}(\mathbf{x})\ \frac{1}{1 + (f+\ell)\lambda}\ \sigma_1$$

$$= h_{i}^{s}(\mathbf{x})\ \Delta T - w_{i}^{s}(\mathbf{x})\ \frac{f+\ell}{1 + (f+\ell)\lambda}\ \gamma\ \Delta T \tag{4.47}$$

The effective thermal expansion coefficients are, by definition, the average strains resulting from a unit temperature rise for a traction free composite:

$$\alpha_{ij}^{c} = \frac{1}{\Delta T} \langle\epsilon_{ij}\rangle_D = \alpha_{ij}^{m} + \frac{1}{\Delta T}\ (f+\ell)\ \epsilon_{ij}^{*} \tag{4.48}$$

Therefore

$$\alpha^{c} = \alpha^{m} + \frac{f+\ell}{1 + (f+\ell)\lambda}\ \eta \tag{4.49}$$

where η is defined in (4.15) and λ in (4.32).

It is interesting to note that the thermal expansion coefficient obtained by the successive iteration method coincides with the one obtained by modified composite spheres model, which includes the interphase layer. It is also found that for the cylindrical composite without coating (the perfect bonding case), the results obtained by the successive iteration method coincide with the ones obtained by the composite cylinders model. In this dissertation, the results for the transverse thermal expansion coefficient α_T and longitudinal thermal expansion coefficient α_L for the composites with cylindrical coated inclusions are obtained by using the composite cylinders model for simplicity (See Appendix E).

CHAPTER 5

INTERPHASE WITH PROPERTY GRADIENTS

5.1 DESCRIPTION OF THE METHOD

The incorporation of a realistic interphasial model into the micromechanical analysis of composite systems is critical for the understanding of composite behavior. The interphase is usually modeled as a homogeneous region, despite the fact that it may have property gradients.

In this dissertation, the effect of variation of interphase properties is also studied. For the mathematical simplicity, the attention is given to the case of spherical inclusions in a matrix subjected to the hydrostatic stress at the matrix boundary and the uniform temperature change. The successive iteration method (Mori and Wakashima) is used here for the determination of the stresses, effective bulk modulus, and effective thermal expansion coefficients of composite. A power law introduced by Jayaraman et al. (1990) is chosen to simulate the variation of both elastic Young's modulus and coefficient of thermal expansion in the interphase region. The governing field equations are solved directly in the closed form. The influence of various parameters such as interphase thickness and inclusion volume fraction on the local stresses and thermal and elastic constants is studied. It is found that the property gradients have a distinct and important effect on the local stresses and the overall thermal and elastic properties.

Future research may deal with the interphase with other variations of the properties and include other geometric shapes.

Consider a composite with a single spherical coated inclusion embedded in a matrix which is subjected to a hydrostratic stress $\sigma_{xx}^{(0)} = \sigma_{yy}^{(0)} = \sigma_{zz}^{(0)} = \sigma_0$ at infinity and a uniform temperature change. The governing differential equations in terms of displacements for isotropic inclusion and matrix are

$$\frac{d^2u^s}{d\,r^2} + \frac{2}{r}\frac{du^s}{dr} - \frac{2}{r^2}u^s = 0 \qquad s = m,\ f \qquad (5.1)$$

where u is the radial displacement. The general solutions to equation (5.1) are given by

$$u^s(r) = A_s r + \frac{B_s}{r} \qquad (5.2)$$

where s = f, m. A_s and B_s are the constants for phase s which will be determined by the perfect bonding boundary conditions at inclusion-layer and layer-matrix interfaces.

Power variation model

The elastic Young's modulus and the thermal expansion coefficient of the interphase are given by

$$E_\ell\ (r) = P\ r^Q \tag{5.3}$$

$$\alpha_\ell\ (r) = M\ r^N \tag{5.4}$$

where P, Q, M, N are the constants which are evaluated by considering the following conditions:

$$E_\ell = E_f \qquad \alpha_\ell = \alpha_f \qquad \text{at } r = a \tag{5.5}$$

$$E_\ell = E_m \qquad \alpha_\ell = \alpha_m \qquad \text{at } r = b \tag{5.6}$$

The governing differential equations for the interphase region are

$$\frac{d^2u}{d\ r^2} + \frac{(Q+2)}{r}\ \frac{du}{dr} - \frac{2(Q\psi-1)}{r^2}\ u$$

$$= \frac{1}{1-2\nu_\ell}\ (Q+N)Mr^{N-1}\Delta T \tag{5.7}$$

where $\psi = \dfrac{\nu_\ell}{1-\nu_\ell}$. ν_ℓ is the Poisson's ratio of the layer, which is assumed constant for simplicity. The general solutions to equation (5.7) are given by

$$u(r) = A_\ell r^{\lambda 1} + B_\ell r^{\lambda 2} + C_\ell r^{N+1} \tag{5.8}$$

where

$$\lambda 1,\ \lambda 2 = \frac{1}{2}\ \{\ -(Q+1)\ \pm \sqrt{\ Q^2+2Q(1-4Q)+9\ }\ \} \tag{5.9}$$

and

$$C_\ell = \frac{(Q+N)M}{(1-2\nu_\ell)[\ (N+1)N+(Q+2)(N+1)+2(Q\phi-1)]} \tag{5.10}$$

A_ℓ and B_ℓ are the unknown constants. These constants will be determined from the boundary conditions. The boundary conditions for this problem are the perfect bonding conditions at interfaces

$$\sigma^f_{rr} = \sigma^\ell_{rr} \qquad u^f_r = u^\ell_r \qquad \text{at } r = a \tag{5.11}$$

$$\sigma^\ell_{rr} = \sigma^m_{rr} \qquad u^\ell_r = u^m_r \qquad \text{at } r = b \tag{5.12}$$

and applied loading (hydrostatic stress)

$$\sigma^m_{rr} = \sigma_0 \qquad \text{as } r \rightarrow \infty \tag{5.13}$$

where a is the radius of inclusion, while b is the outside radius of the layer.

The effective bulk modulus is evaluated by setting ΔT in Equation (5.7) to be zero, while the thermal expansion coefficient is calculated by setting σ_0 in Equation (5.13) to be zero.

Then the isolated inclusion solution is iterated and the procedure is similar to the constant interphase case given in Sec. 3.3. Comparing with the results of constant interphase case, in which the Young's modulus and thermal expansion coefficient of the interphase are taken as the average of Equations (5.3) and (5.4) over the interphase volume, it is found that the property gradients at the interphase may have significant effect on the local fields and the thermoelastic properties of the composites.

It might be noted that the present results coincide with the ones obtained by the modified composite spheres model, which includes the variable interphase.

CHAPTER 6

NUMERICAL RESULTS AND DISCUSSION

6.1 EFFECTIVE ELASTIC MODULUS

In the numerical results presented in this paper the inclusions are assumed to be cylindrical or spherical in shape. They are uniformly but not periodically distributed in the matrix. The effect of inclusion interaction is accounted for by using the successive iteration method based on Mori-Tanaka theory. The details of derivations are given in Chapters 3, 4 and Appendices A-D. The predicted elastic constants are the transverse shear and bulk moduli for the unidirectional composites reinforced with aligned cylindrical fibers and the shear and bulk moduli for the composites reinforced with spherical particles. In the calculations presented, the Poisson's ratio of the constituents is taken as 0.3 and the ratio of stiffness $\Gamma = G_f/G_m = 10$ unless it is specified otherwise.

6.1.1 UNIDIRECTIONAL COMPOSITES

A unidirectional composite reinforced with aligned cylindrical fibers of circular cross-section is considered. The transverse shear and bulk moduli are investigated for the two interfacial models.

The two interfacial models discussed in Chapter 3 and Chapter 4 can reduce to the case of two phase composite with the perfect bonding at the inclusion matrix interface. Therefore, initially, as a check of the present predictions we compare our results for the effective shear modulus for the perfect bonding case with the three phase model (Christensen and Lo, 1979), self-consistent scheme (Hill, 1965, Budiansky, 1965), and Hill-Hashin's upper and lower bounds (Hill, 1964; Hashin, 1965). As it is seen from Figure 6.1, the present solution coincides with Hill-Hashin's lower bound for the composites with stiffer inclusions. It is also found that for the composites with softer inclusions, the result coincides with Hill-Hashin's upper bound. Also, it is seen from Figure 6.1 that our result lies closely with the result given by the three phase model. The effective elastic moduli obtained by the self-consistent scheme are somewhat higher for stiffer inclusions and lower for softer inclusions.

The expressions of the bulk modulus and shear modulus for perfect bonding case are obtained, by using the successive iteration method, as follows:

$$\frac{K_c}{K_m} = \frac{(G_m + K_m) + (\frac{K_f}{K_m} - 1)(K_m + fG_m)}{(G_m + K_m) + (1 - f)(K_f - K_m)} \tag{6.1}$$

$$\frac{G_c}{G_m} = \frac{\frac{G_m}{G_f} + \kappa_m + f\left(1 - \frac{G_m}{G_f}\right)}{\frac{G_m}{G_f} + \kappa_m + \kappa_m f\left(\frac{G_m}{G_f} - 1\right)} \tag{6.2}$$

where G and K are transverse shear and bulk moduli. It is found that the transverse shear modulus given by the Equation (6.1) coincides the one given by composite cylinders model (Hashin, 1965). However, the composite cylinders model does not yield a solution for the tranverse shear modulus, whereas the Mori-Tanaka mathod does give the solution as given by Equation (6.2).

The effect of interface on the effective elastic moduli is studied in Figures (6.2)-(6.10). Figure (6.2) represents the nondimensional transverse shear modulus of composite G_c/G_m versus fiber volume fraction f for the flexible interface model for changing $\tilde{k} = k\ a\ /\ G_m$, where "a" is the radius of the fiber, G_m is the shear modulus of the matrix, and k is the tangential spring constant given in Equation (3.5). In the numerical calculations, the normal spring constant $\tilde{g} = g\ a\ /G_m$ is taken as infinity in order to avoid the overlapping of the material. Figure (6.3) supplements Figure (6.2) by showing the change of the effective shear modulus with $1/\tilde{k}$ for $f = 0.5$. When $\tilde{k}$ is infinite, G_c corresponds to perfect bonding case, then the behavior rapidly changes and finally G_c approaches asymptotically the solution for pure sliding case. Note that $\tilde{k}$ has a significant effect on G_c. The effective shear modulus decreases as $\tilde{k}$ decreases, therefore the weaker the bond, the lower the effective shear modulus. At the limit case $\tilde{k} = 0$ (pure sliding), the shear modulus decreases considerably in comparison with the perfectly bonded case. The similar behavior is observed in Figure (6.4), which illustrates the effect of parameter $\tilde{g}$ on the effective transverse bulk modulus K_c. Note that K_c is independent of $\tilde{k}$ due to symmetry. Figure (6.5) shows that the spring constant $\bar{k}$ has an important effect on the interfacial shear stress.

Figures (6.6)-(6.10) illustrate the numerical results for the interphase model. Figure (6.6) shows G_c versus f and G_ℓ/G_m for $\Gamma = G_f/G_m = 10$ and $\ell/f = 0.01$. Similarly as for the previous model, the weaker the interface, or the lower the stiffness of the interphase region, the lower the effective shear modulus. This effect becomes more pronounced as the inclusion volume fraction increases. It is interesting to observe that for certain interphase stiffness ($G_\ell/G_m = 0.0055$ for this case), the stiff fiber combined with the soft interphase results the effective shear modulus of the composite to be the same as those of the pure matrix material. Also, the soft interphase may cause the effective modulus to become lower than that of the matrix. For example, when $G_\ell/G_m = 0.001$, the increase in f will only further reduce G_c, as shown in Figure (6.6). Since the interface plays an important role in transfering the load from the matrix to inclusions, when $G_\ell \rightarrow 0$, the inclusions will not contribute toward the reinforcement. In Figure (6.6), the volume fraction of inclusion is taken theoretically to $f \approx 1$ for completeness, but in the typical composite this value never reached. Compared with the case when $G_\ell/G_m = 1$, which corresponds to the case when there is no layer (classical perfect bonding case), the stiffer interphase will improve the elastic properties of composite. Similar phenomenon is observed in Figure (6.7), which illustrates K_c/K_m vs. f for $\Gamma = 10$ and

$\ell/f = 0.01$ with changing K_ℓ/K_m. Figure (6.8) gives K_c/K_m vs. K_ℓ/K_m and ℓ/f for fixed inclusion volume fraction $f = 0.2$. Again it is observed that the stiffer layer will improve the effective properties, while the softer layer with larger thickness will significantly reduce the elastic constants. Note that the thickness of interphase has a significant effect on the effective properties if the layer is softer than the matrix. Figures (6.9)-(6.10) show the transverse shear modulus and transverse bulk modulus vs. f for the composite materials given in Table (6.1).

Table 6.1

Material properties of several coated-fiber composites

System	E_L (Gpa)	E_T (Gpa)	G_L (Gpa)	G_T (Gpa)	α_T $(10^{-6}/^{o}C)$	α_L $(10^{-6}/^{o}C)$	f
1 Nicalon fiber	172.38	172.38	71.78	71.78	3.8	3.8	0.4
Carbon coating	34.48	34.48	14.34	14.34	3.3	3.3	0.01616
LAS matrix	103.43	103.43	43.09	43.09	2.8	2.8	0.58384
2 Tungsten fiber	345.0	345.0	135.0	135.0	5.0	5.0	0.4
Carbon coating	34.48	34.48	14.34	14.34	3.3	3.3	0.0107
Nickel matrix	214.0	214.0	81.6	81.6	13.3	13.3	0.5893
3 SiC fiber	431.0	431.0	172.0	172.0	4.86	4.86	0.4
Carbon coating	34.48	34.48	14.34	14.34	3.3	3.3	0.0107
Titanium aluminate matrix	96.5	96.5	37.1	37.1	9.25	9.25	0.5893

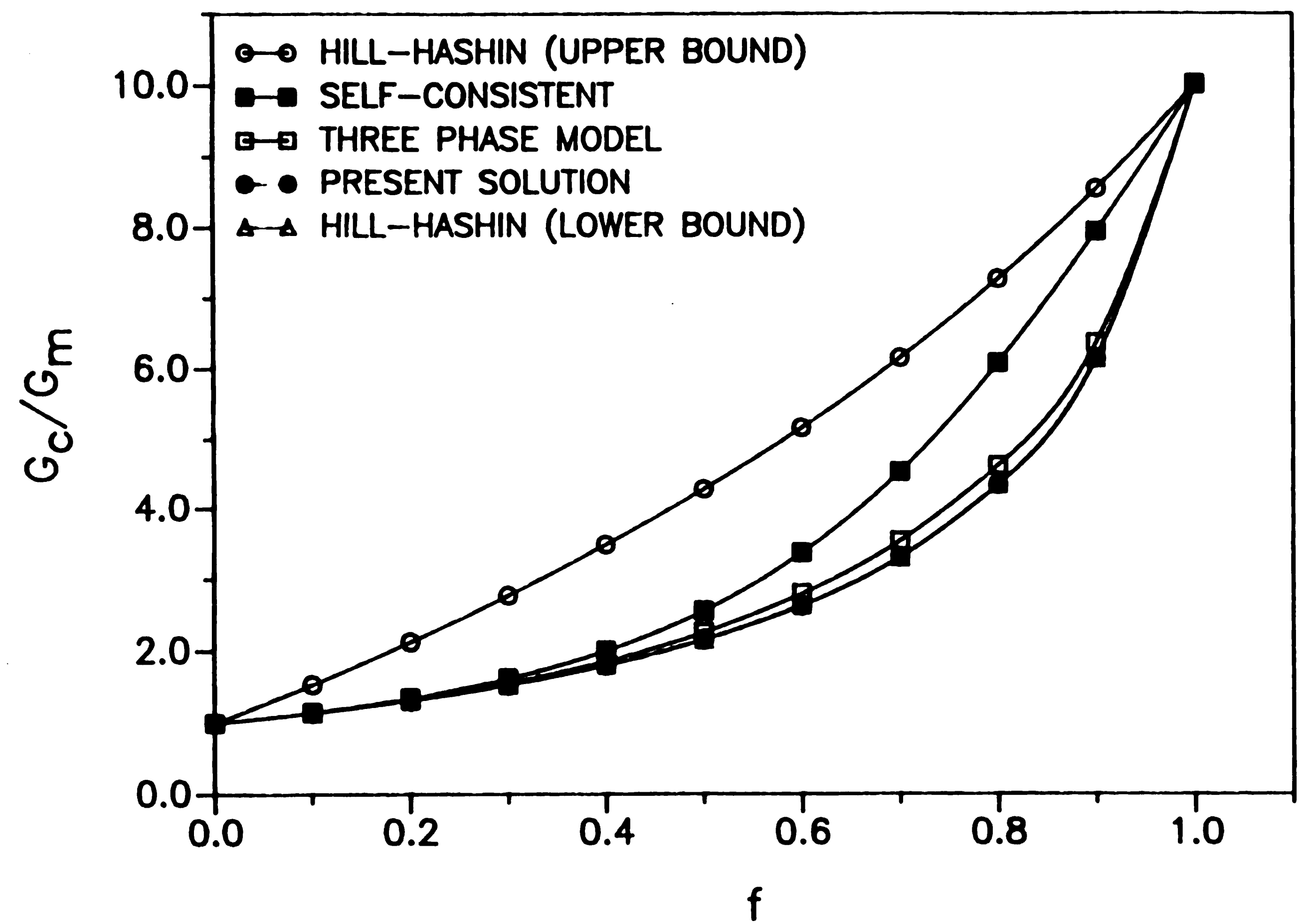

Figure 6.1 Effective transverse shear modulus G_c/G_m vs. fiber volume fraction f when G_f/G_m = 10 (perfect bonding, 2D)

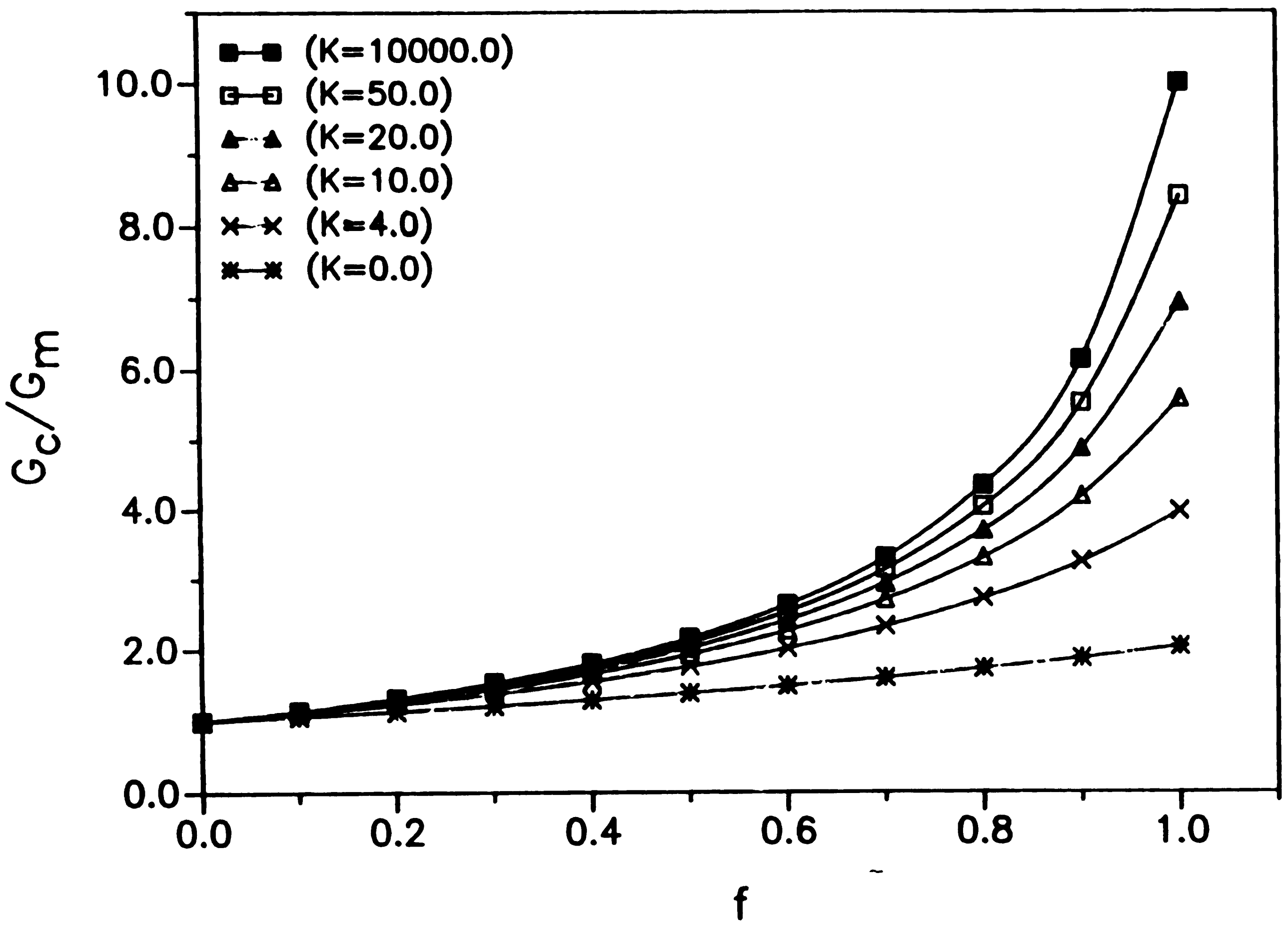

Figure 6.2 G_c/G_m vs. f for changing $\tilde{k} = ka/G_m$ when $G_f/G_m = 10$

(the flexible interface model, 2D)

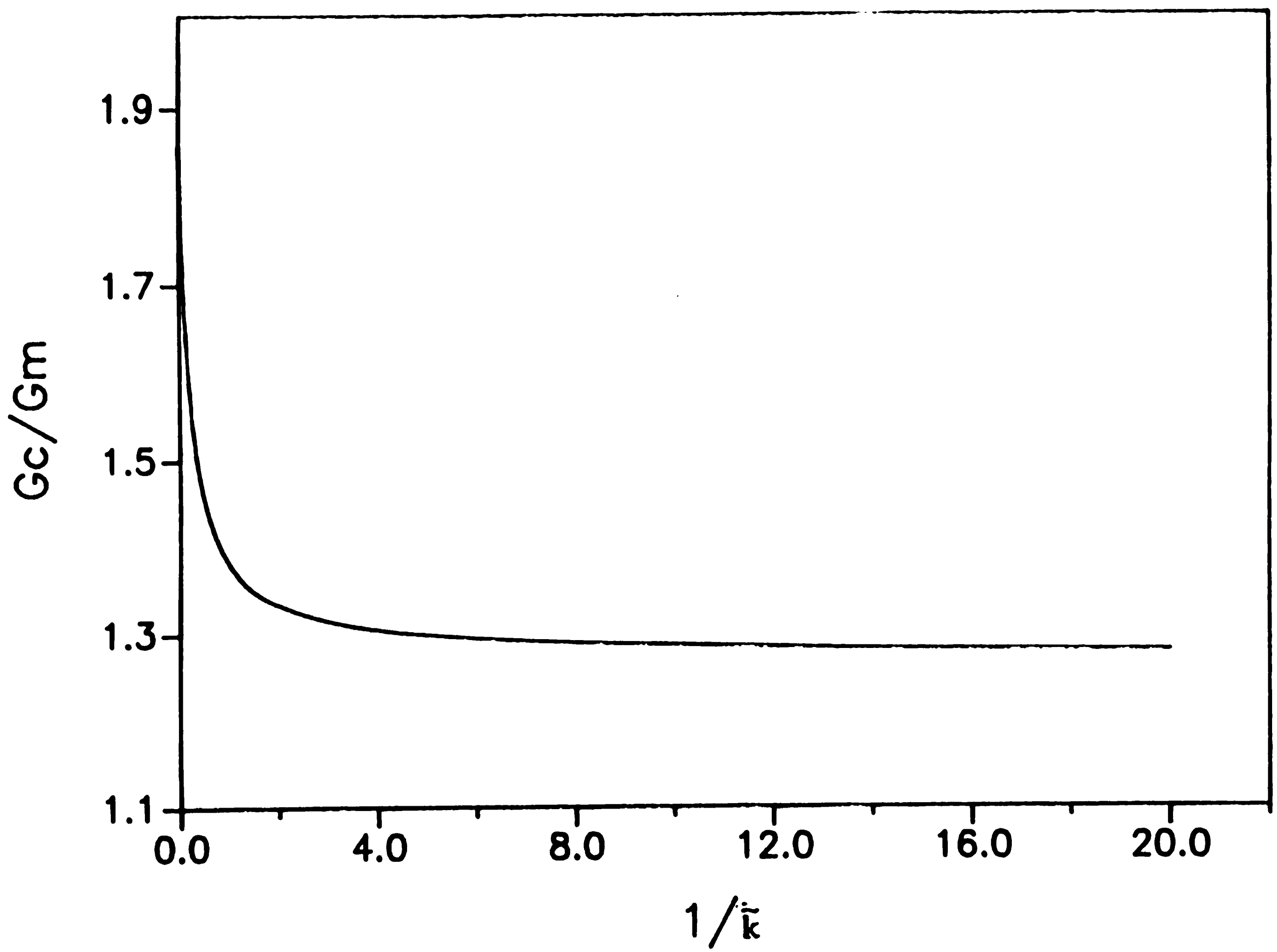

Figure 6.3 G_c/G_m vs. $1/\tilde{k}$ for G_f/G_m - 10 and f - 0.5

(the flexible interface model, 2D)

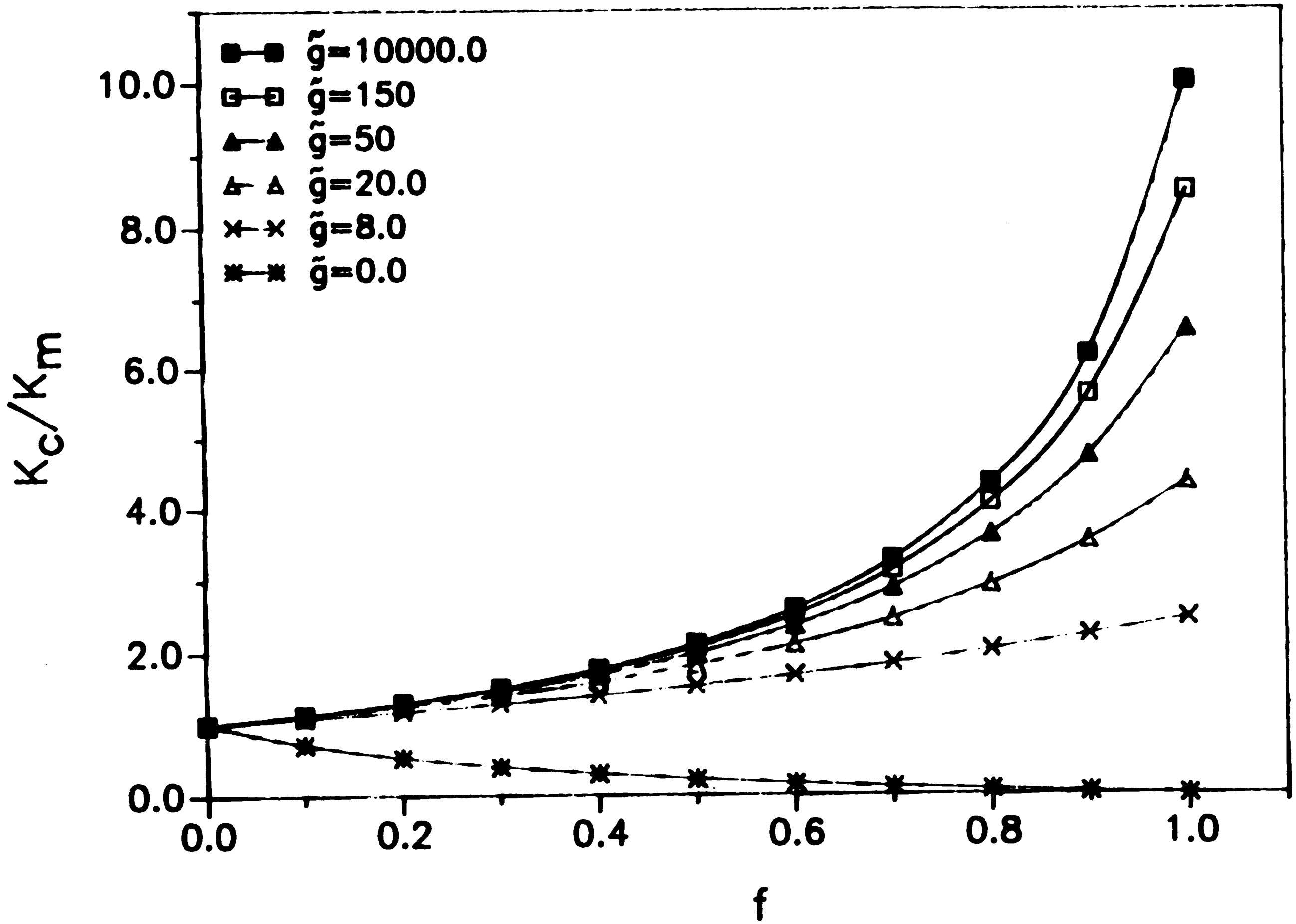

Figure 6.4 Transverse effective bulk modulus K_c/K_m vs. f for changing $\tilde{g}$ (the flexible interface model, 2D)

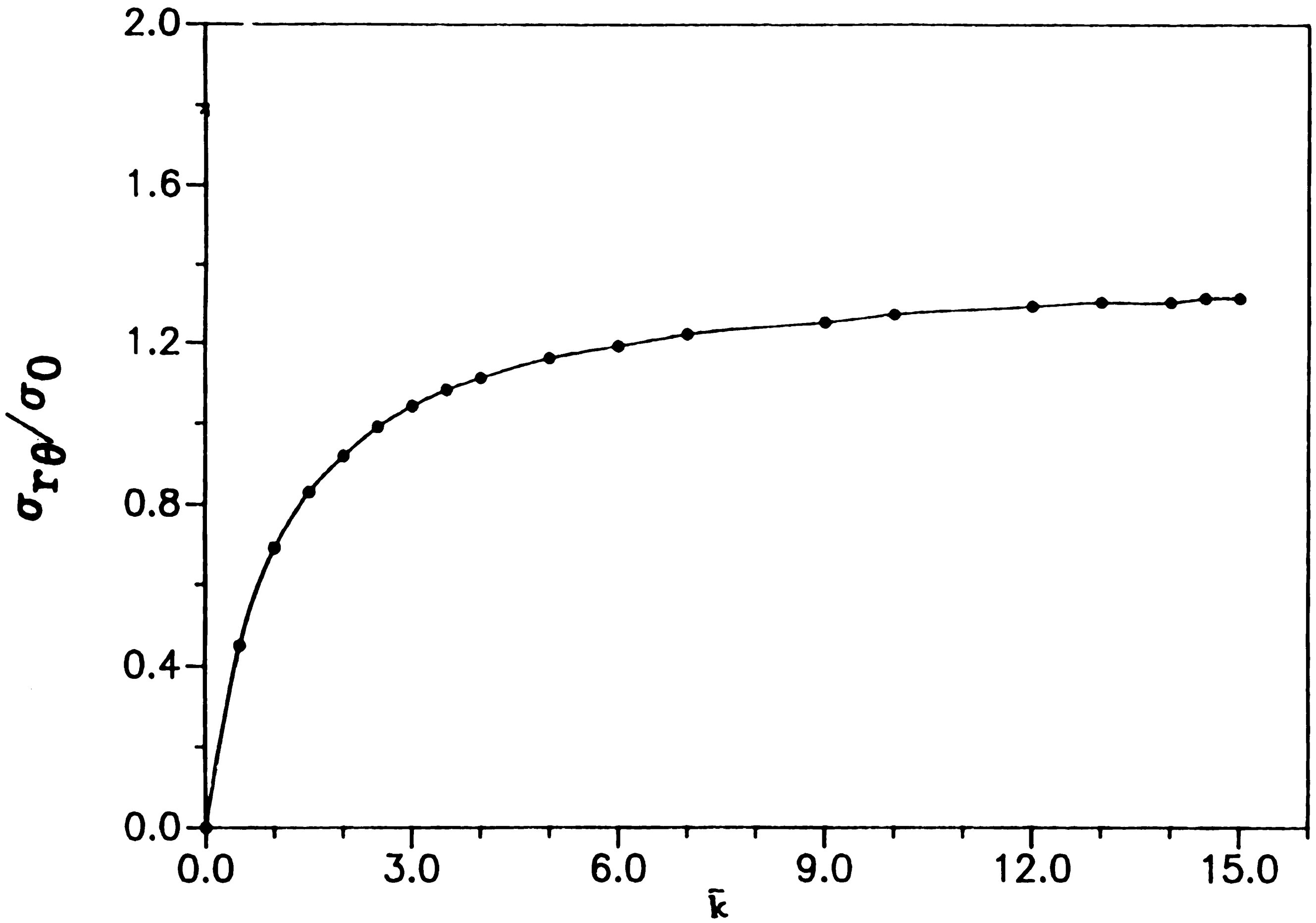

Figure 6.5 Interfacial shear stress vs. $\tilde{k}$

(the flexible interface model, 2D)

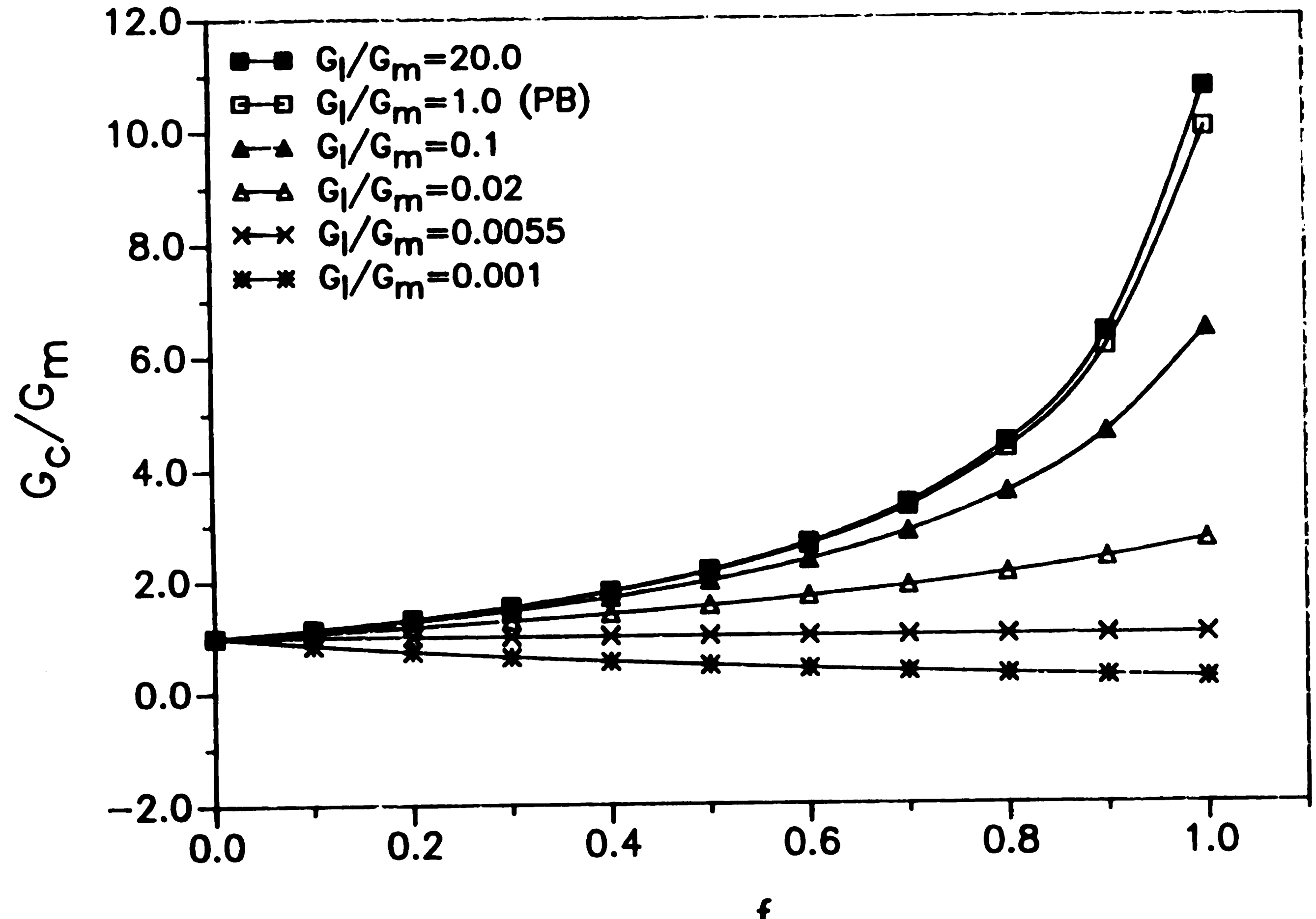

Figure 6.6 G_c/G_m vs. f for G_f/G_m = 10, ℓ/f = 0.01 and changing G_ℓ/G_m (the interphase model, 2D)

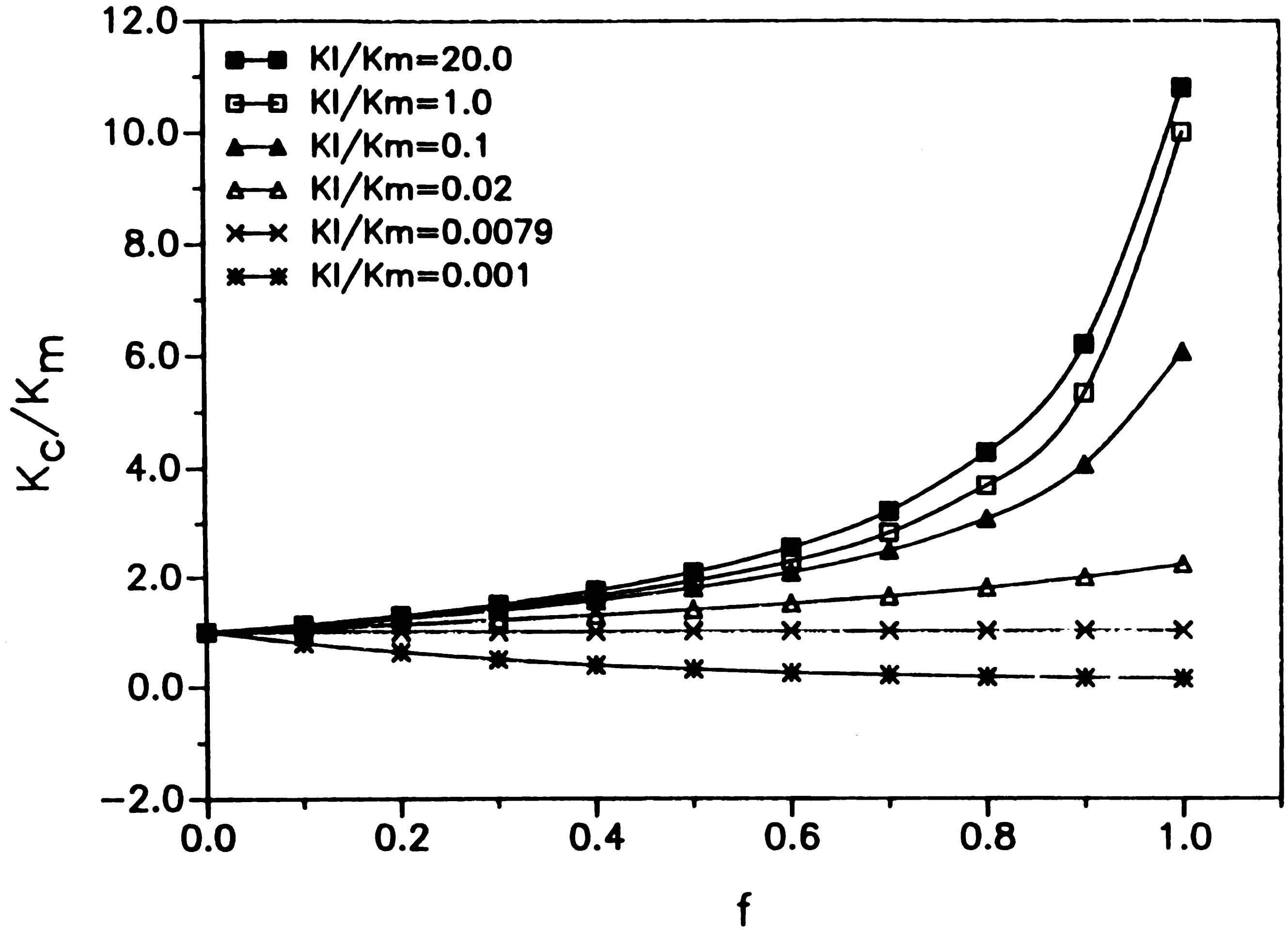

Figure 6.7 K_c/K_m vs. f for K_f/K_m = 10. ℓ/f = 0.01 and changing K_ℓ/K_m (the interphase model, 2D)

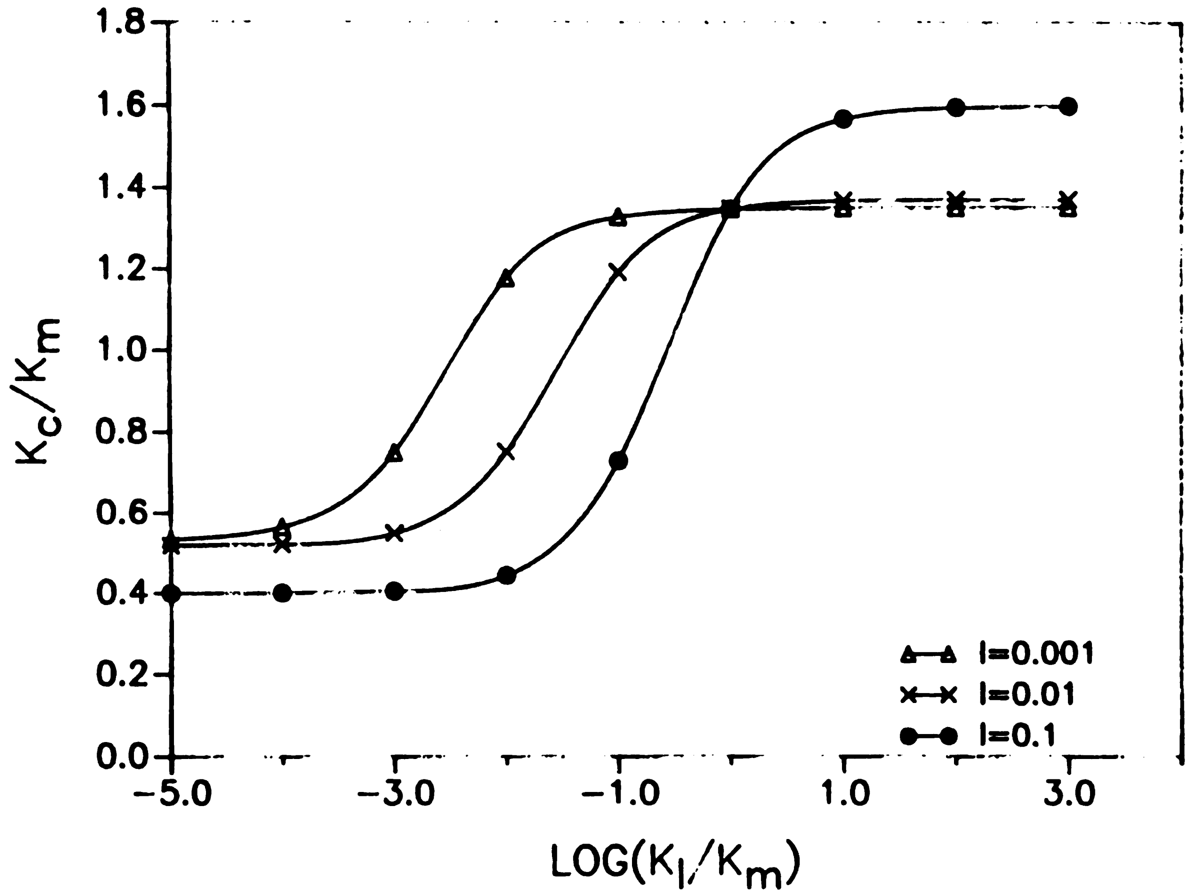

Figure 6.8 K_c/K_m vs. K_ℓ/K_m for changing ℓ/f when f = 0.2 and K_m/K_f<<1 (the interphase model, 2D)

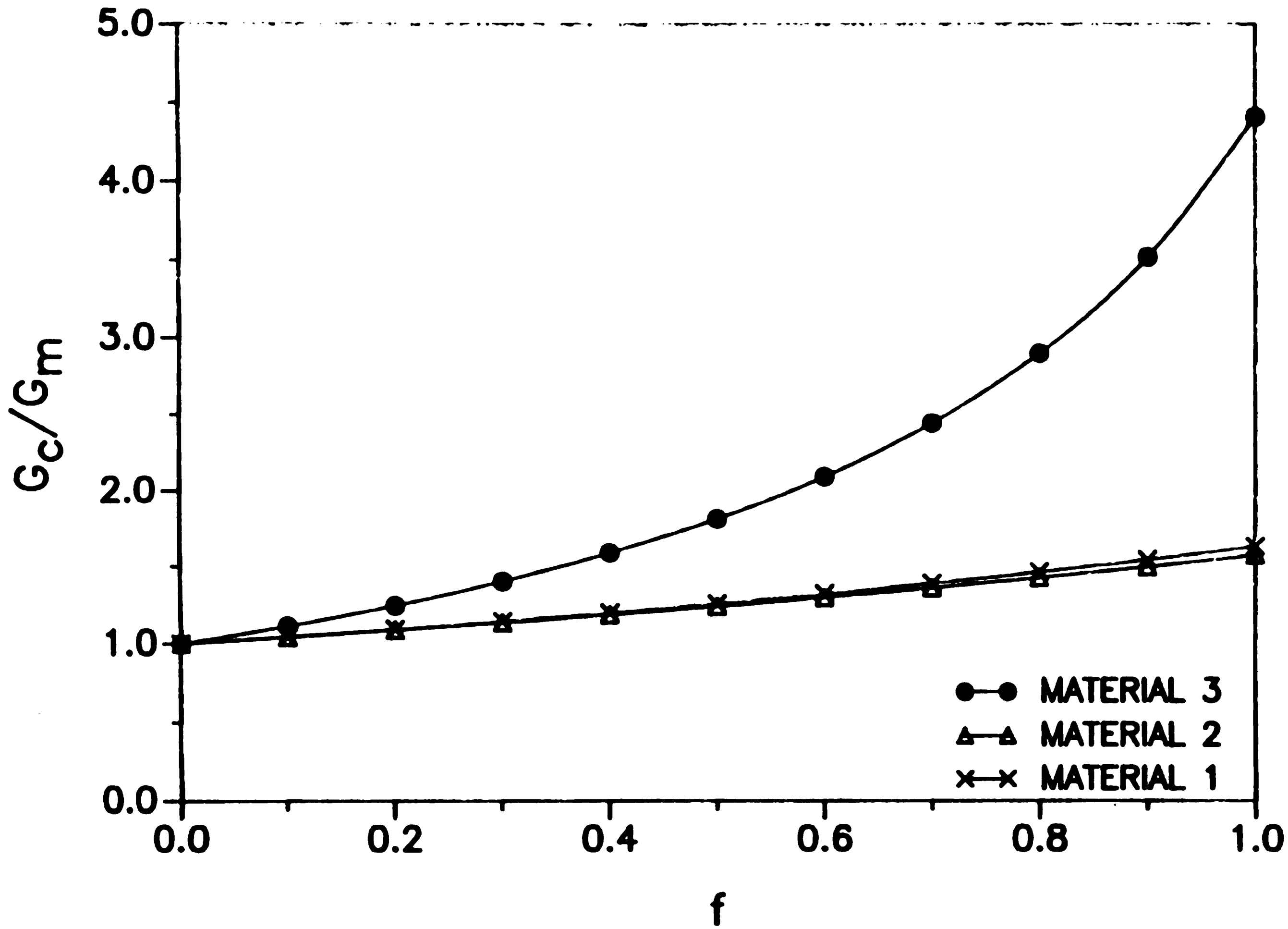

Figure 6.9 G_c/G_m vs. f for coated-fiber composite materials when $\ell/f = 0.01$ (the interphase model, 2D, data taken from table 6.1)

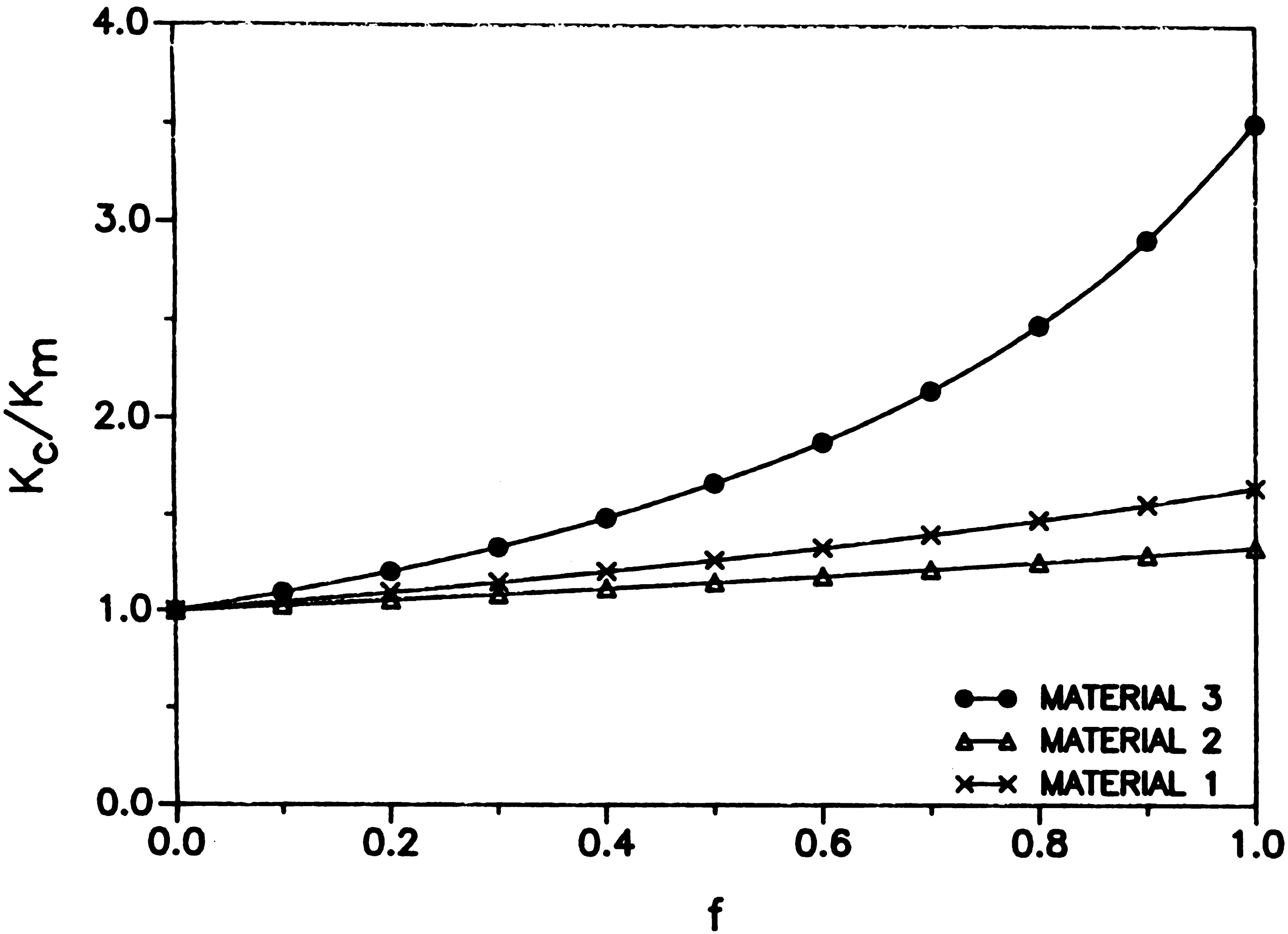

Figure 6.10 K_c/K_m vs. f for coated-fiber composite materials when $\ell/f = 0.01$ (the interphase model, 2D, data taken from table 6.1)

6.1.2 PARTICULATE COMPOSITES

The numerical results in this section include the effective bulk and shear moduli of composites reinforced with spherical inclusions. In the calculations presented, Poisson's ratio of inclusions, layers and matrix is taken as 0.3 unless it is specified otherwise. The two interfacial models considered in this dissertation can reduce to the case of two phase composite with the perfect bonding at the inclusion matrix interface. The results of both models coincide in this case. As a check of the present prediction, we compare our results with Hashin-Shtrikman upper and lower bounds (Hashin and Shtrikman, 1963) and three phase model (Christensen and Lo, 1979). As is seen from Figure 6.11, the present solution coincides with Hashin-Shtrikman's lower bound for the composites with stiffer inclusions. It may also be mention that the present solution coincides with Hashin-Shtrikman's upper bound for the composites with softer inclusions.

The perfect bonding results obtained by the current method are expressed as:

$$\frac{K_c}{K_m} = \frac{(3K_f+4G_m)K_m+4fG_m(K_f-K_m)}{(3K_f+4G_m)K_m+3K_mf(K_m-K_f)} \tag{6.3}$$

$$\frac{G_c}{G_m} = \frac{6G_f(K_m+2G_m)+G_m(9K_m+8G_m)+f(G_m-G_f)(6K_m+12G_m)}{6G_f(K_m+2G_m)+G_m(9k_m+8G_m)-f(G_m-G_f)(9K_m+8G_m)} \tag{6.4}$$

where G and K are shear and bulk moduli. It is found that the effective bulk modulus given by the Equation (6.3) coincides the one obtained by the composite spheres model (Hashin, 1962). However, the composite spheres model does not yield a solution for the effective shear modulus, whereas the Mori-Tanaka mathod does give the solution as given by Equation (6.4).

Figure 6.12 represents the ratio of effective shear moduli G_c/G_m versus inclusion volume fraction f for the flexible interface model for changing $\tilde{k} = k\ a/G_m$. Note that the parameter $\tilde{k}$ has a significant effect on G_c. The effective shear modulus decreases as $\tilde{k}$ decreases, therefore the weaker the bond the lower the effective shear modulus. At the limit case

$\tilde{k} = 0$ (pure sliding), the shear modulus decreases considerably in comparison with the perfectly bonded case.

Figure 6.13 supplements Figure 6.12 by showing the change of the effective shear modulus with $1/\tilde{k}$ for $f = 0.4$. When $\tilde{k}$ approaches infinity, G_c corresponds to the perfect bonding case, then the behavior rapidly changes and finally G_c approaches asymptotically the solution for pure sliding case. The results obtained by Benveniste (1985) and Mal and Rose (1974) are given here for comparison. The similar behavior is observed in Figure 6.14, which illustrates the effect of parameter $\tilde{g}$ on the effective bulk modulus K_c. Note that K_c is independent of $\tilde{k}$ due to symmetry.

Figures 6.15-6.18 illustrate the numerical results for the interphase model.

Figure 6.15 shows G_c/G_m versus f and G_ℓ/G_m for $\Gamma = 10$ and $\ell/f = 0.01$, where G_ℓ is the shear modulus of the layer. Similarly as for the previous model, the weaker the interface, or the lower the stiffness of the interphase, the lower the effective shear modulus. It is interesting to observe that the softer interphase may cause the effective modulus to become lower than that of the matrix. Similar phenomenon is observed in Figure 6.16, which illustrates K_c/K_m vs. f for $\Gamma = 10$, $\ell/f = 0.01$ and change K_ℓ/K_m, where K_ℓ is the bulk modulus of the layer. Figure 6.17 gives K_c/K_m vs. K_ℓ/K_m and ℓ/f for the fixed inclusion volume fraction $f = 0.2$ and $K_m/K_f \ll 1$. Again it is observed that the stiffer layer will improve the effective properties, while the softer layer with larger thickness will have a significant effect on the effective properties if the layer is softer than the matrix. For a nearly incompressible interlayer and matrix, $K_\ell \gg G_\ell$, $K_m \gg G_m$, the results coincide with Equation 1b in the paper of Maurer (1986) as show in Figure 6.18.

Comparing Figure 6.12 with Figure 6.15, it is found that the parameter $\tilde{k}$ of the flexible interface model has the similar effect on the effective shear modulus as G_ℓ/G_m of the interphase model, i.e. the increasing of $\tilde{k}$ or G_ℓ/G_m will increase the effective modulus of the composites. The fact that the interphase model gives the effective modulus lower than that of the matrix for soft interlayer, which will not happen for flexible interface model even for the pure sliding, is because of the effect of the layer's thickness. Since in the flexible interface model it is assumed that the interface is very thin film while in the interphase model certain thickness has been given (ℓ/f = 0.01), which amplifies the effect of weak interface.

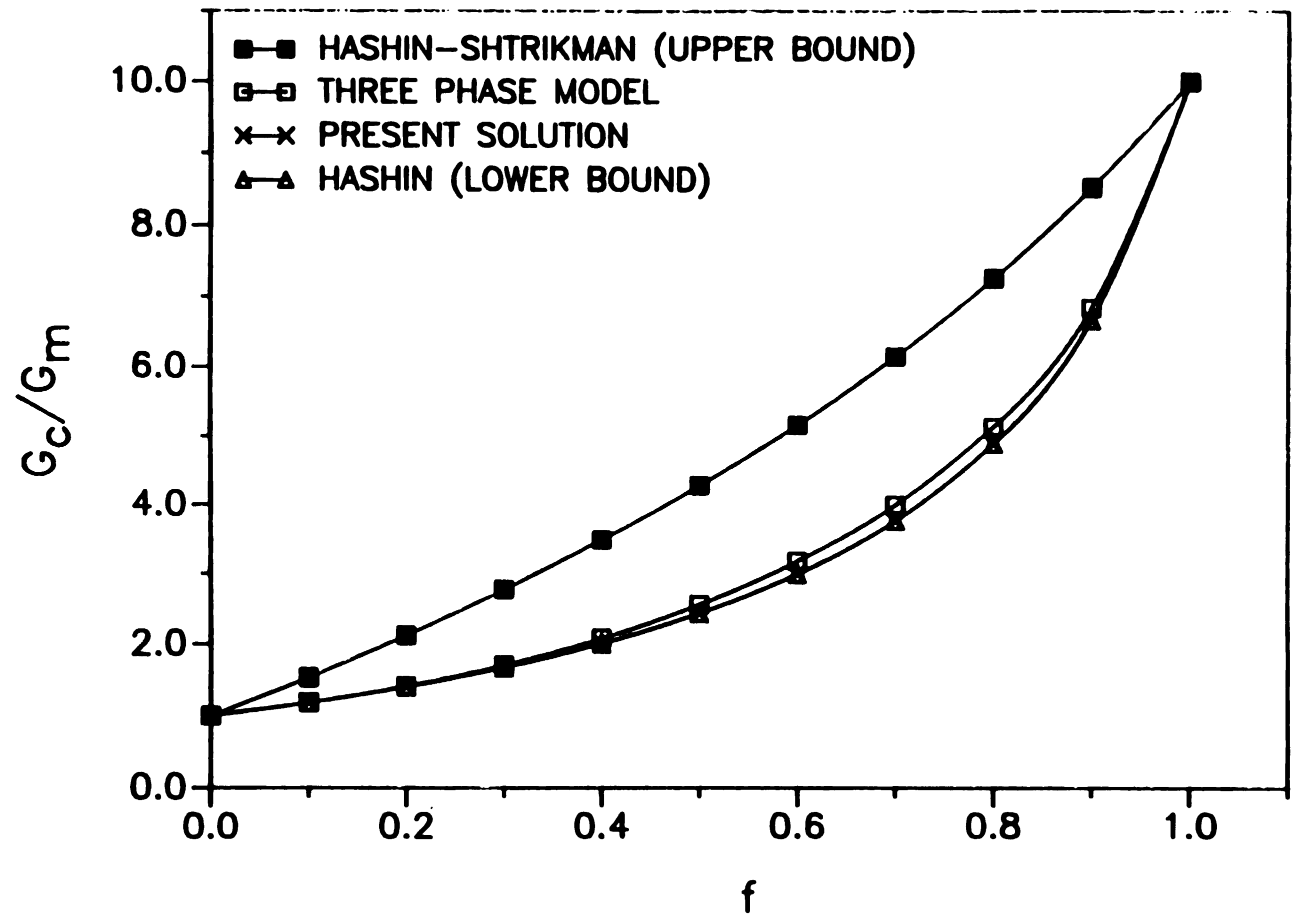

Figure 6.11 G_c/G_m vs. f when $G_f/G_m = 10$ (PB, 3D)

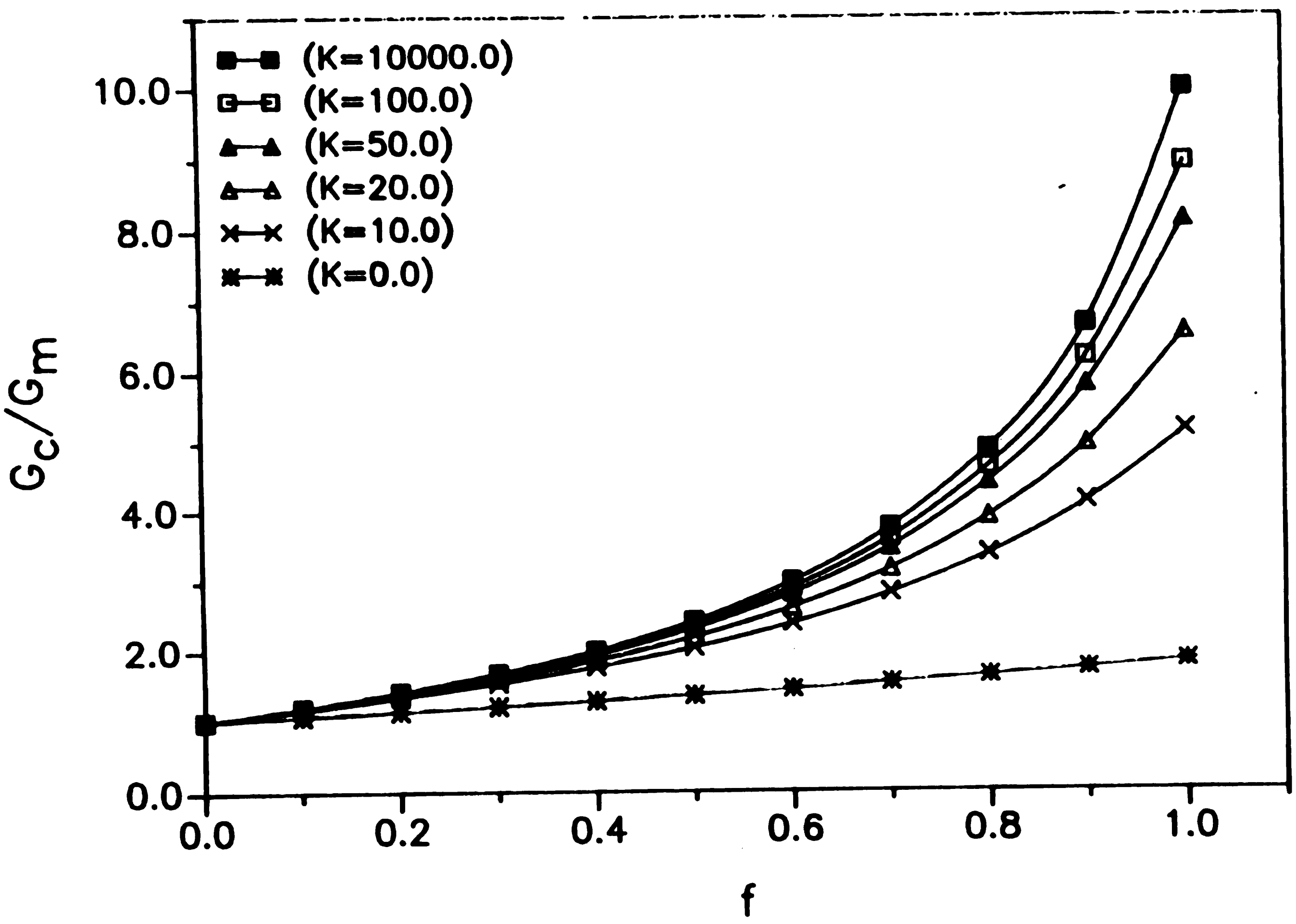

Figure 6.12 G_c/G_m vs. f for changing $\tilde{k} = ka/G_m$ when $\tilde{g} \to \infty$ $G_f/G_m = 10$ (the flexible interface model, 3D)

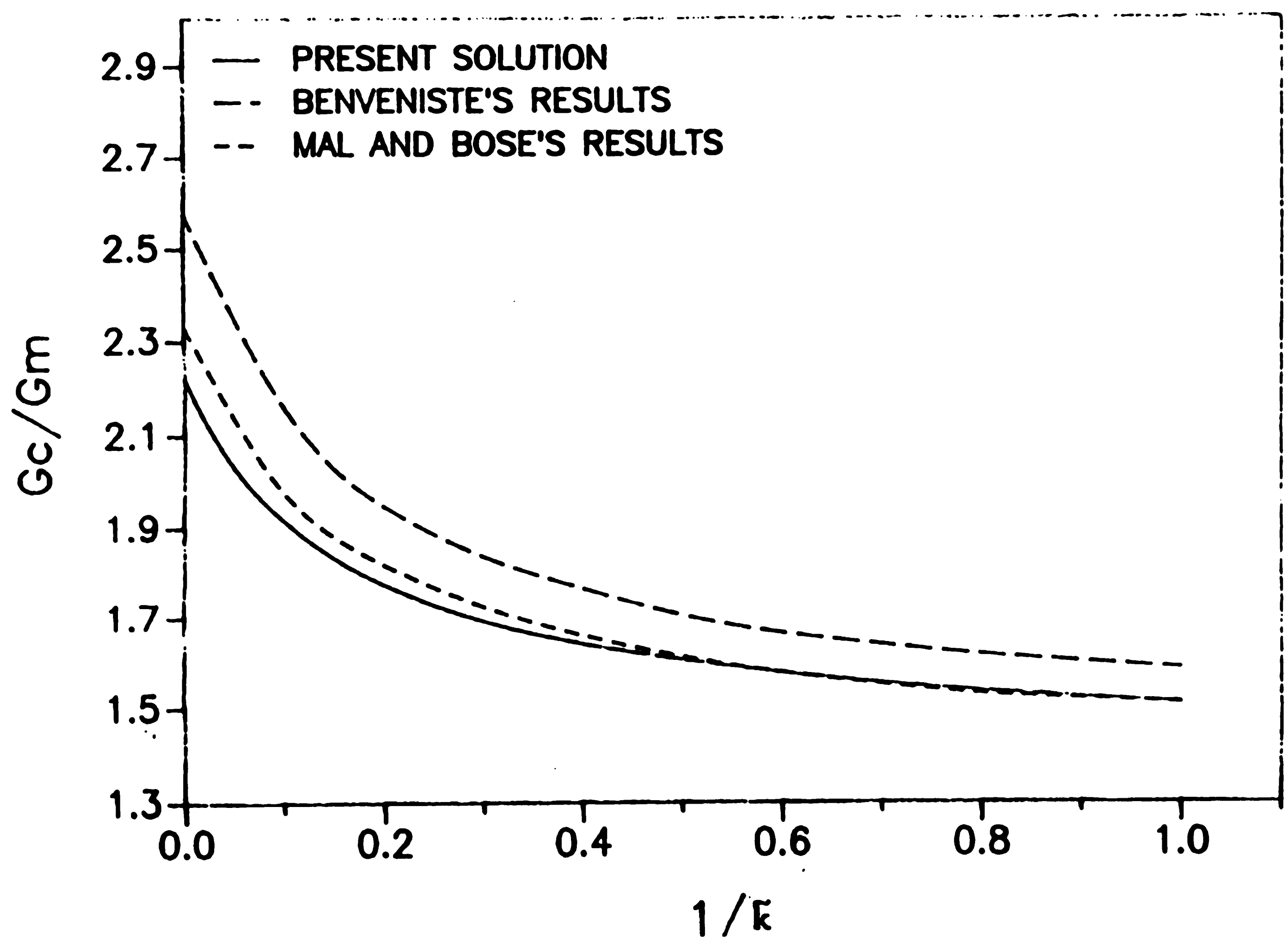

Figure 6.13 G_c/G_m vs. $1/\bar{k}$ for $G_f/G_m = 10$ and $f = 0.4$

(the flexible interface model, 3D)

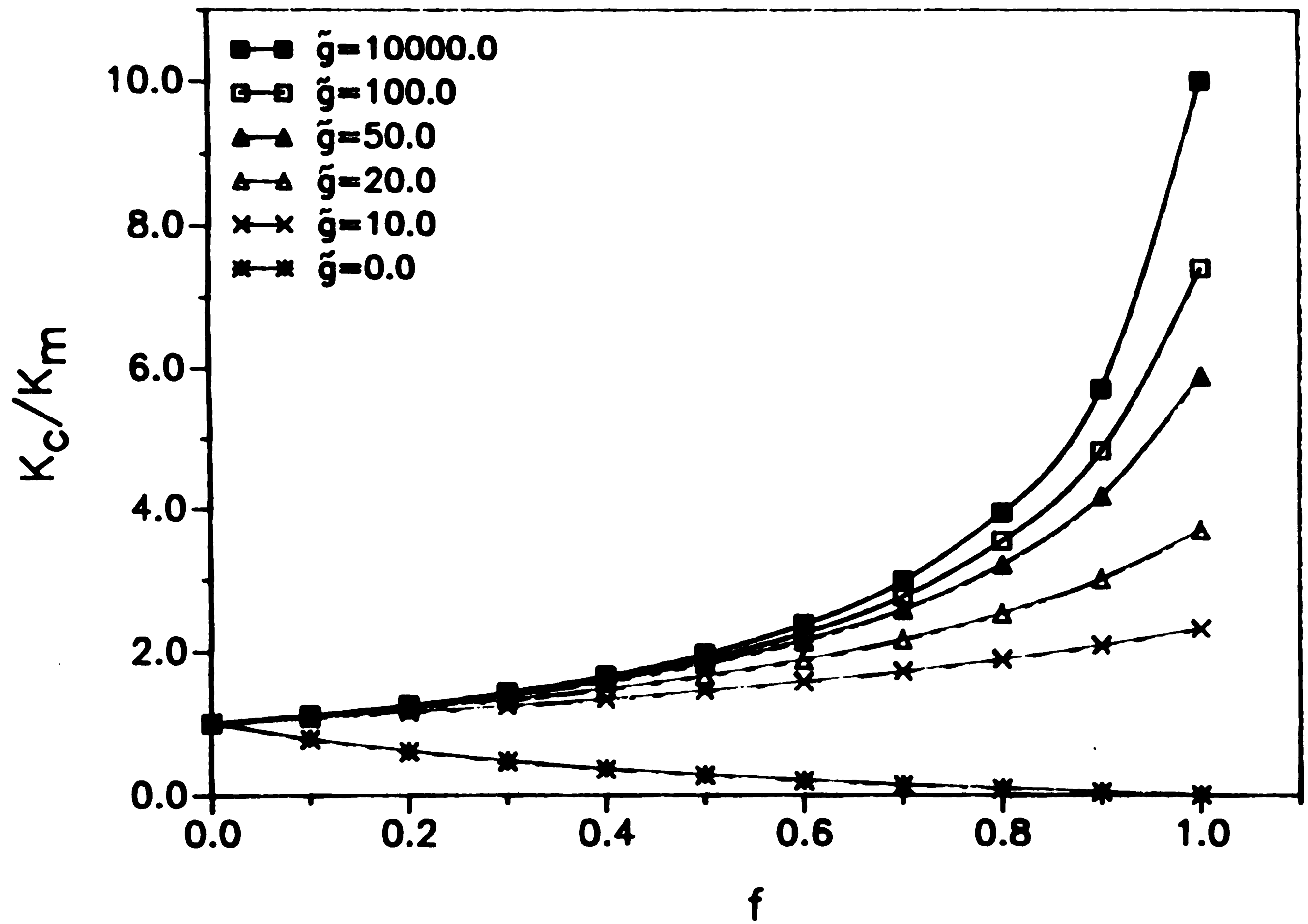

Figure 6.14 Effective bulk modulus K_c/K_m vs. f for changing $\tilde{g}$

(the flexible interface model, 3D)

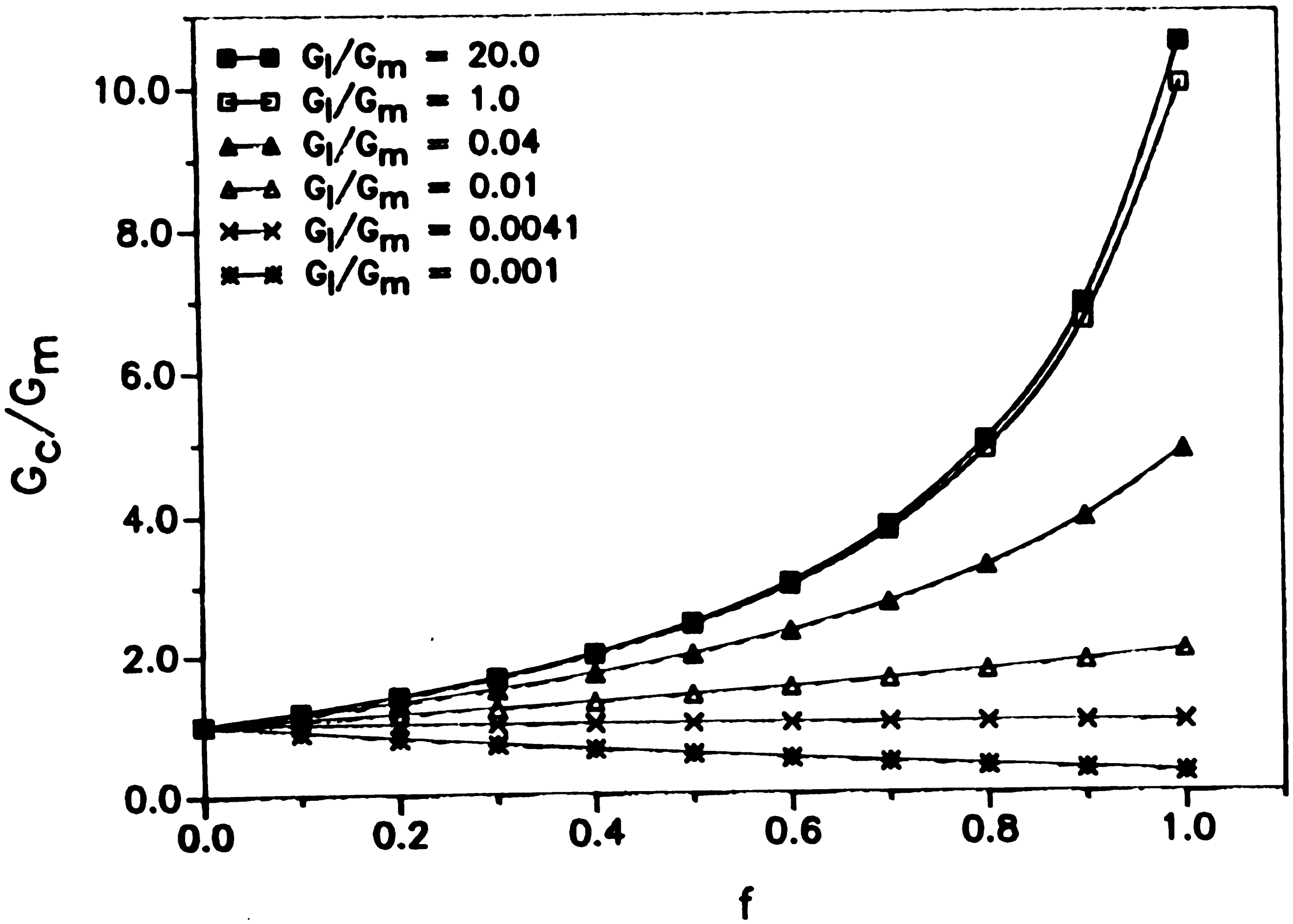

Figure 6.15 G_c/G_m vs. f for $G_f/G_m = 10$, $\ell/f = 0.01$ and changing G_ℓ/G_m (the interphase model, 3D)

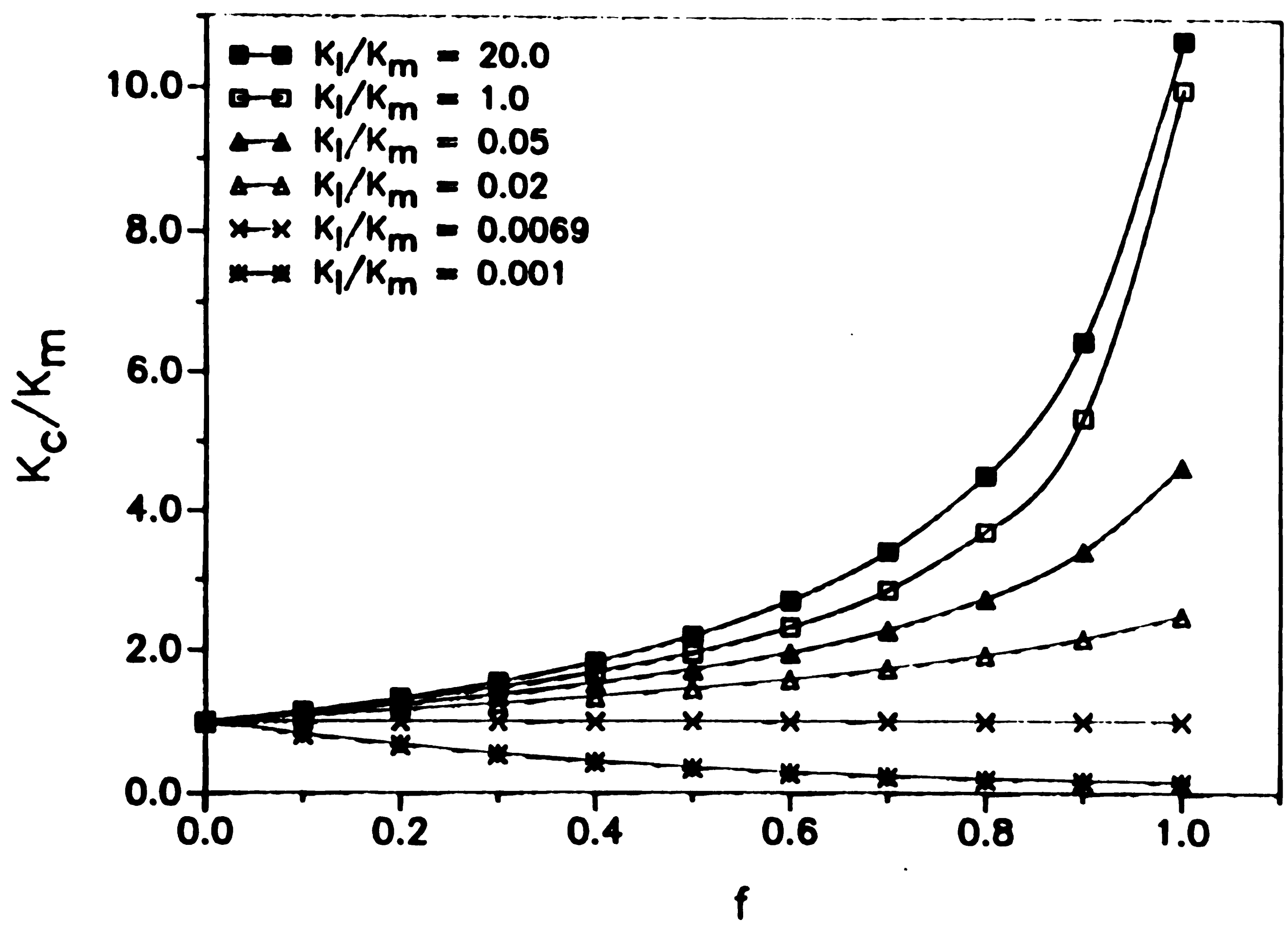

Figure 6.16 K_c/K_m vs. f for K_f/K_m = 10. ℓ/f = 0.01 and changing K_ℓ/K_m (the interphase model, 3D)

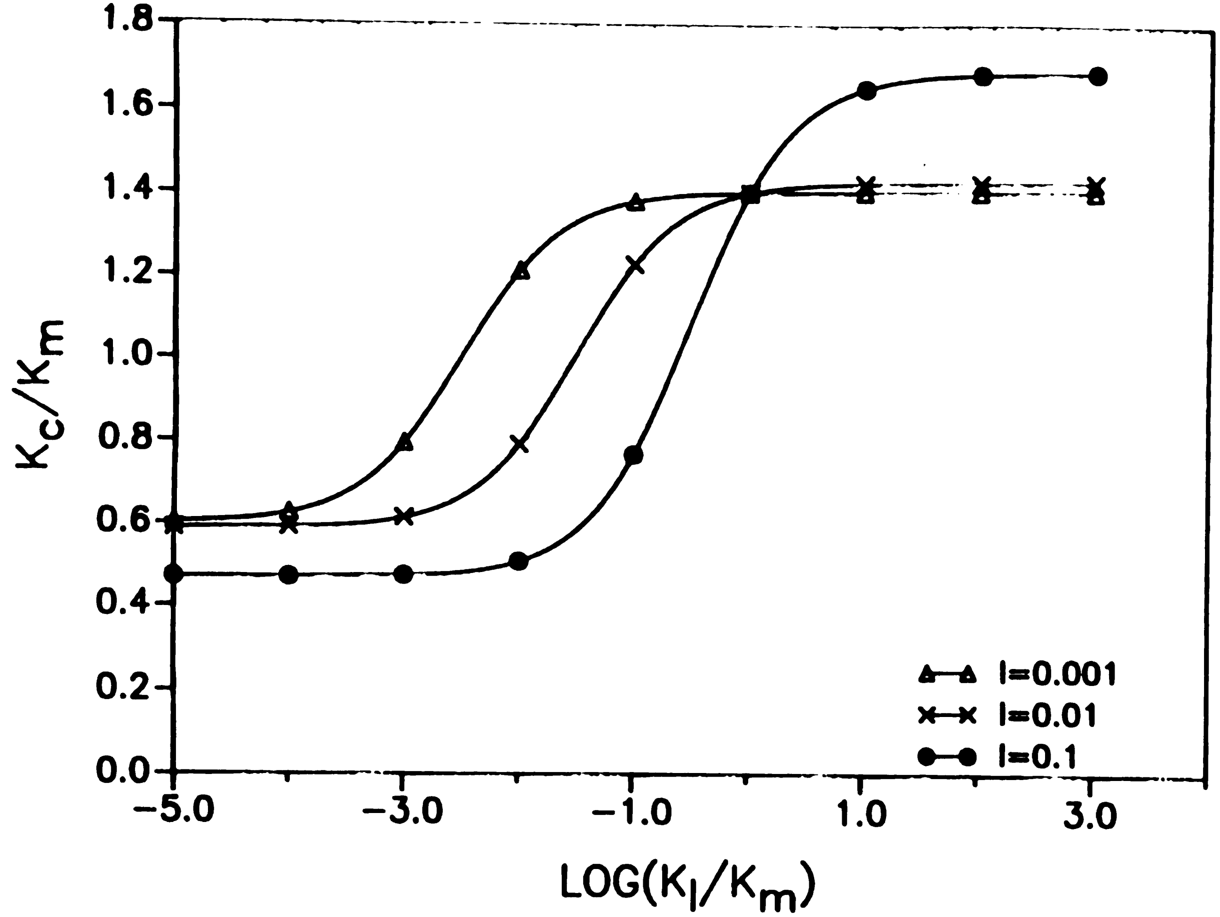

Figure 6.17 K_c/K_m vs. K_ℓ/K_m for changing ℓ/f when $K_m/K_f << 1$ and $f = 0.2$ (the interphase model, 3D)

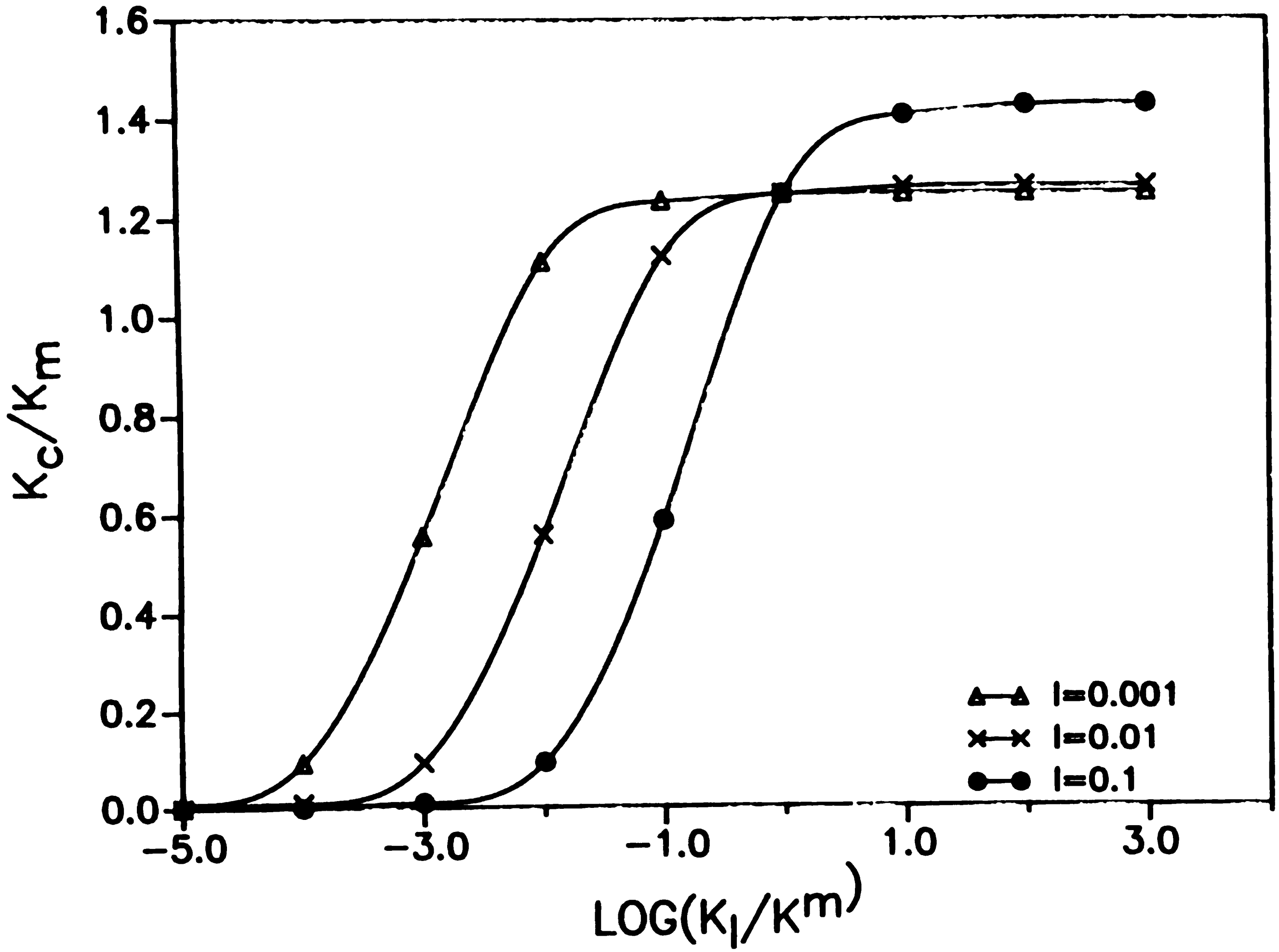

Figure 6.18 K_c/K_m vs. K_ℓ/K_m for changing ℓ/f when K_m/K_f << 1 and f = 0.2 (the interphase model, 3D)

6.2 THERMAL STRESS AND THERMAL EXPANSION COEFFICIENTS

6.2.1 UNIDIRECTIONAL COMPOSITES

Figures 6.19-6.20 show the effective thermal expansion coefficents versus Poisson's ratio of the matrix ν_m for the perfect bonding case. It is interesting to find that the Poisson's ratio of the matrix has a significant effect on the transverse thermal expansion coefficent but insignificant effect on the longitudinal thermal expansion coefficient. Note that when ν_m is great, the transverse thermal expansion coefficent has a peak at $f \approx 0.1$. This increase of α_{cT} can be interpreted in the following manner. The fibers constrain the thermal expansion of the matrix in the longitudinal direction. This constraining (compression) of the matrix in the longitudinal direction is accompanied by the transverse strain components (extensions) equal to the product of the compression strain and Poisson's ratio. At the small volume fraction of fibers, this extension of matrix has more effect on the overall expansion of the composite in the transverse direction than the small thermal expansion of fibers. Therefore in the transverse direction, α_{cT} becomes effectively larger even than that of the matrix.

Figures 6.21-6.23 show the thermal stress versus the radial distance for changing of the interphase thickness with the given composite materials given in Table 6.1. It is seen that the interphase thickness may have significant effect on the stress field in the composite.

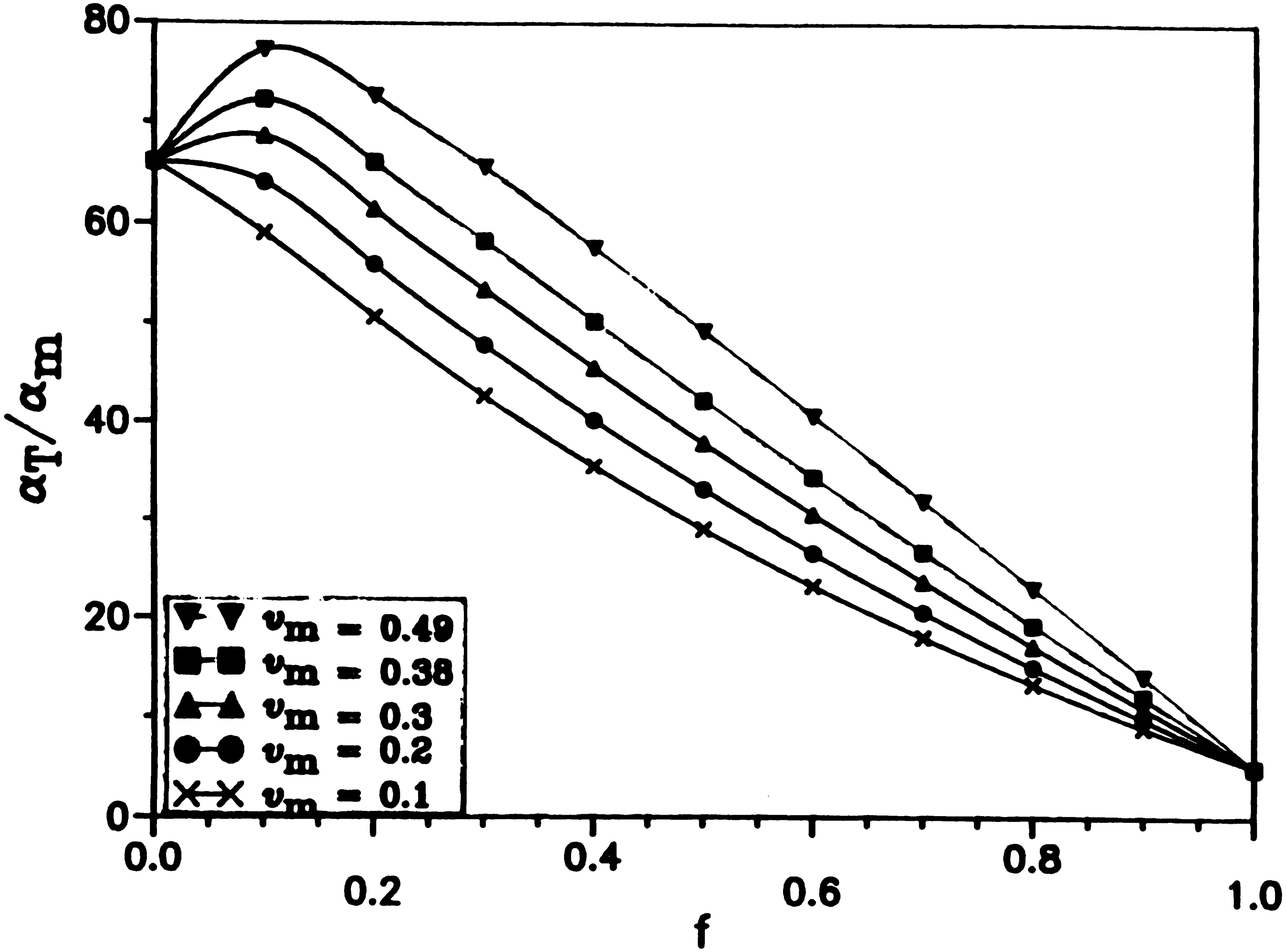

Figure 6.19 Transverse thermal expansion coefficient α_{cT}/α_m vs. f for changing Poisson's ratio of matrix ν_m (PB, 2D)

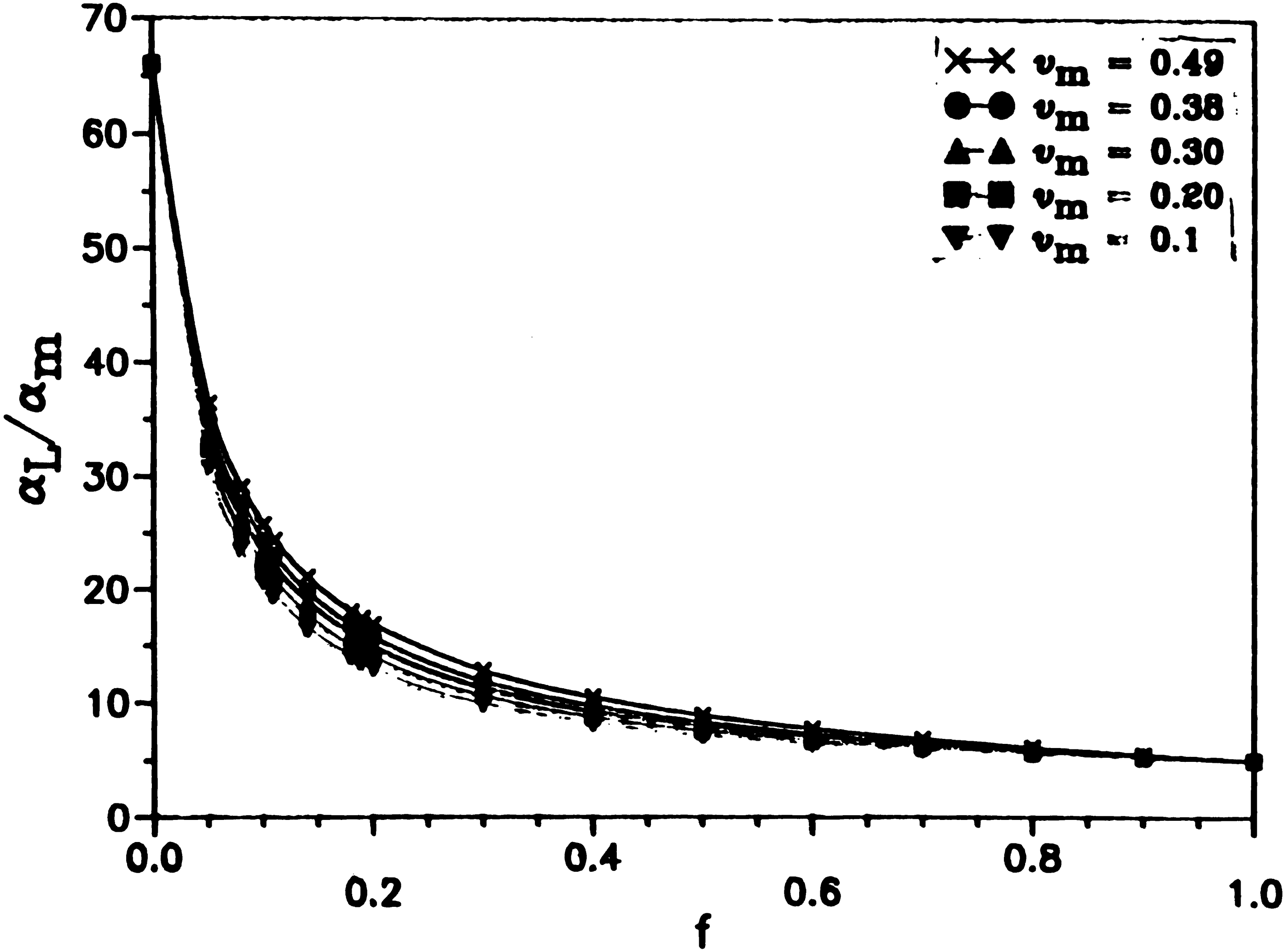

Figure 6.20 Longitudinal thermal expansion coefficient α_{cL}/α_m vs. f for changing Poisson's ratio of matrix ν_m (PB, 2D)

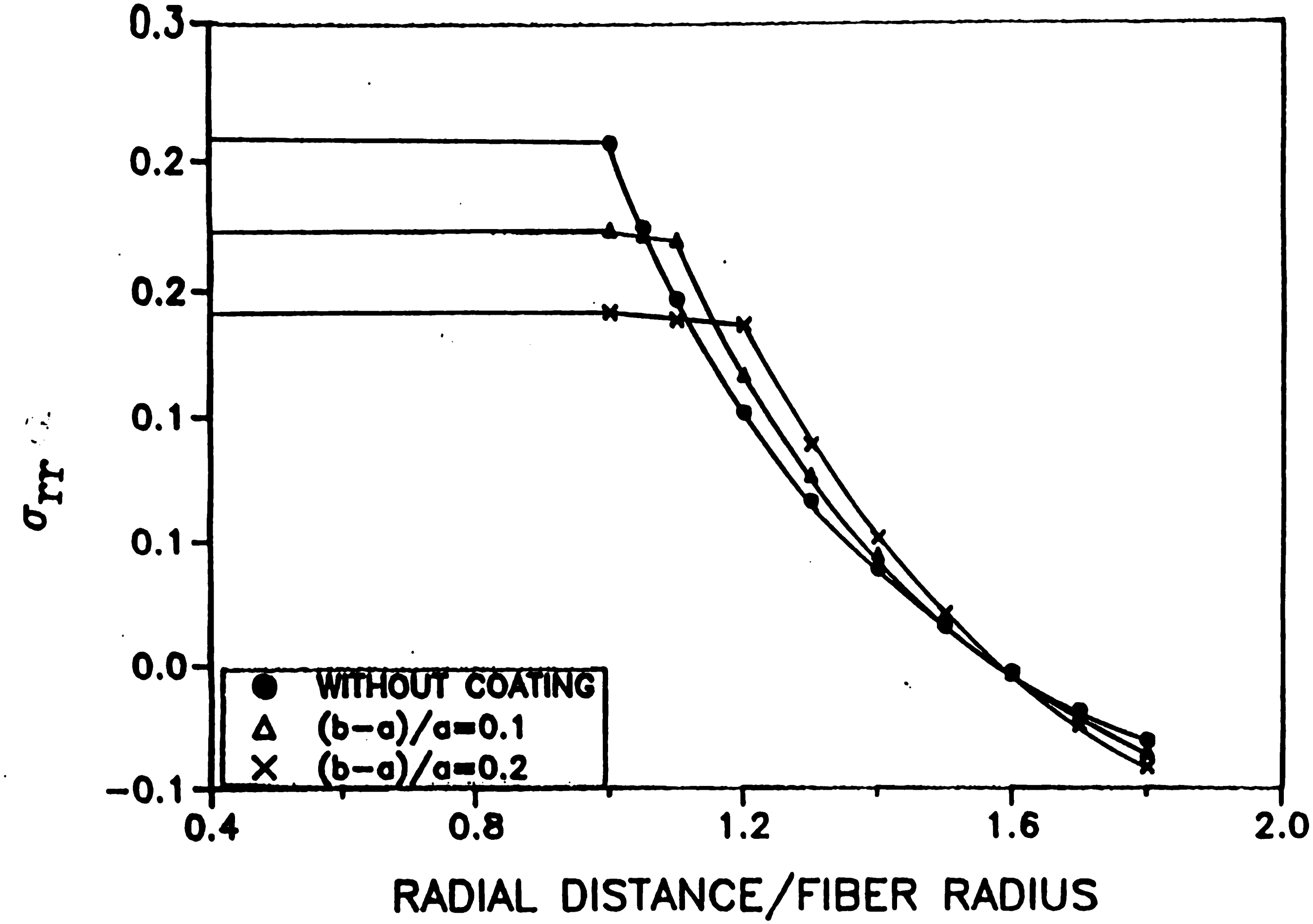

Figure 6.21 Radial stress σ_{rr} vs. radial distance for material 3 with changing of the interphase thickness (the interphase model, 2D)

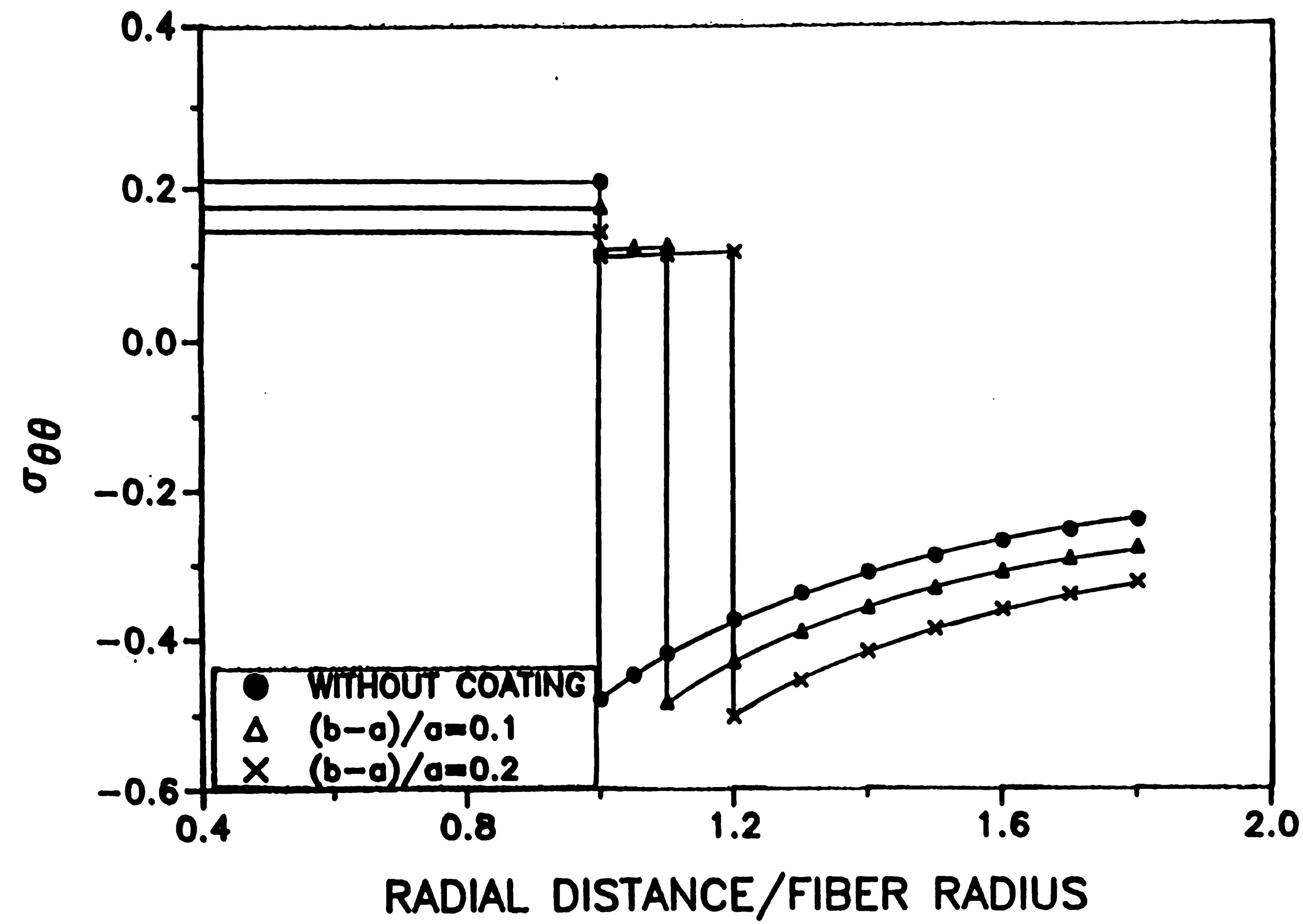

Figure 6.22 Hoop stress $\sigma_{\theta\theta}$ vs. radial distance for material 3 with changing of the interphase thickness (the interphase model, 2D)

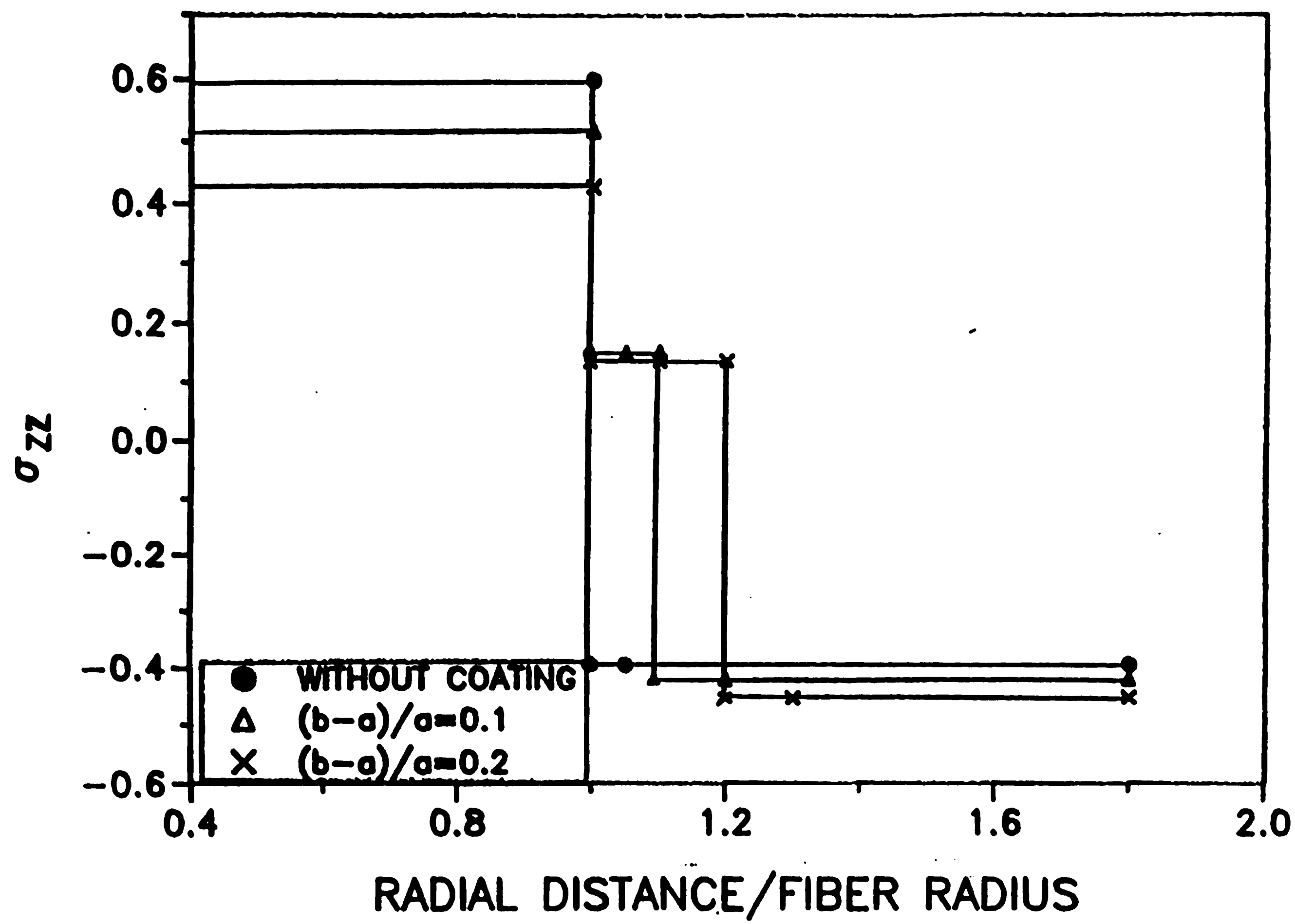

Figure 6.23 σ_{zz} vs. radial distance for material 3 with changing of the interphase thickness (the interphase model, 2D)

6.2.2 PARTICULATE COMPOSITES

Figures 6.24-6.28 represent the ratio of thermal expansion coefficient $\alpha^{\ell} / \alpha^{m}$ versus inclusion volume fraction f.

In Figure 6.24 , the elastic properties of the interlayer are given to be the same as those of the matrix, while the thermal expansion coefficient of the layer is varied from lower to higher values. The numerical results show that the lower ratio of $\alpha_{\ell} / \alpha_{m}$ yields the lower α_{c} / α_{m}.

Figures 6.25 and 6.26 both show the effect of the interlayer thickness on the α_{c} / α_{m}. In Figure 6.25, $\alpha_{\ell} / \alpha_{m}$ is given as 0.5, while in Figure 6.26 it is given as 10. The results show that when $\alpha_{\ell} / \alpha_{m}$ is higher than 1, the thicker layer will yield higher α_{c} / α_{m}, and when $\alpha_{\ell} / \alpha_{m}$ is lower than 1, the results will be opposite.

Figures 6.27 and 6.28 both shows the effect of interlayer stiffness on the α_{c}/ α_{m} when $\alpha_{\ell} / \alpha_{m}$ equals to one. Figure 6.27 shows that when $K_{\ell} / K_{m} > 1$, the increase in K_{ℓ} / K_{m} will cause increase in α_{c} / α_{m}. It is interesting to note that when $K_{\ell}/K_{m} < 1$, the increase in K_{ℓ}/K_{m} will result again in the higher α_{c}/α_{m} (Figure 6.28). It implies that $K_{\ell}/K_{m} = 1$ may give the lower bound of α_{c}/α_{m}.

Figure 6.29 represents the radial stress at inclusion-interphase interface versus K_{ℓ}/K_{m}. It is observed that the change of α_{ℓ}/α_{m} has a significant influence on the interfacial stress. The higher ratio of α_ℓ/α_m will cause higher interfacial stress. The increase of K_ℓ/K_m will also increase the interfacial stress when $\alpha_\ell/\alpha_m>1$. However, when $\alpha_\ell/\alpha_m<1$ and $K_\ell/K_m>1$, there will be no effect on the interfacial stress as K_ℓ/K_m changes. When $K_\ell/K_m=0$, which corresponds to the debonding at the interface, the interfacial stress σ_{rr} becomes zero as expected. One can also see that the solution for single coated inclusion gives upper bound of the actual stress as expected.

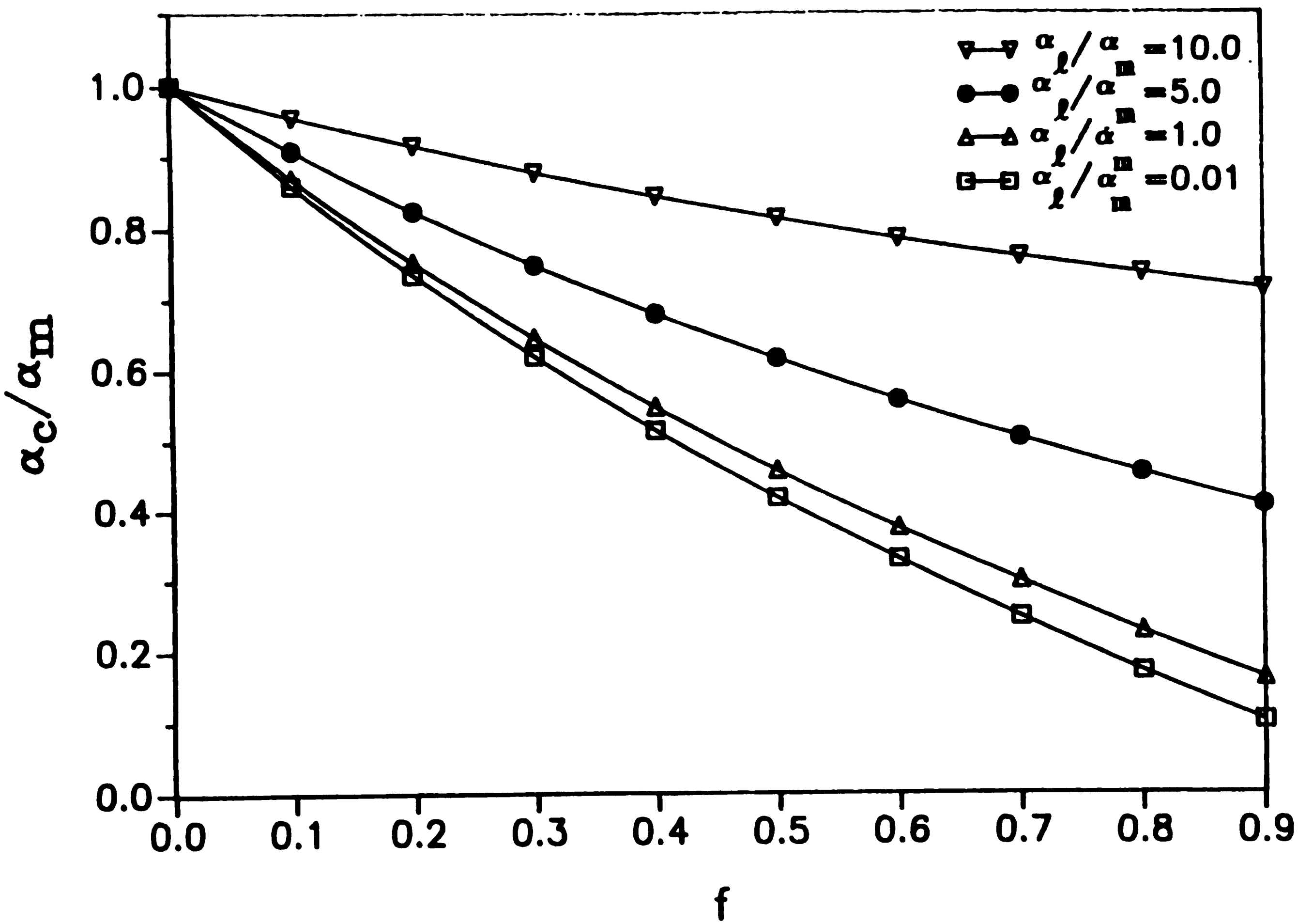

Figure 6.24 Thermal expansion coefficient α_c/α_m vs. f for changing of α_ℓ/α_m (the interphase model, 3D)

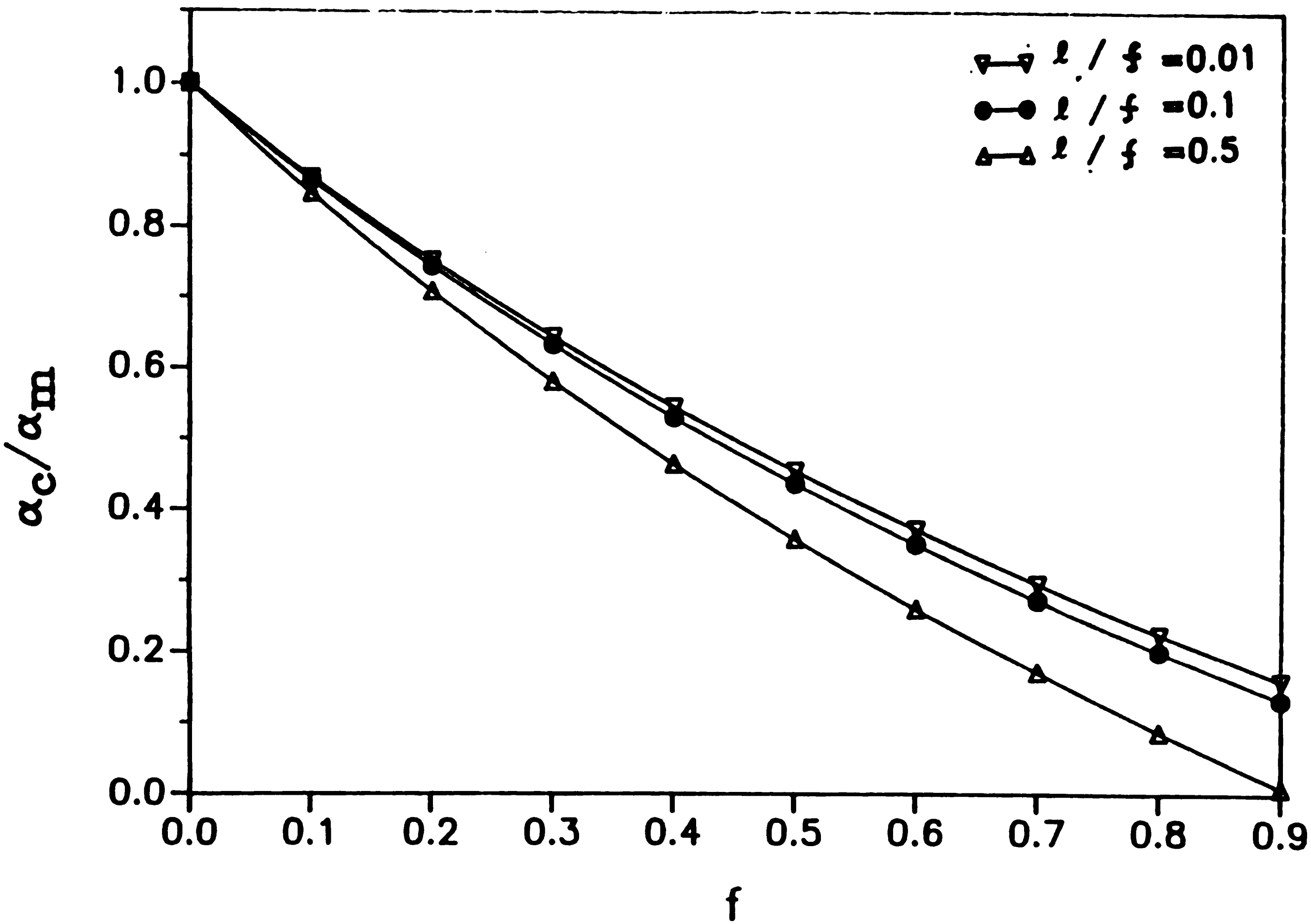

Figure 6.25 Thermal expansion coefficient α_c/α_m vs. f when α_ℓ/α_m = 0.5 and changing ℓ/f (the interphase model, 3D)

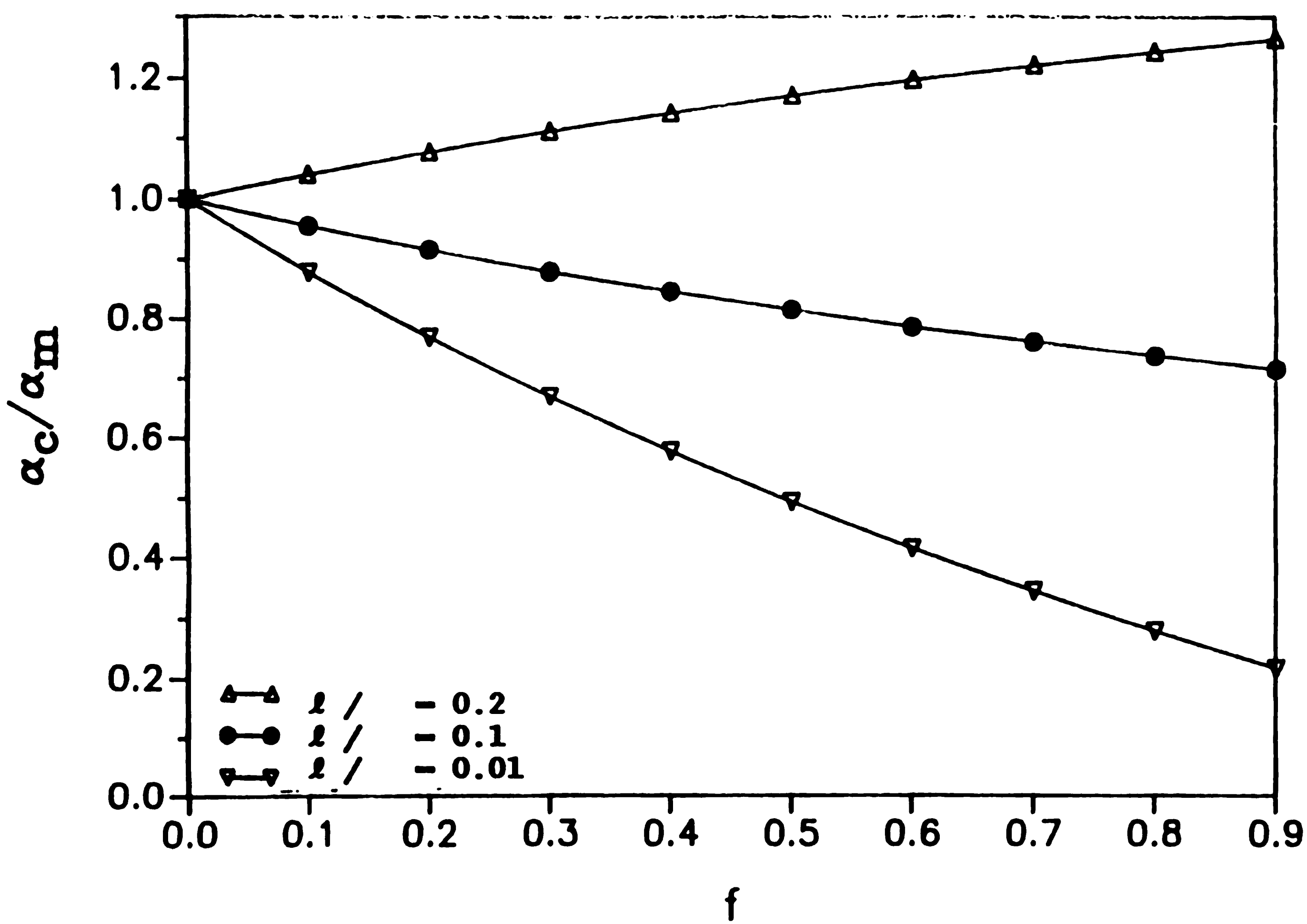

Figure 6.26 Thermal expansion coefficient α_c/α_m vs. f when α_ℓ/α_m = 10 and changing ℓ/f (the interphase model, 3D)

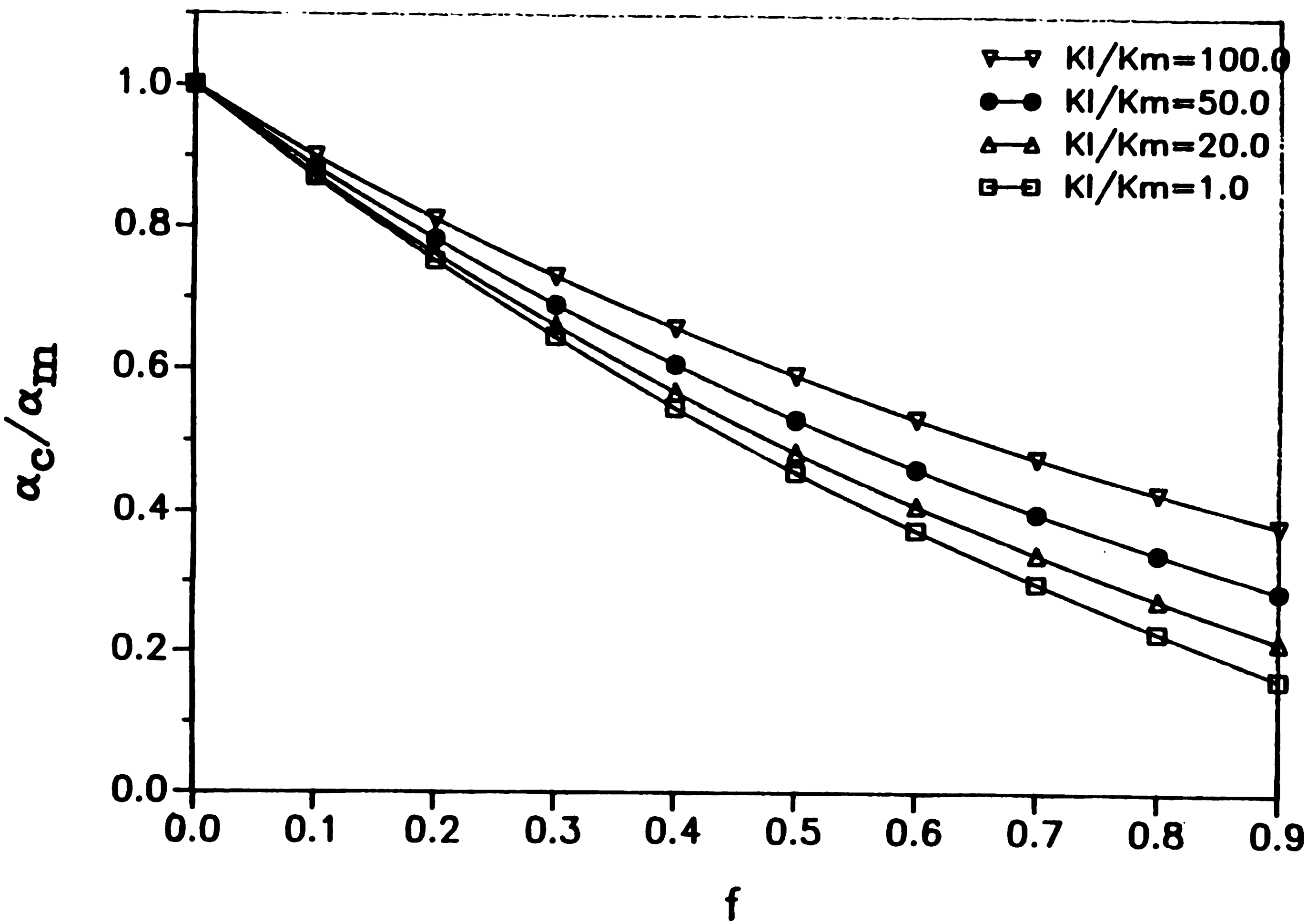

Figure 6.27 Thermal expansion coefficient α_c/α_m vs. f for changing of K_ℓ/K_m when $K_\ell/K_m > 1$ (the interphase model, 3D)

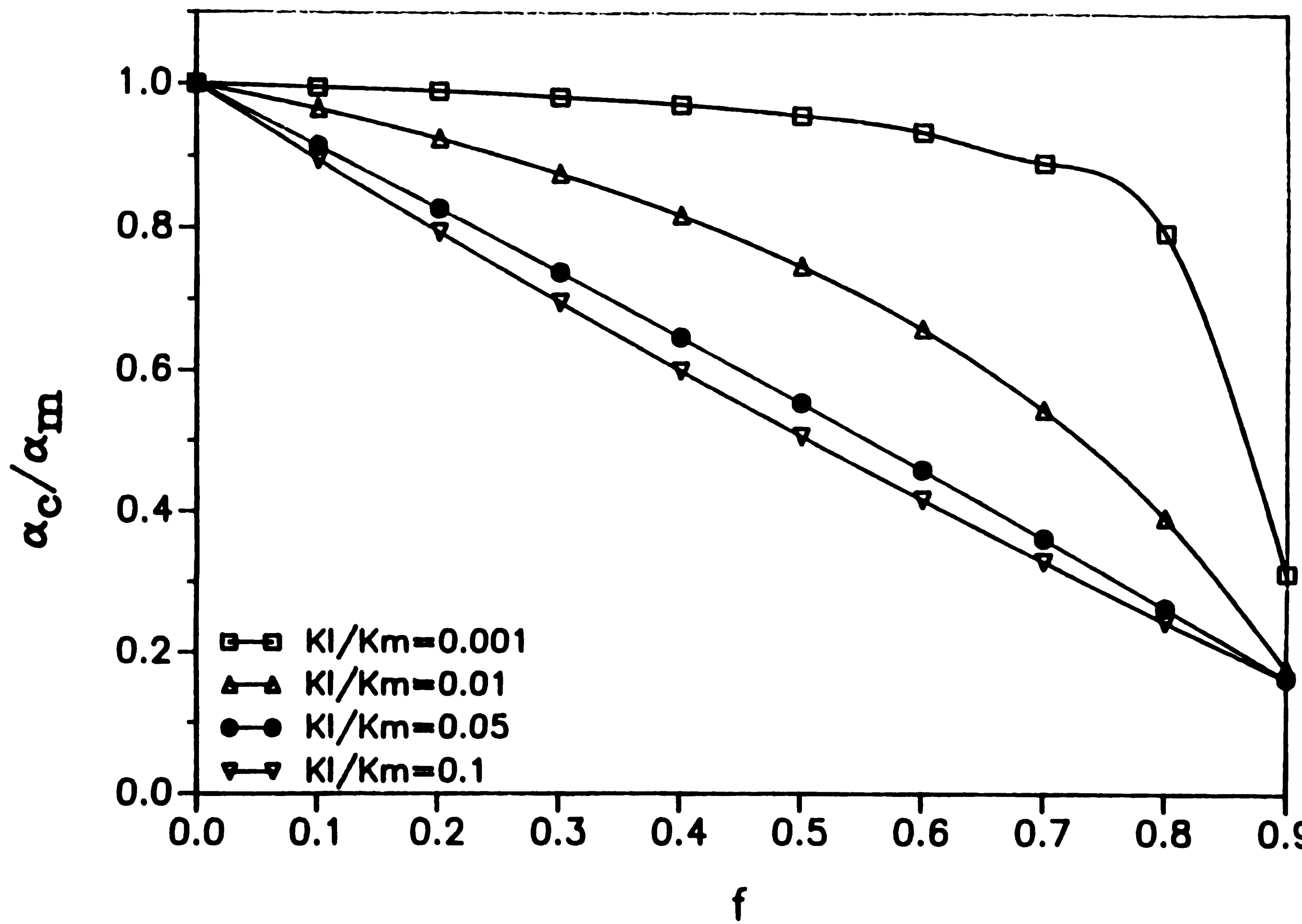

Figure 6.28 Thermal expansion coefficient α_c/α_m vs. f for changing of K_ℓ/K_m when $K_\ell/K_m < 1$ (the interphase model, 3D)

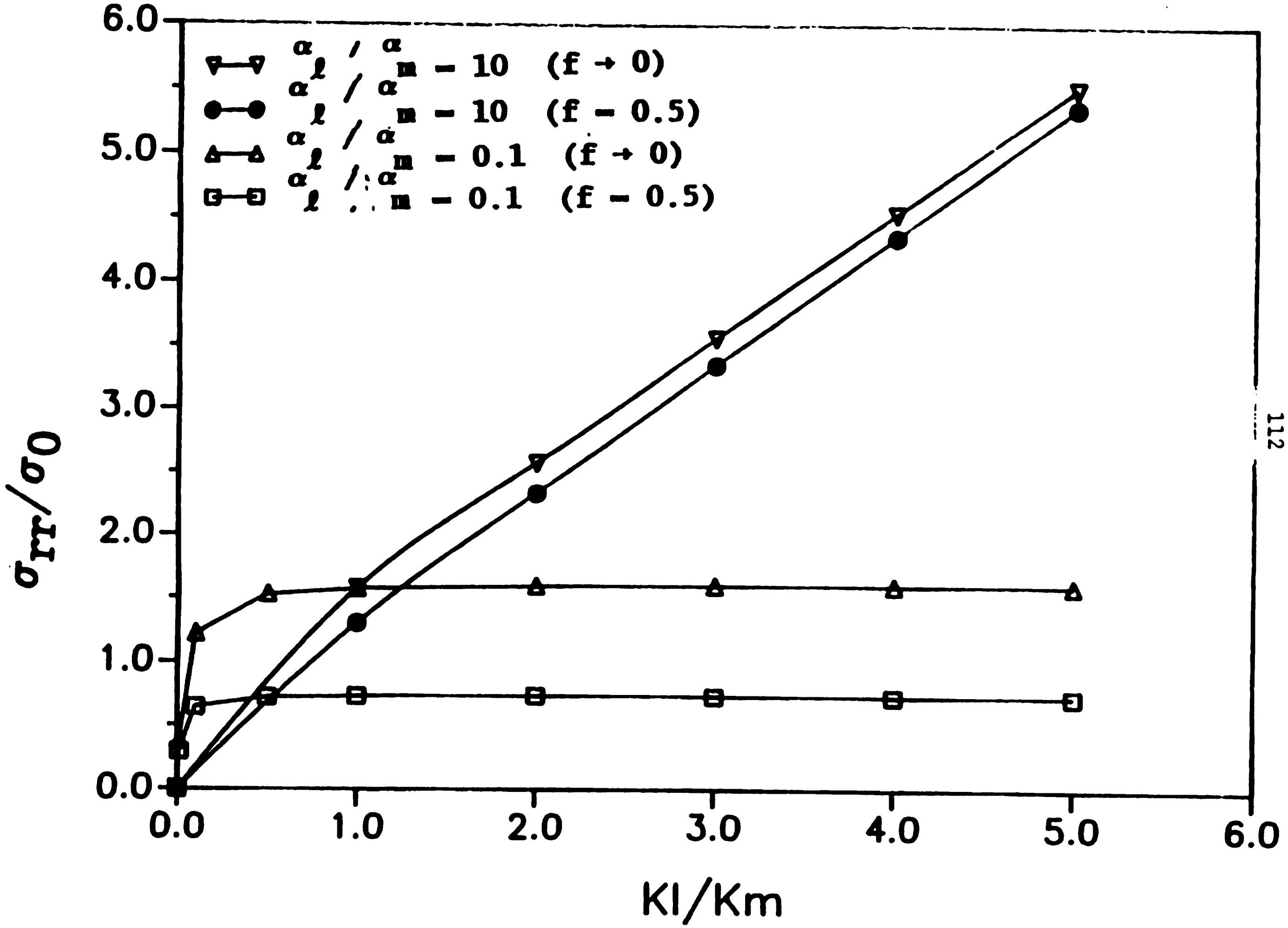

Figure 6.29 Radial stress σ_{rr} at inclusion-interphase interface versus K_ℓ/K_m for changing f and α_ℓ/α_m (the interphase model, 3D)

Figure 6.30 shows the interphase modulus variation following the power law. The same variation is given to the interphase thermal expansion coefficient. Figures 6.31-6.33 show the stresses and the effective thermoelastic properties of the composite with properties gradation at the interphase.

Figures 6.31 and 6.32 show the stresses along the radial direction of the concentric sphere for f and (b-a)/a equal to 0.3 and 0.14 respectively. The stresses are normalized by $\sigma_0 = K_m \alpha_m \Delta T$. The radial distance is normalized by the radius of the matrix, d. The result of the variable layer are compared with those of the constant layer in which the interphase properties are taken as the average of Equation (5.3) and (5.4) over the interphase volume, and the perfect bonding result. It is clear seen that continuous variations in the interphase elastic modulus and the thermal expansion coefficient affect the stress states in all the constituents even though the effects are more pronounced in the inclusions and the interphase. This is very important since the composite failure often initiated in the interphase.

Figure 6.33 shows the effective bulk modulus K_c/K_m versus inclusion volume fraction f. It is observed that the variable layer has significant effect on the effective bulk modulus when the interphase is thick but insignificant effect when the interphase is thin. It is also seen from the figure that when $K_f/K_m > 1$, the thicker layer will increase the effective bulk modulus.

Figure 6.34 shows the effective thermal expansion coefficient α_c/α_m versus f. It is observed that the variable layer has insignificant effect on the effective thermal expansion coefficient. It is also seen from the figure that wheb $\alpha_f/\alpha_m < 1$, the thick layer will decrease the effective thermal expansion coefficient.

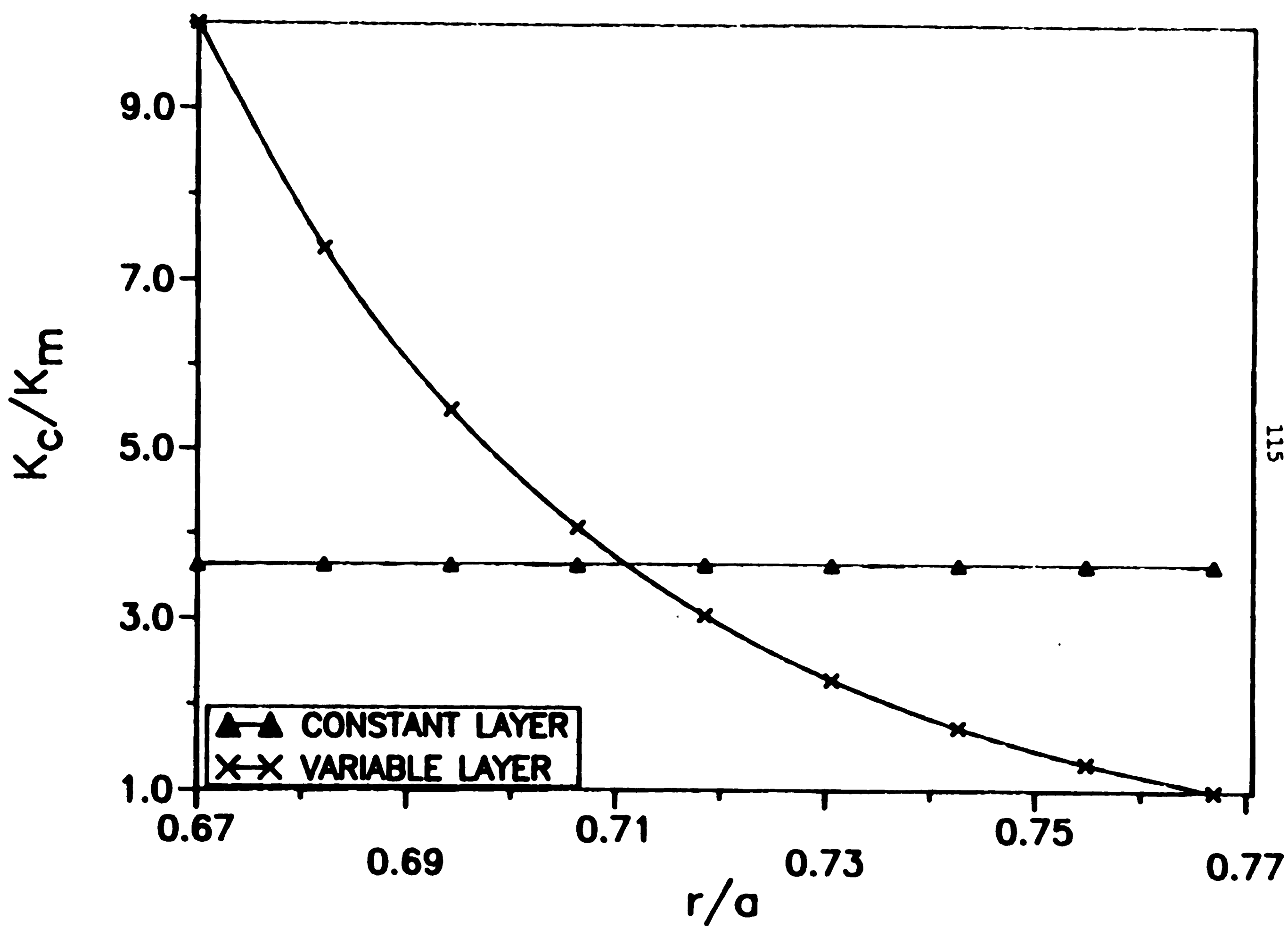

Figure 6.30 Interphase modulus variation

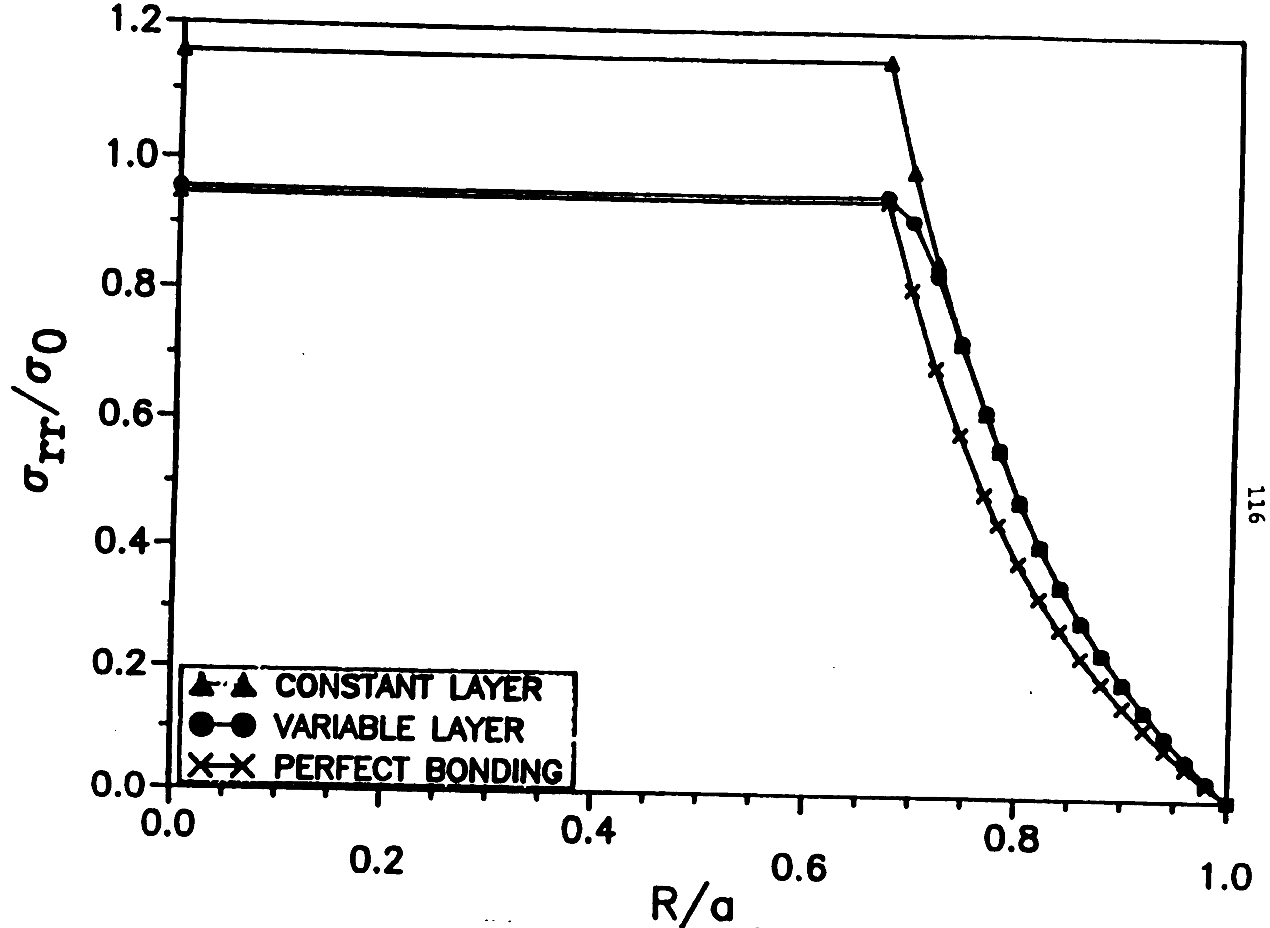

Figure 6.31 Radial stress σ_{rr}/σ_0 vs. radial distance r/a with f = 0.3 (b-a)/a = 0.14 (the interphase model, 3D)

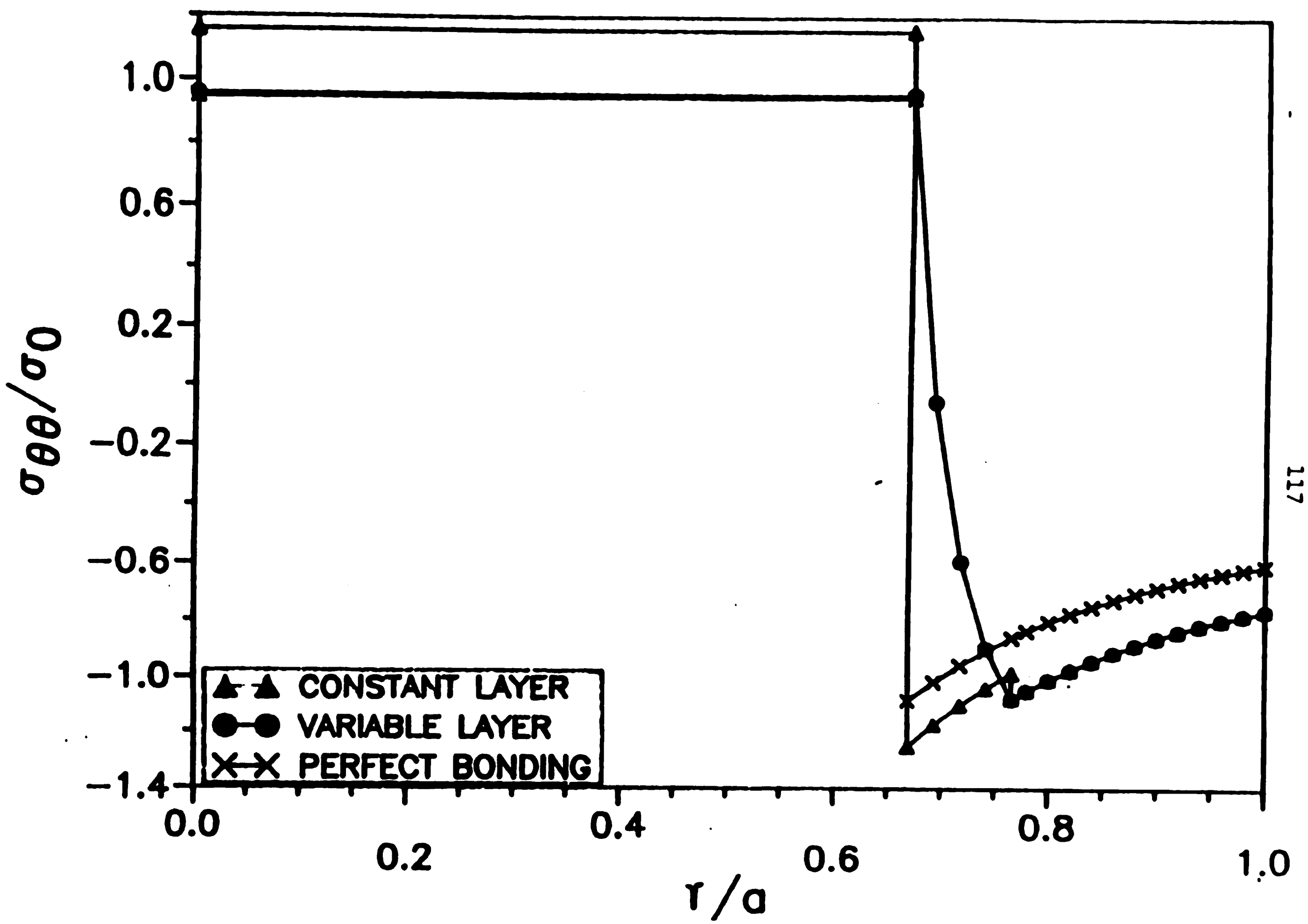

Figure 6.32 Hoop stress $\sigma_{\theta\theta}/\sigma_0$ vs. radial distance r/a

with f = 0.3, (b-a)/a = 0.14 (the interphase model, 3D)

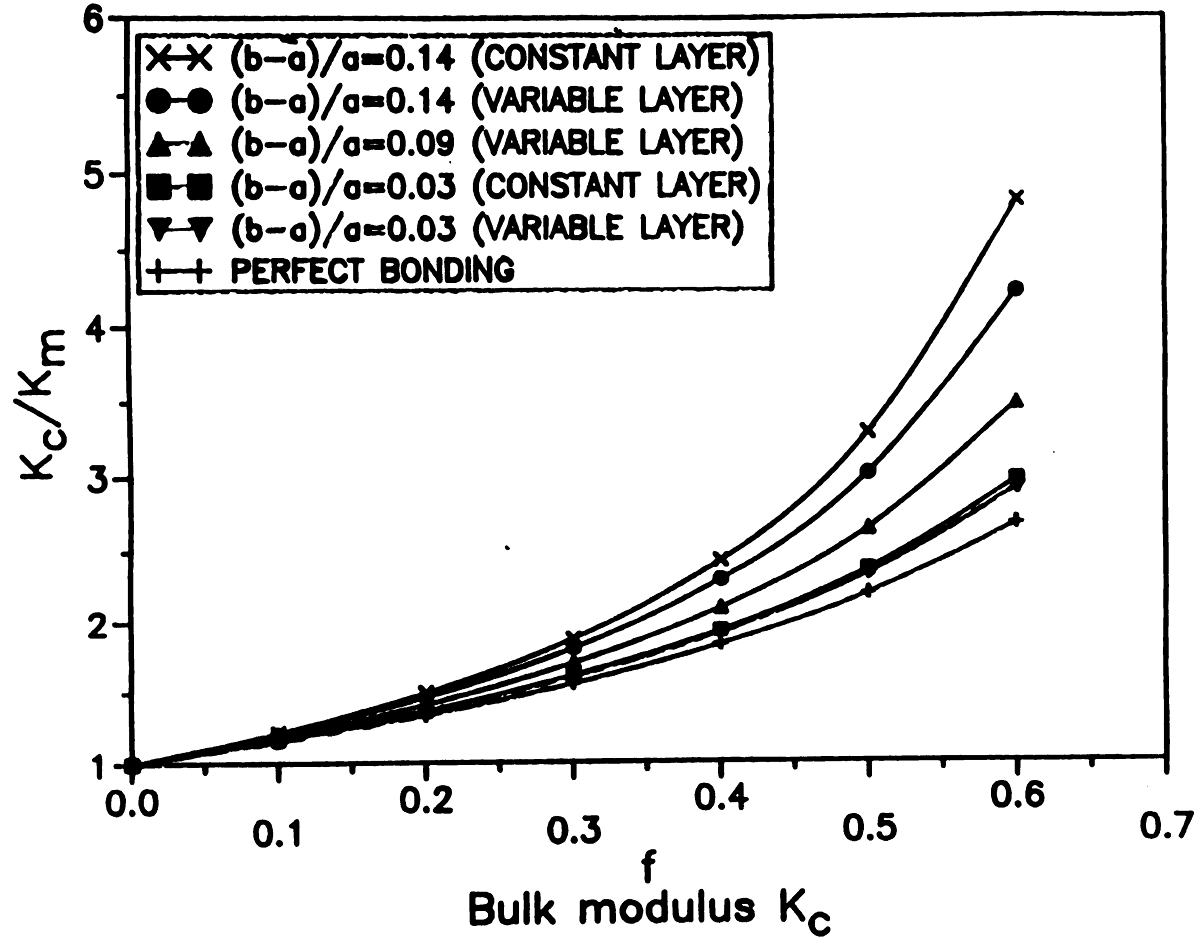

Figure 6.33 Bulk modulus K_c/K_m vs. f for changing the interphase thickness

(the interphase model, 3D)

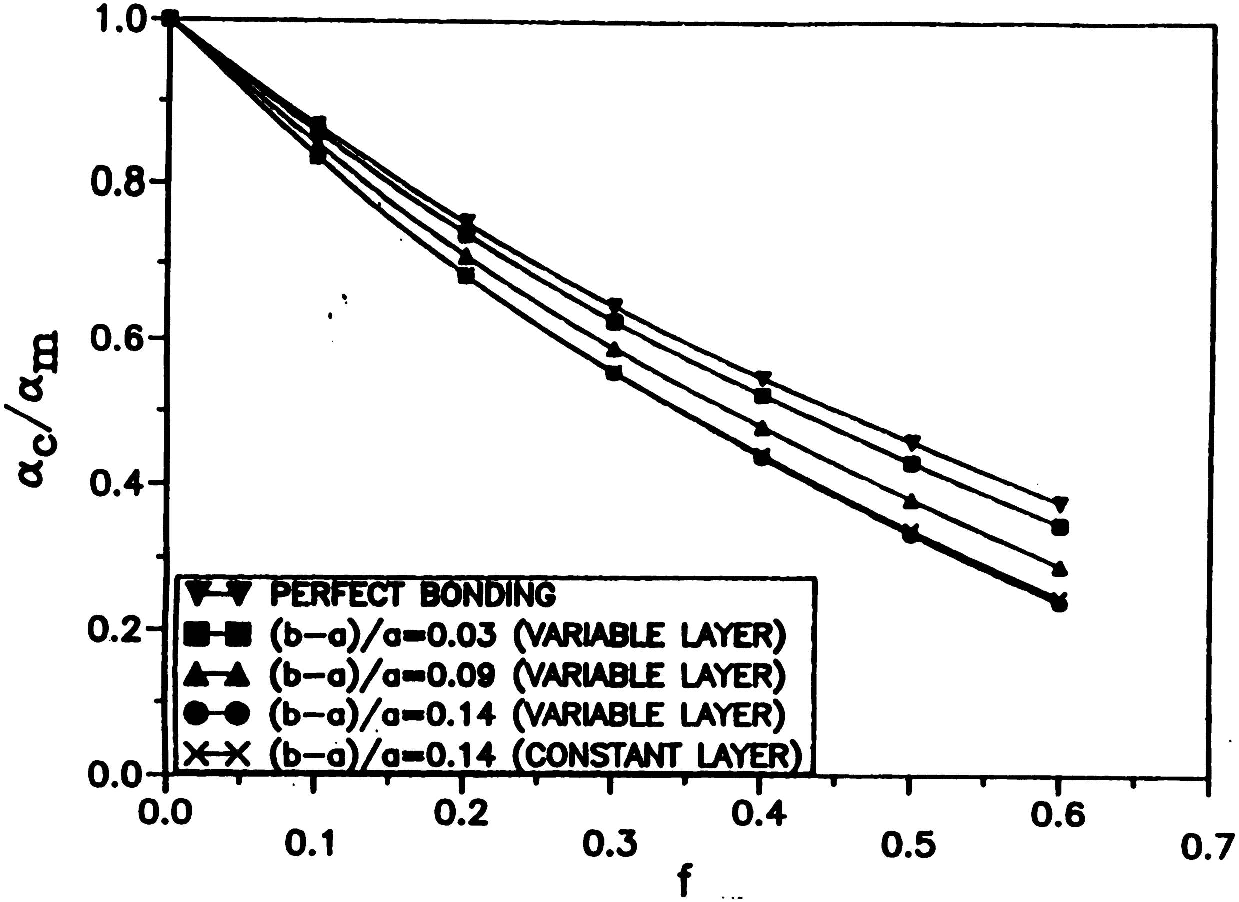

Figure 6.34 Thermal expansion coefficient α_c/α_m vs. f for changing the variable interphase thickness (the interphase model)

CHAPTER 7

CONCLUSIONS

The local stress and displacement fields and the effective thermo-mechanical properties of composite materials are predicted for composites with imperfectly bonded interface by considering the two interfacial models: the flexible interface model and the interphase model. The closed form solutions are obtained. The results from this study are compared to the perfect bonding results, bounds, and other analytical results. It is observed that the interface behavior may have significant effect on the stress field, effective elastic moduli and thermal expansion coefficients.

APPENDIX A

TRANSVERSE SHEAR MODULUS (2D) THE INTERPHASE MODEL AND THE FLEXIBLE INTERFACE MODEL

A1. A SINGLE CYLINDRICAL INCLUSION

A1.1 The interphase model

Consider a cylindrical coated inclusion embedded in a matrix is subjected to a uniform transverse shear stress $\sigma_{12}^{(0)} = \tau_0$ at infinity. Both the inclusion and the matrix are assumed isotropic. The displacement fields are as follows:

a) in the inclusion $(r < a)$

$$\frac{2G_f u_r^f}{\tau_0} = [\ (\kappa_f - 3)A_f r^3 - 2B_f r\ ]\ \sin 2\theta \tag{A1-1}$$

$$\frac{2G_f u_\theta^f}{\tau_0} = -\ [\ (\kappa_f + 3)A_f r^3 + 2B_f r\ ]\ \cos 2\theta \tag{A1-2}$$

b) in the layer $(a < r < b)$

$$\frac{2G_\ell u_r^\ell}{\tau_0} = [\ (\kappa_\ell - 3)A_\ell r^3 - 2B_\ell r$$

$$+ (\kappa_\ell+1)C_\ell\frac{1}{r} + 2D_\ell\frac{1}{r^3} \,]\, \sin 2\theta \tag{A1-3}$$

$$\frac{2G_\ell u_\theta^\ell}{\tau_0} = -\,[\; (\kappa_\ell+3)A_\ell r^3 + 2B_\ell r$$

$$- (\kappa_\ell-1)C_\ell\frac{1}{r} + 2D_\ell\frac{1}{r^3} \,]\, \cos 2\theta \tag{A1-4}$$

c) in the matrix $\quad (r > b)$

$$\frac{2G_m u_r^m}{\tau_0} = [\; r + (\kappa_m+1)C_m\frac{1}{r} + 2D_m\frac{1}{r^3} \,]\, \sin 2\theta \tag{A1-5}$$

$$\frac{2G_m u_\theta^m}{\tau_0} = [\; r + (\kappa_m-1)C_m\frac{1}{r} - 2D_m\frac{1}{r^3} \,]\, \cos 2\theta \tag{A1-6}$$

In the notation used

$$\kappa_i = \begin{cases} 3-4\nu_i & \text{for plane strain} \\ (3-\nu_i)/(1+\nu_i) & \text{for plane stress} \end{cases} \qquad (i = f, \ell, m)$$

where ν_i is the Poisson's ratio of the components and G_i are the transverse shear moduli. "a" is radius of inclusion and "b" is outside radius of layer. Subscripts or superscripts f, ℓ, m correspond to the fiber, layer, and matrix, respectively. A_f, B_f, A_ℓ, B_ℓ, C_ℓ, D_ℓ, C_m and D_m are the constants determined from the perfect bonding boundary conditions, involving continuity of tractions and displacements at fiber-layer and layer-matrix interfaces:

$$\sigma^{f}_{rr} = \sigma^{\ell}_{rr} \quad u^{f}_{r} = u^{\ell}_{r} \quad u^{f}_{\theta} = u^{\ell}_{\theta} \qquad \text{at } r = a \tag{A1-7}$$

$$\sigma^{\ell}_{rr} = \sigma^{m}_{rr} \quad u^{\ell}_{r} = u^{m}_{r} \quad u^{\ell}_{\theta} = u^{m}_{\theta} \qquad \text{at } r = b \tag{A1-8}$$

where "a" is the radius of the fiber and "b" is the outside radius of the layer. Note that the boundary condition $\sigma^{m}_{rr} = \tau_0$ at infinity is satisfied automatically by Equation (A1-5)-(A1-6).

A1.2 The flexible interface model

Consider a cylindrical inclusion embedded in a matrix is subjected to a uniform shear stress $\sigma^{(0)}_{12} = \tau_0$ at infinity. Both inclusion and the matrix are assumed to be isotropic. The displacement fields are given as follows:

a) in the inclusion (r < a)

$$\frac{2G_f u^f_r}{\tau_0} = [\ (\kappa_f - 3)A_f r^3 - 2B_f r\ ]\ \sin 2\theta \tag{A1-9}$$

$$\frac{2G_f u^f_\theta}{\tau_0} = -\ [\ (\kappa_f + 3)A_f r^3 + 2B_f r\ ]\ \cos 2\theta \tag{A1-10}$$

b) in the matrix (r < a)

$$\frac{2G_m u_r^m}{\tau_0} = \left[r + (\kappa_m+1)C_m\frac{1}{r} + 2D_m\frac{1}{r^3} \right] \sin 2\theta \tag{A1-11}$$

$$\frac{2G_m u_\theta^m}{\tau_0} = \left[r + (\kappa_m-1)C_m\frac{1}{r} - 2D_m\frac{1}{r^3} \right] \cos 2\theta \tag{A1-12}$$

where A_f, B_f, C_m and D_m are the constants determined from the interface boundary conditions as follows:

$$\sigma_{r\theta}^f = \sigma_{r\theta}^m = k\ (u_\theta^m - u_\theta^f) \qquad \text{at } r = a \tag{A1-13}$$

$$\sigma_{rr}^f = \sigma_{rr}^m = g\ (u_r^m - u_r^f) \qquad \text{at } r = a \tag{A1-14}$$

where "a" is the radius of the fiber, and "k" and "g" are the spring constants . Note that the boundary condition $\sigma_{rr}^m = \tau_0$ at infinity is automatically satisfied by Equations (A1-11)-(A1-12).

A2. Effective shear modulus for the composites with finite volume fraction of inclusions

A2.1 The interphase model

The single inclusion solution obtained in section A1 is used in the successive iteration scheme (Mori and Wakashima, 1990), which is based on Mori-Tanaka average field theory (Mori and Tanaka, 1973), to evaluate the effective shear moduli. Considering the average strain in the composite, the effective shear modulus is obtained as:

$$\frac{1}{G_c} = \frac{1}{G_m} + \frac{f+\ell}{1 + (f+\ell)\alpha}\beta \tag{A2-1}$$

where

$$\beta = \frac{a^2}{b^2}[3A_f a^2 + 2B_f](\frac{1}{G_m} - \frac{1}{G_f})$$

$$+ \frac{1}{b^2}[3A_\ell(b^4-a^4) + 2B_\ell(b^2-a^2)](\frac{1}{G_m} - \frac{1}{G_\ell}) \tag{A2-2}$$

$$\alpha = -\frac{a^2}{b^2}[3A_f a^2 + 2B_f]$$

$$-\frac{(b^2-a^2)}{b^2}[3A_\ell(b^2+a^2) + 2B_\ell)] - 1 \tag{A2-3}$$

A2.2 The flexible interface model

The single inclusion solution obtained in section A1 is used in the iteration scheme (Mori and Wakashima, 1990), which is based on Mori-Tanaka average field theory (Mori and Tanaka, 1973), to evaluate the effective shear modulus.

Considering the average strain in the composite, the effective shear modulus is obtained as:

$$\frac{1}{G_c} = \frac{1}{G_m} + \frac{f}{1 + f\alpha}\beta \tag{A2-4}$$

where

$$\beta = [\ 3A_f a^2 + 2B_f](\frac{1}{G_m} - \frac{1}{G_f})$$

$$+ \frac{1}{2G_m} [\ 1 + \frac{2G_m}{a^2} (\kappa_m - 1) - \frac{2D_m}{a^2}\ ]$$

$$- \frac{1}{2G_m} [\ A_f a^2(\kappa_f + 3) + 2B_f] \qquad \text{(A2-5)}$$

$$\alpha = - [3A_f a^2 + 2B_f] - 1 \qquad \text{(A2-6)}$$

APPENDIX B

TRANSVERSE BULK MODULUS (2D)
THE INTERPHASE MODEL AND
THE FLEXIBLE INTERFACE MODEL

B1. A single cylindrical inclusion

B1.1 The interphase model

Consider a single coated fiber is embedded in the matrix. All of the constitutes are linear elastic and isotropic materials. The applied loading is the transverse hydrostatic stress $\sigma_{xx}^{(0)} = \sigma_{yy}^{(0)} = \sigma_0$ at infinity. The plane elasticity problem yields the following displacement fields:

a) in the inclusion (r < a)

$$\frac{2G_f u_r^f}{\sigma_0} = A_f\,(\kappa_f - 1)r \qquad \text{(B1-1)}$$

b) in the layer (a < r < b)

$$\frac{2G_\ell u_r^\ell}{\sigma_0} = A_\ell(\kappa_\ell - 1)r - \frac{B_\ell}{r} \qquad \text{(B1-2)}$$

c) in the matrix $(r > b)$

$$\frac{2G_m u_r^m}{\sigma_0} = \frac{1}{2}(\kappa_m - 1)r - \frac{B_m}{r} \tag{B1-3}$$

The other displacement components vanish due to symmetry.

A_f, A_ℓ, B_ℓ and B_m are the constants determined from the perfect bonding boundary conditions:

$$\sigma_{rr}^f = \sigma_{rr}^\ell \qquad u_r^f = u_r^\ell \qquad \text{at } r = a \tag{B1-4}$$

$$\sigma_{rr}^\ell = \sigma_{rr}^m \qquad u_r^\ell = u_r^m \qquad \text{at } r = b \tag{B1-5}$$

Note that the boundary condition $\sigma_{rr}^m = \sigma_0$ at infinity is automatically satisfied by Equation (B1-3).

B1.2 The flexible interface model

Consider a single fiber is embedded in the matrix. Both fiber and matrix are isotropic materials. The applied loading is the transverse hydrostatic stress $\sigma_{xx}^{(0)} = \sigma_{yy}^{(0)} = \sigma_0$ at infinity. The plane elasticity problem yields the following displacement fields:

a) in the inclusion $(r < a)$

$$\frac{2G_f u_r^f}{\sigma_0} = A_f\,(\kappa_f-1)r \qquad \text{(B1-6)}$$

b) in the matrix $\qquad (r > a)$

$$\frac{2G_m u_r^m}{\sigma_0} = \frac{1}{2}\,(\kappa_m-1)r - \frac{B_m}{r} \qquad \text{(B1-7)}$$

The other displacement components vanish due to symmetry.

A_f, B_m are the constants determined from the interfacial boundary conditions:

$$\sigma_{rr}^f = \sigma_{rr}^m = g\,(\,u_r^m - u_r^f\,) \qquad \text{at } r = a \qquad \text{(B1-8)}$$

Note that the boundary condition $\sigma_{rr}^m = \sigma_0$ at infinity is satisfied automatically by Eqn.(B1-7). Note also that the transverse bulk modulus for this case is independent of the interface sliding parameter k due to symmetry.

B2. Effective bulk modulus of the composites with finite inclusion volume fraction

B2.1 The interphase model

The effective transverse bulk modulus is defined by using the concept of the average strain in the composite, i.e:

$$\frac{1}{K_c} = \frac{1}{K_m} + \frac{f+\ell}{1 + (f+\ell)\alpha} 2 \beta \qquad \text{(B2-1)}$$

where K_i $(i=f,\ell,m)$ are the transverse bulk moduli of components defined as $K_i = 2G_i/(\kappa_i - 1)$.

$$\beta = \left[\frac{f}{f+\ell} \left(\frac{1}{K_f} - \frac{1}{K_m}\right) A_f + \frac{\ell}{f+\ell} \left(\frac{1}{K_\ell} - \frac{1}{K_m}\right) A_\ell \right] \qquad \text{(B2-2)}$$

$$\alpha = \frac{f}{f+\ell} 2A_f + \frac{\ell}{f+\ell} 2A_\ell - 1 \qquad \text{(B2-3)}$$

B2.2 The flexible interface model

The effective transverse bulk modulus is defined by using the concept of the average strain in the composite, i.e:

$$\frac{1}{K_c} = \frac{1}{K_m} + \frac{f}{1 + f\alpha} 2 \beta \qquad \text{(B2-4)}$$

where

$$\beta = \left[\left(\frac{1}{K_f} - \frac{1}{K_m} \right) A_f + \left(\frac{1}{2K_m} - \frac{B_m}{2G_m a^2} - \frac{A_f}{K_f} \right) \right] \qquad \text{(B2-5)}$$

$$\alpha = 2A_f - 1 \tag{B2-6}$$

APPENDIX C

EFFECTIVE SHEAR MODULUS (3D) THE INTERPHASE MODEL AND THE FLEXIBLE INTERFACE MODEL

C1. A single spherical inclusion

C1.1 The interphase model

When a spherical inclusion in an infinite matrix subjected to a uniform shear stress τ_0 at infinity, the displacement fields are as follows:

a) in the inclusion $(r < a)$

$$\frac{2G_f u_r^f}{\tau_0} = (A_f r - \frac{6\nu_f}{1-2\nu_f} B_f r^3) \sin^2\theta \sin 2\phi \tag{C1-1}$$

$$\frac{2G_f u_\theta^f}{\tau_0} = (A_f r - \frac{7-4\nu_f}{1-2\nu_f} B_f r^3) \sin\theta \cos\theta \sin 2\phi \tag{C1-2}$$

$$\frac{2G_f u_\phi^f}{\tau_0} = (A_f r - \frac{7-4\nu_f}{1-2\nu_f} A_f r^3) \sin\theta \cos 2\phi \tag{C1-3}$$

b) in the layer $(a < r < b)$

$$\frac{2G_\ell u_r^\ell}{\tau_0} = \left(A_\ell r - \frac{6\nu_\ell}{1-2\nu_\ell} B_\ell r^3 \right.$$

$$\left. + \frac{3C_\ell}{r^4} + \frac{5-4\nu_\ell}{1-2\nu_\ell} \frac{D_\ell}{r^2} \right) \sin\theta \sin 2\phi \tag{C1-4}$$

$$\frac{2G_\ell u_\theta^\ell}{\tau_0} = \left(A_\ell r - \frac{7-4\nu_\ell}{1-2\nu_\ell} B_\ell r^3 \right.$$

$$\left. - \frac{2C_\ell}{r^4} + \frac{2}{r^2} D_\ell \right) \sin\theta \cos\theta \sin 2\phi \tag{C1-5}$$

$$\frac{2G_\ell u_\phi^\ell}{\tau_0} = \left(A_\ell r - \frac{7-4\nu_\ell}{1-2\nu_\ell} B_\ell r^3 \right.$$

$$\left. - \frac{2C_\ell}{r^4} + \frac{2}{r^2} D_\ell \right) \sin\theta \cos 2\phi \tag{C1-6}$$

c) in the matrix $(r > b)$

$$\frac{2G_m u_r^m}{\tau_0} = \left(r + \frac{3C_m}{r^4} + \frac{5-4\nu_m}{1-2\nu_m} \frac{D_m}{r^2} \right) \sin^2\theta \sin 2\phi \tag{C1-7}$$

$$\frac{2G_m u_\theta^m}{\tau_0} = \left(r - \frac{2C_m}{r^4} + \frac{2D_m}{r^2} \right) \sin\theta \cos\theta \sin 2\phi \tag{C1-8}$$

$$\frac{2G_m u_\phi^m}{\tau_0} = \left(r - \frac{2C_m}{r^4} + \frac{2D_m}{r^2} \right) \sin^2 \theta \cos 2\phi \tag{C1-9}$$

where A_f, B_f, A_ℓ, B_ℓ, C_ℓ, D_ℓ, C_m, D_m are the constants determined from the perfect bonding condition at the particle-layer and layer-matrix interfaces,

i.e.

At $r = a$

$$\sigma_{rr}^f = \sigma_{rr}^\ell \qquad u_r^f = u_r^\ell \tag{C1-10}$$

$$\sigma_{r\theta}^f = \sigma_{r\theta}^\ell \qquad u_\theta^f = u_\theta^\ell \tag{C1-11}$$

At $r = b$

$$\sigma_{rr}^\ell = \sigma_{rr}^m \qquad u_r^\ell = u_r^m \tag{C1-12}$$

$$\sigma_{r\theta}^\ell = \sigma_{r\theta}^m \qquad u_\theta^\ell = u_\theta^m \tag{C1-13}$$

C1.2 The flexible interface model

Consider a single elastic inclusion of radius "a", embedded in an elastic matrix, subjected to the shear stress $\sigma_{xy}^{(0)} = \tau_0$ applied at infinity. The displacement fields are:

a) in the inclusion $(r < a)$:

$$\frac{2G_f u_r^f}{\tau_0} = (A_f r - \frac{6\nu_f}{1-2\nu_f} B_f r^3) \sin^2\theta \sin 2\phi \qquad \text{(C1-14)}$$

$$\frac{2G_f u_\theta^f}{\tau_0} = (A_f r - \frac{7-4\nu_f}{1-2\nu_f} B_f r^3) \sin\theta \cos\theta \sin 2\phi \qquad \text{(C1-15)}$$

$$\frac{2G_f u_\theta^f}{\tau_0} = (A_f r - \frac{7-4\nu_f}{1-2\nu_f} B_f r^3) \sin\theta \cos 2\phi \qquad \text{(C1-16)}$$

b) in the matrix $(r > a)$:

$$\frac{2G_m u_r^m}{\tau_0} = (r + C_m \frac{3}{r^4} + D_m \frac{5-4\nu_m}{1-2\nu_m} \frac{1}{r^2}) \sin^2\theta \sin 2\phi \qquad \text{(C1-17)}$$

$$\frac{2G_m u_\theta^m}{\tau_0} = (r - C_m \frac{2}{r^4} + D_m \frac{2}{r^2}) \sin\theta \cos\theta \sin 2\phi \qquad \text{(C1-18)}$$

$$\frac{2G_m u_\theta^m}{\tau_0} = - (r - C_m \frac{2}{r^4} + D_m \frac{2}{r^2}) \sin\theta \cos 2\phi \qquad \text{(C1-19)}$$

where G and ν are the shear modulus and Poisson's ratio, respectively, and the subscripts and superscripts m and f denote the matrix and inclusion. A_f, B_f, C_m, D_m are the constants determined from the following boundary conditions at $r = a$:

$$u_r^f = u_r^m \qquad\qquad \sigma_{rr}^f = \sigma_{rr}^m \tag{C1-20}$$

$$\sigma_{r\theta}^f = \sigma_{r\theta}^m = k\ (u_\theta^m - u_\theta^f) \tag{C1-21}$$

where k is the tangential spring constant, which characterizes the flexibility of the interface at tangential direction. In the limit cases, when $k \to \infty$ we have perfectly bonded interface and when $k \to 0$ pure sliding case exists. In order to avoid the overlaping phenomenon, the normal spring constant g is chosen to be $g \to \infty$ in this case. Note that the Equations (C1-17) - (C1-19) satisfy automatically the boundary condition at infinity, i.e.

$$\sigma_{12}^m = \tau_0$$

C2. Effective shear modulus for the composites with finite volume fraction of inclusions

C2.1 The interphase model

The effective shear modulus is defined by using the concept of the average strain in the composite as

$$\frac{1}{G_c} = \frac{1}{G_m} + \frac{f+\ell}{1+(f+\ell)\alpha}\,\beta \tag{C2-1}$$

where

$$\beta = \frac{f}{f+\ell}\left[A_f - \frac{21}{5(1-2\nu_f)}\,B_f a^2\right]\left(\frac{1}{G_f} - \frac{1}{G_m}\right) \tag{C2-2}$$

$$+ \frac{\ell}{f+\ell}\left[A_\ell - \frac{21(b^5-a^5)}{5(1-2\nu_\ell)(b^3-a^3)}\,B_\ell\right]\left(\frac{1}{G_\ell} - \frac{1}{G_m}\right) \tag{C2-3}$$

$$\alpha = \frac{f}{f+\ell}\left[A_f - \frac{21}{5(1-2\nu_f)}\,B_f a^2\right]$$

$$+ \frac{\ell}{f+\ell}\left[A_\ell - \frac{21(b^5-a^5)}{5(1-2\nu_\ell)(b^3-a^3)}\,B_\ell\right] - 1 \tag{C2-4}$$

C2.2 The flexible interface model

The effective shear modulus is defined by using the concept of the average strain in the composite as

$$\frac{1}{G_c} = \frac{1}{G_m} + \frac{f}{1+f\alpha}\,\beta \tag{C2-5}$$

where

$$\beta = \frac{1}{5} \left(\frac{1}{G_m} \left[5 + \frac{16-20\nu_m}{1-2\nu_m} \frac{D_m}{a^3}\right] - \frac{1}{G_f} \left[5A_f - \frac{21}{1-2\nu_f} B_f a^2\right] \right) + \left[A_f - \frac{21}{5(1-2\nu_f)} B_f a^2\right] \left(\frac{1}{G_f} - \frac{1}{G_m} \right) \tag{C2-6}$$

$$\alpha = A_f - \frac{21}{5(1-2\nu_f)} B_f a^2 - 1 \tag{C2-7}$$

It is observed that for a homogenous material containing pure sliding inclusions, the results obtained from Equation (C2-5) coincide with Equation (51) in Shibata et al. (1990).

APPENDIX D

EFFECTIVE BULK MODULUS (3D)

THE INTERPHASE MODEL AND

THE FLEXIBLE INTERFACE MODEL

D1. A single spherical inclusion

D1.1 The interphase model

When a single coated inclusion in an infinite matrix subjected to the hydrostatics stress $\sigma_{xx}^{(0)} = \sigma_{yy}^{(0)} = \sigma_{zz}^{(0)} = \sigma_0$, the displacement fields are:

a) in the inclusion (r > a)

$$\frac{2G_f u_r^f}{\sigma_0} = A_f (\kappa_f - 1) r \tag{D1-1}$$

b) in the layer (a < r < b)

$$\frac{2G_\ell u_r^\ell}{\sigma_0} = A_\ell (\kappa_\ell - 1) r - \frac{B_\ell}{r^2} \tag{D1-2}$$

c) in the matrix (r > b)

$$\frac{2G_m u_r^m}{\sigma_0} = \frac{1}{3}(\kappa_m - 1)r - \frac{B_m}{r^2} \qquad \text{(D1-3)}$$

The other displacement components vanish due to symmetry.

A_f, A_ℓ, B_ℓ and B_m are the constants determined from the perfect bonding boundary conditions:

$$\sigma_{rr}^f = \sigma_{rr}^\ell \qquad u_r^f = u_r^\ell \qquad \text{at } r = a \qquad \text{(D1-4)}$$

$$\sigma_{rr}^\ell = \sigma_{rr}^m \qquad u_r^\ell = u_r^m \qquad \text{at } r = b \qquad \text{(D1-5)}$$

Note that the boundary condition $\sigma_{rr}^m = \sigma_0$ at infinity is satisfied automatically by Equation (D1-3).

D1.2 The flexible interface model

When a single inclusion in an infinite matrix is subjected to the hydrostatic stress $\sigma_{xx}^{(0)} = \sigma_{yy}^{(0)} = \sigma_{zz}^{(0)} = \sigma_0$, the displacement fields are:

a) in the inclusion $(r < a)$

$$\frac{2G_f u_r^f}{\sigma_0} = A_f\,(\kappa_f - 1)r \qquad \text{(D1-6)}$$

b) in the matrix $(r > a)$

$$\frac{2G_m u_r^m}{\sigma_0} = \frac{1}{3}(\kappa_m - 1) r - \frac{B_m}{r^2} \qquad \text{(D1-7)}$$

The other displacement components vanish due to symmetry. A_f, B_m are the constants determined from the interfacial boundary conditions:

$$\sigma_{rr}^f = \sigma_{rr}^m = g\,(\,u_r^m - u_r^f\,) \qquad \text{at } r = a \qquad \text{(D1-8)}$$

Note that the boundary condition $\sigma_{rr}^m = \sigma_0$ at infinity is satisfied automatically by Equation (D1-7). Note also that the transverse bulk modulus in this case is independent of the interface-sliding-parameter k due to symmetry.

D2. Effective bulk modulus of the composites with finite inclusion volume fraction

D2.1 The interphase model

The effective transverse bulk modulus are defined by using the concept of the average strain in the composite, i.e:

$$\frac{1}{K_c} = \frac{1}{K_m} + \frac{f+\ell}{1 + (f+\ell)\alpha}\, 2\,\beta \qquad \text{(D2-1)}$$

where K_s ($s = f, \ell, m$) are the bulk moduli of components.

$$\beta = \left[\frac{f}{f+\ell} \left(\frac{1}{K_f} - \frac{1}{K_m}\right) A_f + \frac{\ell}{f+\ell} \left(\frac{1}{K_\ell} - \frac{1}{K_m}\right) A_\ell \right] \tag{D2-2}$$

$$\alpha = \frac{f}{f+\ell} 3A_f + \frac{\ell}{f+\ell} 3A_\ell - 1 \tag{D2-3}$$

D2.2 The flexible interface model

The effective transverse bulk modulus are defined by using the concept of the average strain in the composite, i.e:

$$\frac{1}{K_c} = \frac{1}{K_m} + \frac{f}{1 + f\alpha} 2 \beta \tag{D2-4}$$

where

$$\beta = \left[\left(\frac{1}{K_f} - \frac{1}{K_m} \right) A_f + \left(\frac{1}{3K_m} - \frac{B_m}{2G_m a^3} - \frac{A_f}{K_f} \right) \right] \tag{D2-5}$$

$$\alpha = 3A_f - 1 \tag{D2-6}$$

APPENDIX E

EFFECTIVE THERMAL EXPANSION COEFFICIENTS (2D)

THE INTERPHASE MODEL

E1. The stress and displacement fields

The thermal expansion coefficients for perfectly bonded fiber composites have been derived by using both the successive iteration method (Mori and Wakashima, 1990) and the modified composite cylindrical model (which include interphase), it is found that both method give the same results.

The effective thermal expansion coefficients for the composite with coated fiber are derived here by using modified composite cylindrical model for symplicity.

When a coated fiber in a matrix is subjected to a uniform temperature change ΔT, the displacement and stress fields in the cylindrical coordinates (r, θ, z) are:

a) in the fiber ($r < a$)

$$u_r^f = A_f \, r \tag{E1.1}$$

$$u_z^f = \epsilon_0 \, z \tag{E1.2}$$

$$\sigma_{rr}^{f} = \frac{E_f}{(1+\nu_f)(1-2\nu_f)} A_f + \frac{\nu_f E_f}{(1+\nu_f)(1-2\nu_f)} \epsilon_0$$

$$- \left[\frac{E_f}{(1+\nu_f)(1-2\nu_f)} \alpha_T^f + \frac{\nu_f E_f}{(1+\nu_f)(1-2\nu_f)} \alpha_L^f \right] \Delta T \qquad \text{(E1.3)}$$

$$\sigma_{\theta\theta}^{f} = \frac{E_f}{(1+\nu_f)(1-2\nu_f)} A_f + \frac{\nu_f E_f}{(1+\nu_f)(1-2\nu_f)} \epsilon_0$$

$$- \left[\frac{E_f}{(1+\nu_f)(1-2\nu_f)} \alpha_T^f + \frac{\nu_f E_f}{(1+\nu_f)(1-2\nu_f)} \alpha_L^f \right] \Delta T \qquad \text{(E1.4)}$$

$$\sigma_{zz}^{f} = \frac{2\nu_f E_f}{(1+\nu_f)(1-2\nu_f)} A_f + \frac{E_f(1-\nu_f)}{(1+\nu_f)(1-2\nu_f)} \epsilon_0$$

$$- \left[\frac{2\nu_f E_f}{(1+\nu_f)(1-2\nu_f)} \alpha_T^f + \frac{E_f(1-\nu_f)}{(1+\nu_f)(1-2\nu_f)} \alpha_L^f \right] \Delta T \qquad \text{(E1.5)}$$

b) in the layer ($a < r < b$)

$$u_r^{\ell} = A_{\ell}\, r + \frac{B_{\ell}}{r} \qquad \text{(E1.6)}$$

$$u_z^{\ell} = \epsilon_0\, z \qquad \text{(E1.7)}$$

$$\sigma_{rr}^{\ell} = \frac{E_{\ell}}{(1+\nu_{\ell})(1-2\nu_{\ell})} A_{\ell} - \frac{E_{\ell}}{(1+\nu_{\ell})} \frac{B_{\ell}}{r^2} + \frac{\nu_{\ell} E_{\ell}}{(1+\nu_{\ell})(1-2\nu_{\ell})} \epsilon_0$$

$$- \left[\frac{E_\ell}{(1+\nu_\ell)(1-2\nu_\ell)} \alpha_T^\ell + \frac{\nu_\ell E_\ell}{(1+\nu_\ell)(1-2\nu_\ell)} \alpha_L^\ell \right] \Delta T \qquad \text{(E1.8)}$$

$$\sigma_{\theta\theta}^\ell = \frac{E_\ell}{(1+\nu_\ell)(1-2\nu_\ell)} A_\ell + \frac{E_\ell}{(1+\nu_\ell)} \frac{B_\ell}{r^2} + \frac{\nu_\ell E_\ell}{(1+\nu_\ell)(1-2\nu_\ell)} \epsilon_0$$

$$- \left[\frac{E_\ell}{(1+\nu_\ell)(1-2\nu_\ell)} \alpha_T^\ell + \frac{\nu_\ell E_\ell}{(1+\nu_\ell)(1-2\nu_\ell)} \alpha_L^\ell \right] \Delta T \qquad \text{(E1.9)}$$

$$\sigma_{zz}^\ell = \frac{2\nu_\ell E_\ell}{(1+\nu_\ell)(1-2\nu_\ell)} A_\ell + \frac{E_\ell(1-\nu_\ell)}{(1+\nu_\ell)(1-2\nu_\ell)} \epsilon_0$$

$$- \left[\frac{2\nu_\ell E_\ell}{(1+\nu_\ell)(1-2\nu_\ell)} \alpha_T^\ell + \frac{E_\ell(1-\nu_\ell)}{(1+\nu_\ell)(1-2\nu_\ell)} \epsilon_0 \right] \Delta T \qquad \text{(E1.10)}$$

c) in the matrix (r > b)

$$u_r^m = A_m\, r + \frac{B_m}{r} \qquad \text{(E1.11)}$$

$$u_z^m = \epsilon_0\, z \qquad \text{(E1.12)}$$

$$\sigma_{rr}^m = \frac{E_m}{(1+\nu_m)(1-2\nu_m)} A_m - \frac{E_m}{(1+\nu_m)} \frac{B_m}{r^2} + \frac{\nu_m E_m}{(1+\nu_m)(1-2\nu_m)} \epsilon_0$$

$$- \left[\frac{E_m}{(1+\nu_m)(1-2\nu_m)} \alpha_T^m + \frac{\nu_m E_m}{(1+\nu_m)(1-2\nu_m)} \alpha_L^m \right] \Delta T \qquad \text{(E1.13)}$$

$$\sigma^{m}_{\theta\theta} = \frac{E_m}{(1+\nu_m)(1-2\nu_m)} A_m + \frac{E_m}{(1+\nu_m)} \frac{B_m}{r^2} + \frac{\nu_m E_m}{(1+\nu_m)(1-2\nu_m)} \epsilon_0$$

$$- \left[\frac{E_m}{(1+\nu_m)(1-2\nu_m)} \alpha^{m}_{T} + \frac{\nu_m E_m}{(1+\nu_m)(1-2\nu_m)} \alpha^{m}_{L} \right] \Delta T \qquad \text{(E1.14)}$$

$$\sigma^{m}_{zz} = \frac{2\nu_m E_m}{(1+\nu_m)(1-2\nu_m)} A_m + \frac{E_m(1-\nu_m)}{(1+\nu_m)(1-2\nu_m)} \epsilon_0$$

$$- \left[\frac{2\nu_m E_m}{(1+\nu_m)(1-2\nu_m)} \alpha^{m}_{T} + \frac{E_m(1-\nu_m)}{(1+\nu_m)(1-2\nu_m)} \epsilon_0 \right] \Delta T \qquad \text{(E1.15)}$$

where E and ν are the Yonng's modulus and Poisson's ratio, and the subscripts and superscripts f, ℓ and m denote the fiber, layer and matrix. A_f, A_ℓ, B_ℓ, A_m, B_m and ϵ_0 are the unknown constants to be determined from the following boundary conditions:

$$\sigma^{f}_{rr} = \sigma^{\ell}_{rr} \qquad u^{f}_{r} = u^{\ell}_{r} \qquad \text{at } r = a \qquad \text{(E1.16)}$$

$$\sigma^{\ell}_{rr} = \sigma^{m}_{rr} \qquad u^{\ell}_{r} = u^{m}_{r} \qquad \text{at } r = b \qquad \text{(E1.17)}$$

$$\sigma^{m}_{rr} = 0 \qquad \text{at } r = d \qquad \text{(E1.18)}$$

$$\sigma^{f}_{zz}(\text{area of fiber}) + \sigma^{\ell}_{zz}(\text{area of interfacial layer})$$

$$+ \sigma^{m}_{zz}(\text{area of matrix}) = 0 \qquad \text{(E1.19)}$$

where "a" is the radius of the fiber, "b" and "d" are the outside radius of the layer and matrix. Note that Equation (E1.19) is due to the fact of force equilibrium in the z-direction.

E2. Effective thermal expansion coefficients

The effective thermal expansion coefficients are, by definition, the average strains resulting from a unit temperature rise for a traction free composite. Therefore, α_L^c and α_T^c are given as follows

$$\alpha_L^c = \frac{\epsilon_0}{\Delta T} \tag{E2.1}$$

$$\alpha_T^c = \frac{u_m/r_m}{\Delta T} = \frac{A_m+B_m/r_m^2}{\Delta T} \tag{E2.2}$$

APPENDIX F

EFFECTIVE THERMAL EXPANSION COEFFICIENTS (3D)
THE INTERPHASE MODEL

F1. A single spherical inclusion

When an isolated spherical coated inclusion in a matrix is subjected to a uniform temperature change ΔT, the displacement and stress fields in the spherical coordinates (r, θ, ϕ) are:

a) in the inclusion $(r < a)$

$$u_r^f = (A_f r)\Delta T \tag{F1.1}$$

$$\sigma_{rr}^f = \sigma_{\theta\theta}^f = \sigma_{\phi\phi}^f = 3K_f(A_f - \alpha_f)\Delta T \tag{F1.2}$$

b) in the layer $(a < r < b)$

$$u_r^\ell = \left(A_\ell r + \frac{B_\ell}{r^3} \right)\Delta T \tag{F1.3}$$

$$\sigma_{rr}^\ell = \left(3K_\ell A_\ell - 4G_\ell \frac{B_\ell}{r^3} - 3K_\ell \alpha_\ell \right)\Delta T \tag{F1.4}$$

$$\sigma_{\theta\theta}^\ell = \sigma_{\phi\phi}^\ell = \left(3K_\ell A_\ell + 2G_\ell \frac{B_\ell}{r^3} - 3K_\ell \alpha_\ell \right)\Delta T \tag{F1.5}$$

c) in the matrix $(r > b)$

$$u_r^m = \frac{B_m}{r^3} \Delta T \tag{F1.6}$$

$$\sigma_{rr}^m = (- 4G_m \frac{B_m}{r^3} - 3K_m\alpha_m)\Delta T \tag{F1.7}$$

$$\sigma_{\theta\theta}^m = \sigma_{\phi\phi}^m = (2G_m \frac{B_m}{r^3} - 3K_m\alpha_m)\Delta T \tag{F1.8}$$

where K and G are bulk and shear moduli. The other displacement and stress components are zero due to symmetry. A_f, A_ℓ, B_ℓ and B_m are the unknown constants to be determined from the perfect bonding boundary conditions at the particle-layer and layer-matrix interfaces:

$$\sigma_{rr}^f = \sigma_{rr}^\ell \qquad u_r^f = u_r^\ell \qquad \text{at } r = a \tag{F1.9}$$

$$\sigma_{rr}^\ell = \sigma_{rr}^m \qquad u_r^\ell = u_r^m \qquad \text{at } r = b \tag{F1.10}$$

$$\sigma_{rr}^m = 0 \qquad \text{as } r \to \infty \tag{F1.11}$$

where "a" is the radius of the particle, and "b" is the outside radius of the layer. Note that the stresses in the matrix are chosen such that the condition of vanishing tractions at infinity is automatically satisfied (A_m is taken as zero).

The zeroth-order solution is given as:

$$\epsilon_0^* = \eta \, \Delta T \tag{F1.12}$$

$$\Delta\sigma_0 = \gamma \; \Delta T \tag{F1.13}$$

where ϵ_0^* and $\Delta\sigma_0$ are the zeroth-order eigenstrain and the average stress disturbance in the coated inclusion, respectively.

$$\eta = \frac{f}{f+\ell}\left(\frac{1}{K_f} - \frac{1}{K_m}\right)K_f A_f + \frac{\ell}{f+\ell}\left(\frac{1}{K_\ell} - \frac{1}{K_m}\right)K_\ell A_\ell$$
$$+ \frac{f}{f+\ell}(\alpha^f - \alpha^m) + \frac{\ell}{f+\ell}(\alpha^\ell - \alpha^m) \tag{F1.14}$$

and

$$\gamma = \frac{f}{f+\ell} \; 3K_f \; A_f + \frac{\ell}{f+\ell} \; 3K_\ell \; A_\ell \tag{F1.15}$$

F2. First order solution

Consider an isolated spherical coated inclusion subjected to hydrostatic stress $\sigma_{xx}^{(1)} = \sigma_{yy}^{(1)} = \sigma_{zz}^{(1)} = \sigma_1$ at infinity. The stress and displacement fields are as follows:

a) in the inclusion $(r < a)$

$$u_r^f = (A_f' \; r)\sigma_1 \tag{F2.1}$$

$$\sigma_{rr}^f = \sigma_{\theta\theta}^f = \sigma_{\phi\phi}^f = (3K_f A_f')\sigma_1 \tag{F2.2}$$

b) in the layer $(a < r < b)$

$$u_r^{\ell} = (A_{\ell}' r + \frac{B_{\ell}'}{r^3})\sigma_1 \tag{F2.3}$$

$$\sigma_{rr}^{\ell} = (3K_{\ell}A_{\ell}' - 4G_{\ell}\frac{B_{\ell}'}{r^3})\sigma_1 \tag{F2.4}$$

$$\sigma_{\theta\theta}^{\ell} = \sigma_{\phi\phi}^{\ell} = (3K_{\ell}A_{\ell}' + 2G_{\ell}\frac{B_{\ell}'}{r^3})\sigma_1 \tag{F2.5}$$

c) in the matrix $(r > b)$

$$u_r^{m} = (A_m' r + \frac{B_m'}{r^3})\sigma_1 \tag{F2.6}$$

$$\sigma_{rr}^{m} = (3K_m A_m' - 4G_m \frac{B_m'}{r^3})\sigma_1 \tag{F2.7}$$

$$\sigma_{\theta\theta}^{m} = \sigma_{\phi\phi}^{m} = (3K_m A_m' + 2G_m \frac{B_m'}{r^3})\sigma_1 \tag{F2.8}$$

A_f', A_{ℓ}', A_m', B_{ℓ}' and B_m' are the constants determined from the following boundary conditions:

$$\sigma_{rr}^{f} = \sigma_{rr}^{\ell} \qquad u_r^{f} = u_r^{\ell} \qquad \text{at } r = a \tag{F2.9}$$

$$\sigma_{rr}^{\ell} = \sigma_{rr}^{m} \qquad u_r^{\ell} = u_r^{m} \qquad \text{at } r = b \tag{F2.10}$$

$$\sigma_{rr}^{m} = \sigma_1 \qquad \text{as } r \to \infty \tag{F2.11}$$

where

$$\beta = \frac{f}{f+\ell} \left(\frac{1}{K_f} - \frac{1}{K_m} \right) K_f A_f' + \frac{\ell}{f+\ell} \left(\frac{1}{K_\ell} - \frac{1}{K_m} \right) K_\ell A_\ell' \tag{F2.12}$$

and

$$\lambda = \frac{f}{f+\ell} 3K_f A_f' + \frac{\ell}{f+\ell} 3K_\ell A_\ell' \tag{F2.13}$$

F3. Effective thermal expansion coefficient of the composites with finite inclusion volume fraction

The effective thermal expansion coefficients are, by definition, the average strains resulting from a unit temperature rise for a traction free composite

$$\alpha_{ij}^c = \frac{1}{\Delta T} \langle \epsilon_{ij} \rangle_D = \alpha_{ij}^m + \frac{1}{\Delta T} (f+\ell)\ \epsilon_{ij}^* \tag{F2.14}$$

therefore

$$\alpha^c = \alpha^m + \frac{f+\ell}{1 + (f+\ell)\lambda}\ \eta \tag{F2.15}$$

where η is defined in (F1.15) and λ in (F2.13).

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