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MODELING OF DRIVELINE NOISE USING
STATISTICAL ENERGY ANALYSIS

presented by

NELSON SCOTT EMERY

has been accepted towards fulfillment
of the requirements for

MASTERS degree in MECH. ENG.

Clark J. Radcliffe
Major professor

Date August 12, 1991

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MODELING OF DRIVELINE NOISE USING STATISTICAL ENERGY ANALYSIS

By

Nelson Scott Emery

A THESIS

Submitted to
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in partial fulfillment of the requirements
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ABSTRACT

MODELING OF DRIVELINE NOISE USING STATISTICAL ENERGY ANALYSIS

By

Nelson Scott Emery

Noise and vibration levels of mechanical systems are historically modeled using differential equation techniques. These modeling techniques have problems describing systems with large numbers of modes. Statistical Energy Analysis is a useful technique for predicting noise and vibration levels for systems with large numbers of modes. Software was developed to model complex multi-modal systems using Statistical Energy Analysis. The utility of the software is demonstrated with the development of an automobile noise model. The computer predictions are compared with experimental results. The noise and vibration levels computed by the software are typically within 10 dB of the experimental results for the modeled components.

DEDICATION

To my parents, Bill and Rachel Emery

ACKNOWLEDGEMENTS

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NOMENCLATURE

$<p^2>$	= mean-square pressure, expected value (N^2/m^4)
$<v^2>$	= mean-square velocity, expected value (m^2/sec^2)
[N]	= loss and coupling loss factor matrix (dimensionless)
A	= FPL_FPL coupling loss factor intermediate calculation variable
A	= plate surface area (m^2)
A_{door}	= surface area of door (m^2)
A_{hatch}	= surface area of hatch (m^2)
A_p	= plate surface area (m^2)
A_{pi}	= plate i surface area (m^2)
A_{roof}	= surface area of roof (m^2)
A_T	= *body panels* combined surface area (m^2)
A_{wind}	= surface area of windows (m^2)
A_{xb}	= beam cross sectional area (m^2)
B	= FPL_FPL coupling loss factor intermediate calculation variable
bet(i)	= FPL_FPL coupling loss factor intermediate calculation variable
C	= FPL_FPL coupling loss factor intermediate calculation variable
c	= speed of sound (m/sec)
C_3	= FPL_BEAM coupling loss factor intermediate calculation variable
C_4	= FPL_BEAM coupling loss factor intermediate calculation variable
c_a	= speed of sound in air (m/sec)
c_{fb}	= beam flexural wave speed (m/sec)

c_{fi}	= element i flexural wave speed (m/sec)
c_l	= longitudinal wave speed (m/sec)
c_l	= plate longitudinal wave speed (m/sec)
c_{lb}	= beam longitudinal wave speed (m/sec)
clf_f	= flexural coupling loss factor
clf_t	= torsional coupling loss factor
c_{li}	= longitudinal wave speed of element i (m/sec)
c_{lp}	= plate longitudinal wave speed (m/sec)
c_t	= torsional wave speed (m/sec)
c_{tb}	= beam torsional wave speed (m/sec)
D	= FPL_FPL coupling loss factor intermediate calculation variable
$deti$	= determinant of matrix i
D_i	= inner diameter beam with circular cross section (m)
D_o	= outer diameter beam with circular cross section (m)
E	= FPL_FPL coupling loss factor intermediate calculation variable
E_b	= beam Young's modulus (GPa)
$E_i(\omega)$	= subsystem total energy (joules-sec/rad)
$E_j(\omega)$	= subsystem total energy (joules-sec/rad)
E_p	= plate Young's modulus (GPa)
F	= FPL_FPL coupling loss factor intermediate calculation variable
f	= center frequency (Hz)
f_c	= critical frequency (Hz)
F_l	= FPL_BEAM coupling loss factor calculation constant
G	= shear modulus (GPa)
γ	= angle between plate and beam in FPL_BEAM
G_b	= beam shear modulus (GPa)
h	= plate or beam thickness (m)

h_{door} = plate thickness of door (m)

h_{equiv} = *body panels* total plate thickness (m)

h_{hatch} = plate thickness of hatch (m)

h_p = plate thickness (m)

h_{roof} = plate thickness of roof (m)

h_{wind} = plate thickness of windows (m)

k = radius of gyration (m)

k_b = beam radius of gyration (m)

k_p = plate radius of gyration (m)

k_{tb} = beam torsional radius of gyration (m)

l = beam length (m)

l = FPL_FPL joint length (m)

l = plate length (m)

m = structure mass (kg)

m_b = beam mass (kg)

m_s = plate structural mass (kg)

m_s = structural mass (kg)

N = Mode count per frequency band (modes)

n = modal density (modes-sec/rad)

n = modal density (modes-sec/rad)

n_a = acoustic element modal density (modes-sec/rad)

n_a = modal density of an acoustic element (modes-sec/rad)

n_{door} = modal density of door (modes-sec/rad)

n_{fb} = beam flexural modal density (modes-sec/rad)

n_{fp} = plate flexural modal density (modes-sec/rad)

n_{hatch} = modal density of hatch (modes-sec/rad)

$n_j(\omega)$ = modal density of subsystem "j" (modes-sec/rad)

n_{roof} = modal density of roof (modes-sec/rad)

n_s = modal density of a structural element (modes-sec/rad)

n_s = structural element modal density

n_{tb} = beam torsional modal density

n_{tot} = *body panels* total modal density (modes-sec/rad)

n_{wind} = modal density of windows (modes-sec/rad)

P = plate perimeter edge length (m)

p = number of connection points in FPL_PLB connector

P_{diss_i} = power dissipated by subsystem "i" (Watts)

P_{in_i} = power input to subsystem "i" (Watts)

P_{jk} = power transferred from subsystem "j" to "k" (Watts)

P_r = plate perimeter (m)

Q = Quality factor

r = wave number ratio (dimensionless)

r_{bi} = bending rigidity of element i

R_{equiv} = equivalent radius (m)

r_{hw} = ratio of beam thickness to beam width (dimensionless)

R_{rad} = plate radiation resistance (kg-rad/sec)

tao = FPL_FPL coupling loss factor intermediate calculation variable

tb = FPL_FPL coupling loss factor intermediate calculation variable

tf = FPL_FPL coupling loss factor intermediate calculation variable

T_R = Reverberation time (sec)

V = acoustic volume (m^3)

w = plate width (m)

x_a = FPL_BEAM coupling loss factor intermediate calculation variable

x_{a2} = FPL_BEAM coupling loss factor intermediate calculation variable

x_b = FPL_BEAM coupling loss factor intermediate calculation variable

x_c = FPL_BEAM coupling loss factor intermediate calculation variable

z_b = beam impedance

z_{bf} = beam flexural impedance

z_{bm} = beam moment impedance

z_{bt} = beam torsional impedance

z_p = plate impedance

z_{pf} = plate flexural impedance

z_{pm} = plate moment impedance

Greek Letters

α = $(f/f_c)^{1/2}$ (dimensionless)

β = plate edge condition (dimensionless)

Δf = frequency band centered at f (Hz)

γ = angle between plate and beam in FPL_BEAM

η = loss factor

η_{as} = acoustic to structural element coupling loss factor

h_i = thickness of element i

$\eta_i(\omega)$ = subsystem i loss factor (dimensionless)

η_{ij} = i to j coupling loss factor

η_{pb} = plate to beam coupling loss factor

η_{sa} = structure to acoustic element coupling loss factor

K_b = beam wave number (m^{-1})

K_i = element i wave number (m^{-1})

K_p = plate wave number (m^{-1})

λ_a = wavelength of sound in acoustic volume (m)

λ_c = wavelength of sound at critical frequency (m)

μ = angle between plate and beam in FPL_BEAM (degrees)

θ = angle between two coupled plates (degrees)

ρ = density (kg/m^3)

ρ_a = density of air (kg/m^3)

ρ_b = beam mass density (kg/m^3)

ρ_ι = element i mass density (kg/m^3)

ρ_{lb} = beam lineal density (kg/m^3)

ρ_p = plate mass density (kg/m^3)

σ_{rad} = plate radiation efficiency

ω = band center frequency (rad/sec)

ζ = damping ratio (dimensionless)

INTRODUCTION

The customer's perception of automobile quality is directly linked to vehicle interior noise amplitude and quality. Noise amplitudes are quantified by sound pressure levels. Noise quality is related to the sound spectrum. Sound level measurements such as A-weighted sound pressure levels attempt to adjust these levels to be more representative of human hearing perception. In 1990, the Toyota Motor Company introduced the Lexus LS400 with an advertising campaign focusing on the automobile's interior sound quality. Although other vehicles had similar low sound pressure levels, the Lexus was chosen quietest (Consumer Reports, June 1990) based on hearing perception.

Automobile noise is typically modeled using ordinary differential equation models based on structural analysis. Finite element analysis, a differential-equation-based modeling technique, models a system's natural frequencies and mode shapes. The number of modes present in complex systems becomes large at high frequencies. For example, the modal density of an acoustic volume increases at an exponential rate as frequency increases (Figure 1). When analyzing systems with large numbers of modes, differential equation methods require many calculations, are sensitive to small variations in model parameters, and generate large amounts of data. Due to the limits of differential equation methods, vehicle noise problems are difficult to analyze with limited information available from early design specifications and are typically addressed in the latter stages of the design process when production prototypes are available for testing.

Statistical Energy Analysis is a method of modeling steady state energy storage and power flow in systems with large numbers of modes per frequency bandwidth. For complex systems, predictions of sound pressure levels as a function of frequency can be made early in the design process because Statistical Energy Analysis is not sensitive to design details. Since Statistical Energy Analysis produces results in the frequency domain, both noise amplitude and quality are predicted.

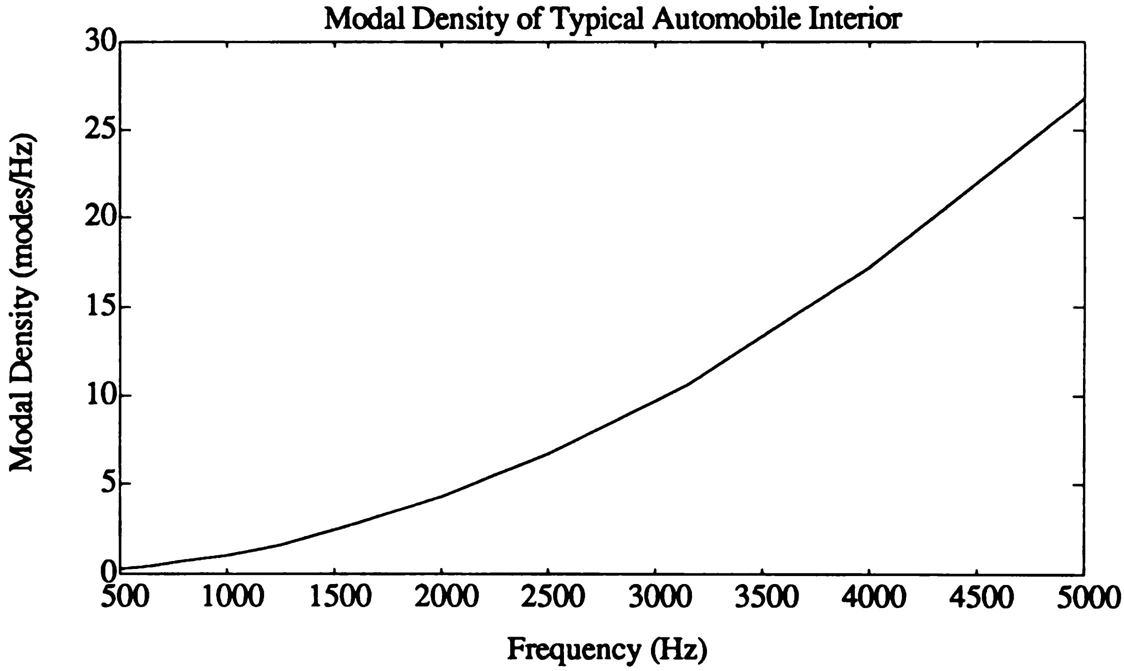

Figure 1: Acoustic Modal Density of a Typical Automobile Interior versus Frequency

Software was developed to evaluate structural systems using Statistical Energy Analysis (See Appendices). As an example of the software's capabilities, a model is built to predict the noise level spectrum of a typical automobile interior. Vehicle noise is generally broken into three components: road, wind, and driveline. This study focuses on determining the requirements for Statistical Energy Analysis (SEA) modeling of driveline noise for a typical automobile. A model of a typical automobile which emphasizes driveline noise is developed and evaluated. In addition, an experiment to determine model parameters and to verify the modeling process was conducted and the results are reported.

STATISTICAL ENERGY ANALYSIS

Statistical Energy Analysis is a method for modeling energy storage and power flow in vibrational and acoustical systems. Elements store energy. Connectors transfer energy between elements. External sources such as turbulent flow, acoustic excitation, and mechanical vibration provide energy to the system (Lyon 1975, 10; Beranek 1971, 297). The mean squared velocities or accelerations of structural elements and the mean squared pressure levels of acoustic elements can be calculated from element energy levels.

Statistical Energy Analysis was developed as a 'simple' approach to modeling complex multi-modal systems (Ungar 1967, 626). SEA is robust in that it is not sensitive to design details. This is an advantage over other methods because the engineer is often required to make response predictions at a point in the design process when little detail is available. SEA provides a framework for modeling based on fundamental parameters such as average panel thickness and damping (Lyon 1975, 4). There are two general categories in an SEA model: energy storage and energy transfer (Lyon 1975, 12). Energy is stored in model elements. Energy transfer includes energy lost to element internal damping and energy transferred between elements.

Details of SEA theory are best illustrated with the simple vehicle acoustic energy transmission model shown below in Figure 2. This figure shows a conceptual model of coarse road noise transmitted into the vehicle interior. The conceptual model shown will be used to predict acoustic response of the vehicle's interior. Boxes 1-3 represent three vehicle subsystem models: the tires, the suspension/body structure and the interior acoustic volume. The variables, P, represent power flows into, through, and dissipated by each subsystem.

The power flow of each vehicle subsystem must satisfy a steady-state flow balance.

$$P_{in_1} = P_{diss\ 1} + P_{12} + P_{13} \tag{1a}$$

$$P_{in_2} = P_{diss\ 2} - P_{12} + P_{23} \tag{1b}$$

$$P_{in_3} = P_{diss\ 3} - P_{23} - P_{13} \tag{1c}$$

where: P_{in_i} = power input to subsystem "i"

P_{diss_i} = power dissipated by subsystem "i"

P_{jk} = power transferred from subsystem "j" to "k"

In this example, P_{12} represents the structural power flow from the tire structure to the suspension and body structure while P_{13} represents the airborne acoustic power directly to the vehicle interior.

The steady-state power dissipated in each subsystem, P_{diss_i}, in a 1 rad/sec band centered at frequency ω can be written in terms of that subsystem's energy in that band, band frequency, and internal loss factor.

$$P_{diss_i} = \omega\eta_i E_i \tag{2}$$

where: ω = band center frequency (rad/sec)

$\eta_i(\omega)$ = subsystem loss factor (dimensionless)

$E_i(\omega)$ = subsystem total energy (joules-sec/rad)

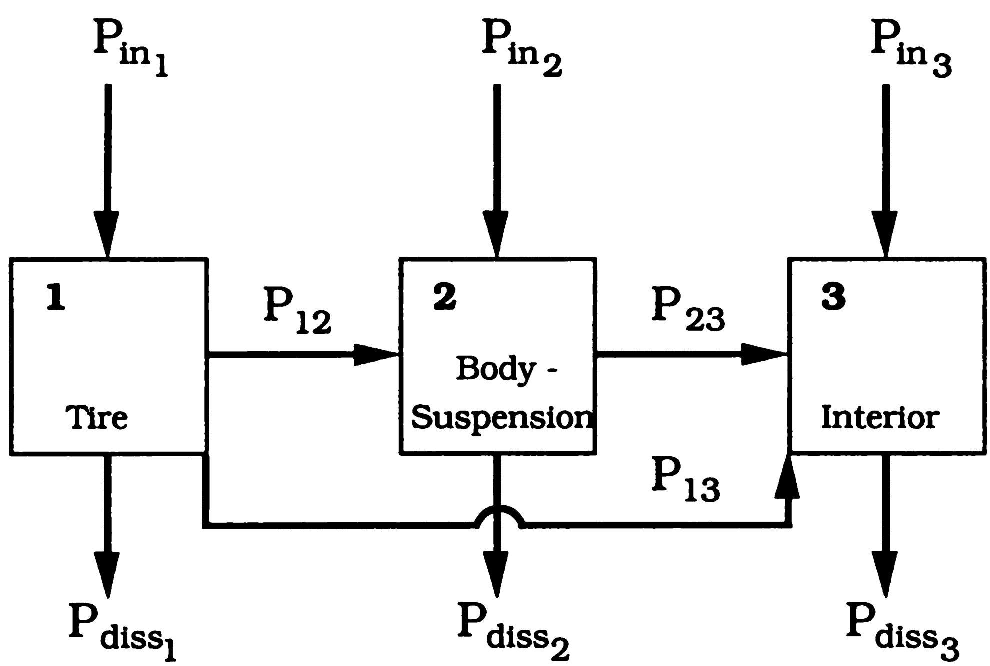

Figure 2: Simplified Conceptual SEA Model of Vibration and Sound Transmission from a Coarse Road to Vehicle Interior

The ability to compute the steady-state energy in a 1 rad/sec band centered at frequency, ω, (Crocker and Price 1969, 472) is a central feature of SEA theory developed by Lyon, Scharton and Newland in 1962-1968 (Lyon 1975, 7-10). The theory states that the steady-state power transfer in such a narrow frequency band is such that the energies in each mode of both systems are equalized in that frequency band.

$$P_{jk} = \omega\eta_{jk} n_j \left(\frac{E_j}{n_j} - \frac{E_k}{n_k} \right) \tag{3}$$

where: P_{jk} = power transferred from subsystem "j" to "k"

ω = band center frequency (rad/sec)

$\eta_{jk}(\omega)$ = subsystem coupling loss factor (dimensionless)

$E_j(\omega)$ = subsystem total energy (joules-sec/rad)

$n_j(\omega)$ = modal density of subsystem "j" (modes-sec/rad)

At high frequencies where the modal density is high in any system's spectrum, the above result is important because it will be used to compute the subsystem RMS pressure or velocity implied by the system's total energy. At these frequencies of high modal density, there is no comparable way of predicting subsystem response from the consideration of individual modes because of the model and numerical accuracies required in modal analysis. The result in equation (3) is therefore uniquely valuable at high frequencies but has been shown to apply to low frequencies as well. Even the assumption of relatively weak coupling between subsystems in the original derivation has been shown to be unnecessary (Lyon 1975, 8) and the result is valid for strong linear and/or non-linear coupling (Newland 1965, 199). Modal densities can be accurately estimated from geometrical parameters and/or measured by counting resonances in laboratory verification studies. In fact, obtaining accuracy in modal density estimates at any frequency is typically much easier than obtaining the same degree of accuracy in the computed properties of individual modes such as natural frequencies in that frequency range.

The system of linear equations giving the energies, $E_j(\omega)$, in terms of the input powers, P_{in_j}, can be found by substituting equations (2) and (3) into equation (1a-c) and rearranging. The result is:

$$[N]\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \left(\frac{1}{\omega}\right)\begin{bmatrix} P_{in_1} \\ P_{in_2} \\ P_{in_3} \end{bmatrix} \tag{4}$$

where:

$$[N] = \begin{bmatrix} (\eta_1 + \eta_{12} + \eta_{13}) & \left(-\eta_{12}\left(\frac{n_1}{n_2}\right)\right) & \left(-\eta_{13}\left(\frac{n_1}{n_3}\right)\right) \\ (-\eta_{12}) & \left(\eta_2 + \eta_{12}\left(\frac{n_1}{n_2}\right) + \eta_{23}\right) & \left(-\eta_{23}\left(\frac{n_2}{n_3}\right)\right) \\ (-\eta_{13}) & (-\eta_{23}) & \left(\eta_3 + \eta_{13}\left(\frac{n_1}{n_3}\right) + \eta_{23}\left(\frac{n_2}{n_3}\right)\right) \end{bmatrix}$$

The above equations can be solved for steady-state subsystem energy as a function of frequency given values for the loss factors, η.

The mean-square velocities for vibrating subsystems and mean-square pressures for acoustic subsystems can be directly related to steady-state subsystem energies, E_j.

$$\langle v^2 \rangle = \left(\frac{1}{m}\right) E_j \text{ for structures} \tag{5a}$$

$$\langle p^2 \rangle = \left(\frac{\rho c^2}{V}\right) E_j \text{ for acoustic systems} \tag{5b}$$

where: $<v^2>$ = mean-square velocity, expected value (m^2/sec^2)

$<p^2>$ = mean-square pressure, expected value (N^2/m^4)

E_j = $E_j(\omega)$ = subsystem total energy

m = structure mass (kg)

ρ = fluid density (kg/m^3)

c = speed of sound (m/sec)

V = acoustic volume (m^3)

The loss factor values required to compute the above steady-state mean-square response variables can be either estimated using conventional modeling methods and/or measured once prototype subsystems are available. The subsystem loss factor, η_j, is simply the reciprocal of the electrical engineer's quality factor, Q, or twice the mechanical engineer's damping ratio, ζ. Typical values for the loss factor, η_j, are between 0.001 and 0.1. The coupling loss factor, η_{jk}, is also related to other familiar modeling parameters. The coupling loss factor for acoustic power flow between two rooms is the transmission loss of the wall familiar to acoustic engineers while the coupling loss factor between a plate and an acoustic volume can be directly related to the plate's radiation efficiency, σ_{rad}. It can be seen that the estimation of SEA parameters is no more difficult than the estimation of more conventional modeling parameters such as structural mass, stiffness and damping.

THE MODELING PROCESS

Creating an SEA model requires model subsystem identification, subsystem type classification, and subsystem parameter quantification. Model subsystems are elements and connectors. Elements are classified as acoustic volumes, flat plates, beams, etc. Connector classifications are: acoustic volume to flat plate, flat plate to beam, flat plate to flat plate, etc. Subsystem parameters include volumes, surface areas, plate thicknesses, damping ratios, and reverberation times.

The model is evaluated over a specified frequency range. The frequency range is typically in 1/3 octave bandwidths. Power inputs are specified explicitly as functions of frequency or implicitly by specifying the average response of one or more elements.

MODELING AUTOMOTIVE DRIVELINE NOISE

Modeling the response of an automobile interior to driveline vibration illustrates the application of the modeling methodology and software. An SEA model consists of energy storage elements and the power flow paths between them. The energy source for the driveline model is the *engine*. The acoustic response of the *interior* is the desired model output. The *interior* and *engine* are the first model elements identified. The model building methodology is to determine how energy is transferred from the *engine* to the *interior*. Energy storing subsystems, elements, and the paths between them, connectors, are identified by tracing the power flow paths from the *engine* to the *interior*. Energy is transmitted from the *engine* to the *interior* through both structureborne and airborne paths. One example of a structureborne path is from the *engine* to *frame structure* to *floor pan* to *interior*. An example of an airborne path is from the *engine* to *under car volume* to *floor pan* to *interior*. Identifying these two paths creates three model elements: *frame structure*, *floor pan*, and *under car volume* (Figure 3). In addition, five model connectors are created: *engine to frame structure*, *frame structure to floor pan*, *floor pan to interior*, *engine to under car volume*, and *under car volume to floor pan*. The remaining model elements and connectors are identified in the same manner (Figure 4).

The *body panels* element required in the model combines the effects of the windshield, hatch glass, door glass, and roof into a single element. Modeling these components individually would have created 4 to 5 elements and 5 or more connectors. Since all of these elements can be modeled as flat plates, the model size is reduced by combining their effects.

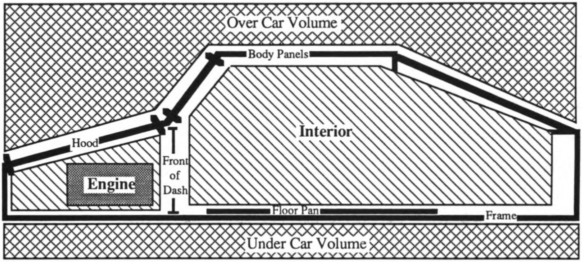

Figure 3: Schematic of the Statistical Energy Analysis Model showing general element layout

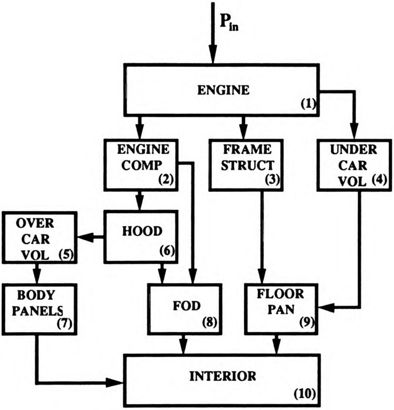

Figure 4: Automobile Statistical Energy Analysis Model showing energy storage and power connection between them

SEA element and connector classification of the 10 elements and 13 connectors is the next phase of the modeling process. For this example, only a few element types were to be developed: acoustic volumes, flat plates, and beams. The associated connector types developed are: acoustic volume to flat plate, flat plate to acoustic volume, flat plate to flat plate, flat plate to beam, and flat plate to parallel beam. Tables 1 and 2 show the model element and connector classifications respectively.

Table 1: Automobile Model Statistical Energy Analysis Element Classification

Automobile Model Element	Element Type
engine compartment *under car volume* *over car volume* *interior*	acoustic volume
engine *hood* *front of dash* *body panels* *floor pan*	flat plate
frame structure	beam

Element parameters vary depending on the element type. For acoustic volumes, the volume, surface area, edge length, and reverberation time are required. The plate parameters are thickness, length, width, perimeter edge length, mass, longitudinal wave speed, Young's modulus, and damping ratio. The beam parameters are length, width, thickness, mass, longitudinal wave speed, Young's modulus, shear modulus, and damping ratio. For beams with circular cross sections, width and thickness are replaced by inner and outer diameters. The damping ratios and reverberation times are used in loss factor calculations. Element masses are used in calculating element energy levels and RMS velocities or accelerations.

Table 2: Automobile Model Statistical Energy Analysis Connector Classification

Automobile Model Connectors	Connector Type
engine to engine compartment *engine to under car volume* *hood to over car volume* *body panels to interior* *front of dash to interior* *floor pan to interior*	flat plate to acoustic volume
engine compartment to hood *engine compartment to front of dash* *under car volume to floor pan* *over car volume to body panels*	acoustic volume to flat plate
hood to front of dash	flat plate to lat plate
engine to frame	flat plate to beam
floor pan to frame	flat plate to parallel beam

Connector parameters are required to compute coupling loss factors. Acoustic volume to flat plate and flat plate to acoustic volume connectors require the plate edge condition, β. β is defined as (Lyon 1975, 300):

$$\beta = \begin{cases} 1 & \text{simply supported} \\ 2 & \text{clamped} - \text{clamped} \\ \sqrt{2} & \text{realistic cases} \end{cases} \qquad (6)$$

For flat plate to flat plate connections, the angle between the two plates and the length of the connection joint must be known. For flat plate to beam, two angles which describe the position of the beam relative to the plate are required. Flat plate to parallel beam is modeled as a 'point connection,' with the number of connection points specified. The *frame structure* properties are determined from an equivalent rectangular beam structure which is converted to an equivalent pair of circular beams for final model evaluation because only flat plate to circular beam connection models were available.

The parameters of the system are used to evaluate the three SEA values: modal densities, coupling loss factors, and loss factors. While modal densities and coupling loss factors are related to system and element geometry, loss factors must be derived from experimentally determined values such as acoustic absorption coefficient, reverberation time, and damping ratio. Parameter accuracy depends on the model, the modeler's knowledge of the system being modeled, and the modeler's experience developing SEA models.

The *body panels* element parameter evaluation is more involved than the evaluation of the other elements. The *body panels* element includes the windshield, roof, rear window, and door glasses, and is to be modeled as a single flat plate element. The individual components of the *body panels* element are made of glass and steel which have the same longitudinal wave speed (Lyon 1975, 282), but do not have the same Young's modulus. In addition, the element's components do not have equivalent plate thicknesses.

Parameter evaluation is facilitated by examining the related model equations. The equation for calculating the modal density of a flat plate is (Lyon 1975, 282):

$$n = \frac{\sqrt{3}A}{hc_l} \tag{7}$$

where: A = plate surface area (m^2)

h = plate thickness (m)

c_l = plate longitudinal wave speed (m/sec)

The only paths to and from the *body panels* connect to acoustic volumes. The equation for the plate to volume coupling loss factor which characterizes energy transfer between elements is (Lyon 1975, 300):

$$\eta_{sa} = R_{rad}/\omega m_s \tag{8}$$

where: R_{rad} = plate radiation resistance (kg-rad/sec)

m_s = plate structural mass (kg)

ω = band center frequency (rad/sec)

The volume to plate coupling loss factors are found from the relationship (Lyon 1975, 300):

$$\eta_{as} = \eta_{sa}(n_s/n_a) \tag{9}$$

where n_s and n_a are the structural and acoustic volume modal densities respectively. Acoustic waves in a fluid travel at the same speed regardless of frequency (Crocker and Kessler 1982, 63-66). In contrast, high frequency bending waves in a structure like a panel travel at faster speeds than low frequency waves. The transfer of energy between a volume and a panel is maximized at the frequency where acoustic waves in the volume and bending waves in the panel have the same propagation velocity. This frequency is the critical frequency of the panel (Beranek 1971, 275),

$$f_c = \frac{c^2}{1.8138h\, c_l} \tag{10}$$

where: c = speed of sound (m/sec)

h = plate thickness (m)

c_l = plate longitudinal wave speed (m/sec)

The radiation resistance, R_{rad}, is evaluated as follows (Maidanak 1962, 817-818):

$$R_{rad} = A\rho c\begin{Bmatrix} \left[(\lambda_a\lambda_c/A)g_1(f/f_c)+(P\lambda_c/A)g_2(f/f_c)\right]\beta, & f < f_c \\ \left[(l/\lambda_c)^{\frac{1}{2}}+(w/\lambda_c)^{\frac{1}{2}}\right]\beta & f = f_c \\ \left[1-f_c/f\right]^{-\frac{1}{2}}, & f > f_c \end{Bmatrix} \tag{11}$$

where: λ_c = wavelength of sound at critical frequency (m)

λ_a = wavelength of sound in acoustic volume (m)

ρ = density of air (kg/m^3)

β = edge condition [See equation (6)]

A = plate surface area (m^2)

P = plate perimeter edge length (m)

c = speed of sound in air (m/sec)

f = center frequency (Hz)

l = plate length (m)

w = plate width (m)

and

$$g_1(f/f_c) = \begin{Bmatrix} (4/\pi^4)(1-2\alpha^2)\big/\alpha(1-\alpha^2)^{\frac{1}{2}}, & f < \frac{1}{2}f_c \\ 0, & f > \frac{1}{2}f_c \end{Bmatrix} \tag{12}$$

$$g_2(f/f_c) = \frac{\left\{(1-\alpha^2)\ln[(1+\alpha)/(1-\alpha)]+2\alpha\right\}}{4\pi^2(1-\alpha^2)^{\frac{3}{2}}} \tag{13}$$

with

$$\alpha = (f/f_c)^{\frac{1}{2}} \tag{14}$$

The surface area appears in calculations of the modal density (7) and the radiation resistance (11). Equation (11), the radiation resistance equation, affects the calculated flow of energy between the plate and surrounding acoustic volumes. It is desired to model the combined effects of the individual components. Therefore the component areas are summed to generate a total plate surface area, A_T. The plate mass is used in calculating the loss factor (8) and in calculating the plate's RMS velocity (5a). To model combined plate effects, the individual component masses are summed. The plate thickness is used in evaluating the modal density (7) and in evaluating the critical frequency (10). The definition of modal density is n = N/Δf, where N is the total number of modes in the frequency bandwidth Δf. It is observed that the total number of modes in the *body panels* element is equal to the sum of the modes of the individual elements (Lyon 1975, 288). Therefore:

$$n_{tot} = n_{wind} + n_{roof} + n_{hatch} + n_{door} \tag{15}$$

where n_{tot}, n_{wind}, n_{roof}, n_{hatch}, n_{door} are the total,window, roof, hatch glass, and door glass modal densities respectively. The relationship for composite modal density is derived here by substituting equation (7) into equation (15) and noting that c_l is the same for steel and glass,

$$n_{tot} = \frac{\sqrt{3}A_T}{h_{equiv}c_l} = \frac{\sqrt{3}}{c_l}\left[\frac{A_{wind}}{h_{wind}} + \frac{A_{roof}}{h_{roof}} + \frac{A_{hatch}}{h_{hatch}} + \frac{A_{door}}{h_{door}}\right] \tag{16}$$

where A_{wind}, A_{roof}, A_{hatch}, A_{door}, h_{wind}, h_{roof}, h_{hatch}, and h_{door} are the windshield, roof, hatch, and door, surface areas and thicknesses respectively. h_{equiv} is the equivalent thickness for the composite element which will yield the element's total modal density. Solving for h_{equiv} yields:

$$h_{equiv} = \frac{A_T}{\left[\frac{A_{wind}}{h_{wind}} + \frac{A_{roor}}{h_{roof}} + \frac{A_{hatch}}{h_{hatch}} + \frac{A_{door}}{h_{door}}\right]} \tag{17}$$

The plate perimeter , length , and width are used in calculating the radiation resistance (11). The perimeter edge length models the edge effects of the plate. Since each component of the *body panels* element is considered to act independently, the perimeter is taken as the sum of the individual component edge lengths. The length and width are defined as the square root of A_T. The Young's modulus does not enter into any of the calculations for this model. However, should Young's modulus be required it can be calculated as an area weighted average.

Acoustic volume parameters for the *over car* and *under car volumes* are evaluated from vehicle geometry. Since the *over car* and *under car volumes* do not have definite boundaries, they could be considered infinite in volume and surface area. To practically model the system these values must be finite; thus volume boundaries are defined. The *under car volume* boundaries are defined as the road (ground), the car underside, and the imaginary edge created by extending the car's lower outside edge to the ground. The car top, the car top's mirror image 0.6m above the car, and the edges created, form the *over car volume*. The length 0.6m is chosen because the experimental measurements of sound pressure level were taken 0.3m above the car. The volumes, surface areas, and edge lengths can be estimated from the defined volume boundaries.

Reverberation times and damping ratios are determined from experimental measurements or estimates based on engineering experience since theoretical models do not exist. Since this model is of a production vehicle, some of these values could be measured. It has been suggested that these parameters be determined by removing elements from systems, providing excitation to these elements, removing excitation, and measuring the element response decay rate. For a complex system, this can be difficult. In the driveline model, removal of the *body panels* element is virtually impossible. It is also difficult to separate acoustical elements from systems. In general, the effects of neighboring elements cannot be eliminated from reverberation time measurements.

For the driveline model, only the reverberation time of the *interior* was measured. The remaining reverberation times and damping ratios were estimated based on experience. Since these values were not accurately determined, they were adjusted to match the model results more closely with experimental results. Two purposes are served by making such adjustments. First, it provides a way to evaluate the validity of the model itself. For example, when adjusting the *frame structure* damping ratio, it was found that a large damping ratio was required to produce reasonable results. This fact led to the discovery that the *frame structure* had been incorrectly modeled. The second benefit is that potential values for future models are determined.

LABORATORY MEASUREMENT OF AUTOMOBILE RESPONSE

Data were collected from a typical automobile operating in first gear at 3000 RPM with a 100lb tractive force on a dynamometer. These conditions were chosen to emphasize the noise created by the driveline. The majority of the car's interior was removed to facilitate acquisition of data. The car's dash board, steering wheel, head liner, rear seat lateral cushion, and hatch carpeting were not removed.

Acceleration data were measured using PCB accelerometers model 303A, with a Kistler Pieztron Coupler 5122. A-weighted sound pressure levels were taken using B&K 4144 omni-directional microphones with a B&K WB1057 microphone amplifier. The data were recorded on a TEAC RD-111T PCM Data Recorder with a TEAC TZ-314FA Antialiasing

Filter Unit. For each set of data, a thirty second sample was taken and then averaged to give an RMS acceleration or pressure value for each measurement location.

The desired result was to characterize the steady state energy content of the ten model elements. For each structural element (i.e. *floor pan*, *hood*, *body panels*, etc.) several acceleration measurements where taken. For example, eight accelerometer readings where taken on the *hood*, *front of dash* (toe pan), roof, door glass, windshield, and hatch glass. For the *floor pan*, a larger number of measurements were taken to get a more accurate sample of the element's response. The *floor pan* was broken into five sections with six to eight measurements taken for each section. For the sound pressure measurements, two to four measurements were taken for each volume. The *interior* and *engine compartment* were each measured in two locations. Four measurements were taken for the *under car* and *over car volumes*.

The raw vibration and pressure data were analyzed using a B&K 2133 spectrum analyzer and reduced to power spectrums in 1/3 octave bandwidths. The acceleration data were calibrated in dB re 20 microvolts, with 1 volt equal to 1 g. Sound pressure levels were calibrated in standard A-weighted dB format. For the purpose of SEA, it is desired to have a characteristic energy spectrum for each element. With the collected data, several sets of data for each element were available. To reduce this data to a single spectrum for each element, it is necessary to average the data. First, the acceleration data were reduced to characteristic acceleration plots for each structural element. The data were first converted to voltages (or g's since 1 g equals 1 volt). Next, the root mean square of the data for a given element at each frequency was calculated to give a characteristic value for each structural element. The RMS acceleration values are used as inputs and expected outputs for the driveline model. A similar procedure was followed for reducing the sound pressure data. The data were first converted to pressures, root mean squared values calculated, and then converted to dB.

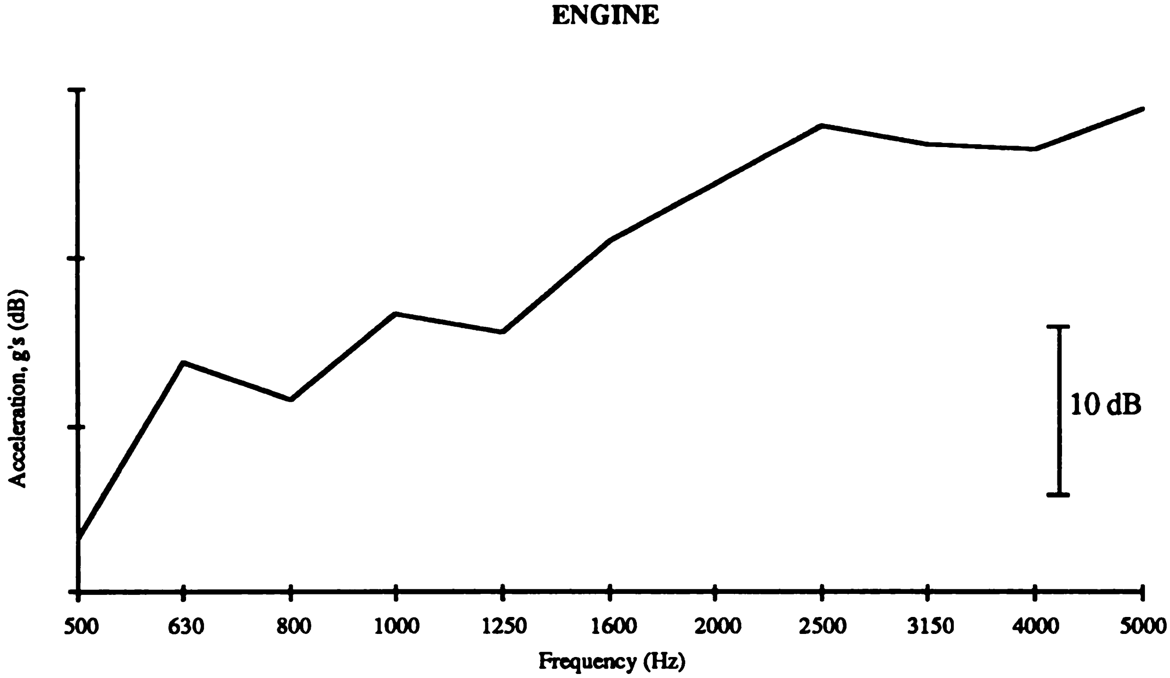

Figure 5: RMS Engine Accelerations versus Frequency - Used as element model energy input

RESULTS

The model was evaluated from 500 to 5000 Hz at 1/3 octave bandwidth intervals using the SEA modeling software developed . For the driveline model, energy enters the system via the *engine*. The *engine* input is modeled by specifying the *engine's* energy response spectrum which is available from experimental data (Figure 5). The results of the model analysis are modal densities, loss factors, coupling loss factors, and steady state energy levels.

The first analysis of the model used estimates of damping and reverberation times based on engineering experience. For the *interior*, the first run modeled values are within 7 dB of the RMS experimental values (Figure 6). Considering the lack of model refinement, the modeled *interior* response is quite satisfying. However, the results of the entire model must be analyzed before the model can be judged correct. The *interior* models a pressure

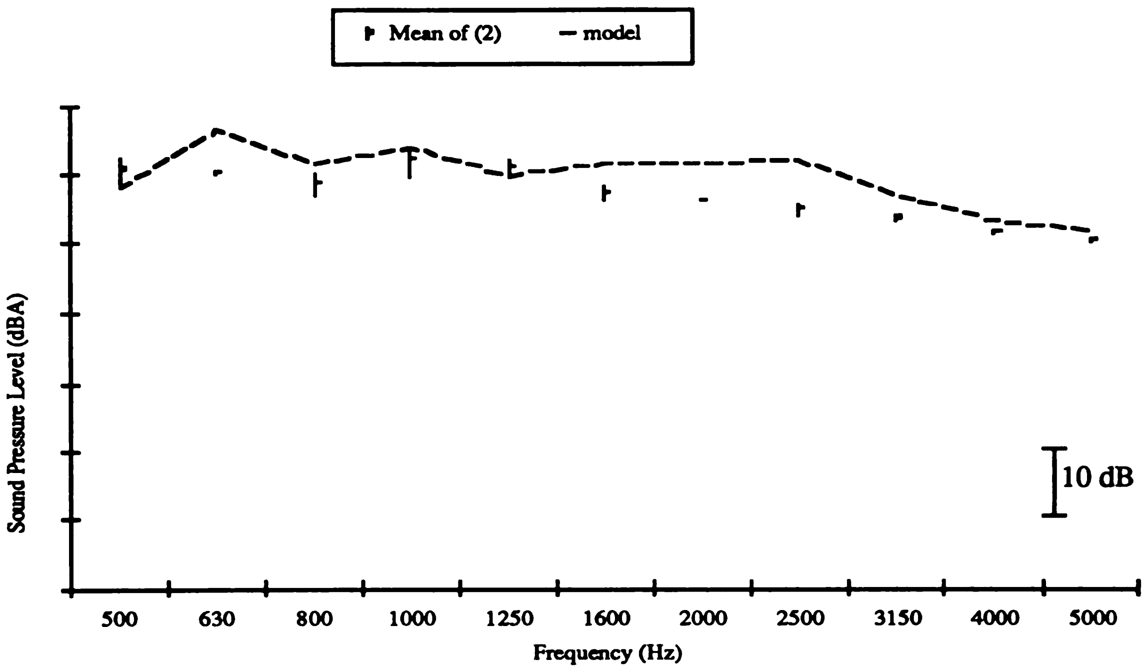

Figure 6: Automobile Driveline Model Interior Response - Experimental and Predicted Values - Run 1

response that is very close to the measured values, while the results of the *over car volume* model are very different from those measured (Figure 7). Due to the severe discrepancy in the model and experimental results of some elements, an attempt to improve the model results was made. The parameters of damping ratio, reverberation time, and the size of the *over car volume* were altered in an attempt to improve the results. Even with a reverberation time $1.0x10^6$ seconds a difference of 8 to 25 dB between modeled values and experimental values existed. This led to the discovery that the *hood* to *over car volume* connector had not been included in the model database. Run nine included the *hood* to *over car volume* connector in the model database and good results were obtained with an *over car volume* reverberation time of 50 seconds. This reverberation time is rather large, but it should be noted that the theoretical model used for modeling the *over car volume* is not reperesentative of the actual *over car volume*. The *over car volume* will not have the modal interaction that a room or automobile interior might have. For run nine, the *interior* response has decreased slightly, but is within 10 dBA of the RMS experimental values (Figure 9). The remaining model element results were typically within the range of the experimental values (Figure 10).

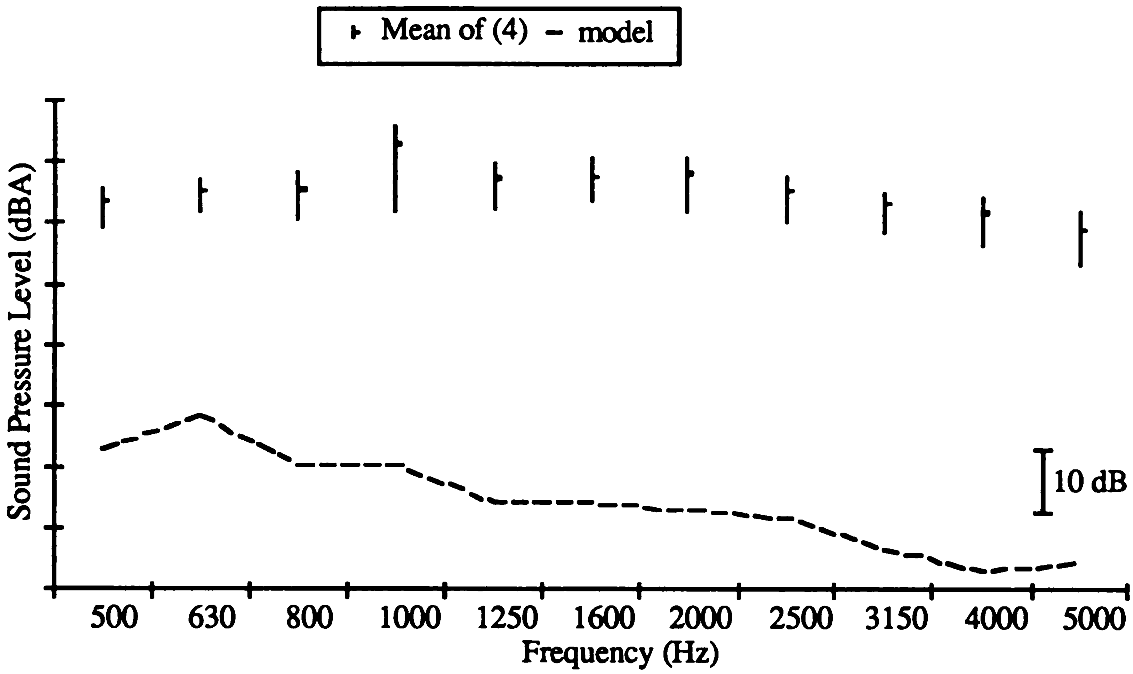

Figure 7: Automobile Driveline Model Over Car Volume Response - Run 1

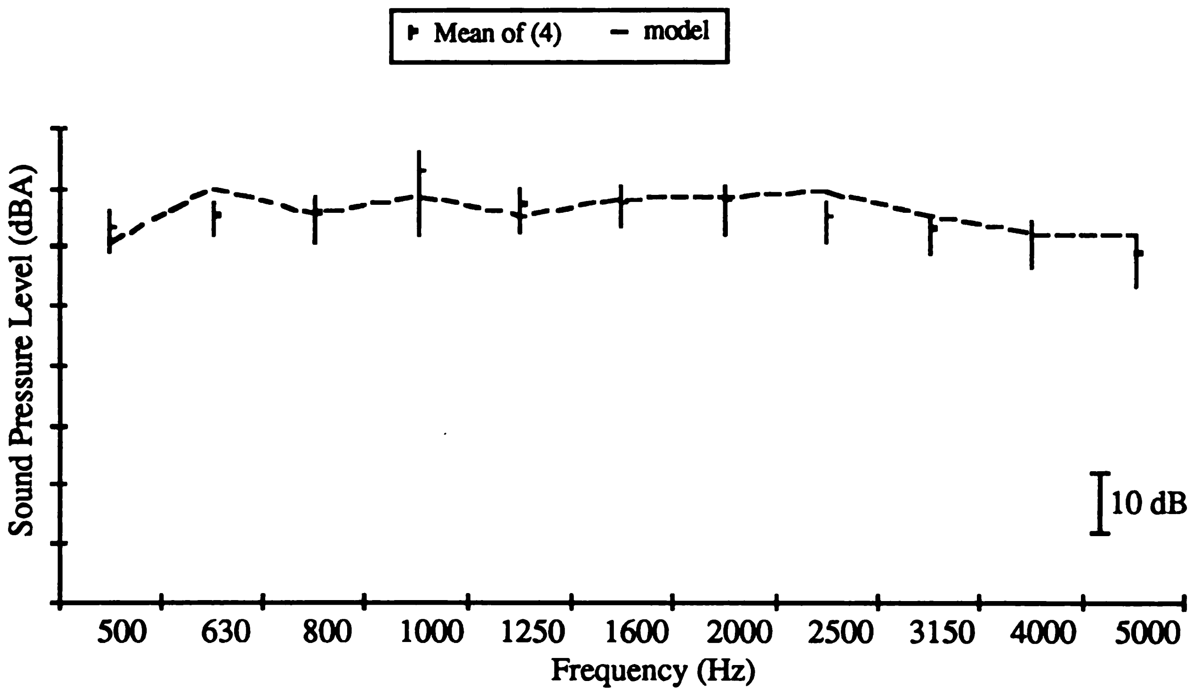

Figure 8: Automobile Driveline Model Over Car Volume Response - Run 9

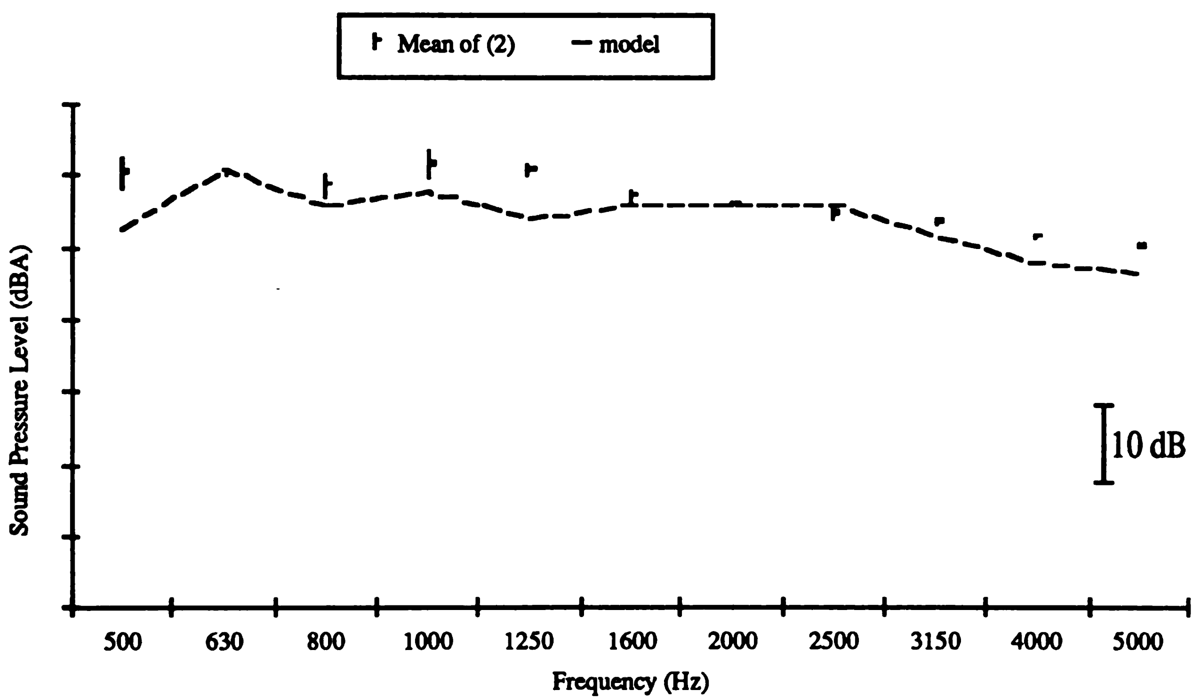

Figure 9: Automobile Driveline Model Interior Response - Run 9

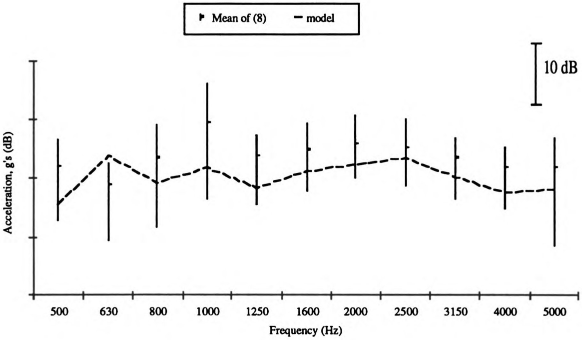

Figure 10: Automobile Driveline Model Front of Dash Response - Run 9

CONCLUSIONS

The ability to model automobile drive line noise has been demonstrated. Although a rough model was used, relatively accurate values were predicted for most elements. The modeled values were generally in the range of the experimental values.

The reverberation times and damping ratios which were not determined experimentally have reasonable values based on engineering experience, with the exception of the *over car volume*. There are many possible explanations for the large *over car volume* reverberation time. It is possible that a path to the volume was not identified. Perhaps the contribution of unmodeled exhaust noise is large, or the theoretical model used for the volume does not accurately model the volume. Further investigation into this problem and its solutions is recommended.

Refinement of the model involves breaking the system into more elements, identifying new flow paths, and developing new theoretical elements to model automotive components. Elements such as the *body panels* and *floor pan* can be divided into and modeled as multiple elements. Creating more elements will create more flow paths. Elements such as *engine* do not necessarily fit into the categories of beam, volume, or flat plate; thus new element models need to be developed.

SEA provides a method for modeling the high frequency response of automotive systems. The model developed uses rough estimates of vehicle parameters similar to values that would be available in the early stages of vehicle design. The modeled vehicle response results are typically within the range of the experimental values and predict the general trends of most elements. This model incorporated a very limited range of model elements and connectors. With the development of new element and connector models, refinement of the model should produce vastly improved results. The high frequency analysis capabilities of SEA and the low frequency range of finite element methods could be combined to provide an acoustic model which represents the full frequency range of an interior's acoustic response.

APPENDIX A: VARPS

INTRODUCTION

VARPS, Vibro Acoustic Response Prediction System, evaluates user defined system models using Statistical Energy Analysis. A model database file in ASCII format is created from VARPS element models, connector models, and model input nodes using a text editor such as EDT. VARPS supported elements and connectors are shown in Tables A-1 and A-2. The model input nodes are shown in Table A-3. VARPS outputs modal densities, loss factors, coupling loss factors, and RMS responses in tabular form.

Table A-1: VARPS Supported Elements

Element	**VARPS *type***
acoustic volume	AVOL or AVOL2
flat plate	FPLATE
beam	BEAM

Table A-2: VARPS Supported Connectors

Connector	**VARPS *type***
acoustic volume to flat plate	AVOL_FPL or AVOL_FPL2
flat plate to acoustic volume	FPL_AVOL or FPL_AVOL2
flat plate to flat plate	FPL_FPL
flat plate to beam	FPL_BEAM
flat plate to parallel beam	FPL_PLB

Table A-3: VARPS Model Inputs

Inputs	**VARPS *type***
frequency	FREQ
power	POW
energy	ENRG

GENERAL MODEL DATA BASE ENTRY

Each VARPS model database entry is called a node. A VARPS model node has the four general fields: *type, name, location,* and *parameters*. The general form of a data base node is:

type *name* *location* *parameters*

type must be one of the types shown in Tables A-1, A-2, and A-3. *name* is a user specified node name used in displaying the final results. *location* describes the elements topological position in the model. For each element, *location* is a single, unique integer. For a connector, *location* is two integer numbers separated by a comma. The first number indicates the element *from* which energy flows and the second indicates the element *to* which energy flows. For example, the entry 1,2 indicates a connector from element 1 to element 2. It is important that the user be sure that the *from* and *to* elements exist and are of the type specified by the connector *type*. The software does not currently check connectors for correct element reference, thus incorrectly designating the location will cause the program to give incorrect results. The number and type of *parameters* vary depending on the element.

CREATING A MODEL

Creating a VARPS model consists of dividing the system being modeled into elements and connectors, determining model power inputs, classifying the elements and connectors as VARPS supported types, and determining the required model parameters. (See individual element and connector write ups below.) After determining the required model data, the model data base is built using any text editor. File names must have a "dat" file extension. The order of data base entries is elements, connectors, frequency vector, and power/energy inputs (Table A-4). Elements must be in numerical order with element 1 appearing before element 2, element 2 appearing before element 3, etc. Connectors must be listed after the elements they connect are listed. The model must include one frequency vector and one or more energy/power inputs. (See Model Inputs for details on frequency, power, and energy inputs.)

Table A-4: VARPS Model Database Format

Elements . . .
Connectors . . .
Frequency
Power or Energy Inputs . . .

Data Base Rules

1. One or more spaces separate database fields. **Tabs must not be used anywhere in the database.**
2. A single comma separates field values.
3. For node entries exceeding 80 characters or for node entries being continued on the next line, an '-' is placed at the end of the line being continued.
4. In the database, Elements must be listed in order with respect to spatial location. For example, element 1 must appear before element 2. Element 1 appearing after element 2 will cause errors in model evaluation.
5. Elements must be listed before they are used in any connectors.
6. File names must have "dat" as the file extension. i.e. 'model.dat'.

EVALUATING THE MODEL

Type "@VARPS" at the system prompt to begin model evaluation. VARPS will prompt for a file name. Enter the name of the data base file to be evaluated. VARPS will generate modal densities, loss factors, coupling loss factors, and RMS responses in tabular form. The response data is produced in standard sound pressure levels for volumes and in RMS accelerations for structural elements. To quit VARPS, type "EXIT" at the file name prompt.

DATA BASE NODES - SUMMARY

Table A-5: VARPS Database Node Summary

AVOL	*name*	*location*	*volume,surface area,edge length,-reverberation time*
AVOL2	*name*	*location*	*volume,surface area,edge length,-reverberation time*
FPLATE	*name*	*location*	*thickness,surface area,mass,-edge length,longitudinal wave speed,damping ratio, length,width,-Young's modulus*
BEAM	*name*	*location*	*1,length,mass,Young's modulus,-shear modulus,longitudinal wave speed,damping ratio,width,thickness*
BEAM	*name*	*location*	*2,length,mass,Young's modulus,-shear modulus,longitudinal wave speed,damping ratio,inner diameter,outer diameter*
FPL_AVOL	*name*	*to,from*	β
FPL_AVOL2	*name*	*to,from*	β
AVOL_FPL	*name*	*to,from*	β
AVOL_FPL2	*name*	*to,from*	β
FPL_FPL	*name*	*to,from*	θ,*joint length*
FPL_BEAM	*name*	*to,from*	γ,μ
FPL_PLB	*name*	*to,from*	*number of connection points*
FREQ	*name*	*location*	*f1,f2,f3,f4,f5,......*
POW	*name*	*location*	*POW(f1),POW(f2), POW(f3), ...*
ENRG	*name*	*location*	*ENRG(f1),ENRG(f2),.....*

ELEMENT MODELS

AVOL and AVOL2 (Sheet 1 of 2)

Table A-6: AVOL and AVOL2 Element Parameters

Name	Units
volume	m^3
surface area	m^2
edge length	m
reverberation time	seconds

DATA BASE ENTRY FORMAT

AVOL *name* *location* *volume,surface area,edge length,-*
reverberation time

or

AVOL2 *name* *location* *volume,surface area,edge length,-*
reverberation time

DATA BASE ENTRY EXAMPLE

AVOL VOLA 1 0.125,1.5,6.0,1.77

or

AVOL2 VOLA 1 0.125,1.5,6.0,1.77

DESCRIPTION

AVOL and AVOL2 model acoustic volumes such as rooms or car interiors. Both element models require the same database entries. AVOL2 models the element identically to the way in which VAPEPS, an SEA modeling package available from NASA, models an acoustic volume. The modal density, ***n***, is (Lyon 1975, 281):

AVOL and AVOL2 (Sheet 2 of 2)

$$n = \frac{4\pi f^2 V}{c^3} + \frac{\pi f A}{2c^2} + \frac{l}{8c} \tag{A-1}$$

where: f = band center frequency (Hz)

V = volume (m^3)

A = volume surface area (m^2)

l = perimeter edge length (m)

c = speed of sound of fluid in volume (m/sec)

AVOL2 ignores the second and third terms of equation (A-1). This is acceptable when frequency is high or a large volume is being evaluated. The loss factor for an acoustic volume can be modeled using the reverberation time, T_R (Lyon 1975, 264):

$$\eta = {}^{2.2}\!/_{fT_R} \tag{A-2}$$

where f is defined as before.

FPLATE (Sheet 1 of 2)

Table A-7: FPLATE Element Parameters

Name	**Units**
thickness	m
surface area	m^2
mass	kg
edge length	m
longitudinal wave speed	m/second
damping ratio	dimensionless
length	m
width	m
Young's modulus	GPa

DATA BASE ENTRY FORMAT

FPLATE *name* *location* *thickness,surface area,mass,-*
edge length,longitudinal wave speed,damping ratio,length,width,Young's modulus

DATA BASE ENTRY EXAMPLE

FPLATE PLATE 2 0.0075,0.25,5.08125,2.0,5082.3,0.00005,-
0.5,0.5,70.0

DESCRIPTION

FPLATE models flat plate elements such as walls or floor pans. The modal density, n, is (Lyon 1975, 282):

$$n = \frac{\sqrt{3}A}{hc_l} \tag{A-3}$$

where: A = plate surface area (m^2)

h = plate thickness (m)

c_l = longitudinal wave speed (m/sec)

FPLATE (Sheet 2 of 2)

For steel, aluminum, and glass the longitudinal wave speed is 5181.6 m/sec (Lyon 1975, 282). The loss factor of a flat plate is modeled as a function of the plate's damping ratio, ζ (Lyon 1975, 264).

$$\eta = 2.0\zeta \tag{A-4}$$

BEAM (Sheet 1 of 3)

Table A-8: BEAM Element Parameters

Name	Units
type	rectangular = 1 circular = 2
length	m
mass	kg
Young's modulus	GPa
shear modulus	GPa
longitudinal wave speed	m/second
damping ratio	dimensionless
width or inner diameter	m
thickness or outer diameter	m

DATA BASE ENTRY FORMAT

Rectangular Cross Section

BEAM *name location 1,length,mass,Young's modulus,shear modulus,,- longitudinal wave speed,damping ratio,width,thickness*

Circular Cross Section

BEAM *name location 2, length, mass,Young's modulus,shear modulus, longitudinal wave speed,damping ratio,inner diameter,outer diameter*

DATA BASE ENTRY EXAMPLE

BEAM FRAME 3 2,1.7272,94.29,190.0,75.0,5181.67,- 1.5,2.77,2.8787

BEAM (Sheet 2 of 3)

DESCRIPTION

BEAM models structural elements such as frame structures. A BEAM element can have a solid rectangular or a circular cross section. Circular cross sections may be tubular or solid. For both cross sections, length, mass, Young's modulus, Shear modulus, longitudinal wave speed, and critical damping ratio are specified. For rectangular cross sections, the beam width and thickness are specified. Inner and outer diameters are specified for circular cross sections with the inner diameter being 0.0 for solid cross sections. The modal density equation is (Lyon 1975, 288-289; VAPEPS):

$$n = \frac{l}{\sqrt{kc_l\omega}} + \frac{2l}{c_t} \qquad \text{(A-5)}$$

where l = the beam length (m)

k = radius of gyration (m)

c_l = longitudinal wave speed (m/sec)

c_t = torsional wave speed (m/sec)

ω = band center frequency (rad/sec)

The first term in equation (A-5) is the flexural modal density and the second term is the torsional modal density. c_t is calculated from the following equation for circular cross sections (VAPEPS):

$$c_t = \sqrt{G/\rho} \qquad \text{(A-6)}$$

where: G = shear modulus (Pa)

ρ = mass density (kg/m^3)

For rectangular cross sections (VAPEPS):

$$c_t = \frac{r_t}{\rho J} \qquad \text{(A-7)}$$

where: J = polar moment of inertia (m^4)

BEAM (Sheet 3 of 3)

ρ = beam mass density (kg/m^3)

and

$$r_t = Gt_1$$

with t_1 evaluated as follows (VAPEPS):

$$t_1 = \left(A_{xb}^2 r_{hw}^{-C_1}\right)C_2$$

where A_{xb} is the beam cross sectional area and r_{hw} is the ratio of the beams thickness to its width. If $r_{hw} < 1.0$, $r_{hw} = 1.0/r_{hw}$.

C_1 and C_2 are evaluated as follows (VAPEPS):

$$C_1, C_2 = \begin{Bmatrix} 0.2901, 0.141 & 1.0 < r_{hw} < 2.0 \\ 0.7603, 0.194 & 2.0 < r_{hw} < 6.0 \\ 0.9101, 0.2537 & 6.0 < r_{hw} < 10.0 \\ 1.0, 0.333 & 10.0 < r_{hw} \end{Bmatrix} \quad \text{(A-8)}$$

The internal loss factor is (Lyon 1975, 264):

$$\eta = 2.0\zeta \quad \text{(A-9)}$$

where ζ is the critical damping ratio.

CONNECTOR MODELS

FPL_AVOL and FPL_AVOL2 (Sheet 1 of 3)

Table A-9: FPL_AVOL and FPL_AVOL2 Connector Parameters

Name	Units
β(plate edge condition)	dimensionless

DATA BASE ENTRY FORMAT

FPL_AVOL *name* *to,from* β

or

FPL_AVOL2 *name* *to,from* β

DATA BASE ENTRY EXAMPLES

FPL_AVOL FPAN_INTER 3,4 1.41412

or

FPL_AVOL2 FPAN_INTER 3,4 1.41412

DESCRIPTION

FPL_AVOL and FPL_AVOL2 model flat plate to acoustic volume connections such as walls to rooms or floor pans to automobile interiors. FPL_AVOL2 provides results matching VAPEPS, an SEA modeling package available from NASA. The model inputs are connector name, connectivity, and β. The connector name is user selected and does not affect model evaluation. Connectivity is input as two integers separated by a comma. The first number is the flat plate location and the second is the acoustic volume location. Suggested values for β, the plate edge condition, are (Lyon 1975,300):

$$\beta = \begin{cases} 1 & \text{simply supported} \\ 2 & \text{clamped} - \text{clamped} \\ \sqrt{2} & \text{realistic cases} \end{cases} \tag{A-10}$$

FPL_AVOL and FPL_AVOL2 (Sheet 2 of 3)

Other required parameters are obtained from the associated FPLATE and AVOL element database entries.

COUPLING LOSS FACTOR CALCULATION

The coupling loss factor is (Lyon 1975,300):

$$\eta_{sa} = R_{rad}/\omega m_s \tag{A-11}$$

where η_{sa} indicates the coupling loss factor from the structural element to the acoustic volume element. R_{rad}, is the radiation resistance, ω is the band center frequency in radians and m_s is the plate structural mass.

For FPL_AVOL (Maidanak 1962, 818):

$$R_{rad} = A_p \rho_a c_a * \begin{Bmatrix} \left[(\lambda_a \lambda_c/A_p) g_1(f/f_c) + \left(P_r \lambda_c/A_p\right) g_2(f/f_c)\right]\beta, & f < f_c \\ \left[(l/\lambda_c)^{\frac{1}{2}} + (h/\lambda_c)^{\frac{1}{2}}\right]\beta & f = f_c \\ \left[1 - f_c/f\right]^{-\frac{1}{2}}, & f > f_c \end{Bmatrix} \tag{A-12}$$

and for FPL_AVOL2 (VAPEPS):

$$R_{rad} = A_p \rho_a c_a * \begin{Bmatrix} \left[\dfrac{2.0\lambda_c P}{\pi^2 A_p} \sin^{-1}(\alpha)\right]\beta, & f < f_k \\ \left[\left(P_r \lambda_c/A_p\right) g_2(f/f_c)\right]\beta, & f_k < f < f_c \\ \left[(l/\lambda_c)^{\frac{1}{2}} + (h/\lambda_c)^{\frac{1}{2}}\right]\beta & f = f_c \\ \left[1 - f_c/f\right]^{-\frac{1}{2}}, & f > f_c \end{Bmatrix} \tag{A-13}$$

where

$$g_1(f/f_c) = \begin{Bmatrix} (4/\pi^4)(1 - 2\alpha^2)\Big/\alpha(1 - \alpha^2)^{\frac{1}{2}}, & f < \dfrac{1}{2} f_c \\ 0, & f > \dfrac{1}{2} f_c \end{Bmatrix} \tag{A-14}$$

FPL_AVOL and FPL_AVOL2 (Sheet 3 of 3)

and

$$g_2(f/f_c) = \frac{\left\{(1-\alpha^2)\ln[(1+\alpha)/(1-\alpha)] + 2\alpha\right\}}{4\pi^2(1-\alpha^2)^{\frac{3}{2}}} \tag{A-15}$$

with

$$\alpha = (f/f_c)^{\frac{1}{2}} \tag{A-16}$$

where: ρ = density of air (kg/m^3)

C = speed of sound (m/sec)

f = band center frequency (Hz)

f_c = critical frequency (Hz)

and f_k, the wave number frequency, is the frequency at which $\frac{1}{2}k_a * d_{avg} = 1.0$. k_a is the acoustic wave number and d_{avg}, the average plate dimension, is $d_{avg} = (l + w)/2.0$. l and w are the plate's length and width respectively. The critical frequency and wave number frequency are (Beranek 1971,270):

$$f_c = \frac{c^2}{1.8138 h_p c_l} \tag{A-17}$$

and

$$f_k = \frac{c}{\pi d_{avg}} \tag{A-18}$$

where: h_p = plate thickness (m)

c_l = plate longitudinal wave speed (m/sec)

AVOL_FPL and AVOL_FPL2 (Sheet 1 of 1)

Table A-10: AVOL_FPL and AVOL_FPL Connector Parameters

Name	Units
β(plate edge condition)	dimensionless

DATA BASE ENTRY FORMAT

AVOL_FPL *name* *location* β

or

AVOL_FPL2 *name* *location* β

DATA BASE ENTRY EXAMPLE

AVOL_FPL VOLA_PLATE 1,2 1.4142

or

AVOL_FPL2 VOLA_PLATE 1,2 1.4142

DESCRIPTION

Modeling the connectivity between an acoustic volume and a flat plate requires the same parameters as the acoustic volume to flat plate connector. It is calculated using the relationship (Lyon 1975, 300):

$$\eta_{as} = \eta_{sa}(n_s/n_a) \tag{A-19}$$

where n_s and n_a are the structural and acoustical modal densities respectively.

FPL_FPL (Sheet 1 of 6)

Table A-11: FPL_FPL Connector Parameters

Name	**Units**
θ(angle between plates)	degrees
l(joint length)	m

DATA BASE ENTRY FORMAT

FPL_FPL *name* *location* θ,*joint length*

DATA BASE ENTRY EXAMPLE

FPL_FPL HOOD_FOD 6,8 45.0,0.25

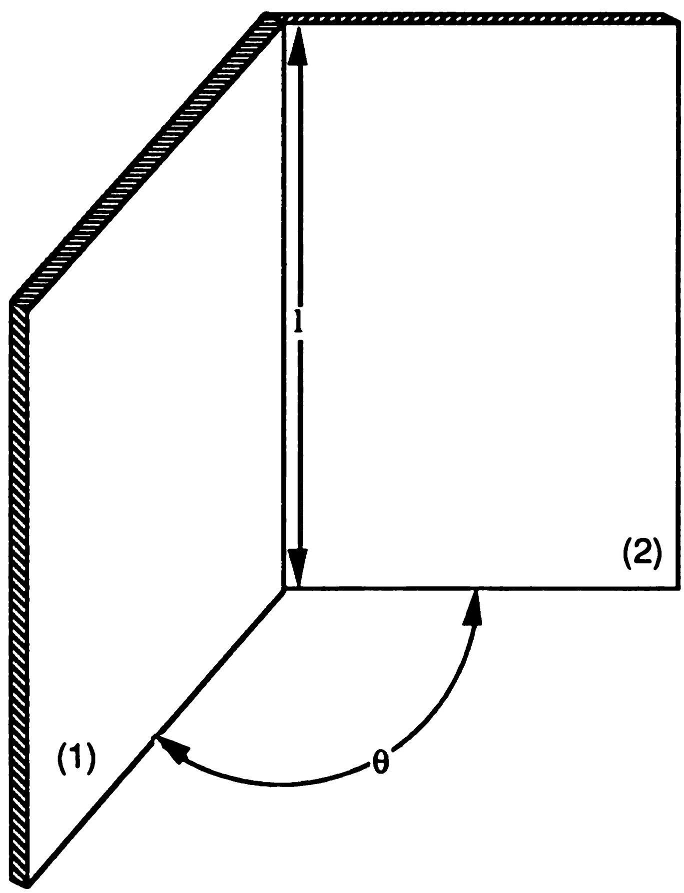

Figure A-1: FPL_FPL Connector Diagram.

FPL_FPL (Sheet 2 of 6)

DESCRIPTION

FPL_FPL models flat plate to flat plate connections (Figure A-1). The angle between the plates, θ, and the length of the joint, *l*, are needed to model the connection. Other required parameters are obtained from the FPLATE elements this connector links. The coupling loss factor equations obtained from VAPEPS source code are:

$$\eta_{12} = \frac{2l(tao)}{\pi K_1 A_{p1}} \tag{A-20}$$

where: l = joint length (m)

A_{p1} = plate 1's surface area (m^2)

K_1 = plate 1's acoustic wave number which is evaluated as follows:

$$K_1 = \left(\frac{\sqrt{12}\omega}{h_1 C_{l1}} \right)^{1/2} \tag{A-21}$$

where: ω = the band center frequency (rad/sec)

h_1 = plate 1 thickness (m)

c_{l1} = plate 1 longitudinal wave speed (m/sec)

tao is evaluated as follows:

$$tao = (tf + tb)/3.0 \tag{A-22}$$

where:

$$tf = \frac{\left(\left|(\det 1/\det 2)\right|\right)^2 r * (psi)}{2.0} \tag{A-23}$$

and

$$tb = \frac{\left(\left|(\det 3/\det 4)\right|\right)^2 r * (psi)}{2.0} \tag{A-24}$$

with

FPL_FPL (Sheet 3 of 6)

$$r = K_2/K_1 \tag{A-26}$$

where K_2, the wave number of plate 2, is evaluated using equation (A-21).

$$psi = r^2 r_{b_2}/r_{b_1} \tag{A-27}$$

r_{b_2} and r_{b_1}, the bending rigidity of plates 1 and 2 respectively, are evaluated from:

$$r_{b_i} = \frac{Eh_i^3}{12.0} \tag{A-28}$$

and

$$\det 1 = |matrix1| \tag{A-29}$$

$$\det 2 = |matrix2| \tag{A-30}$$

$$\det 3 = |matrix3| \tag{A-31}$$

$$\det 4 = |matrix4| \tag{A-32}$$

where *matrix1*, *matrix2*, *matrix3*, and *matrix7* are shown in Table A-12.

Table A-12: Matrices for evaluating FPL_FPL loss fators

$$matrix1 = \begin{bmatrix} -1.0+0.0i & 1.0+0.0i & 1.0+0.0i & psi/2.0+0.0i \\ 0.0+1.0i & 1.0+0.0i & 0.0+1.0i & -r+0.0i \\ [1.0+bet(2)]+0.0i & 1.0+bet(2)i & [bet(2)-1.0]+0.0i & \cos(\theta)-(c_{f2}/c_{l2})\cos(\theta)i \\ A+0.0i & A+0.0i & -A+0.0i & B+Ci \end{bmatrix}$$

$$matrix2 = \begin{bmatrix} -1.0+0.0i & 1.0+0.0i & -psi/2.0+0.0i & psi/2.0+0.0i \\ 0.0+1.0i & 1.0+0.0i & 0.0-ri & -r+0.0i \\ [1.0+bet(2)]+0.0i & 1.0+bet(2)i & [1.0-(c_{f2}/c_{l2})]\cos(\theta)+0.0i & \cos(\theta)-(c_{f2}/c_{l2})\cos(\theta)i \\ A+0.0i & A+0.0i & [B+C]+0.0i & B+Ci \end{bmatrix}$$

$$matrix3 = \begin{bmatrix} -1.0+0.0i & 1.0+0.0i & 1.0+0.0i & psi/2.0+0.0i \\ 0.0+1.0i & 1.0+0.0i & 0.0+1.0i & -r+0.0i \\ [1.0+bet(2)]+0.0i & 1.0+bet(2)i & [bet(2)-1.0]+0.0i & \cos(\pi-\theta)-(c_{f2}/c_{l2})\cos(\pi-\theta)i \\ D+0.0i & D+0.0i & -D+0.0i & E+Fi \end{bmatrix}$$

$$matrix4 = \begin{bmatrix} -1.0+0.0i & 1.0+0.0i & -psi/2.0+0.0i & psi/2.0+0.0i \\ 0.0+1.0i & 1.0+0.0i & 0.0-ri & -r+0.0i \\ [1.0+bet(2)]+0.0i & 1.0+bet(2)i & [1.0-(c_{f2}/c_{l2})]\cos(\pi-\theta)+0.0i & \cos(\pi-\theta)-(c_{f2}/c_{l2})\cos(\pi-\theta)i \\ D+0.0i & D+0.0i & [E+F]+0.0i & E+Fi \end{bmatrix}$$

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The matrix variables are:

$$A = \left(1.0 + \frac{\rho_2 h_2 c_{l2}}{\rho_1 h_1 c_{l1}}\right)\cos(\theta) \tag{A-33}$$

$$B = \left[\left(1.0 + \frac{\rho_2 h_2 c_{l2}}{\rho_1 h_1 c_{l1}}\right)\cos(\theta) - \cos(\theta)\right]\cos(\theta) + 1.0 \tag{A-34}$$

$$C = bet(1)[\sin(\theta)]^2 \tag{A-35}$$

$$D = \left(1.0 + \frac{\rho_2 h_2 c_{l2}}{\rho_1 h_1 c_{l1}}\right)\cos(\pi - \theta) \tag{A-36}$$

$$E = \left[\left(1.0 + \frac{\rho_2 h_2 c_{l2}}{\rho_1 h_1 c_{l1}}\right)\cos(\pi - \theta) - \cos(\pi - \theta)\right]\cos(\pi - \theta) + 1.0 \tag{A-37}$$

$$F = bet(1)[\sin(\pi - \theta)]^2 \tag{A-38}$$

and

$$bet(2) = \frac{c_{f1}\rho_1 h_1}{c_{l2}\rho_2 h_2} \tag{A-40}$$

where: c_{fi} = flexural wave speed of plate i (m/sec)

c_{li} = longitudinal wave speed of plate i (m/sec)

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ρ_i = the mass density of plate i (kg/m^3)

h_i = thicknesses of plate i (m)

θ = joint angle (degrees)

FPL_BEAM (Sheet 1 of 8)

Table A-13: FPL_BEAM Connector Parameters

Name	Units
γ (angle between plate and beam - See Figure A-2)	degrees
μ (angle between plate and beam - See Figure A-2)	degrees

DATA BASE ENTRY FORMAT

FPL_BEAM name location γ,μ

DATA BASE ENTRY EXAMPLE

FPL_BEAM ENG_FRAME 1,3 0.0,90.0

FPL_BEAM (Sheet 2 of 8)

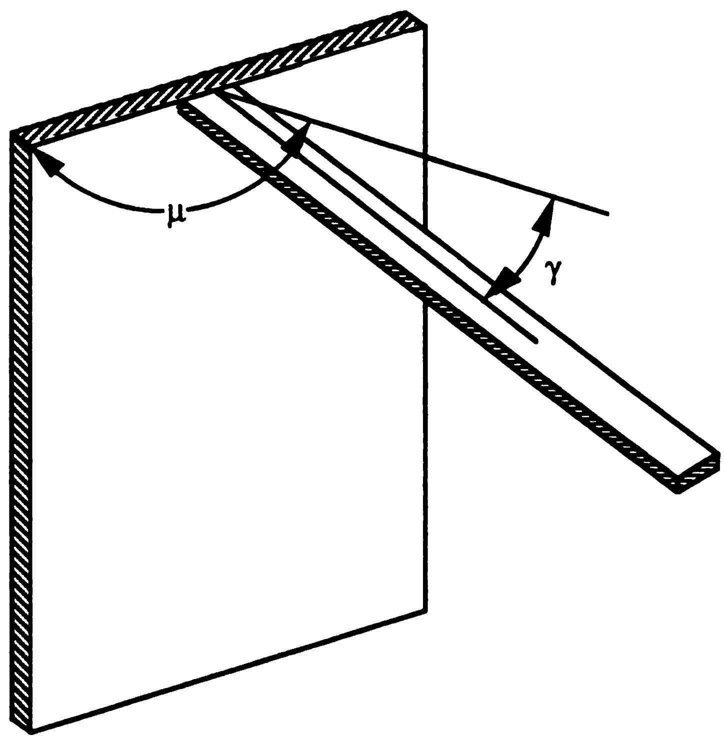

Figure A-2: FPL_BEAM Connector Diagram

DESCRIPTION

FPL_BEAM models the connection between flat plates and beams attached at acute angles (Figure A-2). In addition to the parameters obtained from the FPLATE and BEAM element models, the angles γ and μ must be specified. The equations for the coupling loss factor were obtained from VAPEPS source code. The loss factor is evaluated by

FPL_BEAM (Sheet 3 of 8)

$$\eta_{pb} = \frac{(2.0C_3C_4)\left[real(z_p)\right]}{\omega m_b\left(1.0 + n_{tb}/n_{fb}\right)} \tag{A-41}$$

where: ω = band center frequency (rad/sec)

m_b = beam mass (kg)

n_{tb} = beam torsional modal density (modes/Hz)

n_{fb} = beam flexural modal density (modes/Hz)

z_p = plate impedance

with:

$$n_{fb} = \frac{l}{\sqrt{\omega k_b c_{lb}}} \tag{A-42}$$

where: l = beam length (m)

ω = band center frequency (rad/sec)

k_b = beam radius of gyration (m)

c_{lb} = beam longitudinal wave speed (m/sec)

with:

$$k_b = \frac{\sqrt{D_o^2 + D_i^2}}{4.0} \tag{A-43}$$

for circular cross sections. And

$$k_b = \frac{h_b}{\sqrt{12.0}} \tag{A-44}$$

for rectangular cross sections.

where: D_o = beam outer diameter (m)

D_i = beam inner diameter (m)

h_b = beam thickness (m)

FPL_BEAM (Sheet 4 of 8)

$$n_{tb} = 2.0l/c_{tb} \tag{A-45}$$

where: l = beam length (m)

c_{tb} = beam torsional wave speed (m/sec)

with:

$$c_{tb} = \sqrt{G_b/\rho_b} \tag{A-46}$$

where: G_b = beam shear modulus (Pa)

ρ_b = beam density (kg/m^3)

$$C_3 = \left|z_b/\left(z_b + z_p\right)\right|^2 \tag{A-47}$$

where: z_b = beam flexural impedance

z_p = plate impedance

with:

if γ = 90.0° or μ = 0.0°, then

$$z_b = z_{bf}$$

else

$$z_b = \sqrt{\frac{1.0}{1.0 + x_{a2}}\left(z_{bt}^2 + x_{a2} z_{bf}^2\right)} \tag{A-48}$$

where: z_{bt} = beam torsional impedance

z_{bf} = beam flexural impedance

with:

$$z_{bf} = \frac{\rho_{lb}\left(k_b c_{lb}\right)^2}{c_{fb}}(1.0 - 1.0i) \tag{A-49}$$

where: ρ_{lb} = beam linear density (kg/m)

k_b = beam radius gyration (m)

FPL_BEAM (Sheet 5 of 8)

c_{lb} = beam longitudinal wave speed (m/sec)

c_{tb} =beam torsional wave speed (m/sec)

with:

$$\rho_{lb} = \rho_b A_{xb} \tag{A-50}$$

where: r_b = beam density (kg/m^3)

A_{xb} = beam x-sectional area (m^2)

and k_b evaluated by Equations (A-43) and (A-44) and c_{tb} evaluated by equation (A-46).

and

$$z_{bt} = \rho_{lb} k_{tb}^2 c_{tb} \tag{A-51}$$

where: ρ_{lb} = beam linear density (kg/m)

k_{tb} = beam torsional radius of gyration

c_{tb} =beam torsional wave speed (m/sec)

with:

$$k_{tb} = \frac{4.0 k_b}{\sqrt{8.0}} \tag{A-52}$$

for circular cross sections. And

$$k_{tb} = \frac{t_1}{A_{xb}} \tag{A-53}$$

for rectangular cross sections.

where: A_{xb} = beam cross sectional area (m^2)

and:

$$t_1 = \left(A_{xb}^2 r_{hw}^{-C_1}\right) C_2 \tag{A-54}$$

where: r_{hw} = the ratio of the beams thickness to its width. If $r_{hw} < 1.0$, $r_{hw} = 1.0/r_{hw}$..

FPL_BEAM (Sheet 6 of 8)

and C_1 and C_2 are evaluated as follows:

$$C_1, C_2 = \begin{Bmatrix} 0.2901, 0.141 & 1.0 < r_{hw} < 2.0 \\ 0.7603, 0.194 & 2.0 < r_{hw} < 6.0 \\ 0.9101, 0.2537 & 6.0 < r_{hw} < 10.0 \\ 1.0, 0.333 & 10.0 < r_{hw} \end{Bmatrix}$$

and

$$x_{a2} = \frac{[\cos(\gamma)\cos(\mu)]^2 + \sin^2(\gamma)}{[\cos(\gamma)\cos(\mu)]^2} \qquad \text{(A-55)}$$

$$C_4 = \frac{x_a(n_{tb}/n_{fb})}{k_{tb}^2} + \frac{x_b K_b \sqrt{n_{tb}/n_{fb}}}{k_{tb}} + x_c K_b^2 \qquad \text{(A-56)}$$

where: n_{fb} = beam flexural modal density (modes/Hz)

n_{tb} = beam torsional modal density (modes/Hz)

k_{tb} = beam torsional radius of gyration (m)

K_b = beam wave number (m^{-1})

with n_{fb}, n_{tb}, and k_{tb} are evaluated as before in equations (A-42), (A-45), (A- 52), and (A- 53) respectively.

and with:

$$x_a = \cos^2(\gamma) \qquad \text{(A-57)}$$

if $\gamma = 0^\circ$ and $\mu = 90^\circ$, then

$$x_b = 2.0\cos(\gamma)\sin(\mu)\left\{\sqrt{[\cos(\gamma)\cos(\mu)]^2 + \sin^2(\gamma)} - 1.0\right\} \qquad \text{(A-58)}$$

and

$$x_c = [\cos(\gamma)\cos(\mu)]^2 + \sin^2(\gamma) + 1.0 \qquad \text{(A-59)}$$

FPL_BEAM (Sheet 7 of 8)

else

$$x_b = \frac{2.0\cos(\gamma)\sin(\mu)\sin^2(\gamma)}{\sqrt{[\cos(\gamma)\cos(\mu)]^2 + \sin^2(\gamma)}} \tag{A-60}$$

and

$$x_c = [\cos(\gamma)\cos(\mu)]^2 + \sin^2(\gamma) + \frac{[\cos^2(\gamma)\sin(\mu)\cos(\mu)]^2}{[\cos(\gamma)\cos(\mu)]^2 + \sin^2(\gamma)} \tag{A-61}$$

and:

$$K_b = \sqrt{\frac{\omega}{k_b c_{lb}}} \tag{A-62}$$

where: k_b = beam radius of gyration (m)

c_{lb} = beam longitudinal wave speed (m/sec)

with k_b evaluated as before.

and

$$z_p = \frac{2.0 br_p (1.0 + F_l)}{\omega\left\{0.25 + \left[-1.0\ln\left(K_p R_{equiv}\right)/\pi + 0.9095 F_l \left(h_p/\pi R_{equiv}\right)^2\right] i\right\}} \tag{A-63}$$

where: h_p = plate thickness (m)

br_p = plate bending rigidity

F_l = moment impedance constant

R_{equiv} = Equivalent radius (m)

K_p = plate wave number

with:

$$br_p = E_p h_p^3 / 12.0 \tag{A-64}$$

where: E_p = plate Youngs modulus (Pa)

FPL_BEAM (Sheet 8 of 8)

h_p = plate thickness (m)

$$F_l = 1.0 \tag{A-65}$$

$$R_{equiv} = \sqrt{A_{xb}/\pi} \tag{A-66}$$

where: A_{xb} = beam cross sectional area (m^2)

and

$$K_p = \sqrt{\omega/k_p c_{lp}} \tag{A-67}$$

where: k_p = plate radius of gyration (m)

c_{lp} = plate longitudinal wave speed (m/s)

with:

$$k_p = h_p/\sqrt{12} \tag{A-68}$$

where: h_p = plate thickness (m)

FPL_PLB (Sheet 1 of 6)

Table A-14: FPL_PLB Connector Parameters

Name	**Units**
number of connection points	dimensionless

DATABASE ENTRY FORMAT

FPL_PLB *name location number of connection points*

DATABASE ENTRY EXAMPLE

FPL_PLB FPAN_FRAME 9,3 10.0

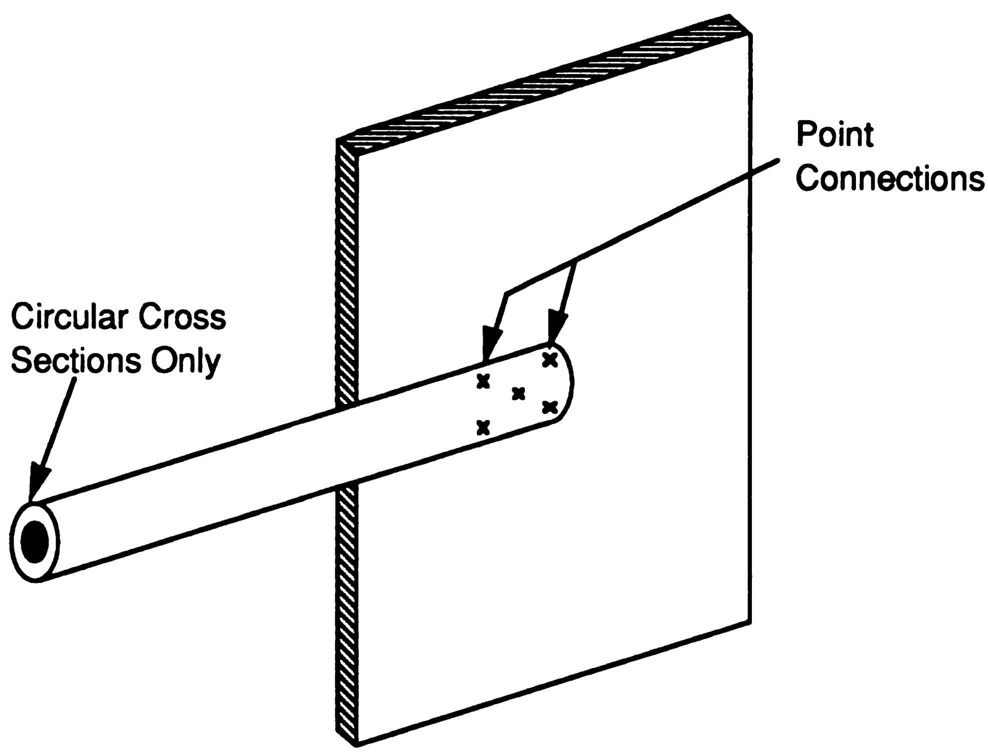

Figure A-3: FPL_PLB Connector Diagram

FPL_PLB (Sheet 2 of 6)

DESCRIPTION

FPL_PLB models connections between flat plates and parallel beams with circular cross sections (Figure A-3). The equations for evaluating FPL_PLB were obtained from VAPEPS source code. The model requires the number of connection points between the beam and the plate. The coupling loss factor is:

$$\eta_{pb} = p * clf_f \frac{n_{fb}}{n_{fp}} + clf_t \frac{n_{tb}}{n_{fp}} \qquad \text{(A-69)}$$

where: p = number of connection points

clf_t = torsional coupling loss factor

clf_f = flexural coupling loss factor

n_{tb} = beam torsional modal density (modes/Hz)

n_{fb} = beam flexural modal density (modes/Hz)

n_{fp} = plate flexural modal density (modes/Hz)

with:

$$n_{fb} = \frac{l}{\sqrt{\omega k_b c_{lb}}} \qquad \text{(A-70)}$$

where: l = beam length (m)

ω = band center frequency (rad/sec)

k_b = beam radius of gyration (m)

c_{lb} = beam longitudinal wave speed (m/sec)

with:

$$k_b = \frac{\sqrt{D_o^2 + D_i^2}}{4.0} \qquad \text{(A-71)}$$

where: D_o = outer diameter (m)

D_i = inner diameter (m)

FPL_PLB (Sheet 3 of 6)

$$n_{tb} = 2.0l/c_{tb} \tag{A-72}$$

where: l = beam length (m)

c_{tb} = beam torsional wave speed (m/sec)

with:

$$c_{tb} = \sqrt{G_b/\rho_b} \tag{A-73}$$

where: G_b = beam shear modulus (Pa)

ρ_b = beam density (kg/m^3)

with:

$$G_b = E_b/2.0 \tag{A-74}$$

where: E_b = beam Young's modulus (Pa)

$$n_{fp} = \frac{\sqrt{3}A_p}{h_p c_{lp}} \tag{A-75}$$

where: A_p = plate area (m^2)

h_p = plate thickness (m)

c_{lp} = plate longitudinal wave speed (m/sec)

$$clf_t = \frac{\left\{\left[\left|z_{bt}/(z_{bt} + z_{bm})\right|^2\right] real\left(z_{pm}\right)\right\}}{2.0\omega m_b k_{tb}^2} \tag{A-76}$$

where: z_{bt} = beam torsional impedance

z_{bm} = beam moment impedance

z_{pm} = plate moment impedance

ω = band center frequency (rad/sec)

k_{tb} = beam torsional radius of gyration (m)

m_b = beam mass (kg)

with:

FPL_PLB (Sheet 4 of 6)

$$k_{tb} = \sqrt{\frac{D_o^2 + D_i^2}{8.0}} \tag{A-77}$$

where: D_o = beam outer diameter (m)

D_i = beam inner diameter (m)

$$z_{bm} = \frac{2.0 A_{xb} E_b k_b^2 \mathrm{K}_b}{\omega}(1.0 + 1.0i) \tag{A-78}$$

where: ω = band center frequency (rad/sec)

A_{xb} = beam cross sectional area (m^2)

E_b = beam Young's modulus (Pa)

k_b = beam radius of gyration (m)

K_b = beam wave number (m^{-1})

with:

$$\mathrm{K}_b = \sqrt{\omega / k_b c_{lb}} \tag{A-79}$$

where: ω = band center frequency (rad/sec)

k_b = beam radius of gyration (m)

c_{lb} = beam longitudinal wave speed (m/sec)

with k_b evaluated as before.

$$z_{bt} = 2.0 A_{xb} k_{tb}^2 \sqrt{G_b \rho_b} \tag{A-80}$$

where: A_{xb} = beam cross sectional area (m^2)

G_b = beam shear modulus (Pa)

k_{tb} = beam torsional radius of gyration (m)

ρ_b = beam density (kg/m^3)

with k_{tb} evaluated as before.

and:

FPL_PLB (Sheet 5 of 6)

$$z_{pm} = \frac{4.0br_p}{\omega\left(0.25 + \left\{\left[1.0 - \ln\left(2.0K_pD_o\right)\right]/\pi\right\}i\right)} \tag{A-81}$$

where: br_p = plate bending rigidity (N-m)

ω = band center frequency (rad/sec)

D_o = beam outer diameter (m)

K_p = plate wave number (m^{-1})

with:

$$br_p = E_p h_p^3/12.0 \tag{A-82}$$

where: E_p = plate Young's modulus (Pa)

h_p = plate thickness (m)

and

$$K_p = \sqrt{\omega/k_p c_{lp}} \tag{A-83}$$

where: ω = band center frequency (rad/sec)

k_p = plate radius of gyration (m)

c_{lp} = plate longitudinal wave speed(m/sec)

with:

$$k_p = h_p/\sqrt{12} \tag{A-84}$$

where: h_p = plate thickness (m)

and Equation (A-85) is:

$$clf_f = \frac{\left\{\left|z_{bf}/\left(z_{bf}+z_{pf}\right)\right|^2 + \left[\left|z_{bf}/\left(z_{bf}+z_{pf}\right)\right|^2\right]z_{pf} + K_b^2\left[\left|z_{bm}/\left(z_{bm}+z_{pm}\right)\right|^2\right]real\left(z_{pm}\right)\right\}}{2.0\omega m_b}$$

where: z_{bf} = beam flexural impedance

z_{bm} = beam moment impedance

z_{pf} = plate flexural impedance

z_{pm} = plate moment impedance

FPL_PLB (Sheet 6 of 6)

ω = band center frequency (rad/sec)

m_b = beam mass (kg)

K_b = beam wave number (m^{-1})

with:

$$z_{bf} = \frac{2.0 A_{xb} E_b k_b^2 K_b^3}{\omega}(1.0 + 1.0i) \tag{A-86}$$

where: ω = band center frequency (rad/sec)

A_{xb} = beam cross sectional area (m^2)

E_b = beam Young's modulus (Pa)

k_b = beam radius of gyration (m)

K_b = beam wave number (m^{-1})

with k_b and K_b evaluated as before.

$$z_{pf} = 8.0 c_{lp} k_p \rho_{sp} \tag{A-87}$$

where: c_{lp} = plate longitudinal wave speed (m/sec)

k_p = plate radius of gyration (m)

ρ_{sp} = plate surface density (kg/m)

with:

$$\rho_{sp} = \rho_p A_p \tag{A-88}$$

where: ρ_p = plate density (kg/m^3)

A_p = plate surface area (m^2)

and k_b is evaluated as before.

and z_{bm}, z_{pm}, and k_b are evaluated as before.

OTHER MODELING ELEMENTS

Designation of the model elements and model connectors does not complete the model specification process. In addition, the frequency range and power input or an element energy response spectrum must be specified. VARPS has three model elements for specifying these values: FREQ,POW, and ENRG.

FREQ (Sheet 1 of 1)

Table A-15: Frequency Node Parameters

Name	**Units**
fi (frequency)	(Hz)

DATA BASE ENTRY FORMAT

FREQ *name* *location* *f1,f2,f3,f4,f5,......*

DATA BASE ENTRY EXAMPLE

FREQ TEST 1 200.,250.,315.,400.,500.,630.,800.,-
1000.,1250.,1600.,2000.,2500.,3150.,4000.,5000.

DESCRIPTION

The frequency element allows the user to specify the frequency analysis vector. It's *location* can be specified as any existing element. Specifying a new element number for frequency will cause program errors. Only one frequency vector is specified for proper model evaluation. The standard 1/3 octave center frequencies in Hertz are:

1.0,1.25,1.60,2.0,2.5,3.15,4.0,5.0,6.3,8.0,10.0,12.5,16.0,20.0,25.0,31.5,40.0,50.0,
63.0,80.0,100.0, etc.

POW (Sheet 1 of 1)

Table A-16: Power Node Parameters

Name	**Units**
POW(fi)	g's - structural dB - acoustical

DATA BASE ENTRY FORMAT

POW *name* *location* *POW(f1), POW(f2), POW(f3), ...*

DATA BASE ENTRY EXAMPLE

POW INPUT 1 0.9,0.75,0.85,0.67,0.75,0.70,0.78,0.85,-
0.89,0.93,0.98,0.95,0.90,0.87,0.91

DESCRIPTION

The power element is used to specify power inputs into the model. The *location* field specifies the element at which power is entering the model. For example, if power is entering the model at an element called *engine* and the location of *engine* is 1, the *location* of the POW element is also 1. The number of power levels in the power input spectrum must be the same as the number of frequencies in the frequency node. Use of the power input is optional. If a POW element is not specified, at least one ENRG element must be present in the model. Power levels for structural elements (plates and beams) are specified g's and levels for acoustic elements are specified as sound pressure levels.

ENRG (Sheet 1 of 1)

Table A-17: Energy Node Parameters

Name	**Units**
ENRG(fi)	g's - structural dB - acoustical

DATA BASE ENTRY FORMAT

ENRG *name location ENRG(f1),ENRG(f2),ENRG(f3),.....*

DATA BASE ENTRY EXAMPLE

ENRG INPUT 1 100.,100.,100.,100.,100.,100.,100.,100.,-
100.,100.,100.,100.,100.,100.,100.

DESCRIPTION

An energy level in one or more elements can be specified. The particular element is specified in the location field. The number of energy levels must be identical to the number of frequencies specified. An energy input is not required if at least one input power vector has been defined. Energy levels for structural elements (plates and beams) are specified as RMS accelerations in g's and levels for acoustic elements are specified as sound pressure levels.

APPENDIX B: VARPS VERIFICATION AND EXAMPLE MODEL

INTRODUCTION

The SEA modeling process is best illustrated with a simple example. A simple three element model is evaluated. To verify the accuracy of VARPS, results are compared with results from VAPEPS, an SEA modeling program available from NASA (VAPEPS). It is found that the VARPS results for this model match the VAPEPS results to four significant figures. The VARPS and VAPEPS model results are presented below.

THE MODEL

The theoretical three element model shown in Figure B-1 is evaluated. The model consists of two identical reverberant volumes separated by an aluminum plate. Each volume is an air filled cube with sides 0.5m long. The aluminum plate is 0.5m X 0.5m with a thickness of 7.5mm. The model parameters are shown in Tables B-1 and B-2.

The system SEA model (Figure B-2) consists of two acoustical volume elements and a single flat plate element. Power flows between VolA and the Plate through a connector modeled as an acoustic volume to a flat plate. Power flows from the Plate to VolB through a connector modeled as a flat plate to acoustic volume. The power input to the model is P_{in1} which is specified indirectly by setting the sound pressure level response spectrum in VolA to a constant 100 dB. VARPS solves for the input power spectrum required to generate the 100 dB response.

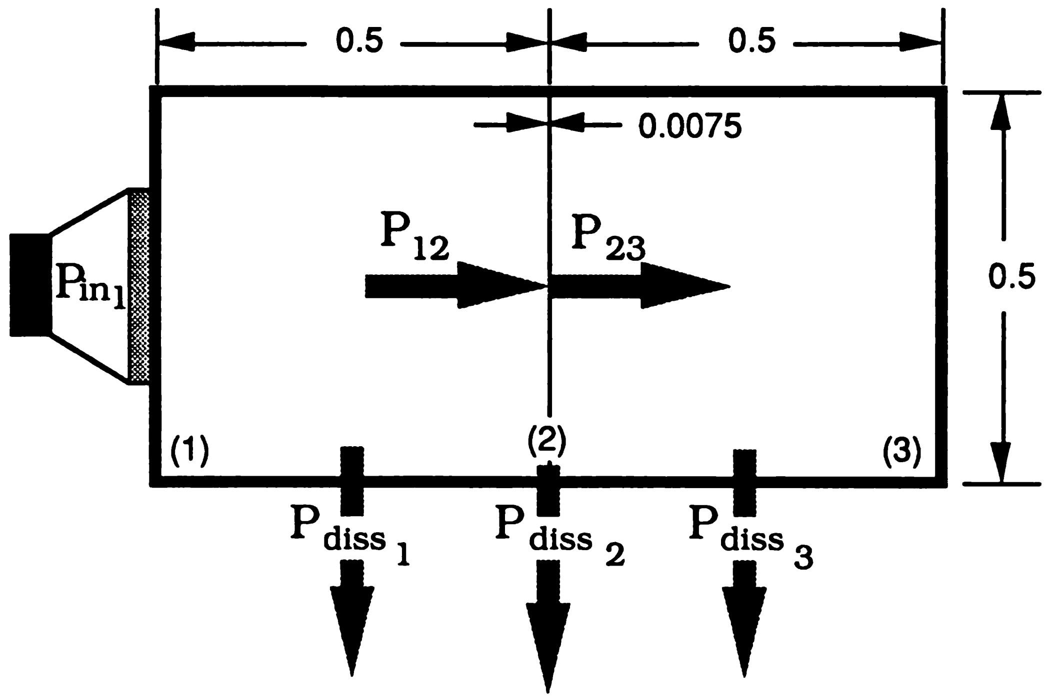

Figure B-1: Volume-Plate-Volume: Example System Conceptual Model

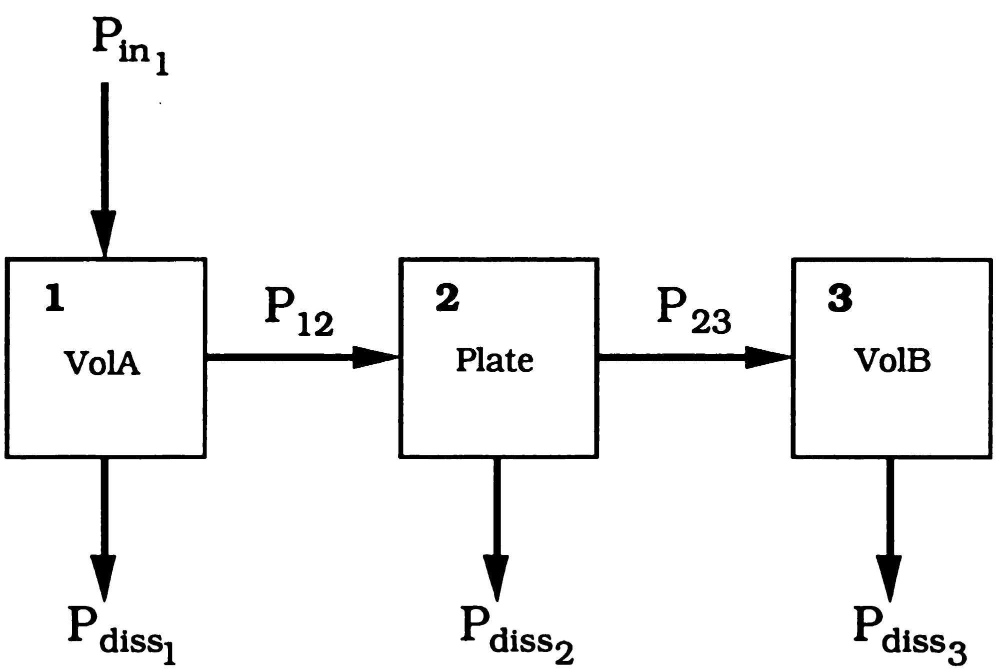

Figure B-2: Statistical Energy Analysis Model for Volume-Plate-Volume Model

Table B-1: Acoustic Volume Parameters for Volume-Plate-Volume Model

VOLUME:	0.125 m^3
SURFACE AREA:	1.5 m^2
EDGE LENGTH:	3.0 m
*REVERBERATION TIME:	6.69719 sec
DENSITY OF AIR:	1.244186 kg/m^3
**SPEED OF SOUND IN AIR:	344 m/sec
**ACOUSTIC ABSORPTION COEFFICIENT:	0.007567

*VARPS only
**VAPEPS only

Table B-2: Plate Parameters for Volume-Plate-Volume Model

THICKNESS:	0.0075 m
SURFACE AREA:	0.25 m^2
*PLATE MASS:	5.19375 kg
PLATE PERIMETER:	2.0 m
WAVE SPEED:	5181.6 m/s
*CRITICAL DAMPING RATIO:	0.00005
LENGTH	0.5 m
WIDTH	0.5 m
YOUNGS MODULUS:	70 GPa
**PLATE MASS DENSITY:	2710.0 kg/m^3
**PLATE SURFACE DENSITY:	20.33 kg/m^2
**DAMPING LOSS FACTOR:	0.0001
**TOTAL LENGTH OF DISCONTINUITY:	2.0 m
**SURFACE MASS DENSITY:	20.775 kg/m^2
**NON-STRUCTURAL MASS:	0.0
PLATE EDGE CONDITION	$\sqrt{2}$

*VARPS only
**VAPEPS only

THE VARPS MODEL DATA BASE

Table B-3 is the VARPS model database. It should be noted that the entries such as FPLATE, FREQ, and ENRG which require two lines have a '-' as the last character in the first line. The element type AVOL2 and connector types AVOL_FPL2 and FPL_AVOL2 are modeling nodes that best match VAPEPS results. The VARPS models AVOL,

AVOL_FPL, and FPL_AVOL are more representative of available theory. Further details of the database elements, parameters, and format are found in Appendix B.

Table B-3: The Model Database

```
AVOL2      VOLA         1    0.125,1.5,6.0,1.77
FPLATE     PLATE        2    0.0075,0.25,5.08125,2.0,5082.3,-
0.00005,0.5,0.5,70.0
AVOL2      VOLB         3    0.125,1.5,6.0,1.77
AVOL_FPL2  VOLA_PLATE   1,2 1.4142
FPL_AVOL2  PLATE_VOLB   2,3 1.4142
FREQ       TEST         1    200.,250.,315.,400.,500.,630.,-
800.,1000.,1250.,1600.,2000.,2500.,3150.,4000.,5000.
ENRG       INPUT        1    100.,100.,100.,100.,100.,100.,-
100.,100.,100.,100.,100.,100.,100.,100.,100.
```

EVALUATING THE MODEL

The model is evaluated using VARPS and VAPEPS. The motivation for evaluating this model with VARPS and VAPEPS is to check the accuracy of the VARPS predictions versus VAPEPS predictions. It is assumed that the VAPEPS results are accurate representations of current SEA theory. No experimental results are presented since this evaluation is of the software's accuracy and not an evaluation of SEA theory.

The database in Table B-3 is placed in a file with the name 'example.dat.' Running the file 'example.dat' in VARPS provides modal densities, loss factors, coupling loss factors, and model response. The same outputs are produced using VAPEPS.

VARPS EXAMPLE RUN

A sample VARPS run of the example is shown in Table B-4. The predicted response of the Plate element, the specified VolA level, and the predicted response of VolB shown in Figures B-3 and B-4 match the VAPEPS results to four significant figures.

Table B-4: Volume-Plate-Volume Example Run

<table>
<tr><th>PROGRAM OUTPUT</th><th>COMMENTS</th></tr>
<tr><td>_CLVAX1:: ed example.dat

AVOL2   VOLA   1 0.125,1.5,6.0,1.77
FPLATE   PLATE   2 0.0075,0.25,5.08125,2.0,5181.6,0.00005,-
0.5,0.5,70.0
AVOL2   VOLB   3 0.125,1.5,6.0,1.77
AVOL_FPL2 VOLA_PLATE 1,2 1.41421
FPL_AVOL2 PLATE_VOLB 2,3 1.41421
FREQ   FREQUENCY   1 200.,250.,315.,400.,500.,630.,800.,-
1000.,1250.,1600.,2000.,2500.,3150.,4000.,5000.
ENRG   INPUT_SPEC   1 100.,100.,100.,100.,100.,100.,100.,-
100.,100.,100.,100.,100.,100.,100.,100.</td><td>Create data file ‘example.dat’ using VMS EDT editor.</td></tr>
<tr><td>[EOB]</td><td>End of file mark - not input by user.</td></tr>
<tr><td></td><td>Type <CTRL+Z> to enter line mode.</td></tr>
<tr><td>*exit
USER3:[EMERY.SEA]EXAMPLE.DAT;4 10 lines</td><td>Enter ‘exit’ to leave EDT and save file.</td></tr>
<tr><td>_CLVAX1:: @VARPS</td><td>Start VARPS</td></tr>
<tr><td>Enter VARPS filename <EXAMPLE.DAT> (Type EXIT to get out):
example.dat</td><td>Enter name of data file and hit

<RETURN>.</td></tr>
</table>

Table B-4: cont'd

<table>
<tr><th>PROGRAM OUTPUT</th><th>COMMENTS</th></tr>
<tr><td><pre>
          Welcome to VARPS 1.0

                    MODAL DENSITIES

FREQUENCY | VOLA| PLATE| VOLB|

200.00 | 1.543493e-03| 1.114232e-02| 1.543493e-03|
250.00 | 2.411707e-03| 1.114232e-02| 2.411707e-03|
315.00 | 3.828826e-03| 1.114232e-02| 3.828826e-03|
     .   .   .   .
     .   .   .   .
     .   .   .   .
4000.00 | 6.173970e-01| 1.114232e-02| 6.173970e-01|
5000.00 | 9.646829e-01| 1.114232e-02| 9.646829e-01|

                       LOSS FACTORS

FREQUENCY | VOLA| PLATE| VOLB|

200.00 | 6.214689e-03| 1.000000e-04| 6.214689e-03|
250.00 | 4.971751e-03| 1.000000e-04| 4.971751e-03|
315.00 | 3.945834e-03| 1.000000e-04| 3.945834e-03|
     .   .   .   .
     .   .   .   .
     .   .   .   .
4000.00 | 3.107345e-04| 1.000000e-04| 3.107345e-04|
5000.00 | 2.485876e-04| 1.000000e-04| 2.485876e-04|
</pre></td><td>Modal densities are output first.

Loss factors are output second.</td></tr>
</table>

Table B-4: cont'd

PROGRAM OUTPUT	COMMENTS

PROGRAM OUTPUT

```
                    COUPLING LOSS FACTORS

FREQUENCY     |   VOLA_PLATE|   PLATE_VOLB|
------------------------------------------
200.00        | 2.002657e-02| 2.774186e-03|
250.00        | 6.783928e-03| 1.468353e-03|
315.00        | 4.022390e-03| 1.382211e-03|
     .               .              .
     .               .              .
     .               .              .
4000.00       | 1.984992e-05| 1.099887e-03|
5000.00       | 9.499316e-06| 8.224347e-04|
```

COMMENTS: Coupling loss factors are output third.

PROGRAM OUTPUT

```
                            SOLUTION

FREQUENCY     |VOLA         |PLATE         |VOLB         |
    Hz        |      dB     |   m/s**2     |      dB     |
---------------------------------------------------------
200.00        | 1.000000e+02| 2.446322e+00| 9.777836e+01|
250.00        | 1.000000e+02| 2.260289e+00| 9.587748e+01|
315.00        | 1.000000e+02| 2.204432e+00| 9.507903e+01|
     .               .              .              .
     .               .              .              .
     .               .              .              .
4000.00       | 1.000000e+02| 1.936713e+00| 8.470794e+01|
5000.00       | 1.000000e+02| 1.911508e+00| 8.246865e+01|

Enter VARPS filename <EXAMPLE.DAT> (Type EXIT to get out): exit
 CLVAX1::
```

COMMENTS: The steady state solution is output last.

COMMENTS: Enter 'exit' to leave VARPS.

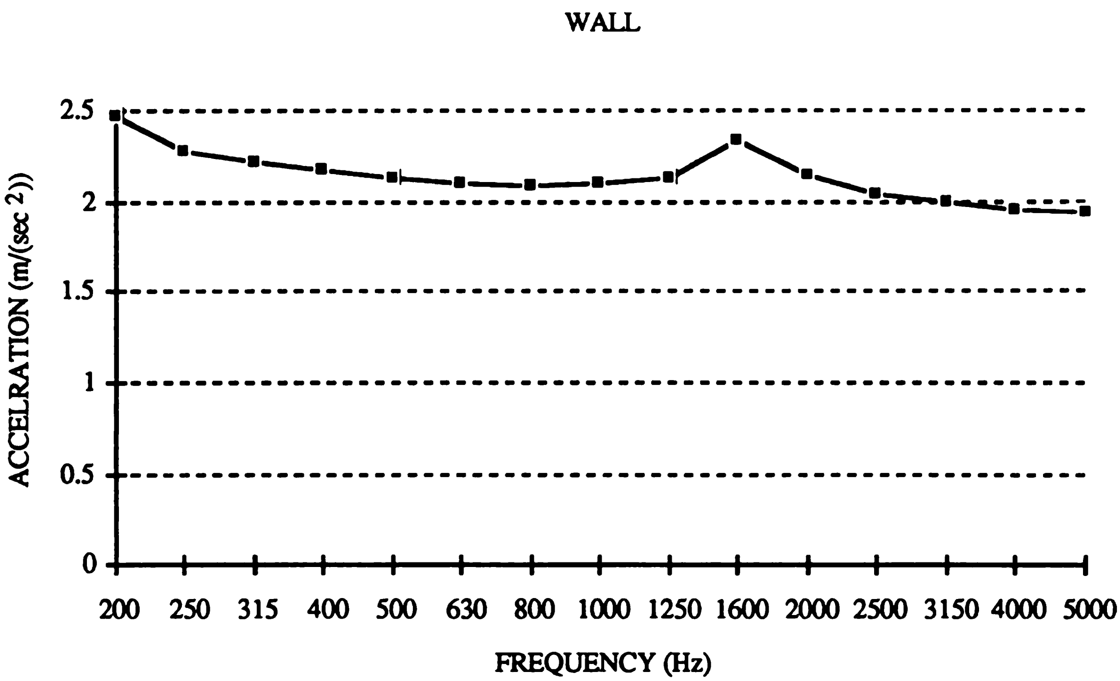

Figure B-3: Plate - Predicted RMS Acceleration (VARPS and VAPEPS)

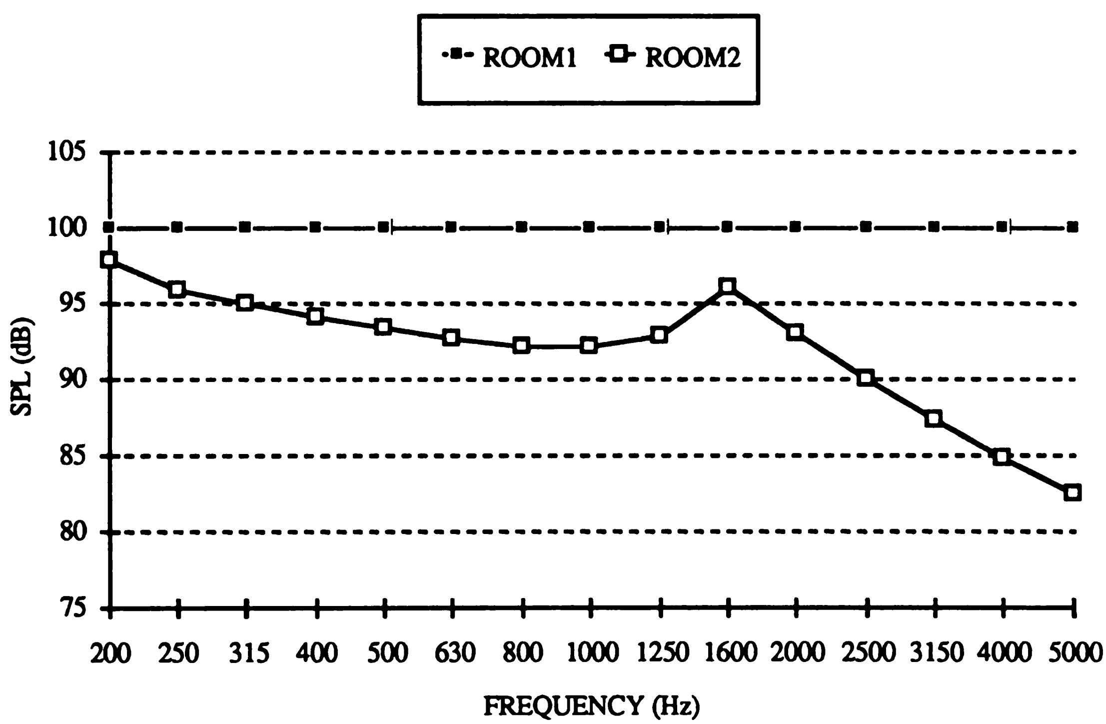

Figure B-4: VolA and VolB - Predicted Sound Pressure Levels (VARPS and VAPEPS)

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