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PROBABILISTIC RELIABILITY EVALUATION
OF NAVIGATION STRUCTURES

By

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ABSTRACT

PROBABILISTIC RELIABILITY EVALUATION OF NAVIGATION STRUCTURES

By

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The inland navigation system plays an important role in the nation's economy. In the United States of America, many structures in the navigation system are near or have exceeded their design service life and major rehabilitations are desperately needed. As funds for maintenance and rehabilitation are always limited, it is necessary to find a rational method to prioritize improvements and better allocate the fund.

The existing procedure of allocating funds for the maintenance and rehabilitation of navigation structures is mainly based on deterministic methods and decisions on a case by case basis. Since deterministic methods cannot incorporate the numerous uncertainties in engineering practice, the calculated factor of safety and assessment of its adequacy may be overly dependent on assumptions used for analysis. This research was to develop a probabilistic procedure which can rationally evaluate the reliability of navigation structures, thus helping prioritize the system.

The proposed procedure copes with many uncertainties by treating the analysis parameters involved as random variables. These random variables were described by their statistical moments and characterized by basic statistical and specially designed methods, based on all available information. The reliability index was used to measure the reliability because it is consistent, invariant and dimensionless. First-order second-moment approximate methods were employed to calculate the reliability index.

Numerous engineering application examples were provided in this research. The reliability of structures, chosen from existing locks and dams on the Monongahela River, Pennsylvania and the Tombigbee River, Alabama, USA, was evaluated with respect to their performances in overturning, sliding and bearing. Alternative performance functions and probabilistic methods were compared. A method to express hydrostatic uplift force as a random variable was developed. The examples proved that the proposed probabilistic procedure is very useful in the reliability evaluation of navigation structures and can be used to prioritize the navigation system. Clearly stated concepts and carefully examined simple methods make this procedure highly applicable in engineering practice.

Recommendations, including the target reliability index for navigation structures, are offered for the implementation of the proposed procedure in design practice.

DEDICATION

To my father and mother

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findings and conclusions or recommendations expressed in this study are those of the author and do not necessarily reflect the view of the Corps of Engineers.

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LIST OF SYMBOLS AND ABBREVATIONS

ROMMAN LETTERS

B	Width of base
c, c_s	Cohesion of soils
$\tilde{C}$	Nominal values of capacity
C_c	Coefficient of consolidation
$Cov(X, Y)$	Covariance of X and Y
D	Demand or load. Depth of overburden soil layer
$D_{s,h}$	Lateral earth force
D_{soil}	Normal earth pressure
$\tilde{D}$	Nominal values of demand
e	Eccentricity of the load with respect to geometric base width
E	Coefficient of uplift force
E_s	Young's modulus
$E[X]$	Expected value of random variable X
$f(x)$	Probability density function of X
$f_{C,D}(x,y)$	Joint probability density functions of capacity C and demand D
F_D	Cumulative probability function of D
F_{impact}	Lateral impact force
F_{wind}	Wind force
$F(X)$	Performance function

$F(\pm,\pm,\pm,\ldots\pm)$	Values of F at certain points at which the X_i have values $X_i \pm \sigma_{X_i}$
G_s	Shear modulus
H_H	Upper pool level
H_L	Lower pool level
I	Impact value
k	Coefficient of permeability
K, K_h	Coefficient of lateral earth pressure
K_0	Coefficient of lateral earth pressure at-rest
K_a	Coefficient of active earth pressure
K_p	Coefficient of passive earth pressure
k_M	Mutiplying constant of design value of material strength
k_{S_i}	Mutiplying constant of load intensity
L	Nominal live load effect due to traffic loadings. Length of sliding surface of base
M^*	Design value of material strength
m_c	Soil compressibility ratio
m_M	Mean of design value of material strength
M_O	Overturning moment
M_R	Resisting moment
M_u	Moment caused by uplift force U
n	Soil parameter
N	Number of data points
N'	Effective normal force
Nc, Nq, N_γ	Bearing capacity factors for a strip load, corresponding to cohension resistance, overburden pressure and base friction resistance, respectively
$P(f)$, p_f	Probability of failure

P_{anchor}	Anchor force
p_i	Frequency or probability of x_i, probability concentration coefficients of points
P_R	Probability of safety
P_{hydro}	Hydraulic driving force
$P_{s,v}$	Vertical wall shear force
PI	Plasticity index
PC%	Percentage of base which in compression
q_0	Effective overburden pressure
Q	Normal component of the ultimate bearing capacity
Q_m	Mean load effect
R	Resistance
R_m	Resistance
R_n	Nominal resistance
R_s	Lateral earth force, resistance
S	Load
S_i*	Load intensity
$S(X)$	Safe region of random variable space
T	Shear force underneath the base
U	Hydraulic uplift force on the structure base
$U(X)$	Failure region of random variable space
V_Q	Coefficients of variation of load
V_R	Coefficients of variation of resistance
$Var(X)$	Variance of X
w	Water content
X	Population, random variable
X_R	Location of the effective resultant base force

Y	Population, random variable
z	Depth
Z	Section modulus

GREEK LETTERS

(Upper Case)

Δ	Inclination angle of resultant force on the base
Γ	Load factor
Φ	Resistance factor
$\Phi(\beta)$	Cumulative probability function of the standard normal variate X for $X \geq \beta$
$\Phi^{-1}(\bullet)$	Inverse of the cumulative distribution of standard normal distribution functio
$\Theta(T)$	Cumulative probability function of θ(t)

(Lower Case)

α	Slope of wall back
β	Reliability index. Slope of the backfill
β_{toe}	Overturning reliability index for resultant location at toe of base criterion
$\beta_{B/2}$	Overturning reliability index for one half of base in compression riterion
$\beta_{3B/4}$	Overturning reliability index for resultant location at toe of base riterion
β_{B}	Overturning reliability index for resultant location at toe of base riterion
β_{FS}	Reliability index of factor of safety
δ	Wall friction angle
ϕ	Internal friction angle of soils
γ	Unit weight of backfill, unit weight of base

$\gamma_{concrete}$	Unit weight of concrete
γ_{soil}	Unit weight of soil
γ_w	Unit weight of water
μ	Poisson's ratio
μ_X	Expected value of random variable X
μ_X^k	The k^{th} moment of random variable X
λ	Central safety factor
η	Coefficient of group efficiency
θ	Slope of the surface of the overburden soil
$\theta(t)$	Probability density function of time t
ζ's	Factors related to depth, inclination of load, tilt of base and ground slope
ρ_{XY}	Coefficient of correlation of random variables X and Y
σ_M	Standard deviation of design value of material strength
σ_n	Normal stress
σ_{S_i}	Standard deviation of load intensity
σ_x	Standard deviation of X
τ	Shear stress

Abbrevations

ASCE	American Sociaty of Civil Engineers
ASCE-EMD	Engineering Mechanics Division of ASCE
cpf	Cumulative probability function
EL.	Elevation
exp	Exponenatial
FS	Factor of Safety
FOSM	First Order Second Moment

GR	Group Reliability
LRFD	Load and Resistance Factor Design
OCR	Over Consolidation Ratio
pdf	Probability density function
PEM	Point Estimate Method
PPM	Paired Points Method
PRA	Probabilistic Risk Assessment
R.F	Rating factor
SCM	System Characteristic Models
SM	Safety margin
USCOLD	U.S. Committee On Large Dams

SPECIAL EXPRESSIONS

Hat $\overline{\ }$	Mean value
{ }	Vector
[ ]	Matrix
$[\ ]^T$	Transpose of matrix
$[\ ]^{-1}$	Reverse of matrix

Chapter I

INTRODUCTION

1.1 Probabilistic Methods in Civil Engineering

Since its birth in the seventeenth century, with studies on dice by Pascal and others in the 1650's and the first published paper about probability by Bernoulli in 1731, probability theory has been continuously improved and has become accepted in more and more scientific and technology fields, especially into the 20th century. With the growing interest of its application in engineering fields, today probabilistic methods have become very useful and familiar tools for civil engineers.

1.1.1 Rationale for Use of Probabilistic Method

The need for probabilistic methods in civil engineering arises from by the fact that uncertainties exist throughout design, construction and operation of a project: For a number of engineering projects, little or no previous experience exists with certain specific project aspects; current design procedures often fall short of expectations in new or alien situations. In the case of geotechnical applications, engineers can never know with certainty what the precise foundation soil conditions are.

1.1.1.1 Uncertainties

Countless uncertainties exist in our daily life. With no exception, uncertainties dominate all engineering practices, and perhaps, civil engineering is the engineering field which faces more uncertainties than others as it is commonly involved with unique, non-replicate structures.

Uncertainties exist throughout civil engineering, such as:

1. Uncertainty in engineering materials. There are many uncertainties among materials used in engineering projects, especially for soils, because of

- imperfections and faults existing in all engineering materials;
- no two manufactured objects being exactly the same;
- the dimensions of materials not being exact; and
- great variations of earth material.

Perhaps soil is the most uncertain engineering material considering its structural and mechanical properties. The uncertainties come from its complex aggregations of discrete particles in arrays, shapes, sizes and orientations. The uncertainties appear as

- no general theory can fully describe the mechanical properties of soil;
- no drilling log can fully represent site characteristics;
- no set of soil test data can be obtained without

significant variation; and

- no one can perfectly control the quality of earth structures.

2. Uncertainties in loading determination. Engineers can never completely know actual loading conditions and the induced loads in civil engineering systems, such as

- flow of surface water and ground water;
- frequency and intensity of earthquakes;
- action and variability of wind and/or waves;
- ice and snow load;
- freezing and thawing;
- vibrations and shock;
- traffic loads;
- chemical and environmental factors; etc.

3. Uncertainties in the expression of material properties and behavior. First, geotechnical engineers can never expect to get real "undisturbed" samples from the field, therefore, testing results cannot completely represent the real field condition and neither can the site characterization. Second, testing methods and results are always inconsistent and many have considerable variation. Third, design formulas are not always accurate models of the specific condition under study.

1.1.1.2 The shortcomings of deterministic methods

So far, deterministic methods still play a big role in civil engineering practice, and they are the only basis of design criteria in many projects. One very often used term, the conventional design criterion and safety measure related to a specific event (usually a failure), is the so called "Factor of Safety" (FS).

The factor of safety is the ratio of nominal values of capacity, $\tilde{C}$, and demand, $\tilde{D}$, of a structure or system. It is expressed as

$$FS = \frac{\tilde{C}}{\tilde{D}} \tag{1•1}$$

where $\tilde{C}$ is some nominal capacity, typically less than the expected capacity, and $\tilde{D}$ is some nominal demand, typically greater than the expected demand.

The criteria of factor of safety are allowable values which are usually learned from previous experience for the considered structure or system in its anticipated environment.

There are severe drawbacks of using factor of safety as an assessment of the risk of failure:

1. The primary deficiency of using factor of safety as a safety measurement is that all parameters in calculations must be assigned as single, precise values, therefore, no uncertainty can be directly considered. A great deal of engineers' personal experience must be involved to select the values, and then once done, the information leading to the selection is lost to the analysis;

2. Multiple safety criteria must be set up for one component of a structure or system subject to different failure modes in order to make the factor of safety satisfy the

criteria based on the "rule of thumb". For example, some often used criteria of FS for slope stability of earth dams are: FS_{min} =1.3 for the end-of-construction condition; 1.05 for the rapid drawdown condition. For the same dam, 10.0 may be used for piping in silty soils and 10-20 for the length of an impervious upstream blanket cutoff.

3. The factor of safety itself may not be sufficient enough to make design procedures adequate and reliable and it is not a rational reliability measurement. Because the engineers' personal experience plays an important role in determining the design parameter values, for similar structures or systems, their reliability may be quite different even if they have the same value of factor of safety. Also, for similar structures or systems, the one which has higher value of factor of safety may not be more reliable than the one which has lower value of factor of safety.

Since the deterministic method cannot give rational safety or reliability evaluation of structures or systems, and it is realized there is no "absolute" safety in reality, the application of probabilistic method in civil engineering is a must.

1.1.2 Applications of Probabilistic Method

Applying probabilistic methods in civil engineering has been called for since the 1940s (e.g. Freudenthal, 1947 and others[41,42,43,89]), but the probabilistic method was not really employed in civil engineering practice until the 1970s.

Since the late 1960's, the application of probabilistic methods has grown rapidly in engineering fields as more and more people realized the importance and necessity of such applications, and more studies have been done. Although the ASCE-EMD (Engineering Mechanics Division of ASCE) specialty conference on probabilistic concepts and methods in engineering held in 1969 caught more attention on this subject, the milestone is the first international conference on "Application of Statistics and Probability in Soil and Structural Engineering" in 1971. Since then the probabilistic method has spread in the civil engineering field. It should be also mentioned that four text publications, by Benjamin and Cornell (1970[15]), Ang and Tang (1975[7], 1984[8]) and Harr (1987[53]), are important references on statistical and probabilistic applications for civil engineers.

It was no surprise to see the quick and early application of probabilistic methods in power systems, especially in nuclear power plant design and analysis. This is because the great safety concern of nuclear power plant from all levels of the society, and the tragic results from several nuclear power plants accidents, such as the nuclear reactor fire in Windscale, UK, 1957 (17 delayed deaths); the Three Mile Island accident, USA, 1979 (1 delayed death) and the Chernobyl accident in USSR, 1986 (at least 31 delayed deaths, not accounting for the long term radiation pollution effect). After the WASH-1400 report, which applied the probabilistic risk assessment to nuclear power plant safety, by U.S Nuclear Regulatory Commission in 1975, more applications were carried out. In the early 1980's, ASCE formed a committee on certification of uncertainties for dynamic analysis of nuclear structures and materials. As the result of effort by this committee, later on, related regulations were established[57,77]. In the same period, a multi-country joint research project on probabilistic risk assessment of nuclear power plants was started, which was sponsored and coordinated by the Joint Research Centre, Commission of the European Communities (JRC), to set up a systematical and feasible method for performing probabilistic risk assessment of nuclear power plants. It is still an ongoing project [3].

Another good example involves application of probabilistic methods to highway systems. The highway transportation administration conducted a multi-year National Corporative Highway Research Program in 1980s. In this program probabilistic methods were employed to evaluate the load capability of existing highway bridges[80].

Many applications of probabilistic methods occur in the structural and geotechnical engineering fields. In structural engineering, items such as lifeline structures, offshore and marine structures, and steel building structures are targets for application especially when seismic, wind or wave loads are involved. In geotechnical engineering, applications involve geostatistics, safety of hydaulic engineering systems, stability of earth structures, foundation settlement prediction, quality control, etc.

Today, although probabilistic methods are used in many aspects in civil engineering, some new design and analysis regulations which request probabilistic methods are still needed to speed this application.

1.2 Applying Probabilistic Methods to the Reliability Evaluation of Navigation Systems

1.2.1 Current Condition of US Nation's Navigation System

Inland waterway transportation is a significant segment of the national economy. For example, the United States' inland waterways carry 16 per cent of the nation's intercity freight, and it is a cheap way to move products. But, the navigation system in the United States is old[106]: Locks range in age from less than three to more than 150 years old and the median age of all chambers is 37 years; over 40 per cent are more than 50 years old – near or beyond their design service life – an item of significant concern. As the navigation structures continue aging, more expenditures are needed to maintain their satisfactory performance and to conduct necessary major rehabilitation or replacement projects, otherwise, more shut down incidents on the waterway would be expected. Figure 1-1, based on the report of the Subcommittee of Dam Incidents and Accidents of the Committee on Dam Safety of the U.S. Committee on Large Dams, USCOLD (1988)[66], shows that failures and accidents of dams are somewhat uniformly distributed but more accidents happen at the age of 20 to 80 years. Note that this data is for dams

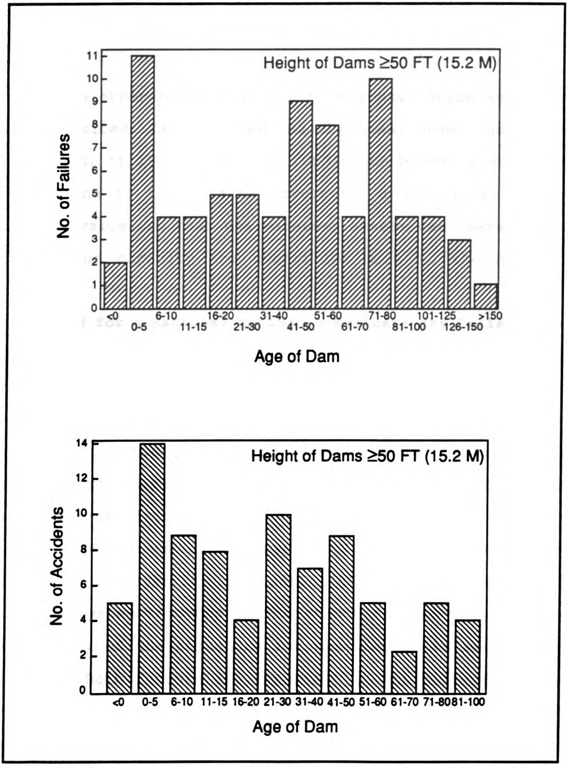


Figure 1-1 Numbers of Failures and Accidents vs. Age of Dam (After USCOLD, 1988)

in general, not particularly for navigation dams on inland waterways or even federally designed and operated structures.

On the other hand, funding for the navigation system is always limited, far from meeting the need. Under these circumstances, it is crucial that funding be properly allocated – according to the current structural reliability and the economic value, so that the waterway system can operate with its maximum capacity and contribute more to the nation's economy.

1.2.2 Need for a Rational Method to Evaluate the Reliability of Navigation Systems

To best allocate funds for rehabilitation, structures and their specific deficiencies within the whole navigation system need to be prioritized. There are two major factors which affect this prioritization: their economic value and structural condition.

The economic value, or capacity of a navigation structure or system, involves the combination of traffic volume, delay time and system throughput. The structural condition is the reliability of the structure for designed satisfactory performance. Both factors are important in the prioritization, and an optimum balance must be reached. The economic value or capacity can be evaluated by other economic analysis methods, but a rational method which can be used to evaluate the comparative structural reliability of different navigation structures had not been developed prior to this study.

So far, the structural reliability of the nation's navigation system has not been evaluated though some individual work has been done recently[78, 115]. Funding allocations have been on the traditional routine and case-by-case basis, and the structural reliability evaluation was totally deterministic. Considering that locks and dams were built several decades ago and there is often no or very little new available information about these structures; also, the conventional analysis methods cannot give a "true" reliability evaluation, therefore, a new method must be established to rationally evaluate the comparative reliability of navigation structures and prioritize the system for rehabilitation.

1.2.3 Feasibility of Applying Probabilistic Methods for Navigation Structural Reliability Analysis

Can a procedure which must take many significant uncertainties into consideration be developed? Is there a way to quantitatively evaluate the reliability of a navigation structure or system by using some not totally unfamiliar and not too complicated methods? The answer is: Yes, by applying probability theory. This is because the concept of engineering reliability theory is based on the fact that nothing is "absolute"; it aims to find the inside rule and to predict the possible outcomes for a specific performance or a random event from its messy appearance, and there have been broad existing applications of this theory in engineering practice.

Before discussing the use of probabilistic methods for evaluating the reliability of navigational structures, it is worthwhile to examine the difficulties of its application in engineering practice. These difficulties can be summarized below.

One difficulty is the "Chuan tong guan nian" (a Chinese phrase, meaning the inertia of traditional mind). Like any new concept, new ideas or new methods being accepted, the main resistance of applying the probabilistic method in civil engineering practice was not from the imperfection of this new arrival but from the traditional mind. Engineers are used to the conventional method and may not be familiar with probability theory. Although some engineers agree that there are uncertainties in engineering practice, they insist that "the factor of safety itself takes care of the uncertainty", rather than realizing the shortcomings of deterministic methods (e.g. the uncertainty is not the same for all variables) and exploring the necessity of this application.

A second difficulty was the computational problem with applying probability theory to engineering practice because of the lack of efficient approximate methods and high speed, high capacity computers. Also, the probabilistic analysis and design procedures were not ready for engineers' use; for a long time the probabilistic method only gave engineers qualitative results.

A third difficulty is the limitation of probability theory in engineering applications. In real cases studied, the calculated reliability of a structure or system by a simplified method (to avoid computational difficulty) sometimes depends on the definition of failure event or analysis method but the reliability of a structure or system should be unique both in the physical and theoretical sense.

A fourth difficulty is the absence or lack of data. There has not been enough data collected from engineering practice and some load factors and parameters involved in engineering design and analysis cannot be statistically characterized (although this should not be a reason for not applying statistical methods in engineering practice);

A fifth difficulty has been the fact that some earlier application examples gave "unreasonable" analysis results which discouraged application of probability theory in engineering practice. For example, in one earlier application[71], the analysis result showed that an embankment had FS=1.13 but with possibility of failure, *P(f)*, of 0.4. Although that embankment failed, some people thought the *P(f)* = 0.4 was too high to be reasonable, therefore, this method could not be trusted.

After decades of research work, some of the difficulties listed above have been overcome: better computational methods have been established, high quality computers have been produced and are improving day by day; some of these limitations have been removed; and more important, many engineers have realized the need of probabilistic methods in engineering practice and have accepted the concept. In the past twenty years, probability theory has been successfully applied to many engineering fields, including civil engineering.

Without depending on the an engineer's personal experience, the probabilistic method can rationally evaluate the reliability of an engineering project or system with an accounting for many uncertainties. Furthermore, a dimensionless reliability measurement used in probabilistic method, the "Reliability Index", β, can give a fair, quantitative reliability measurement for a structure or system regardless of the structural characterization, precise analysis form and the choice of the basic variables.

Based on all facts considered, an analysis procedure which is based on probability theory and the commonly practiced design and analysis concepts in civil engineering field is suitable for navigation system prioritization. The study reported herein describes the implementation part of this task and focuses on the structural reliability evaluation of concrete gravity monoliths.

Chapter II

LITERATURE REVIEW

2.1 Early Application of Probabilistic Method in Civil Engineering

The application of probabilistic methods in civil engineering can be traced back to the 1940's. During World War II, statistical methods were first used in the aeronautical field (1942)[41] for aircraft structural analysis because so many aircraft were produced and lot of testing and loading data were generated. The probabilistic method was used on statistical rationalization of strength factors.

In 1947, Freudenthal published a paper titled "The Safety of Structures"[41], which is the earliest published paper applying probability concepts in civil engineering. Realizing that many uncertainties exist in civil engineering design and analysis, such as uncertainty of loading, imperfection of manufactured products, imperfection of intellectual concept, as well as the imperfection of human observations and actions, Freudenthal tried to set up a rational method to evaluate the safety of structures based on probability theory. In this publication, the basic probability concepts were well discussed and the method of applying those concepts to loads, structural materials and the safety measurement – factor of safety, FS – were studied. Although this paper was not a real case application, it explored the feasibility of applying probabilistic methods in civil engineering practice and paved the road for late work.

In 1956, Freudenthal further developed the probabilistic method in structural safety analysis. In his paper[42], the probability concept was applied to load analysis, structural analysis and the definition of failure. The idea of "Margin of Safety" (or safety margin, SM), which is an often-used concept in probabilistic methods today, was clearly presented and the probability distributions of load and resistance were also discussed.

Entering the 1960's, more research work on the application of probabilistic methods in civil engineering was conducted. In another paper by Freudenthal (1961[43]), he extended the previous work and emphasized reliability in conjunction with the factor of safety and the reliability of a structural system consisting of more than one component. Later on, the ASCE formed the Task Committee on Factor of Safety with the purpose of developing

> "a widespread interest in the topic of motivating additional engineers to study the problem of developing a rational procedure for determining the factor of safety of structures, and provide guidance by suggesting and illustrating techniques that may be of considerable value in studying certain phases of the safety

(reliability) problem".

A final report of this committee was published in 1966 by Freudenthal, Garrelts and Shinozuka[44].

At the same time, application of probabilistic methods in civil engineering was also studied in Canada, European countries and elsewhere. In 1957, Pugsley in his paper, "Concepts of Safety in Structural Engineering" applied probability theory in the structural safety analysis process[89].

In 1967, Cornell studied the complexity of loading and resisting forces in structural systems[25] — both are randomly varying in time and space and there is dependency between them. Trying to make reliability methods feasible in structural system analysis, Cornell studied the bounds on the reliability of structural systems, based on which the reliability computation can be greatly simplified. In two other papers by Cornell (1969)[26,27], especially in the final report of International Association for Bridge and Structural Engineering, the suggested "Second Moment" reliability analysis method made the application of probabilistic methods in civil engineering more practicable. The second moment method uses the first two statistical moments, the mean or expected value and the variance, of random variables to characterize the components of a system. Furthermore, the whole system's statistical character can be described without the specific distribution knowledge of its components.

Ang and Amin in 1968[4] tried to give a mathematical derivation of reliability of structures and structural systems in terms of risk function and reliability function. Although no new concept was raised, their work gave insight on the risk function and the correlation of forces induced in the structure components and the member strengths.

Beginning in the 1970's, more and more methodology studies have been conducted and real case applications of probabilistic methods in civil engineering were carried out. In the respect of theoretical and methodology studies, the works by Ang, Lind and Hasofer and Lind need to be specially mentioned.

In paper of 1971 by Lind[69], "Consistent Partial Safety Factors", based on conventional structural safety design standards, an attempt at making the factor of safety consistent by employing probability concepts was made. In this study, the postulates on which the new safety factor formats were based were well discussed. These postulates and related formats are:

1. Load Postulate — The load on a section is the sum of separate load effects of random magnitude;

2. Strength Postulate — Strength is a random variable reflecting many chance effects;

3. Design Method — Design is to satisfy certain code format.

4. Principle of Constant Reliability — The design values are to be chosen such that the reliability as nearly as possible is constant over the domain of load influence

coefficients.

5. Postulate of data sufficiency — The available data on any random variable are sufficient to characterize its probability distribution with acceptable accuracy for code calibration.

6. ISO Format — The design value of material strength, M*, and load intensity, S_i*, shall equal prescribed constant multiples of the standard deviation below and above the mean, respectively. i.e.

$$M^* = m_M - k_M\sigma_M \tag{2•1}$$

$$S_i^* = m_{S_i} + k_{S_i}\sigma_{S_i} \tag{2•2}$$

in which the k terms are constants.

7. Cornell's Format — (a) The reliability is characterized with sufficient accuracy by the mean and standard deviation of safety margin; and (b) the effects that influence the section strength, other than the material strength, can be separated into two independent random variables representing errors in analysis and fabrication, respectively.

8. Linearized Factor Format — Each partial safety factor is to be a linear function of the mean and standard deviation of the variable in question.

Examples of applying second moment theory and the safety index was illustrated in this paper.

Ang in 1973[5] first declared the "safety index" (later and more often called as "reliability index"), β, as a "statistically consistent indicator of reliability" and the expressions of the safety indices with respect to different distributions, normal or lognormal, of load and resistance variables. In his study, the safety margin was defined as

$$SM = R - D \tag{2•3}$$

for normal distribution, and

$$SM = \ln R - \ln D = \ln(R/D) \tag{2•4}$$

for lognormal distribution. Where R is resistance and D is demand or load. These definitions were accepted by many people thereafter.

Hasofer and Lind gave a clear and rigorous mathematical definition of reliability of design and analysis for structures in their paper of 1974[55], which built a solid theoretical base for the probabilistic method application. In their study, after carefully analyzing the fundamental meaning of second-moment reliability in multivariate of reliability, the reliability measurement –"reliability index" – was well defined. Based on the mathematics derivation, the expressions of reliability index were used for normal and lognormal assumptions, as well as for the failure criterion of

$$(R - ZS) < 0$$

case, where R is the resistance, Z and S are section modulus and load, respectively. In this paper, the Taylor's series approach was employed to simplify the complex and sometimes even intractable statistical calculation.

After near 40 years preparation, the application of probabilistic method in civil engineering stepped beyond its infancy period and is now quickly growing.

2.2 State-Of-The-Art

The first international conference on application of statistics and probability in soil and structural engineering in 1971 marked a new period of the application of probabilistic and statistical methods; a shift from methodology development to engineering application in civil engineering. Although there is still some theoretical work where improvements can be made, such as how to handle the dependency of random variables involved and how to find reliability calculation methods which are simple, easy to use and accurate, today, the applications of probabilistic method can be seen in almost all civil engineering fields.

2.2.1 Applications of Probabilistic Method in Structural Safety

Probabilistic methods in civil engineering were first applied to structural safety problems because of the great safety concerns of society and the great efforts of developing the theory of structural analysis.

2.2.1.1 Seismic risk assessment

An early application of the probabilistic methods was in seismic risk assessment of structures. The reason is obvious: an earthquake is almost a totally uncertain event with respect to its time of occurrence and scale, therefore, the seismic force acting on the structure is a random process and the structural response is not deterministic.

Many seismic risk assessments of structures studies have been for nuclear power plant safety analysis. Besides the electrical part of the power plant, civil engineers use Probabilistic Risk Assessment (PRA) method to evaluate the safety of nuclear facilities. The earliest application of the PRA method was represented in the WASH-1400 report by U.S Nuclear Regulatory Commission in 1975. In the early 1980's, an ongoing multi-country joint research project about the PRA on nuclear power plant was conducted and coordinated by the Commission of the European Communities Joint Research Centre (JRC) and this project produced a useful guide for nuclear industry. At the same time, the committee on Dynamic Analysis of the Committee on Nuclear Structures and Materials of the Structural Division of the ASCE sponsored a working group to study applying probabilistic methods in the seismic analysis and design of nuclear facilities and a guidance book was published thereafter[57,77].

The PRA method is based on a "divide-and-conquer" approach, which first divides the basic system failure model into component events – usually expressed by an event tree or fault tree, then an overall probability of failure can be computed from an aggregation of these individual probability components. This method was easily accepted by the public because it gives the results in the form of quantity (probability of failure) and the mathematical techniques are relatively well formulated, despite the fact that this method does not consider correlations between many components and it requires some "precise" values of the "uncertain" probability of failure for each component.

As a complement, some people discussed another method, called SCM (System Characteristic Models) method, in the later 1980's (Pidgeon, Turner and Blockley[86]). The SCM method served to identify a set of characteristics associated with the system, such as technical problems, individual human, organizational or institutional problems, which are significant indicators of a potential failure in an ongoing technology activity, and used these indicators to evaluate the reliability of this system. This method can cope with complex interaction between technology and social and politics as well as avoid the philosophical error of "over-precise" in the PRA method, but this method has not been as well developed as the PRA method.

The seismic risk assessment is also applied on the response of structures to seismic loading[9]. Since the 1980's, many studies have been on life line structures[48],[51], high buildings and bridges safety analysis but they were more focused on the loading characterization and structural response.

2.2.1.2 Dynamic response of structures

Beside the seismic load, many other dynamic loads, such as wind, wave and traffic loads, are also random factors. In structural dynamic response analysis, some research work which considered those random dynamic loads has been carried out for offshore structures[46], marine structures, high buildings, etc. by applying probabilistic methods. It should be pointed out that among these studies, many analysis results were in terms of statistical moments for particular response quantities (such as acceleration, velocity, displacement, etc.) rather than the reliability of structures or system.

2.2.1.3 The LRFD procedure

The LRFD (Load and Resistance Factor Design) procedure is a relatively well developed probabilistic method in structural engineering and it is a good method for structural design.

In the early probabilistic method application, a method called "second moment reliability code format" was suggested by some researchers. After the earlier draft of this idea by Basilar in 1960[14], Cornell developed and clarified this second moment reliability analysis method in 1969[26], followed by Schorn and Lind (1974)[98], Hasofer and Lind (1974)[55] as well as Ravindra, Lind and Siu (1974)[91].

In 1978, Ravindra and Galambos published a paper "Load and Resistance Factor Design for Steel" which renamed this second moment analysis procedure as LRFD[92].

In the LRFD procedure, the statistical characteristics of load and resistance are first determined then the limit state is specified as

$$\Phi R_n \geq \sum_{k=1}^{j} \Gamma_k Q_{km} \qquad (2\cdot5)$$

where

R_n—nominal resistance;
Φ—resistance factor;
Q_m—mean load effect; and

Γ—load factor.

The subscript k is the load component sequence. If only dead and live loads are involved, then

$$\sum_{k=1}^{j} \Gamma_k Q_{km} = \Gamma_D Q_{Dm} + \Gamma_L Q_{Lm} \tag{2•6}$$

where the subscripts D and L correspond to dead and live loads, respectively.

By the so called "first-order second-moment" (FOSM) analysis method, the reliability index, β, can be determined. For the simple case of independent load, Q_m, and resistance, R_m, with lognormal distribution assumption of R_m/Q_m

$$\beta \approx \frac{ln(\frac{R_m}{Q_m})}{\sqrt{V_R^2 + V_Q^2}} \tag{2•7}$$

where V_R and V_Q are coefficients of variation of R_m and Q_m, respectively.

The central safety factor, λ, can be defined as

$$R_m \geq \lambda Q_m \tag{2•8}$$

By using a linear approximation for $\sqrt{V_R^2 + V_Q^2}$ and redefining eqn. (2•8) as

$$\lambda_R R_m \geq \lambda_Q Q_m \tag{2•9}$$

where

$$\lambda_R = exp(-\alpha\beta V_R) \tag{2•10}$$

and

$$\lambda_Q = exp(\alpha\beta V_Q) \tag{2•11}$$

are resistance and load safety factor, respectively. $\alpha \approx 0.75$

by linear approximation.

If the target reliability index β is chosen, then the design resistance can be determined in accordance with the loading conditions.

2.2.1.4 Other structural analysis

Probabilistic analysis methods are also used in other aspects of structural analysis, such as the deterioration of structural elements, stochastic and time varying fatigue of structures, assessment of damaged structures, nonlinear structural response, the effect of imperfection of structure materials, etc. It is interesting to note that in 1988, Bakouros[13] used discriminant analysis, a statistical method which is often used in social science, to evaluate the safety of North Sea offshore pipeline. Also, some people are trying to apply fuzzy sets theory to model uncertainties (Chiang *et al* 1988[22]).

2.2.2 Reliability Evaluation of the Safety of Highway Systems

The probabilistic method is broadly used in the reliability evaluation of the safety of highway system in the past ten years. Recently, a very good example is the safety evaluation of highway bridges.

Besides some individual research work[38,45], in the early 1980's, the Transportation Research Board of the National Research Council conducted a study under the national cooperative highway research program to evaluate the reliability of existing highway bridges. A report was published in 1987 titled "Load Capacity Evaluation of Existing Bridges" by

Moses and Verma[80].

This national cooperative research program investigated numerous existing steel and prestressed concrete highway bridges, gave the rating for those bridges according to operating and inventory, evaluated the safety of the structures in terms of reliability index β and provided target reliability level for highway bridge design.

The basic evaluation process of this study was by the LRFD method defined by

$$\Phi R_n > \Gamma_D D + \Gamma_L L(R.F)(1+I) \tag{2•12}$$

where

$R.F$ – rating factor,
R_n – nominal member capacity,
D – dead load effect,
L – nominal live load effect due to traffic loadings,
I – impact value,
Γ_D – dead load factor.
Γ_L – live load factor,

and Φ – resistance or capacity reduction factor with

$$\Phi = (R_m/R_n)\ exp(-0.55\beta V_R) \tag{2•13}$$

where

R_m – true mean resistance,
R_n – nominal resistance in the code's strength formula,
V_R – coefficient of variation and
β – safety index (or reliability index).

The rating factor, R.F, can be expressed as

$$R.F = \frac{\Phi R_n - \Gamma_D D}{\Gamma_L L(1+I)} \tag{2•14}$$

This program studied loading model, resistance and load factors and criteria for expressing safety or reliability level. Based on the investigation, target reliability indices were proposed, the correlations between the factors, deduction for deteriorated sections, corrosion effect, as well as the sensitivity of *R.F* with respect to V_R and β were examined.

2.2.3 Applications of Probabilistic Method in Geotechnical Engineering

Since soil is the most uncertain engineering material in civil engineering practice, the probabilistic method has very broad potential in geotechnical engineering field. Although its application has not played the role as it should today, probabilistic methods have touched many aspects of geotechnical engineering.

2.2.3.1 Statistical characterization of testing data

Statistical characterization of testing data is a basic application of statistics and is the necessary step for the reliability analysis of structures. So far, statistical methods, or so called "geostatistics", have been used to describe shear strength test data, conventional lab and field test data. By geostatistics study, soil profiles with the consideration of the effect of spatial variation were better described, estimators for soil parameters were better defined and Bayes' theory can be used to update parameters' probability distribution[56].

2.2.3.2 Slope stability analysis

Much research and many real case applications have been done on earth slope stability since the early 1970's. The paper by Lambe, Marr and Silva (1981)[65], described a comprehensive safety program for dams and natural slopes. After two decades' investigation, by comparing predicted performance with measured performance, a conclusion was reached that geotechnical engineers can predict the performance of structures and foundations with the accuracy of one-half to one order of magnitude. This program provided guidance for design and evaluation of actual performances of geotechnical facilities. Furthermore, risk analysis was employed in this program based on event tree concept to generate numerical assessment of safety.

The application of probability methods to slope stability analysis is mainly on statistically characterizing some particular parameters involved, such as soil strength parameters, c and ϕ (or $\tan\phi$), pore pressure, u, etc. The methods for slope stability analysis are the most commonly used methods in engineering practice, such as Bishop's simplified method, Spencer's method, and Morgenstern and Price's method. The probabilistic calculation methods are Taylor's series approach, Point Estimate Method (PEM), Monte Carlo simulation and direct integration (very few in real application). Quite a few people gave contributions in this respect, such as McMahon (1971), Yucemen, Tang and Ang (1973), Matsuo and Kuroda (1974), Wen (1974), Alonso (1976), Catalan and Cornell (1976), Chowdbury, Chowdbury and Grivas, Chowdbury and Tang (1980, 1982, 1985, 1987[24]), Alfaro and Harr (1981), Bergado and Anderson (1983), Bowles *et al* (1983)[20], Wolff (1985[122], 1991[124]), Matsuo (1987)[73], Lahlaf and Marciano (1988)[64], Resheidat (1988)[93] and Yucemen and Al-Homoud (1988)[121] among others.

The spatial effect of soil properties on slope stability was studied by Vanmarcke (1977[117], 1980[118]), Li and White (1987)[67] and others. Non-circular failure surfaces and progressive failure have been other subjects of slope stability study (Oboni and Bourdeau 1983, Wolff and Harr 1987[123]). Also some attention was given on sliding surface location (Cherubini 1987) and landslide risk assessment (Pack et al 1987[84]).

2.2.3.3 Settlement and consolidation prediction

Soil deformation is an important aspect in foundation design but analysis methods are far from accurate because of the complex composite structure and mechanical behavior of soil materials. Folayan, Hoeg and Benjamin first used probabilistic methods to evaluate the settlement of soils by treating the soil compressibility ratio, m_c as random variable in 1969[36]. Baecher and Ingra (1981[12]) studied one and two dimensional settlement problems with a similar approach. Bourdeau (1986, 1987[17]) focused on the probabilistic analysis of settlements in loose particulate media by applying diffusion theory. As engineering applications, shallow foundation settlement (Usmen, Wang and Cheng 1987[116]), differential settlement of an oil storage tank (Ozawa and Suzuki 1987[83]), and a spread footing on sand (Russell, Denis and Byrne 1987[97]), are good examples. In the study of pier tip deflection due to lateral force by Drumm, Bennett and Oakley

(1990[32]), a three-point PEM reliability calculation method was used.

Soil consolidation is another topic of settlement but most such studies were on the theoretical side. Harr (1977)[52], Athanasiou-Grivas and Harr (1978)[11] applied probability concepts to the one dimensional diffusion equation to study soil consolidation, and by treating it as random walk expanded the study into a 3-D problem. Freeze (1977[40]), Chang (1985[21]) and Koppula (1987[60]) reported similar studies.

2.2.3.4 Probabilistic design

Probabilistic design usually includes reliability analysis of an as-designed structure or system and determination of design parameters based on the reliability analysis results and reliability criteria. Although there is a lack of systematic probabilistic design procedures in geotechnical aspects of structure design, some individual studies or applications are good starts.

In 1969, Tang, Shah and Benjamin[104] used probabilistic methods to characterize load factors and studied the relationships of load factor, the reliability of structure and load factor versus cost, as well as how to apply these results to design. Tang in 1981[105] further outlined the probabilistic characterization method for static, occupancy, extraordinary loads (e.g. snow, wind, wave, earthquake), soil induced pore pressure and seepage forces in design. In the study by Wu and Kraft (1976[128]), a design procedure based on reliability analysis was described. Matsuo (1976[72],1987[73]) studied the reliability in embankment design and Duckstein and Bogardi (1981[30]) studied the probabilistic hydraulic engineering design for flood levees. Smith (1981[102]) gave examples of bearing capacity of soil for retaining walls, Goni and Hadj-Hamou (1988[47]) also studied this subject. Kay (1982[59]) applied probabilistic methods to pile foundation design and Hamer and Vrijling (1986)[50] used probability concepts in storm surge barrier design process; from the plan proposal to technical design. Other probabilistic design applications have been on wood structures (Ellingwood, Hendrickson and Murphy 1987[33]), braced excavations (Halakeyama and Yasuda, 1987[49], Kuwahara and Yamamoto 1987[63]), and pile capacity (Madhav and Ramakrishna 1988[70]). Also needing to be mentioned is the application of probability theory on Mohr failure criteria by Kreuzer and Bury in 1989[61].

By examining all studies listed, it may be important to note that a key point related to probabilistic design is the corresponding criteria setting (may be different from the conventional ones). It is the criteria that controls the design, but criteria are determined by both technology and social factors. For future application of probabilistic method in engineering design, suitable new criteria which are based on reliability theory are desperately needed and more effort should be made, although it takes time to place new criteria in regulations or standards.

2.2.3.5 Navigation structure analysis

In the past ten years, there were few attempts on the use of probabilistic methods in evaluating the performance of navigation structures and systems. In 1987, Benjamin & Associates Inc. studied the probabilistic risk assessment of Emsworth locks and dam on Ohio river for the Corps of Engineers[78]. In this study, probabilistic risk analysis was performed to assess the likelihood of failure (closure) of the locks and dam for the twenty-five year period from 1986 to 2010. The analysis method used in this study was failure tree and the emphasis was on the economic effect of the lock and dam system. Because of insufficient data, the time factor could not be fully studied. In the "Report on Major Rehabilitation, Lock and Dam No. 25" (1992[115]) by Corps of Engineers, Lower Mississippi Valley Division, probabilistic methods were also used to evaluate the reliability of lock structures in the terms of probability of failure. Wolff and Wang (1992)[125] applied probabilistic methods to evaluate the reliability of navigation structures. This study, also sponsored by the Corps of Engineers, investigated several typical gravity monoliths at navigation locks and dams on Monongahela River in Pennsylvania and on the Tombigbee River in Alabama and the reliability of structures was measured by reliability index. Much of the work described herein was performed in connection with this study.

Having no intention to exhaustively summarize all its applications, it is clear that the probabilistic method has been developing and has been applied in civil engineering successfully for several decades. Nevertheless, there is still much room for improvement and this application should be broadened and standardized. Therefore, the probabilistic method application can also be used for the assessment of the relative reliability of navigation structures and used to help prioritize major rehabilitations of navigation structures.

Chapter III

THEORETICAL BACKGROUND

3.1 Probabilistic Methods Used in Civil Engineering Field

Applications of probabilistic methods in civil engineering involve several components. These include statistical data processing, reliability analysis, or risk assessment of individual structural elements and analysis of systems of components.

3.1.1 Statistical Data Processing

In engineering design, especially in geotechnical design of structures, values for parameters are obtained from laboratory or field tests. Since there typically exists great variation among testing data caused by many uncertainties, statistical methods are easily accepted by engineers. For navigation structures, it will be necessary to mathematically characterize the uncertainty or variability of parameters related to the safety of the structure.

Statistical processing of test data is the basic and first step in structural reliability evaluation procedure. The theories used in this step are basic statistical theories and engineers usually are only interested in the first several statistical moments of random variables and the distributions of the random variables.

3.1.1.1 Statistical moments

For a population *X*, with samples $[x_1, x_2, ..., x_n]$ (e.g. soil strength data from direct shear test), its statistical characteristics can be described by the central moments (refer to Harr[53], Ang and Tang[7] and other standard statistics text books). The k^{th} moment, μ_X^k, can be expressed as

$$\mu_X^k = E[(X - \mu_X)^k] \tag{3•1}$$

where μ_X is the expected value, or weighted arithmetic mean of *X*.

For discrete distribution

$$\mu_X = E[X] = \sum_i^n x_i\, p_i \tag{3•2}$$

and for continuous distribution

$$\mu_X = E[X] = \int_{-\infty}^{\infty} x\, f(x)\; dx \tag{3•3}$$

where

p_i –frequency or probability of x_i and

$f(x)$–probability density function (pdf) of *X*.

The *X* is also called a *random variable* which assumes values determined by its probability distribution over certain range.

In statistical analysis, the unbiased estimation of the expected value, $\bar{X}$, of a data set X is defined as

$$\bar{X} = \frac{1}{N}\sum_i^n x_i \tag{3•4}$$

The expected value, E[X], of a random variable is mathematically equivalent to the mean, $\bar{X}$, of a data set.

The second moment of *X*, or the variance of *X*, *Var(X)*, is another important statistical characteristic which describes the variation of the random variable. For population X, its variance can be expressed as

$$Var(X) = \frac{1}{N}\sum_{i}^{n}(x_i - \bar{X})^2 = \frac{1}{N}\left(\sum_{i}^{n}x_i^2 - \bar{X}^2\right) \tag{3•5}$$

and the unbiased estimation of the variance of X is

$$Var(X) = \frac{1}{N-1}\sum_{i}^{n}(x_i - \bar{X})^2 = \frac{1}{N-1}\left(\sum_{i}^{n}x_i^2 - \bar{X}^2\right) \tag{3•6}$$

The standard deviation of X, a measure of dispersion and having same unit as the random variable, is

$$\sigma_X = \sqrt{Var(X)} \tag{3•7}$$

and the coefficient of variation, V_X, a non-dimensional measure of dispersion, is

$$V_X = \frac{\sigma_X}{\bar{X}} \tag{3•8}$$

The V_x is usually expressed by percentage.

For two correlated random variables, X, and Y, the dependency can be described by their covariance, $Cov(X,Y)$, which is defined by

$$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] \tag{3•9}$$

The coefficient of correlation, ρ_{XY}, a non-dimensional measure of the correlation, is

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X\sigma_Y} \tag{3•10}$$

In navigation structural reliability analysis, some random variables are correlated with each other. For example, the shear strength parameters of foundation material, cohesion c and internal friction angle ϕ (or $\tan\phi$) are usually negatively correlated and their correlation can be represented by $\rho_{c,\phi}$ (or $\rho_{c,\tan\phi}$).

3.1.1.2 Probability distributions

In probability calculations, the probability distributions of the random variables determines the probability outcome. Although every random variable has its own probability distribution, two commonly assumed distributions in structural reliability evaluation are the normal distribution and the lognormal distribution.

The normal distribution is a very commonly assumed probability distribution and the central limit theorem supports this popularity. According to the central limit theorem, for any event, if it is composed of a sum of independent random variables with the same distribution, when the number of the random variables is large enough, then the probability distribution of the event approaches normal. Furthermore, even if the random variables are not strictly independent and do not have exactly the same distribution, the density function of the event still approaches normal. The normal distribution can be expressed as

$$f(x) = \frac{1}{\sigma_x\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2\right] \qquad -\infty \le x \le \infty \qquad (3\text{•}11)$$

If $Y = lnX$ and Y is normally distributed, then X is lognormally distributed. The lognormal distribution is in the form of

$$f(x) = \frac{1}{\sigma_Y x\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{lnx-\mu_Y}{\sigma_Y}\right)^2\right] \quad 0 \le x \le \infty \qquad (3\bullet12)$$

Because the lognormal distribution permits no negative values of X but has a lower bound of zero, which has good physical meaning, the lognormal distribution is also very often used in engineering practice.

3.1.2 Probabilistic Method Levels

The probability method levels, which are cataloged according to the degree of involvement of probabilistic theory, are from level 0 to level 3[108].

Level 0 — Deterministic, or no probability theory involved;

Level 1 — Semi-probabilistic. A single simple characteristic value is connected to each uncertainty variable and the safety of structures is estimated by a set of partial coefficients which provide a design basis. When determining the partial coefficients, some probabilistic methods may be involved. For example, in a dam design, a factor of safety may be preset for different loading conditions (e.g. end-of-construction, sudden drawdown, partial pool, etc.), but all parameters involved are precise values. The preset factors of safety may be obtained statistically based on many real cases and experience;

Level 2 — Approximate probabilistic. A safety coefficient is calculated with the help of means and variances of the design parameters. Then the probability of safety (or probability of failure) is calculated from the mean and variance of this coefficient, together with a hypothesis of the probability distribution.

Level 3 — Full probabilistic. Based on complete statistic knowledge of all the random variables (factors, parameters) involved in the safety of structure, the estimation of safety is given through the probability of occurrence of a particular state (safe or unsafe) of the structure.

Today, most engineering applications are at level 2.

3.2 Reliability Evaluation

3.2.1 Reliability of Structures

The reliability of a structure, or characterization of its safety, is not an "absolute" term (the definition of safety is "freedom from danger"), but a relative matter. In a broad sense, the reliability of a structure (or system) can be described as the probability that the structure will maintain a satisfied specific performance, or not fail, during a specified time period under given environmental and operational conditions.

The quantitative measurements of reliability are in terms of probability of safety (or probability of failure) and reliability index.

3.2.2 Reliability Measurements

3.2.2.1 Probability of safety

The probability of safety of a structure (or system) is a measurement of reliability of structure, which is expressed by the probability that the structure satisfied certain operating requirement, or, in a more mathematical manner, the performance function is better than or equal to the given critical value(s).

The mathematical expression of probability of an event, say, $X \geq c$ with $X = [x_1, x_2, \ldots x_n]$ and $c = [c_1, c_2, \ldots c_n]$ is

$$p_R(X \geq c) = P(X \geq c) \qquad \text{(for discrete distribution)}$$

$$= \int \cdots \int_c^{\infty} f(x_1, x_2, \ldots, x_n)\, dx_1 dx_2 \ldots dx_n$$

$$\text{(for continuous distribution)} \qquad (3 \cdot 13)$$

In engineering practice, if the joint probability density functions of capacity C and demand D, $f_{C,D}(x,y)$, are known, then the probability of safety can be expressed as (in two random variables case)

$$p_R \equiv p(safe) = P(C \geq D)$$

$$= \int_{-\infty}^{\infty} \int_0^{(y-x)} f_{C,D}(x,y)\, dx dy \qquad (3 \cdot 14)$$

and the probability of failure, p_f, is

$$p_f \equiv p_f(failure) = P(C \leq D) = 1 - p_R$$

$$= \int_{-\infty}^{\infty} \int_{(x-y)}^{0} f_{C,D}(x,y)\, dx dy \qquad (3 \cdot 15)$$

For independent C and D with pdfs $f_C(y)$ and $f_D(x)$, respectively, then

$$p_R = \int_0^{\infty} \left[\int_0^{y} f_D(x)\, dx \right] f_C(y)\, dy$$

$$= \int_0^{\infty} F_D(y)\, f_C(y)\, dy \qquad (3 \cdot 16)$$

which is a convolution integral where F_D is the cumulative probability function (cpf) of *D*.

Similarly, $p_f = 1 - p_R$.

Two special cases arise. If *C* and *D* are normally distributed, then

$$p_R = \Phi\left(\frac{\mu_C - \mu_D}{\sqrt{\sigma_C^2 + \sigma_D^2 - 2\rho_{c,d}\sigma_C\sigma_D}}\right) \tag{3•17}$$

where

- μ_C and μ_D are expected values of *C* and *D*, respectively;
- σ_C and σ_D are standard deviations of *C* and *D*, respectively;
- $\rho_{c,d}$ is the coefficient of covariance of *C* and *D*.

and $\Phi(\beta)$ is the cumulative probability function of the standard normal variate *X* for $X \geq \beta$.

For uncorrelated *C* and *D*

$$p_R = \Phi\left(\frac{\mu_C - \mu_D}{\sqrt{\sigma_C^2 + \sigma_D^2}}\right) \tag{3•18}$$

For a lognormal distribution of independent random variables *C* and *D*, the probability of safety is

$$p_R = \Phi\left(\frac{ln\left(\frac{\mu_C}{\mu_D}\frac{\sqrt{1 + V_D^2}}{\sqrt{1 + V_C^2}}\right)}{\sqrt{ln(1 + V_C^2)(1 + V_D^2)}}\right) \tag{3•19}$$

where V_C *and* V_D are coefficients of variation of *C* and *D*, respectively.

If V_C *and* V_D are small ($\leq$ 30%), then eqn. (3•19) can be approximated as

$$p_R \approx \Phi\left(\frac{ln\left(\frac{\mu_C}{\mu_D}\right)}{\sqrt{V_C^2 + V_D^2}}\right) \qquad (3\bullet20)$$

3.2.2.2 Reliability index β

The reliability index, β, is another often used and very useful probability measurement. The general definition of reliability index can be summarized as the result of the following:

Let $X = (x_1, x_2, ..., x_n)$ be the vector of random variables relevant to the analysis/design problem with the criterion $F(X) < 0$, where F(X) is the performance function formulated by subtracting the demand from the capacity of the underlying problem. The space of x can be divided into a safe region, $S(X)$ and a failure region, $U(X)$, separated by $F(X) = 0$ (limit state). Then[55]

1. Make an orthogonal transformation of the variables x_i to a new set of variables $Y = (Y_1, Y_2, ..., Y_n)$ such that the Y_i are independent to each other if x_i are correlated;

2. Normalize variables by $y_i = (Y_i - \mu_{Y_i})/\sigma_{Y_i}$ where μ_{Y_i} and σ_{Y_i} are expected values and standard deviations of Y_i;

3. Transfer $F(X)$ to new space y. Corresponding to the new safe and failure regions, $S(y)$ and $U(y)$, the shortest distance from origin to the failure region is the reliability index β.

For a special case, only two variables, C and D, and the safe and failure regions are divided by $F(C-D)=0$, the illustration of reliability index β is as shown in Figure 3-1.

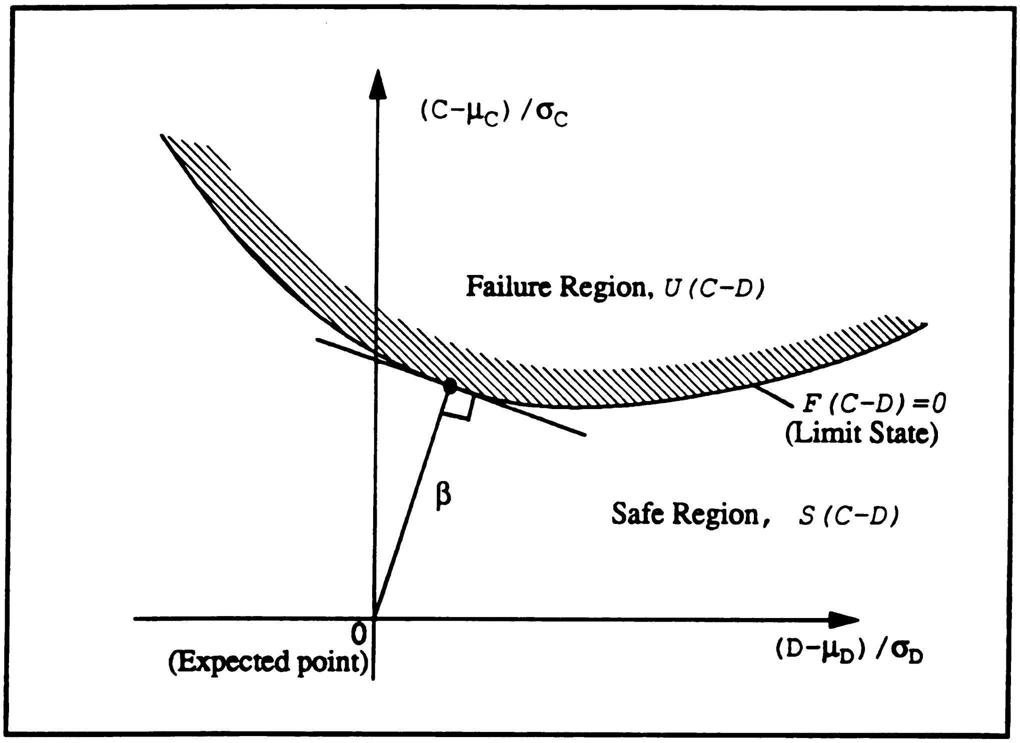


Figure 3-1 Reliability Index β of Two Random Variables

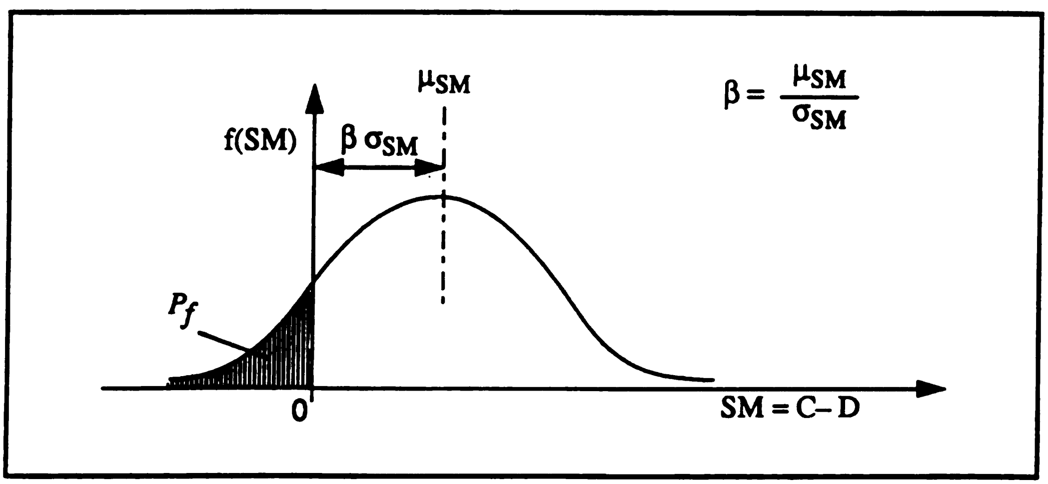


Figure 3-2 Reliability Index β of Safety Margin ***SM***

In engineering reliably analysis, the safety margin, *SM*, is a commonly used variable, it defined as

$$SM = C - D \tag{3•21}$$

and

$$\mu_{SM} = \mu_C - \mu_D \tag{3•22}$$

$$\sigma_{SM} = \sqrt{\sigma_C^2 + \sigma_D^2 - 2\rho_{c,d}\sigma_C\sigma_C} \tag{3•23}$$

If the limit state corresponds to the condition SM=0, then the reliability index is defined as shown in Figure 3-2.

Note that the P_f in Figure 3-2 is the probability of failure, or the probability of capacity being less then demand.

The advantages of using the reliability index as the measurement of structural safety are mainly the following:

1. The reliability index β itself is a dimensionless quantitative value, which is the distance from expected condition to the limit state (or edge of the failure region) in the units of standard deviation of the performance function. In the reliability sense, the greater the value of β is, the safer the structure is;

2. Freedom of choice of basic random variables. There is no restriction of how to chose the random variables in the performance function, it only depends on the statistical properties of those variables;

3. Invariance. It does not depend on the precise analytical form of the criterion, $F(X)<0$, or, in other words, the expression of performance function. For example, one not need specify which variables are loads and which are

resistances as long as the variables involved in the performance function remain the same in the analysis procedure; and

4. Consistency. No matter how many random variables are involved and how their properties change, the definition of reliability index β is always valid.

3.2.3 Reliability Calculations

3.2.3.1 Calculation of probability of safety

The calculation of probability can be carried out by an exact method, approximate methods and simulation methods.

Direct Integration Method – This method is the so called "exact" method which directly integrates the performance function, *F(X)*, over a certain interval for the given problem (e.g. for safety margin, *SM*, integration interval is *SM*>0 for probability of safe). The mathematical expression is:

$$P_R(X \geq X_0) = \iint \cdots \int_{X_0}^{\infty} F(X)\, dx_1 dx_2 \ldots dx_n \qquad (3 \cdot 24)$$

where X_0 is the lower bound of interval, a constant vector.

The direct integration method can give an accurate measurement of probability if the probability joint density distribution function is known. But in most cases, the probability distribution is unknown (though very often people assume the random variables are normally, multivariate normally, or lognormally distributed). Even if the distribution function is known, usually the integration does not have a closed form solution, therefore, it must be carried out by

numerical methods and this is not always an easy task.

Monte Carlo Simulation. This simulation method first generates random variables according to their probability distributions, then simulates the probability function (or process) as if collecting actual event outcomes. The statistical results, such as the failure frequency, the probability of failure, can be calculated based on the simulations.

The principle of simulation is simple and clear. The results can be very accurate and are visible when graphically represented as a histogram, frequency distribution, etc. The usual techniques of simulation are the inverse transform method, the acceptance-rejection method and the composition method (details of these methods can be found in some text books, e.g. Harr[53], Hart [54], etc.).

There are some drawbacks about the Monte Carlo simulation method. (1) The distribution of random variables and the performance function must be known precisely (or assumed) before the simulation can be performed; (2) It is not easy to obtain the inverse of the cumulative probability function for some distributions although numerical techniques may help; (3) Great computing time is requested if one wants to get accurate result. For example, for a function with two random variables, about 16,000 trials are needed in order to have 99% confidence in reliability of the simulation.

In navigation structural reliability analysis, the actual distributions of the random variables involved usually are unknown and the performance functions are usually complex, therefore, it is not desirable to apply the Monte Carlo simulation method in real case studies. Although some new simulation techniques have been developed recently, such as important sampling methods which can greatly reduce the computation time, the Monte Carlo simulation method still needs to be improved for practice use.

Approximate Method. The approximate method uses some mathematical approximation to calculate the probability quantities of performance functions. This method is on the probabilistic analysis level 2. Two often used approximate methods in engineering reliability analysis are the Taylor's series method and the point estimate method (PEM).

1. Taylor's Series Method

This method is base on expanding the performance function about its mean (expected value) according to Taylor's series theory and expressing its mean and variance in terms of the expected values and variances of individual random variables involved.

This is an attractive method in reliability analysis calculation especially when used in the first order-second moment method. This method has the advantages of (1) knowledge of probability distributions of variables is not needed; (2) usually only up to the second moments of the random variables are of the interest; (3) the computation is simple but the calculation results are fairly accurate especially when the performance function is linear; and (4) the influence of each random variable on the statistical moments of performance function (i.e. its contribution to the overall uncertainty of the performance function) can be easily pinpointed.

The expected value and variance of a performance function *F(X)* can be easily obtained by Taylor's series method as (only first order terms are considered.):

$$E[F(X)] \approx F(\bar{x}_i) \tag{3•25}$$

$$Var(F) \approx \sum_{i=1}^{n} \left(\frac{\partial F}{\partial x_i}\right)^2 Var(x_i) + 2 \sum_{i=1, i<j}^{n} \frac{\partial F}{\partial x_i}\frac{\partial F}{\partial x_j} Cov(x_i x_j) \tag{3•26}$$

where

$F(\bar{x}_i)$ indicates the value of performance function at values of $\bar{x}_i$; and

$\frac{\partial F}{\partial x_i}$ are the first partial derivatives of *F(X)* with respect to x_i.

Note that all values of x_i in the calculation are their expected values. For proof of eqn. (3•25) and eqn. (3•26) see Appendix A.

The first derivatives can be carried out either by mathematical definition or by approximation, which is

$$\frac{\partial}{\partial x_i}F(X) \approx \frac{F(X|\, x_i + \Delta x_i) - F(X|\, x_i - \Delta x_i)}{2\Delta x_i} \tag{3•27}$$

where $F(X|x_i \pm \Delta x_i)$ is the value of F at its mean except the x_i has the value of $\mu_{x_i} \pm \Delta x_i$.

It should be pointed out that Δx_i can be any number and the smaller the better, according to the mathematical definition of the derivative. In practice, the standard deviation of x_i, σ_{x_i}, may be chosen as Δx_i (suggested by Mlakar (1990), personal communication). By choosing one standard deviation of the random variable as Δx_i, some non-linearity of the performance function may be included in the calculation but the derivatives become a secant rather than a tangent of the function with respect to random variable x_i at range of $\mu_{x_i} \pm \sigma_{x_i}$.

Like any other methods, the Taylor's series method has its disadvantages: (1) in order to carry out Taylor's series method, the derivatives of reliability function must be first determined (not always a easy job though approximation can make it easier); (2) In practice, many performance functions have a certain degree of nonlinearity and the covariances of the random variables in the performance function are usually unknown, plus the terms higher than second order usually are ignored, which sometimes can lead to a large error. (3) Expanding the Taylor's series should be about the "design point" and about its mean is only a approximation. If these two points are far apart, then the accuracy of this method will be poor and this is often the case for a high nonlinear performance function.

2. Point Estimate Method (PEM)

This method is based on discretizing the random variable probability distribution from its whole region into two (or more[32]) points along its own dimension with weighted probability concentration values. In this fashion, the reliability function can be approximately represented by a few weighted points over all dimensions and the mean and variance of this function can be easily estimated.

In the *PEM*, the first two moments of performance function $F(X)$ can be expressed as[95,96]

$$E[F] = \sum P_i \; F(\pm,\pm,\pm,\ldots\pm) \tag{3•28}$$

$$E[F^2] = \sum P_i \; F^2(\pm,\pm,\pm,\ldots\pm) \tag{3•29}$$

and

$$Var(F) = E[F^2] - (E[F])^2 \tag{3•30}$$

where

P_i are probability concentration coefficients of points; and

$F(\pm,\pm,\pm,\ldots\pm)$ are the values of F at certain points at which the X_i have values $X_i \pm \sigma_{X_i}$.

The correlations between random variables are reflected in P_i which are related to the coefficients of correlation.

Like the Taylor's series method, the PEM is another attractive reliability calculation method, because that (1) no precise probability distribution information is necessary; (2) only first two moments of random variables are needed (or three, if desired, for cases with few variables where more accurate result required); (3) correlations between random variables can be easily included; (4) it can be easily used for a complex performance function. One does not even need to know the function form – it could be a table or observation of a process; (5) the computation is simple and easy to program; and (6) as the weighted point values are spanned over the performance function domain on a certain interval, some nonlinearity effect of the function is counted, therefore, it gives more accurate results.

On the other hand, in the PEM, for a performance function which has more than 2 variables, the skewnesses (third moment) of each variable are not able to be simply taken into consideration, therefore, the results may loss certain accuracy. Also, for each analysis calculating round, the values of performance function must be determined for 2^N points (if this function contains N random variables). If the function is very complex and N is large, then the computation will be time consuming.

A general comparison of reliability calculation methods is summarized in Table 3-1.

Although approximative methods have some disadvantages, such as: a significant error may be introduced if the performance function is highly nonlinear; different results may be yielded for different mechanically equivalent formulation of the same problem; and the information on distribution of the random variables is totally ignored, their simplicity, feasibility and good accuracy make them easily adapted for engineering practice.

3.2.3.2 Calculations of β

The reliability index β can be calculated either from its general definition or by an approximate method.

1. General Definition Approach

To find the "exact" value of reliability index, the performance function, F(X), must be first established as well as the criterion C. Then performing transformation to

Table 3-1 Comparison of Reliability Calculation Method

Method	Concept	Mathematical Work Needed	Probabilistic Properties Needed	Computation	Accuracy	Computer Programming
Direct Integration	Determining probability of failure (safe)	Integration	Distribution functions (pdf)	Complex usually numerical	Very accurate if pdf is true	Not simple
Monte Carlo Simulation	Generating random variables, Simulation	Determining the inverse of cumulating function	Distribution or cumulative functions	Not complex but time consuming	Accuracy increases with square of trials	Not always simple
Taylor's Series Approach	Taylor's series expansion	Derivatives of performance or distribution functions	Means and variances	Simple	Approximate but good	Not difficult but need calculate derivatives
Point Estimate Method	Discretizing distribution function into concentrated points	Determining weight coefficient and point values	Means and variances	Simple	Approximate but good	Not difficult, can be easily changed for different cases

create a set of uncorrelated new variables, Y and normalize them by their expected values and standard deviations, i.e.

$$Y_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \tag{3•31}$$

By the limit state concept, Let

$$F(Y) = 0 \tag{3•32}$$

define the boundary of the safe and failure regions. The shortest distance from origin to the boundary defined by eqn. (3•32) is the value of β. The corresponding point, Y_0, on the boundary is called the "design point" (this definition is first defined by Hasofer and Lind in 1974). It is not easy to find the shortest distance from a given point to a space which contains many variables (multiple dimensions), very often mathematical iteration must be applied.

In engineering reliability analysis, the performance function usually can be summarized as the function of two general random variables – load, D, and capacity (resistance), C, although D and C are function of many other random variables. In this case, closed form solutions of reliability exist for two probability distributions (also see discussion in section 3.2.2.2).

For a normal distribution of C and D, by using the safety margin concept

$$\beta = \frac{\mu_C - \mu_D}{\sqrt{\sigma_C^2 + \sigma_D^2 - 2\rho_{c,d}\sigma_C\sigma_D}} \quad \text{if } C \text{ and } D \text{ are correlated}$$

$$\beta = \frac{\mu_C - \mu_D}{\sqrt{\sigma_C^2 + \sigma_D^2}} \qquad \text{if } C \text{ and } D \text{ are independent} \quad (3\bullet 33)$$

For a lognormal distribution of C and D (independent), by the factor of safety concept, $FS=C/D$

$$\beta = \frac{ln\left(\frac{\mu_C\sqrt{1+V_D^2}}{\mu_D\sqrt{1+V_C^2}}\right)}{\sqrt{ln\,(1+V_C^2)\,(1+V_D^2)}} \approx \frac{ln\left(\frac{\mu_C}{\mu_D}\right)}{\sqrt{V_C^2+V_D^2}} \qquad (3\bullet 34)$$

The approximation is valid for V_C and V_D less than 30%.

2. Approximative Methods

In engineering practice, the first order—second moment method can be used to approximate the reliability index. The basic idea of approximative methods is to linearize nonlinear performance function then to determine the reliability index β.

If the performance function is in terms of the safety margin, then the means and standard deviations of C and D can be calculated by any method — usually the Taylor's series method and PEM, then eqn. (3•33) can be used.

If the performance function is in terms of the factor of safety, there are two ways to calculate the reliability index, one is by first calculating the means and standard deviations of C and D separately and then use eqn. (3•34) to determine the reliability index; the other one is to first find the mean, μ_{FS}, and standard deviation, σ_{FS}, for FS (treating $FS = C/D$ as a whole), then the reliability index,

β, can be calculated by

$$\beta = \frac{\mu_{FS} - 1}{\sigma_{FS}} \tag{3•35}$$

if it is assumed the factor of safety normally distributed; or

$$\beta = \frac{ln\left(\frac{\mu_{FS}}{\sqrt{1 + V_{FS}^2}}\right)}{\sqrt{ln(1 + V_{FS}^2)}} \approx \frac{ln(\mu_{FS})}{\sqrt{V_{FS}^2}} = \frac{ln(\mu_{FS})}{V_{FS}} \tag{3•36}$$

if it is assumed the factor of safety is lognormally distributed. In eqn. (3•36) V_{FS} is the coefficient of variation of the factor of safety and the first two moments of FS can be determined by any suitable method.

For other assumed probability distributions of the factor of safety, the equivalent reliability index (referencing to the normal distribution) can be determined by

$$\beta = \Phi^{-1}(p_R(FS \geq 1)) \tag{3•37}$$

where

$\Phi^{-1}(\bullet)$ is the inverse of the cumulative distribution of standard normal distribution function; and

$p_R(FS \geq 1)$ is the probability of safety for $FS \geq 1$.

It needs to be pointed out that the reliability index obtained by linearization of nonlinear function methods is different from that obtained by Hasofer and Lind's definition. This is because the approximative methods will depend on the choice of point X_0, about which the function is expanded, and the choice of performance function, while the

Hasofer and Lind's definition relates β to failure surface (the boundary of failure and safe) rather than the performance function, so the reliability index defined by Hasofer and Lind is invariant. The design point usually may not be the expected point of $\bar{X}$ (at which the performance function has its mean value) although they may be pretty close. In engineering applications, approximative methods are often used, and the "design" point may not be easily obtained, therefore, herein the design point is approximated by the expected point.

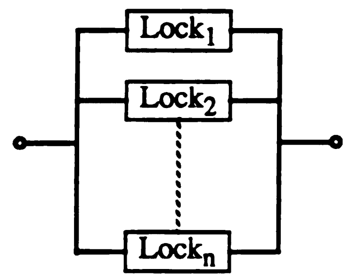
3.2.4 System reliability

System reliability is the overall reliability of a system which consists of many components. The system reliability is determined by the reliability of individual components and their configurations, under certain environmental and operation conditions for a specific criterion. For example, a lock consists of several walls and the walls are formed by many lock monoliths, the overall reliability of the lock is determined by the individual reliabilities of lock walls and monoliths and the system configuration.

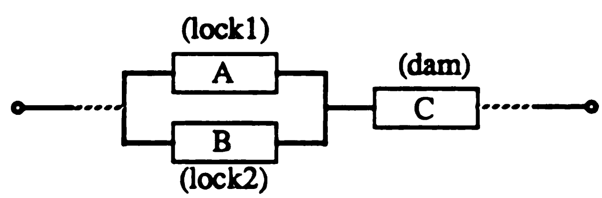
3.2.4.1 Series system

A series system is a basic simple system whose components are in series, such as a gate hoist chain (see Figure 3-3(a)). For a series system, the system is safe only when all components work. In general, if the probability of safety is R_i and the probability of failure is F_i for each components, then the probability of safety for the system is

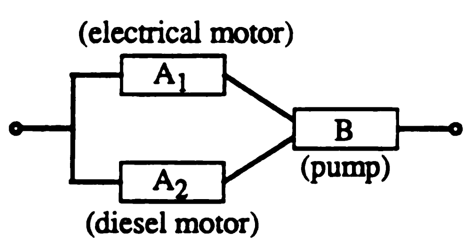
a. Series system (gate hoist chain)



b. Parallel system (multiple locks)



c. Parallel-Series system (multiple locks and dam)



d. Standby system (dewater system with)

Figure 3-3 Some Typical System Configurations

$$P_R = \prod_{i=1}^{n} R_i \tag{3•38}$$

and $P_f = 1 - P_R$.

3.2.4.2 Parallel system

A parallel system is another basic simple system whose components are in parallel, such as a set of multiple locks (see Figure 3-3(b)). If any component works in a parallel system, this whole system is safe, or, only if all components fail then the system will fail. So

$$P_f = \prod_{i=1}^{n} F_i = \prod_{i=1}^{n} (1-R_i)$$

therefore

$$P_R = 1-P_f = 1-\prod_{i=1}^{n} F_i = 1-\prod_{i=1}^{n} (1-R_i) \tag{3•39}$$

3.2.4.3 Series-Parallel system configuration

When the components of the system are configurated in the combination of series and parallel, the system is a mixed series and parallel system, such as two locks and dam system (see Figure 3-3(c)). For mixed systems, the whole system can be divided into several subsystems in such a way that the subsystems are simple series or parallel systems, and those subsystems are in simple series or parallel. After finding the probability of safety or probability of failure for each subsystem, eqn. (3•38) and eqn. (3•39) can be used to calculate the overall reliability of the system.

For the system in Figure 3-3(c), three components, A, B (two locks) and C (dam), exist. Since only if both locks are out of order or the dam part fails this locks and dam system will fail, therefore, this system is a parallel-series system. The two locks form a simple parallel subsystem (say, D) with reliability of

$$R_D = 1 - \prod_{i=1}^{2} (1 - R_i)$$
$$= 1 - (1 - R_A)(1 - R_B) = R_A + R_B - R_A R_B \qquad (3\bullet 40)$$

As D and C are components in simple series, the system reliability is

$$P_R = \prod_{i=1}^{2} R_i = R_D R_C = [R_A + R_B - R_A R_B] R_C \qquad (3\bullet 41)$$

3.2.4.4 Standby systems

A standby system is a pseudo-parallel subsystem, only when the normal operating component fails the spare (standby) component starts working. For example, a dewatering system in a lock and dam system, usually an electrical motor will drive a pump but a diesel motor (or another electrical motor) will be in standby position. For the system in Figure 3-3(d), the probability of failure is (suppose A_2 is spare component and B is switching operation)

$$P_f = P(A_1 \text{ and } A_2 \text{ both fails, } B \text{ works}) + P(A_1 \text{ and } B \text{ fail})$$
$$= (1 - R_{A_1})(1 - R_{A_2})R_B + (1 - R_{A_1})(1 - R_B)$$
$$= (1 - R_{A_1})(1 - R_{A_2}R_B) \qquad (3\bullet 42)$$

and

$$P_R^* = 1 - P_f = 1 - (1 - R_{A_1})(1 - R_{A_2}R_B)$$
$$= R_{A_1} + R_{A_2}R_B - R_{A_1}R_{A_2}R_B \qquad (3\bullet 43)$$

If the reliability of the switch is R_B^* under normal operating condition, then the overall system reliability is condition, then the overall system reliability is

$$P_R = P_R^* \quad R_B^*$$
$$= (R_{A_1} + R_{A_2}R_B - R_{A_1}R_{A_2}R_B)R_B^* \qquad (3 \cdot 44)$$

3.2.4.5 Other system configurations

Real systems are often in a mixed configuration. For the mixed configuration systems, each subsystem can be first transferred to a equivalent single component with its probability of safe or failure, then the reliability of the whole system can be determined by a simplified equivalent system which is usually a simple series or parallel system.

3.2.5 Fault Tree

The *fault tree* and *event tree* are two techniques often used in system risk assessment. The difference between a fault tree and an event tree is that the event tree method uses "forward logic", that is: it starts by assuming either a normally operating system or a given initiating event and by means of a binary logic (fail/not fail) diagram, then charts all possible system states. The fault tree uses "backward logic": it first identifies the "given" event, i.e. failure event, then traces back all possible "basic" events which contribute to the failure. For a navigation structure or system, there are many events which could result in the failure of the structure or system, and many natural or human factors, subsystem or component failure can also make the system fail. For example, foundation failure, structural failure, sliding, gate machinery failure, strong earthquake, severe flood or drought, a runaway barge, etc. can all make tree or event tree methods can be used to assess the risk of navigation system.

The procedure of building a fault tree is the following (the event tree method has similar procedure)[108 and others]:

1. Draw flow graphs, system logic diagrams to show the relationship of all components and events of the system;

2. Identify the different failures and modes of failure that can occur at the component, subsystem and system level;

3. Evaluate the direct and consequential effect of those failure;

4. Build a logical tree-type network which relates the various failure events both causal and consequential. In this logic tree, all components are linked by *AND* and *OR* logic gates to represent the fault relationship of the system under study;

5. Evaluate the probability of failure of the system for each failure event and failure mode.

To develop a fault tree, the whole system must be first divided into many components and the probabilities of failure (or safe) of each component must be known, then the quantitative analysis of the failure can be carried out. Because of this requirement, the fault tree (event tree) method has its limitations since sometimes it may be not easy to divide the system into reasonable subsystems or components and their "precise" probability of safety (or failure) cannot be easily determined. Also, the fault tree method can not take the dependency of components (or events) into consideration.

3.2.6 Reliability Index of Systems

A major item of interest is assessing the "overall" reliability of a structure and understanding how changes in component reliability affect the overall reliability. Although it may be argued that if the smallest reliability of all individual components is found then this reliability can be used as the base of design or rehabilitation decision, actually, this smallest reliability may not represent the reliability of the whole system. The overall reliability may be greater or smaller than the smallest individual reliability depending on the configuration of the system.

If the first two statistical moments of the performance function of a navigation system are known (obtained by any probabilistic analyzing methods described before), then the calculation of the reliability index of the system will be the same as that for single component. If the probability of failure or reliability of components are known, since the whole system can be modeled as a system with mixed components configuration, the reliability index of the system can be determined by the methods discussed in section 3.2.4.

If applying fault tree method, the "minimal cut set", or the smallest unreducible collection of basic events (components not functioning) required to insure occurence of the top event (system failure), must be first determined, then the reliability of this minimal cut set can be found. The calculated reliability index, β, of this set is the reliability index of the system for that specific event or failure mode.

3.2.7 Effect of Time Factor

The time factor is an important factor in navigation structural reliability evaluation because of the following:

1. The longer the service life is for a structural system, the more risk exposure it will take, such as earthquake, flood, etc.;

2. Stress relaxing and strain creeping phenomena exist in engineering structures and materials; and

3. The deterioration of engineering materials may get worse as time spans.

3.2.7.1 Reliability of system with time dependence

Generally, the time factor can be considered as a single function which is affected by several different sources, or can account for the effect from each source separately. For example, if the performance function (the joint probability function of capacity and demand) is also a function of time, t, that is, $f_{C,D} = f_{C,D}(x,y,t)$ in mathematics expression, then the reliability of the system over a time period, 0 to T, with criterion, $C>D$, is

$$P_R(t>T) = \int_T^{\infty}\left\{\int_{-\infty}^{\infty}\int_0^{(y-x)} f_{C,D}(x,y,t)\,dxdy\right\}dt \qquad (3\bullet 45)$$

If the joint probability function can be written as

$$f_{C,D}(x,y,t) = f_{C,D}(x,y)\theta(t) \qquad (3\bullet 46)$$

where $\theta(t)$ is probability density function with variable time, t, then the eqn. (3•45) can be written as

$$P_R(t > T) = \int_0^T P_R \theta(t)\,dt = P_R \Theta(T) \qquad (3 \cdot 47)$$

where $\Theta(T)$ is the integration of $\theta(t)$ over the time span or the cumulative probability function of $\theta(t)$. Note that the function $\theta(t)$ must be normalized so that the product of $f_{C,D}(x,y)\theta(t)$ satisfies the basic requirement of a joint probability function.

If the $\theta(t)$ is not a probability density function but a function which reflects the property changes of random variables as time passes by, then the reliability of the system at time T is

$$P_R(t=T) = \int_{-\infty}^{\infty} \int_0^{(y-x)} f_{C,D}(x, y, T)\,dx\,dy \qquad (3 \cdot 48)$$

or
$$P_R(t=T) = P_R \theta(T) \qquad (3 \cdot 49)$$

if the joint density function can be expressed by eqn. (3•46).

Another probabilistic description of failure of a system or single component which considers time factor, is the so called *hazard function*, or age-dependent failure rate. In navigation system structural reliability analysis, it is difficult to define the hazard function of the system so this concept may not be easily applied.

Chapter IV

APPLICATION OF RELIABILITY ANALYSIS TO NAVIGATION STRUCTURES

4.1 Introduction

The reliability of navigation structures is a very important factor in the decision making process for allocating funding for navigation system's rehabilitation. An analysis procedure which can provide a quantitative and rational reliability evaluation is desired. As numerous uncertainties are present in the structural safety analysis process and these lead to the shortcomings associated with traditional deterministic methods, a probabilistic reliability evaluation procedure is a solution.

A probabilistic method for reliability analysis to navigation structure must have the following capacities:

1. It must be able to evaluate the reliability of structures under different loading and hazard conditions;

2. It must take most of the uncertainties existing in particular cases into consideration;

3. It must give a quantitive comparative measurement of structural reliabilities, both for single components and systems (e.g. single monolith and a lock system) not just some fuzzy words such as "good", "bad", "better", etc.;

4. It must be suitable for use in engineering practice,

not too complex but sufficiently accurate; and

5. It should be applicable, in principle, to both structural safety analysis and design.

To fulfill this task, the failure event characteristics, loading conditions, and uncertainties must first be identified, then the reliability criteria must be specified and the quantitative reliability measurement must be calculated.

4.2 Identification of Failures

To evaluate the reliability of a structure or system, the potential failure modes must be first identified. The identification of failures includes failure specification and detection.

Failure is a relative matter depending on its definition. As the reliability is defined as the freedom from failure of a component or system while maintaining a specific performance, failure is usually defined as a condition where a system cannot operate satisfactorily during a specified time period under given operating condition. For a lock and dam system, the typical navigation structure, the possible types of failure include:

Foundation failure;

Structural failure;

Spillway failure;

Overtopping;

Piping;

Sliding; and

Other modes.

According to the survey conducted by the Subcommittee of Dam Incidents and Accidents of the Committee on Dam Safety of the U.S. Committee on Large Dams in 1988, for 516 dam failures investigated, the percentage of incident types by causes was

Foundation failure:	10.1%
Structural failure:	28.5%
Spillway failure:	21.1%
Overtopping:	14.3%
Piping:	11.6%
Sliding:	7.6%
Unknown:	6.7%

The data shows that the foundation failure, structural failure and spillway failure caused about 60% of the total dams incidents. For navigation locks and dam structures, usually much lower than the large dams (less than 50 feet high), foundation failure, structural failure, sliding and overturning would be the failure modes of most interest.

Failures can be initiated by natural causes and/or by human errors. Human error is a very uncertain factor and it is difficult to account for in structural reliability analysis. Also because good regulations can reduce the possible human errors, the structural reliability analysis techniques developed herein will focus on the "natural" causes.

To find out the possible failure events for a navigation structural system, three sources can be used: 1. Observable through inspection; 2. Detected by investigations; and 3. Predicted from past occurrences.

In the analysis, different events on the same structure, similar events on the structure system and the combinations of events, as well as the failure sequence and probability need to be carefully examined.

4.3 Identification of Hazards

Every failure that has occurred must have its causes, common or unique, man-made or natural. These causes initiated the failures and were responsible for its consequences. For example, heavy rain may cause severe flood leading to dam overtopping and also cause spillway failure; a severe earthquake will generate additional shear force on the foundation of a lock and dam, it may cause stresses which exceed the soil shear strength and foundation failure may occur, therefore, the configuration of lock monoliths may be distorted. As the consequence of these hazards, the lock and dam may not be able to function normally.

There are many events which can initiate failures. To identify these events, or to identify the hazard for the object studied is another necessary step in reliability analysis. For gravity monoliths at navigation structures, the possible events are instability for resisting sliding or overturning; loads exceeding the bearing capacity of foundation; severe earthquake and flood; etc.

4.4 Identification of Random Variables

In structural reliability analysis, many uncertainties are involved, or, many *random variables* are involved in *performance functions* of structures. The random variables are usually related to loads and resistances, either explicitly or implicitly, and they need to be identified in order to determine the moments or distribution of performance functions.

4.4.1 Loadings

When considering the reliability of structures, the loading conditions immediately surface. There are different types of loadings which can be distinguished from the terms of their measurements or from the sources.

Based on the measurements, the loadings can be divided as[8,54,105]:

1. Loading which is based on measurements of load intensity without regard for the time frequency of occurrence, such as dead loads (gravity loads) and live load (contents of a building);

2. Loading which is based on measurements obtainable at prescribed periodic time intervals, such as wind, snow, ice, storm induced wave, traffic, etc.; and

3. Loading which is based on infrequent measurements which are not obtainable in a foreseeable period of time, such as earthquakes, tornadoes, hurricanes, etc.

If loadings are divided according to their sources, they can be categorized as:

Static loads;

Dynamic loads; and

Environmental loads.

There are many uncertainties among the loading conditions; not to mention the uncertainties in dynamic loads, even the static loads are not "exactly" known. Some loading conditions themselves are random variables, some are functions of other random variables. For reliability analysis of navigation structures, without analyzing their significance, the possible random variables involved can be discussed as the following.

Static loads. Although static loads have less uncertainty as they are produced by the weight of all permanent structural and nonstructural components of a structure, they still relate to some random variables: the dead weight of lock monolith is function of unit weight of concrete and its dimensions. The dimensions change as the results of erosion and deterioration, especially for old structures, and the unit weight of concrete, $\gamma_{concrete}$, is a random variable; the unit weight of soil, γ_{soil}, as well as the weight of other structures on the locks and dam, also have their uncertainties.

Among the static loads, loads induced by earth materials need to be specially mentioned because they are significant in navigation structures, especially for retaining walls. Such loads (or resistances) are determined by soil or rock's material and mechanical properties, and the soil properties may vary widely because of the complexity of its mechanical and chemical mechanism. So soil parameters, such as the soil unit weight, γ_{soil}, the water content, w, plasticity index, PI, Young's modulus, E_s, shear modulus, G_s, Poisson's ratio, μ, damping ratio, λ, coefficient of lateral earth pressure, K, coefficient of permeability, k, soil compressibility ratio, m_c, coefficient of consolidation, C_c, internal friction angle, ϕ_s, cohesion, c_s, etc. are all random variables regardless whether they are involved in load or resistance.

Dynamic loads. Dynamic loads are clearly random variables, not only because of uncertainties in their amplitudes and frequencies but also because of the uncertainties in the structural response analysis formulations. The dynamic loads involved may be the impact load produced by barges, the hawser force, traffic on the locks and dams, load induced by motors operating, etc.

Environmental loads. The environmental loads, or extraordinary loads are cause by nature, such as earthquake, flooding, wind, snow and ice. Corrosion and deterioration are also caused by nature—chemical reaction or water excision, but they can be treated as indirect loads because they will negatively impact on structure resistance. All environmental loads are random variables which can be represented by some corresponding quantities, such as the maximum ground acceleration induced by earthquake, maximum air pressure induced by wind, and so forth.

4.4.2 Resistances

Structures can safely perform normal functions because they normally have the capacity of counteracting all loads acting on them, or there is enough resistance from the structure and its foundation.

The capacity of structure is not a simple value or figure, it contains many uncertainties as mentioned before. The resistances of a structure are determined by its design, structural materials, construction quality, environmental conditions, operation conditions and loading conditions.

It is easy to see that there are many random variables in structural design and construction quality because the design and construction quality themselves are uncertain factors which depends on the theoretical concepts and the skill and experiences of human beings. In specific structural safety analysis, as the structure is already designed and constructed, usually only the construction quality factor can be taken into consideration which will be reflected in structural strength parameters.

The random variables related to structure materials and environmental conditions are the unit weight of the materials and their strength parameters. For rock and soil, the material shear strength is usually represented by the parameters of cohesion, c, and the internal friction angle, ϕ. The environmental conditions are described in terms of the degree of erosion and corrosion of the structure.

The loading conditions affect the resistance, because some loads act in such a way that they make the structure unsafe with respect to one aspect but make the structure safer against another. For example, the self weight of a lock monolith is a load which will tend to make foundation fail with respect to the bearing capacity of the foundation, but it is also a resistance which will tend to make the monolith stable when considering sliding and overturning aspects. So the random variables involved in loads sometimes may also appear in resistance under different performance modes.

4.4.3 Hydraulic Forces

Hydraulic forces are special but typical forces associated with navigation structures. Hydraulic forces are caused by pool levels and saturation levels of backfill. In fact, the pool levels around locks and dam, as well as the saturation levels, are fluctuating all the time. Without any doubt, these levels are random variables and they greatly affect the hydraulic loading condition and the effective soil shear strength. The pool levels and saturation levels also affect seepage conditions which are an important factor in piping failure of dams. As various pool levels occur, which change the hydraulic forces involved, the reliability of the analyzed structures will also vary, so the pool levels and/or saturation level of soil are basic random variables.

A related random variable, the hydraulic uplift force on the structure base, U, was a primary focus of this study (which will be discussed later) because this force plays an important role in structural reliability under some circumstances. Hydraulic uplift force is determined by pool levels, the geometry and relative permeability of base material and the compression area of the base.

4.4.4 Significance and Dependence of Random Variables

Although all variables involved in the structural reliability evaluation process actually can be seen as random variables (in the sense of that there is nothing "absolute" in the world), the influences of each random variable are different. Usually, if a variable significantly influences the performance of a structure and has great variation or uncertainty, this random variable should be defined as a random variable (e.g. soil strength parameters, c and $\tan\phi$). Conversely, if a variable has very small variation (e.g. the unit weight of water) or has great variation but only a small influence on the performance (e.g. hawser force in sliding stability analysis), then this variable can be fairly treated as a constant. The criterion of assigning variables as random or constant in reliability analysis is to make the analysis simple but with maximum practical accuracy. In the real case analysis examples (Chapter V), these effects will be examined.

Many random variables are, in reality, dependent on each other (not independent). The dependences may exist with respect to space, time, or the physical or mechanical properties of random variables. For instance, the mechanical properties of soil at different locations is correlated with space (distance), while the saturation level of backfill soil is closely correlated with pool levels both in time and space (height). In many cases, the inter-dependencies of random variables greatly affect the probabilistic behavior of those variables, the performance of structures where those random variables are involved, and, of course, the reliability analysis results. For example, usually the soil strength parameters, c and $\tan\phi$, are negatively correlated by their mechanical properties. If this correlation is not counted, greater variance will result and lower reliability will be calculated for the structures. It may be so conservative as to indicate unreliability for a structure that is, in fact, quite reliable. Unfortunately, the dependencies of random variables are not always known and their involvement will make the analysis process more complicated. If dependencies are to be considered in analysis, a transformation may be necessary to obtain a new set of equivalent independent random variables to make the analysis simpler. This transformation method will be discussed later.

4.4.5 Exchangeability of Load and Resistance and Its Limitation

The reliability of a system (or structure) depends only on its properties, environmental conditions, the time of interest and the criteria applied. It should be totally independent of the analysis methods and procedure — at least in the view of probability theory and in common physical sense. This means that the variables involved in the structural reliability analysis should be able to be freely defined either as "loads" or as "resistances". In other words, a random variable can be defined as a resisting force or a negative loading and this exchange should not affect the analysis results.

In reality, the invariance of probabilistic structure reliability is valid only if strict conditions are met: the probabilistic properties of every random variable are known, the performance function which represents the mathematical and/or logical relation of those random variables is exact, the joint probability distribution is explicit, and the calculations involved are exact. Obviously, one could not find even a single real case which satisfies all those requirements; therefore, to make the probabilistic evaluation feasible, the loads and resistances need to be pre-identified and the definitions must be kept consistent through out the analysis process.

4.5 Data Characterization

In the probabilistic method, the probabilistic, or statistical properties of random variables must be known. These properties basically include the mean (or expected) value, the variance, the coefficient of skewness, etc., of random variables, and the covariances among them. If the probability distribution function (or probability frequency function) of every random variable and the joint distribution function between random variables are known, these parameters can be exactly estimated by applying the corresponding statistical definitions. In engineering practice, more often, the statistical properties of random variables are obtained through the data which are from laboratory or field testing, field investigation, or even from engineers' experience. In order to make the analysis more reasonable, the required data must be carefully collected and reasonably characterized.

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4.5.1 Data Collection

Data collection is an easily ignored problem although it may greatly affect the reliability of analysis itself because sometimes reliability data are less reliable than some other sorts of data.

The basic concerns of data collection are objectives and the form of the data[103]. The objectives will affect the data required and the variable model to be considered, so they must be closely related to the problem studied and the factors or variables involved. The form of the data is a useful tool to validate the data and check for consistencies and any abnormalities. A good form can best describe the data context which should be clearly defined, especially for the historic data.

For the failure type or failure event data, the context should include (if applicable)

(1) Frequencies of failures and failure initiating events;

(2) Consequences and scale of failures.

(3) Time between failures for a repairable system, may include repair times, etc.;

(4) Multiple failure time; etc.

To collect data for structural condition and specific parameters, information may include:

(1) All related lab test and/or field test data, sorted by the test type, material type and testing condition, etc.;

(2) Structure condition survey, including history and

current conditions of operating, maintenance, structure and foundation appearance.

(3) Data from other sites or projects which are similar to the object studied.

After collecting the data, the data characterization needs to be carried out in order to perform probabilistic reliability analysis.

4.5.2 Data Characterization

Data characterization is another significant component in reliability analysis.

Data characterization includes the determination of statistical properties for random variables, establishment of the random variable model (or distribution), as well as quantification of the dependency between the variables.

For the failure types and the failure initiating event data, usually only the probabilistic model needs to be set up. For example, if the collected data gives the information about the frequency and scale of severe earthquakes at the site of interest, then a probabilistic model of earthquake event can be set up which describes the probability of earthquake occurence as function of the amplitude of maximum ground motion and time span. This model may be a Poisson process.

To determine the statistical properties for an individual random variable from the collected data, in the form of parameters, is not usually difficult. Conventional statistical method can be used to obtain the first three or four probabilistic moments, i.e. the mean or expected value, the variance, the coefficient of skewness (reflecting the asymmetry of the probability distribution) and the coefficient of kurtosis (reflecting the peakedness or flatness of the probability distribution). The probability distribution of a random variable is important only if exact and Monte Carlo methods are to be used in the reliability analysis. To determine the appropriateness of an assumed probability distribution, the goodness-of-fit test is a tool. It should be pointed out that when trying to find a suitable distribution for the data sets, some "bad" points" may need to be removed. In this case, one must be very careful not to change the real property of the data, or in other words, the distribution function should fit the data, not the data fit to the function.

For a data set which contains two or more random variables, a suitable model, or a bivariate distribution, needs to be established in order to properly describe the properties of, and relationship between, those random variables. To fulfill this task, some numerical method, such as regression method, or other special method can be employed. To explain how this objective can be achieved, an example will be given in next section.

4.5.3 Example of Data Characterization – Soil Shear Strength Parameters

To illustrate the data characterization of a data set which contains two random variables, the example of determining the statistical properties of soil (or rock) shear strength parameters, c and $\tan\phi$, from direct shear testing data is described below.

Soil (or rock) shear strength parameters are usually determined by direct shear tests which give the data as peak and/or residual shear stress, τ, versus normal stress, σ_n. For sliding analysis of navigation structures, the shear strength parameters c and $\tan\phi$ (or ϕ) are key parameters, therefore, the statistical moments of c and $\tan\phi$ (or ϕ) from the data set $\tau(\sigma_n)$ must be determined.

As accepted by engineering practice, the Mohr-Coulomb theory can be used to express the relationship of τ and σ_n which contains the parameters c and $\tan\phi$ (or ϕ):

$$\tau = c + \sigma_n \tan\phi \qquad (4\cdot1)$$

Notice that the eqn. (4•1) can be written as

$$y = b_0 + b_1 x \qquad (4\cdot2)$$

where

$$y = \tau,\ b_0 = c,\ b_1 = \tan\phi \text{ and } x = \sigma_n$$

Since the Mohr-Coulomb equation is a linear single variable function, the linear regression method seems a natural choice.

Applying linear regression method[31], the statistical parameters of c and $\tan\phi$ can be calculated by

$$\tan\phi = \frac{\Sigma(\sigma_{n,i} - \overline{\sigma}_n)(\tau_i - \overline{\tau})}{\Sigma(\sigma_{n,i} - \overline{\sigma}_n)^2} \qquad (4\cdot3)$$

and

$$c = \bar{\tau} - (\overline{\sigma_n})\,(\tan\phi) \qquad (4\bullet4)$$

where

$\overline{\sigma_n}$ and $\bar{\tau}$ are the mean values of τ and σ_n, respectively.

The covariance matrix of c and $\tan\phi$ is

$$Var\begin{bmatrix} c \\ \tan\phi \end{bmatrix} = \begin{bmatrix} Var(c) & Cov(c, \tan\phi) \\ Cov(c, \tan\phi) & Var(\tan\phi) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\sigma_\tau^2 \Sigma \sigma_{n,i}^2}{N\Sigma(\sigma_{n,i} - \overline{\sigma_n})^2} & \dfrac{-\sigma_\tau^2 \overline{\sigma_n}}{\Sigma(\sigma_{n,i} - \overline{\sigma_n})^2} \\ \dfrac{-\sigma_\tau^2 \overline{\sigma_n}}{\Sigma(\sigma_{n,i} - \overline{\sigma_n})^2} & \dfrac{\sigma_\tau^2}{\Sigma(\sigma_{n,i} - \overline{\sigma_n})^2} \end{bmatrix} \qquad (4\bullet5)$$

where N is the number of data points and σ_τ^2 is the variance of τ.

Since the σ_τ^2 is not known, the estimate of σ_τ^2, s^2, can be used in eqn. (4•5) with the definition of

$$\sigma^2 \approx s^2 = \frac{\Sigma\tau_i^2 - (\Sigma\tau_i)^2 / N}{(N-2)} \qquad (4\bullet6)$$

where $(N-2)$ is the number of degrees of freedom.

The results from the eqn. (4•3) and eqn. (4•4) can be used as the mean or expected values of c and $\tan\phi$ and their variances, the covariance can be determined by eqn. (4•5).

The statistical moments of c and $\tan\phi$ can also be determined by another method which may be called the *Paired Points Method* (PPM). This method is based on the consideration of that if the Mohr-Coulomb equation holds and the testing samples are the same type of soil (or rock) and collected from the same site, then the shear strength parameters, *c* and *tan*ϕ, should be similar, of course, with certain variations. The variations may be caused by the spacial effect, sampling method, testing method and process, data measurement error, etc. Based on the Mohr-Coulomb's equation, for any pair of data, as long as the normal stress σ_n is different, one set of *c* and *tan*ϕ can be solved from the first order two variables equation group:

$$\begin{pmatrix}\tau_1\\ \tau_2\end{pmatrix} = \begin{bmatrix}1 & \sigma_{n,1}\\ 1 & \sigma_{n,2}\end{bmatrix}\begin{pmatrix}c_i\\ (\tan\phi)_i\end{pmatrix} \qquad (4\bullet7)$$

and the solution is

$$\begin{pmatrix}c_i\\ (\tan\phi)_i\end{pmatrix} = \begin{bmatrix}1 & \sigma_{n,1}\\ 1 & \sigma_{n,2}\end{bmatrix}^{-1}\begin{pmatrix}\tau_1\\ \tau_2\end{pmatrix} = \frac{1}{(\sigma_{n,2}-\sigma_{n,1})}\begin{bmatrix}\tau_1\sigma_{n,2}-\tau_2\sigma_{n,1}\\ \tau_2-\tau_1\end{bmatrix} \qquad (4\bullet8)$$

For N sets of $\tau(\sigma_n)$ data, M possible data pairs can be combined, therefore, M sets of new data of *c* and *tan*ϕ can be generated. After eliminating the data which do not have physical meaning (e.g. negative values), the *c* and *tan*ϕ data set is ready to be analyzed by basic statistical methods.

The concepts of linear regression method and paired point method are illustrated in Figure 4-1.

In the above example, the two data characterization methods have their own advantages and disadvantage. The linear regression method is a well published method used in statistical procedures and several commercial software packages are available. Also, this method gives the smallest variance

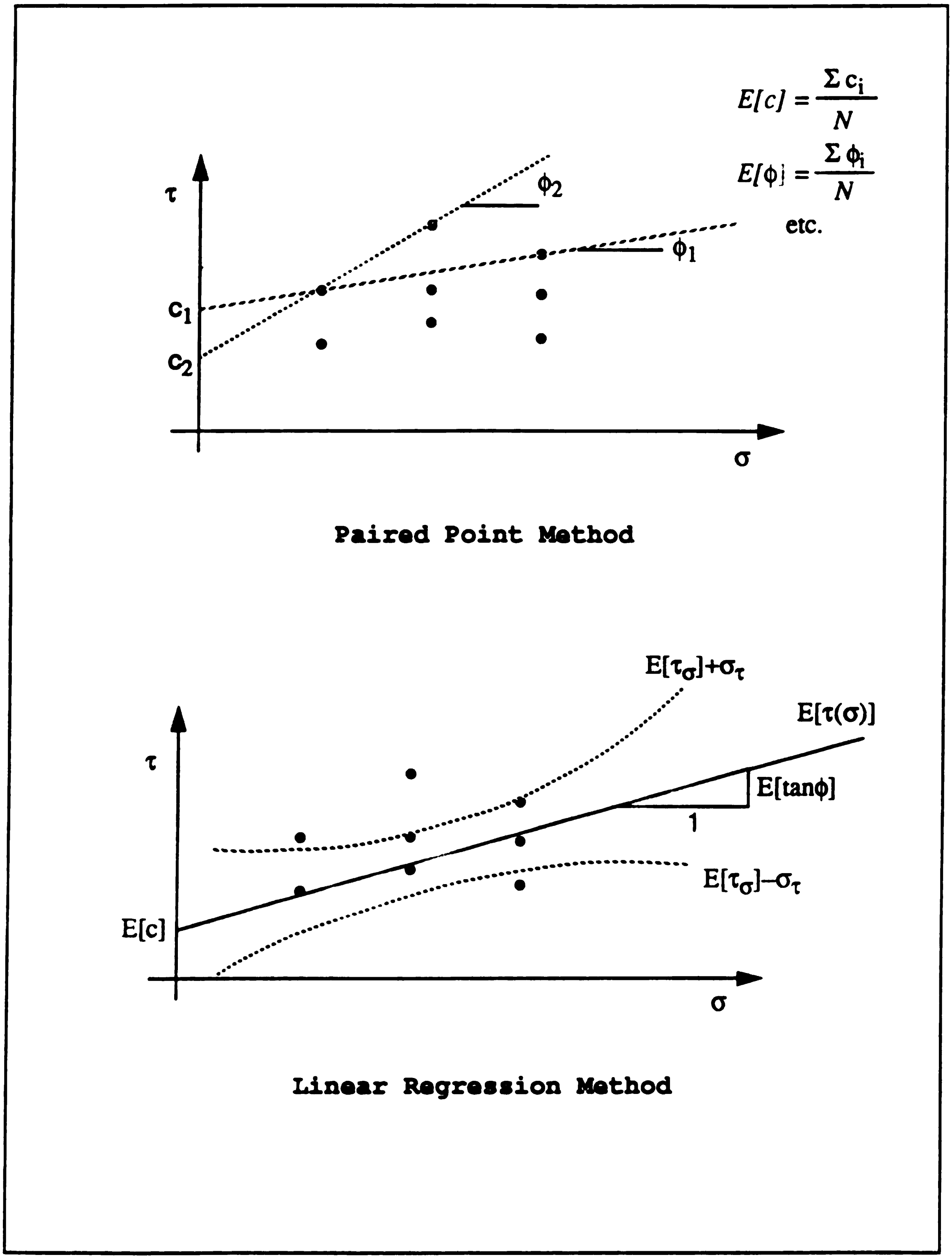


Figure 4-1 Concepts of Linear Regression Method and Paired Point Method in Soil shear Strength Parameters Determination

at the mean normal stress and increases the variance with distance from the mean — which makes sense at the physical meaning point of view. But this method can only determine statistical moments up to the second order and the variances and covariance are based on the estimate of variance of τ, which includes the data variation and regression error, therefore, more error is introduced. Another drawback is that this method can only solve for $tan\phi$, not the internal friction angle ϕ, so it needs to be transformed from $\tan\phi$ to ϕ when an analysis needs ϕ instead of $\tan\phi$. Approximative methods of conversions between ϕ and $\tan\phi$ are described in Appendix B.

The PPM is a straight forward method, therefore it is easy to understand. By PPM the moments, $tan\phi$ and ϕ can be determined separately because every $(tan\phi)_i$ corresponds to a ϕ_i. If more different normal stresses were used in the test, more data sets of $\tan\phi$ and ϕ can be generated by PPM, therefore, more information can be picked up. But the associated problem is that as the size of the data sets increases, the number of combinations may increase greatly (depending on the normal stresses used and the data size). Also, the solved c and $tan\phi$ (ϕ) pairs need to be examined and the unseasonable ones eliminated, although a simple statement in a computer program can take care this problem. It needs to be pointed out that the PPM works well in "reasonable" consistent materials or test data but may not work for "erratic" material or a set of test data with few and scattered points.

4.5.4 Transforming Dependent Random Variables to Independent

The dependency of random variables is an important factor in probabilistic analysis and it will make the analysis more complex. To account for the dependency and simplify the analysis, especially when many dependent random variables are involved, a transformation method, which will transfer the correlated variables to a new set of independent random variables, can be employed.

If a set of dependent random variables, *X*, can be transformed to a new set of independent random variable, *Y*, then the relation can be written as

$$\{X\} = [C]\{Y\} \tag{4•9}$$

and the expected value and covariance matrices are

$$\{E[X]\} = [C]\ \{E[Y]\} \tag{4•10}$$

and

$$\{S_X\} = [C]\{S_Y\}[C]^T \tag{4•11}$$

where

$[C]$ is the transformation matrix,
$\{E[X]\}$ and $\{E[Y]\}$ are mean value vectors of X and Y, respectively,
$\{S_X\}$ and $\{S_Y\}$ are covariance matrices of X and Y, respectively, and
$[C]^T$ is the transpose of $[C]$.

Since the $\{S_X\}$ is a symmetric matrix, it can be decomposed, by the Choleski decomposition method (which can be found in some text books, e.g. Hart [54]),

$$\{S_X\} = [C]\{S_Y\}[C]^T \tag{4•12}$$

where

$\{S_Y\}$ is a diagonal matrix, as needed, and
$[C]$ is a lower triangular matrix with 1s in the principal diagonal.

The elements of $\{S_Y\}$ and $[C]$ are

$$S_{Y,11} = S_{X,11}$$

$$C_{ii} = 1 \qquad i = 1, 2, \ldots, n$$

$$C_{j1} = S_{X,1j}/S_{Y,11} \quad j \geq 2$$

$$S_{Y,ii} = S_{X,ii} - \sum_{l=1}^{i-1} C_{il}^2 S_{Y,ll} \qquad i \geq 2 \text{ and}$$

$$C_{ji} = \frac{1}{S_{Y,ii}} \left[S_{X,ij} - \sum_{l=1}^{i-1} C_{il} C_{jl} S_{Y,ll} \right] \qquad i \geq 2, \; j \geq i+1 \quad (4 \bullet 13)$$

After solving $\{S_Y\}$ and $[C]$, the expected value of Y is ready to be calculated by

$$\{E[Y]\} = [C]^{-1} \{E[X]\} \qquad (4 \bullet 14)$$

where $[C]^{-1}$ is the inverse of $[C]$ and $E[X]$ is the expected value matrix of dependent random variables X. The covariance matrix of independent random variables Y, transformed from X, is $\{S_Y\}$.

After the transformation, the new set of random variables, Y, and the corresponding variances (note that the covariances are zero for Y) should be used in the place of X in the analysis.

The dependency and independency transformation is an important step in probabilistic analysis, especially when many correlated random variables are involved and a method which "models" correlation, such as Monte Carlo simulation, is to be used. But it is not really necessary if only few pairs of correlated random variables are involved and simplified method, such as first order—second moment method, is going to be employed.

4.6 Reliability of An Individual Structure or Component

A navigation structure usually is a system of components. To evaluate the reliability of a navigation structure, for example, a locks and dam system, the reliability of individual components of the system must be first evaluated. To develop a practical methodology, the reliability of a simplified but typical lock monolith will first be assessed. The cross-section and the general loading conditions of common monolith structures are illustrated in Figure 4-2 and the notations in this figure will be discussed in the following sections. Random variables will be characterized and the reliability index, β, chosen as the reliability measurement, will be calculated by two first order-second moment approximative methods — Taylor's series method and point estimate method.

4.6.1 Performance Functions

To evaluate the reliability of a monolith, its potential failure modes under different loading, or operating conditions must be identified, the performance functions with associated random variables and criteria must be first determined and characterized,.

For a monolith, the main structural safety aspects are sliding stability, overturning stability and bearing capacity. Although the seepage conditions and foundation settlement are important for earth embankments, and the failure of structural members (such as the failure of lock gate) is another mode needing to be considered, the methodology studied here will be limited to performance of gravity structures, founded on rock foundations, with respect to sliding, overturning stability and bearing capacity.

4.6.2 Sliding Stability

In order to determine the performance function of sliding stability (the basic analysis concept used by Corps of Engineers[112] will be adopted), the driving and resisting forces and the underlying random variables must be first identified. It is worthwhile to point out again, because approximative methods are to be used in the analysis, the loads and resistances must be predefined, otherwise different reliability indices may be calculated under the same loading condition for the some performance mode. But the positive direction of forces can be defined arbitrarily.

4.6.2.1 Identification of force types

The forces acting on a structure can be classified into driving force and resisting force as often used in engineering practices.

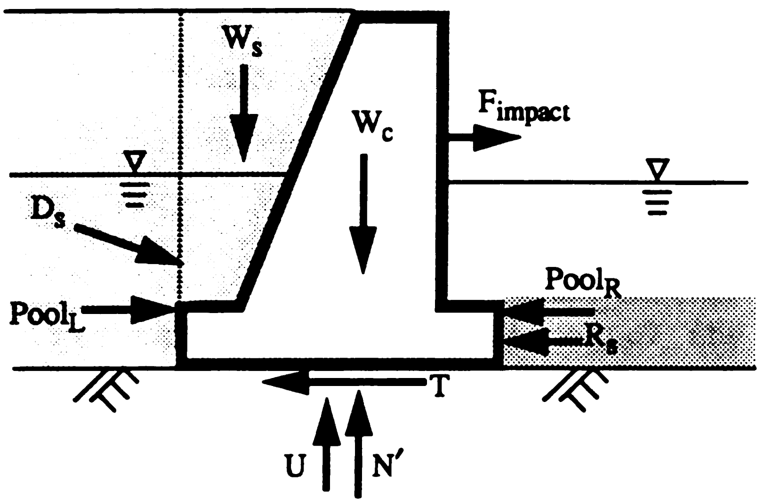
Driving Forces

Driving forces, usually loads, are forces that tend to make a structure unstable or unsafe. Referring to Figure 4-2, driving forces include (if left to right is defined as the positive direction for driving forces):

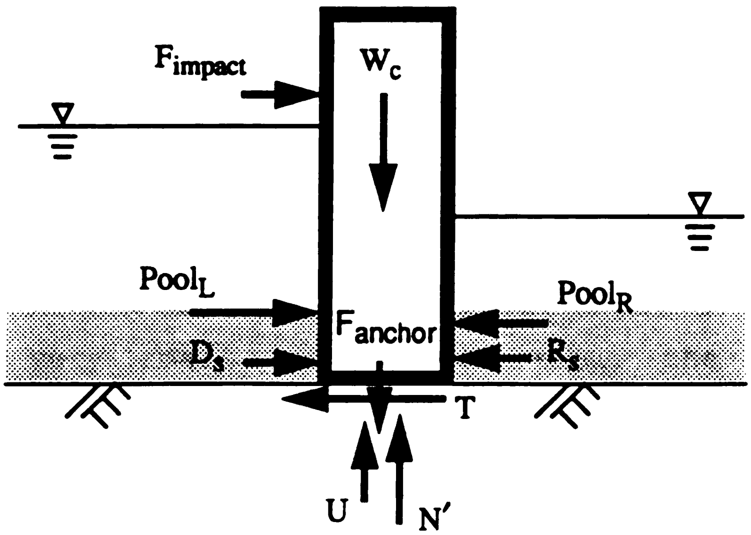
F_{impact} – Lateral impact force produced by barge impact or hawser force;

$D_{s,h}$ – Lateral earth force from left side of structure (horizontal component of D_{soil});

$Pool_L$ – Hydrostatic force on the left side and



a) Guide wall



b) Lock monolith

Figure 4-2 Cross-Section and General Loading Conditions of Monolith of Locks (Simplified)

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U – Hydrostatic uplift force under the base of structure.

Among these forces, the F_{impact}, D_{soil} and U can be defined as driving forces and are quite straight forward, but it needs to be further examined as to whether it is suitable to define $Pool_L$ alone as a driving force without considering its counterpart, $Pool_R$.

Pool always exists associated with navigation structures, and, in most cases, on both sides of the structure. Considering these levels, or the water table behind retaining walls, hydrostatic pressure differential nearly always exists across a structure. Should the hydrostatic forces on different sides be considered as driving and resisting forces separately, or should the hydrostatic pressure *differential* be considered as driving force?

If the hydrostatic forces on either side of structure are defined separately as driving and resisting forces, which certainly has its physical meaning, but one may face a not uncommon situation: for a tall structure with a high pool level but very small pool differential, the hydrostatic forces are the dominant forces, therefore, the ratio of resisting force to driving force will approach to 1, or the factor of safety, *FS*, will be close to 1.0. Although the structure is indeed very safe (from both physical and reliability analysis argument), the *FS* value may not satisfy the conventional design criteria (which probably will make some engineers uncomfortable).

To avoid this undesirable situation, defining the *hydrostatic pressure differential* as a driving force will be more appropriate for the following reasons:

(1) It will give more reasonable conventional analysis results (*FS*); and

(2) It will not significantly affect the reliability analysis results, because of the very small variation of unit weight of water and the great correlation between the pool (or water) levels at two side of structure.

The hydraulic driving force, P_{hydro}, in this study is defined as

$$P_{hydro} = Pool_L - Pool_R \qquad (4\bullet15)$$

Resisting Forces

Similar to defining driving forces, all the forces which tend to keep structure from being unstable, or resist driving forces, can be defined as resisting forces. Once again, depending on the particular case considered, the positive direction of resisting forces can be arbitrary but is opposite the positive driving direction.

The resisting forces are

R_s — Lateral earth force (from right side of structure in this case);

T — Shear force underneath the base which resists sliding.

The T is usually expressed by Mohr-Coulomb theory

$$T = Lc_{base} + N' \tan \phi'_{base} \qquad (4\bullet16)$$

where

L—length of sliding surface of base, may or may not equal to the length of base, B;

c_{base} – cohesion of base material;

N' – effective normal force = $\sum Normal\ forces - U$; and

ϕ'_{base} – effective internal friction angle of base material

Among these forces, the T is unquestionably a resisting force as it always has the opposite direction of the driving force. Note that the sum of normal forces includes vertical wall shear force, $P_{s,v}$, and anchor force, P_{anchor}, if they are applicable. Although the R_s could be considered as a negative part of driving force with $D_{s,h}$ together, considering that the uncertainties of R_s and $D_{s,h}$ may be much different due to strain mobilization, it is more straight forward and has better physical meaning to define the R_s as a resisting force.

4.6.2.2 Identification of random variables

After defining the driving and resisting forces, the random variables involved need to be identified.

The impact force, F_{impact}, can be seen as a line force with great uncertainty, so itself is a random variable.

$D_{s,h}$ is a function of variables: height of soil layer(s), $h_{s,d}$, effective unit weight of soil, γ_s, and coefficient of lateral earth pressure, K_h. It can be expressed, if only one layer is involved and it is entirely below water, as

$$D_{s,h} = \frac{h_{s,d}^2 \gamma_s}{2} K_h \qquad (4 \bullet 17)$$

Among these variables, γ_s is a random variable and K_h is a function of other random variables (the K_h will be discussed later). The $h_{s,d}$ can be fairly treated as a deterministic variable.

If the pool (or saturation) levels are H_H and H_L, corresponding to upper and lower pools, respectively, then the deferential water force is

$$P_{hydro} = \frac{H_H^2 - H_L^2}{2}\gamma_w \quad (4\bullet18)$$

where γ_w can be seen as a deterministic value but H_H and H_L usually are correlated random variables.

In the function T expression, ϕ'_{base} or $\tan\phi'_{base}$ is a random variable as well as c_{base}. The L usually is determined by the effective base and is a function of other random variables. The effective base pressure, N' is a function of other variables and it can be expressed as

$$N' = \sum Normal\ forces - U$$
$$= W_c + W_s + P_{s,v} + P_{anchor} - U \quad (4\bullet19)$$

where

W_c is the product of the unit weight of monolith material, usually concrete, $\gamma_{concrete}$, and its volume, V_{con};

W_s is the prodect of the unit weight of soil whose weight acts above the base of monolith, and its volume, V_{soil};

$P_{s,v} = D_{s,h}\tan\delta$ is the component of wall friction force which is normal to the base and is a function of random variables of γ_s, K_h and angle of wall-soil friction, δ;

P_{anchor} is the normal holding force by anchors which usually is a function of the area of anchor, A_{anchor}, γ_s, K_h, ϕ_s and breakout factor for anchor, N_q (the anchor holding force which will be discussed later); and

U is the hydraulic uplift force which will be discussed in next section.

Among the variables involved in the N' expression, $\gamma_{concrete}$, γ_s, ϕ_s, δ, K_h, N_q, H_H, H_L, L, and U are random variables and the rest can be treated as deterministic parameters.

R_s is a function of the same random variables as $D_{s,H}$ but the coefficient of lateral earth pressure may be different.

Each of these random variables has its own probability distribution and is not necessarily normally distributed. If an "exact" method is going to be used, these probability distributions must first be determined; if an approximate method is going to be employed, only their first and second moments are of interest.

4.6.2.3 Uplift force U

Since pool differential exists around navigation structures, there is uplift force underneath the structures. The uplift force U can be expressed by

$$U = \frac{[2H_L + (1-E)(H_H - H_L)]}{2} B\gamma_w \qquad (4\bullet20)$$

where E is the coefficient of uplift force which reflects the drainage condition, structure-foundation joint condition, permeability and geometry of base material and how much the base is under compression. As all of those factors are varying widely from case to case, the E certainly is a random variable and is distributed over values -1.0 and +1.0. The probability distribution of E can be determined based on observed data or assumption. The expected (mean) value of E should be determined with accordance to the effective base (the area of base which is in compression). If the percentage of base which in compression is denoted as PC%, then the expected value of E is

$$E[E] = PC\%/100 - 1.0 \qquad (4 \bullet 21)$$

From this equation, it is clear that if the base is not 100 percent in compression then the E value will be negative; On the other hand, as PC% can not be greater than 100, when the whole base is in compression, the mean value of E should be determined by field conditions and/or engineer's judgement. E[E]=0 may be a good estimation if no additional information is available. If the approximation method is used in the reliability analysis, the other statistic properties of E, such as the standard deviation, can also be estimated by judgement or common sense unless field data indicate otherwise.

The relationships for expected value of E and PC% and uplift force U as function of E are illustrated in Figure 4-3 and Figure 4-4. Note that the relationship of E[E] and PC% may not be linear in the real case analysis when PC% is in the range of [0, 100], and E[E] should be determined by iterating the performance function of overturning analysis.

The moment of uplift force, M_U, about the toe of structures can be expressed as

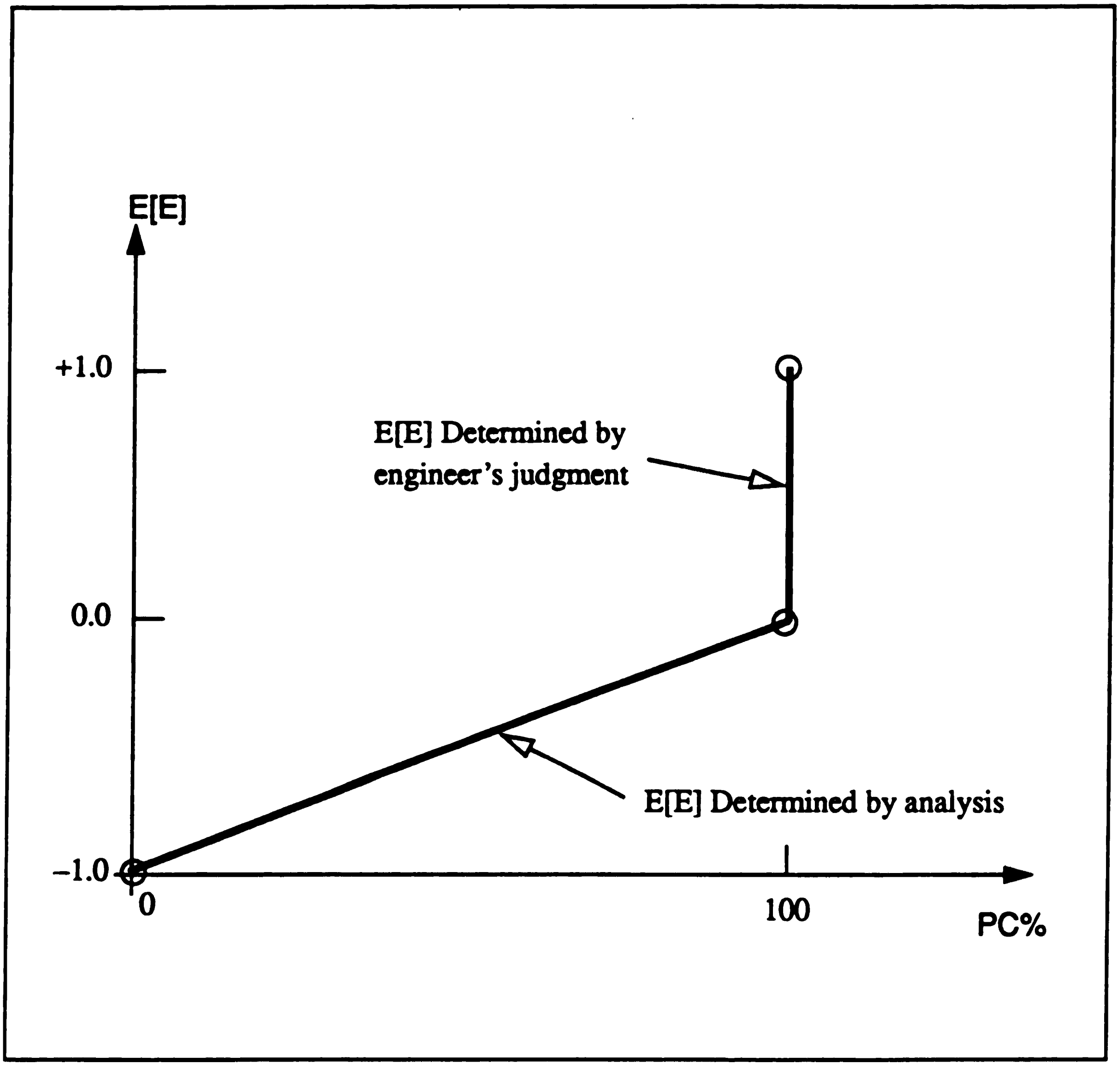


Figure 4-3 Relationship between E[E] and Percent of Base in Compression, PC%

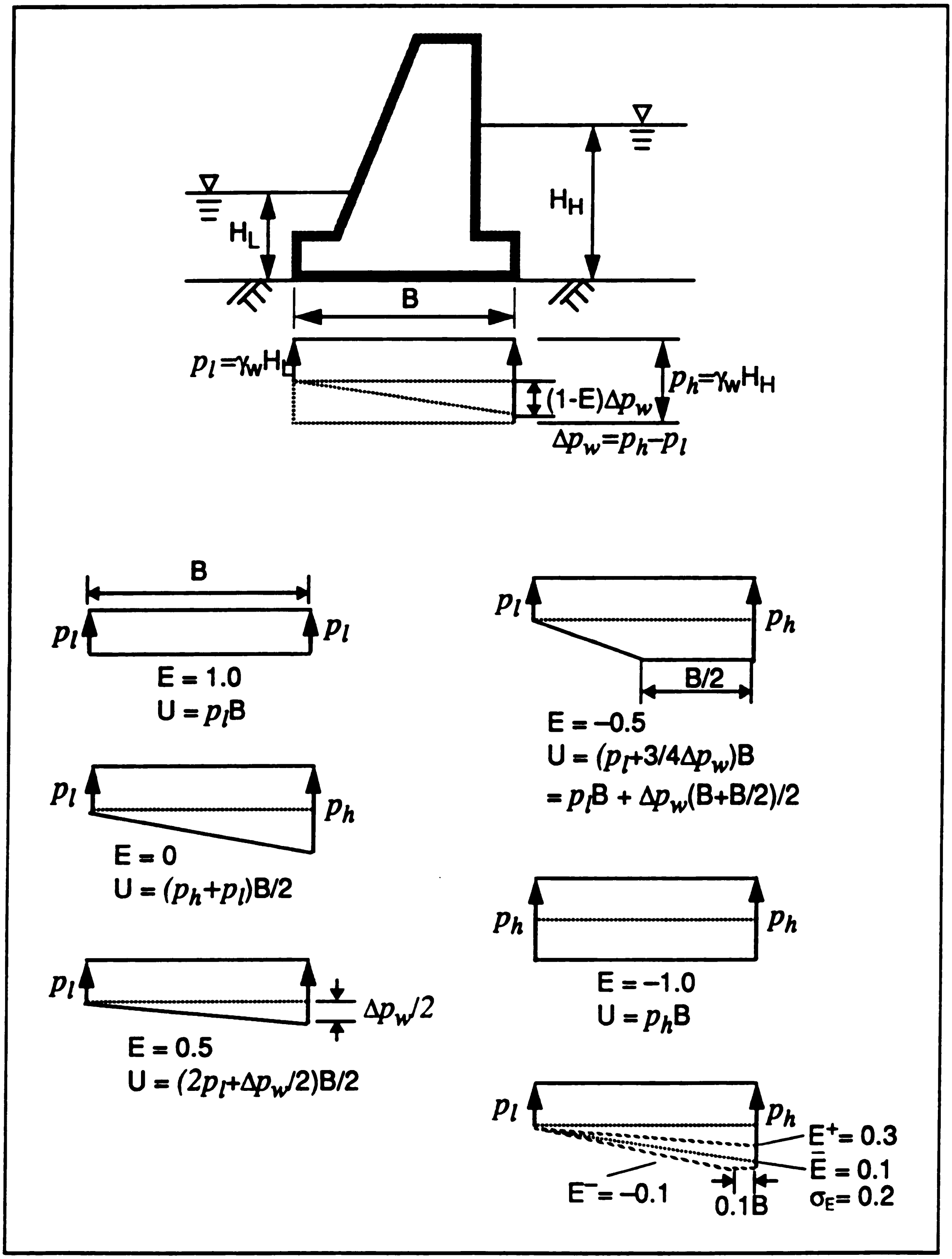
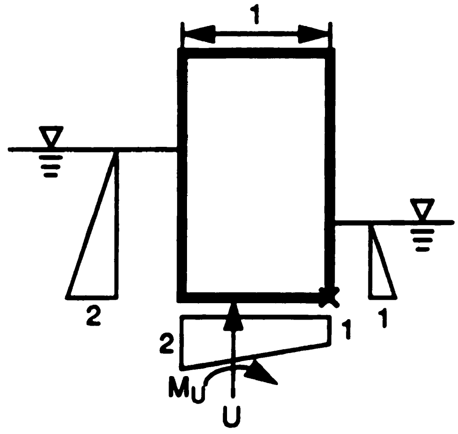


Figure 4-4 Definition of Hydraulic Uplift Force and E Factor



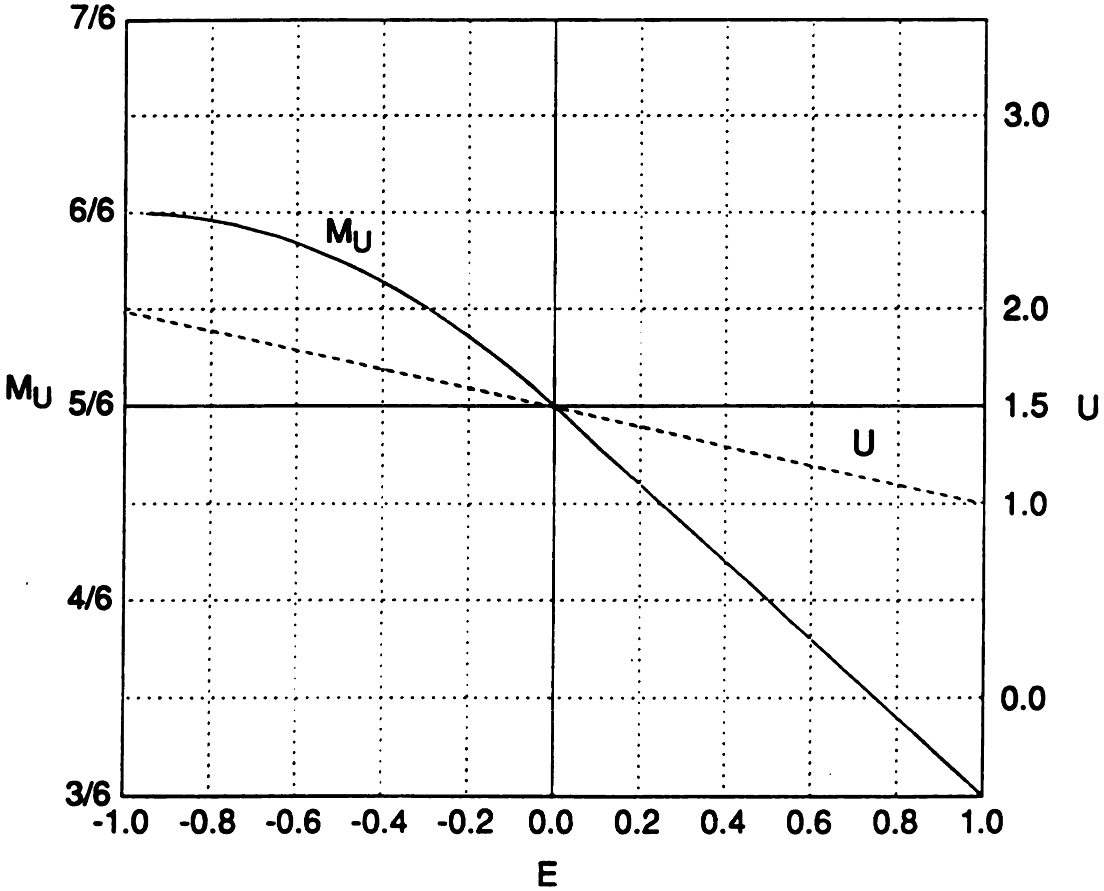


Figure 4-5 Uplift Force U and its Moment M_U versus E Factor

$$M_U = \frac{B^2\gamma_w}{6}[(2H_H + H_L) - 2E(H_H - H_L)] \qquad E\geq 0$$

$$= \frac{B^2\gamma_w}{6}[3H_L + (H_H - H_L)(-E^2 + 2|E| + 2)] \qquad E<0 \qquad (4\bullet22)$$

The uplift force U and its moment M_u as function of E is illustrated in Figure 4-5. It clearly shows the nonlinearity of M_u with negative E values.

4.6.2.4 Coefficient of lateral earth pressure

The lateral earth pressure is a random variable in stability analysis; it plays an important role in deterministic analysis and likewise in probabilistic reliability analysis. It is important to choose an appropriate coefficient of lateral earth pressure in stability analyses.

If there is no wall movement (either retaining wall or lock monolith) involved, the soil at both side of structure can be considered as "at-rest". Without field measurements, the coefficient of lateral earth pressure *at-rest*, K_0, is indeterminate, but it can be approximated by the following:

For normally consolidated cohesionless soils:

$$K_0 = \frac{1-\sin\phi'}{1+\sin\phi'}\left(1+\frac{2}{3}\sin(\phi')\right) \approx 1-\sin\phi' \quad \text{Jaky (1944)} \qquad (4\bullet23)$$

$K_0 = 1 - 1.003\sin\phi'$ Mayne and Kulhawy (1982) (4•24)

For normally consolidated clay:

$K_0 = 0.95 - \sin\phi'$ Brooker and Ireland (1965) (4•25)

$K_0 = 0.4 + 0.007PI$ *for* $0 \leq PI < 40$

Brooker and Ireland (1965) (4•26)

$K_0 = 0.64 + 0.001PI \quad for\ 40 \leq PI < 80$

Brooker and Ireland (1965) (4•27)

$K_0 = 0.19 + 0.233 log PI$ Alpan (1967) (4•28)

where PI is plasticity index.

For overconsolidated soils, K_0 can expressed as

$$K_0 = K_{0,nc}\ OCR^n$$ Schmidt (1966) (4•29)

where OCR is over consolidation ratio and n is a soil parameter varying with soil type. Mayne and Kulhawy suggested that $n \approx sin\phi'$ in 1982.

For simplicity, Jaky's equation $K_0 = 1 - sin\phi'$ is usually used in engineering practice.

When performing an overturning analysis (also referring to the procedure used by the Corps of Engineers[113]), the K_0 condition is often assumed to simulate the worst condition. In sliding analysis, the extreme condition is that the structure sliding is initiated by the driving force but within the limit state, or the earth pressure is in "active" state.

If wall movement is involved, the "active-earth-pressure" and "passive-earth-pressure" should be used in the analysis. To apply the active-earth-pressure concept, two assumptions need to be satisfied: (1) no soil compaction effect is present which creates excessive stresses; and (2) wall rotation (or translation) is sufficient to fully mobilize the "active" zone and the shear stresses along the rupture surface. Usually the wall movement required is from 0.001 to 0.05 times the wall height, for soils from dense

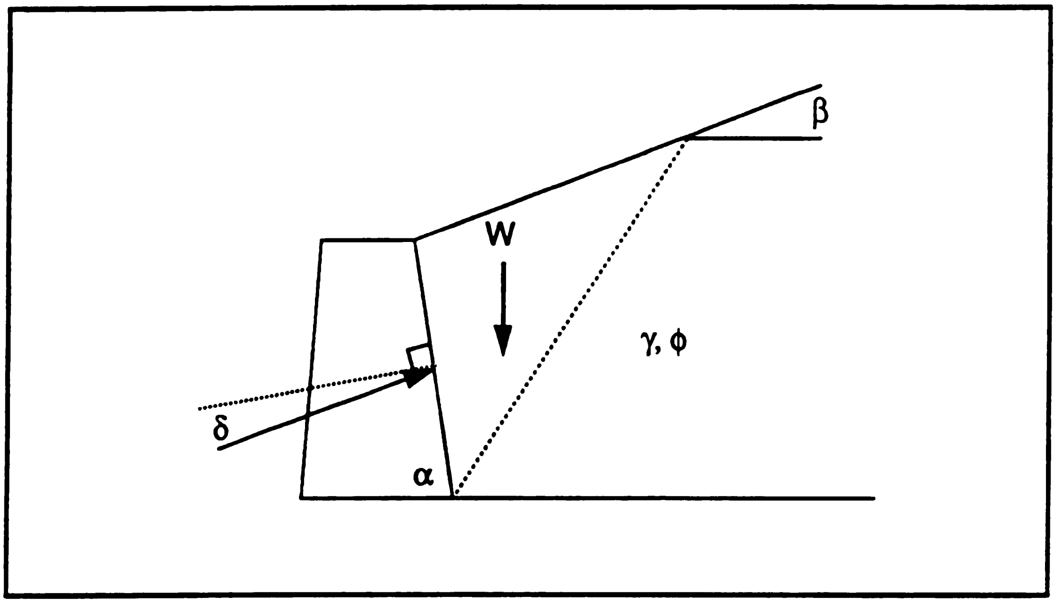


Figure 4-6 Coulomb's Active-Earth-Pressure Assumptions

sand to soft clay.

For active and passive earth pressure, p_a and p_p, the Coulomb's theory will be applied which defines (referring to Figure 4-6) for cohesionless soils:

$$\begin{bmatrix} p_a \\ p_p \end{bmatrix} = \gamma z \frac{1}{\sin\alpha\cos\delta} \begin{bmatrix} K_a \\ K_p \end{bmatrix} \tag{4•30}$$

$$\begin{bmatrix} K_a \\ K_p \end{bmatrix} = \frac{\sin^2(\alpha \pm \phi)\cos\delta}{\sin\alpha\sin(\alpha \mp \delta)\left[1 \pm \sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi \mp \beta)}{\sin(\alpha \mp \delta)\sin(\alpha+\beta)}}\right]^2} \tag{4•31}$$

where

K_a coefficient of active earth pressure;

K_p coefficient of passive earth pressure;

β slope of the backfill;

ϕ internal friction angle of the backfill soil;

α slope of wall back;

δ friction angle of wall and backfill;

z depth of soil below the backfill top; and

γ unit weight of backfill.

For cohesive soils:

$$\begin{bmatrix} p_a \\ p_p \end{bmatrix} = \gamma z \begin{bmatrix} K_a \\ K_p \end{bmatrix} \mp 2c \begin{bmatrix} \sqrt{K_a} \\ \sqrt{K_p} \end{bmatrix} \qquad (4\bullet32)$$

where

$$\begin{bmatrix} K_a \\ K_p \end{bmatrix} = \tan^2(45° \mp \frac{\phi}{2}) \qquad (4\bullet33)$$

In engineering design and analysis, the cohesion of backfill is usually treated as zero. For walls with a vertical back and negligible backfill slope, eqn. (4•33) can be used.

4.6.2.5 Holding capacity of anchors

Anchors are often used in lock structures to increase the stability of the monoliths. If an anchor is terminated in bedrock (the usual case for most navigation structures) and its holding force is monitored by an instrument, the single anchor force, P_{anchor}, as well as the anchor group force, $P_{(anchor)g}$, can be statistically determined from field measurement data. It should be noted that the anchor force is time dependent because of the stress relaxation and creep strain.

If the anchor is rooted in soil, its uplift holding capacity can be estimated by[28]

$$P_{anchor} = (\gamma_s A_{anchor} H) N_q \tag{4•34}$$

where

γ_s is the unit weight of soil;

A_{anchor} is the area of anchor;

H is soil thickness above the anchor end plate; and

N_q is the breakout factor of anchor, which is function of the anchor shape, ratio of H to the anchor's dimension, B, and the internal friction angle of soil, ϕ_{soil}.

The N_q can be determined either mathematically or from tables or charts.

The group holding capacity is

$$P_{anchor,g} = \eta \sum P_{anchor} \tag{4•35}$$

where η is the group efficiency, usually ≤ 1.0.

For the time dependence of anchor force, if the anchor's tension or its holding ability is adjustable, the anchor force can be seen as time independent, otherwise, the anchor force can be expressed as (also referring to Mirza and MacGregor, 1979[74], and Mirza and *et al*, 1980[75])

$$P_{anchor}(t) = (\gamma_s A_{anchor} H) N_q exp(-\lambda t^{\alpha\lambda}) \tag{4•36}$$

where

t is time, and

α, λ are parameters which are determined by the anchor and soil properties.

4.6.2.6 Group reliability of anchors

For a group of anchors, the group reliability (*GR*), i.e. how many anchors in the group are normally functioning, will certainly affect the capacity of the anchored foundation, as well as the safety of the related structure.

It can be argued that because of uncertainties in manufacture, construction, loading and operating conditions, each anchor in a group will perform differently, therefore, the anchor group reliability is a random event (or a random variable). A similar argument and discussion can be also applied to pile foundations.

The mechanism of group reliability is quite complex and has not been fully understood. To simplify the problem and try to find a feasible analysis method, several assumptions are needed. First, only two states are defined for performance of an individual anchor, either normally working or failure; second, assume that failure of the anchors is independent to each other. Then, if the probability of n anchors not functioning at the same time is P_f*(n anchors failure)*, to determine the group reliability, *GR*, one should also consider the probability of a specific geometrical configuration representing n possible failed anchors in the group. The configuration of failed anchors will affect the foundation stability, the consequence of the same number of anchors failure to structural safety may be totally different if these anchors are located at different positions. For example, assume that 10 anchors, in a two row anchored foundation with 10 anchors per row, failed, if the 10 failed anchors happen to be all at one row, the structure on this foundation may be unsafe with respect to some performances, such as overturning stability; but if the 10 failed piles are evenly distributed within the group, the foundation may still be stable. To evaluate the group reliability for a anchored (or a pile) foundation, the most unreliable condition — under which the combination of the possibility of n anchors failure and their geometrical configuration would lead most unsafe result for the structure, must be examined.

The probability of n anchors failing at the same time, $P_f(n\ anchors\ failure)$, can be determined by assuming a beta distribution with range from 0 to N (for an anchor group with N anchors), with reasonable expected value and variance. Other distributions can also be used as long as they fairly represent the problem. Since the configurations of n anchors failure is an n-out-of-N problem, the possibility of a particular configuration of n anchors in the group of N anchors is

$$P(A\ particular\ configuration\ of\ n\ anchors) = P(n|N)$$
$$= 1/\binom{N}{n} = \frac{n!(N-n)!}{N!} \qquad (4{\bullet}37)$$

The product of $P_f(n\ anchors\ failure)$ and $P(n|N)$ is the probability of failure of the anchor group under a particular geometrical configuration of n failed anchors, that is

$$(P_f(group\ failure))_n = P_f(n\ anchors\ failure)P(n|N) \qquad (4{\bullet}38)$$

Since the final result of the analysis is the structural reliability which is affected by the performance of the anchor group, the group reliability should be related to the reliability of the structure, therefore, the corresponding probability of structural failure,

P_F(Structure | n anchors fail), also should be evaluated. The group reliability under the n anchors failure condition, $(GR)_n$, can be defined as

$$(GR)_n = 1 - P_f(n\ anchors\ failure) P(n|N)(P_F)$$
$$= 1 - P_f(n\ anchors\ failure) / \binom{N}{n} (P_F) \qquad (4{\bullet}39)$$

and the group reliability *GR* can be expressed by

$$GR = \min\ [(GR)_n]$$
$$= 1 - \max\ [P_f(n\ anchors\ failure) / \binom{N}{n} (P_F)] \qquad (4{\bullet}40)$$

If the *P_f(n anchors failure)* is unknown but it is assumed that all anchors in the foundation have the same reliability, *R*, and are independent of each other, which is a typical binomial problem, the possibility of *n* anchors failing will be

$$f(n\ anchors\ failure)$$
$$= \binom{N}{n} (1-R)^n R^{N-n} = \frac{N!}{n!(N-n)!}((1-R)^n R^{N-n}) \qquad (4{\bullet}41)$$

As an example, consider an anchored foundation with 20 anchors, 10 anchors/row, with single anchor reliability *R*=0.8. Assume that the possibility of foundation failure, *P_F (Structure|10 anchors fail)*, is 1.0 if 10 anchors fail all in the same row, while *P_F (Structure|2 anchors fail)* = 0.001 for two anchors failure at one edge of foundation, then by applying eqn. (4•37) to eqn. (4•41)

$$(GR)_{10} = 1 - P_f(10\ anchors\ failure) \times P(10|20) \times P_F(\text{Structure}|10\ \text{anchors fail})$$
$$= 1 - (1-0.8)^{10}(0.8)^{10}(1.0) = 1 - 1.1\times10^{-8}$$

Note that the $\binom{N}{n}$ term cancelled out. While

$$(GR)_2 = 1 - P_f(2\ \text{anchors failure}) \times P(2|20) \times P_F(\text{Structure}|2\ \text{anchors fail})$$
$$= 1 - (1-0.8)^2 (0.8)^{18} (0.001) = 1 - 7.2\times10^{-7}$$

therefore, $GR = (GR)_2$, or in other words, assuming 2 anchors failing should be considered in the reliability analysis, rather than the 10 anchors failure assumption because of the greater value of group reliability.

Note that: (1) the worst possible condition for the anchor group, concerning its effect on safety of the structure, may not be easily determined because this can be done only when all possible number anchors fail and the associated consequence – in terms of the structure's reliability, with respect to their geometrical configurations – are examined; (2) assuming that each anchor has the same reliability R and is independent to each other does not represent reality, therefore, when applying the method outlined above, the probability distribution of n anchors failing at the same time must be carefully examined.

For Engineering application, assume that each anchor (pile) row (or column) in the foundation forms an independent group, then the problem will be greatly simplified, because the binomial distribution can be used as an approximation and the first two statistical moments of the anchor group can be estimated.

For a binomial distribution of N anchors, with single anchor reliability R, probability density function of capacity f(p) and its expected value μ_{anchor}, the expected group capacity of the N anchors, $E[N_g]$, is

$$E[N_g] = NR\mu_{anchor} \tag{4•42}$$

and the group variance of N anchors, $Var(N_g)$, can be estimated by

$$Var(N_g) = NR(1-R)\mu^2_{anchor} \tag{4•43}$$

This approximation holds because that at each point along the n (number of anchors), the probability distribution values of the group capacity, $f(N_g)$, can be approximated by mean values of $f(N_g)$ about that point as if it were a standard binomial distribution function. The first two moments of N_g can be determined. Figure 4-7 gives a graphic explanation.

Note that eqn. (4•43) is not exact because it omits the variation of single anchor and does not consider the correlation between anchors in the group.

4.6.2.7 The performance function

After identifying the random variables involved and defining the driving and resisting forces, the performance function for sliding stability of a monolith can be expressed according to the criteria (or measurements).

1. Reliability measurement by the factor of safety, FS

$$FS = R/D = (\text{Total resisting force}) \,/\, (\text{Total driving force})$$

$$= \frac{R_s + T}{D_{s,h} + P_{hydro} + F_{impact}} \tag{4•44}$$

All variables were previously defined.

The criterion for the factor of safety is

$$FS \geq 1.0$$

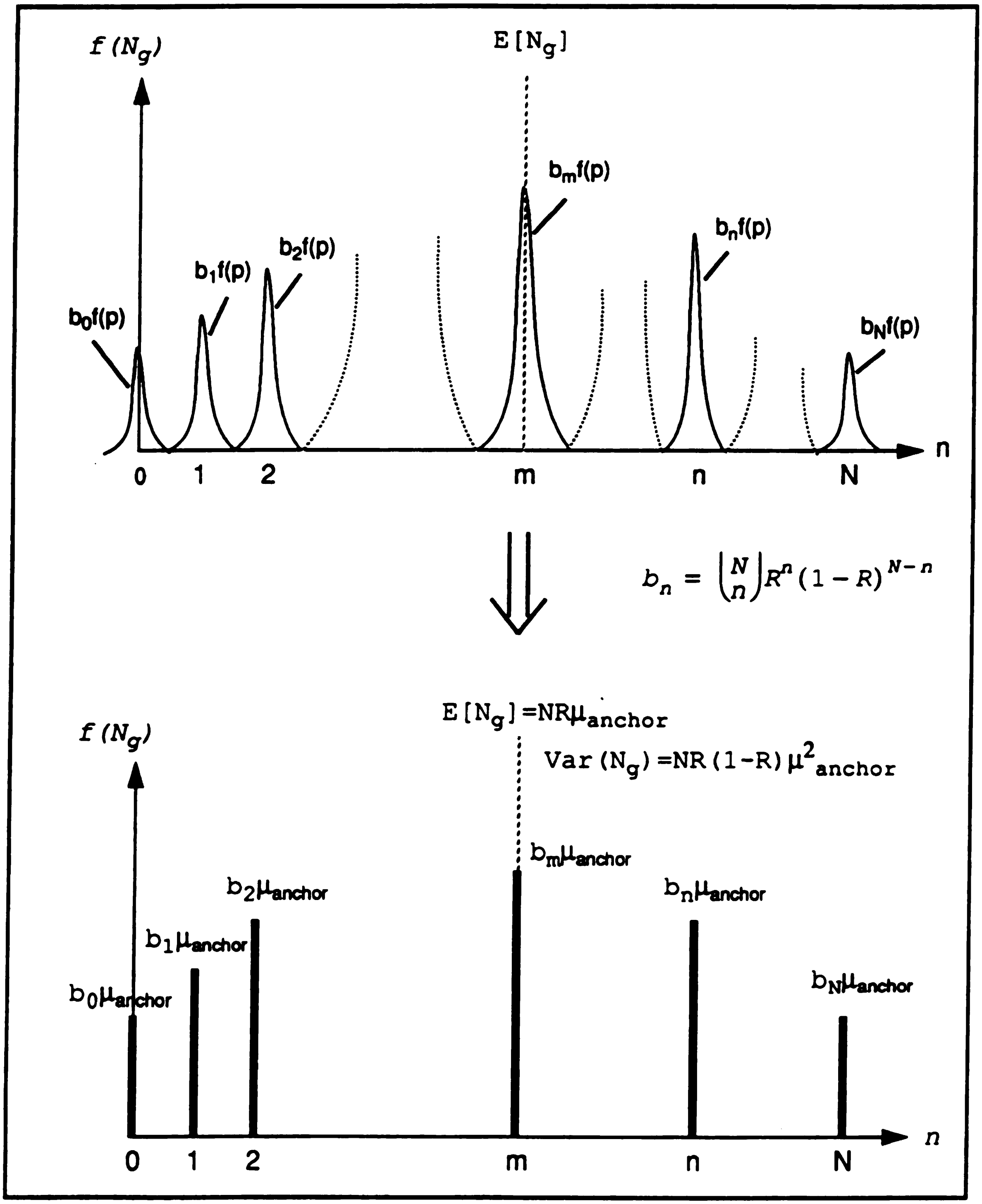


Figure 4-7 Anchor Group Probability Distribution Approximation

2.Reliability measurement by safety margin, SM

$SM = R - D$

$= (R_s + T) - (D_{s,h} + P_{hydro} + F_{impact})$ (4•45)

and the criterion is

$R \geq D$

The calculation of reliability index, β, can be carried out by the method described in section 3.2.3.2. Usually a lognormal distribution will be assumed for FS measurement and a normal distribution will be applied for SM measurement.

In engineering structural analyses, some terms in eqn. (4•44) and eqn. (4•45) may not present; in this case the only change in the analysis is to omit these non-existing terms and complete the process to find the reliability index for the performance mode of interest.

4.6.3 Overturning Stability

To evaluate the overturning stability of a navigation structure, the related criteria and performance functions must be first determined.

In engineering practice, two stability measurements, the location of the effective resultant base force, X_R, and the factor of safety, *FS* are used[113]. The resultant location is the ratio of the sum of moments about the rotating point to the effective normal force, defined as

$$X_R = \frac{M_R - M_O}{N'} \qquad (4\bullet 46)$$

and the factor of safety is the ratio of resisting moment to overturning moment, defined as

$$FS = M_R/M_O \tag{4•47}$$

The often used criteria for these two measurements will be discussed later accordingly.

4.6.3.1 Moments types and random variables

The definitions of driving and resisting moments, as well as the random variables involved, are very similar to that defined in sliding stability analysis but without the force(s) related to base shear strength. Compared to sliding stability analysis, there are two points that should be mentioned: (1) The lateral earth pressure should be treated as at-rest because the active-earth-pressure state is not the worst case in the overturning problem; (2) The normal (or vertical) wall friction force may play a relatively important role in the overturning performance mode; without taking this friction force into consideration the structural reliability may be underestimated.

4.6.3.2 Choice of moment center

As the rotating forces are measured in terms of moments, the point about which moments are taken will affect the results. In an overturning stability analysis, X_R is measured by lineal length unit, any point (or line, in 2-D problem) within or along the structure's base can be chosen as the rotating point without affecting the final result. Where FS is the criterion and since both M_R and M_O are in units of (force•length) or (force•length/ (unit length)), the choice for the rotating point will affect the results if there is any force acting perpendicular to the base.

In foundation reliability analysis, the limit state concept is usually applied, and in the physical sense, the concern for overturning is that the structure will overturn to one side, or will rotate about an edge of the base. The toe (or heel) is the logical choice of the rotating point. This pre-chosen rotating point can also make the reliability analysis result consistent, not because the factor of safety will be comparable for similar structures but also because the approximation method used in reliability calculation will be affected by the definition of the performance functions to some degree.

4.6.3.3 The performance functions

As two measurements will be used in an overturning stability analysis, the performance functions will be discussed separately.

Location of effective resultant base force, X_R

Considering all forces involved in an overturning mode for a typical navigation structure, the performance function for location of effective resultant base force, X_R, can be expressed as

$$X_R = \frac{M_R - M_O}{N'}$$

$$= \frac{(M_c + M_s + M_{s,v} + M_{anchor}) - (M_{s,h} + M_{hydro} + M_{impact} + M_{uplift})}{N'} \qquad (4 \cdot 48)$$

where

M_c moment of monolith material, usually concrete, about rotating point;

M_s moment of soil on the monolith about rotating point;

$M_{s,v}$ moment generated by the vertical wall friction force about rotating point;

M_{anchor} moment of anchor(s) about rotating point;

$M_{s,h}$ moment of backfill soil about rotating point;

M_{hydro} moment of pool differential about rotating point;

M_{impact} moment caused by impact force about rotating point;

M_{uplift} moment of uplift force about rotating point; and

N' effective normal base force.

Among those terms in eqn. (4•48) the M_{hydro} can be expressed as

$$\frac{H_H^3 - H_L^3}{6}\gamma_w \tag{4•49}$$

the M_{uplift} should be determined according to the distribution of uplift force (see section 4.6.2.3) and N' is the same as defined in eqn. (4•19).

Considering that X_R is a relative distance and N' can be located at any point along the base, even outside of the base, assuming X_R normally distributed is more reasonable than the lognormal distribution assumption. This definition of X_R is illustrated in Figure 4-8.

The performance criterion of X_R can be set as $X_R>0$ according to the limit state theory.

Since X_R represents the location of a resultant base force, it may be better to relate X_R to PC% (percentage of base which is in compression). If the toe is chosen as the

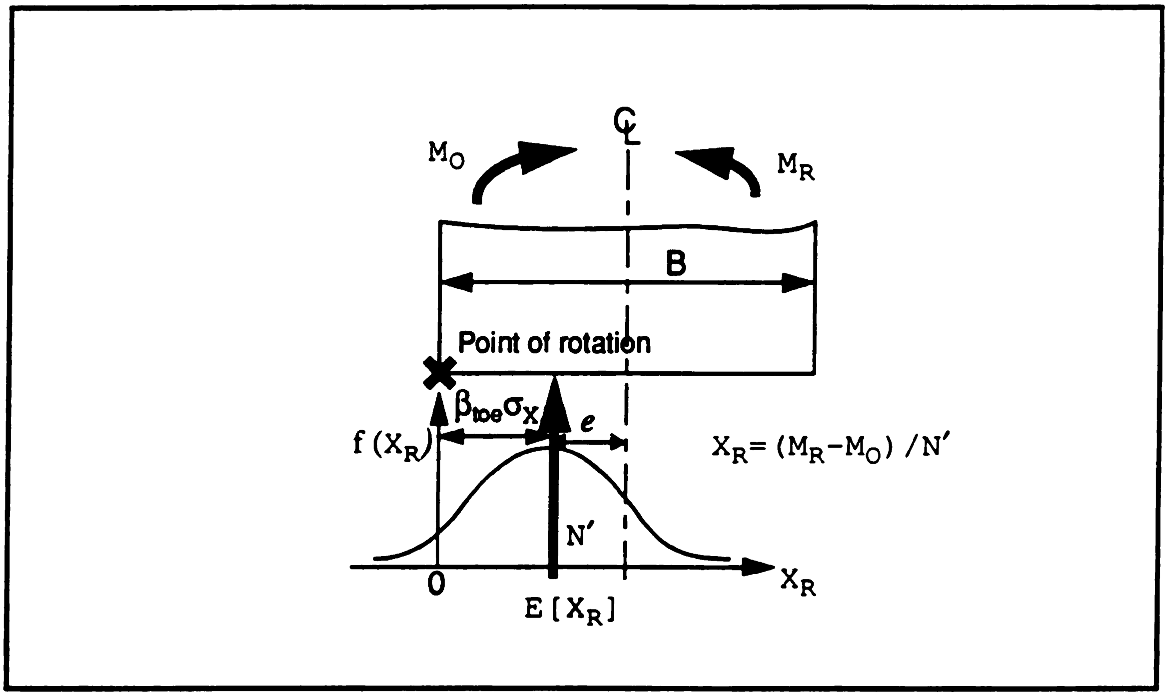


Figure 4-8 Location of Effective Resultant Base Force, X_R, and its Distribution

moment center, then the relationship

$$PC\% = 3X_R/B(100), \quad PC\% \leq 100 \tag{4•50}$$

will hold, where B is the base length.

Note that using PC% as an overturning reliability measurement is purely because some current regulations require checking this values (e.g. by Corps of Engineers[113]) and it does not strictly reflect the overturning stability. Usually the requirement is 100 per cent base compression for soil and normal rock foundation, and 50-75 per cent for special cases. If one takes the base compression requirement as C% and the turning point is about toe, the criterion of PC% can be defined as

$$PC\% = \frac{3X_R}{B}(100) \geq C\% \tag{4•51}$$

Since a normal distribution is more reasonable for this criterion, the corresponding reliability index will be

$$\beta_{PC\%} = \frac{3X_R - (C\%/100)B}{3\sigma_{X_R}} \tag{4•52}$$

The term of (C%/100)B in eqn. (4•52) can be replaced in terms of fraction of base length, that is, (CB), where C may be any positive fraction numbers, then this criterion can be written as

$$X_R \geq C(B/3) \tag{4•53}$$

and

$$\beta_{CB} = \frac{X_R - C(B/3)}{\sigma_{X_R}} \tag{4•54}$$

Note that C may have values of 1/2, 3/4 and 1, which will correspond to situations where one-half of, three-quarter of and the full base is in compression, respectively. The C can also have a zero value, which represents that resultant base force is located on the toe – it is the limit state of structure keeping balance or the performance criterion of X_R (shown in Figure 4-8).

As the PC% criterion is used to limit compressive stresses in the foundation rather than directly reflecting the overturning stability, this criterion is not recommended as a reliability measurement but it can be used to determine the uplift force distribution and associated uplift factor, E.

Factor of safety, FS

The performance function of *FS* can be expressed as

$$FS = \frac{M_c + M_s + M_{s,v} + M_{anchor}}{M_{s,h} + M_{hydro} + M_{impact} + M_{uplift}} \tag{4•55}$$

All terms in eqn. (4•55) are the same as that in eqn. (4•48) except that the effective normal force term is not present as it would shift to the toe and pass through the moment center if overturning began.

The criterion for *FS* is $FS \geq 1.0$.

4.6.4 Bearing Capacity

4.6.4.1 Generalized bearing capacity equation

Bearing capacity is another safety aspect of concerned to engineers. In practice, the ultimate foundation bearing capacity is often determined by the so called generalized bearing capacity equation as expressed by (Meyerhof, 1963)

$$Q = \bar{B}\left[(\zeta_c\zeta_{cd}\zeta_{ci}\zeta_{ct}\zeta_{cg}cN_c) + (\zeta_q\zeta_{qd}\zeta_{qi}\zeta_{qt}\zeta_{qg}q_0N_q) + \frac{1}{2}(\zeta_\gamma\zeta_{\gamma d}\zeta_{\gamma i}\zeta_{\gamma t}\zeta_{\gamma g}\bar{B}\gamma N_\gamma)\right] \tag{4•56}$$

and the safety factor for bearing capacity is

$$FS = \frac{Q}{N'} = \frac{\text{Normal component of the ultimate bearing capacity}}{\text{Effective normal force applied to the base of structure}} \tag{4•57}$$

where

$\bar{B}$ – effective width of the base,

$$\bar{B} = B - 2e = B - 2\left(\frac{B}{2} - X_R\right) = 2X_R \tag{4•58}$$

$X_R = \Sigma M/\Sigma N'$ is the location of resultant force at the base with reference point at toe as defined in overturning stability analysis;

B – width of the geometric base;

e – eccentricity of the load with respect to geometric base width;

c – cohesion parameter of the foundation material;

ζ's – factors related to depth, inclination of load, tilt of base and ground slope;

N_c, N_q, N_γ – bearing capacity factors for a strip load, corresponding to cohesion resistance, overburden pressure and base friction resistance;

q_0 – effective overburden pressure on the plane passing through the base of the footing;

$q_0 = D\gamma'_{soil}$

where

D–depth of overburden soil layer; and

γ'_{soil} – effective unit weight of overburden soil;

and

γ – effective unit weight of the foundation material, $\gamma_{buoyant}$ below water table and γ_{moist} above water table.

The bearing capacity factors were defined by Meyerhof (1963) and Vesic (1975). These factors are function of the internal friction angle of the foundation material, ϕ, effective base length, *L*, inclination angle of resultant force on the base, Δ, slope of the footing base, α, and the slope of the overburden soil surface, θ. The Δ is defined by

$$\Delta = \tan^{-1}(\Sigma H/\Sigma N') \qquad (4{\cdot}59)$$

where

ΣH is the sum of horizontal forces; and

$\Sigma N'$ is the sum of effective normal forces applied on the footing;

So the ultimate bearing capacity, Q, is a function of many variables, that is

$$Q = f\ (X_R,\ c,\ \phi,\ \gamma,\ D,\ B,\ L, \gamma'_{soil},\ \Delta,\ \alpha,\ \theta)$$

and X_R and Δ are functions of other variables.

4.6.4.2 Performance function and random variables

The random variables involved in a bearing capacity analysis are similar to that in a sliding and overturning stability analyses. Since some variables can be reasonably treated as deterministic variables, such as D, B, L (dimensional parameters) and γ_w, as well as α and θ (if involved), therefore, the Q is a function of random variables

$$Q = f\ (X_R,\ c,\ \phi,\ \gamma, \gamma'_{soil},\ \Delta)$$

Among the "new" random variables, X_R and Δ can be treated either as a individual random variables or as explicit functions of other random variables. If the new random variables are going to be directly used in analysis, the correlations between these "new" variables and "primary" variables should be considered unless these correlations are insignificant.

The generalized bearing capacity equation is quite complex and is not generally suitable for all type of foundations (for example, it may not fairly represent the bearing capacity for rock foundations, see discussion in the following section), so when this equation is used to determine the bearing capacity of a foundation, caution must be taken.

The performance function for FS can be written as

$$FS = \frac{\overline{B}\left[(\zeta_c\zeta_{cd}\zeta_{ci}\zeta_{ct}\zeta_{cg}cN_c) + (\zeta_q\zeta_{qd}\zeta_{qi}\zeta_{qt}\zeta_{qg}q_0N_q) + \frac{1}{2}(\zeta_\gamma\zeta_{\gamma d}\zeta_{\gamma i}\zeta_{\gamma t}\zeta_{\gamma g}\overline{B}\gamma N_\gamma)\right]}{N'} \tag{4•60}$$

and all variables should be substituted into eqn. (4•60). Note that N' is the same as that defined in the overturning stability analysis.

The criterion is $FS > 1.0$.

In bearing capacity analysis, for simplicity, the Terzaghi bearing capacity equation for a continuous footing under general shear failure of foundation

$$Q_D = \overline{B}(cN_c + \gamma DN_q + \gamma\frac{\overline{B}}{2}N_\gamma) \tag{4•61}$$

can be used as a alternative where all variables are defined as before.

4.6.4.3 Nonlinearity of Q as function of ϕ

Although the bearing capacity Q is a function of many variables, the base material internal friction angle ϕ is the dominant variable and the Q is a high nonlinear function of ϕ. It is easy to observe that the nonlinearity of Q is basically caused by the three bearing capacity factors, N_c, N_q and N_γ (the ζ's usually assume values near 1.0). The bearing capacity factors versus ϕ is illustrated in Figure 4-9.

It is clearly shown that the nonlinearity of bearing capacity factors increases dramatically when the internal friction angle increases beyond 40 degrees. The value of factor N_γ increases the most among all three factors because N_γ contains the term of tan(1.4ϕ) which will approach infinity when ϕ increases to about 64.3 degrees.

It is not uncommon in practice that the internal friction angle, ϕ, is greater than 45° for rock foundations, therefore, the factor N_γ defined by Meyerhof:

$$N_\gamma = (N_q - 1)\tan(1.4\phi) \tag{4•62}$$

will achieve very unreasonable values. For example, a rock foundation has an internal friction angle of 52° with a 30% variation. If the Point Estimate Method is used in reliability analysis, at $\mu_\phi + \sigma_\phi$ point, the factor N_γ will have value

$$\begin{aligned} N_\gamma &= (N_q-1)\tan[1.4(\mu_\phi + \sigma_\phi)] \\ &= (N_q-1)\tan[1.4(52°+15.6°)] \\ &= (N_q-1)\tan(94.64°) = -12.321(N_q-1) \end{aligned}$$

which has no physical meaning. To avoid this unwanted situation, the N_γ defined by Caquot and Kerisel (1953) and Vesic (1973) (denoted as "N_γ defined by Vesic *et al*" in the following) may be used:

$$N_\gamma = 2(N_q+1)\tan(\phi) \tag{4•63}$$

The comparison of two N_γ definitions is shown in Figure 4-10. Note that the factors N_γ, defined by Meyerhof and Vesic *et al*, have a similar behavior when ϕ is less than the "limiting angle", 64.3°, but the Vesic *et al* defined N_γ makes better physical sense when ϕ is greater than 64.3°. In the case of high internal friction angle of foundation material, the Vesic *et al* defined N_γ is preferred but unreasonably high values will still be obtained.

The nonlinearity of Q as a function ϕ can not be overlooked, because it will mean: (1) the suitability of

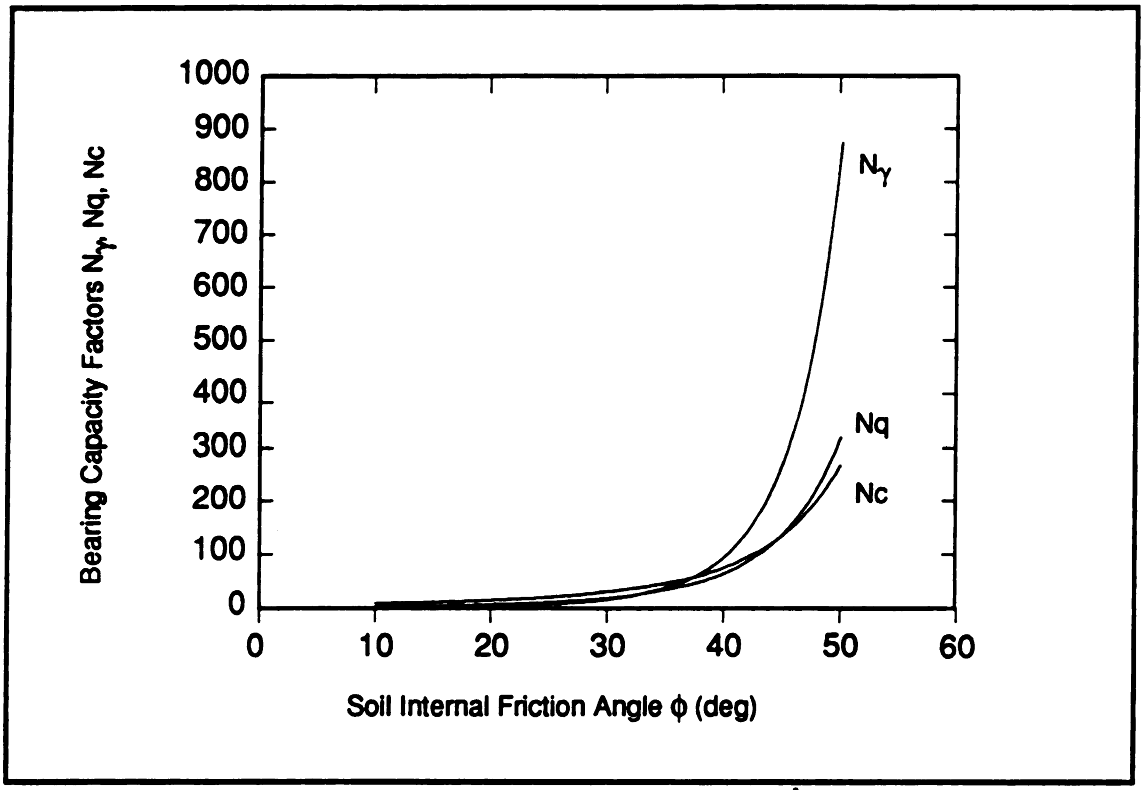


Figure 4-9 Nonlinearity of Bearing Capacity Factors vs. ϕ

applying generalized bearing capacity equation in analysis would be in question if the base material has high internal friction angle (say, greater then 45 degree); (2) simplified methods in reliability index calculation, such as the first order second moment (FOSM) method, will lead to greater error since nonlinearty can not be well accounted for by these methods and the design point may be far apart from the mean of the performance function.

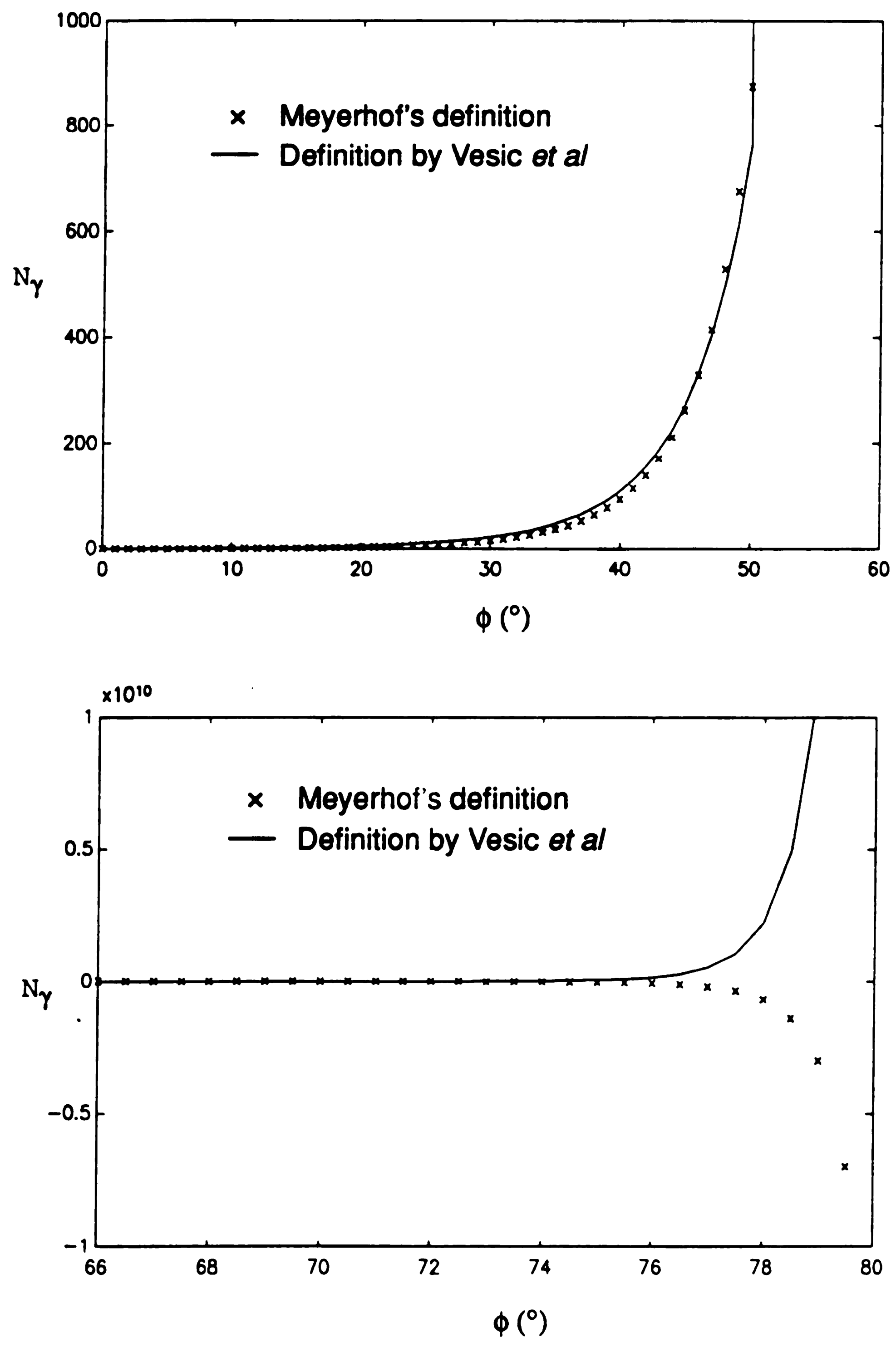


Figure 4-10 Comparison of Two Different Definitions of Bearing Capacity Factor N_γ

4.6.5 Foundation Settlement

If the navigation structures are based on soil foundations, especially when a soft soil layer is involved, the foundation settlement is another structural safety aspect that needs to be analyzed.

The settlements of soil usually can be divided as two different types, elastic (or immediate) settlement and plastic (or consolidation) settlement. Although a complete understanding of these settlements has not yet been achieved, some commonly used methods in engineering practice can be applied in the foundation settlement reliability analysis.

Since the settlement of existing navigation structures can usually be observed and this study mainly focuses on the reliability analysis of existing structures, no details will be discussed herein.

4.6.6 Stability of Pile Foundation

Piles, like anchors, are very often used in foundations for navigation structures so as to improve foundation stability.

Many uncertainties affect pile capacity: besides defects in the pile material, quality of construction of the pile foundation, the inherent spatial variability of soil properties within the soil medium, imperfection of theoretical formulas and correlation between piles also give great influence.

Several aspects of pile foundation may need to be given special attention, such as negative skin friction, effect of pile length on the pullout resistance and the capacity of the pile foundation resisting lateral loads, the settlement of pile groups, and the effect of pile group reliability on structural safety.

Because of the different behavior and complexity, reliability analysis of structures with pile foundations needs special study and it is beyond the scope of this project. The pile group reliability (*GR*), could be determined following the approach discussed for anchor groups in section 4.6.2.6.

When evaluating the reliability of navigation structures, including retaining walls and lock monoliths, the reliabilities of other structures, such as the lock gate, also should be studied. Since these structures are more "structural engineering" related and much research work and applications have been done, no further discussion will be given herein.

4.7 Reliability of Structural Systems

After examining the reliability of individual structures (or components) in a system, the reliability of the structural system is ready to be evaluated.

To evaluate the reliability of a structural system, first each individual component in the system must be represented in the terms of failure probability, then either the complex system must be simplified into a equivalent simple series or parallel system or using the fault tree technique to define the minimal-cut-set. The probability of the whole system can be calculated by using the methods discussed in section 3.2.4 and section 3.2.5, and the equivalent reliability index can be determined if assume normal distribution.

4.8 Reliability Prediction with Time Factor

All engineering facilities have certain period of service life, or in other words, the reliability of a structure tends to decrease as time goes on. To predict the reliability of a structure, the time factor must be taken into consideration.

As mentioned before, the decrease in reliability as a function of time must be caused by deterioration of materials, stress relaxation, strain and material creep and the effect of repeated load. If the time factor can not be expressed separately by a time function, the reliability evaluation will be much more complicated. But if the time factor can be described as a pure time, *t*, function (at least approximately), say $\theta(t)$, then the reliability of a structure or a system at time T can be evaluated by

$$R_{predict}(t=T) = R_{current}\,\Theta(T) \qquad (4{\bullet}64)$$

where $\Theta(T)$ is the integration of $\theta(t)$ in the time domain from current time to time T or just the value of $\theta(t)$ at time T (see section 3.2.7). Conveniently, the reliability index can be easily calculated by multiplying the $\Theta(T)$ to the β which was obtained from previous analysis.

It should be pointed out that all underlying random variables actually are time dependent; however, since there is no practical way to determine this dependency (lack of data and not fully understood mechanism), only approximate estimation of time factor are feasible.

Chapter V

EXAMPLES OF NAVIGATION STRUCTURE RELIABILITY EVALUATION

5.1 Introduction

To illustrate the feasibility of applying probabilistic analysis methods to navigation structures, as well as to discover the potential problems, selected navigation structures – locks and dams on the Monongahela River, Pennsylvania, USA and the Tombigbee River, Alabama, USA – were analyzed by the methods aforementioned in Chapter IV. Because of limited data and information available, the focus was on the reliability evaluation with respect to the specific performance modes of sliding stability, overturning stability and bearing capacity. The reliability measurement was the reliability index, β, calculated by both the Taylor's series and the point estimate approximative methods. Several performance functions were considered. Also, to simplify the problem and suit engineering applications, all studies were limited to two dimensional problems. To obtain the analysis results, extensive customized computer programing on workstations and microcomputers was performed.

The individual structures studied were selected from concrete gravity monoliths at Locks and Dam No. 2, 3 and 4 on the Monongahela River, and Demopolis Locks and Dam on the

Tombigbee River. These monoliths represent some typical navigation structures: a landside lock wall, a middle lock wall, a dam pier, and foundations with and without anchors.

5.2 Overturning Analysis

Two performance functions were considered for overturning (or rotational) stability: the *factor of safety, FS,* and the *location of effective resultant base force,* X_R (refer to Wolff and Wang[125]).

The factor of safety is a common widely used criterion but the specific definition may vary considerably. In the following examples, FS is defined by

$$FS = \Sigma M_R / \Sigma M_O \tag{5•1}$$

where

ΣM_R – summation of the resisting moments about the rotation point (in 2-D problem about the rotation line, usually the toe); and

ΣM_O – summation of the overturning moments about the rotation point.

The components of M_R and M_O were described in detail in section 4.6.3.3.

The location of effective resultant base force, X_R, is another often used criterion of rotational stability in engineering practice. It is defined as

$$X_R = \frac{\Sigma M_R - \Sigma M_O}{N'} \tag{5•2}$$

where

ΣM_R, ΣM_O – defined as before. Note that these moments do not include the moment produced by the effective base resultant force; and

N' – normal component of the effective base resultant force.

The criterion for FS is simple, that is, FS≥1.0 (by limiting state theory) and a lognormal distribution assumption of FS seems reasonable (based on its nonnegative property). Therefore, the reliability index of FS is

$$\beta_{FS} = \frac{ln\left(\frac{\mu_{FS}}{\sqrt{1+V_{FS}^2}}\right)}{\sqrt{ln(1+V_{FS}^2)}} = \frac{E[lnFS]}{\sigma_{lnFS}} \tag{5•3}$$

If the percentage of the base in compression, PC%, is used as overturning stability measurement and the criterion is in terms of a fraction of base length, that is, PC% > CB, (B is the length of base and C is a fraction which may have value from 0 to 1), since $3X_R$ is the length of base in compression (rotating about toe and $3X_R \le B$), then this criterion and the corresponding reliability index are

$$X_R \geq CB/3 \tag{5•4}$$

and

$$\beta_{CB} = \frac{E[X_R] - CB/3}{\sigma_{X_R}} \tag{5•5}$$

Note that the X_R is assumed normally distributed and this reliability index gives an indication of how certain that the effective base resultant force is located at the distance of (CB/3) measured from the toe.

In the following examples, both criteria, FS and X_R, will be considered. The reliability index, corresponding to C=0, or the resultant force located at the toe, will be denoted as β_{toe}; the reliability indices for 1/2, 3/4 and full base in compression will be denoted by $\beta_{B/2}$, $\beta_{3B/4}$ and β_B, respectively.

5.2.1 Locks and Dam No.2 Monolith M-16, Monongahela River

5.2.1.1 Introduction

Locks and Dam No. 2 are located at mile 11.2 above the mouth of the Monongahela River. The structure includes a concrete overflow dam and two lock chambers and was originally placed in operation in 1905. During the period of 1949 through 1953, the dam was shortened and new locks were constructed. The dam is founded on pile and timber cribbing; the locks are founded on sedimentary rock comprised of sandstone, siltstone, shale and clay shale.

M-16 is a shale-founded gravity monolith forming a part of the middle wall between the two chambers. A cross-section through the monolith is shown in Figure 5-1. The section contains two openings: a filling and emptying culvert and a pipe gallery.

The water levels considered for analysis are (elevation in feet above sea level):

Case	Upper Pool	Lower Pool
Normal Operating (A)	718.7	710.0
Maintenance (A)	718.7	691.5
High Water (A)	729.0	723.5
Normal Operating (B)	725.5	716.0
Maintenance (B)	724.7	691.5

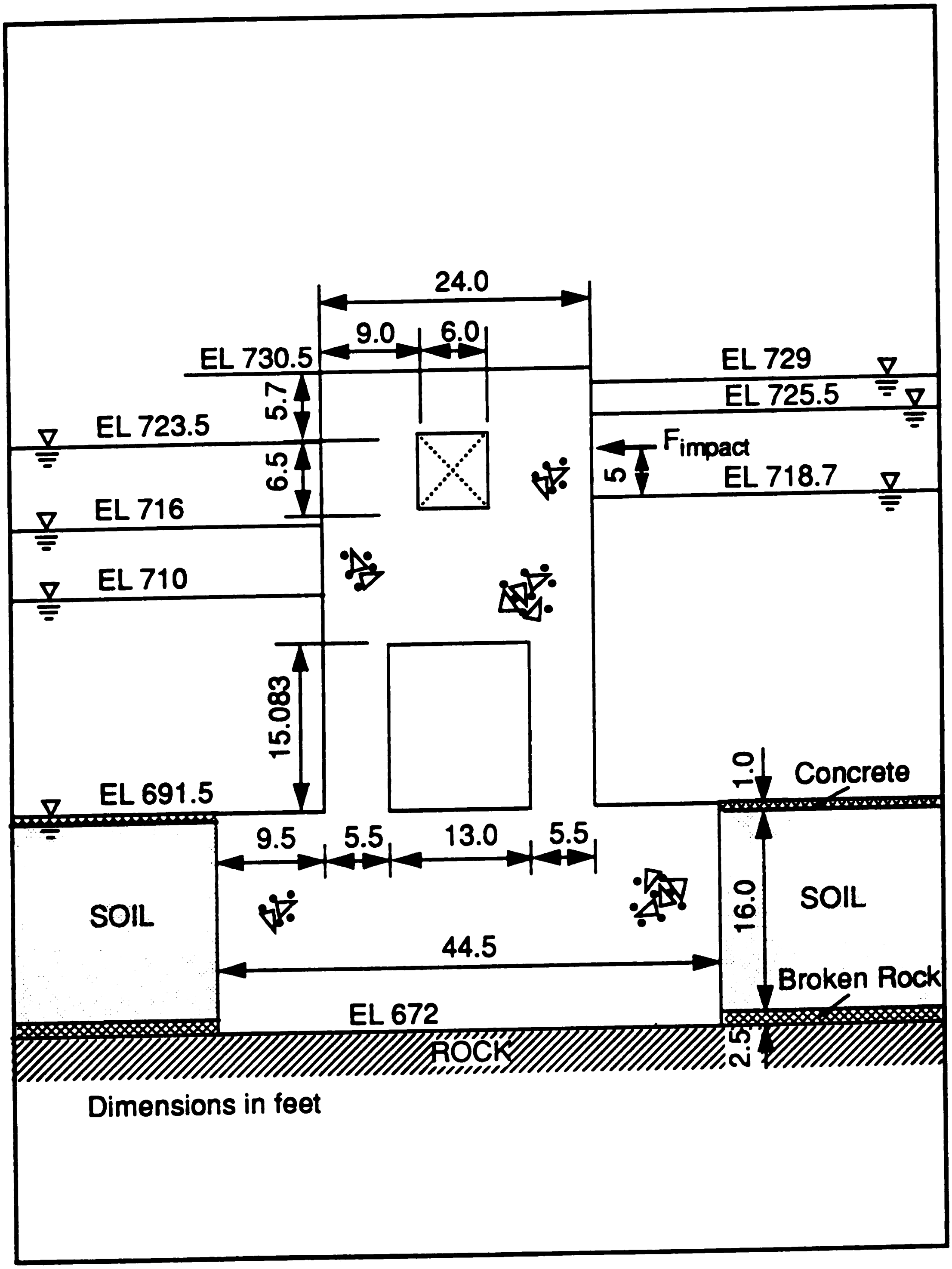


Figure 5-1 Locks & Dam No.2, Monolith M-16, Cross-Section

Table 5-1 Random Variables for Overturning Analysis – Locks and Dam No. 2, Monolith M-16

Variable	μ	σ	V (%)
γ_{soil}	0.0755 (kcf)	0.003775 (kcf)	5.0
ϕ'_{soil}	33 °	3.3 °	10.0
$\gamma_{concrete}$	0.15 (kcf)	0.0075 (kcf)	5.0
F_{impact}	1.0 (kips/ft)	0.5 (kips/ft)	50.0
Uplift parameter, E	Varies if PC% <100; 0.0, if PC% 100)	0.2	—

5.2.1.2 Random variables

The random variables used in an overturning analysis, with their statistical properties (first two moments), are listed in Table 5-1. These were assigned based on the data and information from the condition survey of Locks and Dam No. 2[126]. Note that the coefficients of variation or standard deviations in Table 5-1 were also based on other studies and references (e.g. Harr[53]), as well as common engineering judgment. Since the mean value of the factor E is implicitly related to the percentage of base in compression, therefore it is reasonable to determine μ_E by iteration (refer to section 4.6.2.3). A standard deviation of 0.2 was assigned for the E factor based on engineer's judgment.

5.2.1.3 Performance functions

The expressions of M_R, M_O and N' are the following (with upper pool water head H_H and lower pool water head H_L).

$$M_O = M_{pool} + M_{soil,D} + M_{uplift} + M_{impact}$$

$$= \frac{[H_H^3 - H_L^3]}{6}\gamma_w + \frac{(17)^2}{2}(\frac{17}{3} + 2.5)\gamma'_{soil}K_0 +$$

$$+ \frac{B^2\gamma_w}{6}[(2H_H + H_L) - 2E(H_H - H_L)] + (H_H+5)F_{impact}$$

$$= \frac{[H_H^3 - H_L^3]}{6}\gamma_w + 1180.08\gamma'_{soil}(1-\sin\phi_{soil}) +$$

$$+ 330.04\,[(2H_H + H_L) - 2E(H_H - H_L)] + (H_H+5)F_{impact} \quad (5\bullet6)$$

$$M_R = M_{concrete} + M_{water} + M_{soil,R}$$

$$=34377.239\gamma_{con} + (H_H-19.5)(11.0)\gamma_w(11.0/2+33.5) +$$

$$+ (H_L-19.5)(9.5)\gamma_w(9.5/2) + 1180.08\gamma'_{soil}(1-\sin\phi_{soil}) \quad (5\bullet7)$$

$$N' = W_{concrete} + W_w - U$$

$$= 1568.671\,\gamma_{con} + 22.825 - \frac{(H_h + H_l) - (H_h - H_l)E}{2}\gamma_w B \quad (5\bullet8)$$

The location of effective resultant base force, X_R, is

$$X_R = \frac{M_R - M_O}{N'}$$

$$= (M_{concrete} + M_{water} - M_{pool} - M_{uplift} - M_{impact})/N' \quad (5\bullet9)$$

and the related performance function is

$$X_R \geq C(B/3) \qquad C = 0,\ 1/2,\ 3/4 \text{ and } 1 \quad (5\bullet10)$$

Note that the moments are in units of kips-ft/ft, forces are in the unit of kip/ft and dimensions are in the unit of ft. For C = 0, 1/2, 3/4 and 1, the corresponding reliability indices are β_{toe}, $\beta_{B/2}$, $\beta_{3B/4}$ and β_B, respectively.

The performance function of FS is

$$FS = M_R / M_O \tag{5•11}$$

The criteria are as mentioned before. The free body diagram for the maintenance condition (B) is shown in Figure 5-2.

5.2.1.4 Analysis results

The analysis results are presented in Table 5-2.

The analysis results show that the monolith M-16 is very safe from overturning because of high reliability indices β_{toe} and β_{FS}. Although the location of effective resultant base force may not be within the middle 1/3 of the base (or the base may not be 100 per cent in compression, e.g. under maintenance condition (B)), it is almost certain that it is within the base – the $\beta_{3B/4}$ is greater than 0 and the β_{toe} is in the range of 11 to 57. Note that the change in reliability indices indicate that they are very sensitive to water level difference. This is because the hydraulic force on monolith M-16 greatly affects the mean (or expected) value of the performance functions but has no effect on their variances since the unit weight of water was treated as a deterministic value.

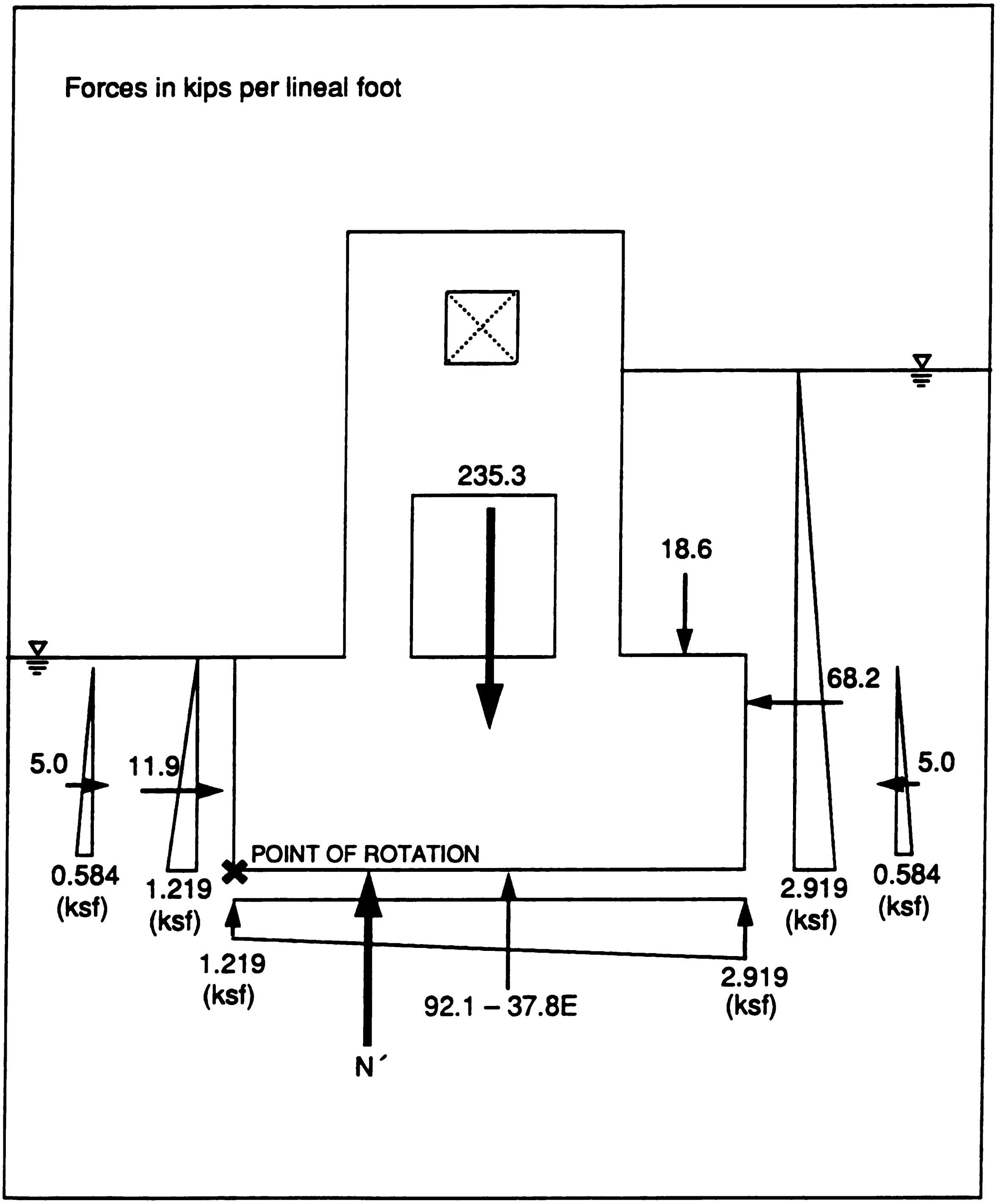


Figure 5-2 Locks and Dam No. 2, Monolith M-16, Free Body Diagram, Overturning Analysis–Maintenance Condition (B)

Table 5-2 Overturning Reliability Analysis Results – Locks and Dam No. 2, Monolith M-16

Mode (Pools)	Method	$E[X_R]$ (ft)	σ_{X_R}	β_{toe}	$\beta_{B/2}$	$\beta_{3B/4}$	β_B	E[FS]	σ_{FS}	β_{FS}
Normal (A) (718.7 /710)	Taylor's Series	18.50	0.344	53.76	32.21	21.43	10.66	1.896	0.090	13.49
	PEM	18.50	0.335	55.20	33.07	22.01	10.95	1.899	0.089	13.66
Normal (B) (725.5 /716)	Taylor's Series	16.81	0.500	33.62	18.79	11.37	3.95	1.655	0.068	10.92
	PEM	16.80	0.492	34.17	19.09	11.55	4.00	1.645	0.074	11.05
Maint. (A) (718.7/ 691.5)	Taylor's Series	15.89	0.778	20.42	10.89	6.12	1.36	1.767	0.141	7.11
	PEM	15.90	0.729	21.80	11.63	6.55	1.46	1.781	0.138	7.44
Maint. (B) (724.7/ 691.5)	Taylor's Series	12.11	1.068	11.34	4.39	0.92	-2.55	1.422	0.095	5.23
	PEM	12.10	1.094	11.07	4.28	0.89	-2.50	1.430	0.097	5.29
High Water (729 /723.5)	Taylor's Series	18.35	0.321	57.20	34.08	22.52	10.96	1.655	0.068	12.29
	PEM	18.34	0.316	57.93	34.50	22.79	11.08	1.656	0.068	12.35

Note: 1. During calculation the mean values of uplift factor E were determined by iteration but assigned as 0 when E greater than 0.

2. Shaded numbers are the reliability indices which are smaller than 4.0 (same for the tables thereafter).

5.2.2 Locks and Dam No.3 Monolith M-20, Monongahela River

5.2.2.1 Introduction

Built in 1905 and put in operation in 1907, the Locks and Dam No. 3 is the oldest locks and dam in the Pittsburgh District of the US Army, Corps of Engineers. Locks and Dam No. 3 are located at mile 23.8 above the mouth of the Monongahela River and upstream of Elizabeth, Pennsylvania. Because of a severe degree of deterioration of the locks and dam structures, a major rehabilitation was implemented in 1978 - 1980. This structure includes a concrete overflow dam with a fixed crest and two lock chambers. The dam is founded on piles driven into the river bottom alluvium and the locks and upper guide wall monoliths are founded on sedimentary rock which is composed of several feet of hard shale or siltstone, then a few feet of black carbonaceous fissile shale, then six to twelve inches of coal, another few feet of black shale, and several feet of limestone. The rock is weathered and badly fractured for an average depth of 8.5 feet below the concrete.

Monolith M-20 is a gravity monolith forming a part of the middle wall between the two chambers. The monolith section contains a pipe gallery and 2 emptying ports with diameter 4.5 ft. As one of the measures provided in the 1978 rehabilitation to improve stability, anchors were installed in the lock walls. The cross-section of the M-20 is shown in Figure 5-3.

The water levels for analysis are (elevations):

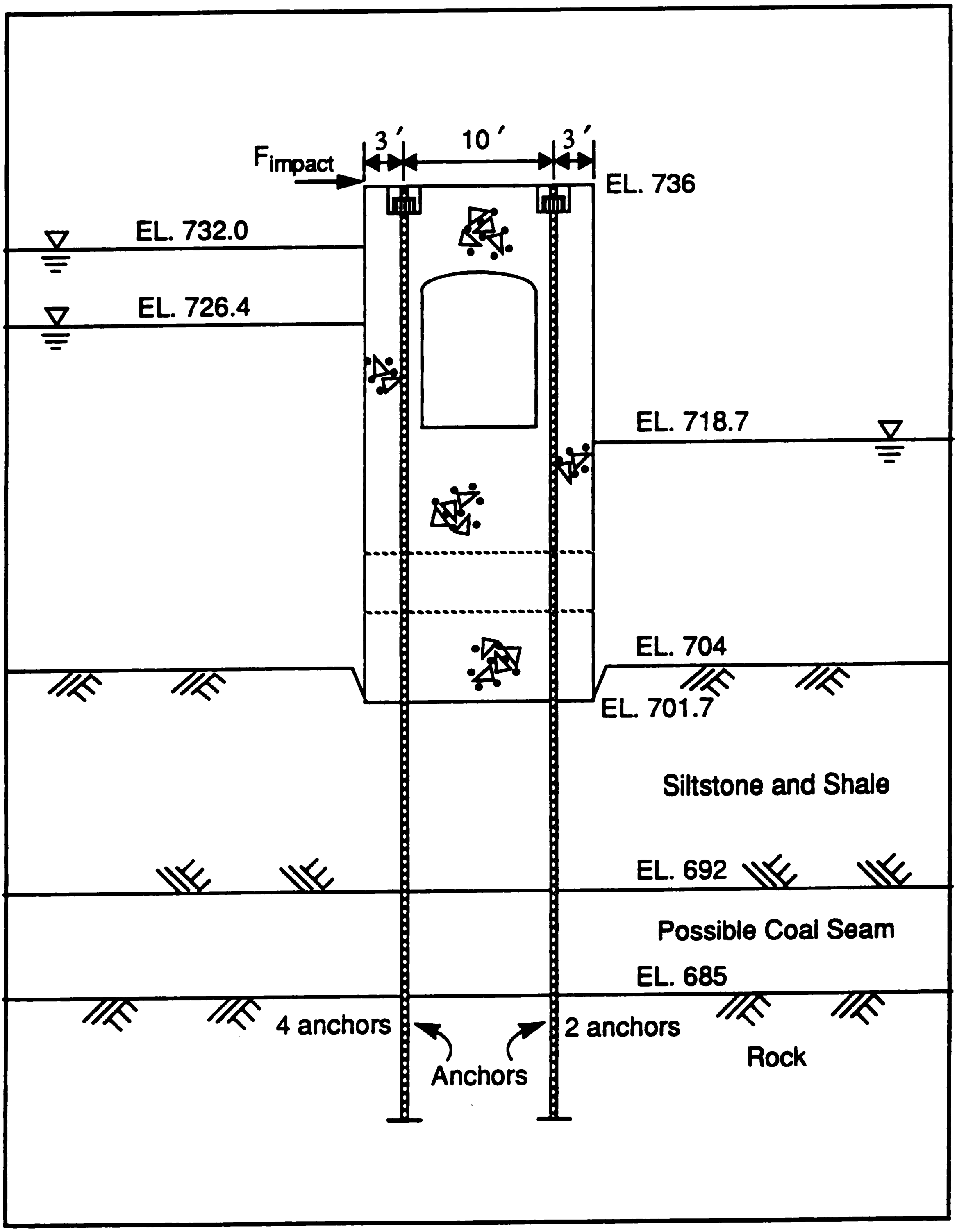


Figure 5-3 Locks and Dam No. 3, Monolith M-20, Cross-Section

Case	Upper Pool	Lower Pool
Normal Operating	726.9	718.7
Maintenance (A)	726.4	701.7 (lock dewatered)
Maintenance (B)	732.0	701.7 (lock dewatered)
High Water (A)	732.8	726.4

5.2.2.2 Random variables

The random variables involved in overturning stability analysis are: unit weight of concrete, $\gamma_{concrete}$, Impact force, F_{impact}, elevation of base, ElE_{base}, uplift factor E and anchor force, F_{anchor}.

Similar to that in the Locks and Dam No. 2, M-16 overturning analysis, the statistical properties of $\gamma_{concrete}$, F_{impact}, ElE_{base} were determined from testing results and/or by engineer's judgement based on observations and/or experience. The mean value of the E factor will be determined by iteration. According to the condition survey of Locks and Dam No.3[111], the anchors installed on M-20 are prestressed steel bars anchored in rock with possibly poor bonding. The prestress force was 112.5 kips and was regularly monitored at the time of installation. Six anchors were installed in monolith M-20, four on the riverside and two on the landside. Of the four on the riverside, one failed during installation, and another one on the landside was also found not functional. Based on this performance Corps of Engineers' personnel have some doubt as to whether the remaining anchors are functional. Interviews with engineers knowledgable about the installation led to the assumption that the reliability of each single anchor is 0.5 and the anchors are independent of each other. The anchors were divided into two independent groups, the riverside group and the landside group, to simplify the group effect.

The field survey report and construction records also indicated that the base elevation of the specific monoliths is uncertain and could vary from about 700 ft to 703 ft in the middle lock wall. The reported base elevation of M-20 is 701.7 ft. Considering that the monolith is only about 34 feet tall, uncertainty regarding the base elevation may affect the overturning stability. To reflect this uncertainty, an expected value of 701.7 ft and standard deviation of 0.3 ft were assigned to the base elevation of M-20 in this analysis.

The statistical properties of random variables used in the overturning analysis are listed in Table 5-3.

5.2.2.3 Performance functions

The overturning performance functions with respect to two different criteria are the following:

$$M_O = M_{pool} + M_{impact} + M_{uplift}$$

$$= \frac{[H_H^3 - H_L^3]}{6}\gamma_w + (H_H + 5)F_{impact} + \frac{B^2\gamma_w}{6}[(2H_H + H_L) - 2E(H_H - H_L)] \quad (5\bullet 12)$$

where

$$H_H = ELE_{upper\ pool} - ELE_{base} \text{ and}$$

$$H_L = ELE_{lower\ pool} - ELE_{base}$$

$$M_R = A_{con}\gamma_{concrete}\ B/2 + W_W B/2 + T_{river}(13.0) + T_{land}(3.0) \quad (5\bullet 13)$$

where

Table 5-3 Random Variables for Overturning Analysis – Locks and Dam No. 3, Monolith M-20

Variable	μ	σ	V (%)
$\gamma_{concrete}$	0.15 (kcf)	0.0075 (kcf)	5.0
ELE_{base}	701.7 (ft)	0.3 (ft)	—
F_{anchor} $P_r = 1$ $P_r = 0$	112 (kips/anchor) 0	2.24 (kips/anchor 0	2.0 0
F_{impact}	0.80 (kips/ft)	0.40 (kips/ft)	50.0
Uplift parameter, E	Varies, if PC%<100) 0.0, if PC% 100)	0.2	—

A_{con}—section area of concrete part of the monolith;

W_w—weight of water in the emptying ports, kips/ft;

T_{river}, T_{land} = [(# of piles)*F_{anchor}/(length of base)]

– holding forces of anchor groups on riverside and landside, respectively, kips/ft.

The statistical properties of anchor groups is based on the binomial distribution assumption and the concept discussed in section 4.6.2.6 was used. Thus

$$E[T_{river}] = NR\mu_{F_{anchor}} \qquad \sigma_{T_{river}} = \sqrt{R(1-R)N}\mu_{F_{anchor}} \quad (5\bullet14)$$

$$E[T_{land}] = MR\mu_{F_{anchor}} \qquad \sigma_{T_{land}} = \sqrt{R(1-R)M}\mu_{F_{anchor}} \quad (5\bullet15)$$

where

N, M – number of anchors on riverside and landside, respectively;

R – reliability of single anchor; and

$\mu_{F_{anchor}} = E[F_{anchor}]$ – mean value of single anchor force.

$$N' = W_{concrete} + W_w - U + T_{river} + T_{land}$$

$$= A_{con}\gamma_{concrete} + W_w - \frac{(H_h + H_l) - (H_h - H_l)E}{2}\gamma_w B + T_{river} + T_{land} \quad (5\cdot16)$$

The definitions of X_R and FS, as well as the corresponding criteria are the same as before.

The free body diagram for the maintenance condition (A), with 3 anchors on riverside and 1 anchor on landside, is shown in Figure 5-4.

5.2.2.4 Analysis results

The overturning analysis results are listed in Table 5-4.

In the results table, "3+1" represents the condition of 3 anchors at riverside plus 1 anchor at landside functioning; "3+1 R=0,5" implies the same anchor configuration but 0.5 reliability is assumed for each single anchor. The mean and standard deviation of the number of working anchors follows the binomial distribution for each side of the wall is also assumed.

The analysis results indicate that the reliability of monolith M-20 with respect to overturning is not as great as might be desired because under maintenance condition (B) the reliability indices are small (less than 3.0) even when 4+2 anchors are installed but with 0.5 reliability. Therefore, some remedial measures may be needed to improve the overturning stability of the lock wall, or at least some

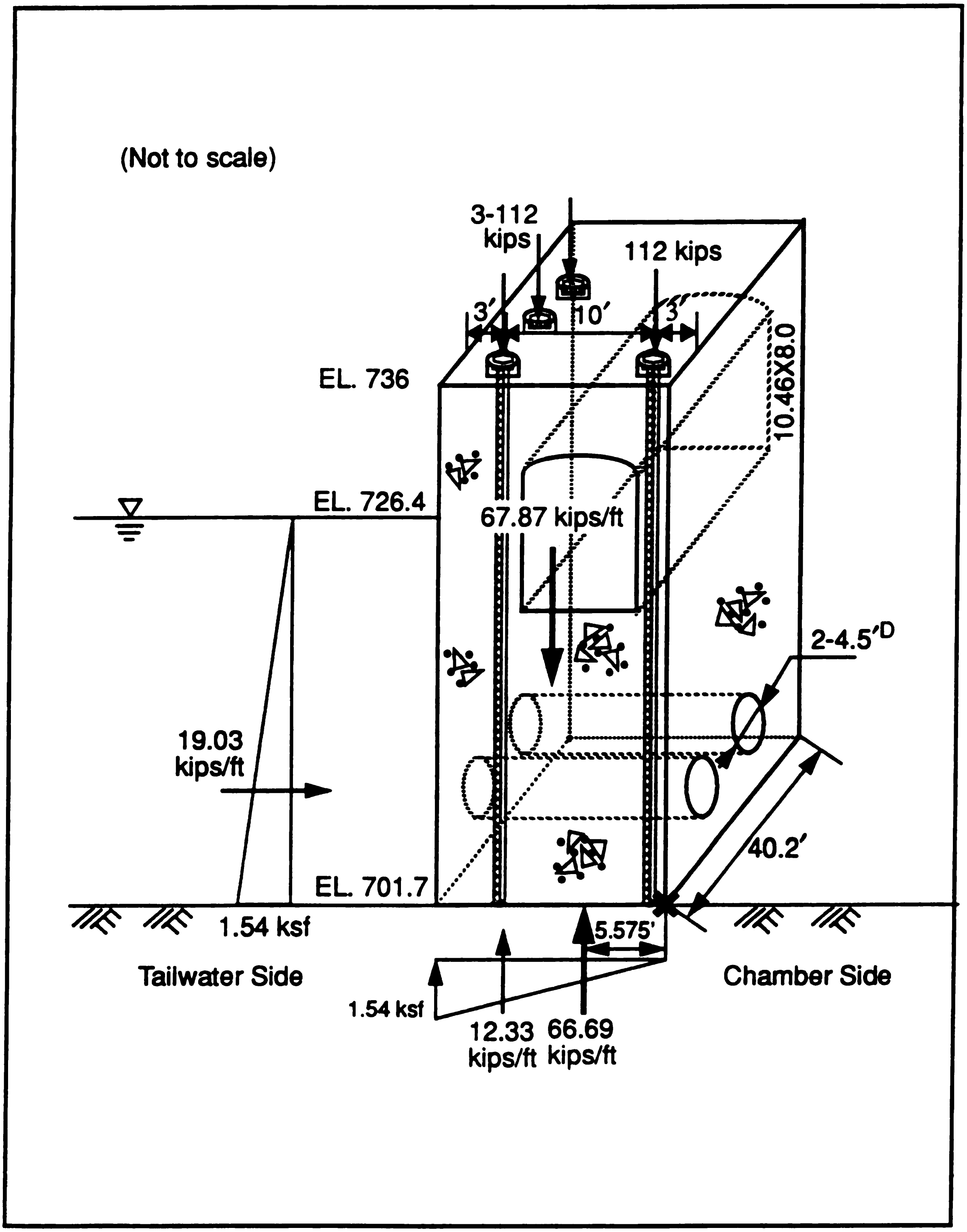


Figure 5-4 Locks and Dam No. 3, Monolith M-20, Free Body Diagram, Overturning Analysis - Maintenance Condition (A), 3+1 Anchors

Table 5-4 Overturning Reliability Analysis Results – Locks and Dam No. 3, Monolith M-20

Mode (Pools)	Anchor	$E[X_R]$ (ft)	σ_{X_R} (ft)	β_{toe}	$\beta_{B/2}$	$\beta_{3B/4}$	β_B	E[FS]	σ_{FS}	β_{FS}
Normal	No anchors	4.793	.358	13.39	5.94	2.22	-1.51	1.70	.114	7.85
(726.9	3+1	5.914	.247	23.91	13.13	7.74	2.35	2.09	.130	11.84
/718.7)	3+1 R=0.5	5.433	.456	11.92	6.06	3.14	0.22	1.91	.158	7.73
Maint. (A)	No Anchors	4.373	.379	11.55	4.51	0.99	-2.54	1.75	.167	5.87
(726.4	3+1	5.574	.276	20.19	10.53	5.70	0.87	2.29	.195	9.71
/701.7	3+1 R=0.5	5.073	.445	11.39	5.40	2.41	-0.58	2.04	.211	6.88
High Water	No Anchors	3.849	.368	10.45	3.21	-0.41	-4.03	1.39	.072	6.28
(732.8	3+1	5.367	.187	28.77	14.48	7.33	0.18	1.72	.077	12.05
/726.4)	3+1 R=0.5	4.707	.509	9.24	4.01	1.39	-1.23	1.55	.110	6.18
	No Anchors	0.309	.735	.42	-3.21	-5.02	-6.84	1.04	.298	0.01
Maint.	3+1	2.638	.426	6.19	-0.07	-3.20	-6.23	1.49	.296	1.94
(B) (732.0	3+1 R=0.5	1.624	.781	2.08	-1.33	-3.04	-4.75	1.25	.314	0.80
/701.7)	4+2	3.188	.380	8.40	1.37	-2.14	-5.65	1.66	.297	2.78
	4+2 R=0.5	2.010	.763	2.63	-0.86	-2.61	-4.35	1.34	.322	1.10

temporary reinforcement methods must be employed when the lock is under maintenance operation. In fact this has been the case, and the walls have been supported by struts during dewatering.

It is also noted that in most cases, if the reliability index is relatively "high" (say, greater than 4.0) with respect to the FS criterion, it also tends to be high with respect to "overturning about toe" criterion (i.e. β_{toe}); but other criteria related to the percent base in compression may have much smaller (even negative) values. This indicates that the factor of safety and overturning about toe criteria are directly related to overturning reliability because they reflect the "limit state".

5.2.3 Locks and Dam No. 4, Dam Pier 3, Monongahela River

5.2.3.1 Introduction

Locks and Dam No. 4 are located at mile 41.5 on the Monongahela River between Charleroi and Monessen, Pennsylvania. The Locks and Dam were reconstructed from an earlier structure in 1931-1932. In 1967, the dam was again reconstructed to raise the pool six (6) feet. A related reconstruction of the lock was completed in 1964.

Dam pier monolith 3 is a heavy concrete structure. Its base passes through 25 to 34 feet of river alluvium and about 9 feet of clayey shale above its foundation elevation. The survey showed good contact between the rock and concrete and the rock bed is in good condition. As the dam pier is a high structure (127 feet high), the wind force is an additional concern.

The cross-section of the Pier 3 is shown in Figure 5-5.

The water levels (elevations) for analysis are the following:

Water levels:

Case	Upper Pool	Lower Pool
Normal Operating	743.5	726.9
Maintenance	743.5	726.9

The normal operating condition is the usually-prevailing condition at the structure. The maintenance condition has the same pool levels but one gate bay is dewatered for gate maintenance, which reduces the total weight of the monolith due to removal of the downward pool pressure on the pier base.

5.2.3.2 **Random variables**

The random variables involved in the overturning analysis are listed in Table 5-5. The unit weight and internal friction angle of soil, γ_{soil} and ϕ_{soil}, are used to determine the at-rest pressure due to the alluvium. The dam pier weight, which includes a 30 foot wide base, 10 foot wide pier stem and one half of two bridge spans, is modified by a factor f_w that reflects the uncertainty in dead weight. The statistical moments of wind force are based on the survey report and judgment. Note that the impact force is assumed to be a head-on impact and thus has a higher expected value than that for lock wall.

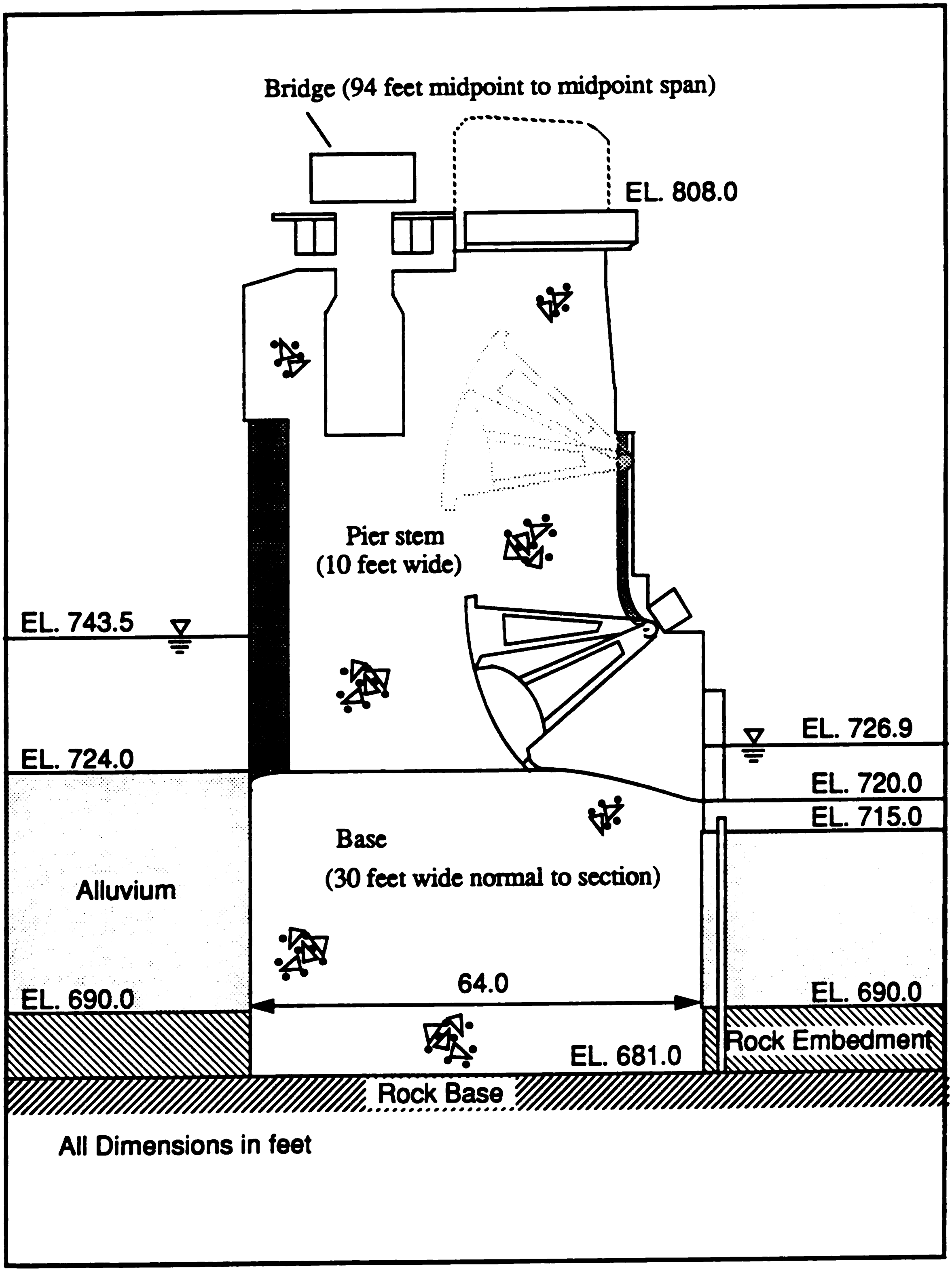


Figure 5-5 Locks and Dam No. 4, Dam Pier 3, Cross-Section

Table 5-5 Random Variables for Overturning Analysis – Locks and Dam No. 4, Dam Pier 3

Variable	μ	σ	V%
γ_{soil}	0.125 (kcf)	0.0065 (kcf)	5.0
ϕ_{soil}	32 (°)	3.2 (°)	10.0
Weight factor, f_w	1.0	0.05	5.0
F_{impact}	5.0 (kips/ft)	2.5 (kips/ft)	50.0
F_{wind}	0.03 (ksf)	0.006 (ksf)	20.0
Uplift factor, E	Varies for PC%<100 0.0 for PC% 100	0.2	—

5.2.3.3 Performance functions

The overturning performance functions with respect to two different criteria are the following.

$$M_O = M_{pool} + M_{impact} + M_{wind} + M_{uplift} + M_{D,soil}$$

$$= \frac{[H_H^3 - H_L^3]}{6}\gamma_w L + H_H L_F F_{impact} + A_{wind} F_{wind} +$$

$$+ \frac{B^2 \gamma_w}{6}[(2H_H + H_L) - 2E(H_H - H_L)]\,L + \frac{25^3}{6}\gamma'_{soil} K_0 L \quad (5\bullet17)$$

where

H_H, H_L – upper and lower pool heads, respectively;

L – length of the pier;

L_F – length of impact force;

A_{wind} – area of wind load;

$\gamma'_{soil} = \gamma_{soil} - \gamma_w$ – effective unit weight of soil; and

$K_0 = 1 - \sin(\phi_{soil})$ – coefficient of earth pressure at-rest.

$$M_R = M_W f_w + M_{R,soil}$$

$$= M_W f_w + \frac{34^3}{6} \gamma'_{soil} K_0 L \qquad (5 \cdot 18)$$

where

M_W – moment caused by total weight (concrete, etc.) of the pier.

$$N' = W f_W - U$$

$$= W f_W - \frac{(H_h + H_l) - (H_h - H_l) E}{2} \gamma_w B L \qquad (5 \cdot 19)$$

where W is the total dead weight of the pier. Note that the moment is in the unit of kips-ft and force is in the unit of kips.

The performance functions are

$$FS = M_R / M_O$$

and

$$X_R \geq C(B/3)$$

and the associated criteria are the same as that used before.

The free body diagram of Locks and Dam No. 4, Pier 3 under maintenance condition is shown in Figure 5-6.

5.2.3.4 Analysis results

The overturning analysis results of Locks and Dam No. 4, Pier 3 under normal operating and maintenance conditions are listed in Table 5-6.

The results showed that the dam pier 3 is very reliable

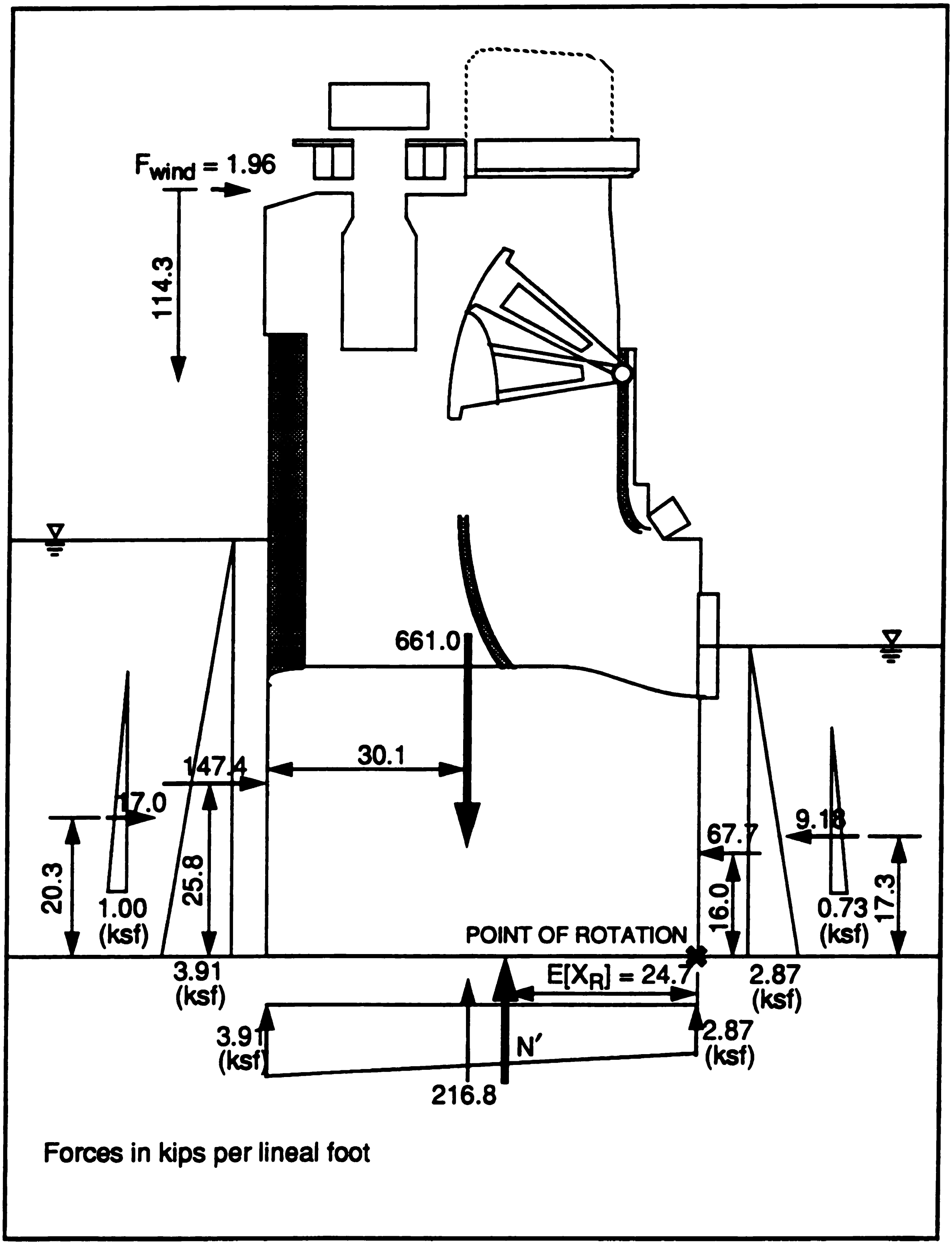


Figure 5-6 Locks and Dam No. 4, Dam Pier 3, Free Body Diagram Overturning Analysis - Maintenance Condition

Table 5-6 Overturning Reliability Analysis Results — Locks and Dam No. 4, Dam Pier 3

Mode (Pools)	Method	$E[X_R]$ ft	σ_{X_R}	β_{toe}	$\beta_{B/2}$	$\beta_{3B/4}$	β_B	E[FS]	σ_{FS}	β_{FS}
Normal (743.5	Taylor's Series	24.734	1.361	18.18	10.34	6.42	2.50	1.93	0.136	9.36
/726.9)	PEM	24.676	1.373	17.98	10.21	6.32	2.44	1.94	0.137	9.38
Maint. (743.5	Taylor's Series	27.602	0.551	50.10	30.74	21.06	11.38	2.18	0.124	13.63
/726.9)	PEM	27.560	0.556	49.60	30.40	20.81	11.21	2.18	0.124	13.64

for resisting overturning because of high reliability indices. It should be pointed out that one purpose of choosing this structure as an example was for calibration because the engineering survey and normal operation showed that this structure is in very good condition. The analysis results confirmed the real condition.

Since wind load was one of the concerns for this tall structure, its effect can be determined by examining the variation contributed from the wind load to X_R and FS. From the analysis results by Taylor's series method, the variation components from all random variables are listed in Table 5-7 and Table 5-8.

It is clear that among all random variables, the total dead weight, impact force and uplift force variables control

Table 5-7 Variation Components of X_R and FS. Locks and Dam No. 4, Dam Pier 3, Overturning, Normal Operating

Variable X_i	X_R		FS	
	Var(X_i)	Per cent	Var(X_i)	Per cent
Weight	0.48194	26.03	9.293E-03	50.21
γ_{soil}	0.00072	0.04	6.684E-06	0.04
ϕ_{soil}	0.00073	0.04	6.786E-06	0.04
F_{wind}	0.01072	0.58	5.734E-05	0.31
F_{impact}	1.28159	69.22	6.852E-03	37.01
E	0.07581	4.09	2.294E-03	12.39
Total	1.85151	100	1.851E-02	100

Table 5-8 Variation Components of X_R and FS. Locks and Dam No. 4, Dam Pier 3, Overturning, Maintenance

Variable X_i	X_R		FS	
	Var(X_i)	Per cent	Var(X_i)	Per cent
Weight	0.24128	79.49	1.176E-02	76.53
γ_{soil}	6.814E-04	0.22	1.078E-05	0.07
ϕ_{soil}	6.917E-04	0.23	1.095E-05	0.07
F_{wind}	0.01017	3.35	8.741E-05	0.57
F_{impact}	0.0	0.0	0.0	0.0
E	0.050713	16.71	3.498E-03	22.76
Total	0.30354	100	1.851E-02	100

the reliability of this structure. Although the variation of weight is very small, assumed 5 percent, it is the dominant variable in the analysis because the gravity force is much greater than other forces. But, if it were assumed that the weight is a deterministic variable, then the uplift force, impact force and the wind force would be the most important random variables which would contribute most of the uncertainty to the overturning reliability.

5.2.4 Demopolis Locks and Dam, Monolith L-17

5.2.4.1 Introduction

Demopolis Locks and Dam is located on the Tombigbee River at navigation mile 213.2, about 3.6 miles below the confluence of the Tombigbee and Black Warrior River at Demopolis, Alabama. The structure was completed in 1955. It was found during a periodic inspection that the saturation level in the backfill behind the land side lock wall was higher than that assumed in the design, and a similar structure already had experienced wall cracks. Based on the deterministic analysis results, remedial action was undertaken to improve rotational stability and 20 feet of backfill was removed as proposed by Mobile District, Corps of Engineers[114].

L-17 is one of the monoliths in the landside wall. It was founded seven (7) feet below the top of a chalk layer. The backfill consists of a medium to high plasticity clay down to elevation 47 and then a silty, clayey sand down to elevation 13. The original backfill extended to the top of the lock wall. The remedial action included removing 20 feet of backfill and providing a drainage system.

The cross-section of monolith L-17 is shown in Figure 5-7 where the dashed line represents the outline of 20 feet backfill removed.

The pool levels used in analysis are the following.

Case	Water in Chamber (Elevation)	Water in Backfill (Elevation)
Before removal of backfill:		
Normal Operating	33.0	68.0
Maintenance	13.0	68.0
High Water	83.0	84.0
After removal of backfill:		
Normal Operating	33.0	61.0
Maintenance	13.0	61.0
High Water	83.0	84.0

5.2.4.2 Random variables

The statistical properties of the random variables used in an overturning analysis are listed in Table 5-9. All parameters are based on the test data or field measurements except the wall friction angle δ, which is determined by judgment. Four different assumptions were made for the expected value of δ to assess the importance of this variable. The expected saturation elevation of backfill, H_{sat}, are based on the field observation when the backfill was at the top of the lock wall; in analysis, the water levels in

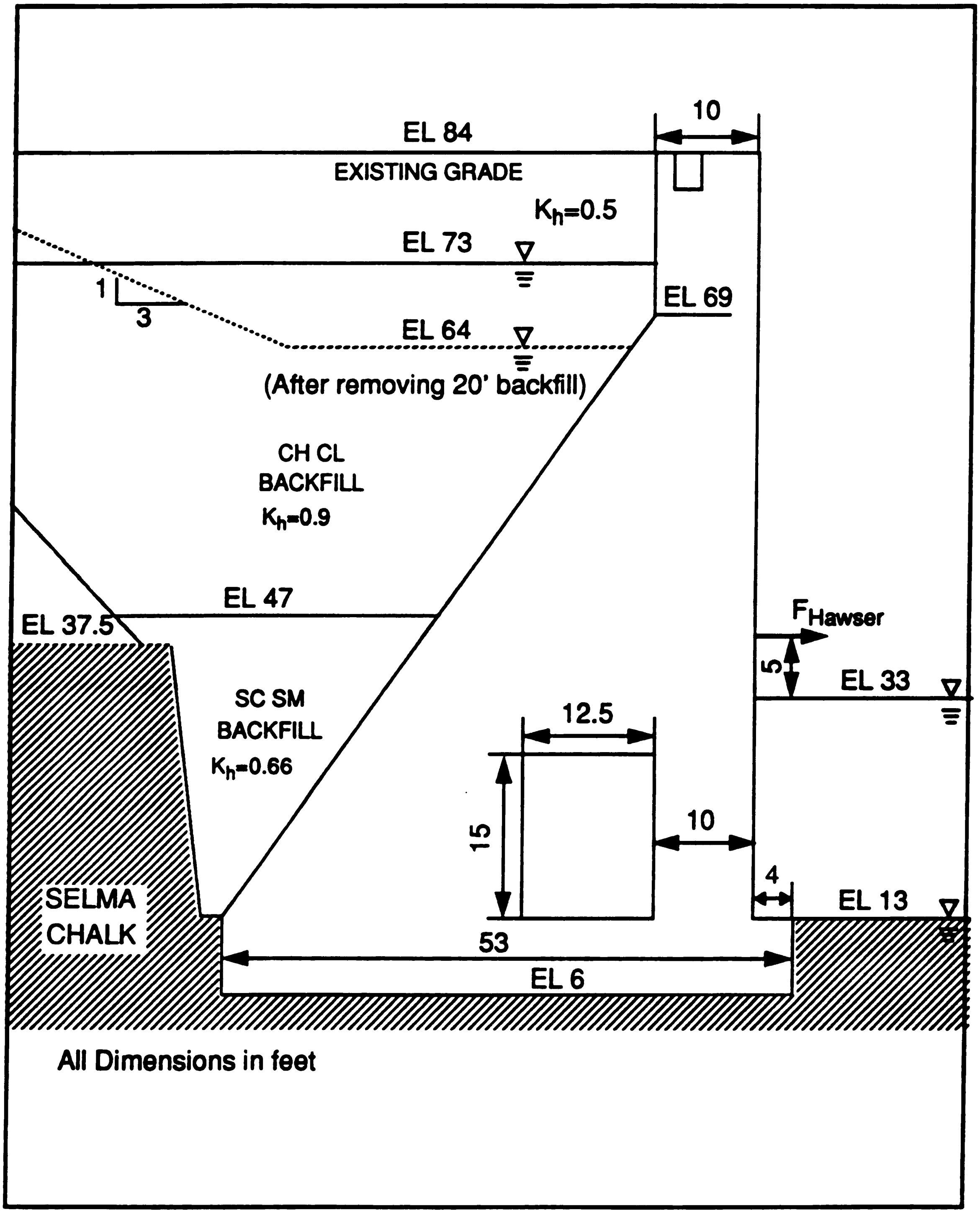


Figure 5-7 Demopolis Locks and Dam, Monolith L-17, Cross-Section

Table 5-9 Random Variables for Overturning Analysis – Demopolis Locks and Dam, Monolith L-17

Variable	μ	σ	V%
$\gamma_{concrete}$	0.15 (kcf)	0.0075 (kcf)	5.0
γ_{soil}	0.125 (kcf)	0.00625 (kcf)	5.0
ϕ_{base}	30 (°)	11.46 (°)	38.2
α_K ($K_h = \alpha_K K'_h$)	1.0	0.1	10.0
F_{impact}	1.0 (kips/ft)	0.5 (kips/ft)	50.0
H_{sat} (Original backfill)	68 (ft)	1.7(ft).	—
H_{sat} (Remove 20' backfill)	61 (ft)	1.5(ft).	—
Wall friction, δ	12.0 (°)	3.0 (°)	25.0
Uplift factor, E	Varies for PC%<100 0.0 for PC% 100	0.2	—

the backfill are the same as the saturation levels and have the same variation. The coefficients of lateral earth pressure are based on the report by Mobile District of Corps of Engineers[114]. To simplify the analysis, it is assumed that the different backfill soil layers, with different coefficients of earth pressure, K'_h, are fully correlated; or in other words, they are varying simultaneously and in the exactly same manner. The variation of these coefficients is assumed to be 10% of their expected values and a factor, α_K, with mean 1.0 will be used as a multiplier to the "nominal" earth pressure coefficient K'_h to obtain K_h.

5.2.4.3 Performance functions

The performance functions with respect to two different overturning criteria are the following.

$$M_O = M_{pool} + M_{impact} + M_{uplift} + M_{D,soil}$$

$$= \frac{[H_H^3 - H_L^3]}{6}\gamma_w + (H_H+5)F_{impact} + M_{uplift} + M_{D,soil} \qquad (5\bullet 20)$$

where

$$M_{uplift} = \frac{B^2\gamma_w}{6}[(2H_H + H_L) - 2E(H_H - H_L)] \quad if\ E \quad 0$$

$$= \frac{B^2\gamma_w}{6}[3H_L + (H_H - H_L)(-E^2 + 2|E| + 2)] \quad if\ E<0$$

$M_{D,soil}$ – driving moment produced by backfill soil;

$$M_{D,soil} = M_H^1 + M_H^2 + M_H^3$$

where

$$M_H^1 = D^1{}_{h,soil}L_{h1} + D^2{}_{h,soil}L_{h2} + D^3{}_{h,soil}L_{h3}$$
$$D^1{}_{h,soil} = (Ele_{fill}-H_{sat})^2/2\gamma_{soil}(0.5\alpha_K)$$
$$L_{h1} = (Ele_{fill}-H_{sat})/3 + (H_{sat}-6)$$

$$M_H^2 = D^{2,1}{}_{h,soil}L_{h2,1} + D^{2,2}{}_{h,soil}L_{h2,2}$$
$$D^{2,1}{}_{h,soil} = (Ele_{fill}-H_{sat})(H_{sat}-47)\gamma_{soil}(0.9\alpha_K)$$
$$D^{2,2}{}_{h,soil} = (H_{sat}-47)^2(\gamma_{soil}-\gamma_w)/2(0.9\alpha_K)$$
$$L_{h2,1} = (H_{sat}-47)/2+(47-6)$$
$$L_{h2,2} = (H_{sat}-47)/3+(47-6)$$

$$M_H^3 = D^{3,1}{}_{h,soil}L_{h3,1} + D^{3,2}{}_{h,soil}L_{h3,2}$$

$$D^{3,1}{}_{h,soil} = [(Ele_{fill}-H_{sat})\gamma_{soil} + (H_{sat}-47)(\gamma_{soil}-\gamma_w)(47-6)](0.9\alpha_K)$$
$$D^{3,2}{}_{h,soil} = (47-6)^2(\gamma_{soil}-\gamma_w)/2(0.9\alpha_K)$$

$$L_{h3,1} = (47-6)/2$$

$$L_{h3,2} = (47-6)/3$$

where

Ele_{fill} — backfill elevation.

$$M_R = V_{con}L_{con} + M_{water} + V_{soil}L_{soil} + M_{R,soil} + M_{wall}$$

where

$V_{con} = A_{con}\gamma_{concrete} = (1985.5)\gamma_{concrete}$ — weight of concrete per foot, kips/ft;

$L_{con} = 21.107$ — moment arm of V_{con}, ft;

M_{water} — moment by the water in culvert kips-ft/ft;

$V_{soil} = A_{soil}\gamma_{soil}$ — weight of soil per foot on the monolith, changes when backfill level changes, kips/ft;

L_{soil} — moment arm of V_{soil}, ft;

$M_{R,soil} = (13-6)^3/6\ (\gamma_{soil}-\gamma_w)(1-\sin\phi_{base})$ — resisting moment by overburden soil, kips-ft/ft;

$M_{wall} = D_{h,soil}\ (\tan\delta)B$ — resisting moment produced by wall friction, kips-ft/ft; where

$$D_{h,soil} = D^1{}_{h,soil} + (D^{2,1}{}_{h,soil} + D^{2,2}{}_{h,soil}) + (D^{3,1}{}_{h,soil} + D^{3,2}{}_{h,soil})$$

$$N' = V_{con} + V_{water} + V_{soil} + V_{wall} - U$$

$$= V_{con} + V_{water} + V_{soil} + D_{h,soil}(\tan\delta) - U$$

where

V_{water} — weight of water in culvert per foot, kips/ft;

$V_{wall} = D_{h,soil}(\tan\delta)$ — vertical component of wall friction force; and

U — uplift force as defined before.

The reliability criteria are the same as that used in previous examples.

A free body diagram under maintenance condition with original backfill level is shown in Figure 5-8.

5.2.4.4 Analysis results

The overturning analysis results, under different operating condition, with and without 20 feet backfill removal are listed in Table 5-10.

The analysis results show that, before backfill removal, monolith L-17 had a low reliability regarding overturning stability even under the normal operating condition. Although it is quite certain that the resultant location is within the monolith base, this certainty will reduce if the wall friction angle is smaller than the assumed value. After 20 feet of backfill was removed, then the monolith became much more reliable, the reliability index increased to about 6 for the FS criterion and it is almost certain that about 75% of the base will be in compression.

5.2.4.5 Effects of backfill level and wall friction

In an analysis, the wall friction angle, δ, must be assumed by engineering judgment if no field data are available. For this study, four assumptions were considered. Also, the analysis results showed that part of the backfill did, in fact, need to be removed in order to increase the overturning reliability. As to how the wall friction and the backfill removal will affect the reliability of monolith L-17, the further analysis results, listed in Table 5-11 and plotted in Figure 5-9 to Figure 5-11 give some insight.

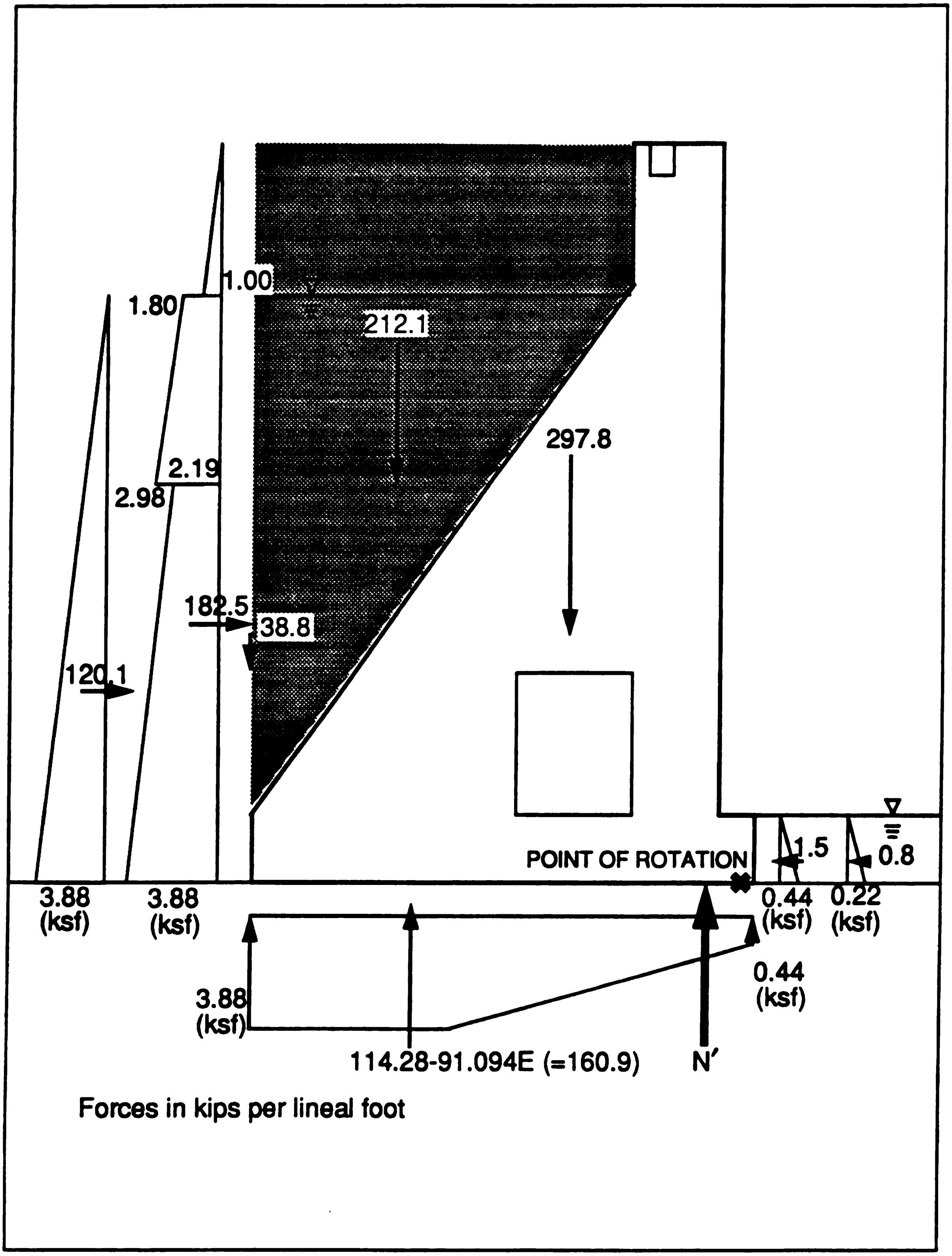


Figure 5-8 Demopolis Locks and Dam, Monolith L-17, Free Body Diagram - Overturning Analysis, Maintenance Condition, No Backfill Removed

Table 5-10 Overturning Reliability Analysis Results — Demopolis Locks and Dam, Monolith L-17

Backfill	Model	$E[X_R]$ (ft)	σ_{X_R}	β_{toe}	$\beta_{B/2}$	$\beta_{3B/4}$	β_B	E[FS]	σ_{FS}	β_{FS}
Original backfill	Normal	9.493	1.715	5.54	0.38	-2.19	-4.77	1.28	0.071	4.46
	Maint.	8.626	1.756	4.91	-0.12	-2.63	-5.15	1.26	0.074	3.88
20' removed	Normal	15.643	0.953	16.41	7.15	2.51	-2.12	1.63	0.094	8.42
	Maint.	14.734	1.107	13.31	5.33	1.34	-2.65	1.63	0.128	6.26

From the results, it is clear that lowering backfill level increases the overturing stability of the monolith L-17 and it seems that if more than 15 feet backfill is removed, the β_{FS} will be greater than 4.0 and the compression area will be 50 to 75 percent of the base. The wall friction angle also plays an important role in the reliability. The reliability indices increase when assumed wall friction angle increases (a negative reliability index reflects the certainty of failing to satisfy the criterion), the more backfill present, the more significant this effect is. The wall friction force also greatly affects the active base area: the values of percentage of base which is in compression will increase about 0.2 to 2 times when wall friction angle increases from 0 to 12 degree.

Table 5-11 Overturning Reliability Analysis Results — Demopolis Locks and Dam, Monolith L-17, Maintenance Condition

Backfill Level	Wall Friction (°)	$E[X_R]$ (ft)	σ_{X_R}	β_{toe}	$\beta_{B/2}$	$\beta_{3B/4}$	β_B	E[FS]	σ_{FS}	β_{FS}
No backfill removal	0.0	2.927	2.077	1.41	-2.84	-4.97	-7.10	1.07	0.055	1.29
	6.0	5.917	2.086	2.84	-1.40	-3.51	-5.63	1.16	0.069	2.46
	12.0	8.626	1.756	4.91	-0.12	-2.63	-5.15	1.26	0.074	3.88
	18.0	11.145	1.532	7.28	1.51	-1.37	-4.26	1.37	0.082	5.19
10' backfill removed	0.0	5.120	1.510	3.39	-2.46	-5.38	-8.31	1.12	0.048	2.73
	6.0	7.291	1.560	4.67	-0.99	-3.82	-6.65	1.19	0.060	3.51
	12.0	9.317	1.388	6.71	0.35	-2.83	-6.01	1.27	0.068	4.48
	18.0	11.251	1.282	8.78	1.89	-1.56	-5.00	1.36	0.079	5.26
15' backfill removed	0.0	8.798	1.256	7.01	-0.03	-3.54	-7.06	1.26	0.066	4.49
	6.0	10.574	1.322	8.00	1.32	-2.02	-5.36	1.35	0.080	4.94
	12.0	12.247	1.222	10.03	2.79	-0.82	-4.44	1.43	0.092	5.57
	18.0	13.862	1.163	11.92	4.33	0.53	-3.27	1.53	0.107	6.07
20' backfill removed	0.0	11.925	1.098	10.86	2.82	-1.21	-5.23	1.44	0.094	5.62
	6.0	13.364	1.163	11.49	3.90	0.10	-3.70	1.54	0.112	5.87
	12.0	14.734	1.107	13.31	5.33	1.34	-2.65	1.63	0.128	6.26
	18.0	16.068	1.077	14.91	6.71	2.62	-1.48	1.74	0.147	6.56
30' backfill removed	0.0	16.506	0.901	18.32	8.52	3.61	-1.29	1.96	0.192	6.83
	6.0	17.411	0.947	18.39	9.06	4.40	-0.27	2.07	0.216	6.90
	12.0	18.158	0.905	20.06	10.30	5.42	0.54	2.15	0.228	7.16
	18.0	18.853	0.864	21.82	11.60	6.48	1.37	2.21	0.235	7.46

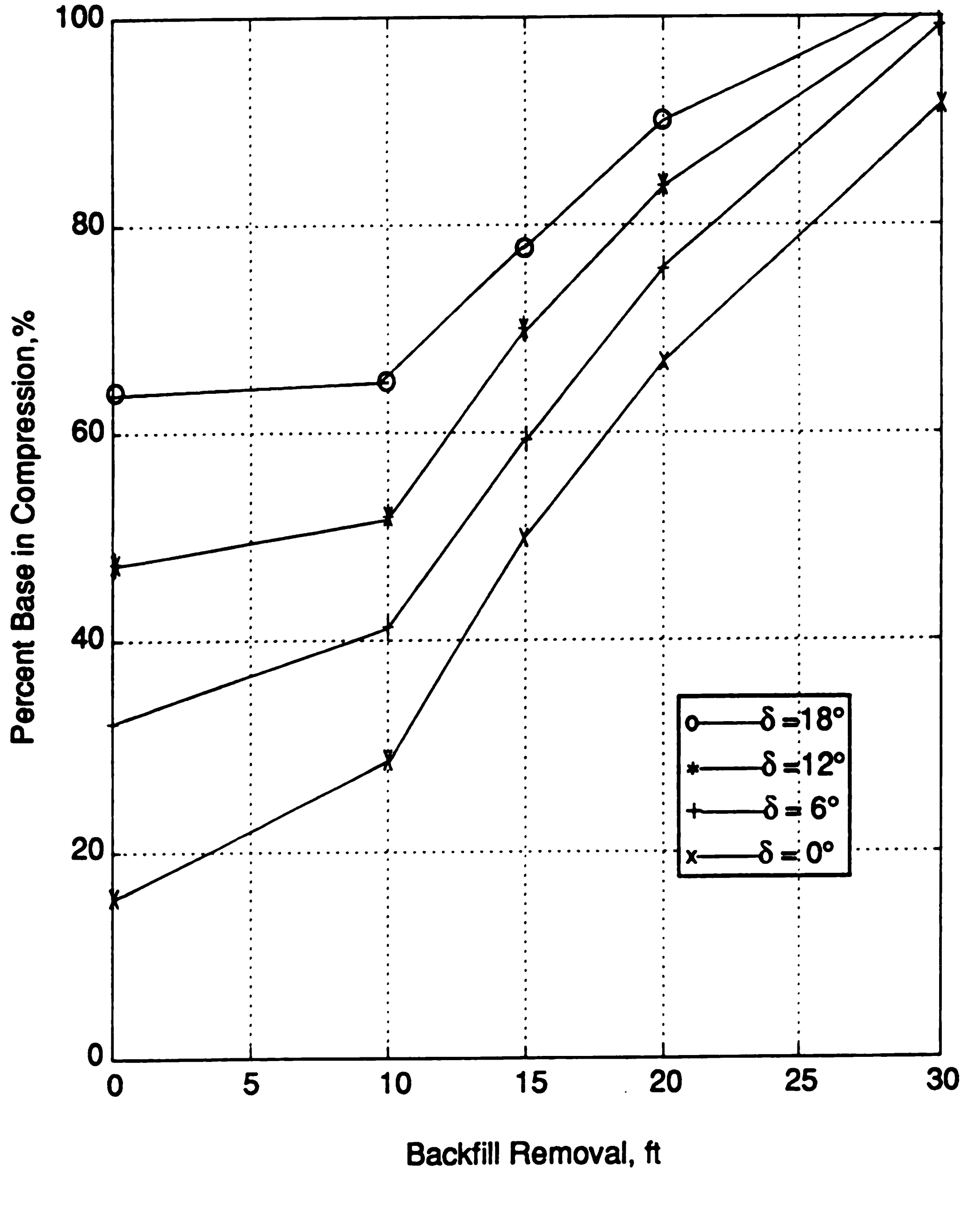


Figure 5-9 Demopolis Locks and Dam, Monolith L-17, Overturning Analysis, Percent Base in Compression versus Backfill Removal

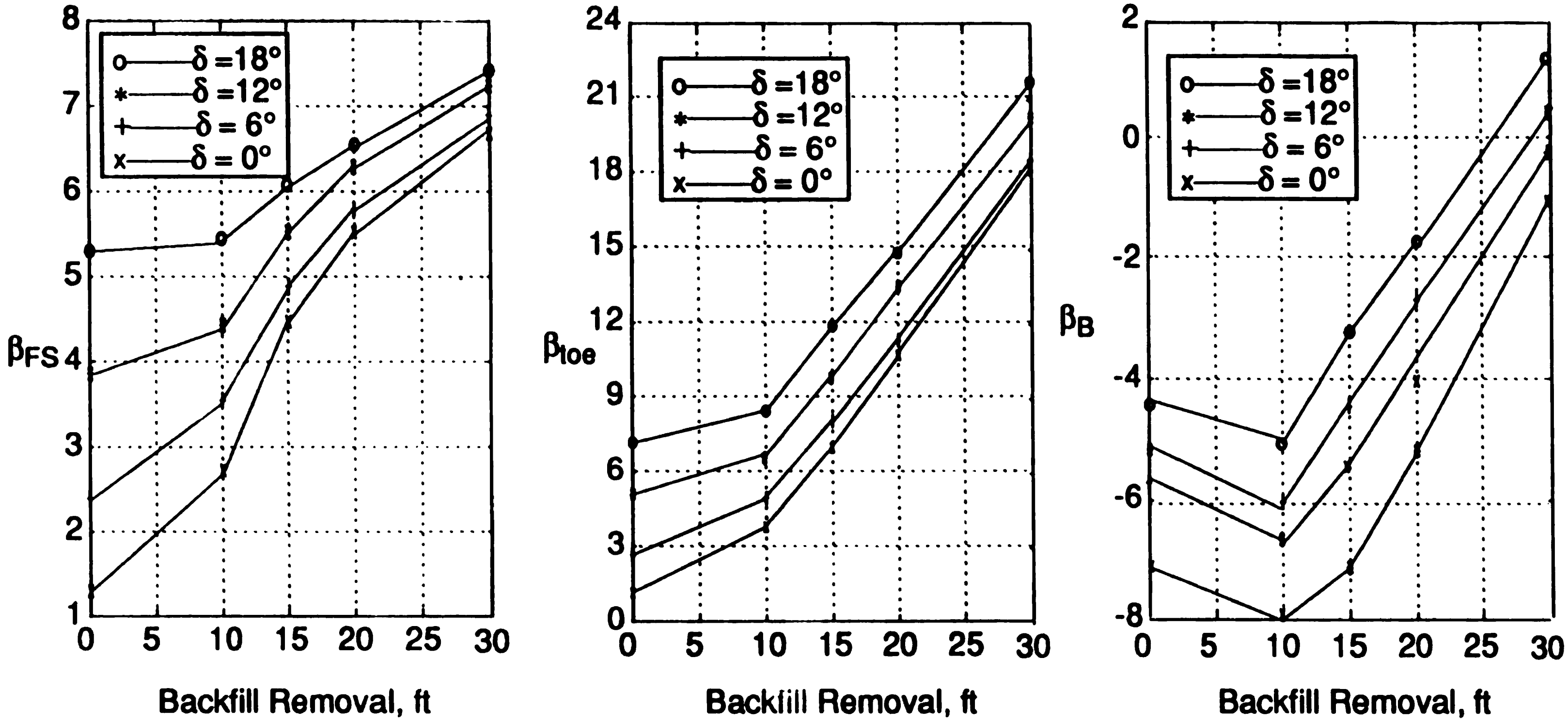


Figure 5-10 Demopolis Locks and Dam, Monolith L-17, Overturning Stability β_{FS}, β_{toe} and β_B versus Backfill Removal

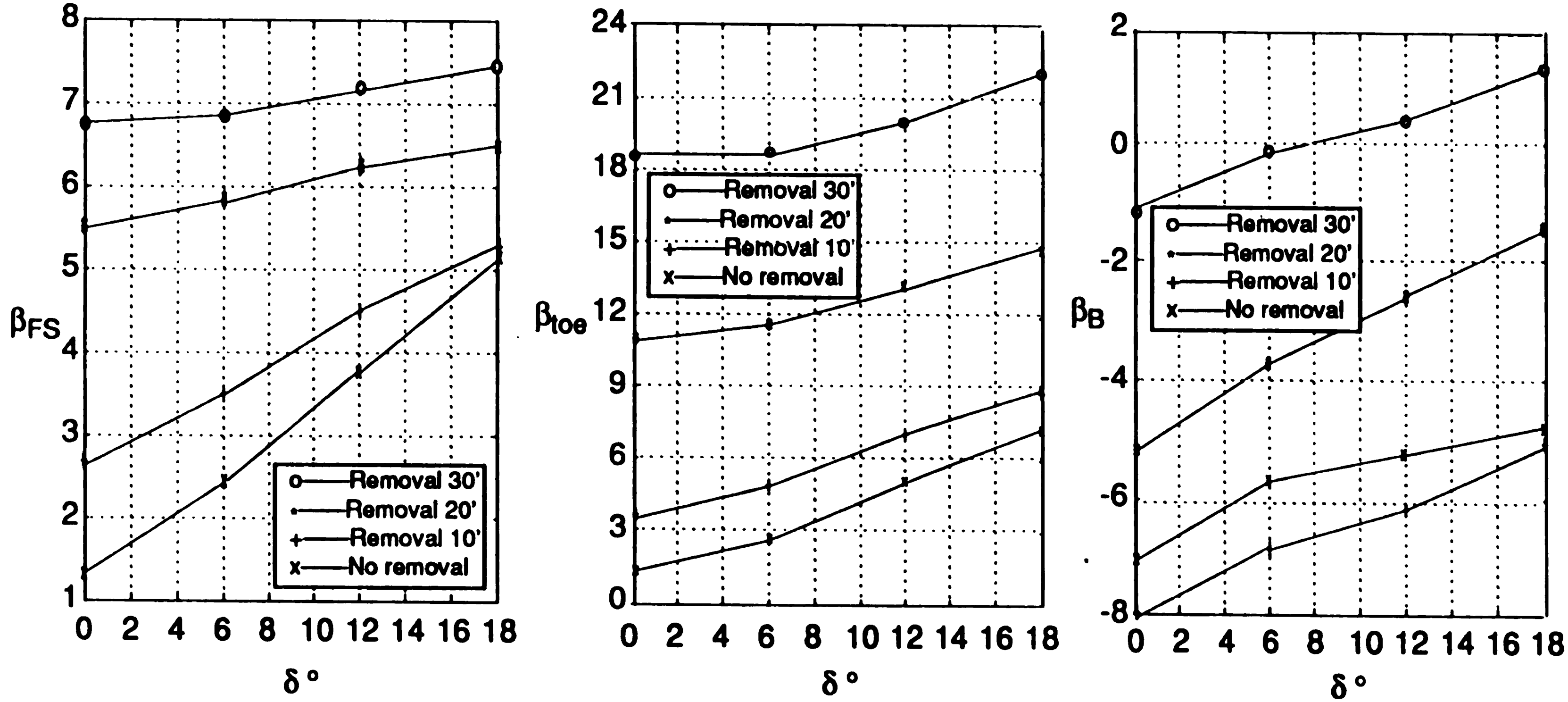


Figure 5-11 Demopolis Locks and Dam, Monolith L-17, Overturning β_{FS}, β_{toe} and β_B versus Wall Friction Angle

5.3 Sliding Stability

The reliability analysis procedure with respect to sliding stability has been discussed in section 4.6.2. The criterion used for sliding reliability analysis will be the factor of safety, FS, greater than or equal to 1.0, i.e. $FS \geq 1.0$, defined by

$$FS = R/D = (Total\ resisting\ force) / (Total\ driving\ force)$$

and the lognormal definition of β will be used.

In engineering practice, the definition of the sliding driving and resisting forces varies widely. In this study, the total resisting force consists of shear resisting force underneath the base, T, and overburden soil resisting force, R_s, if it is available. The total driving force is taken as the sum of all other horizontal forces acting on the structure. Since the limit state concept is commonly used in engineering practice, the "active-passive" earth pressure state will be assumed. This definition is a "simple" one.

It is well known that the shear resisting force is related to the base area of structure which is in compression and the shear strength properties of the base material. The Mohr-Coulomb's theory, therefore, is used to determine the base sliding shear resistance, T, which can be expressed by

$$T = Lc'_{base} + N' \tan\phi'_{base}$$

$$= 3X_R c'_{base} + N' \tan\phi'_{base} \qquad (5 \cdot 21)$$

where

$L=3X_R$ – length of sliding surface of base in compression and X_R is the location of effective base resultant force measured from the toe. $L \leq B$;

N' – effective normal force acting on the base; and

c'_{base} and ϕ'_{base} – cohesion and effective internal friction angle of base material, respectively.

The location of resultant X_R can be determined from overturning analysis but the shear strength properties of base material need to be carefully estimated.

5.3.1 Shear Strength of Base Material

The methods discussed in section 4.5.3 can be used to determine the shear strength properties of base material.

Based on the laboratory direct shear test results, the shear strength properties of foundation rock from Locks and Dam No. 2, No.3 and No.4 were determined and are listed in Table 5-12 to Table 5-15. In these tables, the "LR" represents Linear Regression method and "PP" represents Paired Points method (details see section 4.5.3).

The results show quite a large variation of shear strength parameters among different rock types. By the combination of analysis results and engineering judgment, the shear strength parameters recommended for Locks and Dam No. 2, No.3 and No.4 are listed in Table 5-16. Among those parameters, the first two statistical moments for ϕ were converted from that for $\tan\phi$ when applying the linear regression method (see Appendix B for details). It is noted that the linear regression method and paired points method gave similar results in most of the case, but the linear regression method always gave a high coefficient of correlation – about -0.9. The difference of the expected value and variance are

Table 5-12 Shear Strengths of Base Material, Locks and Dam No. 2

Material	Strength	Method	E[c] (ksf)	σ_c	E[tanϕ]	σ_{tanf}	$\rho_{c,tanf}$	E[ϕ] (°)	σ_ϕ	$\rho_{c,\phi}$
Soft-Moderate Hard Shale	Peak	LR	10.88	7.42	2.23	0.61	-0.89	51.8		
	Peak	PP	10.92	6.37	1.228	0.684	-0.642	47.16	14.584	-0.822
	Residual	c=0			0.878	0.453		38.57	16.150	
Moderate Hard Shale	Peak	LR	14.66	16.52	2.67	1.32	-0.91	69.45		
	Peak	PP	18.44	13.15	2.68	0.999	-0.719	67.16	13.704	-0.651
	Residual	c=0			0.798	0.420		36.21	13.888	
All Data	Peak	LR	12.52	11.08	2.08	0.81	-0.90	64.3		
	Peak	PP	15.02	10.90	2.02	1.123	-0.238	58.07	15.382	-0.177
	Residual	c=0			0.838	0.421		37.39	13.918	

Table 5-13 Shear Strengths of Base Material, Locks and Dam No. 3

Material	Strength	Method	E[c] (ksf)	σ_c	E[tanϕ]	σ_{tanf}	$\rho_{c,tanf}$	E[ϕ] (°)	σ_ϕ	$\rho_{c,\phi}$
Shale	Peak	LR	20.68	9.10	0.800	0.887	-0.92	38.67		
	Peak	PP	10.24	7.53	1.602	1.208	-0.322	49.01	21.912	-0.315
	Residual	c=0			0.538	0.160		27.85	6.920	

Table 5-14 Shear Strengths of Base Material, Locks and Dam No. 4

Material	Strength	Method	E[c] (ksf)	σ_c	E[tanϕ]	σ_{tanf}	$\rho_{c,tanf}$	E[ϕ] (°)	σ_ϕ	$\rho_{c,\phi}$
Moderate Hard Grey Shale	Peak	LR	16.89	27.69	1.708	1.813	-0.943	59.66		
	Peak	PP	12.16	3.95	1.347	0.263	-0.961	52.96	5.455	-0.978
	Residual	c=0			0.492	0.083		26.10	3.927	
Moderate Hard Grey Shale	Peak	LR	9.59	8.94	1.554	0.639	-0.915	57.24		
	Peak	PP	11.76	11.05	1.588	0.730	-0.921	53.66	17.051	-0.97
	Residual	c=0			0.510	0.077		26.91	3.625	
Hard Shale	Peak	LR	16.07	42.52	2.625	2.733	-0.926	69.15		
	Peak	PP	19.23	19...77	2.625	1.319	-0.961	65.82	11963	-0.999
	Residual	c=0			0.577	0.244		29.27	10.917	
All Moderate Hard Shales	Peak	LR	10.58	9.48	1.700	0.659	-0.923	59.54		
	Peak	PP	11.25	9.03	1.571	0.639	-0.90	54.33	14.101	-0.969
	Residual	c=0			0.504	0.076		26.66	3.573	

Table 5-15 Shear Strengths of Base Material, Locks and Dam No. 2 and No. 4

Material	Strength	Method	E[c] (ksf)	σ_c	E[tanϕ]	σ_{tanf}	$\rho_{c,tanf}$	E[ϕ] (°)	σ_ϕ	$\rho_{c,\phi}$
All Data	Peak	LR	20.24	8.41	1.201	0.616	-0.914	50.22		
	Peak	PP	15.58	13.16	1.44	0.889	-0.599	48.92	18.771	-0.762
	Residual	c=0			0.667	0.336		31.93	11.31	
All Data w/o Outlier	Peak	LR	19.84	7.14	1.090	0.524	-0.910	47.46		
	Peak	PP	14.76	11.13	1.366	0.762	-0.767	48.39	17.992	-0.833
	Residual	c=0			0.667	0.336		31.93	11.32	

Table 5-16 Shear Strengths of Base Material Recommended in Reliability Analysis

Strength	Locks and Dams	E[c] (ksf)	σ_c	E[tanϕ]	σ_{tanf}	$\rho_{c,tanf}$	E[ϕ] (°)	σ_ϕ	$\rho_{c,\phi}$
Peak	All	11.0	7.70	1.50	0.675	-0.70	52.4	12.898	-0.70
Residual	No. 2			0.80	0.40		35.93	13.970	
	No. 3			0.54	0.16		27.85	6.92	
	No. 4			0.50	0.25		21.98	11.459	
	All			0.64	0.32		30.50	13.0	

partially because the paired points method eliminated the calculated data sets which have no physical meaning. The high negative correlation given by linear regression method infers that the error induced, by this method, in predicting the shear strength at a given normal stress is much smaller than the error in the appropriate c or ϕ value (reference Wolff and Wang [125] for more discussion).

Figure 5-12 illustrates the direct shear test data of Locks and Dam No. 3, linear regression results and the shear strength parameters used in analysis. Note that the "possible" curve segments were based on the rock mechanics concept that the shear failure envelope can be approximated by a bilinear model which reflects the undulation or waviness along the rock joint surface[29,110].

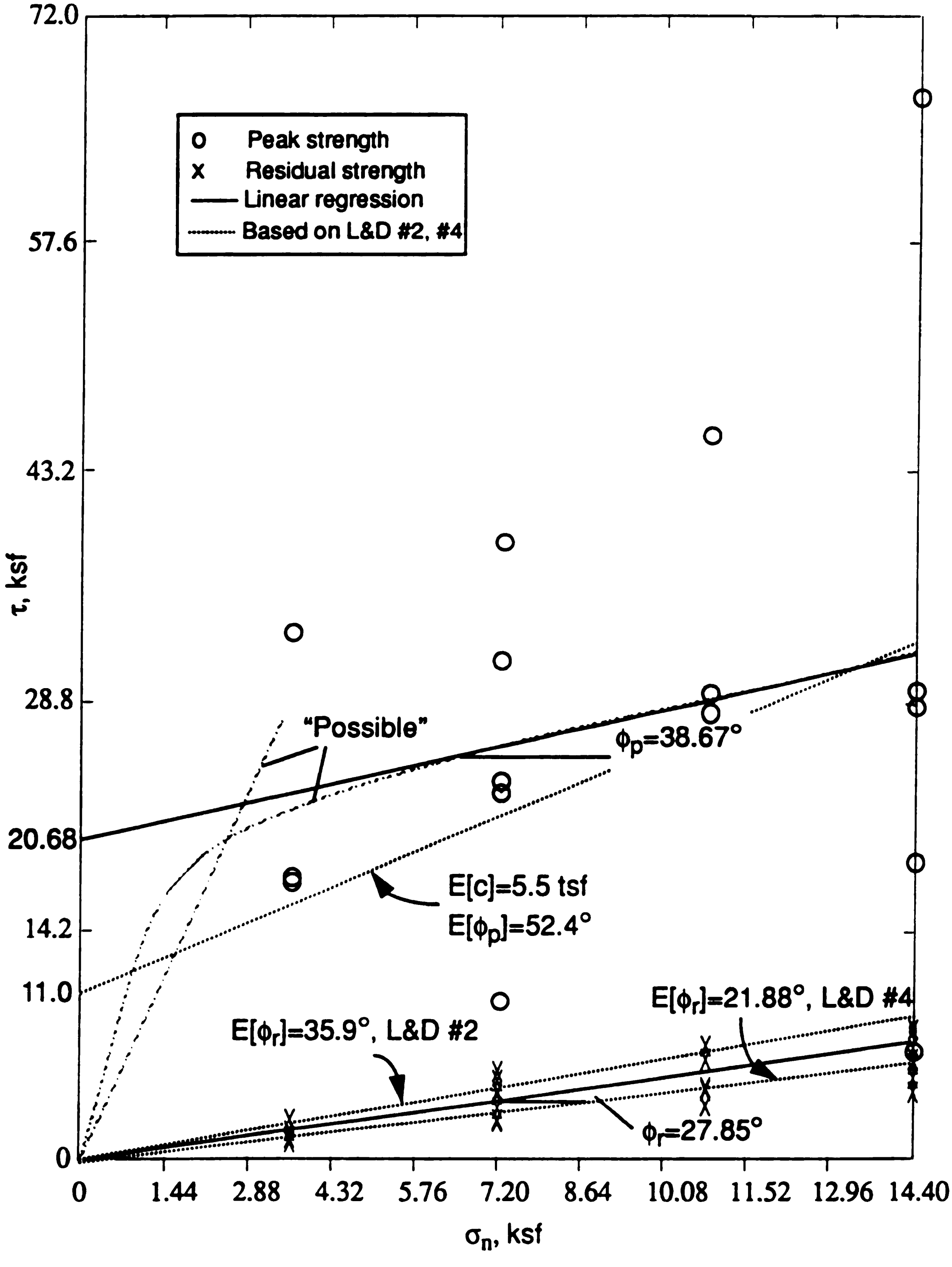


Figure 5-12 Shear Strength of Base Material, Locks and Dam No. 3, Based on Direct Shear Test Results

5.3.2 Locks and Dam No.2 Monolith M-16, Monongahela River

The structural and loading conditions of Locks and Dam M-16 have been described in section 5.2.1 and the distance of effective resultant base force from the toe, obtained from overturning analysis results of M-16, will be used to calculate the length of sliding surface.

5.3.2.1 Random variables

All random variables involved in overturning analysis will be used in sliding analysis, plus the shear strength parameters of base material. The properties of those random variables are listed in Table 5-17.

5.3.2.2 Performance functions

The performance functions related to R and D. R is composed by R_{soil} and T, where

$$T = Lc_{base} + N'\tan\phi'_{base} = 3X_R c_{base} + N'\tan\phi'_{base} \qquad (5\cdot22)$$

Note that $L = 3X_R \leq B$ (B, length of base) is the length of sliding surface of the base. Therefore

$$R = T + R_{soil} = c_{base}L + N'\tan\phi + R_{soil} \quad \text{(kips/ft)} \qquad (5\cdot23)$$

where

R_{soil} is the resistance caused by overburden soil which can be expressed as

$$R_{soil} = \frac{(19.5)^2}{2}\gamma'_{soil}K_p = 190.125\gamma'_{soil}K_p \text{ (kips/ft)}$$

where K_p is coefficient of passive earth pressure and defined as

$$K_p = \tan^2(45^\circ + \phi_{soil}/2)$$

The D, with upper pool water head H_H and lower pool water

Table 5-17 Random Variables for Sliding Analysis – Locks and Dam No. 2, Monolith M-16

Variable	μ	σ	V (%)
c_{base} (Peak)	11.0 (ksf)	7.70 (ksf)	70.0
$\tan\phi_{base}$ (Peak)	1.50	0.675	45.0
			$\rho_{c,\tan\phi}$ = -0.70
c_{base} (Residual)	0.0	0.0	—
$\tan\phi_{base}$ (Residual)	0.80	0.40	50.0
γ_{soil}	0.0755 (kcf)	0.003775 (kcf)	5.0
ϕ'_{soil}	33 °	3.3 °	10.0
$\gamma_{concrete}$	0.15 (kcf)	0.0075 (kcf)	5.0
F_{impact}	1.0 (kips/ft)	0.5 (kips/ft)	50.0
Uplift parameter, E	Varies for PC%<100, 0.0 for PC% 100	0.2	—

head H_L, can be expressed by

$$D = D_{soil} + D_{pool}$$

$$= \frac{(19.5)^2}{2}\gamma'_{soil}K_a + D_{pool}$$

$$= 190.125\gamma'_{soil}K_a + \frac{(H_H^2 - H_L^2)}{2}\gamma_w \qquad \text{(kips/ft)} \quad (5\bullet24)$$

where

$K_a = \tan^2(45° - \phi_{soil}/2)$ is coefficient of active earth pressure.

N' is the sum of normal forces defined by

$$N' = V_{concrete} + V_w - U$$

$$= A_{con}\gamma_{concrete} + A_w\gamma_w - \frac{(H_h + H_l) - (H_h - H_l)E}{2}\gamma_w B \quad \text{(kips/ft)}$$

where

A_{con} is the concrete section area of monolith;

A_w is the area of water acting on the monolith;

U is hydraulic uplift force.

Other variables were defined before. Finally

$FS = R/D$

The free body diagram for maintenance condition is shown in Figure 5-13.

5.3.2.3 Analysis results

Sliding analysis results of Locks and Dam No. 2 Monolith M-16 are listed in Table 5-18.

The analysis results show that the monolith M-16 is very safe with respect to sliding failure. If the foundation rock is solid (peak shear strength condition), then the reliability indices will be greater than 6 under any operating circumstances; if the connection of base and foundation is very weak and previous sliding or material weathering of the foundation exists (residual shear strength condition), the reliability index is still greater than 2 even under the worst operating condition (maintenance condition B).

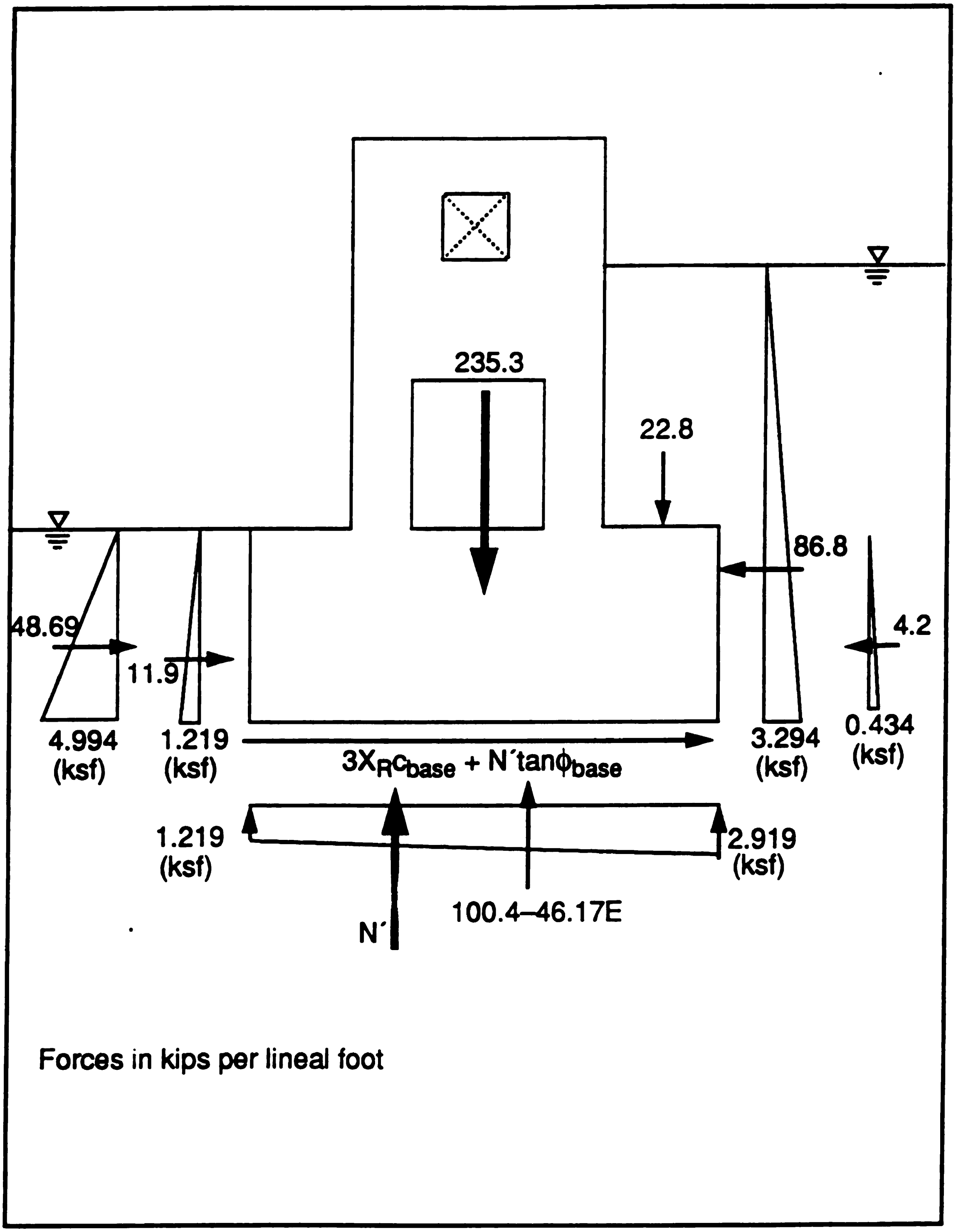


Figure 5-13 Locks and Dam No. 2, Monolith M-16, Free Body Diagram – Sliding Analysis, Maintenance Condition

Table 5-18 Sliding Analysis Results — Locks and Dam No. 2 Monolith M-16

Strength	Pool	Method	E[FS]	σ_{FS}	β_{FS}
Peak	Normal (A)	Taylor's	28.96	10.165	9.70
		PEM	28.97	10.170	9.70
	Normal (B)	Taylor's	22.47	8.223	8.60
		PEM	22.48	8.245	8.59
	Maintenance (A)	Taylor's	13.47	4.488	7.85
		PEM	13.48	4.488	7.86
	Maintenance (B)	Taylor's	10.01	3.485	6.64
		PEM	10.01	3.490	6.64
	High Water	Taylor's	33.13	12.419	9.47
		PEM	33.15	12.423	9.47
Residual	Normal (A)	Taylor's	6.70	2.513	5.06
		PEM	6.73	2.519	5.08
	Normal (B)	Taylor's	5.01	1.820	4.40
		PEM	5.03	1.844	4.37
	Maintenance (A)	Taylor's	3.25	1.240	3.01
		PEM	3.26	1.242	3.02
	Maintenance (B)	Taylor's	2.33	0.869	2.15
		PEM	2.33	0.875	2.15
	High Water	Taylor's	7.25	2.639	5.44
		PEM	7.28	2.647	5.46

5.3.3 Locks and Dam No.3 Monolith M-20, Monongahela River

Monolith M-20 of Locks and Dam No. 3 was described for its overturning analysis. In sliding analysis, some analysis results, such as the resultant location, were directly borrowed and the shear strength properties of foundation rock were taken the same as that mentioned in section 5.3.1.

5.3.3.1 Random variables

The random variabels used in sliding analysis of Locks and Dam No.3 Monolith M-20 are listed in Table 5-19. Pool levels are the same as that used in overturning analysis.

5.3.3.2 Performance function

The performance function, for criterion F=R/D, is the following:

$$R = 3X_R c_{base} + N' \tan(\phi_{base})$$

where

N' is the effective normal resultant force as defined in overturning analysis (see section 5.2.2), which is

$$N' = W_{concrete} + W_w - U + T_{river} + T_{land}$$

$$= A_{con}\gamma_{concrete} + W_w - \frac{(H_h + H_l) - (H_h - H_l)E}{2}\gamma_w B + T_{river} + T_{land}$$

$$D = D_{pool} + F_{impact}$$

$$= \frac{H_H^2 - H_L^2}{2}\gamma_w + F_{impact}$$

and $FS = R/D$. The unit of R and D is kips/ft.

The free body diagram of sliding analysis of monolith M-20, under maintenance condition without anchors, is illustrated in Figure 5-14.

Table 5-19 Random Variables for Sliding Analysis - Locks and Dam No. 3, Monolith M-20

Variable	μ	σ	V (%)
c_{base} (Peak)	11.0 (ksf)	7.70 (ksf)	70.0
ϕ_{base} (Peak)	52.4 (°)	12.9 (°)	24.6
			$\rho_{c,\tan\phi} = -0.70$
c_{base} (Residual)	0.0	0.0	0.0
ϕ_{base} (Residual)	30.5 (°)	13.0 (°)	42.6
$\gamma_{concrete}$	0.15 (kcf)	0.0075 (kcf)	5.0
ELE_{base}	701.7 (ft)	0.3 (ft)	—
F_{anchor}	112 (kips/anchor)	2.24 (kips/anchor)	2.0
F_{impact}	0.80 (kips/ft)	0.40 (kips/ft)	50.0
Uplift parameter, E	Varies, if PC%<100) 0.0, if PC% 100)	0.2	—

5.3.3.3 Analysis results

The sliding analysis results of Locks and Dam No. 2, monolith M-20 are listed in Table 5-20.

The results show that the anchors are needed for monolith M-20 if dewatered under adverse water levels, even if the contact of the monolith base and foundation is good and the base rock is solid. Without anchors, the reliability index is only 1.09 under maintenance condition (B), regarding to

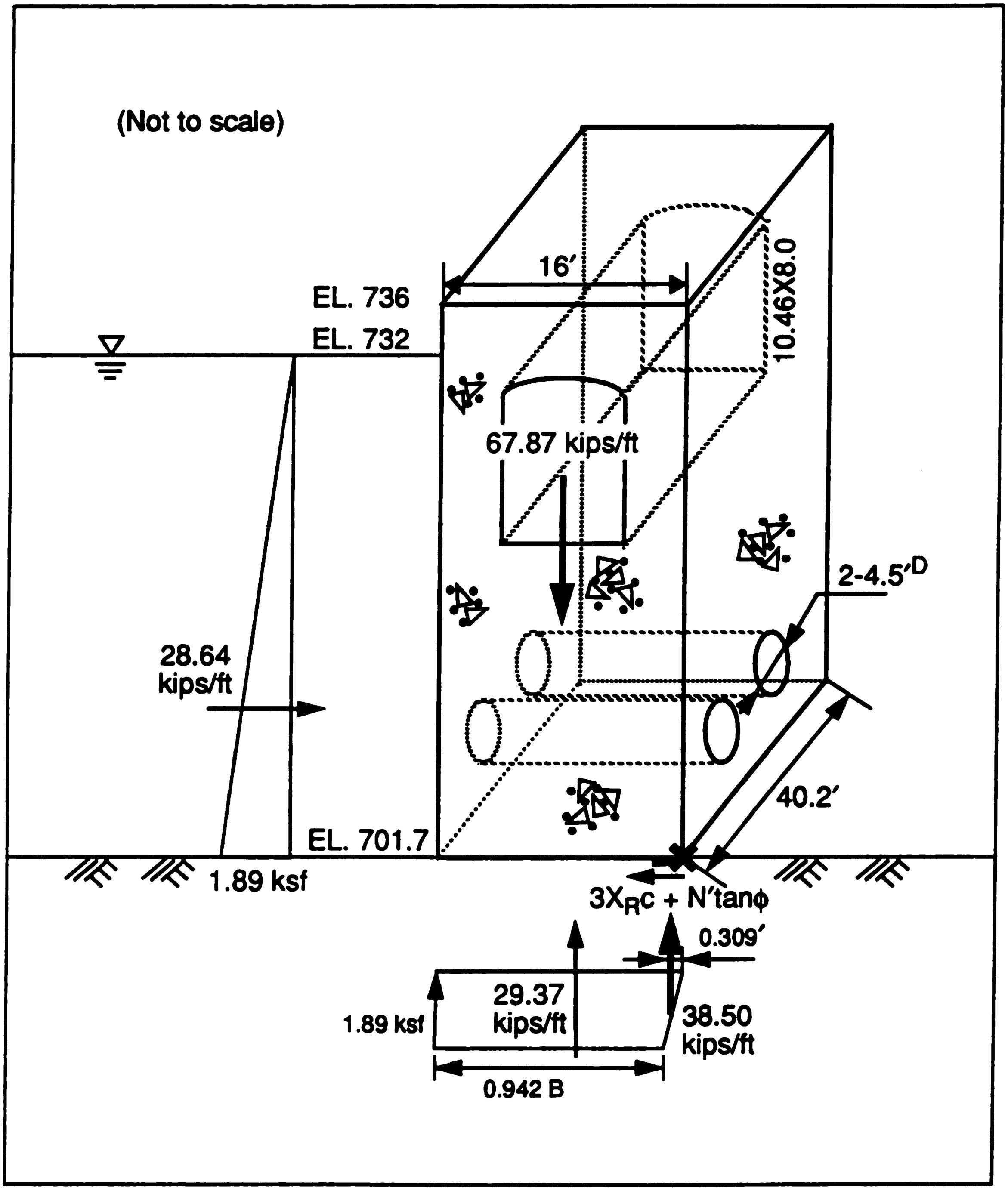


Figure 5-14 Locks and Dam No. 3, Monolith M-20, Sliding Analysis – Maintenance Condition (B), No Anchors

Table 5-20 Sliding Analysis Results – Locks and Dam No.3 Monolith M-20

Strength	Pool	Anchors	E[FS]	σ_{FS}	β_{FS}
Peak	Normal Operating	No	19.36	8.320	6.99
		3+1, R=0.5	21.76	9.296	7.32
		3+1	23.72	10.022	7.61
	Maintenance (A)	No	11.22	4.286	6.37
		3+1, R=0.5	12.92	4.945	6.74
		3+1	14.21	5.412	7.03
	Maintenance (B)	No	2.10	1.236	1.09
		3+1, R=0.5	4.04	1.218	4.58
		3+1	5.59	1.651	5.08
	High Water	No	16.05	6.692	6.73
		3+1, R=0.5	19.30	8.144	7.11
		3+1	21.95	9.254	7.43
Residual	Normal Operating	No	2.40	1.307	1.47
		3+1, R=0.5	2.70	1.468	1.70
		3+1	2.98	1.618	1.90
	Maintenance (A)	No	1.65	0.903	0.72
		3+1, R=0.5	1.87	1.022	0.97
		3+1	2.06	1.122	1.17
	Maintenance (B)	No	0.79	0.448	-0.71
		3+1, R=0.5	0.98	0.548	-0.29
		3+1	1.16	0.638	0.02
	High Water	No	2.11	1.155	1.20
		3+1, R=0.5	2.43	1.327	1.49
		3+1	2.75	1.492	1.73

sliding stability. On the other hand, if residual shear strength represents actual foundation strength, anchoring will not help much, therefore, other method(s) must be considered to increase its sliding stability.

5.3.4 Locks and Dam No.3 Monolith L-8, Monongahela River

5.3.4.1 Introduction

Locks and Dam No. 3 were previously described in section 5.2.2. Monolith L-8 is a gravity monolith forming a part of the landside upper guide wall. A cross-section through the monolith is shown in Figure 5-15. The monolith is founded at elevation 709.1, apparently on weathered siltstone. Monolith L-8 is relatively slender, being 27 feet tall and only 14 feet wide at the base and the backfill soil is the dominant load.

The water levels (elevation) selected for the analysis are the following:

Case	Upper Pool	Water in Backfill
Normal Operating	726.9	728.9
High Water (A)	732.9	734.9

The normal operating condition represents the usually-prevailing conditions at the lock. The high water condition corresponds to water levels just before the lock would go out of operation. As the L-8 is an upper guidewall monolith, the maintenance condition is not applicable. The water level in

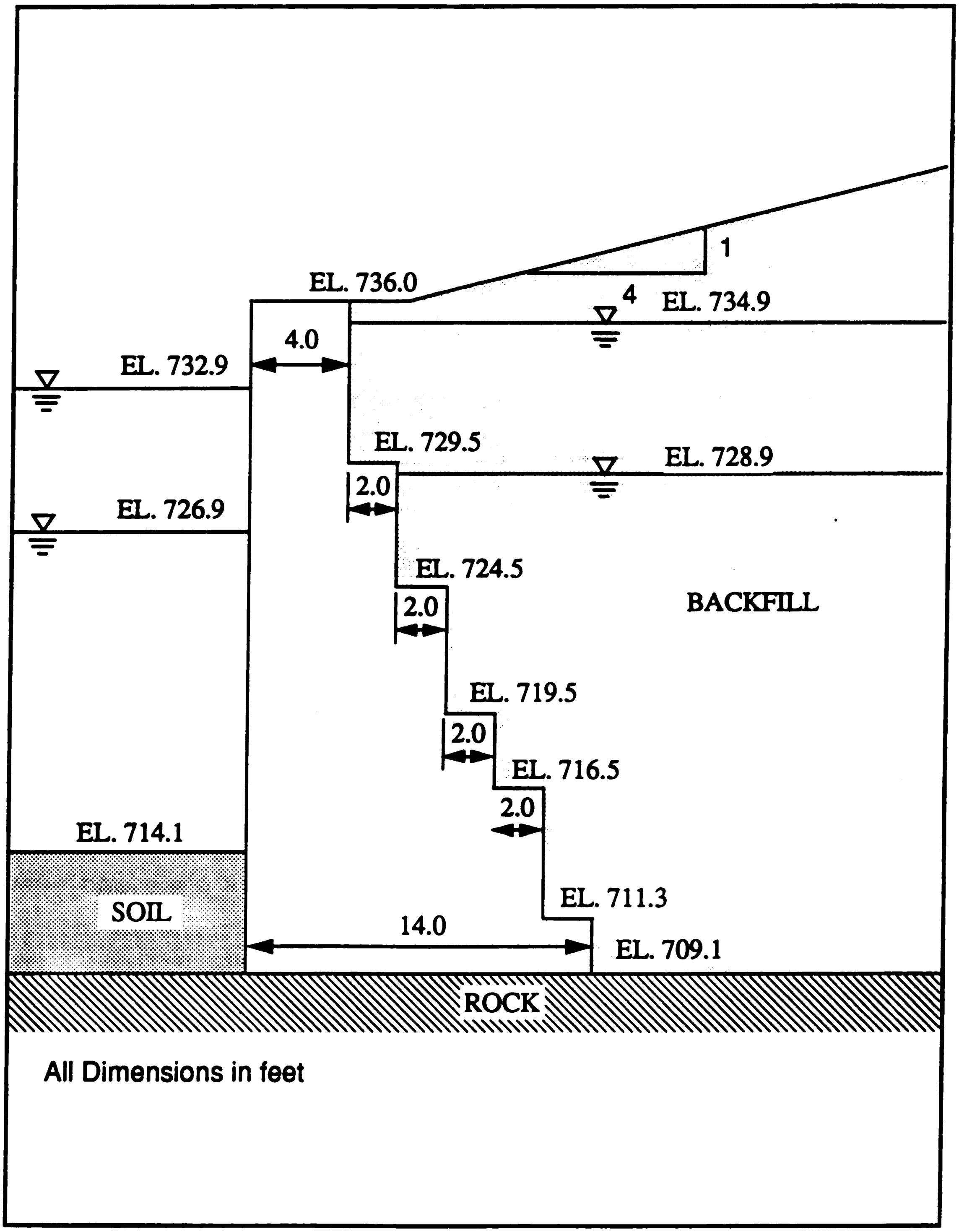


Figure 5-15 Locks and Dam No. 3, Monolith L-8, Cross-Section

the backfill was assumed to be 2.0 ft higher than the upper pool level for the analysis.

5.3.4.2 Random variables

The random variables involved in the sliding analysis of monolith L-8 are listed in Table 5-21. These random variables are similar to those in other monolith analysis but wall friction angle and the saturation level (water level in backfill) are two special random variables for landwall monoliths. In this analysis, the wall friction angle is based on the judgment that the common assumption of no vertical shear is unlikely to be representative of actual conditions, and the developed vertical shear friction angle may be considerably less than the soil internal friction angle, i.e. $\delta \ll \phi$. Also as assumed, the saturation level of backfill is 2.0 ft higher than the pool level and has a 1.0 ft standard deviation.

The shear strength parameters of foundation material were the same as that used for monolith M-20 of Locks and Dam 3.

5.3.4.3 Performance function

Very similar to the performance functions of sliding analysis for other monolith, the resisting and driving forces on the monolith L-8 can be expressed by:

$$D = D_{pool} + D_{soil} \tag{5•25}$$

where

$$D_{pool} = \mathrm{H}_{pool1} - \mathrm{H}_{pool2} = \frac{H_H^2 - H_L^2}{2}\gamma_w$$

Table 5-21 Random Variables for Sliding Analysis – Locks and Dam No. 3, Monolith L-8

Variable	μ	σ	V (%)
c_{base} (Peak)	11.0 (ksf)	7.70 (ksf)	70.0
ϕ_{base} (Peak)	52.4 (°)	12.9 (°)	24.6
			$\rho_{c,\tan\phi} = -0.70$
c_{base} (Residual)	0.0	0.0	0.0
ϕ_{base} (Residual)	30.5 (°)	13.0 (°)	42.6
$\gamma_{concrete}$	0.145 (kcf)	0.00725 (kcf)	5.0
Backfill saturation level, Ele_{sat}	2.0+Pool level (ft)	1.0 (ft)	—
γ_{soil}	0.13 (kcf)	0.0065 (kcf)	5.0
ϕ_{soil}	30.0 (°)	3.0 (°)	10.0
Wall friction, δ	12.0 (°)	3.0 (°)	25.0
F_{Hawser}	1.0 (kips/ft)	0.50 (kips/ft)	50.0
Uplift parameter, E	Varies, if PC%<100) 0.0, if PC% 100)	0.2	—

$$D_{soil} = (D_{soil,1} + D_{soil,2})K_a$$

where

$$D_{soil,1} = \frac{(738 - Ele_{sat})^2}{2}\gamma_{soil}$$

$$D_{soil,2} = (738 - Ele_{sat})(Ele_{sat} - 709.1)\gamma_{soil} +$$

$$+ \frac{(Ele_{sat} - 709.1)^2}{2}\gamma_{soil}$$

and

$K_a = tan^2(45° - \phi_{base}/2)$ is the coefficient of active lateral earth pressure.

It should be pointed out that the slope of the backfill surface was not taken into account for simplifying the calculation.

$$R = 3X_R c_{base} + N' tan(\phi_{base}) + R_{soil}$$

where

$N' = V_{concrete} + V_{soil} + V_{wall\ friction} - U$ is effective normal force on the base,

where

$V_{concrete} = A_{con}\gamma_{concrete}$ is the weight of monolith per foot;

$V_{soil} = A_{soil}\gamma_{soil}$ is the weight of soil on the monolith per foot;

$V_{wall\ friction} = D_{soil} tan\,\delta$ is the vertical component of wall friction force per foot; and

$$U = \frac{(H_h + H_l) - (H_h - H_l)E}{2}\gamma_w B \;;$$

and

$$R_{soil} = (714.1 - 709.1)^2/2(\gamma_{soil}\ K_p)$$

where

$K_p = tan^2(45° + \phi_{base}/2)$ is the coefficient of passive lateral earth pressure.

Table 5-22 Sliding Analysis Results – Locks and Dam No.3 Monolith L-8

Strength	Pool	E[FS]	σ_{FS}	β_{FS}
Peak	Normal	5.24	3.677	2.30
	High water	4.46	3.912	1.60
Residual	Normal	1.47	0.771	0.54
	High water	1.45	0.741	0.53

Other variables were defined before.

A free body diagram is shown in Figure 5-16.

5.3.4.4 Analysis results

The sliding analysis of Locks and Dam No. 3, Monolith L-8 are listed in Table 5-22.

The reliability indices of factor of safety of sliding for L-8 show that the land wall monolith is not very reliable with respect to resisting sliding even the base rock is in good condition. Therefore, some remedy may be needed, such as removing part of the backfill if a higher reliability (e.g. $\beta \geq 4.0$) is desired.

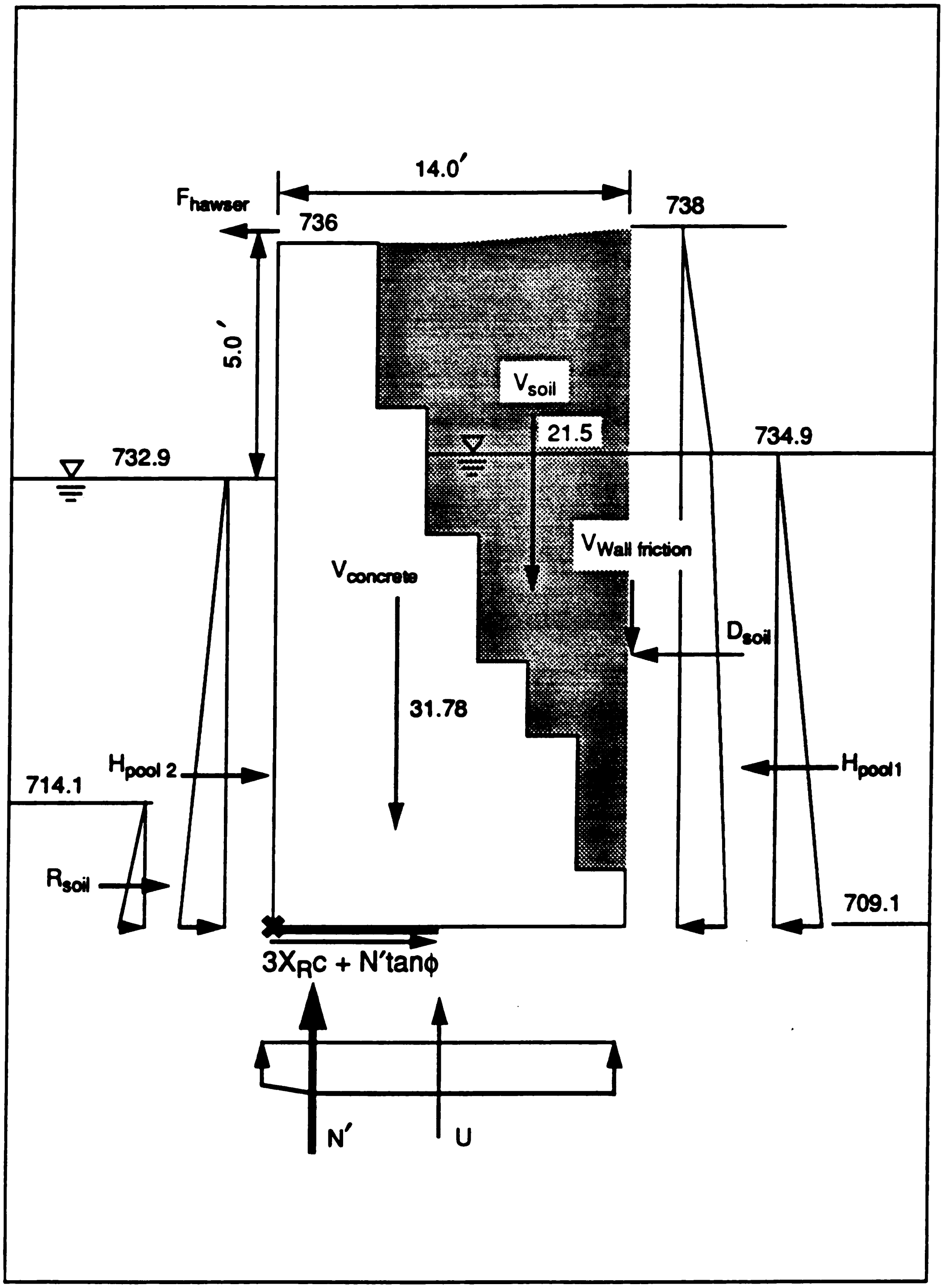


Figure 5-16 Locks and Dam No. 3, Monolith L-8, Free Body Diagram - Sliding Analysis

5.4 Bearing Capacity

Bearing capacity i another important safety aspect for locks and dams, especi. ly if these structures are built on soft soil foundations.

In the bearing capacity reliability analysis, the generalized bearing capacity equation, which is often used in engineering practices, will be employed. Without considering the slope of the ground, shape of the base and tilt of the base, the factor of safety of bearing capacity can be expressed by

$$FS = \frac{\bar{B}\left[\zeta_{cd}\zeta_{ci}cN_c + \zeta_{qd}\zeta_{qi}q_0N_q + \frac{1}{2}\left(\zeta_{\gamma d}\zeta_{\gamma i}\bar{B}\gamma N_\gamma\right)\right]}{N'} \qquad (5 \cdot 26)$$

where

$\bar{B} = 2X_R$ is effective width of the base, where X_R is measured from the toe of the base;

c is cohesion parameter of the foundation material;

$q_0 = D\gamma'_{soil}$ is effective overburden pressure on the plane passing through the base of the footing;

γ is effective unit weight of the foundation material; and

Nc, Nq, N_γ, ζ's are factors of bearing capacity. The definitions of these factors were explained in section 4.6.4.1

As mentioned before, if locks and dam are located on rock foundation, there are some difficulties to performing bearir capacity reliability analysis, because:

(1) the bearing capacity factor N_c, N_q and N_γ are highly .-linear functions of internal friction angle of founda- n material, ϕ, and their values become extremely large as

ϕ value exceeds 45 degree (see section 4.6.4.3). This non-linearity greatly affects the accuracy of the approximative methods used in reliability evaluation practices;

(2) the factor of safety calculated by conventional method (using nominal or expected values) may be extremely high for rock foundation where foundation failure seems very much unlikely.

As rock foundations are involved in the following bearing capacity analysis examples, bearing capacity factor N_γ will be used as defined by Vesic *et al*[52] and the analysis procedure will follow that discussed in section 4.6.4. To reduce the calculations, some analysis results from overturning analysis, e.g. the resultant location X_R will be used as a new independent random variable.

5.4.1 Locks and Dam No.2 Monolith M-16, Monongahela River

Locks and Dam No. 2 and M-16 have been described in detail in section 5.2.1. The bearing capacity of monolith M-16 will be evaluated for 5 different loading conditions and considering both peak and residual strength of foundation. Although it can be argued that the residual strength may not relevant to bearing capacity, for some real field condition ...ng residual strength may be more appropriate.

...1.1 Random variables

The random variables used in capacity analysis are listed ... Table 5-23. Note that the distances of resultant force ...om t e toe, X_R, under different operating conditions are

Table 5-23 Random Variables for Bearing Capacity Analysis – Locks and Dam No. 2, Monolith M-16

Variable	μ	σ	V (%)
c_{base} (Peak)	11.0 (ksf)	7.70 (ksf)	70.0
ϕ_{base} (Peak)	52.4 (°)	12.9 (°)	24.6
			$\rho_{c,\tan\phi} = -0.70$
c_{base} (Residual)	0.0	0.0	0.0
ϕ_{base} (Residual)	30.5 (°)	13.0 (°)	42.6
γ_{base}	0.147 (kcf)	0.00735 (kcf)	5.0
γ_{soil}	0.0755 (kcf)	0.003775 (kcf)	5.0
ϕ'_{soil}	33 (°)	3.3 (°)	10.0
$\gamma_{concrete}$	0.15 (kcf)	0.0075 (kcf)	5.0
F_{impact}	1.0 (kips/ft)	0.5 (kips/ft)	50.0
Uplift parameter, E	Varies	0.2	—
Resultant location, X_R	Varies, based on overturning analysis	Varies	Varies

obtained from overturning analysis.

5.4.1.2 Performance function

As the generalized bearing capacity equation will be used in analysis, no particular performance function needs to be generated but the inclination angle of the resultant must be determined during the analysis, and this angle is a function of several random variables which are involved in determining the resultant. For monolith M-16, the inclination angle, Δ, can be expressed as

$$\Delta = \tan^{-1}(\Sigma H/\Sigma N')$$

$$= \tan^{-1}\left(\frac{D_{pool} + F_{impact}}{W_{concrete} + W_w - U}\right) \qquad (5\bullet 27)$$

where

$D_{pool} = \frac{(H_h^2 - H_l^2)}{2}\gamma_w$ is the horizontal hydraulic force on the monolith;

F_{impact} is the impact force, if applicable, under concerned operating condition; and

$N' = W_{concrete} + W_w - U$ is the normal component of resultant force,

where

$W_{concrete}$ is the weight of the monolith body;

W_w is the weight of water in the culvert of the monolith if applicable; and

$U = \frac{(H_h + H_l) - (H_h - H_l)E}{2}\gamma_w B$ is hydraulic uplift force under the base.

Note that all forces are in the unit of kips/ft.

5.4.1.3 Analysis results

The reliability analysis results of bearing capacity of Locks and Dam No.2, M-16 are listed in Table 5-24.

The analysis results show that

1. The expected values of the factor of safety are extremely high (as high as 28626 under high water condition by PEM method!). Those high values of factor of safety by no mean imply that the monolith is absolutely reliable because of error induced by high non-linearity of performance function;

Table 5-24 Bearing Capacity Analysis Results – Locks and Dam No. 2 Monolith M-16

Strength	Pool	Method	E[FS]	σ_{FS}	β_{FS}
Peak	Normal (A)	Taylor's	1626.6	3650.71	4.84
		PEM	24854.5	26556.34	11.16
	Normal (B)	Taylor's	1446.9	3213.75	4.79
		PEM	21369.3	23090.84	10.90
	Maintenance (A)	Taylor's	987.1	2165.64	4.53
		PEM	13731.21	15021.90	10.30
	Maintenance (B)	Taylor's	651.1	1391.28	4.29
		PEM	8348.6	9545.78	9.42
	High Water	Taylor's	1875.6	4209.97	4.95
		PEM	28626.1	30613.79	11.32
Residual	Normal (A)	Taylor's	10.9	20.30	1.34
		PEM	44.7	43.05	4.28
	Normal (B)	Taylor's	8.8	16.45	1.15
		PEM	36.3	35.04	4.03
	Maintenance (A)	Taylor's	4.8	9.15	0.66
		PEM	20.9	20.07	3.35
	Maintenance (B)	Taylor's	2.7	4.58	0.27
		PEM	11.1	10.45	2.62
	High Water	Taylor's	12.9	23.80	1.48
		PEM	52.1	50.25	4.47
$E[\phi]=30.5°$ $\sigma_\phi=13.0°$ $E[c]=11.0ksf$ $\sigma_c=7.7$ ksf $\rho_{c,\phi}=-0.7$	Normal (A)	Taylor's	88.5	77.47	5.57
		PEM	141.6	140.54	5.57
	Normal (B)	Taylor's	80.0	69.39	5.47
		PEM	125.4	126.36	5.35
	Maintenance (A)	Taylor's	55.8	47.51	5.08
		PEM	82.3	84.56	4.77
	Maintenance (B)	Taylor's	38.3	32.02	4.64
		PEM	54.7	58.31	4.16
	High Water	Taylor's	102.1	89.49	5.75
		PEM	163.6	162.27	5.75

2. The reliability indices of the factor of safety with respect to bearing capacity of M-16 are greater than 4 in most of the cases, with great variation – about 100% in most cases, which is also caused by the high nonlinearity of the generalized bearing capacity function. If using residual strength is not proper for in-situ condition (which may result low reliability index – as low as 0.27 under maintenance condition (B) by Taylor's series method), the "reliable" conclusion may be drawn;

3. The Taylor's series method and point estimate method gave quite different results. The differences are mainly from the high nonlinearity of the performance function and the high value of the internal friction angle of the foundation material. This suggests that expanding Taylor's series about the expected values of the random variables is not suitable for a high nonlinear performance function and other methods (such as higher order approximation, Monte Carlo simulation, etc.) may need be employed.

4. As another comparison, an assumed strength parameter set, which has the residual strength internal friction angle but the peak strength cohesion parameters, was used to evaluate the reliability of bearing capacity (see Table 5-24). Once again, the results showed some difference between the two approximative methods, especially on the expected value, but the reliability indices are in very good agreement. This result may be of evidence that the reliability index is indeed a good reliability measurement.

5.4.2 Demopolis Locks and Dam, Monolith L-17

The description of Demopolis Locks and Dam, Monolith L-17 was given in section 5.2.4. The water levels will be the same as that used in overturning analysis and the resultant locations will be directly used in bearing capacity analysis. Choosing Demopolis Locks and Dam, Monolith L-17 is to see whether the backfill level will affect the bearing capacity.

5.4.2.1 Random variables

The random variables used in this analysis are listed in Table 5-25. The shear strength parameters of foundation material are based on the laboratory direct shear test results of Demopolis Locks and Dam with referencing the rock shear strength of locks and dams on Monongahela River (Locks and Dam No.2, 3 and 4).

5.4.2.2 Performance function

Similar to Locks and Dam No. 2, Monolith M-16 whose bearing reliability was analyzed in section 5.4.1, only the inclination angle of resultant force need to be specially mentioned. For Demopolis Locks and Dam, L-17, the Δ can be expressed by

$$\Delta = \tan^{-1}(\Sigma H/\Sigma N')$$

$$= \tan^{-1}\left(\frac{D_{pool} + F_{impact} + D_{h,soil} - R_{soil}}{V_{con} + V_{water} + V_{soil} + V_{wall} - U}\right) \qquad (5\bullet 28)$$

where

$$D_{h,soil} = D^{1}{}_{h,soil} + (D^{2,1}{}_{h,soil} + D^{2,2}{}_{h,soil}) + + (D^{3,1}{}_{h,soil} + D^{3,2}{}_{h,soil})$$

is the lateral earth pressure by backfill. The

Table 5-25 Random Variables for Bearing Capacity Analysis – Demopolis Locks and Dam, Monolith L-17

Variable	μ	σ	V%
$\gamma_{concrete}$	0.15 (kcf)	0.0075 (kcf)	5.0
γ_{soil}	0.125 (kcf)	0.00625 (kcf)	5.0
γ_{base}	0.125 (kcf)	0.00625 (kcf)	5.0
c_{base} (Peak)	30.0 (ksf)	21.0 (ksf)	70.0
ϕ_{base} (Peak)	30 (°)	11.46 (°)	38.2
			$\rho_{c,\phi} = -0.70$
c_{base} (Residual)	0.0 (ksf)	0.0 (ksf)	0.0
ϕ_{base} (Residual)	25.2 (°)	8.02 (°)	31.8
K_h	1.0	0.1	10.0
F_{impact}	5.0 (kips/ft)	2.5 (kips/ft)	50.0
H_{sat}	68 (ft)	6.8 (ft).	10.0
Wall friction, δ	12.0 (°)	3.0 (°)	25.0
Uplift factor, E	Varies for PC%<100 0.0 for PC% 100	0.2	—
Resultant location, X_R	Varies, based on overturning analysis	Varies	Varies

$D^{i,j}_{h,soil}$ are defined in section 5.2.4;

R_{soil} = $(13-6)^2/2(\gamma_{soil}-\gamma_w)(1-\sin\phi_{base})$ is the lateral force by overburden soil;

V_{con} = $A_{con}\gamma_{concrete}=(1985.5)\gamma_{concrete}$ is the weight of concrete per foot;

V_{water} is the weight of water in culvert per foot;

V_{soil} = $A_{soil}\gamma_{soil}$ is the weight of soil per foot on the monolith, changes when backfill level changes;

V_{wall} = $D_{h,soil}\tan\delta$ is vertical component of wall friction force; and

U —uplift force as defined before.

Note that all forces are in the unit of kips/ft and their definition were described in detail in section 5.2.4.

5.4.2.3 Analysis results

The analysis results of bearing capacity of monolith L-17, Demopolis Locks and Dam are listed in Table 5-26.

If only the reliability of bearing capacity of monolith L-17 is considered, based on the analysis results the backfill level will not affect its reliability as long as the foundation rock is in good condition. This conclusion may not be unexpected if considering that the backfill level will mainly affects the inclination angle of resultant force and the effective base width in the performance function. Note that the reliability indices are even smaller for 20 feet backfill removal condition. This is because although the expected values of FS increased, the variances gained more since when active base increases the uncertainties brought by cohesion c_{base} and X_R also increase.

Table 5-26 Bearing Capacity Analysis Results – Demopolis Locks and Dam Monolith L-17 (Taylor's Series Method)

Backfill level	Strength	Pool	E[FS]	σ_{FS}	β_{FS}
Top of Monolith	Peak	Normal	37.01	13.273	10.21
		Maintenance	33.85	10.649	11.31
	Residual	Normal	0.26	0.084	-4.44
		Maintenance	0.17	0.072	-4.55
20 feet of backfill Removed	Peak	Normal	75.27	42.322	7.98
		Maintenance	68.08	26.508	11.05
	Residual	Normal	0.65	0.378	-1.07
		Maintenance	0.57	0.237	-1.61

Chapter VI

SUMMARY AND DISCUSSIONS

6.1 Introduction

Based on the procedures and examples of navigation structural reliability analyses discussed and illustrated in Chapter IV and Chapter V, many observations may be made and a number have been discussed by Wolff and Wang[125]. Some important ones are summarized in this chapter.

6.2 Importance of Random Variables

There are many variables involved in the reliability analysis. Although all variables actually are random, in the strict sense, only the variables which not only have relatively great uncertainty (compared with others) and play a significant role in the performance need to be treated as random variables. The criterion of selecting random variables is that of retaining an acceptable degree of accuracy but making the analysis process as simple as possible.

Usually the same random variable may play different roles in different performance modes, but attention should be paid to the random variables which are relatively important.

6.2.1 The Effect of Hydraulic Uplift Factor E on Reliability Index

The hydraulic uplift force, U, is a special but important loading force involved in stability analysis of navigation structures. In reliability analysis, costly mistakes would be made if this force is ignored. Unfortunately, it is not a simple matter to precisely know how the uplift force distributes along the base as well as its magnitude. To account those uncertainties, a random variable, the uplift factor, E, was defined and introduced in this reliability analysis. This variable was first suggested by Wolff (1991) and its definition and treatment in the analysis was extended and refined by Wolff and the writer. The uplift factor E reflects the drainage and base-foundation contact condition, the uplift force distribution and the active base length (or area). The analysis results showed that this random variable played an important role in the reliability evaluation of gravity structures, especially with respect to overturning stability. To reveal the effect of E on the reliability index, additional overturning reliability analyses were performed for Locks and Dam No.2 monolith M-16, Monongahela River. The analysis results are listed in Table 6-1 and shown in Figure 6-1 to Figure 6-3. In those analyses, only maintenance condition (B) (pool elevations: 724.7/691.5 ft) was examined with varying mean and standard deviation of E.

It is clear that the uplift factor E indeed affects the overturning reliability of lock monolith by both its mean value and standard deviation. For example, when the mean value of E increases from 0 to 0.8, the reliability index will increases about 2 times; when E[E] is greater than 0,

Table 6-1 Effect of Uplift Factor E on Overturning Analysis Locks and Dam No. 2, Monolith M-16, Maintenance Condition (B)

E[E]	σ_E	$E[X_R]$ (ft)	σ_{X_R}	β_{toe}	E[FS]	σ_{FS}	β_{FS}
-0.8	0.1	11.44	1.042	10.98	1.29	0.055	6.00
-0.6	0.1	11.25	0.964	11.66	1.32	0.058	6.24
-0.4	0.1	11.48	0.905	12.69	1.36	0.063	6.59
-0.2	0.1	12.05	0.859	14.02	1.42	0.070	7.04
0.0	0.1	12.83	0.838	15.30	1.50	0.081	7.44
0.2	0.1	13.76	0.724	19.01	1.61	0.089	8.49
0.4	0.1	14.59	0.629	23.21	1.73	0.100	9.52
0.6	0.1	15.34	0.549	27.96	1.88	0.112	10.51
0.8	0.1	16.02	0.481	33.29	2.05	0.128	11.44
-0.8	0.2	11.44	1.105	10.35	1.29	0.057	5.84
-0.6	0.2	11.25	0.965	11.66	1.32	0.064	5.65
-0.4	0.2	11.48	0.972	11.82	1.36	0.076	5.44
-0.2	0.2	12.05	1.059	11.38	1.42	0.093	5.25
0.0	0.2	12.83	1.197	10.72	1.50	0.119	5.04
0.2	0.2	13.76	1.051	13.09	1.61	0.135	5.62
0.4	0.2	14.59	0.929	15.70	1.73	0.153	6.16
0.6	0.2	15.34	0.826	18.57	1.88	0.177	6.63
0.8	0.2	16.02	0.739	21.69	2.05	0.208	7.04
-0.8	0.4	11.44	1.327	8.62	1.29	0.062	5.32
-0.6	0.4	11.25	0.967	11.63	1.32	0.083	4.32
-0.4	0.4	11.48	1.203	9.55	1.36	0.114	3.59
-0.2	0.4	12.05	1.629	7.40	1.42	0.155	3.13
0.0	0.4	12.83	2.085	6.15	1.50	0.212	2.80
0.2	0.4	13.76	1.851	7.43	1.61	0.242	3.09
0.4	0.4	14.59	1.654	8.82	1.73	0.279	3.34
0.6	0.4	15.34	1.486	10.32	1.88	0.327	3.56
0.8	0.4	16.02	1.343	11.93	2.05	0.388	3.73

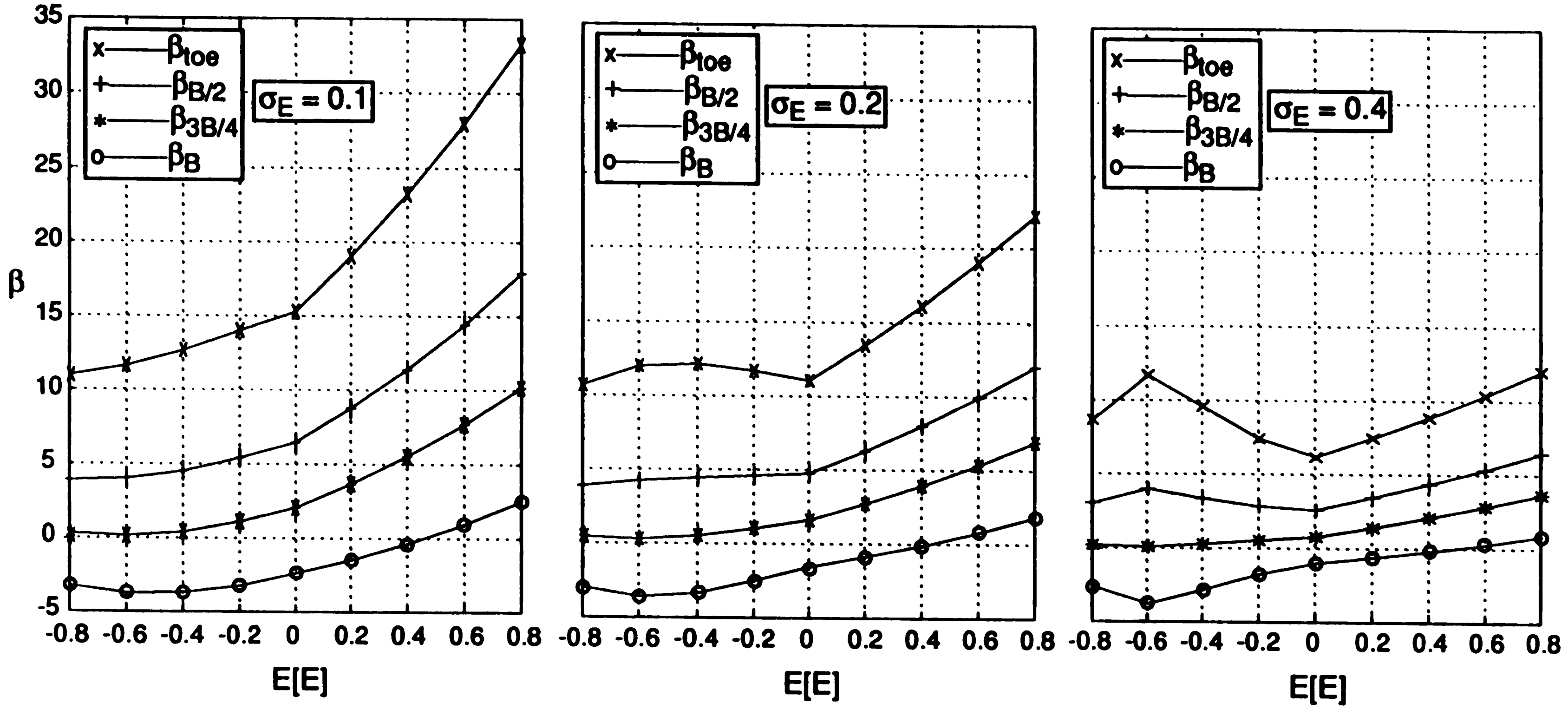


Figure 6-1 Effect of Uplift Factor E, Locks and Dam No. 2, Monolith M-16, Overturning Analysis, β versus E[E] with $\sigma_E = 0.1, 0.2$ and 0.4

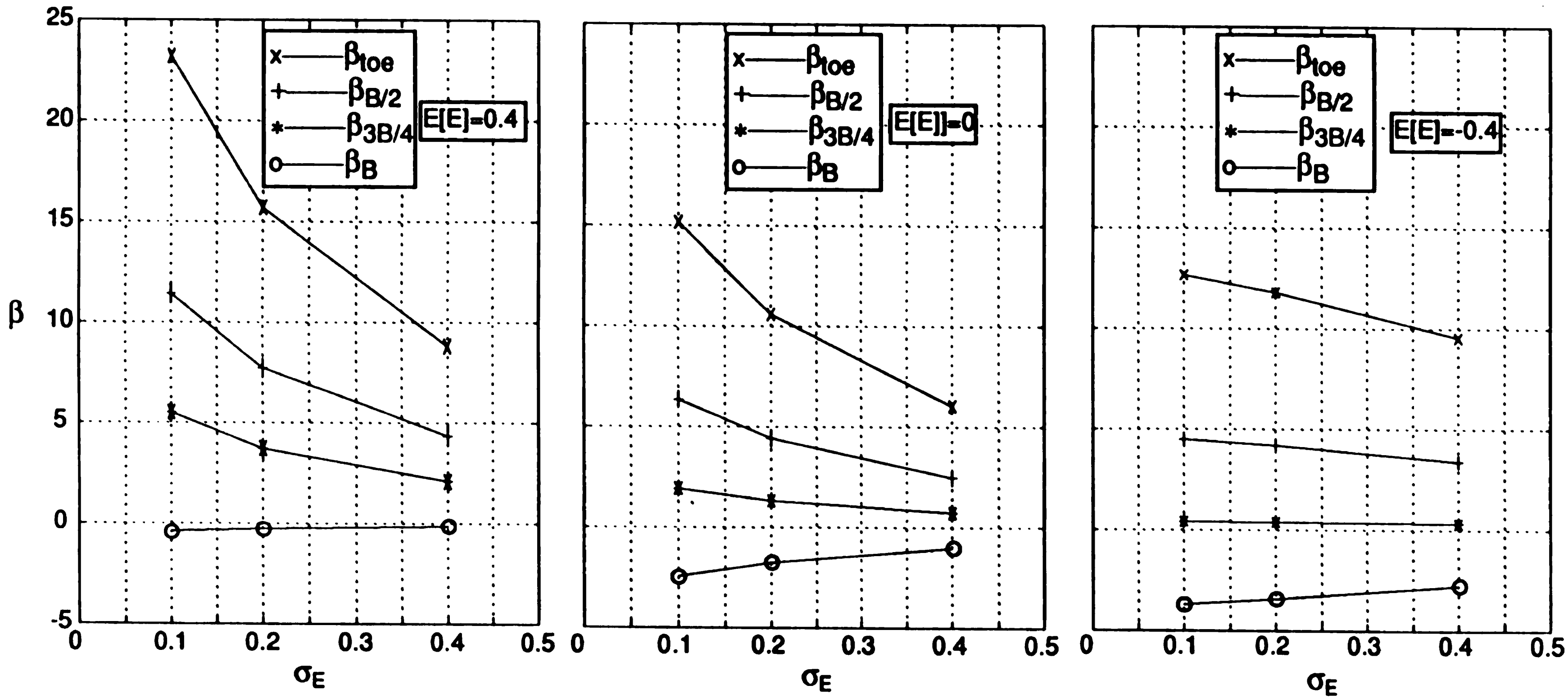


Figure 6-2 Effect of Uplift Factor E, Locks and Dam No. 2, Monolith M-16, Overturning Analysis, β versus σ_E with E[E] = 0.4, 0 and −0.4

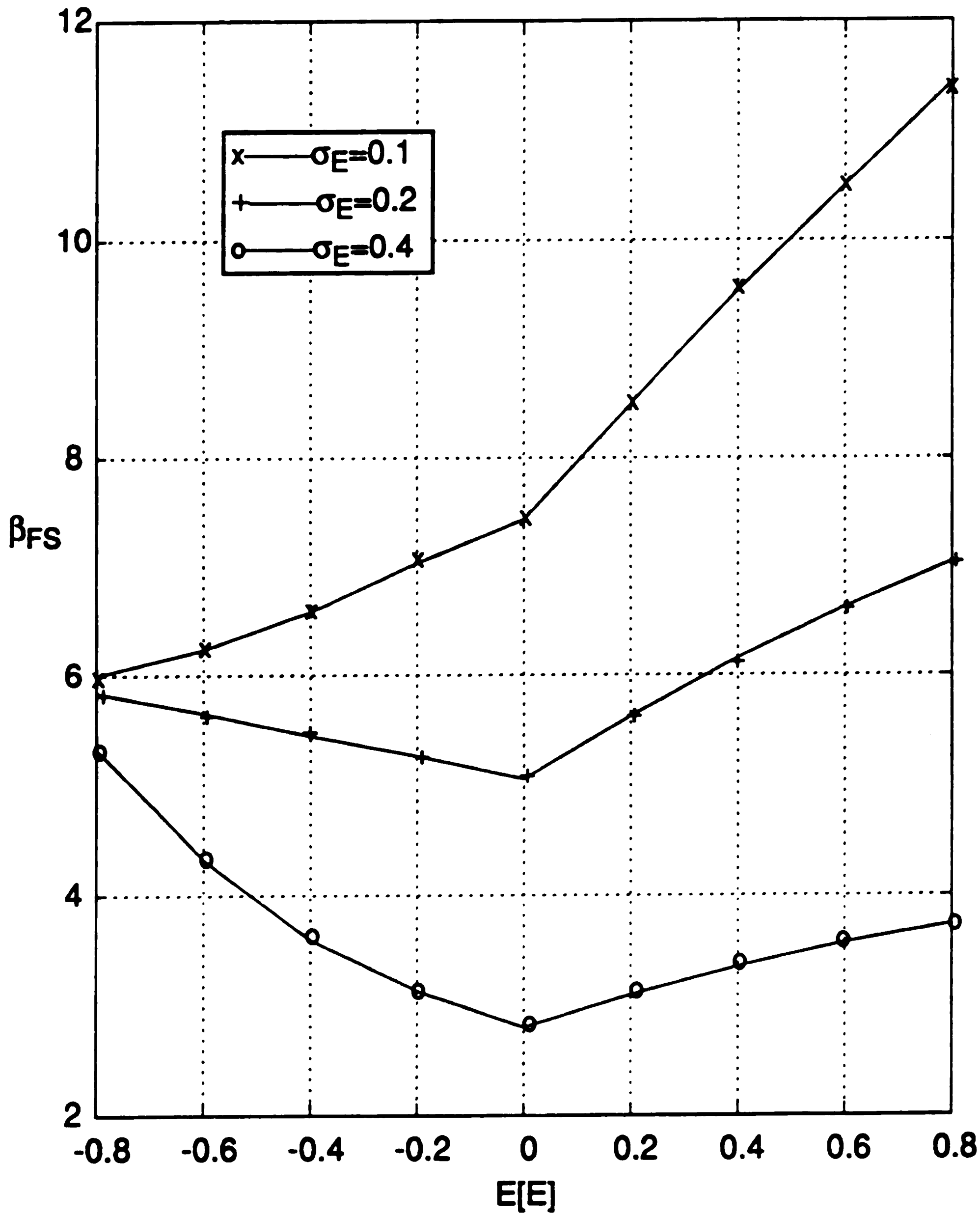


Figure 6-3 Effect of Uplift Factor E, Locks and Dam No. 2, Monolith M-16, Overturning Analysis, β_{FS} versus E[E]

the reliability index will increase about 3 times if the standard deviation of E decreases from 0.4 to 0.1, but when E[E] is less than 0 (or in other words, the base is not 100% in compression), this effect is not very significant.

It may be interesting to note that the β_{FS} (as well as β_{toe}) may have its smallest value when E[E] equals to 0 and with greater uncertainty (say, $\sigma_E \geq 0.2$). This may be explained by the following:

1. When the expected value of E is zero but with large variance, it represents the situation that whether the base is fully or partially in compression is very uncertain. As this uncertainty is relatively greater than that when E either takes a positive value (full base in compression) or negative values (part of base in compression), therefore, the reliability index at E[E]=0 is also relatively smaller;

2. The reliability index is related to the variance of performance function and this variance is determined by the sum of the products of the square of first derivatives and variances of random variables involved. The components in the overturning performance functions which contain the uplift factor E are uplift force U and its moment M_U. Figure 4-5 in section 4.6.2.3 clearly shows that the first derivative of U is a constant regardless the E value; while for M_U, its first derivative is constant when E takes values from 1.0 to 0, then the derivative becomes smaller and smaller as E decreases from 0 to -1.0. At E=-1.0 the first derivative is 0. This is one of the reasons why the variances of

performance functions increase when E increases.

3. The performance functions are nonlinear functions of E, therefore, their expected values and variances increase nonlinearly as expected value of E increases. Since the reliability index is determined by these two nonlinear quantities, expected values and variances of performance functions, for some combinations of expected value and variance of E, the reliability index has a minimum value.

6.2.2 Effect of Wall Friction Angle on Reliability Index

Backfill is usually placed along the landside wall of locks and it is reasonable to assume a vertical friction force develops between the lock wall and the backfill. This friction force usually can be expressed by the product of lateral earth force and the tangent of "wall friction angle". The wall friction force is a random variable, not only because the unit weight of soil and the coefficient of lateral earth pressure have great uncertainties but also because the wall friction angle itself is far from certain.

Wall friction angle is affected by the internal friction angle of backfill, the roughness of the wall, settlement of the backfill, ground water level, etc. There is no accurate formula to calculate this angle and it is difficult to measure it in situ, although many people think its upper limit is about 2/3 of the internal friction angle of backfill. Notice that the wall friction angle was not taken into consideration in some navigation structure performance analyses, different values of wall friction angle were considered for overturning analysis of Demopolis Locks and Dam, monolith L-17 to examine its effect on the structural reliability. The analysis results shown in Table 5-10, Figure 5-10 and Figure 5-11, in section 5.2.4. The analysis results indicated that ignoring the wall friction angle will result in a conservative design and reliability evaluation. The wall friction angle with mean value of 12 degree and 25% variation seems a good assumption.

6.2.3 Effect of Correlation Between Shear Strength Parameters

It is well known that there is correlation between soil shear strength parameters, cohesion c, and internal friction angle ϕ (or $\tan\phi$). Although this correlation has not been totally understood, much research work (e.g. Wolff, 1985[122], etc.) suggested that most likely this correlation is negative, and this study again confirms this.

The correlation between random variables will not affect the nominal value of the factor of safety but may greatly affect the structures' reliability. To examine this effect, additional sliding analyses of Locks and Dam No.3, Monolith M-20, Monongahela River were performed. In those analysis only maintenance condition (A) (pool levels: 726.4/710.7) with peak strength parameters was examined with varying the coefficient of correlation, $\rho_{c,\phi}$, of c and ϕ.

The analysis results are shown in Table 6-2 and Figure 6-4. The results clearly show that the correlation between shear strength parameters will not affect the mean value of the factor of safety, as expected, but will affect the

Table 6-2 Effect of Correlation of Shear Strength Parameters on Sliding Analysis – Locks and Dam No.3 Monolith M-20, Maintenance (A), Peak Shear Strength, 3+1 Anchors

$\rho_{c,\phi}$	E[FS]	σ_{FS}	β_{logFS}
-1.0	13.60	4.565	7.77
-0.8	13.60	5.175	6.91
-0.6	13.60	5.696	6.29
-0.4	13.60	6.173	5.81
-0.2	13.60	6.616	5.43
0.0	13.60	7.031	5.12
0.2	13.60	7.423	4.86
0.4	13.60	7.795	4.63
0.6	13.60	8.151	4.43
0.8	13.60	8.491	4.26
1.0	13.60	8.818	4.11

variance of the factor of safety and the reliability index. In the given example, when the coefficient of correlation changes from 0 to -1.0, the standard deviation decreases about 1.5 times and the reliability index increases about 1.5 times; when the coefficient of correlation varied from 0 to +1.0, the standard deviation increases about 1.25 times and the reliability index decreases about 1.25 times. If one assume $\rho_{c,\phi}$ = -0.70, then the reliability index increases

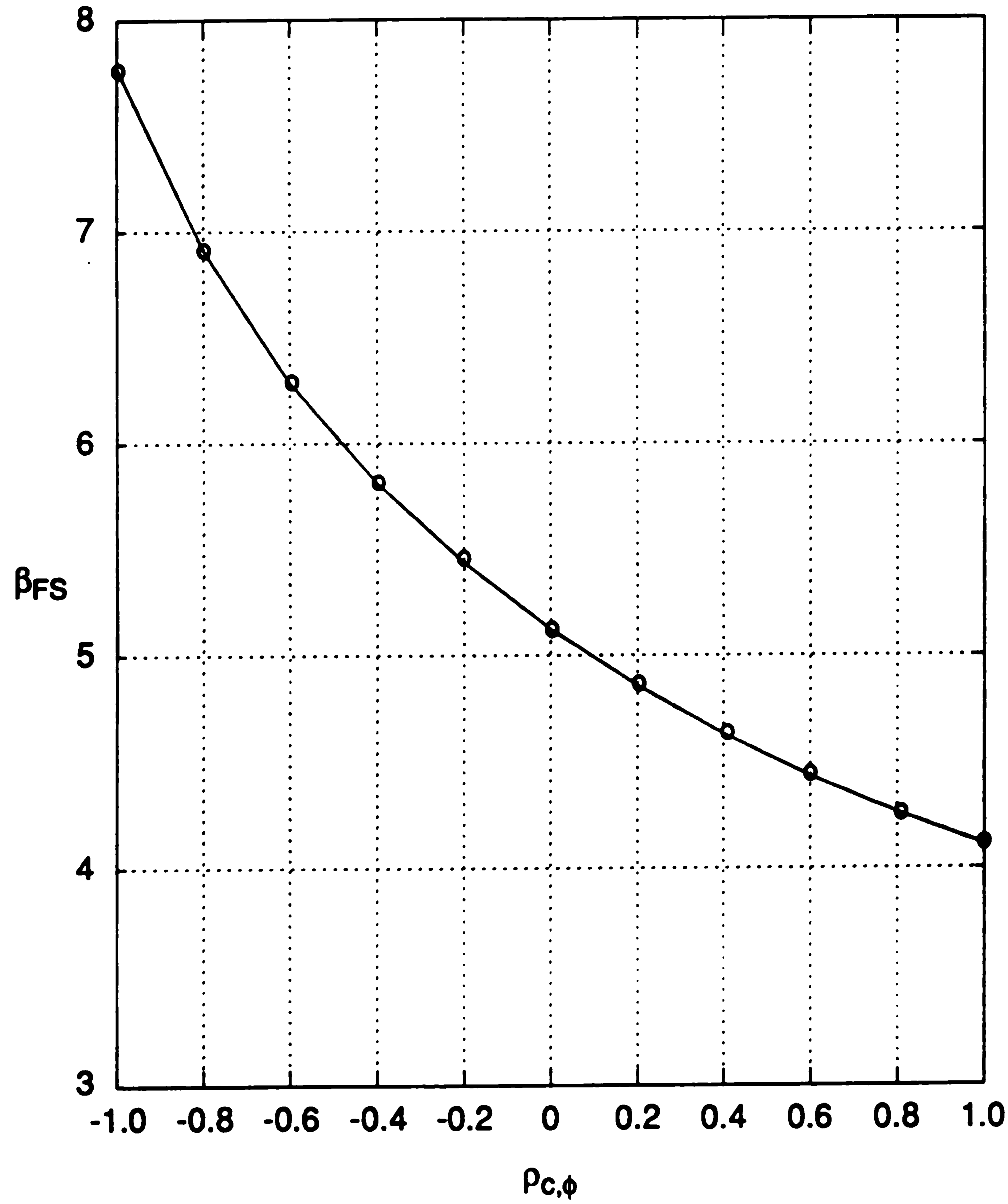


Figure 6-4 Effect of Coefficient of Correlation of c and ϕ on Sliding Reliability Index. Locks and Dam No.3 Monolith M-20, Maintenance (A), Peak Shear Strength, 3+1 Anchors

about 1.3 times as compared with using $\rho_{c,\phi} = 0$.

6.3 Comparison of Factor of Safety and Their Corresponding Reliability Indices

Factor of safety is the "safety" measurement in conventional structural analysis and it has major shortcomings as previously discussed in Chapter I. The reliability index copes many uncertainties existing in real engineering practice and gives better and more rational reliability measurement. Figure 6-5 to Figure 6-7 display the relation of mean values of factor of safety versus the reliability indices with respect to overturning and sliding performances by summary of the analysis results conducted in this study. As most of the mean values were obtained by Taylor's series method, that is, the mean values of all variables were used in the calculations, therefore, these comparison are very close to comparing the reliability indices with their nominal values of factor of safety.

The plots show that:

1. Although normally the higher the value of factor of safety, the higher the value of its reliability index, in some cases, the same value of factor safety may correspond to different reliability indices;

2. For certain performance functions, e.g. the overturning problem, a small change of factor of safety may mean significant changes of the reliability index. In other words, the reliability index is very sensitive to the factor of safety for some performance functions. It is easy to see that

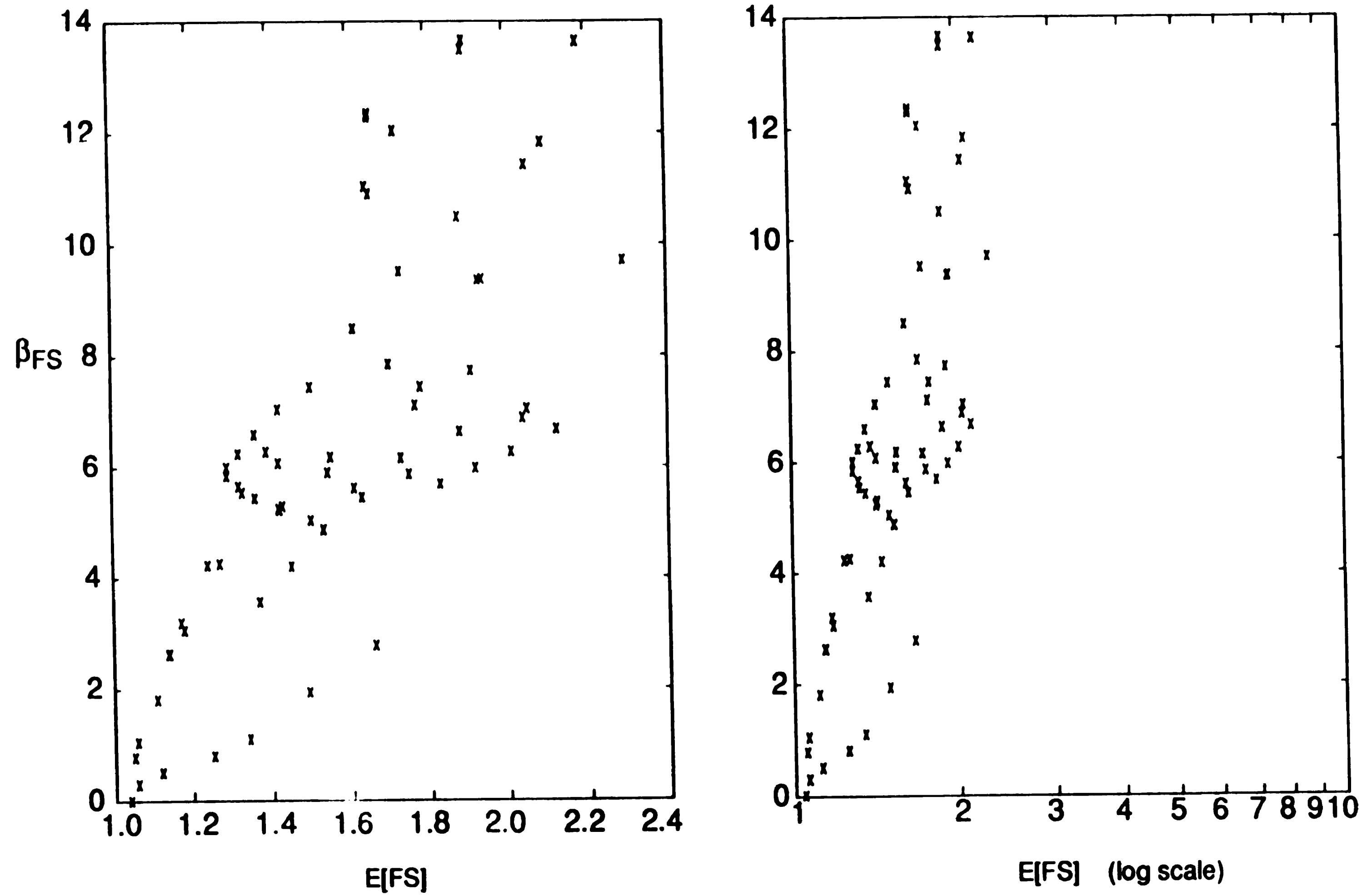


Figure 6-5 Reliability Index versus Factor of Safety. Overturning Analysis Results

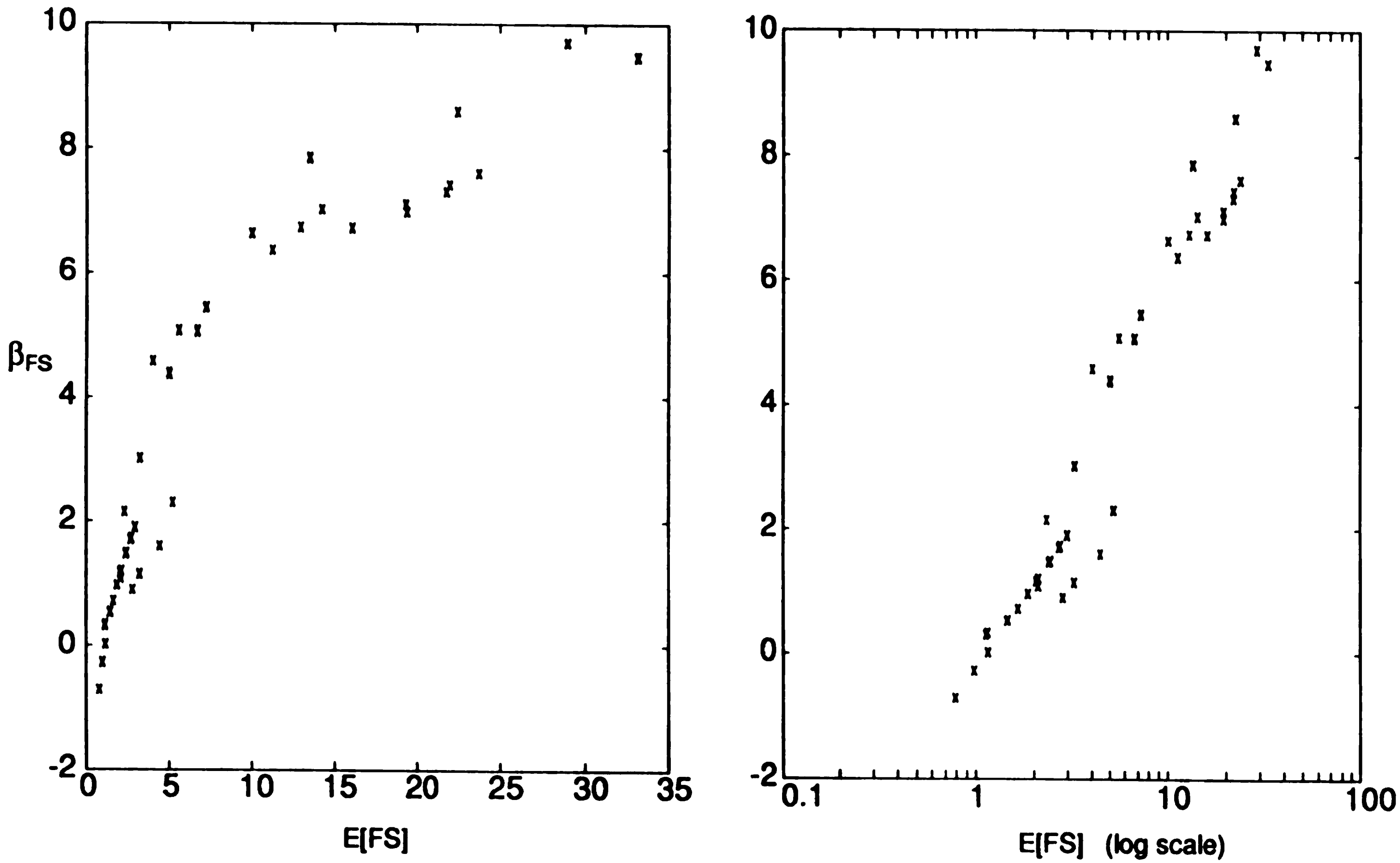


Figure 6-6 Reliability Index versus Factor of Safety. Sliding Analysis Results

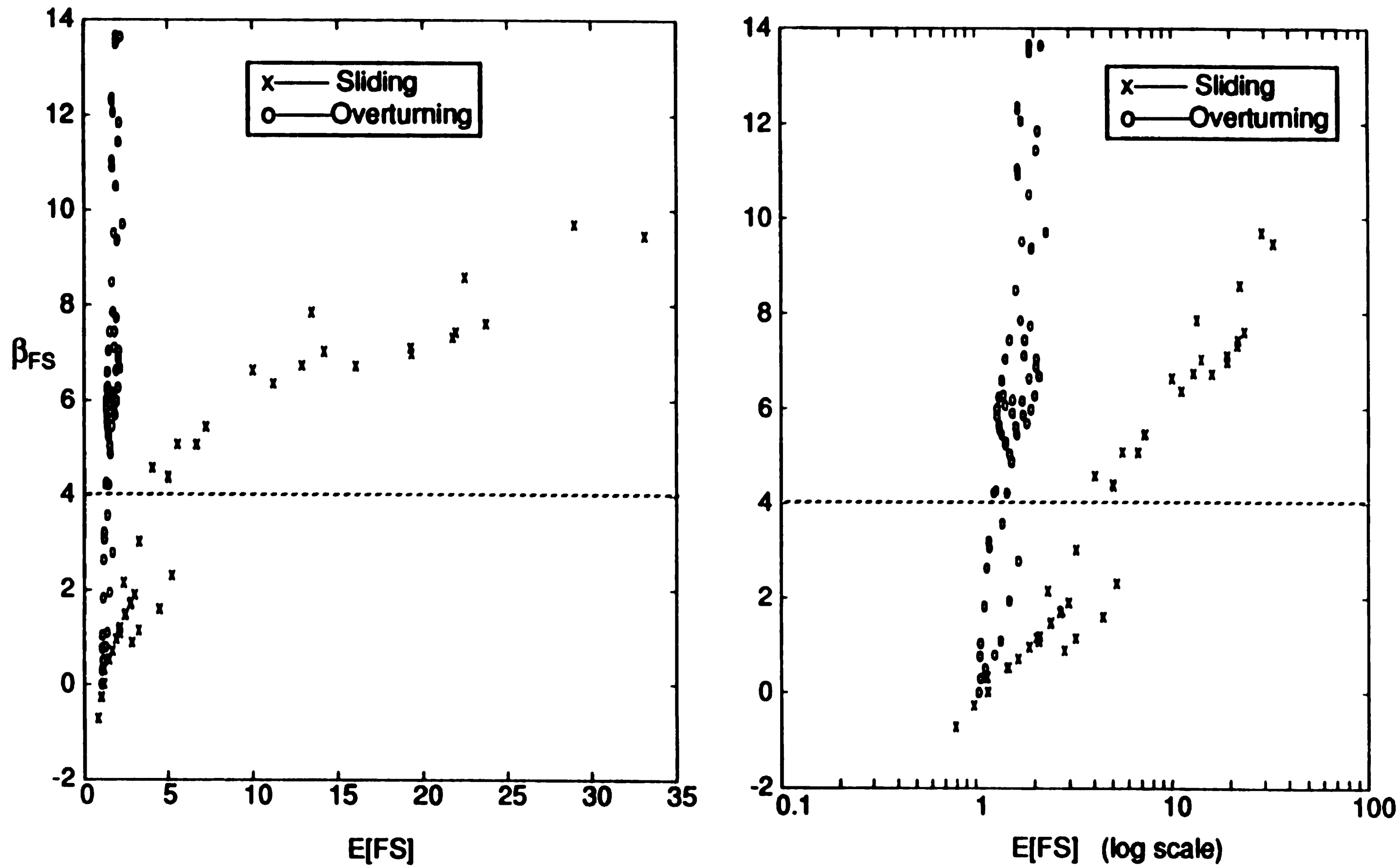


Figure 6-7 Reliability Index versus Factor of Safety

from the given figures, for a structure with a factor of safety of 1.20, the corresponding reliability index was 3.0, but when the factor of safety increased to 1.25, the reliability index could increase to 4.0. On the other hand, as the factor of safety of a structure increases from 2.0 to 4.0 for sliding problem, the corresponding reliability index may increase only from 1.0 to 2.5. This, once again, indicates that the reliability index is a more "robust" reliability measurement;

3. Factor of safety and reliability index have a log-linear relationship because of the lognormal distribution assumption. This is expected since for the factors of safety, with small variance, the reliability index can be approximated by

$$\beta \approx ln(E[FS])/V_{FS} \tag{6•1}$$

where V_{FS} is the coefficient of variation of factor of safety.

6.4 Comparison of Taylor' Series Method and Point Estimate Method

In reliability analysis, the Taylor's series method and the Point Estimate Method (PEM) are two often used approximative methods. In Chapter III, the advantages and disadvantages of these two methods were discussed. As a comparison, Figure 6-8 plots the reliability indices of factor of safety by these two methods for sliding and overturning analysis (some reliability indices included are from other analysis results produced by this study but not illustrated in

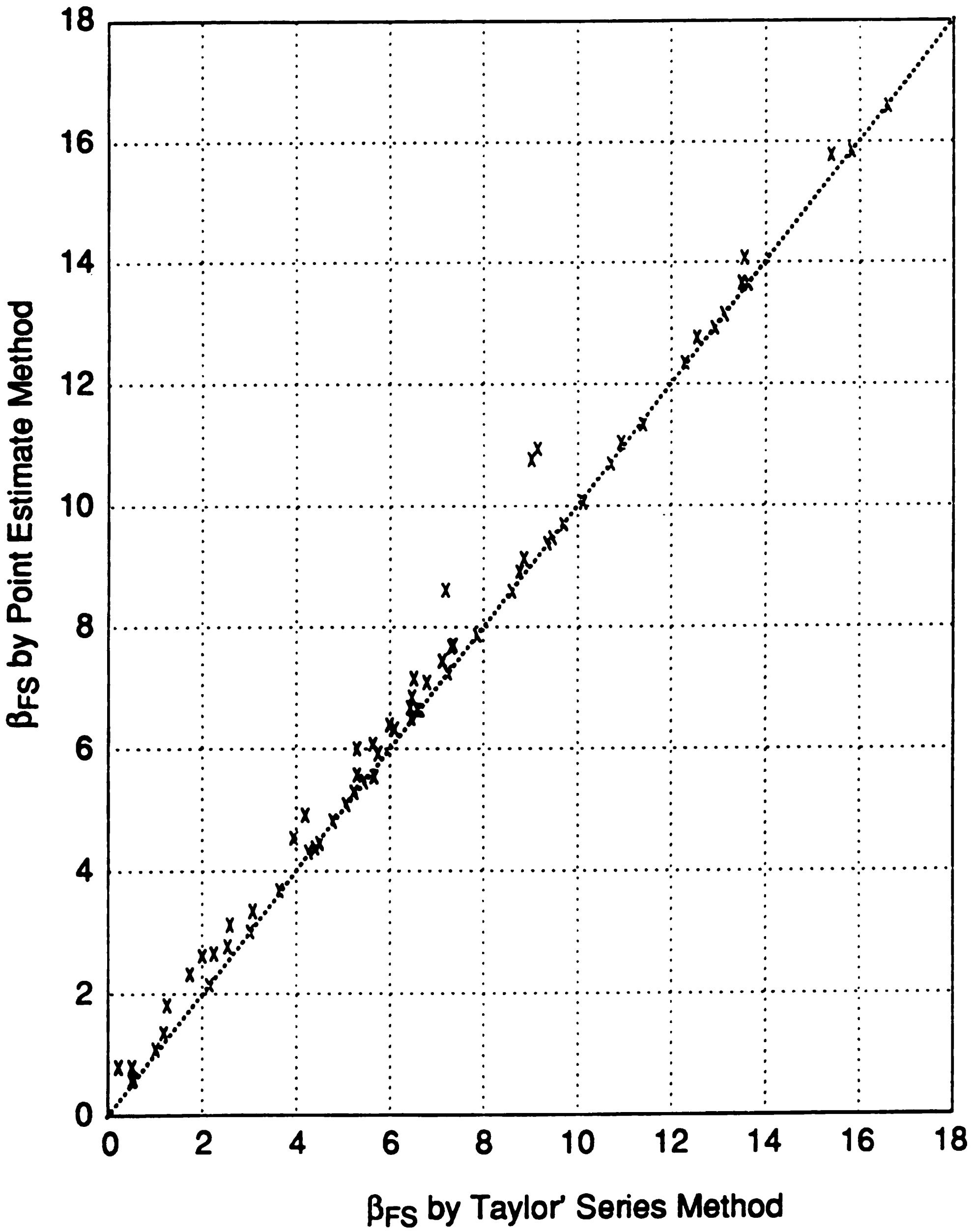


Figure 6-8 β_{FS} by Point Estimate Method versus β_{FS} by Taylor's Series Method

Chapter V.)

The comparison indicates that the Taylor's series method and the point estimate method gave almost the exactly same results though the reliability indices obtained by PEM may be slightly higher for the functions considered (sliding and bearing).

It needs to be pointed out that the bearing capacity analysis results showed much greater difference between these two methods. This difference is mainly caused by high nonlinearity of the performance function, and the higher order derivatives of the function were totally ignored when using Taylor's series method in the given examples. For example, the expected values of a performance functions calculated by Taylor's series approach were

$$E[FS] \approx FS(\bar{X})$$

but by PEM were

$$E[FS] = \frac{FS_{+} + FS_{-}}{2}$$

where FS_+ and FS_- represent the values of $FS(\bar{X}) \pm \sigma_{FS}$, respectively. As the point estimate method includes part of the nonlinearity of the functions, it may give a better estimate of reliability index.

6.5 Criteria for Overturning Stability Analysis

There are two often used criteria for overturning analysis: location of effective resultant base force X_R and factor of safety FS.

The resultant location criterion usually, in practice, is related to the area of base which is in compression and the requirements are different for different foundations and structures. For example, the Corps of Engineers[113] requires that generally the structures' base should be 100 percent in compression for soil and rock foundations, but for special cases, this requirement may be reduced to 75 percent or even less. In the analysis examples, four reliability indices, β_{toe}, $\beta_{B/2}$, $\beta_{3B/4}$ and β_B, which correspond to resultant force located at the toe, and 1/2, 3/4 and full base compression, respectively, were considered, and all these reliability indices are based on normal distribution assumption. The β_{toe} reflects how much the resisting moment exceeds the overturning moment scaled by the vertical component of resultant force, therefore, it basically represents the limit state of overturning stability, $M_R=M_O$.

The factor of safety criteria is based on the limit state of overturning stability and its reliability index β_{FS} is based on lognormal distribution assumption.

Since in overturning analysis, only the resisting overturning of structures is the concern, reliability indices β_{toe} and β_{FS} may be better measurements because of their theoretical basis. Although the percentage of base in compression is important for navigation structural safety, it may be more proper to be considered in other safety analysis, such as the bearing capacity analysis and monolith base structural (usually concrete body) analysis.

6.6 Group Reliability and Configurations of Anchors and Piles

Anchors and piles are often used in locks and dams foundations. In reliability analysis, special attention should be given to anchor and pile groups. This is not only because the group effect of anchors and piles is not yet fully understood even in deterministic methods, but also because the reliability of each single anchor or pile is seldom known and the configuration of failed anchors or piles will greatly affect the reliability of the structure. Therefore, the group reliability of anchors and piles should be properly determined and the method discussed in section 4.6.2.6 may be used as a first approximation.

Chapter VII

CONCLUSIONS AND SUGGESTIONS

7.1 Conclusions

After discussing analyses procedures and example illustrations, the following conclusions may be drawn for reliability evaluation of navigation structures.

7.1.1 The Suggested Methods Can be Used in Practice

The reliability analysis methods described in Chapter IV and V have been shown to be useful to evaluate the reliability of existing navigation structures, locks and dams. The discussed procedures are the combination of probabilistic methods and commonly-used structural analysis concepts in civil engineering, therefore, they are easily understood and suited for use in engineering applications. By using the suggested methods, remedial actions or rehabilitation at navigation structures can be rationally prioritized and limited funds may be better allocated. As a point in fact, they are already being used by the Corps of Engineers based on the results of this and related studies.

7.1.2 Using Reliability Index as Measurement

The reliability index β is a more consistent and rational measurement of the reliability of structures compared to the conventional measurement – the factor of safety. The reliability index incorporates more information, it considers the uncertainties in the structural materials, foundation materials, loads, construction quality and design and analysis formulas, and it gives more realistic and reasonable structural safety evaluations. Reliability index calculation is independent of the random variables and the precise values of those variables selected in the analysis, therefore, it much less relies on the arbitrary single values set by judgment. The reliability index is a dimensionless number regardless of the analysis object and the type of performance, and this numerical quantity indicates the degree of reliability and gives rational analysis results. As a contrast, factor of safety itself cannot actually take uncertainties into account; whether the factor of safety is properly obtained or not, it heavily relies on the experience of engineers in assuming single values for variables. Therefore, the value of the factor of safety does not truly reflect the reliability of structures. Also, the "safety" standard measured by the factor of safety is tied to a specific type of performance though it is also dimensionless.

7.1.3 Pre-Defining and Characterizing Random Variables

Loads, resistances and random variables involved in performance functions need to be predefined and characterized, especially if approximative methods are to be used.

The choice of random variables will affect the accuracy and ease of calculation of the reliability analysis. The variables which weigh more by their roles in the performance function and with greater uncertainty should be defined as random variables; otherwise, the variables can be fairly treated as deterministic ones to simplify the analysis but without losing much accuracy.

Based on the analysis examples, the uplift factor E and the shear strength parameters are important random variables and their statistical properties must be carefully determined. This study shows that

1. The E factor greatly affects the role played by hydraulic uplift force in navigation structural safety. It is better to use an iterative procedure in overturning analysis to find the active base (percent of base which is in compression) then in turn to determine the mean value of E. If the whole base is in compression, a positive value can be assigned to the mean of E when the base drainage conditions are known, otherwise a mean value equal to 0 can be used with some conservatism.

2. The shear strength parameter, cohesion c and internal friction angle ϕ(or tanϕ), can be determined based on laboratory test results, usually from direct shear test results for rock foundations. The correlation between these two shear strength parameters is important and will greatly affect the reliability analysis results, especially for sliding analysis. Note that spatial correlation also exists in soil shear strength, which needs to be considered and studied, but the correlation between c and ϕ mainly reflects the bias of applying Mohr-Coulomb's theory on soil strength.

Both the linear regression method and paired point method can be used to determine the statistical properties of the shear strength parameters but adjustment may be needed if the linear regression method is used to determine the covariance of c and tanϕ — it usually gives large negative values.

For anchored and pile foundations, the group reliability of anchors or piles is another important factor which will affect the reliability of structures. For an approximation, the binomial distribution may be applied for certain configurations of anchor or pile groups.

7.1.4 Clearly Defining Performance Functions and Criteria

In order to evaluate the reliability of navigation structures, proper performance functions and their criteria must to be clearly defined. Performance functions are the functions which represent the structural safety aspects of concern, and commonly used engineering design and analysis concepts can be chosen as these functions. The suitable criteria can be either based on the limit state theory or specific requirements, such as the percentage of base in compression.

Theoretically, the reliability index is independent of how the resistance and loads are defined but it is related to the performance function and the criteria. Although the reliability of a structure should be unique as long as all factors involved are kept the same (all loads and resistances, for instance), by using approximative methods in

Table 7-1 Sliding Analysis Results 2—Locks and Dam No.3 Monolith L-8

Strength	Pool	$E[FS^1]$	σ_{FS^1}	β_{FS^1}	$E[FS^2]$	σ_{FS^2}	β_{FS^2}
Peak	Normal	5.24	3.677	2.30	5.97	4.373	2.40
	High water	4.46	3.912	1.60	5.15	4.747	1.70
Residual	Normal	1.47	0.771	0.54	1.56	0.907	0.55
	High water	1.45	0.741	0.53	1.54	0.893	0.53

Note: FS^1—overburden force as resistance.
FS^2—overburden force as negative driving force.

reliability index calculation, different expressions for the same performance of structure may lead to different results. For example, in sliding analysis, the reliability index of factor of safety will be different for defining all horizontal forces as driving force and the base shear force T as a resisting force, from defining overburden soil resistance plus base shear force T as resisting force. Table 7-1 gives a numerical example. Although the reliability indices are very similar, they did show some difference (note greater difference among the factors of safety).

From the examples of overturning analysis in Chapter V, it is clear that different criteria will result in different reliability indices. For consistency, the criterion based on the factor of safety is recommended because of the better physical definition and more reasonable distribution assumption (lognormal distribution) of the factor of safety.

7.1.5 Simplified Methods Are Suitable

Simplified methods are suitable for navigation structural reliability analysis. The simplified methods are the reliability analysis methods discussed and illustrated in Chapter IV and Chapter V, including the definition of the performance functions and calculation method. It has been shown that both the Taylor's series approach and the point estimate method are simple, easy to apply and can give good reliability estimates. It may be worthwhile to point out that Taylor's method may have some calculational advantage if more than 3 random variables are involved and it can trace the variation contributions from each random variable to the performance; on the other hand, the point estimate method may be able to "pick up" more nonlinearity of the performance functions and make better reliability evaluations in some cases.

The reliability of rock foundations with respect to bearing capacity needs to be further studied and higher order moments or other methods may need to be used in the calculation because of the high nonlinearity of commonly used performance function.

7.1.6 Categorizing Structures by Reliability Index

Since all analysis examples in Chapter V were chosen from real existing structures, based on the field conditions of those structures and the reliability analysis results, opinions may be rendered on how structures can be categorized.

Note that these are the conclusions of the author and do not necessarily reflect the policy of the Corps of Engineers. It may be concluded that for overturning and sliding, the reliability of structures can be categorized by their reliability index:

$\beta \geq 4.0$	Structure is apparently highly reliable;
$3.0 \leq \beta < 4.0$	Structure may be marginally reliable but additional data, test or investigations should be considered to determine if uncertainty in the relevant parameters can be reduced.
$\beta < 3.0$	The structure is comparatively less reliable than well-performing structures and should be given a high priority for investigation and possible remedial action.

In connection with the probability of failure, $\beta = 4.0$ corresponds to about 3 out of 10,000 chance of failure if normal distribution on ln(FS) is assumed (see Table 7-2).

Furthermore, improvements in the structural reliability of navigation system can be prioritized by the reliability indices of the structures in the system.

7.2 Recommendations and Suggestions

Based on the findings and conclusions, some suggestions and recommendations for further research are listed as the following.

7.2.1 Recommendations

1. Probabilistic method based analysis procedures should be employed to evaluate the reliability of navigation structures and prioritize improvement to the system. This procedure should include data collection (field survey and test,

Table 7-2 Probability of Failure versus Some Typical Reliability Indices for Normal Distribution

β	*P (Failure)*
0.0	0.5
0.5	0.3085
1.0	0.1587
2.0	0.02275
3.0	0.00135
4.0	3.167×10^{-4}
5.0	2.868×10^{-7}
6.0	9.867×10^{-10}

laboratory test, operation record, maintenance record, etc.) and data characterization by the means of statistics, performance functions and associated suitable criteria identification, related random variables determination, and reliability index calculation. The procedures discussed in Chapter IV and Chapter V can be directly used.

2. Target β values should be used as a new design criterion for design of new navigation structures. Setting the target β_{FS}=4.0 may be reasonable for this purpose. As the reliability index is defined by

$$\beta_{FS} = \frac{ln\left(\frac{\mu_{FS}}{\sqrt{1+V_{FS}^2}}\right)}{\sqrt{ln(1+V_{FS}^2)}} \approx \frac{ln(\mu_{FS})}{V_{FS}}$$

where

$\mu_{FS} = E[C]/E[D]$ is the mean value of factor of safety, or the ratio of the expected value of capacity to the expected value of demand; and

$V_{FS} = \mu_{FS}/\sigma_{FS}$ is the coefficient of variation of factor of safety.

Equation 7•1 can be rewritten as

$$ln(\mu_{FS}) = ln\ (E[C]/E[D]) \approx V_{FS}\beta_{FS} \qquad (7\bullet1)$$

or

$$E[C]/E[D] \approx \exp(V_{FS}\beta_{FS}) \qquad (7\bullet2)$$

Finally,

$$E[C] = \alpha E[D] \qquad (7\bullet3)$$

where

$$\alpha \approx \exp(V_{FS}\beta_{FS}) \qquad (7\bullet4)$$

can be called reliability coefficient.

Since in the design, E[D] usually is known and the V_{FS} can be first assumed based on other information, therefore, reliability coefficient α and E[C] can be determined and then the structure can be designed. As the V_{FS} is assumed at first, the designed structure needs to be analyzed based on the performance functions and the design may need be changed; after a few iterations, a structure which satisfies the target reliability index can be put on blueprint.

7.2.2 Suggestions

The study of reliability evaluation of navigation system is far from completed. It is suggested that the following studies be carried out:

1. The response of navigation structures to earthquake and other dynamic loads. Since earthquake may be an important loading source in certain areas, and other dynamic forces, such as wind, tide and traffic loads, may also affect the safety of structures, it is important to understand the dynamic response of locks and dams to those dynamic loads in terms of probabilistic reliability.

2. Characterization of correlations between random variables. Although by using the suggested reliability analysis method, which defines the "factor of safety" as a functional variable, one does not need worry about the correlation between the demand D and capacity C, there certainly exist some correlations between random variables within the D and C. Beside the shear strength parameters c and ϕ, for example, the upper pool and lower pool levels, the saturation level and pool level may be also related each other. Understanding and characterizing these correlations between random variables will reduce some uncertainties and improve the accuracy of reliability evaluation.

3. Bearing capacity of rock foundation. The shear strength of rock is different from that of soil, and the generalized bearing capacity equation may not be suitable for rock foundation bearing capacity analysis for usually high

internal friction angle of rock materials. Study is needed to find a better method to characterize the shear parameters of rocks and to form a better analysis formula.

4. Foundation settlement. The foundation settlement is another aspect which is of great concern of the safety of locks and dams, especially when locks and dams are not built on rock foundations.

5. More study on anchor and pile foundations. Anchor and pile foundations are two very common types of foundation in navigation structures, but the group behavior of anchors or piles, especially their individual and group reliabilities, is far from totally understood. Only after the reliability of anchor and pile foundations being correctly estimated, can the reliabilities of structures built on them be well evaluated.

6. System reliability of locks and dams. To evaluate the overall reliability and prioritized navigation system, the system reliability of locks and dams must be evaluated. That is, not only the individual structural reliability, but also the entire locks and dam system's reliability needs to be evaluated. In the further study, as discussed in Chapter IV, the system configuration model, events consideration, probability of failure for each element in the system and overall reliability of the system need to be investigated. The control reliability index, reliability index of overall system or the individual structures which has minimum reliability index value, needs to be determined. Other factors, such as

the economic value, also need to be taken into consideration for the system prioritization.

7. Reliability as function of time. Similar to other civil engineering facilities, locks and dam have certain service life and the reliability of navigation structures changes as time passes. To predict the reliability of navigation structures, and make long term rehabilitation plans, the reliability of navigation structures and the system must be studied as a function of time.

8. Three dimensional structural reliability analysis. The study carried out herein was focused on two dimensional problems. Since the real structures are in three dimensions and the reliability of three-dimensional analysis may be different from that in two-dimensional analysis, the difference of two-dimensional and three-dimensional reliability must be compared to check and improve the suggested reliability evaluation procedure.

9. Develop an expert system. The final product of the navigation system reliability evaluation study should be an expert system. Based on all data and knowledge, stored by experts and freshly input by users, in the knowledge base, using intelligent analysis procedures, this expert system could give a good reliability evaluation of navigation structures and systems considering all possible factors involved, therefore, it will help prioritize navigation system and optimize the function of the system to benefit the nation's economy.

APPENDIX A

DERIVATION OF FIRST TWO MOMENTS FOR A FUNCTION WITH CORRELATED MULTIPLE VARIABLES IN TAYLOR'S SERIES EXPRESSION

Appendix A

Derivation of First Two Moments for a Function with Correlated Multiple Variables in Taylor's Series Expression

For a function

$$F = F(x_1, x_2, \ldots, x_i, \ldots, x_n) = F(X) \quad i=1,2,\ldots, n \tag{1}$$

where x_i are random variables, the Taylor's series expansion of F about the means of x_i (denoted by $\overline{x_i}$), omitting the terms with higher than second order of derivatives of F, is

$$F \approx F(\bar{X}) + \sum_{i=1}^{n} \frac{\partial F}{\partial x_i}(x_i - \bar{x}_i) + \frac{1}{2} \sum_{i,j=1}^{n} \frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j}) \tag{2}$$

Then the expected value of F is

$$E[F] \approx E\left[F(\bar{X}) + \sum_{i=1}^{n} \frac{\partial F}{\partial x_i}(x_i - \bar{x}_i) + \frac{1}{2} \sum_{i,j=1}^{n} \frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j})\right] \tag{3}$$

Since

$$E[C] = C \qquad C \text{ is constant} \tag{4}$$

$$E[aX + bY] = aE[X] + bE[Y] \qquad a \text{ and } b \text{ are constant} \tag{5}$$

and

$$E[(X - \bar{X})(Y - \bar{Y})] = Cov(X,Y) \quad \text{for } X \neq Y$$
$$= Cov(X,X) = Var(X) \quad \text{for } X = Y \tag{6}$$

so

$E[(x_i - \bar{x}_i)] = 0$ and

$$E[F] \approx F(\bar{X}) + \frac{1}{2} \sum_{i,j=1}^{n} \frac{\partial^2 F}{\partial x_i \partial x_j} Cov(x_i x_j) \tag{7}$$

or

$$E[F] \approx F(\bar{X}) + \frac{1}{2}\sum_{i=1}^{n}\frac{\partial^2 F}{\partial x_i^2}Var(x_i) + \sum_{i=1, i<j}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}Cov(x_i x_j) \quad (8)$$

Denote that

$$F_A = \frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}Cov(x_i x_j) \quad (9)$$

then

$$E[F] = F(\bar{X}) + F_A \quad (10)$$

The variance of F is

$$Var(F) = E[(F - E[F])^2]$$

$$\approx E[\{ \left(F(\bar{X}) + \sum_{i=1}^{n}\frac{\partial F}{\partial x_i}(x_i - \bar{x}_i) + \frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j})\right)$$

$$- (F(\bar{X}) + F_A)\}^2]$$

$$= E\left[\left(\sum_{i=1}^{n}\frac{\partial F}{\partial x_i}(x_i - \bar{x}_i) + \frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j}) - F_A\right)^2\right]$$

$$= E\left[\left(\sum_{i=1}^{n}\frac{\partial F}{\partial x_i}(x_i - \bar{x}_i)\right)^2 + \left(\frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j})\right)^2 + (F_A)^2\right.$$

$$+ 2\left(\sum_{i=1}^{n}\frac{\partial F}{\partial x_i}(x_i - \bar{x}_i)\right)\left(\frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j})\right)$$

$$\left.-2F_A\left(\sum_{i=1}^{n}\frac{\partial F}{\partial x_i}(x_i - \bar{x}_i)\right) - 2F_A\left(\frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j})\right)\right] \quad (11)$$

Since

$$E\left[\sum_{i=1}^{n}\frac{\partial F}{\partial x_i}(x_i - \bar{x}_i)\right] = 0 \qquad \text{and}$$

$$E\left[\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j})\right] = 2F_A$$

define that

$$A = E\left[\left(\sum_{i=1}^{n} \frac{\partial F}{\partial x_i}(x_i - \bar{x}_i)\right)^2\right] \tag{12}$$

$$B = E\left[\frac{1}{4}\left(\sum_{i,\,j=1}^{n} \frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j})\right)^2\right] \tag{13}$$

and $C = E\left[\left(\sum_{i=1}^{n} \frac{\partial F}{\partial x_i}(x_i - \bar{x}_i)\right)\left(\sum_{i,\,j=1}^{n} \frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j})\right)\right]$ (14)

then eqn (11) can be written as

$$Var(F) \approx A + B + C - (F_A)^2 \tag{15}$$

Let's examine A, B and C.

$$A = E\left[\left(\sum_{i=1}^{n} \frac{\partial F}{\partial x_i}(x_i - \bar{x}_i)\right)^2\right]$$

$$= \sum_{i,\,j=1}^{n} \left(\frac{\partial F}{\partial x_i}\right)\left(\frac{\partial F}{\partial x_j}\right) Cov(x_i x_j)$$

$$= \sum_{i=1}^{n} \left(\frac{\partial F}{\partial x_i}\right)^2 Var(x_i) + 2 \sum_{i=1,\, i<j}^{n} \left(\frac{\partial F}{\partial x_i}\right)\left(\frac{\partial F}{\partial x_j}\right) Cov(x_i x_j) \tag{16}$$

$$B = \frac{1}{4} \; E\left[\left(\sum_{i,\,j=1}^{n} \frac{\partial^2 F}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j})\right)^2\right]$$

$$= \frac{1}{4}\Bigg\{ \sum_{i,\,j=1}^{n} \left(\frac{\partial^2 F}{\partial x_i^2}\right)\left(\frac{\partial^2 F}{\partial x_j^2}\right) E[(x_i - \overline{x_i})^2 (x_j - \overline{x_j})^2]$$

$$+2 \sum_{i=1,\, i<j}^{n} \sum_{s=i,\, s<t}^{n} \frac{\partial^2 F}{\partial x_i x_j} \frac{\partial^2 F}{\partial x_s x_t} E[(x_i - \overline{x_i})(x_j - \overline{x_j})(x_s - \overline{x_s})(x_t - \overline{x_t})] \Bigg\}$$

$$\approx \frac{1}{4}\sum_{i=1}^{n}\left(\frac{\partial^2 F}{\partial {x_i}^2}\right)^2 E[(x_i-\overline{x_i})^4] \tag{17}$$

$$C = E\left[\left(\sum_{i=1}^{n}\frac{\partial^2 F}{\partial {x_i}^2}(x_i-\overline{x_i})\right)\left(\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}(x_i-\overline{x_i})(x_j-\overline{x_j})\right)\right]$$

$$= \sum_{i,j=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)\left(\frac{\partial^2 F}{\partial {x_j}^2}\right)E[((x_i-\overline{x_i})^2)(x_j-\overline{x_j})] \quad +$$

$$+\ 2\sum_{i=1}^{n}\ \sum_{s<i,s<t}^{n}\left(\frac{\partial F}{\partial x_i}\right)\left(\frac{\partial^2 F}{\partial x_s \partial x_t}\right)E[(x_i-\overline{x_i})(x_s-\overline{x_s})(x_t-\overline{x_t})]$$

$$\approx \sum_{i=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)\left(\frac{\partial^2 F}{\partial {x_i}^2}\right)E[(x_i-\overline{x_i})^3] \tag{18}$$

Finally the expression of variance of F is

$$Var(F) \approx \sum_{i,j=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)\left(\frac{\partial F}{\partial x_j}\right)Cov(x_i x_j) + \frac{1}{4}\sum_{i=1}^{n}\left(\frac{\partial^2 F}{\partial {x_i}^2}\right)^2 E[(x_i-\overline{x_i})^4]$$

$$+ \sum_{i=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)\left(\frac{\partial^2 F}{\partial {x_i}^2}\right)E[(x_i-\overline{x_i})^3] - \frac{1}{4}\left(\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}Cov(x_i x_j)\right)^2 \tag{19}$$

If x_i are normally distributed, then

$E[(x_i - \overline{x_i})^3] = 0$ and $E[(x_i - \overline{x_i})^4] = 3$, therefore

$$Var(F) \approx \sum_{i,j=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)\left(\frac{\partial F}{\partial x_j}\right)Cov(x_i x_j) + \frac{3}{4}\sum_{i=1}^{n}\left(\frac{\partial^2 F}{\partial {x_i}^2}\right)^2 -$$

$$-\frac{1}{4}\left(\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}Cov(x_i x_j)\right)^2 \tag{20}$$

The approximation for B and C can be obtained by proving that if using Taylor's series approach up to second order, the $E[\Sigma]$ terms involved have the properties of

$$E[(x_i - \overline{x_i})(x_j - \overline{x_j})(x_s - \overline{x_s})] \approx 0 \tag{21}$$

$$E[(x_i - \overline{x_i})^2 (x_j - \overline{x_j})] \approx 0 \tag{22}$$

$$E[(x_i - \overline{x_i})^2 (x_j - \overline{x_j})^2] \approx 0 \tag{23}$$

$$E[(x_i - \overline{x_i})^3 (x_j - \overline{x_j})] \approx 0 \tag{24}$$

and

$$E[(x_i - \overline{x_i})^2 (x_j - \overline{x_j})(x_s - \overline{x_s})] \approx 0 \tag{25}$$

In engineering practice, usually the function F does not have a second derivative with respect to some random variables and their third and fourth moments are usually unknown (though we can assume some typical probability distributions), therefore, the commonly used Taylor's series approach expressions for a function F with correlated random variables are

$$E[F] \approx F(\bar{X}) + \frac{1}{2}\sum_{i=1}^{n}\frac{\partial^2 F}{\partial x_i^2}Var(x_i) + \sum_{i=1,\, i<j}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}Cov(x_i x_j) \approx F(\bar{X}) \tag{26}$$

and

$$Var(F) \approx \sum_{i,\, j=1}^{n}\frac{\partial F}{\partial x_i}\frac{\partial F}{\partial x_j}Cov(x_i x_j)$$

$$= \sum_{i=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)^2 Var(x_i) + 2\sum_{i=1,\, i<j}^{n}\frac{\partial F}{\partial x_i}\frac{\partial F}{\partial x_j}Cov(x_i x_j) \tag{27}$$

APPENDIX B

STATISTICAL CONVERSIONS BETWEEN tan ϕ AND ϕ

Appendix B

Statistical Conversions between tan ϕ and ϕ

In reliability analysis of earth structures, the internal friction angle of earth material, ϕ, is a important random variable. In structural analysis, sometime the angle ϕ is needed, sometimes the tanϕ is actually in use. It is not unusual that the statistical moments, very often the first two moments — mean and standard deviation, only either for ϕ or tanϕ are known but both of them are needed. The conversions between tan ϕ and ϕ will be discussed as the following.

Methods of Conversion

In practice, two approximative methods can be used for converting ϕ and tan ϕ, namely, Taylor's series method and PEM (Point Estimate Method).

— Taylor's Series Method

As usual only first two statistic moments are known, if variable Y is the function of variable X, the Taylor's series expansion about the mean of X, $\bar{X}$, omitting the terms higher than the second order, is

$$Y = Y(X) \approx Y(\bar{X}) + Y'(X-\bar{X}) + \frac{1}{2}Y''(X-\bar{X})^2$$

The first two statistic moments of Y are

$$E[Y] \approx Y(\bar{X}) + \frac{1}{2}Y'Var(X)$$

and $$Var(Y) \approx (Y')^2 Var(X)$$

— PEM (Point Estimate Method)

The formulas for PEM are

$$E[Y] \approx \frac{Y(\bar{X}+\sigma_X) + Y(\bar{X}-\sigma_X)}{2}$$

$$E[Y^2] \approx \frac{(Y(\bar{X}+\sigma_X))^2 + (Y(\bar{X}-\sigma_X))^2}{2}$$

and $$Var(Y) = E(Y^2) - (E(Y))^2$$

Converting from ϕ to $\tan\phi$

Suppose that $E(\phi) = \bar{\phi}$ and $Var(\phi)$ are known.

– *Taylor's series method*

$$\tan(\phi) \approx \bar{\phi} + \sec^2(\bar{\phi})(\phi-\bar{\phi}) + \sec^2(\bar{\phi})\tan(\bar{\phi})(\phi-\bar{\phi})^2$$

$$E[\tan\phi] \approx \tan(\bar{\phi}) + \sec^2(\bar{\phi})\tan(\bar{\phi})\,Var(\phi)$$

and

$$Var(\tan\phi) \approx \sec^4(\bar{\phi})\,Var(\phi)$$

Example:

Let $\bar{\phi} = 33°$, $c.o.v = 10\%$, or $Var(\phi) = \left(\frac{3.3°\pi}{180°}\right)^2 = 0.003317$

then

$$E[\tan\phi] \approx \tan 33° + \sec^2(33°)\tan(33°)(0.003317) = 0.65247$$

and

$$Var(\tan\phi) \approx \sec^4(33°)(0.003317) = 0.00670$$

or $\sigma_{\tan\phi} = 0.08188$ $\quad c.o.v = 12.55\%$

– *PEM*

Example: $\bar{\phi} = 33°$, $c.o.v = 10\%$

$$E[\tan\phi] \approx \frac{\tan(33° + 3.3°) + \tan(33° - 3.3°)}{2} = 0.65248 \quad \text{and}$$

$$Var(\tan\phi) \approx \frac{\tan^2(33° + 3.3°) + \tan^2(33° - 3.3°)}{2} - (0.65248)^2 = 0.00674$$

and $\sigma_{tan\phi} = 0.08210$ $\quad c.o.v = 12.58\%$

Converting from tanϕ to ϕ

Suppose that $E[\tan\phi] = \overline{\tan\phi} = \bar{X}$ and $Var(\tan\phi)$ are known.

– *Taylor's series method*

Let $\tan\phi = X$ then $\phi = \tan^{-1}X$, therefore

$$\phi \approx \tan^{-1}\bar{X} + \frac{1}{1+\bar{X}^2}(X - \bar{X}) - \frac{\bar{X}}{(1+\bar{X}^2)^2}(X - \bar{X})^2$$

$$E[\phi] \approx \tan^{-1}\bar{X} - \frac{\bar{X}}{(1+\bar{X}^2)^2} var(X) \qquad \text{and}$$

$$Var(\phi) \approx \left(\frac{1}{1+\bar{X}^2}\right)^2 var(X)$$

Example:

$\overline{\tan\phi} = \bar{X} = 1.50$ $\qquad \sigma_{\tan\phi} = 0.675$

then

$$E[\phi] \approx \tan^{-1}(1.50) - \frac{1.50}{(1+(1.50)^2)^2}(0.675)^2 = 0.9181 \text{ (rad.)}$$
$$= 52.60 \text{ (deg.)}$$

$$Var(\phi) \approx \frac{1}{(1+(1.5)^2)^2}(0.675)^2 = 0.043136 \quad (\text{rad.}^2)$$

or $\sigma_\phi = 0.20769$ (rad.) $\quad c.o.v = 22.62\%$

– *PEM*

Example:

$\overline{\tan\phi} = \bar{X} = 1.50 \qquad \sigma_{\tan\phi} = 0.675$

$$E[\phi] \approx \frac{\tan^{-1}(1.5+0.675)+\tan^{-1}(1.5-0.675)}{2} = 0.9148 \text{ (rad.)}$$

$$= 52.42 \text{ (deg.)}$$

$$Var(\phi) \approx \frac{(\tan^{-1}(1.5+0.675))^2+(\tan^{-1}(1.5-0.675))^2}{2} - 0.9148^2$$

$$= 0.050679 \text{ (rad.}^2\text{)}$$

or $\sigma_\phi = 0.22512$ (rad.) $c.o.v = 24.61\%$

Correlation Between c − tan ϕ and c − ϕ

Usually there is correlation between cohesion, c, and ϕ or c and $\tan\phi$. The coefficient of correlation, ρ, will change but very small when converting from ϕ to $\tan\phi$ or from $\tan\phi$ to ϕ.

Proof by Taylor's series approximation:

Generally, if $Y = Y(Z)$ and $Cov(X,Z)$ is known, and

$$E[Y(Z)] = \bar{Y} \approx Y(\bar{Z}) + Y''Var(Z)/2 = Y(\bar{Z}) + Y_A$$

then

$$Cov(X,Y(Z)) = E[(X-\bar{X})(Y(Z)-\bar{Y})]$$

$$\approx E[(X-\bar{X})(Y(\bar{Z})+Y'(Z-\bar{Z})+Y''(Z-\bar{Z})^2/2 - Y(\bar{Z}) - Y_A)]$$

$$= E[(X-\bar{X})(Y'(Z-\bar{Z})+Y''(Z-\bar{Z})^2/2 - Y_A)]$$

$$= E[(X-\bar{X})Y'(Z-\bar{Z})] + E[(X-\bar{X})Y''(Z-\bar{Z})^2/2] - E[(X-\bar{X})Y_A]$$

$$\approx Y'Cov(X,Z) \qquad (*)$$

Proof of (*):

In eqn. (*), the third term, $E[(X-\bar{X})Y_A] = 0$, the first term

$$E[(X-\bar{X})Y'(Z-\bar{Z})] = Y'E[(X-\bar{X})(Z-\bar{Z})] = Y' \, Cov(X,Z)$$

and the second term

$$E[(X-\bar{X})Y''(Z-\bar{Z})^2/2] = (Y''/2)E[(X-\bar{X})(Z-\bar{Z})^2] \approx 0 \qquad (**)$$

Proof of (**):

$$E[(X-\bar{X})(Z-\bar{Z})^2]$$

$$= E[(X-\bar{X})(Z-\bar{Z})(Z-\bar{Z})]$$

$$=E[(X-\bar{X})(Z-\bar{Z})Z - \bar{Z}(X-\bar{X})(Z-\bar{Z})]$$

$$=E[(X-\bar{X})(Z-\bar{Z})Z] - \bar{Z}E[(X-\bar{X})(Z-\bar{Z})]$$

$$= E[XZ^2 - \bar{Z}XZ - \bar{X}Z^2 + \bar{X}\bar{Z}Z] - \bar{Z}Cov(X,Z)$$

By Taylor's series:

$$XZ^2 \approx \bar{X}\bar{Z}^2 + \bar{Z}^2(X-\bar{X}) + 2\bar{X}\bar{Z}(Z-\bar{Z}) + \bar{X}(Z-\bar{Z})^2 + 2\bar{Z}(X-\bar{X})(Z-\bar{Z})$$

$$\therefore \quad E[XZ^2] \approx \bar{X}\bar{Z}^2 + \bar{X}E[(Z-\bar{Z})^2] + 2\bar{Z}E[(X-\bar{X})(Z-\bar{Z})]$$

$$= \bar{X}\bar{Z}^2 + \bar{X}Var(Z) + 2\bar{Z}Cov(X,Z)$$

Also $\quad E[\bar{Z}XZ] = \bar{Z}E[XZ] = \bar{Z}\,(Cov(X,Z) + \bar{X}\bar{Z})$

$$= \bar{Z}\,Cov(X,Z) + \bar{X}\bar{Z}^2$$

$$E[\bar{X}Z^2] = \bar{X}E[Z^2] = \bar{X}(Var(Z) + \bar{Z}^2) = \bar{X}Var(Z) + \bar{X}\,\bar{Z}^2$$

then

$$E[(X-\bar{X})(Z-\bar{Z})^2]$$

$$= E[XZ^2 - \bar{Z}XZ - \bar{X}Z^2 + \bar{X}\bar{Z}Z] - \bar{Z}Cov(X,Z)$$

$$\approx \bar{X}\bar{Z}^2 + \bar{X}Var(Z) + 2\bar{Z}Cov(X,Z) - \bar{Z}\,Cov(X,Z) - \bar{X}\bar{Z}^2$$

$$- \bar{X}Var(Z) - \bar{X}\,\bar{Z}^2 + \bar{X}\bar{Z}^2 - \bar{Z}Cov(X,Z)$$

$$=0$$

so

$$Cov(X,Y(Z)) = E[(X-\bar{X})(Y-\bar{Y})] \approx Y'Cov(X,Z)$$

and

$$\rho_{X,Y} = \frac{Cov(X, Y(Z))}{\sigma_X \sigma_Y} = \frac{Y' Cov(X, Z)}{\sigma_X \sigma_Y} = \frac{Y' \rho_{XZ} \sigma_X \sigma_Z}{\sigma_X \sigma_Y} = \frac{Y' \rho_{XZ} \sigma_Z}{\sigma_Y}$$

//

Example:

$E[c] = 11.0, \quad \sigma_c = 7.7$

$E[\tan\phi] = 1.50, \ \sigma_{\tan\phi} = 0.675$

$\rho_{c,\tan\phi} = -0.70$

Let $\phi = \tan^{-1}(\tan\phi)$

$$\phi' = \frac{1}{1 + (\overline{\tan\phi})^2} = \frac{1}{1 + (1.5)^2} = \frac{1}{3.25}$$

then

$$\rho_{c,\phi} = \frac{\phi' \rho_{c,\tan\phi} \sigma_{\tan\phi}}{\sigma_\phi} = \frac{(-0.70)(0.675)}{(3.25)(0.2077)} = -0.69997 \approx -0.70$$

Note that the σ_ϕ was converted from $\tan\phi$ by Taylor's series approximation (see previous example).

In practice, assume that $\rho_{c,\phi} \approx \rho_{c,\tan\phi}$ is acceptable.

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